# ECE 340: Semiconductor Electronics

## Chapter 3: Energy bands and charge carriers in semiconductors (part III)

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# Outline

- Drift of carriers in electric and magnetic fields
  - Conductivity and mobility
  - Drift and resistance
  - Effect of temperature and doping on mobility
- Invariance of the Fermi level at equilibrium

# Factors that influence current or cargo flow

Number of the carriers

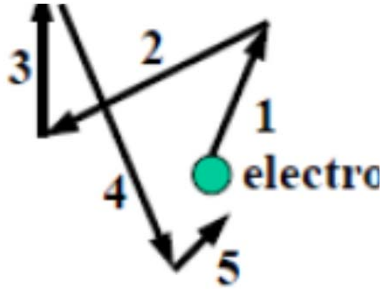
Carrier velocity (how mobile they are, interruptions on the road)

# Thermal velocity

- In thermal equilibrium, carriers are not sitting still, but rather move about with random velocities.

- The mean-square thermal velocity of the electrons is related to temperature by:

$$\frac{1}{2}m_n^* v_{th}^2 = \frac{3}{2}kT$$




**$m_n^*$ is the effective mass of conduction-band electrons**
**k: Boltzmann's constant**

- At *T=300K*: $v_{th} = 2.3 \times 10^7 cm \cdot s^{-1}$

- The net current in any direction is zero, if no electric field is applied

# Carrier Scattering in thermal motion

- Mobile electrons in Silicon lattice move in random directions with many collisions:
  - collide with the lattice
  - collide among themselves
  - scatter by ionized impurity atoms

- **Mean free time between collision: $\tau_{cn}$**

# Mean free path

- **Mean free path, characteristic length of thermal motion:**

$$\lambda = v_{th}\tau_{cn}$$

- **For Si at 300K:**

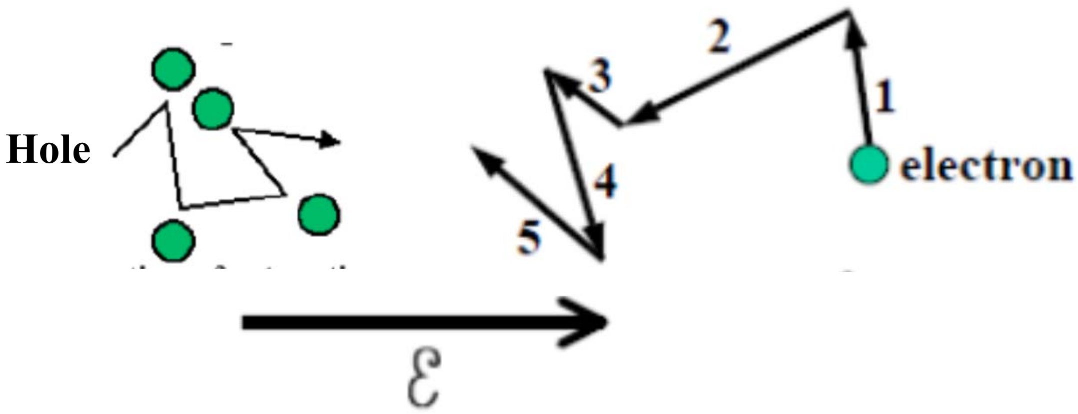
$$\tau_{cn} \approx 10^{-14} \sim 10^{-13} s$$

$$v_{th} \approx 2.3 \times 10^{7} cm \cdot s^{-1}$$

$$\lambda \approx 1 \sim 10 nm$$

# Carrier Motion in Electric Field

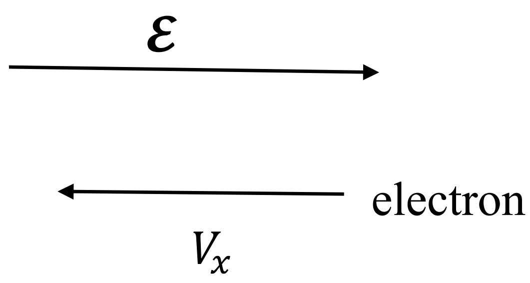
- If apply a small electric field to the lattice:
  - Holes move in the direction of the electric field
  - Electrons move in the opposite direction of the electric field
- Motion is highly non-directional on a local scale, but has a net direction on a macroscopic scale.
- Net motion of charged particles give rise to a current.



# Acceleration and deceleration process

- **Acceleration by electric field:**

$$-nq\mathcal{E} = \frac{dP_x}{dt}$$




- **Deceleration due to collisions:**

$$-\frac{dN(t)}{dt} = \frac{1}{\tau}N(t) \quad \Rightarrow N(t) = N_0\, e^{-t/\tau}$$

$$dP_x = -P_x\frac{dt}{\tau}$$

- **At steady state, the sum of acceleration and deceleration effects is zero:**

$$-nq\mathcal{E} - \frac{P_x}{\tau} = 0$$

Average momentum per electron: $\langle p_x \rangle = \dfrac{P_x}{n} = -q\tau\mathcal{E}$

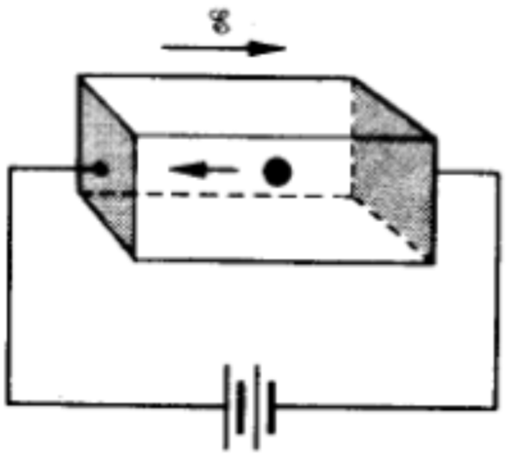
# Drift Velocity

- **Drift velocity: the average net velocity, $v_d$ [cm/s].**

$$V_d = \frac{\langle p_x \rangle}{m^*}$$

⇨ **Electron** $$v_{d\_n} = -\frac{q\tau_{cn}}{m_n^*}\mathcal{E}$$

**Hole** $$v_{d\_p} = \frac{q\tau_{cp}}{m_p^*}\mathcal{E}$$



# Mobility

- Define mobility:

$$\mu_n = \frac{q\boldsymbol{\tau}_{\boldsymbol{cn}}}{\boldsymbol{m}_{\boldsymbol{n}}^*}$$ **Electron mobility**

$$\mu_p = \frac{q\boldsymbol{\tau}_{\boldsymbol{cp}}}{\boldsymbol{m}_{\boldsymbol{p}}^*}$$ **Hole mobility**

