Diversity techniques

Combating the effects of fading

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A review of probability methods for diversity analysis

Contiuous random variables

We (normally) characterize a continuous random variable X by means of:

- The probability density function:

$$f_X(x), \quad -\infty < x < \infty \tag{1}$$

- The cumulative distribution function:

$$F_X(x) = \int_{-\infty}^{x} f_X(z)\, dz \tag{2}$$

- The moment generating function:

$$M_X(s) = \mathbb{E}\left\{e^{sX}\right\} = \int_{-\infty}^{\infty} e^{sX} f_X(x)\, dx = \mathcal{L}\left\{f_X\right\}|_{s=-s}, \tag{3}$$

where $\mathcal{L}\{\cdot\}$ is the Laplace transform.

Example: The gamma distribution

We define a Gamma distributed random variable as a random variable X having a probability density function of the form:

$$f_X(x) = \frac{1}{\Gamma(m)\rho^m} x^{m-1} e^{-\frac{x}{\rho}}, x \geq 0, \tag{4}$$

where m is the shape parameter, ρ the scale parameter, and $\Gamma(m)$ the complete Gamma function.

Cumulative distribution function of a Gamma distributed random variable:

$$F_X(x) = \frac{1}{\Gamma(m)} \int_0^x \frac{z^{m-1}}{\rho^m} e^{-\frac{z}{\rho}} dz = \frac{1}{\Gamma(m)} \int_0^{x/v} v^{m-1} e^{-v} dv = \frac{\gamma(m,x)}{\Gamma(m)}, \tag{5}$$

where we have introduced the lower incomplete Gamma function:

$$\gamma(m,x) = \int_0^x z^{m-1} e^{-z} dz. \tag{6}$$

Example: The gamma distribution

Calculating the moment generating function:

$$\begin{aligned} M_X(s) &= \frac{1}{\Gamma(m)\rho^m}\int_0^\infty x^{m-1}e^{-\frac{x}{\rho}}e^{sx}dx \\ &= \frac{1}{\Gamma(m)\rho^m}\int_0^\infty x^{m-1}e^{-\left(\frac{1}{\rho}-s\right)x}dx \\ &= \frac{1}{\Gamma(m)\rho^m}\frac{1}{\left(\frac{1}{\rho}-s\right)^m}\int_0^\infty \left(\frac{1}{\rho}-s\right)^m x^{m-1}e^{-\left(\frac{1}{\rho}-s\right)x}dx \\ &= \frac{1}{\Gamma(m)(1-\rho s)^m}\int_0^\infty z^m e^{-z}dz = \frac{1}{(1-\rho s)^m}, \end{aligned} \tag{7}$$

where we have used the fact that by definition:

$$\Gamma(m) = \int_0^\infty z^{m-1}e^{-z}dz. \tag{8}$$

Functions of a random variable

If X is a continuous random variable and $Y = g(X)$ a transform of it, we can write the probability density function of Y as:

$$f_Y(y) = \sum_{i=1}^{n} \frac{f_X(x_i)}{|g'(x_i)|}, \tag{9}$$

where $x_1, \ldots, x_n$ are the real roots of the equation

$$y = g(x_1) = \ldots = g(x_n). \tag{10}$$

Proof: We start by calculating the probability that the value of Y falls in an infinitesimally small interval starting from the point y, where we wish to calculate the probability density function. This probability can be expressed as:

$$\Pr(y \leq Y \leq y + dy) = f_Y(y)\, dy. \tag{11}$$

Using the fact that $Y = g(X)$, this probability can then be expressed as the probability that X belongs in an infinitesimally small interval starting or ending at one of the points $x_1, \ldots, x_n$.

As a result, we have that:

$$\Pr\left(y \leq Y \leq y + dy\right) = \sum_{i=1}^{n} \Pr\left(x_i \leq X \leq x + dx_i\right). \tag{12}$$

where we assume that $x_1, \ldots, x_k$ are such that $dx_1, \ldots, dx_k$ are greater than zero, and that $x_{k+1}, \ldots, x_n$ are such that $dx_{k+1}, \ldots, dx_n$ are less than zero.

