ECE 340: Semiconductor Electronics

Chapter 4: Excess Carriers in Semiconductors (part II: diffusion)

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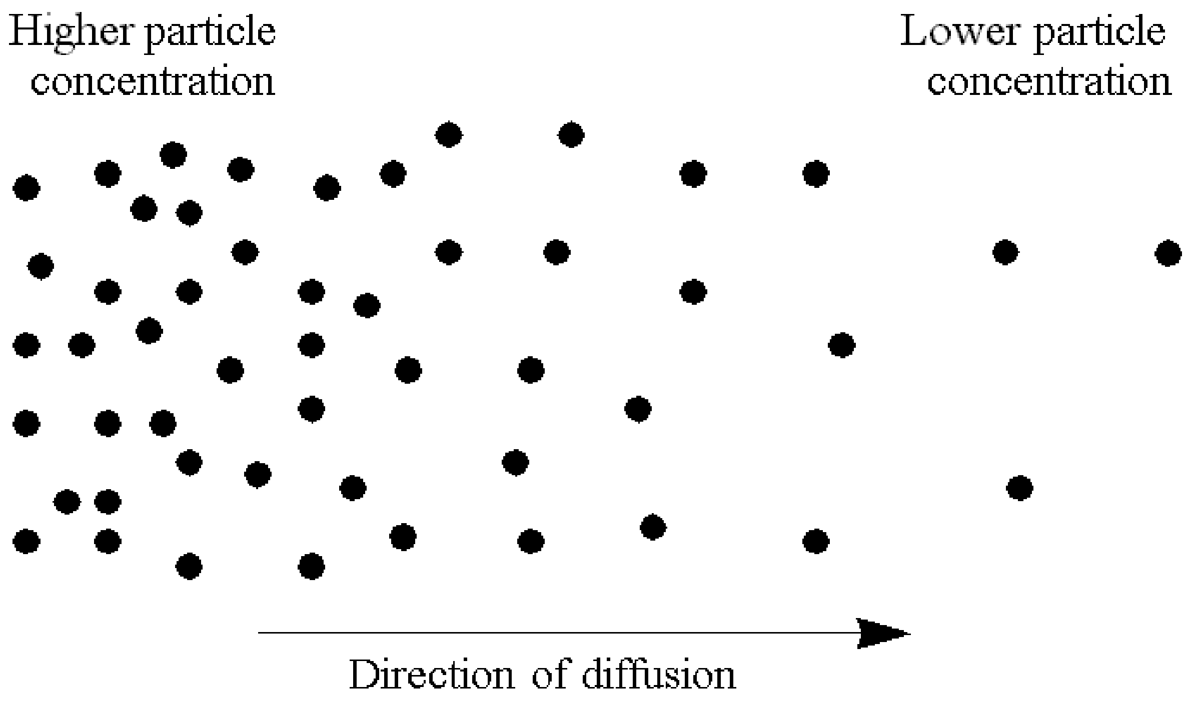
Outline

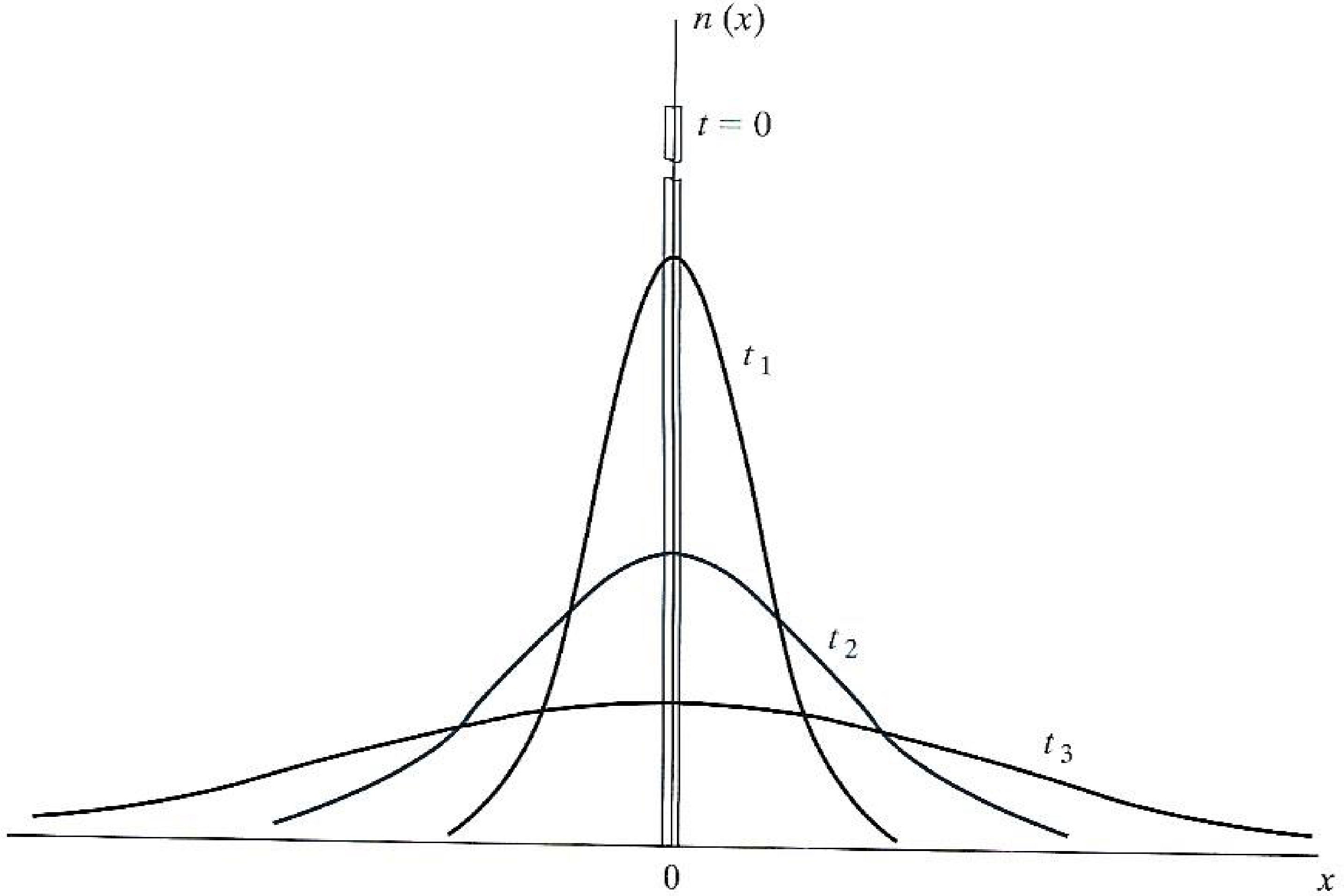
- Diffusion of carriers
  - Diffusion process
  - Diffusion and drift of carriers; built-in fields
  - Diffusion and recombination; the continuity equation
  - Steady state carrier injection; diffusion length
  - The Haynes-Shockley experiment

