1. Modeling Low Reynolds Number Incompressible Flows Using SPH. (Morris et al. 1997)

1.1 Couette flow.

There are two infinite plates located at y=0 and y=L in Couette flow respectively. The system is initially at rest. At time $t = 0$, the upper plate moves at constant velocity V_0 parallel to the x-axis. The flow was simulated using SPH for $\nu = 10^{-6} m^2 s^{-1}$, $L = 10^{-3} m$, $\rho = 10^3 kgm^{-3}$, $V_0 = 1.25 \times 10^{-5} ms^{-1}$, and with 50 particles spanning the channel. This corresponds to a Reynolds 1.25×10^{-2}, using $Re = V_0 L/\nu$. Figure 1 shows the initial particle position for Couette flow. Figure 2 shows a comparison between velocity profiles obtained using series solution and SPH at several times including the steady state solution.

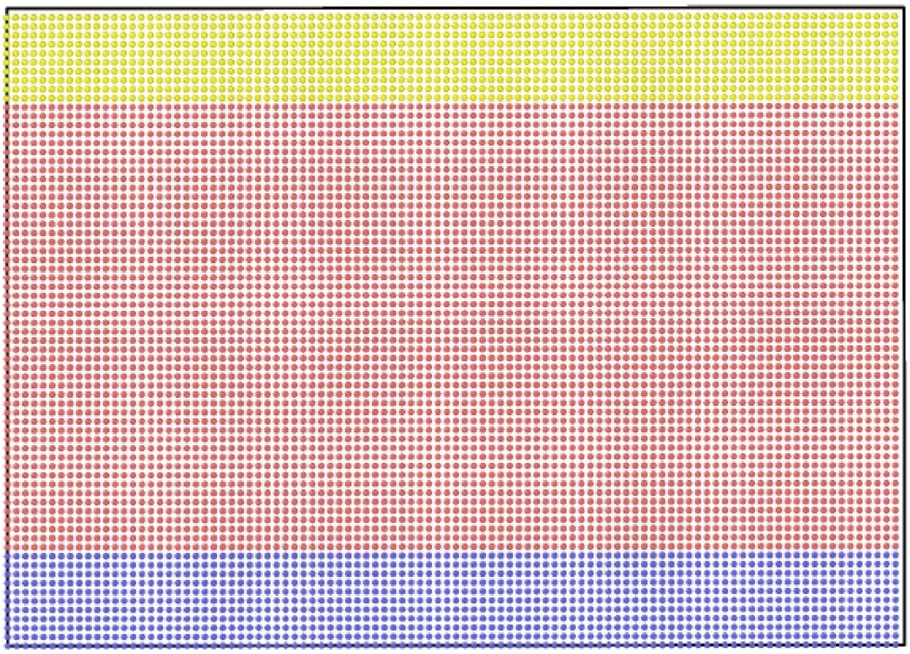

Fig.1. Initial particle position for Couette flow.

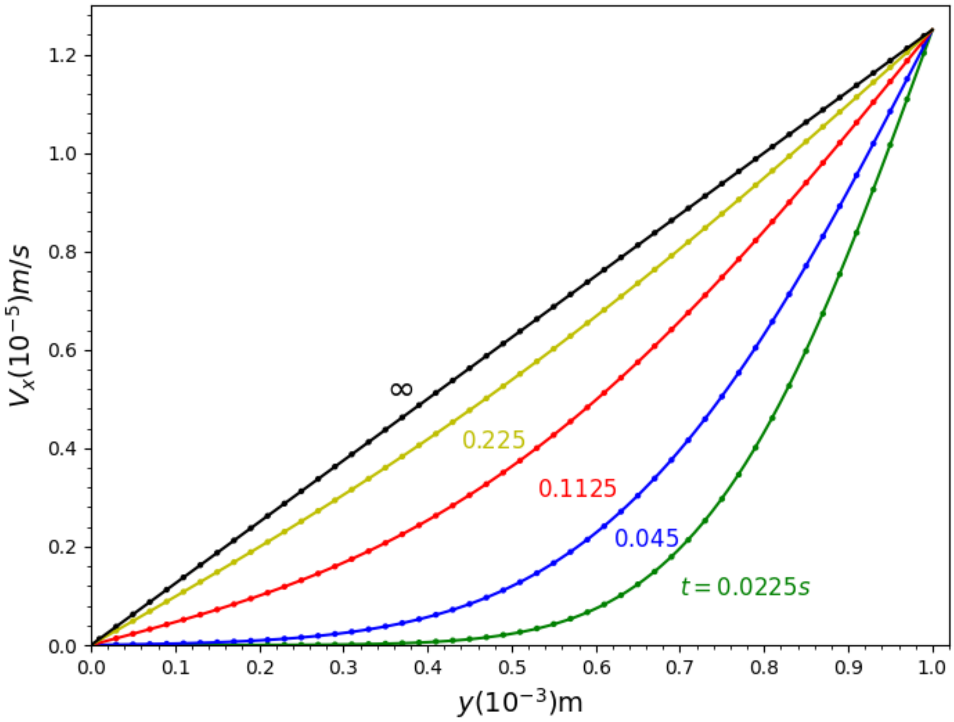

Fig.2. Comparison of SPH and series solutions for Couette flow (Re = 1.25e-2).

1.2 Poiseuille flow

The infinite plates at y=0 and y=L are stationary. The fluid is initially at rest and is driven by an applied body force F parallel to the x-axis for $t \gg 0$. The $\nu = 10^{-6} m^2 s^{-1}$, $L = 10^{-3} m$, $\rho = 10^3 kgm^{-3}$, $F = 10^{-4} ms^{-2}$ in Poiseuille flow. The initial position is same as Couette flow. This corresponds to a peak fluid velocity $V_0 = 1.25 \times 10^{-5}$ so that the $Re = 1.25 \times 10^{-2}$. Figures 3 shows a comparison between velocity profiles obtained using series solution and SPH at several times including the steady state solution.

