ECE 340 | **Homework Solution #6** | Fall 2015

Homework Solution #6

DUE: FRIDAY, OCTOER 16, 2015

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1. DIFFUSION IN P-TYPE SI

(A).

$$D_p = \frac{kT}{q} \times \mu_p = 0.0259 \times 500 = 12.95\, cm/s$$
$$L_p = \sqrt{D_p \tau_p} = \sqrt{12.95 \times 10^{-10}} = 3.60 \times 10^{-5}\, cm$$

(B).

$$\delta p = \Delta p e^{-x/L_p} \tag{1.1}$$

$$Q_p = qA \int_0^{\infty} \Delta p e^{-x/L_p} dx = qAL_p \Delta_p$$

(C).

$$I_p(x) = -qAD_p \frac{\partial \delta_p}{\partial x} = qA \frac{D_p}{L_p} \Delta_p e^{-x/L_p} \tag{1.2}$$

Therefore, we can see that the maximum hole diffusion current occurs at x = 0. The maximum hole diffusion current is then:

$$I_p = qA \frac{D_p}{L_p} \Delta p$$

(D).

$$p(x) = p_o + \delta_p(x)$$

Assuming low level injection and $p_o = N_a$

At x = 10^{-5} cm

$$p(10^{-5} cm) = N_a + \delta_p(10^{-5} cm)$$
$$E_i - E_{fp} = kT \times ln(\frac{p(10^{-5} cm)}{n_i}) = 0.0259\ ln(\frac{1.379 \times 10^{17}}{1.5 \times 10^{10}}) = 0.417 eV$$

$$E_c - E_{fp} = 1.1/2eV + 0.415eV = 0.967eV$$

2. Doping Gradient and Energy Band Bending

(A). Since $N_o \gg n_i$

$$n(x) = N_d(x) = N_o e^{(-x/x_o)}$$

$$\frac{1}{q}\frac{\partial E_i}{\partial x} = \frac{-D_n}{\mu_n}\frac{1}{n(x)}\frac{\partial n(x)}{\partial x} \tag{2.1}$$

$$\frac{\partial E_i}{\partial x} \propto \frac{-1}{n(x)}\frac{\partial n(x)}{\partial x}$$

$$E_i \propto \int \frac{-1}{n(x)}\frac{\partial n(x)}{\partial x}dx = \frac{x}{x_o} \quad \text{linear relationship with x}$$

Alternatively, from the doping concentration:

$$n = N_0 exp(-x/x_0) = n_i exp(\frac{E_f - E_i}{KT})$$

$$\rightarrow \frac{E_f - E_i}{KT} + \frac{x}{x_0} = ln(\frac{N_0}{n_i})$$

Therefore the separation between Fermi level and intrinsic level has a linear relationship to x.
Notice that the electric field is independent of position, x, i.e. the electrical field is uniform.

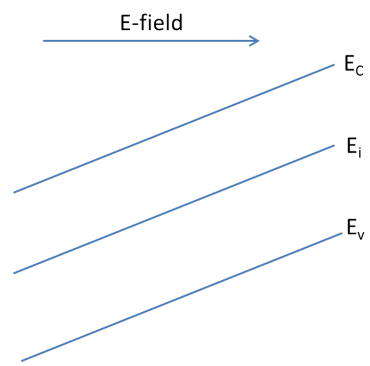

Figure 2.1: Energy Band Diagram for the given N_d

Note the direction of electric field points from the lower end to the higher end of the energy band (think of how electron/hole would flow).

(B). Since $N_o \gg n_i$, similarly

$$p(x) = N_a(x) = P_o e^{(-x/x_o)}$$

$$\frac{1}{q}\frac{\partial E_i}{\partial x} = \frac{D_p}{\mu_p}\frac{1}{p(x)}\frac{\partial p(x)}{\partial x} \tag{2.2}$$

$$\frac{\partial E_i}{\partial x} \propto \frac{1}{p(x)}\frac{\partial p(x)}{\partial x}$$

$$E_i \propto \int \frac{1}{p(x)}\frac{\partial p(x)}{\partial x}dx = \frac{-x}{x_o} \quad \text{linear relationship with x}$$

Alternatively, from the doping concentration:

$$p = N_a = P_0 exp(-x/x_0) = n_i exp(\frac{E_i - E_f}{kT})$$

$$\rightarrow \frac{E_i - E_f}{kT} = ln(\frac{P_0}{n_i}) - x/x_0$$

The electric field is uniform.

