ELEC-E7120 - Wireless Systems (Fall 2023)

Weekly Exercise Session #4

Unit IV. Cellular Systems

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Solutions for Homework 3

Unit III. Wireless link

Homework 3

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3.1. Capacity of AWGN channel

The capacity in an AWGN channel is given by:

$$C_{AWGN} = W\ log_2\left(1 + \frac{P}{N_0 W}\right)\ \textit{[bit/s]},$$

where P [W] is the received signal power, W [Hz] is the communication bandwidth, and N_0 [W/Hz] is the Power Spectral Density (PSD) of the additive background noise (at the receiver side).

a) Find the spectral efficiency of the AWGN channel for a fixed received signal power and PSD of additive noise, when the bandwidth grows large (i.e., when $W \to \infty$). What kind of behavior are you able to observe in the so-called power limited region?

b) Consider now the case when the bandwidth remains fixed, and the power grows large. Determine the spectral efficiency of the AWGN channel under an infinite signal power (i.e., when $P \to \infty$). What kind of behavior is now observed in the so-called bandwidth limited region?

c) Let us assume that the bandwidth of the AWGN channel is $W = 40\ MHz$, the total received signal power $P = 5\ mW$, and noise PSD $N_0 = 2 \times 10^{-9}\ W/Hz$. How much does capacity increase by doubling the received power? What happens to the channel capacity when the channel bandwidth is doubled? What looks more convenient from a Wireless System designer perspective? Is this system working in the bandwidth limited or power limited region? Please, justify your answer shortly.

d) Let us now consider that bandwidth of the AWGN channel is reduced to $W = 1\ MHz$, total received signal power increased to $P = 100\ mW$, and noise PSD maintained at $N_0 = 2 \times 10^{-9}\ W/Hz$. How much does capacity increase by doubling the received power? How much does capacity increase by doubling the channel bandwidth? What looks more convenient from a Wireless System designer perspective? Is this system working in the bandwidth limited or power limited region? Please, give a brief justification of your observation.

Problem 3.1

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Considering the next formulas:

AWGN Channel Capacity

$$\boxed{C_{AWGN} = W \cdot \log_2(1 + SNR)}$$

Signal – to – Noise ratio

$$\boxed{SNR = \frac{P}{N_0 W}}$$

Spectral Efficiency

$$\boxed{\eta = \frac{C_{AWGN}}{W}}$$

And observe that

$$\log_2(1+x) \approx x \cdot \log_2(e) \quad ; \quad \text{when } x \approx 0$$

$$\log_2(1+x) \approx \log_2(x) \quad ; \quad \text{when } x \gg 1$$

; where **x = SNR**

[1]

a) SNR is low (SNR≈0), when W → ∞:

$$SNR = \frac{P}{N_0 W} \approx 0$$

So, we can define the channel capacity as:

$$\boxed{C_{AWGN} \approx W \cdot SNR \cdot \log_2(e) \approx \frac{P}{N_0}\log_2(e)}$$

Thereby, the **spectral efficiency** is:

$$\eta = \frac{C_{AWGN}}{W} = \lim_{W\to\infty} \frac{P}{N_0 W}\log_2(e) = 0$$

b) SNR is high (SNR ≫1), when P → ∞:

$$SNR = \frac{P}{N_0 W} \gg 1$$

So, we can define the channel capacity as:

$$\boxed{C_{AWGN} \approx W \cdot \log_2(SNR) \approx W \cdot \log_2\left(\frac{P}{N_0 W}\right)}$$

Thereby, the **spectral efficiency** is:

$$\eta = \frac{C_{AWGN}}{W} = \lim_{P\to\infty} \log_2\left(\frac{P}{N_0 W}\right) = \log_2(\infty) = \infty$$

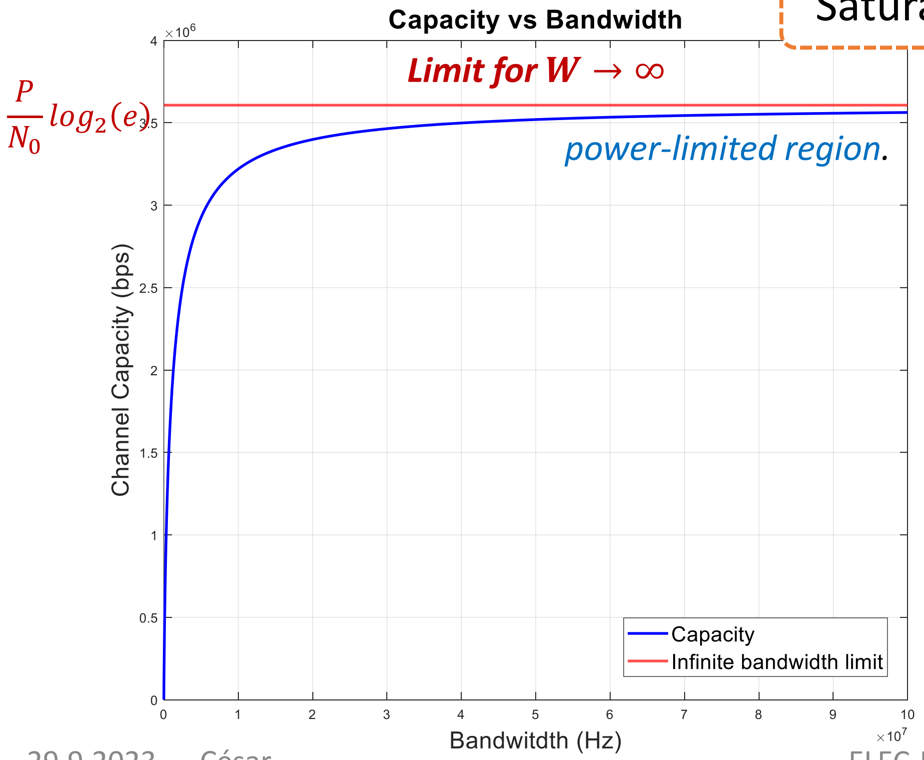
[1] *Fundamentals of Wireless Communication*, 2005. Available at: *https://web.stanford.edu/~dntse/Chapters_PDF/Fundamentals_Wireless_Communication_chapter5.pdf*

