Essentials of MOSFETs

Unit 2: Essential Physics of the MOSFET

Lecture 2.4: The Square Law MOSFET

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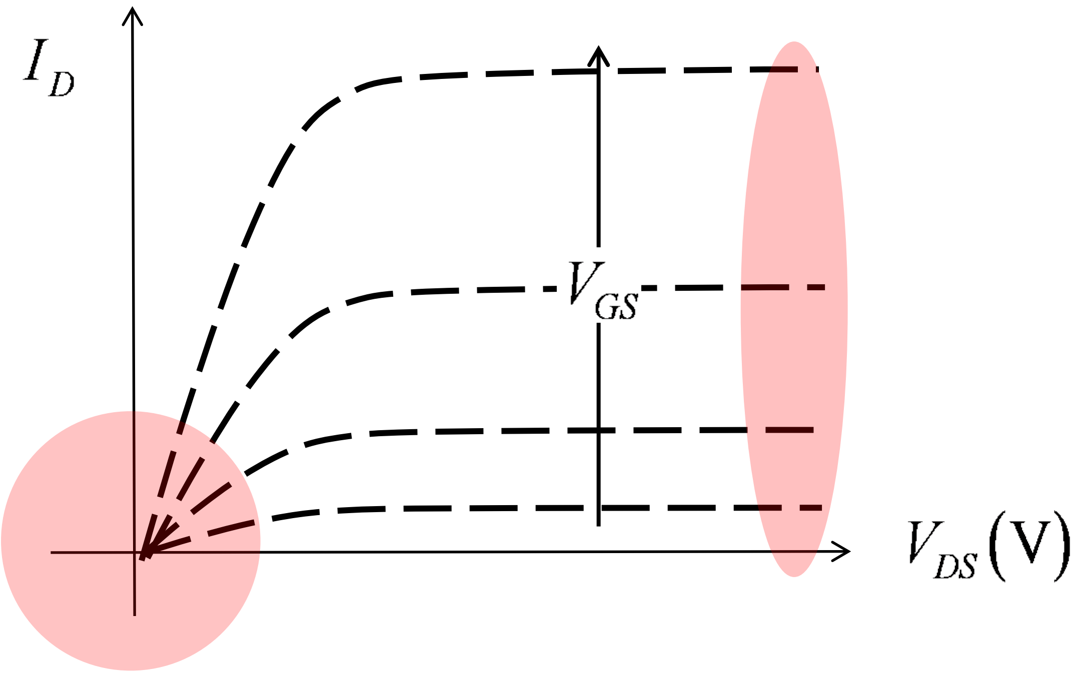
Square law MOSFET theory

