

1. SCHOTTKY BARRIERS

(A). The potential barrier can be found by noting that the potential exists whenever dissimilar metal and semiconductor bands are joined. By the requirement that Fermi levels must align at equilibrium (as always), it follows that the junction be discontinuous in band energies. Thus, the following relation holds:

$$\Phi_B = \Phi_m - \chi \Rightarrow \Phi_B = 5\text{eV} - 4.05\text{eV} = 0.95\text{eV} \quad (1.1)$$

The contact potential is the intrinsic potential between the semiconductor junction energies and the semiconductor energies far away from the junction.

$$V_0 = \Phi_B - (E_C - E_F) = \Phi_m - \chi - \left(\frac{E_g}{2} - k_B T \ln\left(\frac{1 \times 10^{18}}{1.5 \times 10^{10}}\right) \right) = 0.95\text{eV} - 0.56\text{eV} + 0.463\text{eV} = 0.853\text{eV} \quad (1.2)$$

The depletion width extends until the semiconductor conduction band, E_C , drops to a difference of V_0 below the junction value. This creates the following band diagram in Figure 1.1, with the metal on the left and semiconductor on the right:

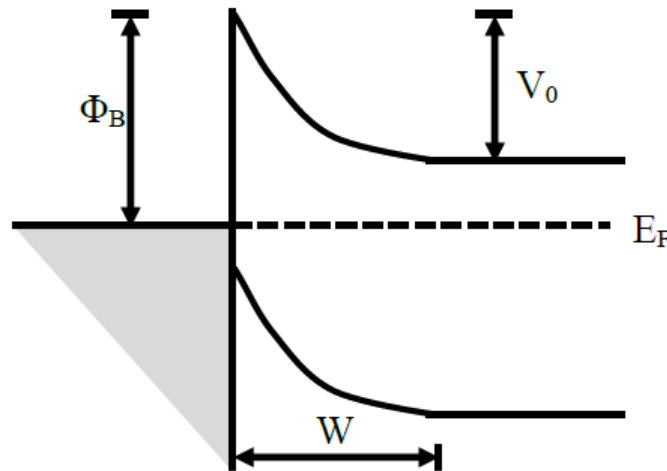


Figure 1.1: Schottky contact between a metal and semiconductor.

(B). For an applied bias of +0.4V, this shifts up the semiconductor conduction band relative to the metal's Fermi level, thereby only changing the value of the contact potential. Thus, the potential (Schottky) barrier between metal and semiconductor is left unchanged by an applied positive bias.

$$\text{Potential Barrier: } 0.95\text{eV} \text{ (unchanged)} \quad (1.3)$$

$$\text{Contact Potential: } V_0 - 0.4\text{eV} = 0.453\text{eV} \quad (1.4)$$

This creates the following band diagram in Figure 1.2, which is similar to the previous but with a reduced barrier, improving conduction.

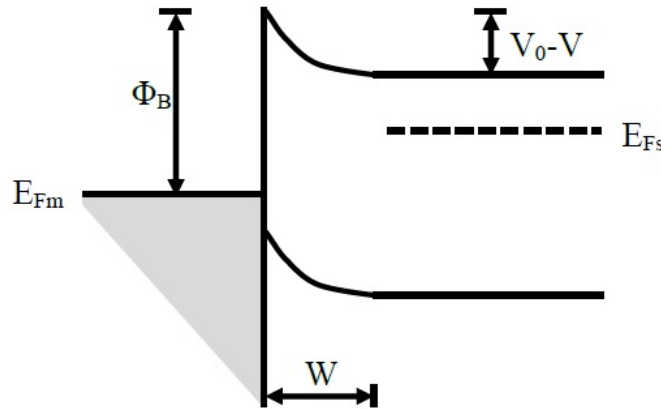


Figure 1.2: Schottky contact between a metal and semiconductor with applied bias of +0.4V

For an applied bias of -1V, this shifts down the semiconductor conduction band relative to the metal's Fermi level, once again only affecting the contact potential and leaving the potential (Schottky) barrier intact.

$$\text{Potential Barrier: } 0.95\text{eV} \text{ (unchanged)} \quad (1.5)$$

$$\text{Contact Potential: } V_0 + 1\text{eV} = 1.853\text{eV} \quad (1.6)$$

This corresponds with another shifted band diagram shown in Figure 1.3, where an even stronger barrier is created further reducing conduction.

