# Unit 4:
# Transmission Theory of the MOSFET

# **Lecture 4.5: Transmission Theory of the MOSFET**

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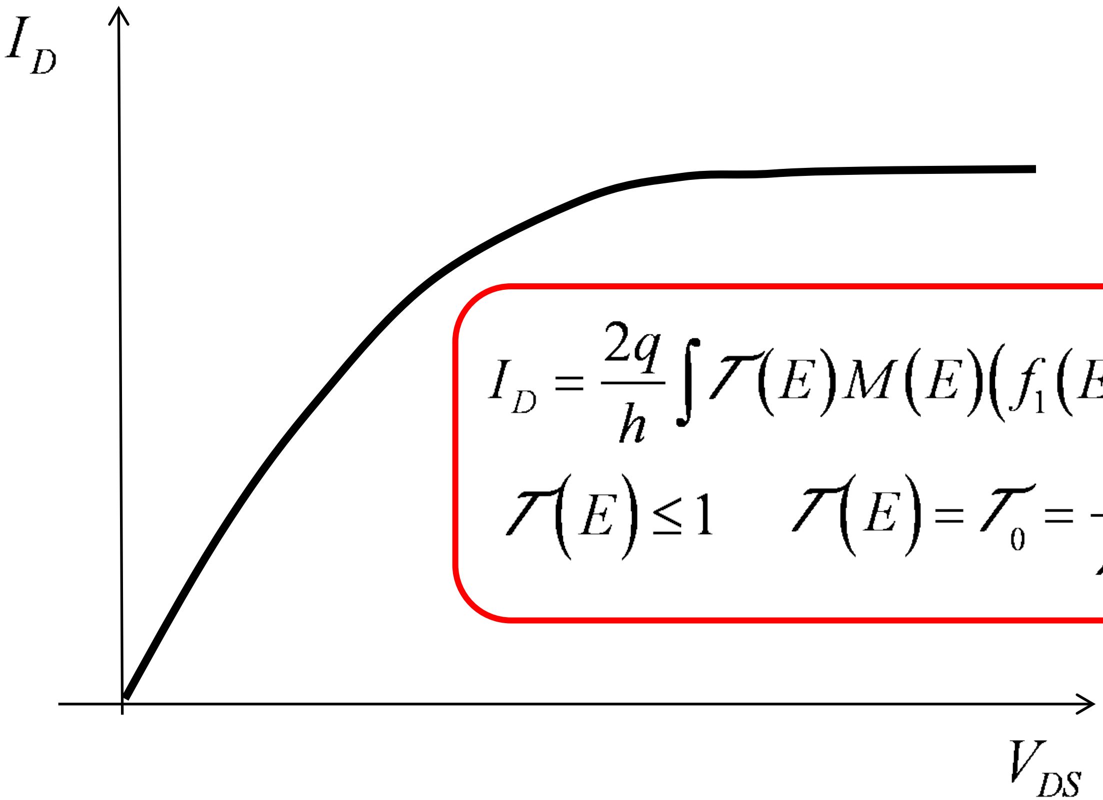
# Transmission theory



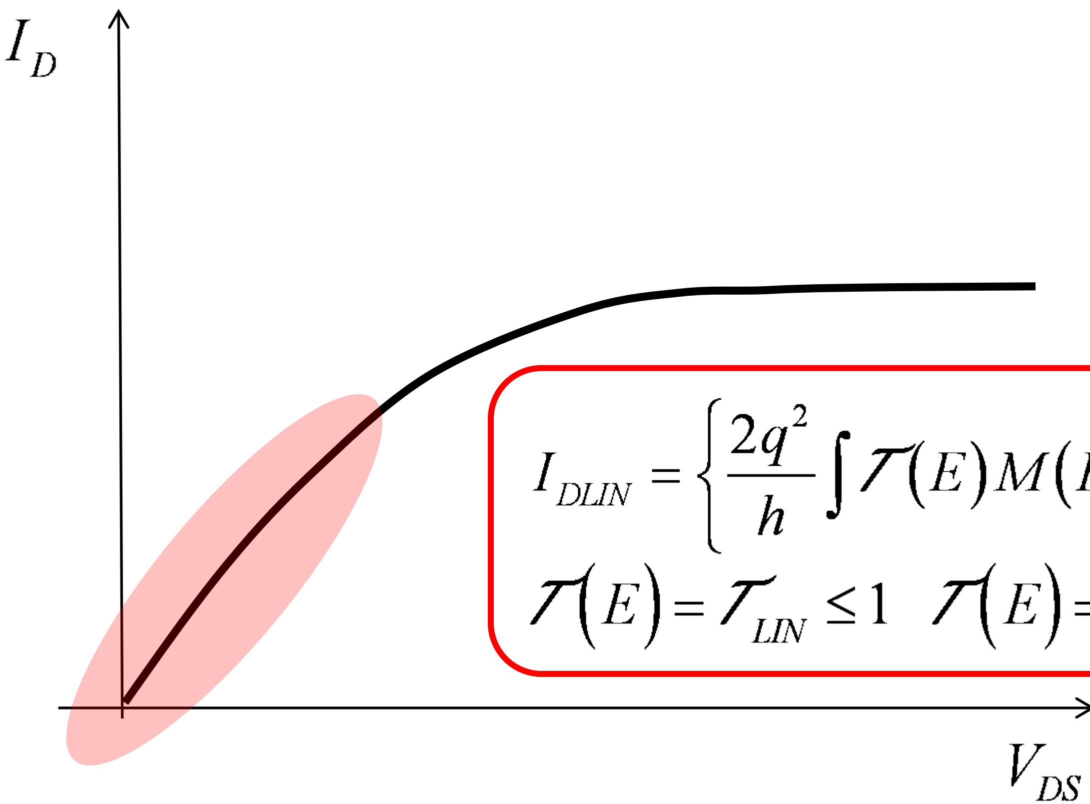

$$I_D = \frac{2q}{h}\int \mathcal{T}(E)M(E)\big(f_1(E)-f_2(E)\big)dE$$

$$\mathcal{T}(E)\leq 1 \qquad \mathcal{T}(E)=\mathcal{T}_0=\frac{\lambda_0}{\lambda_0+L}$$

1) Linear region

2) Saturation region

# 1) Linear region




$$I_{DLIN} = \left\{ \frac{2q^2}{h} \int \mathcal{T}(E) M(E) \left( -\frac{\partial f_0}{\partial E} \right) dE \right\} V_{DS}$$

$$\mathcal{T}(E) = \mathcal{T}_{LIN} \leq 1 \quad \mathcal{T}(E) = \mathcal{T}_0 = \lambda_0 / (\lambda_0 + L)$$

$$I_{DLIN} = W \left( \frac{\upsilon_T}{2(k_B T/q)} \right) |Q_n| V_{DS} \rightarrow W \mathcal{T}_{LIN} \left( \frac{\upsilon_T}{2(k_B T/q)} \right) |Q_n| V_{DS} \quad \textbf{?}$$

