1 Model

The equation of $n_\varepsilon(x,\theta)$

$$\begin{cases} -D(\theta)\Delta_x n_\varepsilon - \varepsilon^2\Delta_\theta n_\varepsilon = n_\varepsilon(K(x)-\rho_\varepsilon(x)), \\ \rho_\varepsilon = \int_0^1 n_\varepsilon(x,\theta)d\theta \end{cases} \tag{1}$$

Set $n_\varepsilon(x,\theta) = W_\varepsilon(x,\theta)e^{u(\theta)/\varepsilon}$, then

$$\begin{aligned} &\nabla_x n_\varepsilon = \nabla_x W_\varepsilon\ e^{u/\varepsilon}, \\ &\Delta_x n_\varepsilon = \Delta_x W_\varepsilon\ e^{u/\varepsilon}, \\ &\nabla_\theta n_\varepsilon = \nabla_\theta W_\varepsilon\ e^{u/\varepsilon} + W_\varepsilon\ e^{u/\varepsilon}\frac{\nabla_\theta u}{\varepsilon}, \\ &\Delta_\theta n_\varepsilon = \Delta_\theta W_\varepsilon\ e^{u/\varepsilon} + 2\nabla_\theta W_\varepsilon\ e^{u/\varepsilon}\ \frac{\nabla_\theta u}{\varepsilon} + W_\varepsilon e^{u/\varepsilon}\frac{|\nabla_\theta u|^2}{\varepsilon^2} + W_\varepsilon e^{u/\varepsilon}\frac{\Delta_\theta u}{\varepsilon} \end{aligned} \tag{2}$$

and (1) will be

$$\begin{aligned} &-D(\theta)\Delta_x W_\varepsilon\ e^{u/\varepsilon} - \varepsilon^2\left(\Delta_\theta W_\varepsilon\ e^{u/\varepsilon} + 2e^{u/\varepsilon}\ \frac{\nabla_\theta W_\varepsilon\cdot\nabla_\theta u}{\varepsilon} + W_\varepsilon e^{u/\varepsilon}\frac{|\nabla_\theta u|^2}{\varepsilon^2} + W_\varepsilon e^{u/\varepsilon}\frac{\Delta_\theta u}{\varepsilon}\right) \\ &= W_\varepsilon e^{u/\varepsilon}(K(x)-\rho_\varepsilon(x)), \\ &-D(\theta)\Delta_x W_\varepsilon\ e^{u/\varepsilon} - \varepsilon^2\Delta_\theta W_\varepsilon\ e^{u/\varepsilon} - 2e^{u/\varepsilon}\ \nabla_\theta W_\varepsilon\cdot\nabla_\theta u\varepsilon - W_\varepsilon e^{u/\varepsilon}|\nabla_\theta u|^2 - W_\varepsilon e^{u/\varepsilon}\Delta_\theta u\varepsilon \\ &= W_\varepsilon e^{u/\varepsilon}(K(x)-\rho_\varepsilon(x)), \end{aligned} \tag{3}$$

i.e.,

$$-D(\theta)\Delta_x W_\varepsilon - \varepsilon^2\Delta_\theta W_\varepsilon - 2\varepsilon\nabla_\theta W_\varepsilon\cdot\nabla_\theta u - W_\varepsilon|\nabla_\theta u|^2 - W_\varepsilon\Delta_\theta u\varepsilon = W_\varepsilon(K(x)-\rho_\varepsilon(x)), \tag{4}$$

$$\begin{cases} -D(\theta)\Delta_x W_\varepsilon - \varepsilon^2\Delta_\theta W_\varepsilon - 2\varepsilon\nabla_\theta W_\varepsilon\cdot\nabla_\theta u = W_\varepsilon(K(x)-\rho_\varepsilon(x)), \\ -|\nabla_\theta u|^2 - \varepsilon\Delta_\theta u = 0 \end{cases} \tag{5}$$

$$\begin{cases} -D(\theta)\Delta_x W_\varepsilon - \varepsilon^2\Delta_\theta W_\varepsilon - 2\varepsilon\nabla_\theta W_\varepsilon\cdot\nabla_\theta u = W_\varepsilon(K(x)-\rho_\varepsilon(x)) + W_\varepsilon H, \\ -|\nabla_\theta u|^2 - \varepsilon\Delta_\theta u = -H \end{cases} \tag{6}$$

