ECE 340: Semiconductor Electronics

Chapter 7: Narrow-based diode

Solid State Electronic Devices (Streetman): § 7

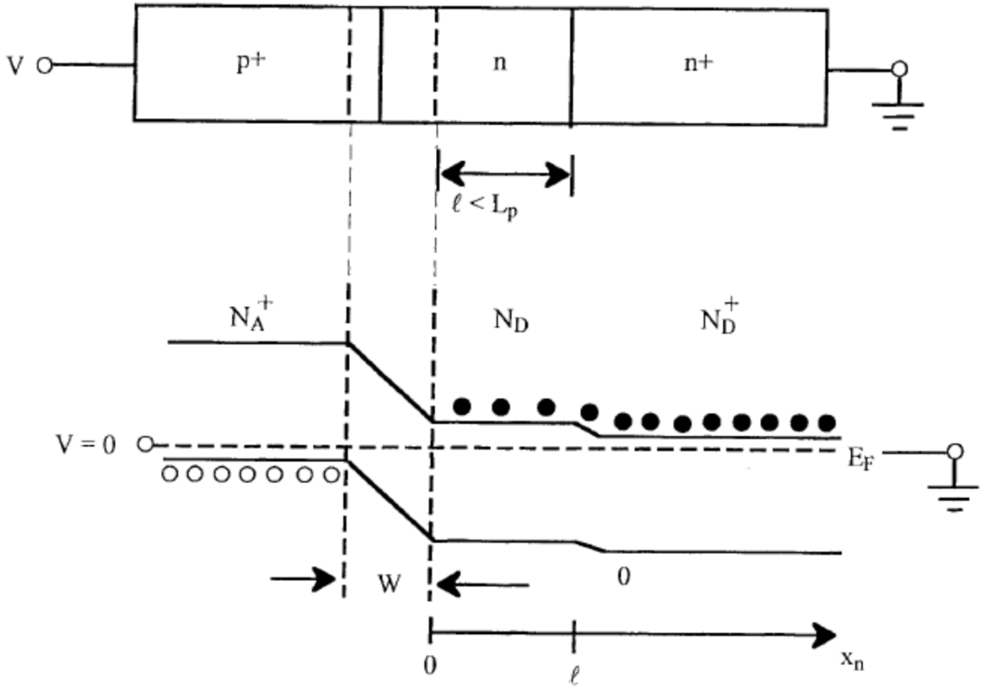
Narrow-base diode handout

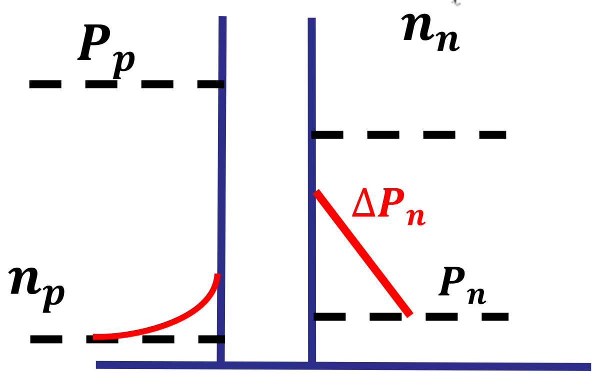
Wenjuan Zhu

Outline

- Narrow-base diode
- Bipolar junction transistor
  - Fundamentals of BJT operation and Amplification with BJTs
  - Minority carrier distributions and terminal currents
  - Generalized biasing
  - Switching
  - Normal mode operation
- Common-emitter amplifier and small-signal current gain

Narrow base diode





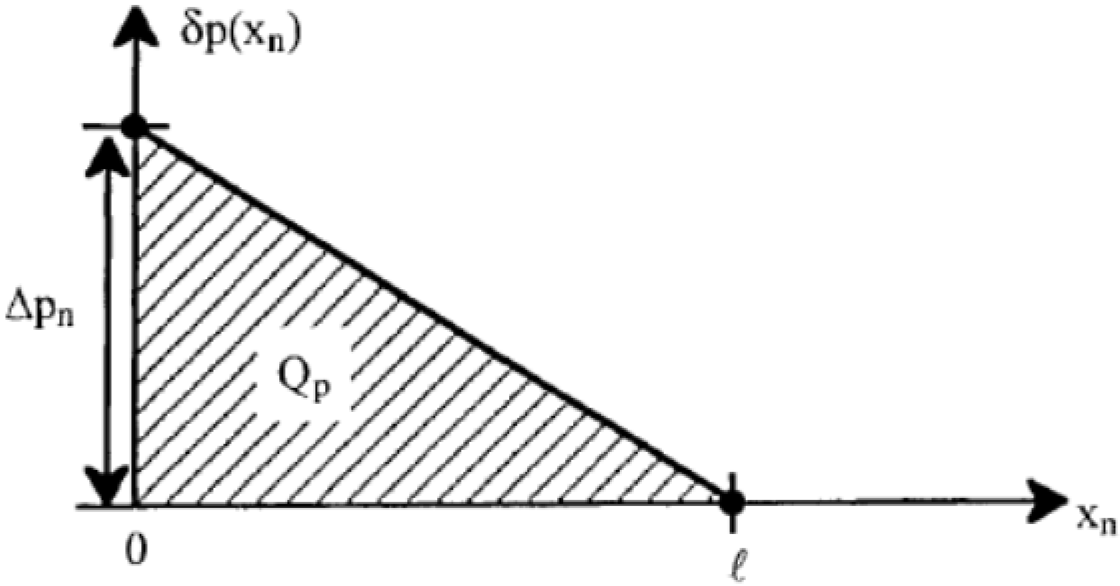
- The base region is lightly doped, and the length is much smaller than the minority carrier diffusion length: $\ell \ll L_p$
- Holes entering the n+ region are assumed to recombined instantly

Boundary condition:

$$\delta p(x_n = 0) = \Delta p_n = p_n(e^{\frac{qV}{kT}} - 1)$$

$$\delta p(x_n = \ell) \approx 0$$

Straight-line approximation



- **If** $\ell \ll L_p$, most of the injected minority holes will diffuse across the n type base without recombining until they hit the n+ contact, then:

$$\boxed{\frac{-dp(x_n)}{dx_n} \approx \frac{\Delta p_n}{\ell}}$$

$$\boxed{Q_p = \frac{1}{2} qA\ell\Delta p_n}$$

Majority and minority current

- The hole current in the n-region:

$$I_p(\boldsymbol{x_n}) = AJ_p(\boldsymbol{diff}) = -AqD_p \frac{dp(x_n)}{dx_n} \approx \boldsymbol{q}AD_p \frac{\boldsymbol{\Delta p_n}}{\ell}$$

Higher curren

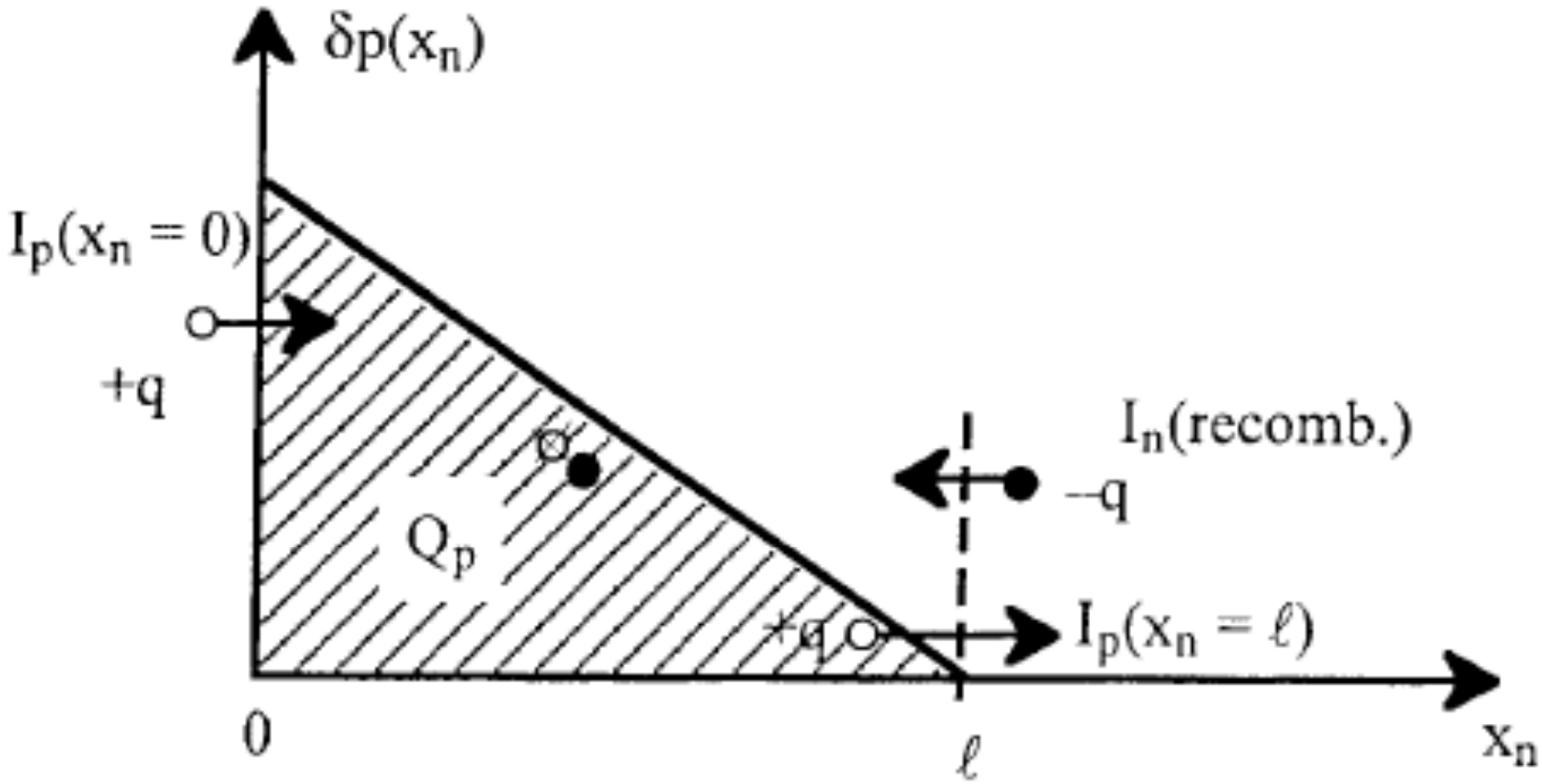
- The electron recombination current in n region:

$$I_n(\boldsymbol{recomb.}) = \frac{\boldsymbol{Q_p}}{\boldsymbol{\tau_p}} = \frac{\boldsymbol{qA\ell}}{\boldsymbol{2\tau_p}} \Delta \boldsymbol{p_n}$$

Or $$I_n(\boldsymbol{recomb.}) = I_p(\boldsymbol{x_n} = \boldsymbol{0}) \left(\frac{\ell^{\boldsymbol{2}}}{\boldsymbol{2L_p^2}} \right)$$

When $\ell \ll \boldsymbol{L_p}$, $I_n(\boldsymbol{recomb.}) \approx \boldsymbol{0}$

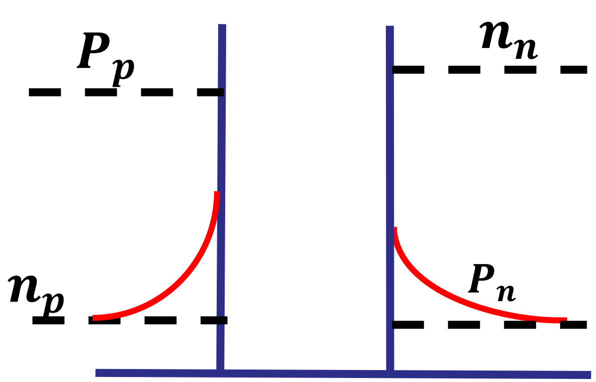
Current component in narrow-based diode



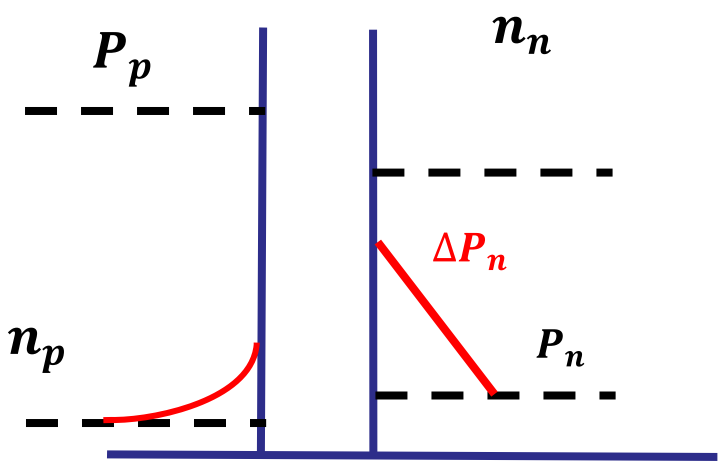
$$I_p(x_n = 0) = I_p(x_n = \ell) + I_n(recomb.)$$

Current density impact

Conventional p-n diode



Narrow base diode



Higher current density

- The slope of the excess carrier concentration is much higher for the narrow base diode than the conventional junction
- The current density in the narrow base diode is therefore much higher (same voltage produces more current)

Exact solution to the 1D diffusion equation

- Diffusion equation:

$$\frac{d^2\delta p}{dx^2} = \frac{\delta p}{L_p^2}$$

- Solution:

$$\delta p(\boldsymbol{x_n}) = C_1 e^{-x_n/L_p} + C_1 e^{x_n/L_p}$$

- **Boundary condition:**

$$\boldsymbol{\delta p(x_n = 0) = \Delta p_n = p_n\,(e^{\frac{qV}{kT}} - 1)}$$

$$\boldsymbol{\delta p(x_n = \ell) \approx 0}$$

Exact solution

- Excess hole concentration:

$$\delta p(\boldsymbol{x_n}) = \Delta \boldsymbol{p_n} \frac{e^{(\ell - \boldsymbol{x_n})/L_p} - e^{(\boldsymbol{x_n} - \ell)/L_p}}{e^{\ell/L_p} - e^{-\ell/L_p}}$$

