Unit 4: Transmission Theory of the MOSFET

Lecture 4.2: Landauer at Low and High Bias

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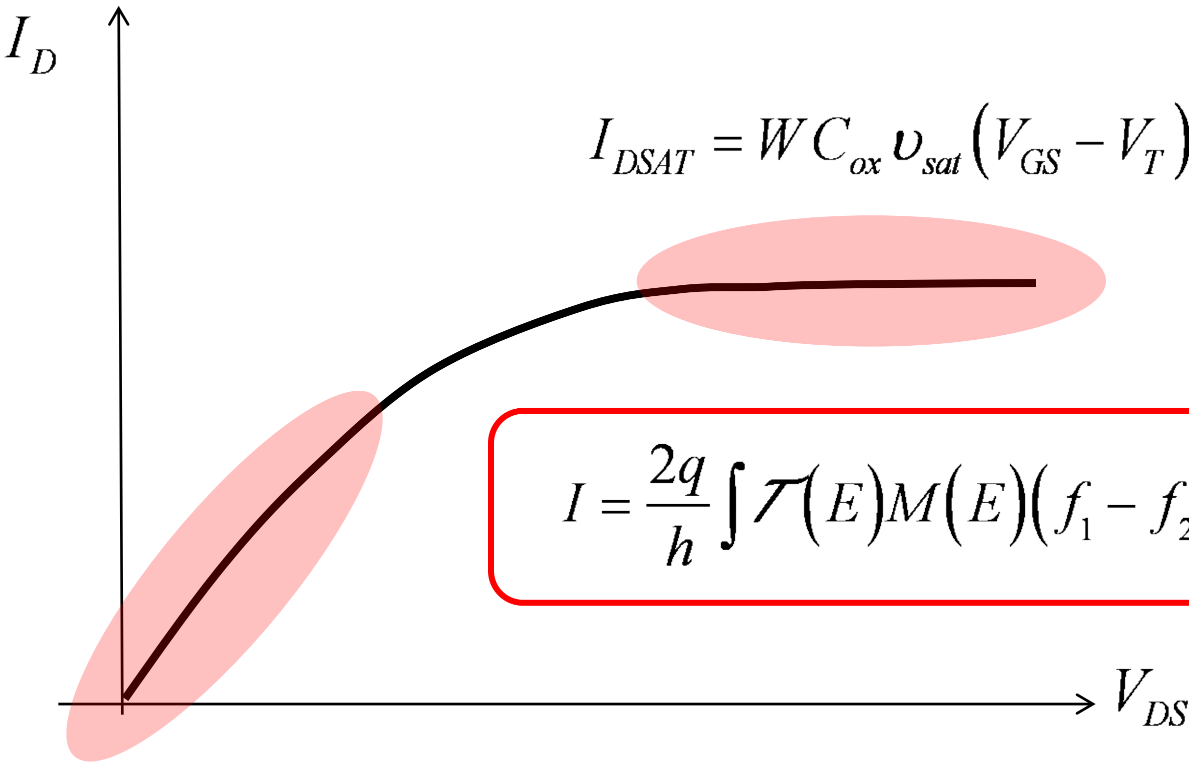
Low and high bias Landauer expressions

$$I_{DSAT} = W C_{ox} \upsilon_{sat} \left(V_{GS} - V_T \right) \rightarrow ?$$



$$\boxed{I = \frac{2q}{h} \int \mathcal{T}\left(E\right) M\left(E\right) \left(f_1 - f_2\right) dE}$$

$$I_{DLIN} = \frac{W}{L} \mu_n C_{ox} \left(V_{GS} - V_T \right) V_{DS} \rightarrow ?$$

1) Low bias

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)\left(\underline{f_1(E) - f_2(E)}\right)dE$$

Fermi window

$$f_1(E) = \frac{1}{1+e^{(E-E_{F1})/k_BT}} = f_0(E)$$

$$\delta E_F = -qV$$

$$f_2(E) \approx f_1(E) + \frac{\partial f_1}{\partial E_F}\delta E_F$$

$$f_2(E) \approx f_1(E) + \left(-\frac{\partial f_1}{\partial E}\right)\delta E_F$$

$$f_1(E) - f_2(E) \approx -\left(-\frac{\partial f_1}{\partial E}\right)\delta E_F$$

