ELEC-E7120 - Wireless Systems (Fall 2023)

Weekly Exercise Session #5

Unit V. Wireless Systems

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Solutions for Homework 4

Unit IV. Cellular systems

Homework 4

4.1. Effect of the Frequency Reuse Factor in a Cellular network

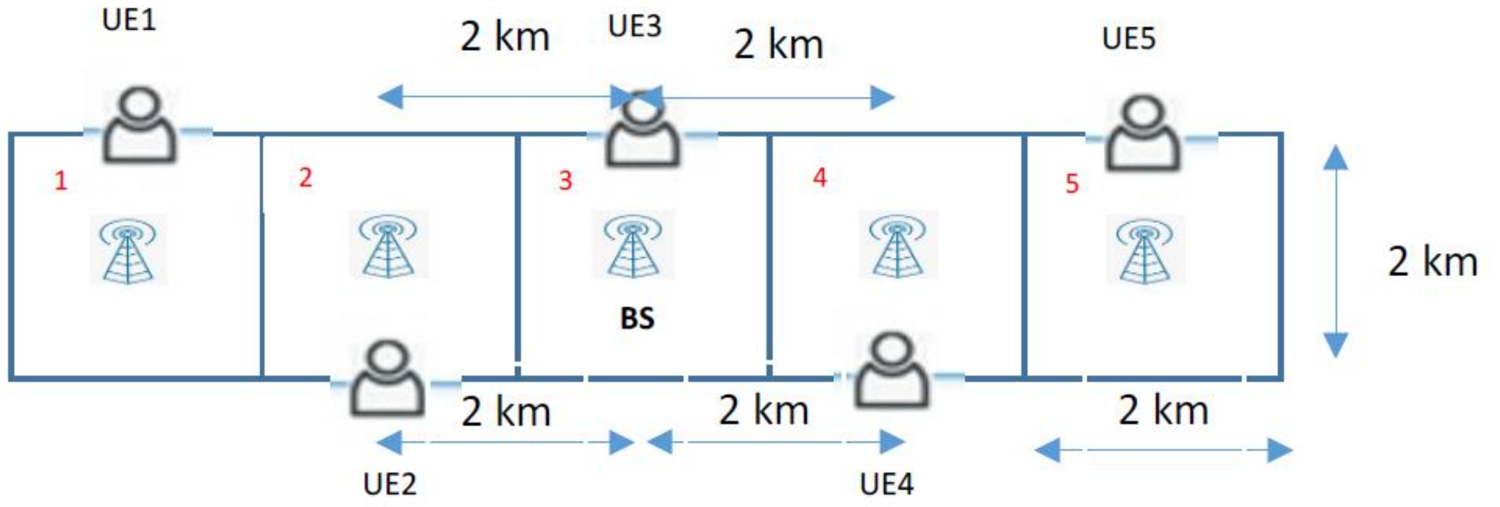

Figure 1. Simplified cellular network that provides coverage along a long-horizontal stripe (e.g., next to a highway). Cells are assumed to be squares of size 2 km x 2 km, with one base station in the center. For simplicity, only one mobile user (UE) is assumed per cell, whose location is precisely north (up) in cells with odd indexes and south (down) in cells with even indexes. If in your opinion there is any missing information, please propose a given value for the missing parameter using your common sense.

Given: Pt = 40 W, bandwidth = 10 Mhz, noise power spectral density = -174 dBm/Hz, L(d) = 137.4 + 35.2log10(d)

Compute: (a) SINR, SIR for UE 3 for frequency reuse factors 1, 2 (b) Data rate for reuse factor 1, 2 (c) Comment on which frequency reuse is better, and draw your conclusions

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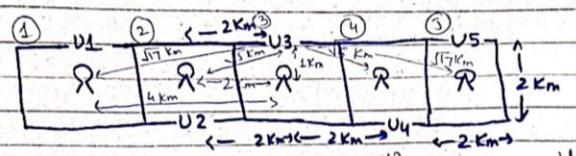

∴ Use pythagoras theorem

$hyp^2 = Perp^2 + base^2$

$hyp = \sqrt{Perp^2 + base^2}$

$= \sqrt{1^2 + 2^2}$

$= \sqrt{1+4}$

$hyp = \sqrt{5}$ Km

a)