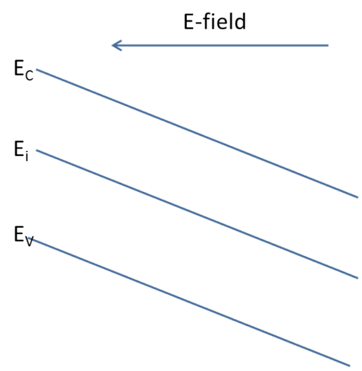

Figure 2.2: Energy Band Diagram for the given N$_a$

(C). (i)

$$n(x) = N_d(x) = -\frac{N_o}{5}\frac{x}{x_o} + N_o$$

$$\frac{1}{q}\frac{\partial E_i}{\partial x} = \frac{-D_n}{\mu_n}\frac{1}{n(x)}\frac{\partial n(x)}{\partial x} \tag{2.3}$$

$$\frac{\partial E_i}{\partial x} \propto \frac{-1}{n(x)}\frac{\partial n(x)}{\partial x}$$

$$E_i \propto \int \frac{-1}{n(x)}\frac{\partial n(x)}{\partial x}dx = -ln(5x_o - x) \quad \text{natrual log relationship with x}$$

Alternatively,

$$n = N_d = N_0 - N_0\frac{x}{5x_0} = n_i exp(\frac{E_f - E_i}{kT})$$

$$\rightarrow \frac{E_i - E_f}{kT} \propto -ln(A - Bx)$$

Then from the slope of band diagram we can extract electric field strength.

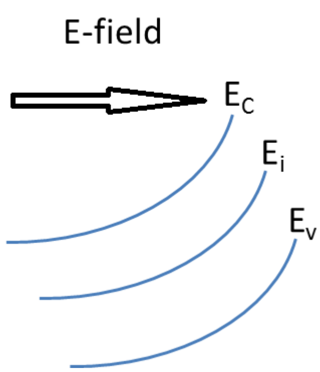

Figure 2.3: Energy Band Diagram for the given N_d

The electric field is not uniform. It is lower towards x=0 end.
(ii)

$$p(x) = N_a(x) = -\frac{P_o}{5}\frac{x}{x_o} + P_o$$

$$\frac{1}{q}\frac{\partial E_i}{\partial x} = \frac{-D_p}{\mu_p}\frac{1}{p(x)}\frac{\partial p(x)}{\partial x} \tag{2.4}$$

$$\frac{\partial E_i}{\partial x} \propto \frac{-1}{p(x)}\frac{\partial p(x)}{\partial x}$$

$$E_i \propto \int \frac{1}{p(x)} \frac{\partial p(x)}{\partial x} dx = ln(5x_o - x) \quad \text{natrual log relationship with x}$$

Alternatively,

$$p = N_a = P_0 - P_0 \frac{x}{5x_0} = n_i exp(\frac{E_i - E_f}{kT})$$

$$\rightarrow \frac{E_i - E_f}{kT} \propto ln(A - Bx)$$

Then from the slope of band diagram we can extract electric field strength.

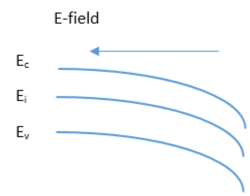

Figure 2.4: Energy Band Diagram for the given N$_a$

The electrical field is not uniform. It is higher towards x=0 end.

(D). Linear doping profile does not produce uniform electrical field. Due to the exponential dependence between energy difference and carrier density, linear doping profile does not produce straight energy band. We know electrical field is the negative gradient of potential. Applying it to energy band, the derivative of energy band diagram gives us the strength of electrical field, and the direction points from low to high. Therefore, flat energy band indicates zero electrical field. Straight but tilted energy band indicates uniform electrical field. Curved energy band indicates non-uniform electrical field.

