HW3_STAT5000

Xingyu Chen

September 11, 2021

Due through Canvas on Friday September 17, 2021 @ 11:59 PM. Instructions for "theoretical" questions: Answer all of the following questions. The "theoretical" problems should be neatly numbered, written out or typed, and solved. This homework has NO computational section. It has 14 lettered questions and no stand- alone questions, so self-grade should be out of 28 total.

Total Score: 23/28

1 Theoretical Questions

1.

(a) The CU Boulder triathlon team has 12 women and 9 men. The team is going to a race and can only enter 5 participants. What is the probability of randomly selecting a race squad of 5 participants with exactly 3 women. Q1: 2/2

Answer:
P(exactly 3 women) = 12C3 * 9C2 / 21C5 = 220 * 36 / 20349 = 880/2261 = 38.9%

(b) Find the probability that that at least two people in a room of 45 have the same birthday. Assume that all possible birthdays are equally likely, and ignore leap year. Q2: 2/2

Answer:
P'(same birth among 45 people) = 365 * 364 … (365-45+1) / 365^45 = 0.0590241 ;
P(same birth among 45 people) = 1 - P'(same birth among 45 people) = 1- 0.0590241 = 94.0976%

(c) Three dice are thrown. What is the probability that a sum of 12 appears on the faces? What is the probability that a sum of 13 appears? Q3: 2/2

Answer:
P(sum of 12) = (11C2 - P(1,1,10) - P(1,2,9) - P(2,2,8) - P(1,3,8) - P(2, 3, 7) - P(1, 4, 7))/ 6^3 = (55- 3 - 6 - 3 - 6 - 6 - 6)/216 = 25 / 216 = 11.6%
P(sum of 13) = (12C2 - P(1,1,11) - P(1,2,10) - P(2,2,9) - P(1,3,9) - P(2, 3, 8) - P(1, 4, 8) - P(3, 3 ,7) - P(2, 4, 7) - P(1, 5, 7))/ 6^3 = (66-3-6-3-6-6-6-3-6-6)/216 = 21 / 216 = 9.7%

2. What does it mean for one event C to cause another event E—for example, smoking (C) to cause cancer (E)? There is a long history in philosophy, statistics, and science of trying to clearly analyze the concept of a cause. One tradition says that causes raise the probability of their effects; we may write this symbolically is $P(E|C) > P(E)$. (1)

(a) Does equation (1) imply that $P(C|E) > P(C)$? If so, prove it. If not, give a counter example. Q4: 1/2

Answer:

Yes.

We have $P(E|C) = \frac{P(E\cap C)}{P(C)} > P(E)$, thus, $P(E \cap C) > P(E) * P(C)$.

Finally, $P(C|E) = \frac{P(C\cap E)}{P(E)} = \frac{P(E\cap C)}{P(E)} > P(C)$.

(b) Another way to formulate a probabilistic theory of causation is to say that $P(E|C) > P(E|C^c)$. (2)

Show that equation (1) implies equation (2). Q5: 1/2

Answer:

$P(E|C^c) = \frac{P(E\cap C^c)}{P(C^c)} = \frac{P(E)P(C^c)}{1-P(C)} = \frac{P(E)(1-P(C))}{1-P(C)} = P(E)$;

So $P(E|C) > P(E|C^c)$, the equation (1) implies equation (2).

(c) Let C be a the drop in the level of mercury in a barometer and let E be a storm. Briefly describe why this leads to a problem with using equation (1) (or equation (2)) as a theory of causation. Q6: 1/2

Answer:

From equation (1), we know $P(E|C) = \frac{P(E\cap C)}{P(C)} = \frac{0}{P(C)} = 0$, it is impossible for P(C) less than 0.

