MPC: Homework Part

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1 Question 1

1.1 Dimensions of Matrices

- F: $N_p \times 1$;
- H_u: $N_p \times N_p$;
- H_e: $N_p \times N_p$;

1.2 Expressions of Matrices

$$F = \begin{pmatrix} a & a^2 & a^3 & ... & a^{N_p} \end{pmatrix}^{\mathrm{T}} \tag{1.1}$$

$$H_u = \begin{pmatrix} b_u & 0 & 0 & 0 \\ ab_u & b_u & 0 & 0 \\ \vdots & \ddots & \ddots & 0 \\ a^{N_p-1}b_u & \cdots & ab_u & b_u \end{pmatrix} \tag{1.2}$$

$$H_e = \begin{pmatrix} b_e & 0 & 0 & 0 \\ ab_e & b_e & 0 & 0 \\ \vdots & \ddots & \ddots & 0 \\ a^{N_p-1}b_e & \cdots & ab_e & b_e \end{pmatrix} \tag{1.3}$$

2 Question 2

The outside temperature has an obvious effect on this system because every time when you open or close the refrigerator, there will exist an heat exchange. And if it is equal to 0, the expression corresponds to a system which has no connection with the outside. Therefore, it should not be 0.

3 Question 3

3.1 $N_p = 1$

When N_p is equal to 1, the loss function is:

$$J = (T(k+1) - 3)^2 + \lambda u(k)^2 \tag{3.1.1}$$

Combined with the description of the system:

$$T(k+1) = aT(k) + b_u u(k) + b_e T_o(k) \tag{3.1.2}$$

The loss function is rewritten as;

$$J = (aT(k) + b_u u(k) + b_e T_o(k) - 3)^2 + \lambda u(k)^2 \tag{3.1.3}$$

Let us consider the first order condition:

$$\frac{\partial J}{\partial u} = 2b_u(aT(k) + b_u u(k) + b_e T_o(k) - 3) + 2\lambda u(k) = 0 \tag{3.1.4}$$

Then we get:

$$u(k) = \frac{b_u}{\lambda + b_u^2} \times (3 - aT(k) - b_e T_o(k)) \tag{3.1.5}$$

Further:

$$T(k+1) = a(1 - \frac{b_u^2}{\lambda + b_u^2})T(k) + (1 - \frac{3b_u^2}{\lambda + b_u^2})b_e T_e(k) + \frac{3b_u^2}{\lambda + b_u^2} \tag{3.1.6}$$

Therefore, to satisfy the stability of this closed loop system:

$$|a(1 - \frac{b_u^2}{\lambda + b_u^2})|<1 \tag{3.1.7}$$

3.2 General Case

Generally, the loss function could be expressed like this:

$$J = (FT(k) + H_u U(K) + H_e T_o(K) - W)^T (FT(k) + H_u U(K) + H_e T_o(K) - W) + \lambda U^T U \tag{3.2.1}$$

The first-order condition:

$$2(H_u^T H_u + \lambda I)U(K) + 2H_u^T (FT(k) + H_e T_o(K) - W) = 0 \tag{3.2.2}$$

Then, we get the relation between U and $T(k)$:

$$U^*(K) = (H_u^T H_u + \lambda I)^{-1} H_u^T (W - FT(k) - H_e T_o(K)) \tag{3.2.3}$$

Finally:

$$T(K+1) = (I-(H_u^T H_u+\lambda I)^{-1} H_u^T)FT(k)+(I-(H_u^T H_u+\lambda I)^{-1} H_u^T)H_e T_o(K)+(H_u^T H_u+\lambda I)^{-1} H_o^T W \tag{3.2.4}$$

$$Norm(EigenValue((I - (H_u^T H_u + \lambda I)^{-1} H_u^T)F)) < 1 \tag{1}$$

4 Question 4

4.1 Explicit expression of $u^*(K|k)$

The explicit expression of $U^*(K|k)$ is exactly shown in equation 3.2.3

4.2 Relations among $u(k)$, u_{min}, u_{max} and $u^*(k|k)$

- if $u^*>u_{max}$: $u(k) = u_{max}$;
- if $u_{min} \leq u^* \leq u_{max}$: $u(k) = u^*(k|k)$;
- if $u^*<u_{min}$: $u(k) = u_{min}$;

5 Question5

Ever time when the disturbance happens, it would cause a significant loss of power. Supervisory control over the disturbance could be added to decrease the energy loss.

6 Question 6

Let's define:

$$T^+ = FT(k) + H_u U(K) + H_e T_o(K) - W \tag{6.1}$$

Then we define the new variable:

$$Z = \begin{pmatrix} U(K) \\ T^+ \end{pmatrix} \tag{6.2}$$

The initial problem could be rewritten as:

$$J = \frac{1}{2} Z^T H_z Z \tag{6.3}$$

where

$$H_z = \begin{pmatrix} R_U & 0 \\ 0 & Q_T \end{pmatrix} \tag{6.4}$$

with

$$R_U = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \lambda \end{pmatrix} \quad Q_T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{6.5}$$

The constraint is:

$$(-H_U \quad I_{N_p \times N_p}) Z = FT(k) + H_e T_o(K) - W \tag{6.6}$$

7 Question 7

Possible reason 1: quadprog() returns negative value means that the optimisation failed.

Function help: `https://ww2.mathworks.cn/help/optim/ug/quadprog.html`

Possible reason 2:

8 Question 8

8.1 Applicability of LP

We define: $c^T = (1, 1, ..., 1)_{1 \times N_p}$ and the objective function is:

$$J_2 = \sum_{i=0}^{N_p - 1} u(k+i|k) = c^T U(K) \tag{8.1}$$

with the constraint:

$$H_u U(K) \leq T_{max}(K+1|k) - H_e T_o(K) - FT(k) \tag{8.2}$$

$$lb = u_{min} \times c, ub = u_{max} \times c \tag{8.3}$$

8.2 Potential Problems

This control law might lead to the loss of stability of temperature.
Also,the optimiser tempeture is alway 0 whatever the initial value To is.
Possible solution: add an extra penalty part in the loss function J_2 to achieve the stability.

9 Question 9

Compare these three strategies above and make a balance.

10 Question 10