Essentials of MOSFETs

Unit 4: Transmission Theory of the MOSFET

Lecture 4.4: Velocity at the Virtual Source

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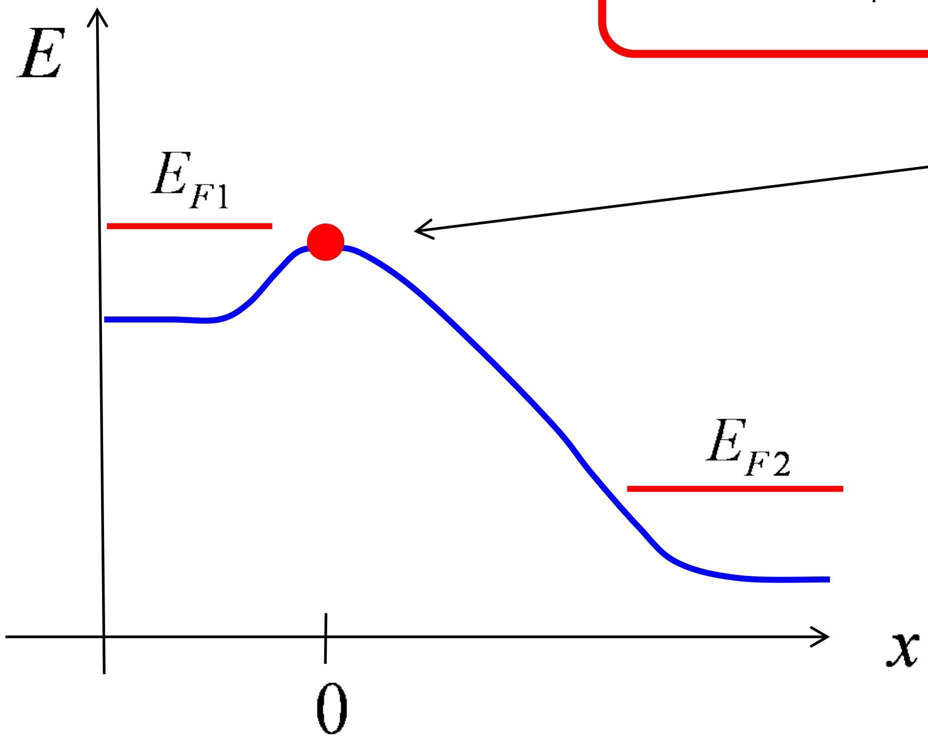
Focus on the top of the barrier (the VS)

$$\boxed{I_D = W|Q_n|\langle \upsilon_x \rangle}$$



1) We know the charge at the VS.

2) What is the velocity at the VS?

3) How does it depend on V_{DS} and V_{GS}?

Approach

Definition of current:

$$I_D = W\left|Q_n\left(x=0, V_{GS}, V_{DS}\right)\right|\left\langle \upsilon_x\left(x=0, V_{GS}, V_{DS}\right)\right\rangle$$

