RL@Coursera
week 5

Policy gradient methods

Yandex
Data Factory

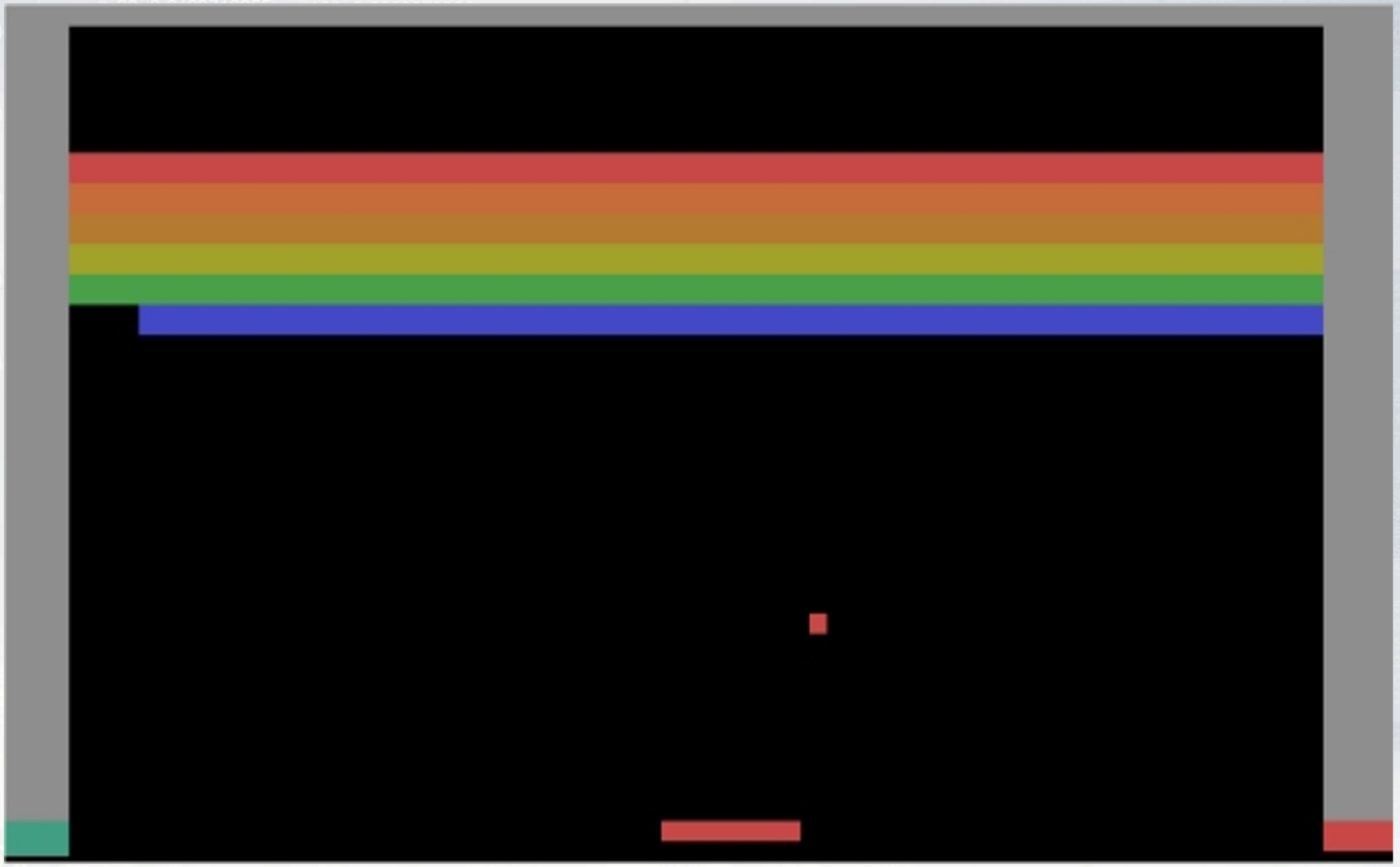
LAMBDA

Small experiment

The next slide contains a question