3. THE DIFFUSION EQUATION

(A). From Eqn. 4-36 of the book:

$$\delta p = \Delta p e^{-x/L_p}$$

The calculation of L_p requires the hole diffusion coefficient:

$$D_p = kT\mu_p/q = 0.0259\,V \cdot 600\,cm^2/V - s = 15.54\,cm^2/s$$

Therefore:

$$L_p = \sqrt{D_p \tau_p} = \sqrt{15.54\,cm^2/s \cdot 2E - 10\,s} = 5.57E - 5\,cm$$

For $x = L_p/2$:

$$\delta p(L_p/2) = 7E15 \cdot exp(-L_p/2L_p) = 7E15 \cdot e^{-1/2} = 4.25E15\, cm^{-3}$$

For $x = 10 \cdot Lp$:

$$\delta p(10L_p) = 7E15 \cdot exp(-10L_p/L_p) = 7E15 \cdot e^{-10} = 3.18E11\, cm^{-3}$$

(B). To calculate the current due to the diffusion of holes, use Eqn. 4-40 from the book:

$$J_p(x) = I_p(x)/A = q\frac{D_p}{L_p}\delta p(x)$$

So:

$$I_p(x) = qA\frac{D_p}{L_p}\delta p(x)$$

For $x = L_p/2$:

$$I_p(L_p/2) = 1.6E-19 \cdot \pi \cdot \left(\frac{50E-4}{2}\right)^2 \cdot \frac{15.54}{5.57E-5} \cdot 4.25E15 = 3.725E-3\,A = 3.725\,mA$$

For $x = 10 \cdot L_p$:

$$I_p(10L_p) = 1.6E-19 \cdot \pi \cdot \left(\frac{50E-4}{2}\right)^2 \cdot \frac{15.54}{5.57E-5} \cdot 3.18E11 = 2.787E-7\,A = 278.7\,nA$$

(C). Beginning with the diffusion equation for holes (Eqn. 4-33b in the book):

$$\partial\delta p_n/\partial t = D_p\partial^2\delta p_n/\partial x^2 - \delta p_n \tau_{pn}$$

We first note that since steady state conditions are assumed:

$$\partial\delta p_n/\partial t$$

Also, since we were told to neglect generation-recombination effects:

$$\delta p_n/\tau_{pn} \to 0$$

Leaving us with:

$$\partial^2\delta p_n/\partial x^2 = 0$$

The general solution to this equation is:

$$\delta p_n(x) = A + Bx$$

Applying the boundary condition: $\delta p_{n(}0) = \Delta p_n$:

$$\Delta p_n = A$$

Applying the second boundary condition:

$$0 = \Delta p_n + BL \rightarrow B = -\Delta p_n / L$$

We finally see that the solution of the diffusion equation under these conditions is:

$$\delta p_n(x) = \Delta p_n - \frac{\Delta p_n}{L} x = \Delta p_n (1 - x/L)$$

(D). This approximation is only valid for semiconductors with lengths much smaller than the diffusion length of the carrier. If this is the case, the assumption that generation-recombination is negligible is acceptable since carriers will diffuse across the semiconductor in a time much shorter than their carrier lifetime. This approximation will be seen in the analysis of the minority carrier concentrations as a function of position in the base region of a bipolar junction transistor (BJT) in Chapter 7. However, it is important to note that an exact form in this narrow semiconductor case can be derived even if you assume a finite value of $\delta p_n / \tau_p$ (since on average there will still be some recombination events even in a very thin semiconductor layer). The functional form will be a sum of exponentials, which will also be derived in Chapter 7.

4. More on doping profile and electric field

(A). Total current (electron current, since it is n-doped) is:

$$J_n = J_n^{diff} + J_n^{drift} = qD_n \frac{dn(x)}{dx} + q\mu_n n(x) E(x) = 0$$

$$\rightarrow E(x) = -\frac{D_n}{\mu_n} \frac{1}{n(x)} \frac{dn(x)}{dx} = \frac{kT}{q} \frac{2\alpha x exp(-\alpha x^2)}{exp(-\alpha x^2)} = \frac{kT}{q} 2\alpha x$$

Note the opposite sign between $E(x)$ and $\frac{dn(x)}{dx}$.

(B). If $\alpha = 9E7\, cm^{-2}$:

$$E(x) = \frac{kT}{q} 2\alpha x = 0.026 \times 2 \times 9E7 \cdot x = 4.68E6 \cdot x\, V/cm$$

$$E(x) = 1E4\, V/cm \rightarrow x = 0.00214\, cm = 21.4\, \mu m$$

(C). The field is zero at $x = 0$ because $\frac{dn(x)}{dx} = -N_0 2\alpha x exp(-\alpha x^2) = 0$ at $x = 0$. The carrier concentration has no gradient at this point, meaning the system is static: no carrier gradient, no diffusion, no charge imbalance, no electric field.