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## Abstract

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1.

The equation of  $n_\varepsilon(x, \theta)$

$$\begin{cases} -D(\theta)\Delta_x n_\varepsilon - \varepsilon^2 \Delta_\theta n_\varepsilon = n_\varepsilon(K(x) - \rho_\varepsilon(x)), \\ \rho_\varepsilon = \int_0^1 n_\varepsilon(x, \theta) d\theta \end{cases} \quad (1.1)$$

Set  $n_\varepsilon(x, \theta) = W_\varepsilon(x, \theta)e^{u(\theta)/\varepsilon}$ , then

$$\begin{aligned} \nabla_x n_\varepsilon &= \nabla_x W_\varepsilon e^{u/\varepsilon}, \\ \Delta_x n_\varepsilon &= \Delta_x W_\varepsilon e^{u/\varepsilon}, \\ \nabla_\theta n_\varepsilon &= \nabla_\theta W_\varepsilon e^{u/\varepsilon} + W_\varepsilon e^{u/\varepsilon} \frac{\nabla_\theta u}{\varepsilon}, \\ \Delta_\theta n_\varepsilon &= \Delta_\theta W_\varepsilon e^{u/\varepsilon} + 2\nabla_\theta W_\varepsilon e^{u/\varepsilon} \frac{\nabla_\theta u}{\varepsilon} + W_\varepsilon e^{u/\varepsilon} \frac{|\nabla_\theta u|^2}{\varepsilon^2} + W_\varepsilon e^{u/\varepsilon} \frac{\Delta_\theta u}{\varepsilon} \end{aligned} \quad (1.2)$$

and (1.1) will be

$$\begin{aligned} & -D(\theta)\Delta_x W_\varepsilon e^{u/\varepsilon} - \varepsilon^2 \left( \Delta_\theta W_\varepsilon e^{u/\varepsilon} + 2e^{u/\varepsilon} \frac{\nabla_\theta W_\varepsilon \cdot \nabla_\theta u}{\varepsilon} + W_\varepsilon e^{u/\varepsilon} \frac{|\nabla_\theta u|^2}{\varepsilon^2} + W_\varepsilon e^{u/\varepsilon} \frac{\Delta_\theta u}{\varepsilon} \right) \\ &= W_\varepsilon e^{u/\varepsilon} (K(x) - \rho_\varepsilon(x)), \\ & -D(\theta)\Delta_x W_\varepsilon e^{u/\varepsilon} - \varepsilon^2 \Delta_\theta W_\varepsilon e^{u/\varepsilon} - 2e^{u/\varepsilon} \nabla_\theta W_\varepsilon \cdot \nabla_\theta u - W_\varepsilon e^{u/\varepsilon} |\nabla_\theta u|^2 - W_\varepsilon e^{u/\varepsilon} \Delta_\theta u \\ &= W_\varepsilon e^{u/\varepsilon} (K(x) - \rho_\varepsilon(x)), \end{aligned} \quad (1.3)$$

i.e.,

$$-D(\theta)\Delta_x W_\varepsilon - \varepsilon^2 \Delta_\theta W_\varepsilon - 2\varepsilon \nabla_\theta W_\varepsilon \cdot \nabla_\theta u - W_\varepsilon |\nabla_\theta u|^2 - W_\varepsilon \Delta_\theta u = W_\varepsilon (K(x) - \rho_\varepsilon(x)), \quad (1.4)$$

$$\begin{cases} -D(\theta)\Delta_x W_\varepsilon - \varepsilon^2 \Delta_\theta W_\varepsilon - 2\varepsilon \nabla_\theta W_\varepsilon \cdot \nabla_\theta u = W_\varepsilon (K(x) - \rho_\varepsilon(x)), \\ -|\nabla_\theta u|^2 - \varepsilon \Delta_\theta u = 0 \end{cases} \quad (1.5)$$

$$\begin{cases} -D(\theta)\Delta_x W_\varepsilon - \varepsilon^2 \Delta_\theta W_\varepsilon - 2\varepsilon \nabla_\theta W_\varepsilon \cdot \nabla_\theta u = W_\varepsilon (K(x) - \rho_\varepsilon(x)) + W_\varepsilon H, \\ -|\nabla_\theta u|^2 - \varepsilon \Delta_\theta u = -H \end{cases} \quad (1.6)$$

