Cluster Growth using the DLA Model

By Tahoe, Ksenia, and Xinmeng

In this presentation:

- 2D Random Walks
- The 1D Diffusion Equation
- Diffusion Limited Aggregation Model for Cluster Growth

2D Random Walks

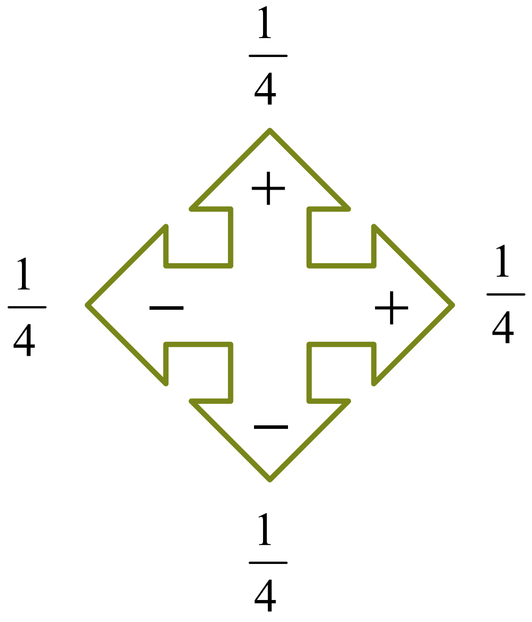
Analytical parameters for a 2D random walk

$$\langle x \rangle = \sum_{i=1}^{N} \langle x_i \rangle = \sum_{i=1}^{N} [0.25 \times (-1) + 0.25 \times 1] = 0$$

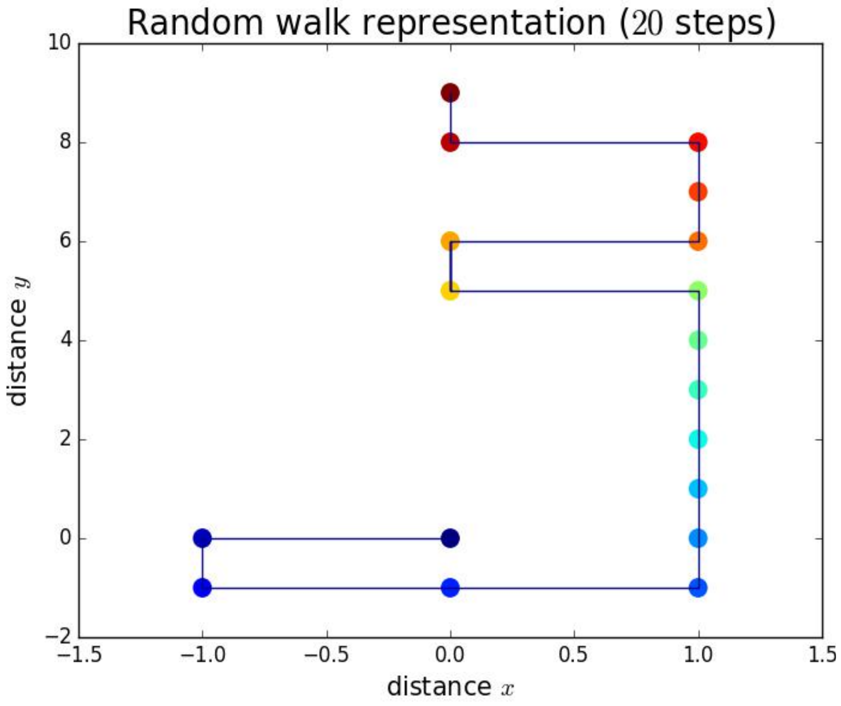
$$\langle x^2 \rangle = \langle \sum_{i=1}^{N} x_i^2 \rangle + \langle \sum_{i \neq j} x_i x_j \rangle = N \langle x_i^2 \rangle$$

