Unit 4:
Transmission Theory of the MOSFET

Lecture 4.1:
The Landauer Approach

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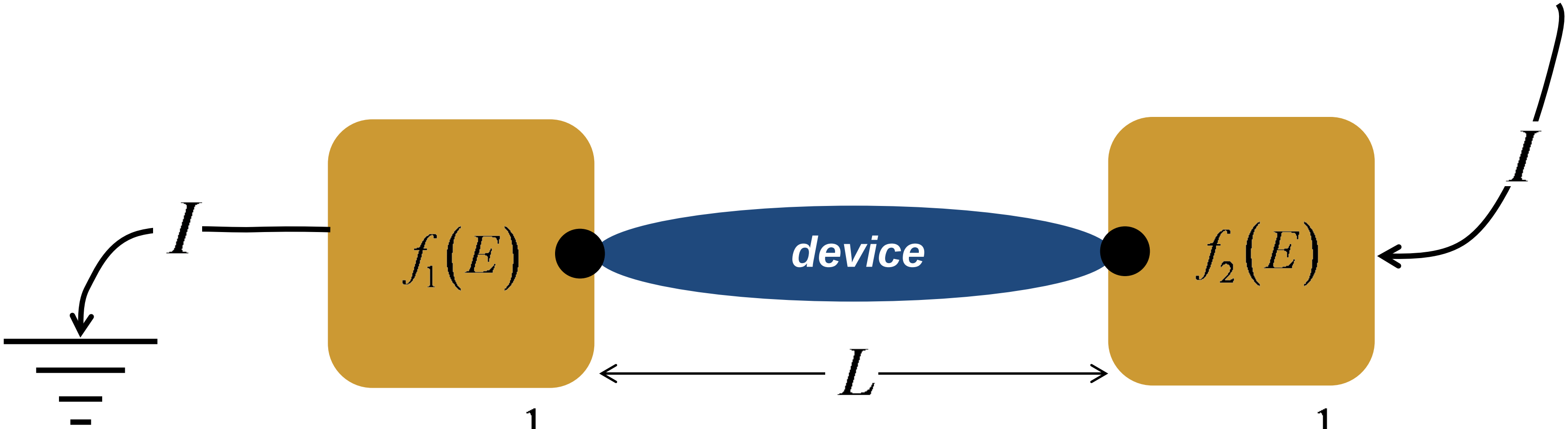
Introduction

The Landauer Approach is a simple and very physical way to describe electron transport from the ballistic to diffusive limits (i.e. from short to long channel MOSFETs).

Our description of this approach will be very simple and intuitive. Those who want a deeper understanding, should consult:

Supriyo Datta, *Lessons from Nanoelectronics*, 2nd Ed., Part A: Basic Concepts, World Scientific Publishing Co., Singapore, 2017.

Current in a nano device



$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}}$$

$$f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

How does the current that flows in contact 2, depend on the voltages on the two contacts?

Current

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)\left(f_1(E) - f_2(E)\right)dE$$

Fundamental constants

Transmission:

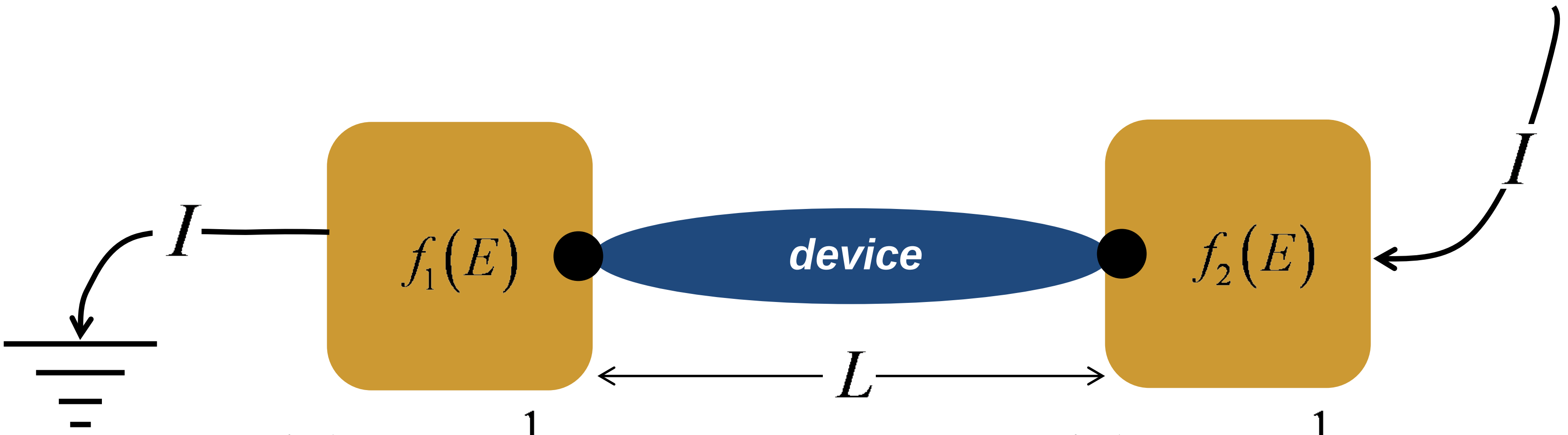
$0 < \mathcal{T}(E) \leq 1$

No. of Channels

Ideal "Landauer" contacts

Can be derived from rigorous transport theory (the Boltzmann equation), but this expression is intuitively easy to understand.

