nonlocal MPO and SLM optimization

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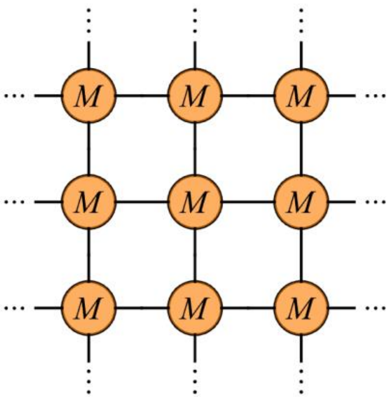
AFIsing resident entropy



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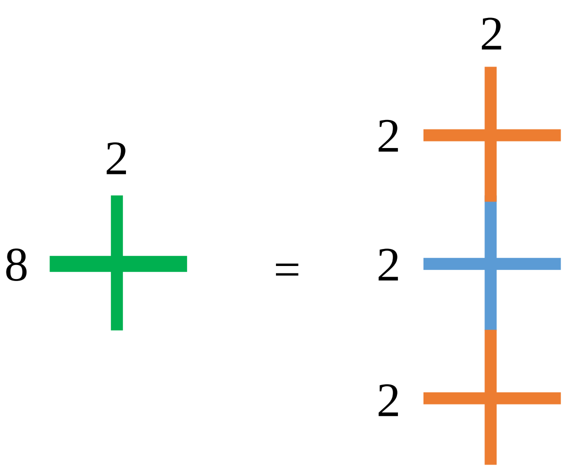
十[1,1,2,2]=1.0
十[1,1,1,1]=1.0
十[1,2,1,1]=1.0
十[2,2,2,2]=1.0
十[2,1,2,2]=1.0

Non-Hermitian and non-normal

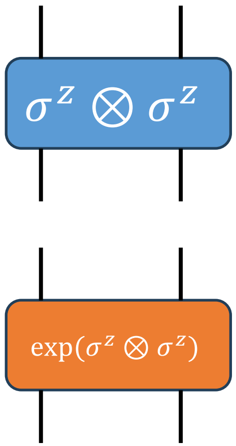
十′ ← P十P^{-1}

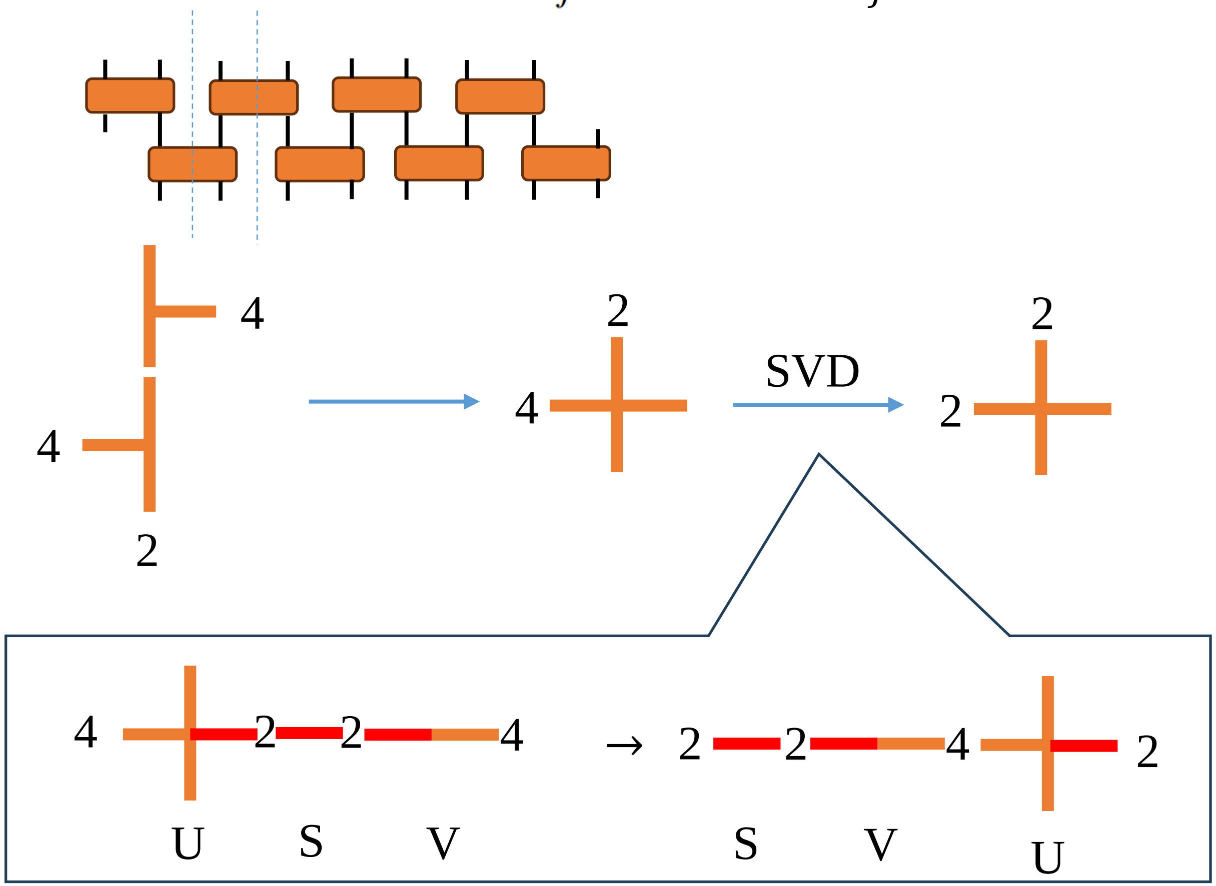
$$\mathcal{P} = \exp(\tau Q) = \exp(-\tau \sum_j \sigma_j^z \sigma_{j+1}^z)$$



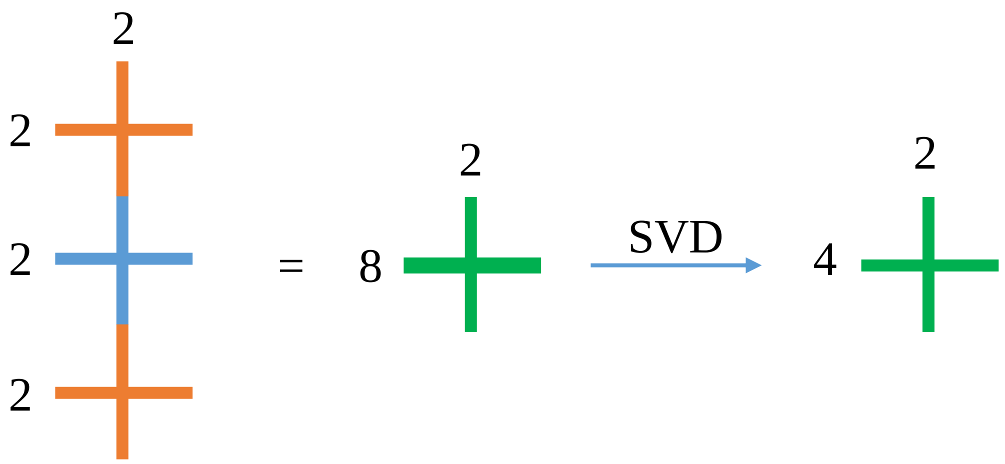
vertical nonlocal gauge P

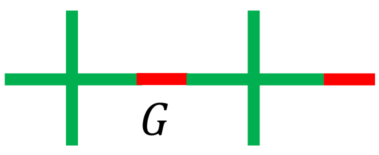
$$\mathcal{P} = \exp(\tau Q) = \exp\left(-\tau \sum_j \sigma_j^z \sigma_{j+1}^z\right) = \prod_j \exp(-\tau \sigma_j^z \sigma_{j+1}^z)$$





