4.4 PCR

May 26, 2025

```
[1]: import numpy as np
    import pandas as pd
    import math
    import sklearn
    import sklearn.preprocessing
    import datetime
    import os
    import matplotlib.pyplot as plt
    from sklearn.linear_model import LinearRegression
    from sklearn.decomposition import PCA
    from sklearn.metrics import mean_squared_error
    import warnings
    warnings.simplefilter(action='ignore', category=Warning)
    import seaborn as sns
    sns.set()
    pd.options.mode.chained_assignment = None # default='warn'
     #Program that executes the rolling window train-val-test split
    %run TimeBasedCV.ipynb
[2]: #Read in the cleansed and winsorised data
    df = pd.read_csv(r"C:\Users\krist\Documents\Data\ger_factor_data_from2003.csv",_
      →dtype ={"comp_tpci": str}, parse_dates =["eom"])
    #Convert to float 32 (format needed for the most ML models)
    df[df.columns[2:]] = df[df.columns[2:]].astype('float32')
     #Sort observations by date and stock id
    df = df.sort_values(by = ['eom', 'id'], ascending = True)
    df.head()
[2]:
                     id
                                         prc
                                                                 ret
                                                                       ret_exc \
    0 comp_001166_02W 2003-01-31 11.486148
                                                567.074646 -0.018948 -0.019948
    1 comp_001661_01W 2003-01-31 35.853958
                                               5221.931641 0.005815 0.004815
    2 comp_004367_02W 2003-01-31 36.068653
                                               4139.599121 -0.103801 -0.104801
    3 comp_004925_02W 2003-01-31 29.735168 15302.482422 -0.070425 -0.071425
    4 comp_005959_01W 2003-01-31 39.181721
                                               6355.691406 -0.085813 -0.086813
```

```
ret_exc_lead1m
                  ret_6_1 ret_12_1 tax_gr1a ... cowc_gr1a
                                                                pi_nix \
0
       -0.089504 -0.229560 -0.353251 -0.020556
                                                    0.041844
                                                              3.815513
1
        0.108428   0.129149   -0.035109   -0.033306   ...
                                                  -0.019596 1.284930
2
        0.127227 -0.020286  0.087151 -0.037240
                                                    0.026114 1.725004
3
        0.064832 0.046064 0.023265 -0.007344 ...
                                                   -0.000724 1.961724
        0.001832 0.123795 0.012811 -0.012241
                                                   -0.014214 1.069862
                                cop_atl1 prc_highprc_252d
                                                              ocf_at \
   ret_6_0
             ret_1_0
                        noa_at
0 -0.244158 -0.018948  0.692706  0.093015
                                                  0.398130 -0.007700
1 0.135715 0.005815 0.751569 0.164124
                                                  0.918059
                                                           0.096164
2 -0.121981 -0.103801 0.961141 0.240563
                                                  0.675468 0.070724
3 -0.027605 -0.070425 0.466852 0.221983
                                                  0.861333 0.084234
4 0.027359 -0.085813 0.635742 0.108581
                                                  0.721256 0.069331
  dbnetis_at netdebt_me
                0.135368
0
    0.085202
1
                0.200036
    0.063714
2
   -0.000607
                0.410184
3
   -0.008142
               -0.075232
                0.500641
   -0.038594
```

[5 rows x 55 columns]

```
[3]: #Copy date variable and stock id to use them as multindex

df["id2"] = df["id"].copy()

df["eom2"] = df["eom"].copy()

df = df.set_index(['eom2','id2'])

#Make a copy of the "me" variable (market equity) before rank standartization

to use afterwards for value weighting

df["me2"] = df["me"].copy()
```

To investigate the potential heterogeneity in model predictability, a subgroup analysis for small (the bottom 30% stocks by market equity each month) and large (the top 30% stocks each month) stocks is reported.

0.0.1 Rank standardization in interval [-1,1]

A rank standardization procedure following Kelly, Pruitt, and Su (2019) and Freyberger, Neuhierl, and Weber (2020) is employed. The stock characteristics are ranked month-by-month cross-

sectionally and the ranks are mapped into the [-1,1] interval, thereby transforming features into a uniform distribution and increasing the insensitivity to outliers

1 Principal component regression

1.1 All firms

```
y = df["ret_exc_lead1m"]
#Empty containers to save results from each window
predictions = []
y_test_list =[]
dates = []
dic_r2_all = {}
# Model's complexity: dictionary to save the number of components over time
numpc time = {}
# List of prespecified values to use to determine the optimal number of \Box
\hookrightarrow components
numpc = [1,2,3,5,7,9,11,15,17,22,25,29,33,40,45,49]
# Empty container to save the objective loss function (mean squared errors) for
→each number of components
mse = np.full((len(numpc),1),np.nan, dtype = np.float32)
for train_index, val_index, test_index in tscv.split(X, first_split_date=u
 datetime.date(2008,1,31), second_split_date= datetime.date(2010,1,31)):
   X_train = X.loc[train_index].drop('eom', axis=1)
   y_train = y.loc[train_index]
   X_val = X.loc[val_index].drop('eom', axis=1)
   y_val = y.loc[val_index]
           = X.loc[test_index].drop('eom', axis=1)
   y_test = y.loc[test_index]
    ⇔dimention reduction using Principal
    # component analysis and than proceed with prediction using linear
 \rightarrowregression
    #Loop over the list containing potential number of components, fit on the
 →training sample and use
   #validation set to generate predictions
   for i in range(len(numpc)):
       pca_val = PCA(n_components = numpc[i])
       # Run PCA producing the reduced variable X_reduced_val containing i_{\sqcup}
 \hookrightarrow components
```

```
X_reduced_train = pca_val.fit_transform(X_train)
        X_reduced_val = pca_val.transform(X_val)
        # Create linear regression object and fit
        line_fitter_val = LinearRegression()
        line_fitter_val.fit(X_reduced_train, y_train)
        # Predict using the reduced variable
        Yval_predict=line_fitter_val.predict(X_reduced_val)
        #calculate mean squared error for each potential value of the numpcu
 \hookrightarrowhyperparameter
        mse[i,0] = np.sqrt(mean_squared_error(y_val, Yval_predict))
    #The optimal value of the numpc hyperparameter is the value that causes the
 → lowest loss
    optim_numpc = numpc[np.argmin(mse)]
    \#Fit again using the train and validation set and the optimal numpcu
 \hookrightarrow parameter
    pca = PCA(n_components = optim_numpc)
    principalComponents = pca.fit_transform(np.concatenate((X_train, X_val)))
    X_reduced_test = pca.transform(X_test)
    line_fitter = LinearRegression()
    line fitter.fit(principalComponents, (np.concatenate((y train, y val))))
    #Use test set to generate final predictions using the redused variable
    preds=line_fitter.predict(X_reduced_test)
    \#Save predictions, dates and the true values of the dependent variable to \sqcup
 \hookrightarrow list
    predictions.append(preds)
    dates.append(y_test.index)
    y_test_list.append(y_test)
    #Calculate OOS model performance the for current window
    r2 = 1-sum(pow(y test-preds,2))/sum(pow(y test,2))
    #Save OOS model performance and the respective month to dictionary
    dic_r2_all["r2." + str(y_test.index)] = r2
    # Save the number of components to inspect model's complexity over time
    numpc_time["numpc." + str(y_test.index)] = optim_numpc
#Concatenate to get results over the whole OOS test period (Jan 2010-Dec 2019)
predictions_all= np.concatenate(predictions, axis=0)
y_test_list_all= np.concatenate(y_test_list, axis=0)
dates_all= np.concatenate(dates, axis=0)
```

```
#Calculate OOS model performance over the entire test period in line with Gu et_\( \text{al } (2020) \)

R200S_PCR = 1-sum(pow(y_test_list_all-predictions_all,2))/\( \text{sum}(pow(y_test_list_all,2)) \)

print("R200S principal component regression: ", R200S_PCR)

Train period: 2003-01-31 - 2008-01-31 ,val period: 2008-01-31 - 2010-01-31 ,
```

```
Train period: 2003-01-31 - 2008-01-31 , val period: 2008-01-31 - 2010-01-31 ,
Test period 2010-01-31 - 2011-01-31 # train records 37757 ,# val records 17356 ,
# test records 8974
Train period: 2004-01-31 - 2009-01-31 , val period: 2009-01-31 - 2011-01-31 ,
Test period 2011-01-31 - 2012-01-31 # train records 40925 ,# val records 17143 ,
# test records 9537
Train period: 2005-01-31 - 2010-01-31 , val period: 2010-01-31 - 2012-01-31 ,
Test period 2012-01-31 - 2013-01-31 # train records 42278 ,# val records 18511 ,
# test records 9107
Train period: 2006-01-31 - 2011-01-31 , val period: 2011-01-31 - 2013-01-31 ,
Test period 2013-01-31 - 2014-01-31 # train records 43971 ,# val records 18644 ,
# test records 9636
Train period: 2007-01-31 - 2012-01-31 , val period: 2012-01-31 - 2014-01-31 ,
Test period 2014-01-31 - 2015-01-31 \# train records 45267 , \# val records 18743 ,
# test records 9919
Train period: 2008-01-31 - 2013-01-31 , val period: 2013-01-31 - 2015-01-31 ,
Test period 2015-01-31 - 2016-01-31 \# train records 44974, \# val records 19555,
# test records 9260
Train period: 2009-01-31 - 2014-01-31 , val period: 2014-01-31 - 2016-01-31 ,
Test period 2016-01-31 - 2017-01-31 # train records 45423 ,# val records 19179 ,
# test records 9109
Train period: 2010-01-31 - 2015-01-31 , val period: 2015-01-31 - 2017-01-31 ,
Test period 2017-01-31 - 2018-01-31 # train records 47173 ,# val records 18369 ,
# test records 9667
Train period: 2011-01-31 - 2016-01-31 , val period: 2016-01-31 - 2018-01-31 ,
Test period 2018-01-31 - 2019-01-31 \# train records 47459 , \# val records 18776 ,
# test records 9569
Train period: 2012-01-31 - 2017-01-31 , val period: 2017-01-31 - 2019-01-31 ,
Test period 2019-01-31 - 2020-01-31 # train records 47031 ,# val records 19236 ,
# test records 8639
R200S principal component regression: 0.008067757591002311
```

2 Big only

```
features = df_top.columns[~df_top.columns.
 →isin(['id',"id2","me2","prc","eom2","ret","ret_exc","ret_exc_lead1m"])].
→tolist()
X = df top[features]
y = df_top["ret_exc_lead1m"]
#Empty containers to save results from each window
predictions_top = []
y_test_list_top =[]
dates_top = []
dic_r2_all_top = {}
numpc = [1,2,3,5,7,9,11,15,17,22,25,29,33,40,45,49]
mse = np.full((len(numpc),1),np.nan, dtype = np.float32)
for train_index, val_index, test_index in tscv.split(X, first_split_date=_

datetime.date(2008,1,31), second_split_date= datetime.date(2010,1,31)):

   X_train = X.loc[train_index].drop('eom', axis=1)
   y_train = y.loc[train_index]
   X_val = X.loc[val_index].drop('eom', axis=1)
   y_val = y.loc[val_index]
   X test = X.loc[test index].drop('eom', axis=1)
   y_test = y.loc[test_index]
   for i in range(len(numpc)):
       pca_val = PCA(n_components = numpc[i])
       X_reduced_train = pca_val.fit_transform(X_train)
       X_reduced_val = pca_val.transform(X_val)
       line_fitter_val = LinearRegression()
       line_fitter_val.fit(X_reduced_train, y_train)
       Yval_predict=line_fitter_val.predict(X_reduced_val)
       mse[i,0] = np.sqrt(mean_squared_error(y_val, Yval_predict))
   optim_numpc = numpc[np.argmin(mse)]
   pca = PCA(n_components = optim_numpc)
   principalComponents = pca.fit_transform(np.concatenate((X_train, X_val)))
   X_reduced_test = pca.transform(X_test)
   line_fitter = LinearRegression()
   line_fitter.fit(principalComponents, (np.concatenate((y_train, y_val))))
   preds=line_fitter.predict(X_reduced_test)
```

```
predictions_top.append(preds)
    dates_top.append(y_test.index)
    y_test_list_top.append(y_test)
    r2 = 1-sum(pow(y_test-preds,2))/sum(pow(y_test,2))
    dic_r2_all_top["r2." + str(y_test.index)] = r2
predictions_all_top_top= np.concatenate(predictions_top, axis=0)
y_test_list_all_top= np.concatenate(y_test_list_top, axis=0)
dates_all_top= np.concatenate(dates_top, axis=0)
R200S_PCR_top = 1-sum(pow(y_test_list_all_top-predictions_all_top_top,2))/
  →sum(pow(y_test_list_all_top,2))
print("R200S principal component regression Big only: ", R200S_PCR_top)
Train period: 2003-01-31 - 2008-01-31 , val period: 2008-01-31 - 2010-01-31 ,
Test period 2010-01-31 - 2011-01-31 \# train records 11301 , # val records 5199 ,
# test records 2687
Train period: 2004-01-31 - 2009-01-31 , val period: 2009-01-31 - 2011-01-31 ,
Test period 2011-01-31 - 2012-01-31 # train records 12252 ,# val records 5135 ,
# test records 2855
Train period: 2005-01-31 - 2010-01-31 , val period: 2010-01-31 - 2012-01-31 ,
Test period 2012-01-31 - 2013-01-31 # train records 12661 ,# val records 5542 ,
# test records 2728
Train period: 2006-01-31 - 2011-01-31 , val period: 2011-01-31 - 2013-01-31 ,
Test period 2013-01-31 - 2014-01-31 # train records 13170 ,# val records 5583 ,
# test records 2885
Train period: 2007-01-31 - 2012-01-31 , val period: 2012-01-31 - 2014-01-31 ,
Test period 2014-01-31 - 2015-01-31 \# train records 13558 , # val records 5613 ,
# test records 2969
Train period: 2008-01-31 - 2013-01-31 , val period: 2013-01-31 - 2015-01-31 ,
Test period 2015-01-31 - 2016-01-31 # train records 13469 ,# val records 5854 ,
# test records 2772
Train period: 2009-01-31 - 2014-01-31 , val period: 2014-01-31 - 2016-01-31 ,
Test period 2016-01-31 - 2017-01-31 \# train records 13603 , # val records 5741 ,
# test records 2729
Train period: 2010-01-31 - 2015-01-31 , val period: 2015-01-31 - 2017-01-31 ,
Test period 2017-01-31 - 2018-01-31 \# train records 14124 , # val records 5501 ,
# test records 2894
Train period: 2011-01-31 - 2016-01-31 ,val period: 2016-01-31 - 2018-01-31 ,
Test period 2018-01-31 - 2019-01-31 # train records 14209 ,# val records 5623 ,
# test records 2864
Train period: 2012-01-31 - 2017-01-31 , val period: 2017-01-31 - 2019-01-31 ,
```

```
Test period 2019-01-31 - 2020-01-31 # train records 14083 ,# val records 5758 , # test records 2587 R200S principal component regression Big only: 0.006243024175388245
```

