STA457 Project

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2025-03-29

```
library(dplyr)
library(tidyverse)
library(readr)
library(lubridate)
library(forecast)
```

```
## Warning: package 'forecast' was built under R version 4.3.3
```

```
library(astsa)
```

```
## Warning: package 'astsa' was built under R version 4.3.3
```

```
library(tseries)
```

```
## Warning: package 'tseries' was built under R version 4.3.3
```

```
library(mgcv)
library(Metrics)
```

```
## Warning: package 'Metrics' was built under R version 4.3.3
```

```
library(ggplot2)
library(xgboost)
```

```
## Warning: package 'xgboost' was built under R version 4.3.3
```

```
# library(XGBClassifier)
library(rugarch)
```

```
## Warning: package 'rugarch' was built under R version 4.3.3
```

1. EDA

```
price = read.csv("./Daily Prices_ICCO.csv")
weather = read.csv("./Ghana_data.csv")
USD_GHS_Historical_Data = read.csv("./USD_GHS Historical Data.csv")
```

1.1 Clean Data

```
weather <- weather |> dplyr::select(DATE, TAVG)
exchangerate <- USD_GHS_Historical_Data |> dplyr::select(Date, Price)
```

```
colnames(price)[colnames(price) == 'ICCO.daily.price..US..tonne.'] <- 'Daily_Price'
colnames(weather)[colnames(weather) == 'DATE'] <- 'Date'
```

```
colnames(weather)[colnames(weather) == 'TAVG'] <- 'Avg_Temp'
colnames(exchangerate)[colnames(exchangerate) == 'Price'] <- 'exchange_rate'
```

1.2 Check duplicated values

```
price |> group_by(Date) |> filter(n() > 1) |> ungroup()
```

```
## # A tibble: 8 x 2
##   Date       Daily_Price
##   <chr>      <chr>
## 1 31/01/2024 4,798.20
## 2 31/01/2024 10,888.05
## 3 30/01/2024 4,775.17
## 4 30/01/2024 10,676.42
## 5 09/01/2024 4,171.24
## 6 09/01/2024 4,171.24
## 7 15/12/2023 4,272.15
## 8 15/12/2023 4,272.15
```

```
price <- price |> filter(!(Date == "31/01/2024" & Daily_Price == "10,888.05"))
price <- price |> filter(!(Date == "30/01/2024" & Daily_Price == "10,676.42"))
price <- distinct(price)
```

1.3 Convert to Time Series Data

1.3.1 price Dataset

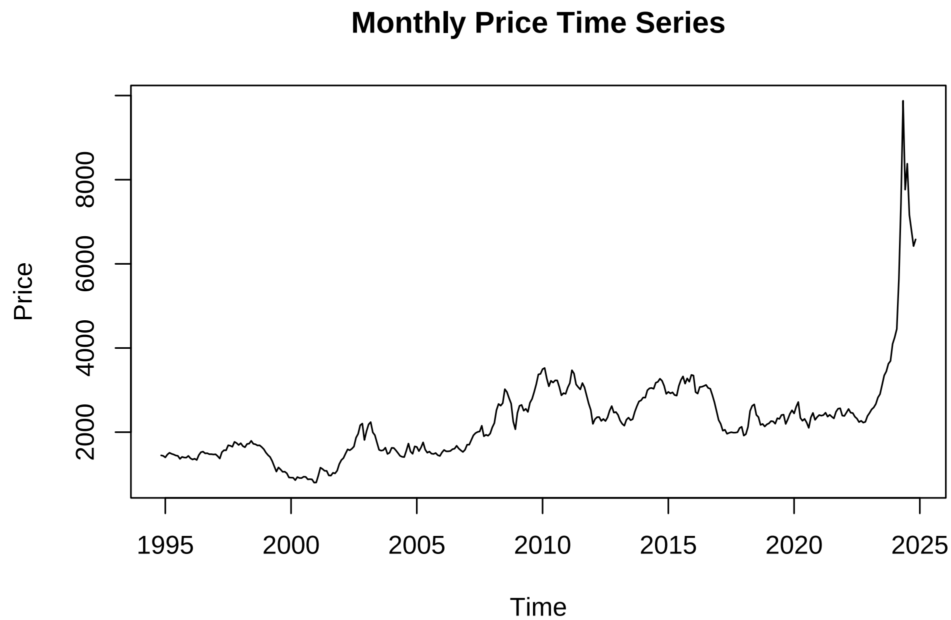
```
price$Date <- as.Date(price$Date, format="%d/%m/%Y")
price$Daily_Price <- as.numeric(gsub(",", "", price$Daily_Price))
price_month <- price |> mutate(Time = floor_date(Date, "month")) |> group_by(Time) |>
  summarise(month_Price = mean(Daily_Price, na.rm = TRUE)) |> ungroup()
```

```
summary(price)
```

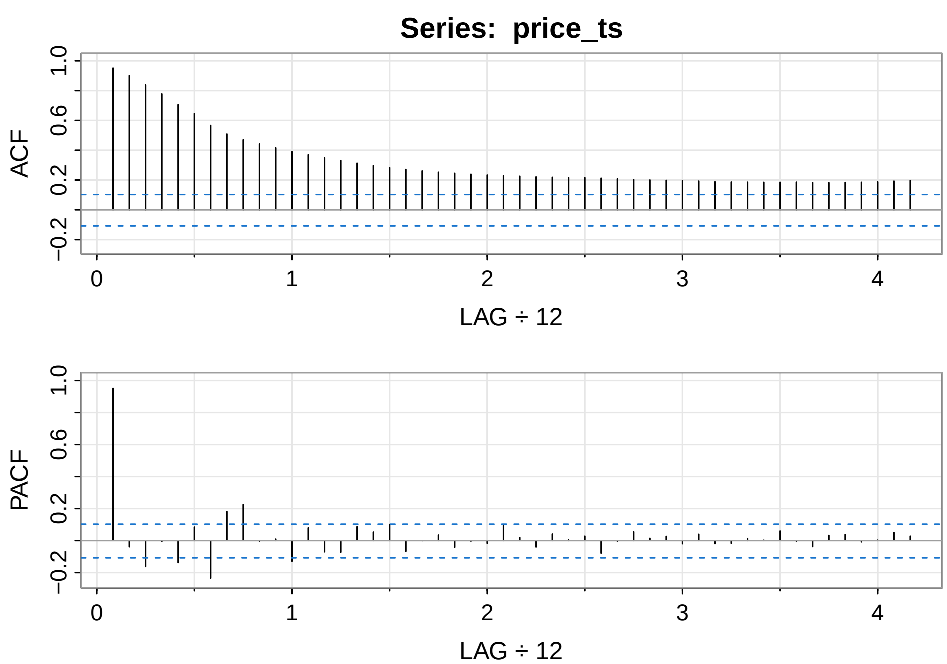
```
##       Date             Daily_Price
##  Min.   :1994-10-03   Min.   :  774.1
##  1st Qu.:2002-05-16   1st Qu.: 1557.8
##  Median :2009-12-17   Median : 2202.0
##  Mean   :2009-12-17   Mean   : 2350.1
##  3rd Qu.:2017-07-24   3rd Qu.: 2738.1
##  Max.   :2025-02-27   Max.   :11984.7
```

```
price_ts <- ts(price_month$month_Price, start = c(1994, 11), end = c(2024, 11), frequency = 12)
```

```
plot(price_ts, main="Monthly Price Time Series", ylab="Price", xlab="Time")
```




```r
acf2(price_ts, 50)
```




```
##       [,1]  [,2]  [,3]  [,4]  [,5] [,6]  [,7] [,8] [,9] [,10] [,11] [,12] [,13]
## ACF  0.95  0.90  0.84  0.78  0.71 0.65  0.57 0.51 0.47  0.44  0.42  0.39  0.37
## PACF 0.95 -0.04 -0.16 -0.01 -0.14 0.08 -0.24 0.18 0.23  0.00  0.01 -0.13  0.08
##      [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25]
## ACF   0.35  0.33  0.31  0.30  0.28  0.27  0.26  0.25  0.25  0.24  0.23  0.23
## PACF -0.07 -0.07  0.09  0.05  0.10 -0.07  0.00  0.03 -0.04  0.00 -0.02  0.10
##      [,26] [,27] [,28] [,29] [,30] [,31] [,32] [,33] [,34] [,35] [,36] [,37]
## ACF   0.23  0.22  0.22  0.22  0.22  0.21  0.21  0.20  0.20  0.20  0.20  0.19
## PACF  0.02 -0.04  0.04  0.00  0.03 -0.08  0.00  0.05  0.01  0.03 -0.02  0.04
##      [,38] [,39] [,40] [,41] [,42] [,43] [,44] [,45] [,46] [,47] [,48] [,49]
## ACF   0.19  0.19  0.19  0.18  0.18  0.19  0.18  0.18  0.18  0.18  0.19  0.19
## PACF -0.02 -0.02  0.01  0.00  0.06  0.00 -0.04  0.03  0.04 -0.01  0.00  0.05
```

```
##      [,50]
## ACF   0.20
## PACF  0.03
```

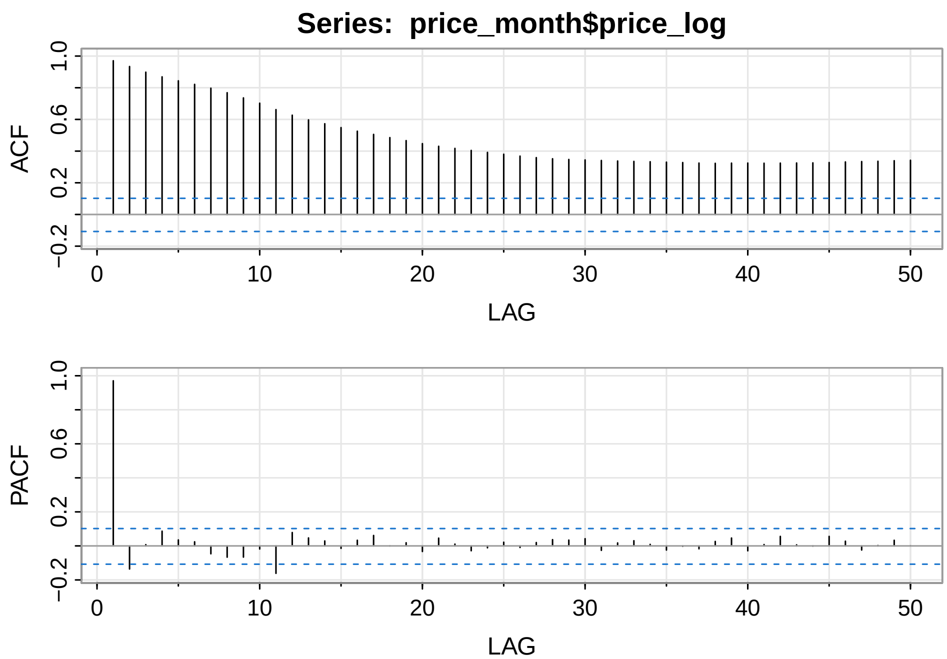
```r
ndiffs(price_ts)
```

```
## [1] 1
```

```r
price_month$price_log <- log(price_month$month_Price)
adf.test(price_month$price_log)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  price_month$price_log
## Dickey-Fuller = -1.736, Lag order = 7, p-value = 0.6883
## alternative hypothesis: stationary
```

