Linear Regerssion

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```
library(dplyr)
library(caret)
library(ggplot2)
library(rjags)
library(coda)
```

Data Preparation

We first prepare the data for each cluster.

```
aggregated_data <- read.csv("AggregatedData1.csv")</pre>
daily_data <- read.csv("Daily_AggregatedData1.csv")</pre>
cluster1 <- aggregated_data[, -c(2,3,4,5,6)]</pre>
cluster2 <- aggregated_data[, -c(1,3,4,5,6)]</pre>
cluster3 \leftarrow aggregated_data[, -c(1,2,4,5,6)]
cluster4 <- aggregated_data[, -c(1,2,3,5,6)]</pre>
cluster5 <- aggregated_data[, -c(1,2,3,4,6)]</pre>
cluster6 <- aggregated_data[, -c(1,2,3,4,5)]</pre>
cluster1_trans <- cluster1 %>%
  mutate(Cluster.1_lag1 = lag(Cluster.1, n = 1)) %>%
  mutate(
    tod_poly1 = tod,
    tod_poly2 = tod^2,
    tod_poly3 = tod^3,
    tod_poly4 = tod^4,
    weekend_dummy = ifelse(weekend == "TRUE", 1, 0),
    toy_sin = sin(toy),
    toy_cos = cos(toy)
  ) %>%
  na.omit()
cluster2_trans <- cluster2 %>%
  mutate(Cluster.2_lag1 = lag(Cluster.2, n = 1)) %>%
  mutate(
    tod_poly1 = tod,
    tod_poly2 = tod^2,
   tod_poly3 = tod^3,
    tod_poly4 = tod^4,
    weekend_dummy = ifelse(weekend == "TRUE", 1, 0),
    toy_sin = sin(toy),
```

```
toy_cos = cos(toy)
  ) %>%
 na.omit()
cluster3_trans <- cluster3 %>%
  mutate(Cluster.3_lag1 = lag(Cluster.3, n = 1)) %>%
 mutate(
   tod poly1 = tod,
   tod_poly2 = tod^2,
   tod_poly3 = tod^3,
   tod_poly4 = tod^4,
   weekend_dummy = ifelse(weekend == "TRUE", 1, 0),
   toy_sin = sin(toy),
   toy_cos = cos(toy)
  ) %>%
 na.omit()
cluster4_trans <- cluster4 %>%
  mutate(Cluster.4_lag1 = lag(Cluster.4, n = 1)) %>%
 mutate(
   tod_poly1 = tod,
   tod_poly2 = tod^2,
   tod_poly3 = tod^3,
   tod poly4 = tod^4,
   weekend_dummy = ifelse(weekend == "TRUE", 1, 0),
   toy_sin = sin(toy),
   toy_cos = cos(toy)
  ) %>%
 na.omit()
cluster5_trans <- cluster5 %>%
  mutate(Cluster.5_lag1 = lag(Cluster.5, n = 1)) %>%
  mutate(
   tod_poly1 = tod,
   tod_poly2 = tod^2,
   tod_poly3 = tod^3,
   tod_poly4 = tod^4,
   weekend_dummy = ifelse(weekend == "TRUE", 1, 0),
   toy_sin = sin(toy),
   toy_cos = cos(toy)
  ) %>%
 na.omit()
cluster6_trans <- cluster6 %>%
 mutate(Cluster.6_lag1 = lag(Cluster.6, n = 1)) %>%
  mutate(
   tod_poly1 = tod,
   tod_poly2 = tod^2,
   tod_poly3 = tod^3,
   tod_poly4 = tod^4,
   weekend_dummy = ifelse(weekend == "TRUE", 1, 0),
   toy_sin = sin(toy),
```

