Special Instructions: You must write your name and netid on every sheet of the exam.

ECE 340 –Fall 2015

Name Solutions

Net ID#________________

Section________________

ECE 340 – EXAM II
Thursday November 5th, 2015 7:00 – 8:00 p.m.

100 Noyes Laboratory Sections A (Feng), E (Bayram), X (Dallesasse)
314 Altgeld Hall Sections B (Zhu), D (Kim)

NOTE: This is a closed book and closed notes exam. No calculators are allowed. The exam consists of five problems. Unless stated otherwise, do your work on the page of the problem and if necessary on the preceding blank page. ***Be sure to explicitly show the units in your work, as well as in your answers. Circle your answer. Be neat!***

If we cannot read or follow your work, you get zero credit!

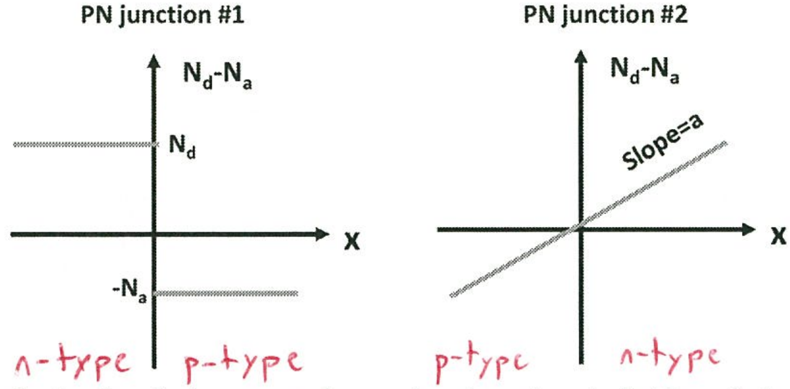
For each problem, you must show complete work and **indicate your reasoning.** No credit will be given if you do not show the complete work *and describe your procedure*, even if the answer is correct. Write your name, ID#, and section on this page and **sign below**.

	Prob.#1		Prob.#2		Prob.#3		Prob.#4		Prob.#5	
	20 pts.		20 pts.		20 pts.		20 pts.		20 pts.	
a	10		7		7		7		6	
b	4		7		7		7		6	
c	6		6		6		6		8	
d	---		---		---		---		---	
e	---		---		---		---		---	
f	---		---		---		---		---	
g	---		---		---		---		---	
T	20		20		20		20		20	

Signature:____________________ Your Exam Score:________

1. pn Junctions at Equilibrium and Contact Potential

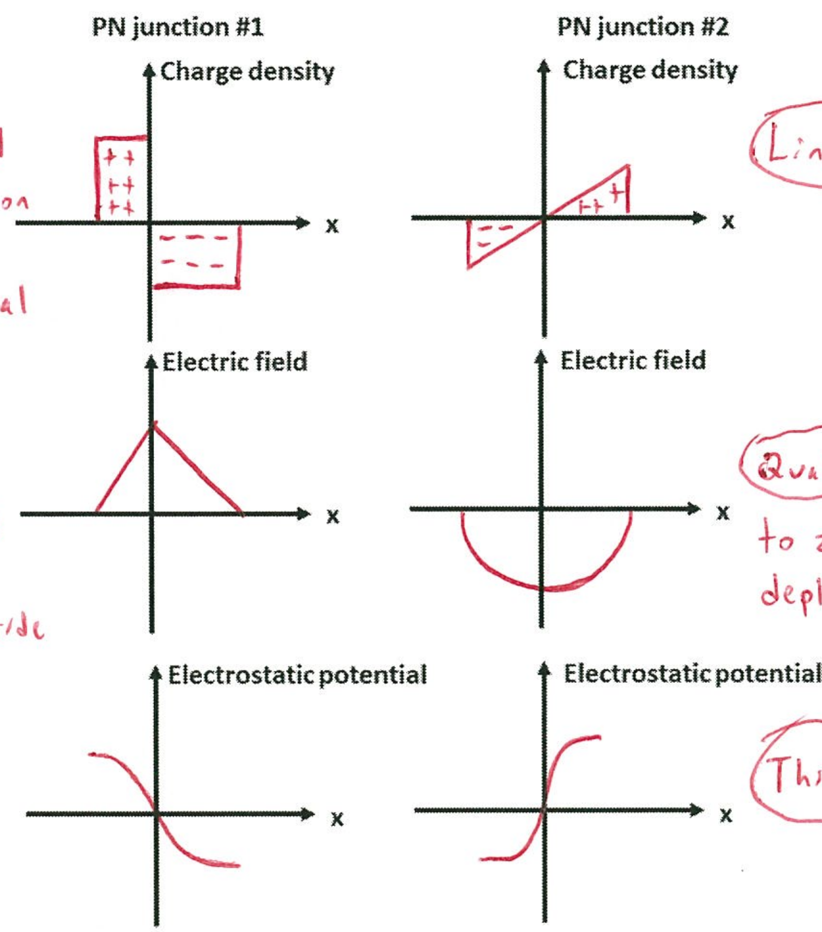
Consider two pn junctions with following doping profiles:



a) Please qualitatively sketch the space charge density, electric field, and electrostatic potential of these pn junctions. Please specify if they are constant, varying linearly, quadratically, to the third power, or exponentially with position x. Hint: Think about how these terms were derived. (10 pts)

Constant and total charge on either side must be equal

Linear



Linear and points from n-side to p-side

Quadratic and must go to zero at edge of depletion region

Quadratic

Third Power

b) Use arrows to indicate the directions and relative magnitudes of the drift and diffusion currents in these pn junctions at equilibrium. (4 pts)

PN junction #1

	Electron	Hole
Diffusion current	←	←
Drift current	→	→

Current Direction

PN junction #2

	Electron	Hole
Diffusion current	→	→
Drift current	←	←

Current Direction

c) Consider a pn junction with doping concentration N_d in n type region, N_a in p type region, and a contact potential V_0. Which of following changes will increase the contact potential (choose all options that may apply)? Please explain why you choose these options. (6 pts)

i) Increase the donor doping concentration N_d in the n type material.
ii) Reduce the acceptor doping concentration N_a in the p type material.
iii) Increase the bandgap of the p and n material, while keeping N_d and N_a the same.
iv) Decrease the temperature.

i) $V_o = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$, if N_D increases then V_o increases

iii) An increase in E_g results in a decrease in n_i: $n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT}$

By the equation in (i), if n_i decreases and N_A and N_D remain the same, V_o increases

iv) A decrease in temperature results in a decrease in n_i. Even if carrier freeze out is taken into effect, E_{Fp} approaches E_v as $T \to 0$ on the p-side and E_{Fn} approaches E_c as $T \to 0$ on the n-side. Since $qV_o = (E_{Fn} - E_i) + (E_i - E_{Fp})$, V_o increases

2. pn Junction Electrostatics

A semi-infinite abrupt n^+ - p junction has the n-side 9 times more heavily doped than the p-side. For this device the contact potential V_0 is 0.8V and the depletion region width W_0 at equilibrium is 260 nm.

Please answer the following questions:

a) What is the extension (i.e., length) of the depletion region on each side of the junction at equilibrium? Note: You should be able to calculate the values. (7 pts)

$$W_o = X_{no} + X_{po} = X_{no} + X_{no}\left(\frac{N_D}{N_A}\right) = 10\,X_{no}$$

9

$$X_{no} = \frac{W_o}{10} = 26\,\text{nm}$$

260 nm

$$X_{po} = \frac{9}{10} W_o = 234\,\text{nm}$$

b) What is the maximum electric field strength $\mathcal{E}_0$ within the depletion region? Note: Write an expression in the simplest possible form. (7 pts)

$$\frac{1}{2} W_o \vec{E}_o = V_o$$

$$\Rightarrow \vec{E}_o = \frac{2V_o}{W_o} = \frac{1.6\,V}{260\,nm} = \frac{1.6\,V}{2.6\cdot 10^{-7}\,m}$$

