Linear algebra HW1
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Strang

P.29 #3 Solve these three equations for y_1, y_2, y_3 in terms of c_1, c_2, c_3:

$$Sy = c, \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

Write the solution y as a matrix $A = S^{-1}$ times the vector c. Are the columns of S independent or dependent?

In order to compute $A = S^{-1}$, we can instead compute $SS^{-1} = I$. Using Gaussian elimination, we could computed A by creating an augumented matrix that has S on its left part and indentity matrix on the right part. Turning the left part indentity matrix will leave the right part become s^{-1}

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

Substract the 3^{rd} row by the 2^{nd} and substract the 2^{nd} row by the 1^{st} row gives us

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

As we can see, the left part is already the identity, so

$$A = S^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Therefore, $y = AC = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$. By the rule of matrix multiplication, we can get

$$\begin{cases} y_1 = c1 \\ y_2 = -c1 + c2 \\ y_3 = -c_2 + c3 \end{cases}$$

The columns in S are independent because there is no way to construct one column using other two columns in S.

p.41 #5

$$x + y + z = 2$$
$$x + 2y + z = 3$$
$$2x + 3y + 2z = 5$$

(Fill in the blanks question) If x, y, z satisfy the first two equations then they also satisfy the 3_{rd} equation because the 3_{rd} equation could be obtained by adding the 1_{st} equation and the 2_{nd} equation.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 30 \\ 1 \\ -29 \end{bmatrix}$$

Axler

p.11 #3 Suppose a and b are real numbers, not both 0. Find real numbers c and d such that $\frac{1}{a+bi} = c + di$.

$$\frac{1}{a+bi} \times \frac{a-bi}{a-bi} = \frac{a-bi}{a^2-bi^2} = \frac{a-bi}{a^2+b^2}$$

$$c = \frac{a}{a^2+b^2}, d = \frac{-b}{a^2+b^2}$$

p.11 #3 Find two distinct square roots of i.

assume $\sqrt{i} = a + bi$

squaring both sides gives us $i = a^2 + 2abi - b^2$

Since the imaginary parts are equal, $2ab = 1, a^2 = b^2$

So $a = b = \pm\frac{\sqrt{2}}{2}$.

Therefore the two square roots of i are $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ and $-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$