$$\boxed{f_1(E) - f_2(E) = \left(-\frac{\partial f_0}{\partial E}\right)(qV)}$$

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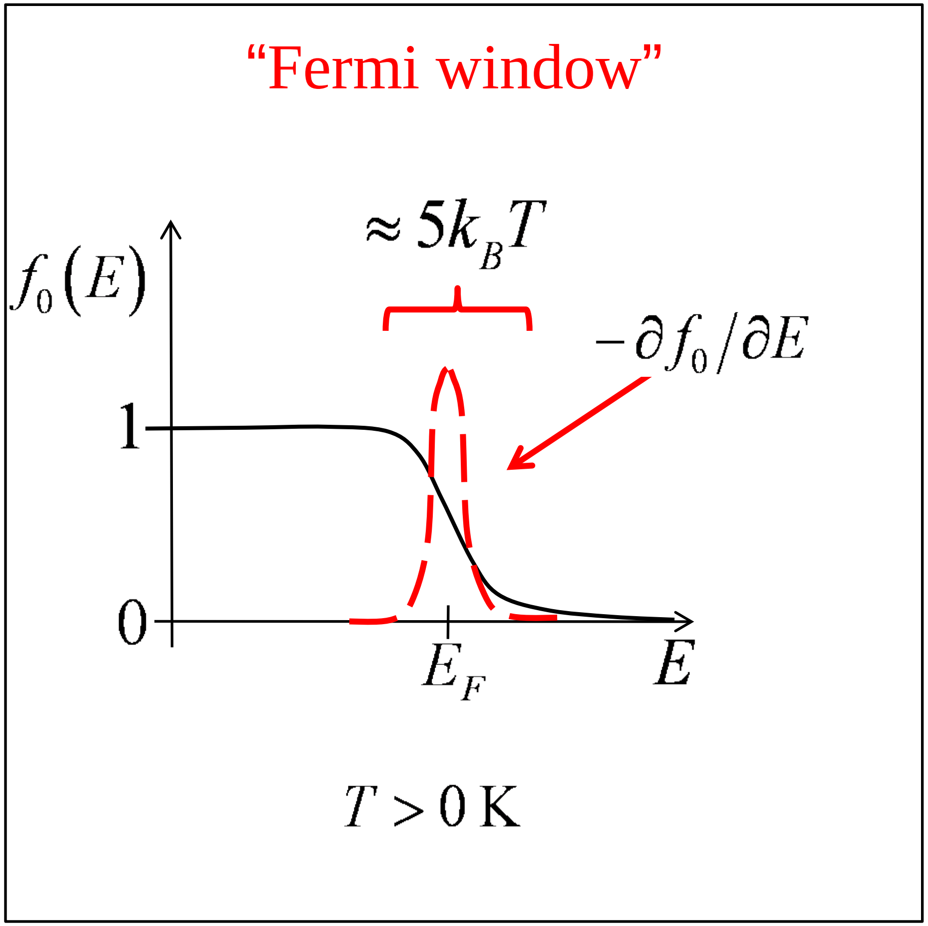
Fermi window: Low bias

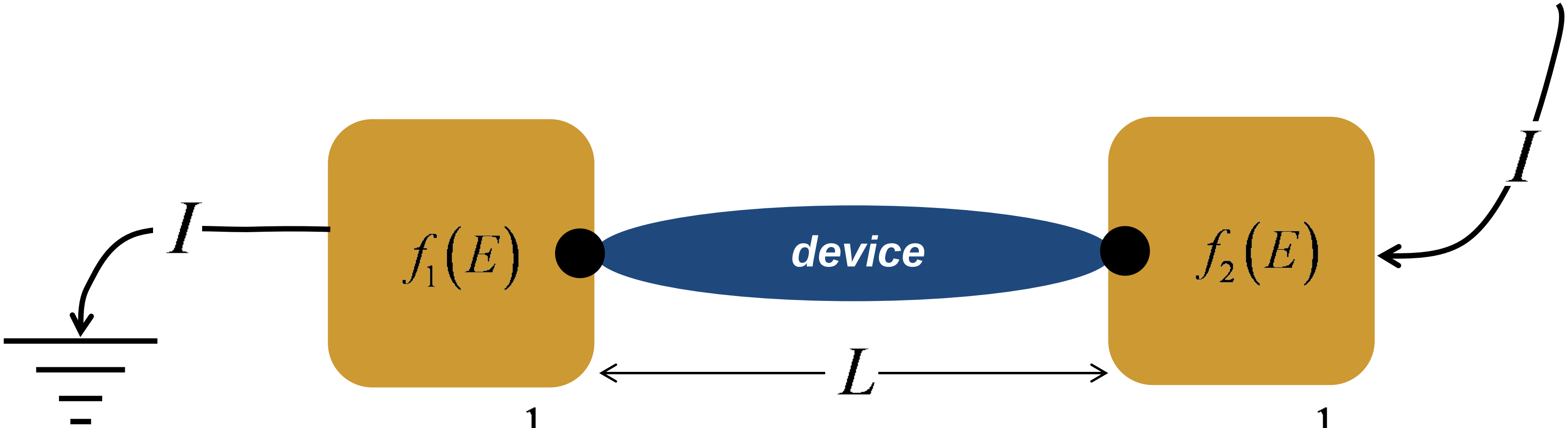


$$W_F(E) = \left(-\frac{\partial f_0}{\partial E}\right)$$

$$\int W_F(E)\,dE = 1$$

(window function)

