Homework1

Xueying_Li&Huiqiong_Liao

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Abstract

We use Monte Carlo method to estimate the value of the distribution of N(0,1) at $n \in \{10^2, 10^3, 10^4\}$ at $t \in \{0.0, 0.67, 0.84, 1.28, 1.65, 2.32, 2.58, 3.09, 3.72\}$. Then we form the table including the mean value of 100 times experiments and the true value. Also, we generate the box plots of the bias at all t. $\Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$This is a template mainly designed for data science lab projects. In this template, we review most common components of a single R Markdown document with the power of the **bookdown** package and demonstrate their basic usage through examples.

The application of Monte Carlo methods

The distribution function of $N(0, 1)$,

$$\Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy, \tag{1.1}$$

We use the formular below to estimate the value of the formular 1.1.

$$\hat{\Phi}(t) = \frac{1}{n} \sum_{i=1}^{n} I(X_i \leq t), \tag{1.2}$$

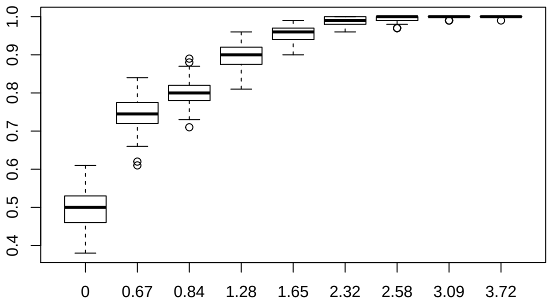
where X_i's are iid $N(0, 1)$ variables. We wrote the function phi_cal to calculate the $\hat{\Phi}(t)$ given specific t and n. The R code chunk like this:

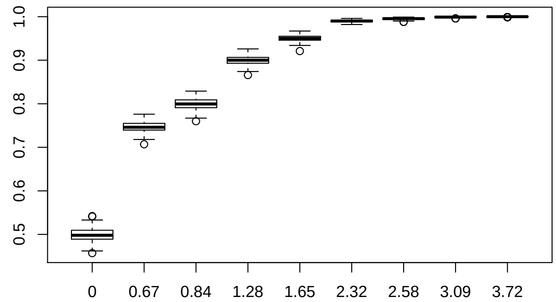
```r
phi_cal <- function(t,n){
  x <- rnorm(n,0,1)
  I_x <- array(n)
  for (i in 1:n) {
    if(x[i]>t)
      I_x[i] <- 0
    else
      I_x[i] <- 1
  }
  phi <- sum(I_x)/n
  return(phi)
}
```

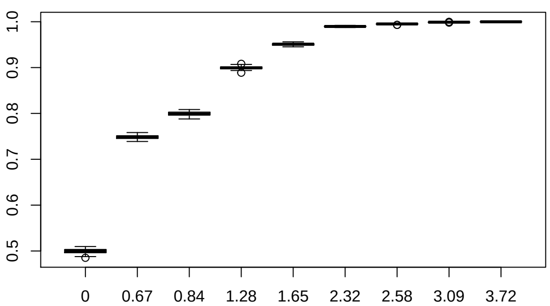
Then similarly, we wrote the function phi_real to get the real value of the cdf of $N(0, 1)$. The R code chunk like this:

```r
phi_real <- function(t){
  y <- pnorm(t,0,1)
  return(y)
}
```

The next step is to experiment for 100 times and draw the box plts of the bias at all t. The results are shown as the follow 3 pictures.







Then we calculate the mean of the **phi_cal**. We use a table to present out results compared with the real value of the normal distribution, whose first three lines corespond to n = 100, 1000, 10000 respectively and the last line of table is the real value. Here is the table:

	0	0.67	0.84	1.28	1.65	2.32	2.58	3.09	3.72
n=100	0.501300	0.7530000	0.8021000	0.9041000	0.9507000	0.9914000	0.995900	0.9990000	0.9998000
n=1000	0.500560	0.7483900	0.7994300	0.9019300	0.9510400	0.9898700	0.994930	0.9988600	0.9998900
n=10000	0.499588	0.7477820	0.7994170	0.8994280	0.9509850	0.9899150	0.995065	0.9989880	0.9998910
real_value	0.500000	0.7485711	0.7995458	0.8997274	0.9505285	0.9898296	0.995060	0.9989992	0.9999004