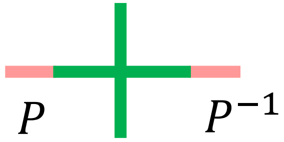
Horizontal local gauge





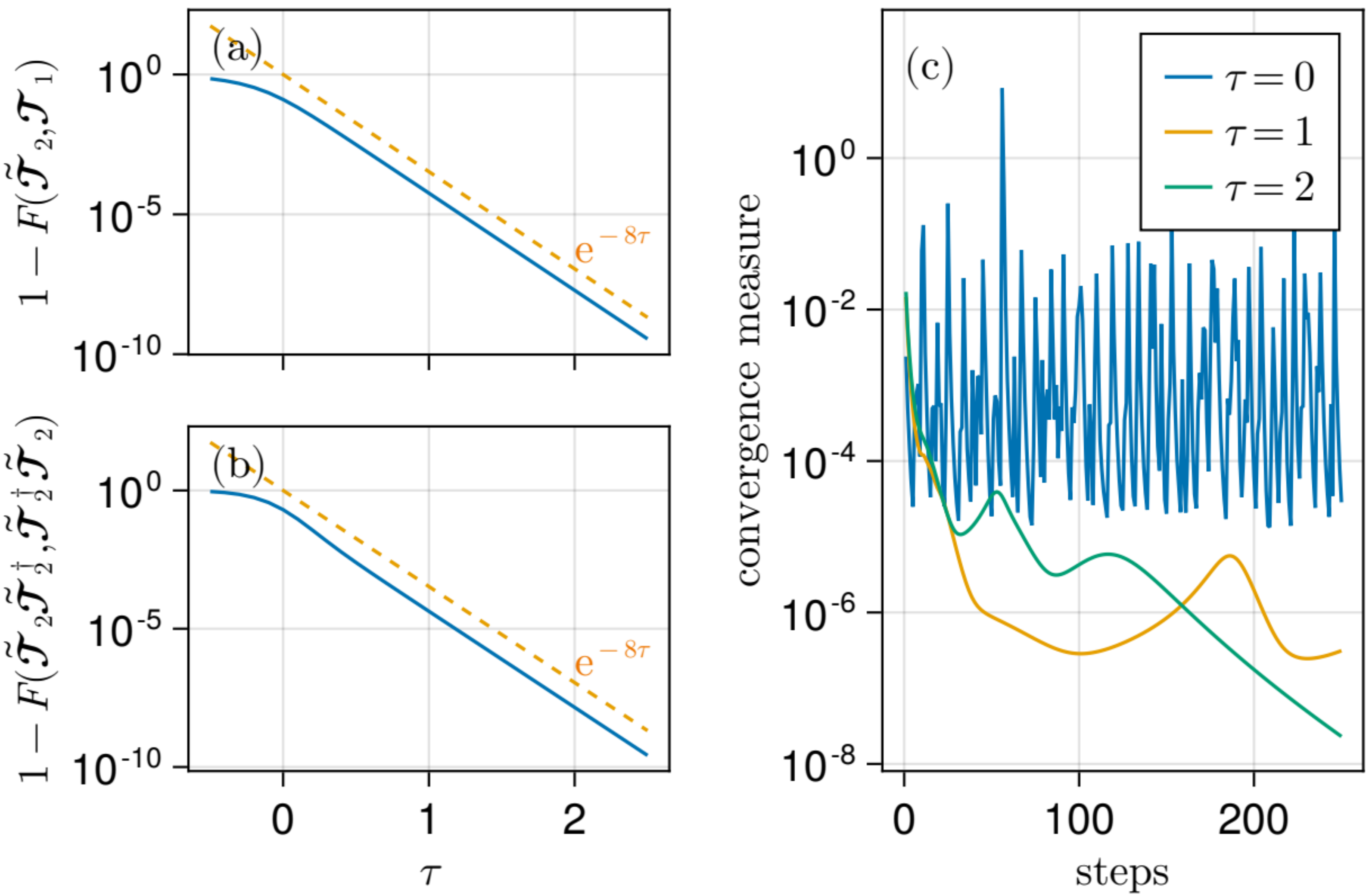
$G = e^p = U\Lambda U'$ $\quad P = \sqrt{\Lambda}U'$

$p \leftarrow p + p'$ $\quad P^{-1} = U\sqrt{\Lambda^{-1}}$

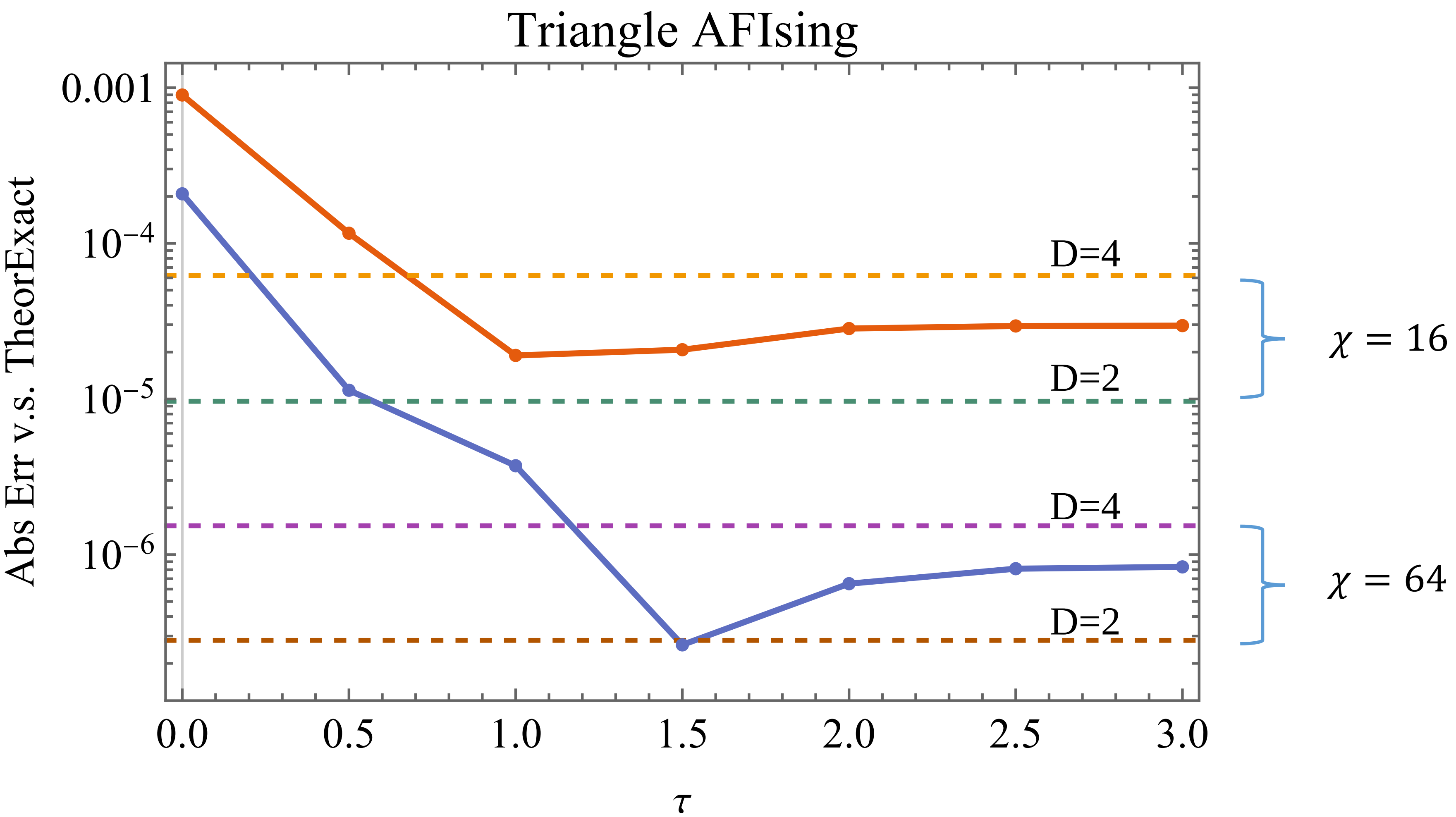


Precondition is important!

