Unit 4: Transmission Theory of the MOSFET

Lecture 4.3: The Ballistic MOSFET

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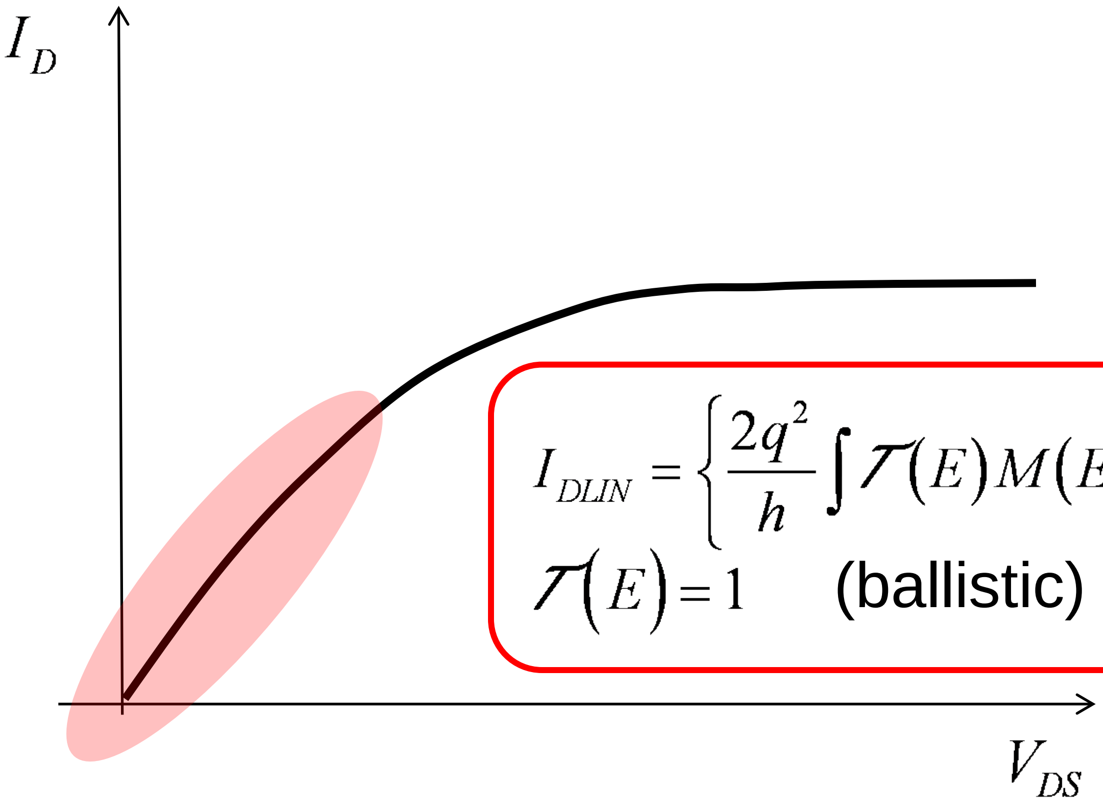
1) Linear region



$$I_{DLIN} = \left\{ \frac{2q^2}{h} \int \mathcal{T}(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \right\} V_{DS}$$

$\mathcal{T}(E) = 1$ (ballistic)

$$I_{DLIN} = \frac{W}{L}\mu_n C_{ox}\left(V_{GS} - V_T\right)V_{DS} \rightarrow$$ **?**

Linear region with MB statistics (i)

$$I_{DLIN} = G_{CH} V_{DS}$$

$$G_{CH} = \frac{2q^2}{h} \int_{E_C}^{\infty} \mathcal{T}(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

(See Sections 13.3 and 15.4 of FoN lecture notes for the complete derivation.)

$$M(E) = W g_V \frac{\sqrt{2m^*(E - E_C)}}{\pi\hbar} \quad \text{(2D)}$$

$$\mathcal{T}(E) = 1$$

$$f_0(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}} \approx e^{(E_F - E)/k_B T}$$

$$n_S = N_{2D} e^{(E_F - E_C)/k_B T} \quad \text{(nondegenerate)}$$

$$N_{2D} = \left(g_V \frac{m^*}{\pi\hbar^2} k_B T \right)$$

Linear region with MB statistics (ii)

$$G_{CH} = \frac{2q^2}{h}\int_{E_C}^{\infty} \mathcal{T}(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$G_{CH} = W(qn_S)\frac{\upsilon_T}{2(k_BT/q)} \qquad \upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}}$$