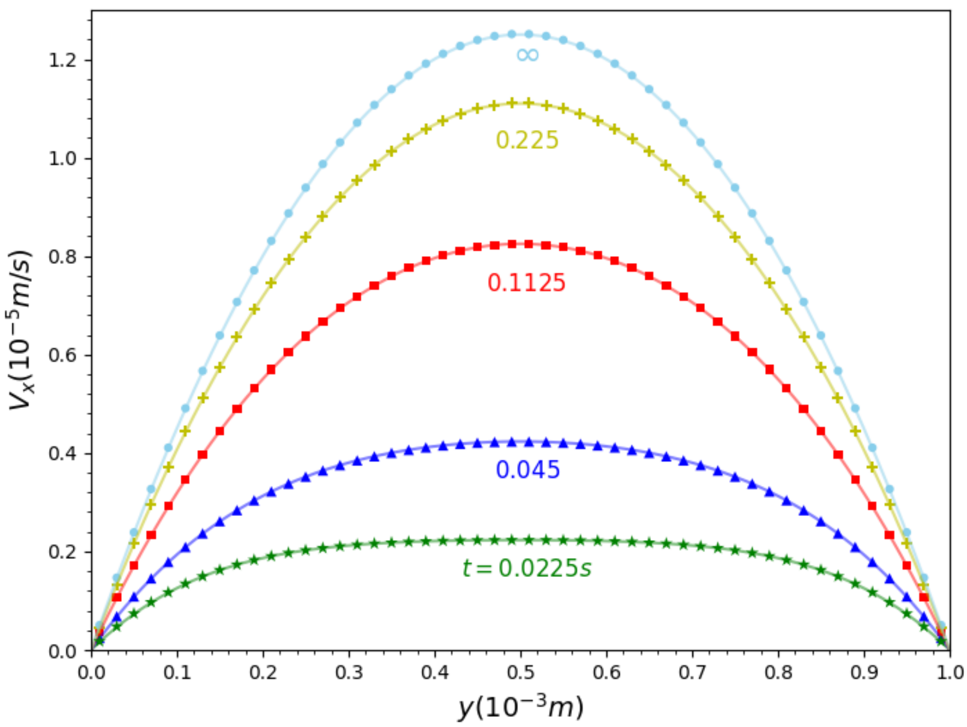

Fig.3. Comparison of SPH and series solutions for Poiseuille flow (Re = 1.25e-2).

1.3 Flow through a Periodic Lattice of Cylinders

In these cases, flow is driven by a pressure gradient and periodic boundary conditions are applied to an infinite periodic array. Figure 4 shows the sketch of flow through a cylinder and four paths for SPH and FEM.

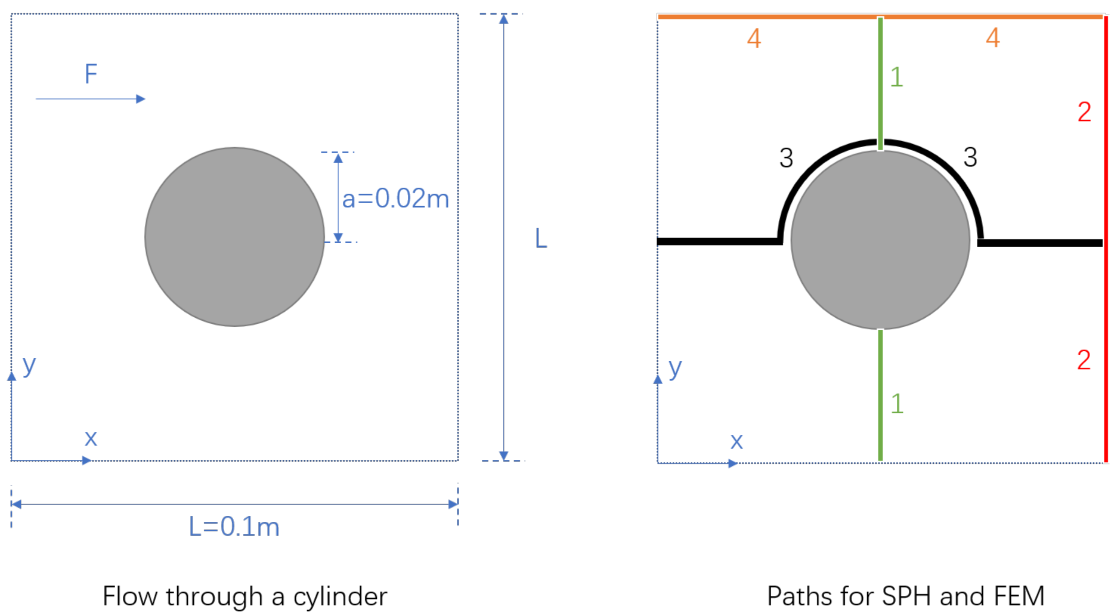

Fig.4. Single cylinder within a periodic lattice and paths for comparison of SPH and FEM solutions.

1.3.1 Low Reynolds Number

Periodic flow past a cylinder was simulated using SPH. For $L = 0.1m$, $\nu = 10^{-6}$, $a =$

$2 \times 10^{-2} m$, $\mathrm{F} = 1.5 \times 10^{-7}$, and $\mathrm{c} = 5.77 \times 10^{-4} ms^{-1}$. taking the velocity scale to be $V_0 = 5 \times 10^{-5} ms^{-1}$gives Re = 1. Figure 5 shows the simulation using approximately 3000 particles placed on a hexagonal lattice with a nearest neighbor distance of 0.002m.

