Lab Report

on

Reflection Coefficient Measurements using Generator and Spectrum Analyzer and VNA

Preliminary exercises

Aalto University

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1 Introduction

In electrical systems, there is always transfer of power from source to load. At every connector some of the power is passed forward and some of the power is reflected back. A connection quality is characterized by the amount of energy reflected from the connection. In an optimal case none of the power is reflected. Usually the amount of reflection is measured by a circuit analyzer or by a cable radar. In this laboratory work we measure reflection coefficient by using a signal generator and spectrum analyzer.

2 Transmission line Analysis

2.1 Wave Phenomenon

At low frequencies a circuit can be analyzed by its lumped model. A lumped model replaces a spatially distributed circuit by its topological models of its discrete components. As such it is valid only when the studied signal wavelength is much larger than the physical length of the studied circuit. If the length of the circuit conductors is comparable to the wavelength of the underlying electrical signal, the assumptions of lumped circuits analysis becomes invalid. For instance, in that case while measuring current in a cable we can't assume that all the measurements along the cable are taken at the same time. This assumptions can be proved false with the analysis of the set up shown in figure 1.

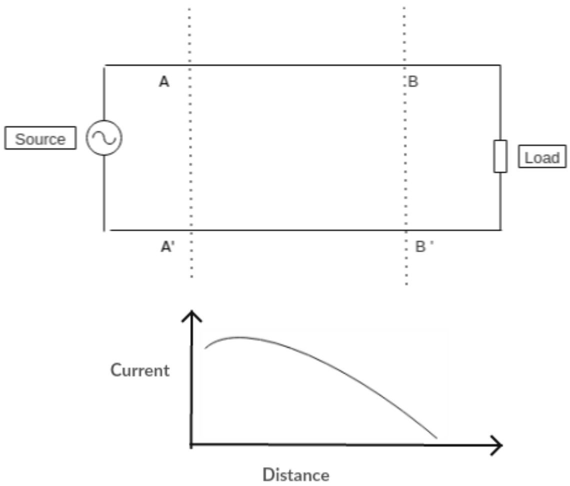

Figure 1: Typical Electrical Set up

In the figure 1 between source and load are marked two points AA' and BB'. Distance between the points is l and the speed of the signal is ϑ. Given the signal frequency *'f'* the transit time between AA' and BB' is

$$TransitTime(\tau) = \frac{Distance}{Speed} = \frac{l}{\vartheta} \tag{1}$$

and time period of signal is,

$$T = \frac{1}{f}. \tag{2}$$

If T $\backsim \tau$, then the signal value doesn't vary only as a function of time but also as a function of position (see figure 1). Because the signal varies also the current in line between AA' and BB' varies and therefore lumped circuit model is not applicable.
When distances are in order of wavelength we have to treat the **current and voltages as waves** and in this case the current and voltage values depend on measurement position in the transmission line [1] . This phenomenon is known as wave phenomenon.

2.2 Characteristic Impedance

If the circuit size is comparable to the wavelength the analysis has to consider the elements physical dimensions. The impact of individual circuit elements has to be computed based on their distances. The analysis can still be simplified by splitting the transmission line into small section (with size Δz). Within one of such section we can still use lumped circuit analysis (figure 2). A typical transmission line characteristics are expressed as a functions of resistive, capacitive, inductive and conductance elements.

