A Context and Preference-aware Neural Network for Auction Design

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Abstract

The central issue of auction design is to devise a mechanism that maximizes expected revenue while ensuring incentive-compatible. In recent years, remarkable advancements have been achieved in the exploration of optimal auction design using deep learning techniques. However, prevailing researches mainly focus on the basic characteristics of bidders and items, overlooking deeper contextual information and latent bidder preferences underlying the bids. To overcome these limitations, we propose a **C**ontext and **P**reference-aware Neural Network for **A**uction **D**esign (CPAD). Specifically, we initially design a context-aware encoder to automatically extract augmented contextual features, and then transform them into a joint representation that captures the interactions between bidders and items using a self-attentive layer. Subsequently, we construct a preference network from the bid information and design a preference-aware encoder to learn bidders' preference. Finally, we integrate the preference information into the processes of allocation and payment computation. Extensive experimental results show that CPAD recovers the known optimal solution in the single-item auction setting and outperforms strong baselines in the multi-item setting. CPAD can find the optimal solution for asymmetry while preserving the permutation equivariance of bids and the context.

Keywords: Auction design, Contextual information, Preference network,

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1. INTRODUCTION

Auctions, as one of the most commonly used market agreements in global trade, play a crucial role in modern economies [1]. Since the mid-1990s, the Federal Communications Commission (FCC) in the United States has been utilizing auctions to regularly allocate broadcasting licenses for electromagnetic spectrum, involving substantial financial transactions [2]. Search engine companies such as Google and Baidu have gained significant revenues by introducing auction mechanisms in the advertising ecosystem [3]. In addition, e-commerce platforms like Amazon and Taobao extensively employ auction mechanisms to serve as intermediaries between bidders and sellers, facilitating transactions. Overall, the contribution of auction mechanisms to the global economy is significant, with estimated annual size ranging from hundreds of billions to trillions of dollars [4, 5].

There are two primary objectives considered in auction mechanisms: maximizing social welfare and maximizing sellers' revenue. The former seeks to maximize the total valuations of the winning bidders from the buyers' perspective [6, 7]. The latter, also referred to as the optimal auction, aims to maximize total revenue from the sellers' perspective [8, 9]. The objective of our auction mechanism design is to maximize sellers' revenue. The central problem in achieving this goal is to design an auction mechanism that satisfies the constraints of Incentive Compatibility (IC) and Individual Rationality (IR), while also generating high expected revenue [10]. Nevertheless, it is imperative to acknowledge that the optimal auctions design is intricate and challenging. Myerson's seminal work [11] in 1981 offered an optimal auction framework for a single-item scenario, but even after forty years, designing revenue-optimal auctions for two bidders and two items is still not fully understood.

In recent years, deep learning has attracted an increasing number of researchers to apply it to auction mechanism design due to its powerful feature of finding global optimal solutions [12–14]. Dütting et al. [15] pioneered an optimal auction mechanism based on deep learning, laying a crucial foundation for research in this field. Subsequently, some researchers further integrated constraints such as fairness, budget, and privacy into the model, and designed various innovative auction mechanisms [16–18]. Although traditional auction mechanisms have achieved performance improvements, they primarily utilize fully connected layers for model design, resulting in limited expressive power and inductive biases. Furthermore, they generally fail to effectively incorporate the contextual information of bidders and items into the mechanism design process. The impact of bidder (item) context and preferences behind bids on auction results is illustrated in Figure 1. In an auction, the item to be auctioned is a post-impressionist oil painting, Sunset, which is highly scarce. Bidder B is a wealthy collector who values the appreciation potential of this painting. Due to his high preference for it, he is willing to pay a high price, even beyond the market valuation. This will significantly increase the probability that Bidder B wins the Sunset.

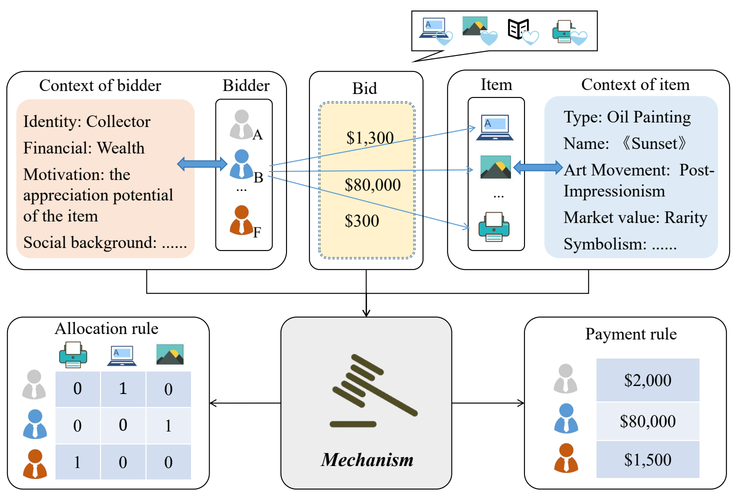

Figure 1: The impact of bidder (item) context and preferences behind bids on auction results.

In order to efficiently explore the potential advantages of incorporating contextual information and bidder preferences, we propose a context and preference-aware neural network for auction design, called CPAD. We first employ contextual enhancement to obtain augmented contextual representations and model the interactions. Secondly, a preference-aware encoder is designed to extract implicit preferences of bidders. Finally, the preference is integrated into the allocation and payment calculations. Importantly, the number of parameters in CPAD remains unaffected by the auction size (i.e., the number of bidders and items). This characteristic endows CPAD with enhanced generalization capabilities, enabling effective handling of auctions across various scales. The main contributions are summarized as follows:

- We propose an innovative method for auction design, which aggregates information from all candidate bidders and items, producing augmented contextual representations for more effective mechanism learning.

- The preference-aware encoder is designed to learn bidders' preferences from the preference network constructed using bid information. The whole mechanism is permutation-equivariant on bids and bidder (item) contexts.

- We conduct extensive experiments, and the results demonstrate that our approach outperforms the baseline in terms of revenue while exhibiting robust generalization capabilities.

The subsequent sections of the paper are structured as follows: In Section 2, we summarize the related work. Then, we provide a problem description for auction design in Section 3. We describe the proposed auction framework in detail in Section 4. In Section 5, we conduct extensive experiments to demonstrate the superiority of the proposed approach. Finally, we conclude the paper in Section 6.

2. RELATED WORK

In this section, we review studies that are pertinent to our work. We summarize the main findings of these studies and clarify the connections between their findings and our research.

2.1. Traditional Auction Mechanisms

Optimal auction design is one of the cornerstones of economic theory. Early auction mechanism design relied heavily on manual work, where designers utilized their experience or intuition to formulate allocation and payment rules to achieve the desired goals. In the 1970s, Vickrey et al. [19] introduced the Vickrey-Clarke-Groves (VCG) mechanism, emphasizing the maximization of social welfare, which ensures that all participants benefit from the total utility of the auction. Auction designers typically concern with social welfare, but in many cases, auctioneers may wish to maximize revenue. In 1981, Myerson [11] pioneered the definition of an optimal strategy-proof auction for selling a single item. However, progress on characterizing strategy-proof, revenue-maximizing auctions outside this setting has been limited. Although some advancements have been made in selling multiple items to a single bidder [20–22], a complete solution has yet to be obtained even for the case of selling two items to two bidders. In more general settings, the complexity of the optimal auction design problem increases dramatically and is difficult to handle with existing theoretical tools. To tackle the intricacies of the optimal auction design problem, researchers have commenced exploring automated methodologies for identifying the most optimal auction mechanism. Conitzer et al. [23] defined the issue as a linear program, but it faced significant scalability challenges due to the exponential increase in the number of constraints with the number of bidders and items [24]. Later, Sandholm et al. [25] developed algorithms for finding optimal auctions that, while scalable, are limited to incentive compatible auctions of specific classes. Unlike the previous work, our proposed model is well scalable and not restricted to specific categories of auctions.

2.2. Deep Learning-based Auction Mechanisms

In recent years, deep learning has attracted the attention of researchers in the field of mechanism design due to its ability to automatically extract hidden features from data [26, 27]. Dütting et al. [15] developed a deep learning-based architecture, RegretNet, to find optimal solutions for multi-item auctions. Different researchers have extended this work from different perspectives, bringing new advances in finding the optimal solution of the auction mechanism. Kuo et al. [16] extended RegretNet by incorporating fairness constraints into the model, effectively addressing the discriminatory nature of resource allocation. Peri et al. [28] were motivated by advertising auctions and encoded socially desirable constraints from data to efficiently capture real human preferences, making the auctions more relevant to real-world application scenarios. Stein et al. [18] extended RegretNet by incorporating privacy-preserving mechanisms that address bidder information leakage while optimizing auction outcomes through neural network approximations. Curry et al. [29] developed a verifiable method for certifying strategyproof auction mechanisms using integer programming. Feng et al. [30] extended RegretNet infrastructure by corporating budget constraints and dealing with Bayesian Incentive Compatible (BIC) and conditional IC constraints to achieve optimal auction design. Rahme et al. [31] proposed EquivariantNet, which utilizes exchangeable matrices to implement symmetric auctions, a special case where both bidders and items are symmetric. Additionally, some researchers have attempted to decompose the context into an auction learning framework to improve the adaptability of the auction mechanism. Golrezaei et al. [32] utilize contextual information to enable dynamic incentive-aware learning for robust pricing strategies. Duan et al. [33] proposed a transformer-based auction model that effectively captures the intricate relationship between bidders and items to achieve optimal auctions. However, methods based on RegretNet [15, 16, 18, 28–30] primarily utilize fully connected layers for model design, which may result in limited expressive capabilities and insufficient inductive bias for auction design. Recent neural auction models [32, 33] that consider context incorporate context merely as auxiliary input features. In contrast, this paper proposes a novel model that learns augmented contextual representations reflecting the overall auction environment and explicitly leverages bidders' bids to capture latent preference information.

