



# [acm 模板]

V2.0.0

作者: [qwb]

2017 年 4 月 14 日

图论	4
1. 强连通分量有向图缩点	4
2. 强连通分量无向图缩点	5
3. 凸多边形平面图转对偶图	5
4. 差分约束	7
5. 点分治	7
6. 匈牙利匹配	7
7. KM 二分图带权匹配	8
8. 欧拉回路 Fleury	9
9. 删 2 条边不连通	9
10. 树中最长路	9
11. 2sat	10
12. $O(1)$ lca	11
13. lca 离线	11
14. lca 在线	13
15. sap 网络流	13
16. dinic 邻接表网络流	14
17. zkw 费用流	15
18. 普通费用流	17
19. Dijkstra	18
20. floyd	19
21. 普通 spfa	19
22. SLF 优化的 spfa	20
23. 有负环的 spfa	21
24. 割顶和桥	21
25. 其他网络流	22
26. 费用流可行流	22
27. 最小割	22
28. 最大权闭合图	22
字符串	23
1. AC 自动机	23
2. 在线 AC 自 X 动机	25
3. 后缀数组	26
4. KMP	27
5. Manacher	28
6. MT 定理	28
7. 二分求 lcp	29
8. 最小表示法	30
数论	30
1. 矩阵快速幂(vector 版)	30
2. 矩阵快速幂(更快)	31
3. $O(1)$ gcd	31
4. 线性基	32
5. k 次幂之和	33
6. 约瑟夫环	34
7. fft	34
8. fwt	35
9. ntt	37
10. Lucas 定理	38
11. 大质数判定	38
12. 康托展开	39
13. 扩展欧几里德	39
14. 欧拉函数	40
15. 求组合数	40
16. 高斯消元(浮点数)	41
17. 高斯消元(任意模数)	42
18. 高斯消元(xor)	43
19. 凸包	44
20. 极角排序	44
21. 集合-莫比乌斯反演	45
22. 集合-莫比乌斯变换	45
23. $O(3^n)$ 枚举子集	45
24. 求 n 以内质数个数	45

25. simpson 定积分	47
26. 中国剩余定理	47
27. 莫比乌斯函数筛法	48
28. 可不互质的中国剩余定理	48
29. 排列组合总结	48
30. 奇怪的公式	49
数据结构	50
1. 左偏树	50
2. ST 表	51
3. 二维 RMQ	52
4. 线段树缩空间	52
5. 曼哈顿最小生成树	52
6. 主席树	54
7. KD 树-子矩阵查询修改	54
8. KD 树	56
9. 离线第 k 大带修改	56
10. 非旋转 Treap	58
11. 平衡堆	59
12. 莫队算法	60
13. 表达式树	61
14. 树链剖分	62
15. 线段树扫描线	64
16. 树状数组	66
17. 树状数组第 k 大	67
18. Splay	67
19. DLX 覆盖	69
动态规划	71
1. TSP	71
2. 四边形不等式优化的石子合并	72
3. 斜率优化(凸包)	72
4. 斜率优化(单调队列)	73
5. 往子集传递值	74
6. 数位 dp	74
7. 区间内 2 个数位异或等于特定值	75
博弈	75
1. 斐波那契博弈	75
2. 威佐夫博弈	75
3. 巴什博弈	76
4. Anti-num 博弈	76
5. Nim 博弈	76
其他杂类	76
6. 大数	76
7. pb_ds 大法	78
8. bitset	79
9. 蔡勒公式	79
10. 第 k 小	79
11. 三分整数	79
12. 求阶乘后缀 0 个数	79
13. DFS 构造矩阵	80
14. 手动扩栈	80
15. 正常的读入挂	80
16. fread 读入挂	80
17. 测试系统类型	80
18. 方阵的循环节	81
19. 字符串分割读入的	81
20. 区间随机数生成	81
21. 祖传头文件	81

## 1. 强连通分量有向图缩点

```

const int MX = 5e3 + 5;
const int INF = 0x3f3f3f3f;
struct Edge {
    int u, v, nxt;
} E[60005];
int Head[MX], erear;
void edge_init() {
    erear = 0;
    memset(Head, -1, sizeof(Head));
}
void edge_add(int u, int v) {
    E[erear].u = u;
    E[erear].v = v;
    E[erear].nxt = Head[u];
    Head[u] = erear++;
}
int n, m, IN[MX], cnt[MX], val[MX];
int bsz, ssz, dsz;
int Low[MX], DFN[MX];
int belong[MX], Stack[MX];
bool inStack[MX];
void Init_tarjan(int n) {
    bsz = ssz = dsz = 0;
    for(int i = 1; i <= n; ++i) Low[i] = DFN[i] = 0;
}
void Tarjan(int u) {
    Stack[++ssz] = u;
    inStack[u] = 1;
    Low[u] = DFN[u] = ++dsz;
    for(int i = Head[u]; ~i; i = E[i].nxt) {
        int v = E[i].v;
        if(!DFN[v]) {
            Tarjan(v);
            Low[u] = min( Low[v], Low[u]);
        } else if(inStack[v]) {
            Low[u] = min( Low[u], DFN[v]);
        }
    }
    if(Low[u] == DFN[u]) {
        ++bsz;
        int v;
        do {
            v = Stack[ssz--];
            inStack[v] = 0;
            belong[v] = bsz;
        } while(u != v);
    }
}
int solve(int n) {
    Init_tarjan(n);
    for (int i = 1; i <= n; i++) {
        if (!DFN[i]) Tarjan(i);
    }
    edge_init();
    for(int i = 0; i < m; i++) {
        int u = E[i].u, v = E[i].v;
        u = belong[u]; v = belong[v];
        if(u != v) {

```

```

        edge_add(u, v);
    }
}
}

```

## 2. 强连通分量无向图缩点

```

int DFN[MX], Low[MX], dsz, tim;
int Stack[MX], inStack[MX], Belong[MX], bsz, ssz;

void trajan(int u, int e) {
    inStack[u] = 1;
    Stack[++ssz] = u;
    DFN[u] = Low[u] = ++dsz;
    for(int i = Head[u]; ~i; i = E[i].nxt) {
        int v = E[i].v;
        if((i ^ 1) == e) continue;
        if(!DFN[v]) {
            trajan(v, i);
            Low[u] = min(Low[u], Low[v]);
        } else if(inStack[v]) {
            Low[u] = min(Low[u], Low[v]);
        }
    }
    if(DFN[u] == Low[u]) {
        bsz++; int v;
        do {
            v = Stack[ssz--];
            inStack[v] = 0;
            Belong[v] = bsz;
        } while(ssz && v != u);
    }
}

void tarjan_solve(int n) {
    dsz = bsz = ssz = 0;
    memset(DFN, 0, sizeof(DFN));
    for(int i = 1; i <= n; i++) {
        if(!DFN[i]) trajan(i, -1);
    }
    /*缩点*/
    edge_init();
    for(int i = 0; i < 2 * m; i += 2) {
        int u = E[i].u, v = E[i].v;
        u = Belong[u]; v = Belong[v];
        if(u == v) continue;
        edge_add(u, v);
        edge_add(v, u);
    }
}

```

## 3. 凸多边形平面图转对偶图

```

/*
1->2->3->...n->1
然后在上面加了 m 条边
把这种图转换成对偶图
区域的个数就是 dsz
做模板的时候忘记初始化了，多组输入注意一下
*/

```

```

int n, m;
vector<int> point[MX];
map<pii, bool> vis;
map<pii, int> id;
int cur[MX], dsz;
int who[MX][2];
struct Data {
    int color;
    vector<int> num;
} data[MX];

int get_id(int u, int v) {
    if(u > v) swap(u, v);
    if(id.count(pii(u, v))) return id[pii(u, v)];
    return -1;
}

int get_next_pos(int pre, int u, int s) {
    if(pre != s) { // 上一条边如果是 s, 不能往回走的
        // 看 u 是否有边直接连到了起点, 有的话就要回起点
        auto p = lower_bound(point[u].begin(), point[u].end(), s);
        if(p != point[u].end() && *p == s) {
            return s;
        }
    }
    while(cur[u] >= 0) {
        int v = point[u][cur[u]];
        if(vis.count(pii(u, v))) {
            cur[u]--;
        }
        break;
    }
    // assert(cur[u] >= 0);
    return point[u][cur[u]];
}

void presolve() {
    for(int i = 1; i <= m; i++) {
        int u, v; // 点的下标都减了 1
        scanf("%d%d", &u, &v); u--; v--;
        point[u].push_back(v);
        point[v].push_back(u);
        if(u > v) swap(u, v);
        id[pii(u, v)] = i;
    }
    for(int i = 0; i < n; i++) {
        point[i].push_back((i + 1) % n);
        point[i].push_back((i - 1 + n) % n);
        sort(point[i].begin(), point[i].end());
    }
    for(int i = 0; i < n; i++) {
        cur[i] = point[i].size() - 1;
    }

    cur[0]--; // 减去 n-1 的
    for(int s = 0; s < n - 1; s++) {
        while(cur[s] >= 0) {
            int u = s, v = point[s][cur[s]];
            if(vis.count(pii(s, v))) {
                cur[s]--; continue;
            }

            dsz++; // 这个时候一定会有区域
            int last = -1;

```

```

while(true) {
    int v = get_next_pos(last, u, s);
    int id = get_id(u, v); // 确定这条边是否是后来加的
    if(id == -1) {
        vis[pri(u, v)] = vis[pri(v, u)] = 1;
    } else {
        vis[pri(u, v)] = 1;
        who[id][who[id][0] ? 1 : 0] = dsz;
    }
    data[dsz].num.push_back(v);
    if(v == s) break;
    last = u; u = v;
}
}
}
}
}

```

## 4. 差分约束

$B - A \leq C$  转换成  $A \rightarrow B$  的边权值为  $C$

求  $B - A$  最大值转换为求  $A \rightarrow B$  最短路

求  $B - A$  最小值转换为求  $B \rightarrow A$  最短路并取负号

如果存在负环，则无解

如果不存在最短路，则无数解

## 5. 点分治

```

/*
记得打上 vis 标记。
点分治常用思路：
1. 先找到重心
2. 维护重心为根的树
3. 若答案以重心为端点时的答案
4. 若答案经过重心时的答案
5. 上述两种情况，要注意两个端点不能同时属于一颗子树
6. 递归
*/
int sum[MX], root, rtsum;
void Tree_G(int u) {
    int mx = 0;
    sum[u] = 1;
    for (int i = Head[u]; ~i; i = E[i].nxt) {
        int v = E[i].v;
        if (vis[v]) continue;
        Tree_G(v);
        sum[u] += sum[v];
        mx = max(mx, sum[v]);
    }
    mx = max(mx, sum[0] - sum[u]);
    if (mx < rtsum) rtsum = mx, root = u;
    vis[u] = 0;
}

```

## 6. 匈牙利匹配

/\*复杂度  $O(VE)$

最小点覆盖 = 最大匹配数

最小边覆盖 = 左右点数 - 最大匹配数

最小路径覆盖 = 点数 - 最大匹配数

最大独立集=点数-最大匹配数

```
*/
int match[MX];
bool vis[MX];
bool DFS(int u) {
    for(int i = Head[u]; ~i; i = E[i].nxt) {
        int v = E[i].v;
        if(!vis[v]) {
            vis[v] = 1;
            if(match[v] == -1 || DFS(match[v])) {
                match[v] = u;
                return 1;
            }
        }
    }
    return 0;
}
int BM(int n) {
    int res = 0;
    memset(match, -1, sizeof(match));
    for(int u = 1; u <= n; u++) {
        memset(vis, 0, sizeof(vis));
        if(DFS(u)) res++;
    }
    return res;
}
```

## 7.KM 二分图带权匹配

```
const int MX = 3e2 + 5;
/* KM 算法
 * 复杂度 O(nx*nx*ny)
 * 求最大权匹配
 * 若求最小权匹配, 可将权值取相反数, 结果取相反数
 * 点的编号从 1 开始
 */
const int INF = 0x3f3f3f3f;
int nx, ny; //两边的点数
int G[MX][MX]; //二分图描述
int linker[MX], lx[MX], ly[MX]; //y 中各点匹配状态, x,y 中的点标号
int slack[MX];
bool visx[MX], visy[MX];

bool DFS(int x) {
    visx[x] = 1;
    for(int y = 1; y <= ny; y++) {
        if(visy[y]) continue;
        int tmp = lx[x] + ly[y] - G[x][y];
        if(tmp == 0) {
            visy[y] = 1;
            if(linker[y] == -1 || DFS(linker[y])) {
                linker[y] = x;
                return 1;
            }
        } else if(slack[y] > tmp) {
            slack[y] = tmp;
        }
    }
    return 0;
}

int KM() {
    memset(linker, -1, sizeof(linker));
    memset(ly, 0, sizeof(ly));
}
```



```

for(int i = 1; i <= nx; i++) {
    lx[i] = -INF;
    for(int j = 1; j <= ny; j++) {
        if(G[i][j] > lx[i]) lx[i] = G[i][j];
    }
}
for(int x = 1; x <= nx; x++) {
    for(int i = 1; i <= ny; i++) slack[i] = INF;
    while(true) {
        memset(visx, 0, sizeof(visx));
        memset(visy, 0, sizeof(visy));
        if(DFS(x)) break;
        int d = INF;
        for(int i = 1; i <= ny; i++) {
            if(!visy[i] && d > slack[i]) d = slack[i];
        }
        for(int i = 1; i <= nx; i++) {
            if(visx[i]) lx[i] -= d;
        }
        for(int i = 1; i <= ny; i++) {
            if(visy[i]) ly[i] += d;
            else slack[i] -= d;
        }
    }
}
int res = 0;
for(int i = 1; i <= ny; i++) {
    if(linker[i] != -1) res += G[linker[i]][i];
}
return res;
}

```

## 8. 欧拉回路 Fleury

/\*删边要注意复杂度，尽量别用标记删除，而是直接删除

无向图满足欧拉回路：度为偶数，或者度为奇数的点个数为 2

有向图满足欧拉回路：入度全部等于出度，或者 1 个点入度-出度=1，一个点出度-入度=1，其他点入度等于出度  
\*/

```

void Fleury(int u) {
    for(int i = Head[u]; ~i; i = Head[u]) {
        Head[u] = E[i].nxt;
        if(!vis[i | 1]) {
            int v = E[i].v;
            vis[i | 1] = 1;
            Fleury(v);
        }
    }
    Path[++r] = u;
}

```

## 9. 删 2 条边不连通

先搞一颗生成树，给每条非树边随机 hash

对于每条树边 hash 值=所有经过他的非树边的 hash 值 xor 和

如果存在两条边的 hash 值相等，则不连通

## 10. 树中最长路

```

int solve(int u, int from, int &ans) {
    int Max1 = 0, Max2 = 0;
    for(int id = Head[u]; ~id; id = Next[id]) {
        int v = E[id].v;
        if(v == from) continue;
    }
}

```

```

    int t = solve(v, u, ans) + 1;

    if(t > Max1) {
        Max2 = Max1;
        Max1 = t;
    } else if(t > Max2) Max2 = t;
}

ans = max(ans, Max1 + Max2);
return Max1;
}
/*调用方法
int ans = 0;
solve(1, -1, ans);
*/

```

## 11.2sat

```

struct Edge {
    int v, nxt;
} E[MX << 1];
int Head[MX][2], erear;
void edge_init() {
    erear = 0;
    memset(Head, -1, sizeof(Head));
}
void edge_add(int z, int u, int v) {
    E[erear].v = v;
    E[erear].nxt = Head[u][z];
    Head[u][z] = erear++;
}
void edge_add(int u, int v) {
    edge_add(0, u, v);
    edge_add(1, v, u);
}
int Stack[MX], Belong[MX], vis[MX], ssz, bsz;
void DFS(int u, int s) {
    vis[u] = 1;
    if(s) Belong[u] = s;
    for(int i = Head[u][s > 0]; ~i; i = E[i].nxt) {
        int v = E[i].v;
        if(!vis[v]) DFS(v, s);
    }
    if(!s) Stack[++ssz] = u;
}
/*得到的 Belong 的拓扑序
把 u 拆成 2 个点，分别表示真和假
如果 a 和 -a 在同一个连通分量里则无解
如果 Belong[a]>Belong[-a]，则 a 为 true
如果 Belong[-a]>Belong[a]，则为 false
tarjan 的 2sat 时，记得是 2 倍点数，切记 MX
A,B 不能同时取 <A,B'><B,A'>
A,B 必须取一个<A',B><B',A>
A,B 必须都取或者都不取 <A,B><B,A><A',B'><B',A'>
必须取 A <A',A>
*/
void tarjan(int n) {
    ssz = bsz = 0;
    for(int i = 1; i <= n; i++) vis[i] = 0;
    for(int i = 1; i <= n; i++) {
        if(!vis[i]) DFS(i, 0);
    }
    for(int i = 1; i <= n; i++) vis[i] = 0;
}

```

```

    for(int i = ssz; i >= 1; i--) {
        if(!vis[Stack[i]]) DFS(Stack[i], ++bsz);
    }
}

```

## 12.0(1)lca

```

struct ST_LCA {
    int loo[MX * 2];
    int first[MX], dfn_to_id[MX], dfn;
    int ST[MX * 2][20], st_len;
    int dist[MX];

    void presolve() {
        dfn = st_len = 0;
        loo[1] = 0; loo[2] = 1;
        for(int i = 3; i < MX * 2; i++) {
            loo[i] = loo[i / 2] + 1;
        }
    }

    void DFS(int u, int f, int d) {
        int now = ++dfn;

        dist[u] = d;
        dfn_to_id[now] = u;
        ST[++st_len][0] = now;
        first[u] = st_len;

        for(int i = Head[u]; ~i; i = E[i].nxt) {
            int v = E[i].v;
            if(v == f) continue;
            DFS(v, u, d + 1);
            ST[++st_len][0] = now;
        }
    }

    void MT_presolve() {
        DFS(1, -1, 0);
        for(int i = 1; (1 << i) <= st_len; i++) {
            for(int j = 1; j + (1 << i) - 1 <= st_len; j++) {
                ST[j][i] = min(ST[j][i - 1], ST[j + (1 << i) - 1][i - 1]);
            }
        }
    }

    int query(int u, int v) {
        int l = first[u], r = first[v];
        if(l > r) swap(l, r);
        int i = loo[r - l + 1];
        return dfn_to_id[min(ST[l][i], ST[r - (1 << i) + 1][i])];
    }

    int distance(int u, int v) {
        int lca = query(u, v);
        return dist[u] + dist[v] - 2 * dist[lca];
    }
} lca;

```

## 13.lca 离线

```

const int MQ = 40000 + 5;
const int MX = 80000 + 5;

struct Edge {
    int v, d;
    Edge(int _v, int _d) {
        v = _v; d = _d;
    }
}

```

```

    }
};
struct Que {
    int id, u, v;
    Que() {}
    Que(int _u, int _v, int _id) {
        u = _u; v = _v; id = _id;
    }
} A[MQ];

int D[MX];
struct LCA {
    int n, ans[MQ]; //答案按照 id 保存在 ans 中
    int P[MX]; bool vis[MX];

    vector<Edge> E[MX];
    vector<Que> Q[MQ];

    void Init(int _n) {
        n = _n;
        memset(ans, -1, sizeof(ans));
        memset(vis, false, sizeof(vis));
        for(int i = 1; i <= n; i++) {
            E[i].clear();
            Q[i].clear();
            P[i] = i;
        }
    }

    void AddQue(int u, int v, int id) {
        Q[u].push_back(Que(u, v, id));
        Q[v].push_back(Que(v, u, id));
    }

    void AddEdge(int u, int v, int d) {
        E[u].push_back(Edge(v, d));
        E[v].push_back(Edge(u, d));
    }

    int Find(int x) {
        return P[x] == x ? x : (P[x] = Find(P[x]));
    }

    void Union(int u, int v) {
        int p1 = Find(u), p2 = Find(v);
        P[p1] = p2;
    }

    /*初始 DFS(root,root)*/
    void DFS(int u, int f, int d) {
        D[u] = d;
        for(int i = 0; i < E[u].size(); i++) {
            int v = E[u][i].v, cost = E[u][i].d;
            if(v != f) DFS(v, u, d + cost);
        }

        vis[u] = 1;
        for(int i = 0; i < Q[u].size(); i++) {
            int v = Q[u][i].v, id = Q[u][i].id;
            if(vis[v]) {
                ans[id] = Find(v);
            }
        }
    }
}