- Mobility can also be expressed as the average particle drift velocity per unit electric field:

$$\mu_n = -\frac{\boldsymbol{v}_{\boldsymbol{d\_n}}}{\boldsymbol{\mathcal{E}}}$$

$$\mu_p = \frac{\boldsymbol{v}_{\boldsymbol{d\_p}}}{\boldsymbol{\mathcal{E}}}$$

**Mobility describes how strongly the motion of an electron is influenced by an applied field.**

# Electron and hole mobility

**μ has the dimension of $v/\mathcal{E}$, unit: $[\frac{cm/s}{V/cm} = \frac{cm^2}{V\cdot s}]$**

Electron and hole mobilities of some undoped (intrinsic) semiconductors at room temperature:

| | **Si** | **Ge** | **GaAs** | **InAs** |
|---|---|---|---|---|
| $\mu_n$ ($cm^2/V\cdot s$) | 1400 | 3900 | 8500 | 30000 |
| $\mu_p$ ($cm^2/V\cdot s$) | 470 | 1900 | 400 | 500 |

$$\boxed{\mu = \frac{q\tau_c}{m^*}}$$

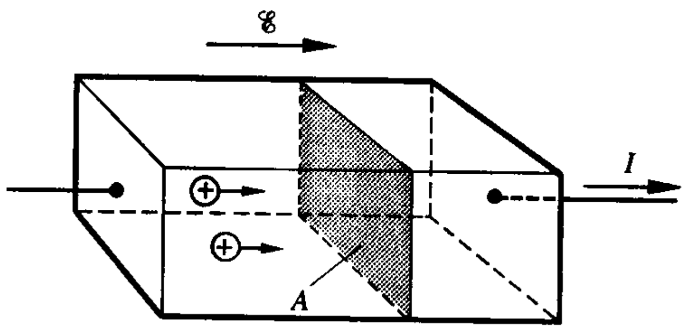
**Mobility is measure of ease of carrier drift:**

- **If $\tau_c$ ↑, longer time between collisions → μ ↑**
- **If $m^*$ ↓, "lighter" particle → μ ↑**

# Current Density

- The electron current density flowing in the direction of the applied field:

**Current density** $=$ **Charge on each carrier** $\times$ **Number of carriers flow through per unit area per unit time**




**Electron:** $$J_n = -qnv_{d\_n} = qn\mu_n\mathcal{E}$$

**Hole:** $$J_p = qpv_{d\_p} = qp\mu_p\mathcal{E}$$

$$\frac{ampere}{cm^2} = coulomb \cdot \frac{1}{cm^3}\frac{cm}{s}$$

# Conductivity

- Total current including both electron and hole:

$$\boxed{J = q(n\mu_n + p\mu_p)\mathcal{E}}$$

- Define conductivity:

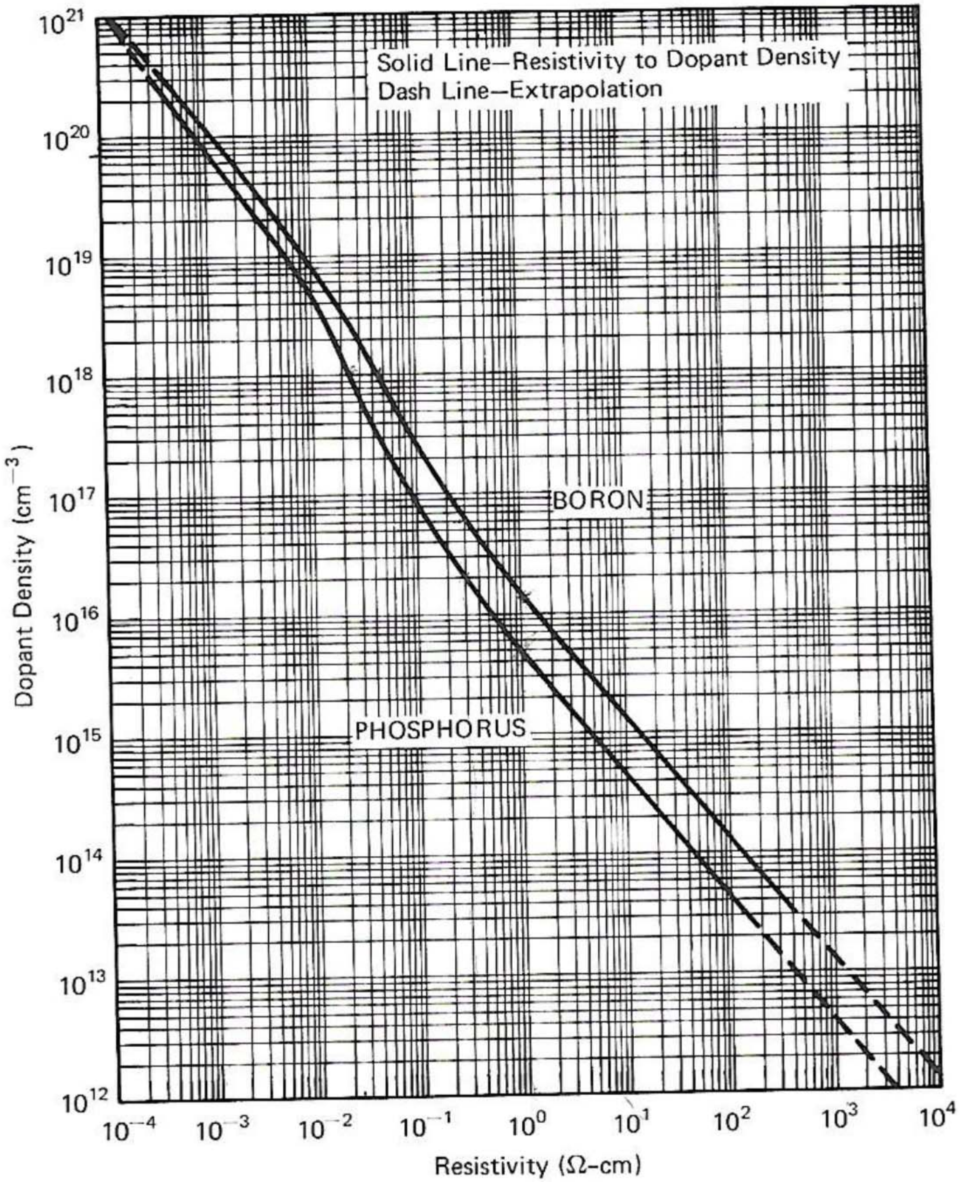
$$\boxed{\sigma = J/\mathcal{E}}$$

$$\Rightarrow \quad \boxed{\sigma = q(n\mu_n + p\mu_p)}$$

# Impact of doping on conductivity/resistivity

- Conductivity: $\boldsymbol{\sigma = J/\mathcal{E}}$
- Resistivity: $\boldsymbol{\rho = \mathcal{E}/J = \frac{1}{\sigma}}$
- Conductivity increase with increasing carrier concentration

$$\boldsymbol{\sigma = q(n\mu_n + p\mu_p)}$$



# Example problem

- For a piece of P-type piece of silicon at 300K, its conductivity is 4 $\Omega^{-1}cm^{-1}$ , assuming the P-type silicon has $\mu_n$ = 1200 $cm^2/V$-s and $\mu_p$ = 300 $cm^2/V$-s, calculate the hole and electron concentration

