P1

First, using AWG to generate a 5MHz sin wave. Then we modulate the sin signal by IQ modulator. The IQ modulator will mix these baseband signals with a local oscillator signal at the center frequency of 400MHz and we can get the 2-tone signal.

To generate an M-tone signal, we use the AWG to generate M distinct sine waves, each with a frequency that the target frequency added or subtracted to central frequency. Then, after modulating the baseband signal by IQ Modulator, we can get the target signal.

P2

(a) r is the amplitute of signal, θ is the phase

(b) $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{x^2}{2\sigma^2})$, $f_Y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{y^2}{2\sigma^2})$

$$f(x,y) = f_X(x) \cdot f_Y(y) = \frac{1}{2\pi\sigma^2} \exp(-\frac{x^2+y^2}{2\sigma^2})$$

(c) $X = r\cos\theta$, $Y = r\cdot\sin\theta$

Jacobian Matrix: $J = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{bmatrix}$, $|J| = r\cos^2\theta + r\sin^2\theta = r$

$$f(r,\theta) = f(x,y) \cdot |J| = \frac{r}{2\pi\sigma^2} \exp(-\frac{r^2}{2\sigma^2})$$

(d) $f(r) = \int_0^{2\pi} f(r,\theta)\, d\theta = \int_0^{2\pi} \frac{r}{2\pi\sigma^2} \exp(-\frac{r^2}{2\sigma^2})\, d\theta = \frac{r}{\sigma^2} \exp(-\frac{r^2}{2\sigma^2})$

θ is uniform distribution; $f(\theta) = \frac{1}{2\pi}$, $\theta \in (0, 2\pi)$

(e) Central Limit Theorem states that the sum of a large number of independent and identically distributed random variables, each with any distribution, will tend to have a normal distribution. Since the in-phase and quadrature-phase components are essentially sums of the cosines and sines of various phases respectively. So the amplitude of I/Q signals tend to be normally distributed due to the central limit theorem.

(e) It will be Rician distribution.

P3

(a) $h(\tau, t) = \alpha_0 \delta(\tau - \tau_0) + \alpha_1 \delta(\tau - \tau_1)$

$\phi(\Delta\tau, 0)$ is the power delay profile

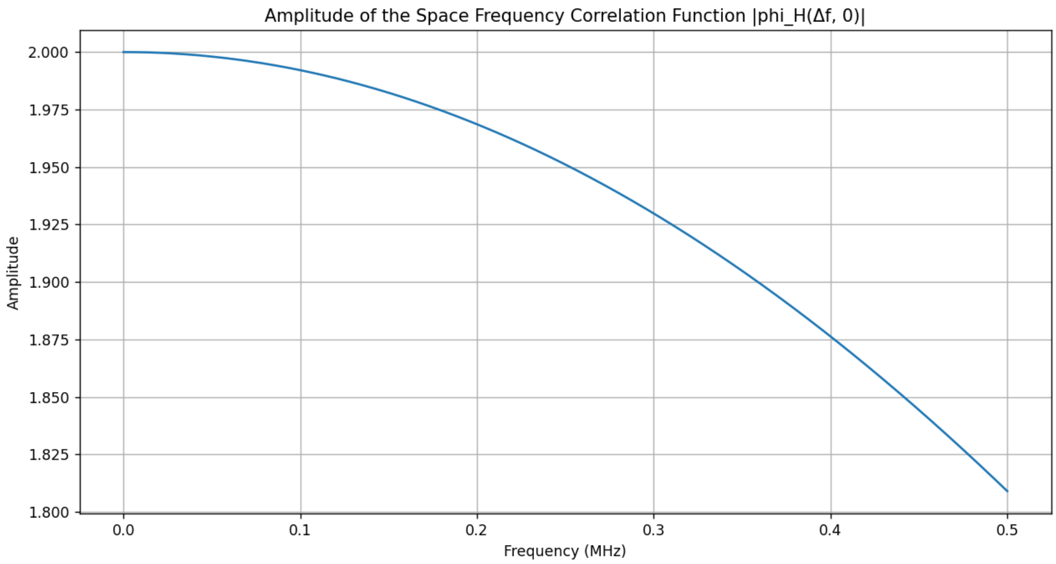
Space-frequency correlation function $\phi_H(\Delta f, 0)$ is the Fourier transform of PDP

$$\phi_H(\Delta f, 0) = \int_{-\infty}^{+\infty} \phi(\Delta\tau, 0) e^{-2\pi\Delta f \tau} d\tau = \bar{\alpha} e^{-j2\pi f \tau_0} + \bar{\alpha} e^{-j2\pi f \tau_1}$$

$$= \bar{\alpha}\left(1 + e^{-j2\pi f \cdot 200ns}\right)$$

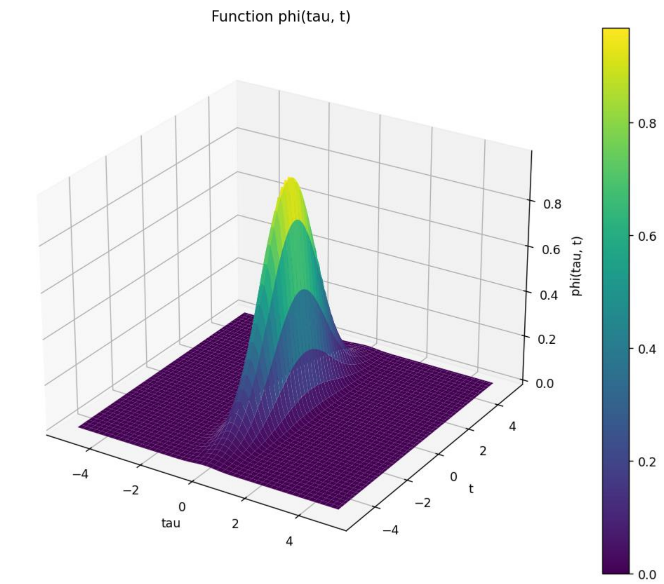
(b) $$|\phi_H(\Delta f, 0)| = \bar{\alpha}\sqrt{(1 + \cos(2\pi f \cdot 200ns))^2 + \sin^2(2\pi f \cdot 200ns)}$$

