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1. We Prove this using contradiction. Assume $\forall e \in C, \forall C \subseteq G : \nexists K_3, e \in K_3$. In other words, for $\forall e \in C, \forall C \subseteq G$, $e \in C_n$, where $n \geq 4$ (note here C_3 and K_3 are equivalent). Thus every edge must be in a cycle that has more than or equal to 4 vertices, and there's no edge within the C_n otherwis there will exist a smaller cycle and some edge e in the graph G will belong to that cycle. Thus we have a cycle of at least 4 vertices that has no chord, which leads to a contradictionn with G is a chordal graph. Thus every edge in every cycle of G is part of a triangle K_3.

2.

We prove this using induction on the number of edges e.

Base Case: $e = 0$. Since G is connected, we have $1 - 0 + 1 = 2$. Thus the base case holds.

Inductive step: Suppose the formula holds for gaphs of less than e-1 edges. Let G be a graph with e edges, show the euler's formula holds for graph G.

Case 1: G does not contain a cycle.

Then G is a tree. It has 1 region since it is connected. It has $v - 1$ edges. Thus $v - (v - 1) + 1 = 2$.

Case 2: G contains at least 1 cycle.

We assume the cycle is C. If we remove an edge e from C, we get a path P and new graph G'. G' has one less region than G by definition and thus we have $r' = r - 1$. with the same number of vertices $v' = v$ and one less edge than G, which means $e' = e - 1$.

Thus the I.H. holds since $v - (e - 1) + (r - 1) = 2 \implies v - e + r = 2$ holds.

Thus we have proved the euler's formula