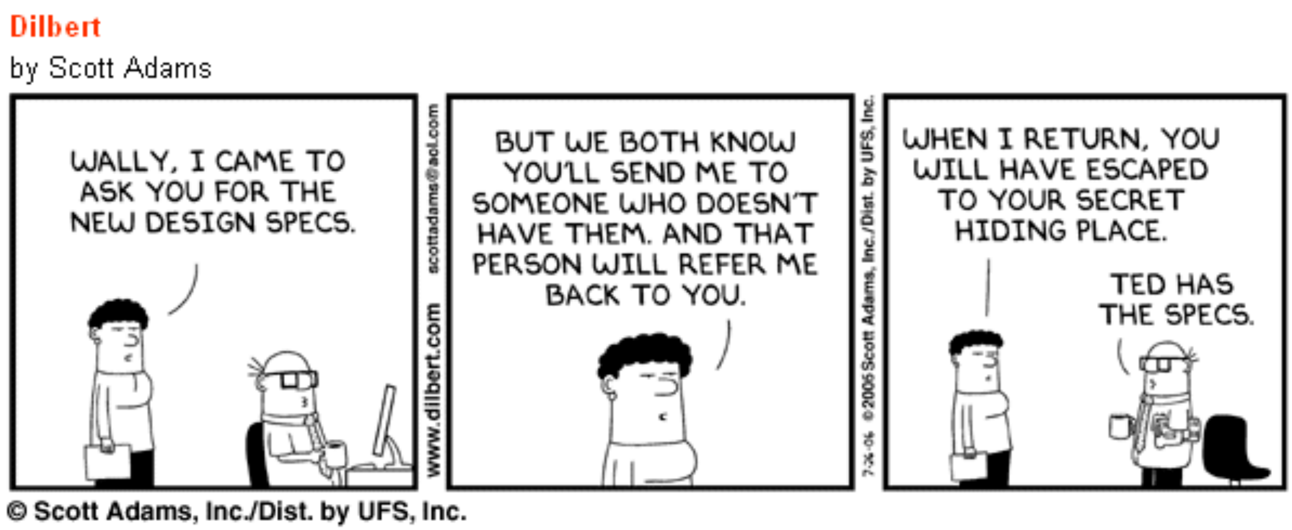
Specifications, continued



Review

- Spec “A is stronger than B” means
  - For every implementation *I*
    - “*I* satisfies A” implies *“I* satisfies B”
    - If the implementation satisfies the stronger spec (A), it satisfies the weaker (B)
    - The opposite is not necessarily true!
  - For every client *C*
    - “*C* meets the obligations of B” implies “*C* meets the obligations of A”
    - If C meets the weaker spec (B), it meets the stronger spec (A)
    - The opposite is not necessarily true
- A **larger world** of implementations satisfy the weaker spec B than the stronger spec A
- Consequently, it is easier to implement a weaker spec!
  - Weaker specs require *more* AND/OR Weaker specs guarantee (promise) *less*

Satisfaction of Specifications

- I is an implementation and S is a specification
- I satisfies S if
  - Every behavior of I is permitted by S
  - No behavior of I violates S
- The statement “I is correct” is meaningless, but often used
- If I does not satisfy S, either or both could be wrong
  - I does something that S doesn’t specify
  - S expects a result that I doesn’t produce
- When I doesn’t satisfy S, it’s usually better to change the program rather than the spec.
- If spec is too complex modify spec

Why Compare Specs?

- Liskov Substitution Principle
  - We want to use a subclass method in place of superclass method
  - Spec of subclass method must be stronger
    - Or at least equally strong
- Which spec is stronger?
  - A procedure satisfying a stronger spec can be used anywhere a weaker spec is required.
- Does the implementation satisfy the specification?

Comparing Specifications

- One way: by hand, examine each clause
- Another way: logical formulas representing the spec
- Use whichever is most convenient

- Comparing specs enables reasoning about substitutability

Exercise

- Specification A:

requires: `a` is non-null and `value` occurs in `a`
modifies: none
effects: none
returns: the smallest index `i` such that `a[i] = value`

- Specification B:

requires: `a` is non-null and `value` occurs in `a` // same as A
modifies: none // same as A
effects: none // same as A
returns: `i` such that `a[i] = value` // fewer guarantees

- Therefore, A is stronger.
- In fact, A's postcondition implies B's postcondition

Example

- Specification B:
  - requires: **`a`** is non-null and **`value`** occurs in **`a`**
  - modifies: none
  - effects: none
  - returns: **`i`** such that **`a[i] = value`**
- Specification A:
  - requires: **`a`** is non-null // fewer conditions!
  - modifies: none // same
  - effects: none // same
  - returns: **`i`** such that **`a[i] = value`** if value occurs in **`a`** and **`i = -1`** if value is not in **`a`** // guarantees more!
- Therefore, A is stronger!

