ECE 340: Semiconductor Electronics

Chapter 3: Energy bands and charge carriers in semiconductors (part II)

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Outline

- Carrier concentrations
  - The Fermi level
  - Electron and hole concentration at equilibrium
  - Temperature dependence of carrier concentration
  - Compensation and space charge neutrality

Carrier concentration

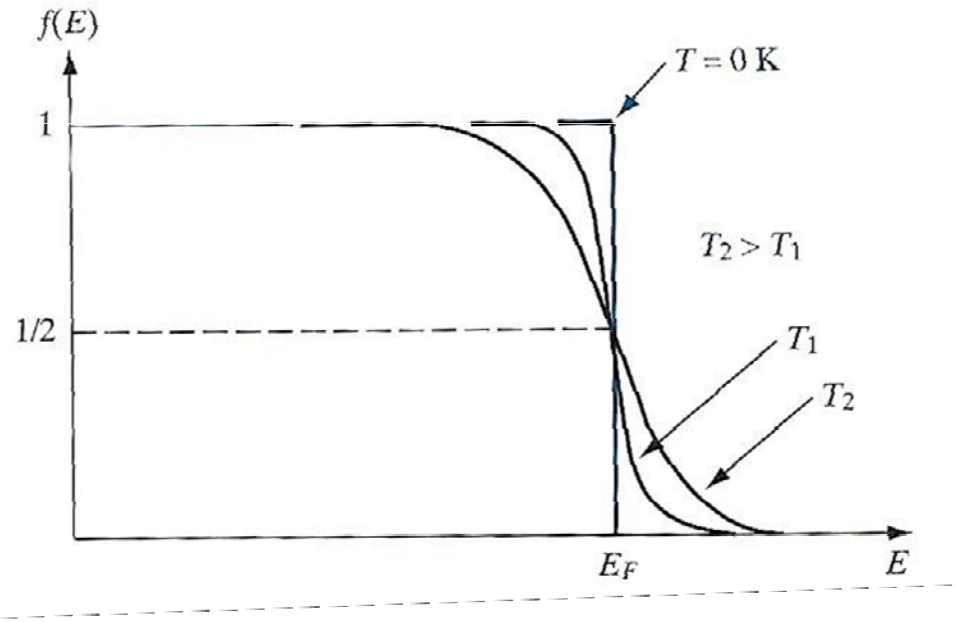
- How to calculate electron (and hole) densities at
  - Any temperature
  - Any doping concentration
  - Any energy level

Carrier density = **density of available states** × **probability of occupation**

Fermi-Dirac distribution

- **Electrons (and holes) obey Fermi-Dirac statistics**

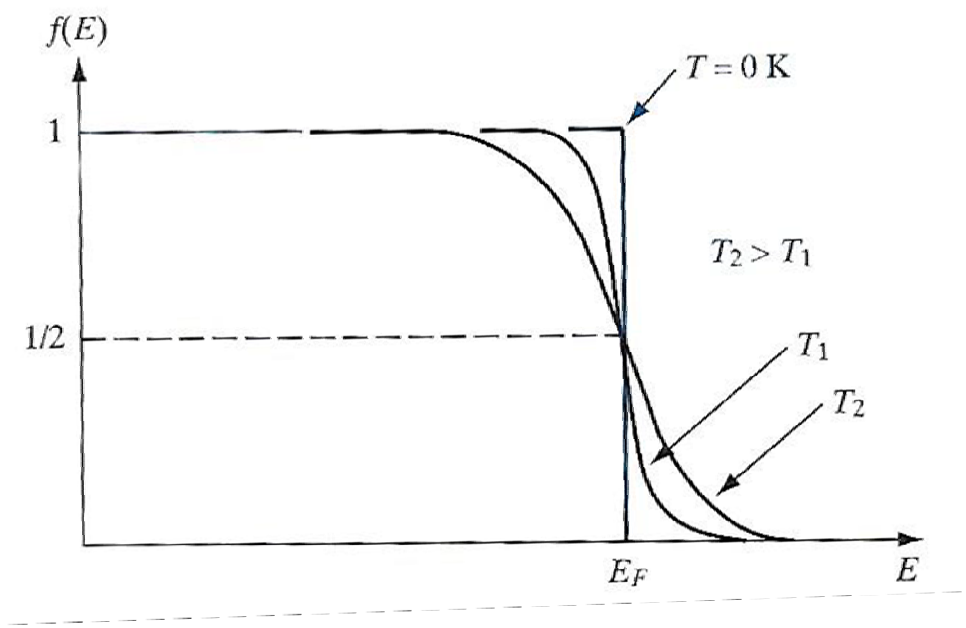
$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$



E_F: Fermi level, at which an energy state has a probability of 1/2 being occupied by an electron
k: Boltzmann constant (8.62×10^{-5} eV/K)
T: temperature in Kelvin (K)

- ***f(E)*** **is the probability that a state at energy E is occupied**
- ***1-f(E)*** **is the probability that a state at energy E is unoccupied**

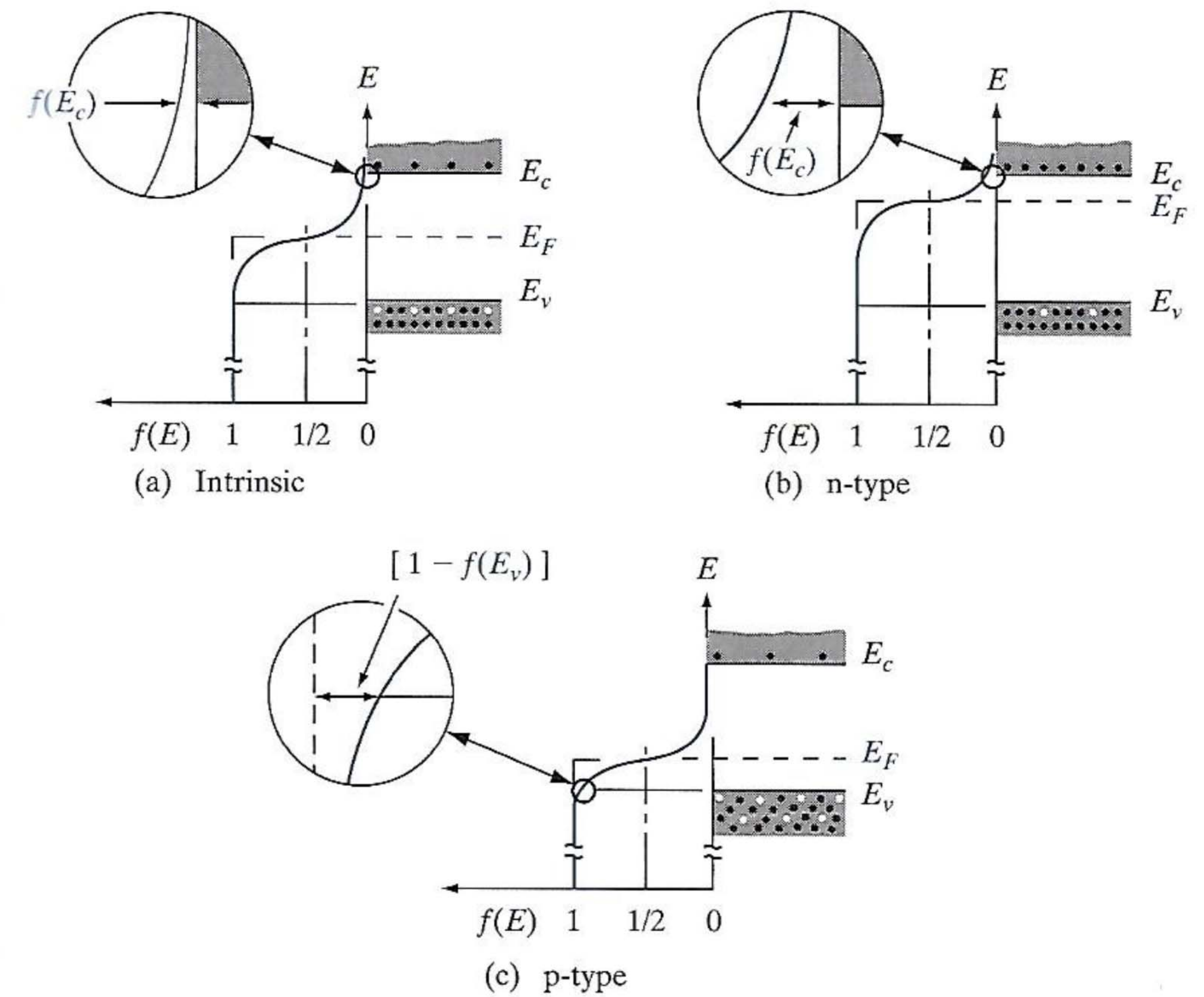
Fermi function at different temperatures



