

$$PID = k \left(1 + \frac{1}{T_i p} + T_d p \right) = k \left(\frac{1 + T_i p + T_i T_d p^2}{T_i p} \right)$$

$$\Delta = T_i^2 - 4 T_i T_d = T_i (T_i - 4 T_d)$$

First case: $T_i > 4 T_d$

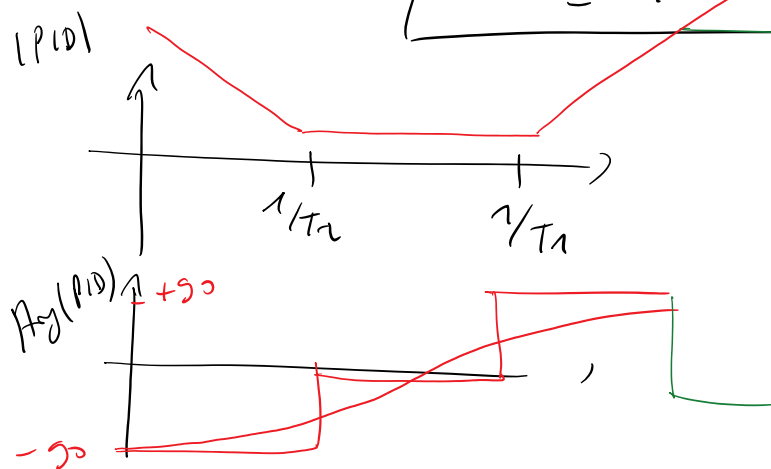
\rightarrow 2 zeros real

$$PID(p) = \frac{k}{T_i p} (1 + T_i p + T_i T_d p^2) = \frac{k}{T_i p} (1 + T_1 p)(1 + T_2 p)$$

$$\boxed{T_i = T_1 + T_2}$$

$$T_i T_d = T_1 T_2 \rightarrow T_d = \frac{T_1 T_2}{T_i} = \frac{T_1 T_2}{T_1 + T_2}$$