3. PRELIMINARIES

In this section, we introduce the auction model and formalize it as a machine learning problem.

3.1. Problem Statement

In a typical auction scenario, let $N = \{1, 2, \ldots, n\}$ denote the set of bidders and $M = \{1, 2, \ldots, m\}$ denote the set of items. The context of bidders and items is denoted by the matrices $CB \in \mathbb{R}^{n \times d_{cb}}$ and $CI \in \mathbb{R}^{m \times d_{ci}}$, respectively. The i-th row cb_i of matrix CB denotes the context feature of the i-th bidder, while the j-th row ci_j of matrix CI denotes the context feature of the j-th item. Both d_{cb} and d_{ci} represent the embedding dimension. Each bidder $i \in N$ has a valuation v_{ij} and a bid b_{ij} for item $j \in M$. We only focus on additive valuations, that is, for a subset $S \subseteq M$, bidder i 's valuation v_{iS} is the sum of the valuations for each item, i.e., $v_{iS} = \sum_{j \in S} v_{ij}$. We denote $v_i = (v_{i1}, v_{i2}, \ldots, v_{im})$ and $b_i = (b_{i1}, b_{i2}, \ldots, b_{im})$ as the valuation and bidding profile of bidder i, respectively. Let $B = (b_1, b_2, \ldots, b_n)$ be the bidding profile, and let B_{-i} be the bidding profile except bidder i.

The auctioneer does not know the valuation profile $V = (v_1, v_2, \ldots, v_n)$, but knows the distribution $D_{V|CB,CI}$. Table 1 summarizes the symbols used in this paper. Given the bidding profile B, the auction mechanism is defined as follows:

Definition 1. An auction mechanism $\mathcal{M}\langle g, p\rangle$ with contexts consists of an allocation rule $g = (g_{ij})_{i \in N, j \in M}$ and a payment rule $p = (p_i)_{i \in N}$.

Table 1: Mathematical symbols

Symbols	Definitions and Descriptions
N	the set of bidders
M	the set of items
v_{ij}	the valuation of item j by bidder i
b_{ij}	the bid of item j by bidder i
cb_i	the context of bidder i
ci_j	the context of item j
a_i	the augmented contextual feature of bidder i
q_j	the augmented contextual feature of item j
$e_i^{(k)}$	the embedding presentation of bidder i in the k-th layer
$e_j^{(k)}$	the embedding presentation of item j in the k-th layer
u_i	the utility of bidder i
W	the weight matrix of the neural network
b	the bias vector of the neural network
σ	the ReLU activation function

Given the bidding profile B, the bidder context CB, and the item context CI, the allocation rule $g : \mathbb{R}^{n\times m} \times \mathbb{R}^{n\times d_{cb}} \times \mathbb{R}^{m\times d_{ci}} \to [0,1]^{n\times m}$ calculates the probability of allocating items to bidders, and the payment rule $p : \mathbb{R}^{n\times m} \times \mathbb{R}^{n\times d_{cb}} \times \mathbb{R}^{m\times d_{ci}} \to \mathbb{R}^n_{\geq 0}$ determines the price that the winner should pay. The aim of each bidder is to optimize their utility, which is defined as follows:

Definition 2. The utility of bidder i under the mechanism $\mathcal{M}\langle g,p\rangle$ is expressed as:

$$u_i\left(v_i, B; CB, CI\right) = \sum_{j=1}^{m} g_{ij}(B, CB, CI)v_{ij} - p_i(B, CB, CI) \tag{1}$$

During the auction, bidders may strategically misreport their valuations to obtain higher revenues, leading to unpredictable outcomes. Therefore, it is crucial for the auction mechanism to satisfy properties, such as IC and IR.

Definition 3. $\mathcal{M}\langle g,p\rangle$ is IC if each bidder can maximize her own utility by truthfully reporting valuation, regardless of the strategies employed by other bidders. Formally,

$$u_i\left(v_i,\left(v_i,B_{-i}\right);CB,CI\right) \geq u_i\left(v_i,\left(b_i,B_{-i}\right);CB,CI\right) \tag{2}$$

Definition 4. $\mathcal{M}\left\langle g,p\right\rangle$ is IR if the utility of bidder $i \in N$ in the auction is non-negative. Formally,

$$u_i\left(v_i,\left(v_i,B_{-i}\right);CB,CI\right) \geqslant 0 \tag{3}$$

The objective of optimal auction design is to maximize the expected revenue for the seller, while ensuring IC and IR constraints. This can be expressed as follows:

$$\begin{gathered}\max \mathbb{E}_{(V,CB,CI)\sim D_{V,CB,CI}} \sum_{i=1}^{n} p_i(V,CB,CI) \\ \text{s.t. } \mathcal{M}\left\langle g,p\right\rangle \text{ satisfy } IC \text{ and } IR\end{gathered} \tag{4}$$

3.2. Optimal Auction as a Learning Problem

We formulate the problem of finding the optimal auction as an optimization problem with constraints using machine learning. To handle the intricate constraints, the auction mechanism is parameterized as $\mathcal{M}\left\langle g^w,p^w\right\rangle$. The goal of the optimal auction is as follows:

$$\begin{gathered}\min_w -\mathbb{E}_{(V,CB,CI)\sim D_{V,CB,CI}}\left[\sum_{i=1}^{n} p_i^w(V,CB,CI)\right] \\ \text{s.t. } \mathcal{M}\left\langle g,p\right\rangle \text{ satisfy } IC \text{ and } IR\end{gathered} \tag{5}$$

where $w \in \mathbb{R}^{d_w}$ represents the machine learning parameters. We use data (including bidding profile, bidder context, and item context) along with stochastic gradient descent to search for parameters that maximize expected revenue.

Given the absence of known characterizations of incentive compatible auction, we resort to employing the more relaxed concept of ex-post regret [15]. The ex-post regret for bidder i is defined as follows:

$$\begin{aligned} rgt_i(w) = E_{(V,CB,CI)\sim D_{V,CB,CI}}\Bigg[&\max_{b_i\in V_i} u_i\left(v_i,(b_i,v_{-i});CB,CI\right) \\ &- u_i\left(v_i,(v_i,v_{-i});CB,CI\right)\Bigg]\end{aligned} \tag{6}$$

Ex-post regret refers to the maximum increase in utility that a bidder could achieve by considering all possible bidding strategies, while fixing the bids of others. It essentially measures the extent to which the auction violates incentive compatibility. The machine learning form of the optimal auction can be further formalized as:

$$\begin{aligned} \min_{w} -\mathbb{E}_{(V,CB,CI)\sim D_{V,CB,CI}} \left[\sum_{i=1}^{n} p_i^w(V,CB,CI)\right] \\ \text{s.t. } rgt_i(w) = 0, \forall i \in N. \end{aligned} \tag{7}$$

Theorem 1. *Let $D_{V|CB,CI}$ denote the conditional distribution of valuation profiles V given the bidder context CB and the item context CI. Then the following are equivalent:*

1) $\mathcal{M}\langle g,p\rangle$ is DSIC.

2) For every bidder $i \in N$, the expected ex-post regret satisfies $rgt_i(w) = 0$.

See Appendix A for detailed proofs.

4. METHODOLOGY

In this section, we first present an overview of the CPAD architecture. Then, we delve into each core component of CPAD. Finally, we discuss the model optimization process.

4.1. Overall of Architecture

The proposed CPAD model, as depicted in Figure 2, comprises three key components: 1) Contextual Enhancement: A context-aware encoder is employed to extract the contextual information of bidders and items, which is attached as an enhancement feature to the representation of each bidder and item. Then, the interactions between bidders and items are modeled through self-attention layers to learn comprehensive contextual representations. 2) Preference-aware encoder: We design a preference-aware encoder to effectively capture the bidder's preference for items. Specifically, we first construct a preference network based on the bidder's behavior. Subsequently, we learn the representations of bidders and items through the message propagation mechanism. Finally, the bidder's preference is obtained using the inner product operation. 3) Calculate allocation and payment: The learned preference is incorporated into the designed output layer, calculating the final allocation result and the corresponding payment.