normality


(a)
(b)
(c)
10^0
10^{-5}
10^{-10}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
$e^{-8\tau}$
$1 - F(\tilde{\mathcal{T}}_2, \mathcal{T}_1)$
$1 - F(\tilde{\mathcal{T}}_2 \tilde{\mathcal{T}}_2^\dagger, \tilde{\mathcal{T}}_2^\dagger \tilde{\mathcal{T}}_2)$
τ
0
1
2
convergence measure
steps
100
200
$\tau = 0$
$\tau = 1$
$\tau = 2$

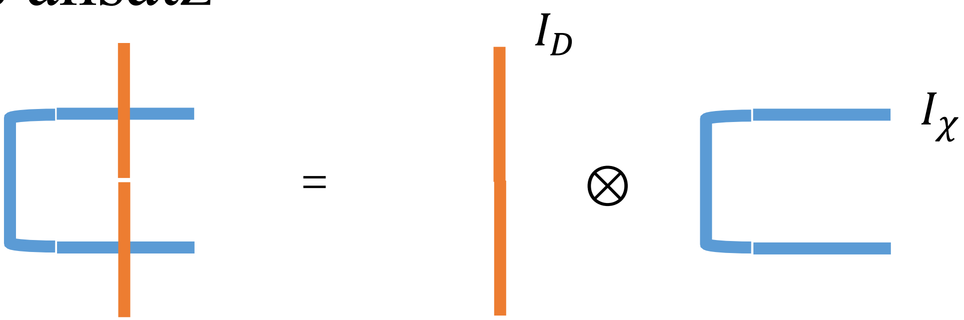
Free energy



Triangle AFIsing
Abs Err v.s. TheorExact
0.001
10^{-4}
10^{-5}
10^{-6}
D=4
D=2
D=4
D=2
$\chi = 16$
$\chi = 64$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
τ

More discuss

- Hard to generalize

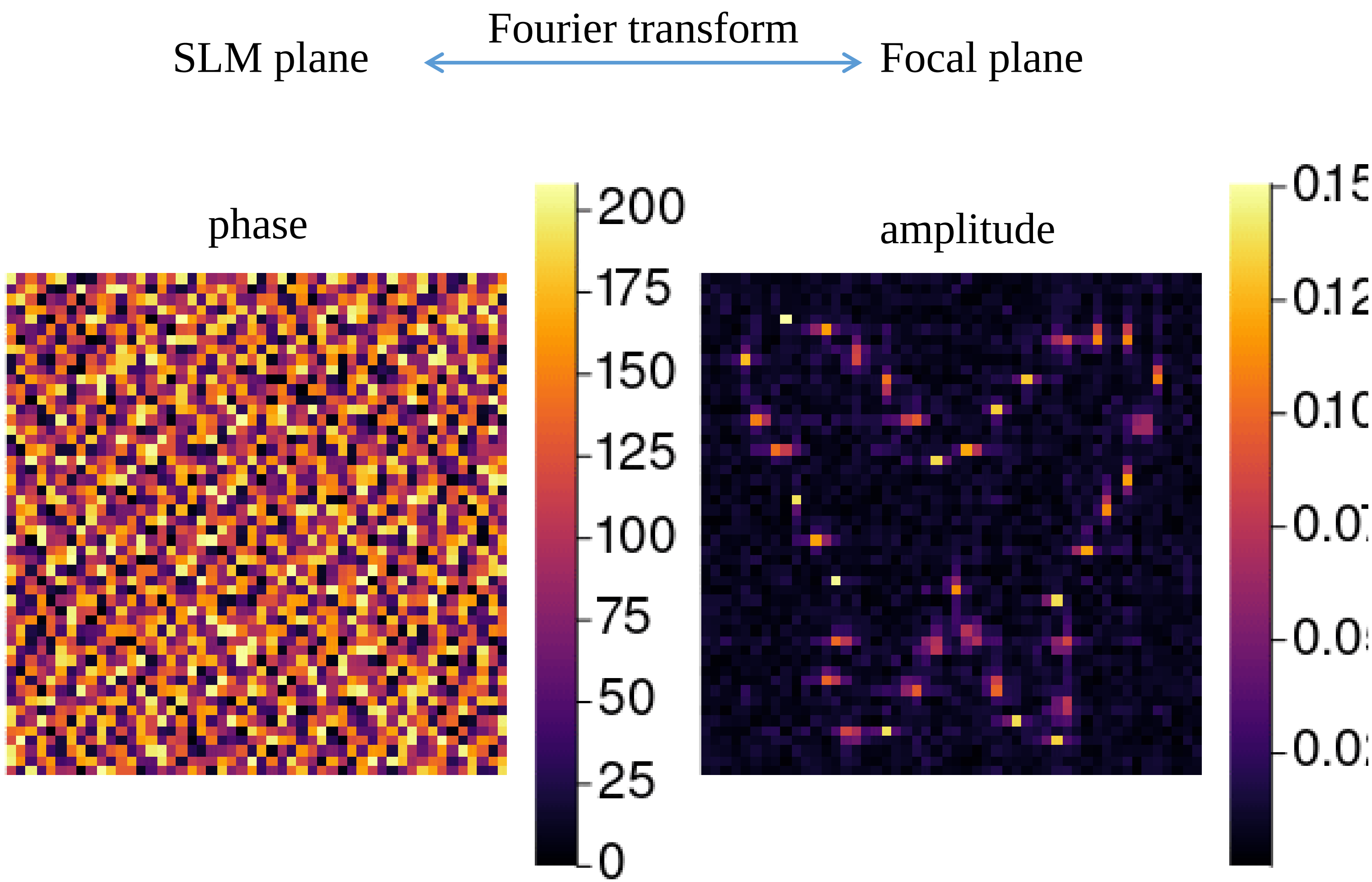
- Qi Yang's ansatz
    - 

    - Good normality but wrong free energy
    - Quasi-fixed point?

