## Unit 2: Essential Physics of the MOSFET

# Lecture 2.3: MOSFET IV Theory

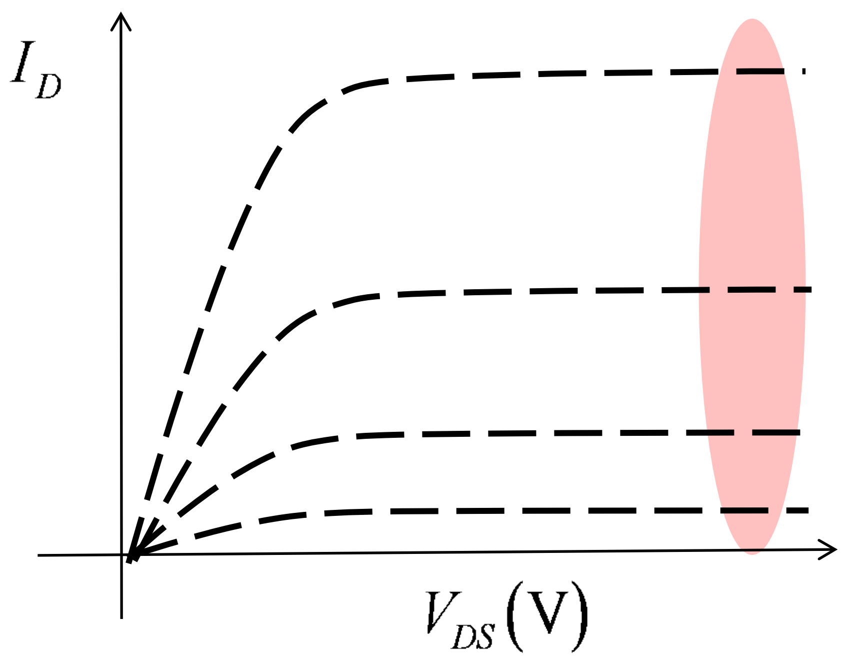
**Mark Lundstrom**

lundstro@purdue.edu
Electrical and Computer Engineering
Purdue University
West Lafayette, Indiana USA

# Long vs. short channel MOSFETs

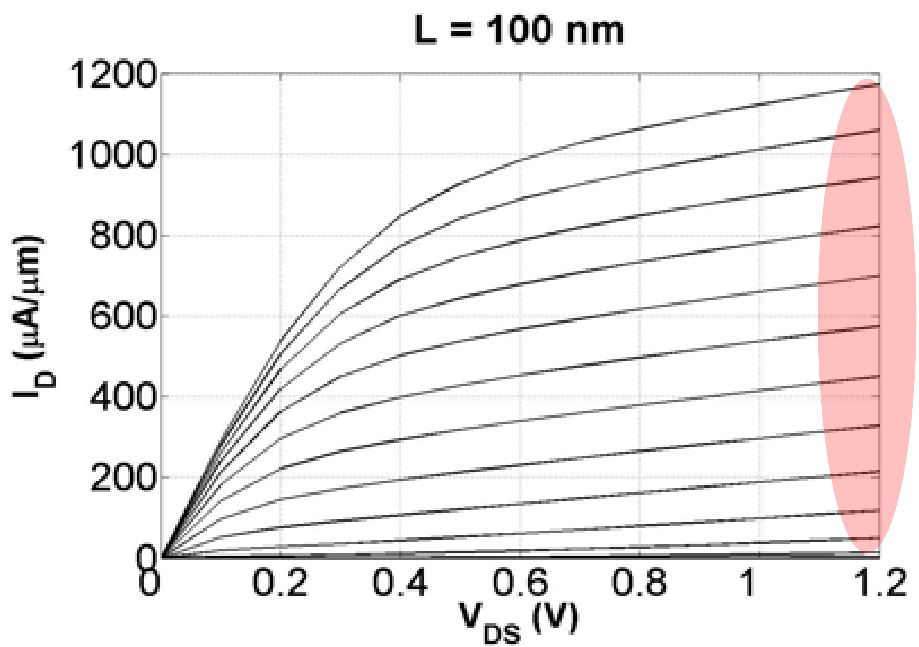
Square Law

$$I_{DSAT} \propto \left(V_{GS} - V_T\right)^2$$



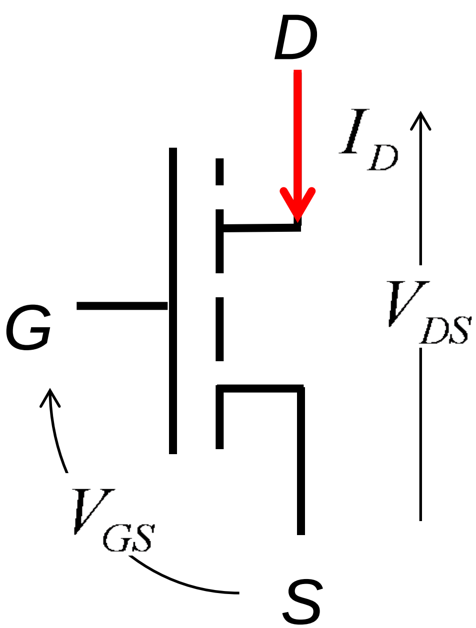

"Velocity saturated"