$$qn_S = -Q_n = C_{inv}(V_{GS} - V_T)$$

$$G_{CH} = W\,C_{inv}(V_{GS} - V_T)\frac{\upsilon_T}{2(k_BT/q)}$$ ✔

$$M(E) = g_V W\frac{\sqrt{2m^*(E-E_C)}}{\pi\hbar}$$

$$\mathcal{T}(E) = 1$$

$$f_0(E) = e^{(E_F - E)/k_BT}$$

$$n_S = N_{2D}e^{(E_F - E_C)/k_BT}$$

$$N_{2D} = \left(g_V\frac{m^*}{\pi\hbar^2}k_BT\right)$$

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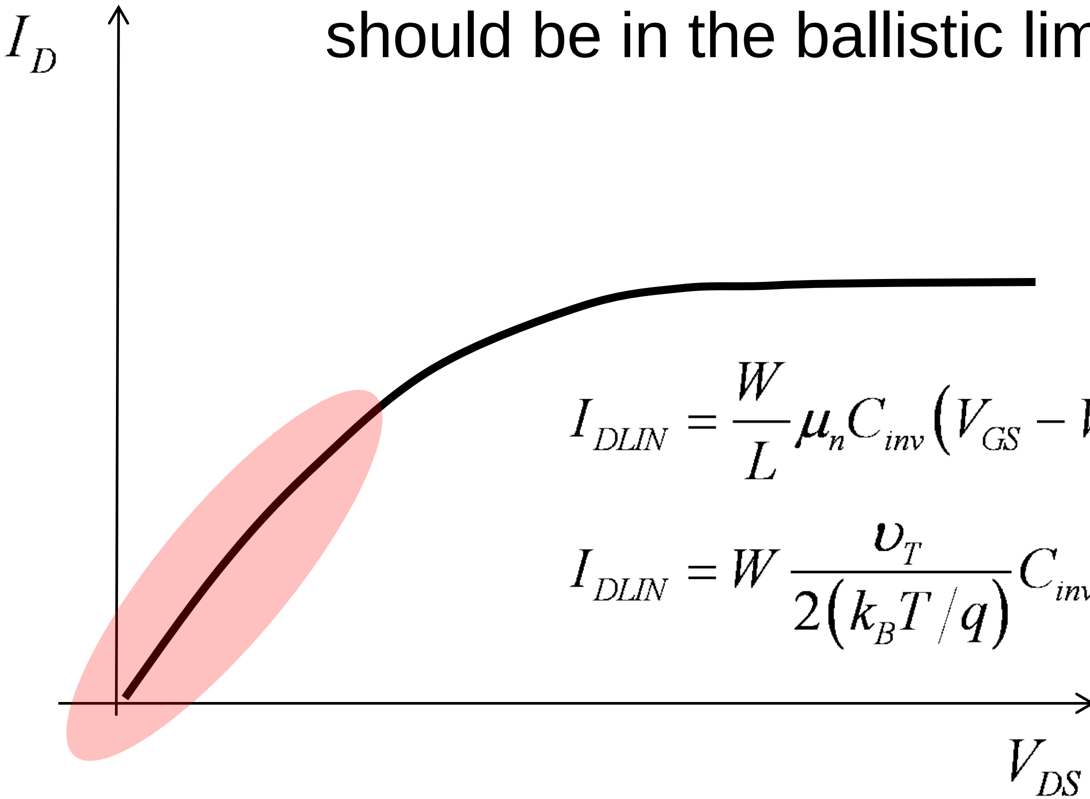
1) Linear region

I_D is independent of L, as it should be in the ballistic limit



$$I_{DLIN} = \frac{W}{L}\mu_n C_{inv}\left(V_{GS}-V_T\right)V_{DS} \rightarrow$$

$$I_{DLIN} = W\frac{\upsilon_T}{2\left(k_B T/q\right)}C_{inv}\left(V_{GS}-V_T\right)V_{DS}$$

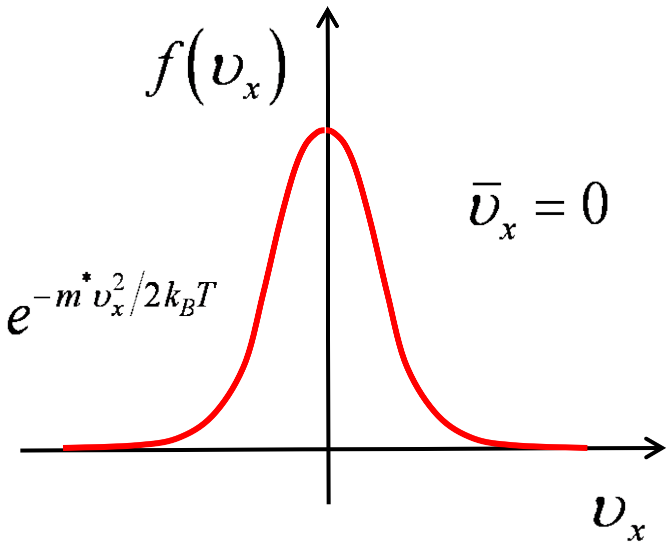
Questions

$$I_{DLIN} = W \frac{\upsilon_T}{2(k_B T/q)} C_{inv} (V_{GS} - V_T) V_{DS} \qquad \upsilon_T = \sqrt{\frac{2k_B T}{\pi m^*}}$$

1) How do we interpret the velocity, v_T?

2) Why does the traditional model, with (W/L) times mobility fit measured data for nanoscale MOSFETs so well?

Equilibrium Maxwellian velocity distribution



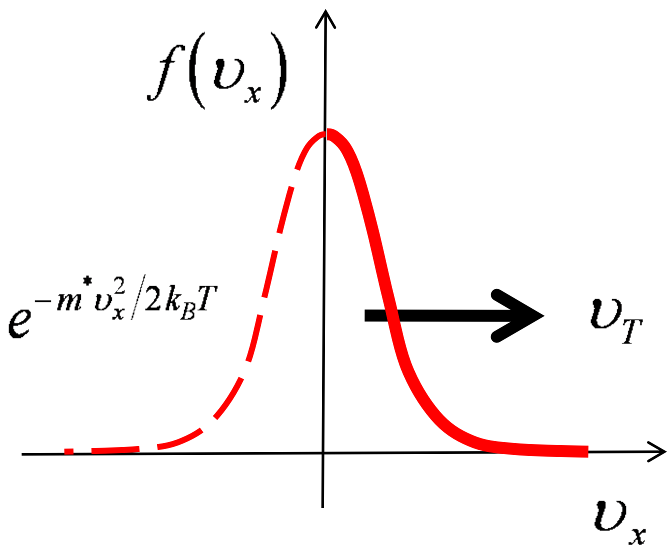
$$f_0(E) \propto e^{-m^*\left(\upsilon_x^2 + \upsilon_y^2\right)/2k_B T}$$

$$f_0(E) = \frac{1}{1 + e^{\left(E - E_F\right)/k_B T}} \approx e^{\left(E_F - E\right)/k_B T}$$

$$E = E_C + \frac{1}{2} m^* \upsilon^2$$

$$f_0(E) = e^{\left(E_F - E_C\right)/k_B T} \times e^{-m^* \upsilon^2 / 2k_B T}$$

Unidirectional thermal velocity



$$\langle\langle \upsilon_x^+ \rangle\rangle = \upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}}$$

(nondegenerate)

average over angle:

$$\langle \upsilon_x^+(E) \rangle = \frac{2}{\pi}\upsilon(E) \qquad \text{(2D)}$$

average over energy:

$$\langle\langle \upsilon_x^+(E) \rangle\rangle = \frac{\int_{E_C}^{\infty} \langle \upsilon_x^+(E) \rangle D_{2D}(E) f_0(E) dE}{\int_{E_C}^{\infty} D_{2D}(E) f_0(E) dE}$$

