Problem 1

Calculate and compare the expected value and standard deviation of price at time t (P_t), given each of the 3 types of price returns, assuming $r_t \sim N(0, \sigma^2)$ and the price at t-1 (P_{t-1}). Simulate each return equation using $r_t \sim N(0, \sigma^2)$ and show the mean and standard deviation match your expectations.

Anwser1

Given that price at t-1 is P_{t-1}, and r_t follows normal distribution with mean equals to zero and variance equals to σ^2, we can calculate the mean and variance based on three types of price returns.

1. Classical Brownian Motion

   $P_t = P_{t-1} + r_t$

   Mean = $P_{t-1} + 0 = P_{t-1}$

   Variance = σ^2

   Std = σ

2. Arithmetic Return System

   $P_t = P_{t-1} * (1 + \mathrm{r_t})$

   Mean = $P_{t-1} * (1 + 0) = P_{t-1}$

   Variance = $\sigma^2 * {P_{t-1}}^2$

   Std = $\sigma * P_{t-1}$

3. Log Return or Geometric Brownian Motion

$$P_t = P_{t-1} * e^{rt}$$

$$P_t \; follows \; LN(\mu + \ln(P_{t-1}), \sigma^2)$$

For a Log-normal distribution, the features are as follows:

$$\begin{aligned}
\mathrm{E}[X] &= e^{\mu+\frac{1}{2}\sigma^2}, \\
\mathrm{E}[X^2] &= e^{2\mu+2\sigma^2}, \\
\mathrm{Var}[X] &= \mathrm{E}[X^2] - \mathrm{E}[X]^2 = (\mathrm{E}[X])^2(e^{\sigma^2}-1) = e^{2\mu+\sigma^2}(e^{\sigma^2}-1), \\
\mathrm{SD}[X] &= \sqrt{\mathrm{Var}[X]} = \mathrm{E}[X]\sqrt{e^{\sigma^2}-1} = e^{\mu+\frac{1}{2}\sigma^2}\sqrt{e^{\sigma^2}-1},
\end{aligned}$$

As a result,

Mean = $e^{\mu+\ln(P_{t-1})+1/2\sigma^2} = P_{t-1} * e^{1/2\sigma^2}$

Variance = $e^{\sigma^2} * (e^{\sigma^2}-1) * {P_{t-1}}^2$

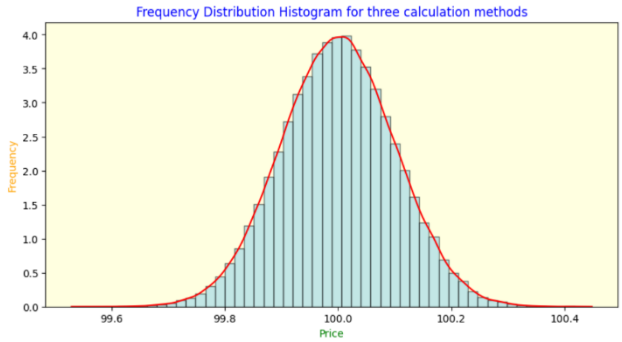
Std = $(e^{\sigma^2} * (e^{\sigma^2}-1))$ ^(1/2) $* P_{t-1}$ = $e^{1/2\sigma^2} * \sqrt{(e^{\sigma^2}-1)} * P_{t-1}$

We can assume that the $P_{t-1} = 100, \; and \; \sigma = 0.10$, so the mean and std for the three calculation ways are as follows (run 100,000 times):

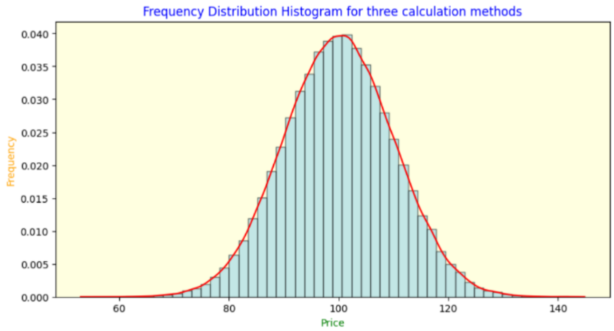
	Classical	Arithmetic	Geometric
Mean Expected	100	100	100.5
Mean Simulated	100.00	100.01	100.51
Std Expected (%)	0.1	10	10.0753
Std Simulated (%)	0.099	9.995	10.0738

As a result, the simulated one match the expected one.

