In [140…
```
import pandas as pd
import numpy as np
from google.colab import drive

import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

import os
import yfinance as yf
```

In [141…
```
drive.mount('/content/drive')
df_call = pd.read_csv('/content/drive/My Drive/CapstoneProject/IVSurfaceWay/Data/2025-08-19spx_call_options.csv')
df_put = pd.read_csv('/content/drive/My Drive/CapstoneProject/IVSurfaceWay/Data/2025-08-19spx_put_options.csv')
```

```
Drive already mounted at /content/drive; to attempt to forcibly remount, call drive.mount("/content/drive", force_remount=True).
```

In [291…
```
PATH = '/content/drive/My Drive/CapstoneProject/IVSurfaceWay/Result'
DATE = '2025-08-19'
```

Data Cleaning

In [142…
```
def get_current_spot_price(date):
  spx = yf.Ticker("^SPX")
  hist = spx.history(start=date) # Get data for July 15, 2024

  spot_price = float(hist['Close'].iloc[0])
  return spot_price

SPOT = get_current_spot_price(DATE)
```

In [143…
```
def clean_df(df, DATE=DATE):
  temp_df = df.copy()
  temp_df["expirationDate"]= pd.to_datetime(temp_df["expirationDate"])
  date = pd.to_datetime(DATE)
  temp_df['T'] = (temp_df["expirationDate"]-date).dt.days
  temp_df.rename(columns = {"strike":"K", "lastPrice":'price', 'impliedVolatility':'IV'}, inplace = True)
  temp_df = temp_df[(temp_df['openInterest'] >= 1) | (temp_df['volume'] >= 1)]
  temp_df = temp_df[(temp_df['K'] <= SPOT*1.25) & (temp_df['K'] >= SPOT*0.75)]
  temp_df = temp_df[temp_df['T'] <= 200]
  temp_df = temp_df[temp_df['IV'] > 0.001]
  return temp_df
```

In [144…
```python
cleaned_call = clean_df(df_call)
cleaned_put = clean_df(df_put)
```

In [145…
```python
cleaned_call.head(1)
```

Out[145…

	contractSymbol	lastTradeDate	K	price	bid	ask	change	percentChange	volume	openInterest	IV	inTheMor
0	SPXW250819C05000000	2025-08-19 15:03:11+00:00	5000.0	1425.9	1402.3	1420.5	152.34998	11.962623	1.0	1.0	1.375003	Tı

