ECE 340 # Homework Solution #11 **Fall 2015**

DUE: FRIDAY, DECEMBER 4, 2015

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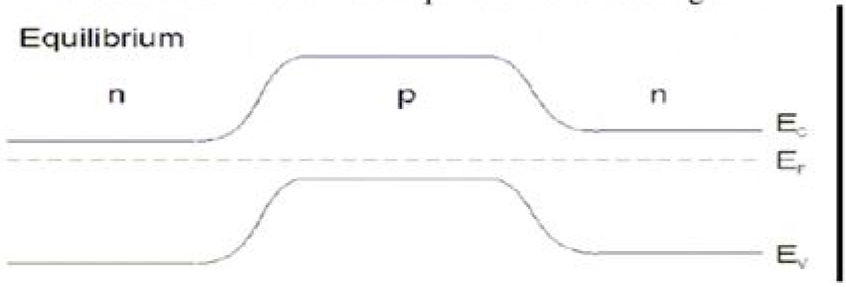
1. BIPOLAR JUNCTION TRANSISTOR (BJT) FUNDAMENTALS

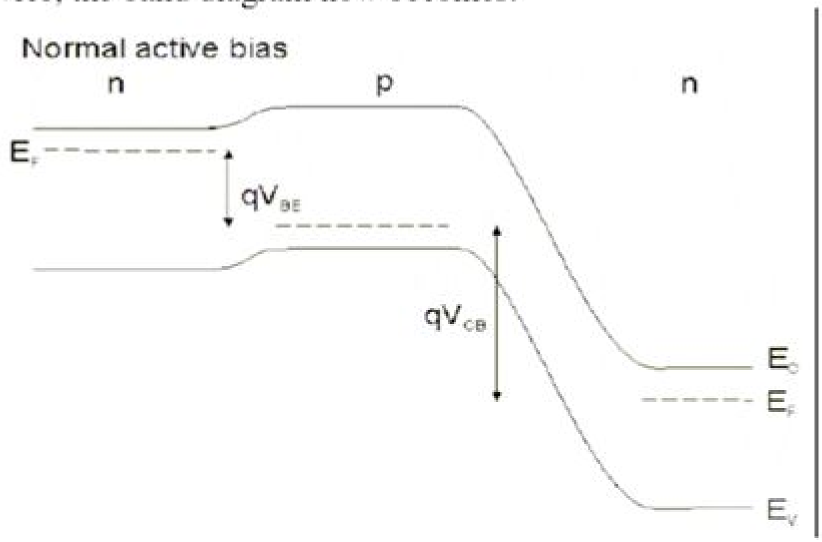
(A). Energy Band Diagram for n-p-n BJT in equilibrium

Drawing a transistor band diagram is essentially like drawing a band diagram for two p-n junctions back to back. First draw the equilibrium band diagram.



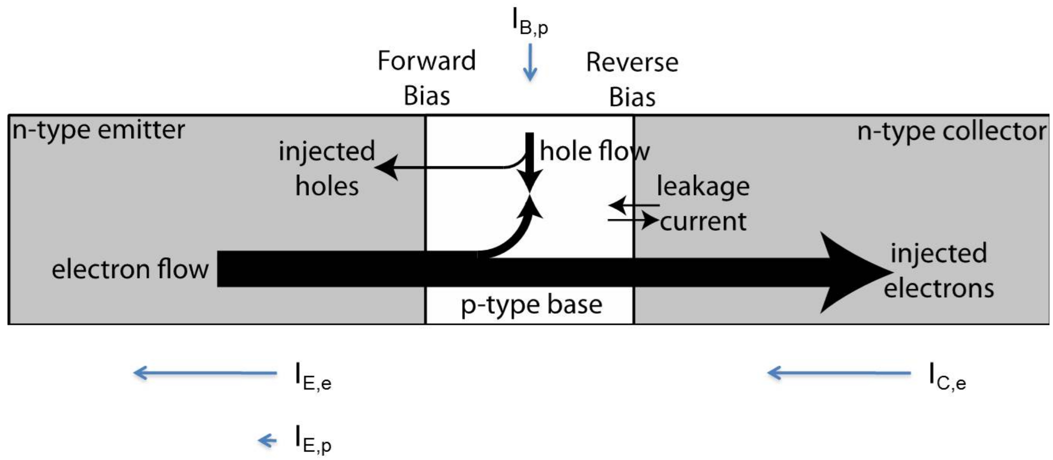
(B). Energy Band Diagram for n-p-n BJT in forward active mode

When the transistor is in active bias or "normal" mode operation, the emitter base junction is forward biased while the base collector junction is reversed biased. After biasing the device, the band diagram now becomes:



B-E junction is forward biased; B-C junction is reversed biased.
If emitter is grounded, $V_{BE} > 0$, $V_{CE} > 0$.