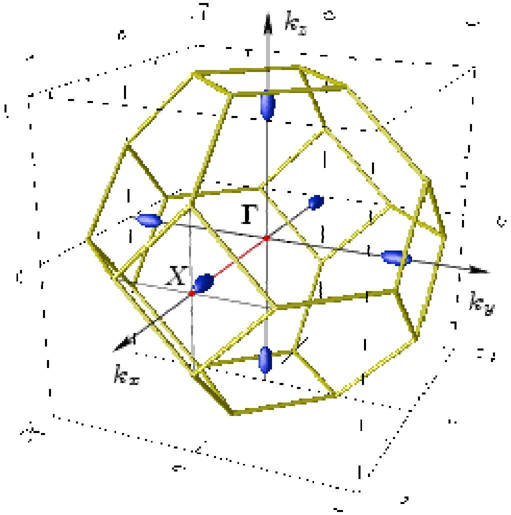
# Example problem

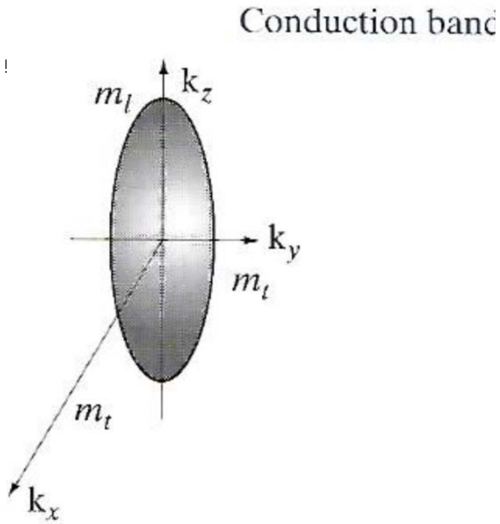
- Ex: What is the hole drift velocity at room temperature in silicon, in a field $\mathcal{E}$ = 1000 V/cm? What is the average time and distance between collisions?

# Effective Mass

## 1. Band curvature Effective Mass:

$$m^* = \frac{\hbar^2}{d^2E/dk^2}$$




## 2. Density of State Effective Mass:

$$m^*_{n\_DOS} = 6^{2/3}(m_l m_t^2)^{1/3}$$

**For density of state and carrier concentration calculation**

$$N_C = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

## 3. Conductivity Effective Mass:

$$\frac{1}{m^*_{n\_cond}} = \frac{1}{3}\left(\frac{1}{m_l} + \frac{2}{m_t}\right)$$

**For conductivity calculation**

$$\mu = \frac{q\boldsymbol{\tau_c}}{m^*}$$

## Effective mass and energy bandgap of Ge, Si and GaAs

| Name | Symbol | Germanium | Silicon | Gallium Arsenide |
|---|---|---|---|---|
| Smallest energy bandgap at 300 K | $E_g$ (eV) | 0.66 | 1.12 | 1.424 |
| **Effective mass for density of states calculations** | | | | |
| Electrons | $m_{e,dos}^*/m_0$ | 0.56 | 1.08 | 0.067 |
| Holes | $m_{h,dos}^*/m_0$ | 0.29 | 0.57/0.81$^2$ | 0.47 |
| **Effective mass for conductivity calculations** | | | | |
| Electrons | $m_{e,cond}^*/m_0$ | 0.12 | 0.26 | 0.067 |
| Holes | $m_{h,cond}^*/m_0$ | 0.21 | 0.36/0.386$^2$ | 0.34 |

$m_0 = 9.11 \times 10^{-31}$ kg is the free electron rest mass.

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  - Drift and resistance
  - Effect of temperature and doping on mobility
- Invariance of the Fermi level at equilibrium

# Drift and resistance

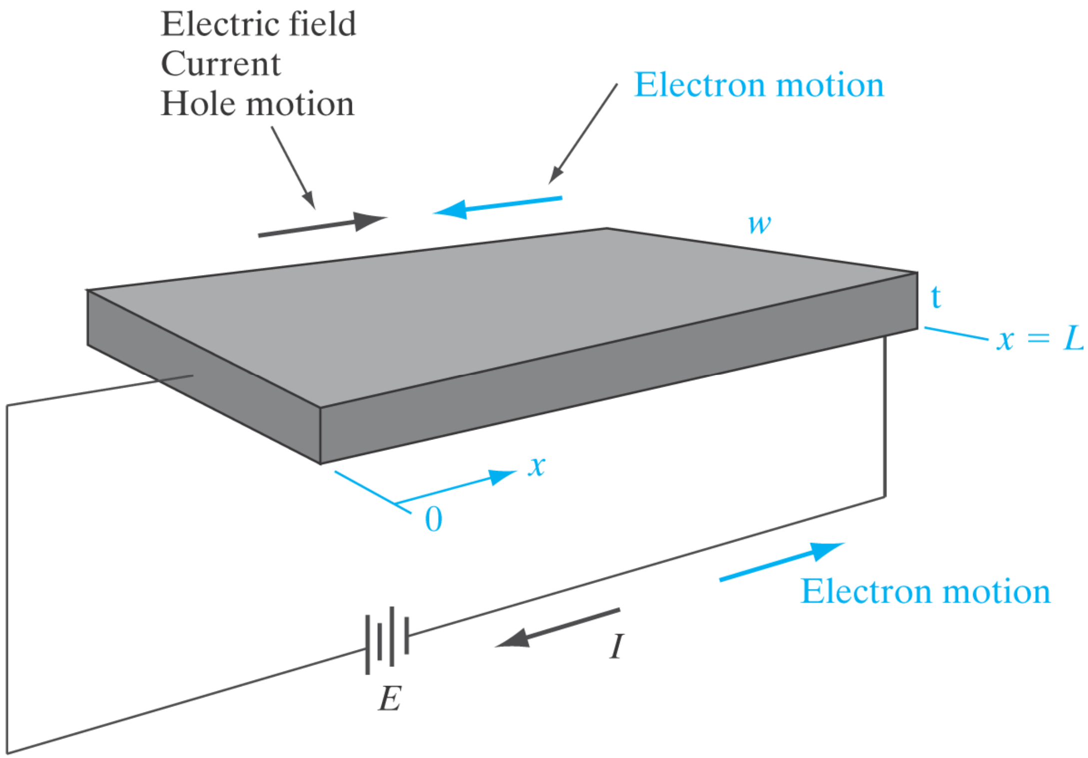
- Ohm's Law

$$I = \frac{V}{R}$$

- Resistance of the bar:

$$\boxed{R = \rho \frac{L}{wt}}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{q(n\mu_n + p\mu_p)}$$