- Hole diffusion current:

$$\boldsymbol{I_p}(\boldsymbol{x_n}) = -\mathrm{qA}\boldsymbol{D_p}\frac{\boldsymbol{d\delta p}(\boldsymbol{x_n})}{\boldsymbol{dx_n}} = \mathrm{qA}\frac{\boldsymbol{D_p}}{\boldsymbol{L_p}}\Delta \boldsymbol{p_n}\frac{e^{(\ell - \boldsymbol{x_n})/L_p} - e^{(\boldsymbol{x_n} - \ell)/L_p}}{e^{\ell/L_p} - e^{-\ell/L_p}}$$

$\boldsymbol{when}\ \ell \ll \boldsymbol{L_p}$:

$$\boldsymbol{I_p}(\boldsymbol{x_n} = \boldsymbol{0}) = \mathrm{qA}\frac{\boldsymbol{D_p}}{\boldsymbol{L_p}}\Delta \boldsymbol{p_n}\ \boldsymbol{ctnh}\left(\frac{\ell}{L_p}\right) \approx \mathrm{qA}\frac{\boldsymbol{D_p}}{\boldsymbol{L_p}}\Delta \boldsymbol{p_n}\left(\boldsymbol{1} + \frac{\ell^2}{\boldsymbol{3L_p^2}}\right)$$

$$\boldsymbol{I_p}(\boldsymbol{x_n} = \ell) = \mathrm{qA}\frac{\boldsymbol{D_p}}{\boldsymbol{L_p}}\Delta \boldsymbol{p_n}\ \boldsymbol{csch}\left(\frac{\ell}{L_p}\right) \approx \mathrm{qA}\frac{\boldsymbol{D_p}}{\boldsymbol{L_p}}\Delta \boldsymbol{p_n}\left(\boldsymbol{1} - \frac{\ell^2}{\boldsymbol{6L_p^2}}\right)$$

Electron current

- The electron current flowing into the base from n+ contact to offset recombination of holes:

$$I_n(\boldsymbol{recomb.}) = I_p(\boldsymbol{x_n} = \mathbf{0}) - I_p(\boldsymbol{x_n} = \ell)$$
$$= \mathrm{qA}\frac{\boldsymbol{D_p}}{\boldsymbol{L_p}}\Delta\boldsymbol{p_n}\left(\frac{\ell^{\mathbf{2}}}{\mathbf{2}\boldsymbol{L_p^2}}\right)$$

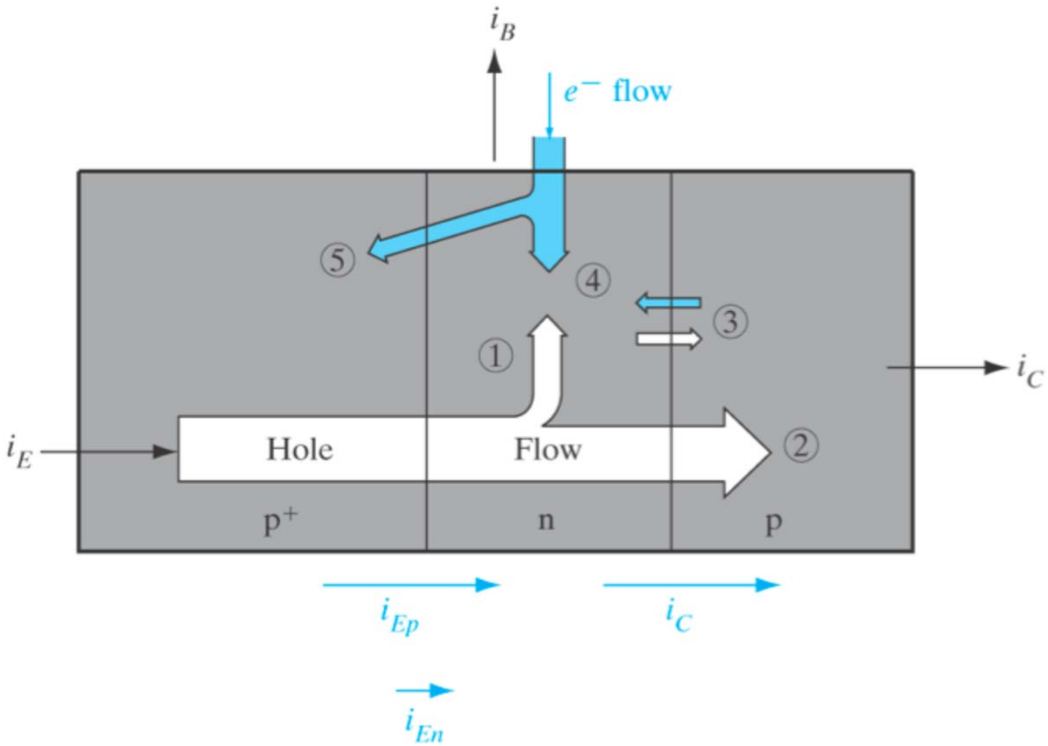
- Electron injection current:

$$I_n(\boldsymbol{inj.}) = \boldsymbol{qA}\frac{\boldsymbol{D_n}}{\boldsymbol{L_n}}\boldsymbol{n_p}(\boldsymbol{e}^{\frac{\boldsymbol{qV}}{\boldsymbol{kT}}} - \mathbf{1})$$

Outline

- Narrow-base diode
- Bipolar junction transistor
  - ⇨ Fundamentals of BJT operation and Amplification with BJTs
  - Minority carrier distributions and terminal currents
  - Generalized biasing
  - Switching
  - Normal mode operation
- Common-emitter amplifier and small-signal current gain

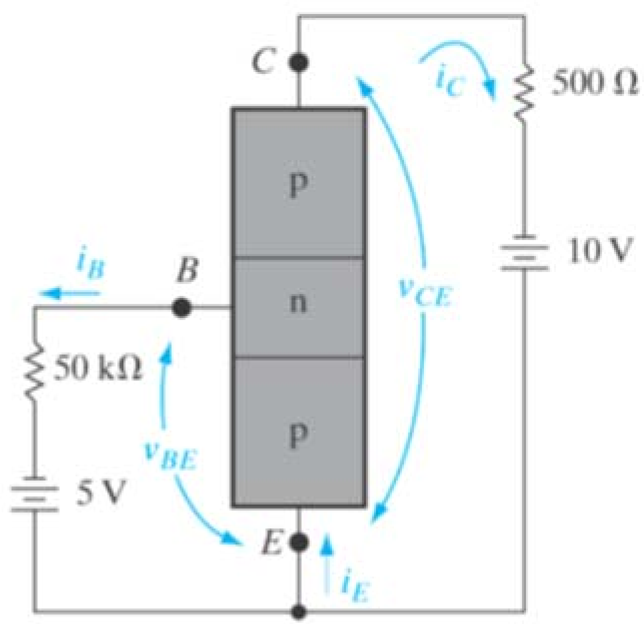
Current flow in pnp transistor



Transistor Current Components

- Hole recombination in base (1)
- Holes injected into reverse-biased base-collector junction (2)
- Thermal generation-recombination current in reverse-biased base-collector junction (3)
- Electron flow into base to compensate for electrons lost due to recombination with injected holes (4)
- Electron flow from n-type base to p^+ emitter under forward bias (5)

Definitions in BJT



Collector current:

$$i_C = B i_{EP}$$ B: Base transport factor

Emitter injection efficiency:

$$\gamma = \frac{i_{EP}}{i_{EP} + i_{En}}$$

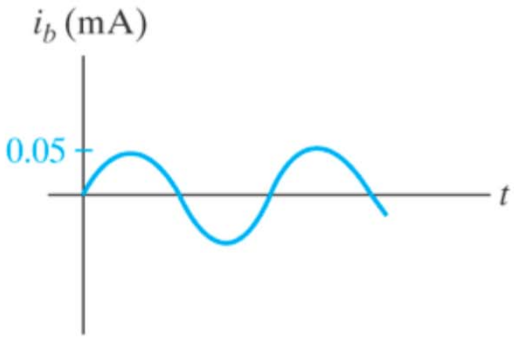
Current transfer ratio:

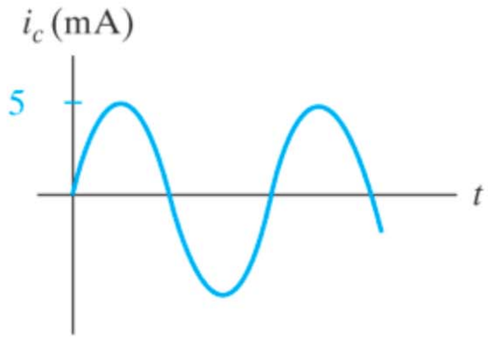
$$\frac{i_C}{i_E} = B\gamma \equiv \alpha$$

Base-to-collector current amplification factor:

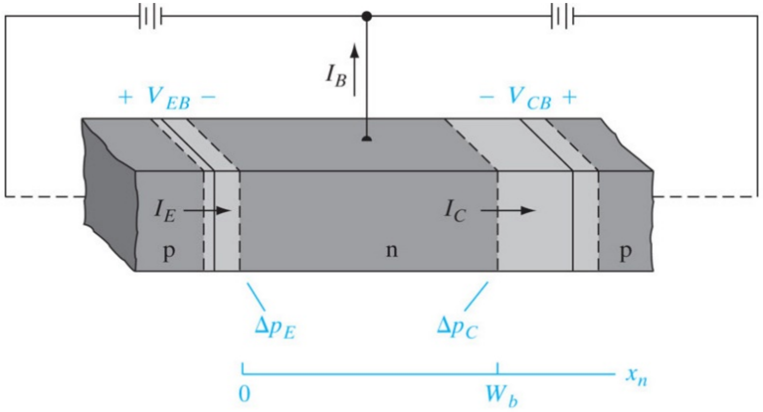
$$\frac{i_C}{i_B} = \frac{\alpha}{1-\alpha} \equiv \beta$$

$$\frac{i_C}{i_B} = \beta = \frac{\tau_p}{\tau_t}$$

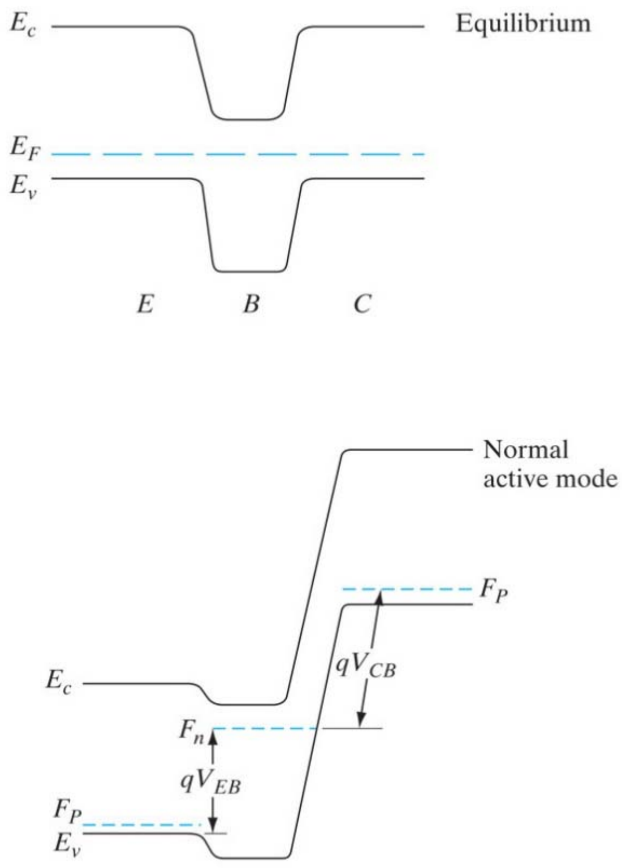




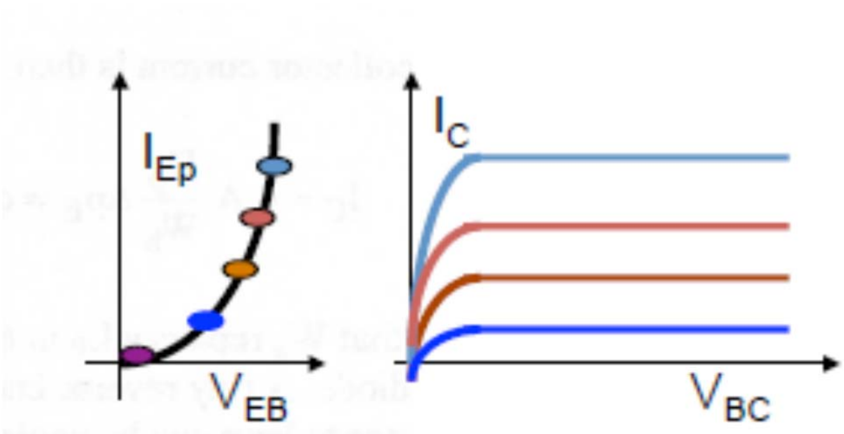
pnp transistor energy band diagram



(a)



(b)



Outline

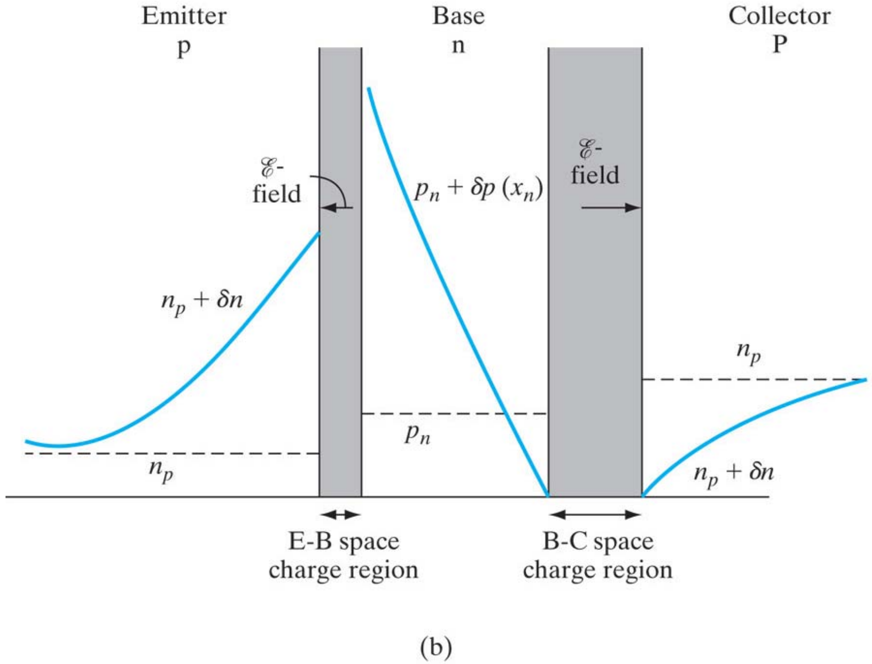
- Narrow-base diode
- Bipolar junction transistor
  - Fundamentals of BJT operation and Amplification with BJTs
  - ⇨ Minority carrier distributions and terminal currents
  - Generalized biasing
  - Switching
  - Normal mode operation
- Common-emitter amplifier and small-signal current gain

