Essentials of MOSFETs

Unit 4:
Transmission Theory of the MOSFET

Lecture 4.8:
Unit 4 Recap

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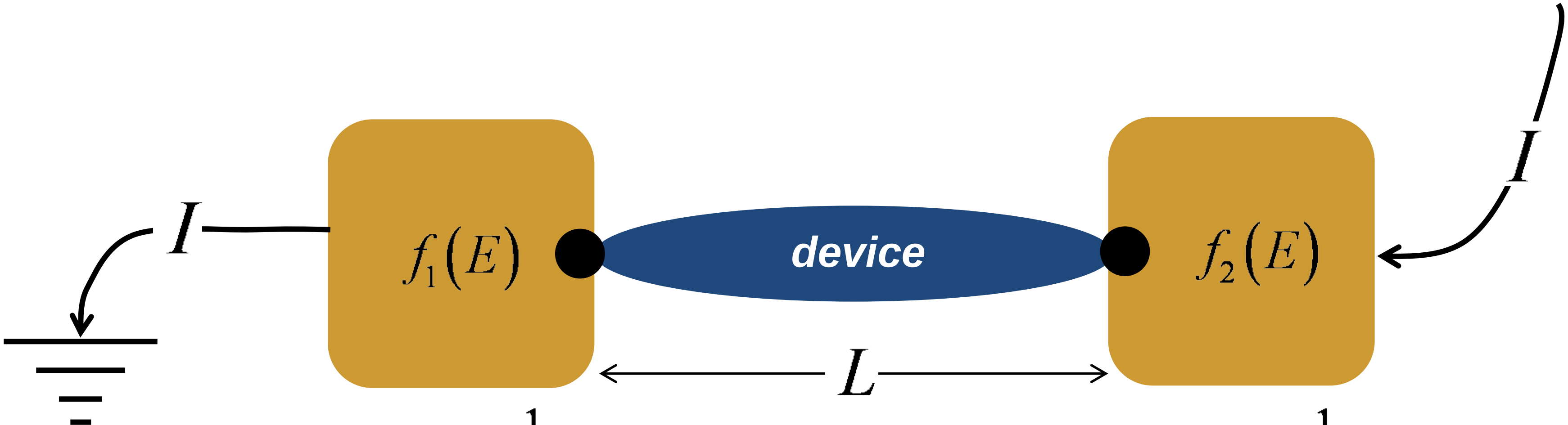
Unit 4 topics

4.1 Landauer Approach
4.2 Landauer at low and High Bias
4.3 The Ballistic MOSFET
4.4 Velocity at the Virtual Source
4.5 Transmission Theory of the MOSFET
4.6 The VS Model Revisited
4.7 Analysis of Experiments

Landauer Approach



$$f_1(E) = \frac{1}{1+e^{(E-E_{F1})/k_BT}} \qquad f_2(E) = \frac{1}{1+e^{(E-E_{F2})/k_BT}}$$

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)\left(f_1 - f_2\right)dE$$

Can be used to describe the current in small and large devices and in short to long devices.

Landauer at low and high bias

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)\left(f_1 - f_2\right)dE$$

1) Linear region: $$I_{DLIN} = \left[\frac{2q^2}{h}\int \mathcal{T}(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE\right]V_{DS}$$

2) Saturation region: $$I_{DSAT} = \frac{2q}{h}\int \mathcal{T}(E)M(E)f_1(E)dE$$

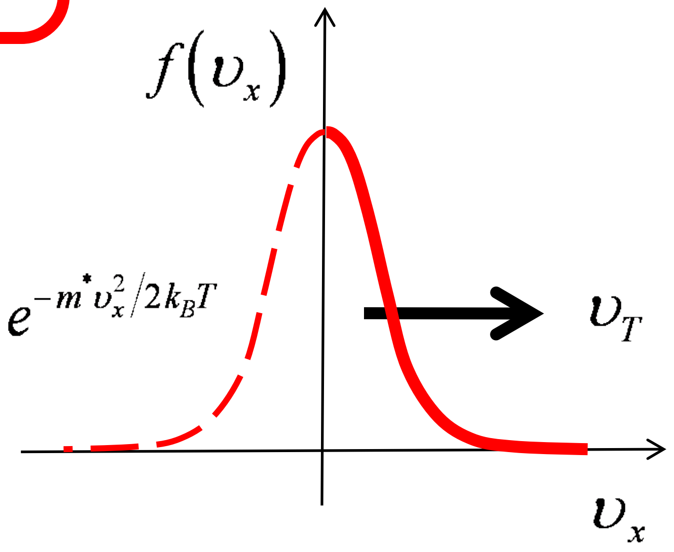
Transmission, MFP, and diffusion coefficient