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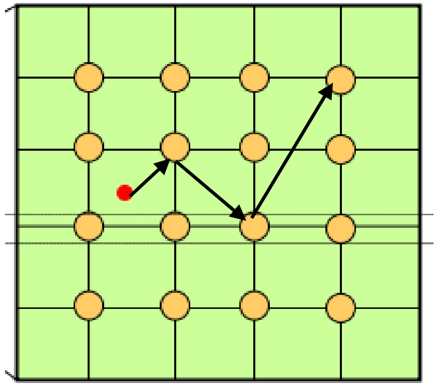
# Scattering and mobility

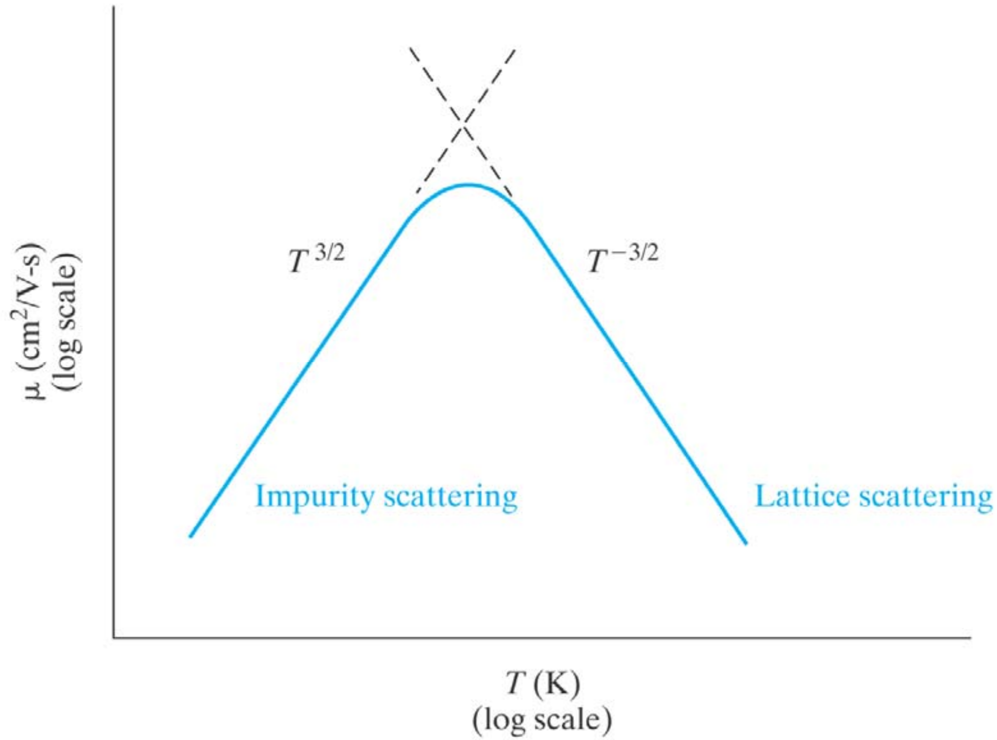
- Two basic types of scattering mechanism that influence mobility:
  - Lattice scattering



  - Impurity scattering

# Temperature dependence of mobility

- As T increase, thermal agitation of the lattice increases, the frequency of lattice scattering event increases → mobility limited by lattice scattering decrease
- At T decrease, thermal motion of the carrier is slower, carrier is scattered more strongly by an charged ion → mobility limited by impurity scattering decrease



# Matthiessen Rule

- The total probability of a carrier is scattered in the time interval $dt$ is the sum of probability of being scattered by each mechanism:

$$\frac{dt}{\tau_c} = \sum_i \frac{dt}{\tau_i}$$

- Therefore the mobility due to two or more scattering mechanisms is:

$$\frac{\mathbf{1}}{\boldsymbol{\mu}} = \frac{\mathbf{1}}{\boldsymbol{\mu}_{\mathbf{1}}} + \frac{\mathbf{1}}{\boldsymbol{\mu}_{\mathbf{2}}} + \cdots$$

- The mechanism causing the lowest mobility value dominants.