The eigenvalue problem

$$-D(\theta)\Delta_x W_\varepsilon - W_\varepsilon (K(x) - \rho_\varepsilon(x)) = W_\varepsilon H, \quad (1.7)$$

$$\rho_\varepsilon(x) = \int_0^1 W_\varepsilon(x, \theta) e^{u(\theta)/\varepsilon} d\theta \quad (1.8)$$

$$\varepsilon \rightarrow 0, \quad \rho_\varepsilon(x) \rightarrow \int_0^1 W_\varepsilon(x, \theta) \delta(\theta - \theta_m) d\theta = \bar{W}, \quad (1.9)$$

where  $\bar{W}(x)$  satisfies

$$-D(\theta_m)\Delta_x \bar{W} = \bar{W}(K(x) - \bar{W})$$

1. initial value  $W_\varepsilon(x, \theta), u(\theta)$ ;
2. calculate  $\rho_\varepsilon$ ;
3. eigenvalue  $H$ ;
4. the second equation, calculate  $u(\theta)$ ;
5. the first equation, calculate  $W_\varepsilon$ .

$$\partial_t n_\varepsilon - D(\theta) \Delta_x n_\varepsilon - \varepsilon^2 \Delta_\theta n_\varepsilon = n_\varepsilon (K(x) - \rho_\varepsilon(x))$$

Set  $n_\varepsilon(x, \theta, t) = W_\varepsilon(x, \theta, t) e^{u(\theta, t)/\varepsilon}$ , then

$$\begin{aligned} \partial_t n_\varepsilon &= \partial_t W_\varepsilon e^{u/\varepsilon} + W_\varepsilon e^{u/\varepsilon} \frac{\partial_t u}{\varepsilon}, \\ \nabla_x n_\varepsilon &= \nabla_x W_\varepsilon e^{u/\varepsilon}, \\ \Delta_x n_\varepsilon &= \Delta_x W_\varepsilon e^{u/\varepsilon}, \\ \nabla_\theta n_\varepsilon &= \nabla_\theta W_\varepsilon e^{u/\varepsilon} + W_\varepsilon e^{u/\varepsilon} \frac{\nabla_\theta u}{\varepsilon}, \\ \Delta_\theta n_\varepsilon &= \Delta_\theta W_\varepsilon e^{u/\varepsilon} + 2 \nabla_\theta W_\varepsilon e^{u/\varepsilon} \frac{\nabla_\theta u}{\varepsilon} + W_\varepsilon e^{u/\varepsilon} \frac{|\nabla_\theta u|^2}{\varepsilon^2} + W_\varepsilon e^{u/\varepsilon} \frac{\Delta_\theta u}{\varepsilon} \end{aligned} \tag{1.10}$$

and

$$\begin{aligned} & \partial_t W_\varepsilon e^{u/\varepsilon} + W_\varepsilon e^{u/\varepsilon} \frac{\partial_t u}{\varepsilon} \\ & - D(\theta) \Delta_x W_\varepsilon e^{u/\varepsilon} - \varepsilon^2 \left( \Delta_\theta W_\varepsilon e^{u/\varepsilon} + 2 e^{u/\varepsilon} \frac{\nabla_\theta W_\varepsilon \cdot \nabla_\theta u}{\varepsilon} + W_\varepsilon e^{u/\varepsilon} \frac{|\nabla_\theta u|^2}{\varepsilon^2} + W_\varepsilon e^{u/\varepsilon} \frac{\Delta_\theta u}{\varepsilon} \right) \\ & = W_\varepsilon e^{u/\varepsilon} (K(x) - \rho_\varepsilon(x)), \\ & \partial_t W_\varepsilon e^{u/\varepsilon} + W_\varepsilon e^{u/\varepsilon} \frac{\partial_t u}{\varepsilon} \\ & - D(\theta) \Delta_x W_\varepsilon e^{u/\varepsilon} - \varepsilon^2 \Delta_\theta W_\varepsilon e^{u/\varepsilon} - 2 e^{u/\varepsilon} \nabla_\theta W_\varepsilon \cdot \nabla_\theta u \varepsilon - W_\varepsilon e^{u/\varepsilon} |\nabla_\theta u|^2 - W_\varepsilon e^{u/\varepsilon} \Delta_\theta u \varepsilon \\ & = W_\varepsilon e^{u/\varepsilon} (K(x) - \rho_\varepsilon(x)), \end{aligned} \tag{1.11}$$

