

# Essentials of MOSFETs

## Unit 1: Transistors and Circuits

### **Lecture 1.5: Compact Models**

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# About “compact models”

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The term, **compact model**, is used in many fields of science and engineering to describe a simple (usually analytical) model - as opposed to a simulation, which is typically a numerical solution to a set of equations.

In electronics, “**compact model**” means a model that describes an electronic device in a form that is suitable for use in numerical circuit simulation programs.

# Two types of compact models

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In the course, we will distinguish between two different types of compact models:

## **1) Compact physical models**

These models aim to describe a device in terms of a few parameters with strong physical significance. These kinds of models are useful for device characterization, process monitoring, and for the conceptual understanding that guides device research.

**Our focus in this course is on this type of compact model.**

# Second type of compact models

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## 2) Compact device models for circuit simulation

These models accurately relate the currents that flow into a device's terminals to the voltages on the leads **in a form suitable for use in numerical circuit simulation programs**. To describe everything relevant to a circuit, these models are more complex, but the core of the model is usually a compact physics model.

These kinds of compact models play a critical role in connecting semiconductor R&D and manufacturing to product design.

# Focus of course and this lecture

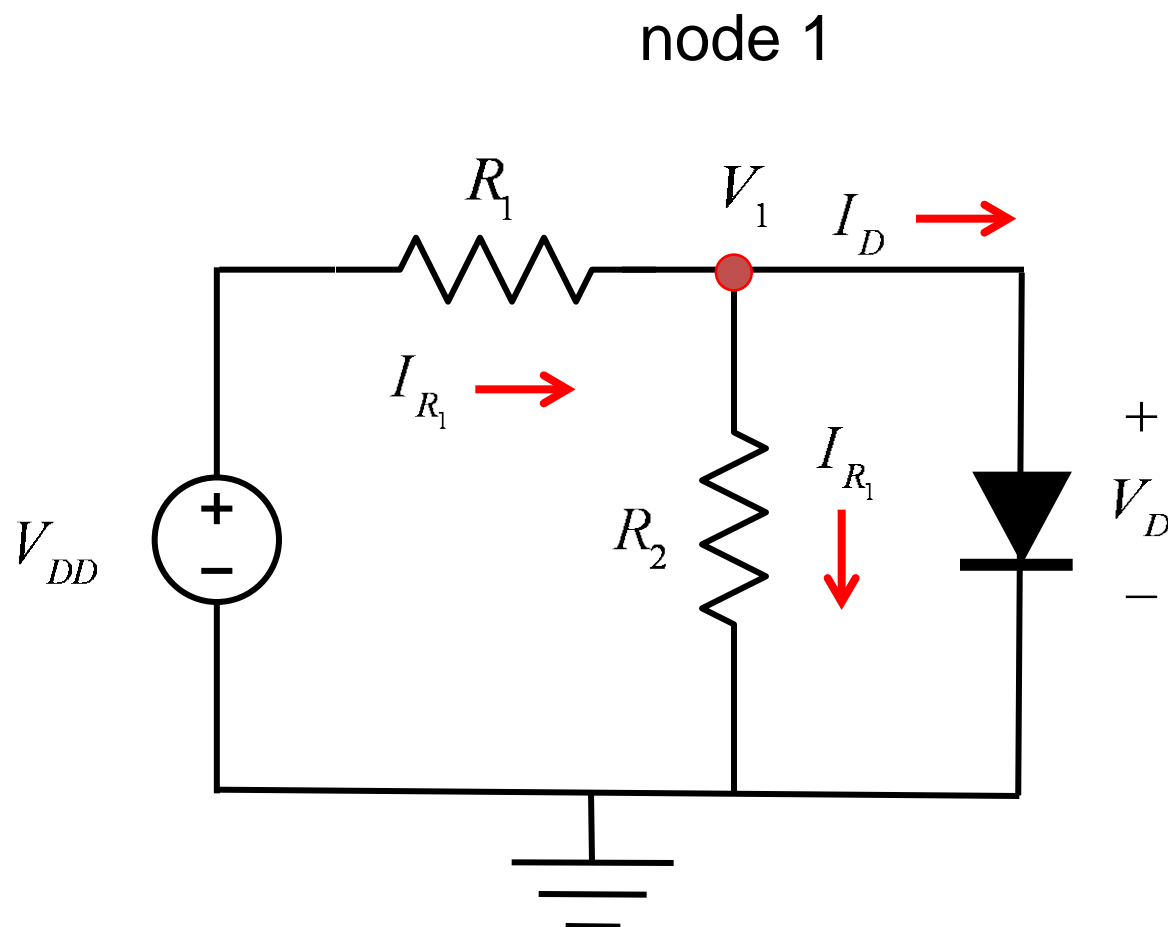
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Our focus in this course is on Type 1 compact models designed to provide insight into the physics of MOSFETs.

