CSCI 2200 HW3
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Problem 15.15 Among 400 students, 150 are in math, 120 are in bio and 50 are math-bio duals. What are the chances a random student is in: (a) math or bio (b) bio and not math (c) neither math nor bio?

(a)$|Math \cup Bio| = |Math| + |Bio| - |Math \cap Bio|$.
Thus

$$\begin{aligned} P(|Math \cup Bio|) &= P(|Math|) + P(|Bio|) - P(|Math \cap Bio|) \\ &= \frac{150}{400} + \frac{120}{400} - \frac{50}{400} \\ &= \frac{11}{20} \end{aligned}$$

(b)$|Math^c \cap Bio - Math| = |Bio - Bio\ capMath|$, since biology is a subset of not math.
Thus

$$\begin{aligned} P(|Math^c \cap Bio|) &= P(|Bio - Bio \cap Math|) \\ &= \frac{70}{400} \\ &= \frac{7}{40} \end{aligned}$$

(c)$|Math \cup Bio|^c = \Omega - |Math \cup Bio|^c$.
Thus

$$\begin{aligned} P(|Math \cup Bio|^c) &= P(\Omega) - P(|Math \cup Bio|^c\,|) \\ &= 1 - \frac{11}{20} \\ &= \frac{9}{20} \end{aligned}$$

Problem 16.37.There are two beavers, brown and black. What are the chances both are male? What if you know: (a) one is male (b) one is male and one is born on a Tuesday (c) one is a male born on a Tuesday? Verify answers with Monte Carlo simulation. How strange, the birthday of a beaver changes the probability of two males.

(a) The sample space is $\{MM, MF, FM\}$

$$P[\text{Both are male}|\text{One is Male}] = \frac{1}{3}.$$

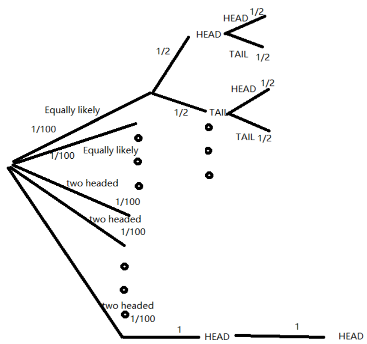
(b). The sample space is $\{MM2, MF2, F2M\}$.

$$P[\text{Both are Male}|\text{One is Male and One is born on Tuesday}] = \frac{1}{3}.$$

(c). The sample space is $\{M1M2, M2M2...., M2M1, ...\}$.

$$P[\text{Both are Male}|\text{One is Male and One is born on Tuesday}] = \frac{13}{27}.$$

Problem 16.42. Five out of 100 coins are two-headed. You randomly pick a coin and flip it "fairly" twice (each side is equally probable). What is the probability to get (a) 2 heads (b) 2 tails (c) matching tosses?



(a). According to the probabability tree, we have $P(2 \text{ heads}) = \sum_{i=1}^{95} \dfrac{1}{400} + \sum_{i=96}^{100} \dfrac{1}{100} = \dfrac{95}{400} + \dfrac{5}{100} = \dfrac{23}{80}$.

(b). According to the probabability tree, we have $P(2 \text{ tails}) = \sum_{i=1}^{95} \dfrac{1}{400} + \sum_{i=96}^{100} 0 = \dfrac{95}{400} = \dfrac{19}{80}$.

(c). According to the probabability tree, we have $P(2 \text{ tosses match}) = \sum_{i=1}^{95} \dfrac{2}{400} + \sum_{i=96}^{100} \dfrac{1}{100} = \dfrac{90}{400} + \dfrac{5}{100} = \dfrac{21}{40}$.

Problem 16.81. You have a fair 5-sided die which can generate one of the numbers $\{1,2,3,4,5\}$ with probability $\frac{1}{5}$ each. You wish to simulate a fair 7-sided die which generates a number in $\{1,2,3,4,5,6,7\}$ with probability $\frac{1}{7}$ each. Give an algorithm to do so, and prove it.

Toss a coint 2 twice, $\forall (i,j) \in \{1...5\} \times \{1...5\}$, we have $(1,1) = 1, (2,2) = 2, (3,3) = 3, (4,4) = 4, (5,5) = 5, (5,1) = 6, (5,2) = 7$. Other wise, we restart.

Prove: Let n be an arbitrary number and $n \in \{1,..,7\}$. We have

$$\begin{aligned} P[n] &= p[n|(1,1)]P[(1,1)] + p[n|(1,2)]P[(1,2)] + ... + p[n|(i,j)]P[(i,j)] + P[n|restart]P[restart] \\ P[n] &= \frac{1}{25}(\text{since there will be only 1 exact match}) + \frac{18}{25}P[n] \\ \frac{17}{25}P[n] &= \frac{1}{25} \\ P[n] &= \frac{1}{7} \end{aligned}$$

Problem 17.35. You have 100 and bet 1 at a time on roulette. You goal is to win 50. Compute the probability that you reach your goal before going bankrupt.

I here assume it is exactly the same roulette as the one on the book.

probability of landing on red (probability of winning) $= \frac{18}{38}$. There are fifty steps away from winning, thus $L = 150$. The formula is $P(k, l, p) = \frac{p^k - p^l}{p^k - 1}$.

Thus we have $P[win] = P(50, 150, p) = \frac{0.9^{150} - 0.9^{50}}{0.9^{150} - 1} = 2.64 \times 10^{-}5$.

Problem 18.59. For 20 fair coin flips, define events A $=$ {equal number of H and T} and B $=$ {first 3 flips are H}. Compute the probabilities: (a) A occurs. (b) B occurs. (c) A and B occur. (d) A or B occur.

(a). We are choosing 10 from 20 $P[A] = \frac{\binom{20}{10}}{2^20} = \frac{183746}{20^20} = 0.17.$

(b). $P[B] = \frac{1}{2^3} = \frac{1}{8}.$

(c). $P[A \cap B] = P[A] \times \binom{17}{7} \times 0.5^7 \times 0.5^{10} = 0.125 \times 0.148 = 0.0185$

(d)$P[A \cup B] = P[A] + P[B] - P[A \cap B] = 0.283.$