Supplementary Material for the paper *Spectral-spatial and Superpixelwise Unsupervised Linear Discriminant Analysis for Feature Extraction and Classification of Hyperspectral Images*

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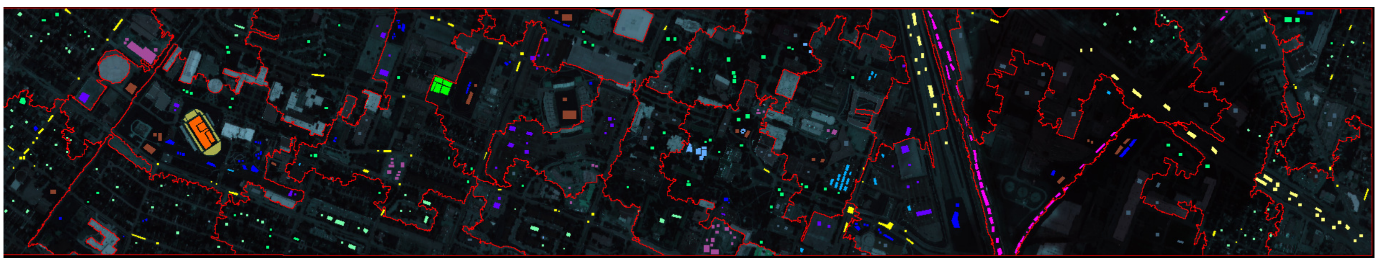
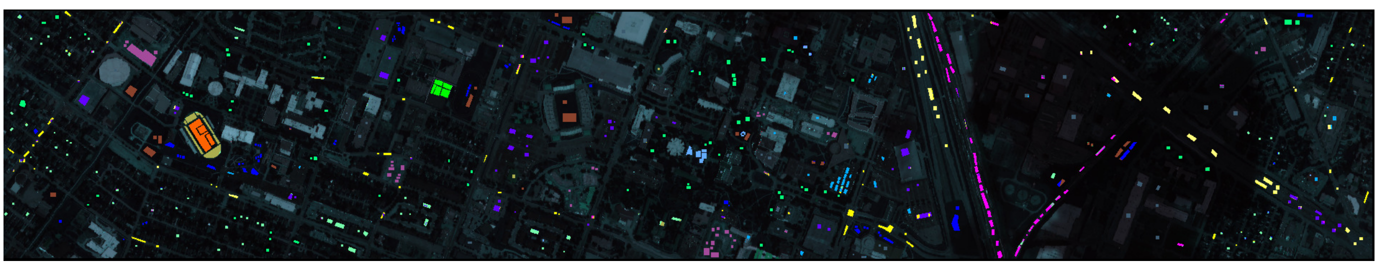
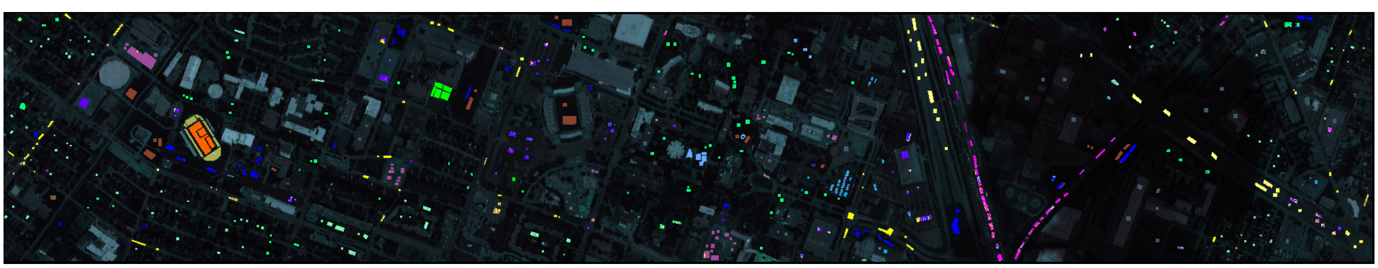
In this supplementary material, we show the classification maps of the proposed S^3-ULDA on Houston 2013 data where 30 labeled samples are randomly picked as the training data and the remaining samples are testing data. The following classification maps in Figure. S1 correspond to results in Table VI in the paper.



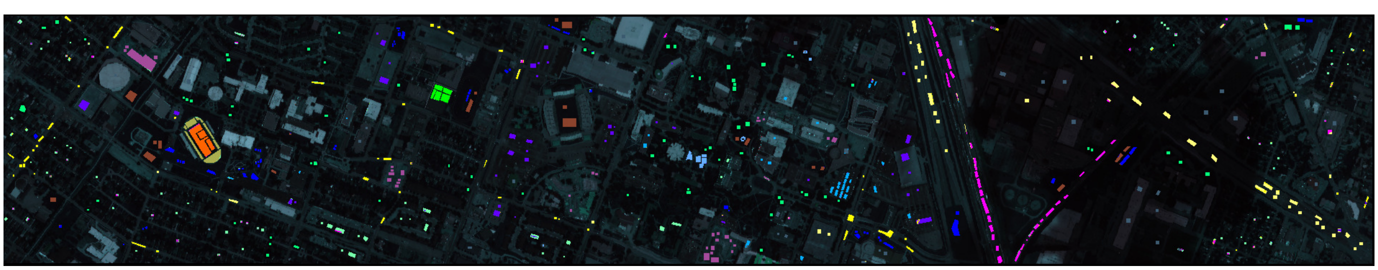
(a) Ground Truth



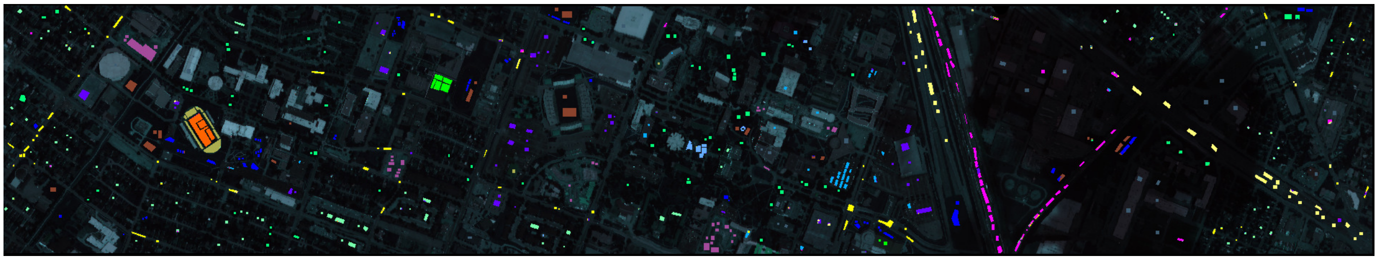
(b) RAW(87.37)



