2A - Automatique

Chapter 3

Control Science (AUT)

Frequency-domain approach, II

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Preamble

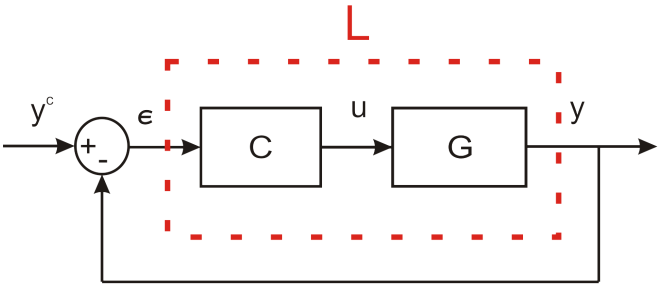
About this course

Course outline

- Stability margins
- Simplified Nyquist Criterion
- About performances
- About errors : steady-state error and dynamic error

Outline

Don't forget the Nyquist Criterion
General forumlation



- Let us denote with P the number of poles of $L(p)$ with a positive real part.

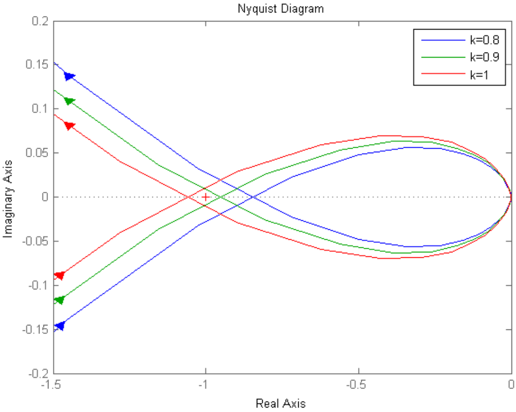
Le critère de Nyquist

The transfer $S = \frac{1}{1+L}$ is asymptotically stable if and only if the Nyquist plot of L encircles P times the point -1 counter-clockwisely !

Introduction Example
Do not go too close to -1

$$L(p) = \frac{k}{p(1+\tau_2 p)(\tau p + 1)} \mathrm{e}^{-Tp}$$

- k gain, τ, τ_2 time constants, T delay.



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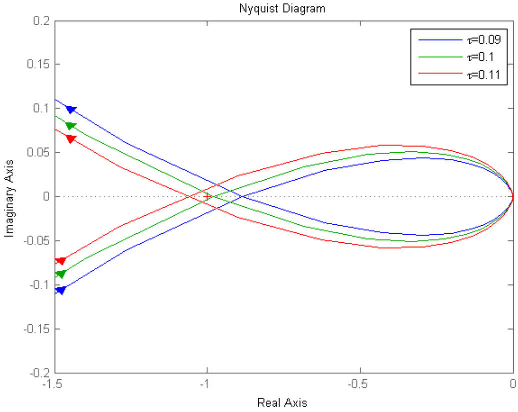

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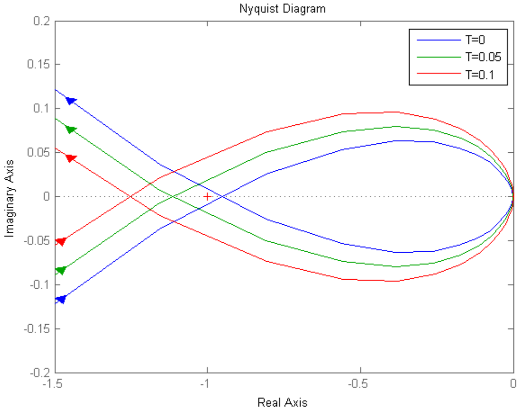
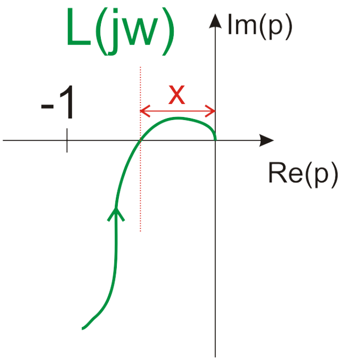


Stability margins
For a robustness issue

Gain margin

- What is the maximum gain by which the system can be multiplied ?
- Stability margin : value in dB



