Unit 3: MOS Electrostatics

Lecture 3.7: The Mobile Charge vs. Gate Voltage

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MOSFET drain current

$$I_{DS}/W = -Q_n(V_{GS})\langle \upsilon_x(V_{DS})\rangle$$

We have been discussing Q_S and Q_D, but we need Q_n as a function of **surface potential** and **gate voltage**.

$$Q_S = Q_D + Q_n \ \ \mathrm{C/cm^2}$$

$Q_n(\psi_S)$ ← - - - last lecture

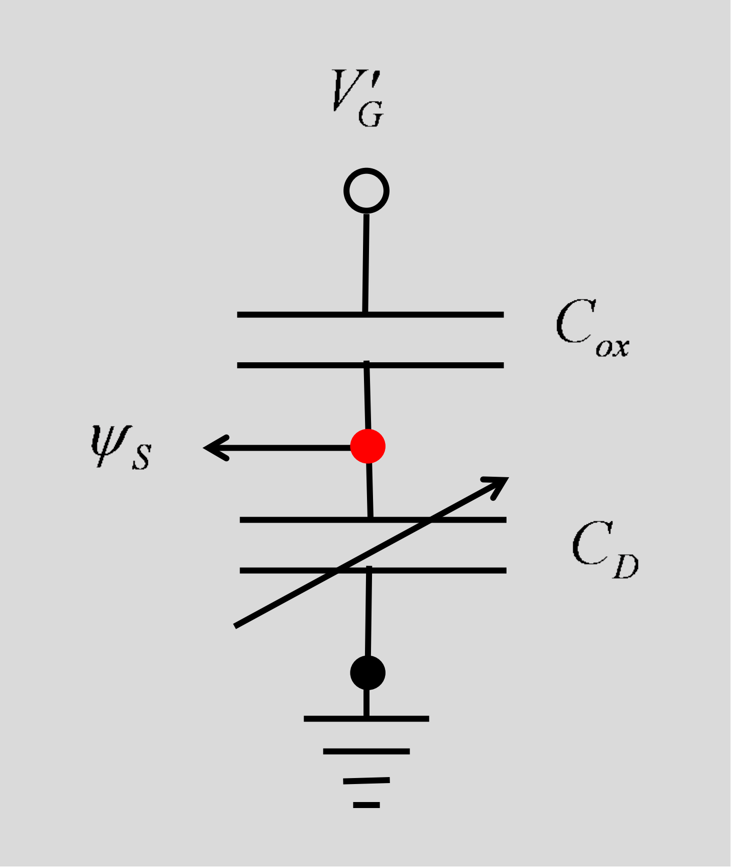
$Q_n(V_G)$ ⟵ **this lecture**

1) Subthreshold charge vs. **gate voltage**

$$V_G' = V_G - V_{FB} = -\frac{Q_S(\psi_S)}{C_{ox}} + \psi_S$$

$$\psi_S = \frac{V_G'}{m}$$

$$m = 1 + C_D / C_{ox}$$



Mobile charge vs. gate voltage: Subthreshold

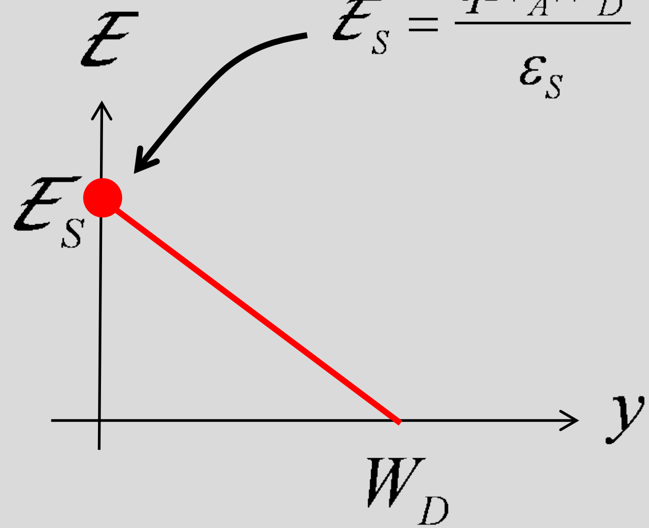
$$C_D = \frac{\varepsilon_S}{W_D}$$

$$\mathcal{E}_S = \frac{qN_A W_D}{\varepsilon_S} = \frac{qN_A}{C_D}$$

$$m = 1 + C_D / C_{ox}$$

$$C_D = (m-1) C_{ox}$$

$$\mathcal{E}_S = \frac{qN_A W_D}{\varepsilon_S}$$



$$\mathcal{E}_S = \frac{qN_A}{(m-1)C_{ox}}$$

Subthreshold charge vs. gate voltage

$$Q_n(\psi_S) = -qn_B e^{q\psi_S/k_BT}\left(\frac{k_BT/q}{\mathcal{E}_S}\right)$$

$$n_B = n_i^2/N_A$$

$$\mathcal{E}_S = \frac{qN_A}{(m-1)C_{ox}}$$

$$Q_n(\psi_S) = -(m-1)C_{ox}\left(\frac{k_BT}{q}\right)\left(\frac{n_i}{N_A}\right)^2 e^{q\psi_S/k_BT}$$

