Wireless Channel
and Multi-Carrier Modulation

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Signal Communications & Embedded Systems

Outline

1. Radio waves
2. Propagation
3. Noise sources
4. Wireless channel vs. AWGN channel
5. Multi-carrier modulation
6. Applications

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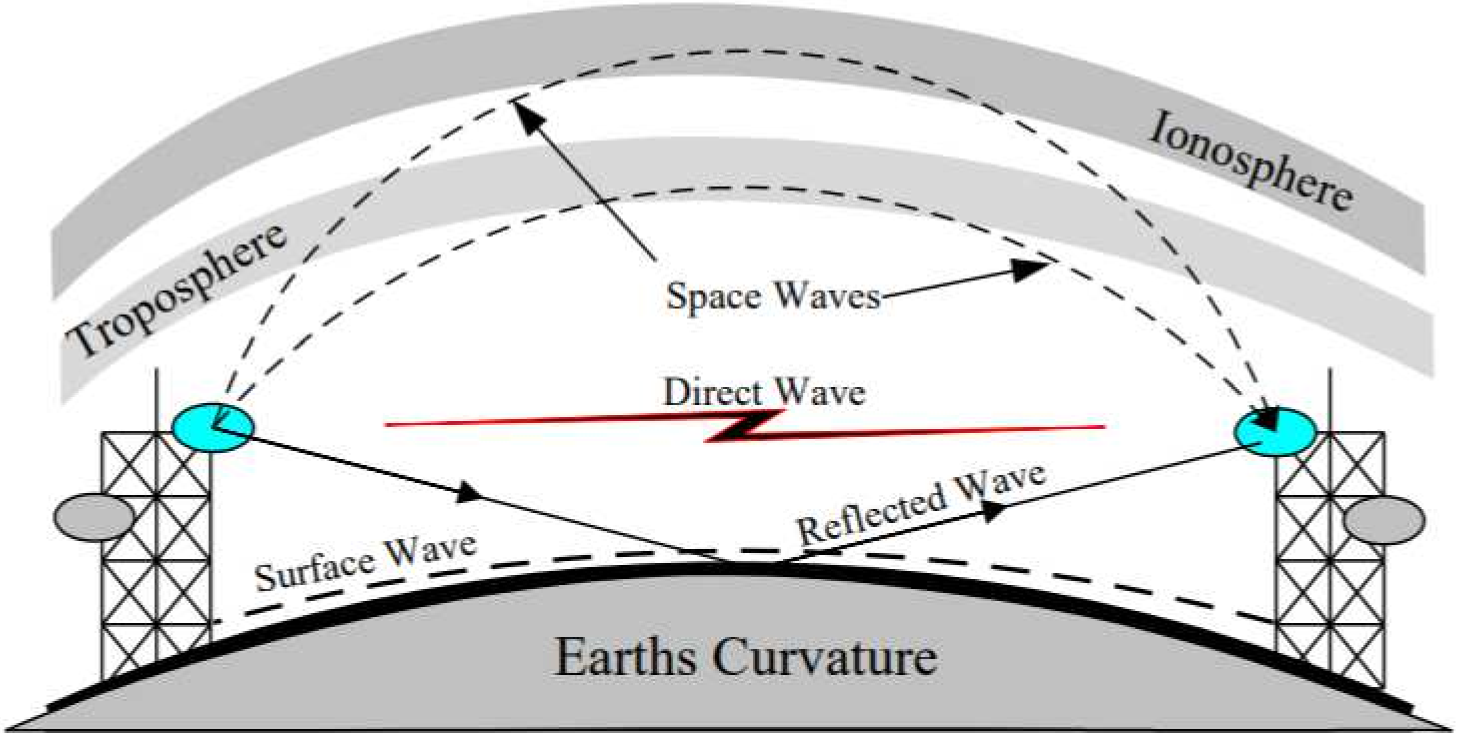
Radio Waves



Ionosphere
Troposphere
Space Waves
Direct Wave
Reflected Wave
Surface Wave
Earths Curvature

Outline

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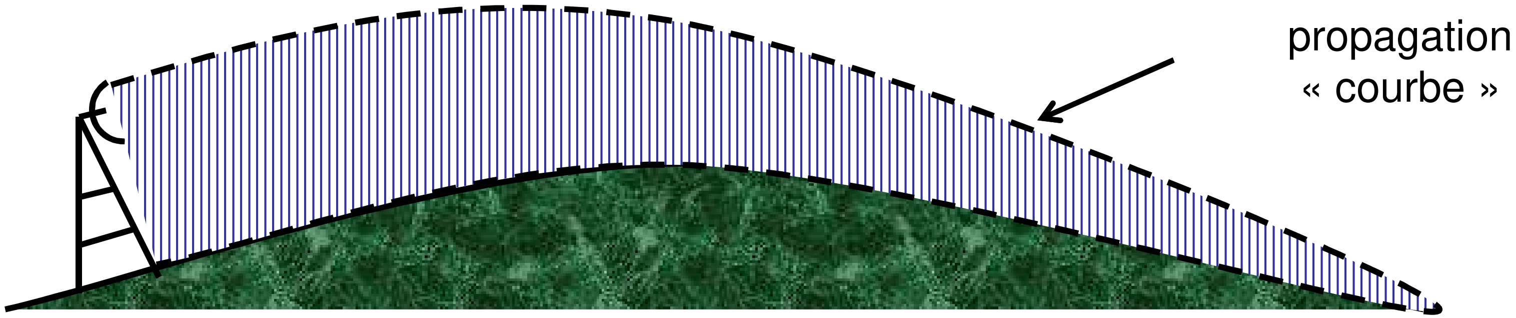
Propagation

Propagation by the ground wave

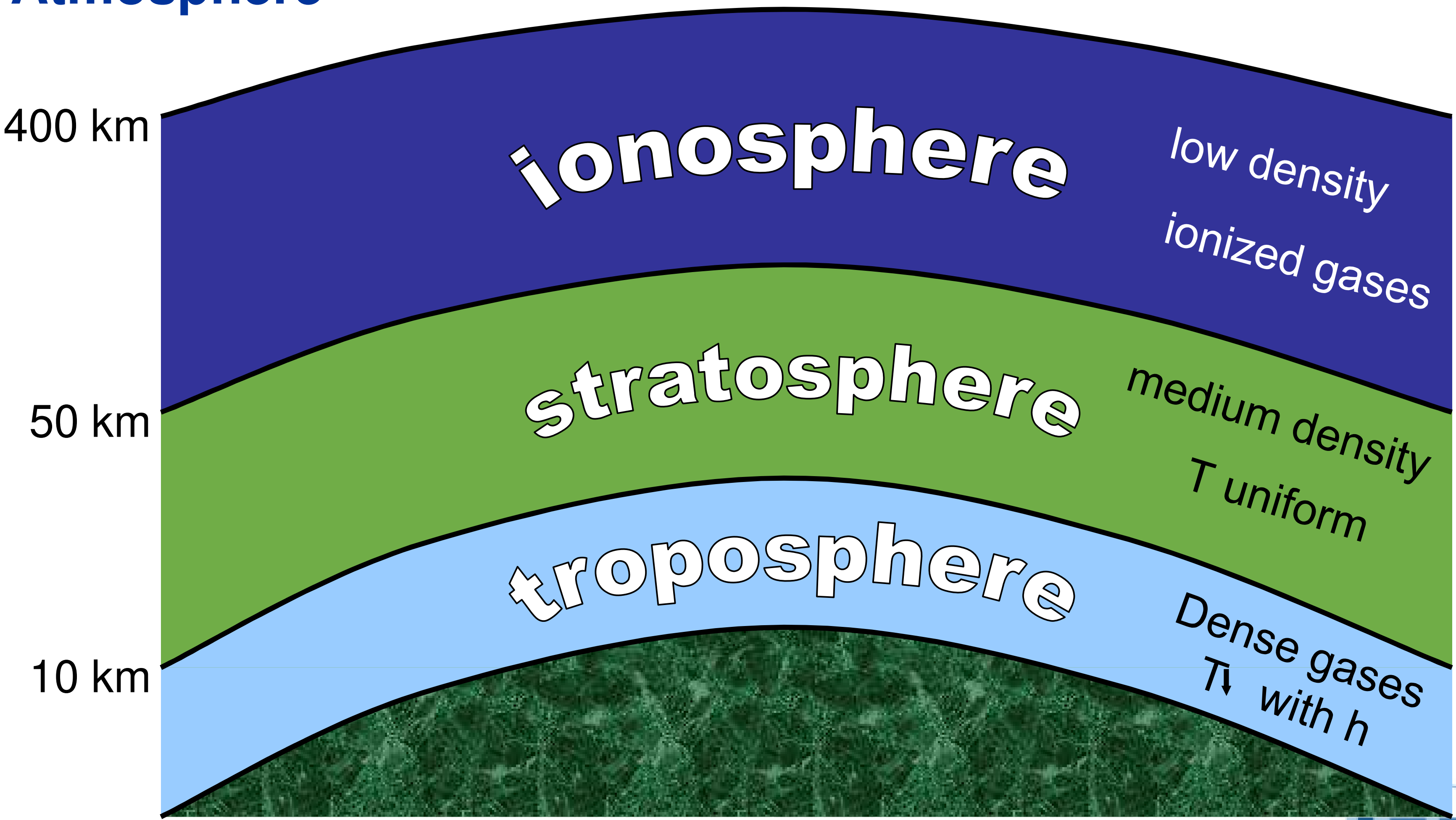
- The wave follows the curvature of the earth. Its range depends on the nature of the ground, the frequency and the transmit power.
- Part of the energy of the surface wave is absorbed by the ground and causes induced currents there; energy absorption is much greater in horizontal polarization.



