2020.09.28 Cours 8

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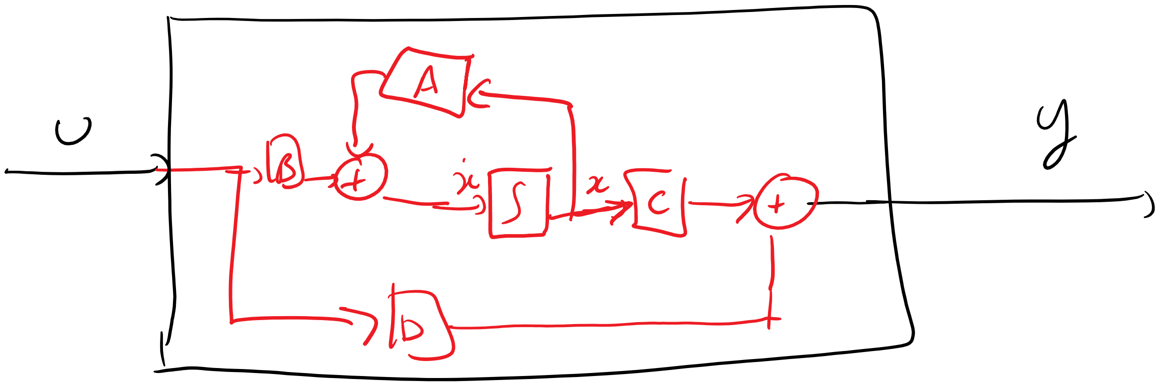
$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \leftarrow$$



$$\dot{x} = f(x, u) \rightarrow (x_e, u_e) \rightarrow f(x_e, u_e) = 0$$

$$x = \tilde{x} + x_e \quad ; \quad u = \tilde{u} + u_e$$

$$\dot{x} = \dot{\tilde{x}} + \dot{x}_e = f(x, u) = f(\tilde{x} + x_e, \tilde{u} + u_e)$$

$$= \dot{\tilde{x}}$$

$$= \underbrace{f(x_e, u_e)}_{=0} + \frac{\partial f}{\partial x}^T\Big|_{\substack{x = x_e \\ u = u_e}} \tilde{x} + \frac{\partial f}{\partial u}^T\Big|_{\substack{x = x_e \\ u = u_e}} \tilde{u}$$

$$\boxed{\dot{\tilde{x}} = \frac{\partial f}{\partial x}^T\Big|_{\substack{x = x_e \\ u = u_e}} \tilde{x} + \frac{\partial f}{\partial u}^T\Big|_{\substack{x = x_e \\ u = u_e}} \tilde{u}}$$

Ex:
$$\begin{cases} \dot{x}_1 = x_1(1 - x_2) \\ \dot{x}_2 = x_2(1 - x_1) \end{cases}$$

$$\left|\ \dot{x}_2 = x_2(1-x_1)\right.$$

Equilibrium points: $x_1(1-x_2)=0$ $\to$ $x_1 = 0 \to x_2 = 0$

$\to$ $x_2 = 1 \to x_1 = 1$

2 couples: $(0,0)$ and $(1,1)$

Jacobian:
$$\begin{cases} \dot{\tilde{x}}_1 = (1-x_2)\,\tilde{x}_1 + (-x_1)\,\tilde{x}_2 \\ \dot{\tilde{x}}_2 = -x_2\,\tilde{x}_1 + (1-x_1)\,\tilde{x}_2 \end{cases}$$

Around $(0,0)$
$$\boxed{\begin{aligned} \dot{\tilde{x}}_1 &= \tilde{x}_1 \\ \dot{\tilde{x}}_2 &= \tilde{x}_2 \end{aligned}} \Rightarrow \dot{\tilde{X}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tilde{X}$$

Around $(1,1)$:
$$\begin{aligned} \dot{\tilde{x}}_1 &= -\tilde{x}_2 \\ \dot{\tilde{x}}_2 &= -\tilde{x}_1 \end{aligned} \Rightarrow \dot{\tilde{X}} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \tilde{X}$$

$$x_p = \phi(t,t_0)\, S(t)$$

$$\dot{x}_p = \dot{\phi}(t,t_0)\, S(t) + \phi(t,t_0)\,\dot{S}(t) = A\,x_p + B\,u(t)$$

$$= \cancel{A(t)\,\phi(t,t_0)\,S(t)} + \phi(t,t_0)\,\dot{S}(t) = \cancel{A\phi S} + B\,u(t)$$

$$\dot{S}(t) = \phi^{-1}(t,t_0)\,B(t)\,u(t)$$

$$S(t) = \int \phi^{-1}(\tau, t_0) B(\tau) u(\tau)\, d\tau$$

$$= \int \phi(t_0, \tau) B(\tau) u(\tau)\, d\tau$$

$$x_p = \phi S = \phi(t, t_0) \int \text{----}$$

$$\boxed{x_p = \int \phi(t, \tau) B(\tau) u(\tau)\, d\tau}$$

general expression of the solution:

$$\boxed{x(t) = \phi(t, t_0)\, x(t_0) + \int_{t_0}^{t} \phi(t, \tau) B(\tau) u(\tau)\, d\tau}$$

Non unicity of S.S.