```

```

        Union(u, f);
    }
} lca;

```

## 14. lca 在线

```

const int M = 30; // n 的 log

int dep[MX], fa[MX][M], n;
void DFS(int u, int _dep, int _fa) {
    dep[u] = _dep; fa[u][0] = _fa;
    for(int i = Head[u]; ~i; i = E[i].nxt) {
        int v = E[i].v;
        if(v == u || v == _fa) continue;
        DFS(v, _dep + 1, u);
    }
}
/*记得构图后要初始化一遍*/
void presolve() {
    DFS(1, 0, 1);
    for(int i = 1; i < M; i++) {
        for(int j = 1; j <= n; j++) {
            fa[j][i] = fa[fa[j][i - 1]][i - 1];
        }
    }
}
/*倍增法要理解对 2 的次方的枚举顺序
如果是要走固定步数，那么顺序枚举与 i 位与为 1 就行
如果是要求一个临界位置，那么要从大到小枚举
*/
int LCA(int u, int v) {
    while(dep[u] != dep[v]) {
        if(dep[u] < dep[v]) swap(u, v);
        int d = dep[u] - dep[v];
        for(int i = 0; i < M; i++) {
            if(d >> i & 1) u = fa[u][i];
        }
    }
    if(u == v) return u;
    for(int i = M - 1; i >= 0; i--) {
        if(fa[u][i] != fa[v][i]) {
            u = fa[u][i];
            v = fa[v][i];
        }
    }
    return fa[u][0];
}

```

## 15. sap 网络流

```

const int MX = 300 + 5;
const int INF = 0x3f3f3f3f;
/*复杂度 O(n^2*m)
下标从 0 开始
*/
int maze[MX][MX];
int gap[MX], dis[MX], pre[MX], cur[MX];

int sap(int start, int end, int nodenum) {
    memset(cur, 0, sizeof(cur));
    memset(dis, 0, sizeof(dis));
    memset(gap, 0, sizeof(gap));
    int u = pre[start] = start, maxflow = 0, aug = -1;

```

```

    gap[0] = nodenum;
    while(dis[start] < nodenum) {
loop:
    for(int v = cur[u]; v < nodenum; v++) {
        if(maze[u][v] && dis[u] == dis[v] + 1) {
            if(aug == -1 || aug > maze[u][v]) aug = maze[u][v];
            pre[v] = u; u = cur[u] = v;
            if(v == end) {
                maxflow += aug;
                for(u = pre[u]; v != start; v = u, u = pre[u]) {
                    maze[u][v] -= aug;
                    maze[v][u] += aug;
                }
                aug = -1;
            }
            goto loop;
        }
    }
    int mindis = nodenum - 1;
    for(int v = 0; v < nodenum; v++) {
        if(maze[u][v] && mindis > dis[v]) {
            cur[u] = v;
            mindis = dis[v];
        }
    }
    if(--gap[dis[u]]) == 0) break;
    gap[dis[u] = mindis + 1]++;
    u = pre[u];
}
return maxflow;
}

```

## 16.dinic 邻接表网络流

```

const int MX = 1e3;
const int MS = 4e5 + 5;
const int INF = 0x3f3f3f3f;

template<class T>
struct Max_Flow {
    int n;
    int Q[MX], sign;
    int head[MX], level[MX], cur[MX], pre[MX];
    int nxt[MS], pnt[MS], E;
    T cap[MS];
    void Init(int n) {
        E = 0;
        this->n = n + 1;
        fill(head, head + this->n, -1);
    }
    void Add(int from, int to, T c, T rw = 0) {
        pnt[E] = to; cap[E] = c; nxt[E] = head[from]; head[from] = E++;
        pnt[E] = from; cap[E] = rw; nxt[E] = head[to]; head[to] = E++;
    }
    bool BFS(int s, int t) {
        sign = t;
        std::fill(level, level + n, -1);
        int *front = Q, *tail = Q;
        *tail++ = s; level[s] = 0;
        while (front < tail && level[t] == -1) {
            int u = *front++;
            for (int e = head[u]; e != -1; e = nxt[e]) {

```

```

        if (cap[e ^ 1] > 0 && level[pnt[e]] < 0) {
            level[pnt[e]] = level[u] + 1;
            *tail++ = pnt[e];
        }
    }
}
return level[s] != -1;
}
void Push(int t, T &flow) {
    T mi = INF;
    int p = pre[t];
    for (int p = pre[t]; p != -1; p = pre[pnt[p ^ 1]]) {
        mi = std::min(mi, cap[p]);
    }
    for (int p = pre[t]; p != -1; p = pre[pnt[p ^ 1]]) {
        cap[p] -= mi;
        if (!cap[p]) {
            sign = pnt[p ^ 1];
        }
        cap[p ^ 1] += mi;
    }
    flow += mi;
}
void DFS(int u, int t, T &flow) {
    if (u == t) {
        Push(t, flow);
        return;
    }
    for (int &e = cur[u]; e != -1; e = nxt[e]) {
        if (cap[e] > 0 && level[u] - 1 == level[pnt[e]]) {
            pre[pnt[e]] = e;
            DFS(pnt[e], t, flow);
            if (level[sign] > level[u]) {
                return;
            }
            sign = t;
        }
    }
}
}
T Dinic(int s, int t) {
    pre[s] = -1;
    T flow = 0;
    while (BFS(s, t)) {
        std::copy(head, head + n, cur);
        DFS(s, t, flow);
    }
    return flow;
}
};
Max_Flow<int>F;

```

## 17.zkw 费用流

```

namespace MCMF {
    int S, T; // 源点, 汇点
    int erar, n;
    int st, en, maxflow, mincost;
    bool vis[MX];
    int Head[MX], cur[MX], dis[MX];
    int roade[MX], roadv[MX], rsz; // 用于打印路径
    const int ME = 4e5 + 5; // 边的数量

    queue<int> Q;
}

```

```

struct Edge {
    int v, cap, cost, nxt, flow;
    Edge() {}
    Edge(int a, int b, int c, int d) {
        v = a, cap = b, cost = c, nxt = d, flow = 0;
    }
} E[ME], SE[ME];

void init(int _n) {
    n = _n, erear = 0;
    for(int i = 0; i <= n; i++) Head[i] = -1;
}

void edge_add(int u, int v, int cap, int cost) {
    E[erear] = Edge(v, cap, cost, Head[u]);
    Head[u] = erear++;
    E[erear] = Edge(u, 0, -cost, Head[v]);
    Head[v] = erear++;
}

bool adjust() {
    int v, min = INF;
    for(int i = 0; i <= n; i++) {
        if(!vis[i]) continue;
        for(int j = Head[i]; ~j; j = E[j].nxt) {
            v = E[j].v;
            if(E[j].cap - E[j].flow) {
                if(!vis[v] && dis[v] - dis[i] + E[j].cost < min) {
                    min = dis[v] - dis[i] + E[j].cost;
                }
            }
        }
    }
    if(min == INF) return false;
    for(int i = 0; i <= n; i++) {
        if(vis[i]) {
            cur[i] = Head[i];
            vis[i] = false;
            dis[i] += min;
        }
    }
    return true;
}

int augment(int i, int flow) {
    if(i == en) {
        mincost += dis[st] * flow;
        maxflow += flow;
        return flow;
    }
    vis[i] = true;
    for(int j = cur[i]; j != -1; j = E[j].nxt) {
        int v = E[j].v;
        if(E[j].cap == E[j].flow) continue;
        if(vis[v] || dis[v] + E[j].cost != dis[i]) continue;
        int delta = augment(v, std::min(flow, E[j].cap - E[j].flow));
        if(delta) {
            E[j].flow += delta;
            E[j ^ 1].flow -= delta;
            cur[i] = j;
            return delta;
        }
    }
    return 0;
}

void spfa() {

```



```

int u, v;
for(int i = 0; i <= n; i++) {
    vis[i] = false;
    dis[i] = INF;
}
Q.push(st);
dis[st] = 0; vis[st] = true;
while(!Q.empty()) {
    u = Q.front(), Q.pop(); vis[u] = false;
    for(int i = Head[u]; ~i; i = E[i].nxt) {
        v = E[i].v;
        if(E[i].cap == E[i].flow || dis[v] <= dis[u] + E[i].cost) continue;
        dis[v] = dis[u] + E[i].cost;
        if(!vis[v]) {
            vis[v] = true;
            Q.push(v);
        }
    }
}
for(int i = 0; i <= n; i++) {
    dis[i] = dis[en] - dis[i];
}
spfa_time_total++;
}
int zkw(int s, int t, int &ret_flow) {
    st = s, en = t;
    spfa();
    mincost = maxflow = 0;
    for(int i = 0; i <= n; i++) {
        vis[i] = false;
        cur[i] = Head[i];
    }
    do {
        while(augment(st, INF)) {
            memset(vis, false, n * sizeof(bool));
        }
    } while(adjust());
    ret_flow = maxflow;
    return mincost;
}
}

```

## 18. 普通费用流

```

const int MX = 400 + 5; //都开4倍把..
const int MM = 400 + 5;
const int INF = 0x3f3f3f3f;

```

```

struct Edge {
    int to, next, cap, flow, cost;
    Edge() {}
    Edge(int _to, int _next, int _cap, int _flow, int _cost) {
        to = _to; next = _next; cap = _cap; flow = _flow; cost = _cost;
    }
} E[MM];

```

```

int Head[MX], tol;
int pre[MX]; //储存前驱顶点
int dis[MX]; //储存到源点 s 的距离
bool vis[MX];
int N; //节点总个数, 节点编号从 0~N-1

```

```

void init(int n) {

```

```

    tol = 0;
    N = n + 2;
    memset(Head, -1, sizeof(Head));
}
void edge_add(int u, int v, int cap, int cost) {
    E[tol] = Edge(v, Head[u], cap, 0, cost);
    Head[u] = tol++;

    E[tol] = Edge(u, Head[v], 0, 0, -cost);
    Head[v] = tol++;
}
bool spfa(int s, int t) {
    queue<int>q;
    for (int i = 0; i < N; i++) {
        dis[i] = INF;
        vis[i] = false;
        pre[i] = -1;
    }
    dis[s] = 0;
    vis[s] = true;
    q.push(s);
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        vis[u] = false;
        for (int i = Head[u]; i != -1; i = E[i].next) {
            int v = E[i].to;
            if (E[i].cap > E[i].flow && dis[v] > dis[u] + E[i].cost) {
                dis[v] = dis[u] + E[i].cost;
                pre[v] = i;
                if (!vis[v]) {
                    vis[v] = true;
                    q.push(v);
                }
            }
        }
    }
    if (pre[t] == -1) return false;
    else return true;
}

//返回的是最大流， cost 存的是最小费用
int minCostMaxflow(int s, int t, int &cost) {
    int flow = 0;
    cost = 0;
    while (spfa(s, t)) {
        int Min = INF;
        for (int i = pre[t]; i != -1; i = pre[E[i ^ 1].to]) {
            if (Min > E[i].cap - E[i].flow)
                Min = E[i].cap - E[i].flow;
        }
        for (int i = pre[t]; i != -1; i = pre[E[i ^ 1].to]) {
            E[i].flow += Min;
            E[i ^ 1].flow -= Min;
            cost += E[i].cost * Min;
        }
        flow += Min;
    }
    return flow;
}

```

## 19.Dijkstra

```

const int dij_v = 3e5;
const int dij_edge = 8e5;
template<class T>
struct Dijkstra {
    struct Edge {
        T w;
        int v, nxt;
    } E[dij_edge << 1];
    typedef pair<T, int> PII;
    int Head[dij_v], erear;
    T d[dij_v], INF;

    void init() {
        erear = 0;
        memset(Head, -1, sizeof(Head));
    }
    void add(int u, int v, T w) {
        E[erear].v = v;
        E[erear].w = w;
        E[erear].nxt = Head[u];
        Head[u] = erear++;
    }
    void run(int u) {
        memset(d, 0x3f, sizeof(d));
        INF = d[0];
        priority_queue<PII, vector<PII>, greater<PII> >Q;

        Q.push(PII(0, u)); d[u] = 0;
        Q.push(PII(A[u], u + n)); d[u + n] = A[u];
        while(!Q.empty()) {
            PII ftp = Q.top(); Q.pop();
            int u = ftp.second;
            if(ftp.first != d[u]) continue;
            for(int i = Head[u]; ~i; i = E[i].nxt) {
                int v = E[i].v; T w = E[i].w;
                if(d[u] + w < d[v]) {
                    d[v] = d[u] + w;
                    Q.push(PII(d[v], v));
                }
            }
        }
    }
};
Dijkstra<LL> dij;

```

## 20.floyd

```

void floyd(int n) {
    for(int k = 1; k <= n; k++) {
        for(int i = 1; i <= n; i++) {
            for(int j = 1; j <= n; j++) {
                d[i][j] = min(d[i][j], d[i][k] + d[k][j]);
            }
        }
    }
}

```

## 21.普通 spfa

/\*

spfa 的优化

SPFA 算法有两个优化算法 SLF 和 LLL：SLF：Small Label First 策略，设要加入的节点是  $j$ ，队首元素为  $i$ ，若  $\text{dist}(j) < \text{dist}(i)$ ，则将  $j$  插入队首，否则插入队尾。LLL：Large Label Last 策略，设队首元素为  $i$ ，队

列中所有  $\text{dist}$  值的平均值为  $x$ ，若  $\text{dist}(i) > x$  则将  $i$  插入到队尾，查找下一元素，直到找到某一  $i$  使得  $\text{dist}(i) \leq x$ ，则将  $i$  出队进行松弛操作。SLF 可使速度提高 15 ~ 20%；SLF + LLL 可提高约 50%。在实际的应用中 SPFA 的算法时间效率不是很稳定，为了避免最坏情况的出现，通常使用效率更加稳定的 Dijkstra 算法。

```

*/
void spfa(int s) {
    queue<int> q;
    for(int i = 1; i <= n; i++) {
        d[i] = INF; vis[i] = 0;
    }
    d[s] = 0; vis[s] = 1; q.push(s);
    while(!q.empty()) {
        int u = q.front(); q.pop(); vis[u] = 0;
        for(int i = Head[u]; ~i; i = E[i].nxt) {
            int v = E[i].v, cost = E[i].cost;
            if(d[u] + cost < d[v]) {
                d[v] = d[u] + cost;
                if(!vis[v]) {
                    vis[v] = 1;
                    q.push(v);
                }
            }
        }
    }
}

```

## 22.SLF 优化的 spfa

```

const int MX = 1e5 + 5;
const int MS = 1e5 + 5;
template<class T>
struct SPFA {
    struct Edge {
        T w;
        int v, nxt;
    } E[MS << 1];
    int Head[MX], erear;
    bool vis[MX];
    T d[MX], INF;
    deque<int> Q;

    void init() {
        erear = 0;
        memset(Head, -1, sizeof(Head));
    }
    void add(int u, int v, T w) {
        E[erear].v = v;
        E[erear].w = w;
        E[erear].nxt = Head[u];
        Head[u] = erear++;
    }

    inline void relax(int u, int v, T w) {
        if(d[u] + w < d[v]) {
            d[v] = d[u] + w;
            if(!vis[v]) {
                if(!Q.empty() && d[v] <= d[Q.front()]) {
                    Q.push_front(v);
                } else Q.push_back(v);
                vis[v] = 1;
            }
        }
    }
    void run(int u) {

```

```

    Q.clear();
    memset(d, 0x3f, sizeof(d)); INF = d[0];
    d[u] = 0; Q.push_back(u); vis[u] = 1;
    while(!Q.empty()) {
        int u = Q.front(); Q.pop_front(); vis[u] = 0;
        for(int i = Head[u]; ~i; i = E[i].nxt) {
            relax(u, E[i].v, E[i].w);
        }
    }
}
};
SPFA<LL> spfa;

```

## 23.有负环的 spfa

```

/*
d[i]为-INF, 说明路径上存在一个负环
d[i]为 INF, 说明不连通
D[i]为其他值, 表示普通的最短路
*/
int d[MX], IN[MX];
void spfa(int n, int op) {
    for(int i = 0; i <= n; i++) {
        d[i] = INF;
        vis[i] = IN[i] = 0;
    }
    queue<int>Q;
    Q.push(op); d[op] = 0;
    while(!Q.empty()) {
        int u = Q.front(); Q.pop();
        vis[u] = 0; dvis[u] = 1;
        for(int i = Head[u]; ~i; i = E[i].nxt) {
            int v = E[i].v, w = E[i].w;
            int s = d[u] == -INF ? -INF : w + d[u];
            if(s < d[v]) {
                d[v] = s;
                if(!vis[v]) {
                    vis[v] = 1; IN[v]++;
                    if(IN[v] > 10) d[v] = -INF; //最差情况为 n, 根据需要修改
                    Q.push(v);
                }
            }
        }
    }
}
}

```

## 24.割顶和桥

```

int Low[MX], DFN[MX], dfs_clock;
int cut[MX];
void tarjan_init() {
    dfs_clock = 0;
    memset(DFN, 0, sizeof(DFN));
    memset(cut, 0, sizeof(cut));
}
/*
桥的性质 lowv>DFN[u]
割点的性质 lowv>=DFN[u]或者为有>=2 儿子的根节点
*/
int tarjan(int u, int e) {
    Low[u] = DFN[u] = ++dfs_clock;
    int child = 0;
    for(int id = Head[u]; ~id; id = Next[id]) {
        int v = E[id].v;

```

```

        if(!DFN[v]) {
            int lowv = tarjan(v, id | 1);
            Low[u] = min(Low[u], lowv);

            if(lowv >= DFN[u]) {
                cut[u] = 1;
            }
            if(lowv > DFN[u]) {
                E[id].sign = 1;
                E[id ^ 1].sign = 1;
            }
            child++;
        } else if((id | 1) != e && DFN[v] < DFN[u]) {
            Low[u] = min(Low[u], DFN[v]);
        }
    }
    if(e == -1 && child == 1) cut[u] = 0;
    return Low[u];
}

void tarjan_run() {
    for(int i = 1; i <= n; i++) {
        if(!DFN[i]) tarjan(i, -1);
    }
}

```

## 25.其他网络流

无源汇有上下界最大流

$du[i] = in[i] - out[i]$ , 入的总流减出的总流

$du[i] > 0$ , 连一条边从  $S$  到  $i$ , 流量为  $du[i]$

$du[i] < 0$ , 连一条边从  $i$  到  $T$ , 流量为  $-du[i]$

最后看总流量是否都等于所有  $du[i] (du[i] > 0)$  之和

先增加一条边从  $T$  到  $S$  没有下界上界  $INF$

然后按无源汇有上下界最大流的方法建图, 然后跑一次最大流

然后把  $T$  到  $S$  的边给拆掉

再跑一次从  $S$  到  $T$  的最大流

把两次得到的最大流相加就是答案

## 26.费用流可行流

方案 1: 如果某一次的最短路费用  $> 0$  时, 直接返回 false

方案 2:  $S \rightarrow S'$ , 费用 0, 流  $INF$ 。  $S' \rightarrow T$ , 费用 0, 流  $INF$ 。用  $S'$  当作新的源点

## 27.最小割

最大流即是最小割。

最小割: 删除某些边, 使得  $s$  和  $t$  不连通, 要求删除的边的容量和最小

## 28.最大权闭合图

能求一个闭合图, 使得点权值之和最大。

最大权闭合图建图:

如果要选  $a$ , 就必须先选  $b, c, d$ , 那么就要连  $a \rightarrow b, a \rightarrow c, a \rightarrow d$ , 容量为  $INF$