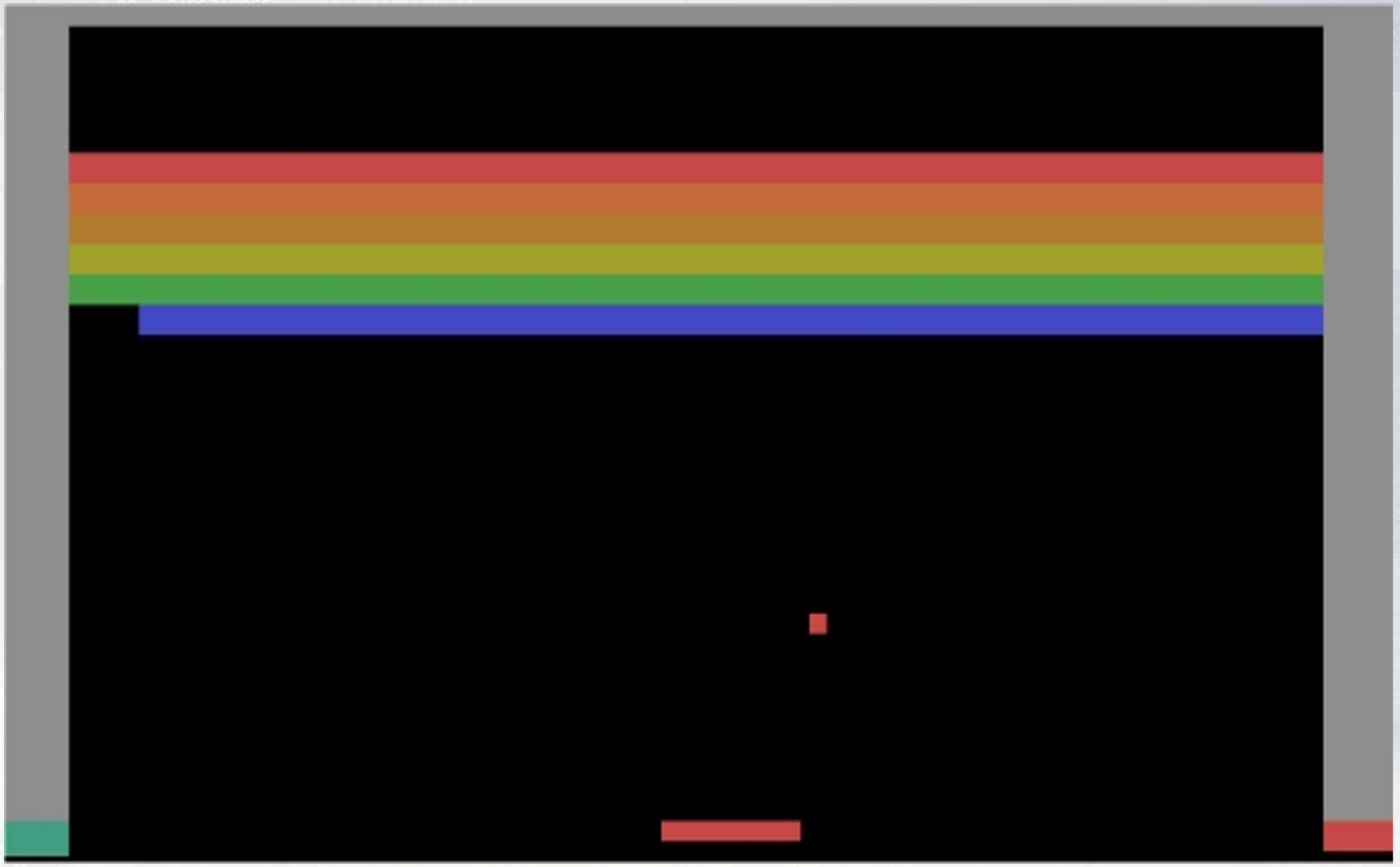
Please respond as fast as you can!

Small experiment



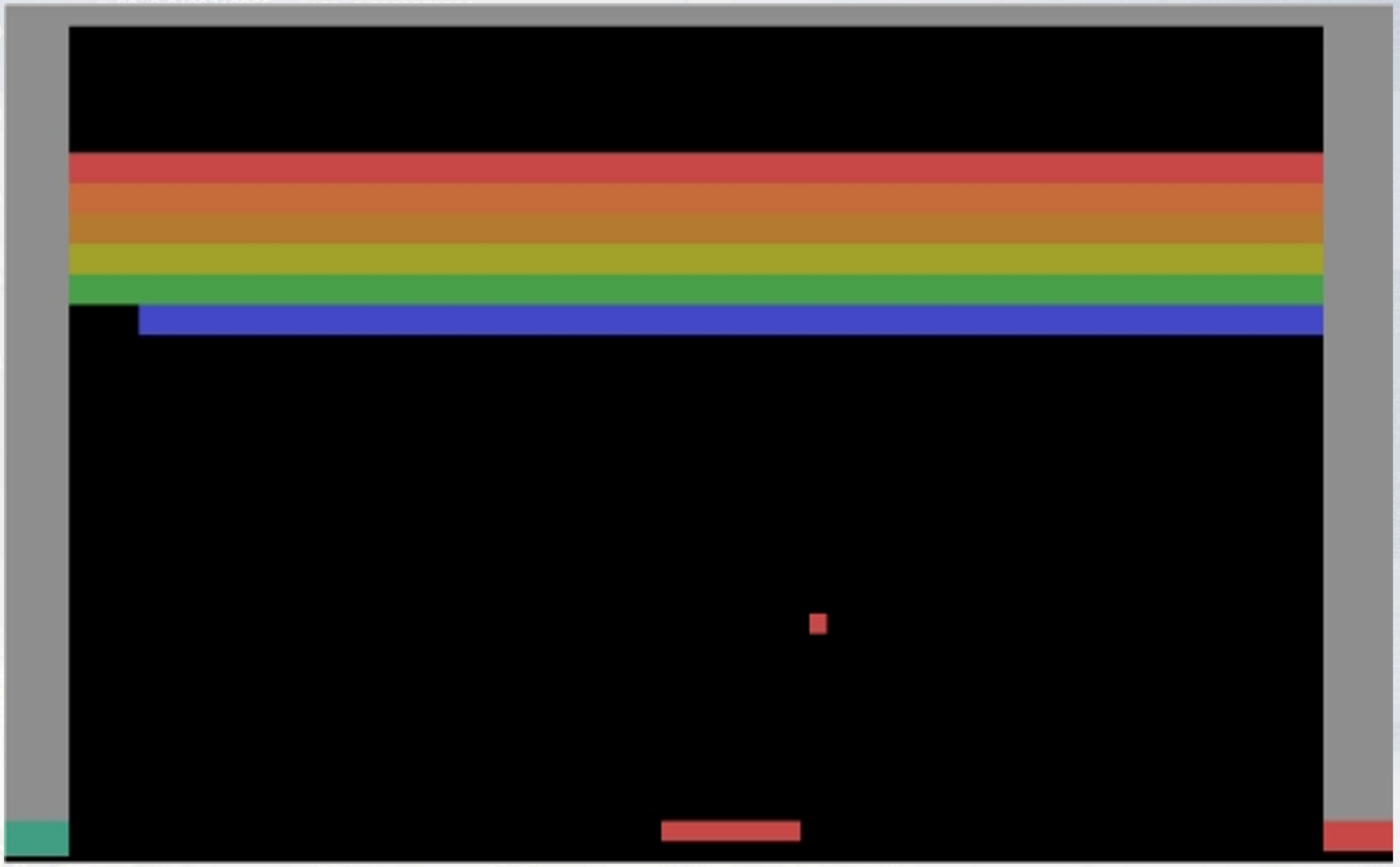
Left or Right?