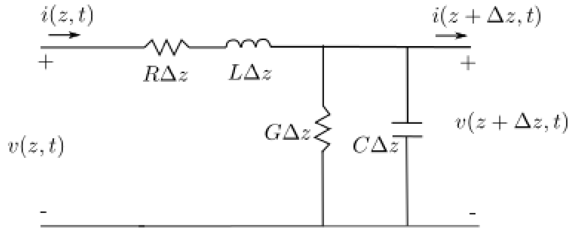

Figure 2: Small Section of transmission line

The parameters in the above figure are defined as follows:

R resistance per unit length for the transmission line (Ohms/meter)

L inductance per unit length for the tx line (Henries/meter)

G conductance per unit length for the tx line (Siemans/meter)

C capacitance per unit length for the tx line (Farads/meter)

The current and voltage of the circuit are expressed as

$$\Delta V = -(R\Delta z + j\omega L\Delta z)I \tag{3}$$
$$\Delta I = -(G\Delta z + j\omega C\Delta z)V \tag{4}$$

From equations 3 and 4, we can derive the **telegrapher's equation**

$$\frac{d^2V}{dz^2} = (R + j\omega L)(G + j\omega C)V = \gamma^2 V \tag{5}$$
$$\frac{d^2I}{dz^2} = (R + j\omega L)(G + j\omega C)I = \gamma^2 I \tag{6}$$

From telegrapher's equation 5 & 6, we can derive

$$V(z,t) = V^+\epsilon^{(j\omega t - \gamma z)} + V^-\epsilon^{(j\omega t + \gamma z)} \tag{7}$$
$$I(z,t) = I^+\epsilon^{(j\omega t - \gamma z)} + I^-\epsilon^{(j\omega t + \gamma z)} \tag{8}$$

$V^+\epsilon^{(j\omega t - \gamma z)}$ is a complex quantity. The real portion of it, $V^+\cos(\omega t - \beta z)$, describes the forward propagating wave. Similarly, real component of other half of the equation,$V^-\epsilon^{(j\omega t + \gamma z)}$ is $V^-\cos(\omega t + \beta z)$, describes backward propagating wave.

The solutions to the equations 7 or 8, unpacks the complex nature of the voltages (or currents) in the transmissions lines. It illustrates also the reason of **standing waves in a cable**. Standing waves are mix of **forward traveling voltage wave (V^+) from source to load and backward traveling voltage wave(V^-) from load to source**. The interference pattern of these two waves defines the value of the voltage (or current) at any point of the transmission line. As the signal travels in forward and backward direction, it experiences crest and trough, thus **different impedance at all points of the transmission line**. As per conventional definition, we can find out the impedance experienced by the forward traveling wave as a ratio of voltages and current wave.

Repeating the same for backward traveling waves, we have

$$\text{Characteristic Impedance } Z_o = \frac{V^+}{I^+} = \frac{V^-}{I^-} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \tag{9}$$

$$\text{and Propagation constant}(\gamma) = (\alpha + j\beta) = \sqrt{(R + j\omega L).(G + j\omega C)} \tag{10}$$

From the real portion(α) of propagation constant, we can estimate the decay rate of the amplitude of the wave. The transmission line is more lossy the larger α is. The imaginary portion (β) describes the number of oscillations of the wave as a function of position in the transmission line. We can use characteristic impedance to derive the instantaneous impedance experienced by the signal at any position of the line.

2.3 Reflection Coefficient

The reflection coefficient is the ratio of reflected or backward traveling wave to transmitted or forward traveling wave. It is used to **understand the implications of the impedance discontinuity due to the wave phenomenon**.
Relationship between impedance at any point of transmission line(Z_L) and characteristic impedance Z_0 is

$$Z_L = \frac{1 + \tau_L}{1 - \tau_L} Z_0 \tag{11}$$

where τ_L represents the reflection coefficient at any point of the transmission line and is defined as

$$\tau_L = \frac{V^- \epsilon^{\gamma l}}{V^+ \epsilon^{-\gamma l}} \tag{12}$$

This newly derived term as a function of Z_L, helps us understand how to design systems which can extract maximum power from the source. Few characteristics of the reflection coefficient are

a) τ_L can never be greater than 1, as the reflected energy can only be derived from the transmitted one. Thus it is always less than the forward traveling wave $sourcevoltage = V^+ + V^-$ and $(\tau_L)_{MAX} = 1$.