Current for a small voltage difference



$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}} \qquad f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) (f_1 - f_2) dE$$

$$f_1(E) - f_2(E) \rightarrow \left(-\frac{\partial f_1}{\partial E} \right) (qV) \Rightarrow I = GV$$

Small bias conductance

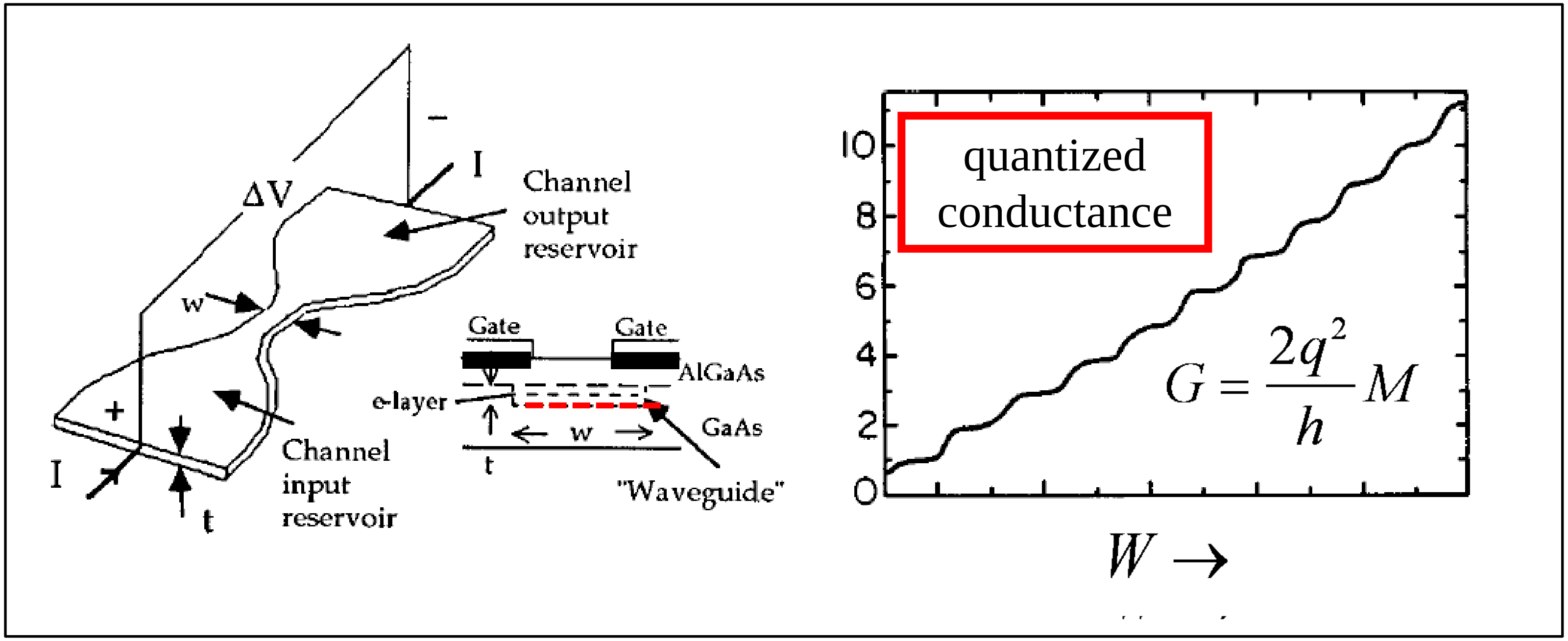
$$I = GV \qquad \mathrm{A}$$

$$G = \frac{2q^2}{h}\int \mathcal{T}(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \qquad \mathrm{S}$$

$\mathcal{T}(E) = 1$ $\qquad \left(-\frac{\partial f_0}{\partial E}\right) = \delta(E_F)$ $\qquad \rightarrow \quad G = \frac{2q^2}{h}M(E_F)$