However, we also have that:

$$\Pr\left(x_i \leq X \leq x + dx_i\right) = f_X\left(x_i\right)|dx_i|, \tag{13}$$

where due to the fact that $y = g\left(x_i\right)$, we have that $dy = g'\left(x_i\right) dx_i$. We therefore obtain that:

$$\begin{aligned} \Pr\left(y \leq Y \leq y + dy\right) &= \sum_{i=1}^{n} \Pr\left(x_i \leq X \leq x + dx_i\right) = \sum_{i=1}^{n} f_X\left(x_i\right)|dx_i| \\ &= \sum_{i=1}^{n} \frac{f_X\left(x_i\right)}{|g'\left(x_i\right)|} dy. \end{aligned} \tag{14}$$

Example

For a positive valued random variable X and a positive constant c, find the distribution of $Y = cX^2$.

Solution: Applying the above result, and noticing that for the fuction $g(x) = cx^2$ (where $dg(x)/dx = 2cx$) is an invertible function, which allows as to write the solution of the equation $y = cx^2$ as $x = \sqrt{y/c}$, we obtain that:

$$f_Y(y) = \frac{f_X\left(\sqrt{y/c}\right)}{2c\sqrt{y/c}} = \frac{f_X\left(\sqrt{y/c}\right)}{2\sqrt{cy}}. \tag{15}$$

Application

Let us consider a Rayleigh distributed random variable:

$$f_X(x;\sigma) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}, x \geq 0, \tag{16}$$

where σ the parameter of the Rayleigh distibution and define the variable $Y = cX^2, \ \ c > 0$. We then obtain that:

$$\begin{aligned} f_Y\left(y\right) &= \frac{f_X\left(\sqrt{y/c}\right)}{2c\sqrt{y/c}} = \frac{f_X\left(\sqrt{y/c}\right)}{2\sqrt{cy}} = \frac{\sqrt{y/c}}{2\sigma^2 c\sqrt{y/c}} e^{-\left(\sqrt{y/c}\right)^2/(2\sigma^2)} \\ &= \frac{1}{2c\sigma^2} e^{-y^2/(2\sigma^2)}. \end{aligned} \tag{17}$$

Mutliple continuous random variables

We can characterize a set of continuous random variables by means of:

- The joint probability density function:

$$f_{X_1,\ldots,X_n}\left(x_1,\ldots,x_n\right). \tag{18}$$

- The joint cumulative distribution function:

$$F_{X_1,\ldots,X_n}\left(x_1,\ldots,x_n\right)=\Pr\left(X_1\le x_1,\ldots,X_n\le x_n\right) \tag{19}$$

- Their joint moment generating function:

$$M_{X_1,\ldots,X_n}\left(s_1,\ldots,s_n\right)=\mathbb{E}\left\{e^{\sum_{i=1}^{n}s_iX_i}\right\}. \tag{20}$$

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Functions of more than one random variables

For the purposes of analyzing diversity systems, we will consider the following types of functions of random variables

- Sums of random variables: If $Y = \sum_{i=1}^{n} X_i$, we have that:

$$M_Y(s) = \mathbb{E}\left\{e^{\sum_{i=1}^{n} sX_i}\right\} = M_{X_1,\ldots,X_n}(s,\ldots,s). \tag{21}$$

- Maximum of a random variable: If $Y = \max\{X_1,\ldots,X_n\}$, then:

$$F_Y(y) = \Pr(Y \leq y) = \Pr(X_1 \leq y,\ldots,X_n \leq y). \tag{22}$$

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Analyzing sums of random variables

- In case of independent random variables:

$$
\begin{aligned}
M_Y(s) &= \mathbb{E}\left\{e^{\sum_{i=1}^{n} sX_i}\right\} = \mathbb{E}\left\{\prod_{i=1}^{n} sX_i\right\} \\
&= \prod_{i=1}^{n} \mathbb{E}\left\{e^{sX_i}\right\} = \prod_{i=1}^{n} M_{X_i}(s) .
\end{aligned}
\tag{23}
$$

- The PDF of Y can be found by using methods to invert the MGF of Y.
- Alternatively, given that convolution of two random variables results, in multiplying their PDFs, we can express the PDF of the sum of two random variables as the convolution of their PDF.
- For certain cases of correlated random variables where the joint MGF (and as a result $M_Y(s)$) are known, the same process can be also applied for determining the PDF of a sum of correlated random variables.