(C). Based on the band diagram for a metal-*n*-type junction, we can see that a Schottky diode can emerge whenever the work function of the metal is larger than the electron affinity of the semiconductor. This effectively pulls down the band, and inhibits current flow by creating a wall between the charge carriers on either side.

If we think of an electron as a "ball" and a hole as a "bubble", electrons will want to diffuse down any potential boundaries to the lowest point (from high to low in E_C) whereas holes want the reverse and wish to ascend (from low to high in E_V). From the left, this means that the metal is able to disperse its holes into the semiconductor, yet its free electrons are unable to conduct rightwardly because of a larger potential energy wall stopping electron flow (i.e. current). This causes an accumulation of (-) carriers on the metal side of the junction. On the other side of the junction, the Schottky barrier acts to pinch off the Valence band at the metal contact, creating an upward crevice that holes drift into but cannot escape. This causes an accumulation of (+) carriers at the edge of the junction for the semiconductor. Beyond the depletion width, the background doping once again trumps the junction behaviors and thus presents an excess of (-) carriers, as we'd expect. Overall, we then see a separation of carriers as negative-positive-negative from left to right based on the Schottky barrier selectively providing barriers to carrier flow.

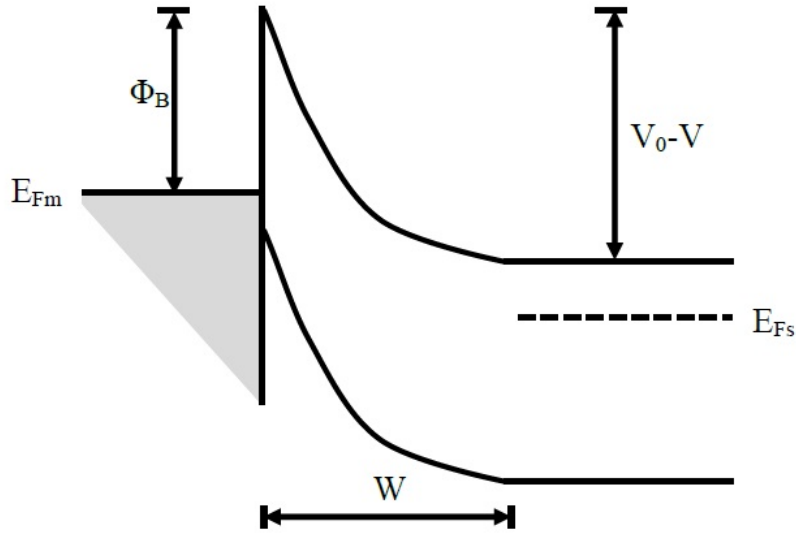


Figure 1.3: Schottky contact between a metal and semiconductor with applied bias of -1V

We would expect this behavior of charge separation to remain for our proposed biases of 0.4V and -1V, since in both cases the potential barrier is still insurmountable by the charge carriers. For a positive bias greater than V_0 , however, this would allow carriers to overcome the potential barrier and conduct. Similarly, if we apply a large enough reverse bias, the potential barrier width becomes so thin that an appreciable amount of charge carrier tunneling occurs, causing conduction yet again.