$$I_{DSAT} \propto \left(V_{GS} - V_T\right)$$



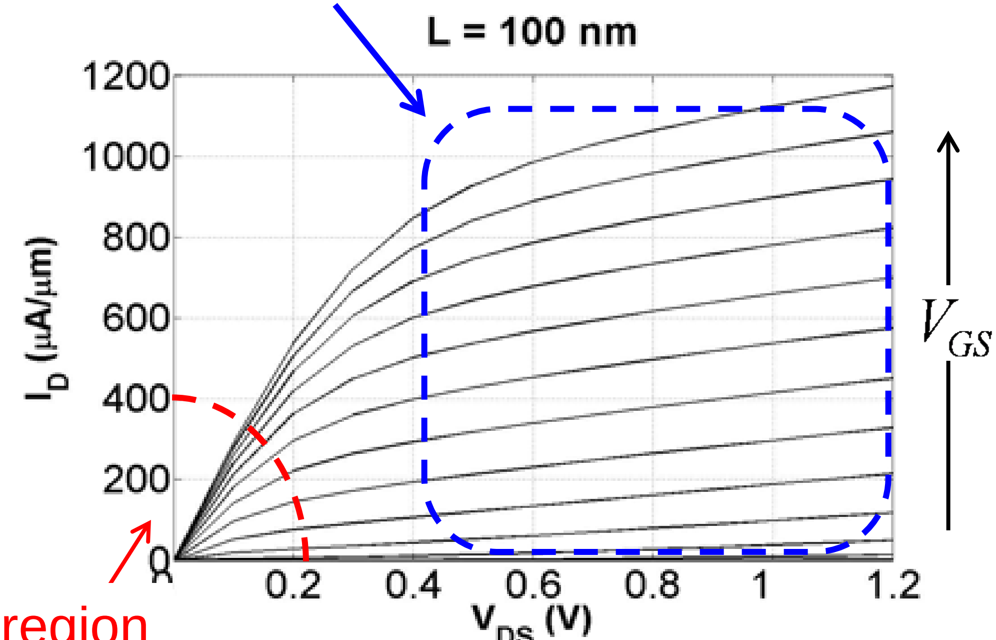
# MOSFET IV characteristic

circuit symbol




gate-voltage controlled current source

2) saturation region

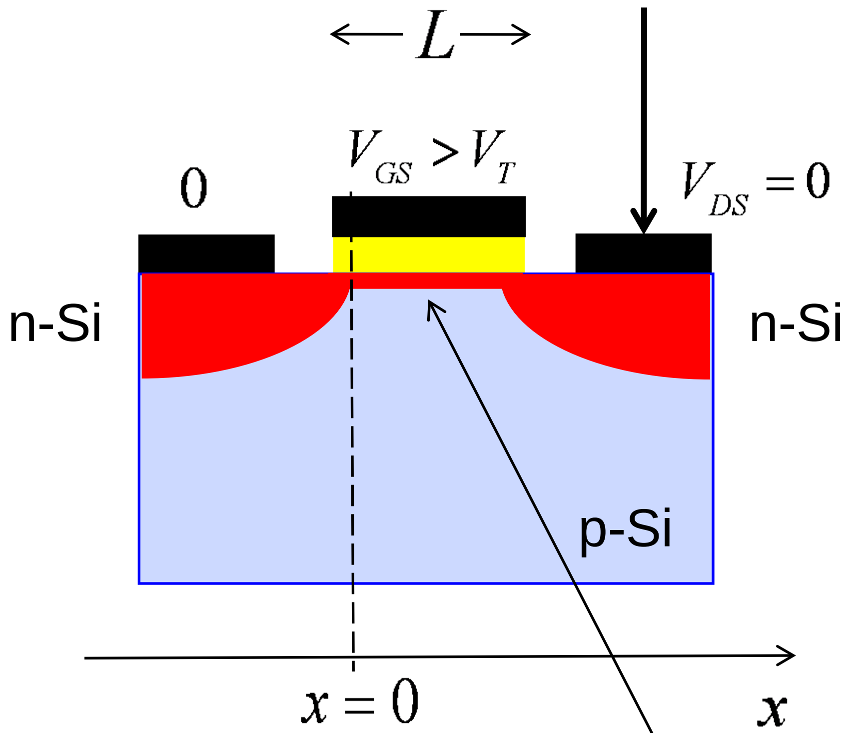


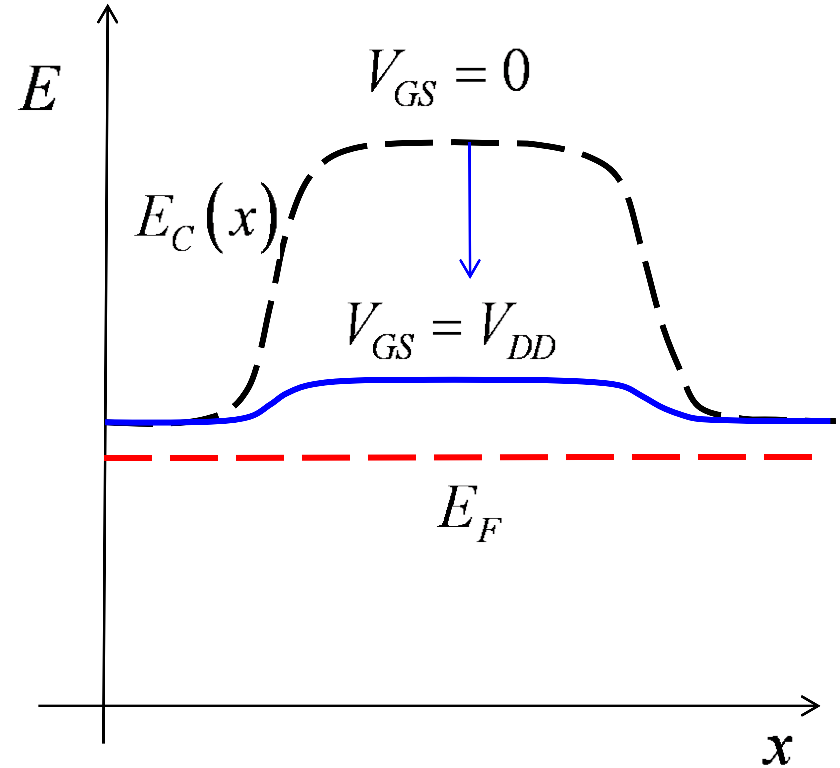

1) linear region

gate-voltage controlled resistor

(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

# MOSFET e-band (equilibrium)

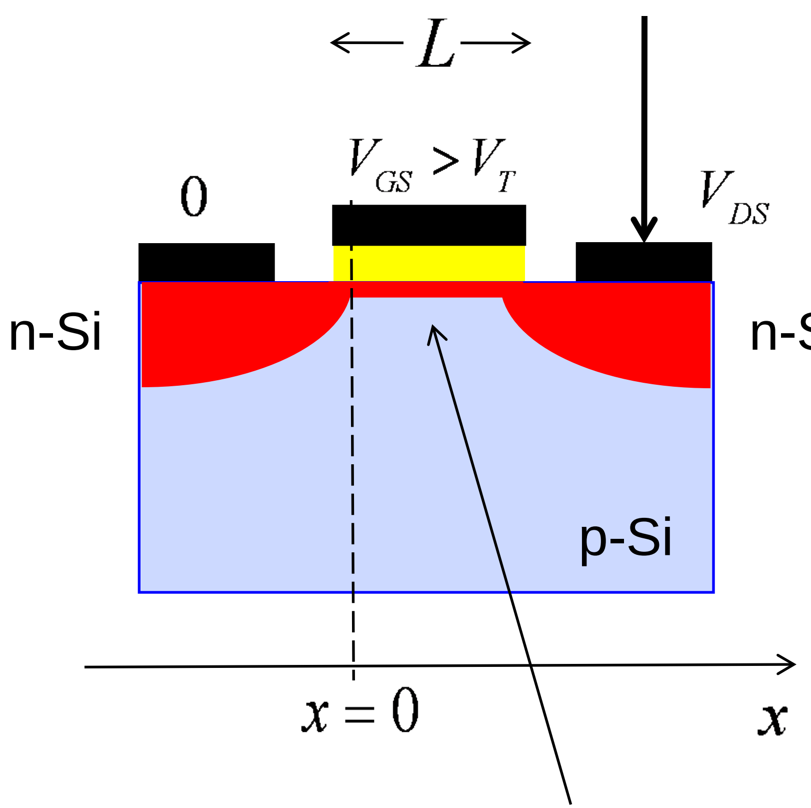


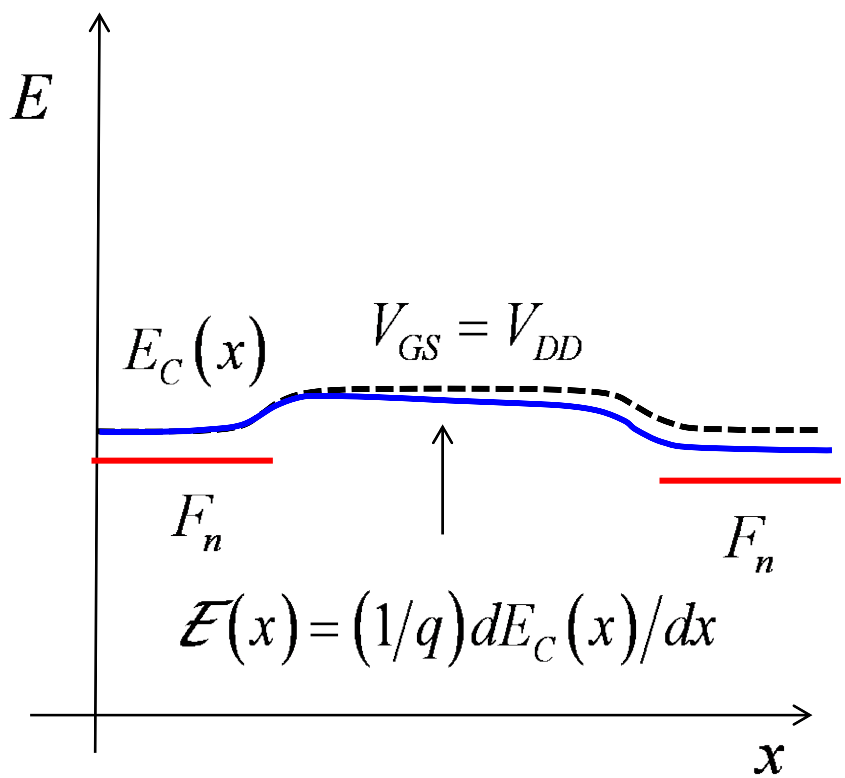




$$C \equiv \frac{Q}{V}$$

$$Q_n \approx -C_{ox}\left(V_{GS} - V_T\right) \quad \mathrm{C/cm^2}$$

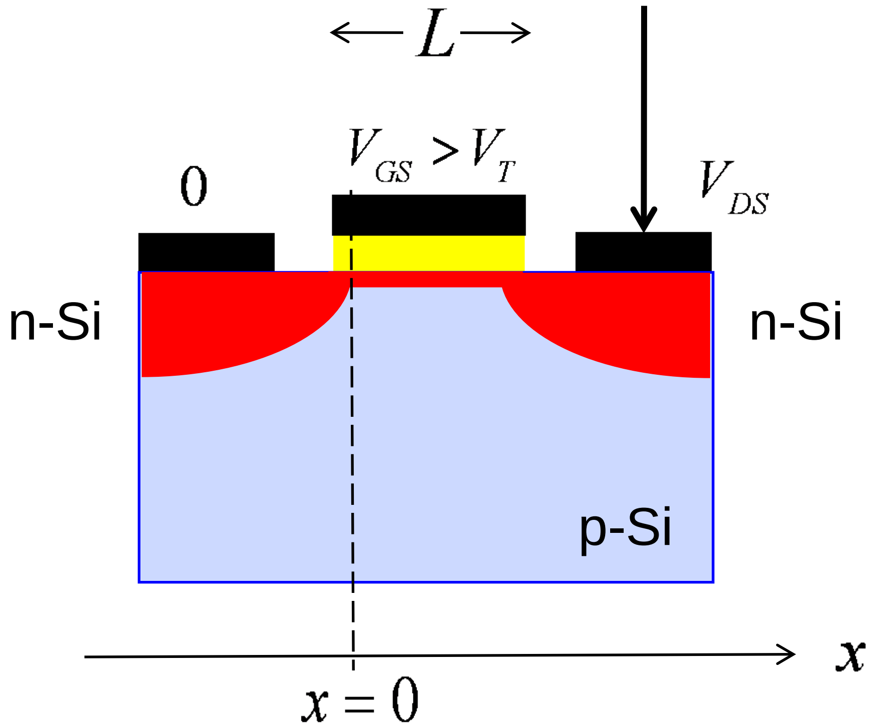
# MOSFET e-band (high $V_{GS}$, low $V_{DS}$)




$$Q_n \approx -C_{ox}\left(V_{GS} - V_T\right) \quad \mathrm{C/cm^2} \qquad \mathcal{E}_x = -V_{DS}/L$$

