Simulation on 2D Ising model

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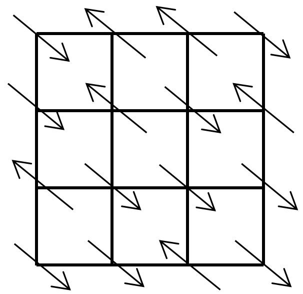
Motivation

What are the statistical mechanical properties of 2D ising model ?

Order parameter: *Magenetization,which is the average value of the spin.*



$$M = \frac{1}{N}\sum_{i=1}^{N}\sigma_i$$

Model

2D Ising model:

- Interactions: $H(\sigma) = -\sum_{\langle i,j\rangle} J_{ij}\sigma_i\sigma_j - \mu\sum_j h_j\sigma_j$
- Simplifications: $H(\sigma) = -\sum_{\langle i,j\rangle} J\sigma_i\sigma_j$
- Boundary Condition:***Periodic boundary Condition.***
- Initialization:***A random initial state.***
- Lattice coordination number:5;8;16;32;64;100

Algorithm

The **Metropolis Method:**

1.Select a spin at random,and calculate the contribution to the energy involving this spin.

2.Flip the value of the spin,and calculate the new contribution.

3.If the new energy is less,keep the flipped value.

4.If the new energy is more,accept the flip with probability:

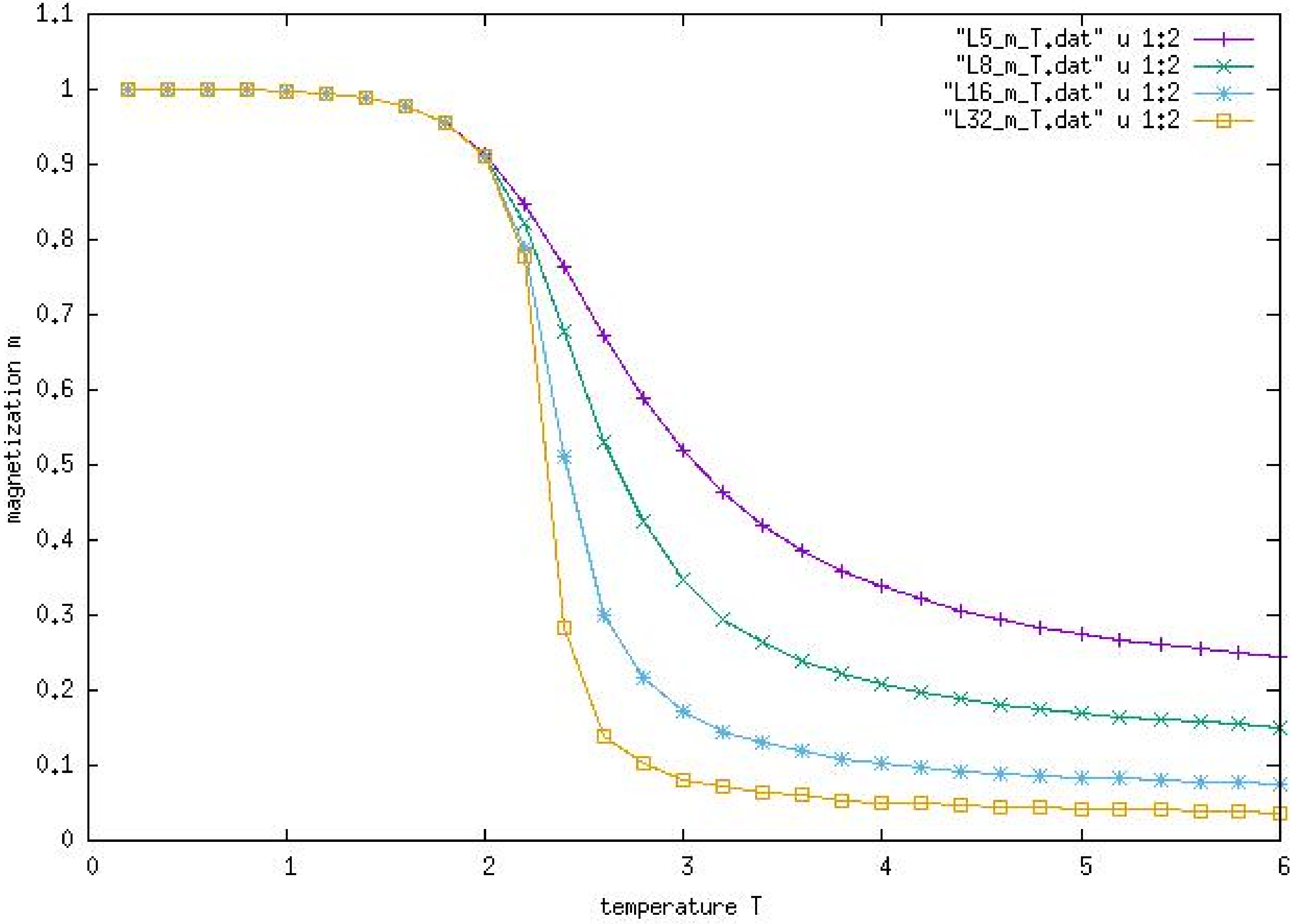
$$acc(o \rightarrow n) = \exp\{-\beta[H(n) - H(o)]\}$$

5.Repeat.

✓ *The change in energy only depends on the value of the spin and its nearest graph neighbors.*

Results

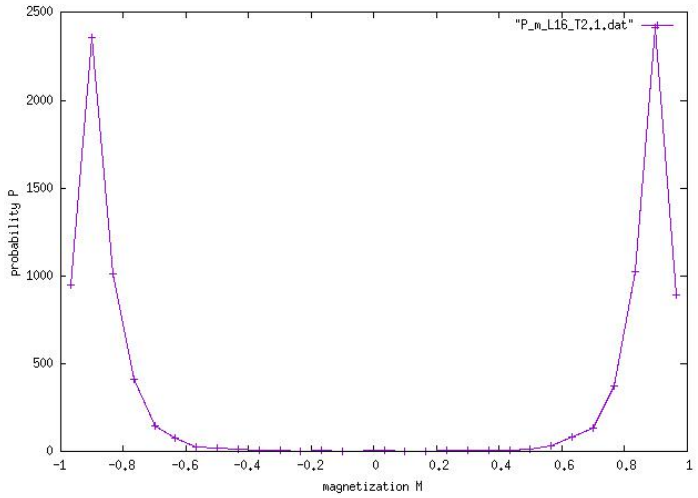
m(T)

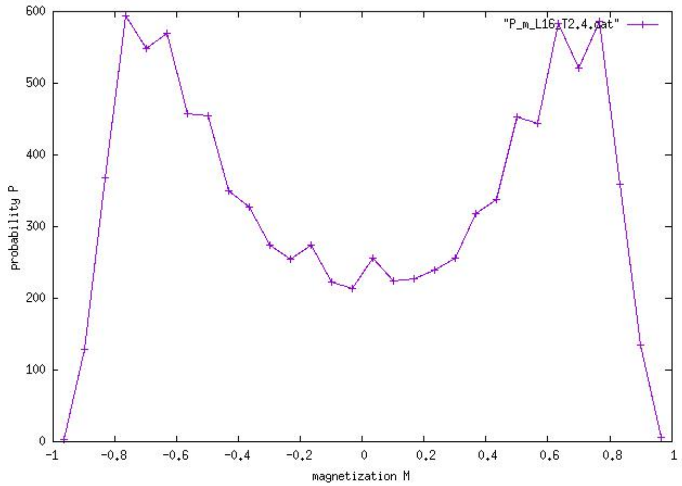


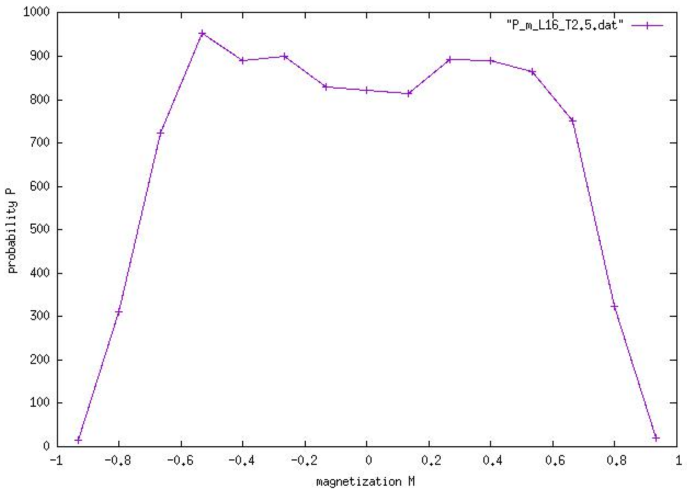
"L5_m_T.dat" u 1:2
"L8_m_T.dat" u 1:2
"L16_m_T.dat" u 1:2
"L32_m_T.dat" u 1:2
magnetization m
temperature T
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6

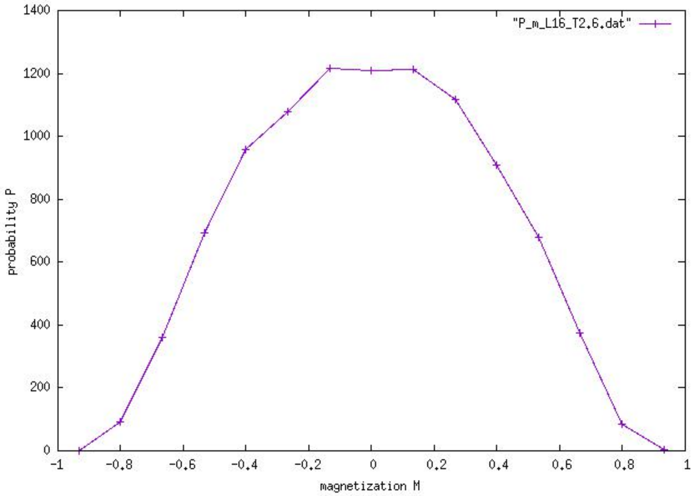
Results

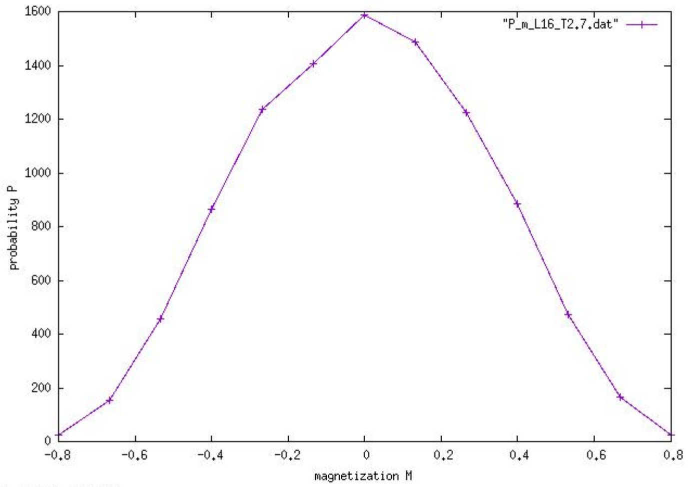
Distributions of M vary with T:

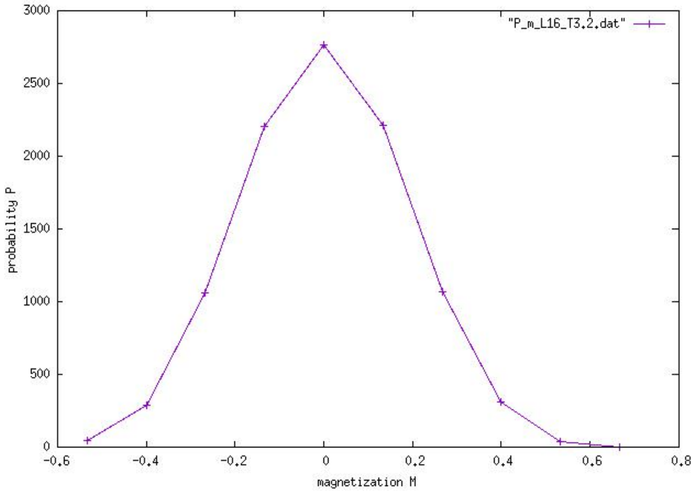








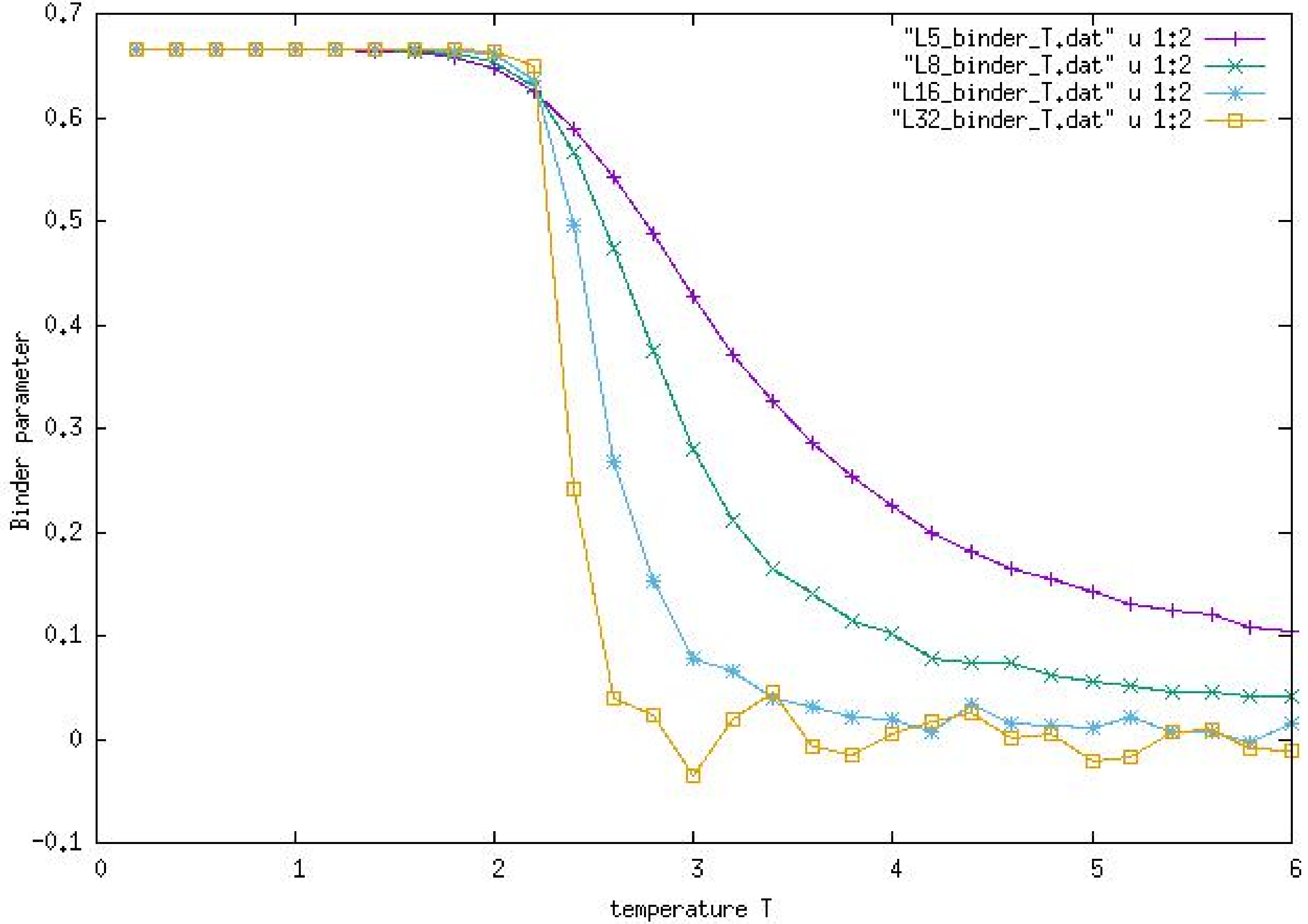




Results

Binder parameter

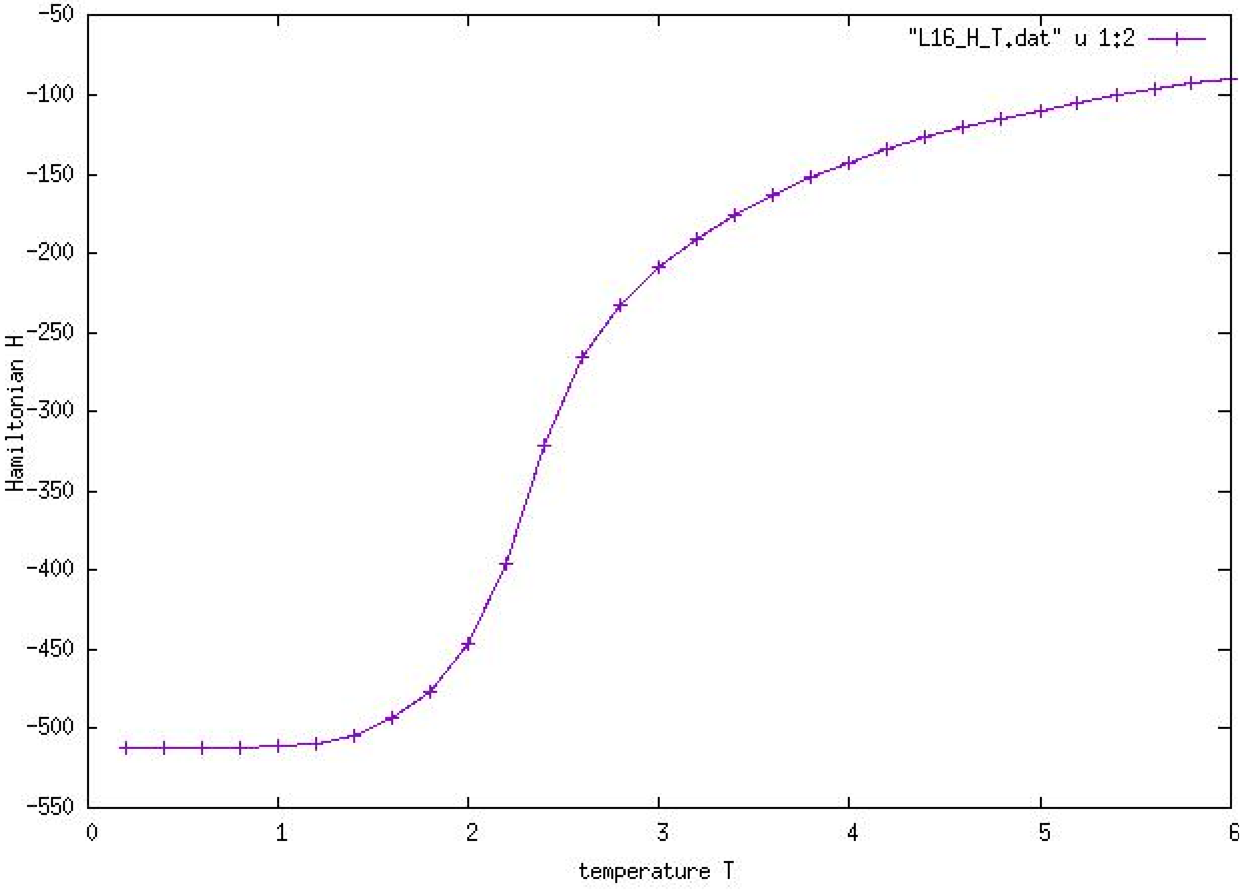
$$U_L = 1 - \frac{\left\langle S^4 \right\rangle_L}{3\left\langle S^2 \right\rangle_L^2}$$



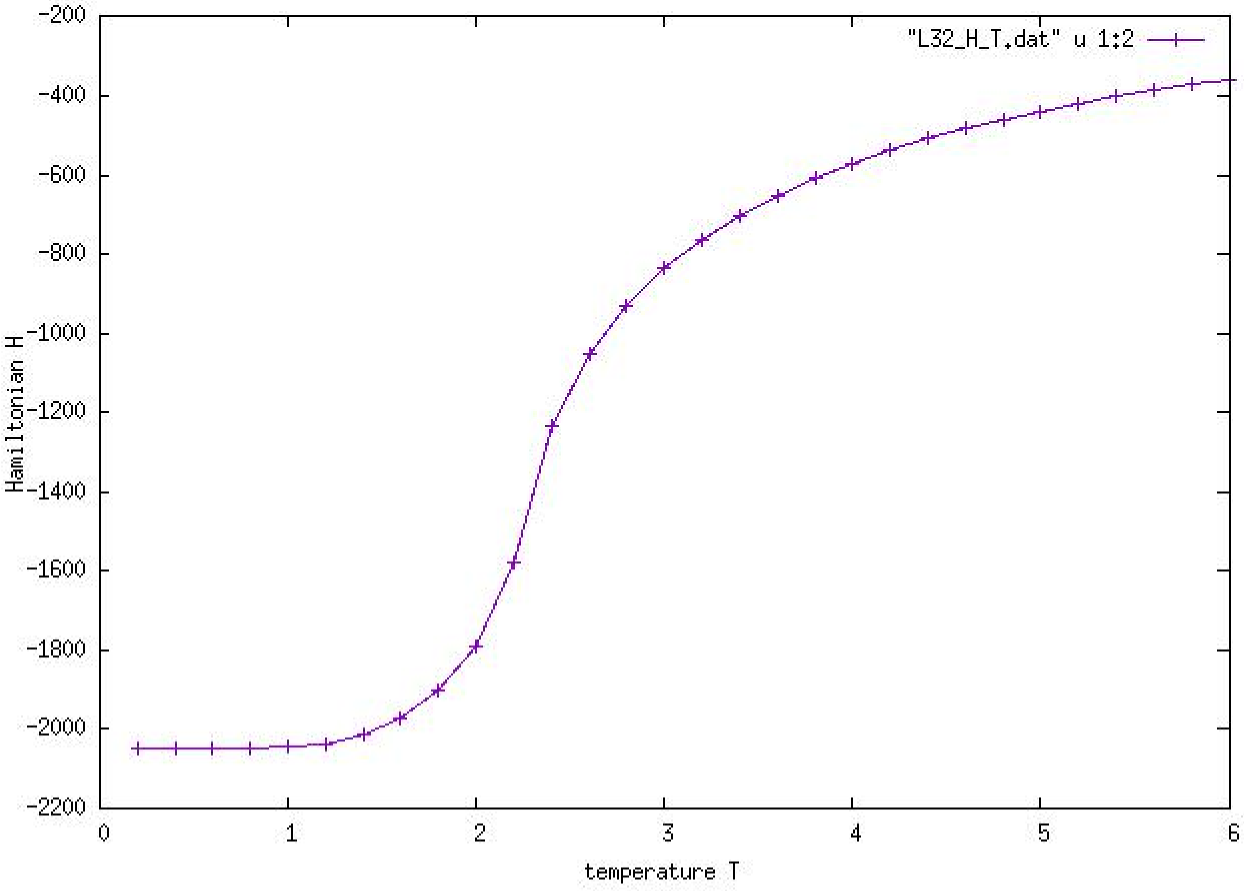
Results

Average energy:

L=16



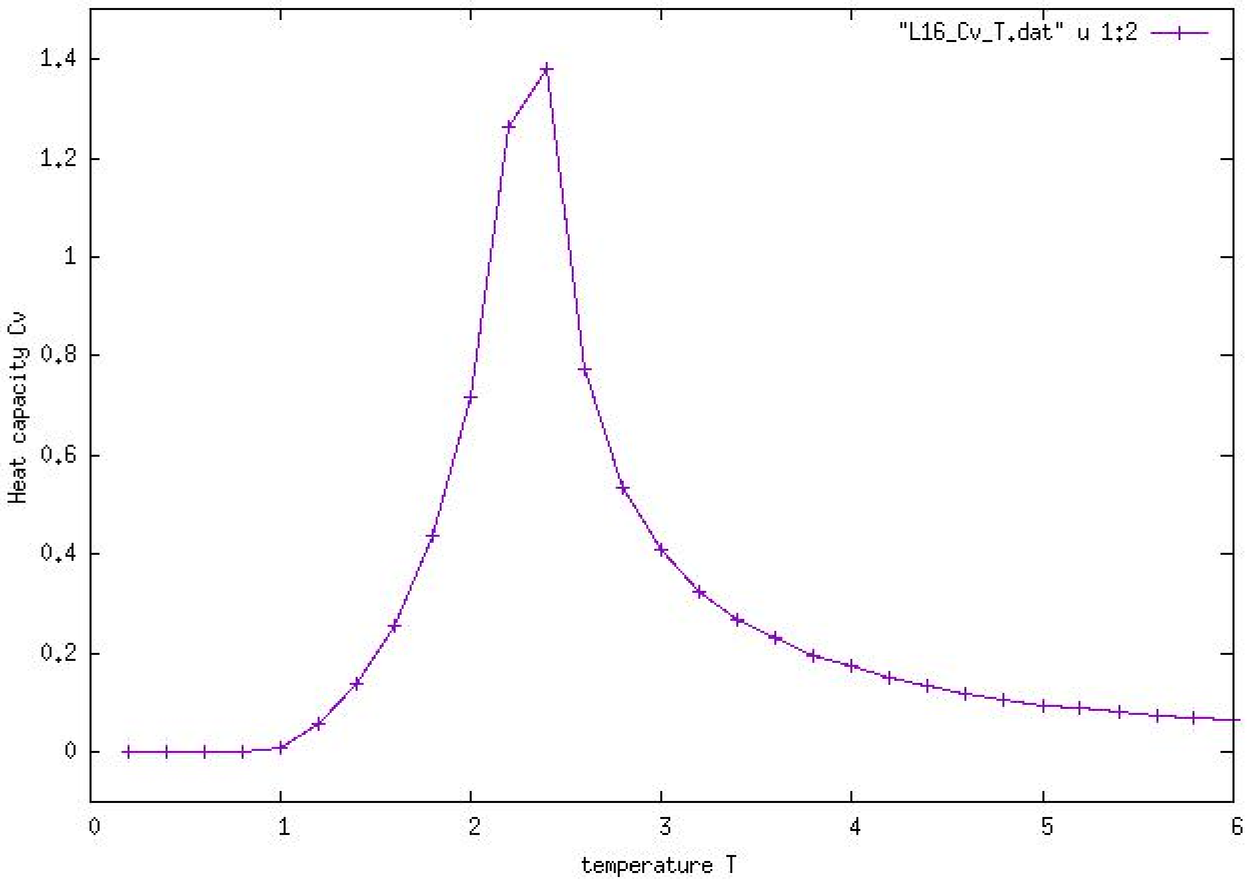
L=32



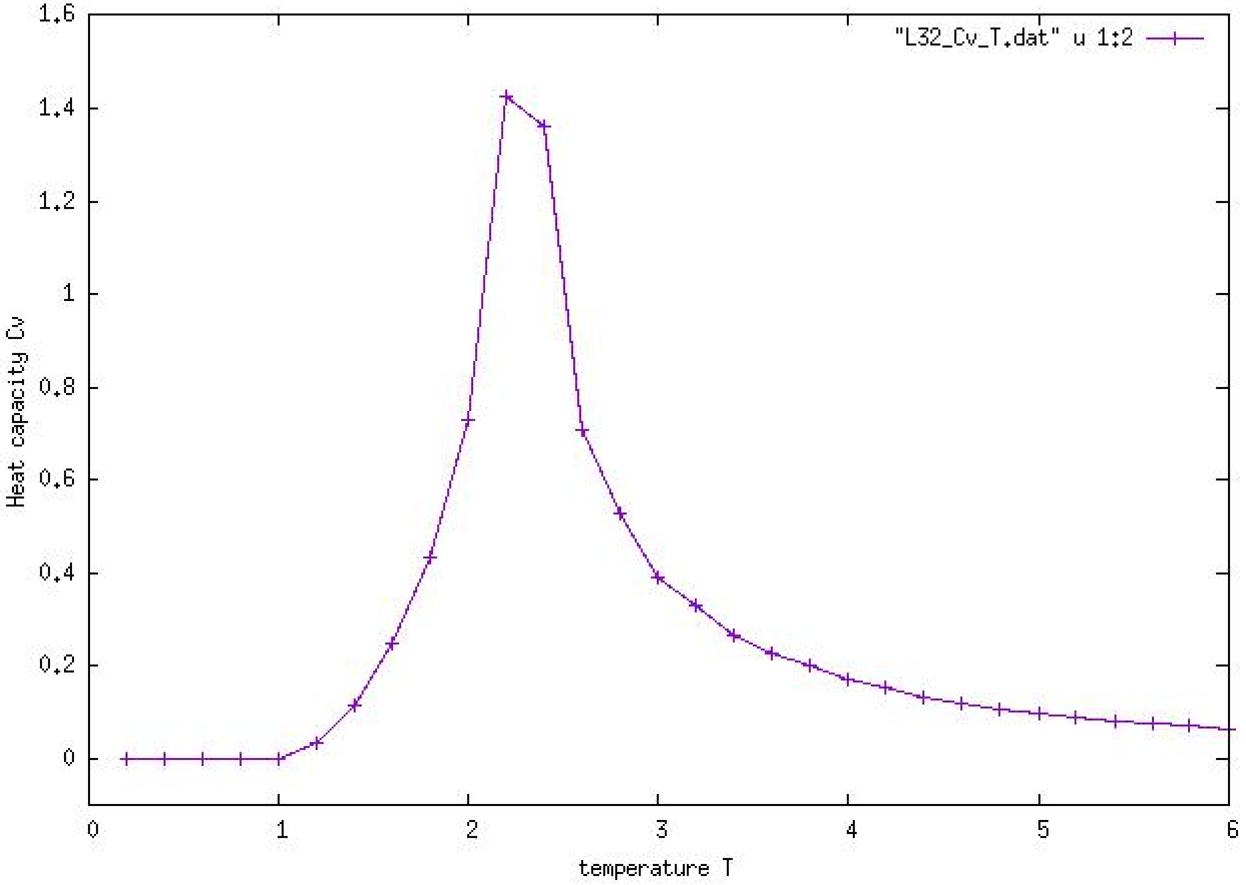
Results

the specific heat: $c = \frac{\beta^2}{N}(\langle E^2 \rangle - \langle E \rangle^2)$