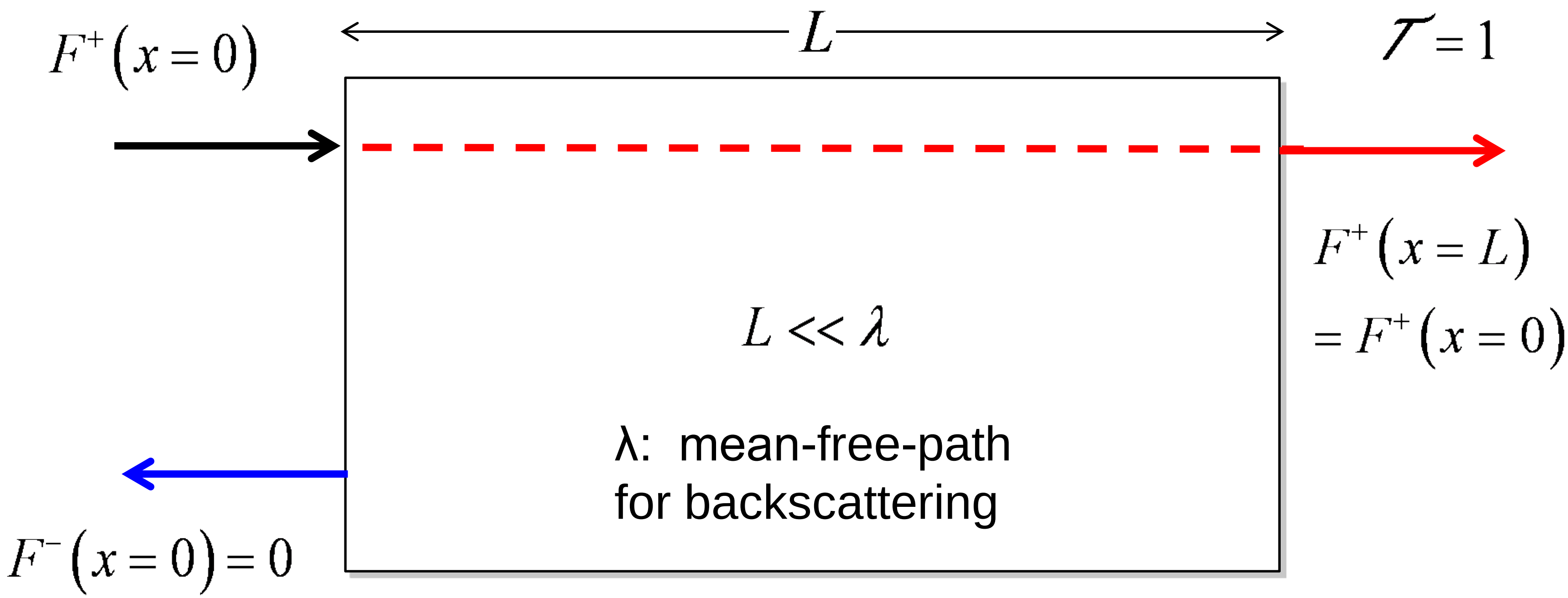
What is transmission?



$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}} \qquad f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

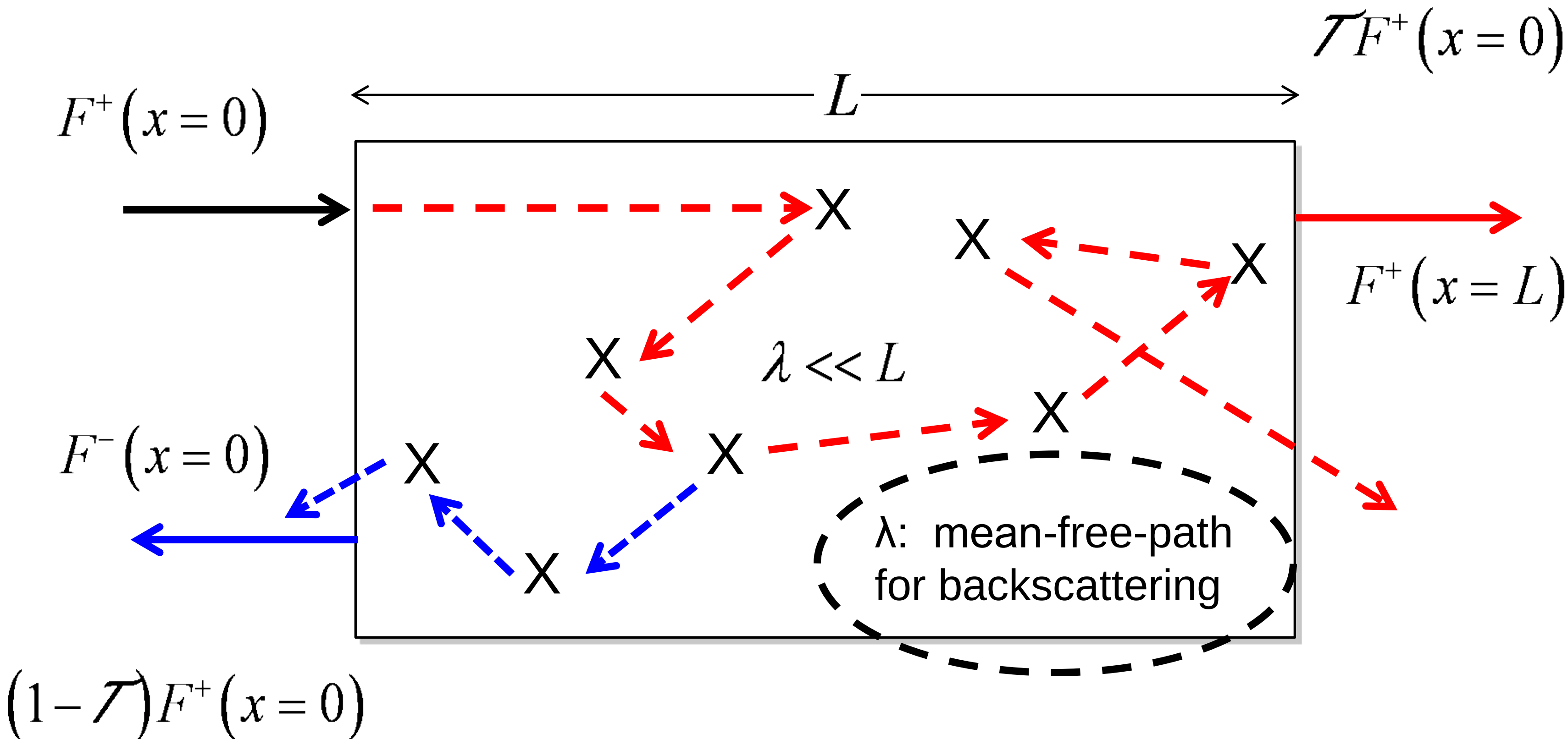
$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) \left(f_1 - f_2\right) dE$$