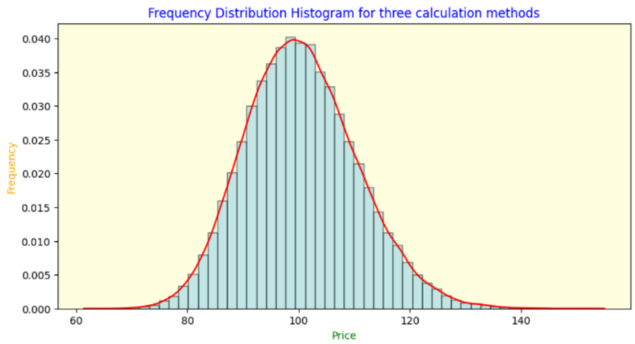
Classical



Arithmetic



Geometric



Problem 2

Implement a function similar to the "return_calculate()" in this week's code. Allow the user to specify the method of return calculation.

Use DailyPrices.csv. Calculate the arithmetic returns for all prices.

You own 1 share of META. Remove the mean from the series so that the mean(META)=0

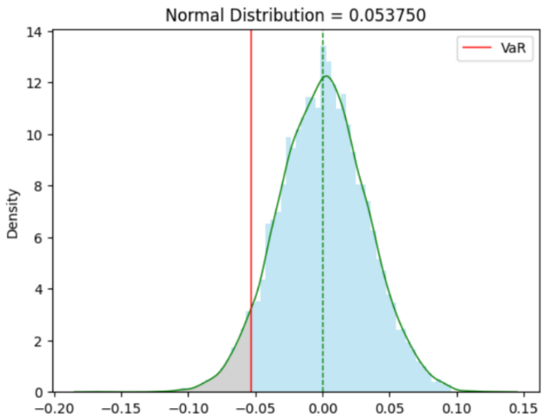
Calculate VaR

1. Using a normal distribution.
2. Using a normal distribution with an Exponentially Weighted variance ($\lambda = 0.94$)
3. Using a MLE fitted T distribution.
4. Using a fitted AR(1) model.
5. Using a Historic Simulation.

Compare the 5 values.

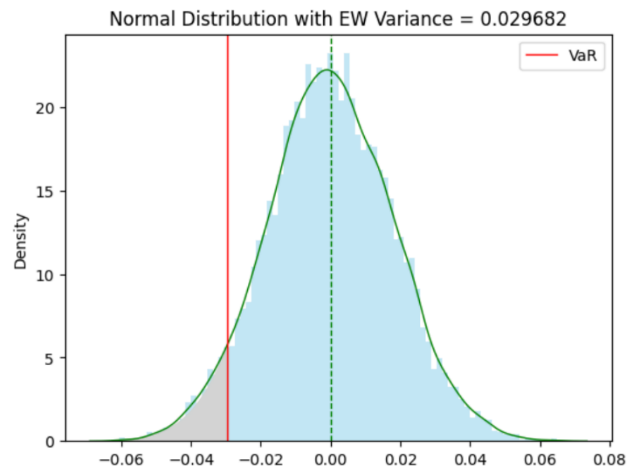
First of all, the way I present VaR in this question is by percentage, which means the percentage of the maximum loss given the α= 5%.

1. Normal distribution



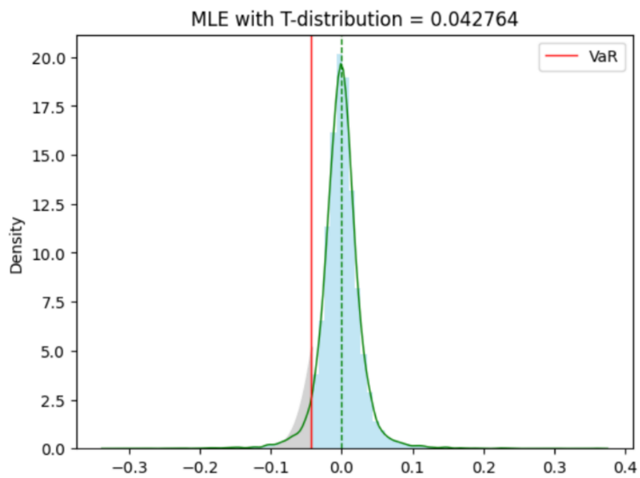
Using normal distribution method is a sample way by calculating the variance of the META returns, and then simulating the data set with normal distribution, in which mean =0, and variance the variance of the META returns. By setting the grey area = 5%, we can get the Var of normal distribution equals to 5.375%.