b) If $\tau_L = 0$ at any point along the line there is absence of reflected wave. This case occurs when **the terminated load is equal to characteristic impedance or** Z_0.This can be proved by plugging $\tau_L = 0$ in the equation 16,

$$Z_L = Z_0 \tag{13}$$

This is referred as the **Matched Load** condition, which ensures **completer transfer of power from source to load and homogeneous experience of characteristic impedance at any point of the transmission line.** This trick can be used by all **measuring devices to correctly ascertain the source power** and by the **load to absorb maximum power** possible.

c) If $\tau_L = 1$ at any point, it implies that the forward traveling wave reflects totally, thus completely failing in transfer of power from source to that point in the transmission line. This occurs, **when the transmission line is not terminated or short circuited**. Also, here the **signal experiences infinite impedance, thus complete reflection**. It can be proved by plugging $\tau_L = 1$ in the equation 11,

$$Z_L = \infty. \tag{14}$$

This feature can be used for steering power from one port to another (assuming if the transmission coefficient between the two is not very high).

3 Characterization of High Frequency Devices

To completely describe electrical behavior of a linear device we need easily measurable parameters. In low frequency operation we typically measure voltages and currents at the input and output ports of a device (such as H, Y, Z parameters). Similarly we can measure there parameters at high frequencies.

3.1 S parameters

As the wave phenomenon kicks in, it makes it difficult to measure total currents or voltages. It is because they become very sensitive to the location or distance from source or load. Therefore, we characterize devices by forward-backward traveling waves magnitude and phase (such parameters are called scattering parameters or in short S parameters). S parameters is used to describe a multi port device at a certain characteristic impedance Z_0 and frequency(ω). They are easily convertible to to H, Y and Z parameters and their measurement does require loads at the at the ports of the measured device.

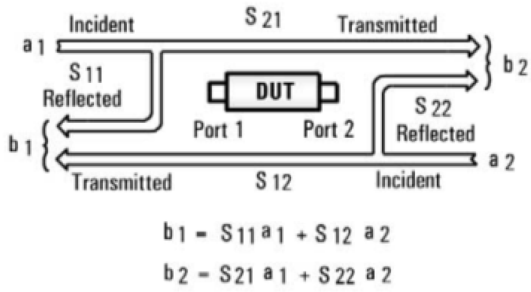

Figure 3: S parameters [2]

A N port device is characterized by N^2 S parameters which are denoted as S_{XY}, where X represents the port from which signal emerges and Y represents the input port. Incident voltage traveling waves are usually denoted by $a_{portnumber}$ and output are by $b_{portnumber}$ (figure 3).

3.2 Measuring S parameters

S parameters are linear and complex quantities, expressed as magnitude-phase pairs or real-imaginary pairs. Usually only the magnitude is used to quantify relevant terms using S parameters such as insertion loss or return loss. For a typical two port device, the S parameters are defines as follows

a) $S_{11} \rightarrow$ **Forward reflection Coefficient**. Defined as the ratio of reflected voltage traveling wave to the incident voltage traveling wave at port 1.

b) $S_{22} \rightarrow$ **Reverse reflection Coefficient**. Defined as the ratio of reflected voltage traveling wave to the incident voltage traveling wave at port 2.

c) $S_{21} \rightarrow$ **Forward transmission Coefficient**. Defined as the ratio of transmitted voltage traveling wave to port 2 to the incident voltage traveling wave at port 1.

d) $S_{12} \rightarrow$ **Reverse transmission Coefficient**. Defined as the ratio of transmitted voltage traveling wave to port 1 to the incident voltage traveling wave at port 2.

In order to measure S_{21} or **Forward parameters** as shown in figure 4:

1) We need to terminate the output port 2 by the load which is equivalent to the Z_0 or characteristic impedance. This ensures, there is no incident wave from the output port.

2) We place the source at port 1.

3) We measure S_{21}.