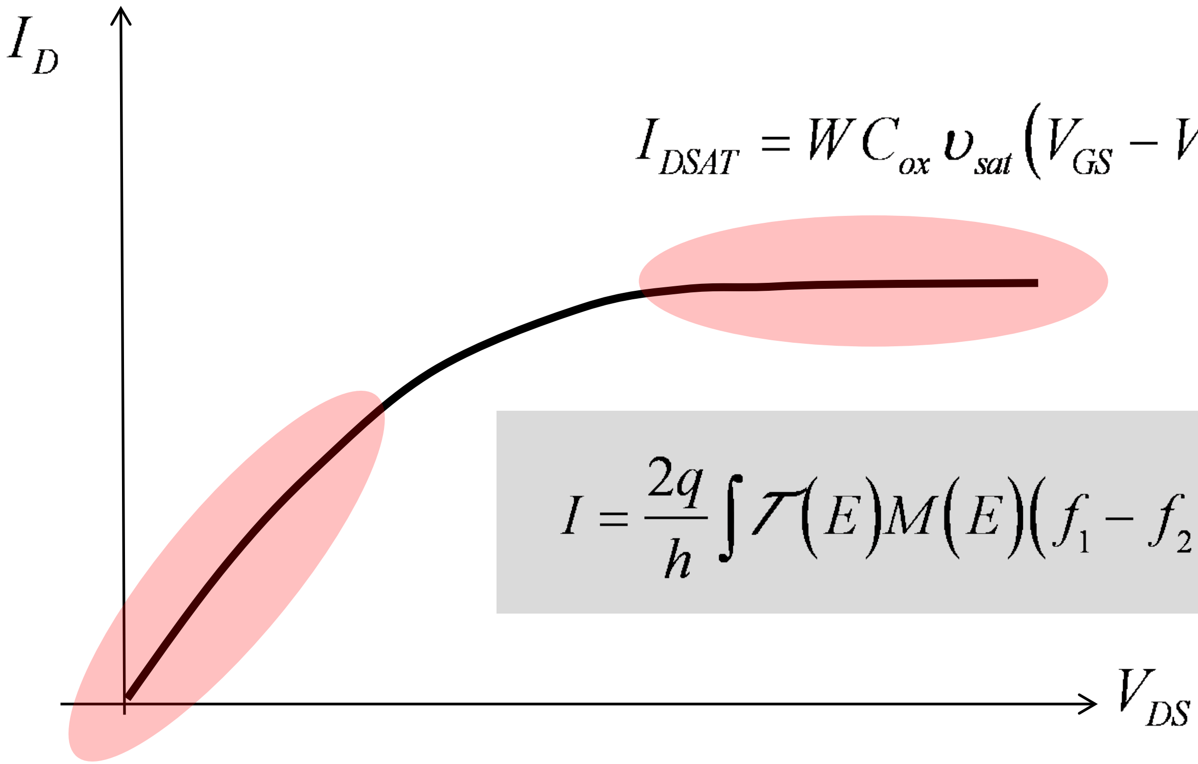
(ballistic) (T ≈ 0 K)

Quantized conductance



D. Holcomb, *American J. Physics*, **67**, pp. 278-297 1999.
Data from: B. J. van Wees, et al., *Phys. Rev. Lett*. **60**, 848851, 1988.

1) Linear Current in the Landauer Approach



$$I_{DSAT} = W C_{ox} \upsilon_{sat} \left(V_{GS} - V_T\right)$$

$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) \left(f_1 - f_2\right) dE$$

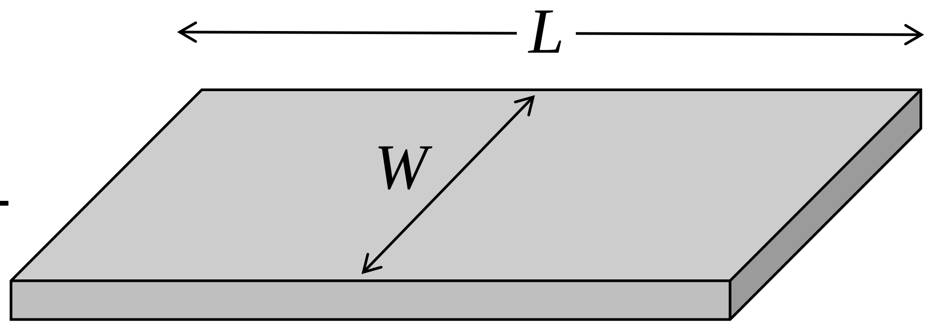
$$I_{DLIN} = \frac{W}{L} \mu_n C_{ox} \left(V_{GS} - V_T\right) V_{DS}$$

$$\boxed{I_{DLIN} = \left[\frac{2q^2}{h} \int \mathcal{T}(E) M(E) \left(-\frac{\partial f_0}{\partial E}\right) dE \right] V_{DS}}$$

Aside: Bulk semiconductors

Before we consider the high bias case, let’s consider a bulk semiconductor (many MFP’s long in both directions).