0.8V

$$= 6.15\cdot 10^{6}\,\frac{V}{m}$$

$$\frac{\cancel{16}\ 8}{\cancel{26}\ 1.3}\cdot 10^{6}\,\frac{V}{m}$$

1.3) 8.0 = 06.15

78

20

13

70

65

5

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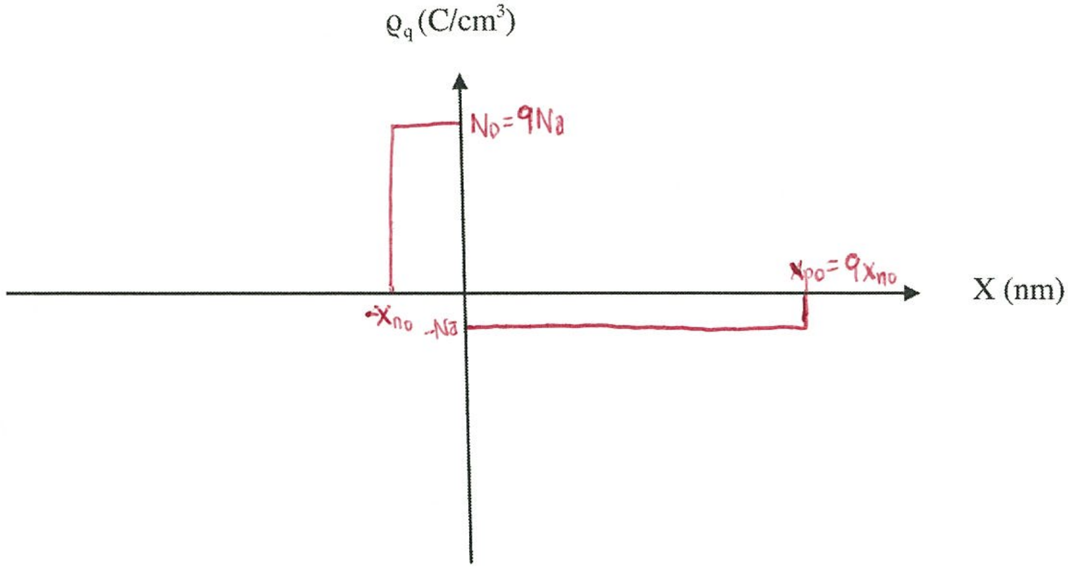
c) Plot the space charge distribution within the depletion region of the n^+ - p junction. Assume that donors (N_d) and acceptors (N_a) are fully ionized and that N_a is known. (6 pts)



3. Injection and Currents in pn Junctions

Assume that we have a silicon diode with n-doping of 10^{17} cm^{-3} and p-doping of 10^{17} cm^{-3}. If we increase the n-doping to 10^{19} cm^{-3} and p-doping to 10^{19} cm^{-3};

a) What would happen to the magnitude of the reverse-bias saturation current? Circle one ONLY. Explain briefly. (7 pts)

INCREASE DECREASE STAY SAME

When in reverse bias, $|J|$ depends on minority carrier concentration alone.

$$J_S = q\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right) = q\left(\frac{L_p}{\tau_p}p_n + \frac{L_n}{\tau_n}n_p\right)$$

since $p_p\uparrow, n_n\uparrow$, $p_n\downarrow = \dfrac{n_i^2}{n_n\uparrow}$ where $n_n = N_D$ (increasing)

$n_p\downarrow = \dfrac{n_i^2}{p_p\uparrow}$ where $p_p = N_A$ (increasing)

therefore, $|J_S| = q\left(\frac{L_p}{\tau_p}p_n\downarrow + \frac{L_n}{\tau_n}n_p\downarrow\right)$

so the magnitude of J_{sat} reduces, meaning $|I_{sat}|$ decreases for a constant volume.

b) What would happen to the magnitude of the breakdown voltage? Circle one ONLY. Explain briefly. (7 pts)