In [146…
```python
cleaned_put.head(1)
```

Out[146…

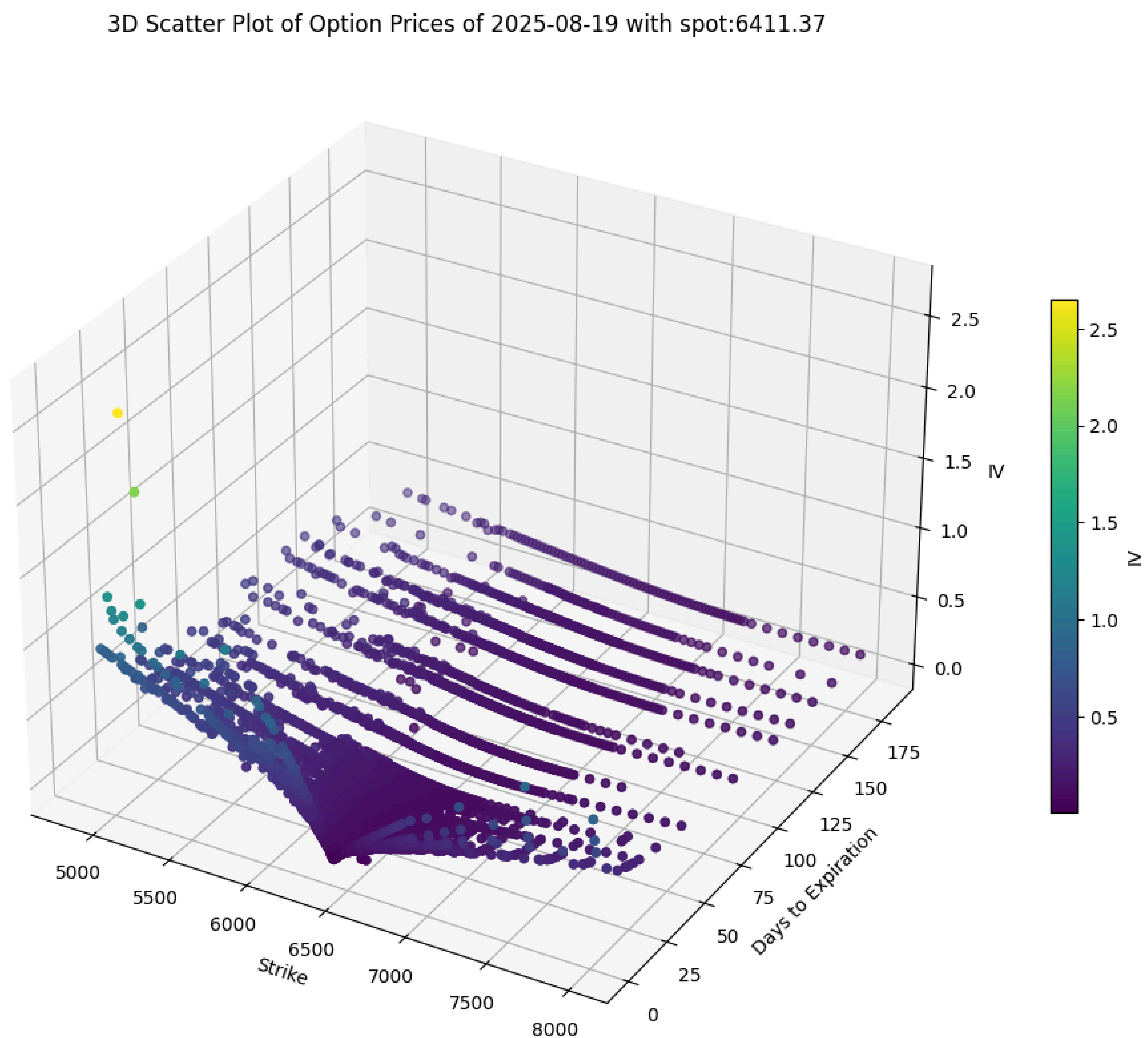
	contractSymbol	lastTradeDate	K	price	bid	ask	change	percentChange	volume	openInterest	IV	inTheMoney	con
12	SPXW250819P05000000	2025-08-14 19:30:15+00:00	5000.0	0.05	0.0	0.05	0.0	0.0	47.0	936.0	1.359378	False	

In [304…
```python
# graph_raw
def graph_raw(datadf, Z='IV', fst_axis=None, second_axis=None,
              title='3D Scatter Plot of Option Prices', spot=SPOT,
              path=None, date=None, DATE=DATE):
  # datadf["expirationDate"]= pd.to_datetime(datadf["expirationDate"])
  # date = pd.to_datetime(DATE)
  # datadf['T'] = (datadf["expirationDate"]-date).dt.days
  # datadf['K'] = datadf['strike']

  X_vals = datadf['K']
  Y_vals = datadf['T']
  Z_vals = datadf[Z]

  fig = plt.figure(figsize=(14, 10))
  ax = fig.add_subplot(111, projection='3d')
  ax.scatter(X_vals, Y_vals, Z_vals, c=Z_vals, cmap='viridis')

  ax.set_xlabel('Strike')
  ax.set_ylabel('Days to Expiration')
  ax.set_zlabel('IV')
  ax.set_title(f'{title} of {date} with spot:{round(spot, 2)}')
  fig.colorbar(ax.scatter(X_vals, Y_vals, Z_vals, c=Z_vals, cmap='viridis'),
             ax=ax, shrink=0.5, label=Z)
  ax.view_init(fst_axis, second_axis)

  if path is not None:
```

```python
    plt.savefig(path + f'/{date}BlendedIVScatter.png')
  else:
    plt.show()
```

