

$$1. I_a = \int x \ln x \, dx$$

$$\begin{aligned} & \int x \ln x \, dx \\ &= \frac{\ln x}{2} x^2 - \int \frac{x^2}{2} \frac{1}{x} + C \\ &= \frac{\ln x}{2} x^2 - \int \frac{x}{2} + C \\ &= \frac{\ln x}{2} x^2 - \frac{x^2}{4} + C \end{aligned}$$

$$2. I_b = \int \frac{x}{x^2 + 3} \, dx$$

$$\begin{aligned} & \int \frac{x}{x^2 + 3} \, dx \\ \text{Let } u &= x^2 + 3, \text{ then } du = 2x \, dx \\ &= \frac{1}{2} \int \frac{1}{u} \, du + C \\ &= \frac{1}{2} \ln u + C \\ &= \frac{1}{2} \ln(x^2 + 3) + C \end{aligned}$$

$$3. I_c = \int_0^2 t e^{-t} \, dt$$

$$\begin{aligned} & \int_0^2 t e^{-t} \, dt \\ &= -t e^{-t} \Big|_0^2 + \int_0^2 e^{-t} \, dt \\ &= -2e^{-2} - e^{-t} \Big|_0^2 \\ &= 1 - 3e^{-2} \end{aligned}$$

$$4. y^2 + 2y + t^3 + \sin(t) - 3 = 0$$

$$\begin{aligned} y^2 + 2y + t^3 + \sin(t) - 3 &= 0 \\ y^2 + 2y &= 3 - \sin(t) - t^3 \\ y^2 + 2y + 1 &= 4 - \sin(t) - t^3 \\ (y + 1)^2 &= 4 - \sin(t) - t^3 \\ y &= \\ \sqrt{4 - \sin(t) - t^3} - 1 \end{aligned}$$

5.

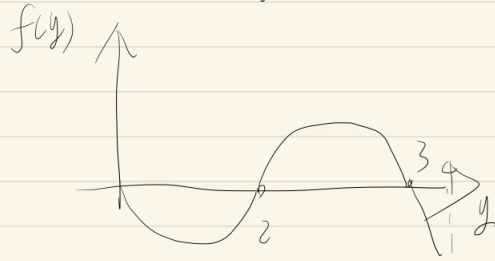
$$f(y) = y(y-2)(3-y)$$

roots: 0, 2, 3

$$y \in (0, 2): f(y) < 0$$

$$y \in (2, 3): f(y) > 0$$

$$y \in (3, 4): f(y) < 0$$



$$y' > 0 \text{ for } 2 < y < 3$$