

How to use U1 symmetry in specific given symmetry model?

tensor  $\rightarrow$  U1 tensor  $\begin{matrix} 0,1 \\ \text{---} \\ 0,1 \end{matrix}$  contraction gr vamps

AD

particle number  $\begin{cases} \text{finite size} & p=0, 1, 2, \dots \\ \text{infinite size} & p=\frac{1}{2} \quad p=1 \quad p=\frac{7}{8} \dots \end{cases}$   
 $S_2 \quad S_2 = 0, \pm 1, \pm 2$

How to set iMPS or iPEPS blocks and their environment?

(1) finite size 1-D MPS  $\begin{matrix} 2 & 0,1 & 2 \\ \text{---} & & \\ & 0,1 & \\ & 2 & \end{matrix} \rightarrow \text{---|---|---|---|}$

$q_1 \rightarrow q_3$   $f$ : fluctuation  
 $f = \sum_i q_i^{\text{out}} - \sum_i q_i^{\text{in}}$

$$f = q_3 + q_3 - q_1$$

$f = -1$   $\begin{matrix} \text{---} & 0 \\ | & \\ 0 & \end{matrix}$   $2 \times 2 \times 2 = 8$

$f = 0$   $\begin{matrix} 0 & \text{---} & 0 \\ | & & | \\ 0 & & 0 \end{matrix}$   $\begin{matrix} 1 & \text{---} & 0 \\ | & & | \\ 0 & & 0 \end{matrix}$   $\begin{matrix} 1 & \text{---} & 1 \\ | & & | \\ 0 & & 0 \end{matrix}$   $3$   
 $f = 1$   $\begin{matrix} 0 & \text{---} & 0 \\ | & & | \\ 1 & & 0 \end{matrix}$   $\begin{matrix} 0 & \text{---} & 1 \\ | & & | \\ 0 & & 1 \end{matrix}$   $\begin{matrix} 1 & \text{---} & 1 \\ | & & | \\ 1 & & 1 \end{matrix}$   $1 \times 1 \times 1$   
 $f = 2$   $\begin{matrix} 0 & \text{---} & 1 \\ | & & | \\ 1 & & 1 \end{matrix}$   $\begin{matrix} 0 & \text{---} & 0 \\ | & & | \\ 0 & & 1 \end{matrix}$   $\begin{matrix} 1 & \text{---} & 0 \\ | & & | \\ 0 & & 1 \end{matrix}$

$\begin{matrix} 0 & 0 & 0 \\ | & & | \\ 0 & & 1 \end{matrix}$

2-site period boundary

$d=2$   $\begin{matrix} 0 & 0 \\ | & | \\ 0 & 0 \end{matrix}$   $\begin{matrix} 0 & 1 \\ | & | \\ 0 & 1 \end{matrix}$   $\begin{matrix} 1 & 0 \\ | & | \\ 1 & 0 \end{matrix}$   $\begin{matrix} 1 & 1 \\ | & | \\ 1 & 1 \end{matrix}$

$p=1$   $f=1$   $f=0$   
 $\begin{matrix} 0 & 1 & 0 \\ | & | & | \\ 0 & 1 & 0 \end{matrix}$   $\begin{matrix} 0 & 1 \\ | & | \\ 0 & 1 \end{matrix}$   $\begin{matrix} 0 & 1 \\ | & | \\ 0 & 1 \end{matrix}$   
 $f=0$   $f=1$   
 $\begin{matrix} 1 & 0 & 1 \\ | & | & | \\ 1 & 0 & 1 \end{matrix}$   $8 \rightarrow 2$

(2) infinite size  $N \rightarrow \infty$

$p=1$   $\text{---|---|---|---|}$   $\begin{matrix} 0 & \text{---} & 0 \\ | & & | \\ 0 & & 0 \end{matrix}$

