Reinforcement learning
week 1

Why should you care

Yandex
Data Factory

LAMBDA

Supervised learning

Given:

- objects and answers (x, y)
- algorithm family $a_\theta(x) \to y$
- loss function $L(y, a_\theta(x))$

Find: $$\theta' \leftarrow argmin_\theta L(y, a_\theta(x))$$

Supervised learning

Given:

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Find: $$\theta' \leftarrow argmin_\theta L(y, a_\theta(x))$$

Supervised learning

Given:

- objects and answers (x, y) — [banner,page], ctr
- algorithm family $a_\theta(x) \to y$ — linear/ tree / NN
- loss function $L(y, a_\theta(x))$ — MSE, crossentropy

Find: $$\theta' \leftarrow argmin_\theta L(y, a_\theta(x))$$

Supervised learning

Supervised learning

Great… except if we have no reference answers

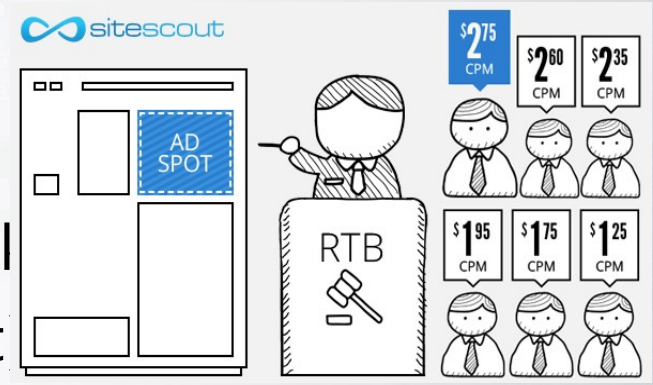
Great… except if we have no reference answers

Online Ads

Great… except if we have no reference answers

We have:

- YouTube at your disposal
- Live data stream
- (banner & video features, #clicl
- (insert your favorite ML toolkit



We want:

- Learn to pick relevant ac

Ideas?

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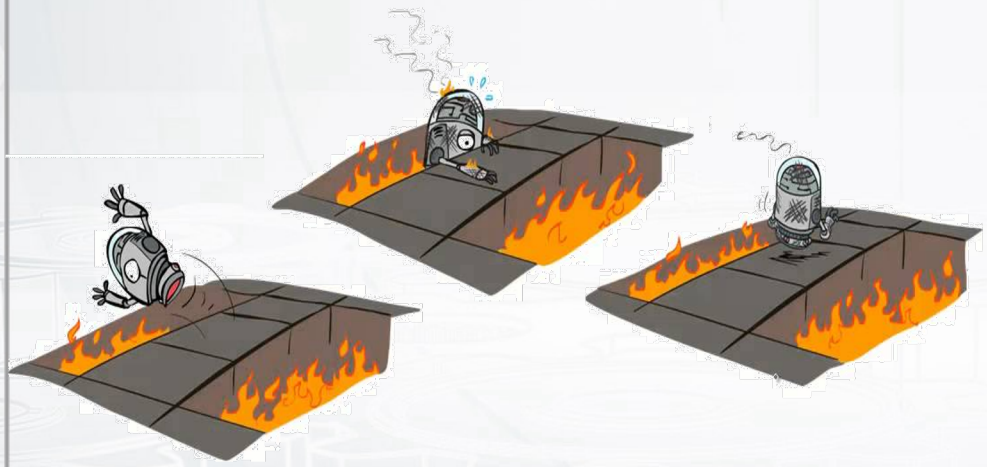
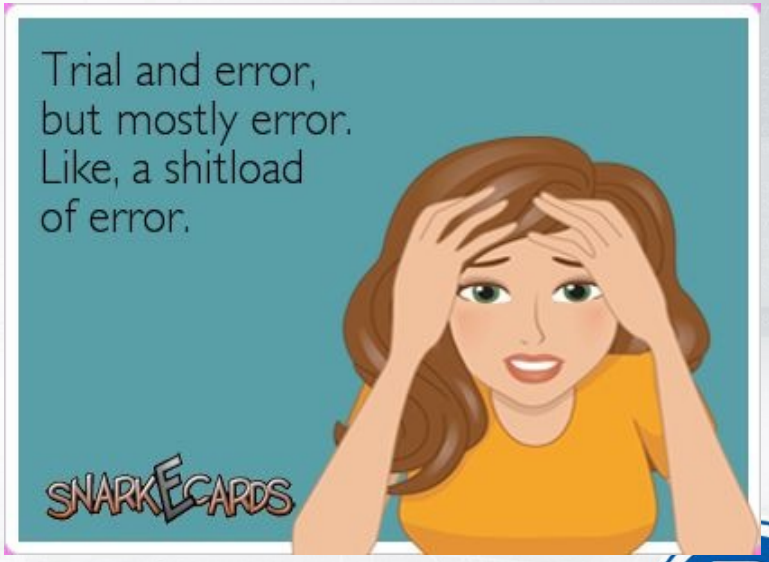
Duct tape approach

Common idea:

- Initialize with naïve solution
- Get data by trial and error and error and error and error
- Learn (situation) → (optimal action)
- Repeat





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Problems

Problem 1:

- What exactly does the “optimal action” mean?

Extract as much money as you can right now

Make user happy so that he would visit you again

Problems

Problems

Problem 2:

- If you always follow the “current optimal” strategy, you may never discover something better.

- If you show the same banner to 100% users, you will never learn how other ads affect them.

Ideas?

Duct tape approach

IF IN CAN'T BE FIXED
E
WITH DUCT TAPE

THEN YOU'RE NOT
USING ENOUGH DUCT
TAPE

Reinforcement learning

What-what learning?

Supervised learning	Reinforcement learning
Learning to approximate reference answers	Learning optimal strategy by trial and error
Needs correct answers	Needs feedback on agent's own actions
Model does not affect the input data	Agent can affect it's own observations

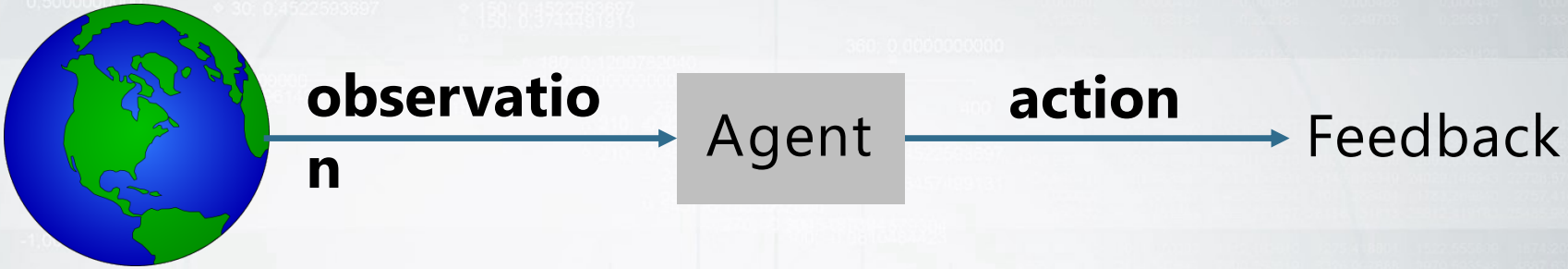
What-what learning?

	Reinforcement learning
	Learning optimal strategy by trial and error
	Needs feedback on agent's own actions
Model does not affect the input data	Agent can affect it's own observations

What-what learning?

Unsupervised learning	Reinforcement learning
Learning underlying data structure	Learning optimal strategy by trial and error
No feedback required	Needs feedback on agent's own actions
Model does not affect the input data	Agent can affect it's own observations

What is: bandit



Examples:

- banner ads (RTB)
- recommendations
- medical treatment

What is: bandit

observation

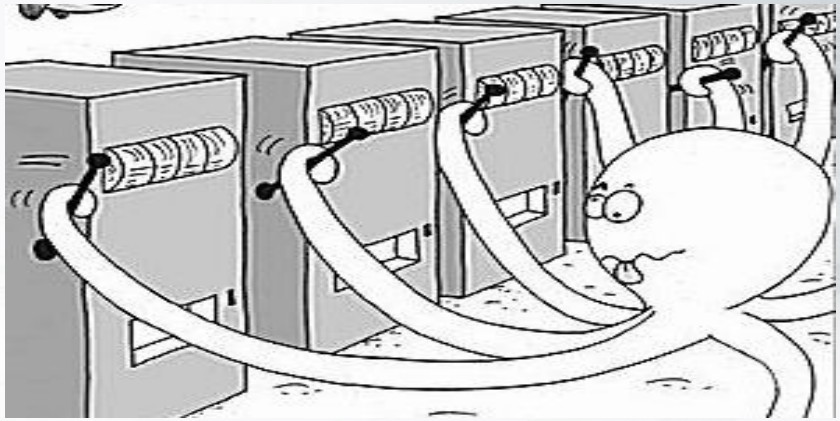
Agent

action

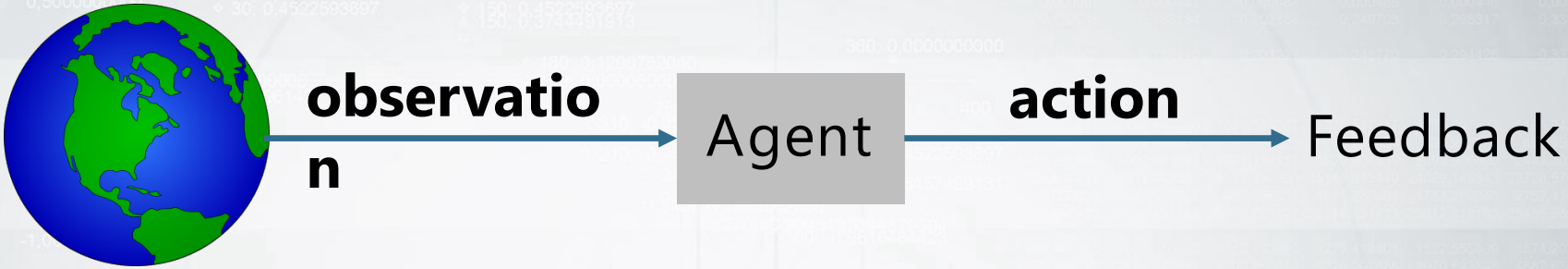
Feedback

Examples:

- banner ads (RTB)
- recommendations
- medical treatment



What is: bandit



Examples:

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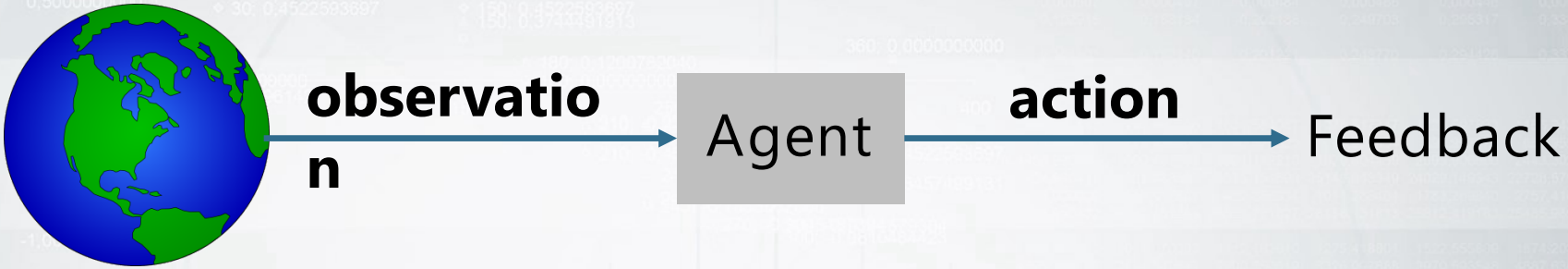
What is: bandit



Examples:

- banner ads (RTB)
- recommendations
- medical treatment

Q: what's observation, action and feedback in the banner ads problem?

What is: bandit

observation

user features
time of year
trends

Agent

action

show
banner

Feedback

click,
money

Examples:

- banner ads (RTB)
- recommendations
- medical treatment

What is: bandit

observation

user features
time of year
trends

Agent

action

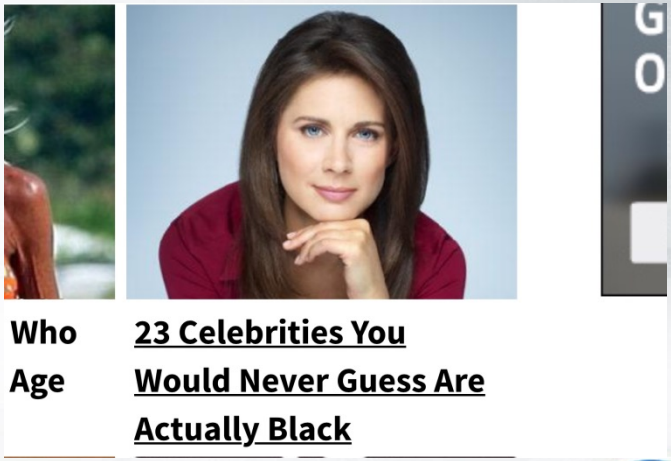
show banner

Feedback

click, money

Q: You're Google/Yandex/Youtube. There's a kind of banners that would have great click rates: the “clickbait”.

Is it a good idea to show clickbait?



What is: bandit

observation

Agent

action

Feedback

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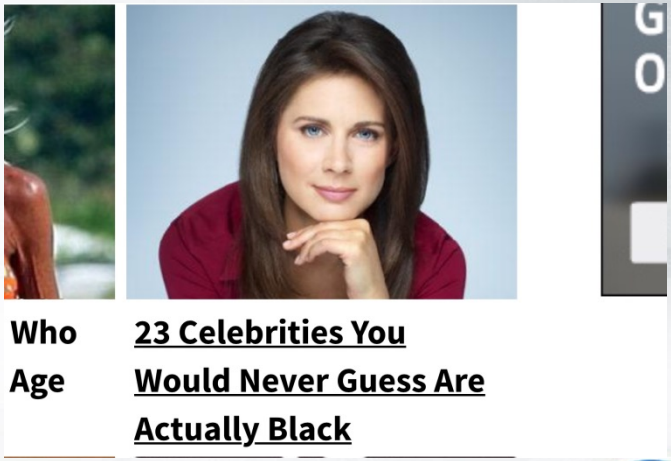
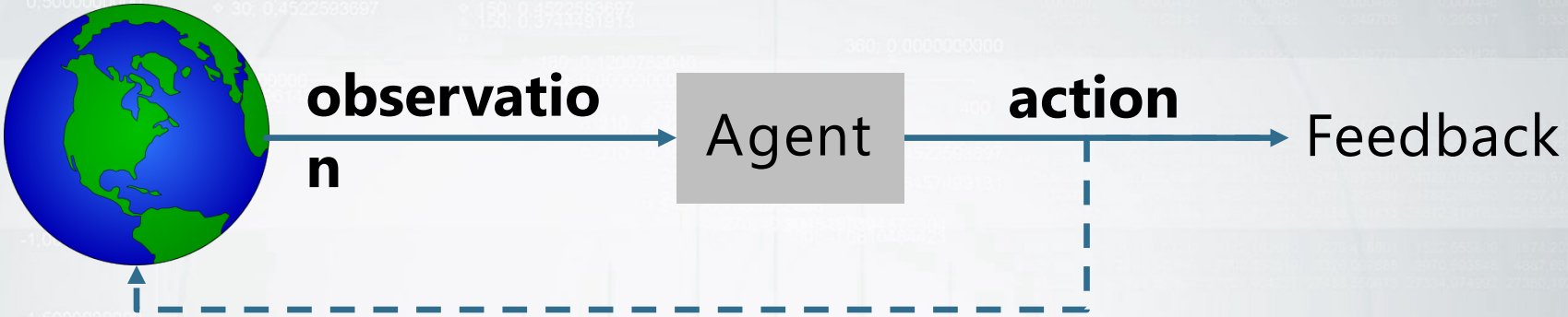


What is: bandit



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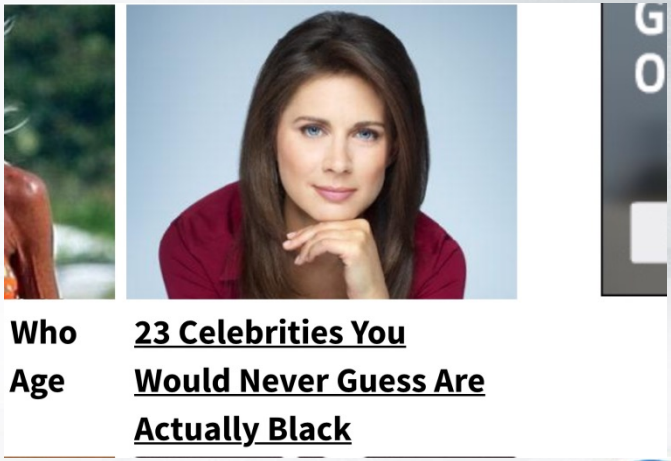
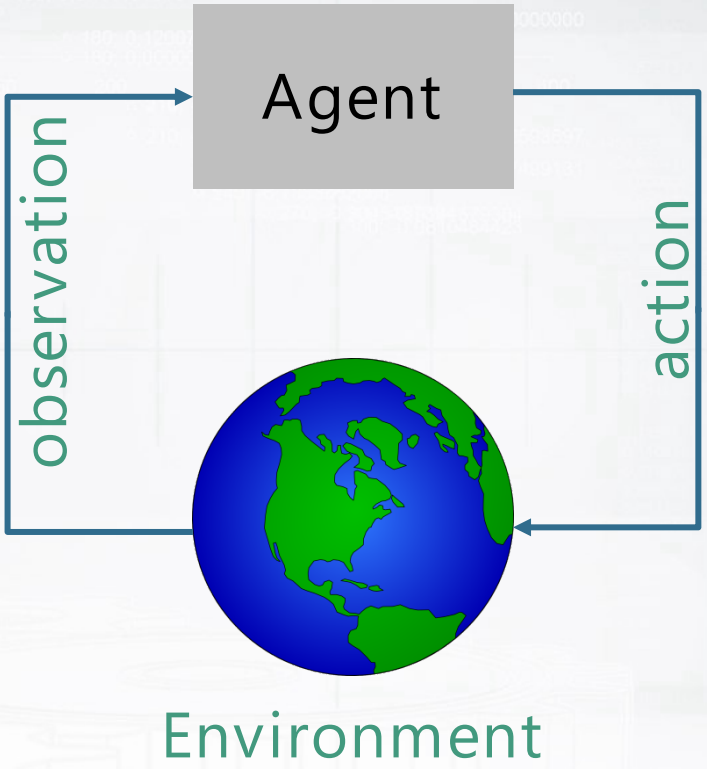