Small experiment



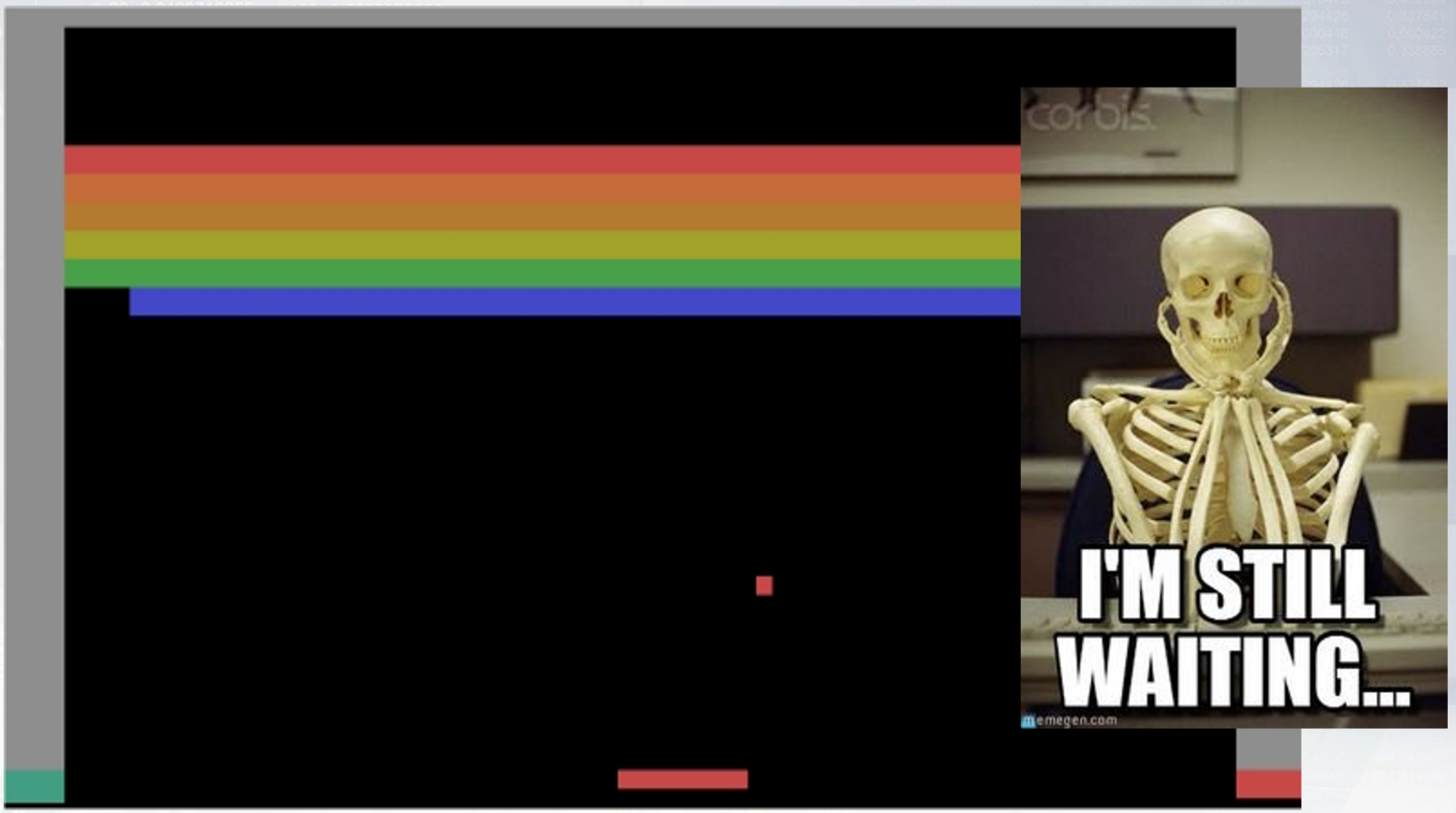
Right! Ready for next one?

