Judges' Commentary: Moving North

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Introduction

Problem A of the 2020 MCM asked contestants to examine the impact of rising global ocean temperatures on the Scottish North Atlantic fishery. Teams focused on two specific fisheries, herring and mackerel (a fishery is a collection of fish of a given species and the area that they inhabit). The goal of the problem was to investigate potential changes to the migration of herring and mackerel as global ocean temperatures rise. Teams were asked to make suggestions to Scottish fishing companies for moving their operations, updating fishing vessels, and changing business practices in response to the fish movement.

Teams focused on this real-world problem in two distinct stages:

- First, they built a mathematical model to link global ocean temperatures to the local migration of herring and mackerel near Scotland. Most teams first modeled the ocean surface temperature and then linked fish migration to optimal temperature ranges for each species, giving a prediction for the fishery location in the future.
- Next, teams took a wide range of approaches to analyze the impact that the fishery movement would have on local fishing operations. Teams

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were expected to make practical suggestions for the fishing operations based on clearly explained models.

We discuss the questions posed, provide an overview of how teams approached each part of the problem, and comment on issues that arose in the models developed and results presented. Finally, we provide advice and address a variety of important topics highlighted by this year's judges.

This article does not provide an overview of the judging process. We highly recommend that advisors and students read commentaries from previous years [Shannon 2016; Wilhelm 2017; Bieri and Black 2018; Bieri 2019] for more information about the judging process and other important aspects of the contest.

The Modeling Problem

We discuss the problem statement and an overview of the approaches used by teams in the modeling process. Judges are aware of the time and resource constraints that teams face when participating in the contest, and we make sure that judging is based on effort rather than on judges' expectations. As part of this process, judges read the problem statement carefully, read a random sample of papers, and work from a broad common rubric that is based on what teams were able to achieve.

The Questions

The problem asked teams to address four distinct problem areas and to summarize their results in an article for *Hook Line and Sinker* magazine.

1. Teams needed to build a mathematical model to identify the most likely locations for two species of fish, Scottish mackerel and herring, over the next 50 years, assuming that water temperatures will continue to change enough to cause population movement.

2. Based on their location projections, teams were asked to predict the best-case, worst-case, and most-likely scenario for when the populations would be too far away for small fishing companies—with limited financial resources to invest in new equipment—to harvest.

3. Teams were then expected to use the results from their model to identify and assess practical and economically-attractive strategies for the small fishing companies. They could consider relocation of assets, a change in fishing vessels, or other options.

4. In their final modeling step, teams were asked to address how their proposed strategy for the small fishing companies would be affected if a proportion of the fishery moved into the territorial waters of another

country, defined as coastal waters extending at most 12 nautical miles (22.2 km; 13.8 mi) from the baseline (usually the mean low-water mark) of a coastal state.

The intent of the *Hook Line and Sinker* magazine article was to help fishermen understand the seriousness of the problem of rising ocean temperatures and fish migration, and to explain how the team's proposed solution(s) would improve future business prospects.

Models

Part I: Water Temperature and Fish Migration

There were two elements to this part of the problem. Teams were asked first to predict water temperature changes over the next 50 years, and then to use their results to predict fish migration.

Teams used various statistical models to forecast sea surface temperature (SST) based on historical data. Many used an Autoregressive Integrated Moving Average (ARIMA) [Bisgard and Kulahci 2011] or a Grey Prediction Model [Deng 1989]. Some teams predicted SST changes using latitude alone. There were even good examples of simple regression models and curve-fitting to achieve temperature predictions. Most of the top papers developed a two-dimensional SST model and explained the steps that they took to obtain a map of sea surface temperatures.

Next, teams used their SST models to predict the movement of the fish population. Teams ran simulations or used cellular automata models with a Moore-neighborhood [Ermentrout and Edelstein-Keshet 1993] to predict how the fish would react to changes in SST. One of the Outstanding papers used an agent-based model and NetLogo to predict fish migration.

It was important to the judges that teams made an effort to model both species of fish. Herring and mackerel have different habitable temperature requirements and thus different migration paths as sea surface temperatures change. This difference in turn affects how the fishing companies would need to react to improve their future business prospects. Treating herring and mackerel separately also served as a way for the top teams to sanity-check their model that linked temperature change to migration.