如果一个位置费用为正, 那么  $op \rightarrow u$ , 容量为费用

如果一个位置费用为负, 那么  $u \rightarrow ed$ , 容量为费用取反

最后的最大闭合权值答案就是正权之和 - 最大流

残余网络中的点, 就是要被删除的点。

## 1. AC 自动机

```

/*基础代码*/
/*MX 为总长度*/
const int MX = 500000 + 5;

struct AC_machine {
    int rear, root;
    int Next[MX][26], Fail[MX], End[MX];

    void Init() {
        rear = 0;
        root = New();
    }

    int New() {
        rear++;
        End[rear] = 0;
        for(int i = 0; i < 26; i++) {
            Next[rear][i] = -1;
        }
        return rear;
    }

    void Add(char*A) {
        int n = strlen(A), now = root;
        for(int i = 0; i < n; i++) {
            int id = A[i] - 'a';
            if(Next[now][id] == -1) {
                Next[now][id] = New();
            }
            now = Next[now][id];
        }
        End[now]++;
    }
} AC;
/*状态自动机 build*/
void Build() {
    queue<int>Q;
    Fail[root] = root;
    for(int i = 0; i < 4; i++) {
        if(Next[root][i] == -1) {
            Next[root][i] = root;
        } else {
            Fail[Next[root][i]] = root;
            Q.push(Next[root][i]);
        }
    }
}

while(!Q.empty()) {
    int u = Q.front(); Q.pop();

    //注意这一句话根据不同的情况修改
    if(End[Fail[u]]) End[u] = 1;
    for(int i = 0; i < 4; i++) {
        if(Next[u][i] == -1) {
            Next[u][i] = Next[Fail[u]][i];
        } else {
            Fail[Next[u][i]] = Next[Fail[u]][i];
        }
    }
}

```

```

        Q.push(Next[u][i]);
    }
}
}
}
/*字符串匹配 build*/
void Build() {
    queue<int>Q;
    Fail[root] = root;
    for(int i = 0; i < 26; i++) {
        if(Next[root][i] == -1) {
            Next[root][i] = root;
        } else {
            Fail[Next[root][i]] = root;
            Q.push(Next[root][i]);
        }
    }

    while(!Q.empty()) {
        int u = Q.front();
        Q.pop();

        for(int i = 0; i < 26; i++) {
            if(Next[u][i] == -1) {
                Next[u][i] = Next[Fail[u]][i];
            } else {
                Fail[Next[u][i]] = Next[Fail[u]][i];
                Q.push(Next[u][i]);
            }
        }
    }
}
}
/*匹配串出现了几个*/
int Query(char *S) {
    int n = strlen(S), now = root, ret = 0;
    for(int i = 0; i < n; i++) {
        now = Next[now][S[i] - 'a'];
        int temp = now;

        while(temp != root) {
            ret += End[temp];
            //如果要多次匹配，下面改成 vis 标记
            End[temp] = 0;
            temp = Fail[temp];
        }
    }
    return ret;
}
}
/*每个字符串出现次数*/
void Query(char *S) {
    int n = strlen(S), now = root;
    for(int i = 0; i < n; i++) {
        now = Next[now][S[i] - 'a'];
        int temp = now;

        while(temp != root) {
            if(End[temp]) ans[End[temp]]++;
            temp = Fail[temp];
        }
    }
}
}
/*防重叠匹配出现次数*/
void Query(char *S) {

```



```

int n = strlen(S), now = root, ret = 0;

for(int i = 0; i < n; i++) {
    now = Next[now][S[i] - 'a'];
    int temp = now;

    while(temp != root) {
        if(End[temp] && (last[End[temp]] == -1 || last[End[temp]] + len[temp] <= i)) {
            ans[End[temp]]++;
            last[End[temp]] = i;
        }
        temp = Fail[temp];
    }
}
}

```

## 2. 在线 AC 自动机

```

/*下标从 1 开始*/
const int MG = 30;
const int MX = 6e5 + 5;
struct Trie_graph_online {
    int nxt[MX][26], Fail[MX], End[MX], sz;
    int root[MG], gsize[MG], gsz, ssz;
    string str[MX];
    int val[MX];

    void Init() {
        sz = gsz = ssz = 0;
    }
    int New() {
        End[++sz] = 0;
        for(int i = 0; i < 26; i++) nxt[sz][i] = 0;
        return sz;
    }
    void DealNxt(int root) {
        queue<int> Q;
        Fail[root] = root;
        for(int i = 0; i < 26; i++) {
            if(!nxt[root][i]) {
                nxt[root][i] = root;
            } else {
                Fail[nxt[root][i]] = root;
                Q.push(nxt[root][i]);
            }
        }
        while(!Q.empty()) {
            int u = Q.front(); Q.pop();
            End[u] += End[Fail[u]];
            for(int i = 0; i < 26; i++) {
                if(!nxt[u][i]) {
                    nxt[u][i] = nxt[Fail[u]][i];
                } else {
                    Fail[nxt[u][i]] = nxt[Fail[u]][i];
                    Q.push(nxt[u][i]);
                }
            }
        }
    }
    void Rebuild(int l, int r, int &root) {
        root = New();
        for(int i = l; i <= r; i++) {
            int len = str[i].length();

```

```

        int rt = root;
        for(int j = 0; j < len; j++) {
            int id = str[i][j] - 'a';
            if(!nxt[rt][id]) nxt[rt][id] = New();
            rt = nxt[rt][id];
        }
        End[rt] += val[i];
    }
    DealNxt(root);
}
void Add(char s[], int x) {
    str[++ssz] = string(s);
    val[ssz] = x;

    gsize[++gsz] = 1; root[gsz] = ++sz;
    while(gsz >= 2 && gsize[gsz] == gsize[gsz - 1]) {
        gsz--; gsize[gsz] *= 2;
    }
    sz = root[gsz] - 1;
    Rebuild(ssz - gsize[gsz] + 1, ssz, root[gsz]);
}
int Query_each(int root, char s[], int len) {
    int now = root, ret = 0;
    for(int i = 0; i < len; i++) {
        now = nxt[now][s[i] - 'a'];
        ret += End[now];
    }
    return ret;
}
int Query(char s[]) {
    int ret = 0, len = strlen(s);
    for(int i = 1; i <= gsz; i++) {
        ret += Query_each(root[i], s, len);
    }
    return ret;
}
} AC;

```

### 3. 后缀数组

```

/*
复杂度 O(nlogn)
n 自动+1 无需再管，返回的 SA,R,H 的下标都是 0~n
其中多包括了一个空字符串
SA 后缀数组，R 名次数组，H 高度数组
H[i]表示 SA[i]和 SA[i-1]的 lcp
*/
char s[MX];
int SA[MX], R[MX], H[MX];
int wa[MX], wb[MX], wv[MX], wc[MX];
int cmp(int *r, int a, int b, int l) {
    return r[a] == r[b] && r[a + 1] == r[b + 1];
}
void Suffix(char *r, int m = 128) {
    int n = strlen(r) + 1;
    int i, j, p, *x = wa, *y = wb, *t;
    for(i = 0; i < m; i++) wc[i] = 0;
    for(i = 0; i < n; i++) wc[x[i]] = r[i]++;
    for(i = 1; i < m; i++) wc[i] += wc[i - 1];
    for(i = n - 1; i >= 0; i--) SA[--wc[x[i]]] = i;
    for(j = 1, p = 1; p < n; j *= 2, m = p) {
        for(p = 0, i = n - j; i < n; i++) y[p++] = i;
        for(i = 0; i < n; i++) if(SA[i] >= j) y[p++] = SA[i] - j;
    }
}

```

```

        for(i = 0; i < n; i++) wv[i] = x[y[i]];
        for(i = 0; i < m; i++) wc[i] = 0;
        for(i = 0; i < n; i++) wc[wv[i]]++;
        for(i = 1; i < m; i++) wc[i] += wc[i - 1];
        for(i = n - 1; i >= 0; i--) SA[--wc[wv[i]]] = y[i];
        for(t = x, x = y, y = t, p = 1, x[SA[0]] = 0, i = 1; i < n; i++) {
            x[SA[i]] = cmp(y, SA[i - 1], SA[i], j) ? p - 1 : p++;
        }
    }
    int k = 0; n--;
    for(i = 0; i <= n; i++) R[SA[i]] = i;
    for(i = 0; i < n; i++) {
        if(k) k--;
        j = SA[R[i] - 1];
        while(r[i + k] == r[j + k]) k++;
        H[R[i]] = k;
    }
}

```

## 4. KMP

```

int Next[MX], n;
void GetNext() {
    Next[0] = 0;
    for(int i = 1; i < n; i++) {
        int j = Next[i - 1];
        while(j && S[i] != S[j]) j = Next[j - 1];
        Next[i] = S[i] == S[j] ? j + 1 : 0;
    }
}
/*求前缀 i 循环节最长长度*/
int GetCir(int p) {
    return (p + 1) % (p - Next[p] + 1) == 0 ? p - Next[p] + 1 : p + 1;
}

/*会有重叠部分*/
int Next[MX];

int KMP(char *A, char *B) {
    int m = strlen(A), n = strlen(B);

    Next[0] = 0;
    for(int i = 1; i < n; i++) {
        int k = Next[i - 1];
        while(B[i] != B[k] && k) k = Next[k - 1];
        Next[i] = B[i] == B[k] ? k + 1 : 0;
    }

    int ans = 0, j = 0;
    for(int i = 0; i < m; i++) {
        while(A[i] != B[j] && j) j = Next[j - 1];
        if(A[i] == B[j]) j++;
        if(j == n) ans++;
    }
    return ans;
}

/*不会有重叠部分*/
int Next[MX];

int KMP(char *A, char *B) {
    int m = strlen(A), n = strlen(B);

```

```

Next[0] = 0;
for(int i = 1; i < n; i++) {
    int k = Next[i - 1];
    while(B[i] != B[k] && k) k = Next[k - 1];
    Next[i] = B[i] == B[k] ? k + 1 : 0;
}

int ans = 0, j = 0;
for(int i = 0; i < m; i++) {
    while(A[i] != B[j] && j) j = Next[j - 1];
    if(A[i] == B[j]) j++;
    if(j == n) ans++, j = Next[j - 1];
}
return ans;
}

```

## 5. Manacher

```

const int MAX = 110000 + 10;
char s[MAX * 2];
int p[MAX * 2];

/*
首先, i>=2 的 p 才有意义
p[i]-1 为以 i 为中心的回文长度
p[i]/2 表示回文半径
i%2==0 表示这个位置为字符, i/2-1 表示原字符串的位置
i%2==1 表示为字符中间, 这两边的字符在原字符串的位置分别为 i/2-1 和 i/2
*/
int manacher(char *s){
    int len = strlen(s), id = 0, ans = 0;
    for(int i = len; i >= 0; i--) {
        s[i + i + 2] = s[i];
        s[i + i + 1] = '#';
    }
    s[0] = '*';
    for(int i = 2; i < 2 * len + 1; ++i) {
        if(p[id] + id > i) p[i] = min(p[2 * id - i], p[id] + id - i);
        else p[i] = 1;
        while(s[i - p[i]] == s[i + p[i]]) p[i]++;
        if(id + p[id] < i + p[i]) id = i;
        ans = max(ans, p[i] - 1);
    }
    return ans;
}

```

## 6. MT 定理

```

/*
url:http://www.spoj.com/problems/HIGH/
Matrix-Tree 定理的裸题
构造方法: C 矩阵=D 矩阵-G 矩阵, D[i][i]表示 i 的度, 其他位置为 0
G[i][j]=1 表示 i 和 j 之间有一条边。
之后, 我们用高斯消元去求 C 的其中一个余子式的行列式就行了。
为了方便, 我们通常取(n-1,n-1)的余子式。
这里有几个要注意的地方:
1. 如果最后的答案非常大, 要取模, 就把高斯消元的除法改成逆元
2. 如果我们消元的那个位置 A[i][i]为 0, 应该及时返回 0, 否则就会除以 0
3. 得多留意重边之类的处理。
*/

```

```

const int MX = 10 + 5;
const int INF = 0x3f3f3f3f;
const int mod = 1e9 + 7;
const double eps = 1e-8;

typedef double Matrix[MX][MX];

int n, m;
Matrix C;
int G[MX][MX], D[MX][MX];

double det(Matrix A, int n) {
    double ret = 1;
    int i, j, k, r;
    for(i = 0; i < n; i++) {
        r = i;
        for(j = i + 1; j < n; j++) {
            if(fabs(A[j][i]) > fabs(A[r][i])) r = j;
        }
        if(r != i) for(j = 0; j < n; j++) swap(A[r][j], A[i][j]);
        if(fabs(A[i][i]) < eps) return 0;

        for(k = i + 1; k < n; k++) {
            double f = A[k][i] / A[i][i];
            for(j = i; j < n; j++) A[k][j] -= f * A[i][j];
        }
        ret = ret * A[i][i];
    }
    return ret;
}

int main() {
    int T; //FIN;
    scanf("%d", &T);
    while(T--) {
        memset(D, 0, sizeof(D));
        memset(G, 0, sizeof(G));
        scanf("%d%d", &n, &m);
        for(int i = 1; i <= m; i++) {
            int u, v;
            scanf("%d%d", &u, &v);
            if(u == v) continue;
            u--; v--;
            G[u][v] = G[v][u] = 1;
            D[u][u]++; D[v][v]++;
        }
        for(int i = 0; i < n; i++) {
            for(int j = 0; j < n; j++) {
                C[i][j] = D[i][j] - G[i][j];
            }
        }
        printf("%.0f\n", fabs(det(C, n - 1)));
    }
    return 0;
}

```

## 7. 二分求 lcp

```

const int seed = 131;
typedef unsigned long long ULL;
ULL fac[MX], pre[MX];
char A[MX];

```

```

void presolve() {
    fac[0] = 1;
    for(int i = 1; i < MX; i++) {
        fac[i] = fac[i - 1] * seed;
    }
}

bool check(int a, int b, int l) {
    ULL left = pre[a + l - 1] - pre[a - 1] * fac[l];
    ULL right = pre[b + l - 1] - pre[b - 1] * fac[l];
    return left == right;
}

int lcp(int n, int a, int b) {
    pre[0] = 0;
    for(int i = 1; i <= n; i++) {
        pre[i] = pre[i - 1] * seed + A[i];
    }

    int l = 0, r = n, m;
    while(l <= r) {
        m = (l + r) >> 1;
        if(check(a, b, m)) l = m + 1;
        else r = m - 1;
    }
    return l - 1;
}

```

## 8. 最小表示法

```

int solve(char *s, int l) {
    int i = 0, j = 1, k = 0, t;
    while(i < l && j < l && k < l) {
        t = s[(i + k) >= l ? i + k - 1 : i + k] - s[(j + k) >= l ? j + k - 1 : j + k];
        if(!t) k++;
        else {
            if(t > 0) i = i + k + 1;
            else j = j + k + 1;
            if(i == j) j++;
            k = 0;
        }
    }
    return min(i, j);
}

```

## 数论

### 1. 矩阵快速幂(vector 版)

```

typedef vector<int> vec;
typedef vector<vec> mat;
mat mat_mul(mat &A, mat &B) {
    mat C(A.size(), vec(B[0].size()));
    for(int i = 0; i < A.size(); i++) {
        for(int j = 0; j < B[0].size(); j++) {
            for(int k = 0; k < B.size(); k++) {
                C[i][j] = ((LL)A[i][k] * B[k][j] + C[i][j]) % mod;
            }
        }
    }
    return C;
}

mat mat_pow(mat A, LL n) {
    mat B(A.size(), vec(A.size()));
    for(int i = 0; i < A.size(); i++) B[i][i] = 1;
}

```

```

while(n) {
    if(n & 1) B = mat_mul(B, A);
    A = mat_mul(A, A);
    n >>= 1;
}
return B;
}
/*初始化矩阵*/
mat A(n, vec(n));

```

## 2. 矩阵快速幂(更快)

```

const int matX = 1e2 + 5;
const int mod = 1e9 + 7;
struct Matrix {
    int n, m, s[matX][matX];
    Matrix(int n, int m): n(n), m(m) {
        for(int i = 0; i < n; i++) {
            for(int j = 0; j < m; j++) s[i][j] = 0;
        }
    }
    Matrix operator*(const Matrix &P)const {
        Matrix ret(n, P.m);
        for(int i = 0; i < n; i++) {
            for(int k = 0; k < m; k++) {
                if(s[i][k]) {
                    for(int j = 0; j < P.m; j++) {
                        ret.s[i][j] = ((LL)s[i][k] * P.s[k][j] + ret.s[i][j]) % mod;
                    }
                }
            }
        }
        return ret;
    }
    Matrix operator^(const LL &P)const {
        LL num = P;
        Matrix ret(n, m), tmp = *this;
        for(int i = 0; i < n; i++) ret.s[i][i] = 1;
        while(num) {
            if(num & 1) ret = ret * tmp;
            tmp = tmp * tmp;
            num >>= 1;
        }
        return ret;
    }
};

```

## 3. $O(1)$ gcd

```

namespace qwb_gcd {
    const int MX = 33000 + 5; //最大值
    const int MP = 1e4 + 5; //1e6 时约 79000 个
    const int MK = 180; //sqrt(MX)

    int g[MK][MK];
    int prime[MP], prear;
    int pmin[MX], s[MX][3];
    bool not_prime[MP];

    void init() {
        for(int i = 1; i < MK; i++) {
            for(int j = 0; j < i; j++) {
                if(!j) g[i][j] = i;
                else g[i][j] = g[j][i % j];
            }
        }
    }
}

```

```

    }
}

prear = 0;
not_prime[1] = 1;
for(int i = 2; i < MX; i++) {
    if(!not_prime[i]) {
        pmin[i] = i;
        prime[++prear] = i;
    }
    for(int j = 1; j <= prear && i * prime[j] < MX; j++) {
        not_prime[prime[j]*i] = 1;
        pmin[prime[j]*i] = prime[j];
        if(i % prime[j] == 0) break;
    }
}

s[1][0] = s[1][1] = s[1][2] = 1;
for(int i = 2; i < MX; i++) {
    for(int j = 0; j < 3; j++) s[i][j] = s[i / pmin[i]][j];
    if(s[i][0]*pmin[i] < MK) s[i][0] *= pmin[i];
    else if(s[i][1]*pmin[i] < MK) s[i][1] *= pmin[i];
    else s[i][2] *= pmin[i];
}
}

int gcd(int x, int y) {
    if(!x || !y) return x + y;
    if(x < MK && y < MK) return g[x][y % x];

    int ret = 1, d;
    for(int i = 0; i < 3; i++) {
        if(s[x][i] == 1) continue;
        if(s[x][i] < MK) d = g[s[x][i]][y % s[x][i]];
        else if(y % s[x][i] == 0) d = s[x][i];
        else d = 1;
        ret *= d; y /= d;
    }
    return ret;
}
}

```

## 4. 线性基

/\*复杂度  $n \log n$

能求出 A 数组的线性基，并保存到 p 中

要注意 A 数组的数据范围

\*/

```

void Guass_base() {
    memset(P, 0, sizeof(P));
    for(int i = 1; i <= n; i++) {
        for(int j = 62; j >= 0; j--) {
            if(!(A[i] >> j & 1)) continue;
            if(!P[j]) {
                P[j] = A[i]; break;
            }
            A[i] ^= P[j];
        }
    }
}
}

