

1 Chapter 1

- Energy E of a photon of light in eV:

$$\lambda = \frac{1.24 \text{ eV}}{E}$$

- Distance D between adjacent planes in cubic lattices:

$$D = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

- Angle between 2 Miller index directions A and B :

$$\cos(\theta) = \frac{A \bullet B}{|A||B|}$$

2 Chapter 2

- Planck relationship:

$$E = hv = \left(\frac{h}{2\pi}\right)(2\pi v) = \hbar\omega$$

- Classical energy of a particle:

$$\frac{1}{2}mv^2 = \frac{1}{2} \frac{m^2 v^2}{m} = \frac{\rho^2}{2m}$$

- De Broglie:

$$\lambda = \frac{h}{\rho} = \frac{h}{mv} \Rightarrow \rho = \frac{h}{\lambda} = \frac{h}{2\pi} \frac{2\pi}{\lambda} = \hbar k$$

- Momentum or energy in terms of k can be derived by combining De Broglie with the classical energy of a particle:

$$E = \hbar\omega = \frac{\rho^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

- Rydberg Constant:

$$R = 109,678 \text{ cm}^{-1}$$

- Lyman:

$$v = cR\left(\frac{1}{1^2} - \frac{1}{n^2}\right), n = 2, 3, 4, \dots$$

- Balmer:

$$v = cR\left(\frac{1}{2^2} - \frac{1}{n^2}\right), n = 3, 4, 5, \dots$$

- Paschen:

$$v = cR\left(\frac{1}{3^2} - \frac{1}{n^2}\right), n = 4, 5, 6, \dots$$

- Postulate for Bohr:

$$\rho_0 = n\hbar, n = 1, 2, 3, 4, \dots$$

- Finding radial forces on orbiting electron:
(Electrical force toward nucleus) = (Equivalent force in terms of radial acceleration)

$$-\frac{q^2}{kr^2} = -\frac{mv^2}{r}$$

$$\rho_0 = n\hbar = mvr \Rightarrow mv^2 = \frac{m^2 v^2}{m} = \frac{n^2 \hbar^2}{mr^2}$$

$$\frac{q^2}{kr^2} = \frac{1}{mr} \frac{n^2 \hbar^2}{r^2} \Rightarrow r_n = \frac{kn^2 \hbar^2}{mq^2}$$

$|r_n$ is the radius of the n th orbit

$$-\frac{q^2}{kr^2} = -\frac{mv^2}{r} \Rightarrow \frac{n\hbar}{rm} \Rightarrow \frac{q^2}{kn\hbar}$$

by subbing r from above.

\Rightarrow K.E. of $e^- =$

$$\frac{1}{2}mv^2 = \frac{mq^4}{2k^2n^2\hbar^2}$$

P.E. of $e^- =$

$$-\frac{q^2}{kr_n} = -\frac{mq^4}{k^2n^2\hbar^2}$$

by subbing r

Total Energy of $e^- =$

$$E_n = KE = PE = -\frac{mq^4}{2k^2n^2\hbar^2} = -KE$$

- Energy difference between orbits:

$$E_{n2} - E_{n1} = \frac{mq^4}{2k^2\hbar^2} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

- Frequency of light given by a transition between orbits:

$$V_{21} = \left[\frac{mq^4}{2k^2\hbar^2} \right] \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

- Heisenberg uncertainty principle:

$$(\Delta x)(\Delta p_x) \geq \frac{\hbar}{2} \quad (1)$$

$$(\Delta E)(\Delta t) \geq \frac{\hbar}{2} \quad (2)$$

2.1 Quantum Mechanics

Classical Variable	\rightarrow	Quantum Operator
x	\rightarrow	x
$f(x)$	\rightarrow	$f(x)$
$\rho(x)$	\rightarrow	$\frac{\hbar}{j} \frac{\partial}{\partial x}$
E	\rightarrow	$-\frac{\hbar}{j} \frac{\partial}{\partial t}$

- Normalization of the probability density (the wave function is the probability density):

$$\int_{-\infty}^{\infty} \Psi^* \Psi dx dy dz = 1 \quad (3)$$

- Time averaged expectation of the particle state

$$\langle Q \rangle = \int_{-\infty}^{\infty} \Psi^* Q_{op} \Psi dx dy dz \quad (4)$$