# Ballistic to diffusive linear current

$$I_{DLIN} = W\mathcal{T}_{LIN}\left(\frac{\upsilon_T}{2(k_BT/q)}\right)|Q_n|V_{DS}$$

$$\mathcal{T}_{LIN} = \frac{\lambda_0}{\lambda_0 + L}$$

$$I_{DLIN} = \frac{W}{\lambda_0 + L}\underbrace{\left(\frac{\upsilon_T\lambda_0}{2(k_BT/q)}\right)}_{\mu_n}|Q_n|V_{DS}$$

$$I_{DLIN} \propto \frac{W}{\lambda_0 + L} \text{ not } \frac{W}{L}$$

$$I_{DLIN} = \frac{W}{L + \lambda_0}\mu_n|Q_n|V_{DS} \qquad L \rightarrow L + \lambda_0$$

Lundstrom: 2018

# Alternative formulation

$$I_{DLIN} = \frac{W}{L+\lambda_0}\mu_n |Q_n| V_{DS} \qquad \mathcal{T}_{LIN} = \frac{\lambda_0}{\lambda_0 + L}$$

$$I_{DLIN} = \frac{W}{L}\left(\frac{L}{L+\lambda_0}\right)\mu_n |Q_n| V_{DS} \qquad \mu_n = \left(\frac{\upsilon_T \lambda_0}{2(k_B T/q)}\right)$$

$$I_{DLIN} = \frac{W}{L}\left(\frac{1}{1+\lambda_0/L}\right)\mu_n |Q_n| V_{DS} \qquad \frac{\lambda_0}{L\mu_n} = \frac{1}{\mu_B}$$

$$I_{DLIN} = \frac{W}{L}\left(\frac{1}{1/\mu_n + \lambda_0/(L\mu_n)}\right)|Q_n| V_{DS} \qquad \mu_B = \left(\frac{\upsilon_T L}{2(k_B T/q)}\right)$$

# Apparent mobility

$$I_{DLIN} = \frac{W}{L}\left(\frac{1}{1/\mu_n + 1/\mu_B}\right)|Q_n| V_{DS} \qquad \mu_n = \left(\frac{\upsilon_T \lambda_0}{2(k_B T/q)}\right) \quad \mu_B = \left(\frac{\upsilon_T L}{2(k_B T/q)}\right)$$

$$\boxed{\begin{gathered} I_{DLIN} = \frac{W}{L}\mu_{app}|Q_n| V_{DS} \\ \frac{1}{\mu_{app}} = \frac{1}{\mu_n} + \frac{1}{\mu_B} \end{gathered}}$$

# Example

Estimate the apparent mobility for a 22 nm N-MOSFET.

$$\mu_n \approx 200 \text{ cm}^2/\text{V-s}$$

$$\mu_B = \frac{\upsilon_T L}{2 k_B T / q}$$

$$\upsilon_T = \sqrt{\frac{2 k_B T}{\pi m_t^*}} = 1.2 \times 10^7 \text{ cm/s}$$

$$\mu_B \approx 500 \text{ cm}^2/\text{V-s}$$

$$\frac{1}{\mu_{app}} = \frac{1}{\mu_n} + \frac{1}{\mu_B} = \frac{1}{200} + \frac{1}{500}$$

$$\mu_{app} \approx 140 \ \frac{\text{cm}^2}{\text{V-s}}$$

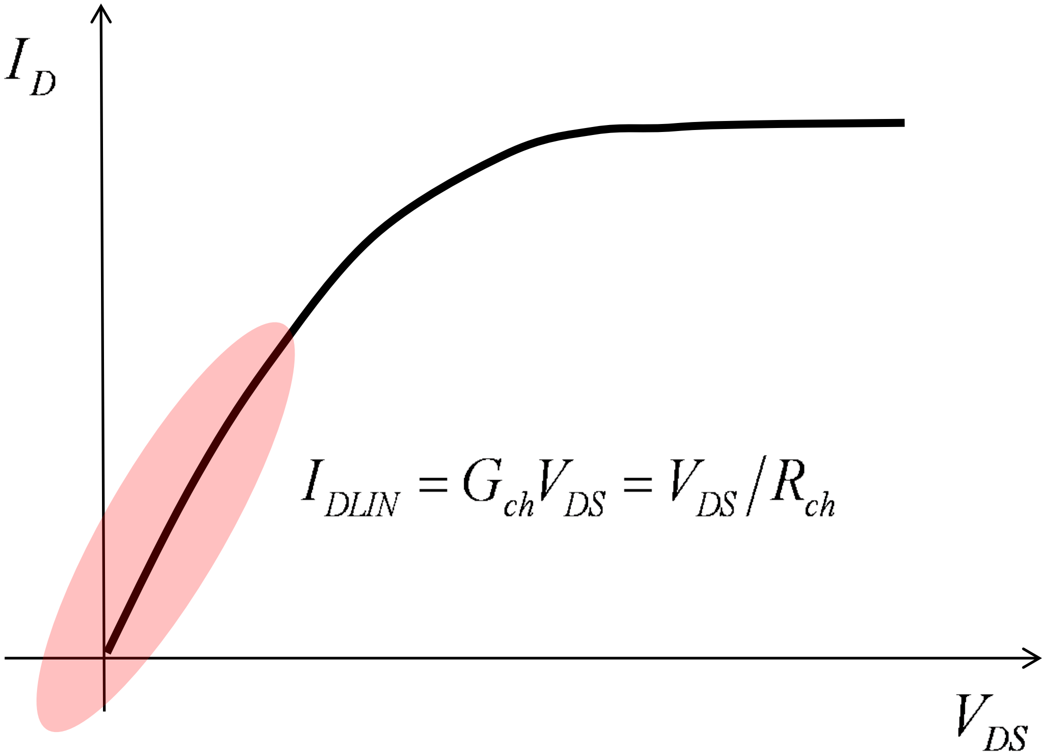
This device operates in the quasi-ballistic regime.

(Assumes confinement in the <100> direction.)

# Exercise

Repeat the previous exercise for a MOSFET with a 10 nm long channel (assume the same diffusive mobility).