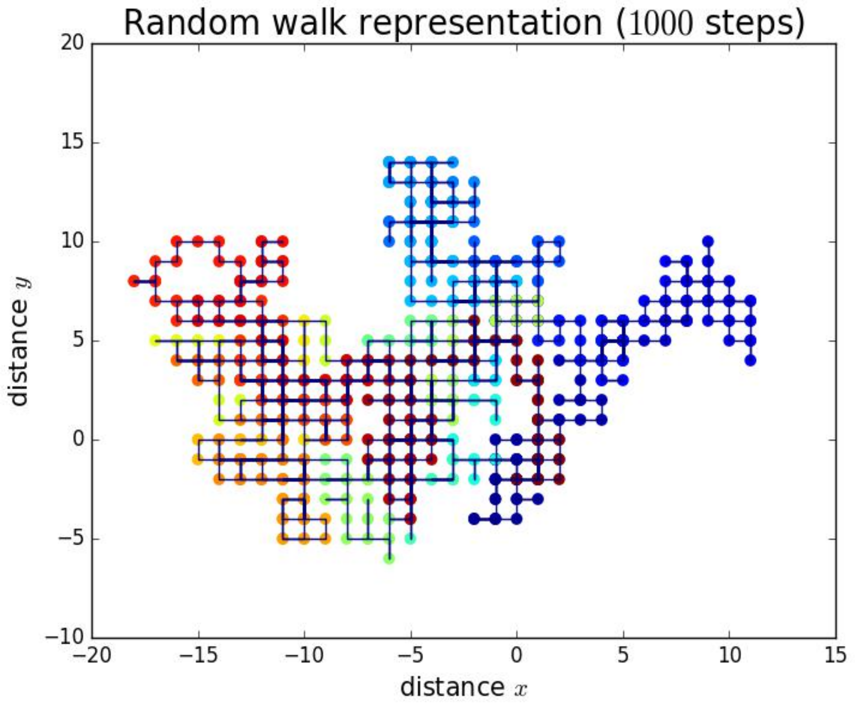
$$= N(0.25 \times 1^2 + 0.25 \times (-1)^2) = 0.5N$$

$$\langle d^2 \rangle = \langle x^2 + y^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle = 0.5N + 0.5N = N$$