Small experiment



What's Q(s,right) under gamma=0.99?

Small experiment



What's Q(s,right) under gamma=0.99?

Approximation error

$$L \approx \; E[Q(s_t, a_t) - (r_t + \gamma \cdot max_{a'})]^2$$

Simple 2-state world

	True	(A)	(B)
Q(s0,a0)	1	1	2
Q(s0,a1)	2	2	1
Q(s1,a0)	3	3	3
Q(s1,a1)	100	50	100

Trivia: Which prediction is better (A/B)?

Approximation error

$$L \approx \; E[Q(s_t, a_t) - (r_t + \gamma \cdot max_{a'})]^2$$

Simple 2-state world

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		better policy	less MSE

Approximation error

$$L \approx \; E[Q(s_t, a_t) - (r_t + \gamma \cdot max_{a'})]^2$$

Simple 2-state world

	True	**(A)**	**(B)**
Q(s0,a0)	1	1	2
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Q(s1,a0)	3	3	3
Q(s1,a1)	100	50	100
	Q-learning will prefer worse policy (B)!	**better policy**	**less MSE**

Conclusion

- Often computing q-values is harder than picking optimal actions!
- We could avoid learning value functions by directly learning agent's policy $\pi_\theta(s|a)$

Q: what algorithm works that way?

Conclusion

- Often computing q-values is harder than picking optimal actions!
- We could avoid learning value functions by directly learning agent's policy $\pi_\theta(s|a)$

Q: what algorithm works that way?

(of those we studied)

Conclusion

- Often computing q-values is harder than picking optimal actions!
- We could avoid learning value functions by directly learning agent's policy $\pi_\theta(s|a)$

Q: what algorithm works that way?

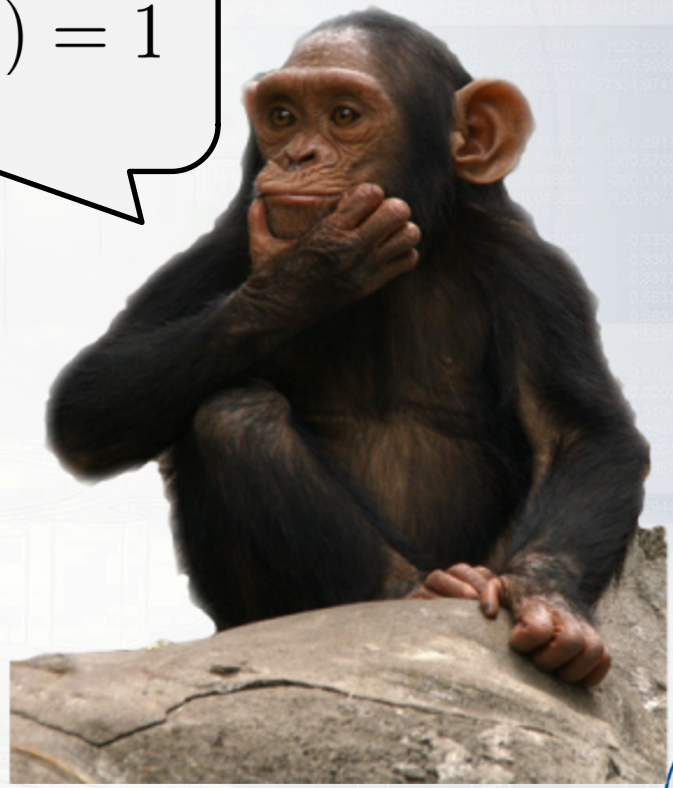
e.g. crossentropy method

NOT how humans survived

argmax
[Q(s,pet the tiger)
Q(s,run from tiger)
Q(s,provoke tiger)
Q(s,ignore tiger)]

how humans survived

$$\pi(run|s) = 1$$



Policies

In general, two kinds

Deterministic policy

$$a = \pi_\theta(a|s)$$

Stochastic policy

$$a \backsim \pi_\theta(a|s)$$

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Q: Any case where stochastic is better?

