Exercise 2

A medical doctor is only allowed to prescribe 3 possible drugs out of 6 available ones to a patient. The expected benefit of the i-th drug with $i \in \{1, \ldots, 6\}$ is quantified by a value b_i (for example, related to the viral load after some given period). It is assumed that the benefits of different drugs can be added. Furthermore, the following rules must be applied:

- drugs 1 and 2 are incompatible.
- If drug 2 is prescribed, then drug 3 must be prescribed.
- If drugs 3 and 4 are prescribed, then drug 5 cannot be prescribed.
- If drugs 4 or 5 is prescribed, then drug 6 cannot be prescribed.

Formulate the problem as a binary linear programming problem

Solution

We introduce one decision variable per medicine. Let x_i be the decision variable corresponding to the i-th medicine. We have that:

$$x_i = \begin{cases} 1 & \text{if the } i\text{-th medicine is prescribed} \\ 0 & \text{otherwise} \end{cases}$$

We set as a target to ~~minimize~~ maximize the benefit. If for example b_i is the viral load after the period, we set as a target to: ~~minimize~~:

$$\text{maximize} \sum_{i=1}^{6} b_i x_i$$
$$x \in \{0,1\}^6$$

Subject to:

$$x_1 + x_2 \leq 1$$
$$x_3 \geq x_2$$
$$x_5 \leq 2 - (x_3 + x_4)$$
$$x_6 \leq 1 - x_4$$
$$x_6 \leq 1 - x_5$$