CSCI 2200 HW4
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Problem 7.45: Give recursive definitions for the set S in each of the following cases.
(a) $S = \{0, 3, 6, 9, 12, ...\}$, the multiples of 3.
(b) $S = \{1, 2, 3, 4, 6, 7, 8, 9, 11, ...\}$, the numbers which are not multiples of 5.
(c) $S = \{$all strings with the same number of 0's as 1's$\}$ (e.g. $0011, 0101, 100101$).
(d) The set of odd multiples of 3.
(e) The set of binary strings with an even number of 0's.
(f) The set of binary strings of even length.

(a).$S = \{$The multiples of 3$\}$
1.$0 \in S$
2.$x \in S \implies x + 3 \in S$
3. Nothing else is in S.

(b).$S = \{$The numbers which are not multiples of 5$\}$
1.$1 \in S$
2.$x \in S \implies 2x, 3x, 4x \in S$
4. Nothing else is in S.

(c).$S = \{$All strings with the same number of 0's as 1's$\}$
1.$\epsilon \in S$
2.$x \in S \implies 01x \in S$
3.$x \in S \implies x01 \in S$
4.$x \in S \implies 10x \in S$
5.$x \in S \implies x10 \in S$
6.$x \in S \implies 0x1 \in S$
7.$x \in S \implies 1x0 \in S$
8. Nothing else is in S.

(d).$S = \{$The set of odd multiples of 3$\}$
1.$3 \in S$
2.$x \in S \implies (2k + 1)x \in S$, where $k \in \mathbb{N}$
3. Nothing else is in S.

(e).$S = \{$All strings with an even number of 0's$\}$
1.$\epsilon, 1 \in S$
2.$x \in S \implies 0x0 \in S$
2.$x, y \in S \implies xy \in S$
8. Nothing else is in S.

(f).$S = \{$The set of strings with even length$\}$
1.$\epsilon \in S$
2.$x \in S \implies x00 \in S$.
2.$x \in S \implies x01 \in S$.
2.$x \in S \implies x10 \in S$.
2.$x \in S \implies x11 \in S$.
3. Nothing else is in S.

Problem 8.19(c). prove that every RFBT has an odd number of vertices.
1.$\epsilon \in RFBT$
2.$T_1, T_2 \in RFBT \implies T_1, T2$ form a new RFBT.

Base Case: $\epsilon + F = 3$ vertcies, which is odd.
Inductive step: The property preserves in the child.
(a). $T_1 = \epsilon, T_2 = \epsilon$.
$n = 0$.
The property preserves.

(b) $T_1 = \epsilon, T_2 = F$
$n = 2 \times 1 + 1 = 3$
The property preserves.

(b) $T_1 = F, T_2 = \epsilon$
$n = 2 \times 1 + 1 = 3$
The property preserves.

(b) $T_1 = F, T_2 = F$
$n = 2 \times 2 + 1 = 5$
The property preserves.

Problem 9.3(h)

$$\sum_{i=0}^{n}\sum_{j=i}^{n}(i+j)$$

$\sum_{i=0}^{n}\sum_{j=i}^{n}(i+j)$
$= \sum_{i=0}^{n}(\sum_{j=i}^{n} i + \sum_{j=i}^{n} j)$
$= \sum_{i=0}^{n}(n-i+1)i + (\sum_{j=1}^{n} j - \sum_{j=1}^{i} j)$
$= \sum_{i=0}^{n}(ni - i^2 + i) + (\frac{1}{2}n(n+1)) - \frac{1}{2}i(i+1)$
$= \sum_{i=0}^{n}(ni - i^2 + i) + \sum_{i=0}^{n}(\frac{1}{2}n(n+1)) - \sum_{i=0}^{n} \frac{1}{2}i(i+1)$
$= \sum_{i=0}^{n}(ni - i^2 + i) + (n+1)(\frac{1}{2}n(n+1)) - \frac{1}{2}((n+1)(\frac{1}{2}n + \frac{1}{6}n(n+1)(2n+1)))$
$= n\sum_{i=0}^{n} i - \sum_{i=0}^{n} i^2 + \sum_{i=0}^{n} i + (n+1)(\frac{1}{2}n(n+1)) - -\frac{1}{2}((n+1)(\frac{1}{2}n + \frac{1}{6}n(n+1)(2n+1))$
$= \frac{1}{2}n(n+1)(n+2)$

9.3(l)

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\ln{(ij)}$$

$\sum_{i=1}^{n}\sum_{j=1}^{n}\ln{(ij)}$
$= \sum_{i=1}^{n}(\sum_{j=1}^{n}\ln{(i)} + \sum_{j=1}^{n} ln(j))$
$= \sum_{i=1}^{n}(n\ln(i) + \ln(j)!)$
$= n\ln(i)! + n\ln(j)!$
$= n(\ln(i)! + \ln(j)!)$

Problem 10.9: How many zeros are at the end of 1000! ?

This is equivalent to finding n such that $10^n|1000!$. Numbers in the end only depends on 5 since it is the greatest in the prime pair for 10. Therefore the numbers in the end are given by $\left\lfloor \frac{1000!}{5^1} \right\rfloor + \left\lfloor \frac{1000!}{5^2} \right\rfloor + \left\lfloor \frac{1000!}{5^3} \right\rfloor + \left\lfloor \frac{1000!}{5^4} \right\rfloor + ... = 200 + 48 + 1 + 0 + ... = 249$. Therefore there are 249 zeros in the end of 1000!

Problem 10.41(b)(iii). what is the last digit of $2^{70} + 3^{70}$

2^{70} can be re-written as $(2^2)^{35}$. Therefore the last digit of 2^{70} is 4. Likewise, 3^{70} can be re-written as $(3^2)^{35}$. Therefore the last digit of 3^{70} is 9. Summing them up gives us 13. Therefore the last digit is 3.