$P(X=4) = \frac{1}{6}$

$P(X_1 + X_2 = 4) = \frac{3}{36} = \frac{1}{12}$

$P(\text{fair}) = \{0, 1\}$

$P(\text{rain})$

$P(\text{droppacket}) = \frac{1}{10000}$

lose 1 in 10,000

$P(Y_1 > Y_2)$

$P(\text{universe expands})$

Classical - equally likely

Frequentist - relative frequency

Bayesian

$P(X=4) = \frac{1}{6}$

$P(X_1 + X_2 = 4) = \frac{3}{36} = \frac{1}{12}$

$P(\text{fair}) \quad \in \{0,1\}$

$P(\text{rain})$

$P(\text{drop packet}) = \frac{1}{10000}$ lose 1 in 10,000

$P(Y_1 > Y_2)$

$P(\text{Universe expands})$

if rain win \$4 | if no rain lose \$1 | 4:1

if rain lose \$4 | if no rain win \$1

$P(\text{rain}) \frac{1}{1+4} = \frac{1}{5}$

Expected return

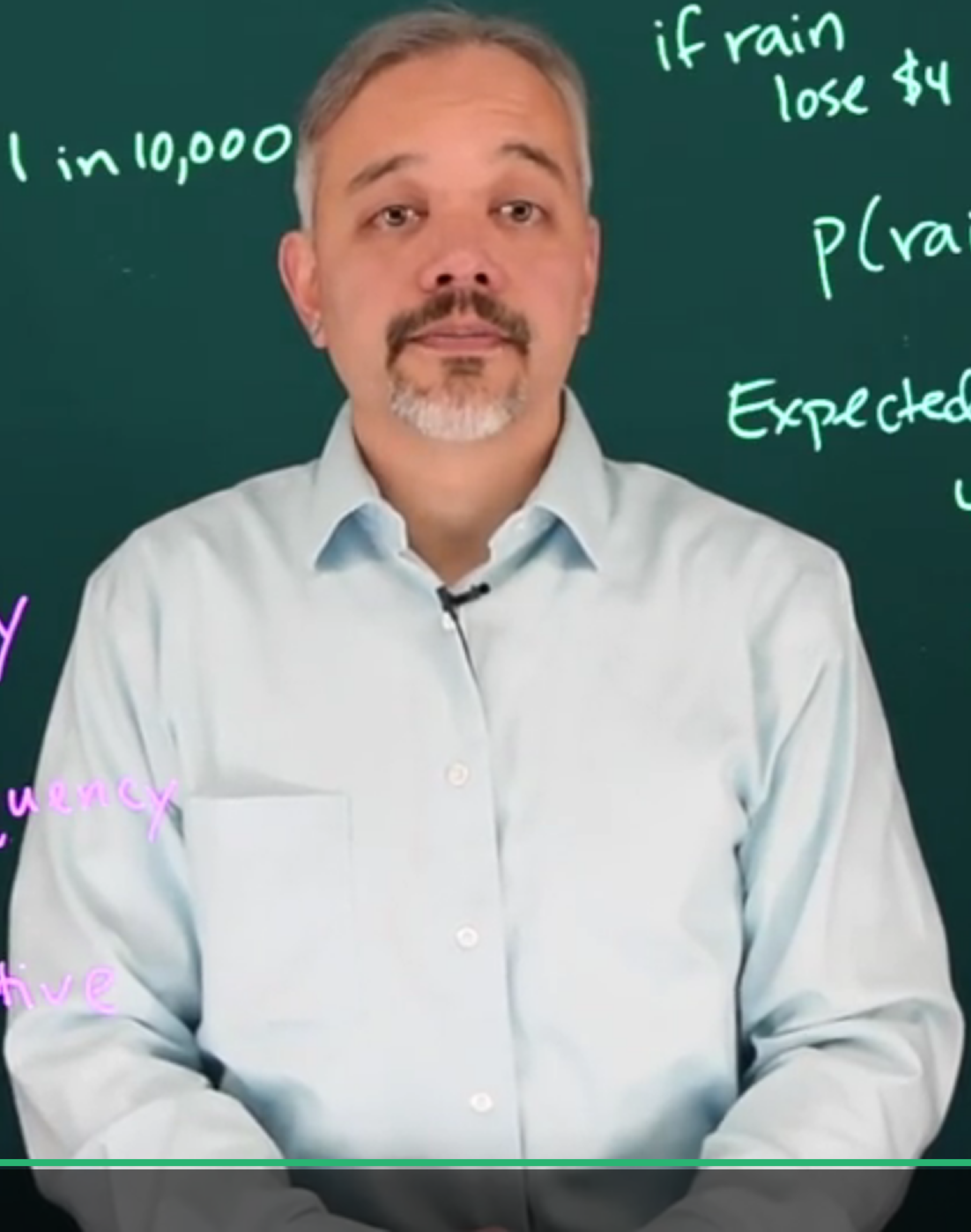
$4\left(\frac{1}{5}\right) - 1\left(\frac{4}{5}\right) = 0$

$1\left(\frac{4}{5}\right) - 4\left(\frac{1}{5}\right) = 0$

Classical – equally likely

Frequentist – relative frequency

Bayesian – personal perspective



$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

30 students
9 females
12 computerscience
of which 4 female

$P(F) = \frac{9}{30} = \frac{3}{10}$ $P(CS) = \frac{12}{30} = \frac{2}{5}$

$P(F\cap CS) = \frac{4}{30} = \frac{2}{15}$

$$P(F|CS) = \frac{P(F\cap CS)}{P(CS)} = \frac{2/15}{2/5} = \frac{1}{3}$$

$$P(F|CS^c) = \frac{P(F\cap CS^c)}{P(CS^c)} = \frac{5/30}{18/30} = \frac{5}{18}$$

Independence

$P(A|B) = P(A)$

then $P(A\cap B) = P(A)P(B)$

$P(F|CS) \neq P(F)$

not independent

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)} = \frac{P(A \cap B)}{P(B)}$$

$$P(+|HIV) = .977$$

$$P(-|\text{no } HIV) = .926$$

$$P(HIV) = .0026$$

$$P(CS|F) = \frac{P(F|CS)P(CS)}{P(F|CS)P(CS) + P(F|CS^c)P(CS^c)}$$

$$= \frac{(\frac{1}{3})(\frac{2}{5})}{(\frac{1}{3})(\frac{2}{5}) + (\frac{5}{18})(\frac{3}{5})} = \frac{4}{9}$$

$$P(CS|F) = \frac{P(CS \cap F)}{P(F)} = \frac{4/30}{9/30} = \frac{4}{9}$$

$$P(HIV|+) = \frac{P(+|HIV)P(HIV)}{P(+|HIV)P(HIV) + P(+|\text{no } HIV)P(\text{no } HIV)}$$