Problem 3.1

a) When received **SNR is small** ($W \to \infty$), the capacity is linear in the received power and **<u>increasing power has a significant effect</u>**, but it is insensitive to bandwidth (i.e., increasing the bandwidth has a small impact on capacity), which is called *power-limited region.*

$$C \approx \frac{P}{N_0} log_2(e)$$

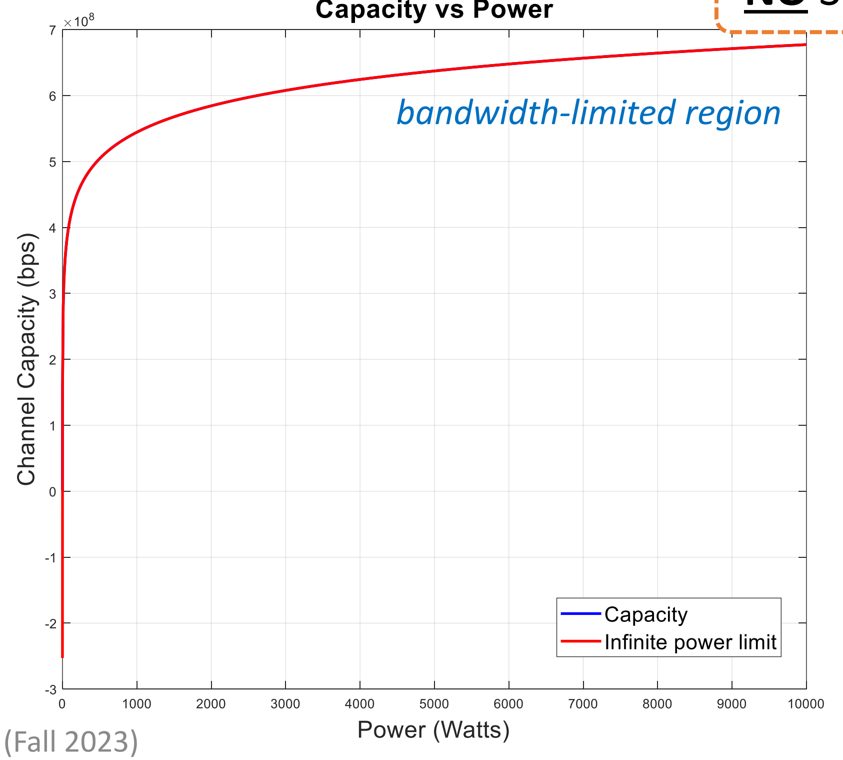
Saturation



b) When received **SNR is large** ($P \to \infty$), the capacity is linear in bandwidth and **<u>increasing bandwidth has a significant effect</u>**, but it increases logarithmically in power with a small impact on capacity, which is called *bandwidth-limited region*

$$C \approx W \cdot log_2\left(\frac{P}{N_0 W}\right)$$

<u>NO</u> Saturation



Problem 3.1

In order to increase the capacity will ***depend on the communication system*** to **determine what is more convenient** to increase either power or bandwidth.

	c) $W = 40\ MHz$ $P = 5\ mW$	**d)** $W = 1\ MHz$ $P = 100\ mW$
Capacity	3.4985 Mbps	5.6724 Mbps
Capacity ($\boldsymbol{doubling\ Power}$)	**6.797 Mbps**	6.6582 Mbps
Capacity ($\boldsymbol{doubling\ Bandwidth}$)	3.5515 Mbps	**9.4009 Mbps**
Region	***Power-limited*** *region*	***Bandwidth-limited*** *region*
More convenient	Increase the **power** signal	Increase the **bandwidth**

AWGN Channel Capacity

$$\boxed{C_{AWGN} = W \cdot log_2\left(1 + \frac{P}{N_0 W}\right)} \quad , \quad \text{where:} \quad N_0 = 2 \times 10^{-9}\ W/Hz$$

Homework 3

3.2. Bit error probability analysis

a) For a Rayleigh fading channel (i.e., when received SNR follows an exponential distribution), and in case of binary signalling, the probability of error (P_e) is given by

$$P_e = \frac{1}{2}\left(1 - \sqrt{\frac{\mathrm{SNR}}{1+\mathrm{SNR}}}\right) \qquad (1)$$

and can be approximated by

$$P_e \approx \frac{1}{4\mathrm{SNR}} \qquad (2)$$

at high SNR values. Show how to obtain approximation in (2) from expression (1). What kind of asymptotic behaviour does it show?

Hint: Use Taylor series expansions.

b) Use the inequality

$$\left(1 - \frac{1}{z^2}\right)\frac{1}{z\sqrt{2\pi}} \leq e^{\frac{z^2}{2}}\, Q(z) \leq \frac{1}{z\sqrt{2\pi}} \qquad (3)$$

to find a suitable approximation for the probability of error in an AWGN channel when SNR grows large (in case of binary signalling). What kind of asymptotic behaviour does it show?

c) Find the SNR values that are required to obtain a bit error probability of 10^(-6) in case of:

- AWGN channel, and
- Rayleigh fading channel.

Assume BPSK signalling and express the final result in dB and linear. Use the expressions derived in (a) and (b). Analyse the obtained results and take out your own conclusions.

Problem 3.2

3.2. Probability of bit error at the receiver for Rayleigh fading is given by

$$P_e = \frac{1}{2}\left(1 - \sqrt{\frac{\mathrm{SNR}}{1+\mathrm{SNR}}}\right) = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1+\mathrm{SNR}^{-1}}}\right). \qquad (1)$$

According to the Taylor series expansion for the square root, i.e.,

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \dots \qquad |x| \le 1, \qquad (2)$$

the following approximation holds:

$$\sqrt{1+\mathrm{SNR}^{-1}} \approx 1 + \frac{1}{2\,\mathrm{SNR}} \qquad \text{for } \mathrm{SNR} \to \infty. \qquad (3)$$

In other words, the first order truncation of the Taylor series becomes accurate enough in the high SNR regime.