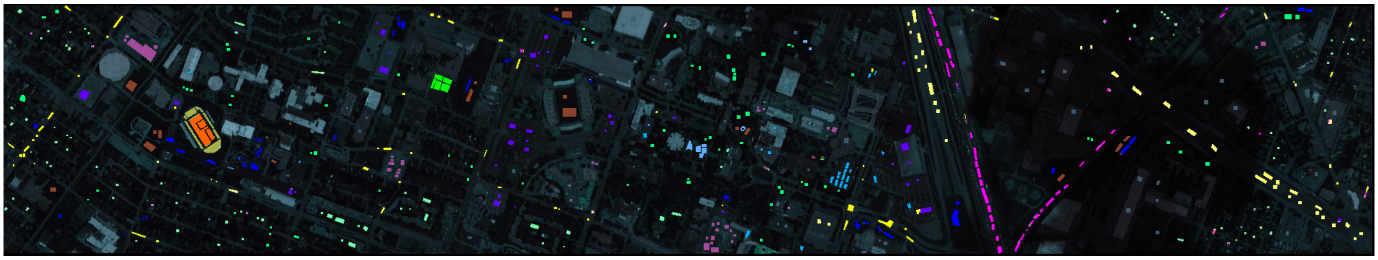
(c) PCA(86.88)



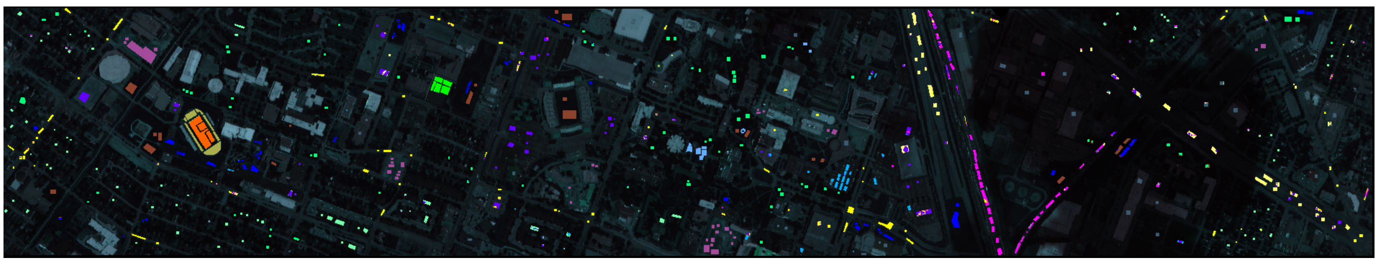
(d) NPE(91.45)



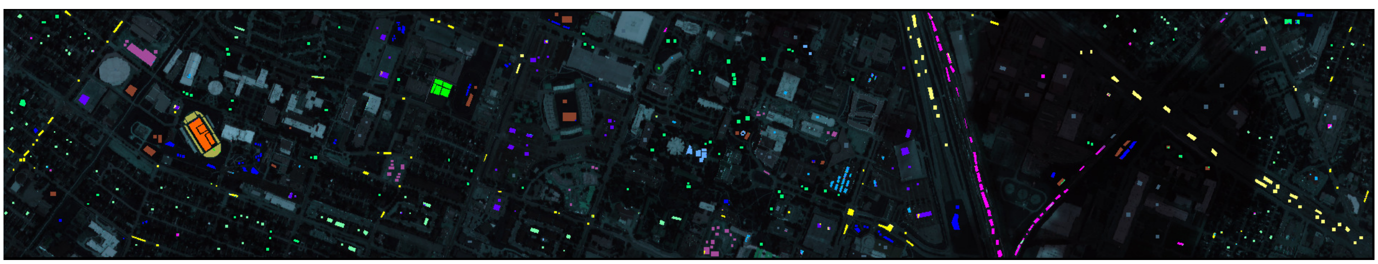
(e) LNSPE(93.12)



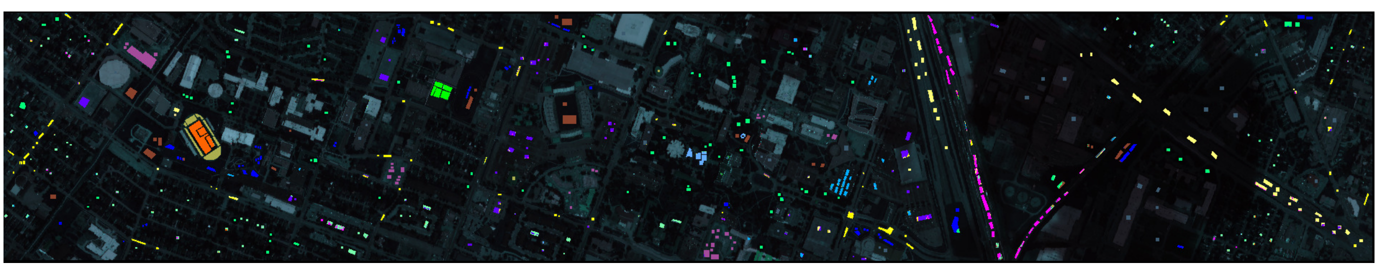
(f) NWFE(92.69)



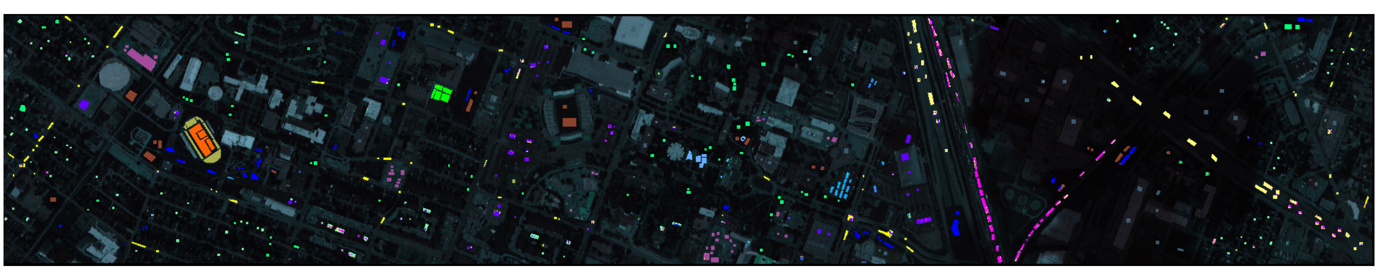
(g) LFDA(89.06)



