Essentials of MOSFETs

Unit 3: MOS Electrostatics

Lecture 3.2: The Depletion Approximation

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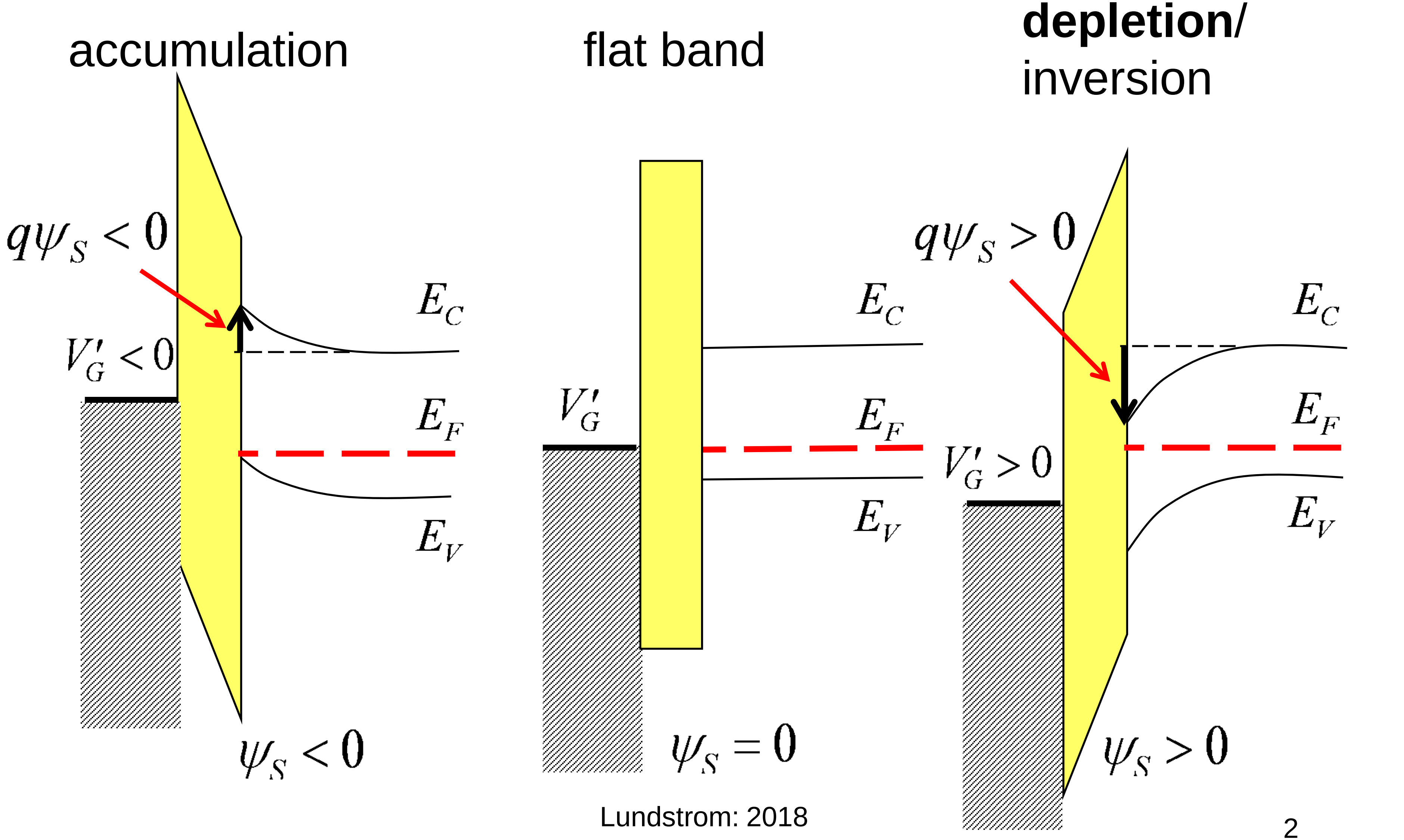
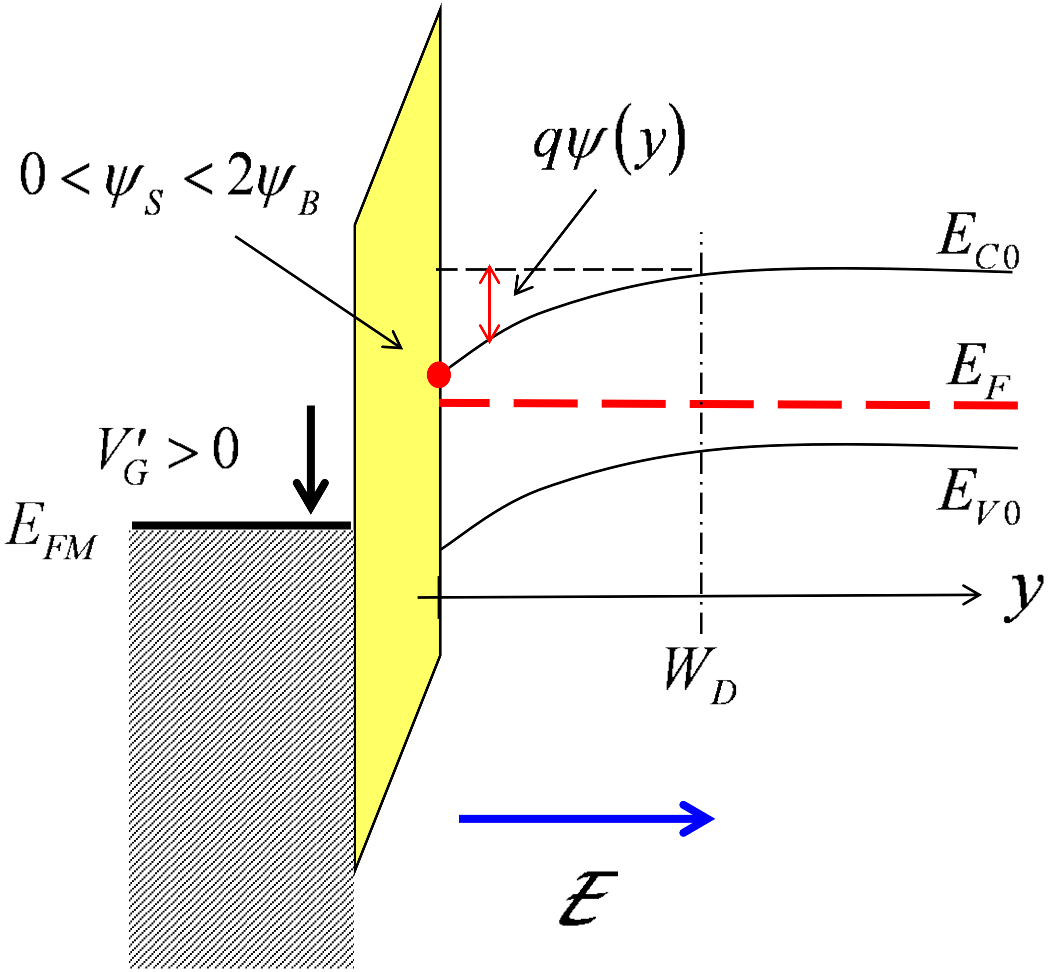
1D MOS electrostatics