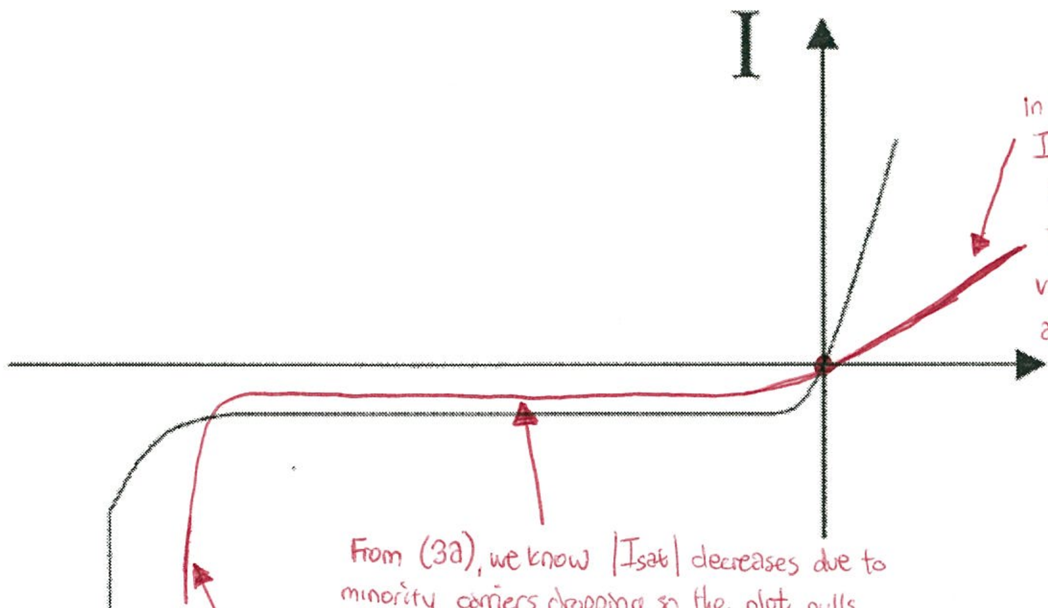
INCREASE DECREASE STAY SAME

Since $W \propto \sqrt{\dfrac{N_A+N_D}{N_AN_D}}$, we know that an increase in doping will decrease our depletion width.

Since doping increases, we can expect the E_{Fp} and E_{Fn} to separate more so from p-side to n-side, causing V_o to increase.

Breakdown voltage will be decreased as we increase the concentration of holes and electrons, for the Zener effect will more readily cause tunneling since the depletion width is reduced and V_o is increased, allowing carriers to slip across a thinner barrier. Thus, we need less applied voltage, $|V_{BR}|$, to align the bands for a Zener breakdown.

c) The following shows the current-voltage plot of the silicon diode with n-doping of 10^{17} cm^{-3} and p-doping of 10^{17} cm^{-3}. Show your approximation of the current-voltage for a silicon diode with n-doping of 10^{19} cm^{-3} and p-doping of 10^{19} cm^{-3}on the same plot. Explain your reasons. (6 pts)



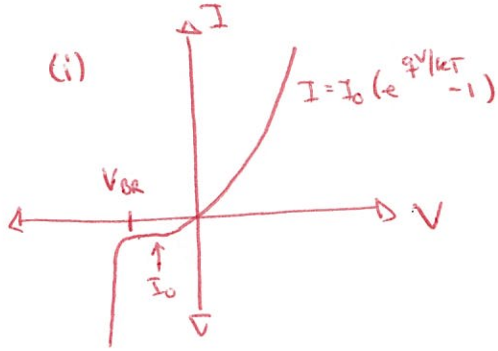
In positive bias; we know $I_0 \simeq I_{sat}$ so the slope will change since $\frac{\partial I}{\partial V} \propto I_0$. This means it will take more (+ voltage applied to surpass V_0 and allow the same current thru

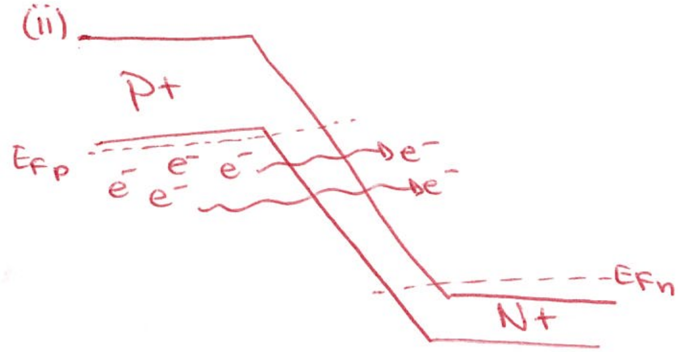
From (3a), we know $|I_{sat}|$ decreases due to minority carriers dropping so the plot pulls upward toward $I=0$ to show a smaller magnitude.