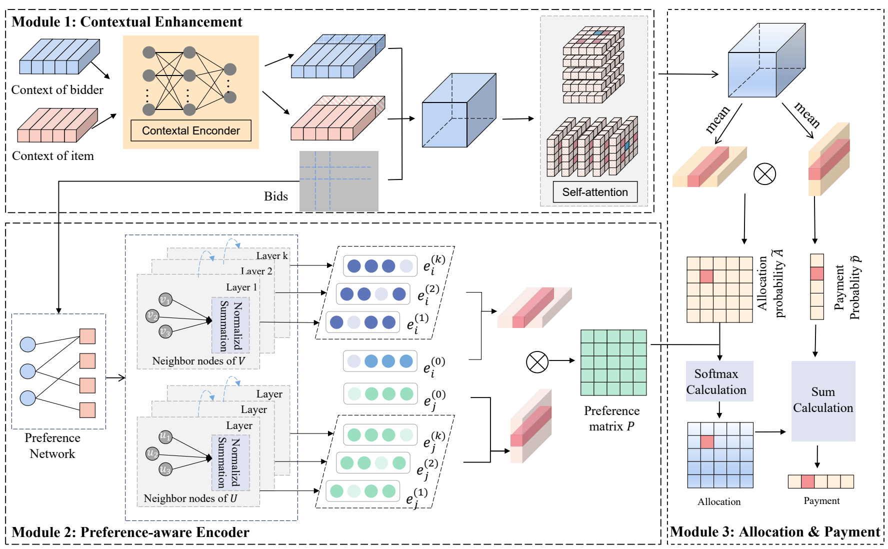

Figure 2: Overall architecture of the CPAD. It contains three components: Contextual Enhancement, Preference-aware Encoder, and Allocation & Payment calculation. The Contextual Enhancement module integrates the deep contextual information, while the Preference-aware Encoder module generates the preference matrix. The Allocation and Payment Calculation module computes allocations and payments by integrating the outputs of the preceding two modules.

4.2. Contextual Enhancement

4.2.1. Context-aware Encoder

To obtain enhanced contextual representations, we design a context-aware encoder that automatically aggregates augmented features from the contexts of all candidate bidders and items, providing richer semantic information for the target bidders or items.

The context-aware encoder takes a set of contextual features from bidders and items as input. It leverages permutation-equivariant operations (shared fully connected network) and symmetric operations (average pooling) to learn and aggregate all contextual features, ensuring that the auction results remain invariant to feature order.

Specifically, we first preprocess the context data to obtain an initial contextual representation $cb_i \in \mathbb{R}^{d_{cb}}$ for each bidder i. Given the contextual features cb_i of the candidate bidder, we utilize shared fully connected network to learn a set of hidden states $h = \{h_i\}_{i=1}^{n}$ for each bidder:

$$h_i = \sigma\left(W cb_i + b\right) \tag{8}$$

where W is the trainable weight matrix, b is a trainable bias vector, and σ is the ReLU activation function.

The set of hidden states is subsequently processed using average pooling:

$$\hat{h}_{-i} = \mathrm{AvgPool}\left(h_{-i}\right) \tag{9}$$

where h_{-i} represents the hidden state of all bidders except i

The final complementary feature h'_i is obtained for each bidder i using another fully connected network:

$$h'_i = \sigma\left(W\hat{h}_{-i} + b\right) \tag{10}$$

Finally, the augmented contextual features for bidder i is obtained by concatenating h'_i with cb_i, yielding $a_i \in \mathbb{R}^{n \times d'_{cb}}$:

$$a_i = [\, cb_i;\, h'_i \,] \tag{11}$$

where $[\,;\,]$ denotes concatenation. Similarly, given the contextual features of the candidate items, the augmented contextual features for item j is obtained as $q_j = [\, ci_j, h'_j \,] \in \mathbb{R}^{m \times d'_{ci}}$.

4.2.2. Self-attention Layer

The self-attention layer converts the augmented contextual features into a unified representation, effectively capturing complex interactions between bidders and items. Specifically, we first construct an initial representation $X = (X_{i,j})_{i \in N, j \in M} \in \mathbb{R}^{n \times m \times \left(1 + d'_{cb} + d'_{ci}\right)}$ of each bidder-item pair, where

$$X_{ij} = [b_{ij}; a_i; q_j] \tag{12}$$

Then, multiple Fully Connected Layers (FCNs) are employed to transform X into a low-dimensional space of dimension d, capturing the internal impact of each bidder-item pair.

$$T = \sigma(WX + b) \in \mathbb{R}^{n \times m \times d} \tag{13}$$

We employ self-attention layers [34] to model the interactions between bidders and items. The higher-order features are acquired through row-wise (column-wise) self-attention layers for each bidder i (item j).

$$\begin{aligned} T_{i,\cdot}^{\text{row}} &= \text{SelfAttention}\left(T_{i,\cdot}\right) \in \mathbb{R}^{m \times d'}, \forall i \in N \\ T_{\cdot,j}^{\text{column}} &= \text{SelfAttention}\left(T_{\cdot,j}\right) \in \mathbb{R}^{n \times d'}, \forall j \in M \end{aligned} \tag{14}$$

where d' represents the dimension of the hidden node. Next, the representations of bidder i and item j are combined to obtain a interaction representation $T_{ij}^{\text{m}} \in \mathbb{R}^{2d'}$:

$$T_{ij}^{\text{m}} := \left[T_{ij}^{\text{row}} ; T_{ij}^{\text{column}}\right] \tag{15}$$

Next, the new unified representation $H \in \mathbb{R}^{n \times m \times d_{out}}$ is obtained by applying FCNs with ReLU activation function on T^{m} :

$$H = \sigma(WT^{\text{m}} + b) \tag{16}$$

By stacking multiple self-attention layers, higher-order interactions between all bidders and items can be effectively modeled.

It is noteworthy that the parameters that need to be optimized in the contextual enhancement include the weights W of the fully connected network, as well as the parameters $W_{\text{query}}^{(h)}, W_{\text{key}}^{(h)}, W_{\text{value}}^{(h)}$ for each head in the Self-Attention. All of these parameters are independent of the number of bidders n and the number of items m.

4.3. Preference-aware Encoder

4.3.1. Constructing Preference Networks

In an auction scenario, bidders' bids contain information about their preferences for items. To effectively utilize this information, we first construct a preference network based on the bidding of bidders.

Specifically, we define the preference intensity $\eta_{i,j}$ of bidder i for item j as follows:

$$\eta_{ij} = \frac{b_{ij}}{\sum_{i=1}^{n} b_{ij}} \tag{17}$$

where b_{ij} represents the bid made by bidder i for item j, and η_{ij} denotes the weight of bidder i's bid in the overall bidding, intuitively reflecting their degree of preference for item j.

Then, we construct a preference graph $G_I^R = (V_I, E_I)$ based on preference intensity relations, where V_I is the set of bidders and items, and E_I is the set of edges connecting bidders and items.

By analyzing the preference network, we can delve into bidders' behavior patterns and effectively capture potential preference structures.

4.3.2. Message Propagation

In order to fully leverage the potential preference information in the bidder's preference network, we design a preference-aware encoder that aims to learn representations with deep preference information from preference network.

To begin with, we generate initial embeddings $e_i^{(0)} = cb_i$ for bidder i and $e_j^{(0)} = ci_j$ for item j. The superscript "(0)" indicates the initial 0-th layer.

We then employ a Graph Convolution Network (GCN) over the bidder–item preference graph to propagate and refine embeddings. Specifically, following the design philosophy of LightGCN by He et al. [35], we adopt a lightweight message-passing layer that aggregates information from 1-hop neighbors, thereby capturing collaborative filtering signals embedded in the graph structure.

$$\begin{aligned} e_i^{(k+1)} &= \sum_{j \in \mathcal{N}_i} \frac{\eta_{ij}}{\sqrt{|\mathcal{N}_i|}\sqrt{|\mathcal{N}_j|}} e_j^{(k)} \\ e_j^{(k+1)} &= \sum_{i \in \mathcal{N}_j} \frac{\eta_{ij}}{\sqrt{|\mathcal{N}_j|}\sqrt{|\mathcal{N}_i|}} e_i^{(k)} \end{aligned} \tag{18}$$

where $\mathcal{N}_i$ and $\mathcal{N}_j$ are the sets of neighbors of bidder i and item j on the preference graph G_I^R respectively, and the symmetric normalization term $\frac{1}{\sqrt{|\mathcal{N}_i||\mathcal{N}_j|}}$ serves as the decay factor.

The representations from different layers highlight messages transmitted through various connections, resulting in differing impacts on user preference. Thus, we concatenate the representations from each layer to form the final embedding for bidders and items.

$$e_i = \sum_{k=0}^{K} \alpha_k e_i^{(k)}; e_j = \sum_{k=0}^{K} \alpha_k e_j^{(k)} \tag{19}$$

where $\alpha_k \geq 0$ represents the contribution of the embedding from the k-th layer to the final representation.