(C). n-p-n BJT carrier flows



(D). No it is impossible. Base current (recombination current) is never zero. There will always be some finite amount of recombination happening in the base region.

Additionally, we can interpret in terms of Eq. 7-7, where the current gain is expressed as the ratio between recombination lifetime and carrier transit time. Both of these quantities are finite. Therefore current gain cannot be made infinitely large.

(E). Electron has higher mobility than hole. Therefore n-p-n BJT is faster than p-n-p BJT.

2. BJT IN EQUILIBRIUM

(A). Emitter (n-type):

$$F_n - E_i = 0.026\, ln(\frac{1E18}{1.5E10}) = 0.467\, eV$$

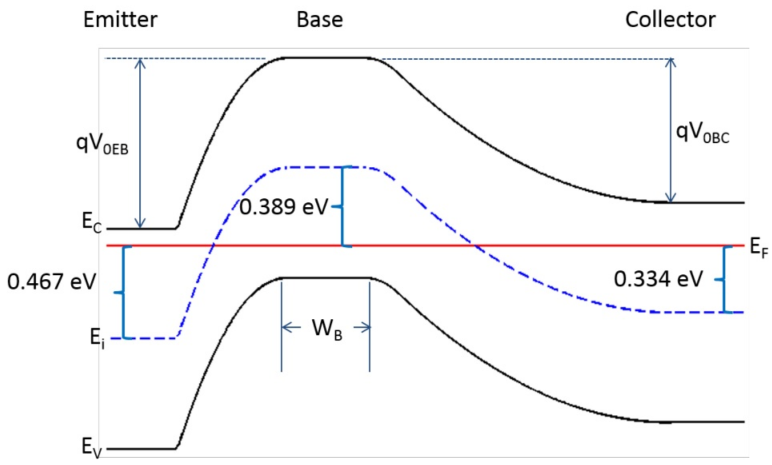
Base (p-type):

$$E_i - F_p = 0.026\, ln(\frac{5E16}{1.5E10}) = 0.389\, eV$$

Collector (n-type):

$$F_n - E_i = 0.026\, ln(\frac{6E15}{1.5E10}) = 0.334\, eV$$

Energy band diagram



(B). The build-in potential is obtained first:

$$V_{BE} = 0.467 + 0.389 = 0.856\,V$$

$$V_{BC} = 0.389 + 0.334 = 0.723\,V$$

At B-E junction:

$$W_{BE} = [\frac{2\times 11.8\times 8.85\times 10^{-14}\times 0.856}{1.6\times 10^{-19}}(\frac{1}{1\times 10^{18}}+\frac{1}{5\times 10^{16}})]^{1/2} = 0.153\,\mu m$$

The base region accounts for $0.153\times\frac{1E18}{1E18+5E16} = 0.145\,\mu m$
At B-C junction:

$$W_{BE} = [\frac{2\times 11.8\times 8.85\times 10^{-14}\times 0.723}{1.6\times 10^{-19}}(\frac{1}{6\times 10^{15}}+\frac{1}{5\times 10^{16}})]^{1/2} = 0.420\,\mu m$$

