2020.09.29 Cours 9

mardi 29 septembre 2020 08:30

$$H_1 = \frac{K}{p(1+2p)} \rightarrow$$

Controller Form:

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ 0 & -\frac{1}{2} \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$

$$y = \begin{pmatrix} \frac{K}{2} & 0 \end{pmatrix} x$$

$$\mathcal{C} = \begin{pmatrix} 0 & 1 \\ 1 & -1/2 \end{pmatrix} \quad \mathcal{O} = \begin{pmatrix} K/2 & 0 \\ 0 & \frac{K}{2} \end{pmatrix}$$

controlable and observable (if $K \neq 0$)

trivial case

$\rightarrow$ observer Form:

$$\dot{x} = \begin{pmatrix} 0 & 0 \\ 1 & -\frac{1}{2} \end{pmatrix} x + \begin{pmatrix} K/2 \\ 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} 0 & 1 \end{pmatrix} x$$

$$\mathcal{C} = \begin{pmatrix} K/2 & 0 \\ 0 & -\frac{K}{2} \end{pmatrix} ; \quad \mathcal{O} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

same conclusion (almost ...)

$$H_2(p) = \frac{a_0 + a_1 p}{b_0 + b_1 p + p^2}$$

Controller form:

$$\begin{cases} \dot{x} = \begin{pmatrix} 0 & 1 \\ -b_0 & -b_1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \\ y = \begin{pmatrix} a_0 & a_1 \end{pmatrix} x \end{cases}$$

$$\mathcal{C} = \begin{pmatrix} 0 & 1 \\ 1 & -b_1 \end{pmatrix} \Rightarrow \operatorname{rank}(\mathcal{C}) = 2 \quad : \text{controlable !}$$

$$\mathcal{O} = \begin{pmatrix} a_0 & a_1 \\ -a_1 b_0 & a_0 - a_1 b_1 \end{pmatrix} \Rightarrow \operatorname{rank}(\mathcal{O}) = 2 \text{ except if}$$

$$\det(\mathcal{O}) = 0 \Rightarrow \boxed{a_0^2 - a_0 a_1 b_1 + a_1^2 b_0 = 0}$$

$\rightarrow$ observer form:

$$\begin{cases} \dot{x} = \begin{pmatrix} 0 & -b_0 \\ 1 & -b_1 \end{pmatrix} x + \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} u \\ y = \begin{pmatrix} 0 & 1 \end{pmatrix} x \end{cases}$$

$$\begin{cases} \dot{x} = \ldots \\ y = \begin{pmatrix} 0 & 1 \end{pmatrix} x \end{cases}$$

$$\mathcal{C} = \begin{pmatrix} a_0 & -b_0 a_1 \\ a_1 & a_0 - a_1 b_1 \end{pmatrix} \Rightarrow \operatorname{rank}(\mathcal{C}) = 2 \text{ except if}$$

$$\det(\mathcal{C}) = 0 \Rightarrow \boxed{a_0^2 - a_0 a_1 b_1 + b_0 a_1^2 = 0}$$

$$\mathcal{O} = \begin{pmatrix} 0 & 1 \\ 1 & -b_1 \end{pmatrix} \Rightarrow \operatorname{rank}(\mathcal{O}) = 2 \text{ observable.}$$

$$\dot{x} = \begin{pmatrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & 0 & 1 \\ -a_0 & \cdots & & -a_{n-1} \end{pmatrix} x + \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} u$$

$$u = -Kx = -\begin{pmatrix} k_0 & \cdots & k_{n-1} \end{pmatrix} x$$

$$A - BK = \begin{pmatrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & 0 & 1 \\ -a_0 & \cdots & & -a_{n-1} \end{pmatrix} - \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} k_0 & \cdots & k_{n-1} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & 0 & 1 \\ (-a_0 - k_0) & (-a_1 - k_1) & \cdots & (-a_{n-1} - k_{n-1}) \end{pmatrix}$$

