

1. FERMI-DIRAC STATISTICS

From equation 3-21 in the textbook:

$$n_i = N_c e^{-(E_c - E_i)/kT} \quad (1.1)$$

$$p_i = N_v e^{-(E_i - E_v)/kT} \quad (1.2)$$

We know that at equilibrium, the intrinsic electron and hole concentrations are equal ($n_i = p_i$).

Starting from:

$$n_0 = N_c e^{-(E_c - E_F)/kT} \quad (1.3)$$

$$p_0 = N_v e^{-(E_F - E_v)/kT} \quad (1.4)$$

We obtain:

$$n_0 = N_c e^{-(E_c - E_F)/kT} = N_c e^{-(E_c - E_i)/kT} \times e^{-(E_i - E_F)/kT} = n_i e^{(E_F - E_i)/kT} \quad (1.5)$$

$$p_0 = N_v e^{-(E_F - E_v)/kT} = N_v e^{-(E_i - E_v)/kT} \times e^{-(E_F - E_i)/kT} = n_i e^{(E_i - E_F)/kT} \quad (1.6)$$

E_F relative to E_i for Si at room temperature (300K) ($n_0 = 10^{15} \text{ cm}^{-3}$, $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$, $kT = 0.0259 \text{ eV}$) can be calculated as follows:

$$E_F - E_i = kT \ln\left(\frac{n_0}{n_i}\right) = 0.288 \text{ eV} \quad (1.7)$$

2. INTRINSIC CARRIER CONCENTRATION VS. TEMPERATURE 1

(A). For silicon, $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$. Using the mass action law:

$$\begin{aligned} np &= n_i^2 \rightarrow p = \frac{n_i^2}{n} \\ p &= \frac{(1.5 \times 10^{10} \text{ cm}^{-3})^2}{2 \times 10^{13} \text{ cm}^{-3}} = 1.13 \times 10^7 \text{ cm}^{-3} \end{aligned} \quad (2.1)$$

(B). From Eq. 3-25a of the text, we know that

$$n = n_i e^{(E_F - E_i)/kT} \quad (2.2)$$

Rearranging to solve for the Fermi level location, we see that

$$E_F - E_i = kT \ln \frac{n}{n_i} = 0.186 \text{ eV} \quad (2.3)$$

(c). The energy band diagram for this structure is shown in Fig. 2.1.

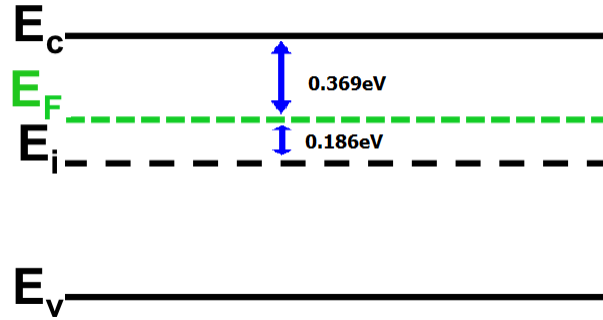


Figure 2.1: Real-space energy band diagram for n -type silicon at 300K.

(D). From Fig. 3-17, we find that $n_i = 8 \times 10^{12} \text{ cm}^{-3}$ at 400K. The higher intrinsic carrier concentration means that we can expect to have more of both electrons and holes than at 300K with the same doping level. To solve for carrier concentration, we start with the charge neutrality condition (Eq. 3-27):

$$n + N_A^- = p + N_D^+ \quad (2.4)$$

And we also know that the relationship between the equilibrium electron and hole concentrations (Eq. 3-24):

$$np = n_i^2 \rightarrow n = \frac{n_i^2}{p} \quad (2.5)$$

Substituting (2.4) into (2.5) we get the following:

$$\frac{n_i^2}{p} + N_A^- = p + N_D^+ \quad (2.6)$$

$$n_i^2 + p(N_A^-) = p^2 + p(N_D^+) \quad (2.7)$$

$$p^2 + p(N_D^+ - N_A^-) - n_i^2 = 0 \quad (2.8)$$

Applying the quadratic formula we get:

$$p = \frac{-(N_D^+ - N_A^-) \pm \sqrt{(N_D^+ - N_A^-)^2 + 4n_i^2}}{2} \quad (2.9)$$