But everyone involved in electronics should also know a bit about the Type 2 compact models for circuit simulation.

This lecture will introduce a few key considerations for Type 2 models. The course will then focus on Type 1 models for understanding, and we'll return at the end of the course to say a little more about Type 2 compact models.

# Compact models for numerical circuit analysis



KCL:

$$I_{R_1} - I_{R_2} - I_D = 0$$

$$I_{R_1} = (V_{DD} - V_1) / R_1$$

$$I_{R_2} = V_1 / R_2$$

$$I_D(V_1) = ?$$

We need a  
compact model!

# Numerical circuit analysis

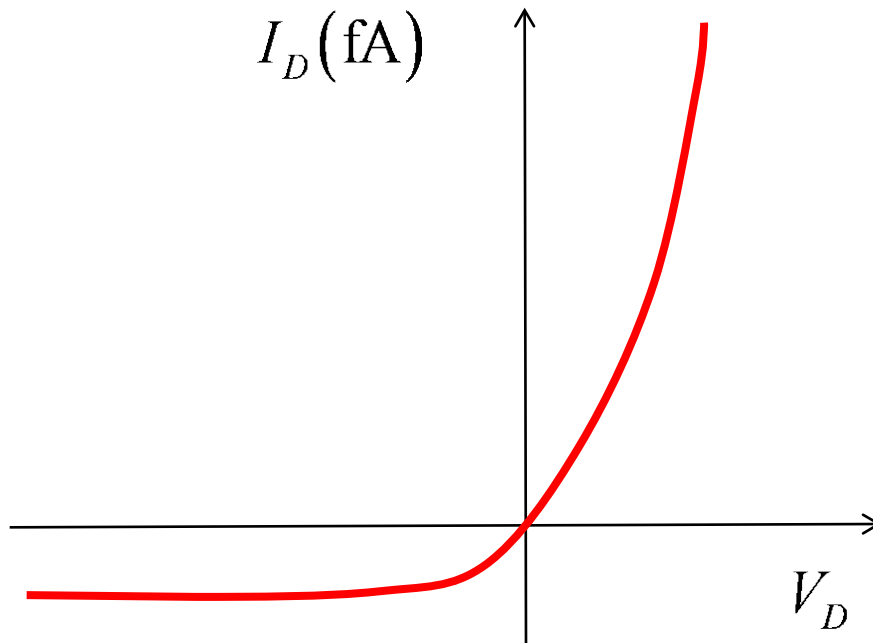
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Kirchoff's Current Law (KCL) gives one equation in one unknown  $V_1$ , the voltage at node 1:

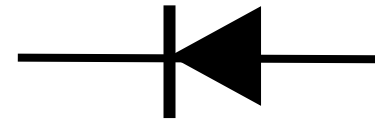
$$f(V_1) = (V_{DD} - V_1)/R_1 - V_1/R_2 - I_D(V_1) = 0$$

The diode current is a non-linear function of the voltage across its terminals; a numerical solution will be needed, but first, we need a compact model for  $I_D(V_1)$ .