Excess carrier in the base region

Excess hole concentration in the base at emitter side and collector side:

$$\Delta p_E = p_n(e^{qV_{EB}/kT} - 1) \approx p_n e^{qV_{EB}/kT}$$

$$\Delta p_C = p_n(e^{qV_{CB}/kT} - 1) \approx -p_n$$



(b)

Diffusion equation:

$$\frac{d^2\delta p(x_n)}{dx_n^2} = \frac{\delta p}{L_p^2}$$

Solution:

$$\delta p(x_n) = C_1 e^{-x_n/L_p} + C_2 e^{x_n/L_p}$$

Boundary condition:

$$\delta p(x_n = 0) = C_1 + C_2 = \Delta p_E$$

$$\delta p(x_n = W_b) = C_1 e^{W_b/L_p} + C_2 e^{-W_b/L_p} = \Delta p_C$$

Excess hole distribution

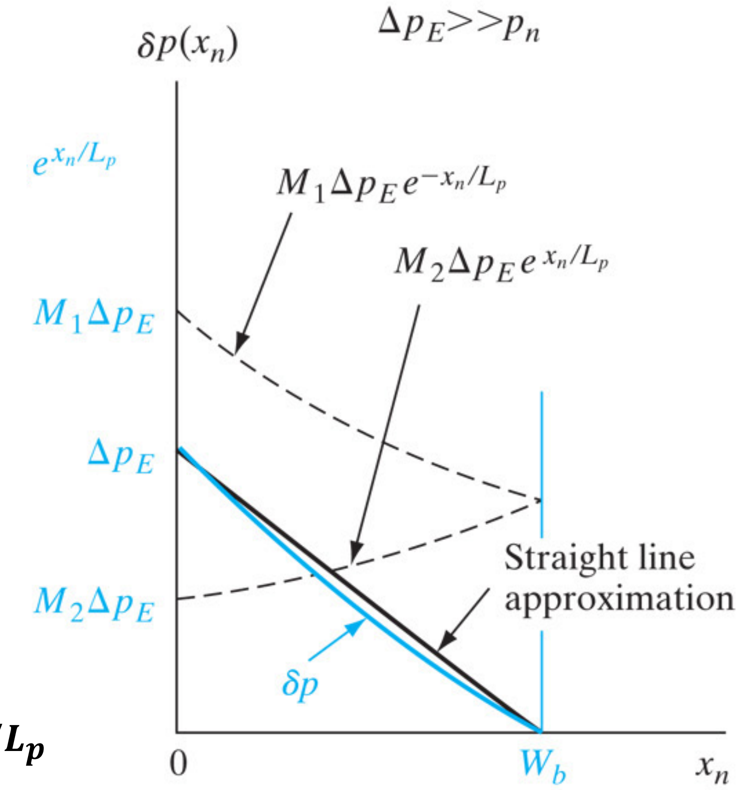
The parameters:

$$\Rightarrow \quad C_1 = \frac{\Delta p_C - \Delta p_E e^{-W_b/L_p}}{e^{W_b/L_p} - e^{-W_b/L_p}}$$

$$C_2 = \frac{\Delta p_E e^{W_b/L_p} - \Delta p_C}{e^{W_b/L_p} - e^{-W_b/L_p}}$$

Assume the collector junction is strongly reverse biased and $p_n \approx 0$, i.e. $\Delta p_C \approx 0$, then the access hole concentration distribution:

$$\delta p(x_n) = \Delta p_E \frac{e^{W_b/L_p} e^{-x_n/L_p} - e^{-W_b/L_p} e^{x_n/L_p}}{e^{W_b/L_p} - e^{-W_b/L_p}}$$



Terminal current

Hole diffusion current:

$$\boldsymbol{I_p(x_n)} = -\mathrm{qA}\boldsymbol{D_p}\frac{\boldsymbol{d\delta p(x_n)}}{\boldsymbol{dx_n}}$$

Hole component of the emitter current:

$$\boldsymbol{I_{Ep}} = \boldsymbol{I_p(x_n = 0)} = \mathrm{qA}\frac{\boldsymbol{D_p}}{\boldsymbol{L_p}}(\boldsymbol{C_2}\text{-}\boldsymbol{C_1})$$

Collector current:

$$\boldsymbol{I_C} = \boldsymbol{I_p(x_n = W_b)} = \mathrm{qA}\frac{\boldsymbol{D_p}}{\boldsymbol{L_p}}\left(\boldsymbol{C_2 e^{-W_b/L_p}} - \boldsymbol{C_1 e^{W_b/L_p}}\right)$$

Substitute $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$:

$$\boldsymbol{I_{Ep}} = \boldsymbol{I_p(x_n = 0)} = \mathrm{qA}\frac{\boldsymbol{D_p}}{\boldsymbol{L_p}}\left[\frac{\Delta \boldsymbol{p_E}\left(\boldsymbol{e^{W_b/L_p}} + \boldsymbol{e^{-W_b/L_p}}\right) - \Delta \boldsymbol{p_C}}{\boldsymbol{e^{W_b/L_p}} - \boldsymbol{e^{-W_b/L_p}}}\right]$$

Hyperbolic functions

$$\sinh(x) = \frac{1}{2}\left(e^{x} - e^{-x}\right)$$

$$\cosh(x) = \frac{1}{2}\left(e^{x} + e^{-x}\right)$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

$$\mathrm{sech}(x) = \frac{1}{\cosh(x)}$$

$$\mathrm{csch}(x) = \frac{1}{\sinh(x)}$$

$$\mathrm{ctnh}(x) = \frac{1}{\tanh(x)}$$

Terminal current

Emitter, collector and base current:

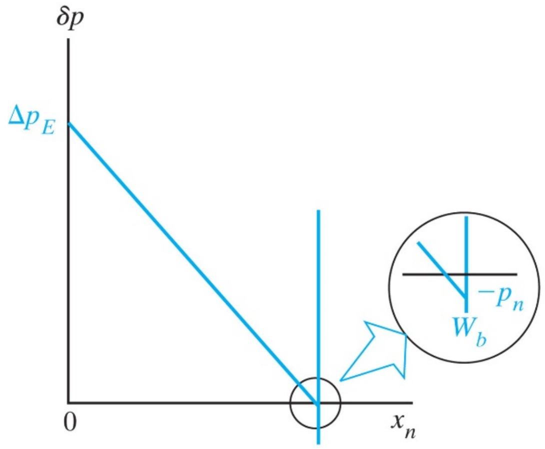
$$I_{Ep} = \text{qA}\frac{D_p}{L_p}\left(\Delta p_E\, ctnh\frac{W_b}{L_p} - \Delta p_C csch\frac{W_b}{L_p}\right)$$

$$I_C = \text{qA}\frac{D_p}{L_p}\left(\Delta p_E\, csch\frac{W_b}{L_p} - \Delta p_C ctnh\frac{W_b}{L_p}\right)$$

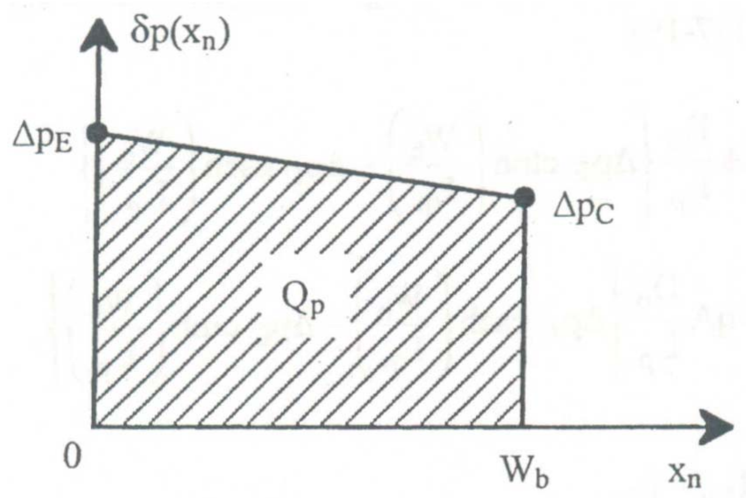
$$I_B = I_E - I_C \approx \text{qA}\frac{D_p}{L_p}\left[(\Delta p_E + \Delta p_C) tanh\frac{W_b}{2L_p}\right]$$

Various bias condition

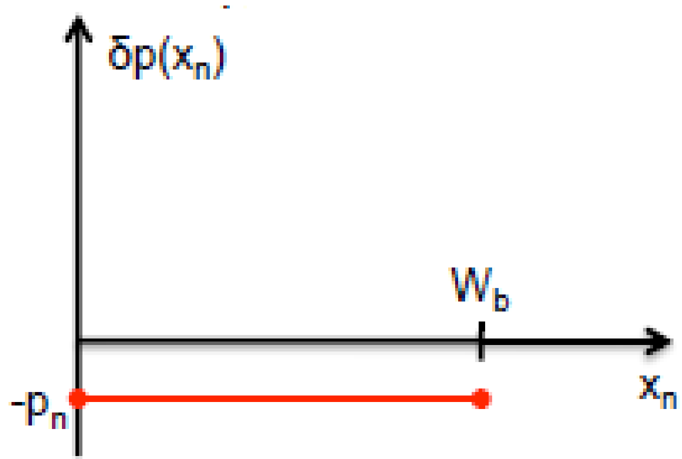
Normal bias condition:
Emitter-base forward bias
Base-collector reverse bias



Both junction forward biased



Both junction reverse biased



Normal bias condition

- If the collector junction is reverse biased and $\boldsymbol{p_n \approx 0}$, i.e. $\Delta \boldsymbol{p_C \approx 0}$, then the terminal currents are:

$$\boldsymbol{I_E} = \mathrm{qA}\frac{\boldsymbol{D_p}}{\boldsymbol{L_p}}\left(\Delta \boldsymbol{p_E}\, \boldsymbol{ctnh}\frac{\boldsymbol{W_b}}{\boldsymbol{L_p}}\right)$$

$$\boldsymbol{I_C} = \mathrm{qA}\frac{\boldsymbol{D_p}}{\boldsymbol{L_p}}\left(\Delta \boldsymbol{p_E}\, \boldsymbol{csch}\frac{\boldsymbol{W_b}}{\boldsymbol{L_p}}\right)$$

$$\boldsymbol{I_B} = \boldsymbol{I_E} - \boldsymbol{I_C} \approx \mathrm{qA}\frac{\boldsymbol{D_p}}{\boldsymbol{L_p}}\left(\Delta \boldsymbol{p_E}\boldsymbol{tanh}\frac{\boldsymbol{W_b}}{\boldsymbol{2L_p}}\right)$$

Hyperbolic function expansion

$$\operatorname{sech} y = 1 - \frac{y^2}{2} + \frac{5y^4}{24} - \dots$$

$$\operatorname{ctnh} y = \frac{1}{y} + \frac{y}{3} - \frac{y^3}{45} + \dots$$

$$\operatorname{csch} y = \frac{1}{y} - \frac{y}{6} + \frac{7y^3}{360} - \dots$$

$$\tanh y = y - \frac{y^3}{3} + \dots$$

Normal bias condition

- Use the first order approximation, then the currents:

$$\boldsymbol{I_E} = \mathrm{qA}\frac{\boldsymbol{D_p}}{\boldsymbol{L_p}}\Delta\boldsymbol{p_E}\left(\frac{\boldsymbol{L_b}}{\boldsymbol{W_p}} + \frac{\boldsymbol{W_b}}{\boldsymbol{3L_p}}\right)$$

$$\boldsymbol{I_C} = \mathrm{qA}\frac{\boldsymbol{D_p}}{\boldsymbol{L_p}}\Delta\boldsymbol{p_E}\left(\frac{\boldsymbol{L_b}}{\boldsymbol{W_p}} - \frac{\boldsymbol{W_b}}{\boldsymbol{6L_p}}\right)$$

$$\boldsymbol{I_B} \approx \mathrm{qA}\frac{\boldsymbol{D_p}}{\boldsymbol{L_p}}\Delta\boldsymbol{p_E}\frac{\boldsymbol{W_b}}{\boldsymbol{2L_p}}$$

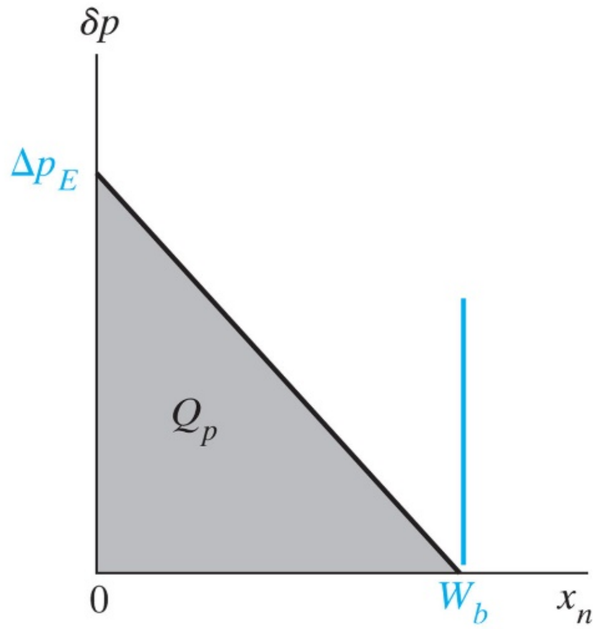
Charge control model

- The hole concentration in the base:

$$Q_p = \frac{1}{2} qA\Delta p_E W_b$$

- The base current:

$$I_B \approx \frac{Q_p}{\tau_p} = \frac{qA\Delta p_E W_b}{2\tau_p}$$

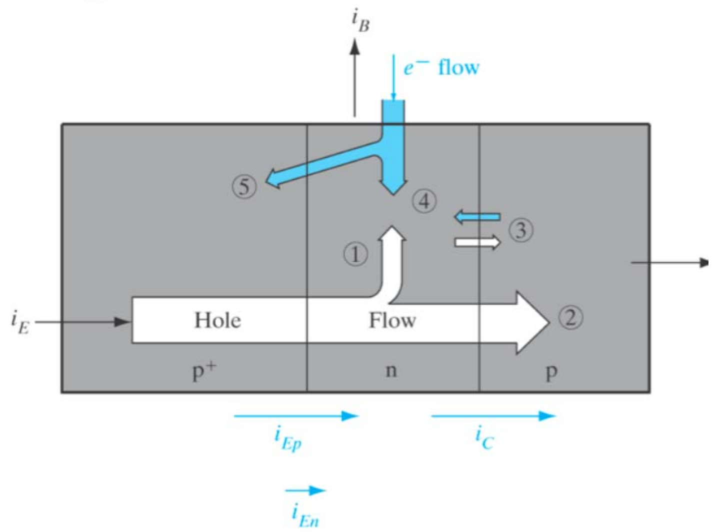


Emitter efficiency

If emitter efficiency $\gamma = \frac{i_{EP}}{i_{EP}+i_{En}} < 1$,

$$I_{En} = \mathrm{qA}\frac{D_n^P}{L_n^P} n_p e^{qV_{EB}/kT} \qquad \text{For } V_{EB} \gg kT/q$$