Frequency reuse 1

Interference contributed by all other cells. So

$$SINR = \frac{P_{rx}}{Noise + (I_2 + I_1 + I_4 + I_5)}$$

$$P_{rx} = P_t - P_L = 10\log_{10}(40\times10^3) - [137.4 + 35.2\log_{10}(1)]$$

$hyp^2 = \sqrt{Perp^2 + base^2}$

$= \sqrt{1^2 + 4^2}$

$= \sqrt{1+16}$

$hyp = \sqrt{17}$ Km

$P_{rx} = -91.38$ dBm

$\boxed{P_{rx} = 7.28\times10^{-10}\, mW}$

$N_0 = -174\ dBm/Hz = 3.98\times10^{-18}\ mW/Hz$

$Noise = N_0 \times B$

$= 3.98\times10^{-18} \times 10\times10^6$

$\boxed{Noise = 3.98\times10^{-11}\, mW}$

* Cell (2) and cell (4) are at same distance so both have same interference contribution

$I_2 = I_4 = P_t - P_L$

$= 10\log_{10}(40\times10^3) - [137.4 + 35.2\log_{10}(\sqrt{5})]$

$I_2 = I_4 = -103.68\ dBm = \boxed{4.29\times10^{-11}\, mW}$

* Cell (1) and cell (5) are at same distance so

$I_1 = I_5 = P_t - P_L$

$= 10\log_{10}(40\times10^3) - [137.4 + 35.2\log_{10}(\sqrt{17})]$

$I_1 = I_5 = -113.03\ dBm = \boxed{4.98\times10^{-12}\, mW}$

$$SINR = \frac{P_{rx}}{Noise + (I_2 + I_1 + I_4 + I_5)}$$

$$= \frac{7.28\times10^{-10}}{3.98\times10^{-11} + (2\times4.29\times10^{-11} + 2\times4.98\times10^{-12})}$$

$= 5.44$ W

$\boxed{SINR = 7.30\ dB}$

$$SIR = \frac{P_{rx}}{\cancel{Noise}\ I_2 + I_1 + I_4 + I_5}$$

$SIR = 7.60$ W

$\boxed{SIR = 8.81\ dB}$

Frequency reuse 1/2

Interference contribution by every other cell (~~frequency~~ bandwidth is divide into 2 parts so same resource are used by every other cell).

$$SINR = \frac{P_{Rx}}{Noise + I_5 + I_1}$$

∴ $Noise = N_0 \times B/2$

$= 3.98\times10^{-18} \times \frac{10\times10^6}{2}$

$= 1.99\times10^{-11}\ mW$

$$= \frac{7.28\times10^{-10}}{1.99\times10^{-11} + (2\times4.98\times10^{-12})}$$

$SINR = 24.38$ W

$\boxed{SINR = 13.87\ dB}$

$$SIR = \frac{P_{Rx}}{I_5 + I_1} = \frac{7.28\times10^{-10}}{2\times4.98\times10^{-12}}$$

$= 73.09$ W

$\boxed{SIR = 18.64\ dB}$

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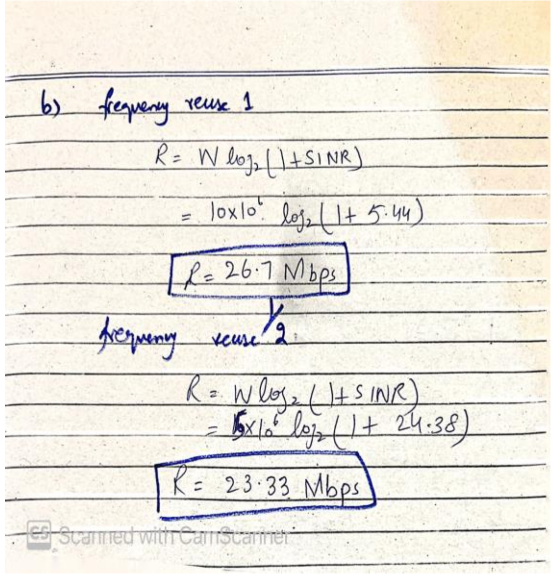


