CSCI 2200 HW3
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Problem 19.20. A keychain has 10 similar keys. You are fumbling in the dark trying each key in a random order to open your appartment door . What is the expected number of keys you try before you unlock the door?

Let x denote the number of trials before you unlock the door. Then we have $E[X] = \sum_{x \in \omega} xP(x) = 0 \times \frac{1}{10} + 1 \times \frac{1}{10} + 2 \times \frac{1}{10} + ... + 10 \times \frac{1}{10} = \frac{1}{10} \times \sum_{i=0} 10i = 5.5$

Problem 19.21. You randomly guess every answer on a multiple choice exam with 50 questions and 4 possible answers per question. What is the expected number of questions you answer correctly?

Let x denote the number of expected questions you answer correctly. Then we have $E[X] = np = \frac{1}{4} \times 50 = 12.5$.

Problem 19.38. A box has 1024 fair and 1 two-headed coin. You pick a coin randomly, make 10 flips and get all H.
(a) You flip the same coin you picked 100 times. What is the expected number of H?

Let X denote the expected number of H, we have $E[X] = P(fair)E[X|fair] + P(two - headed)E[X|two - headed] = \frac{9}{10} \times \frac{100+1}{2} + \frac{1}{10} \times 100 = 55.45$

(b) You flip the same coin you picked unitl you get H. What is the expected number of flips you make?

Let X denote the expected number of trails to get an H, we have $E[X] = P(fair)E[X|fair] + P(two - headed)E[X|two - headed] = \frac{9}{10} \times 2 + \frac{1}{10} \times 0 = \frac{18}{10} = 1.8$

Problem 20.11. Ten sailors return from shore and sleep randomly in their ten bunks (one sailor per bunk).
(a) Let X be the number of sailors in the correct bunk. Compute (i) P[X = 10] (ii) P[X = 9] (iii) P[X = 8].
(b) Compute the expected number of sailors in the correct bunk, that is E[X].

(a).
(i).$P[X = 10] = \frac{1}{10!} = \frac{1}{3628800} = 2.755 \times 10^{-7}$
(ii) $P[X = 9] = 0$ Only one person sleeping on the wrong bed is impossible
(iii) $P[X = 8] = \frac{1}{8!} \sum_{i=0}^{2} \frac{(-1)^i}{i!} = 1.24 \times 10^{-5}$

(b).
$E[X] = \sum_{x \in \omega} xP(x) = 2 \times \frac{P_{10}^{5}}{10!} + ... + 10 \times \frac{P_{10}^{0}}{10!}$(add them up when the remainder of i divides 2 is 0). we calculate using the following code

```
def factorial(x):
    if (x == 0):
        return 1
    else:
        return x*factorial(x-1)
sums = 0
for i in range(1,11):
    if (i % 2 == 0):
        sums += i * ((factorial(10)/factorial(i)) / factorial(10))
print(sums)
```

which gives us 1.175.

Problem 20.23. Five students independently get a random number in 1, . . . , 10. A score is increased for every pair of student whose numbers agree. Find the expected score when:
(a) For every pair of students whose numbers agree, the score is increased by 1.
Since after one person chooses, there are only four people left, and the probability of forming a pair with the one person is $\frac{1}{10}$ for all the four students. Let X denote the expected score, and thus we have

$$E[X] = \frac{5 \times 4 \times \frac{1}{10}}{2} = 1$$

(b) For every pair of students whose numbers agree, the score is increased by the number the pair has.
On average(the expected value), every pair has a number of $1 \times \frac{1}{10} + 2 \times \frac{1}{10} + ... + 10 \times \frac{1}{10} = 5.5$ Thus

$$E[X] = \frac{5.5 \times 5 \times 4 \times \frac{1}{10}}{2} = 5.5$$

(c) Generalize (a) and (b) to n students independently getting a number in 1, . . . , k.

$$E[X] = \frac{\frac{n+1}{2} \times n \times (n-1) \times \frac{1}{k}}{2}$$

Problem 20.20(j).20 kids stand in line. A random pair of adjacent standing kids pair up and sit. This continues until no more pairs can be formed. What is the expected number of unpaired kids?

Let $S(n)$ denote the number of students standing. For 20 people, we have $S(20) = \frac{1}{19}S(0) + S(18) + \frac{1}{19}S(1) + S(17) + \frac{1}{19}S(18) + S(0)$. With base case $n = 1$ and 3 return 1 and $n = 0$ and 2 return 0. According to python, we have $S(20) = 2.977$.