L=16

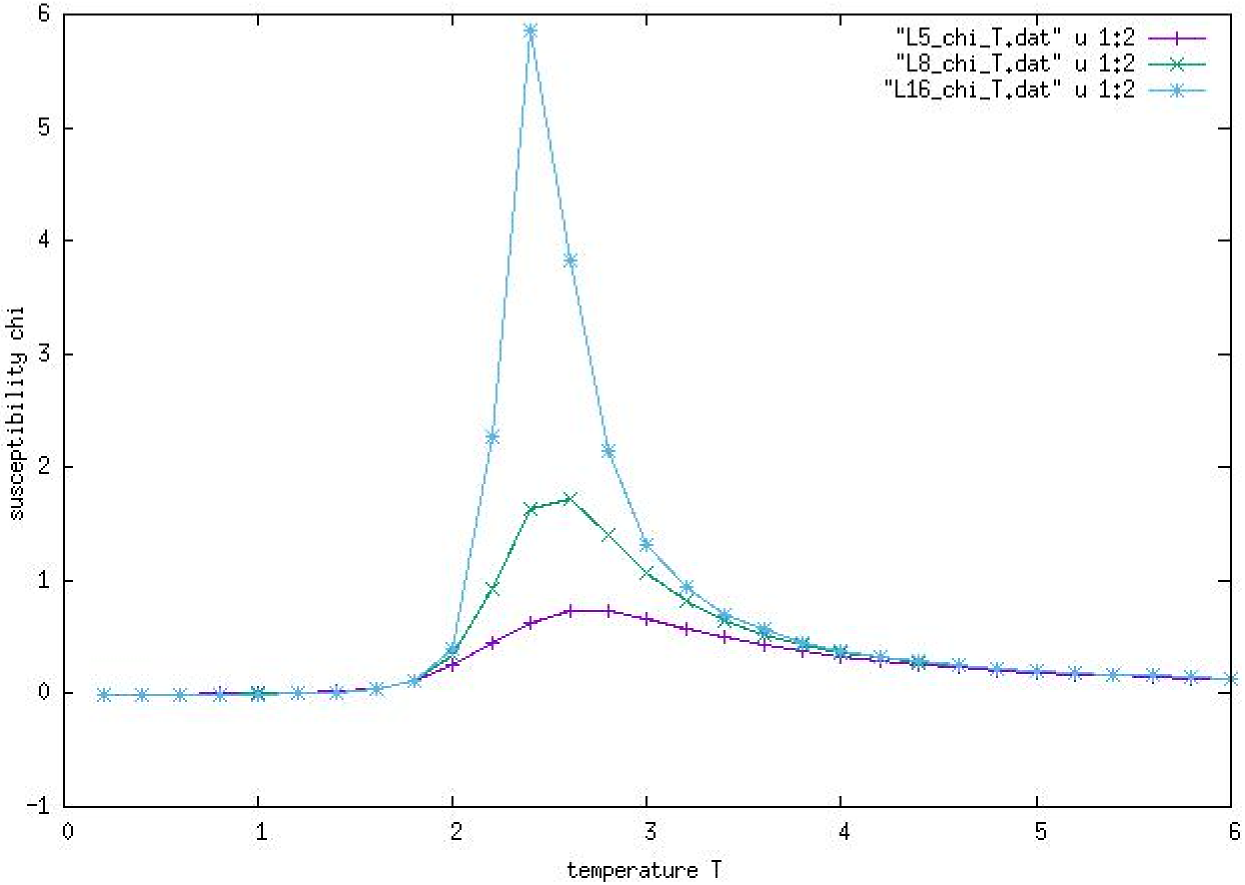


L=32



Results

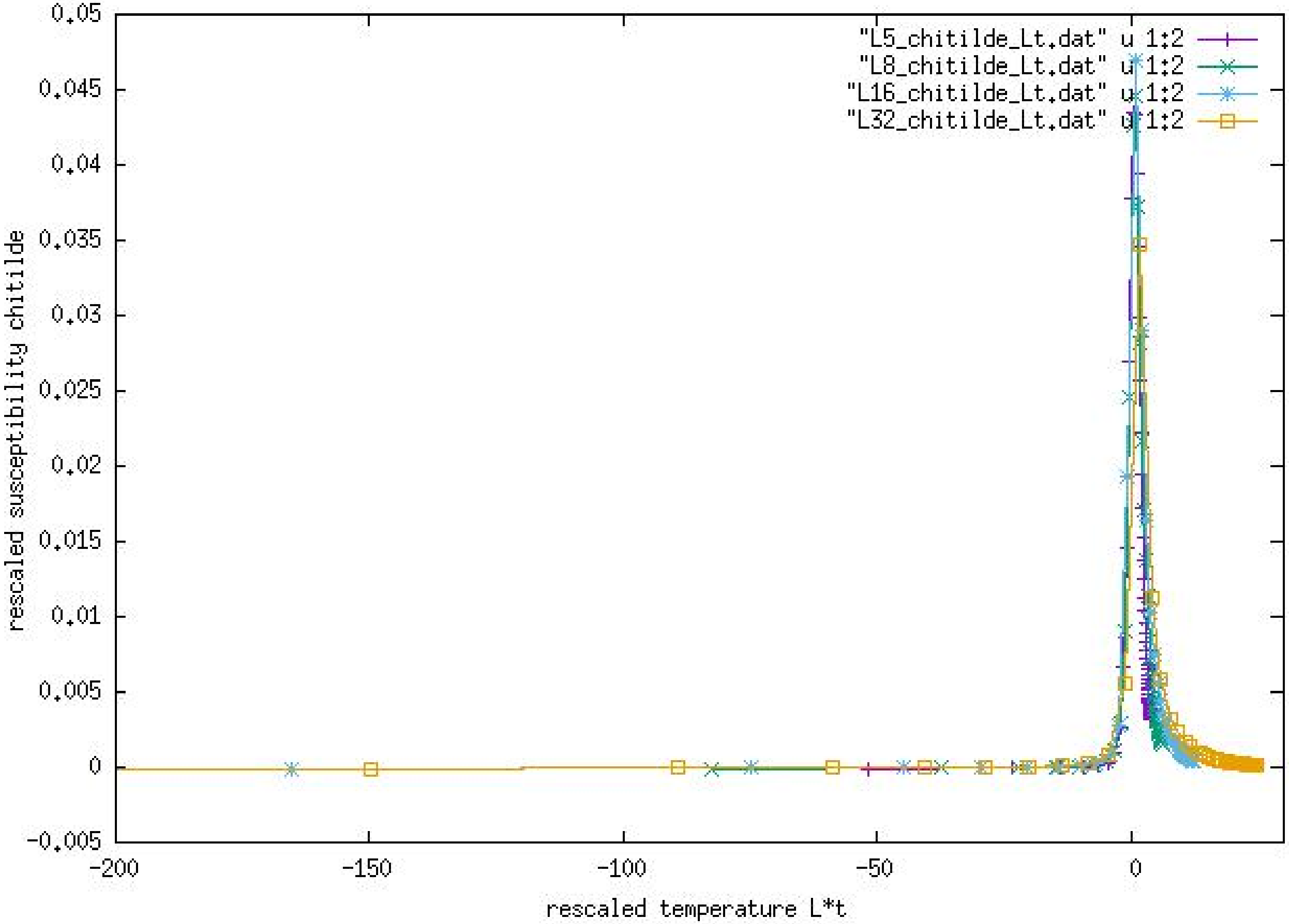
$$\chi = \beta N \left(\left\langle m^2 \right\rangle - \left\langle m \right\rangle^2 \right)$$



Results

Finite-size scaling

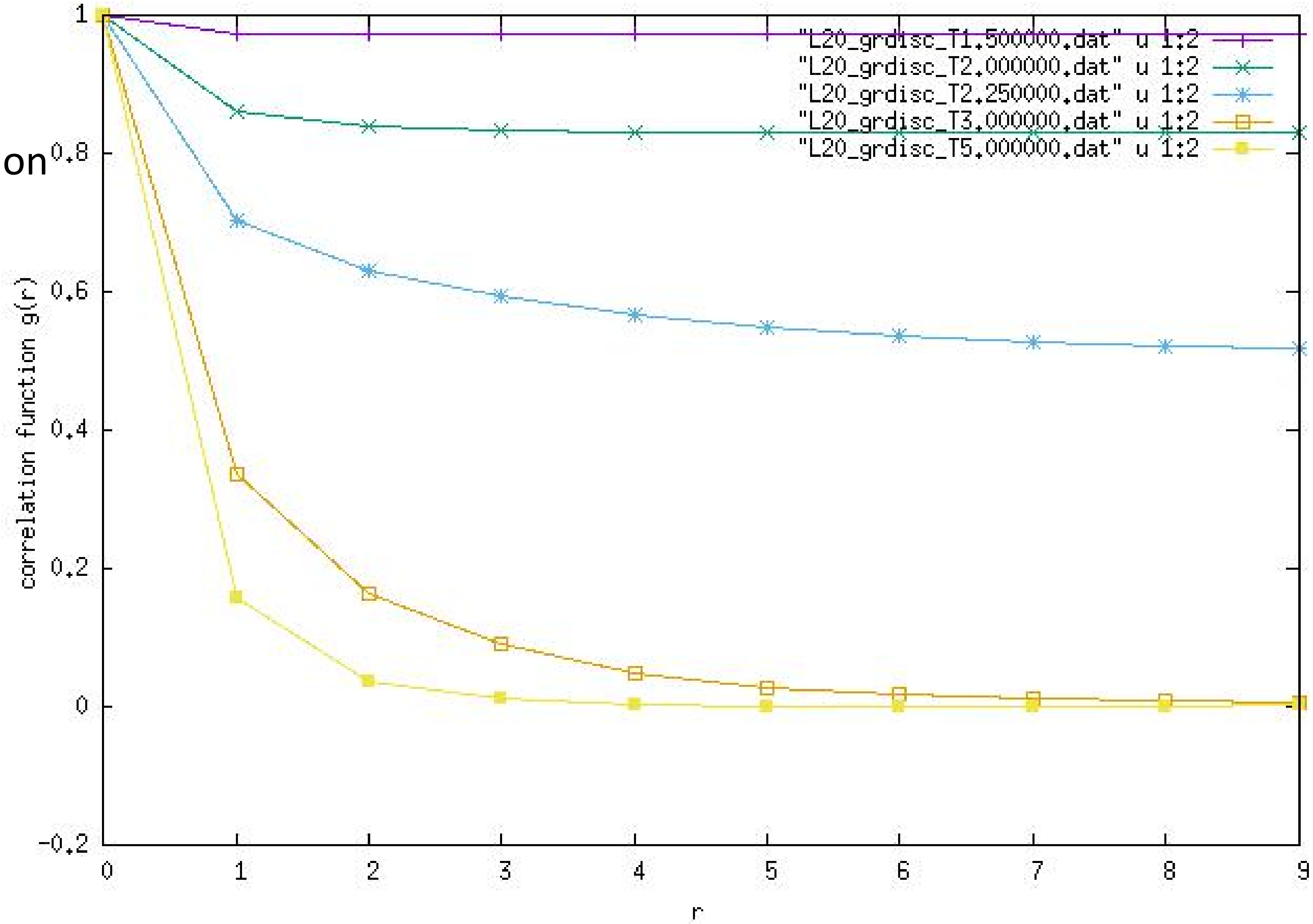
$$\tilde{\chi}\left(L^{1/\nu} \frac{T - T_C}{T} \right)$$



Results

disconnected correlation function g(r)

L=20



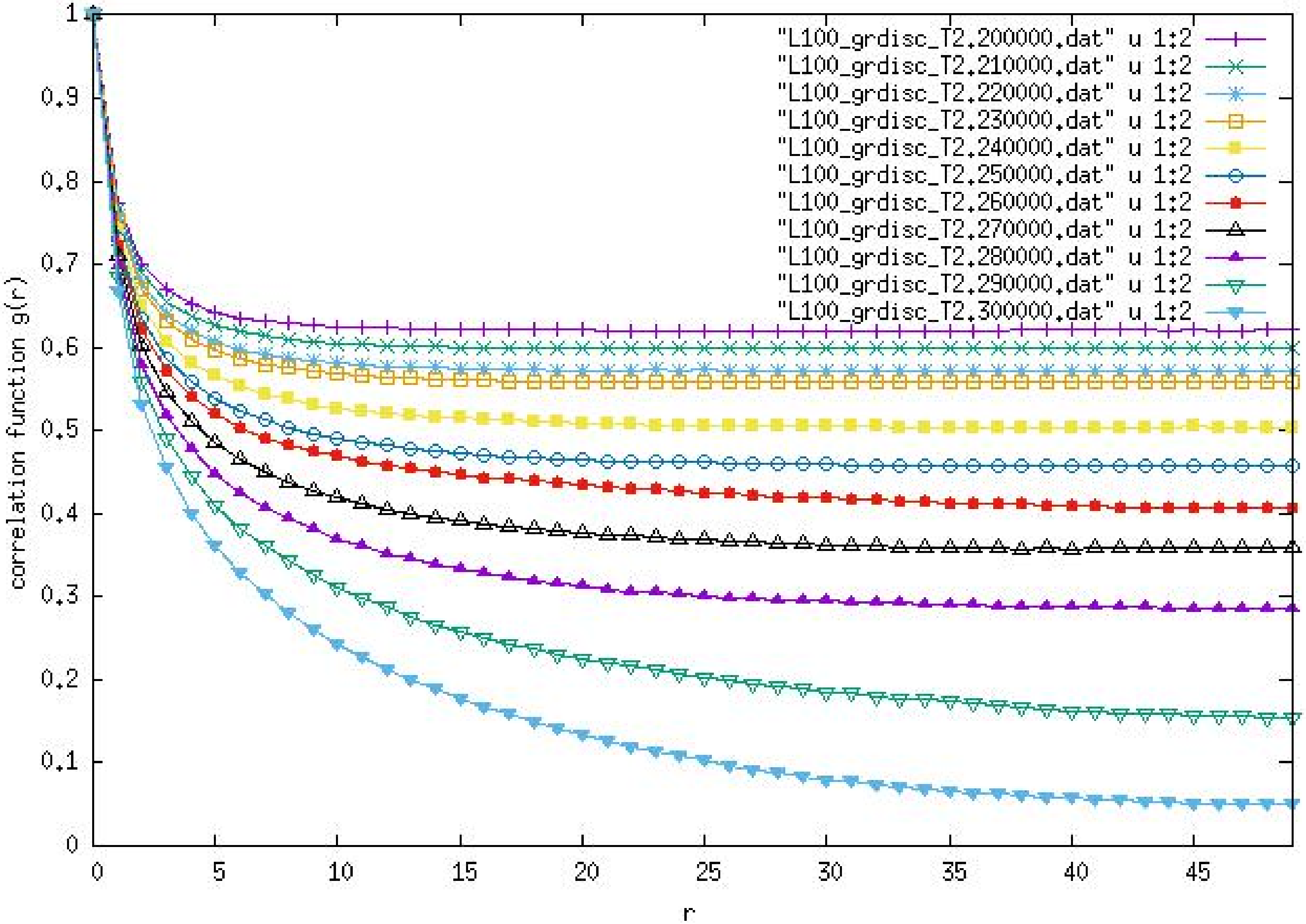
Averaging the 2D radii (i,j) into distance bins around the nearest integer to distance sqrt(i*i+j*j).

Normalizing g(r) by the actual number of particles.