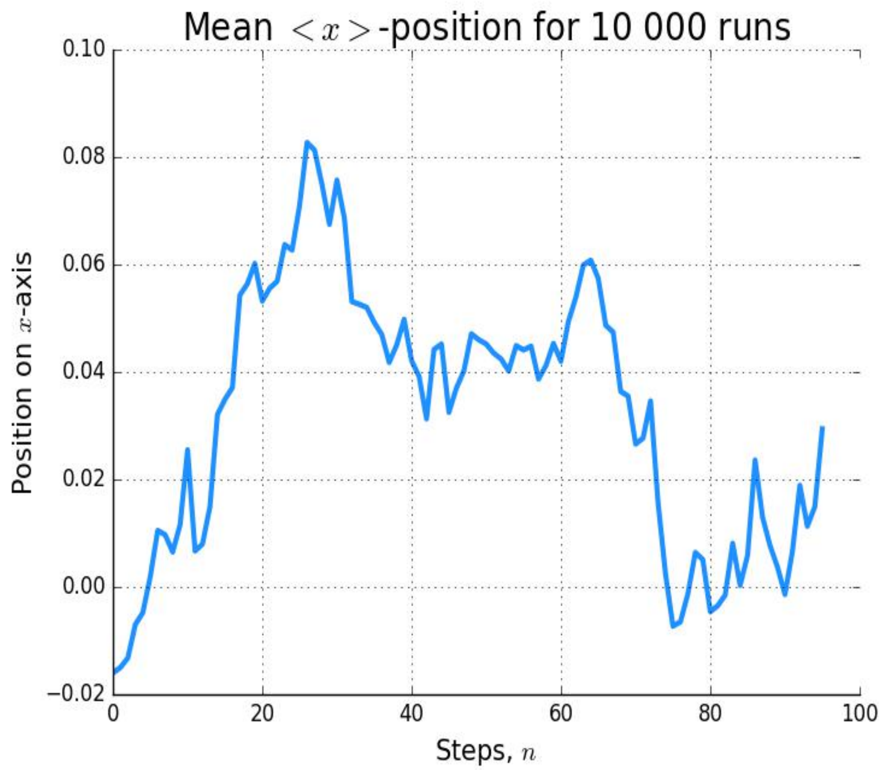
Simulation of 2D Random Walk

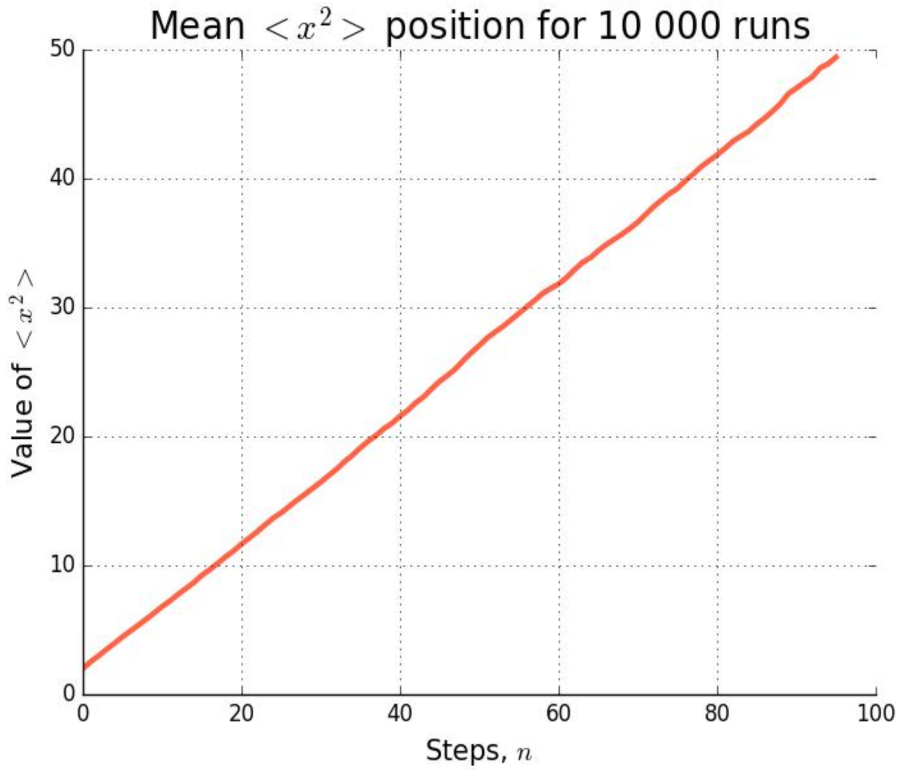




- **Define** random-walk function using 4 thresholds in random numbers among (0,1)
- **Record** position and displacement information for each step
- **Average** over 10,000 different walkers

