Essentials of MOSFETs

Unit 3: MOS Electrostatics

Lecture 3.6: The Mobile Charge vs. Surface Potential

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MOSFET drain current

$$I_{DS}/W = -Q_n\left(V_{GS}\right)\left\langle \upsilon_x\left(V_{DS}\right)\right\rangle$$

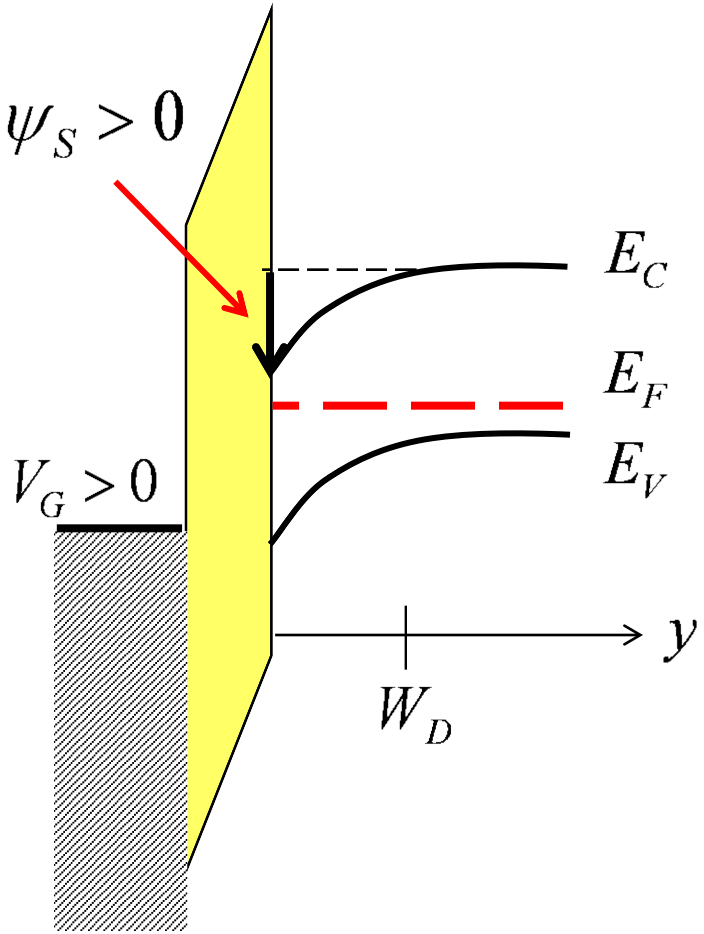
We have been discussing Q_S and Q_D, but we need $\boldsymbol{Q_n}$ as a function of **surface potential** and **gate voltage**.

$$Q_S = Q_D + Q_n \quad \mathrm{C/cm^2}$$

$Q_n\left(\psi_S\right) \longleftarrow$ **this lecture**

$Q_n\left(V_G\right) \dashleftarrow$ next lecture

Mobile charge (per cm^3) vs.depth

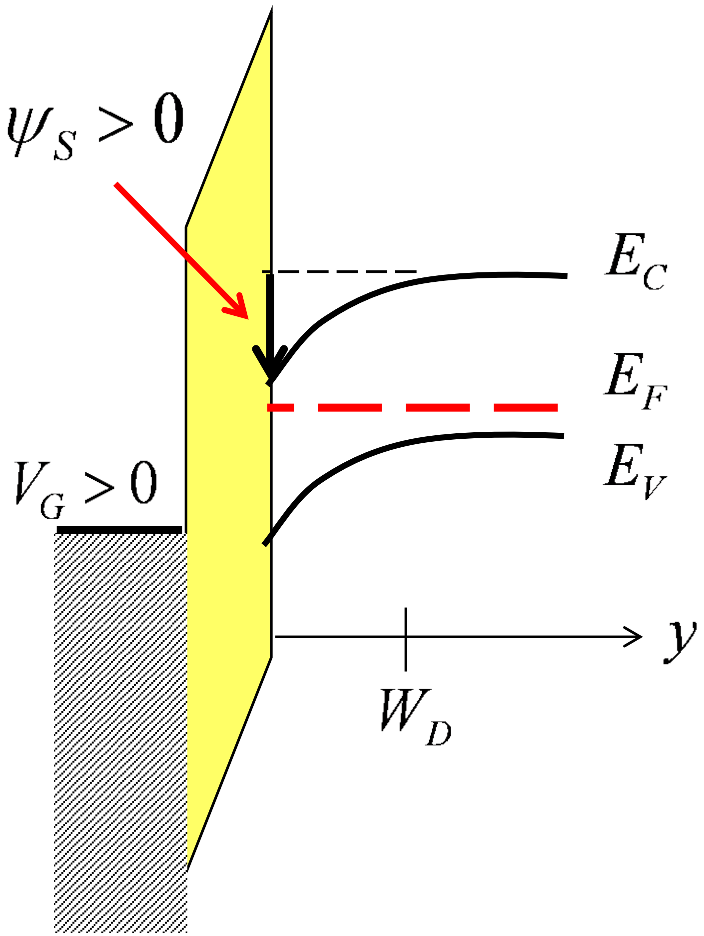


$$n_0(y) = N_C e^{(E_F - E_C(y))/k_B T} \text{ cm}^{-3}$$

$$n_0(y) = n_B \times e^{q\psi(y)/k_B T}$$

$$n_B = \frac{n_i^2}{N_A}$$

Mobile sheet charge (per cm^2)



$$Q_n = -q\int_0^\infty n(y)\,dy$$

$$= -q\int_0^\infty n_B\, e^{q\psi(y)/k_BT}\,dy$$

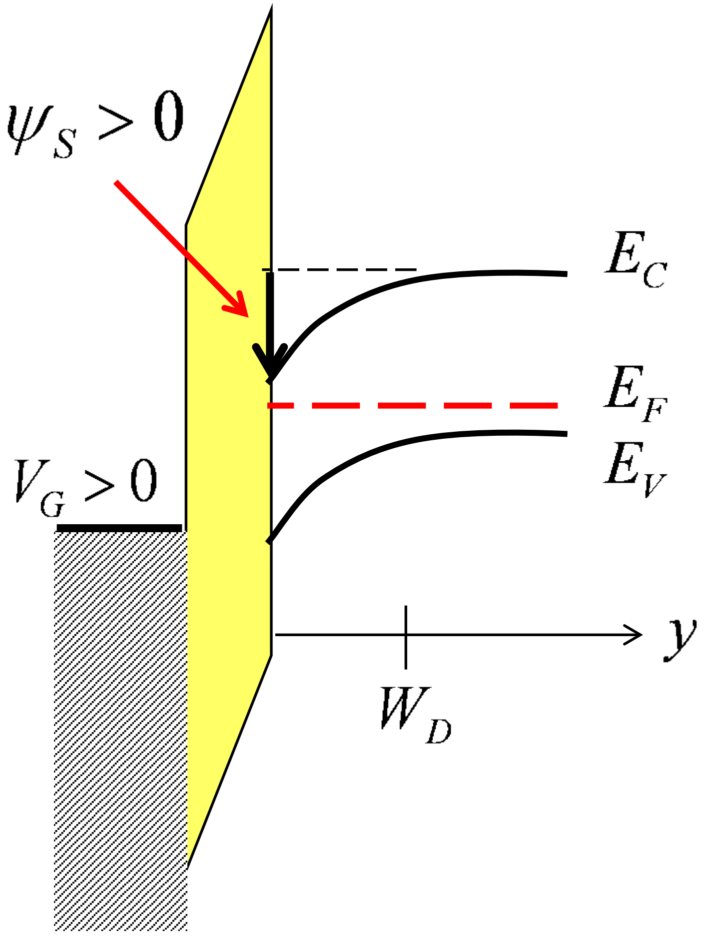
$$= -qn_B\int_{\psi_S}^0 e^{q\psi(y)/k_BT}\,\frac{dy}{d\psi}\,d\psi$$