- Suitable for long distance transmissions
- Frequencies > 2 MHz
- Example: AM radio

Propagation

Atmosphere



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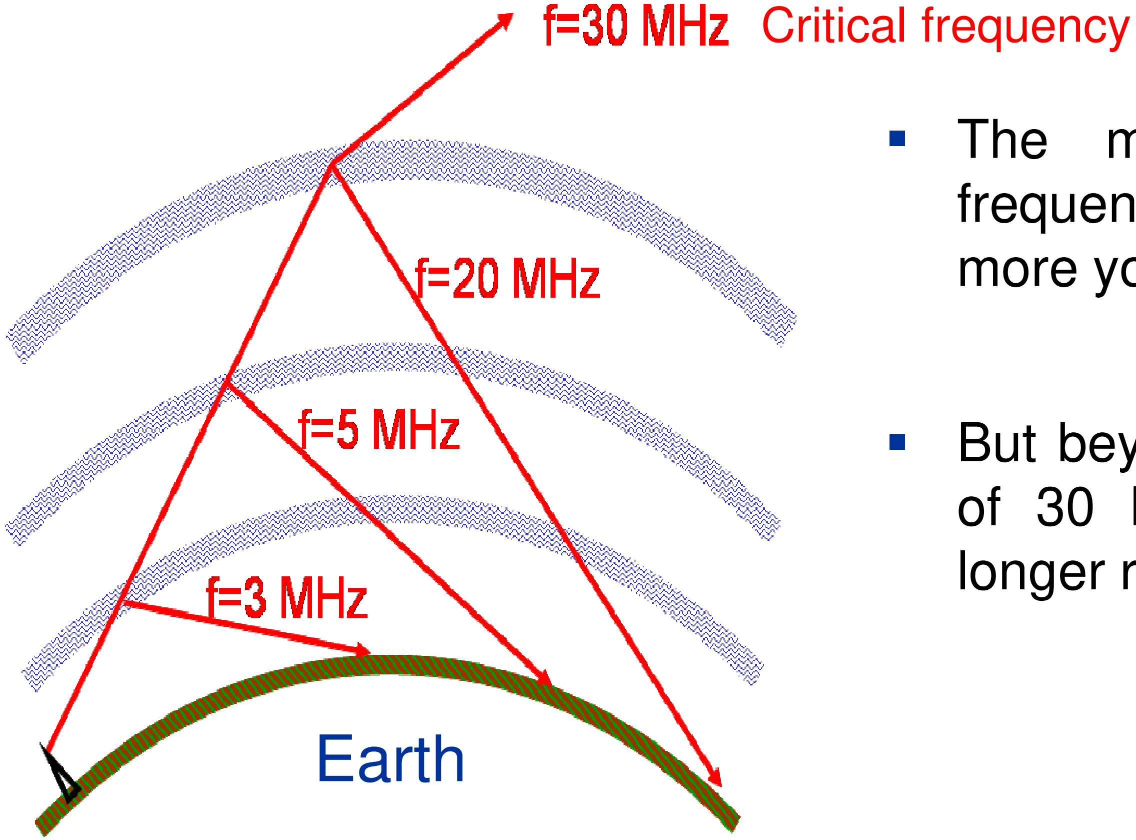
Propagation

Atmospheric propagation



- The more you use a high frequency (so a short wave), the more you increase the range.
- But beyond the critical frequency of 30 MHz, the waves are no longer reflected.

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Propagation

Attenuation – At the transmistter

- Assume an isotropic radiation. Radiates power equally in all directions.
- Does not exist in reality. A mathematical construct to compare other antennas to.
- Assume all of the transmitter power goes into space.

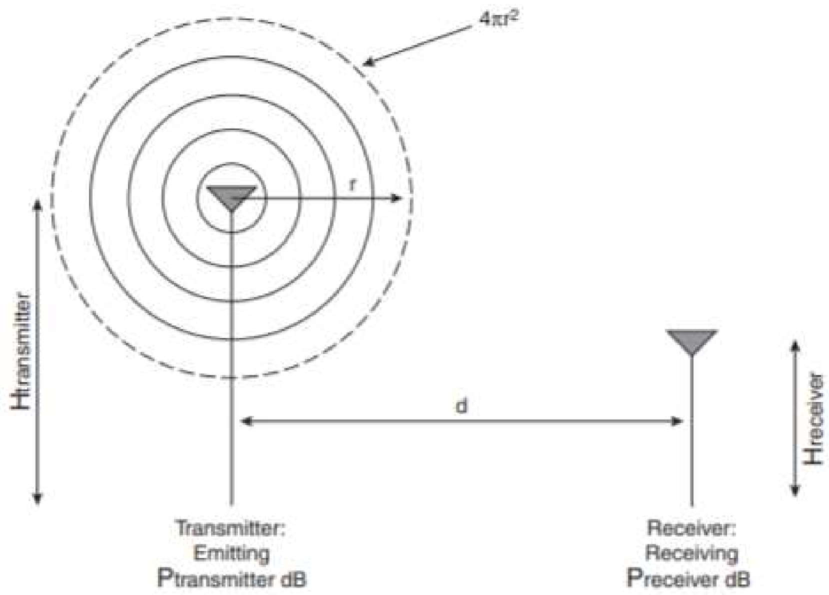
Propagation

Attenuation – Between transmitter and receiver

- Signal expands in all directions.
- At some distance, *d*, signal covers a sphere with surface area:

$$S = 4\pi d^2$$

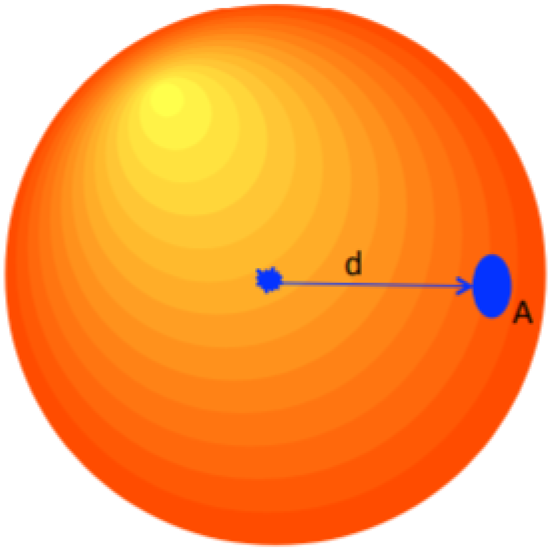
- Power density, P_s: $P_s = \frac{P_t}{S} = \frac{P_t}{4\pi d^2}$



Propagation

Attenuation – At the receiver

- Aperture : How much of the signal sphere is “captured” by the receiver antenna.
- For isotropic antenna, aperture is expressed as an area: $A = \frac{\lambda^2}{4\pi}$



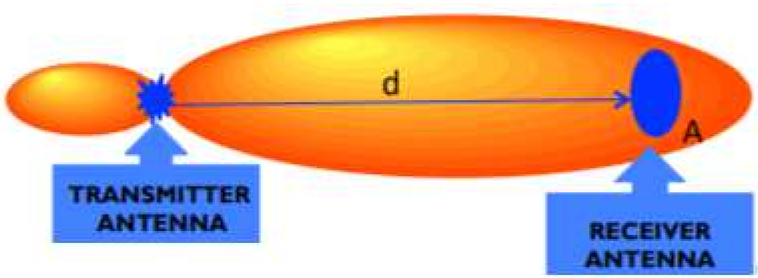
Propagation

Attenuation – Free space loss (1)

- Signal power at the receiver: $P_r = AP_s = \frac{P_t \lambda^2}{(4\pi d)^2}$

➔ Basic link equation for isotropic antennas

- Antenna Gain
  - Antenna is a passive device – cannot add power and may have losses.
  - Gain is power increased in one direction at the expense of it in another.
  - Same power over smaller area



Propagation

Attenuation – Free space loss (2)

- Link equation with antenna gains:

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi d)^2}$$

- Tradeoffs:
  - Higher frequency = lower receive power
  - But easier to build high gain antennas at higher frequency
  - Also lower noise at higher frequency

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Noise Sources

Classification

- Terrestrial, mostly lightning (HF)
- Extra-terrestrial, mostly the sun (VHF through microwaves)
- Man-made (possible at all frequencies, but usually low frequency)
- Thermal (all frequencies)
- Quantizing (only in digital signal processing)
- Circuit

Noise Sources

Thermal or Johnson noise

- Dependent on:
  - Absolute Temperature, T (Kelvin)
  - Bandwidth, B (Hz)
  - Boltzmann constant (k)

$$P_n = 4kTB, \text{ with } k = 1{,}38.10^{-23} \text{ joules/°K}$$

Noise Sources

Circuit noise

- From active devices
- Can be slightly above thermal noise power to many times thermal noise power.
- Careful design can minimize circuit noise.