Similarly, for **Reverse parameters**,

1) We terminate the output port 1 by the load which is equivalent to the Z_0 or characteristic impedance.

2) We place the source at port 2.

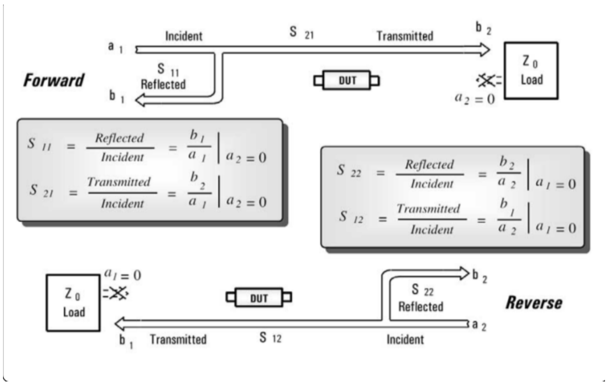

Figure 4: Measuring S parameters[2]

3) We measure S_{12}.

Notice that we don't **directly measure reflection parameters** and the transmission parameters are **indirectly derived from the insertion loss**.

3.2.1 Insertion Loss & Isolation

Insertion Loss-: is defined as the amount of signal attenuation or power loss at certain frequency, as a device is introduced in the transmission lines. Generally it is defined between the coupled ports of the device. It can be used for designing devices for suitable applications. For e.g In two way power dividers, insertion loss is expected to be 3 dB between input ports and output ports, thus ensuring equal division of power.

The insertion loss for two port is defined as

$$Insertionloss(Forward) = -20log_{10}|S_{21}|, a_2 = 0, a_1 = Input \tag{15}$$

$$Insertionloss(Reverse) = -20log_{10}|S_{12}|, a_1 = 0, a_2 = Input \tag{16}$$

Isolation -: is defined as the amount of signal passing between non coupled ports of a device. It is generally very high in magnitude (usually isolation is around 20 dB). Some microwave devices take advantage of isolation. For instance, for connecting transmitter and receiver to a single antenna we separate them by circulator. Circulator has high impedance between transmitter output and receiver input port and in in this way helps to protect receiver input.

Isolation for two port is defined, when **one of the port is not terminated by matched load, thus ensuring reflection or incident wave from that port**.

$$Isolation = -20log_{10}|S_{12}|, a_2 = Notmatched, a_1 = Input \tag{17}$$

$$Isolation = -20log_{10}|S_{21}|, a_1 = Notmatched, a_2 = Input \tag{18}$$

4 Microwave Devices

4.1 Circulator

A coaxial circulator is passive 3 or 4 port device, commonly used to isolate one signal from other. It couples the incident wave on certain port to another port and decouples the remaining port in a prescribed order.

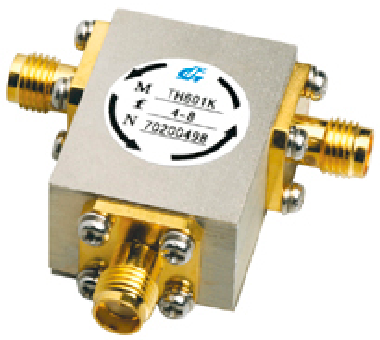

Figure 5: Coaxial Circulator

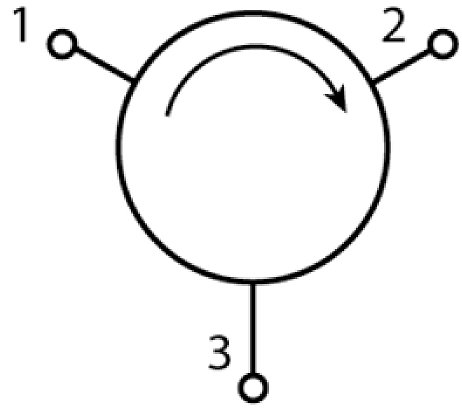

Figure 6: Schematic diagram of Circulator

General properties of Circulator are described as follows

a) An ideal Circulator is a lossless and matched at all ports. It means no power is absorbed by the circulator (power is not converted to heat). There is no reflection from any port i.e. if a port is terminated by Z_0 the reflection coefficient from the port is zero,

b) In a clockwise manner, the wave incident at port 1 is coupled with port 2 but isolated from port 3. Similarly, the incident wave at port 2 is coupled with port 3 but isolated from port 1.