The eigenvalue problem

$$-D(\theta)\Delta_x W_\varepsilon - W_\varepsilon(K(x)-\rho_\varepsilon(x)) = W_\varepsilon H, \tag{7}$$

$$\rho_\varepsilon(x) = \int_0^1 W_\varepsilon(x,\theta)e^{u(\theta)/\varepsilon}d\theta \tag{8}$$

$$\varepsilon \to 0, \qquad \rho_\varepsilon(x) \to \int_0^1 W_\varepsilon(x,\theta)\delta(\theta-\theta_m)d\theta = \bar{W}, \tag{9}$$

where $\bar{W}(x)$ satisfies

$$-D(\theta_m)\Delta_x \bar{W} = \bar{W}(K(x) - \bar{W})$$

1. initial value $W_\varepsilon(x,\theta), u(\theta)$;
2. calculate ρ_ε;
3. eigenvalue H;
4. the second equation, calculate $u(\theta)$;
5. the first equation, calculate W_ε.

$$\partial_t n_\varepsilon - D(\theta)\Delta_x n_\varepsilon - \varepsilon^2\Delta_\theta n_\varepsilon = n_\varepsilon(K(x)-\rho_\varepsilon(x))$$

Set $n_\varepsilon(x,\theta,t) = W_\varepsilon(x,\theta,t)e^{u(\theta,t)/\varepsilon}$, then

$$\begin{aligned}
&\partial_t n_\varepsilon = \partial_t W_\varepsilon e^{u/\varepsilon} + W_\varepsilon e^{u/\varepsilon}\frac{\partial_t u}{\varepsilon},\\
&\nabla_x n_\varepsilon = \nabla_x W_\varepsilon\ e^{u/\varepsilon},\\
&\Delta_x n_\varepsilon = \Delta_x W_\varepsilon\ e^{u/\varepsilon},\\
&\nabla_\theta n_\varepsilon = \nabla_\theta W_\varepsilon\ e^{u/\varepsilon} + W_\varepsilon\ e^{u/\varepsilon}\frac{\nabla_\theta u}{\varepsilon},\\
&\Delta_\theta n_\varepsilon = \Delta_\theta W_\varepsilon\ e^{u/\varepsilon} + 2\nabla_\theta W_\varepsilon\ e^{u/\varepsilon}\ \frac{\nabla_\theta u}{\varepsilon} + W_\varepsilon e^{u/\varepsilon}\frac{|\nabla_\theta u|^2}{\varepsilon^2} + W_\varepsilon e^{u/\varepsilon}\frac{\Delta_\theta u}{\varepsilon}
\end{aligned} \tag{10}$$

and

$$\begin{aligned}
&\partial_t W_\varepsilon e^{u/\varepsilon} + W_\varepsilon e^{u/\varepsilon}\frac{\partial_t u}{\varepsilon}\\
&- D(\theta)\Delta_x W_\varepsilon\ e^{u/\varepsilon} - \varepsilon^2\left(\Delta_\theta W_\varepsilon\ e^{u/\varepsilon} + 2e^{u/\varepsilon}\ \frac{\nabla_\theta W_\varepsilon\cdot\nabla_\theta u}{\varepsilon} + W_\varepsilon e^{u/\varepsilon}\frac{|\nabla_\theta u|^2}{\varepsilon^2} + W_\varepsilon e^{u/\varepsilon}\frac{\Delta_\theta u}{\varepsilon}\right)\\
&= W_\varepsilon e^{u/\varepsilon}(K(x)-\rho_\varepsilon(x)),\\
&\partial_t W_\varepsilon e^{u/\varepsilon} + W_\varepsilon e^{u/\varepsilon}\frac{\partial_t u}{\varepsilon}\\
&- D(\theta)\Delta_x W_\varepsilon\ e^{u/\varepsilon} - \varepsilon^2\Delta_\theta W_\varepsilon\ e^{u/\varepsilon} - 2e^{u/\varepsilon}\ \nabla_\theta W_\varepsilon\cdot\nabla_\theta u\varepsilon - W_\varepsilon e^{u/\varepsilon}|\nabla_\theta u|^2 - W_\varepsilon e^{u/\varepsilon}\Delta_\theta u\varepsilon\\
&= W_\varepsilon e^{u/\varepsilon}(K(x)-\rho_\varepsilon(x)),
\end{aligned} \tag{11}$$

