Essentials of MOSFETs

Unit 3: MOS Electrostatics

Lecture 3.9: The VS Model Revisited

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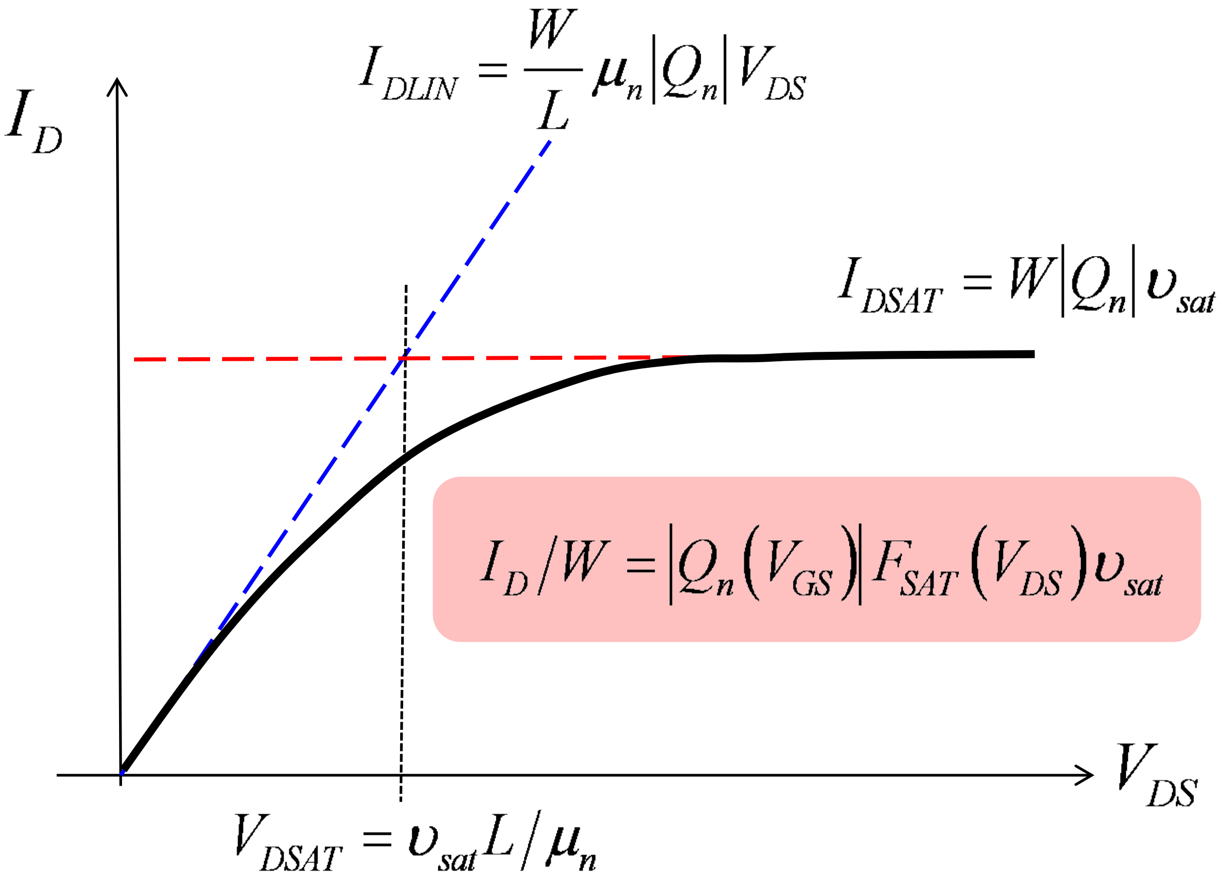
Level 0 VS Model



Level 0 VS model

1) $I_D/W = \left|Q_n\left(V_{GS}\right)\right|\left\langle \upsilon_x\left(V_{DS}\right)\right\rangle$

2) $Q_n\left(V_{GS}\right) = -C_{ox}\left(V_{GS} - V_T\right) \qquad \left(V_{GS} > V_T\right)$

