CentraleSupélec

2A - Automatique

Chapter 2

Control Science (AUT)

Frequency-domain approach

September 2019

Romain Bourdais
CentraleSupélec
romain.bourdais@centralesupelec.fr

Preamble

About this course

Course outline

- Effect of loop closing
- Concept of sensitivities
- Stability and the Nyquist Criterion

Preamble

Introduction example 1

We want to control the position of a motor to a setpoint y^c, with a position sensor. The model is as follows :

$$y = \frac{G_0}{p(1+\tau_1 p)(1+\tau_2 p)}(u - w)$$

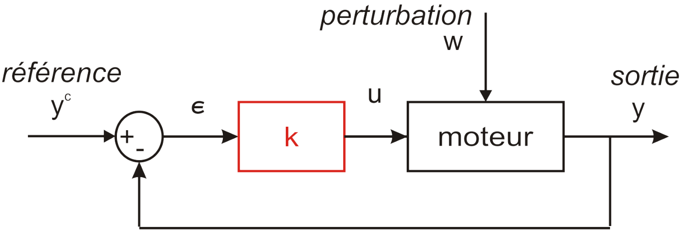
- u : supply voltage, w : disturbance input, y : angular position
- G_0 : speed gain, τ_1 electrical time constant, τ_2 mechanical time constant

What can we tell about this system ?

- Unstable (BIBO) in open loop : there is an integrator
- The slightest disturbance causes the motor to deviate infinitely far from its setpoint

Preamble
Introduction example 1

- We then propose to control the motor, by a control $u = k(y^c - y)$, with k a gain to adjust



What does intuition tell us ?

- The higher the k, the better the performance will be
- But the higher the k, the greater the control effort will be

Preamble
Introduction example 1

- And why not a small simulation study ? *Matlab is coming . . .*

The code

The model :

- go=5 ; tau1=0.03 ; tau2=0.005 ;
- p=tf('p') ;
- sys=go/(p*(1+tau1*p)*(1+tau2*p)) ;

Feedback - relationship between output and setpoint :

- k=1 ;
- sysbf=feedback(k*sys,1) ;
- step(sysbf,1)

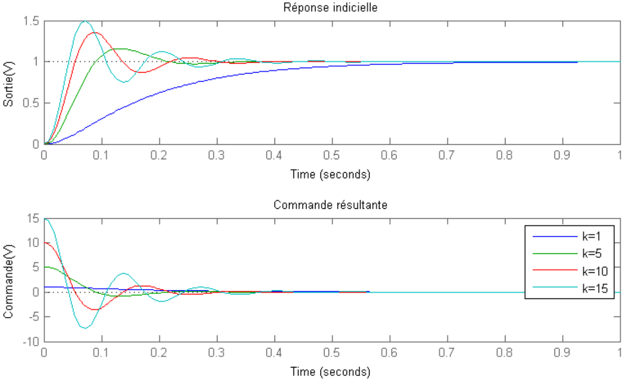
Feedback - relationship between control input and setpoint :

- k=1 ;
- sysbfcom=feedback(k,sys) ;
- step(sysbfcom,1)

Preamble

Introduction example 1

- Effect of k



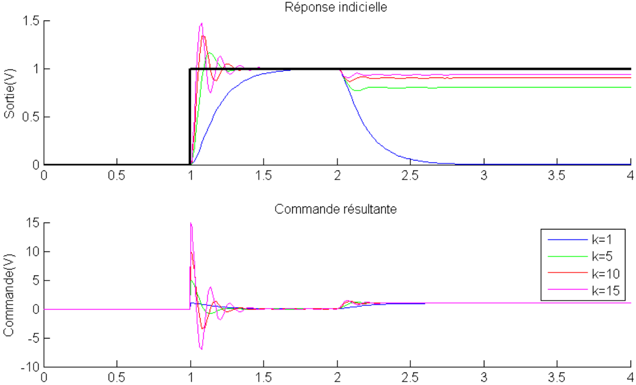
Question

- Steady-State error $\frac{\varepsilon}{Y_c}$?

