2A - Automatique

Chapter 4

Control Science (AUT)

Frequency-domain approach, Design Methods, I

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Preamble
About this course

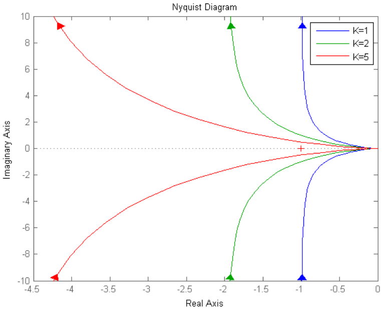
Course Outline

- Nichols plot
- Specifications
- Control design - Series action

Outline

Introduction Example
The Limitations of the Nyquist plot

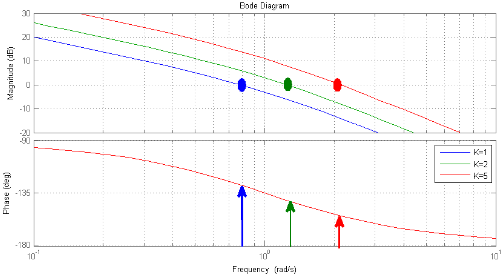
$$L(p) = K\frac{1}{p(1+\tau p)}$$



- Multiply by K : homothety in the Nyquist plot
- Reading the margins. Yes, we can ! But it is not the most appropriate tool
- What about bandwidth ?

Introduction Example
Switching to Bode. (systematic for minimum phase systems)

$$L(p) = K\frac{1}{p(1+\tau p)}$$

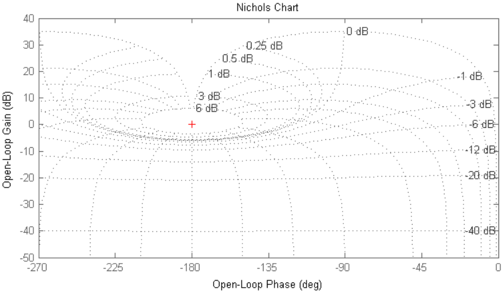


- Very convenient to read the margins and see the evolution according to ω
- The phase margin gives an idea of the closed-loop behaviour (overshoot)
- ω_0 is an approximation of the bandwidth

The Nichols plot (le lieu de Black-Nichols)

Abacus : link between L and $\frac{1}{1+L}$

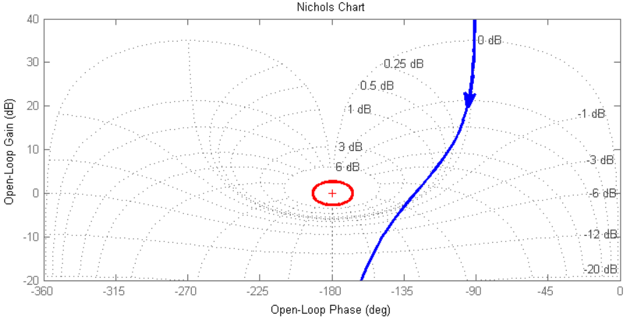
- Abscissa axis : phase of $L(j\omega)$ in degrees
- Y-axis : Gain of $L(j\omega)$ in dB



- We draw the curve $L(j\omega)$, parametric curve in ω
- The chart gives the isogains of the closed-loop (and also the isopahses)

The Nichols plot (le lieu de Black-Nichols)
What kind of data ?

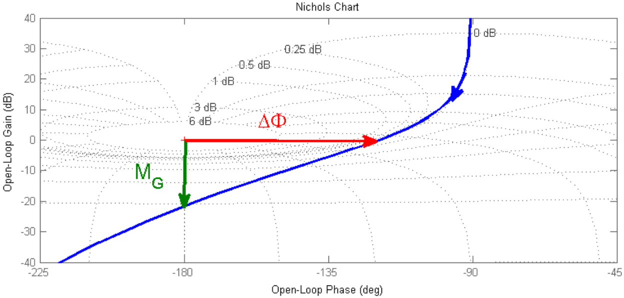
- Stability : (systematic for minimum phase systems)



- We leave the critical point on the right when we drive in the direction of increasing *omega*.

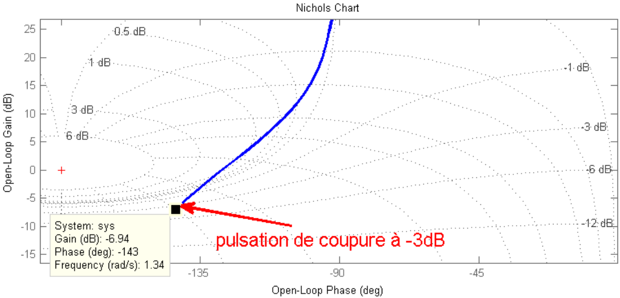
The Nichols plot (le lieu de Black-Nichols)
What kind of data ?