Strong Versus Weak Specifications

- double sqrt(double x)

  A. @requires x>= 0
  @return y such that |y^2 – x| <= 1

  B. @requires none
  @return y such that |y^2 – x| <= 1
  @throws IllegalArgumentException if x < 0

  C. @requires x>= 0
  @return y such that |y^2 – x| <= 0.1

- Which are stronger?

Comparing Specifications

Most of our specification comparisons will be informal

A is stronger than B if

A’s precondition is weaker than B’s
(keeping postcondition the same)

- Requires less of client

Or

A’s postcondition is stronger than B’s
(keeping precondition the same)

- Guarantees more to client

Or

A’s precondition is weaker than B’s
AND
A’s postcondition is stronger than B’s

Comparing by Logical Formulas

- Specification S1 is stronger than S2 iff
  - For all implementations I, (I satisfies S1) => (I satisfies S2)
  - The set of implementations that satisfy S1 is a *subset* of the set of implementations satisfying S2.
- If each specification is a logical formula
  - S1 => S2
- Comparison using logical formulas is precise but can be difficult to carry out.
- It is often difficult to express all preconditions and postconditions with precise logical formulas!

Implication Truth Table

Truth Tables for Connectives

P	Q	$P \wedge Q$	$P \vee Q$	$P \rightarrow Q$
True	True	True	True	True
True	False	False	True	False
False	True	False	True	True
False	False	False	False	True

S1	S2	S1=>S2
T	T	T
T	F	F
F	T	T
F	F	T

Comparing by Logical Formulas

- S1 is stronger than S2
- (x is an element of set of programs satisfying S1) => (x is an element of the set of programs satisfying S2)
  - the set of programs satisfying S1 is a subset of the set of programs satisfying S2
  - "A is a subset of B" if and only if every element of *A* also belongs to *B*
- An implementation I that satisfies S1 also satisfies S2
- If (I satisfies S1) => (I satisfies S2) is false
  - Then S1 does not imply S2, or S1 is not stronger than S2.
- If I does not satisfy S1, all bets are off. I might or might not satisfy S2.
  - See http://press.princeton.edu/chapters/s8898.pdf

Comparing by Logical Formulas

- Let Spec A: $\{P_A\}$ **code** $\{Q_A\}$,
  Spec B: $\{P_B\}$ **code** $\{Q_B\}$.

We say code satisfies a specification with precondition P and postcondition Q iff {P} **code** {Q} Hoare triple is true.

Do not confuse it with P => Q.

e.g., { true } `x = 1;` { x = 1 } is true, but

true => x = 1 is false.

Comparing by Logical Formulas

- Let Spec A: $\{P_A\}$ **code** $\{Q_A\}$,
  Spec B: $\{P_B\}$ **code** $\{Q_B\}$.

The following are equivalent:

- $P_B \Rightarrow P_A$ and $Q_A \Rightarrow Q_B$
- A is stronger than B
- A => B

Example Revisited: int find(int[] a, int val)

```
int find(int[] a, int value) {
    for (int i=0; i<a.length; i++) {
        if (a[i] == value) return i;
    }
    return -1;
}
```

- Specification B:
  - requires: **`a`** is non-null and **`value`** occurs in **`a`**
  - returns: **`i`** such that **`a[i] = value`**
- Specification A:
  - requires: **`a`** is non-null
  - returns: **`i`** such that **`a[i] = value`** or **`i = -1`** if value is not in **`a`**

Be careful with specifications!

returns: **`i`** such that **`a[i] = value`** or **`i = -1`** if **`value`** is not in **`a`**

Let **P** = "val occurs in a",

Q = "return i s.t. a[i] = val"

R = "return -1"

$$(P \Rightarrow Q) \lor (!P \Rightarrow R)$$

$$= (!P \lor Q) \lor (P \lor R)$$

$$= Q \lor R$$

"or" would allow us to write a method that always returns -1!

Be careful with specifications!

returns: **`i`** such that **`a[i] = value`** or **`i = -1`** if **`value`** is not in **`a`**

We really mean: "**`i`** such that **`a[i] = value`** if **`value`** is in **`a`**, **AND** **`i = -1`** if **`value`** is not in **`a`**".