(Exercise 12.2, p. 189, of FoN)

Ballistic mobility

$$I_{DLIN} = \frac{W}{L}\mu_n C_{inv}\left(V_{GS} - V_T\right)V_{DS}$$

traditional

?

$$I_{DLIN} = W\left(\frac{\upsilon_T}{2\left(k_B T/q\right)}\right)C_{inv}\left(V_{GS} - V_T\right)V_{DS}$$

Landauer

$$I_{DLIN} = \frac{W}{L}\left(\frac{\upsilon_T L}{2\left(k_B T/q\right)}\right)C_{inv}\left(V_{GS} - V_T\right)V_{DS}$$

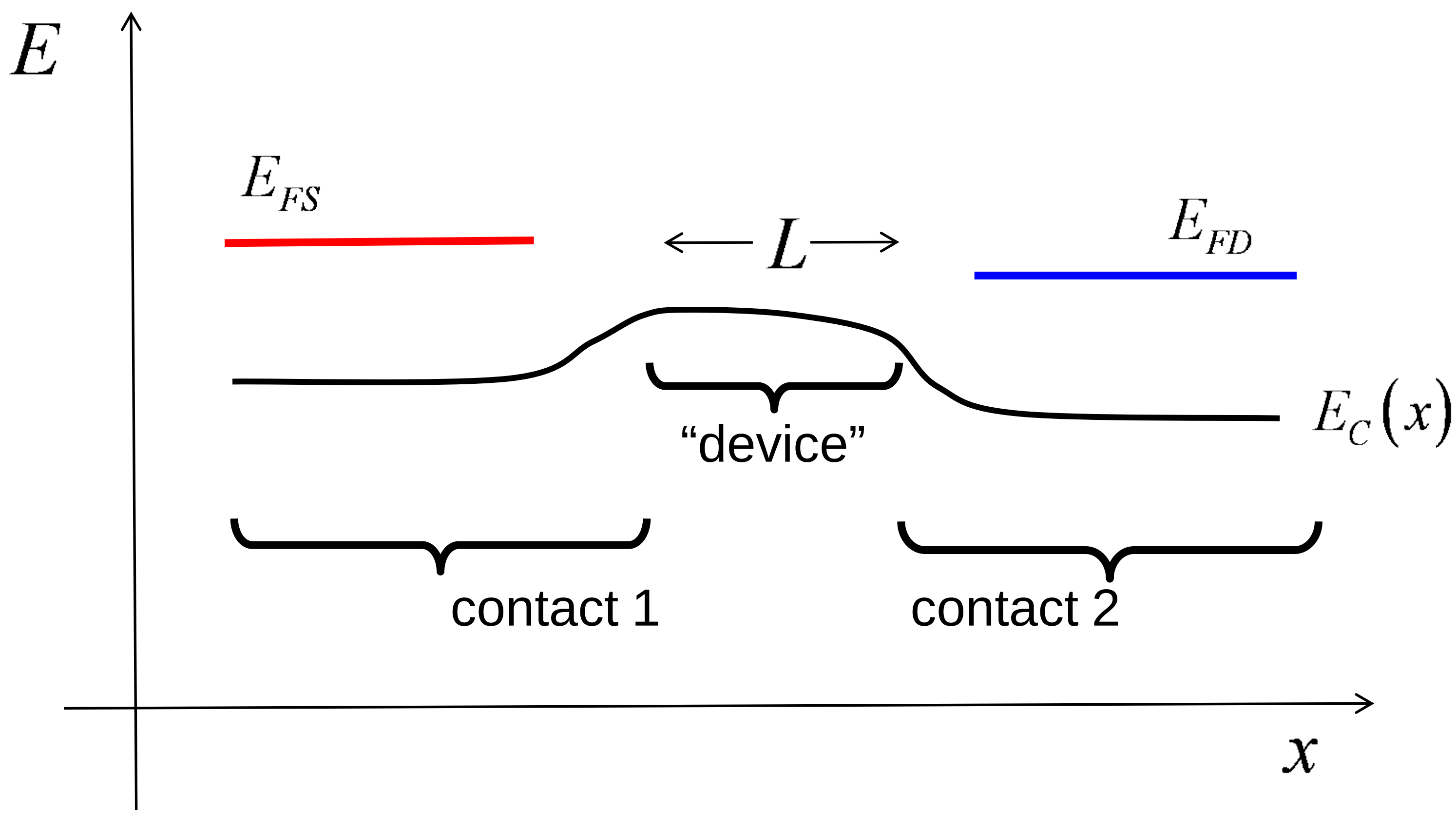
$$\boxed{\mu_B \equiv \frac{\upsilon_T L}{2\left(k_B T/q\right)}}$$

"ballistic mobility"

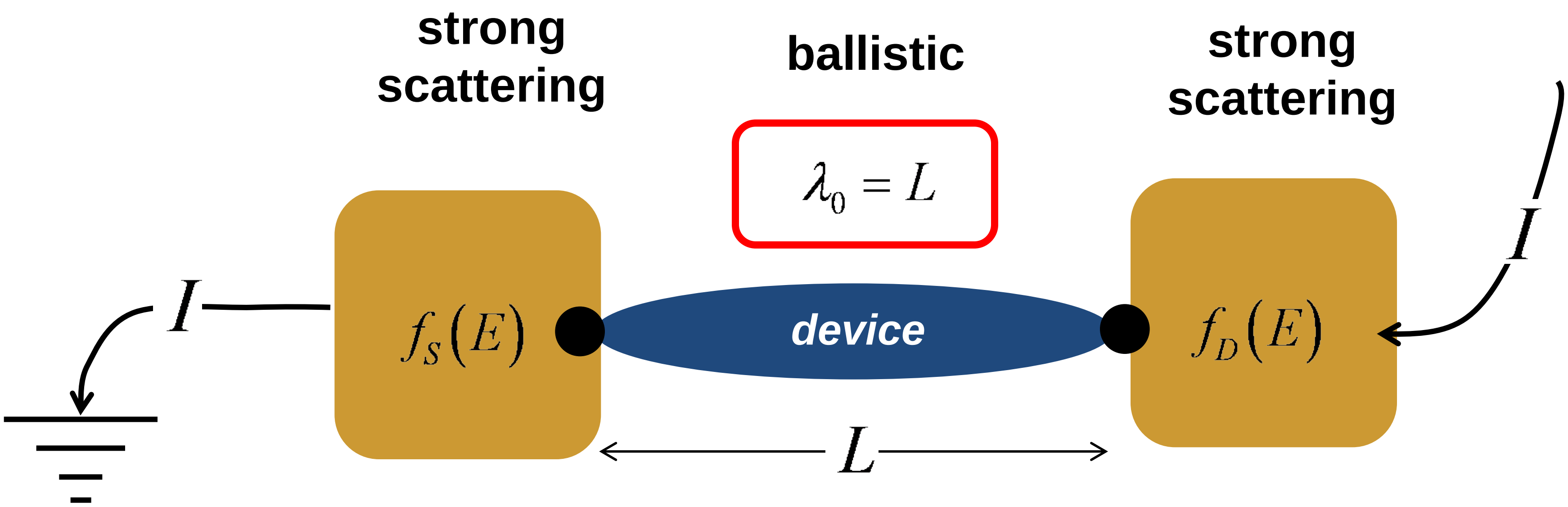
$$\boxed{\mu_n = \frac{\upsilon_T \lambda_0}{2\left(k_B T/q\right)}}$$