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$$\psi_S = V_G'/m$$

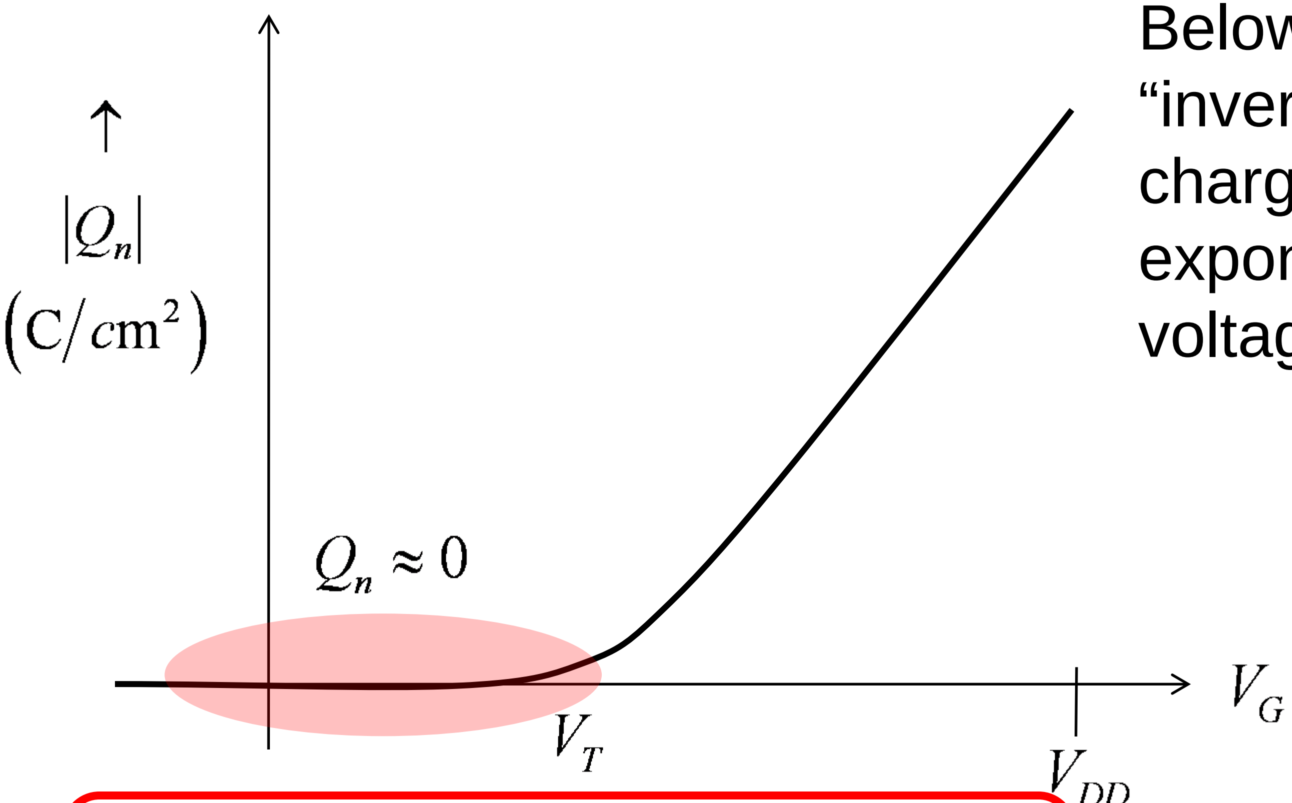
$$\psi_B = \frac{k_BT}{q}\ln\left(\frac{N_A}{n_i}\right)$$

$$\left(\frac{n_i}{N_A}\right)^2 = e^{-2q\psi_B/k_BT}$$

$$2\psi_B = V_T'/m$$

$$\left(\frac{n_i}{N_A}\right)^2 = e^{-qV_T/mk_BT}$$

Subthreshold charge vs. gate voltage



Below threshold, the "inversion" layer charge increases exponentially with gate voltage.

$$Q_n(V_G) = -(m-1)C_{ox}\left(\frac{k_B T}{q}\right)e^{q(V_G - V_T)/mk_B T}$$

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2) Mobile charge vs. gate voltage: Above threshold

$$V_G = V_{FB} - \frac{Q_S(\psi_S)}{C_{ox}} + \psi_S$$

$$V_G = V_{FB} - \frac{Q_D(\psi_S)}{C_{ox}} - \frac{Q_n(\psi_S)}{C_{ox}} + \psi_S$$