3 Small only

```
[13]: tscv = TimeBasedCV(train_period=60,
                         val_period=24,
                         test_period=12,
                         freq='months')
      features = df_bottom.columns[~df_bottom.columns.
       →isin(['id',"id2","me2","prc","eom2","ret","ret_exc","ret_exc_lead1m"])].
       →tolist()
      X = df bottom[features]
      y = df_bottom["ret_exc_lead1m"]
      #Empty containers to save results from each window
      predictions_bottom = []
      y_test_list_bottom =[]
      dates bottom = []
      dic_r2_all_bottom = {}
      numpc = [1,2,3,5,7,9,11,15,17,22,25,29,33,40,45,49]
      mse = np.full((len(numpc),1),np.nan, dtype = np.float32)
      for train index, val index, test_index in tscv.split(X, first_split_date=_
       datetime.date(2008,1,31), second_split_date= datetime.date(2010,1,31)):
          X_train = X.loc[train_index].drop('eom', axis=1)
          y_train = y.loc[train_index]
          X_val = X.loc[val_index].drop('eom', axis=1)
          y_val = y.loc[val_index]
                  = X.loc[test_index].drop('eom', axis=1)
          y_test = y.loc[test_index]
          for i in range(len(numpc)):
              pca_val = PCA(n_components = numpc[i])
              X_reduced_train = pca_val.fit_transform(X_train)
              X_reduced_val = pca_val.transform(X_val)
              line_fitter_val = LinearRegression()
              line_fitter_val.fit(X_reduced_train, y_train)
```

```
Yval_predict=line_fitter_val.predict(X_reduced_val)
        mse[i,0] = np.sqrt(mean_squared_error(y_val, Yval_predict))
    optim_numpc = numpc[np.argmin(mse)]
    pca = PCA(n_components = optim_numpc)
    principalComponents = pca.fit_transform(np.concatenate((X_train, X_val)))
    X_reduced_test = pca.transform(X_test)
    line fitter = LinearRegression()
    line_fitter.fit(principalComponents, (np.concatenate((y_train, y_val))))
    preds=line fitter.predict(X reduced test)
    predictions_bottom.append(preds)
    dates_bottom.append(y_test.index)
    y_test_list_bottom.append(y_test)
    r2 = 1-sum(pow(y_test-preds,2))/sum(pow(y_test,2))
    dic_r2_all_bottom["r2." + str(y_test.index)] = r2
predictions_all_bottom_bottom= np.concatenate(predictions_bottom, axis=0)
y_test_list_all_bottom= np.concatenate(y_test_list_bottom, axis=0)
dates_all_bottom= np.concatenate(dates_bottom, axis=0)
R200S_PCR_bottom =
 -1-sum(pow(y_test_list_all_bottom-predictions_all_bottom_bottom,2))/
 ⇒sum(pow(y_test_list_all_bottom,2))
print("R200S principal component regression Small only: ", R200S_PCR_bottom)
Train period: 2003-01-31 - 2008-01-31 , val period: 2008-01-31 - 2010-01-31 ,
Test period 2010-01-31 - 2011-01-31 \# train records 11301 , # val records 5199 ,
# test records 2687
Train period: 2004-01-31 - 2009-01-31 , val period: 2009-01-31 - 2011-01-31 ,
Test period 2011-01-31 - 2012-01-31 \# train records 12252 , # val records 5135 ,
# test records 2855
Train period: 2005-01-31 - 2010-01-31 ,val period: 2010-01-31 - 2012-01-31 ,
Test period 2012-01-31 - 2013-01-31 \# train records 12661 , # val records 5542 ,
# test records 2728
Train period: 2006-01-31 - 2011-01-31 , val period: 2011-01-31 - 2013-01-31 ,
Test period 2013-01-31 - 2014-01-31 \# train records 13170 , # val records 5583 ,
Train period: 2007-01-31 - 2012-01-31 ,val period: 2012-01-31 - 2014-01-31 ,
Test period 2014-01-31 - 2015-01-31 \# train records 13558 , # val records 5613 ,
```

```
Train period: 2008-01-31 - 2013-01-31 ,val period: 2013-01-31 - 2015-01-31 ,
     Test period 2015-01-31 - 2016-01-31 # train records 13469 ,# val records 5854 ,
     # test records 2772
     Train period: 2009-01-31 - 2014-01-31 , val period: 2014-01-31 - 2016-01-31 ,
     Test period 2016-01-31 - 2017-01-31 \# train records 13603 , # val records 5741 ,
     # test records 2729
     Train period: 2010-01-31 - 2015-01-31 ,val period: 2015-01-31 - 2017-01-31 ,
     Test period 2017-01-31 - 2018-01-31 \# train records 14124 , # val records 5501 ,
     # test records 2894
     Train period: 2011-01-31 - 2016-01-31 , val period: 2016-01-31 - 2018-01-31 ,
     Test period 2018-01-31 - 2019-01-31 # train records 14209 ,# val records 5623 ,
     # test records 2864
     Train period: 2012-01-31 - 2017-01-31 , val period: 2017-01-31 - 2019-01-31 ,
     Test period 2019-01-31 - 2020-01-31 \# train records 14083 , # val records 5758 ,
     # test records 2587
     R200S principal component regression Small only: 0.010347082445304134
[12]: #Generate table containing the R2oos measures for all three datasets
      chart = np.array([[R200S_PCR],
                        [R200S_PCR_top],
                        [R200S_PCR_bottom]])
      r2oos_pcr = pd.DataFrame(chart, columns=['PCR'],
                                    index=["Full sample", "Large firms", "Small___

firms"])

      r2oos_pcr
[12]:
                        PCR
     Full sample 0.008066
     Large firms 0.006243
     Small firms 0.010347
[13]: #Save the model's performance measures to compare with other models later.
      r2oos_pcr.to_csv(r'C:\Users\krist\Documents\Data\r2oos_pcr.csv')
     3.0.1 Time-varying model complexity
[24]: # Convert dictionary containing the number of components over time in dataframe
      pd.DataFrame(numpc_time.items())
      numpc_table =pd.DataFrame(numpc_time.items(), columns=['Identifier',_

    'num_comp'])
      numpc_table['Identifier'] = numpc_table['Identifier'].astype(str)
```

test records 2969

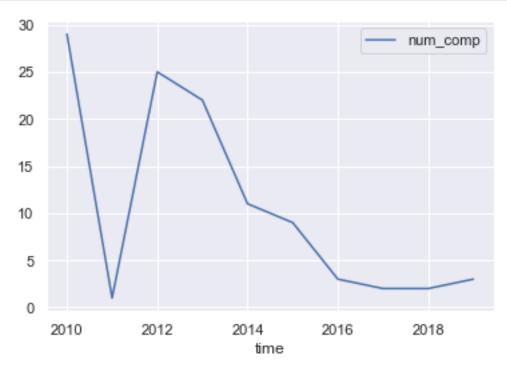
numpc_table["time"] = numpc_table["Identifier"].str[20:30]

numpc_table["time"] = numpc_table["time"].dt.year

numpc_table["time"] = pd.to_datetime(numpc_table["time"], utc = False)

```
numpc_table.drop(["Identifier"], axis = 1, inplace = True)

#Plot time-varying model complexity
numpc_table.set_index('time').plot();
#plt.xticks(rotation=45);
```



```
[25]: #Save the model's complexity to compare with other models later.

numpc_table.to_csv(r'C:\Users\krist\Documents\Data\model_complexity\comp_pcr.

ocsv')
```