# Simple, compact model for a diode



$$I_D = I_S \left( e^{qV_D/k_B T} - 1 \right)$$



-  $V_D$  +

“Shockley diode equation”

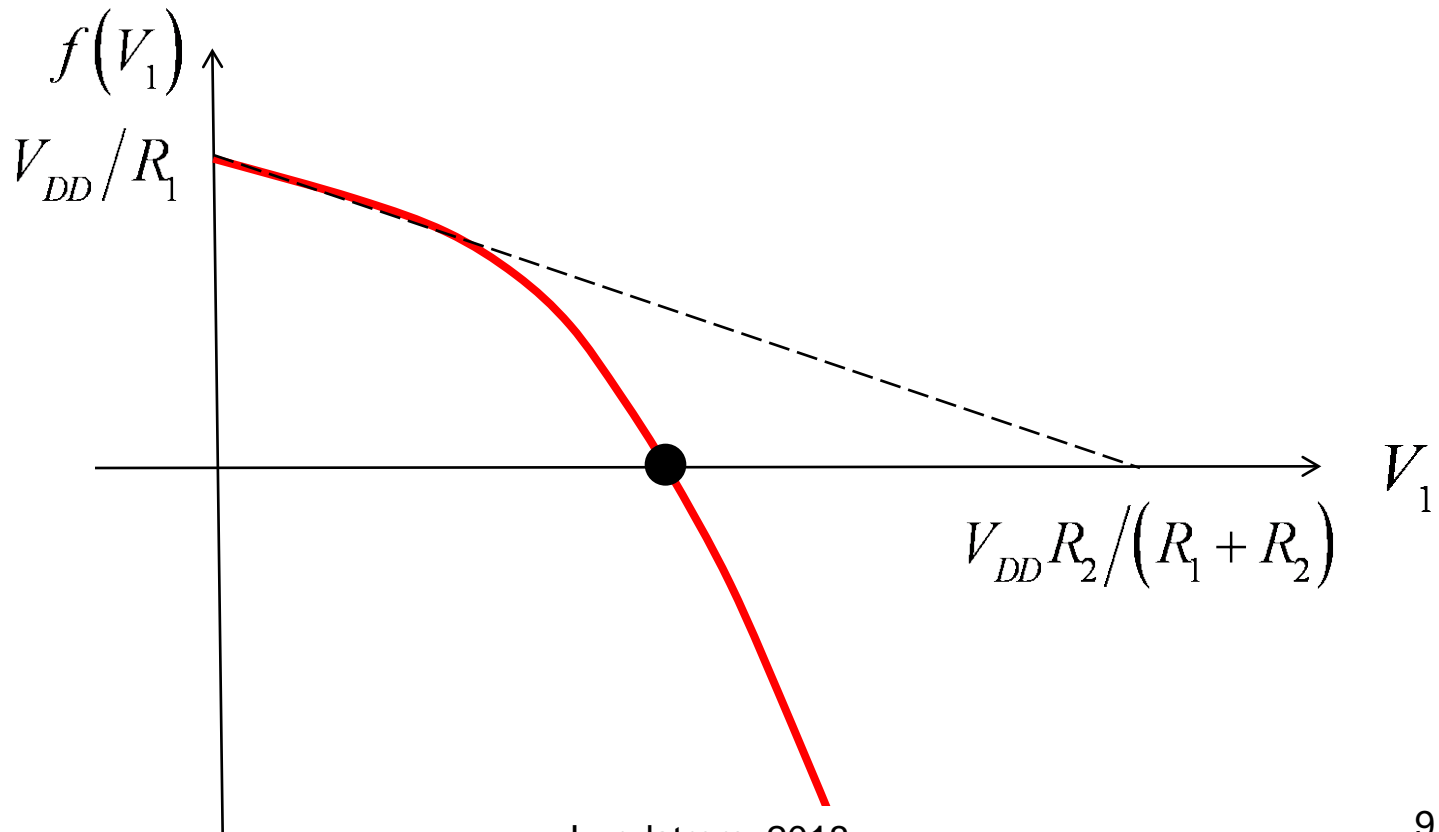
$$I_S = Aq \left( \frac{n_i^2}{N_D} \sqrt{\frac{D_p}{\tau_p}} + \frac{n_i^2}{N_A} \sqrt{\frac{D_n}{\tau_n}} \right)$$

(Additional physical effects would be included to describe real diodes.)