Contact 1



Contact 2

2D conductor

$$G = \sigma_S \frac{W}{L} \qquad \sigma_S = G\frac{L}{W} \quad \Omega/\square \qquad \sigma_S \equiv n_S q \mu_n$$

Conductivity (bulk)

$$G = \frac{2q^2}{h}\int \mathcal{T}(E) M(E)\left(-\frac{\partial f_0}{\partial E}\right) dE \qquad \sigma_S = G\frac{L}{W}$$

$$\sigma_S = \frac{2q^2}{h}\int \left[\mathcal{T}(E) L\right] \left| M(E)/W \right| \left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$\mathcal{T}(E) = \frac{\lambda(E)}{\lambda(E)+L} \rightarrow \frac{\lambda(E)}{L}$$ diffusive

$$M(E) = W\frac{\sqrt{2m^*\left(E-E_C\right)}}{\pi\hbar}$$ 2D

Sheet conductivity

$$\boxed{\sigma_S = \frac{2q^2}{h}\int \lambda(E)\left(M(E)/W\right)\left(-\frac{\partial f_0}{\partial E}\right)dE}$$

$$\lambda(E) = \lambda_0 \qquad M(E)/W = \frac{\sqrt{2m^*\left(E - E_C\right)}}{\pi\hbar} \qquad f_0(E) = \frac{1}{1 + e^{(E-E_F)/k_BT}}$$

$$\sigma_S = \frac{q^2}{h}\lambda_0 \frac{\sqrt{2\pi m^* k_B T}}{\pi\hbar} e^{(E_F - E_C)/k_BT}$$ (non-degenerate)

$$n_S = \frac{m^* k_B T}{\pi\hbar^2} e^{(E_F - E_C)/k_BT}$$

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Sheet conductivity

$$\sigma_S = \frac{q^2}{h}\lambda_0 \frac{\sqrt{2\pi m^* k_B T}}{\pi\hbar} e^{(E_F - E_C)/k_B T} \equiv n_S q \mu_n$$

$$n_S = \frac{m^* k_B T}{\pi\hbar^2} e^{(E_F - E_C)/k_B T}$$

$$\mu_n = \frac{\upsilon_T \lambda_0}{2(k_B T/q)}$$

$$\upsilon_T = \sqrt{\frac{2k_B T}{\pi m^*}} \text{ m/s}$$

uni-directional thermal velocity (non-degenerate)

$$\frac{D_n}{\mu_n} = \frac{k_B T}{q}$$

(Einstein relation)

$$\boxed{D_n = \frac{\upsilon_T \lambda_0}{2} \text{ cm}^2/\text{s}}$$

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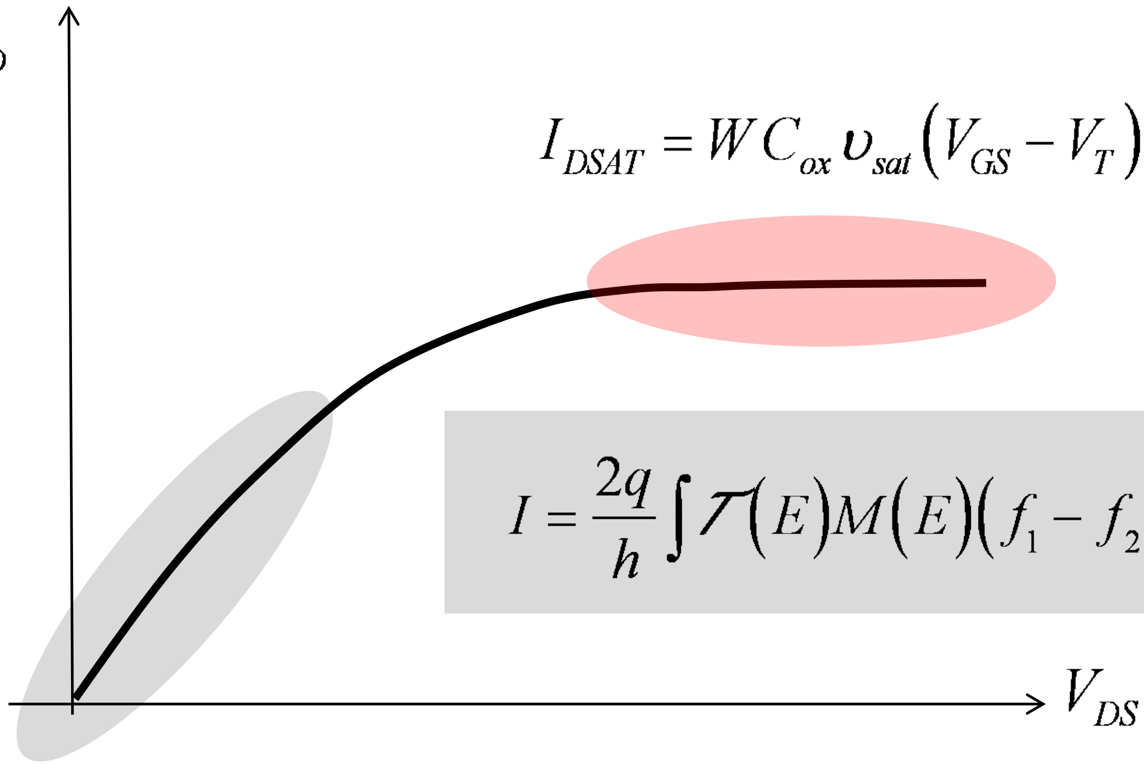
2) Saturation Current in the Landauer Approach