- Margins : straight reading



The Nichols plot (le lieu de Black-Nichols)
What kind of data ?

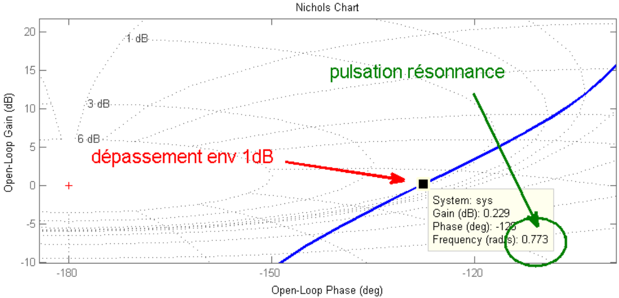
- Bandwidth at $-x$ dB



- We will look for the frequency for which the curve cuts the corresponding isogain

The Nichols plot (le lieu de Black-Nichols)
What kind of data ?

- Overshoot

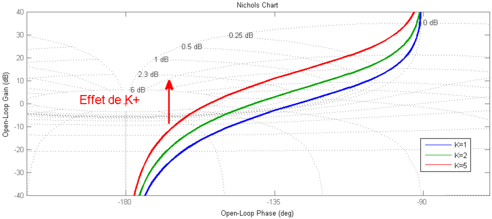


- We will look for the largest positive isogain tangent to the curve
- Also gives the resonance frequency

Introduction Example

Switching to Bode. (systematic for minimum phase systems)

$$L(p) = K \frac{1}{p(1+\tau p)}$$



- Very convenient to read the margins and see the evolution according to *omega*
- We also characterize the overshooting
- Give information on the key frequencies

Comparison
3 analysis tools

- The main principle : analysis of L to deduce properties of $\frac{1}{1+L}$

Nyquist

- Required in the general case
- Possible reading of margins, but changes difficult to anticipate

Black-Nichols

- Systematic if minimum phase system
- Gives the frequency behavior of the closed-loop system
- Matlab required because of the parameterized curve

Bode

- Systematic if minimum phase system
- Gives the frequency behavior of the closed-loop system
- Easy to read because *omega* is explicitly visible

Outline

The elements of a specification

Stability - Accuracy - Performance - Robustness

The easy ones :

- Stability : necessary
- Accuracy - steady-state error : depends on the number of integral actions
- Accuray - dynamic error : the gain has to be high enough at specific frequencies
- Robustness : the margins have to be high enough

The less easy ones :

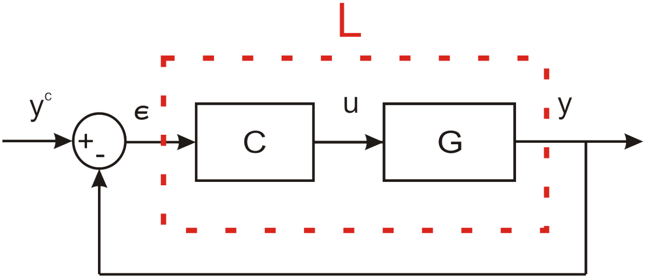
- Bandwidth : don't try to make yourself bigger than the beef !
- The overshoot : yes, no, a little bit

The hard ones :

- Constraints fulfilment. (Actuator saturation, . . .)

Outline

The structure of the serial corrector
No surprise



- G analysis
- Comparison with the specifications
- Determine C to modify where necessary

Various steps

This order is not the only solution

1 : Performance : improving the compromise between speed and stability

- High-frequency action
- Choice of the desired bandwidth
- Choice of the desired phase margin

2 : Accuracy :

- Low-frequency action
- Choice of the number of integral actions
- Adjustment of the gain to certain frequencies

First part : Performance
Case study 1 : presentation

$$G(p) = \frac{1}{p(1+0.3p)(1+0.05p)}$$

Specification

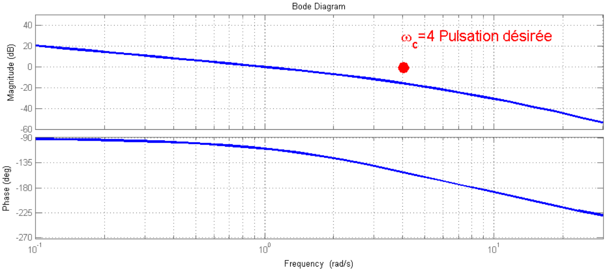
- Bandwidth : $\omega_c = 4 \text{rad.s}^{-1}$
- Overshoot below 10%

Preliminary analyses : B.