Transmission (ballistic)



ballistic transport: $\mathcal{T}=1$

Transmission (diffusive)



"diffusive transport" $\mathcal{T} << 1$

Transmission (general)

$$\mathcal{T}(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

λ is the "mean-free-path for backscattering"

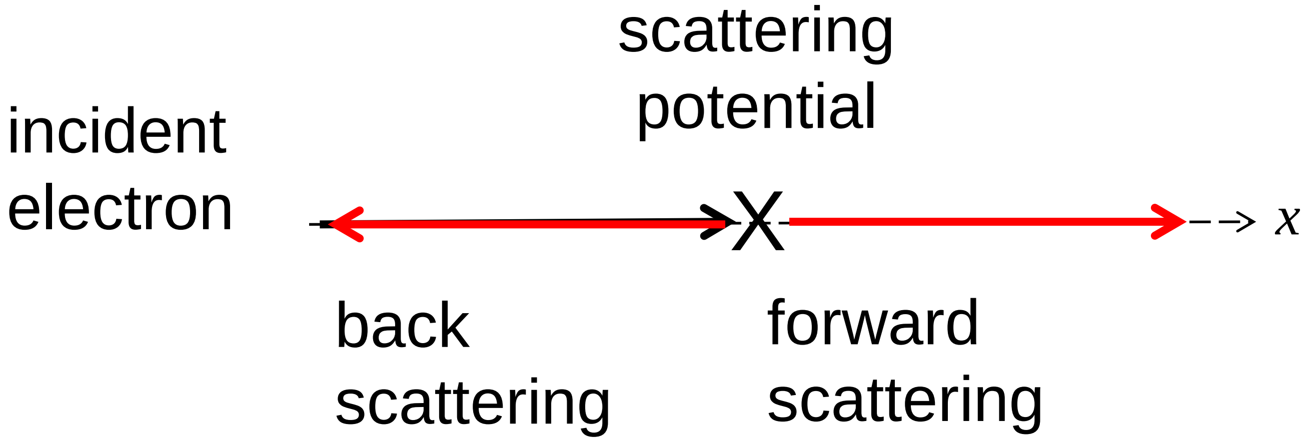
This expression can be <u>derived</u> with relatively few assumptions.

