AD-BC-VUMPS

Xingyu Zhang 2021.4.30

Contents

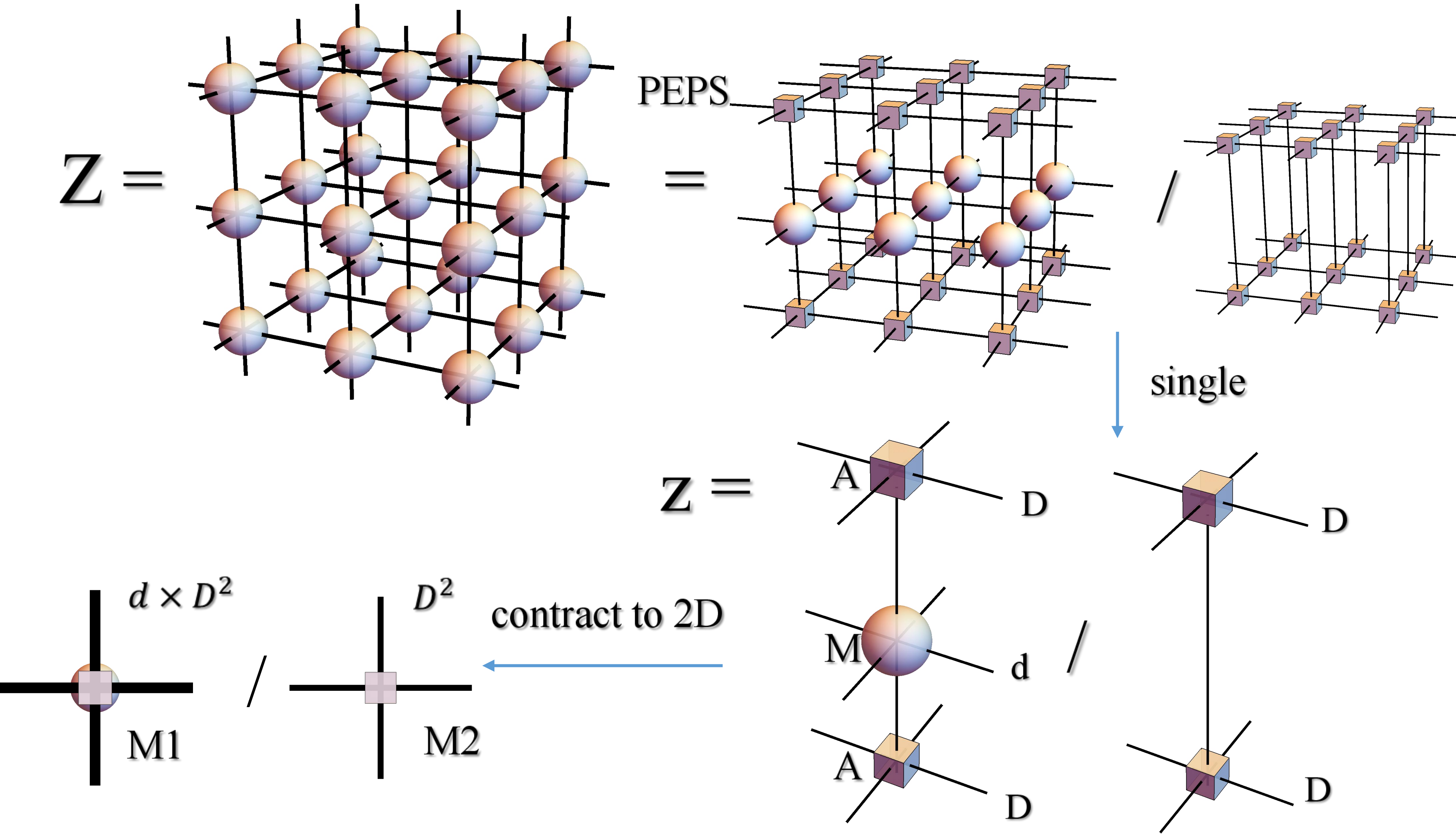
Why AD? → AD-VUMPS

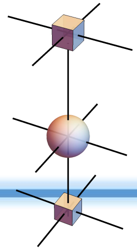
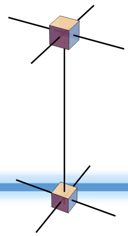
big cell

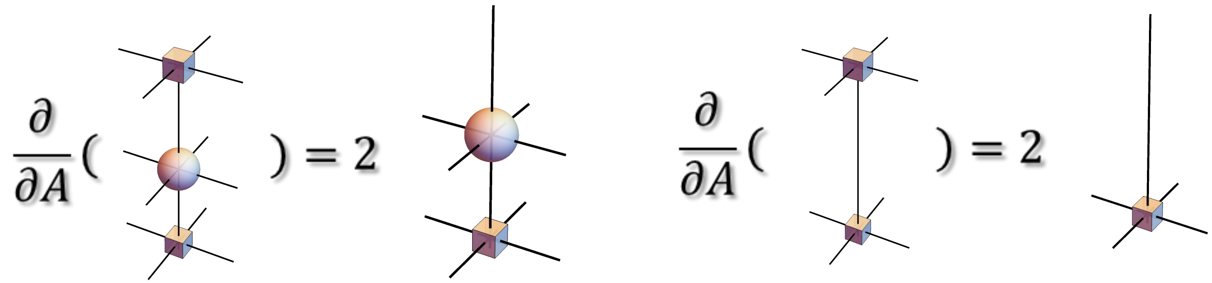
AD-BC-VUMPS

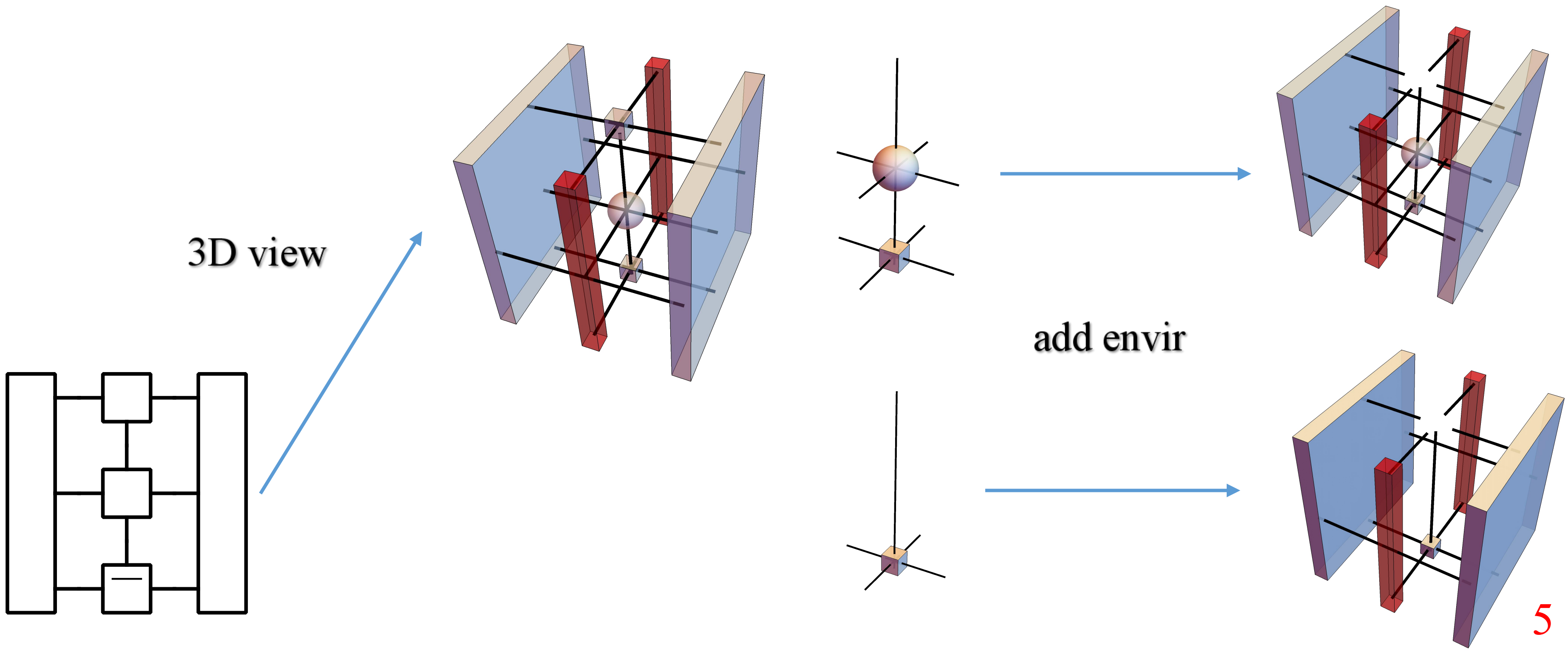
Why AD?

3D ising contract



Gradient to  and 

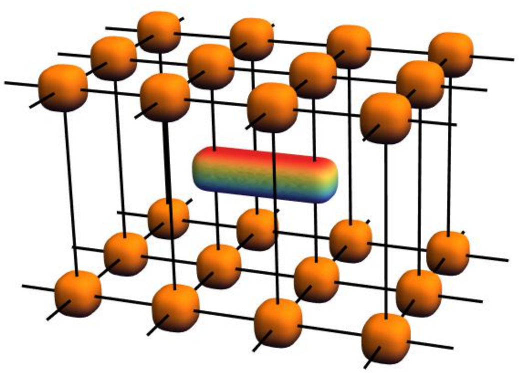


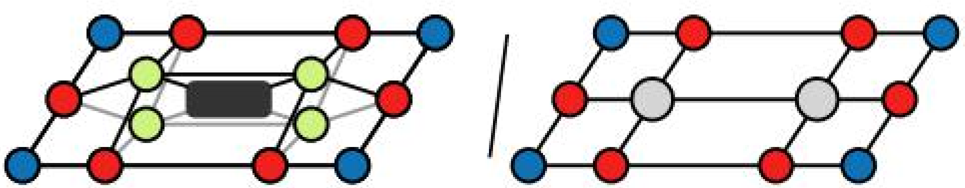


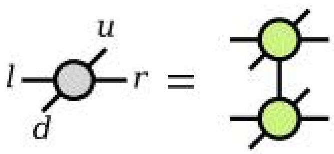
Quantum case

- 2D Energy

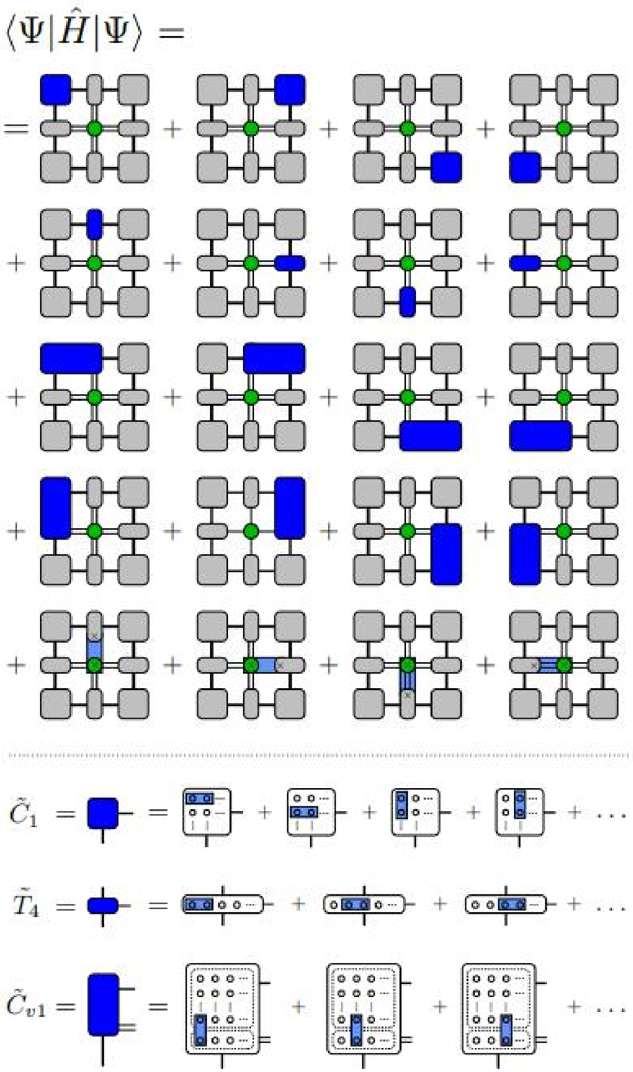
$$\min_A E(A) = \min_A \frac{\langle \Psi(A) | \hat{H} | \Psi(A) \rangle}{\langle \Psi(A) | \Psi(A) \rangle}$$

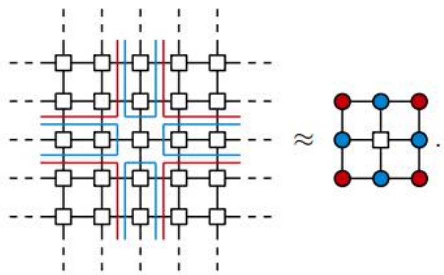


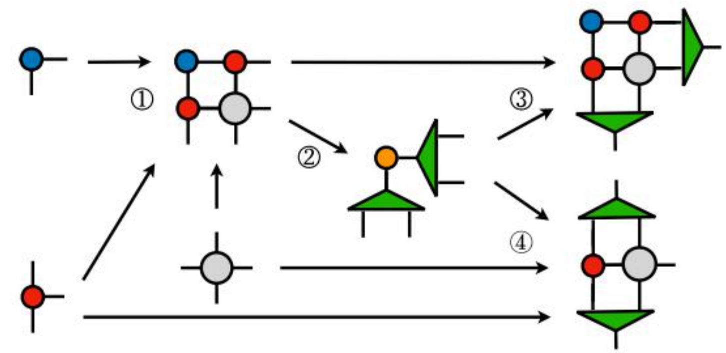
$\mathcal{L} =$ 

$T_{uldr} =$ 

CTMRG







- Experimental optimization method

(1) Compute $E(1)$ (corresponding to the previous energy with the old tensor $A' = A$) and $E(0.5)$ (corresponding to the energy with $A' = \tilde{A}$).

(2) If $E(0.5) < E(1)$, take $A' = \tilde{A}$ as the solution and exit.

(3) Define an initial step size Δ_0 (e.g., $\Delta_0 = 0.1$) and a tiny step size h (e.g., $h = 10^{-4}$).

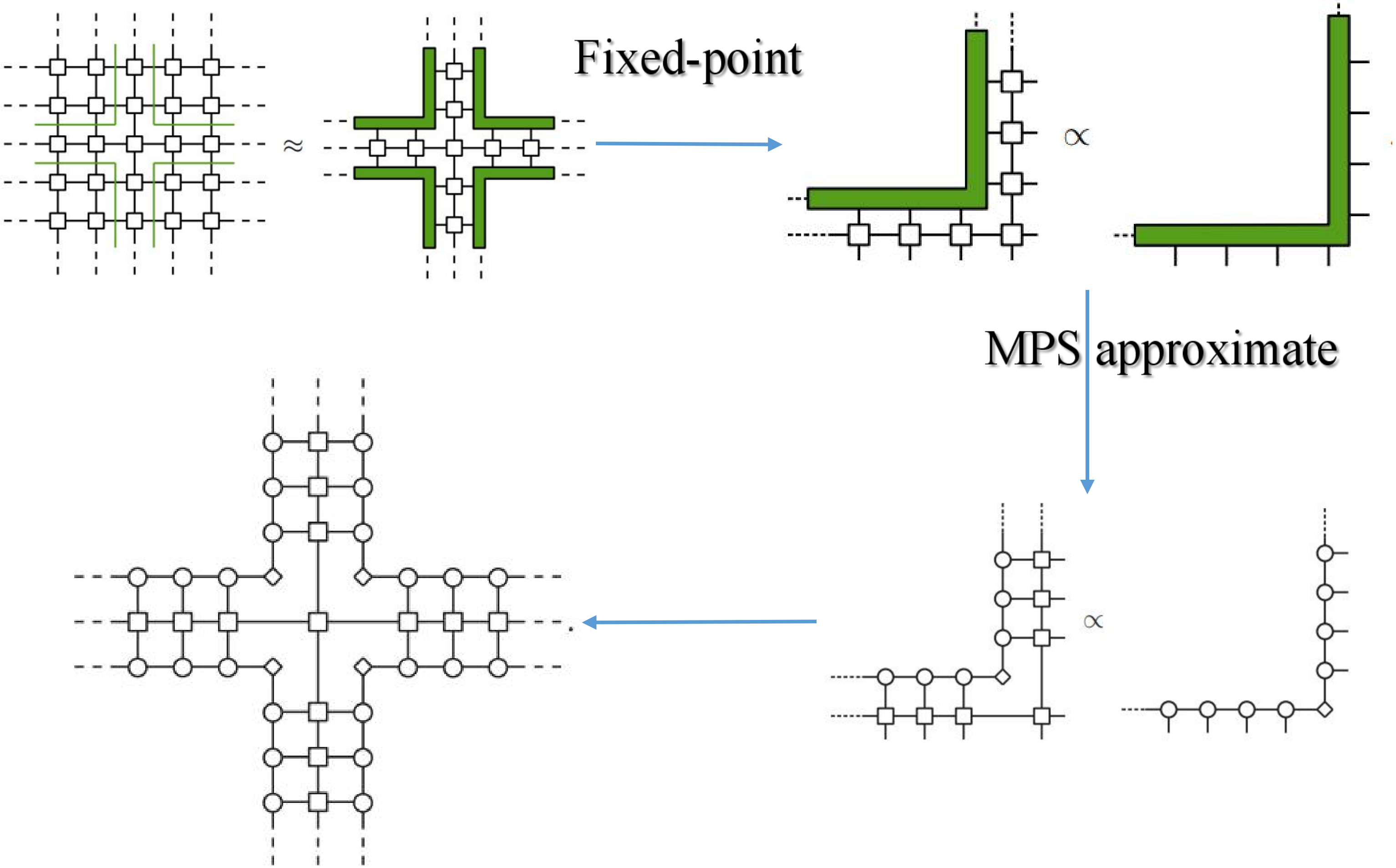
(4) If $E(1+h) < E(1)$, set $\Delta = \Delta_0$, else $\Delta = -\Delta_0$.

(5) For *iter* $= 1$ to *maxiter*

(a) If $E(1+\Delta) < E(1)$, accept solution [44] with $\lambda = 1 + \Delta$ and exit.

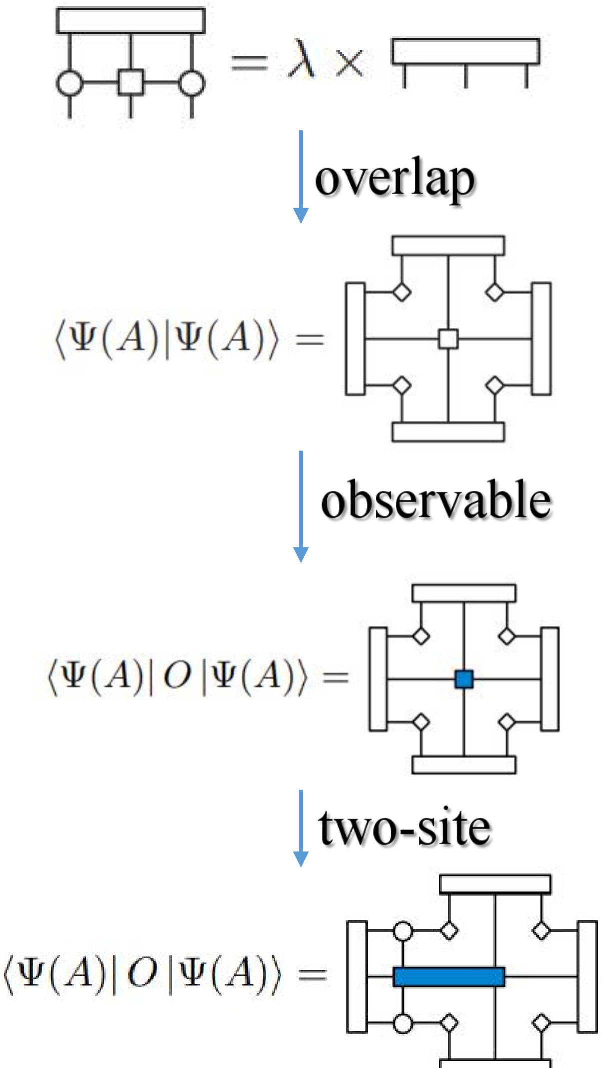
(b) Else $\Delta = \Delta/2$.