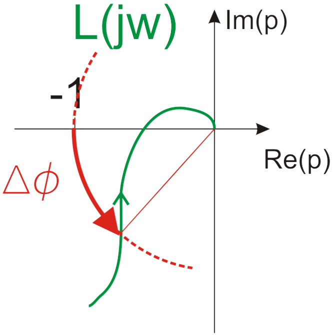
$$\arg[L(j\omega_1)] = -180$$

- $|L(j\omega_1)| = x$
- The gain margin is :

$$\Delta G = -20\log(x)$$

Phase margin

- What is the maximum phase shift allowed ?
- Phase margin : value in degrees



- $|L(j\omega_0)| = 1$
- The phase margin is

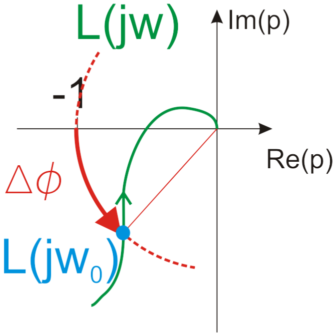
$$\Delta\Phi = \arg[L(j\omega_0)] - (-180)$$

Stability margins

For a robustness issue

Delay margin

- What is the maximum delay allowed ?
- Delay margin : value in seconds



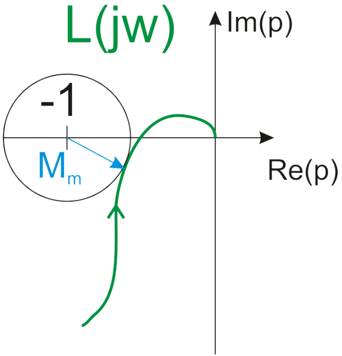
- $|L(j\omega_0)| = 1$
- $\Delta\Phi$: the phase margin (en radians !)
- The delay margin is :

$$\Delta\tau = \frac{\Delta\Phi}{\omega_0}$$

Stability margins
For a robustness issue

Modulus margin

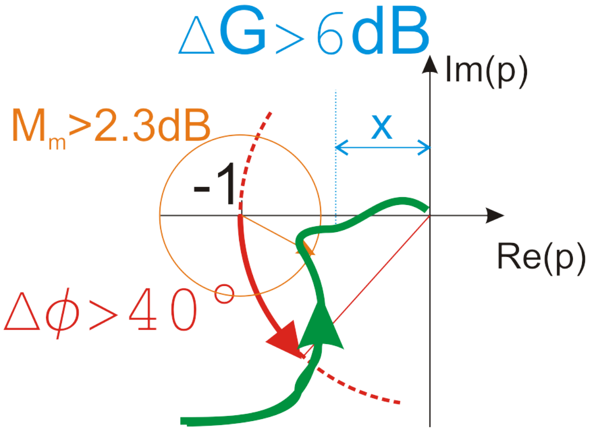
- The least known, but still the most important for robustness !
- Modulus margin : corresponds to the maximum of the complementary sensitivity function



- The modulus margin is :

$$M_m = \max_{\omega} |T(jw)|$$

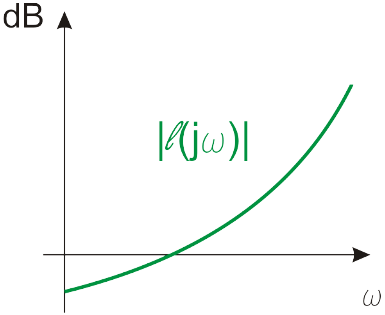
The Performance Robustness template
The *relative* importance of margins



The Performance Robustness template
Uncertainty handling

- Let us define an upper bound of the relative uncertainty :

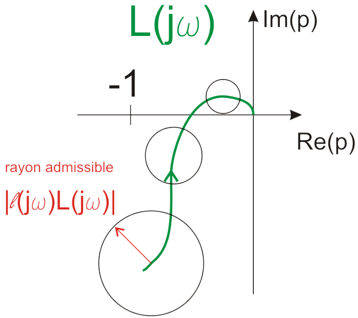
$$\left|\frac{\Delta L(j\omega)}{L(j\omega)}\right| \leq |l(j\omega)|$$



- The more the frequency increases, the more noise there is.