"diffusive mobility"

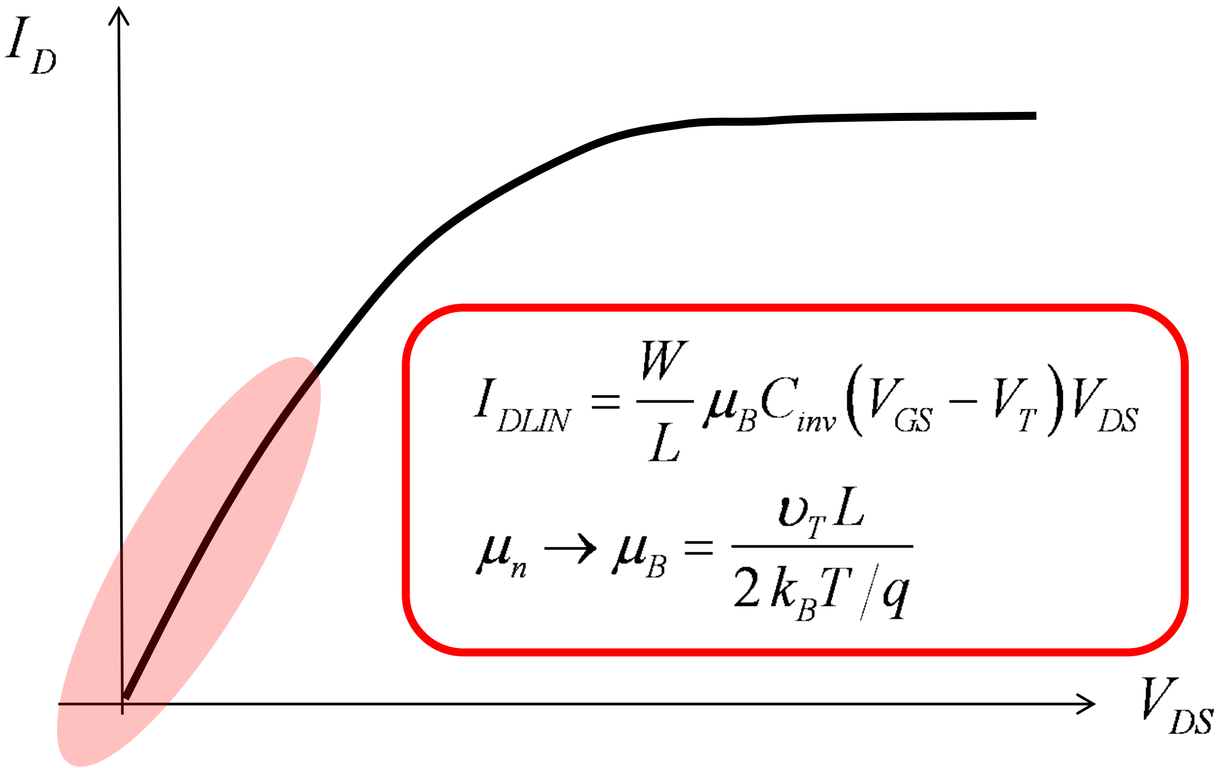
Low bias energy band diagram



MFP in a ballistic device



1) Linear region summary

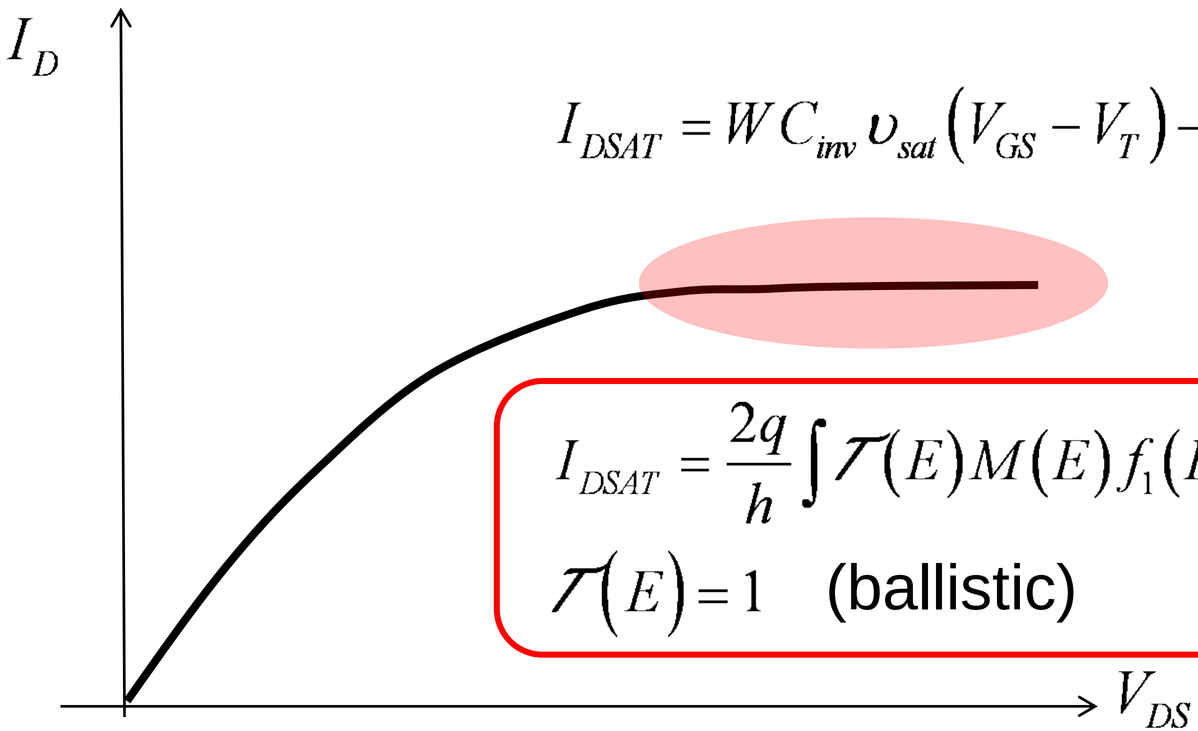


$$I_{DLIN} = \frac{W}{L}\mu_B C_{inv}\left(V_{GS} - V_T\right)V_{DS}$$

$$\mu_n \to \mu_B = \frac{\upsilon_T L}{2\,k_B T/q}$$

