Math 54 Lec3

Vectors

With any collection of vectors: $\{v_1, v_2, \dots, v_n\}$, consider a linear combination of these vectors: $c_1v_1 + c_2v_2 + \cdots + c_nv_n$ where each $c_i \in \mathbb{R}$.

Do these create the "full space"? If not, then what?

Say you have n vectors in $\mathbb{R}^m$. Consider the matrix A, $m \times n$, whose columns are those n vectors. Can these vectors have a linear combination b, for some b in $\mathbb{R}^m$?

That boils down to: is the system

$$A\mathbf{x} = \mathbf{b}$$

consistent or not?

For $\{v_1, v_2, \dots, v_n\}$, the set of linear combinations of the v_is are the **span** of the set.

$A\mathbf{x}$'s meaning:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & \cdots & \cdots & a_{mn} \end{bmatrix}$$

Four equivalent statements:

1. $\forall b \in \mathbb{R}^m$, $A\mathbf{x} = \mathbf{b}$ has a solution
2. Each $b \in \mathbb{R}^m$ is a linear combinations of columns of A
3. Columns of A span $\mathbb{R}^m$
4. A has a pivot position in every row

If it doesn't span $\mathbb{R}^m$, how do we find/express the span, i.e. find which b for which $A\mathbf{x} = \mathbf{b}$ is possible.

Homogeneous System

$$A\mathbf{x} = \mathbf{0}$$

Put A in row echelon form to check the solution set.