# Solving the circuit equation

$$f(V_1) = (V_{DD} - V_1)/R_1 - V_1/R_2 - I_s(e^{qV_1/k_B T} - 1) = 0$$



# Newton's method

$$f(V_1) = (V_{DD} - V_1)/R_1 - V_1/R_2 - I_S(e^{qV_1/k_B T} - 1) = 0$$

1) Guess the voltage at node 1 and then correct the guess:

$$V_1^1 \quad f(V_1^1) \neq 0 \quad f(V_1^1 + \delta V) = 0$$

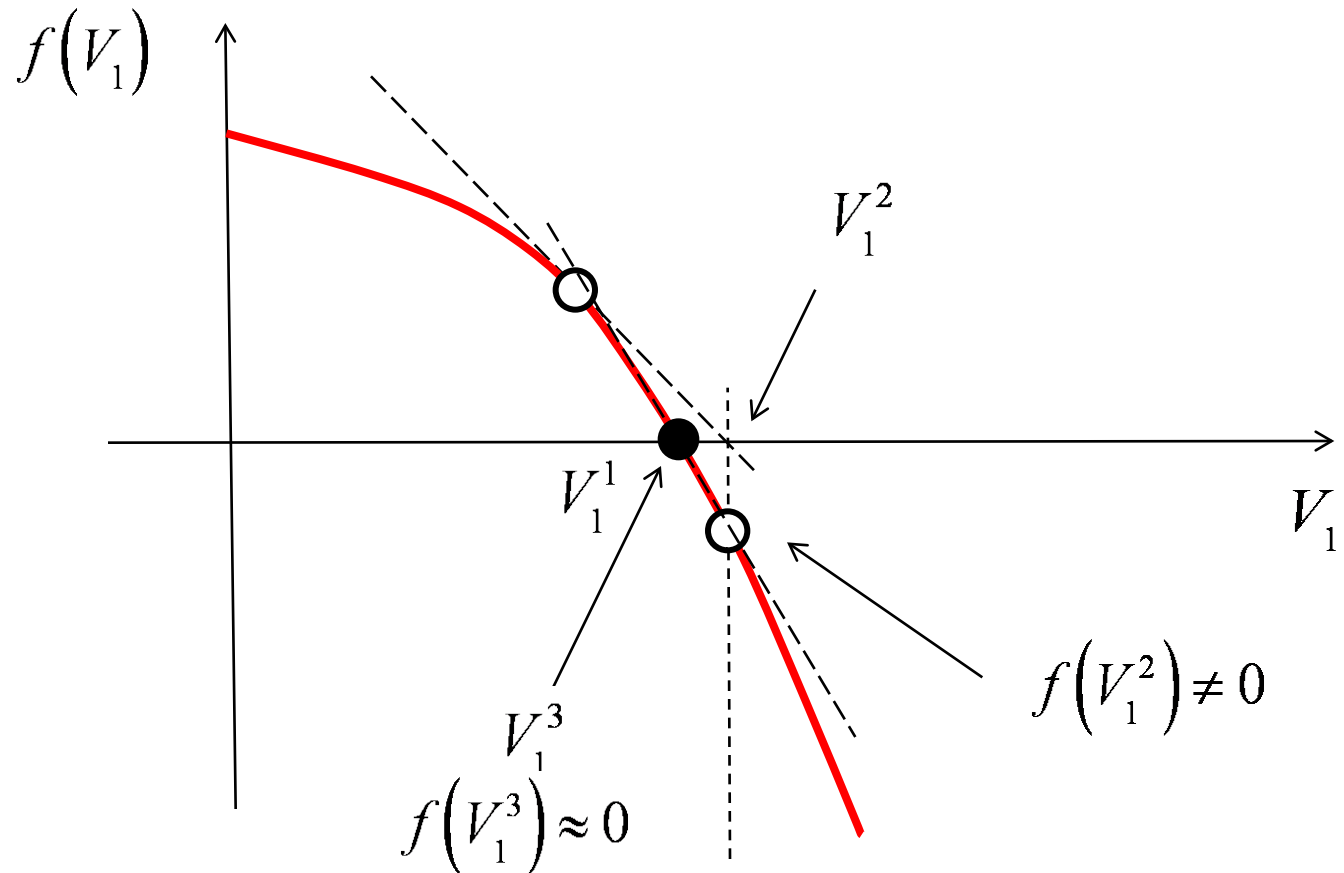
2) Expand  $f(V_1)$  in a Taylor series:

$$f(V_1^1 + \delta V) \approx f(V_1^1) + \left. \frac{df}{dV} \right|_{V_1^1} \delta V = 0 \quad \delta V = - \frac{f(V_1^1)}{\left. df/dV \right|_{V_1^1}}$$