$$I_{DSAT} = W C_{ox} \upsilon_{sat} \left(V_{GS} - V_T \right) \rightarrow ?$$

$$I = \frac{2q}{h} \int \mathcal{T}\left(E\right) M\left(E\right)\left(f_1 - f_2\right) dE$$

$$I_{DLIN} = \frac{W}{L} \mu_n C_{ox} \left(V_{GS} - V_T \right) V_{DS}$$

$$\boxed{I_{DLIN} = \left[\frac{2q^2}{h} \int \mathcal{T}\left(E\right) M\left(E\right)\left(-\frac{\partial f_0}{\partial E} \right) dE \right] V_{DS}}$$

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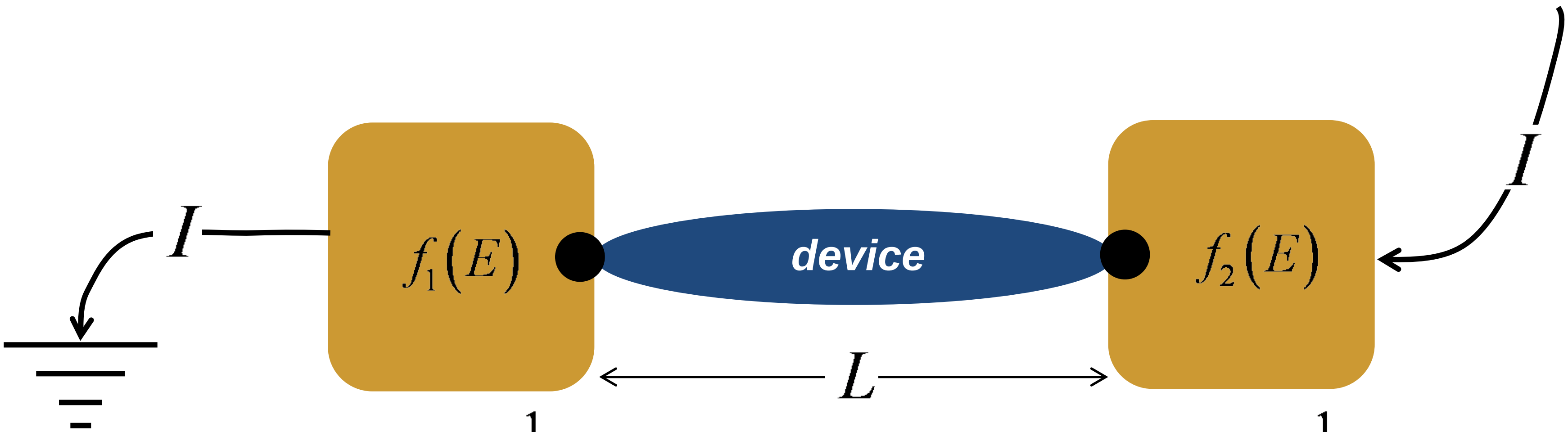
Current for a large voltage difference