2. P-N DIODE

(A). First we recognize that the power extracted from the P-N diode corresponds to the product of the diode current equation (Eq. 8-2a) and the voltage:

$$P = I(V) \cdot V = I_{th}V(e^{\frac{qV}{k_B T}} - 1) - I_{op}V \quad (2.1)$$

To maximize power, we find a voltage V_{max} such that the slope $\frac{dP}{dV}|_{V=V_{max}} = 0$. Applying the chain rule,

$$\frac{dP}{dV} = I_{th}(e^{\frac{qV}{k_B T}} - 1) + I_{th}V(\frac{q}{k_B T}e^{\frac{qV}{k_B T}}) - I_{op} = 0 \quad (2.2)$$

Substitute in $V=V_{max}$,

$$I_{th}(e^{\frac{qV_{max}}{k_B T}} - 1) + I_{th}V_{max}(\frac{q}{k_B T}e^{\frac{qV_{max}}{k_B T}}) - I_{op} = 0 \quad (2.3)$$

Therefore,

$$(1 + \frac{q}{k_B T}V_{max})e^{\frac{qV_{max}}{k_B T}} = 1 + \frac{I_{op}}{I_{th}} \quad (2.4)$$

(B). The above cannot be solved symbolically for V_{max} , so instead it must be solved as a transcendental equation. An easy way to solve a transcendental is graphically by looking for the intersection between the two functional conditions. This can be done by plotting the left side and right side of the above equation on your calculator or in MATLAB, then finding the coordinates for where they intersect (Figure 2-1). Your x-axis will be in Volts so your x-component of the intersection will give you V_{max} .

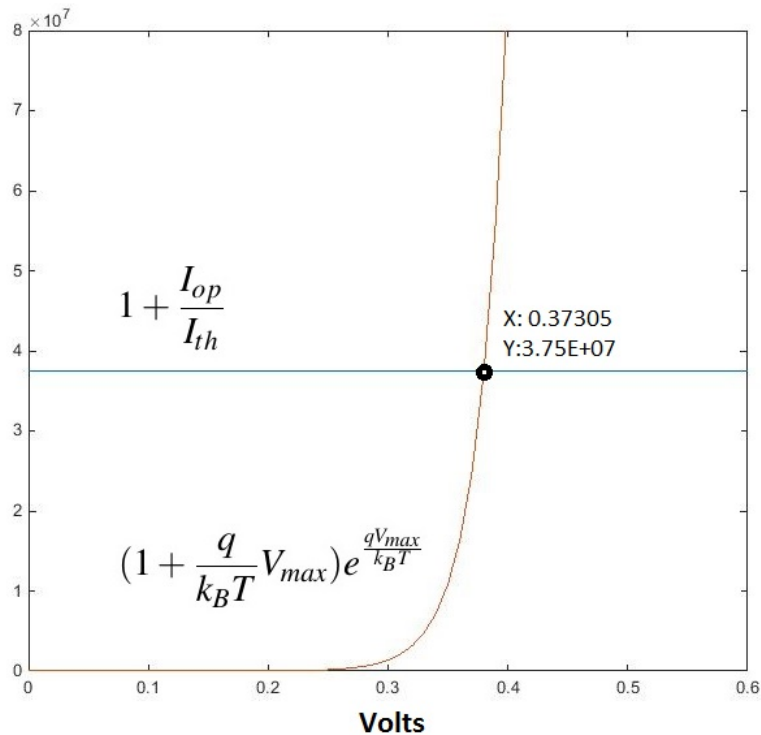


Figure 2.1: Intersection of functions maximizing power

The value of V_{max} is then easily found to be **0.373V**. By plugging into the diode equation, we can then find current.

$$I_{max} = I_{th} e^{\frac{qV_{max}}{k_B T}} - I_{op} \Rightarrow I_{max} = \mathbf{-138mA} \quad (2.5)$$

The maximum power P_m from the diode is thus $P=IV=0.373V \cdot -138mA = -0.0514 \text{ Watts} = \mathbf{-51.4 \text{ mWatts}}$.

Note that this is a negative value for Power, which tells us that it is not Power consumed or dissipated by the diode, but rather power gained and put into the system from solar energy.

(C). The fill factor is defined as the ratio of the maximum power over the product of the open circuit voltage, V_{oc} , and the short circuit current, I_{sc} (where $I_{sc}=I_{op}$).