$$= \frac{(.977)(.0026)}{(.977)(.0026) + (1-.926)(1-.0026)}$$

$$= .033$$

Bernoulli

$X \sim B(p)$

$P(X=1) = p$

$P(X=0) = 1-p$

$$f(X=x|p) = f(x|p) = p^x (1-p)^{1-x} I_{\{x \in \{0,1\}\}}(x)$$

Expected value

$$E[X] = \sum_x x P(X=x) = (1)p + (0)(1-p) = p$$

$$Var(X) = p(1-p)$$

Binomial

$X \sim Bin(n,p)$

$$P(X=x|p) = f(x|p) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$\binom{n}{x} = \frac{n!}{x!(n-x)!} \quad \text{for } x \in \{0,1,\dots,n\}$$

$$E[X] = np \quad Var(X) = np(1-p)$$

$$X \sim U[0,1]$$

$$f(x) = \begin{cases} 1 & \text{if } x \in \boxed{[0,1]} \\ 0 & \text{otherwise} \end{cases}$$

$$= I_{\{0 \le x \le 1\}}(x)$$

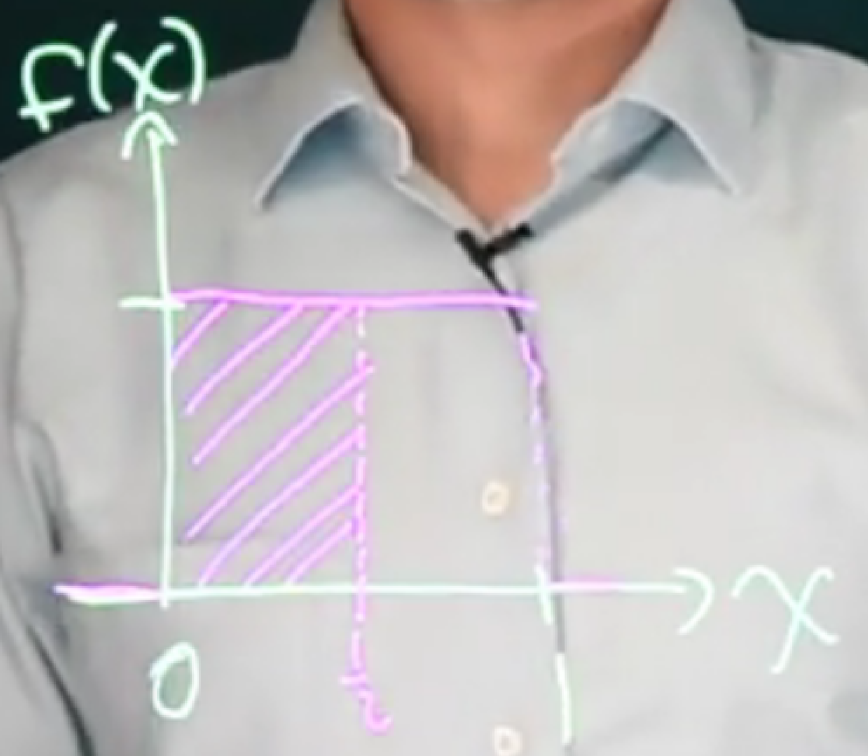
$$P(0 < x < \tfrac{1}{2}) = \int_0^{1/2} f(x)\,dx = \int_0^{1/2} dx = \frac{1}{2}$$

$$P(0 \le x \le \tfrac{1}{2}) = \frac{1}{2}$$

$$P(x = \tfrac{1}{2}) = 0$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$f(x) \geq 0$$



$$E[X] = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$E[g(X)] = \int g(x) f(x)\,dx$$

$$E[cX] = cE[X] \qquad E[X+Y] = E[X] + E[Y]$$

$$\text{if } X \perp Y, \text{ then } E[XY] = E[X]E[Y]$$

Exponential

$X \sim \text{Exp}(\lambda)$

$f(x|\lambda) = \lambda e^{-\lambda x}$ for $x \geq 0$

$E[X] = \frac{1}{\lambda}$ $\quad Var(X) = \frac{1}{\lambda^2}$

Normal

$X \sim N(\mu, \sigma^2)$

$f(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}(x-\mu)^2\right\}$

$E(X) = \mu$ $\quad Var(X) = \sigma^2$

Uniform

$X \sim [\theta_1, \theta_2]$

$f(x|\theta_1,\theta_2) = \frac{1}{\theta_2 - \theta_1} I_{\{\theta_1 \leq x \leq \theta_2\}}$

44H 56T

$X_i \sim B(p)$

By CLT $\displaystyle\sum_{i=1}^{100} X_i \;\dot{\sim}\; N\big(100p,\, 100p(1-p)\big)$

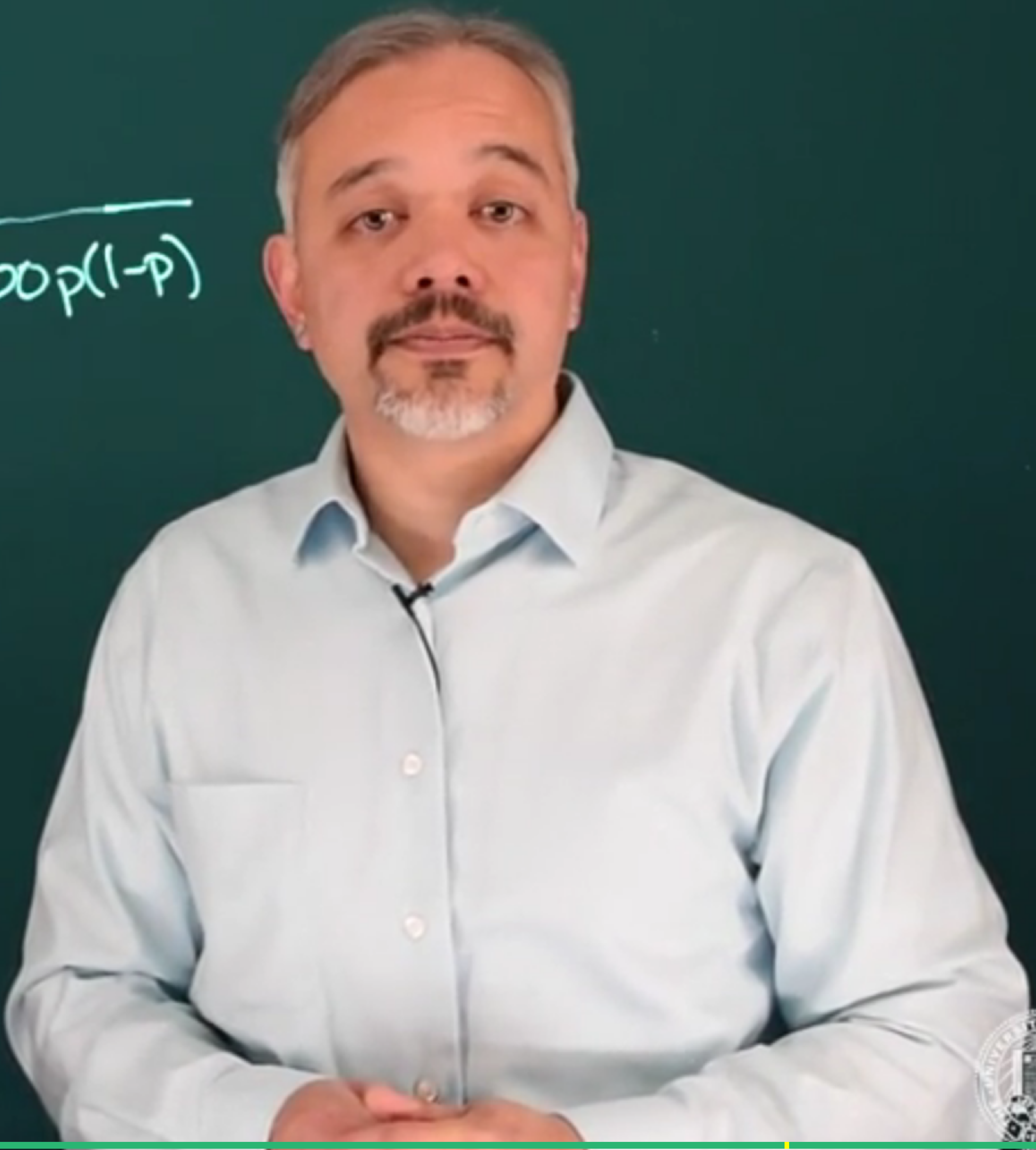
$100p - 1.96\sqrt{100p(1-p)}$ and $100p + 1.96\sqrt{100p(1-p)}$