- Classical energy of a partical:

$$KE + PE = E \Rightarrow \frac{1}{2}mv^2 + V = E \quad (5)$$

$$\frac{1}{2}mv^2 = \frac{1}{2} \frac{(mv)^2}{m} = \frac{1}{2} \frac{\rho^2}{m} = \frac{\rho^2}{2m} \Rightarrow \frac{\rho^2}{2m} + V = E \quad (6)$$

$$\rho \rightarrow \frac{\hbar}{j} \frac{\partial}{\partial x}, E \rightarrow -\frac{\hbar}{j} \frac{\partial}{\partial t}$$

$$\Rightarrow \frac{-1}{2m} \hbar \frac{\partial^2}{\partial x^2} \Psi(x, t) + V(x) \Psi(x, t) = \frac{-\hbar}{j} \frac{\partial \Psi(x, t)}{\partial t}$$

where

$$\left(\frac{\partial}{\partial x} \right)^2 \rightarrow \frac{\partial^2}{\partial x^2}, j^2 = -1$$

- Wave function in 3D then:

$$\frac{-\hbar}{2m} \nabla^2 \Psi + V \Psi = \frac{-\hbar}{j} \frac{\partial \Psi}{\partial t} \Rightarrow \nabla^2 \Psi = \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} \quad (7)$$

- Separation of variables:

$$\frac{-\hbar}{2m} \frac{\partial^2 \Psi(x, t)}{\partial x^2} + V(x) \Psi(x, t) = \frac{-\hbar}{j} \frac{\partial \Psi(x, t)}{\partial t} \quad (8)$$

\Rightarrow

$$-\frac{\hbar}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} \phi(t) + V(x) \psi(x) \phi(t) = -\frac{\hbar}{j} \psi(x) \frac{\partial \phi}{\partial t} \quad (9)$$

\Rightarrow

$$\frac{d\phi(t)}{dt} + \frac{j}{\hbar} E \phi(t) = 0 \quad (10)$$

(time dependent portion)

$$-\frac{\hbar}{2m} \frac{d^2 \psi(x)}{dx^2} + V(x) \psi(x) = E \psi(x) \quad (11)$$

(time independent portion)

- $E \equiv$ equivalent constant, corresponds to total energy of the particle
- Wave function as linear combination of various eigenfunctions

$$\psi(x, t) = \sum_n C_n \Psi_n e^{-j \frac{E_n}{\hbar} t} \Rightarrow E_n \equiv nth \text{ prefactor} \quad (12)$$

- Infinite potential well

$$V(x) = \begin{cases} 0 & , x \neq 0 \text{ and } x \neq L \\ \infty & , x = 0 \text{ or } x = L \end{cases}$$

\Rightarrow

$$\left(\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \right) \psi(x) = E \psi(x) \Rightarrow \frac{d^2 \psi(x)}{dx^2} = \frac{2m}{\hbar^2} E \psi(x) \quad (13)$$

\Rightarrow

$$\frac{d^2 \psi(x)}{dx^2} + \frac{2m}{\hbar^2} E \psi(x) = 0 \quad (14)$$

\Rightarrow

$$\psi(x) = \{ \sin(kx), \cos(kx) \} \Rightarrow k = \frac{\sqrt{2mE}}{\hbar} \quad (15)$$

$$\cos(kx) \neq 0 \text{ when } x = 0 \Rightarrow A \sin(kx) \text{ is our solution. } \Rightarrow k = \frac{\sqrt{2mE}}{\hbar} \quad (16)$$

$$\text{when potential is 0 at } x = 0 \text{ and } L, k = \frac{n\pi}{L}, n = \{1, 2, 3, \dots\} \quad (17)$$

$$\text{therefore: } \frac{\sqrt{2mE_n}}{\hbar} = \frac{n\pi}{L} \Rightarrow E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \quad (18)$$

- A is found by normalizing the probability density integral

$$\int_{-\infty}^{\infty} \Psi^* \Psi dx = \int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = A^2 \frac{L}{2} \text{review trig calc and show the process here} \quad (19)$$

Set the above to 1 to find A

$$\frac{A^2 L}{2} = 1 \Rightarrow A = \sqrt{\frac{2}{L}} \quad (20)$$

$$\text{Therefore: } \psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}\right) \text{ for an infinite well} \quad (21)$$