◆ $$V_T = V_{FB} - \frac{Q_D(2\psi_B)}{C_{ox}} + 2\psi_B$$

$$V_G = V_{FB} - \frac{Q_D(2\psi_B + \delta\psi_S)}{C_{ox}} - \frac{Q_n(2\psi_B + \delta\psi_S)}{C_{ox}} + 2\psi_B + \delta\psi_S \qquad \delta\psi_S << 2\psi_B$$

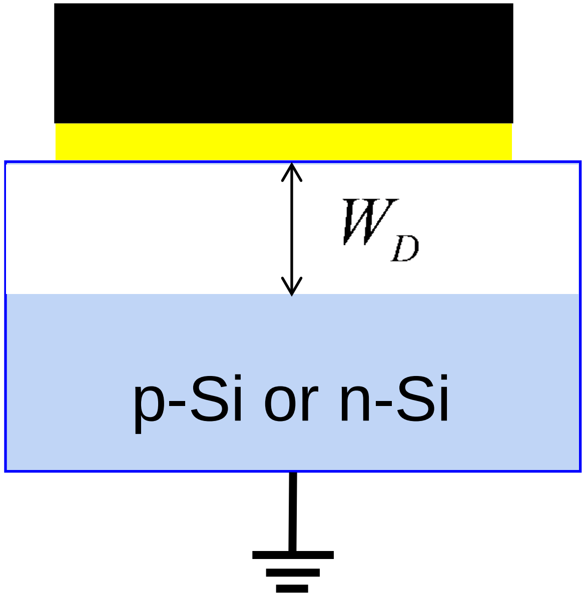
$$V_G \approx V_{FB} - \frac{Q_D(2\psi_B)}{C_{ox}} - \frac{Q_n(2\psi_B + \delta\psi_S)}{C_{ox}} + 2\psi_B$$

◆ ◆ ◆

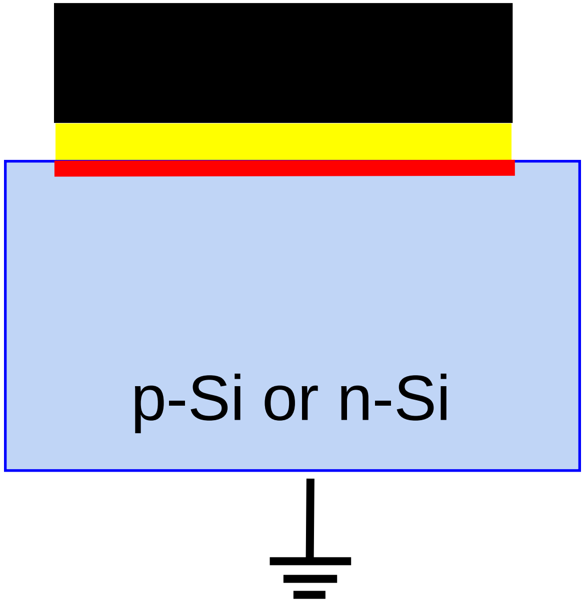
$$\boxed{Q_n \approx -C_{ox}(V_G - V_T)}$$

About the gate capacitance

$V_G < V_T$



$V_G >> V_T$



$$\frac{1}{C_G} = \frac{1}{C_{ox}} + \frac{1}{C_S} \qquad C_S = C_D = \frac{\varepsilon_S}{W_D}$$

$$C_G(\text{depl}) < C_{ox}$$

$$\frac{1}{C_G} = \frac{1}{C_{ox}} + \frac{1}{C_S} \quad C_S = C_S(\text{inv}) \equiv \frac{\varepsilon_S}{t_{inv}}$$

$$C_G(\text{inv}) < C_{ox}$$

Semiconductor capacitance

$$\frac{1}{C_G} = \frac{1}{C_{ox}} + \frac{1}{C_S} \qquad C_S \equiv -\frac{dQ_S}{d\psi_S}$$

Depletion:

$$Q_S \approx Q_D$$

$$C_S = C_D = \frac{\varepsilon_S}{W_D}$$

Strong inversion:

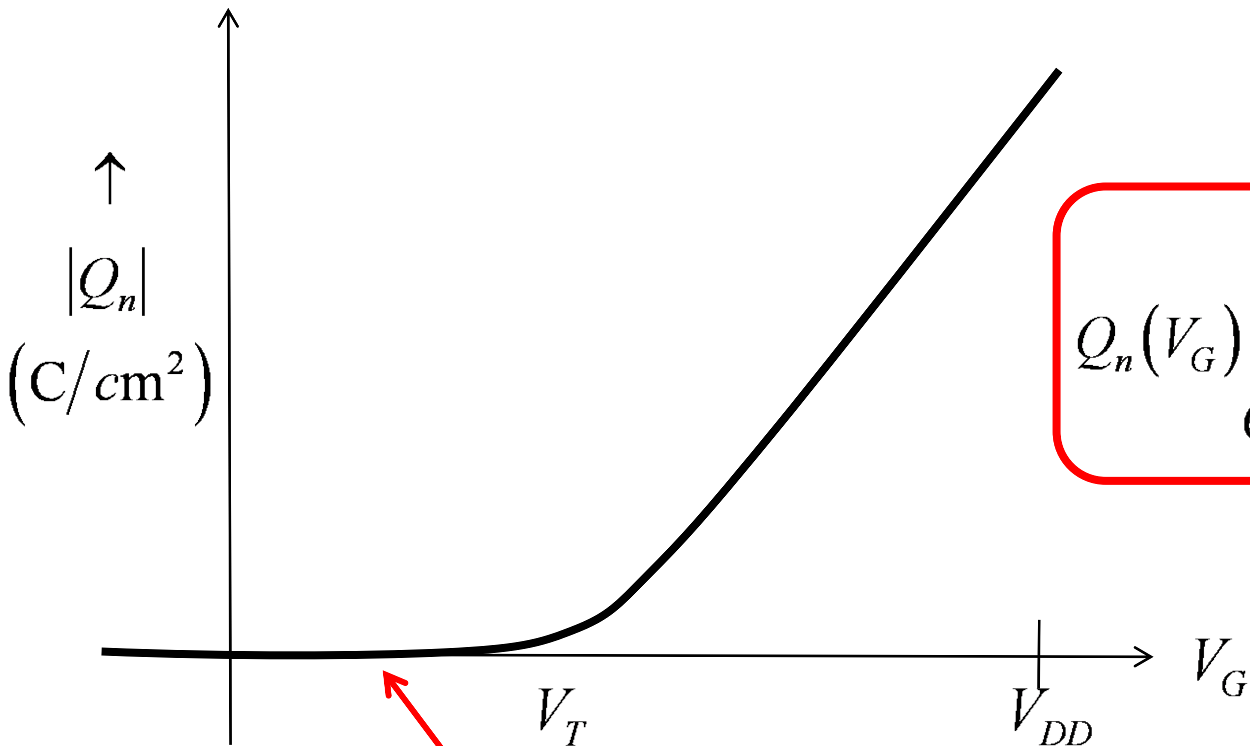
$$Q_S \approx Q_n \propto e^{q\psi_S/2k_BT}$$

$$C_S = \frac{|Q_n|}{2k_BT/q}$$

$$C_S(\text{inv}) \equiv \frac{\varepsilon_S}{t_{inv}}$$

$$t_{inv} << W_D$$

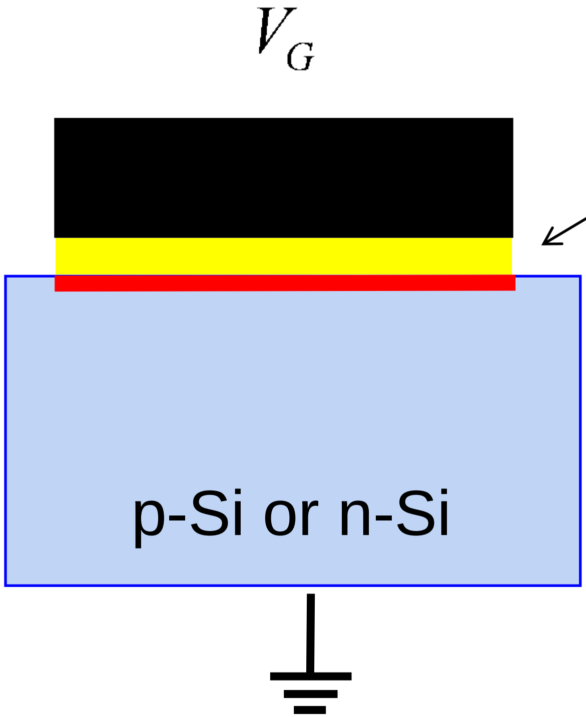
Mobile charge vs. gate voltage



$$V_G >> V_T$$
$$Q_n(V_G) = -C_{inv}(V_G - V_T)$$
$$C_{inv} < C_{ox}$$

$$V_G < V_T: \quad Q_n(V_G) = -(m-1)C_{ox}\left(\frac{k_B T}{q}\right)e^{q(V_G - V_T)/mk_B T}$$

Equivalent Oxide Thickness (EOT)



high-k gate insulators

$\kappa_{ins}, t_{ins} \qquad \kappa_{ins} > \kappa_{ox}$

$$C_{ins} \equiv \frac{\kappa_{ins}\varepsilon_0}{t_{ins}} \qquad C_{ins} \equiv \frac{\kappa_{ox}\varepsilon_0}{EOT}$$

$$\boxed{EOT \equiv \frac{\kappa_{ox}}{\kappa_{ins}} t_{ins}}$$

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Capacitance Equivalent Thickness (CET)

$$\frac{1}{C_G(\mathrm{inv})} = \frac{1}{C_{ox}} + \frac{1}{C_S(\mathrm{inv})}$$

EOT

$$C_G(\mathrm{inv}) = \frac{C_{ox} C_S(\mathrm{inv})}{C_{ox} + C_S(\mathrm{inv})} < C_{ox}$$

$$\boxed{C_G(\mathrm{inv}) \equiv \frac{\kappa_{ox}\varepsilon_0}{CET}}$$

Note: We will frequently refer to the gate capacitance in inversion, $C_G(\mathrm{inv})$, as just C_{inv}. Remember that $C_S(inv)$ is just part of the total gate capacitance in inversion..