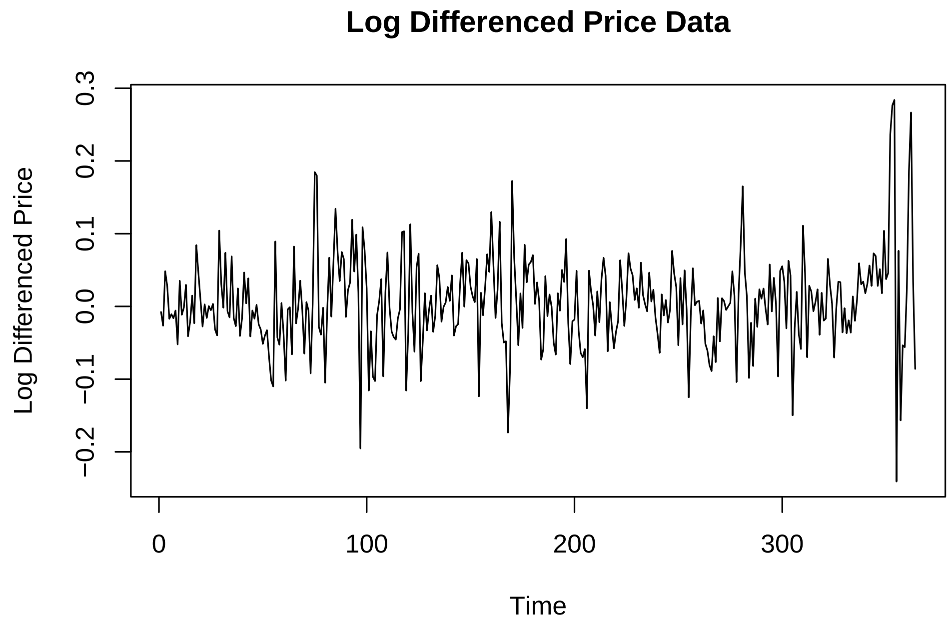
```r
acf2(price_month$price_log, 50)
```




```
##      [,1]  [,2] [,3] [,4] [,5] [,6]  [,7]  [,8]  [,9] [,10] [,11] [,12] [,13]
## ACF  0.97  0.93 0.90 0.87 0.84 0.82  0.80  0.77  0.74  0.70  0.66  0.63  0.60
## PACF 0.97 -0.14 0.01 0.09 0.04 0.02 -0.05 -0.07 -0.07 -0.02 -0.16  0.08  0.05
##      [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25]
## ACF   0.57  0.55  0.53  0.51  0.49  0.47  0.45  0.43  0.42  0.40  0.39  0.38
## PACF  0.03 -0.01  0.03  0.06  0.00  0.02 -0.03  0.05  0.01 -0.03 -0.01  0.02
##      [,26] [,27] [,28] [,29] [,30] [,31] [,32] [,33] [,34] [,35] [,36] [,37]
## ACF   0.37  0.36  0.35  0.35  0.34  0.34  0.34  0.34  0.33  0.33  0.33  0.32
## PACF -0.01  0.02  0.04  0.03  0.04 -0.03  0.02  0.03  0.01 -0.02  0.00 -0.02
##      [,38] [,39] [,40] [,41] [,42] [,43] [,44] [,45] [,46] [,47] [,48] [,49]
## ACF   0.32  0.32  0.32  0.32  0.32  0.33  0.33  0.33  0.33  0.33  0.34  0.34
## PACF  0.03  0.05 -0.03  0.01  0.06  0.01  0.00  0.06  0.03 -0.02  0.00  0.03
##      [,50]
## ACF   0.34
## PACF  0.00
```

Hence, we want to difference the price data.

```
diff_log_price = diff(price_month$price_log)
ts.plot(diff_log_price, main = "Log Differenced Price Data", ylab = "Log Differenced Price")
```

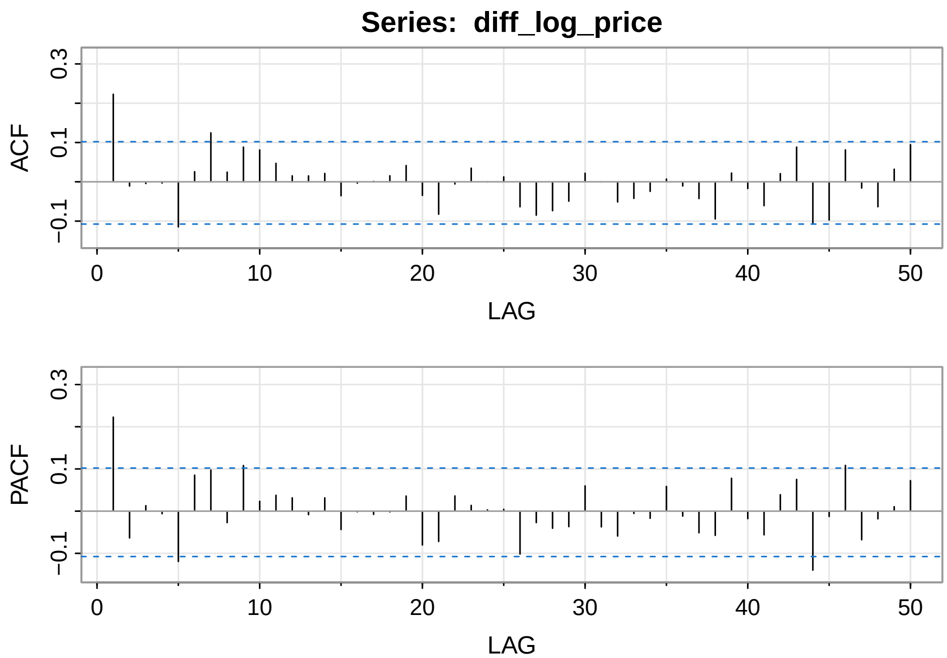



```
adf.test(diff_log_price)
```

```
## Warning in adf.test(diff_log_price): p-value smaller than printed p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  diff_log_price
## Dickey-Fuller = -6.1385, Lag order = 7, p-value = 0.01
## alternative hypothesis: stationary
```

```
acf2(diff_log_price, 50)
```




```
##      [,1]  [,2] [,3]  [,4]  [,5] [,6] [,7]  [,8] [,9] [,10] [,11] [,12] [,13]
## ACF  0.22 -0.01 0.00  0.00 -0.11 0.03 0.12  0.02 0.09  0.08  0.05  0.02  0.02
## PACF 0.22 -0.06 0.01 -0.01 -0.12 0.09 0.10 -0.03 0.11  0.02  0.04  0.03 -0.01
##      [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25]
## ACF   0.02 -0.04     0  0.00  0.02  0.04 -0.03 -0.08 -0.01  0.03     0  0.01
## PACF  0.03 -0.04     0 -0.01  0.00  0.04 -0.08 -0.07  0.04  0.01     0  0.00
##      [,26] [,27] [,28] [,29] [,30] [,31] [,32] [,33] [,34] [,35] [,36] [,37]
## ACF  -0.06 -0.09 -0.07 -0.05  0.02  0.00 -0.05 -0.04 -0.02  0.01 -0.01 -0.04
## PACF -0.10 -0.03 -0.04 -0.04  0.06 -0.04 -0.06 -0.01 -0.02  0.06 -0.01 -0.05
##      [,38] [,39] [,40] [,41] [,42] [,43] [,44] [,45] [,46] [,47] [,48] [,49]
## ACF  -0.09  0.02 -0.02 -0.06  0.02  0.09 -0.11 -0.10  0.08 -0.02 -0.06  0.03
## PACF -0.06  0.08 -0.02 -0.06  0.04  0.08 -0.14 -0.01  0.11 -0.07 -0.02  0.01
##      [,50]
## ACF   0.09
## PACF  0.07
```

1.3.2 ghana Dataset

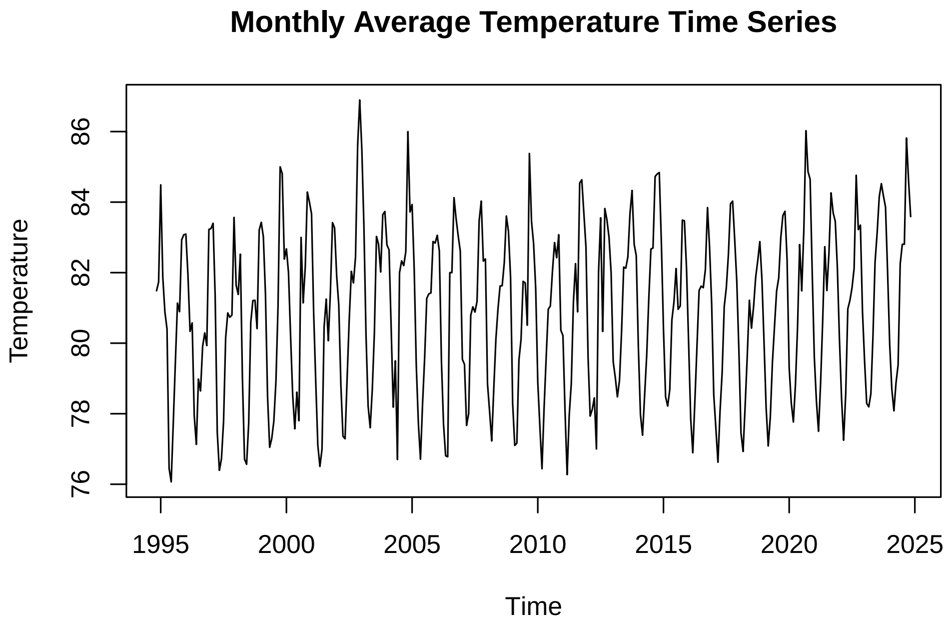
```
weather$Date <- as.Date(weather$Date)
weather$Avg_Temp <- as.numeric(gsub("", "", weather$Avg_Temp))
weather_month <- weather |> mutate(Time = floor_date(Date, "month")) |> group_by(Time) |>
  summarise(Avg_Temp = mean(Avg_Temp, na.rm = TRUE)) |> ungroup()
```

```
summary(weather_month)
```

```
##       Time               Avg_Temp    
##  Min.   :1990-01-01   Min.   :76.07  
##  1st Qu.:1998-09-23   1st Qu.:78.90  
##  Median :2007-07-16   Median :81.20  
##  Mean   :2007-06-22   Mean   :80.97  
##  3rd Qu.:2016-03-08   3rd Qu.:82.82  
##  Max.   :2024-11-01   Max.   :86.90  
```

```
weather_ts <- ts(weather_month$Avg_Temp, start = c(1994, 11), end = c(2024, 11), frequency = 12)
```

```
ts.plot(weather_ts, main="Monthly Average Temperature Time Series", ylab="Temperature", xlab="Time")
```



1.3.3 exchange Data

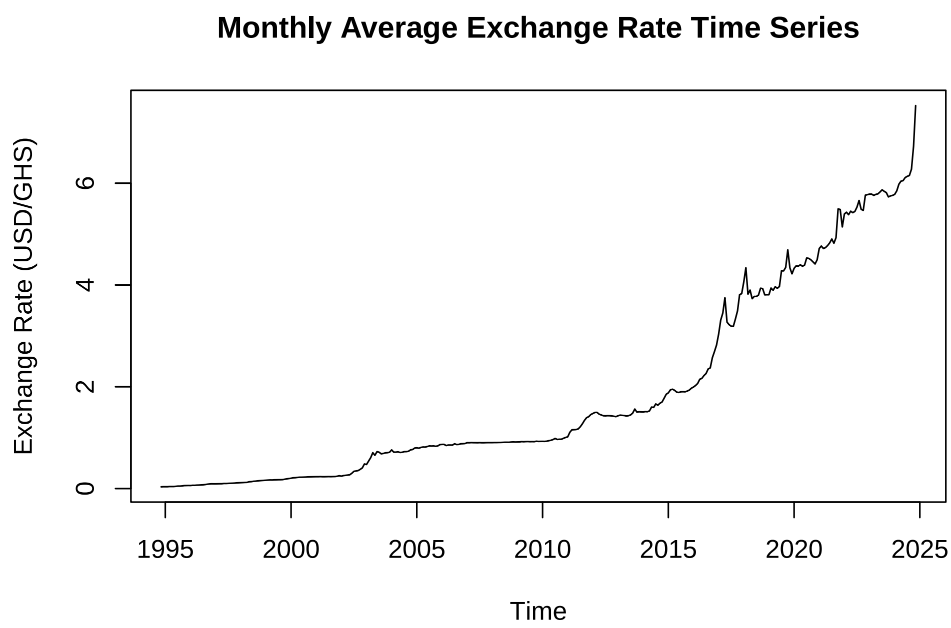
```
exchangerate$Date <- as.Date(exchangerate$Date)
exchangerate$exchange_rate <- as.numeric(gsub("", "", exchangerate$exchange_rate))
rate_month <- exchangerate |> mutate(Time = floor_date(Date, "month")) |> group_by(Time) |>
  summarise(exchange_rate = mean(exchange_rate, na.rm = TRUE)) |> ungroup()
```

```
summary(exchangerate)
```