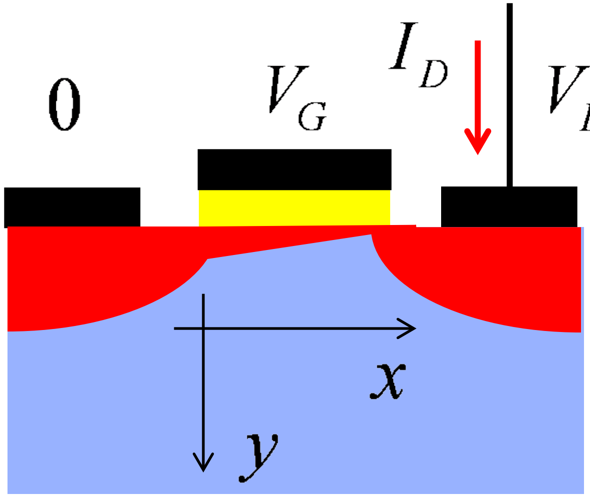


$$I_D = \frac{W}{L}\mu_n C_{ox}\left(V_{GS} - V_T\right)V_{DS}$$

$$I_D = \frac{W}{2L}\mu_n C_{ox}\left(V_{GS} - V_T\right)^2$$

Lundstrom: 2018

I-V formulation



$$I_D = W\left|Q_n(x)\right|\upsilon_x(x) \text{ Amperes}$$

$$\upsilon_x(x) = -\mu_n \mathcal{E}_x(x) = \mu_n\, dV/dx$$

$$I_D = W\left|Q_n(x)\right|\mu_n \frac{dV}{dx}$$

$$I_D dx = W\left|Q_n(V)\right|\mu_n dV$$

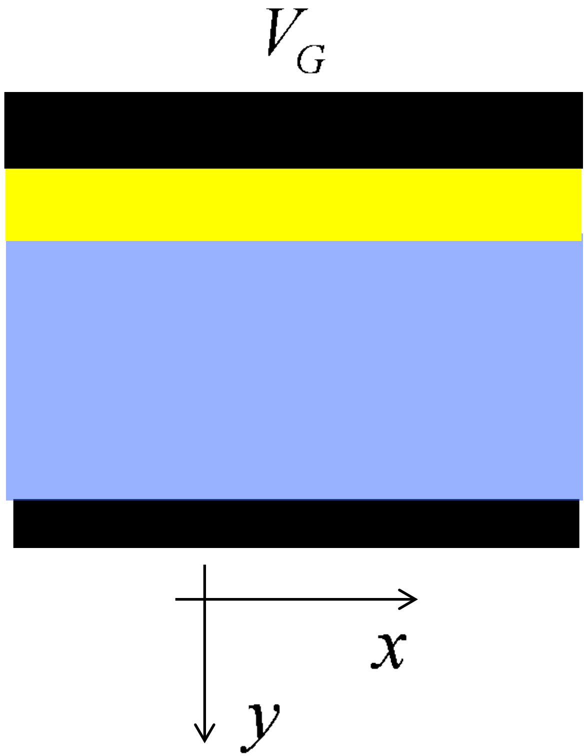
$$\boxed{I_D = -\frac{W}{L}\mu_n \int_0^{V_{DS}} \left|Q_n(V)\right| dV}$$

to include diffusion:

$$-\frac{dV}{dx} \rightarrow \frac{d(F_n/q)}{dx}$$

Lundstrom: 2018

1D MOS capacitor



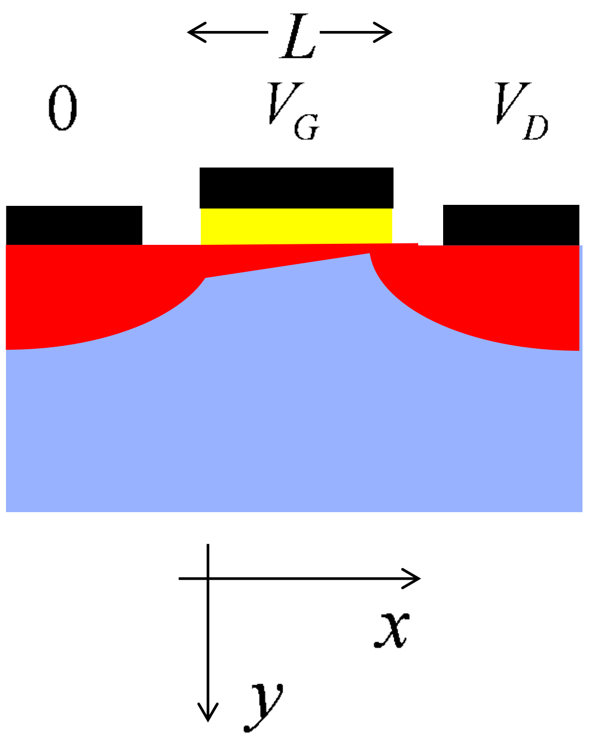
$$V_G - V_T \leq 0$$

$$Q_n \approx 0$$

$$V_G - V_T > 0$$

$$Q_n = -C_{ox}\left(V_G - V_T\right)$$

Gradual channel approximation



for $0 \le x \le L$

$$V = V(x)$$

$$V(0) = 0 \quad V(L) = V_D$$

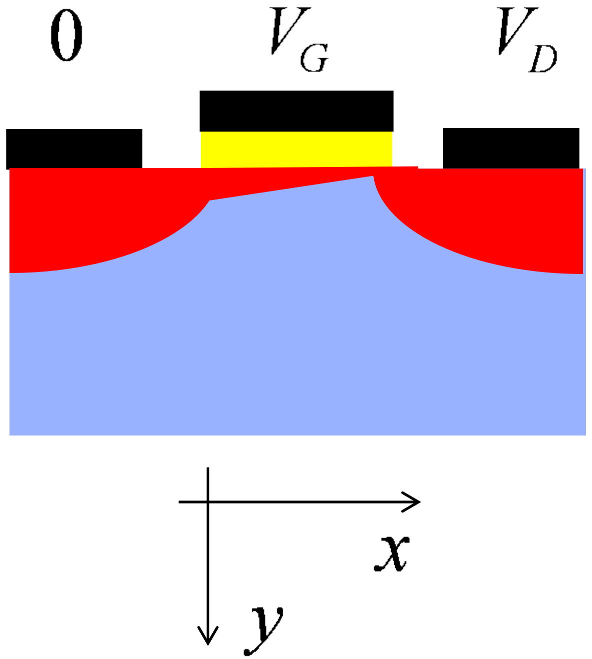
1D MOS-C: $Q_n = -C_{ox}\left(V_G - V_T\right)$