```



## 5. k 次幂之和

```
/*
求(1^k+2^k+3^k+...+n^k)%mod
复杂度约为 O(klogMOD)
*/
const int MX = 1e6 + 10;
const int mod = 1e9 + 7;
struct Lagrange {
    short factor[MX];
    int P[MX], S[MX], ar[MX], inv[MX];

    inline LL power(LL a, LL b) {
        LL res = 1;
        while (b) {
            if (b & 1) res = res * a % mod;
            a = a * a % mod;
            b >>= 1;
        }
        return res;
    }

    int lagrange(LL n, int k) {
        if (!k) return n % mod;

        int i, j, x, res = 0;
        if (!inv[0]) {
            for (i = 2, x = 1; i < MX; i++) x = (long long)x * i % mod;
            inv[MX - 1] = power(x, mod - 2);
            for (i = MX - 2; i >= 0; i--) inv[i] = ((long long)inv[i + 1] * (i + 1)) % mod;
        }

        k++;
        for (i = 0; i <= k; i++) factor[i] = 0;
        for (i = 4; i <= k; i += 2) factor[i] = 2;
        for (i = 3; (i * i) <= k; i += 2) {
            if (!factor[i]) {
                for (j = (i * i), x = i << 1; j <= k; j += x) {
                    factor[j] = i;
                }
            }
        }

        for (ar[1] = 1, ar[0] = 0, i = 2; i <= k; i++) {
            if (!factor[i]) ar[i] = power(i, k - 1);
            else ar[i] = ((LL)ar[factor[i]] * ar[i / factor[i]]) % mod;
        }

        for (i = 1; i <= k; i++) {
            ar[i] += ar[i - 1];
            if (ar[i] >= mod) ar[i] -= mod;
        }
        if (n <= k) return ar[n];

        P[0] = 1, S[k] = 1;
        for (i = 1; i <= k; i++) P[i] = ((LL)P[i - 1] * ((n - i + 1) % mod)) % mod;
        for (i = k - 1; i >= 0; i--) S[i] = ((LL)S[i + 1] * ((n - i - 1) % mod)) % mod;

        for (i = 0; i <= k; i++) {
            x = (LL)ar[i] * P[i] % mod * S[i] % mod * inv[k - i] % mod * inv[i] % mod;
            if ((k - i) & 1) {
                res -= x;
                if (res < 0) res += mod;
            }
        }
    }
};
```

```

        } else {
            res += x;
            if (res >= mod) res -= mod;
        }
    }
    return res % mod;
}
} lgr;

```

## 6. 约瑟夫环

```

/*
F[n] = (F[n - 1] + m) % n, F[1] = 0
返回的下标从 0 开始, 复杂度大约为 O(m)*/
int Joseph(int n, int m) {
    if(n == 1) return 0;
    if(m == 1) return n - 1;
    LL pre = 0; int now = 2;
    while(now <= n) {
        if(pre + m >= now) {
            pre = (pre + m) % now;
            now++;
        } else {
            int a = now - 1 - pre, b = m - 1;
            int k = a / b + (a % b != 0);
            if(now + k > n + 1) k = n + 1 - now;
            pre = (pre + (LL)m * k) % (now + k - 1);
            now += k;
        }
    }
    return pre;
}

```

## 7. fft

```

const double PI = acos(-1.0);
struct complex {
    double r, i;
    complex(double _r = 0.0, double _i = 0.0) {
        r = _r; i = _i;
    }
    complex operator +(const complex &b) {
        return complex(r + b.r, i + b.i);
    }
    complex operator -(const complex &b) {
        return complex(r - b.r, i - b.i);
    }
    complex operator *(const complex &b) {
        return complex(r * b.r - i * b.i, r * b.i + i * b.r);
    }
};

void change(complex y[], int len) {
    int i, j, k;
    for(i = 1, j = len / 2; i < len - 1; i++) {
        if(i < j) swap(y[i], y[j]);
        k = len / 2;
        while(j >= k) {
            j -= k;
            k /= 2;
        }
        if(j < k) j += k;
    }
}

void fft(complex y[], int len, int on) {

```

```

change(y, len);
for(int h = 2; h <= len; h <= 1) {
    complex wn(cos(on * 2 * PI / h), sin(on * 2 * PI / h));
    for(int j = 0; j < len; j += h) {
        complex w(1, 0);
        for(int k = j; k < j + h / 2; k++) {
            complex u = y[k];
            complex t = w * y[k + h / 2];
            y[k] = u + t;
            y[k + h / 2] = u - t;
            w = w * wn;
        }
    }
}
if(on == -1) {
    for(int i = 0; i < len; i++) {
        y[i].r /= len;
    }
}
}
void solve(int n) {
    for(len = 1; len < 2 * n; len <= 1);
    for(int i = 0; i < len; i++) {
        a[i] = 条件 ? complex(A[i], 0) : complex(0, 0);
        b[i] = 条件 ? complex(B[i], 0) : complex(0, 0);
    }
    fft(a, len, 1); fft(b, len, 1);
    for(int i = 0; i < len; i++) {
        a[i] = a[i] * b[i];
    }
    fft(a, len, -1);
    for(int i = 0; i < len; i++) {
        int t = a[i].r + 0.5;
        if(t) printf("[%d]%d\n", i, t);
    }
}
}

```

## 8. fwt

```

/*
复杂度  $O(n \log n)$ ,  $n$  为区间长度
 $n$  必须为 2 的  $n$  次幂
使用方法：
fwt(a, 0, n-1); fwt(b, 0, n-1);
for(int i=0; i<n-1; i++) a[i]=a[i]*b[i]%mod;
fwt(a, 0, n-1);
之后 a 数组就是答案
模数只是为了防止爆 long long
如果答案不会爆 long long, 可以不模数
*/
LL inv2 = power(2, mod - 2);
void fwt_xor(LL a[], int l, int r) {
    if (l == r) return;
    int mid = (l + r) >> 1;
    fwt_xor(a, l, mid);
    fwt_xor(a, mid + 1, r);
    int len = mid - l + 1;
    for (int i = l; i <= mid; ++i) {
        LL x1 = a[i];
        LL x2 = a[i + len];
        a[i] = (x1 + x2) % mod;
        a[i + len] = (x1 - x2 + mod) % mod;
    }
}

```

```

    }
}
void ifwt_xor(LL a[], int l, int r) {
    if (l == r) return;
    int mid = (l + r) >> 1;
    int len = mid - l + 1;
    for (int i = l; i <= mid; ++i) {
        LL y1 = a[i];
        LL y2 = a[i + len];
        a[i] = (y1 + y2) * inv2 % mod;
        a[i + len] = ((y1 - y2 + mod) % mod * inv2) % mod;
    }
    ifwt_xor(a, l, mid);
    ifwt_xor(a, mid + 1, r);
}

void fwt_and(LL a[], int l, int r) {
    if (l == r) return;
    int mid = (l + r) >> 1;
    fwt_and(a, l, mid);
    fwt_and(a, mid + 1, r);
    int len = mid - l + 1;
    for (int i = l; i <= mid; ++i) {
        LL x1 = a[i];
        LL x2 = a[i + len];
        a[i] = (x1 + x2) % mod;
        a[i + len] = x2 % mod;
    }
}
void ifwt_and(LL a[], int l, int r) {
    if (l == r) return;
    int mid = (l + r) >> 1;
    int len = mid - l + 1;
    for (int i = l; i <= mid; ++i) {
        LL y1 = a[i];
        LL y2 = a[i + len];
        a[i] = (y1 - y2 + mod) % mod;
        a[i + len] = y2 % mod;
    }
    ifwt_and(a, l, mid);
    ifwt_and(a, mid + 1, r);
}

void fwt_or(LL a[], int l, int r) {
    if (l == r) return;
    int mid = (l + r) >> 1;
    fwt_or(a, l, mid);
    fwt_or(a, mid + 1, r);
    int len = mid - l + 1;
    for (int i = l; i <= mid; ++i) {
        LL x1 = a[i];
        LL x2 = a[i + len];
        a[i] = x1 % mod;
        a[i + len] = (x2 + x1) % mod;
    }
}
void ifwt_or(LL a[], int l, int r) {
    if (l == r) return;
    int mid = (l + r) >> 1;
    int len = mid - l + 1;
    for (int i = l; i <= mid; ++i) {
        LL y1 = a[i];
        LL y2 = a[i + len];
    }
}

```

```

        a[i] = y1 % mod;
        a[i + len] = (y2 - y1 + mod) % mod;
    }
    ifwt_or(a, 1, mid);
    ifwt_or(a, mid + 1, r);
}

```

## 9. ntt

```

const int MX = 5e5;
const int g = 3;//3
const LL MOD = 40531930642382849LL;//(479<<21)+1

LL qp[40];
LL x1[MX], x2[MX];

LL multi(LL x, LL y, LL mod) {
    return (x * y - (LL)(x / (long double)mod * y + 1e-3) * mod + mod) % mod ;
}

LL power(LL x, LL y, LL P) {
    LL ans = 1;
    while(y > 0) {
        if(y & 1)ans = multi(ans, x, P) % P;
        x = multi(x, x, P) % P;
        y >>= 1;
    }
    return ans;
}

/*记得一定要init()*/
void init() {
    for(int i = 0; i < 33; i++) {
        int t = 1 << i;
        qp[i] = power(g, (MOD - 1) / t, MOD);
    }
}

void rader(LL F[], int len) {
    int j = len / 2;
    for(int i = 1; i < len - 1; i++) {
        if(i < j)swap(F[i], F[j]);
        int k = len / 2;
        while(j >= k) {
            j -= k;
            k >>= 1;
        }
        if(j < k)j += k;
    }
}

void ntt(LL F[], int len, int t) {
    int id = 0;
    rader(F, len);
    for(int h = 2; h <= len; h <= 1) {
        id++;
        for(int j = 0; j < len; j += h) {
            LL E = 1;
            for(int k = j; k < j + h / 2; k++) {
                LL u = F[k] % MOD;
                LL v = multi(E, F[k + h / 2], MOD);
                F[k] = (u + v) % MOD;
                F[k + h / 2] = ((u - v) + MOD) % MOD;
                E = multi(E, qp[id], MOD);
            }
        }
    }
}

```

```

    }
    if(t == -1) {
        for(int i = 1; i < len / 2; i++) swap(F[i], F[len - i]);
        LL inv = power(len, MOD - 2, MOD);
        for(int i = 0; i < len; i++) F[i] = multi(F[i] % MOD, inv, MOD);
    }
}

void solve(int n) {
    //len 为长度, len1 为 2 的幂的长度
    int len = n, len1 = 1;
    while(len1 < 2 * len) len1 *= 2;
    ntt(x1, len1, 1); ntt(x2, len1, 1);
    for(int i = 0; i < len1; i++) {
        x1[i] = multi(x1[i], x2[i], MOD);
    }
    ntt(x1, len1, -1);
}

```

## 10. Lucas 定理

```

int fac[mod + 7];
void init() {
    fac[0] = 1;
    for (int i = 1; i <= mod; ++i) fac[i] = (LL)fac[i - 1] * i % mod;
}

LL power(LL a, LL b) {
    LL x = a % mod, ret = 1;
    while (b) {
        if (b & 1) ret = ret * x % mod;
        x = x * x % mod;
        b >>= 1;
    }
    return ret;
}

LL C(int n, int m, int mod) {
    return m > n ? 0 : fac[n] * power((LL)fac[m] * fac[n - m], mod - 2) % mod;
}

LL Lucas(LL n, LL m, int mod) {
    return m ? (LL)C(n % mod, m % mod, mod) * Lucas(n / mod, m / mod, mod) % mod : 1;
}

```

## 11. 大质数判定

```

LL multi(LL a, LL b, LL mod) {
    LL ret = 0;
    while(b) {
        if(b & 1) ret = ret + a;
        if(ret >= mod) ret -= mod;

        a = a + a;
        if(a >= mod) a -= mod;
        b >>= 1;
    }
    return ret;
}

LL power(LL a, LL b, LL mod) {
    LL ret = 1;
    while(b) {
        if(b & 1) ret = multi(ret, a, mod);
        a = multi(a, a, mod);
        b >>= 1;
    }
}

```

```

    return ret;
}

bool Miller_Rabin(LL n) {
    LL u = n - 1, pre, x;
    int i, j, k = 0;
    if(n == 2 || n == 3 || n == 5 || n == 7 || n == 11) return true;
    if(n == 1 || (! (n % 2)) || (! (n % 3)) || (! (n % 5)) || (! (n % 7)) || (! (n % 11))) return
false;
    for(; !(u & 1); k++, u >>= 1);
    srand(time(NULL));
    for(i = 0; i < 5; i++) {
        x = rand() % (n - 2) + 2;
        x = power(x, u, n);
        pre = x;
        for(j = 0; j < k; j++) {
            x = multi(x, x, n);
            if(x == 1 && pre != 1 && pre != (n - 1))
                return false;
            pre = x;
        }
        if(x != 1) return false;
    }
    return true;
}

```

## 12.康托展开

```

int F[100];

void init() {
    F[0] = 1;
    for(int i = 1; i <= 9; i++) F[i] = F[i - 1] * i;
}

/*下标从 0 开始,返回值也从 0 开始*/
int Contor(int A[], int n) {
    int ret = 0;
    for(int i = 0; i < n; i++) {
        int cnt = 0;
        for(int j = i + 1; j < n; j++) {
            if(A[i] > A[j]) cnt++;
        }
        ret += F[n - i - 1] * cnt;
    }
    return ret;
}

```

## 13.扩展欧几里德

/\*可以得到  $x \geq \text{bound}$  时的  $x$  和  $y$ , 返回 true 表示有解  
 否则无解, 我只想问这个模板无脑调用有木有~  
 但是不同的题目特判不同, 有的地方记得还是特判, 比如  $a$  和  $b$  的正负和是否为 0~\*/

```

LL exgcd(LL a, LL b, LL &x, LL &y) {
    if(b == 0) {
        x = 1; y = 0;
        return a;
    }
    LL r = exgcd(b, a % b, x, y);
    LL t = y;
    y = x - a / b * y;
    x = t;
    return r;
}

```

```

}
bool solve(LL a, LL b, LL c, LL bound, LL &x, LL &y) {
    LL xx, yy, d = exgcd(a, b, xx, yy);
    if(c % d) return false;

    xx = xx * c / d; yy = yy * c / d;
    LL t = (bound - xx) * d / b;

    x = xx + b / d * t;
    if(x < bound) {
        t++;
        x = xx + b / d * t;
    }
    y = yy - a / d * t;
    return true;
}

```

## 14.欧拉函数

```

/*单个点的欧拉函数 O(sqrt(n))*/
LL euler(LL n) {
    LL ans = n;
    for(int i = 2; (LL)i * i <= n; i++) {
        if(n % i == 0) {
            ans -= ans / i;
            while(n % i == 0) n /= i;
        }
    }
    if(n > 1) ans -= ans / n;
    return ans;
}

/*线性筛 O(nlogn)*/
void phi_init() {
    memset(phi, 0, sizeof(phi));
    phi[1] = 1;
    for(int i = 2; i < MX; i++) if(!phi[i]) {
        for(int j = i; j < MX; j += i) {
            if(!phi[j]) phi[j] = j;
            phi[j] = phi[j] / i * (i - 1);
        }
    }
}

```

## 15.求组合数

利用递推公式

```

const int MX = 1000;
LL C[MX][MX];

C[0][0] = 1;
for(int i = 1; i < MX; i++) {
    C[i][0] = C[i][i] = 1;
    for(int j = 1; j < i; j++) {
        C[i][j] = (C[i-1][j-1] + C[i-1][j]) % mod;
    }
}

/*利用费马小定理*/
const int MX = 1000000 + 5;
const int mod = 1e9 + 7;

LL F[MX], invF[MX];

```



```

LL power(LL a, LL b) {
    LL ret = 1;
    while(b) {
        if(b & 1) ret = (ret * a) % mod;
        a = (a * a) % mod;
        b >>= 1;
    }
    return ret;
}

void init() {
    F[0] = 1;
    for(int i = 1; i < MX; i++) {
        F[i] = (F[i - 1] * i) % mod;
    }
    invF[MX - 1] = power(F[MX - 1], mod - 2);
    for(int i = MX - 2; i >= 0; i--) {
        invF[i] = invF[i + 1] * (i + 1) % mod;
    }
}

LL C(int n, int m) {
    if(n < 0 || m < 0 || m > n) return 0;
    if(m == 0 || m == n) return 1;
    return F[n] * invF[n - m] % mod * invF[m] % mod;
}

LL A(int n, int m) {
    if(n < 0 || m < 0 || m > n) return 0;
    return F[n] * invF[n - m] % mod;
}

```

## 16.高斯消元(浮点数)

```

const double exps = 1e-8;
typedef vector<double> vec;
typedef vector<vec> mat;

int dcmp(double x) {
    if(fabs(x) < exps) return 0;
    return x < 0 ? -1 : 1;
}

void guass(mat &A, int m, int n) {
    for(int i = 0; i < m; i++) {
        int pv = i, id;
        for(int j = 0; j <= n; j++) {
            for(int k = i + 1; k < m; k++) {
                if(fabs(A[k][j]) > fabs(A[pv][j])) {
                    pv = k;
                }
            }
            if(dcmp(A[pv][j])) break;
        }
        swap(A[i], A[pv]);

        for(id = 0; id <= n && !dcmp(A[i][id]); id++);
        if(id > n) return;

        for(int j = i + 1; j < m; j++) {
            if(!dcmp(A[j][id])) continue;

            double f = A[j][id] / A[i][id];

```

```

        for(int k = id + 1; k <= n; k++) A[j][k] -= A[i][k] * f;
        A[j][id] = 0;
    }
}
}
/*-1 无解, 0 多组解, 1 唯一解*/
int solve(mat &A) {
    int m = A.size(), n = A[0].size() - 1;
    guass(A, m, n);

    int r1 = 0, r2 = 0;
    for(int i = 0; i < m; i++) {
        bool sign = true;
        for(int j = 0; j <= n; j++) {
            if(dcmp(A[i][j])) {
                r2++;
                if(j < n) r1++;
                sign = false;
                break;
            }
        }
        if(sign) break;
    }

    if(r1 != r2) return -1;
    if(r1 == r2 && r1 != n) return 0;
    for(int i = n - 1; i >= 0; i--) {
        A[i][n] /= A[i][i];
        for(int j = i - 1; j >= 0; j--) A[j][n] -= A[i][n] * A[j][i];
    }
    return 1;
}

```

## 17.高斯消元(任意模数)

///以下代码是用高斯消元求同余方程组

```

ll exgcd(ll a, ll b, ll &x, ll &y) {
    if(!b) {
        x = 1;
        y = 0;
        return a;
    } else {
        ll r = exgcd(b, a % b, y, x);
        y -= x * (a / b);
        return r;
    }
}

ll lcm(ll a, ll b) {
    ll x = 0, y = 0;
    return a / exgcd(a, b, x, y) * b;
}

int A[MX][MX], free_x[MX], x[MX];
void Gauss(int n, int m) {
    int r, c;
    for(r = 0, c = 0; r < n && c < m; c++) {
        int maxr = r;
        for(int i = r + 1; i < n; i++) if(abs(A[i][c]) > abs(A[maxr][c])) maxr = i;
        if(maxr != r) for(int i = c; i <= m; i++) swap(A[r][i], A[maxr][i]);
        if(!A[r][c]) continue;
        for(int i = r + 1; i < n; i++) if(A[i][c]) {
            ///这里要保证运算都是整数, 所以要求最小公倍数
            ///范围不大可以用 int, int 运算更快
            ll d = lcm(A[i][c], A[r][c]);

```

```

        ll t1 = d / A[i][c], t2 = d / A[r][c];
        for(int j = c; j <= m; j++)
            A[i][j] = ((A[i][j] * t1 - A[r][j] * t2) % mod + mod) % mod;
    }
    r++;
}
for(int i = r; i < n; i++) if(A[i][m]) return ;
///这里保证是没有自由变元的情况下。
///有自由变元的时候,不一定是x[i] 对应 A[i][m]应该找那一行最开始的一列不为0的那个
for(int i = r - 1; i >= 0; i--) {
    x[i] = A[i][m];
    for(int j = i + 1; j < m; j++) {
        x[i] = ((x[i] - A[i][j] * x[j]) % mod + mod) % mod;
    }
    ll x1 = 0, y1 = 0;
    ///这里是用exgcd求逆元,也可以用费马小定理求,如果mod是素数
    ll d = exgcd(A[i][i], mod, x1, y1);
    x1 = ((x1 % mod) + mod) % mod;
    x[i] = x[i] * x1 % mod;
}
}
void Gauss_init() {
    memset(A, 0, sizeof(A));
    memset(free_x, 0, sizeof(free));
    memset(x, 0, sizeof(x));
}