Observe $\sum x_i = 44$ $\quad \hat{p} = \frac{44}{100} = .44$

CI $\quad 44 \pm 1.96\sqrt{44(.56)} = 44 \pm 9.7$

$(34.3, 53.7)$

95% confident $p \in (.343, .537)$



$Y_i \sim B(\theta)$ $\quad$ $Y_i \overset{iid}{\sim} B(\theta)$

$P(Y_i = 1) = \theta$

$$P(\underset{\sim}{Y} = \underset{\sim}{y} \mid \theta) = P(Y_1 = y_1, Y_2 = y_2, \ldots, Y_n = y_n \mid \theta)$$

$$= P(Y_1 = y_1) \cdots P(Y_n = y_n \mid \theta)$$

$$= \prod_{i=1}^{n} P(Y_i = y_i \mid \theta) = \prod_{i=1}^{n} \theta^{y_i} (1-\theta)^{1-y_i}$$

Likelihood $\quad L(\theta \mid \underset{\sim}{y}) = \prod_{i=1}^{n} \theta^{y_i} (1-\theta)^{1-y_i}$

MLE $\quad \hat{\theta} = \text{argmax}\, L(\theta \mid \underset{\sim}{y})$

$$\ell(\theta) = \log L(\theta \mid \underset{\sim}{y})$$

$$\ell(\theta) = \log \left[\prod \theta^{y_i} (1-\theta)^{1-y_i} \right]$$

$$= \sum \log \left[\theta^{y_i} (1-\theta)^{1-y_i} \right]$$

$$= \sum \left[y_i \log \theta + (1-y_i) \log(1-\theta) \right]$$

$$= \left(\sum y_i \right) \log \theta + \left(\sum (1-y_i) \right) \log(1-\theta)$$

7:01 / 7:06

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$$\ell'(\theta) = \frac{1}{\theta}\sum y_i - \frac{1}{1-\theta}\sum(1-y_i) \overset{\text{set}}{=} 0$$

$$\Rightarrow \frac{\sum y_i}{\hat{\theta}} = \frac{\sum(1-y_i)}{1-\hat{\theta}}$$

$$\Rightarrow \hat{\theta} = \frac{1}{n}\sum y_i = \hat{p} = \frac{72}{400} = .18$$

Approx CI $\hat{\theta} \pm 1.96\sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}}$

$$\hat{\theta} \,\dot{\sim}\, N\left(\theta, \frac{1}{I(\hat{\theta})}\right)$$

$$I(\theta) = \frac{1}{\theta(1-\theta)}$$

$$\ell(\theta) = \log L(\theta|\underset{\sim}{y})$$

$$\ell(\theta) = \log\left[\prod \theta^{y_i}(1-\theta)^{1-y_i}\right]$$

$$= \sum \log\left[\theta^{y_i}(1-\theta)^{1-y_i}\right]$$

$$= \sum\left[y_i\log\theta + (1-y_i)\log(1-\theta)\right]$$

$$= \left(\sum y_i\right)\log\theta + \left(\sum(1-y_i)\right)\log(1-\theta)$$

$$X_i \overset{iid}{\sim} \text{Exp}(\lambda)$$

$$f(\underset{\sim}{x}|\lambda) = \prod_{i=1}^{n} \lambda e^{-\lambda x_i} = \lambda^n e^{-\lambda \sum x_i}$$