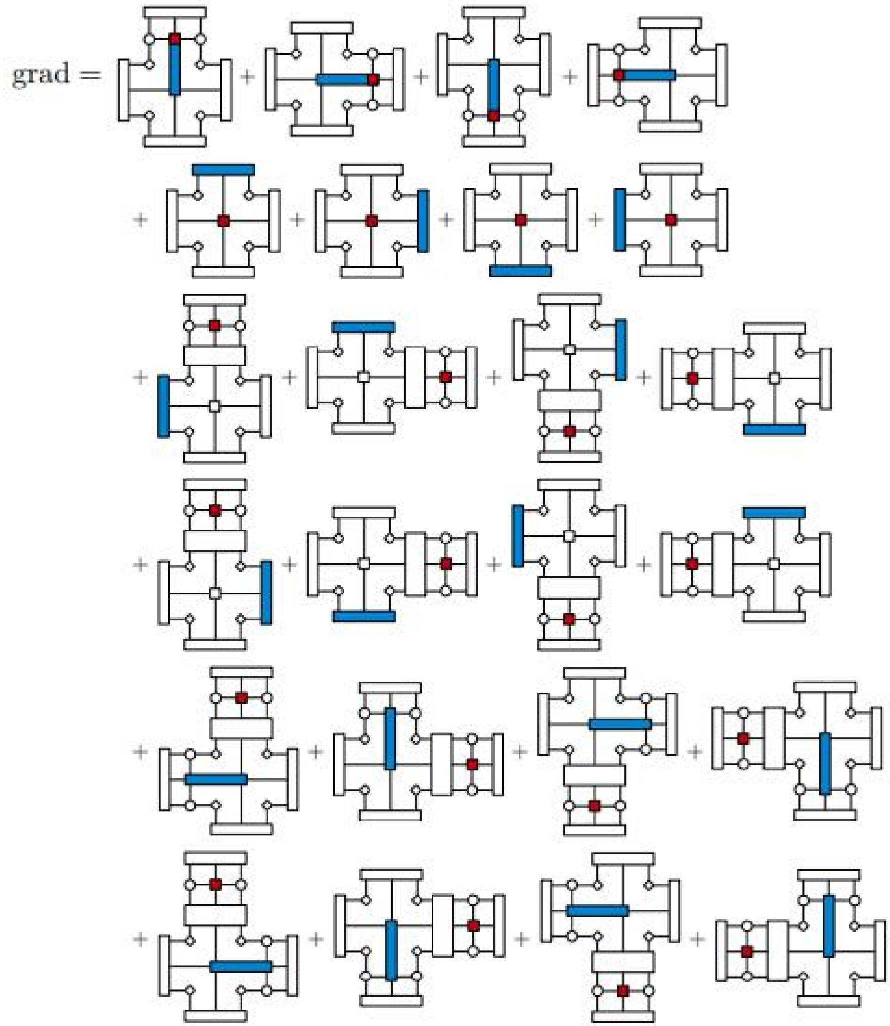
Channel environments



PRB 94, 155123 (2016)

Gradient

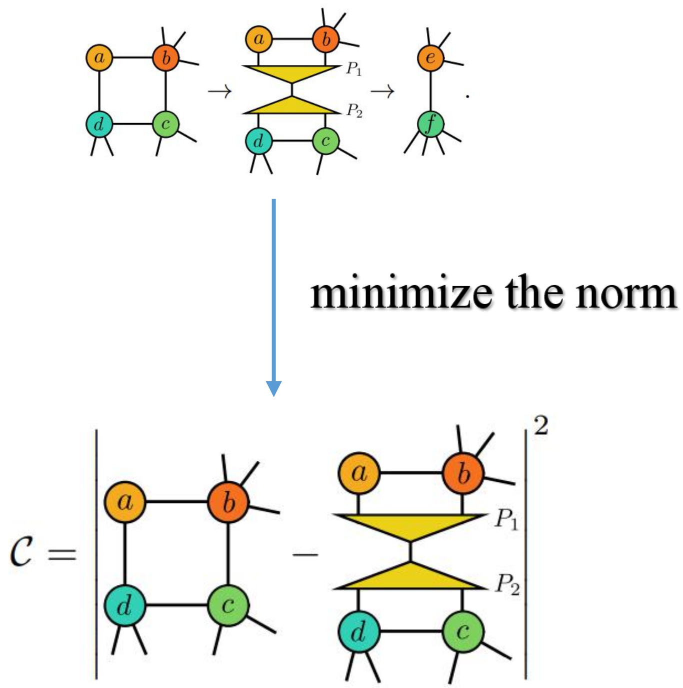


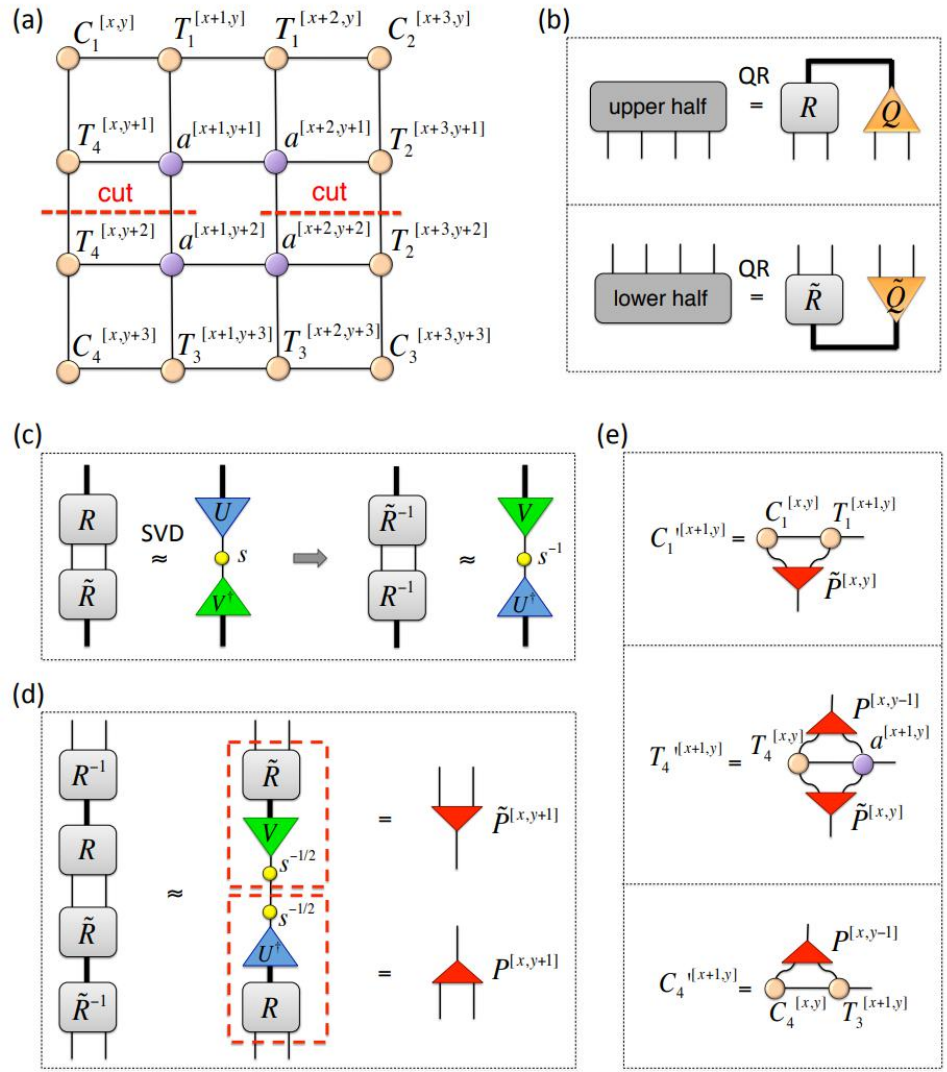


PRB 94, 155123 (2016)

Big Cell CTMRG

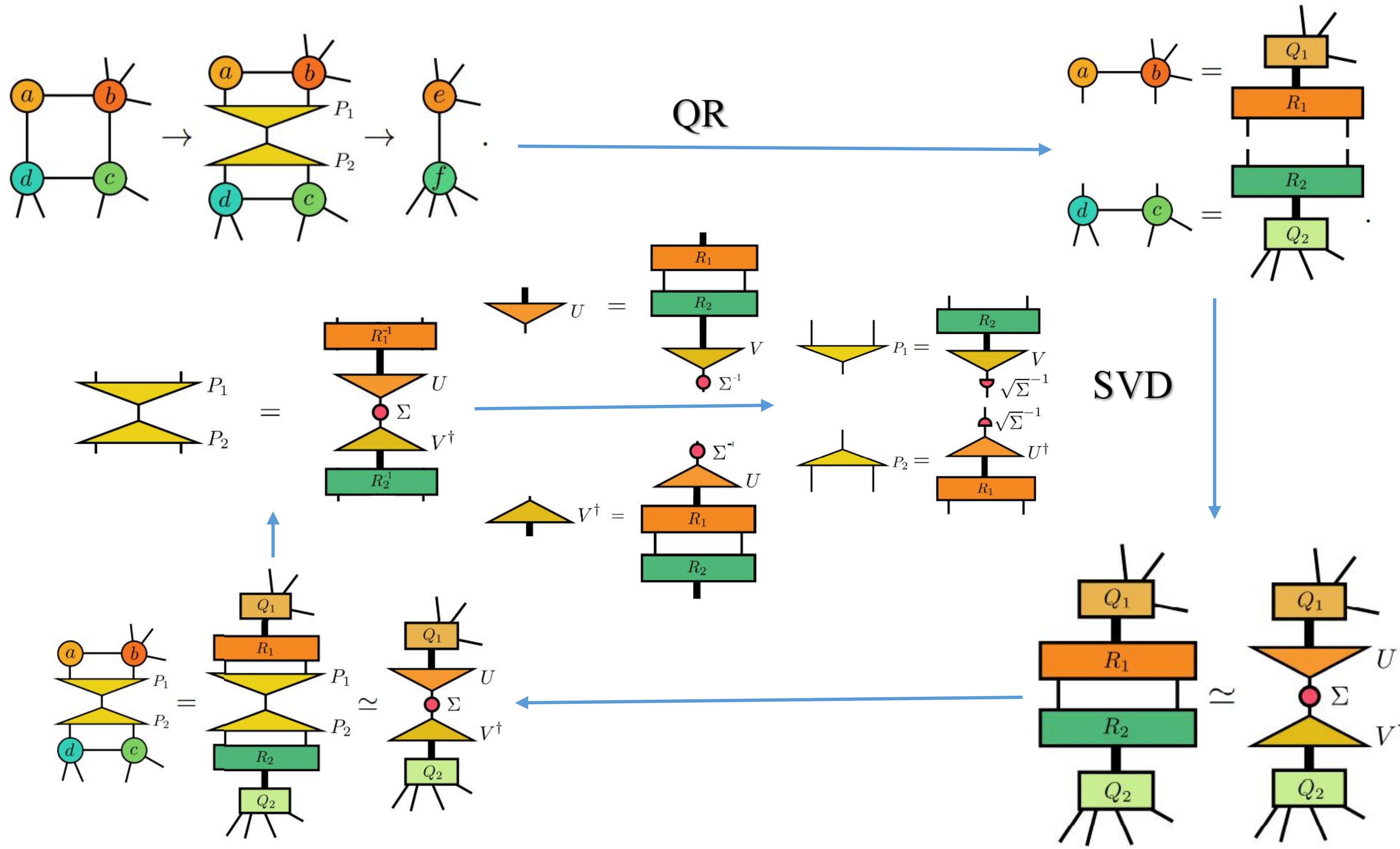
- General case





https://journals.aps.org/prl/supplemental/10.1103/PhysRevLett.113.046402/suppl.pdf

Get P_1 and P_2

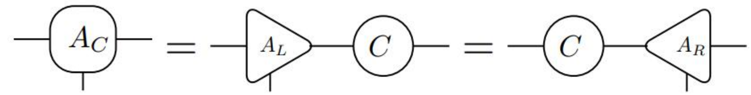


https://arxiv.org/abs/1905.02351

AD-VUMPS

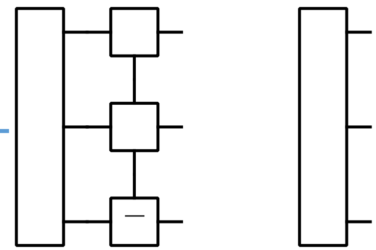
VUMPS-algorithm

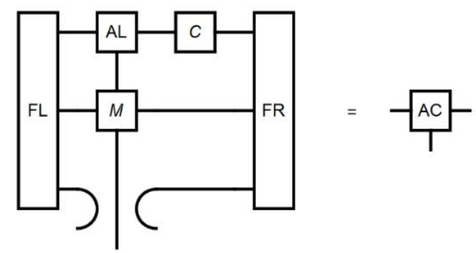
\# Initial orthonormal form MPS
M = classical2Disingmpo(β; J = 1.0, h = 0.)
AL,C = leftorth (A)
AR = rightorth (AL,C)



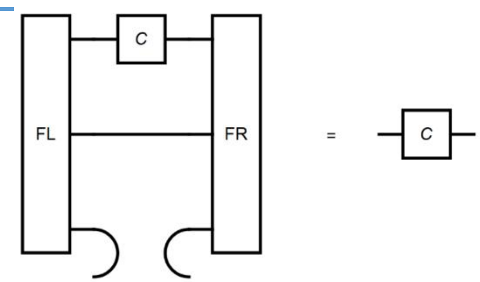
while error(AC,FL,FR,M) > tol_error

\# Get environment
FL = leftenv(AL, M)
FR = rightenv(AR, M)

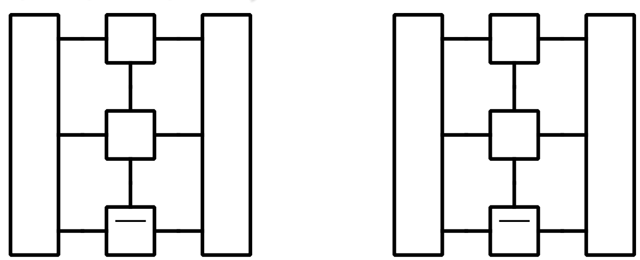




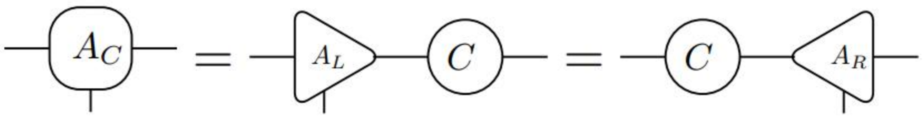
\# Get orthonormal form
AC, C = vumpsstep(AL, C, AR, M, FL, FR)



\# calculate observable
Obs = obser(O, AC, M, FL, FR)



QR to avoid inverse



$$A_C = Q_{A_C} \cdot R_{A_C}$$
$$C = Q_C \cdot R_C$$

$$Q^T = Q^{-1}$$

$$\downarrow$$

$$\begin{aligned} A_L &= A_C \cdot C^{-1} \\ &= Q_{A_C} \cdot R_{A_C} \cdot R_C^{-1} \cdot Q_C^{-1} \\ &= Q_{A_C} \cdot R_{A_C} \cdot R_C^{-1} \cdot Q_C^T \\ &\approx Q_{A_C} \cdot Q_C^T \end{aligned}$$

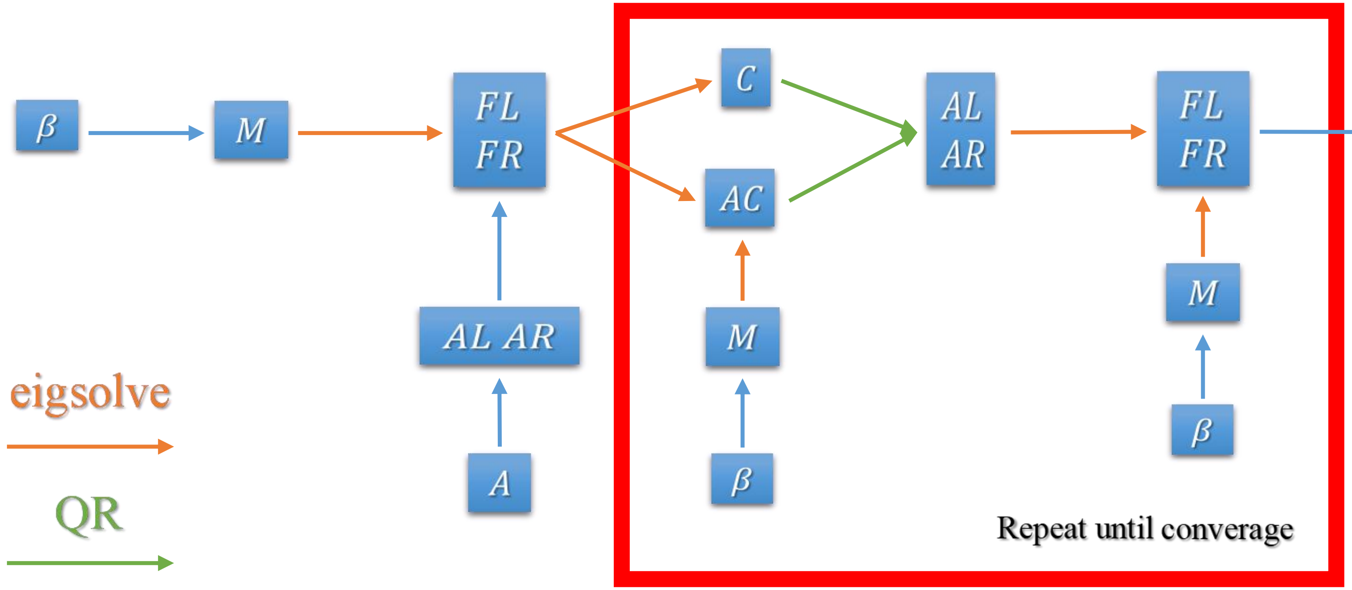
Error = R_{A_C} - R_C

$$\downarrow$$

$$A_L^T \cdot A_L = Q_C \cdot Q_{A_C}^T \cdot Q_{A_C} \cdot Q_C^T = 1$$

inverse and orthogonality at the same time

Computation Graphs



Adjoint of eigsolve

- $$l^T A = \lambda l^T, \quad A r = \lambda r, \quad l^T r = 1,$$

$$(A - \lambda I)\xi_l = (1 - r l^T)\bar{l}, \quad l^T \xi_l = 0$$
$$(A^T - \lambda I)\xi_r = (1 - l r^T)\bar{r}, \quad r^T \xi_r = 0$$

$$\longrightarrow \bar{A} = \bar{\lambda} l r^T - l \xi_l^T - \xi_r r^T$$

gauge invariant $\downarrow$

$$l^T \bar{l} = r^T \bar{r} = 0$$

$$(A - \lambda I)\xi_l = \bar{l}, \quad l^T \xi_l = 0$$
$$(A^T - \lambda I)\xi_r = \bar{r}, \quad r^T \xi_r = 0$$

Only l

$$\boxed{(A - \lambda I)\xi_l = \bar{l}, \quad l^T \xi_l = 0}$$
$$(A^T - \lambda I)\xi_r = 0, \quad r^T \xi_r = 0$$

$$\boxed{\bar{A} = -l \xi_l^T}$$

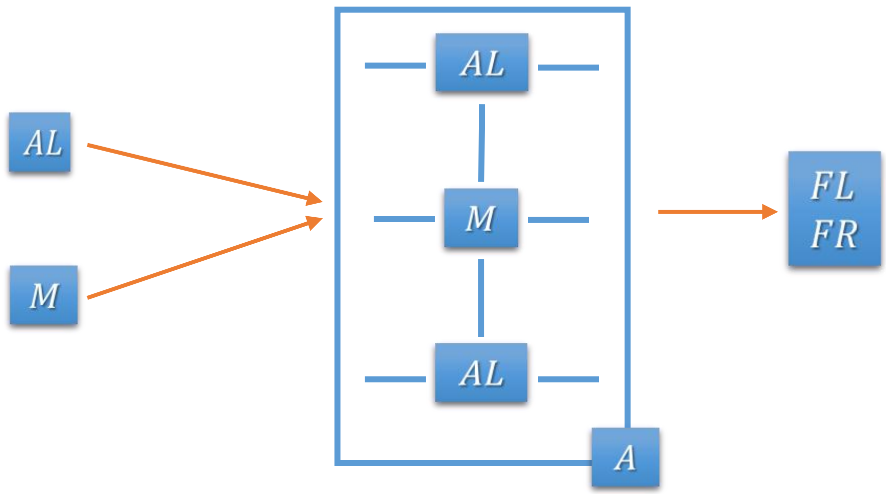
Only r

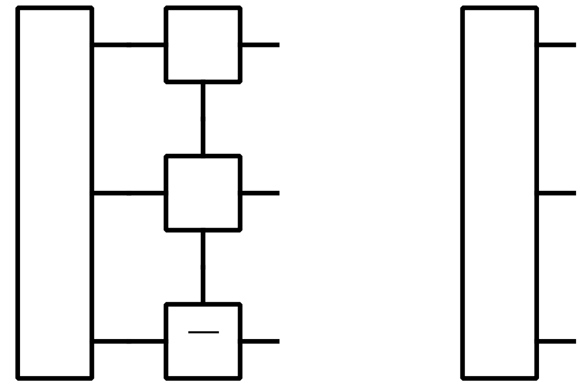
$$(A - \lambda I)\xi_l = 0, \quad l^T \xi_l = 0$$
$$\boxed{(A^T - \lambda I)\xi_r = \bar{r}, \quad r^T \xi_r = 0}$$

$$\boxed{\bar{A} = -\xi_r r^T}$$

PRB 101, 245139 (2020)

Adjoint of VUMPS envirment





$$(A-\lambda I)\xi_l = \bar{l}, \quad l^T \xi_l = 0$$

$$\bar{A} = -\, l\xi_l^T$$

$$\overline{AL} = \bar{A}\cdot \frac{\partial A}{\partial AL}$$

$$\bar{M} = \bar{A}\cdot \frac{\partial A}{\partial M}$$

```
                    ┌── AL ──┐
                    │   │    │
dM    =   - FL ──       ── ξl
                    │   │    │
                    └── AL ──┘


                    ┌──    ──┐              ┌── AL ──┐
                    │   │    │              │   │    │
dAL   =   - FL ── M ── ξl     -        FL ── M ── ξl
                    │   │    │              │   │    │
                    └── AL ──┘              └──    ──┘
```