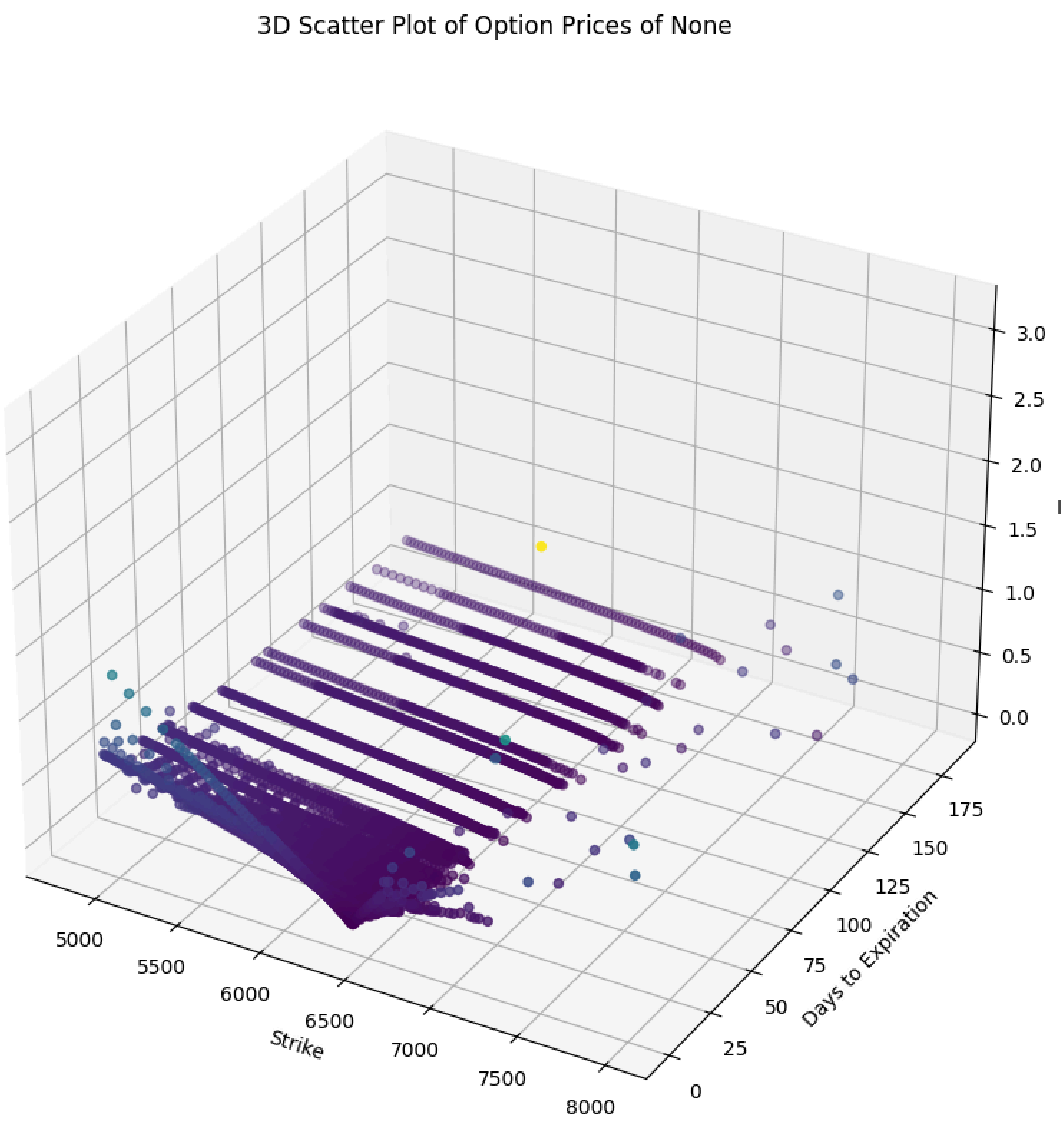
In [300…

```python
graph_raw(cleaned_call)
```