and

$$\begin{cases}
\partial_t W_\varepsilon - D(\theta)\Delta_x W_\varepsilon - \varepsilon^2\Delta_\theta W_\varepsilon - 2\varepsilon\nabla_\theta W_\varepsilon\cdot\nabla_\theta u = W_\varepsilon(K(x)-\rho_\varepsilon(x)),\\
\dfrac{\partial_t u}{\varepsilon} - |\nabla_\theta u|^2 - \varepsilon\Delta_\theta u = 0
\end{cases} \tag{12}$$

so

$$\begin{cases}
\partial_t W_\varepsilon - D(\theta)\Delta_x W_\varepsilon - \varepsilon^2\Delta_\theta W_\varepsilon - 2\varepsilon\nabla_\theta W_\varepsilon\cdot\nabla_\theta u = W_\varepsilon(K(x)-\rho_\varepsilon(x)) + W_\varepsilon H,\\
\dfrac{\partial_t u}{\varepsilon} - |\nabla_\theta u|^2 - \varepsilon\Delta_\theta u = -H
\end{cases} \tag{13}$$

where H and ρ_ε are obtained by the eigenvalue problem

$$-D(\theta)\Delta_x W_\varepsilon - W_\varepsilon(K(x)-\rho_\varepsilon(x)) = W_\varepsilon H, \tag{14}$$

and

$$\rho_\varepsilon(x) = \int_0^1 W_\varepsilon(x,\theta)e^{u(\theta)/\varepsilon}d\theta \tag{15}$$

The step to solve the problem:

- initial value $t = 0$: $W_\varepsilon(x, \theta, 0), u(\theta, 0)$;
- calculate $\rho_\varepsilon(x, 0)$;
- solve the eigenvalue problem and obtain H;
- solve the second equation, obtain $u(\theta)$;
- solve the first equation, obtain W_ε.

Discretization

1D:

$$\rho_\varepsilon(x_j) = \int_0^1 W_\varepsilon(x_j, \theta) e^{u(\theta)/\varepsilon} d\theta = \sum_{i=1}^{M} \int_{\theta_{i-1}}^{\theta_i} W_\varepsilon(x_j, \theta) e^{u(\theta)/\varepsilon} d\theta \tag{16}$$

The choose of numerical integration?

The eigenvalue problem: Sturm-Liouville theory. We obtain $H_0, H_1, \cdots, H_M$.

$$-D(\theta) \frac{W_\varepsilon(x_{j+1}, \theta) - 2W_\varepsilon(x_j, \theta) + W_\varepsilon(x_{j-1}, \theta)}{\delta x^2} - W_\varepsilon(x_j, \theta)(K(x_j) - \rho_\varepsilon(x_j)) = W_\varepsilon(x_j, \theta) H, \qquad j = 1, 2, \cdots, \tag{17}$$

with the Neumann boundary condition:

$$W_\varepsilon(x_0, \theta_j) = W_\varepsilon(x_1, \theta_j), \qquad W_\varepsilon(x_N, \theta_j) = W_\varepsilon(x_{N-1}, \theta_j)$$

the matrix:

$$-D(\theta)\frac{1}{\delta x^2}(-2diag(ones(n,1),0)+diag(ones(n-1,1),1)+diag(ones(n-1,1),-1))-diag(K_j-\rho_j)$$

i.e.

$$-D(\theta)\frac{1}{\delta x^2}\begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 1 & -2 & 1 & \cdots & 0 \\ 0 & 1 & -2 & \cdots & 0 \\ & & \cdots & & \\ 0 & \cdots & 1 & -2 & 1 \\ 0 & \cdots & 0 & 1 & -1 \end{bmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_{N-2} \\ w_{N-1} \end{pmatrix} - \begin{bmatrix} K_1 - \rho_1 & 0 & \cdots & 0 \\ 0 & K_2 - \rho_2 & \cdots & 0 \\ & & \ddots & \\ 0 & 0 & \cdots & K_{N-1} - \rho_{N-1} \end{bmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_{N-2} \\ w_{N-1} \end{pmatrix} \tag{18}$$

Then we solve the second equation by finite difference method:

$$\frac{\partial_t u(\theta, t)}{\varepsilon} - |\nabla_\theta u(\theta, t)|^2 - \varepsilon \Delta_\theta u(\theta, t) = -H(\theta) \tag{19}$$