1) Diffusive: $L >> \lambda \quad \mathcal{T} = \frac{\lambda}{L} << 1$

2) Ballistic: $L << \lambda \quad \mathcal{T} = 1$

$$\lambda(E) \neq \upsilon(E)\tau(E) = \Lambda$$

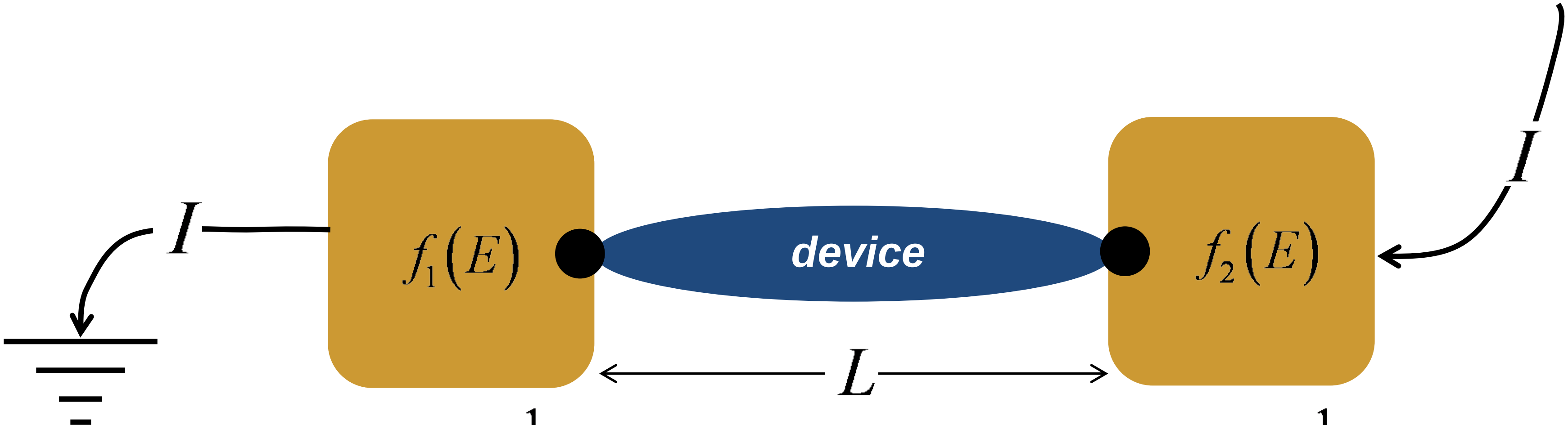
MFP for **backscattering** in 1D



If we assume that the scattering is ***isotropic*** (equal probability of scattering forward or back) then average time between **backscattering** events is 2 | .

$$\lambda(E) = 2\upsilon(E)\tau(E) \qquad \left\{\Lambda(E) = \upsilon(E)\tau(E)\right\}$$

What is a channel?



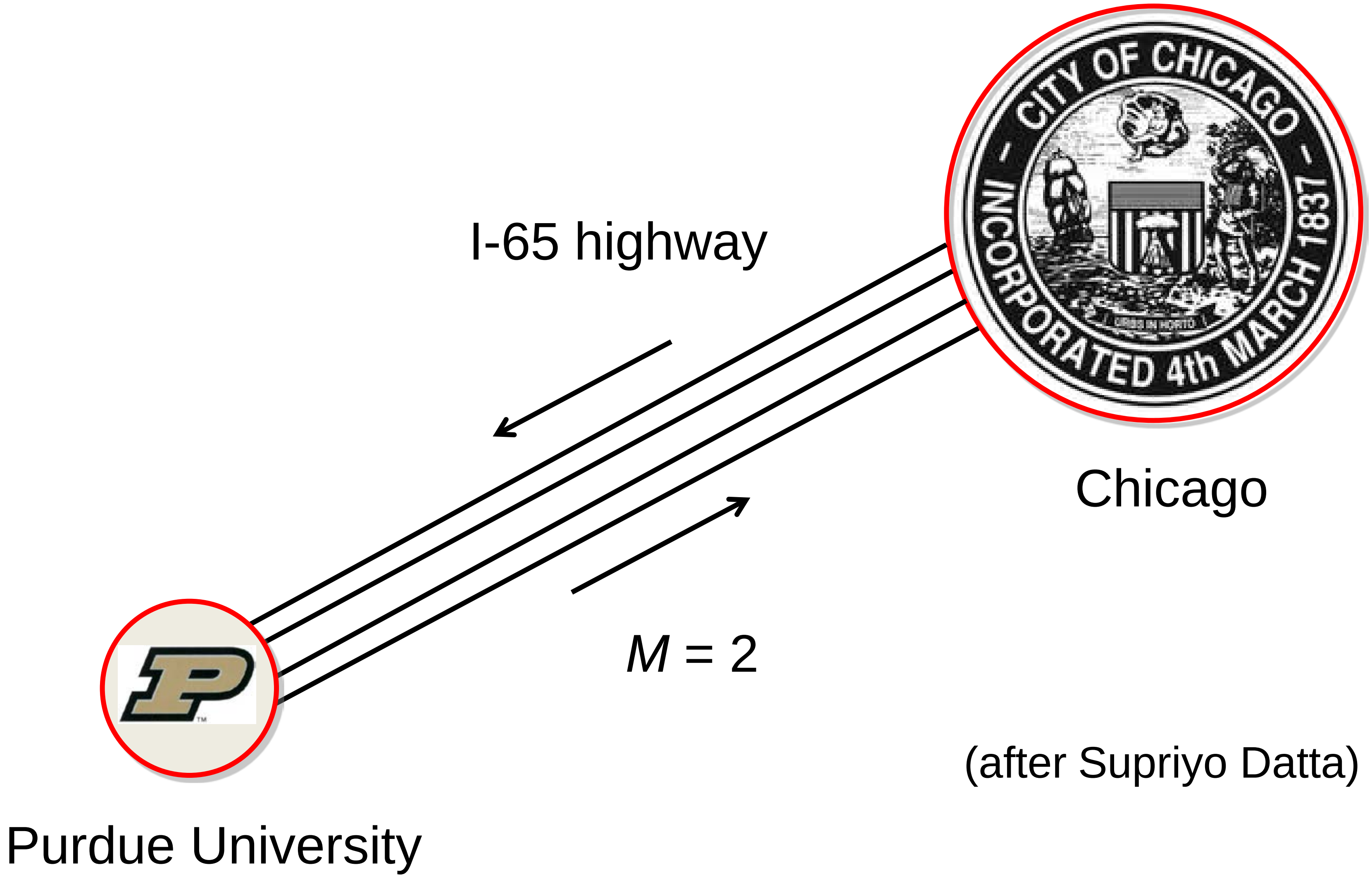
$$f_1(E) = \frac{1}{1 + e^{(E-E_{F1})/k_BT}}$$

$$f_2(E) = \frac{1}{1 + e^{(E-E_{F2})/k_BT}}$$

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)(f_1 - f_2)dE$$

(channels are also called “modes”)