$V_T = V_{T0} - \delta V_{DS}$

$Q_n\left(V_{GS}\right) = 0 \qquad \left(V_{GS} \leq V_T\right)$

3) $\left\langle \upsilon\left(V_{DS}\right)\right\rangle = F_{SAT}\left(V_{DS}\right)\upsilon_{sat}$

4) $F_{SAT}\left(V_{DS}\right) = \dfrac{V_{DS}/V_{DSAT}}{\left[1+\left(V_{DS}/V_{DSAT}\right)^{\beta}\right]^{1/\beta}}$

5) $V_{DSAT} = \upsilon_{sat}L/\mu_n$

Lundstrom: 2018

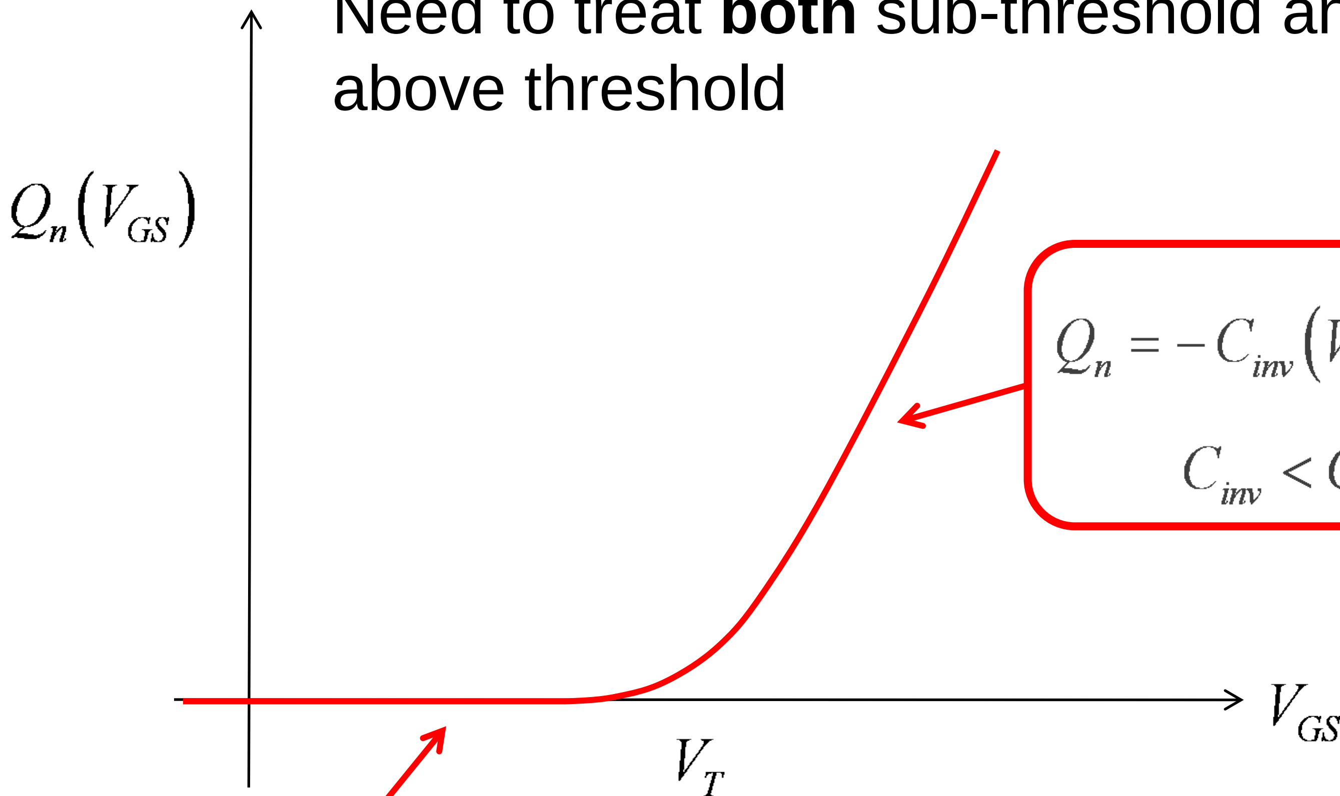
There are only 8 device-specific parameters in this model:

$$C_{ox}, V_{T0}, \delta, \upsilon_{sat}, \mu_n, L$$

$$R_{SD} = R_S + R_D, \beta$$

Improving the VS model: inversion charge

Need to treat **both** sub-threshold and above threshold

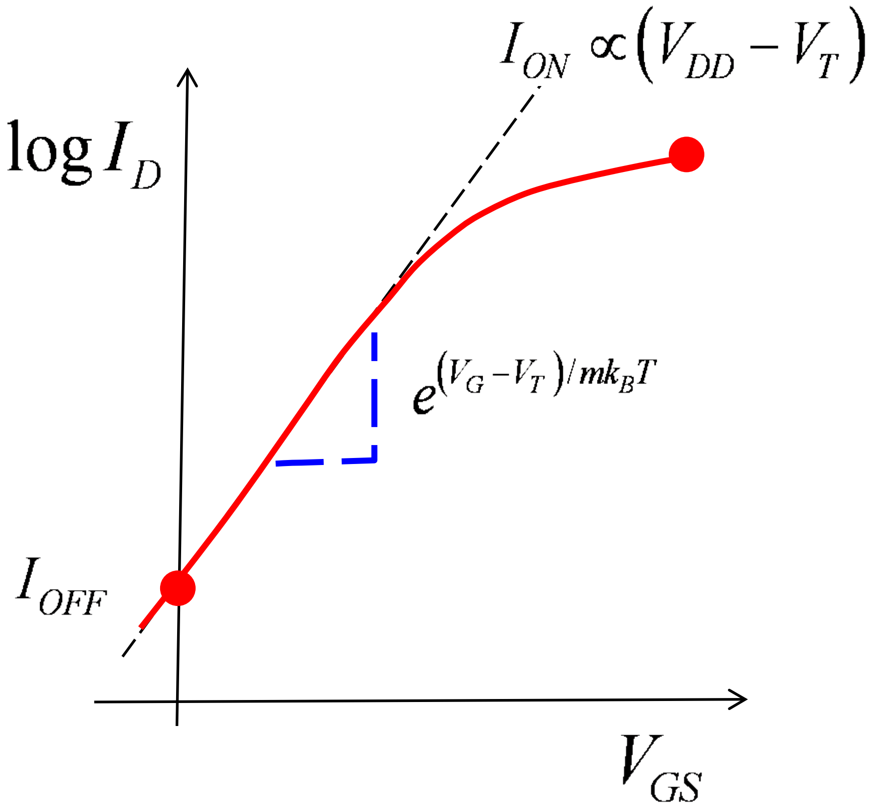


$$Q_n = -C_{inv}\left(V_{GS} - V_T\right)$$

$$C_{inv} < C_{ox}$$

$$Q_n\left(V_{GS}\right) = -(m-1)C_{ox}\left(\frac{k_B T}{q}\right)e^{q\left(V_{GS}-V_T\right)/mk_B T}$$

But first: the subthreshold current



1) subthreshold swing
2) off-current
3) on-current

$$Q_n\left(V_{GS}\right) \propto e^{q\left(V_{GS}-V_T\right)/mk_BT}$$

$$I_D \propto Q_n\left(V_{GS}\right)$$

$$I_D \propto e^{q\left(V_{GS}-V_T\right)/mk_BT}$$

Subthreshold swing

$$I_D \propto e^{q\psi_S/k_BT} \qquad \psi_S = V_{GS}/m \qquad m \geq 1$$

$$\ln I_D = \frac{\psi_S}{(k_BT/q)} + c \qquad \log_{10} I_D = \frac{\psi_S}{2.3(k_BT/q)} + \frac{c}{2.3}$$

$$\frac{\partial(\log_{10} I_D)}{\partial V_{GS}} = \frac{\partial(\log_{10} I_D)}{\partial \psi_S} \times \frac{\partial \psi_S}{\partial V_{GS}} = \frac{1}{2.3(k_BT/q)} \times \frac{1}{m} \qquad \frac{\text{Decades of } I_D}{\text{Volts of } V_{GS}}$$

$$\boxed{SS = \left(\frac{\partial(\log_{10} I_D)}{V_{GS}}\right)^{-1} = 2.3m(k_BT/q)\ \frac{\text{mV}}{\text{dec}}}$$