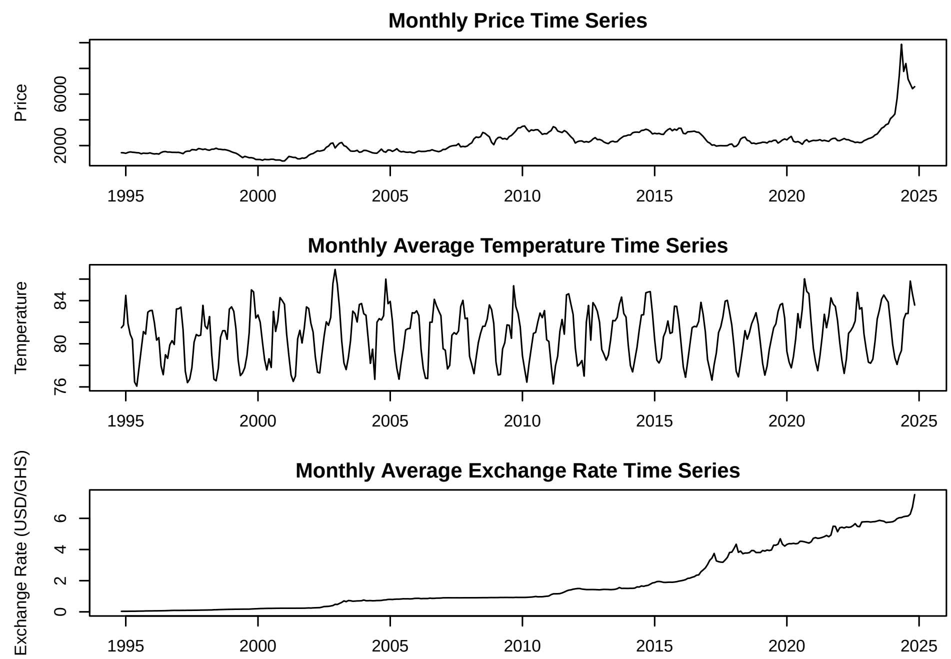
```
##       Date             exchange_rate
##  Min.   :1992-03-01   Min.   : 0.0338
##  1st Qu.:2000-06-01   1st Qu.: 0.5400
##  Median :2008-09-01   Median : 1.1595
##  Mean   :2008-08-31   Mean   : 2.8314
##  3rd Qu.:2016-12-01   3rd Qu.: 4.2805
##  Max.   :2025-03-01   Max.   :16.2500
```

```
rate_ts <- ts(rate_month$exchange_rate, start = c(1994, 11), end = c(2024, 11), frequency = 12)
```

```
ts.plot(rate_ts, main="Monthly Average Exchange Rate Time Series", ylab="Exchange Rate (USD/GHS)", xlab
```

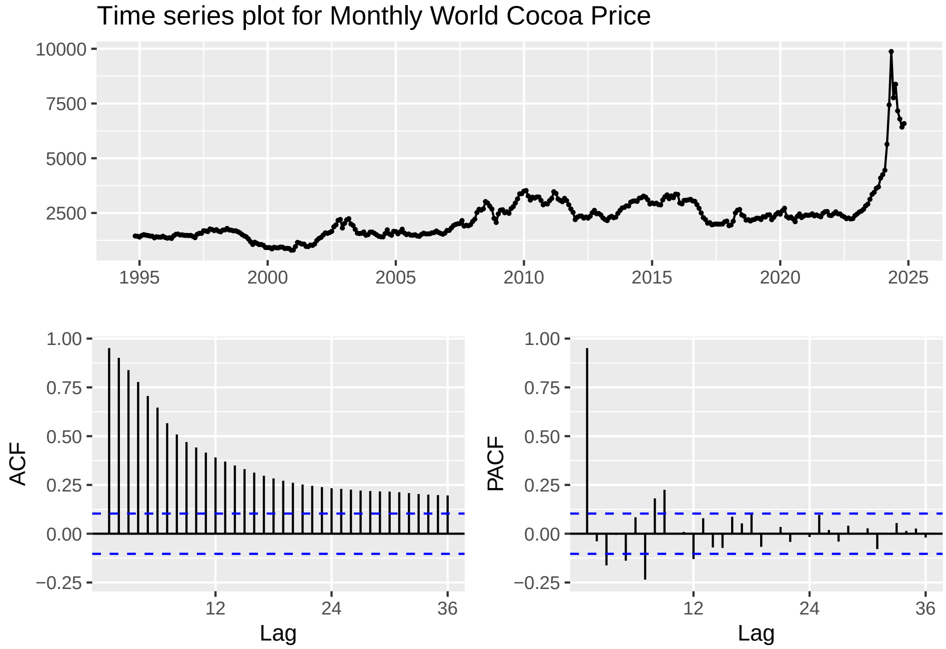



```r
par(mfrow=c(3,1), mar = c(3, 4, 2, 2))
# price
plot(price_ts, main="Monthly Price Time Series", ylab="Price", xlab="Time")
#temperature
ts.plot(weather_ts, main="Monthly Average Temperature Time Series", ylab="Temperature", xlab="Time")
# exchange rate
ts.plot(rate_ts, main="Monthly Average Exchange Rate Time Series", ylab="Exchange Rate (USD/GHS)", xlab=
```

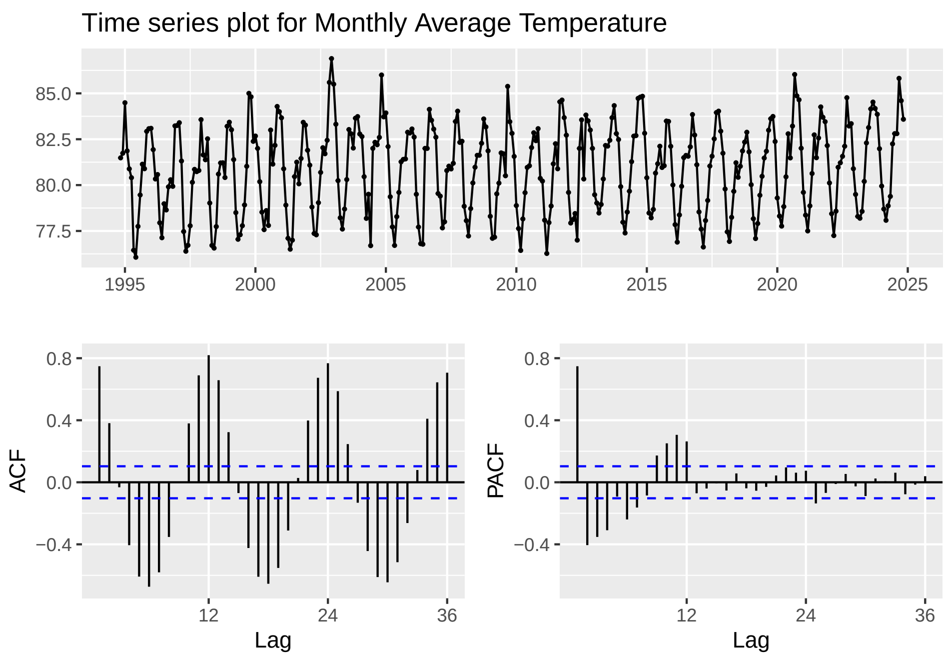


1.4 Time series plots for data

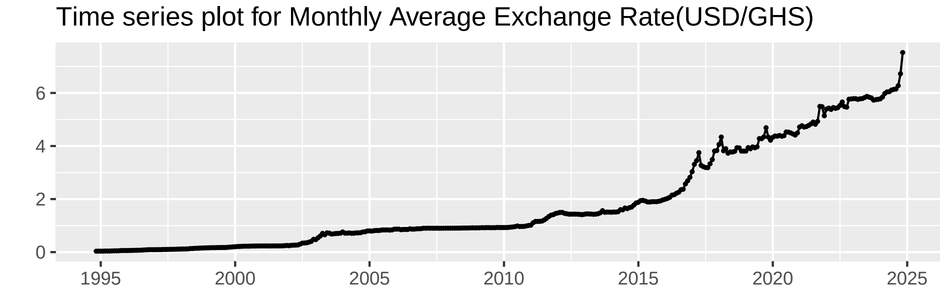
```r
ggtsdisplay(price_ts, main="Time series plot for Monthly World Cocoa Price")
```

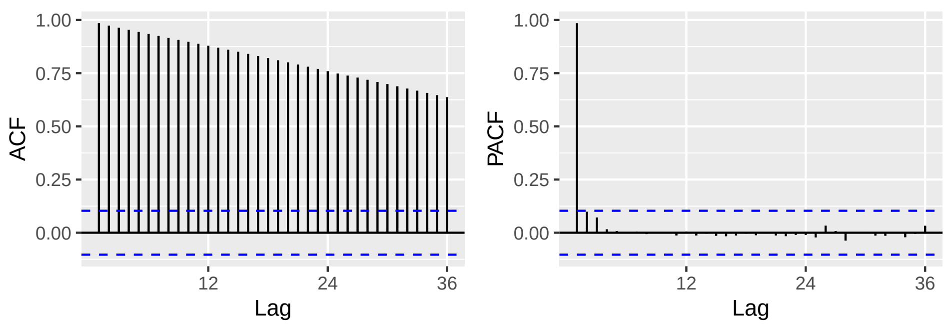



```r
ggtsdisplay(weather_ts, main="Time series plot for Monthly Average Temperature")
```

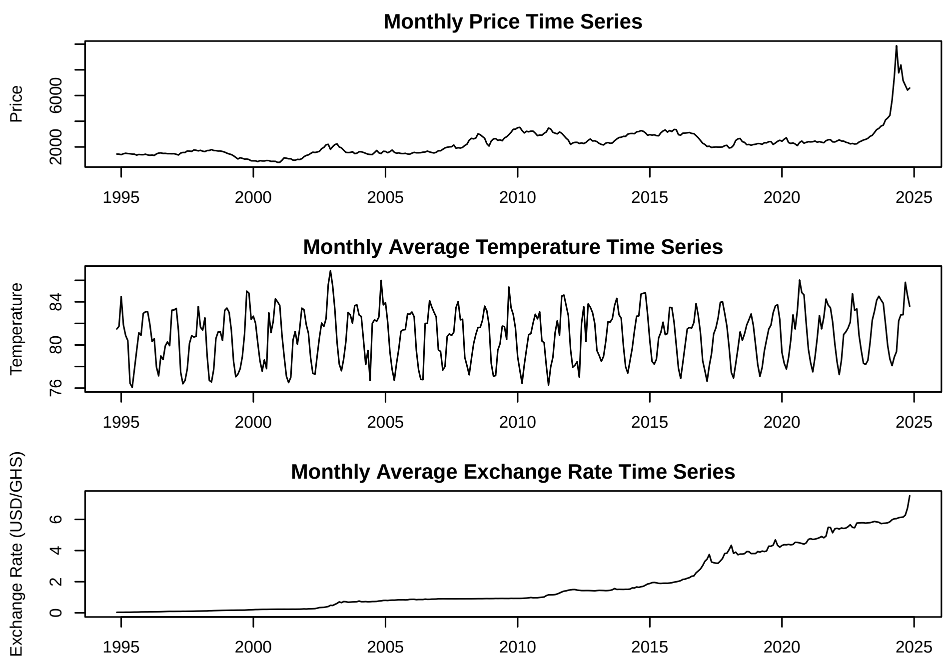



```r
ggtsdisplay(rate_ts, main="Time series plot for Monthly Average Exchange Rate(USD/GHS)")
```






```r
par(mfrow=c(3,1), mar = c(3, 4, 2, 2))
# price
plot(price_ts, main="Monthly Price Time Series", ylab="Price", xlab="Time")
#temperature
ts.plot(weather_ts, main="Monthly Average Temperature Time Series", ylab="Temperature", xlab="Time")
# exchange rate
ts.plot(rate_ts, main="Monthly Average Exchange Rate Time Series", ylab="Exchange Rate (USD/GHS)", xlab
```