Channels are like lanes on a highway



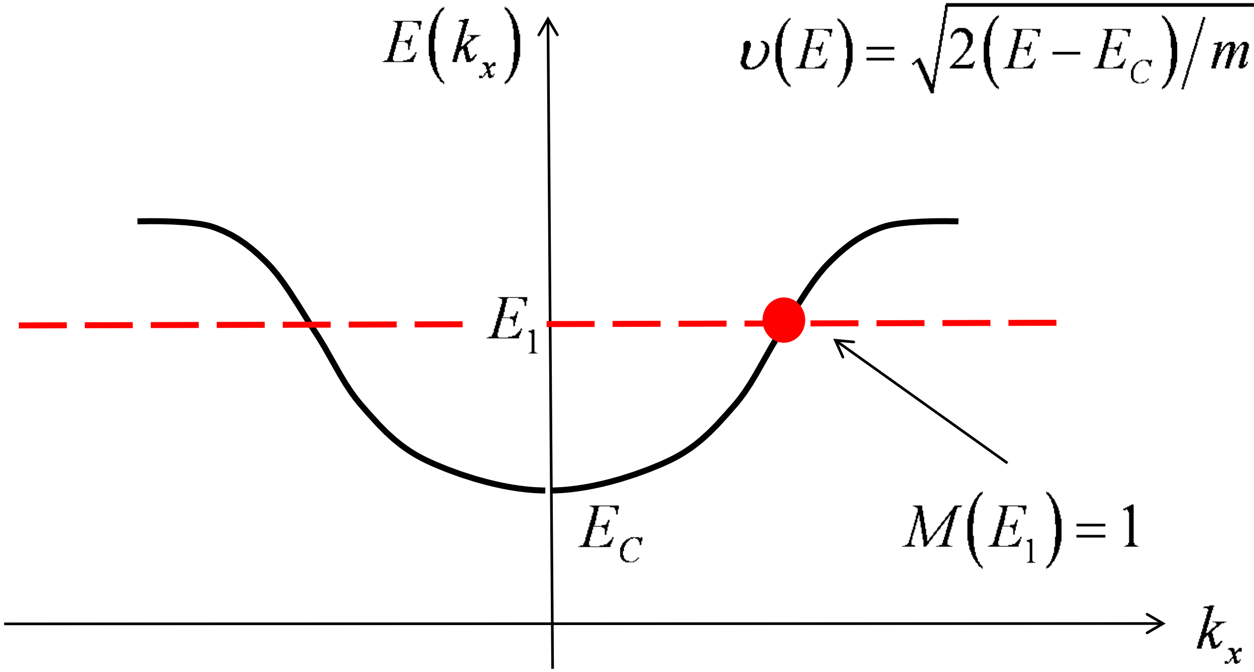
(after Supriyo Datta)

Channels (modes) from *E*(*k*)

A channel is a state with a velocity.

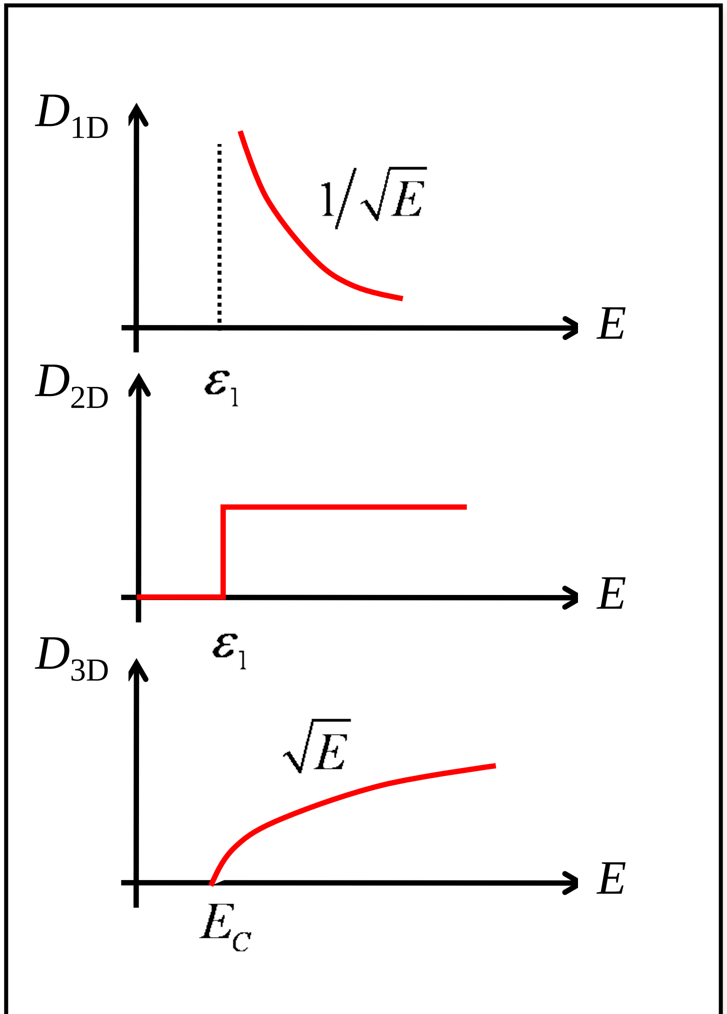
$$M(E) = \frac{h}{4}\left\langle \upsilon_x^+(E)\right\rangle D_{1D}(E)$$