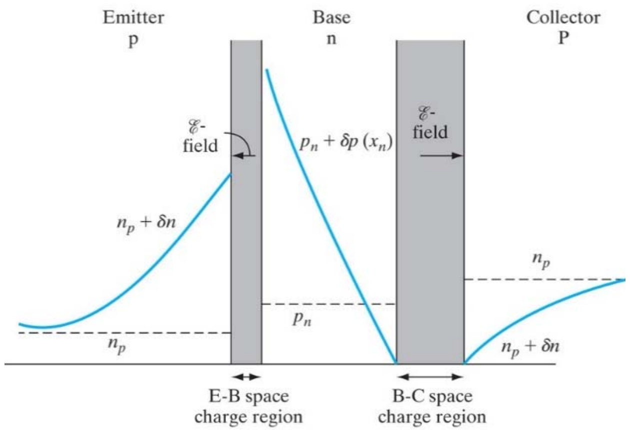
$$I_{EP} \approx \mathrm{qA}\frac{D_p^n}{L_p^n} ctnh\frac{W_b}{L_p} p_n e^{qV_{EB}/kT}$$



$$\Rightarrow \gamma = \frac{i_{EP}}{i_{EP}+i_{En}} = \left[1+\frac{i_{EN}}{i_{EP}}\right]^{-1} = \left[1+\frac{\frac{D_n^P}{L_n^P} n_p}{\frac{D_p^n}{L_p^n} p_n} tanh\frac{W_b}{L_p}\right]$$

since $\quad \frac{n_p}{p_n} = \frac{n_n}{p_p} \qquad \frac{D_n^P}{D_p^n} = \frac{\mu_n^P}{\mu_p^n}$

$$\Rightarrow \gamma = \left[1+\frac{L_p^n n_n \mu_n^P}{L_n^P p_p \mu_p^n} tanh\frac{W_b}{L_p}\right]^{-1} \approx \left[1+\frac{W_b n_n \mu_n^P}{L_n^P p_p \mu_p^n}\right]^{-1}$$



(b)

BJT parameters

Base transport factor:

$$B = \frac{I_C}{I_{Ep}} = \frac{\text{csch}\, W_B/L_p}{ctnh\, W_B/L_p} = sech\frac{W_B}{L_p}$$

Current transfer ratio:

$$\boldsymbol{\alpha = \frac{i_C}{i_E} = B\gamma}$$

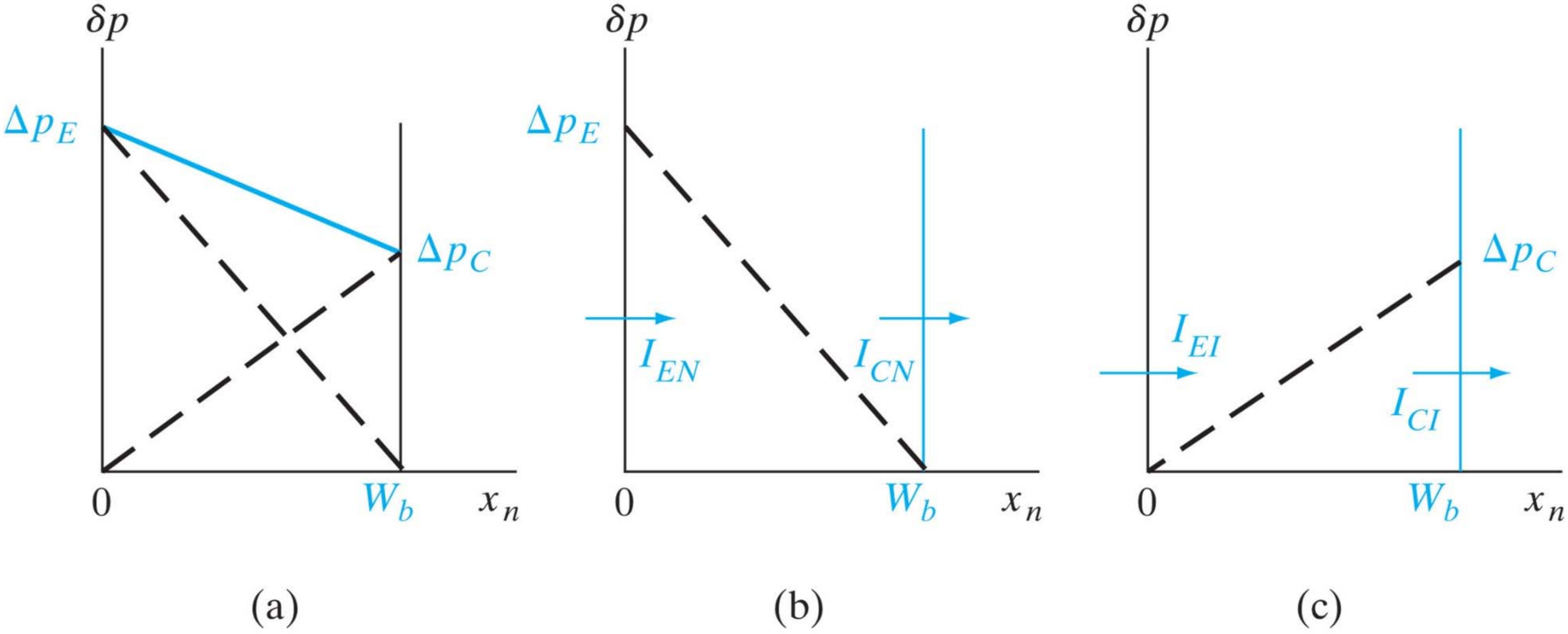
Base-to-collector current amplification factor:

$$\boldsymbol{\beta = \frac{i_C}{i_B} = \frac{\alpha}{1-\alpha}}$$

Outline

- Narrow-base diode
- Bipolar junction transistor
  - Fundamentals of BJT operation and Amplification with BJTs
  - Minority carrier distributions and terminal currents
  - ⇨ Generalized biasing
  - Switching
  - Normal mode operation
- Common-emitter amplifier and small-signal current gain

Coupled diode model



- If the both the emitter and collector junction of a transistor are forward biased, the hole distribution in the base has two component: normal mode components and inverted mode component.

Eber-Moll equation

- The total current can be abstained by superposition of the 2 components:

$$I_E = I_{EN} + I_{EI} = I_{ES}\left(\boldsymbol{e}^{\boldsymbol{qV_{EB}/kT}} - 1\right) - \alpha_I I_{CS}\left(\boldsymbol{e}^{\boldsymbol{qV_{CB}/kT}} - 1\right)$$

$$I_C = I_{CN} + I_{CI} = \alpha_N I_{ES}\left(\boldsymbol{e}^{\boldsymbol{qV_{EB}/kT}} - 1\right) - I_{CS}\left(\boldsymbol{e}^{\boldsymbol{qV_{CB}/kT}} - 1\right)$$

Where I_{Es} and I_{CS} are the magnitude of the emitter and collector saturation current with the other junction shorted
α_N and α_I are the ratios of the collected current to injected current in each mode

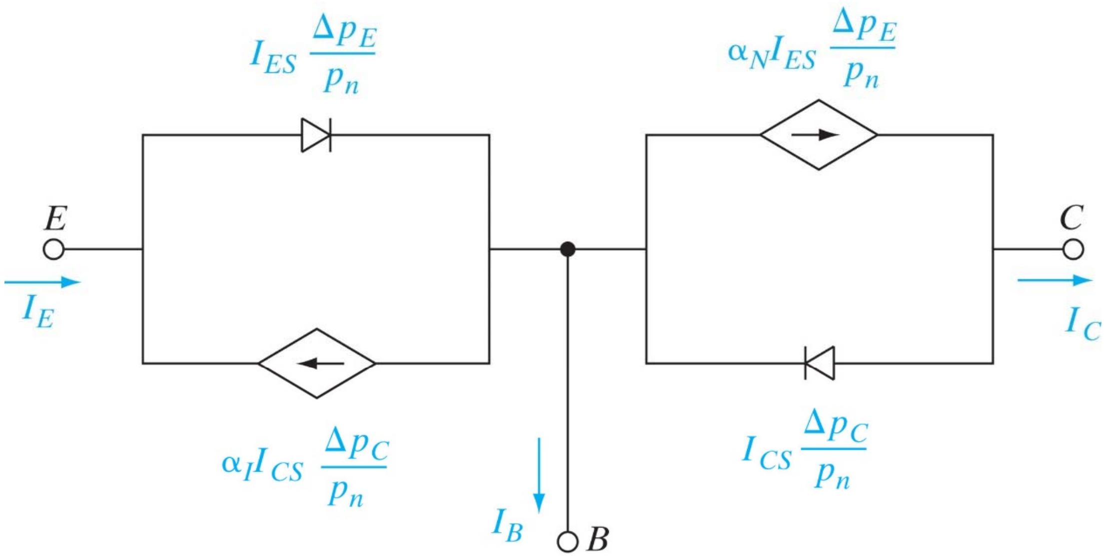
Note For Example:

$$I_{ES} \approx qA\left(\frac{D_p}{W_b}p_n + \frac{D_n^E}{L_n^E}n_p^E\right) \text{ for } W_b \ll L_p$$

Equivalent circuit synthesizing the Ebers-Moll equation

$$I_E = I_{Es}\frac{\Delta p_E}{p_n} - \alpha_I I_{CS}\frac{\Delta p_C}{p_n}$$

$$I_C = \alpha_N I_{Es}\frac{\Delta p_E}{p_n} - I_{CS}\frac{\Delta p_C}{p_n}$$

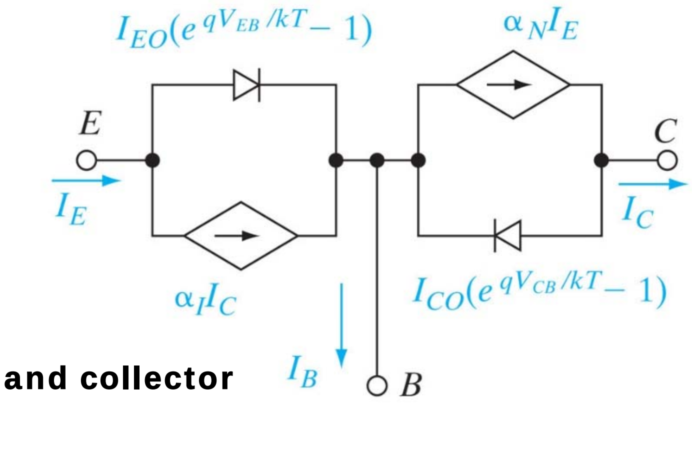


$$I_B = (1-\alpha_N)\,I_{ES}\frac{\Delta p_E}{p_n} + (1-\alpha_I)\,I_{CS}\frac{\Delta p_C}{p_n}$$

Equivalent circuit in terms of terminal current and open circuit saturation currents

$$I_E = \alpha_I I_C + I_{EO}\left(e^{qV_{CB}/kT} - 1\right)$$
$$I_C = \alpha_N I_E - I_{CO}\left(e^{qV_{CB}/kT} - 1\right)$$

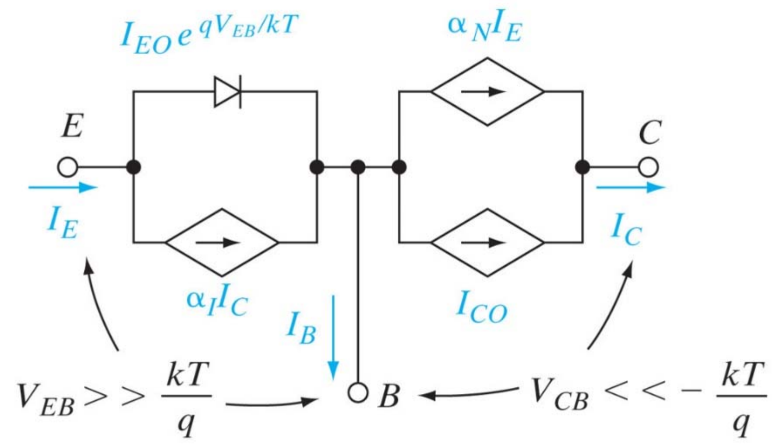
Where I_{EO} and I_{CO} are the magnitude of the emitter and collector saturation current with the other junction open.



