Homework Solution #5

1. Mobility Contributions

(A). From Eqn. 3-45 of the text,

$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \dots \tag{1.1}$$

Since the problem only considers contributions from impurity scattering and phonon scattering, the expression for the total mobility is given as

$$\boxed{\mu = \left[\frac{1}{\mu_I} + \frac{1}{\mu_{AC}}\right]^{-1}} \tag{1.2}$$

At lower temperature ranges, impurity scattering dominates as $\mu_I \propto T^{3/2}$. At higher temperatures, phonon scattering dominates as $\mu_{AC} \propto T^{-3/2}$.

A sketch of mobility vs. temperature is shown in Fig. 1.1.

(B). In the low temperature range, fewer of the donors are ionized, leading to a smaller number of ionized impurities. Thus, the mobility contribution from impurity scattering increases. This effect is shown by the red line in Fig. 1.1.

(C). Since optical phonon scattering is proportional to T^{-2}, the effect is most noticeable in the high temperature range. This effect is shown by the green line in Fig. 1.1.

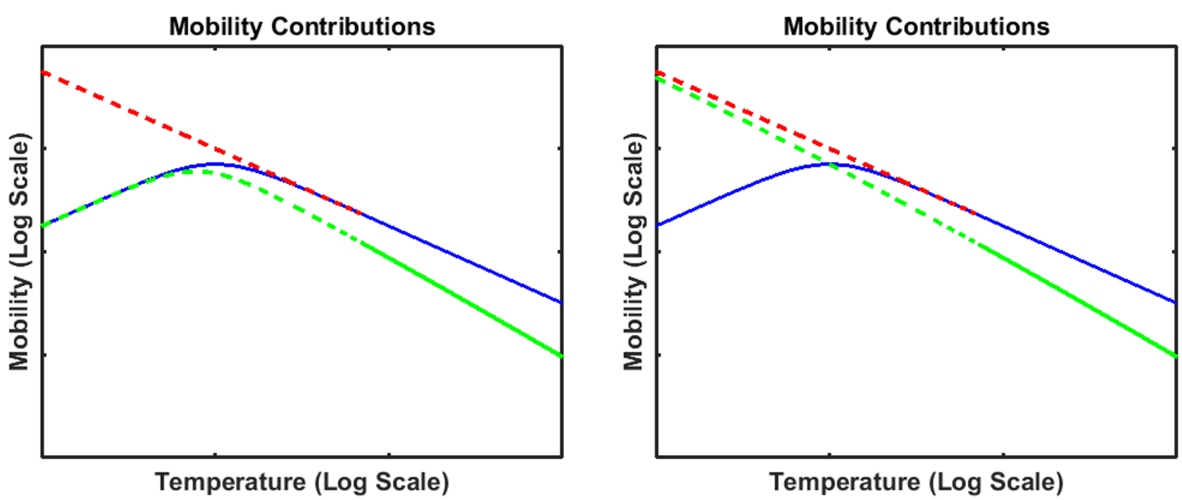

Figure 1.1: *Blue:* Sketch of mobility as a function of temperature for part (a). *Red:* Sketch of mobility vs. temperature including carrier freeze-out for part (b). *Green:* Sketch of mobility vs. temperature including optical phonons for part (c). The alternate sketch to the right is in the case the student included carrier freeze out along with the optical phonons. Both answers will be accepted.

2. Excess Carrier Recombination Times

(A). The semiconductor is p-type. This clear both because the hole concentration $p(t)$ does not change noticeably when the sample is illuminated and because the hole concentration remains high long after the excess carriers have decayed. The doping level is $N_a = 2 \times 10^{16}$ cm^{-3}.

To find the carrier lifetime, we consider the equation from Fig. 4-7 on p. 131 of the text

$$\ln \delta n = \ln \Delta n - t/\tau \tag{2.1}$$

Using the carrier concentration given in the problem at 20 μs, we find

$$\tau_n = \tau_p = 10\ \mu\text{s} \tag{2.2}$$

The optical generation rate is given by

$$g_{op} = \frac{\Delta n}{\tau_n} = 1 \times 10^{19}\ \text{cm}^{-3}\text{s}^{-1} \tag{2.3}$$

The minority carrier concentration, then, can be described by the equation

$$\delta n = 1 \times 10^{14} \exp(-t/10) \tag{2.4}$$

where t is in μs.