$$\approx \frac{qn_B}{\mathcal{E}_S}\int_{\psi_S}^0 e^{q\psi(y)/k_BT}\,d\psi$$

$$Q_n \approx -qn_B e^{q\psi_S/k_BT}\left(\frac{k_BT/q}{\mathcal{E}_S}\right)$$

$$Q_n \approx -qn(0)\times t_{inv}$$

Mobile sheet charge (per cm^2)

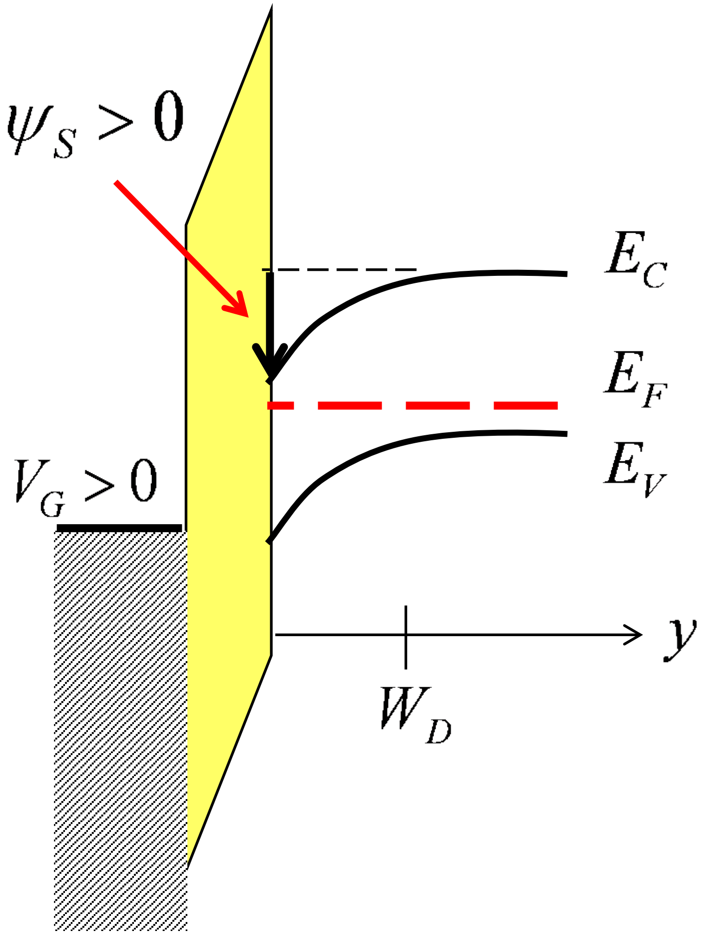


$$Q_n(\psi_S) \approx -q n_B e^{q\psi_S/k_BT}\left(\frac{k_BT/q}{\mathcal{E}_S}\right)$$

valid above ***or*** below threshold

(if we are careful)

Mobile sheet charge: **below threshold**



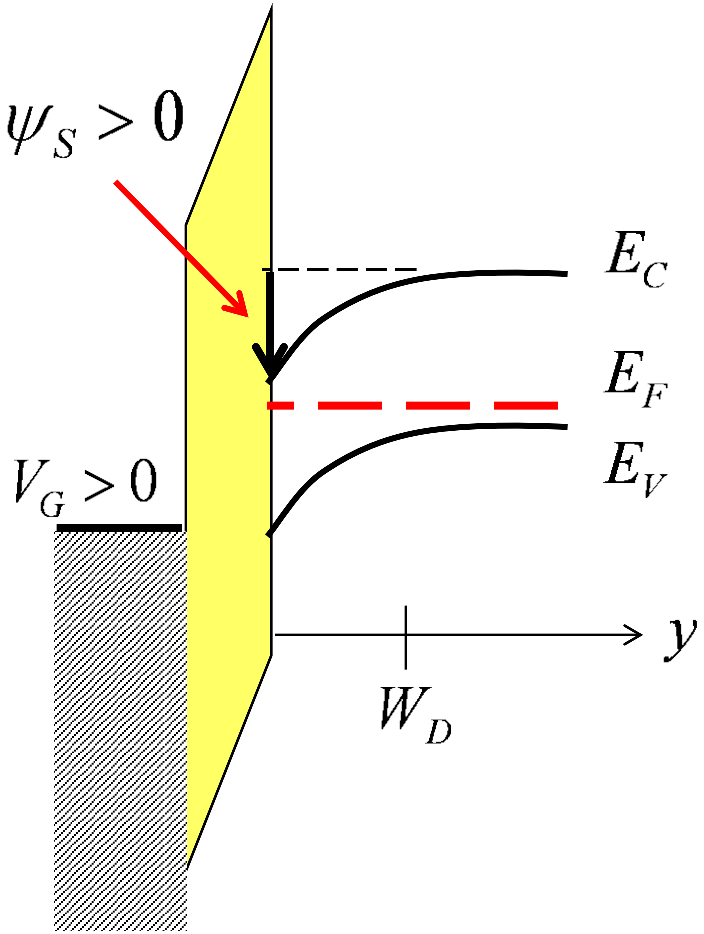
$$Q_n(\psi_S) \approx -qn_B e^{q\psi_S/k_BT}\left(\frac{k_BT/q}{\mathcal{E}_S}\right)$$

$$\mathcal{E}_S = \left(2qN_A\psi_S/\varepsilon_S\right)^{1/2}$$

$$Q_n(\psi_S) \approx -\frac{n_i^2 k_BT/N_A}{\left(2qN_A\psi_S/\varepsilon_S\right)^{1/2}} e^{q\psi_S/k_BT}$$

$$\boxed{Q_n(\psi_S) \propto e^{q\psi_S/k_BT}}$$

Mobile sheet charge: **above threshold**



$$Q_n(\psi_S) \approx -q n_B e^{q\psi_S/k_BT}\left(\frac{k_BT/q}{\mathcal{E}_S}\right)$$