Example: Finding the PDF of the sum of two Gaussian random variables

Let us assume that X is a Gaussian random variable having of zero mean and unit variance. We then have that:

$$\begin{aligned} M_X(s) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\frac{x^2}{2}} e^{sx} dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\left(\frac{x^2}{2} - sx + \frac{s^2}{2}\right) + \frac{s^2}{2}} dx \\ &= \frac{e^{s^2/2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(x-s)^2/2} dx = e^{\frac{s^2}{2}}. \end{aligned} \tag{24}$$

By properties of Gaussian random variables, we know that if X is a zero mean and unit variance Gaussian, it holds that $Z = \sigma X + \mu$ is also Gaussian with mean μ and variance σ. The MGF is then equal to:

$$M_Z(s) = \mathbb{E}\left\{e^{Zs}\right\} = \mathbb{E}\left\{e^{\sigma X s + \mu s}\right\} = e^{\mu s}\mathbb{E}\left\{e^{X\sigma s}\right\} = e^{\mu s} M_X(\sigma s) = e^{\mu s + \frac{\sigma^2 s^2}{2}}. \tag{25}$$

Example: Finding the PDF of the sum of two Gaussian random variables

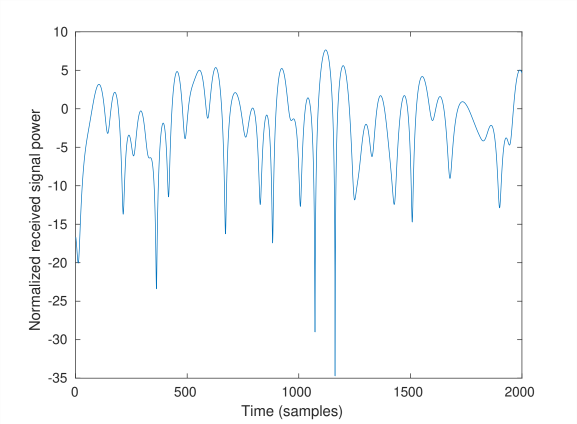
Given the above result, if $Z_i, i = 1, 2$, are independent Gaussian random variables having mean values $\mu_i, i = 1, 2$ and variances $\sigma_i^2, i = 1, 2$, defing $Y = Z_1 + Z_2$ we have that:

$$M_{Y=Z_1+Z_2}(s) = e^{\mu_1 s + \frac{\sigma_1^2 s^2}{2}} e^{\mu_2 s + \frac{\sigma_2^2 s^2}{2}} = e^{(\mu_1+\mu_2)s + \frac{\left(\sigma_1^2+\sigma_2^2\right)s}{2}}. \tag{26}$$

Conclusion: The sum of two Gaussian independent random variables is a Gaussian random variable with mean $\mu_1 + \mu_2$ and variance $\sigma_1^2 + \sigma_2^2$.

Summarizing the effects of wireless communications systems

What does a faded signal look like?



What is the effect of fading on the ergodic capacity?

- Definition of ergodic capacity: Assuming that the channel SNR is a random variable taking independent from one channel realization to the next one, the maximum achievable transmission rate is the ergodic capacity defined as

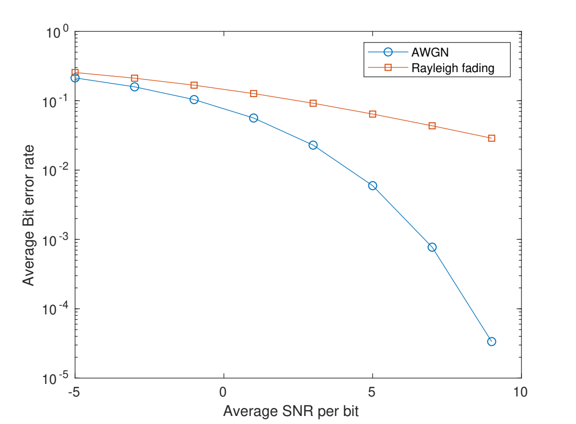
$$\bar{C} = \mathbb{E}\left\{\log_2\left(1+\gamma\right)\right\} \tag{27}$$

