

最小二乘拟合问题的求解

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最小二乘拟合四次多项式

$$P(x) = c_0 + c_1x + c_2x^2 + c_3x^3 + c_4x^4$$

当 $w_k \equiv 1$, 正规方程组简化为 $G^T Gc = G^T y$.

这时若取 $\varphi_i = x^i$ ($i = 0, 1, \dots, n$), 则

$$G = \begin{pmatrix} 1 & x_1 & \cdots & x_1^n \\ 1 & x_2 & \cdots & x_2^n \\ \vdots & \vdots & & \vdots \\ 1 & x_m & \cdots & x_m^n \end{pmatrix}, \quad (\varphi_i, \varphi_j) = \sum_{k=1}^m x_k^i \cdot x_k^j = \sum_{k=1}^m x_k^{i+j}.$$

故

$$\begin{pmatrix} \sum_{k=1}^m 1 & \sum_{k=1}^m x_k & \cdots & \sum_{k=1}^m x_k^n \\ \sum_{k=1}^m x_k & \sum_{k=1}^m x_k^2 & \cdots & \sum_{k=1}^m x_k^{n+1} \\ \vdots & \vdots & & \vdots \\ \sum_{k=1}^m x_k^n & \sum_{k=1}^m x_k^{n+1} & \cdots & \sum_{k=1}^m x_k^{2n} \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_n \end{pmatrix} = \begin{pmatrix} \sum_{k=1}^m y_k \\ \sum_{k=1}^m x_k y_k \\ \vdots \\ \sum_{k=1}^m x_k^n y_k \end{pmatrix}$$

In [1]:

```
import numpy as np

# 算例
X = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9]
Y = [5.1234, 5.3057, 5.5687, 5.9375, 6.4370, 7.0978, 7.9493, 9.0253, 10.3627]
Y = np.array(Y)
#print(Y)
```

In [2]:

```

M = 9
N = 5
G = np.zeros((M,N))
for i in range(M):
    for j in range(N):
        G[i,j] = X[i]**j

print(G)

```

```

[[1.000e+00 1.000e-01 1.000e-02 1.000e-03 1.000e-04]
 [1.000e+00 2.000e-01 4.000e-02 8.000e-03 1.600e-03]
 [1.000e+00 3.000e-01 9.000e-02 2.700e-02 8.100e-03]
 [1.000e+00 4.000e-01 1.600e-01 6.400e-02 2.560e-02]
 [1.000e+00 5.000e-01 2.500e-01 1.250e-01 6.250e-02]
 [1.000e+00 6.000e-01 3.600e-01 2.160e-01 1.296e-01]
 [1.000e+00 7.000e-01 4.900e-01 3.430e-01 2.401e-01]
 [1.000e+00 8.000e-01 6.400e-01 5.120e-01 4.096e-01]
 [1.000e+00 9.000e-01 8.100e-01 7.290e-01 6.561e-01]]

```

In [3]:

```

c = np.linalg.inv(G.T@G) @ G.T@Y
print('c:', c)

print('最小二乘拟合四次多项式为: \n p = %s + %s*x + %s*x^2 + %s*x^3 + %s*x^4'%(c[0],c[1]

```

```
c: [5.00097222 0.99268907 2.01064782 3.00333463 0.99096737]
```

最小二乘拟合四次多项式为:

```

p = 5.000972222298487 + 0.99268907027583*x + 2.010647823986403*x^2 +
3.003334628477546*x^3 + 0.9909673659544183*x^4

```

给定算例的最小二乘拟合四次多项式为

$$P(x) = 5.00097222 + 0.99268907x + 2.01064782x^2 + 3.00333463x^3 + 0.99096737x^4$$

误差分析

In [4]:

```

R = Y - G@c
print(R)
#误差
R_norm2 = np.linalg.norm(R,2)
print(R_norm2)

```

```

[-5.00389310e-05  1.51825874e-04 -1.54118173e-04  1.66316962e-04
 -3.31002365e-04  2.31468522e-04  1.52952622e-04 -2.45648746e-04
  7.82440615e-05]
0.0005744292123220798

```

最小二乘拟合四次多项式的误差为

$$R = 0.0005744292123220798 < 10^{-3}$$

误差在可接受范围内。