2. Normal distribution with EW variance



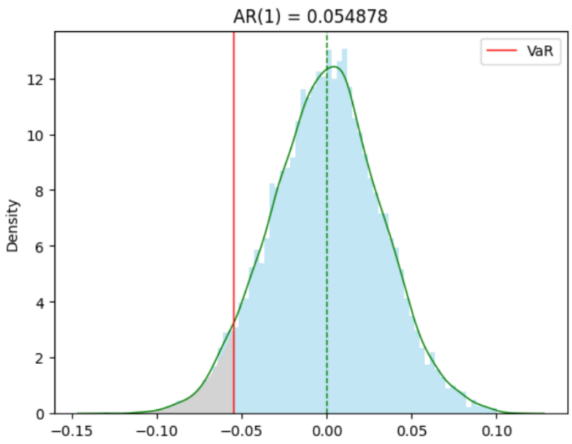
Pretty Similar to the first method, but when calculating the variance of the META returns, we use exponentially weighted method to do that, which will give recent data a lot more weight, and the mean is same = 0. The result can be seen in the picture that Var = 2.9682%.

3. MLE fitted T distribution



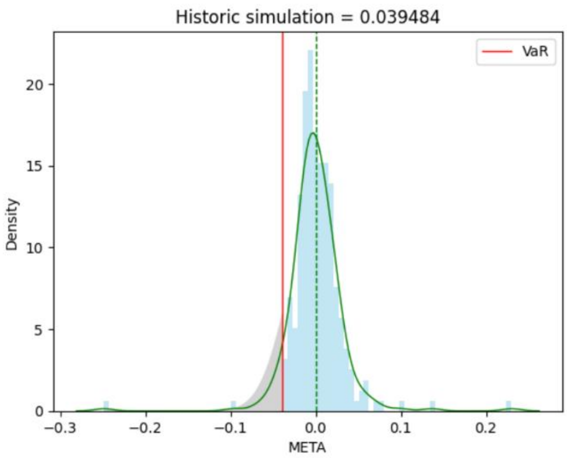
In Maximum Likelihood Estimation (MLE), we set the degrees of freedom, location, and scale parameters, and iterate to minimize the negative log-likelihood function to find the optimal values for these parameters. Then, we can use these optimal parameters to estimate returns and assess the maximum acceptable risk, thereby determining the VAR. As we can see, the Var = 4.2764%.

4. Fitted AR (1)



Firstly, we can fit the given returns data using an AR (1) model. From the model, we obtain the autoregressive coefficient and constant term, along with the standard deviation of the model residuals (sigma). Then, using these parameters, we simulate a large number of potential future return, each simulation predicts the next period's return using the last actual return, the autoregressive coefficient, and random normal distribution noise. Finally, we calculate the VaR at a specified confidence level (alpha), which is the negative value below the corresponding percentile of the simulated returns. This method considers the time series' autocorrelation, making it suitable for assessing the risk of returns or other financial time series data. The Var is 5.4062%.

5. Historic simulation



The historical simulation is basically using the given return value to find the var. We don't make any data preparation to the data, and then making the grey area to 5% to get the Var = 3.9484%.

Method	Var
Normal distribution	5.375%
Normal distribution with EW	2.9682%
MLE with T-distribution	4.2764%
AR (1)	5.4062%
Historical Simulation	3.9484%

The data provided shows a comparison of Value at Risk estimates obtained from different statistical methods. The Normal distribution method yields a higher VaR (5.375%), which may not account for skewness or excess kurtosis present in financial returns. The Exponentially Weighted (EW) Normal distribution method, which assigns more weight to recent data, shows a much lower VaR (2.9682%), suggesting it may be more sensitive to recent volatility. The MLE with T-distribution has a VaR (4.2764%) that is lower than the standard Normal and AR(1) models but higher than the EW method, indicating it accounts for fat tails in the data better. The AR(1) model, which considers autocorrelation in the data, has the highest VaR (5.4062%), possibly reflecting the impact of serial correlation on forecasted risk. The Historical Simulation method, which doesn't assume a specific distribution, presents a moderate VaR (3.9484%), reflecting the actual observed variation in the data. Differences in VaR estimates are due to the various ways each method processes and weighs historical data and statistical properties of the return series.