Example

On-current conditions: $Q_n = -q \times 10^{13}\ \mathrm{C/cm^2}$

Oxide thickness: $t_{ox} = 1\ \mathrm{nm} = 10^{-7}\ \mathrm{cm}$ $\kappa_{ox} = 3.9$

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = 3.45 \times 10^{-6}\ \mathrm{F/cm^2}$$

$$C_S(\mathrm{inv}) = \frac{-Q_n}{2(k_B T/q)} = 30.8 \times 10^{-6}\ \mathrm{F/cm^2}$$

$$\frac{1}{C_G(\mathrm{inv})} = \frac{1}{C_{ox}} + \frac{1}{C_S(\mathrm{inv})}$$

$$= \frac{1}{3.45 \times 10^{-6}} + \frac{1}{30.8 \times 10^{-6}}$$

$$C_G(\mathrm{inv}) = 3.10 \times 10^{-6}\ \mathrm{F/cm^2}$$

$$C_G(\mathrm{inv}) = C_{inv} = 0.90 C_{ox}$$

$$C_G(\mathrm{inv}) \equiv \frac{\kappa_{ox}\varepsilon_0}{CET} \quad CET = 1.1\ \mathrm{nm}$$

Why is it hard to bend the bands more than $2\psi_B$?

below threshold:

$$C_S = \frac{d(-Q_D)}{d\psi_S} = C_D < C_{ox}$$

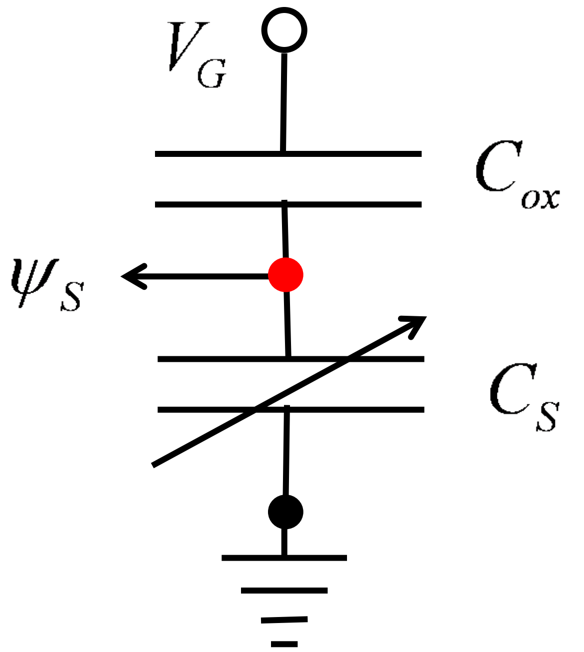
$$m = 1 + C_S / C_{ox} \approx 1 - 1.4$$

above threshold:

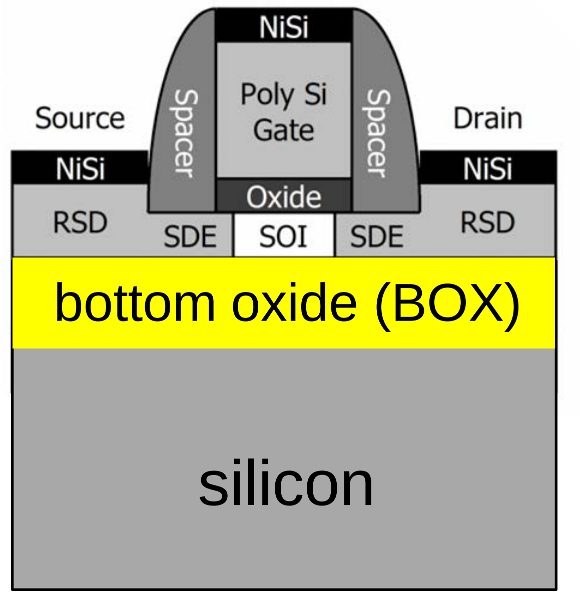
$$C_S = \frac{\varepsilon_S}{t_{inv}} >> C_{ox}$$

$$m >> 1$$

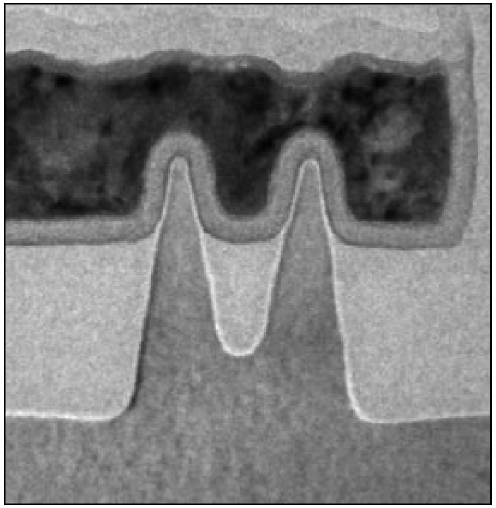
$$\Delta\psi_S = \Delta V_G \frac{C_{ox}}{C_{ox} + C_S} = \frac{\Delta V_G}{m}$$