- Parabolic Potential Well (simple harmonic oscillator)

$$V(x) = kx^2, E_n = \left(n + \frac{1}{2}\right) \hbar \omega \quad (22)$$

- Coulombic Potential Well

$$\frac{d^2}{d\phi^2}\Phi + m^2\Phi = 0 \Rightarrow \Phi_m(\phi) = Ae^{jm\phi} \quad (23)$$

$$1 = \int_0^{2\pi} \Psi^* \Psi d\psi \Rightarrow \int_0^{2\pi} \Phi_m^*(\phi) \Phi_m(\phi) d\phi = 1 \quad (24)$$

$$\Rightarrow A^2 \int_0^{2\pi} e^{-jm\phi} e^{jm\phi} d\phi \quad (25)$$

$$\Rightarrow A^2 \int_0^{2\pi} d\phi = 2\pi A^2 \quad (26)$$

$$\Rightarrow A = \frac{1}{\sqrt{2\pi}} \Rightarrow \Phi_m(\phi) = \frac{1}{\sqrt{2\pi}} e^{jm\phi} \quad (27)$$

$$\ni m = \{\dots, -3, -2, -1, 0, 1, \dots\}$$

similar for $\Theta_l(\theta)$ and $R_n(r)$

- Atomic numbers describing allowable states in a hydrogen atom

$$n = 1, 2, 3, \dots$$

$$l = 0, 1, 2, 3, \dots, n-1$$

$$m(m_l) = -l, \dots, -2, -1, 0, 1, 2, \dots, l$$

$$s(m_s) = \pm \frac{\hbar}{2}$$

n	l	m	$\frac{s}{\hbar}$	Allowable states in subshell	Allowable states in complete shell
1	0	0	$\pm \frac{1}{2}$	2	2
2	0	0	$\pm \frac{1}{2}$	2	
2	1	-1	$\pm \frac{1}{2}$		8
2	1	0	$\pm \frac{1}{2}$	6	
2	1	1	$\pm \frac{1}{2}$		
3	0	0	$\pm \frac{1}{2}$	2	18
3	1	-1	$\pm \frac{1}{2}$		
3	1	0	$\pm \frac{1}{2}$	6	18
3	1	1	$\pm \frac{1}{2}$		
3	2	-2	$\pm \frac{1}{2}$		
3	2	-1	$\pm \frac{1}{2}$		
3	2	0	$\pm \frac{1}{2}$	10	18
3	2	1	$\pm \frac{1}{2}$		
3	2	2	$\pm \frac{1}{2}$		

- n is the principle atomic number
- l is the number that determines s,p,d,f,g,...

3 Chapter 3

- Equilibrium number of EHP's in pure Si at room temp:

$$10^{10} \frac{EHP}{cm^3}$$

- Si atom density in pure Si at room temp:

$$5 * 10^{22} \frac{atoms}{cm^3}$$

- Relationship of (E,k) for a free electron related to electron mass

$$\frac{d^2 E}{dk^2} = \frac{\hbar^2}{m} \Rightarrow m^* = \frac{\hbar^2}{\frac{d^2 E}{dk^2}} \text{ so the curvature of the band determines the } e^- \text{ effective mass} \quad (28)$$

- Velocity of an electron (v) is the group velocity of a quantum mechanical electron wavepacket

$$v = \frac{d\omega}{dk} = \left(\frac{1}{\hbar}\right) \frac{dE}{dk} \quad (29)$$

- Rate of recombination of electrons and holes r_i is proportional to equilibrium concentrations of electrons and holes n_0 and p_0

$$r_i = \alpha_r n_0 p_0 = \alpha_r n_i^2 = g_i \quad (30)$$

- Calculating the approximate energy required to excite a donor electron into conduction band.

Consider the loosely bound electron as circling the tightly bound “core” electrons.

Magnitude of the “ground state” is:

$$E = \frac{mq^4}{2k^2\hbar^2} \ni k = 4\pi\epsilon_0\epsilon_r, m \equiv m_n^* \text{ (conductivity effective mass)} \quad (31)$$

- Radius of of electron orbit around donor, assuming ground state. Equation was pulled from chapter 2

$$r = \frac{4\pi\epsilon_r\epsilon_0\hbar^2}{m_n^*q^2} \quad (32)$$

- Intrinsic carrier concentration n_i of Si is about 10^{10} cm^{-3} at room temperature

3.1 Carrier Concentrations

- Standard Fermi Equation:

$$f(E) = \frac{1}{1 + e^{\frac{(E-E_F)}{kT}}} \ni k \text{ is the Boltzman constant } k = 8.62 * 10^{-5} \frac{eV}{K} = 1.38 * 10^{-23} \frac{J}{K} \quad (33)$$

- Calculate the number of electrons and holes in a semiconductor when you know the density of states in valence and conduction bands are known

- Concentration of electrons in the conduction band by integral:

$$n_0 = \int_{E_c}^{\infty} f(E)N(E)dE \ni N(E)d(E) \text{ is the density of states } (cm^{-3}) \text{ in the energy range of } dE \quad (34)$$

