Participation2

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Recall that in Poisson regression we model $y|x \sim Poisson(\exp(\theta^T x))$.

1. Give the expression for $P_\theta(y_i|x_i)$.
   since $\lambda = \exp(\theta^T x)$, we have the following observations:

$$P_\theta(y_i = 0|x_i) = e^{-\exp(\theta^T x_i)} \frac{\exp(\theta^T x_i)^0}{0!}$$
$$P_\theta(y_i = 1|x_i) = e^{-\exp(\theta^T x_i)} \frac{\exp(\theta^T x_i)^1}{1!}$$
$$P_\theta(y_i = 2|x_i) = e^{-\exp(\theta^T x_i)} \frac{\exp(\theta^T x_i)^2}{2!}$$

   Thus we have

$$P_\theta(y_i|x_i) = e^{-\exp(\theta^T x_i)} \frac{\exp(\theta^T x_i)^{y_i}}{y_i!}$$

2. State, in as simple a form you can manage, the optimization problem for finding an estimate $\hat{\theta}$ of θ by using MLE for Poisson regression.

   Solution is on page Two!

$$
\begin{aligned}
\mathcal{L}(\theta) &= \frac{1}{n}\sum_{i=1}^{n} -\log P_\theta(y_i|x_i) \\
&= \frac{1}{n}\sum_{i=1}^{n} -\log(e^{-\exp(\theta^T x_i)}\frac{\exp(\theta^T x_i)^{y_i}}{y_i!}) \\
&= \frac{1}{n}\sum_{i=1}^{n} -(\log(e^{-\exp(\theta^T x_i)}) + \log(\frac{\exp(\theta^T x_i)^{y_i}}{y_i!})) \\
&= \frac{1}{n}\sum_{i=1}^{n} -(-e^{\theta^T x_i} + \log(\exp(\theta^T x_i)^{y_i}) - \log(y_i!)) \\
&= \frac{1}{n}\sum_{i=1}^{n} e^{\theta^T x_i} - y_i \log(\exp(\theta^T x_i)) + \log(y_i!) \\
&= \frac{1}{n}\sum_{i=1}^{n} e^{\theta^T x_i} - y_i \theta^T x_i + \log(y_i!)
\end{aligned}
$$

Thus we have

$$
\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \frac{1}{n}\sum_{i=1}^{n} e^{\theta^T x_i} - y_i \theta^T x_i + \log(y_i!)
$$