$$\boxed{T_d^{-1} = T_1^{-1} + T_2^{-1}}$$

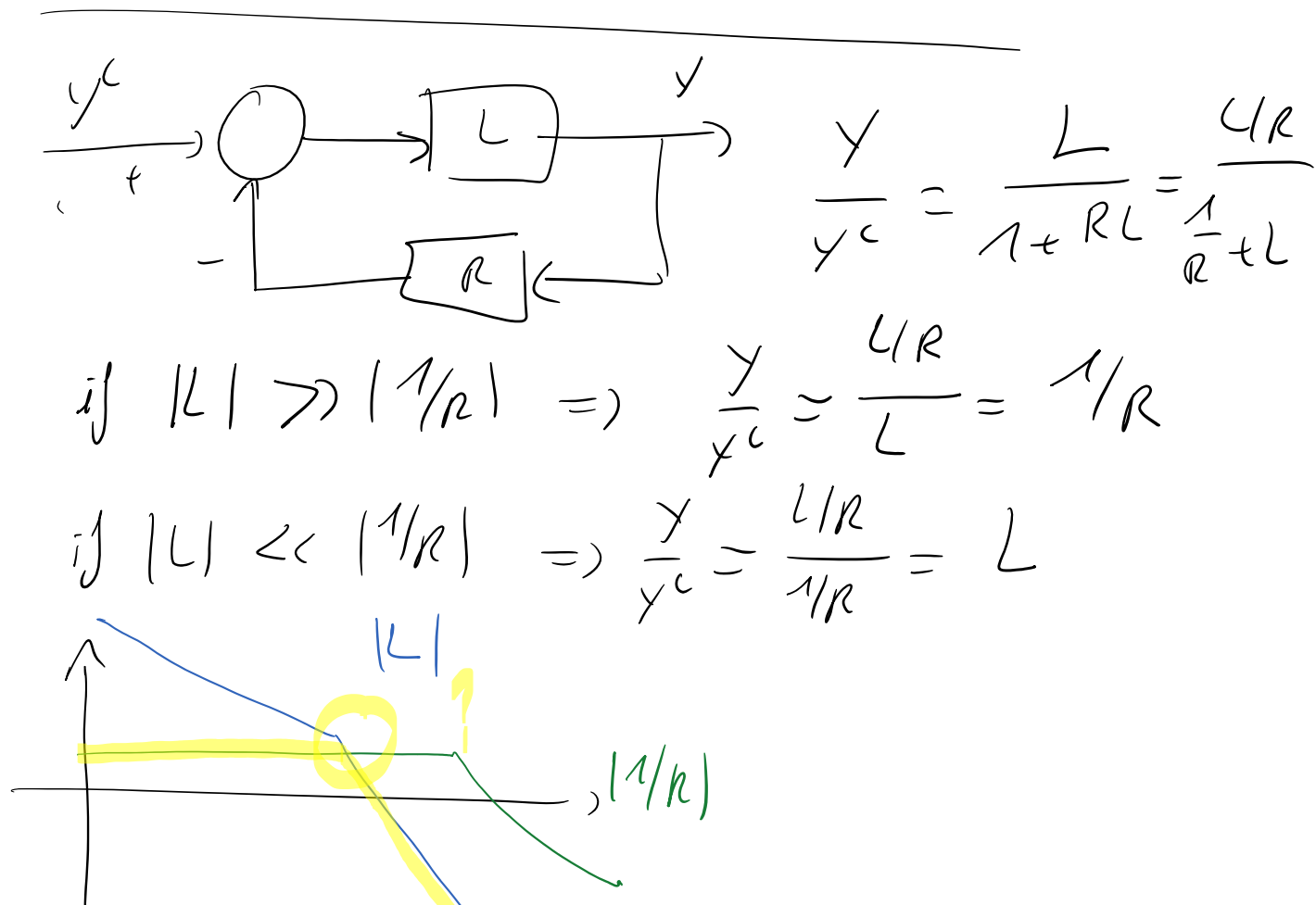
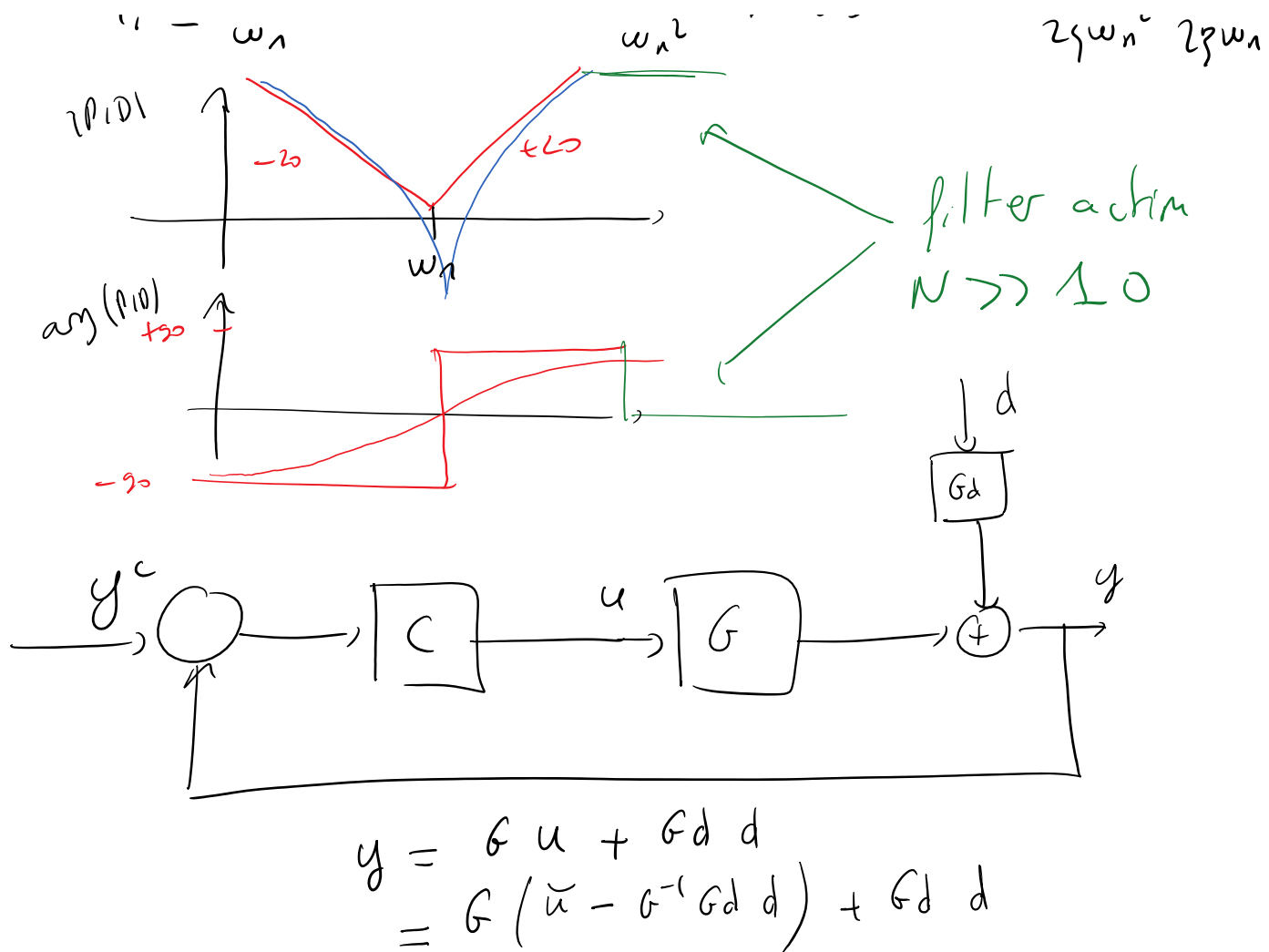


End case: $T_i < 4 T_d \rightarrow$ 2 zeros complex conjugate.

$$\frac{k}{T_i p} (1 + T_i p + T_i T_d p^2) = \frac{k}{T_i p} \left(1 + \frac{2\zeta}{\omega_n} p + \frac{p^2}{\omega_n^2} \right)$$

$$T_i = \frac{2\zeta}{\omega_n}$$

$$T_i T_d = \frac{1}{\omega_n^2} \rightarrow T_d = \frac{1}{T_i \omega_n^2} = \frac{\omega_n}{2\zeta \omega_n^2} = \frac{1}{2\zeta \omega_n}$$



|