SLM flow

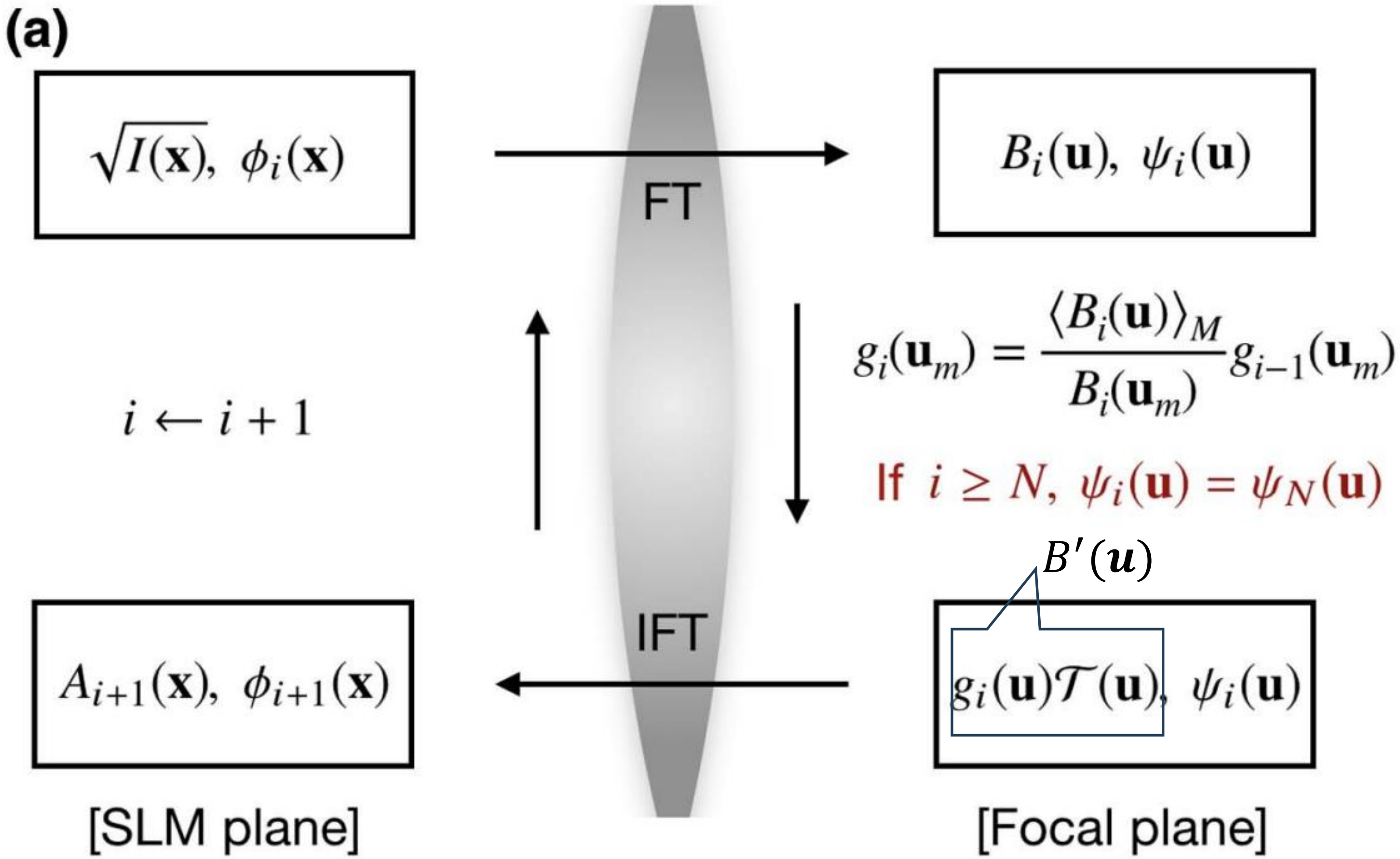
Uniform optical focus arrays

- spatial light modulators (SLM)



WGS algorithm

weighted-Gerchberg–Saxton



Donggyu Kim, Opt. Lett. 44, 3178-3181 (2019)

How to get $\frac{d\phi}{dt}$ from $\frac{du}{dt}$?

implicit function theorem

Continuous fourier transformation

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{-2\pi i(ux+vy)}$$

$trap\ N_u \times N_v$ $\leq$ $SLM\ N_x \times N_y$

$$F_{uv} = \sum_y \left(\sum_x X_{ux} f_{xy} \right) Y_{vy}$$

$$f_{xy} = \sum_v \left(\sum_u X^*_{ux} F_{uv} \right) Y^*_{vy}$$

$$X_{ux} = e^{-2\pi i u x}$$

$$Y_{vy} = e^{-2\pi i v y}.$$

$$F_j = \sum_y \left(\sum_x X_{jx} f_{xy} \right) Y_{jy}$$

$$f_{xy} = \sum_j \left(X^*_{jx} F_j \right) Y^*_{jy}$$

$$X_{jx} = e^{-2\pi i u_i x}$$

$$Y_{jy} = e^{-2\pi i v_i y},$$

Fixed point

- $\phi^* = f(\phi^*, B', u)$

$$\begin{tikzcd} A, \phi^* \arrow[r, "\mathrm{FT}"] & B, \psi \arrow[d] \arrow[r, "\mathrm{iFT}"] & \phi^* \\ & B' & \end{tikzcd}$$

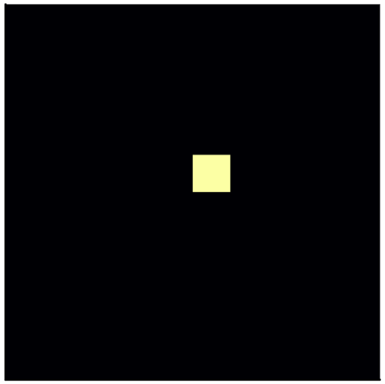
$$\frac{d\phi^*}{dt} = \frac{\partial f}{\partial \phi^*}\frac{d\phi^*}{\partial t} + \frac{\partial f}{\partial B'}\frac{dB'}{dt} + \frac{\partial f}{\partial u}\frac{du}{dt}$$

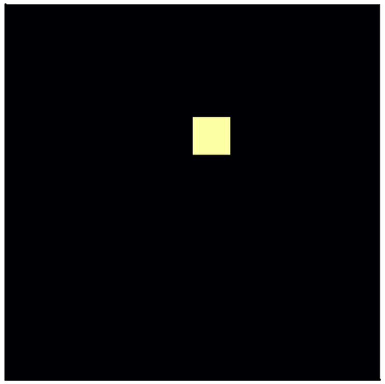
$$\left(1 - \frac{\partial f}{\partial \phi^*}\right)\frac{d\phi^*}{dt} = \frac{\partial f}{\partial B'}\frac{dB'}{dt} + \frac{\partial f}{\partial u}\frac{du}{dt}$$

Forward AD + Linear solve

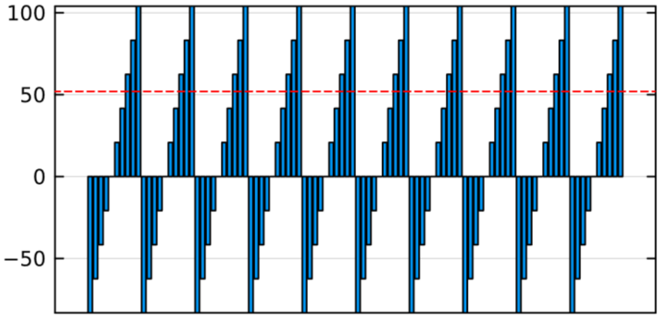
Test: move one point (0.1)