ballistic limit: μ_B diffusive limit: μ_n

2) Saturation region

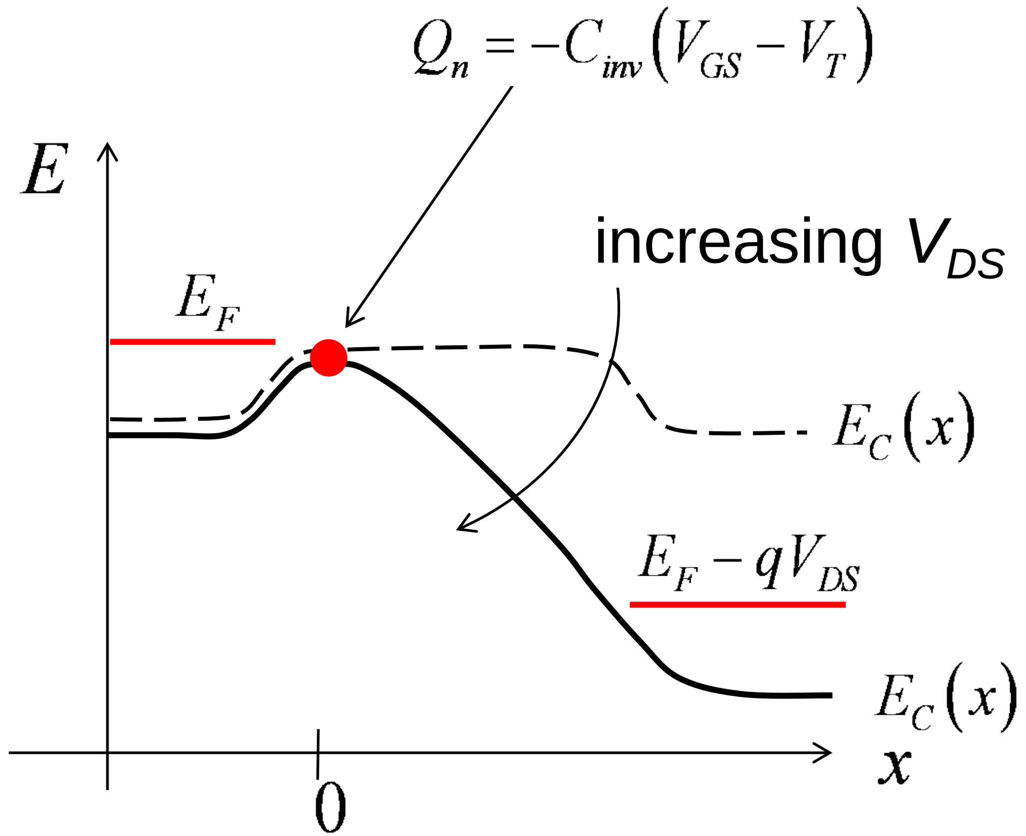


$$I_{DSAT} = W C_{inv} \upsilon_{sat} \left(V_{GS} - V_T\right) \rightarrow \ \textbf{?}$$

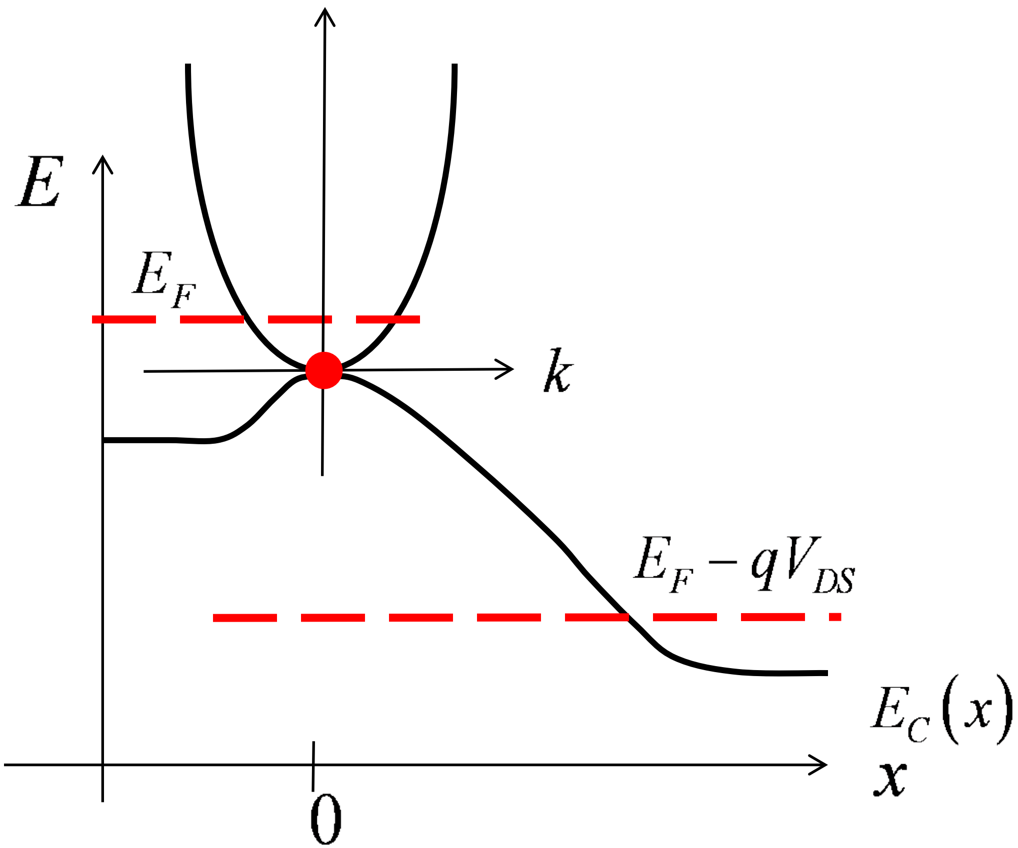
$$I_{DSAT} = \frac{2q}{h} \int \mathcal{T}(E) M(E) f_1(E) dE$$