Next, we construct the embedding matrices for all bidders and all items by stacking individual embeddings column-wise:

$$\begin{aligned} E^{\text{bidder}} &= [\, e_1 \ e_2 \ \cdots \ e_n \,] \\ E^{\text{item}} &= [\, e_1 \ e_2 \ \cdots \ e_m \,] \end{aligned} \tag{20}$$

where $E^{\text{bidder}} \in \mathbb{R}^{d \times n}$ and $E^{\text{item}} \in \mathbb{R}^{d \times m}$ denote the embedding matrices of all bidders and all items, respectively.

Finally, we employ the inner product operation to obtain the bidders' preference matrix $P \in \mathbb{R}^{n \times m}$:

$$P = (E^{bidder})^{\top} E^{item} \tag{21}$$

The Preference-aware Encoder is designed to extract implicit preferences of bidders through their bidding behaviors, constructed based on a preference network gathered from the bids. The complete procedure is presented in Algorithm 1 in Appendix C.

4.4. Allocation and Payment

We perform row and column means on the output $H \in \mathbb{R}^{n \times m \times d_{out}}$ from the final self-attention layer to obtain the bidder feature $H^b \in \mathbb{R}^{n \times d_{out}}$ and the item feature $H^i \in \mathbb{R}^{m \times d_{out}}$, respectively.

$$\begin{aligned} H^b &= \frac{1}{m} \sum_{j=1}^{m} H_{:,j,:} \\ H^i &= \frac{1}{n} \sum_{i=1}^{n} H_{i,:,:} \end{aligned} \tag{22}$$

For allocation, we calculate the initial allocation probability $\widetilde{A} \in \mathbb{R}^{n\times m}$ via the inner product.

$$\widetilde{A} = H^b \cdot (H^i)^\top \tag{23}$$

Then, we perform the element-wise product to obtain A:

$$A = \left[\widetilde{A} \odot P; 0^m\right] \tag{24}$$

where $\odot$ represents element-wise multiplication; 0^m represents a virtual bidder (indexed as $n+1$), denoted by the all-zero vector with dimension m, ensuring that the item may not be allocated to any individual. The intuition behind this operation is that the allocation probability depends not only on the bidders' bids but also on their preferences.

To ensure the feasibility of allocation, i.e., that each item has a probability of being allocated to a bidder no greater than 1, we apply the softmax activation function on each column of $A \in \mathbb{R}^{(n+1)\times m}$. The allocation result g^w is given by:

$$g^w_{\cdot j} = \text{Softmax}\left(A_{\cdot j}\right) \tag{25}$$

Here, we have $\sum_{i=1}^{n+1} g^w_{ij} = 1$. For payments, we perform a sigmoid operation on the bidder feature H^b to obtain the payment probability $\tilde{p}^w \in (0,1)^n$, ensuring that the auction satisfies IR.

$$\tilde{p}^w_i = \text{Sigmoid}\left(\frac{1}{d_{out}} \sum_{j=1}^{d_{out}} H^b_{ij}\right) \tag{26}$$

where $\tilde{p}^w_i$ represents the probability that bidder i is required to pay fees to the seller.

Given the allocation g^w_{ij} and the payment $\tilde{p}^w_i$, we can calculate the actual payment that the bidder needs to pay.

$$p^w_i(B, CB, CI) = \tilde{p}^w_i(B, CB, CI) \sum_{j=1}^{m} g^w_{ij}(B, CB, CI) b_{ij} \tag{27}$$

The module 1 comprises a context-aware encoder and a self-attention network, where the former learns augmented contextual features in a permutation-equivalent manner, while the latter acquires new unified representations by aggregating information from all input embeddings, thereby maintaining permutation-equivariance. The module 2 utilizes the inherent disorder of the graph to acquire bidders' preferences, ensuring that the final output remains consistent to input ordering. The combination of these properties ensures that the output of CPAD remains consistent regardless of the order in which input elements are presented, thereby making CPAD permutation-equivariant.

Definition 5. The auction mechanism $\mathcal{M}\langle g, p\rangle$ is said to be permutation-equivariant if for any two permutation matrices $\Pi_n \in \{0,1\}^{n\times n}$ and $\Pi_m \in \{0,1\}^{m\times m}$, the allocation function satisfies

$$g\left(\Pi_n B \Pi_m, \Pi_n CB, \Pi_m^T CI\right) = \Pi_n g(B, CB, CI)\Pi_m\,,$$

and the payment function satisfies

$$p\left(\Pi_n B \Pi_m, \Pi_n CB, \Pi_m^T CI\right) = \Pi_n p(B, CB, CI)\,,$$

when given inputs including arbitrary bids B, bidder context CB and item context CI.

Theorem 2. *The mechanism $\mathcal{M}\langle g, p\rangle$ is IR if, for each bidder i, the allocation network outputs $g_{ij} \in [0,1]$ and the payment network outputs a fractional payment $\tilde{p}_i \in [0,1]$ such that the payment is*

$$p_i(B, CB, CI) = \tilde{p}_i^w(B, CB, CI)\sum_{j=1}^{m} g_{ij}(B, CB, CI) b_{ij},$$

Under these conditions, for all bid profiles B, bidder context CB, and item context CI,

$$u_i\left(v_i, B; CB, CI\right) = \sum_{j=1}^{m} g_{ij}(B, CB, CI) v_{ij} - p_i(B, CB, CI) \geq\ 0$$

so the mechanism is IR.

See Appendix B for detailed proofs.

4.5. Optimization and Training

The objective of our mechanism design is to maximize sellers' revenue. To accomplish this objective, we adhere to the standard machine learning pipeline and set the loss function to maximize empirical revenue. However, the nonlinearity of the constraints tends to lead to non-convexity in the solution space, which complicates the optimization problem with constraints. Therefore, we use the augmented Lagrangian method to transform the constrained optimization problem into an unconstrained problem.

$$\begin{aligned}\mathcal{L}(w, \lambda) = -\frac{1}{|\mathcal{D}|}\sum_{\ell=1}^{|\mathcal{D}|}\sum_{i=1}^{n} p_i^w\left(V^{(\ell)}, CB^{(\ell)}, CI^{(\ell)}\right) + \\ \sum_{i=1}^{n}\lambda_i \widehat{rgt}_i(w) + \frac{\rho}{2}\sum_{i=1}^{n}\left(\widehat{rgt}_i(w)\right)^2\end{aligned} \tag{28}$$

where $\widehat{rgt}_i(w) = \frac{1}{|\mathcal{D}|}\sum_{l \in \mathcal{D}} rgt_i(w)$ is the empirical regret of the i-th bidder, $\mathcal{D}$ is the training dataset, λ is the Lagrangian multiplier, and $\rho > 0$ is a fixed parameter that controls the quadratic penalty.

Throughout the training procedure, the parameters w and Lagrange multiplier λ are iteratively updated to minimize the loss function. The updating process is as follows:

$$\begin{aligned}& w^{t+1} \in \operatorname{argmin}_w \mathcal{L}(w, \lambda) \\ & \lambda_i^{t+1} = \lambda_i^t + \rho\widehat{rgt}_i\left(w^{t+1}\right), \forall i \in N\end{aligned} \tag{29}$$

where t is the number of iterations. In fact, appropriately increasing the parameter ρ can reduce the number of iterations and accelerate convergence to the optimal solution. However, if parameter ρ is too large, the optimal result may not be achieved. The value of ρ is typically set to a smaller value in order to ensure its convergence within a specified number of iterations. For a detailed explanation of the optimization and training process, see Appendix C.

5. Time Complexity Analysis

In this section, we analyze the time complexity of the CPAD. The complexity analysis focuses on the primary components of the algorithm, including the contextual enhancement, preference-aware encoder, and allocation and payment calculation.

First, the contextual enhancement component operates on a set of contextual features for each bidder and item. Assuming there are n bidders and m items, feature extraction involves a single forward pass through a neural network with $O(d)$ parameters, where d is the dimensionality of the contextual features. Additionally, the encoder utilizes self-attention mechanisms to model interactions, which introduces a complexity of $O(n^2 \cdot d)$ for calculating attention weights and updating feature representations. Thus, the total time complexity for this component can be expressed as $O(nm \cdot d + n^2 \cdot d)$.

Next, the preference-aware encoder processes bidding information to learn bidders' preferences. The message propagation over the preference graph runs for K layers, aggregating information from neighbors. For K iterations, the complexity of this component is $O(K \cdot (n + m))$, as each node must update its preference representation based on its neighbors.

Subsequently, the allocation and payment Calculation component computes the allocation probabilities for each bidder-item pair. The complexity is proportional to the number of bidders and items, yielding $O(nm)$ for allocation and $O(n)$ for payment calculation. Combining these complexities, the overall time complexity for the CPAD model can be summarized as $O(nm \cdot (d + K) + n^2 \cdot d + nm)$.