  where γ is the received signal SNR.
- Jensen inequality: For a strictly concave function $f\left(\cdot\right)$, for any random variable x, it holds that $\mathbb{E}\left\{f\left(x\right)\right\} \leq f\left(\mathbb{E}\left\{x\right\}\right)$.
- Ergodic capacity is upper bounded by AWGN capacity:

$$\mathbb{E}\left\{\log_2\left(1+\gamma\right)\right\} \leq \log_2\left(1+\bar{\gamma}\right). \tag{28}$$

- Given a channel with $\mathbb{E}\left\{\gamma\right\} = \bar{\gamma}$, the ergodic rate is maximized if $\gamma = \bar{\gamma}$.
- To maximize the ergodic rate we need to eliminate deep fades.

What is the effect of fading on the BER/SER?

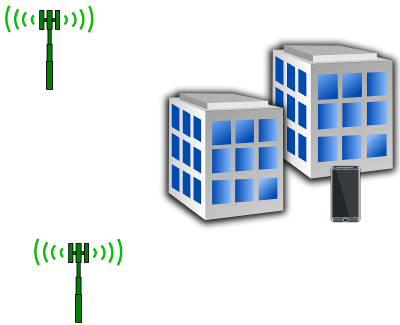


The concept of diversity

Diversity techniques

- Key idea: Transmit the same information over multiple paths (diversity branches) and combine them the signals of the different branches to create a signal where the effects of fading are reduced.
- If the diversity branches are characterized by independent fading, the probability of concurrent deep fading in all branches is reduced.
- Two types of diversity can be considered:
    - Macrodiversity: Aims at combating the effects of shadowing.
    - Microdiversity: Aims at combating the effects of multipath fading.

Macrodiversity



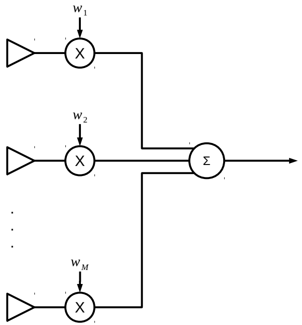
As shadowing effects can be correlated for tens of meters, macrodiversity requires base station cooperation.

Microdiversity

Microdiversity

- Frequency diversity: The same data stream is transmitted over multiple frequency channels.
  - Inefficient utilization of bandwidth resources: The inefficiency becomes larger as we increase the number of diversity branches.
  - Received power needs to be split on the two frequency bands.
- Time diversity: The same data symbols are transmitted at different times.
  - Decoding delay: We need to wait in order to receive all signals corresponding to a specific data stream in order to decode.
  - In case of high coherence time, the time separation between transmissions of the same data stream need to large enough in order to have independent fading effects on the different transmission. The decoding delay may be large!
  - Time slots that could be used for transmitting new data are wasted;
- Spatial diversity: The transmitter and/or receiver uses multiple transmit antennas to transmit/receive the same information signal.
  - No time/bandwidth resources are wasted.
  - As we are trying to increase the number of diversity branches more hardware is required.

(Spatial) receive diversity



- The signal reaching the m-th antenna is expressed as: $y_m(t) = \alpha_m e^{j\theta_m} s(t) + n_m(t)$
- The combiner output is expressed as $y = \sum_{m=1}^{M} w_m y_m$.
- Different rules can be found for selecting weights $w_m, m = 1, \ldots, M$.