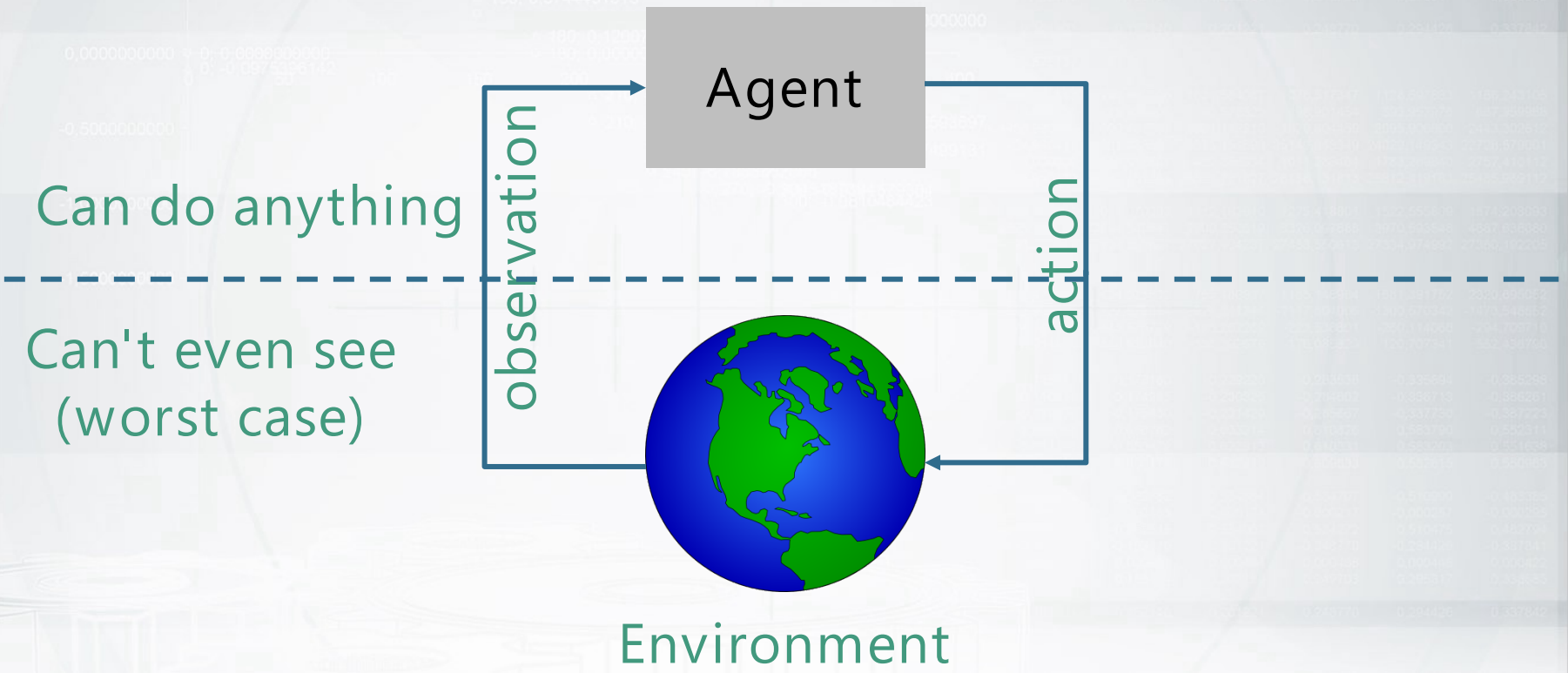
Is it a good idea to show clickbait?

No, no one will trust you after

What is: decision process



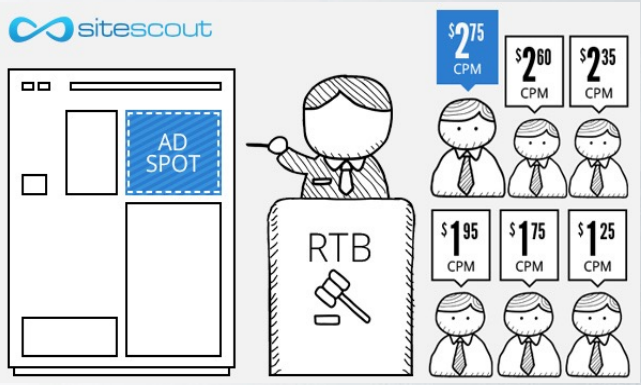
What is: decision process



Reality check: web

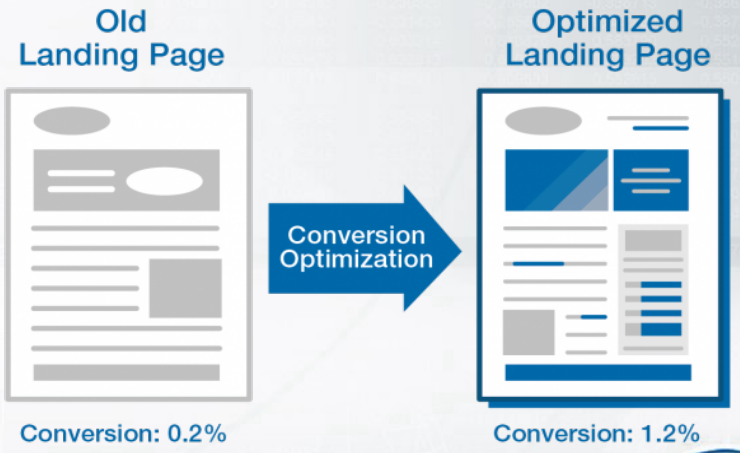
Cases:

- Pick ads to maximize profit
- Design landing page to maximi
  user retention
- Recommend movies to users
- Find pages relevant to queries



Example:

- Observation – user features
- Action – show banner #i
- Feedback – did user click?



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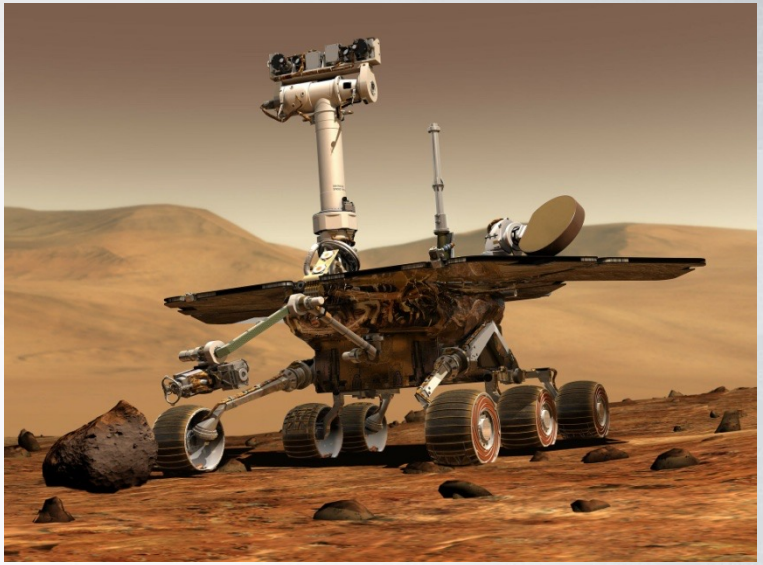
Reality check:

Reality check: dynamic systems

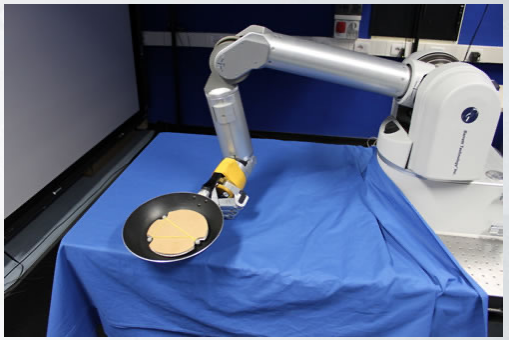


Reality check: dynamic systems

Reality check: dynamic systems

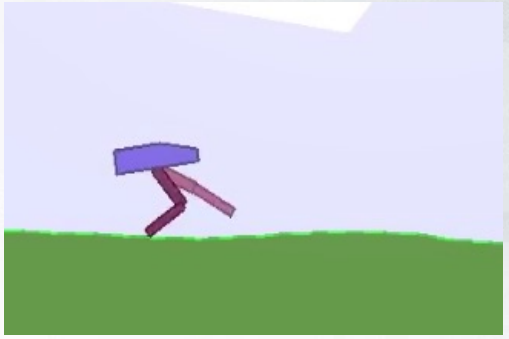
Cases:

- Robots
- Self-driving vehicles
- Pilot assistant
- More robots!



