Steepest Gradient Method

Problem: $\min_{x \in \mathbb{R}^n} f(x)$

where f is continuously differentiable on $\mathbb{R}^n$.

One strategy: line search

1. At the k-th iterate $x^{(k)}$, find a descent direction $p^{(k)}$

   i.e. $\langle p^{(k)}, \nabla f(x^{(k)}) \rangle < 0$

2. Search a stepsize $\alpha_k > 0$, such that

   $$f(x^{(k)} + \alpha_k p^{(k)}) < f(x^{(k)})$$

3. update the iterate:

   $$x^{(k+1)} = x^{(k)} + \alpha_k p^{(k)}$$

Steepest gradient descent method is a line-search type method.

1. It uses $p^{(k)} = -\nabla f(x^{(k)})$ as the search direction.

   Remark: if $\nabla f(x^{(k)}) \neq 0$, then $p^{(k)}$ is a descent direction,

   because $\langle p^{(k)}, \nabla f(x^{(k)}) \rangle = -\|\nabla f(x^{(k)})\|^2 < 0$