The base region accounts for $0.420\times\frac{6E15}{6E15+5E16} = 0.045\,\mu m$
The quasi-neutral base width is:

$$4 - 0.145 - 0.045 = 3.81\,\mu m$$

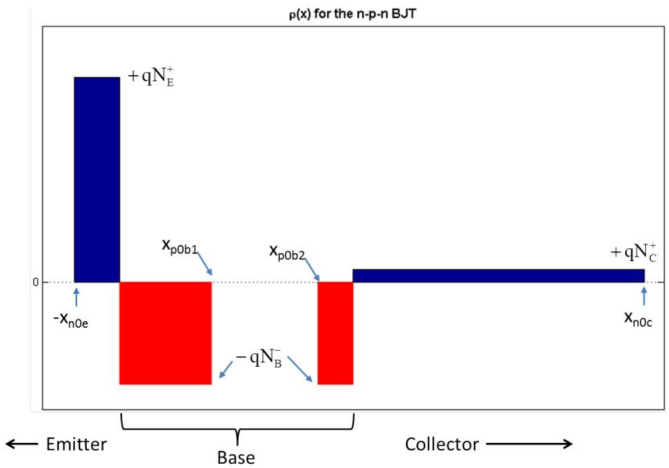
(C). The correct order is charge distribution -> electric field -> potential.

The charge distribution is straightforward. In ECE340 we made the simple approximation that the depletion region boundary is clearly defined, within which the charge distribution is uniform.

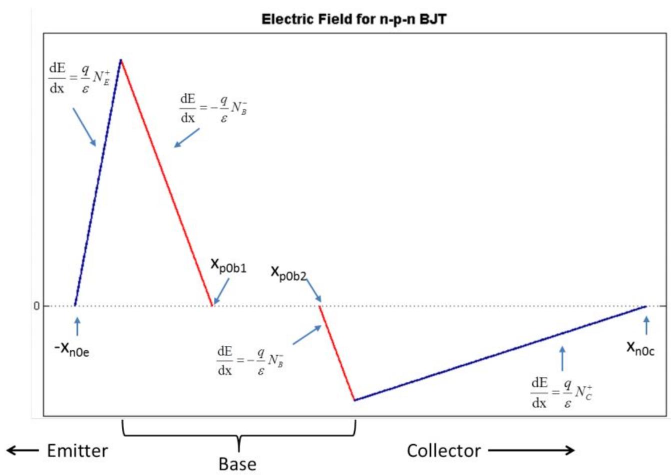
The electric field can derived from the charge distribution. It only exists in the depletion region (where the band diagram is not flat). It is a piece-wise linear function with a maximum at the junction point, which is exactly the same dielectric slab problem studied in ECE329.

The electrostatic potential (or just the potential. "Electrostatic" means the field does not change over time.) is the integral of the electric field. Thus we expect a quadratic function in the depletion region given the linearity of the electric field function.

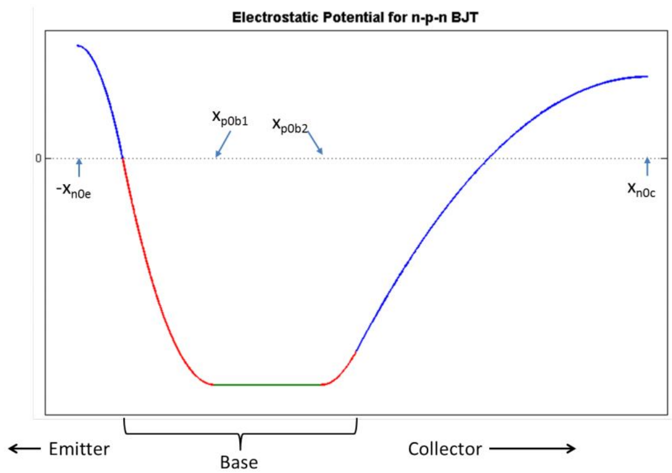
Plot of charge distribution:



Plot of electric field:



Plot of electrostatic potential:



3. BJT IN FORWARD ACTIVE MODE

(A). With bias, the Fermi levels are separated. $V_{BE} = 0.3\,V$ means the B-E junction is forward biased. $V_{BC} = -0.2\,V$ means the B-C junction is reverse biased.