# Channel resistance




$$I_{DLIN} = \frac{W}{L}\left(\frac{1}{1/\mu_n + 1/\mu_B}\right)|Q_n| V_{DS}$$

$$G_{ch} = \frac{W}{L}\left(\frac{1}{1/\mu_n + 1/\mu_B}\right)|Q_n|$$

$$R_{ch} = \frac{1}{G_{ch}} = \left(\frac{1/\mu_n + 1/\mu_B}{|Q_n|}\right)\frac{L}{W}$$

$$R_{ch} = R_{diff} + R_B$$

# Channel resistance

$$R_{ch} = \frac{1}{G_{ch}} = \left( \frac{1/\mu_n + 1/\mu_B}{|Q_n|} \right) \frac{L}{W} \qquad \mu_n = \left( \frac{\upsilon_T \lambda_0}{2(k_B T/q)} \right) \quad \mu_B = \left( \frac{\upsilon_T L}{2(k_B T/q)} \right)$$

$$R_{ch} = R_{diff} + R_B \qquad R_{diff} = \left( \frac{1}{\mu_n |Q_n|} \right) \frac{L}{W} \qquad R_B = \left( \frac{1}{\mu_B |Q_n|} \right) \frac{L}{W}$$

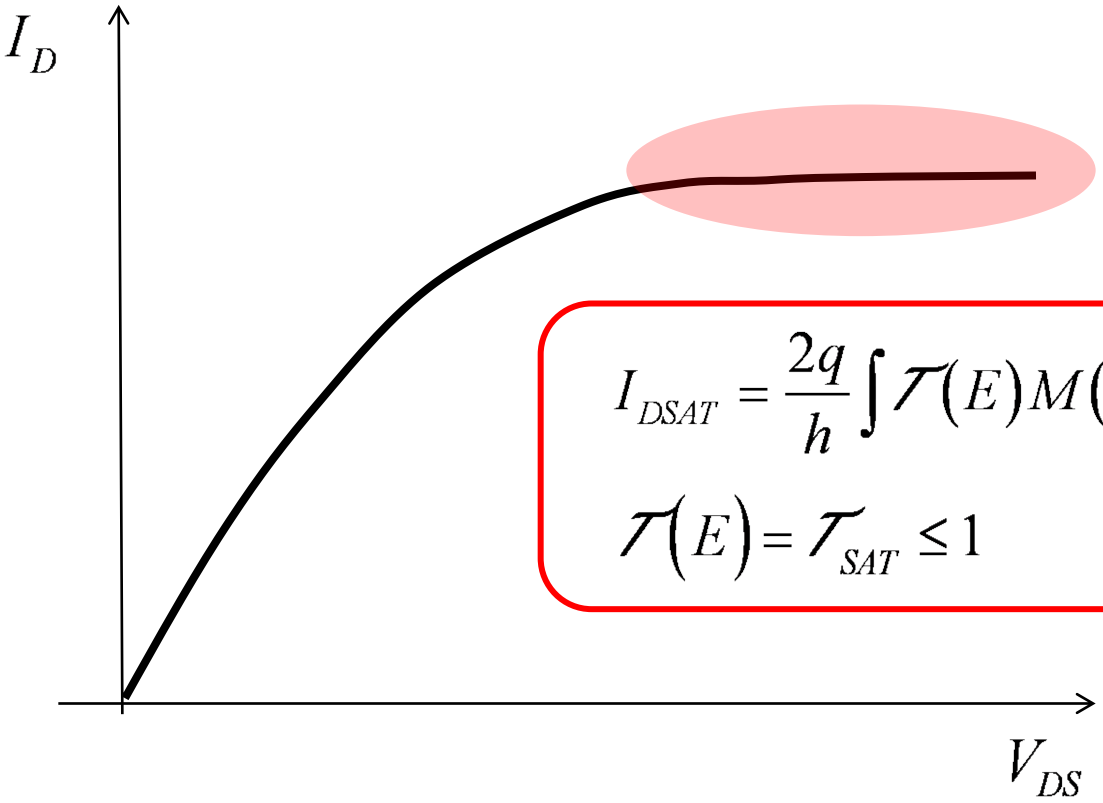
$$R_B = \left( \frac{2 k_B T/q}{\upsilon_T |Q_n|} \right) \frac{1}{W}$$