$$\text{GCA}: \mathcal{E}_x << \mathcal{E}_y$$

$$V_T \rightarrow V_T\left(x\right) = V_T + V\left(x\right)$$

$$\boxed{V_G - V_T > 0 \qquad Q_n(x) = -C_{ox}\left[V_{GS} - V_T - V(x)\right]}$$

IV relation



$$I_D = \frac{W}{L}\mu_n \int_0^{V_{DS}} \left|Q_n(V)\right| dV$$

$$I_D = \mu_n C_{ox} \frac{W}{L} \int_0^{V_D} \left[V_{GS} - V_T - V\right] dV$$

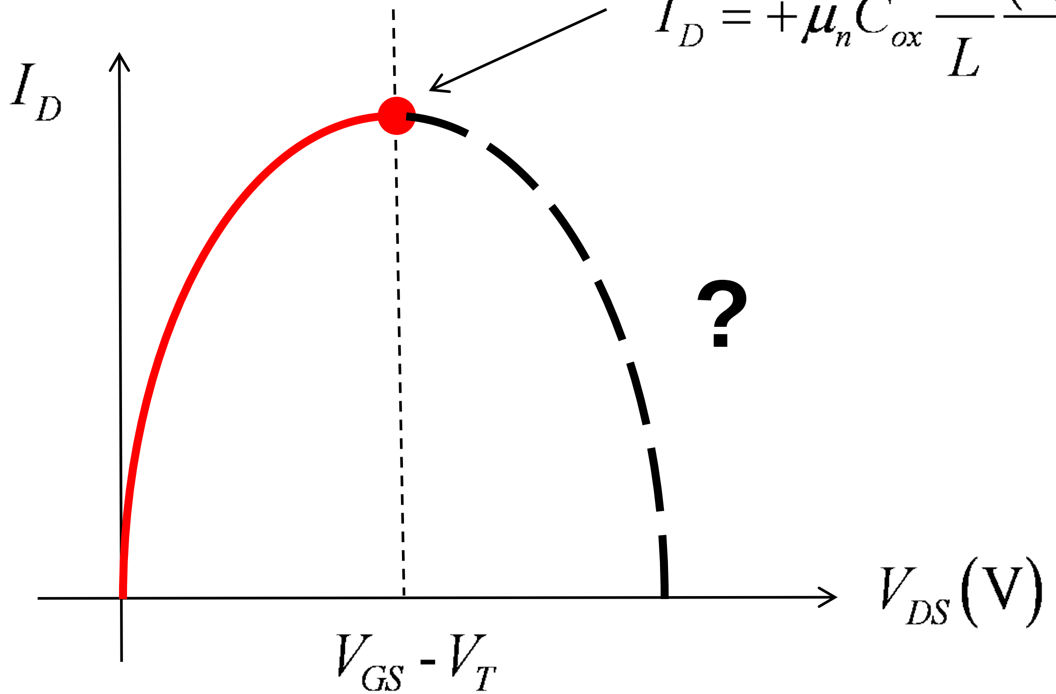
$$I_D = \mu_n C_{ox} \frac{W}{L} \left[\left(V_{GS} - V_T\right)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

(triode region of operation)

Lundstrom: 2018

Beyond pinch-off?