Policies

In general, two kinds

Deterministic policy

$$a = \pi_\theta(a|s)$$

Stochastic policy

$$a \backsim \pi_\theta(a|s)$$

e.g. rock-paper-scissors

Q: Any case where stochastic is better?

Policies

In general, two kinds

Deterministic policy

$$a = \pi_\theta(a|s)$$ ← same action each time

Stochastic policy

$$a \backsim \pi_\theta(a|s)$$ ← sampling takes care of exploration

Policies

In general, two kinds

Deterministic policy

$$a = \pi_\theta(a|s)$$ ← same action each time

Stochastic policy

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Q: how to represent policy in continuous action space?

Policies

In general, two kinds

Deterministic policy

$$a = \pi_\theta(a|s)$$ ← same action each time

Stochastic policy

$$a \backsim \pi_\theta(a|s)$$ ← sampling takes care of exploration

categorical, normal, mixture of normal, whatever

Two approaches

- **Value based:**

Learn value function $Q_\theta(s,a)$ or $V_\theta(s)$

Infer policy $\pi(a|s) = \#[a = \underset{a}{argmax} Q_\theta(s,a)]$

- **Policy based:**

Explicitly learn policy $\pi_\theta(s,a)$ or $\pi_\theta(s) \to a$

Implicitly maximize reward over policy

Recap: crossentropy method

- Initialize policy params $\theta_0 \leftarrow random$
- Loop:
  - Sample N sessions
  - elite = take M best sessions and concatenate

$$\theta_{i+1} = \theta_i + \alpha \bigtriangledown \sum_i \log \pi_{\theta_i}(a_i|s_i) \cdot [s_i, a_i \in Elite$$

Policy gradient main idea

Why so complicated?

We'd rather simply maximize G over pi!

Objective

- Expected reward:

$$J = \underset{\substack{s \backsim p(s) \\ a \backsim \pi_\theta(s|a) \\ \dots}}{E} R(s, a, s', a', \dots)$$

- Expected discounted reward:

$$J = \underset{\substack{s \backsim p(s) \\ a \backsim \pi_\theta(s|a)}}{E} G(s, a)$$

Objective

- Expected reward:

$R(z)$ setting

$$J = \mathop{E}_{\substack{s \backsim p(s) \\ a \backsim \pi_\theta(s|a) \\ \dots}} R(s, a, s', a', \dots)$$

- Expected discounted reward:

$$G(s,a) = r + \gamma \cdot G(s', a')$$

$$J = \mathop{E}_{\substack{s \backsim p(s) \\ a \backsim \pi_\theta(s|a)}} G(s, a)$$

Objective

- Consider an 1-step process for simplicity

$$J = \underset{\substack{s \backsim p(s) \\ a \backsim \pi_\theta(s|a)}}{E} R(s,a)$$

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$$J = \underset{\substack{s \backsim p(s) \\ a \backsim \pi_\theta(s|a)}}{E} R(s,a) = \int_s p(s) \int_a \pi_\theta(a|s)\ R(s,a)\ da\ ds$$

Objective

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state visitation frequency
(may depend on policy)

Reward for 1-step session

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state visitation frequency
(may depend on policy)

Reward for 1-step session

Q: how do we compute that?

Objective

- Consider an 1-step process for simplicity

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Reward for agent's action

$$J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} R(s,a)$$

sample N sessions by following $\pi_\theta(a|s)$

Objective

- Consider an 1-step process for simplicity

$$J = \underset{\substack{s \backsim p(s) \\ a \backsim \pi_\theta(s|a)}}{E} R(s,a) = \int_s p(s) \int_a \pi_\theta(a|s)\ R(s,a)\ da\ ds$$

Reward for agent's action

$$J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} R(s,a)$$

sample N sessions

Can we optimize policy now?

Objective

- Consider an 1-step process for simplicity

$$J = \underset{\substack{s \backsim p(s) \\ a \backsim \pi_\theta(s|a)}}{E} R(s,a) = \int_s p(s) \int_a \pi_\theta(a|s)\ R(s,a)\ da\ ds$$

parameters "sit" here

$$J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} R(s,a)$$

We don't know how to compute dJ/dtheta

Optimization

- Finite differences
  - Change policy a little, evaluate

$$\bigtriangledown J \approx \frac{J_{\theta+\epsilon} - J_{\theta}}{\epsilon}$$

- Stochastic optimization
  - Good old crossentropy method
  - Maximize probability of “elite” actions

Optimization

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Optimization

- Finite differences
  - Change policy a little, evaluate

$$\bigtriangledown J \approx \frac{J_{\theta+\epsilon} - J_\theta}{\epsilon}$$

VERY noizy, especially if both Js are sampled

- Stochastic optimization
  - Good old crossentropy method
  - Maximize probability of “elite” actions

stochastic MDPs;
Throws a most sessions away

Objective

- Consider an 1-step process for simplicity

$$J = \underset{\substack{s \backsim p(s) \\ a \backsim \pi_\theta(s|a)}}{E} R(s,a) = \int_s p(s) \int_a \pi_\theta(a|s)\ R(s,a)\ da\ ds$$