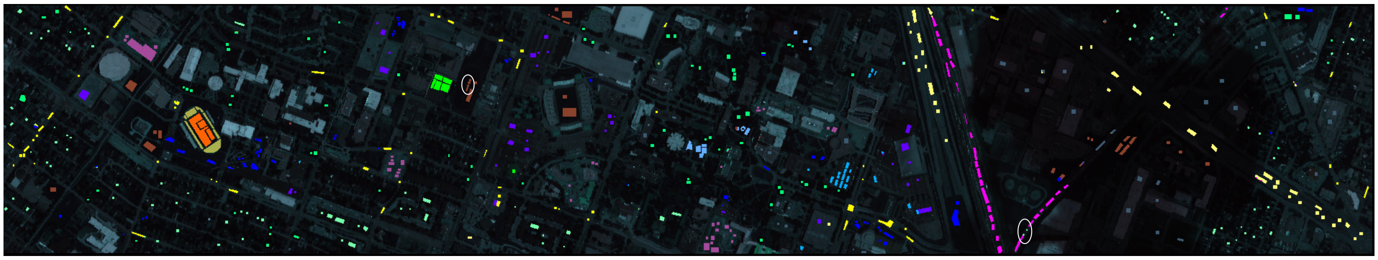
(h) OTVCA(92.34)



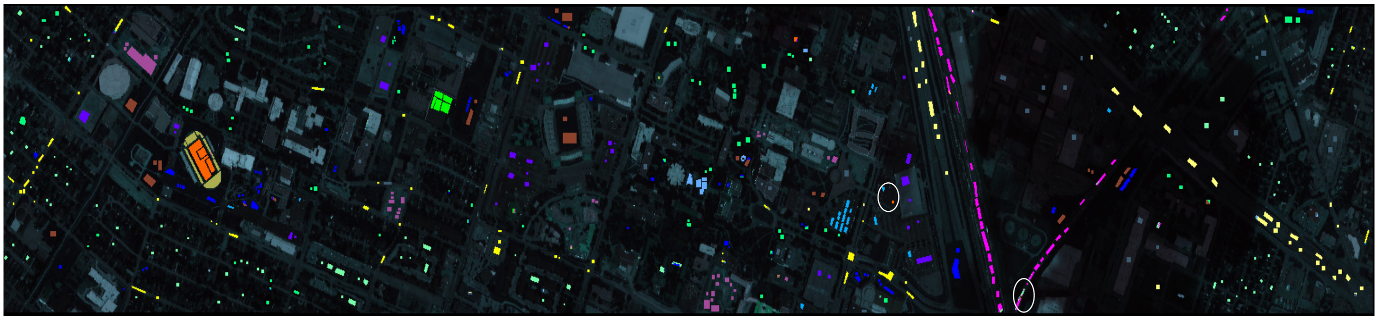
(i) 3DCNN(89.34)



(j) FuNet-C(87.98)

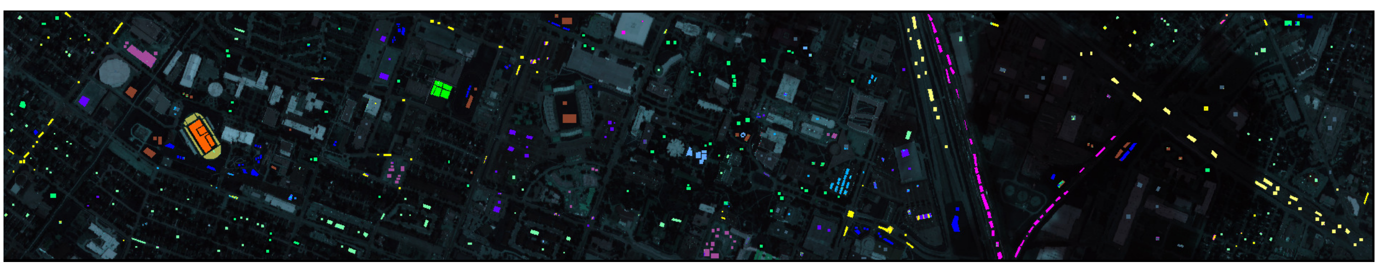


(k) GiGCN(86.03)

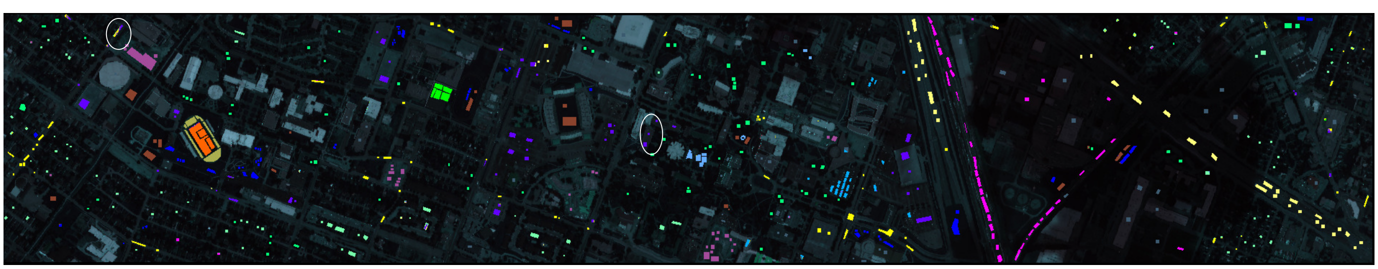


(l) DSGSF(92.73)