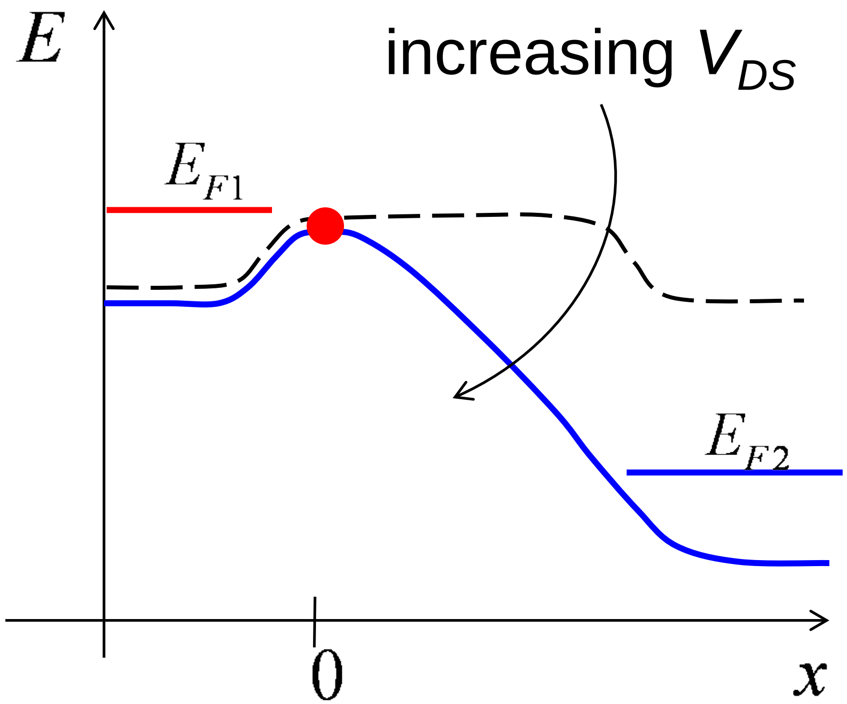
+

Ballistic IV:

$$I_D = W\left|Q_n\left(x=0, V_{GS}, V_{DS}\right)\right|\upsilon_T\left(\frac{1-e^{-qV_{DS}/k_BT}}{1+e^{-qV_{DS}/k_BT}}\right)$$

(assumes nondegenerate conditions)