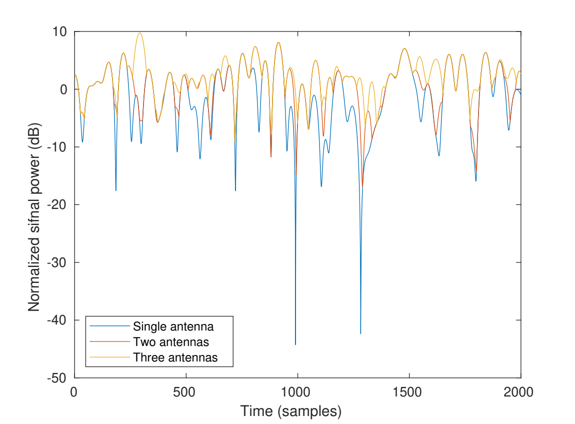
Selection combining

- The SNR (per symbol) on the m-th branch is equal to:

$$\gamma_m = \alpha_m^2 \frac{E_s}{N_0}. \tag{29}$$

- Let m be an index of an antenna, for which $\gamma_m = \max\{\gamma_1, \ldots, \gamma_M\}$.
- We output the signal corresponding to his signal, setting $w_m = 1$ and the remaining weights equal to zero.
- In some cases, selection combining may be implemented using only one receiver that is switched each time to the selected antenna branch. However, in several cases one receiver per antenna may be required in order to monitor the SNR on all branches.

Selection combining



As the number of branches increases, the signal power variability is reduced.

Selection combining: SNR distribution

$$F_{\gamma}(\gamma_0) = \Pr(\gamma_1 \leq \gamma_0, \ldots, \gamma_M \leq \gamma_0) \tag{30}$$

- Assuming independent fading:

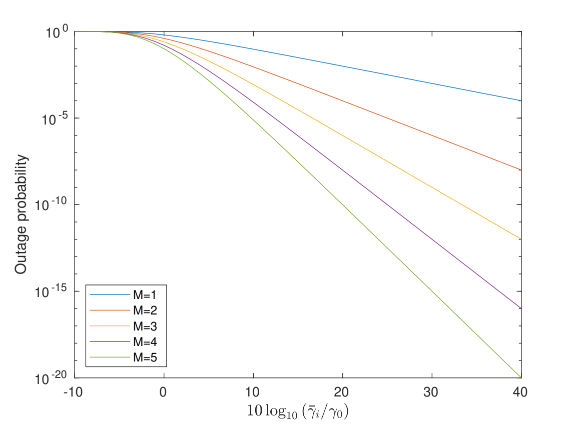
$$F_{\gamma}(\gamma_0) = \prod_{m=1}^{M} \Pr(\gamma_m \leq \gamma_0) \tag{31}$$

- Assuming i.i.d. Rayleigh fading with $\mathbb{E}\{\gamma_m\} = \bar{\gamma}_m$, $F_{\gamma}(\gamma_0)$ becomes:

$$P_{\gamma}(\gamma_0) = \left(1 - e^{-\frac{\gamma_0}{\bar{\gamma}_i}}\right)^M, \quad \gamma_0 \geq 0. \tag{32}$$

- Outage probability $P_{out}(\gamma_0)$ can be calculated as $F_{\gamma}(\gamma_0)$.

Selection combining: Outage probability



The benefit of extra antennas becomes less important as the total number of antennas increases.

Threshold combining

- Only one of the diversity branches is used for reception.
- Diversity branches are scanned in sequential order.
- The first diversity branch having an SNR above a threshold γ_T is selected.
- After choosing a branch, we keep receiving using this branch as long as the SNR stays higher than γ_T.
- Switch and stay combining: If the SNR on the current branch becomes less than the predetermined threshold we switch randomly to another one.
- Advantage: No dedicated receiver per antenna is necessary.

Threshold combining: Outage analysis

- Assuming i.i.d. fading and only two diversity branches the CDF of the SNR is given as:

$$P_\gamma(\gamma_0) = \begin{cases} P_{\gamma_1}(\gamma_T)\, P_{\gamma_2}(\gamma_0), & 0 \le \gamma_0 < \gamma_T \\ \Pr(\gamma_T \le \gamma_1 \le \gamma_0) + P_{\gamma_1}(\gamma_T)\, P_{\gamma_2}(\gamma_0), & \gamma_0 \ge \gamma_T. \end{cases} \tag{33}$$

- Assuming Rayleigh fading with average SNR $\bar{\gamma}$ on each branch:

$$P_\gamma(\gamma_0) = \begin{cases} 1 - e^{-\frac{\gamma_T}{\bar{\gamma}}} - e^{-\frac{\gamma_0}{\bar{\gamma}}} + e^{-\frac{\gamma_T+\gamma_0}{\bar{\gamma}}}, & 0 \le \gamma_0 < \gamma_T \\ 1 - 2e^{-\frac{\gamma_0}{\bar{\gamma}}} + e^{-\frac{\gamma_T+\gamma_0}{\bar{\gamma}}}, & \gamma_0 \ge \gamma_T. \end{cases} \tag{34}$$

- With appropriate selection of γ_T threshold combining may result in the same outage probability as selection combining.