- No poles with a positive real part (except $p = 0$)
- Minimum phase system
- So we can do the analysis in the Bode plot

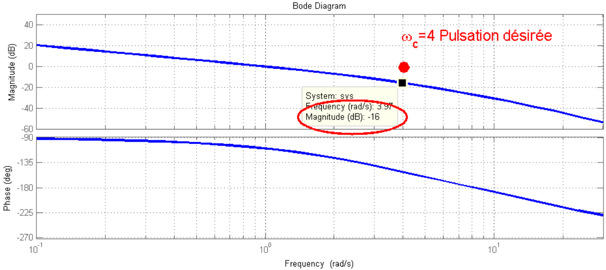
First part : Performance
Case study 1 : Bode diagrams

$$G(p) = \frac{1}{p(1 + 0.3p)(1 + 0.05p)}$$



First part : Performance
Case study 1 : control setup

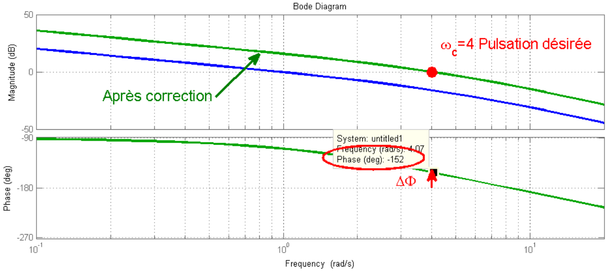
$$G(p) = \frac{1}{p(1 + 0.3p)(1 + 0.05p)}$$



- 16dB is required
- $k = 10^{\frac{16}{20}}$

First part : Performance
Case study 1 : analysis of the closed loop system

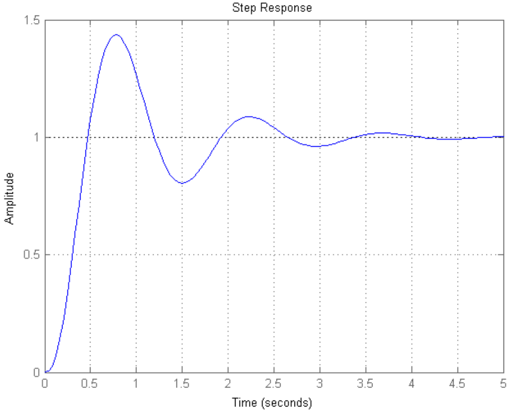
$$L(p) = kG(p)$$



- Insufficient phase margin : only about 30 degrees : at least 60 degrees would be required
- We will not be able to act on the overshoot !
- We need to increase the phase margin

First part : Performance
Case study 1 : temporal checking

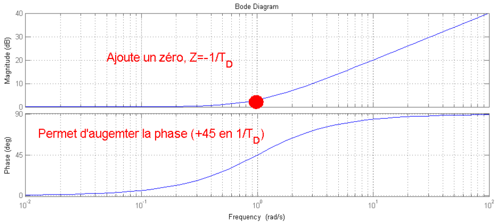
$$\frac{Y}{Y^c}(p) = \frac{kG(p)}{1 + kG(p)}$$



- Indeed, no surprise, there is too much overshoot

First part : Performance

Second proposition : proportional and derivative action $C(p) = k(T_D p + 1)$



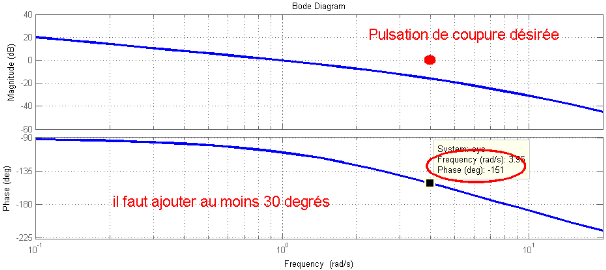
Control analysis

- Adds a negative real zero in $-\frac{1}{T_D}$
- Can restore up to +90 phase (one decade after $\frac{1}{T_D}$)
- Add gain for the high frequencies : be careful with noise !
- Feasibility ?