# MOSFET IV



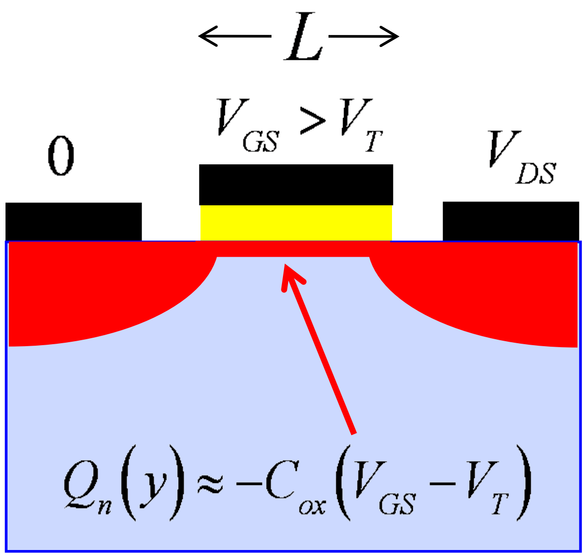

Current is charge per unit time

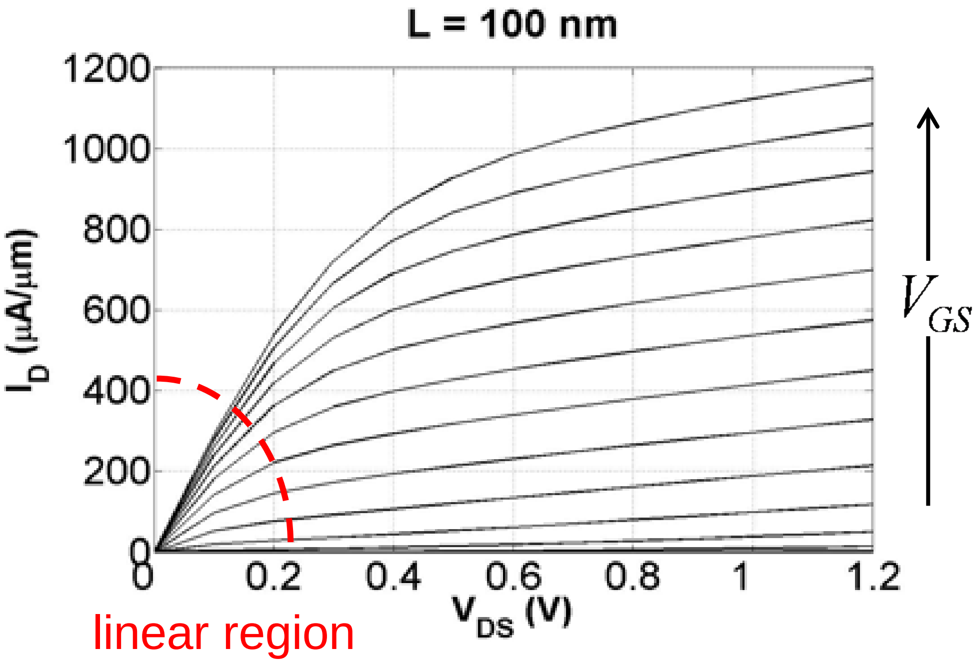
$$I_D = -W\,Q_n(x)\langle \upsilon_x(x)\rangle$$

MOS electrostatics

$$\begin{cases} Q_n \approx -C_{ox}\left(V_{GS} - V_T\right) & \mathrm{C/cm^2} \\ Q_n \approx 0 & \left(V_{GS} < V_T\right) \\ C_{ox} = \dfrac{K_O \varepsilon_0}{x_o} & \mathrm{F/cm^2} \end{cases}$$

# MOSFET IV: low $V_{DS}$




$$I_D = -W\,Q_n(x)\langle \upsilon_x(x)\rangle$$

$$Q_n = -C_{ox}(V_{GS} - V_T)$$

$$\langle \upsilon_x \rangle = -\mu_n \mathcal{E}_x$$

$$\mathcal{E}_x = -V_{DS}/L$$

$$\boxed{I_D = \frac{W}{L}\mu_n C_{ox}(V_{GS} - V_T)V_{DS}}$$ ✓

# Mobility and effective mobility

The mobility of carriers in the channel of a MOSFET is often called the effective mobility.  It is lower than the mobility of electrons in bulk silicon.