Maximum ratio combining

- Given that the signal on the m-th branch is given as:

$$y_m(t) = \alpha_m e^{j\theta_m} s(s) + n_m(t), \tag{35}$$

  we select $w_m = \alpha_m e^{-\jmath\theta_m}$.
- Diversity branches characterized by higher SNR are weighted with larger weights.
- It can be proven that, among all combiner methods, it results in the maximum possible SNR.

Maximum ratio combining: Deriving the SNR expression

- The signal after combining is expressed as:

$$y = \sum_{m=1}^{M} w_m y_m(t) = \sum_{m=1}^{M} \alpha_m^2 s(t) + \sum_{m=1}^{M} \alpha_m e^{-j\theta_m} n_m(t)\,. \tag{36}$$

- The noise variance after combining is equal to:

$$\sigma_{MRC}^2 = \sum_{m=1}^{M} \alpha_m^2 \sigma^2. \tag{37}$$

- The signal power P_{MRC} after combining is equal to:

$$P_{MRC} = \left(\sum_{m=1}^{M} \alpha_m^2\right)^2 P_s \tag{38}$$

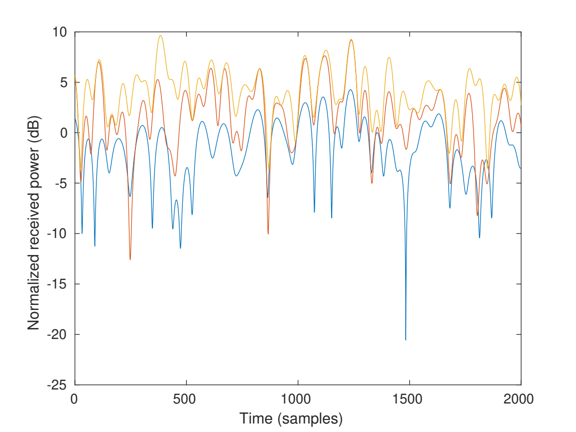
- The signal SNR is finally written as:

$$\gamma = \sum_{m=1}^{M} \alpha_m^2 \frac{P_s}{\sigma^2} \tag{39}$$

- Similarly, the SNR per symbol is given as:

$$\gamma_{MRC} = \sum_{m=1}^{M} \alpha_m^2 \frac{E_s}{N_0}. \tag{40}$$

Maximum ratio combining



Maximum ratio combining: SNR distribution

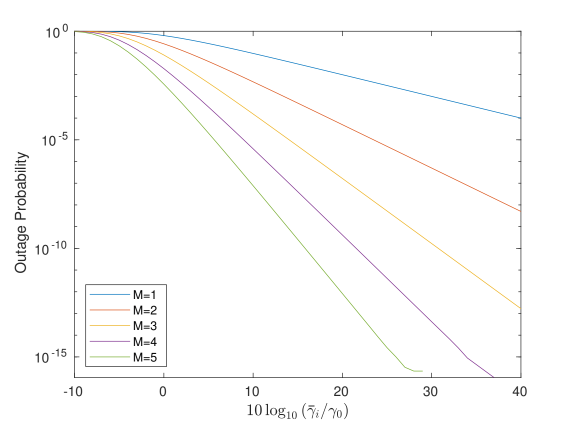
- Assuming i.i.d. Rayleigh fading, with average SNR $\bar{\gamma}$ per branch, the SNR is a chi-squared random variable with $2M$ degrees of freedom and average SNR $\bar{\gamma}_{MRC} = M\bar{\gamma}$, and its PDF is:

$$p_{\gamma_{MRC}}(\gamma) = \frac{\gamma^{M-1} e^{-\frac{\gamma}{\bar{\gamma}}}}{\bar{\gamma}^{M}(M-1)!}, \gamma \geq 0. \tag{41}$$