Adjoint of QR

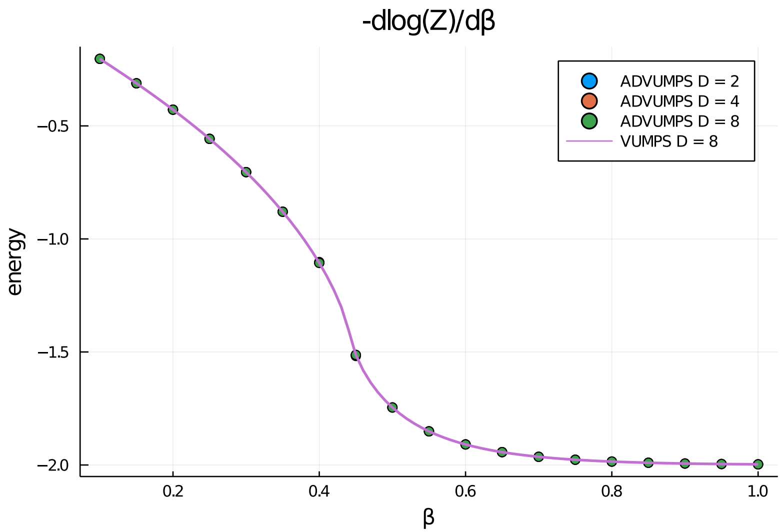
- $A = QR$

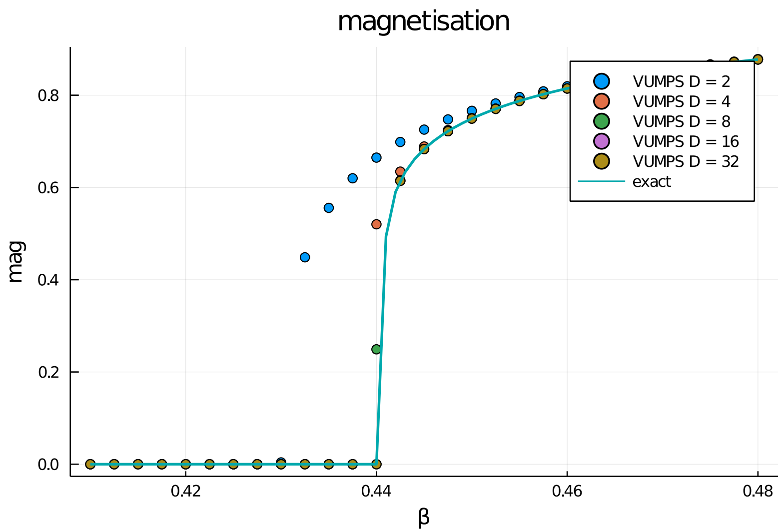
  $\bar{A} = [\bar{Q} + Q\texttt{copyltu}(M)]R^{-T}$

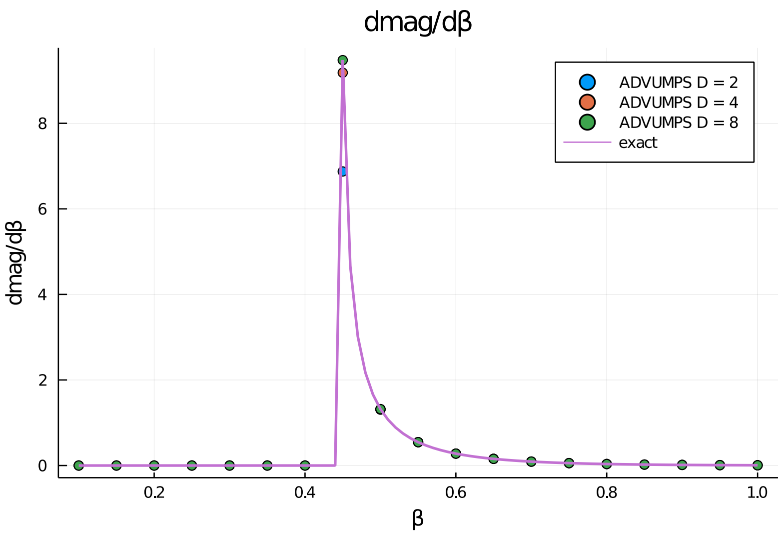
  $$M = R\bar{R}^T - \bar{Q}^T Q$$
  $$[\texttt{copyltu}(M)]_{ij} = M_{\max(i,j),\min(i,j)}$$

- Trick: add $\delta = 10^{-6}$ to R's diagonal element for the stability of inverse

2D Classical Ising Model







https://github.com/XingyuZhang2018/ADVUMPS.jl

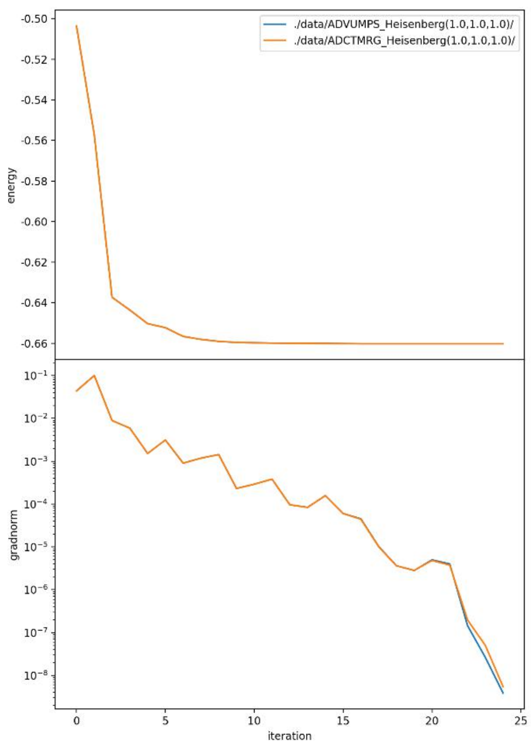
Finding the Ground State of infinite 2D Heisenberg model

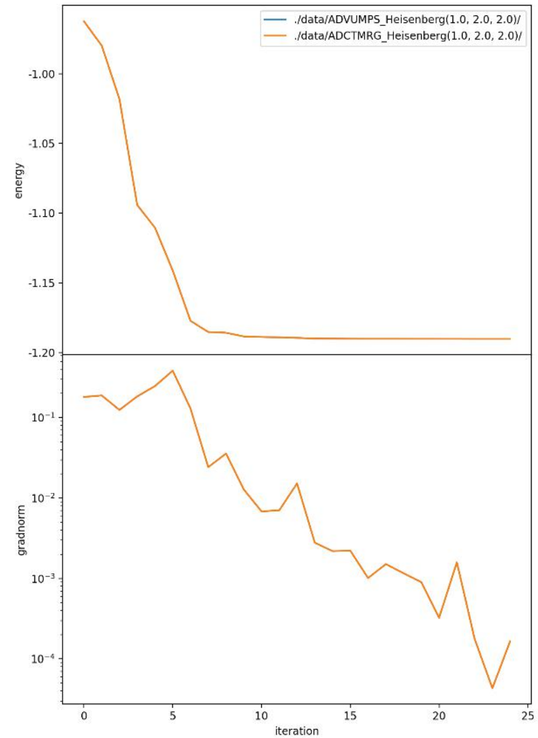
ADVUMPS vs ADCTMRG $D = 2$ $\chi = 20$ $\delta = 10^{-12}$

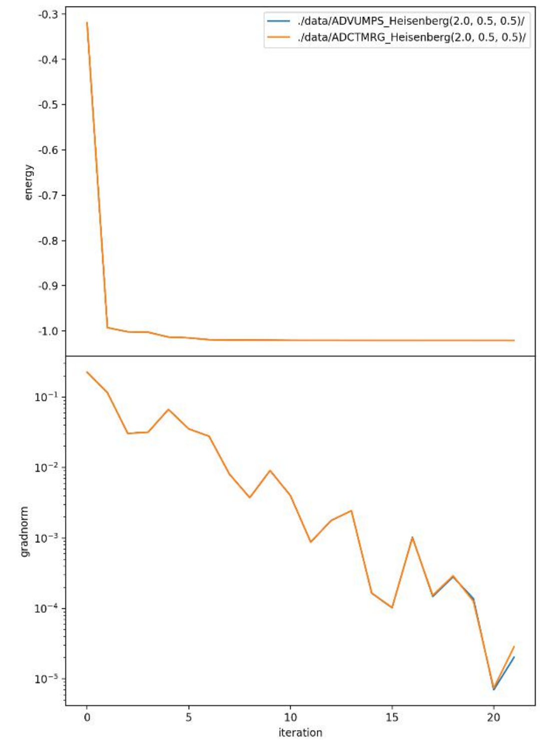
33.76s vs 15.62s

29.11s vs 15.58s

16.61s vs 4.94s







Difference ~1e-14

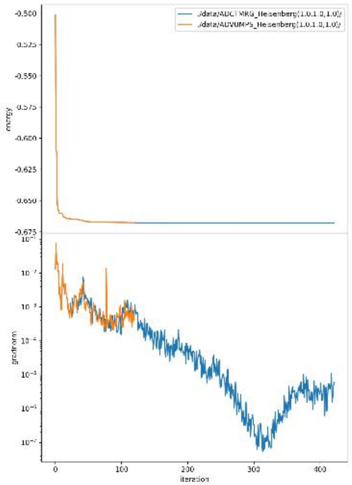
Difference ~1e-9

Difference ~1e-9

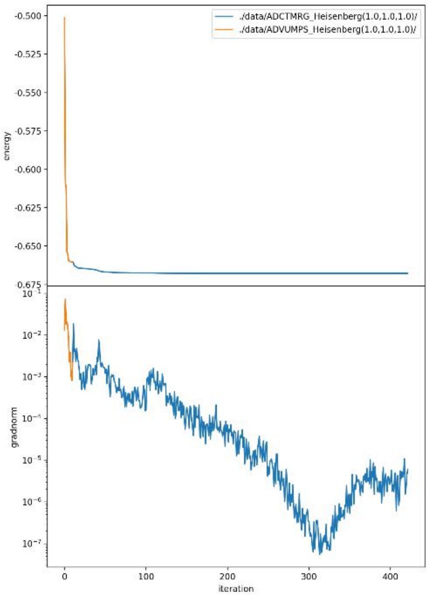
		Time			Allocations		
Tot / % measured:		8.66s / 100%			7.54GiB / 100%		
Section	ncalls	time	%tot	avg	alloc	%tot	avg
backward	61	6.22s	72.1%	102ms	6.11GiB	81.0%	103MiB
forward	61	2.41s	27.9%	39.5ms	1.43GiB	19.0%	24.0MiB

		Time			Allocations		
Tot / % measured:		30.2s / 100%			26.2GiB / 100%		
Section	ncalls	time	%tot	avg	alloc	%tot	avg
backward	62	22.8s	75.8%	368ms	20.1GiB	76.6%	332MiB
forward	62	7.30s	24.2%	118ms	6.13GiB	23.4%	101MiB

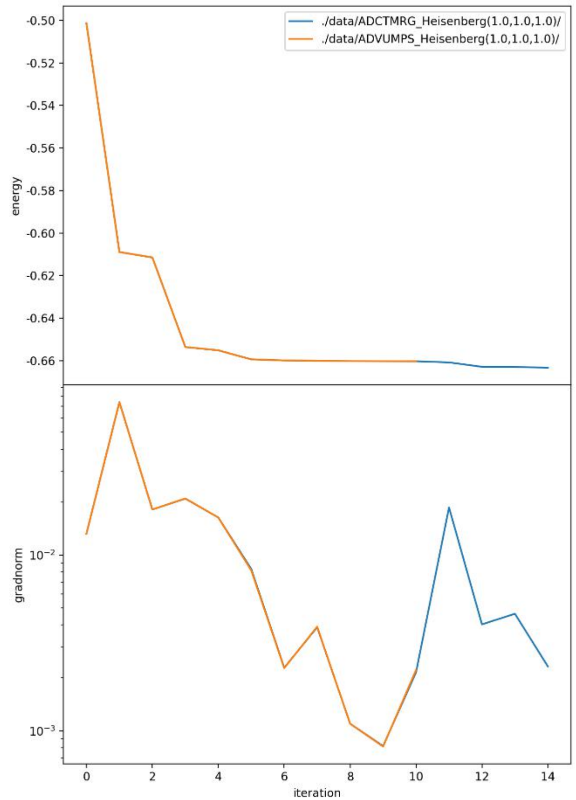
Problem of D=3



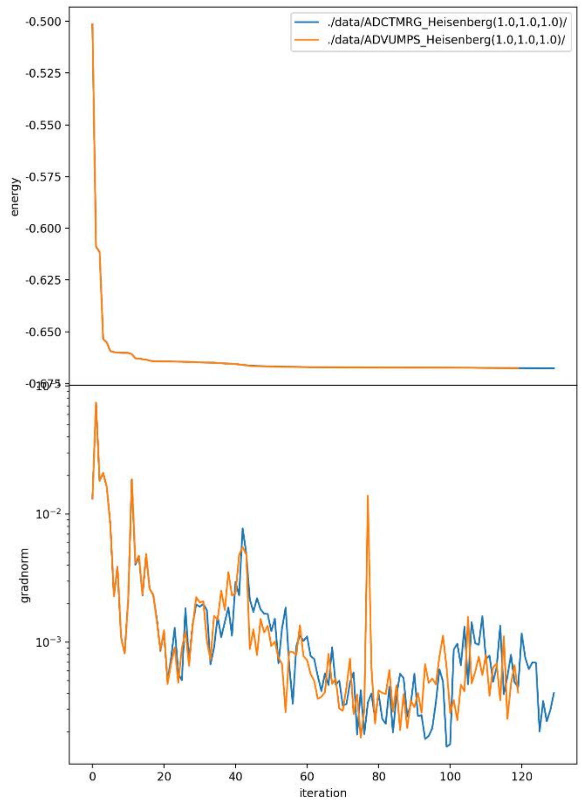
- QR isn't stable for R^{-1}



tol=1e-10



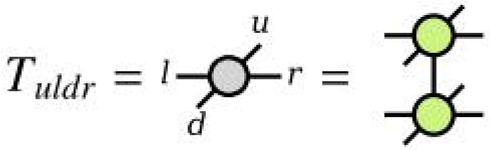
tol=1e-20



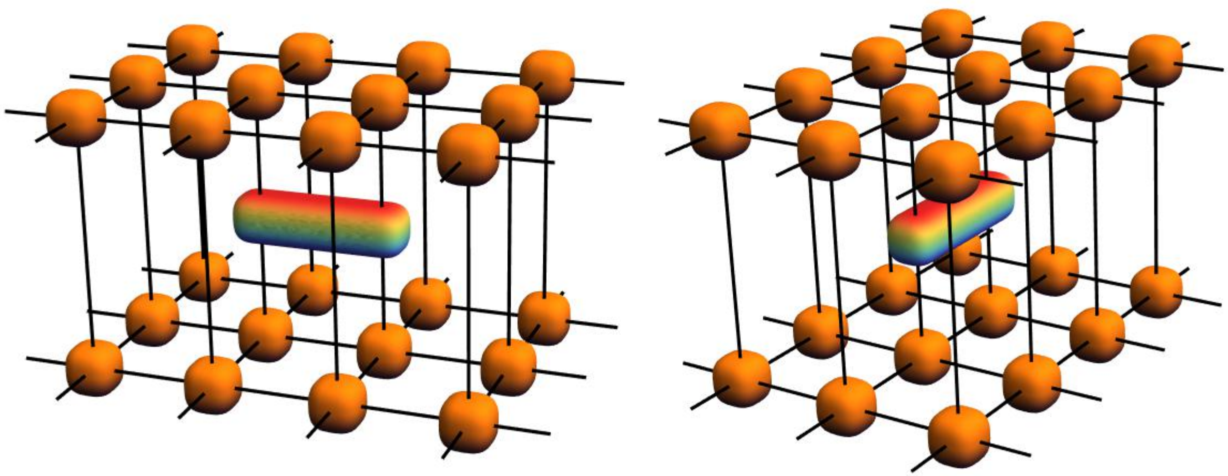
Problem of iPEPS symmetry

- When only require up and down symmetry, Heisenberg(1,1,1)

  $E_0 = -0.85 < -0.66$

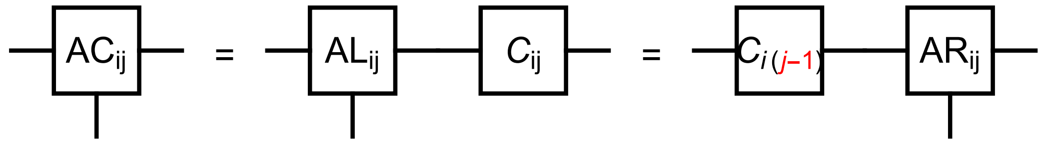


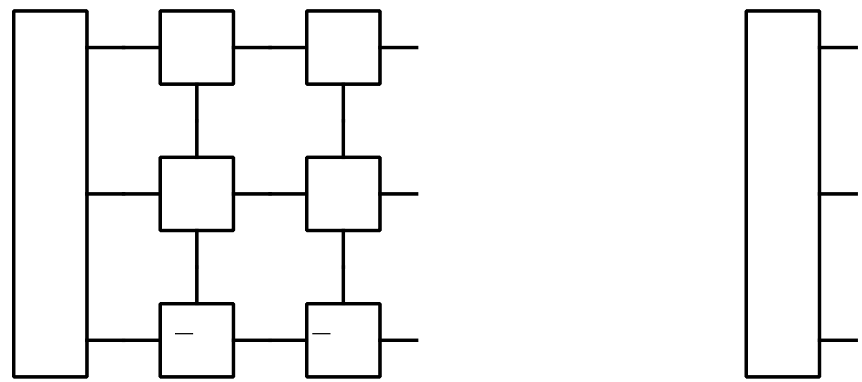
- Vertical and horizontal bond energy should be optimized at the same time



AD-BC-VUMPS

BCVUMPS-algorithm



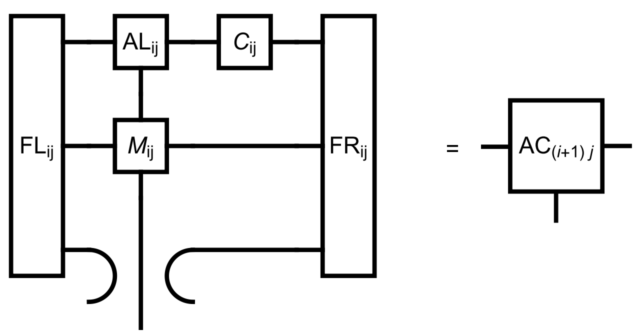


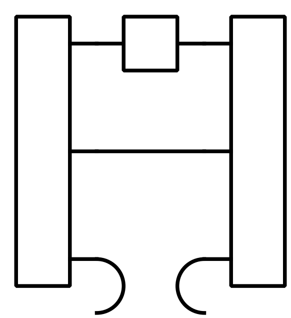
while error(AC,FL,FR,M) > tol_error

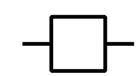
Get environment
AL[i, j] = leftenv(AL[i , :], M [i , :])
AR[i, j]= rightenv(AR[i , :], M [i , :])

Get orthonormal form
AC[i, j], C[i, j] = vumpsstep(AL[:, j], C[:, j], AR[:, j], M[:, j], FL[:, j], FR[:, j])

# calculate observable
Obs = obser(O, AC, M, FL, FR)



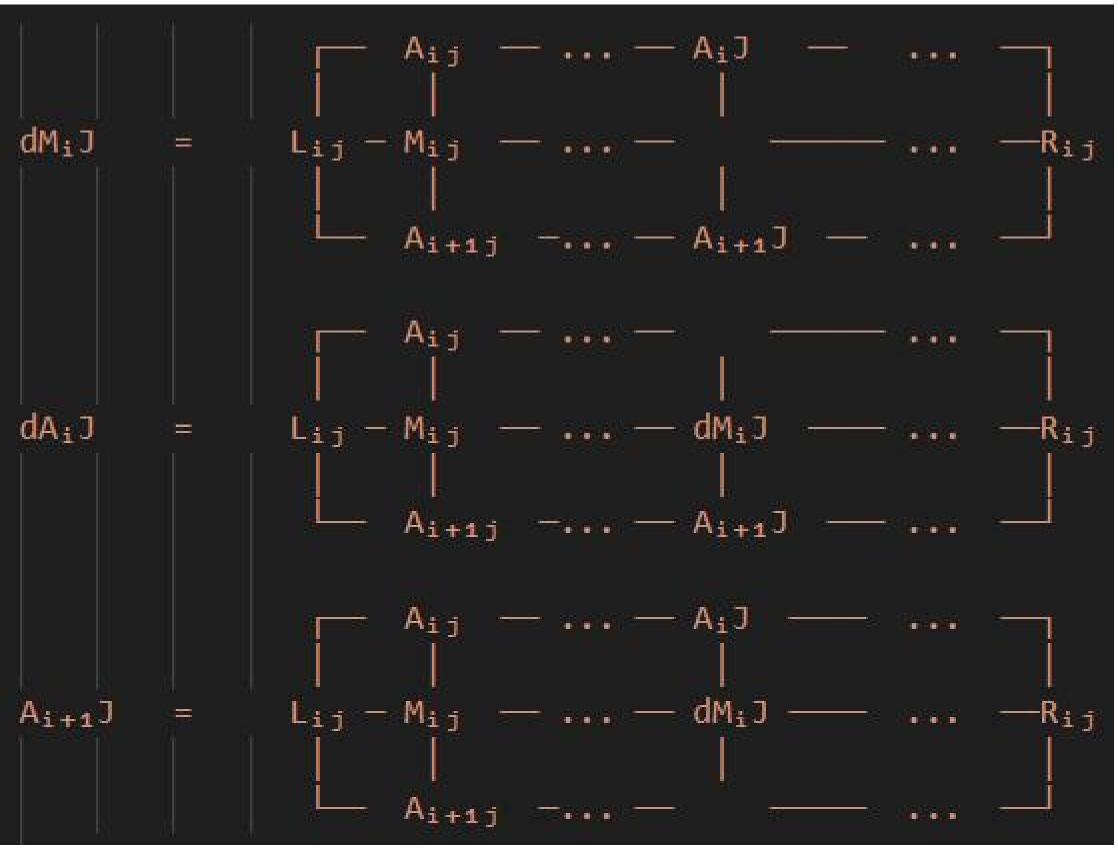


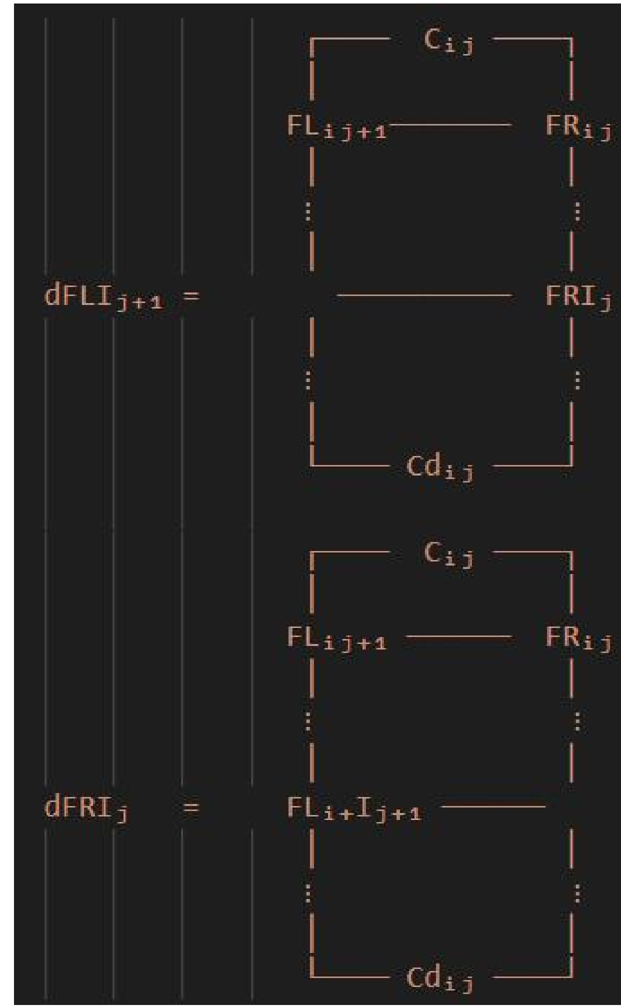


AD for array of array

```
# example to solve differential of array of array
# use `[]` list then reshape
A = Array{Array,2}(undef, 2, 2)
for j = 1:2,i = 1:2
    A[i,j] = rand(2,2)
end
function foo2(x)
    # B[i,j] = A[i,j].*x   # mistake
    B = reshape([A[i].*x for i=1:4],2,2)
    return sum(sum(B))
end
@test Zygote.gradient(foo2, 1)[1] ≈ num_grad(foo2, 1)
```

Be careful of index and replaced function!