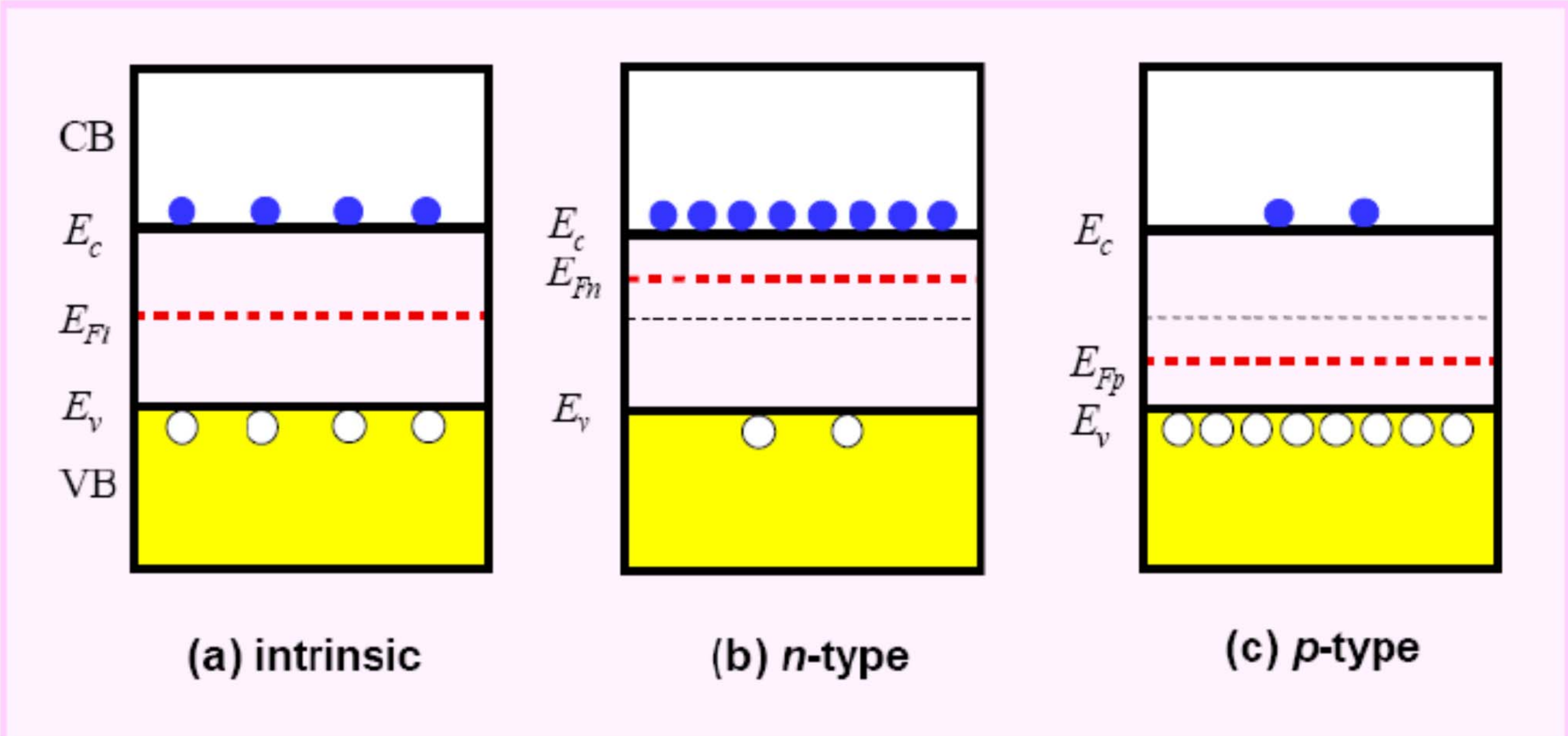
- **At T=0K,** occupancy is "digital": No occupation of states above E_f and complete occupation of states below E_f.
- **At T>0K,** occupation probability is reduced with increasing energy. $f(E=E_f) = 1/2$ regardless of temperature.

- The Fermi function is symmetrical about E_F, i.e. $f(E_F + \Delta E) = 1 - f(E_F - \Delta E)$

Fermi distribution function applied to semiconductors



E_F in Energy Band Diagram



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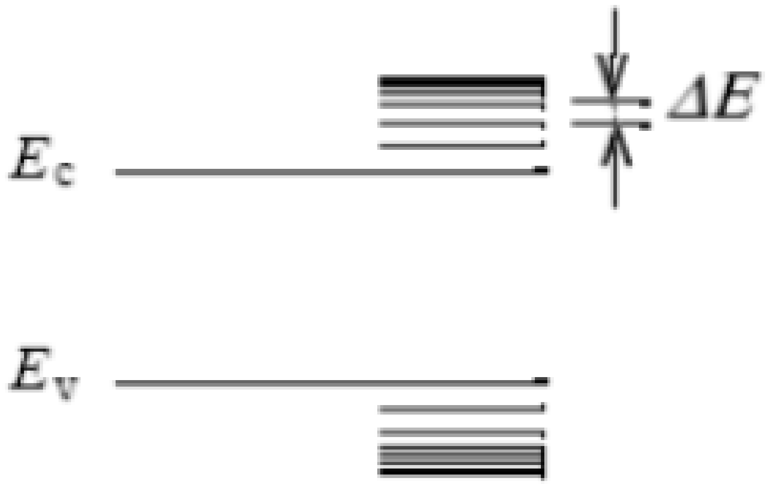
Density of State concept