Figure 6 shows a comparison of velocity profiles obtained using SPH and FEM for paths 1 and 2 defined in Fig. 4. Corresponding contour plots of velocity magnitude are shown in Fig. 7. And the dynamic pressure along paths 3 and 4 are shown in Fig.8.

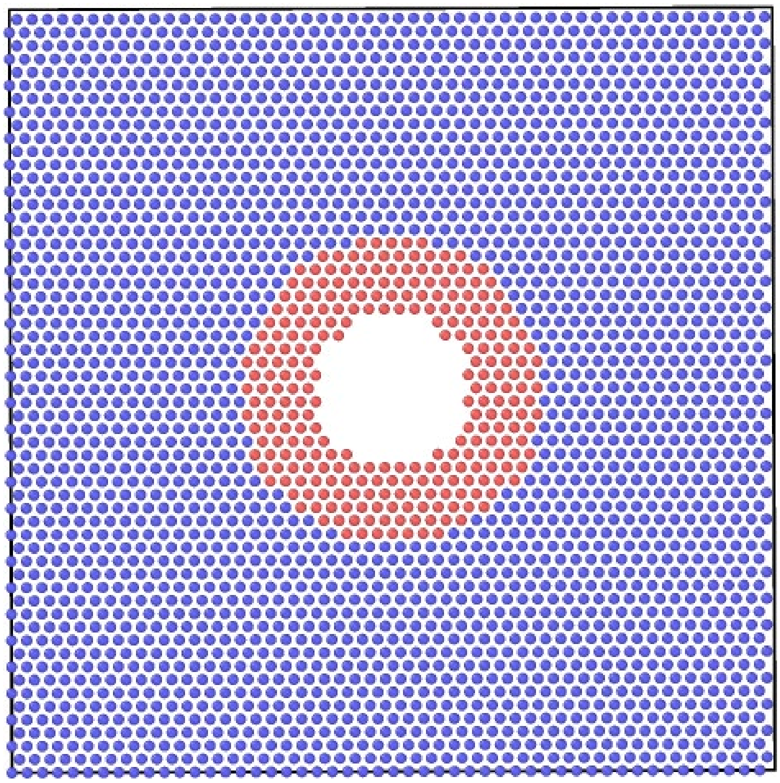

Fig.5. Initial particle positions

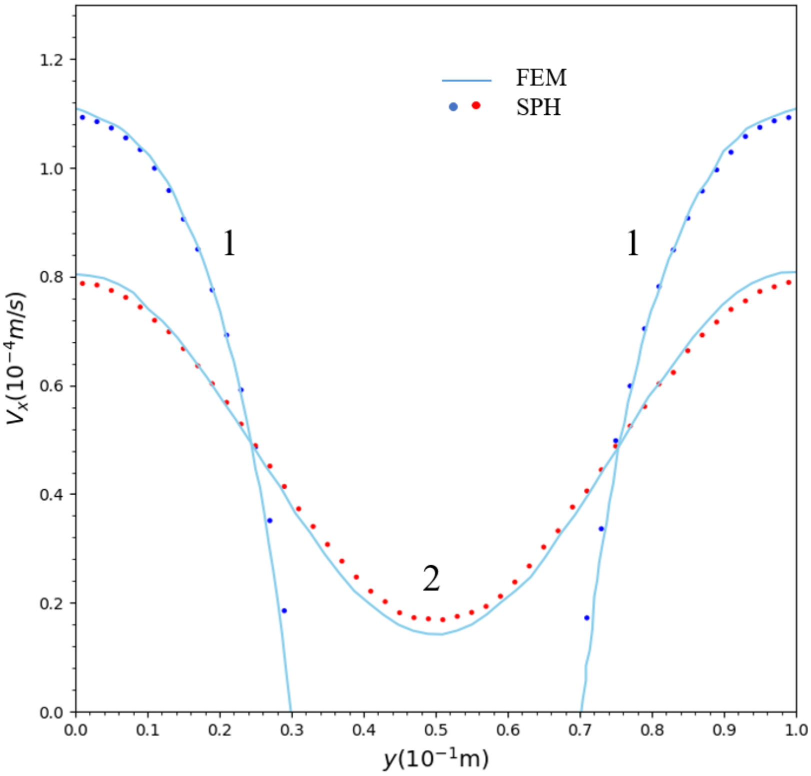

Fig.6. Comparison of SPH and FEM velocity profiles along paths 1 and 2 for Re = 1.

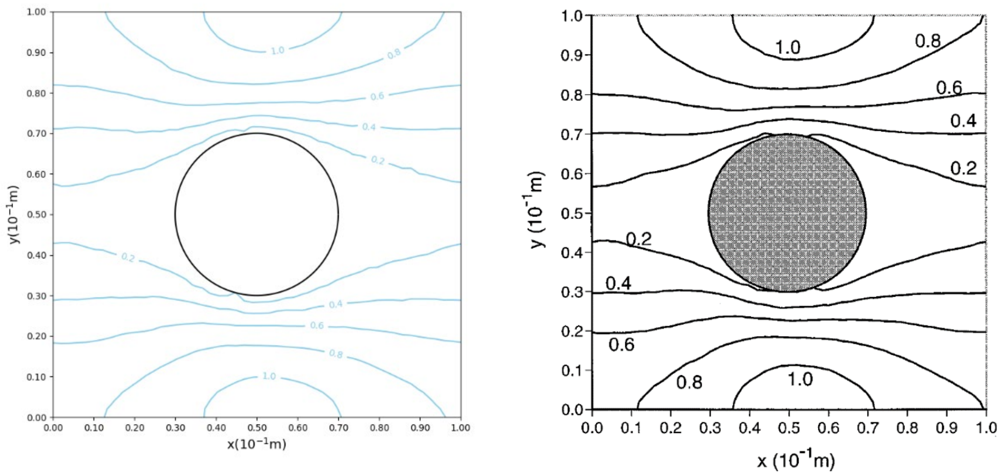

Fig.7. Contour plots of Velocity magnitude using SPH and FEM for Re = 1(contour lines are labeled in units of 10^{-4}m/s).