```
toy_cos = cos(toy)
) %>%
na.omit()
```

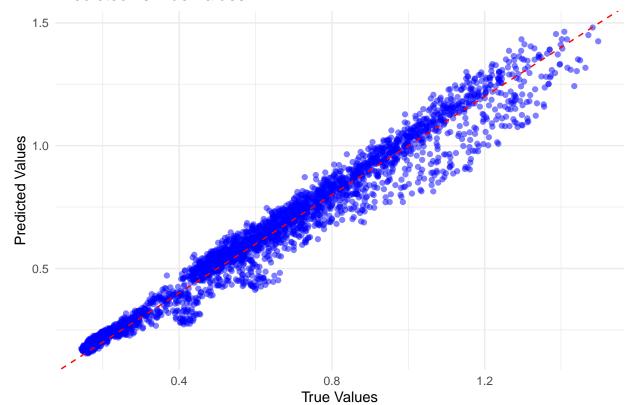
Clusrer 1

Linear Regression Model for Cluster 1

On the basis of cluster1_trans, we fit a linear regression model and evaluate its performance:

```
# Split the data into training and testing sets
train_index_1 <- 1:floor(0.8 * nrow(cluster1_trans))</pre>
train_data_1 <- cluster1_trans[train_index_1, ]</pre>
test_data_1 <- cluster1_trans[-train_index_1, ]</pre>
# Fit the linear regression model
linear_model_1 <- lm(Cluster.1 ~ Cluster.1_lag1 + temp + tod_poly1 + tod_poly2 + tod_poly3 + tod_poly4
```

```
# Make predictions on the testing set
test_data_1$predictions <- predict(linear_model_1, newdata = test_data_1)</pre>
# Calculate prediction error metrics
mae_1 <- mean(abs(test_data_1$Cluster.1 - test_data_1$predictions))</pre>
mse_1 <- mean((test_data_1$Cluster.1 - test_data_1$predictions)^2)</pre>
rmse_1 <- sqrt(mse_1)</pre>
# Print the results
cat("Mean Absolute Error (MAE):", mae_1, "\n")
## Mean Absolute Error (MAE): 0.04186565
cat("Mean Squared Error (MSE):", mse_1, "\n")
## Mean Squared Error (MSE): 0.003343571
cat("Root Mean Squared Error (RMSE):", rmse_1, "\n")
## Root Mean Squared Error (RMSE): 0.05782362
# Plot predicted vs true values
ggplot(test_data_1, aes(x = Cluster.1, y = predictions)) +
  geom_point(color = 'blue', alpha = 0.5) +
  geom_abline(intercept = 0, slope = 1, color = 'red', linetype = "dashed") +
 labs(title = "Predicted vs True Values",
       x = "True Values",
       y = "Predicted Values") +
  theme minimal()
```



```
summary(linear_model_1)
```

```
##
## Call:
## lm(formula = Cluster.1 ~ Cluster.1_lag1 + temp + tod_poly1 +
       tod_poly2 + tod_poly3 + tod_poly4 + weekend_dummy + toy_sin +
##
##
       toy_cos, data = train_data_1)
##
## Residuals:
##
        Min
                   1Q
                         Median
                                       3Q
## -0.165153 -0.026932 -0.008028 0.017576 0.300232
##
## Coefficients:
##
                   Estimate Std. Error t value Pr(>|t|)
                 -3.660e-02 1.999e-02 -1.831
## (Intercept)
                                               0.0671 .
## Cluster.1_lag1 9.265e-01
                            3.346e-03 276.867
                                               < 2e-16 ***
## temp
                 -9.932e-04 1.286e-04 -7.725 1.2e-14 ***
## tod_poly1
                  2.293e-02 6.069e-04 37.780
                                               < 2e-16 ***
## tod_poly2
                 -1.602e-03 5.276e-05 -30.367
                                               < 2e-16 ***
## tod_poly3
                 5.057e-05 1.632e-06 30.989
                                               < 2e-16 ***
## tod_poly4
                 -5.649e-07 1.682e-08 -33.597
                                               < 2e-16 ***
                 1.519e-03 9.309e-04
## weekend_dummy
                                         1.632
                                                 0.1027
## toy_sin
                 -9.073e-03 9.531e-03 -0.952
                                                 0.3411
## toy cos
                 -3.571e-02 1.860e-02 -1.920
                                                 0.0549 .
## ---
```