b) frequency reuse 1

$R = W \log_2(1+SINR)$

$= 10\times10^6 \log_2(1+5.44)$

$\boxed{R = 26.7\ \text{Mbps}}$

frequency reuse 1/2

$R = W \log_2(1+SINR)$

$= 5\times10^6 \log_2(1+24.38)$

$\boxed{R = 23.33\ \text{Mbps}}$

Note: A freuquency reuse (FR) factor of 1/2 is not possible to implement in a 2-D network. It only works in this toy example in which networks are deployed in a 1-D setting. The only number of cluster sizes that are available for a 2-D deployment verify the relation $N = i^2 + j^2 + i * j$, for $i, j = 0, 1, 2, ...$

That is, N = 1, 3, 4, 7, ….

c) Conclusion

- Conclusion drawn is from a network-level perspective, a frequency reuse factor of 1 (also known as universal frequency reuse) outperforms a frequency reuse factor of 1/2 in terms of achievable data rate (Shannon Capacity)
- Despite having lower SINR and SIR values, Frequency Reuse 1 offers a significantly higher capacity of 26.7 Mbps compared to 23.3 Mbps for Frequency Reuse ½ because the available bandwidth per cell is higher..
- Capacity is a crucial performance metric, representing the maximum data transfer rate the network can support. Therefore, conclusion drawn is Frequency Reuse 1 is the preferred choice for better network-level performance, as it can handle higher data transfer rates. This is the reason why FR = 1/7 (1G); FR = 1/3 (2G); FR = 1 (3G, 4G, 5G, …)
- The choice of the optimal frequency reuse factor can depend on various factors, including network topology and traffic patterns. If our aim is to improve the SINR/SIR, Frequency Reuse 1/2 appears to offer better performance because in Frequency reuse 1 as all base stations are using the same bandwidth resources, and thus causing interference in adjacent cells.

Homework 4

4.2. Sectorization principle and directive antennas

Consider the communication system as shown in Figure 2, composed of two base stations (BSs) and a mobile station (MS). Assume two cases, which are differentiated as follows:

- **Case-A:** Both BSs do not apply sectorisation, deploying **omnidirectional antennas** with 2.1 dBi gain in the horizontal plane.
- **Case-B:** Both BSs do apply sectorization using **directional antennas**. Here, the main direction of the irradiated power in each BS is denoted by a solid arrow (black) below.

The aim is to **determine the maximum SINR (in dB)** that would be feasible at the MS in **both cases** when the average path loss attenuation (in dB scale) is given by

$$L(d) = 137.4 + 35.2 \cdot log_{10}(d) \qquad d > 0,$$

where d is the distance between transmitter and receiver in kilometers.

For sake of simplicity, we assume that both BSs apply the same transmit power. For the sectorization case (Case-B), we consider that the antenna gain pattern at both BSs attains the following form:

$$G(\theta) = G_{max} + max\left\{-12\left(\frac{\theta - \theta_0}{\theta_{3dB}}\right)^2, -G_{fb}\right\}$$

where θ is the angle of arrival/departure [degrees], $G_{max} = 16\ dBi$ is the maximum antenna gain, θ_0 is the main direction of the irradiated power [degrees], $\theta_{3dB} = 60°$ is the bean width at $3\ dB$, and $G_{fb} = 25\ dB$ is the front-to-back ratio for the antenna.

Estimate the feasible spectral efficiency (i.e., date rate per unitary bandwidth) that the mobile user can achieve in downlink. When computing this value, assume that the thermal noise power is negligible with respect to the co-channel interference power, and that co-channel interference coming from adjacent cells is treated as AWGN (i.e., R = W*log2(1+ SINR)).