(B). After 10 μs, the minority carrier concentration is given by

$$\delta n = \Delta n \exp(-t/10) = \frac{1}{e} 1 \times 10^{14} = 3.69 \times 10^{13}\ \text{cm}^{-3} \tag{2.5}$$

Then the quasi-Fermi for electrons is given as

$$n = n_0 + \delta n \approx \delta n = n_i e^{(F_n - E_i)/kT} \rightarrow F_n - E_i = 0.202\ \text{eV} \tag{2.6}$$

For the quasi-Fermi level for holes, we can ignore the excess carriers because the doping is so high. Therefore,

$$p = p_0 + \delta p \approx N_a = n_i e^{(E_i - F_p)/kT} \rightarrow E_i - F_p = 0.365\ \text{eV} \tag{2.7}$$

(C). Using the equation found in part (a), we can write

$$\delta n = 1.5 \times 10^{10} = 1 \times 10^{14} \exp(-t/10) \rightarrow t = 88.0\ \mu\text{s} \tag{2.8}$$

3. Steady-State Illumination

(A). To find the carrier lifetime, we use the relation

$$\tau_n = \tau_p = \frac{1}{\alpha_r (n_0 + p_0)} \tag{3.1}$$

Since the silicon is heavily p-type, $p_0 \approx N_a$. Therefore,

$$\tau_{n,p} = \frac{1}{1\times 10^{11}\left(1\times 10^{17}\right)} = \boxed{1\ \mu\text{s}} \tag{3.2}$$

The thermal generation rate is given as

$$g_i = \alpha_r n_i^2 = \alpha_r n_0 p_0 = \boxed{2.25\times 10^9\ \text{cm}^{-3}\text{s}^{-1}} \tag{3.3}$$

(B). The excess carrier concentration is given as

$$\delta n = \delta p = g_{op}\tau_n = \boxed{2\times 10^{19}\cdot 1\ \mu\text{s} = 2\times 10^{13}\ \text{cm}^{-3}} \tag{3.4}$$

(C). The power absorbed can be written as

$$P = g_{op}E_{photon}V = 2\times 10^{19}\left(q\frac{1.24}{0.80}\right)(0.1\ \text{cm})^2(4\ \mu\text{m}) = \boxed{19.9\ \mu\text{W}} \tag{3.5}$$

To find the fraction of power lost to heat, note that all of the photon's energy above the band gap is lost to heat as the electron settles to the edge of the conduction band. Therefore,

$$f_{th} = \frac{E_{photon} - E_g}{E_{photon}} = \boxed{28.4\%} \tag{3.6}$$

4. PHOTOCONDUCTIVITY

(A). To find the current, we must first find the electric field across the sample, given as

$$E = \frac{V}{d} = \frac{10\ \text{V}}{1\ \text{mm}} = 100\ \text{V/cm} \tag{4.1}$$

From the plot on p. 107 of the text, the mobility for electrons is given as $\mu_n = 6\times 10^3$ and the mobility for holes is $\mu_p = 3\times 10^2$. Because the hole concentration is negligible compared to the electron concentration,

$$J = q\left(\mu_n n\right) = 9612\ \text{A/cm}^2 \tag{4.2}$$

To find the total current, then,

$$I = J\times A = \boxed{19.2\ \text{mA}} \tag{4.3}$$

(B). To find the current, it is first necessary to find the excess carrier concentration

$$\delta_n = \delta_p = g_{op}\tau_{n,p} \tag{4.4}$$

Then, the current density is calculated in the same way as above, except the excess minority carrier must be included

$$J = q\left(\mu_n(n+\delta n) + \mu_p(p+\delta_p)\right)E \tag{4.5}$$

Then, the current can be found as I = 23.7 mA. Therefore, the change in current is

$$\Delta J = 23.7\text{ mA} - 19.2\text{ mA} = \boxed{4.5\text{ mA}} \tag{4.6}$$

5. Drift and Diffusion

(A). The electric field across the sample causes the holes to drift in the positive x direction and, at the same time, the holes diffuse symmetrically away from high concentration areas. Thus, as the center of the peak drifts due to the electric field, the peak decreases and and the pulse widens due to diffusion. A sketch of the pulse at the indicated times is shown in Fig. 5.1.