```
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.048 on 13428 degrees of freedom
## Multiple R-squared: 0.9653, Adjusted R-squared: 0.9652
## F-statistic: 4.145e+04 on 9 and 13428 DF, p-value: < 2.2e-16
```

Analysis

Coefficients:

- Cluster.1_lag1: The coefficient is very high and significant, indicating a strong autocorrelation
- temp: The negative coefficient suggests that an increase in temperature slightly decreases Cluster.1.
- tod_poly1, tod_poly2, tod_poly3, and tod_poly4: These terms capture the non-linear effect of the time of day. Significant coefficients indicate a polynomial relationship with Cluster.1.
- weekend_dummy: The positive and significant coefficient indicates that Cluster.1 is higher on weekends compared to weekdays.
- toy_sin and toy_cos: These coefficients are not significant, suggesting that the trigonometric transformation of the time of year might not be a strong predictor for Cluster.1.

• Model Fit:

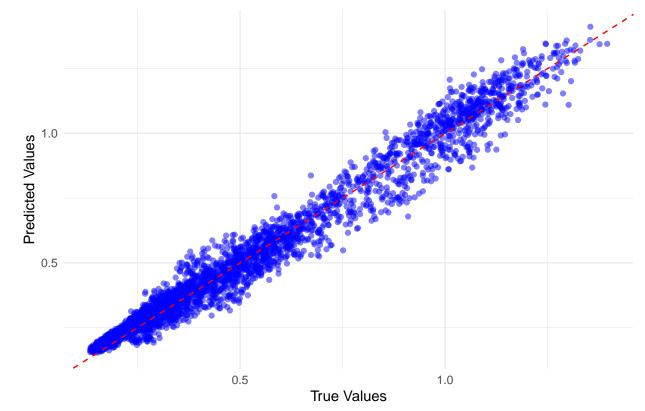
- The R-squared value of 0.9623 indicates that the model explains 96.23\% of the variance in Cluster.1, which suggests a very good fit.

Cluster 2

Linear Regression Model for Cluster 2

```
# Split the data into training and testing sets
train_index_2 <- 1:floor(0.8 * nrow(cluster2_trans))</pre>
train data 2 <- cluster2 trans[train index 2, ]</pre>
test_data_2 <- cluster2_trans[-train_index_2, ]</pre>
# Fit the linear regression model
linear_model_2 <- lm(Cluster.2 ~ Cluster.2_lag1 + temp + tod_poly1 + tod_poly2 + tod_poly3 + tod_poly4</pre>
```

```
# Make predictions on the testing set
test_data_2$predictions <- predict(linear_model_2, newdata = test_data_2)</pre>
# Calculate prediction error metrics
mae_2 <- mean(abs(test_data_2$Cluster.2 - test_data_2$predictions))</pre>
mse_2 <- mean((test_data_2$Cluster.2 - test_data_2$predictions)^2)</pre>
rmse 2 <- sqrt(mse 2)</pre>
# Print the results
cat("Mean Absolute Error (MAE):", mae 2, "\n")
## Mean Absolute Error (MAE): 0.03925669
cat("Mean Squared Error (MSE):", mse 2, "\n")
## Mean Squared Error (MSE): 0.002707433
```



Linear Regression Model Summary

summary(linear_model_2)

```
##
## Call:
## lm(formula = Cluster.2 ~ Cluster.2_lag1 + temp + tod_poly1 +
       tod_poly2 + tod_poly3 + tod_poly4 + weekend_dummy + toy_sin +
##
       toy_cos, data = train_data_2)
##
##
## Residuals:
##
         Min
                   1Q
                          Median
                                        ЗQ
                                                 Max
## -0.196463 -0.024745 -0.005851 0.020245 0.263215
##
```

