MKT440/31
Pricing Analytics
Spring A 2020

Project 2:
Kiwi Bubbles

Due: Thursday, 14 Febuary 2020

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Question 3 Logit model without segmentation

In question 3, we assume that beta1 and beta0j and identical across customers.
First, we calculate the coefficient using the simple multinomial logit model.

```
### Convert data to mlogit.data form.
mlogitdata=mlogit.data(data,id="id",varying=4:7,choice="choice",shape="wide")
#Run MLE.
mle= gmnl(choice ~  price, data = mlogitdata)
summary(mle)
coef=mle$coefficients
beta1 <- coef[4]
beta0KB <- coef[1]
beta0KR <- coef[2]
beta0MB <- coef[3]
```

Simple Multinomial Logit Model Result :

```
KB:(intercept) KR:(intercept) MB:(intercept)          price
      4.253157       4.362403       4.204396      -3.737931
```

Second, we calculate the elasticity for KB, KR, and MR.

```
demand1 = function(priceKB,priceKR,priceMB,beta0KB,beta0KR,beta0MB,beta1){
  prob1=exp(beta0KB+beta1*priceKB)/(1+exp(beta0KB+beta1*priceKB)+exp(beta0KR+beta1*priceKR)+exp(beta0MB+beta1*priceMB))
  return(prob1)
}

demand2 = function(priceKB,priceKR,priceMB,beta0KB,beta0KR,beta0MB,beta1){
  prob2=exp(beta0KR+beta1*priceKR)/(1+exp(beta0KB+beta1*priceKB)+exp(beta0KR+beta1*priceKR)+exp(beta0MB+beta1*priceMB))
  return(prob2)
}

demand3 = function(priceKB,priceKR,priceMB,beta0KB,beta0KR,beta0MB,beta1){
  prob3=exp(beta0MB+beta1*priceMB)/(1+exp(beta0KB+beta1*priceKB)+exp(beta0KR+beta1*priceKR)+exp(beta0MB+beta1*priceMB))
  return(prob3)
}

elac.KB <- -beta1*average.KB*(1-demand1(average.KB,average.KR,average.MB,beta0KB,beta0KR,beta0MB,beta1))
elac.KR <- -beta1*average.KR*(1-demand2(average.KB,average.KR,average.MB,beta0KB,beta0KR,beta0MB,beta1))
elac.MB <- -beta1*average.MB*(1-demand3(average.KB,average.KR,average.MB,beta0KB,beta0KR,beta0MB,beta1))
```

Third, we calculate the cross-elasticity for KB, KR, and MR.

```
elac.KBKR <- -beta1*average.KR*(demand2(average.KB,average.KR,average.MB,beta0KB,beta0KR,beta0MB,beta1))
elac.KBMB <- -beta1*average.MB*(demand3(average.KB,average.KR,average.MB,beta0KB,beta0KR,beta0MB,beta1))
elac.KRMB <- -beta1*average.MB*(demand3(average.KB,average.KR,average.MB,beta0KB,beta0KR,beta0MB,beta1))
elac.KRKB <- -beta1*average.KB*(demand1(average.KB,average.KR,average.MB,beta0KB,beta0KR,beta0MB,beta1))
elac.MBKB <- -beta1*average.KB*(demand1(average.KB,average.KR,average.MB,beta0KB,beta0KR,beta0MB,beta1))
elac.MBKR <- -beta1*average.KR*(demand2(average.KB,average.KR,average.MB,beta0KB,beta0KR,beta0MB,beta1))
```

*Elasticities Before Segmentation

```
      KB       KR       MB
4.257847 4.13127 4.069547
```

By observing the elasticity, we find out that the elasticity for KB is the highest, indicating that customers are more price-sensitive when buying KB. The product with customers that are second rank price-sensitive is KR, and MB has the product with the least price-sensitive customers.

*Cross-elasticities Before Segmentation

```
    KBKR      KBMB      KRMB      KRKB      MBKB      MBKR
1.019923 0.9601564 0.9601564 0.9054743 0.9054743 1.019923
```

From the cross-price elasticity result, we can first notice that when we are considering how the percent change in choice probability of KB, the elasticity of KRKB and MBKB is the same. The result is reasonable, since we are considering the effect on KB, when we change the price of other two products.

The highest cross-elasticity is for KR (KBKR/MBKR). When KR increases its price by 1%, either MR or KB increases its price by 1.019% (1.019/1). This result indicates that KR is the product that affect the other products the most.