# Effect of impurity concentration on mobility

- Higher the doping concentration,

→higher the impurity scattering rate

→ lower mobility

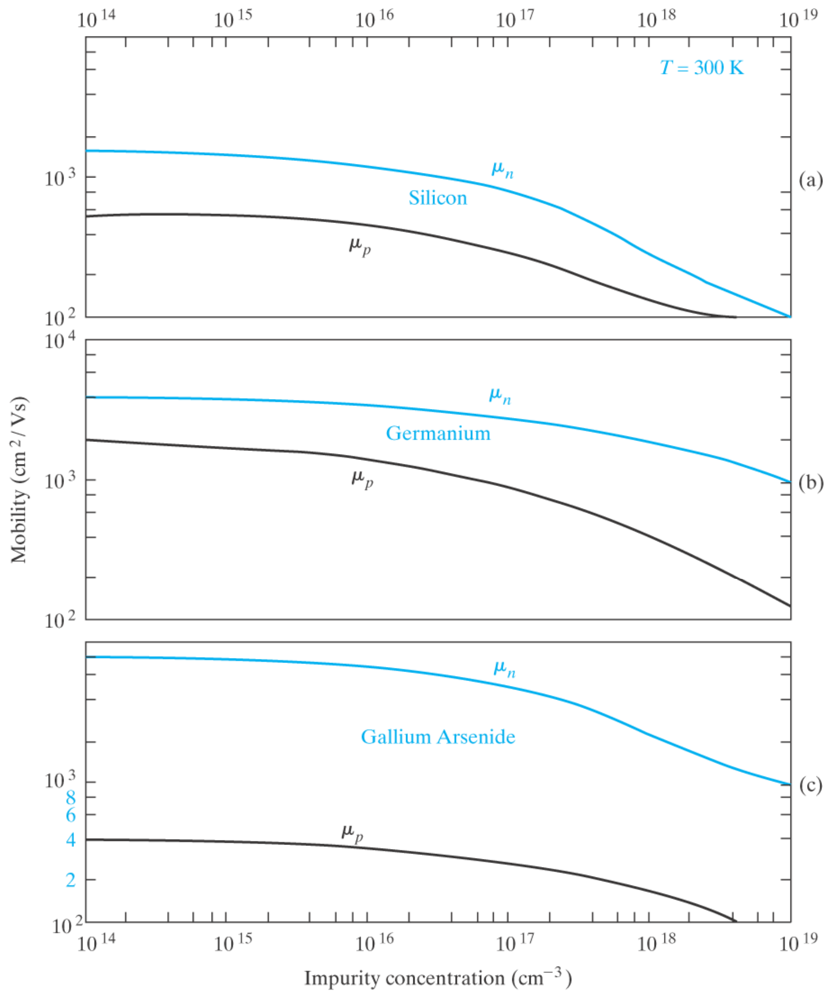
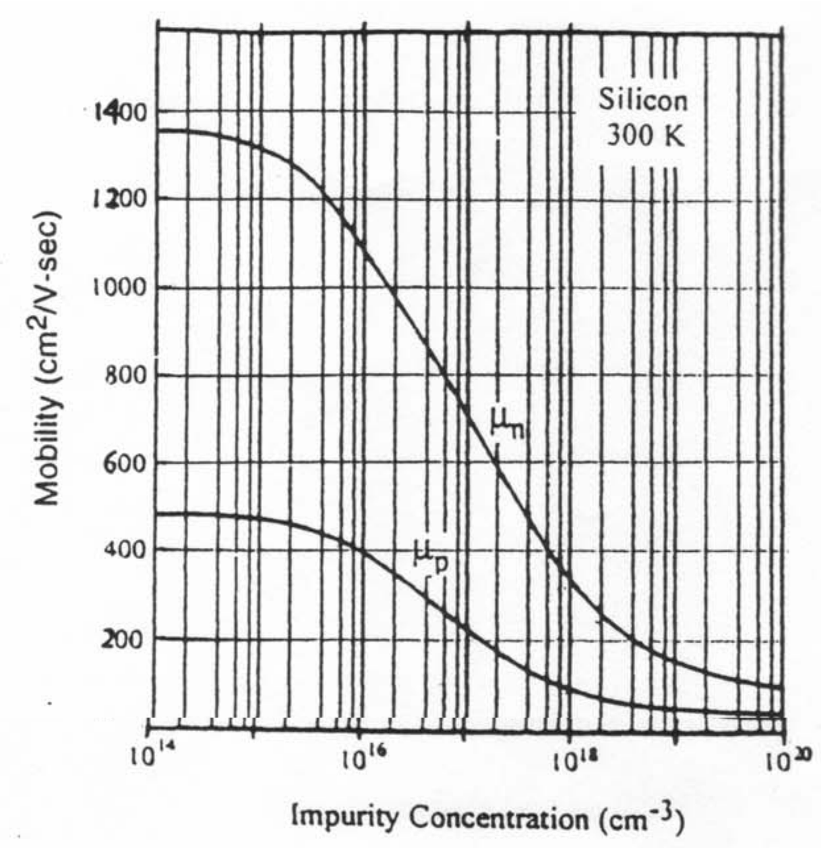
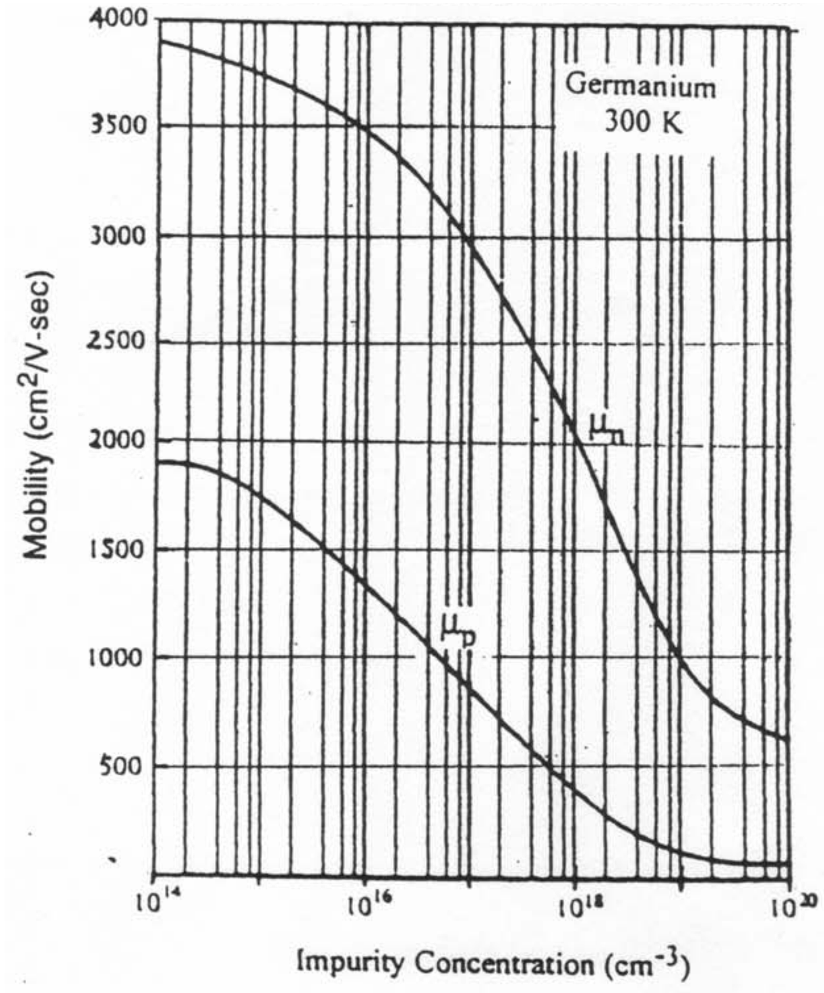



Figure 3.23

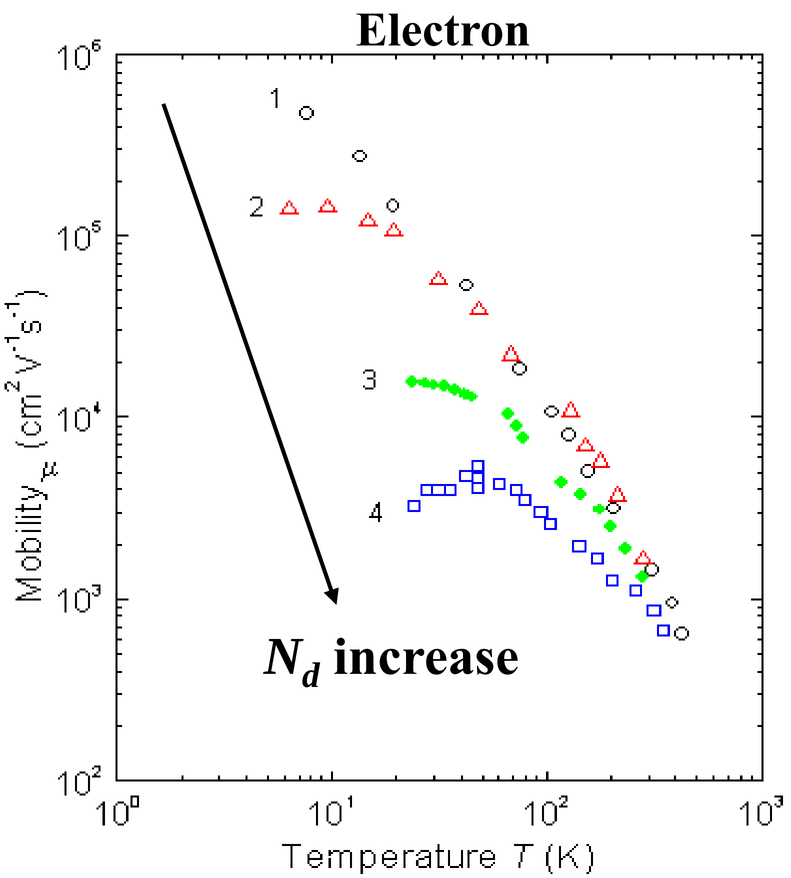
Variation of mobility with total doping impurity concentration $\boxed{(N_a + N_d)}$ for Ge, Si, and GaAs at 300 K.