c) Circulator is a non reciprocal device. It contains ferrite materials and forward transmission coefficients and reverse transmission coefficients are not the same.

d) Applying all above described properties to S parameters we can conclude that

   - As all the ports are matched, $s_{11} = s_{22} = s_{33} = 0$.
   - As the ports are strictly coupled in a clockwise manner, only $s_{21}, s_{13} and s_{32}$ have any value.
   - As it is also lossless. Columns of S parameters add to 1. for e.g

$$|s_{11}|^2 + |s_{21}|^2 + |s_{31}|^2 = 1 \tag{19}$$

$$|s_{12}|^2 + |s_{22}|^2 + |s_{32}|^2 = 1 \tag{20}$$

   - we get ideal circulator as ,

$$S = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

e) A circulator behaves as a isolator, if one of it's port is terminated with a matched load. For e.g, if port 3 is terminated with 50 ohms, circulator behaves likes a two port device, with all power transfer from port 1 to port 2.

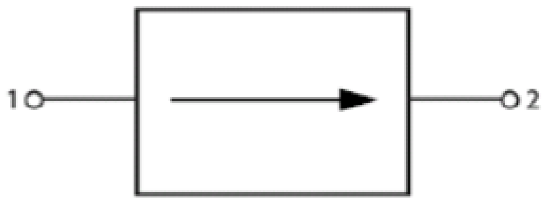

Figure 7: Circulator as Isolator

4.2 Four Way Coaxial Power Splitter/Combiner

Power splitter is a passive device, used to combine power from many ports or split power equally to many ports. Ideally, the output for a splitter output signals have same phase and amplitude. (or in case of combiner output is delivered as vector combination of input signals.

In figure 8 is 2 splitter used in measurements. The schematic diagram of the device is shown in figure 9.

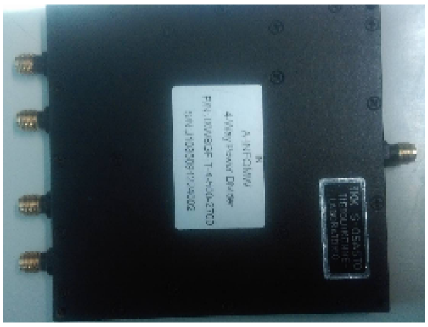

Figure 8: Coaxial 4 Way Power Splitter/Power Combiner

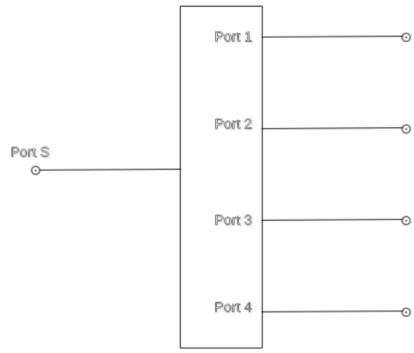

Figure 9: Schematic diagram of Power Splitter

A two way splitter has a theoretical insertion loss around 3 dB. The insertion loss increases with the number of ports, thus being 6dB for 4 ports. According, if power is supplied at to port S of a four way splitter the output ports have power reduced by 6 dB. Impedance mismatch at ports impacts the power appearing at ports.

5 Measurements and confidence intervals

Because of the noise we can not measure exactly. The measured value is only approximation of the wanted subject of measurement, or **measurand**. Therefore we have to described also uncertainty of the measured value.