Diffusion process

- particles diffuse from regions of higher concentration to regions of lower concentration region, due to random thermal motion



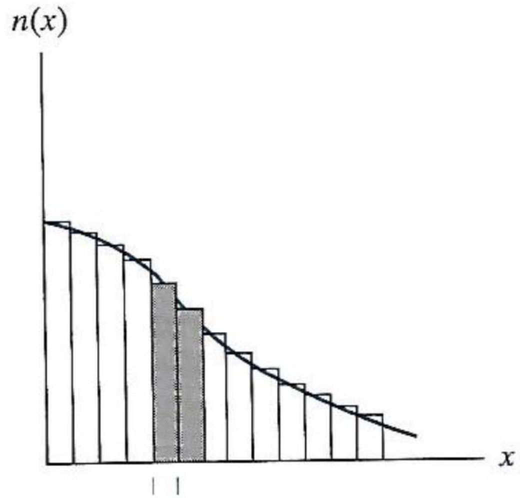
Diffusion of a pulse of electrons



- A pulse of excess electron injected at x=0 and t=0 will spread out in time.

- The net number of electrons passing x_0 from left to right in one mean free time is

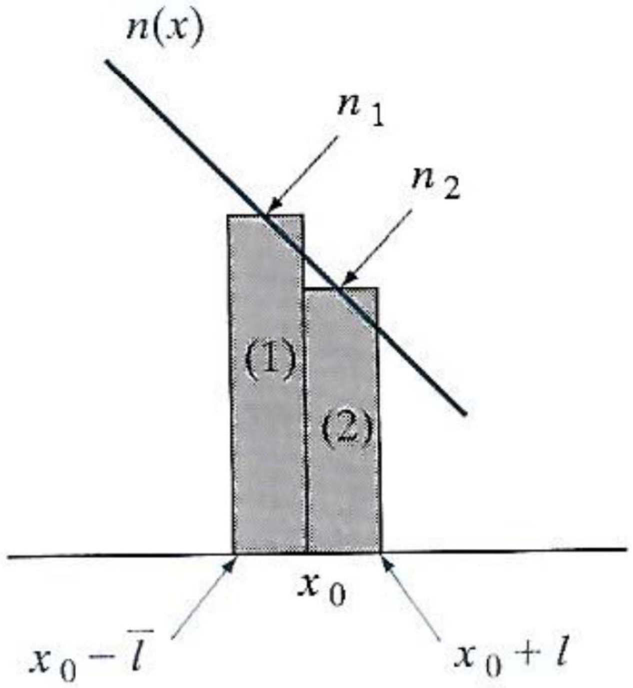
$$\frac{1}{2}(n_1 l A) - \frac{1}{2}(n_2 l A)$$



- The rate of electron flow in the +x direction per unit area:

$$\Phi_n(x_0) = \frac{l}{2\tau}(n_1 - n_2)$$

where $n_1 - n_2 \approx \frac{-dn(x)}{dx} l$



Electron flux density

- Thus the electron flux density:

$$\Phi_n(x_0) = \frac{-l^2}{2\tau} \frac{dn(x)}{dx}$$

Define $D_n = \frac{l^2}{2\tau}$ **Diffusion coefficient**

- The electron and hole flux density:

$$\Phi_n(x_0) = -D_n \frac{dn(x)}{dx}$$

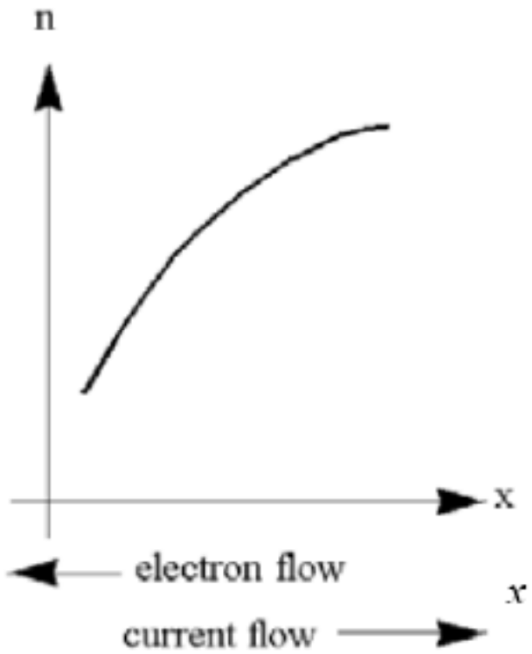
$$\Phi_p(x_0) = -D_p \frac{dp(x)}{dx}$$

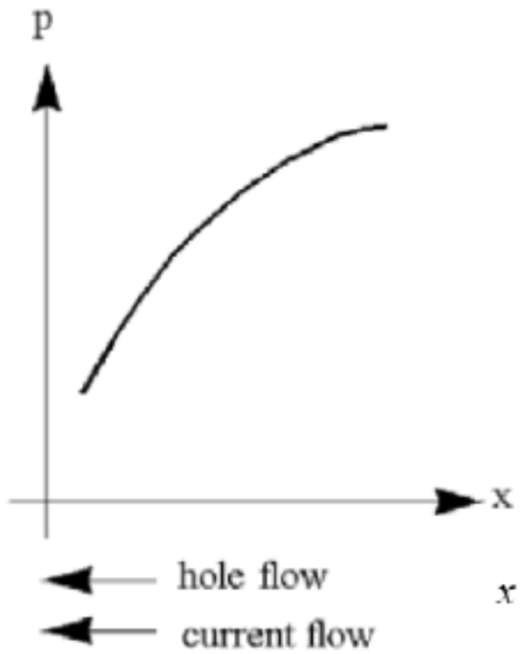
Diffusion current

- Electron and hole diffusion current:

$$J_n(diff.) = -(-q)D_n\frac{dn(x)}{dx} = +qD_n\frac{dn(x)}{dx}$$

$$J_p(diff.) = -(+q)D_p\frac{dp(x)}{dx} = -qD_p\frac{dp(x)}{dx}$$





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Diffusion and drift current

- If an electric field is present in addition to the carrier gradient, the electron and hole current density:

$$\boxed{J_n(x) = q\mu_n n(x)\mathcal{E}(x) + qD_n \frac{dn(x)}{dx}}$$

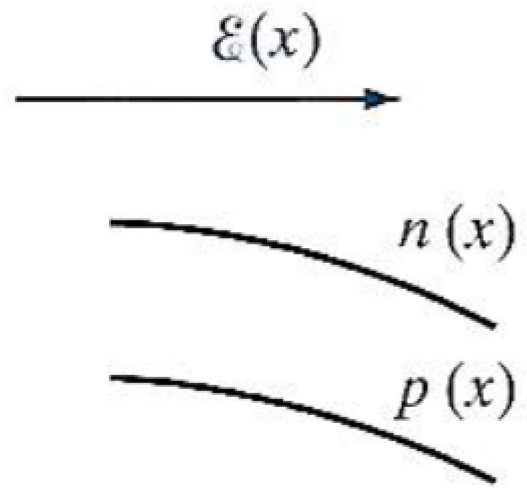
drift **diffusion**

$$\boxed{J_p(x) = q\mu_p p(x)\mathcal{E}(x) - qD_p \frac{dp(x)}{dx}}$$

- The total current density:

$$\boxed{J(x) = J_n(x) + J_p(x)}$$

Drift and diffusion directions for electrons and holes



	Electron	**Hole**
Diffusion	$\dashrightarrow$ $\Phi_n(diff.)$	$\dashrightarrow$ $\Phi_p(diff.)$
	$\longleftarrow$ $J_n(diff.)$	$\longrightarrow$ $J_p(diff.)$
Drift	$\dashleftarrow$ $\Phi_n(drift.)$	$\dashrightarrow$ $\Phi_p(drift.)$
	$\longrightarrow$ $J_n(drift.)$	$\longrightarrow$ $J_p(drift.)$

Current contribution from minority carrier

- Minority carriers can contribute significantly to the current through diffusion, since diffusion current is proportional to the gradient of concentration, instead of carrier concentration.
- Minority carrier typically do not contribute much to drift current.

$$\boxed{J_n(x) = q\mu_n \boldsymbol{n(x)} \mathcal{E}(x) + qD_n \frac{\boldsymbol{dn(x)}}{\boldsymbol{dx}}}$$

drift **diffusion**

Relation of electric field and electron energy

Electrostatic potential:

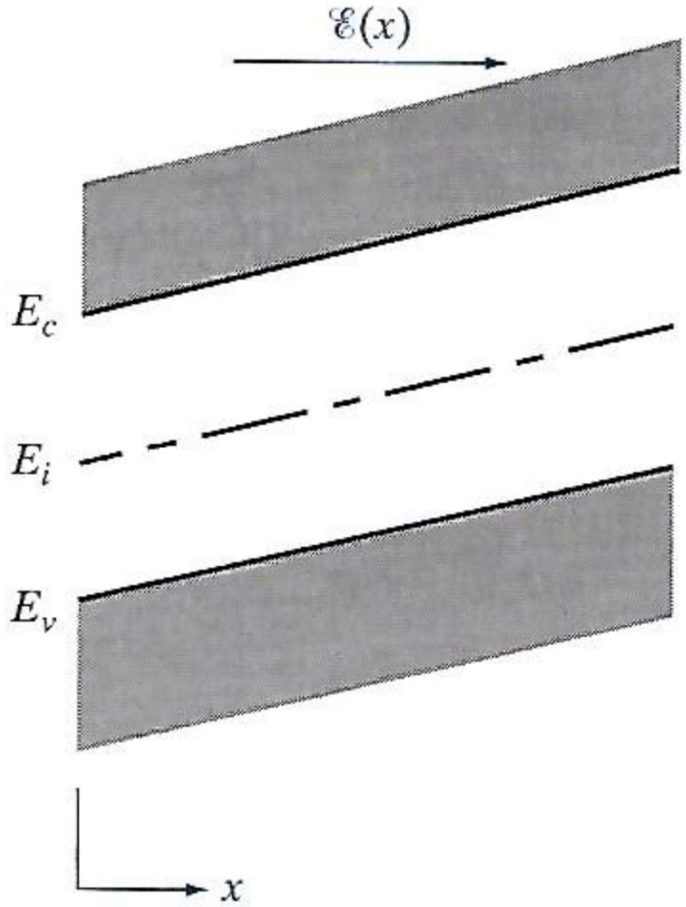
$$V(x) = \frac{E(x)}{-q}$$

electron potential energy

Definition of electric field:

$$\mathcal{E} = -\frac{dV(x)}{dx}$$

$$\Rightarrow \mathcal{E} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$



Balance of diffusion and drift at equilibrium

- At equilibrium, no net current flows in a semiconductor, $J_p = 0$, $J_n = 0$:

$$J_p(x) = q\mu_p p(x)\mathcal{E}(x) - qD_p \frac{dp(x)}{dx} = 0$$

$$\Rightarrow \quad \mathcal{E}(x) = \frac{D_p}{\mu_p}\frac{1}{p(x)}\frac{dp(x)}{dx}$$

using $p_0 = n_i e^{(E_i - E_F)/kT}$

$$\mathcal{E}(x) = \frac{D_p}{\mu_p}\frac{1}{kT}\left(\underbrace{\frac{dE_i}{dx}}_{= q\mathcal{E}(x)} - \underbrace{\frac{dE_F}{dx}}_{=0}\right)$$