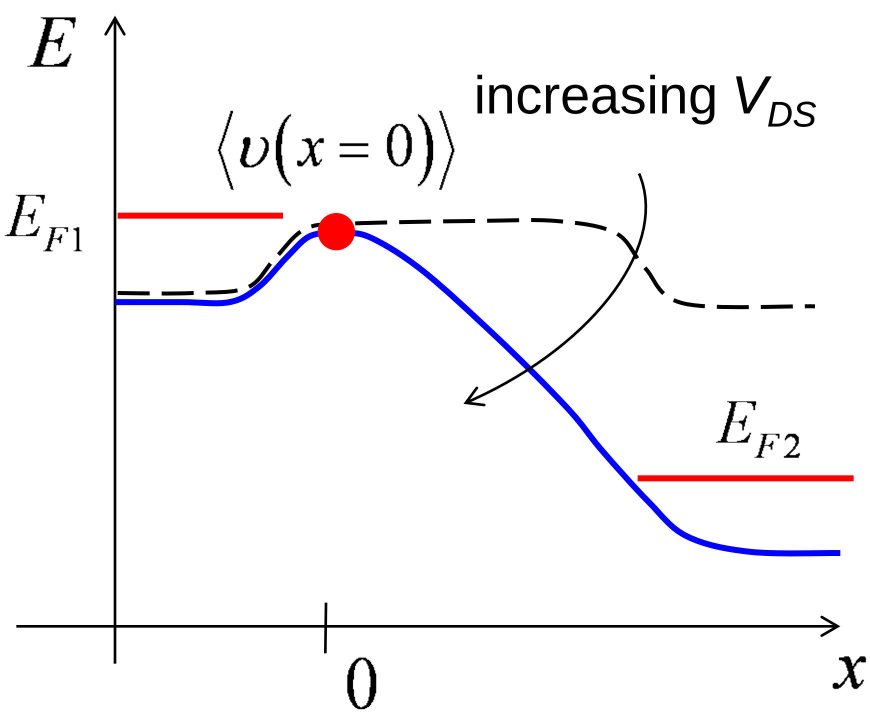
Average velocity at the VS

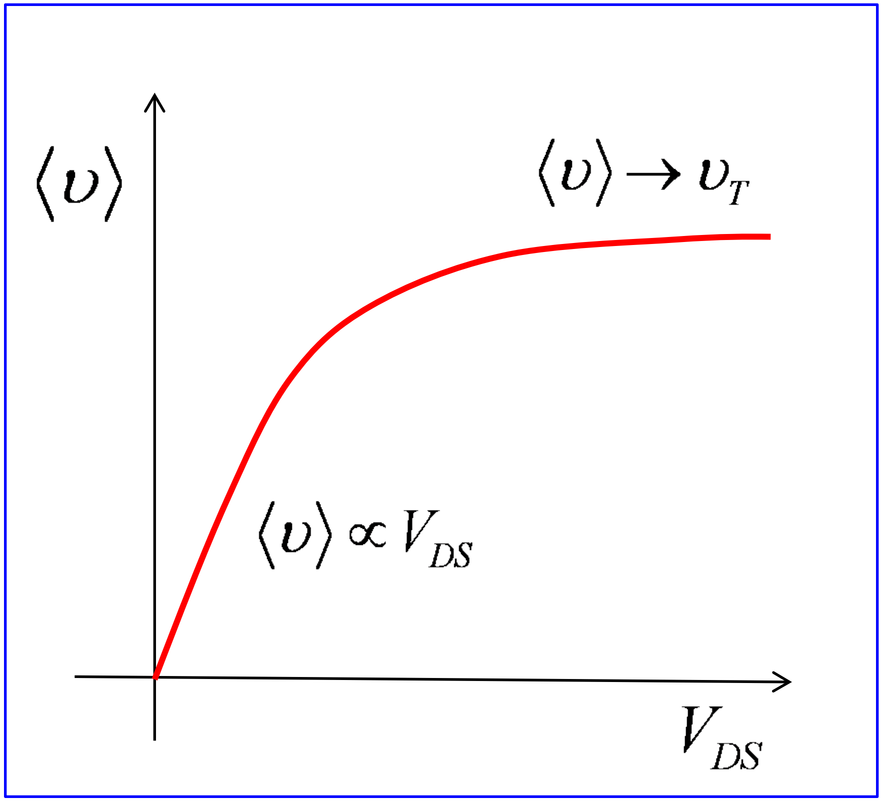


$$\left\langle \upsilon_x (x = 0) \right\rangle = \upsilon_T \frac{\left(1 - e^{-qV_{DS}/k_B T}\right)}{\left(1 + e^{-qV_{DS}/k_B T}\right)}$$

(nondegenerate carrier statistics)

Velocity vs. V_{DS}





$$\langle \upsilon_x (x=0) \rangle = \upsilon_T \frac{\left(1 - e^{-qV_{DS}/k_BT}\right)}{\left(1 + e^{-qV_{DS}/k_BT}\right)}$$

Velocity for small V_{DS}

$$\langle \upsilon(x=0) \rangle = \upsilon_T \frac{\left(1 - e^{-qV_{DS}/k_BT}\right)}{\left(1 + e^{-qV_{DS}/k_BT}\right)}$$

$$V_{DS} << k_BT/q \qquad e^x \approx 1 + x$$

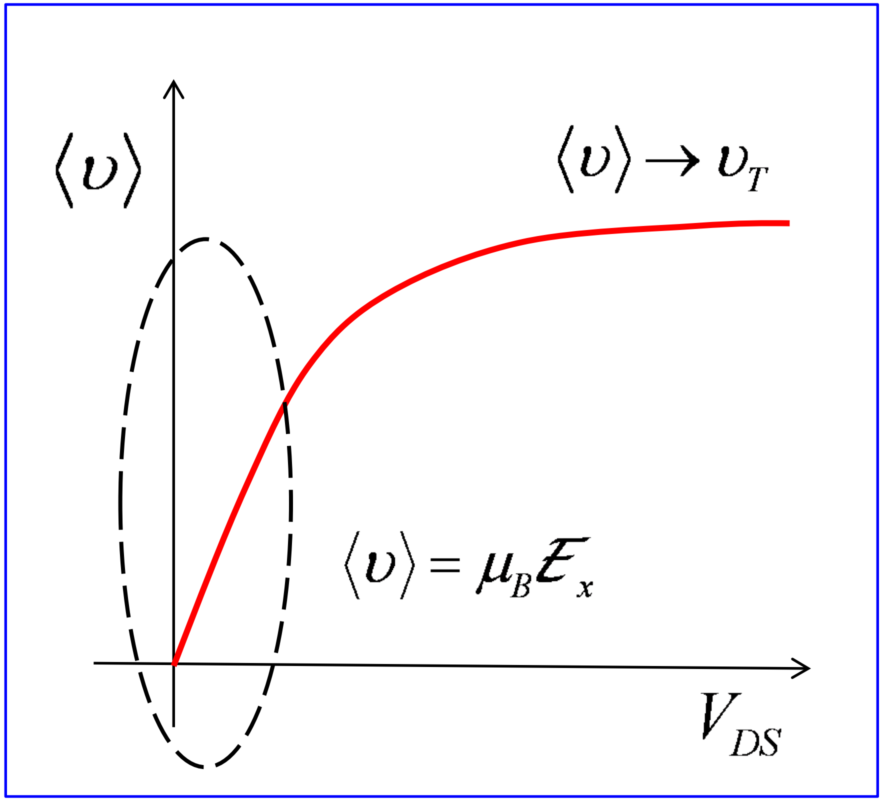
$$\langle \upsilon(x=0) \rangle = \frac{\upsilon_T}{2(k_BT/q)} V_{DS}$$

