1 Chapter 1

- Energy E of a photon of light in eV:

$$\lambda = \frac{1.24eV}{E}$$

- Distance D between adjacent planes in cubic lattices:

$$D = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

- Angle between 2 Miller index directions A and B:

$$\cos\theta = \frac{A \bullet B}{|A||B|}$$

2 Chapter 2

- Planck relationship:

$$E = hv = (\frac{h}{2\pi})(2\pi v) = \hbar\omega$$

- Classical energy of a particle:

$$\frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{\rho^2}{2m}$$

- De Broglie:

$$\lambda = \frac{h}{\rho} = \frac{h}{mv} \Rightarrow \rho = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda} = \hbar k$$

- Momentum or energy in terms of k can be derived by combining De Broglie with the classical energy of a particle:

$$E = \hbar\omega = \frac{\rho^2}{2m} = \frac{\hbar^2k^2}{2m}$$

- Rydberg Constant:

$$R = 109,678cm^{-1}$$

- Lyman:

$$v = cR(\frac{1}{1^2} - \frac{1}{n^2}), n = 2, 3, 4, ...$$

- Balmer:

$$v = cR(\frac{1}{2^2} - \frac{1}{n^2}), n = 3, 4, 5...$$

- Paschen:
$$v = cR(\frac{1}{3^2} - \frac{1}{n^2}), n = 4, 5, 6, ...$$
- Postulate for Bohr:
$$\rho_0 = n\hbar, n = 1, 2, 3, 4, ...$$
- Finding radial forces on orbiting electron:
(Electrical force toward nucleus) = (Equivalent force in terms of radial acceleration)
$$-\frac{q^2}{kr^2} = -\frac{mv^2}{r}$$
$$\rho_0 = n\hbar = mvr \Rightarrow mv^2 = \frac{m^2v^2}{m} = \frac{n^2\hbar^2}{mr^2}$$
$$\frac{q^2}{kr^2} = \frac{1}{mr}\frac{n^2\hbar^2}{r^2} \Rightarrow r_n = \frac{kn^2\hbar^2}{mq^2}$$
$|r_n$ is the radius of the nth orbit
$$-\frac{q^2}{kr^2} = -\frac{mv^2}{r} \Rightarrow \frac{n\hbar}{rm} = \frac{q^2}{kn\hbar}$$
by subbing r from above.

$\Rightarrow$ K.E. of e^- =
$$\frac{1}{2}mv^2 = \frac{mq^4}{2k^2n^2\hbar^2}$$

P.E. of e^- =
$$-\frac{q^2}{kr_n} = -\frac{mq^4}{k^2n^2\hbar^2}$$
by subbing r

Total Energy of e^- =
$$E_n = KE = PE = -\frac{mq^4}{2k^2n^2\hbar^2} = -KE$$
- Energy difference between orbits:
$$E_{n2} - E_{n1} = \frac{mq^4}{2k^2\hbar^2}(\frac{1}{n_1^2} - \frac{1}{n_2^2})$$
- Frequency of light given by a transition between orbits:
$$V_{21} = \left[\frac{mq^4}{2k^2\hbar^2 h}\right](\frac{1}{n_1^2} - \frac{1}{n_2^2})$$

- Heisenberg uncertainty principle:
  $(\Delta x)(\Delta \rho_x) \geq \frac{\hbar}{2}$
  $(\Delta E)(\Delta t) \geq \frac{\hbar}{2}$

2.1 Quantum Mechanics

Classical Variable	$\rightarrow$	Quantum Operator
x	$\rightarrow$	x
$f(x)$	$\rightarrow$	$f(x)$
$\rho(x)$	$\rightarrow$	$\frac{\hbar}{j}\frac{\partial}{\partial x}$
E	$\rightarrow$	$-\frac{\hbar}{j}\frac{\partial}{\partial t}$

- Normalization of the probability density (the wave function is the probability density):

$$\int_{-\infty}^{\infty} \Psi^* \Psi dxdydz = 1 \tag{1}$$

- Time averaged expectation of the particle state

$$< Q >= \int_{-\infty}^{\infty} \Psi^* Q_{op} \Psi dxdydz \tag{2}$$

- Classical energy of a partical:

$$KE + PE = E \Rightarrow \frac{1}{2}mv^2 + V = E$$

$$\frac{1}{2}mv^2 = \frac{1}{2}\frac{(mv)^2}{m} = \frac{1}{2}\frac{\rho^2}{m} = \frac{\rho^2}{2m} \Rightarrow \frac{\rho^2}{2m} + V = E \tag{4}$$

$$\rho \rightarrow \frac{\hbar}{j}\frac{\partial}{\partial x}, E \rightarrow -\frac{\hbar}{j}\frac{\partial}{\partial t}$$

$$\Rightarrow \frac{-1}{2m}\hbar\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t) = \frac{-\hbar}{j}\frac{\partial \Psi(x,t)}{\partial t}$$

where

$$(\frac{\partial}{\partial x})^2 \rightarrow \frac{\partial^2}{\partial x^2}, j^2 = -1$$