Einstein relation

$$\Rightarrow \quad \boxed{\frac{D}{\mu} = \frac{kT}{q}} \qquad \textbf{Einstein relation}$$

- This equation is valid for either carrier type.
- At room T, D/μ =0.026V

Built-in electric field

- At equilibrium, concentration gradients result in built-in fields, such that the drift current exactly cancels out the diffusion current:

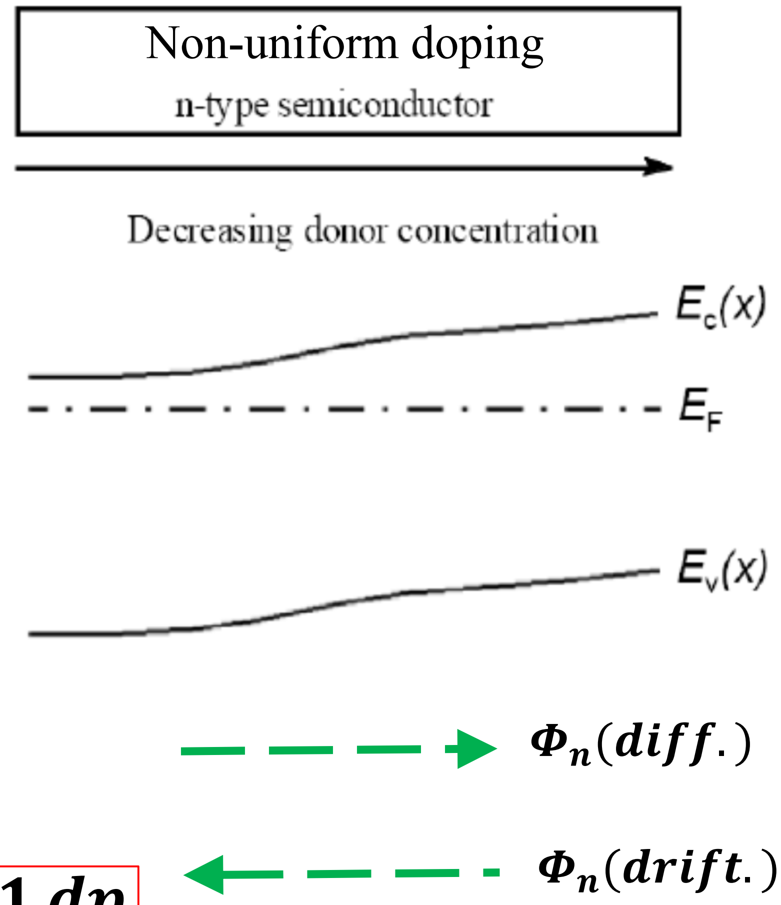
$$n = N_c e^{-(E_c - E_F)/kT}$$

$$\frac{dn}{dx} = -\frac{N_c}{kT} e^{-\frac{E_c - E_F}{kT}} \frac{dE_c}{dx}$$

$$= -\frac{n}{kT}\frac{dE_c}{dx}$$

$$= -\frac{n}{kT} q\mathcal{E}$$

$$\Rightarrow \boxed{\mathcal{E} = -\frac{kT}{q}\frac{1}{n}\frac{dn}{dx}} \qquad \boxed{\mathcal{E} = \frac{kT}{q}\frac{1}{p}\frac{dp}{dx}}$$



Potential Difference Due to Carrier Concentration Gradient

- The ratio of carrier densities (n, p) at two points depends exponentially on the potential difference between these points:

$$E_F - E_{i1} = kT\ln\left(\frac{n_1}{n_i}\right) \quad => \quad E_{i1} = E_F - kT\ln\left(\frac{n_1}{n_i}\right)$$

$$\text{Similarly, } E_{i2} = E_F - kT\ln\left(\frac{n_2}{n_i}\right)$$

$$\text{Therefore} \quad E_{i1} - E_{i2} = kT\left[\ln\left(\frac{n_2}{n_i}\right) - \ln\left(\frac{n_1}{n_i}\right)\right] = kT\ln\left(\frac{n_2}{n_1}\right)$$

$$V_2 - V_1 = \frac{1}{q}(E_{i1} - E_{i2}) = \frac{kT}{q}\ln\left(\frac{n_2}{n_1}\right)$$

Gradual-variation, Quasi-neutrality

- Majority carrier distribution does not differ much from the donor (or acceptor) distribution, so that the semiconductor region is nearly neutral or quasi-neutral. $\boldsymbol{n \approx N_d}$, or $\boldsymbol{p \approx N_a}$.
- This quasi-neutrality approximation is more valid for slowly varying dopant densities. Then:

$$\mathcal{E} = \frac{kT}{q}\frac{1}{N_a}\frac{dN_a}{dx}$$

$$\mathcal{E} = -\frac{kT}{q}\frac{1}{N_d}\frac{dN_d}{dx}$$

Example

- An intrinsic Si sample is doped with donors from one side such that $N_d = N_0 exp(\frac{-x}{\lambda})$.

