ASSIGNMENT 1

Xinshi Wang, 661975305

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1 Problem 1

1. all string of a's and b's, beginning and ending with one a
2. all string of a's and b's
3. all string of a's and b's. ending with aaa, aab, aba, or abb
4. all string of a's and b's with exactly 3 b's
5. all string of a's and b's with even number of a's and b's

2 Problem 2

1. 21 is 10101 in binary form
   C $\rightarrow$ A1
   $\rightarrow$ B01
   $\rightarrow$ B101
   $\rightarrow$ A0101
   $\rightarrow$ 10101

2. We can prove it using induction.
   Base case: C $\rightarrow$ 0, then 0 mod 3 $\equiv$ 0.
   Induction hypothesis 1: C_n mod 3 = 0
   Induction Step:
   Branch 1: $C_{n+1} \rightarrow$ C0 $\implies C_{n+1} = C \times 2$ (Property of Binary number) $\implies C_{n+1}$ mod 3 $\equiv$ 0.
   Branch 2: $C_{n+1} \rightarrow$ A 1
   Case 1 (branch for A): A $\rightarrow$ 1 $\implies C_{n+1} =$ 11C $\implies C_{n+1}$ mod 3 $\equiv$ 0 (difference between 1 at odd and even positions remain unchanged) $\implies$ A $\equiv$ 1 (mod 3)
   Induction hypothesis: $A_n \equiv 1$ (mod 3).
   Case 2 (branch for A): $A_{n+1} \rightarrow$ C1 $\implies C_{n+1} =$ C11 $\implies C_{n+1} =$ C $\times$4 +3 $\implies C_{n+1}$ mod 3 $\equiv$ 0. $C_1 = C \times 2 \implies C1 \equiv 1$ (mod 3)
   Case 3 (branch for A): $A_{n+1} \rightarrow$ B 0
   Case 1 (branch for B): B $\rightarrow$ A 0 $\implies C_{n+1} =$ A 0 0 1 $= A \times 8 + 1$ Since $A_n \equiv 1(mod 3)$ in previous case, we have $A \times 8 + 1 = 1 \times 8 + 1 \equiv$ mod 3.
   Induction Hypothesis: $B_n = 2$(mod 3)

Case 2 (branch for B): $B_{n+1} \to$ B 1 $\implies C_{n+1} =$ B 1 0 1 $= B \times 8 + 5$ Since $B \equiv 2(mod3)$ in previous case, we have $B \times 8 + 5 = 2 \times 8 + 5 \equiv 0$ mod 3. $B_{n+1} =$ B1 = 2B+1 $\implies B_{n+1} \equiv 2$ (mod 3).

Case 3 (branch for B): B $\to$ A 0 $\implies C_{n+1} =$ A 0 0 1 $= A \times 8 + 1$ Since $A \equiv 1(mod3)$ in previous case, we have $A \times 8 + 1 = 1 \times 8 + 1 \equiv$ mod 3. $A_{n+1} =$ B1- =4B+2 = 8+2 $\equiv$ 1 (mod 3)

Thus all cases hold for B branch

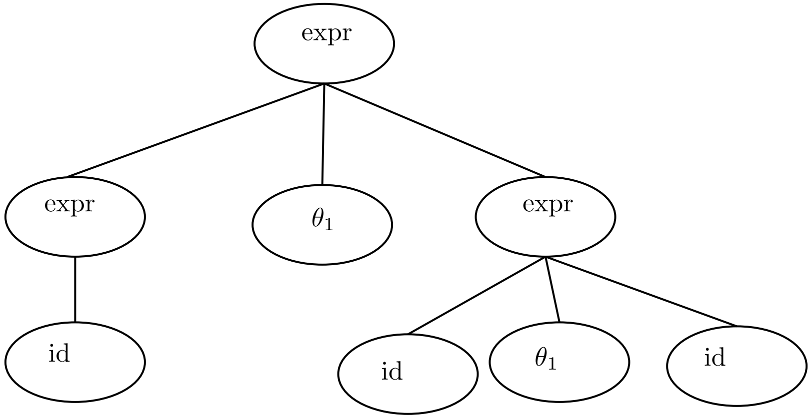
Thus all cases hold for A branch

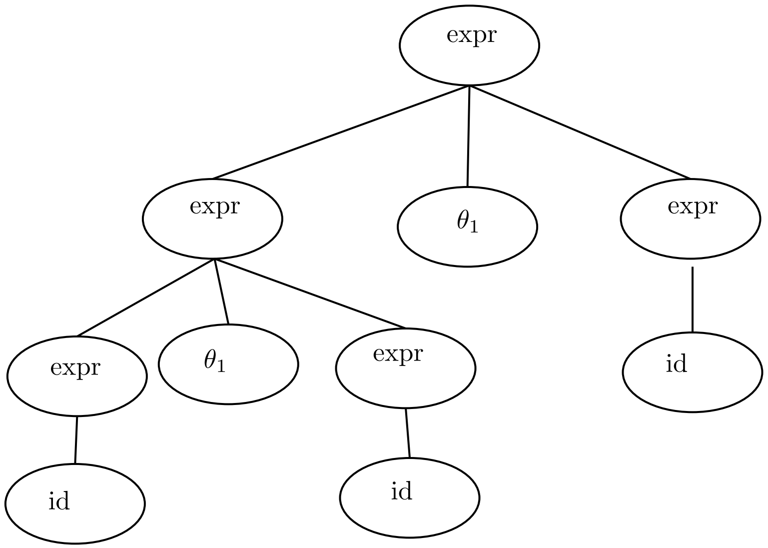
Thus all cases hold for C branch

3. In problem 2, we prove that C,A,and B could produce numbers equal to 0 mod 3, 1 mod3, and 2 mod 3, which are a disjoint set of numbers. Then S $\to$ C $\|$ A $\|$ B will generate any numbers. S-A-B would then equal to C. Thus C must contain all the numbers equal to 0 mod 3. Thus such grammar would generate all the numbers.

