Introduction

LTV channel

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1 Introduction

This laboratory work is mostly about linear time-variant (LTV) systems. Mobile radio channel is an archetypal example of an LTV system.

The goal of this laboratory work is to gather an understanding of how LTV systems are characterised by their system functions.

The purpose of this document is to present an overview of certain topics covered in the laboratory exercise, not to reproduce material already covered in literature and lecture notes. Section two discusses deterministic and stochastic LTV system functions and section three touches on measurement of impulse response of an LTV system.

2 Radio channel characterization

2.1 Signal fading

In this document we subscribe ray type radio signal propagation modeling. The signal can propagate directly from transmitter to receiver or it can pass through some obstacle. For instance it could reflect from a wall. Different paths from transmitter to receiver have different length an therefore have different propagation delays. If the propagation delay difference is less than the reverse of receiver bandwidth receiver is not able to separate the rays and they sum together at the receiver front end. The rays sum together with random phase. If the phases are aligned the received signal is strong if they are opposite the signals cancel out. The phase at what the ray arrives at the receiver depends on the path length, reflected material properties etc. If user moves the phases of the rays also change. The resulting signal strength change variation is called fading. The rate of the fading depends on user speed.

We characterize the fading by computing the correlation of the fading channel time series values. The time of the correlation value where the signal has not significantly reduced with respect to correlation value (for instance at 3 dB level) is called coherence time T_d. Fourier transform of the correlation time series gives Doppler spectrum. The with of the main lobe in the Doppler spectrum is called Doppler spread B_d.

2.2 Multi tap signal

If the path difference is more than the separable resolution at the receiver the receiver is capable to identify different paths. Unfortunately in a communication system signal from the delayed path could overlap with already next symbol arriving at the shorter path causing inter-symbol interference. A non fading multi tap channel is characterized with its impulse response

$$h(\tau) = \sum_n a_n \delta(\tau - \tau_n) \tag{1}$$

where a_n is the amplitude of the n-th path and τ_n is delay of the n-th path with respect to the first path.(First path delay is $\tau_0 = 0$).

The time delay between the arrival of the first path and the last path is called delay spread T_m. We can take Fourier transform of the channel impulse response. The bandwidth of the main lobe of this Fourier transform is called coherence bandwidth B_m. Usually this bandwidth is measured as 3 dB bandwidth of the main lobe. In coherence bandwidth we can assume that the channel is flat ie. the spectrum has zeros - multi path fades.

If the multi path components amplitudes are constant the channel is called time invariant. It is modeled as linear time invariant process.

2.3 Multi tap fading signals

Each tap in multi tap channel response could contain multiple rays. Because of that each tap will have its own fading. Such channel is random time varying channel. It is characterized by computing auto-correlation in delay direction and in time varying direction (the details are explained in the next section).

If the multi path components amplitudes are changing over time the channel is called time invariant. It is modeled as linear time variant LTV process.

If the taps fading are uncorrelated such auto-correlation function in delay spread direction has a simple form

$$\phi_h(\tau, 0) = \sum_n \alpha_n \delta(\tau - \tau_n) \tag{2}$$

where now α_n is average fading amplitude of the tap n.

3 Radio channel measurements

For characterizing a channel we have to measure what multipath component it has and how the components amplitudes are changing over time.

We measure the components by sending from the transmitter a sequence with good correlation properties - the sounding signal. At the receiver we correlate the received signal with the sounding signal. If the signal correlation resolution is short enough we will have a peak at each multi path component.

For estimating how multi path components amplitude change over longer time we send sounding signal periodically. After correlation at the receiver we will have periodic peaks of multi path components. Amplitude change of those peaks gives the coherence time.

4 Radio channel modeling as LTV system

4.1 Deterministic LTV channel

Figure 1 gives an example of a real - valued time-variant impulse response $h(\tau, t)$ at the output of an channel at time instants t_1, t_2, t_3. The periodicity the sounding signal is sent is $t_2 - t_1$. In the figure all the time units have been normalized. The response of the first impulse response is observed at time $t_1 = 0$. It is clearly different from the response observed at time $t_2 = 1$. In the time-invariant case the