The Performance Robustness template
The impact on the Nyquist plot

- We have $L = CG$, C controller : no uncertainty
- All the incertitude on ΔL comes from ΔG



- We must have :

$$|l(j\omega)||L(j\omega)| \leq |L(j\omega) - (-1)|$$

- Soit :

$$\left|\frac{L(j\omega)}{1 + L(j\omega)}\right| \leq \frac{1}{|l(j\omega)|}$$

Is there any bluffing ?

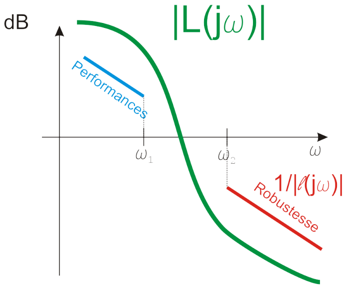
-

The Performance Robustness template
Conclusion

$$\color{red}{|T(j\omega)|} = \left|\frac{L(j\omega)}{1+L(j\omega)}\right| \leq \frac{1}{|l(j\omega)|}$$

- For high frequencies : $|l(j\omega)|$ is huge
- We must have (using **THE APPROXIMATION**)

$$|L(j\omega)| \leq \frac{1}{|l(j\omega)|}$$



Outline

The minimum phase systems case
When Nyquist can be simplified

Minimum phase system

Let us consider $G(p) = \frac{B(p)}{A(p)}$. G is said to be a minimum phase system if :

- Stable and no null real part poles other than $p = 0$
- No zero with a positive real part(sauf $p = 0$)
- The transfer is never zero or infinite for any finite frequency

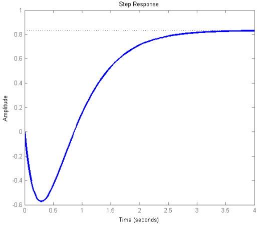


FIGURE – Nonminimum phase system

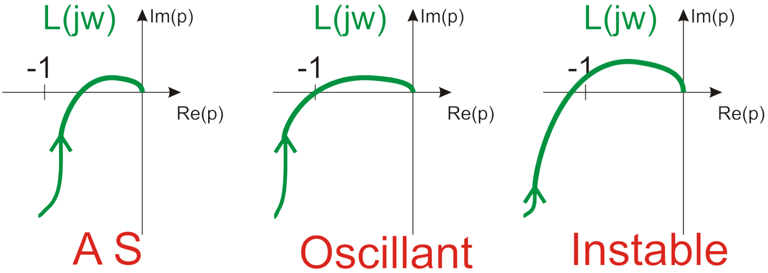
The minimum phase systems case

When Nyquist can be simplified

The simplified Nyquist criterion (critère du revers)

- Let us cionsider $L(p)$ a minimum phase syste
- The closed-loop system is asymptotically stable if the -1 point is not surrounded.

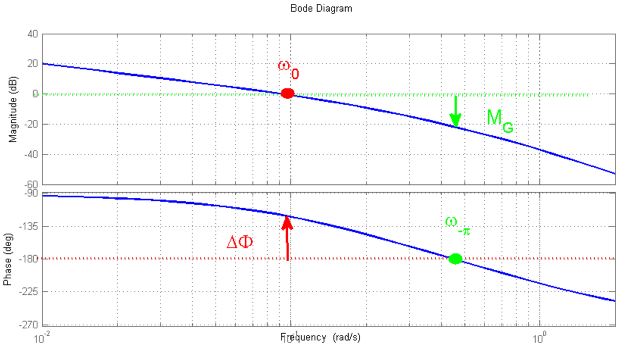
- It is even simpler : You don't have to draw the complete Nyquist plot : only part 1 ($p = j\omega$) allows you to conclude : you have to leave the -1 point on your left when you walk the curve in the direction ω increasing.



Let us go in the Bode diagrams

BE CAREFUL if not using minimum phase systems !

- « leave the -1 point on your left »
- The margins have to be positive



The counter example of a system that is nevertheless stable

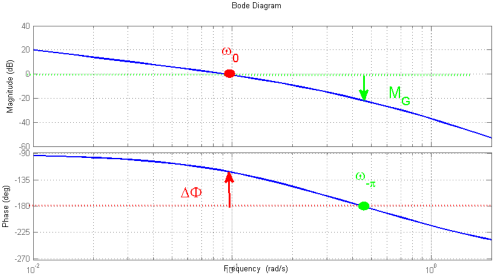
Do not forget to check the zeros . . .

$$L(p) = K\frac{p-10}{p(p+2)(p+5)}$$

Outline

A first remark on performance
Just to start the reasoning

- Let us draw the Bode diagrams $L(p)$
- We deduce from this the stability of $\frac{1}{1+L(p)}$ and $\frac{L(p)}{1+L(p)}$ (minimum phase case !)