$$\langle \upsilon(x=0) \rangle = \left(\frac{\upsilon_T L}{2(k_BT/q)}\right)\left(\frac{V_{DS}}{L}\right)$$

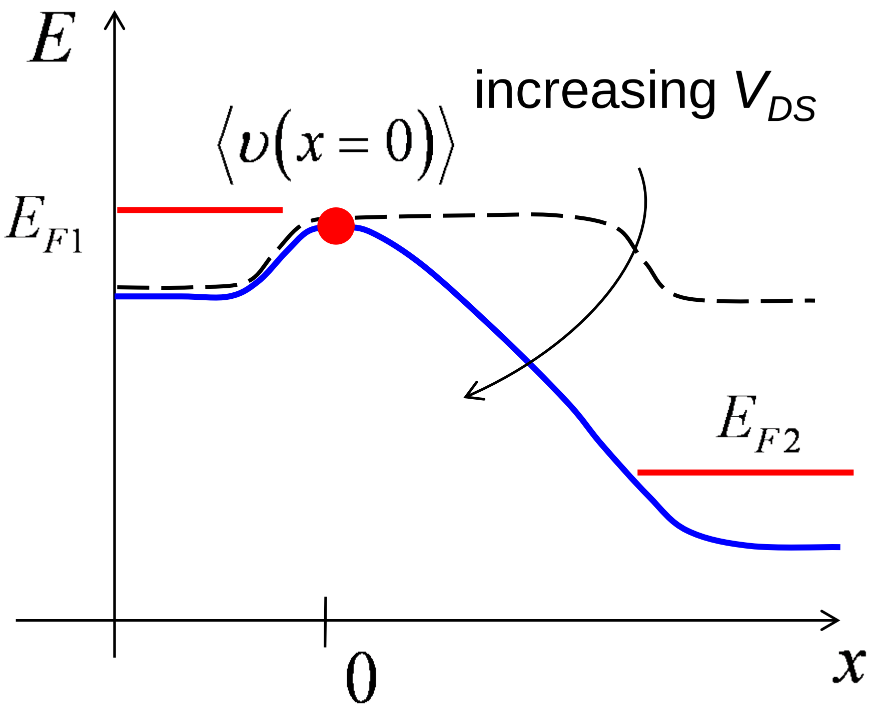
$$\mu_B \equiv \frac{\upsilon_T L}{2(k_BT/q)} \quad \text{cm}^2/\text{V-s}$$