In 1D case,

$$\frac{\partial_t u(\theta, t)}{\varepsilon} - (\partial_\theta u(\theta, t))^2 - \varepsilon \partial_{\theta\theta} u(\theta, t) = -H(\theta) \tag{20}$$

and the forward Euler,

$$\frac{u(\theta_i,t_{m+1})-u(\theta_i,t_m)}{\varepsilon\tau}-\left(\frac{u(\theta_{i+1},t_m)-u(\theta_i,t_m) or u(\theta_i,t_m)-u(\theta_{i-1},t_m)}{\delta\theta}\right)^2$$
$$-\varepsilon\frac{u(\theta_{i+1},t_{m+1})-2u(\theta_i,t_{m+1})+u(\theta_{i-1},t_{m+1})}{\delta\theta^2}=-H(\theta_i) \tag{21}$$

i.e.

$$\frac{u(\theta_i,t_{m+1})-u(\theta_i,t_m)}{\varepsilon\tau}-\varepsilon\frac{u(\theta_{i+1},t_{m+1})-2u(\theta_i,t_{m+1})+u(\theta_{i-1},t_{m+1})}{\delta\theta^2}=-H(\theta_i)+dthetaupwind^2,$$
$$u(\theta_i,t_{m+1})-u(\theta_i,t_m)-\varepsilon^2\tau\frac{u(\theta_{i+1},t_{m+1})-2u(\theta_i,t_{m+1})+u(\theta_{i-1},t_{m+1})}{\delta\theta^2}=\varepsilon\tau(-H(\theta_i)+dthetaupwind^2),$$
$$u(\theta_i,t_{m+1})+\frac{2\varepsilon^2\tau}{\delta\theta^2}u(\theta_i,t_{m+1})-\frac{\varepsilon^2\tau}{\delta\theta^2}u(\theta_{i+1},t_{m+1})-\frac{\varepsilon^2\tau}{\delta\theta^2}u(\theta_{i-1},t_{m+1})=u(\theta_i,t_m)+\varepsilon\tau(-H(\theta_i)+dthetaupwin \tag{22}$$

obtain $u(\theta_i,t_{m+1})$, and we solve the first equation

$$\partial_t W_\varepsilon(x,\theta,t)-D(\theta)\partial_{xx}W_\varepsilon(x,\theta,t)-\varepsilon^2\partial_{\theta\theta}W_\varepsilon-2\varepsilon\partial_\theta W_\varepsilon\ \partial_\theta u=W_\varepsilon(K(x)-\rho_\varepsilon(x))+W_\varepsilon H, \tag{23}$$

which is discreted as

$$\frac{W_\varepsilon(x_j,\theta_i,t_{m+1})-W_\varepsilon(x_j,\theta_i,t_m)}{\tau}-D(\theta)\frac{W_\varepsilon(x_{j+1},\theta_i,t_{m+1})-2W_\varepsilon(x_j,\theta_i,t_{m+1})+W_\varepsilon(x_{j-1},\theta_i,t_{m+1})}{\delta x^2}$$
$$-\varepsilon^2\frac{W_\varepsilon(x_j,\theta_{j+1},t_{m+1})-2W_\varepsilon(x_j,\theta_j,t_{m+1})+W_\varepsilon(x_j,\theta_{j-1},t_{m+1})}{\delta\theta^2}$$
$$-2\varepsilon dwupwind\frac{u(\theta_{j+1},t_{m+1})-u(\theta_{j-1},t_{m+1})}{2\delta\theta}$$
$$=W_\varepsilon(K(x)-\rho_\varepsilon(x))+W_\varepsilon H, \tag{24}$$

i.e.