$$\boxed{\mathcal{T}_0 = \frac{\lambda_0}{\lambda_0 + L}}$$

$$D_n = \frac{k_B T}{q}\mu_n$$

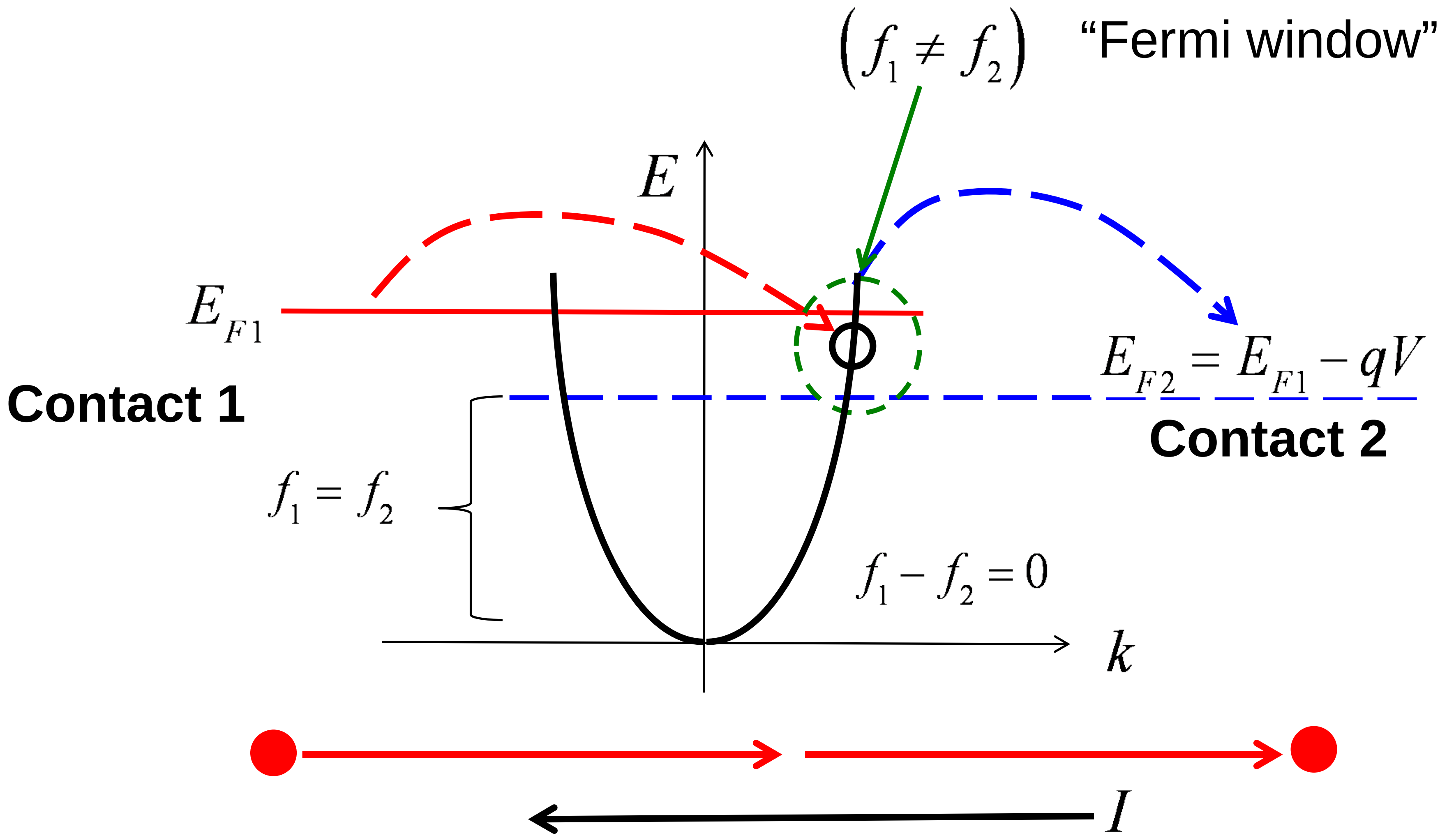
Einstein relation

$$D_n = \frac{\upsilon_T \lambda_0}{2}\ \mathrm{cm}^2/\mathrm{s}$$

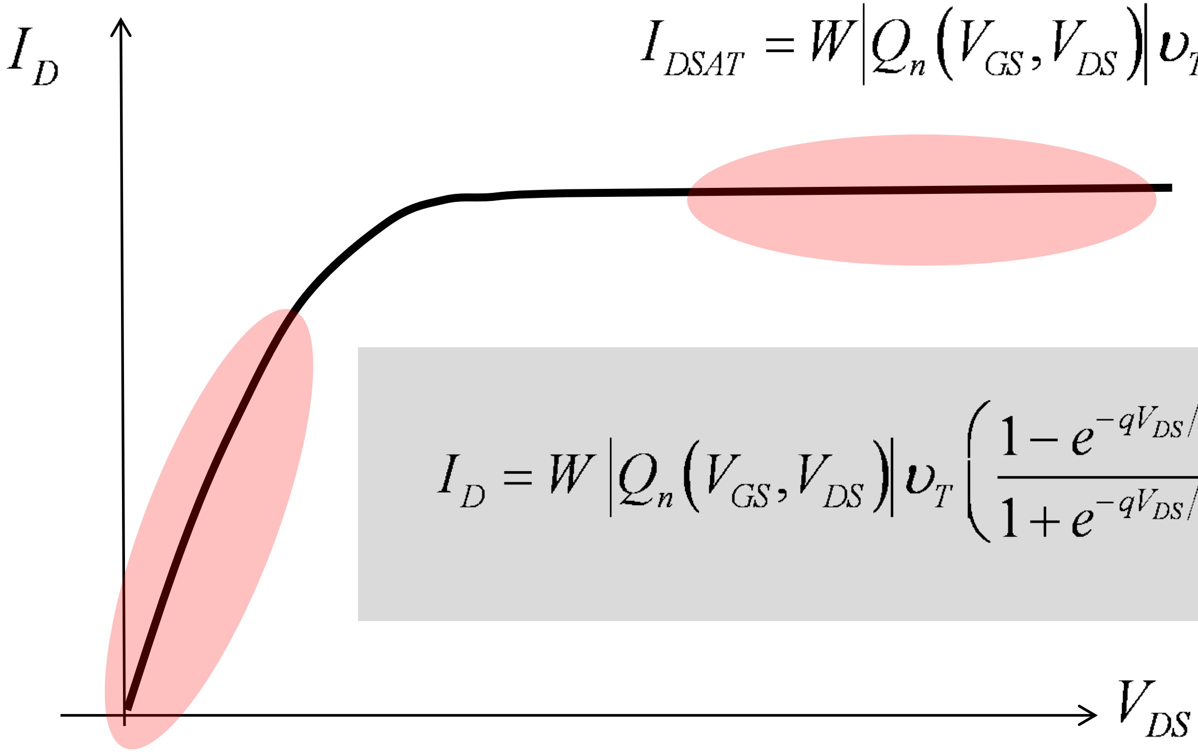


$$\langle\langle \upsilon_x^+ \rangle\rangle = \upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}}$$

Femi window and current flow



The Ballistic MOSFET