(a)

Normal biasing:

$$I_E = \alpha_I I_C + I_{EO} e^{qV_{CB}/kT}$$
$$I_C = \alpha_N I_E + I_{CO}$$



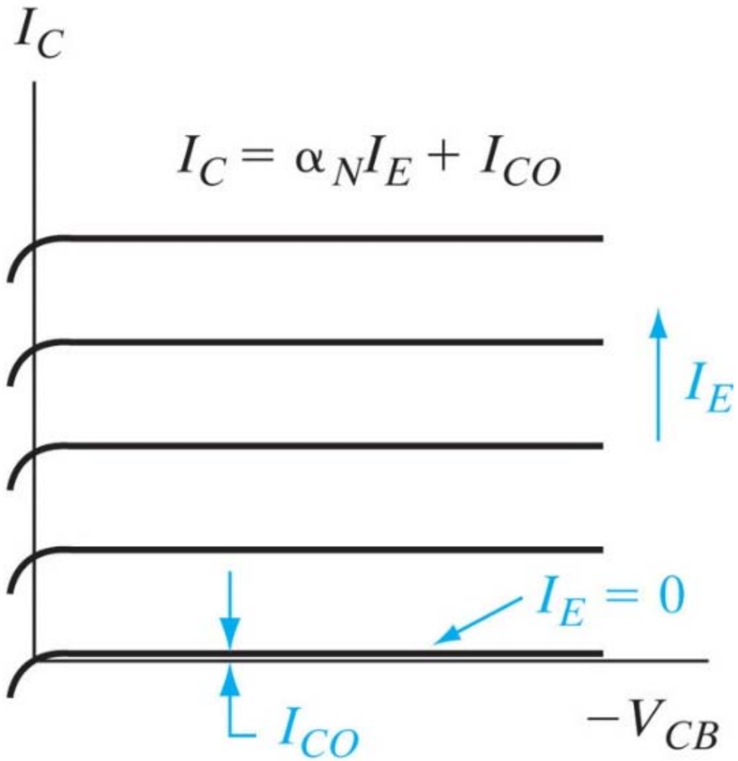
(b)

Collector characteristics with normal biasing

Normal biasing:

$$I_E = \alpha_I I_C + I_{EO} e^{qV_{CB}/kT}$$

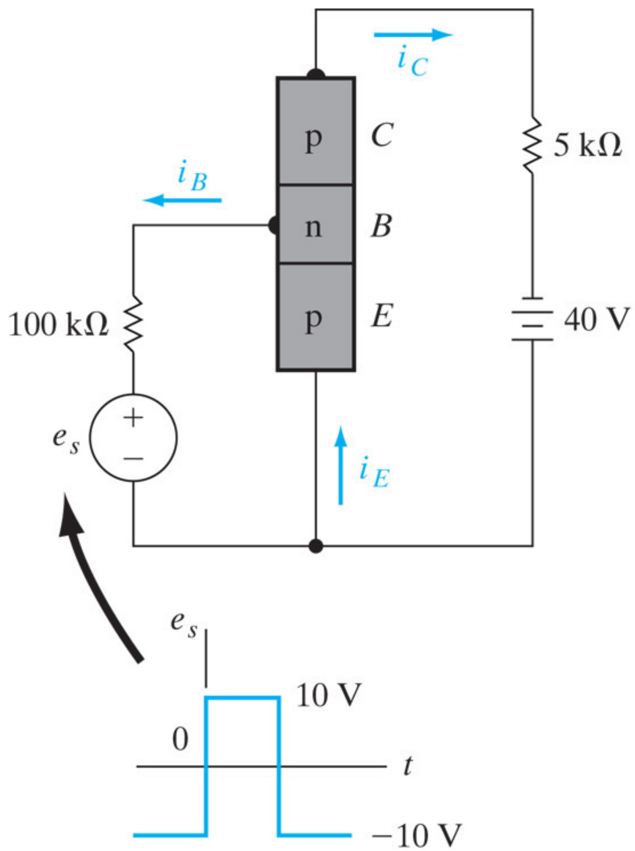
$$I_C = \alpha_N I_E + I_{CO}$$

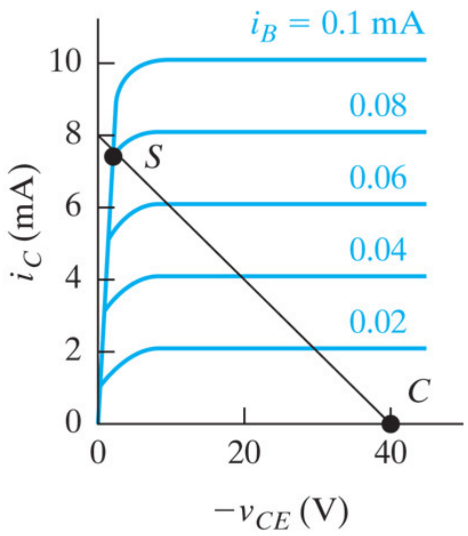


Outline

- Narrow-base diode
- Bipolar junction transistor
  - Fundamentals of BJT operation and Amplification with BJTs
  - Minority carrier distributions and terminal currents
  - Generalized biasing
  - Switching
  - Normal mode operation
- Common-emitter amplifier and small-signal current gain

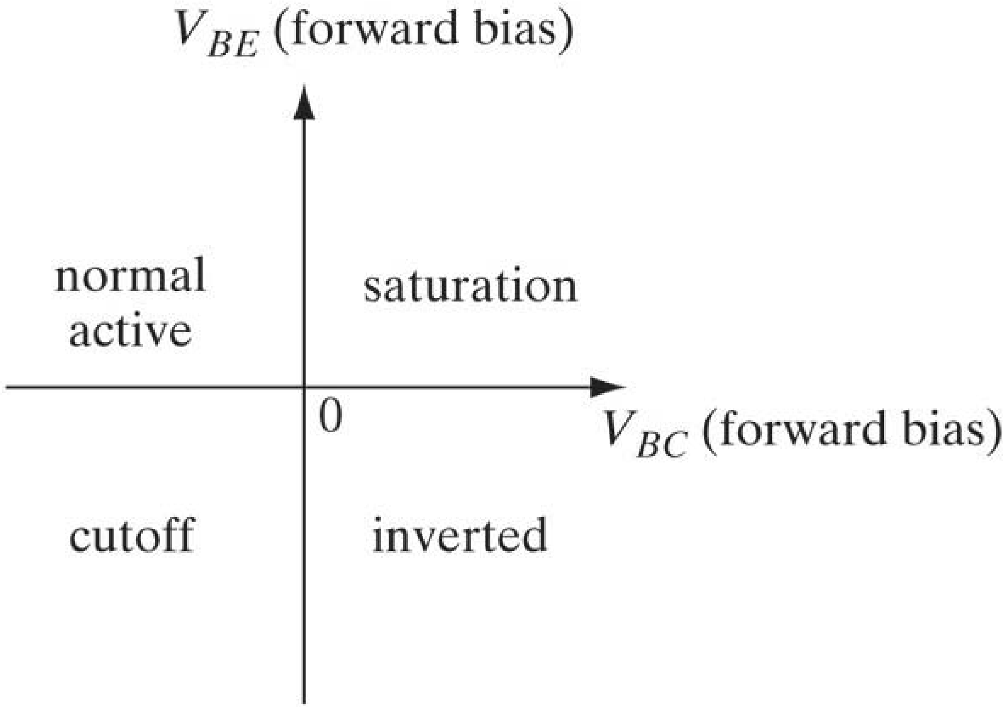
Switching circuit





- Between C and S point: normal active mode
- Point C: off state
- Point S: saturation regime

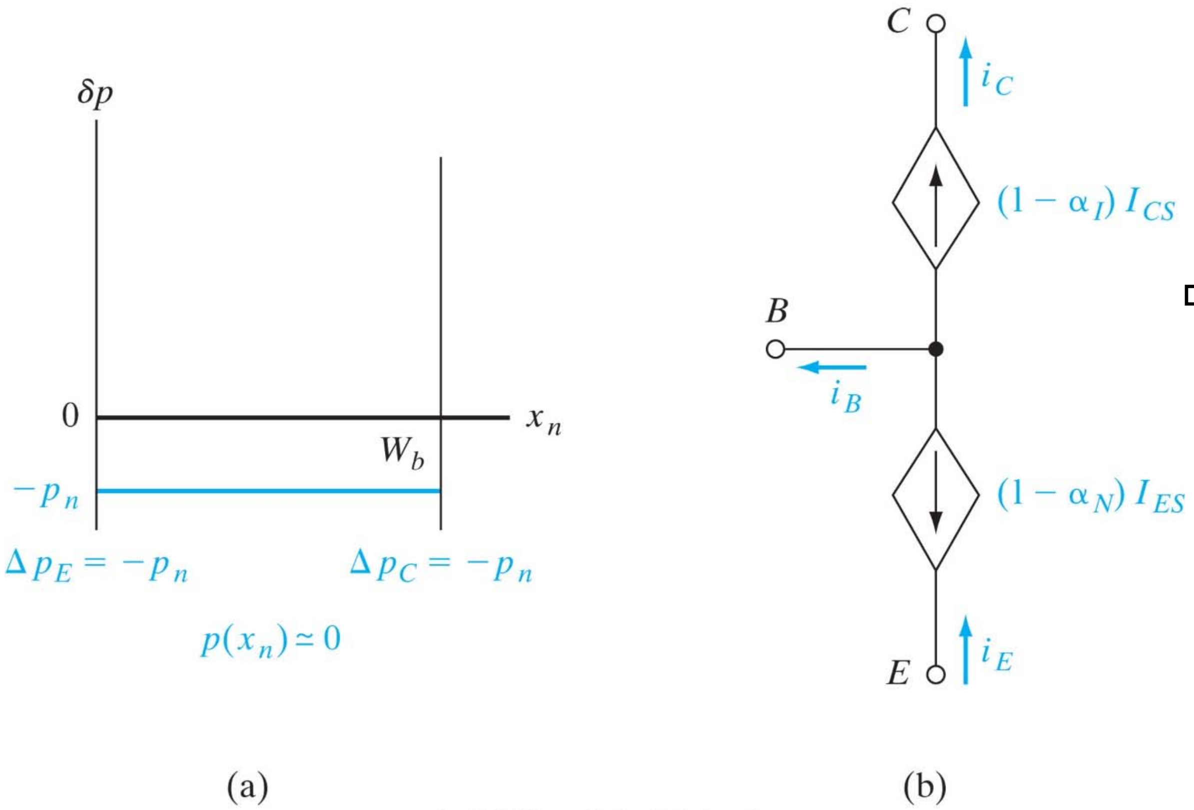
Operating regimes of a BJT



(c)

Cut-off regime

- Both junction are reverse biased:



$$\frac{\Delta p_E}{p_n} \approx \frac{\Delta p_C}{p_n} \approx -1$$

$$\Rightarrow \quad i_E = -I_{ES} + \alpha_I I_{CS} = (1-\alpha_N) I_{ES}$$

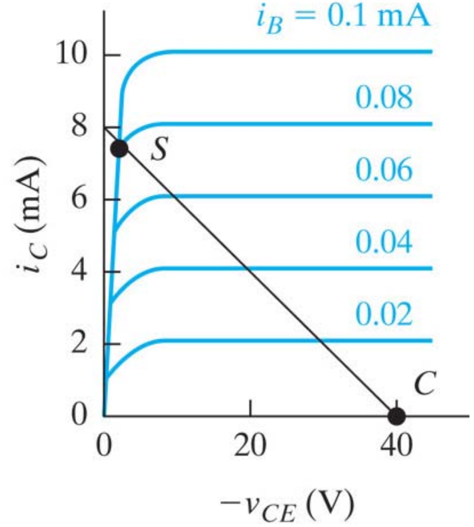
$$i_C = -\alpha_N I_{ES} + I_{CS} = (1-\alpha_I) I_{CS}$$

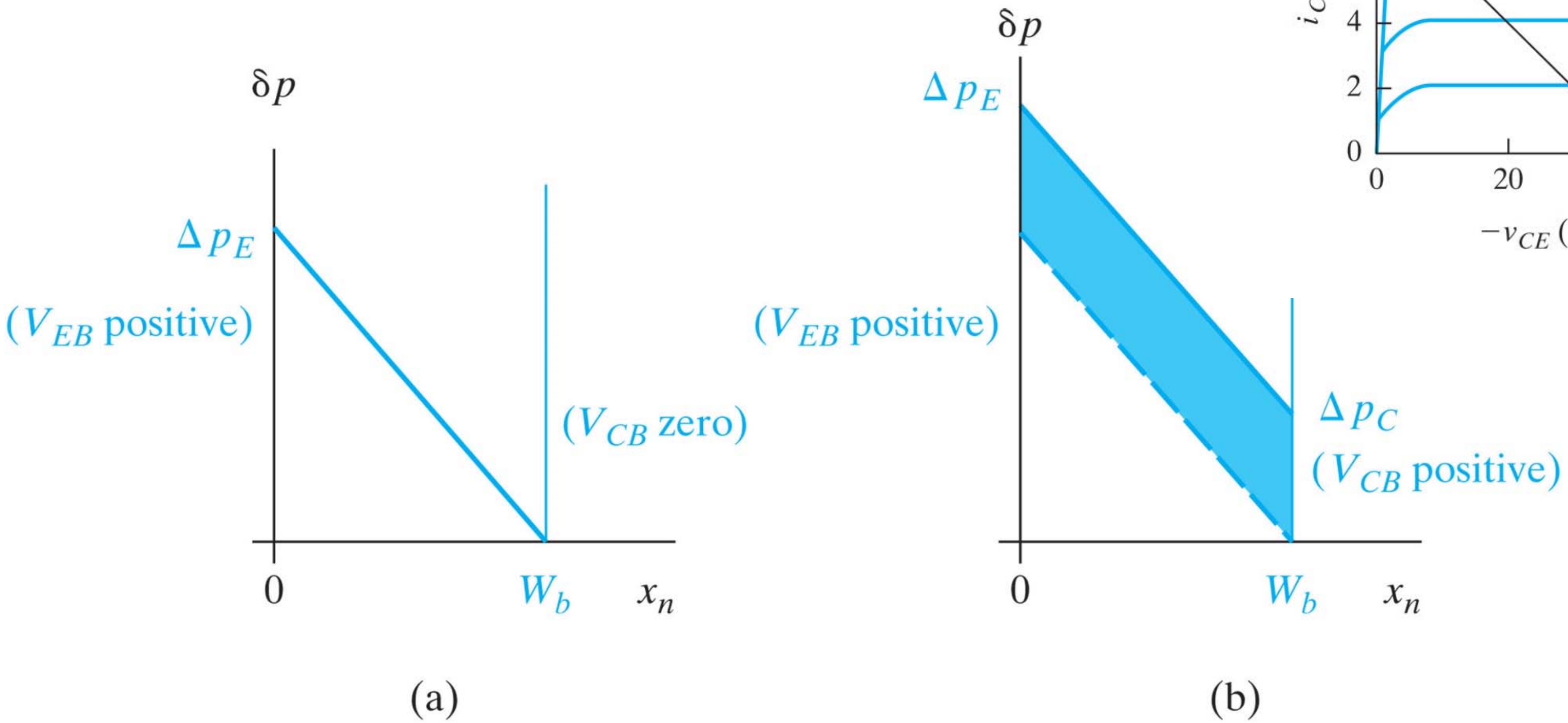
$$i_B = i_E - i_C = (1-\alpha_N) I_{ES} - (1-\alpha_I) I_{CS}$$

Note: $i_E,\ i_C,\ i_B \approx 0$

Saturation regime

- Both junction are forward biased:





- At saturation: $\Delta p_C \geq 0$
- An increase in the area under the Δp_C distribution increase i_B

Example

A symmetrical p$^+$-n-p$^+$ bipolar transistor has the following properties:

	Emitter	Base
$A = 10^{-4}\ \text{cm}^2$	$N_a = 10^{17}$	$N_d = 10^{15}\ \text{cm}^{-3}$
$W_b = 1\ \mu\text{m}$	$\tau_n = 0.1\ \mu\text{s}$	$\tau_p = 10\ \mu\text{s}$
	$\mu_p = 200$	$\mu_n = 1300\ \text{cm}^2/\text{V-s}$
	$\mu_n = 700$	$\mu_p = 450\ \text{cm}^2/\text{V-s}$

(a) Calculate the saturation current $I_{ES} = I_{CS}$.