- Concentration of electrons in the conduction band by using *Effective Density of states*:

$$n_0 = N_c f(E_c) \ni N_c \text{ is the effective density of states at } E_c \quad (35)$$

- When $E-E_F$ is several kT we can simplify the equation by recognizing that the exponential dominates the denominator

$$f(E_c) = \frac{1}{1 + e^{\frac{(E-E_F)}{kT}}} \Rightarrow e^{\frac{(E-E_F)}{kT}} \quad (36)$$

$$n_0 = N_c e^{-\frac{E_c-E_F}{kT}} \quad (37)$$

- Effective Density of States is:

$$N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{\frac{3}{2}} \ni m_n^* \equiv \text{the density of states effective mass for electrons} \quad (38)$$

- Density of States Effective Mass for Electrons is:

$$(m_n^*)^{\frac{3}{2}} = 6(m_i m_t^2)^{\frac{1}{2}} \quad (39)$$

- So by similar reasoning, the concentration of holes in the valence band is:

$$p_0 = N_v [1 - f(E_v)] \quad (40)$$

where N_v is the effective density of states for the valence band.

- The probability of finding an empty state at E_v is:

$$1 - f(E_v) = 1 - \frac{1}{1 + e^{\frac{E_v - E_F}{kT}}} \simeq e^{-\frac{(E_F - E_v)}{kT}} \quad (41)$$

- So the concentration of holes in the valence band is:

$$p_0 = N_v e^{-\frac{E_F - E_v}{kT}} \quad (42)$$

- With effective density of states in the valence band:

$$N_v = 2 \left(\frac{2\pi m_p^* kT}{h^2} \right)^{\frac{3}{2}} \quad (43)$$

- For *intrinsic material*, E_F is at some intrinsic level E_i near the middle of the band gap. If $N_v = N_c$ then it actually is the middle of the band gap. Usually there is a difference in effective masses, though so there is usually a difference.
- Intrinsic electron and hole concentrations are:

$$n_i = N_c e^{-\frac{E_c - E_i}{kT}}, p_i = N_v e^{-\frac{E_i - E_v}{kT}} \quad (44)$$

- The product of n_0 and p_0 at equilibrium is a constant for a particular material and temperature, even if the doping is varied

$$n_0 p_0 = (N_c e^{-\frac{E_c - E_i}{kT}})(N_v e^{-\frac{E_i - E_v}{kT}}) = N_c N_v e^{-\frac{E_c - E_v}{kT}} = N_c N_v e^{-\frac{E_g}{kT}} \quad (45)$$

$$n_i p_i = (N_c e^{-\frac{E_c - E_i}{kT}})(N_v e^{-\frac{E_i - E_v}{kT}}) = N_c N_v e^{-\frac{E_g}{kT}} \quad (46)$$

$$\boxed{n_i = p_i \Rightarrow \sqrt{N_c N_v} e^{-\frac{E_g}{2kT}}, \quad n_0 p_0 = n_i^2} \quad (47)$$

$$\boxed{n_0 = n_i e^{-\frac{E_F - E_i}{kT}}} \quad (48)$$

$$\boxed{p_0 = n_i e^{-\frac{E_i - E_F}{kT}}} \quad (49)$$

3.2 Temperature Dependence of Carrier Concentrations

The exponential temperature dependence on the intrinsic carrier concentration dominates the other factors:

$$n_i(T) = 2 \left(\frac{2\pi kT}{h^2} \right)^{\frac{3}{2}} (m_n^* m_p^*)^{\frac{3}{4}} e^{-\frac{E_g}{2kT}} \quad (50)$$

3.3 Compensation and Space Charge Neutrality

- Compensation changes the effective number of donors:

$$N_{d_e} = N_d - N_a \text{ when the number of donors exceeds the number of acceptors} \quad (51)$$

$$N_{a_e} = N_a - N_d \text{ when the number of acceptors exceeds the number of donors} \quad (52)$$

- Space Charge Neutrality dictates that the number of positive charges (holes and ionized donor atoms) must balance the number of negative charges (electrons and ionized acceptor atoms):

$$p_0 + N_d^+ = n_0 + N_a^- \Rightarrow n_0 = p_0 + (N_d^+ - N_a^-) \quad (53)$$

- For a material that is doped n-type and the material is in the extrinsic region, we can approximate by recognizing that the donors dominate:

$$n_0 \simeq N_d - N_a \quad (54)$$

4 Drift of Carriers in Electric and Magnetic Fields

4.1 Conductivity and Mobility