$$\frac{W_\varepsilon(x_j,\theta_i,t_{m+1})-W_\varepsilon(x_j,\theta_i,t_m)}{\tau}-D(\theta)\frac{W_\varepsilon(x_{j+1},\theta_i,t_{m+1})-2W_\varepsilon(x_j,\theta_i,t_{m+1})+W_\varepsilon(x_{j-1},\theta_i,t_{m+1})}{\delta x^2}$$
$$-\varepsilon^2\frac{W_\varepsilon(x_j,\theta_{j+1},t_{m+1})-2W_\varepsilon(x_j,\theta_j,t_{m+1})+W_\varepsilon(x_j,\theta_{j-1},t_{m+1})}{\delta\theta^2}$$
$$=2\varepsilon dwupwind\frac{u(\theta_{j+1},t_{m+1})-u(\theta_{j-1},t_{m+1})}{2\delta\theta}+W_\varepsilon(K(x)-\rho_\varepsilon(x))+W_\varepsilon H, \tag{25}$$

$$W_\varepsilon(x_j,\theta_i,t_{m+1})-\frac{\tau D}{\delta x^2}[W_\varepsilon(x_{j+1},\theta_i,t_{m+1})-2W_\varepsilon(x_j,\theta_i,t_{m+1})+W_\varepsilon(x_{j-1},\theta_i,t_{m+1})]$$
$$-\frac{\varepsilon^2\tau}{\delta\theta^2}[W_\varepsilon(x_j,\theta_{j+1},t_{m+1})-2W_\varepsilon(x_j,\theta_j,t_{m+1})+W_\varepsilon(x_j,\theta_{j-1},t_{m+1})]$$
$$=\tau(2\varepsilon dwupwind\frac{u(\theta_{j+1},t_{m+1})-u(\theta_{j-1},t_{m+1})}{2\delta\theta}+W_\varepsilon(K(x)-\rho_\varepsilon(x))+W_\varepsilon H)+W_\varepsilon(x_j,\theta_i,t_m), \tag{26}$$

$$
\begin{aligned}
&W_\varepsilon(x_j,\theta_i,t_{m+1}) - \frac{\tau D}{\delta x^2}W_\varepsilon(x_{j+1},\theta_i,t_{m+1}) + \frac{2\tau D}{\delta x^2}W_\varepsilon(x_j,\theta_i,t_{m+1}) - \frac{\tau D}{\delta x^2}W_\varepsilon(x_{j-1},\theta_i,t_{m+1})\\
&-\frac{\varepsilon^2\tau}{\delta\theta^2}W_\varepsilon(x_j,\theta_{i+1},t_{m+1}) + \frac{2\varepsilon^2\tau}{\delta\theta^2}W_\varepsilon(x_j,\theta_i,t_{m+1}) - \frac{\varepsilon^2\tau}{\delta\theta^2}W_\varepsilon(x_j,\theta_{i-1},t_{m+1})\\
&=\tau(2\varepsilon dwupwind\frac{u(\theta_{i+1},t_{m+1})-u(\theta_{i-1},t_{m+1})}{2\delta\theta} + W_\varepsilon(K(x)-\rho_\varepsilon(x)) + W_\varepsilon H) + W_\varepsilon(x_j,\theta_i,t_m),
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
&W_\varepsilon(x_j,\theta_i,t_{m+1}) + \frac{2\tau D}{\delta x^2}W_\varepsilon(x_j,\theta_i,t_{m+1}) + \frac{2\varepsilon^2\tau}{\delta\theta^2}W_\varepsilon(x_j,\theta_i,t_{m+1})\\
&-\frac{\tau D}{\delta x^2}W_\varepsilon(x_{j+1},\theta_i,t_{m+1}) - \frac{\tau D}{\delta x^2}W_\varepsilon(x_{j-1},\theta_i,t_{m+1})\\
&-\frac{\varepsilon^2\tau}{\delta\theta^2}W_\varepsilon(x_j,\theta_{i+1},t_{m+1}) - \frac{\varepsilon^2\tau}{\delta\theta^2}W_\varepsilon(x_j,\theta_{i-1},t_{m+1})\\
&=\tau(2\varepsilon dwupwind\frac{u(\theta_{i+1},t_{m+1})-u(\theta_{i-1},t_{m+1})}{2\delta\theta} + W_\varepsilon(K(x)-\rho_\varepsilon(x)) + W_\varepsilon H) + W_\varepsilon(x_j,\theta_i,t_m),
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
&\partial_t n_\varepsilon - D(\theta)\Delta_x n_\varepsilon - \varepsilon^2 \Delta_\theta n_\varepsilon = n_\varepsilon(K(x) - \rho_\varepsilon(x)),\\
&\rho_\varepsilon = \int_0^1 n_\varepsilon(x,\theta)d\theta
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
&\frac{n_\varepsilon(x_j,\theta_i,t^{m+1}) - n_\varepsilon(x_j,\theta_i,t^m)}{\delta t} - D(\theta_i)\frac{n_\varepsilon(x_{j+1},\theta_i,t_{m+1}) - 2n_\varepsilon(x_j,\theta_i,t_{m+1}) + n_\varepsilon(x_{j-1},\theta_i,t_{m+1})}{\delta x^2}\\
&-\varepsilon^2 \frac{n_\varepsilon(x_j,\theta_{i+1},t_{m+1}) - 2n_\varepsilon(x_j,\theta_i,t_{m+1}) + n_\varepsilon(x_j,\theta_{i-1},t_{m+1})}{\delta\theta^2} - n_\varepsilon(x_j,\theta_i,t_{m+1})\left(K(x_j) - \rho_\varepsilon(x_j)\right) = 0
\end{aligned} \tag{30}
$$