$$L(\lambda|\underset{\sim}{x}) = \lambda^n e^{-\lambda \sum x_i}$$

$$\ell(\lambda) = n \log \lambda - \lambda \sum x_i$$

$$\ell'(\lambda) = \frac{n}{\lambda} - \sum x_i \overset{set}{=} 0$$

$$\Rightarrow \hat{\lambda} = \frac{n}{\sum x_i} = \frac{1}{\bar{x}}$$

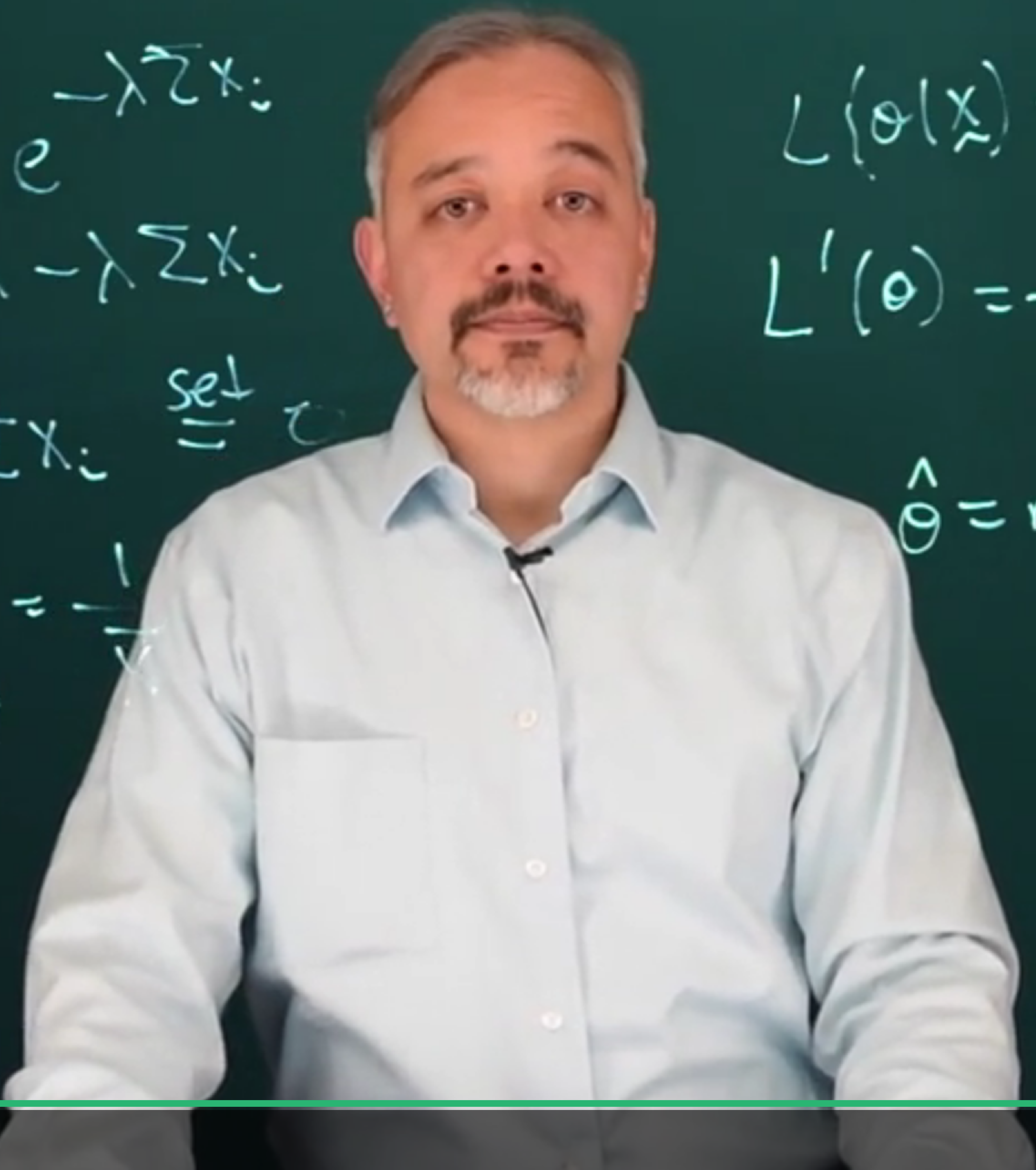
$$X_i \overset{iid}{\sim} U[0, \theta]$$

$$f(\underset{\sim}{x}|\theta) = \prod_{i=1}^{n} \frac{1}{\theta} I_{\{0 \le x_i \le \theta\}}$$

$$L(\theta|\underset{\sim}{x}) = \theta^{-n} I_{\{0 \le \min x_i \le \max x_i \le \theta\}}$$

$$L'(\theta) = -n\theta^{-(n+1)} I_{\{0 \le \min x_i \le \underline{\max x_i \le \theta}\}}$$

$$\hat{\theta} = \max x_i$$



4:16 / 4:21

$$\Theta = \{\text{fair}, \text{loaded}\}$$

$$X \sim \text{Bin}(5, ?)$$

$$f(x|\theta) \begin{cases} \binom{5}{x}\left(\frac{1}{2}\right)^5 & \text{if } \theta=\text{fair} \\ \binom{5}{x}(.7)^x(.3)^{5-x} & \text{if } \theta=\text{loaded} \end{cases}$$

$$= \binom{5}{x}(.5)^5 I_{\{\theta=\text{fair}\}} + \binom{5}{x}(.7)^x(.3)^{5-x} I_{\{\theta=\text{loaded}\}}$$

$X=2$

$$f(\theta|X=2) = \begin{cases} 0.3125 & \text{if } \theta=\text{fair} \\ 0.1323 & \text{if } \theta=\text{loaded} \end{cases}$$

MLE $\hat{\theta} = \text{fair}$

$$P(\theta=\text{fair}|X=2)$$

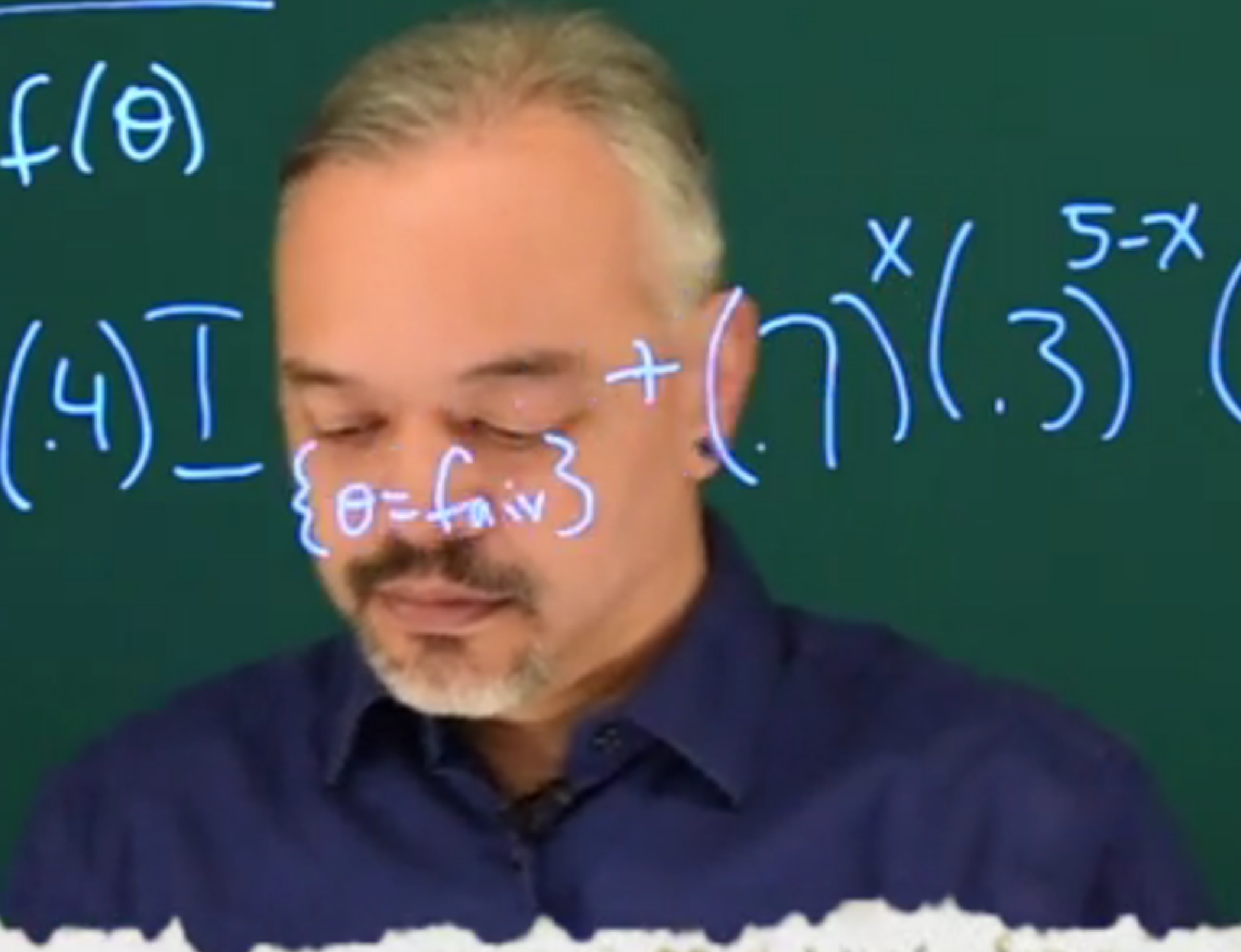
$$= P(\theta=\text{fair})$$

$$\in \{0,1\}$$

Prior $P(\text{loaded}) = .6$

$$f(\theta|x) = \frac{f(x|\theta)f(\theta)}{\sum_\theta f(x|\theta)f(\theta)}$$

$$= \binom{5}{x}\left[\left(\frac{1}{2}\right)^5(.4)I_{\{\theta=\text{fair}\}} + (.7)^x(.3)^{5-x}(.6)I_{\{\theta=\text{loaded}\}}\right]$$