Example:

- Observation: sensor feed
- Action: voltage sent to motors
- Feedback: how far did it move
- forward before falling



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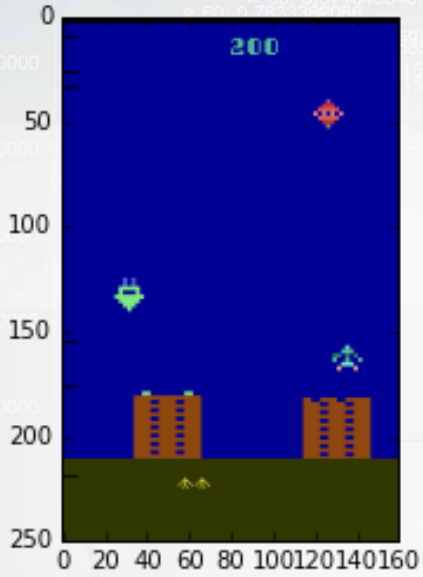
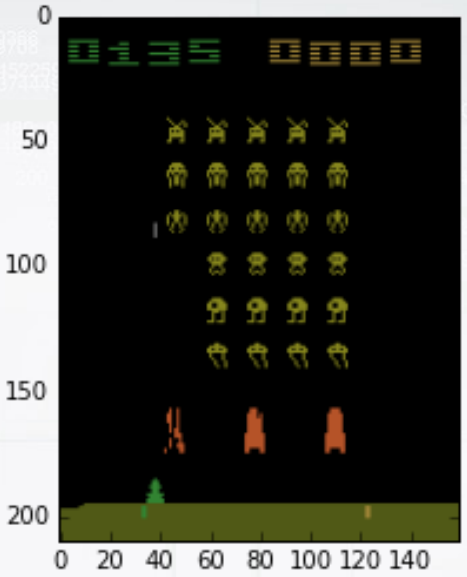
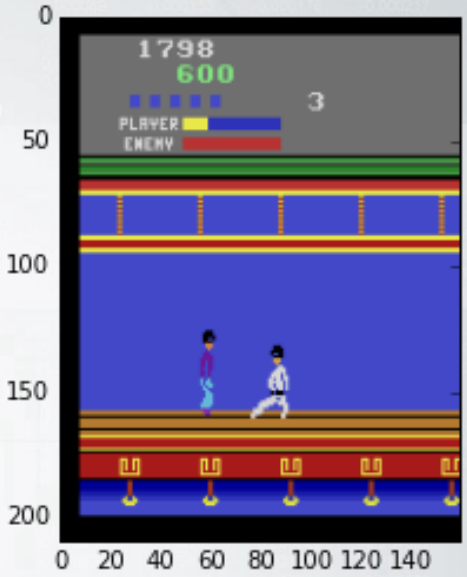
~~Reality~~ check:

~~Reality~~ check: videogames

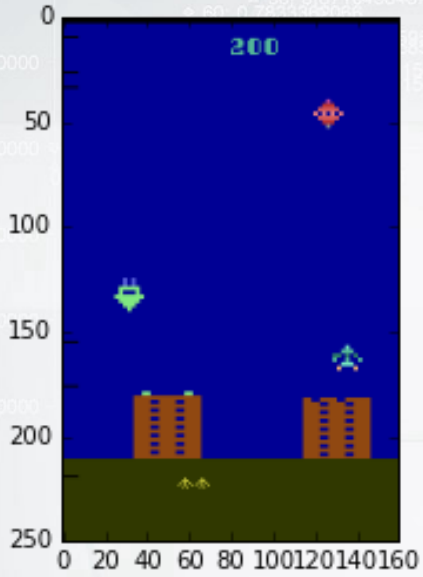


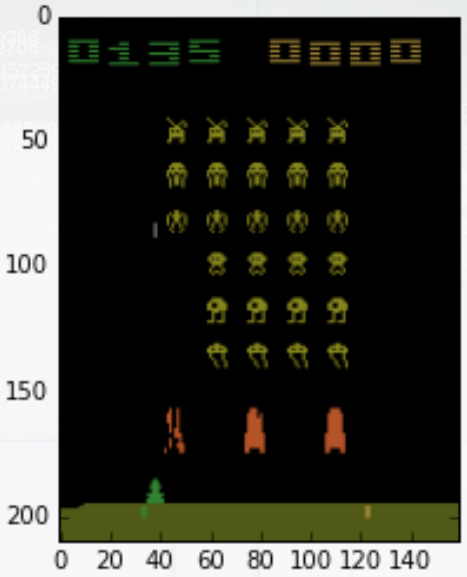


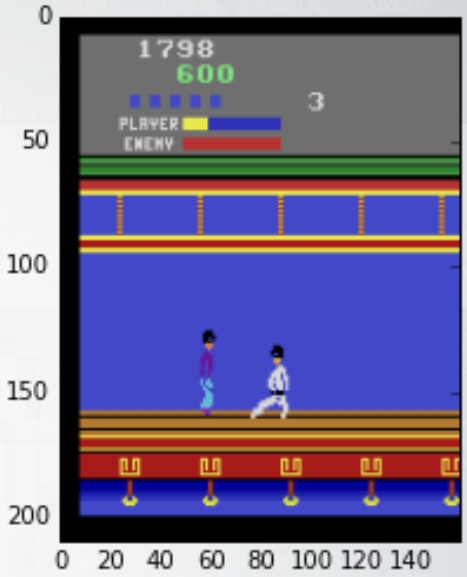


Q: What are observations, actions and feedback?

~~Reality~~ check: videogames





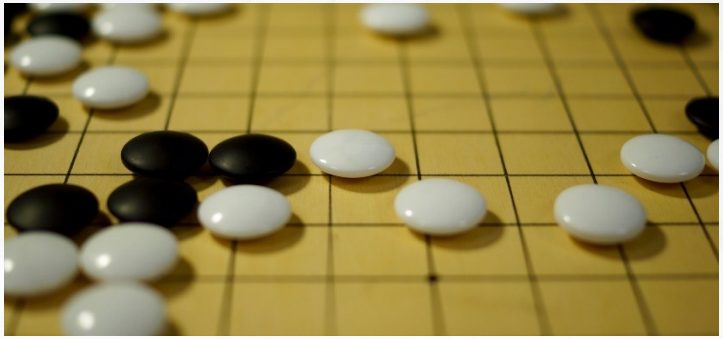


Q: What are observations, actions and feedback?

Other use cases

Personalized medical treatment

Even more games (Go, chess, etc)

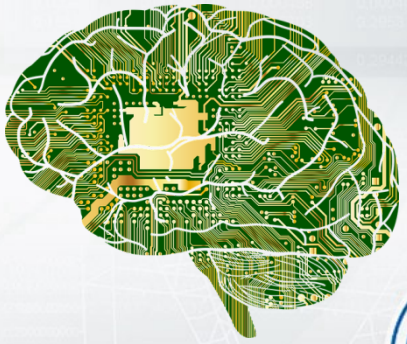


Q: What are observations, actions and feedback?

Other use cases

Other use cases

- Conversation systems
  - learning to make user happy
- Quantitative finance
  - portfolio management
- Deep learning
  - optimizing non-differentiable loss
  - finding optimal architecture



The MDP formalism

Markov Decision Process

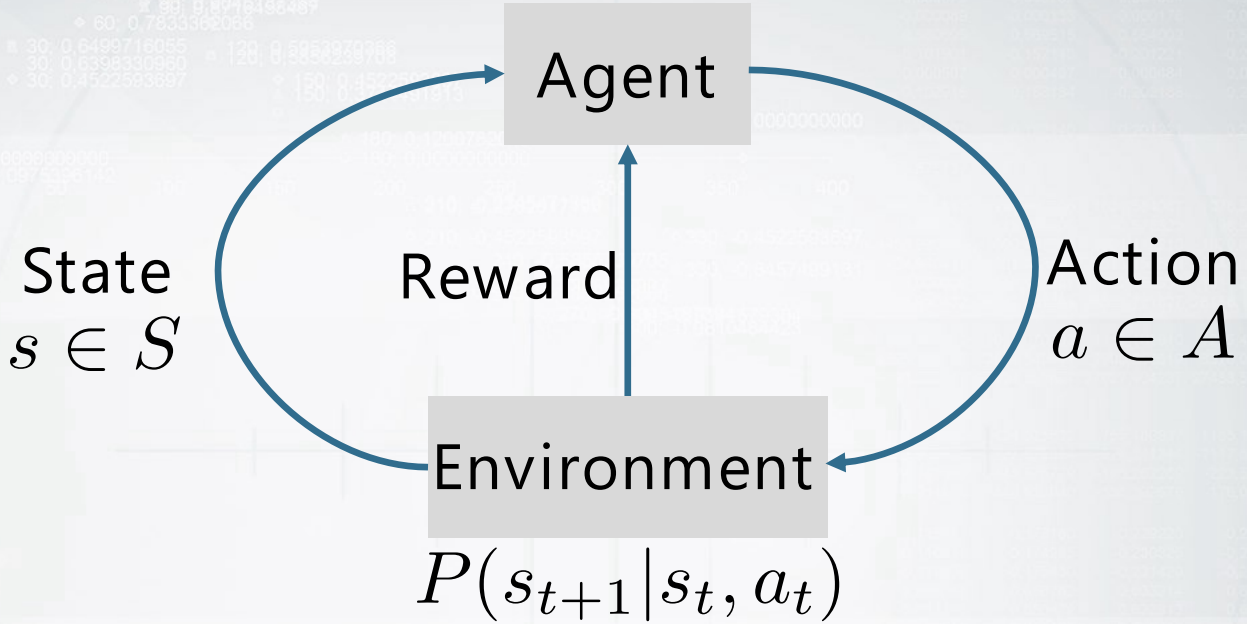
- Environment states: $s \in S$
- Agent actions: $a \in A$
- Rewards $r \in \mathbb{R}$