Part II: Operational Lifespan for Small Fishing Companies

To model the radial distance that small fishing boats could travel from their current ports to harvest and deliver quality fish, many groups used boat velocity and estimated the amount of time that a catch would remain fresh. They then paired the resulting distance with their fish migration model to see when the fish would effectively be out of range. To predict how the quality (i.e., freshness) of the catch deteriorates over time, some teams turned to food-control literature and employed a nutrient loss model [Labuza et al. 1978].

Part III: Operational Changes for Small Fishing Companies

The teams that stood out in this part considered several different options such as company relocation, using new fishing boats, and exploring their own solutions, such as harvesting a new species of fish. For these operational decisions, many teams performed a cost-benefit analysis or used an analytic hierarchy process (AHP) [Thomas and Doherty 1980]. In AHP models, teams evaluated alternate strategies (e.g., relocation and new fishing boats) based on criteria such as economic and technical feasibility. Successful modelers who used methods such as the AHP also included enough information to justify the weights in the process; otherwise, an AHP model is just an arbitrary set of weights.

Judges did not expect teams to do a full economic analysis at the micro level of fishing operations; some teams spent too much time doing this. Judges recommend that teams include in their model only the things that are very likely to impact the final result.

Part IV: Complications Due to Territorial Waters

The best teams addressed every part of the problem, even if only briefly, as was usually the case for this territorial waters question. Some teams determined the time when fish would move into territorial waters, and then reassessed their result from Part II of predicting when fishing companies can no longer harvest under their current operations. Other successful teams modified their AHP model by adjusting the weights assigned to the factors being considered by the fishing companies.

Part V: Article for *Hook Line and Sinker*

Teams were successful in this part of the problem if their *Hook Line and Sinker* article was well written for a general audience and included specific recommendations but omitted the technical details of the model. Technical language should be reserved for the executive summary and main report. In this day and age, it is so important for mathematical modelers to be able to present their results to a non-technical audience. Finalists stood out by succeeding in both the mathematical modeling and general communication aspects of the problem.

Model Development and Presenting Results

Building a mathematical model is a balancing act between adding in a sufficient amount of detail to make the model applicable to real life and leaving out enough detail, through careful assumptions, so that the model is simple enough to obtain results. Many teams developed models that were too complex for them to answer all of the questions in a reasonable amount of time. It might have been beneficial for these teams to start with a simpler model and try to answer the questions with their simple model, before adding in complexity. The top teams built a strong but focused initial model with reasonable assumptions and then used that model to inform later parts of the problem. This year, the model for temperature change and migration was the foundation on which all other parts of the problem were built.

Every year, more teams are using pre-packaged numerical and statistical algorithms. This has led to great results in some cases, but in other cases to "black box" models that fall far from the intent of the contest. It is extremely important that teams do some exploratory data analysis to support their choice of algorithm and to explain how their computational techniques are working. A good approach is to show a flow diagram for the major mathematical steps in the algorithm and discuss why the model or technique is applicable to the current system. If using machine learning techniques, explain the parameter choices; if using statistical models, underlying assumptions must be justified by the data, and the data sources must be documented.

One way to stand out among the final papers in the competition is to add something unique to your model and to be self-reflective about your modeling process. Top teams often proposed options not specifically mentioned in the problem statement, such as exploring the strategy of fishing haddock, or including a comprehensive introspection and identifying the relative weaknesses of their model. There is a lot of value in knowing your model limitations.

Finally, in most years of the contest, teams are asked to write both an executive summary and a general-audience article or letter. Decisions about the top teams often come down to these components. Remember that the audience for your executive summary is fellow mathematical modelers, whereas the additional article or letter is usually for a general audience. Both of these documents should contain results. The executive summary can include more jargon and should explain the top methods and results from your paper. The general audience piece should superficially explain the modeling process and communicate the top results to people who do not have a mathematics background. This year, very few teams did well on both of these tasks.

Other Comments

We discuss a number of important but subtle topics that arose during the final judging process. These topics, along with themes discussed in previous judges' commentaries [Shannon 2016; Wilhelm 2017; Bieri and Black 2018; Bieri 2019], should serve as a resource for contestants. Other recommended resources are the past Outstanding papers and problems published in this *Journal*.

