Math 54 Lec6

Linear Transformation

one-to-one

$T:\mathbb{R}^n\to\mathbb{R}^m$ linear transformation with associated matrix A. This is one-to-one if and only if $A\mathbf{x}=\mathbf{0}$ has only the trivial solution.

Which means this is one-to-one exactly when the columns of A are linearly independent.

onto

This is onto exactly when the columns of A span $\mathbb{R}^m$. (i.e. $A\mathbf{x}=\mathbf{b}$ has solution for $\forall\mathbf{b}\in\mathbb{R}^m$)

composition of linear transformations

$S:\mathbb{R}^p\to\mathbb{R}^n, T:\mathbb{R}^n\to\mathbb{R}^m$ are linear transformations. Then $T\circ S:\mathbb{R}^p\to\mathbb{R}^m$ is linear transformation as well.

$S\circ T$ might not be defined (need $p=m$)

Proof :

$$\begin{aligned}
T\circ S(c\mu+d\nu) &= T[S(c\mu+d\nu)]\\
&= T[c\cdot S(\mu)+d\cdot S(\nu)]\\
&= c\cdot[T\circ S](\mu)+d\cdot[T\circ S](\nu)\\
&= c\cdot(T\circ S)(\mu)+d\cdot(T\circ S)(\nu)\\
&\Rightarrow T\circ S \text{ is a linear transformation}
\end{aligned}$$

Matrix Multiplication

terminology

Say

$$\begin{aligned}
A &= [a_{ij}]_{1\leqslant i\leqslant m,1\leqslant j\leqslant n}\\
&= \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n}\\ a_{21} & a_{2}2 & \dots & a_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ a_{m1} & \dots & \dots & a_{mn}\end{bmatrix}
\end{aligned}$$

$a_{11}, a_{22}, \ldots a_{ii}, \ldots$ are the diagonal entries.

When A is a square matrix, a **diagonal matrix** is any matrix where any non-diagonal entry is 0.

$A + B$ only defined if A and B are same size.

$A \cdot B$ only defined if # of **columns** of A is same as # of **row** in B.

Computation of AB