Results

disconnected
correlation function
g(r)

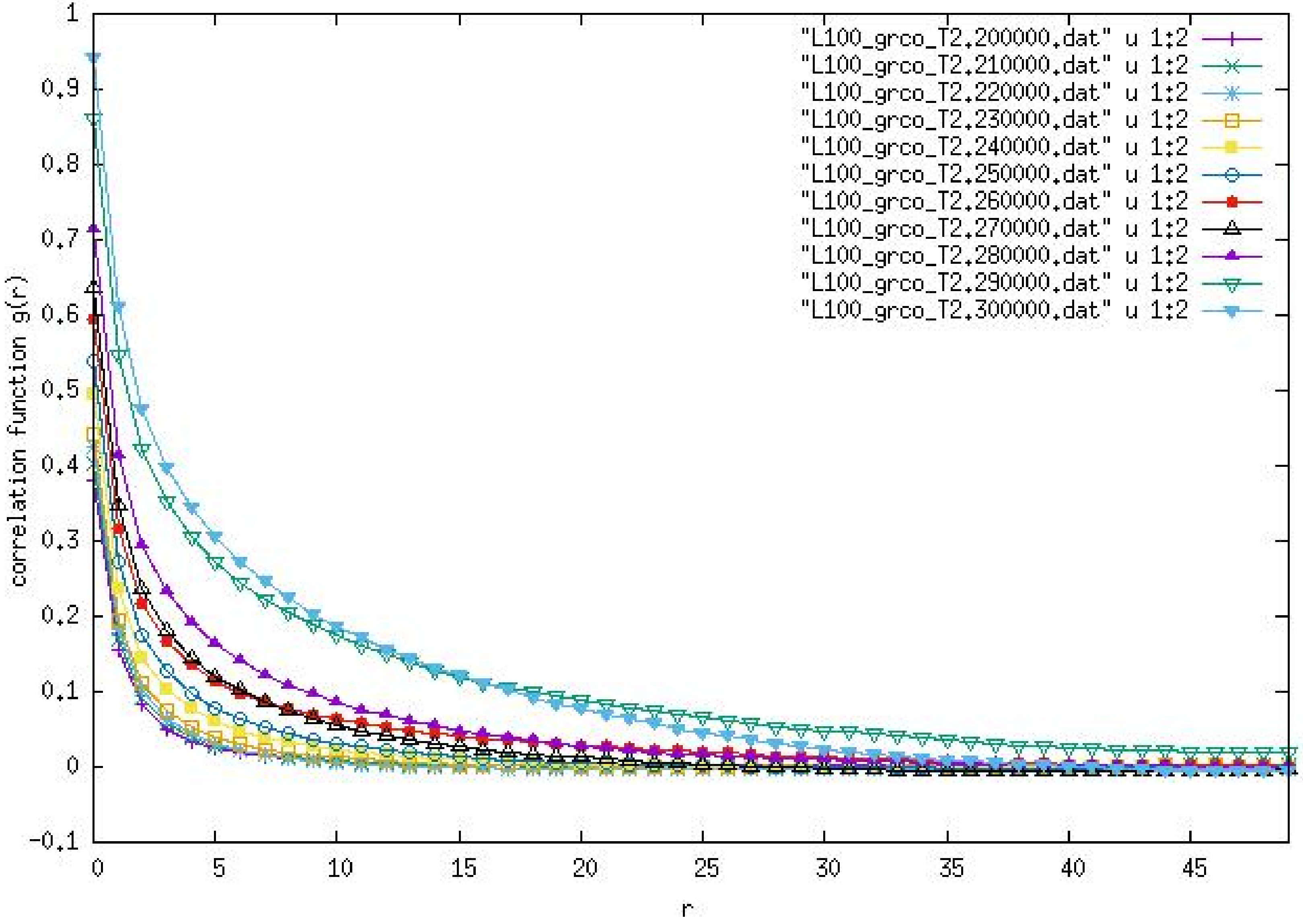
L=100



Results

connected
correlation function
g(r)

L=100

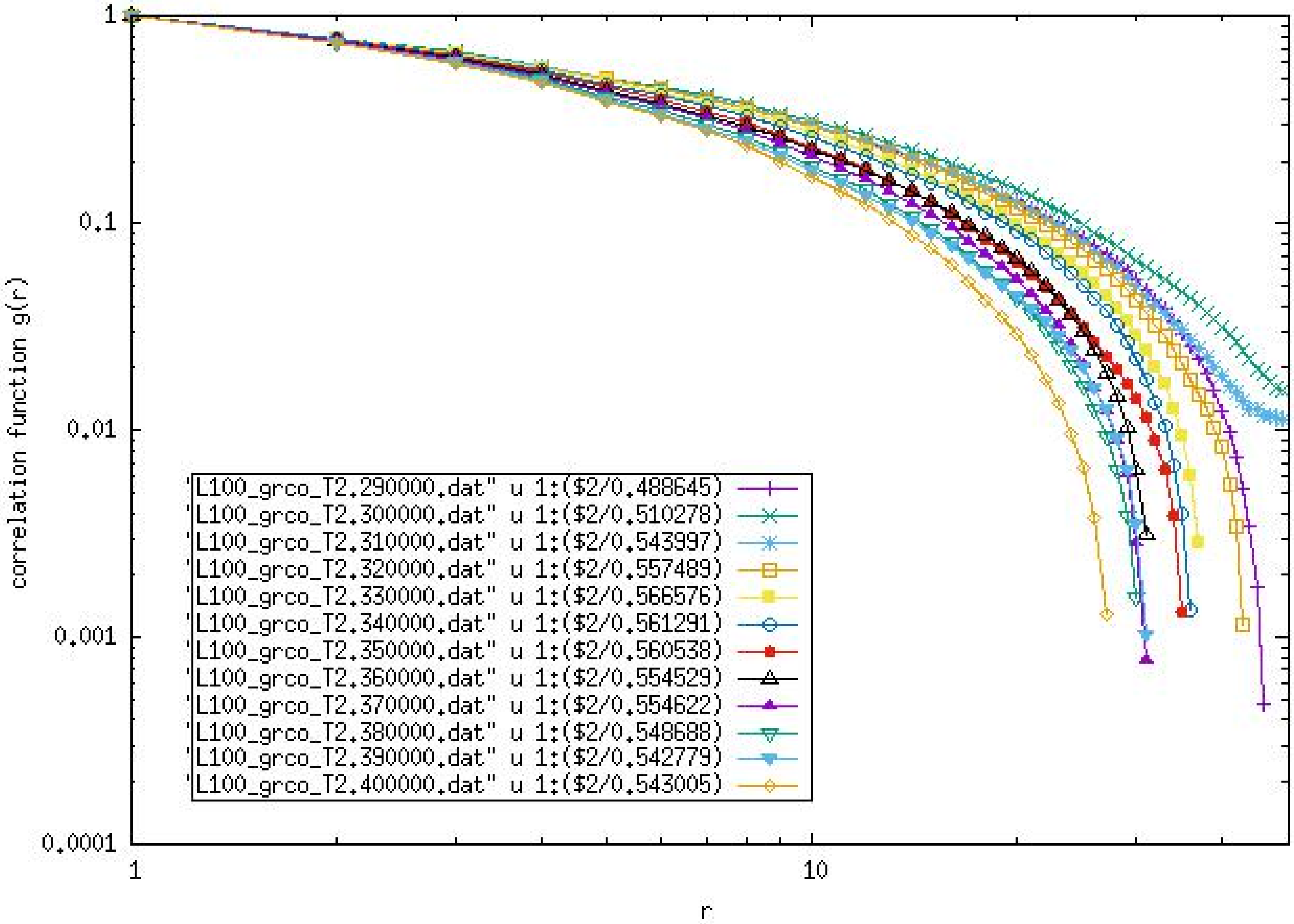


Results

Correlation function g(r)

L=100

We can make sure that the critical point is about T=2.30 through this plot.



Results

T=2.30

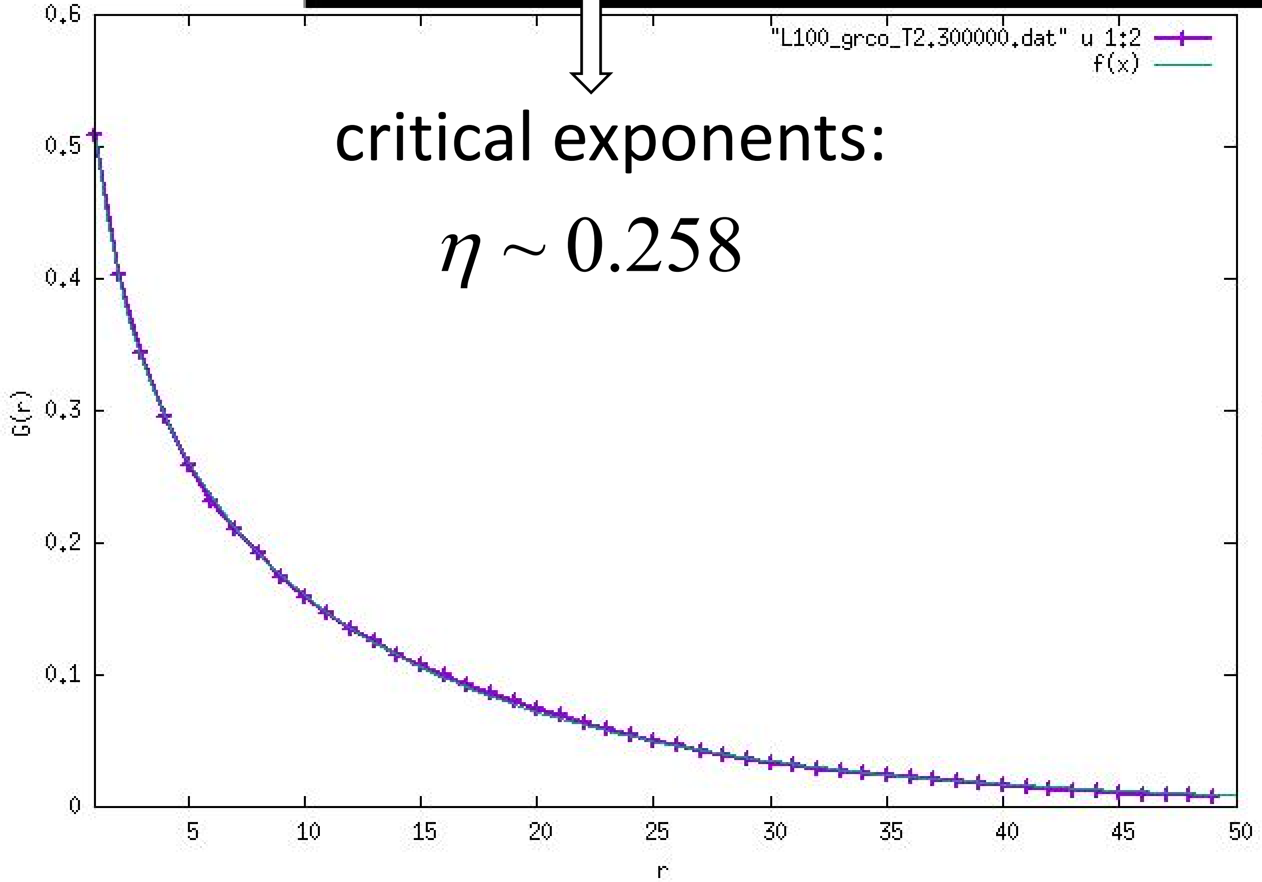
```
Final set of parameters            Asymptotic Standard Error
=======================            ==========================
a               = 0.258027         +/- 0.003514     (1.362%)
b               = 16.0057          +/- 0.1476       (0.9221%)
c               = 0.544159         +/- 0.001488     (0.2735%)
```