3.0.2 Variable Importance

Variable importance is defined following Gu et al. (2020), i.e., for each model, all values of a given predictor are set to zero and the reduction in predictive R2OOS is calculated. Then, the absolute reductions in predictive R2OOS are normalized to sum to 1, indicating the relative contribution of each variable to a model. In contrast to Gu et al. (2020), variable importance is calculated based on the last rolling window observation sample and does not represent an average over the whole sample.

```
[39]: # R200S based on the last rolling window when all variables are included features = df.columns[~df.columns.

→isin(['id',"id2","eom","eom2","me2","prc","ret","ret_exc","ret_exc_lead1m","year"])].

→tolist()

df["year"] = df["eom"].dt.year
```

```
X train = df[features].loc[(df["year"]>=2012) & (df["year"]<=2016)]</pre>
      y_train = df["ret_exc_lead1m"].loc[(df["year"]>=2012) & (df["year"]<=2016)]</pre>
      X_{val} = df[features].loc[(df["year"]>=2017) & (df["year"]<=2018)]
      y_val = df["ret_exc_lead1m"].loc[(df["year"]>=2017) & (df["year"]<=2018)]</pre>
      numpc = [1,2,3,5,7,9,11,15,17,22,25,29,33,40,45,49]
      mse = np.full((len(numpc),1),np.nan, dtype = np.float32)
      for i in range(len(numpc)):
          pca_val = PCA(n_components = numpc[i])
          X_reduced_train = pca_val.fit_transform(X_train)
          X_reduced_val = pca_val.transform(X_val)
          line_fitter_val = LinearRegression()
          line_fitter_val.fit(X_reduced_train, y_train)
          Yval_predict=line_fitter_val.predict(X_reduced_val)
          mse[i,0] = np.sqrt(mean_squared_error(y_val, Yval_predict))
      optim_numpc = numpc[np.argmin(mse)]
      pca = PCA(n components = optim numpc)
      principalComponents = pca.fit_transform(np.concatenate((X_train, X_val)))
      line fitter = LinearRegression()
      line_fitter.fit(principalComponents, (np.concatenate((y_train, y_val))))
      preds=line fitter.predict(principalComponents)
      R200S_all = 1-sum(pow(np.concatenate((y_train, y_val))-preds,2))/sum(pow(np.
       ⇔concatenate((y_train, y_val)),2))
      print(R200S_all)
     0.008041589337524191
[40]: # Generate a separate dataframe for each independent variable, where all values
       ⇔of that variable are set to zero
      for j in features:
          globals()['df_' + str(j)] = df.copy()
          globals()['df_' + str(j)][str(j)] = 0
[41]: \#e.g., df_ret_12_1 is a dataframe where all values of the ret_12_1 variable are
       ⇔set to zero
      df_ret_12_1
[41]:
                                               id
                                                          eom
                                                                    prc
                                                                               me \
      eom2
      2003-01-31 comp_001166_02W comp_001166_02W 2003-01-31 -0.376652 0.162996
                 comp_001661_01W comp_001661_01W 2003-01-31 0.427313 0.718062
```

```
comp_004367_02W
                            comp_004367_02W 2003-01-31 0.433921
                                                                 0.674009
           comp_004925_02W
                            comp_004925_02W 2003-01-31
                                                        0.295154
                                                                  0.881057
           comp_005959_01W
                            comp_005959_01W 2003-01-31
                                                        0.477974
                                                                  0.762115
2020-12-31 comp_333885_01W
                            comp_333885_01W 2020-12-31 0.367156 0.484646
           comp_334302_02W
                            comp_334302_02W 2020-12-31 -0.842457 -0.487316
           comp 335706 02W
                            comp_335706_02W 2020-12-31 -0.722296 -0.618158
           comp_340115_01W
                            comp_340115_01W 2020-12-31 -0.392523 -0.025367
           comp 340153 01W
                            comp 340153 01W 2020-12-31 0.138852 0.711615
                                 ret
                                       ret exc ret exc lead1m
                                                                 ret 6 1 \
           id2
eom2
2003-01-31 comp_001166_02W -0.018948 -0.019948
                                                     -0.089504 -0.665198
           comp_001661_01W 0.005815 0.004815
                                                      0.108428 0.669604
           comp_004367_02W -0.103801 -0.104801
                                                      0.127227
                                                                0.114537
           comp_004925_02W -0.070425 -0.071425
                                                      0.064832
                                                                0.392070
           comp_005959_01W -0.085813 -0.086813
                                                      0.001832
                                                                0.643172
2020-12-31 comp_333885_01W
                            0.128645
                                      0.128545
                                                      -0.037398 -0.957276
           comp_334302_02W
                                                     -0.033670 -0.420561
                            0.097961 0.097861
           comp_335706_02W
                            0.114381
                                      0.114281
                                                      0.089684 0.097463
           comp 340115 01W
                            0.155813
                                                      0.009796
                                                                0.001335
                                     0.155713
           comp_340153_01W 0.238817 0.238717
                                                      0.006943 0.001335
                            ret_12_1 tax_gr1a
                                                    ret_6_0
                                                              ret 1 0 \
eom2
           id2
2003-01-31 comp_001166_02W
                                   0 -0.726872 ... -0.669604 -0.312775
           comp 001661 01W
                                   0 -0.845815 ... 0.577093 -0.110132
           comp_004367_02W
                                   0 -0.894273
                                                ... -0.356828 -0.766520
           comp_004925_02W
                                   0 -0.453744 ... 0.096916 -0.612335
           comp_005959_01W
                                   0 -0.599119 ... 0.259912 -0.691630
                                       ... ...
2020-12-31 comp_333885_01W
                                   0 -0.951936
                                               ... -0.853138
                                                             0.591455
           comp_334302_02W
                                   0 0.540721 ... -0.268358
                                                             0.417891
                                   0 0.311081 ... 0.236315
           comp_335706_02W
                                                             0.511348
           comp_340115_01W
                                   0 0.871829 ... 0.001335
                                                             0.687583
                                   0 0.847797 ... 0.001335
           comp 340153 01W
                                                             0.879840
                              noa at cop atl1 prc highprc 252d
                                                                    ocf at \
eom2
           id2
2003-01-31 comp 001166 02W
                            0.585903 -0.735683
                                                       -0.859031 -0.797357
           comp_001661_01W
                            0.696035 -0.392070
                                                        0.607930 0.286344
           comp_004367_02W
                            0.881057 0.458150
                                                       -0.365639 -0.110132
           comp_004925_02W -0.264317
                                                        0.422907 0.149780
                                     0.330396
           comp_005959_01W
                            0.458150 -0.669604
                                                       -0.255507 -0.118943
2020-12-31 comp_333885_01W
                            0.457944 0.815754
                                                       -0.319092 0.877170
```

```
comp_334302_02W 0.791722 0.345794
                                                     0.754339 0.054740
          comp_335706_02W -0.241656 -0.935915
                                                    -0.313752 -0.917223
          comp_340115_01W -0.415220 -0.436582
                                                     0.001335 0.025367
          comp_340153_01W -0.815754 -0.636849
                                                     0.001335 -0.129506
                                                          me2 year
                          dbnetis_at netdebt_me
eom2
          id2
2003-01-31 comp_001166_02W
                            0.735683 -0.149780
                                                   567.074646 2003
                                                  5221.931641 2003
          comp 001661 01W
                            0.647577
                                       0.145374
          comp_004367_02W
                           -0.207048
                                       0.339207
                                                  4139.599121 2003
                                       -0.709251 15302.482422 2003
          comp 004925 02W
                           -0.370044
          comp_005959_01W
                           -0.656388
                                       0.436123 6355.691406 2003
2020-12-31 comp_333885_01W
                           -0.930574
                                       -0.316422 10829.783203 2020
          comp_334302_02W
                                                   297.482697 2020
                            0.692924
                                       -0.252336
          comp_335706_02W
                            0.164219
                                       -0.730307
                                                   196.243454 2020
          comp_340115_01W
                            0.642190
                                      0.562083 1799.958496 2020
                                       -0.578104 26788.144531 2020
          comp_340153_01W
                           -0.110814
```

[156948 rows x 57 columns]

```
[42]: # calculate R200S based on the last rolling window for each dataframe and save
      ⇔to dictionary
     dic = \{\}
     numpc =np.arange(2, 20, 1).tolist()
     mse = np.full((len(numpc),1),np.nan, dtype = np.float32)
     for j in features:
        df_var = globals()['df_' + str(j)]
        X_train = df_var[features].loc[(df_var["year"]>=2012) &__
      y_train = df_var["ret_exc_lead1m"].loc[(df_var["year"]>=2012) &__
      X_val = df_var[features].loc[(df_var["year"]>=2017) &__
      y_val = df_var["ret_exc_lead1m"].loc[(df_var["year"]>=2017) &__
      for i in range(len(numpc)):
            pca_val = PCA(n_components = numpc[i])
            X_reduced_train = pca_val.fit_transform(X_train)
            X_reduced_val = pca_val.transform(X_val)
            line_fitter_val = LinearRegression()
```

```
line_fitter_val.fit(X_reduced_train, y_train)
              Yval_predict=line_fitter_val.predict(X_reduced_val)
              mse[i,0] = np.sqrt(mean_squared_error(y_val, Yval_predict))
          optim_numpc = numpc[np.argmin(mse)]
          pca = PCA(n components = optim numpc)
          principalComponents = pca.fit_transform(np.concatenate((X_train, X_val)))
          line fitter = LinearRegression()
          line_fitter.fit(principalComponents, (np.concatenate((y_train, y_val))))
          preds=line fitter.predict(principalComponents)
          R200S_var = 1-sum(pow(np.concatenate((y_train, y_val))-preds,2))/sum(pow(np.

→concatenate((y_train, y_val)),2))
          dic['R200S_' + str(j)] = R200S_var
[43]: # dictionary, containing OOS predictive R2, when the all values of a given_
       ⇔predictor are set to zero
      dic
[43]: {'R200S me': 0.007882220039139964,
       'R200S_ret_6_1': 0.008586615999087055,
       'R200S ret 12 1': 0.010370354590459319,
       'R200S_tax_gr1a': 0.007978826799963645,
       'R200S be me': 0.008288464095017045,
       'R200S_debt_me': 0.008631007521485379,
       'R200S ni me': 0.007984778018852445,
       'R200S_sale_gr3': 0.007874781125965735,
       'R200S_sale_gr1': 0.00784479918724379,
       'R200S_sale_me': 0.008299388634497529,
       'R200S_lnoa_gr1a': 0.0078019205425843285,
       'R200S_inv_gr1a': 0.007898867883029737,
       'R200S_oaccruals_at': 0.008941151477232179,
       'R200S_taccruals_at': 0.007747618429569503,
       'R200S_be_gr1a': 0.007906715343042992,
       'R200S_taccruals_ni': 0.00910221401604916,
       'R200S_ebit_sale': 0.007962942569583187,
       'R200S sale bev': 0.007977620633951465,
       'R200S_age': 0.007941142206918772,
       'R200S beta 60m': 0.00804514105843046,
       'R200S rmax1 21d': 0.007815170517090797,
       'R200S_bidaskhl_21d': 0.007816242034879517,
       'R200S_ret_9_1': 0.008364546881691193,
       'R200S_ret_12_7': 0.00883128264109212,
       'R200S_ni_be': 0.009038454681636221,
       'R200S_ocf_debt': 0.007985318772707695,
       'R200S_cash_gr1a': 0.00780605712852267,
       'R200S_at_gr1': 0.008163130085626302,
```

```
'R200S_chcsho_12m': 0.008045981914818978,
       'R200S ca cl': 0.00847370291000693,
       'R200S_ivol_capm_252d': 0.007807258237678361,
       'R200S_debt_gr1': 0.007652747313190433,
       'R200S_debtlt_gr1a': 0.0077522462576546936,
       'R200S_ope_be': 0.009064548721107624,
       'R200S caliq cl': 0.008412886534385211,
       'R200S_dsale_drec': 0.008101990268522652,
       'R200S f score': 0.007948187101653792,
       'R200S rvol 21d': 0.007790027924323817,
       'R200S gp bev': 0.007979135066951026,
       'R200S_cowc_gr1a': 0.008304409487430053,
       'R200S_pi_nix': 0.008129826232805115,
       'R200S_ret_6_0': 0.008641354157683634,
       'R200S_ret_1_0': 0.008078274095004057,
       'R200S_noa_at': 0.00798900050450091,
       'R200S_cop_atl1': 0.007862696111906153,
       'R200S_prc_highprc_252d': 0.008398339916934505,
       'R200S_ocf_at': 0.00784796720004255,
       'R200S_dbnetis_at': 0.007704580123266247,
       'R200S_netdebt_me': 0.008513387574519138}
[44]: # Convert dictionary to dataframe
      pd.DataFrame(dic.items())
      imp=pd.DataFrame(dic.items(), columns=['Feature', 'R200S'])
      # Feature: name of the variable whose values are set to zero
      imp["Feature"] = imp["Feature"].str[6:]
      # Calculate reduction in predictive R200S
      imp["red_R200S"] = R200S_all -imp["R200S"]
      imp["var imp"] = imp["red R200S"]/sum(imp["red R200S"])
      imp=imp.sort_values(by = ['var_imp'], ascending = False)
      imp
[44]:
                  Feature
                              R200S red_R200S
                                                 var_imp
      2
                 ret_12_1 0.010370 -0.002329 0.316949
      15
             taccruals_ni 0.009102 -0.001061 0.144353
      33
                   ope_be 0.009065 -0.001023 0.139227
      24
                    ni_be 0.009038 -0.000997 0.135675
             oaccruals at 0.008941 -0.000900 0.122432
      12
      23
                 ret 12 7 0.008831 -0.000790 0.107479
      41
                  ret 6 0 0.008641 -0.000600 0.081629
      5
                  debt_me 0.008631 -0.000589 0.080221
      1
                  ret 6 1 0.008587 -0.000545 0.074179
      48
               netdebt_me 0.008513 -0.000472 0.064213
      29
                    ca_cl 0.008474 -0.000432 0.058811
      34
                  calig cl 0.008413 -0.000371 0.050534
```

```
9
                   sale_me
                            0.008299
                                      -0.000258
                                                  0.035087
      4
                     be_me
                            0.008288
                                      -0.000247
                                                  0.033600
      27
                            0.008163
                                      -0.000122 0.016542
                    at_gr1
      40
                    pi_nix 0.008130
                                      -0.000088 0.012009
                dsale_drec
      35
                            0.008102
                                      -0.000060 0.008221
                   ret 1 0
      42
                            0.008078
                                      -0.000037
                                                  0.004993
      28
                chcsho 12m
                            0.008046
                                      -0.000004
                                                 0.000598
      19
                  beta 60m
                            0.008045
                                      -0.000004
                                                  0.000483
      43
                    noa_at
                            0.007989
                                       0.000053 -0.007157
      25
                  ocf debt
                            0.007985
                                       0.000056 -0.007659
      6
                     ni_me
                            0.007985
                                       0.000057 -0.007732
      38
                    gp_bev
                            0.007979
                                       0.000062 -0.008500
      3
                  tax_gr1a
                            0.007979
                                       0.000063 -0.008542
      17
                  sale_bev
                                       0.000064 -0.008706
                            0.007978
      16
                 ebit_sale
                            0.007963
                                       0.000079 -0.010704
      36
                   f_score
                            0.007948
                                       0.000093 -0.012712
                            0.007941
      18
                                       0.000100 -0.013671
                       age
      14
                   be_gr1a
                            0.007907
                                       0.000135 -0.018357
      11
                  inv_gr1a
                            0.007899
                                       0.000143 -0.019425
      0
                           0.007882
                                       0.000159 -0.021690
                        me
      7
                  sale_gr3
                           0.007875
                                       0.000167 -0.022703
      44
                  cop_atl1
                            0.007863
                                       0.000179 -0.024348
      46
                    ocf_at
                            0.007848
                                       0.000194 -0.026352
                  sale_gr1
      8
                            0.007845
                                       0.000197 -0.026783
      21
              bidaskhl_21d 0.007816
                                       0.000225 -0.030670
      20
                 rmax1_21d 0.007815
                                       0.000226 -0.030816
            ivol_capm_252d
      30
                           0.007807
                                       0.000234 -0.031893
                 cash_gr1a
      26
                            0.007806
                                       0.000236 -0.032056
      10
                 lnoa_gr1a
                                       0.000240 -0.032619
                            0.007802
      37
                  rvol_21d
                            0.007790
                                       0.000252 -0.034238
      32
               debtlt_gr1a
                            0.007752
                                       0.000289 -0.039380
                                       0.000294 -0.040010
      13
              taccruals_at
                            0.007748
      47
                dbnetis_at
                            0.007705
                                       0.000337 -0.045868
                  debt_gr1 0.007653
                                       0.000389 -0.052922
      31
[45]: # Plot variable importance measures for the top-20 most influential variables
      fea_imp_graph = imp.sort_values(['var_imp', 'Feature'], ascending=[True,_
       →False]).iloc[-20:]
      _ = fea_imp_graph.plot(kind='barh', x='Feature', y='var_imp', figsize=(20, 10))
      plt.title('PCR')
      plt.show()
```