independent of channel length

$$\boxed{R_{ch}(L) = R_{diff}(L) + R_B}$$

Lundstrom: 2018

# 2) Saturation region



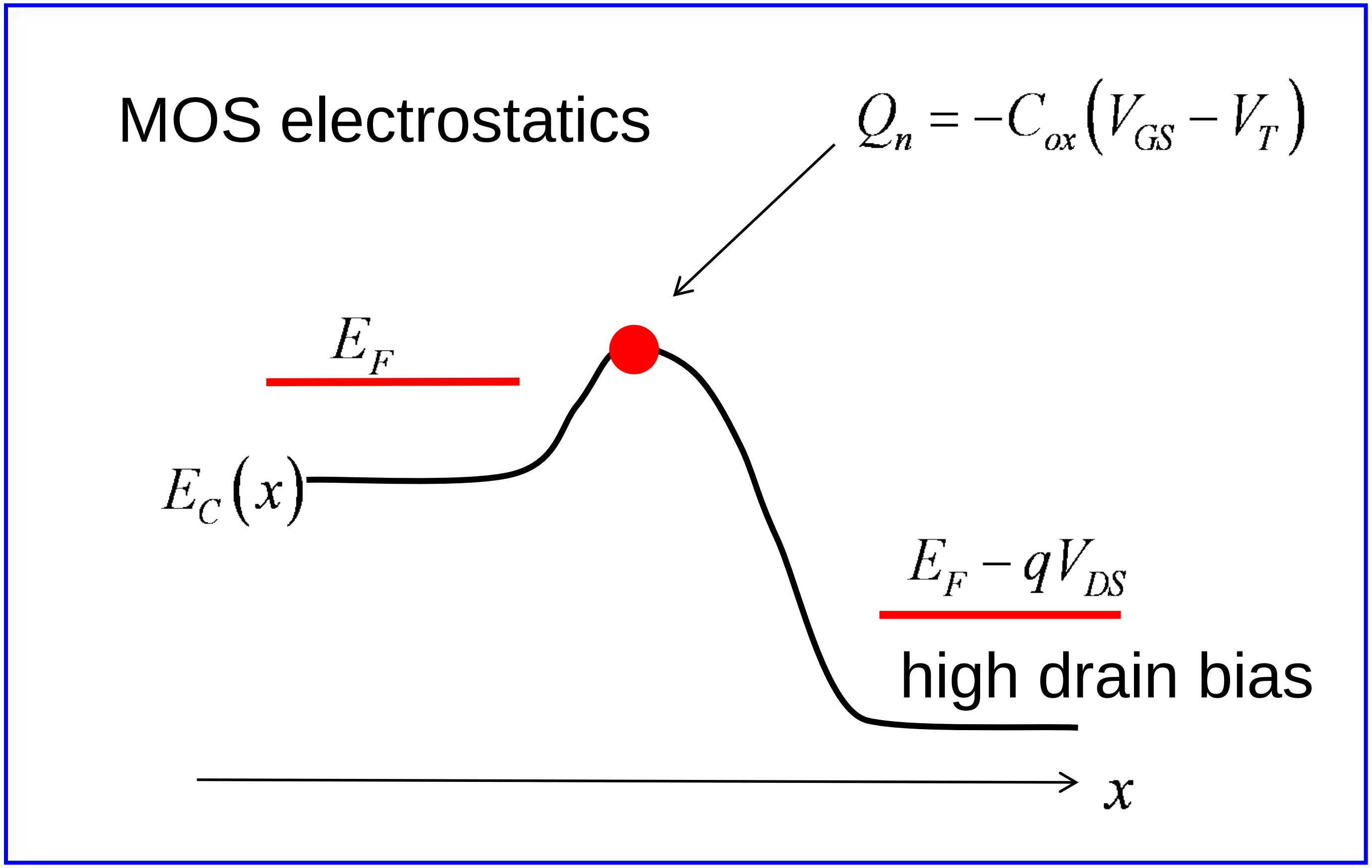

$$I_{DSAT} = \frac{2q}{h}\int \mathcal{T}(E)M(E)f_1(E)dE$$

$$\mathcal{T}(E) = \mathcal{T}_{SAT} \leq 1$$

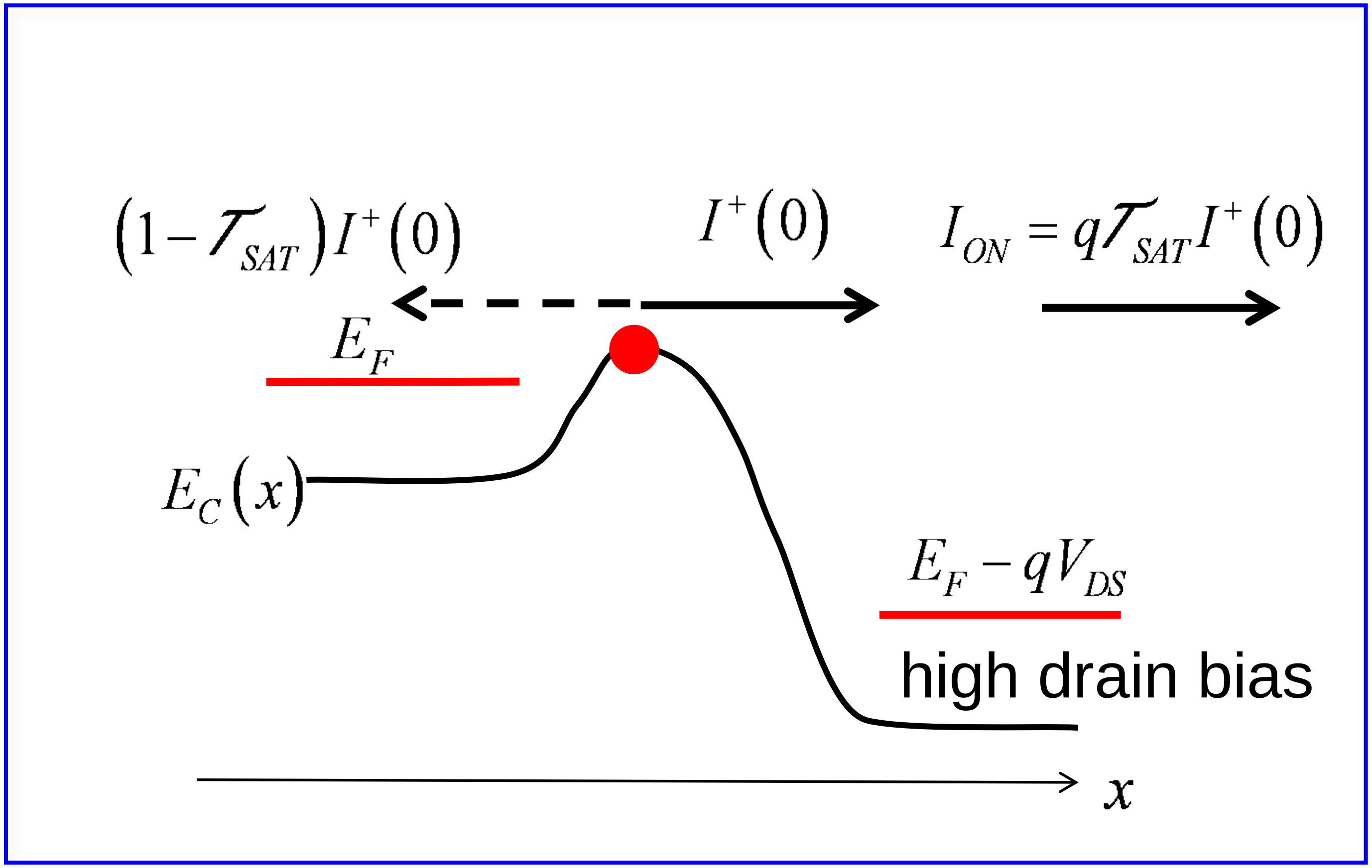
$$I_{DSAT} = W|Q_n|\upsilon_T \rightarrow \mathcal{T}_{SAT} W|Q_n|\upsilon_T$$ **?**