Subthreshold swing

$$S = 2.3m\left(k_B T/q\right)\ \frac{\text{mV}}{\text{dec}} \qquad m \geq 1$$

$$m \approx 1.1 - 1.4 \text{ typically}$$

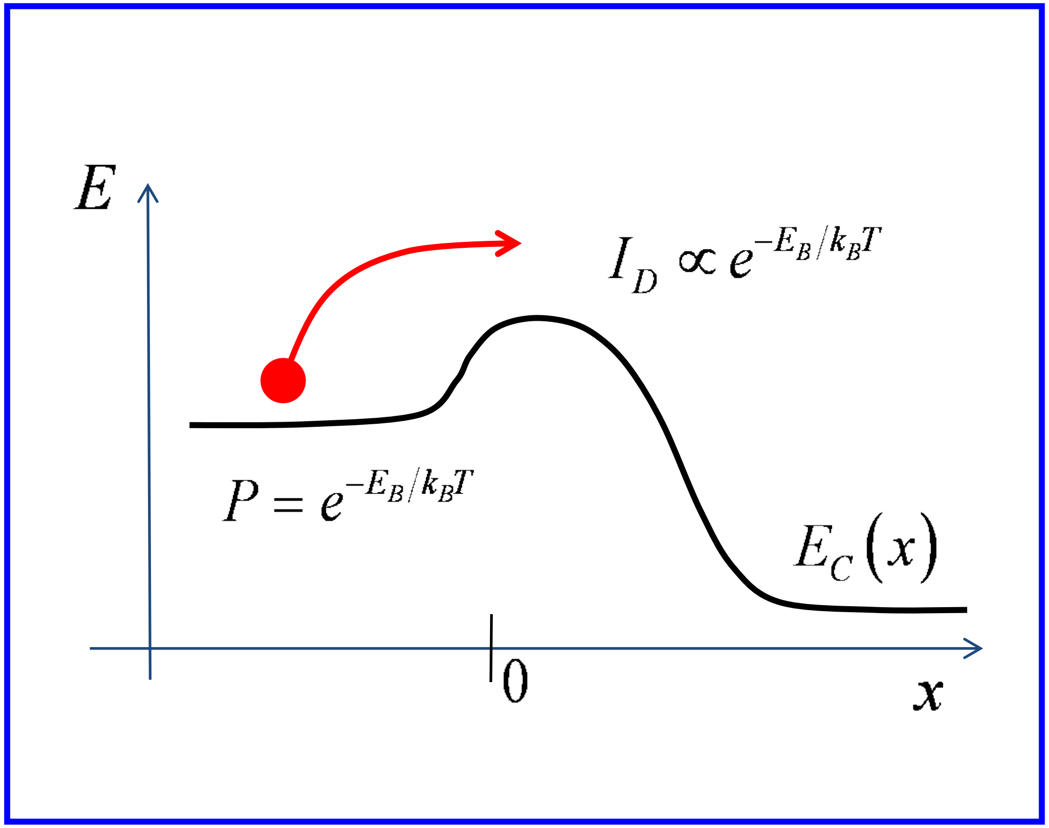
$$SS > 60\ \frac{\text{mV}}{\text{dec}}\ (T = 300K)$$

$$SS < 100\ \frac{\text{mV}}{\text{dec}}\ \text{(typically)}$$

Why is a small *SS* important?