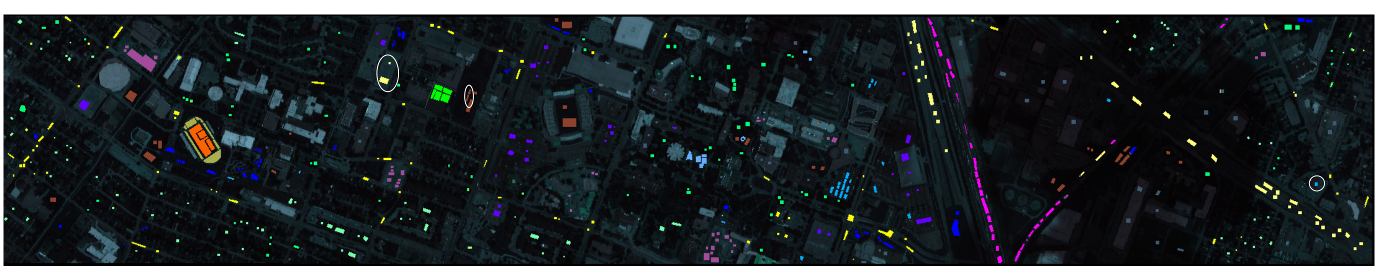
(m) C-SS-MTr(91.95)



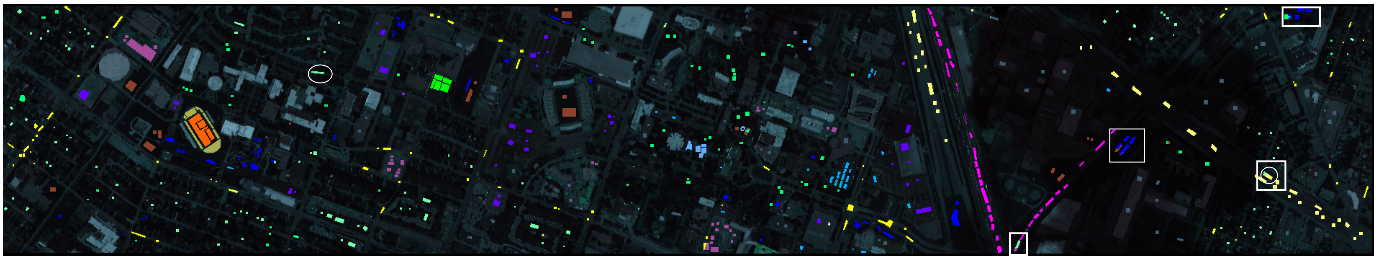
(n) SuperPCA(87.74)



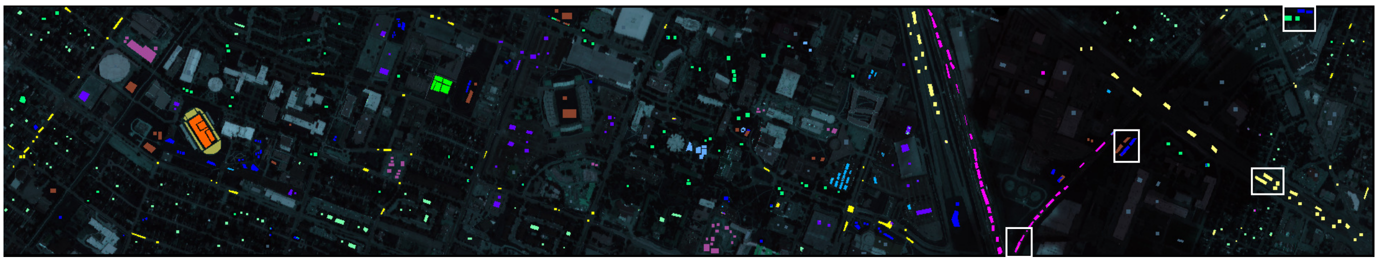
(o) S^3-PCA(91.71)



(p) FG_SuULDA(93.35)



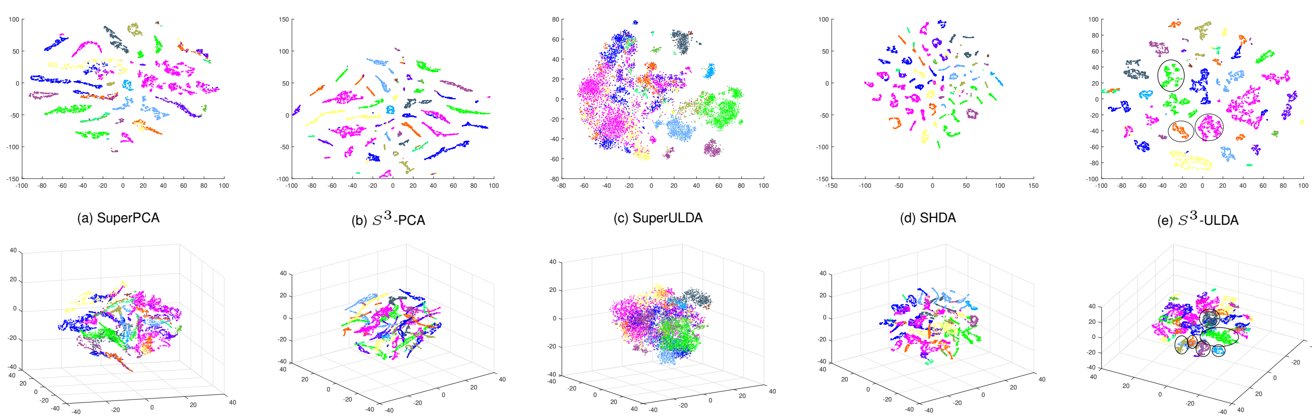
(q) SHDA(93.48)



(r) S^3-ULDA(94.64)

Fig. S1. Classification maps of the Houston 2013 from different models. (a) Ground Truth. (b) Raw. (c) PCA. (d) NPE. (e) LNSPE. (f) NWFE. (g) LFDA. (h) OTVCA. (i) DSGSF. (j) C-SS-MTr. (k) 3DCNN. (l) FuNet-C. (m) GiGCN. (n) SuperPCA. (o) S^3-PCA. (p) FG_SuULDA. (q) SHDA. (r) S^3-ULDA.

Secondly, in addition to the previous experiment of 3D data visualization based on PCA, we also provide 2D and 3D data visualization based on t-SNE for state-of-the-art superpixel-based feature reduction models such as SuperPCA, S^3-PCA, SuperULDA (which can be seen as FG_SuULDA without 3D the flexible Gabor filter), SHDA and the proposed S^3-ULDA. For the two fused superpixelwise feature reduction models S^3-PCA and the proposed S^3-ULDA, we first reduce the dimensionality of the three considered HSIs to 30 and then classic t-SNE is performed to further reduce the 30-dimensional features to 2D and 3D feature space for visualization purposes. All labeled data in the considered HSIs are used for creating the 2D and 3D data visualization and the results is in Figures S2-S4. Similar to the data visualization results based on PCA, the results from t-SNE could also demonstrate that the proposed S^3-ULDA is superior to the comparative models, because in the learnt 2D and 3D feature space, the samples belonging to the same class stay closer, while data from different classes appear away from other classes (especially in the circled areas, which means that the extracted low-dimensional features from the proposed S^3-ULDA are more discriminative than those from the comparative superpixel-based DR models).