Depletion



$$\rho(y) = q\left[p(y) - n(y) - N_A^-\right]$$

$y < W_D$:

$$p(y) << N_A^-$$

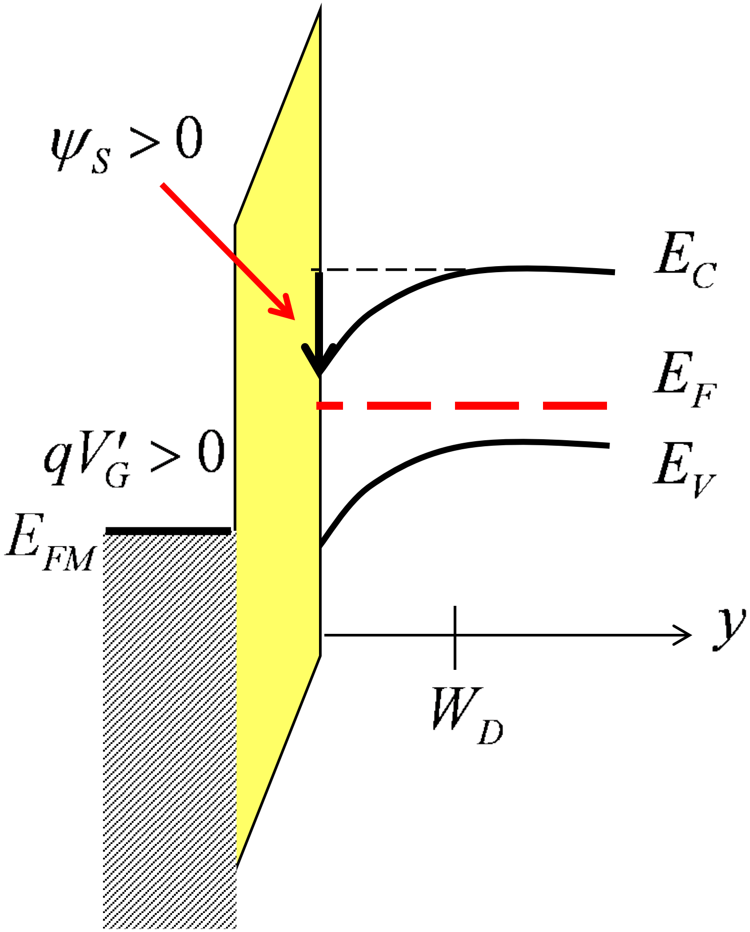
$$n(y) << N_A^-$$

$$\rho(y) \approx -qN_A$$

$y \geq W_D$:

$$\rho(y) \approx 0$$

Poisson equation

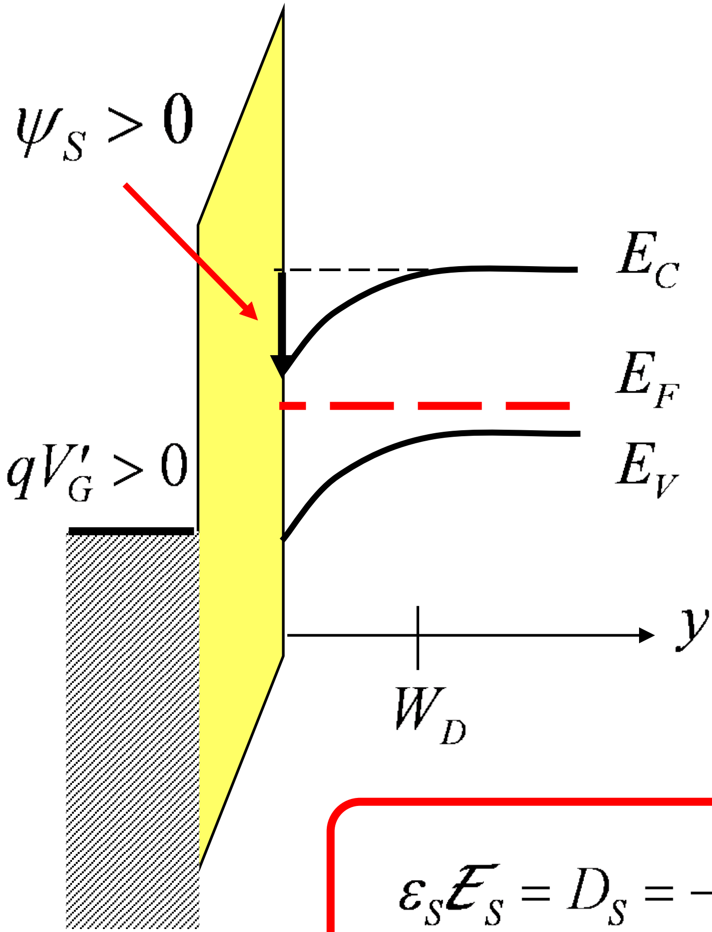


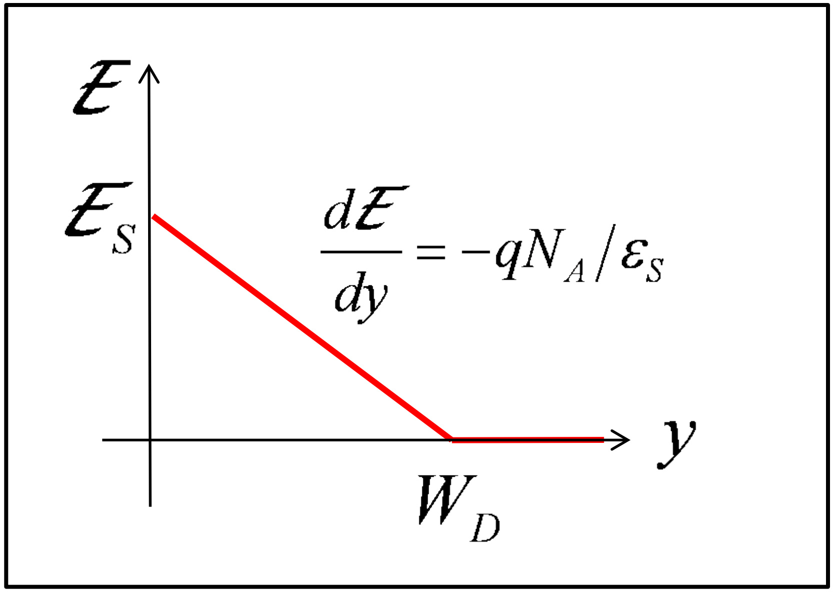
$$\boxed{\frac{d\,D(y)}{dy} = \rho(y)}$$

$$\frac{d\mathcal{E}}{dy} = \frac{\rho(y)}{\varepsilon_S} = -\frac{qN_A}{\varepsilon_S}$$