- Wave function in 3D then:

$$\frac{-\hbar}{2m}\nabla^2\Psi + V\Psi = \frac{-\hbar}{j}\frac{\partial \Psi}{\partial t} \ni \nabla^2\Psi = \frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2} \tag{5}$$

- Separation of variables:

$$\frac{-\hbar}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t) = \frac{-\hbar}{j}\frac{\partial \Psi(x,t)}{\partial t} \tag{6}$$

$$\Rightarrow$$

$$-\frac{\hbar}{2m}\frac{\partial^2\psi(x)}{\partial x^2}\phi(t)+V(x)\psi(x)\phi(t)=-\frac{\hbar}{j}\psi(x)\frac{\partial\phi}{\partial t} \tag{7}$$

$$\Rightarrow$$

$$\frac{d\phi(t)}{dt}+\frac{j}{\hbar}E\phi(t)=0 \tag{8}$$

(time dependent portion)

$$-\frac{\hbar}{2m}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x) \tag{9}$$

(time independent portion)

- E $\equiv$ equivalent constant, corresponds to total energy of the particle
- Wave function as linear combination of various eigenfunctions

$$\psi(x,t)=_n C_n\Psi_n e^{-j\frac{E_n}{\hbar t}} \ni E_n \equiv nth\ prefactor \tag{10}$$

- Infinite potential well

$$V(x)=\left\{\begin{array}{ll} 0 & ,x\neq 0 and x \neq L \\ \infty & \quad ,x=0 or x = L \end{array}\right.$$

$$\Rightarrow$$

$$(\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x))\psi(x)=E\psi(x)\Rightarrow\frac{d^2\psi(x)}{dx^2}=\frac{2m}{\hbar}E\psi(x) \tag{11}$$

$$\Rightarrow$$

$$\frac{d^2\psi(x)}{dx^2}+\frac{2m}{\hbar^2}E\psi(x)=0 \tag{12}$$

$$\Rightarrow$$

$$\psi(x)=\{\sin(kx),\cos(kx)\}\ni k=\frac{\sqrt{2mE}}{\hbar} \tag{13}$$

$$\cos(kx)0 when x=0\Rightarrow A\sin(kx) is our solution. \ni k=\frac{\sqrt{2mE}}{\hbar} \tag{14}$$

$$when potential is 0 at x=0 and L, k=\frac{n\pi}{L},n=\{1,2,3,...\} \tag{15}$$

$$therefore:\frac{\sqrt{2mE_n}}{\hbar}=\frac{n\pi}{L}\Rightarrow E_n=\frac{n^2\pi^2\hbar^2}{wmL^2} \tag{16}$$

- A is found by normalizing the probability density integral

$$\int_{-\infty}^{\infty} \Psi^* \Psi dx = \int_0^L A^2 \sin^2(\frac{n\pi x}{L}) dx = A^2 \frac{L}{2} review\ trig\ calc\ and\ show\ the\ process\ here \tag{17}$$

Set the above to 1 to find A

$$\frac{A^2 L}{2} = 1 \Rightarrow A = \sqrt{\frac{2}{L}} \tag{18}$$

$$Therefore:\ \psi = \sqrt{\frac{2}{L}} \sin(\frac{n\pi}{L})\ for\ an\ infinite\ well \tag{19}$$

- Parabolic Potential Well (simple harmonic oscillator)

$$V(x) = kx^2, E_n = (n + \frac{1}{2})\hbar\omega \tag{20}$$

- Coulombic Potential Well

$$\frac{d^2}{d\phi^2}\Phi + m^2\Phi = 0 \Rightarrow \Phi_m(\phi) = Ae^{jm\phi} \tag{21}$$

$$1 = \int_0^{2\pi} \Psi^* \Psi d\psi \Rightarrow \int_0^{2\pi} \Phi_m^*(\phi)\Phi_m(\phi) d\phi = 1 \tag{22}$$

$$\Rightarrow A^2 \int_0^{2\pi} e^{-jm\phi} e^{jm\phi} d\phi \tag{23}$$

$$\Rightarrow$$

$$A^2 \int_0^{2\pi} d\phi = 2^2 \tag{24}$$

$$\Rightarrow$$

$$A = \frac{1}{\sqrt{2\pi}} \Rightarrow \Phi_m(\phi) = \frac{1}{\sqrt{2\pi}} e^{jm\phi} \tag{25}$$

$$\ni m = \{..., -3, -2, -1, 0, 1, ...\}$$

similar for $\Theta_l(\theta) and R_n(r) Atomic numbers describing allowable states in a hydrogen atom$
n = 1,2,3,...
l=0,1,2,3,...,n-1
m (m$_l$) = $-l, ..., -2, -1, 0, 1, 2, ..., l$
$s(m_s) = \frac{\pm\hbar}{2}$