Dynamics: $P(s_{t+1}|s_t, a_t)$

The MDP formalism

Markov Decision Process

The MDP formalism

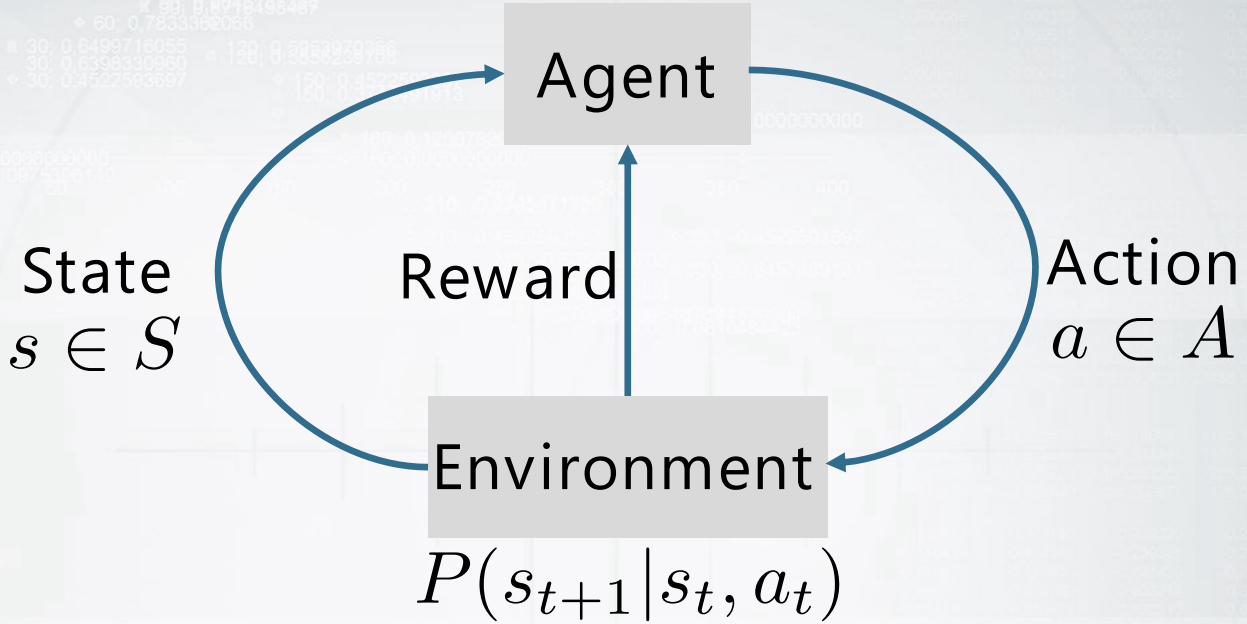


Markov Decision Process

Markov assumption

$$P(s_{t+1}|s_t, a_t, s_{t-1}, a_{t-1}) = P(s_{t+1}|s_t, a_t)$$

The MDP formalism



Markov Decision Process

Markov assumption

$$P(s_{t+1}|s_t, a_t, s_{t-1}, a_{t-1}) = P(s_{t+1}|s_t, a_t)$$

Total reward

Total reward for session:

$$R = \sum_t r_t$$

Agent's policy:

$$\pi(a|s) = P(take\ action\ a|in\ state\ s)$$

Problem: find policy with highest reward:

$$\pi(a|s): E_\pi[R] \to max$$

Objective

The easy way:

$E_\pi R$ is an expected sum of rewards that agent with policy π earns per session

The hard way:

$$\underset{s_0 \backsim p(s_0), a_0 \backsim \pi(a|s_0), s_1 r_0 \backsim P(s', r|s, a),}{E} \cdots \underset{s_n, r_n \backsim \pi(a|s_{n-1})}{E} [r_0 + r_1 + \cdots + r_n]$$

How do we solve it?

Policy = probability of

General idea:

Play a few sessions

↓

Update your policy

↓

Repeat

Crossentropy method

Intialize policy

Repeat:

- Sample N[100] sessions
- Pick M[25] best sessions, called **elite** sessions
- Change policy so that it prioritizes actions from elite sessions

Tabular crossentropy method

- Policy is a matrix

$$\pi(a|s) = A_{s,a}$$

- Sample N games with that policy
- Get M best sessions (elites)

$$Elite = [(s_0, a_0), (a_1, a_1), (s_2, a_2), \ldots, (s_k, a_k)]$$

Tabular crossentropy method

- Policy is a matrix

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Tabular crossentropy method

- Policy is a matrix

$$\pi(a|s) = A_{s,a}$$

- Sample N games with that policy
- Take M best sessions (elite)
- Aggregate by states

$$\pi(a|s) = \frac{\sum_{s_t,a_t \in Elite}[s_t = s][a_t = a]}{\sum_{s_t,a_t \in Elite}[s_t = s]}$$

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Tabular crossentropy method

- Policy is a matrix

$$\pi(a|s) = A_{s,a}$$

- Sample N games with that policy
- Take M best sessions (elite)
- Aggregate by states

$$\pi(a|s) = \frac{took\ a\ at\ s}{was\ at\ s}$$

In M best games

Smoothing

- If you were in some state only once, you only take this action now.
- Apply smoothing

$$\pi(a|s) = \frac{[\ took\ a\ at\ s] + \lambda}{[\ was\ at\ s] + \lambda \cdot N_{actions}}$$

In M best games

Alternative idea: smooth updates:

$$\pi_{i+1}(a|s) = \alpha \cdot \pi_{opt} + (1 - \alpha)\pi_i(a|s)$$

Stochastic MDPs

- If there's randomness in environment, algorithm will prefer "lucky" sessions.
- Training on lucky sessions is no good

- **Solution:** sample action for each state and run several simulations with these state-action pairs. Average the results.

Grim reality

If your environment has infinite/large state space

Approximate crossentropy method

- Policy is approximated
  - Neural network predicts $\pi_W(a|s)$ given s
  - Linear model / Random Forest / ...

Can't set $\pi(a|s)$ explicitly

All state-action pairs from M best sessions

$$Elite = [(s_0, a_0), (s_1, a_1), (s_2, a_2), \ldots, (s_k, a_k)]$$

Approximate crossentropy method

Approximate crossentropy method

Neural network predicts $\pi_W(a|s)$ given s

All state-action pairs from M best sessions

$$Best = [(s_0, a_0), (s_1, a_1), (s_2, a_2), \ldots, (s_k, a_k)]$$

Maximize likelihood of actions in "best" games

$$\pi = \underset{\pi}{\operatorname{argmax}} \sum_{s_i, a_i \in Elite} \log \pi(a_i|s_i)$$

Approximate crossentropy method

Neural network predicts $\pi_W(a|s)$ given s

All state-action pairs from M best sessions

$$Best = [(s_0, a_0), (s_1, a_1), (s_2, a_2), \ldots, (s_k, a_k)]$$

Maximize likelihood of actions in “best” games

Approximate crossentropy method

Neural network predicts $\pi_W(a|s)$ given s

All state-action pairs from M best sessions

$$Best = [(s_0, a_0), (s_1, a_1), (s_2, a_2), \ldots, (s_k, a_k)]$$

Maximize likelihood of actions in "best" games conveniently,

nn.fit(elite_states,elite_actions)

Approximate crossentropy method

Approximate crossentropy method

- Initialize NN weights $W_0 \leftarrow random$
- Loop:
  - Sample N sessions
  - elite = take M best sessions and concatenate

$$W_{i+1} = W_i + \alpha \nabla \left[\sum_{s_i, a_i \in Elite} \log \pi_{W_i}(a_i|s_i) \right]$$

Approximate

- Initialize NN weights $W_0 \leftarrow random$
- Loop:
  - Sample N sessions
  - elite = take M best sessions and concatenate

$$W_{i+1} = W_i + \alpha \nabla \left[\sum_{s_i, a_i \in Elite} \log \pi_{W_i}(a_i|s_i) \right]$$

Approximate (deep) version

- Initialize NN weights $W_0 \leftarrow random$

```
model =
MLPClassifier()
```

- Loop:
  - Sample N sessions
  - elite = take M best sessions and concatenate

$$W_{i+1} = W_i + \alpha \nabla \left[\sum_{s_i, a_i \in Elite} \log \pi_{W_i}(a_i|s_i) \right]$$

```
model.fit(elite_states,elite_acti
ons)
```

Continuous action spaces

- Continuous state space
- Model $\pi_W(a|s) = N(\mu(s), \sigma^2)$
  - Mu(s) is neural network output
  - Sigma is a parameter or yet another network output
- Loop:
  - Sample N sessions
  - elite = take M best sessions and concatenate
  -

$$W_{i+1} = W_i + \alpha \nabla \left[\sum_{s_i, a_i \in Elite} \log \pi_{W_i}(a_i|s_i) \right]$$

What changed?