The open circuit is found by setting current equal to zero (Eq. 8-3a):

$$V_{oc} = \frac{k_B T}{q} \ln\left(\frac{I_{op}}{I_{th}} + 1\right) = 0.443V \quad (2.6)$$

Thus, the fill factor is:

$$FF = \frac{|P_m|}{V_{oc}I_s} = \frac{0.0514}{.443V \times 0.150A} = 0.774 \quad (2.7)$$

(D). For a particular photogenerated current produced by a solar cell, we can derive the corresponding number of photons detected by recognizing that each electron contributing to current matches to one incident photon.

$$\frac{0.150\text{Amps}}{q} = 9.375 \cdot 10^{17} \text{photons per second absorbed} \quad (2.8)$$

Each of these photons creates an EHP, yet not every incident photon is absorbed. The quantum efficiency describes the overall percentage of photons absorbed and converted to electricity by the solar cell. Therefore,

$$\frac{9.375 \cdot 10^{17}}{0.31} = 3.024 \cdot 10^{18} \text{total incident photons per second} \quad (2.9)$$

(E). If we have a metal with a work function at $q\Phi_M=3.0\text{eV}$, we can apply a simple concept to see whether the contact this metal produces will be rectifying or Ohmic in nature.

For a p-type attached to a metal whose work function is **smaller** than its electron affinity, this means that pinning the Fermi levels of the two materials at equilibrium would move the Si upwards in its band diagram relative to the metal. By doing this, the Valence band of the metal (at E_{F_m}) runs into a potential barrier at the junction, preventing majority carrier hole diffusion into the semiconductor. This then separates charge out into three pooled regions resembling a p-n+ junction. Therefore, a metal-p-type junction where $\Phi_M < \chi$ creates a rectifying contact.

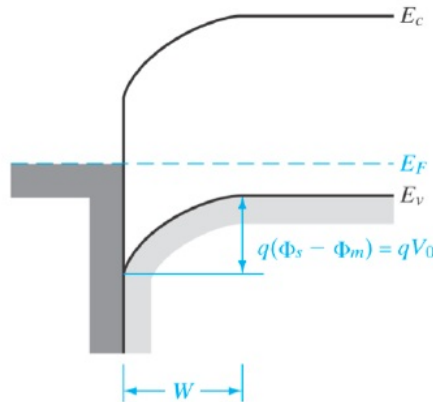


Figure 2.2: P-type Rectifying contact

For a n-type attached to the same metal, pinning the Fermi levels together at equilibrium again causes the Si band diagram to shift upward relative to the metal. Yet, since electrons are the majority carrier in this case, we look to the Conduction band instead. Since the Si band is now above the Conduction band of the metal (at E_{F_m}), no potential barrier exists for the electron so majority carrier flow can diffuse leftward into the metal, now only limited in conduction by its own Ohmic resistivity. Therefore, a metal-n-type junction where $\Phi_M < \chi$ creates an Ohmic contact.

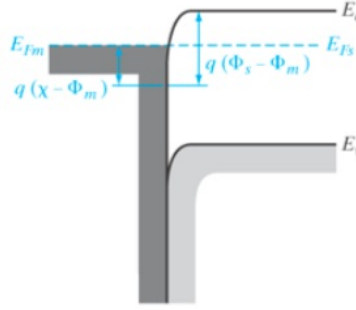


Figure 2.3: N-type Ohmic contact

3. MOS-CAPACITOR

(A). The maximum value for the depletion width can be found from Eq. 6-31. Note that all length units must be kept common, so cm is used.

$$W_m = 2\sqrt{\frac{\epsilon_{Si} kT \ln\left(\frac{N_d}{n_i}\right)}{q^2 N_d}} = 2\sqrt{\frac{11.8 \cdot 8.85 \times 10^{-14} \cdot 0.025 \ln\left(\frac{1 \times 10^{19}}{1.5 \times 10^{10}}\right)}{1.6 \times 10^{-19} \cdot 1 \times 10^{19}}} = 1.15 \times 10^{-6} \text{ cm} \quad (3.1)$$