Up: Discrete

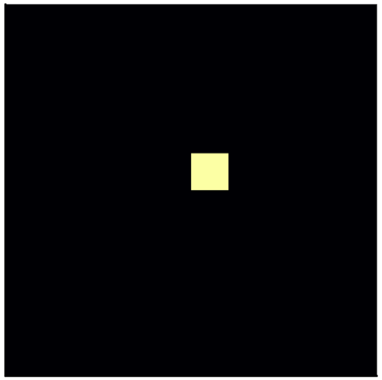


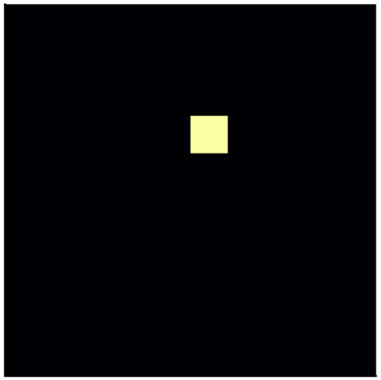


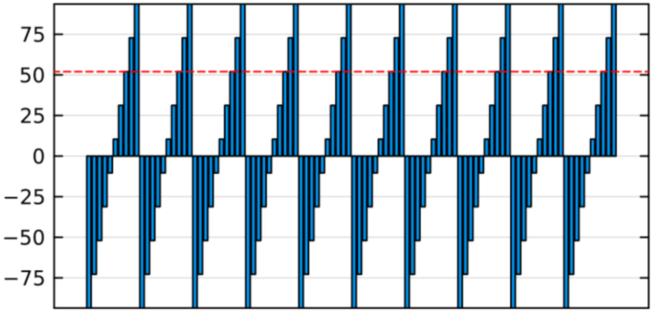
$\Delta\phi$



Down: Current flow

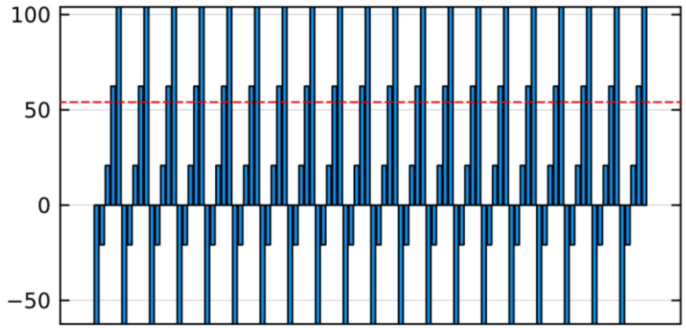


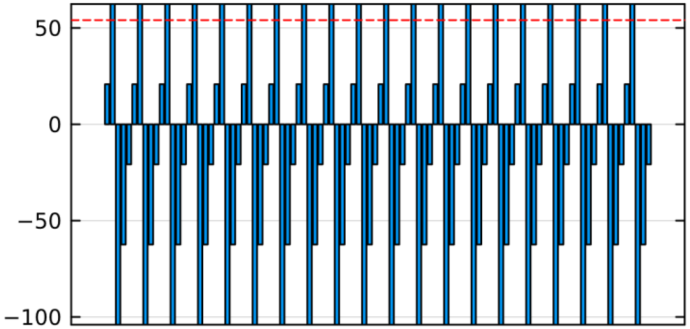




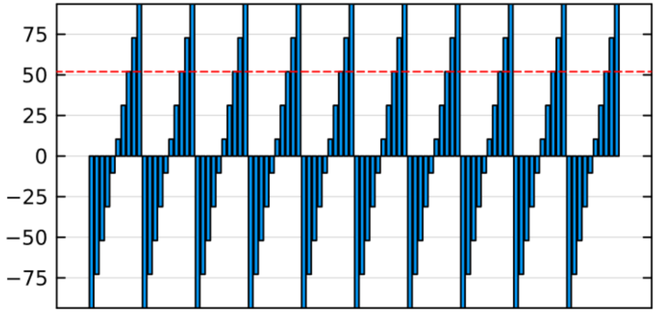
Test: move one point (0.2)