**Example:**

Consider an ultra-thin body (UTB) Si on Insulator (SOI) MOSFET with an undoped channel.

For undoped bulk Si: $\mu_n = 1360 \ \ \mathrm{cm^2/V\text{-}s}$

For the UTB MOSFET: $\mu_n = 200 - 300 \ \ \mathrm{cm^2/V\text{-}s}$

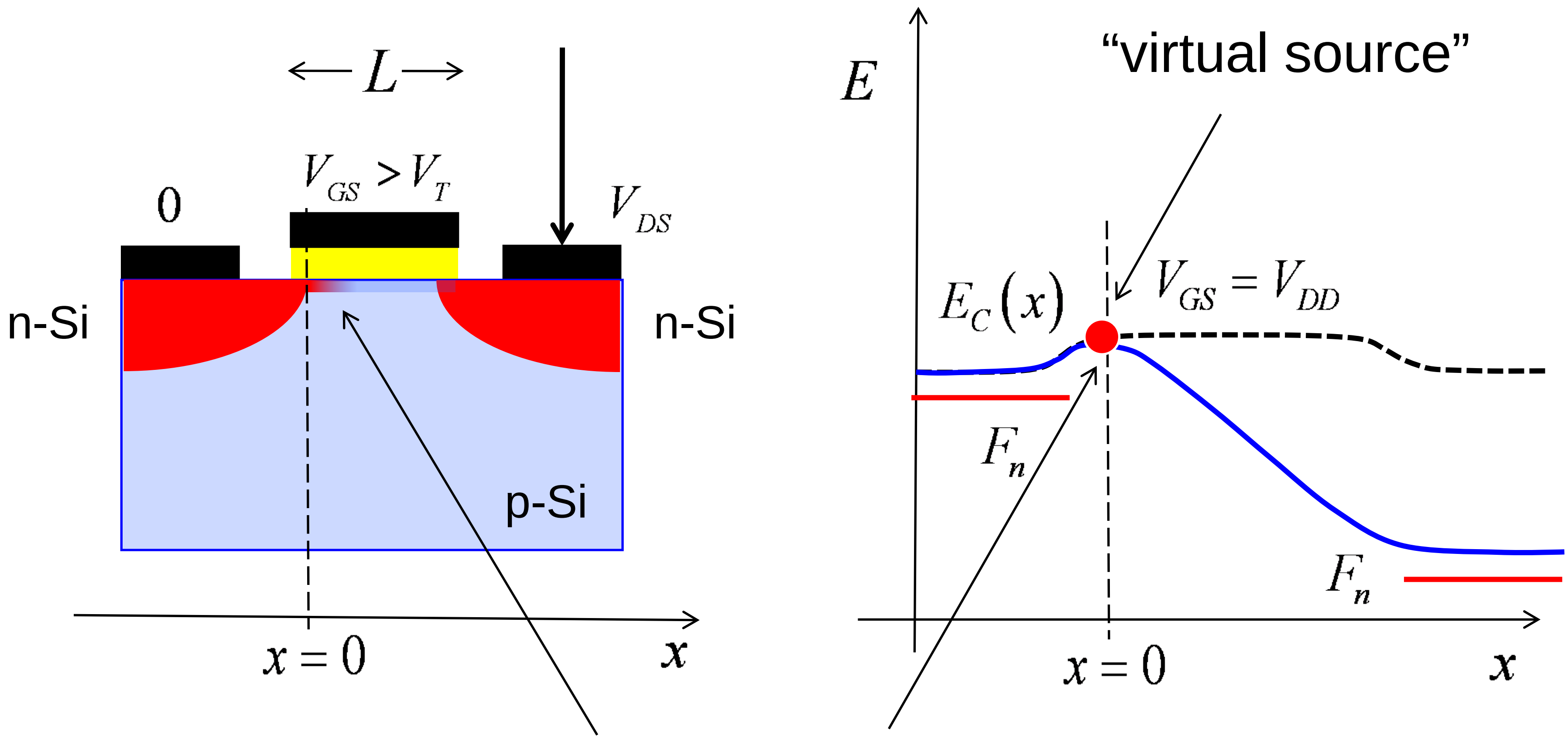
The lower mobility is due to **surface roughness scattering** at the Si/oxide interface.

# Outline

1) Linear region

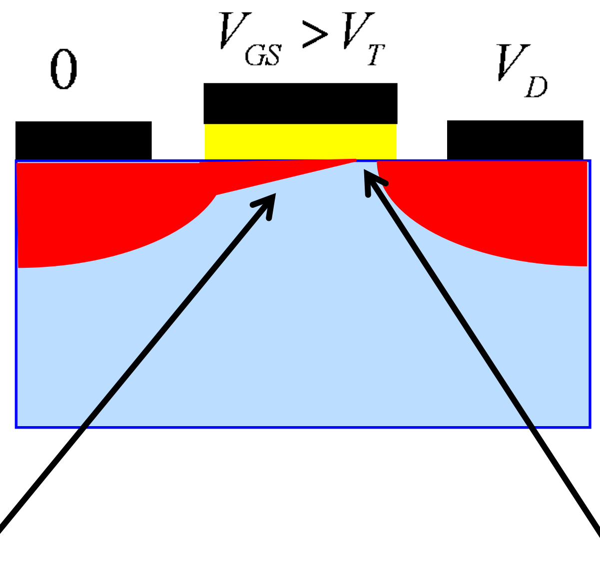
**2) Saturation region**
  -classical pinch off model
  -velocity saturation model