Finally, combining (1) with (3), it is possible to observe that

$$P_e = \frac{1}{2}\left(1 - \frac{1}{1+\frac{1}{2\,\mathrm{SNR}}}\right) = \frac{1}{2}\left(1 - \frac{2\,\mathrm{SNR}}{2\,\mathrm{SNR}+1}\right) \approx \frac{1}{4\,\mathrm{SNR}}. \qquad (4)$$

Problem 3.2

3.2. Probability of bit error at the receiver for Rayleigh fading is given by

We will find a suitable approximation for the prbability of error in AWGN channel when SNR is large by using the given inequality as ;

$$\left(1-\frac{1}{z^2}\right)\frac{1}{z\sqrt{2\pi}} \le e^{\frac{z^2}{2}}Q(z) \le \frac{1}{z\sqrt{2\pi}} \tag{5}$$

According to the lecture slides, the bit error probability for of an AWGN channel is given

$$P_e = Q\left(\sqrt{2\,\mathrm{SNR}}\right). \tag{6}$$

In addition, according to (5), it is possible to see that

$$\left(1-\frac{1}{z^2}\right)\frac{1}{z\sqrt{2\pi}}e^{-\frac{z^2}{2}} \le Q(z) \le \frac{1}{z\sqrt{2\pi}}e^{-\frac{z^2}{2}} \tag{7}$$

holds. Note that both, the upper and the lower bound become asymptotically tight as z grows large (i.e., when $z \to \infty$).

Therefore, it is possible to conclude that

$$Q(z) \approx \frac{1}{z\sqrt{2\pi}}e^{-\frac{z^2}{2}} \qquad \text{when } z \to \infty, \tag{8}$$

or equivalently,

$$Q(z) \le e^{-\frac{z^2}{2}} \qquad \text{when } z \to \infty. \tag{9}$$

Finally, plugging equations (8) and (9) in (6), we get the following expressions:

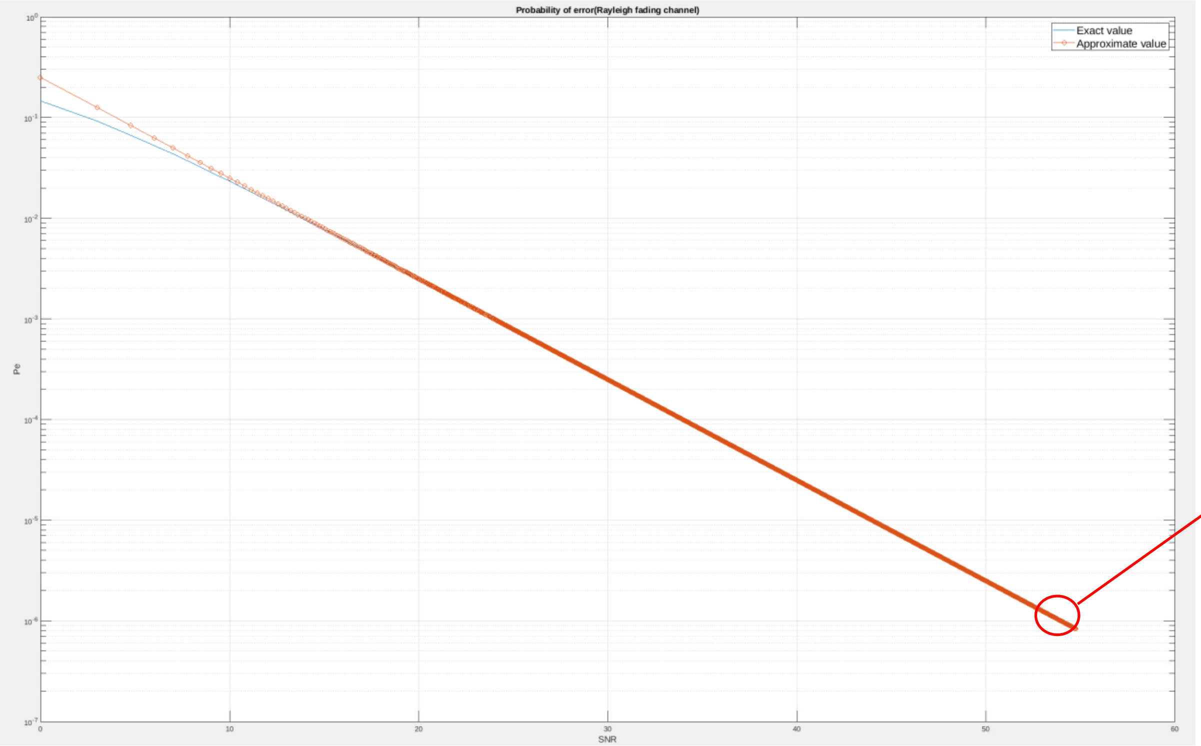
$$P_e \approx \frac{1}{\sqrt{4\pi\,\mathrm{SNR}}}e^{-\mathrm{SNR}} \qquad \text{when SNR} \to \infty, \tag{10}$$