With the same interpretation, we can also conclude that KB's price is least responsive to other products' price.

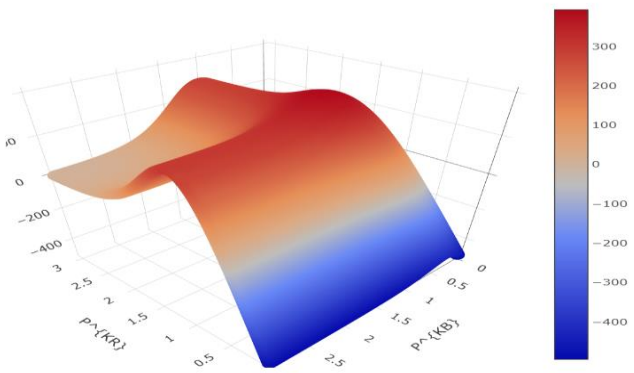
Fourth, we calculate optimal prices for KB and KR , when Mango price is $1.43.

```
### Finding optimal price
#Unit cost
uc=0.5;

# Optimal price of KR and KB
profit=function(priceKB,priceKR,priceMB,beta0KB,beta0KR,beta0MB,beta1){
  profitKB=1000*demand1(priceKB,priceKR,priceMB,beta0KB,beta0KR,beta0MB,beta1)*(priceKB-0.5)
  profitKR=1000*demand2(priceKB,priceKR,priceMB,beta0KB,beta0KR,beta0MB,beta1)*(priceKR-0.5)
  return(cbind(profitKB,profitKR))
}
#Choose space of prices to search for the optimal price over
pricespace=seq(0,3,0.01)
#Because we search over two dimensions, create complete combination
#of the two prices
pricespace2=expand.grid(pricespace,pricespace)
profitmat=matrix(0L,nrow(pricespace2),1)
for (i in 1:nrow(pricespace2)){
  profitmat[i]=sum(profit(pricespace2[i,1],pricespace2[i,2],1.43,beta0KB,beta0KR,beta0MB,beta1))
}

xaxis=list(title="P^{KB}")
yaxis=list(autorange = "reversed",title="P^{KR}")
zaxis=list(title="Profit")
p=plot_ly(x=pricespace2[,1],y=pricespace2[,2],z=as.numeric(profitmat),
          type="scatter3d",mode="markers",
          marker = list(color = as.numeric(profitmat), colorscale = c('#FFE1A1', '#683531'), showscale = TRUE))%>%
  layout(scene=list(xaxis=xaxis,yaxis=yaxis,zaxis=zaxis))%>%
  config(mathjax = 'cdn')
p
pricespace2[profitmat==max(profitmat)]
max(profitmat)
```

Optimal Profit 3D Model:

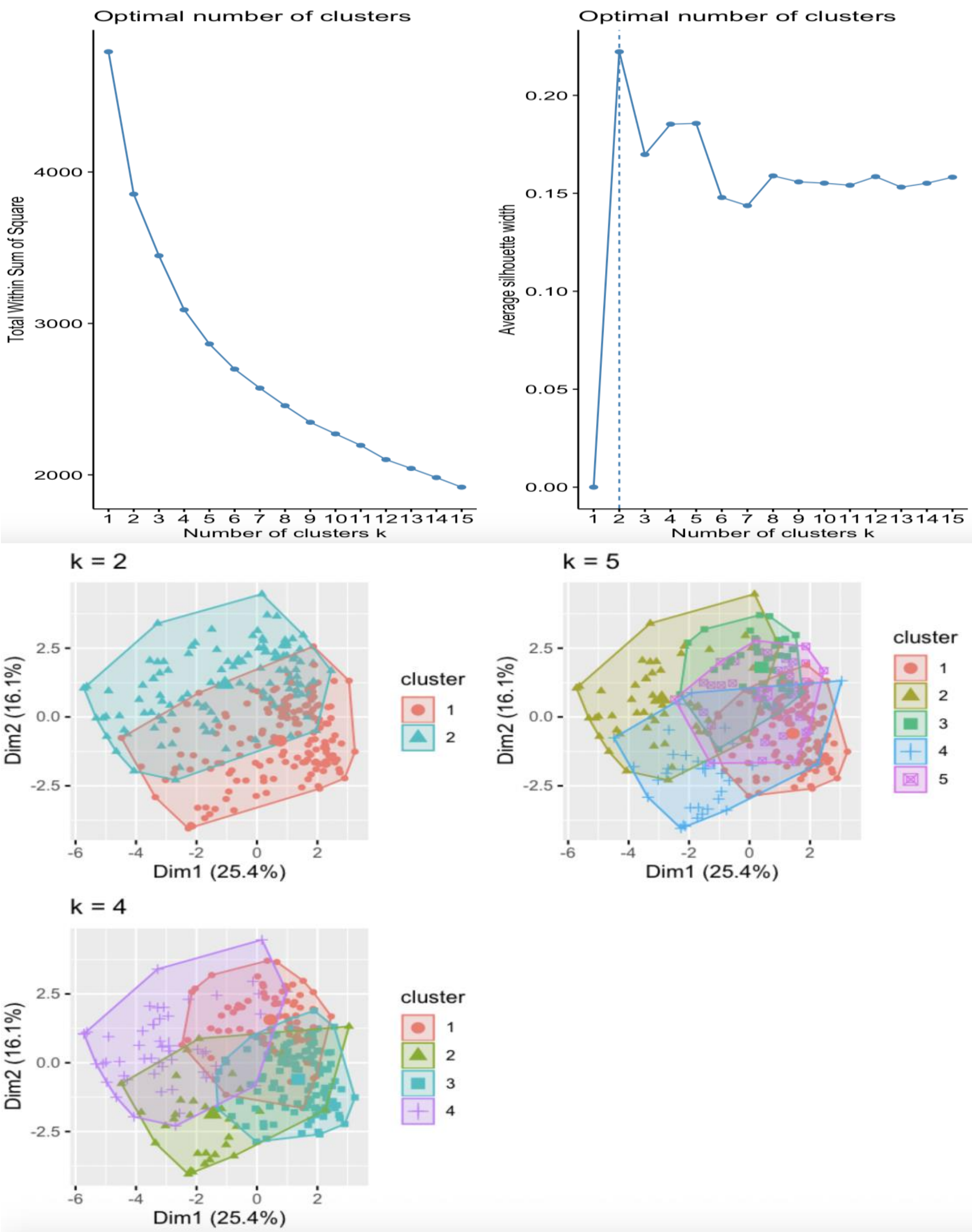


The optimal price for KB and KR are both $1.16, giving highest profit at $393.4.

Question 4 Logit model with segmentation

We used two methods to calculate and double check the best k-means.

The first method is “the elbow method” in pricing course, and the second method is the function in analytics application & design course. Both methods show that the best k-means we should select is k=2. However, the number of segment is not enough when k=2 since there are too many overlapping areas. Those areas are hard to cluster. That’s why we choose the second best choice, which is k=4. We can see that when k=4, overlapping area of every two clusters becomes smaller. For those customers who still don’t fit in any cluster, we put them into the fifth cluster, so we have 5 segments in total.



Then we calculate segment shares and coefficients for different segments

```
> seg.share
         1          2          3          4
0.09749304 0.19498607 0.35097493 0.14484680 0.21169916
```

Segment 1 occupies 9.7%.
Segment 2 occupies 19.5%.
Segment 3 occupies 35.1%.
Segment 4 occupies 14.5%.
Segment 5 (NA) occupies 21.2%.

Coefficients of each product in each segment are listed as follows.

	segment	intercept.KB	intercept.KR	intercept.MB	price.coef
1	1	4.149431	3.947833	3.948372	-3.774973
2	2	4.085952	4.316191	4.577172	-3.798219
3	3	3.864525	4.347728	4.017122	-3.600028
4	4	4.335498	4.674025	4.111057	-3.682239
5	5	5.117430	4.509340	4.544963	-4.062526

<u>*Elasticities Before Segmentation</u>

```
      KB       KR       MB
4.257847 4.13127 4.069547

    KBKR      KBMB      KRMB      KRKB      MBKB     MBKR
1.019923 0.9601564 0.9601564 0.9054743 0.9054743 1.019923
```

<u>*Elasticities After Segmentation</u>

```
      KB       KR       MB
4.254761 4.077948 4.061342

     KBKR      KBMB      KRMB      KRKB      MBKB     MBKR
0.9965285 0.9580629 0.9624656 0.8835981 0.8973701 1.016712
```

Like the information above, at the average price based on the data own elasticities of KB, KR, MB is 4.2547, 4.077, 4.061. Also, cross elasticities with the segmentation are 0.9965, 0.9580, 0.9624, 0.8835, 0.8973, 1.0167.