**This is wrong!**

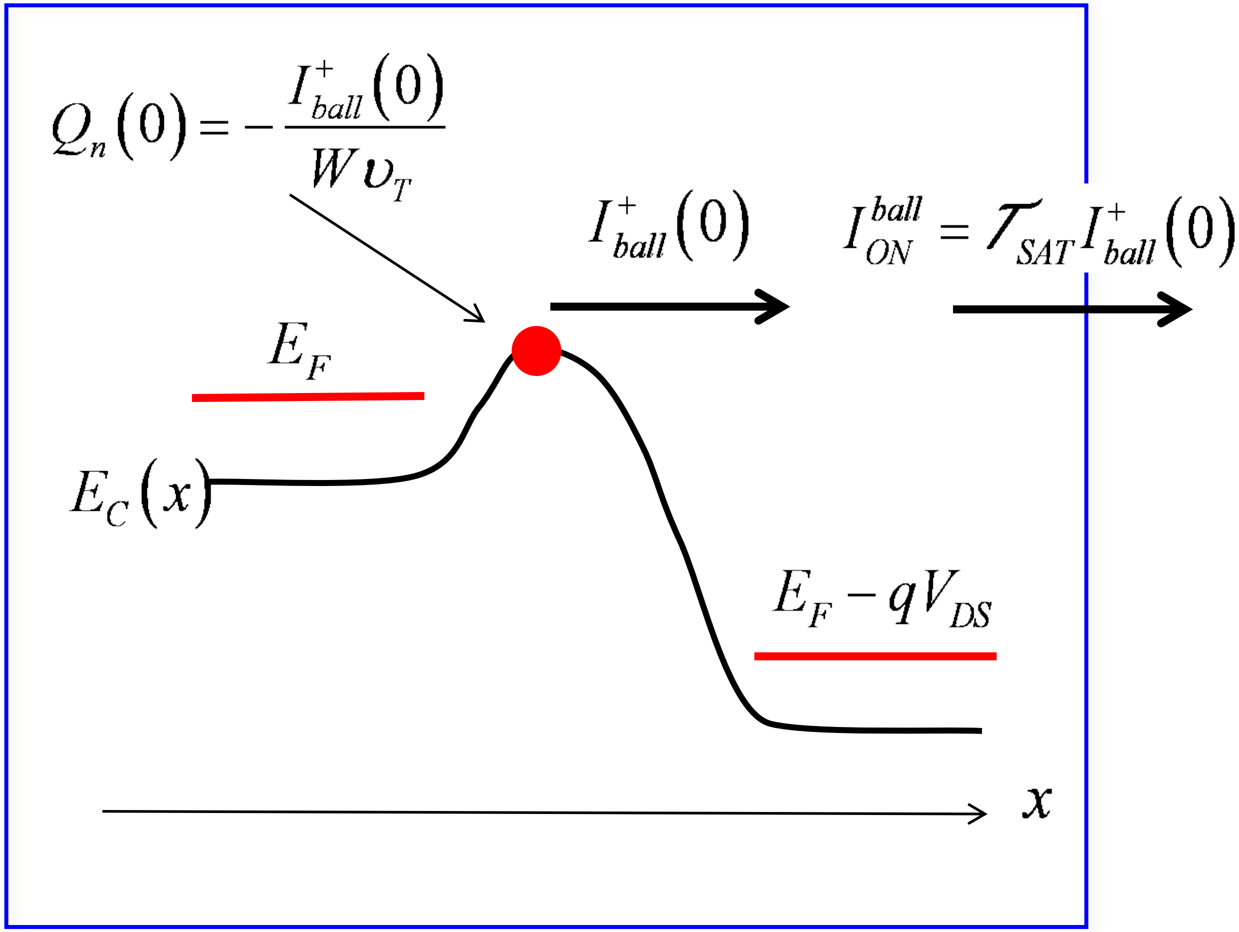
# Focus on the VS



# On-current and transmission



# Ballistic case first



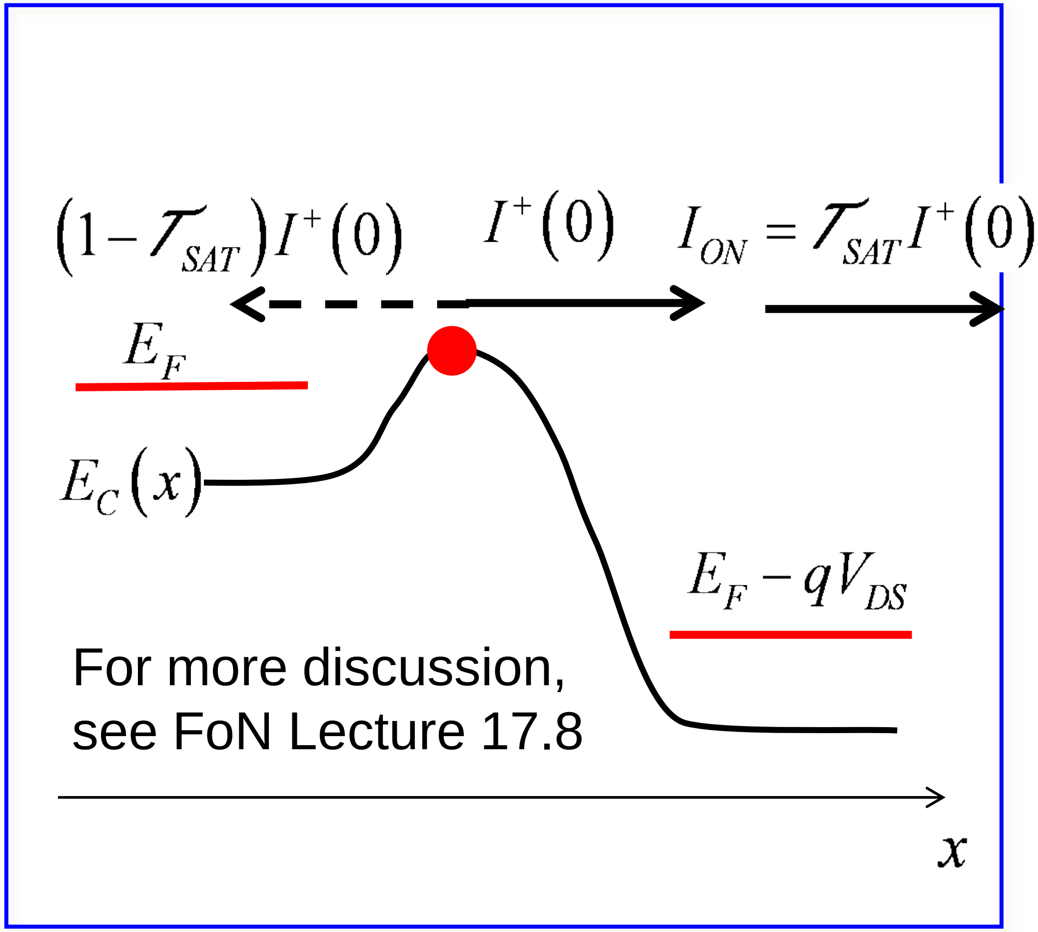
# Now include scattering

$$Q_n(0) = -\frac{I_{ball}^+(0)}{W\upsilon_T}$$

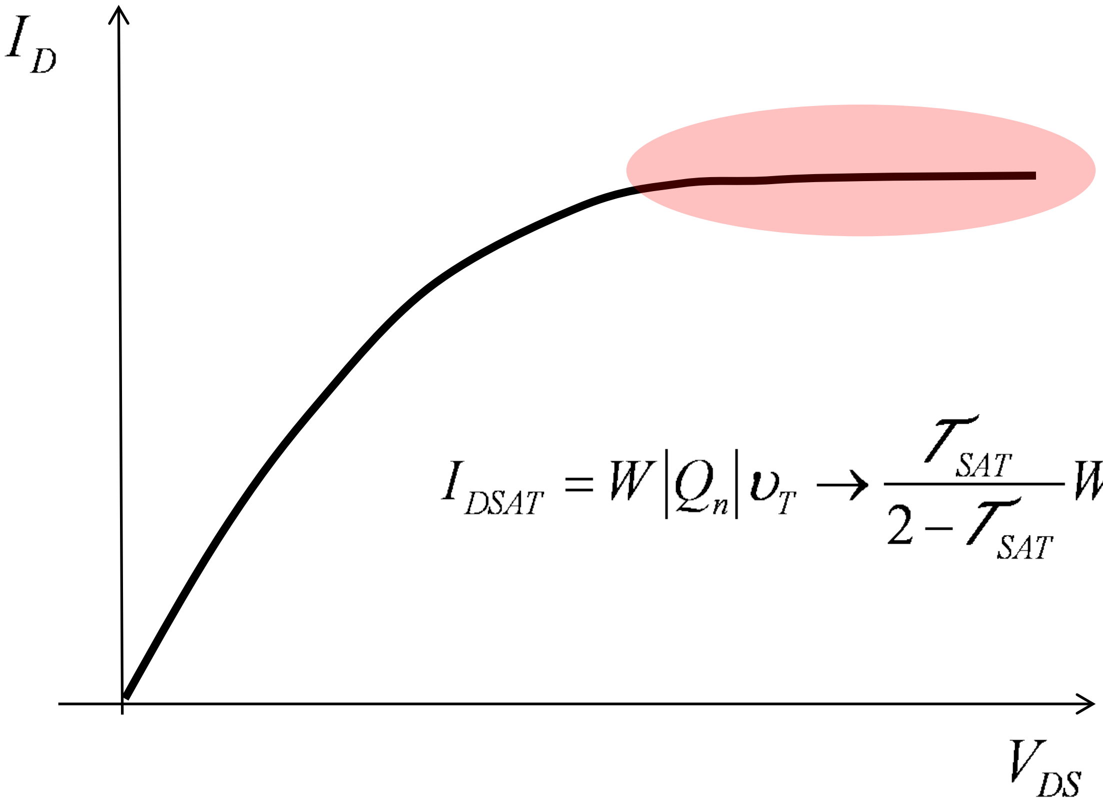
$$Q_n(0) = -\frac{I^+(0) + (1-\mathcal{T}_{SAT})I^+(0)}{W\upsilon_T}$$