We can assume that the doping level is roughly equal to the majority carrier concentration at 300K:

$$(N_D^+ - N_A^-) \approx n_{300K} = 2 \times 10^{13} \text{ cm}^{-3} \quad (2.10)$$

Plugging in the values for $(N_D^+ - N_A^-)$ and n_i we get:

$$p = 2.81 \times 10^{12} \text{ cm}^{-3} \quad (2.11)$$

Then we can get the Fermi level position:

$$E_F - E_i = kT \ln \frac{n_i}{p} = 0.036 \text{ eV} \quad (2.12)$$

The energy band diagram for this structure is shown in Fig. 2.2.

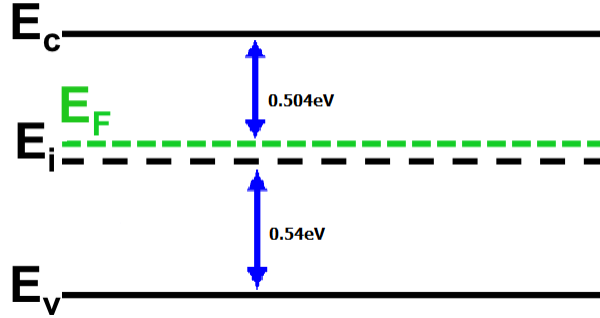


Figure 2.2: Real-space energy band diagram for *n*-type silicon at 400K.

3. INTRINSIC CARRIER CONCENTRATION VS. TEMPERATURE 2

(A). Reading from the figure, $n_{i,GaAs} = n_{i,Ge,300K}$ at $T \approx 430K$ and $n_{i,Si} = n_{i,Ge,300K}$ at $T \approx 600K$.

(B). The intrinsic carrier concentration in a semiconductor material is related to the bandgap by Eq. 3-26. Neglecting the differences in carrier effective masses, the ratio between intrinsic carrier concentrations becomes:

$$\frac{n_{iA}}{n_{iB}} = \frac{e^{-E_{gA}/2kT}}{e^{-E_{gB}/2kT}} = e^{(1.5-2.5)/(2 \times 0.0259)} = 4.13 \times 10^{-9} \quad (3.1)$$

4. CARRIER CONCENTRATION OF COMPENSATED SEMICONDUCTORS

We start with the charge neutrality condition (Eq. 3-27):

$$n_0 + N_A^- = p_0 + N_D^+ \quad (4.1)$$

And we also know that the relationship between the equilibrium electron and hole concentrations (Eq. 3-24):

$$n_0 p_0 = n_i^2 \rightarrow n_0 = \frac{n_i^2}{p_0} \quad (4.2)$$

Substituting (4.2) into (4.1) we get the following:

$$p_0 + N_D^+ = \frac{n_i^2}{p_0} + N_A^- \quad (4.3)$$

$$p_0(p_0 - N_A^- + N_D^+) = n_i^2 \quad (4.4)$$

$$p_0^2 + (N_D^+ - N_A^-)p_0 - n_i^2 = 0 \quad (4.5)$$

Since (4.5) is a quadratic with respect to p_0 , we can use the quadratic formula to find p_0 . Note that for room temperature (300K), it is a good approximation to assume that all donors and acceptors are ionized ($N_D^+ = N_D$, $N_A^- = N_A$). Afterwards, we can use (4.2) to find n_0 . Since the quadratic equation yields two roots, for every set of N_D , N_A we will have two sets of p_0 , n_0 . Only one of these two sets will satisfy the charge neutrality condition (4.1) and therefore is the correct answer.

