LTV channel

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1 Preliminary exercises

P1 How can you generate a 2-tone signal centered at $f_c = 400MHz$ and having frequencies $f_l = 395MHz$ and $f_h = 405MHz$, given that you have an arbitrary waveform generator (AWG) an IQ modulator at your disposal. AWG can generate any waveform whose maximum frequency is less than 100 MHz. How can you generate M-tone signal, where M can be $M = 2, 4, 5, ...$

P2 Rayleigh probability density function $f(r)$ is the amplitude distribution of a complex Gaussian signal.

$$f(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} u(r) \tag{1}$$

where $u(r)$ is the unit step function.

a) Present the complex signal $X + j \cdot Y$ in polar form $r \cdot e^{j\theta}$. How can you interpret r and θ?

b) If the random variables X and Y are independent and have zero-mean Gaussian distribution with the same variance σ^2 (this is "complex Guassian"). Write out the joint probability density function $f(x, y)$?

c) Do a change of variables $X = r\cos(\theta)$ and $Y = r\sin(\theta)$ and derive from $f(x, y)$ $f(r, \theta)$. This is the joint distribution of the amplitude and phase of a complex Gaussian signal at the given time instant.

d) Find the Rayleigh pdf $f(r)$ by integrating $0 \leq \theta < 2\pi$ away from $f(r, \theta)$. What is the pdf of the phase, i.e. $f(\theta)$?

e) How can the assumption that X and Y are Gaussian be justified in a multipath fading scenario?

f) Comment only: What distribution would $f(r)$ be if X and Y are not zero-mean (but still independent Gaussians)?

P3 Assume a two tap channel with independently Rayleigh fading taps. $\tau_0 = 0$ and $\tau_1 = 200$ ns. After correlation in τ direction this channel has impulse response

$$h(\tau, t) = \alpha_0 \delta(\tau - \tau_0) + \alpha_1 \delta(\tau - \tau_1) \tag{2}$$

where α_0, α_1 are Rayleigh fading amplitudes of the taps and $\delta(.)$ is Dirac delta function.

$$\phi(\Delta\tau, 0) = E[h(\tau, t)h^*(t + \Delta\tau, t)] = \bar{\alpha}_0 \delta(\tau - \tau_0) + \bar{\alpha}_1 \delta(\tau - \tau_1) \tag{3}$$

where average amplitudes are equal $\bar{\alpha}_0 = \bar{\alpha}_1 = \bar{\alpha}$.

a) Derive the space frequency correlation function $\phi_H(\Delta f, 0)$.

b) Derive amplitude of the space frequency correlation function as $|\phi_H(\Delta f, 0)|$

c) Plot the function for frequency interval $0 \ldots 5$ MHz.

P4 A linear time-variant (LTV) system is described by correlation of its impulse response correlation function

$$\phi(\tau, t) = e^{-\pi\left(\frac{\tau^2}{m^2} + \frac{t^2}{n^2}\right)} \tag{4}$$

where parameter $n = 4m$. Compute and sketch the following functions:

a) The function itself $\phi(\tau, t)$. (Set τ to be on x axis and t on y axis and the amplitude ϕ on z axis).

b) The frequency response $\phi_H(f, t)$ that is Fourier transform of the function with respect to τ. (for that take Fourier transform with respect to τ and assume that t is constant).

P5 Consider the channel correlation function in previous problem [P4] with $n = 5 \cdot 10^{-4}$. Assume the channel response width is set to be at the 3 dB level from the peak. Estimate the following parameters

a) The delay spread T_m
b) The coherence bandwidth B_m
c) Verify the relationship $B_m \approx 1/T_m$
a) The coherence time T_d
b) The Doppler bandwidth B_d
c) Verify the relationship $B_d \approx 1/T_d$

P6 Explain briefly the following concepts:

a) Flat fading and frequency-selective fading
b) Slow fading and fast fading
c) Spread factor