The denominator is missing here, the full expression should be:

$$\frac{\binom{5}{x}\left[\left(\frac{1}{2}\right)^5(.4)I_{\{\theta=\text{fair}\}} + (.7)^x(.3)^{5-x}(.6)I_{\{\theta=\text{loaded}\}}\right]}{\binom{5}{x}\left[\left(\frac{1}{2}\right)^5(.4) + (.7)^x(.3)^{5-x}(.6)\right]}$$

Prior $P(\text{loaded}) = .6$

$$f(\theta|x) = \frac{f(x|\theta)f(\theta)}{\sum_\theta f(x|\theta)f(\theta)}$$

$$= \binom{5}{x}\left[\left(\tfrac{1}{2}\right)^5(.4)\,I_{\{\theta=\text{fair}\}} + (.7)^x(.3)^{5-x}(.6)\,I_{\{\theta=\text{loaded}\}}\right]$$

$$f(\theta|X=2) = \frac{.0125\,I_{\{\theta=\text{fair}\}} + .0079\,I_{\{\theta=\text{loaded}\}}}{.0125+.0079} = .612\,I_{\{\theta=\text{fair}\}} + .388\,I_{\{\theta=\text{loaded}\}}$$

$$P(\theta=\text{loaded}|X=2) = .388$$

$P(\theta=\text{loaded}) = \frac{1}{2} \Rightarrow P(\theta=\text{loaded}|X=2) = .297$

$P(\theta=\text{loaded}) = .9 \Rightarrow P(\theta=\text{loaded}|X=2) = .792$

$$f(\theta|y) = \frac{f(y|\theta)f(\theta)}{f(y)} = \frac{f(y|\theta)f(\theta)}{\int f(y|\theta)f(\theta)\,d\theta} = \frac{\text{likelihood} \times \text{prior}}{\text{normalizing constant}} \propto \text{likelihood} \times \text{prior}$$

$\theta \sim U[0,1]$ $\quad f(\theta) = I_{\{0\le\theta\le1\}}$

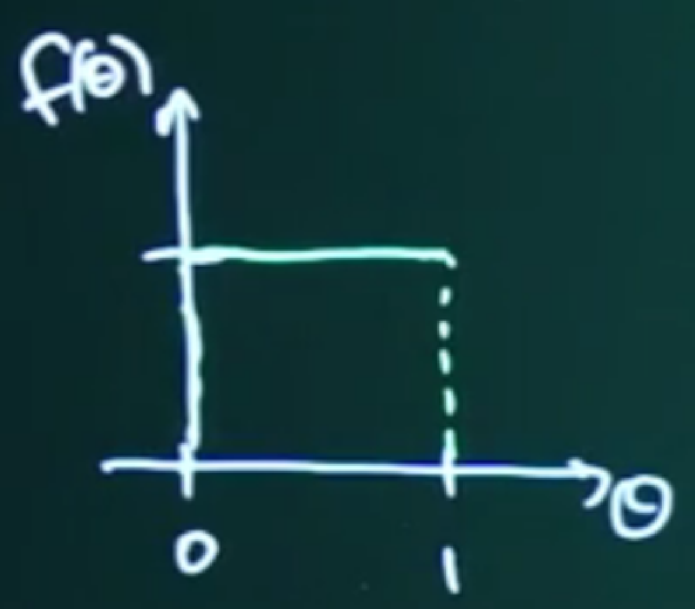
$f(\theta|y)$

$$f(\theta|Y=1) = \frac{\theta^1(1-\theta)^0 I_{\{0\le\theta\le1\}}}{\int_{-\infty}^{\infty}\theta^1(1-\theta)^0 I_{\{0\le\theta\le1\}}\,d\theta} = \frac{\theta I_{\{0\le\theta\le1\}}}{\int_0^1 \theta\,d\theta} = 2\theta I_{\{0\le\theta\le1\}}$$

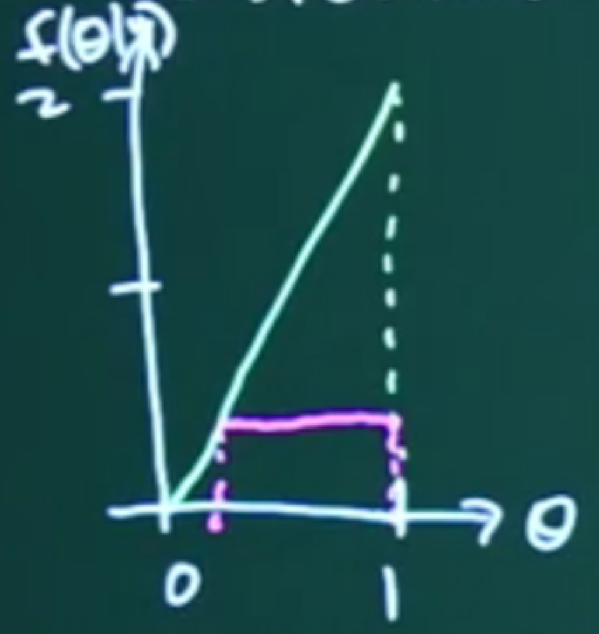
$$f(\theta|y) \propto f(y|\theta)f(\theta) \propto \theta I_{\{0\le\theta\le1\}} = 2\theta I_{\{0\le\theta\le1\}}$$

I mixed my notation a bit in this last line. Clearly $\theta \neq 2\theta$. What I intended is that $f(\theta|y) = 2\theta I_{\{0\le\theta\le1\}}$. The terms written in between are just proportional, without the normalizing constant, and the normalizing constant is put back at the end, so that at the end we do have equality with the original left-hand side. And to be completely correct here, I really should write this as $f(\theta|Y=1) = 2\theta I_{\{0\le\theta\le1\}}$, rather than writing this as $f(\theta|y)$.