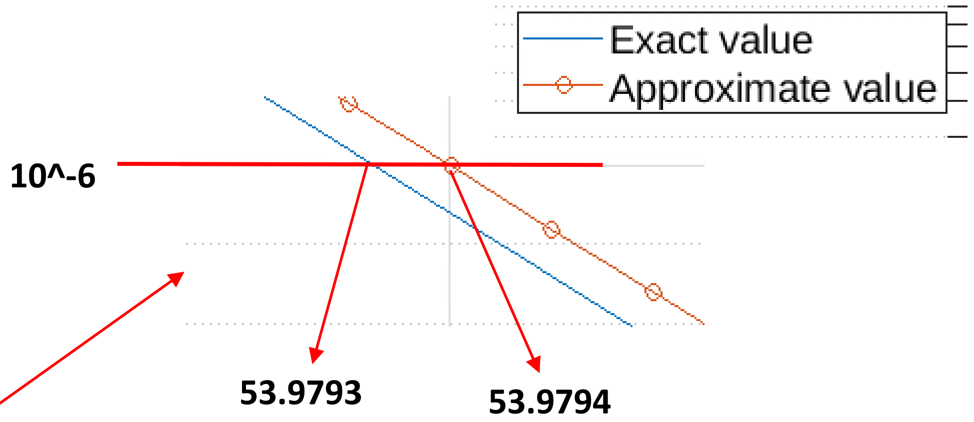
and

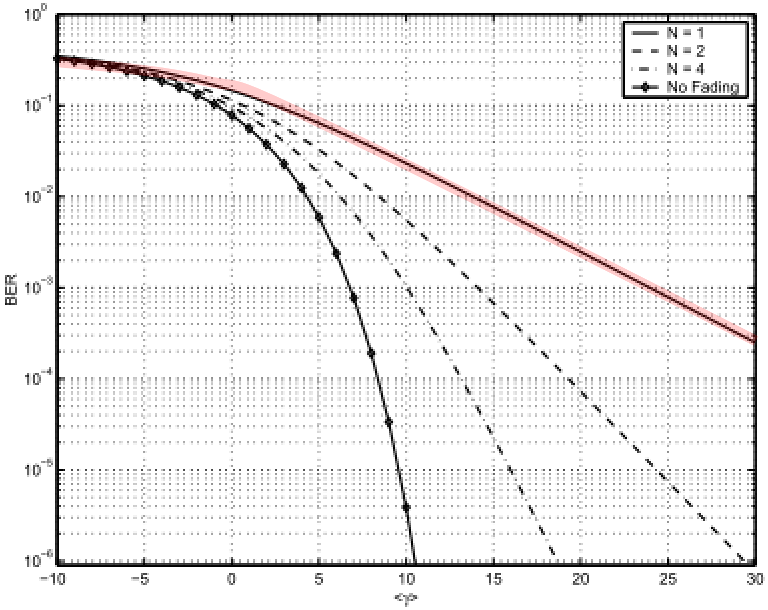
$$P_e \le e^{-\mathrm{SNR}} \qquad \text{when SNR} \to \infty. \tag{11}$$

