

1. CONDUCTIVITY

(A).

$$\sigma = \frac{3(\Omega \times cm^{-1})}{d} \int_0^\infty e^{-x^2/2} dx \quad (1.1)$$

$$\int_0^\infty e^{-x^2/2} dx = \sqrt{\frac{\pi}{2}} \mu m \quad (1.2)$$

$$\sigma = \frac{3(\Omega \times cm^{-1})}{2\mu m} \cdot \sqrt{\frac{\pi}{2}} \mu m = 1.88(\Omega \times cm^{-1}) \quad (1.3)$$

(B). Since this is an N-type semiconductor, we know that $n \gg p$ so the conductivity due to the holes is negligible compared to the electrons. Because of this, $\sigma \approx q\mu_n N_D$.

$$n \approx N_D = \sigma / q\mu_n = \frac{1.88(\Omega \times cm^{-1})}{1.6 \times 10^{-19} \cdot 1450 cm^2 / V \cdot s} = 8.1 \times 10^{15} cm^{-3} \quad (1.4)$$

$$p = n_i^2 / n = (1.5 \times 10^{10})^2 / 8.1 \times 10^{15} = 2.77 \times 10^4 cm^{-3} \quad (1.5)$$

2. RESISTANCE UNDER DOPING

(A). To find resistance, we know that:

$$R = \frac{\rho \cdot L}{A} = \frac{L}{A} \cdot \frac{1}{\sigma} = \frac{L}{q(\mu_n \cdot n + \mu_p \cdot p) \cdot A} \quad (2.1)$$

$n \approx N_D = 2.5 \times 10^{17} cm^{-3}$, this is much larger than n_i so we can ignore p as $p = \frac{n_i^2}{n} \ll n$
 $\mu_n \approx 600 cm^2 / V \cdot s$ at impurity concentration of $2.5 \times 10^{17} cm^{-3}$ of Si (Figure 3-23 in the text)

$$R = \frac{0.25}{1.6 \times 10^{-19} \times (600 \cdot 2.5 \times 10^{17}) \cdot 5 \times 10^{-4}} = 20.83 \Omega \quad (2.2)$$

$$I = \frac{V}{R} = \frac{12V}{20.83 \Omega} = 0.576A \quad (2.3)$$

(B). To find p_o , we note the space charge neutrality equation:

$$p_o + N_d^+ = n_o + N_a^- \rightarrow p_o^2 + (N_d^+ - N_a^-)p_o - n_o p_o = 0 \rightarrow p_o^2 + (N_d^+ - N_a^-)p_o - n_i^2 = 0 \quad (2.4)$$

$$p = 1.0002 \times 10^{12}, -2.24 \times 10^8 \quad (2.5)$$

The negative root doesn't make physical sense, therefore we can see that the exact carrier concentration is fairly close to the approximate solution. We can also see that $p \gg n$.

$\mu_p \approx 550 cm^2 / V \cdot s$ at total impurity concentration of 3.99×10^{14} (Ga: $2 \times 10^{14} cm^{-3}$ + As: $1.99 \times 10^{14} cm^{-3}$)

$$R = \frac{0.25}{1.6 \times 10^{-19} \times (550 \cdot 1.0002 \times 10^{12}) \cdot 5 \times 10^{-4}} = 5.67 \times 10^6 \Omega \quad (2.6)$$

$$I = \frac{V}{R} = \frac{12V}{5.67 \times 10^6 \Omega} = 2.12 \mu A \quad (2.7)$$

(C). For case (a),

$\mu_n \approx 600 \text{ cm}^2/\text{V} \cdot \text{s}$ at impurity concentration of $2.5 \times 10^{17} \text{ cm}^{-3}$

$E = 12 \text{ V}/0.25 \text{ cm} = 48 \text{ V/cm}$

$$v_d = \mu_n \cdot E = 600 \text{ cm}^2/\text{V} \cdot \text{s} \cdot 48 \text{ V/cm} = 2.88 \times 10^4 \text{ cm/s} = 288 \text{ m/s} \quad (2.8)$$

For case (b),

$\mu_n \approx 1500 \text{ cm}^2/\text{V} \cdot \text{s}$ at impurity concentration of $3.99 \times 10^{14} \text{ cm}^{-3}$

$E = 12 \text{ V}/0.25 \text{ cm} = 48 \text{ V/cm}$

$$v_d = \mu_n \cdot E = 1500 \text{ cm}^2/\text{V} \cdot \text{s} \cdot 48 \text{ V/cm} = 7.2 \times 10^4 \text{ cm/s} = 720 \text{ m/s} \quad (2.9)$$

(D). **The "How?":** The resistance varies with doping since it is dependent on resistivity, ρ , as $R = \rho L/A$, where resistivity is the inverse of conductivity, σ , and L and A are the length and area respectively. Based on Eq. 3-40a in the text, conductivity can be written in terms of the fundamental charge of an electron, q , multiplied by the total carrier concentration and the carrier mobility. From Figure 3-23 in the text, we can see that as concentration of impurities (i.e. dopants) increases, the mobility for both holes and electrons decreases.