(c)



P4

(a)



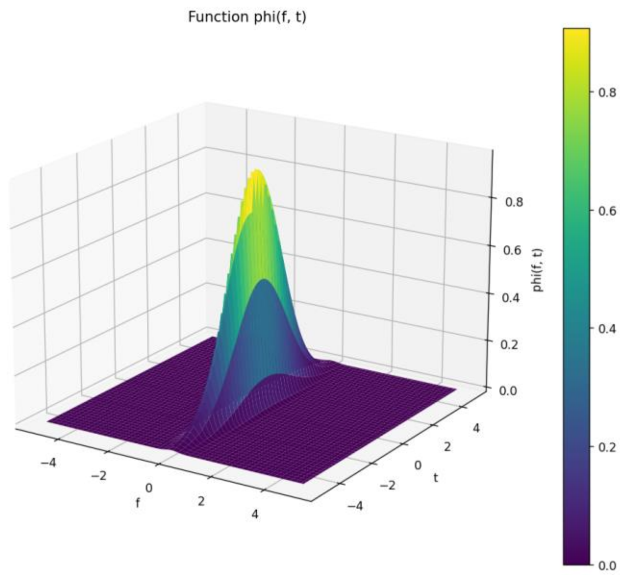
(b)

$$\phi(\tau,t) = e^{-\pi\left(\frac{\tau^2}{m^2}+\frac{t^2}{n^2}\right)}$$

Fourier transform with respect to τ: $\Phi(f,t) = e^{-\pi\frac{t^2}{n^2}} \int_{-\infty}^{\infty} e^{-\pi\frac{\tau^2}{m^2}} e^{-2\pi j f\tau}\, d\tau$

$$= e^{-\pi\frac{t^2}{n^2}} \cdot m \cdot e^{-m^2\pi^2 f^2}$$

$$n = 4m \Rightarrow \Phi(f,t) = m\, e^{-m^2\pi^2 f^2 - \pi\frac{t^2}{16m^2}}$$



P5

(a) delay spread:

mean delay: $\mu_\tau = 0$

$$T_{RMS} = \sqrt{\int_{-\infty}^{\infty} (\tau - \mu_\tau)^2 e^{-\frac{\tau}{m^2}\tau^2}}$$

$$= \frac{m}{\sqrt{2\pi}} = 3.133 \times 10^{-4}$$

RMS delay spread is 3.133×10^{-4}

(b) coherence bandwidth:

$$\Phi(f) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow f = \frac{1}{m\pi}\sqrt{\frac{\ln 2}{2}}$$

$$B_m = 2f = \frac{1}{m\pi}\sqrt{2 \cdot \ln 2} \approx 3 \times 10^3$$

(c) $\frac{1}{T_{RMS}} \approx 3.19 \times 10^3$, $B_m \approx 3 \times 10^3$

$$\Rightarrow B_m \approx \frac{1}{T_{RMS}}$$

(d) coherent time:

$$e^{-\pi \frac{t^2}{n^2}} \approx \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad t = n \cdot \sqrt{\frac{\ln 2}{2\pi}}$$

$$T_d = 2t = n \cdot \sqrt{\frac{2\ln 2}{\pi}} = 3{,}32 \times 10^{-4}$$

(e) doppler bandwidth:

Fourier transform of $\phi(\tau, t)$ with respect to t:

$$S(\tau, \rho) = e^{-\pi \frac{\tau^2}{m^2}} \cdot \int_{-\infty}^{\infty} e^{-\pi(\frac{t^2}{n^2})} e^{-2\pi j \rho t} dt$$

$$= n \cdot e^{-\pi n^2 \rho^2 - \pi \frac{\tau^2}{m^2}}$$

$$S(\rho) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad n \cdot e^{-\pi n^2 \rho^2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \rho = \frac{1}{n\sqrt{\pi}} \sqrt{\frac{\ln 2}{2} - \ln n}$$

$$B_d = \rho = 3{,}18 \times 10^3$$

(f) $\frac{1}{T_d} = 3 \times 10^3$, $B_d = 3{,}18 \times 10^3$

$$B_d \approx \frac{1}{T_d}$$

P6

(a)

Flat fading channel means all frequency components of the signal undergo the relatively constant magnitude of fading. Flat fading affects the signal uniformly across its entire bandwidth.

Frequency-sensitive fading means fading happens when different frequency components of the signal experience different levels of fading.

(b)

Slow fading means that the channel maintains a relatively constant value for a period of time that is much greater than the symbol time.

Slow fading refers to a relatively large change in the channel in a time greater than the symbol time.

(c)

The spread factor is the ratio of the bandwidth of the spread signal to the bandwidth of the original signal. A higher spread factor indicates a wider spread of the signal across the frequency band, which enhances resistance to interference and eavesdropping, and improves signal robustness.