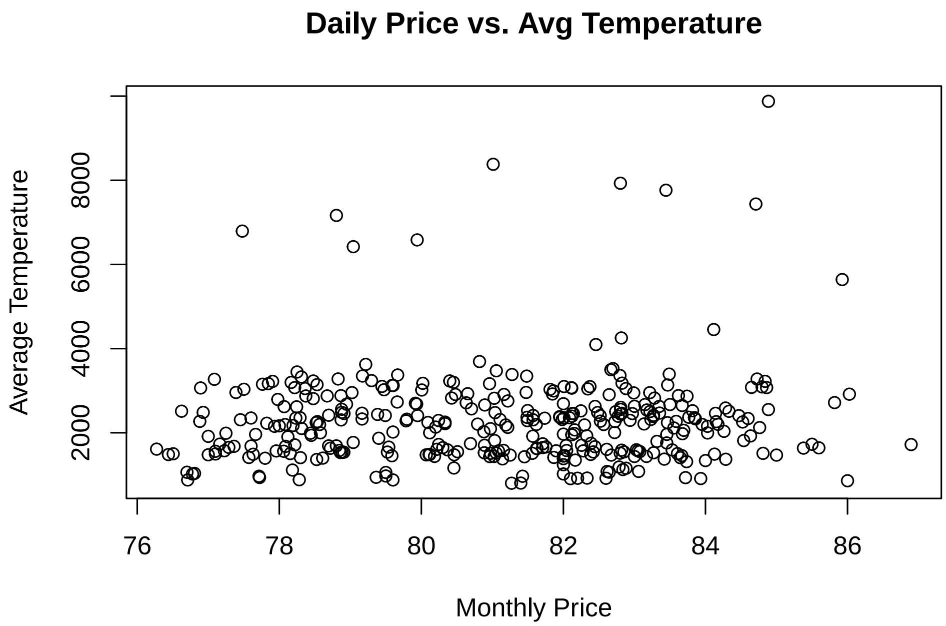


1.5 Combine and Split data

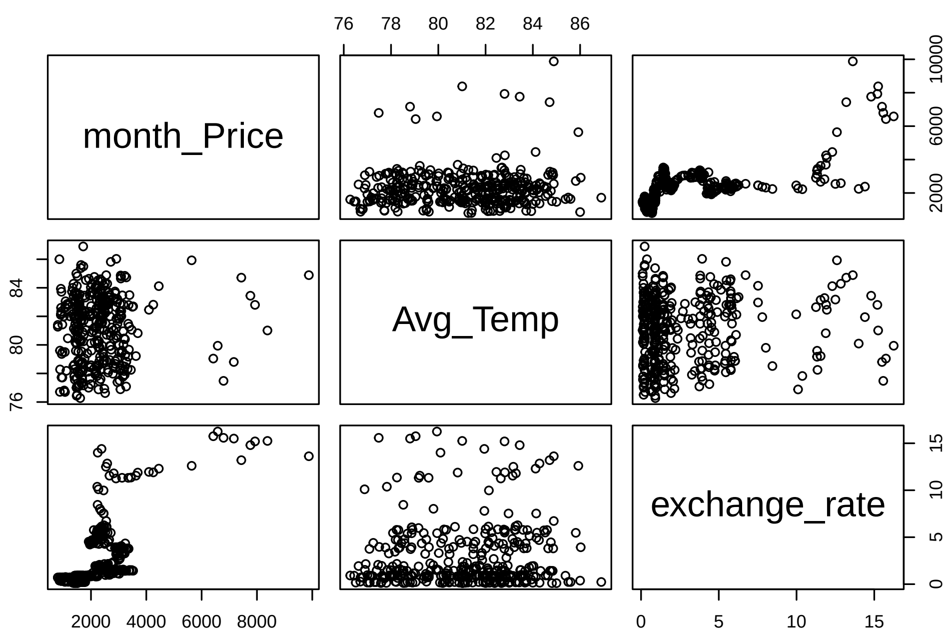
```r
data <- price_month |> left_join(weather_month, by = "Time") |> left_join(rate_month, by = "Time")
data <- data |> mutate(log_price = log(month_Price), diff_log_price =
```

```r
                    c(NA, diff(price_month$price_log))) |> drop_na()
data <- data |> dplyr::select(Time, Avg_Temp, exchange_rate, diff_log_price, log_price, month_Price)

data$Time <- as.Date(data$Time)
```

```r
plot(data$Avg_Temp, data$month_Price, xlab = "Monthly Price", ylab = "Average Temperature",
     main = "Daily Price vs. Avg Temperature")
```




```r
pairs(data[, c("month_Price", "Avg_Temp", "exchange_rate")])
```




```r
data <- data[order(data$Time), ]
cutoff <- floor(0.7 * nrow(data))
trainSet <- data[1:cutoff, ]
testSet <- data[(cutoff+1):nrow(data), ]
```

```r
data_train_ts <- ts(trainSet$diff_log_price, frequency = 12)
```

1.6 Stationarity check and Decomposition

```r
adf.test(data$month_Price)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  data$month_Price
## Dickey-Fuller = -1.7041, Lag order = 7, p-value = 0.7017
## alternative hypothesis: stationary
```

```r
adf.test(data$log_price)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  data$log_price
## Dickey-Fuller = -2.3875, Lag order = 7, p-value = 0.4133
## alternative hypothesis: stationary
```

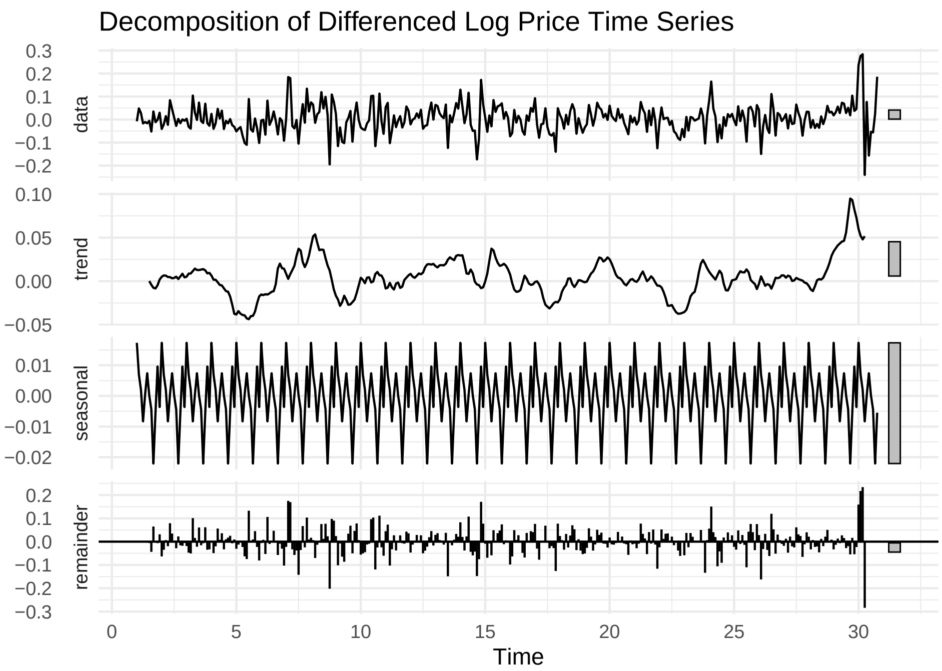
```r
adf.test(data$diff_log_price)
```

```
## Warning in adf.test(data$diff_log_price): p-value smaller than printed p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  data$diff_log_price
## Dickey-Fuller = -6.2103, Lag order = 7, p-value = 0.01
## alternative hypothesis: stationary
```

Since only the diff_log_price is stationary, we choose differenced monthly log price when fitting the model.

```r
diff_price_ts <- ts(data$diff_log_price, frequency = 12)
autoplot(decompose(diff_price_ts, type="additive")) +
  ggtitle("Decomposition of Differenced Log Price Time Series") +
  theme_minimal()
```



2. Method

2.1 ETS Model

ETS is a purely univariate model and cannot directly handle external regressors.

2.1.1 Fit Model

```
ets_model <- ets(data_train_ts, model = "ANA")
ets_zmodel <- ets(data_train_ts, model = "ZZZ") # Automatically selects best model
summary(ets_model)
```

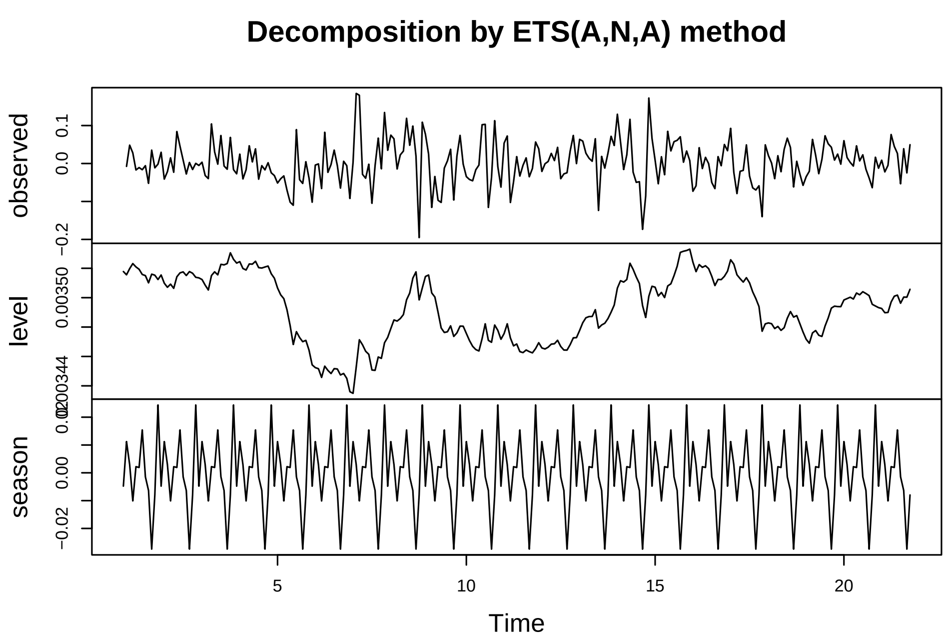
```
## ETS(A,N,A)
##
## Call:
## ets(y = data_train_ts, model = "ANA")
##
##   Smoothing parameters:
##     alpha = 1e-04
##     gamma = 1e-04
##
##   Initial states:
##     l = 0.0035
##     s = -0.0048 0.0244 -0.008 -0.0274 -0.0064 -0.0014
##            0.0154 0.0019 0.0022 -0.0101 0.0029 0.0112
##
##   sigma:  0.057
##
##       AIC      AICc       BIC
## -36.76439 -34.71311  16.05752
##
## Training set error measures:
```

```
##                              ME     RMSE        MAE      MPE     MAPE      MASE
## Training set -0.000482889 0.05534 0.04218605 116.6964 180.0324 0.6834589
##                      ACF1
## Training set 0.1729102
```

```r
summary(ets_zmodel)
```

```
## ETS(A,N,N)
##
## Call:
## ets(y = data_train_ts, model = "ZZZ")
##
##   Smoothing parameters:
##     alpha = 1e-04
##
##   Initial states:
##     l = 0.0029
##
##   sigma:  0.0569
##
##       AIC      AICc       BIC
## -48.96308 -48.86552 -38.39869
##
## Training set error measures:
##                              ME       RMSE        MAE      MPE     MAPE     MASE
## Training set 1.567182e-05 0.05666171 0.04285329 109.1957 114.6766 0.694269
##                      ACF1
## Training set 0.1682833
```