- **Density of state $N(E)$: number of available state per unit of volume per unit of energy (unit: cm^{-3}eV^{-1})**

- **$N(E)dE$: number of available state per unit of volume lying in the energy range between E and $\mathrm{E}+dE$ (unit: cm^{-3})**



Density of states in 3D solid

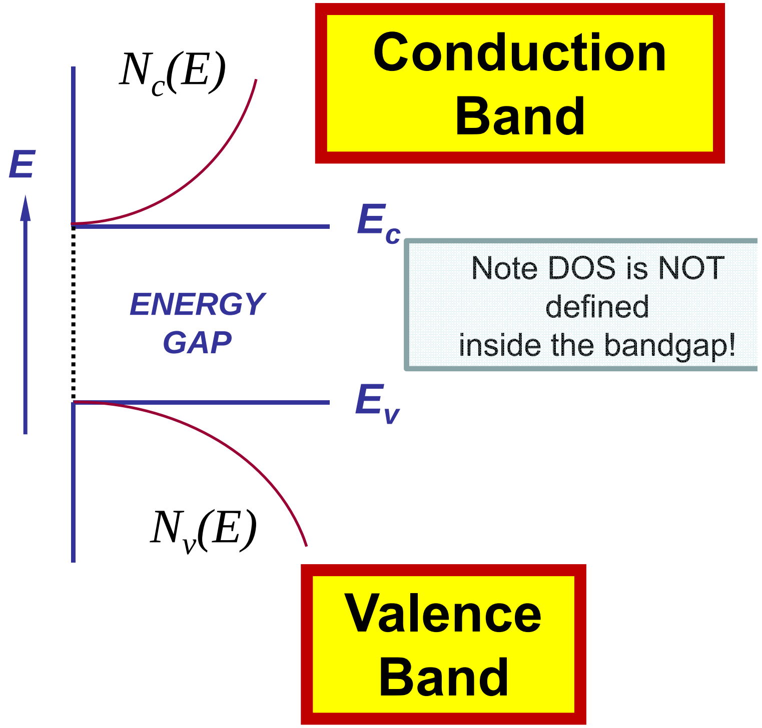
- In conduction band ($E>E_c$):

$$N_c(E) = \frac{\sqrt{2}}{\pi^2}\left(\frac{m_n^*}{\hbar^2}\right)^{\frac{3}{2}}(E - E_c)^{1/2}$$

- In valence band ($E<E_v$):

$$N_v(E) = \frac{\sqrt{2}}{\pi^2}\left(\frac{m_p^*}{\hbar^2}\right)^{\frac{3}{2}}(E - E_v)^{1/2}$$

- Most important feature is ~$E^{1/2}$ (more states at higher E)



Notice in 3D: $DOS \propto \sqrt{E}$

Carrier density calculation

- Electron concentrations in conduction band:

$$n_0 = \int_{E_c}^{\infty} f(E) N_c(E) dE$$

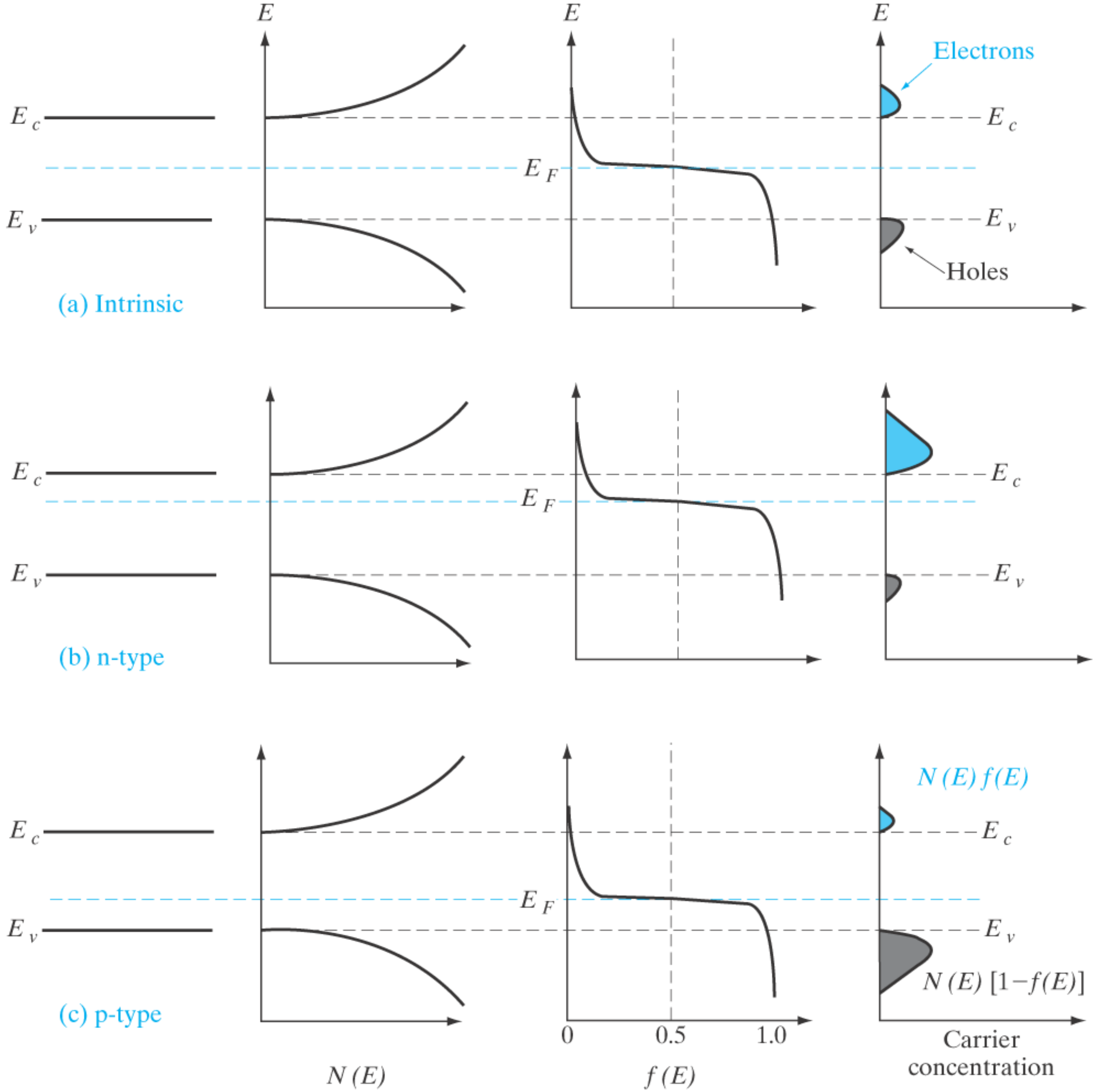
Probability the state is occupied by electrons