P => Q ^ !P => R is equivalent to: (P ^ Q) v (!P ^ R) v (Q ^ R)

In our case, "P => Q ^ !P => R" and "(P ^ Q) v (!P ^ R)" are equivalent since "Q^R" is false (return -1 and return a value >= 0 cannot both be true.)

So, we could also say: "(i such that a[i] = value **and** value is in a) **or** (i = -1 **and** value is not in a)**".**

Example: int find(int[] a, int val)

- Specification B:

  requires: **a** is non-null and **val** occurs in **a** **[P_B]**

  returns: **i** such that **a[i] = val** **[Q_B]**

- Specification A:

  requires: **a** is non-null **[P_A]**

  returns: **i** such that **a[i] = val** if value **val** occurs in **a** and **-1** if value **val** does not occur in **a** **[Q_A]**

Clearly, P_B => P_A.

Q_A states "**val** occurs in **a** => returns **i** such that **a[i]=val** AND **val** does not occur in **a** => returns **-1**"

Q_B can be logically rewritten as: "**val** occurs in **a** => returns **i** such that **a[i]=val** AND **val** does not occur in **a** => returns **anything**." (violated precondition allows anything.)

Comparing postconditions

- Q_B (postcondition of Spec B)

  i such that **a[i] == value** can be written (due to the precondition) as:

  value is in **a** => **i** such that **a[i] == value**

  && **value** is not in **a** => true

- Q_A (postcondition of Spec A)

  value is in **a** => **i** such that **a[i] == value**

  && **value** is not in **a** **=>** -1=i

- Which is stronger, Q_B or Q_A?

Q_B and Q_A are **NOT** :

Q_{B2}: {0 <= i < a.length}
Q_{A2}: {-1 <= i < a.length}

For these, Q_{B2} => Q_{A2}, i.e., Q_{B2} is stronger

Comparing by Logical Formulas

Let $A = \{P_A\}$ `code` $\{Q_A\}$,

$B = \{P_B\}$ `code` $\{Q_B\}$ be Hoare triples.

A is stronger than B if and only if P_A is weaker than P_B and Q_A is stronger than Q_B, i.e.,

- $A \Rightarrow B \iff (P_B \Rightarrow P_A \wedge Q_A \Rightarrow Q_B)$.

$A \Rightarrow B$ means that any `code` satisfying A also satisfies B.

Example: int find(int[] a, int val)

- Specification B:
  requires: **a** is non-null and **val** occurs in **a** **[P_B]**
  returns: **i** such that **a[i] = val** **[Q_B]**
- Specification A:
  requires: **a** is non-null **[P_A]**
  returns: **i** such that **a[i] = val** if value **val** occurs in **a** and **-1** if value **val** does not occur in **a** **[Q_A]**

- P_B requires more of the caller than P_A. That is, $\mathbf{P_B \Rightarrow P_A}$.
- Q_A promises more to the caller than Q_B (Q_B does not promise anything if **val** does not occur in **a**; e.g., code satisfying B could return -99.). That is, $\mathbf{Q_A \Rightarrow Q_B}$.

Example: int find(int[] a, int val)

- Specification B:
  requires: **`a`** is non-null and **`val`** occurs in **`a`** **[P_B]**
  returns: **`i`** such that **`a[i] = val`** **[Q_B]**
- Specification A:
  requires: **`a`** is non-null **[P_A]**
  returns: **`i`** such that **`a[i] = val`** if **`val`** occurs in **`a`** and **`-1`** if **`val`** does not occur in **`a`** **[Q_A]**

Intuition: **Q_B** should really be thought of as:

`i` such that **`a[i] = val`** if **`val`** occurs in **`a`**

Thus, it's still OK to substitute A for B.

Exercise: int find(int[] a, int val)
Sort specifications in order of strength

- Specification B:
  requires: **`a`** is non-null and **`val`** occurs in **`a`** **[P_B]**
  returns: **`i`** such that **`a[i] = val`** **[Q_B]**
- Specification A:
  requires: **`a`** is non-null **[P_A]**
  returns: **`i`** such that **`a[i] = val`** if **`val`** occurs in **`a`** and **`-1`** if **`val`** does not occur in **`a`** **[Q_A]**
- Specification C:
  requires: none **[P_C]**
  returns: **`i`** such that **`a[i] = val`** if **`val`** occurs in **`a`** and **`-1`** if **`val`** does not occur in **`a`** **[Q_C]**
  throws: **`NullPointerException`** if **a** is **null** **[Q_C]**