3D Scatter Plot of Option Prices of 2025-08-19 with spot:6411.37
Strike
5000
5500
6000
6500
7000
7500
8000
Days to Expiration
0
25
50
75
100
125
150
175
IV
0.0
0.5
1.0
1.5
2.0
2.5
IV
0.5
1.0
1.5
2.0
2.5

In [149…

```
graph_raw(cleaned_put)
```



3D Scatter Plot of Option Prices of None
Strike
Days to Expiration
I
5000
5500
6000
6500
7000
7500
8000
0
25
50
75
100
125
150
175
0.0
0.5
1.0
1.5
2.0
2.5
3.0

IV blending

In [293…

```
def blend_data(cleaned_calls, cleaned_puts, spot=SPOT, path=None, date=None):
    df_call = cleaned_calls[['K', 'T', 'IV', 'openInterest']].copy()
    df_put = cleaned_puts[['K', 'T', 'IV', 'openInterest']].copy()

    # Far-from-spot slices
    sliced_call = df_call[df_call['K'] >= spot * 1.02]
    sliced_put = df_put[df_put['K'] <= spot * 0.98]

    # Near-the-money slices (±2%)
    small_sliced_call = df_call[(df_call['K'] <= spot * 1.02) & (df_call['K'] >= spot * 0.98)]
    small_sliced_put = df_put[(df_put['K'] <= spot * 1.02) & (df_put['K'] >= spot * 0.98)]

    # Merge near-the-money calls and puts on strike & maturity
    merged_df = pd.merge(
        small_sliced_call, small_sliced_put,
        on=['K', 'T'], suffixes=('_call', '_put')
    )

    # Weighted IV
    merged_df['IV'] = (
        merged_df['IV_call'] * merged_df['openInterest_call'] +
        merged_df['IV_put'] * merged_df['openInterest_put']
    ) / (merged_df['openInterest_call'] + merged_df['openInterest_put'])

    # Keep only desired columns
    merged_df = merged_df[['K', 'T', 'IV', 'openInterest_call', 'openInterest_put']]

    # Final blended dataset: far calls + merged + far puts
    final_df = pd.concat([sliced_call, merged_df, sliced_put], ignore_index=True)

    # Add call and put price to the final_df based on cleaned_call and cleaned_puts
    final_df = pd.merge(
      final_df,
      cleaned_calls[['K', 'T', 'price']].rename(columns={'price': 'call_price'}),
      on=['K', 'T'],
      how='left'
    )

    final_df = pd.merge(
      final_df,
```

```python
        cleaned_puts[['K', 'T', 'price']].rename(columns={'price': 'put_price'}),
        on=['K', 'T'],
        how='left'
    )

    if path is not None:
        final_df.to_excel(path + f'/{date}BlendedIV.xlsx')

    return final_df
```

In [296…

```python
blended_df = blend_data(cleaned_call, cleaned_put, path=PATH, date=DATE)
blended_df
```