Fully depleted ultra thin body (UTB) MOS structures



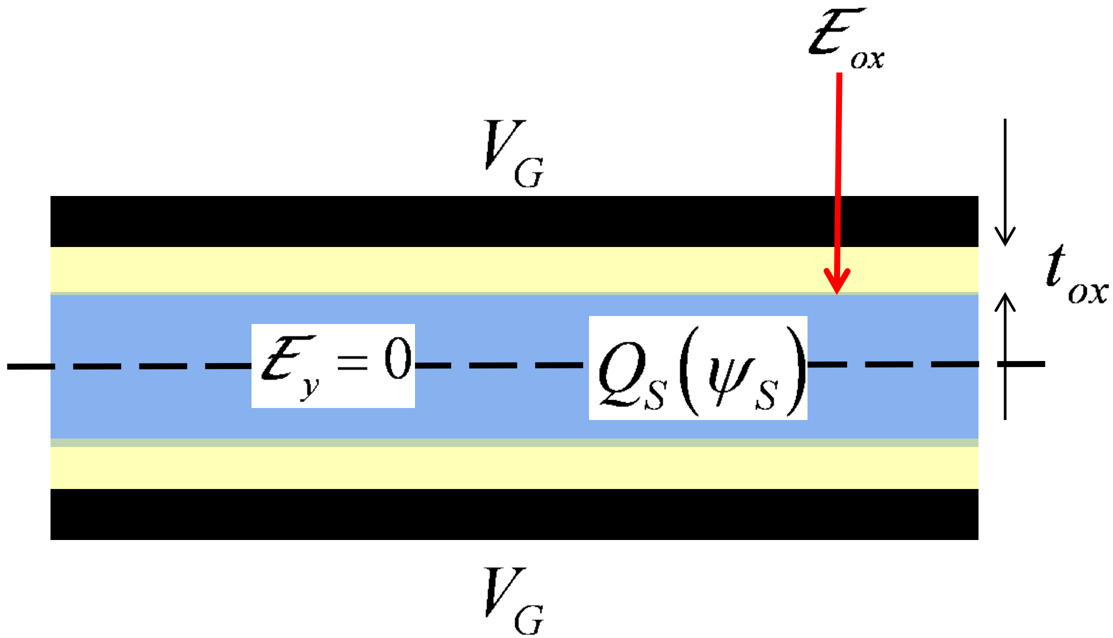
(ETSOI: Source: IBM, 2009)



(FinFET: Source: Intel, 2015)

Does anything change for these UTB MOS structures?

3) Mobile charge vs. gate voltage: Subthreshold UTB



$$\varepsilon_{ox}\mathcal{E}_{ox} = -Q_S(\psi_S)/2$$

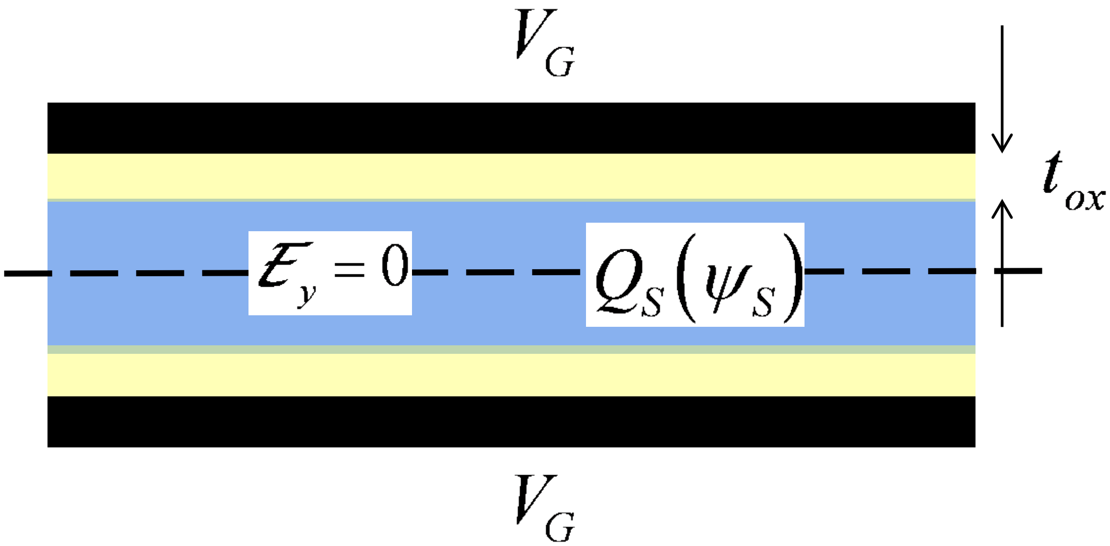
$$\Delta V_{ox} = \mathcal{E}_{ox} t_{ox}$$
$$= -Q_S(\psi_S)/2C_{ox}$$