As we know from (3b), the $|V_{BR}|$ breakdown voltage is decreased since an increased carrier concentration requires us to apply less negative voltage for the same effect. Since this is $|V_{BR}|$ decreasing, the breakdown shifts toward $V=0$.

4. Reverse Bias with Breakdown and Junction Capacitance

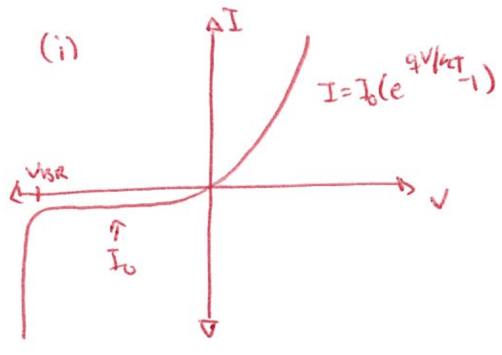
a) Draw i) an I-V and ii) an energy band diagram and explain Zener breakdown. (7 pts)

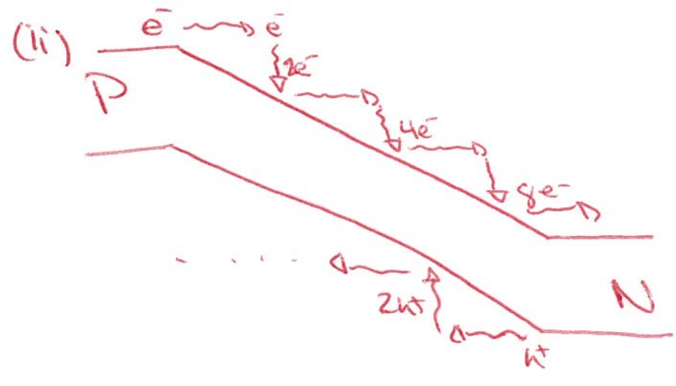




(iii) Electron on p-side tunneling to n-side conduction band to provide breakdown current

b) Draw i) an I-V and ii) an energy band diagram and explain Avalanche breakdown. (7 pts)





$V_{BR}(AV) > V_{BR}(Z)$

(iii) Low doping so large depletion width with high electric field causes carriers to have high kinetic energy. KE high enough for impact ionization, carriers multiply and provide breakdown current.

c) Given 3 silicon diodes – a p^+n^+ diode, a p^+n diode, and a pn diode ("+" indicates heavy doping), list the diodes in order from greatest reverse-bias capacitance to smallest reverse-bias capacitance. Assume that the n-doping level is the same for the second two diodes. Explain your answer. Avoid circular arguments. (6 pts)

Reverse bias capacitance is junction capacitance, given as

$$C = \frac{\epsilon A}{W}$$ where W is depletion width.

Higher doping gives smaller depletion width, so

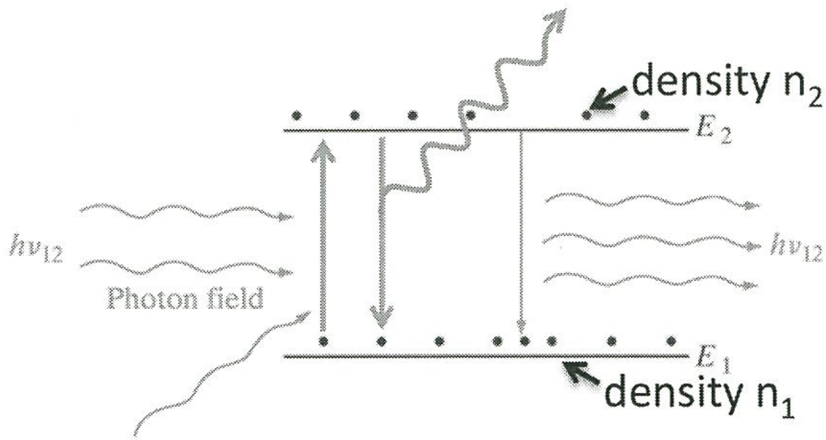
$p^+n^+ \rightarrow$ smallest W $\rightarrow$ highest C