Wish list:

- Analytical gradient
- Easy/stable approximations

Logderivative trick

Simple math

$$\bigtriangledown \log \pi(z) = ???$$

(try chain rule)

Logderivative trick

Simple math

$$\bigtriangledown \log \pi(z) = \frac{1}{\pi(z)} \cdot \pi(z)$$

$$\pi \cdot \bigtriangledown \log \pi(z) = \bigtriangledown \pi(z)$$

Policy gradient

Analytical inference

$$J = \int_s p(s) \int_a \pi_\theta(a|s)\ R(s,a)\ da\ ds$$

$$\pi \cdot \bigtriangledown \log \pi(z) = \bigtriangledown \pi(z)$$

Policy gradient

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$$J = \int_s p(s) \int_a \pi_\theta(a|s) \bigtriangledown \log \pi_\theta(a|s)\ R(s,a)\ da\ ds$$

Trivia: Anything curious about that formula?

Policy gradient

Analytical inference

$$J = \int_s p(s) \int_a \pi_\theta(a|s)\ R(s,a)\ da\ ds$$

$$\pi \cdot \bigtriangledown \log \pi(z) = \bigtriangledown \pi(z)$$

$$J = \int_s p(s) \int_a \pi_\theta(a|s) \bigtriangledown \log \pi_\theta(a|s)\ R(s,a)\ da\ ds$$

that's expectation

:)

Discounted reward case

Replace R with Q :)

True action value a.k.a. $E(G[s,a])$

$$J = \int_s p(s) \int_a \pi_\theta(a|s)\ Q(s,a)\ da\ ds$$

$$\pi \cdot \bigtriangledown \log \pi(z) = \bigtriangledown \pi(z)$$

$$J = \int_s p(s) \int_a \pi_\theta(a|s) \bigtriangledown \log \pi_\theta(a|s)\ Q(s,a)\ da\ ds$$

that's expectation :)

Policy gradient (REINFORCE)

- Policy gradient

$$\bigtriangledown J = \underset{\substack{s \backsim p(s) \\ a \backsim \pi_\theta(s|a)}}{E} \bigtriangledown \log \pi_\theta(a|s) \cdot Q(s,a)$$

- Approximate with sampling

$$J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \bigtriangledown \log \pi_\theta(a|s) \cdot Q(s,a)$$

REINFORCE algorithm

- Initialize NN weights $\theta_0 \leftarrow random$

- Loop:
  - Sample N sessions **z** under current $\pi_\theta(a|s)$
  - Evaluate policy gradient

$$J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \bigtriangledown \log \pi_\theta(a|s) \cdot Q(s,a)$$

Ascend $$\theta_{i+1} \leftarrow \theta_i + \alpha \cdot \bigtriangledown J$$

REINFORCE algorithm

- Initialize NN weights $\theta_0 \leftarrow random$

Q: is it on- or off-policy?

- Loop:
  - Sample N sessions **z** under current $\pi_\theta(a|s)$
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REINFORCE algorithm

- Initialize NN weights $\theta_0 \leftarrow random$

actions under current policy

= on-policy

- Loop:
  - Sample N sessions **z** under current $\pi_\theta(a|s)$
  - Evaluate policy gradient

$$J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \bigtriangledown \log \pi_\theta(a|s) \cdot Q(s,a)$$

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Ascend $$\theta_{i+1} \leftarrow \theta_i + \alpha \cdot \bigtriangledown J$$

What is better for learning:

random action in good state or

great action in bad state?

REINFORCE baseline

- Initialize NN weights $\theta_0 \leftarrow random$

- Loop:
  - Sample N sessions **z** under current $\pi_\theta(a|s)$
  - Evaluate policy gradient

$$J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \bigtriangledown \log \pi_\theta(a|s) \cdot Q(s,a)$$

Ascend $$\theta_{i+1} \leftarrow \theta_i + \alpha \cdot \bigtriangledown J$$

$$Q(s,a) = V(s) + A(s,a)$$

Actions influence $A(s,a)$ only, so $V(s)$ is irrelevant

REINFORCE baseline

- Initialize NN weights $\theta_0 \leftarrow random$

- Loop:
  - Sample N sessions **z** under current $\pi_\theta(a|s)$
  - Evaluate policy gradient

$$J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \bigtriangledown \log \pi_\theta(a|s) \cdot (Q(s,a) - b(s))$$

Ascend $$\theta_{i+1} \leftarrow \theta_i + \alpha \cdot \bigtriangledown J$$

Anything that doesn't depend on action

ideally, $b(s) = V(s)$

Actor-critic

- Learn both $V(s)$ and $\pi_\theta(a|s)$
- Hope for best of both worlds :)

image: youtube: mr tivikov

Advantage actor-critic

- Idea: learn both $\pi_\theta(a|s)$ and $V_\theta(s)$
- Use $V_\theta(s)$ to learn $\pi_\theta(a|s)$ faster!

Non-**trivia:** how can we estimate $A(s,a)$ from (s,a,r,s') and V- function?