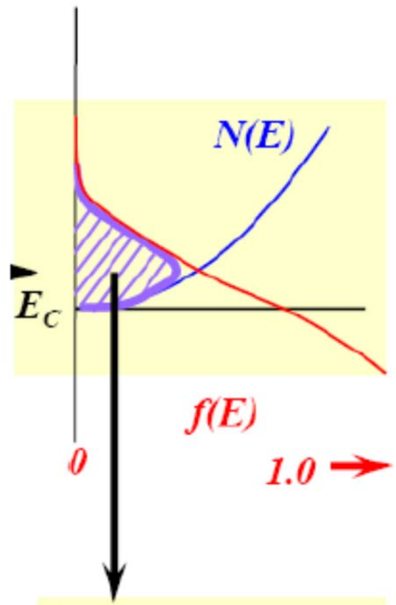
- Hole concentrations in valence band:

$$p_0 = \int_{-\infty}^{E_v} \underbrace{[1 - f(E)]} N_v(E) dE$$

Probability the state is occupied by holes

Carrier Concentration in Semiconductors





$$\int_{E_C}^{E_{top}} N(E)f(E)dE$$

Fermi function, density of state and carrier concentration

- **In the gap, density of state *N(E)=0* → carrier centration=0**

- **At *T=0K*, *f(E)=0* at conduction band --> no electrons**
  **
1-f(E)=0 at valence band → no holes**

- **At high T, in the conduction or valence band, both density of state and Fermi function are finite → finite carriers**

Maxwell-Boltzmann Approximation

$$\text{If } E - E_F > 3kT,\ \ f(E) \approx e^{-(E-E_F)/kT}$$

$$\text{If } E_F - E > 3kT,\ \ f(E) \approx 1 - e^{E-E_F/kT}$$

Carrier Concentration

- If E_F is well inside the band gap, ($E_v+3kT<E_F<E_c-3kT$), by using Boltzmann approximation, we get:

Electron concentration:

$$\boxed{n_0 \approx N_C e^{-(E_C-E_F)/kT}}$$

where $N_C = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$ Effective density of state in conduction band

Hole concentration:

$$\boxed{p_0 \approx N_V e^{-(E_F-E_V)/kT}}$$

where $N_v = 2(\frac{2\pi m_p^* kT}{h^2})^{3/2}$ Effective density of state in valence band

Intrinsic carrier concentration

- **np product**

$$n_0 p_0 = N_c \, N_v e^{-E_g/kT}$$

- For Intrinsic material $n_i = p_i$

$$\Rightarrow n_i p_i = n_i^2 = N_c \, N_v e^{-E_g/kT}$$

$$\Rightarrow n_i = \sqrt{N_c N_v} e^{-E_g/2kT}$$

For silicon, at room T, $n_i = 1.5 \times 10^{10} cm^{-3}$

$$\Rightarrow \boxed{n_0 p_0 = n_i^2}$$

- **Express carrier concentration using intrinsic carrier concentration**

$$\boxed{n_0 \approx N_C e^{-(E_C - E_F)/kT}}$$

$$\boxed{p_0 \approx N_V e^{-(E_F - E_V)/kT}}$$

$$n_i = N_c e^{-(E_c - E_i)/kT}$$

$$p_i = N_v e^{-(E_i - E_v)/kT}$$

$$\Downarrow$$

$$\boxed{n_0 = n_i e^{(E_F - E_i)/kT}}$$

$$\boxed{p_0 = n_i e^{(E_i - E_F)/kT}}$$

Intrinsic level

For intrinsic material:

$$n_i = N_c e^{-(E_c - E_i)/kT} = N_v e^{-(E_i - E_v)/kT} = p_i$$

$$\Rightarrow \quad \boldsymbol{E_i = \frac{E_c + E_v}{2} + \frac{kT}{2} ln\left(\frac{N_v}{N_c}\right)} \quad \text{or} \quad \boldsymbol{E_i = \frac{E_c + E_v}{2} + \frac{3kT}{4} ln\left(\frac{m_p^*}{m_n^*}\right)}$$

If $N_v = N_c$ $(i.e.\ \ m_p^* = m_n^*), then\ E_i = \frac{E_c + E_v}{2}$, i.e at mid-gap

If $N_v > N_c, \left(i.e.\ \ m_p^* > m_n^*\right), will\ E_i$ above or below the mid-gap?

If $N_v < N_c, \left(i.e.\ \ m_p^* < m_n^*\right), will\ E_i$ above or below the mid-gap?

Example problem 1

An intrinsic Silicon wafer has $1x10^{10}$ cm^{-3} holes. When $1x10^{18}$ cm^{-3} donors are added, what is the new hole concentration?

Solution to problem 1

An intrinsic Silicon wafer has 1×10^{10} cm^{-3} holes. When 1×10^{18} cm^{-3} donors are added, what is the new hole concentration?

$$if \quad N_D \gg N_A \quad and \quad N_D \gg n_i$$

$$n \cong N_D \qquad and \qquad p \cong \frac{n_i^2}{N_D}$$

$$n \cong N_D = 10^{18} \; cm^{-3}$$

$$p = \frac{n_i^2}{n} = \frac{\left(10^{10}\right)^2}{10^{18}} \; cm^{-3} = 100 \; cm^{-3}$$

Example problem 2

Silicon doped with 10^{16} Boron atoms per cm^3. What are the hole & electron concentrations at room temperature? (assume lights off). Is this *n*- or *p*-type material? Where is the Fermi level E_F with respect to the other energy bands?

Hint:

$$n = n_i e^{(E_f - E_i)/kT}$$

$$p = n_i e^{(E_i - E_f)/kT}$$

$$n_i = 1.5 \times 10^{10} cm^{-3}$$

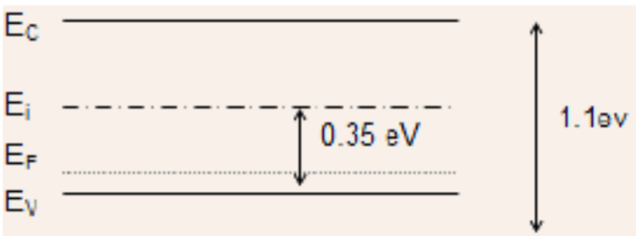
Solution to problem 2

Silicon doped with 10^{16} Boron atoms per cm^3. What are the hole & electron concentrations at room temperature? (assume lights off). Is this *n*- or *p*-type material? Where is the Fermi level E_F with respect to the other energy bands?

$$n = n_i e^{(E_f - E_i)/kT}$$

$$p = n_i e^{(E_i - E_f)/kT}$$

Since B (trivalent) is a p-type dopant in Si, hence, the material will be predominantly p-type, and since $N_A >> n_i$, therefore, p_0 will be approximately equal to N_A, and $n_0 = n_i^2/p_0 = 2.25 \times 10^4\ cm^{-3}$.