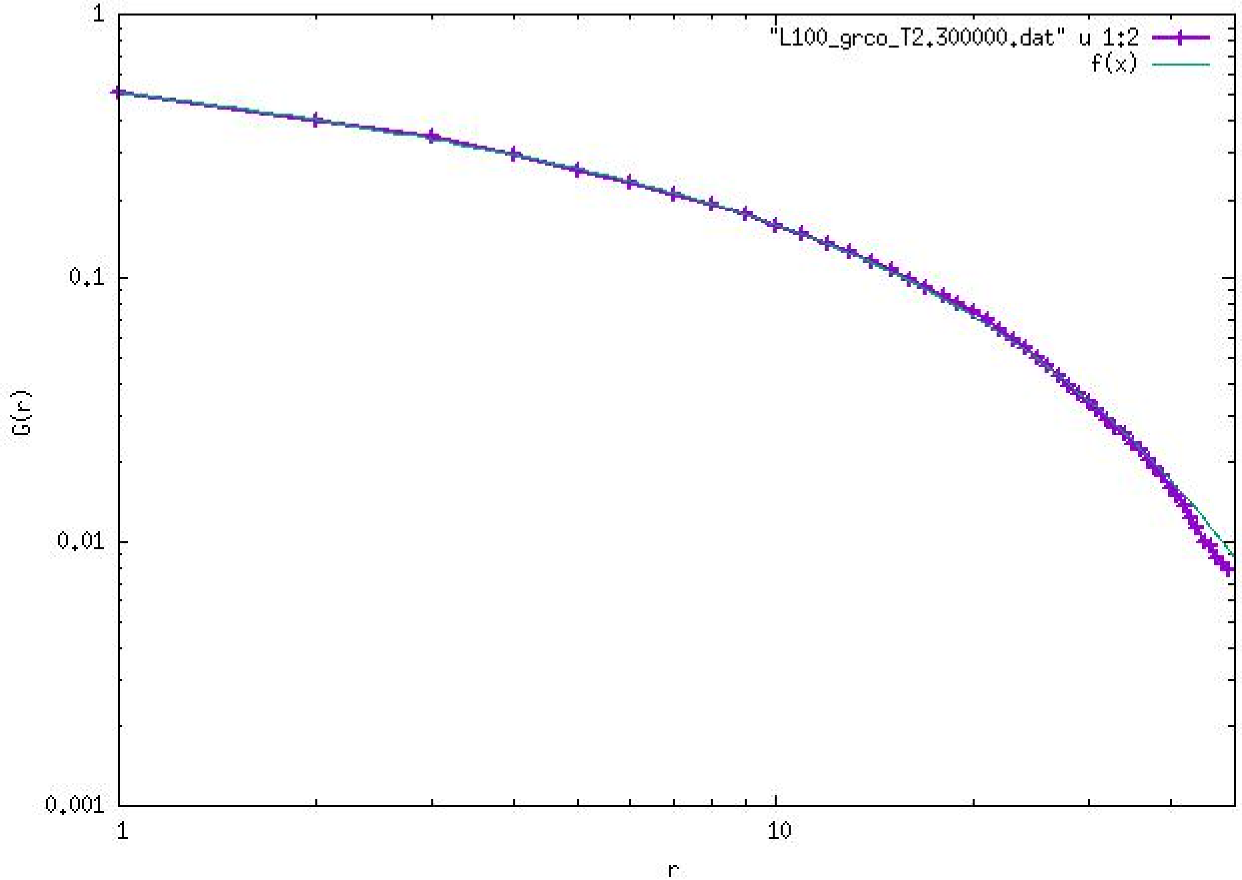
fitting by

$$g(r) \sim r^{-(d-2+\eta)} \cdot \exp(-r / \xi(T))$$

critical exponents:

$$\eta \sim 0.258$$



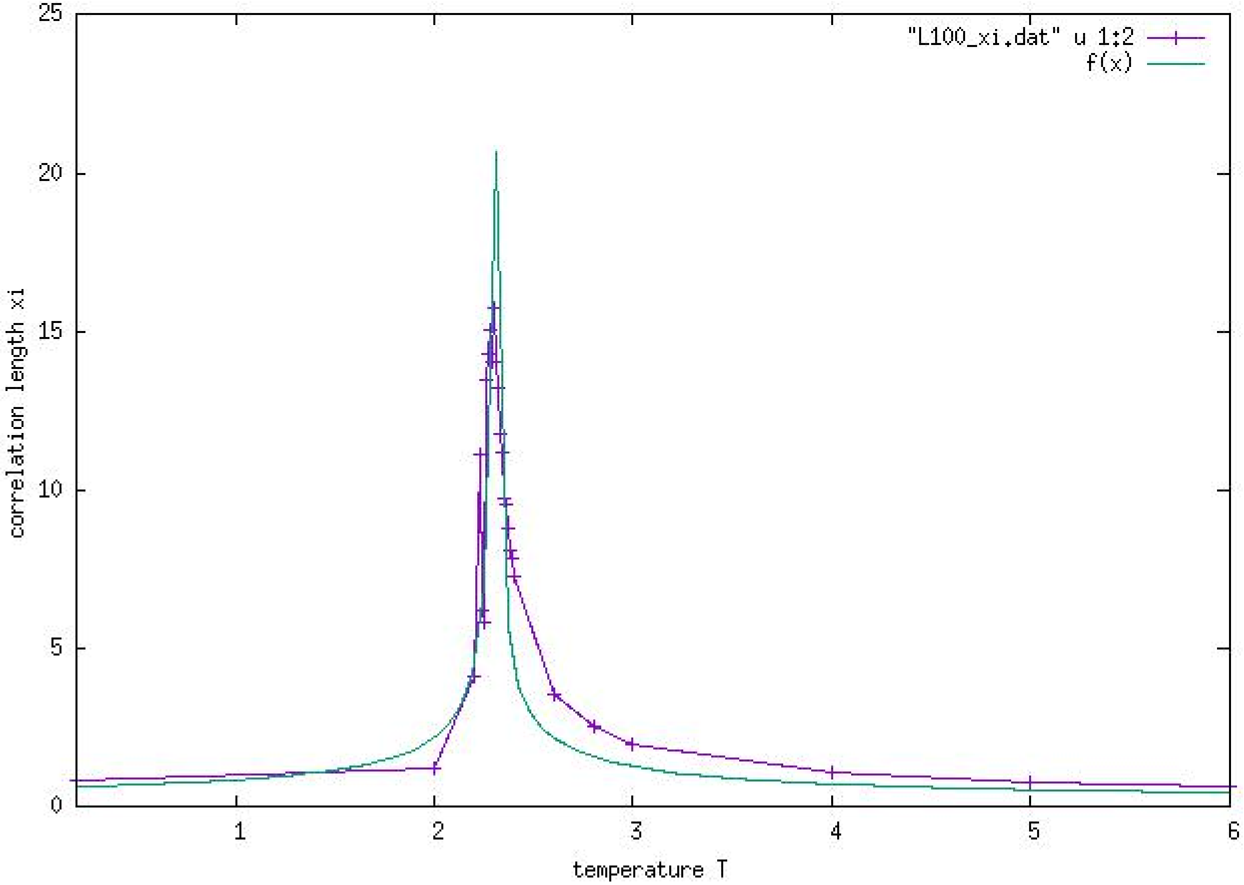


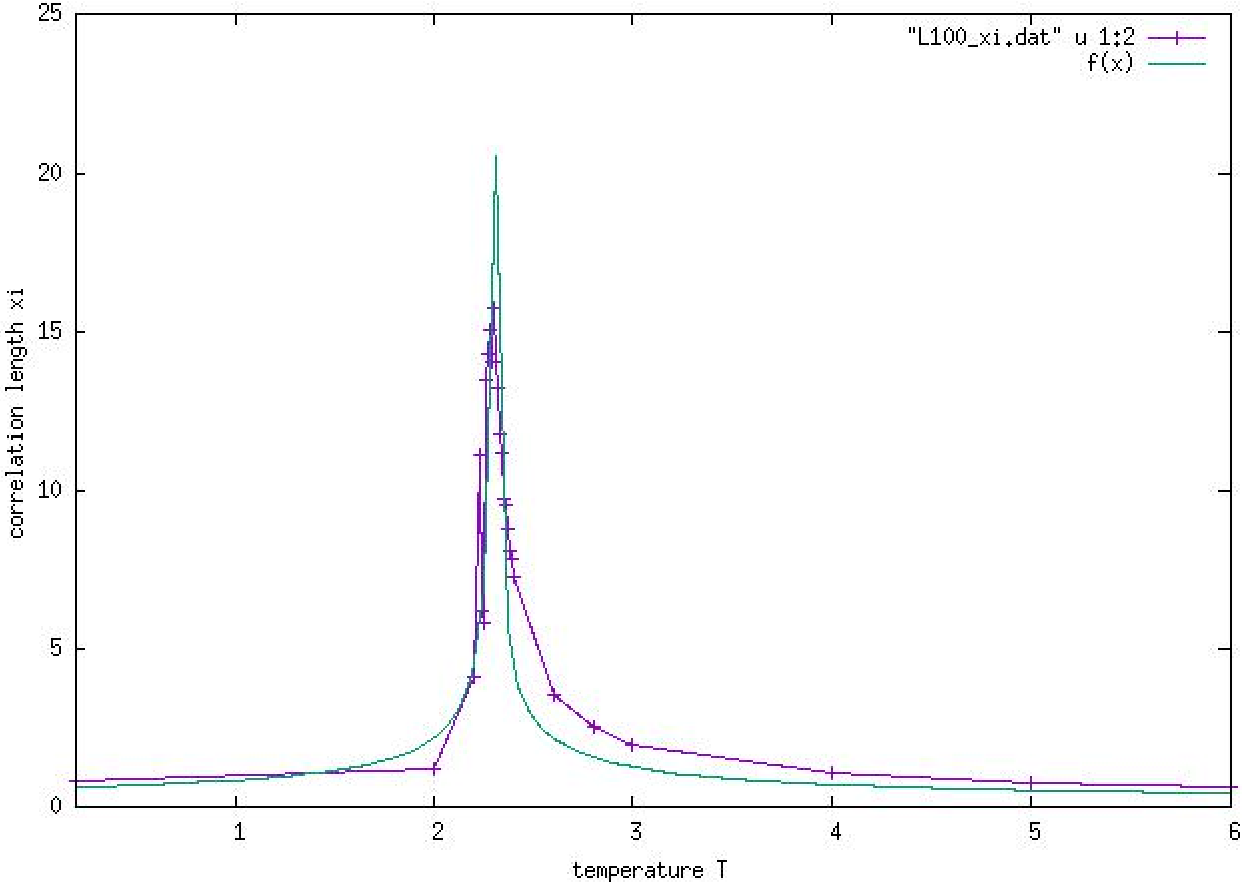
Results

$\xi(T)$

```
Final set of parameters
===========================
a                = 0.644238
```

```
Final set of parameters
===========================
a                = 0.643256
```





Results

Autocorrealation function: $\chi(t)=\int dt'\left[m(t')-\langle m\rangle\right]\left[m(t'+t)-\langle m\rangle\right]=\int dt'\left[m(t')m(t'+t)-\langle m\rangle^{2}\right]$