$$I_{DSAT} = W\left|Q_n\left(V_{GS}, V_{DS}\right)\right|\upsilon_T$$

$$I_D = W\left|Q_n\left(V_{GS}, V_{DS}\right)\right|\upsilon_T\left(\frac{1-e^{-qV_{DS}/k_BT}}{1+e^{-qV_{DS}/k_BT}}\right)$$

$$I_{DLIN} = \frac{W}{L}\mu_B\left|Q_n\left(V_{GS}, V_{DS}\right)\right|V_{DS} \qquad \mu_B = \frac{\upsilon_T L}{2k_BT/q}$$

Ballistic vs. diffusive mobility

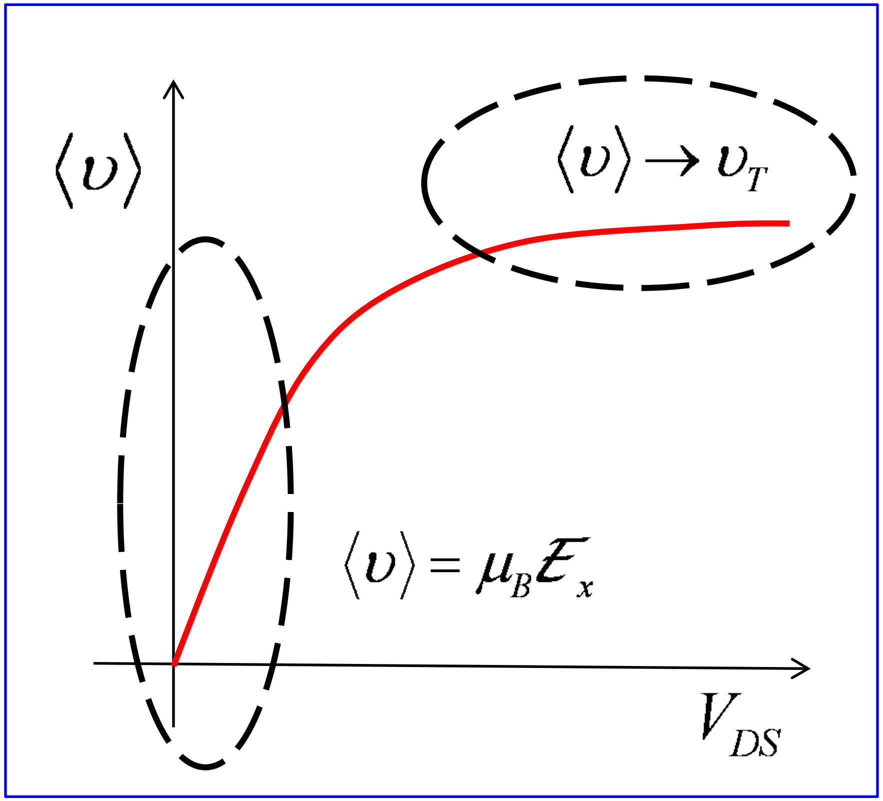
$$\mu_B \equiv \frac{\upsilon_T L}{2(k_B T/q)}$$