(a) Find an expression for the built-in field Ɛ(x) at equilibrium over the range for which $N_d \gg n_i$?

(b) Sketch a band diagram and indicate the direction of Ɛ

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Recall: Direct recombination and thermal generation

$$\frac{dn(t)}{dt} = \alpha_r n_i^2 - \alpha_r n(t)p(t)$$

Carrier concentration change rate

Thermal generation rate

Recombination rate

$$\boldsymbol{\frac{dn}{dt} = -\frac{\delta n(t)}{\tau_n}}$$

Where $\tau_n = (\alpha_r p_0)^{-1}$,

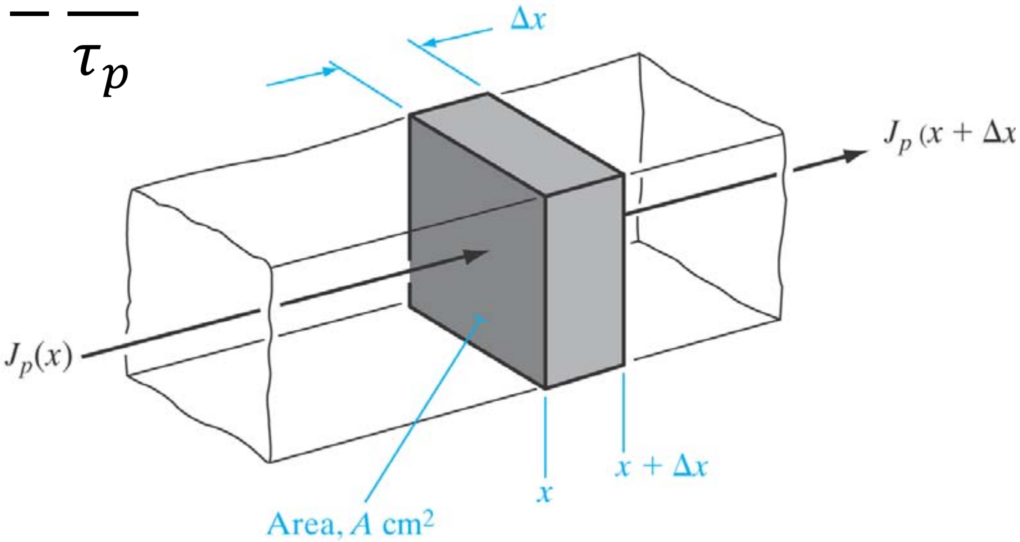
Note: in this case, assume excess carrier is uniformly distributed in the semiconductor and there is no electric field, i.e. diffusion and drift current are not considered.

Diffusion and recombination

If we consider both carrier flow by drift/diffusion and thermal generation/recombination process, then:

Rate of hole build up = (**# of carriers flow IN per unit volume** - **# of carriers flow OUT per unit volume**) + (**Thermal generation rate** - **Recombination rate**)

$$\frac{\partial p}{\partial t} = \frac{1}{q} \cdot \frac{J_p(x) - J_p(x + \Delta x)}{\Delta x} - \frac{\delta p}{\tau_p}$$



Continuity equation for holes and electrons

As Δx approaches zero:

$$\frac{\partial p}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p}$$ **continuity equation for holes**

$$\frac{\partial \delta n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n}$$ **continuity equation for electrons**

Continuity equation if all things considered

$$\frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p} + g_{op}$$

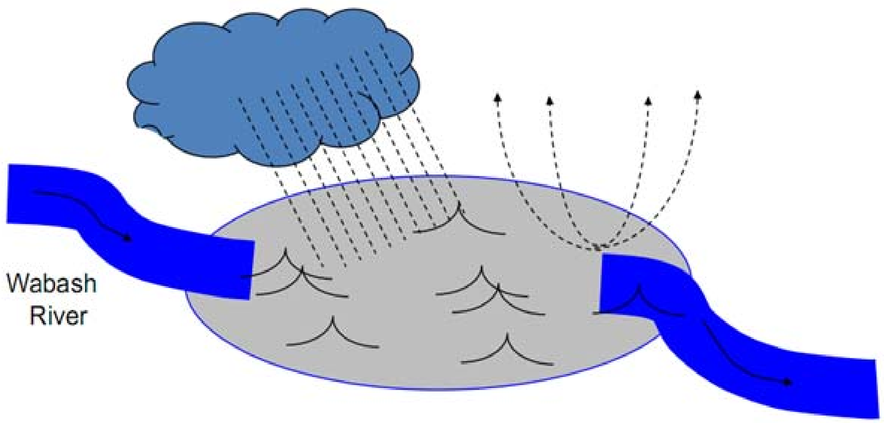
$$\frac{\partial \delta n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n} + g_{op}$$

Carrier concentration changing rate

Carrier flow in-out rate

Thermal Generation Recombination rate

Optical generation rate



Rate of increase of water level in lake = (in flow - outflow) + rain - evaporation

$$\frac{\partial p}{\partial t} = \frac{1}{q}\nabla \bullet J_p \quad + g_N \quad - r_N$$

Common simplifications

- Steady State $\dfrac{d\Delta n}{dt} \longrightarrow 0$
- No concentration gradient $D_N \dfrac{\partial^2 \Delta n_p}{\partial x^2} \longrightarrow 0$
- No drift Current $E = 0$
- No thermal R-G $\dfrac{\Delta n}{\tau_n} \longrightarrow 0$
- No Light $G_L \longrightarrow 0$

Steady state: *n(x)* is time **invariant**
Transient state: *n(x)* is time **dependent**

Diffusion equation (transient state)

- When the current is carried by diffusion only (negligible drift), using diffusion current:

$$J_n(diff.) = qD_n \frac{\partial \delta n}{\partial x}$$

- We obtain the diffusion equation for electrons:

$$\boxed{\frac{\partial \delta n}{\partial t} = D_n \frac{\partial^2 \delta n}{\partial x^2} - \frac{\delta n}{\tau_n}}$$

