CSCI 2200 HW3
Wang Xinshi

Problem 15.15 Among 400 students, 150 are in math, 120 are in bio and 50 are math-bio duals. What are the chances a random student is in: (a) math or bio (b) bio and not math (c) neither math nor bio?

(a)$|Math \cup Bio| = |Math| + |Bio| - |Math \cap Bio|$.
    Thus

$$P(|Math \cup Bio|) = P(|Math|) + P(|Bio|) - P(|Math \cap Bio|)$$
$$= \frac{150}{400} + \frac{120}{400} - \frac{50}{400}$$
$$= \frac{11}{20}$$

(b)$|Math^c \cap Bio - Math| = |Bio - Bio\ capMath|$, since biology is a subset of not math.
    Thus

$$P(|Math^c \cap Bio|) = P(|Bio - Bio \cap Math|)$$
$$= \frac{70}{400}$$
$$= \frac{7}{40}$$

(c)$|Math \cup Bio|^c = \Omega - |Math \cup Bio|^c$.
    Thus

$$P(|Math \cup Bio|^c) = P(\Omega) - P(|Math \cup Bio|^c|)$$
$$= 1 - \frac{11}{20}$$
$$= \frac{9}{20}$$

Problem 16.55 (Clinical tests). Chances are 80% that an untreated person gets acne during a month's observation. Chances are 40% an acne drug works. If the drug works, 90% of acne cases are suppressed. Patients arrive (one per month), are randomly either given the drug (treated group) or not (control group), and then observed for a month.

(a) What are the chances the first patient to develop acne is a control patient?

(b) What are the chances the drug is effective if the first patient to develop acne is a control patient?

(c) What are the chances the drug is effective if the first patient to develop acne is a treated patient?

(a).

$$P[\text{patient to develop acne} \cap \text{is control group}] = P[\text{patient to develop acne}|\text{is control group}]P[\text{is control group}].$$
$$= 0.8 * 0.5 = 40\%$$

(b).

$$P[\text{drug is effective}|\text{first patient is develop group}] =$$

Problem 16.42. Five out of 100 coins are two-headed. You randomly pick a coin and flip it "fairly" twice (each side is equally probable). What is the probability to get (a) 2 heads (b) 2 tails (c) matching tosses?
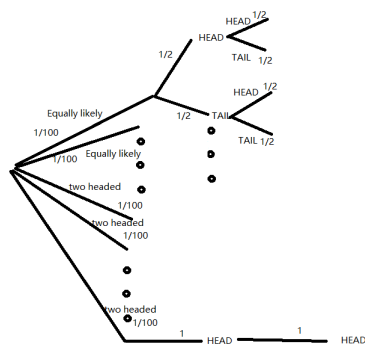
(a). According to the probabability tree, we have $P(2 \text{ heads}) = \sum_{i=1}^{95} \frac{1}{400} + \sum_{i=96}^{100} \frac{1}{100} = \frac{95}{400} + \frac{5}{100} = \frac{23}{80}$.

(b). According to the probabability tree, we have $P(2 \text{ tails}) = \sum_{i=1}^{95} \frac{1}{400} + \sum_{i=96}^{100} 0 = \frac{95}{400} = \frac{19}{80}$.

(c). According to the probabability tree, we have $P(2 \text{ tosses match}) = \sum_{i=1}^{95} \frac{2}{400} + \sum_{i=96}^{100} \frac{1}{100} = \frac{90}{400} + \frac{5}{100} = \frac{21}{40}$.

Problem 16.81. You have a fair 5-sided die which can generate one of the numbers $\{1, 2, 3, 4, 5\}$ with probability $\frac{1}{5}$ each. You wish to simulate a fair 7-sided die which generates a number in $\{1, 2, 3, 4, 5, 6, 7\}$ with probability $\frac{1}{7}$ each. Give an algorithm to do so, and prove it.

Toss a coint 2 twice, $\forall (i, j) \in \{1...5\} \times \{1...5\}$, we have $(1, 1) = 1, (2, 2) = 2, (3, 3) = 3, (4, 4) = 4, (5, 5) = 5, (5, 1) = 6, (5, 2) = 7$. Other wise, we restart.

Prove: Let n be an arbitrary number and $n \in \{1, .., 7\}$. We have

$$P[n] = p[n|(1, 1)]P[(1, 1)] + p[n|(1, 2)]P[(1, 2)] + ... + p[n|(i, j)]P[(i, j)] + P[n|restart]P[restart]$$

$$P[n] = \frac{1}{25}(\text{since there will be only 1 exact match}) + \frac{18}{25}P[n]$$

$$\frac{17}{25}P[n] = \frac{1}{25}$$

$$P[n] = \frac{1}{7}$$

Problem 17.35. You have 100 and bet 1 at a time on roulette. You goal is to win 50. Compute the probability that you reach your goal before going bankrupt.

I here assume it is exactly the same roulette as the one on the book.

probability of landing on red (probability of winning) $= \dfrac{18}{38}$. There are fifty steps away from winning, thus $L = 150$. The formula is $P(k, l, p) = \dfrac{p^k - p^l}{p^k - 1}$.

Thus we have $P[win] = P(50, 150, p) = \dfrac{0.9^{150} - 0.9^{50}}{0.9^{150} - 1} = 2.64 \times 10^-5$.

Problem 18.82. You have N coins in your pocket. N is unknown and has a Poisson PDF, $P[N = k] = \dfrac{e^{-2}2^k}{k!}$. You toss all the coins. What is the PDF for the number of H you get?

$P[N = k] = \dfrac{e^{-2}2^k}{k!}$. Since the probability of getting head is one half, thus we have

$P[\dfrac{N}{2} = k] = P[N = 2k] = \dfrac{e^{-2}2^{2k}}{2k!}$.