$$I^+(0) = \frac{I_{ball}^+(0)}{(2-\mathcal{T}_{SAT})}$$

$$I_{ON} = \left(\frac{\mathcal{T}_{SAT}}{2-\mathcal{T}_{SAT}}\right) I_{ball}^+(0)$$



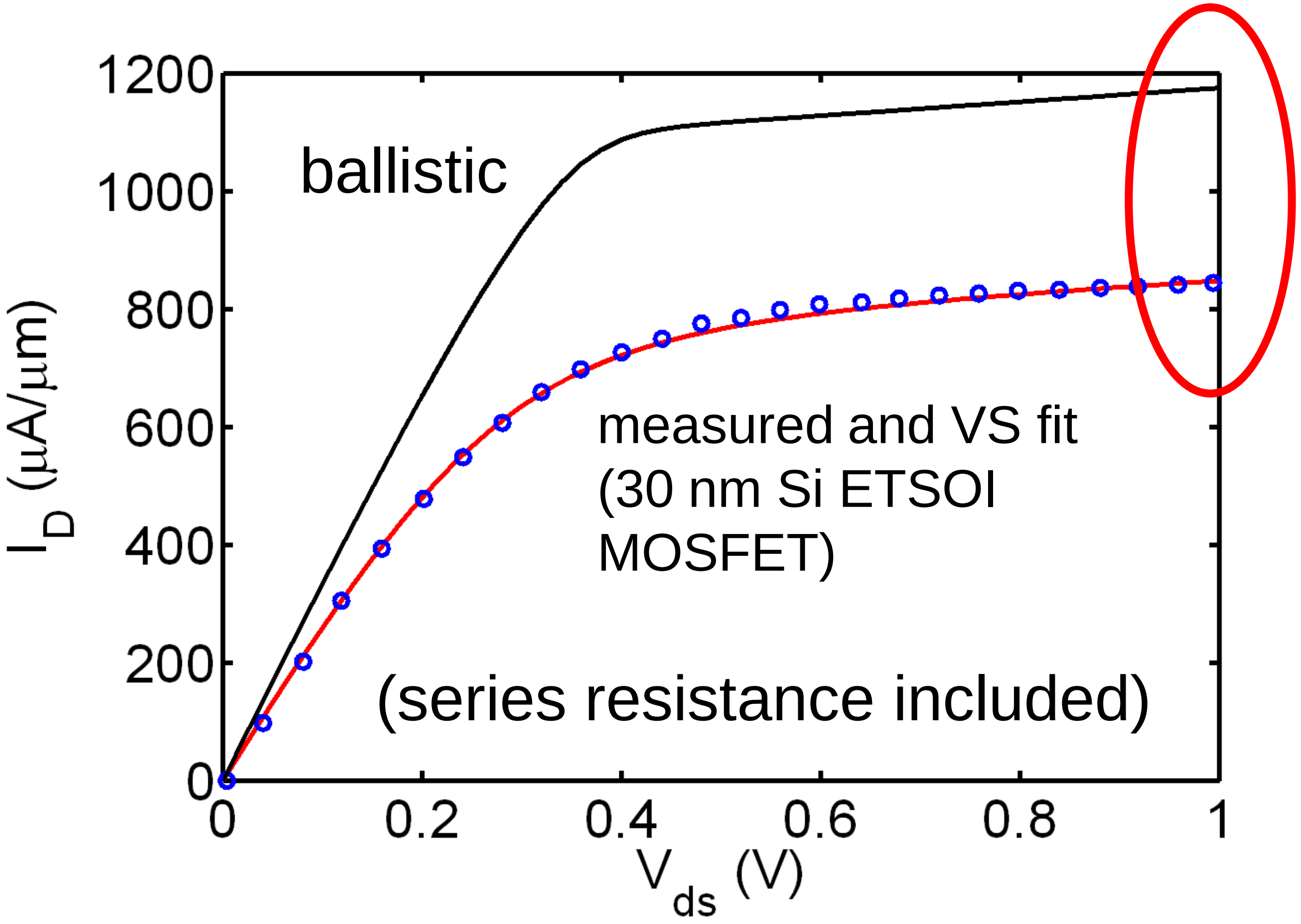
# 2) Saturation region




$$I_{DSAT} = W\left|Q_n\right|\upsilon_T \rightarrow \frac{\mathcal{T}_{SAT}}{2-\mathcal{T}_{SAT}} W\left|Q_n\right|\upsilon_T$$

The extra factor accounts for MOS electrostatics. The charge at the VS is determined by electrostatics – not backscattering.

# Transmission in saturation




$$\frac{I_{ON}}{I_{ON}^{ball}} = 0.70$$

$$\frac{I_{DSAT}}{I_{DSAT}^{ball}} = \left(\frac{\mathcal{T}_{SAT}}{2-\mathcal{T}_{SAT}}\right)$$

$$\mathcal{T}_{SAT} = 0.82$$

Fig. 15.2 *Fundamentals of Nanotransistors*, World Scientific Lecture Notes, 2015.