First part : Performance

Second proposition : tuning $C(p) = k(T_D p + 1)$

$$G(p) = \frac{1}{p(1+0.3p)(1+0.05p)}$$

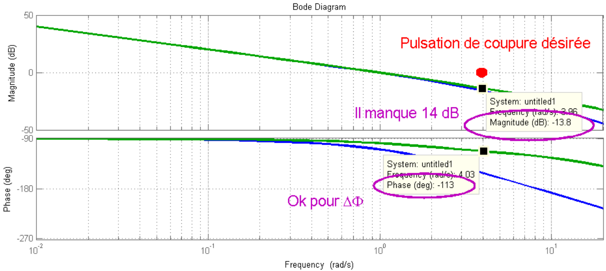


- At least 30 degrees must be added for the phase margin
- Roughly $\frac{1}{T_D} \approx 4$

First part : Performance

Second proposition : tuning $C(p) = k(T_D p + 1)$

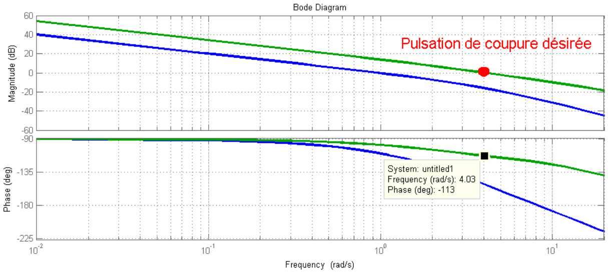
$$L(p) = (1 + T_D p)G(p)$$



- We need to add some gain
- $k = 10^{\frac{14}{20}}$

First part : Performance
Second proposition : tuning $C(p) = k(T_D p + 1)$

$$L(p) = k(1 + T_D p)G(p)$$

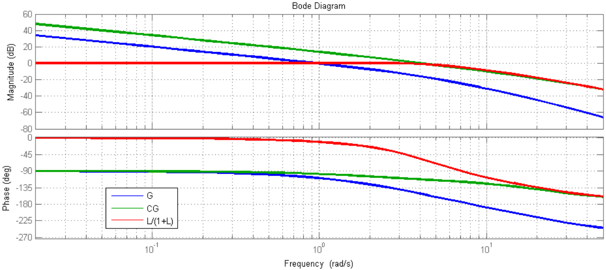


- It looks nice !
- Let's check !

First part : Performance

Second proposition : frequency behavior checking $C(p) = k(T_D p + 1)$

$$L(p) = k(1 + T_D p)G(p)$$

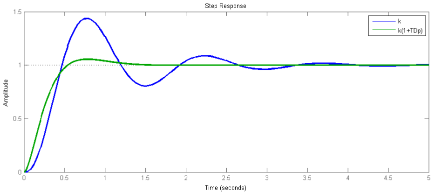


- Ok for the bandwidth
- And for the temporal behavior ?

First part : Performance

Second proposition : temporal behavior checking $C(p) = k(T_D p + 1)$

$$\frac{Y}{Y^c}(p) = \frac{k(1 + T_D p)G(p)}{1 + k(1 + T_D p)G(p)}$$



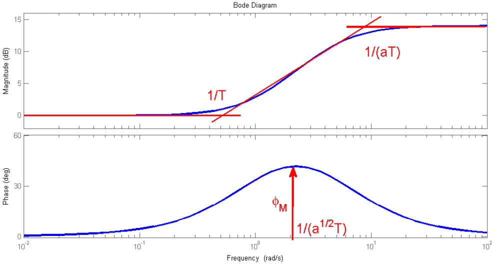
- Ok for the overshoot

Where are the traps ?

- Be careful because the gain in HF increases !
- What about the feasibility of the control action ?
- Adding a filtering action : $C(p) = k\frac{1+T_D p}{1+\frac{T_D}{N}p}$, with N very large

First part : Performance

Third proposition : lead-phase action $C(p) = k\frac{1+Tp}{1+aTp}$, $a < 1$ (avance de phase)



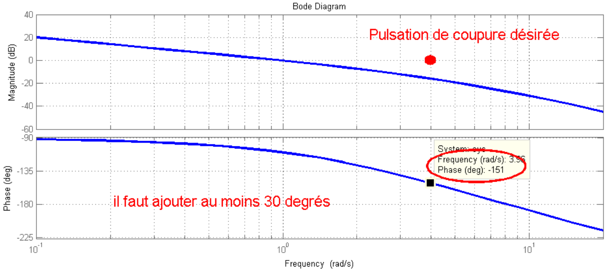
Control analysis

- Adds a negative real zero : $-\frac{1}{T}$
- Adds a negative real pole : $-\frac{1}{aT}$
- Increase the phase, with a maximum for the frequency : $\frac{1}{\sqrt{a}T}$
- $a = \frac{1-\sin \phi_M}{1+\sin \phi_M}$