```
## Coefficients:
##
                  Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.276e-01 1.877e-02 -6.799 1.10e-11 ***
## Cluster.2_lag1 9.571e-01 2.936e-03 326.014 < 2e-16 ***
## temp
                -9.467e-04 1.217e-04 -7.778 7.87e-15 ***
## tod_poly1
## tod_poly2
                3.788e-02 5.538e-04 68.407 < 2e-16 ***
                -3.201e-03 4.786e-05 -66.870 < 2e-16 ***
                1.053e-04 1.510e-06 69.782 < 2e-16 ***
## tod_poly3
## tod_poly4
                -1.148e-06 1.579e-08 -72.691 < 2e-16 ***
## weekend_dummy 1.161e-03 8.849e-04 1.312
                                                0.190
## toy_sin
                 1.321e-02 8.965e-03 1.474
                                                0.141
                 5.188e-03 1.752e-02 0.296
                                                0.767
## toy_cos
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.04535 on 13428 degrees of freedom
## Multiple R-squared: 0.9669, Adjusted R-squared: 0.9669
## F-statistic: 4.359e+04 on 9 and 13428 DF, p-value: < 2.2e-16
```

Linear Regression Model for Cluster 3

```
# Split the data into training and testing sets
train_index_3 <- 1:floor(0.8 * nrow(cluster3_trans))
train_data_3 <- cluster3_trans[train_index_3, ]
test_data_3 <- cluster3_trans[-train_index_3, ]

# Fit the linear regression model
linear_model_3 <- lm(Cluster.3 ~ Cluster.3_lag1 + temp + tod_poly1 + tod_poly2 + tod_poly3 + tod_poly4</pre>
```

```
# Make predictions on the testing set
test_data_3$predictions <- predict(linear_model_3, newdata = test_data_3)

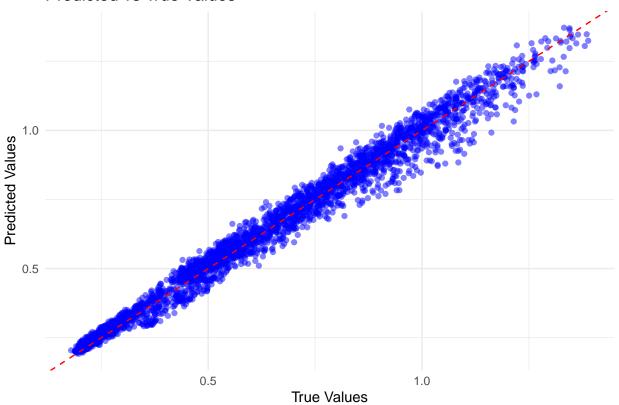
# Calculate prediction error metrics
mae_3 <- mean(abs(test_data_3$Cluster.3 - test_data_3$predictions))
mse_3 <- mean((test_data_3$Cluster.3 - test_data_3$predictions)^2)
rmse_3 <- sqrt(mse_3)

# Print the results
cat("Mean Absolute Error (MAE):", mae_3, "\n")

## Mean Absolute Error (MSE): 0.0290271
cat("Mean Squared Error (MSE): ", mse_3, "\n")

## Mean Squared Error (MSE): 0.001502561
cat("Root Mean Squared Error (RMSE): ", rmse_3, "\n")

## Root Mean Squared Error (RMSE): 0.03876289</pre>
```



```
summary(linear_model_3)
##
## Call:
## lm(formula = Cluster.3 ~ Cluster.3_lag1 + temp + tod_poly1 +
##
      tod_poly2 + tod_poly3 + tod_poly4 + weekend_dummy + toy_sin +
##
       toy_cos, data = train_data_3)
##
## Residuals:
                          Median
                    1Q
                                                 Max
## -0.118353 -0.018677 -0.002823 0.016148 0.198274
##
## Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
##
                  -2.665e-02 1.459e-02 -1.827 0.06779 .
## (Intercept)
```

```
## Cluster.3_lag1 9.349e-01 3.359e-03 278.292 < 2e-16 ***
                -7.840e-04 8.744e-05 -8.966 < 2e-16 ***
## temp
## tod_poly1
                1.486e-02 5.223e-04 28.443 < 2e-16 ***
                -7.698e-04 4.278e-05 -17.994 < 2e-16 ***
## tod_poly2
## tod_poly3
                 2.094e-05 1.248e-06 16.775 < 2e-16 ***
## tod_poly4
             -2.247e-07 1.237e-08 -18.173 < 2e-16 ***
## weekend_dummy 1.569e-03 6.391e-04 2.455 0.01409 *
                 -1.191e-02 6.667e-03 -1.786 0.07406 .
## toy_sin
## toy_cos
                 -3.624e-02 1.299e-02 -2.790 0.00529 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.03262 on 13428 degrees of freedom
## Multiple R-squared: 0.9797, Adjusted R-squared: 0.9797
## F-statistic: 7.193e+04 on 9 and 13428 DF, p-value: < 2.2e-16
```

Linear Regression Model for Cluster 4