This indicates that the computational cost scales proportionally with the number of bidders and items while being influenced by the dimensionality of the features, the number of iterations for the preference-aware encoder, and the interactions modeled by self-attention layers.

6. EXPERIMENTS

In this section, we conduct a series of experiments including comparative, generalization and ablation experiments to verify the effectiveness of CPAD. The experiments are conducted on a server equipped with an NVIDIA RTX 3090 GPU, an AMD Ryzen 9 5900X CPU, and 64GB of RAM. The operating system is Ubuntu 20.04. The experiments are implemented using the PyTorch 2.0.1 framework, with GPU acceleration provided by CUDA 12.1 and cuDNN 8.7. All experiments are performed and tested under the aforementioned software and hardware environment.

6.1. Baselines

- **Myerson** [11]: The method incentivizes participants to truthfully report their valuations for items, pioneering a solution to the single-item optimal auction problem.

- **RegretNet** [15]: It utilizes fully-connected neural networks to develop a generalized end-to-end model for addressing multi-item auction problem.
- **Item-Myerson**: A robust baseline utilized by Dutting et al. [15], where Myerson auction is applied individually for each item.
- **EquivariantNet** [31]: This is an auction mechanism based on permutation equivariance for the special case where bidders are anonymous and items are symmetric.
- **CITransNet** [33]: This is a context-based auction model that utilizes transformers to effectively model bidder-item interactions.
- **CIRegretNet**: A powerful baseline used in CITransNet, which utilizes RegretNet as the interaction layer in CITransNet to construct optimal auctions.
- **CIEquivariantNet**: A powerful baseline used in CITransNet, which utilizes EquivariantNet as the interaction layer in CITransNet to construct optimal auctions.

6.2. Evaluation Metrics

We employ two metrics to evaluate the performance of each method: 1) Average of all bidders' empirical revenue $rev = \frac{1}{|S|}\sum_{\ell=1}^{|S|}\sum_{i=1}^{n} p_i$; 2) Average of all bidders' empirical ex-post regrets $rgt = \frac{1}{n}\sum_{i=1}^{n}\widehat{rgt}_i$. Each bidder's empirical regret is determined through 1000 iterations of gradient ascent on their bids. The gradient ascent is executed 200 times using various initial bids b_i', and we record the largest regret.

6.3. Comparison Results

6.3.1. Single-item Auctions

The purpose of our experiments in the single-item auction is to demonstrate that CPAD can successfully recover near-optimal solutions, as defined by Myerson. The specific settings are as follows:

(I) There 3 bidders with discrete contexts $CB = \{1, 2, \ldots, 5\}$ and 1 item with discrete context $CI = \{1\}$. The bidder i's valuation $v_i \sim \mathcal{N}\left(\frac{cb_i}{6}, 0.1\right)$ for item.

(II) There are 3 bidders with discrete contexts $CB = \{1, 2, \ldots, 5\}$ and 1 item with discrete context $CI = \{1, 2\}$. When $ci_1 = 1$, bidder i 's valuation $v_i \sim \mathcal{N}\left(\frac{cb_i}{6}, 0.1\right)$ for item, and bidder i 's valuation $v_i \sim e\left(\frac{i}{6}\right)$ for item when $ci_1 = 2$.

(III) There are 5 bidders with continuous contexts $CB = [-1, 1]^{10}$ and 1 item with continuous context $CI = [-1, 1]^{10}$. The bidder i's valuation $v_i \sim U\left[0, \mathrm{Sigmoid}\left(cb_i^T ci_j\right)\right]$ for item.

Table 2: Experimental results of our proposed CPAD compared to other baselines in single-item settings.

Dist.	Myerson		RegretNet		EquivariantNet		CITransNet		CPAD	
	rev	*rgt*	*rev*	*rgt*	*rev*	*rgt*	*rev*	*rgt*	*rev*	*rgt*
3×1 (I)	0.594	-	0.517	< 0.001	0.498	< 0.002	0.592	< 0.001	**0.594**	< 0.001
3×1 (II)	0.456	-	0.412	< 0.001	0.403	< 0.001	0.454	< 0.001	**0.455**	< 0.001
5×1 (III)	0.366	-	0.329	< 0.001	0.311	< 0.001	0.364	< 0.001	**0.365**	< 0.001

The results of comparing CPAD with other baselines in a single-item scenario are presented in Table 2. The EquivariantNet performs the worst among all methods, which indicates that designing asymmetric solutions has a positive impact on improving revenues. Compared to RegretNet, CITransNet and CPAD are able to achieve higher seller revenues while maintaining lower regret, reflecting the significance of incorporating contextual information into the model architecture. Our proposed CPAD consistently outperforms other baselines and is able to reach the Myerson optimal solution. The reason behind this phenomenon can be attributed to the contextual enhancement module effectively acquiring comprehensive information regarding contextual induction bias.

6.3.2. Multi-item Auctions

To further demonstrate the potential of our proposed CPAD in a multi-item auction scenario, we conduct experiments to compare CPAD with other baseline models. The considered multi-item settings are as follows:

(IV) There are 2 bidders with discrete contexts $CB = \{1, 2, \ldots, 10\}$ and 5 items with discrete contexts $CI = \{1, 2, \ldots, 10\}$. Both the context is uniformly sampled. The bidder i's valuation $v_i \sim \mathcal{N}\left(\frac{(x_i+y_j) \bmod 10+1}{11}, 0.05\right)$ for item.

(V) There are 3 bidders and 10 items. Both the context and valuation are drawn as setting IV.

(VI) There are 5 bidders and 10 items. Both the context and valuation are drawn as setting IV.

(VII) There are 2 bidders with continuous contexts $CB = [-1,1]^{10}$ and 5 items with continuous context $CI = [-1,1]^{10}$. Both the context is uniformly sampled. The bidder i's valuation $v_i \sim U\left[0, \mathrm{Sigmoid}\left(cb_i^T ci_j\right)\right]$.

(VIII) There are 3 bidders and 10 items. Both the context and valuation are drawn as setting VII.

(IX) There are 5 bidders and 10 items. Both the context and valuation are drawn as setting VII.

Table 3: Experimental results of our proposed CPAD compared to other baselines in multi-item settings.

Dist.	Item-Myerson		CIRegretNet		CIEquivariantNet		CITransNet		CPAD	
	rev	*rgt*	*rev*	*rgt*	*rev*	*rgt*	*rev*	*rgt*	*rev*	*rgt*
2×5 (IV)	2.821	-	2.803	< 0.001	2.841	< 0.002	2.916	< 0.001	**3.011**	< 0.001
3×10 (V)	6.509	-	5.846	< 0.001	6.703	< 0.001	6.872	< 0.001	**7.039**	< 0.001
5×10 (VI)	7.376	-	6.339	< 0.015	7.602	< 0.001	7.778	< 0.001	**7.973**	< 0.001
2×5 (VII)	1.071	-	1.104	< 0.001	1.147	< 0.002	1.177	< 0.001	**1.227**	< 0.001
3×10 (VIII)	2.793	-	2.424	< 0.003	2.872	< 0.002	2.918	< 0.001	**3.016**	< 0.001
5×10 (IX)	3.684	-	2.999	< 0.001	3.806	< 0.002	3.899	< 0.001	**4.023**	< 0.001

The results of comparing our proposed CPAD with other baselines in the multi-item settings are presented in Table 3. It is evident that with the increasing scale, models with permutation-equivariance (such as CIEquivariantNet, CITransNet, and CPAD) exhibit significantly superior performance compared to CIRegretNet, which lacks this feature. This observation suggests that incorporating permutation-equivariance into the model enhances the effective utilization of the inherent data structure. In all setting environments, the CPAD consistently achieves optimal revenue while maintaining low regret values. This is attributed to the fact that CPAD fully exploits contextual information and bidder preferences, enabling it to make more realistic decisions.

In addition, we compared the running time of our CPAD with the CITransNet method. We conducted experiments based on the setting V. The results are shown in Table 4. We find that due to the exponential growth of the problem, the running time of our proposed method surpassed the baseline, reaching 54 hours, even under a modest setting.

Table 4: Revenue, regret, and run-time for CPAD and CITransformer under setting V.