$$\varepsilon_S \mathcal{E}_S = -Q_S(\psi_S) = -Q_D(\psi_S) - Q_n(\psi_S)$$

1) Below threshold:

$$Q_S(\psi_S) \approx Q_D(\psi_S)$$

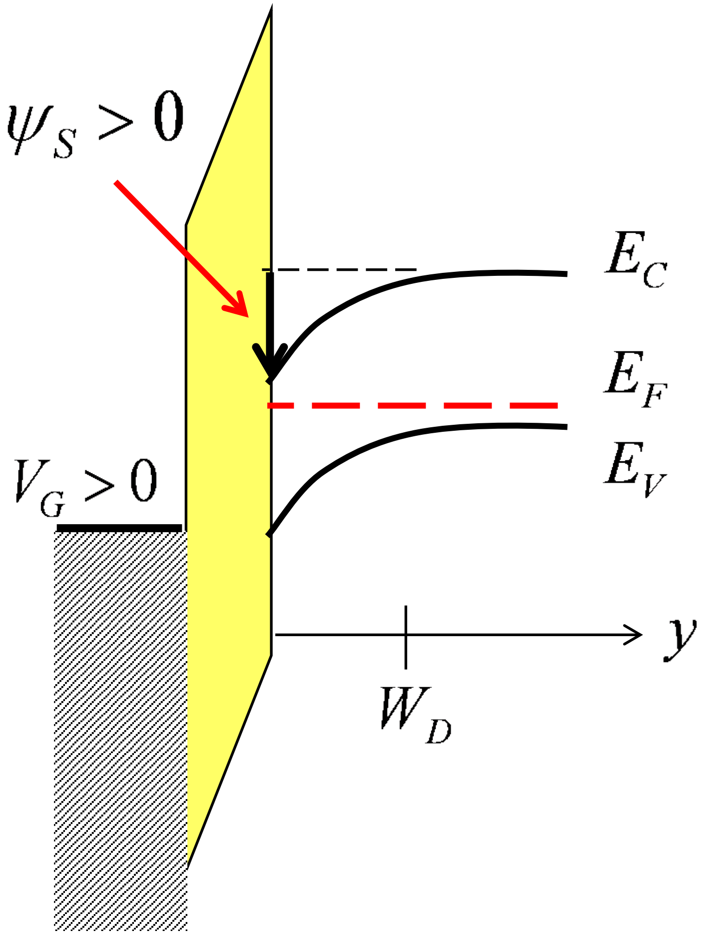
$$\mathcal{E}_S = \left(2qN_A\psi_S/\varepsilon_S\right)^{1/2}$$

2) Above threshold:

$$Q_S(\psi_S) \approx Q_n(\psi_S)$$

$$\varepsilon_S \mathcal{E}_S \approx -Q_n(\psi_S)$$

Mobile sheet charge: **above threshold**



$$Q_n(\psi_S) \approx -qn_B e^{q\psi_S/k_BT}\left(\frac{k_BT/q}{\bar{\mathcal{E}}_S}\right)$$

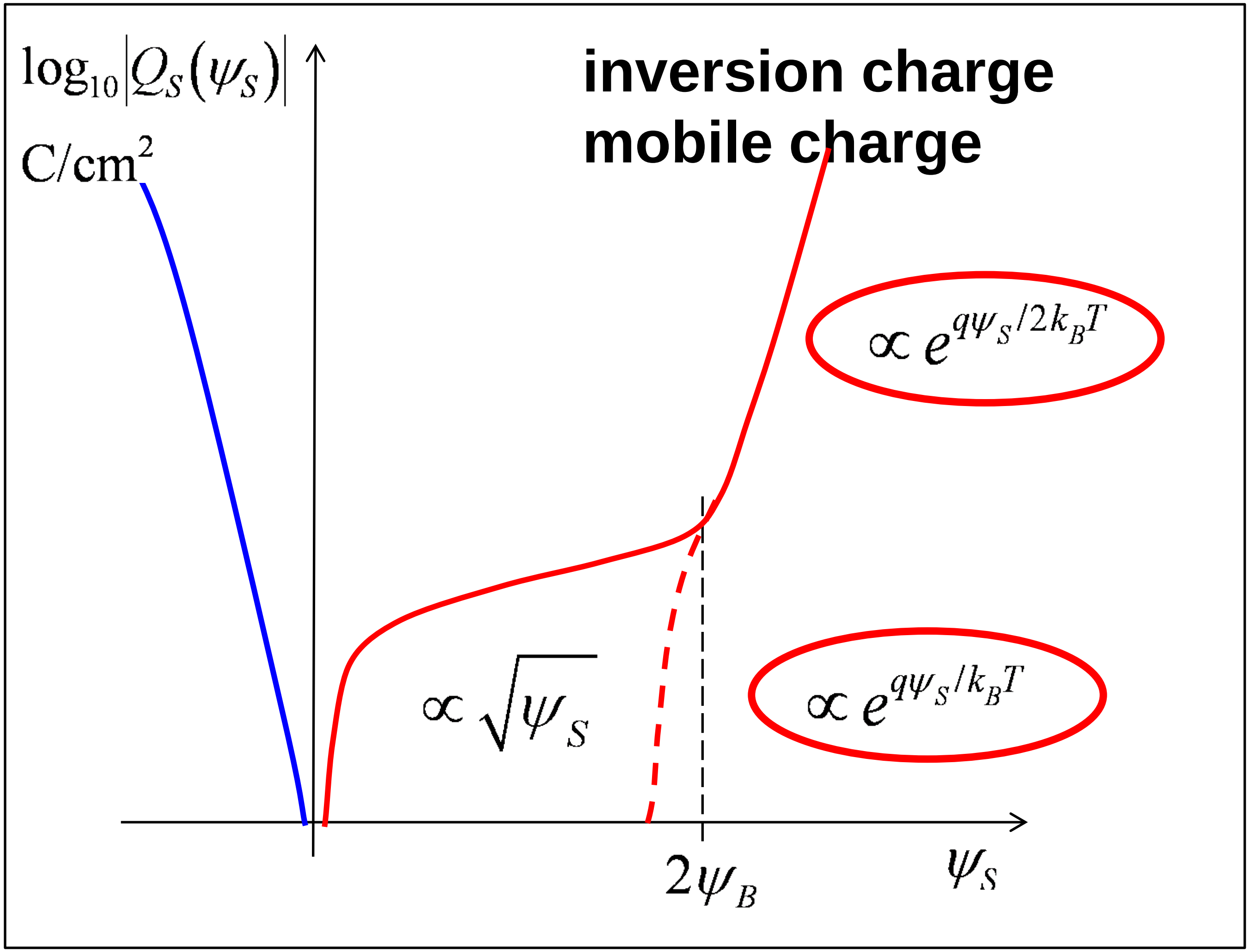
$$\mathcal{E}_S \to \bar{\mathcal{E}}_S \qquad \bar{\mathcal{E}}_S = \frac{Q_n(\psi_S)}{2\varepsilon_S}$$