"ballistic mobility"

$$\mu_n = \frac{\upsilon_T \lambda_0}{2(k_B T/q)}$$

"diffusive mobility"

Ballistic velocity vs. V_{DS} at the VS

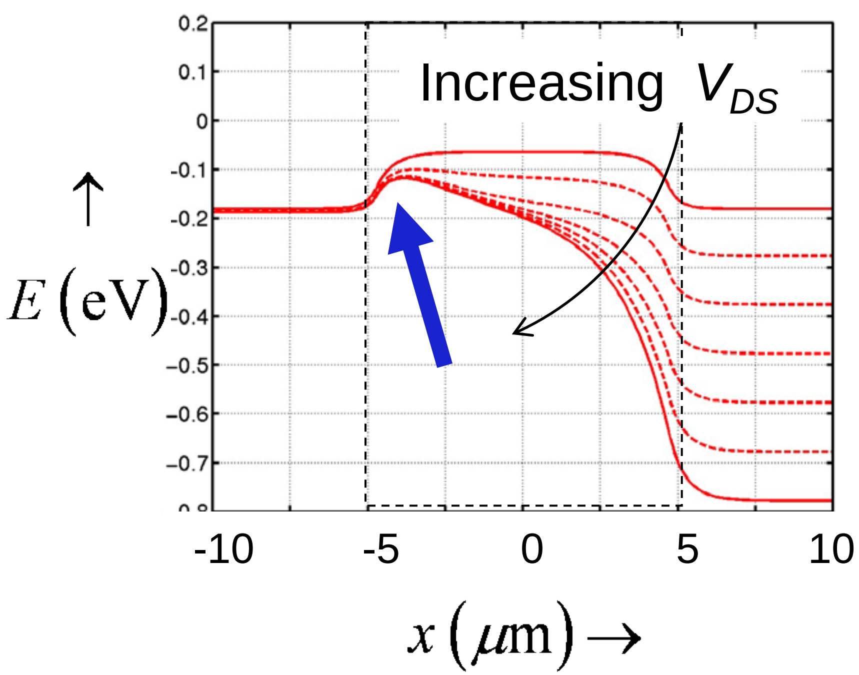


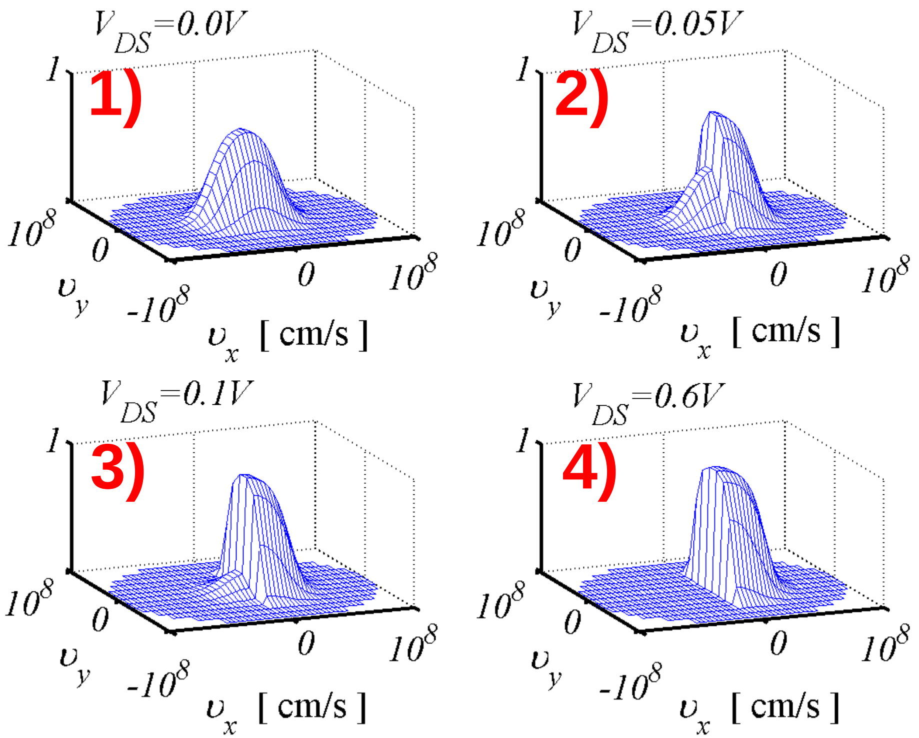
Velocity saturation with no scattering!