$E_i - E_F = kT \ln(p_0/n_i) = 0.026 \ln[10^{16}/(1.5 \times 10^{10})] = 0.35\ eV$

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- Carrier concentrations
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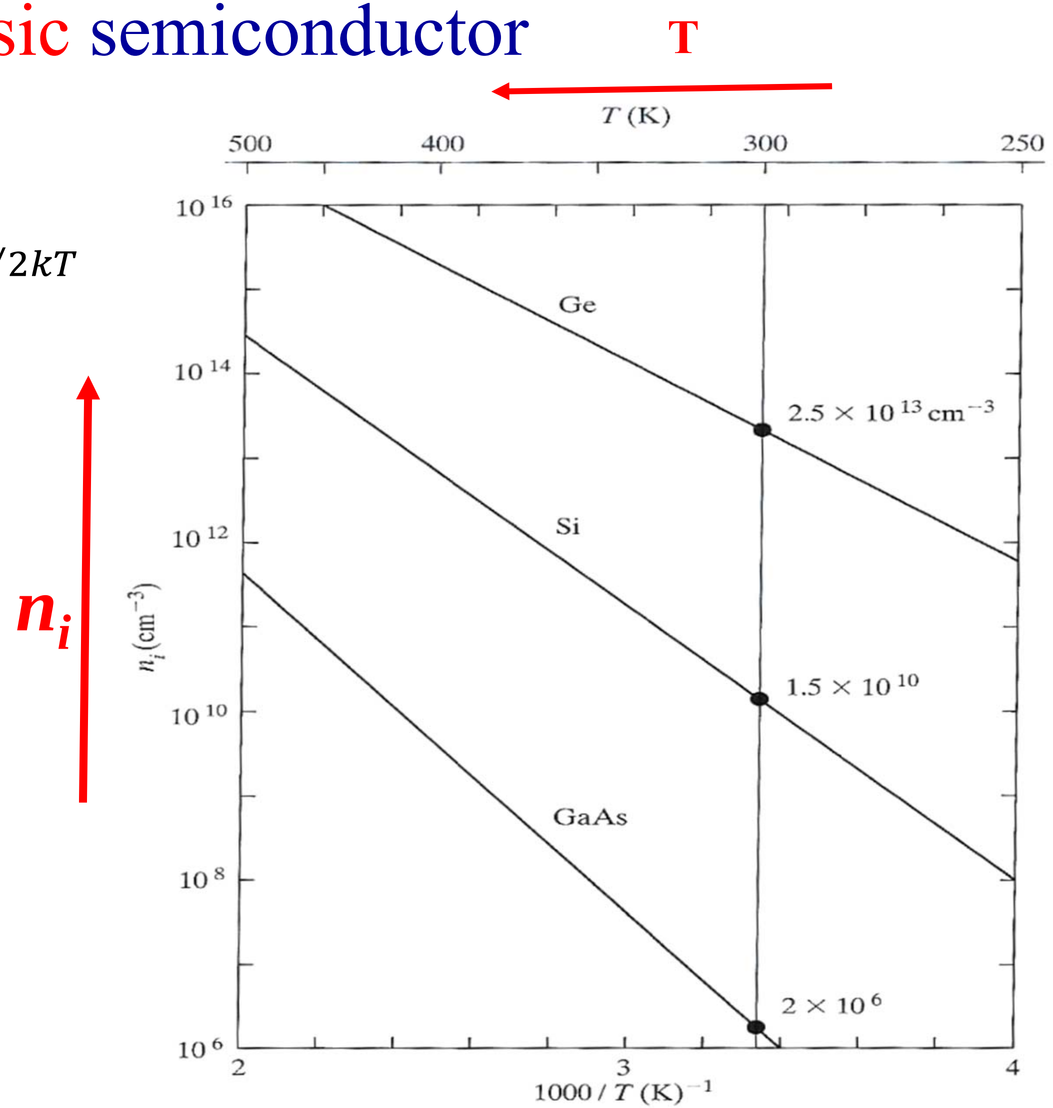
Temperature dependence of carrier concentration (1) –intrinsic semiconductor

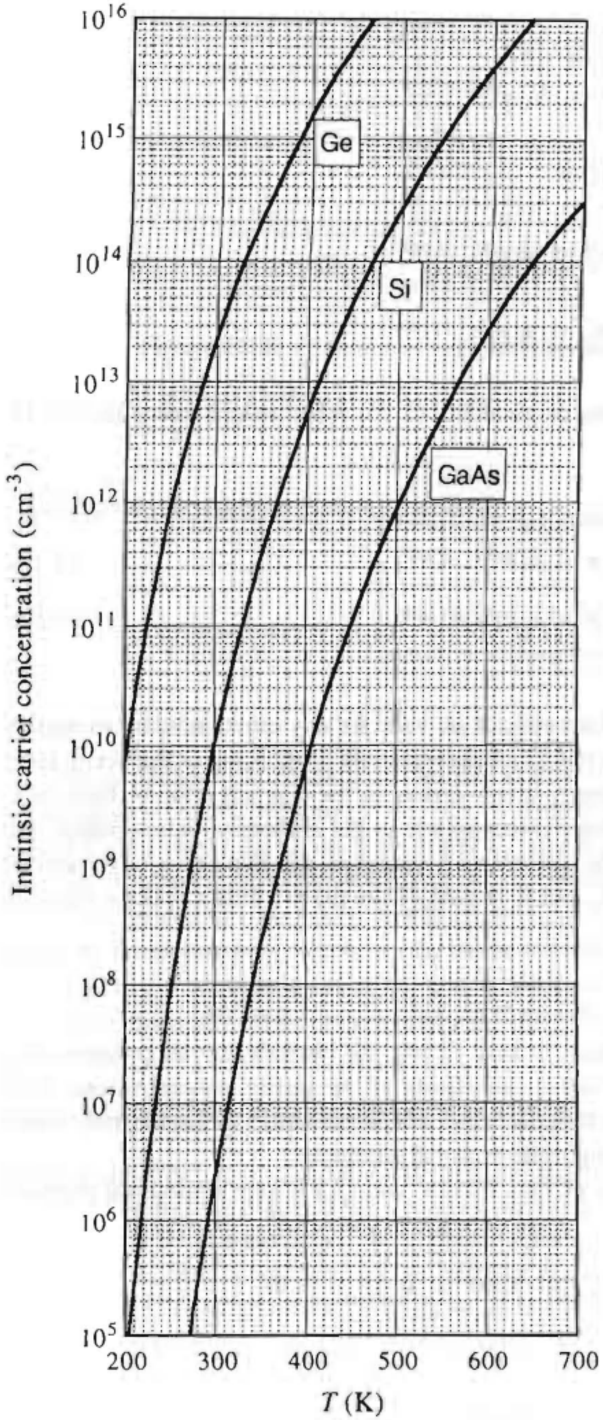
Intrinsic carrier concentration:

$$n_i = 2\left(\frac{2\pi kT}{h^2}\right)^{\frac{3}{2}} (m_n^* m_p^*)^{3/4} e^{-E_g/2kT}$$

- T increases → n_i increase
- E_g decrease → n_i increase
- m_n^* and m_p^* are density-of-state effective mass

Question: (1) Does density of state change with T?
(2) Does E_g change with T?
(2) why plot n_i vs 1000/T?