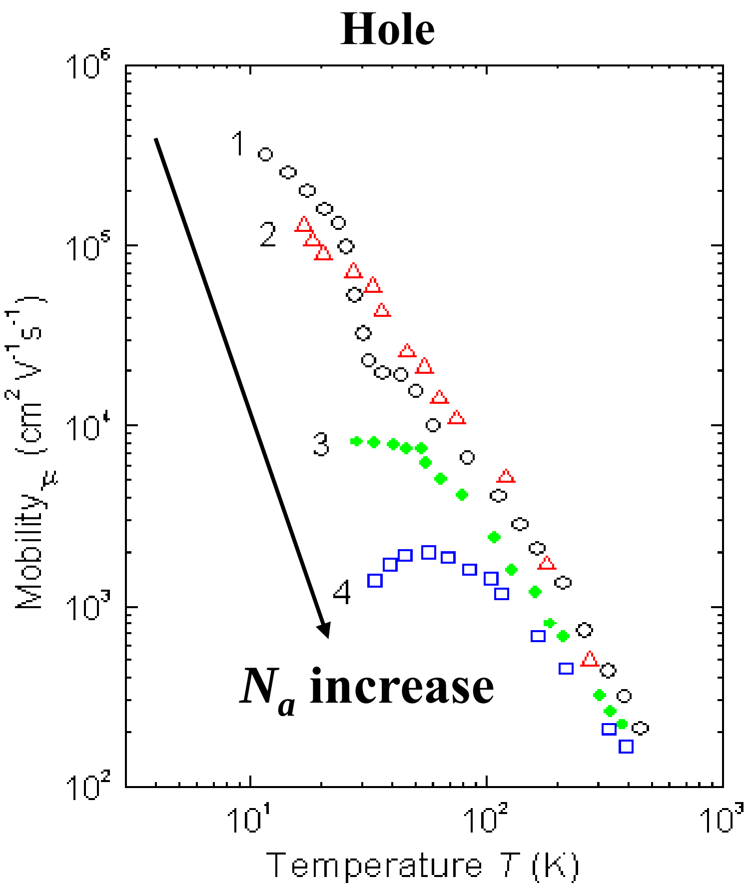
- In linear scale, from the ECE 340 course web site:



# Mobility versus temperature for different doping levels




- Impurity ↑ → $\mu$ ↓, due to impurity scattering;
- At low T, $T$ ↑→ $\mu$ ↑ due to impurity scattering;
  At high T, $T$ ↑→ $\mu$ ↓ due to lattice scattering;

# Resistivity dependence on doping

- For n type material:

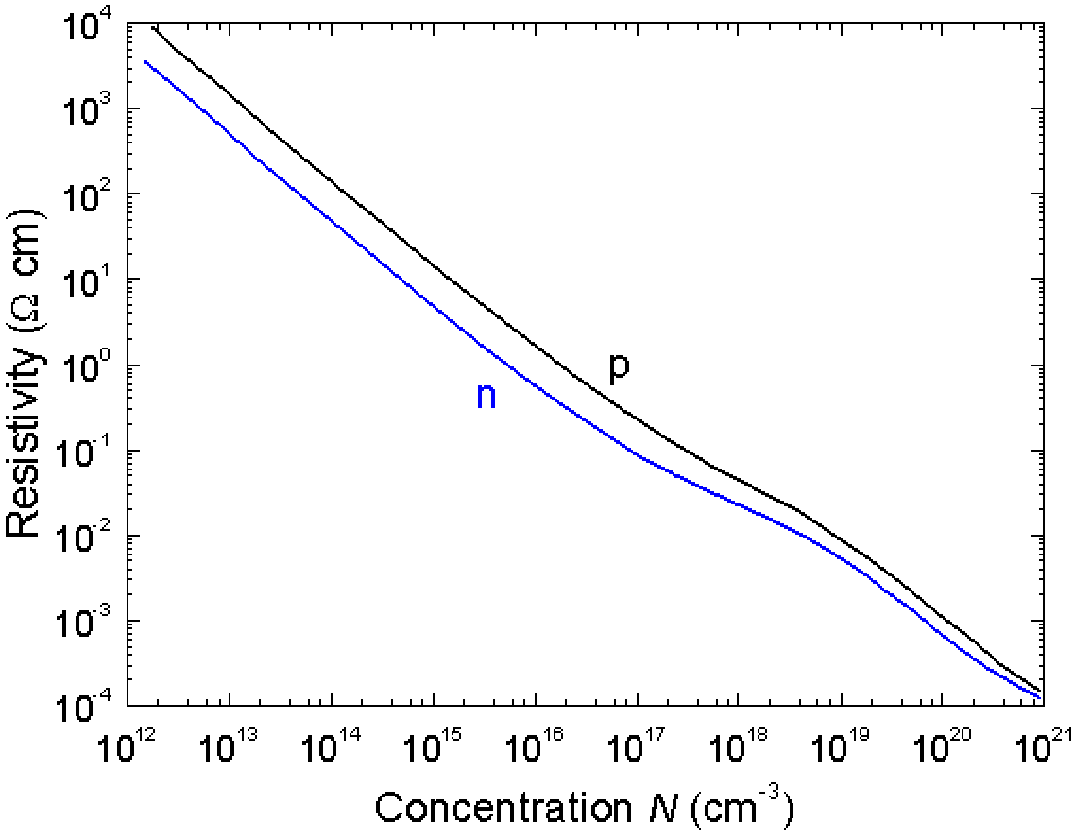
$$\boldsymbol{\rho = \frac{1}{qn\mu_n}}$$

- For p type material:

$$\boldsymbol{\rho = \frac{1}{qp\mu_p}}$$

- Doping ↑→ $n$ (or p) ↑, ($\mu$ ↓ slightly) → $\rho$ ↓

- We have control over resistivity via doping!