and

$$\begin{cases} \partial_t W_\varepsilon - D(\theta) \Delta_x W_\varepsilon - \varepsilon^2 \Delta_\theta W_\varepsilon - 2 \varepsilon \nabla_\theta W_\varepsilon \cdot \nabla_\theta u = W_\varepsilon (K(x) - \rho_\varepsilon(x)), \\ \frac{\partial_t u}{\varepsilon} - |\nabla_\theta u|^2 - \varepsilon \Delta_\theta u = 0 \end{cases} \tag{1.12}$$

so

$$\begin{cases} \partial_t W_\varepsilon - D(\theta) \Delta_x W_\varepsilon - \varepsilon^2 \Delta_\theta W_\varepsilon - 2 \varepsilon \nabla_\theta W_\varepsilon \cdot \nabla_\theta u = W_\varepsilon (K(x) - \rho_\varepsilon(x)) + W_\varepsilon H, \\ \frac{\partial_t u}{\varepsilon} - |\nabla_\theta u|^2 - \varepsilon \Delta_\theta u = -H \end{cases} \tag{1.13}$$

where  $H$  and  $\rho_\varepsilon$  are obtained by the eigenvalue problem

$$-D(\theta) \Delta_x W_\varepsilon - W_\varepsilon (K(x) - \rho_\varepsilon(x)) = W_\varepsilon H, \tag{1.14}$$

and

$$\rho_\varepsilon(x) = \int_0^1 W_\varepsilon(x, \theta) e^{u(\theta)/\varepsilon} d\theta \tag{1.15}$$

The step to solve the problem:

- initial value  $t = 0$ :  $W_\varepsilon(x, \theta, 0), u(\theta, 0)$ ;
- calculate  $\rho_\varepsilon(x, 0)$ ;
- solve the eigenvalue problem and obtain  $H$ ;
- solve the second equation, obtain  $u(\theta)$ ;
- solve the first equation, obtain  $W_\varepsilon$ .

Discretization

1D:

$$\rho_\varepsilon(x_j) = \int_0^1 W_\varepsilon(x_j, \theta) e^{u(\theta)/\varepsilon} d\theta = \sum_{i=1}^M \int_{\theta_{i-1}}^{\theta_i} W_\varepsilon(x_j, \theta) e^{u(\theta)/\varepsilon} d\theta \quad (1.16)$$

The choose of numerical integration?

The eigenvalue problem: Sturm-Liouville theory. We obtain  $H_0, H_1, \dots, H_M$ .

$$-D(\theta) \frac{W_\varepsilon(x_{j+1}, \theta) - 2W_\varepsilon(x_j, \theta) + W_\varepsilon(x_{j-1}, \theta)}{\delta x^2} - W_\varepsilon(x_j, \theta)(K(x_j) - \rho_\varepsilon(x_j)) = W_\varepsilon(x_j, \theta)H, \quad j = 1, 2, \dots, N-1 \quad (1.17)$$

with the Neumann boundary condition:

$$W_\varepsilon(x_0, \theta_j) = W_\varepsilon(x_1, \theta_j), \quad W_\varepsilon(x_N, \theta_j) = W_\varepsilon(x_{N-1}, \theta_j)$$

the matrix:

$$-D(\theta) \frac{1}{\delta x^2} (-2 \text{diag}(\text{ones}(n, 1), 0) + \text{diag}(\text{ones}(n-1, 1), 1) + \text{diag}(\text{ones}(n-1, 1), -1)) - \text{diag}(K_j - \rho_j)$$

i.e.

$$-D(\theta) \frac{1}{\delta x^2} \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 1 & -2 & 1 & \cdots & 0 \\ 0 & 1 & -2 & \cdots & 0 \\ & & \cdots & & \\ 0 & \cdots & 1 & -2 & 1 \\ 0 & \cdots & 0 & 1 & -1 \end{bmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_{N-2} \\ w_{N-1} \end{pmatrix} - \begin{bmatrix} K_1 - \rho_1 & 0 & \cdots & 0 \\ 0 & K_2 - \rho_2 & \cdots & 0 \\ & & \ddots & \\ 0 & 0 & \cdots & K_{N-1} - \rho_{N-1} \end{bmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_{N-2} \\ w_{N-1} \end{pmatrix} \quad (1.18)$$

Then we solve the second equation by finite difference method:

$$\frac{\partial_t u(\theta, t)}{\varepsilon} - |\nabla_\theta u(\theta, t)|^2 - \varepsilon \Delta_\theta u(\theta, t) = -H(\theta) \quad (1.19)$$

