2020.09.30 Cours 11

mardi 29 septembre 2020 13:22

$$\boxed{H(p) = \frac{2}{p^2 - 0{,}18p + 2{,}4}}$$

y and $\dot{y}$ are measured

① Spec 1. 1st-order behavior with $p_1 = -0{,}3$

1.1. State Model

1.2. Properties → controllability

1.3 Pole Placement (what are the expected poles)

1.4 Control Design - $u = -Kx + \ell^R y^R$

→ K

→ ℓ^R

$\boxed{1.1.}$ S. Model : $\boxed{\text{choice } 1}$: controller form.

$$\begin{cases} \dot{x} = \begin{pmatrix} 0 & 1 \\ -2{,}4 & 0{,}18 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \\ y = \begin{pmatrix} 2 & 0 \end{pmatrix} u \end{cases}$$

other option:

$x_1 = y$

$x_2 = \dot{y}$

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -2{,}4 & 0{,}18 \end{pmatrix} x + \begin{pmatrix} 0 \\ 2 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} x$$

$\boxed{1.2}$ Controllability.

$x \in \mathbb{R}^2$

$$\mathcal{C} = (B, AB) = \begin{pmatrix} 0 & 1 \\ 1 & 0{,}18 \end{pmatrix} \Rightarrow \text{rank}(\mathcal{C}) = 2.$$

controllable !

$\boxed{1.3}$ Pole Placement.

$x \in \mathbb{R}^2 \Rightarrow 2$ poles to be placed.

$P_1 = -0{,}3$ (because of the exercice)

$\boxed{P_2 = -3}$ (one decade faster) (personal choice here)

$\boxed{1.4}$ K?

we want the eigenvalues of $A - BK$ equal to $-0{,}3$ and -3.

⇒ the expected characteristic polynomial is :

$$P_c(p) = (p+3)(p+0{,}3) = p^2 + \underline{3{,}3}p + \underline{0{,}9}$$

$$P_c(p) = (p+3)(p+0,3) = p^2 + \underline{3,3}\,p + \underline{0,9}$$

$$(A - BK) = \begin{pmatrix} 0 & 1 \\ -2,4 & 0,18 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} k_0 & k_1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ -2,4-k_0 & 0,18-k_1 \end{pmatrix}$$

$$P_{C_{A-BK}}(p) = p^2 + (\underline{k_1 - 0,18})\,p + (\underline{k_0 + 2,4})$$

$$\boxed{\begin{aligned} k_1 &= 3,3 + 0,18 = 3,48 \\ k_0 &= 0,9 - 2,4 = -1,5 \end{aligned}}$$

The closed-loop is now:

$$\frac{y}{y^R} = \frac{2\,l^R}{p^2 + 3,3p + 0,9} \Rightarrow \boxed{l^R = \frac{0,9}{2} = 0,45}$$

$$\varepsilon = \hat{x} - x$$

$$\dot{\varepsilon} = \dot{\hat{x}} - \dot{x}$$

$$= (A\hat{x} + \cancel{Bu} + L(\underbrace{y}_{Cx} - C\hat{x})) - (Ax + \cancel{Bu})$$

$$= (A - LC)(\hat{x} - x)$$

$$\boxed{\dot{\varepsilon} = (A - LC)\,\varepsilon}$$

$\rightarrow$ eig$(A - LC)$ have to be Re < 0

but eig$(A - LC) = $ eig$\left((A - LC)^T\right)$ duality

$= $ eig$(A^T - C^T L^T)$

(Remind for pole placement: "K" so that $A - BK$ has the eigenvalues we want)

Impact of the real control law: $u = -K\hat{x} + l\,y^R$

$\rightarrow$ augmented state: $X = \begin{pmatrix} x \\ \varepsilon \end{pmatrix}$

$\rightarrow$ Question: what is the $\dot{X}$

$\hookrightarrow$ $\varepsilon = \hat{x} - x$
$\hat{x} = \varepsilon + x$

→ Question: what is the $\dot{X}$? ↳ $\hat{x} = \varepsilon + x$

$$\begin{cases} \dot{X} = \begin{pmatrix} \dot{x} \\ \dot{\varepsilon} \end{pmatrix} = \begin{pmatrix} Ax + B(-K\hat{x} + \rho y^R) = Ax - BK(\varepsilon + x) + B\rho y^R \\ (A - LC)\varepsilon \end{pmatrix} \\ \quad = \begin{pmatrix} A - BK & -BK \\ 0 & A - LC \end{pmatrix} \begin{pmatrix} x \\ \varepsilon \end{pmatrix} + \begin{pmatrix} B\rho \\ 0 \end{pmatrix} y^R \\ y = \begin{pmatrix} C & 0 \end{pmatrix} X \end{cases}$$

<u>Looking for the margins.</u>

we have: $u = -K\hat{x} + \rho y^R$

we need: $u = C(Ry^R - Hy)$

$$\dot{\hat{x}} = (A - LC)\hat{x} + Bu + Ly$$
$$= (A - LC)\hat{x} + (-BK\hat{x}) + B\rho y^R + Ly$$
$$= (A - LC - BK)\hat{x} + B\rho y^R + Ly$$

$\mathcal{L}$ ↳

$$\hat{X} = (pI - (A - LC - BK))^{-1}(B\rho Y^R + LY)$$

$$U = -K\hat{X} + \rho Y^R$$

$$U = -K(pI - (A - LC - BK))^{-1}(B\rho Y^R + LY) + \rho Y^R$$
$$= C(RY^R - HY)$$

$$\boxed{CH = K(pI - (A - BK - LC))^{-1} L}$$

→ The margin can be determined using CHG ← the system