<u>*Difference of own / cross elasticities after segmentation</u>

```
         KB         KR          MB
0.003086506 0.05332133 0.008205379

    KBKR        KBMB         KRMB       KRKB        MBKB        MBKR
0.023394 0.002093559 -0.002309184 0.02187619 0.008104185 0.003210933
```

After segmentation, elasticities of Kiwi Bubble, Kiwi Regular, Mango Bubble are decreased slightly which means that the consumers become less price sensitive. Cross elasticities are generally decreased after the segmentation except for the KRMB's cross elasticity. It seems like they become less close substitutes except for the KRMB. The table above is the difference between before segmentation and after segmentation(before segment elasticity - after segment elasticity).

After the segmentation, It turns out that Mango Bubble is a closer substitute of Kiwi Regular than Kiwi Bubble. And Kiwi Bubble is a closer substitute of Mango Bubble than Kiwi Regular.
The closest substitutes are MBKR which means that when the price of Kiwi Regular moves by 1%, 1.016 percent of the demand of the Mango Bubble will be affected by it.

```
> seg.share
         1          2          3          4 
0.09749304 0.19498607 0.35097493 0.14484680 0.21169916
```

	segment	intercept.KB	intercept.KR	intercept.MB	price.coef
1	1	4.149431	3.947833	3.948372	-3.774973
2	2	4.085952	4.316191	4.577172	-3.798219
3	3	3.864525	4.347728	4.017122	-3.600028
4	4	4.335498	4.674025	4.111057	-3.682239
5	5	5.117430	4.509340	4.544963	-4.062526

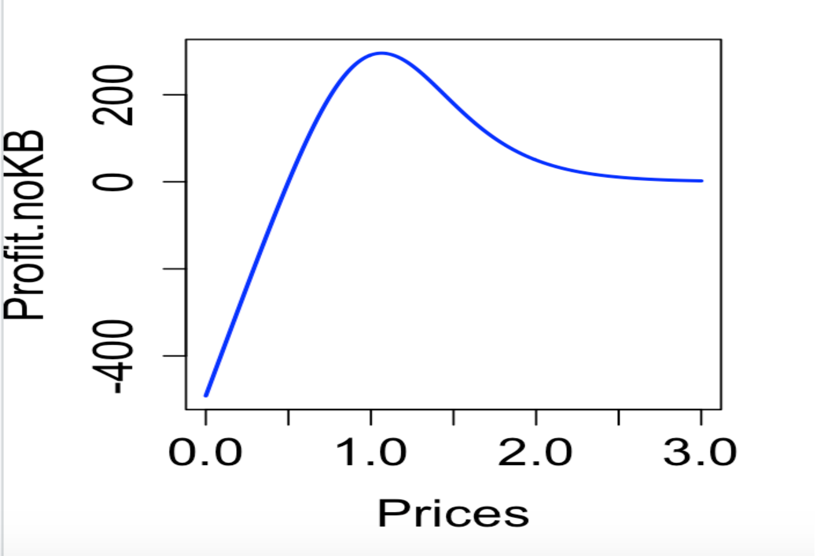
Except for segment 4, the intercept of KR and MB are similar which means that the popularity of them are similar. Therefore, it is reasonable that KB and KR are closet substitutes.

The company should position Kiwi Bubble in segment 1, 5 (around 30%) since people in that segments like Kiwi Bubble the most. And If the company does not launch the Kiwi Bubble in there, its competitor Mango Bubble will take the market share.

<u>*Optimal prices for KB/KR</u>
Supposed that Mango Bubbles is priced at $1.43 and doesn't react to Kiwi's pricing.
Unit cost for all three products is $0.5.
We have two choices now:
Don't launch Kiwi Bubbles
Launch Kiwi Bubbles
Let's discuss which choice is more profitable.