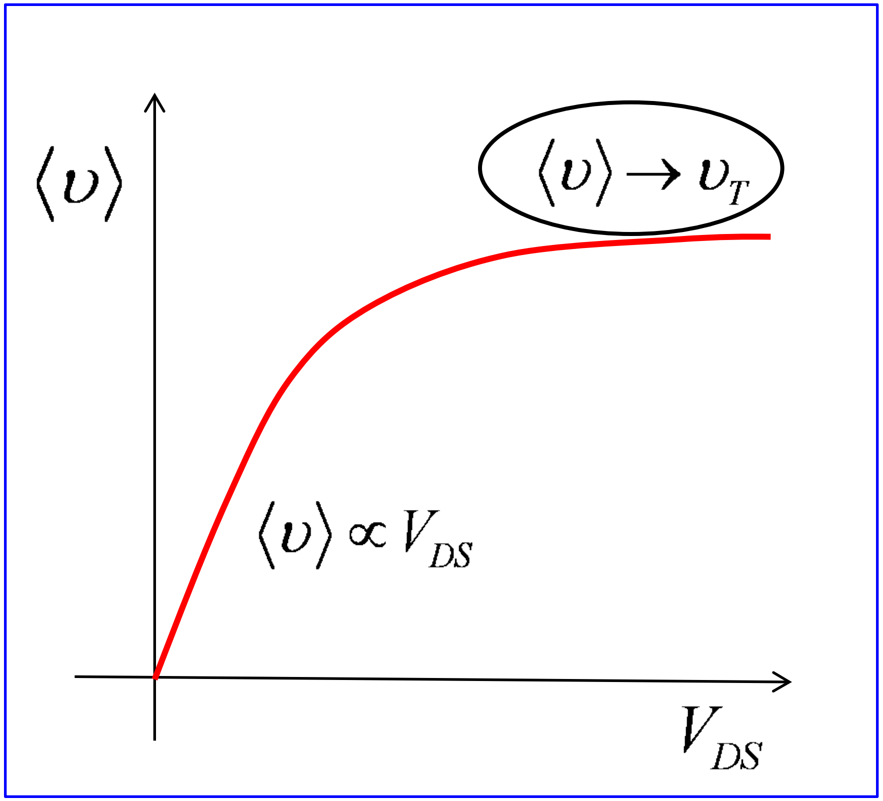
$$\boxed{\langle \upsilon(x=0) \rangle = \mu_B \mathcal{E}_x}$$

“ballistic mobility”



Velocity for large V_{DS}



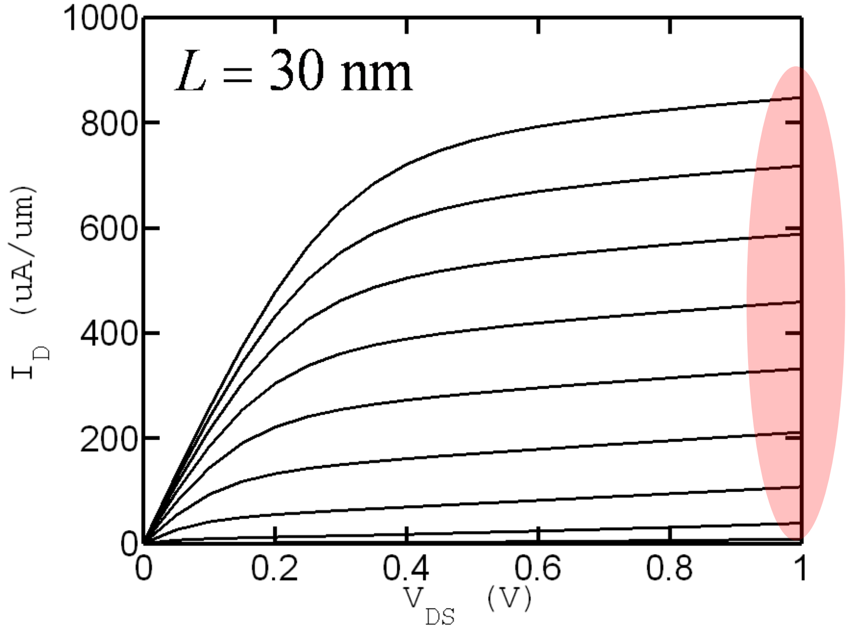


$$\langle \upsilon_x(x=0) \rangle = \upsilon_T \frac{\left(1 - e^{-qV_{DS}/k_BT}\right)}{\left(1 + e^{-qV_{DS}/k_BT}\right)}$$

$$V_{DS} >> k_BT/q$$

The velocity at the VS ***saturates*** in a ballistic MOSFET.

The "signature" of velocity saturation in MOSFETs



$$I_D \propto \left(V_{GS} - V_T\right)^1$$

ETSOI MOSFET data provided by A. Majumdar, IBM Research, 2015.

Physics of velocity saturation

In a long channel MOSFET with a high electric field, the carrier velocity saturates at high drain bias because of **strong scattering**.

It saturates in the high-field region **near the drain**.

In a ballistic MOSFET there is no scattering, but the velocity saturates at high drain bias.