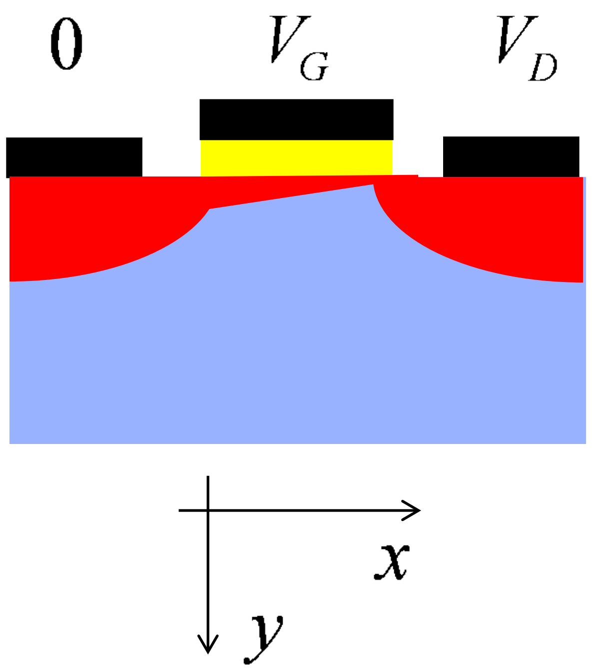
$$I_D = +\mu_n C_{ox} \frac{W}{L} \frac{\left(V_{GS} - V_T\right)^2}{2}$$



$$I_D = +\mu_n C_{ox} \frac{W}{L} \left[\left(V_{GS} - V_T\right) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

Lundstrom: 2018

Pinch-off



$$Q_n(L) = -C_{ox}\left[V_{GS} - V_T - V_D\right]$$

when $V_D = V_{GS} - V_T$,

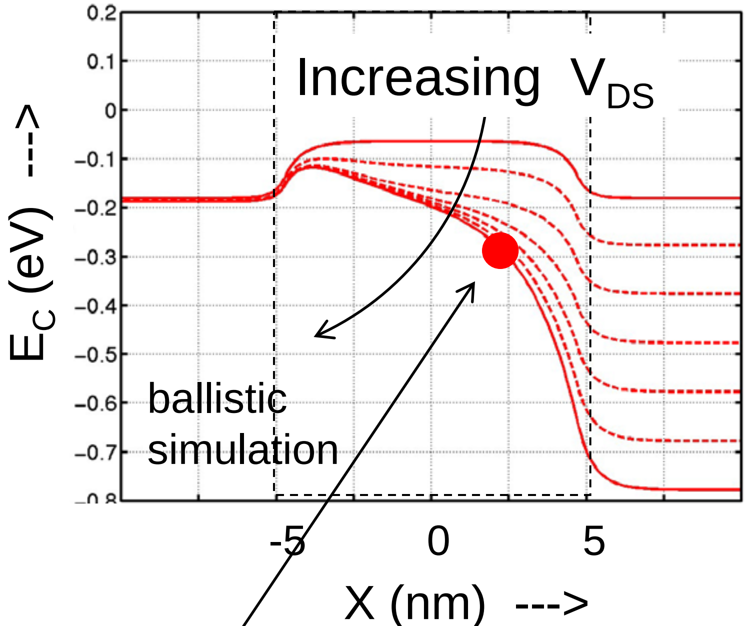
then $Q_n(L) = 0$

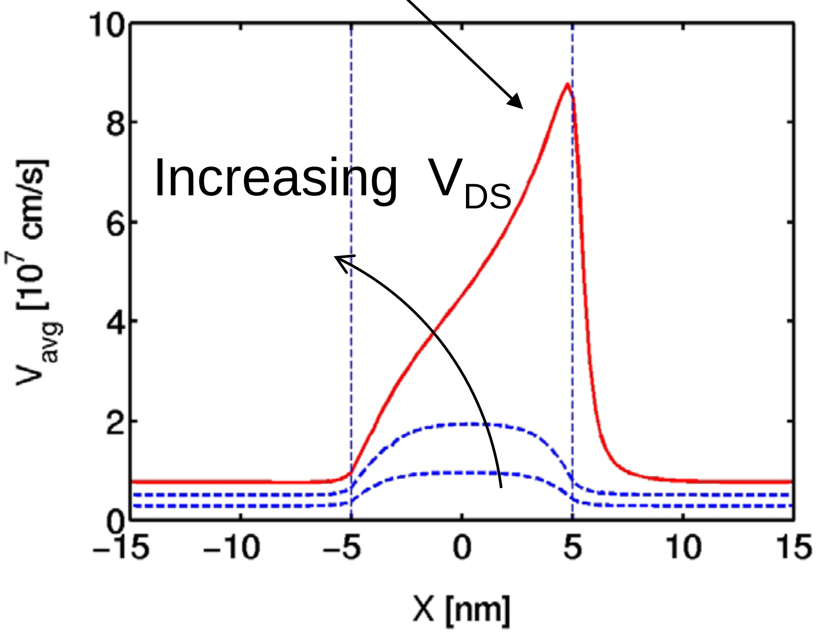
$\mathcal{E}_y >> \mathcal{E}_x$ GCA fails!

but current still flows!