$$V_G = \Delta V_{ox} + \psi_S$$

$$\boxed{V_G = -\frac{Q_S(\psi_S)}{2C_{ox}} + \psi_S}$$

Two gates, twice the capacitance.

Mobile charge vs. gate voltage: Subthreshold UTB



subthreshold

$$V_G = -\frac{Q_S(\psi_S)}{2C_{ox}} + \psi_S \qquad Q_S(\psi_S) \approx 0 \rightarrow V_G \approx \psi_S$$

$$\psi_S = \frac{V_G}{m}$$

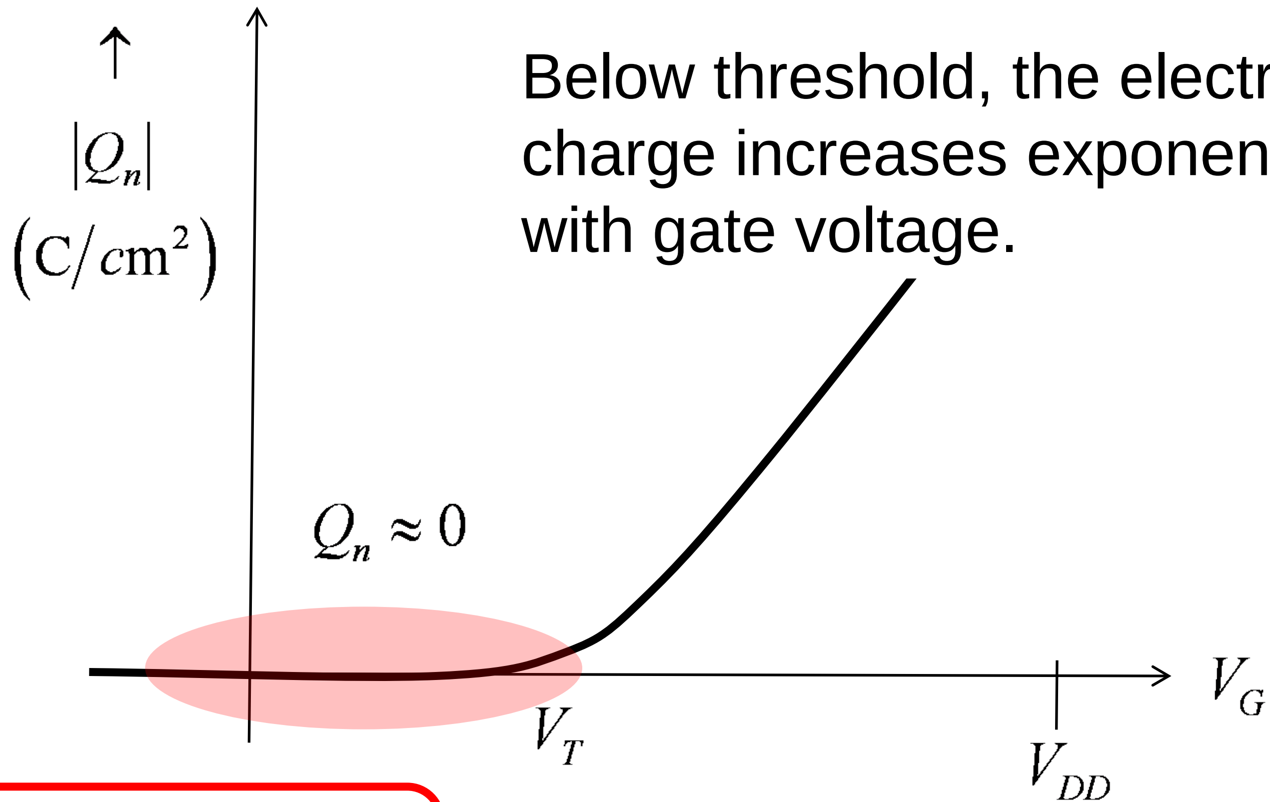
$$\boxed{m = 1}$$

Aside

$$SS = \left(\left. \frac{\partial \log_{10} I_{DS}}{\partial V_{GS}} \right|_{V_{DS}} \right)^{-1} = 2.3m \frac{k_B T}{q}$$

***m* = 1 for fully depleted structures.**

Subthreshold charge vs. gate voltage



Below threshold, the electron charge increases exponentially with gate voltage.