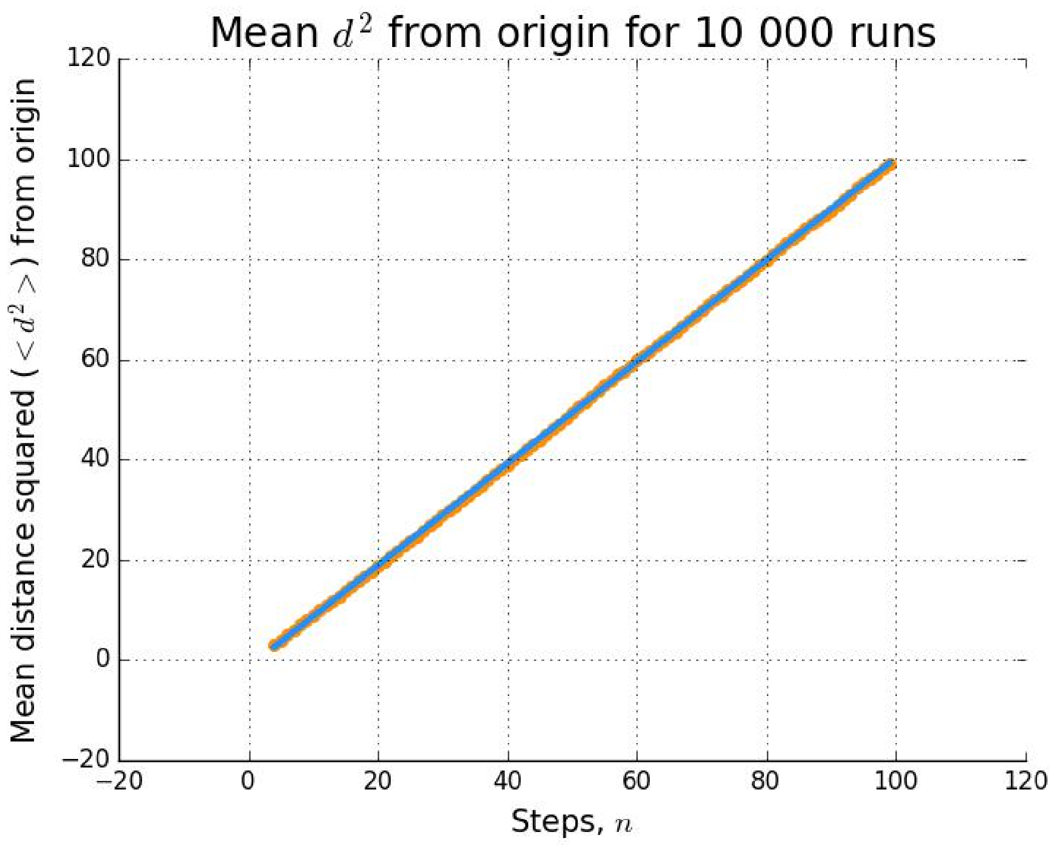
Simulation of 2D Random Walk





$$< x > \approx 0$$

$$< x^2 > \approx 0.5N$$



2D Random Walk is diffusive

- Diffusive motion has

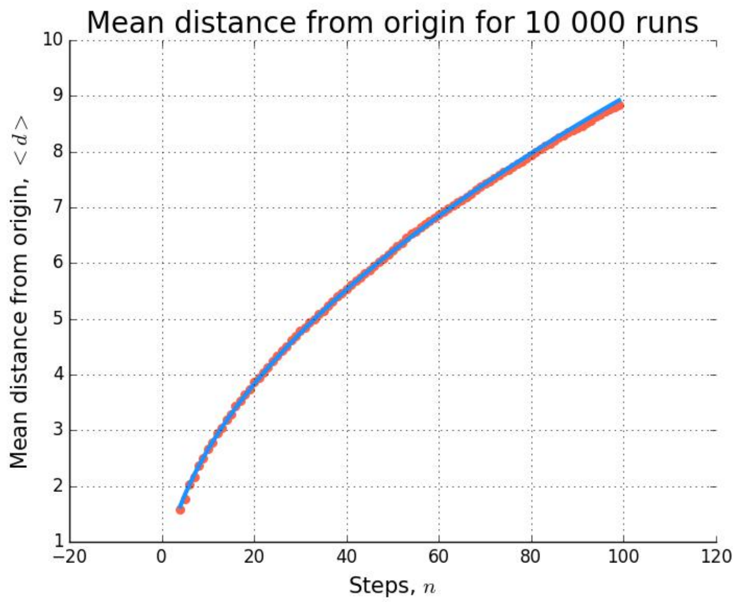
$$<x^2> \propto t (= 2Dt)$$

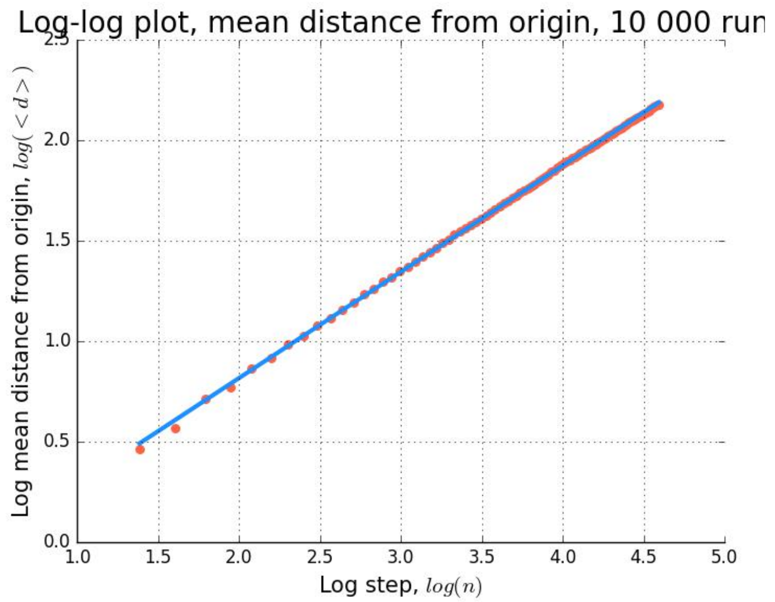
- For 2D R.W. our simulation gives the relation

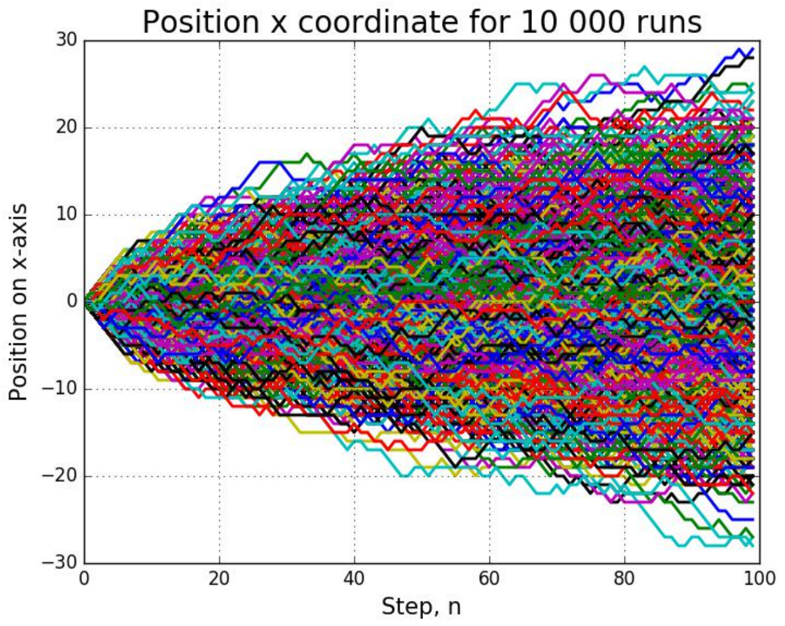
$$<d^2> = n \quad t \sim n$$

- Thus, the diffusion coefficient is given by D=0.25.