Pinch off in a MOSFET

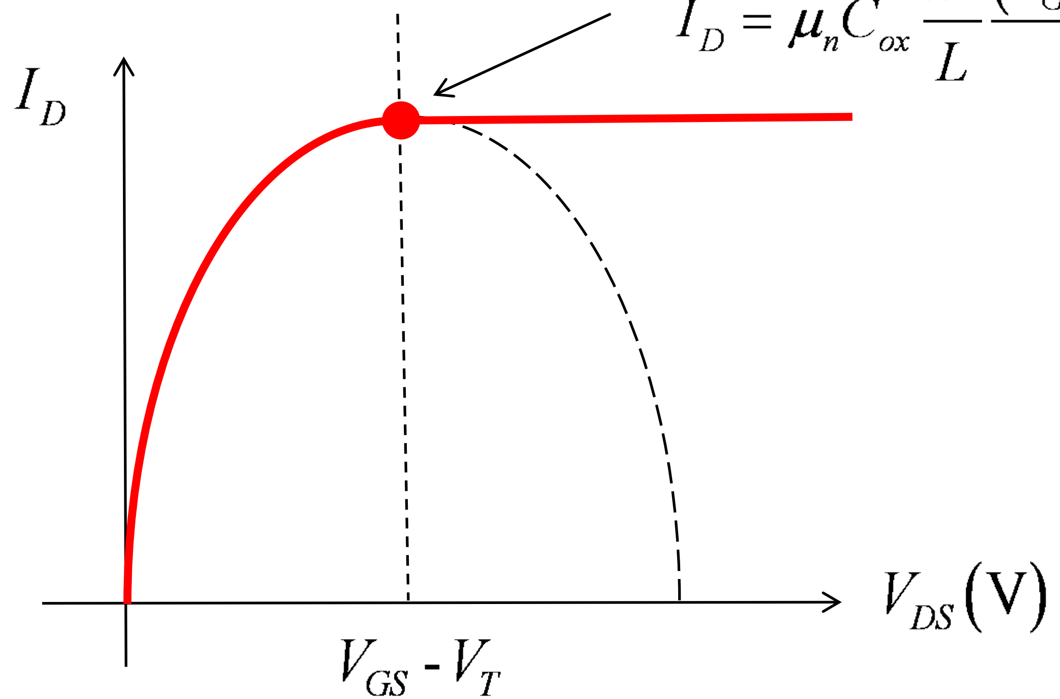
The electron velocity is very high in the pinch-off region. High velocity implies low inversion layer density (because I_D is constant). In the textbook model, we say $Q_i \approx 0$, but it is not really zero - just very small.





pinch-off point: where the electric field along the channel becomes very large. Note that electrons are simply swept across the high-field (pinched-off) portion at very high velocity.

IV beyond pinch-off

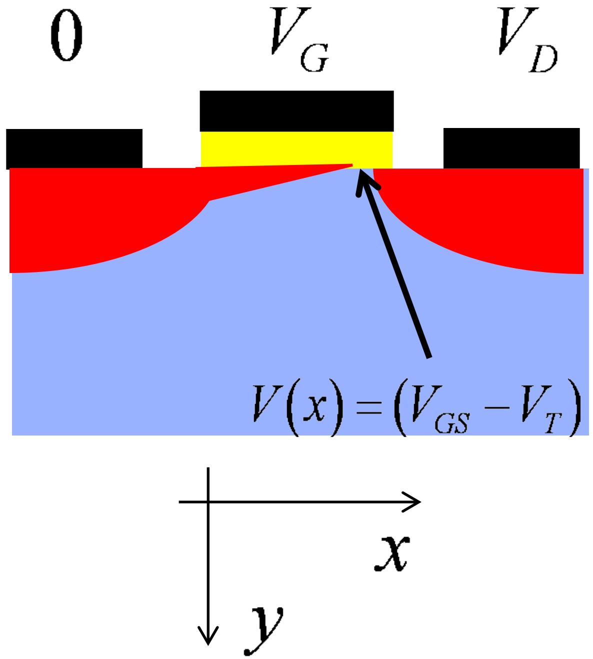


$$I_D = \mu_n C_{ox} \frac{W}{L} \frac{\left(V_{GS} - V_T\right)^2}{2}$$

$$I_D = \mu_n C_{ox} \frac{W}{L} \left[\left(V_{GS} - V_T\right) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