$$
\begin{aligned}
&n_\varepsilon(x_j,\theta_i,t^{m+1}) - D(\theta_i)\delta t\frac{n_\varepsilon(x_{j+1},\theta_i,t_{m+1}) - 2n_\varepsilon(x_j,\theta_i,t_{m+1}) + n_\varepsilon(x_{j-1},\theta_i,t_{m+1})}{\delta x^2}\\
&-\varepsilon^2\delta t\frac{n_\varepsilon(x_j,\theta_{i+1},t_{m+1}) - 2n_\varepsilon(x_j,\theta_i,t_{m+1}) + n_\varepsilon(x_j,\theta_{i-1},t_{m+1})}{\delta\theta^2} - \delta t n_\varepsilon(x_j,\theta_i,t_{m+1})\left(K(x_j) - \rho_\varepsilon(x_j)\right) = n_\varepsilon(x_j
\end{aligned} \tag{31}
$$

$$
\begin{aligned}
&n_\varepsilon(x_j,\theta_i,t^{m+1}) + 2D(\theta_i)\delta t/\delta x^2 n_\varepsilon(x_j,\theta_i,t^{m+1}) - D(\theta_i)\delta t\frac{n_\varepsilon(x_{j+1},\theta_i,t_{m+1}) + n_\varepsilon(x_{j-1},\theta_i,t_{m+1})}{\delta x^2}\\
&+ 2\varepsilon^2\delta t/\delta\theta^2 n_\varepsilon(x_j,\theta_i t_{m+1}) - \varepsilon^2\delta t\frac{n_\varepsilon(x_j,\theta_{i+1},t_{m+1}) + n_\varepsilon(x_j,\theta_{i-1},t_{m+1})}{\delta\theta^2}\\
&- \delta t n_\varepsilon(x_j,\theta_i,t_{m+1})\left(K(x_j) - \rho_\varepsilon(x_j)\right) = n_\varepsilon(x_j,\theta_i,t^m)
\end{aligned} \tag{32}
$$

$$
\rho_\varepsilon = \int_0^1 n_\varepsilon(x,\theta)d\theta = \sum_{i=1}^{M}\int_{\theta_{i-1}}^{\theta_i} n_\varepsilon(x,\theta)d\theta
$$

$$
\begin{aligned}
&- D(\theta)\Delta_x n_\varepsilon - \varepsilon^2 \Delta_\theta n_\varepsilon = n_\varepsilon(K(x) - \rho_\varepsilon(x)),\\
&\rho_\varepsilon = \int_0^1 n_\varepsilon(x,\theta) d\theta
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
&2D(\theta_i)/\delta x^2 n_\varepsilon(x_j, \theta_i, t^{m+1}) - D(\theta_i)\frac{n_\varepsilon(x_{j+1}, \theta_i, t_{m+1}) + n_\varepsilon(x_{j-1}, \theta_i, t_{m+1})}{\delta x^2}\\
&+ 2\varepsilon^2/\delta\theta^2 n_\varepsilon(x_j, \theta_i t_{m+1}) - \varepsilon^2 \frac{n_\varepsilon(x_j, \theta_{i+1}, t_{m+1}) + n_\varepsilon(x_j, \theta_{i-1}, t_{m+1})}{\delta\theta^2}\\
&- n_\varepsilon(x_j, \theta_i, t_{m+1})\left(K(x_j) - \rho_\varepsilon(x_j)\right) = 0
\end{aligned}
\tag{34}
$$