Continuous action spaces

- Continuous state space
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- Loop:
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  -

$$W_{i+1} = W_i + \alpha \nabla \left[\sum_{s_i, a_i \in Elite} \log \pi_{W_i}(a_i|s_i) \right]$$

Continuous action spaces

- Continuous state space model = MLPRegressor()
- Model $\pi_W(a|s) = N(\mu(s), \sigma^2)$
  - Mu(s) is neural network output
  - Sigma is a parameter or yet another network output
- Loop:
  - Sample N sessions
  - elite = take M best sessions and concatenate
  -

$$W_{i+1} = W_i + \alpha \nabla \left[\sum_{s_i, a_i \in Elite} \log \pi_{W_i}(a_i|s_i) \right]$$

model.fit(elite_states, elite_actions)

Nothing!

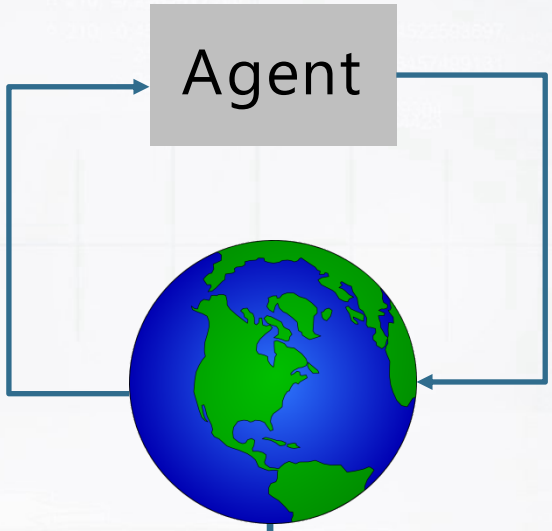
Tricks

- Remember sessions from 3-5 past iterations
  - Threshold and use all of them when training
  - May converge slower if env is easy to solve.

- Regularize with entropy
  - to prevent premature convergence.

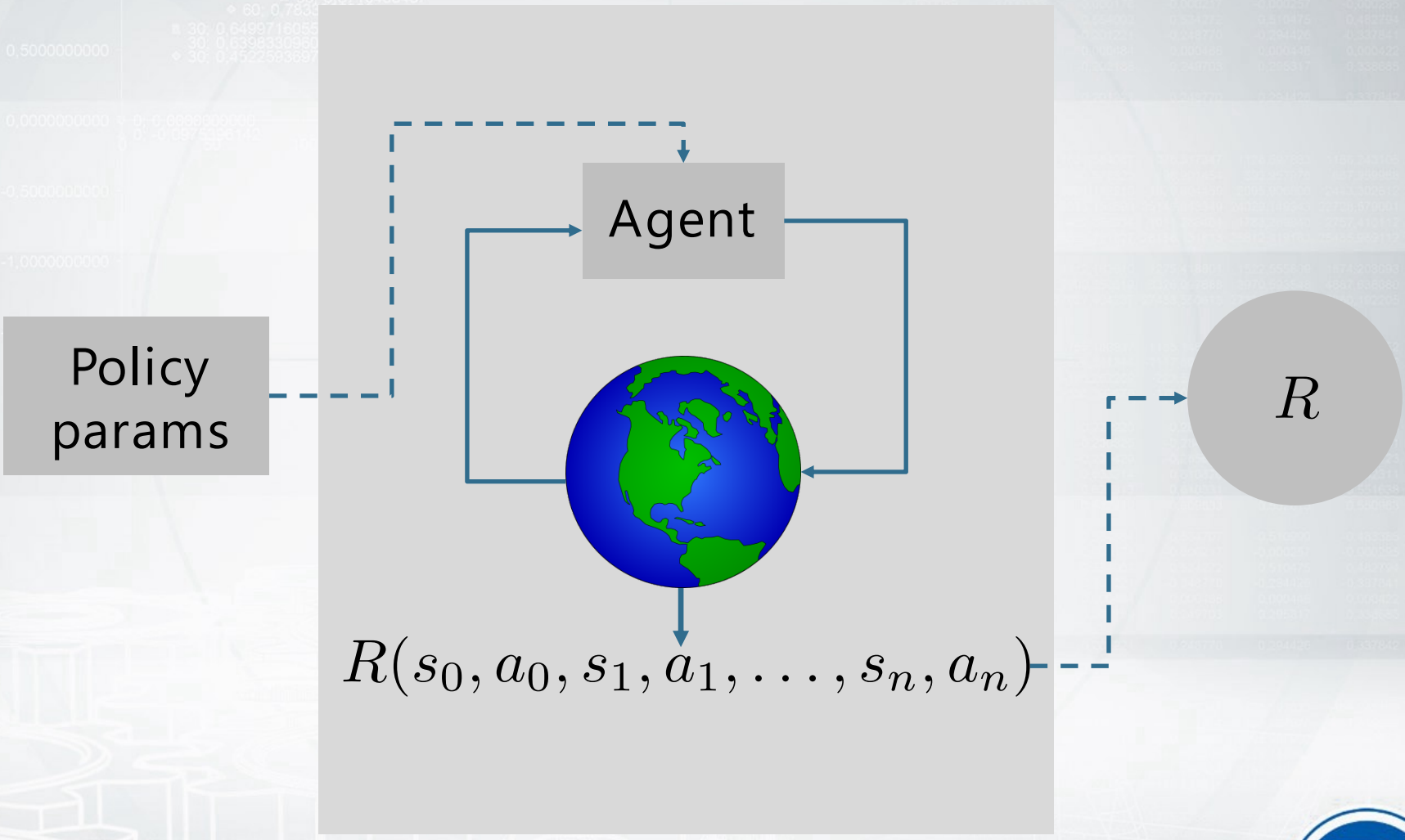
- Parallelize sampling
- Use RNNs if partially-observable (later)