Problem 4.2

Calculating both path loss attenuations according to the distances (where *d* is the distance in kilometers):

Case A & B

$$L(0.3\ km) = 137.4 + 35.2 \cdot log_{10}(0.3\ km) \approx \mathbf{119\ dB}$$
$$L(0.2\ km) = 137.4 + 35.2 \cdot log_{10}(0.2\ km) \approx \mathbf{112.8\ dB}$$

And obtaining the antenna gain pattern for each signal in the **Case-B**:

Case-B

$$G(50°) = 16\ dBi + max\left\{-12\left(\frac{50° - 0°}{60°}\right)^2, -25dB\right\} = \mathbf{7.667\ dBi}$$
$$G(30°) = 16\ dBi + max\left\{-12\left(\frac{30° - 0°}{60°}\right)^2, -25dB\right\} = \mathbf{13\ dBi}$$

Considering *the noise power tends to zero to determine the maximum SINR due to* ***SIR is always an upper bound of the SINR***:

$$SINR \approx SIR[dB] = Desired\ signal\ power - Interference\ power$$

Path loss attenuation (for this problem)

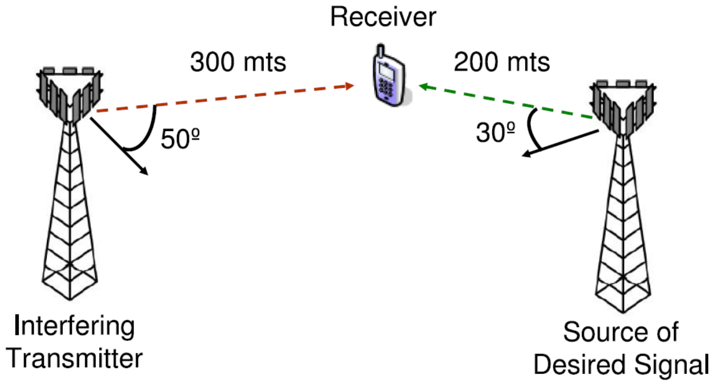
$$L(d)[dB] = 137.4 + 35.2 \cdot log_{10}(d) \qquad d > 0,$$

Antenna gain pattern

$$G(\theta)[dBi] = G_{max} + max\left\{-12\left(\frac{\theta - \theta_0}{\theta_{3dB}}\right)^2, -G_{fb}\right\}$$

Signal-to-Interference plus Noise power Ratio (SINR)

$$SINR[linear] = \frac{Desired\ signal\ power}{Interference\ power + Noise\ power}$$



Case-A

Problem 4.2

To calculate SIR, we define the desired signal power and interference signal power considering the antenna gain and path loss attenuation (**Note**: It is omnidirectional antenna gain):

$$Desired\ signal\ power\ [dB] = Pt_{BS2} - L(0.2\ km) + G(2\ dBi)$$

$$Interference\ power\ [dB] = Pt_{BS1} - L(0.3\ km) + G(2\ dBi)$$

$$Desired\ signal\ power[linear] = \frac{Pt_{BS2}}{L(0.2\ km)} G(2\ dBi)$$

$$Interference\ power[linear] = \frac{Pt_{BS1}}{L(0.3\ km)} G(2\ dBi)$$

Besides, assume that both BSs apply the <u>same transmit power</u>

$$Pt = Pt_{BS1} = Pt_{BS2}$$

Therefore,

$$SIR[dB] = Pt - L(0.2\ km) + G(2\ dBi) - (Pt - L(0.3\ km) + G(2\ dBi))$$

$$SIR[dB] = \mathbf{-112.8\ dB} + \mathbf{2}dBi + \mathbf{119\ dB - 2\ dBi}$$

$$\boxed{SIR[dB] = 6.2\ dB}$$

$$SIR[linear] = \frac{Pt \cdot G(2\ dBi) \cdot L(0.3\ km)}{L(0.2\ km) \cdot Pt \cdot G(2\ dBi)}$$

$$SIR[linear] = \frac{10^{11.9}}{10^{11.28}} = \boxed{4.1687}$$

$$SIR[dB] = 10 \times log_{10}(4.1687)$$

- Then, the spectral efficiency is calculated by:

$$\eta = \frac{C_{AWGN}}{W} = \frac{W \cdot log_2(1 + SIR)}{W} = log_2(1 + 10^{0.62})$$

$$\boxed{\eta = 2.37\ bps/Hz}$$

Case-B

Problem 4.2

To calculate SIR, we define the desired signal power and interference signal power considering the antenna gain and path loss attenuation (**Note**: It is directional antenna gain):