```
# Split the data into training and testing sets
train_index_4 <- 1:floor(0.8 * nrow(cluster4_trans))
train_data_4 <- cluster4_trans[train_index_4, ]
test_data_4 <- cluster4_trans[-train_index_4, ]

# Fit the linear regression model
linear_model_4 <- lm(Cluster.4 ~ Cluster.4_lag1 + temp + tod_poly1 + tod_poly2 + tod_poly3 + tod_poly4

# Make predictions on the testing set
test_data_4$predictions <- predict(linear_model_4, newdata = test_data_4)</pre>
```

```
# Calculate prediction error metrics
mae_4 <- mean(abs(test_data_4$Cluster.4 - test_data_4$predictions))
mse_4 <- mean((test_data_4$Cluster.4 - test_data_4$predictions)^2)
rmse_4 <- sqrt(mse_4)

# Print the results
cat("Mean Absolute Error (MAE):", mae_4, "\n")

## Mean Absolute Error (MSE):", mse_4, "\n")

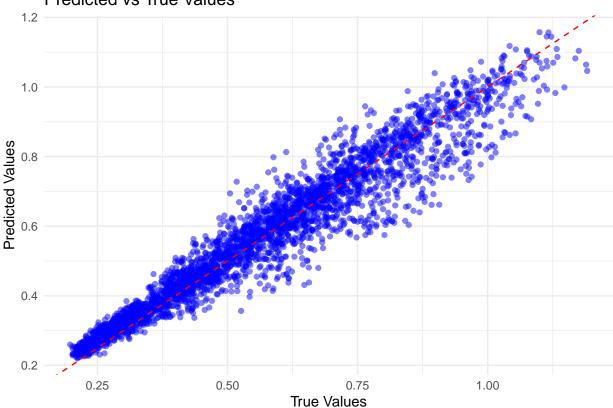
## Mean Squared Error (MSE):", mse_4, "\n")

## Mean Squared Error (MSE): 0.003455547
cat("Root Mean Squared Error (RMSE):", rmse_4, "\n")

## Root Mean Squared Error (RMSE): 0.0587839

# Plot predicted vs true values
ggplot(test_data_4, aes(x = Cluster.4, y = predictions)) +
    geom_point(color = 'blue', alpha = 0.5) +
    geom_abline(intercept = 0, slope = 1, color = 'red', linetype = "dashed") +</pre>
```

```
labs(title = "Predicted vs True Values",
    x = "True Values",
    y = "Predicted Values") +
theme_minimal()
```



```
summary(linear_model_4)
```

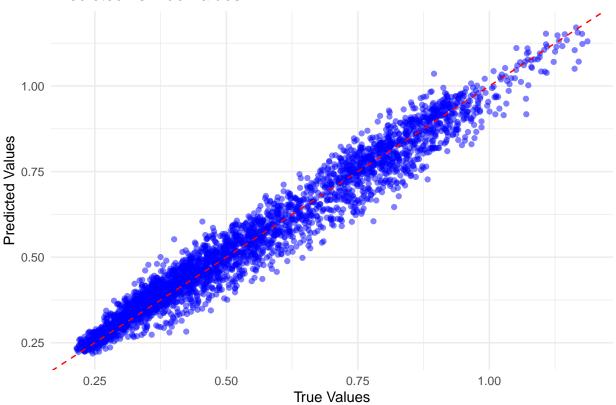
```
##
## Call:
## lm(formula = Cluster.4 ~ Cluster.4_lag1 + temp + tod_poly1 +
      tod_poly2 + tod_poly3 + tod_poly4 + weekend_dummy + toy_sin +
##
##
      toy_cos, data = train_data_4)
##
## Residuals:
##
                   1Q
                         Median
## -0.220407 -0.030273 -0.006641 0.024947 0.279164
##
## Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                  3.487e-02 2.205e-02 1.581 0.11389
## Cluster.4_lag1 8.858e-01 3.853e-03 229.912 < 2e-16 ***
## temp
                 -1.079e-03 1.438e-04 -7.502 6.7e-14 ***
## tod_poly1
                 2.221e-02 6.335e-04 35.058 < 2e-16 ***
                 -1.660e-03 5.618e-05 -29.545 < 2e-16 ***
## tod_poly2
```

Linear Regression Model for Cluster 5