Physics of velocity saturation in ballistic MOSFETs

$f(\upsilon_x, \upsilon_y)$

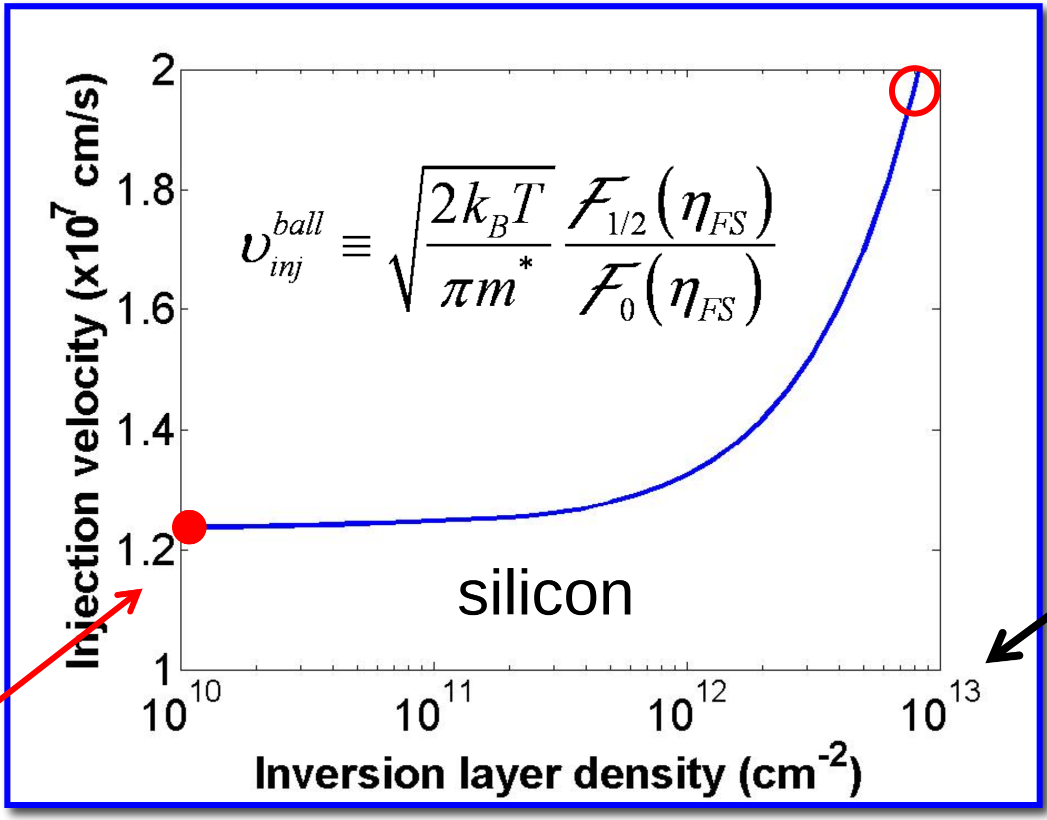
E_X *vs.* x for V_{GS} = 0.5V





(Numerical simulations of an L = 10 nm double gate Si MOSFET from J.-H. Rhew and M.S. Lundstrom, *Solid-State Electron.*, **46**, 1899, 2002)