**Note: this plot do not apply to compensated material**

# Example problem 1

- Ex: What is the hole drift velocity at room temperature in silicon, in a field $\mathcal{E}$ = 1000 V/cm? What is the average time and distance between collisions?

Hint:

| | Si | Ge | GaAs | InAs |
|---|---|---|---|---|
| $\mu_n$ (cm$^2$/V·s) | 1400 | 3900 | 8500 | 30000 |
| $\mu_p$ (cm$^2$/V·s) | 470 | 1900 | 400 | 500 |

# Example problem 2

Consider a Si sample doped with $10^{16}/cm^3$ Boron. What is its resistivity?

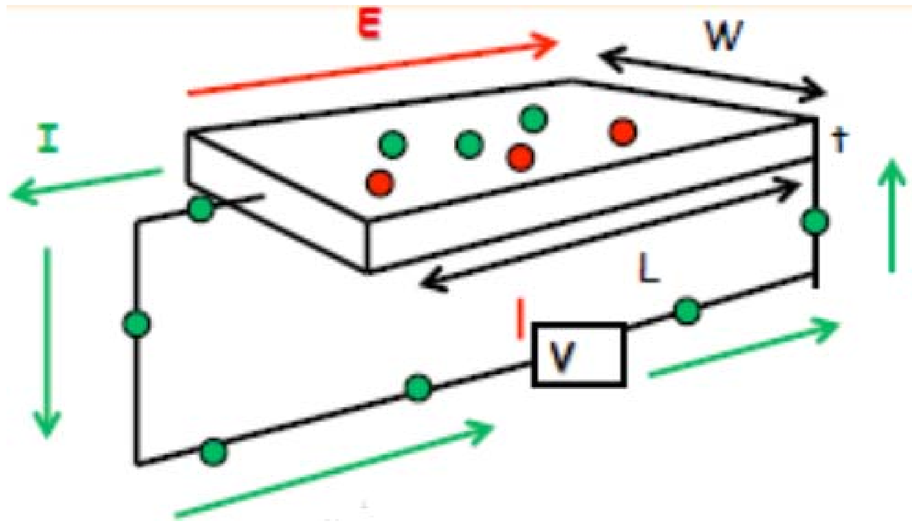
Consider the same Si sample, doped *additionally* with $10^{17}/cm^3$ Arsenic. What is its resistivity?