- ω_0 : gives the phase margin
- There could be a confusing between ω_0 and ω_c ?
- « c »for cut-off frequency

A first remark on performance

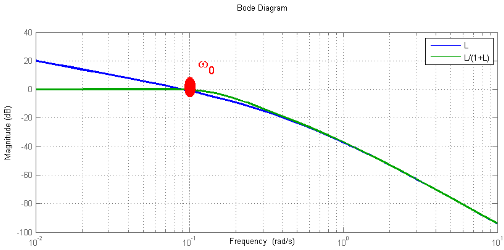
Comparison between L and $\frac{L(p)}{1+L(p)}$

- Everything starts using THE APPROXIMATION :
- for $\omega << \omega_0$:

$$|L(jw)| >> 1, \quad \Longrightarrow \left|\frac{L(p)}{1+L(p)}\right| \approx 1$$

- for $\omega >> \omega_0$:

$$|L(jw)| << 1, \quad \Longrightarrow \left|\frac{L(p)}{1+L(p)}\right| \approx |L(p)|$$

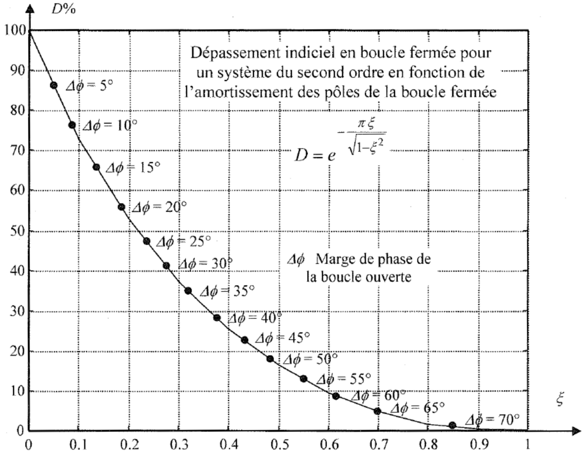


- And what happens when we are close to ω_0 ?

A first remark on performance

Comparison between L and $\frac{L(p)}{1+L(p)}$: the famous Appendix xx

- The closed-loop behaviour is close to that of a second order !



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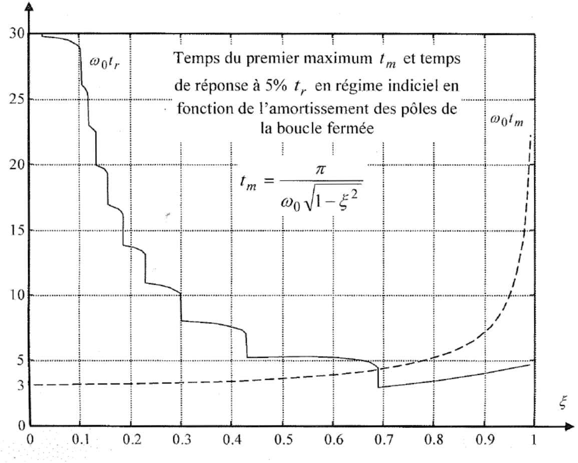

A first remark on performance

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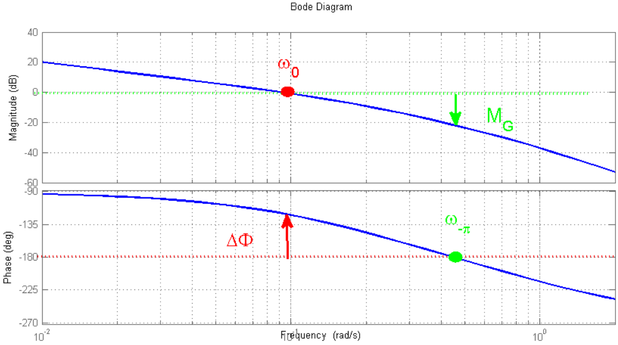
- Should we check ?