- and similarly for holes:

$$\boxed{\frac{\partial \delta p}{\partial t} = D_p \frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau_p}}$$

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Diffusion equation (steady state)

- At steady state, $\frac{\partial \delta n}{\partial t} = 0, \frac{\partial \delta p}{\partial t} = 0$, the diffusion equation become:

$$\boxed{\frac{d^2 \delta n}{dx^2} = \frac{\delta n}{D_n \tau_n} \equiv \frac{\delta n}{L_n^2}}$$

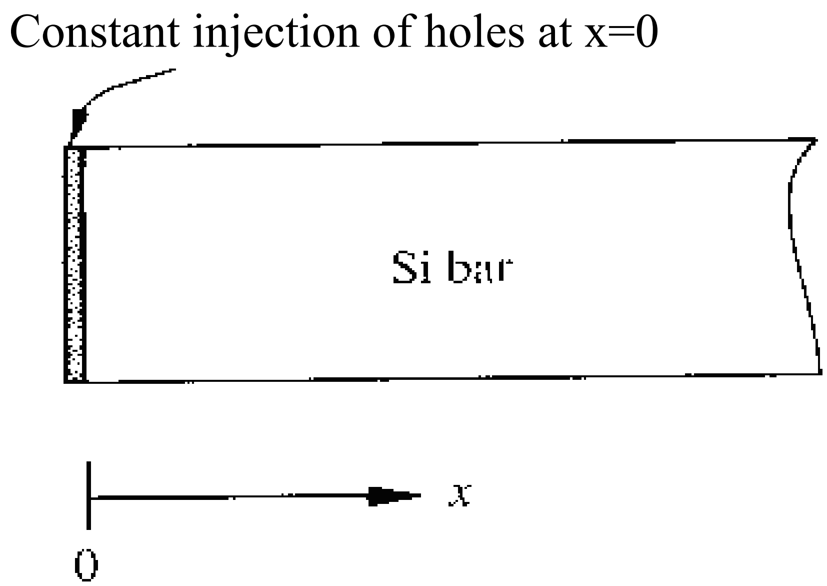
$$\boxed{\frac{d^2 \delta p}{dx^2} = \frac{\delta p}{D_p \tau_p} \equiv \frac{\delta p}{L_p^2}}$$

$L_n \equiv \sqrt{D_n \tau_n}$: electron diffusion length

$L_p \equiv \sqrt{D_p \tau_p}$: hole diffusion length

Steady-state injection

- Consider an example under steady-state illumination:



- Boundary condition: $x = 0, \delta p = \Delta p$

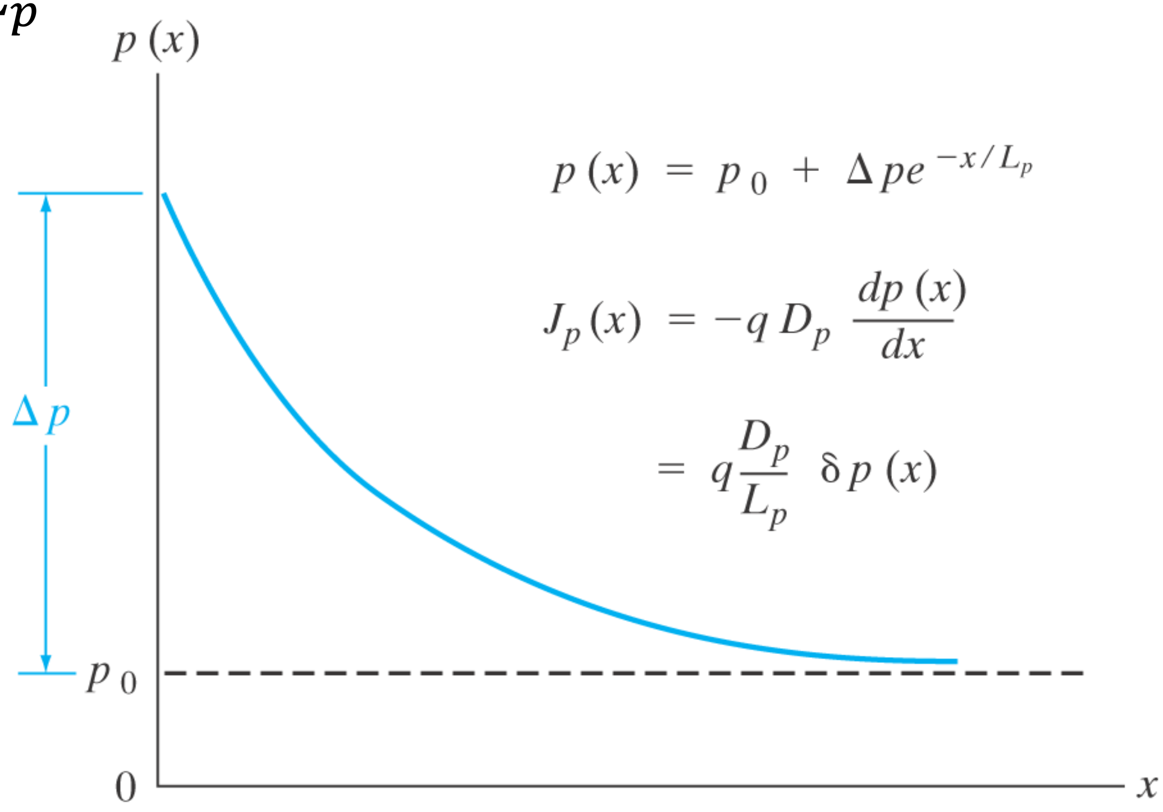
$$x = \infty, \ \delta p = 0$$

Excess carrier concentration

- Solution of the diffusion equation is:

$$\delta p(x) = \Delta p e^{-x/L_p}$$

L_p is the average distance a hole diffuses before recombining



Steady state diffusion current

- The steady state distribution of excess holes cause hole diffusion current:

$$J_p(x) = -qD_p \frac{dp}{dx} = -qD_p \frac{\partial \delta p}{\partial x} = q \frac{D_p}{L_p} \Delta p e^{-x/L_p}$$

$$= q \frac{D_p}{L_p} \delta p(x)$$

The diffusion current at any x is proportional to the excess concentration at that position.