First part : Performance

Third proposition : lead-phase action $C(p) = k\frac{1+Tp}{1+aTp}$, $a < 1$, tuning

$$G(p) = \frac{1}{p(1+0.3p)(1+0.05p)}$$

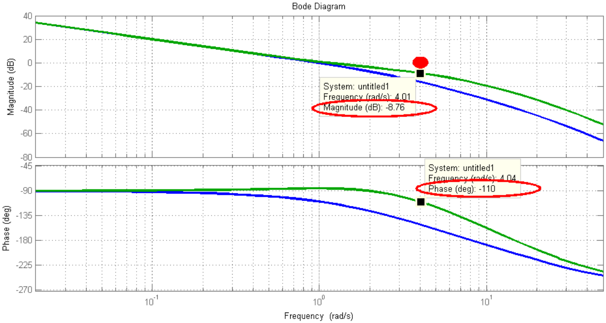


- A minimum of 30 (40) degrees must be added for the phase margin
- $a = \frac{1-\sin\phi_M}{1+\sin\phi_M} \approx 0.2$
- $\omega_c = \frac{1}{\sqrt{aT}}$ gives $T \approx 0.6$

First part : Performance

Third proposition : lead-phase action $C(p) = k\frac{1+Tp}{1+aTp}$, $a < 1$, tuning

$$G(p) = \frac{1}{p(1+0.3p)(1+0.05p)}$$

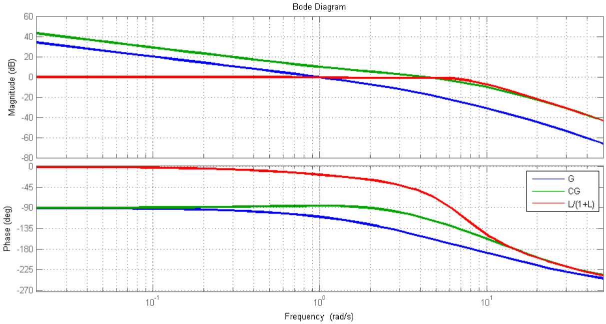


- 9dB is missing
- $k = 10^{9/20}$

First part : Performance

Third proposition : lead-phase action $C(p) = k\frac{1+Tp}{1+aTp}$, $a < 1$, checking

$$G(p) = \frac{1}{p(1+0.3p)(1+0.05p)}$$

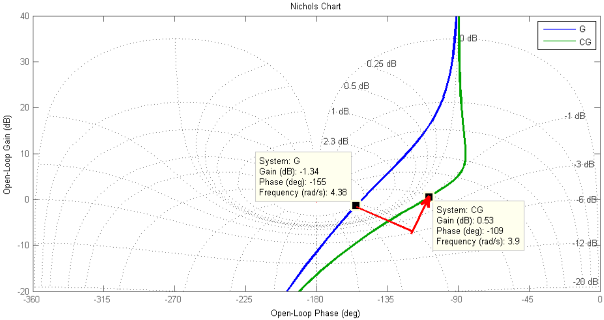


- It looks nice !

First part : Performance

Third proposition : lead-phase action $C(p) = k\frac{1+Tp}{1+aTp}$, $a < 1$, checking

$$G(p) = \frac{1}{p(1+0.3p)(1+0.05p)}$$

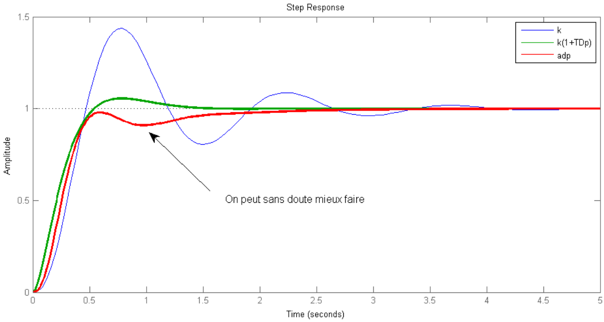


- no bad !
- maybe we could do better (the lead-phase action is too high)

First part : Performance

Third proposition : lead-phase action $C(p) = k\frac{1+Tp}{1+aTp}$, $a < 1$, checking

$$G(p) = \frac{1}{p(1+0.3p)(1+0.05p)}$$



- For a quick adjustment, that's good !