# The injection velocity

$$I_{DSAT} = W|Q_n|\upsilon_{inj} \qquad \upsilon_{inj} = \frac{\mathcal{T}_{SAT}}{2-\mathcal{T}_{SAT}}\upsilon_T$$

1) Traditional (velocity saturation) MOSFET model: $\upsilon_{inj} = \upsilon_{sat}$

2) Ballistic MOSFET model: $\upsilon_{inj} = \upsilon_T$

3) Transmission model: $\upsilon_{inj} = \dfrac{\mathcal{T}_{SAT}}{2-\mathcal{T}_{SAT}}\upsilon_T < \upsilon_T$

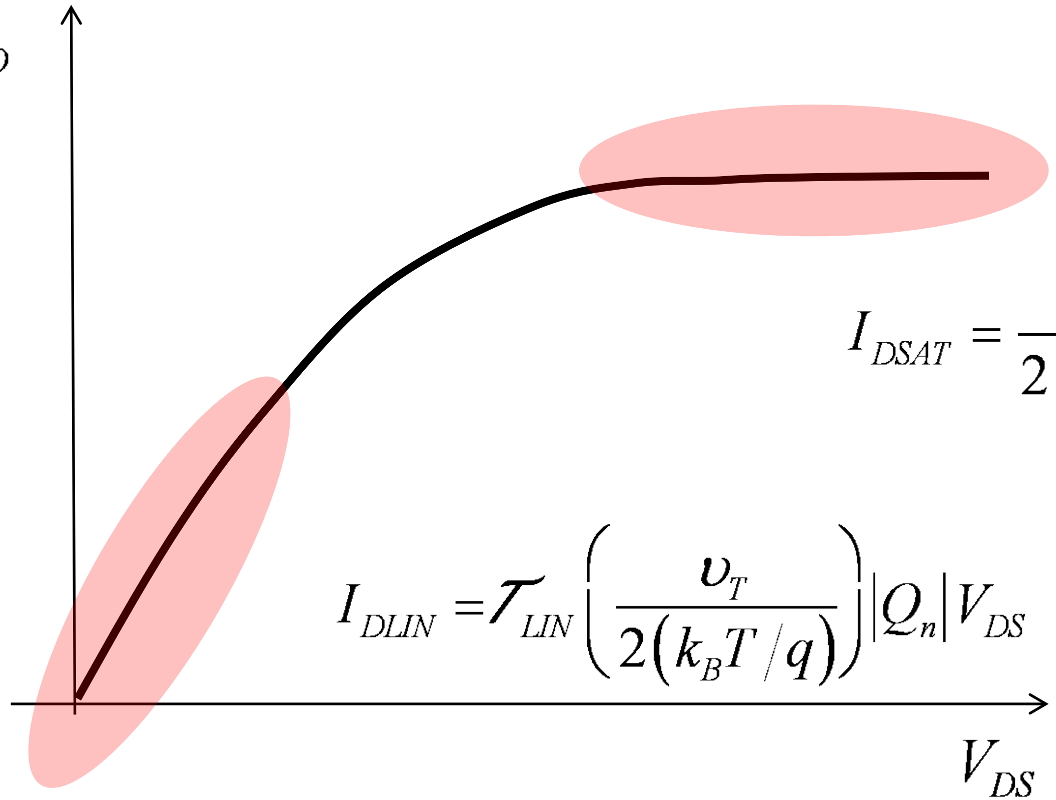
# Example

What is the injection velocity of the *L* = 30 nm ETSOI MOSFET discussed earlier?

$$\mathcal{T}_{SAT} = 0.82 \qquad \upsilon_T = \sqrt{\frac{2k_B T}{\pi m_t^*}} = 1.2 \times 10^7 \text{ cm/s}$$

$$\upsilon_{inj} = \frac{\mathcal{T}_{SAT}}{2 - \mathcal{T}_{SAT}} \upsilon_T = 0.7 \upsilon_T = 0.84 \times 10^7 \text{ cm/s}$$

# Linear and saturation region transmission



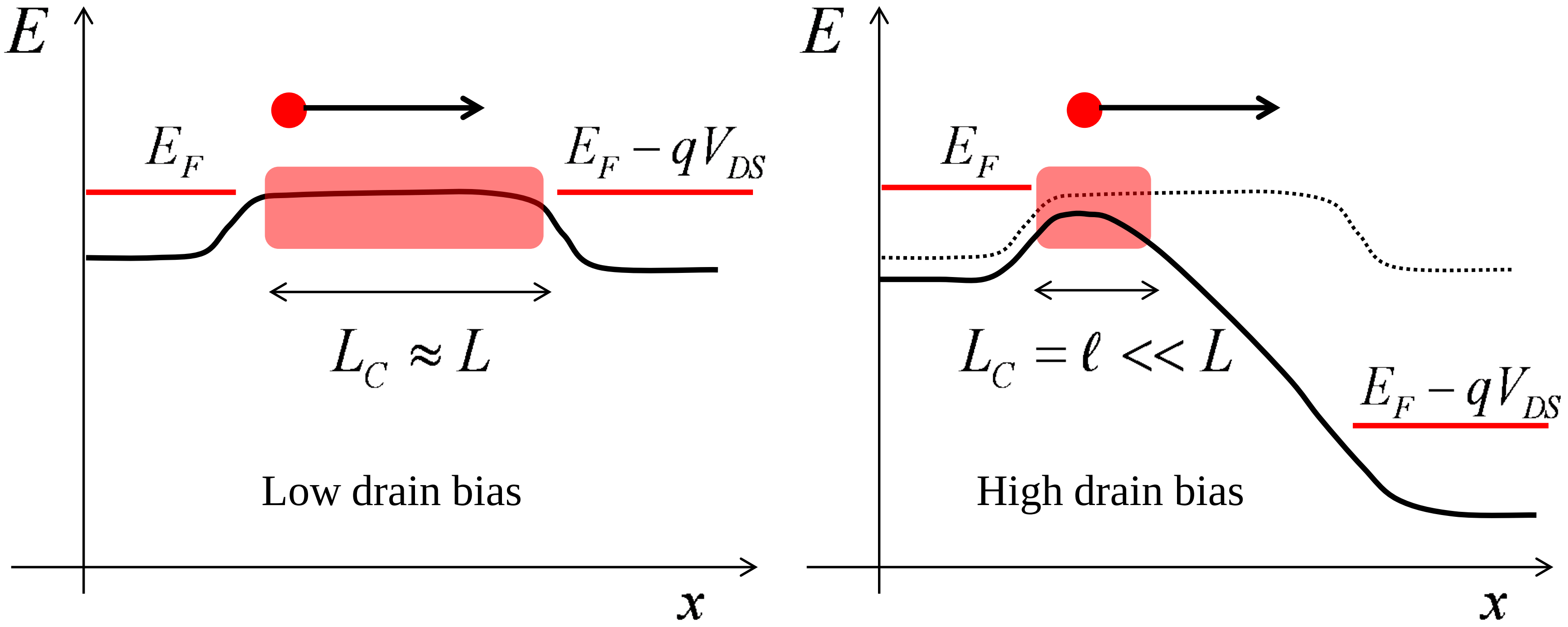

$$I_{DSAT} = \frac{\mathcal{T}_{SAT}}{2-\mathcal{T}_{SAT}} W|Q_n|\upsilon_T$$

$$I_{DLIN} = \mathcal{T}_{LIN}\left(\frac{\upsilon_T}{2(k_B T/q)}\right)|Q_n|V_{DS}$$