Problem 3.2

3.2. Bit error probability analysis



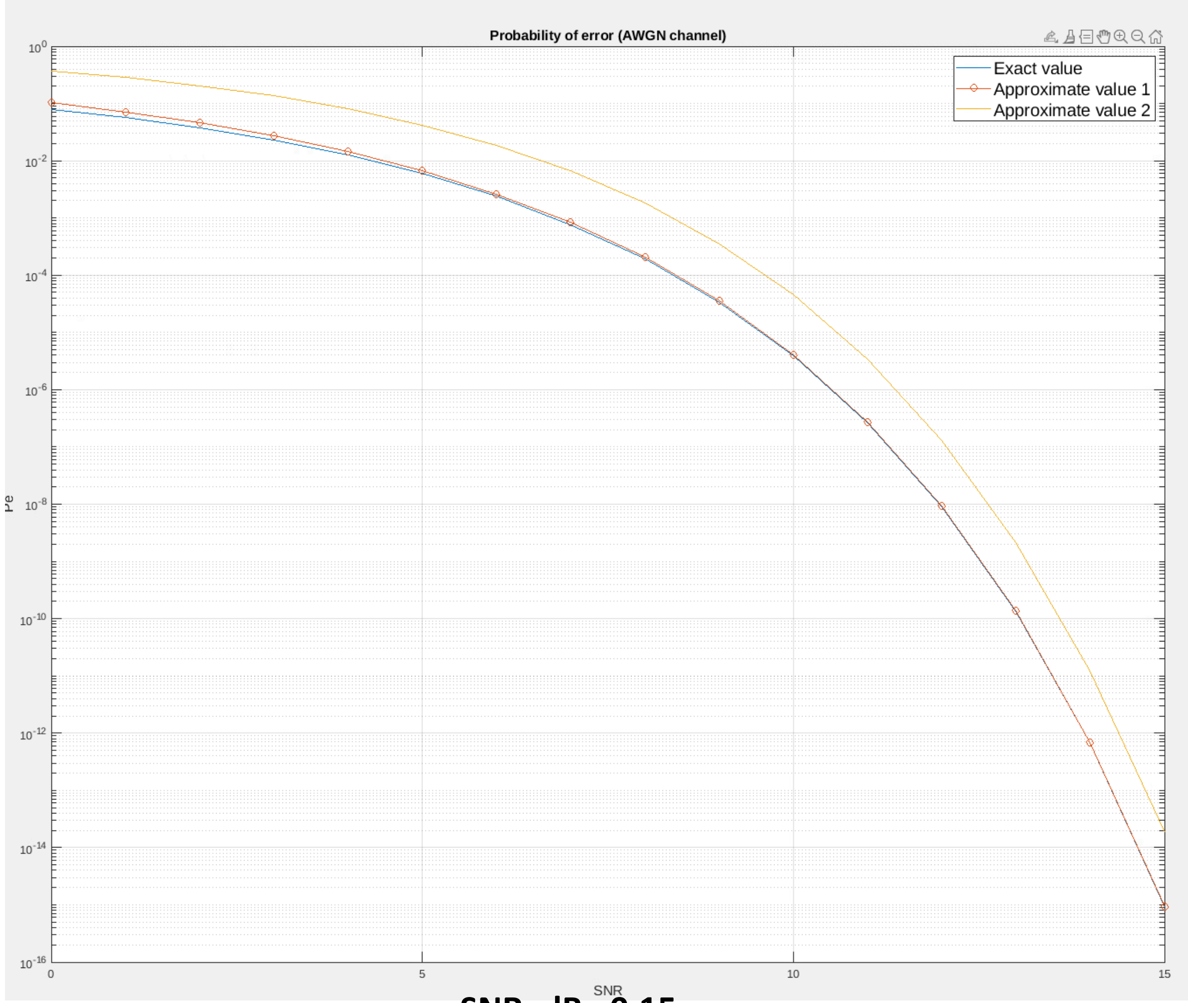




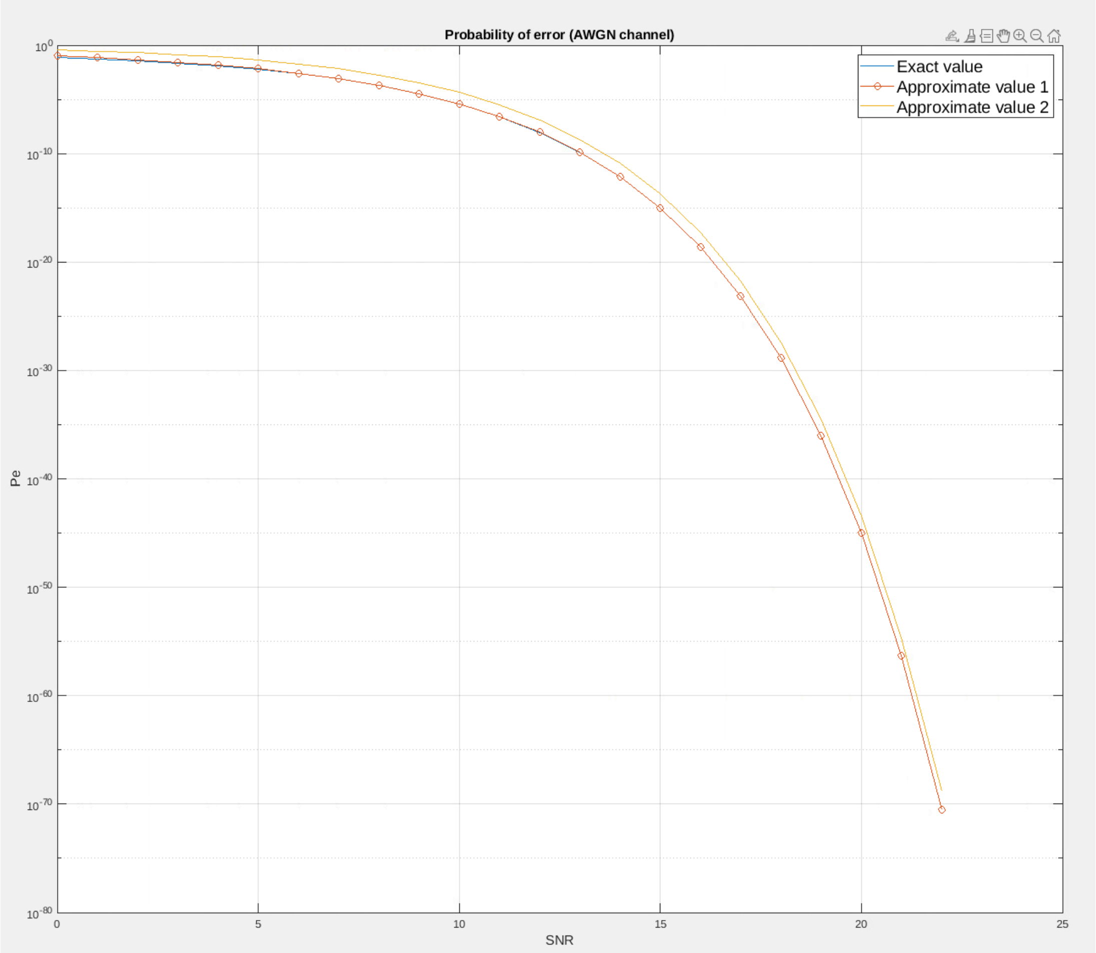
- **For Rayleigh fading channel**
  - **Pe ≈ 1/(4*SNR) implying that SNR ≈ 1/(4Pe)=250000[linear scale] or 53.98dB**