Conclusion

- 3 methods, 2 of which are relevant
- A proportional-derivative action
- Lead-phase action

Expected skills

- Analyze the deficiencies of G to achieve desired performance
- Set up the appropriate control action and tune it
- A proportional-derivative action
- Lead-phase action

Part 2 : Accuracy
Study case 2 : presentation

- The case of a temperature regulator

$$G(p) = \frac{1}{(p+1)(1+0.5p)(1+2p)}$$

Specifications

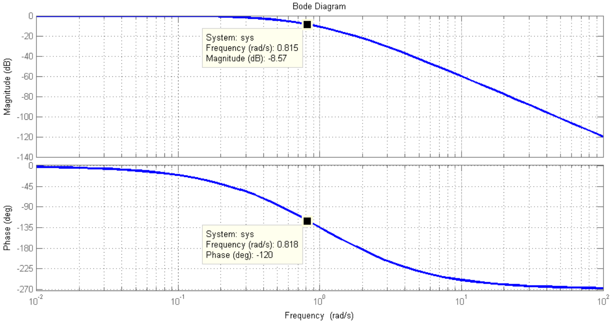
- Steady-State error : below 5%
- Phase margin greater than 50 degrees

Preliminary analysis of G

- No poles with real positive part : but 3 poles close enough : the phase will drop quickly !
- No integral action : a steady-state error is expected
- Minimum phase system : we can work with the Bode plots

Part 2 : Accuracy
Study case 2 : Bode diagrams

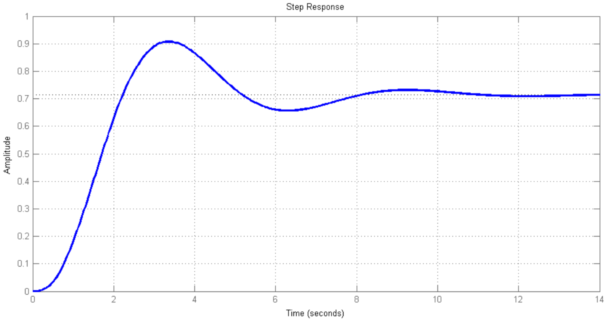
$$G(p) = \frac{1}{(p+1)(1+0.5p)(1+2p)}$$



- we can increase by 8dB, with a simple gain
- $k = 10^{\frac{8}{20}} \approx 2.5$
- Bandwidth : around 0.8 rad.s^{-1}
- Static gain : 2.5 : insufficient for accuracy ! Indeed :

Part 2 : Accuracy
Study case 2 : Bode diagrams

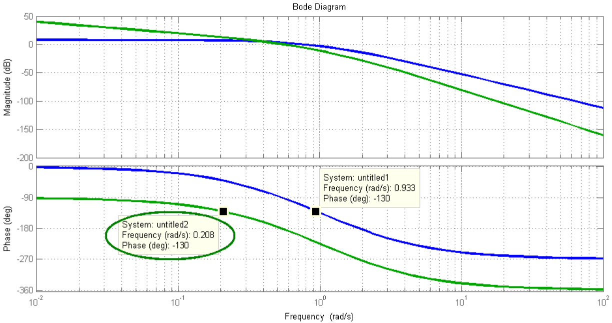
$$L(p) = kG(p)$$



- No surprise : steady-state error is around 30%
- How to improve accuracy WITHOUT changing performance ?

Part 2 : Accuracy

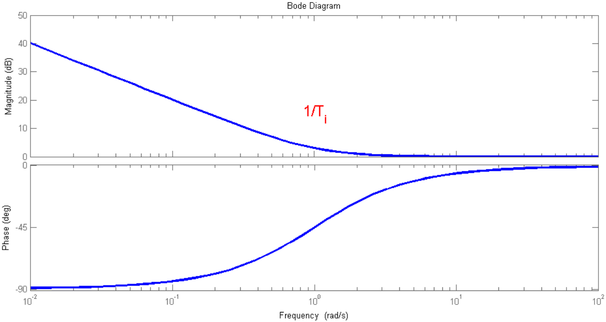
Study case 2 : first correction I : $C(p) = k_I \frac{1}{p}$



- Impossible with the single integral action !

Part 2 : Accuracy

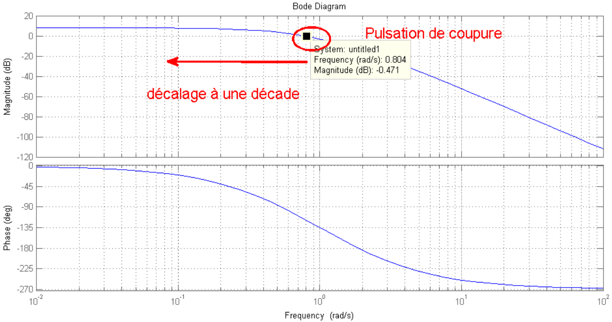
Study case 2 : second correction PI : $C(p) = k_p(1 + \frac{1}{T_i p})$