In 1D case,

$$\frac{\partial_t u(\theta, t)}{\varepsilon} - (\partial_\theta u(\theta, t))^2 - \varepsilon \partial_{\theta\theta} u(\theta, t) = -H(\theta) \quad (1.20)$$

and the forward Euler,

$$\frac{u(\theta_i, t_{m+1}) - u(\theta_i, t_m)}{\varepsilon \tau} - \left( \frac{u(\theta_{i+1}, t_m) - u(\theta_i, t_m) \text{ or } u(\theta_i, t_m) - u(\theta_{i-1}, t_m)}{\delta \theta} \right)^2 - \varepsilon \frac{u(\theta_{i+1}, t_{m+1}) - 2u(\theta_i, t_{m+1}) + u(\theta_{i-1}, t_{m+1})}{\delta \theta^2} = -H(\theta_i) \quad (1.21)$$

i.e.

$$\begin{aligned} \frac{u(\theta_i, t_{m+1}) - u(\theta_i, t_m)}{\varepsilon \tau} - \varepsilon \frac{u(\theta_{i+1}, t_{m+1}) - 2u(\theta_i, t_{m+1}) + u(\theta_{i-1}, t_{m+1})}{\delta \theta^2} &= -H(\theta_i) + d\theta \varepsilon \text{wind}^2, \\ u(\theta_i, t_{m+1}) - u(\theta_i, t_m) - \varepsilon^2 \tau \frac{u(\theta_{i+1}, t_{m+1}) - 2u(\theta_i, t_{m+1}) + u(\theta_{i-1}, t_{m+1})}{\delta \theta^2} &= \varepsilon \tau (-H(\theta_i) + d\theta \varepsilon \text{wind}^2), \\ u(\theta_i, t_{m+1}) + \frac{2\varepsilon^2 \tau}{\delta \theta^2} u(\theta_i, t_{m+1}) - \frac{\varepsilon^2 \tau}{\delta \theta^2} u(\theta_{i+1}, t_{m+1}) - \frac{\varepsilon^2 \tau}{\delta \theta^2} u(\theta_{i-1}, t_{m+1}) &= u(\theta_i, t_m) + \varepsilon \tau (-H(\theta_i) + d\theta \varepsilon \text{wind}^2), \end{aligned} \quad (1.22)$$

obtain  $u(\theta_i, t_{m+1})$ , and we solve the first equation

$$\partial_t W_\varepsilon(x, \theta, t) - D(\theta) \partial_{xx} W_\varepsilon(x, \theta, t) - \varepsilon^2 \partial_{\theta\theta} W_\varepsilon - 2\varepsilon \partial_\theta W_\varepsilon \partial_\theta u = W_\varepsilon(K(x) - \rho_\varepsilon(x)) + W_\varepsilon H, \quad (1.23)$$

which is discretized as

$$\begin{aligned}
& \frac{W_\varepsilon(x_j, \theta_i, t_{m+1}) - W_\varepsilon(x_j, \theta_i, t_m)}{\tau} - D(\theta) \frac{W_\varepsilon(x_{j+1}, \theta_i, t_{m+1}) - 2W_\varepsilon(x_j, \theta_i, t_{m+1}) + W_\varepsilon(x_{j-1}, \theta_i, t_{m+1})}{\delta x^2} \\
& - \varepsilon^2 \frac{W_\varepsilon(x_j, \theta_{j+1}, t_{m+1}) - 2W_\varepsilon(x_j, \theta_j, t_{m+1}) + W_\varepsilon(x_j, \theta_{j-1}, t_{m+1})}{\delta \theta^2} \\
& - 2\varepsilon dwupwind \frac{u(\theta_{j+1}, t_{m+1}) - u(\theta_{j-1}, t_{m+1})}{2\delta \theta} \\
& = W_\varepsilon(K(x) - \rho_\varepsilon(x)) + W_\varepsilon H,
\end{aligned} \tag{1.24}$$

i.e.

$$\begin{aligned}
& \frac{W_\varepsilon(x_j, \theta_i, t_{m+1}) - W_\varepsilon(x_j, \theta_i, t_m)}{\tau} - D(\theta) \frac{W_\varepsilon(x_{j+1}, \theta_i, t_{m+1}) - 2W_\varepsilon(x_j, \theta_i, t_{m+1}) + W_\varepsilon(x_{j-1}, \theta_i, t_{m+1})}{\delta x^2} \\
& - \varepsilon^2 \frac{W_\varepsilon(x_j, \theta_{j+1}, t_{m+1}) - 2W_\varepsilon(x_j, \theta_j, t_{m+1}) + W_\varepsilon(x_j, \theta_{j-1}, t_{m+1})}{\delta \theta^2} \\
& = 2\varepsilon dwupwind \frac{u(\theta_{j+1}, t_{m+1}) - u(\theta_{j-1}, t_{m+1})}{2\delta \theta} + W_\varepsilon(K(x) - \rho_\varepsilon(x)) + W_\varepsilon H,
\end{aligned} \tag{1.25}$$