- The outage probability is expressed as:

$$P_{out}(\gamma_0) = \int_0^{\gamma_0} p_{\gamma_{MRC}} d\gamma = 1 - e^{-\frac{\gamma_0}{\bar{\gamma}}} \sum_{m=1}^{M} \frac{\left(\frac{\gamma_0}{\bar{\gamma}}\right)^{m-1}}{(m-1)!}. \tag{42}$$

Maximum ratio combining: Outage probability



Assuming i.i.d. Rayleigh fading

Note: For large values of M the benefits of an extra antenna are reduced.

Maximum ratio combining: Error rate analysis

- Recall the definition of the average bit error rate:

$$\bar{P}_b = \mathbb{E}\left\{P_b\left(\gamma_{mrc}\right)\right\} \tag{43}$$

where γ_{MRC} the SNR per symbol.

- Focusing on BPSK:

$$P_b\left(\gamma\right) = Q\left(\sqrt{2\gamma}\right). \tag{44}$$

and the following closed form expression can be found for the average bit error rate:

$$\bar{P}_b = \left(\frac{1-\Gamma}{2}\right)^M \sum_{m=0}^{M-1} \binom{M-1+m}{m} \left(\frac{1+\Gamma}{2}\right)^m, \tag{45}$$

with $\Gamma = \sqrt{\bar{\gamma}/(1+\bar{\gamma})}$.

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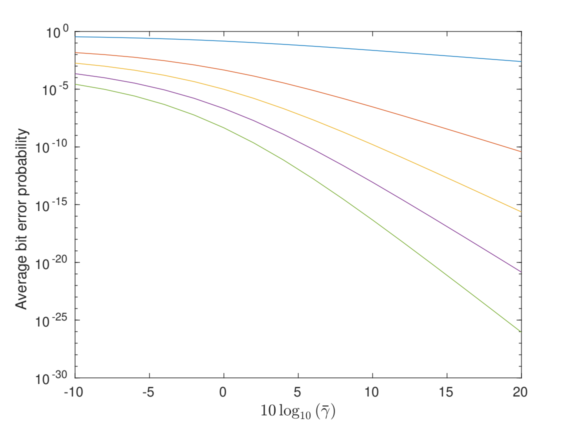
Maximum ratio combining: Error rate analysis



Assuming i.i.d. Rayleigh fading with average SNR per bit $\bar{\gamma}$ for each branch

MRC: Analyzing more sophisticated fading models

- Recall that the SNR of MRC is expressed as:

$$\gamma_{MRC} = \sum_{m=1}^{M} \alpha_m^2 \frac{E_s}{N_0} = \sum_{m=1}^{M} \gamma_m, \tag{46}$$

  where γ_k is the SNR on the k-th branch.

- Assuming independent fading on each branch, the moment generating function of the SNR is expressed as:

$$M_{\gamma_{MRC}}(s) = \prod_{m=1}^{M} M_{\gamma_m}(s). \tag{47}$$

- The MGF can be used in order to analyze the performance of the system.

Equal gain combining

- Given that the signal on the m-th branch is given as:

$$y_m(t) = \alpha_m e^{j\theta_m} s(t) + n_m(t), \tag{48}$$

  we select $w_m = e^{-j\theta_m}$ such as to make sure that the signals are combined in a constructive manner.
- The signal after combining is expressed as:

$$y = \sum_{m=1}^{M} w_m y_m(t) = \sum_{m=1}^{M} \alpha_m s(t) + \sum_{m=1}^{M} e^{-j\theta_m} n_m(t). \tag{49}$$

- The receiver SNR is expressed as:

$$\gamma_{EGC} = \left(\sum_{m=1}^{M} \alpha_m\right)^2 \frac{E_s}{MN_0}. \tag{50}$$

- Only the phase effects of the channels need to be estimated.
- The performance analysis is (normally) non-tractable analytically.