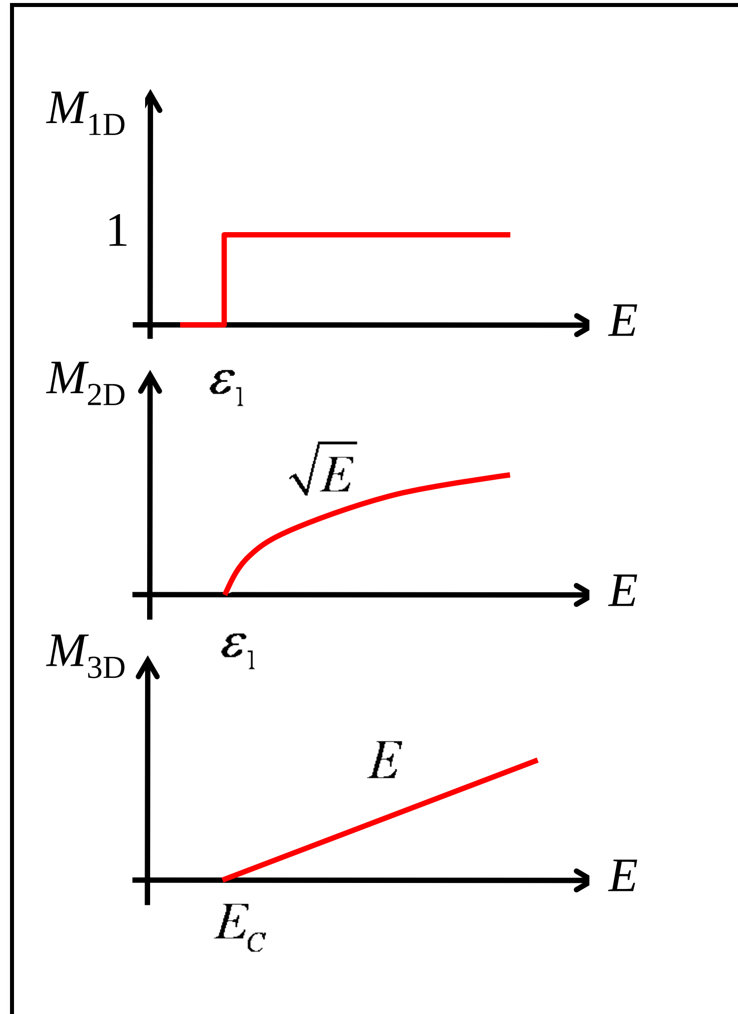
$$\upsilon(E) = \sqrt{2(E - E_C)/m^*}$$



(Easily generalized to arbitrary band structures in 2D and 3D.)

DOS(*E*) vs. *M*(*E*) (parabolic bands)





Transmission, channels, and MFP

$$\mathcal{T}(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

For two-dimensional electrons
(i.e. in the channel of a MOSFET):

$$M(E) = W\frac{\sqrt{2m^*(E - E_C)}}{\pi\hbar}$$

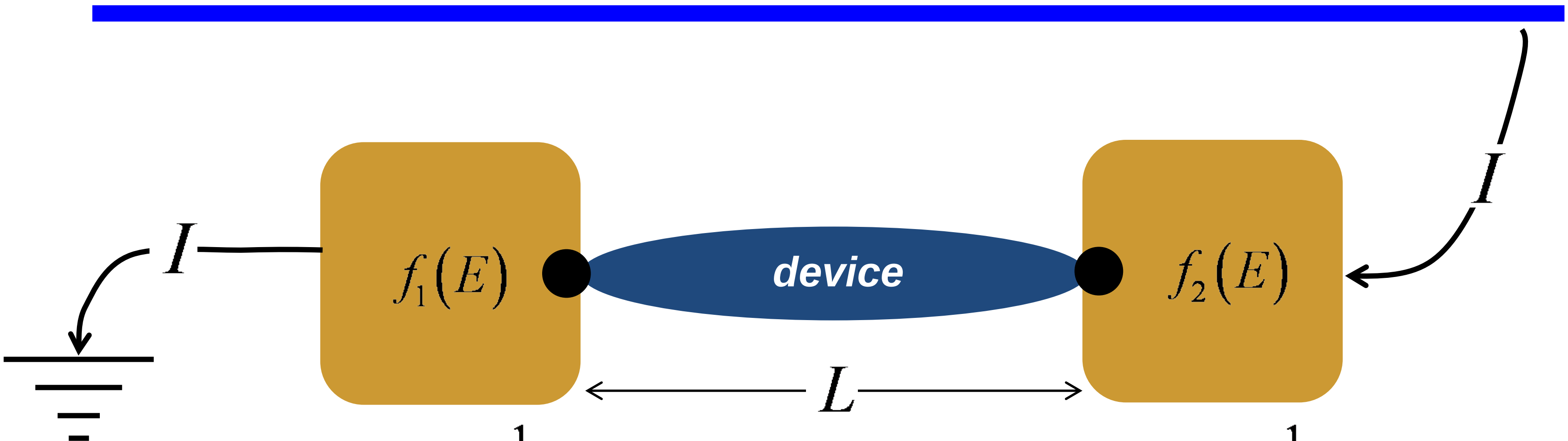
$$M_{2D}(E) = M(E)/W$$

Parabolic bands + large
structures with many channels.