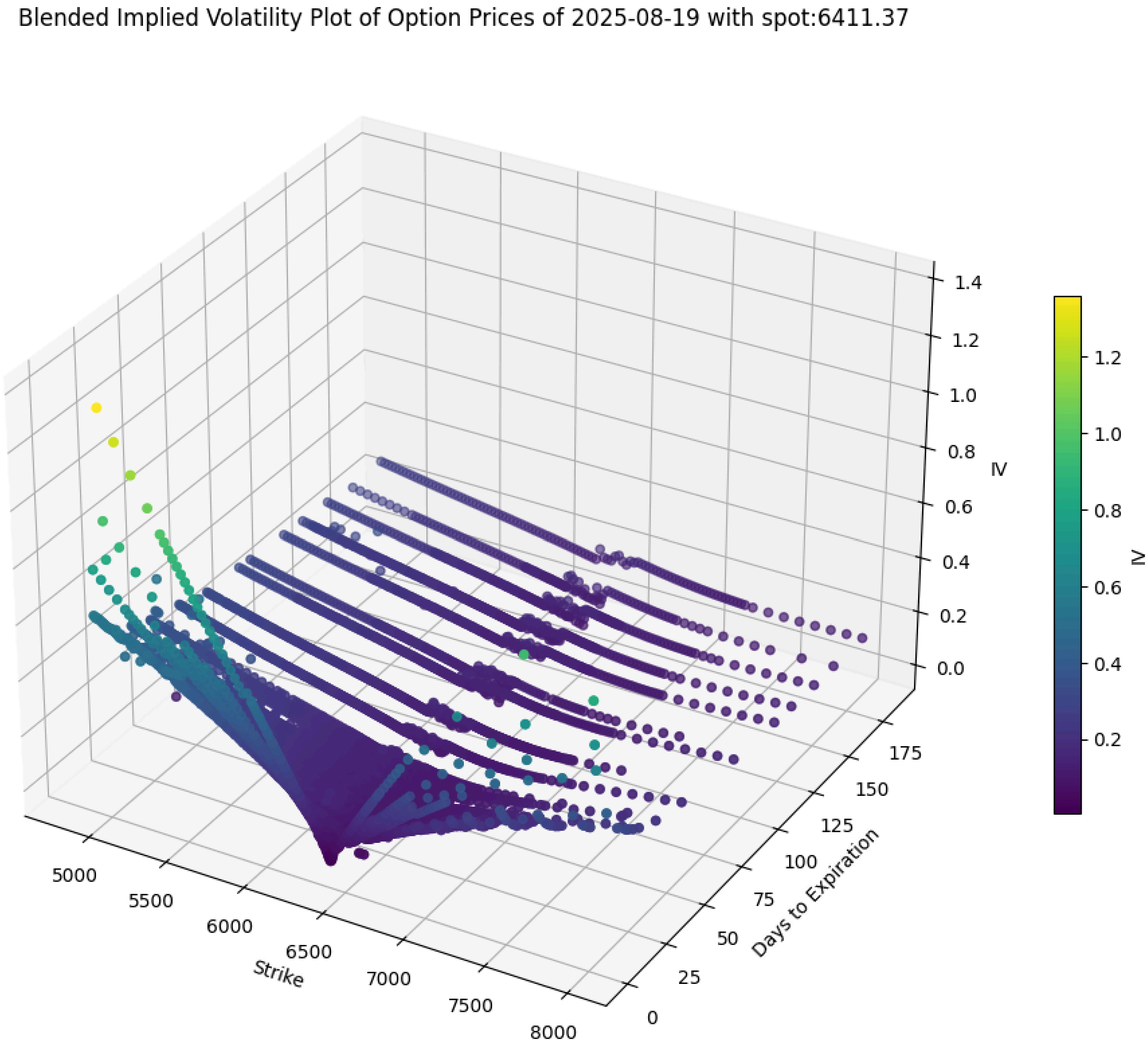
Out[296…

	K	T	IV	openInterest	openInterest_call	openInterest_put	call_price	put_price
0	6540.0	0	0.140634	2251.0	NaN	NaN	0.03	132.32
1	6545.0	0	0.145516	530.0	NaN	NaN	0.03	125.46
2	6550.0	0	0.149911	1188.0	NaN	NaN	0.03	145.80
3	6555.0	0	0.154794	719.0	NaN	NaN	0.03	118.80
4	6560.0	0	0.159676	1241.0	NaN	NaN	0.03	88.39
...	...	...	...	...	...	...	...	...
6350	6175.0	185	0.150090	2100.0	NaN	NaN	459.75	168.80
6351	6200.0	185	0.147361	2915.0	NaN	NaN	518.75	174.79
6352	6225.0	185	0.144605	2343.0	NaN	NaN	460.09	168.84
6353	6250.0	185	0.141871	1491.0	NaN	NaN	452.86	173.10
6354	6275.0	185	0.139037	354.0	NaN	NaN	463.87	188.45