Advantage actor-critic

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Advantage actor-critic

- Idea: learn both $\pi_\theta(a|s)$ and $V_\theta(s)$
- Use $V_\theta(s)$ to learn $\pi_\theta(a|s)$ faster!

$$A(s,a) = Q(s,a) - V(s)$$

$$Q(s,a) = r + \gamma \cdot V(s')$$

$$A(s,a) = r + \gamma \cdot V(s') - V(s)$$

Advantage actor-critic

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Also: n-step version

Advantage actor-critic

- Idea: learn both $\pi_\theta(a|s)$ and $V_\theta(s)$
- Use $V_\theta(s)$ to learn $\pi_\theta(a|s)$ faster!

$$A(s,a) = r + \gamma \cdot V(s') - V(s)$$

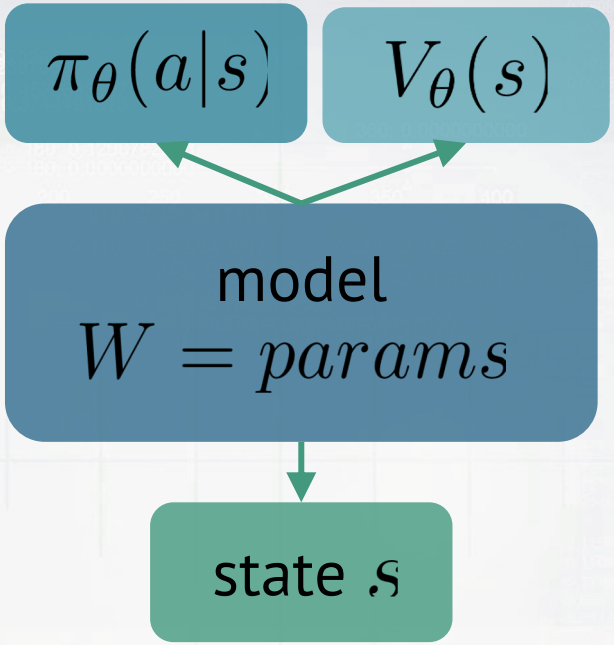
$$Q(s,a) = r + \gamma \cdot V(s')$$

$$\nabla J_{actor} \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \nabla \log \pi_\theta(a|s) \cdot A(s,a)$$

consider const

Trivia: How do we train V then?

Advantage actor-critic



$$\nabla J_{actor} \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \nabla \log \pi_\theta(a|s) \cdot A(s,a)$$

$$L_{critic} \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} (V_\theta(s) - [r + \gamma \cdot V(s')])^2$$

value-based Vs policy-based

Value-based	Policy-based + Actor-critic
Q-learning, SARSA,value-iteration	REINFORCE, Advantage Actor-Critic, Crossentropy Method

value-based Vs policy-based

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Q-learning, SARSA,value-iteration	REINFORCE, Advantage Actor-Critic, Crossentropy Method

Your guess?

Casy Study: A3C

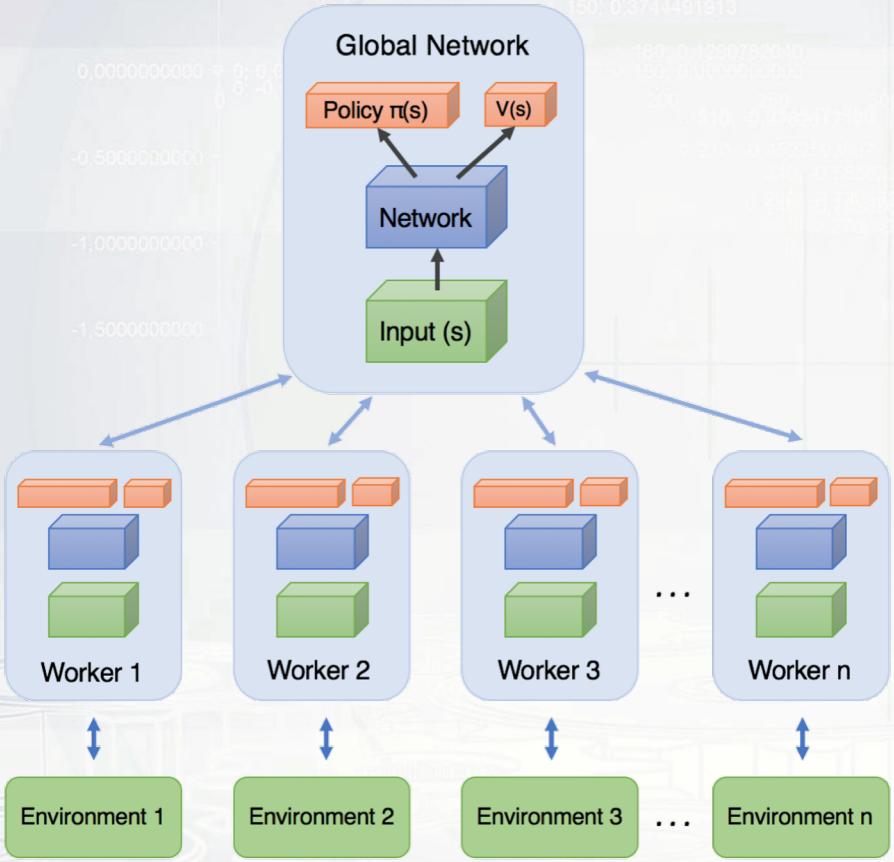
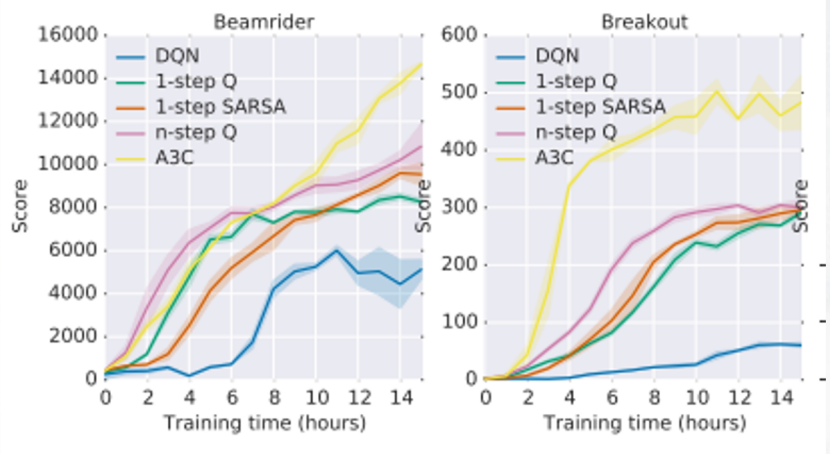