$$\varepsilon_S = \kappa_{Si}\varepsilon_0$$

Electric field





$$\boxed{\varepsilon_S \mathcal{E}_S = D_S = -Q_S}$$

$$\mathcal{E}(y) = \frac{qN_A}{\varepsilon_S}(W_D - y)$$

$$\mathcal{E}_S = \frac{qN_A W_D}{\varepsilon_S} = \frac{-Q_S}{\varepsilon_S}$$

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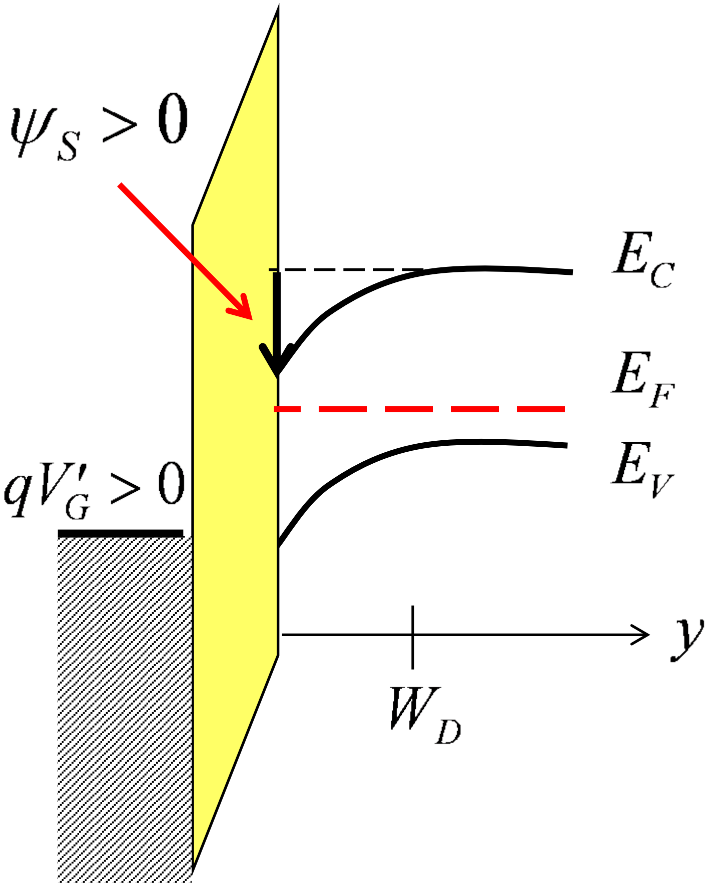
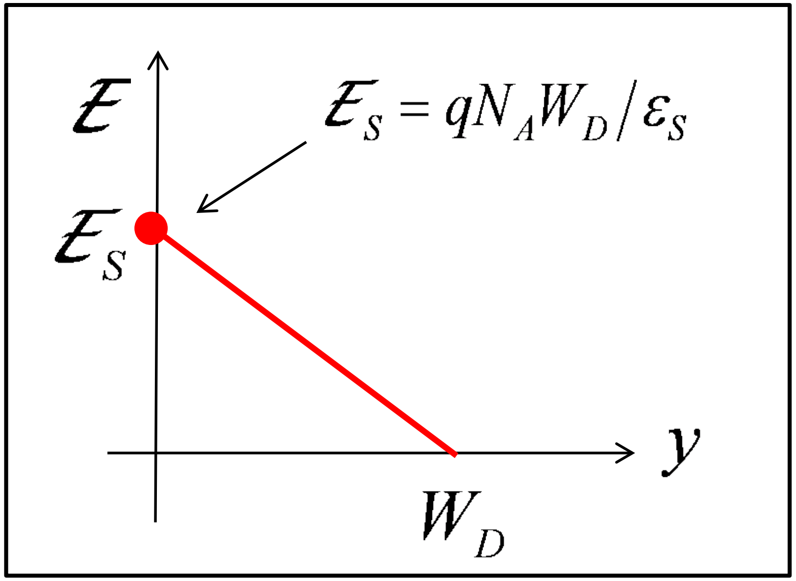
Electrostatic potential