Simulation of 2D Random Walk - Plots of interest







1D Diffusion Equation Solver

Modelling the Equation

Differential Form

- Time dependent diff. eq.

$$\frac{\partial \rho(x,t)}{\partial t} = D\nabla^2 \rho(x,t)$$

- For a box density, later solutions should be Gaussian

$$\rho(x,t) = \frac{1}{\sqrt{2\pi\sigma(t)^2}} \exp\left(-\frac{x^2}{2\sigma(t)^2}\right)$$

$$\sigma(t) = \sqrt{2Dt}.$$

Finite Difference Form

- Convert to an iterable form

$$\rho(x,t) = \rho(i\Delta x, n\Delta t)$$

- Algebraically manipulate formal derivatives

$$\rho(i,n+1) = \rho(i,n) + \frac{D\Delta t}{\Delta x^2}\left(\rho(i+1,n) + \rho(i-1,n) - 2\rho(i,n)\right)$$

- For our purposes, D=2 and stability is guaranteed by

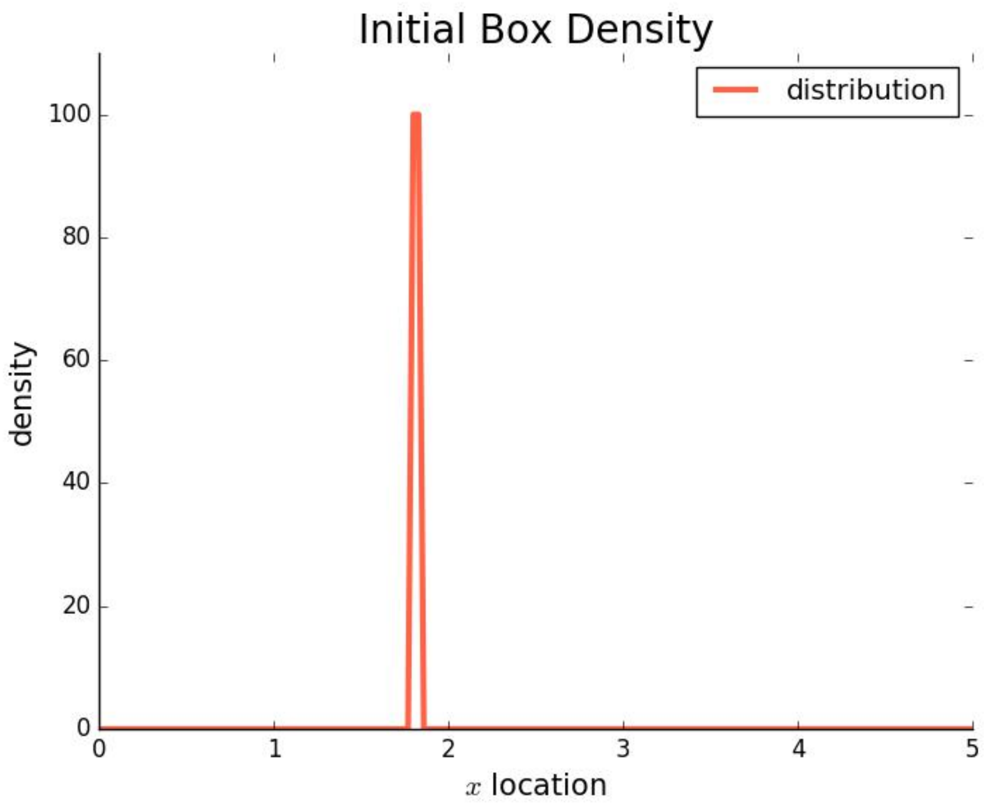
$$\Delta t \leq \frac{(\Delta x)^2}{2D}$$

Variance of a Gaussian – Quick Proof

- Expectation values are of the form: $\langle x \rangle = \int_{-\infty}^{\infty} f(x) x dx$
- Thus, after using the fact that we are dealing with an even function