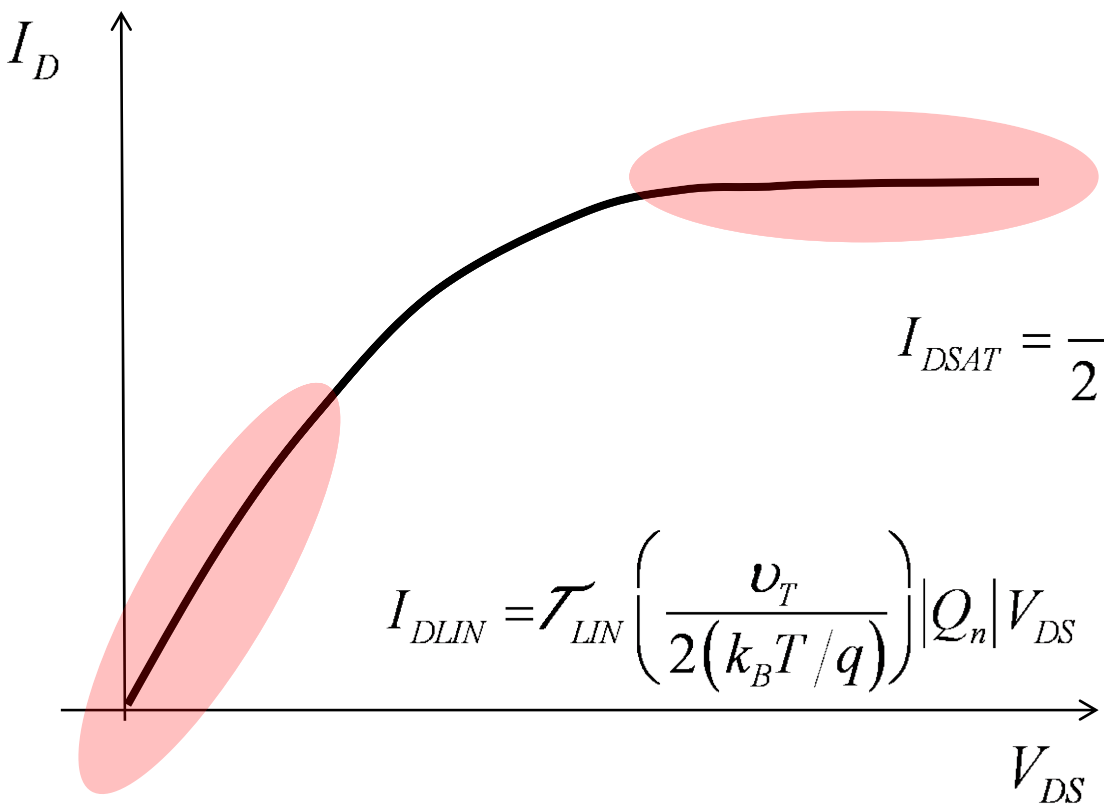
Injection velocity vs. gate voltage



$$\upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}}$$

non-degenerate

Transmission theory of the MOSFET



$$I_{DSAT} = \frac{\mathcal{T}_{SAT}}{2-\mathcal{T}_{SAT}} W\left|Q_n\right|\upsilon_T$$

$$I_{DLIN} = \mathcal{T}_{LIN}\left(\frac{\upsilon_T}{2\left(k_B T/q\right)}\right)\left|Q_n\right|V_{DS}$$

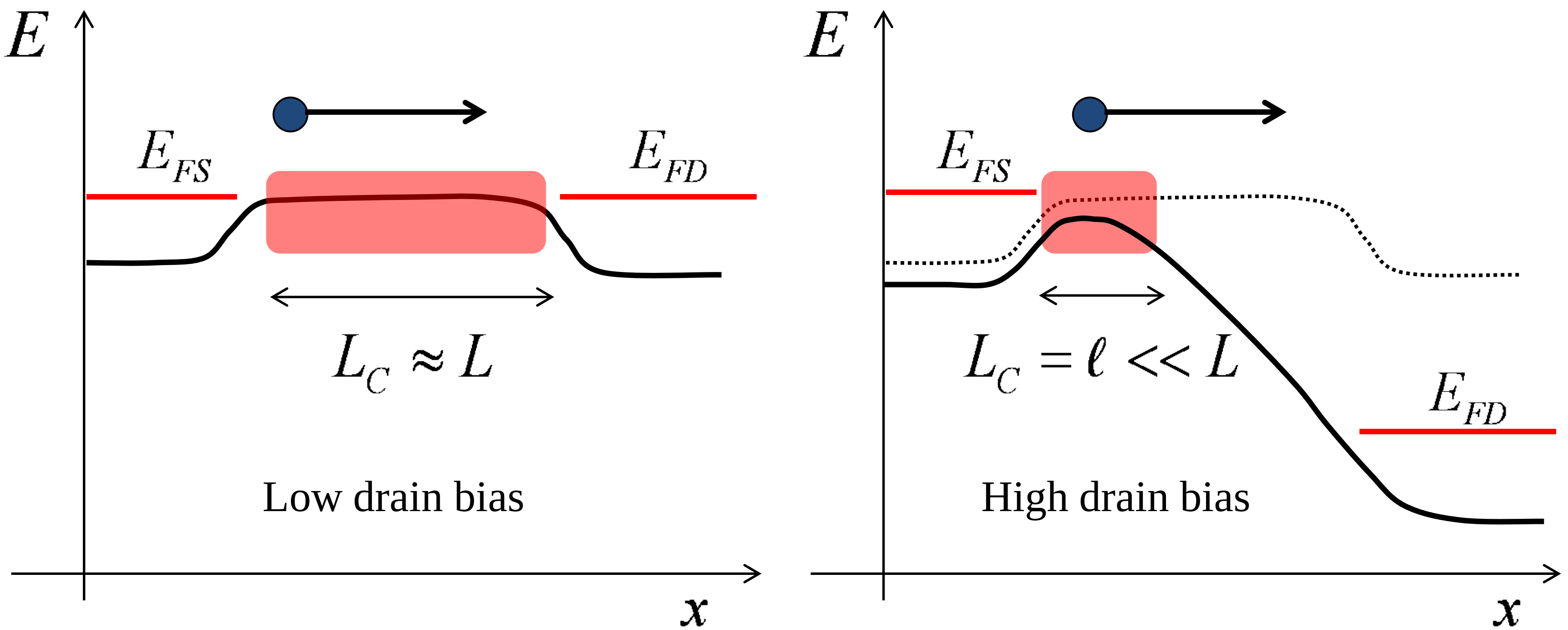
$$\mathcal{T}_{LIN} < \mathcal{T}_{SAT}$$