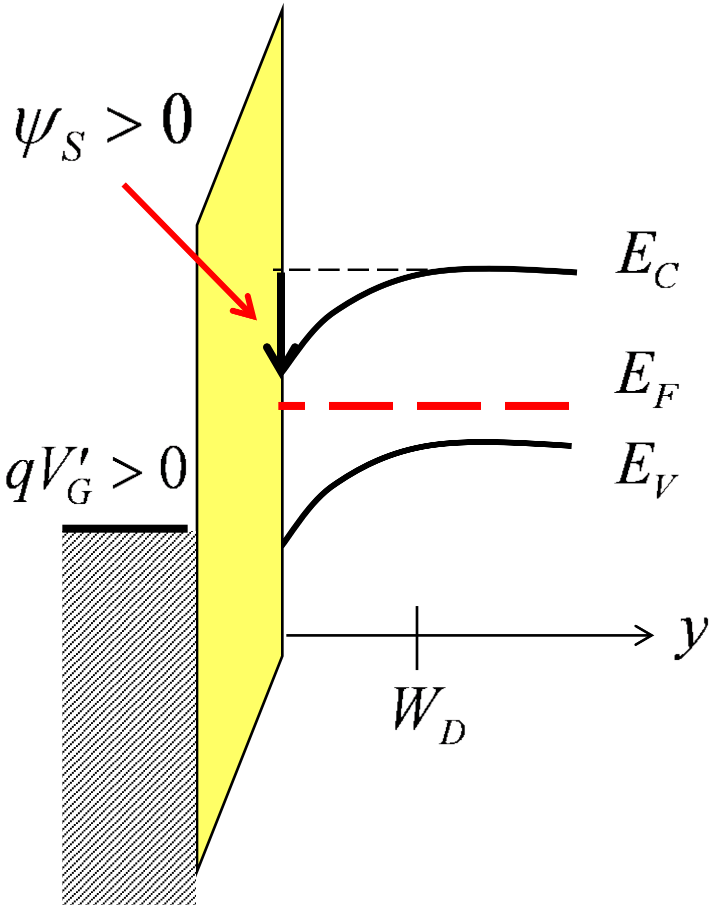


$$\mathcal{E}(y) = -d\psi(y)/dy \qquad \psi(y) = -\int \mathcal{E}(y)\,dy$$

$$\psi_S = \frac{1}{2}\mathcal{E}_S W_D$$

$$\boxed{W_D = \sqrt{2\varepsilon_S \psi_S / qN_A}}$$

Depletion charge per cm^2



$$W_D = \sqrt{2\varepsilon_S \psi_S / qN_A}$$

$$Q_D = -qN_A W_D = -\sqrt{2qN_A \varepsilon_S \psi_S} \text{ C/cm}^2$$

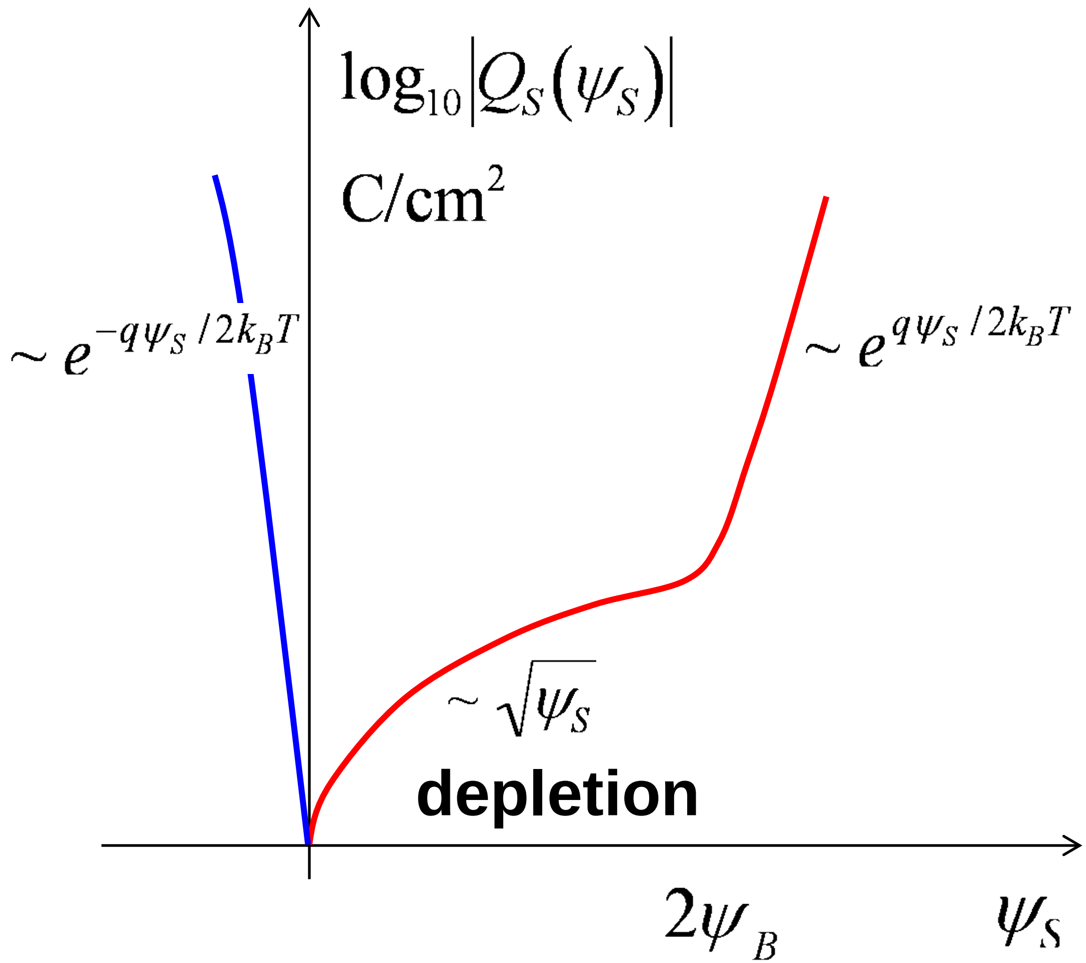
(depletion charge)