3) New guess:

$$V_1^2 = V_1^1 + \delta V \quad f(V_1^2) \neq 0 \quad \rightarrow ? \quad \text{iterate}$$

# Graphical representation



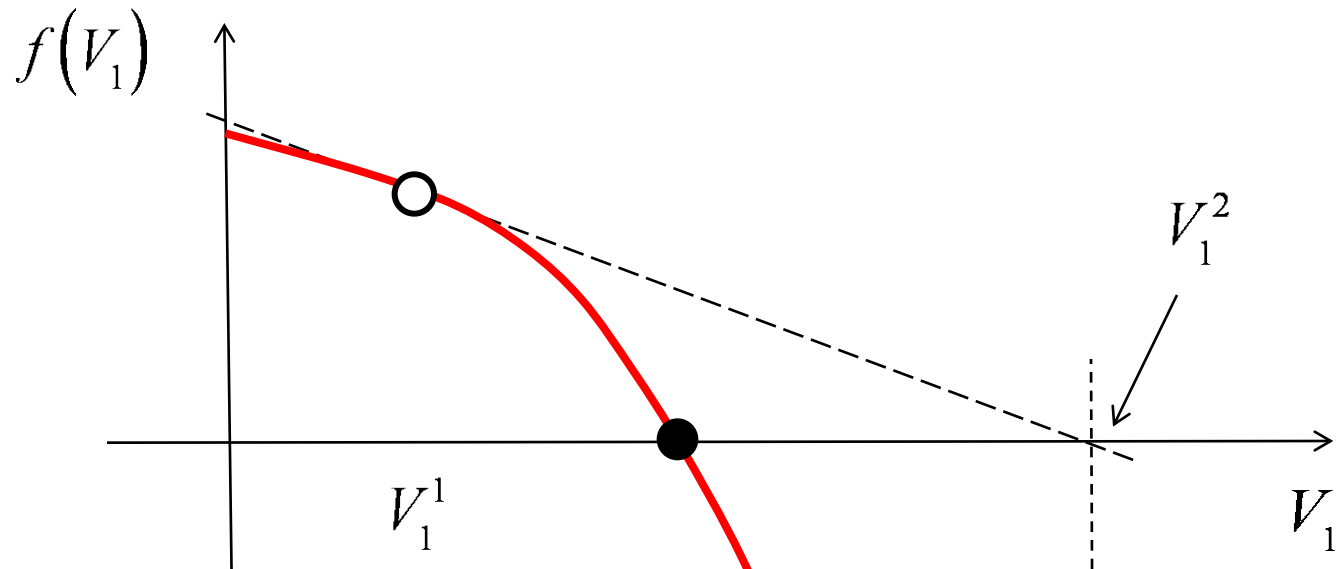
# Newton's method

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$$f(V_1) = (V_{DD} - V_1)/R_1 - V_1/R_2 - I_S \left( e^{qV_1/k_B T} - 1 \right) = 0$$

- 1) kth guess:  $V_1^k$
- 2) Correction:  $\delta V^k = - \frac{f(V_1^k)}{df/dV|_{V_1^k}}$
- 3) New guess:  $V_1^{k+1} = V_1^k + \delta V^k$
- 4) Check convergence:  $f(V_1^{k+1}) \approx 0?$
- 5) Iterate to a pre-determined tolerance