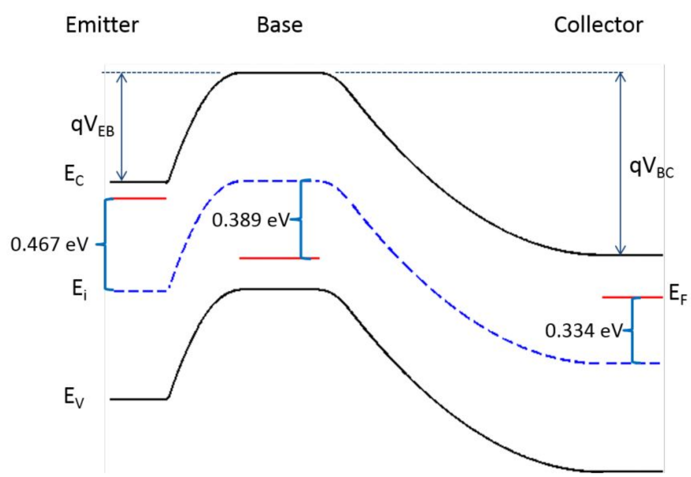
The new potential at B-E junction is:

$$V_{BE} = 0.856 - 0.3 = 0.556\,V$$

The new potential at B-C junction is:

$$V_{BC} = 0.723 + 0.2 = 0.923\,V$$

Plot of forward active mode band diagram:



(B). The calculation is similar to Problem 2.

At B-E junction:

$$W_{BE} = [\frac{2\times 11.8\times 8.85\times 10^{-14}\times 0.556}{1.6\times 10^{-19}}(\frac{1}{1\times 10^{18}}+\frac{1}{5\times 10^{16}})]^{1/2} = 0.123\,\mu m$$

The base region accounts for $0.123\times\frac{1E18}{1E18+5E16} = 0.117\,\mu m$
At B-C junction:

$$W_{BE} = [\frac{2\times 11.8\times 8.85\times 10^{-14}\times 0.923}{1.6\times 10^{-19}}(\frac{1}{6\times 10^{15}}+\frac{1}{5\times 10^{16}})]^{1/2} = 0.474\,\mu m$$

The base region accounts for $0.474\times\frac{6E15}{6E15+5E16} = 0.051\,\mu m$
The quasi-neutral base width is:

$$4-0.117-0.051 = 3.832\,\mu m$$

(C). This will be similar to Problem 2 as well. Note due to the forward bias at B-E junction, the depletion width is reduced, which leads to reduced total charge and reduced electric field. Due to the reverse bias at B-C junction, the depletion width is increased, which leads to increased total charge and increased electric field. The overall effect to the potential (if we treat emitter as ground) is that the collector is at really high potential ($0.5\,V$). This is why we call collector the "collector" (high potential to attract electrons).

Also, don't confuse the electrostatic potential with the "electron potential" (may not be the best phrase) being used in the band diagram. The band diagram is drawn according to "electron potential", i.e. electrons tend to flow from high to low. But electron has negative charge. Thus the lower you go in the band diagram, the lower you go along the electron potential, and the higher you go along the electrostatic potential.

(D). Diffusion length:

$$L_n = L_p = \sqrt{D\tau} = \sqrt{10\times 10^{-6}} = 3.16\times 10^{-3}\,cm = 31.6\,\mu m$$

We can treat is as a narrow base device. The straight-line approximation holds.

In the base region:
The minority electron in equilibrium is:

$$n_p = \frac{(1.5E10)^2}{5E16} = 4500\,cm^{-3}$$

The minority electron under injection is:

$$n_{p,BE} = n_p exp(\frac{qV_{BE}}{kT}) = 4500\times exp(\frac{0.3}{0.026}) = 4.62E8\,cm^{-3}$$

$$n_{p,BC} = n_p exp(\frac{qV_{BC}}{kT}) = 4500\times exp(\frac{-0.2}{0.026}) \approx 0\,cm^{-3}$$

Therefore the charge at BC junction is negligible.

In the emitter region:

The minority hole in equilibrium is:

$$p_{n,BE} = \frac{(1.5E10)^2}{1E18} = 225\, cm^{-3}$$

The minority hole under injection is:

$$p_{n,BE} = p_n exp(\frac{qV_{BE}}{kT}) = 225 \times exp(\frac{0.3}{0.026}) = 2.31E7\, cm^{-3}$$