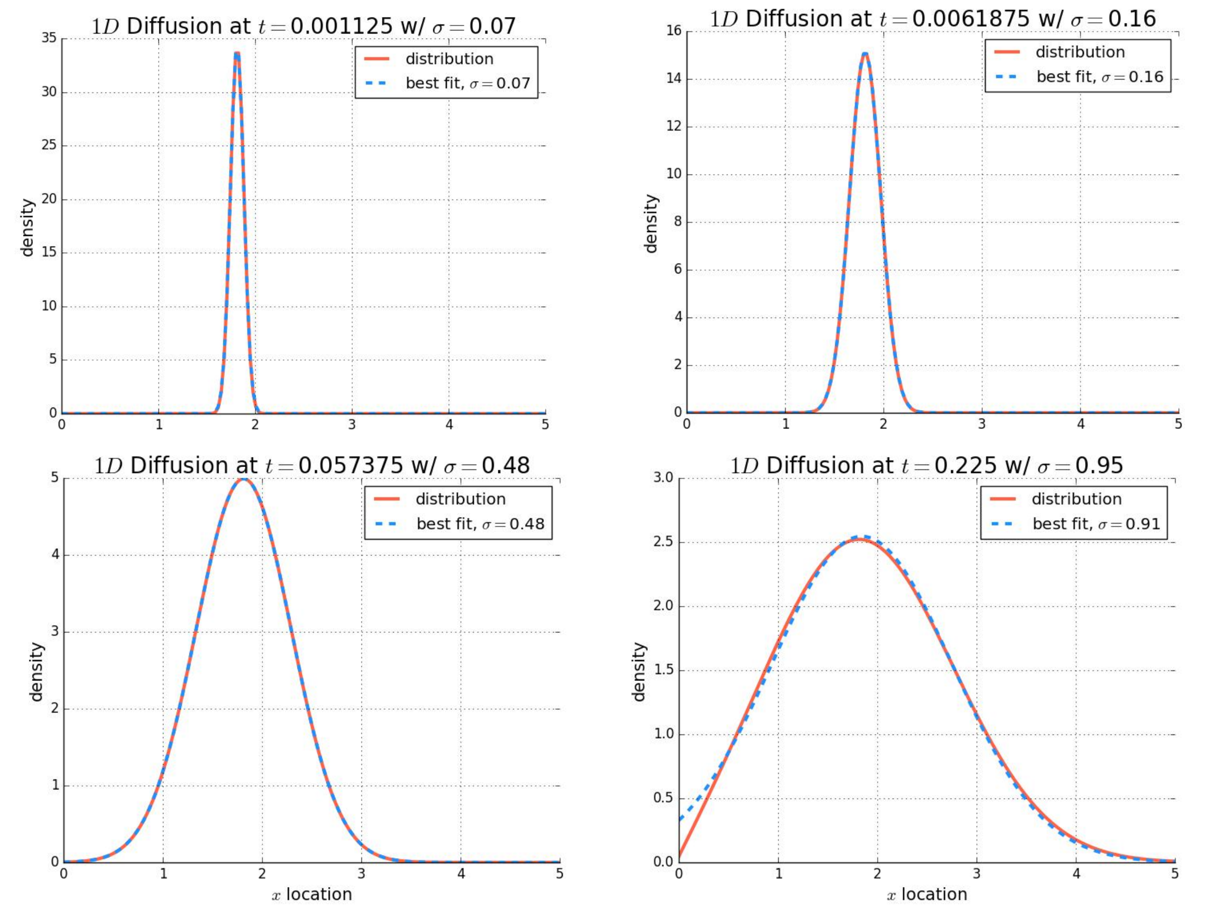
$$\langle x^2 \rangle = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma(t)^2}} \exp\left(-\frac{x^2}{2\sigma(t)^2}\right) x^2 dx$$
$$= 2\int_{0}^{\infty} \frac{1}{\sqrt{2\pi\sigma(t)^2}} \exp\left(-\frac{x^2}{2\sigma(t)^2}\right) x^2 dx.$$

- After a change of variable $\langle x^2 \rangle = \frac{4\sigma^2}{\sqrt{\pi}} \int_{0}^{\infty} x^2 \exp(-x^2) dx$
- Finally, after integrating by parts: $\langle x^2 \rangle = \sigma^2$



Initial Distribution

Important to put the box density far from the edges, otherwise you will affect the Gaussian and make it lopsided.



Four Evolutions in Time

A sigma fit was performed for each time snapshot, and they match very well with the expected:

$$\sigma = \sqrt{2Dt}.$$

Note: Edge effects distorting the fit at later times.