(A). The exact solution is as follows:

$$p_0^2 + (10^{15} - 4 \times 10^{13})p_0 - (2 \times 10^{13})^2 = 0 \quad (4.6)$$

The resulting roots for p_0 are $p_{0,1} = 4.165 \times 10^{11} \text{ cm}^{-3}$ and $p_{0,2} = 9.604 \times 10^{14} \text{ cm}^{-3}$ (take the absolute value of the second root as a negative doping concentration does not make sense).

Next we find n_0 for each case:

$$n_{0,1} = \frac{n_i^2}{p_{0,1}} = \frac{(2 \times 10^{13})^2}{4.165 \times 10^{11}} = 9.604 \times 10^{14} \text{ cm}^{-3} \quad (4.7)$$

$$n_{0,2} = \frac{n_i^2}{p_{0,2}} = \frac{(2 \times 10^{13})^2}{9.604 \times 10^{14}} = 4.165 \times 10^{11} \text{ cm}^{-3} \quad (4.8)$$

Plugging the results in the charge neutrality condition:

$$n_{0,1} + N_A^- = p_{0,1} + N_D^+ \rightarrow 9.604 \times 10^{14} + 1 \times 10^{15} \neq 4.165 \times 10^{11} + 4 \times 10^{13} \quad (4.9)$$

$$n_{0,2} + N_A^- = p_{0,2} + N_D^+ \rightarrow 4.165 \times 10^{11} + 1 \times 10^{15} \approx 9.604 \times 10^{14} + 4 \times 10^{13} \quad (4.10)$$

Since the result from the second root satisfies the charge neutrality condition, we can conclude that $p_{0,2}$, $n_{0,2}$ are the correct solutions.

The approximate solution is as follows:

$$p_{approx} \approx N_A - N_D = 1 \times 10^{15} - 4 \times 10^{13} = 9.6 \times 10^{14} \quad (4.11)$$

$$n_{approx} = \frac{n_i^2}{p_{approx}} = \frac{(2 \times 10^{13})^2}{9.6 \times 10^{14}} = 4.167 \times 10^{11} \text{ cm}^{-3} \quad (4.12)$$

We can calculate the % error for the majority carrier concentration:

$$\%error = \frac{|p_0 - p_{approx}|}{p_0} \times 100\% = \frac{|9.604 \times 10^{14} - 9.604 \times 10^{14}|}{9.604 \times 10^{14}} \times 100\% = 0.042\% \quad (4.13)$$

The results for parts (A), (B), and (C) are tabulated below:

Part	$N_A \text{ (cm}^{-3}\text{)}$	$N_D \text{ (cm}^{-3}\text{)}$	$p_0 \text{ (cm}^{-3}\text{)}$	$n_0 \text{ (cm}^{-3}\text{)}$	$p_{approx} \text{ (cm}^{-3}\text{)}$	$n_{approx} \text{ (cm}^{-3}\text{)}$	% error
(A)	1×10^{15}	4×10^{13}	9.604×10^{14}	4.165×10^{11}	9.604×10^{14}	4.167×10^{11}	0.042
(B)	1×10^{14}	4×10^{13}	6.606×10^{13}	6.056×10^{12}	6×10^{13}	6.667×10^{13}	9.173
(C)	4.1×10^{13}	4×10^{13}	2.051×10^{13}	1.951×10^{13}	1.00×10^{12}	4×10^{14}	95.124

For part (A), we see that the approximation yields values very close to the exact solution (thus the approximation is valid).

For part (B), even though a % error of 9.173% may be tolerable for the hole concentration, notice how the approximation yields a hole concentration that is LESS than the electron concentration, even though the sample has more acceptors than donors (the approximation is invalid).

For part (C), the approximation yields wholly inaccurate electron and hole concentrations (the approximation is invalid).

In general, the approximation should only be used when $N_D \gg N_A$ or $N_A \gg N_D$.