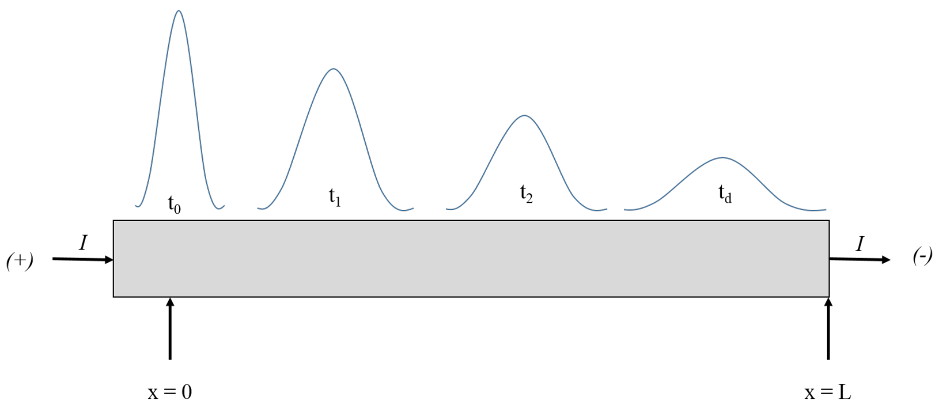

Figure 5.1: Sketch of light pulse as the holes drift in the direction of the electric field and diffuse due to a concentration gradient.

(B). To find the hole mobility, we first need to find the electric field in the sample.

$$E = \frac{V}{d} = \frac{9\text{ V}}{0.5\text{ cm}} = 18\text{ V/cm} \tag{5.1}$$

Then, the hole velocity can be written in terms of the mobility as

$$v = \mu_p E \tag{5.2}$$

and in terms of t_d as

$$v = L/t_x \tag{5.3}$$

Therefore,

$$\mu_p = \frac{L}{E \cdot d} = \boxed{163.4\text{ cm}^2/\text{Vs}} \tag{5.4}$$

(C). To find the corresponding value Δx where the function drops to $1/e$ of its peak value,

$$\frac{1}{e}\hat{\partial p} = \delta p(x,t)\,\Big|_{x=\Delta x/2} \rightarrow \frac{1}{e}\hat{\partial p} = \hat{\partial p}\exp\left(-(\Delta x/2)^2/4D_p t_d\right) \tag{5.5}$$

Solving for Δx, we find

$$\Delta x = \boxed{\sqrt{16 D_p t_d}} \tag{5.6}$$

The hole drift velocity can be rewritten to express the width Δx in terms of a t by

$$v_d = \frac{\Delta x}{\Delta t} \to \Delta x = \Delta t v_d = \boxed{\Delta t \frac{L}{t_d}} \tag{5.7}$$

(D). Using the results from the previous part, we can write the diffusion coefficient in terms of the detection time t_d and the pulse width Δx as

$$\Delta x = \sqrt{16 D_p t_d} = \Delta t \frac{L}{t_d} \to D_p = \frac{(\Delta t)^2 L^2}{16 t_d^3} = \boxed{2.86 \text{ cm}^2/\text{s}} \tag{5.8}$$

(E). Using the Einstein relations

$$\frac{D}{\mu} = \frac{kT}{q} \tag{5.9}$$

and rearranging for temperature,

$$T = \frac{qD}{k\mu} = \boxed{205 \text{ K}} \tag{5.10}$$