Setting	Method	rev	rgt	Run-time
V	CITransNet	2.914	< 0.002	27 hrs
	CPAD	3.024	< 0.003	54 hrs

6.3.3. Generalization Experiments

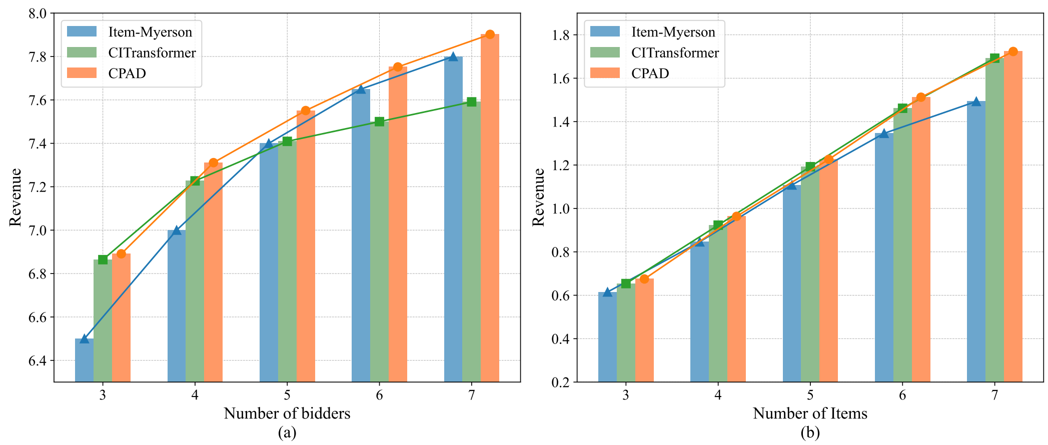

Figure 3: Results of generalization experiments. (a) Choose the setting IV (3×10) for training each model and evaluate their performance across different numbers of bidders; (b) Choose the setting V (2×5) for training each model and evaluate their performance across different numbers of items.

To demonstrate the generalization capability of CPAD, we train it alongside two baselines, Item-Myerson and CITransNet, and evaluate their performance in diverse scenarios beyond the experimental settings. The evaluation experiment is feasible because the size of CPAD parameters is independent of the number of bidders and items. Figure 3a illustrates the performance of different models with a fixed number of items. We train each model in setting V and evaluate their performance in settings with varying numbers of bidders, while keeping the number of items fixed at 10. The results show an increasing trend in revenues across all models as the number of bidders increases, with CPAD consistently outperforming baselines. Figure 3b illustrates the performance of different models with a fixed number of bidders. We train each models in setting VII and generalize it to settings with different number of items. We find that CPAD consistently outperforms baselines while retaining low regret. Our proposed CPAD exhibits excellent generalization capability.

6.3.4. Ablation Analysis

To gain a deeper insight into the proposed CPAD model, we conduct an ablation study. We compare the complete CPAD with three variant versions, which are defined as follows:

- CPAD-d: We remove the contextual enhancement from CPAD, i.e., the model only considers bids as inputs and does not take into account the impact of context on the auction.

- CPAD-p: We remove the preference-aware encoder, i.e., the model does not construct a preference network from bids, and allocations no longer take preference effects into account.

- CPAD-a: We remove both the contextual enhancement module and the preference-aware encoder from CPAD.

Figure 4 illustrates the comparison results of CPAD and its variants in different settings. We can clearly observe that the removal of contextual enhancement leads to a significant decrease in performance, possibly because contextual information provides additional insights for bidders to formulate more reasonable strategies. Furthermore, although the preference-aware encoder is beneficial to the auction mechanism, its impact remains relatively modest. The reason for this result may be due to the fact that, after undergoing contextual learning on a substantial number of samples, the model is able to capture shallow features of bidder preferences. The performance of CPAD is outstanding, indicating that contextual enhancement and bidder preferences play a critical role in optimizing auction design as well as improving revenues.

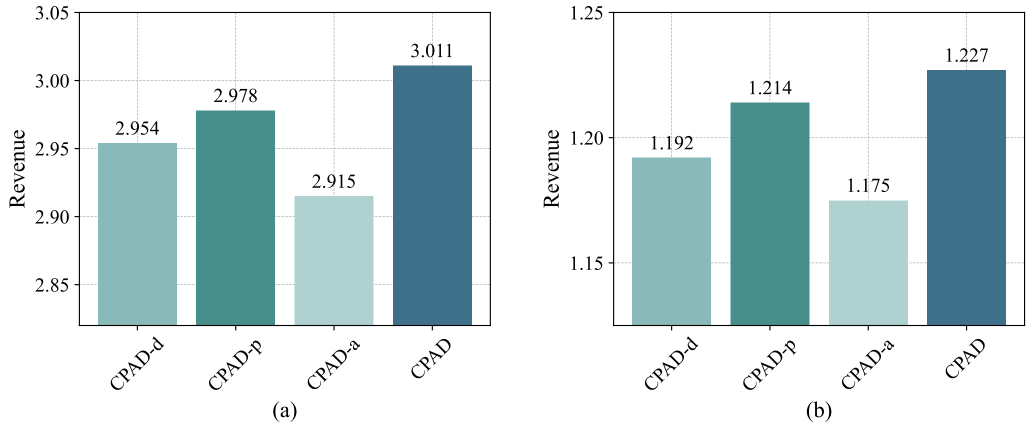

Figure 4: Comparative results of CPAD and its variants in different settings. (a) Comparing the revenues of CPAD with the three variants in setting IV (2×5). (b) Comparing the revenues of CPAD with the three variants in setting VII (2×5).

6.3.5. Simulated Realistic Scenario

To approximate a realistic auction environment, we conduct experiments in a 5×1 setting in which five candidate bidders compete for one item. The experiments cover both continuous and discrete contexts, with bid distributions consistent with Setting IV (discrete) and Setting VII (continuous). The complete configurations for the bidder contexts and the item contexts are provided in Appendix D. Table 5 lists the Top-3 bidders in descending order of allocation probability. The ranking indicates that bidder 2 achieves the highest allocation probability, followed by bidder 4 and bidder 1. This result can be attributed to the attention mechanism effectively capturing bidder-item interaction patterns. In addition, since bids are sampled from distributions conditioned on both bidder and item contexts, the preference-aware encoder can accurately model preference information. We further compare CPAD with the classic optimal auction mechanism Myerson, and the experimental results show that the revenues achieved by CPAD approach the theoretical upper bound established by Myerson.

7. CONCLUSION

In this paper, we innovatively introduce CPAD, a context and preference-aware neural network for auction design. Our proposed model is permutation-equivariant on bids and contexts, a property that enhances the generalization

Table 5: Example top-3 allocation in a 5×1 auction: bidders ranked by allocation probability (highest to lowest), with brief rationales.

Rank	Bidder	Allocation Prob.	Brief Rationale
1	Bidder 2	0.45	Strong preference for certified, high-integrity items aligns directly with the item's characteristics.
2	Bidder 4	0.31	Agile and opportunity-seeking behavior matches the item's short-term growth potential.
3	Bidder 1	0.13	Reliable and risk-moderate traits fit partially but are less responsive to near-term opportunities.

capability of the mechanism. Through extensive experiments conducted in both single-item and multi-item auction scenarios, we have verified that our method recovers known optimal solutions and exhibits significant advantages over existing methods such as CITransNet. Furthermore, the ablation study provides valuable insights into each module's contribution to the CPAD. In the future, we will further investigate methods for incorporating more intricate social relationships and additional factors into the auction process, aiming to devise auction mechanisms that are more intelligent and adaptable.

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References

[1] R. Cassady, Auctions and auctioneering, University of California Press, 2021.

[2] K. Leyton-Brown, P. Milgrom, I. Segal, Economics and computer science of a radio spectrum reallocation, Proceedings of the National Academy of Sciences 114 (28) (2017) 7202–7209.

[3] B. Aslam, H. Karjaluoto, Digital advertising around paid spaces, e-advertising industry's revenue engine: A review and research agenda, Telematics and Informatics 34 (8) (2017) 1650–1662.

[4] Alphabet, Alphabet announces first quarter 2024 results, https://abc.xyz/assets/91/b3/3f9213d14ce3ae27e1038e01a0e0/2024q1-alphabet-earnings-release-pdf.pdf (2024).

[5] Baidu, Baidu announces first quarter 2024 results, https://ir.baidu.com/news-releases/news-release-details/baidu-announces-first-quarter-2024-results (2024).

[6] Y. Jiao, P. Wang, D. Niyato, B. Lin, D. I. Kim, Toward an automated auction framework for wireless federated learning services market, IEEE Transactions on Mobile Computing 20 (10) (2020) 3034–3048.

[7] W. Zhong, K. Xie, Y. Liu, C. Yang, S. Xie, Multi-resource allocation of shared energy storage: A distributed combinatorial auction approach, IEEE transactions on smart grid 11 (5) (2020) 4105–4115.

[8] X. Chen, Y. Deng, G. Zhu, D. Wang, Y. Fang, From resource auction to service auction: An auction paradigm shift in wireless networks, IEEE Wireless Communications 29 (2) (2022) 185–191.

[9] D. Pieńkowski, Selfishness, cooperation, the evolutionary point of view and its implications for economics, Ecological Economics 69 (2) (2009) 335–344.

[10] A. C.-C. Yao, Dominant-strategy versus bayesian multi-item auctions: Maximum revenue determination and comparison, in: Proceedings of the 2017 ACM Conference on Economics and Computation, 2017, pp. 3–20.