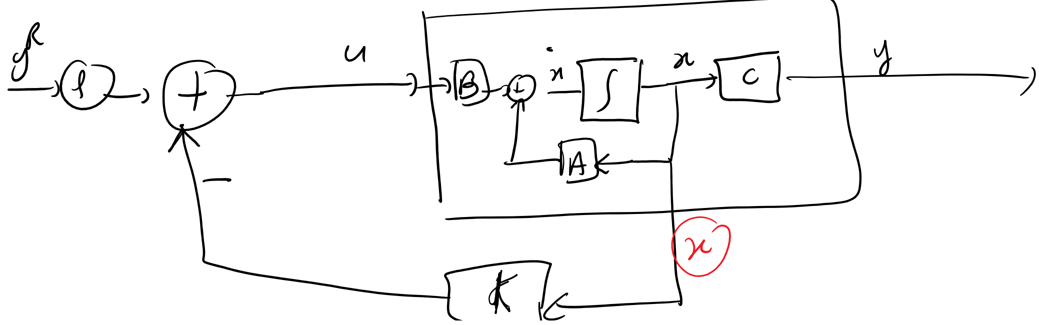
$$p_{A-BK} = (a_0 + k_0) + (a_1 + k_1)p + \cdots (a_{n-1} + k_{n-1}) p^{n-1} + p^n$$

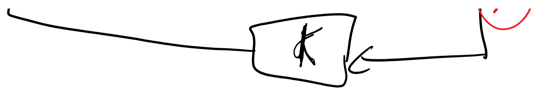
$$= \beta_0 + \beta_1 p + \cdots \beta_{n-1} p^{n-1} + p^n$$

a_i are known
β_i are also known

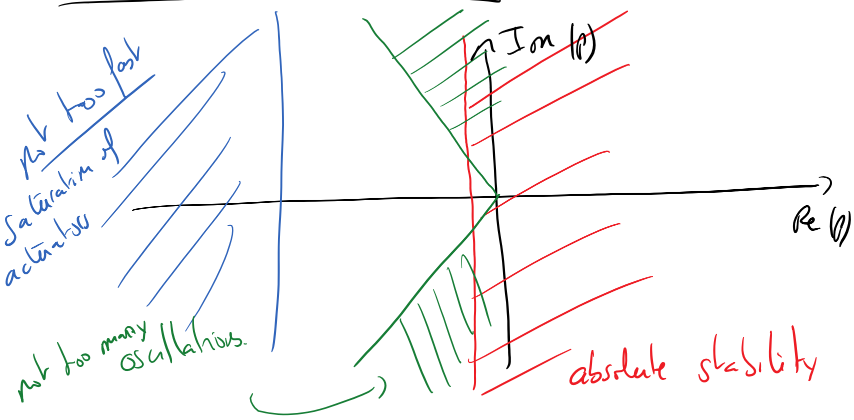
term by term we have: $\boxed{k_i = \beta_i - a_i}$

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \qquad u = l y^R - Kx$$



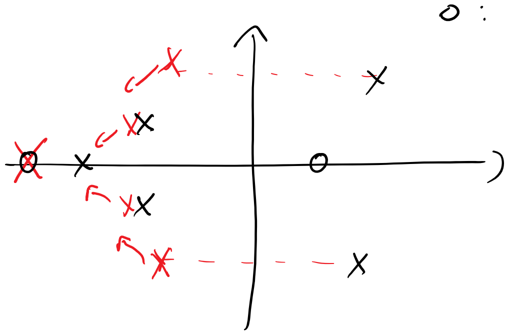


Where to place the poles?



Link with open loop poles:

x : poles of OL
o : zeros of OL
x : poles of C.L.



Example: G ⇒ third order system.

we expect: 2nd order behavior with D = 10% ✓
TR = 5 seconds ✓
No steady-state error.

How to choose the poles.

Frequency specification "open loop design".

→ No steady-state ⇒ Infinite gain when $p=0$ (integral action)

→ $D = 10\% \rightarrow \Delta\phi = 60°$

$\xi = 0,6$

→ $\omega_0 t_R = 5$

$\cancel{p=0}$

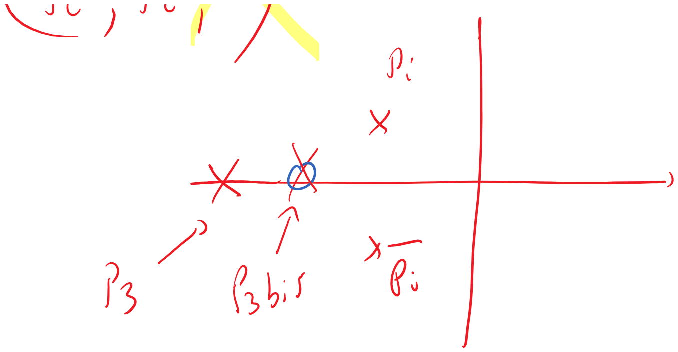
$*$

$$\frac{}{\left(\frac{p^2}{\omega_0^2} + \frac{2\xi}{\omega_0} p + 1\right)}$$

$p_i, \bar{p}_i$

$(p_i, \bar{p}_i, \cancel{0})$

p_i



↳ steady-state error?
↳ ?