Problem 3

Using Portfolio.csv and DailyPrices.csv. Assume the expected return on all stocks is 0.

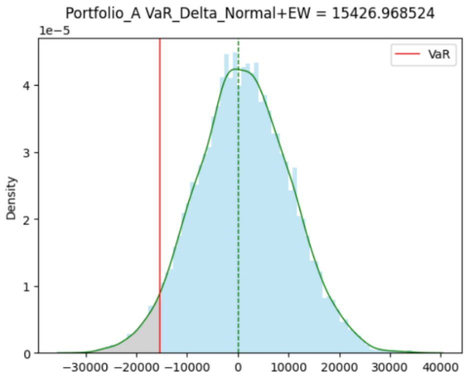
This file contains the stock holdings of 3 portfolios. You own each of these portfolios. Using an exponentially weighted covariance with lambda = 0.94, calculate the VaR of each portfolio as well as your total VaR (VaR of the total holdings). Express VaR as a $.

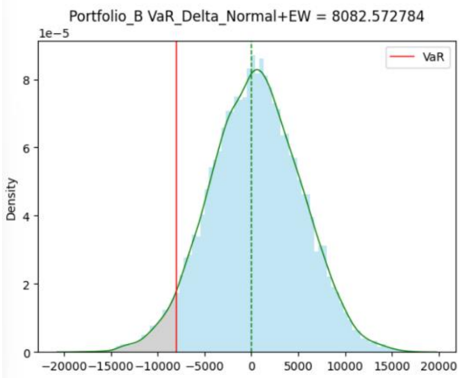
Discuss your methods and your results.

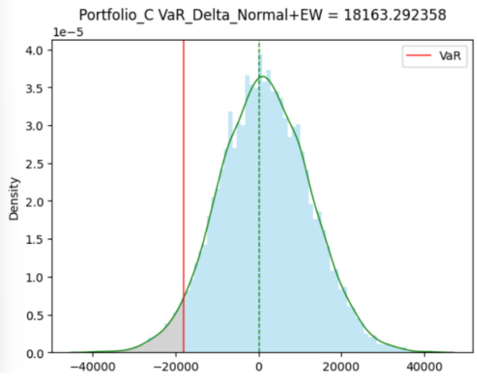
Choose a different model for returns and calculate VaR again. Why did you choose that model? How did the model change affect the results?

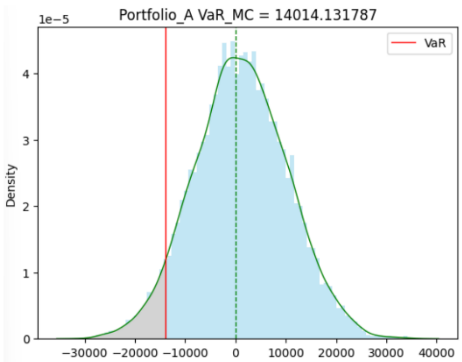
My methods: Similar to the EW part in Question 2, in this question we are going to build normal distribution with EW model to calculate the VaR, but the underlying target is the portfolio instead of the single asset. As a result, we have to calculate the total value of the portfolio based on the number of stocks they include and the price of stocks on each timeline. Then, we can get the returns on each time, and thus calculating the VaR. The result are as follows:

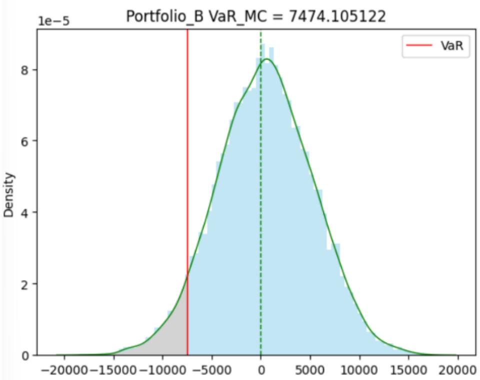
VaR($)	**Normal + EW**	**Monte Carlo**	**Decrease**
A	15426.97	14014.13	9.16%
B	8082.57	7474.11	7.52%
C	18163.29	16285.41	10.34%
Total VaR	38941.38	35642.05	7.31%