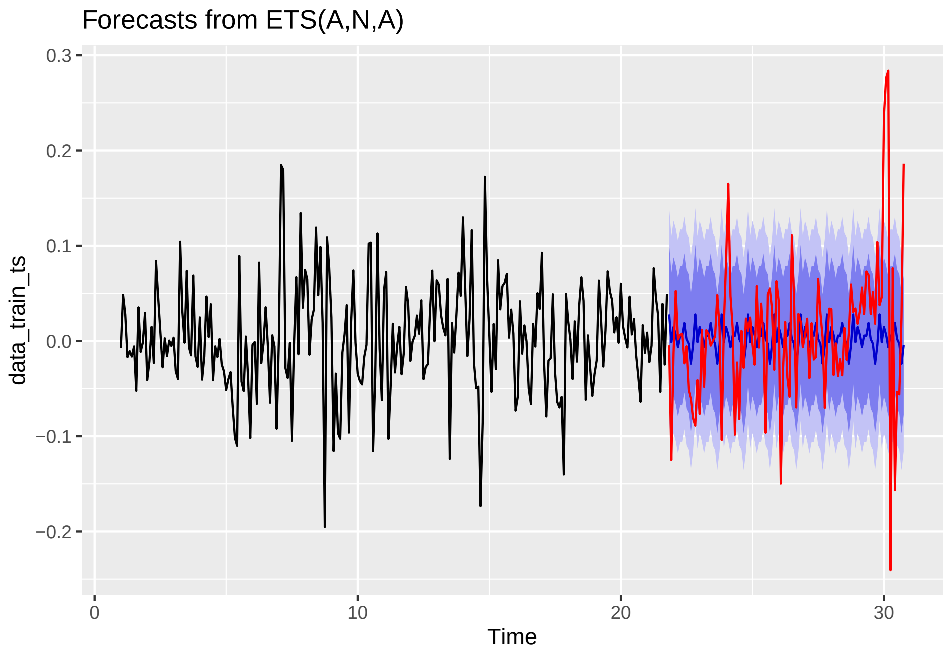
```r
plot(ets_model)
```



2.1.2 Forecasting and Plotting

```r
# Plot using log differenced price
data_test_ts <- ts(testSet$diff_log_price, start = end(data_train_ts) + c(0,1),
```

```r
                   frequency = 12)

h <- nrow(testSet)
forecast_ets <- forecast(ets_model, h = h)

autoplot(forecast_ets) + autolayer(data_test_ts, series = "Actual", color = "red")
```

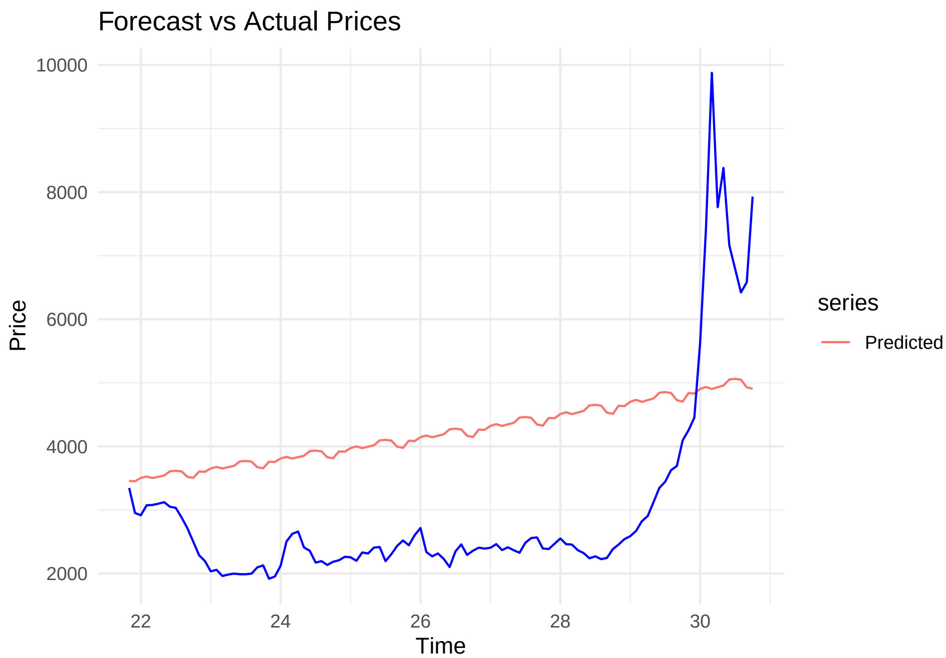


The red line is the observed actual values. The forecasted values are the central blue line within the blue shaded prediction intervals.

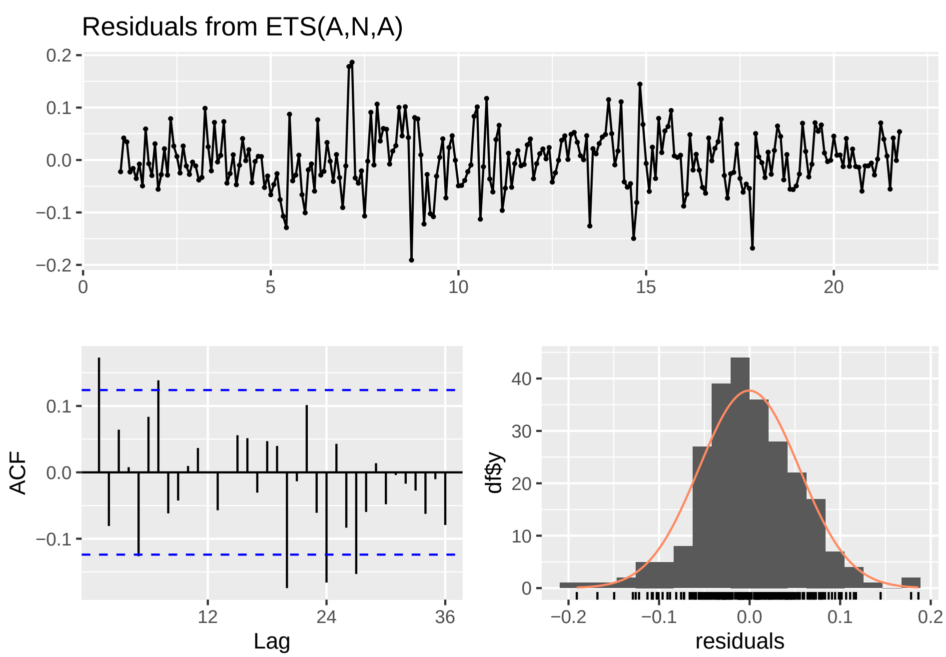
```r
last_log_price <- tail(trainSet$log_price, 1)

# Convert back to actual price
forecasted_price <- exp(cumsum(forecast_ets$mean) + last_log_price)

actual_price <- exp(testSet$log_price)
```

```r
data_test_ts <- ts(testSet$diff_log_price, start = end(data_train_ts) + c(0,1),
                   frequency = 12)

forecast_ets_ts <- ts(forecasted_price, start = start(data_test_ts), frequency = 12)
actual_ets_ts <- ts(actual_price, start = start(data_test_ts), frequency = 12)
```

```r
# Plot using actual price
autoplot(forecast_ets_ts, series = "Predicted") +
  autolayer(actual_ets_ts, series = "Actual", color = "blue") +
  ggtitle("Forecast vs Actual Prices") +
  ylab("Price") +
  xlab("Time") +
  theme_minimal()
```

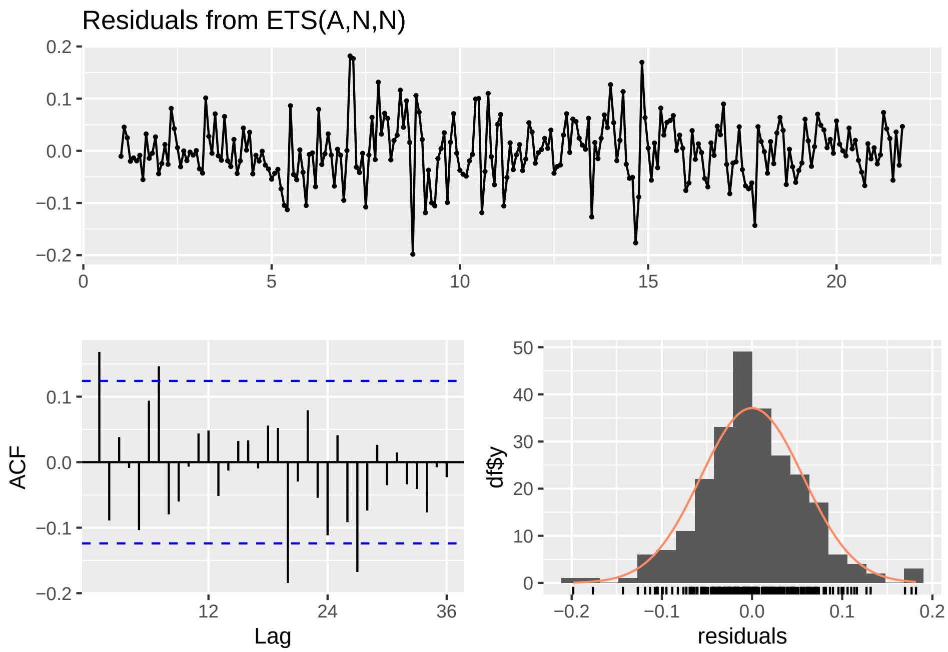



```r
checkresiduals(ets_model)
```




```
##
##  Ljung-Box test
##
## data:  Residuals from ETS(A,N,A)
## Q* = 46.672, df = 24, p-value = 0.003672
##
## Model df: 0.   Total lags used: 24
```

```r
checkresiduals(ets_zmodel)
```




```
##
##  Ljung-Box test
##
## data:  Residuals from ETS(A,N,N)
## Q* = 42.424, df = 24, p-value = 0.01156
##
## Model df: 0.   Total lags used: 24
```

2.2 ARIMAX Model

Recall that in Section 1.3.1, we have tested the acf and adf.test, and determined that we would be using the differenced price data. To fit the trainset, we evaluate p and q for ARIMA model.

```r
adf.test(trainSet$log_price)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  trainSet$log_price
## Dickey-Fuller = -2.5744, Lag order = 6, p-value = 0.334
## alternative hypothesis: stationary
```

Next, we check if applying 1st differencing is good enough

```r
adf.test(diff(trainSet$month_Price))
```

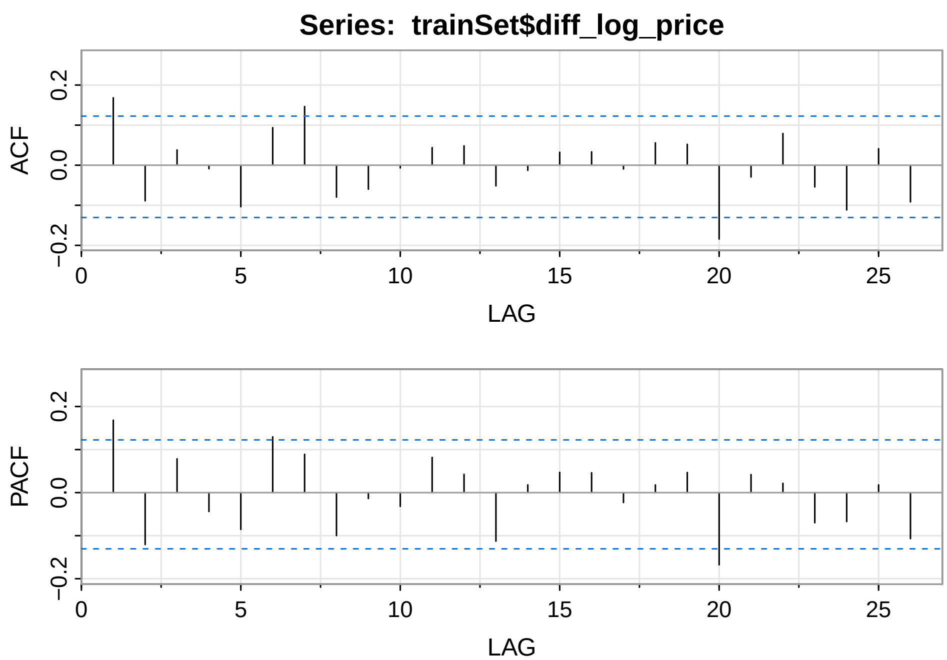
```
## Warning in adf.test(diff(trainSet$month_Price)): p-value smaller than printed
## p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  diff(trainSet$month_Price)
## Dickey-Fuller = -5.2038, Lag order = 6, p-value = 0.01
```

```
## alternative hypothesis: stationary
```

P-value is smaller than 0.01 for differenced log price, and we are

```r
acf2(trainSet$diff_log_price)
```




```
##      [,1]  [,2] [,3]  [,4]  [,5] [,6] [,7]  [,8]  [,9] [,10] [,11] [,12] [,13]
## ACF  0.17 -0.09 0.04 -0.01 -0.10 0.09 0.15 -0.08 -0.06 -0.01  0.04  0.05 -0.05
## PACF 0.17 -0.12 0.08 -0.04 -0.09 0.13 0.09 -0.10 -0.01 -0.03  0.08  0.04 -0.11
##      [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25]
## ACF  -0.01  0.03  0.03 -0.01  0.06  0.05 -0.18 -0.03  0.08 -0.05 -0.11  0.04
## PACF  0.02  0.05  0.05 -0.02  0.02  0.05 -0.17  0.04  0.02 -0.07 -0.07  0.02
##      [,26]
## ACF  -0.09
## PACF -0.11
```