Problem 3.2

3.2. Bit error probability analysis



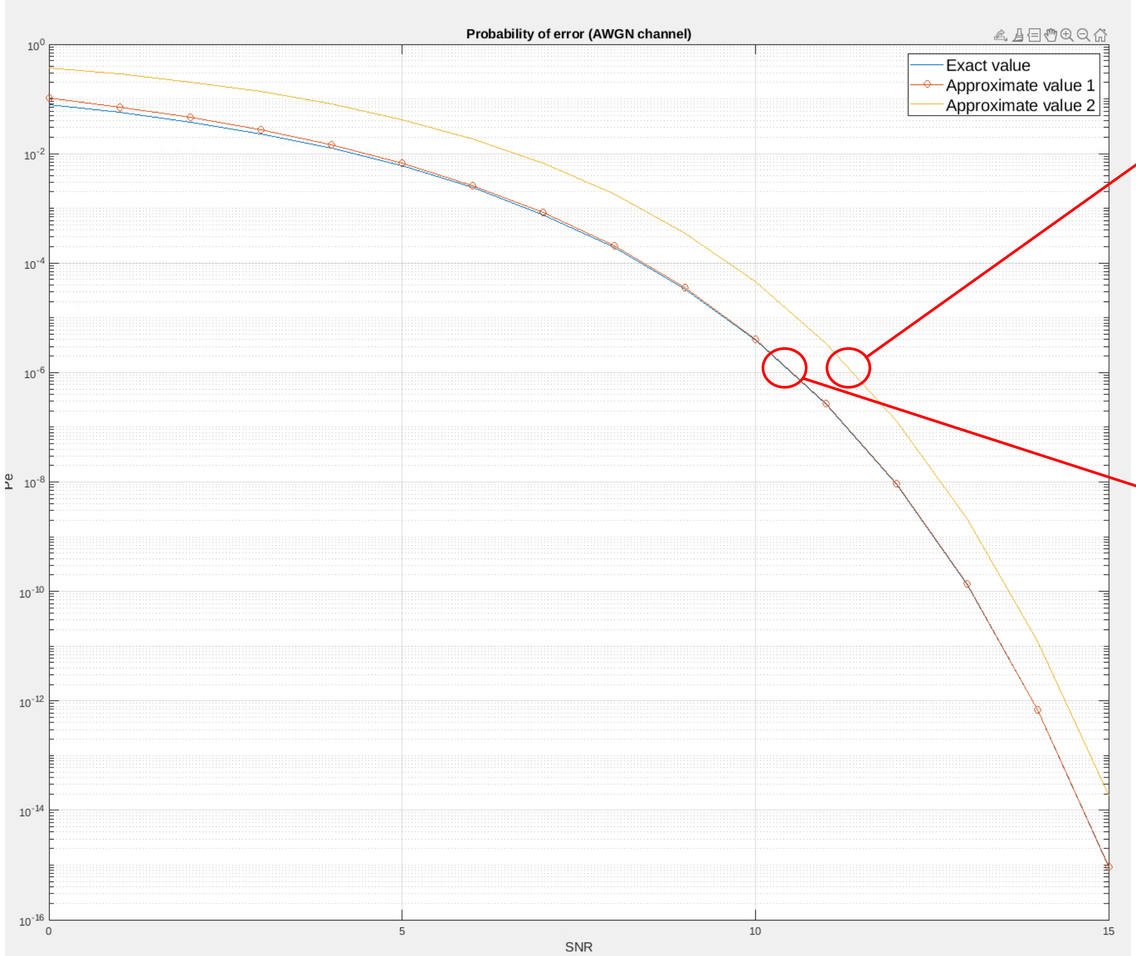
SNR_dB =0:15

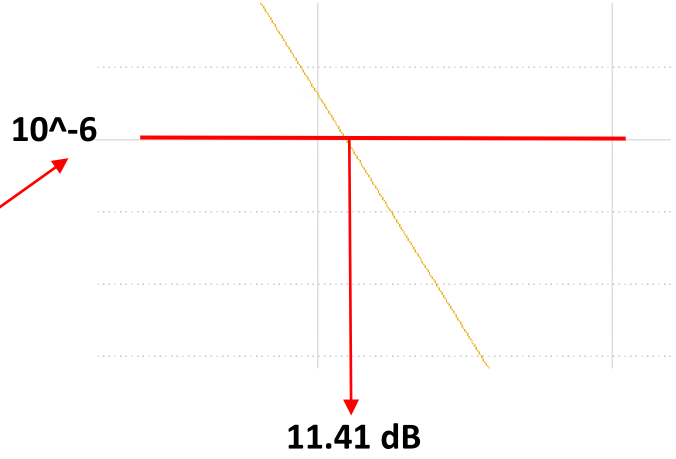


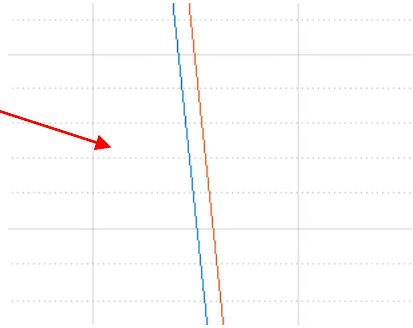
SNR_dB =0:22

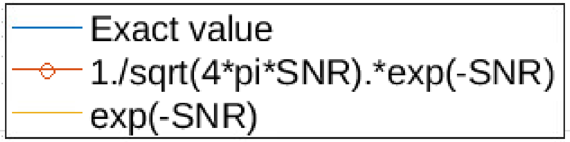
Problem 3.2

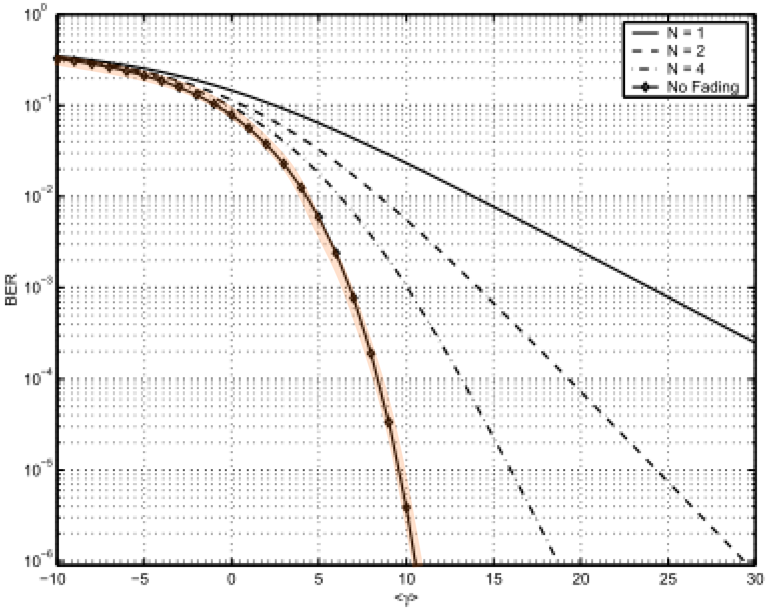
3.2. Bit error probability analysis











SNR_dB =0:15

- For AWGN channel
  - Pe ≤ exp(-SNR) implying that SNR ≥-loge(Pe)=13.82[linear scale] or 11.41 dB

Homework 3

3.3. Cross link interference in TDD system

Consider the parameters below for a scenario where there is a transmission and reception between the base stations (BS) and mobile stations (MS) as shown in Figure 1.