The difference of the measured value from the exact value is called measurement error. The error can be systematic or random. Systematic error remain same over multiple measurements. Random error varies from measurements to measurements. The random error often is described by its distribution. In our measurements we consider additive Gaussian random error model. In that model the exact measurand x is added random noise n that has Gaussian distribution. The received signal y is

$$y = x + n. \tag{21}$$

From this received signal we compute what would be most likely value of x and describe with what confidence we think that it has this value.

In this work we use an approach where we make multiple measurements. We estimate from the measurements the most likely measurand value and corresponding confidence interval. Confidence interval describes how the exact value could deviate from the estimated value. To quote Jerzy Neyman, who employed frequentist inference methods *"Confidence level C defines that if we repeat the experiment many times the interval will contain the true value a fraction C of the time"* . For instance in figure 10 is illustrate a distribution of the measured samples. This data set is close to normal distribution. In the figure we use 95 % confidence interval. The confidence interval describes the lower and higher value between which the measurement is with 95 % probability (confidence).

In order to compute the confidence interval we need to know the distribution of measurements. Often the measurements is well approximated by Gaussian distribution. However, we don't know the means and variance of the distribution. These parameters can be computed from the received samples as estimates. The mean is

$$\hat{x} = \frac{1}{N} \sum_{n=0}^{N-1} y_n \tag{22}$$

where y_n is n-th measured sample.

Estimate of the unbiased variance

$$\hat{s} = \frac{1}{N-1} \sum_{n=0}^{N-1} (y_i - \hat{x})^2 \tag{23}$$

Both estimates $\hat{x}$ and $\hat{s}$ are random variables. If to use them as mean and variance the distribution is not any more Gaussian but Student-t distribution. However, if we have about more than 10 samples the distribution is well approximated by Gaussian distribution.

In this document we estimate confidence interval by assuming a Gaussian distribution. As illustrated in figure 10 X% confidence interval is interval where falls X% probability (In 10 we use 95 % interval). The confidence interval is expressed as limits. We insert the mean and variance estimates into Gaussian distribution equation and compute the corresponding limits.

For confidence interval X% we compute

$$Q(z) = 1 - X \tag{24}$$

where X corresponds to size of the confidence interval (it is not in %) and $Q(x)$ is Q function

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{\frac{u^2}{2}} du \tag{25}$$

After change of variables from zero mean and variance one to estimated mean $\hat{x}$ and variance $\hat{s}$ we get

$$z => \frac{(z' - \hat{x})}{\sqrt{\hat{s}}} \tag{26}$$

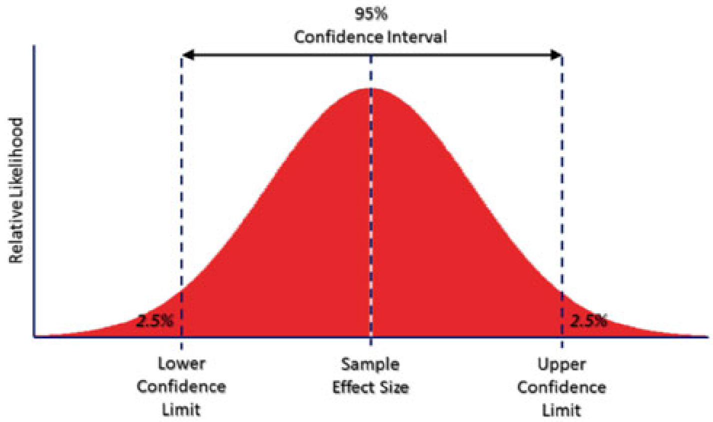

Figure 10: Normal distribution & Confidence levels

Table 1: $\mathrm{invQ}(1-X)$ for various confidence interval values

Confidence Level	Critical Value
99%	2.576
98%	2.326
95%	1.96
90%	1.645

where $z = \mathrm{invQ}(1-X)$. $\mathrm{invQ}(1-X)$ is inverse Q function, it's values can be pre-computed or evaluated by mathematical tools (Matlab for instance). Some of the $\mathrm{invQ}(1-X)$ values for various confidence intervals are given in the table 1.