Prior



Posterior



Equal tailed

$$P(\theta < q \mid Y=1) = \int_0^q 2\theta \, d\theta = q^2$$

$$P(\sqrt{.025} < \theta < \sqrt{.975}) = P(.158 < \theta < .987) = .95$$

HPD $\quad P(\theta > \sqrt{.05} \mid Y=1) = P(\theta > .224 \mid Y=1) = .95$

Prior interval estimates

$$P(.025 < \theta < .975) = .95$$

$$P(\theta > .05) = .95$$

Posterior interval estimates

$$P(.025 < \theta < .975) = \int_{.025}^{.975} 2\theta \, d\theta = .975^2 - .025^2 = .95$$

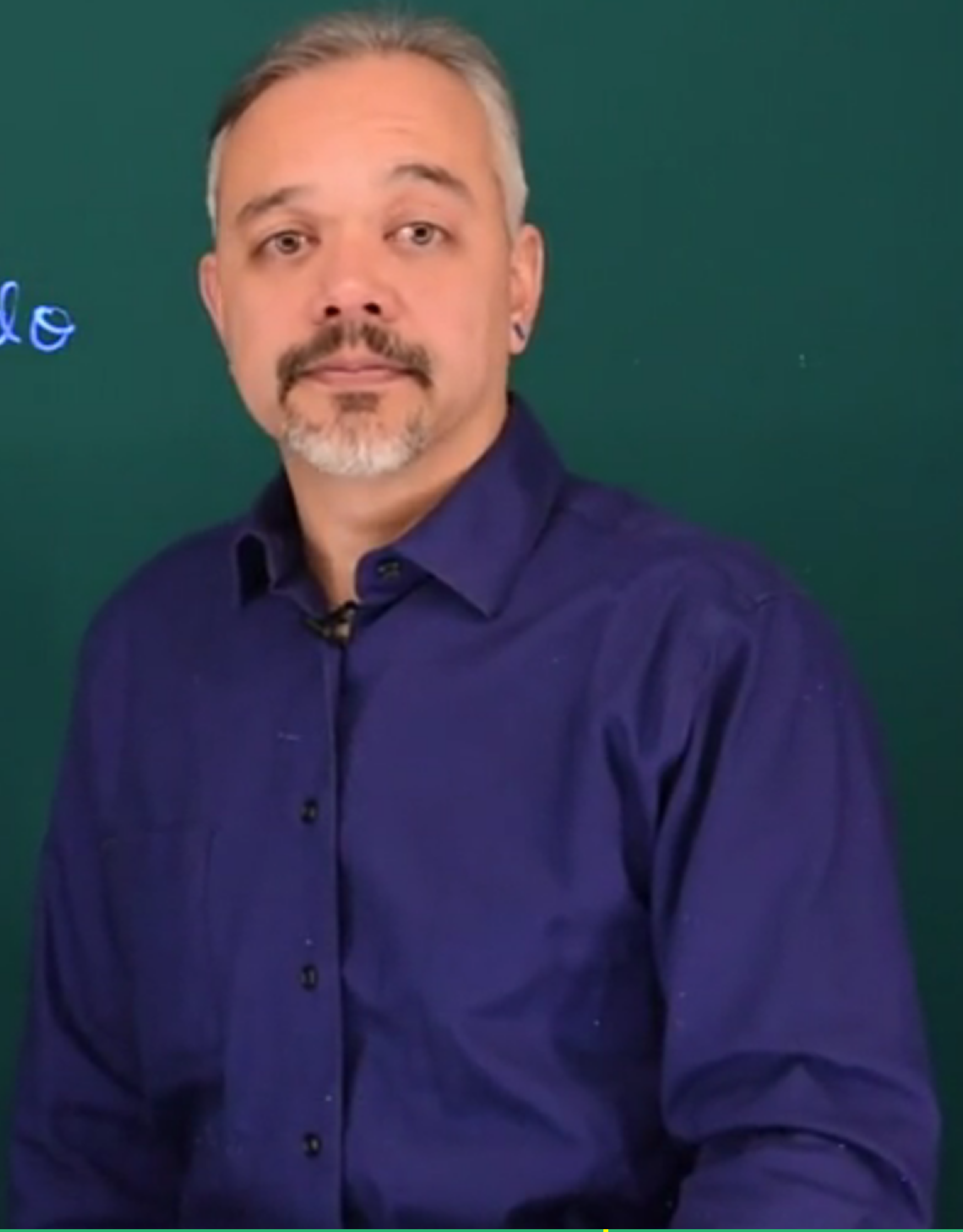
$$P(\theta > .05) = 1 - .05^2 = .9975$$

$$P(\theta \le c) \quad \text{for all } c \in \mathbb{R}$$

$$P(\theta = \tfrac{1}{2}) = 1 \qquad \delta(\tfrac{1}{2})$$

$$f(\theta \mid y) \propto f(y \mid \theta) f(\theta) = f(\theta)$$

$$f(y) = \int f(y \mid \theta) f(\theta)\, d\theta = \int f(y, \theta)\, d\theta$$



$$X = \sum_{i=1}^{10} Y_i$$

$$f(\theta) = I_{\{0 \le \theta \le 1\}}$$

$$f(x) = \int f(x|\theta) f(\theta)\, d\theta = \int_0^1 \frac{10!}{x!(10-x)!} \theta^{x} (1-\theta)^{10-x} (1)\, d\theta$$

$$= \int_0^1 \frac{\Gamma(11)}{\Gamma(x+1)\Gamma(11-x)} \theta^{(x+1)-1} (1-\theta)^{(11-x)-1}\, d\theta$$

$$= \frac{\Gamma(11)}{\Gamma(12)} \int_0^1 \frac{\Gamma(12)}{\Gamma(x+1)\Gamma(11-x)} \theta^{(x+1)-1} (1-\theta)^{(11-x)-1}\, d\theta$$

$$= \frac{\Gamma(11)}{\Gamma(12)} (1) = \frac{10!}{11!} = \frac{1}{11} \quad \text{for } x \in \{0, 1, 2, \dots, 10\}$$

$$n! = \Gamma(n+1)$$

$$z \sim \text{Beta}(\alpha, \beta)$$

$$f(z) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} z^{\alpha-1} (1-z)^{\beta-1}$$