```

## 18.高斯消元(xor)

```

int gauss(int equ, int var) {
    int max_r, col, k;
    for(k = 0, col = 0; k < equ && col < var; k++, col++) {
        max_r = k;
        for(int i = k + 1; i < equ; i++) {
            if(A[i][col] > A[max_r][col]) {
                max_r = i;
            }
        }
        if(A[max_r][col] == 0) {
            k--;
            continue;
        }
        if(max_r != k) {
            for(int j = col; j < var + 1; j++) {
                swap(A[k][j], A[max_r][j]);
            }
        }
        for(int i = k + 1; i < equ; i++) {
            if(A[i][col] != 0) {
                for(int j = col; j < var + 1; j++) {
                    A[i][j] ^= A[k][j];
                }
            }
        }
    }
    for(int i = k; i < equ; i++) {
        if(A[i][col] != 0) return -1;
    }
    if(k < var) return var - k;

    for(int i = var - 1; i >= 0; i--) {
        for(int j = i + 1; j < var; j++) {
            A[i][var] ^= (A[i][j] && A[j][var]);
        }
    }
}

```

```

    }
}
return 0;
}

```

## 19.凸包

```

struct Node {
    double x, y;
    bool operator<(const Node&b) const {
        if(x == b.x) return y < b.y;
        return x < b.x;
    }
} P[MX], R[MX];

double cross(Node a, Node b, Node c) {
    return ((b.x - a.x) * (c.y - a.y) - (c.x - a.x) * (b.y - a.y));
}

int convex(int n) {
    int m = 0, k;
    sort(P, P + n);
    for(int i = 0; i < n; i++) {
        while(m > 1 && cross(R[m - 1], P[i], R[m - 2]) <= 0) m--;
        R[m++] = P[i];
    }

    k = m;
    for(int i = n - 2; i >= 0; i--) {
        while(m > k && cross(R[m - 1], P[i], R[m - 2]) <= 0) m--;
        R[m++] = P[i];
    }
    if(n > 1) m--;
    return m;
}

```

## 20.极角排序

```

struct Point {
    LL x, y;
    Point(LL _x = 0, LL _y = 0) {
        x = _x; y = _y;
    }
    Point operator-(const Point &P) const {
        Point ret(x - P.x, y - P.y);
        return ret;
    }
    LL operator^(const Point &P) const {
        return x * P.y - y * P.x;
    }
    LL operator*(const Point &P) const {
        return x * P.x + y * P.y;
    }
} P[MX], W[MX], pc;
int n;

inline int get_seg(Point a) {
    if(a.x > 0 && a.y >= 0) return 1;
    if(a.x <= 0 && a.y > 0) return 2;
    if(a.x < 0 && a.y <= 0) return 3;
    return 4;
}
/*pc 为当前排序的中心*/

```

```
bool cmp(const Point &a, const Point &b) {
    if(a.x == pc.x && a.y == pc.y) return 1;
    if(b.x == pc.x && b.y == pc.y) return 0;
    int u = get_seg(a - pc), v = get_seg(b - pc);
    if(u == v) return ((a - pc) ^ (b - pc)) > 0;
    return u < v;
}
```

## 21.集合-莫比乌斯反演

```
/*
莫比乌斯反演复杂度  $O(n \log n)$ 
若原先为整个集合的所有子集的答案之和
通过反演后,就能得到本身的答案
*/
for(int i = 0; i < n; i++) {
    for(int s = 0; s < 1 << n; s++) {
        if(s >> i & 1) continue;
        dp[s | (1 << i)] -= dp[s];
    }
}
```

## 22.集合-莫比乌斯变换

```
/*
莫比乌斯变换复杂度  $O(n \log n)$ 
可以将子集的答案求和,加到自己上面
*/
for(int i = 0; i < n; i++) {
    for(int s = 0; s < 1 << n; s++) {
        if(s >> i & 1) continue;
        dp[s | (1 << i)] += dp[s];
    }
}
```

## 23. $O(3^n)$ 枚举子集

```
for(int i = s; i; i = (i - 1) & s) {

}
```

## 24.求 $n$ 以内质数个数

```
/*
使用前,先 init()
n 可以等于  $1e11$ 
lehmer_pi(n)求  $n$  以内质数的个数
*/
const int N = 5e6 + 2;
const int M = 7;
const int PM = 2 * 3 * 5 * 7 * 11 * 13 * 17;
struct prime_cnt {
    bool np[N];
    int p[N], pi[N];
    int getprime() {
        int cnt = 0;
        np[0] = np[1] = true;
        pi[0] = pi[1] = 0;
        for(int i = 2; i < N; ++i) {
            if(!np[i]) p[++cnt] = i;
            pi[i] = cnt;
        }
    }
}
```

```

        for(int j = 1; j <= cnt && i * p[j] < N; ++j) {
            np[i * p[j]] = true;
            if(i % p[j] == 0) break;
        }
    }
    return cnt;
}

int phi[PM + 1][M + 1], sz[M + 1];
void init() {
    getprime();
    sz[0] = 1;
    for(int i = 0; i <= PM; ++i) phi[i][0] = i;
    for(int i = 1; i <= M; ++i) {
        sz[i] = p[i] * sz[i - 1];
        for(int j = 1; j <= PM; ++j) {
            phi[j][i] = phi[j][i - 1] - phi[j / p[i]][i - 1];
        }
    }
}

int sqrt2(LL x) {
    LL r = (LL)sqrt(x - 0.1);
    while(r * r <= x) ++r;
    return int(r - 1);
}

int sqrt3(LL x) {
    LL r = (LL)cbrt(x - 0.1);
    while(r * r * r <= x) ++r;
    return int(r - 1);
}

LL getphi(LL x, int s) {
    if(s == 0) return x;
    if(s <= M) return phi[x % sz[s]][s] + (x / sz[s]) * phi[sz[s]][s];
    if(x <= p[s]*p[s]) return pi[x] - s + 1;
    if(x <= p[s]*p[s]*p[s] && x < N) {
        int s2x = pi[sqrt2(x)];
        LL ans = pi[x] - (s2x + s - 2) * (s2x - s + 1) / 2;
        for(int i = s + 1; i <= s2x; ++i) {
            ans += pi[x / p[i]];
        }
        return ans;
    }
    return getphi(x, s - 1) - getphi(x / p[s], s - 1);
}

LL getpi(LL x) {
    if(x < N) return pi[x];
    LL ans = getphi(x, pi[sqrt3(x)]) + pi[sqrt3(x)] - 1;
    for(int i = pi[sqrt3(x)] + 1, ed = pi[sqrt2(x)]; i <= ed; ++i) {
        ans -= getpi(x / p[i]) - i + 1;
    }
    return ans;
}

LL lehmer_pi(LL x) {
    if(x < N) return pi[x];
    int a = (int)lehmer_pi(sqrt2(sqrt2(x)));
    int b = (int)lehmer_pi(sqrt2(x));
    int c = (int)lehmer_pi(sqrt3(x));
    LL sum = getphi(x, a) + LL(b + a - 2) * (b - a + 1) / 2;
    for(int i = a + 1; i <= b; ++i) {
        LL w = x / p[i];
        sum -= lehmer_pi(w);
        if(i > c) continue;
        LL lim = lehmer_pi(sqrt2(w));
    }
}

```

```

        for (int j = i; j <= lim; j++) {
            sum -= lehmer_pi(w / p[j]) - (j - 1);
        }
    }
    return sum;
}
} prime;

```

## 25.simpson 定积分

```

double f(double x) {
    return 2 * x;
}

double simpson(double a, double b) {
    double c = a + (b - a) / 2;
    return (f(a) + 4 * f(c) + f(b)) * (b - a) / 6;
}

double asr(double a, double b, double eps, double A) {
    double c = a + (b - a) / 2;
    double L = simpson(a, c), R = simpson(c, b);
    if(fabs(L + R - A) <= 15 * eps) return L + R + (L + R - A) / 15.0;
    return asr(a, c, eps / 2, L) + asr(c, b, eps / 2, R);
}

/*求区间[a,b]的定积分,精度为 eps*/
double asr(double a, double b, double eps) {
    return asr(a, b, eps, simpson(a, b));
}

```

## 26.中国剩余定理

```

LL exgcd(LL a, LL b, LL &x, LL &y) {
    if(b == 0) {
        x = 1; y = 0;
        return a;
    }
    LL r = exgcd(b, a % b, x, y);
    LL t = y;
    y = x - a / b * y;
    x = t;
    return r;
}

LL multi(LL a, LL b, LL mod) {
    LL ret = 0;
    while(b) {
        if(b & 1) {
            ret = ret + a;
            if(ret >= mod) ret -= mod;
        }
        a = a + a;
        if(a >= mod) a -= mod;
        b >>= 1;
    }
    return ret;
}

/*x % m = a*/
LL ex_crt(int n, LL m[], LL a[]) {
    LL M = 1, d, y, x = 0;
    for(int i = 0; i < n; i++) M *= m[i];
    for(int i = 0; i < n; i++) {
        LL w = M / m[i];
        d = exgcd(m[i], w, d, y);
        y = (y % m[i] + m[i]) % m[i];
        x = ((x + multi(multi(a[i], w, M), y, M)) % M + M) % M;
    }
}

```

```

    }
    return x;
}

```

## 27.莫比乌斯函数筛法

```

bool vis[MX];
int prime[MX], mu[MX], tot;

void miu_init() {
    memset(vis, 0, sizeof(vis));
    mu[1] = 1; tot = 0;
    for(int i = 2; i < MX; i++) {
        if(!vis[i]) {
            prime[tot++] = i;
            mu[i] = -1;
        }
        for(int j = 0; j < tot; j++) {
            if(i * prime[j] >= MX) break;
            vis[i * prime[j]] = 1;
            if(i % prime[j] == 0) {
                mu[i * prime[j]] = 0;
                break;
            } else {
                mu[i * prime[j]] = -mu[i];
            }
        }
    }
}

```

## 28.可不互质的中国剩余定理

```

int n;
LL m[MX], r[MX];
LL exgcd(LL a, LL b, LL &x, LL &y) {
    if(b == 0) {
        x = 1; y = 0;
        return a;
    }
    LL r = exgcd(b, a % b, x, y);
    LL t = y;
    y = x - a / b * y;
    x = t;
    return r;
}
pair<LL, LL> ex_crt() {
    LL M = m[1], R = r[1], x, y, d;
    for(int i = 2; i <= n; i++) {
        d = exgcd(M, m[i], x, y);
        if((r[i] - R) % d) return make_pair(-1, -1);
        x = (r[i] - R) / d * x % (m[i] / d);
        R = R + x * M;
        M = M / d * m[i];
        R %= M;
    }
    R = R > 0 ? R : R + M;
    return make_pair(R, M);
}

```

## 29.排列组合总结

1) 球同，盒同，无空箱  
 $dp[n-m][m]$ ,  $dp$  同第 2 种情况,  $n \geq m$   
 $0, n < m$



2) 球同, 盒同, 允许空箱

$dp[n][m] = dp[n][m-1] + dp[n-m][m], n \geq m$

$dp[n][m] = dp[n][m-1], n < m$

边界  $dp[k][1] = 1, dp[1][k] = 1, dp[0][k] = 1$

3) 球同, 盒不同, 无空箱

$C(n-1, m-1), n \geq m$

$0, n < m$

4) 球同, 盒不同, 允许空箱  $C(n+m-1, m-1)$

5) 球不同, 盒相同, 无空箱 第二类斯特林数  $dp[n][m]$

$dp[n][m] = m * dp[n-1][m] + dp[n-1][m-1], 1 \leq m < n$

$dp[k][k] = 1, k \geq 0$

$dp[k][0] = 0, k \geq 1$

$dp[n][m] = 0, n < m$

6) 球不同, 盒相同, 允许空箱  $\sum dp[n][i], 1 \leq i \leq m$

7) 球不同, 盒不同, 无空箱  $dp[n][m] * fact[m]$

8) 球不同, 盒不同, 允许空箱  $power(m, n)$

### 30. 奇怪的公式

$GCD(a, b, c) = 1$ , 则必然有  $ax + by + cz = 1$ , 与扩展欧几里德的原理是一样的

若有  $GCD(x, n) = 1$ , 那么在一个圈中隔点报数必能全部报完

$x \leq 1e9$ , 则说明最多只会由 9 个质数组成

与  $n$  互质的所有数 ( $< n$ ) 的和为  $n * \phi(n) / 2$ , 要注意  $n = 1$  时

错排公式  $F[i] = (i-1) * (F[i-1] + F[i-2])$

其中边界  $F[1] = 0, F[2] = 1$

一些质数 999983

关于组合数和杨辉三角的性质

$C(m+1, n) / C(m, n) = (m+1) / (m+1-n)$

杨辉三角

```

      1
     1 1
    1 2 1
   1 3 3 1
  1 4 6 4 1
 1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
```

初始值为 1, 每次都是前面的数字之和, 假如做  $k$  次, 会得到一个矩阵

$C(k-1, k-1)$       0      0      0..

$C(k, k-1)$        $C(k-1, k-1)$       0      0..

$C(k+1, k-1)$        $C(k, k-1)$        $C(k-1, k-1)$       0..

.....

$C(k+n-2, k-1)$        $C(k+n-3, k-1)$ .....

卡特兰数定义:  $F(n) = C(2n, n) / (n+1)$

$F(n) = C(2n, n) - C(2n, n-1)$   
 $F(n) = F(0) * F(n-1) + F(1) * F(n-2) + \dots + F(n-1) * F(0), n \geq 1, F(0) = F(1) = 1$   
 递推公式  $F(n) = F(n-1) * (4n-2) / (n+1), F(1) = 1$

斐波那契数列

$F[1] = F[2] = 1, F[n] = F[n-1] + F[n-2]$   
 $F[n] = 1/\sqrt{5} * (\text{pow}((1+\sqrt{5})/2, n) - \text{pow}((1-\sqrt{5})/2, n));$   
 奇项求和  $F[1] + F[3] + F[5] + \dots + F[2n-1] = F[2n]$   
 偶项求和  $F[2] + F[4] + F[6] + \dots + F[2n] = F[2n+1] - 1$   
 全部求和  $F[1] + F[2] + \dots + F[n] = F[n+2] - 1$   
 平方求和  $F[1]^2 + F[2]^2 + \dots + F[n]^2 = F[n] * F[n+1]$   
 两倍关系  $F[2*n] / F[n] = F[n-1] + F[n+1]$   
 其他关系  $F[n-1] * F[n+1] - F[n] * F[n] = (-1)^n$   
 $F[1] + 2 * F[2] + 3 * F[3] + \dots + n * F[n] = n * F[n+2] - F[n+3] + 2$   
 $F[m]F[n] + F[m-1]F[n-1] = F[m+n-1]$   
 $F[m]F[n+1] + F[m-1]F[n] = F[m+n]$   
 前一项/后一项=黄金分割数  
 杨辉三角每行相加等于斐波那契数列  
 斐波那契数列个位数每 60 一循环

平方剩余：存在一个整数  $x$  使得  $x * x \% p = a$   
 如果  $p$  是奇质数，则  $a$  平方剩余当且仅当  $\text{power}(a, (p-1)/2, p) == 1$   
 且在  $1, 2, \dots, p-1$  中恰好有  $(p-1)/2$  个数是平方剩余的

对于一般的数论题，所以通常取一个比较小的质数  $p$ ，然后开始找规律

$F = (a + \sqrt{b})^n$   
 可以写成递推式  $F(n) = a * F(n-1) + (b + a * \sqrt{b}) * F(n-2)$

海伦公式

$p = (a+b+c)/2$   
 $S = \sqrt{p * (p-a) * (p-b) * (p-c)}$

求一个  $n$  个数  $A_i$ ，有多少个数与  $x$  互质：  
 对于  $A_i$ ，设  $t$  为  $A_i$  的约数，那么  $\text{cnt}[t]++$   
 枚举  $x$  的约数记为  $t$ ，然后求  $\sum \text{cnt}[t] * \text{miu}[t]$ ，即与  $x$  互质的个数  
 其中  $\text{miu}$  为莫比乌斯函数

## 数据结构

### 1. 左偏树

```

/*复杂度
取最小 O(1)
合并 O(logn)
这个是最小堆，求最大堆改 merge 即可
*/
const int MX = 1000 + 5;

struct Data {
    int l, r, key, dist;
} D[MX << 1];
int rear, root;

int lt_init() {
    rear = root = 0;
    D[0].dist = -1;
}

int lt_new(int _key = 0) {
    rear++;
    D[rear].l = D[rear].r = 0;
  
```

```

    D[rear].key = _key;
    D[rear].dist = 0;
    return rear;
}
int lt_merge(int r1, int r2) {
    if(!r1) return r2;
    if(!r2) return r1;
    if(D[r1].key > D[r2].key) {
        swap(r1, r2);
    }
    D[r1].r = lt_merge(D[r1].r, r2);
    if(D[D[r1].l].dist < D[D[r1].r].dist) {
        swap(D[r1].l, D[r1].r);
    }
    D[r1].dist = D[D[r1].r].dist + 1;
    return r1;
}
int lt_pop(int &rt) {
    int ret = D[rt].key;
    rt = lt_merge(D[rt].l, D[rt].r);
    return ret;
};
void lt_push(int &rt, int key) {
    rt = lt_merge(rt, lt_new(key));
}

/*使用的时候
lt_init();
lt_push(rt,1);
*/

```

## 2. ST 表

```

/*可以从 0 也可以从 1 开始,30 应该能使用 100w 以内的*/
int A[MX];
int MIN[MX][30], MAX[MX][30];

void RMQ_init(int n) {
    for(int i = 0; i < n + 1; i++) {
        MAX[i][0] = MIN[i][0] = A[i];
    }
    for(int j = 1; (1 << j) <= n + 1; j++) {
        for(int i = 0; i + (1 << j) - 1 < n + 1; i++) {
            MAX[i][j] = max(MAX[i][j - 1], MAX[i + (1 << (j - 1))][j - 1]);
            MIN[i][j] = min(MIN[i][j - 1], MIN[i + (1 << (j - 1))][j - 1]);
        }
    }
}

int RMQ_min(int L, int R) {
    int k = 0;
    while((1 << (k + 1)) <= R - L + 1) k++;
    return min(MIN[L][k], MIN[R - (1 << k) + 1][k]);
}

int RMQ_max(int L, int R) {
    int k = 0;
    while((1 << (k + 1)) <= R - L + 1) k++;
    return max(MAX[L][k], MAX[R - (1 << k) + 1][k]);
}

```

### 3. 二维 RMQ

```
/*
傻逼二维 RMQ 超级大常数
理论支持下标从 0 开始
n 是一维的大小, m 是二维的大小
P 是 MX 的 log 大小
*/
int n, m;
int dp[MX][MX][P][P];
void umax(int &a, int b) {
    a = max(a, b);
}
void RMQ_init() {
    for(int p = 0; (1 << p) <= n; p++) {
        for(int q = 0; (1 << q) <= m; q++) {
            int l1 = 1 << p, l2 = 1 << q;
            for(int i = 1; i + l1 - 1 <= n; i++) {
                for(int j = 1; j + l2 - 1 <= m; j++) {
                    if(!p && !q) continue;
                    int t1 = max(p - 1, 0), t2 = max(q - 1, 0);
                    umax(dp[i][j][p][q], dp[i][j][t1][t2]);
                    umax(dp[i][j][p][q], dp[i][j + l2 - (1 << t2)][t1][t2]);
                    umax(dp[i][j][p][q], dp[i + l1 - (1 << t1)][j][t1][t2]);
                    umax(dp[i][j][p][q], dp[i + l1 - (1 << t1)][j + l2 - (1 << t2)][t1][t2]);
                }
            }
        }
    }
}
int RMQ(int x1, int y1, int x2, int y2) {
    int l1 = x2 - x1 + 1, l2 = y2 - y1 + 1;
    int p = 0, q = 0, ret = 0;
    while((1 << (p + 1)) <= l1) p++;
    while((1 << (q + 1)) <= l2) q++;
    l1 = 1 << p, l2 = 1 << q;
    umax(ret, dp[x1][y1][p][q]);
    umax(ret, dp[x1][y2 - l2 + 1][p][q]);
    umax(ret, dp[x2 - l1 + 1][y1][p][q]);
    umax(ret, dp[x2 - l1 + 1][y2 - l2 + 1][p][q]);
    return ret;
}
```