# What can go wrong?

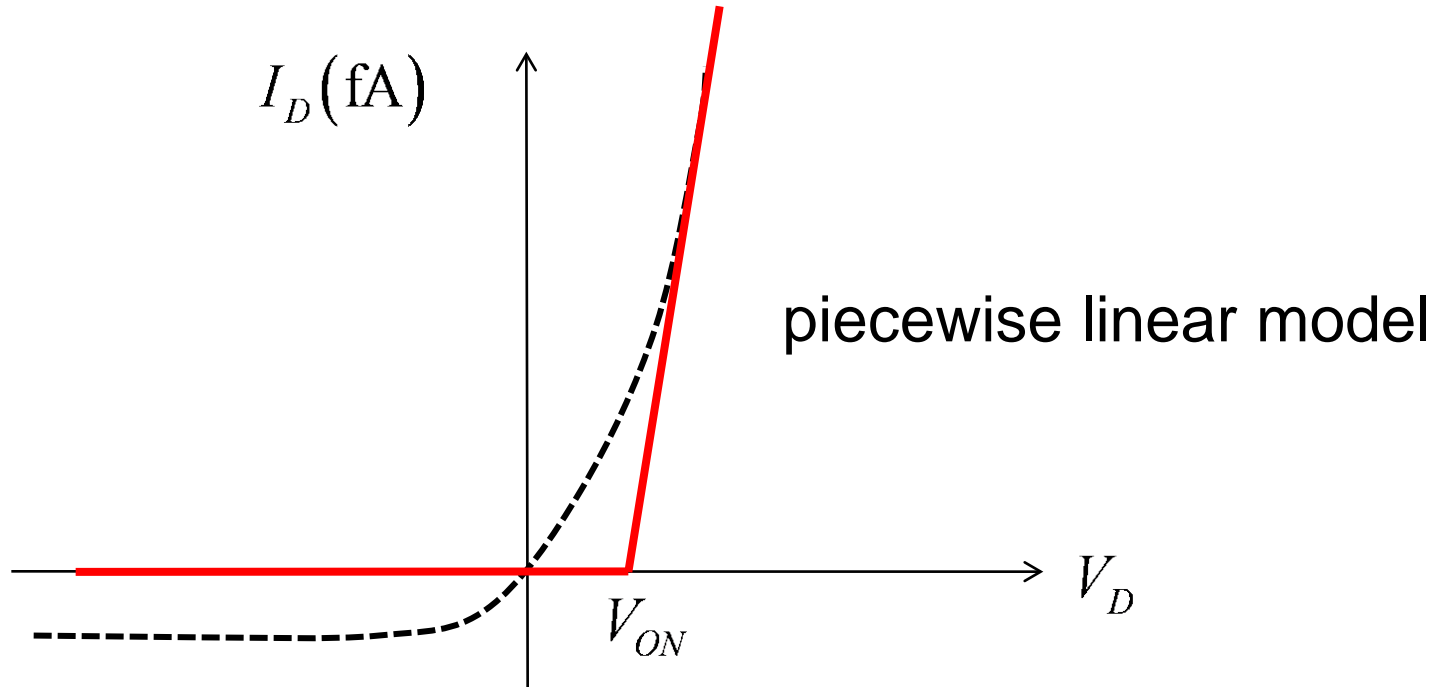


The compact model must describe the device outside the range of voltages that will occur in practice.