Preamble

Introduction example 1

- Effect of k and a disturbance occurs at time $t = 2$ seconds

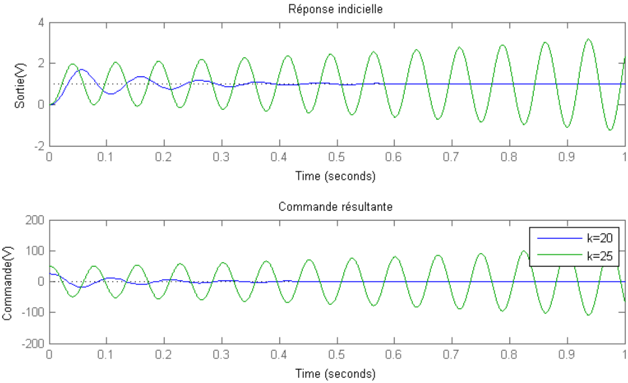


Question

- Steady-State error $\frac{\varepsilon}{D}$?

Preamble
Introduction example 1

- Effect of k, with k very large



Question

- Unstability ?

Preamble
Introduction example 1 - Conclusions

What to remember

- Effect of feedback and proportional action
- The steady-state error is small when k is huge, but the control action is important
- Danger : unstability
- A simple proportional control k is (often) insufficient for the stability-precision trade-off

Skills - supposed to be known

- Definition of a system in Matlab
- The final value theorem ! Beware of the area of convergence.
- Stability analysis (Routh criterion)

Feedback configurations

Writing games and Schema

- In a general point of view, we can use 2 transfer functions in the control structure :
$$u = C_c(p)y^c - C(p)y$$
- Which can be rewritten into the RST structure :
$$u = \frac{Ty^c - Ry}{S}$$

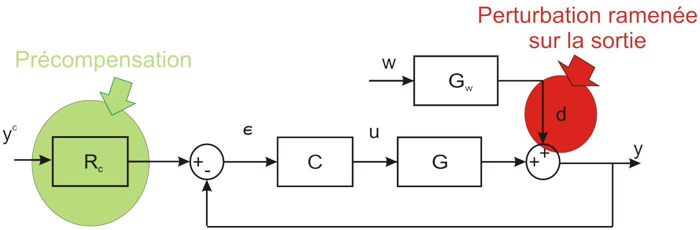
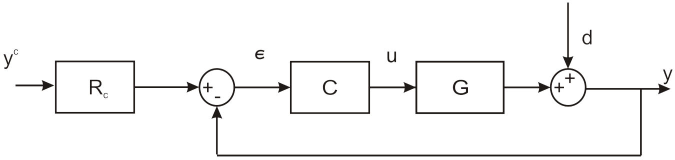


FIGURE – THE schéma

Sensitivities
Disturbance sensitivity



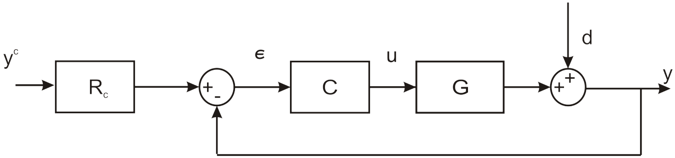
- The transfer between the disturbance d and the error ε :

$$\varepsilon = \frac{-1}{1 + CG} d$$

Sensitivity : definition

$$S = \frac{1}{1 + CG}$$

Sensitivities
Model uncertainty sensitivity



- We can define the transfer between the reference y^c and the output y : $H = \frac{y}{y^c} = \frac{R_c CG}{1+CG}$
- What happens if a small variation ΔG is applied on the model. What is ΔH ?

Sensitivity - property

$$\frac{\Delta H}{H} = S \frac{\Delta G}{G}$$

- We want to find C so that S is as small as possible !

Sensitivities

Complementary sensitivity

- Making S close to 0 is equivalent to making $(1-S)$ close to 1

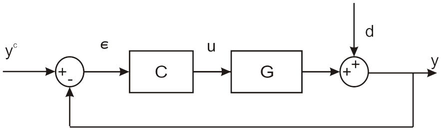
Complementary sensitivity : definition

$$T = 1 - S = \frac{CG}{1+CG}$$

- We have

$$H = \frac{R_c CG}{1+CG} = TR_c$$

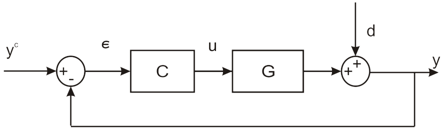
- The R_c pre-compensation is not necessarily required.
- We can therefore choose $R_c = 1$ and we get the simplified schema :



Sensitivities

A major remark : $R_c = 1$

- We often reason about



- We have $\varepsilon = S(y^c - d)$
- Reference and disturbance play almost the same role on the deviation
- In the exercises : the main specifications are on $\frac{y}{y^c}$ and therefore T but S is hidden ! In engineering problems, it is on S that the main specifications are imposed
- $R_c = 1$ is not absolute law . . .