-0.000357

-0.000263

-0.000323 0.043955

0.048554

0.035770

45

22

39

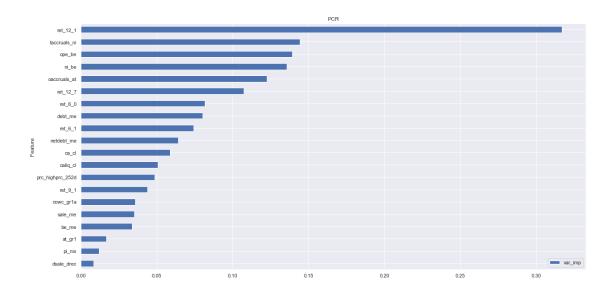
prc_highprc_252d 0.008398

0.008365

0.008304

ret_9_1

cowc_gr1a



```
[46]: var_imp_pcr=imp[["Feature", "var_imp"]]
var_imp_pcr.to_csv(r'C:\Users\krist\Documents\Data\variable

→importance\var_imp_pcr.csv', index = False)
```

4 Machine learning portfolios

Explore whether predictability translates into portfolio gains

```
[9]: #Generate a results dataframe containing the model predictions (yhat) and the
     \hookrightarrow true\ values\ (y\_true)
     #of the dependent variable for each stock in each month
     yhat = predictions_all.tolist()
     y_true = y_test_list_all.tolist()
     i = dates_all.tolist()
     results = pd.DataFrame(
         {'identifier': i,
          'yhat': yhat,
          'y_true': y_true
     results["identifier"] = results["identifier"].astype("str")
     results["date"] = results["identifier"].str[12:22]
     results["id"] = results["identifier"].str[36:51]
     results.drop(["identifier"],axis = 1, inplace=True)
     results['date'] = pd.to_datetime(results['date'], format='%Y-%m-%d')
     results['MonthYear'] = results['date'].dt.to_period('M')
     results = results.sort_values(by = ['date', 'id'], ascending = True)
```

```
results = results.set_index(['MonthYear','id'])
      results
 [9]:
                                                          date
                                     yhat
                                             y_true
     MonthYear id
      2010-01
                comp 001166 02W 0.015672 0.035512 2010-01-31
                comp 001661 01W
                                0.013568 -0.041371 2010-01-31
                comp_002410_04W
                                0.011608 -0.053836 2010-01-31
                comp_002597_02W 0.010799 0.001862 2010-01-31
                comp_003820_01W
                                0.004853 0.078967 2010-01-31
      2019-12
                comp_330716_01W -0.000303 -0.012904 2019-12-31
                comp_331115_01W 0.001667 0.005400 2019-12-31
                comp_332311_01W 0.004555 -0.051340 2019-12-31
                comp_333885_01W 0.011020 0.003528 2019-12-31
                comp_334036_02W -0.000860 -0.100876 2019-12-31
      [93417 rows x 3 columns]
[10]: # Save results to use for Diebold-Mariano test
      pcr= results.reset_index()
      pcr.to_csv(r'C:\Users\krist\Documents\Data\Predictions\pcr.csv', index = False)
 [5]: | ## Import the original "me" avriable (before rank standartization) to use for
      ⇔the value weighting scheme
      unscaled_data = df[["id", 'eom', "me2", "ret", "ret_exc"]].copy()
      unscaled_data['MonthYear'] = unscaled_data['eom'].dt.to_period('M')
      unscaled_data.drop('eom', axis=1, inplace=True)
      unscaled_data = unscaled_data.set_index(['MonthYear','id'])
      unscaled_data
 [5]:
                                          me2
                                                          ret_exc
                                                    ret
     MonthYear id
      2003-01
                comp 001166 02W
                                  567.074646 -0.018948 -0.019948
                comp_001661_01W
                                  5221.931641 0.005815 0.004815
                comp_004367_02W
                                  4139.599121 -0.103801 -0.104801
                comp_004925_02W
                                15302.482422 -0.070425 -0.071425
                comp_005959_01W
                                  6355.691406 -0.085813 -0.086813
      2020-12
                comp_333885_01W
                                 10829.783203 0.128645 0.128545
                comp_334302_02W
                                   297.482697 0.097961 0.097861
                comp_335706_02W
                                   196.243454 0.114381 0.114281
                comp_340115_01W
                                  1799.958496 0.155813 0.155713
                comp_340153_01W 26788.144531 0.238817 0.238717
      [156948 rows x 3 columns]
```

```
[6]: # Import price data in t+1 and ret t+1 --> Import from the original dataframe
     # (otherwise there are missing values because shares with a price < $5 have_
     \hookrightarrowbeen excluded)
     price_data= pd.read_csv(r"C:\Users\krist\Documents\Data\ger_factor_data.csv",__

dtype ={"comp_tpci": str}, parse_dates =["eom"])
     price_data["eom"] = pd.to_datetime(price_data["eom"], utc = False)
     # Only the data between 2003 and 2020 is needed
     price_data["year"] = price_data["eom"].dt.year
     price_data = price_data.loc[(price_data["year"]>=2003) &__
      ⇔(price_data["year"]<=2020)]
     price_data = price_data[["id", 'eom', 'prc', "ret"]]
     \#Generate a MonthYear variable (monthly observations are given, the day is not \sqcup
      \rightarrowneeded)
     price_data['MonthYear'] = price_data['eom'].dt.to_period('M')
     price_data.drop('eom', axis=1, inplace=True)
     #The original dataframe is not winsorized --> winsorize at the 1\% and 99\%
     ⇔level
     price = ["prc"]
     ret = ["ret"]
     price_data["prc"] = price_data[price].apply(lambda x: x.clip(*x.quantile([0.01,
      ⇔0.99])))
     price data["ret"] = price data[ret].apply(lambda x: x.clip(*x.quantile([0.01, 0.
      →99])))
     #Shift to get the price of each stock in month t+1
     price_data['prc_t+1'] = price_data.groupby(['id'])['prc'].shift(-1)
     # Shift to get return for each stock in month t+1
     price_data["ret_t+1"]=price_data.groupby(['id'])['ret'].shift(-1)
     price_data.drop('ret', axis=1, inplace=True)
     price_data = price_data.set_index(['MonthYear','id'])
     price_data
[6]:
                                             prc_t+1
                                                       ret_t+1
                                      prc
    MonthYear id
     2003-01
               comp_001166_02W 11.486148 10.468435 -0.088604
     2003-02
               comp_001166_02W
                                10.468435
                                           10.107208 -0.034506
     2003-03
               comp_001166_02W
                               10.107208
                                           13.610050 0.346569
     2003-04
              comp_001166_02W
                               13.610050
                                           14.518828 0.066773
    2003-05
              comp_001166_02W
                                14.518828
                                           15.212792 0.047798
     2020-11
               comp_340115_01W 14.831518 17.142462 0.155813
     2020-12
               comp_340115_01W 17.142462
                                                           NaN
                                                 {\tt NaN}
     2020-10
               comp_340153_01W 21.909940 29.758645 0.358226
```

```
2020-12
              comp_340153_01W
                               36.865510
                                                NaN
                                                          NaN
     [274031 rows x 3 columns]
[7]: #Merge with the results dataframe
    bigdata = pd.merge(results, unscaled data,left index=True, right index=True)
    bigdata = pd.merge(bigdata, price_data,left_index=True, right_index=True)
    bigdata = bigdata.reset_index()
     # Original "me" (before rank normalization) to use for the value weighting
      ⇔scheme
    bigdata.rename(columns={'me2': 'me'}, inplace=True)
     #Final df containing all information needed for the portfolio creation
    bigdata
[7]:
          MonthYear
                                                               date
                                  id
                                          yhat
                                                  y_true
    0
            2010-01 comp_001166_02W 0.015672 0.035512 2010-01-31
    1
            2010-01 comp 001661 01W 0.013568 -0.041371 2010-01-31
            2010-01 comp 002410 04W
    2
                                      0.011608 -0.053836 2010-01-31
    3
            2010-01 comp 002597 02W 0.010799 0.001862 2010-01-31
    4
            2010-01 comp_003820_01W 0.004853 0.078967 2010-01-31
    93412
            2019-12 comp_330716_01W -0.000303 -0.012904 2019-12-31
    93413
            2019-12 comp_331115_01W 0.001667 0.005400 2019-12-31
    93414
            2019-12 comp 332311 01W 0.004555 -0.051340 2019-12-31
    93415
            2019-12 comp_333885_01W 0.011020 0.003528 2019-12-31
    93416
            2019-12 comp_334036_02W -0.000860 -0.100876 2019-12-31
                      me
                               ret
                                     ret_exc
                                                    prc
                                                           prc_t+1
                                                                     ret_t+1
    0
             1233.673950 -0.099572 -0.099572 22.730003
                                                         23.537181 0.035512
    1
             6796.776367 0.036020 0.036020 22.730003 21.789641 -0.041371
    2
           132047.843750 -0.014941 -0.014941
                                               9.453457
                                                          8.812790 -0.053836
    3
            18138.388672  0.035186  0.035186  13.191743  13.057401  0.001862
    4
              225.897369 -0.125756 -0.125756
                                               6.339377
                                                          6.839981
                                                                    0.078967
    93412
               33.930836 0.066508 0.065108
                                               6.169243
                                                          6.097655 -0.011604
    93413
               36.342445 0.005615 0.004215 18.171224 18.292966 0.006700
    93414
            13412.494141 -0.020668 -0.022068 26.824988
                                                         25.482656 -0.050040
    93415
             7151.834473 0.251631 0.250231 35.759173
                                                         35.931819 0.004828
    93416
              189.330505 0.377943 0.377359 21.424097 19.290764 -0.099576
```

22

[93417 rows x 11 columns]

[8]:

2020-11

comp_340153_01W

29.758645

36.865510 0.238817

```
⇔values)
      bigdata.isna().sum()
 [8]: MonthYear
      id
                    0
     vhat
                    0
     y_true
                    0
      date
                    0
                    0
     me
     ret
                    0
     ret_exc
     prc
                    0
     prc_t+1
                   12
     ret t+1
                   12
      dtype: int64
 [9]: # Drop the twelve observations where prc t+1 and ret t+1 are still missing
      bigdata = bigdata.dropna()
      len(bigdata)
 [9]: 93405
[10]: # Calculate the 1-month risk-free rate as the average monthly difference
       ⇔between returns and excess returns
      bigdata["risk_free_rate"]=bigdata["ret_t+1"]-bigdata["y_true"]
      bigdata["risk_free_rate"] =bigdata.groupby('MonthYear')["risk_free_rate"].
       ⇔transform('mean')
[11]: | # Create a variable "NumMonth", indicating the number of the respectie month:
      # January 2010 = 1,..., December 2019 = 120
      bigdata["MonthYear1"] = bigdata["MonthYear"].copy()
      bigdata["MonthYear"] = bigdata["MonthYear"].astype("int")
      bigdata["NumMonth"] = bigdata["MonthYear"] -479
      bigdata["NumMonth"].unique()
[11]: array([ 1,
                    2,
                         3,
                              4,
                                   5,
                                        6,
                                             7,
                                                  8,
                                                       9,
                                                           10,
                                                                11,
                                                                     12,
                                                                           13,
                            17,
                                       19,
                                            20,
                                                 21,
                   15,
                        16,
                                  18,
                                                      22,
                                                           23,
                                                                24,
                                                                      25,
              14,
              27,
                   28,
                        29,
                             30,
                                  31,
                                       32,
                                            33,
                                                 34,
                                                      35,
                                                           36,
                                                                37,
                                                                      38,
              40,
                   41,
                        42,
                             43,
                                  44,
                                       45,
                                            46,
                                                 47,
                                                      48,
                                                           49,
                                                                50,
                                                                     51,
              53,
                   54,
                        55,
                             56,
                                  57,
                                       58,
                                            59,
                                                 60,
                                                      61,
                                                           62,
                                                                63, 64,
                                                                          65,
                        68, 69,
                                  70,
                                      71,
                                            72,
                                                73,
                                                      74,
                                                           75,
                                                                76, 77,
              66,
                   67,
              79, 80,
                        81, 82, 83,
                                       84,
                                            85,
                                                 86,
                                                      87,
                                                           88,
                                                                89, 90,
              92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104,
             105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117,
             118, 119, 120], dtype=int64)
```