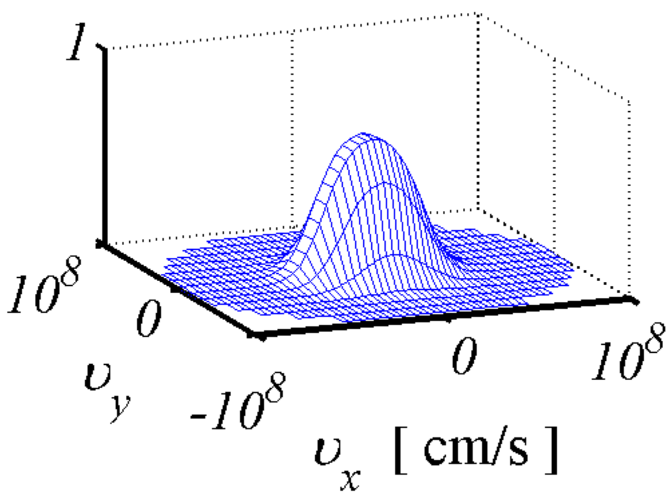
It saturates at the VS, where the E-field is zero.

What is the physics of velocity saturation in a ballistic MOSFET?

Equilibrium Maxwellian velocity distribution

$$f_0(E) = \frac{1}{1 + e^{(E-E_F)/k_BT}} \qquad f_0(E) \approx e^{-(E-E_F)/k_BT} \qquad E = E_C + \frac{m^* \upsilon^2}{2} \qquad \upsilon^2 = \upsilon_x^2 + \upsilon_y^2$$

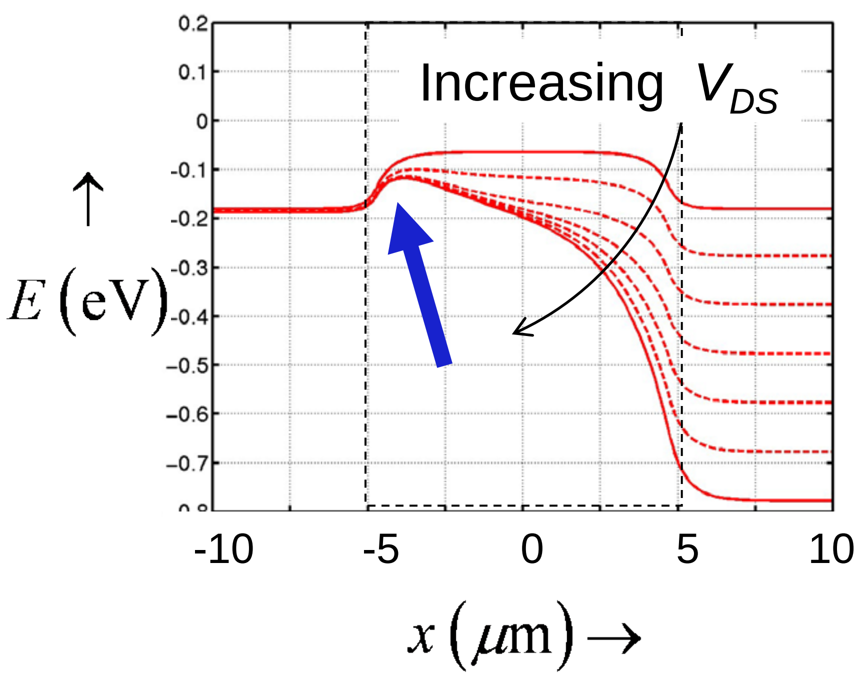
$$f_0(\upsilon_x, \upsilon_y) \propto e^{-m^*\left(\upsilon_x^2+\upsilon_y^2\right)/2k_BT}$$