$$\begin{aligned}
& W_\varepsilon(x_j, \theta_i, t_{m+1}) - \frac{\tau D}{\delta x^2} [W_\varepsilon(x_{j+1}, \theta_i, t_{m+1}) - 2W_\varepsilon(x_j, \theta_i, t_{m+1}) + W_\varepsilon(x_{j-1}, \theta_i, t_{m+1})] \\
& - \frac{\varepsilon^2 \tau}{\delta \theta^2} [W_\varepsilon(x_j, \theta_{j+1}, t_{m+1}) - 2W_\varepsilon(x_j, \theta_j, t_{m+1}) + W_\varepsilon(x_j, \theta_{j-1}, t_{m+1})] \\
& = \tau(2\varepsilon dwupwind \frac{u(\theta_{j+1}, t_{m+1}) - u(\theta_{j-1}, t_{m+1})}{2\delta \theta} + W_\varepsilon(K(x) - \rho_\varepsilon(x)) + W_\varepsilon H) + W_\varepsilon(x_j, \theta_i, t_m),
\end{aligned} \tag{1.26}$$

$$\begin{aligned}
& W_\varepsilon(x_j, \theta_i, t_{m+1}) - \frac{\tau D}{\delta x^2} W_\varepsilon(x_{j+1}, \theta_i, t_{m+1}) + \frac{2\tau D}{\delta x^2} W_\varepsilon(x_j, \theta_i, t_{m+1}) - \frac{\tau D}{\delta x^2} W_\varepsilon(x_{j-1}, \theta_i, t_{m+1}) \\
& - \frac{\varepsilon^2 \tau}{\delta \theta^2} W_\varepsilon(x_j, \theta_{i+1}, t_{m+1}) + \frac{2\varepsilon^2 \tau}{\delta \theta^2} W_\varepsilon(x_j, \theta_i, t_{m+1}) - \frac{\varepsilon^2 \tau}{\delta \theta^2} W_\varepsilon(x_j, \theta_{i-1}, t_{m+1}) \\
& = \tau(2\varepsilon dwupwind \frac{u(\theta_{i+1}, t_{m+1}) - u(\theta_{i-1}, t_{m+1})}{2\delta \theta} + W_\varepsilon(K(x) - \rho_\varepsilon(x)) + W_\varepsilon H) + W_\varepsilon(x_j, \theta_i, t_m),
\end{aligned} \tag{1.27}$$

$$\begin{aligned}
& W_\varepsilon(x_j, \theta_i, t_{m+1}) + \frac{2\tau D}{\delta x^2} W_\varepsilon(x_j, \theta_i, t_{m+1}) + \frac{2\varepsilon^2 \tau}{\delta \theta^2} W_\varepsilon(x_j, \theta_i, t_{m+1}) \\
& - \frac{\tau D}{\delta x^2} W_\varepsilon(x_{j+1}, \theta_i, t_{m+1}) - \frac{\tau D}{\delta x^2} W_\varepsilon(x_{j-1}, \theta_i, t_{m+1}) \\
& - \frac{\varepsilon^2 \tau}{\delta \theta^2} W_\varepsilon(x_j, \theta_{i+1}, t_{m+1}) - \frac{\varepsilon^2 \tau}{\delta \theta^2} W_\varepsilon(x_j, \theta_{i-1}, t_{m+1}) \\
& = \tau(2\varepsilon dwupwind \frac{u(\theta_{i+1}, t_{m+1}) - u(\theta_{i-1}, t_{m+1})}{2\delta \theta} + W_\varepsilon(K(x) - \rho_\varepsilon(x)) + W_\varepsilon H) + W_\varepsilon(x_j, \theta_i, t_m),
\end{aligned} \tag{1.28}$$

$$\begin{aligned}\partial_t n_\varepsilon - D(\theta)\Delta_x n_\varepsilon - \varepsilon^2 \Delta_\theta n_\varepsilon &= n_\varepsilon (K(x) - \rho_\varepsilon(x)), \\ \rho_\varepsilon &= \int_0^1 n_\varepsilon(x, \theta) d\theta\end{aligned}\tag{1.29}$$