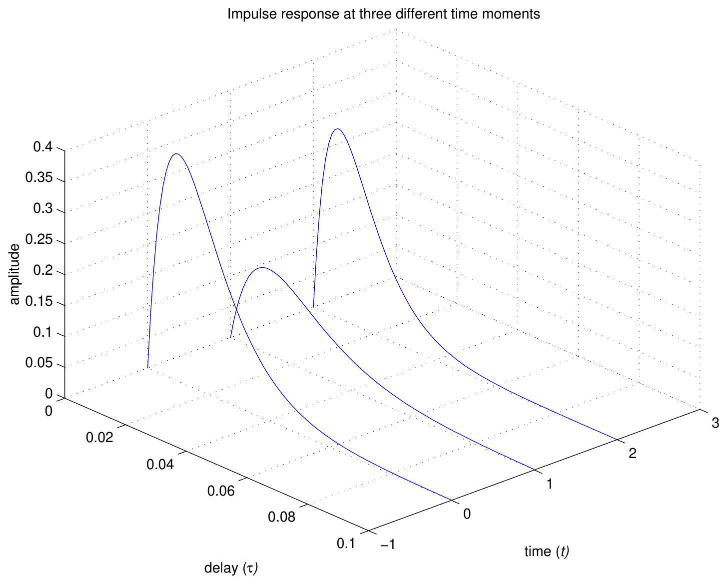

Figure 1: An example of LTV system impulse response.

frequency response $H(f)$ of the channel can be obtained by Fourier transforming the impulse response $h(\tau)$. Generalizing this to the time-variant case, we can obtain the time-variant frequency response $H(f, t)$ by transforming $h(\tau, t)$ with respect to the τ variable. Fourier transforming $h(\tau, t)$ with respect to t produces a function $h(\tau, \phi)$ that depicts the frequency spread of the channel for each delay τ . In context of radio channels ϕ is often called Doppler variable. Variables $\tau \Leftrightarrow f$ and $t \Leftrightarrow \phi$ are dual variables as shown in Figure 2. Function $h(\tau, \phi)$ is often called Doppler-delay function.

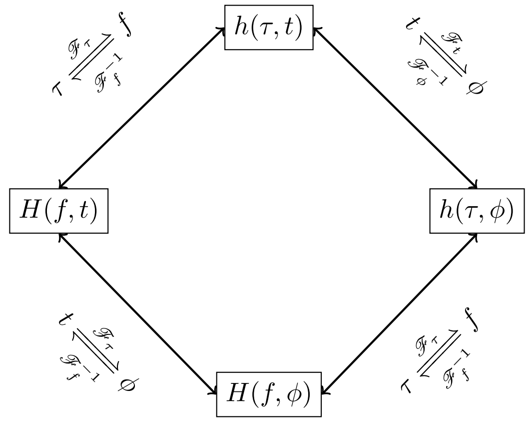

Figure 2: Deterministic LTV system functions and their relationships.

5 Statistical modeling of LTV channel

In the previous section the LTV system functions were assumed to be deterministic. Often the LTV systems encountered in practice can only be modeled by treating the system as a *randomly* time-variant linear system. Mobile radio channel (or a tropo- scatter channel) is the standard example of an LTV system that can only be described statistically.

The intuitive for statistical treatment is model where a tap amplitude is composed of a constant term and time varying term with mean zero. In this case if we compute statistics like average the varying part is averaged out we are left with the mean part only.

The auto-correlation function is by far the most widely used statistical description of an LTV system function. For example, the auto-correlation function of the randomly time-variant complex impulse response is defined as

$$\Phi_h(\tau_1, \tau_2, t_1, t_2) = E\left[h(\tau_1, t_1) h^*(\tau_2, t_2)\right] \tag{3}$$

Even this simple 2-nd order statistic is often too difficult to handle in practice and simplifications are almost always required.

Assuming that $h(\tau, t)$ is wide-sense stationary in the t (time) variable leads to

$$E\left[h(\tau_1, t_1) h^*(\tau_2, t_1 + \Delta t)\right] = \Phi_h(\tau_1, \tau_2, \Delta t) \tag{4}$$

Furthermore, assuming that $h(\tau, t)$ is uncorrelated in the τ (delay) variable results in

$$E\left[h(\tau_1, t_1) h^*(\tau_2, t_1 + \Delta t)\right] = \Phi_h(\tau_1, \Delta t)\sigma(\tau_2 - \tau_1) \tag{5}$$

The left-hand side is usually denoted $\Phi(\tau, \Delta t)\sigma(\tau)$. The delta function states that the values of the impulse response are uncorrelated between any two delay values. Real-world measurements have proven that this assumption cannot be justified for most mobile radio channels. The reason is that the practical measurement systems are always bandlimited and that introduces correlation between the taps. Nevertheless, in analysis and simulation the uncorrelatedness assumption is still often made because it simplifies the situation.