Black box optimization setup

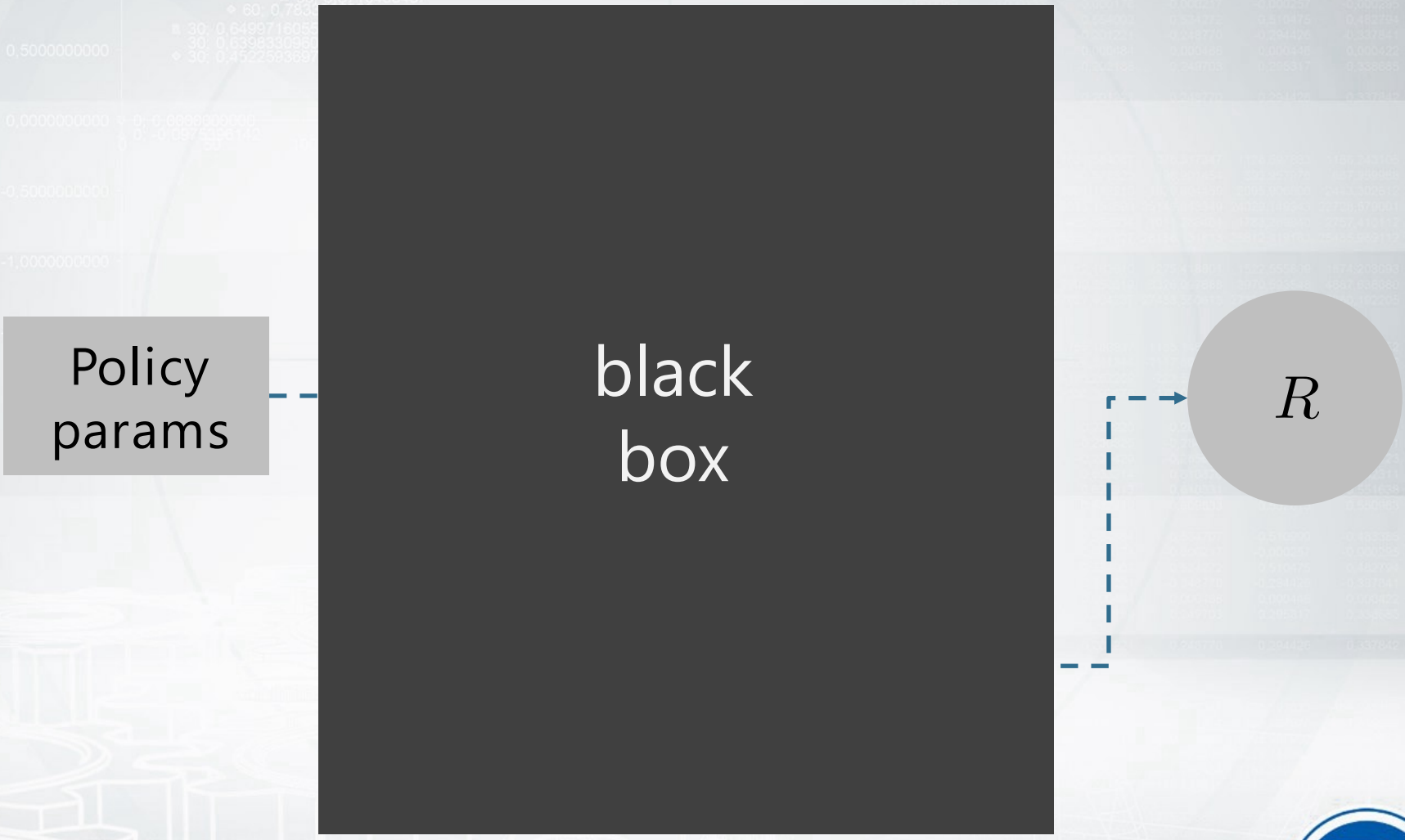


$$R(s_0, a_0, s_1, a_1, \ldots, s_n, a_n)$$

Black box optimization setup



Black box optimization setup



Evolution Strategies

- Introduce distribution over weights

$$\theta \backsim N(\theta|\mu, \sigma^2)$$

- Maximize expected reward

$$J = \underset{N(\theta|\mu,\sigma^2)}{E} R$$

Evolution Strategies

- Introduce distribution over weights

$$\theta \backsim N(\theta|\mu, \sigma^2)$$

other $P(\theta)$ will work as well

- Maximize expected reward

$$J = \underset{N(\theta|\mu,\sigma^2)}{E} R$$

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$$J = \underset{\theta \backsim N(\theta|\mu,\sigma^2)}{E} \; \underset{\tau(\theta)=s,a,s',a',\ldots}{E} R(\tau)$$

Evolution Strategies

Evolution Strategies

- Expected reward (using math. expectation)

$$J = \underset{\theta \backsim N(\theta|\mu,\sigma^2)}{E} \; \underset{\tau(\theta)=s,a,s',a',\ldots}{E} R(\tau)$$

Q: How can we estimate J in practice?
for large/infinite state space

Evolution Strategies

- Expected reward (using math. expectation)

$$J = \underset{\theta \backsim N(\theta|\mu,\sigma^2)}{E} \; \underset{\tau(\theta)=s,a,s',a',\ldots}{E} R(\tau)$$

Evolution Strategies

- Expected reward (using math. expectation)

$$J = \underset{\theta \backsim N(\theta|\mu,\sigma^2)}{E} \; \underset{\tau(\theta)=s,a,s',a',\ldots}{E} R(\tau)$$

- Approximate with sampling

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{\tau_i=s,a,s',a',\ldots} R(\tau_i)$$

Sample θ from $\theta \backsim N(\theta|\mu,\sigma^2)$

Evolution Strategies

- Expected reward (using math. expectation)

- Approximate with sampling

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{\tau_i = s, a, s', a', \ldots} R(\tau_i)$$

Sample θ from $\theta \backsim N(\theta | \mu, \sigma^2)$

Evolution Strategies

- Expected reward (using math. expectation)

$$J = \underset{N(\theta|\mu,\sigma^2)}{E} \; \underset{\tau_i = s,a,s',a',\ldots}{E} R(s,a,s',a',\ldots)$$

- Approximate with sampling

sample full episodes

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{\tau_i = s,a,s',a',\ldots} R(\tau_i)$$

Sample θ from

$$\theta \backsim N(\theta|\mu,\sigma^2)$$

Evolution Strategies

- Expected reward (using math. expectation)

Evolution Strategies

- Expected reward (using math. expectation)

$$J = \int N(\theta|\mu, \sigma^2) \cdot \int P(\tau|\theta) R(\tau) d\tau d\theta$$

- What we need

$$\nabla J = \frac{\partial J}{\partial}$$

Q: you compute ∇J w.r.t. what?

Evolution Strategies

- Expected reward (using math. expectation)

$$J = \int N(\theta|\mu, \sigma^2) \cdot \int P(\tau|\theta) R(\tau) d\tau d\theta$$

- What we need

Evolution Strategies

- Expected reward (using math. expectation)

$$J = \int N(\theta|\mu, \sigma^2) \cdot \int P(\tau|\theta) R(\tau) d\tau d\theta$$

- What we need

$$\nabla J = \frac{\partial J}{\partial \mu, \partial \sigma^2}$$

Gradient w.r.t. distribution parameters

Evolution Strategies

- Expected reward (using math. expectation)

$$J = \int N(\theta|\mu, \sigma^2) \cdot \int P(\tau|\theta) R(\tau) d\tau d\theta$$

- What we need

Evolution Strategies

- Expected reward (using math. expectation)

$$J = \int N(\theta|\mu, \sigma^2) \cdot \int P(\tau|\theta) R(\tau) d\tau d\theta$$

- What we need

$$\nabla J = \int \nabla [N(\theta|\mu, \sigma^2)] \cdot \int P(\tau|\theta) R(\tau) d\tau d\theta$$

Evolution Strategies

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- Expected reward (using math. expectation)

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- What we need

$$\nabla J = \int \nabla [N(\theta|\mu, \sigma^2)] \cdot \int P(\tau|\theta) R(\tau) d\tau d\theta$$

Can't estimate by sampling!

$\nabla N(\theta|\mu, \sigma)$ is not a distribution

Logderivative trick

Simple math

$$\nabla \log f(x) =$$

Logderivative trick

Simple math

$$\nabla \log f(x) = \frac{1}{f(x)} \cdot \nabla f(x)$$

$$f(x) \cdot \nabla \log f(z) = \nabla f(z)$$

Logderivative trick

Logderivative trick

Analytical inference

$$\nabla J = \int \nabla[N(\theta)|\mu, \sigma^2)] \cdot \int P(\tau|\theta) R(\tau) d\tau d\theta$$

$$\nabla N(\theta|\mu, \sigma^2) = N(\theta|\mu, \sigma^2) \cdot \nabla \log N(\theta|\mu, \sigma^2)$$

Analytical inference

Evolution Strategies

Analytical inference

$$\nabla J = \int [N(\theta|\mu, \sigma^2) \cdot \nabla \log N(\theta|\mu, \sigma^2)] \cdot \int P(\tau|\theta) R(\tau) d\tau d\theta$$

Q: How can we estimate ∇J in now?

Evolution Strategies

Analytical inference

Evolution Strategies

Analytical inference

$$\nabla J = \mathop{E}_{\theta \backsim N(\theta|\mu,\sigma^2)} \nabla \log N(\theta|\mu,\sigma^2) \cdot \mathop{E}_{\tau(\theta)=s,a,s',\ldots} R(\tau)$$