### 4. 线段树缩空间

```
int ID(int l, int r) {
    return l + r | l != r;
}
```

### 5. 曼哈顿最小生成树

```
const int MS = 4000 + 5;
const int MID = 2000 + 2;
const int MX = 1e5 + 5;
const int INF = 0x3f3f3f3f;
#define lson l,m,rt<<1
#define rson m+1,r,rt<<1|1
int n, k, sz;
int MIN[MS << 2], id[MS << 2];
int P[MX];
int find(int x) {
    return P[x] == x ? x : (P[x] = find(P[x]));
}
```

```

}
struct point {
    int x, y, id;
    bool operator<(const point &P)const {
        if(x == P.x) return y - x < P.y - P.x;
        else return x < P.x;
    }
} A[MX];
struct Edge {
    int u, v, cost;
    bool operator<(const Edge &P)const {
        return cost < P.cost;
    }
} E[MX];
void push_up(int rt) {
    if(MIN[rt << 1] < MIN[rt]) {
        MIN[rt] = MIN[rt << 1];
        id[rt] = id[rt << 1];
    }
    if(MIN[rt << 1 | 1] < MIN[rt]) {
        MIN[rt] = MIN[rt << 1 | 1];
        id[rt] = id[rt << 1 | 1];
    }
}
void update(int p, int x, int uid, int l, int r, int rt) {
    if(l == r) {
        if(x < MIN[rt]) {
            MIN[rt] = x;
            id[rt] = uid;
        }
        return;
    }
    int m = (l + r) >> 1;
    if(p <= m) update(p, x, uid, lson);
    else update(p, x, uid, rson);
    push_up(rt);
}
PII query(int L, int R, int l, int r, int rt) {
    if(L <= l && r <= R) {
        return PII(MIN[rt], id[rt]);
    }
    int m = (l + r) >> 1; PII ret(INF, -1);
    if(L <= m) ret = min(ret, query(L, R, lson));
    if(R > m) ret = min(ret, query(L, R, rson));
    return ret;
}
/*读入到A数组中*/
void build() {
    sz = 0;
    for(int w = 0; w <= 3; w++) {
        if(w == 1 || w == 3) {
            for(int i = 1; i <= n; i++) {
                swap(A[i].x, A[i].y);
            }
        }
        if(w == 2) {
            for(int i = 1; i <= n; i++) {
                A[i].x = -A[i].x;
            }
        }
    }
    sort(A + 1, A + 1 + n);
    memset(MIN, INF, sizeof(MIN));
    for(int i = n; i >= 1; i--) {

```

```

        PII p = query(A[i].y - A[i].x + MID, MS - 1, 1, MS - 1, 1);
        if(p.first != INF) {
            sz++;
            E[sz].u = A[i].id; E[sz].v = p.second;
            E[sz].cost = p.first - A[i].x - A[i].y;
        }
        update(A[i].y - A[i].x + MID, A[i].x + A[i].y, A[i].id, 1, MS - 1, 1);
    }
}
}
int MST() {
    int cnt = 0, ret = 0;
    sort(E + 1, E + 1 + sz);
    for(int i = 1; i <= n; i++) P[i] = i;
    for(int i = 1; i <= sz; i++) {
        int u = E[i].u, v = E[i].v;
        int p1 = find(u), p2 = find(v);
        if(p1 != p2) {
            cnt++; P[p2] = p1;
            ret += E[i].cost;
        }
    }
}
}

```

## 6. 主席树

```

int A[MX], B[MX], rear;
int S[MX << 2], ls[MX << 2], rs[MX << 2], o[MX], sz;
void push_up(int rt) {
    S[rt] = S[ls[rt]] + S[rs[rt]];
}
void build(int l, int r, int &rt) {
    rt = ++sz;
    if(l == r) {
        S[rt] = 0;
        return;
    }
    int m = (l + r) >> 1;
    build(l, m, ls[rt]); build(m + 1, r, rs[rt]);
    push_up(rt);
}
void update(int pos, int l, int r, int pre, int &rt) {
    rt = ++sz;
    if(l == r) {
        S[rt] = S[pre] + 1;
        return;
    }
    int m = (l + r) >> 1;
    ls[rt] = ls[pre]; rs[rt] = rs[pre];
    if(pos <= m) update(pos, l, m, ls[pre], ls[rt]);
    else update(pos, m + 1, r, rs[pre], rs[rt]);
    push_up(rt);
}
int query(int k, int l, int r, int pre, int rt) {
    if(l == r) return l;
    int m = (l + r) >> 1, num = S[ls[rt]] - S[ls[pre]];
    if(k <= num) return query(k, l, m, ls[pre], ls[rt]);
    else return query(k - num, m + 1, r, rs[pre], rs[rt]);
}

```

## 7. KD 树-子矩阵查询修改

/\*通常关于曼哈顿距离的，都可以把图像旋转 45 度，之后就变成了矩阵了！\*/

```

struct Node {

```

```

    int xy[2];
    int minx, maxx;
    int miny, maxy;
    int f, id, ls, rs;
    int val, sum;
} A[MX];
int kd_cmp, d;
int xl, xr, yl, yr;

bool cmp(const Node &a, const Node &b) {
    return a.xy[kd_cmp] < b.xy[kd_cmp];
}
inline void umax(int &a, int b) {
    a = max(a, b);
}
inline void umin(int &a, int b) {
    a = min(a, b);
}
void push_up(int rt) {
    A[rt].minx = A[rt].maxx = A[rt].xy[0];
    A[rt].miny = A[rt].maxy = A[rt].xy[1];
    if(A[rt].ls) {
        umin(A[rt].minx, A[A[rt].ls].minx);
        umax(A[rt].maxx, A[A[rt].ls].maxx);
        umin(A[rt].miny, A[A[rt].ls].miny);
        umax(A[rt].maxy, A[A[rt].ls].maxy);
    }
    if(A[rt].rs) {
        umin(A[rt].minx, A[A[rt].rs].minx);
        umax(A[rt].maxx, A[A[rt].rs].maxx);
        umin(A[rt].miny, A[A[rt].rs].miny);
        umax(A[rt].maxy, A[A[rt].rs].maxy);
    }
}
/*build(1,n,0,0);*/
int build(int l, int r, int w, int fa) {
    int m = (l + r) >> 1; kd_cmp = w;
    nth_element(A + l, A + m, A + r + 1, cmp);
    Rank[A[m].id] = m;
    A[m].val = A[m].sum = 0; A[m].f = fa;
    A[m].ls = l != m ? build(l, m - 1, !w, m) : 0;
    A[m].rs = r != m ? build(m + 1, r, !w, m) : 0;
    push_up(m);
    return m;
}
int query(int rt) {
    if(A[rt].minx > xr || A[rt].maxx < xl || A[rt].miny > yr || A[rt].maxy < yl)
        return 0;
    if(xl <= A[rt].minx && A[rt].maxx <= xr && yl <= A[rt].miny && A[rt].maxy <= yr)
        return A[rt].sum;
    int ret = 0;
    if(xl <= A[rt].xy[0] && A[rt].xy[0] <= xr && yl <= A[rt].xy[1] && A[rt].xy[1] <= yr)
        ret += A[rt].val;
    if(A[rt].ls) ret += query(A[rt].ls);
    if(A[rt].rs) ret += query(A[rt].rs);
    return ret;
}
void update(int rt, int x) {
    A[rt].val += x;
    while(rt) {
        A[rt].sum += x;
        rt = A[rt].f;
    }
}

```

```
}
```

## 8. KD 树

```
struct Point {
    int xy[2], l, r, id;
    void read(int i) {
        id = i;
        scanf("%d%d", &xy[0], &xy[1]);
    }
} P[MX];
int cmpw; LL ans;
int idx[MX];

bool cmp(const Point &a, const Point &b) {
    return a.xy[cmpw] < b.xy[cmpw];
}

int build(int l, int r, int w) {
    int m = (l + r) >> 1; cmpw = w;
    nth_element(P + l, P + m, P + 1 + r, cmp);
    idx[P[m].id] = m;
    P[m].l = l != m ? build(l, m - 1, !w) : 0;
    P[m].r = r != m ? build(m + 1, r, !w) : 0;
    return m;
}

LL dist(LL x, LL y = 0) {
    return x * x + y * y;
}

void query(int rt, int w, LL x, LL y) {
    LL temp = dist(x - P[rt].xy[0], y - P[rt].xy[1]);
    if(temp) ans = min(ans, temp);
    if(P[rt].l && P[rt].r) {
        bool sign = !w ? (x <= P[rt].xy[0]) : (y <= P[rt].xy[1]);
        LL d = !w ? dist(x - P[rt].xy[0]) : dist(y - P[rt].xy[1]);
        query(sign ? P[rt].l : P[rt].r, !w, x, y);
        if(d < ans) query(sign ? P[rt].r : P[rt].l, !w, x, y);
    } else if(P[rt].l) query(P[rt].l, !w, x, y);
    else if(P[rt].r) query(P[rt].r, !w, x, y);
}

int rt = build(1, n, 0);
for(int i = 1; i <= n; i++) {
    ans = 1e18;
    query(rt, 0, P[idx[i]].xy[0], P[idx[i]].xy[1]);
    printf("%I64d\n", ans);
}
```

## 9. 离线第 k 大带修改

```
const int MX = 4e5 + 5;
const int INF = 0x3f3f3f3f;

int sum[MX], flag[MX], n, DFN;
int val[MX], ans[MX], tmp[MX];
struct Data {
    int op, id, x, y, k, cnt;
    Data() {}
    Data(int _op, int _id, int _x, int _y, int _k) {
        op = _op; id = _id;
        x = _x; y = _y; k = _k;
    }
} A[MX], T1[MX], T2[MX];
void add(int x, int y, int id) {
    for(int i = x; i <= n; i += i & -i) {
```



```

        if(flag[i] != id) flag[i] = id, sum[i] = 0;
        sum[i] += y;
    }
}
int ask(int x, int id) {
    int ret = 0;
    for(int i = x; i; i -= i & -i) {
        if(flag[i] == id) ret += sum[i];
    }
    return ret;
}
void solve(int L, int R, int l, int r) {
    if(L > R) return;
    if(l == r) {
        for(int i = L; i <= R; i++) {
            if(A[i].op == 3) ans[A[i].id] = 1;
        }
        return;
    }

    int m = (l + r) >> 1; DFN++;
    for(int i = L; i <= R; i++) {
        if(A[i].op == 1 && A[i].y <= m) add(A[i].x, 1, DFN);
        if(A[i].op == 2 && A[i].y <= m) add(A[i].x, -1, DFN);
        if(A[i].op == 3) tmp[i] = ask(A[i].y, DFN) - ask(A[i].x - 1, DFN);
    }
    int l1 = 0, l2 = 0;
    for(int i = L; i <= R; i++) {
        if(A[i].op == 3) {
            if(A[i].cnt + tmp[i] >= A[i].k) T1[++l1] = A[i];
            else A[i].cnt += tmp[i], T2[++l2] = A[i];
        } else if(A[i].y <= m) T1[++l1] = A[i];
        else T2[++l2] = A[i];
    }
    for(int i = 1; i <= l1; i++) A[L + i - 1] = T1[i];
    for(int i = 1; i <= l2; i++) A[L + l1 + i - 1] = T2[i];
    solve(L, L + l1 - 1, l, m);
    solve(L + l1, R, m + 1, r);
}

int main() {
    //FIN;
    while(~scanf("%d", &n)) {
        int sz = 0, Q, qsz = 0, Max = 0; DFN = 0;
        memset(sum, 0, sizeof(sum));

        for(int i = 1; i <= n; i++) {
            scanf("%d", &val[i]);
            A[++sz] = Data(1, -1, i, val[i], -1);
            Max = max(Max, val[i]);
        }
        scanf("%d", &Q);
        for(int i = 1; i <= Q; i++) {
            int op, x, y, k;
            scanf("%d%d%d", &op, &x, &y);
            if(op == 1) {
                A[++sz] = Data(2, -1, x, val[x], -1);
                A[++sz] = Data(1, -1, x, y, -1);
                Max = max(Max, y);
                val[x] = y;
            } else {
                qsz++;
                scanf("%d", &k);
            }
        }
    }
}

```

```

        A[++sz] = Data(3, qsz, x, y, k);
        A[sz].cnt = 0;
    }
}
solve(1, sz, 0, Max);

for(int i = 1; i <= qsz; i++) {
    printf("%d\n", ans[i]);
}
}
return 0;
}

```

## 10.非旋转 Treap

```

/*这个是敌兵布阵，非旋转 treap 主要是运用 Cut 和 Merge*/
const int MX = 1e5 + 5;
struct Node {
    Node *ch[2];
    int val, r, sum, sz;
} MEMO[MX], *null, *root;
int tot = 0;
void push_up(Node *o) {
    if(o == null) return;
    o->sz = o->ch[0]->sz + o->ch[1]->sz + 1;
    o->sum = o->ch[0]->sum + o->ch[1]->sum + o->val;
}
void New(Node *&o, int val = 0) {
    o = &MEMO[tot++];
    o->ch[0] = o->ch[1] = null;
    o->sz = 1; o->r = rand();
    o->sum = o->val = val;
}
void Cut(Node *o, Node *&a, Node *&b, int p) {
    if(o->sz <= p) a = o, b = null;
    else if(p == 0) a = null, b = o;
    else {
        if(o->ch[0]->sz >= p) {
            b = o;
            Cut(o->ch[0], a, b->ch[0], p);
        } else {
            a = o;
            Cut(o->ch[1], a->ch[1], b, p - o->ch[0]->sz - 1);
        }
        push_up(o);
    }
}
void Merge(Node *&o, Node *a, Node *b) {
    if(a == null) o = b;
    else if(b == null) o = a;
    else {
        if(a->r > b->r) {
            o = a;
            Merge(o->ch[1], a->ch[1], b);
            push_up(o);
        } else {
            o = b;
            Merge(o->ch[0], a, b->ch[0]);
            push_up(o);
        }
    }
}
}

```

```

void Init() {
    srand(time(NULL));
    tot = 0;
    New(null);
    null->sz = 0;
    root = null;
}
void Insert(int p, int x) {
    Node *a, *b, *c;
    Cut(root, a, b, p);
    New(c, x);
    Merge(a, a, c);
    Merge(root, a, b);
}
void Update(int p, int x) {
    Node *a, *b, *c;
    Cut(root, a, b, p - 1);
    Cut(b, b, c, 1);
    b->val += x;
    push_up(b);
    Merge(b, b, c);
    Merge(root, a, b);
}
int Query(int L, int R) {
    Node *a, *b, *c;
    Cut(root, a, b, R);
    Cut(a, a, c, L - 1);
    int ans = c->sum;
    Merge(a, a, c);
    Merge(root, a, b);
    return ans;
}
int main() {
    //FIN;
    int T, n, ansk = 0;
    scanf("%d", &T);
    while(T--) {
        Init();
        printf("Case %d:\n", ++ansk);
        scanf("%d", &n);
        for(int i = 1; i <= n; i++) {
            int t; scanf("%d", &t);
            Insert(i - 1, t);
        }

        char op[10]; int x, y;
        while(scanf("%s", op), op[0] != 'E') {
            scanf("%d%d", &x, &y);
            if(op[0] == 'Q') printf("%d\n", Query(x, y));
            else if(op[0] == 'A') Update(x, y);
            else Update(x, -y);
        }
    }
    return 0;
}

```

## 11.平衡堆

```

int rear;
int S[MX << 2];
/*这个是最小堆*/
void push(int x) {

```

```

    int now = ++rear, pre = now >> 1;
    S[now] = x;
    while(pre && S[pre] > S[now]) {
        swap(S[now], S[pre]);
        now = pre, pre = now >> 1;
    }
}

void pop() {
    S[1] = S[rear--];

    int now = 1;
    while((now << 1) <= rear) {
        int lt = now << 1, rt = now << 1 | 1;
        if(rt <= rear) {
            if(S[lt] >= S[now] && S[rt] >= S[now]) break;
            if(S[now] >= S[lt] && S[rt] >= S[lt]) swap(S[now], S[lt]), now = lt;
            else swap(S[now], S[rt]), now = rt;
        } else {
            if(S[lt] > S[now]) break;
            swap(S[now], S[lt]), now = lt;
        }
    }
}
}

```

## 12.莫队算法

```

const int MX = 5e4 + 5;
const int MP = 1e6 + 5;
const int MQ = 2e5 + 5;

int n, unit, Qt;
LL ans[MQ];
int vis[MP], A[MX];

struct Que {
    int L, R, id;
    bool operator<(const Que &b)const {
        if(L / unit == b.L / unit) {
            if(R == b.R) return L < b.L;
            return R < b.R;
        }
        return L / unit < b.L / unit;
    }
} Q[MQ];

void solve() {
    LL sum = 0;
    int L = 1, R = 0, c = 1;
    while(c <= Qt) {
        while(Q[c].L < L) {
            vis[A[--L]]++;
            if(vis[A[L]] == 1) {
                sum += A[L];
            }
        }
        while(Q[c].R > R) {
            vis[A[++R]]++;
            if(vis[A[R]] == 1) {
                sum += A[R];
            }
        }
        while(Q[c].L > L) {

```

```

        vis[A[L]]--;
        if(!vis[A[L]]) {
            sum -= A[L];
        }
        L++;
    }
    while(Q[c].R < R) {
        vis[A[R]]--;
        if(!vis[A[R]]) {
            sum -= A[R];
        }
        R--;
    }
    ans[Q[c++].id] = sum;
}
}

int main() {
    int T;
    scanf("%d", &T);
    while(T--) {
        memset(vis, 0, sizeof(vis));
        scanf("%d", &n);
        unit = sqrt(n + 0.5);

        for(int i = 1; i <= n; i++) {
            scanf("%d", &A[i]);
        }

        scanf("%d", &Qt);
        for(int i = 1; i <= Qt; i++) {
            scanf("%d%d", &Q[i].L, &Q[i].R);
            Q[i].id = i;
        }
        sort(Q + 1, Q + 1 + Qt);

        solve();
        for(int i = 1; i <= Qt; i++) {
            printf("%I64d\n", ans[i]);
        }
    }
    return 0;
}

```

### 13.表达式树

```

const int MX = 1e5 + 5;

char op[MX];
int lch[MX], rch[MX], s[MX], r;

int build(char *S, int L, int R) {
    int c[] = { -1, -1}, p = 0, u;

    int sum = 0, sign = true;
    for(int i = L; i <= R; i++) {
        if(isdigit(S[i])) sum = sum * 10 + S[i] - '0';
        else {
            sign = false;
            break;
        }
    }
}

```

```

    if(sign) {
        u = ++r;
        op[u] = '.';
        s[u] = sum;
        return u;
    }

    for(int i = L; i <= R; i++) {
        switch(S[i]) {
            case '(': p++; break;
            case ')': p--; break;
            case '+': case '-': if(!p) c[0] = i; break;
            case '*': case '/': if(!p) c[1] = i; break;
        }
    }

    if(c[0] < 0) c[0] = c[1];
    if(c[0] < 0) u = build(S, L + 1, R - 1);
    else {
        u = ++r;
        op[u] = S[c[0]];
        lch[u] = build(S, L, c[0] - 1);
        rch[u] = build(S, c[0] + 1, R);
    }
    return u;
}

double solve(int u) {
    if(op[u] == '.') return s[u];
    double al = solve(lch[u]), ar = solve(rch[u]);
    switch(op[u]) {
        case '+': return al + ar;
        case '-': return al - ar;
        case '*': return al * ar;
        case '/': return al / ar;
    }
}

```

## 14.树链剖分

```

/*点更新*/
int fa[MX], top[MX], siz[MX], son[MX], dep[MX], id[MX], rear;
/*fa 父节点, top 重链开头起点, siz 子树大小, son 重儿子, dep 深度, id 新编号*/

/*第一次 DFS 找到重边, 并维护好 siz, son, fa, dep*/
void DFS1(int u, int f, int d) {
    fa[u] = f; dep[u] = d;
    son[u] = 0; siz[u] = 1;
    for(int i = Head[u]; ~i; i = E[i].nxt) {
        int v = E[i].v;
        if(v == f) continue;
        DFS1(v, u, d + 1);
        siz[u] += siz[v];
        if(siz[son[u]] < siz[v]) {
            son[u] = v;
        }
    }
}

/*将重边编号好, 维护 id*/
void DFS2(int u, int tp) {
    top[u] = tp;
    id[u] = ++rear;
    if(son[u]) DFS2(son[u], tp);
}