#Check for missing values (the original df has not yet been cleaned of missing \Box

```
[12]: # Create a separate dataframe for each month, e.g.:
      # df_1=January 2010, df_2 = February 2010,..., df_120 = December 2019
      for i in bigdata["NumMonth"].unique():
          globals()['df_' + str(i)] = bigdata[bigdata["NumMonth"]==i]
[13]: # Assign decile ranks to the stocks within each month based on the predicted
       ⇔excess returns:
      # 1) Generate ranks starting from the lowest predicted excess return to the
       \hookrightarrowhighest
      for i in bigdata["NumMonth"].unique():
          globals()['df_' + str(i)]["rank"]= globals()['df_' + str(i)]['yhat'].
       →rank(method='first')
      # 2)Split ranks into 10 deciles to get decile rank
      # DecileRank = 0: stoks with lowest predicted excess returns
      # DecileRank =9: stoks with highest predicted excess returns
      for i in bigdata["NumMonth"].unique():
          globals()['df_' + str(i)]["DecileRank"]=pd.qcut(globals()['df_' + L
       str(i)]['rank'].values, 10, labels = False)
      #Drop normal rank, retain only decile ranks
      for i in bigdata["NumMonth"].unique():
           globals()['df_' + str(i)].drop('rank', axis=1, inplace=True)
[14]: # Create a separate dataframe for each decile rank and each month, e.q.:
      # df_1_0: the bottom decile in January 2010,
      # d_19: the top decile in January 2010,...,
      # df_120_0: the bottom decile in December 2019,
      # df_120_9: the top decile in December 2019
      for i in bigdata["NumMonth"].unique():
          for j,g in globals()['df_' + str(i)].groupby('DecileRank'):
              globals()['df_' + str(i) + "_" + str(j)] = g
[15]: # All stocks are sorted into deciles based on their predicted returns for the
       \rightarrownext month.
      # Create 10 dataframes showing the portfolio composition of each decile_
       ⇔portfolio over time
      for j in np.arange(0,10,1):
          globals()['rank_' + str(j)] = pd.concat([globals()['df_1_'+ str(j)],__
       \rightarrowglobals()['df_2_'+ str(j)]], axis=0)
      # Generate 10 Dataframes for the 10 Decile portfolios 0-9: rank_9: top⊔
       →portfolio, rank_0: bottom portfolio
```

```
for i in np.arange(2,120,1):
         for j in np.arange(0,10,1):
            globals()['rank_' + str(j)] = pd.concat([globals()['rank_' + str(j)],__
      \neg globals()['df' + str(i+1) + "' + str(j)]], axis = 0)
[16]: # We get 10 separate dataframes (rank_0, rank_1,..., rank_9), showing the_
      \hookrightarrowportfolio
     # composition of each decile portfolio over time
     #e.q.: compostion of the top decile portfolio over the entire OOS period (Janu
      →2010-Dec 2019):
     rank 9
[16]:
           MonthYear
                                 id
                                                           date \
                                        yhat
                                                y_true
     11
                 480
                     comp_007152_01W 0.025378 0.006088 2010-01-31
                 480
                     comp 010846 09W
     15
                                     0.023357 -0.014764 2010-01-31
     21
                 480
                     480
     26
                     27
                 480
                     599
                     93335
     93337
                 599
                     comp_317329_02W 0.013119 0.008100 2019-12-31
                 599
                     comp 318659 01W 0.013972 -0.148323 2019-12-31
     93345
     93350
                 599
                     comp_319865_01W 0.015502 0.074787 2019-12-31
     93390
                 599
                     comp 326765 01W 0.014321 -0.020752 2019-12-31
                                  ret exc
                                                        prc t+1
                                                               ret t+1 \
                    me
                            ret
                                                prc
     11
            5164.601074 -0.065609 -0.065609
                                           22.438058
                                                      22.574668 0.006088
     15
           52754.355469 -0.052386 -0.052386
                                           30.765442
                                                      30.035846 -0.014764
     21
            2671.458984 0.006921 0.006921
                                           40.510844
                                                      39.101210 -0.034796
     26
           32227.082031 -0.079324 -0.079324
                                                      28.807106 -0.053604
                                           30.438742
           13871.247070 -0.090486 -0.090486
     27
                                           59.876529
                                                      50.542136 -0.155894
            3372.102051 0.024598 0.023198
                                            8.749108
                                                       8.148685 -0.068627
     93335
     93337
            2280.453613 0.099088 0.097688
                                            8.973444
                                                       9.057790 0.009400
     93345
            6149.301758 0.025512 0.024112
                                           55.343714
                                                      47.206938 -0.147023
     93350
            3019.115234 0.215243 0.213843 100.637172 108.294358 0.076087
     93390 48030.359375 -0.009722 -0.011122
                                           48.030358
                                                      47.096072 -0.019452
           risk free rate MonthYear1 NumMonth DecileRank
     11
                 0.000353
                            2010-01
                                          1
                                                     9
     15
                 0.000353
                            2010-01
                                          1
                                                     9
     21
                 0.000353
                            2010-01
                                          1
     26
                 0.000353
                            2010-01
                                          1
     27
                 0.000353
                            2010-01
                                          1
     93335
                 0.001325
                            2019-12
                                        120
     93337
                 0.001325
                            2019-12
                                        120
```

93345	0.001325	2019-12	120	9
93350	0.001325	2019-12	120	9
93390	0.001325	2019-12	120	9

[9386 rows x 15 columns]

5 Portfolio performance

Next, the performance of the prediction-sorted portfolios is calculated over the 10-year out-of-sample testing for both equal and value weighting schemes

The excess return of each decile portfolio in month t is modeled as the weighted sum of each stocks's excess return in month t:

$$R_{P,t} = \sum_{i=1}^{N} w_{i,t} * r_{i,t}$$

```
[19]: # weighted return per stock in t+1 (to use for the sharpe ratio)

for j in np.arange(0,10,1):
    globals()['rank_' + str(j)]['return_stock_ew'] = globals()['rank_' +_
    str(j)]["ret_t+1"]*globals()['rank_' + str(j)]["eq_weights"]

    globals()['rank_' + str(j)]['return_stock_vw'] = globals()['rank_' +_
    str(j)]["ret_t+1"]*globals()['rank_' + str(j)]["me_weights"]
```

```
[20]: # Portfolio excess return in t+1 for j in np.arange(0,10,1):
```

```
globals()['rank_' + str(j)]['excess_return_portfolio_ew'] =__
       oglobals()['rank_' + str(j)].groupby('MonthYear')["excess_return_stock_ew"].

→transform('sum')
          globals()['rank ' + str(j)]['excess return portfolio vw'] = 
       Globals()['rank_' + str(j)].groupby('MonthYear')["excess_return_stock_vw"].

→transform('sum')
[21]: # Portfolio return in t+1 (to use for the sharpe ratio)
      for j in np.arange(0,10,1):
          globals()['rank_' + str(j)]['return_portfolio_ew'] = globals()['rank_' +

str(j)].groupby('MonthYear')["return_stock_ew"].transform('sum')

          globals()['rank_' + str(j)]['return_portfolio_vw'] = globals()['rank_' +__
       str(j)].groupby('MonthYear')["return stock vw"].transform('sum')
[22]: # Weighted predicted excess return per stock in t+1
      for j in np.arange(0,10,1):
          globals()['rank_' + str(j)]['pred_excess_return_stock_ew'] =__

¬globals()['rank_' + str(j)]["yhat"]*globals()['rank_' + str(j)]["eq_weights"]

          globals()['rank_' + str(j)]['pred_excess_return_stock_vw'] =__
       Globals()['rank_' + str(j)]["yhat"]*globals()['rank_' + str(j)]["me_weights"]
[23]: # Portfolio predicted excess return in t+1
      for j in np.arange(0,10,1):
          globals()['rank_' + str(j)]['pred_excess_return_portfolio_ew'] =__

¬globals()['rank_' + str(j)].
       agroupby('MonthYear')["pred_excess_return_stock_ew"].transform('sum')
          globals()['rank_' + str(j)]['pred_excess_return_portfolio_vw'] =__
       groupby('MonthYear')["pred_excess_return_stock_vw"].transform('sum')
[24]: # Generate dataframes, containing the portfolio returns on mohtly basis for
      ⇔each decile portfolio
      \# e.g., montly\_rank\_0: dataframe, containing only the monthly portfolio excess_{\sqcup}
      ⇔returns (predicted and real)
      # for the bottom rank
      for j in np.arange(0,10,1):
          globals()['montly_rank_' + str(j)] = globals()['rank_' +_{\sqcup}]
       ⇔str(j)][["MonthYear1", "DecileRank",
                                                                           ш

¬"excess_return_portfolio_ew",
       ⇔"excess_return_portfolio_vw",

¬"pred_excess_return_portfolio_ew",
```

```
"pred_excess_return_portfolio_vw",

"return_portfolio_ew",

"return_portfolio_vw"]]

for j in np.arange(0,10,1):
    globals()['montly_rank_' + str(j)]=globals()['montly_rank_' + str(j)].

drop_duplicates()
    globals()['montly_rank_' + str(j)]=globals()['montly_rank_' + str(j)].

set_index("MonthYear1")
```

- Calculate time-series averages of both predicted and realized excess returns for each decile portfolio
- Calculate the standard deviations and the annualized sharpe ratios based on the realized excess returns
- Get measures for both equal- and value-weighted portfolios

5.0.1 Sharpe Ratio = Portfolio return-Risk Free Rate Standard deviation of portfolio return = $\frac{R_p - R_F}{\sigma_P}$ = Portfolio excess return Std(portfolio return)

```
[25]: for j in np.arange(0,10,1):
          #Time-series average of realized excess returns
         globals()["ew_mean_return_rank_" + str(j)] = globals()['montly_rank_' +
       str(j)]["excess_return_portfolio_ew"].mean()
          globals()["vw_mean_return_rank_" + str(j)] = globals()['montly_rank_' + __
       str(j)]["excess return portfolio vw"].mean()
          #Time-series average of predicted excess returns
         globals()["ew_mean_pred_return_rank_" + str(j)] = globals()['montly_rank_'u
       str(j)]["pred_excess_return_portfolio_ew"].mean()
          globals()["vw_mean_pred_return_rank_" + str(j)]= globals()['montly_rank_'_
       str(j)]["pred_excess_return_portfolio_vw"].mean()
          #Standard deviation of realized excess returns
         globals()["std_ew_rank_" + str(j)] = globals()['montly_rank_' +
       str(j)]["excess_return_portfolio_ew"].std()
         globals()["std_vw_rank_" + str(j)]= globals()['montly_rank_' +_

str(j)]["excess_return_portfolio_vw"].std()
          #Annualized sharpe ratio of realized excess returns
         globals()["sharpe_ew_rank_" + str(j)]= (globals()['montly_rank_' +__
       Str(j)]["excess_return_portfolio_ew"].mean()/globals()['montly_rank_' +
       str(j)]["return_portfolio_ew"].std())* np.sqrt(12)
          globals()["sharpe vw rank " + str(j)]= (globals()['montly rank ' +1]
       str(j)]["excess_return_portfolio_vw"].mean()/globals()['montly_rank_' +_
       str(j)]["return_portfolio_vw"].std())* np.sqrt(12)
```

5.0.2 Zero net investment long-short portfolio

A zero-investment portfolio is a portfolio that has a net value of zero when the portfolio is assembled, and therefore requires an investor to take no equity stake in the portfolio. - e.g. short sell **USD** 1,000 worth of the bottom portfolio, and use the proceeds to purchase **USD** 1,000 worth of the top portfolio - this results in a net value of zero

$$\begin{split} &\sum_{i=1}^{N} w_{i, \text{ long}} \ = 1 \\ &\sum_{i=1}^{N} w_{i, \text{ short}} \ = -1 \\ &\sum_{i=1}^{N} w_{i, \text{ long}} \ * \ \mathbf{prc}_{i, \text{ long}} \ = \sum_{i=1}^{N} w_{i, \text{ short}} \ * \ \mathbf{prc}_{i, \text{ short}} \end{split}$$

Value Zero Net Portfolio in $t = \sum_{i=1}^{N} w_{i, \text{ long}} * \text{prc}_{i, \text{ long}} - \sum_{i=1}^{N} w_{i, \text{ short}} * \text{prc}_{i, \text{ short}} = 0$