Converting PSoft Specs into Logical Formulas

- PSoft specification

  requires: R

  modifies: M

  effects: E

is equivalent to this logical formula

{R} `code` {E ^ (nothing but M is modified)}

throws and returns are absorbed into effects E

Convert Spec to Formula, step 1: absorb throws and returns into effects

- PSoft specification convention

  requires: (unchanged)

  modifies: (unchanged)

  effects:

  returns: absorbed into “effects”

  throws:

Convert Spec to Formula, step 1: absorb throws and returns into effects

- **`set`** method from **`java.util.ArrayList<T>`**

`T set(int index, T element)`

requires: true

modifies: this[index]

effects: $this_{post}[index] = element$

throws: IndexOutOfBoundsException if index < 0 || index ≥ size

returns: $this_{pre}[index]$

Absorb effects, returns and throws into new <u>effects:</u>

E= if index < 0 || index ≥ size then
throws IndexOutOfBoundsException
else
$this_{post}[index] = element$ and returns $this_{pre}[index]$

Convert Spec to Formula, step 2: Convert into Formula

- **`set`** from **`java.util.ArrayList<T>`**

`T set(int index, T element)`

requires: true

modifies: this[index]

effects: E = if index < 0 || index ≥ size then
throws IndexOutOfBoundsException
else
$this_{post}$[index] = element and returns $this_{pre}$[index]

Denote effects expression by E. Resulting formula is:

{true} code { (E ^ (forall i ≠ index, $this_{post}[i] = this_{pre}[i]$)) }

Stronger Specification

- S1 is stronger than S2 iff

$\{R_1\}$ code $\{E_1 \wedge (\text{only } M_1 \text{ is modified})\}$
$\Rightarrow$
$\{R_2\}$ code $\{E_2 \wedge (\text{only } M_2 \text{ is modified})\}$

iff $R_2 \Rightarrow R_1 \wedge (E_1 \wedge (\text{only } M_1 \text{ is modified}) \Rightarrow (E_2 \wedge (\text{only } M_2 \text{ is modified}))$

if $R_2 \Rightarrow R_1 \wedge E_1 \Rightarrow E_2 \wedge (\text{only } M_1 \text{ is modified}) \Rightarrow (\text{only } M_2 \text{ is modified})$

iff $R_2 \Rightarrow R_1 \wedge E_1 \Rightarrow E_2 \wedge (M_1 \subseteq M_2)$

Stronger Specification

- S1 is stronger than S2 if $R_2 \Rightarrow R_1 \wedge E_1 \Rightarrow E_2 \wedge (M_1 \subseteq M_2)$

- A stronger specification:
  - Requires less
  - Guarantees more
  - Modifies less

Exercise

- Convert PSoft spec into logical formula

```
public static int binarySearch(int[] a,int key)
```

requires: `a` is sorted in ascending order and a is non-null

modifies: none

effects: none

returns: i such that a[i] = key if such an i exists; -1 otherwise

effects: E: if key occurs in a then returns i such that a[i] = key else returns -1.

E more formally:

E = 0 <= index => index < a.Length && a[index] = value
^ index < 0 ==> forall k :: 0 <= k < a.Length ==> a[k] != value

{ sorted(a) ^ a != null } **code** { E ^ (forall i :: 0 <= i < a.Length, a_{pre}[i] = a_{post}[i]) }

Exercise

```
static void listAdd2(List<Integer> lst1,
                     List<Integer> lst2)
```

requires: `lst1`, `lst2` are non-null. `lst1` and `lst2` are same size.
modifies: `lst1`
effects: i-th element of `lst1` is replaced with the sum of
i-th elements of `lst1` and `lst2`
returns: none

{ (lst1 != null ^ lst2 != null ^ lst1.length = lst2.length) } **code**
{ (forall i :: 0 <= i < lst1.length => $lst1_{post}[i] = lst1_{pre}[i] + lst2_{pre}[i]$)
^ (forall i :: 0 <= i < lst2.length => $lst2_{post}[i] = lst2_{pre}[i]$) }

Exercise

```
private static void swap(int[] a, int i, int j)
```

requires: **a** non-null, **0<=i,j<a.length**

modifies: **a[i]** and **a[j]**

effects: $a_{post}[i]=a_{pre}[j]$ and $a_{post}[j]=a_{pre}[i]$

returns: none

```
static void swap(int[] a, int i, int j) {
     int tmp = a[j];
     a[j] = a[i];
     a[i] = tmp;
}
```