Cluster Growth using DLA

DLA Cluster

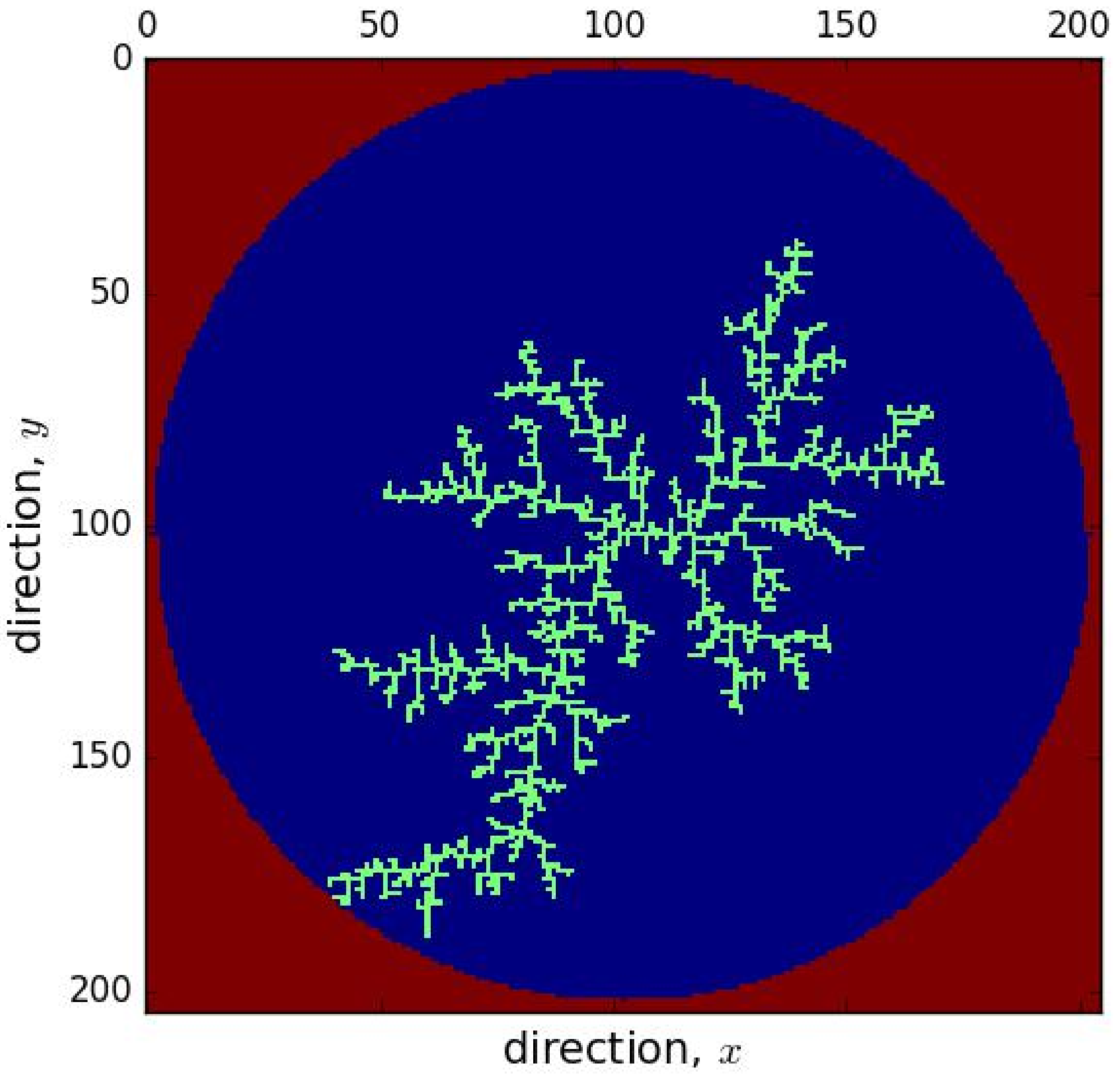
Many large open spaces, irregular perimeter