- Integral action + a real zero $-\frac{1}{T_i p}$
- Infinite gain at low frequency

Part 2 : Accuracy
Study case 2 : second correction PI, tuning

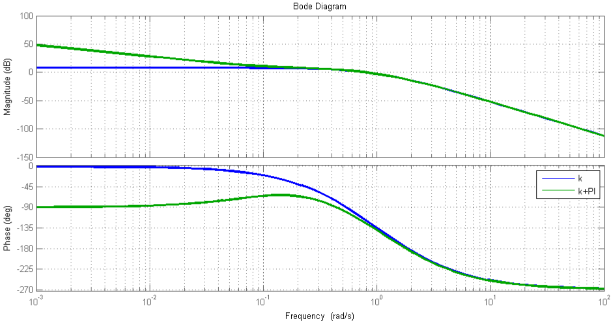
$$L(p) = \frac{1 + T_i p}{T_i p} kG(p)$$



- In order not to affect performance : we can roughly work at a decade before
- This gives the value of T_i

Part 2 : Accuracy
Study case 2 : second correction PI, Bode analysis

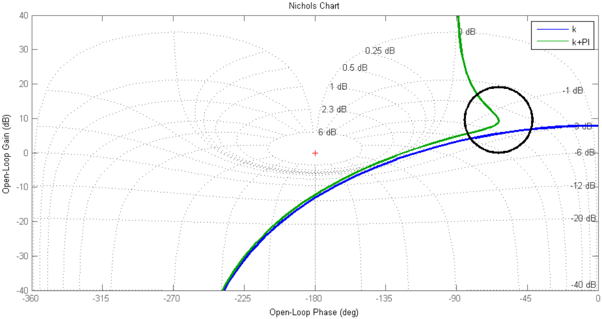
$$L(p) = \frac{1 + T_i p}{T_i p} kG(p)$$



- No change in the cut-off frequency
- Integral action for the low-frequency

Part 2 : Accuracy
Study case 2 : second correction PI, Nichols analysis

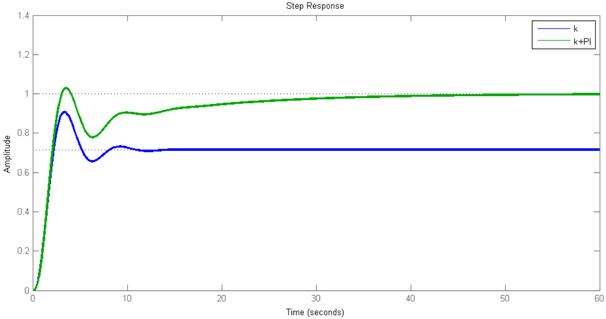
$$L(p) = \frac{1 + T_i p}{T_i p} kG(p)$$



- Setting not optimized : we must be able to do better

Part 2 : Accuracy
Study case 2 : second correction PI, Nichols analysis

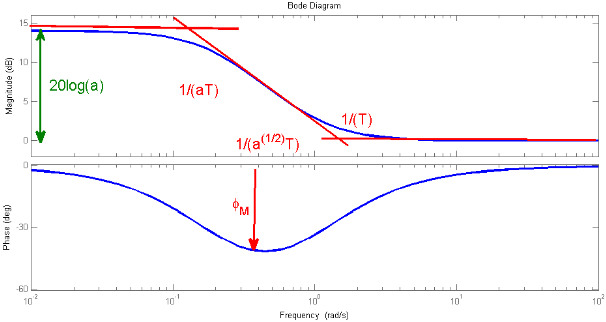
$$L(p) = \frac{1 + T_i p}{T_i p} kG(p)$$



- We fulfilled the specifications, but we can do better

Part 2 : Accuracy

Study case 2 : third correction : lag-phase action $C(p) = a\frac{1+Tp}{1+aTp}$, $a > 1$

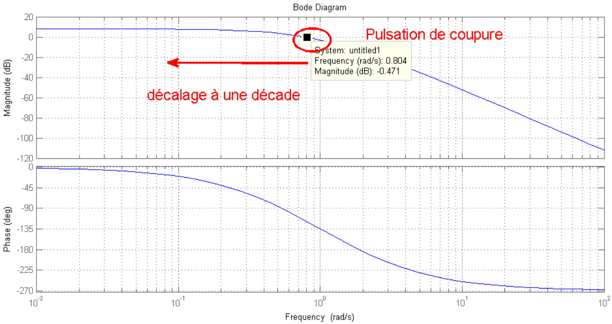


- This is the dual of the lead-phase action
- First a pole, then a zero
- The max of the phase action occurs at $\frac{1}{\sqrt{a}T}$