Si

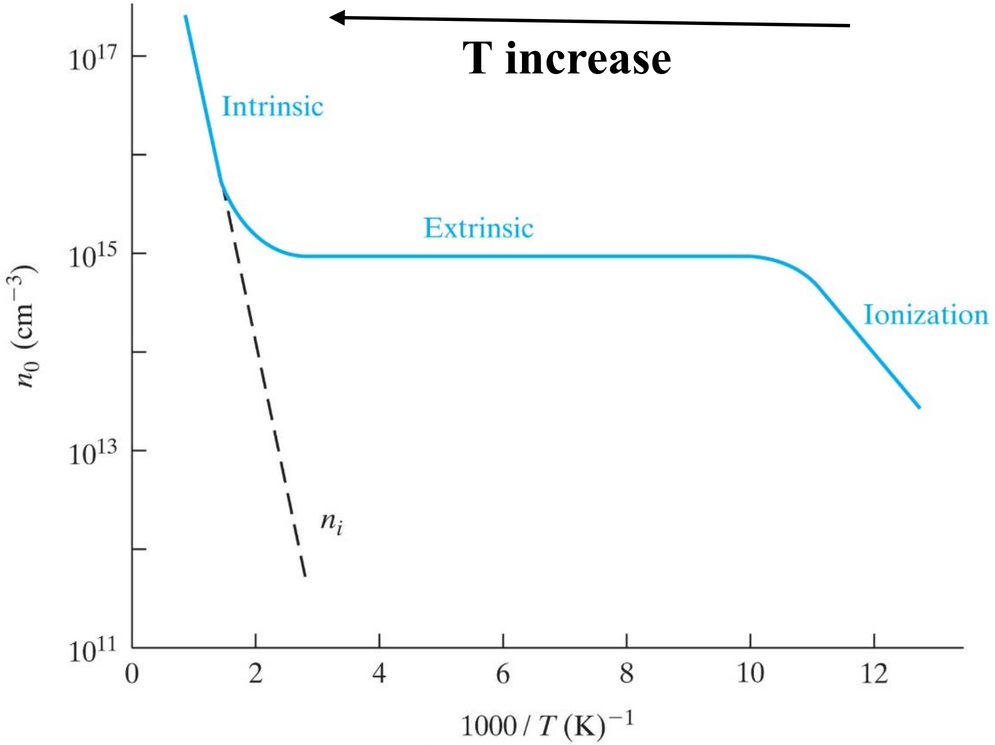
T(°C)	n_i(cm^{-3})
0	8.86×10^8
5	1.44×10^9
10	2.30×10^9
15	3.62×10^9
20	5.62×10^9
25	8.60×10^9
30	1.30×10^{10}
35	1.93×10^{10}
40	2.85×10^{10}
45	4.15×10^{10}
50	5.97×10^{10}
300 K	1.00×10^{10}

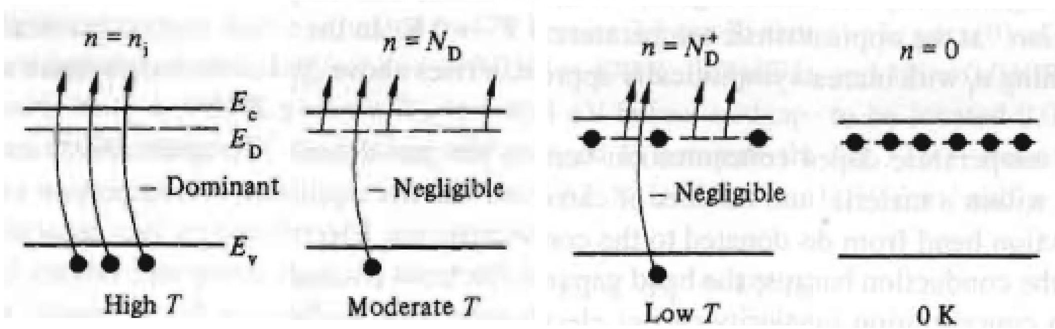
GaAs

T(°C)	n_i(cm^{-3})
0	1.02×10^5
5	1.89×10^5
10	3.45×10^5
15	6.15×10^5
20	1.08×10^6
25	1.85×10^6
30	3.13×10^6
35	5.20×10^6
40	8.51×10^6
45	1.37×10^7
50	2.18×10^7
300 K	2.25×10^6

- Plot $\log_{10}$ of n_i vs. T
- n_i is very temperature-sensitive! Ex: in Silicon:
  - While T = 300 → 330 K (10% increase)
  - $n_i = \sim 10^{10} \rightarrow \sim 10^{11}$ cm^{-3} (10x increase)

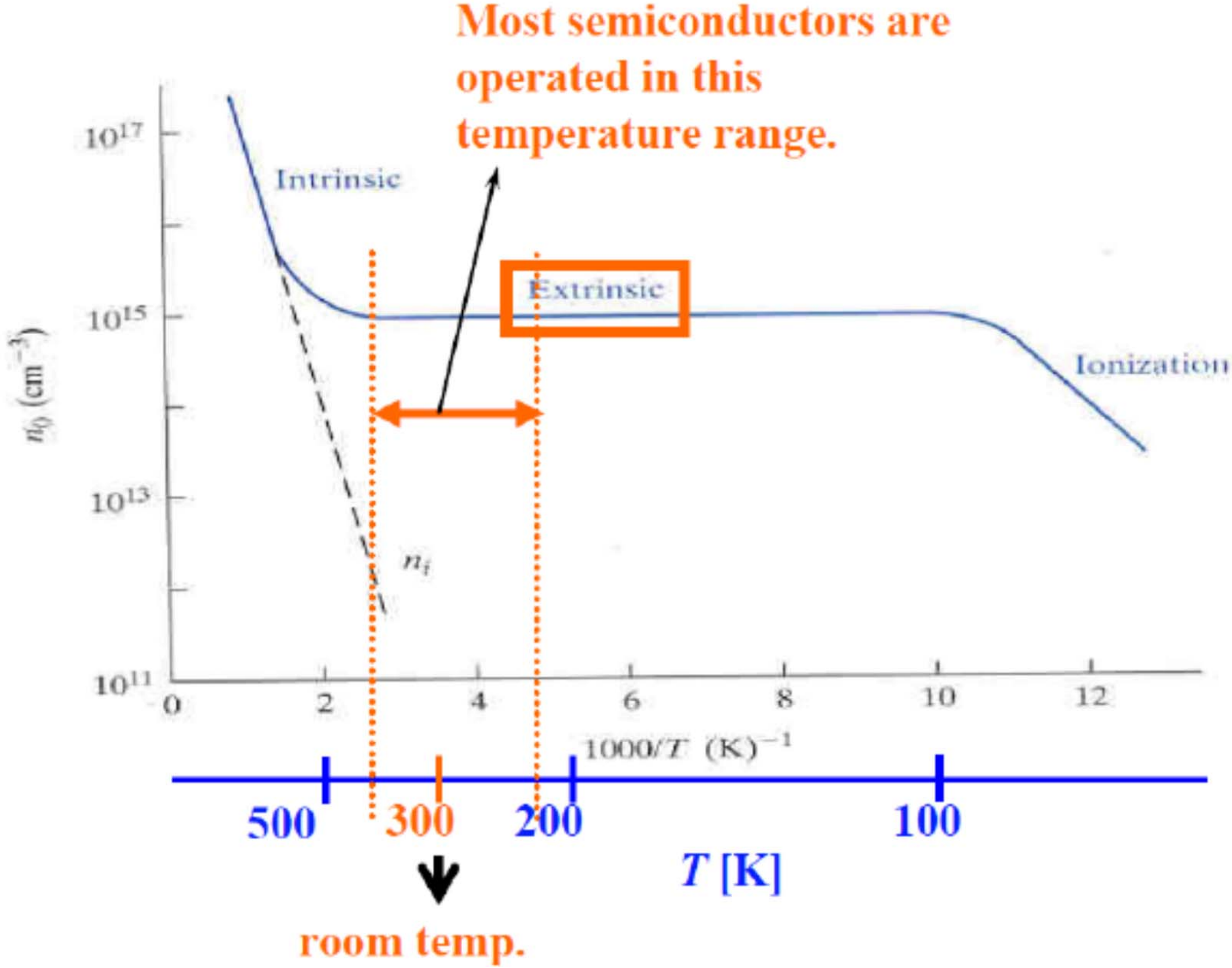
Temperature dependence of carrier concentration (1) –Extrinsic semiconductor