(a) SuperPCA (b) S^3-PCA (c) SuperULDA (d) SHDA (e) S^3-ULDA

(f) SuperPCA (g) S^3-PCA (h) SuperULDA (i) SHDA (j) S^3-ULDA

Fig. S2. 2D and 3D data visualization of Indian Pines data using SuperPCA, S^3-PCA, SuperULDA, SHDA and S^3-ULDA based on T-SNE.

Thirdly, we conduct extra experiments based on the simple 1NN(kNN with $k = 1$) classifier to show the discriminability of the learnt low-dimensional features of HSIs. We keep the optimal parameters from Section VI-B unchanged. The experimental results in The OAs obtained from several feature extraction algorithms (combined with the 1NN classifier) as well as state-of-the-art deep learning models with different training samples on the three considered data sets is listed in Table I. What we can conclude from the results is that the proposed S^3-ULDA could significantly outperform the comparative classic DR models and state-of-the-art superpixels based DR models such as S^3-PCA and SHDA in terms of OAs based on the learnt low-dimensional feature subspace and the simple classification model 1NN. Also, compared to the deep learning models with

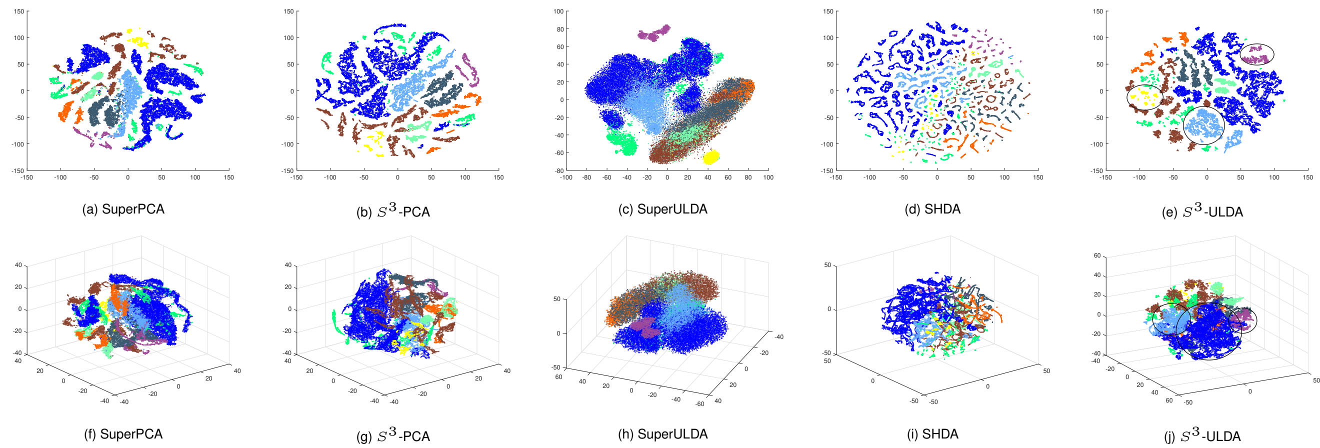


Fig. S3. 2D and 3D data visualization of University of Pavia data using SuperPCA, S^3-PCA, SuperULDA, SHDA and S^3-ULDA based on T-SNE.

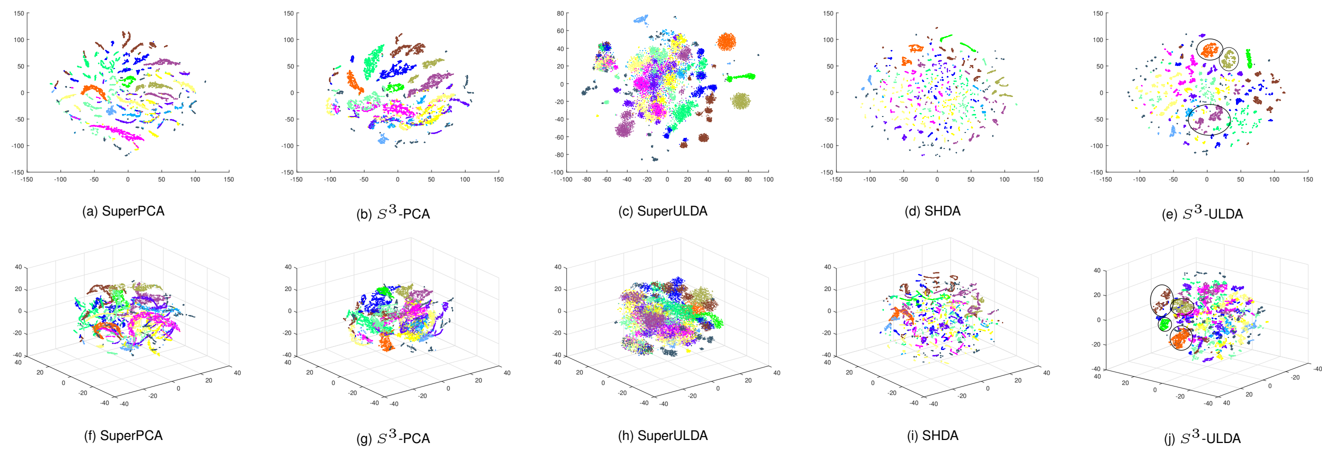


Fig. S4. 2D and 3D data visualization of Houston 2013 data using SuperPCA, S^3-PCA, SuperULDA, SHDA and S^3-ULDA based on T-SNE.

embedded classifiers such as softmax, the proposed S^3-ULDA with 1NN classifier could be superior to some extent, especially in the Indian Pines and University of Pavia data, which demonstrate the effectiveness of the novel model.

TABLE I

OA OBTAINED BY SEVERAL FEATURE EXTRACTION ALGORITHMS (COMBINED WITH THE 1NN CLASSIFIER) AS WELL AS STATE-OF-THE-ART DEEP LEARNING MODELS WITH DIFFERENT TRAINING SAMPLES ON THE THREE CONSIDERED DATA SETS. BEST RESULTS IN BOLD.