[11] R. B. Myerson, Optimal auction design, Mathematics of operations research 6 (1) (1981) 58–73.

[12] Z. Zhang, A survey of online auction mechanism design using deep learning approaches, arXiv preprint arXiv:2110.06880 (2021).

[13] L. Liang, H. Ye, G. Yu, G. Y. Li, Deep-learning-based wireless resource allocation with application to vehicular networks, Proceedings of the IEEE 108 (2) (2019) 341–356.

[14] C. Wang, D. Deng, L. Xu, W. Wang, Resource scheduling based on deep reinforcement learning in uav assisted emergency communication networks, IEEE Transactions on Communications 70 (6) (2022) 3834–3848.

[15] P. Dütting, Z. Feng, H. Narasimhan, D. Parkes, S. S. Ravindranath, Optimal auctions through deep learning, in: International Conference on Machine Learning, PMLR, 2019, pp. 1706–1715.

[16] K. Kuo, A. Ostuni, E. Horishny, M. J. Curry, S. Dooley, P.-y. Chiang, T. Goldstein, J. P. Dickerson, Proportionnet: Balancing fairness and revenue for auction design with deep learning, arXiv preprint arXiv:2010.06398 (2020).

[17] K. Zhu, Y. Xu, Q. Jun, D. Niyato, Revenue-optimal auction for resource allocation in wireless virtualization: A deep learning approach, IEEE Transactions on Mobile Computing 21 (4) (2020) 1374–1387.

[18] A. Stein, A. Schwarzschild, M. Curry, T. Goldstein, J. Dickerson, Neural auctions compromise bidder information, arXiv preprint arXiv:2303.00116 (2023).

[19] W. Vickrey, Counterspeculation, auctions, and competitive sealed tenders, The Journal of finance 16 (1) (1961) 8–37.

[20] A. M. Manelli, D. R. Vincent, Bundling as an optimal selling mechanism for a multiple-good monopolist, Journal of Economic Theory 127 (1) (2006) 1–35.

[21] C. Daskalakis, A. Deckelbaum, C. Tzamos, Strong duality for a multiple-good monopolist, in: Proceedings of the Sixteenth ACM Conference on Economics and Computation, 2015, pp. 449–450.

[22] G. Pavlov, Optimal mechanism for selling two goods, The BE Journal of Theoretical Economics 11 (1) (2011) 0000102202193517041664.

[23] V. Conitzer, T. Sandholm, Self-interested automated mechanism design and implications for optimal combinatorial auctions, in: Proceedings of the 5th ACM Conference on Electronic Commerce, 2004, pp. 132–141.

[24] M. Guo, V. Conitzer, Computationally feasible automated mechanism design: General approach and case studies, in: Proceedings of the AAAI Conference on Artificial Intelligence, Vol. 24, 2010, pp. 1676–1679.

[25] T. Sandholm, A. Likhodedov, Automated design of revenue-maximizing combinatorial auctions, Operations Research 63 (5) (2015) 1000–1025.

[26] N. Golowich, H. Narasimhan, D. C. Parkes, Deep learning for multi-facility location mechanism design., in: IJCAI, 2018, pp. 261–267.

[27] Y. Zhan, J. Zhang, An incentive mechanism design for efficient edge learning by deep reinforcement learning approach, in: IEEE INFOCOM 2020-IEEE conference on computer communications, IEEE, 2020, pp. 2489–2498.

[28] N. Peri, M. Curry, S. Dooley, J. Dickerson, Preferencenet: Encoding human preferences in auction design with deep learning, Advances in Neural Information Processing Systems 34 (2021) 17532–17542.

[29] M. Curry, P.-Y. Chiang, T. Goldstein, J. Dickerson, Certifying strategyproof auction networks, Advances in Neural Information Processing Systems 33 (2020) 4987–4998.

[30] Z. Feng, H. Narasimhan, D. C. Parkes, Deep learning for revenue-optimal auctions with budgets, in: Proceedings of the 17th international conference on autonomous agents and multiagent systems, 2018, pp. 354–362.

[31] J. Rahme, S. Jelassi, J. Bruna, S. M. Weinberg, A permutation-equivariant neural network architecture for auction design, in: Proceedings of the AAAI conference on artificial intelligence, Vol. 35, 2021, pp. 5664–5672.

[32] N. Golrezaei, A. Javanmard, V. Mirrokni, Dynamic incentive-aware learning: Robust pricing in contextual auctions, Advances in Neural Information Processing Systems 32 (2019).

[33] Z. Duan, J. Tang, Y. Yin, Z. Feng, X. Yan, M. Zaheer, X. Deng, A context-integrated transformer-based neural network for auction design, in: International Conference on Machine Learning, PMLR, 2022, pp. 5609–5626.

[34] A. Vaswani, Attention is all you need, Advances in Neural Information Processing Systems (2017).

[35] X. He, K. Deng, X. Wang, Y. Li, Y. Zhang, M. Wang, Lightgcn: Simplifying and powering graph convolution network for recommendation, in: Proceedings of the 43rd International ACM SIGIR conference on research and development in Information Retrieval, 2020, pp. 639–648.

Appendix A. Proof of Theorem 1

Proof. The proof of Theorem 1 proceeds in two steps:

$(1 \Rightarrow 2)$ Suppose the mechanism is DSIC. Fix any bidder i, any (v_i, v_{-i}), and any misreport $v_i' \in V_i$. By DSIC,

$$u_i\big(v_i, (v_i, v_{-i}); CB, CI\big) \;\geq\; u_i\big(v_i, (v_i', v_{-i}); CB, CI\big).$$

Hence, for this (v_i, v_{-i}),

$$\max_{v_i' \in V_i} \big\{ u_i\big(v_i, (v_i', v_{-i}); CB, CI\big) - u_i\big(v_i, (v_i, v_{-i}); CB, CI\big) \big\} \leq 0.$$

Since the left-hand side is a maximum of deviations, it is always ≥ 0. Therefore it must equal 0 for every (v_i, v_{-i}), and taking expectations yields $\mathrm{rgt}_i(w) = 0$.

$(2 \Rightarrow 1)$ Suppose $\mathrm{rgt}_i(w) = 0$ for every i. Fix any i and define, for $(v_i, v_{-i}) \in V$,

$$\Delta_i(v_i, v_{-i}) \;:=\; \max_{v_i' \in V_i} \Big(u_i\big(v_i, (v_i', v_{-i}); CB, CI\big) - u_i\big(v_i, (v_i, v_{-i}); CB, CI\big) \Big).$$

Then $\Delta_i(v_i, v_{-i}) \geq 0$ for all (v_i, v_{-i}) and

$$\mathbb{E}_{v \sim D_{V|CB,CI}} \big[\Delta_i(v_i, v_{-i}) \big] \;=\; \mathrm{rgt}_i(w) \;=\; 0.$$

A nonnegative random variable has zero expectation if and only if it equals zero almost surely. Thus $\Delta_i(v_i, v_{-i}) = 0$ for F-almost every (v_i, v_{-i}).

Because F has full support on V, the set on which $\Delta_i > 0$ cannot contain any open set; hence by continuity assumptions, we obtain that in fact $\Delta_i(v_i, v_{-i}) = 0$ for all $(v_i, v_{-i}) \in V$. Consequently, for any (v_i, v_{-i}) and any $v_i' \in V_i$,

$$u_i\big(v_i, (v_i, v_{-i}); CB, CI\big) \;\geq\; u_i\big(v_i, (v_i', v_{-i}); CB, CI\big),$$

which is exactly the DSIC condition. Therefore the mechanism is DSIC.

Appendix B. Proof of Theorem 2

Proof. Start from the definition of utility:

$$u_i\left(v_i, B; CB, CI\right) = \sum_{j=1}^{m} g_{ij}(B, CB, CI) v_{ij} - p_i(B, CB, CI).$$

By the architectural definitions,

$$p_i(B, CB, CI) = \tilde{p}_i(B, CB, CI) \sum_{j=1}^{m} g_{ij}(B, CB, CI) b_{ij}.$$

According to the valuation upper bound property, we have:

$$v_i \ \geq \ b_i$$

Substituting into the utility expression gives

$$\begin{aligned} u_i(v_i, b) &= \sum_{j=1}^{m} g_{ij}(B, CB, CI)\, v_{ij} \ - \ \tilde{p}_i(B, CB, CI) \sum_{j=1}^{m} g_{ij}(B, CB, CI) b_{ij} \\ &\geq \sum_{j=1}^{m} g_{ij}(B, CB, CI)\, b_{ij} \ - \ \tilde{p}_i(B, CB, CI) \sum_{j=1}^{m} g_{ij}(B, CB, CI) b_{ij} \\ &= \big(1 - \tilde{p}_i(B, CB, CI)\big) \sum_{j=1}^{m} g_{ij}(B, CB, CI) b_{ij}. \end{aligned}$$

Since $\tilde{p}_i \in [0, 1]$, we have $1 - \tilde{p}_i \geq 0$. Because $g_{ij} \in [0, 1]$ and $b_{ij} \geq 0$, it follows that $\sum_{j=1}^{m} g_{ij}\, b_{ij} \geq 0$. Therefore,

$$u_i(v_i, b) \ = \ \big(1 - \tilde{p}_i\big) \sum_{j=1}^{m} g_{ij}\, b_{ij} \ \geq \ 0,$$

which proves that the mechanism does not charge bidder i more than her (reported) value for the allocation, i.e., the mechanism is individually rational.