```

```

        for(int i = Head[u]; ~i; i = E[i].nxt) {
            int v = E[i].v;
            if(v == fa[u] || v == son[u]) continue;
            DFS2(v, v);
        }
    }
    /*用来给点编号, 以及建立线段树, 要用 id 编号建树*/
    void HLD_presolve() {
        rear = 0;
        DFS1(1, 0, 1);
        DFS2(1, 1);
        for(int i = 1; i <= rear; i++) {
            A[id[i]] = B[i];
        }
        build(1, rear, 1);
    }
    /*修改, 要注意使用 id 编号修改*/
    void HLD_update(int x, int d) {
        update(id[x], d, 1, rear, 1);
    }
    /*路径查询*/
    int HLD_query(int u, int v) {
        int tp1 = top[u], tp2 = top[v];
        int sum = 0;
        while(tp1 != tp2) {
            if(dep[tp1] < dep[tp2]) {
                swap(u, v);
                swap(tp1, tp2);
            }
            sum += query(id[tp1], id[u], 1, rear, 1);
            u = fa[tp1]; tp1 = top[u];
        }

        if(dep[u] > dep[v]) swap(u, v);
        sum += query(id[u], id[v], 1, rear, 1);
        return sum;
    }

    /*边更新*/

    /*节点 1 不使用, 建树要小心
    一般边使用更深的那个点的 id 编号来表示
    */
    void HLD_presolve() {
        rear = 0;
        DFS1(1, 0, 1);
        DFS2(1, 1);
        for(int i = 0; i < 2 * (rear - 1); i += 2) {
            int u = E[i].u, v = E[i].v;
            if(dep[u] < dep[v]) swap(u, v);
            A[id[u]] = E[i].cost;
        }
        A[1] = -INF;
        build(1, rear, 1);
    }
    /*找到对应边的更深的点的 id 编号*/
    void HLD_update(int x, int d) {
        x = (x - 1) * 2;
        int u = E[x].u, v = E[x].v;
        if(dep[u] < dep[v]) swap(u, v);
        update(id[u], d, 1, rear, 1);
    }
}

```

```

/*注意最后一个查询与单点更新的区别以及 u==v 就需要返回 x*/
int HLD_query(int u, int v) {
    int tp1 = top[u], tp2 = top[v], ans = -INF;
    while(tp1 != tp2) {
        if(dep[tp1] < dep[tp2]) {
            swap(u, v);
            swap(tp1, tp2);
        }
        ans = max(ans, query(id[tp1], id[u], 1, rear, 1));
        u = fa[tp1]; tp1 = top[u];
    }
    if(u == v) return ans;
    if(dep[u] > dep[v]) swap(u, v);
    ans = max(ans, query(id[son[u]], id[v], 1, rear, 1));
    return ans;
}

/*路径合并更新*/
int HLD_query(int u, int v) {
    int tp1 = top[u], tp2 = top[v], ret = 0;
    int lastla[2] = { -1, -1}, lastra[2] = { -1, -1}, la[2] = { -1, -1}, ra[2] = { -1, -1};
    while(tp1 != tp2) {
        if(dep[tp1] >= dep[tp2]) {
            ret += query(id[tp1], id[u], 1, rear, 1, la[0], ra[0]);
            if(ra[0] == lastla[0]) ret--;
            lastla[0] = la[0]; lastra[0] = ra[0];
            u = fa[tp1]; tp1 = top[u];
        } else {
            ret += query(id[tp2], id[v], 1, rear, 1, la[1], ra[1]);
            if(ra[1] == lastla[1]) ret--;
            lastla[1] = la[1]; lastra[1] = ra[1];
            v = fa[tp2]; tp2 = top[v];
        }
    }
    if(dep[u] <= dep[v]) {
        ret += query(id[u], id[v], 1, rear, 1, la[1], ra[1]);
        if(ra[1] == lastla[1]) ret--;
        if(la[1] == lastla[0]) ret--;
    } else {
        ret += query(id[v], id[u], 1, rear, 1, la[0], ra[0]);
        if(ra[0] == lastla[0]) ret--;
        if(la[0] == lastla[1]) ret--;
    }
    return ret;
}

```

## 15.线段树扫描线

```

/*基本模板*/
int const MX = 1e3 + 5;

int rear, cnt[MX << 2];
double A[MX], S[MX << 2];

struct Que {
    int d;
    double top, L, R;
    Que() {}
    Que(double _top, double _L, double _R, int _d) {
        top = _top; L = _L; R = _R; d = _d;
    }
    bool operator<(const Que &b)const {
        return top < b.top;
    }
}

```



```

    }
} Q[MX];

int BS(double x) {
    int L = 1, R = rear, m;
    while(L <= R) {
        m = (L + R) >> 1;
        if(A[m] == x) return m;
        if(A[m] > x) R = m - 1;
        else L = m + 1;
    }
    return -1;
}

void push_up(int l, int r, int rt) {
    if(cnt[rt]) S[rt] = A[r + 1] - A[l];
    else if(l == r) S[rt] = 0;
    else S[rt] = S[rt << 1] + S[rt << 1 | 1];
}

void update(int L, int R, int d, int l, int r, int rt) {
    if(L <= l && r <= R) {
        cnt[rt] += d;
        push_up(l, r, rt);
        return;
    }

    int m = (l + r) >> 1;
    if(L <= m) update(L, R, d, lson);
    if(R > m) update(L, R, d, rson);
    push_up(l, r, rt);
}

int main() {
    int n, ans = 0;
    //freopen("input.txt", "r", stdin);
    while(~scanf("%d", &n), n) {
        rear = 0;
        memset(cnt, 0, sizeof(cnt));
        memset(S, 0, sizeof(S));

        for(int i = 1; i <= n; i++) {
            double x1, y1, x2, y2;
            scanf("%lf%lf%lf%lf", &x1, &y1, &x2, &y2);
            Q[i] = Que(y1, x1, x2, 1);
            Q[i + n] = Que(y2, x1, x2, -1);

            A[++rear] = x1; A[++rear] = x2;
        }
        sort(Q + 1, Q + 1 + 2 * n);
        sort(A + 1, A + 1 + rear);
        rear = unique(A + 1, A + 1 + rear) - A - 1;

        double ans = 0, last = 0;
        for(int i = 1; i <= 2 * n; i++) {
            ans += (Q[i].top - last) * S[1];
            update(BS(Q[i].L), BS(Q[i].R) - 1, Q[i].d, root);
            last = Q[i].top;
        }
        printf("Test case #%d\n", ++ans);
        printf("Total explored area: %.2lf\n\n", ans);
    }
    return 0;
}

```

```

}

/*只覆盖一次*/
void push_up(int l, int r, int rt) {
    if(cnt[rt]) {
        S1[rt] = A[r + 1] - A[1];
        if(cnt[rt] == 1) S2[rt] = S1[rt] - S1[rt << 1] - S1[rt << 1 | 1];
        else S2[rt] = 0;
    } else if(l == r) S1[rt] = S2[rt] = 0;
    else {
        S1[rt] = S1[rt << 1] + S1[rt << 1 | 1];
        S2[rt] = S2[rt << 1] + S2[rt << 1 | 1];
    }
}

/*覆盖次数>=2*/
void push_up(int l, int r, int rt) {
    if(cnt[rt]) {
        S1[rt] = A[r + 1] - A[1];
        if(cnt[rt] == 1) {
            S2[rt] = S1[rt << 1] + S1[rt << 1 | 1];
        } else S2[rt] = S1[rt];
    } else if(l == r) S1[rt] = S2[rt] = 0;
    else {
        S1[rt] = S1[rt << 1] + S1[rt << 1 | 1];
        S2[rt] = S2[rt << 1] + S2[rt << 1 | 1];
    }
}

/*按优先级覆盖*/
void push_up(int l, int r, int rt) {
    if(cnt[rt][3]) {
        S[rt][1] = S[rt][2] = 0;
        S[rt][3] = A[r + 1] - A[1];
    } else if(cnt[rt][2]) {
        S[rt][3] = S[rt << 1][3] + S[rt << 1 | 1][3];
        S[rt][2] = A[r + 1] - A[1] - S[rt][3];
        S[rt][1] = 0;
    } else if(cnt[rt][1]) {
        S[rt][3] = S[rt << 1][3] + S[rt << 1 | 1][3];
        S[rt][2] = S[rt << 1][2] + S[rt << 1 | 1][2];
        S[rt][1] = A[r + 1] - A[1] - S[rt][3] - S[rt][2];
    } else if(l == r) S[rt][1] = S[rt][2] = S[rt][3] = 0;
    else {
        S[rt][1] = S[rt << 1][1] + S[rt << 1 | 1][1];
        S[rt][2] = S[rt << 1][2] + S[rt << 1 | 1][2];
        S[rt][3] = S[rt << 1][3] + S[rt << 1 | 1][3];
    }
}

```

## 16.树状数组

```

struct BIT {
    int cid[MX], tim;
    int cnt[MX], n;
    void init(int _n) {
        n = _n; tim++;
    }
    void clear() {
        tim++;
    }
    void update(int p, int x) {
        for(; p <= n; p += p & -p) {

```

```

        if(cid[p] != tim) {
            cid[p] = tim;
            cnt[p] = 0;
        }
        cnt[p] += x;
    }
}
int sum(int p) {
    int ret = 0;
    for(; p; p -= p & -p) {
        if(cid[p] == tim) {
            ret += cnt[p];
        }
    }
    return ret;
}
int query(int l, int r) {
    if(l > r) return 0;
    return sum(r) - sum(l - 1);
}
} bit;

```

## 17.树状数组第 k 大

```

int siz[MX], n;
void add(int x) {
    for(; x <= n; x += x & -x) {
        siz[x]++;
    }
}
int kth(int k) {
    int cur = 0;
    for(int i = 1 << 20; i; i >>= 1) {
        if(cur + i <= n && k - siz[cur + i] > 0) {
            k -= siz[cur + i];
            cur += i;
        }
    }
    return cur + 1;
}

```

## 18.Splay

```

int size[MX];
int num[MX], col[MX], n, m;
int son[MX][2], fa[MX], root, sz;
void Link(int x, int y, int c) {
    fa[x] = y; son[y][c] = x;
}
void push_up(int rt) {
    size[rt] = size[son[rt][0]] + size[son[rt][1]] + 1;
}
void push_down(int rt) {
    if(col[rt]) {
        col[son[rt][0]] ^= 1;
        col[son[rt][1]] ^= 1;
        swap(son[rt][0], son[rt][1]);
        col[rt] = 0;
    }
}
void Rotate(int x, int c) {
    int y = fa[x];
    push_down(y); push_down(x);
    Link(x, fa[y], son[fa[y]][1] == y);
}

```

```

    Link(son[x][!c], y, c);
    Link(y, x, !c);
    push_up(y);
}
/*把节点 x 旋转到 g 的下面*/
void Splay(int x, int g) {
    push_down(x);
    while(fa[x] != g) {
        int y = fa[x], cx = son[y][1] == x, cy = son[fa[y]][1] == y;
        if(fa[y] == g) Rotate(x, cx);
        else {
            if(cx == cy) Rotate(y, cy);
            else Rotate(x, cx);
            Rotate(x, cy);
        }
    }
    push_up(x);
    if(!g) root = x;
}
void NewNode(int f, int &rt) {
    rt = ++sz;
    fa[rt] = f, size[rt] = 1;
    son[rt][0] = son[rt][1] = col[rt] = 0;
}
/*把第 k 个找出来, 放到 g 的下面*/
int Select(int k, int g) {
    int rt = root;
    while(size[son[rt][0]] != k) {
        if(size[son[rt][0]] > k) rt = son[rt][0];
        else k -= size[son[rt][0]] + 1, rt = son[rt][1];
        push_down(rt);
    }
    Splay(rt, g);
    return rt;
}
void Build(int l, int r, int &rt, int f) {
    if(l > r) return;
    int m = (l + r) >> 1, t;
    NewNode(f, rt); num[rt] = m;
    Build(l, m - 1, son[rt][0], rt);
    Build(m + 1, r, son[rt][1], rt);
    push_up(rt);
}
void Prepare(int n) {
    sz = 0;
    NewNode(0, root); num[1] = 0;
    NewNode(root, son[root][1]); num[2] = 0;
    Build(1, n, son[2][0], 2);
    Splay(3, 0);
}
void Print(int rt, int &DFN){
    if(!rt) return;
    push_down(rt);
    Print(son[rt][0], DFN);
    if(num[rt]) printf("%d%c", num[rt], ++DFN == n ? '\n' : ' ');
    Print(son[rt][1], DFN);
}
void Flip(int l, int r){
    Select(l - 1, 0);
    Select(r + 1, root);
    col[son[son[root][1]][0]] ^= 1;
}
/*剪断[a,b]放到 c 后面*/

```

```

void Cut(int a, int b, int c){
    Select(a - 1, 0);
    Select(b + 1, root);
    int w = son[son[root][1]][0];
    son[son[root][1]][0] = 0;
    Splay(son[root][1], 0);
    Select(c, 0);
    Select(c + 1, root);
    son[son[root][1]][0] = w;
    Splay(son[root][1], 0);
}
/*平衡树操作*/

void NewNode(int f, int x, int &rt) {
    rt = ++sz;
    fa[rt] = f, size[rt] = 1;
    son[rt][0] = son[rt][1] = 0;
    num[rt] = x;
}
int Kth(int k) {
    int rt = root;
    while(size[son[rt][0]] != k) {
        if(size[son[rt][0]] > k) rt = son[rt][0];
        else k -= size[son[rt][0]] + 1, rt = son[rt][1];
    }
    Splay(rt, 0);
    return num[rt];
}
void Prepare(int n) {
    sz = 0;
    NewNode(0, -INF, root);
    NewNode(root, INF, son[root][1]);
    push_up(root);
}
void Insert(int x) {
    int rt = root;
    while(true) {
        int nxt = x > num[rt];
        if(!son[rt][nxt]) {
            NewNode(rt, x, son[rt][nxt]);
            Splay(sz, 0); return;
        }
        rt = son[rt][nxt];
    }
}

```

## 19.DLX 覆盖

```

/*精确覆盖*/
struct DLX {
    int m, n;
    int H[MX], S[MX];
    int Row[MN], Col[MN], rear;
    int L[MN], R[MN], U[MN], D[MN];

    void Init(int _m, int _n) {
        m = _m; n = _n;
        rear = n;
        for(int i = 0; i <= n; i++) {
            S[i] = 0;
            L[i] = i - 1;
            R[i] = i + 1;
            U[i] = D[i] = i;
        }
    }
}

```

```

    }
    L[0] = n; R[n] = 0;
    for(int i = 1; i <= m; i++) {
        H[i] = -1;
    }
}

void Link(int r, int c) {
    int rt = ++rear;
    Row[rt] = r; Col[rt] = c; S[c]++;

    D[rt] = D[c]; U[D[c]] = rt;
    U[rt] = c; D[c] = rt;
    if(H[r] == -1) {
        H[r] = L[rt] = R[rt] = rt;
    } else {
        int id = H[r];
        R[rt] = R[id]; L[R[id]] = rt;
        L[rt] = id; R[id] = rt;
    }
}

void Remove(int c) {
    R[L[c]] = R[c]; L[R[c]] = L[c];
    for(int i = D[c]; i != c; i = D[i]) {
        for(int j = R[i]; j != i; j = R[j]) {
            D[U[j]] = D[j]; U[D[j]] = U[j];
            S[Col[j]]--;
        }
    }
}

void Resume(int c) {
    for(int i = U[c]; i != c; i = U[i]) {
        for(int j = L[i]; j != i; j = L[j]) {
            D[U[j]] = U[D[j]] = j;
            S[Col[j]]++;
        }
    }
    R[L[c]] = L[R[c]] = c;
}

bool Dance(int cnt) {
    if(R[0] == 0) return true;

    int c = R[0];
    for(int i = R[0]; i != 0; i = R[i]) {
        if(S[i] < S[c]) c = i;
    }

    Remove(c);
    for(int i = D[c]; i != c; i = D[i]) {
        for(int j = R[i]; j != i; j = R[j]) Remove(Col[j]);

        int r = Row[i];
        /*保存方案*/
        if(Dance(cnt + 1)) return true;

        for(int j = L[i]; j != i; j = L[j]) Resume(Col[j]);
    }
    Resume(c);
    return false;
}

```

```

} G;

/*重复覆盖*/
void Remove(int c) {
    for(int i = D[c]; i != c; i = D[i]) {
        R[L[i]] = R[i]; L[R[i]] = L[i];
    }
}

void Resume(int c) {
    for(int i = U[c]; i != c; i = U[i]) {
        R[L[i]] = L[R[i]] = i;
    }
}

int h() {
    int ret = 0;
    memset(vis, 0, sizeof(vis));
    for(int c = R[0]; c != 0; c = R[c]) {
        if(!vis[c]) {
            ret++;
            vis[c] = 1;
            for(int i = D[c]; i != c; i = D[i]) {
                for(int j = R[i]; j != i; j = R[j]) {
                    vis[Col[j]] = 1;
                }
            }
        }
    }
    return ret;
}

void Dance(int cnt) {
    if(cnt + h() >= ans) return;
    if(R[0] == 0) {
        ans = min(ans, cnt);
        return;
    }

    int c = R[0];
    for(int i = R[0]; i != 0; i = R[i]) {
        if(S[i] < S[c]) c = i;
    }

    for(int i = D[c]; i != c; i = D[i]) {
        Remove(i);
        for(int j = R[i]; j != i; j = R[j]) Remove(j);
        Dance(cnt + 1);
        for(int j = L[i]; j != i; j = L[j]) Resume(j);
        Resume(i);
    }
}

```

## 动态规划

### 1. TSP

//w是距离,n是除了起点以外的数量,0为原点

```

int TSP() {
    memset(dp, 0x3f, sizeof(dp));

```

```

for(int S = 0; S <= (1 << n) - 1; S++) {
    for(int i = 1; i <= n; i++) {
        if(S & (1 << (i - 1))) {
            if(S == (1 << (i - 1))) dp[i][S] = w[0][i];
            else for(int j = 1; j <= n; j++) {
                if(S & (1 << (j - 1)) && j != i) {
                    dp[i][S] = min(dp[i][S], dp[j][S ^ (1 << (i - 1))] + w[j][i]);
                }
            }
        }
    }
}

int ret = INF;
for(int i = 1; i <= n; i++) {
    ret = min(ret, dp[i][(1 << n) - 1] + w[0][i]);
}

/*
若不需要回到起点，只需要全部走完，那么直接这样写
int ret=INF;
for(int i=1;i<=n;i++){
    ret=min(ret,dp[i][(1<n)-1]);
}
也就不用需要加上了那w[0][i]而已
*/
return ret;
}

```

## 2. 四边形不等式优化的石子合并

```

S[0] = 0;
for(int i = 1; i <= n; i++) {
    scanf("%d", &t);
    dp[i][i] = 0;
    K[i][i] = i;
    S[i] = S[i - 1] + t;
}

for(int l = 2; l <= n; l++) {
    for(int i = 1; i <= n - l + 1; i++) {
        for(int j = K[i][i + l - 1]; j <= K[i + 1][i + l - 1]; j++) {
            int temp = dp[i][j] + dp[j + 1][i + l - 1] + S[i + l - 1] - S[i - 1];
            if(temp < dp[i][i + l - 1]) {
                dp[i][i + l - 1] = temp;
                K[i][i + l - 1] = j;
            }
        }
    }
}

printf("%d\n", dp[1][n]);

```

## 3. 斜率优化(凸包)

如果最后的表达式中，得到  $k > s$ ， $k$  表示斜率， $s$  为某个数  
那么我们就维护上凸包。

从左往右的上凸包

```

struct Point {
    LL x, y;
    Point() {}
    Point(LL _x, LL _y) {