$$\mathcal{T}(E) = 1$$ (ballistic)

Focus on the VS (the top of the barrier)



Electron density at the VS



$$n_S^+ = \frac{N_{2D}}{2} e^{(E_F - E_C)/k_B T}$$

$$n_S^- = \frac{N_{2D}}{2} e^{(E_F - qV_{DS} - E_C)/k_B T} \approx 0$$

$$q\left(n_S^+ + n_S^-\right) \approx q n_S^+ = -Q_n$$

$$Q_n = -C_{inv}\left(V_{GS} - V_T\right)$$

$$q n_S^+ = C_{inv}\left(V_{GS} - V_T\right) = q n_S$$

Saturation region with MB statistics

$$I_{DSAT} = \frac{2q}{h} \int_{E_C}^{\infty} \mathcal{T}(E) M(E) f_1(E) dE$$

$$M(E) = g_V W \frac{\sqrt{2m^*(E - E_C)}}{\pi\hbar}$$

$$\mathcal{T}(E) = 1$$

$$f_1(E) = f_0(E) = e^{(E_F - E)/k_B T}$$

$$\upsilon_T = \sqrt{2k_B T / \pi m^*}$$

$$n_S = \frac{N_{2D}}{2} e^{(E_F - E_C)/k_B T}$$

(See Sections 13.4 and 15.4 of FoN lecture notes for the complete derivation.)