Datasets	T.N.s/C	RAW	PCA	NPE	LNSPE[7]	NWFE[2]	LFDA[9]	OTVCA[4]	3DCNN[18]
Indian Pines	5	46.19%±2.36%	46.51%±2.67%	52.89%±2.78%	55.06%±3.36%	63.68%±2.54%	49.89%±4.33%	53.59%±2.22%	46.73%±3.38%
	10	51.36%±2.57%	51.56%±2.58%	60.90%±2.47%	63.30%±2.75%	71.37%±3.46%	50.73%±3.49%	62.06%±0.99%	56.27%±3.89%
	20	56.10%±1.35%	56.33%±1.36%	67.77%±1.89%	70.76%±1.45%	75.31%±1.49%	80.06%±2.25%	68.34%±1.29%	69.50%±3.22%
	30	58.49%±0.88%	58.72%±0.83%	72.28%±1.33%	74.90%±1.47%	82.19%±1.15%	83.72%±1.61%	72.44%±0.87%	75.33%±1.90%
	40	60.05%±0.91%	60.39%±0.75%	74.70%±1.12%	77.16%±1.13%	84.24%±1.25%	86.51%±1.16%	74.34%±0.86%	79.78%±2.97%
	50	61.51%±1.02%	61.77%±0.87%	76.67%±1.12%	79.12%±0.98%	86.06%±1.28%	87.68%±0.95%	76.03%±0.67%	81.84%±2.04%
	60	62.54%±1.28%	62.75%±1.26%	78.25%±0.87%	80.77%±0.66%	87.46%±1.06%	88.62%±0.88%	77.50%±0.83%	83.37%±1.99%
University of Pavia	5	58.84%±4.64%	58.84%±4.66%	61.35%±4.63%	67.70%±5.16%	73.52%±3.37%	77.69%±3.05%	63.11%±5.55%	63.91%±4.89%
	10	63.27%±2.66%	63.26%±2.61%	66.38%±3.43%	73.02%±3.78%	77.81%±3.29%	81.81%±2.15%	68.13%±2.90%	72.26%±3.41%
	20	67.26%±1.82%	67.24%±1.78%	73.30%±1.59%	81.14%±1.80%	84.08%±2.44%	84.09%±1.72%	73.69%±1.89%	75.26%±2.20%
	30	68.89%±1.46%	68.86%±1.36%	75.42%±1.98%	82.96%±2.07%	85.23%±2.59%	84.73%±1.71%	75.99%±1.57%	78.49%±1.32%
	40	70.55%±1.11%	70.50%±1.10%	76.99%±1.86%	84.57%±1.89%	86.62%±2.11%	85.53%±1.73%	78.25%±1.94%	81.17%±1.78%
	50	71.13%±0.92%	71.04%±0.86%	78.57%±1.76%	85.92%±1.60%	87.97%±1.63%	85.97%±1.78%	79.07%±1.37%	82.96%±1.71%
	60	72.22%±0.10%	72.09%±1.03%	80.39%±1.69%	87.48%±1.19%	86.61%±1.64%	80.49%±1.72%	80.48%±1.71%	84.97%±2.19%
Houston 2013	5	64.63%±2.96%	64.57%±3.01%	62.68%±2.19%	68.76%±2.64%	72.55%±2.98%	59.36%±3.51%	67.98%±1.97%	70.41%±2.39%
	10	72.05%±1.75%	71.98%±1.81%	70.10%±1.74%	77.51%±1.95%	80.43%±2.05%	62.14%±3.18%	75.40%±1.71%	79.29%±2.78%
	20	78.04%±0.78%	77.96%±0.77%	77.06%±0.90%	84.00%±0.75%	86.72%±0.86%	73.94%±2.43%	82.23%±1.07%	85.51%±1.57%
	30	81.36%±0.63%	81.25%±0.56%	82.14%±0.63%	87.75%±0.74%	90.27%±0.83%	82.18%±1.43%	86.11%±0.72%	87.92%±1.64%
	40	83.38%±0.68%	83.27%±0.66%	84.54%±0.67%	89.88%±0.69%	92.08%±0.79%	85.74%±1.00%	88.42%±0.61%	90.12%±1.74%
	50	84.86%±0.35%	84.77%±0.32%	86.51%±0.75%	91.49%±0.54%	93.54%±0.75%	88.19%±0.82%	89.80%±5.63%	91.70%±1.18%
	60	86.01%±0.55%	85.89%±0.48%	87.66%±0.63%	92.48%±0.66%	94.34%±0.59%	90.18%±0.82%	90.87%±0.61%	92.22%±1.72%

