Semidefinite Programming

Applications in approximating NP-Complete problems & Matrix Completetion

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Presentation Overview

Low rank matrices

Given an incomplete matrix, can we recover the missing values?

		-1		
			1	
1	1	-1	1	-1
1				-1
		-1		

1	1	-1	1	-1
1	1	-1	1	-1
1	1	-1	1	-1
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1	1	-1	1	-1

Low rank matrices

Yes!

Given:

- The matrix is low rank*
- We have enough sample data

Note: This does not apply to *all* low-rank matrices. But most.

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Why low-rank matrices?

Why is this useful

1. **Netflix** has an incomplete set of user preferences based off their past watch history. Can they use this information to recommend new movies?
2. **Recommendation Engine:** The netflix problem can be extended to general recommendation engines where a vendor knows some of the user preferences.
3. **Images:** We will give an example of recovering a corrupted image using matrix completetion

Relaxing Matrix Completetion to SDP

Suppose we have a low rank matrix $\mathbf{M}$. We have a set of location Ω describing our sampling. That is, if $(i, j) \in \Omega$, we observe entry M_{ij}. Given $\mathbf{M}$ is low rank, it seems resonable that we would like to solve the following optimization problem

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$$\begin{array}{ll} \text{minimize} & \operatorname{rank}(\mathbf{X}) \\ \text{subject to} & X_{ij} = M_{ij} \quad (i, j) \in \Omega \\ & \mathbf{X} \in \mathbb{R}^{n \times n} \end{array}$$

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But...

Rank is not a convex. This turns out to be an NP-Hard Problem.

Introduce the nuclear norm

Nuclear Norm

The nuclear norm is a close approximation of the rank.

The nuclear norm of a matrix **X** is defined as the sum of the eigenvalues.

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For a symmetric positive semi-definite (SPSD) matricies, the nuclear norm is equal to the trace.

A better relaxation

What if our matrix is not SPSD

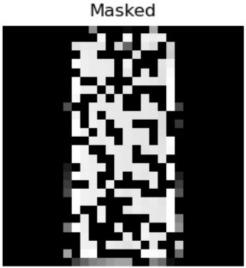
- We introduce two matricies $\mathbf{W}_1$ and $\mathbf{W}_2$

A better relaxation

$$\begin{aligned} \text{minimize} \quad & \operatorname{trace}(\mathbf{W}_1) + \operatorname{trace}(\mathbf{W}_2) \\ \text{subject to} \quad & X_{ij} = M_{ij} \quad (i,j) \in \Omega \\ & \begin{bmatrix} \mathbf{W}_1 & \mathbf{X} \\ \mathbf{X}^\top & \mathbf{W}_2 \end{bmatrix} \succeq 0 \end{aligned}$$

Fashion-MNIST

55% of data



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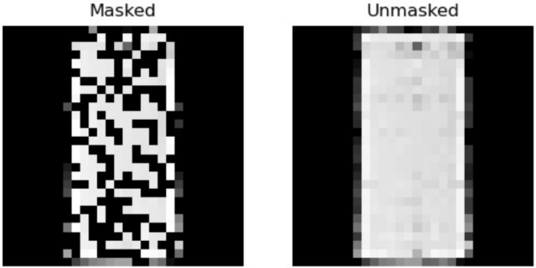

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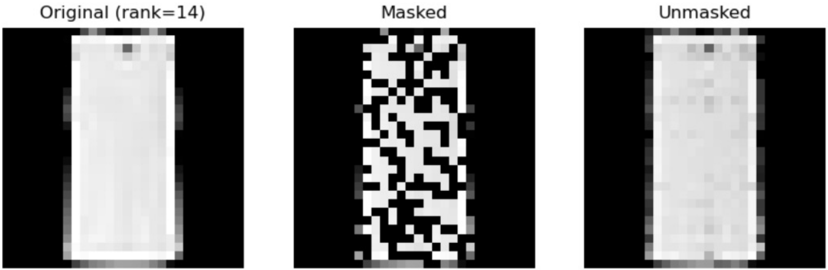
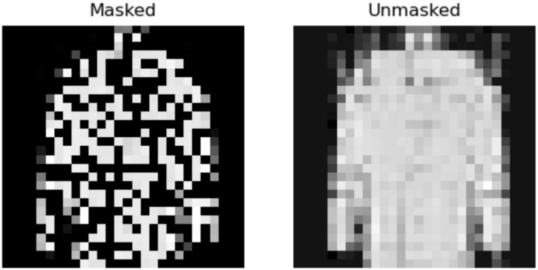


Fashion-MNIST

50% of data

50% of data



Fashion-MNIST

50% of data

Citing References

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Smith, 2022, Kennedy, 2023].

References

John Smith (2022)
Publication title
Journal Name 12(3), 45 – 678.

Annabelle Kennedy (2023)
Publication title
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