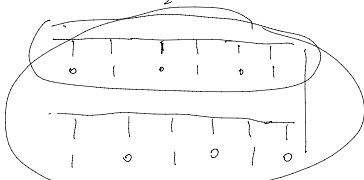
$p=0$   $\text{---|---|---|---|}$   $\begin{matrix} 0 & \text{---} & 0 \\ | & & | \\ 0 & & 0 \end{matrix}$

$p=\frac{1}{2}$   $\text{---|---|---|---|}$   $\begin{matrix} 0 & 1 \\ | & | \\ 0 & 1 \end{matrix}$

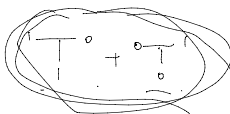
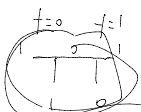
1 2

single qubit

Σ

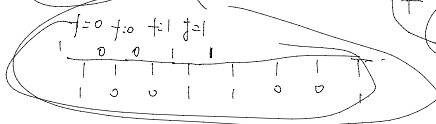
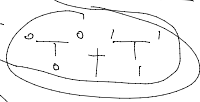


UI tensor



$$\frac{f=0}{f=1} > f=0$$

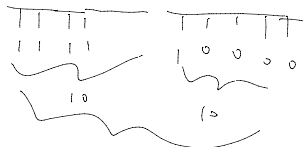
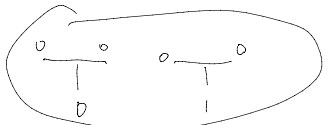
+



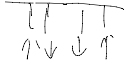
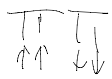
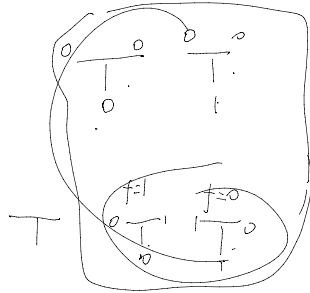
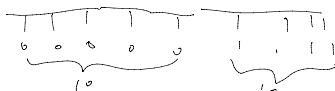
$p = \frac{1}{2}$



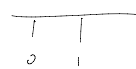
T



20



T



specific

p

mix

single

f=0

$$A \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

QR

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Q

x

R

SVD

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \times \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

1/1

1/1

1/1

1/1

1/1

1/1

$$= \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} - & - & - \end{pmatrix} \times \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$$

$$U \quad \times \quad S \quad \times \quad V^T$$

$f=0,1$

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$Q \quad R$$

shift

$$0,1 \rightarrow -1,1$$

$$f=0,1 \Rightarrow f=0$$

$$0 \begin{array}{|c|c|} \hline f=1 & f=0 \\ \hline 1 & 1 \\ \hline \end{array}$$

$$\rightarrow 0 \begin{array}{|c|c|} \hline f=0 & f=0 \\ \hline 1 & 1 \\ \hline \end{array}$$

$$p = \frac{1}{2}$$

$$S_2 = 0$$

$$S_2 = \frac{1}{2} \quad 0,1$$

$$p = \frac{1}{2}$$

$$p = \frac{1}{2}$$

$$S_2 =$$

$$0 \rightarrow 1$$

$$0 \rightarrow 1$$

$$-2$$

$$\begin{array}{|c|c|c|c|c|} \hline -2 & -1 & 0 & 1 & 2 \\ \hline \end{array} \quad A \quad \text{xxx}$$

$$\begin{array}{|c|c|c|c|c|} \hline -2 & -1 & 0 & 1 & 2 \\ \hline -2 & & & & \\ -1 & & & & \\ 0 & & & & \\ 1 & & & & \\ 2 & & & & \end{array}$$

$$\begin{array}{|c|c|c|c|c|} \hline -2 & -1 & 0 & 1 & 2 \\ \hline -2 & & & & \\ -1 & & & & \\ 0 & & & & \\ 1 & & & & \\ 2 & & & & \end{array}$$

$$\begin{array}{|c|c|c|} \hline -2 & -1 & 0 \\ \hline \end{array} \quad \begin{array}{|c|c|c|} \hline -2 & -1 & 0 \\ \hline \end{array} \quad \begin{array}{|c|c|c|} \hline -2 & -1 & 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline -2 & -1 & 0 \\ \hline \end{array} \quad \begin{array}{|c|c|c|} \hline -2 & -1 & 0 \\ \hline \end{array} \quad \begin{array}{|c|c|c|} \hline -2 & -1 & 0 \\ \hline \end{array}$$