$$Q_S = Q_D + Q_n \text{ C/cm}^2$$

(total charge in semiconductor)

$$\boxed{|Q_S| \approx |Q_D| \sim \sqrt{\psi_S}}$$

MOS electrostatics



Example

P-type Si doped at: $N_A = 10^{18}\ \text{cm}^{-3}$

$T = 300\ \text{K}$

$n_i\left(300\ \text{K}\right) = 10^{10}\ \text{cm}^{-3}$

$\kappa_{Si} = 11.8$

$\psi_S = 0.5\ \text{V}$

$k_B T/q = 0.026\ \text{V}$

Find:

i) the width of the depletion layer

ii) the electric field at the surface

Example

P-type Si doped at: $N_A = 10^{18}\ \text{cm}^{-3}$

$T = 300\ \text{K}$ $\quad$ $n_i(300\ \text{K}) = 10^{10}\ \text{cm}^{-3}$

$\psi_S = 0.5\ \text{V}$

1) Check to see if we are in depletion or inversion.

$$\psi_S < 2\psi_B ?$$

$$\psi_B = \frac{k_B T}{q} \ln\left(\frac{N_A}{n_i}\right) = 0.026 \ln\left(\frac{10^{18}}{10^{10}}\right) = 0.48\ \text{V}$$

$$\psi_S < 2\psi_B ?$$

$$0.5 < 0.96\ \text{V}$$

depletion ✓

Depletion layer thickness

P-type Si doped at: $N_A = 10^{18}\ \text{cm}^{-3}$

$T = 300\ \text{K}$

$n_i\left(300\ \text{K}\right) = 10^{10}\ \text{cm}^{-3}$

$\kappa_{Si} = 11.8$

$\psi_S = 0.5\ \text{V}$

$k_B T/q = 0.026\ \text{V}$

$$W_D = \sqrt{2\varepsilon_S \psi_S / qN_A}$$

$$W_D = \sqrt{2\left(11.8\right)\left(8.854\times10^{-12}\right)\left(0.5\right)/\left[\left(1.6\times10^{-19}\right)10^{24}\right]}$$

$$W_D = 25.6\ \text{nm}$$

$$W_D = 25.6\times10^{-9}\ \text{m}$$

Electric field at the surface

P-type Si doped at: $N_A = 10^{18}\ \text{cm}^{-3}$

$T = 300\ \text{K}$

$n_i\left(300\ \text{K}\right) = 10^{10}\ \text{cm}^{-3}$

$\kappa_{Si} = 11.8$

$\psi_S = 0.5\ \text{V}$

$k_B T / q = 0.026\ \text{V}$

$$\psi_S = \frac{1}{2}\mathcal{E}_S W_D$$

$$\mathcal{E}_S = \frac{2\psi_S}{W_D} = \frac{2\left(0.5\right)}{25.6 \times 10^{-9}}$$