Linear vs. saturation region transmission

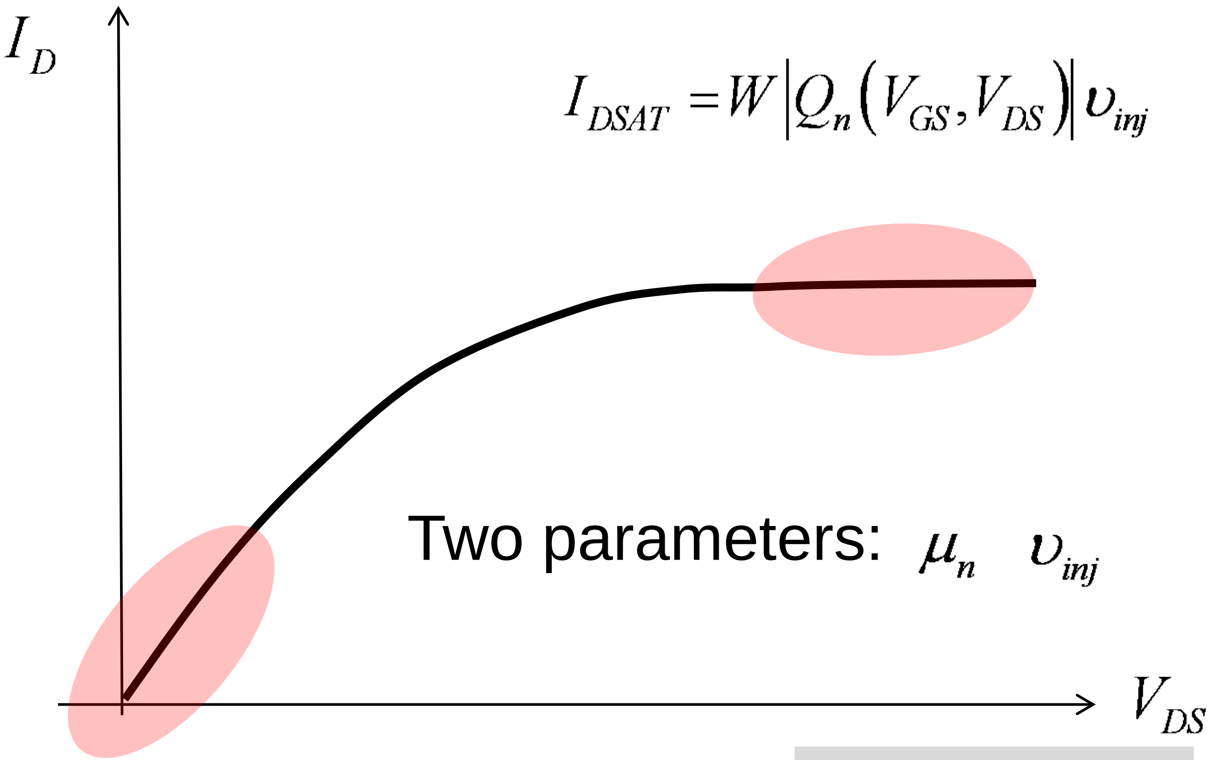
$$\mathcal{T}_{LIN} = \frac{\lambda_0}{\lambda_0 + L}$$

$$\boxed{\mathcal{T}(V_{DS}) = \frac{\lambda_0}{\lambda_0 + L_C(V_{DS})}}$$

$$\mathcal{T}_{SAT} = \frac{\lambda_0}{\lambda_0 + \ell}$$



Alternative formulation



$$I_{DSAT} = W\left|Q_n\left(V_{GS}, V_{DS}\right)\right|\upsilon_{inj}$$

Two parameters: $\mu_n \quad \upsilon_{inj}$

$$\upsilon_{inj} = \left(\frac{\mathcal{T}_{SAT}}{2-\mathcal{T}_{SAT}}\right)\upsilon_T$$

$$\mathcal{T}_{SAT} = \frac{\lambda_0}{\lambda_0 + \ell}$$

$$\ell \ll L$$

$$I_{DLIN} = W\mu_{app}\left|Q_n\left(V_{GS}, V_{DS}\right)\right|V_{DS}$$

$$\frac{1}{\mu_{app}} = \frac{1}{\mu_n} + \frac{1}{\mu_B}$$

Level 2 VS model

1) $I_D/W = \left|Q_n\left(V_{GS}, V_{DS}\right)\right| \left\langle \upsilon_x\left(V_{DS}\right)\right\rangle$