2.2.1 Fit ARIMAX Model

```r
xreg_matrix <- cbind(trainSet$Avg_Temp, trainSet$exchange_rate)
colnames(xreg_matrix) <- c("Avg_Temp", "exchange_rate")
arimax_model <- Arima(trainSet$log_price, order=c(1,1,1), xreg = xreg_matrix)
summary(arimax_model)
```

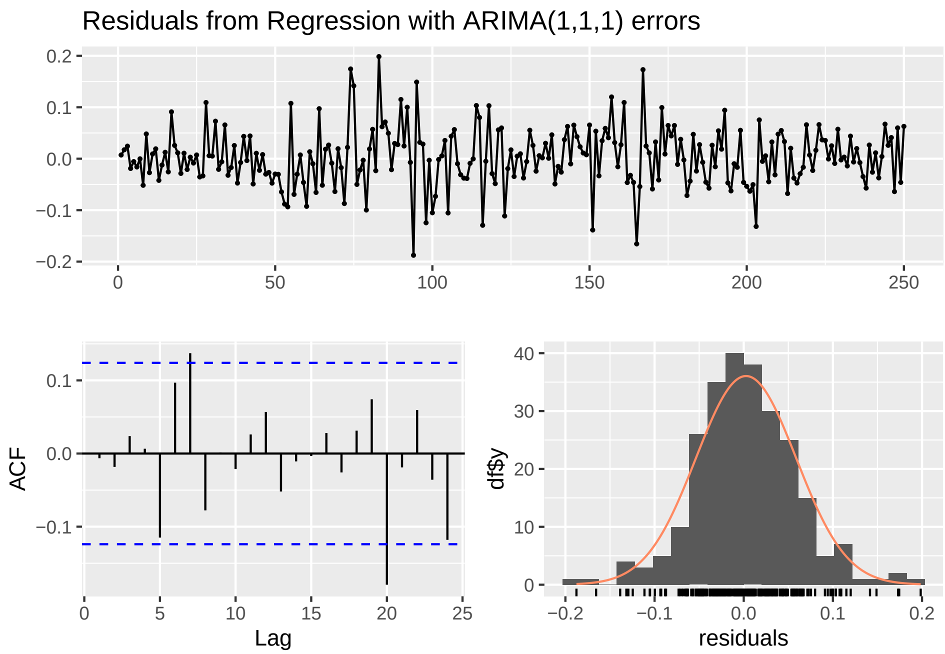
```
## Series: trainSet$log_price
## Regression with ARIMA(1,1,1) errors
##
## Coefficients:
##           ar1     ma1  Avg_Temp  exchange_rate
##       -0.2875  0.5124    0.0010         0.0296
## s.e.   0.1971  0.1743    0.0022         0.0485
##
## sigma^2 = 0.003219:  log likelihood = 363.12
## AIC=-716.24   AICc=-715.99   BIC=-698.65
##
```

```
## Training set error measures:
##                       ME       RMSE        MAE        MPE      MAPE      MASE
## Training set 0.002593798 0.05617033 0.04245345 0.03138562 0.5658759 0.9793631
##                      ACF1
## Training set -0.006404069
```

```r
AIC(arimax_model)
```

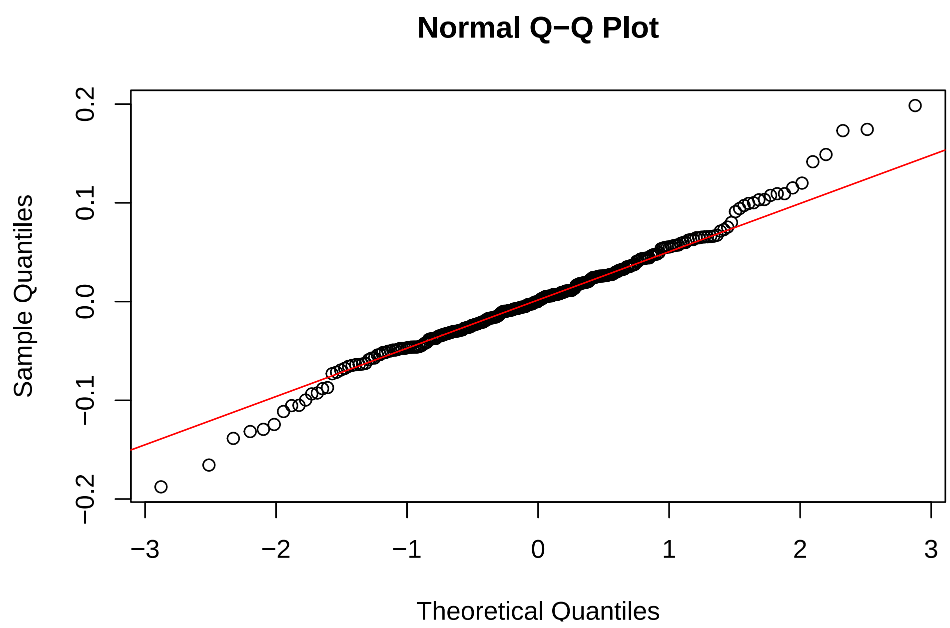
```
## [1] -716.2394
```

```r
checkresiduals(arimax_model)
```



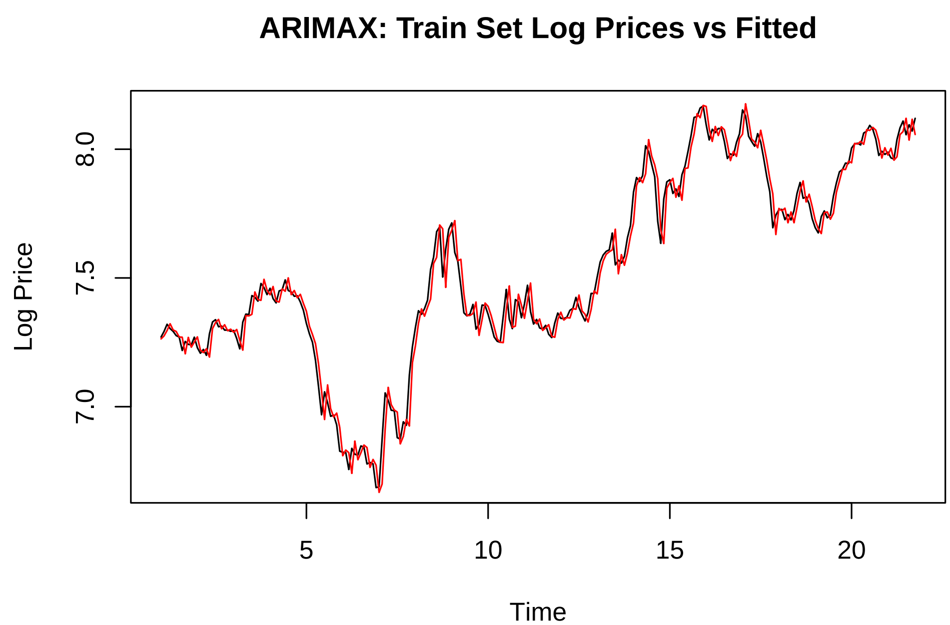

```
##
##  Ljung-Box test
##
## data:  Residuals from Regression with ARIMA(1,1,1) errors
## Q* = 12.635, df = 8, p-value = 0.125
##
## Model df: 2.   Total lags used: 10
```

```r
qqnorm(arimax_model$residuals)
qqline(arimax_model$residuals, col="red")
```



Fail to reject H_0, hence residuals of this plot do not show significant autocorrelation. - QQ-plot shows: …. - ACF shows: … - Residuals shows: …

```
arimax_train_ts <- ts(trainSet$log_price, start = start(min(trainSet$Time)), frequency = 12)
arimax_fitted_ts <- ts(fitted(arimax_model), start = start(min(trainSet$Time)), frequency = 12)
plot(arimax_train_ts, type='l', col='black', main="ARIMAX: Train Set Log Prices vs Fitted",
     ylab="Log Price", xlab="Time")
lines(arimax_fitted_ts, col='red')
```



The ARIMAX model fit the trainSet very accurately.

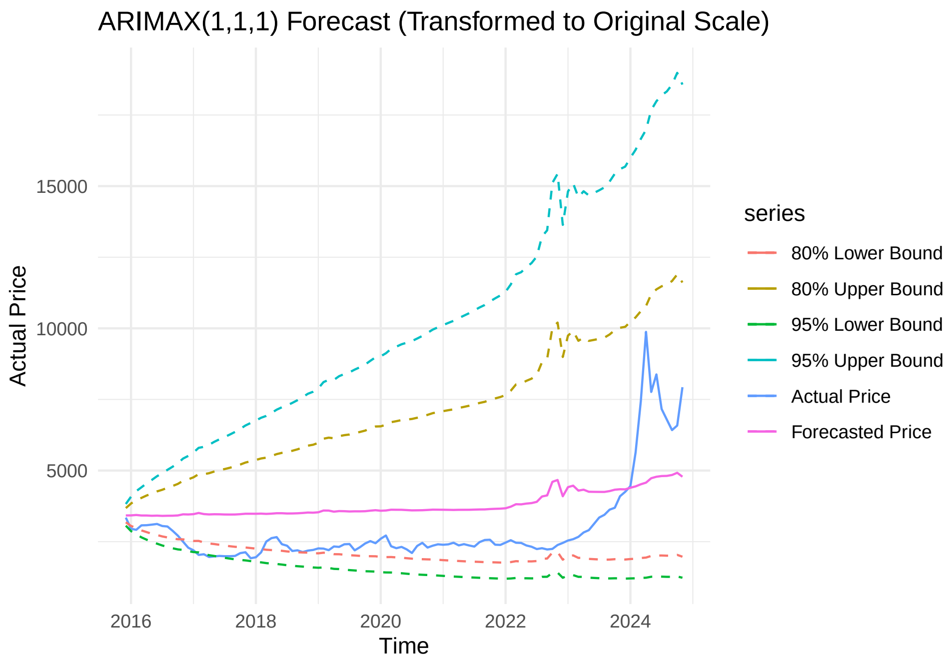
2.2.2 Forecasting With ARIMAX Model

Next we try to fit this ARIMAX model to testing set.

```
forecast_arimax_xreg <- cbind(testSet$Avg_Temp, testSet$exchange_rate)
colnames(forecast_arimax_xreg) <- c("Avg_Temp", "exchange_rate")
forecast_arimax <- forecast(arimax_model, xreg=forecast_arimax_xreg, h=nrow(testSet))
```

Then we convert the log prediction back to original price.

```
start_year <- format(min(testSet$Time), "%Y")
start_month <- format(min(testSet$Time), "%m")
start_arimax_test = c(as.numeric(start_year), as.numeric(start_month))

actual_price_arimax <- testSet$month_Price
forecasted_price_arimax <- exp(forecast_arimax$mean)
forecasted_arimax_lower95 <- exp(forecast_arimax$lower[,2])
forecasted_arimax_lower80 <- exp(forecast_arimax$lower[,1])
forecasted_arimax_upper95 <- exp(forecast_arimax$upper[,2])
forecasted_arimax_upper80 <- exp(forecast_arimax$upper[,1])

actual_arimax_ts <- ts(actual_price_arimax, start = start_arimax_test, frequency = 12)
forecast_arimax_ts <- ts(forecasted_price_arimax, start = start_arimax_test, frequency = 12)
forecasted_arimax_lower80_ts <- ts(forecasted_arimax_lower80, start = start_arimax_test, frequency = 12
forecasted_arimax_lower95_ts <- ts(forecasted_arimax_lower95, start = start_arimax_test, frequency = 12
forecasted_arimax_upper80_ts <- ts(forecasted_arimax_upper80, start = start_arimax_test, frequency = 12
forecasted_arimax_upper95_ts <- ts(forecasted_arimax_upper95, start = start_arimax_test, frequency = 12


# Plot with proper transformation
autoplot(actual_arimax_ts, series="Actual Price") +
  autolayer(forecast_arimax_ts, series="Forecasted Price") +
  autolayer(forecasted_arimax_lower95_ts, series="95% Lower Bound", linetype="dashed") +
  autolayer(forecasted_arimax_lower80_ts, series="80% Lower Bound", linetype="dashed") +
  autolayer(forecasted_arimax_upper80_ts, series="80% Upper Bound", linetype="dashed") +
  autolayer(forecasted_arimax_upper95_ts, series="95% Upper Bound", linetype="dashed") +
  ggtitle("ARIMAX(1,1,1) Forecast (Transformed to Original Scale)") +
  ylab("Actual Price") +
  xlab("Time") +
  theme_minimal()
```




```
accuracy(forecast_arimax$mean, testSet$log_price)
```

```
## [1] 0
```

```
accuracy(forecast_arimax_ts, actual_arimax_ts)
```

```
## [1] 0
```

80% lower and upper bound from forecasts create a tighter bounds for the forecasting the actual price as shown in graph. However, since our price data is non-stationary, and there is a sudden increase towards the end, the ARIMAX model, which relies on historical patterns, struggles to capture this trend, leading to poorer performance on the test set.

