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HW01

Problem 1

1. $\{x \text{ is even} \wedge y = x + 1\}$

2. $\{1 \leq x \leq 3\}$

3. $\{x > 0 \wedge y > 0\}$

4. $\{x \text{ is divisible by } 50\}$

5. None

6. None

Problem 2

1. Invalid; a negative number times a positive number will always produce a negative number and it will never be 0. To make it correct, we need to change $\{y \leq 0\}$ into $\{y < 0\}$

2. Valid;

3. Invalid; we know from code that $i = i + 1$ and $j = j - 1$. Adding those up gives us $i + j = i + j$. From the precondition we know $i + j \neq 0$. Thus to make it correct, the post condition should also be $i + j \neq 0$.

4. Invalid; the else case is $x \leq y$ instead of $x < y$. Thus, the postcondition should be $\{(m = x \wedge x > y) \vee (m = y \wedge x \leq y)\}$

Problem 3

1. A code E: possibly invalid. There's no correlation between A and E.

2. C code D: possibly invalid. C is a weaker precondition compared to F. We want a stronger one.

Problem 4

1. $\{x > 0\}$

x=10;

$\{x = 10\}$

y=20-x;

$\{x = 10 \wedge y = 20 - x\} \implies \{y = 10 \wedge x = 10\}$;

z=y+4;

$\{z = y + 4 \wedge y = 10\} \implies \{y = 10 \wedge x = 10 \wedge z = 14\}$;

y=0;

$\{y = 0\} \implies \{y = 0 \wedge x = 10 \wedge z = 14\}$;

2. $\{|x| > 11\}$

x=-x;

$\{|x| > 11 \wedge x = -x\} \implies \{x > 11 \vee x < -11\}$

x=x*x;

$\{x = x * x \wedge x > 11 \vee x < -11\} \implies \{x > 121\}$;

x=x+1;

$\{x = x + 1 \wedge x > 121\} \implies \{x > 122\}$;

3. $\{|x| < 5\}$

if $(x > 0)$ {

$\{|x| < 5 \wedge x > 0\} \implies \{0 < x < 5\}$

$y = x + 2;$

$\{0 < x < 5 \wedge y = x + 2\} \implies \{2 < y < 7\}$

} else {

$\{|x| < 5 \wedge x \leq 0\} \implies \{-5 < x \leq 0\}$

y=x-1;

$\{2 < y < 7 \wedge y = x - 1\} \implies \{-6 < y \leq -1\}$

}

$\{-6 < y \leq -1 \vee 2 < y < 7\}$

Problem 5

1. $\{wp(x=-5, y>-2x) \implies (y>10)\}$
   $x=-5;$
   $\{wp(z=2*x+y, z>0) \implies (2x+y>0)=(y>-2x)\}$
   $z=2*x+y;$
   $\{z>0\};$

2. $\{wp(code, x>0 \lor x\le 0)=\{x>7 \lor x<-3\}\}$
   if $(x>0)\{$
   $\{wp(x=x+6, x>7)=(x+6>7)=(x>1)\}$
   $x=x+6;$
   $\}$ else $\{$
   $\{wp(x=4-x, x>7)=(4-x>7)=(x<-3)\}$
   $x=4-x;$
   $\}$
   $\{x>7\}$

3. $\{wp(\text{if else statements and codes}, x>0)=(x>3)\lor(x\ge -4)\}$
   if $(x>4)\{$
   $\{wp(x=x-3, x>0)=(x-3>0)=(x>3)\}$
   $x=x-3;$
   $\}$ else $\{$
   $\{wp((x<-4)\text{ do } x=x+3 \land (x\ge -4)\text{ do } (x=x+1), x>0)=(x<-4\land x>-3)\lor(x\ge -4\land x>-1)=(x\ge -4)\}$
   if $(x<-4)\{$
   $\{wp(x=x+3, x>0)=(x>-3)\}$
   $x=x+3$
   $\}$else $\{$
   $\{wp(x=x+1, x\ge -4)=(x+1>0)=(x>-1)\}$
   $x=x+1$
   $\}$
   $\}$
   $\{x>0\}$

4. $\{wp(x=y+2, x>2y-1)=(y+2>2y-1)=(y<3)\}$
   $x=y+2;$
   $\{wp(z=x+1, z>2y)=(x+1>2y)=(x>2y-1)\}$
   $z=x+1;$
   $\{z>2y\};$

5. $\{wp(x \geq 0$ do $z = x \vee x < 0$ do $z = x+1, z \neq 0) = (x \geq 0 \wedge x \neq 0) \vee (x < 0 \wedge x \neq -1) = -1 < x < 0\}$

if $(x \geq 0)$

$\{wp(z = x, z \neq 0) = (x \neq 0)\}$

z=x;

else

$\{wp(z = x+1, z \neq 0) = (x+1 \neq 0) = (x \neq -1)\}$

z=x+1;

$\{z \neq 0\}$

Problem 6

1. $\{x_{pre} < y_{pre}\}$

$\{wp(w = y, w = y_{pre} = x_{post} \wedge y_{post} = x_{pre}) = w = y_{pre} = x_{post} = y_{post} = x_{pre}\}$

$w = y;$

$\{wp(z = x + y - w, w = y_{pre} = x_{post} \wedge y_{post} = x_{pre}) = (z = w - w + y_{post}) = (z = x_{pre})\}$

$z = x + y - w;$

$\{wp(x = w, x_{post} = y_{pre} \wedge y_{post} = x_{pre}) = (w = y_{pre} = x_{post}) \wedge y_{post} = x_{pre}\}$

$x = w;$

$\{x_{post} = y_{pre} \wedge y_{post} = x_{pre}\}$

InSufficient, we need to make sure $x_{pre} = y_{pre}$. The given condition is weaker than $x_{pre} = y_{pre}$.

2 $\{(x = y) \vee (x \neq y \wedge y > 0)\}$

$\{wp((x_1 == y)$ do $x_2 = 0 \vee (x_1! = y)$ do $x_2 = x_1 * y, x_2 \leq y) = (x_1 = y \wedge y \geq 0) \vee (x_1 \neq y \wedge x_1 \leq 1)\}$

if $(x == y)$;

$\{wp(x_2 = 0, x_2 \leq y) = (y \geq 0)\}$

$x = 0;$

else

$\{wp(x_2 = x_1 * y, x_2 \leq y) = (x_1 * y \leq y) = (x_1 \leq 1)\}$

$x = x * y;$

$\{x \leq y\}$

Insufficient. The given condition is weaker. We also need to make sure $\{x \leq 1\}$ and $y \geq 0$.