AD-BC-VUMPS

Contents

some tricks → AD-VUMPS

- ✔ TFising
- ✔ Heisenberg

AD-BC-VUMPS

- ✔ 2×2 Heisenberg
- ✔ Honeycomb Kitaev

Subtle VUMPS → Honeycomb Kitaev

Some argument for up and down VUMPS

some Tricks

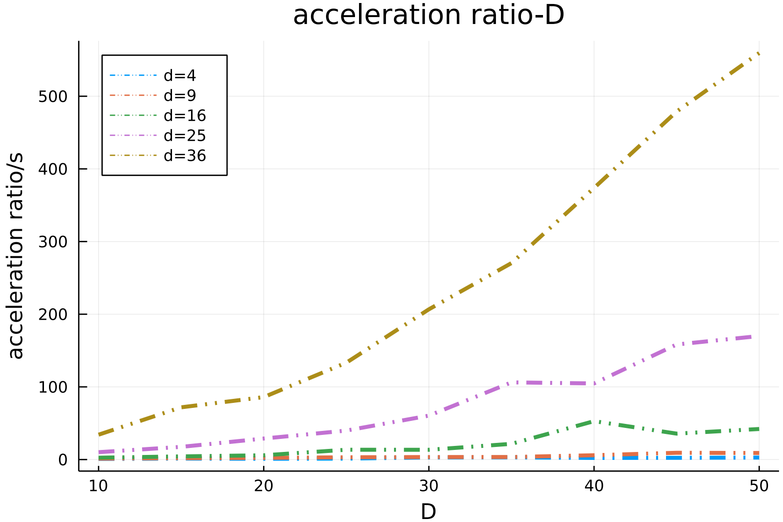
some tricks to accelerate

- Save vumps environment
- Optimize the contract order
- Set maxiter = 1 for linsolve but eigsolve not
- GPU

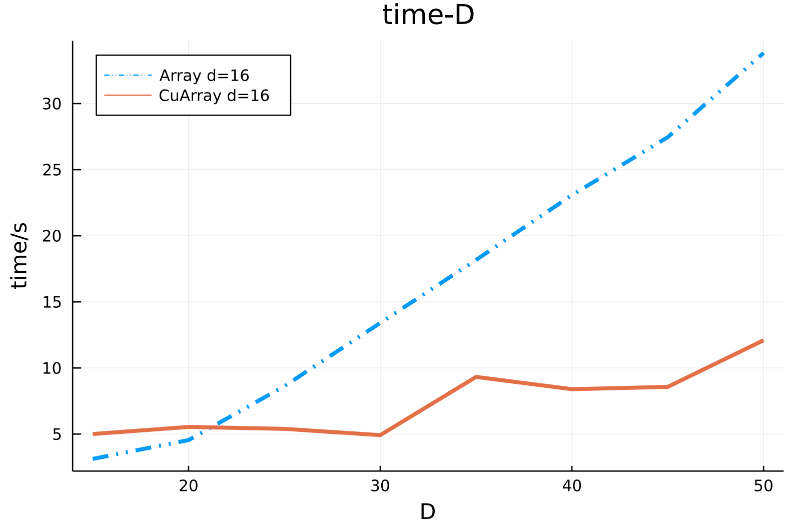
For Julia OMEinsum

optimize_greedy(; method=MinSpaceDiff())

OMEinsum



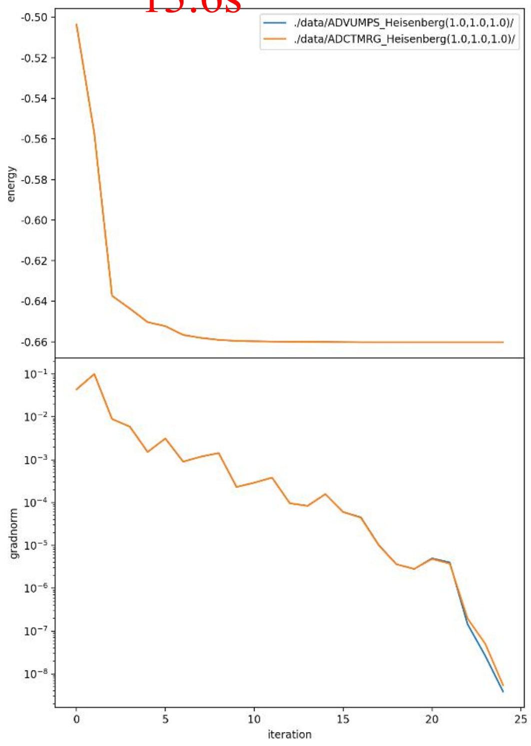
KrylovKit

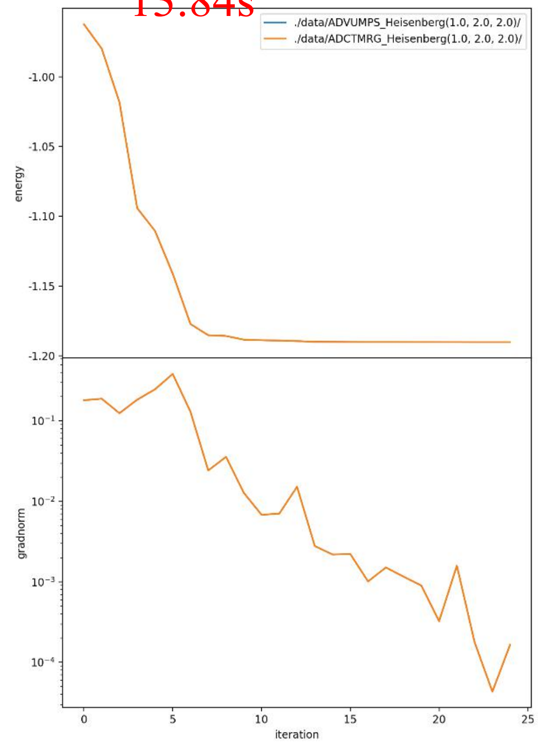


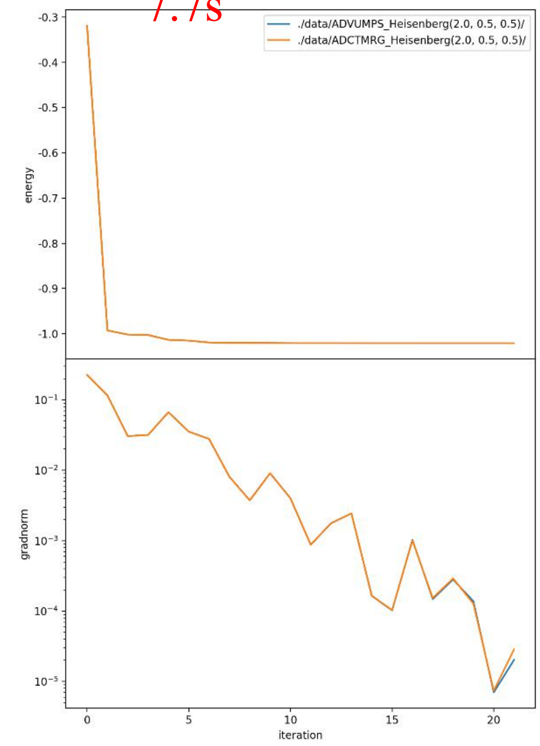
Finding the Ground State of infinite 2D Heisenberg model

ADVUMPS vs ADCTMRG $D = 2\ \chi = 20\ \delta = 10^{-12}$

33.76s vs 15.62s	29.11s vs 15.58s	16.61s vs 4.94s
13.6s	13.84s	7.7s







Difference ~1e-14	Difference ~1e-9	Difference ~1e-9

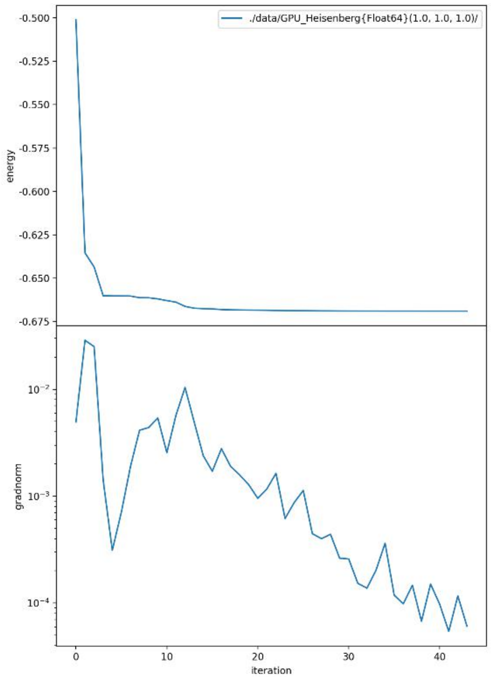
Backward	9.91s 72.3%	10.7s 77.1%	6.14s 76.9%
forward	3.79s 27.7%	3.19s 22.9%	1.84s 23.1%

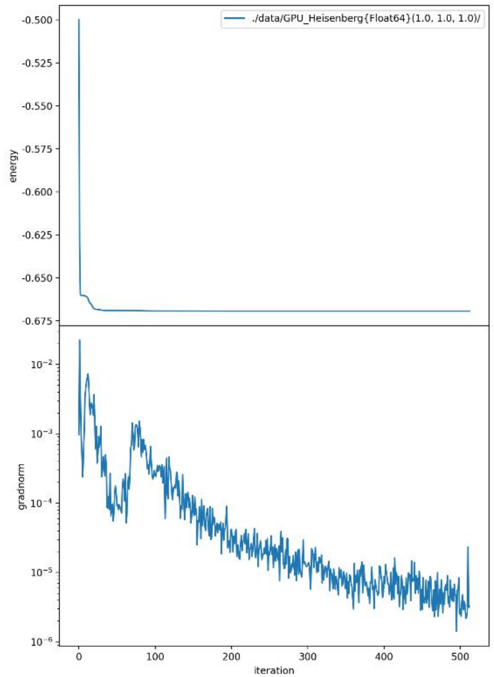
Heisenberg model $D = 4,5,6$

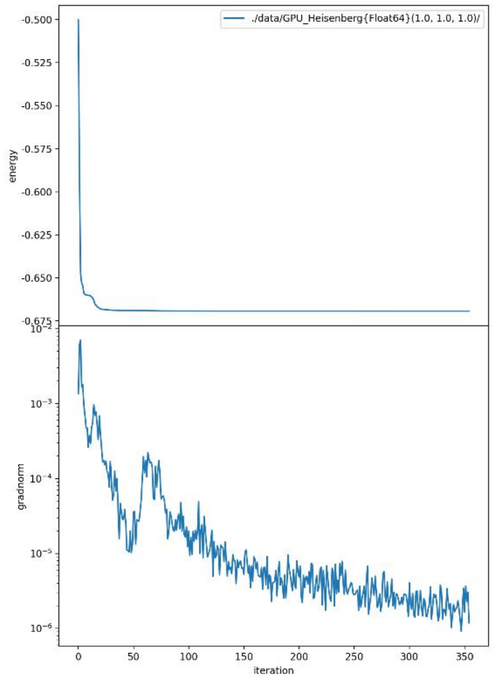
one RTX2060-6G

D	χ	iteration	time	E	error
4	30	44	22.7 min	-0.6689534	7.3×10^{-4}
5	30	513	6.6h	-0.6693341	1.6×10^{-4}
6	30	355	16.1h	-0.6693380	1.56×10^{-4}

MC: -0.6694421







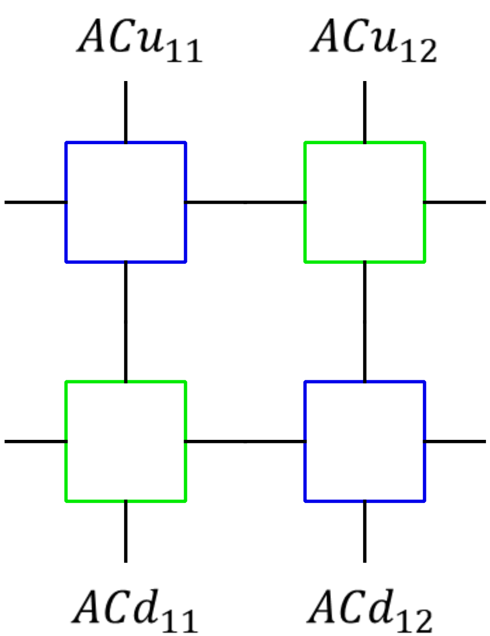
2×2 AD-BC-VUMPS

symmetry environment

- $H = \sum_{ij} S_i^z S_j^z + \frac{1}{2}(S_i^+ S_j^- + S_i^- S_j^+)$

$$\Big\downarrow \quad S_j \leftarrow \sigma_x S_j \sigma_x'$$

- $H = -\sum_{ij} S_i^z S_j^z + \frac{1}{2}(S_i^+ S_j^+ + S_i^- S_j^-)$



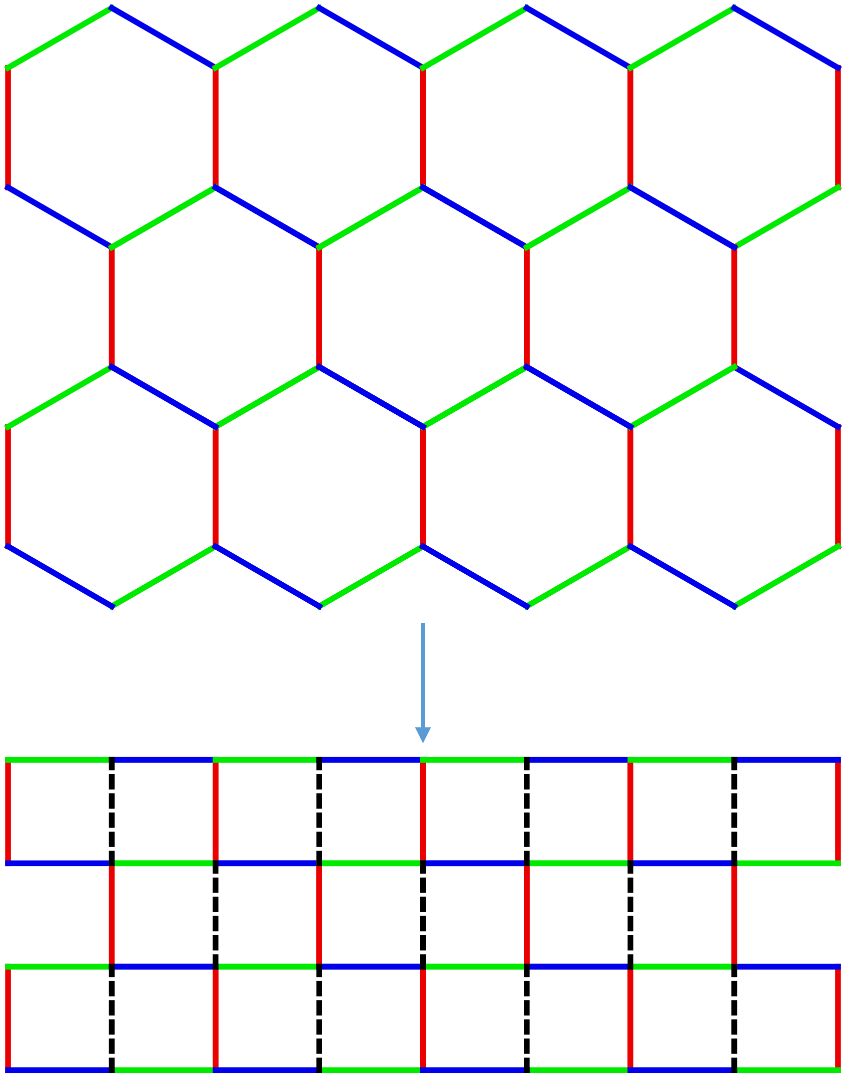
Up-and-down-symmetry assumption

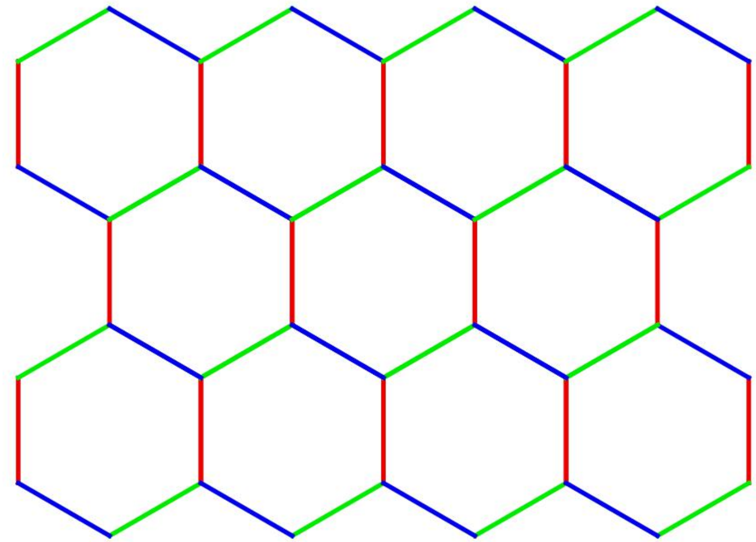
$ACd_{11} = ACu_{12}$

$ACd_{12} = ACu_{11}$

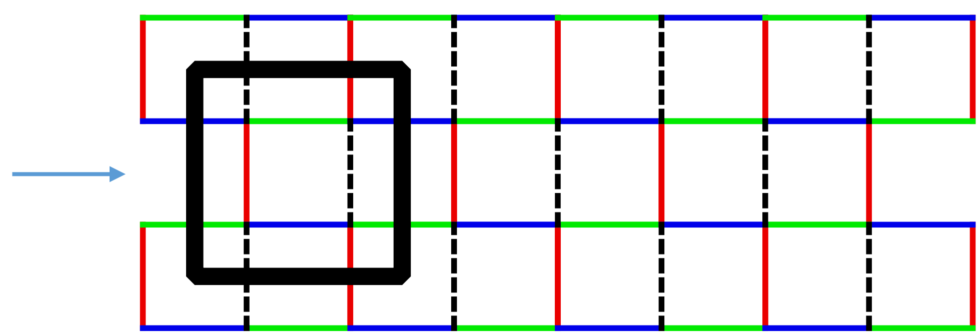
D	χ	iteration	time	E
2	20	25	122.9s	-0.6602265
4	32	134	5.5h	-0.6689648
4	32	86	2.5h	-0.6689539
4	30	513	22.7 min	-0.6689534

Honeycomb lattice to square

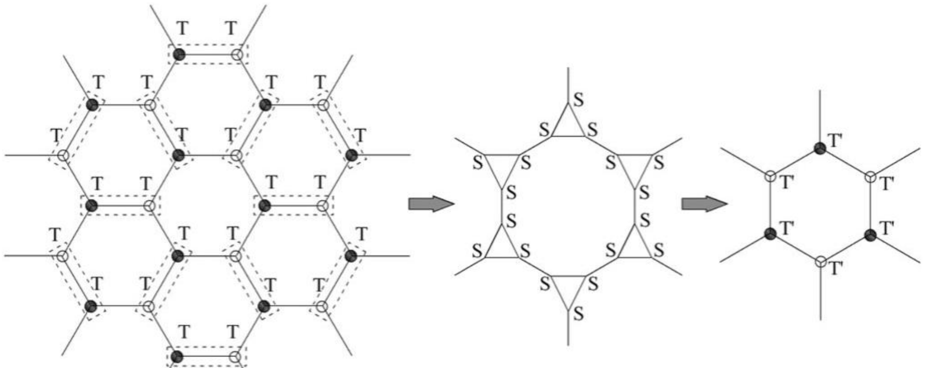




2 × 2 AB iPEPS assumption



Honeycomb Heisenberg



(a) (b) (c)

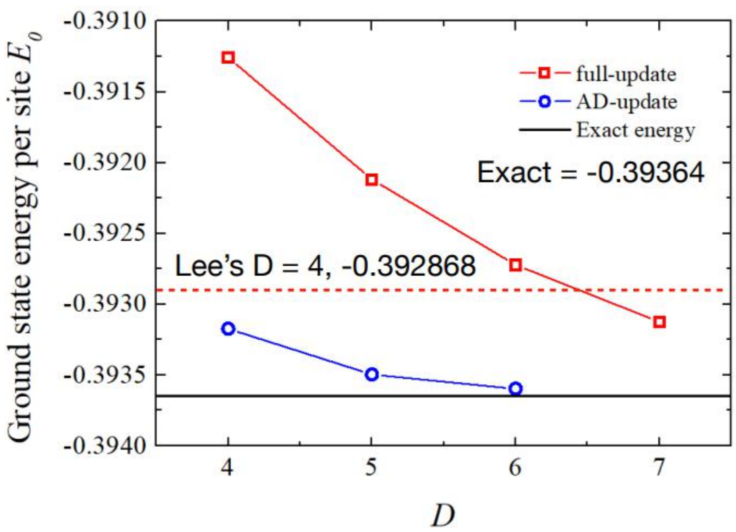
D	χ	E	M
2	20	-0.5400	0.243
3	20	-0.5430	0.222
4	20	-0.5440	0.200

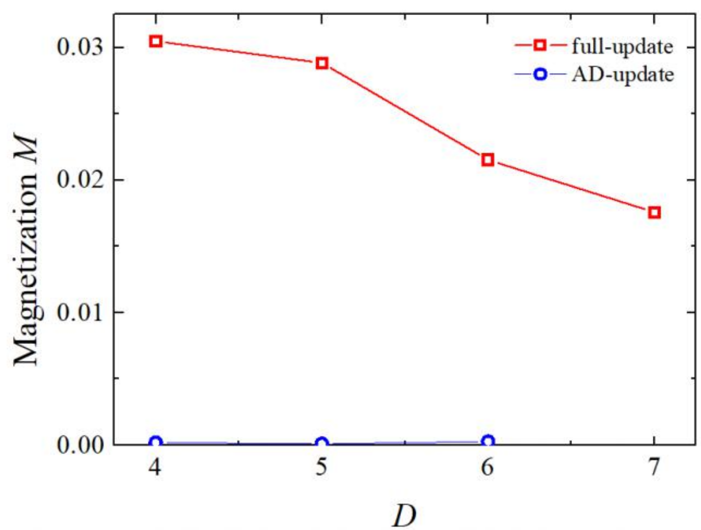
D	E	M
3	−0.5365	0.249
4	−0.5456	0.228
5	−0.5488	0.220
6	−0.5513	0.206
7	−0.5490	0.216
8	−0.5506	0.212

Method	E	M
Spin wave [12]	−0.5489	0.24
Series expansion [13]	−0.5443	0.27
Monte Carlo [14]	−0.5450±0.001	0.22±0.03
Ours $D = 8$	−0.5506	0.21 ± 0.01

Honeycomb Kitaev

Iregui, *et. al.*, PRB 90, 195102 (2014)





CTMRG + imaginary-time evolution

vs.