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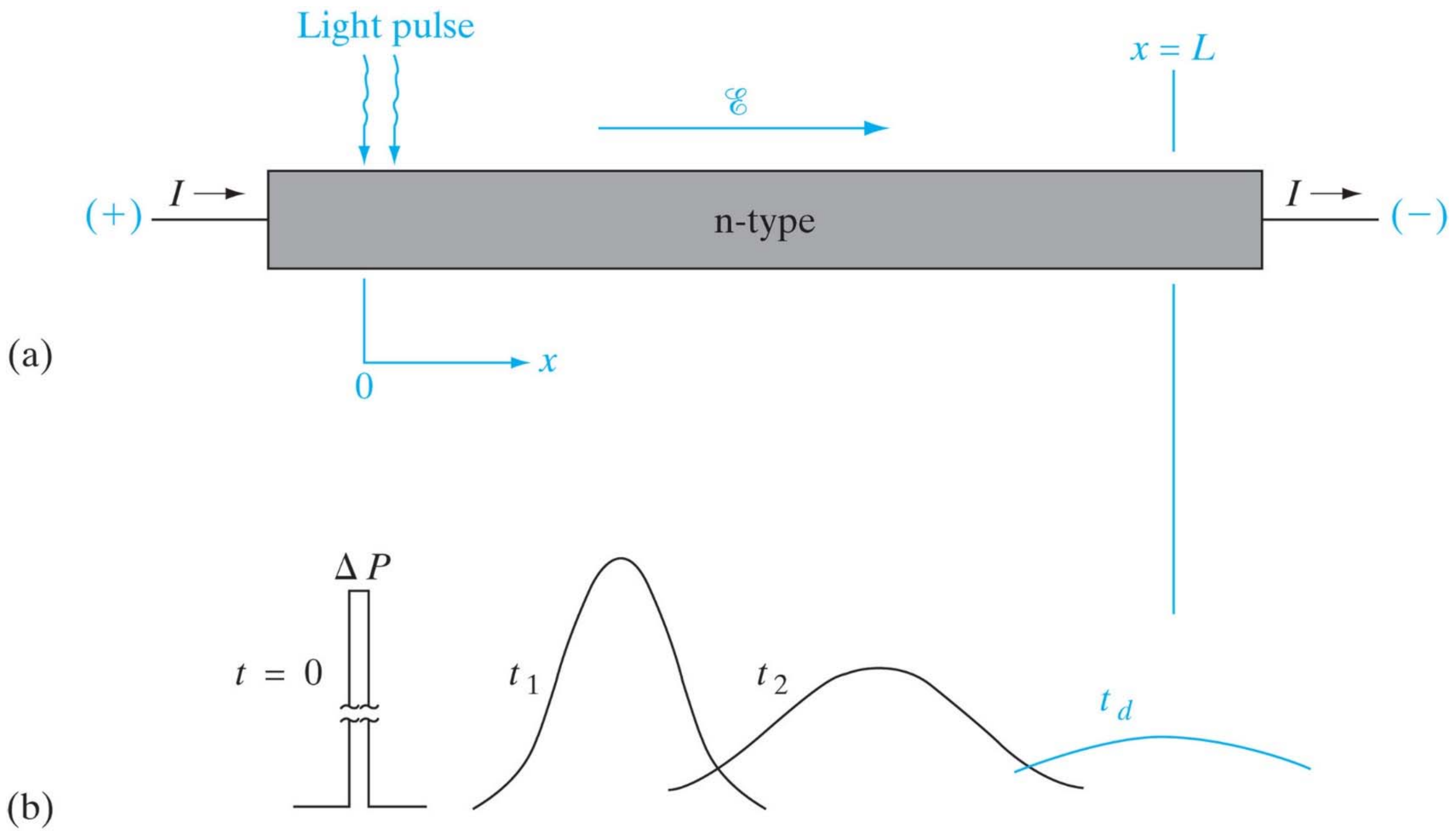
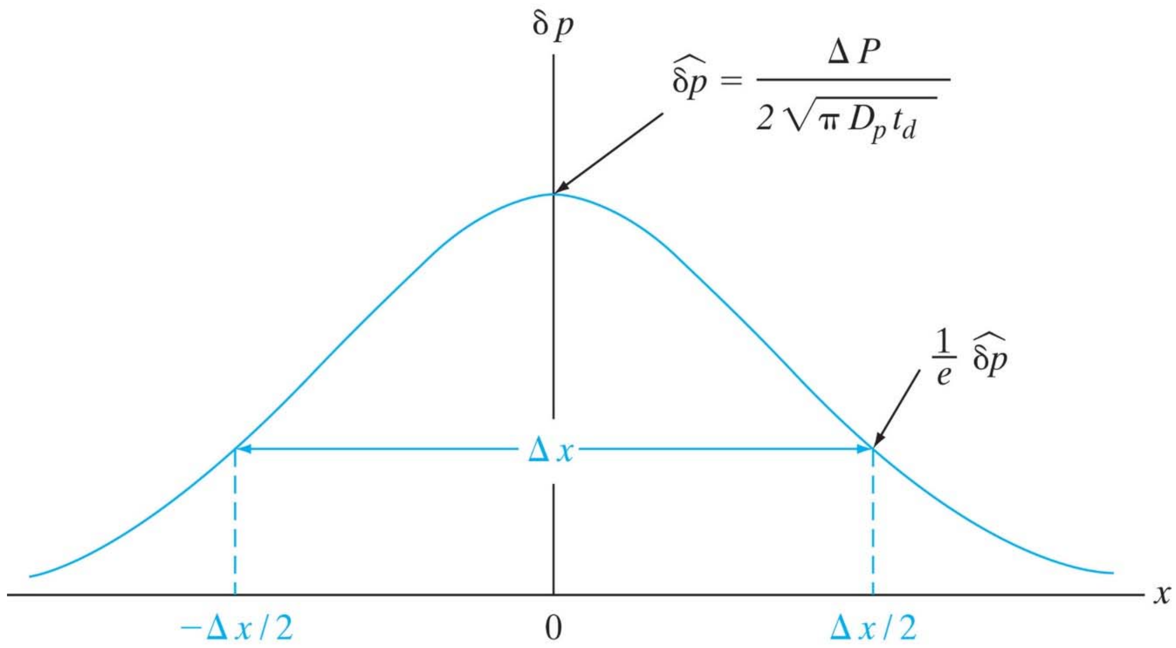
Drift and diffusion of a hole pulse in n type bar