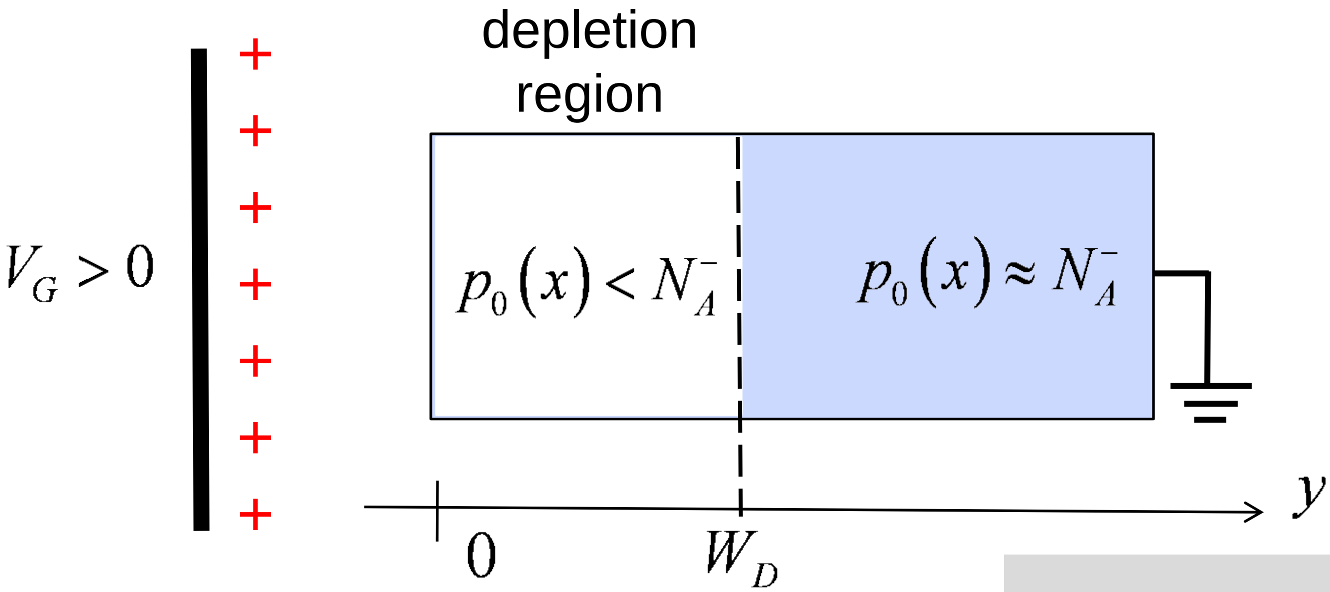
$$\mathcal{E}_S = 3.9 \times 10^5\ \frac{\text{V}}{\text{cm}}$$

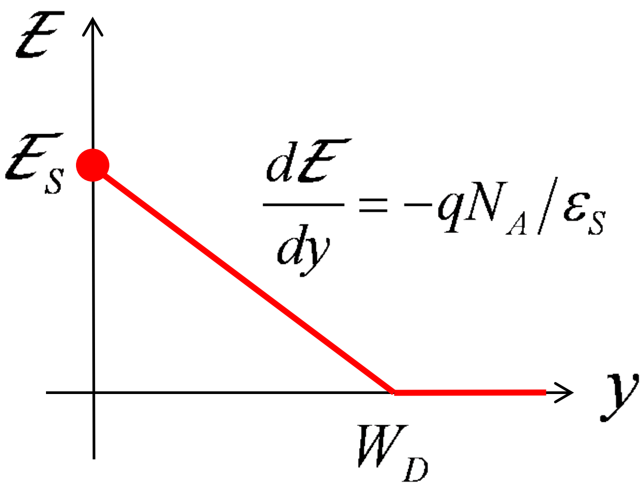
$$\mathcal{E}_S = \frac{2\psi_S}{W_D}$$

$$\mathcal{E}_S = 3.9 \times 10^7\ \frac{\text{V}}{\text{m}}$$

$$\mathcal{E}_S = 390\ \frac{\text{kV}}{\text{cm}}$$

Summary





$$W_D = \sqrt{2\varepsilon_S \psi_S / qN_A}\ \text{m}$$

$$\mathcal{E}_S = \frac{2\psi_S}{W_D}\ \text{V/m}$$

$$Q_D = -\sqrt{2qN_A \varepsilon_S \psi_S}\ \text{C/m}^2$$

Next topic

Given a surface potential, we can compute the electric field and depletion layer thickness (if we are in depletion), but what gate voltage produced this surface potential?

That is the subject of the next lecture.