TEBD + AD

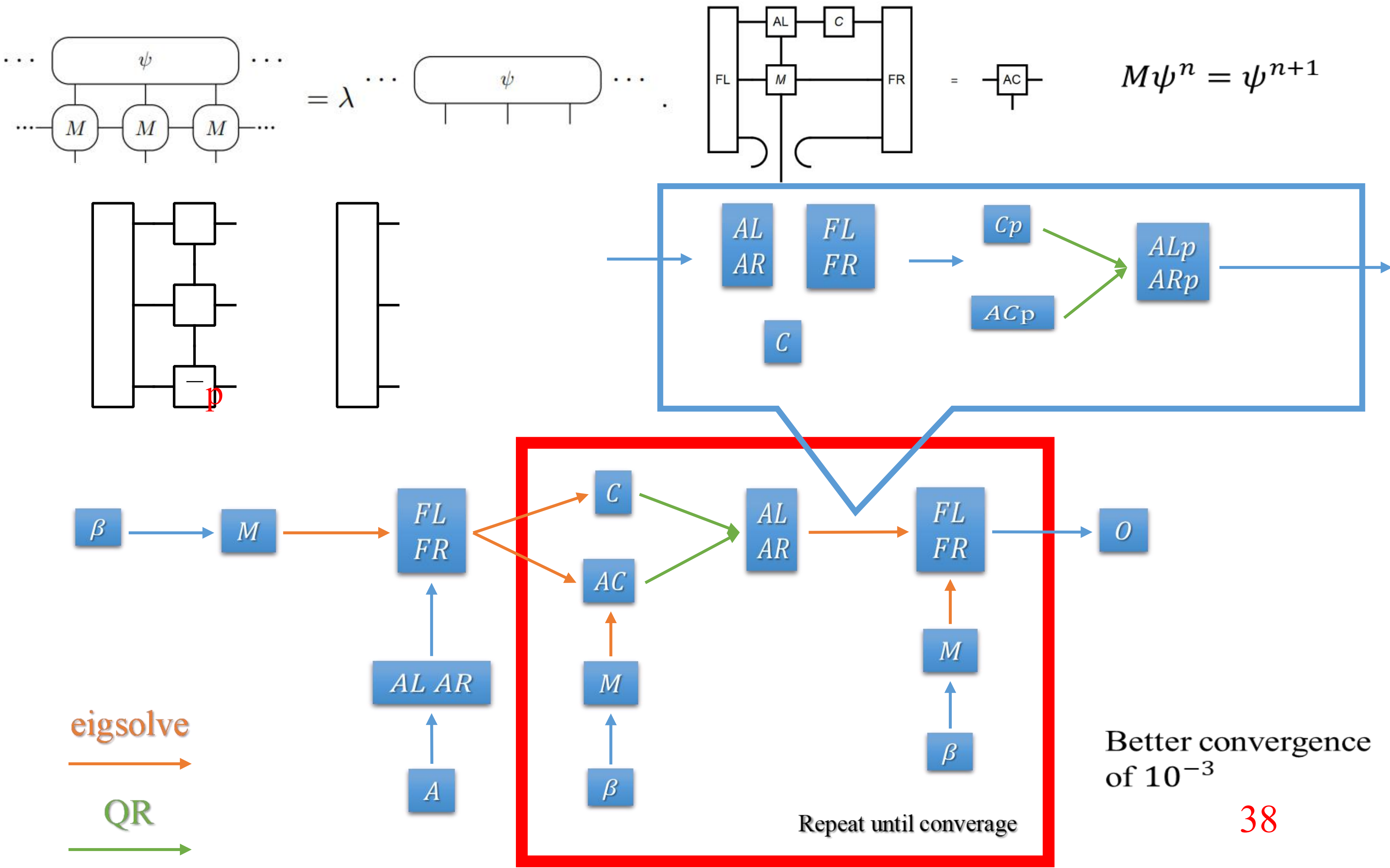
```
D  chi        En                     M
3  50    -0.3795995949421732    0.27211012905099785
4  80    -0.3931735662139908    0.0001872923118720
5  100   -0.3934970101949562    0.0000972301264798
6  100   -0.3935976940110233    0.0002659684803263
```

Theoretical value: −0.3936   0

D	χ	E	M
2	20	-0.3759726	
4	20	-0.3914..	0.02..

VUMPS doesn't converge for error ≈ 0.1

Computation Graphs



result

D	chi	En	M
3	50	-0.3795995949421732	0.27211012905099785
4	80	-0.3931735662139908	0.0001872923118720
5	100	-0.3934970101949562	0.0000972301264798
6	100	-0.3935976940110233	0.0002659684803263

D	χ	E	M
4	20	-0.3933225282386825	0.0005
4	30	-0.39343584624314554	0.002

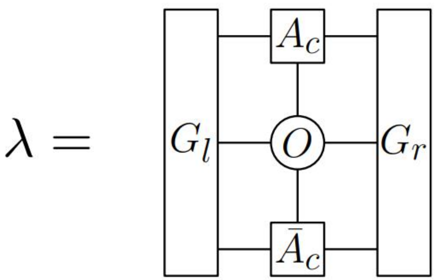
VUMPS up error ≈ 1e-6
down error ≈ 1e-6

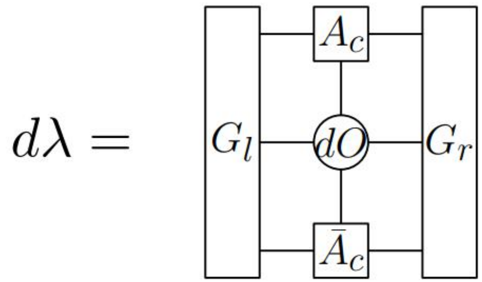
VUMPS up error ≈ 1e-7
down error ≈ 1e-2

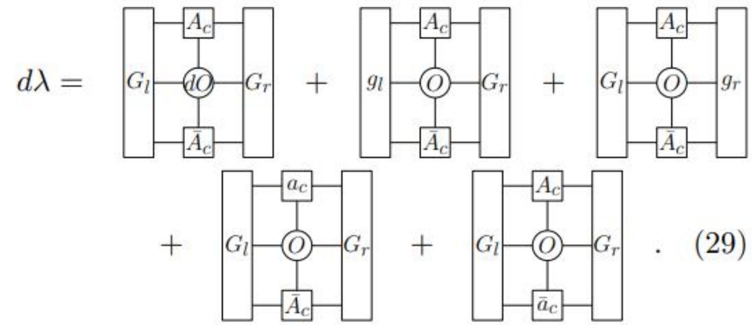
$$E = -0.3933 \sim -0.3929$$

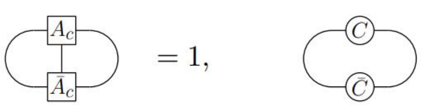
Some arguments for up and down VUMPS

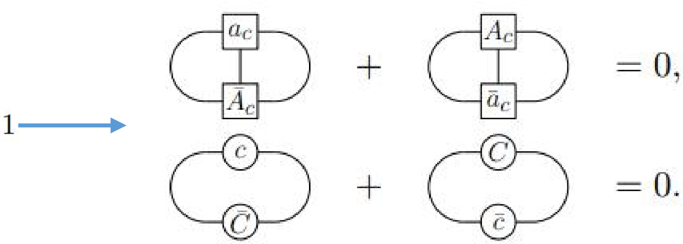
Differential for observable

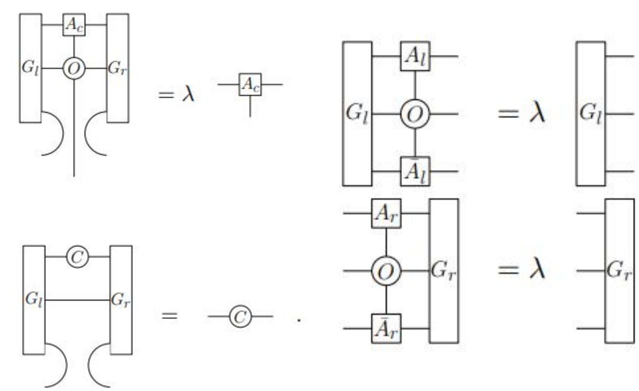




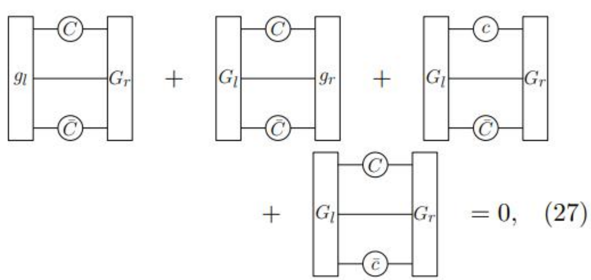


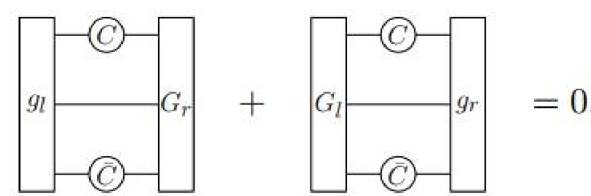






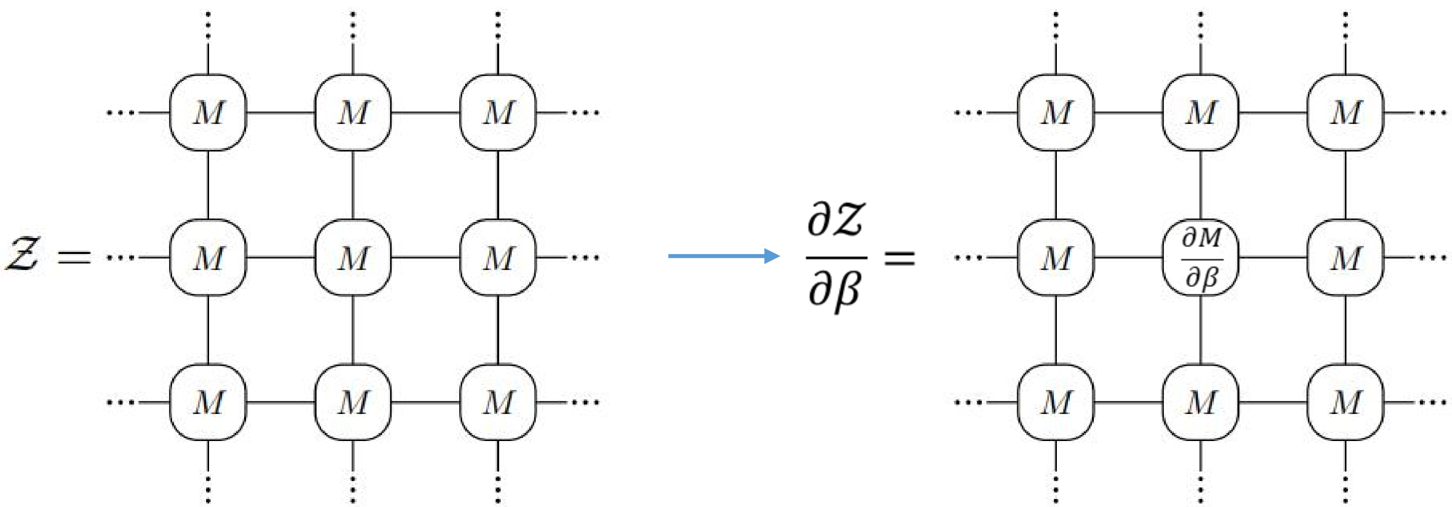
normalization of G_l and G_r

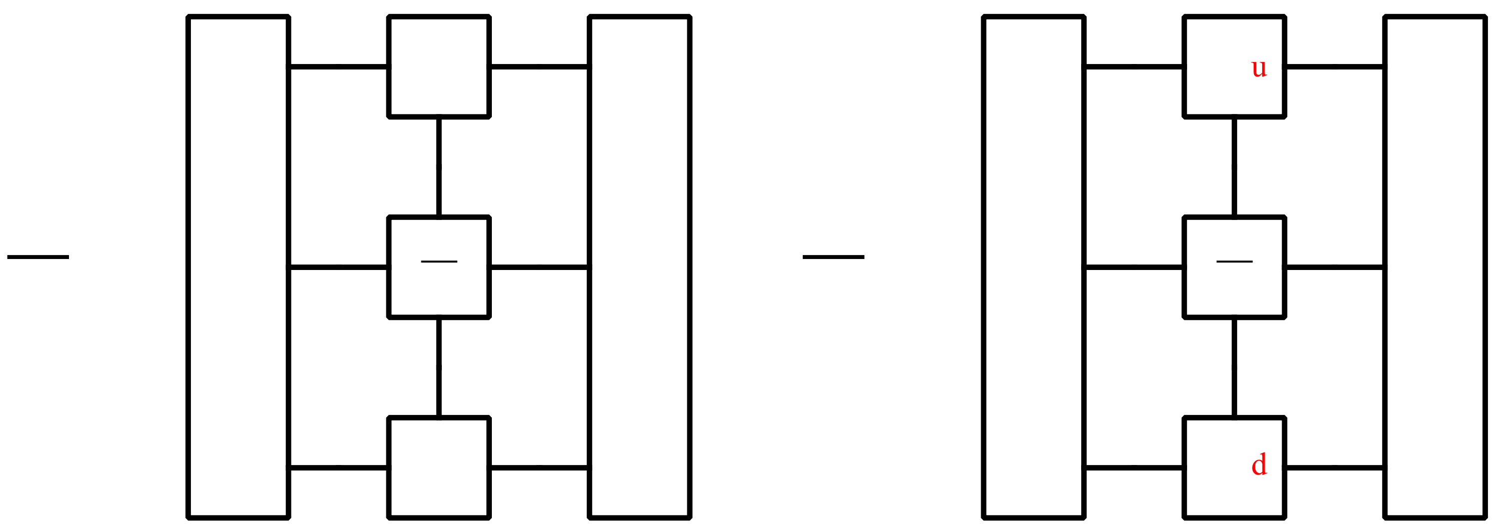




arXiv:2102.03143v2

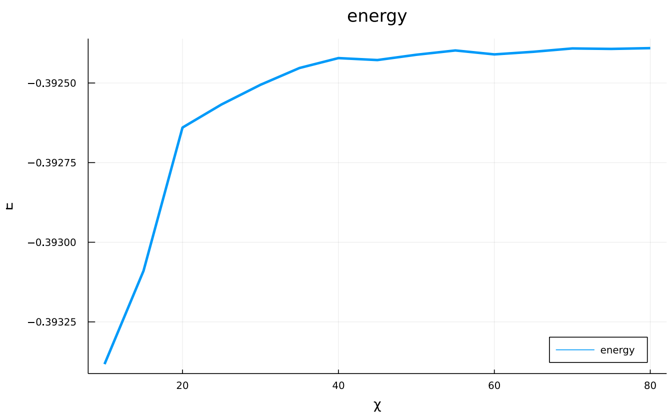
Contradictory result for up and down

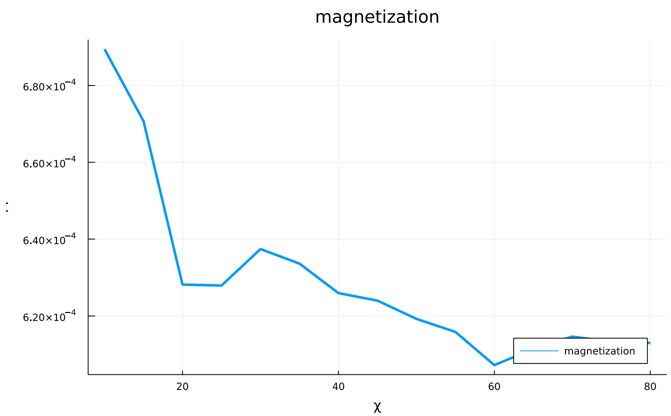




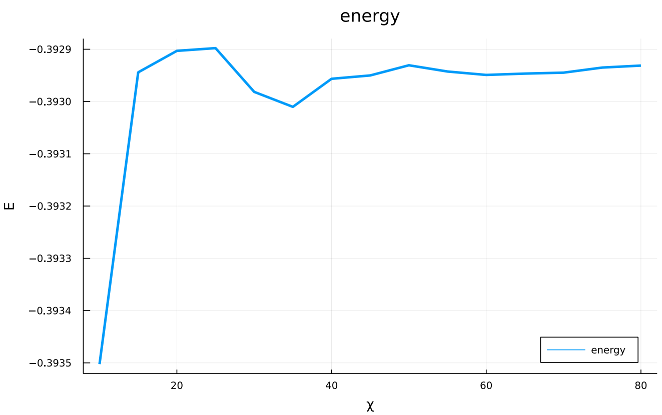
$E - \chi$

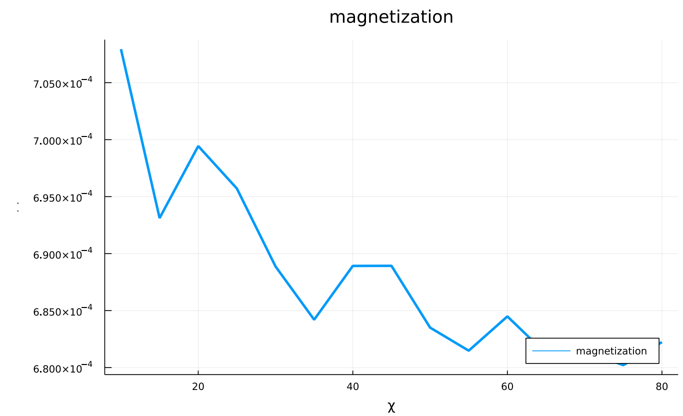
$D = 4\ \chi = 20$





$D = 4\ \chi = 80$





Optimize $\langle\phi|H|\psi\rangle/\langle\phi|\psi\rangle$

Question: how to find ϕ is in a nonorthogonal space of ψ?