For doped materials with only donors or acceptors, this drop in mobility is however still orders of magnitude less than the carrier concentration, so conductivity will still rise and thus resistance will fall. For compensation-doped materials, the combined impurity concentrations create a larger drop in mobility while the total carrier concentration is reduced due to the offsetting effect of having both electrons and holes. So the mobility dominates and thus conductivity drops so resistance rises.

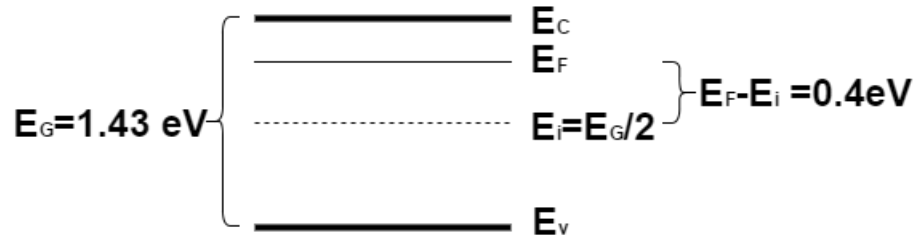
The "Why?": Why this relation occurs is based on the notion of electron-impurity scattering events. Since electrons contain a finite mass and thus a momentum, the likelihood that they repel off an ionized dopant (i.e. Coulombic interaction) and lose their energy to scattering increases with higher concentrations of carriers. This is known as "impurity scattering".

For doped materials with only donors or acceptors, each dopant swapped into the lattice creates an extra possible scatter site for mobile electrons, yet since the dopant also brings with it additional electrons (holes) for conduction, resistance still drops. For compensation-doped materials, an even larger number of dopants are swapped into the lattice as possible scatter sites, though having both additional holes brought with the acceptors and additional electrons brought with the donors has a canceling effect. Thus, for compensated wafers, you get the increase in scattering without the larger increase in carrier conduction, causing resistance to increase.

This scattering mean time, \hat{t} , multiplied by the charge-to-mass ratio of a carrier is what gives us mobility.

3. INVARIANCE

(A). This is depicted as:



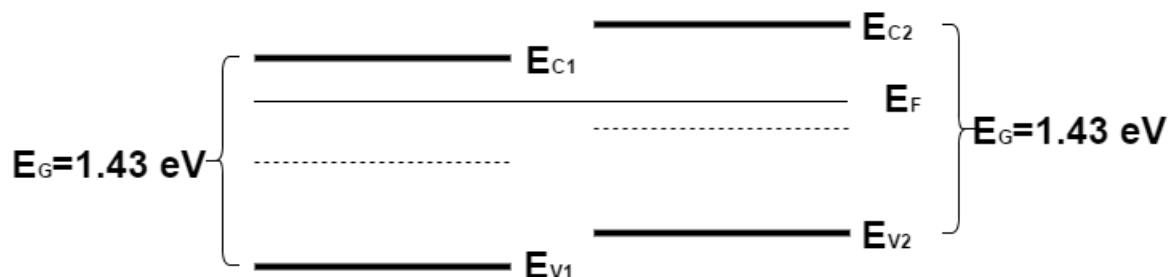
We can first recall by looking up in a table that GaAs has a bandgap of $E_g = 1.43\text{eV}$, which means $E_c - E_v = E_g$. We can assume the intrinsic energy level sits in the center at the bandgap, so $E_i = E_g/2$. Since we have been given information about the Fermi level in that $E_f - E_i = 0.4\text{eV}$, we can easily deduce that this means the Fermi level sits closer in the bandgap to the conduction band, E_c .

(B). When two semiconductors come together, they create what's known as a junction, which will be discussed in great detail later in the course. A separate set of Fermi-Dirac distribution functions and available energy states will occur for each material, yet at thermal equilibrium no net current or charge transport can carry electrons from one semiconductor to the other. Because of this, there is no change in energy due to charge carrier transport. Since the Fermi level above $T=0$ Kelvin can be generally thought of as the chemical potential, or the additional energy in acquiring a single electron, this means that no net changes can occur in occupation statistics, $f(x)$, or total density of states, N , for either material so the Fermi levels must be equal across the junction. Thus,

$$E_{F1} = E_{F2} \quad (3.1)$$

This tells us that what is known for sure is that the Fermi levels must line up and be a constant horizontal line.

Below is a simplified depiction of what we could expect in such a junction, showing the Fermi levels aligned.