Emitter current:

$$I_{En} = qA\frac{D_n}{W_B}n_{p,BE} = 1.6\times10^{-19}\times10^{-5}\times\frac{10}{3.83\times10^{-4}}\times4.62\times10^{8} = 1.93\times10^{-11}\,A$$

$$I_{Ep} = qA\frac{D_n}{L_p}p_{n,BE} = 1.6\times10^{-19}\times10^{-5}\times\frac{10}{3.16\times10^{-3}}\times2.31\times10^{7} = 1.17\times10^{-13}\,A$$

$$I_E = I_{En} + I_{Ep} = 1.942\times10^{-11}\,A$$

Base current:

$$I_{B,rec} = \frac{Q_B}{\tau_n} = qA\frac{n_{p,BE}W_B}{2}\frac{1}{\tau_n} = 1.6\times10^{-19}\times10^{-5}\times\frac{4.62\times10^{8}\times3.83\times10^{-4}}{2}\frac{1}{10^{-6}} = 1.42\times10^{-13}\,A$$

$$I_B = I_{B,rec} + I_{Ep} = 2.59\times10^{-13}\,A$$

Collector current:

$$I_C = I_{En} - I_{B,rec} = 1.916\times10^{-11}\,A$$

We can then find:

$$\beta = \frac{I_C}{I_B} = \frac{1.916\times10^{-11}}{2.59\times10^{-13}} = 74.0$$

$$\alpha = \frac{\beta}{1+\beta} = 0.9867$$

4. Short base transistor

(A). It is better to use short base.

In order to improve β, the base recombination current should be minimized. Conceptually, in a short base BJT, the carrier can transit from the emitter to the collector in a very short time, making recombination in the base region less likely to happen. Mathematically, in a short base BJT, the carrier distribution profile approximates a straight line, which is the solution to the diffusion equation when we neglect the recombination term.

(B). For a long base:

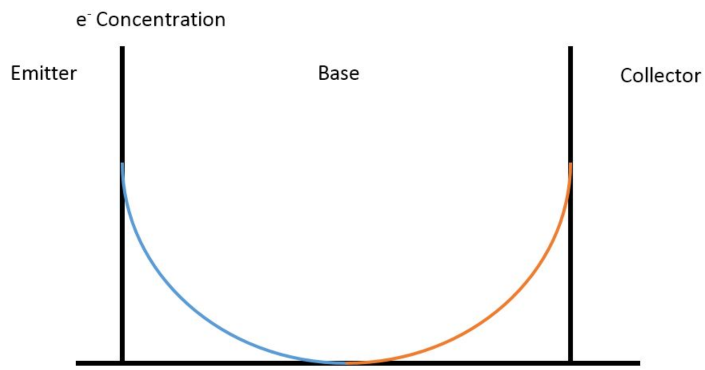
$$Q_n = qA\Delta n_p L_n[-exp(-x/L_n)]|_0^{10L_n} = qA\Delta n_p L_n[-(0)-(-e^{-0})] = qA\Delta n_p L_n$$

For a short base:

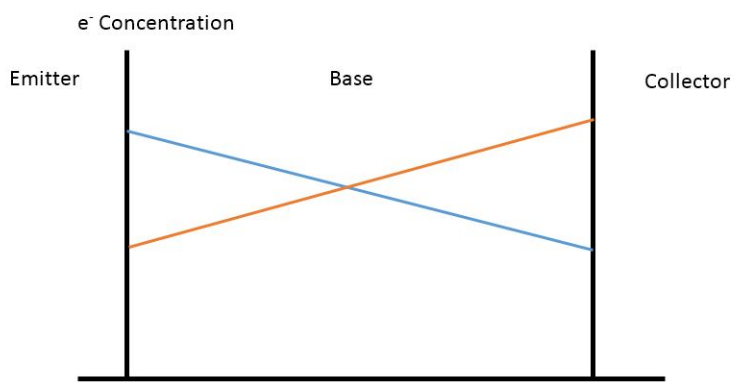
$$Q_n = qA\frac{1}{2}\Delta n_p W_B = qA\frac{1}{2}\Delta n_p \frac{L_n}{10} = \frac{1}{20}qA\Delta n_p L_p$$

The ratio is 20 : 1.

(C). Long base:



Note there are injections from both sides
Short base:



Note there are injections from both sides

(D). The advantage of low base doping is the reduced recombination in the base region. The disadvantage is that low base doping produces a large RC constant at the B-E junction, which will limit the high frequency performance.