- Assume Si sample doped with $N_D = 10^{15}$ cm^{-3} (n-type)
- Recall the band diagram, including the donor level.
- Note three distinct regions:
  - Low, medium, and high-temperature

Operating Temperature of Semiconductor



Real example of operating temperature

MAXIMUM RATINGS (T_C = 25°C unless otherwise noted)

Rating	Symbol	Value	Unit
Drain–to–Source Voltage	V_{DSS}	400	Vdc
Drain–to–Gate Voltage (R_{GS} = 1.0 MΩ)	V_{DGR}	400	Vdc
Gate–to–Source Voltage — Continuous	V_{GS}	±20	Vdc
Drain Current — Continuous — Continuous @ 100°C — Single Pulse ($t_p \le 10\ \mu s$)	I_D I_D I_{DM}	10 6.0 40	Amps Apk
Total Power Dissipation Derate above 25°C	P_D	125 1.00	Watts W/°C
Total Power Dissipation @ T_A = 25°C, when mounted with the minimum recommended pad size		2.5	Watts
Operating and Storage Temperature Range	T_J, T_{stg}	– 55 to 150	°C
Single Pulse Drain–to–Source Avalanche Energy — Starting T_J = 25°C (V_{DD} = 25 Vdc, V_{GS} = 10 Vpk, I_L = 10 Apk, L = 10 mH, R_G = 25 Ω)	E_{AS}	[illegible]	mJ
Thermal Resistance — Junction to Case — Junction to Ambient — Junction to Ambient, when mounted with the minimum recommended pad size	$R_{\theta JC}$ $R_{\theta JA}$ $R_{\theta JA}$	1.00 62.5 50	°C/W
Maximum Lead Temperature for Soldering Purposes, 1/8″ from case for 10 seconds	T_L	260	°C

Designer's Data for "Worst Case" Conditions — The Designer's Data Sheet permits the design of most circuits entirely from the information presented. SOA Limit curves — representing boundaries on device characteristics — are given to facilitate "worst case" design.

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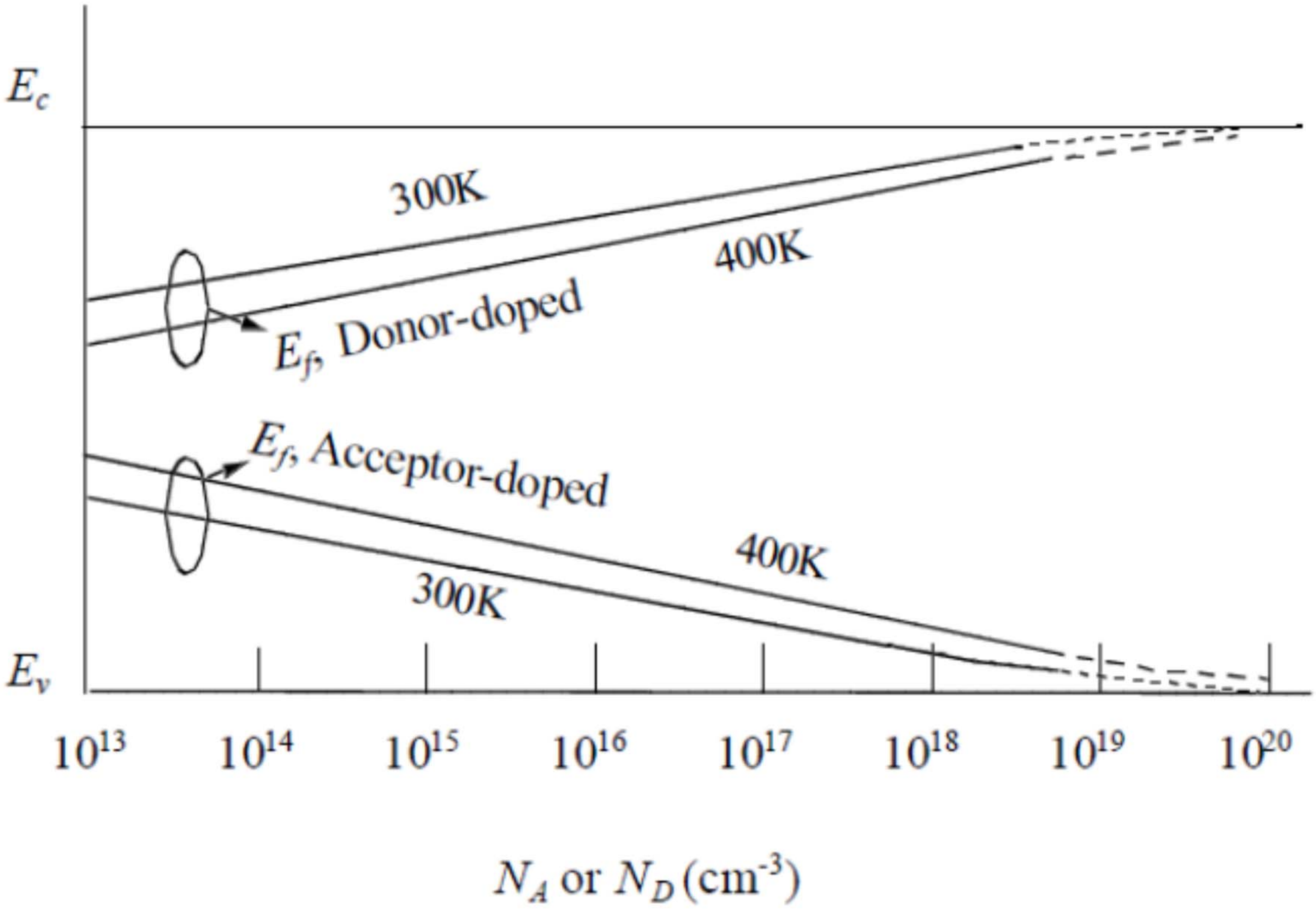
Operating temperature