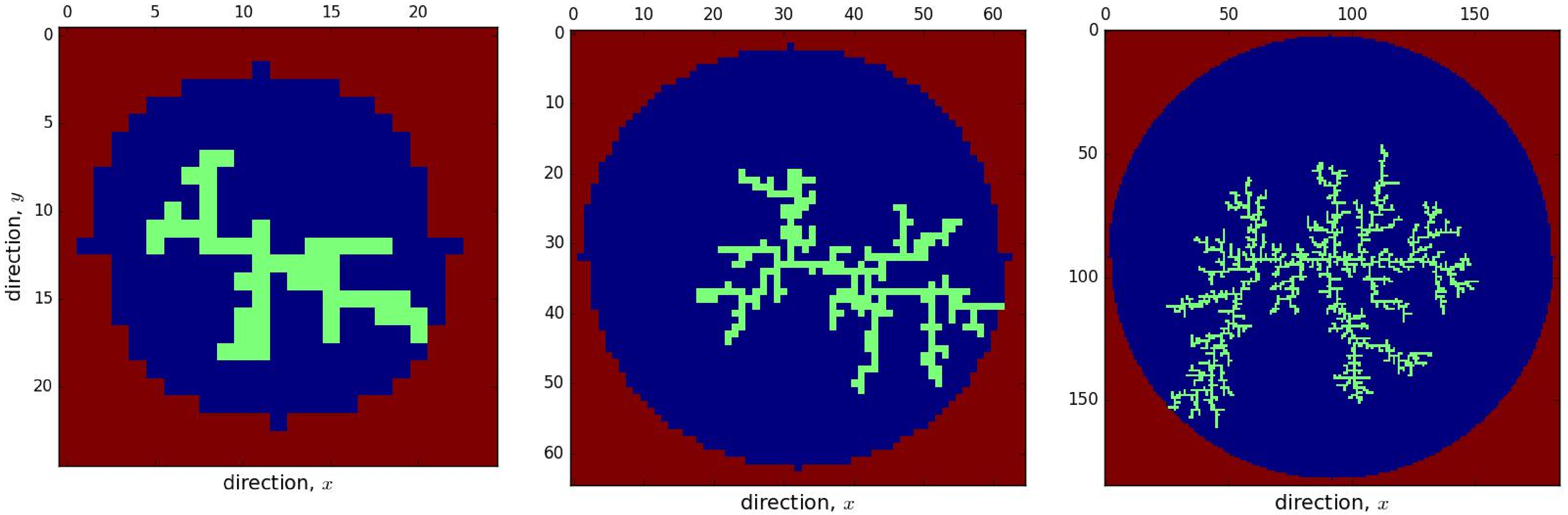
$$m(r) \sim r^{d_f}$$

$$d_f = fractal\ dimensionality$$

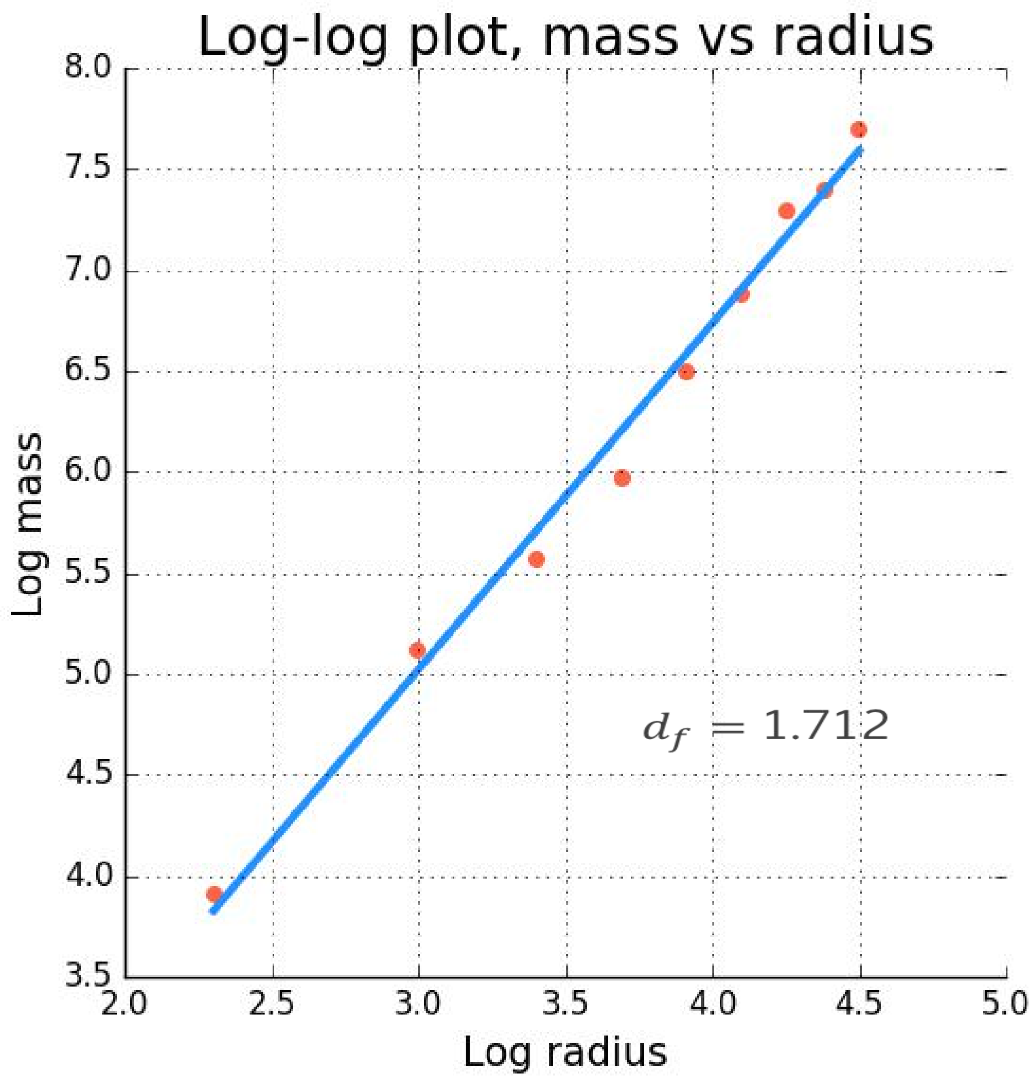
$$d_f = 1\ line, or\ curve$$

$$d_f = 2\ solid\ disk$$





0
5
10
15
20
direction, y
direction, x
0
10
20
30
40
50
60
direction, x
0
50
100
150
direction, x



Fractal dimensionality

$$\log m = d_f \log(r)$$

In 10 runs fractal dimensionality:

2.067, 1.653, 1.877, 1.833, 1.607, 1.832,1.782, 1.712, 1.843, 1.862

mean=1.807

Thank you for your time!