General Modeling Comments

Teams with Outstanding papers exemplify these qualities:

- They find a way to add unique ideas to their model or combine existing models in an unique way;
- they clearly communicate the various aspects of the modeling process;
- they do research into the problem that allows for clearly-formulated assumptions and a well-written introduction;
- they take care to include the most important figures, parameters, and equations in their paper; and
- they address every part of the problem.

Unique ideas "add value" to the model, creating a new and interesting way to think about a problem. These ideas should not be unnecessarily complicated and could be as simple as layering two existing models to create something new. Adding your team's special spin to a problem, and explaining it well, goes a long way in the eyes of the final judges.

One of the most important skills for a mathematical modeler is the ability to communicate mathematics to others. Explain why you are choosing your approach and why your assumptions make sense. Define all parameters or variables used. Explain the equations presented. In fact, another mathematician should be able to replicate your model after reading your paper. If you use an existing model, make sure to include a citation that a reader could use to learn more about the approach. If you present a graph or table, describe in words what the reader should be looking for in the data presented.

Research into the problem should not be underestimated. Some teams make use of a "Background" or "Literature Review" section that is helpful in setting the stage. More importantly, teams need to carefully choose assumptions and include justifications, along with citations if appropriate. It is okay to choose values arbitrarily for some parameters. But be up-front and clear that your choices are part of the assumptions, and then examine the sensitivity of your results to changes in the parameter values.

Sometimes it is hard to decide which parts of your model "make the cut" at the end of a project. Quite often there are large parts of your effort that are not included in the final write-up. The modeling paper should include all of the important methods and results, so you can show the reader your best. Avoid spending a lot of time writing about something that did not work or is not used for answering the problem questions. Condensing your results into a few high-impact graphs and tables is also a way to stand out.

Judges understand that teams are under time pressure. We are always impressed with how much the teams accomplish in just one weekend. This being said, make sure to address every part of the problem. Teams should carefully budget their time. Plan ahead for when you will stop spinning your wheels on one part of the model and move on. Hitting all of the parts of the problem at a decent depth will get you farther than focusing on only one part in extreme detail. Teams who use a simpler model, demonstrate their modeling process, and address all of the questions are ranked higher than teams with a very complicated model who only address one or two parts of the problem.

Sensitivity

An essential part of the modeling process is assessing how well the model actually works. This is why performing a sensitivity analysis is important. Highly successful teams investigate a wide range of parameters and re-interpret their solution based on their sensitivity analysis.

An analysis of how modifying a parameter changes your final results is one of the best ways to show your model in use. Not only can you test your choice of parameters but you can also show that your model applies to a wide range of values, environments, or applications.

Sensitivity analysis does not have to be done as a separate section at the very end. In some cases, it is more appropriate to do sensitivity analysis throughout the paper as each part of the model is introduced. We also recommend to perform a sanity check throughout the modeling process to quickly evaluate whether your result can possibly be true.

Writing and Formatting

Formatting, writing, and communicating mathematics is a vital part of mathematical modeling. Clarity of writing allows the audience to read the paper with ease from start to finish without having to "flip" back and forth between pages. Proper grammar and spelling along with clear overall structure is critical. Here is a quick list of some things to avoid when writing a mathematical paper:

- Misspellings. Remember that sometimes a spell-checker auto-corrects to the wrong word.
- Incomplete sentences or poor structure. Often this can be avoided by reading your paper out loud before submitting it, and having every member of the team do a final read-through of the paper.
- Equations without a written description.
- Inconsistent fonts throughout the paper.
- Too many words in boldface.
- Figures so small that the font is illegible.

Conclusion

This year's MCM Problem A asked teams to model fisheries under changing sea temperatures and to develop practical and economically attractive strategies for smaller fishing companies to take in response to these changes.

Most teams built successful models for predicting how a fish population will move in response to changes in sea surface temperature. The best approaches considered all parts of the problem, rationalized their choices, and expanded and tested their model in each part. Outstanding teams had clear explanations of their modeling process and were effective in communicating their results both to fellow mathematicians and to a general audience.

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