Datasets	T.N.s/C	FuNet-C[19]	GiGCN[20]	DSGSF[23]	C-SS-MTr[26]	SuperPCA[30]	S^3-PCA[16]	FG_SuULDA[15]	SHDA[31]	S^3-ULDA
Indian Pines	5	49.45%±2.63%	77.25%±3.93%	52.03%±5.78%	59.74%±3.94%	61.31%±4.61%	70.50%±4.82%	73.81%±4.78%	78.37%±3.60%	**82.71%±3.17%**
	10	58.80%±1.97%	82.69%±3.86%	77.65%±1.56%	74.21%±3.11%	78.23%±4.04%	84.41%±3.49%	73.81%±4.78%	86.11%±3.63%	**87.93%±3.13%**
	20	68.59%±2.49%	88.55%±2.05%	87.81%±1.33%	82.99%±2.12%	89.78%±2.44%	93.18%±1.60%	82.70%±4.85%	93.82%±1.39%	**94.43%±1.47%**
	30	73.24%±1.15%	90.38%±1.07%	90.73%±0.79%	87.27%±2.00%	93.20%±1.21%	95.16%±1.08%	90.32%±1.07%	95.59%±0.93%	**96.04%±0.91%**
	40	77.03%±1.04%	91.52%±1.22%	93.02%±0.69%	91.92%±0.81%	94.43%±0.96%	95.56%±1.01%	93.24%±0.52%	96.04%±0.77%	**96.59%±1.03%**
	50	79.08%±1.29%	91.32%±1.49%	94.50%±0.91%	92.14%±1.88%	94.77%±0.89%	95.89%±1.09%	94.64%±0.81%	96.27%±0.67%	**97.04%±0.81%**
	60	80.40%±1.66%	91.70%±0.94%	95.77%±0.88%	95.02%±0.76%	95.18%±0.77%	96.49%±0.84%	96.18%±0.42%	96.52%±0.53%	**97.45%±0.66%**
University of Pavia	5	69.87%±5.34%	**86.72%±4.57%**	59.60%±0.56%	71.67%±2.92%	68.82%±4.24%	81.81%±3.55%	74.03%±6.22%	80.71%±4.08%	85.63%±3.60%
	10	77.33%±3.45%	**94.03%±2.57%**	64.20%±0.43%	80.86%±3.19%	81.55%±2.58%	89.18%±2.79%	84.14%±2.62%	88.96%±3.70%	93.47%±2.82%
	20	83.61%±2.53%	96.43%±1.50%	82.68%±2.70%	90.28%±2.87%	87.54%±1.05%	93.69%±2.07%	89.50%±2.76%	93.17%±2.18%	**96.78%±0.96%**
	30	86.60%±1.57%	97.26%±1.18%	89.45%±0.98%	93.41%±1.26%	89.64%±0.90%	95.37%±0.97%	92.35%±0.92%	94.35%±1.98%	**97.67%±0.73%**
	40	88.56%±1.29%	97.53%±0.86%	94.79%±0.34%	95.45%±0.89%	90.90%±1.03%	95.89%±1.13%	93.55%±0.53%	95.87%±0.84%	**97.88%±0.62%**
	50	90.04%±1.10%	97.74%±0.57%	95.71%±0.23%	96.60%±0.99%	91.83%±1.19%	96.45%±1.28%	94.39%±0.61%	96.17%±0.85%	**98.34%±0.41%**
	60	91.40%±0.91%	97.96%±0.62%	96.19%±0.46%	96.95%±0.72%	92.45%±1.21%	96.82%±1.22%	95.34%±0.61%	96.49%±0.63%	**98.62%±0.33%**
Houston 2013	5	70.79%±2.40%	**73.72%±0.41%**	56.42%±2.11 %	70.61%±3.31%	63.15%±2.88%	67.10%±1.85%	67.76%±3.25%	67.90%±3.29%	72.37%±2.94%
	10	79.71%±0.84%	**81.89%±2.37%**	70.69%±2.19%	81.15%±3.07%	72.75%±2.06%	76.11%±1.74%	78.88%±2.18%	77.51%±2.70%	81.77%±1.83%
	20	85.07%±1.35%	87.86%±1.84%	85.05%±1.20%	**89.16%±1.34%**	80.36%±1.34%	83.43%±0.99%	86.23%±1.40%	85.89%±1.93%	88.27%±1.40%
	30	87.73%±0.81%	91.02%±1.11%	85.48%±1.43%	92.47%±1.35%	85.51%±1.07%	88.23%±0.86%	90.81%±0.90%	90.53%±0.94%	**92.39%±0.74%**
	40	89.34%±1.08%	92.51%±0.89%	87.68%±1.19%	94.35%±0.69%	88.25%±0.87%	90.68%±0.67%	93.39%±0.74%	92.65%±1.19%	94.40%±0.67%
	50	91.04%±0.70%	93.31%±0.49%	92.83%±1.15%	**96.04%±0.49%**	90.54%±0.58%	92.62%±0.52%	94.88%±0.85%	94.24%±1.09%	**95.82%±0.63%**
	60	92.11%±0.50%	93.79%±1.28%	95.77%±0.95%	**97.29%±0.34%**	91.70%±0.63%	93.85%±0.58%	95.89%±0.92%	95.49%±0.70%	96.87%±0.58%

In the end, we also provide extra experiments in Table II to see the changes of all the comparative models under weak and strong noise levels. To add noise to HSIs, we use Matlab function imnoise() to add Gaussian noise with variances 0.0001 and 0.001 to original spectral features of HSI data to obtain the noisy HSI data with weak and strong noise levels.

1) hsidata3D=imnoise(hsidata3D,'gaussian',0,0.0001);
2) hsidata3D=imnoise(hsidata3D,'gaussian',0,0.001);

TABLE II

OA OBTAINED BY SEVERAL FEATURE EXTRACTION ALGORITHMS (COMBINED WITH THE SVM CLASSIFIER) AS WELL AS STATE-OF-THE-ART DEEP LEARNING MODELS WITH DIFFERENT NOISE LEVELS AND DIFFERENT TRAINING SAMPLES ON THE THREE CONSIDERED DATA SETS. BEST RESULTS IN BOLD.