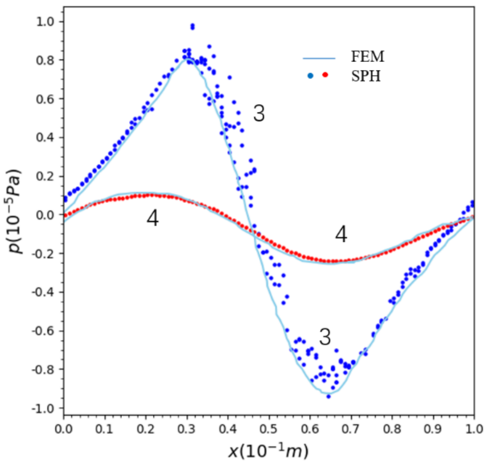

Fig.8. comparison of SPH and FEM pressure profiles along paths 3 and 4 for Re = 1.

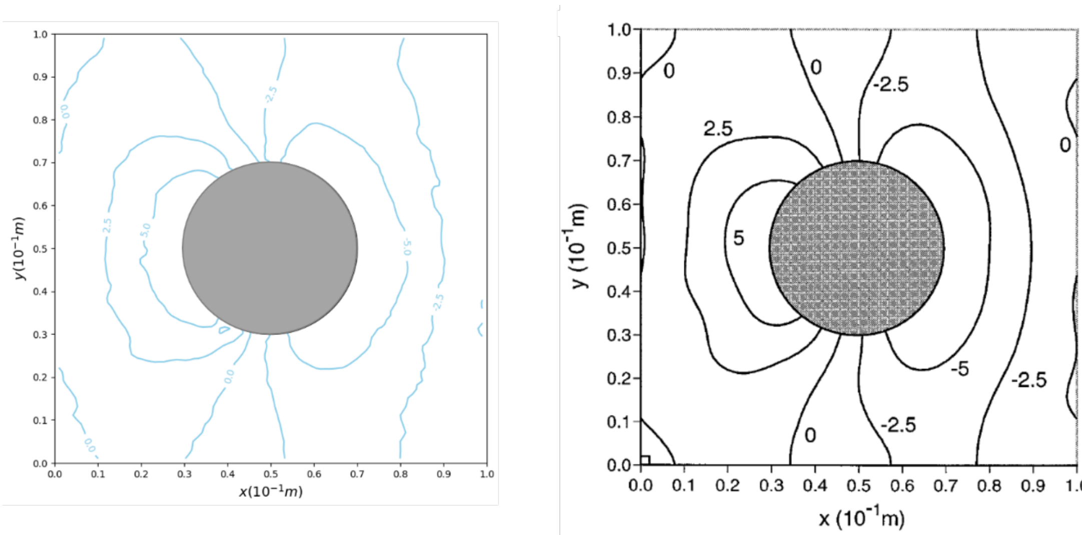

Fig.9. Contour plots of pressure using SPH and FEM for Re = 1(contour lines are labeled in units of 10^{-6}Pa).

1.3.2 Very Low Reynolds Number

Flow through a periodic lattice of cylinders was also solved for $L = 0.1\text{m}$, $\nu = 10^{-4}$, $a = 2 \times 10^{-2}m$, $F = 5 \times 10^{-5}ms^{-1}$, and $c = 1 \times 10^{-2}ms^{-1}$.Taking $V_0 = 1.5 \times 10^{-4}ms^{-1}$, this gives Re = 0.03. As in the previous case, the calculation results of velocity and pressure are shown in the figure 10 -13.

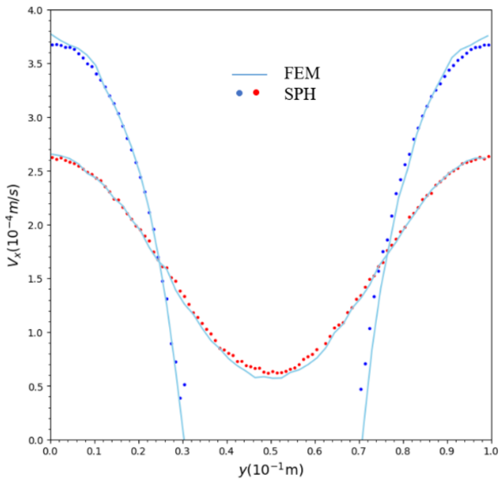

Fig.10. Comparison of SPH and FEM velocity profiles along paths 1 and 2 for Re = 0.03.