Outline

1. Radio waves
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Example : designing a system

4. Wireless channel vs. AWGN channel
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Designing a System - Example

Situation

- F = 400 mHz.
- P_e <= 10^{-6}
- Range = 5 km max.
- Using PSK, data rate = 50 Kbaud.
- Required transmitter power = ?

Designing a System - Example

Noise at the receiver

- Bandwidth = 100 kHz
- Temperature = 300 K
- Antenna gains of 1
- Assume average receiver with circuit noise = 2x thermal noise.

$$P_n = 8kTB = 3{,}3.\,10^{-15}$$

Designing a System - Example

Solution

- Required SNR $$10^{-6} = \frac{1}{2}\text{erfc}\left(\frac{\sqrt{SNR}}{2\sqrt{2}}\right)$$

$$SNR = 90{,}4 \ (19{,}6 \ dB)$$

- Required receiver power

$$P_r = 90{,}4 \times 3{,}3 \times 10^{-15} = 3{,}0.10^{-13} W$$

- And finally back to link equation, the required transmitter power is given by:

$$3{,}0.10^{-13} = \frac{P_t G_t G_r \lambda^2}{(4\pi d)^2}$$

$$P_t = 209mW$$

… not a whole lot, but more than the USRP can deliver.

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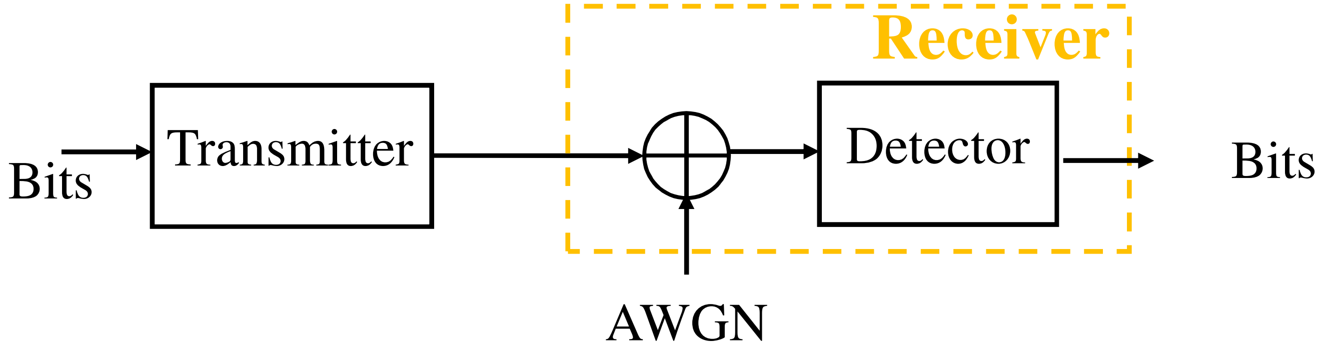
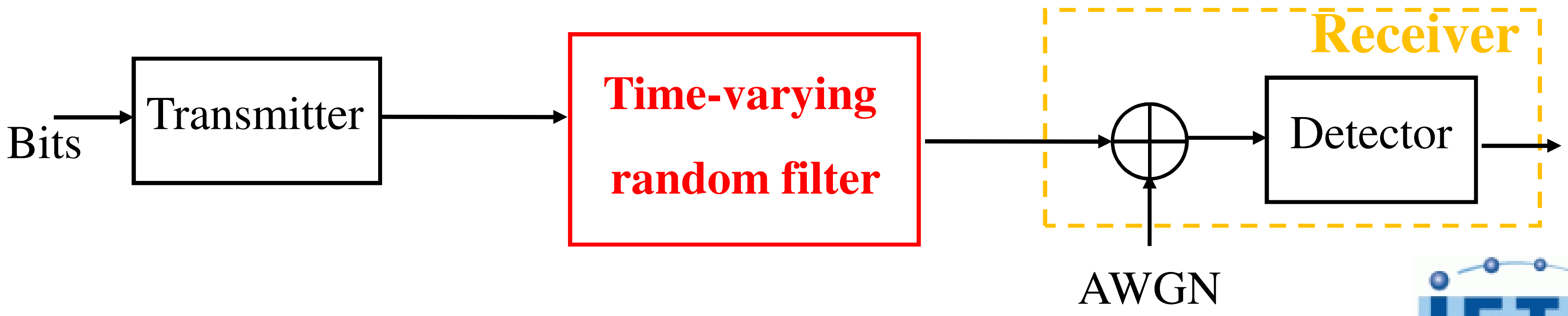
Wireless channel vs. AWGN

AWGN Channel

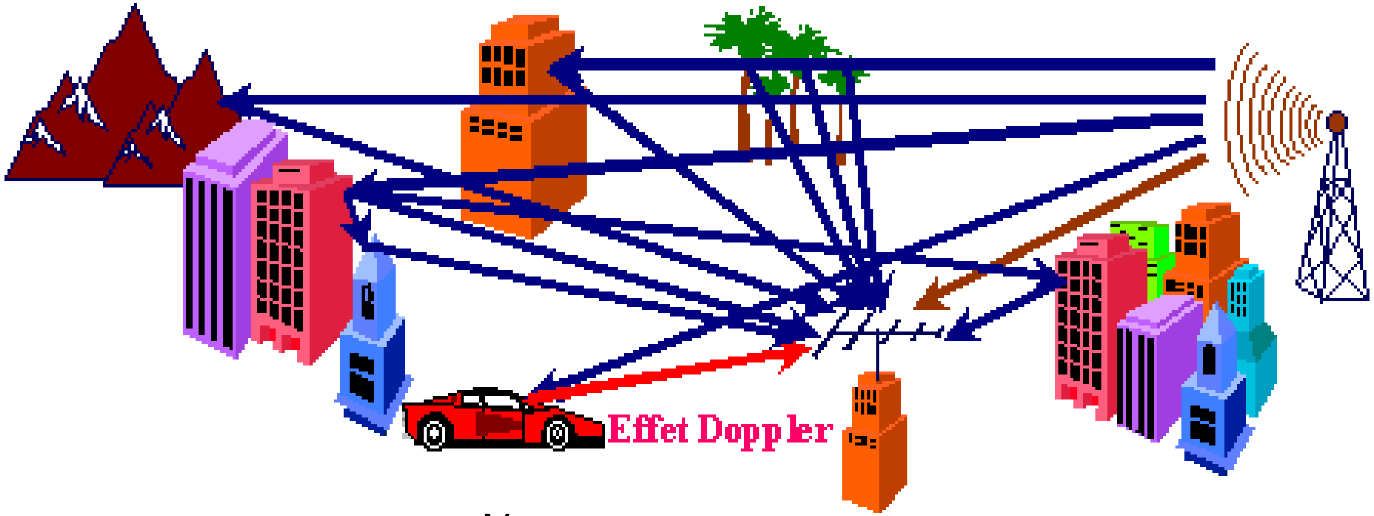


Wireless Communication Channel



Wireless channel vs. AWGN

Multi-path channel



$$r(t) = \sum_{n=0}^{N-1} h_n s(t - \tau_n) + \upsilon(t)$$

n^{th} path attenuation

n^{th} path delay

No line of sight → Rayleigh distribution

line of sight → Rice distribution

Wireless channel vs. AWGN

Fading distribution

- In a classical environment (theoretical perfect model) , noise is generally considered to be AWGN (Additive White Gaussian Noise), modeled by the normal distribution.
- In the case of a fading channel, the probability density of the attenuations will follow a law of the form:

$$f_h(h) = \frac{h}{\sigma^2} e^{-\left(\frac{h^2+A^2}{2\sigma^2}\right)} I_0\left(\frac{hA}{\sigma^2}\right) \quad \text{Rice distribution}$$