$$\lambda(E) = \frac{\pi}{2}\Lambda(E)$$

$$\Lambda(E) = \upsilon(E)\tau(E)$$

Fermi window

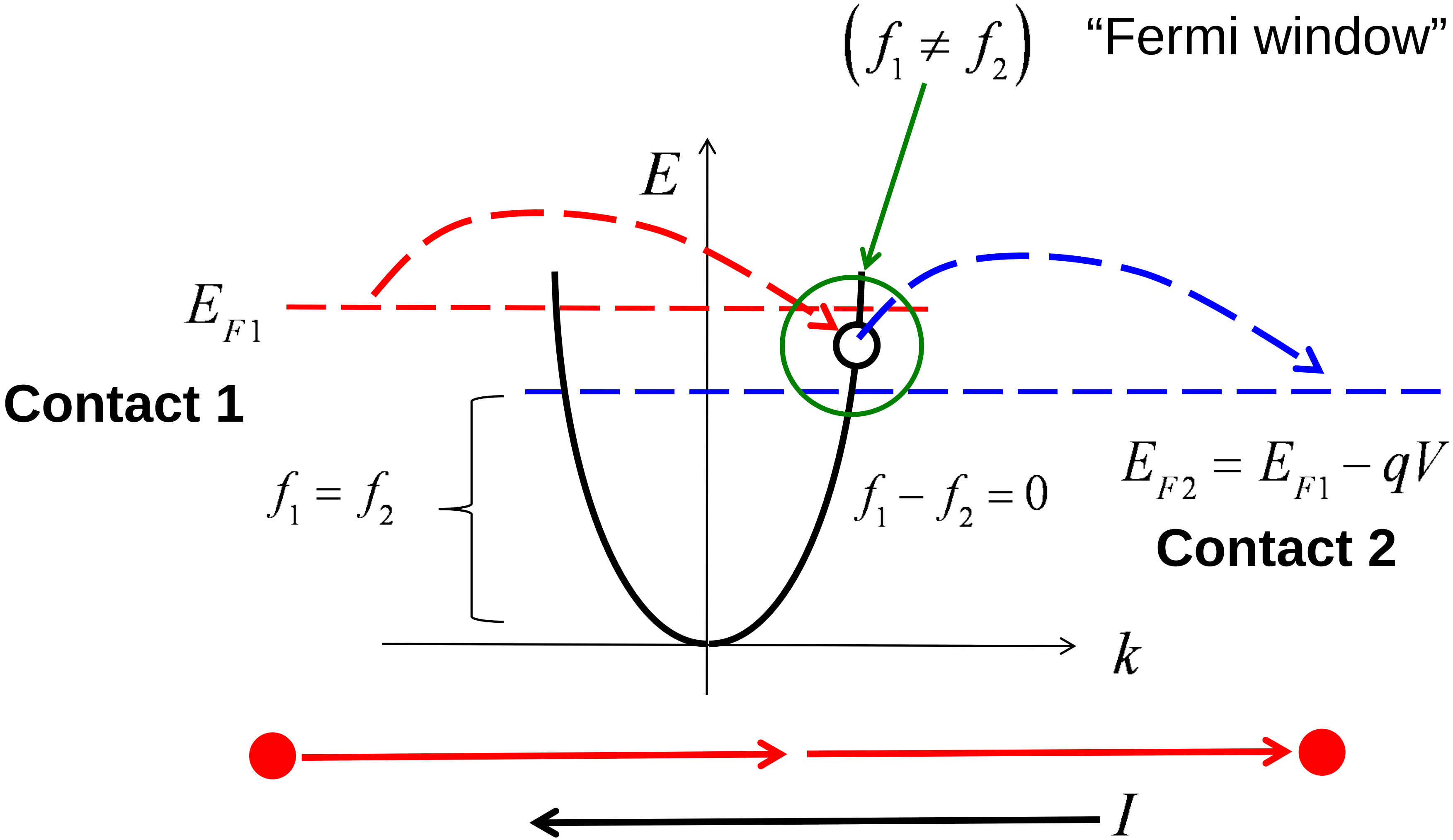


$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}} \qquad f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