- When do we need higher operating temperature for electronics?
  - Car, airplane engine monitor/control
  - Geothermal equipment
  - Oil field down-hole drilling
- How to increase operating temperature?
  - Raise doping
  - Use large band gap material → reduce intrinsic carrier density n_i

Dependence of E_F on Temperature and Doping



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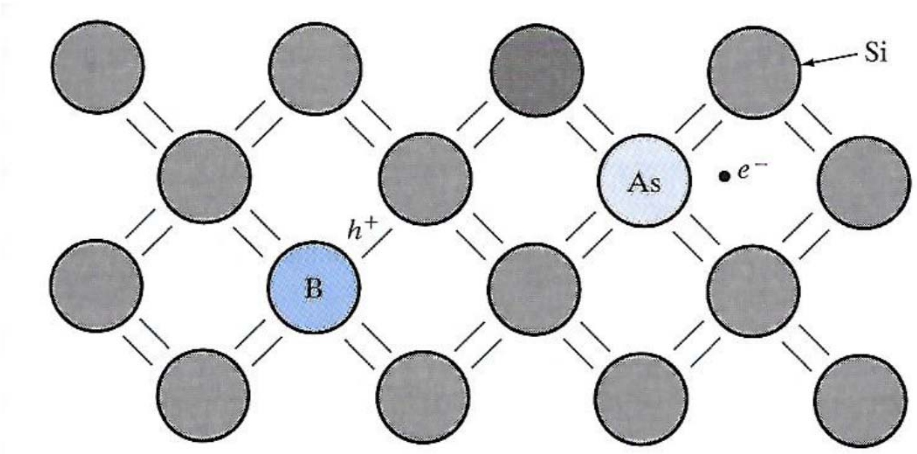
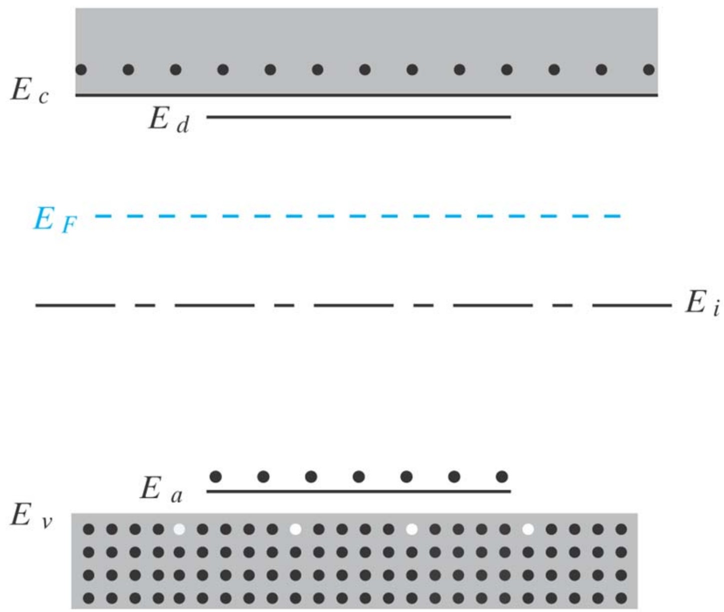
Compensation process

- So far, we assumed material is either just n- or p-doped and life was simple. At most moderate temperatures:
  - $n_0 \approx N_d$
  - $p_0 \approx N_a$

- What if a piece of Si contains BOTH dopant types? This is called compensation.





Space Charge Neutrality

- More generally, we must have charge neutrality in the material, i.e. positive charge = negative charge:

$$p_0 + N_d^+ = n_0 + N_a^-$$

- If all the impurities are ionized ($N_d^+ = N_d$, N_a^-=N_a):

$$p_0 + N_d = n_0 + N_a$$

- If the material is doped n type (n_0>>p_0): $n_0 \approx N_d - N_a$
- If the material is doped p type (p_0>>n_0): $p_0 \approx N_a - N_d$
- If N_d=N_a, the material is back to intrinsic: $n_0 = p_0 = n_i$

Carrier concentration: general case

$$\boldsymbol{p_0 + N_d = n_0 + N_a}$$

$$\Rightarrow \quad n_i^2/n_o + N_d = n_0 + N_a$$

$$n_o^2 - (N_d - N_a)n_o - n_i^2 = 0$$

$$\Downarrow$$

$$\boxed{\boldsymbol{n_o = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}}}$$

$$\boxed{\boldsymbol{p_o = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}}}$$

and $\boxed{n_0 p_0 = n_i^2}$

If $\boldsymbol{N_d - N_a \gg n_i}$, **then** $\boldsymbol{n_o \approx N_d - N_a}$
If $\boldsymbol{N_d \gg n_i}$ **and** $\boldsymbol{N_d >> N_a}$, **then** $\boldsymbol{n_o \approx N_d}$
$\boldsymbol{p_o \approx n_i^2/N_d}$

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Example of Heavy Doping

An intrinsic Silicon wafer has $1\text{x}10^{10}$ cm^{-3} holes. When $1\text{x}10^{18}$ cm^{-3} donors are added, what is the new hole concentration?

$$if \quad N_D \gg N_A \quad and \quad N_D \gg n_i$$

$$n \cong N_D \quad and \quad p \cong \frac{n_i^2}{N_D}$$

$$n \cong N_D = 10^{18}\ cm^{-3}$$

$$p = \frac{n_i^2}{n} = \frac{\left(10^{10}\right)^2}{10^{18}}\ cm^{-3} = 100\ cm^{-3}$$

Example of Heavy Doping

An intrinsic Silicon wafer has $1x10^{10}$ cm^{-3} holes. When $1x10^{18}$ cm^{-3} donors are added, what is the new hole concentration?

Example of Both Donors and Acceptors

An intrinsic Silicon wafer has $1x10^{10}$ cm^{-3} holes. When $1x10^{18}$ cm^{-3} acceptors and $8x10^{17}$ cm^{-3} donors are added, what is the new hole concentration?

Example of Light Doping and High T

An intrinsic Silicon wafer at 470K has 1×10^{14} cm^{-3} holes. When 1×10^{14} cm^{-3} acceptors are added, what is the new electron and hole concentrations?

Example of High Temperature

An intrinsic Silicon wafer at 600K has $4x10^{15}$ cm-3 holes. When $1x10^{14}$ cm^{-3} acceptors are added, what is the new electron and hole concentrations?