(d) Let A, C, and E be events. If $P(E|A \cap C) = P(E|C)$, then C is said to screen A off from E. Suppose that $P(E \cap C) > 0$. Show that screening off is equivalent to saying that $P(A \cap E|C) = P(A|C)P(E|C)$. What does this latter equation say in terms of independence? Q7: 1/2

Answer:

$P(E|A \cap C) = P(E|C)$;

$\frac{P(E\cap A\cap C)}{P(A\cap C)} = \frac{P(E\cap C)}{P(C)}$;

$\frac{P(A\cap E\cap C)}{P(C)} = \frac{P(E\cap C)P(A)}{P(C)}$;

$\frac{P(A\cap E\cap C)}{P(C)} = \frac{P(A\cap C)}{P(C)}\frac{P(E\cap C)}{P(C)}$;

$P(A \cap E|C) = P(A|C)P(E|C)$.

This latter equation saying that the natural consequences of C being a cause of A and of E.

(e) Now let A be a the drop in the level of mercury in a barometer, E be a storm, and C be a drop in atmospheric pressure. Does the result from part (d) help fix the problem suggested in part (c)? Q8: 1/2

Answer:

Yes, the result from part (d) saying that the natural consequences of E being a cause of A and of E.

3. Suppose a particular crime is committed in Jerry's apartment. We'd like to know if Kramer is guilty of the crime. We are torn as to whether we think he is guilty: we think it's equally likely that he guilty or not guilty. Suppose that, in similar situations, we know that if a suspect is guilty, 85% of the time their finger prints are found at the scene, and, we know that if a suspect is not guilty 30% of the time their finger prints are found at the scene.

(a) What is the probability that Kramer's finger prints are found at the scene? Q9: 2/2
Answer:
P(GF) = 85%,
P(G'F) = 30%,
P(Kramer's F) = 0.5 * 0.85 + 0.5 * 0.3 = 23/40 = 0.575 = 57.5%

(b) If Kramer's finger prints are found at the scene, how likely is it that he is guilty? Q10: 2/2
Answer:
P = 0.5 * 0.85 / 0.575 = 17/23 = 73.9%

(c) If Kramer's finger prints are not found at the scene, how likely is it that he is guilty? Q11: 2/2
Answer:
P = 0.5 * (1 - 0.85) / (1 - 0.575) = 3/17 = 17.6%

4. The game of Yahtzee is played with five fair dice. The goal is to roll certain 'hands', such as Yahtzee (all five dice showing the same number) or a full house (three of a kind and two of a kind). In the first round of a player's turn, the player rolls all five dice. Based on the outcome of that roll, the player has a second and third round, where he/she can then choose to re-roll any subset of the dice to get a desired hand.

(a) What is the probability of rolling a Yahtzee on the first round? Q12: 2/2
Answer:
P(Yahtzee) = 6 / 6^5 = 1/1296 = 0.077%

(b) A small straight is defined as having 4 of the 5 dice all in a row (for example, {1, 2, 3, 4, 6}). What is the probability of rolling a small straight on the first round? Q13: 2/2
Answer:
P(small straight) = (P(distinct) + P(not distinct)) / 6^5
= ((P(1,2,3,4,6) + P(1,3,4,5,6)) + P(not distinct)) / 6^5
= ((2 * 5!) + (P(1,2,3,4) * 4 + P(2,3,4,5) * 4 + P(3,4,5,6) * 4)) / 6^5
= ((2 * 5!) + (3 * 4 * 5P5 / 2)) / 6^5
= (2 * 5! + 12 * 60) / 6^5
= 960 / 7776
= 10/81
= 12.3%

(c) Suppose that, on the second round, the dice are {2, 3, 4, 6, 6}. You decide to re-roll both sixes in the third round. What is the probability that you roll either a small straight or a large straight (a large straight is where all five dice are in a row)? Q14:

2/2

Answer:

P(small U large straight) = P(2,3,4,5, no 6 or 1) + P(1,2,3,4, no 5) + P(2,3,4,5,6) + P(1,2,3,4,5) / 6^2

= (7 + 9 + 2 + 2) / 6^2

= 5/9

= 55.5%