(b) With $V_{EB} = 0.3$ V and $V_{CB} = -40$ V, calculate the base current I_B, assuming perfect emitter injection efficiency.

(c) Calculate the base transport factor B, emitter injection efficiency γ, and amplification factor β, assuming that the emitter region is long compared with L_n.

Solution

In the base,

$$p_n = n_i^2/n_n = (1.5 \times 10^{10})^2/10^{15} = 2.25 \times 10^5$$

$$D_p = 450(0.0259) = 11.66, L_p = (11.66 \times 10^{-5})^{1/2} = 1.08 \times 10^{-2}$$

$$W_b/L_p = 10^{-4}/1.08 \times 10^{-2} = 9.26 \times 10^{-3}$$

$$\begin{aligned} I_{ES} = I_{CS} &= qA(D_p/L_p)p_n \operatorname{ctnh}(W_b/L_p) \\ &= (1.6 \times 10^{-19})(10^{-4})(11.66/1.08 \times 10^{-2}) \\ &\quad (2.25 \times 10^5) \operatorname{ctnh} 9.26 \times 10^{-3} \\ &= 4.2 \times 10^{-13}\ \text{A} \end{aligned}$$

$$\Delta p_E = p_n e^{qV_{EB}/kT}, \Delta p_C \approx 0$$

$$\Delta p_E = 2.25 \times 10^5 \times e^{(0.3/0.0259)} = 2.4 \times 10^{10}$$

$$I_B = qA(D_p/L_p)\Delta p_E \tanh(W_b/2L_p)$$

or

$$I_B = \frac{Q_b}{\tau_p} = qAW_b\Delta p_E/2\tau_p = \mathbf{1.9 \times 10^{-12}\ A}$$

Solution

In the emitter,

$$D_n = 700(0.0259) = 18.13$$

$$L_n = (18.13 \times 10^{-7})^{1/2} = 1.35 \times 10^{-3}$$

$$I_{En} = \frac{qAD_n^E}{L_n^E} n_p^E e^{qV_{EB}/kT}$$

$$I_{Ep} = \frac{qAD_p^B}{L_p^B} p_n^B \operatorname{ctnh} \frac{W_b}{L_p^B} e^{qV_{EB}/kT}$$

$$\gamma = \frac{I_{Ep}}{I_{En} + I_{Ep}} = \left[1 + \frac{I_{En}}{I_{Ep}}\right]^{-1}$$

Solution

$$\gamma = \left[1 + \frac{D_n^E / L_n^E\, n_p^E}{D_p^B / L_p^B\, n_n^B} \tanh \frac{W_b}{L_p}\right]^{-1} \qquad \left(\text{use } \frac{n_p^E}{p_n^B} = \frac{n_n^B}{p_p^E}\right)$$

$$= \left[1 + \frac{18.13 \times 1.08 \times 10^{-2} \times 10^{15}}{11.66 \times 1.35 \times 10^{-3} \times 10^{17}} \tanh 9.26 \times 10^{-3}\right]^{-1} = \mathbf{0.99885}$$

$$B = \operatorname{sech} \frac{W_b}{L_p} = \operatorname{sech} 9.26 \times 10^{-3} = \mathbf{0.99996}$$

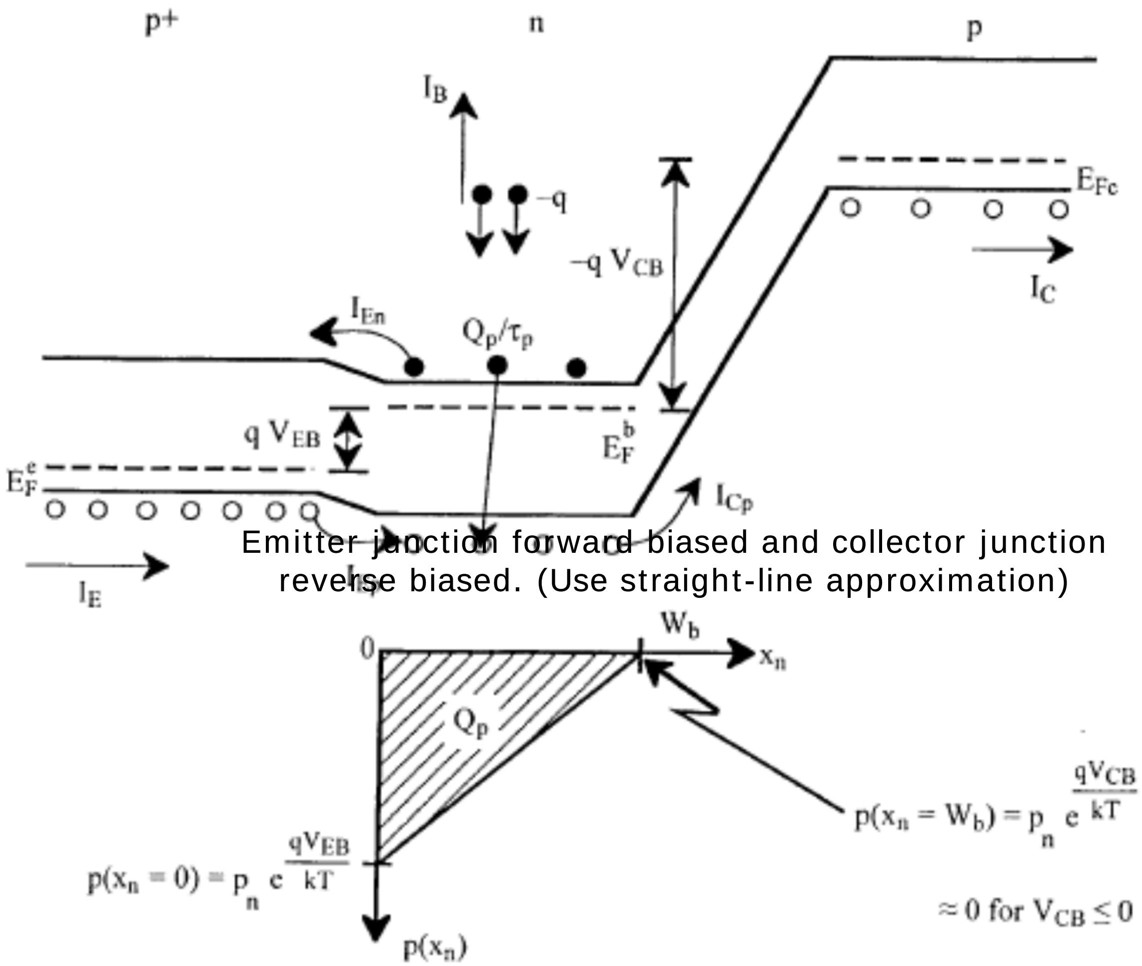
$$\alpha = B\gamma = (0.99885)(0.99996) = \mathbf{0.9988}$$

$$\beta = \frac{\alpha}{1 - \alpha} = \frac{0.9988}{0.0012} = \mathbf{832}$$

Outline

- Narrow-base diode
- Bipolar junction transistor
  - Fundamentals of BJT operation and Amplification with BJTs
  - Minority carrier distributions and terminal currents
  - Generalized biasing
  - Switching
  - ⇨ Normal mode operation
- Common-emitter amplifier and small-signal current gain

pnp normal mode



- Emitter junction forward biased and collector junction reverse biased. (Use straight-line approximation)

pnp normal mode terminal currents

- Collector current

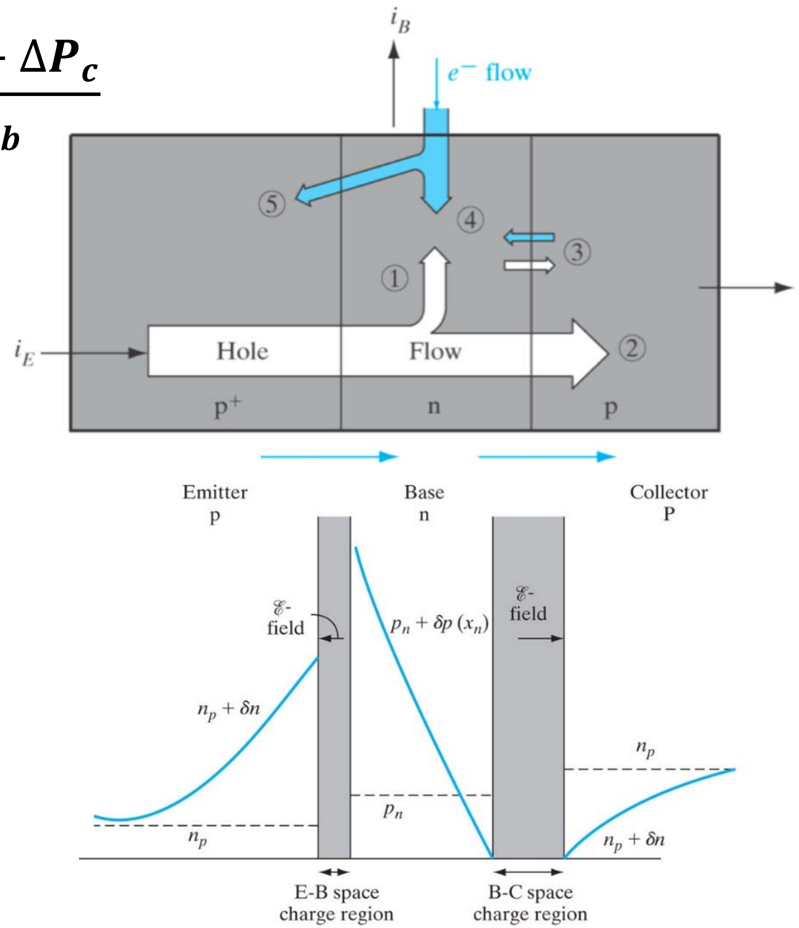
$$I_C \approx I_{Cp} = -qAD_p \frac{dp(x_n)}{dx_n}\Big|_{x_n=W_b} = qAD_p \frac{\Delta P_E - \Delta P_c}{W_b}$$

$$I_C \approx qA\frac{D_p}{W_b}\Delta p_E = qA\frac{D_p}{W_b}p_n\left(e^{qV_{EB}/KT} - 1\right)$$

- Emitter current:

$$I_E \approx I_{Ep} + I_{En} = I_{ES}\left(e^{qV_{EB}/KT} - 1\right)$$

$$\text{where } I_{ES} \approx \text{qA}\left(\frac{D_p}{W_b}p_n + \frac{D_n^E}{L_n^E}n_p^E\right)$$



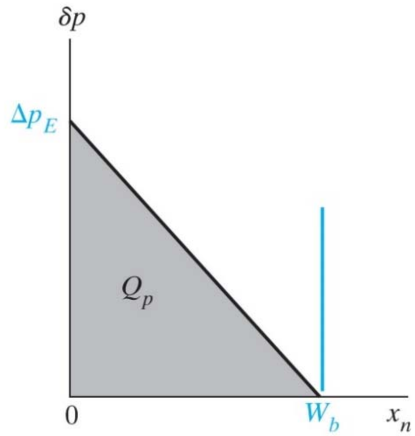
(b)

DC bias levels and terminal currents

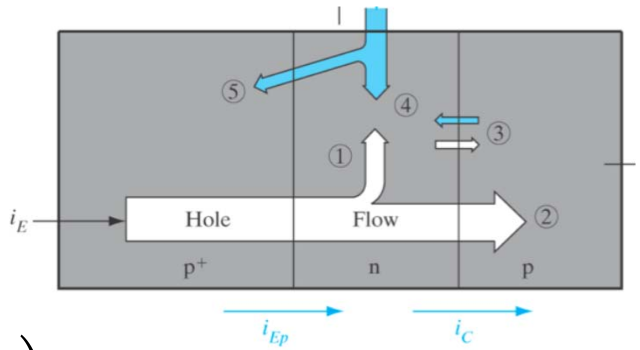
- Stored charge:

$$Q_p \approx \frac{1}{2} \mathrm{q}AW_b \Delta p_E$$

- Base current:



$$I_B(recomb.) = \frac{Q_p}{\tau_p} = \frac{\mathbf{q}AW_b}{2\tau_p} p_n\left(e^{qV_{EB}/KT} - 1\right)$$



$$I_B(inj.) = \frac{\mathbf{q}AD_n^E}{L_n^E} n_p^E\left(e^{qV_{EB}/KT} - 1\right)$$

$$I_B \approx I_B(recomb.) + I_B(inj.) = \mathbf{q}A\left(\frac{W_b}{2\tau_p} p_n + \frac{D_n^E}{L_n^E} n_p^E\right)\left(e^{qV_{EB}/KT} - 1\right)$$

- Current gain, (when emitter injection efficiency $\gamma \to 1$):