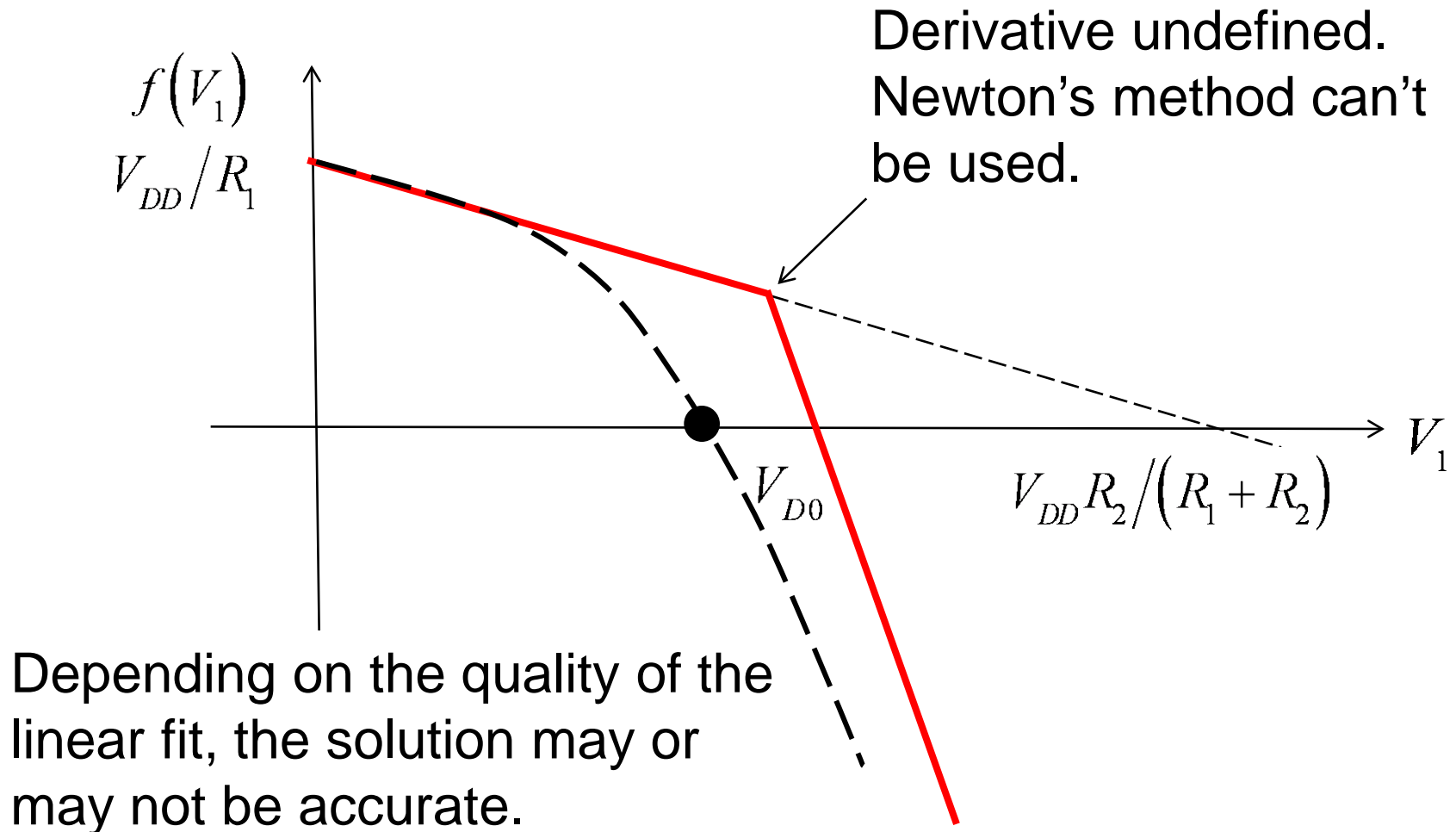
Numbers that are too large or too small for the computer to represent must be avoided.

$$V_1^2 \rightarrow \infty$$

# A simpler compact model



# A simpler compact model



# Some requirements of a compact circuit model

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Must accurately describe the electrical performance of a device for all intended applications.

- DC
- Small-signal AC
- Transient
- Noise analysis

Must do this over a wide range of biases and temperatures.

Must describe transistors of any size.

Must be computationally efficient and robust for use in numerical circuit simulation.



# The need for a physics-based compact model

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Physics-based model relate the needs of designers to the manufacturing process.

Generally results in the fewest number of model parameters and simplifies model calibration.

These models are suitable for statistical and predictive modeling.

In addition to a strong physical core, a compact model must satisfy certain mathematical requirements.

# Mathematical requirements of a compact model

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For numerical convergence, the first derivative with respect to terminal voltages must be continuous. (For small signal simulation, the second and third order derivatives must be continuous. Model with derivatives of all orders are desired.)

The model must be well-behaved outside the range of expected operating voltages.

Model simplicity is necessary to minimize solution time.

## For more information

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On the mathematical aspects of compact models and circuit simulation:

J. Roychowdhury, *Numerical Simulation and Modeling of Electronic and Biochemical Systems*, now Publishers, Inc., Hanover, MA, 2009.

On industrial strength compact models for MOSFETs:

Y. Tsividis and C. McAndrew, *Operation and Modeling of the MOS transistor*, 3<sup>rd</sup> ed., Oxford Univ. Press, New York, 2011.

# Summary

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Type 1 compact models succinctly describe the essential physics of a device.

Type 2 compact models are used by designers – to design circuits and to communicate with manufacturing.

The core of a Type 2 model should be a physics-based (Type 1) compact model.

Type 2 models must satisfy the needs of designers as well as the mathematical constraints of the circuit simulator.