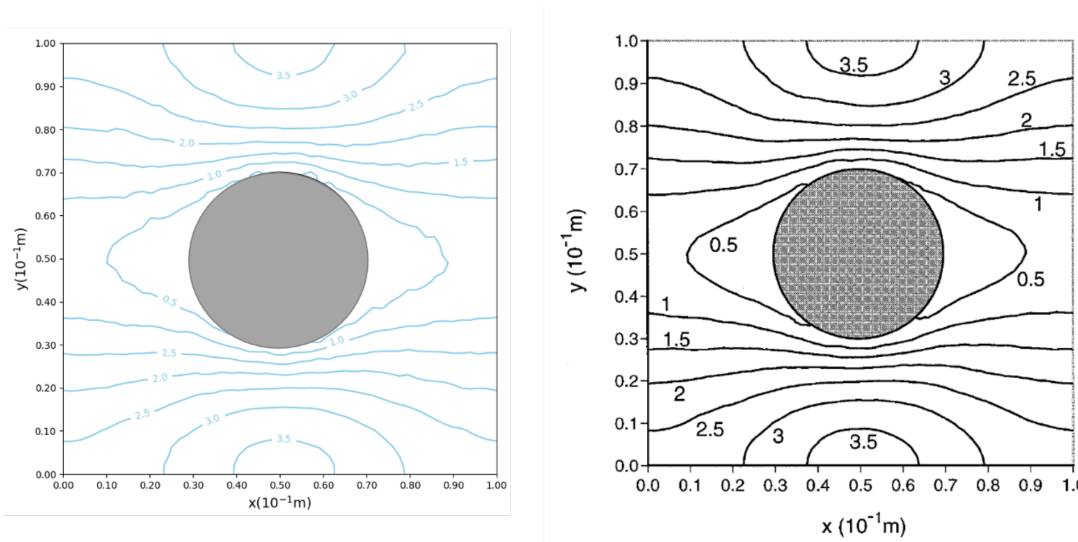

Fig.11. Contour plots of Velocity magnitude using SPH and FEM for Re = 0.03(contour lines are labeled in units of 10^{-4}m/s).

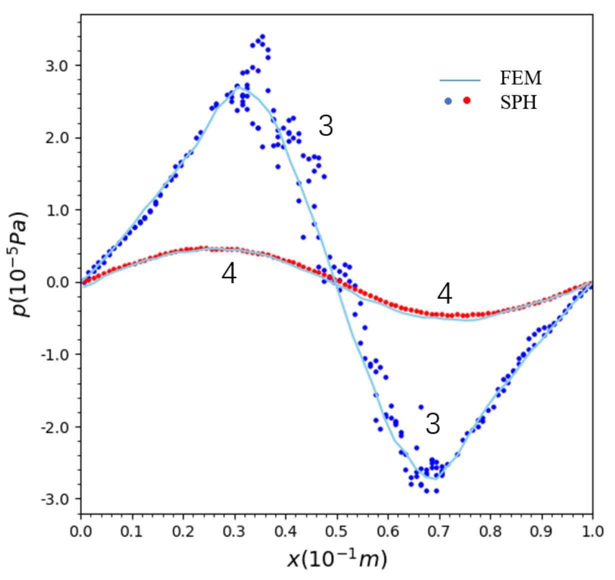

Fig.12. Comparison of SPH and FEM pressure profiles along paths 3 and 4 for Re = 0.03.

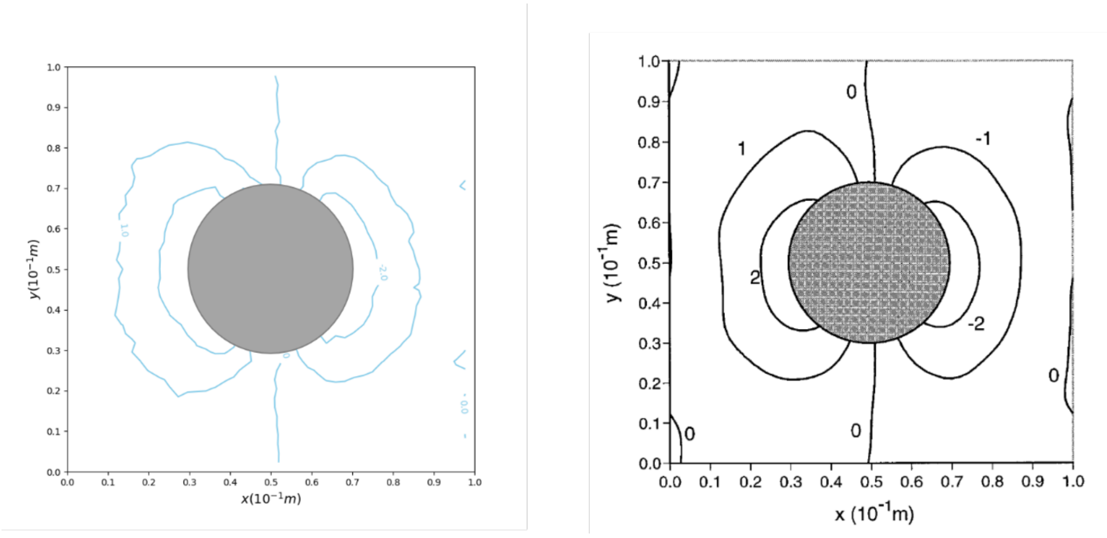

Fig.13. Contour plots of pressure using SPH and FEM for Re = 0.03(contour lines are labeled in units of 10^{-3}Pa).

2. A transport-velocity formulation for smoothed particle hydrodynamics. (Adami et al. 2013)

2.1 Taylor–Green vortex at Re = 100

I reproduced the result of Adami et al. We simulate the two-dimensional Taylor–Green flow at Re = 100 to show that our new approach does not suffer from particle clustering. At t=0 we initialize the velocity of the particles with the analytical solution using a reference velocity of U=1. We vary the initial particle spacing to study the influence of the resolution and use $\Delta x = 0.02$ (50×50 particles), $\Delta x = 0.01$ (100×100 particles) and $\Delta x = 0.005$ (200×200 particles). Fig. 14 shows several snapshots of the particle distribution for the Taylor–Green vortex at Re = 100 using 2500 particles (50×50). A comparison of the decay of the maximum velocity is shown in Fig. 15, where the decay and error of the maximum velocity over time are shown.