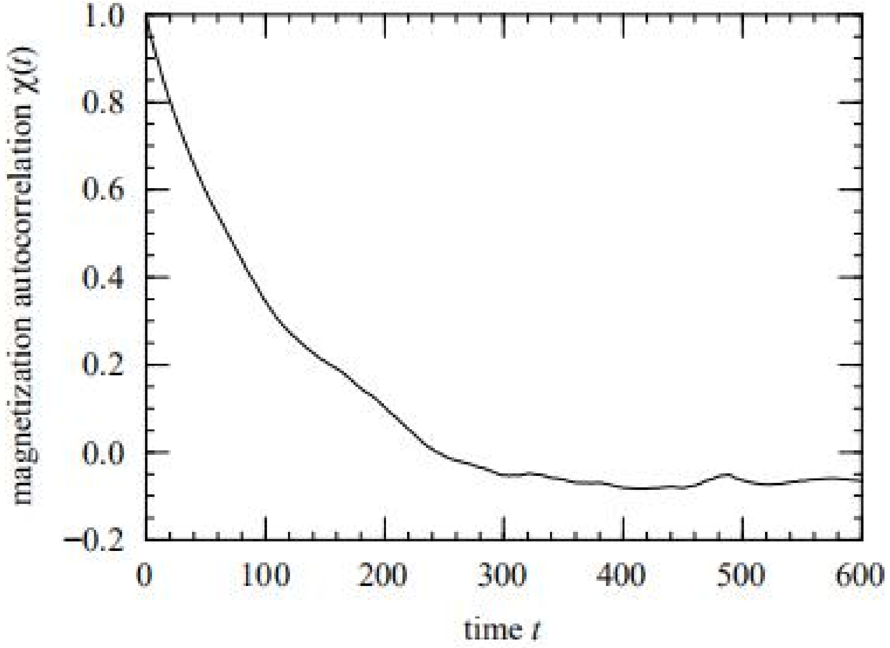
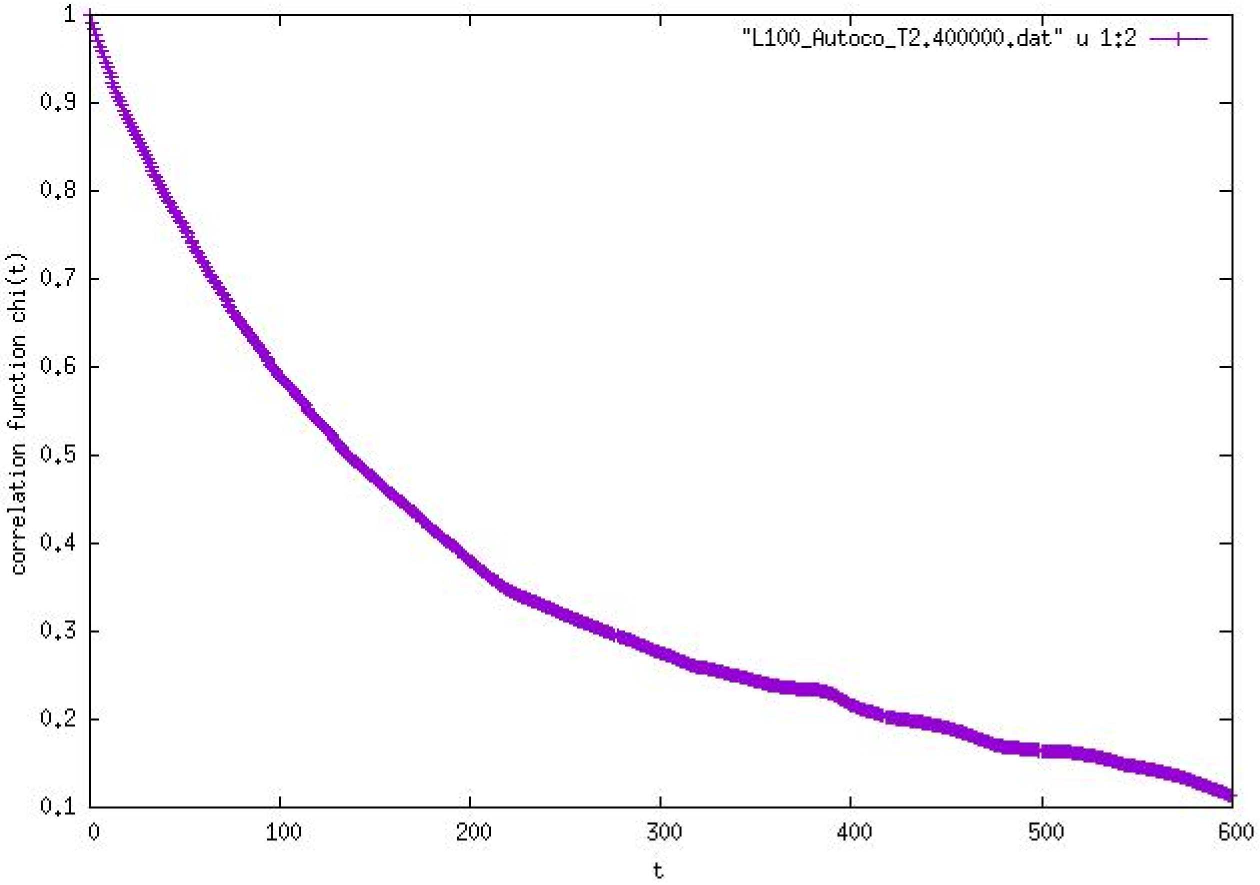


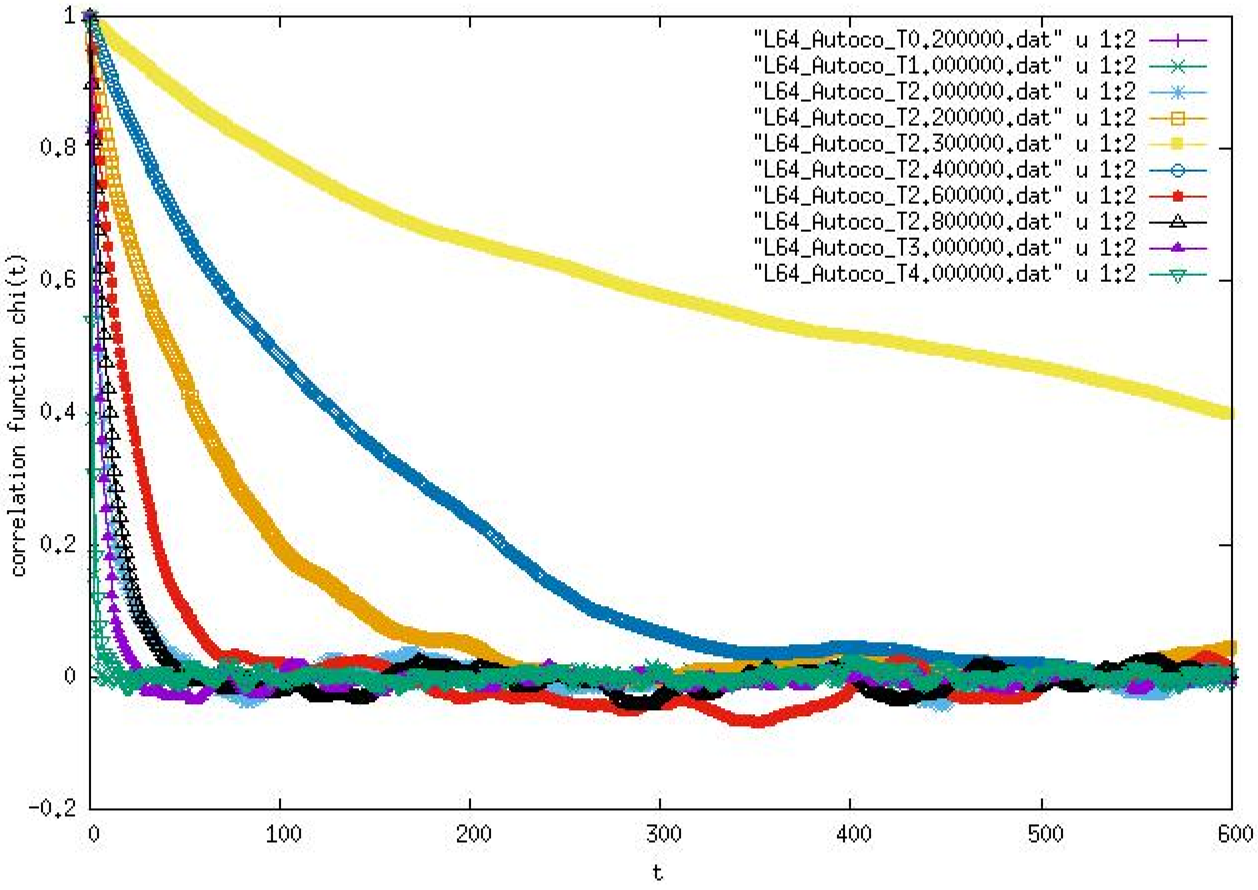
FIGURE 3.5 The magnetization autocorrelation function $\chi(t)$ for a two-dimensional Ising model at temperature $T = 2.4$ on a square lattice of 100×100 sites with $J = 1$ simulated using the Metropolis algorithm of Section 3.1. Time is measured in Monte Carlo steps per site.

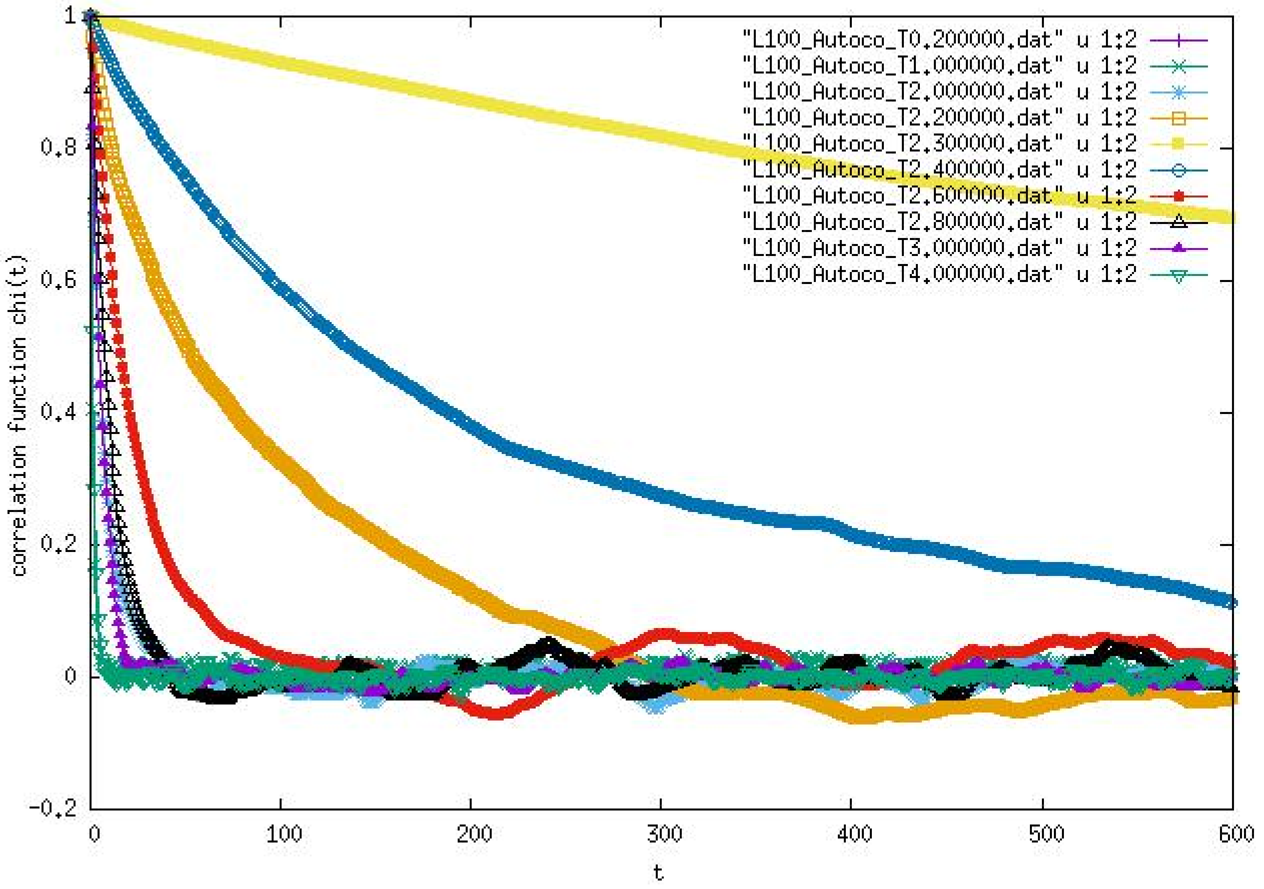


Ref:M. E. J. Newman and G. T. Barkema. Monte Carlo Methods in Statistical Physics[M]. Oxford University Press, 1999