Saturation region with MB statistics

$$I_{DSAT} = \frac{2q}{h}\int_{E_C}^{\infty} \mathcal{T}(E)M(E)f_1(E)dE$$

$$I_{DSAT} = W\left(qn_S\right)\upsilon_T$$

$$qn_S = -Q_n = C_{inv}\left(V_{GS} - V_T\right)$$

$$I_{DSAT} = WC_{inv}\left(V_{GS} - V_T\right)\upsilon_T \quad ✔$$

$$M(E) = g_V W \frac{\sqrt{2m^*\left(E - E_C\right)}}{\pi\hbar}$$

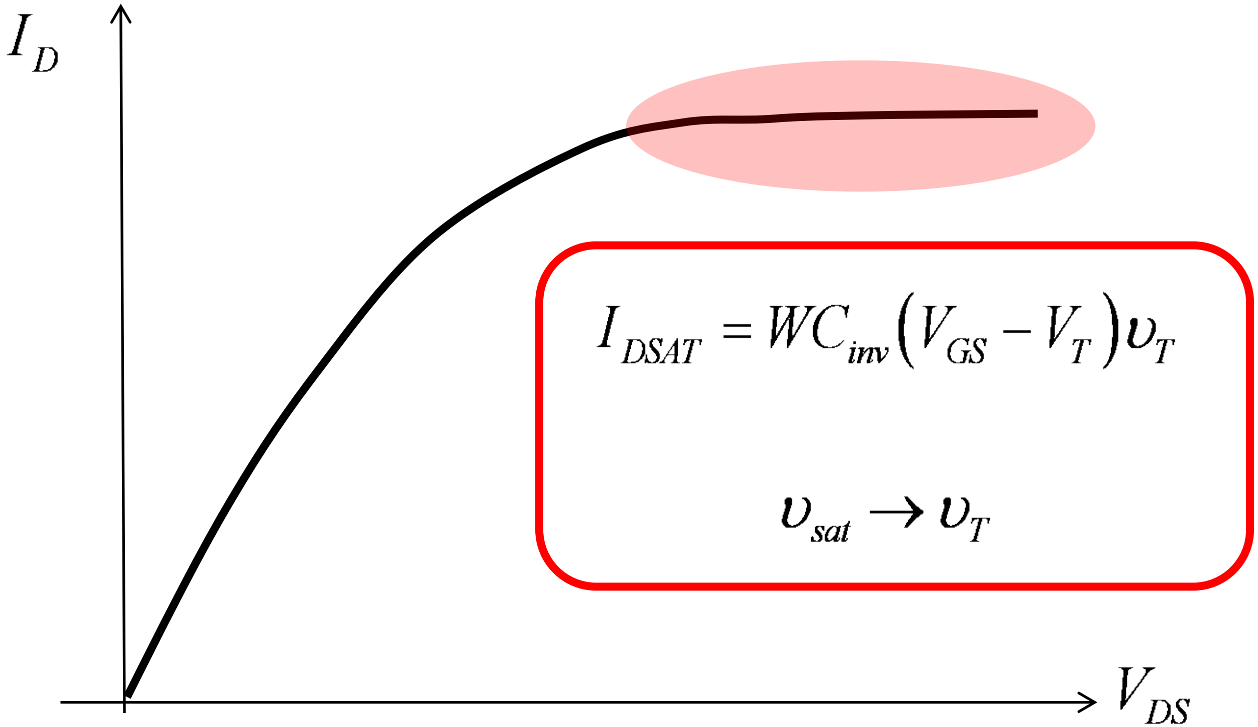
$$\mathcal{T}(E) = 1$$

$$f_1(E) = f_0(E) = e^{\left(E_F - E\right)/k_BT}$$

$$\upsilon_T = \sqrt{2k_BT/\pi m^*}$$

$$n_S = \frac{N_{2D}}{2}e^{\left(E_F - E_C\right)/k_BT}$$

2) Saturation region summary



$$I_{DSAT} = WC_{inv}\left(V_{GS} - V_T\right)\upsilon_T$$

$$\upsilon_{sat} \rightarrow \upsilon_T$$

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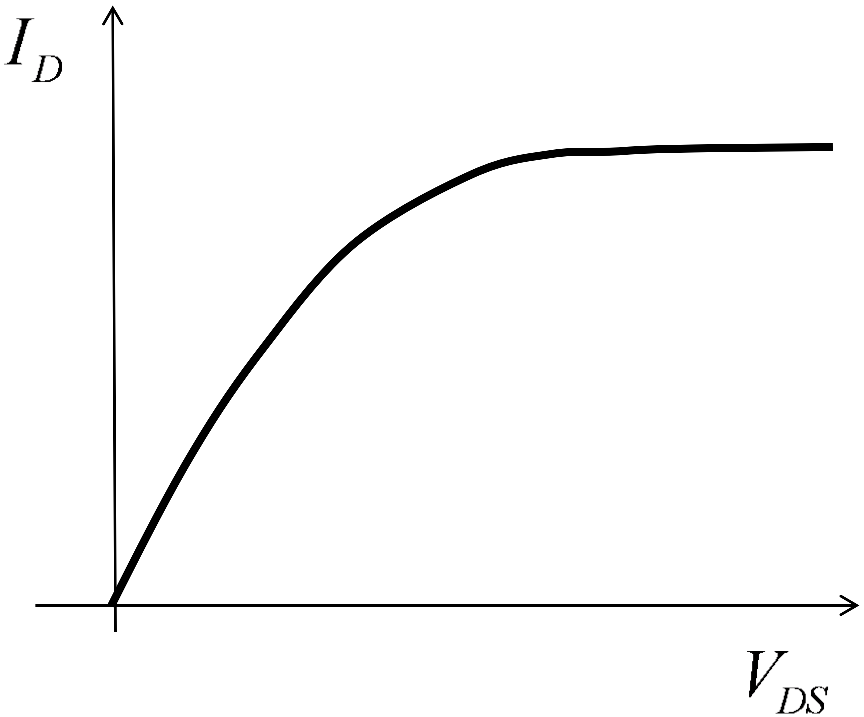
3) Ballistic MOSFET: full V_{DS} range



$$I_D = \frac{2q}{h}\int \mathcal{T}(E)M(E)\left(f_1 - f_2\right)dE$$

$$I_D = I_{S\to D} - I_{D\to S}$$

$$I_{S\to D} = -W\,Q_n^+\,\upsilon_T$$

$$I_{D\to S} = -W\,Q_n^-\,\upsilon_T$$

$$I_D = -W\,\upsilon_T\left(Q_n^+ - Q_n^-\right)$$
$$= -W\,\upsilon_T Q_n^+\left(1 - Q_n^-/Q_n^+\right)$$

Full V_{DS} expression

$$I_D = -W\,\upsilon_T Q_n^+\left(1 - Q_n^-/Q_n^+\right)$$

$$Q_n = Q_n^+ + Q_n^- = Q_n^+\left(1 + Q_n^-/Q_n^+\right)$$

$$Q_n^+ = \frac{Q_n}{\left(1 + Q_n^-/Q_n^+\right)}$$

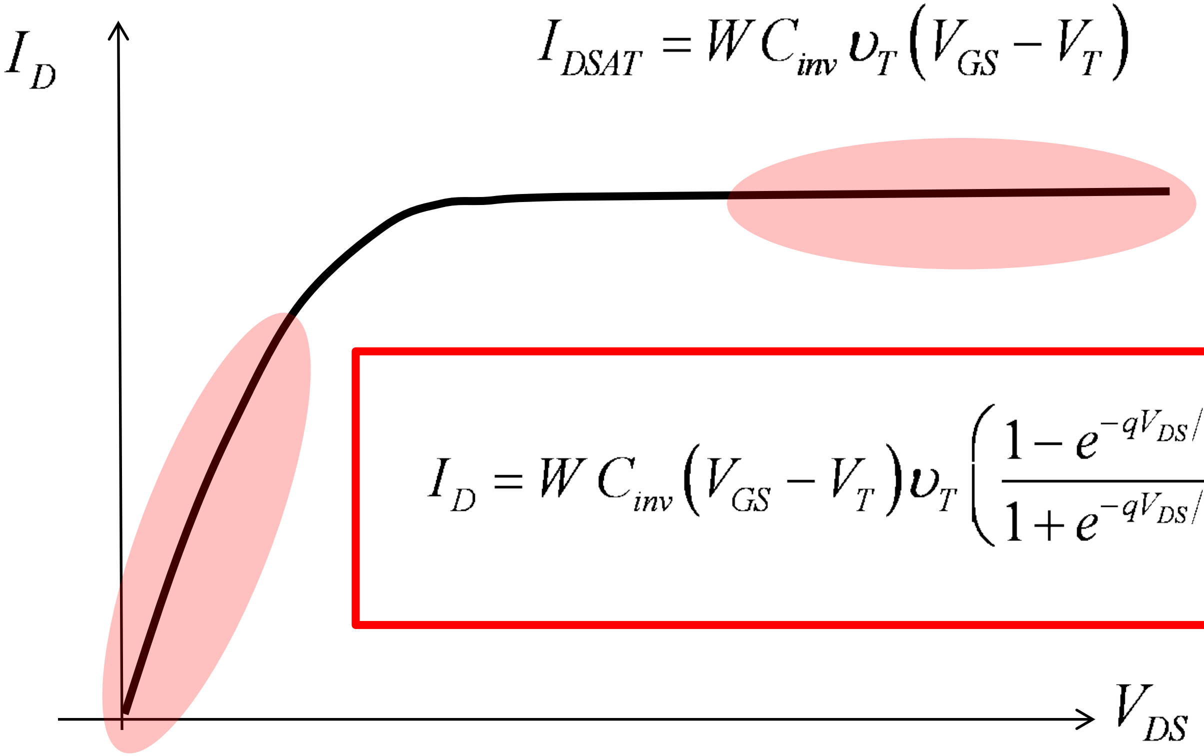
$$I_D = -W\,\upsilon_T Q_n \frac{\left(1 - Q_n^-/Q_n^+\right)}{\left(1 + Q_n^-/Q_n^+\right)}$$