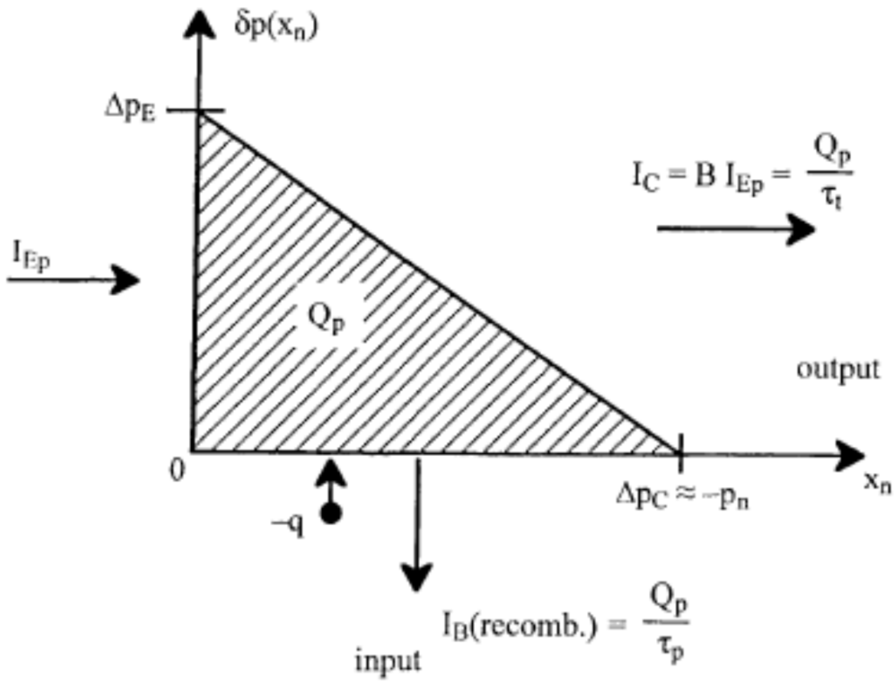
$$\beta^{\gamma \to 1} = \frac{I_C}{I_B(recomb.)} = \frac{2D_p\tau_p}{W_b^2} = \frac{2L_p^2}{W_b^2}$$

Current amplification

- Based on space-charge neutrality:

$$I_B(recomb.) = \frac{Q_p}{\tau_p}$$

$$I_C = \frac{Q_p}{\tau_t}$$



- Current gain:

$$\Rightarrow \quad \beta^{\gamma\to 1} = \frac{I_C}{I_B(recomb.)} = \frac{\tau_p}{\tau_t}$$

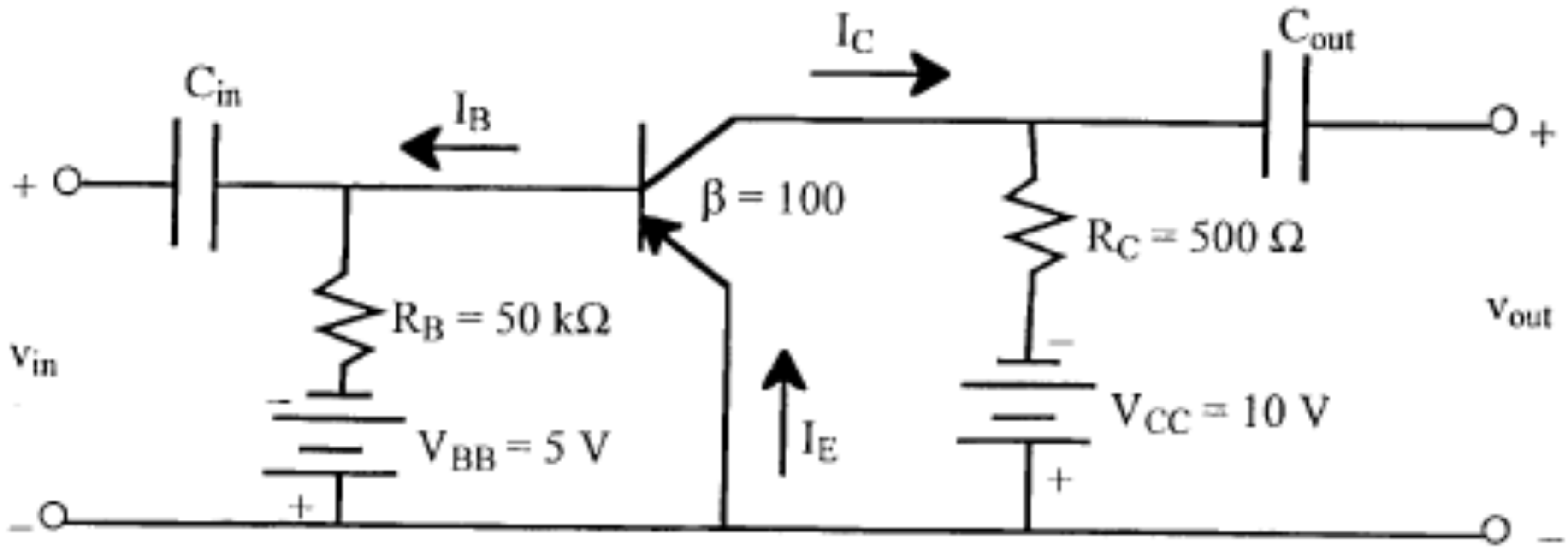
Compare with $\quad \beta^{\gamma\to 1} = \frac{2L_p^2}{W_b^2}$

$$\Rightarrow \quad \tau_t = \frac{W_b^2}{2D_p}$$

Outline

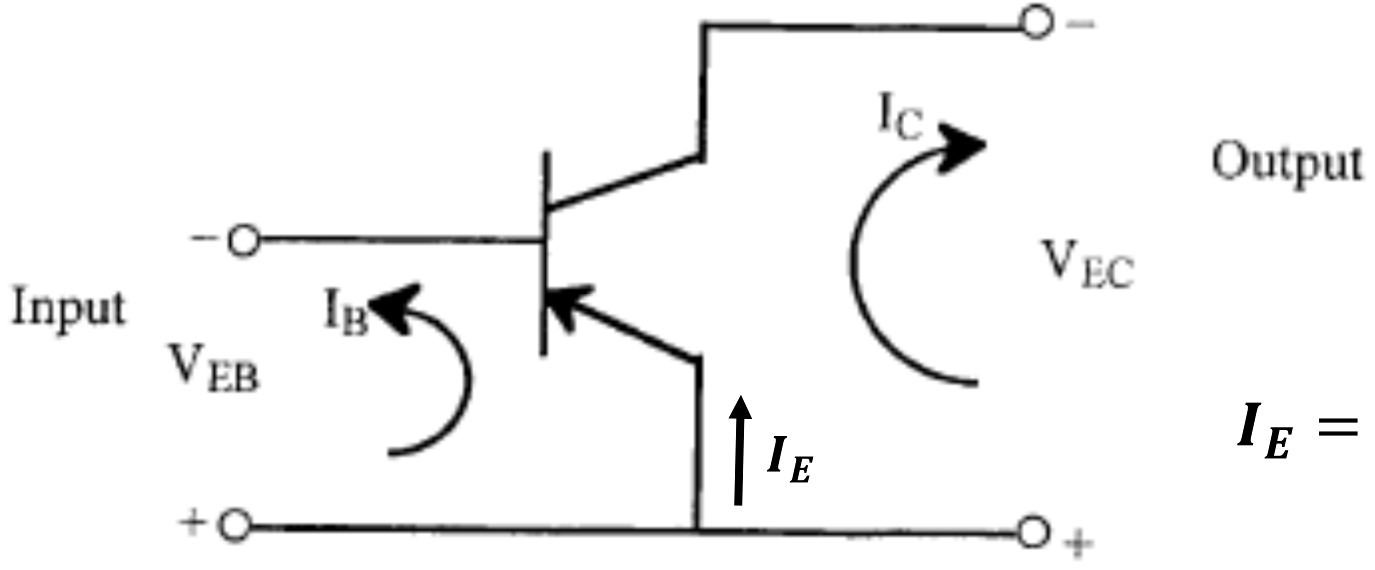
- Narrow-base diode
- Bipolar junction transistor
  - Fundamentals of BJT operation and Amplification with BJTs
  - Minority carrier distributions and terminal currents
  - Generalized biasing
  - Switching
  - Normal mode operation
- ⇨ Common-emitter amplifier and small-signal current gain

Common Emitter Amplifier



- **Coupling capacitors on the input and output**
  - **block DC, pass AC**
- **For the common-emitter amplifier, input is applied to the base and taken from the collector**

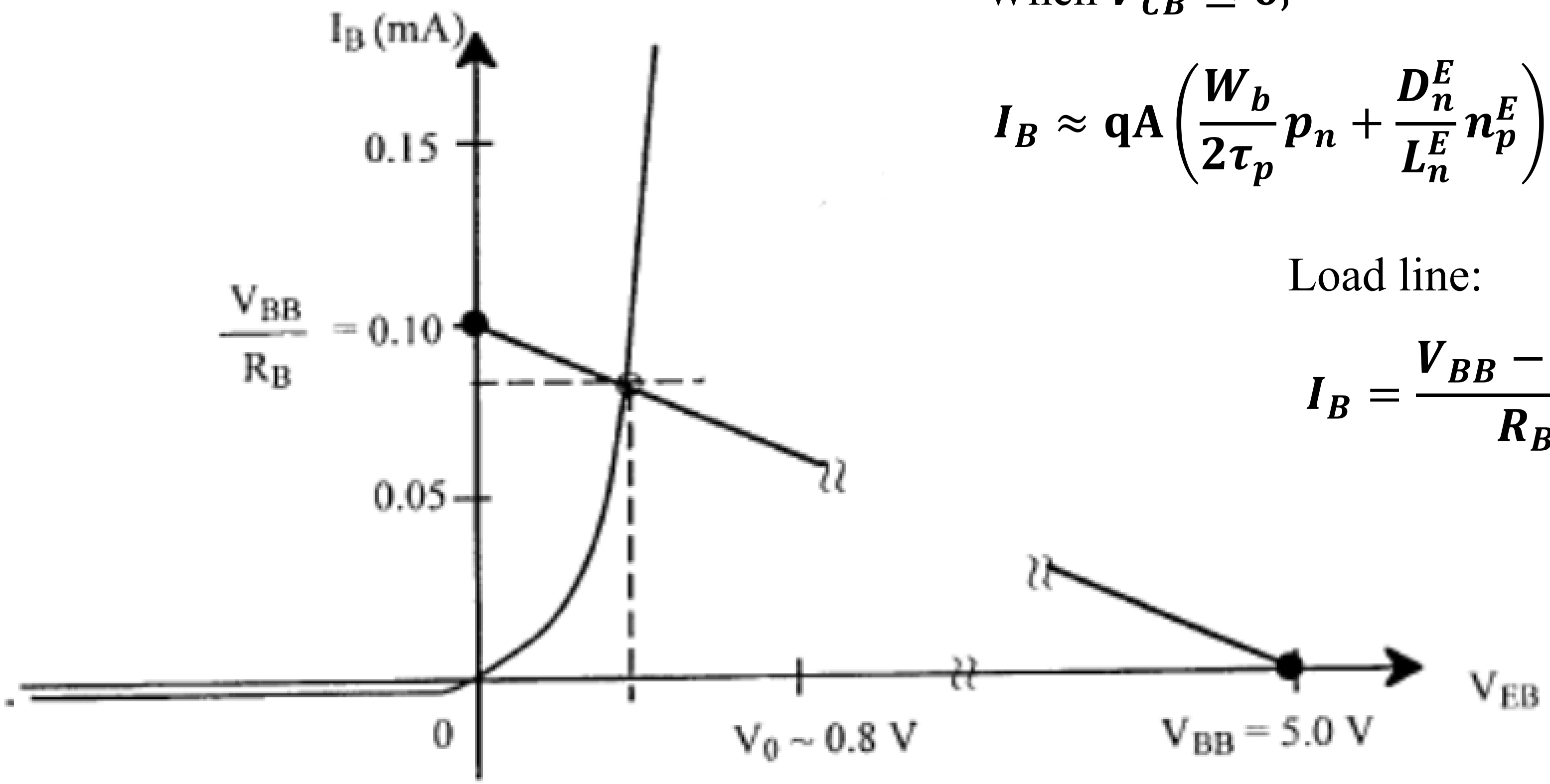
Standard notion for pnp transistor



$$I_E = I_B + I_C$$

- **Emitter identified by an arrow that points in the direction of emitter current flow**
  - **pnp, arrow points to base, holes injected into base**
  - **npn, arrow points away from base, electrons injected into base (but current is in the opposite direction)**

Input characteristic



When $V_{CB} \leq 0$,

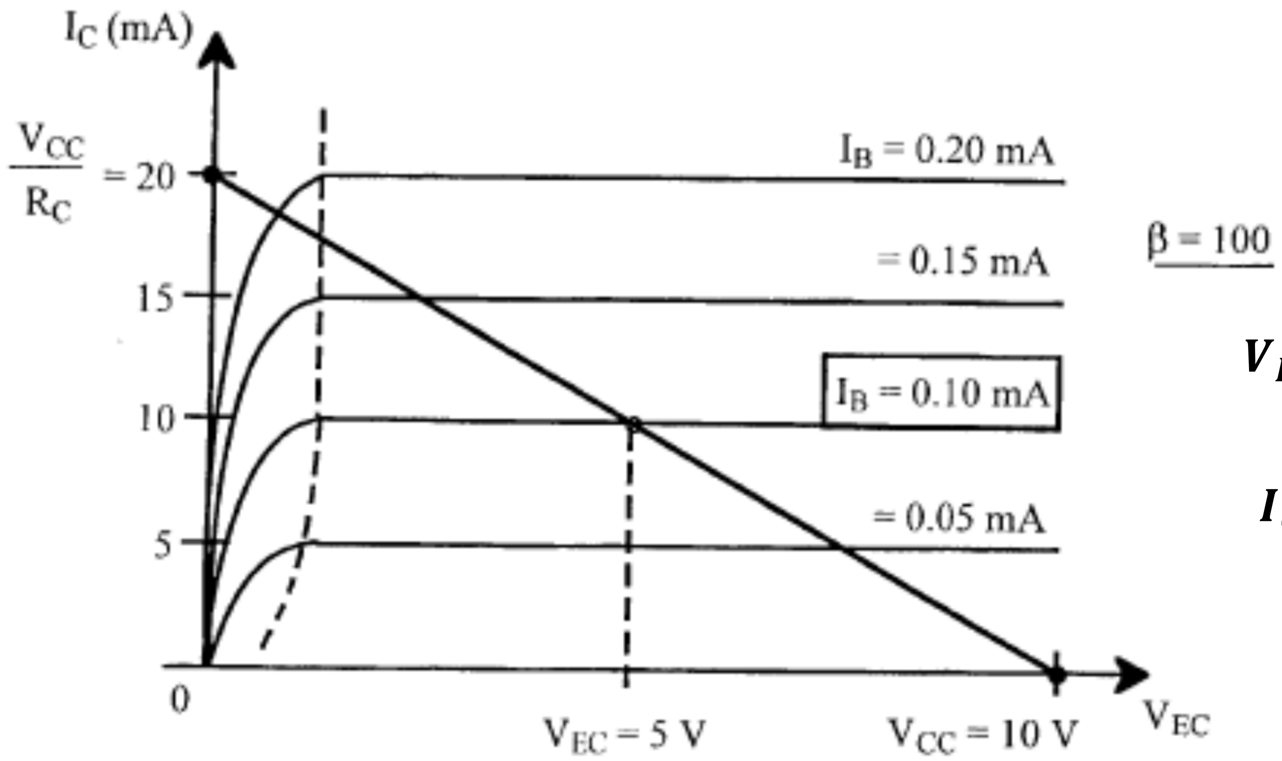
$$I_B \approx qA\left(\frac{W_b}{2\tau_p}p_n + \frac{D_n^E}{L_n^E}n_p^E\right)\left(e^{qV_{EB}/KT} - 1\right)$$