Return Zero Net Portfolio in $t = \sum_{i=1}^{N} w_{i, \text{ long}} * \text{ret }_{i, \text{ long}} - \sum_{i=1}^{N} w_{i, \text{ short}} * \text{ret }_{i, \text{ short}}$ [26]: # For the zero-net-investment long-short portfolio the top (long) and ⇒bottom(short) decile portfolios are needed long_monthly = rank_9[["NumMonth", "MonthYear1", "DecileRank", "] ⇔"excess_return_portfolio_ew", "pred excess return portfolio vw", "return portfolio ew", "return portfolio vw"]].drop duplicates() short_monthly = rank_0[["NumMonth", "MonthYear1", "DecileRank", u ⇔"excess_return_portfolio_ew", "pred_excess_return_portfolio_vw", "return_portfolio_ew", "return portfolio vw"]].drop duplicates() # Create a column, indication the stategy long_monthly["Strategy"] = "long" short_monthly["Strategy"] = "short" # Merge to get the zero net investment portfolio zeronet_monthly= pd.concat([long_monthly, short_monthly]) zeronet_monthly = zeronet_monthly.sort_values(by = ['NumMonth', "Strategy"]) zeronet_monthly["return_portfolio_vw"] = zeronet_monthly["return_portfolio_vw"]. ⇔astype('float64') #Create two new columns containing the exess return of the portfolio and ⇔initially set the values to zero.

```
zeronet_monthly["excess_return_zeronet_ew"] =0
zeronet_monthly["excess_return_zeronet_vw"] =0
# excess return zeronet in t = (weigted\ excess\ return\ long\ in\ t) - (weigted\ long\ t)
\hookrightarrow excess return short in t)
for i in range(0, len(zeronet monthly)):
    if zeronet monthly.iloc[i,9] == "long":
        zeronet_monthly.iloc[i, -2] = zeronet_monthly.iloc[i,__
 →3]-zeronet_monthly.iloc[i+1, 3]
    else:
        zeronet_monthly.iloc[i, -2] = zeronet_monthly.iloc[i-1,__
 →3]-zeronet monthly.iloc[i, 3]
for i in range(0, len(zeronet_monthly)):
    if zeronet_monthly.iloc[i,9] == "long":
        zeronet_monthly.iloc[i, -1] = zeronet_monthly.iloc[i,__
 →4]-zeronet_monthly.iloc[i+1, 4]
    else:
        zeronet_monthly.iloc[i, -1] = zeronet_monthly.iloc[i-1,__
 →4]-zeronet monthly.iloc[i, 4]
#Create two new columns containing predicted the exess return of the portfoliou
 and initially set the values to zero.
zeronet monthly["pred excess return zeronet ew"] =0
zeronet_monthly["pred_excess_return_zeronet_vw"] =0
\# predicted excess return zeronet in t = (weighted predicted excess return longular)
 \rightarrow in t) - (weighted predicted excess return short in t)
for i in range(0, len(zeronet_monthly)):
    if zeronet_monthly.iloc[i,9] == "long":
        zeronet_monthly.iloc[i, -2] = zeronet_monthly.iloc[i,_
 →5]-zeronet_monthly.iloc[i+1, 5]
    else:
        zeronet_monthly.iloc[i, -2] = zeronet_monthly.iloc[i-1,__
 →5]-zeronet_monthly.iloc[i, 5]
for i in range(0, len(zeronet_monthly)):
    if zeronet_monthly.iloc[i,9] == "long":
        zeronet_monthly.iloc[i, -1] = zeronet_monthly.iloc[i,__
 →6]-zeronet_monthly.iloc[i+1, 6]
    else:
        zeronet_monthly.iloc[i, -1] = zeronet_monthly.iloc[i-1,__
 →6]-zeronet_monthly.iloc[i, 6]
#Create two new columns containing return of the portfolio and initially set _{f \sqcup}
 ⇔the values to zero.
```

```
zeronet_monthly["return_zeronet_ew"] =0
     zeronet_monthly["return_zeronet_vw"] =0
     \# return zeronet in t = (weighted return long in <math>t) - (weighted return short in t)
     for i in range(0, len(zeronet_monthly)):
         if zeronet_monthly.iloc[i,9] == "long":
             zeronet_monthly.iloc[i, -2] = zeronet_monthly.iloc[i,_
       →7]-zeronet_monthly.iloc[i+1, 7]
         else:
             zeronet_monthly.iloc[i, -2] = zeronet_monthly.iloc[i-1,__
       →7]-zeronet_monthly.iloc[i, 7]
     for i in range(0, len(zeronet_monthly)):
         if zeronet_monthly.iloc[i,9] == "long":
             zeronet_monthly.iloc[i, -1] = zeronet_monthly.iloc[i,__
       →8]-zeronet_monthly.iloc[i+1, 8]
         else:
             zeronet_monthly.iloc[i, -1] = zeronet_monthly.iloc[i-1,__
       →8]-zeronet_monthly.iloc[i, 8]
[27]: #Only the measures at portfolio level are needed
     zeronet_monthly = zeronet_monthly[['NumMonth', 'MonthYear1',_
      'excess_return_zeronet_vw',_

    'pred_excess_return_zeronet_vw','return_zeronet_ew',
                                       'return zeronet vw']].drop duplicates()
     zeronet_monthly
[27]:
            NumMonth MonthYear1 excess_return_zeronet_ew \
     11
                   1
                       2010-01
                                                0.025036
     735
                   2
                       2010-02
                                                0.073855
     1458
                   3 2010-03
                                                0.024195
     2227
                   4 2010-04
                                                0.005667
     2992
                   5 2010-05
                                               0.006859
     89857
                 116 2019-08
                                              -0.002446
     90561
                 117
                      2019-09
                                               0.029029
                 118 2019-10
                                               0.021879
     91269
     91981
                 119 2019-11
                                               0.006756
     92694
                 120 2019-12
                                               0.015541
            excess_return_zeronet_vw pred_excess_return_zeronet_ew \
                           0.013763
                                                         0.030796
     11
     735
                          -0.008665
                                                         0.029260
```

```
1458
                       0.001179
                                                       0.028525
2227
                       0.077138
                                                       0.030340
2992
                       0.032550
                                                       0.029577
89857
                      -0.003758
                                                       0.016334
90561
                      -0.018734
                                                       0.016095
91269
                       0.040320
                                                       0.016195
91981
                       0.042114
                                                       0.016111
92694
                       0.051998
                                                       0.015917
       pred_excess_return_zeronet_vw return_zeronet_ew return_zeronet_vw
11
                            0.027712
                                                0.022192
                                                                    0.013920
735
                            0.026742
                                                0.072079
                                                                   -0.008560
1458
                            0.025629
                                                0.024195
                                                                    0.001179
2227
                            0.027746
                                                0.011387
                                                                    0.078579
2992
                            0.025736
                                                0.005963
                                                                    0.032525
89857
                                               -0.004718
                                                                   -0.004014
                            0.016093
90561
                            0.016314
                                                0.029029
                                                                   -0.018734
91269
                            0.016032
                                                0.018938
                                                                    0.039263
91981
                                                0.004733
                                                                    0.042020
                            0.015523
92694
                            0.015503
                                                0.012242
                                                                    0.049725
```

[120 rows x 8 columns]

```
[28]: #Calculate zero-net portfolio performance measures
                                                                                      ш
      #Time-series average of realized excess returns
      ew mean return zeronet = zeronet monthly["excess return zeronet ew"].mean()
      vw mean return zeronet= zeronet monthly["excess return zeronet vw"].mean()
      #Time-series average of predicted excess returns
      ew mean pred_return zeronet = zeronet monthly["pred_excess_return_zeronet_ew"].
       ⊶mean()
      vw_mean_pred_return_zeronet = zeronet_monthly["pred_excess_return_zeronet_vw"].
       ⊶mean()
      #Standard deviation of realized excess returns
      std_ew_zeronet = zeronet_monthly["excess_return_zeronet_ew"].std()
      std_vw_zeronet = zeronet_monthly["excess_return_zeronet_vw"].std()
      #Annualized sharpe ratio of realized excess returns
      sharpe_ew_zeronet = (zeronet_monthly["excess_return_zeronet_ew"].mean()/

¬zeronet_monthly["return_zeronet_ew"].std())* np.sqrt(12)

      sharpe_vw_zeronet = (zeronet_monthly["excess_return_zeronet_vw"].mean()/
       ezeronet monthly["return zeronet vw"].std())* np.sqrt(12)
```

5.0.3 Generate table containing the performance measures for each decile portfolio and for the long-short portfolio for each weignting sheme respectively

Equally weighted table:

```
[29]: chart_np = np.array([[ew_mean_pred_return_rank_0, ew_mean_return_rank_0,_
       ⇒std ew rank 0, sharpe ew rank 0],
                           [ew_mean_pred_return_rank_1, ew_mean_return_rank_1,__
       ⇒std_ew_rank_1, sharpe_ew_rank_1],
                           [ew_mean_pred_return_rank_2, ew_mean_return_rank_2,__
       std_ew_rank_2, sharpe_ew_rank_2],
                           [ew_mean_pred_return_rank_3, ew_mean_return_rank_3,__
       ⇔std_ew_rank_3, sharpe_ew_rank_3],
                           [ew mean pred return rank 4, ew mean return rank 4,

→std_ew_rank_4, sharpe_ew_rank_4],
                           [ew_mean_pred_return_rank_5, ew_mean_return_rank_5,_
       ⇒std_ew_rank_5, sharpe_ew_rank_5],
                           [ew_mean_pred_return_rank_6, ew_mean_return_rank_6,__
       →std_ew_rank_6, sharpe_ew_rank_6],
                           [ew_mean_pred_return_rank_7, ew_mean_return_rank_7,__
       ⇔std_ew_rank_7, sharpe_ew_rank_7],
                           [ew_mean_pred_return_rank_8, ew_mean_return_rank_8,_
       ⇒std_ew_rank_8, sharpe_ew_rank_8],
                           [ew_mean_pred_return_rank_9, ew_mean_return_rank_9,__
       ⇒std ew rank 9, sharpe ew rank 9],
                           [ew_mean_pred_return_zeronet, ew_mean_return_zeronet,_
       ⇔std_ew_zeronet, sharpe_ew_zeronet]])
      ew_df = pd.DataFrame(chart_np, columns=['Pred', 'Real', 'Std', 'Sharpe'],
                                   index=['Low (L)', '2', '3', '4', __
       ew_df['Pred'] = pd.Series(["{0:.2f}%".format(val * 100) for val in_
       ⇔ew_df['Pred']], index = ew_df.index)
      ew df['Real'] = pd.Series(["\{0:.2f\}%".format(val * 100) for val in_1
       ⇔ew_df['Real']], index = ew_df.index)
      ew_df['Std'] = pd.Series(["{0:.2f}%".format(val * 100) for val in_{\sqcup}])
       ⇔ew_df['Std']], index = ew_df.index)
      ew_df['Sharpe'] = pd.Series([("%.2f" % round(val, 2)) for val in_
       ⇔ew_df['Sharpe']], index = ew_df.index)
      ew_df
```

```
[29]:
                Pred
                        Real
                                Std Sharpe
     Low (L)
               -0.58% -0.53% 4.76% -0.37
               -0.11%
                      0.16% 4.45%
     2
                                    0.12
     3
                0.14%
                      0.25% 4.50%
                                     0.19
     4
               0.33%
                      0.49% 4.31%
                                     0.39
     5
               0.51%
                       0.63% 4.36%
                                     0.49
```

```
6
          0.67%
                  0.72% 4.33%
                                 0.57
7
          0.84%
                  0.83% 4.49%
                                 0.64
8
          1.02%
                  1.00% 4.37%
                                 0.79
                  1.17% 4.32%
9
          1.23%
                                 0.93
High (H)
          1.62%
                  1.51% 4.53%
                                 1.14
                  2.04% 2.38%
H-L
          2.20%
                                 2.87
```

Value weighted table

```
[30]: chart_np = np.array([[vw_mean_pred_return_rank_0, vw_mean_return_rank_0, u
       ⇒std_vw_rank_0, sharpe_vw_rank_0],
                           [vw mean pred return rank 1, vw mean return rank 1,
       →std_vw_rank_1, sharpe_vw_rank_1],
                           [vw_mean_pred_return_rank_2, vw_mean_return_rank_2,__
       std_vw_rank_2, sharpe_vw_rank_2],
                           [vw_mean_pred_return_rank_3, vw_mean_return_rank_3,__
       std_vw_rank_3, sharpe_vw_rank_3],
                           [vw_mean_pred_return_rank_4, vw_mean_return_rank_4,__
       →std_vw_rank_4, sharpe_vw_rank_4],
                           [vw_mean_pred_return_rank_5, vw_mean_return_rank_5,_
       std_vw_rank_5, sharpe_vw_rank_5],
                           [vw_mean_pred_return_rank_6, vw_mean_return_rank_6, u
       ⇒std_vw_rank_6, sharpe_vw_rank_6],
                           [vw_mean_pred_return_rank_7, vw_mean_return_rank_7,__

std_vw_rank_7, sharpe_vw_rank_7],
                           [vw_mean_pred_return_rank_8, vw_mean_return_rank_8,_
       ⇒std_vw_rank_8, sharpe_vw_rank_8],
                           [vw_mean_pred_return_rank_9, vw_mean_return_rank_9,__
       ⇒std_vw_rank_9, sharpe_vw_rank_9],
                           [vw_mean_pred_return_zeronet, vw_mean_return_zeronet,_
       ⇔std_vw_zeronet, sharpe_vw_zeronet]])
     vw_df = pd.DataFrame(chart_np, columns=['Pred', 'Real', 'Std', 'Sharpe'],
                                   index=['Low (L)', '2', '3', '4', _
      vw_df['Pred'] = pd.Series(["{0:.2f}%".format(val * 100) for val in_
       ⇔vw_df['Pred']], index = vw_df.index)
     vw_df['Real'] = pd.Series(["{0:.2f}%".format(val * 100) for val in_
       ⇔vw_df['Real']], index = vw_df.index)
     vw_df['Std'] = pd.Series(["{0:.2f}%".format(val * 100) for val in_{\sqcup}])
       ⇔vw_df['Std']], index = vw_df.index)
     vw_df['Sharpe'] = pd.Series([("%.2f" % round(val, 2)) for val in_
       →vw_df['Sharpe']], index = vw_df.index)
     vw df
```

```
[30]:
                 Pred
                          Real
                                  Std Sharpe
     Low (L)
                -0.48%
                       -0.35% 6.43%
                                      -0.19
      2
                -0.10%
                        0.24% 5.36%
                                       0.16
                0.14%
                        0.21% 5.29%
                                        0.13
      3
      4
                0.34%
                        0.48% 4.93%
                                       0.34
                0.50%
                        0.48% 4.94%
                                       0.34
      5
      6
                0.67%
                        0.56% 4.84%
                                       0.40
      7
                0.84%
                        0.55% 4.87%
                                       0.39
                1.02%
                        0.86% 4.70%
      8
                                       0.63
      9
                1.23%
                        0.88% 4.67%
                                       0.65
                1.59%
                        0.75% 4.31%
                                        0.60
     High (H)
     H-L
                2.07%
                        1.10% 4.44%
                                        0.86
```

```
[31]: #Save to compare with the other ML models

ew_df.to_csv(r'C:\Users\krist\Documents\Data\decile_portfolios_results\pcr_ew.

→csv')

vw_df.to_csv(r'C:\Users\krist\Documents\Data\decile_portfolios_results\pcr_vw.

→csv')
```

