Judges' Commentary: Fungi

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Introduction

Problem A of the 2021 MCM asked participants to build a model that describes the breakdown of organic matter by the activity of fungal communities.

The goals of the problem included

- linking the breakdown of ground litter and woody fibers to the growth of fungi,
- analyzing how different climates and rapid fluctuations in climate might impact the fungal community,
- modeling the interactions between multiple species of fungi, and
- discussing the effect of biodiversity.

Teams were provided a summary of a research article by Lustenhouwer et al. [2020] that explored how several fungal traits affect the decomposition of wood and how different fungi are better able to adapt to changes in moisture and temperature.

In their approach to this problem, teams focused on a few distinct stages.

- First, teams built a model of fungal growth, which is often measured by hyphae, or the cells that branch out to form the structure of a fungus. The hyphal extension rate and the density of hyphe in a given volume are common measures of fungal growth. This growth, which can be impacted by moisture, temperature, and limited resources, is linked to the breakdown of organic material as part of the carbon cycle.
- Next, teams investigated how different kinds of fungi react to different environments including arid, semi-arid, temperate, arboreal, and tropical climates.

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- Then the analysis was expanded to investigate the interaction between species, including the impact of biodiversity and both short- and long-term trends of fungal persistence.
- Finally, teams wrote a two-page article to discuss recent developments in our understanding of the role that fungi play in ecological systems, and to communicate their results in a way that would be appropriate for an introductory college-level biology textbook.

The goal of this commentary is to take a look at the questions posed in Problem A of this year's MCM contest. We provide an overview of student approaches to each part of the problem and discuss important feedback as it pertains to issues with some of the models developed and results presented. We hope that this will provide insight into some of the topics that were most important in this year's judging. Each year, the commentary focuses on just a subset of important aspects that set apart the best modeling papers; consequently, the commentary does not represent a complete overview of all important aspects of the judging process. We encourage teams to read commentaries from previous years [Wilhelm 2017; Bieri 2018; Bieri 2019; Bieri 2020] to get a broad overview of the aspects important to the contest and to the judging process.

The Modeling Problem

Judges are aware of the time and resource constraints that student teams face when participating in the contest, and we make sure that judging is based on student effort rather than judges' expectations. As part of this process, judges read the problem statement carefully, read a random sample of student papers, and work from a broad common rubric based on what student teams were able to achieve. We are also aware of the impacts that the covid pandemic has had on a team's ability to work together, since many teams competing in this year's contest needed to do so virtually.

The Questions

The MCM Problem A asked students to address five distinct problem areas, to summarize their results in an article for an introductory college-level biology textbook, and to write an executive summary for the judges. Most teams divided the process into four parts.

I Breakdown of Organic Matter

- Teams needed to build a mathematical model to describe the breakdown of ground litter and woody fibers through fungal activity.

II Species Interaction

- Teams were asked to expand their model to incorporate the interactions between different species of fungus, which have different growth characteristics and tolerances.

III Analysis

- Teams were expected to use their model to describe the interactions between species including both short- and long-terms trends, the sensitivity to rapid fluctuations in the environment, and the overall impact of changing atmospheric trends. The goal was to assess the impact of variation in local weather patterns.
- Teams should have included predictions about the advantages and disadvantages for combinations of fungal species and the likelihood of persistence for a range of environments including arid, semi-arid, temperate, arboreal, and tropical.
- In their final modeling step, teams were asked to address how the diversity of fungal communities impacts the overall efficiency of a system in the breakdown of organic material, and to investigate the importance of biodiversity.

IV Textbook Article and Executive Summary.

- The intent of the two-page textbook article was to communicate the most important results of their model and their understanding of the role that fungi play in ecological systems, at a level appropriate for college-level introductory biology students.
- As always, teams were expected to create an executive summary of their modeling process and results.

Models

Part I: The Breakdown of Organic Material

Many teams approached the first part of the problem by assuming a logistic growth model. Some teams modeled the density of the fungi in the medium, while others modeled the area covered by radial growth of the hyphe. It was important when modeling radial growth to consider the fact that area and radius are not linearly dependent. Teams that accounted for this type of important spatial logic stood out to the judges. Most teams parametrized the growth rate and other aspects using the given data and a regression analysis for various types of fungi. It was important that teams based parameter values on data, or minimally, explained their rationale for each parameter value (or range) used.

A few teams that took rarer approaches. For example, some teams used a Gompertz model for fungal population growth [Winsor 1932], and a few teams took a chemical modeling approach that included the analysis of how woody fibers decompose given temperature and moisture levels. A number of teams attempted cellular-automata approaches [Ermentrout and Edelstein-Keshet 1993], but many proved that it was difficult to formulate initialization and transition rules that were truly connected to the real-world information. In an effort to improve the assumptions of the logistic model, some teams used curve fitting and real data to make the intrinsic growth rate a well-reasoned function of the temperature and moisture.