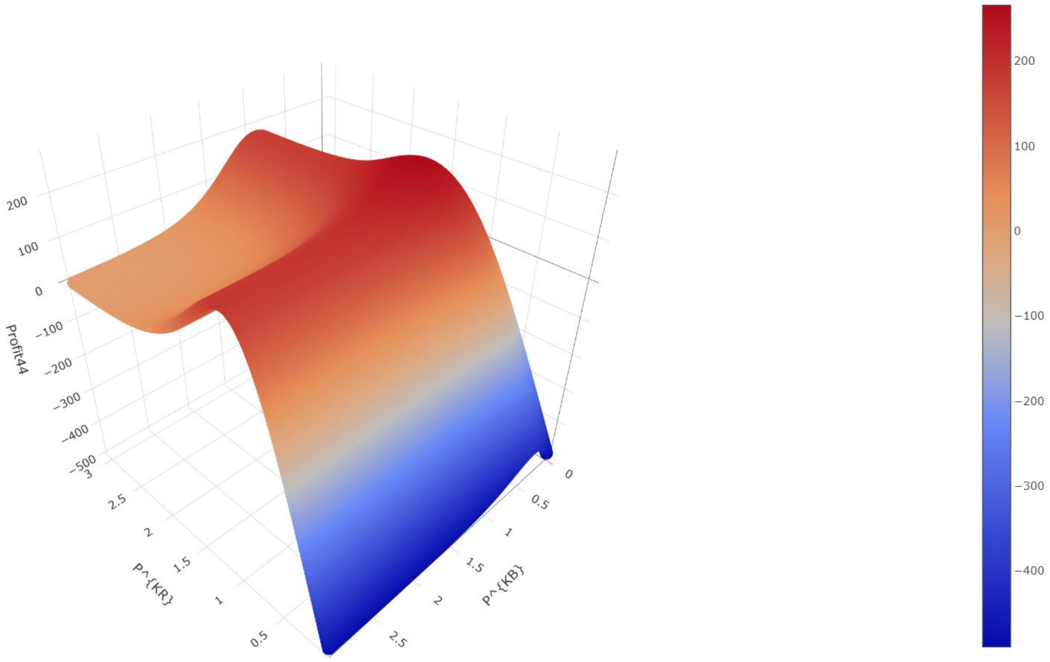
Don’t launch Kiwi Bubbles

If we don’t launch Kiwi Bubbles and launch Kiwi Regular only, the profit-maximized price for KR is $1.06, the maximum profit is $295.5. At the same time, our competitor MB can gain a profit of $106.9.



Launch Kiwi Bubbles

However, if we launch Kiwi Bubbles and Kiwi Regular at the same time, the maximum prices for KR and KB are both $1.16, gaining the maximum profit of $393.8. In this situation, our competitor MB can only gain a profit of $92, which is smaller after the introduction of KB.



*How does the profit of Kiwi and Mango change as Kiwi launches KB? Why (why not)?

The reason why we get this result is that:

```
> seg.share
         1          2          3          4
0.09749304 0.19498607 0.35097493 0.14484680 0.21169916
```

Segment 3>Segment 5>Segment 2>Segment 4>Segment 1

	segment	intercept.KB	intercept.KR	intercept.MB	price.coef
1	1	4.149431	3.947833	3.948372	-3.774973
2	2	4.085952	4.316191	4.577172	-3.798219
3	3	3.864525	4.347728	4.017122	-3.600028
4	4	4.335498	4.674025	4.111057	-3.682239
5	5	5.117430	4.509340	4.544963	-4.062526

If we don't launch KB, the whole market only has KR and MB two products. It is a 1vs1 competition. We can see that consumers in Segment 3 and Segment 4 prefer KR very much since the differences between intercept.KR and intercept.MB are quite large. As Segment 3 and Segment 4 occupy about 50% of the whole market, it is reasonable that we could gain more profit than our competitor.

If we launch KB, now it becomes a 2vs1 competition. Previously, we lost a large number of customers in Segment 1 and Segment 5 who prefer MB than KR. However, customer inside these 2 segments prefer KB most now. Therefore, we obtain customers who used to purchase MB to buy our new launched product KB. Also, consumers in Segment 3 and Segment 4 still prefer KR most so that the cannibalization are unlikely to happen. Only consumers in Segment 2 prefer our competitor MB most. Because of our introduction of KB, our products combination becomes more competitive and we squeeze our competitor's market share. Thus, even we increase our price from $1.06 to $1.16, in comparison with the previous scenario that we do not launch KB, we can still make more profit while our competitor will face the decline of its profit.

Based on results and reasons above, we should launch Kiwi Bubbles to capture market share from our competitors and maximize our profits.

Question 5 Understanding strategic responses

*No segmentation situation.