image: http://bit.ly/2z0kBNu

A popular implementation of advantage actor-critic using neural net agent

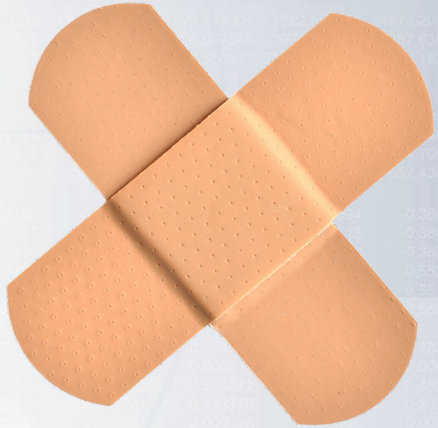
- No experience replay
- Many parallel sessions
- Asynchronous updates

Casy Study: A3C results



Duct tape zone

- V(s) errors less important than in Q-learning
  - actor still learns even if critic is random, just slower
- Regularize with entropy
  - to prevent premature convergence
- Learn on parallel sessions
  - Or super-small experience replay
- Use logsoftmax for numerical stability



value-based Vs policy-based

Value-based	Policy-based + Actor-critic
Q-learning, SARSA, value-iteration	REINFORCE, Advantage Actor-Critic, Crossentropy Method
Solves harder problem	Solves easier problem
Explicit exploration	Innate exploration & stochasticity
Evaluates states & actions	Easier for for continuous actions
Easier to train off-policy	Compatible with supervised learning

Supervised pre-training

Reinforcement learning usually takes longto find optimal policy completely from scratch.

We can use existing knowledge to help it!

- Human experience
- Known heuristic
- Previous system

Supervised pre-training

- Supervised learning:

$$\nabla llh = \mathop{E}_{x, y_{opt} \backsim D} \nabla \log P_\theta(y_{opt}|x)$$

- Policy gradient:

$$\nabla J = \mathop{E}_{\substack{s \backsim d(s) \\ a \backsim \pi(a|obs(s))}} \nabla \log \pi_(a|s) \cdot Q(s,a)$$

Supervised pre-training

- Supervised learning:

$$\nabla llh = \underset{s, a_{opt} \backsim D}{E} \nabla \log \pi_\theta(a_{opt}|s)$$

- Policy gradient:

$$\nabla J = \underset{\substack{s \backsim d(s) \\ a \backsim \pi(a|obs(s))}}{E} \nabla \log \pi_(a|s) \cdot Q(s,a)$$

Supervised pre-training

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- Policy gradient:

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Q: what's different? (apart from $Q(s,a)$)

Supervised pre-training

- Supervised learning:

reference

$$\nabla llh = \mathop{E}_{s, a_{opt} \backsim D} \nabla \log \pi_\theta(a_{opt}|s)$$

- Policy gradient:

$$\nabla J = \mathop{E}_{\substack{s \backsim d(s) \\ a \backsim \pi(a|obs(s))}} \nabla \log \pi_(a|s) \cdot Q(s,a)$$

generated

More out there

This domain is huge!

- Trust Region Policy optimization
  - policy gradient on steroids

- Deterministic policy gradients
  - Off-policy & less variance

- Many many more!
  - Research happens as you read this line

Let's code!