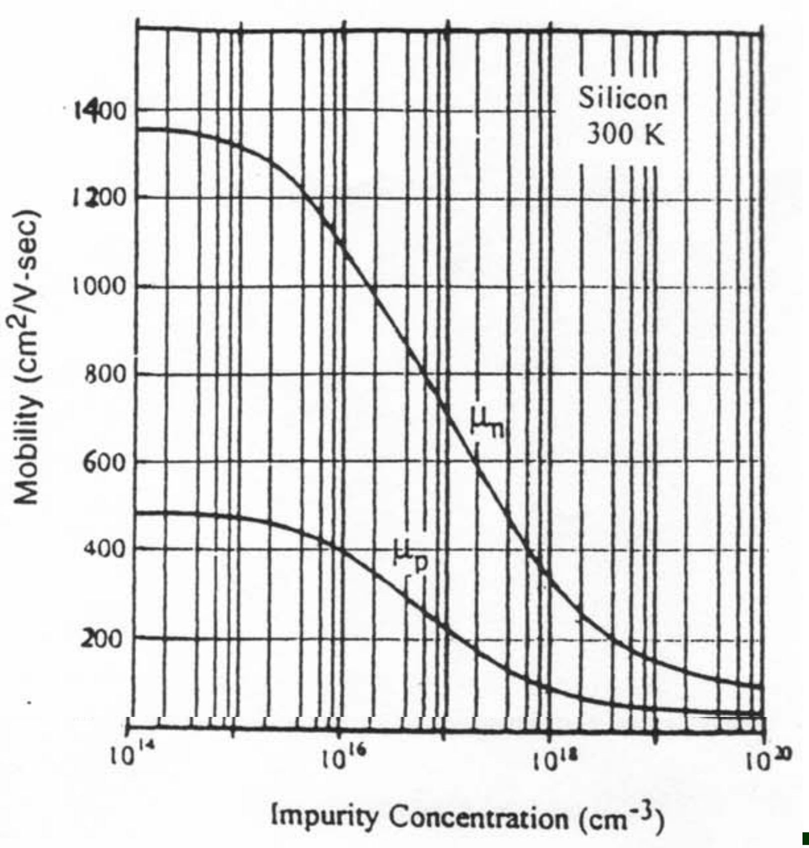
# Example problem 3

Consider a silicon bar

L=0.1cm
A=100 μm$^2$
Nd=$10^{17}$cm$^{-3}$

Find the current with 10V applied at 300K



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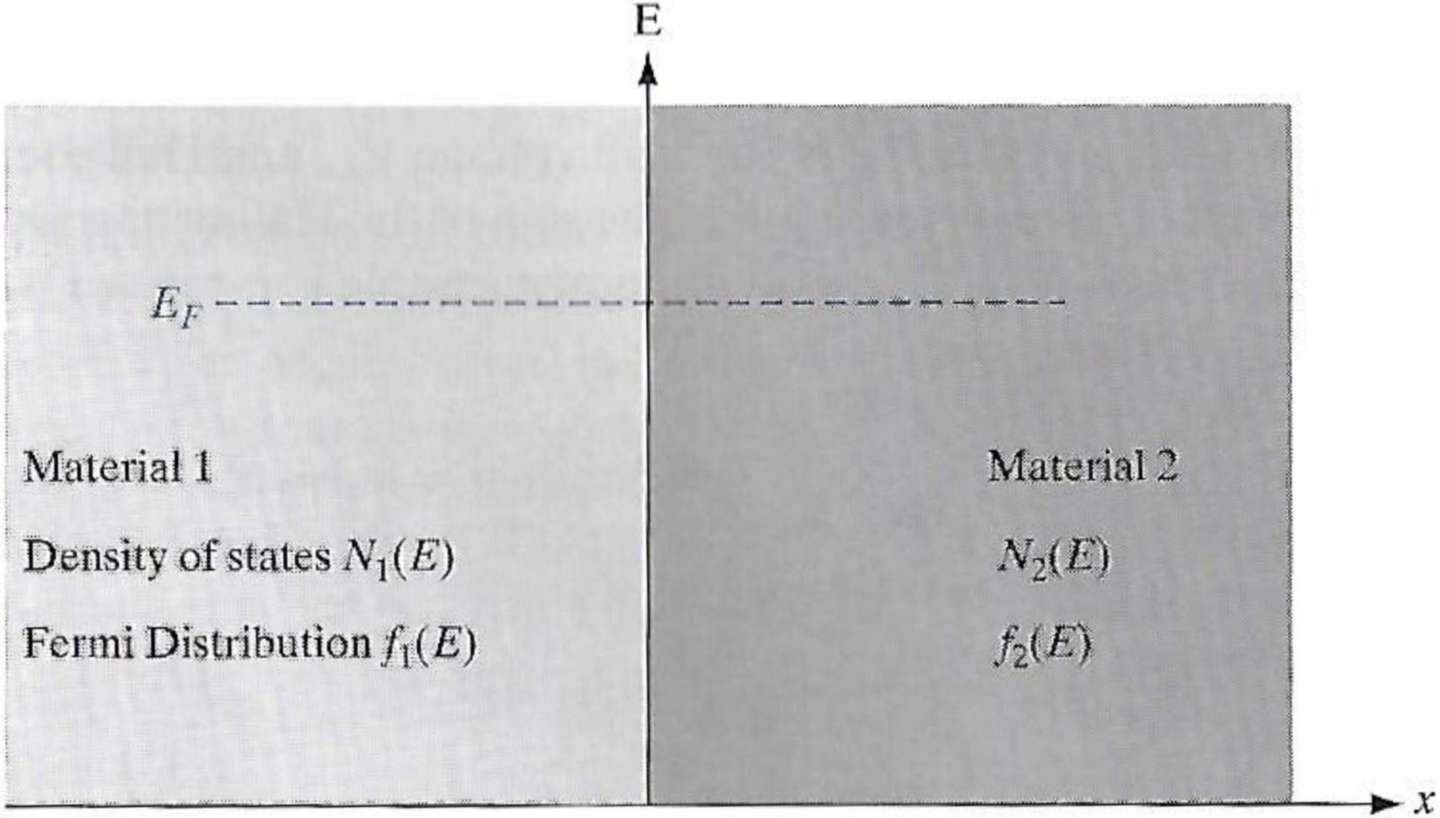
# Fermi level invariance

- Invariance of the Fermi level at equilibrium
  - No discontinuity or gradient can arise in the equilibrium Fermi level



# Fermi level invariance

- At energy E the rate of transfer of electrons is:

$$\text{rate from 1 to 2} \propto N_1(E)f_1(E) \cdot N_2(E)[1 - f_2(E)]$$

$$\text{rate from 2 to 1} \propto N_2(E)f_2(E) \cdot N_1(E)[1 - f_1(E)]$$

At equilibrium, there is no current, i.e. no net charge transport, no net transfer of energy:

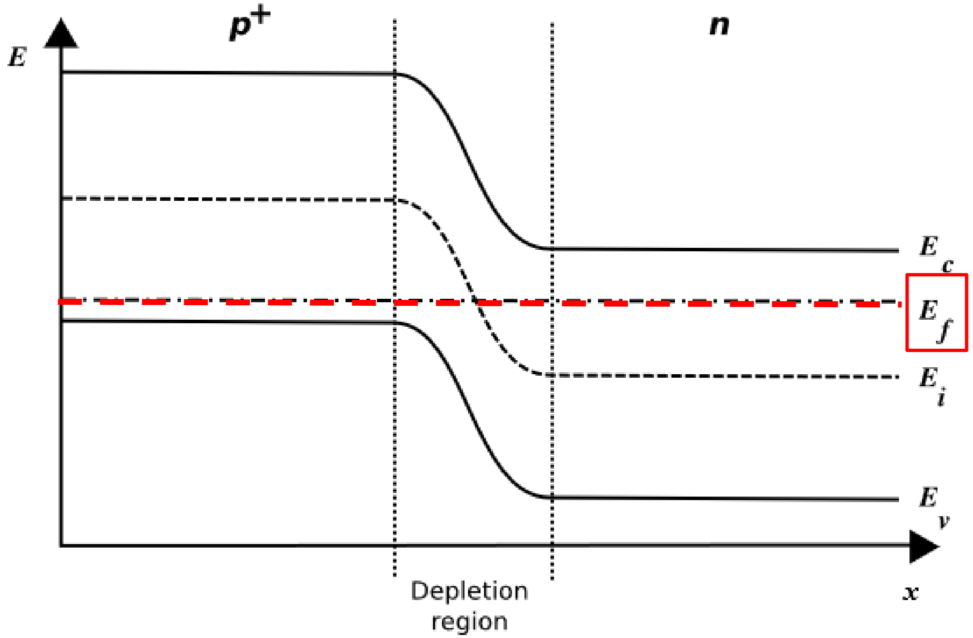
$$N_1(E)f_1(E) \cdot N_2(E)[1 - f_2(E)] = N_2(E)f_2(E) \cdot N_1(E)[1 - f_1(E)]$$

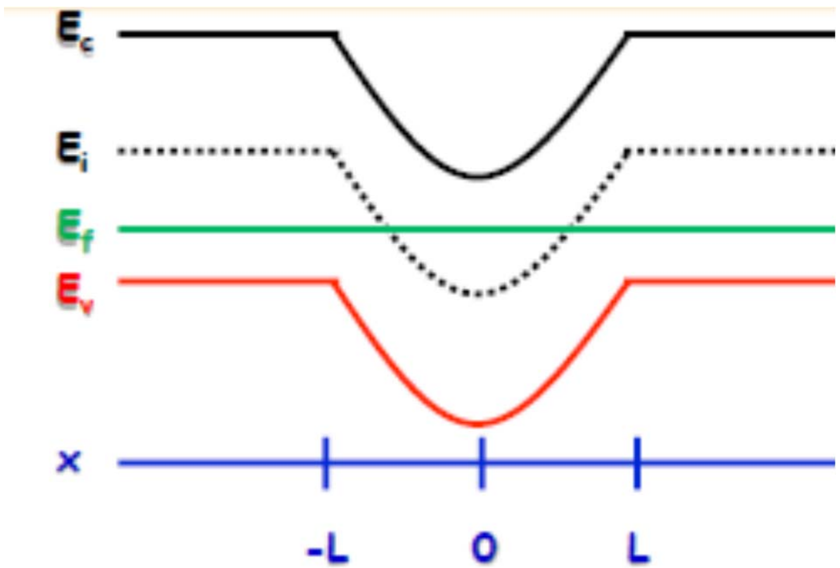
$$\Rightarrow \quad f_1(E) = f_2(E)$$

$$\Rightarrow \quad E_{F1} = E_{F2}$$

More generally: $\boxed{\dfrac{dE_F}{dx} = 0}$

# Example energy diagram at equilibrium




- At equilibrium, Fermi level is **FLAT**, i.e. no gradient.