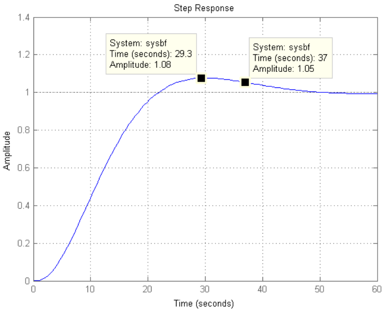


- What is the value of ω_0 ?
- What is the phase margin ?
- What performance could we expect ?

A first remark on performance

Comparison between L and $\frac{L(p)}{1+L(p)}$: the famous Appendix xx

- The stepresponse :

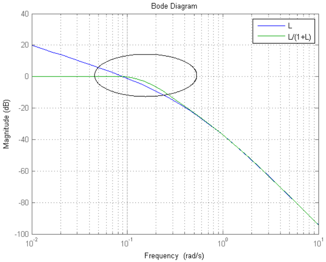


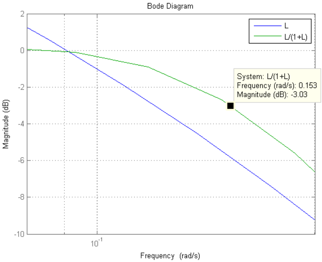
- Not so bad ...
- Gives an estimation of the bandwidth

A first remark on performance

Comparison between L and $\frac{L(p)}{1+L(p)}$: the famous Appendix xx

- An explanation on the differences :

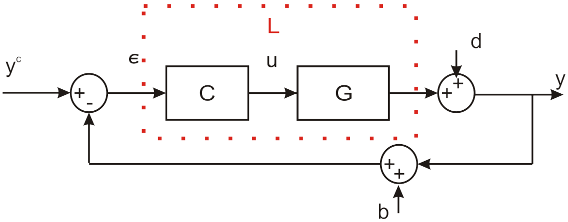




Outline

Steady-state error
Please, forget all the « types »

- Let us define $L(p) = K \frac{N(p)}{p^m D(p)}$, with $N(0) = D(0) = 1$



- m : number of integral actions
- Objective : to understand the impact of m on steady-state error if the reference y^c is a step/ramp, ... with and disturbances d and b.
- What is the limit $\lim_{t \to +\infty} \varepsilon(t)$?
- Using the final value theorem (If the stability is ensured !)

Steady-state error
Your turn to play

Dynamic error
The sinusoidal case

- Objective : compute the error in sinusoidal mode, without disturbances.
- $y^c(t) = Y_0 \sin(\omega t)$
- Only makes sense if *omega* is within the bandwidth of the closed-loop system !

Dynamic error

The non-sinusoidal case

- Can we have an estimation of the dynamic error if the reference is not sinusoidal ?

The equivalent sinusoid

Let us consider $y^c(t)$ an almost random signal :

- Its first-order derivative is bounded by $\dot{y}_c^M$
- Its second-order derivative is bounded by $\ddot{y}_c^M$

We then build a sinusoidal signal $\tilde{y}^c(t) = Y_0 \sin(\omega_0 t)$ that has the same characteristics on its derivatives. A good approximation of the error is the dynamic error of the system in response to this equivalent signal.

Outline

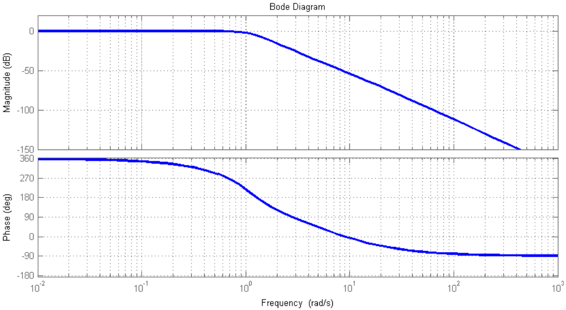
Review of the course

Expected skills

- Read the stability margins - in Nyquist, in Bode
- Understanding the Performance-Robustness trade-off
- Using the simplified Nyquist criterion
- Determine steady-state and dynamic errors

For training purposes
Your turn to play

$$L(p) = K \frac{-0.2p + 1}{(p+1)(0.1p+1)(p^2/1.1^2 + p/1.1 + 1)}$$



- Discuss stability in terms of K.
- For $K = 10$, give the gain margin and the phase margin of the system