- The bandwidth is 10 MHz, and one user is served per cell. Mobile station 1 (MS1) is served by BS1, while MS2 is served by BS2.
- Transmit power per MS = 23 dBm. Transmit power per BS = 30 dBm. Noise power = -174 dBm. Distance between BSs = 100m. Distance from BS to MS= 35m.
- MS1 transmits in UL all the time. MS2 uses three different configurations: 1) 100% in UL; 2) 50% in UL and 50% in DL; 3) 100% in DL.

Answer appropriately the following questions:

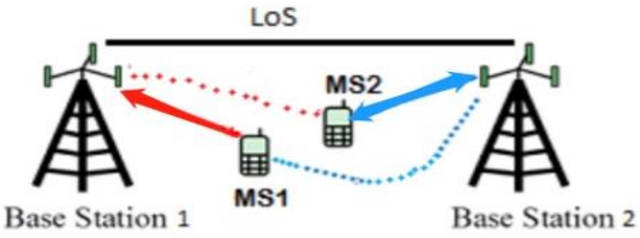
a) What is the maximum data rate that MS1 can achieve in each configuration of the different TDD frame configurations used in cell 2? (Use Shannon’s formula, including the effect of both noise and interference)

b) Estimate the upper bound for the mean data rate that MS 1 can achieve in uplink.

c) Explain, with your own words, the effect of the frame configuration of cell 2 on the data rate of cell 1? Would this effect be observed if, instead of TDD, the duplexing of both uplink and downlink transmissions would take place in the frequency domain?

Use the following path loss models:

- LoS Path Loss: PL(d) = 16.9 log10(d) + 38.8 [dB], ‘d’ in meters
- NLoS Path Loss: PL(d) = 43.3 log10(d) + 17.5 [dB], ‘d’ in meters



Blue and Red Lines Indicate NLoS

Problem 3.3

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Signal-to-Interference plus Noise power Ratio (SINR)

$$SINR = \frac{Desired\ signal\ power}{Interference\ power + Noise\ power}$$

Shannon Capacity

$$C = W \cdot log_2(1 + SINR)$$

Path Loss attenuation

$$PL = Pt - Pr$$

$CONFIGURATION\#1\ : 100\%\ in\ UL\ (MS_2)$

$In\ case\ of\ BS_1\ the\ desired\ signal\ is\ from\ MS_1$

$Desired\ Signal\ Power = Pt_{MS_1} - PL_{NLOS\ at\ d_1} \implies d_1\ is\ distance\ between\ MS_1\ and\ BS_1$

$PL_{NLOS\ at\ d_1} = 43.3 \cdot log_{10}(35) + 17.5 = 84.35\ dB$

$\boxed{Desired\ Signal\ Power = 23.3\ dBm - 84.35\ dB = -61.35\ dBm}$

$When\ MS_2\ is\ transmitting\ Uplink\ signal\ to\ BS_2,\ BS_1\ will\ face\ interference\ from\ MS_2\ as\ shown\ in\ red\ dotted\ line$

$Interference\ Power = Pt_{MS_2} - PL_{NLOS\ at\ d_3} \implies d_3\ is\ distance\ between\ MS_2\ and\ BS_1$

$PL_{NLOS\ at\ d_3} = 43.3 \cdot log_{10}(65) + 17.5 = 95.99\ dB$

$\boxed{Interference\ Power = 23.3\ dBm - 95.99\ dB = -72.99\ dBm}$

$From\ calculations\ we\ can\ see\ that\ Noise\ << Interference\ so$

$SINR \approx SIR$

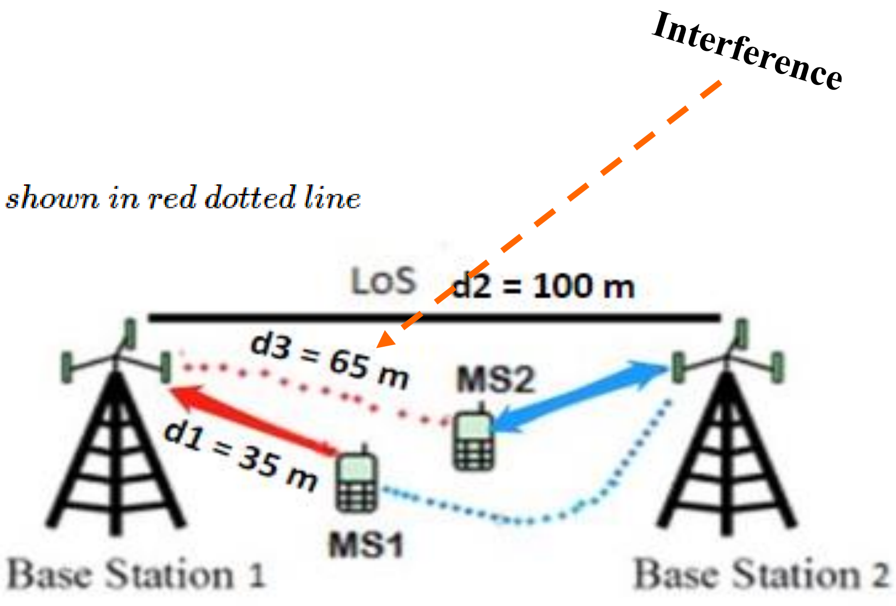
$SIR(dB) = Desired\ signal\ Power - Interference$

$\boxed{SIR = -61.35dBm - (-72.99)dBm = -11.64\ dB}$

$C = W \cdot log_2(1 + SIR)$

$C = 10 \cdot log_2(1 + 10^{-1.164})$

$\boxed{C = 39.62\ Mbps\ \approx 40\ Mbps}$



Blue and Red Lines Indicate NLoS

Problem 3.3

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Signal-to-Interference plus Noise power Ratio (SINR)

$$SINR = \frac{Desired\ signal\ power}{Interference\ power + Noise\ power}$$

Shannon Capacity

$$C = W \cdot log_2(1 + SINR)$$

Path Loss attenuation

$$PL = Pt - Pr$$

$CONFIGURATION\#2\ :\ 100\%\ in\ DL\ (MS_2)$

$In\ case\ of\ BS_1\ the\ desired\ signal\ is\ from\ MS_1$

$Desired\ Signal\ Power = Pt_{MS_1} - PL_{NLOS\ at\ d_1} \implies d_1\ is\ distance\ between\ MS_1\ and\ BS_1$