Contents

Fold method for Heisenberg

D=8

2×2 cell ipeps

complex ADVUMPS

$K-J$ Heisenberg - Kitaev

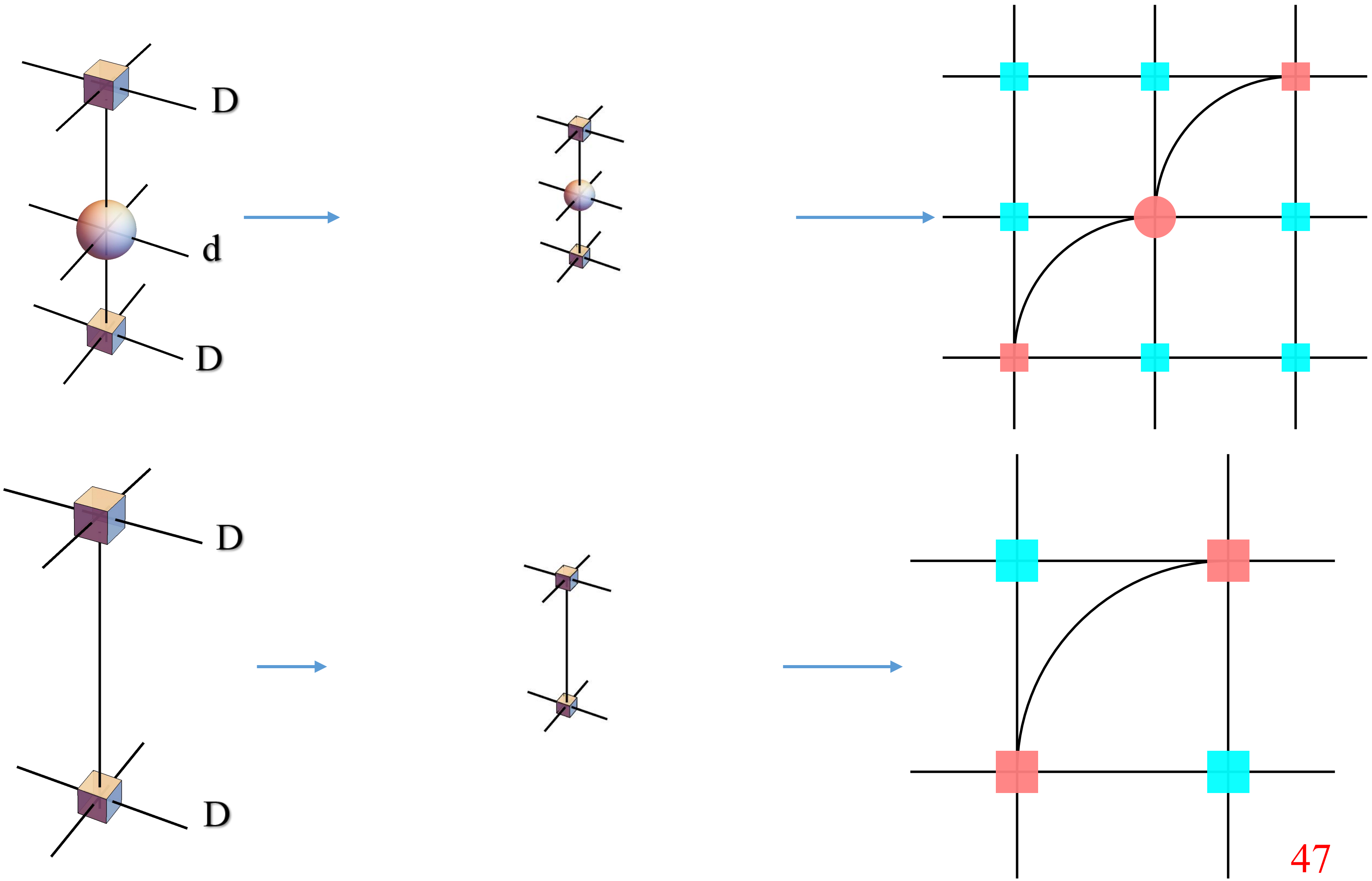
$K-J-\Gamma-\Gamma'$

$K-\Gamma$

1×2 cell ipeps

Fold method for Heisenberg

Fold method



Some details

- Just one side
- FL and FR is simple to get
- $\mathrm{norm}(\rho - \rho') = 0$

Result

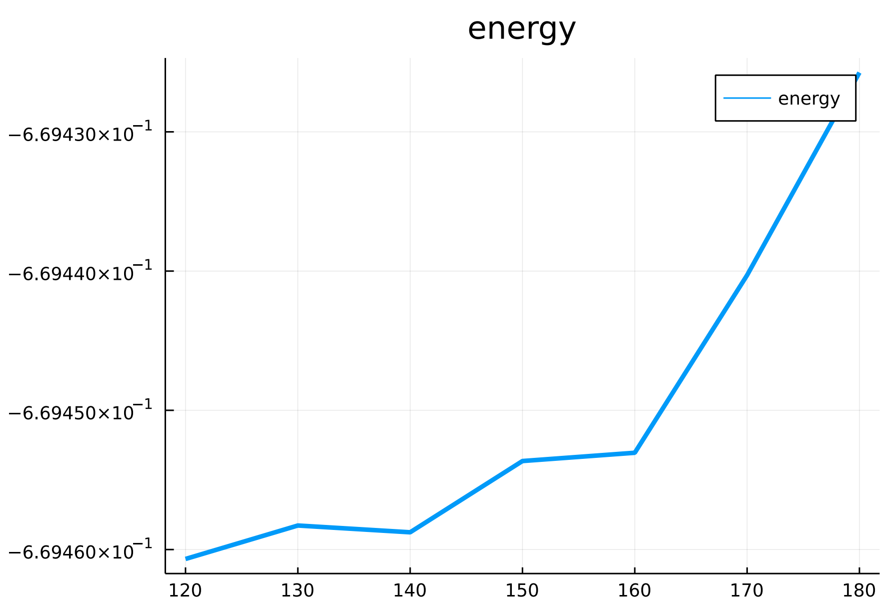
	E
$D = 6\ \chi = 144$	-0.669455693910015
$D = 7\ \chi = 160$	-0.6694572428974581
Liao2019-PRX	-0.6694245109061838
MC	-0.6694421

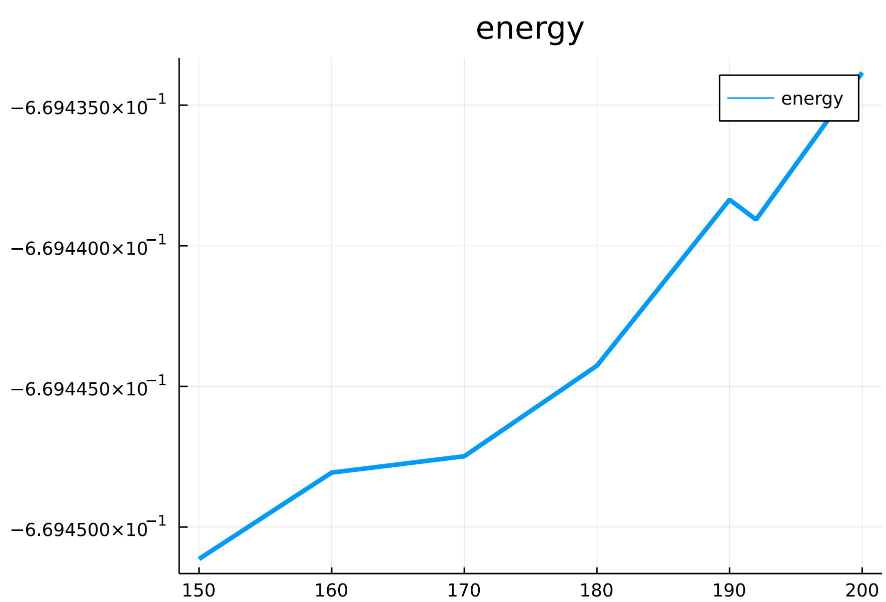
$D = 8$	time	steps	E
$\chi = 64$	1.5h	92	-0.66939453601441
$\chi = 128$	1.9h	75	-0.66940379584180
$\chi = 160$	3.7h	108	-0.66945318815880
$\chi = 192$	10.5h	39	-0.66944042736926

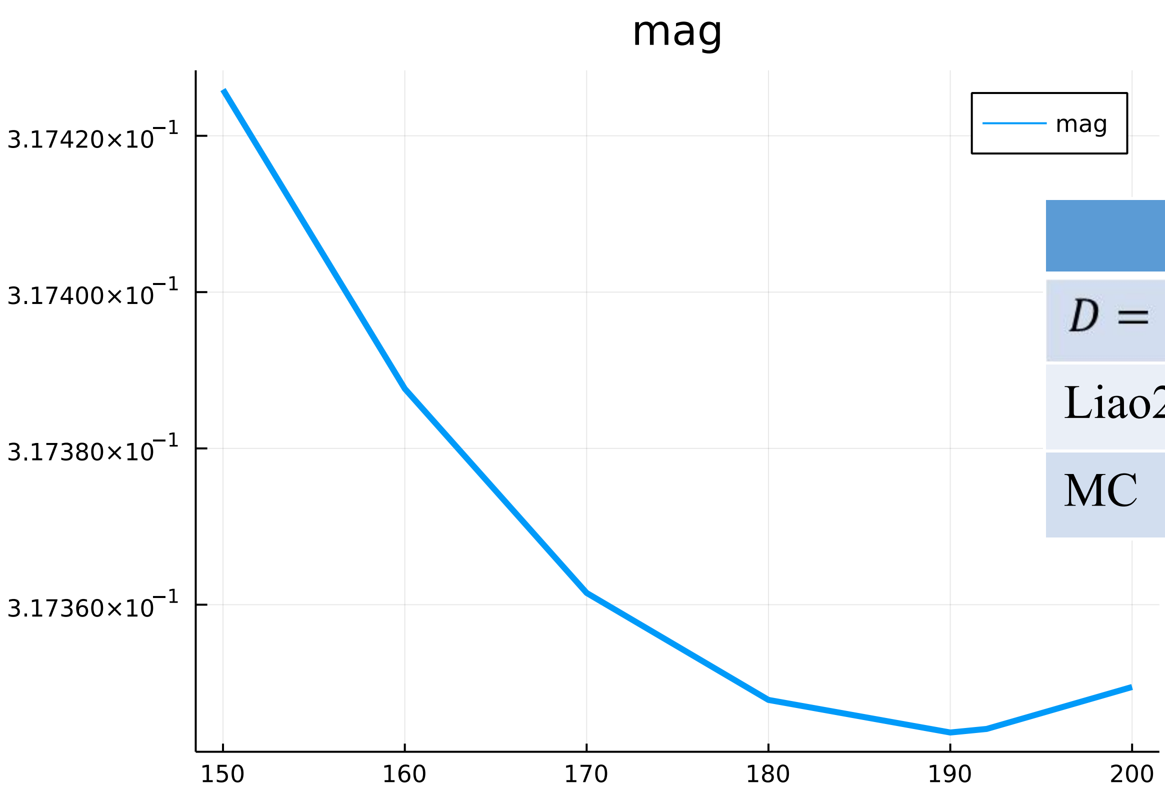
Iteration=50

-0.6694257436261145

-0.669433842168528

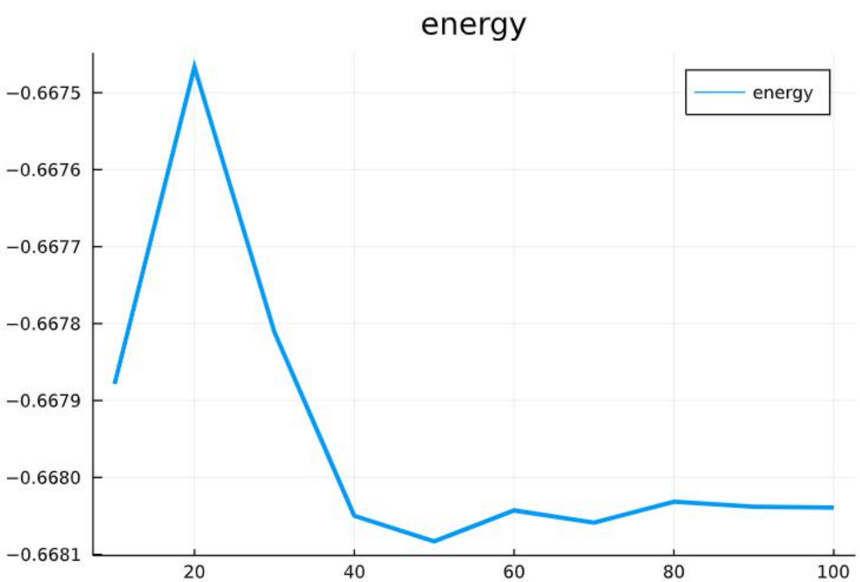




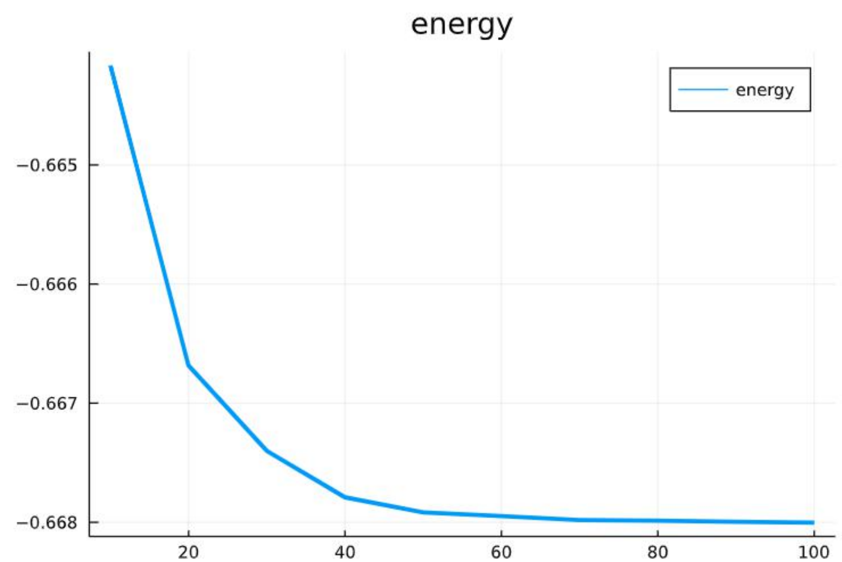


	mag
$D = 8\ \chi = 200$	0.317343647156262
Liao2019-PRX	0.3162526732615075
MC	0.30743

Different FL FR $D = 3$ $\chi = 50$



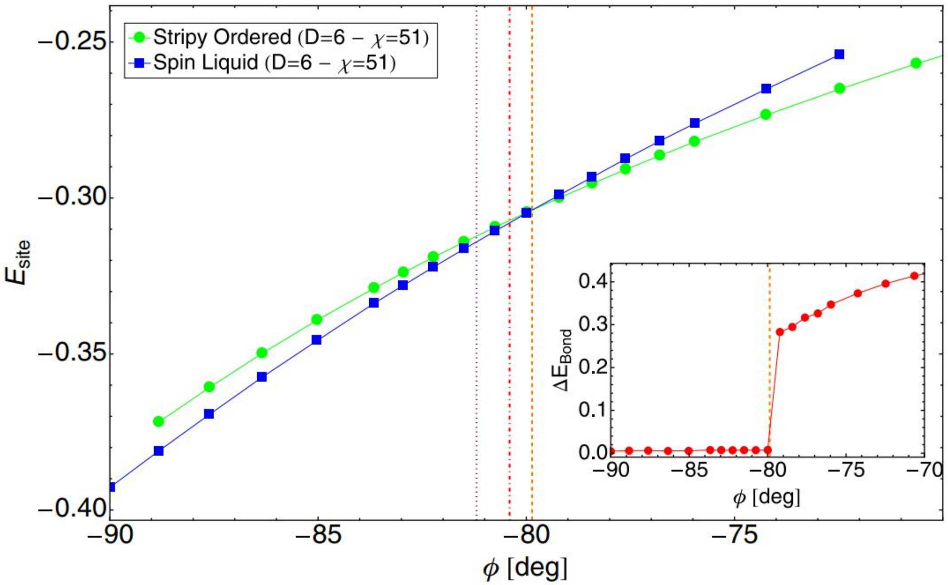
33→4



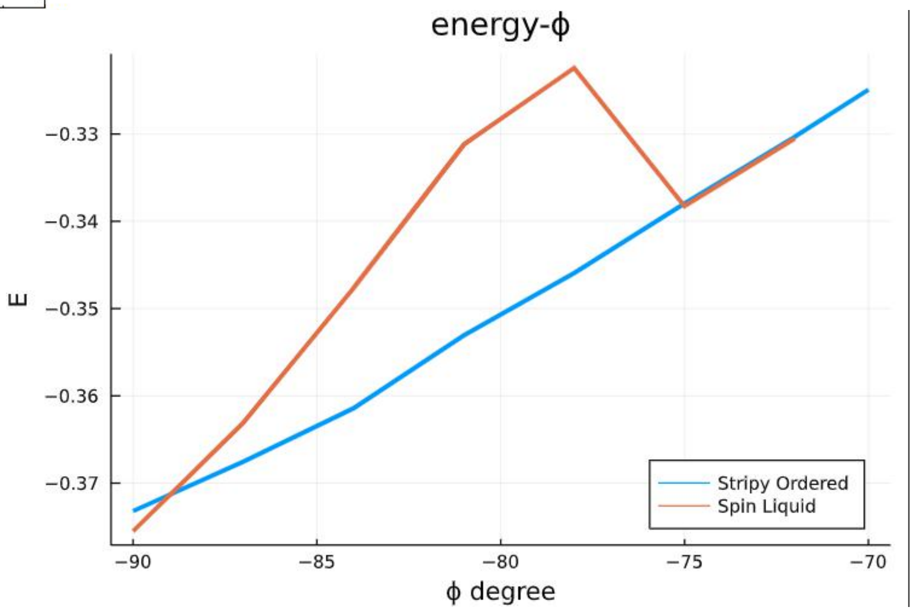
Direct 4

Kitaev-Heisenberg

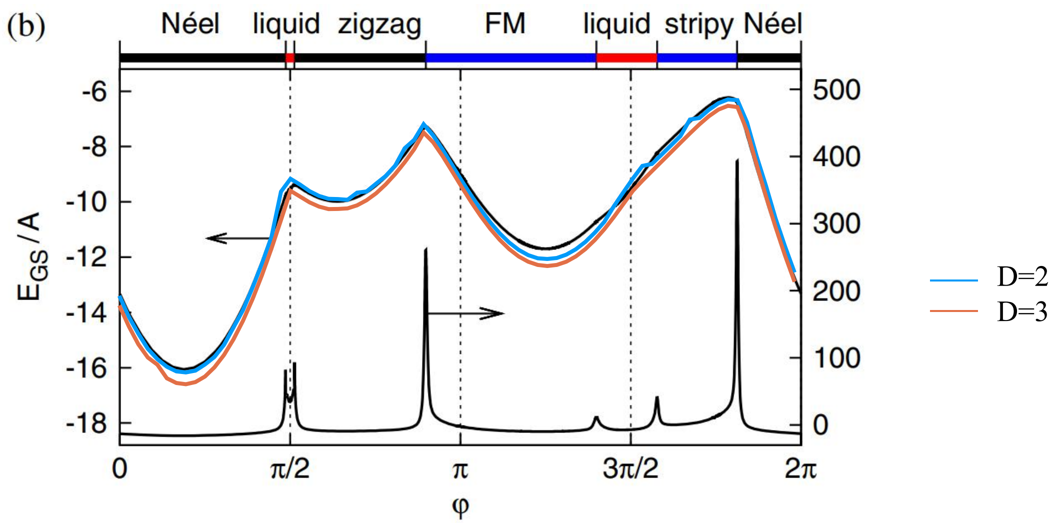
energy

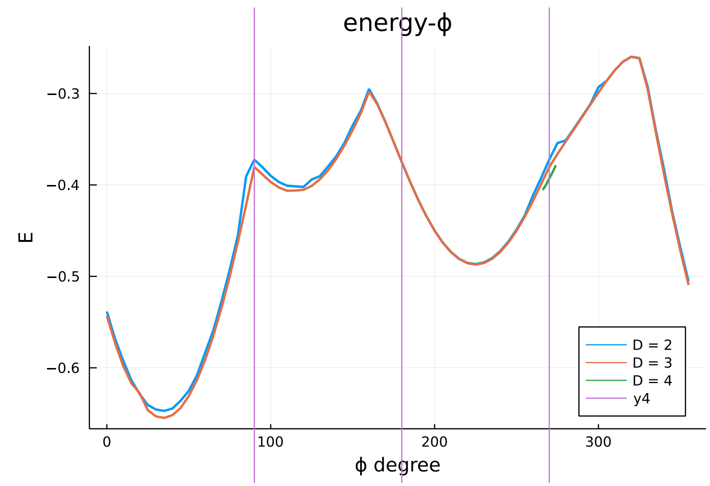

Stripy Ordered (D=6 – χ=51)
Spin Liquid (D=6 – χ=51)
−0.25
−0.30
−0.35
−0.40
E_{site}
−90
−85
−80
−75
ϕ [deg]
0.4
0.3
0.2
0.1
0.0
ΔE_{Bond}
−90
−85
−80
−75
−70
ϕ [deg]

D=2


energy-ϕ
−0.33
−0.34
−0.35
−0.36
−0.37
E
−90
−85
−80
−75
−70
ϕ degree
Stripy Ordered
Spin Liquid

energy and magnetization





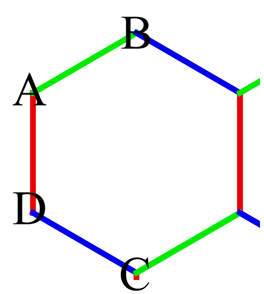
$$O_{\text{mag}} = \sqrt{\frac{1}{4}(\langle\vec{\sigma}_A\rangle^2 + \langle\vec{\sigma}_B\rangle^2 + \langle\vec{\sigma}_C\rangle^2 + \langle\vec{\sigma}_D\rangle^2)},$$

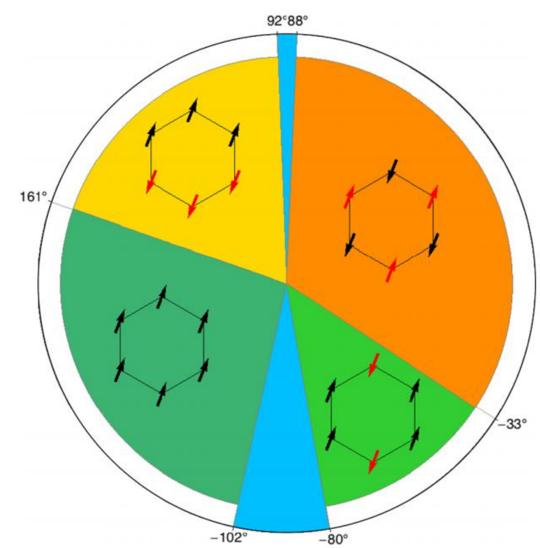
$$O_{\text{ferro}} = \sqrt{\frac{1}{4}(\langle\vec{\sigma}_A\rangle + \langle\vec{\sigma}_B\rangle + \langle\vec{\sigma}_C\rangle + \langle\vec{\sigma}_D\rangle)^2},$$

$$O_{\text{stripy}} = \sqrt{\frac{1}{4}(\langle\vec{\sigma}_A\rangle - \langle\vec{\sigma}_B\rangle - \langle\vec{\sigma}_C\rangle + \langle\vec{\sigma}_D\rangle)^2},$$