```



```

        x = _x; y = _y;
    }
    Point operator-(const Point &P)const {
        return Point(x - P.x, y - P.y);
    }
    LL operator*(const Point &P)const {
        return x * P.y - y * P.x;
    }
} P[MX], W[MX];
LL A[MX];
int n, sz;
LL solve() {
    LL ret = 0; sz = 0;
    for(int i = 1; i <= n; i++) {
        while(sz >= 2 && (P[i] - W[sz]) * (W[sz] - W[sz - 1]) <= 0) sz--;
        W[++sz] = P[i];
        int l = 1, r = sz, m1, m2;
        while(l < r) {
            m1 = (2 * l + r) / 3;
            m2 = (l + 2 * r + 2) / 3;
            if(f(i, W[m1].x) < f(i, W[m2].x)) l = m1 + 1;
            else r = m2 - 1;
        }
        ret = max(ret, f(i, W[l].x));
    }
}
// 从右往左的上凸包
for(int i = n; i >= 1; i--) {
    while(sz >= 2 && (P[i] - W[sz]) * (W[sz] - W[sz - 1]) >= 0) sz--;
}
// 如果最后的表达式中, 得到 k < s, k 表示斜率, s 为某个数
// 那么我们就维护下凸包。
// 从左往右的下凸包
for(int i = 1; i <= n; i++) {
    while(sz >= 2 && (P[i] - W[sz]) * (W[sz] - W[sz - 1]) >= 0) sz--;
}
// 从右往左的下凸包
for(int i = n; i >= 1; i--) {
    while(sz >= 2 && (P[i] - W[sz]) * (W[sz] - W[sz - 1]) <= 0) sz--;
}

```

对于是处理前缀的情况，假如题目要求得到

$$\max(S1[r] - S1[l - 1] - (l - 1) * (S2[r] - S2[l - 1]))$$

设  $l1 < l2$ , 令  $f(l1) < f(l2)$ , 可以得到

$$S2[r] < \frac{((l2-1)S2[l2]-S1[l2-1]) - ((l1-1)S2[l1]-S1[l1-1])}{l2-l1}$$

如果我们把点当作  $(i, (i - 1) * S2[i - 1] - S1[i - 1])$ , 那么其实表达的意思就是，这个位置是我们选择的左区间位置。

如果我们把点当作  $(i, i * S2[i] - S1[i])$ , 那么这个位置  $i$  代表的是  $l - 1$  位置。

(终于能无脑写斜率优化了hhhh)

## 4. 斜率优化(单调队列)

通常斜率优化的代价都是某个平方之类的

假设  $k < j < i$ , 假如  $j$  比  $k$  更优, 列出式子然后能拆成  $(f(j) - f(k)) / (g(j) - f(k)) < h(i)$  的形式

只要是这个形式, 且  $h(i)$  函数单调递增, 就能用斜率优化

最后单调队列中, 左下角是队列首, 右上角是队列尾

从首到尾组成的点斜率越来越大

```

int Q[MX], c, r;
/*分子*/
LL getup(int i, int j) {

```

```

    return dp[j - 1] + A[j] * A[j] - (dp[i - 1] + A[i] * A[i]);
}
/*分母*/
LL getdown(int i, int j) {
    return A[j] - A[i];
}
/*计算 dp 的值*/
LL getdp(int i, int j) {
    return dp[j - 1] + (A[i] - A[j]) * (A[i] - A[j]) + w;
}
for(int i = 1; i <= n; i++) {
    while(r - c + 1 >= 2 && getup(Q[c], Q[c + 1]) <= 2 * A[i]*getdown(Q[c], Q[c + 1])) c++;
    dp[i] = min(getdp(i, Q[c]), dp[i - 1] + w);
    while(r - c + 1 >= 2 && getup(Q[r], i)*getdown(Q[r - 1], Q[r]) <= getdown(Q[r], i)*getup(Q[r - 1], Q[r])) r--;
    Q[++r] = i;
}

```

## 5. 往子集传递值

设  $dp[w][s]$  表示对于低位的  $w$  位, 1 必须是 1, 0 可以是其他的, 对于另外  $20-w$  位, 1 就是 1, 0 就是 0。那么我们可以得到一个转移方程, 如果  $s \gg (w-1) \& 1$ , 那么等于  $dp[w-1][s]$  否则, 就等于  $dp[w-1][s | (1 \ll (w-1))]$

```

/*初始值存在 dp[0][t] 中*/
for(int i = 1; i <= 20; i++) {
    for(int s = 0; s <= w; s++) { //w 所有的二进制状态
        if(s >> (i - 1) & 1) dp[i][s] = dp[i - 1][s];
        else dp[i][s] = dp[i - 1][s] + dp[i - 1][s + (1 << (i - 1))];
    }
}
/*答案存在 dp[20][t] 中*/

```

## 6. 数位 dp

```

inline int func(int s, int x) {
    x = 9 - x;
    for(int i = 0; i <= 9; i++) {
        if(i >= x && (s >> i & 1)) {
            return s ^ (1 << i) ^ (1 << x);
        }
    }
    return s ^ (1 << x);
}
LL DFS(int p, int s, bool limits) {
    if(p == 1) return __builtin_popcount(s); //通常返回 1
    if(!limits && dp[p][s] != -1) return dp[p][s];
    p--; LL ret = 0;
    int bound = limits ? A[p] : 9;
    for(int i = 0; i <= bound; i++) {
        ret += DFS(p, func(s, i), limits & (i == A[p]));
    }
    if(!limits) dp[p + 1][s] = ret;
    return ret;
}
void presolve(LL n) {
    w = 0;
    while(n) {
        A[++w] = n % 10;
        n /= 10;
    }
}
LL solve(LL n) {
    if(n == 0) return 0;
    presolve(n);
}

```

```

LL ret = 0;
for(int i = 1; i <= w; i++) {
    int ed = (i == w ? A[i] : 9);
    for(int j = 1; j <= ed; j++) {
        ret += DFS(i, func(0, j), (i == w && j == A[i]));
    }
}
return ret;
}

```

## 7. 区间内 2 个数位异或等于特定值

```

/*复杂度 O(60*2*2)
可以求出 t1 属于[1,a], t2 属于[1,b]
t1^t2=x 的(t1,t2)的点对数
*/
const int MX = 1e2;
LL dp[MX][2][2];
int na[MX], nb[MX], nx[MX];

LL S(LL a, LL b, LL x) {
    memset(dp, 0, sizeof(dp));
    for(int i = 63; i >= 0; i--) {
        na[i] = a >> i & 1;
        nb[i] = b >> i & 1;
        nx[i] = x >> i & 1;
    }

    dp[63][1][1] = 1;
    for(int i = 62; i >= 0; i--) {
        if(na[i] ^ nb[i] == nx[i]) {
            dp[i][1][1] += dp[i + 1][1][1];

            dp[i][1][0] += dp[i + 1][1][0];
            if(nb[i] && na[i] == nx[i]) dp[i][1][0] += dp[i + 1][1][1];

            dp[i][0][1] += dp[i + 1][0][1];
            if(na[i] && nb[i] == nx[i]) dp[i][0][1] += dp[i + 1][1][1];

            dp[i][0][0] += dp[i + 1][0][0] * 2;
            if(na[i]) dp[i][0][0] += dp[i + 1][1][0];
            if(nb[i]) dp[i][0][0] += dp[i + 1][0][1];
            if(na[i] && nb[i] && !nx[i]) dp[i][0][0] += dp[i + 1][1][1];
        }
    }
    return dp[0][0][0] + dp[0][0][1] + dp[0][1][0] + dp[0][1][1];
}

```

## 博弈

### 1. 斐波那契博弈

题意：1 堆石子  $n$  个，第一个人可以取任意个数但不能全部取完，以后每次拿的个数不能超过上一次对手拿的个数的 2 倍，轮流拿石子，问先手是否必赢  
思路：斐波那契博弈，后手赢的情况的数字会呈现斐波那契数列。

### 2. 威佐夫博弈

题意：轮流取石子。1. 在一堆中取任意个数。2. 在两堆中取相同个数。最后取完的人胜利，问先手是否必赢  
思路：威佐夫博弈，满足黄金分割，且每个数字只会出现在一次。

```

if(a >= b) swap(a, b);
int k = b - a;

```

```
int x = (sqrt(5.0) + 1) / 2 * k, y = x + k;
if(a == x && b == y) printf("0\n");
else printf("1\n");
```

### 3. 巴什博弈

题意：两人竞拍，每次加价的价格在 $[1, n]$ 范围内，第一次 $\geq m$ 的赢

思路：巴什博弈，当  $m \%(n+1) \neq 0$  时，先手赢，否则后手赢

### 4. Anti-num 博弈

SG 函数的求法一模一样，最后如果只有一堆，也能用 SJ 定理

如果为 Anti-Nim 游戏，如下情况先手胜

SG 异或和为 0，且单个游戏的 SG 全部  $\leq 1$

SG 异或不为 0，且存在单个游戏的  $SG > 1$ ，即  $\leq 1$  的个数不等于独立游戏个数

### 5. Nim 博弈

Nim 游戏相当于把独立游戏分开计算 SG 函数，然后再用位异或

$Sg[u] = \text{Mex}(\{\text{后继的集合}\})$  相当于取出最小的集合中不存在的数字，可以发现 mex 的值总是比后继的个数要少而且 vis 数组通常都是开在函数内部，不开在全局变量中，防止冲突。

## 其他杂类

### 6. 大数

```
const int MX = 2500;
const int MAXN = 9999;
const int DLEN = 4;

/*已重载>+-%和 print*/
class Big {
public:
    int a[MX], len;
    Big(const int b = 0) {
        int c, d = b;
        len = 0;
        memset(a, 0, sizeof(a));
        while(d > MAXN) {
            c = d - (d / (MAXN + 1)) * (MAXN + 1);
            d = d / (MAXN + 1);
            a[len++] = c;
        }
        a[len++] = d;
    }
    Big(const char *s) {
        int t, k, index, L, i;
        memset(a, 0, sizeof(a));
        L = strlen(s);
        len = L / DLEN;
        if(L % DLEN) len++;
        index = 0;
        for(i = L - 1; i >= 0; i -= DLEN) {
            t = 0;
            k = i - DLEN + 1;
            if(k < 0) k = 0;
            for(int j = k; j <= i; j++) {
                t = t * 10 + s[j] - '0';
            }
            a[index++] = t;
        }
    }
}
```

```

Big operator/(const int &b)const {
    Big ret;
    int i, down = 0;
    for(int i = len - 1; i >= 0; i--) {
        ret.a[i] = (a[i] + down * (MAXN + 1)) / b;
        down = a[i] + down * (MAXN + 1) - ret.a[i] * b;
    }
    ret.len = len;
    while(ret.a[ret.len - 1] == 0 && ret.len > 1) ret.len--;
    return ret;
}

bool operator>(const Big &T)const {
    int ln;
    if(len > T.len) return true;
    else if(len == T.len) {
        ln = len - 1;
        while(a[ln] == T.a[ln] && ln >= 0) ln--;
        if(ln >= 0 && a[ln] > T.a[ln]) return true;
        else return false;
    } else return false;
}

Big operator+(const Big &T)const {
    Big t(*this);
    int i, big;
    big = T.len > len ? T.len : len;
    for(i = 0; i < big; i++) {
        t.a[i] += T.a[i];
        if(t.a[i] > MAXN) {
            t.a[i + 1]++;
            t.a[i] -= MAXN + 1;
        }
    }
    if(t.a[big] != 0) t.len = big + 1;
    else t.len = big;
    return t;
}

Big operator-(const Big &T)const {
    int i, j, big;
    bool flag;
    Big t1, t2;
    if(*this > T) {
        t1 = *this;
        t2 = T;
        flag = 0;
    } else {
        t1 = T;
        t2 = *this;
        flag = 1;
    }
    big = t1.len;
    for(i = 0; i < big; i++) {
        if(t1.a[i] < t2.a[i]) {
            j = i + 1;
            while(t1.a[j] == 0) j++;
            t1.a[j--]--;
            while(j > i) t1.a[j--] += MAXN;
            t1.a[i] += MAXN + 1 - t2.a[i];
        } else t1.a[i] -= t2.a[i];
    }
    t1.len = big;
    while(t1.a[t1.len - 1] == 0 && t1.len > 1) {
        t1.len--;
        big--;
    }
}

```

```

    }
    if(flag) t1.a[big - 1] = 0 - t1.a[big - 1];
    return t1;
}
int operator%(const int &b)const {
    int i, d = 0;
    for(int i = len - 1; i >= 0; i--) {
        d = ((d * (MAXN + 1)) % b + a[i]) % b;
    }
    return d;
}
Big operator*(const Big &T) const {
    Big ret;
    int i, j, up, temp, temp1;
    for(i = 0; i < len; i++) {
        up = 0;
        for(j = 0; j < T.len; j++) {
            temp = a[i] * T.a[j] + ret.a[i + j] + up;
            if(temp > MAXN) {
                temp1 = temp - temp / (MAXN + 1) * (MAXN + 1);
                up = temp / (MAXN + 1);
                ret.a[i + j] = temp1;
            } else {
                up = 0;
                ret.a[i + j] = temp;
            }
        }
        if(up != 0) {
            ret.a[i + j] = up;
        }
    }
    ret.len = i + j;
    while(ret.a[ret.len - 1] == 0 && ret.len > 1) ret.len--;
    return ret;
}
void print() {
    printf("%d", a[len - 1]);
    for(int i = len - 2; i >= 0; i--) printf("%04d", a[i]);
}
};

```

## 7. pb\_ds 大法

//可并堆测试

#include <ext/pb\_ds/priority\_queue.hpp>

void ceshi\_1() {

//binary\_heap\_tag 一般比 std::priority\_queue 快

//pairing\_heap\_tag 和 std::priority\_queue 启发式合并时, 速度差不多

\_\_gnu\_pbds::priority\_queue<int, less<int>, \_\_gnu\_pbds::pairing\_heap\_tag> Q1, Q2;

Q1.push(1); Q1.push(2); Q1.push(3);

Q2.push(1); Q2.push(2); Q2.push(3);

Q1.join(Q2); //pairing\_heap\_tag 配对堆, zici O(1)合并

while(!Q1.empty()) {

std::printf("%d\n", Q1.top());

Q1.pop();

}

}

//平衡树测试

#include <ext/pb\_ds/assoc\_container.hpp>

#include <ext/pb\_ds/tree\_policy.hpp>

void ceshi\_2() {

//支持 join 和 split

```

typedef __gnu_pbds::tree<int,
    __gnu_pbds::null_type,
    less<int>,
    __gnu_pbds::rb_tree_tag,
    __gnu_pbds::tree_order_statistics_node_update>
    qwb_set;
qwb_set w;
w.insert(1); w.insert(6); w.insert(3); w.insert(100);
auto t = w.find_by_order(1); //查找第 x+1 小的值
int sum = w.order_of_key(101); //比 x 小的有多少个元素
}

```

## 8. bitset

内存占 size/8 字节  
 bitset<128>s; 定义 s 变量  
 s=100; //可以直接赋值  
 s="100010"; //可以赋值字符串  
 s.set(p); //设置 p 位为 1  
 s.reset(p); //设置 p 位为 0  
 s.set(); //全部位设置为 1  
 s.reset(); //全部位设置为 0  
 s.count(); //1 的个数  
 s.flip(); //0 变 1, 1 变 0, 相当于~  
 可以直接使用~|^&符号

## 9. 蔡勒公式

$$\text{Week} = (\text{Day} + 2 * \text{Month} + 3 * (\text{Month} + 1) / 5 + \text{Year} + \text{Year} / 4 - \text{Year} / 100 + \text{Year} / 400) \% 7 + 1$$

i. 该公式中要把 1 月和 2 月分别当成上一年的 13 月和 14 月处理。

例如：2008 年 1 月 4 日要换成 2007 年 13 月 4 日带入公式。

“1” 为星期 1, …… , “7” 为星期日。

## 10. 第 k 小

```

LL l = 1, r = 4e18, m;
while(l <= r) {
    m = (l + r) >> 1;
    if(check(m) >= k) r = m - 1;
    else l = m + 1;
}
check(m) 来求 <=m 的个数

```

## 11. 三分整数

```

while(l < r) {
    int m1 = (2 * l + r) / 3, m2 = (l + 2 * r + 2) / 3;
    if(f(m1) < f(m2)) l = m1 + 1;
    else r = m2 - 1;
}

```

## 12. 求阶乘后缀 0 个数

```

int get(int n) {
    int z = 0;
    while (n > 0) {
        n /= 5;
        z += n;
    }
    return z;
}

```

## 13.DFS 构造矩阵

/\*骨牌覆盖的构造方法\*/

```
void DFS(int a, int b, int l) {
    if(l == m) {
        A[b][a] = 1; //a 对 b 的影响
        return;
    }
    DFS(a << 1, b << 1 | 1, l + 1); //往下放
    DFS(a << 1 | 1, b << 1, l + 1); //不放
    if(l + 2 <= m) DFS(a << 2 | 3, b << 2 | 3, l + 2); //往右放
}
```

## 14.手动扩栈

C++扩栈

```
#pragma comment(linker, "/STACK:102400000,102400000")
```

G++扩栈貌似不可以？

```
int Size = 256 << 20;
char *p = (char*)malloc(Size) + Size;
__asm__("movl %0, %%esp\n" :: "r"(p));
```

## 15.正常的读入挂

```
inline int read() {
    int ret = 0, c, f = 1;
    for(c = getchar(); !(isdigit(c) || c == '-'); c = getchar());
    if(c == '-') f = -1, c = getchar();
    for(; isdigit(c); c = getchar()) ret = ret * 10 + c - '0';
    if(f < 0) ret = -ret;
    return ret;
}
```

## 16.fread 读入挂

```
namespace IO {
    const int MX = 1e7; //1e7 占用内存 11000kb
    char buf[MX]; int c, sz;
    void begin() {
        c = 0;
        sz = fread(buf, 1, MX, stdin);
    }
    inline bool read(int &t) {
        while(c < sz && buf[c] != '-' && (buf[c] < '0' || buf[c] > '9')) c++;
        if(c >= sz) return false;
        bool flag = 0; if(buf[c] == '-') flag = 1, c++;
        for(t = 0; c < sz && '0' <= buf[c] && buf[c] <= '9'; c++) t = t * 10 + buf[c] - '0';
        if(flag) t = -t;
        return true;
    }
}
```

## 17.测试系统类型

```
void test_system() {
#ifdef __linux__
    for(;;);
#else
    int a = 0, b = 1 / a;
#endif
}
```



## 18.方阵的循环节

$$w(n, p) = \prod_{i=0}^{n-1} (p^n - p^i)$$

$$A^t = A^{t \% w(n, p)}$$

只有 p 为质数时候才成立。

## 19.字符串分割读入的

```
/*再乱写就剁手*/
bool read(int &cur, int len, bool sign = 0) {
    for(; cur < len && !check(buf[cur]); cur++);
    if(cur == len) return false;

    int tt = 0;
    for(; cur < len && check(buf[cur]); cur++) {
        if(sign) tmp[tt++] = to_lower(buf[cur]);
        else tmp[tt++] = buf[cur];
    }
    tmp[tt] = 0;
    return true;
}

int cur = 0, len = strlen(buf);
while(read(cur, len)) {
    //tmp 就是已经读入进来的
}
```

## 20.区间随机数生成

```
int Rand(int L, int R) { //区间内随机数生成函数
    return (LL)rand() * rand() % (R - L + 1) + L;
}
```

## 21.祖传头文件

```
#include <map>
#include <set>
#include <cmath>
#include <ctime>
#include <stack>
#include <queue>
#include <cstdio>
#include <cctype>
#include <bitset>
#include <string>
#include <vector>
#include <cstring>
#include <iostream>
#include <algorithm>
#include <functional>
#define fuck(x) cout<<"["<<x<<"]";
#define FIN freopen("input.txt","r",stdin);
#define FOUT freopen("output.txt","w+",stdout);
//#pragma comment(linker, "/STACK:102400000,102400000")
using namespace std;
typedef long long LL;
```

```
typedef pair<int, int> PII;
```