Lundstrom: 2018

Complete IV characteristic



$V_{GS} > V_T$

$V_{DS} < V_{GS} - V_T$

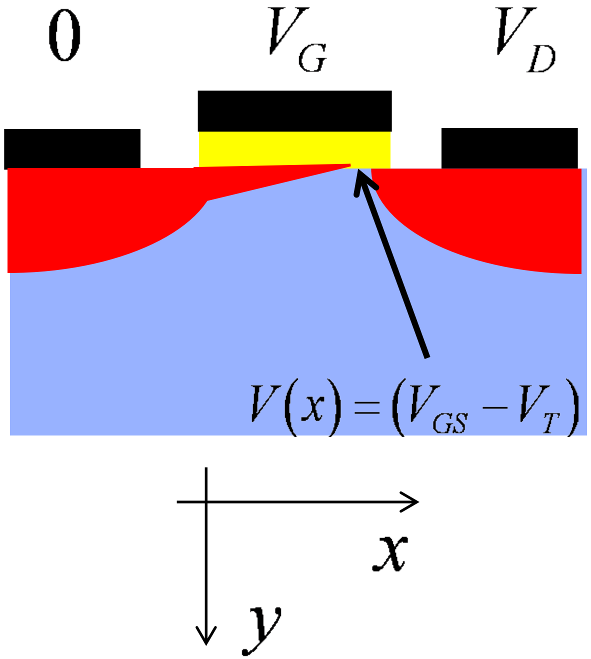
$$I_D = \mu_n C_{ox} \frac{W}{L}\left[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$V_{GS} > V_T$

$V_{DS} > V_{GS} - V_T$

$$I_D = \mu_n C_{ox} \frac{W}{2L}(V_{GS} - V_T)^2$$

Linear region



$$V_{GS} > V_T$$

$$V_{DS} < V_{GS} - V_T$$

$$I_D = \mu_n C_{ox} \frac{W}{L}\left[\left(V_{GS} - V_T\right) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$$V_{GS} > V_T$$

$$V_{DS} << V_{GS} - V_T$$

$$I_D = \mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_T\right) V_{DS}$$

Lundstrom: 2018

The electric field in the channel

small V_{DS}

$$I_D = \mu_n C_{ox} \frac{W}{L}\left(V_{GS} - V_T\right)V_{DS}$$

$$I_D = W\left|Q_n\left(x=0\right)\right|\left\langle \upsilon_x\left(x=0\right)\right\rangle$$

$$I_D = WC_{ox}\left(V_{GS} - V_T\right)\left(-\mu_n \mathcal{E}_x(0)\right)$$

$$\mathcal{E}_x(0) = -\frac{V_{DS}}{L}$$

large V_{DS}

$$I_D = \mu_n C_{ox} \frac{W}{2L}\left(V_{GS} - V_T\right)^2$$

$$I_D = W\left|Q_n\left(x=0\right)\right|\left\langle \upsilon_x\left(x=0\right)\right\rangle$$

$$I_D = WC_{ox}\left(V_{GS} - V_T\right)\left(-\mu_n \mathcal{E}_x(0)\right)$$

$$\mathcal{E}_x(0) = -\frac{\left(V_{GS} - V_T\right)}{2L}$$

Summary

Triode region

$$V_{GS} > V_T$$

$$V_{DS} < V_{GS} - V_T$$

$$I_D = \mu_n C_{ox} \frac{W}{L}\left[\left(V_{GS} - V_T\right)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

Beyond pinch-off region (saturation region)

$$V_{GS} > V_T$$

$$V_{DS} > V_{GS} - V_T$$

$$I_D = \mu_n C_{ox} \frac{W}{2L}\left(V_{GS} - V_T\right)^2$$

Linear region

$$V_{DS} << V_{GS} - V_T$$

$$I_D = \mu_n C_{ox} \frac{W}{L}\left(V_{GS} - V_T\right)V_{DS}$$

Next topic:

Modern MOSFETs are not square law devices, but this example is an illustration of a model that works smoothly from the linear to saturation region.

A full range IV characteristic for velocity saturated MOSFETs can be developed, but it is a bit more complicated.

In the next lecture, we will show how our two-piece velocity saturated model can be easily extended to a full range model.