Results

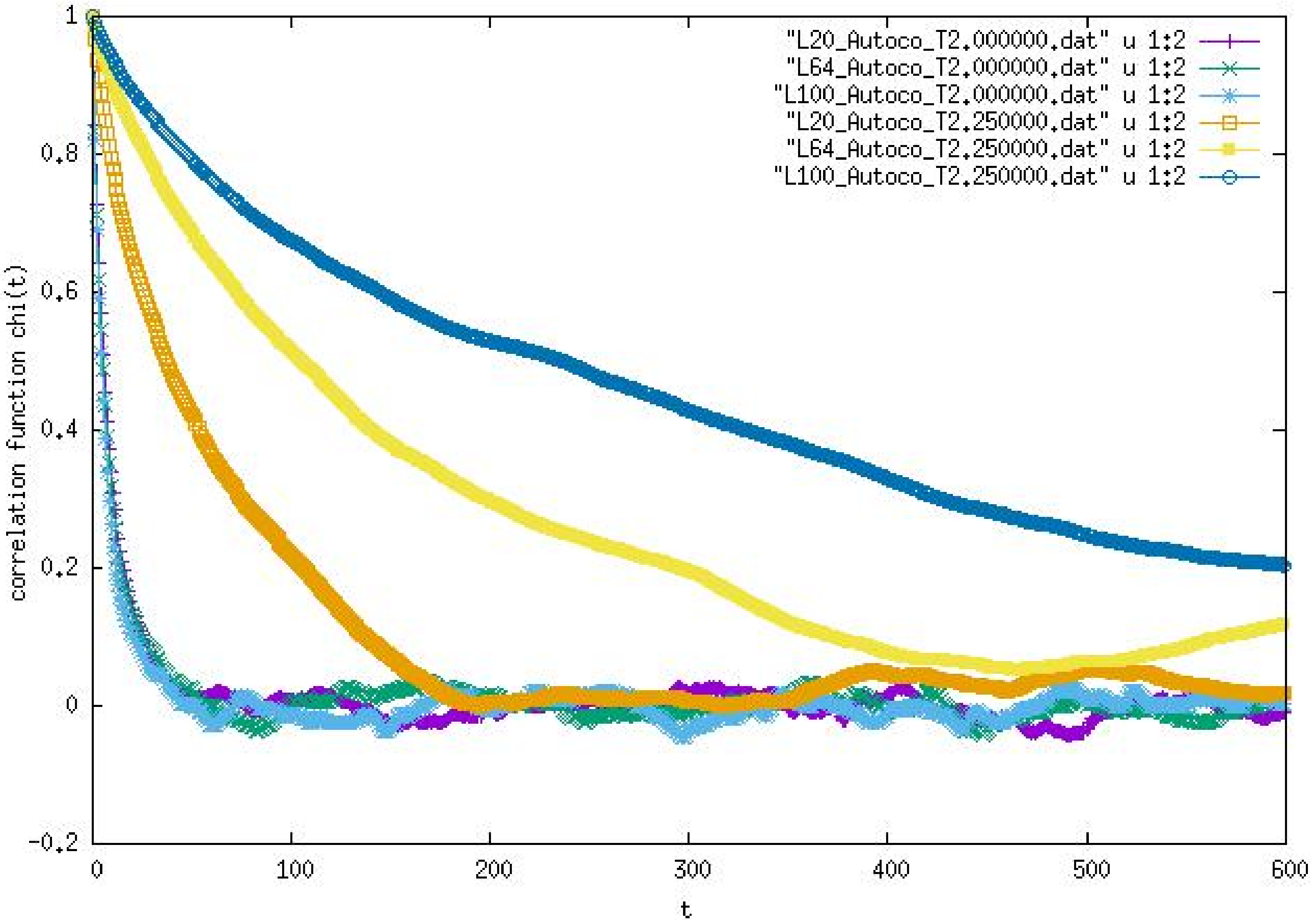
$\chi(t)$ L=64 L=100





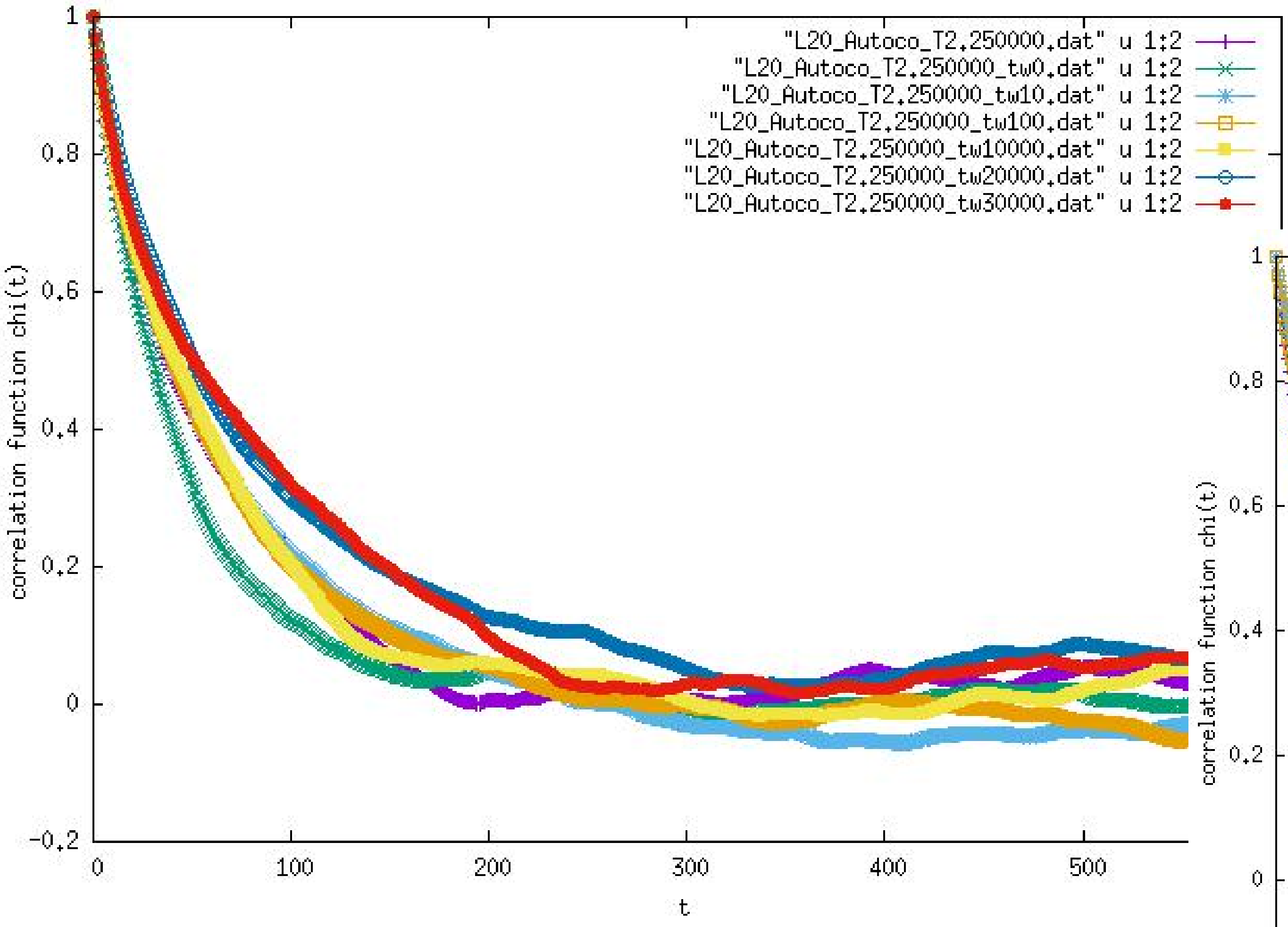
Results

$\chi(t)$



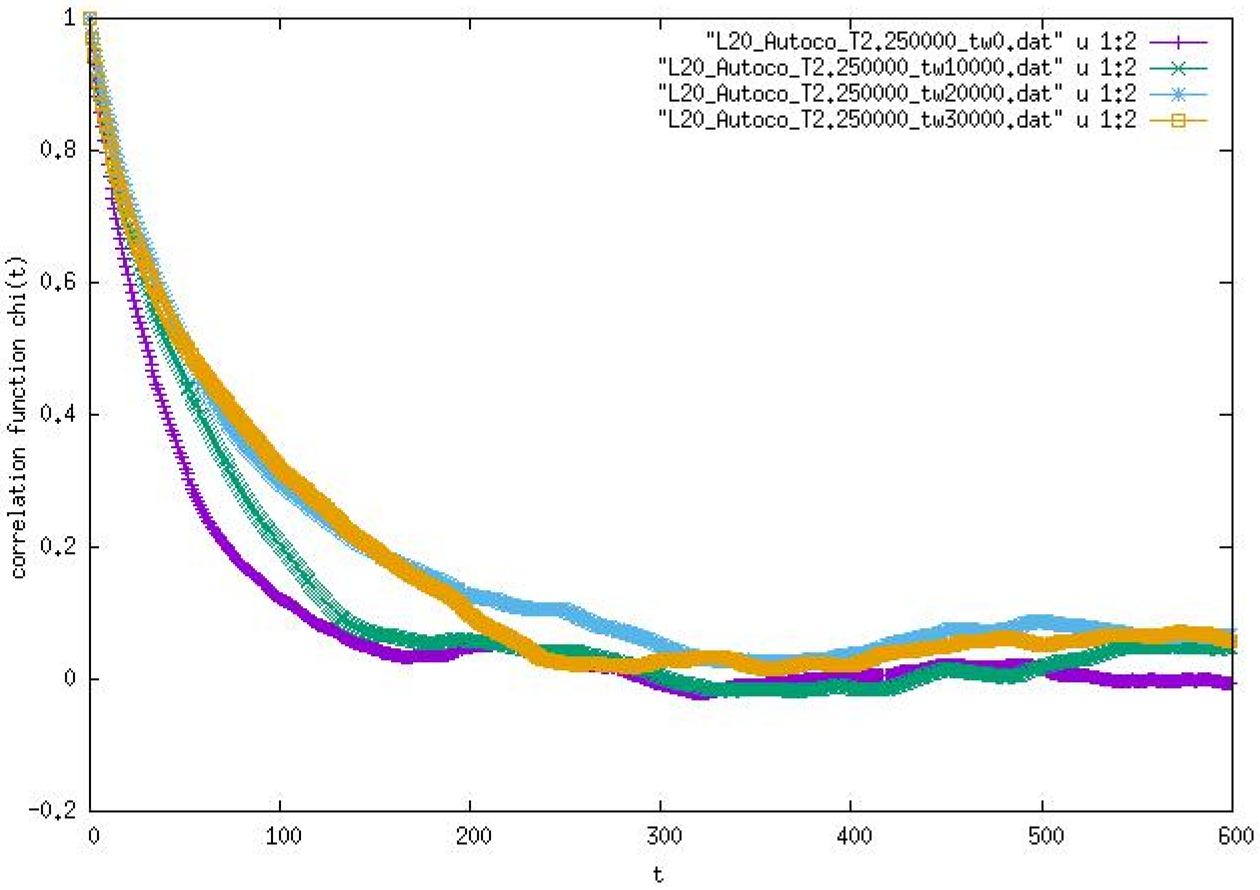
Near the critical point

Results



$\chi(t)$

aging



Thank you!

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