Stability
One definition among others

- Given the transfer $G(p) = \frac{B(p)}{A(p)}$
- $A(p) = a_0 + a_1 p + \ldots + a_{n-1} p^{n-1} + p^n$

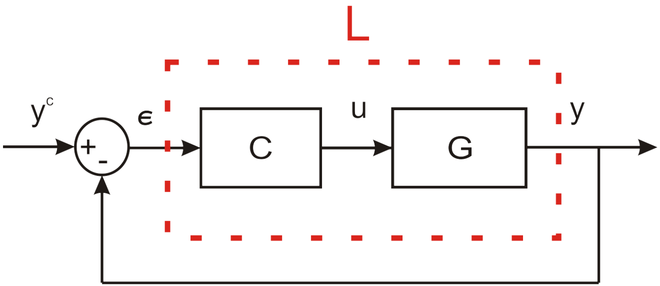
Selected definition
A linear system is asymptotically stable if and only if its impulse response is absolutely integrable.

Characterization
G is AS if and only if all its poles have a strictly negative real part.

- A tool already seen : the Routh criterion (see ST2 Modelisation)
- A useful tool... Matlab

Stability

Open Loop - Closed Loop Relationship



$$\boxed{\text{Relationship between } L \text{ and } \frac{1}{1+L}}$$

Cauchy's theorem

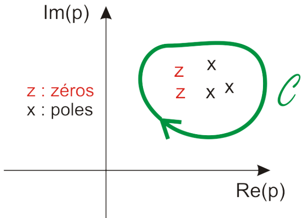
Back to complex functions

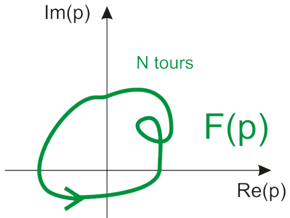
- Let us consider $F(p)$, a meromorph complex function. Let us consider $\mathcal{C}$ a closed contour.
- Z : number of zeros of F, P : number of poles of F inside the closed contour $\mathcal{C}$

Cauchy's theorem

- When p is moving on the contour $\mathcal{C}$, $F(p)$ describes a closed path
- N : number of rotations of $F(p)$ around 0, counted in the same direction of travel.

$$\boxed{N = Z - P}$$

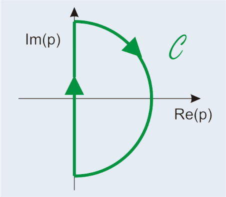




Nyquist Criterion
Cauchy's theorem application to stability analysis

- We look at the transfer $\frac{1}{1+L}$
- Stability condition : no zeros with positive real part for $(1 + L)$

The contour of Bromwich



- The image of the Bromwich's contour by the $1 + L(p)$ function must therefore do : $-P$ turns around 0 !
- How to link this to $L(p)$ and not $1 + L(p)$?

Nyquist Criterion

Cauchy's theorem application to stability analysis : From $1 + L$ to L

- 1st observation :
  - The image of the Bromwich's contour by the $1 + L(p)$ function must therefore do : $-P$ turns around 0 !
  - Consequently, the image of the Bromwich's contour by the $L(p)$ function must therefore do : $-P$ turns around -1 ! (clockwise direction)

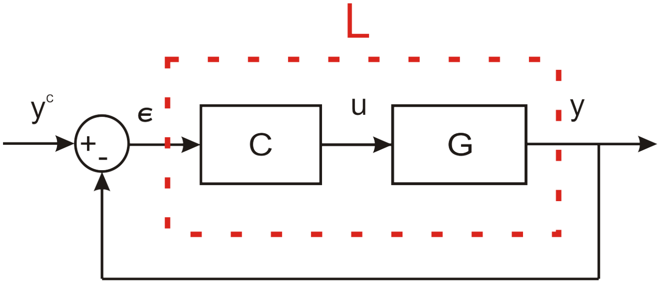
Nyquist plot

The Nyquist plot of L is the image of the Bromwich contour by the $L(p)$ function.

- It is a closed curve.

- 2nd observation :
  - $1 + L$ and L have the same poles
  - P is the number of poles of L with a positive real part

Nyquist Criterion
The criterion, finally, we can get it !

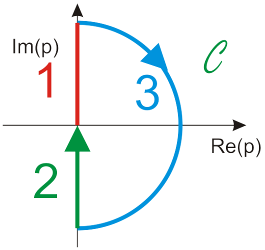


- Let us denote with P the number of poles of $L(p)$ with a positive real part.

Nyquist criterion

The transfer $S = \frac{1}{1+L}$ is asymptotically stable if and only if the Nyquist plot of L encircles P times the point -1 counter-clockwisely !