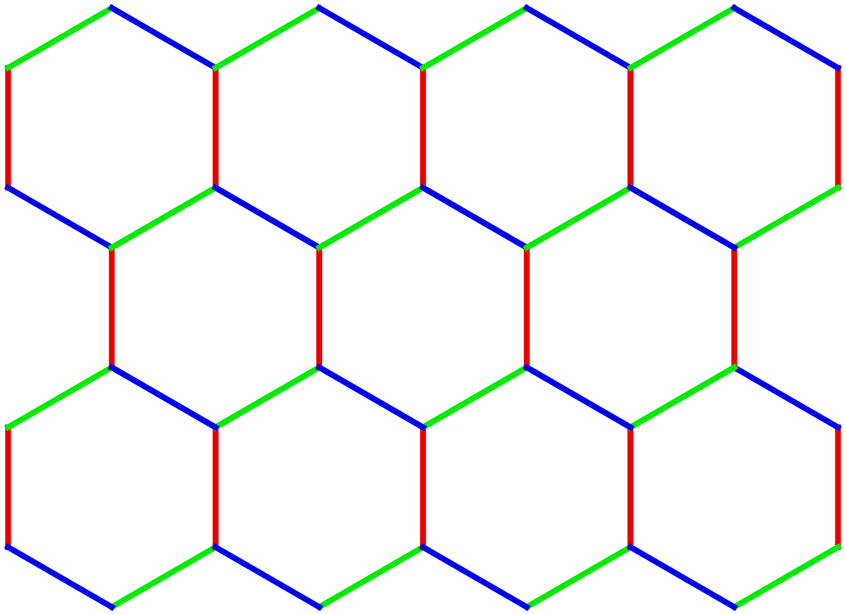
$$O_{\text{zigzag}} = \sqrt{\frac{1}{4}(\langle\vec{\sigma}_A\rangle + \langle\vec{\sigma}_B\rangle - \langle\vec{\sigma}_C\rangle - \langle\vec{\sigma}_D\rangle)^2},$$

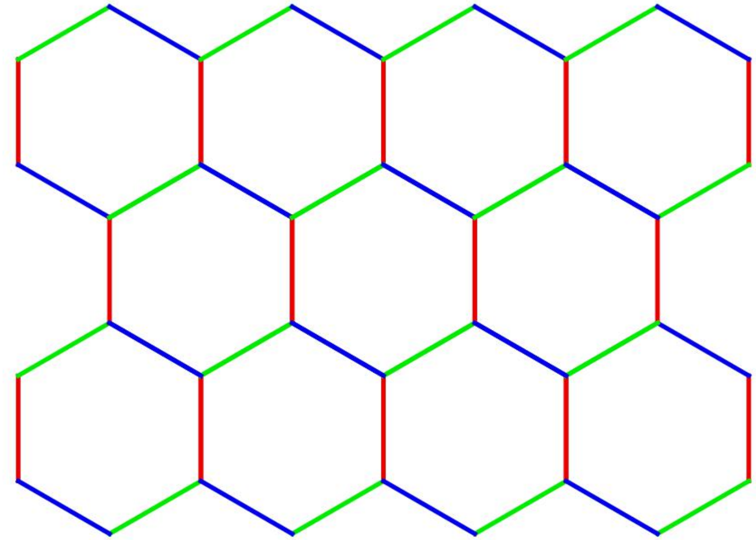
$$O_{\text{Néel}} = \sqrt{\frac{1}{4}(\langle\vec{\sigma}_A\rangle - \langle\vec{\sigma}_B\rangle + \langle\vec{\sigma}_C\rangle - \langle\vec{\sigma}_D\rangle)^2},$$



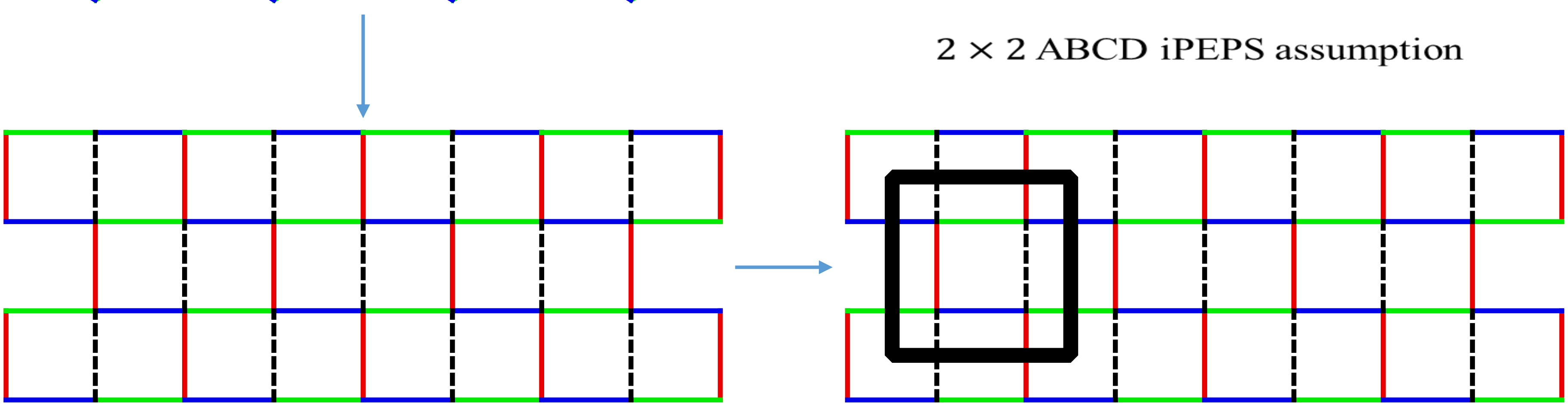


Honeycomb lattice to square

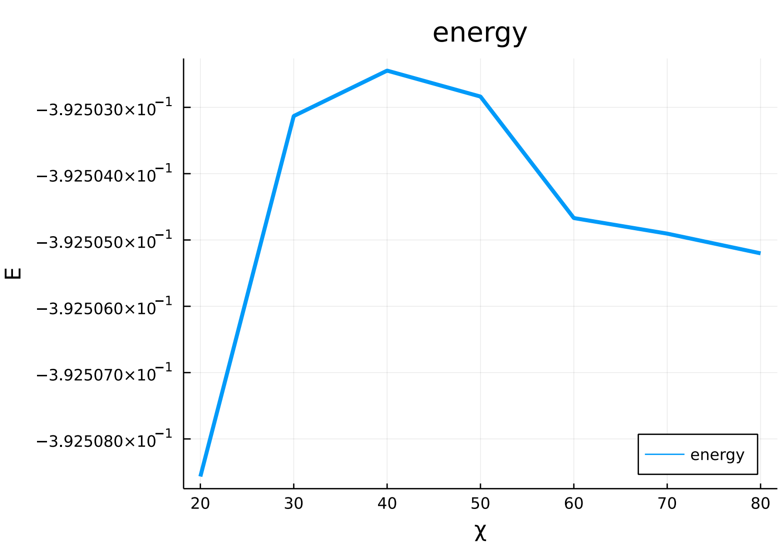




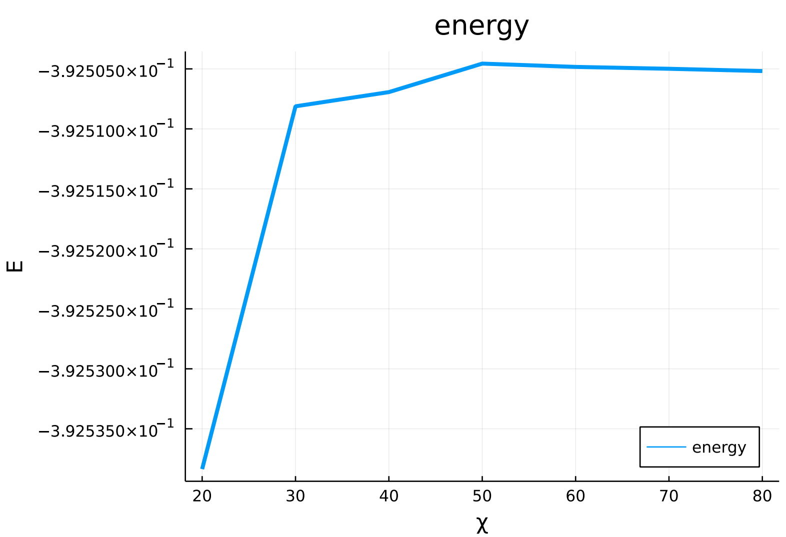
2×2 ABCD iPEPS assumption



$D = 4\ \chi = 50$

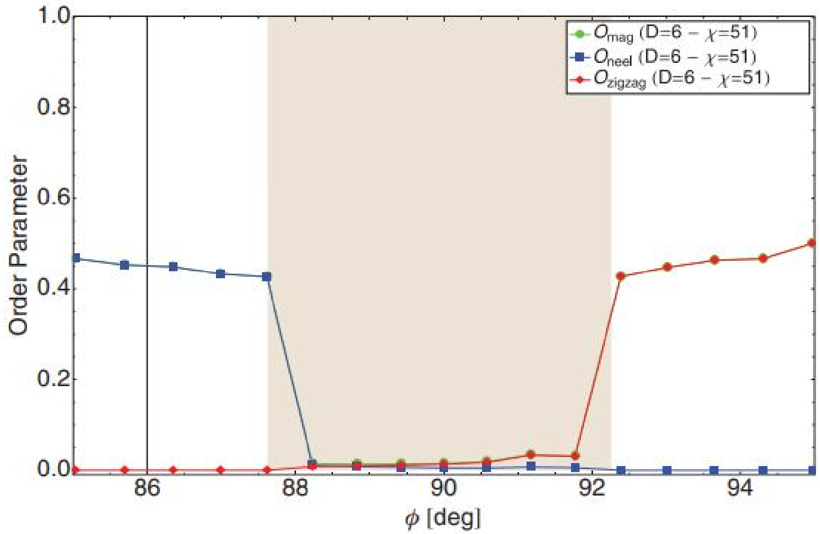


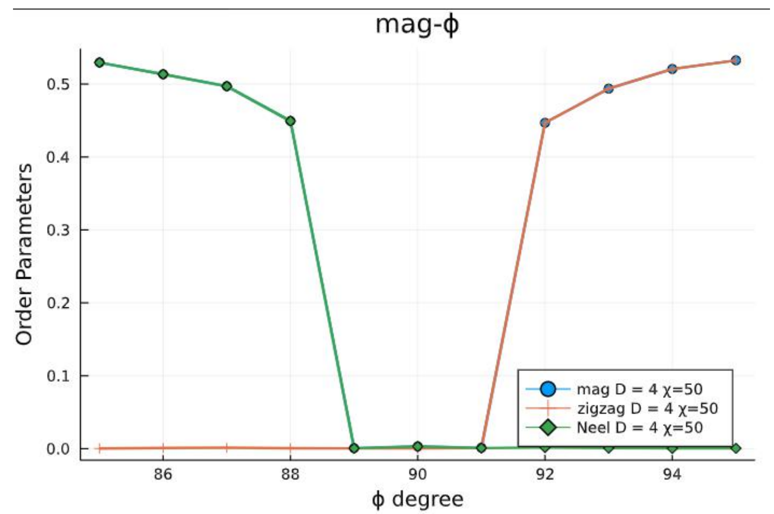
33→4

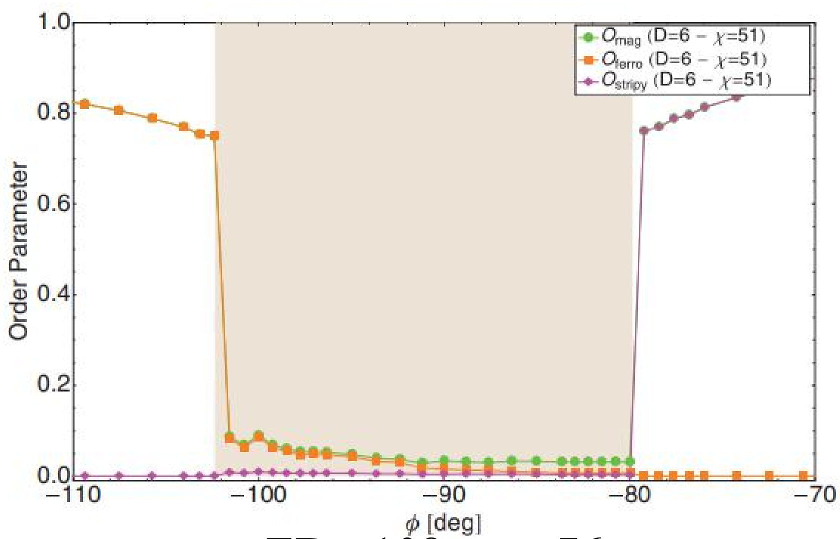


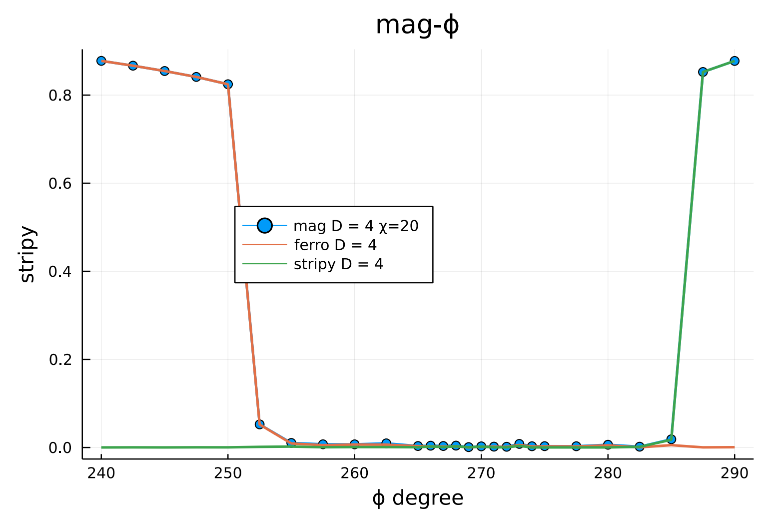
Direct 4

phase transition









ED: -108 -76

Complex AD for $K - J - \Gamma - \Gamma'$

Adjoint of QR

- $A = QR \quad \bar{A} = [\bar{Q} + Q\ \text{Symmetric}(M)]R^{-T}$ $\quad M = R\bar{R}^T - \bar{Q}^T Q$

$$\bar{A} = [\bar{Q} + Q\ \text{Hermitian}(M)]R^{-\prime} \quad M = R\bar{R}' - \bar{Q}' Q$$

- Trick: add $\delta = 10^{-12}$ to R's diagonal element for the stability of inverse

PRX 9, 031041 (2019)

Adjoint of eigsolve $T \to '$

- $l^T A = \lambda l^T, \quad Ar = \lambda r, \quad l^T r = 1,$

$$(A - \lambda I)\xi_l = (1 - r l^T)\bar{l}, \quad l^T \xi_l = 0$$
$$(A^T - \lambda I)\xi_r = (1 - l r^T)\bar{r}, \quad r^T \xi_r = 0$$

$\longrightarrow \bar{A} = \bar{\lambda} l r^T - l \xi_l^T - \xi_r r^T$

gauge invariant $\downarrow$ $l^T \bar{l} = r^T \bar{r} = 0$

$$(A - \lambda I)\xi_l = \bar{l}, \quad l^T \xi_l = 0$$
$$(A^T - \lambda I)\xi_r = \bar{r}, \quad r^T \xi_r = 0$$

Only l

$$\boxed{(A - \lambda I)\xi_l = \bar{l}, \quad l^T \xi_l = 0}$$
$$(A^T - \lambda I)\xi_r = 0, \quad r^T \xi_r = 0$$

$$\boxed{\bar{A} = -l \xi_l^T}$$

Only r

$$(A - \lambda I)\xi_l = 0, \quad l^T \xi_l = 0$$
$$\boxed{(A^T - \lambda I)\xi_r = \bar{r}, \quad r^T \xi_r = 0}$$

$$\boxed{\bar{A} = -\xi_r r^T}$$

Some discuss

- real-valued function is holomorphic function only when function is constant
- Gradient not derivative
  - $\nabla_z f(z) = 2\frac{\partial f}{\partial z^*} = \frac{\partial f}{\partial x} + i\frac{\partial f}{\partial y}$
- Vumps is more stable for complex numbers

$K - J - \Gamma - \Gamma'$

$$H = \sum_{\langle i,j \rangle_\gamma} [K S_i^\gamma S_j^\gamma + J \mathbf{S}_i \cdot \mathbf{S}_j + \Gamma (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) + \Gamma' (S_i^\gamma S_j^\alpha + S_i^\gamma S_j^\beta + S_i^\alpha S_j^\gamma + S_i^\beta S_j^\gamma)],$$

$K = -1 \quad J = -0.1 \quad \Gamma = 0.3 \quad \Gamma' = -0.02$

D=5 χ=50 E = -0.57654(unit cell)

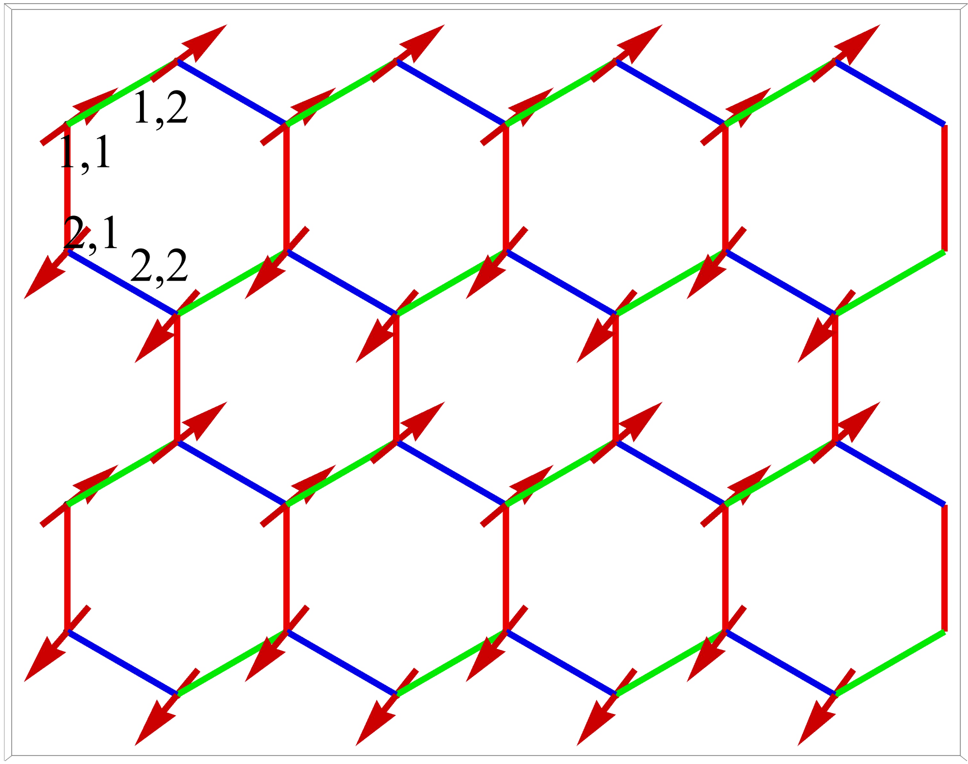
$M = [M_z, M_y, M_z]$

M[1,1] = [0.19091, 0.15684, -0.058433]

M[2,1] = [-0.15873, -0.19096, 0.056480]

M[1,2] = [0.191301, 0.15756, -0.056088]

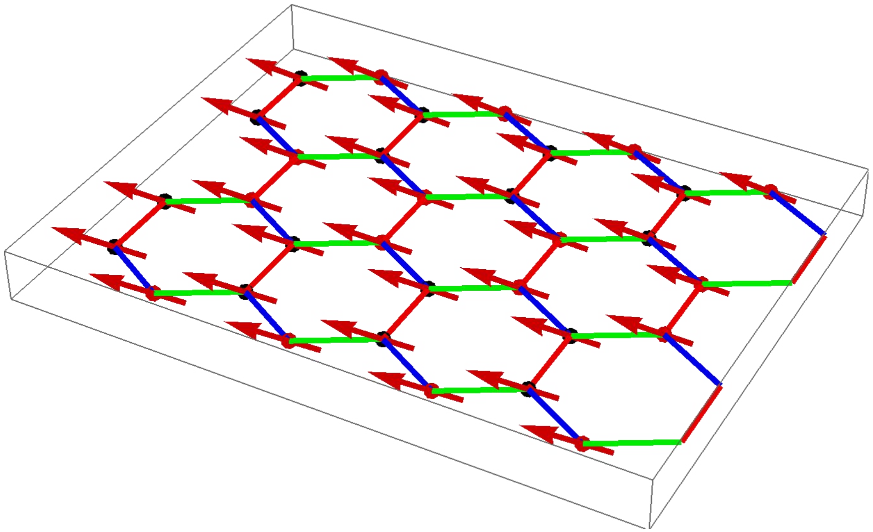
M[2,2] = [-0.15744, -0.19122, 0.057578]



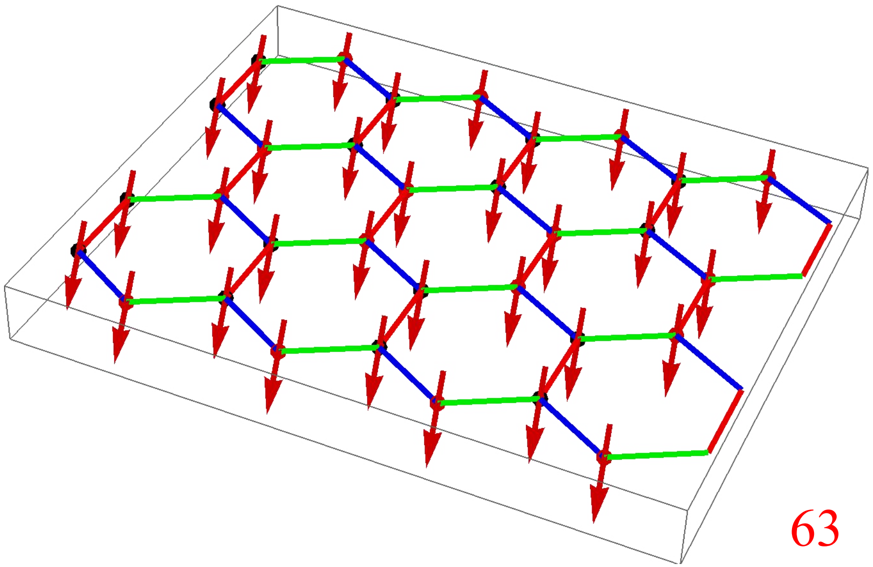
arXiv:2106.04557

Field(h_x, h_y, h_z)