- 6 Additional performance measures of zero net investment longshort portfolio with monthly restructuring, 2010:01-2019:12
- 6.0.1 Drawdown, turnover, and risk-adjusted performance of the long-short portfolio

6.1 Trunover

Portfolio turnover:

6.1.1

$$\mathbf{Turnover} \ = \frac{1}{T} \sum_{t=1}^{T} \left(\sum_{i} \left| w_{i,t+1} - w_{i,t} \right| \right)$$

, where $w_{i,t}$ denotes the weight of stock i in the portfolio in month t.

```
[32]: # Only stock weights are needed for the calculation of the portfolio turnover
long = rank_9.copy()
long=long[["MonthYear1",'NumMonth','id', 'eq_weights', 'me_weights']]
short = rank_0.copy()
short=short[["MonthYear1",'NumMonth','id', 'eq_weights','me_weights']]
short["eq_weights"]=short["eq_weights"]*(-1)
short["me_weights"]=short["me_weights"]*(-1)
```

Create lead weight variables $w_{i,t+1}$ for the equal- uand value-weighting scheme (eq_weights_lead1 and me_weights_lead1)

• lead weight variable = 0, when the stock is not included in the portfolio in the subsequent month

```
[33]: # Equal weights in t+1 for the top decile portfolio
      def create_lead_eq_weights(long):
          mn = long.NumMonth.min()
          mx = long.NumMonth.max() + 1
          d = long.set_index('NumMonth').reindex(pd.RangeIndex(mn, mx,__

¬name='NumMonth'))
          return pd.concat([d, d['eq_weights'].shift(-1).rename('eq_weights_lead1')],__
       ⇒axis=1)
      eq_weights_lead1_long=long.groupby('id')[['NumMonth', 'eq_weights']].
       →apply(create_lead_eq_weights) \
          .sort index(level=[1, 0]).dropna(subset=['eq weights']).reset index()
[34]: #Value weights in t+1 for the top decile portfolio
      def create lead me weights(long):
          mn = long.NumMonth.min()
          mx = long.NumMonth.max() + 1
          d = long.set_index('NumMonth').reindex(pd.RangeIndex(mn, mx,__

¬name='NumMonth'))
          return pd.concat([d, d['me_weights'].shift(-1).rename('me_weights_lead1')],__
       ⇒axis=1)
      me_weights_lead1_long=long.groupby('id')[['NumMonth', 'me_weights']].
       →apply(create_lead_me_weights) \
          .sort_index(level=[1, 0]).dropna(subset=['me_weights']).reset_index()
[35]: # Equal weights in t+1 for the bottom decile portfolio
      def create_lead_eq_weights(short):
          mn = short.NumMonth.min()
          mx = short.NumMonth.max() + 1
          d = short.set_index('NumMonth').reindex(pd.RangeIndex(mn, mx,__

¬name='NumMonth'))
          return pd.concat([d, d['eq_weights'].shift(-1).rename('eq_weights_lead1')],__
       ⇒axis=1)
      eq_weights_lead1_short=short.groupby('id')[['NumMonth', 'eq_weights']].
       →apply(create_lead_eq_weights) \
          .sort_index(level=[1, 0]).dropna(subset=['eq_weights']).reset_index()
[36]: # Value weights in t+1 for the bottom decile portfolio
      def create lead me weights(short):
          mn = short.NumMonth.min()
          mx = short.NumMonth.max() + 1
          d = short.set_index('NumMonth').reindex(pd.RangeIndex(mn, mx,__

¬name='NumMonth'))
```

```
[37]: # Merge lead weight variables to the dataframe containing the top decile.
      \hookrightarrowportfolio
     long=long.set index(['NumMonth','id'])
     eq_weights_lead1_long=eq_weights_lead1_long.set_index(['NumMonth','id'])
     me_weights_lead1_long=me_weights_lead1_long.set_index(['NumMonth','id'])
     long = pd.merge(long,__
       ⇔eq_weights_lead1_long[["eq_weights_lead1"]],left_index=True,⊔
       →right_index=True)
     long = pd.merge(long, )
       →me_weights_lead1_long[["me_weights_lead1"]],left_index=True,__
       →right index=True)
     long=long.reset_index()
      # Merge lead weight variables to the dataframe containing the bottom decile_
       \hookrightarrowportfolio
     short=short.set_index(['NumMonth','id'])
     eq weights lead1 short=eq weights lead1 short.set index(['NumMonth','id'])
     me_weights_lead1_short=me_weights_lead1_short.set_index(['NumMonth','id'])
     short = pd.merge(short,___
       →right index=True)
     short = pd.merge(short,__
       →me_weights_lead1_short[["me_weights_lead1"]],left_index=True,__
       →right_index=True)
     short=short.reset index()
      # Merge to get the zeronet portfolio
     long["Strategy"] = "long"
     short["Strategy"] = "short"
     zeronet= pd.concat([long, short])
     zeronet = zeronet.sort_values(by = ['NumMonth', "id", "Strategy"])
```

• Absolute difference between the weight of the stocks in the two following months $|w_{i,t+1} - w_{i,t}|$

```
[38]: zeronet["abs_ew"] = abs(zeronet["eq_weights_lead1"] -zeronet["eq_weights"])
zeronet["abs_vw"] = abs(zeronet["me_weights_lead1"] -zeronet["me_weights"])
```

• Sum of the absolute differences over all stocks per month $\sum_{i} |w_{i,t+1} - w_{i,t}|$

```
[39]: zeronet["sum_abs_ew"] = zeronet.groupby('NumMonth', sort=False)["abs_ew"].

transform('sum')

zeronet["sum_abs_vw"] = zeronet.groupby('NumMonth', sort=False)["abs_vw"].

transform('sum')
```

• Turnover $= \frac{1}{T} \sum_{t=1}^{T} \left(\sum_{i} |w_{i,t+1} - w_{i,t}| \right)$

```
[41]: # Multiply by 100 to get percent values
turnover_ew_pct = turnover_ew*100
turnover_vw_pct = turnover_vw*100
```

```
[42]: print("Trunover equally weighted: ", turnover_ew_pct) print("Trunover value weighted: ", turnover_vw_pct)
```

Trunover equally weighted: 2.045604785666251 Trunover value weighted: 31.126073201497395

7 Maximum Drawdown

$$\operatorname{MaxDD} \max_{0 \leq t_1 \leq t_2 \leq T} \left(Y_{t_1} - Y_{t_2} \right)$$

, where Y_t is the cumulative log excess return from date 0 through t

MaxDD equally weighted: 6.157246653417059 MaxDD value weighted: 20.64887737270781

7.1 Max 1M loss (%)

• the most extreme negative monthly return

```
[44]: max_1m_loss_ew= (zeronet_monthly["excess_return_zeronet_ew"].min())*(-100)
max_1m_loss_vw= (zeronet_monthly["excess_return_zeronet_vw"].min())*(-100)

print("Max 1M loss equally weighted: ", max_1m_loss_ew)
print("Max 1M loss value weighted: ", max_1m_loss_vw)
```

Max 1M loss equally weighted: 3.4707079091004998 Max 1M loss value weighted: 14.179001083920111

8 Risk-adjusted performance

Mean active return and information ratio relative to DAX (benchmark)

```
[45]: #get DAX price data
     dax = pd.read_csv(r"C:\Users\krist\Documents\Data\Dax monthly.csv",parse_dates_
      dax= dax[dax['Date'] <= '2020-02-01']
     \# Share prices are given on the first day of each month and in the data set at \sqcup
      → the end of the month.
      # Shift by one month to match the months in both datasets
     dax['MonthYear_t+1'] = dax['Date'].dt.to_period('M')
     dax['MonthYear'] = dax["MonthYear t+1"].shift(1)
     dax.drop(["MonthYear_t+1"], axis = 1, inplace = True)
     # We only need the prices from January 2010
     dax= dax[dax['MonthYear'] >= '2010-01']
      # Set MonthYear as index to merge with the portfolio dataframe afterwards
     dax=dax.set_index(["MonthYear"])
      # Get monthly returns
     dax["monthly_return_dax"] = dax["Close"].pct_change()
```

```
[47]: #Merge zeronet and DAX dataframes zeronet_monthly=zeronet_monthly.set_index("MonthYear1")
```

```
[48]: # Calculate portfolio performance measures
     #Time-series average of portfolio excess returns
    mean_ret_ew = dax_zeronet["excess_return_zeronet_ew"].mean()
    mean ret vw = dax zeronet["excess return zeronet vw"].mean()
    mean_ret_dax = dax_zeronet["monthly_excess_return_dax"].mean()
    #Multiply by 100 to get percent values
    mean ret ew pct = mean ret ew*100
    mean_ret_vw_pct= mean_ret_vw*100
    mean_ret_dax_pct = mean_ret_dax*100
    #Standard deviation ofportfolio excess_returns
    std_ew = dax_zeronet["excess_return_zeronet_ew"].std()
    std_vw = dax_zeronet["excess_return_zeronet_vw"].std()
    std_dax = dax_zeronet["monthly_excess_return_dax"].std()
    #Multiply by 100 to get percent values
    std ew pct = std ew*100
    std vw pct = std vw*100
    std_dax_pct = std_dax*100
    #Annualized sharpe ratio of portfolio excess_returns
    sr_vw = ((dax_zeronet["excess_return_zeronet_vw"]).mean() /__
     sr dax = ((dax zeronet["monthly excess return dax"]).mean() /___
      #Multiply by 100 to get percent values
    sr_ew_pct = sr_ew*100
    sr_vw_pct = sr_vw*100
    sr_dax_pcr= sr_dax*100
```

8.0.1 Mean active return: difference between the benchmark return and the portfolio return

```
8.1 Information Ratio = \frac{R_p - R_b}{\sigma_{(R_p - R_b)}}
```

- R_p Return of Portfolio
- R_b Return of Benchmark (Dax)
- $\sigma_{(R_n-R_h)}$ Tracking Error

Mean return [%]

Std [%]

```
[50]: | ir_ew = __
       ⇔((dax_zeronet["excess_return_zeronet_ew"]-dax_zeronet["monthly_excess_return_dax"]).
       →mean() /
       ⇔(dax_zeronet["excess_return_zeronet_ew"]-dax_zeronet["monthly_excess_return_dax"]).
       ⇔std())
      ir_vw =
       ⇔((dax_zeronet["excess_return_zeronet_vw"]-dax_zeronet["monthly_excess_return_dax"]).
       →mean() /
       ⇔(dax_zeronet["excess_return_zeronet_vw"]-dax_zeronet["monthly_excess_return_dax"]).
       ⇒std())
[51]: # This measures we only need to represent for the long-short portfolio
      turnover_dax=0
      ir_dax=0
      mean_act_ret_dax=0
      maxDD dax=0
[52]: # Generate table representing the portfolio performance measures for the equal_1
       ⇔weighting scheme
      ew_chart = np.array([[mean_ret_dax_pct, mean_ret_ew_pct],
                           [std_dax_pct,std_ew_pct],
                            [sr_dax, sr_ew],
                            [maxDD_dax, maxDD_ew_pct],
                            [turnover_dax, turnover_ew_pct],
                            [mean_act_ret_dax, mean_act_ret_ew],
                            [ir_dax, ir_ew]])
      ew_chart = pd.DataFrame(ew_chart, columns=['DAX', 'PCR'],
                              index=["Mean return [%]",'Std [%]',
                                      "Sharpe ratio", "Maximum drawdown [%]", "Turnover
       - [%] ",
                                      "Mean active return", "Information ratio"])
      ew_chart
[52]:
                                 DAX
                                           PCR
```

0.752467 2.039947

4.610969 2.380671

```
Sharpe ratio
      Maximum drawdown [%]
                            0.000000 6.157247
      Turnover [%]
                            0.000000 2.045605
      Mean active return
                            0.000000 0.012836
      Information ratio
                            0.000000 0.240930
[53]: # Generate table representing the portfolio performance measures for the value,
       ⇒weighting scheme
      vw_chart = np.array([[mean_ret_dax_pct, mean_ret_vw_pct],
                            [std_dax_pct,std_vw_pct],
                            [sr dax, sr vw],
                            [maxDD_dax, maxDD_vw_pct],
                            [turnover dax, turnover vw pct],
                            [mean_act_ret_dax, mean_act_ret_vw],
                            [ir dax, ir vw]])
      vw_chart = pd.DataFrame(vw_chart, columns=['DAX', 'PCR'],
                              index=["Mean return [%]",'Std [%]',
                                      "Sharpe ratio", "Maximum drawdown [%]", "Turnover∟
       ⊆ [%] ",
                                      "Mean active return", "Information ratio"])
      vw_chart
[53]:
                                 DAX
                                            PCR
      Mean return [%]
                            0.752467
                                       1.101198
      Std [%]
                            4.610969
                                       4.435544
      Sharpe ratio
                            0.561110
                                       0.856219
      Maximum drawdown [%]
                            0.000000 20.648877
      Turnover [%]
                            0.000000 31.126073
      Mean active return
                            0.000000
                                       0.003464
      Information ratio
                            0.000000
                                       0.045552
[54]: # Save results to compare with the other ML models
      ew_chart.to_csv(r'C:
```

0.561110 2.867392

Cumulative performance of long-only investment portfolio with monthly restructuring, 2010:01-2019:12

→\Users\krist\Documents\Data\performance_zeronet\pcr_zeronet_ew.csv')

→\Users\krist\Documents\Data\performance_zeronet\pcr_zeronet_vw.csv')

vw chart.to csv(r'C:

The H-L returns, tend to be driven more by the long side (top decile) than the short side (bottom decile), so the long-only portfolio performace is also examined separately.