$$Q_n(\psi_S) = -\sqrt{2\varepsilon_S k_BT\left(n_i^2/N_A\right)} \times e^{q\psi_S/2k_BT}$$

$$\boxed{Q_n(\psi_S) \propto e^{q\psi_S/2k_BT}}$$

Lundstrom: 2018

Mobile charge vs. surface potential



Lundstrom: 2018

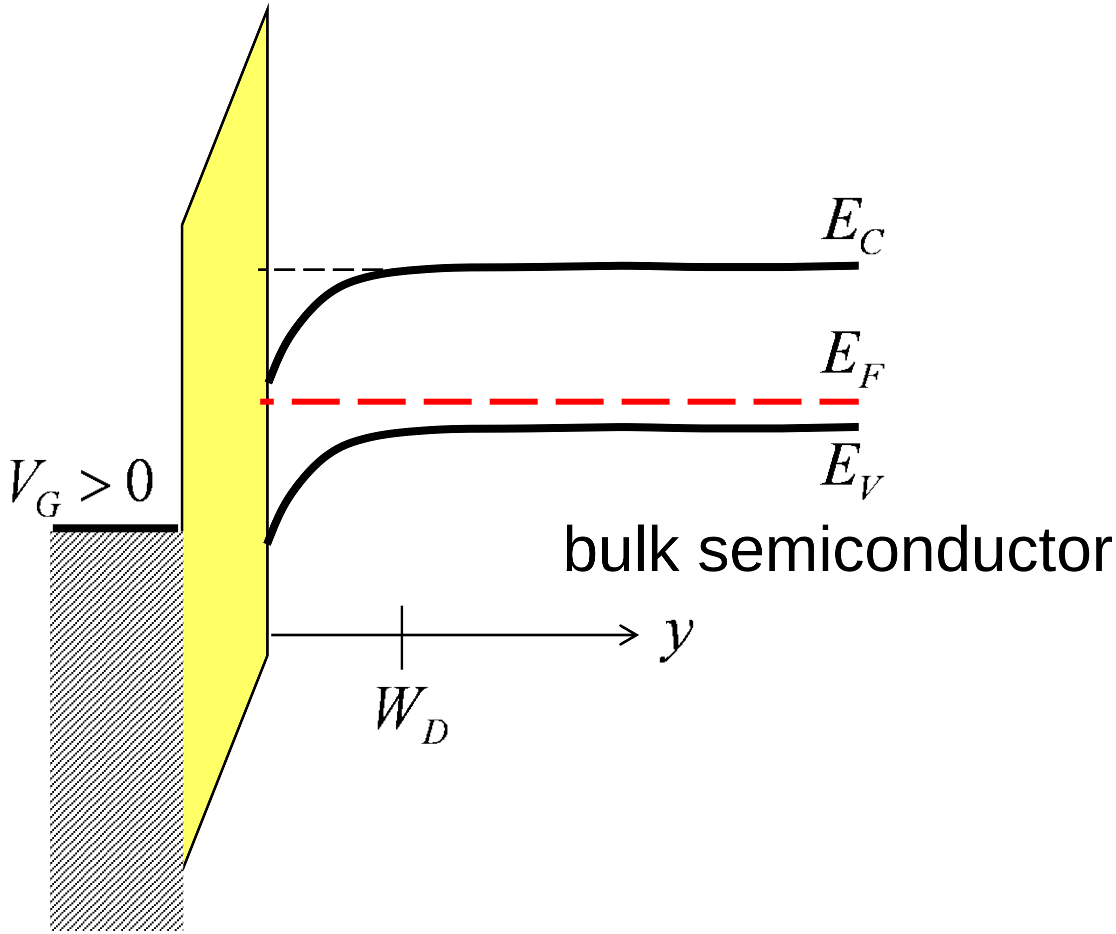
Exact solution

It is possible to solve the problem so that we go smoothly from subthreshold to above threshold.

The exact solution involves solving **the Poisson-Boltzmann equation** as discussed in these notes:

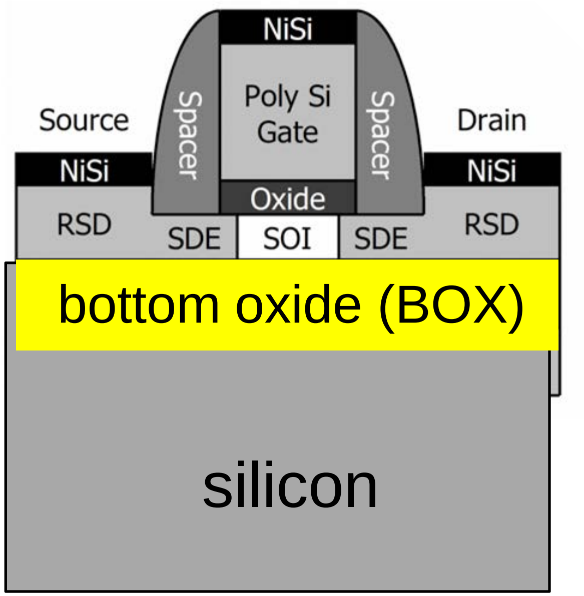
https://nanohub.org/resources/5338

Bulk MOS-C

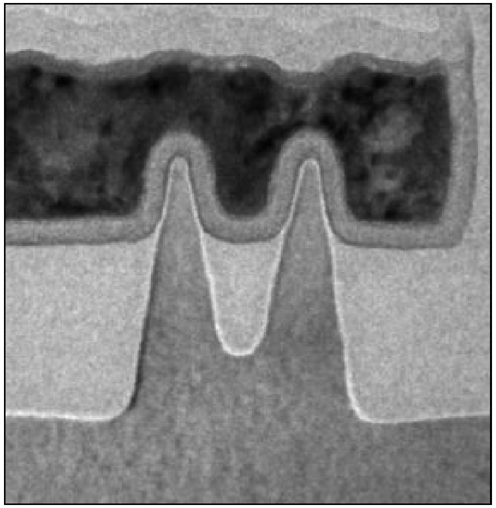


12” wafers are 775 micrometers thick

Fully depleted ultra thin body (UTB) MOS structures

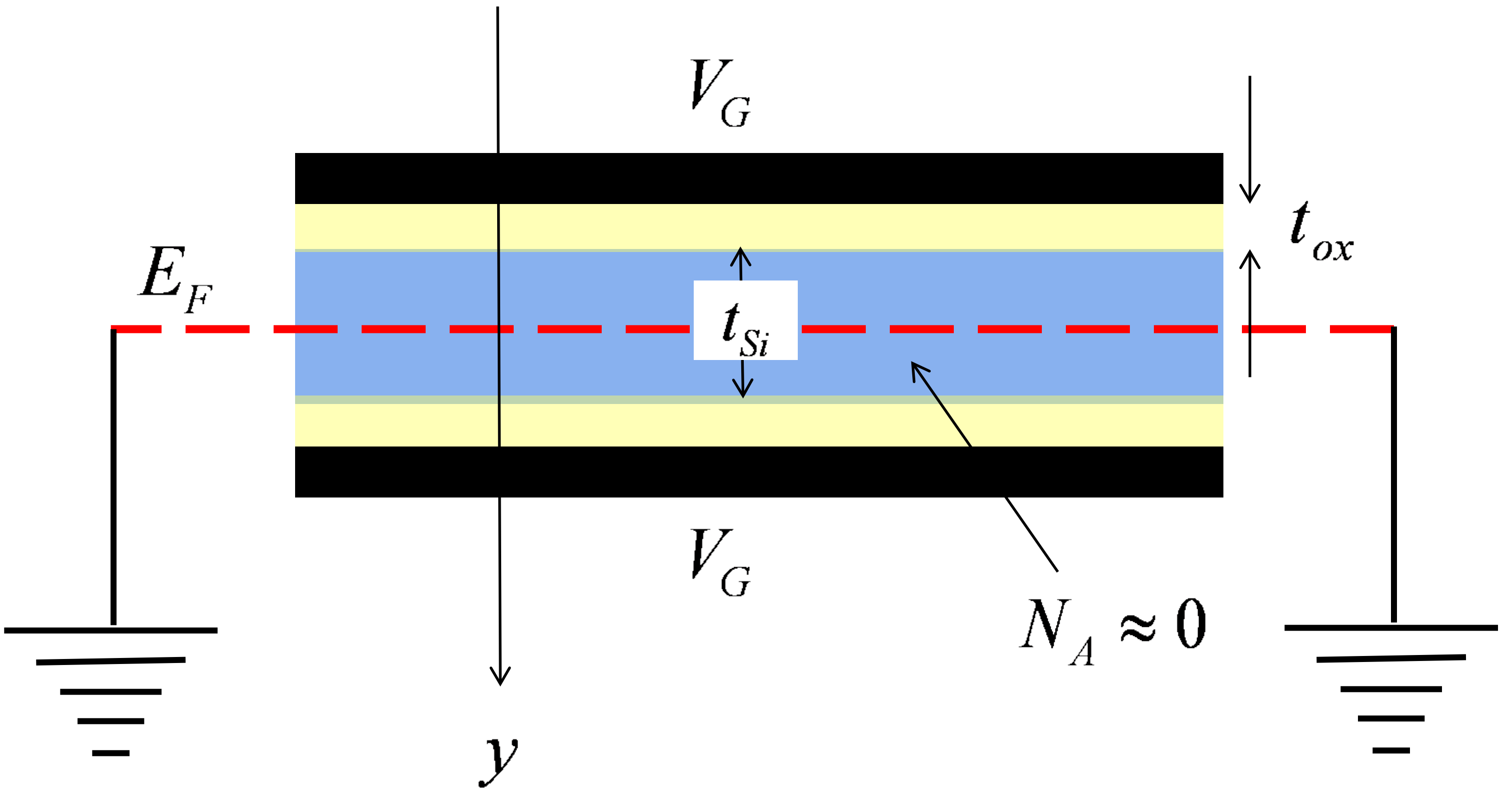


(ETSOI: Source: IBM, 2009)



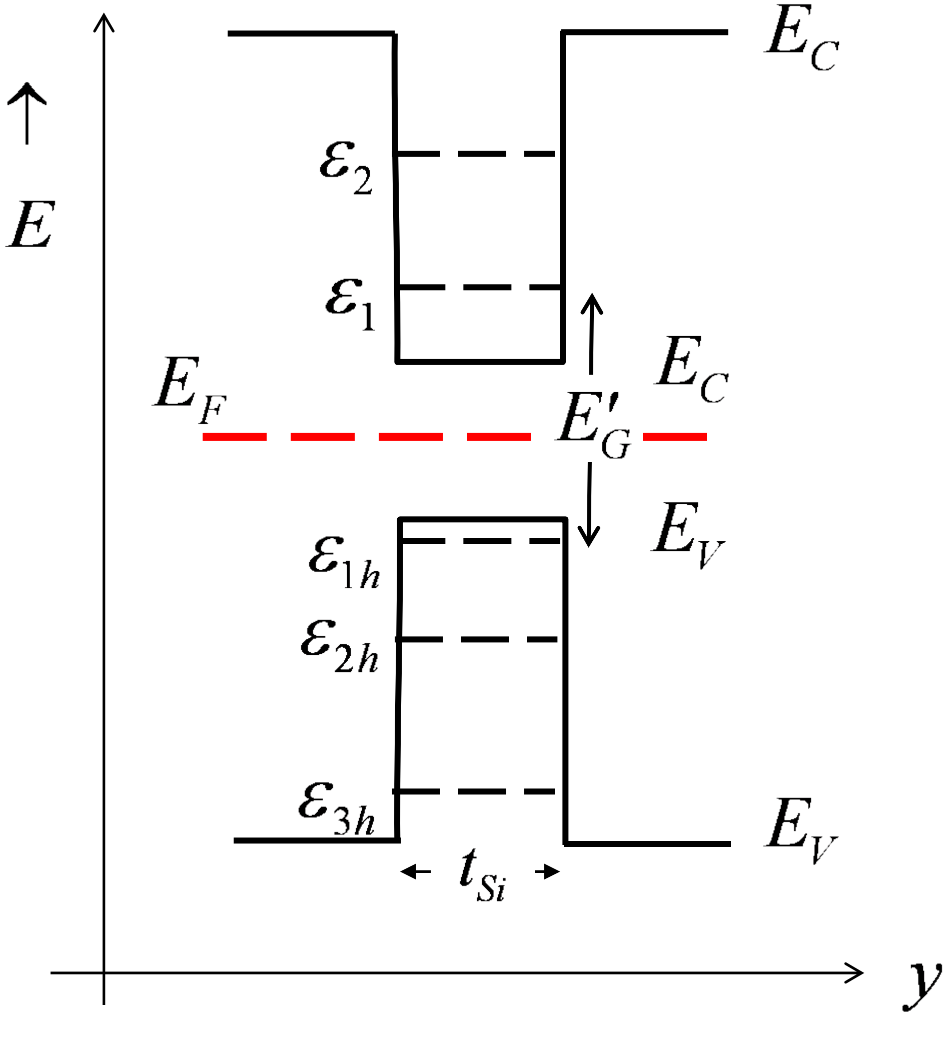
(FinFET: Source: Intel, 2015)

FD UTB double gate-C



We will assume a symmetrical, double gate geometry, which makes this discussion relevant to FinFETs as well.