$$\begin{aligned}& \frac{n_\varepsilon(x_j, \theta_i, t^{m+1}) - n_\varepsilon(x_j, \theta_i, t^m)}{\delta t} - D(\theta_i) \frac{n_\varepsilon(x_{j+1}, \theta_i, t_{m+1}) - 2n_\varepsilon(x_j, \theta_i, t_{m+1}) + n_\varepsilon(x_{j-1}, \theta_i, t_{m+1})}{\delta x^2} \\ & - \varepsilon^2 \frac{n_\varepsilon(x_j, \theta_{i+1}, t_{m+1}) - 2n_\varepsilon(x_j, \theta_i, t_{m+1}) + n_\varepsilon(x_j, \theta_{i-1}, t_{m+1})}{\delta \theta^2} - n_\varepsilon(x_j, \theta_i, t_{m+1}) (K(x_j) - \rho_\varepsilon(x_j)) = 0\end{aligned}\tag{1.30}$$

$$\begin{aligned}& n_\varepsilon(x_j, \theta_i, t^{m+1}) - D(\theta_i) \delta t \frac{n_\varepsilon(x_{j+1}, \theta_i, t_{m+1}) - 2n_\varepsilon(x_j, \theta_i, t_{m+1}) + n_\varepsilon(x_{j-1}, \theta_i, t_{m+1})}{\delta x^2} \\ & - \varepsilon^2 \delta t \frac{n_\varepsilon(x_j, \theta_{i+1}, t_{m+1}) - 2n_\varepsilon(x_j, \theta_i, t_{m+1}) + n_\varepsilon(x_j, \theta_{i-1}, t_{m+1})}{\delta \theta^2} - \delta t n_\varepsilon(x_j, \theta_i, t_{m+1}) (K(x_j) - \rho_\varepsilon(x_j)) = n_\varepsilon(x_j, \theta_i, t^m),\end{aligned}\tag{1.31}$$

$$\begin{aligned}& n_\varepsilon(x_j, \theta_i, t^{m+1}) + 2D(\theta_i) \delta t / \delta x^2 n_\varepsilon(x_j, \theta_i, t^{m+1}) - D(\theta_i) \delta t \frac{n_\varepsilon(x_{j+1}, \theta_i, t_{m+1}) + n_\varepsilon(x_{j-1}, \theta_i, t_{m+1})}{\delta x^2} \\ & + 2\varepsilon^2 \delta t / \delta \theta^2 n_\varepsilon(x_j, \theta_i, t_{m+1}) - \varepsilon^2 \delta t \frac{n_\varepsilon(x_j, \theta_{i+1}, t_{m+1}) + n_\varepsilon(x_j, \theta_{i-1}, t_{m+1})}{\delta \theta^2} \\ & - \delta t n_\varepsilon(x_j, \theta_i, t_{m+1}) (K(x_j) - \rho_\varepsilon(x_j)) = n_\varepsilon(x_j, \theta_i, t^m)\end{aligned}\tag{1.32}$$

$$\rho_\varepsilon = \int_0^1 n_\varepsilon(x, \theta) d\theta = \sum_{i=1}^M \int_{\theta_{i-1}}^{\theta_i} n_\varepsilon(x, \theta) d\theta$$

$$\begin{aligned}
& -D(\theta)\Delta_x n_\varepsilon - \varepsilon^2 \Delta_\theta n_\varepsilon = n_\varepsilon(K(x) - \rho_\varepsilon(x)), \\
& \rho_\varepsilon = \int_0^1 n_\varepsilon(x, \theta) d\theta
\end{aligned} \tag{1.33}$$

$$\begin{aligned}
& 2D(\theta_i)/\delta x^2 n_\varepsilon(x_j, \theta_i, t_{m+1}) - D(\theta_i) \frac{n_\varepsilon(x_{j+1}, \theta_i, t_{m+1}) + n_\varepsilon(x_{j-1}, \theta_i, t_{m+1})}{\delta x^2} \\
& + 2\varepsilon^2/\delta \theta^2 n_\varepsilon(x_j, \theta_i, t_{m+1}) - \varepsilon^2 \frac{n_\varepsilon(x_j, \theta_{i+1}, t_{m+1}) + n_\varepsilon(x_j, \theta_{i-1}, t_{m+1})}{\delta \theta^2} \\
& - n_\varepsilon(x_j, \theta_i, t_{m+1}) (K(x_j) - \rho_\varepsilon(x_j)) = 0
\end{aligned} \tag{1.34}$$