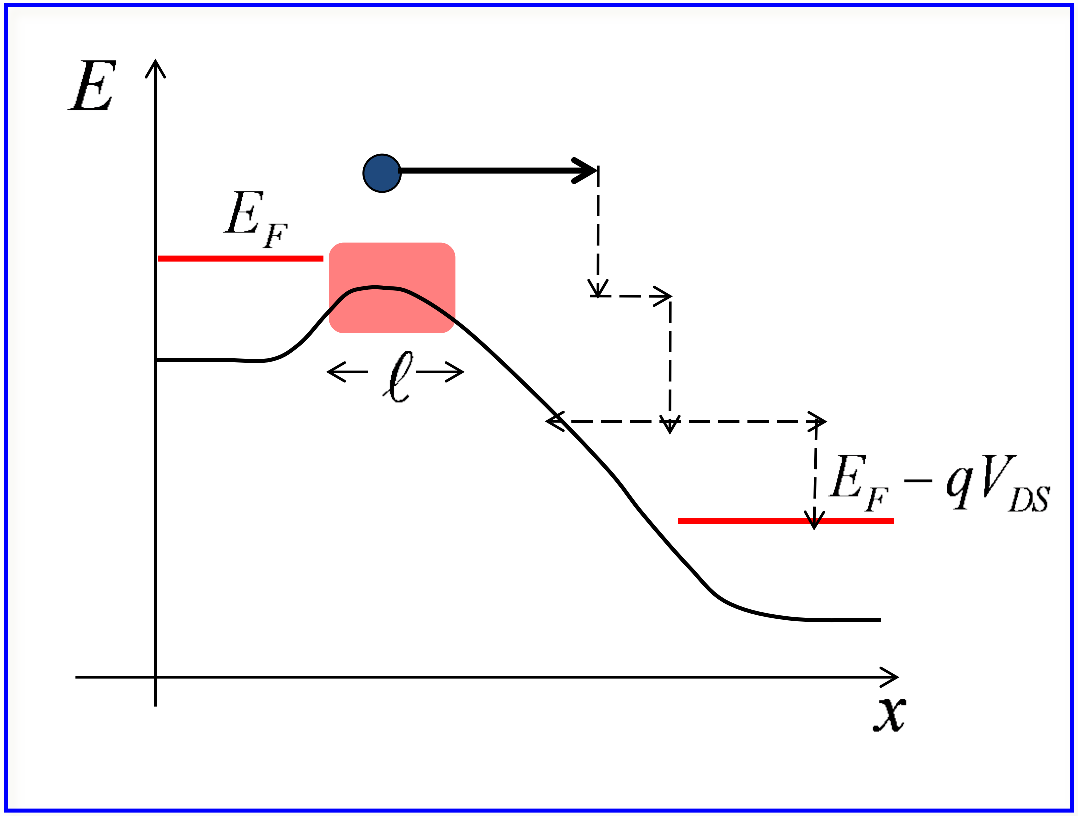
$\mathcal{T}_{LIN} < \mathcal{T}_{SAT}$ Why?

# Scattering under low and high $V_{DS}$

$$\mathcal{T}_{LIN} = \frac{\lambda_0}{\lambda_0 + L} \qquad \mathcal{T}(V_{DS}) = \frac{\lambda_0}{\lambda_0 + L_C(V_{DS})} \qquad \mathcal{T}_{SAT} = \frac{\lambda_0}{\lambda_0 + \ell}$$



# Operation near the "ballistic limit"

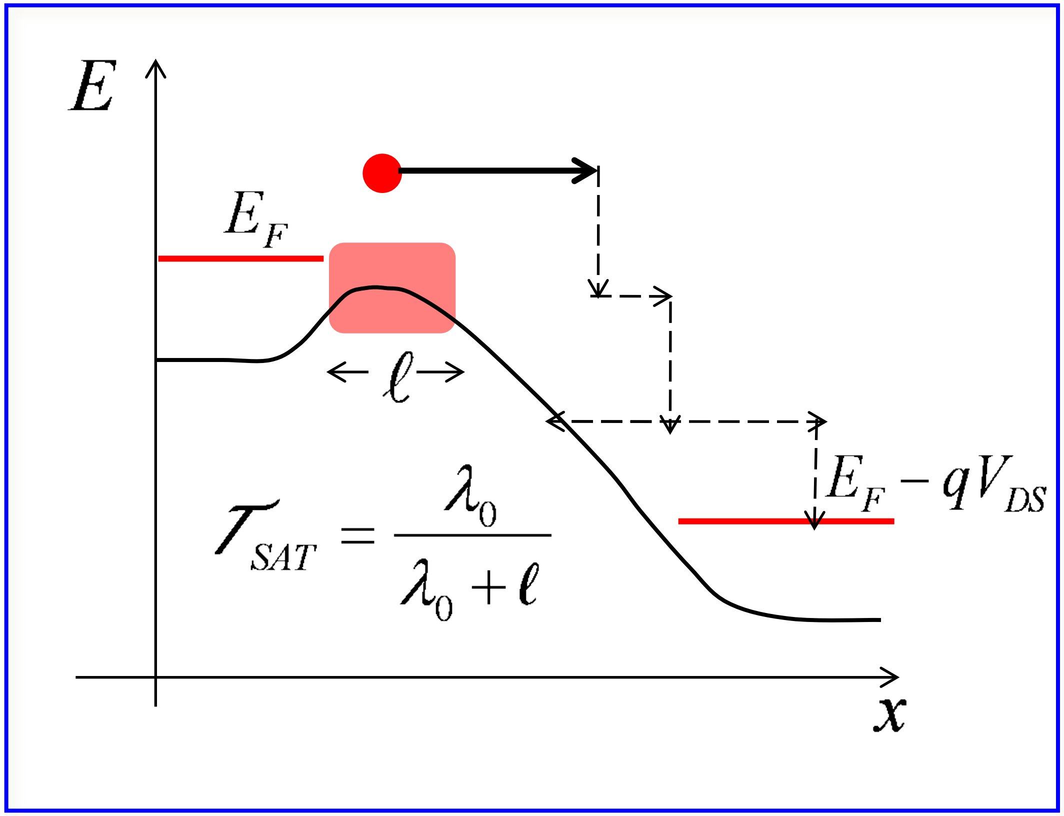



$$\mathcal{T}_{SAT} = \frac{\lambda_0}{\lambda_0 + \ell}$$

Operation near the ballistic limit current just means that $\mathcal{T}_{SAT} \rightarrow 1$,

**it does not imply** that there is little scattering.

# Is mobility relevant at the nanoscale?




- mobility is related to the near-eq. MFP
- backscattering in the critical region is also controlled by the near-eq. MFP.
- mobility determines the on-current
- but the MFP near the drain is very short.

# Summary

- Scattering lowers the drain current.

- Transmission is **higher** under high drain bias than under low drain bias.

- Under low drain bias, transmission is determined by backscattering in the **entire channel**.

- Under high drain bias, transmission is determined by backscattering in a **short, “bottleneck region”** near the top of the barrier.

- **Apparent mobility** and **injection velocity** allow us to use traditional MOSFET theory.

# Next topic

In the next lecture, we will revisit the VS model.