Up: Discrete v.s. Down: Current flow

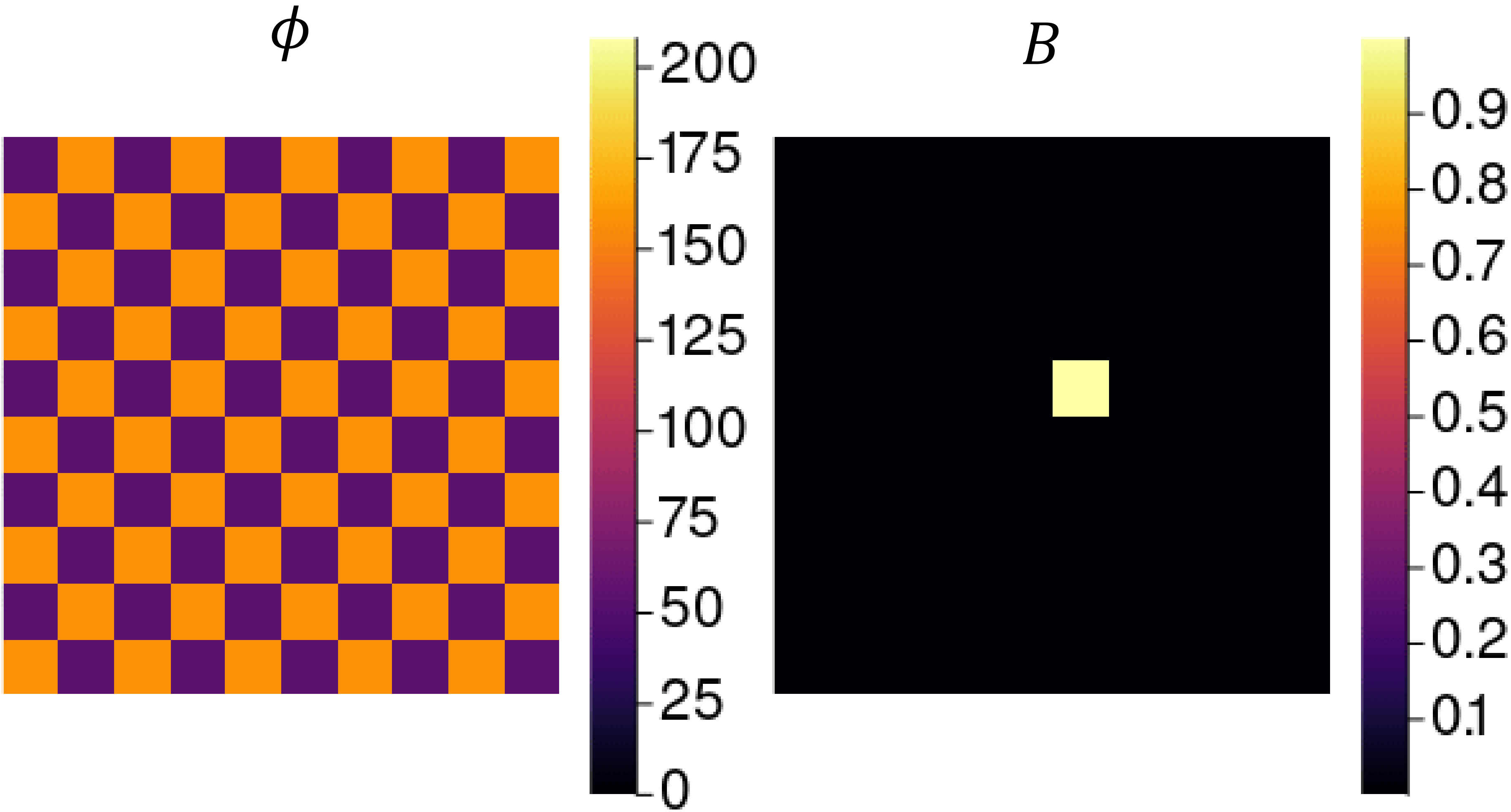




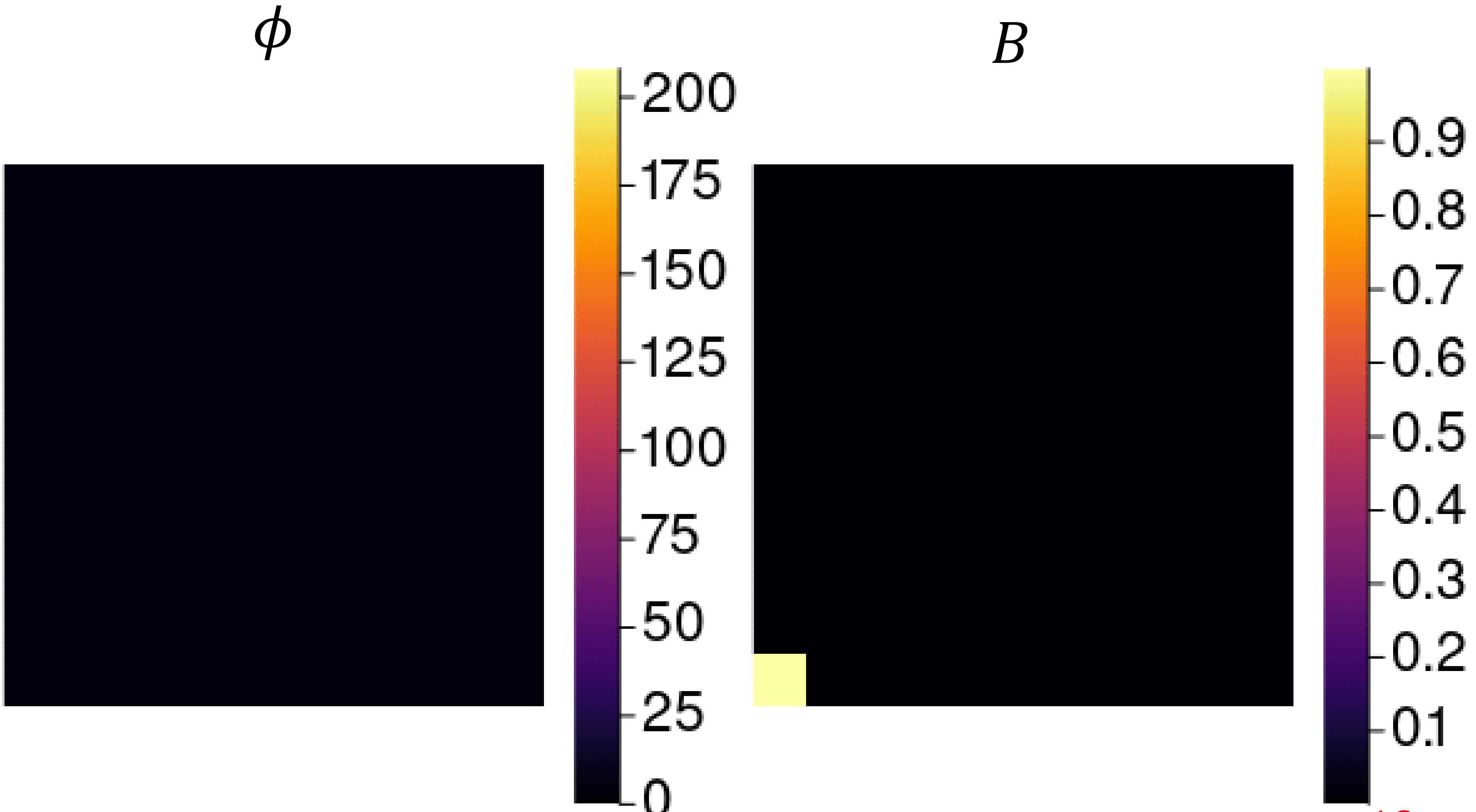
=2×



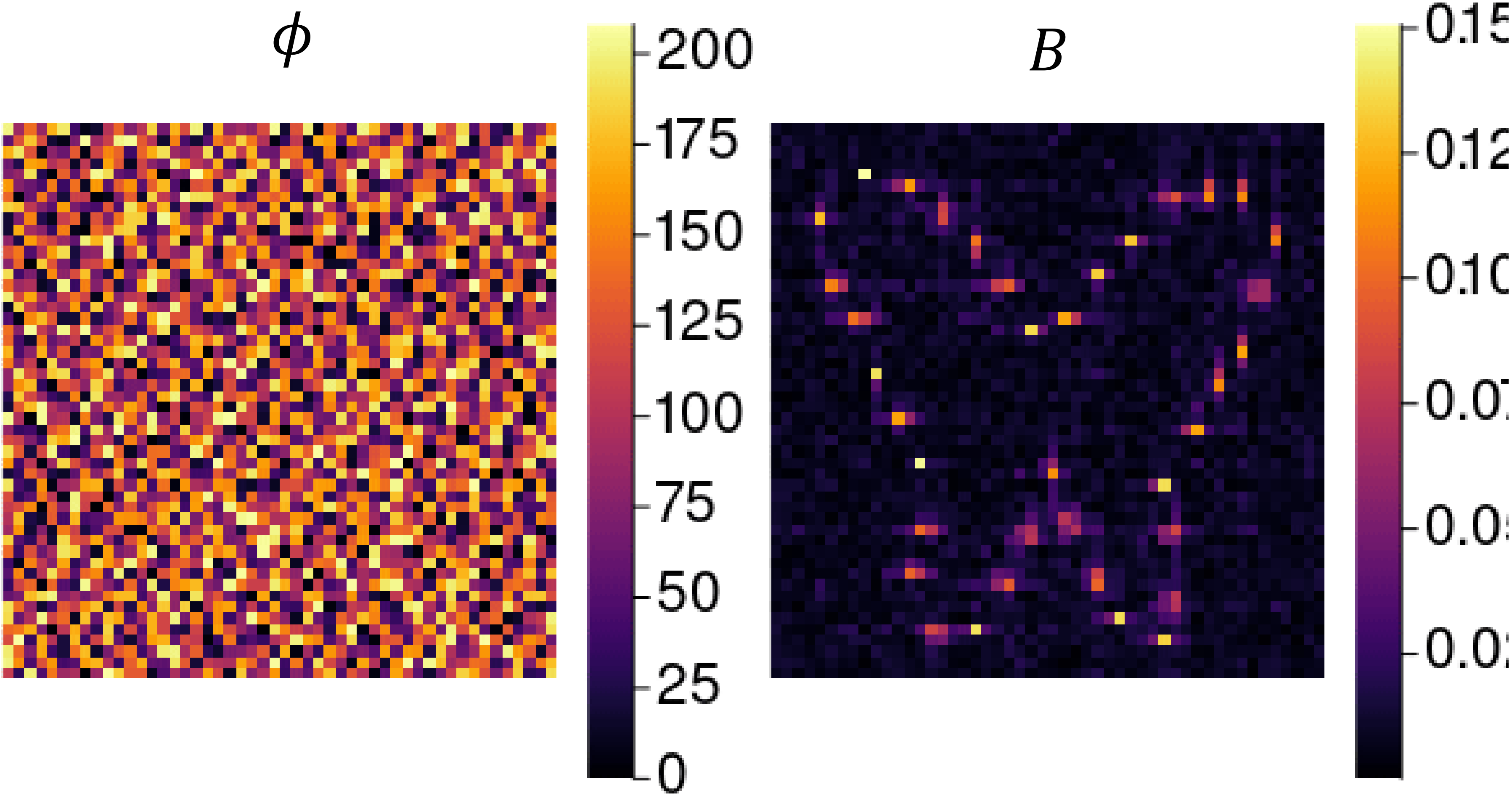
Test: move one point (0.2)