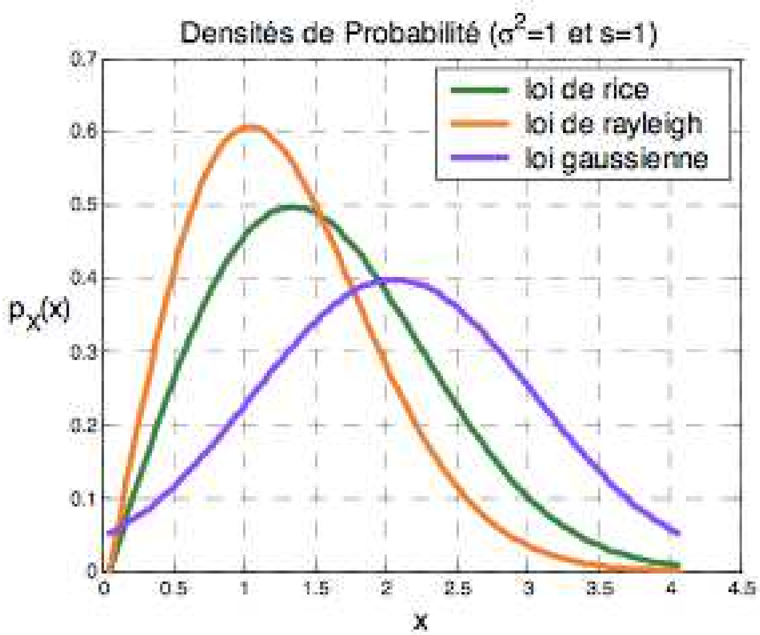
  - If A=0, we come to the Rayleigh distribution

- Rayleigh distribution is the preferred fading model because *it models the most severe conditions with the simplest expression*.

Wireless channel vs. AWGN

Rice vs. Rayleigh

- The Rayleigh model is used more than Rice model for the following reasons:
  - The Rayleigh model corresponds to a propagation without line of sight, and therefore more constraining. This allows to rely on the worst case
  - The Rayleigh model corresponds to many practical cases of propagation, in an urban environment or in indoor propagation (indoor)
  - The mathematical expression of a Rayleigh distribution is simpler than that of a Rice one



Wireless channel vs. AWGN

Channel coherence band and frequency selectivity

- The coherence band B_c gives an approximation of the band on which the channel behaves as a constant gain.
- The coherence band B_c makes it possible to characterize the time spread of the signal received in the frequency domain.
- Principle: Compare B_c to W, the band occupied by the transmitted signal.

	Frequency selectivity
$B_c < W$	Frequency-selective channel
$B_c > W$	Non frequency-selective channel (flat fading)

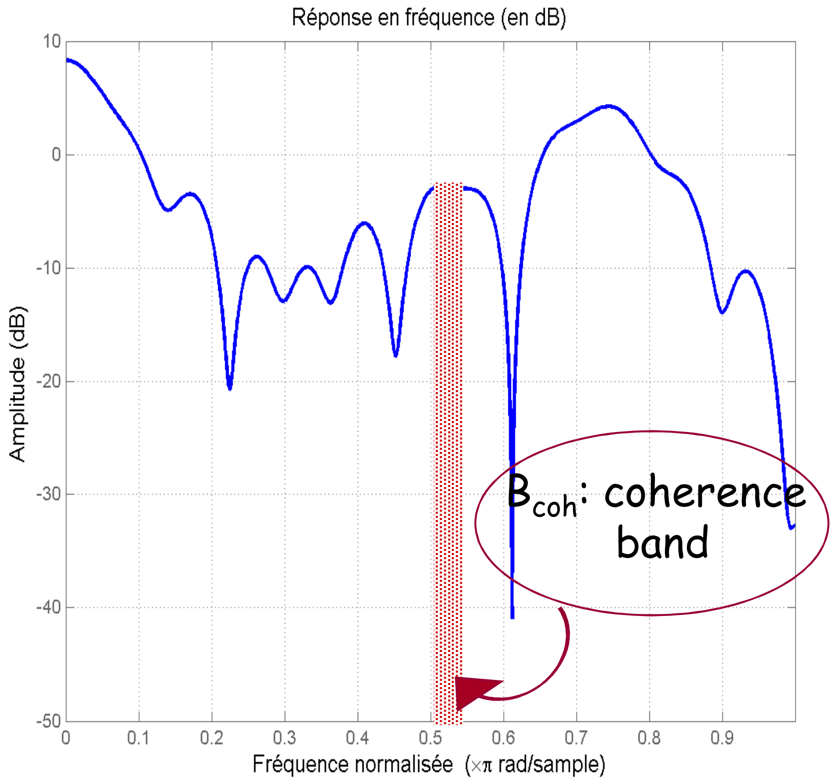
Wireless channel vs. AWGN

Frequency-selective channels ($B_c < W$)

- Interpretation: there are parts of W filtered differently and the channel introduces different gains depending on the frequency

➔Inter Symbol Interference (ISI) due to the channel

$$r(t) = h_n s_n + \sum_{n \neq k} h_n s_{n-k} + \upsilon_n$$



Wireless channel vs. AWGN

Multi-path spreading

- The multi-path spreading T_m gives an approximation of the time during which all the energy used to transmitted a symbol will be received.
- The multi-path spreading T_m allows to characterize the time spread of the signal received in the time domain.
- Principle: compare T_m to T, symbol period.

	Frequency selectivity
$T_m > T$	Frequency-selective channel
$T_m < T$	Non frequency-selective channel (flat fading)

$$\boxed{B_c \approx \frac{1}{T_m}}$$

Wireless channel vs. AWGN

Frequency selectivity in time domain

- Frequency-selective channel ($T_m > T$) :
  - The energy transmitted during a duration T is recovered over a duration greater than T
  - Dispersion of transmitted energy for a symbol beyond the duration of a symbol
  - Consequence : ISI (Inter Symbol Interference)

- Non frequency-selective channel– flat fading ($T_m < T$) :
  - The energy transmitted during a duration T is recovered over a duration less than T
  - No ISI but a risk of destructive combination of paths (phase opposition)
  - Consequence : Possible decrease of SNR

Wireless channel vs. AWGN

Time variation of the channel – Coherence Time

- The coherence time T_c gives an approximation of the time during which the behavior of the channel is relatively constant
- The coherence time T_c allows to characterize the time variation of the channel in the time domain
- Principle: compare T_c to T, symbol period.

	Fading variation
$T_c > T$	Slow Fading
$T_c < T$	Fast Fading

Wireless channel vs. AWGN

Slow fading vs fast fading

- Slow fading :
  - The channel changes but slowly
  - Channel coefficients remain constant all the time of a frame transmission

$$r(t) = \sum_{n=0}^{N-1} h_n s(t - \tau_n) + \upsilon(t)$$

- Fast fading :
  - The channel changes very quickly
  - It is impossible to consider the gains of the paths as constant on an observation window

$$r(t) = \sum_{n=0}^{N-1} h_n(t) s(t - \tau_n(t)) + \upsilon(t)$$

Wireless channel vs. AWGN

Doppler spreading

- Doppler spreading f_d gives an approximation of the band on which the channel spreads the spectral components
- Doppler spreading f_d characterizes the time variation of the channel in the frequency domain
- Principle : compare f_d to W, the band occupied by the transmitted signal.

