Semidefinite Programming

Applications in approximating NP-Complete problems & Matrix Completetion

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Presentation Overview

Low rank matrices

Given an incomplete matrix, can we recover the missing values?

		-1		
			1	
1	1	-1	1	-1
1				-1
		-1		

1	1	-1	1	-1
1	1	-1	1	-1
1	1	-1	1	-1
1	1	-1	1	-1
1	1	-1	1	-1

Yes! Given:

- The matrix is low rank
- We have enough sample data
- and more ... (needle in haystack)

Why is this useful

1. **Netflix** has an incomplete set of user preferences based off their past watch history. They
2. **Recommendation Engine**
3. **Images**

Relaxing Matrix Completetion to SDP

Suppose we have a low rank matrix $\mathbf{M}$. We have a set of location Ω describing our sampling. That is, if $(i,j) \in \Omega$, we observe entry M_{ij}. Given $\mathbf{M}$ is low rank, it seems resonable that we would like to solve the following optimization problem

$$\begin{array}{ll} \text{minimize} & \operatorname{rank}(\mathbf{X}) \\ \text{subject to} & X_{ij} = M_{ij} \quad (i,j) \in \Omega \\ & \mathbf{X} \in \mathbb{R}^{n \times n} \end{array}$$

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But...

Rank is not a convex. This turns out to be an NP-Hard Problem.

Introduce the nuclear norm

Nuclear Norm

The nuclear norm is a close approximation of the rank.

The nuclear norm of a matrix **X** is defined as the sum of the eigenvalues.

$$\|\mathbf{X}\|_* = \sum_{k=1}^{n} \sigma_k \mathbf{X}$$

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For a symmetric positive semi-definite (SPSD) matricies, the nuclear norm is equal to the trace.

A better relaxation

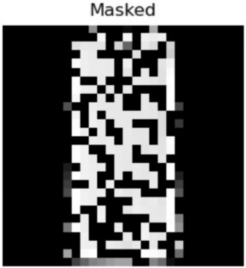
What if our matrix is not SPSD

- We introduce two matricies $\mathbf{W}_1$ and $\mathbf{W}_2$

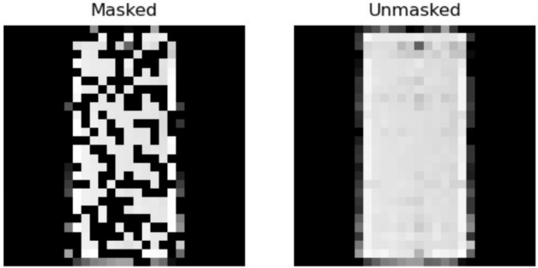
A better relaxation

$$\begin{array}{ll} \text{minimize} & \operatorname{trace}(\mathbf{W}_1) + \operatorname{trace}(\mathbf{W}_2) \\ \text{subject to} & X_{ij} = M_{ij} \quad (i,j) \in \Omega \\ & \begin{bmatrix} \mathbf{W}_1 & \mathbf{X} \\ \mathbf{X}^\top & \mathbf{W}_2 \end{bmatrix} \succeq 0 \end{array}$$

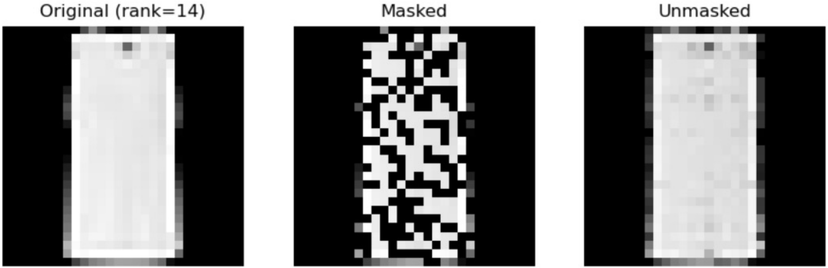
Fashion-MNIST



Fashion-MNIST



Fashion-MNIST



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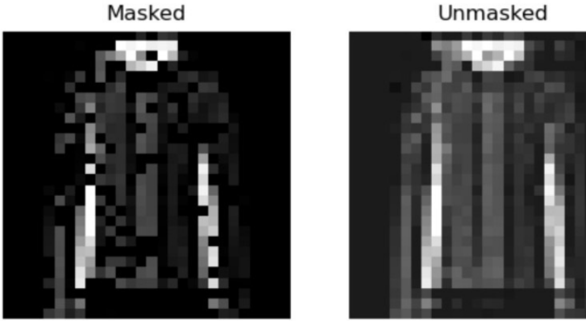
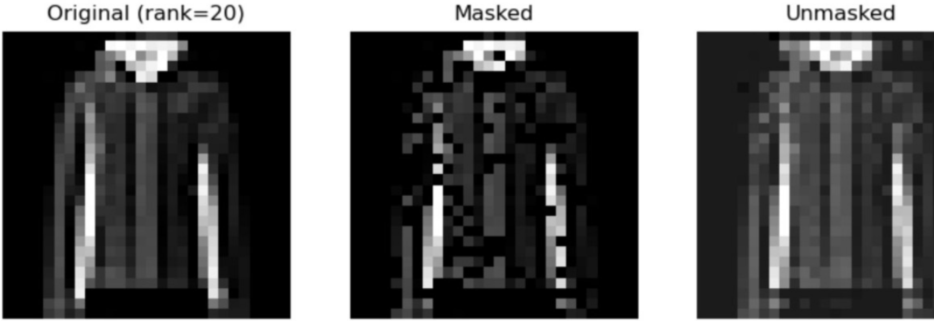


Fashion-MNIST



Lists

Bullet Points and Numbered Lists

- Lorem ipsum dolor sit amet, consectetur adipiscing elit
- Aliquam blandit faucibus nisi, sit amet dapibus enim tempus
    - Lorem ipsum dolor sit amet, consectetur adipiscing elit
    - Nam cursus est eget velit posuere pellentesque
- Nulla commodo, erat quis gravida posuere, elit lacus lobortis est, quis porttitor odio mauris at libero

1. Nam cursus est eget velit posuere pellentesque
2. Vestibulum faucibus velit a augue condimentum quis convallis nulla gravida

Citing References

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Smith, 2022, Kennedy, 2023].

References

John Smith (2022)
Publication title
Journal Name 12(3), 45 – 678.

Annabelle Kennedy (2023)
Publication title
Journal Name 12(3), 45 – 678.

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