FD UTB energy band diagram

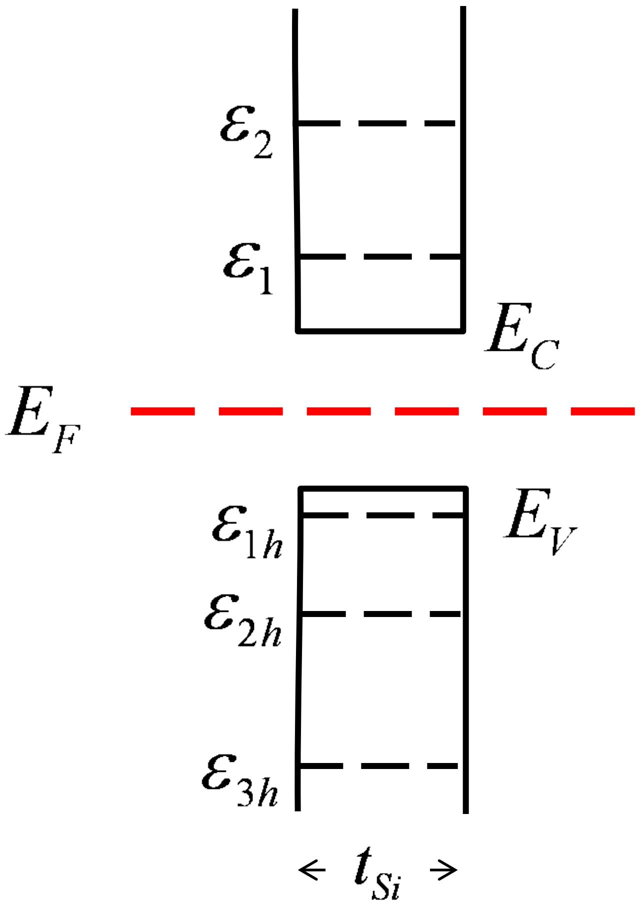


$$\varepsilon_n = \frac{\hbar^2 n^2 \pi^2}{2m^* t_{Si}^2}$$

$$E'_G = E_G + \varepsilon_1 + \varepsilon_{1h}$$

(Neglect band bending, so the potential is constant.)

2D carrier densities



$$n_{S1} = N_C^{2D} \mathcal{F}_0\left(\eta_{F1}\right) \text{ cm}^{-2}$$

$$N_C^{2D} = g_V \frac{m_n^* k_B T}{\pi \hbar^2} \text{ cm}^{-2}$$

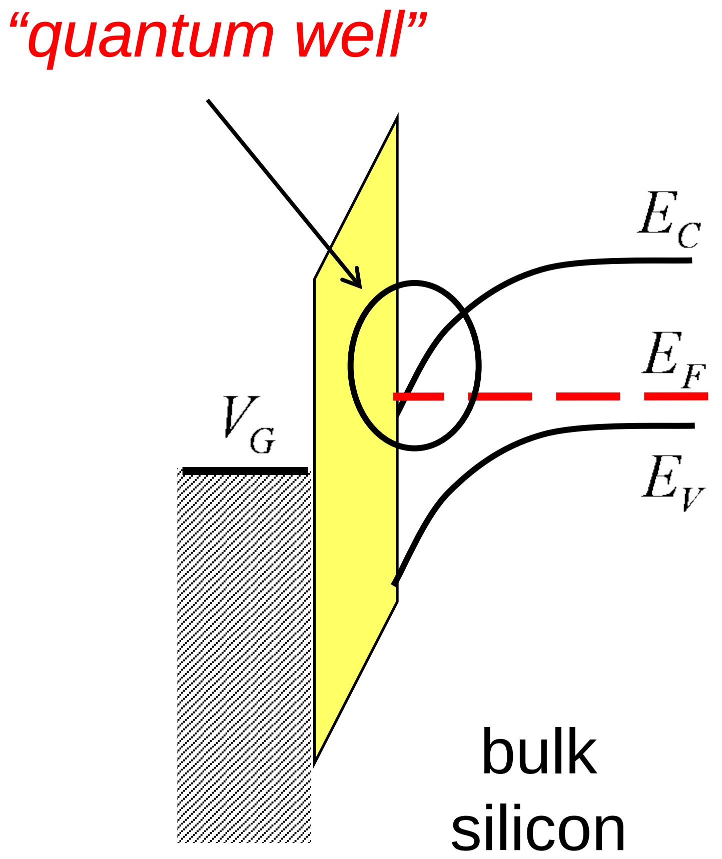
$$\eta_{F1} = \frac{\left(E_F - E_C - \varepsilon_1\right)}{k_B T}$$

Boltzmann statistics:

$$n_{S1} = N_C^{2D} e^{\left(E_F - E_C - \varepsilon_1\right)/k_B T} \text{ cm}^{-2}$$