Based on results from Q4, we plan to sell both KR and KB at the price of $1.16 to compete for MB, which is the at the price of $1.43.

Pricing War Round 1:

```
# Step 1
pricespace = seq(0,3,0.01)
profit1 = 1000*demand3(1.16,1.16,pricespace,beta0KB,beta0KR,beta0MB,beta1)*(pricespace-uc)
pricespace[profit1==max(profit1)]
max(profit1)
```

The competitor responses to our new launch and reduce the price of MB to $0.95 and get its maximum profit $177.6.

Pricing War Round 2:

Reacting to Round 1, we reduce our price to $1.04, getting our maximum profit of $275.76.

```
# Step 2
profit2 = 1000*demand1(pricespace,pricespace,0.95,beta0KB,beta0KR,beta0MB,beta1)*(pricespace-uc)
+1000*demand2(pricespace,pricespace,0.95,beta0KB,beta0KR,beta0MB,beta1)*(pricespace-uc)
pricespace[profit2==max(profit2)]    #1.04
max(profit2)
```

Pricing War Round 3:

Reacting to Round 2, the competitor decreases its price of MB to $0.91, getting its maximum profit of $145.

Pricing War Round 4:

Reacting to Round 3, our price is $1.03 as our competitor will remain its MB price at $0.91. Thus, we achieve the market equilibrium.

```
# Repeat step 1 or step 2
profit3 = 1000*demand3(1.04,1.04,pricespace,beta0KB,beta0KR,beta0MB,beta1)*(pricespace-uc)
pricespace[profit3==max(profit3)]
max(profit3)    # 0.91

profit4 = 1000*demand1(pricespace,pricespace,0.91,beta0KB,beta0KR,beta0MB,beta1)*(pricespace-uc)
+1000*demand2(pricespace,pricespace,0.91,beta0KB,beta0KR,beta0MB,beta1)*(pricespace-uc)
pricespace[profit4==max(profit4)]    #1.03
max(profit4)

profit5 = 1000*demand3(1.03,1.03,pricespace,beta0KB,beta0KR,beta0MB,beta1)*(pricespace-uc)
pricespace[profit5==max(profit5)]
max(profit5)      #0.91
```

Segmentation situation

Pricing War Round 1:

```
# Step 1
profit11 <- 1000*agg_choice3(1.16,1.16,pricespace)*(pricespace-uc)
pricespace[profit11==max(profit11)]   #0.95
max(profit11)  #178.5712
```

The competitor responses to our price by reducing the price of MB from $1.43 to $0.95, hence we get the maximum profit of $178.6.

Pricing War Round 2:

Reacting to Round 1, we lower our price of KB to $1.04 and KR to $1.05, hence our maximum profit is $275.7.

```
# Step 2
pricespace=seq(0,3,0.01)
#Because we search over two dimensions, create complete combination
#of the two prices
pricespace2=expand.grid(pricespace,pricespace)
profit22=matrix(0L,nrow(pricespace2),1)
for (i in 1:nrow(pricespace2)){
  profit22[i]=sum(profit.KB(pricespace2[i,1],pricespace2[i,2],0.95))
}

xaxis=list(title="P^{KB}")
yaxis=list(autorange = "reversed",title="P^{KR}")
zaxis=list(title="Profit22")
p22=plot_ly(x=pricespace2[,1],y=pricespace2[,2],z=as.numeric(profit22),
            type="scatter3d",mode="markers",
            marker = list(color = as.numeric(profit22), colorscale = c('#FFE1A1', '#683531'),
                          showscale = TRUE))%>%
  layout(scene=list(xaxis=xaxis,yaxis=yaxis,zaxis=zaxis))%>%
  config(mathjax = 'cdn')
p22
pricespace2[profit22==max(profit22)]  #KB = 1.04, KR = 1.05
max(profit22) #275.6912
```

Pricing War Round 3:

Reacting to Round 2, the competitor continue lowering the price of MB to $0.92, hence its maximum profit is $148.

Pricing War Round 4:

Reacting to Round 3, we lower our price of KB to $1.03 and KR to $1.04, hence our maximum profit is $265.6.

Pricing War Round 5:

Reacting to Round 4, the competitor continue lowering the price of MB to $0.91, hence its maximum profit is $145.3.

Pricing War Round 6:
Reacting to Round 5, we lower our price of KB to $1.03 and KR to $1.03, hence our maximum profit is $262.2.