# MOSFET e-band (high $V_{GS}$, low $V_{DS}$)




$$Q_n(x=0) \approx -C_{ox}(V_{GS} - V_T) \quad \mathrm{C/cm^2}$$

# MOSFET IV: "pinch-off" at high $V_{DS}$




$$Q_n(x) = -C_{ox}\left(V_{GS} - V_T - V(x)\right)$$

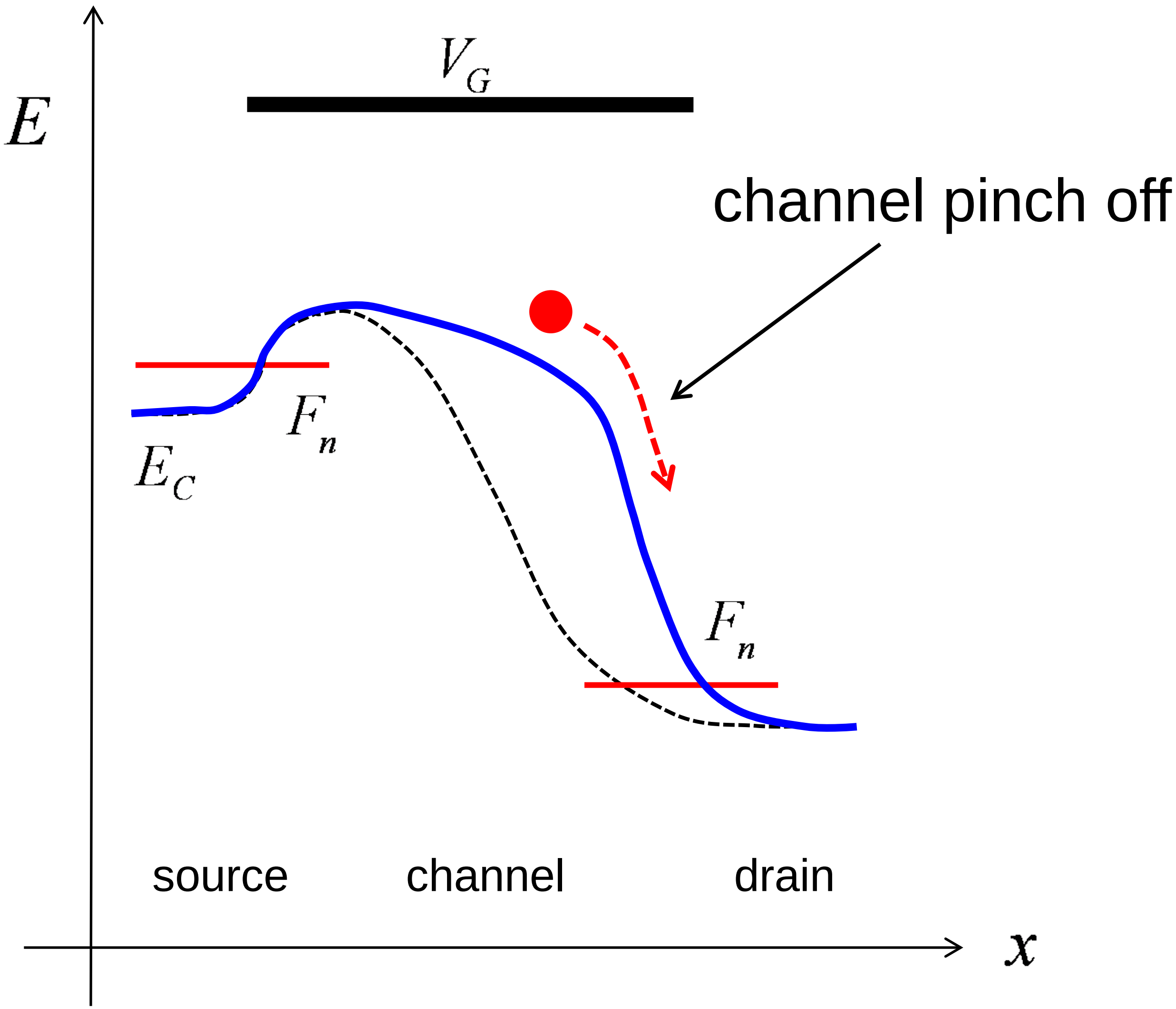
$$V\left(x_{pinch}\right) = \left(V_{GS} - V_T\right)$$

$$Q_n\left(x_{pinch}\right) \approx 0$$

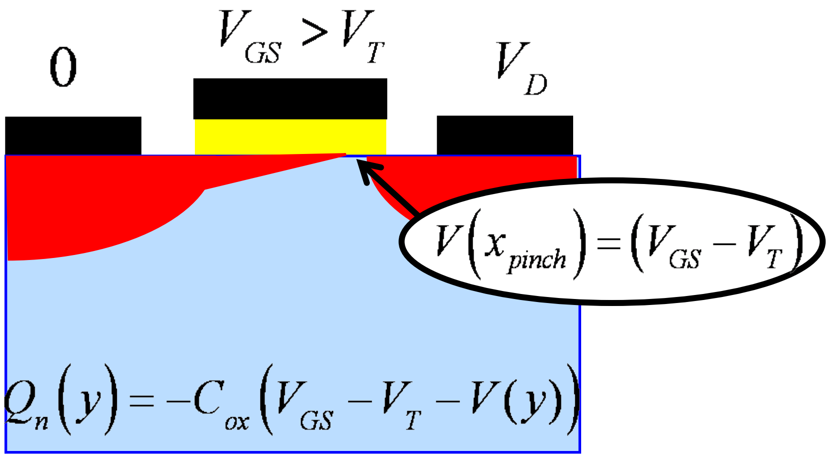
Note: thickness of channel illustrates the areal density of electrons – not the actual thickness.

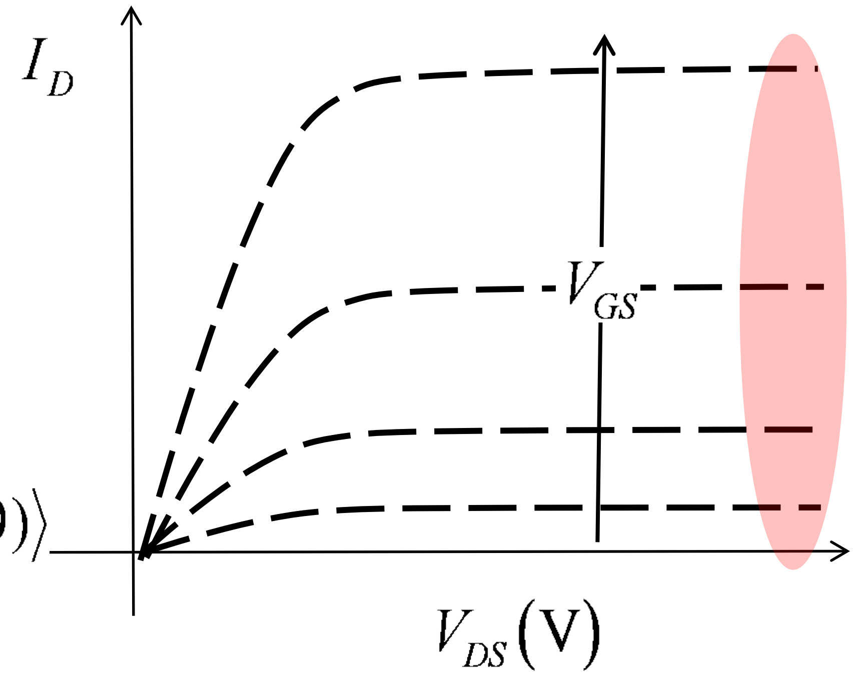
Electric field is very large in the pinch-off region.