After the change of variables the confidence interval limits are

$$\hat{x} \pm z \cdot \hat{s} \tag{27}$$

The algorithm for computing confidence interval is

(1) collect N samples y_n

(2) estimate the mean, $\hat{x}$, and variance $\hat{s}$

(3) select the confidence level (for instance 90 %)

(4) compute the limits z by using invQ function

(5) evaluate from 27

5.1 Example

We estimate a confidence interval by filling table 5.1

Parameter	Sample Size	Mean	Standard Deviation	Confidence Level	Confidence Interval

Assume we have N=10 samples and are interested to estimate the interval where we find a sample with 90% confidence (90 % confidence interval). Assume we have sample size N=10 measurements are

$$y_n = \{9.5731;\ 10.9597;\ 10.2294;\ 9.9801;\ 10.2260;\ 9.9352;\ 9.9607;\ 10.4711;\ 10.4456;\ 10.4482\} \tag{28}$$

We have mean estimate

$$\hat{x} = \frac{1}{N} \sum_{n=0}^{N-1} y_n = 10.22 \tag{29}$$

Variance estimate is

$$\hat{s} = \frac{1}{N-1} \sum_{n=0}^{N-1} \left(y_i - \hat{x}^2\right)^2 = 0.15 \tag{30}$$

For the confidence interval 95% invQ$(1 - 0.95) = z = 1.96$. The interval lower and upper limits are

$$\hat{x} - z \cdot \sqrt{\hat{s}} \quad = \quad 9.93 \tag{31}$$

$$\hat{x} + z \cdot \sqrt{\hat{s}} \quad = \quad 10.51 \tag{32}$$

The result is: mean estimate 10.22 with 90% interval with limits [9.93 10.51].

5.2 Mean estimation given one sample variance

Sometimes we know from distribution the measured samples are derived, ie. we know the mean and variance of one sample. The question we can ask is: if we make multiple measurements (we have multiple samples) and compute the mean over these samples what would be the distribution of the computed value. Formally we can describe the problems as following: we have N samples derived from distribution with mean m and variance s. We compute mean over theses N samples. What is the distribution of this mean? We know that the mean is m. In order to compute describe the distribution we compute the standard deviation of the averaged mean. This is given as $\sqrt{\frac{s}{N}}$. Given this standard deviation we can evaluate now what is the we find a averaged mean value in certain interval as

$$m \quad - \quad z \cdot \sqrt{\frac{s}{N}} \tag{33}$$

$$m \quad + \quad z \cdot \sqrt{\frac{s}{N}} \tag{34}$$

6 PreLab Questions

(1) State the difference between lumped element circuits and distributed element circuits.

(2) Consider a two port device. Is the S_{11} always equal to the reflection coefficient? If not, why?? Provide proof.

Hint: write up two port S parameters equation. Set the ports to be in matched or non matched condition.

(3) For certain frequency f & characteristic impedance Z_0, S parameters of the two port system is defined as

$$S = \begin{bmatrix} 0.1 & 0.8\underline{/90} \\ 0.8\underline{/90} & 0.2 \end{bmatrix}$$

Calculate the reflection coefficient τ, when port 2 is short circuited.

(4) In a class of size 35, the mean score was 8 points for the assignment. The standard deviation (of one student) was found to be 1.5. Calculate the confidence interval for the mean over the class

(a) 95 % confidence level
(b) 99 % confidence level
(c) Which interval is bigger and Why?

References

[1] David M. Pozar. *Microwave engineering.* J. Wiley, 2005.

[2] Agilent Technologies. Agilent network analyzer basics, Jan 2004.