2.3 GARCH Model

ARCH/GARCH Model (Monthly)

```
garch_spec <- ugarchspec(
  variance.model = list(model = "sGARCH", garchOrder = c(1, 1)),
  mean.model = list(armaOrder = c(1, 1), include.mean = TRUE),
  distribution.model = "norm"
)
garch_fit <- ugarchfit(spec = garch_spec, data = trainSet$diff_log_price)
garch_fit
```

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : norm
```

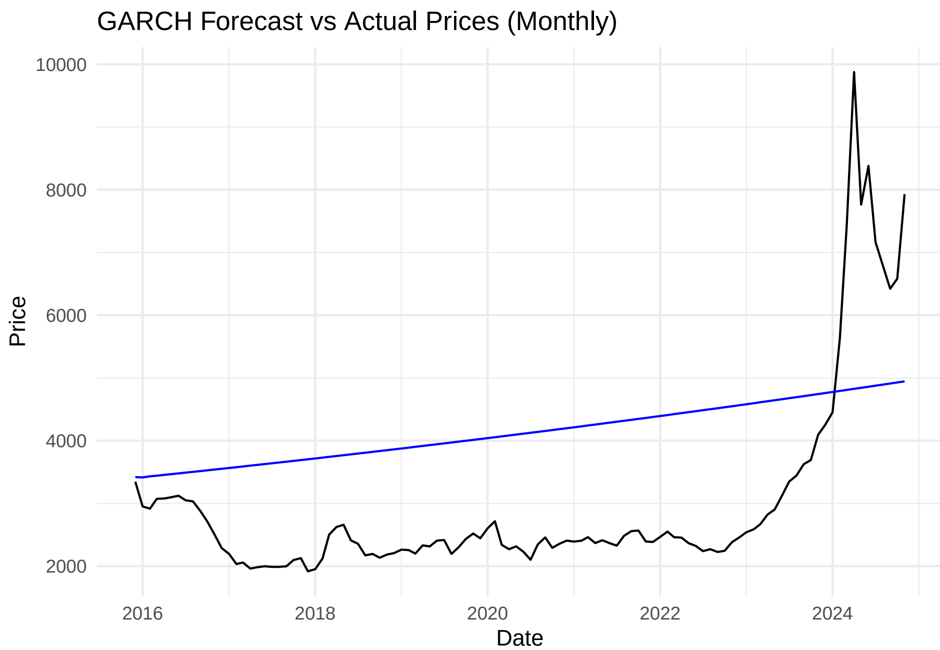
```
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.003483    0.003505  0.99364 0.320399
## ar1    -0.361441    0.218279 -1.65586 0.097750
## ma1     0.530997    0.194176  2.73462 0.006245
## omega   0.000203    0.000140  1.44321 0.148961
## alpha1  0.123035    0.056972  2.15958 0.030805
## beta1   0.817532    0.078868 10.36586 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.003483    0.003503   0.9944 0.320027
## ar1    -0.361441    0.167744  -2.1547 0.031184
## ma1     0.530997    0.145785   3.6423 0.000270
## omega   0.000203    0.000130   1.5581 0.119212
## alpha1  0.123035    0.042833   2.8724 0.004073
## beta1   0.817532    0.068735  11.8939 0.000000
##
## LogLikelihood : 377.0571
##
## Information Criteria
## ------------------------------------
##
## Akaike       -2.9685
## Bayes        -2.8839
## Shibata      -2.9696
## Hannan-Quinn -2.9344
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.2817  0.5956
## Lag[2*(p+q)+(p+q)-1][5]    0.8050  1.0000
## Lag[4*(p+q)+(p+q)-1][9]    4.1960  0.6445
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                  0.0002722  0.9868
## Lag[2*(p+q)+(p+q)-1][5] 3.0820246  0.3923
## Lag[4*(p+q)+(p+q)-1][9] 4.5840493  0.4929
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     2.868 0.500 2.000 0.09035
## ARCH Lag[5]     3.749 1.440 1.667 0.19802
## ARCH Lag[7]     4.452 2.315 1.543 0.28605
##
```

```
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  1.2432
## Individual Statistics:
## mu     0.06455
## ar1    0.05972
## ma1    0.14971
## omega  0.13292
## alpha1 0.10723
## beta1  0.13036
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.49 1.68 2.12
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           1.0710 0.2852
## Negative Sign Bias  0.7394 0.4604
## Positive Sign Bias  0.2215 0.8249
## Joint Effect        2.0718 0.5576
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     26.96       0.1056
## 2    30     26.24       0.6126
## 3    40     40.24       0.4151
## 4    50     54.00       0.2892
##
##
## Elapsed time : 0.04173684
```

2.3.2 GARCH Forecast

```r
garch_forecast <- ugarchforecast(garch_fit, n.ahead = length(testSet$diff_log_price))
last_train_price <- tail(trainSet$month_Price, 1)
garch_forecast_prices <- last_train_price * exp(cumsum(as.numeric(fitted(garch_forecast))))
garch_df <- tibble(
  Date = testSet$Time,
  Price = garch_forecast_prices
)
test_df <- tibble(
  Date = testSet$Time,
  Price = testSet$month_Price
)
```

```r
ggplot() +
  geom_line(data = test_df, aes(x = Date, y = Price), color = "black") +
  geom_line(data = garch_df, aes(x = Date, y = Price), color = "blue") +
  labs(title = "GARCH Forecast vs Actual Prices (Monthly)", y = "Price", x = "Date") +
  theme_minimal()
```



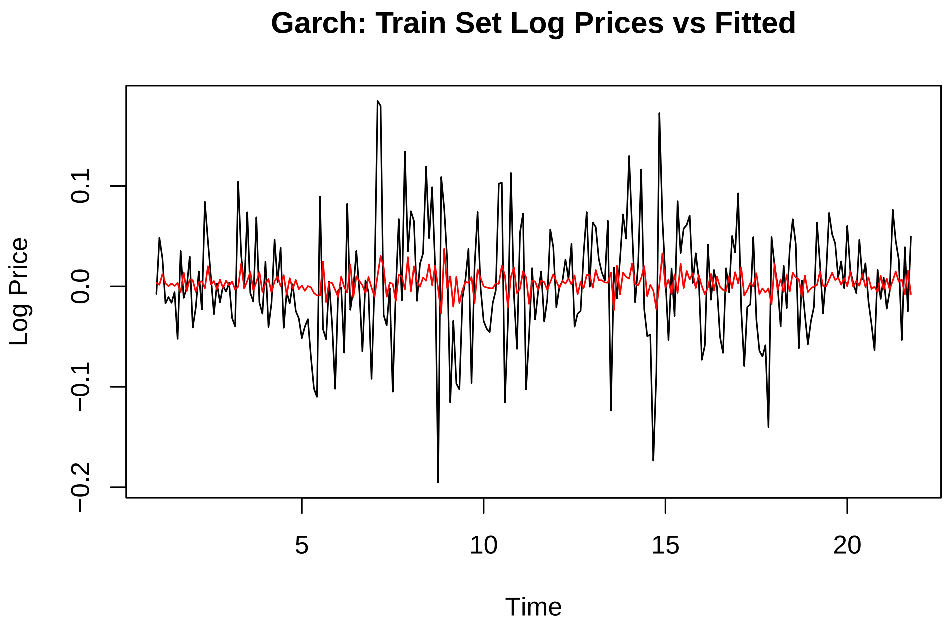

```r
garch_predicted_vol <- sigma(garch_forecast)
actual_vol <- abs(testSet$log_price)  # Assuming you have log-returns
rmse_garch_test <- sqrt(mean((garch_predicted_vol - actual_vol)^2, na.rm = TRUE))
print(rmse_garch_test)
```

```
## [1] 7.869751
```

```r
garch_train_vol <- sigma(garch_fit)
actual_train_vol <- abs(trainSet$log_price)  # Assuming you have log-returns
rmse_garch <- sqrt(mean((garch_train_vol - actual_train_vol)^2, na.rm = TRUE))
print(rmse_garch)
```

```
## [1] 7.488708
```

```r
train_ts <- ts(trainSet$diff_log_price, start = start(min(trainSet$Time)), frequency = 12)
garch_fitted_ts <- ts(fitted(garch_fit), start = start(min(trainSet$Time)), frequency = 12)
plot(train_ts, type='l', col='black', main="Garch: Train Set Log Prices vs Fitted",
     ylab="Log Price", xlab="Time")
lines(garch_fitted_ts, col='red')
```



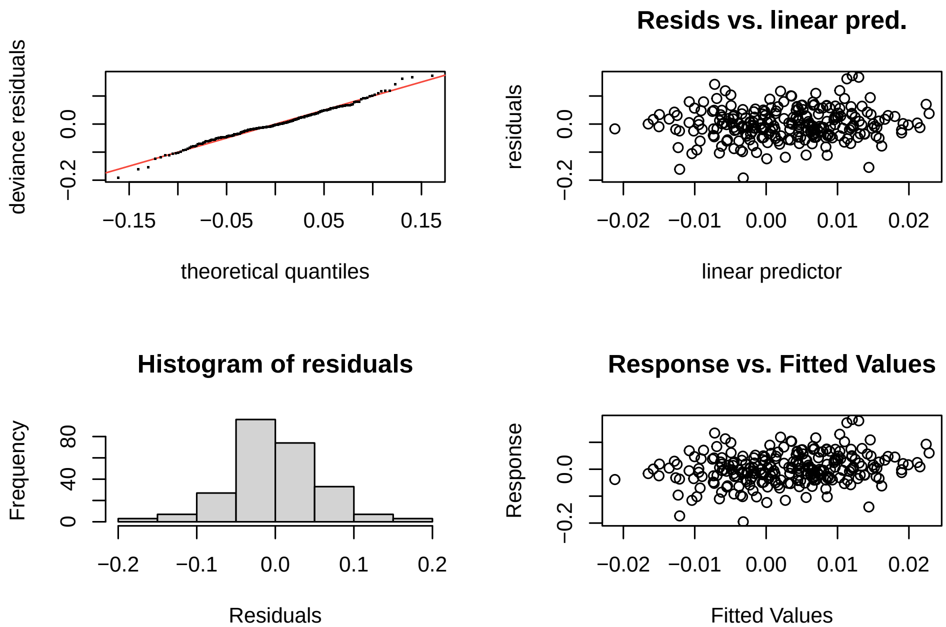
2.5 GAM Model

2.5.1 Fit Model

```
# Uses simple smoothing splines for variables
trainSet$monthFac <- as.factor(format(trainSet$Time, "%m"))
trainSet$month_num <- as.numeric(trainSet$monthFac)

gam_basic <- gam(diff_log_price ~ s(month_num, bs="cc", k=12) + s(Avg_Temp) + s(exchange_rate),
                 data = trainSet, method = "REML")
```