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n	l	m	$\frac{s}{\hbar}$	Allowable states in subshell	Allowable states in complete shell
1	0	0	$\pm\frac{1}{2}$	2	2
2	0	0	$\pm\frac{1}{2}$	2	
2	1	-1	$\pm\frac{1}{2}$		8
2	1	0	$\pm\frac{1}{2}$	6	
2	1	1	$\pm\frac{1}{2}$		
3	0	0	$\pm\frac{1}{2}$	2	18
3	1	-1	$\pm\frac{1}{2}$		
3	1	0	$\pm\frac{1}{2}$	6	18
3	1	1	$\pm\frac{1}{2}$		
3	2	-2	$\pm\frac{1}{2}$		
3	2	-1	$\pm\frac{1}{2}$		
3	2	0	$\pm\frac{1}{2}$	10	18
3	2	1	$\pm\frac{1}{2}$		
3	2	2	$\pm\frac{1}{2}$		

- n is the principle atomic number
- l is the number that determines s,p,d,f,g,...

3 Chapter 3

- Equilibrium number of EHP's in pure Si at room temp:

$$10^{10}\frac{EHP}{cm^3}$$

- Si atom density in pure Si at room temp:

$$5 * 10^{22}\frac{atoms}{cm^3}$$

- Releationship of (E,k) for a free electron related to electron mass

$$\frac{d^2E}{dk^2} = \frac{{\hbar^2}^*}{m} = \frac{\hbar^2}{\frac{d^2E}{dk^2}} \; so\ the\ curvature\ of\ the\ band\ determines\ the\ e^-\ effective\ mass \tag{26}$$

- Velocity of an electron (v) is the group velocity of a quantum mechanical electron wavepacket

$$v = \frac{d\omega}{dk} = (\frac{1}{\hbar})\frac{dE}{dk} \tag{27}$$

- Rate of recombination of electrons and holes r_i is proportional to equilibrium concentrations of electrons and holes n_0 and p_0

$$r_i = \alpha_r n_0 p_0 = \alpha_r n_i^2 = g_i \tag{28}$$

- Calculating the approximate energy required to excite a donor electron into conduction band.

  Consider the loosely bound electron as circling the tightly bound "core" electons.

  Magnitude of the "ground state" is:

$$E = \frac{mq^4}{2k^2\hbar^2} = 4\pi\epsilon_0\epsilon_r, m \equiv m_n^* \ (conductivity\ effective\ mass) \tag{29}$$

- Radius of of electron orbit around donor, assuming ground state. Equation was pulled from chapter 2

$$r = \frac{4\pi\epsilon_r\epsilon_0\hbar^2}{m_n^* q^2} \tag{30}$$

- Intrinsic carrier concentration n_i of Si is about $10^{10} cm^{-3}$ at room temperature

3.1 Carrier Concentrations

- Standard Fermi Equation:

$$f(E) = \frac{1}{1 + e^{\frac{(E-E_F)}{kT}}} \ni k\ is\ the\ Boltzman\ constant\ k = 8.62{*}10^{-5}\frac{eV}{K} = 1.38{*}10^{-23}\frac{J}{K} \tag{31}$$

- Calculate the number of electrons and holes in a semiconductor when you know the density of states in valence and conduction bands are known

  - Concentration of electrons in the conduction band by integral:

$$n_0 = \int_{E_c}^{\infty} f(E)N(E)dE \ni N(E)d(E)\ is\ the\ density\ of\ states\ (cm^{-3})\ in\ the\ energy\ range\ of \tag{32}$$

  - Concentration of electrons in the conduction band by using *Effective Density of states*:

$$n_0 = N_c f(E_c) \ni N_c\ is\ the\ effective\ density\ of\ states\ at\ E_c \tag{33}$$

  - When E-E_F is several kT we can simplify the equation by recognizing that the exponential dominates the denominator

$$f(E_c) = \frac{1}{1 + e^{\frac{(E-E_F)}{kT}}} \Rightarrow e^{\frac{(E-E_F)}{kT}} \tag{34}$$

  E$_c - E_F \frac{}{kT(34)}$

– Effective Density of States is:

$$N_c = 2(\frac{2\pi m_n^* kT}{h^2})^{\frac{3}{2}} \ni m_n^* \equiv \ the\ density\ of\ states\ effective\ mass\ for\ electrons \tag{35}$$

– Density of States Effective Mass for Electrons is:

$$(m_n^*)^{\frac{3}{2}} = 6(m_{.} m_t^2)^{\frac{1}{2}} \tag{36}$$