2) $Q_n\left(V_{GS}, V_{DS}\right) = -C_{inv} m\left(k_B T/q\right) \ln\left(1 + e^{q\left(V_{GS} - V_T + \alpha\left(k_B T_L/q\right)F_f\right)/m k_B T}\right)$

$V_T = V_{T0} - \delta V_{DS}$

3) $\left\langle \upsilon_x\left(V_{DS}\right)\right\rangle = F_{SAT}\left(V_{DS}\right)\upsilon_{inj}$

4) $F_{SAT}\left(V_{DS}\right) = \dfrac{V_{DS}/V_{DSAT}}{\left[1 + \left(V_{DS}/V_{DSAT}\right)^{\beta}\right]^{1/\beta}}$

5) $V_{DSAT} = \dfrac{\upsilon_{inj} L}{\mu_{app}}$

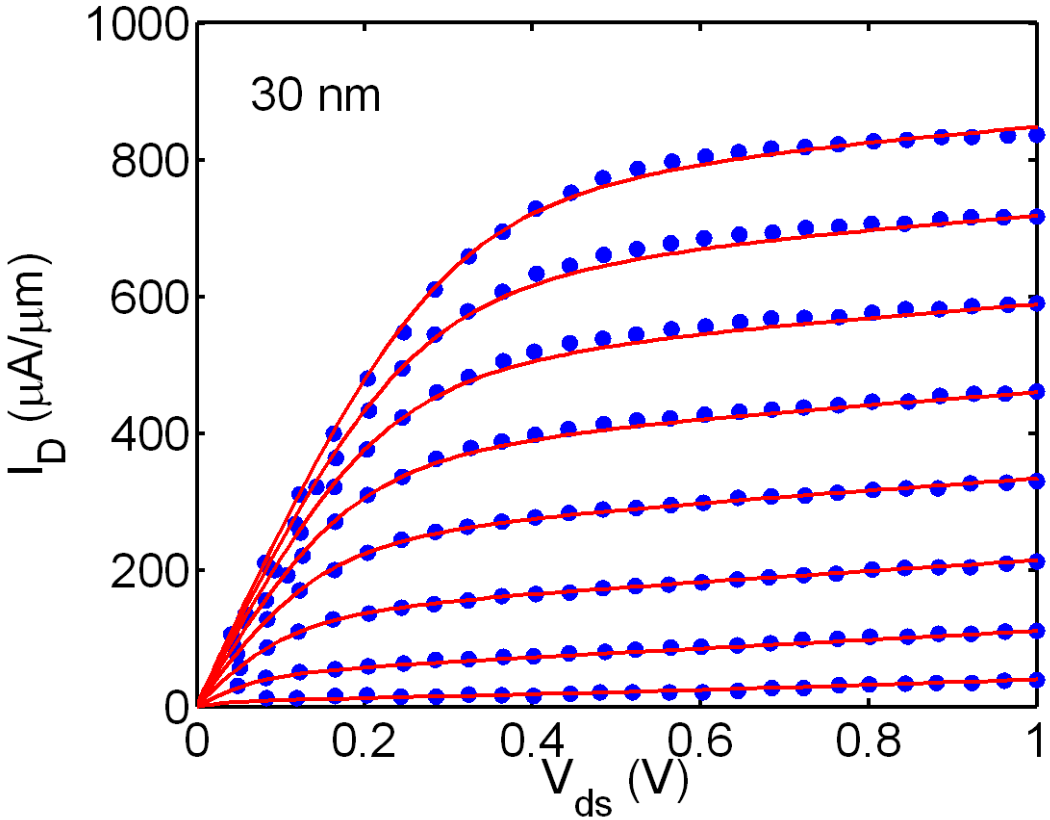
Only 10 device-specific parameters in this model:

$C_{inv}, V_{T0}, \delta, m, \upsilon_{inj}, \mu_{app},$

$L, R_{SD} = R_S + R_D,$

α, β

MVS Fits to experimental Si ETSOI data



$$\mu_{app} = 220 \frac{\text{cm}^2}{\text{V-s}}$$

$$\upsilon_{inj} = 0.82 \times 10^7 \text{ cm/s}$$

$$R_{S0} + R_{D0} = 130 \ \Omega\text{-}\mu\text{m}$$

A. Majumdar and D.A. Antoniadis, "Analysis of Carrier Transport in Short-Channel MOSFETs," *IEEE Trans. Electron. Dev*., **61**, pp. 351- 358, 2014.

MVS analysis of well-tempered MOSFETs

$$\mathcal{T}_{LIN} = \frac{\mu_{app}}{\mu_B} \qquad\qquad \mathcal{T}_{SAT} = \left(\frac{2}{1 + \upsilon_T / \upsilon_{inj}} \right)$$

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