```
# Split the data into training and testing sets
train_index_5 <- 1:floor(0.8 * nrow(cluster5_trans))
train_data_5 <- cluster5_trans[train_index_5, ]
test_data_5 <- cluster5_trans[-train_index_5, ]

# Fit the linear regression model
linear_model_5 <- lm(Cluster.5 ~ Cluster.5_lag1 + temp + tod_poly1 + tod_poly2 + tod_poly3 + tod_poly4</pre>
```

```
# Make predictions on the testing set
test data 5$predictions <- predict(linear model 5, newdata = test data 5)
# Calculate prediction error metrics
mae_5 <- mean(abs(test_data_5$Cluster.5 - test_data_5$predictions))</pre>
mse_5 <- mean((test_data_5$Cluster.5 - test_data_5$predictions)^2)</pre>
rmse 5 <- sqrt(mse 5)
# Print the results
cat("Mean Absolute Error (MAE):", mae_5, "\n")
## Mean Absolute Error (MAE): 0.03355503
cat("Mean Squared Error (MSE):", mse_5, "\n")
## Mean Squared Error (MSE): 0.001909648
cat("Root Mean Squared Error (RMSE):", rmse_5, "\n")
## Root Mean Squared Error (RMSE): 0.04369951
# Plot predicted vs true values
ggplot(test_data_5, aes(x = Cluster.5, y = predictions)) +
  geom_point(color = 'blue', alpha = 0.5) +
  geom_abline(intercept = 0, slope = 1, color = 'red', linetype = "dashed") +
 labs(title = "Predicted vs True Values",
       x = "True Values",
       y = "Predicted Values") +
  theme_minimal()
```



```
summary(linear_model_5)
```

```
##
## Call:
## lm(formula = Cluster.5 ~ Cluster.5_lag1 + temp + tod_poly1 +
      tod_poly2 + tod_poly3 + tod_poly4 + weekend_dummy + toy_sin +
##
##
      toy_cos, data = train_data_5)
##
## Residuals:
                   1Q
##
        Min
                         Median
                                       3Q
## -0.139892 -0.024421 -0.002343  0.020704  0.202596
##
## Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                  8.670e-02 1.768e-02 4.905 9.45e-07 ***
## Cluster.5_lag1 8.866e-01 4.144e-03 213.968 < 2e-16 ***
## temp
                 -1.149e-03 1.040e-04 -11.049 < 2e-16 ***
## tod_poly1
                  4.041e-03 6.185e-04
                                        6.535 6.60e-11 ***
## tod_poly2
                 -9.191e-05 4.936e-05 -1.862
                                                 0.0626 .
                                                 0.2601
## tod_poly3
                  1.671e-06 1.483e-06
                                        1.126
## tod_poly4
                 -1.001e-08 1.514e-08 -0.661
                                                 0.5086
                 3.885e-03 7.573e-04
## weekend_dummy
                                        5.131 2.93e-07 ***
## toy_sin
                 -3.745e-02 7.988e-03 -4.688 2.78e-06 ***
## toy_cos
                 -7.989e-02 1.546e-02 -5.167 2.41e-07 ***
## ---
```

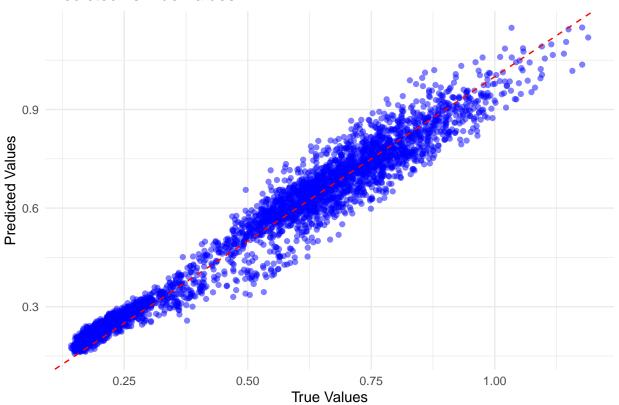
```
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0387 on 13428 degrees of freedom
## Multiple R-squared: 0.9521, Adjusted R-squared: 0.9521
## F-statistic: 2.966e+04 on 9 and 13428 DF, p-value: < 2.2e-16</pre>
```

Linear Regression Model for Cluster 6