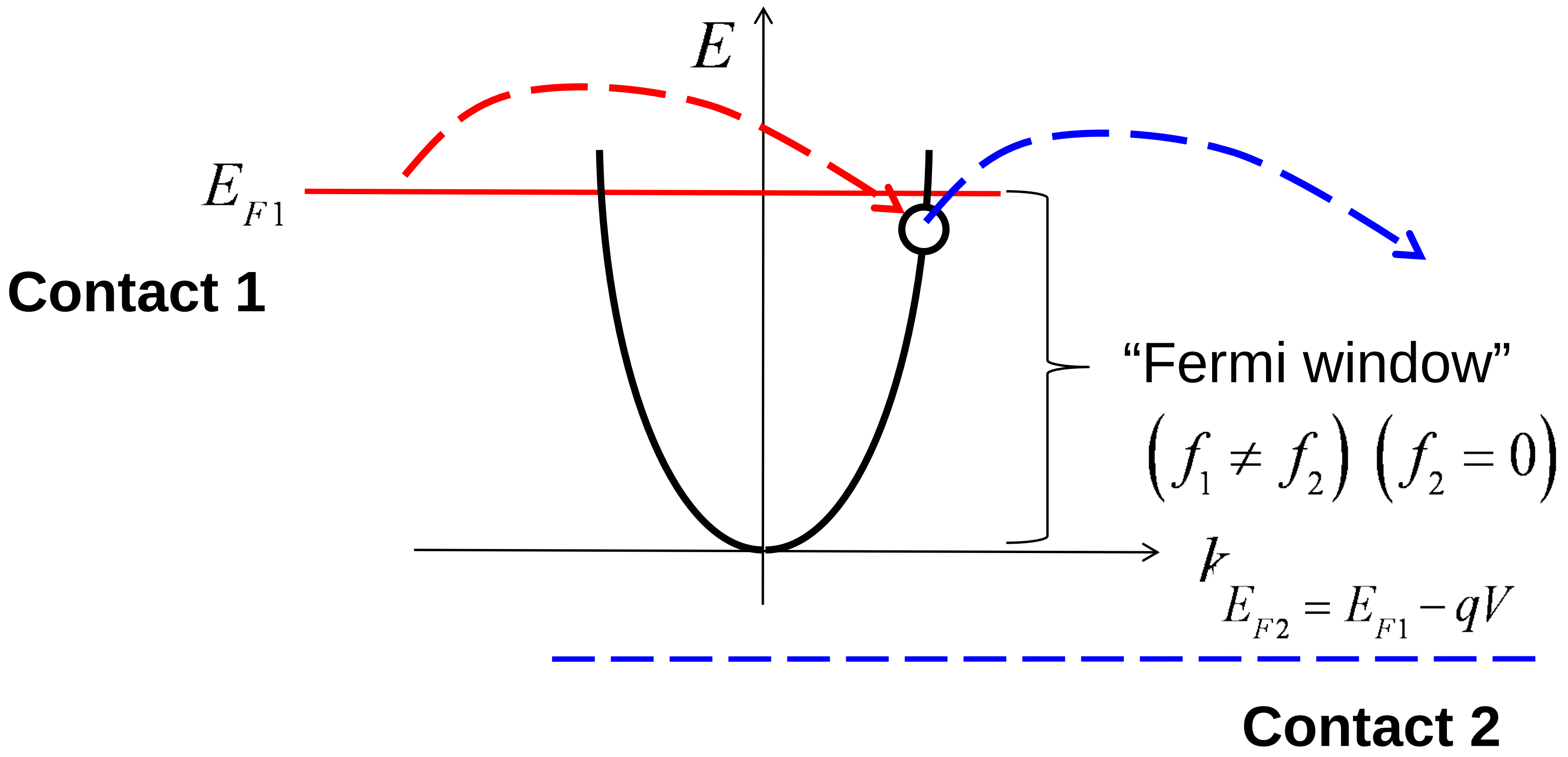


$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}} \qquad f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) \left(f_1 - f_2\right) dE$$

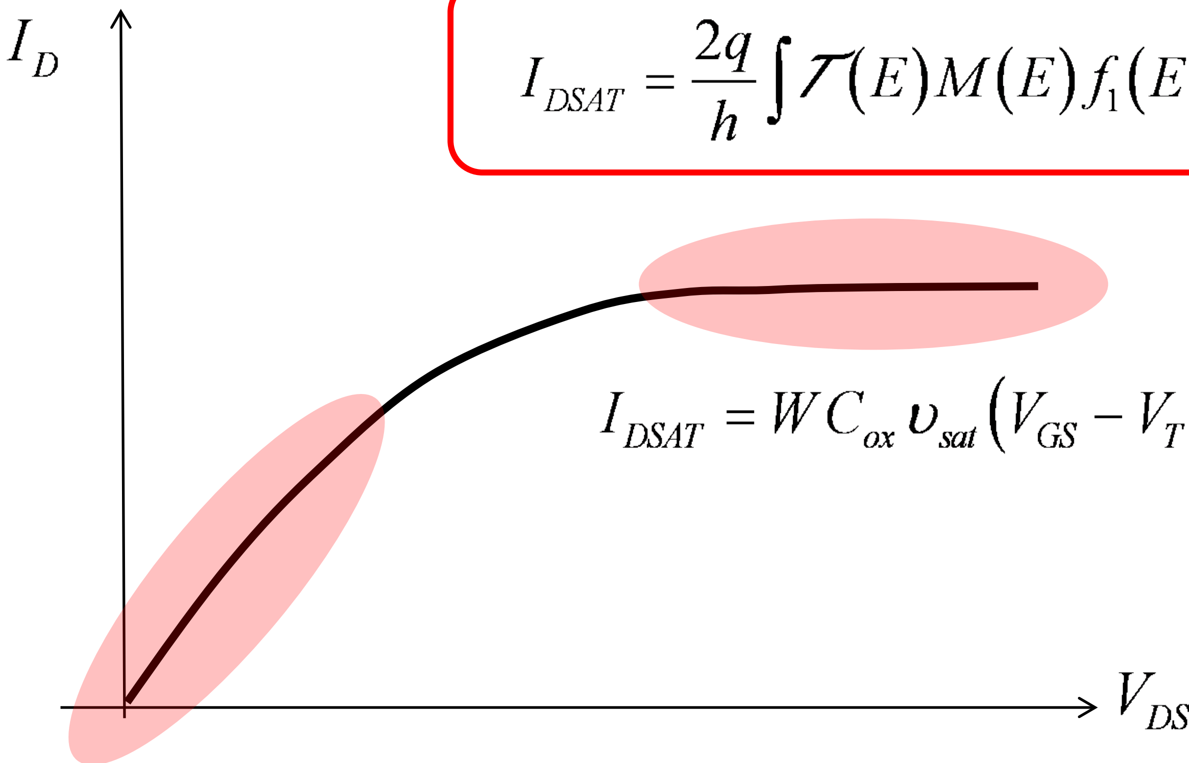
$$f_1 = f_0(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}} \qquad E_{F2} = E_{F1} - qV_D \qquad f_2 = \frac{1}{1 + e^{(E - E_{F1} + qV_D)/k_B T}} \approx 0$$