(B). V_{FB} is found by noting Eq. 6-37, where we can neglect the oxide charge term $\frac{Q_i}{C_i}$:

$$V_{FB} = \Phi_m - \Phi_s = \Phi_m - \left(\chi + \frac{E_g}{2} - (E_F - E_i)\right) = 4.05 - 4.05 - \frac{1.12}{2} + k_B T \ln\left(\frac{1 \times 10^{19}}{1.5 \times 10^{10}}\right) = -0.033 \text{ V} \quad (3.2)$$

V_T can be taken from Eq. 6-38, where we note that this is an n-type substrate p-channel device, so we must adjust signs accordingly:

$$V_T = V_{FB} + 2\Phi_F - \frac{Q_d}{C_i} = V_{FB} - 2 \cdot k_B T \ln\left(\frac{N_D}{n_i}\right) - \frac{\sqrt{4 \cdot q \cdot \epsilon_{Si} \cdot N_D \cdot \Phi_F}}{C_{ox}} \quad (3.3)$$

$$\Rightarrow -0.033 - 1.0524 - \frac{\sqrt{4 \cdot 1.6 \times 10^{-19} \cdot 11.8 \cdot 8.85 \times 10^{-14} \cdot 1 \times 10^{19} \cdot 0.5262}}{4 \cdot 8.85 \times 10^{-14} / 1.5 \cdot 10^{-7}} = -1.88 \text{ V} \quad (3.4)$$

(C). We set equal the equation for capacitance for both the SiO_2 and HfO_2 :

$$C = \frac{\epsilon_{\text{SiO}_2} \epsilon_0 \cdot A}{t_{\text{SiO}_2}} = \frac{\epsilon_{\text{HfO}_2} \epsilon_0 \cdot A}{t_{\text{HfO}_2}} \quad (3.5)$$

$$t_{\text{HfO}_2} = \frac{\epsilon_{\text{HfO}_2} \cdot t_{\text{SiO}_2}}{\epsilon_{\text{SiO}_2}} = \frac{20 \cdot 1.5 \text{ nm}}{4} = 7.5 \text{ nm} \quad (3.6)$$

(D). As we can see, oxide thickness increases to maintain the same capacitance for materials with a higher value of epsilon (i.e. electric permittivity). Previously in industry, scaling up of capacitance in SiO_2 had been

done by scaling down the oxide thickness, yet the limits on tunneling prevent the shrinking of this thickness indefinitely as more charges can seep through as leakage current with too thin a Silicon Dioxide layer. Instead, high-K (high permittivity) dielectrics minimizes this leakage current by allowing larger thicknesses for a given oxide capacitance as compared to SiO₂, allowing capacitance to continue to scale without tunneling.

(E). A gate voltage, V_G , applied to a MOS-Capacitor atop an n-type semiconductor acts to create a build up of charged carriers about the Oxide-Semiconductor junction. This charge build up affects the capacitance and can be divided into the following ranges of behavior, based on the value of V_G :

$V_G < V_T$: Beginning at large negative values of V_G going up to the value of V_T , the applied gate voltage acts on the MOS and creates an inversion layer of carriers inside the semiconductor. For our n-type Si, this means a collection of holes accumulate at the surface of the semiconductor due to the gate voltage's effect on charge separation, creating a p-type inversion layer. This creates what's called strong inversion, where a large insulator capacitance, C_i , is made by a large minority carrier pooling; this happens whenever the gate reverse biases the semiconductor beyond the threshold voltage in an n-type MOS-Capacitor.

$V_T < V_G < V_{FB}$: In this region, the relative biasing of the Oxide-Semiconductor by the gate voltage, V_G , is smaller than for inversion. At and just above V_T , capacitance will drop as the inversion layer of carriers will reduce in relative size; this creates in effect two series capacitors, one from the insulator and from the depletion layer, C_d . Since series capacitors just increase the overall capacitance length, capacitance will drop rapidly in this region. The accumulation of minority carriers in the n-type Si is reduced, as the semiconductor bandgap begins to shift upward with a smaller bias and so the Valence Band potential barrier reduces. This then creates a depletion region, where the semiconductor surface experiences a depletion capacitance just as the increasingly (+) bias begins to allow more majority carriers to accumulate relative to the minority carriers. This region is also sometimes referred to as the weak inversion layer, as the semiconductor bands still are reverse-biased, thoughly decreasingly so.