# “Pinch off” on an energy band diagram



# MOSFET IV: high $V_{DS}$




$$I_D = -W\,Q_n\left(x\right)\left\langle \upsilon_x(x)\right\rangle = W\,Q_n\left(0\right)\left\langle \upsilon_x(0)\right\rangle$$

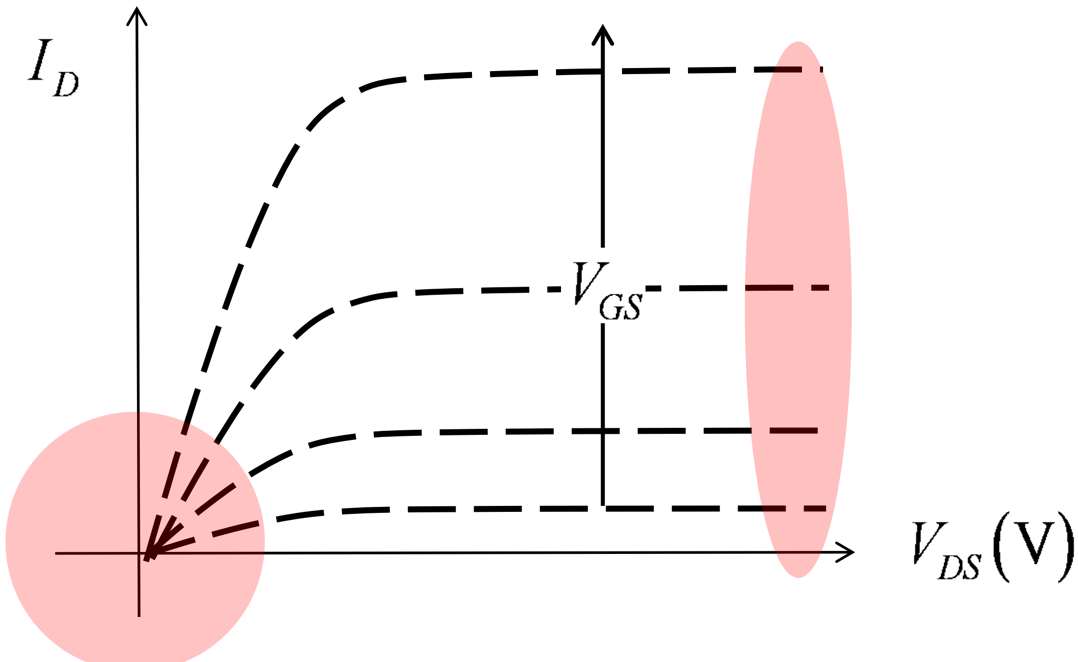
$$Q_n\left(0\right) = -C_{ox}\left(V_{GS} - V_T\right)$$

$$\left\langle \upsilon_x\left(0\right)\right\rangle = -\mu_n \mathcal{E}_x\left(0\right)$$

$$\mathcal{E}_x(0) \approx -V\left(x_{pinch}\right)/L = -\left(V_{GS} - V_T\right)/L$$