A system that is assumed to fulfill both the WSS and US assumption is called a WSSUS system. The WSSUS system functions and their relationships are shown in Figure 3. Many quantities describing the channel characteristics can be computed from the statistical LTV functions. These include coherence bandwidth, coherence time, and Doppler power spectrum. See literature for more details. The statistical LTV functions cannot be determined in practice because the evaluation of the expectation in 3 requires knowledge of the corresponding multi- dimensional probability density function, which in general is unknown. Thus, statistical LTV functions must be estimated from a finite data sample.

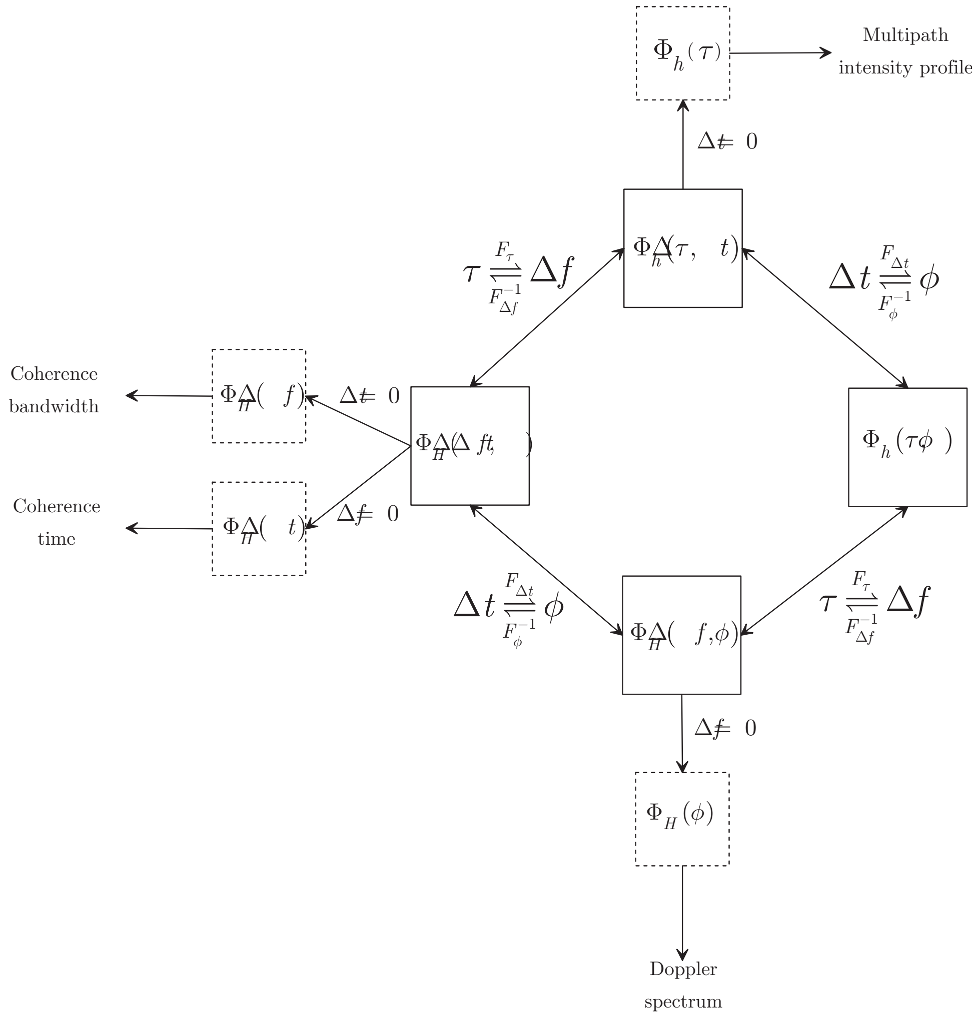

Figure 3: Statistical LTV system functions in the WSSUS case. Adapted from [1].