Drift velocity: $v_d = \dfrac{L}{t_d}$ **Hole mobility:** $\mu_p = \dfrac{v_d}{\mathcal{E}}$

Diffusion of a pulse without drift and recombination

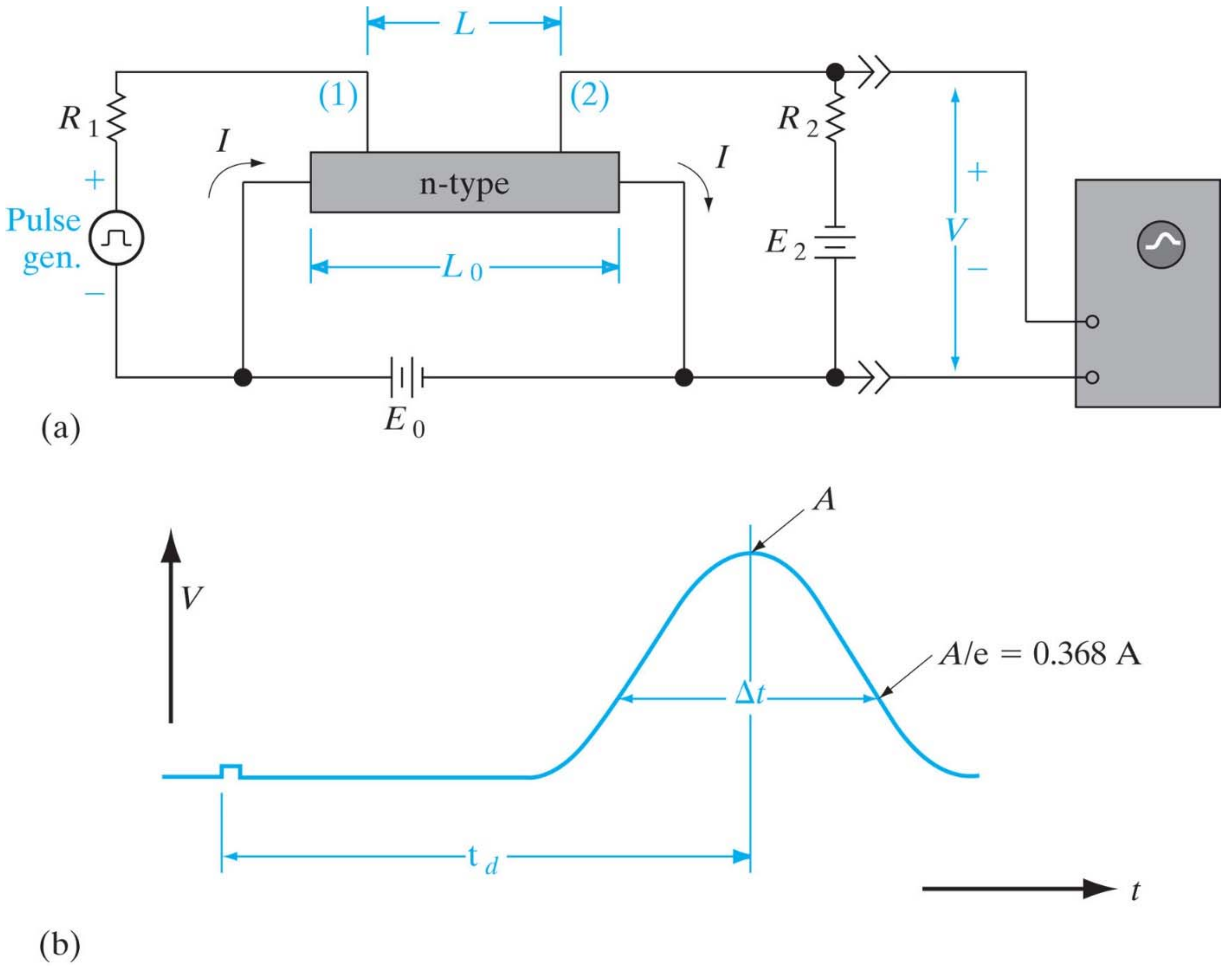


Diffusion equation: $$\frac{\partial \delta p(x,t)}{\partial t} = D_p \frac{\partial^2 \delta p(x,t)}{\partial^2 x}$$

Solution: Gaussian distribution $$\delta p(x,t) = \left[\frac{\Delta P}{2\sqrt{\pi D_p t}}\right] e^{-x^2/4D_p t}$$

Diffusion coefficient: $$D_p = \frac{(\Delta x)^2}{16 t_d}$$

Haynes-Shockley experiment



$$\Delta x = \Delta t \frac{L}{t_d}$$

Example

- A) Calculate minority carrier diffusion length in silicon with $N_D = 10^{16}$ cm^{-3} and $\tau_p = 1$ μs. B) Assuming 10^{15} cm^{-3} excess holes photogenerated at the surface, what is the diffusion current at 1 μm depth?