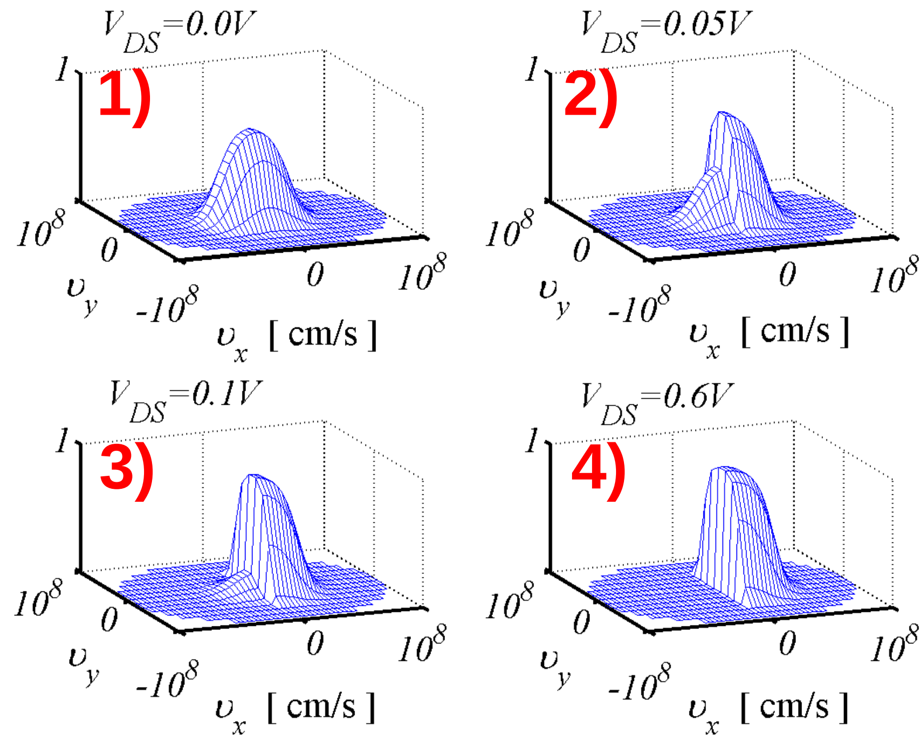
In the bulk, scattering maintains equilibrium.

Filling states at the top of the barrier

E_X *vs*. x for V_{GS} = 0.5V

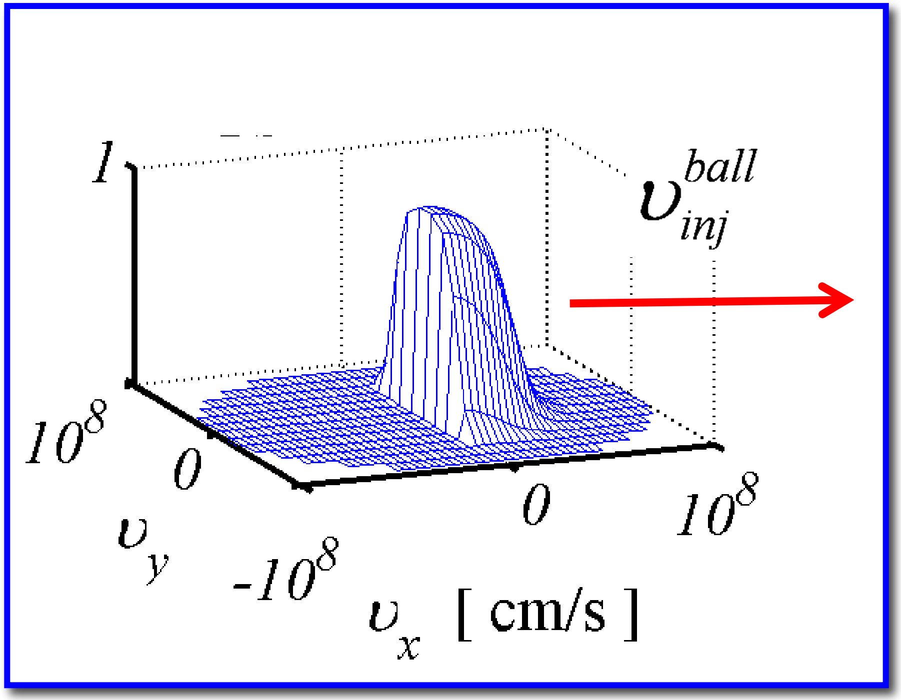


$f(\upsilon_x, \upsilon_y)$



(Numerical simulations of an L = 10 nm double gate Si MOSFET from J.-H. Rhew and M.S. Lundstrom, *Solid-State Electron.*, **46**, 1899, 2002)