Part 2 : Accuracy

Study case 2 : third correction : lag-phase action, tuning

$$L(p) = a\frac{1+Tp}{1+aTp}kG(p)$$

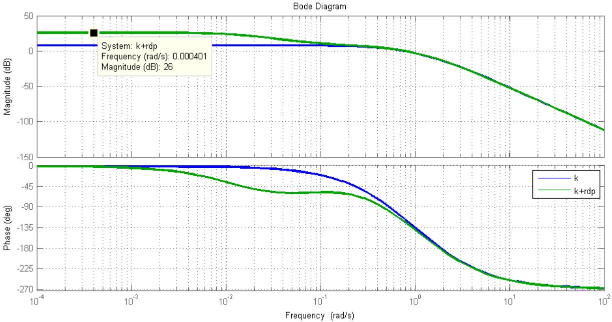


- From the desired steady-state error : the missing gain in BF is calculated
- This gives the *a* parameter
- In order not to affect performance : we work roughly at a decade before
- This gives the *T* parameter

Part 2 : Accuracy

Study case 2 : third correction : lag-phase action, Bode analysis

$$L(p) = a\frac{1 + Tp}{1 + aTp}kG(p)$$

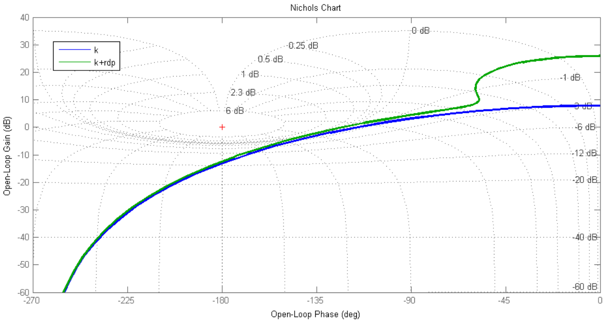


- Ok for the low frequency gain
- Phase modification only on a certain band

Part 2 : Accuracy

Study case 2 : third correction : lag-phase action, Nichols analysis

$$L(p) = a\frac{1+Tp}{1+aTp}kG(p)$$

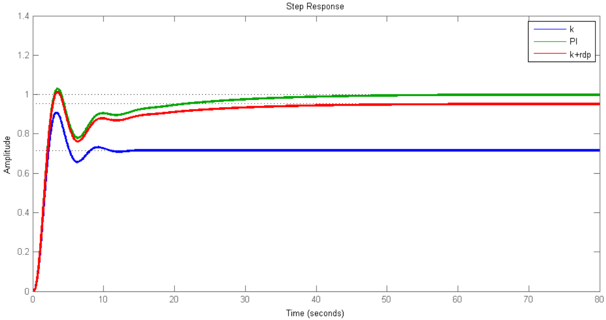


- As before, the tuining is not optimal, but nice enough for this study !

Part 2 : Accuracy

Study case 2 : third correction : lag-phase action, closed-loop behavior

$$L(p) = a\frac{1+Tp}{1+aTp}kG(p)$$



- The specifications are fulfilled
- We could do better, modifying the high frequencies

Part 2 : Accuracy
Conclusions :

- 3 methods, 2 of which are relevant
- Proportional-Integral action
- Lag-phase action

Expected skills

- Analyze the deficiencies of G to achieve desired performance
- Set up the appropriate control action and tune it
- Proportional-Integral action
- Lag-phase action

Outline

1. Introduction
2. Nichols plot
3. Specifications
4. Series action
5. **Conclusion**

Review of the course
Serial actions

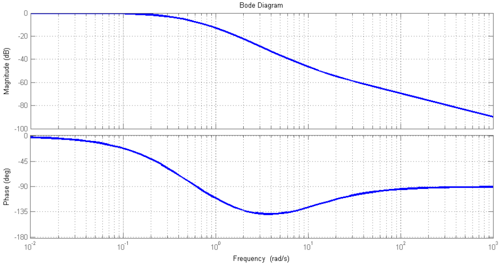
Expected skills

- Understand the different control actions, their influence and parametrization
- Performance
- Accuracy

- These different control actions can of course be combined.

Your turn to play
Just to practice

$$G(p) = \frac{0.1p + 1}{(p+1)(\frac{p}{3}+1)}$$



Specifications

- No overshoot
- No steady-state error for a step reference
- The closed-loop bandwidth has to be approximatively 5 rad.s^{-1}