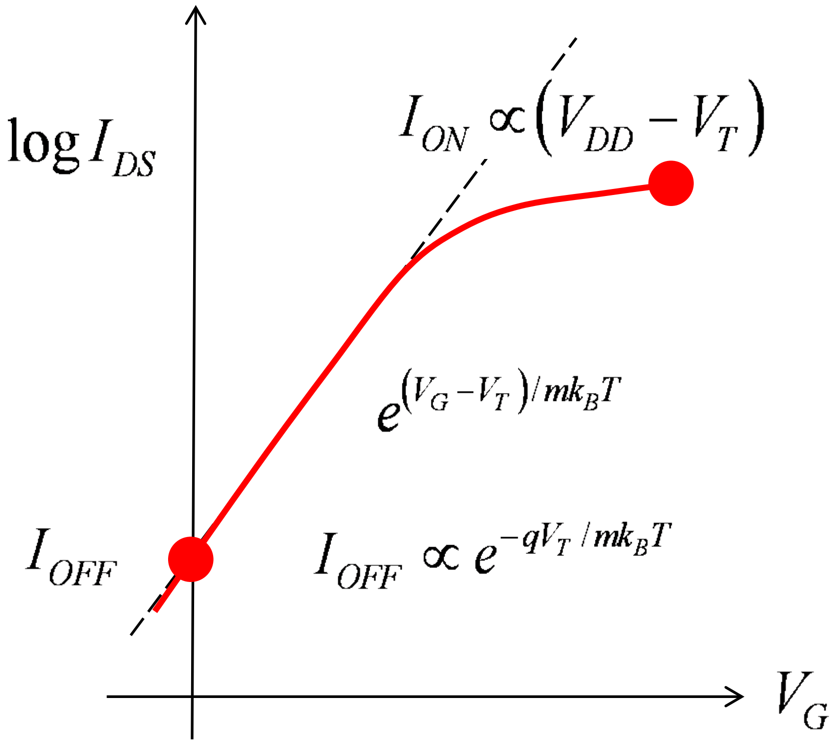
$$P_D \propto V_{DD}^2$$

Why is *S* > 60 mV/decade?

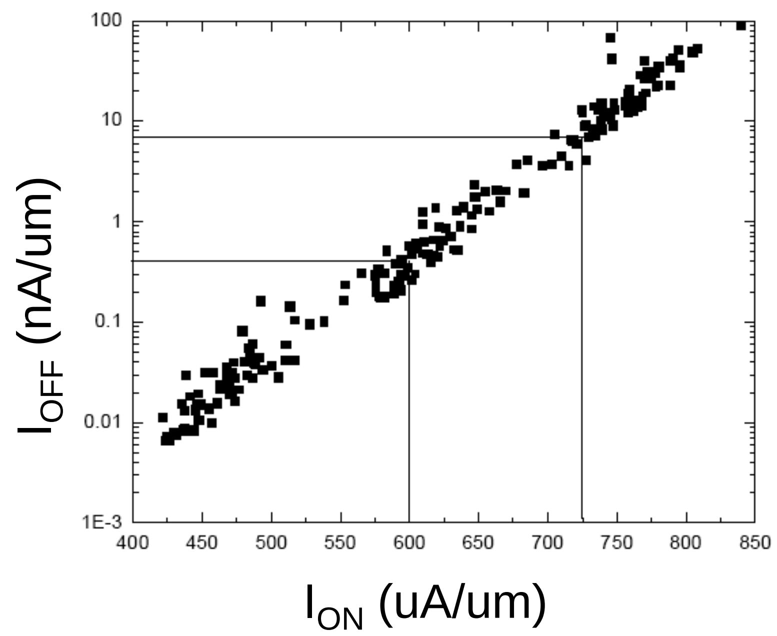


injection of thermal carriers over a barrier

Relation between I_{OFF} and I_{ON}



$$\boxed{\log_{10} I_{OFF} = c_1 + c_2 I_{ON}}$$



A. Steegen, et al., “65nm CMOS Technology for low power applications,” Intern. Electron Dev. Meeting, Dec. 2005.

Back to the VS model

Now let us return to the question at hand:

"How do we describe $Q_n(V_{GS})$ **continuously** below and above threshold?