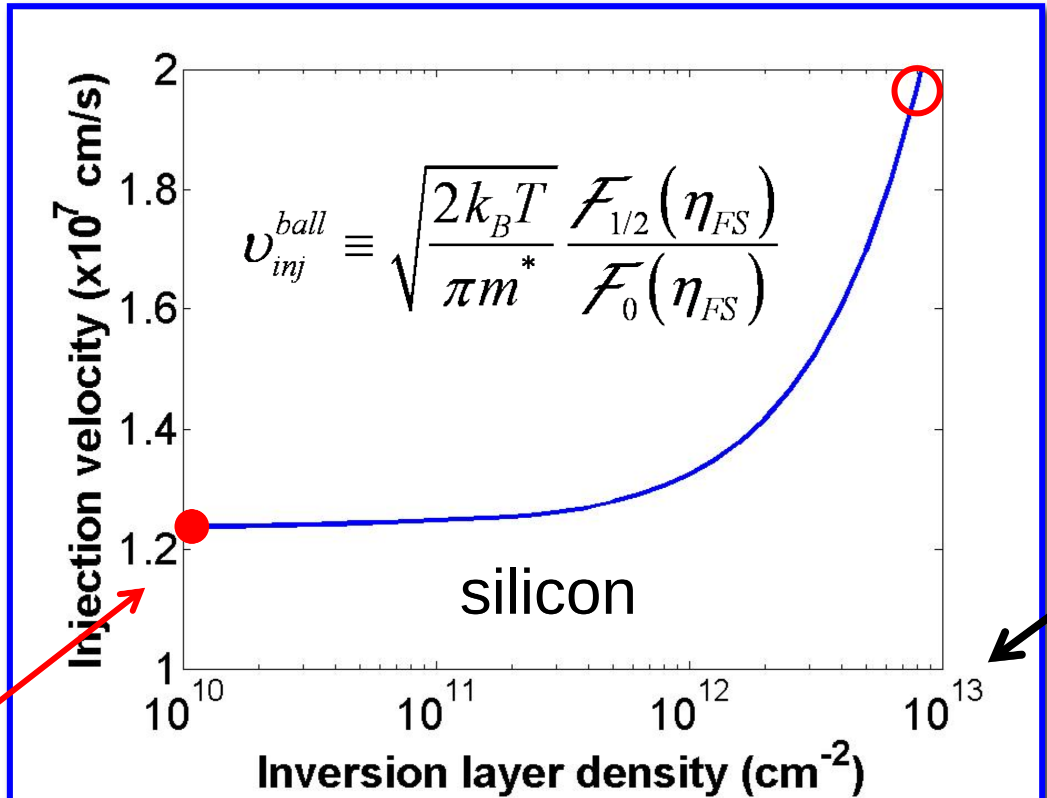
Ballistic injection velocity



$$\langle\langle \upsilon_x(0) \rangle\rangle = \upsilon_{inj}^{ball}(E_{F1}) = \upsilon_T \frac{\mathcal{F}_{1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})}$$

$$\upsilon_{inj}^{ball} = \upsilon_T = \sqrt{\frac{2k_B T}{\pi m^*}}$$ (nondegenerate)

Gate voltage dependent ballistic injection velocity



$$\upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}}$$

non-degenerate

$$m^* = 0.19 m_0$$

For more discussion

To see how Fermi-Dirac statistics are included in the analysis, see:

Lecture 14 in:

Mark Lundstrom, *Fundamentals of Nanotransistors*, World Scientific Publishing Co., Singapore, 2018.

Summary

In a ballistic MOSFET, the velocity saturates for high drain voltages.

But it saturates at the top of the barrier (the VS) where the electric field is zero.

The velocity saturates at the **ballistic injection velocity**, which is a key figure of merit for a transistor.

Next topic

We knew how to treat the diffusive case, where there is a lot of carrier scattering (with the traditional model).

We now know how to treat the ballistic case, where there is no scattering (with the ballistic model).

How do we treat MOSFETs between these two limits?

That is the subject of the next lecture.