```
# Split the data into training and testing sets
train_index_6 <- 1:floor(0.8 * nrow(cluster6_trans))
train_data_6 <- cluster6_trans[train_index_6, ]
test_data_6 <- cluster6_trans[-train_index_6, ]

# Fit the linear regression model
linear_model_6 <- lm(Cluster.6 ~ Cluster.6_lag1 + temp + tod_poly1 + tod_poly2 + tod_poly3 + tod_poly4</pre>
```

```
# Make predictions on the testing set
test_data_6$predictions <- predict(linear_model_6, newdata = test_data_6)</pre>
# Calculate prediction error metrics
mae_6 <- mean(abs(test_data_6$Cluster.6 - test_data_6$predictions))</pre>
mse_6 <- mean((test_data_6$Cluster.6 - test_data_6$predictions)^2)</pre>
rmse 6 <- sqrt(mse 6)
# Print the results
cat("Mean Absolute Error (MAE):", mae_6, "\n")
## Mean Absolute Error (MAE): 0.03879818
cat("Mean Squared Error (MSE):", mse_6, "\n")
## Mean Squared Error (MSE): 0.002503867
cat("Root Mean Squared Error (RMSE):", rmse_6, "\n")
## Root Mean Squared Error (RMSE): 0.05003866
# Plot predicted vs true values
ggplot(test_data_6, aes(x = Cluster.6, y = predictions)) +
  geom_point(color = 'blue', alpha = 0.5) +
  geom abline(intercept = 0, slope = 1, color = 'red', linetype = "dashed") +
  labs(title = "Predicted vs True Values",
       x = "True Values",
       y = "Predicted Values") +
  theme_minimal()
```



```
summary(linear_model_6)
```

```
##
## Call:
## lm(formula = Cluster.6 ~ Cluster.6_lag1 + temp + tod_poly1 +
      tod_poly2 + tod_poly3 + tod_poly4 + weekend_dummy + toy_sin +
##
##
      toy_cos, data = train_data_6)
##
## Residuals:
##
        Min
                   1Q
                         Median
                                       3Q
## -0.213397 -0.027366 -0.004184 0.025137 0.239803
##
## Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                  3.241e-02 2.120e-02
                                        1.529 0.126388
## Cluster.6_lag1 8.936e-01 4.408e-03 202.700 < 2e-16 ***
## temp
                 -1.131e-03 1.308e-04 -8.649 < 2e-16 ***
## tod_poly1
                  1.288e-02
                             7.602e-04 16.943 < 2e-16 ***
## tod_poly2
                 -4.028e-04 7.078e-05 -5.692 1.28e-08 ***
## tod_poly3
                  4.116e-06 2.202e-06
                                        1.870 0.061538 .
## tod_poly4
                 -1.317e-08 2.220e-08 -0.593 0.553046
                 1.425e-03 9.487e-04
## weekend_dummy
                                        1.502 0.133196
## toy_sin
                 -2.796e-02 9.974e-03 -2.803 0.005074 **
## toy cos
                 -6.734e-02 1.937e-02 -3.476 0.000511 ***
## ---
```

```
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.04909 on 13428 degrees of freedom
## Multiple R-squared: 0.9454, Adjusted R-squared: 0.9453
## F-statistic: 2.582e+04 on 9 and 13428 DF, p-value: < 2.2e-16</pre>
```