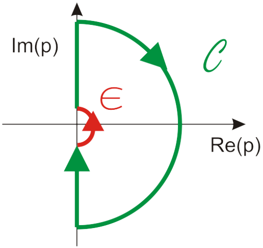
Nyquist plot
How to draw it ?



- Part 1 : $p = jw$: everything is provided by the Bode diagrams !
- Part 2 : $p = -jw$: it is the symmetrical of 1 with respect to the abscissa axis.
- Part 3 : $p = Re^{i\Theta}$, with $R \to \infty$: it's a point for any proper system. (this point is the origin if the system is strictly proper)

Nyquist plot
The little subtility . . .

- Cauchy's theorem : poles and zeros have to be strictly inside the contour.
- What if some poles/zeros are on the Bromwich contour ?



- We make indentations . . . = we go around !
- The Nyquist plot is still a closed curve
- As a consequence :there are infinite phenomena happening on the Nyquist plot (half-turn, turn, 1 turn and a half, . . .)

My first Nyquist
To warm up

$$\frac{K}{1 + \tau p}$$

My first Nyquist
To warm up, the opposite

$$\frac{-K}{1 + \tau p}$$

My first Nyquist
To warm up, with a little help of Matlab

$$\frac{K}{1+\tau p}, \quad \frac{-K}{1+\tau p}$$

The code

Model :

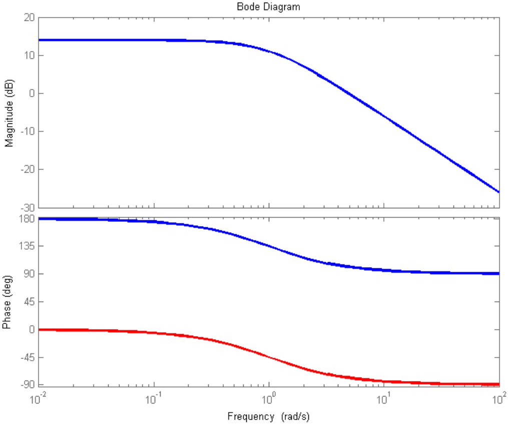
- k=5 ; tau=1 ;
- p=tf('p') ;
- sys=k/(1+tau*p) ;sys2=-k/(1+tau*p) ;

Plots :

- figure
- bode(sys,'r',sys2,'b')
- nyquist(sys,'r',sys2,'b')

My first Nyquist
To warm up, with a little help of Matlab

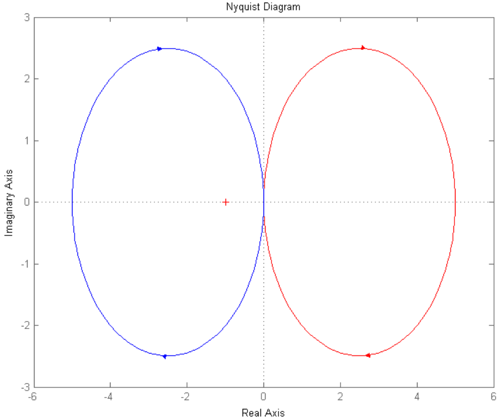
$$\frac{K}{1+\tau p}, \quad \frac{-K}{1+\tau p}$$



My first Nyquist

To warm up, with a little help of Matlab

$$\frac{K}{1+\tau p}, \quad \frac{-K}{1+\tau p}$$



Nyquist for system with integral action

Indentation . . . Bypass ! Bypass ! Half turn

$$\frac{K}{p(1+\tau p)}$$

Nyquist for system with integral action

Indentation . . . Bypass ! Bypass ! Half turn, the opposite

$$\frac{-K}{p(1+\tau p)}$$

Nyquist for system with double integral action

Indentation . . . Bypass ! Bypass ! Full turn

$$\frac{K}{p^2(1+\tau p)}$$

Nyquist - the trap
A matter of good direction

$$\frac{K}{(1-p)(2+p)}$$

$$\frac{-K}{(1-p)(2+p)}$$

Nyquist - the trap
A matter of good direction

$$\frac{K}{(1+p)(2-p)}$$

$$\frac{-K}{(1+p)(2-p)}$$

Nyquist - A last one for the road
The case of the oscillator

$$\frac{K}{(p+2)(p^2+4)}$$

Course report

Expected skills

- Impact of the closed loop on sensitivity
- Drawing the Nyquist plot for a given transfer
- Determine its stability in CL using the Nyquist criterion