3 Problem 3

1. The grammar is ambiguous because there are at least two parse trees for $id\theta_1 id\theta_1 id$.





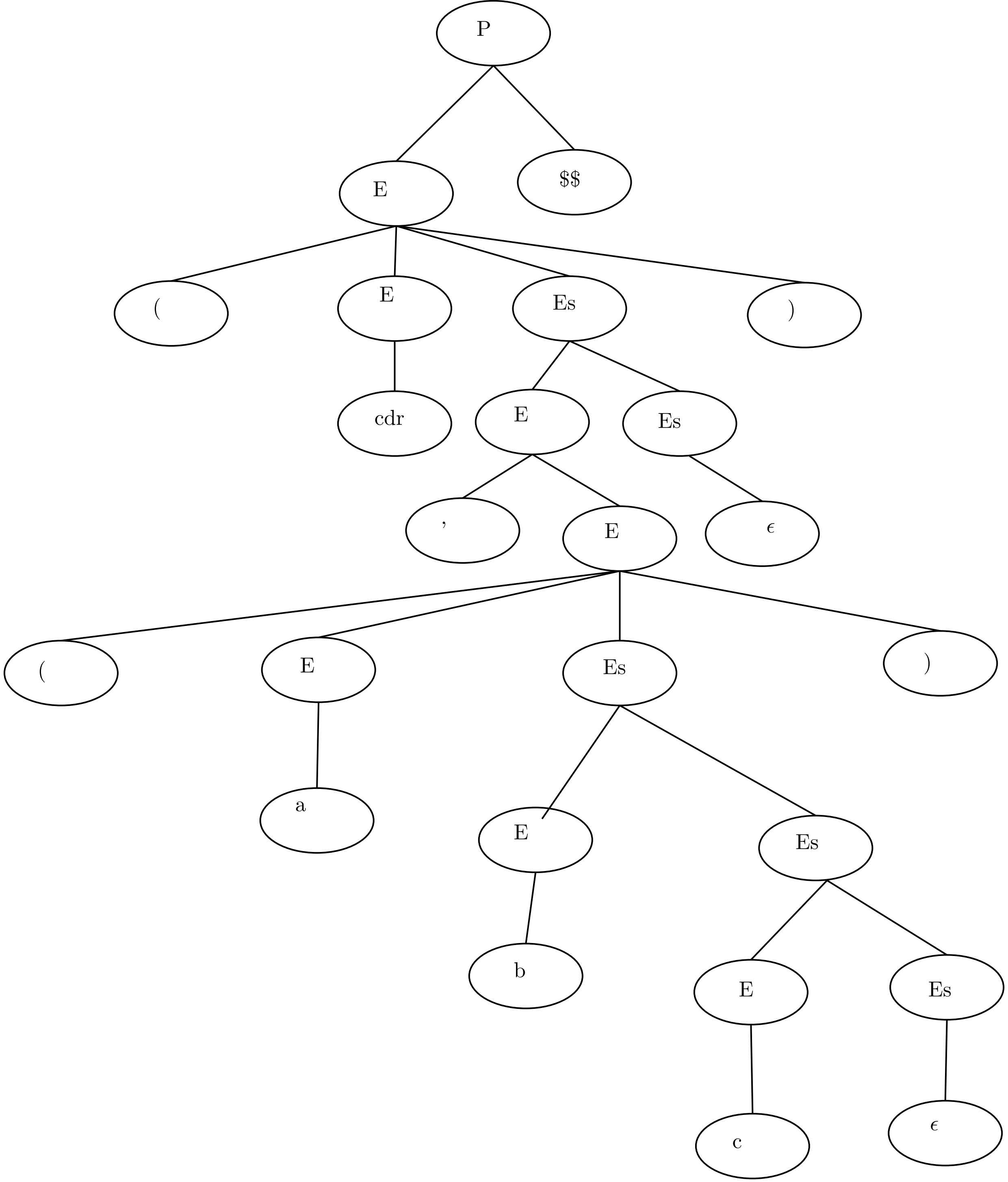
2. We can construct the following:

   expr $\rightarrow expr\theta_1 Terms_1 | Terms_1$
   $Terms_1 \rightarrow Terms_1\theta_2 Terms_2 | Terms_2$
   $Terms_2 \rightarrow Terms_2\theta_3 Terms_3 | Terms_3$
   ...
   $Terms_{n-1} \rightarrow Terms_{n-1}\theta_n Terms_n | Terms_n$
   $Terms_n \rightarrow Terms^* | Terms$
   $Terms \rightarrow id | (expr)$

4 Problem 4

1. Follow(E_s) = {)} Follow(E) = $\{', (,), atom, \$\$\}$ Predict($E_s \rightarrow \epsilon$) = {)}

2. The parse tree is given as the following diagram



P
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E
Es
)
cdr
E
Es
,
E
ϵ
(
E
Es
)
a
E
Es
b
E
Es
c
ϵ

3. P$\rightarrow$E$$
   E$\rightarrow$(EE$_s$)
   $E_s$$\rightarrow$EE$_s$
   $E$$\rightarrow$'E

5 Problem 5

	LL(1)	Ambiguous
(a)	No	No
(b)	Yes	No
(c)	No	Yes
(d)	No	Yes
(e)	Yes	No