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HW01

Problem 1
    1. $\{x$ is even $\land\ y = x + 1\ \}$

    2. $\{1 \leq x \leq 3\}$

    3. $\{x > 0 \land y > 0\}$

    4. $\{x$ is divisible by $50\}$

    5. None

    6. $abs(result * result - x) \leq 0.000001$

Problem 2
    1. Valid;

    2. Valid;

    3. Invalid; we know from code that $i = i + 1$ and $j = j - 1$. Adding those up gives us $i + j = i + j$. From the precondition we know $i + j \neq 0$. Thus to make it correct, the post condition should also be $i + j \neq 0$.

    4. Invalid; the else case is $x \leq y$ instead of $x < y$. Thus, the postcondition should be $\{(m = x \land x > y) \lor (m = y \land x \leq y)\}$

Problem 3
    1. A code E: possibly invalid. There's no correlation between A and E.

    2. C code D: possibly invalid. C is a weaker precondition compared to F. We want a stronger one.

Problem 4
1. $\{x > 0\}$
    x=10;
        $\{x = 10\}$
    y=20-x;
        $\{x = 10 \land y = 20 - x\} \implies \{y = 10 \land x = 10\}$;
    z=y+4;
        $\{z = y + 4 \land y = 10\} \implies \{y = 10 \land x = 10 \land z = 14\}$;
    y=0;
        $\{y = 0\} \implies \{y = 0 \land x = 10 \land z = 14\}$;

1

2. $\{|x| > 11\}$

    x=-x;

        $\{|x| > 11 \wedge x = -x\} \implies \{x > 11 \vee x < -11\}$

    x=x*x;

        $\{x = x * x \wedge x > 11 \vee x < -11\} \implies \{x > 121\};$

    x=x+1;

        $\{x = x + 1 \wedge x > 121\} \implies \{x > 122\};$

3. $\{|x| < 5\}$

    if $(x > 0)$ {

        $\{|x| < 5 \wedge x > 0\} \implies \{0 < x < 5\}$

            $y = x + 2;$

        $\{0 < x < 5 \wedge y = x + 2\} \implies \{2 < y < 7\}$

    } else {

        $\{|x| < 5 \wedge x \leq 0\} \implies \{-5 < x \leq 0\}$

            y=x-1;

      $\{2 < y < 7 \wedge y = x - 1\} \implies \{-6 < y \leq -1\}$

    }

        $\{(-6 < y \leq -1 \wedge -5 < x \leq 0) \vee (2 < y < 7 \wedge 0 < x < 5)\}$

Problem 5

1. $\{wp(x = -5, y > -2x) \implies (y > 10)\}$
   $$x = -5;$$
   $\{wp(z = 2 * x + y, z > 0) \implies (2x + y > 0) = (y > -2x)\}$
   $$z = 2 * x + y;$$
   $\{z > 0\};$

2. $\{wp(code, x > 0 \lor x \le 0) = \{x > 1 \lor x < -3\}\}$
   if $(x > 0)\{$
       $\{wp(x = x + 6, x > 7) = (x + 6 > 7) = (x > 1)\}$
       $x = x + 6;$
   $\}$ else $\{$
       $\{wp(x = 4 - x, x > 7) = (4 - x > 7) = (x < -3)\}$
       $x = 4 - x;$
   $\}$
       $\{x > 7\}$

3. $\{wp(\text{if else statements and codes}, x > 0) = (x > 3) \lor (x \ge -4)\}$
   if $(x > 4)\{$
       $\{wp(x = x - 3, x > 0) = (x - 3 > 0) = (x > 3)\}$
       $x = x - 3;$
   $\}$ else $\{$
       $\{wp((x < -4)$ do $x = x + 3 \land (x \ge -4)$ do $(x = x + 1), x > 0) = (x < -4 \land x > -3) \lor (x \ge -4 \land x > -1) = (x \ge -4)\}$
       if $(x < -4)\{$
           $\{wp(x = x + 3, x > 0) = (x > -3)\}$
           $x = x + 3$
       $\}$else $\{$
           $\{wp(x = x + 1, x \ge -4) = (x + 1 > 0) = (x > -1)\}$
           $x = x + 1$
       $\}$
   $\}$
       $\{x > 0\}$

4. $\{wp(x = y + 2, x > 2y - 1) = (y + 2 > 2y - 1) = (y < 3)\}$
   $$x = y + 2;$$
   $\{wp(z = x + 1, z > 2y) = (x + 1 > 2y) = (x > 2y - 1)\}$
   $$z = x + 1;$$
   $\{z > 2y\};$

5. $\{wp(x \geq 0 \text{ do } z = x \vee x < 0 \text{ do } z = x + 1, z \neq 0) = (x \geq 0 \wedge x \neq 0) \vee (x < 0 \wedge x \neq -1) = -1 < x < 0\}$

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        if (x ≥ 0)
            {wp(z = x, z ≠ 0) = (x ≠ 0)}
        z=x;
        else
            {wp(z = x + 1, z ≠ 0) = (x + 1 ≠ 0) = (x ≠ -1)}
        z=x+1;
        {z ≠ 0}
```

$\{wp(z = x, z \neq 0) = (x \neq 0)\}$

$\{wp(z = x + 1, z \neq 0) = (x + 1 \neq 0) = (x \neq -1)\}$

$\{z \neq 0\}$

Problem 6

1.
$\{x_{pre} < y_{pre}\}$
$\{wp(w = y, w = y_{pre} = x_{post} \wedge y_{post} = x_{pre}) = w = y_{pre} = x_{post} = y_{post} = x_{pre}\}$

$w = y;$
$\{wp(z = x + y - w, w = y_{pre} = x_{post} \wedge y_{post} = x_{pre}) = (z = w - w + y_{post}) = (z = x_{pre})\}$

$z = x + y - w;$
$\{wp(x = w, x_{post} = y_{pre} \wedge y_{post} = x_{pre}) = (w = y_{pre} = x_{post}) \wedge y_{post} = x_{pre}\}$

$x = w;$

$\{x_{post} = y_{pre} \wedge y_{post} = x_{pre}\}$

InSufficient, we need to make sure $x_{pre} = y_{pre}$. The given condition is weaker than $x_{pre} = y_{pre}$.

2
$\{(x = y) \vee (x \neq y \wedge y > 0)\}$
$\{wp((x_1 == y) \text{ do } x_2 = 0 \vee (x_1! = y) \text{ do } x_2 = x_1 * y, x_2 \leq y) = (x_1 = y \wedge y \geq 0) \vee (x_1 \neq y \wedge x_1 \leq 1)\}$

```
        if (x == y);
            {wp(x₂ = 0, x₂ ≤ y) = (y ≥ 0)}
        x = 0;
        else
            {wp(x₂ = x₁ * y, x₂ ≤ y) = (x₁ * y ≤ y) = (x₁ ≤ 1)}
        x = x * y;
        {x ≤ y}
```

$\{wp(x_2 = 0, x_2 \leq y) = (y \geq 0)\}$

$\{wp(x_2 = x_1 * y, x_2 \leq y) = (x_1 * y \leq y) = (x_1 \leq 1)\}$

$\{x \leq y\}$

Insufficient. The given condition is weaker. We also need to make sure $\{x \leq 1\}$ and $y \geq 0$.