Field =[-0.1,-0.1,0.0]
E = -0.717195
M[1,1] = [-0.24242, -0.24161, 0.2105]
M[2,1] = [-0.24139, -0.24225, 0.21112]
M[1,2] = [-0.24199, -0.2417, 0.21105]
M[2,2] = [-0.24179, -0.24223, 0.21081]



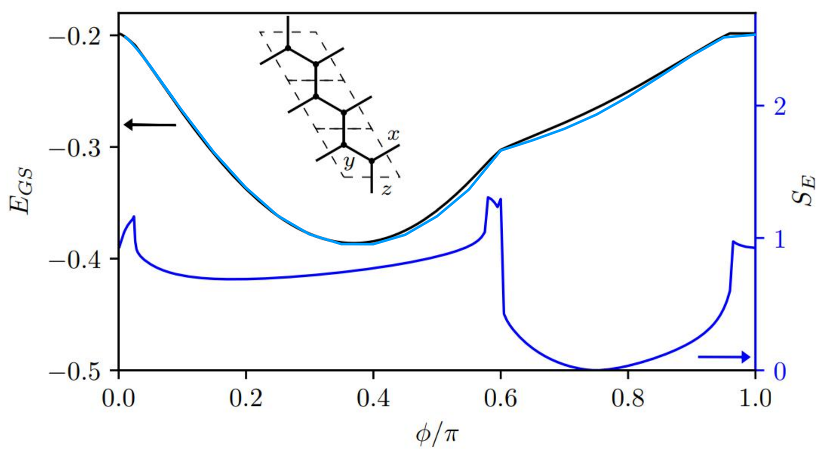
Field =[-0.1,-0.1,-0.1]
E = -0.65990
M[1,1] = [0.04685, -0.22688, -0.2248]
M[2,1] = [0.0485, -0.22538, -0.22486]
M[1,2] = [0.04673, -0.22765, -0.22405]
M[2,2] = [0.04794, -0.22497, -0.2246]



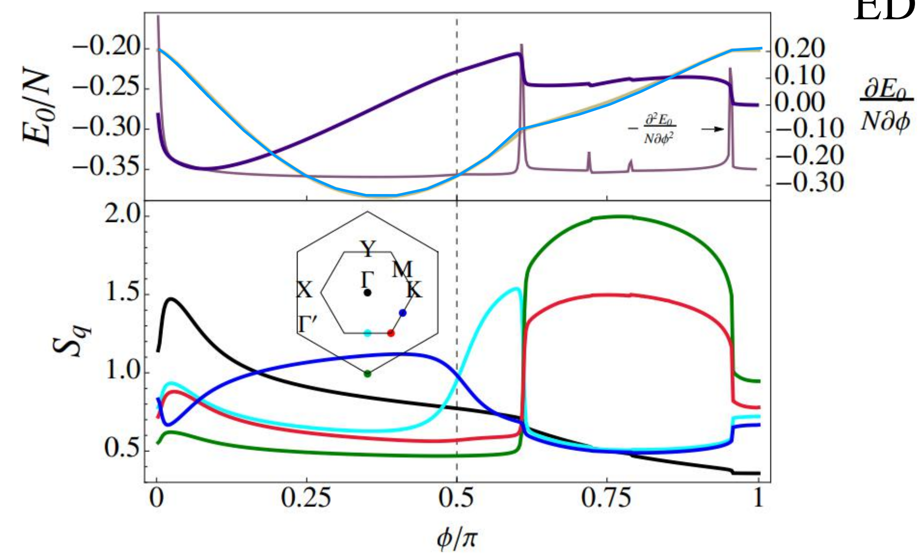
$K - \Gamma$

K = − cos φ and Γ = sin φ

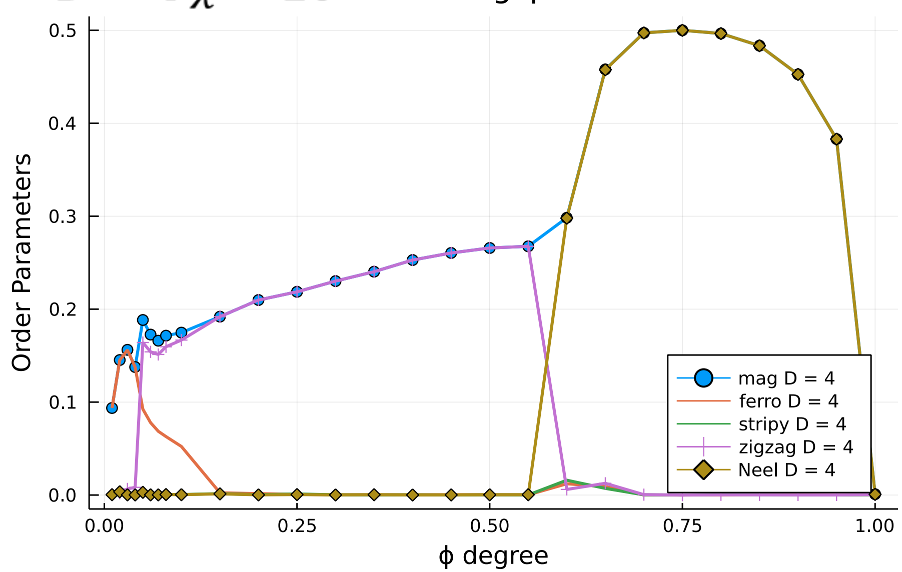
iDMRG



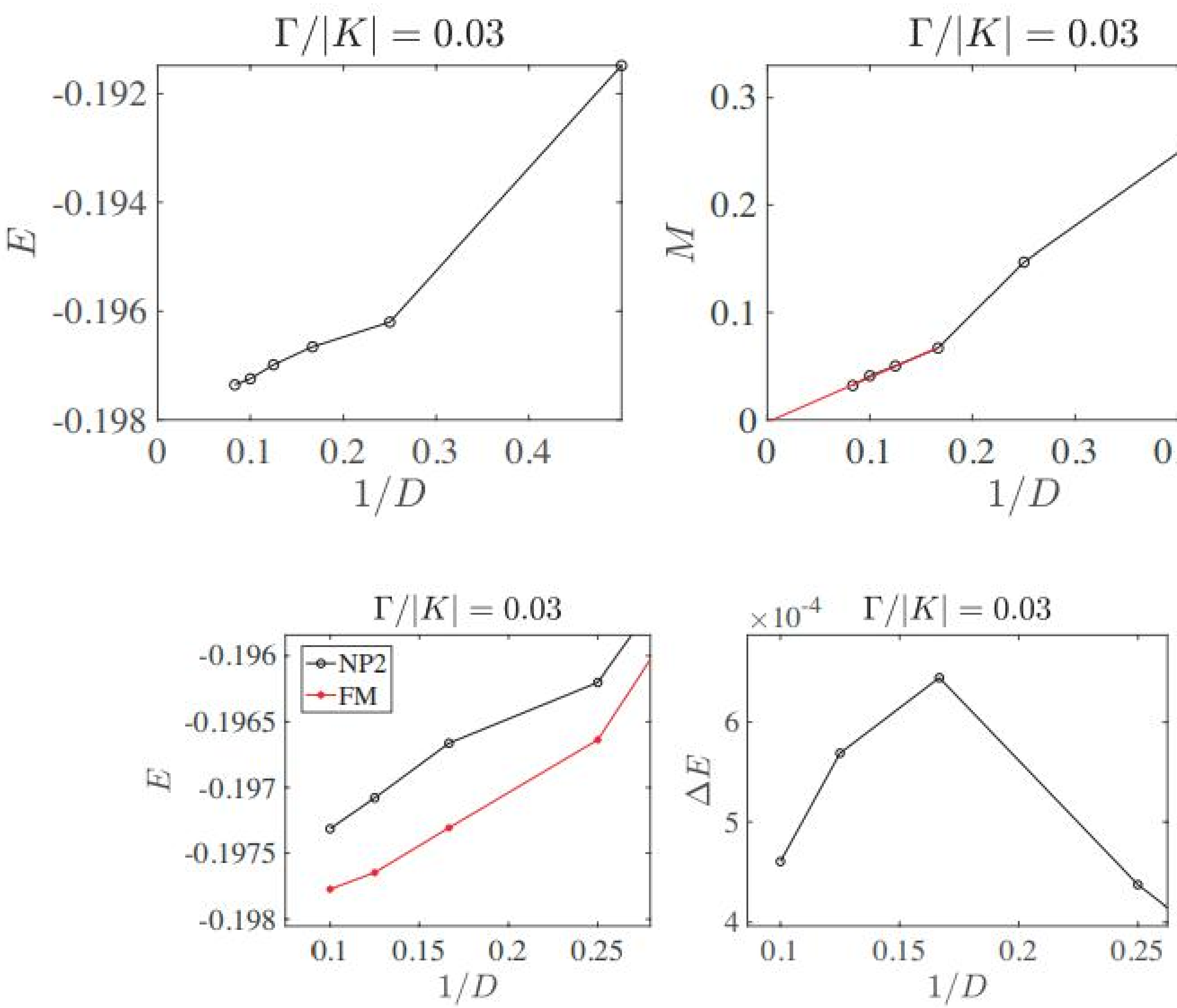
ED



$D = 4\ \chi = 20$ mag-ϕ

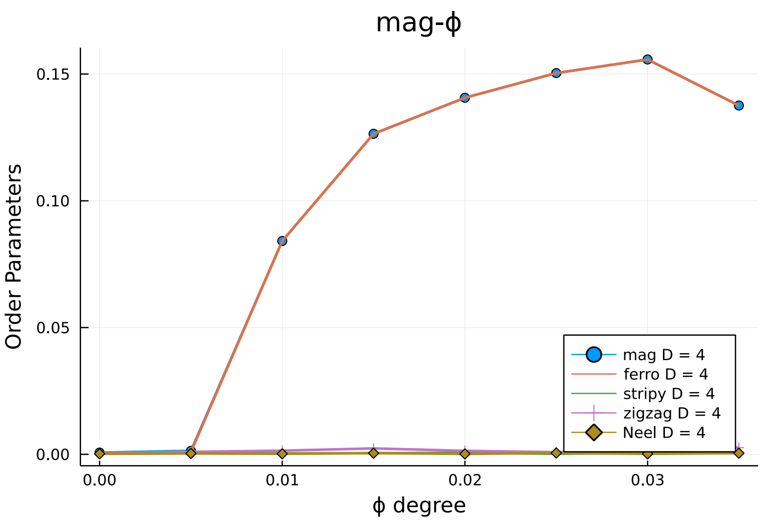


FM or NP?

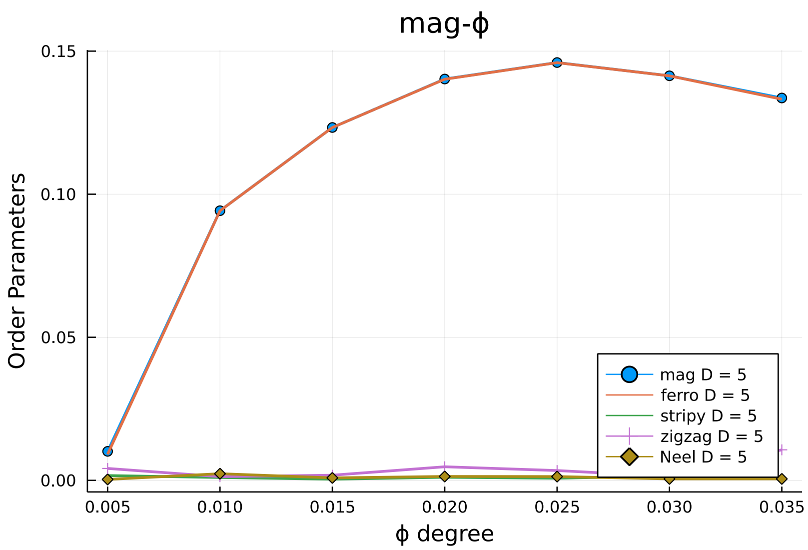

$\Gamma/|K| = 0.03$
E
$1/D$
-0.192
-0.194
-0.196
-0.198
0
0.1
0.2
0.3
0.4
$\Gamma/|K| = 0.03$
M
$1/D$
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
$\Gamma/|K| = 0.03$
NP2
FM
E
$1/D$
-0.196
-0.1965
-0.197
-0.1975
-0.198
0.1
0.15
0.2
0.25
$\Gamma/|K| = 0.03$
$\times 10^{-4}$
ΔE
$1/D$
6
5
4
0.1
0.15
0.2
0.25

$K - \Gamma$

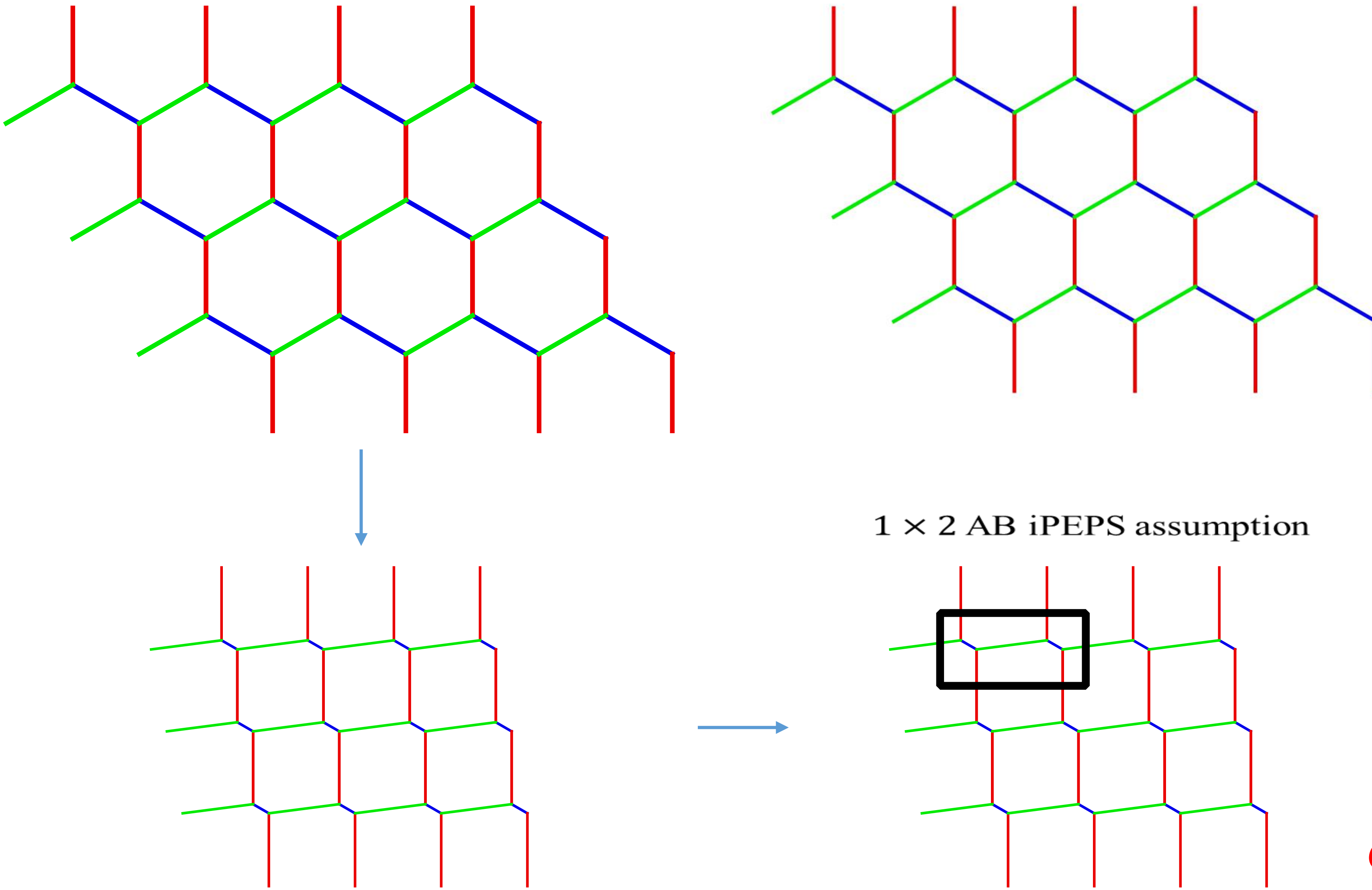
$D = 4\ \chi = 30$ direct 4



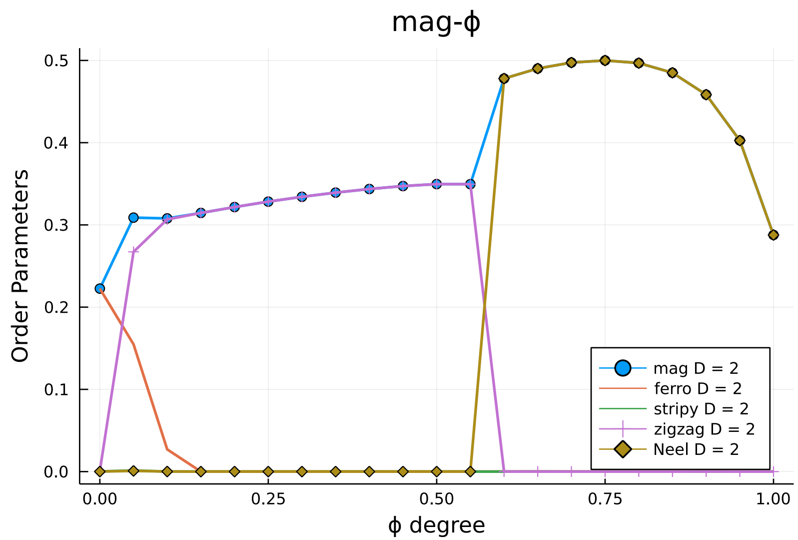
$D = 5\ \chi = 80\ 33 \rightarrow 4$

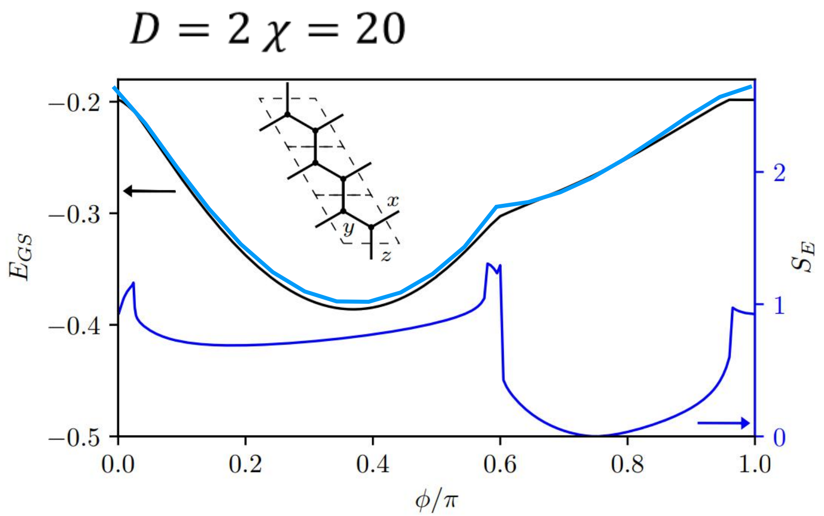


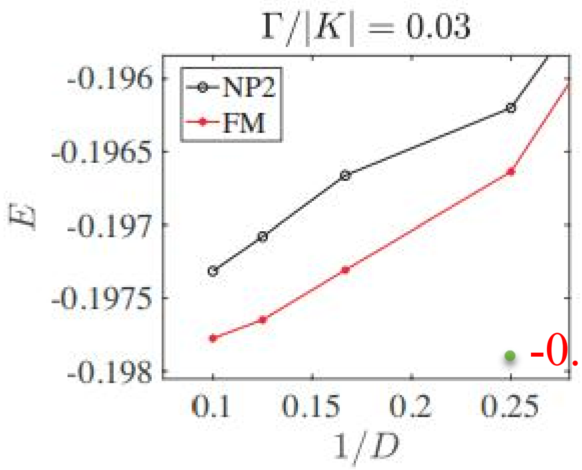
Honeycomb lattice to 1 × 2 cell square



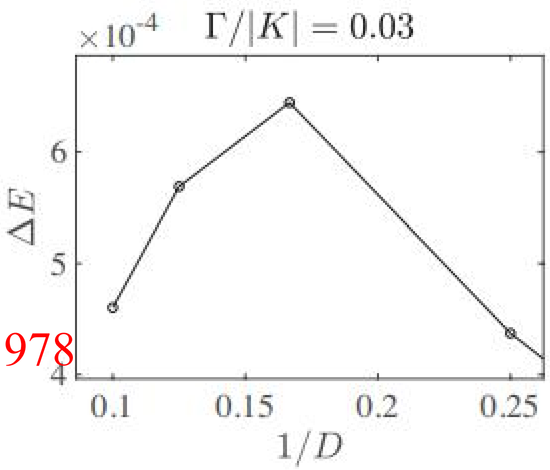
1×2 *cell*







-0.1978



Supplementary Information: Magnetic field induced quantum phases in a tensor network study of Kitaev magnets

extrapolation

D/χ	time	steps	E	mag
2/20	333.7s	33	-0.1941418207761279	0.22714278093309
3/20	23.6min	120	-0.1965226482022378	0.1677384037285575З
4/30	1.3h	151	-0.19781799675211129	0.06719914675549436
4/50	0.7h	63	-0.19851218680018168	
4/80	1h	33	-0.19919462200323318	0.06876717784521033
5/50	2.1h	124	-0.19967101706414678	hy = -0.0615101 hx = -0.0684368 hz = -0.0681459
5/100	1.9h	21	-0.1980928366925278	

ilar parameter
3h for 2×2