6 Measurement example

In this example we measured a time varying two tap channel. The impulse response of the measured signal after correlation with reference sequence is in Figure 5.

For simplifying the measurements we were sending three subsequent copies of the reference sequence waited a bit and send same three copies again. After correlating with the reference sequence in Figure 5 are visible correlation results with this three groups and idle time between them.

From each of the groups we selected the middle impulse response. From three subsequent groups such the impulse responses are in Figure 6. 5 For computing the coherence bandwidths the copies are allocated into matrix form as seen in the Figure 7.

References

[1] L. Ahlin, J. Zander, Principles of Wireless Communications, 2 nd edition, Studentlitteratur, Lund, 1998

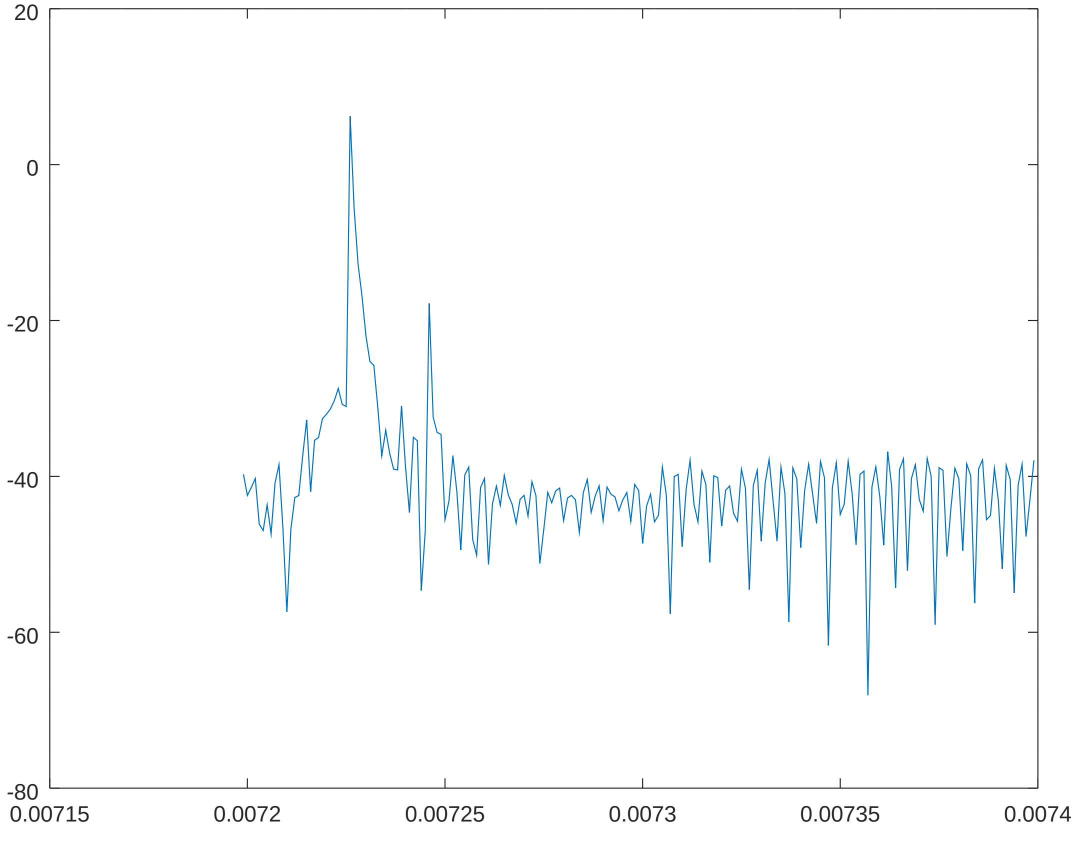

Figure 4: One impulse response.