$$\frac{Q_n^-}{Q_n^+} = e^{-qV_{DS}/k_BT}$$

$$\boxed{I_D = W\,C_{inv}\left(V_{GS} - V_T\right)\upsilon_T\left(\frac{1 - e^{-qV_{DS}/k_BT}}{1 + e^{-qV_{DS}/k_BT}}\right)}$$

Full range ballistic model (nondegenerate)

$$I_{DSAT} = W\,C_{inv}\,\upsilon_T\left(V_{GS} - V_T\right) \qquad V_{DS} >> k_B T/q$$



$$\boxed{I_D = W\,C_{inv}\left(V_{GS} - V_T\right)\upsilon_T\left(\frac{1 - e^{-qV_{DS}/k_B T}}{1 + e^{-qV_{DS}/k_B T}}\right)}$$

$$I_{DLIN} = \frac{W}{L}\mu_B C_{inv}\left(V_{GS} - V_T\right)V_{DS} \qquad \mu_B = \frac{\upsilon_T L}{2\,k_B T/q} \qquad V_{DS} << k_B T/q$$

From subthreshold to above threshold

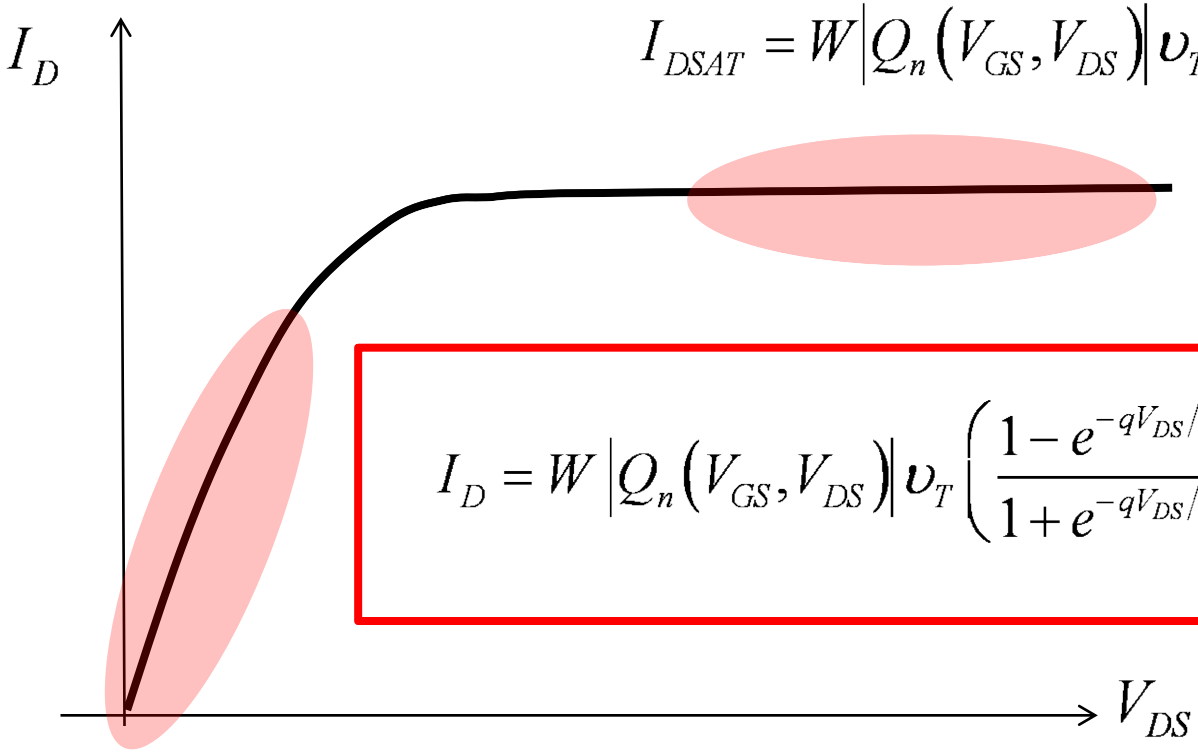
$$V_{GS} >> V_T$$

$$C_{inv}\left(V_{GS} - V_T\right) = \left|Q_n\left(V_{GS}, V_{DS}\right)\right|$$

$$V_T = V_{T0} - \delta V_{DS}$$

From subthreshold to above threshold

$$I_{DSAT} = W\left|Q_n\left(V_{GS}, V_{DS}\right)\right|\upsilon_T$$



$$I_D = W\left|Q_n\left(V_{GS}, V_{DS}\right)\right|\upsilon_T\left(\frac{1-e^{-qV_{DS}/k_BT}}{1+e^{-qV_{DS}/k_BT}}\right)$$

$$I_{DLIN} = \frac{W}{L}\mu_B\left|Q_n\left(V_{GS}, V_{DS}\right)\right|V_{DS} \qquad \mu_B = \frac{\upsilon_T L}{2k_BT/q}$$

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Summary

The traditional, linear region expression for I_D can be extended to the ballistic regime by replacing the mobility with the **ballistic mobility**.

The traditional, saturation region expression for I_D can be extended to the ballistic regime by replacing the high-field saturation velocity with the uni-directional thermal velocity.

Next lecture

$$I_D = W\left|Q_n\left(V_{GS}, V_{DS}\right)\right|\left\langle \upsilon_x\left(V_{GS}, V_{DS}\right)\right\rangle$$

In the next lecture, we will examine the average velocity at the top of the barrier (the VS) vs. gate and drain bias.