Load line:

$$I_B = \frac{V_{BB} - V_{FB}}{R_B}$$

- **Transistor biased in normal mode.**
- **Base and collector currents are independent on the reverse bias across the collector-base junction**
- **Operating point determined using load line.**

Output Characteristic



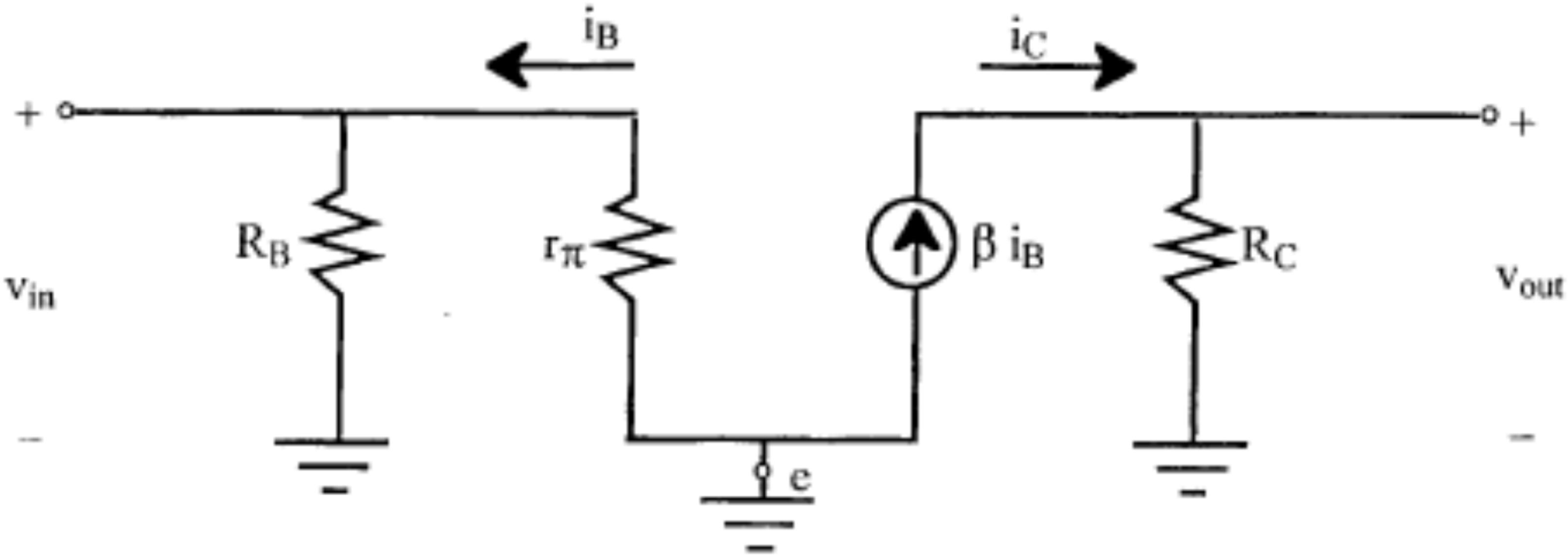
$$V_{EC} = V_{EB} - V_{CB}$$

$$I_C = \frac{V_{cc} - V_{EC}}{R_C}$$

- I_c is plotted as a function of V_{EC} for increasing base current I_B, together with the collector load line, to determine operating points.
- When $V_{EC} > 1V$, $I_c = \beta I_B$

  when V_{EC} approaches 0, V_{CB} approaches V_{EB} and reverse bias on the collector is lost and I_c falls toward zero
- Positioning the operating point midway on the family of curves help to preserve normal model operation over a larger range.

Common-emitter ac equivalent circuit



- AC equivalent circuit model treats coupling capacitor and DC voltage source as short circuits
- A positive voltage applied to the input V_{in} opposes the bias voltage
- Junction resistance r_π is differential resistance of forward-biased junction

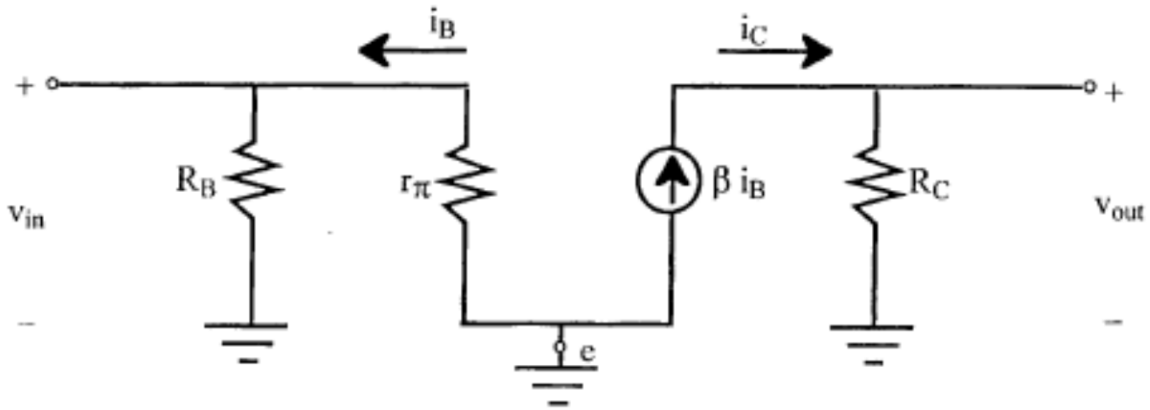
Common-emitter ac equivalent circuit

The differential conductance of the forward biased emitter junction, for a small applied ac signals $\frac{kT}{q} \approx 26mV$, :

$$\frac{\mathbf{1}}{\boldsymbol{r_\pi}} = \frac{\boldsymbol{dI_B}}{\boldsymbol{dV_{EB}}} \approx \frac{\boldsymbol{q}}{\boldsymbol{kT}} \boldsymbol{I_B}$$

The input voltage $\boldsymbol{V_{in}}$ produce a small ac modulation on the base current

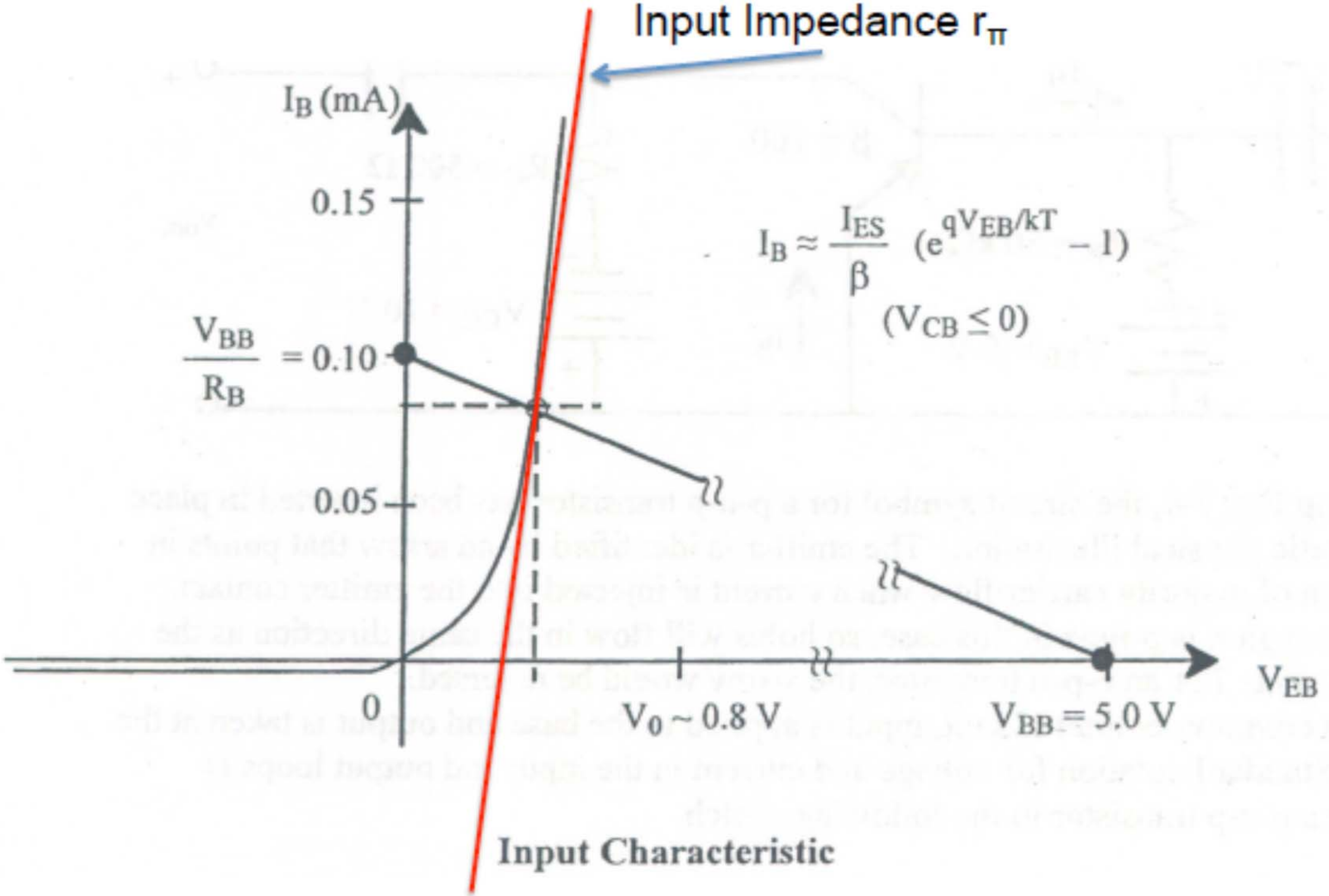
$$\boldsymbol{i_B} = -\frac{\boldsymbol{V_{in}}}{\boldsymbol{r_\pi}}$$



For example in the handout:

$$i_B = -\frac{kT/q}{r_\pi} \approx \frac{0.026V}{0.1mA} = 260\Omega$$

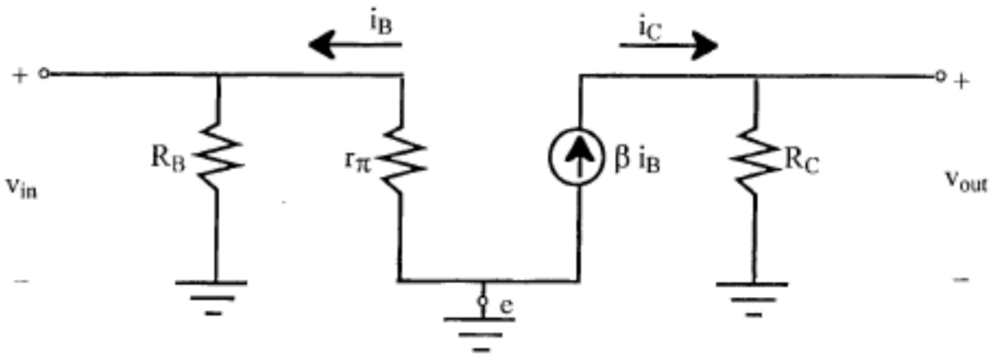
Input impedance



Common-emitter ac equivalent circuit

Induced ac component of collector current is:

$$i_C = \beta i_B = -\ \beta \frac{V_{in}}{r_\pi}$$



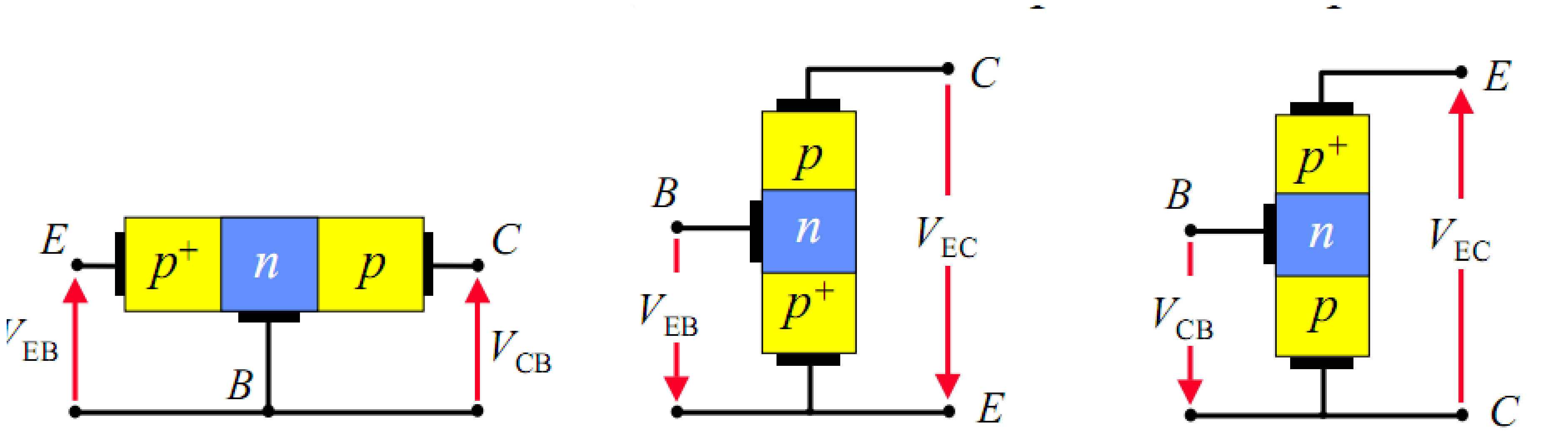
the output voltage under open circuit conditions with no load resistance:

$$V_{out} = i_C R_c = -\beta \frac{R_c V_{in}}{r_\pi}$$

Open circuit voltage gain is:

$$\frac{V_{out}}{V_{in}} = -\beta \frac{R_c}{r_\pi} \approx -100 \times \frac{500\Omega}{260\Omega} \approx -192$$

Types of Amplifier



(a) Common-base (b) Common-emitter (c) Common-collector