	Fading variation
$f_d < W$	Slow Fading
$f_d > W$	Fast Fading

$$\boxed{T_c \approx \frac{1}{f_d}}$$

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Wireless channel vs. AWGN

Summary (1)

Time spreading of the signal	Time variation of the channel
Frequency-selective (ISI) $T_m > T$	Fast fading (PLL failure, high Doppler) : $f_d > W$
(Non selective) Flat fading (decrease of SNR) $T_m < T$	Slow fading (decrease of SNR) $f_d < W$
Frequency-selective (ISI) $B_c < W$	Fast fading (PLL failure, high Doppler) : $T_c > T$
(Non selective) Flat fading (decrease of SNR) $B_c > W$	Slow fading (decrease of SNR) $T_c < T$

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Wireless channel vs. AWGN

Summary (2)

Doppler spreading

Frequency-selctive and fast fading

$$r(t)=\sum_{n=0}^{N-1} h_n(t)s(t-\tau_n(t))+\upsilon(t)$$

Non frequency-selective and fast fading

$$r(t)=h(t)s(t)+\upsilon(t)$$

W

Frequency-selctive and slow fading

$$r(t)=\sum_{n=0}^{N-1} h_n s(t-\tau_n)+\upsilon(t)$$

Non frequency-selective and slow fading

$$r(t)=hs(t)+\upsilon(t)$$

W

Coherence band

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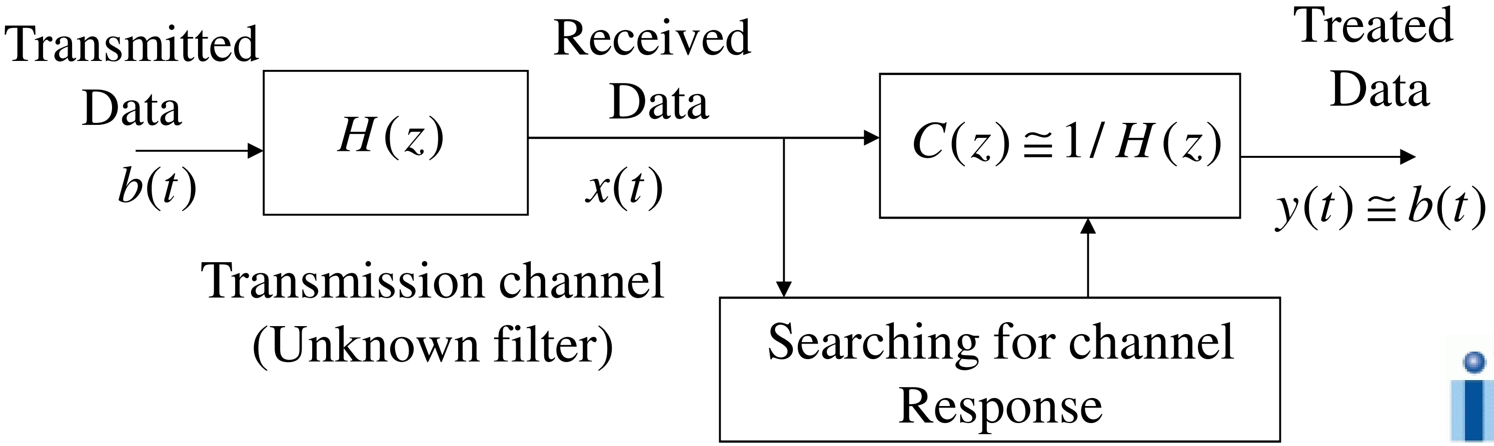
Wireless channel vs. AWGN

Equalization

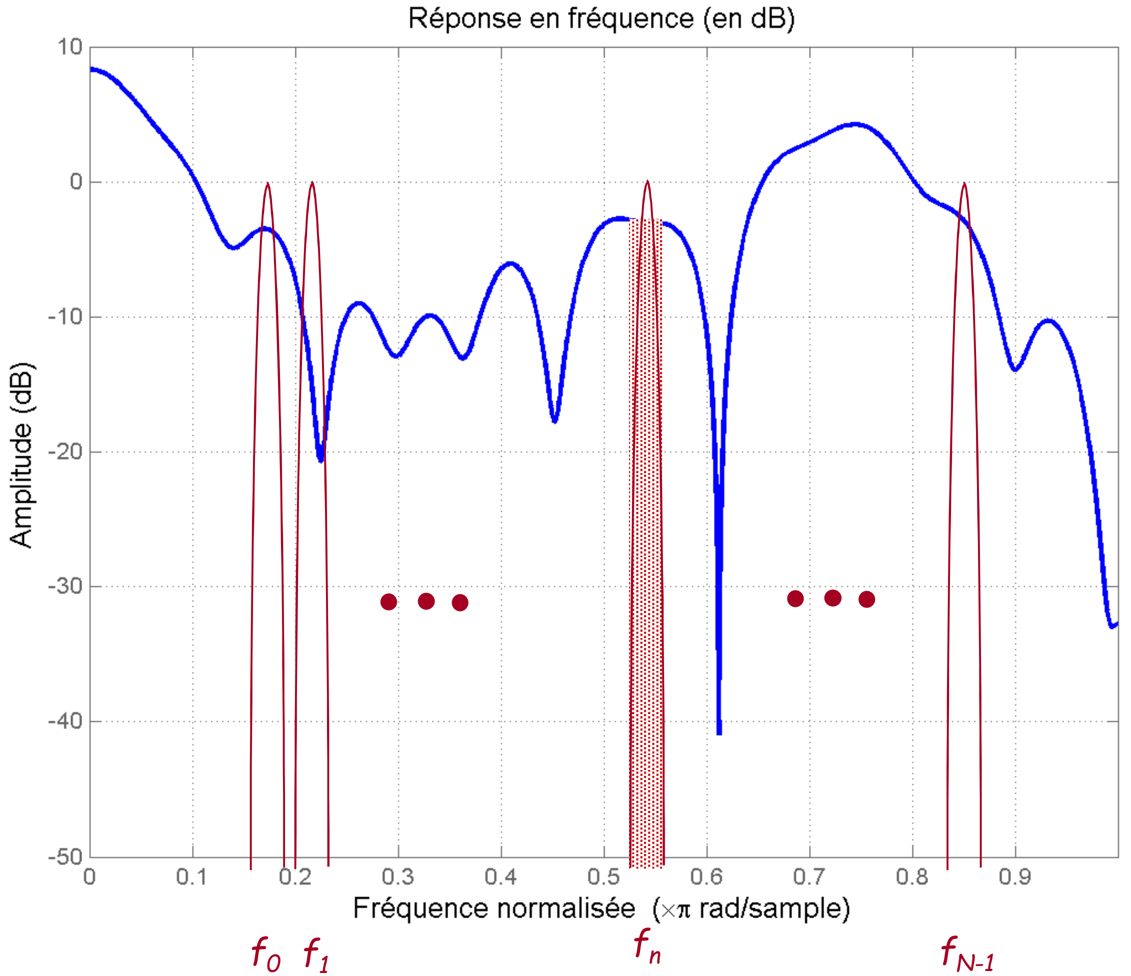
- Compensation for transmission channel impairments
- The channel acts as a linear filter $x(t)=\sum_k m(k)h(t-k)$
- Consists of finding, from the received signal, the characteristics of the inverse filter and applying it to the signal



Outline

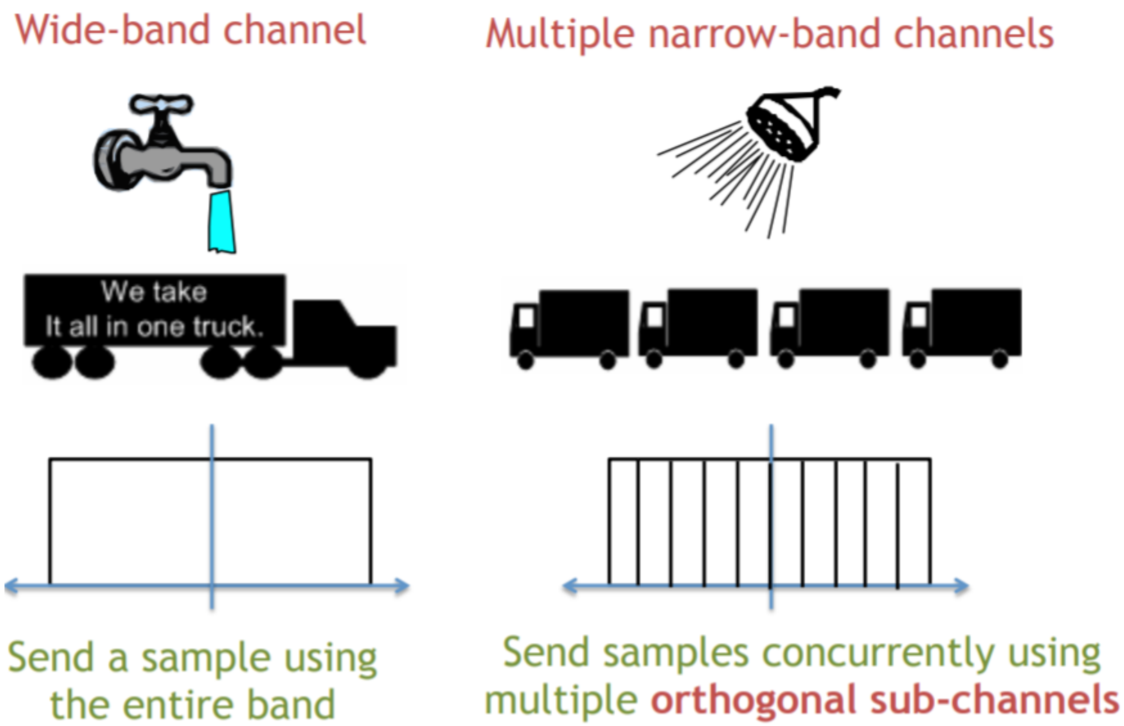
Multi-Carrier modulation

Principle (1)