$V_G > V_{FB}$: As you then continue to sweep the gate bias to values larger than the flat-band voltage, the band diagram now has the Fermi level closer to the Conduction band in the semiconductor. In effect, all minority carrier pooling in the semiconductor vanishes as a contact barrier no longer exists for the Valence band. This then removes the secondary series capacitance associated with the depletion layer, so the capacitance will increase again back to C_i , now attributed to the storage of majority carriers across the oxide. This is known as the accumulation region, where an accumulation of majority carriers (electrons in our case) will be stored in the oxide due to the $>V_{FB}$ forward bias.

4. LASER DESIGN

(A). To find the cavity length for a given adjacent mode spacing, we can use Eq. 8-16 to relate center wavelength to the laser parameters. Since our refractive index is dispersionless, we can leave off the second term.

$$-\Delta\lambda_0 = \frac{\lambda_0^2}{2nL} \Delta m \quad (4.1)$$

Because Δm must equal -1, which corresponds to the integer separation between the central mode and an adjacent

mode, we can solve for L, where $\lambda_0=980\text{nm}$, $\Delta\lambda_0=30\text{nm}$, and $n=3.63$.

$$L = \frac{\lambda_0^2}{2n\Delta\lambda_0} = \frac{(980 \times 10^{-9})^2}{2 \cdot 3.63 \cdot 30 \times 10^{-9}} = 4.41 \times 10^{-6} \text{m} = 4.41 \mu\text{m} \quad (4.2)$$

(B). We know from introductory physics that wavelength and frequency have a relationship, as:

$$v_0 = \frac{c}{\lambda_0} \quad (4.3)$$

where λ_0 is the center wavelength, c is the speed of light, and v_0 is the corresponding center frequency. By taking the derivative of frequency with respect to wavelength,

$$\frac{dv_0}{d\lambda_0} = -\frac{c}{\lambda_0^2} \Rightarrow \Delta v_0 = -\Delta\lambda_0 \frac{c}{\lambda_0^2} \quad (4.4)$$

Plugging in Eq. 8-16, we can then remove the relation to $\Delta\lambda_0$:

$$\Delta v_0 = \left(\frac{\lambda_0^2}{2nL} \Delta m \right) \frac{c}{\lambda_0^2} \Rightarrow \Delta v_0 = \frac{c}{2Ln} \Delta m \quad (4.5)$$

This gives the frequency spacing of each laser mode possible (if we assume a dispersionless medium). As we can see, the relation only depends on the length, the refractive index, the speed of light, and an integer Δm , so **in the frequency domain all modes are evenly spaced.** By then applying the value of L and other constants we know from above, we can find Δv_0 :

$$\Delta v_0 = \frac{3 \times 10^8}{2 \cdot 4.41 \times 10^{-6} \cdot 3.63} \cdot 1 = 9.37 \times 10^{12} \text{ Hz} = 9.37 \text{ THz} \quad (4.6)$$

Note here that Δm is not constrained to being -1 in the frequency domain, since there is no dependence on λ_0 or v_0 in the formula for finding frequency spacing. Generally we pick $\Delta m=1$ to show the difference across one mode separation as a positive frequency spacing.

Again, this equation is only consistent with the assumption of a dispersionless medium, where different wavelengths don't have slightly varying refractive indices. In reality, the refractive index has a downward slope as you increase wavelength so the frequency spacing increases slightly as you "red-shift" to a lower energy photon resonance in your laser cavity.

(C). When the concentration of conduction band electrons surrounding the conduction band edge in a particular physical location is larger than the concentration of valence band electrons surrounding the valence band edge, you get what's called "population inversion". As you forward bias a pn-junction, the depletion region shrinks, causing minority carrier concentrations to rise rapidly; thus, there exists a point where a large concentration of both conduction band electrons and valence band holes will occupy the same spatial location (left-to-right in a band diagram plot). If the two semiconductor regions are doped high enough (in reality, degenerately doped) and then an applied voltage equal to or greater than the band gap is applied across the p-n junction, population inversion can occur.