$$\boxed{I_D = \frac{W}{2L}\mu_n C_{ox}\left(V_{GS} - V_T\right)^2}$$

# The square law MOSFET

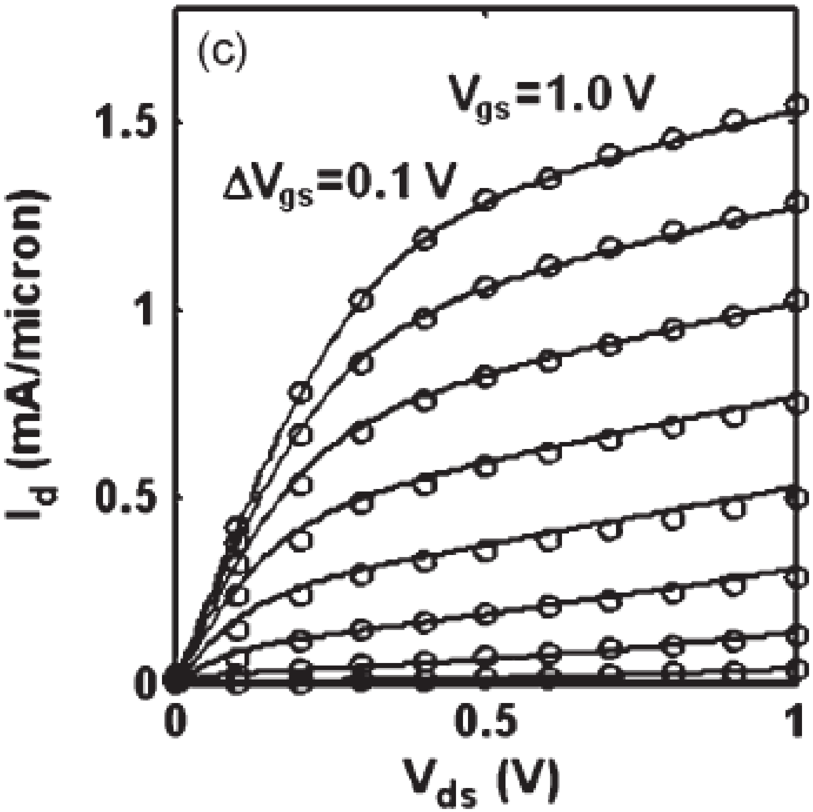



$$I_D = \frac{W}{L}\mu_n C_{ox}\left(V_{GS} - V_T\right)V_{DS}$$

$$I_D = \frac{W}{2L}\mu_n C_{ox}\left(V_{GS} - V_T\right)^2$$

Lundstrom: 2018

# Modern, short channel MOSFETs



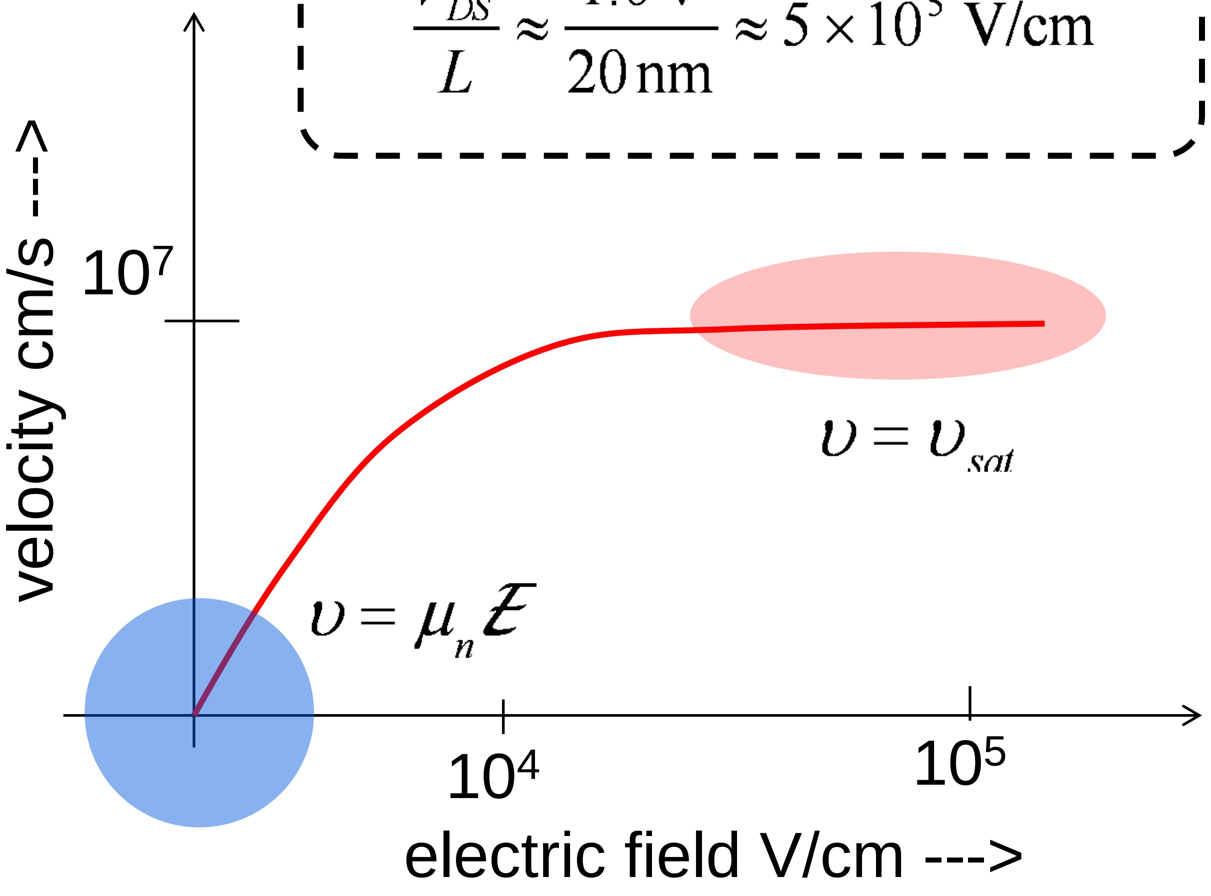

$$I_{DSAT} \propto \left(V_{GS} - V_T\right)$$



Lundstrom: 2018

# High $V_{DS}$: Velocity saturation

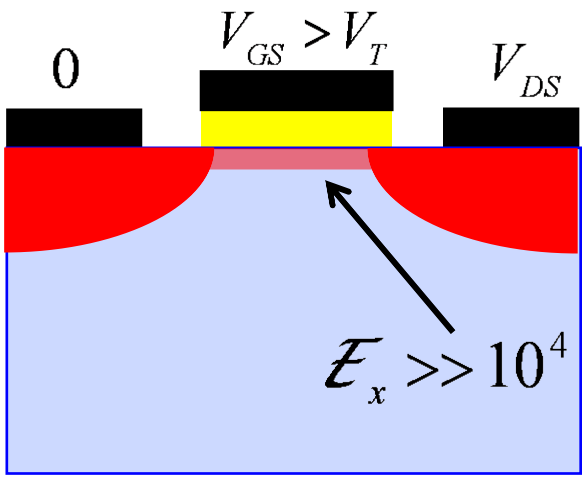
$$\frac{V_{DS}}{L} \approx \frac{1.0\,\mathrm{V}}{20\,\mathrm{nm}} \approx 5 \times 10^{5}\ \mathrm{V/cm}$$

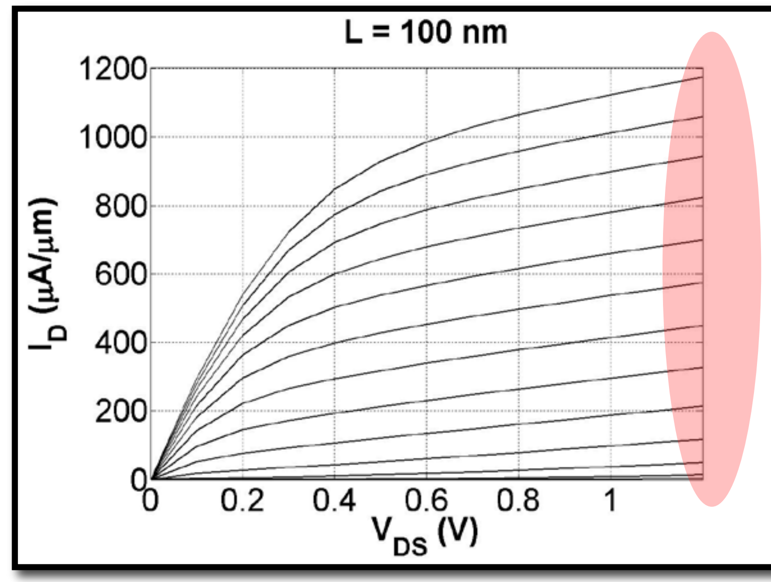





Lundstrom: 2018

# MOSFET IV: velocity saturation




(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

$$I_D = -W\,Q_n(x)\langle \upsilon_x(x)\rangle$$

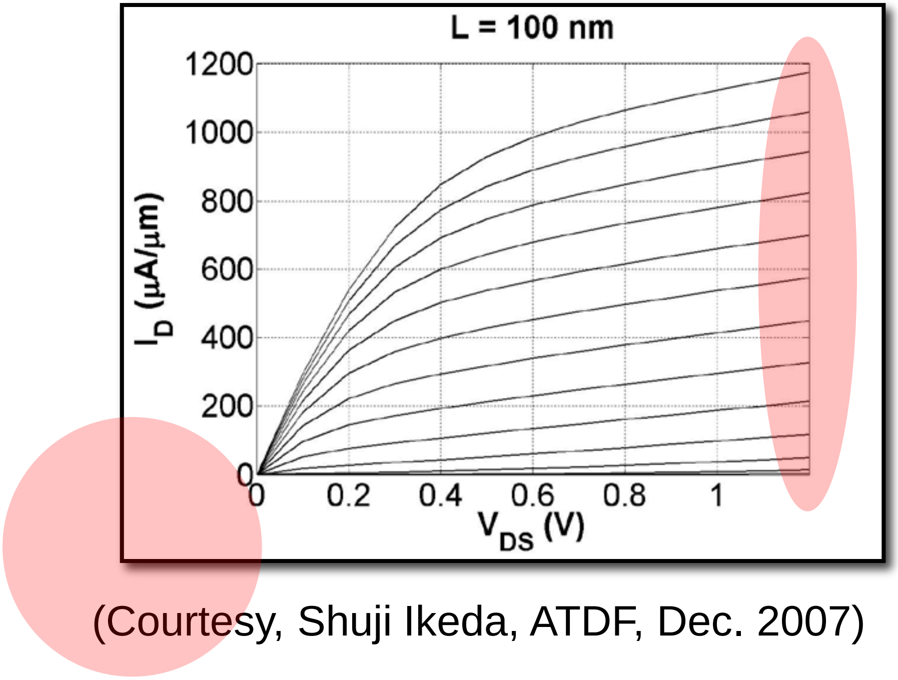
$$Q_n = -C_{ox}\left(V_{GS} - V_T\right)$$