$PL_{NLOS\ at\ d_1} = 43.3 \cdot log_{10}(35) + 17.5 = 84.35\ dB$

$Desired\ Signal\ Power = 23.3\ dBm - 84.35\ dB = -61.35\ dBm$

$When\ MS_2\ is\ transmitting\ Downlink\ signal\ to\ BS_2,\ BS_1\ will\ face\ interference\ from\ BS_2\ as\ shown\ in\ black\ solid\ line$

$Interference\ Power = Pt_{MS_2} - PL_{LOS\ at\ d_2} \implies d_2\ is\ distance\ between\ BS_1\ and\ BS_2$

$PL_{LOS\ at\ d_2} = 16.9 \cdot log_{10}(100) + 38.8 = 72.6\ dB$

$Interference\ Power = 30\ dBm - 72.6\ dB = -42.6\ dBm$

$From\ calculations\ we\ can\ see\ that\ Noise\ << Interference\ so$

$SINR \approx SIR$

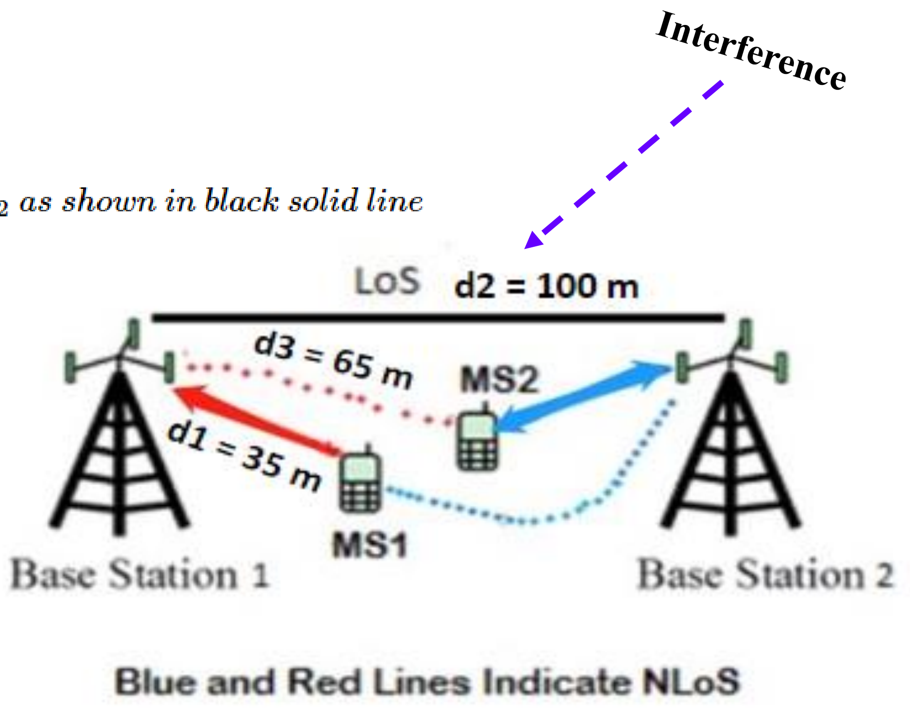
$SIR(dB) = Desired\ signal\ Power - Interference$

$SIR = -61.35dBm - (-42.6)dBm = -18.75\ dB$

$C = W \cdot log_2(1 + SIR)$

$C = 10 \cdot log_2(1 + 10^{-1.875})$

$C = 0.191\ Mbps\ \approx 0.2\ Mbps$



Problem 3.3

Signal-to-Interference plus Noise power Ratio (SINR)

$$SINR = \frac{Desired\ signal\ power}{Interference\ power + Noise\ power}$$

Shannon Capacity

$$C = W \cdot log_2(1 + SINR)$$

Path Loss attenuation

$$PL = Pt - Pr$$

$CONFIGURATION\#3\ :50\%\ in\ UL\ and\ 50\%\ in\ DL(MS_2)$

$In\ this\ case\ 50\%\ from\ configruation\#1\ and\ 50\%\ from\ configuration\#2\ so\ we\ will\ take\ avergae\ of\ both$

$$C = \frac{(40\ Mbps + 0.2\ Mbps)}{2} = 20.1\ Mbps \approx 20\ Mbps$$

MS2 Configuration	MS1 Capacity
100% in UL	40 Mbps
100% in DL	0.2 Mbps
50% in DL and 50% in UL	20 Mbps

Table 1

Part b)

For finding the upper bound for the mean data rate that MS1 can achieve in uplink we can have two possibilities.

- When there is interference the maximum capacity MS1 can have is 40 Mbps as shown in table 1, when MS2 is transmitting 100 % in UL
- When there is no interference then upper bound of MS1 will be shannon formula

  C =W log2(1+SNR)

Problem 3.3

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Part c)

- In a TDD (Time Division Duplexing) system, where UL and DL transmissions share the same frequency channel but occur at different times, BS2's downlink signal have significant impact on the SIR at BS1, leading to interference and potential reductions in MS1's UL data rate.

- If a substantial portion of time is allocated to MS2's downlink traffic, MS1's uplink data rate may suffer.

- In contrast, in an FDD (Frequency Division Duplexing) system with separate frequency bands for UL and DL, there is minimal interference between MS1 and MS2, allowing MS1 to use its full UL capacity without reduction due to BS2's DL transmissions.

- This simultaneous, interference-free transmission in both directions in FDD is more favorable for maximizing data rates compared to the interference-prone nature of TDD.