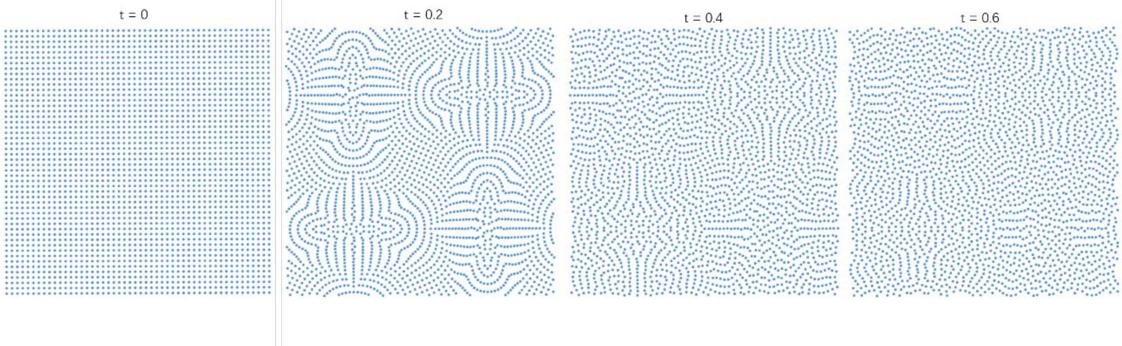

Fig.14. Particle snapshots for the Taylor–Green problem at Re = 100 with a resolution of 50 * 50 particles.

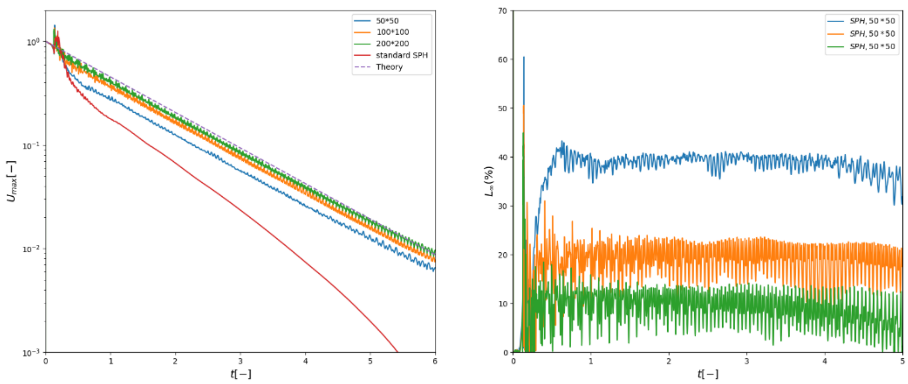

Fig.15. Simulation results for the Taylor–Green problem at Re = 100 with particles initially on a Cartesian lattice.

To avoid the particle rearrangement at the beginning which causes the shift in the maximum velocity, we use the final particle distribution of the previously presented results as initial condition and impose the analytical velocity profile at t=0. Now, the particles are uniformly distributed during the entire simulation as shown in Fig. 16. Velocity field and velocity vectors are shown in Fig.17. Fig. 18 shows the density of all particles at t=2 simply plotted as function of the y-coordinate. The decay of the maximum velocity over time and the relative error for the Taylor–Green flow at Re = 100 using a relaxed particle distribution at t=0 is shown in Fig. 19.

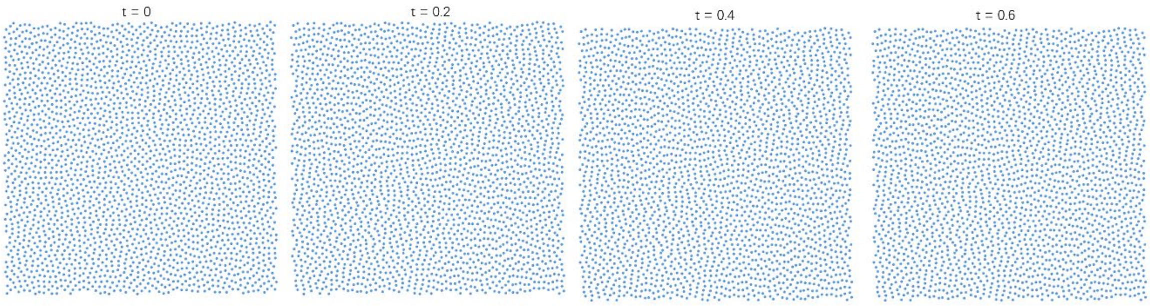

Fig.16. Particle snapshots for the Taylor–Green problem at Re = 100 using 2500 particles.