{ R } **code** { (E ^ (forall k :: k != i,j $a_{post}[k] = a_{pre}[k]$) }

{ a != null ^ 0 <= i,j < a.length } **code**

{ ($a_{post}[i] = a_{pre}[j]$ ^ $a_{post}[j] = a_{pre}[i]$)

^ (forall k :: (0 <= k < a.length ^ k != i ^ k != j) ==> $a_{post}[k] = a_{pre}[k]$) }

Comparison by Logical Formulas

- We often use this equivalence direction:

If P_B => P_A and Q_A => Q_B then A is stronger than B

Comparing Specifications, Review

- It is not easy to compare specifications
- Comparison by hand
  - Easier but can be imprecise
  - It may be difficult to see which of two conditions is stronger
- Comparison by logical formulas
  - Accurate
  - Sometimes, it is difficult to express behaviors with precise logical formulas!

Comparing by Hand

- Requires clause
  - Stronger spec has **fewer** conditions in requires
  - Requires less
- Modifies/effects clause
  - Stronger spec modifies **fewer** objects. Stronger spec guarantees more objects stay unmodified!
- Returns and throws clauses
  - Stronger spec guarantees **more** in returns and throws clauses. They are harder to implement, but easier to use by client
  - When pre-conditions are the same: no new throws in domain
  - When pre-conditions are weaker, it may guarantee more by specific throws. (See e.g., Spec C of `find`.)
- Bottom line: Client code should not be "surprised" by behavior

BallContainer and Box

- Suppose **`Box`** is a subclass of **`BallContainer`**

Spec of BallContainer.add(Ball b)	Spec of Box.add(Ball b)
`boolean add(Ball b)`	**`boolean add(Ball b)`**
requires: **`b`** non-null modifies: **`this`** BallContainer effects: adds **`b`** to this BallContainer if **`b`** not already in returns: true if **`b`** is added false otherwise	requires: **`b`** non-null modifies: **`this`** Box effects: adds **`b`** to this Box if **`b`** is not already in and Box is not full returns: true if **`b`** is added false otherwise

BallContainer and Box

- A client honoring BallContainer’s spec is justified to expect that this will work:

```
BallContainer c = new Box(100);
…
for(int i = 0; i < 20; i++) {
 Ball b = new Ball(10);
 c.add(b)
}
```

- This will fail, but if c is a BallContainer we expect it to work
- Box’ spec <u>is not stronger</u> than BallContainer’s. Thus Box <u>is not substitutable</u> for BallContainer!
- Implementation that satisfies Box specs doesn't satisfy BallContainer specs

BallContainer and Box

- BallContainer.add unconditionally adds the Balls. Box has a condition --- the Box is not full.
- Could a client coding against BallContainer expect to work on Box?
- Is Box guaranteeing more than BallContainer?
  - Box effects are weaker. Box's effects guarantee less.

```
BallContainer.add()
E = if b is_element BallContainer_pre
            return false
     else
       BallContainer_post = BallContainer_pre U b
```

```
Box.add()
E = if b is_element BallContainer_pre
            return false
     else
            if Box.volume_pre >= max_volume
                return false
            else
                Box_post = Box_pre U b
```

Substitutability

- Box is not what we call a true subtype of BallContainer
    - It is more limited than BallContainer.
    - A Box can only hold a limited amount;
    - A user who uses a BallContainer in their code cannot simply substitute a BallContainer with a Box and assume the same behavior in the program.
    - The code may cause the Box to fill up, but they did not have this concern when using a BallContainer.
    - For this reason, it is not a good idea to make Box extend BallContainer.
- Therefore, it is wrong to make Box a subclass of BallContainer

- An object of a true subtype should be able to do everything the superclass object can do and possibly more

Substitutability

- Box is not a true subtype (also called behavioral subtype) of BallContainer
- Bottom line:
  - Box.add() guarantees less
- Therefore, it is wrong to make Box a subclass of BallContainer

- More on substitutability, Java subtypes and true subtypes later

The Weakest Specification

requires: false
// Remember, **false** is the strongest condition of all

modifies: anything

effects: true
// **true** is the weakest condition of all

returns: true

throws: true

(This spec is so weak, it is trivial to implement, but impossible to use.)

The Strongest Specification

requires: true
// Remember, **true** is the weakest condition of all

modifies: none

effects: false
// **false** is the strongest condition of all

returns: false

throws: false

(This spec is so strong, it is impossible to implement with a terminating program.)