Empirical treatment of inversion charge

$$Q_n\left(V_{GS}\right)=-C_{inv}m\left(k_BT/q\right)\ln\left(1+e^{q\left(V_{GS}-V_T\right)/mk_BT}\right)$$

$$Q_n\left(V_{GS}\right)=-\left(m-1\right)C_{ox}\left(k_BT/q\right)e^{q\left(V_{GS}-V_T\right)/mk_BT}$$

correct result

G. T. Wright, “Threshold modelling of MOSFETs for CAD of CMOS VLSI,” *Electron Lett.*, **21**, pp. 223–224, Mar. 1985.

Empirical treatment of inversion charge

$$Q_n\left(V_{GS}\right) = -C_{inv} m\left(k_B T/q\right)\ln\left(1+e^{q\left(V_{GS}-V_T\right)/mk_BT}\right)$$

$V_{GS} > V_T$:

$$\ln\left(1+x\right) \approx \ln\left(x\right)$$

$$Q_n\left(V_{GS}\right) \approx -C_{inv}\left(V_{GS}-V_T\right)$$

$$\boxed{Q_n\left(V_{GS}\right) = -C_{inv}\left(V_{GS}-V_T\right)}$$

correct

G. T. Wright, "Threshold modelling of MOSFETs for CAD of CMOS VLSI," *Electron Lett.*, **21**, pp. 223–224, Mar. 1985.

Empirical treatment of inversion charge

expression used in the MIT VS Model

$$Q_n(V_{GS},V_{DS}) = -C_{inv}m(k_BT/q)\ln\left(1+e^{q\left(V_{GS}-V_T+\alpha(k_BT_L/q)F_f\right)/mk_BT}\right)$$

$$V_T = V_{T0} - \delta V_{DS}$$

DIBL

different V_T's in strong and weak inversion

Ali Khakifirooz, Osama M. Nayfeh, and Dimitri Antoniadis, "A Simple Semi-empirical Short-Channel MOSFET Current–Voltage Model Continuous Across All Regions of Operation and Employing Only Physical Parameters," *IEEE Trans. Electron Devices*, **56**, pp. 1674-1680, 2009.

Level 1 VS model

1) $$I_D/W = \left|Q_n\left(V_{GS}, V_{DS}\right)\right| \left\langle \upsilon_x\left(V_{DS}\right)\right\rangle$$

2) $$Q_n\left(V_{GS}, V_{DS}\right) = -C_{inv} m\left(k_B T/q\right) \ln\left(1 + e^{q\left(V_{GS} - V_T + \alpha\left(k_B T_L/q\right) F_f\right)/m k_B T}\right)$$

$$V_T = V_{T0} - \delta V_{DS}$$

3) $$\left\langle \upsilon_x\left(V_{DS}\right)\right\rangle = F_{SAT}\left(V_{DS}\right) \upsilon_{sat}$$

4) $$F_{SAT}\left(V_{DS}\right) = \frac{V_{DS}/V_{DSAT}}{\left[1 + \left(V_{DS}/V_{DSAT}\right)^{\beta}\right]^{1/\beta}}$$

5) $$V_{DSAT} = \frac{\upsilon_{sat} L}{\mu_n}$$

Only 10 device-specific parameters in this model:

$$C_{inv}, V_{T0}, \delta, m, \upsilon_{sat}, \mu_n,$$
$$L, R_{SD} = R_S + R_D,$$
$$\alpha, \beta$$

Discussion

With this extension (subthreshold to above threshold conduction), the VS model accurately describes modern transistors providing:

1) The high-field saturation velocity is viewed as an empirical, fitting parameter.

2) The mobility of carriers in the inversion is viewed as an empirical, fitting parameter.

3) **But** we will see later, that these empirical parameters can be given a clear, physical interpretation**.**

Download the MVS model at: https://nanohub.org/publications/15

Summary

1) Using a semi-empirical expression for Q_n, we have extended the VS model to treat subthreshold to above threshold.

2) Excellent fits to measured transistor IV characteristics generally result.

3) But the physical understanding of the mobility and saturation velocity *at the nanoscale* needs to be clarified.