Tutorial session 4 - Constraints in continuous optimization

Problem 1

A winner at Euromillions receives a S million prize and decides to live on the investment of her gain. Her welfare at time n+1 is supposed to be proportional to $r_n^{1/2}$ where r_n is the money spent between time n and n + 1. She thus wants to maximize

$$\sum_{n=0}^{N-2} \beta^n \sqrt{r_n}$$

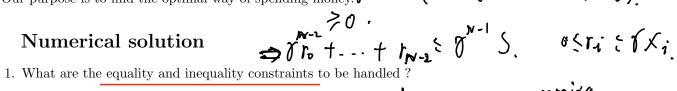
where $N-1 \geq 1$ is her remaining life expectancy and the exponentially decaying term β^n with $\beta \in]0,1[$ accounts for the fact that spending money today is more enjoyable than spending it tomorrow (in other words, the younger, the more fun!). If x_n designates the available amount of money at time $n \in \{0, ..., N-1\}$, we have

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, we have
$$(\forall n \in \{0,\dots,N-2\}) \quad x_{n+1} = \gamma x_n - r_n$$

where $\gamma \in [1, +\infty[$ represents the investment rate.

here $\gamma \in [1, +\infty[$ represents the investment rate. $= r^n \times_1 - (r_n + \gamma r_{n-1} + \gamma r_n)$.

Our purpose is to find the optimal way of spending money. $r^{n+1} = r^n \times_1 - (r_n + \gamma r_{n-1} + \gamma r_n)$.



- 2. Which kind of optimization problem should be solved? non-linear programming
 3. The provided PPXA Matlah function allows us to tackled.
- 3. (The provided PPXA Matlab Junction allows us to tackle the related constrained optimization problem. Use it to solve numerically the problem when N = 100 semesters, S = 130 millions, $\gamma = 1.03$, and $\beta = 0.96$.
- 4. Plot $(x_n)_{0 \le n \le N-1}$ and $(r_n)_{0 \le n \le N-2}$. Compute the associated cost value.
- 5. Compare the optimal strategy with the strategy which would consist in spending half of the money available at each time n.

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- 6. The winner might live longer than expected. As a cautious person, she plans to leave a residual amount of money R at duration N-1. Solve numerically the modified optimization problem when R = 10 millions. XNJ = R

Analytic solution

- Propose a Lagrange formulation of the latter problem.
 - 2. Deduce the closed form expression of the solution.
 - 3. Compare it with the numerical solution.