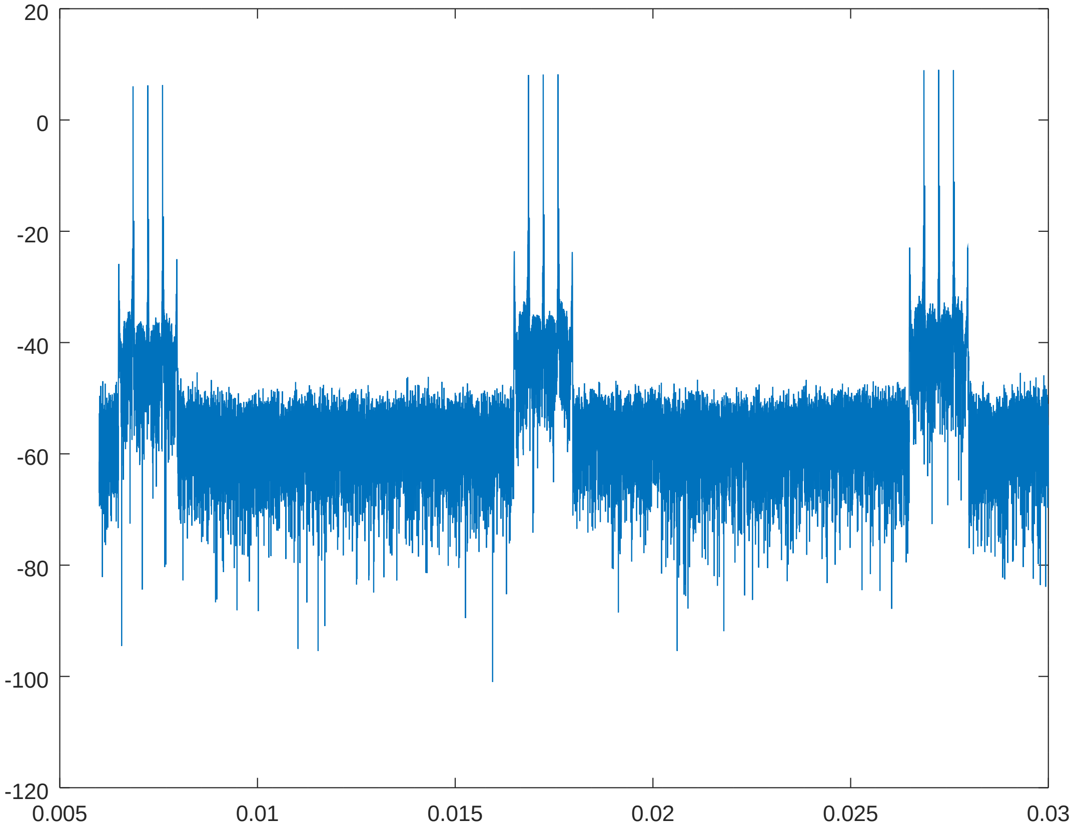

Figure 5: Received signal after correlation.