Datasets	Noise Level	T.N.s/C	RAW	PCA	NPE	LNSPE[7]	NWFE[2]	LFDA[9]	OTVCA[4]	3DCNN[18]	FuNet-C[19]	GiGCN[20]	DSGSF[23]	C-SS-MTr[26]	SuperPCA[30]	S^3-PCA[16]	FG_SuULDA[15]	SHDA[31]	S^3-ULDA
Indian Pines	Week Noise	5	38.26%±5.24%	41.35%±5.12%	51.09%±5.09%	53.85%±6.33%	62.54%±2.27%	47.49%±1.99%	48.89%±5.12%	38.30%±5.03%	41.60%±2.84%	74.72%±2.96%	40.31%±1.59%	58.16%±5.11%	60.98%±5.82%	67.29%±4.51%	73.15%±2.77%	73.23%±5.36%	**77.29%±5.24%**
		10	49.96%±4.55%	52.72%±4.27%	66.58%±2.91%	70.59%±3.87%	73.43%±3.18%	60.85%±3.14%	61.05%±3.78%	47.12%±2.63%	55.48%±1.65%	82.02%±4.52%	65.75%±1.89%	71.32%±3.58%	82.35%±4.96%	83.38%±5.50%	86.37%±2.32%	84.40%±5.30%	**86.65%±4.98%**
		20	57.92%±2.54%	59.01%±2.11%	74.68%±2.91%	81.28%±1.05%	82.33%±1.75%	74.70%±1.60%	69.58%±2.05%	56.57%±2.78%	63.20%±3.18%	87.80%±1.43%	80.43%±2.74%	80.36%±1.87%	90.14%±1.53%	92.33%±2.01%	90.90%±1.23%	91.47%±1.68%	**92.50%±1.40%**
	Strong Noise	5	32.61%±1.54%	35.37%±5.34%	48.26%±5.92%	51.43%±6.81%	55.23%±2.39%	39.18%±3.73%	42.41%±6.10%	30.08%±3.67%	39.29%±1.17%	73.46%±3.31%	33.28%±1.31%	55.21%±2.35%	59.22%±6.05%	65.35%±4.96%	70.62%±3.10%	72.41%±6.43%	**76.37%±5.66%**
		10	36.53%±4.09%	41.58%±3.22%	61.75%±3.07%	67.55%±3.26%	65.39%±2.72%	56.10%±1.71%	53.09%±2.55%	38.05%±4.99%	53.62%±1.78%	81.78%±4.62%	48.85%±1.66%	64.15%±4.05%	81.25%±3.72%	82.31%±4.00%	81.49%±4.49%	83.22%±3.73%	**84.93%±4.57%**
		20	42.67%±1.94%	46.88%±2.12%	69.61%±1.77%	77.18%±2.20%	74.83%±1.96%	70.25%±0.87%	59.16%±1.66%	45.34%±3.14%	60.72%±1.39%	87.57%±1.18%	76.51%±1.72%	74.30%±3.56%	86.95%±1.34%	89.11%±1.74%	89.98%±0.35%	90.41%±2.09%	**91.09%±1.29%**
University of Pavia	Week Noise	5	58.77%±5.51%	61.11%±8.36%	68.07%±6.51%	67.67%±6.46%	70.19%±4.00%	53.81%±4.16%	69.42%±4.27%	59.00%±6.89%	68.75%±4.62%	86.09%±4.12%	41.37%±4.83%	71.15%±6.02%	73.11%±2.61%	83.37%±4.51%	72.97%±4.09%	82.85%±5.61%	**86.64%±3.55%**
		10	65.22%±5.85%	66.48%±5.04%	78.22%±3.87%	78.37%±3.81%	80.62%±3.10%	69.90%±3.66%	77.17%±3.20%	63.58%±1.62%	76.71%±3.82%	93.66%±1.97%	63.58%±1.62%	80.70%±4.66%	82.98%±3.47%	91.39%±3.12%	85.27%±3.23%	90.49%±2.72%	**94.21%±2.75%**
		20	69.40%±2.79%	70.28%±3.10%	83.67%±2.64%	87.30%±1.80%	87.99%±2.25%	78.08%±2.10%	85.15%±2.35%	74.38%±3.99%	82.13%±2.12%	96.01%±0.68%	81.60%±4.81%	88.99%±4.49%	88.20%±2.95%	94.62%±1.53%	93.68%±1.95%	94.59%±1.59%	**97.75%±0.93%**
	Strong Noise	5	56.84%±4.79%	53.95%±4.83%	62.51%±6.59%	65.66%±5.20%	69.79%±4.45%	49.77%±3.89%	57.86%±6.71%	58.94%±4.81%	64.76%±4.63%	81.71%±1111%	38.57%±6.41%	69.88%±4.67%	65.13%±5.08%	77.59%±5.58%	68.79%±4.39%	76.76%±6.69%	**81.79%±4.30%**
		10	59.64%±4.53%	57.13%±5.09%	69.33%±6.04%	76.62%±3.66%	74.32%±3.89%	65.49%±2.82%	64.84%±6.66%	66.96%±2.47%	72.37%±2.23%	**90.69%±1.43%**	61.67%±5.16%	79.73%±3.63%	77.73%±2.84%	86.97%±1.49%	84.90%±2.79%	88.33%±2.53%	90.11%±1.59%
		20	61.80%±1.63%	60.49%±2.60%	74.26%±3.36%	82.15%±2.56%	81.22%±2.49%	71.68%±2.20%	67.32%±3.16%	69.98%±2.83%	75.58%±0.75%	**95.45%±0.68%**	68.15%±2.88%	87.13%±1.77%	82.41%±2.38%	89.67%±2.01%	91.64%±1.95%	91.63%±2.78%	95.22%±1.20%
Houston 2013	Week Noise	5	49.67%±2.71%	54.58%±2.47%	64.74%±3.70%	68.47%±2.69%	67.38%±2.94%	47.89%±1.83%	66.45%±3.09%	65.38%±3.05%	63.72%±3.41%	71.18%±2.70%	51.68%±3.69%	69.08%±2.76%	54.18%±2.98%	60.69%±3.13%	52.72%±2.24%	70.86%±1.63%	**71.87%±2.82%**
		10	56.21%±3.43%	60.16%±2.10%	74.14%±1.54%	79.22%±1.36%	75.22%±1.47%	64.59%±2.11%	79.40%±1.58%	75.42%±2.05%	77.49%±2.29%	82.13%±3.61%	65.73%±1.62%	78.34%±2.70%	64.08%±1.99%	72.16%±1.97%	79.47%±3.52%	81.49%±0.79%	**82.63%±1.95%**
		20	62.93%±1.16%	63.86%±1.32%	79.36%±0.71%	84.74%±0.65%	81.73%±1.45%	73.95%±1.98%	86.64%±1.10%	80.40%±1.91%	81.31%±1.62%	85.44%±1.60%	80.61%±2.32%	86.30%±1.85%	70.64%±1.30%	78.00%±1.03%	87.42%±1.51%	87.54%±0.72%	**88.36%±0.86%**
	Strong Noise	5	39.16%±2.16%	41.33%±1.55%	54.66%±2.26%	60.74%±1.84%	58.23%±1.07%	42.36%±2.50%	49.75%±2.43%	53.90%±4.23%	54.50%±3.07%	65.14%±3.36%	45.22%±4.33%	60.87%±2.08%	42.86%±2.73%	47.18%±2.92%	42.38%±5.69%	**65.74%±2.52%**	63.02%±4.47%
		10	44.56%±2.92%	45.23%±1.62%	63.18%±1.73%	73.29%±1.36%	65.95%±1.09%	57.31%±1.94%	57.51%±0.77%	64.30%±2.85%	65.43%±0.91%	74.69%±2.17%	55.59%±2.39%	70.91%±1.58%	48.94%±1.82%	56.07%±2.26%	73.46%±3.04%	**74.96%±2.01%**	73.76%±1.03%
		20	52.11%±1.50%	49.26%±1.11%	67.06%±0.77%	78.61%±0.50%	72.79%±0.97%	42.36%±2.50%	60.96%±0.68%	70.99%±2.90%	69.16%±1.41%	**83.31%±0.73%**	68.63%±1.47%	80.19%±0.82%	52.26%±0.81%	61.61%±0.62%	82.12%±1.58%	81.28%±1.47%	82.46%±0.71%