9.0.1 Turnover

Trunover equally weighted: 1.0205656330404196 Trunover value weighted: 13.710231781005861

9.0.2 Maximum Drawdown

MaxDD equally weighted: 23.874493011710342 MaxDD value weighted: 26.975509464813076

9.0.3 Max 1M loss (%)

```
[57]: max_1m_loss_ew_long= (long_monthly["excess_return_portfolio_ew"].min())*(-100)
max_1m_loss_vw_long= (long_monthly["excess_return_portfolio_vw"].min())*(-100)

print("Max 1M loss equally weighted: ", max_1m_loss_ew_long)
print("Max 1M loss value weighted: ", max_1m_loss_vw_long)
```

Max 1M loss equally weighted: 10.734665875819012 Max 1M loss value weighted: 10.72582048589566

```
[58]: long_monthly=long_monthly.set_index("MonthYear1")
```

```
[59]: #Time-series average of portfolio excess returns
     mean_ret_vw_long = dax_long["excess_return_portfolio_vw"].mean()
     mean ret dax = dax long["monthly excess return dax"].mean()
     #Multiply by 100 to get percent values
     mean ret ew long pct = mean ret ew long*100
     mean_ret_vw_long_pct= mean_ret_vw_long*100
     mean_ret_dax_pct = mean_ret_dax*100
     #Standard deviation ofportfolio excess returns
     std_ew_long = dax_long["excess_return_portfolio_ew"].std()
     std_vw_long = dax_long["excess_return_portfolio_vw"].std()
     std_dax = dax_long["monthly_excess_return_dax"].std()
     #Multiply by 100 to get percent values
     std_ew_long_pct = std_ew_long*100
     std_vw_long_pct = std_vw_long*100
     std dax pct = std dax*100
     #Annualized sharpe ratio of portfolio excess returns
     sr_ew_long = ((dax_long["excess_return_portfolio_ew"]).mean() /__
      ⇔(dax long["return portfolio ew"]).std()) * np.sqrt(12)
     sr_vw_long = ((dax_long["excess_return_portfolio_vw"]).mean() /__
      sr_dax = ((dax_long["monthly_excess_return_dax"]).mean() /__
      ⇒(dax long["monthly return dax"]).std()) * np.sqrt(12)
     #Multiply by 100 to get percent values
     sr_ew_long_pct = sr_ew_long*100
     sr_vw_long_pct = sr_vw_long*100
     sr_dax_pcr= sr_dax*100
     #Mean active excess return: difference between the benchmark excess return and
      ⇔the portfolio excess return
     mean act ret ew long =
      ⊶mean()
     mean_act_ret_vw_long =_
      Girang dax long ["excess return portfolio vw"] -dax long ["monthly excess return dax"])
      →mean()
     # Calculate the information ratio
```

```
ir_ew_long =_
       ⇔((dax_long["excess_return_portfolio_ew"]-dax_long["monthly_excess_return_dax"]).
       ⊶mean() /

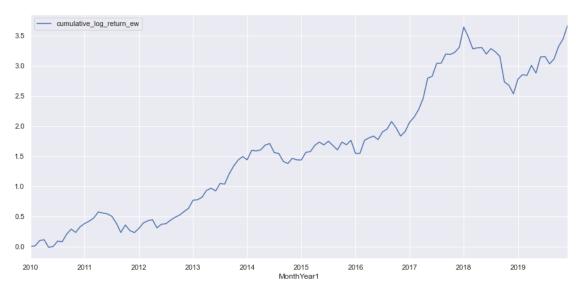
¬(dax_long["excess_return_portfolio_ew"]-dax_long["monthly_excess_return_dax"])
       ⇒std())
      ir_vw_long =__
       →((dax_long["excess_return_portfolio_vw"]-dax_long["monthly_excess_return_dax"]).
       ⊶mean() /
       ⇔(dax_long["excess_return_portfolio_vw"]-dax_long["monthly_excess_return_dax"])
       ⇒std())
[60]: # Generate table representing the portfolio performance measures for the equal.
       ⇔weighting scheme
      ew_long_chart = np.array([[mean_ret_dax_pct, mean_ret_ew_long_pct],
                           [std_dax_pct,std_ew_long_pct],
                           [sr_dax, sr_ew_long],
                           [maxDD_dax, maxDD_ew_pct_long],
                           [turnover_dax, turnover_ew_pct_long],
                           [mean_act_ret_dax, mean_act_ret_ew_long],
                           [ir dax, ir ew long]])
      ew_long_chart = pd.DataFrame(ew_long_chart, columns=['DAX', 'PCR_long'],
                              index=["Mean return [%]",'Std [%]',
                                     "Sharpe ratio", "Maximum drawdown [%]", "Turnover
       - [%] " ,
                                     "Mean active return", "Information ratio"])
      ew_long_chart
[60]:
                                 DAX
                                       PCR_long
     Mean return [%]
                            0.752467
                                       1.510029
     Std [%]
                            4.610969 4.528445
      Sharpe ratio
                            0.561110
                                      1.138538
     Maximum drawdown [%] 0.000000 23.874493
     Turnover [%]
                            0.000000
                                      1.020566
     Mean active return
                            0.000000 0.007637
      Information ratio
                            0.000000 0.207883
[61]: # Generate table representing the portfolio performance measures for the value
       ⇔weighting scheme
      vw_long_chart = np.array([[mean_ret_dax_pct, mean_ret_vw_long_pct],
                           [std_dax_pct,std_vw_long_pct],
                           [sr_dax, sr_vw_long],
```

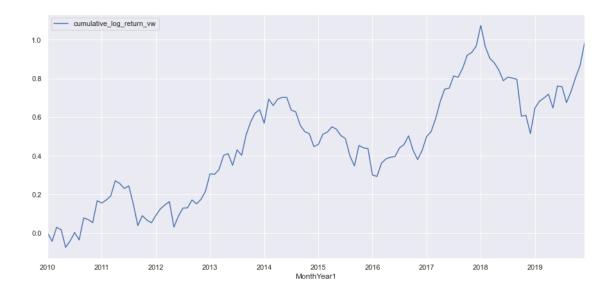
```
[61]:
                                DAX
                                       PCR long
                           0.752467
                                       0.748578
     Mean return [%]
     Std [%]
                                       4.314071
                            4.610969
     Sharpe ratio
                           0.561110
                                      0.600264
      Maximum drawdown [%] 0.000000 26.975509
      Turnover [%]
                            0.000000 13.710232
     Mean active return
                            0.000000
                                      0.000380
      Information ratio
                            0.000000
                                       0.010639
```

10 Cumulative performance graph of expected return-sorted portfolios with monthly restructuring, 2010:01-2019:12

Calculate the cumulative log returns of the long (top decile) and short (bottom decile) portfolios for both weighting schemes. Let's assume one invests 1 USD in January 2010 and buys a fraction of the top decile portfolio.

```
\#Calculate cumulative returns and cumulative log returns for the equal_\sqcup
 ⇔weighting scheme
long["cumulative_return_ew"] = (1 + long["curr_ret_port_ew"]).cumprod()-1
long["cumulative_log_return_ew"] = (1 + np.log(long["curr_ret_port_ew"]+1)).
 # return in January 2010 = 0 (one invests in the end of the month)
long.iloc[0, 2:] = 0
#Plot cumulative log returns for the equal weighting scheme
long[["cumulative_log_return_ew"]].plot(grid=True,figsize=(15, 7))
#Calculate cumulative returns and cumulative log returns for the value
 ⇒weighting scheme
long["cumulative_return_vw"] = (1 + long["curr_ret_port_vw"]).cumprod()-1
long["cumulative_log_return_vw"] = (1 + np.log(long["curr_ret_port_vw"]+1)).
 ⇒cumprod()-1
# Return in January 2010 = O(one invests in the end of the month)
long.iloc[0, 2:] = 0
# Plot cumulative log returns for the value weighting scheme
long[["cumulative_log_return_vw"]].plot(grid=True,figsize=(15, 7));
```



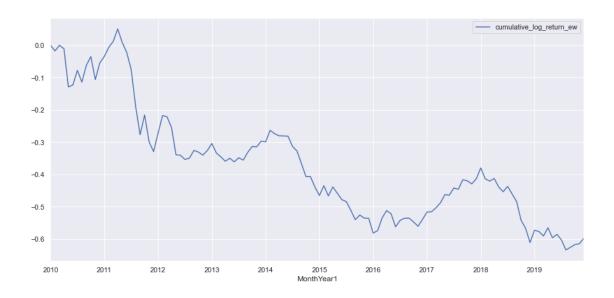


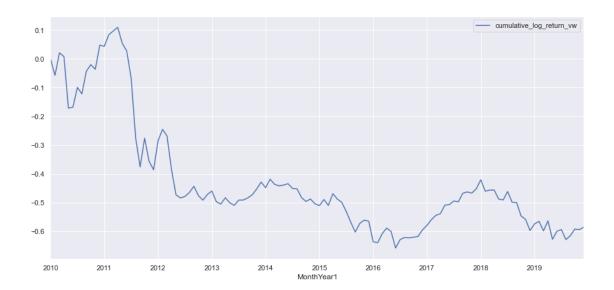
```
[64]: short =rank_0.copy()
      short = short.set_index("MonthYear1")
      short=short[["excess_return_portfolio_ew", "excess_return_portfolio_vw"]].

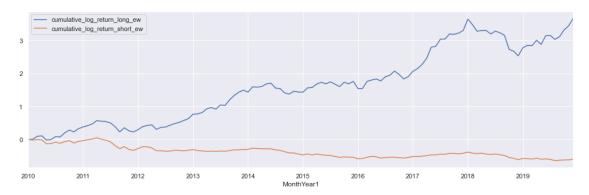
¬drop_duplicates()
      #Shift to get the returns in the current month
      short["curr_ret_port_ew"]=short['excess_return_portfolio_ew'].shift(1)
      short["curr_ret_port_vw"]=short['excess_return_portfolio_vw'].shift(1)
      \#Calculate cumulative returns and cumulative log returns for the equal
       ⇔weighting scheme
      short["cumulative_return_ew"] = (1 + short["curr_ret_port_ew"]).cumprod()-1
      short["cumulative_log_return_ew"] = (1 + np.log(short["curr_ret_port_ew"]+1)).

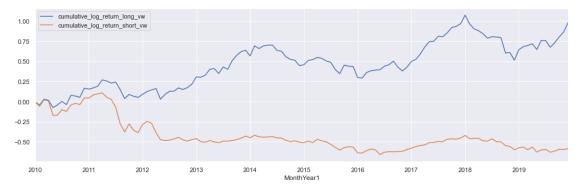
    cumprod()-1

      # return in January 2010 = 0 (one invests in the end of the month)
      short.iloc[0, 2:] = 0
      #Plot cumulative log returns for the equal weighting scheme
      short[["cumulative_log_return_ew"]].plot(grid=True,figsize=(15, 7))
      #Calculate cumulative returns and cumulative log returns for the value
       ⇔weighting scheme
      short["cumulative_return_vw"] = (1 + short["curr_ret_port_vw"]).cumprod()-1
      short["cumulative_log_return_vw"] = (1 + np.log(short["curr_ret_port_vw"]+1)).
       ⇒cumprod()-1
      # Return in January 2010 = O(one invests in the end of the month)
      short.iloc[0, 2:] = 0
      # Plot cumulative log returns for the value weighting scheme
      short[["cumulative_log_return_vw"]].plot(grid=True,figsize=(15, 7));
```









11 DAX Returns

Compare the cumulative returns of both portfolios with DAX cumulative returs over the same period