$$Desired\ signal\ power\ [dB] = Pt_{BS2} - L(0.2\ km) + G(30°)$$

$$Interference\ power\ [dB] = Pt_{BS1} - L(0.3\ km) + G(50°)$$

$$=$$

$$Desired\ signal\ power[linear] = \frac{Pt_{BS2}}{L(0.2\ km)} G(30°)$$

$$Interference\ power[linear] = \frac{Pt_{BS1}}{L(0.3\ km)} G(50°)$$

Besides that, assuming that both BSs apply the same transmit power

$$Pt = Pt_{BS1} = Pt_{BS2}$$

Therefore,

$$SIR[dB] = Pt - L(0.2\ km) + G(30°) - (Pt - L(0.3\ km) + G(50°))$$

$$SIR[dB] = \mathbf{-112.8\ dB + 13\ dBi + 119\ dB - 7.667\ dBi}$$

$$\boxed{\boldsymbol{SIR[dB] = 11.53\ dB}}$$

$$=$$

$$SIR[linear] = \frac{Pt \cdot G(30°) \cdot L(0.3\ km)}{L(0.2\ km) \cdot Pt \cdot G(50°)}$$

$$SIR[linear] = \frac{10^{1.3} \cdot 10^{11.9}}{10^{11.28} \cdot 10^{0.767}} = \boxed{\mathbf{14.223}}$$

$$SIR[dB] = 10 \times log_{10}(14.223)$$

- Then, the spectral efficiency is calculated by:

$$\eta = \frac{C_{AWGN}}{W} = \frac{W \cdot log_2(1 + SIR)}{W} = log_2(1 + 10^{1.153})$$

$$\boxed{\boldsymbol{\eta = 3.9282\ bps/Hz}}$$

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Problem 4.3 (1 point). *Spatial Multiplexing gain when using MIMO technology*

Let us assume that the target peak data rate in a 5G wireless network is 40 Gbps. Assuming a 16 x 8 multilayer MIMO system (i.e., N_t = 16 transmit antennas and N_r = 8 receive antennas), and a communication bandwidth of 500 MHz. Determine the following points:

a) What is the minimum SINR (in dB) that is required per data-stream (layer) to achieve this peak data rate?

b) How does the situation change when the number of transmit antennas increase to N_t=32?

c) How many extra antennas do we need to deploy in the receiver if, for any given reason, the SINR that you computed in item a) drops by 10 dB?

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- **For a MIMO system, we have**
- a)

$$C_{all} = \sum_{i=1}^{min\{N_t,N_r\}} C_i = \sum_{i=1}^{min\{N_t,N_r\}} W * log_2(1+\gamma_i)$$

- **Assuming SNR of all streams are equal, i.e.,** $\gamma_i = \gamma, i = 1, 2, \ldots, N$

$$C_{all} = min\{N_t, N_r\}(W * log_2(1+\gamma))$$

$$\gamma = 2^{\frac{C_{all}}{W*min\{N_t,N_r\}}} - 1 = 2^{\frac{40*10^9}{500*10^6*8}} - 1 = 1023 \approx 30.1[dB]$$

- **b) By increasing the number of transmit antennas Nt = 32, we have Nr = 8**

$$C_{all} = min\{N_t, N_r\}(W * log_2(1+\gamma))$$

$$\gamma = 2^{\frac{C_{all}}{W*min\{N_t,N_r\}}} - 1 = 2^{\frac{40*10^9}{500*10^6*8}} - 1 = 1023 \approx 30.1[dB]$$

- **The required SNR would still remain the same as the amount of independent parallel channels is min(Nt, Nr) = 8**

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- **c) When the SNR drops by 10 dB,**
  - **The minimum required receiver antennas can be computed as**

$$C_{all} = min\{N_t, N_r\}(W * log_2(1+\gamma))$$

$$\gamma_{new} = \gamma - 10[dB] = 30.1 - 10 = 20.1[dB] = 102.33[linear\ scale]$$

$$C_{all} = min\{N_t, N_r\}(W * log_2(1+\gamma)) = min\{N_t, N_r\}(500 * 10^6 * log_2(1+102.33)) = 40 * 10^9$$

$$min\{N_t, N_r\} = \frac{40 * 10^9}{(500 * 10^6 * log_2(1+102.33))} = 11.96 \approx 12$$

  - **The total number of antennas required is 12 from the above expression.**
  - **The extra required antennas is 12-8 = 4 antennas.**