Math 54 Lec7

Recap

Linear transformations $T : \mathbb{R}^n \to \mathbb{R}^m \text{ (matrix } A), S : \mathbb{R}^p \to \mathbb{R}^n \text{ (matrix } B)$

Then $T \circ S$ has matrix $A \cdot B$

Transpose

Very important counterpart of any matrix A, denoted by A^T

This switches the rows and columns of A (i.e. i-th row of A becomes i-th column of A^T). If A is $m \times n$, then A^T is $n \times m$.

properties

- $(A + B)^T = A^T + B^T$
- $(AB)^T = B^T A^T$

Proof of $(AB)^T = B^T A^T$

Consider the ij-th entry in $(AB)^T$. That is the ji-th entry in AB which is the j-th row of A times the i-th column in B.

Now consider the ij-entry in $B^T A^T$. That is the i-th row in B^T(i-th column in B) times j-th column in A^T(j-th column in A).

Identity matrix

For $n \times n$ matrix, I_n is the **identity matrix**, which has 1s for all diagonal entries. 0 for all non-diagonal entries.

I for short if context is clear.

Key property

For any $n \times n$ matrix A, we have $A \cdot I_n = I_n \cdot A = A$.

Inverse (For square matrix)

For an $n \times n$ matrix A, the **"inverse"** of A(if it exists) is denoted A^{-1} has the property that

$$A \cdot A^{-1} = A^{-1} \cdot A = I_n$$

Left inverse of matrix equals to Right inverse of matrix

Say we find B for which $BA = I_n$. Is $AB = I_n$ guaranteed?

Without assuming that say there is also some C where $AC = I_n$

Then $BAC = BI_n \Rightarrow I_nC = B \Rightarrow C = B$

Inverse is unique

Say we have $I_n = AB = AC$

Terminology

A square matrix is **invertible/nonsingular** if it has an inverse, otherwise **singular**.

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then the **determinant** of A is $ad - bc$. And we have

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Naturally requires $ad - bc \neq 0$

If $ad - bc = 0 \rightarrow \frac{a}{b} = \frac{c}{d}$

Properties

- $(A^{-1})^{-1} = A$
- $(AB)^{-1} = B^{-1}A^{-1}$