Multi-Carrier modulation

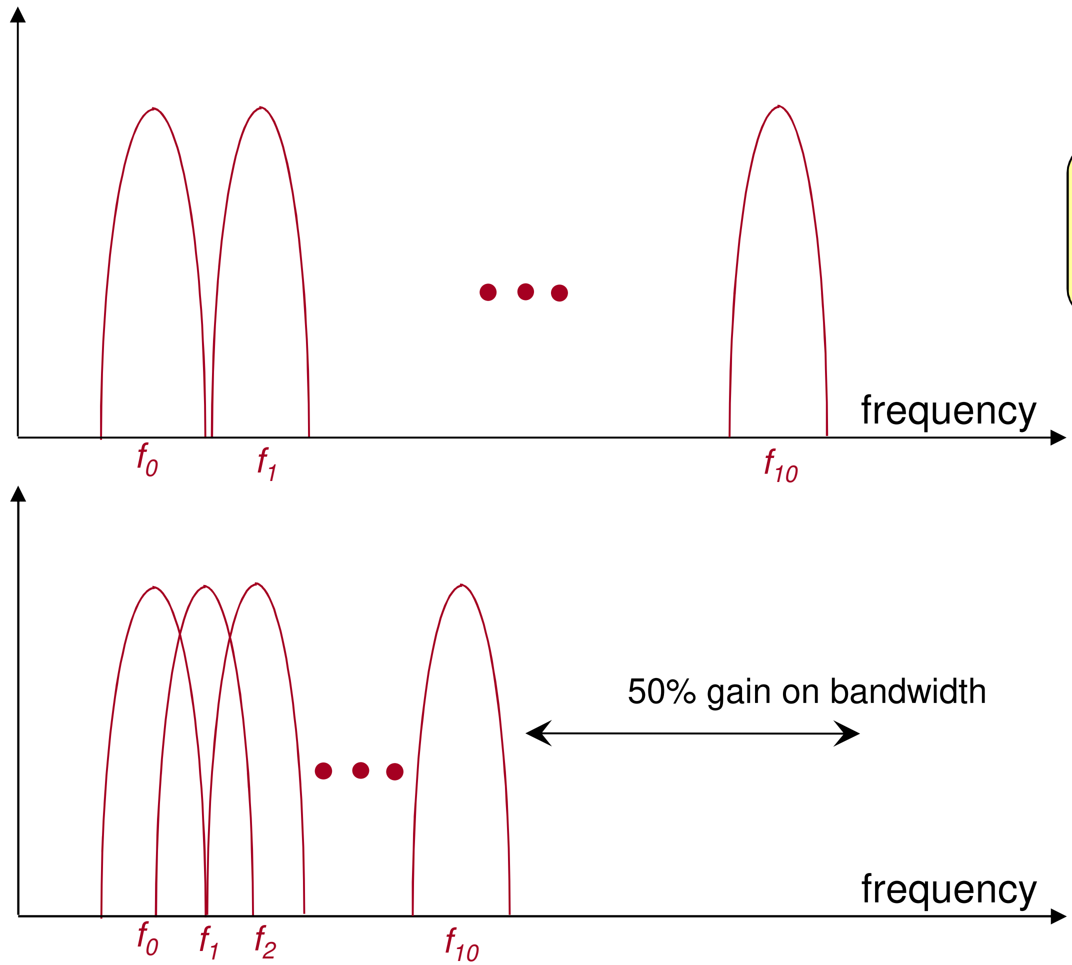
Principle (2)



- Multiple sub-channels (sub-carriers) carry samples sent at a lower rate
  - Almost same bandwidth with wide-band channel
  - Only some of the sub-channels are affected by interferers
    or multi-path effect

Multi-Carrier modulation

Orthogonality



Traditionnal approach

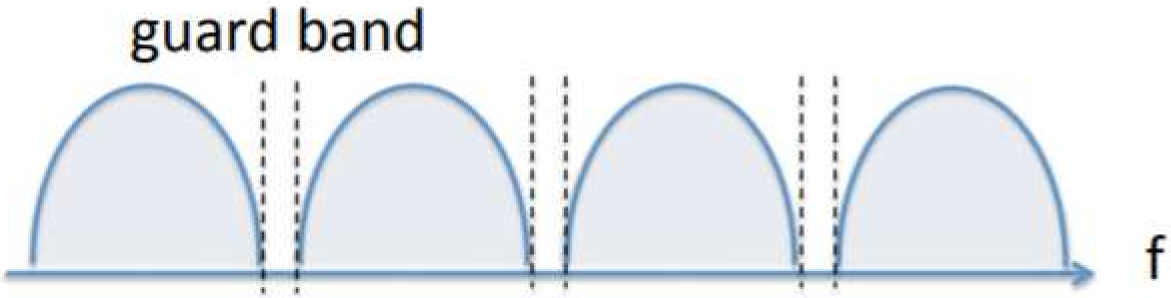
OFDM Concept

Orthogonal Frequency
Division Multiplex

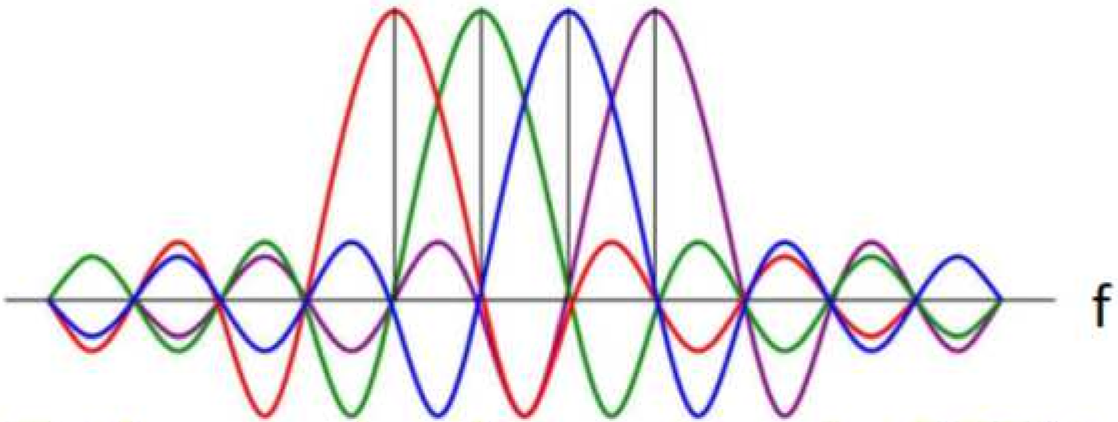
Sub-carrier placed
at multiples of 1/T

Multi-Carrier modulation

Difference between FDM and OFDM



Frequency division multiplexing



Orthogonal sub-carriers in OFDM

Don't need guard bands

Multi-Carrier Modulation

General OFDM expression

$$x_a(t) = \sum_{n=0}^{N-1} \sum_{k \in Z} s_k^{(n)} g_a(t - kT) e^{2i\pi f_n t}$$

Transmitted signal

Sub-carrier symbols

waveform

Notations

T_s : symbol period
N : number of sub-carriers
$T=NTs$: OFDM symbol period
$\sum_{n=0}^{N-1} s_k^{(n)} e^{2i\pi f_n t}$: OFDM symbol

Orthogonality of any two bins

$$\sum_{t=0}^{N-1} e^{-j\,2\pi kt/N} e^{-j\,2\pi pt/N} = 0, \forall p \neq k$$

Multi-Carrier Modulation

OFDM Transmission – Example (1)

- Say we use BPSK and 4 sub-carriers to transmit a stream of samples

  1, 1, -1, -1, 1, 1, 1, -1, 1, -1, -1, -1, -1, 1, -1, -1, -1, 1,…

- Serial to parallel conversion of samples

Frequency-domain signal

	c1	c2	c3	c4
symbol1	1	1	-1	-1
symbol2	1	1	1	-1
symbol3	1	-1	-1	-1
symbol4	-1	1	-1	-1
symbol5	-1	1	1	-1
symbol6	-1	-1	1	1

IFFT →

Time-domain signal

0	2 - 2i	0	2 + 2i
2	0 - 2i	2	0 + 2i
-2	2	2	2
-2	0 - 2i	-2	0 + 2i
0	-2 - 2i	0	-2 + 2i
0	-2 + 2i	0	-2 - 2i