Diagonally move

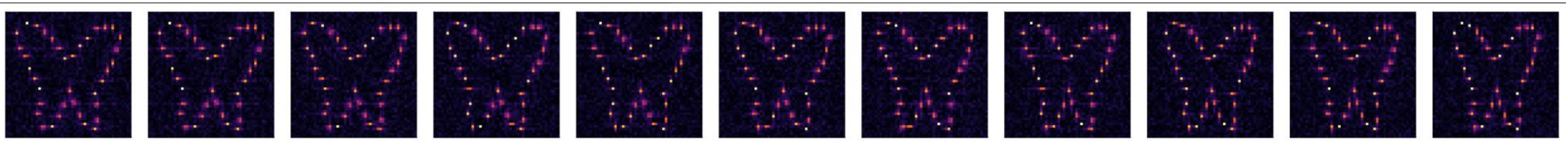


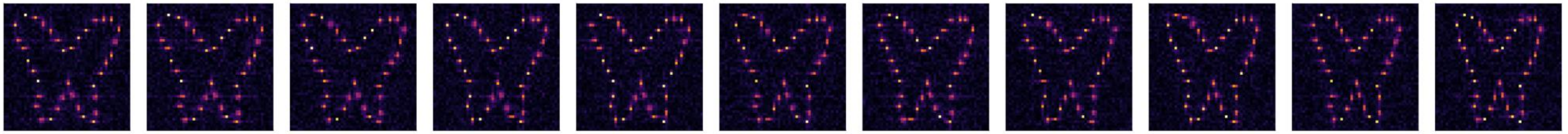
butterfly



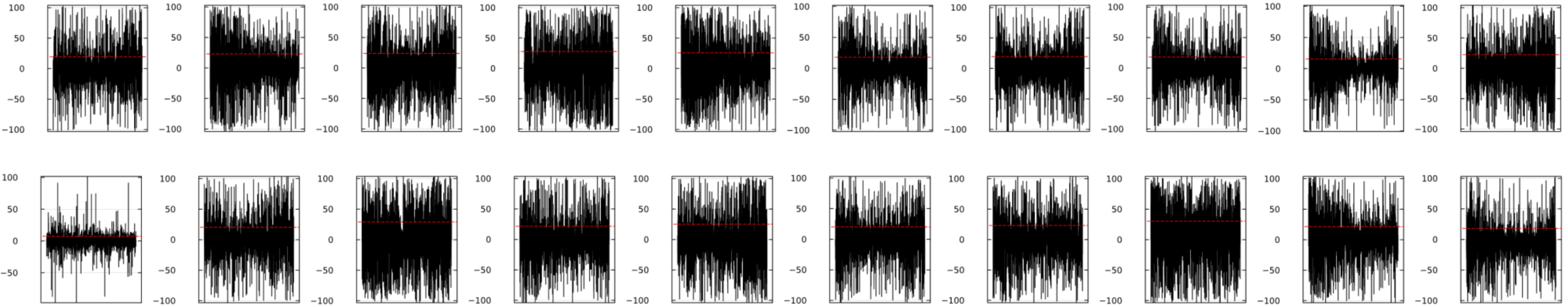
results

Up: Discrete v.s. Down: Current flow



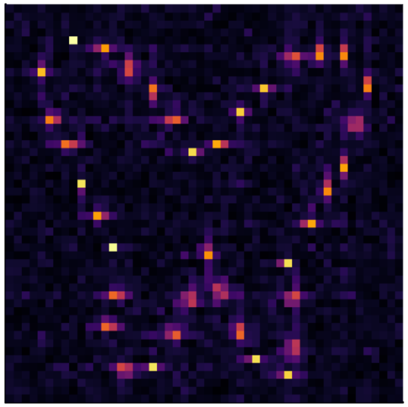


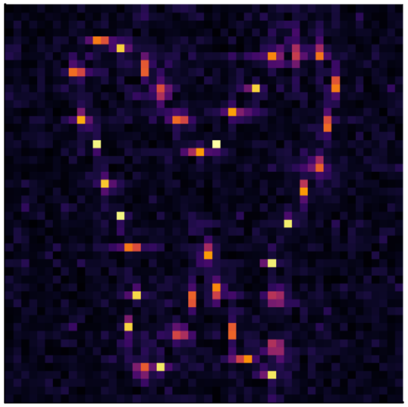
$\Delta\phi$

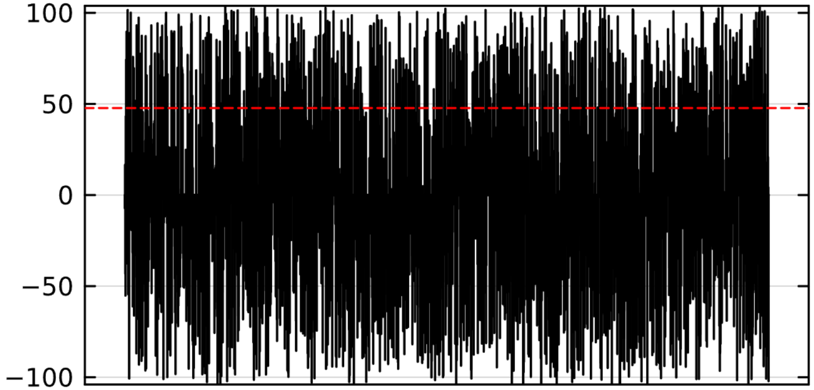


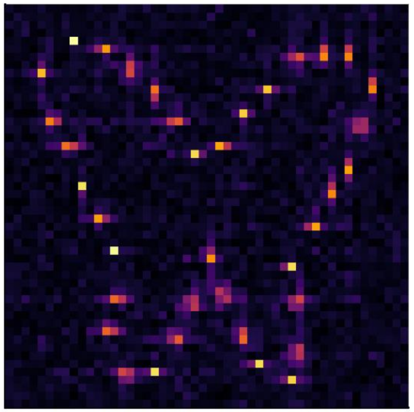
butterfly

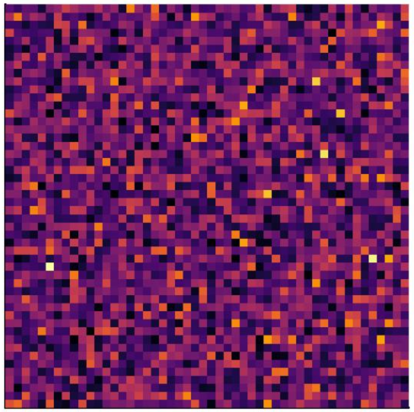
Up: Discrete v.s. Down: Current flow

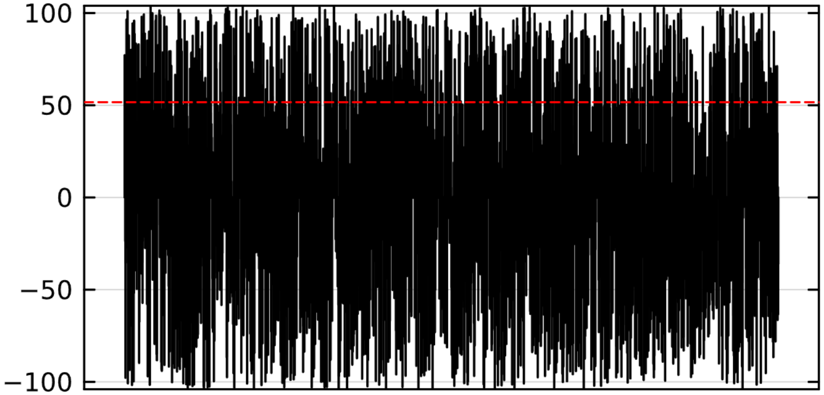






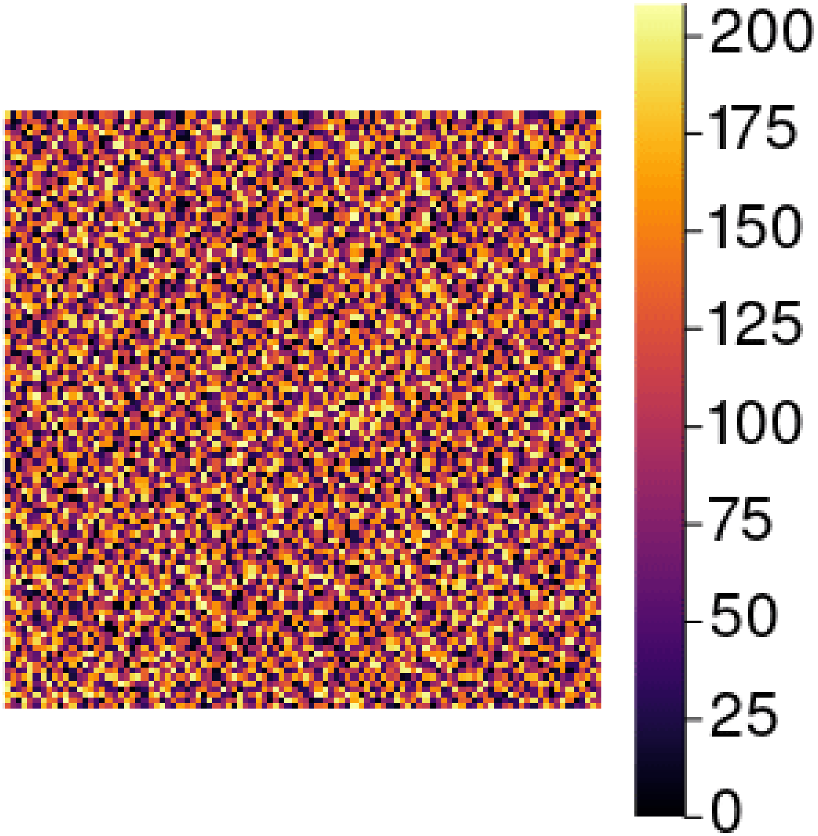


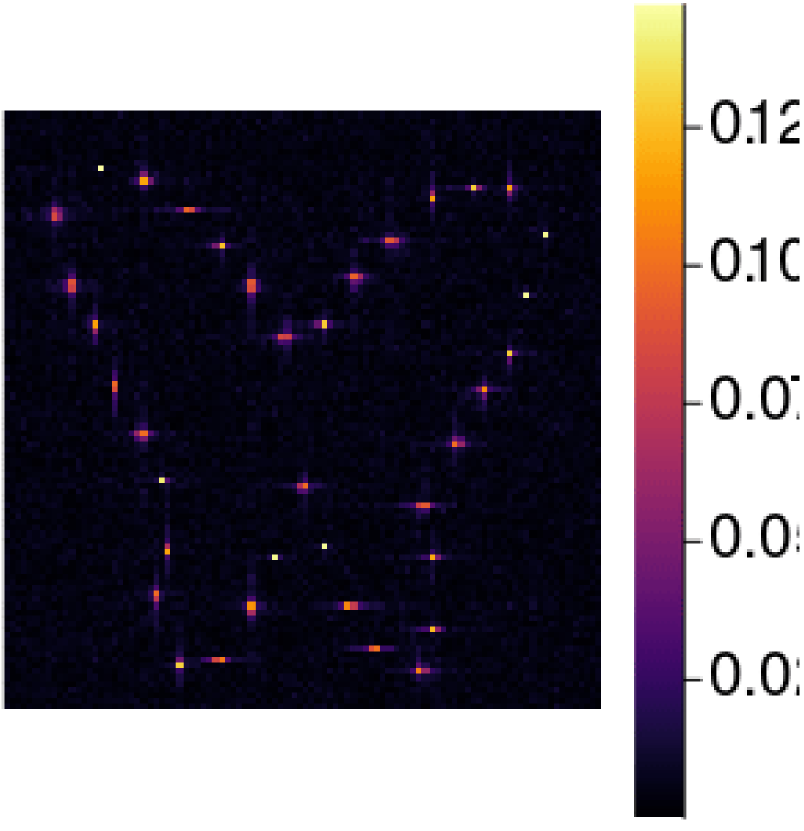




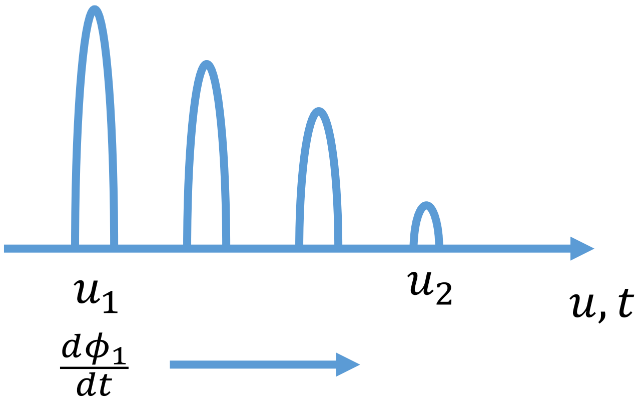
Continuous flow

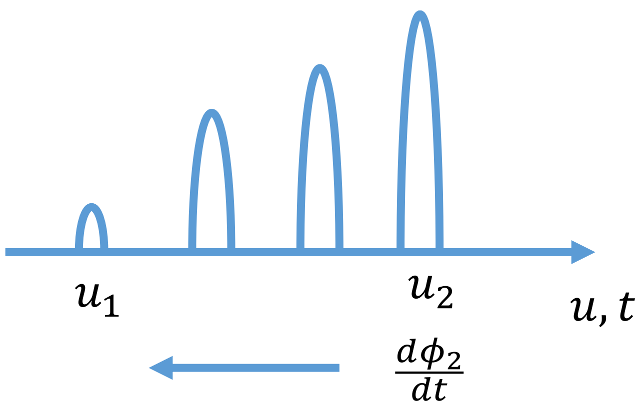
Gradually disappear



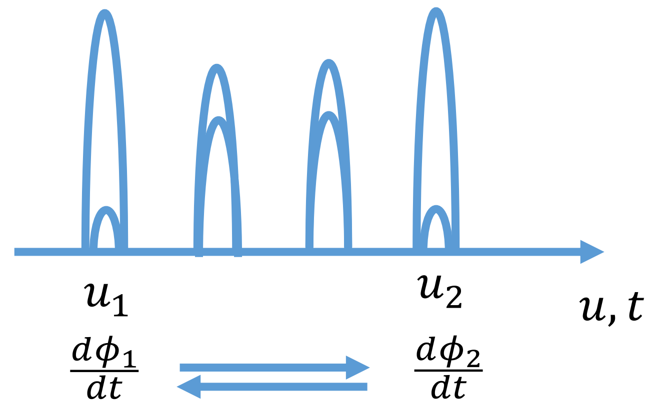


Interpolation between two steps





Combine?



Ignore B!

- $\phi^*, B' = f(\phi^*, B', u)$

$$A,\phi^* \xrightarrow{\text{FT}} B,\boxed{\begin{matrix}\psi\\ B'\end{matrix}} \xrightarrow{\text{iFT}} \phi^*$$

$$B' \leftarrow B' \times \frac{\text{mean}(B)}{B}$$

- $\phi B^* = f(\phi B^*, u)$

$$\frac{d\phi B^*}{dt} = \frac{\partial f}{\partial \phi B^*}\frac{d\phi B^*}{\partial t} + \frac{\partial f}{\partial u}\frac{du}{dt}$$

$$\left(1 - \frac{\partial f}{\partial \phi B^*}\right)\frac{d\phi B^*}{dt} = \frac{\partial f}{\partial u}\frac{du}{dt}$$

ϕ, B' need to include both configuration

Discussion

- There are only disappears without moving for muti-points for short time
- Convergence of the linear solve
- Estimate the moving quality

Thank you for listening!