How current flows



Current in the Landauer Approach

$$I_{DSAT} = \frac{2q}{h}\int \mathcal{T}(E)M(E)f_1(E)\,dE$$



$$I_{DSAT} = W\,C_{ox}\,\upsilon_{sat}\left(V_{GS} - V_T\right)$$

$$I_{DLIN} = \frac{W}{L}\mu_n C_{ox}\left(V_{GS} - V_T\right)V_{DS}$$

$$I_{DLIN} = \left[\frac{2q^2}{h}\int \mathcal{T}(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE\right]V_{DS}$$

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Summary

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)(f_1 - f_2)\,dE$$

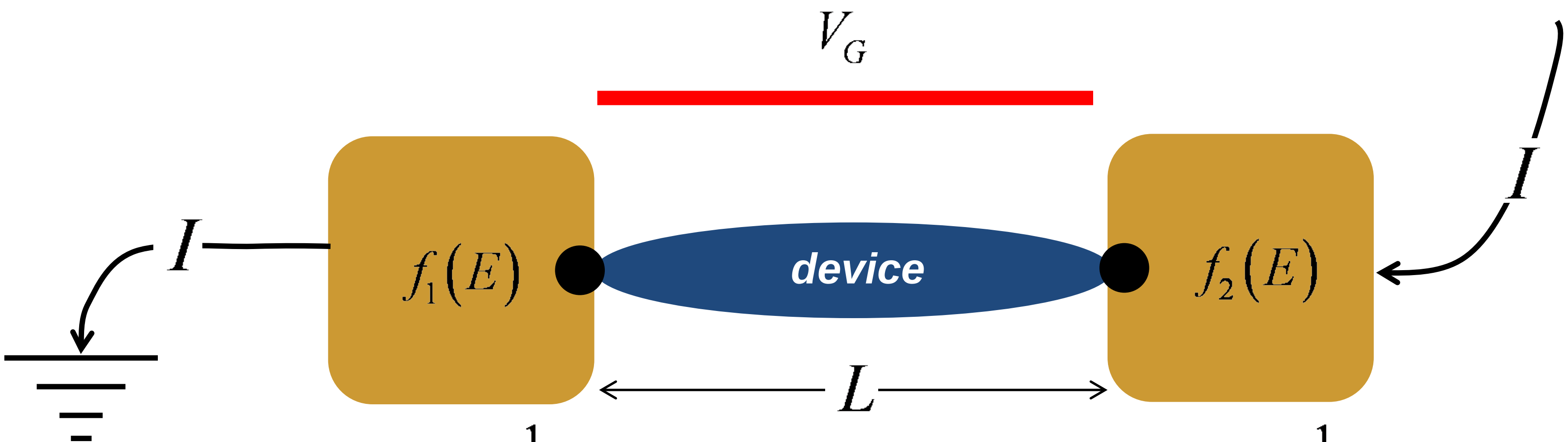
1) Linear region:

$$I_{DLIN} = \left[\frac{2q^2}{h}\int \mathcal{T}(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE\right]V_{DS}$$

2) Saturation region:

$$I_{DSAT} = \frac{2q}{h}\int \mathcal{T}(E)M(E)f_1(E)\,dE$$

Next topic



$$f_1(E) = \frac{1}{1+e^{(E-E_{F1})/k_BT}} \qquad f_2(E) = \frac{1}{1+e^{(E-E_{F2})/k_BT}}$$

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)(f_1 - f_2)\,dE$$

1) Ballistic MOSFET

$\mathcal{T} = 1$

2) MOS electrostatics