Appendix C. Supplementary Algorithms

Appendix C.1. Preference-aware Encoder Algorithm

To enhance readability and facilitate understanding, we present a detailed algorithm 1 for the "Preference-aware Encoder."

The process of building a preference aware encoder is divided into four steps. First, it constructs a preference network based on users' bidding behavior. Second, the embeddings of bidders and items are updated through iterative message propagation processes. Next, it obtains representations of all bidders and items through feature fusion. Finally, a preference matrix is generated to capture the bidders' preferences for various items.

Algorithm 1: Preference-aware Encoder

Input: Bids $b \in \mathbb{R}^{n \times m}$, context features $\{cb\}_{i=1}^{n}$ for bidders, context features $\{ci\}_{j=1}^{m}$ for items.

Output: Bidder preference matrix P.

// Step 1: Construct Preference Network;
for *each bidder* i **do**
  for *each item* j **do**
    $\eta_{ij} \leftarrow \frac{b_{ij}}{\sum_{k=1}^{n} b_{ik}}$;   `// Calculate preference intensity`
  end
end
// Step 2: Message Propagation;
Initialize $e_i^{(0)} \leftarrow c_{bi},\ e_j^{(0)} \leftarrow c_j$, for all bidders $i = 1, \dots, n$ and items $j = 1, \dots, m$.;
for $k = 1, 2, \dots, K$ **do**
  for *each bidder* i **do**
    $e_i^{(k)} \leftarrow \sum_{j \in N(i)} \eta_{ij} \frac{e_j^{(k-1)}}{\sqrt{|N(i)||N(j)|}}$; `// Update bidder embedding`
  end
  for *each item* j **do**
    $e_j^{(k)} \leftarrow \sum_{i \in N(j)} \eta_{ij} \frac{e_i^{(k-1)}}{\sqrt{|N(j)||N(i)|}}$;   `// Update item embedding`
  end
end
Step 3: Feature Fusion;
for *each bidder* i **do**
  $e_i \leftarrow \sum_{k=0}^{K} \alpha_k e_i^{(k)}$;
end
for *each item* j **do**
  $e_j \leftarrow \sum_{k=0}^{K} \alpha_k e_j^{(k)}$;
end
$E^{bidder} \leftarrow \text{Concat}(e_1, e_2, \dots, e_n)$;
$E^{item} \leftarrow \text{Concat}(e_1, e_2, \dots, e_m)$;
// Step 4: Compute Preference Matrix;
$P \leftarrow (E^{bidder})^{\top} E^{item}$;
return P;

Appendix C.2. CPAD Training Algorithm

In this section, we outline the training procedure for the CPAD. This optimization is carried out using an augmented Lagrangian approach to ensure that the model adheres to incentive compatibility and other constraints inherent in auction design.

The algorithm 2 operates on minibatches of auction data, where each bid is accompanied by contextual features. The inner loop of the procedure adjusts misreports iteratively to refine the utility estimates. After this iterative refinement, the algorithm computes the model parameters and the Lagrange multipliers in an alternating manner.

The gradient of $L_\rho(w, \lambda^t)$ for fixed λ^t is given by:

$$\nabla_w \mathcal{L}_\rho(w, \lambda^t) = -\frac{1}{B}\sum_{\ell=1}^{B}\sum_{i\in N}\nabla_w p_i^w(v^{(\ell)}, CB^{(\ell)}, CI^{(\ell)}) + \sum_{i\in N}\sum_{\ell=1}^{B}\lambda_i^t g_{\ell,i} + \rho\sum_{i\in N}\sum_{\ell=1}^{B} r\hat{g}t_i(w) g_{\ell,i},$$

where

$$g_{\ell,i} = \nabla_w \left[\max_{v'^{(\ell)}_i \in V_i} u_i^w\left(v_i^{(\ell)}, (v'^{(\ell)}_i, v_{-i}^{(\ell)}), CB^{(\ell)}, CI^{(\ell)}\right) - u_i^w\left(v_i^{(\ell)}, v^{(\ell)}, CB^{(\ell)}, CI^{(\ell)}\right)\right].$$

The calculation of $\hat{r}g_{t,i}$ and $g_{\ell,i}$ requires maximizing over misreports for each bidder i. We approach this task using gradient ascent for an approximate solution. Specifically, we keep track of misreports $v_i^{(\ell)}$ for each bidder i corresponding to each sample ℓ. During each update of the model parameters w^t, we execute R iterations of gradient ascent to identify the optimal misreport values.

Appendix D. Supplementary Context Examples

In this section, we explicitly specify the continuous and discrete contexts for both bidders and the item to demonstrate the applicability of the proposed model. Table D.6 presents the contextual descriptions of each bidder and the item in natural-language sentences, while Table D.7 provides the corresponding discrete token-style representations.

Algorithm 2: CPAD Training

Input: Minibatches $S_1, \ldots, S_T$ of size B

Parameters: $\forall t \in [T], \rho_t > 0, \gamma > 0, \eta > 0, R \in \mathbb{N}, Q \in \mathbb{N}$;

Initialize: $w^0 \in \mathbb{R}^d$, $\lambda^0 \in \mathbb{R}^n$;

for $t = 0$ *to* T **do**

  Receive minibatch $S_t = \{(v^{(1)}, CB^{(1)}, CI^{(1)}), \ldots, (v^{(B)}, CB^{(B)}, CI^{(B)})\}$

  Initialize misreports ${v'}_i^{(\ell)} \in V_i,\ \forall \ell \in [B],\ i \in N$

  for $r = 0$ *to* R **do**

    $\forall \ell \in [B], i \in N$:

    ${v'}_i^{(\ell)} \leftarrow {v'}_i^{(\ell)} + \gamma \nabla_{v'_i} u_i^w(v_i^{(\ell)}, ({v'}_i^{(\ell)}, v_{-i}^{(\ell)}), CB^{(\ell)}, CI^{(\ell)})$

  end

  Compute regret gradient: $\forall \ell \in [B], i \in N$:

$$g_{\ell,i}^t = \nabla_w \left[u_i^w(v_i^{(\ell)}, ({v'}_i^{(\ell)}, v_{-i}^{(\ell)}), CB^{(\ell)}, CI^{(\ell)}) - u_i^w(v_i^{(\ell)}, v^{(\ell)}, CB^{(\ell)}, CI^{(\ell)}) \right] \Bigg|_{w=w^t}$$

  Update model parameters w^t:

$$w^{t+1} \leftarrow w^t - \eta \nabla_w \mathcal{L}_{\rho_t}(w^t, \lambda^t)$$

  Update Lagrange multipliers once in Q iterations:

  if $t \mod Q = 0$ **then**

    $\lambda_i^{t+1} \leftarrow \lambda_i^t + \rho_t \mathrm{rgti}(w^{t+1}),\ \forall i \in N$

  end

  else

    $\lambda^{t+1} \leftarrow \lambda^t$

  end

end

Table D.6: Continuous context examples. Each bidder and the single item are described by one sentence that summarizes their context.

Entity	**Continuous Context (sentence-level description)**
Bidder 1	A financially steady and risk-moderate participant who prefers verified items but values long-term reliability over short-term profit.
Bidder 2	A growth-oriented bidder with strong budget flexibility and interest in certified, high-integrity items having clear market demand.
Bidder 3	A specialized bidder focusing on niche or collectible items, with selective bidding behavior and moderate liquidity tolerance.
Bidder 4	An agile, opportunity-driven bidder who favors time-sensitive items and accepts higher risk in exchange for fast returns.
Bidder 5	A conservative bidder emphasizing price efficiency and predictable utility rather than premium quality.
Item (single)	A certified, high-integrity item with stable liquidity and observable short-term market growth opportunity.

Table D.7: Discrete context examples. Each bidder and the single item are described by a sequence of keywords representing their context.

Entity	**Discrete Context (semicolon-separated keywords)**
Bidder 1	stable-budget; moderate-risk; reliability-focused; verified-preference; long-term; stable
Bidder 2	growth-oriented; high-budget; certified-item-focused; integrity-seeking; demand-driven; reliable
Bidder 3	niche-collector; specialized; selective; moderate-liquidity; reputation-priority
Bidder 4	agile; opportunity-seeking; time-sensitive; fast-return; risk-tolerant; dynamic
Bidder 5	price-conscious; conservative; predictable; value-oriented; efficiency-driven
Item (single)	certified; high-integrity; liquid; short-term-growth; stable-market