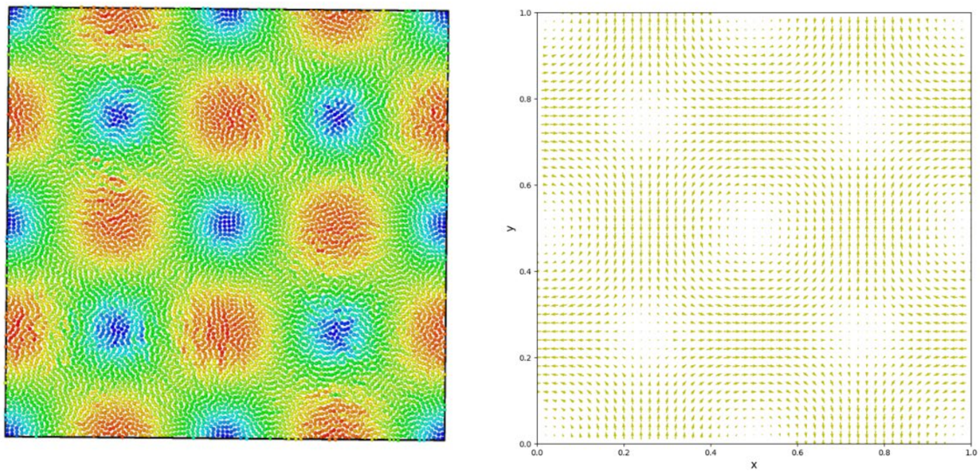

Fig.17. Velocity field and velocity vectors

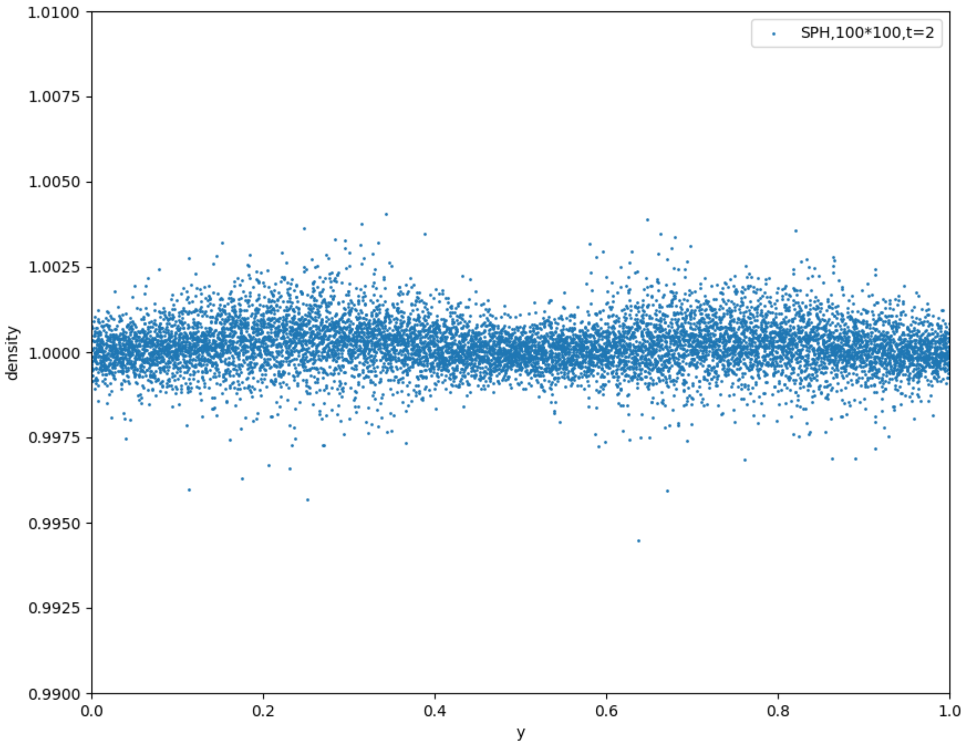

Fig.18.Spatial density variation

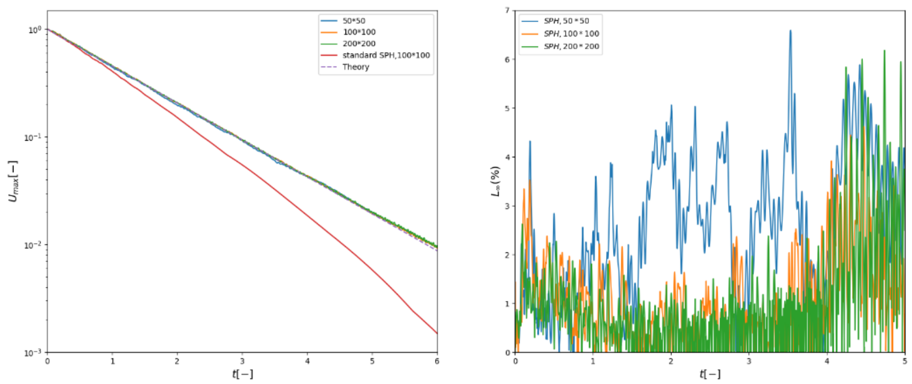

Fig.19. Simulation results for the Taylor–Green problem at Re = 100 using a relaxed initial particle distribution.

2.2 Flow around a periodic lattice of cylinders

This example we study the flow through a periodic lattice of cylinders as presented in Morris et al.) Fig.20. shows two velocity profiles of the axial velocity component $V_x(y)$ as function of the height of the periodic box for the two resolutions. The profiles are taken along vertical lines at x=L/2 (Path 1) and x=L (Path 2), i.e. through the center of the box and at the exit of the domain.

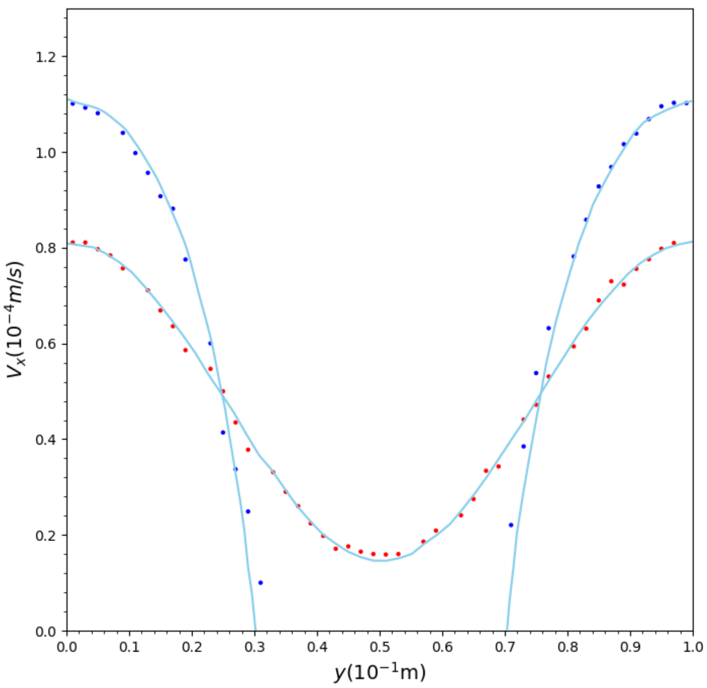

Fig.20. Comparison of SPH results with FEM reference results (Morris et al. 1997) for the flow through a periodic lattice of cylinders at Re = 1.