Part II: Allowing for Species Interaction

Teams needed to extend their ideas from the single species model to a model that allowed for interaction between different types of fungi. Most approaches either used a Lotka-Voltera approach for species interaction or updated their transition assumptions for the cellular-automata approach. Teams needed to make careful assumptions for how the interactions among species affect each population. There were various successful approaches; one used Elo ranking methods [Maynard et al. 2017] to measure the competitive ranking of each species of fungi and thus parametrize the Lotka-Voltera model.

When modeling species interaction, one complication can be the sheer number of interacting species and thus the huge number of parameters needed. One interesting approach to simplify the fungi system was to use clustering analysis to assign the original 34 fungal isolates to subgroups with similar temperature and moisture requirements. Most teams that took this approach used a k-means clustering analysis. It was particularly important for teams to give some logical reasoning behind assumptions, such as number of clusters or the distance metric used. Some teams also used a mass-dependent competition assumption, which implied that competition only started to affect the fungi if total population of all isolates in the system started to exceed a critical mass and thus exceed the limited resource.

Part III: Analysis

Once the basic models were built, teams needed either to solve the model equations or run simulations to answer multiple modeling questions. Many teams used numerical approaches that were built into Matlab or Python. Judges appreciate teams that took the time to specify what solution method and package they used; for example, in Matlab many used `ode45()` but only a few specified that this program uses an explicit Runge-Kutta method and hence is a good default package for solving systems of ordinary differential equations.

In an attempt to consider fluctuations in the environment, seasonality, and both long- and short-term trends, the most successful teams clearly indicated how they were incorporating these fluctuations into their model. One approach was to choose a function, such as a sine wave, to approximate time-dependent fluctuations in temperature and moisture. Another approach was to gather real data about temperature and moisture at a specific location. It was important that teams were clear about how these fluctuations were tied back into their fungi model, such as through the temperature- and moisture-dependent growth rate or transition rules.

Predictions for the advantages and disadvantages for fungi combinations in different environments were often approached by simulating multiple combinations and environmental locations. Teams that were careful about describing this experimental process and presenting their results were most successful in this challenge. Some teams assumed certain parameter ranges for each of the five environments, while others chose specific global locations and used real data. Both approaches were successful as long as the reasoning was well explained.

The final part of the modeling process asked teams to address the issue of biodiversity. Again, teams needed to use their models to run careful experiments to address this question. There was a variety of successful approaches to this problem, ranging from simulating an increasing number of fungi to show how the overall population supports the long-term stability of the population, to exploring how types of fungi, in terms moisture or heat tolerance, contribute to the breakdown of organic material in the presence of environmental fluctuations.

Part IV: Textbook Article and Executive Summary

The executive summary and additional article (this year, a textbook article) are the teams' chance to demonstrate their ability to communicate their model to a variety of audiences in a very compressed space.

Most teams did quite a good job this year on the executive summary. Judges always look for teams to use the executive summary to give a technical overview of their modeling process and—importantly—a summary of the most important modeling results. The most successful teams avoided using the executive summary to simply list the models used and instead included a summary of their thought process about how the model was constructed and what results were achieved.

In contrast to the executive summary, which is intended for a technical audience, the textbook article was intended for an audience of biology undergraduates. This year, more weight was put on this item, since it truly shows how teams can communicate their model to others. Judges also expected an audience-appropriate summary of the model to appear in the textbook article. Communication of the ideas behind—in addition to giving the results of—mathematical models is an important part of professional mathematical modeling. The textbook articles that stood out to the judges gave an appropriate background to the problem and spent most of the two pages describing the model and results from it.

Model Development and Presenting Results

Most multi-species interaction or competition models have a huge number of parameters. This year, that included parameters for such items as growth rates for each fungal isolate, carrying capacities for individual isolates and/or the population as a whole, interaction and competition parameters, and underlying temperature and moisture data. It was vital that teams specified what parameters were used. One goal of a good modeling paper is for it to be reproducible by the reader. Without exact parameter values used, the model cannot be confirmed. Ideally, parameters can be found by referencing research papers or real data; but occasionally it is not possible to find a good parameter value and one must be assumed. This is okay as long as it is clearly stated that the value is an assumption, and a sensitivity analysis of the results to this assumption is performed.

When presenting results for complicated models, teams should consider the best ways to get information across to the readers. The best papers avoided adding many small graphs in favor of fewer high-quality figures. If your graphs or images are too small, the judges cannot interpret the results depicted. Teams that were creative in compiling results from many experimental runs into just one or two graphs often had the space within the 25-page limit to include large high-quality images and a discussion of each image. The font in each graph or image should be approximately the same size as the font in the paper.