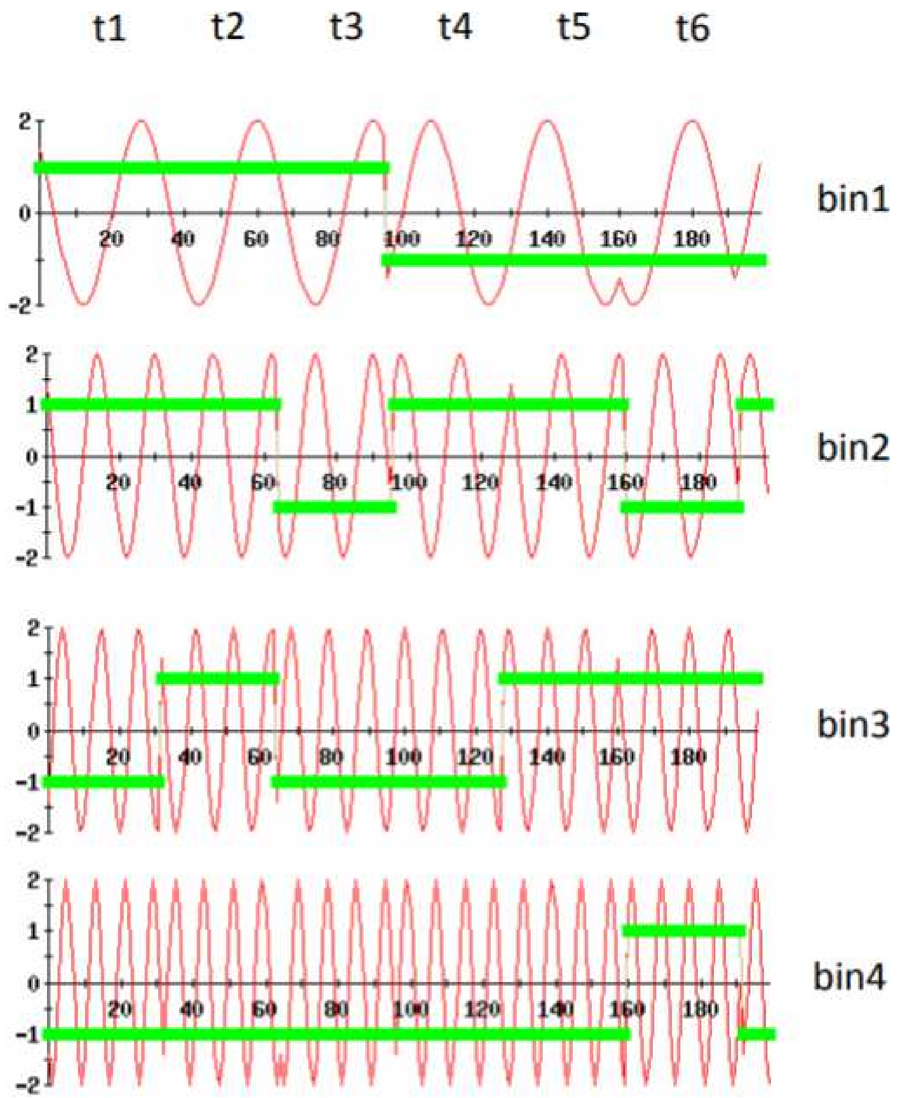
- Parallel to serial conversion, and transmit time-domain samples

  0, 2 - 2i, 0, 2 + 2i, 2, 0 - 2i, 2, 0 + 2i, -2, 2, 2, 2, -2, 0 - 2i, -2,
  0 + 2i, 0, -2 - 2i, 0, -2 + 2i, 0, -2 + 2i, 0, -2 - 2i, …

Multi-Carrier Modulation

OFDM Transmission – Example (2)

symbol1	1	1	-1	-1
symbol2	1	1	1	-1
symbol3	1	-1	-1	-1
symbol4	-1	1	-1	-1
symbol5	-1	1	1	-1
symbol6	-1	-1	1	1

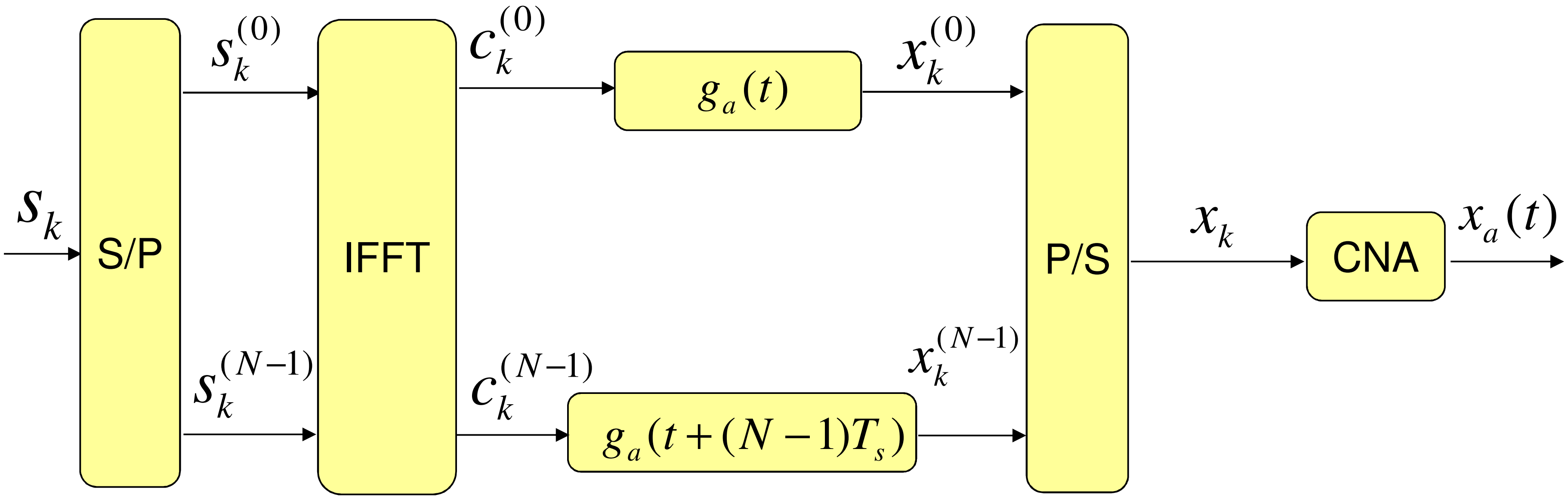


Multi-Carrier Modulation

OFDM Transmitter:

IFFT

frequency-domain samples → time-domain samples

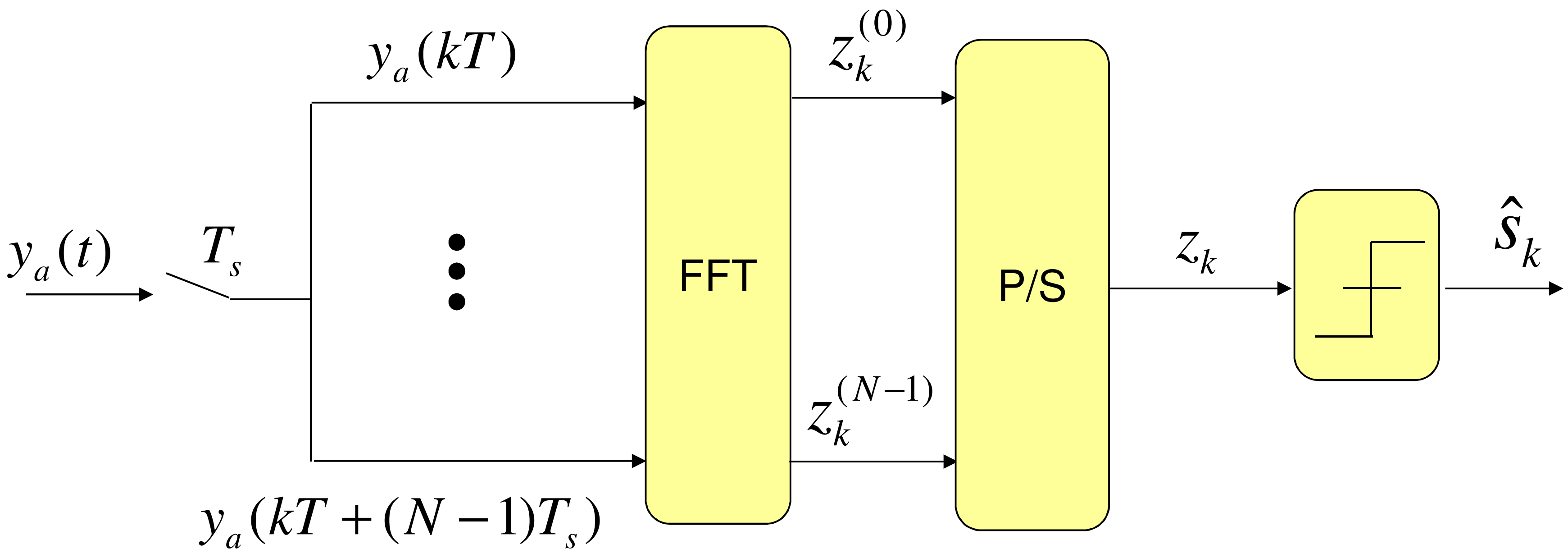


- Simple FFT algorithm: easy to implement (DSP, FPGA, ASIC…)
- One DAC is enough

Multi-Carrier Modulation

OFDM Receiver

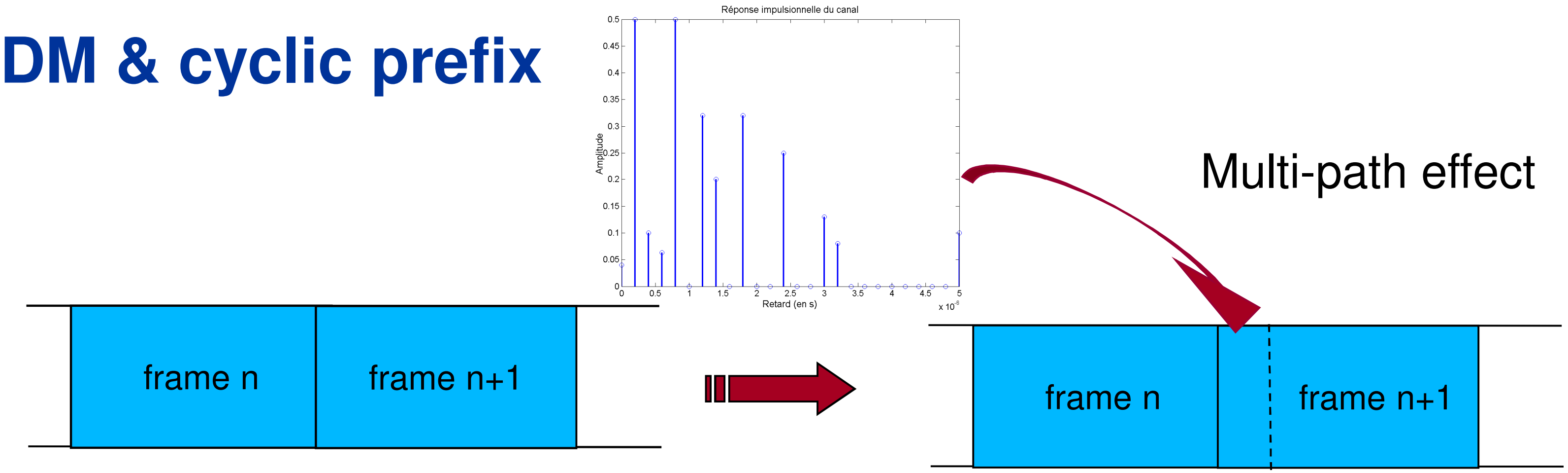
FFT
time-domain samples → frequency-domain samples



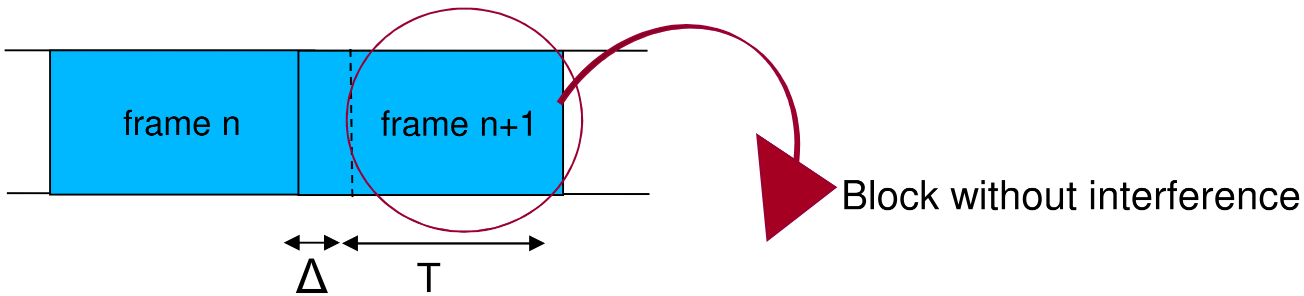
- Very simple receiver: dual transmitter

Multi-Carrier Modulation

OFDM & cyclic prefix



- OFDM frames are delayed because of the channel and overlap in time



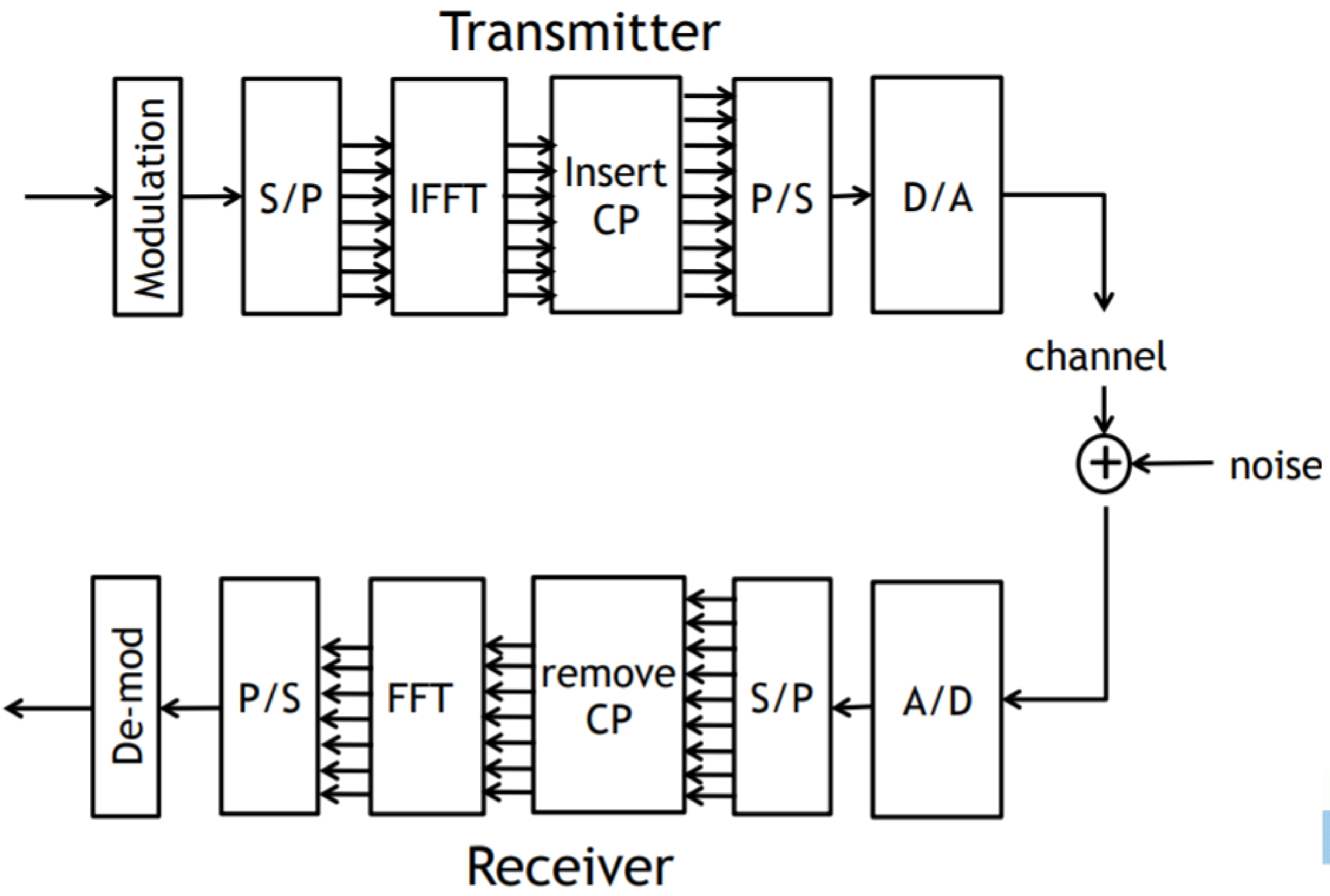
➔ Cyclic prefix $x_k = \left\{ x_k^{(0)}, x_k^{(1)}, \ldots, x_k^{(L-1)}, x_k^{(0)}, x_k^{(1)}, \ldots, x_k^{(N-1)} \right\}$

Cyclic prefix Δ of length L

Output of the FFT inverse

Multi-Carrier Modulation

OFDM diagram



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