$$I = \frac{2q}{h}\int \mathcal{T}(E) M(E)(f_1 - f_2) dE$$

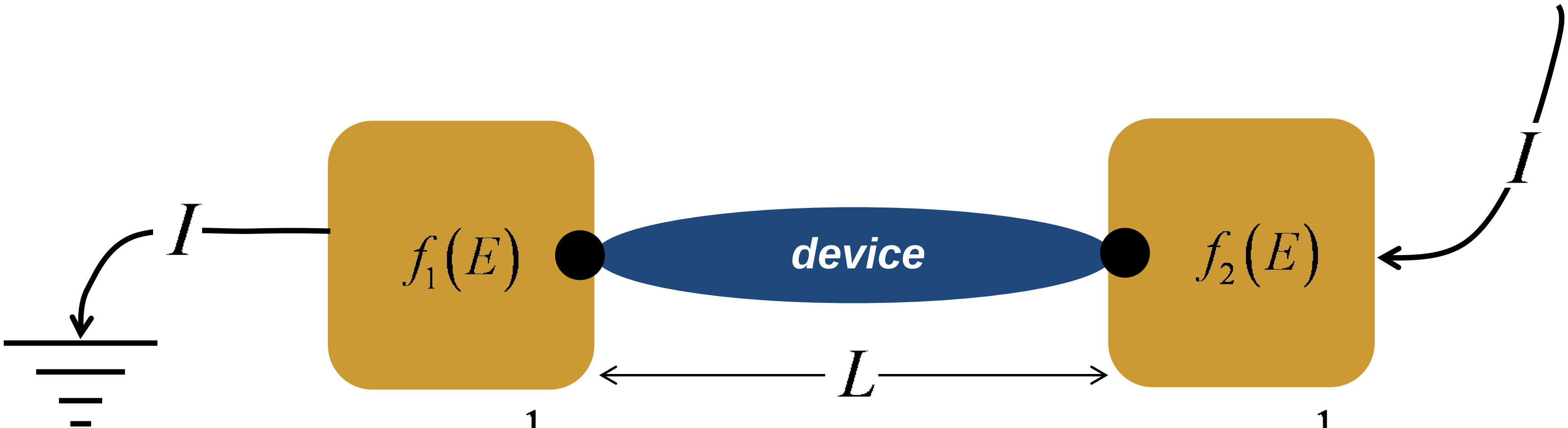
The range of energies over which $(f_1 - f_2) \neq 0$

How current flows (T = 0 K)



Lundstrom: 2018

Summary

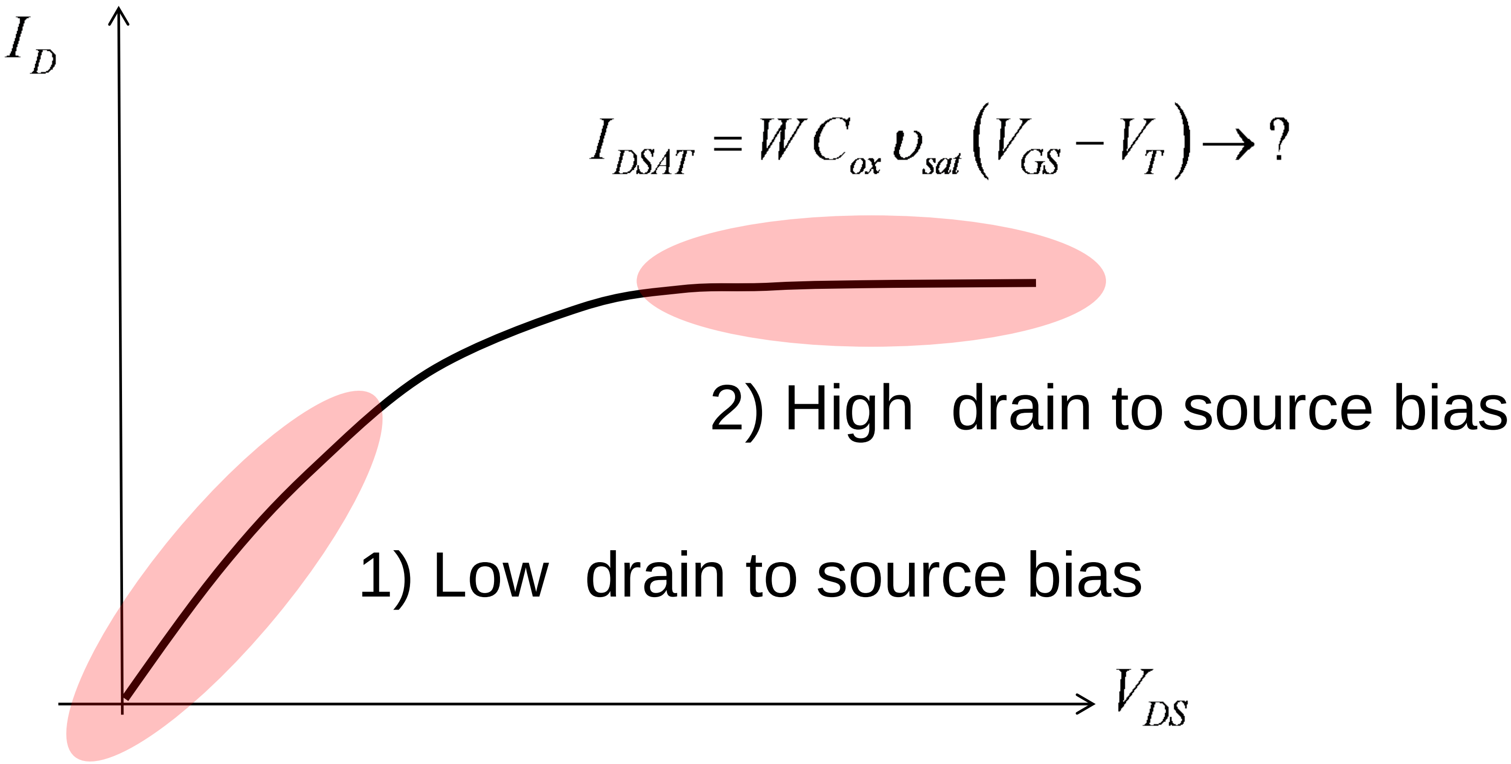


$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}} \qquad f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) \left(f_1 - f_2\right) dE$$

Can be used to describe the current in small and large devices and in short to long devices.

Next topic



$$I_{DLIN} = \frac{W}{L} \mu_n C_{ox} \left(V_{GS} - V_T \right) V_{DS} \rightarrow ?$$