Thus, the forward bias needed for population inversion is:

$$V = \frac{F_n - F_p}{q} > \frac{E_g}{q} \Rightarrow V > \frac{1.24 / .98}{q} \Rightarrow V > 1.26 \text{ Volts} \quad (4.7)$$

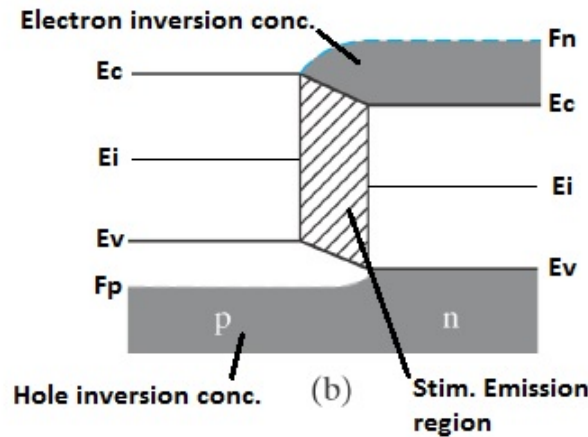


Figure 4.1: PN-diode laser

(D). The above Figure 4.1 shows how a PN-diode laser can be expected to achieve lasing when driven to an appropriate population inversion. Note the inversion layers in both the Conduction and Valence bands, as well as the fact that stimulated emission only occurs within their vertical overlap.

(E). Looking at Eq. 8-8b, we see the ratio between Stimulated and Spontaneous emission, both occurring as a recombination process between an electron in the Conduction band and a hole in the Valence band.

$$\frac{\text{Stimulated emission rate}}{\text{Spontaneous emission rate}} = \frac{B_{21}n_2\rho(\nu_{12})}{A_{21}n_2} = \frac{B_{21}}{A_{21}}\rho(\nu_{12}) \quad (4.8)$$

From this, we can see that the rate of stimulated to spontaneous emission reduces to a ratio between Einstein coefficients and the photon energy density term $\rho(\nu_{12})$, which relates the density of any particular photon frequency that crosses our material bandgap. Since the Einstein coefficients, A_{21} and B_{21} , are constants for a given material, the only parameter that can be changed to affect rate of emission in favor of stimulated emission is the photon energy density. We can think of changing $\rho(\nu_{12})$ by affecting the laser cavity parameters so it selectively increases the energy density for our desired photon frequency. This is done by noting that the walls of a typical laser cavity act as mirrors that reflect or transmit different percentages for any given frequency inside; by adjusting reflectivity of the mirrors so they act as a band-pass that preferentially reflects just the desired frequency, more power at ν_{12} oscillates and amplifies in the cavity while all other frequencies destructively interfere.

So, to change the ratio of stimulated to spontaneous emission, we can adjust the cavity mirrors so only the desired frequency can build up inside the cavity, creating a highly coherent photon density to trump spontaneous emission.

(F). Similarly, we can look at the ratio in Eq. 8-9 that relates the ratio of Stimulated emission, which recombines the EHPs, and Absorption, which separates EHPs to begin with.

$$\frac{\text{Stimulated emission rate}}{\text{Absorption rate}} = \frac{B_{21}n_2\rho(\nu_{12})}{B_{12}n_1\rho(\nu_{12})} = \frac{B_{21}}{B_{12}} \frac{n_2}{n_1} \quad (4.9)$$

Because the Einstein coefficients, B_{21} and B_{12} , are constant for this material, the ratio tells us the only thing we can do to affect the system is increase the concentration in the Conduction band, n_2 , as compared to the Valence band, n_1 . So to change the ratio of Stimulated Emission to Absorption, we can boost the concentration mismatch in favor of the higher state. In practice, we can increase the ratio of $\frac{n_2}{n_1}$ by increasing carrier flow across the junction (to excite more EHPs per second than their lifetime allows to recombine) or by reducing losses to the upper state population.