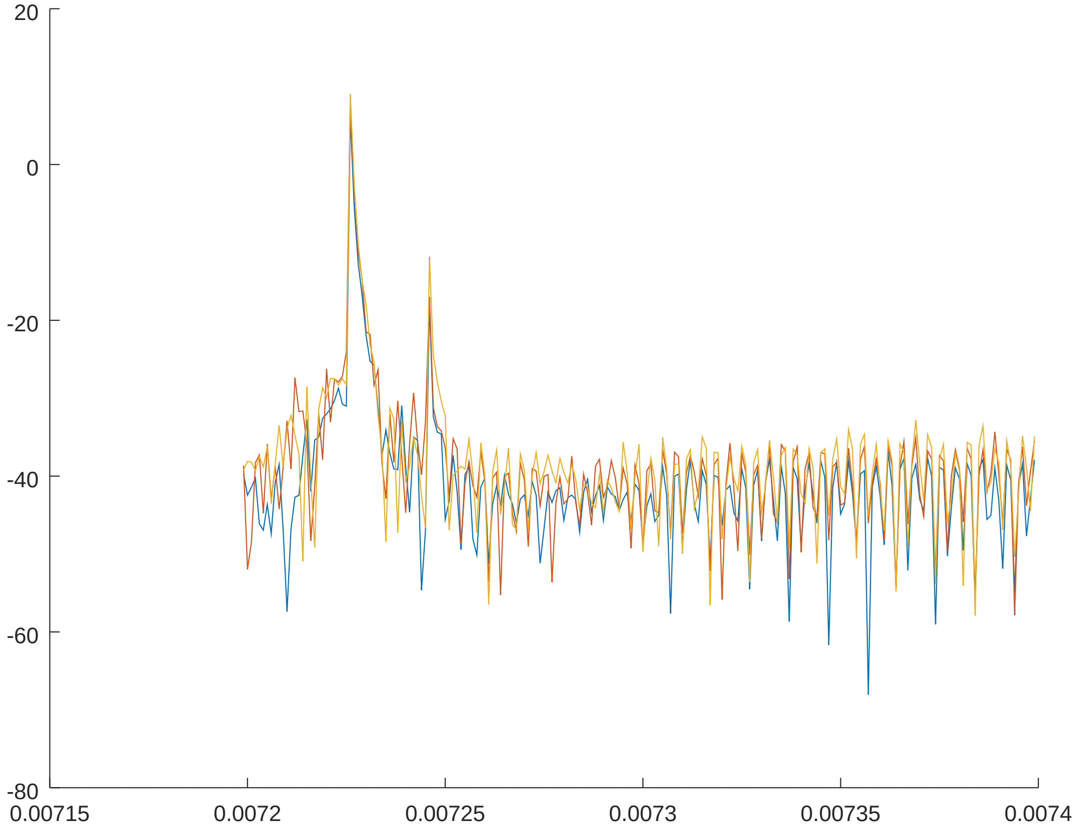

Figure 6: 3 subsequent measured impulse responses.

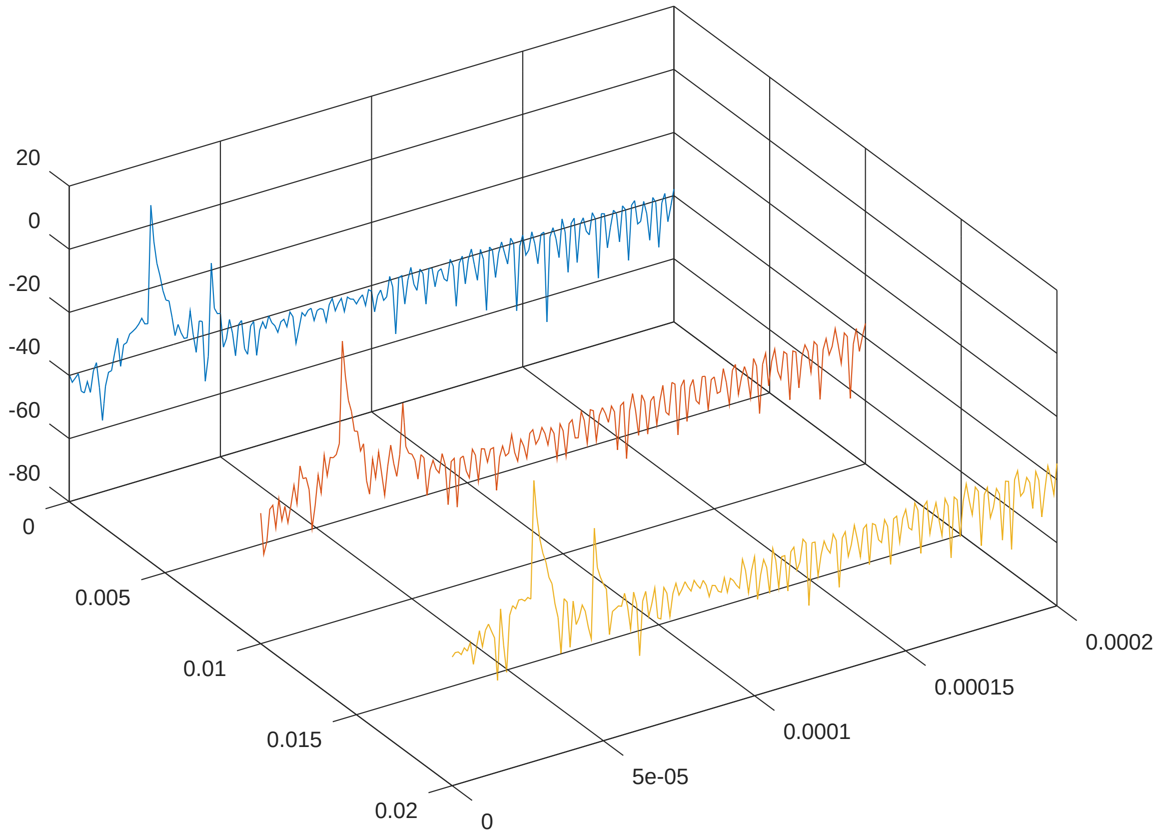

Figure 7: 3D plot of the measured impulse response.