$p^+n \rightarrow$ middle W $\rightarrow$ middle C

$pn \rightarrow$ ~~l~~ largest W $\rightarrow$ smallest C

5. Optoelectronic Devices

Consider a two-state system as depicted below:



a) What terminology (i.e. absorption, etc.) is used to describe the following rates (6 pts):

i) $B_{21}n_2\rho(\nu_{12})$ Terminology: Stimulated emission

ii) $B_{12}n_1\rho(\nu_{12})$ Terminology: Absorption

iii) $A_{21}n_2$ Terminology: Spontaneous emission

b) Write an expression that relates these terms in steady state. (6 pts)

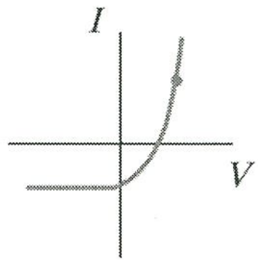
$$B_{12}n_1\rho(\nu_{12}) = B_{21}n_2\rho(\nu_{12}) + A_{21}n_2$$

(Absorption = Stimulated emission + Spontaneous emission)

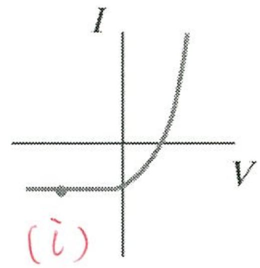
c) In the following diagrams, label the following curves or points:

i) I-V showing operating point for typical photodiode operation. (2 pts)
ii) I-V showing operating point for typical solar cell operation. (2 pts)
iii) Short circuit current I_{sc} (on solar cell I-V). (2 pts)
iv) Open circuit voltage V_{oc} (on solar cell I-V). (2 pts)

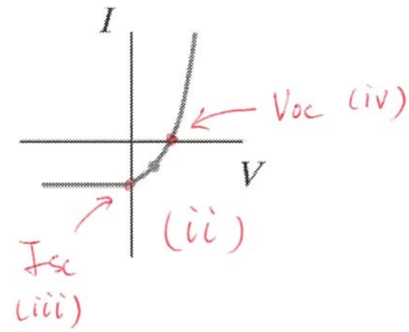
1st quadrant



3rd quadrant



4th quadrant



Useful Formulae

$$N_C = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

$$N_V = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

$$E = -\nabla V$$
$$= \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$

$$V = -\frac{1}{q}\left(E_c - E_{ref}\right)$$

$$J_n(x) = q\mu_n n(x)E(x) + qD_n\frac{dn(x)}{dx}$$

$$J_p(x) = q\mu_p n(x)E(x) - qD_p\frac{dp(x)}{dx}$$

$$-\frac{d^2\mathcal{V}(x)}{dx^2} = \frac{d\mathcal{E}(x)}{dx} = \frac{\rho(x)}{\varepsilon}$$

$$\rho(x) = q\left(p(x) - n(x) + N_d^+(x) - N_a^-(x)\right)$$

$$W = \left[\frac{2\varepsilon(V_0 - V)}{q}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2}$$

$$\Delta p_n = p(x_{n0}) - p_n = p_n\left(e^{qV/kT} - 1\right)$$

$$J_0 = q\left(\frac{D_P}{L_P}p_n + \frac{D_n}{L_n}n_p\right) = q\left(\frac{L_p}{\tau_p}p_n + \frac{L_n}{\tau_n}n_p\right)$$

$$n + N_A^- = p + N_D^+$$

$$np = n_i^2$$

$$J_0 = \frac{I_0}{A}$$

$$J_0 = \frac{I_0}{A}$$

$$J_n^{drift} = -qn\langle v_{dn}\rangle = qn\mu_n E$$

$$\frac{D_N}{\mu_N} = \frac{k_b T}{q}$$

$$\frac{D_P}{\mu_P} = \frac{k_b T}{q}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{q\left(n\mu_n + p\mu_p\right)}$$

$$n = N_c e^{\frac{(E_f - E_c)}{k_b T}}$$

$$p = N_v e^{\frac{(E_v - E_f)}{k_b T}}$$