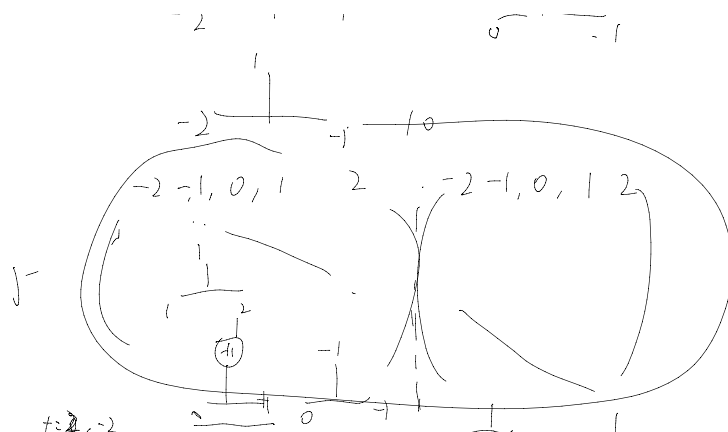
$$S_2 = \begin{array}{|c|c|} \hline f=2 & f=2 \\ \hline \end{array}$$

$$\text{xxx}$$

$$\begin{array}{|c|c|} \hline f=2 & f=2 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline -1 & 0 \\ \hline -2 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 1 & -1 \\ \hline 0 & 1 \\ \hline \end{array}$$



$$f=0 \quad S_2=0$$

$$f=12 \quad S_2=0$$

$$f=2, -2$$

$$f=0 \quad S_2=0$$

$$f=-2, 2 \quad f \uparrow \quad -f \downarrow$$

$$f=-3 \quad p=1$$

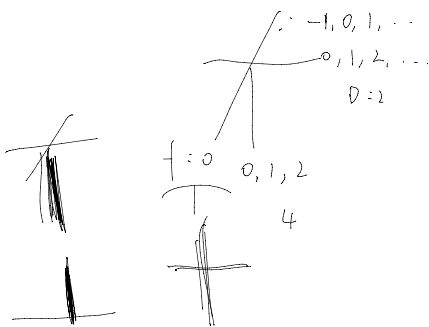
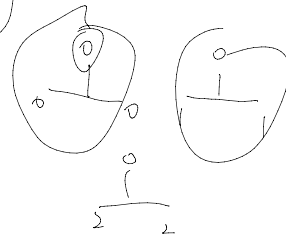
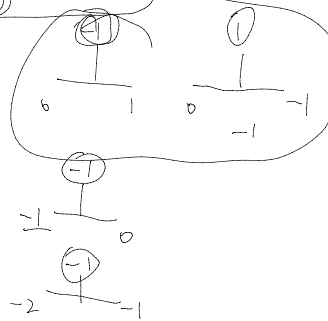
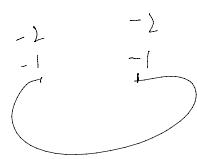
$$f=2$$

$$S_2=0 \quad S_2=0$$

$$-1, 0, 1$$

$$3 \cdot \left(\frac{n}{3}\right)^2$$

$$\left(\frac{n}{3}\right)^2$$



0	0	0
↑	1	+1
↓	1	-1
↑↓	2	0

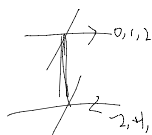
$$q=0 \quad d=1 \quad x=32$$

$$q=1 \quad d=2 \quad x=16$$

$$q=2 \quad d=1$$

$$x=16$$

$$x=16$$



→