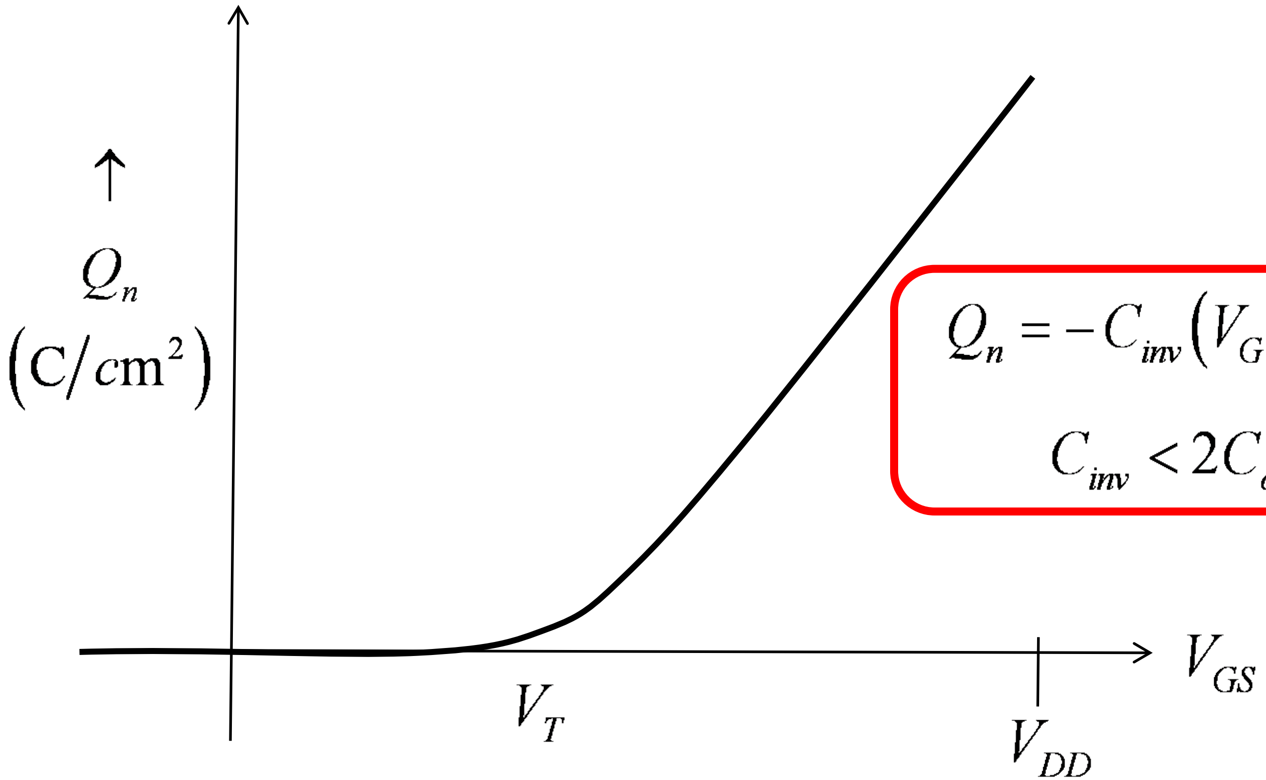
$$\boxed{Q_n(V_G) \propto e^{q(V_G - V_T)/k_B T}}$$

(FD UTB)

$$Q_n(V_G) \propto e^{q(V_G - V_T)/m k_B T}$$

(bulk MOS)

4) **Above threshold** charge vs. gate voltage



$$Q_n = -C_{inv}\left(V_G - V_T\right)$$

$$C_{inv} < 2C_{ox}$$

$$\frac{1}{C_{inv}} = \frac{1}{2C_{ox}} + \frac{1}{C_S(\mathrm{inv})}$$

$$C_S = \frac{d\left(-Q_S\right)}{d\psi_S}$$

Summary

FD UTB

$$V_G << V_T: \quad Q_n(V_G) = -C_Q\left(\frac{k_B T}{q}\right) e^{q(V_G - V_T)/k_B T}$$

$$V_G >> V_T: \quad Q_n(V_G) = -C_{inv}(V_G - V_T) \quad C_{inv} < 2C_{ox}$$

bulk

$$V_G << V_T: \quad Q_n(V_G) = -(m-1)C_{ox}\left(\frac{k_B T}{q}\right) e^{q(V_G - V_T)/mk_B T}$$

$$V_G >> V_T: \quad Q_n = -C_{inv}(V_G - V_T) \qquad C_{inv} < C_{ox}$$

Next topic

We have considered 1D MOS electrostatics – in the direction normal to the channel.

We now understand how the mobile charge varies with gate voltage above and below threshold.

Next Lecture: How does two-dimensional electrostatics affect MOSFETs?