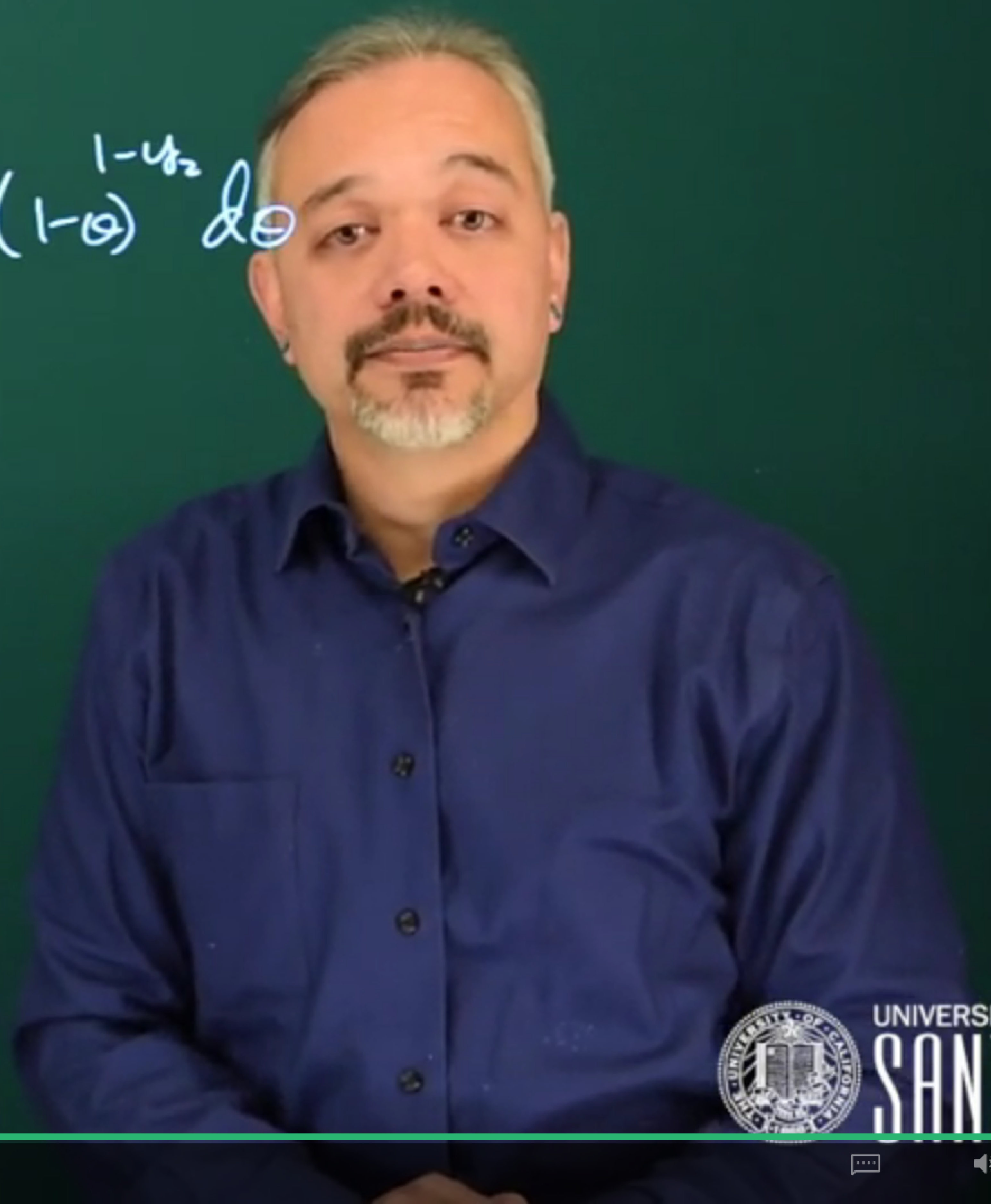
$$f(y_2 \mid y_1) = \int f(y_2 \mid \theta, y_1) f(\theta \mid y_1)\, d\theta$$

$$Y_2 \perp Y_1 \Rightarrow \int f(y_2 \mid \theta) f(\theta \mid y_1)\, d\theta$$

$$f(y_2 \mid Y_1 = 1) = \int_0^1 \theta^{y_2} (1-\theta)^{1-y_2}\, 2\theta\, d\theta = \int_0^1 2\theta^{y_2+1} (1-\theta)^{1-y_2}\, d\theta$$

$$P(Y_2 = 1 \mid Y_1 = 1) = \int_0^1 2\theta^2\, d\theta = \frac{2}{3}$$

$$P(Y_2 = 0 \mid Y_1 = 1) = \frac{1}{3}$$



$$f(\underset{\sim}{y}\mid\theta) = \theta^{\sum y_i}(1-\theta)^{n-\sum y_i} \qquad f(\theta) = I_{\{0\le\theta\le 1\}}$$

$$\theta\mid \underset{\sim}{y} \sim \text{Beta}\left(\sum y_i + 1,\ n-\sum y_i + 1\right)$$

$$f(\theta\mid\underset{\sim}{y}) = \frac{f(\underset{\sim}{y}\mid\theta)f(\theta)}{\int f(\underset{\sim}{y}\mid\theta)f(\theta)\,d\theta} = \frac{\theta^{\sum y_i}(1-\theta)^{n-\sum y_i} I_{\{0\le\theta\le 1\}}}{\int_0^1 \theta^{\sum y_i}(1-\theta)^{n-\sum y_i} I_{\{0\le\theta\le 1\}}\,d\theta}$$

$$= \frac{\theta^{\sum y_i}(1-\theta)^{n-\sum y_i} I_{\{0\le\theta\le 1\}}}{\frac{\Gamma(\sum y_i+1)\Gamma(n-\sum y_i+1)}{\Gamma(n+2)}\int_0^1 \frac{\Gamma(n+2)}{\Gamma(\sum y_i+1)\Gamma(n-\sum y_i+1)}\theta^{\sum y_i}(1-\theta)^{n-\sum y_i}\,d\theta}$$

$$= \frac{\Gamma(n+2)}{\Gamma(\sum y_i+1)\Gamma(n-\sum y_i+1)}\theta^{\sum y_i}(1-\theta)^{n-\sum y_i} I_{\{0\le\theta\le 1\}}$$

$$f(\theta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1} (1-\theta)^{\beta-1} I_{\{0 \le \theta \le 1\}}$$

$$f(\theta \mid \underset{\sim}{y}) \propto f(\underset{\sim}{y} \mid \theta) f(\theta) = \theta^{\sum y_i} (1-\theta)^{n-\sum y_i} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1} (1-\theta)^{\beta-1} I_{\{0 \le \theta \le 1\}}$$

$$\propto \theta^{\alpha+\sum y_i - 1} (1-\theta)^{\beta+n-\sum y_i - 1} I_{\{0 \le \theta \le 1\}}$$

$$\theta \mid \underset{\sim}{y} \sim \text{Beta}\left(\alpha + \sum y_i, \beta + n - \sum y_i\right)$$

$Y_1, \ldots, Y_n \sim B(\theta)$ likelihood

$\theta \mid \alpha, \beta \sim \text{Beta}(\alpha, \beta)$ prior

$\alpha, \beta = \alpha_0, \beta_0$ hyperparameters

Conjugate

$\text{Beta}(\alpha + \sum y_i, \beta + n - \sum y_i)$

95% CI for θ is $\hat{\theta} \pm 1.96\sqrt{\dfrac{\hat{\theta}(1-\hat{\theta})}{n}}$

Prior $\text{Beta}(\alpha, \beta)$

effective sample size of prior is $\alpha+\beta$

mean of Beta is $\dfrac{\alpha}{\alpha+\beta}$

Posterior mean is $\dfrac{\alpha + \sum y_i}{\alpha + \sum y_i + \beta + n - \sum y_i} = \dfrac{\alpha + \sum y_i}{\alpha + \beta + n}$

$$= \frac{\alpha+\beta}{\alpha+\beta+n} \cdot \frac{\alpha}{\alpha+\beta} + \frac{n}{\alpha+\beta+n} \cdot \frac{\sum y_i}{n}$$

posterior mean = prior weight × prior mean + data weight × data mean

$f(\theta)$ observe $y_1, \ldots, y_n$

posterior $\theta \mid y_1, \ldots, y_n$

more data $y_{n+1}, \ldots, y_{n+m}$

yesterday's posterior is today's prior

posterior $\theta \mid y_1, \ldots, y_{n+m}$

$Y_i \sim \text{Pois}(\lambda)$ $\quad f(\underset{\sim}{y}|\lambda) = \dfrac{\lambda^{\sum y_i} e^{-n\lambda}}{\prod_{i=1} y_i!}$ for $\lambda > 0$