```
gam.check(gam_basic)
```

2.5.1.1 Basic Model




```
##
## Method: REML   Optimizer: outer newton
## full convergence after 10 iterations.
## Gradient range [-0.0001167052,0.0001905195]
## (score -350.1765 & scale 0.003138989).
## Hessian positive definite, eigenvalue range [3.540616e-06,123.5188].
## Model rank =  29 / 29
##
## Basis dimension (k) checking results. Low p-value (k-index<1) may
## indicate that k is too low, especially if edf is close to k'.
##
##                      k'   edf k-index p-value
## s(month_num)      10.00  3.05    1.05   0.795
## s(Avg_Temp)        9.00  1.00    1.06   0.820
## s(exchange_rate)   9.00  1.00    0.89   0.045 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(gam_basic)
```

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## diff_log_price ~ s(month_num, bs = "cc", k = 12) + s(Avg_Temp) +
##     s(exchange_rate)
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.002929   0.003543   0.827    0.409
##
## Approximate significance of smooth terms:
##                    edf Ref.df     F p-value
## s(month_num)     3.054 10.000 0.795  0.0278 *
```

```
## s(Avg_Temp)      1.001  1.001 1.289  0.2571
## s(exchange_rate) 1.000  1.000 0.694  0.4055
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.0261   Deviance explained = 4.59%
## -REML = -350.18  Scale est. = 0.003139  n = 250
```

```r
trainSet$dateInt = as.integer(trainSet$Time)
trainSet$Time_num <- as.numeric(trainSet$Time) / 365  # Convert to years
gam_year <- gam(diff_log_price ~ s(month_num, bs="cc", k=12) +
                  sinpi(dateInt / 182.625) + cospi(dateInt / 182.625) +
                  sinpi(dateInt / 91.3125) + cospi(dateInt / 91.3125) +
                  s(Avg_Temp) +  s(exchange_rate), data = trainSet, method = "REML")
```

```r
summary(gam_basic)
```

2.5.1.2 Complex Model

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## diff_log_price ~ s(month_num, bs = "cc", k = 12) + s(Avg_Temp) +
##     s(exchange_rate)
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.002929   0.003543   0.827    0.409
##
## Approximate significance of smooth terms:
##                    edf Ref.df     F p-value
## s(month_num)     3.054 10.000 0.795  0.0278 *
## s(Avg_Temp)      1.001  1.001 1.289  0.2571
## s(exchange_rate) 1.000  1.000 0.694  0.4055
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.0261   Deviance explained = 4.59%
## -REML = -350.18  Scale est. = 0.003139  n = 250
```

```r
summary(gam_year)
```

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## diff_log_price ~ s(month_num, bs = "cc", k = 12) + sinpi(dateInt/182.625) +
##     cospi(dateInt/182.625) + sinpi(dateInt/91.3125) + cospi(dateInt/91.3125) +
##     s(Avg_Temp) + s(exchange_rate)
##
## Parametric coefficients:
```

```
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)             2.936e-03  3.542e-03   0.829   0.4081
## sinpi(dateInt/182.625)  8.070e-03  8.529e-03   0.946   0.3450
## cospi(dateInt/182.625)  9.264e-03  1.058e-02   0.876   0.3820
## sinpi(dateInt/91.3125) -7.343e-05  5.084e-03  -0.014   0.9885
## cospi(dateInt/91.3125)  1.449e-02  6.045e-03   2.397   0.0173 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                        edf Ref.df     F p-value
## s(month_num)     0.0003453 10.000 0.000   0.707
## s(Avg_Temp)      1.0003821  1.001 0.289   0.592
## s(exchange_rate) 1.0000069  1.000 0.820   0.366
##
## R-sq.(adj) =  0.0271   Deviance explained = 5.06%
## -REML = -338.66  Scale est. = 0.0031356  n = 250
```

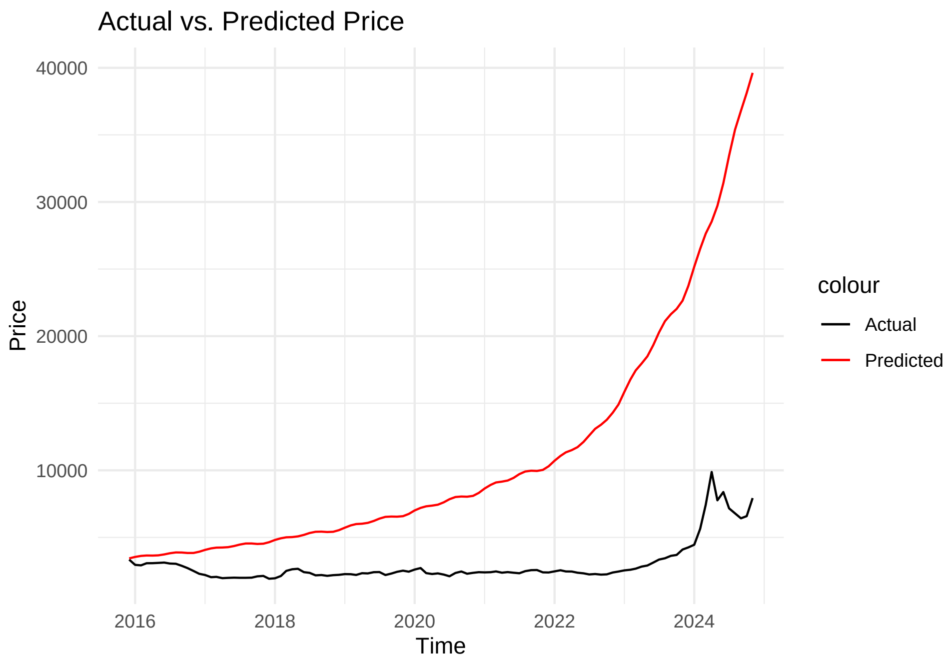
```r
# plot(gam_complex, pages = 1, shade = TRUE)
```

```r
# gam.check(gam_complex)
```

2.5.2 Forecast and Plot

```r
testSet$month_num <- as.numeric(format(testSet$Time, "%m"))
testSet$Time_num <- as.numeric(difftime(testSet$Time, min(trainSet$Time), units="days")) / 365
testSet$dateInt = as.integer(testSet$Time)

testSet$pred_log <- predict(gam_year, newdata = testSet)
testSet$pred_log_price <- last_log_price + cumsum(testSet$pred_log)
testSet$pred_price <- exp(testSet$pred_log_price)

# testSet$pred_price <- exp(log(last_price) + cumsum(testSet$pred_log))
# testSet$pred_price <- exp(cumsum(testSet$pred_log) + last_log_price)
```

```r
ggplot(testSet, aes(x = Time)) +
  geom_line(aes(y = month_Price, color = "Actual")) +
  geom_line(aes(y = pred_price, color = "Predicted")) +
  labs(title = "Actual vs. Predicted Price",
       x = "Time", y = "Price") +
  scale_color_manual(values = c("Actual" = "black", "Predicted" = "red")) +
  theme_minimal()
```




```
rmse(testSet$month_Price, testSet$pred_price)
```

```
## [1] 10954.54
```

2.6 XGBoost Model

```
trainSet$Lag1 <- lag(trainSet$diff_log_price, 1)
trainSet$Lag12 <- lag(trainSet$diff_log_price, 12)
testSet$Lag1 <- lag(testSet$diff_log_price, 1)
testSet$Lag12 <- lag(testSet$diff_log_price, 12)

predictors <- c("Time", "Avg_Temp", "exchange_rate", "Lag1", "Lag12")
target <- "diff_log_price"
```

```
train_data <- trainSet[complete.cases(trainSet[, c(predictors, target)]), ]
test_data <- testSet[complete.cases(testSet[, predictors]), ]

train_data <- train_data |>
  mutate(across(all_of(predictors), as.numeric))
test_data <- test_data |>
  mutate(across(all_of(predictors), as.numeric))

# Convert to DMatrix (XGBoost's optimized format)
dtrain <- xgb.DMatrix(
  data = as.matrix(train_data[, predictors]),
  label = train_data[[target]]
)
```

```
params <- list(
  objective = "reg:squarederror",  # For regression
  eta = 0.05,                      # Learning rate (lower for time series)
  max_depth = 6,                   # Tree depth (avoid overfitting)
  subsample = 0.8,                 # Random subset of data per tree
```

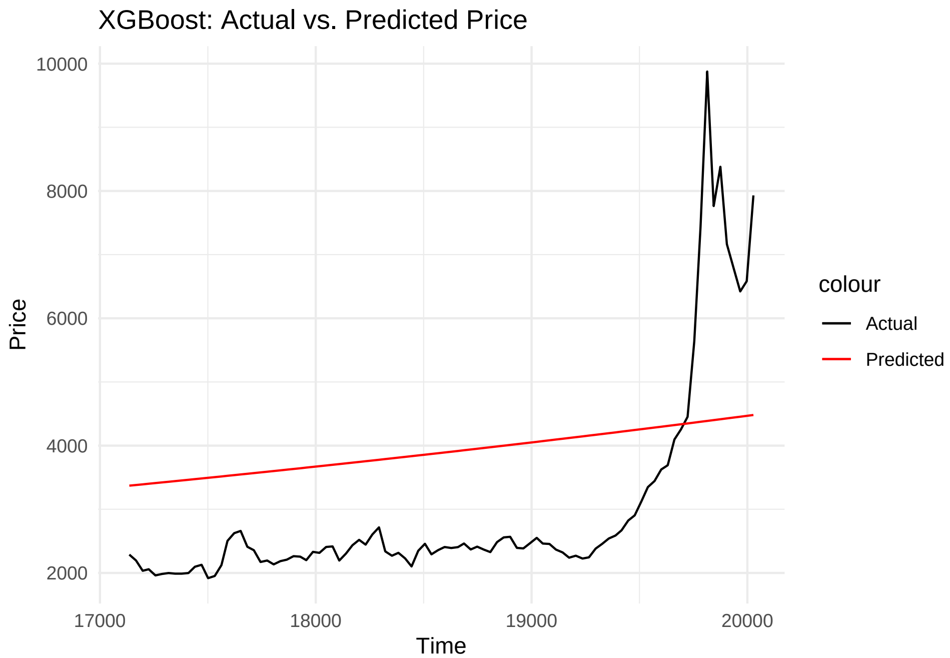
```r
  colsample_bytree = 0.8,           # Random subset of features per tree
  gamma = 1,                        # Minimum loss reduction for splits
  min_child_weight = 5              # Prevent overfitting to small groups
)

set.seed(123)
xgb_model <- xgb.train(
  params,
  data = dtrain,
  nrounds = 1000,                   # Large number (early stopping will handle)
  watchlist = list(train = dtrain),
  early_stopping_rounds = 50,       # Stop if no improvement for 50 rounds
  print_every_n = 10
)
```

```
## [1]  train-rmse:0.475762
## Will train until train_rmse hasn't improved in 50 rounds.
##
## [11] train-rmse:0.289145
## [21] train-rmse:0.179561
## [31] train-rmse:0.117399
## [41] train-rmse:0.084142
## [51] train-rmse:0.068493
## [61] train-rmse:0.061738
## [71] train-rmse:0.059145
## [81] train-rmse:0.058259
## [91] train-rmse:0.057915
## [101]    train-rmse:0.057826
## [111]    train-rmse:0.057774
## [121]    train-rmse:0.057760
## [131]    train-rmse:0.057755
## [141]    train-rmse:0.057752
## [151]    train-rmse:0.057749
## [161]    train-rmse:0.057748
## [171]    train-rmse:0.057748
## [181]    train-rmse:0.057749
## [191]    train-rmse:0.057748
## [201]    train-rmse:0.057748
## [211]    train-rmse:0.057748
## [221]    train-rmse:0.057750
## [231]    train-rmse:0.057749
## [241]    train-rmse:0.057748
## [251]    train-rmse:0.057749
## [261]    train-rmse:0.057748
## Stopping. Best iteration:
## [214]    train-rmse:0.057748
```

```r
dtest <- xgb.DMatrix(as.matrix(test_data[, predictors]))
test_data$pred_diff_log <- predict(xgb_model, dtest)

# Convert to actual price predictions
test_data <- test_data %>%
  mutate(
    pred_log_price = last_log_price + cumsum(pred_diff_log),  # Only if modeling differences
```

```r
    pred_price = exp(pred_log_price)
  )
```

```r
ggplot(test_data, aes(x = Time)) +
  geom_line(aes(y = month_Price, color = "Actual")) +
  geom_line(aes(y = pred_price, color = "Predicted")) +
  labs(title = "XGBoost: Actual vs. Predicted Price",
       x = "Time", y = "Price") +
  scale_color_manual(values = c("Actual" = "black", "Predicted" = "red")) +
  theme_minimal()
```




```r
# xgb = XGBClassifier(random_state=42)
# xgb.fit(predictors, target)
```