① How to go from SS $\longrightarrow$ TF

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \Longleftarrow \mathcal{L}$$

$$pX = A\cancel{X} + BU$$

$$(pI - A)X = BU$$

$$\boxed{X = (pI - A)^{-1} B U}$$

$$\mathcal{L}\left\{\begin{array}{l} y = Cx \\ Y = CX = C(pI-A)^{-1}BU \end{array}\right.$$

$$\boxed{\frac{Y}{U} = C(pI-A)^{-1}B}$$

Change of basis $x = Tx_1$, with T inversible

$$\left\{\begin{array}{l} \dot{x}_1 = A_1 x_1 + B_1 u \\ y = C_1 x_1 \end{array}\right.$$

$$\dot{x} = T\dot{x}_1 \rightarrow \dot{x}_1 = T^{-1}\dot{x}$$
$$= T^{-1}(Ax + Bu)$$
$$= \underbrace{T^{-1}AT}_{A_1}x_1 + \underbrace{T^{-1}B}_{B_1}u$$

$$y = Cx$$
$$= \underbrace{CT}_{C_1}x_1$$

$$\left\{\begin{array}{l} A_1 = T^{-1}AT \\ B_1 = T^{-1}B \\ C_1 = CT \end{array}\right. \qquad \frac{Y}{U} = C_1(pI-A_1)^{-1}B_1$$

$$B_1 = T^{-1} B \qquad \quad = C_1 (pI - A_1)^{-1} B_1$$

$$C_1 = CT \qquad \qquad = C\,(pI - A)^{-1} B.$$

$$C_1 (pI - A_1)^{-1} B_1$$

$$CT\,(pI - T^{-1}AT)^{-1}\, T^{-1} B$$

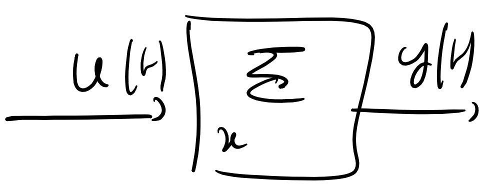
$$CT\,(p T^{-1} T - T^{-1} A T)^{-1}\, T^{-1} B$$

$$CT\,(T^{-1}(pI - A)T)^{-1}\, T^{-1} B$$

$$CT\, T^{-1} (pI - A)^{-1}\, T\, T^{-1} B$$

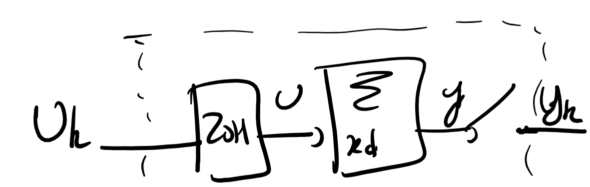
$$\boxed{C \; (pI - A)^{-1} \; B}$$

From continuous-Time SS to discrete-Time SS.



$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$$

Te



$$\begin{cases} x_{k+1} = A_d x_k + B_d u_k \\ y_k = C_d x_k \end{cases}$$

$t \in [t_k, t_{k+1}[$

① $U(t) = U_k = cste$

① $x(t) = e^{A(t-t_0)} x(t_0) + \int_{t_0}^{t} e^{A(t-\tau)} B u(\tau) \, d\tau$

$\begin{cases} t = t_{k+1} \\ t_0 = t_k \end{cases} \quad t_{k+1} = (k+1) T_e \; ; \; t_k = k T_e$

$$x(k+1) = \underbrace{e^{A T_e}}_{A_d} x(k) + \underbrace{\int_{t_k}^{t_{k+1}} e^{A(t_{k+1} - \tau)} d\tau}\, B u_k$$

$$\int_{t_k}^{t_{k+1}} e^{A(t_{k+1} - \tau)} d\tau$$

$\tau = k T_e + \theta$

$$\longrightarrow \int_{0}^{T_e} e^{A(T_e - \theta)} d\theta$$

$\mu = T_e - \theta$

$$\longrightarrow \int_{T_e}^{0} - e^{A\mu} d\mu = \int_{0}^{T_e} e^{A\mu} d\mu$$

$$x(k+1) = \underbrace{e^{A T_e}}_{A_d} x(k) + \underbrace{\int_{0}^{T_e} e^{A\mu} d\mu \, B}_{B_d} \, u(k)$$