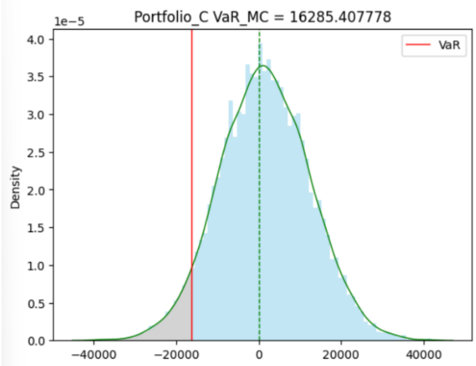

Portfolio_A VaR_Delta_Normal+EW = 15426.968524
1e−5
VaR
Density

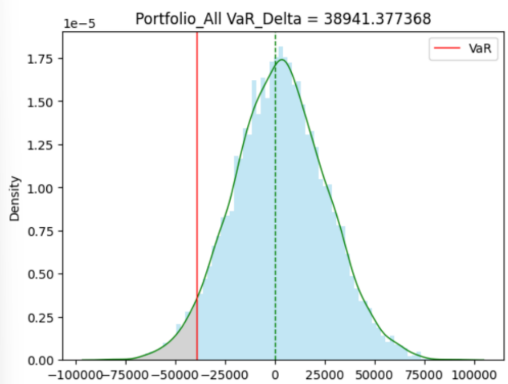

Portfolio_B VaR_Delta_Normal+EW = 8082.572784
1e−5
VaR
Density

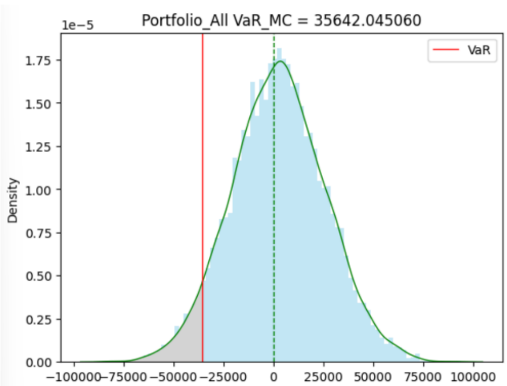

Portfolio_C VaR_Delta_Normal+EW = 18163.292358
1e−5
VaR
Density


Portfolio_A VaR_MC = 14014.131787
1e−5
VaR
Density


Portfolio_B VaR_MC = 7474.105122
1e−5
VaR
Density


Portfolio_C VaR_MC = 16285.407778
1e−5
VaR
Density


Portfolio_All VaR_Delta = 38941.377368
1e−5
VaR
Density


Portfolio_All VaR_MC = 35642.045060
1e−5
VaR
Density

Discuss the result:

Based on the results, Portfolio B has the smallest VaR, significantly smaller than A and C, with A slightly smaller than C. Overall, the portfolio containing all assets has the largest VaR, which is intuitive because of its larger asset volume.

The reason why I choose Monte Carlo and its effectiveness to VaR:

Next, because Monte Carlo simulation is cost-effective and commonly used in risk assessment, especially with large datasets, I employed the Monte Carlo model for VaR calculation once again. From the graph, we can see that for all three portfolios, VaR has decreased. However, in terms of the magnitude of the decrease, we observe significant decreases for Portfolio A and C, while the overall asset decrease is not substantial. Therefore, I believe that as the scale of assets increases, the reduction in accuracy brought about by Monte Carlo simulation diminishes. For Monte Carlo, it can more accurately capture tail risks by simulating a wide range of possible market scenarios, including extreme events that are not well-represented by historical data, and it allows for the modeling of correlations between different market risk factors explicitly and dynamically. There are two reason I believe why the VaR calculated by Monte Carlo is less than the delta with EW method. However, the accuracy of the results depends heavily on the assumptions and models used for the simulation, including the choice of probability distributions for risk factors, which is the potential risk for Monte Carlo we have to know about.