Gamma prior

$\lambda \sim \Gamma(\alpha, \beta) \quad f(\lambda) = \dfrac{\beta^{\alpha}}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda}$

$$f(\lambda|\underset{\sim}{y}) \propto f(\underset{\sim}{y}|\lambda) f(\lambda) \propto \lambda^{\sum y_i} e^{-n\lambda} \lambda^{\alpha-1} e^{-\beta\lambda}$$

$$\propto \lambda^{(\alpha+\sum y_i)-1} e^{-(\beta+n)\lambda}$$

posterior is $\Gamma(\alpha+\sum y_i, \beta+n)$

mean of gamma $\dfrac{\alpha}{\beta}$

posterior mean

$$\frac{\alpha+\sum y_i}{\beta+n} = \frac{\beta}{\beta+n}\cdot\frac{\alpha}{\beta} + \frac{n}{\beta+n}\cdot\frac{\sum y_i}{n}$$

1) prior mean $\dfrac{\alpha}{\beta}$

a) prior std. dev. $\dfrac{\sqrt{\alpha}}{\beta}$

b) effective sample size β

2) vague prior

small $\varepsilon > 0$ $\quad \Gamma(\varepsilon, \varepsilon)$

$$\frac{\varepsilon+\sum y_i}{\varepsilon+n} \approx \frac{\sum y_i}{n}$$

$Y \sim \text{Exp}(\lambda)$

Prior mean $= \frac{1}{10}$

$\Gamma(100, 1000)$

Prior std. dev. $= \frac{1}{100}$

$.1 \pm .02$

$Y = 12$

$$f(\lambda|y) \propto f(y|\lambda) f(\lambda)$$

$$\propto \lambda e^{-\lambda y} \lambda^{\alpha-1} e^{-\beta\lambda}$$

$$\propto \lambda^{(\alpha+1)-1} e^{-(\beta+y)\lambda}$$

$$\lambda|y \sim \Gamma(\alpha+1, \beta+y)$$

$$\Rightarrow \lambda|y \sim \Gamma(101, 1012)$$

Posterior mean $= \frac{101}{1012}$

$= .0998$

$= \frac{1}{10.02}$

$$X_i \overset{iid}{\sim} N(\mu, \sigma_0^2)$$

prior $\mu \sim N(m_0, s_0^2)$

$$f(\mu \mid \underset{\sim}{x}) \propto f(\underset{\sim}{x} \mid \mu) f(\mu)$$

$$\mu \mid \underset{\sim}{x} \sim N\left(\frac{\frac{n\bar{x}}{\sigma_0^2} + \frac{m_0}{s_0^2}}{\frac{n}{\sigma_0^2} + \frac{1}{s_0^2}} , \frac{1}{\frac{n}{\sigma_0^2} + \frac{1}{s_0^2}} \right)$$

posterior mean

$$\frac{\frac{n}{\sigma_0^2}}{\frac{n}{\sigma_0^2} + \frac{1}{s_0^2}} \bar{x} + \frac{\frac{1}{s_0^2}}{\frac{n}{\sigma_0^2} + \frac{1}{s_0^2}} m$$

$$\frac{n}{n + \frac{\sigma_0^2}{s_0^2}} \bar{x} + \frac{\frac{\sigma_0^2}{s_0^2}}{n + \frac{\sigma_0^2}{s_0^2}} m$$

$$X_i \mid \mu, \sigma^2 \overset{iid}{\sim} N(\mu, \sigma^2)$$

$$\mu \mid \sigma^2 \sim N\left(m, \frac{\sigma^2}{w}\right)$$

$$w = \frac{\sigma^2}{\sigma_\mu^2}$$ effective sample size

$$\sigma^2 \sim \Gamma^{-1}(\alpha, \beta)$$

$$\sigma^2 \mid \underset{\sim}{x} \sim \Gamma^{-1}\left(\alpha + \frac{n}{2}, \beta + \frac{1}{2}\sum_{i=1}^{n}(x_i - \bar{x})^2 + \frac{nw}{2(n+w)}(\bar{x} - m)^2\right)$$

$$\mu \mid \sigma^2, \underset{\sim}{x} \sim N\left(\frac{n\bar{x} + wm}{n+w}, \frac{\sigma^2}{n+w}\right)$$

$$\frac{n\bar{x} + wm}{n+w} = \frac{w}{n+w}m + \frac{n}{n+w}\bar{x}$$

$$\mu \mid \underset{\sim}{x} \sim t$$

$Y_i \sim B(\theta)$

$\theta \sim U[0,1] = \text{Beta}(1,1)$

$\text{Beta}(\frac{1}{2},\frac{1}{2})$ $\quad$ $\text{Beta}(.001,.001)$

$\text{Beta}(0,0)$ $\quad$ $f(\theta) \propto \theta^{-1}(1-\theta)^{-1}$

improper prior

$f(\theta|y) \propto \theta^{y-1}(1-\theta)^{n-y-1} \sim \text{Beta}(y, n-y)$

posterior mean $\frac{y}{n} = \hat{\theta}$

$Y_i \overset{iid}{\sim} N(\mu, \sigma^2)$

vague prior $\mu \sim N(0, 1000000^2)$

$f(\mu) \propto 1$

$f(\mu|y) \propto f(y|\mu) f(\mu)$

$\propto \exp\{-\frac{1}{2\sigma^2}\sum(y_i-\mu)^2\}(1)$

$\propto \exp\{-\frac{1}{2\frac{\sigma^2}{n}}(\mu-\bar{y})^2\}$

$\mu|\underset{\sim}{y} \sim N(\bar{y}, \frac{\sigma^2}{n})$

$f(\sigma^2) \propto \frac{1}{\sigma^2}$ $\quad$ $\Gamma^{-1}(0,0)$ $\quad$ $\sigma^2|\underset{\sim}{y} \sim \Gamma^{-1}(\frac{n-1}{2}, \frac{1}{2}\sum(y_i-\bar{y})^2)$

$$Y_i \sim N(\mu, \sigma^2)$$

$$f(\sigma^2) \propto \frac{1}{\sigma^2}$$

$$f(\sigma^2) \propto 1$$

Jeffreys prior $\quad f(\theta) \propto \sqrt{I(\theta)}$

$$Y_i \sim N(\mu, \sigma^2) \qquad f(\mu) \propto 1 \,,\; f(\sigma^2) \propto \frac{1}{\sigma^2}$$

$$Y_i \sim B(\theta) \qquad f(\theta) \propto \theta^{-1/2}(1-\theta)^{-1/2} \sim \text{Beta}\left(\tfrac{1}{2}, \tfrac{1}{2}\right)$$