Then reacting to Round 6, the competitor retains the price of MB at $0.91, so we reach the market equilibrium when KB=$1.03 and KR=$1.03.

```r
# Repeat step 1 or step 2
profit33 <- 1000*agg_choice3(1.04,1.05,pricespace)*(pricespace-uc)
pricespace[profit33==max(profit33)]    #0.92
max(profit33)  #147.9704


pricespace=seq(0,3,0.01)
#Because we search over two dimensions, create complete combination
#of the two prices
pricespace2=expand.grid(pricespace,pricespace)
profit44=matrix(0L,nrow(pricespace2),1)
for (i in 1:nrow(pricespace2)){
  profit44[i]=sum(profit.KB(pricespace2[i,1],pricespace2[i,2],0.92))
}

xaxis=list(title="P^{KB}")
yaxis=list(autorange = "reversed",title="P^{KR}")
zaxis=list(title="Profit44")
p44=plot_ly(x=pricespace2[,1],y=pricespace2[,2],z=as.numeric(profit44),
            type="scatter3d",mode="markers",
            marker = list(color = as.numeric(profit44), colorscale = c('#FFE1A1', '#683531'), showscale = TRUE))%>%
               layout(scene=list(xaxis=xaxis,yaxis=yaxis,zaxis=zaxis))%>%
               config(mathjax = 'cdn')
p44
pricespace2[profit44==max(profit44)]  # KB = 1.03, KR = 1.04
max(profit44) #  265.5745

profit55 <- 1000*agg_choice3(1.03,1.04,pricespace)*(pricespace-uc)
pricespace[profit55==max(profit55)]    #0.91
max(profit55)   # 145.3087


pricespace=seq(0,3,0.01)
#Because we search over two dimensions, create complete combination
#of the two prices
pricespace2=expand.grid(pricespace,pricespace)
profit66=matrix(0L,nrow(pricespace2),1)
for (i in 1:nrow(pricespace2)){
  profit66[i]=sum(profit.KB(pricespace2[i,1],pricespace2[i,2],0.91))
}

xaxis=list(title="P^{KB}")
yaxis=list(autorange = "reversed",title="P^{KR}")
zaxis=list(title="Profit66")
p66=plot_ly(x=pricespace2[,1],y=pricespace2[,2],z=as.numeric(profit66),
            type="scatter3d",mode="markers",
            marker = list(color = as.numeric(profit66), colorscale = c('#FFE1A1', '#683531'), showscale = TRUE))%>%
  layout(scene=list(xaxis=xaxis,yaxis=yaxis,zaxis=zaxis))%>%
  config(mathjax = 'cdn')
p66
pricespace2[profit66==max(profit66)]  # KB = 1.03, KR = 1.03
max(profit66)        #262.1776
```

<u>*Strategic advantage of Kiwi Bubbles</u>
After changing our price several times, we reach the Nash Equilibrium, when MB,KB,and KR all decrease their prices.
In both segmented and non-segmented case, the optimal price for KB and KR are both $1.03, the optimal price for MB is $0.91.
When our competitor lowers its price, we should also lower our price relatively otherwise consumers may willing to buy competitor's product.

No segmentation situation

KB:(intercept)	KR:(intercept)	MB:(intercept)	price
4.253157	4.362403	4.204396	-3.737931

Consumer preference: KR>KB>MB.
When we launch KB, our competitor's product MB becomes the third choice for customers.

Segmentation situation

	segment	intercept.KB	intercept.KR	intercept.MB	price.coef
1	1	4.149431	3.947833	3.948372	-3.774973
2	2	4.085952	4.316191	4.577172	-3.798219
3	3	3.864525	4.347728	4.017122	-3.600028
4	4	4.335498	4.674025	4.111057	-3.682239
5	5	5.117430	4.509340	4.544963	-4.062526

Consumer Preference:
Segment 2 prefer MB
Segment 1 and 5 prefer KB
Segment 3 and 4 prefer KR

When we only have product KR, MB is more preferred by customers in segment 1, 2, and 5 compared to KR. Now we added the product KB that is more popular than MB, we will obtain segment 1, 5 from our competitor's market further.
Moreover, competitor lowers more price ($1.43-$0.91=$0.52) than we do ($1.16-$1.03=$0.13) to attract customers, which indicated that our products are more competitive on other characteristics (except for price).

In sum, launching product KB is beneficial for us both under non-segmentation and segmentation scenarios.