Equal gain combining: Some special cases

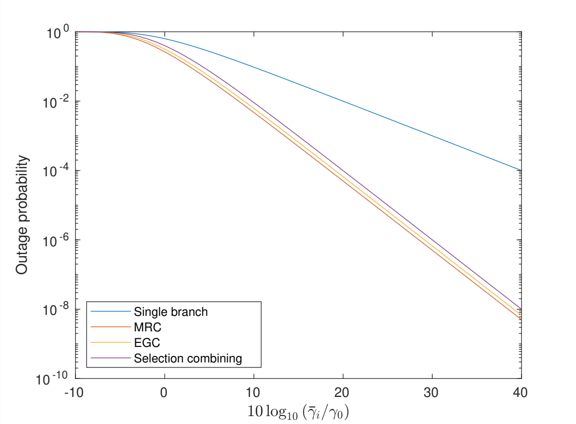
- Restricting ourselves to i.i.d. Rayleigh fading with two diversity branches with average SNR $\bar{\gamma}$ on each branch, some performance measurements can be derived.
- Outage probability:

$$P_{out}\left(\gamma_0\right) = 1 - e^{-2\frac{\gamma_0}{\bar{\gamma}}} - \sqrt{\frac{\pi\gamma_0}{\bar{\gamma}}} e^{-\frac{\gamma_0}{\bar{\gamma}}} \left(1 - 2Q\left(\sqrt{\frac{2\gamma_0}{\bar{\gamma}}}\right)\right) \tag{51}$$

- Average bit error rate of BPSK:

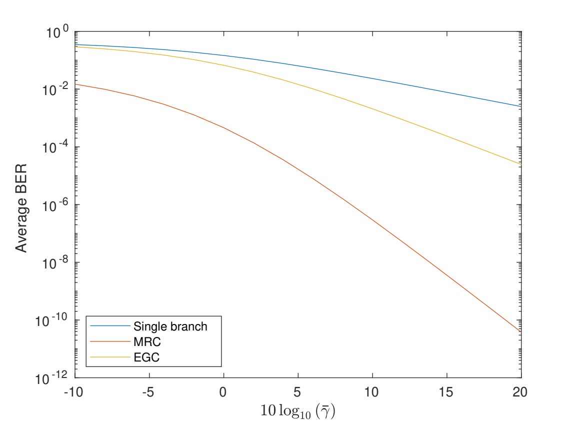
$$\bar{P}_b = 0.5\left(1 - \sqrt{1 - \left(1 + \frac{1}{1+\bar{\gamma}}\right)^2}\right) \tag{52}$$

Outage comparison



Assuming i.i.d.Rayleigh fading with two branches.

BER comparison: MRC vs EGC



Assuming i.i.d.Rayleigh fading with two branches.

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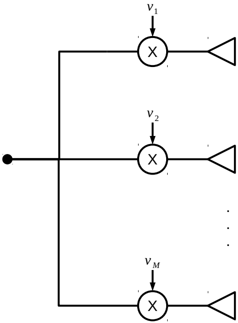
Transmit antenna diversity



By selecting $v_m = \frac{w_m}{\sqrt{\sum_{m=1}^{M} |w_m|^2}}$ the same performance as receive diversity can be achieved.

Drawback: Knowledge of channel information at the transmitted side is required.

Transmit antenna diversity without channel state information

The Alamouti scheme coding process:

- We transmit two symbols in two times slots, using two antennas.
- The symbols are multiplexed in time and antennas.
- During the first time slot:
  - Antenna 1 transmits symbol s_1
  - Antenna 2 transmits symbol s_2
- During the second time slot:
  - Antenna 1 transmits symbol $-s_2^*$
  - Antenna 2 transmits symbol s_1^*

Transmit antenna diversity without channel state information process

The Alamouti scheme decoding process:

- The signal reaching the receiver during the two time slots is expressed as:

$$[y_1, y_2] = [h_1, h_2] \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} + [n_1 + n_2]. \tag{53}$$

- We can form the vector:

$$\underbrace{\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}}_{H} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}. \tag{54}$$

- Note that $H^H H = |h\|^2 I$.
- By multiplying the vector y by H^H the symbol detection problem can be decoupled.
- Each symbol experiences an equivalent SISO model with fading equal to $\|h\|$.