Aalto University
School of Electrical
Engineering

ELEC-E7120 Wireless Systems
Homework for Unit 3

Released on 22.9.2023
Due date 29.9.2023 (by 10:00 AM - Finnish Time)

Guidelines:

- Return the electronic version of your answers before the deadline using the corresponding homework link in MyCourses.
- Homework is individual.
- Some references to do the homework are
  - Slide-sets covered in “Lec5” and “Lec6” (@ MyCourses)

Problem 3.1 (1.5 points). *AWGN channel capacity in different working regimes*

The (Shannon) capacity of an AWGN channel is given by:

$$C_{AWGN} = W\, log_2\left(1 + \frac{P}{N_0 W}\right) \text{ [bit/s]},$$

where P $[W]$ is the received signal power, W $[Hz]$ is the communication bandwidth, and N_0 $[W/Hz]$ is the Power Spectral Density (PSD) of the additive background noise at the receiver side.

a) Find the ***spectral efficiency* of the AWGN channel** for a fixed received signal power and PSD of additive noise, when the bandwidth grows large (i.e., when $\boldsymbol{W} \rightarrow \infty$). What kind of behavior are you able to observe in the so-called ***power-limited region***?

b) Consider now the case when the bandwidth remains fixed, and the power grows large. Determine the ***spectral efficiency* of the AWGN channel** when the received signal power grows large (i.e., when $\boldsymbol{P} \rightarrow \infty$). What kind of behavior is now observed in the so-called ***bandwidth-limited region****?*

c) Let us assume that the bandwidth of the AWGN channel is $\boldsymbol{W = 40\ MHz}$**,** the total received signal power $\boldsymbol{P = 5\ mW}$, and noise PSD $N_0 = 2 \times 10^{-9}\ W/Hz$. How much does capacity increase by doubling the received power? What happens to the channel capacity when the channel bandwidth is doubled? What looks more convenient from a Wireless System designer perspective? Is this system working in the bandwidth-limited or power-limited region? Please, justify your answer briefly.

d) Let us now consider that bandwidth of the AWGN channel is reduced to $\boldsymbol{W = 1\ MHz}$, total received signal power is increased to $\boldsymbol{P = 100\ mW}$, and noise PSD maintained at $N_0 = 2 \times 10^{-9}\ W/Hz$. How much does capacity increase by doubling the received power? How much does capacity increase by doubling the channel bandwidth? What looks more convenient from a Wireless System designer perspective? Is this system working in the bandwidth-limited or power-limited region? Please, give a brief justification of your observation.

Problem 3.2 (1 point). *Bit error probability analysis in AWGN & Rayleigh fading channels*

a) For a **Rayleigh fading channel** (i.e., when received SNR follows an exponential distribution), and in case of binary signalling, it is possible to demonstrate that the probability of error (P_e) is given by the following expression:

$$P_e = \frac{1}{2}\left(1 - \sqrt{\frac{\mathrm{SNR}}{1+\mathrm{SNR}}}\right) \qquad (1)$$

It is possible to demonstrate as well that the exact expression above can be asymptotically approximated by

$$P_e \approx \frac{1}{4\mathrm{SNR}} \qquad (2)$$

at high SNR values (i.e., when SNR $\rightarrow \infty$). Show how to obtain approximation in (2) from expression (1). What kind of asymptotic behaviour does it show?

Hint: Use Taylor series expansions.

b) Use the inequality

$$\left(1 - \frac{1}{z^2}\right)\frac{1}{z\sqrt{2\pi}} \leq e^{\frac{z^2}{2}}\,Q(z) \leq \frac{1}{z\sqrt{2\pi}} \qquad (3)$$

to find a suitable approximation for the probability of error in an **AWGN channel** when SNR grows large in case of binary signalling (BPSK). What kind of asymptotic behaviour does it show?

c) Find the SNR values required to obtain a bit error probability of 10^{-6} in case of:

- AWGN channel, and
- Rayleigh fading channel.

For this, assume BPSK signalling and express the result in both dB and linear. Use the expressions derived in (***a***) and (***b***). Analyse the obtained results and take out your own conclusions.

Problem 3.3 (1.5 points). *Cross link interference in TDD system*

Consider the parameters below for a scenario where there is a transmission and reception between the base stations (BS) and mobile stations (MS) as shown in Figure 1.

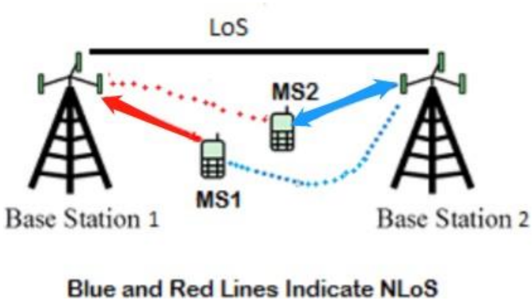

Figure 1: Link between BS 1 and BS 2 is Line-of-Sight (black line). All the other links between BS-MS and MS-MS are Non-Line-of-Sight (blue and red lines).

The bandwidth is 10 MHz, and one user is served per cell. Mobile station 1 (MS1) is served by BS1, while MS2 is served by BS2.

Transmit power per MS = 23 dBm. Transmit power per BS = 30 dBm. Noise power = -174 dBm. Distance between BSs = 100m. Distance from BS to MS= 35m.

MS1 transmits in UL all the time. MS2 uses three different configurations: 1) 100% in UL; 2) 50% in UL and 50% in DL; 3) 100% in DL.

Answer appropriately the following questions:

a) What is the **maximum data rate** that MS1 can achieve in each configuration of the different TDD frame configurations used in cell 2? (Use Shannon's formula, including the effect of both noise and interference)
b) Estimate the **upper bound for the mean data rate** that MS1 can achieve in uplink.
c) Explain, with your own words, the effect of the frame configuration of cell 2 on the data rate of cell 1? Would this effect be observed if, instead of TDD, the duplexing of both uplink and downlink transmissions would take place in the **frequency domain**?

Use the following path loss models:

- LoS Path Loss: PL(d) = 16.9 log10(d) + 38.8 [dB], 'd' in meters
- NLoS Path Loss: PL(d) = 43.3 log10(d) + 17.5 [dB], 'd' in meters