4. EXCITATION AND BAND GAPS

(A). The wavelength is related to energy by $E = hc/\lambda$ which for our use is best written as $E(\text{eV}) = 1.24/\lambda(\mu\text{m})$. Converting the band-gap energies to corresponding wavelengths we see that:

GaN = 364.7 nm (ultraviolet)

$\text{Al}_{0.49}\text{Ga}_{0.51}\text{As}$ = 609.3 nm (orange visible light)

Si = 1107.1 nm (infrared)

A semiconductor will absorb light at higher energies than the band-gap (α increases sharply at the wavelength corresponding to Bandgap energy at room temperature). Shorter wavelengths have higher energy. The LED has a center wavelength of 612 nm (it will have photons of both higher and lower wavelengths in the spectrum).

GaN will appear to transmit the most amount of orange light, followed by $\text{Al}_{0.49}\text{Ga}_{0.51}\text{As}$ and then Si. This is because $\lambda=612$ nm corresponds with 2.02 eV, which is below the band-gap energy for both GaN and $\text{Al}_{0.49}\text{Ga}_{0.51}\text{As}$. $\text{Al}_{0.49}\text{Ga}_{0.51}\text{As}$, however, will be able to absorb more of the higher energy photons emitted by the LED than GaN, since its bandgap is closer to the bandgap, 2.02 eV, of the LED. The Si has a much smaller bandgap than the incident photons so we can assume that it will absorb all of the excitation light. Thus, since the probability of transmission is the complement of absorption, **both GaN and $\text{Al}_{0.49}\text{Ga}_{0.51}\text{As}$ will transmit light.**

(B). For a white light source, the differences in transmission will be more striking between separate semiconductors. White light LEDs typically span a spectrum from around the mid-blue wavelengths ($\lambda=430\text{nm}$) to the short-IR/red ($\lambda=720\text{nm}$), and for this question we are to assume it is uniformly distributed (i.e. the spectrum is flat across the range). Each wafer will absorb the individual photons (light) in the spectrum that can reach energetically above the semiconductor bandgap; $\text{Al}_{0.49}\text{Ga}_{0.51}\text{As}$ will absorb light shorter than the wavelength of 609.3nm, Si will absorb light shorter than the wavelength of 1107.1nm, and GaN will absorb light shorter than the wavelength of 364.7nm.

This means that Si will absorb the entire spectrum of the light and transmit next to nothing. GaN will transmit almost all the light, absorbing only a miniscule amount of the violet end of the light that overlaps with its spectrum and can span its bandgap. $\text{Al}_{0.49}\text{Ga}_{0.51}\text{As}$ is in the middle and will transmit only very little of the light, as its bandgap sits at 2.03 eV ($\lambda=609.3\text{nm}$) and thus will only transmit the wavelengths greater than 609.3nm, roughly a third of the spectrum. So, in order from lowest to highest transmission:

Si transmits the least, followed by $\text{Al}_{0.49}\text{Ga}_{0.51}\text{As}$, followed by GaN with the most transmission.

5. EXCESS CARRIERS AND OPTICAL ABSORPTION

(A). The recombination lifetime for a low-level injection is given as

$$\tau = \frac{1}{\alpha_r(p_o + n_o)} \quad (5.1)$$

$n \approx N_d$. Since this Si wafer is doped with 4×10^{16} donors, this is an n-type wafer where $n \gg p$. We can then ignore p_o .

Because the excess carrier concentration is 100x lower than the background doping concentration, we know that it satisfies low-level injection. So we can solve for α_r using just the doping concentration value, n_o :

$$\alpha_r = \frac{1}{\tau_p(n_o)} = \frac{1}{10 \times 10^{-6} \cdot 4 \times 10^{16}} = 2.5 \times 10^{-12} \text{ cm}^3/\text{s} \quad (5.2)$$

In the case of direct recombination, excess majority carriers (electrons) will decay at the same speed as the minority carriers (holes) the recombination lifetimes of each are equal, as given in the problem.

(B). We are given that $I_o = 20 \times 10^{-2}$, $L=0.5\mu m$, and $\alpha = 2 \times 10^4 cm^{-1}$. From Eq. 4-3,

$$I_t = I_o e^{-\alpha L} = 20 \times 10^{-2} \cdot \exp(-2 \times 10^4 \cdot 5 \times 10^{-5}) = 0.0735W = 7.35mW \quad (5.3)$$

Since this is power transmitted, power absorbed is:

$$I_{abs} = I_o - I_t = 20mW - 7.35mW = 12.64mW \quad (5.4)$$

To find the rate of energy (i.e. Power) given up thermally before recombination, we see how far above the Si bandgap our incident light reached. This fraction of excess energy absorbed, in eV, is:

$$\frac{1.9eV - 1.12eV}{1.9eV} = 0.41 \quad (5.5)$$

Combining the two fraction and the absorbed power, we find the total rate of energy given up thermally before recombination of EHP.

$$0.41 \cdot 12.64 \text{ mW} = 5.189 \text{ mW} \quad (5.6)$$