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quiz05

1.
a. Max = Min = 1

Suppose there are 2 vertices in the beginning. To make those vertices weakly connected we need at least 1 edge. Let us call the vertex with $deg^+(v) = 1$ by v_0 and the other vertex v_1. Let us add a new vertex v_2. To make v_2 weakly connected and $deg^+(v_1) = 1$, we need to add a new edge from v_1 otherwise since we need to make the v_0, v_1, and v_2 weakly connected and $deg^+(v_1) = 1$. Repeat this process until we have n vertices. The last vertex has $deg^+(v) = 0$. Then we need to create an edge with v_{n-1} with $v_i \in \{v_0, ..., v_{n-2}\}$. Therefore there must exists a cycle between v_{n-1} and v_i. Thus the Mimimum number of cycles = 1. Since every vertex v has $deg^+(v) = 1$, we cannot create any more edges. Thus Maximum number of cycles = 1.

b. Max = $\lfloor n/2 \rfloor$

Since $deg^+(v) = 1, \forall v \in V(G)$. we could only form a an edge from v_i to v_j. The maximum number of cycles is attained when v_j also form an edge with v_i since this loop contains the least edges when the number of edges is fixed. Thus there are in total $n/2$ in the case when n is even. When n is odd, no more cycles are formed since it $d^-(v) = 0$. Thus Max = $\lfloor n/2 \rfloor$.

c. Max = n

Since $deg^+(v) = 1, \forall v \in V(G)$. we could only form a an edge from v_i to v_j. The maximum number of cycles is attained when v_i also form an edge with itself since this loop contains the least edges. Thus there are in total n vertices and thus n edges. Thus Max = n.

2.

a.

k = 1, S = $\{e\}$

k'=2, F = $\{ea, eh\}$

b.

k = 2, S = $\{d, i\}$

k' = 3, F = $\{db, di, il\}$