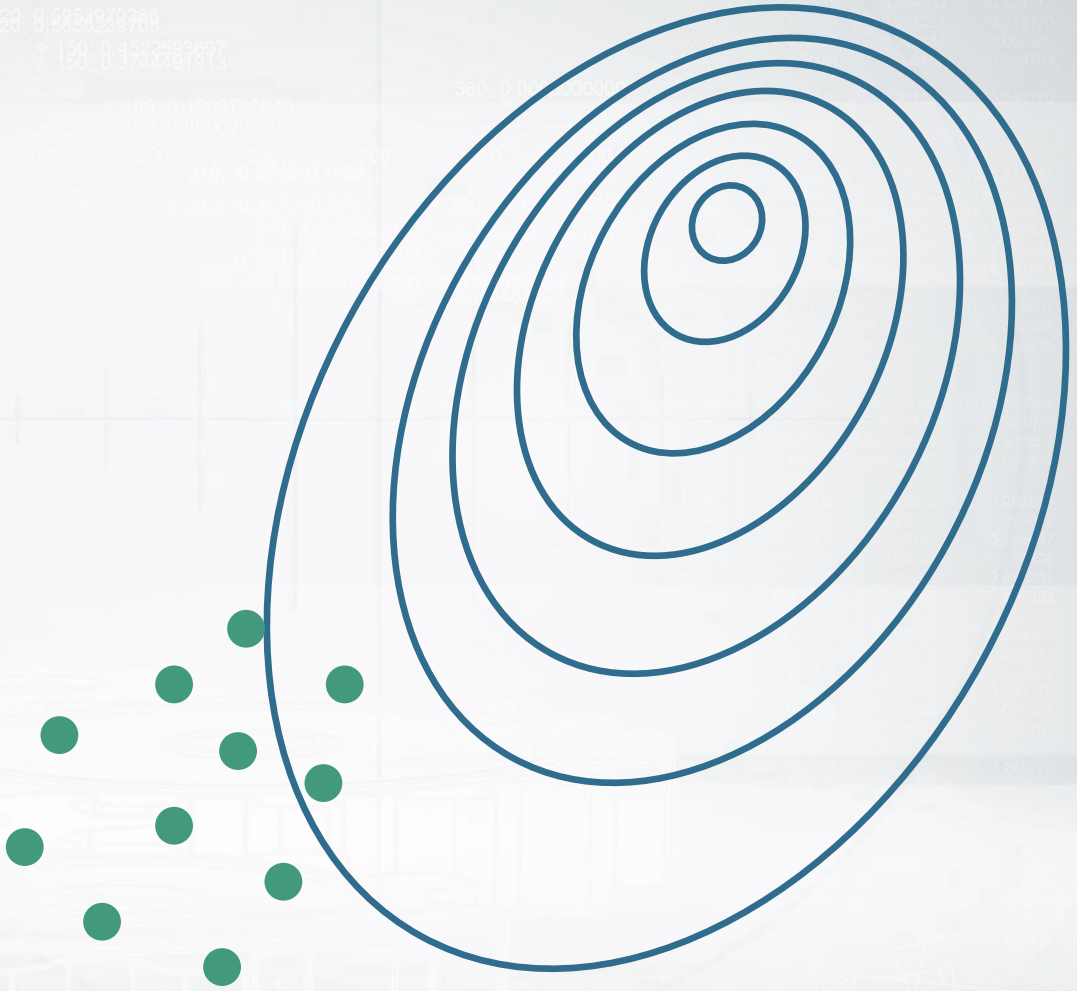
Sampled estimate

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \nabla \log N(\theta|\mu,\sigma^2) \cdot \sum_{s,a \in z_i} R(s,a\ldots)$$

Sample θ from $\theta \backsim N(\theta|\mu,\sigma^2)$

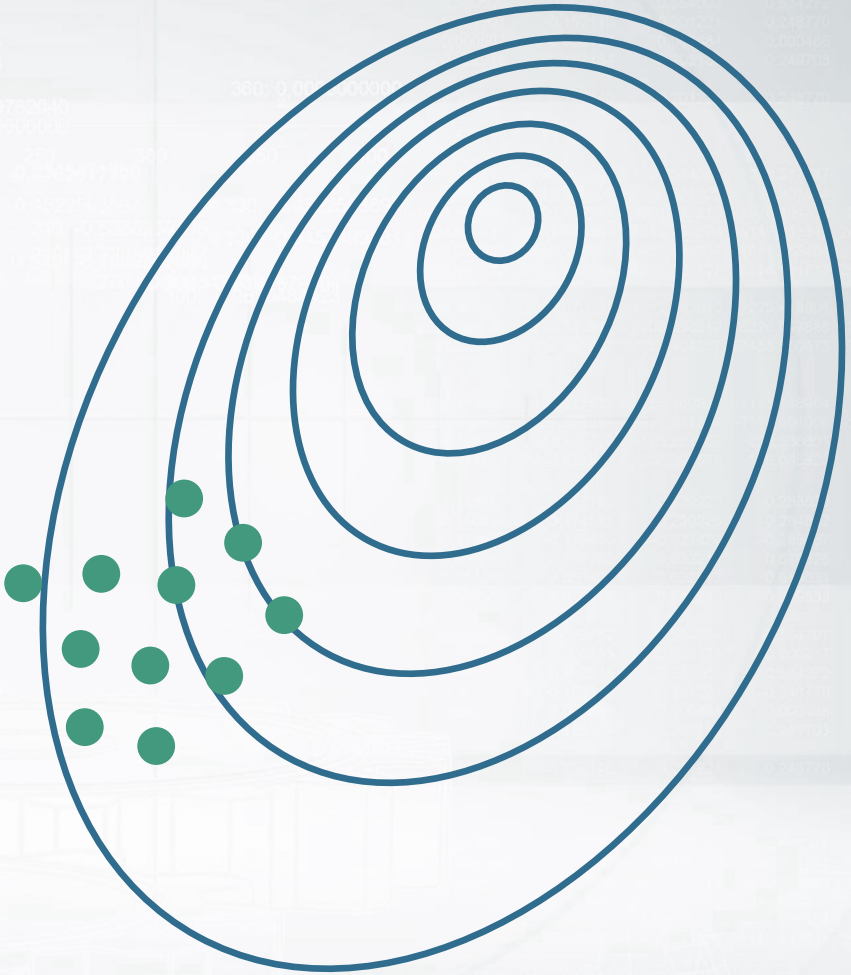
Evolution Strategies

Evolution Strategies



Evolution Strategies

Evolution Strategies



Evolution Strategies

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Evolution Strategies

Algorithm

1. Initialize μ_0, σ_0^2

2. Forever:

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \nabla \log N(\theta|\mu, \sigma^2) \cdot \sum_{s,a,s',\cdots \in \tau_i} R(s, a, s', \dots)$$

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$$\mu = \mu + \alpha \nabla_\mu J \qquad \sigma^2 = \sigma^2 + \alpha \cdot \nabla_{\sigma^2} J$$

Evolution Strategies

Algorithm

1. Initialize μ_0, σ_0^2

2. Forever:

Evolution Strategies

Algorithm

1. Initialize μ_0, σ_0^2

2. Forever: $\theta = \mu + \sigma \cdot \xi : \xi \backsim N(0,1)$

$$\text{sample} \quad \theta_i, \tau_i, R_i : R_i = R(\tau_i) = R(\tau(\theta_i))$$

$$\mu = \mu + \alpha \cdot \frac{1}{N \cdot \sigma} \cdot \sum_i \xi_i \cdot R_i$$

$$\sigma^2 = \dots$$

Reward baselines

TL;DR normalize reward

$$\operatorname{argmax} \underset{N(\theta|\mu,\sigma^2)}{E} R = \operatorname{argmax} \underset{N(\theta|\mu,\sigma^2)}{E} \frac{R-mean}{std}$$

$$A = \frac{R - ER}{VarR}$$

Evolution strategies

Features

- A general black box optimization
- **Easy to implement & scale**

Evolution strategies

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- A general black box optimization
- **Easy to implement & scale**

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \nabla \log N(\theta|\mu, \sigma^2) \cdot \sum_{\tau_i = s, a, s', \ldots} R(\tau_i)$$

Q: You have 1000 CPUs.

Optimize this formula!

Evolution strategies

Features

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- **Easy to implement & scale**

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Evolution strategies

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$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \nabla \log N(\theta|\mu, \sigma^2) \cdot \sum_{\tau_i = s, a, s', \ldots} R(\tau_i)$$

You can compute every session in parallel

Evolution strategies

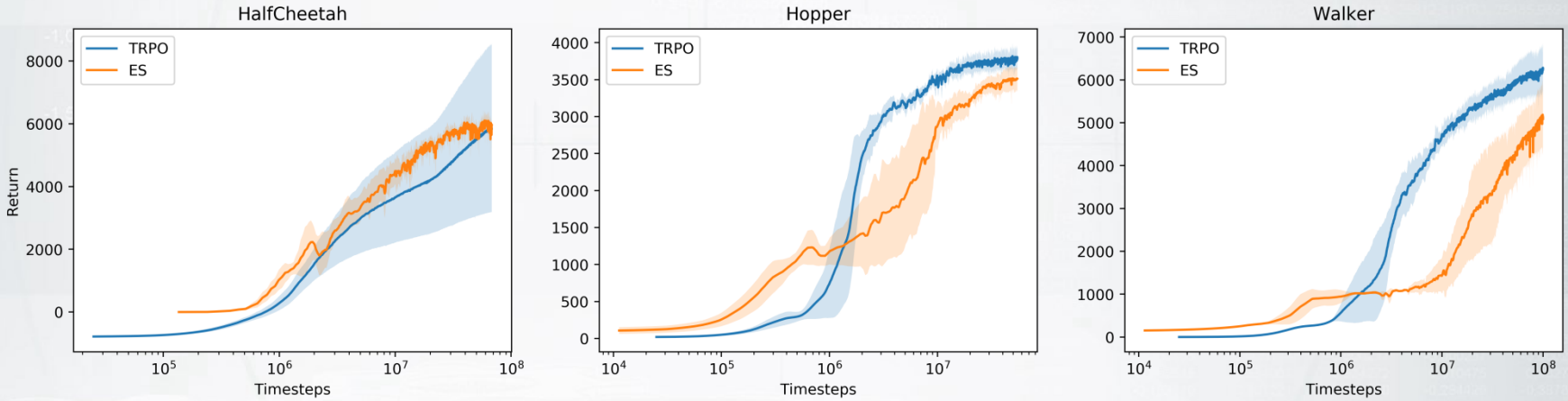
Features

- A general black box optimization

Evolution strategies

Features

- A general black box optimization
- Some results on gym



https://blog.openai.com/evolution-strategies/

Common drawback

Both CEM, ES and all similar methods

- Train from full sessions only
- Require a lot of samples

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