$$\langle \upsilon_x \rangle = \upsilon_{sat}$$

$$\boxed{I_D = W\,C_{ox}\,\upsilon_{sat}\left(V_{GS} - V_T\right)}$$

✓

# The velocity saturated MOSFET

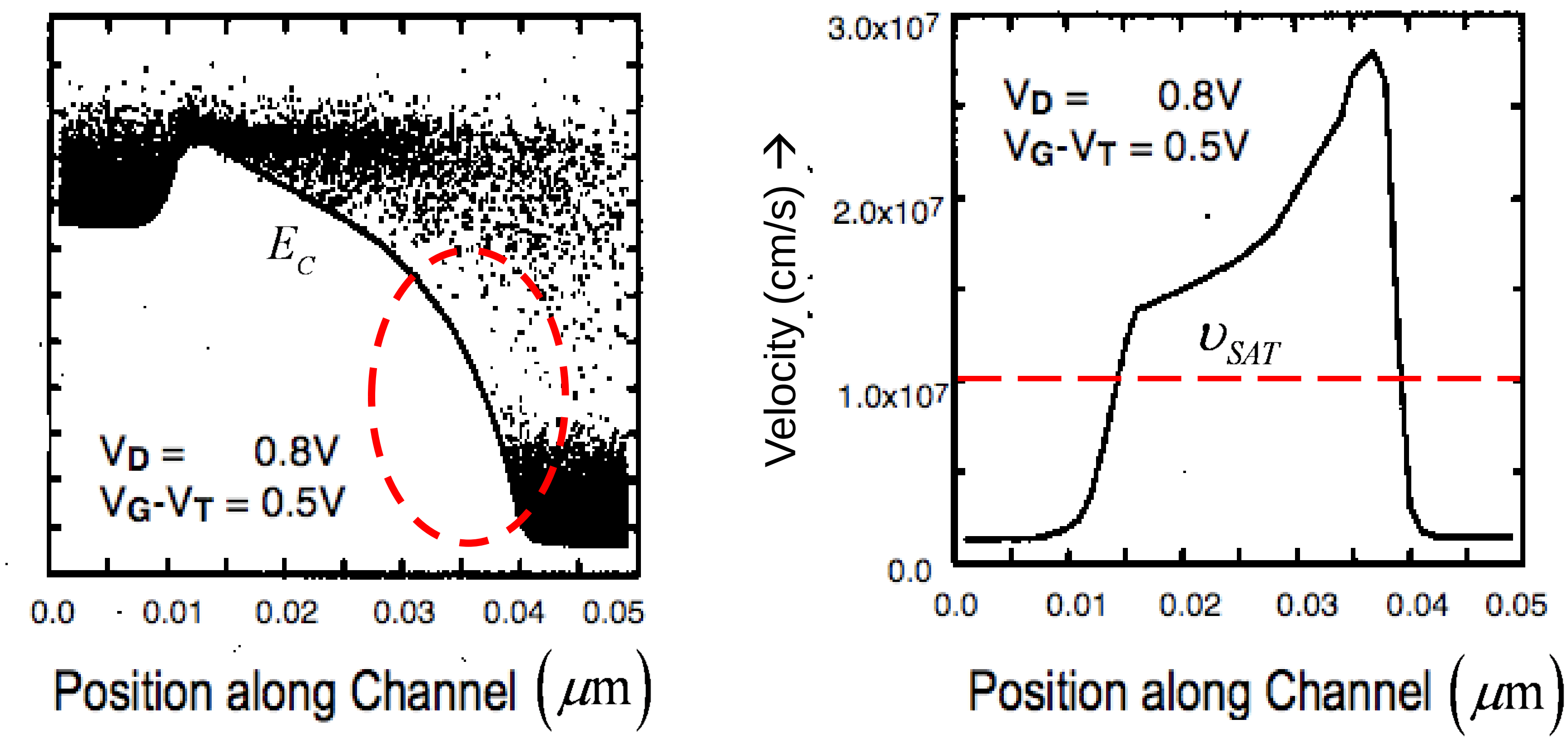



(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

$$I_D = \frac{W}{L}\mu_n C_{ox}\left(V_{GS} - V_T\right)V_{DS}$$

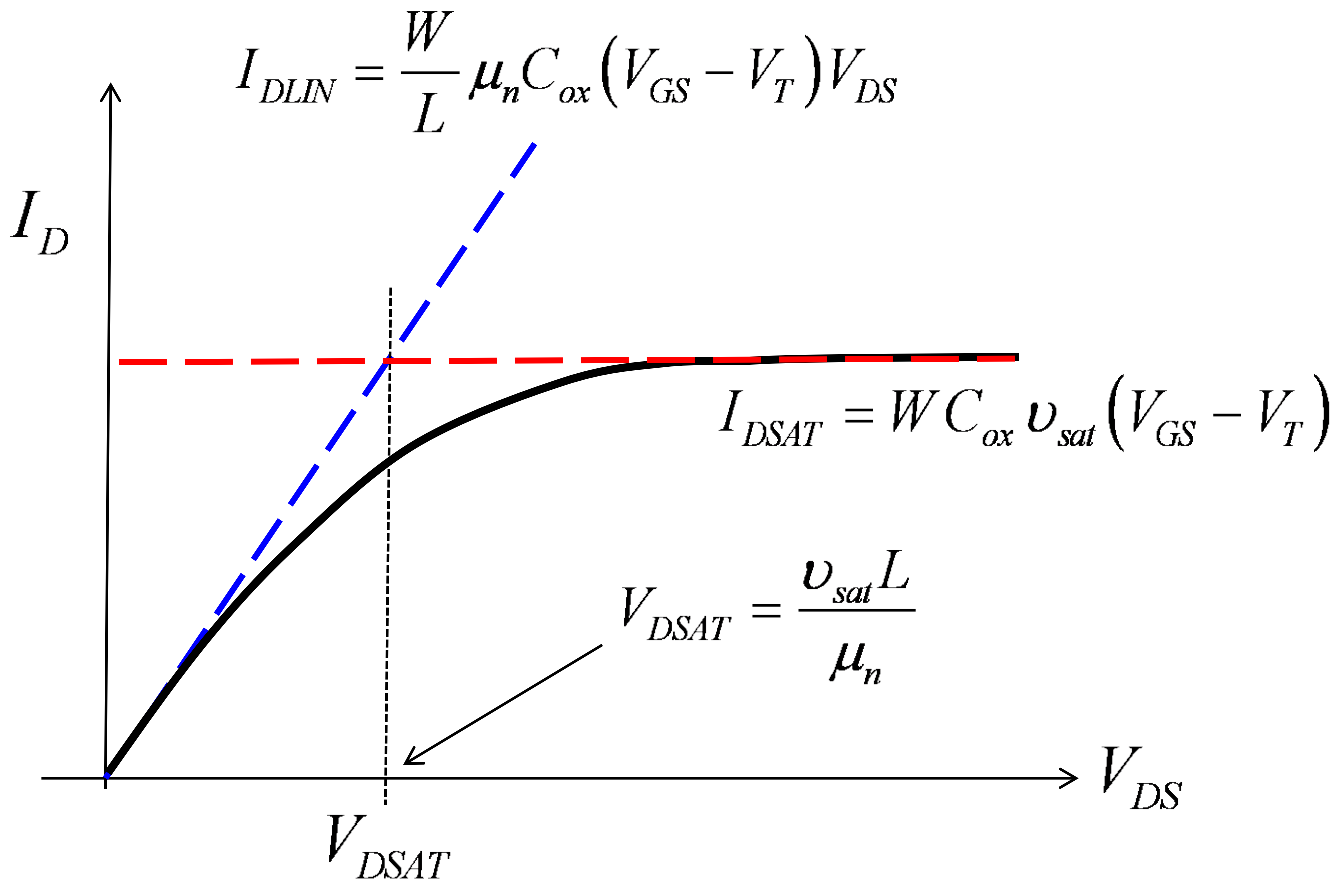
$$I_D = W C_{ox}\,\upsilon_{sat}\left(V_{GS} - V_T\right)$$

# Velocity overshoot




D. Frank, S. Laux, and M. Fischetti, Int. Electron Dev. Mtg., Dec., 1992.

# Piecewise linear model




We have developed a 2-piece approximation to the MOSFET IV characteristic.

# Summary

1) Analytical expression that describe the linear and saturation regions of a MOSFET are easy to develop.

2) The velocity saturation model describes modern transistors.

3) Energy band diagrams, not equations, explain how transistors work.

4) A model that smoothly connects the linear and saturation regions is needed for circuit simulation.

# Next topic:

In the next lecture, we will discuss traditional (square law) MOSFET theory in a way that describes the IV characteristics with simple analytical expressions that **smoothly connect** the linear and saturation regions.