Finally, this year was unique due to the global covid pandemic. Traditionally, judges expect the teams to create models that clearly build upon each other from one question to the next. However, this year, many teams had to collaborate remotely; and in some cases, there was less cohesion between different sections of the paper. Judges were understanding of the unique limitations that some teams faced this year.

Other Comments

In this section we discuss a number of important, but subtle, topics that arose during the final judging process and apply generally from year to year. These topics, along with themes discussed in previous judges' commentaries [Wilhelm 2017; Bieri 2018; Bieri 2019; Bieri 2020], should serve as a resource for contestants. Other recommended resources are the past Outstanding papers and MCM problems published in *The UMAP Journal*. We should emphasize that no paper or model is perfect; every year, the judges are impressed with the accomplishments of the mathematical modeling teams, especially given the short amount of time teams have to create, test, use, and analyze their models. We truly admire and appreciated the work that went into every entry. The following comments are an effort to help teams truly perfect their work.

Page Limits

This year, the modeling paper had a strict 25-page limit that included the executive summary, the modeling project, the 2-page article, and any appendices. This means that teams needed to be selective about how they use the space. Teams that reduced figure sizes to meet page limits were not always rewarded for the included images. If the judges cannot see the graph easily, read its text and labels, and interpret the results, then they will not have a favorable view of the paper. Judges strongly encourage teams instead to find novel and interesting ways to present their results in fewer high-quality graphs or tables. Also, some teams choose to include computer code as part of an appendix to the paper. In most cases, judges do not spend a lot of time looking at verbatim code. Instead of using two pages of appendix for code, teams could give a flow diagram or describe the algorithm created as part of the main text. Teams should also avoid putting important results in an appendix: All important results and ideas should be part of the main document.

Solution Methods and Sensitivity

Something that is becoming more common in the modeling papers each year is the use of "black box" solution methods as part of the modeling process. Examples of this include solving systems of equations using Matlab, but not saying what process or program in Matlab was used. Another example is with the application of statistical and machine-learning models in Python.

Whenever applying statistical methods, it is important to explain why the method is appropriate for the data available. If your team chooses to apply machine-learning methods, you must explain important aspects, such as the structure of the underlying model. For example, in k-means clustering, you should report the number of clusters and distance metric used (and why that number of clusters and why that metric). In neural networks, you should explain your choices behind the network structure and hyperparameters. If you find your paper just saying that the model "was solved" without further explanation, you need to go back and say more about how the model was solved.

Every year, it is important that teams address the sensitivity of their results to the choices made in the modeling process. A question that every mathematical modeler should keep in mind is, "If my calculations were a little bit wrong when finding my parameters, how much would that affect my conclusions?" Save a little bit of time at the end, after you have addressed all the questions in the prompt, to run a few more simulations that can answer this question. A careful analysis of sensitivity can also greatly inform your discussion of strengths and weaknesses, which should truly be a discussion and not just a bulleted list.

Background, Assumptions, and References

An important part of introducing your model is giving your audience a bit of background and clearly stating what global assumptions are going to be applied. Teams are getting better about producing a specific list of assumptions, but each assumption should also include rationale for the assumption and in some cases a citation to a reference to support the rationale. It is important for teams to take some time at the beginning of the project to do a little research and gather resources that support their modeling decisions. Teams should also be careful to cite the sources for any images used in the paper that came from external sources.

Writing and Formatting

Formatting, writing, and communicating mathematics is a vital part of mathematical modeling. Often, the defining line between papers that are identified as Finalists and papers that do not comes down to the clarity and organization of the writing. Remember that it is your job to communicate your modeling process and results in a way the your audience can easily follow. Some things teams should keep in mind include:

- Proofread the paper for misspellings, incomplete sentences, and poor sentence structure. Why should a reader trust your results as reliable if you are sloppy about such aspects? Leave time near the end of the contest to do this and to make necessary corrections, revisions, and changes.
- Check that each equation has a written description, all variables and parameters are defined in the text, and that you include a table that summarizes all variables and parameters, together with their descriptions and their units of measurement.
- Look for formatting issues such as inconsistent fonts or too much boldface.
- Double-check that all of your figures are large enough and clearly labeled in a large-enough font.
- If you used a contest-entry template from a previous year, check that you updated items such as titles and section headings, and remove old content.
- For all sources for quotations, paraphrases of quotations, images, and ideas taken from other sources: Double-check that you cite the source in

the text and give full access details (page numbers for articles or books, URLs for web pages) in the references.

Conclusion

This year's MCM Problem A asked students to model how the growth, interaction, and biodiversity of fungi impacts the breakdown of organic matter. Most teams built successful models for both individual fungi and for fungal interaction. The best papers generally did a great job of clearly explaining the assumptions, their model, and the parameters involved. They also used their model in experiments to address all parts of the problem and communicated their results clearly both to fellow mathematicians and biologists (in the summary and main paper) and to a general audience (in the textbook article).

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