2.3 Lid-driven cavity

Simulation snapshot of the Lid-driven cavity in steady state at Re = 100 is shown in Fig. 21. The colormap shows the magnitude of the velocity ranging from zero (blue) to U_{MAX}(red). The velocity vectors visualize the structure of the flow is shown in

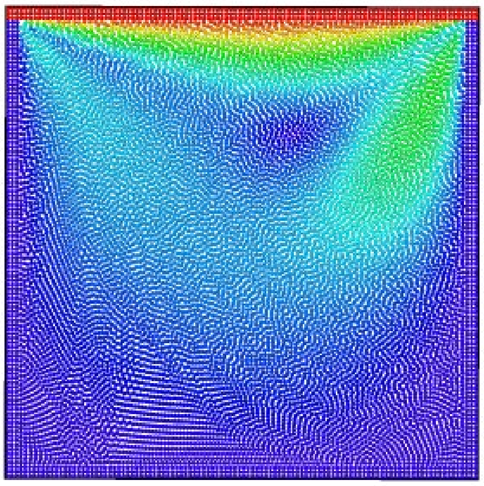

Fig.21.Simulation snapshot of the Lid-driven cavity in steady state (200 × 200 particles).

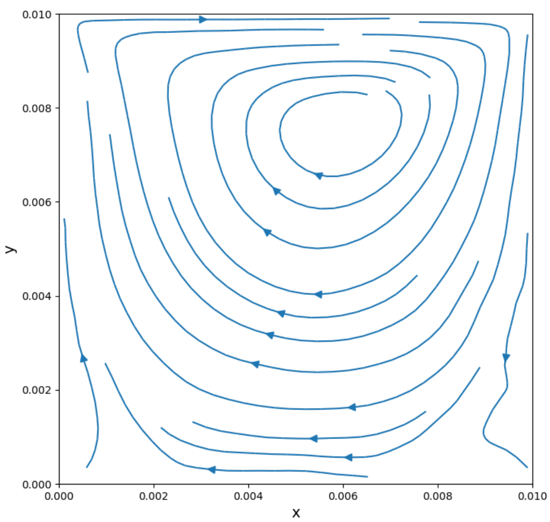

Fig.22. Stream lines of the Lid-driven cavity at Re = 100 (200 × 200 particles).

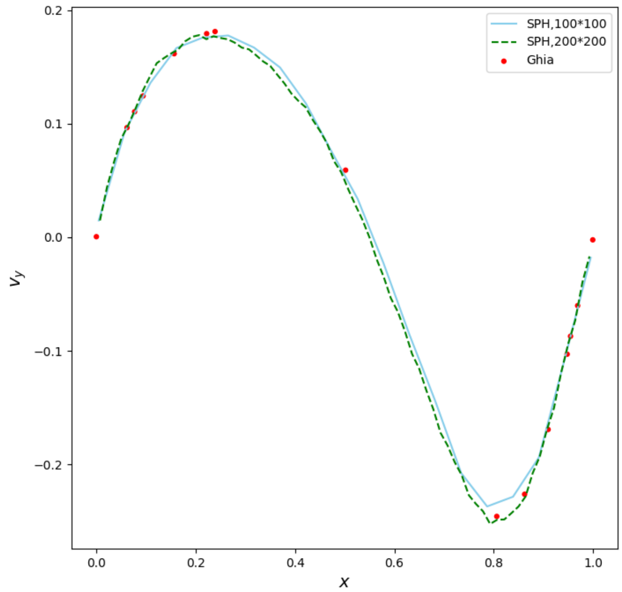

Fig.23. Velocity profiles on the horizontal centerline.

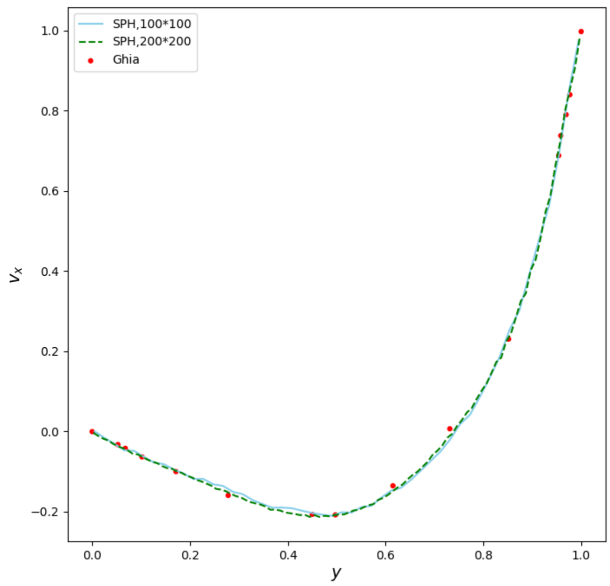

Fig.23. Velocity profiles on the vertical centerline.