Lundstrom: 2018

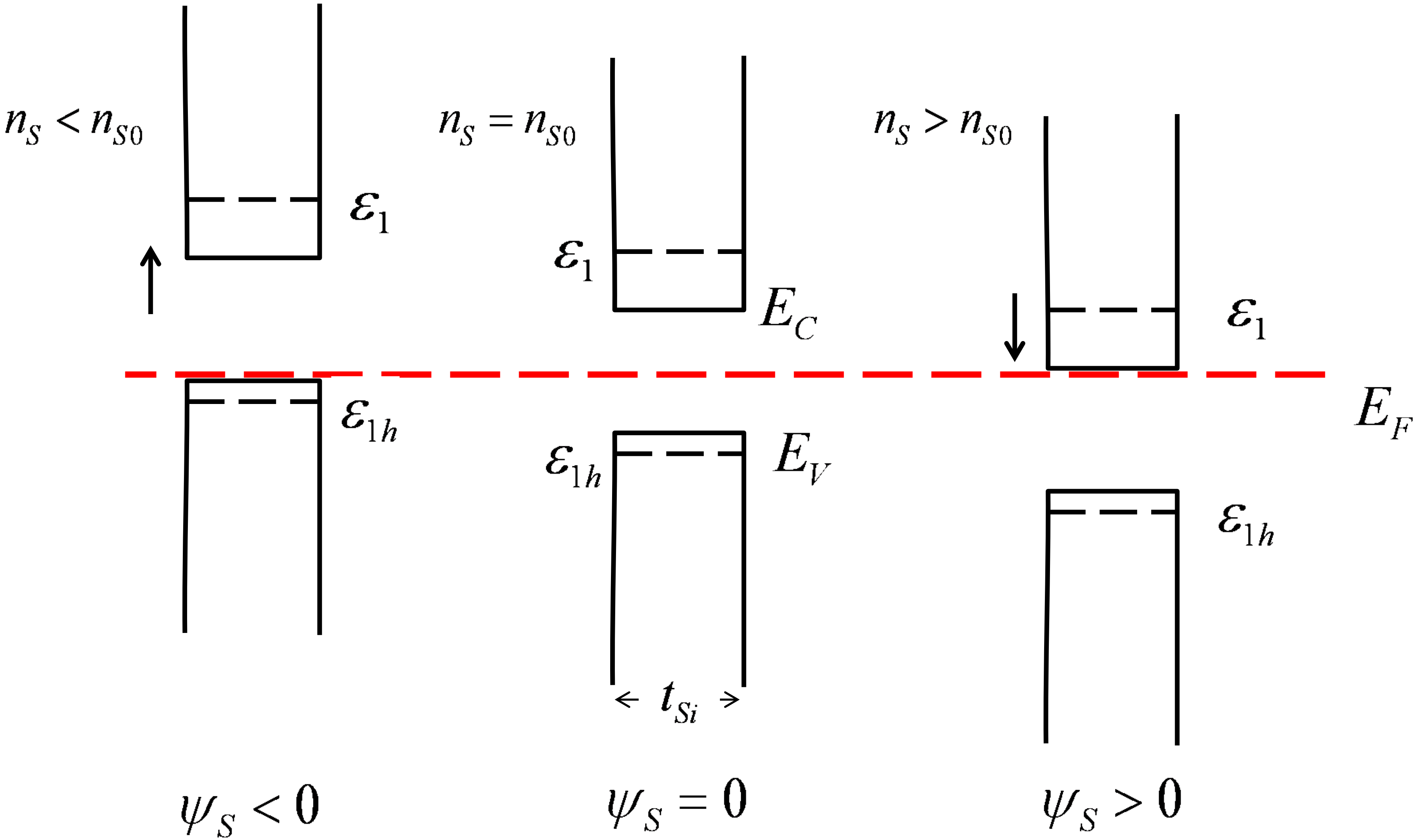
Quantum confinement in a bulk MOS-C



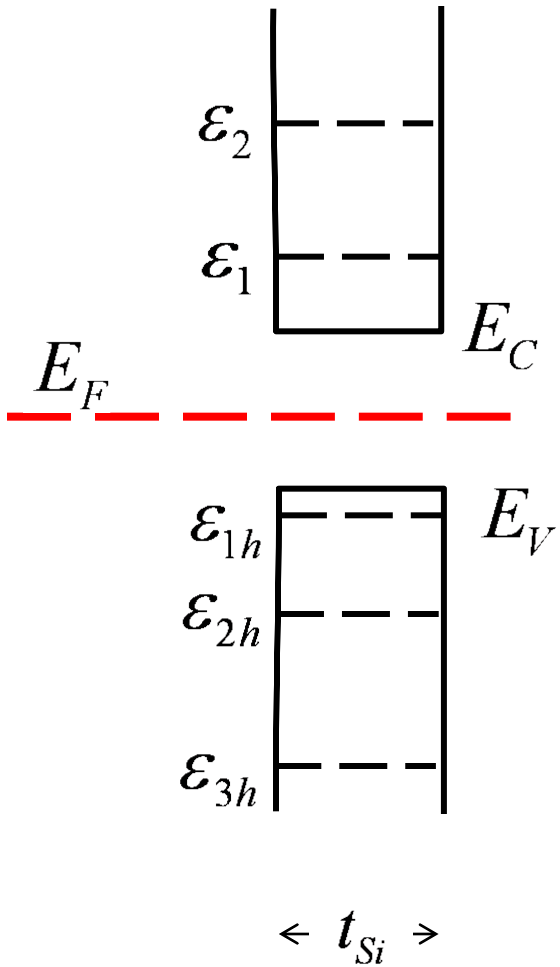
In the bulk, the confining potential is due to electrostatics.

In fully depleted ultra thin body structures, the confining potential is due to the physical structure.

FD UTB for various potentials



Carrier densities and semiconductor potential



$$n_{S1} = N_C^{2D} e^{\left(E_F - E_C - \varepsilon_1\right)/k_B T} \text{ cm}^{-2}$$

$$E_C = E_{C0} - q\psi_S$$

$$n_S = n_{S0} e^{q\psi_S/k_B T}$$

$$p_S = p_{S0} e^{-q\psi_S/k_B T}$$

(These eqns. assume that only 1 subband is occupied.)

Lundstrom: 2018

Mobile sheet charge (per cm^2)

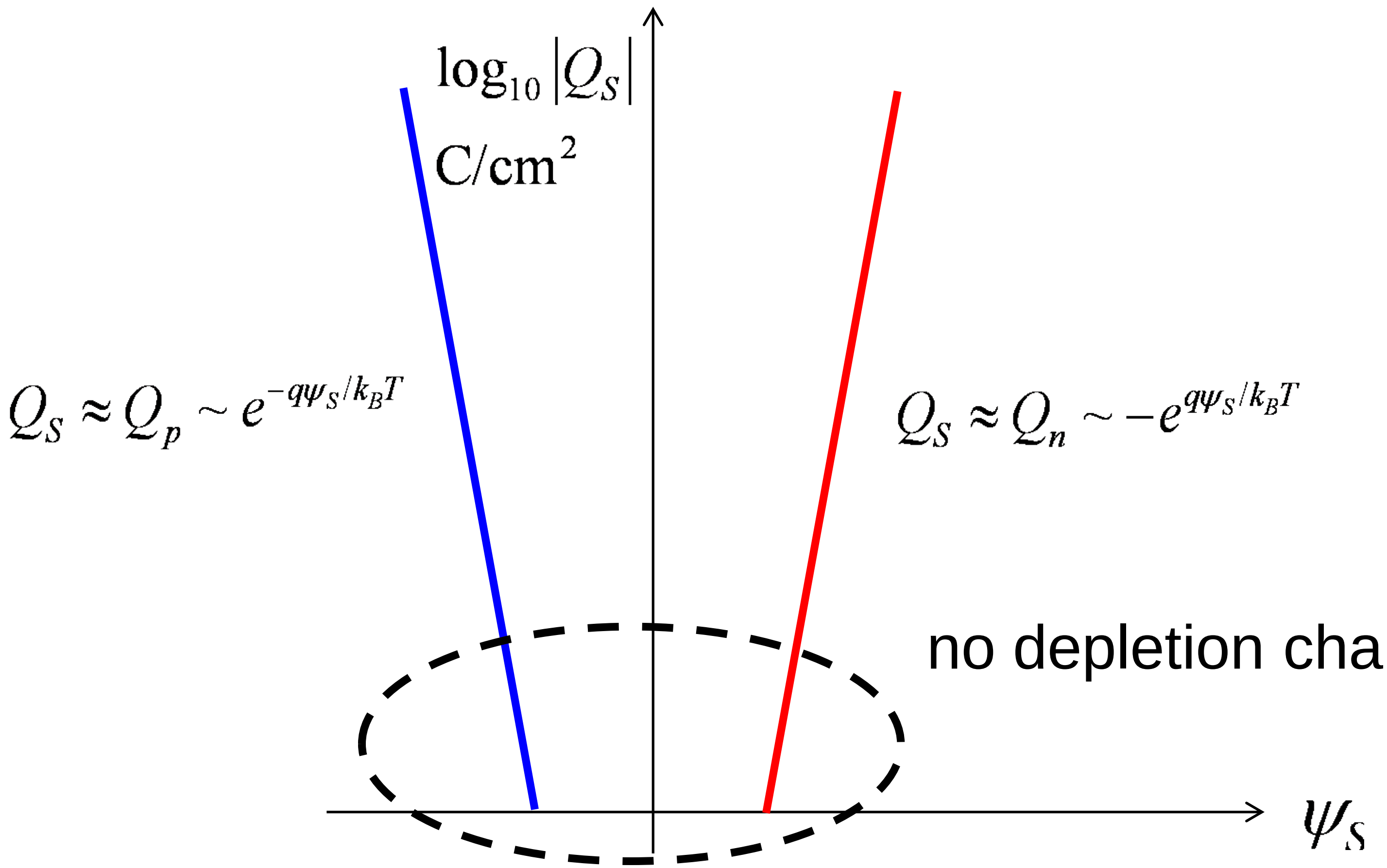
$$Q_S = q\left(p_S - n_S\right) \quad \mathrm{C/cm^2}$$

$$\boxed{Q_n\left(\psi_S\right) = -q n_{S0}\, e^{q\psi_S/k_B T}}$$

Valid above and below threshold.

Charge vs. surface potential

$$Q_S(\psi_S) = q\left(p_{S0}\, e^{-q\psi_S/k_BT} - n_{S0}\, e^{q\psi_S/k_BT}\right)$$



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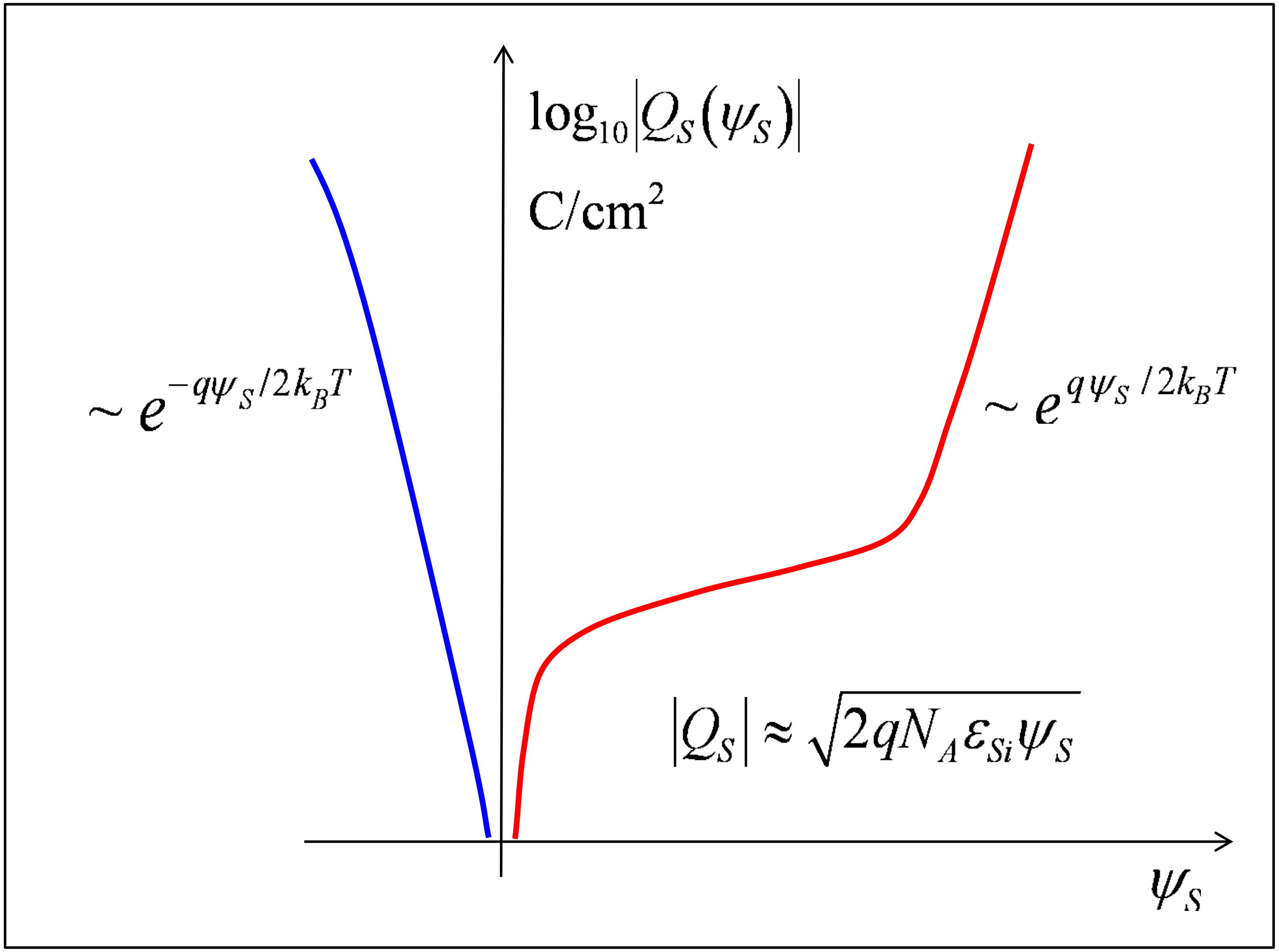
Recall: $Q_S(\psi_S)$ for bulk MOS



Summary

Bulk semiconductor:

$$\psi_S < 2\psi_B: \qquad Q_n(\psi_S) \approx -\left(\frac{n_i^2 k_B T / N_A}{\sqrt{\left(2qN_A\psi_S/\varepsilon_S\right)^{1/2}}}\right) e^{q\psi_S/k_BT}$$

$$\psi_S > 2\psi_B: \qquad Q_n(\psi_S) = -\sqrt{2\varepsilon_S k_B T \left(n_i^2/N_A\right)} \times e^{q\psi_S/2k_BT}$$

Fully depleted, ultra thin body:

$$\psi_S > 0: \qquad Q_n(\psi_S) = -qn_{S0}\, e^{q\psi_S/k_BT}$$

Next topic

$$I_{DS}/W = -Q_n\left(V_{GS}\right)\left\langle \upsilon_x\left(V_{DS}\right)\right\rangle$$

We have been discussing Q_S and Q_D, but we need $\boldsymbol{Q_n}$ as a function of **surface potential** and **gate voltage**.

$$Q_S = Q_D + Q_n \ \ \mathrm{C/cm^2}$$

$Q_n\left(\psi_S\right) \longleftarrow$ this lecture ✓

$Q_n\left(V_G\right) \leftarrow - -$ **next lecture**