6355 rows × 8 columns

In [303…

```python
graph_raw(blended_df, Z='IV', fst_axis=None, second_axis=None,
              title='Blended Implied Volatility Plot of Option Prices', spot=SPOT,
              path=PATH, date=DATE)
```


Blended Implied Volatility Plot of Option Prices of 2025-08-19 with spot:6411.37
Strike
5000
5500
6000
6500
7000
7500
8000
Days to Expiration
0
25
50
75
100
125
150
175
IV
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
IV
0.2
0.4
0.6
0.8
1.0
1.2

ML part

In [165…

```
import torch
from torch import nn
from torch.utils.data import Dataset, DataLoader
import torch.nn.functional as F
from sklearn.preprocessing import MinMaxScaler
from collections import defaultdict

class OptionPriceDataset(Dataset):
    def __init__(self, df):
        self.scaler_X = MinMaxScaler()
        self.scaler_y = MinMaxScaler()
        # Fit and transform inputs
        self.X = self.scaler_X.fit_transform(df[['K', 'T']].values)
        self.y = self.scaler_y.fit_transform(df[['IV']].values)
    def __len__(self):
        return len(self.y)
    def __getitem__(self, idx):
        x = torch.tensor(self.X[idx], dtype=torch.float32)
        y = torch.tensor(self.y[idx], dtype=torch.float32)
        return x, y
    def inverse_transform_y(self, y_scaled_tensor):
        y_np = y_scaled_tensor.detach().cpu().numpy()
        return self.scaler_y.inverse_transform(y_np)

class FeedForwardRegressor(nn.Module):
    def __init__(self, num_inputs, num_hidden, num_outputs=1):
        super().__init__()
        self.net = nn.Sequential(
            nn.Linear(num_inputs, num_hidden),
            nn.Softplus(),
            nn.Linear(num_hidden, num_hidden),
            nn.Softplus(),
            nn.Linear(num_hidden, num_outputs)
        )
    def forward(self, x):
        return self.net(x)
    def loss(self, y_pred, y_true):
        return F.mse_loss(y_pred, y_true)
    def training_step(self, batch):
        X, y = batch
```

```python
        y_hat = self(X)
        return self.loss(y_hat, y)
    def configure_optimizers(self, lr=0.01):
        return torch.optim.Adam(self.parameters(), lr)

class Trainer:
    def __init__(self, model, train_loader, device='cpu', lr=0.01):
        self.model = model.to(device)
        self.train_loader = train_loader
        self.device = device
        self.lr = lr
        self.best_score = float('inf')
        self.best_model_state = None

    def fit(self, epochs=100):
        optimizer = self.model.configure_optimizers(self.lr)
        for epoch in range(epochs):
            self.model.train()
            for batch in self.train_loader:
                optimizer.zero_grad()
                loss = self.model.training_step(batch)
                loss.backward()
                optimizer.step()
            if loss < self.best_score:
                self.best_score = loss
                self.best_model_state = self.model.state_dict()
    def load_best_model(self):
        if self.best_model_state:
            self.model.load_state_dict(self.best_model_state)

def full_train(datadf, epochs=100, learnR=0.01):
    # Create a new model instance each time
    model = FeedForwardRegressor(num_inputs=2, num_hidden=64)

    # Prepare dataset
    train_dataset = OptionPriceDataset(datadf)
    train_loader = DataLoader(train_dataset, batch_size=64, shuffle=True)

    # Train
    trainer = Trainer(model, train_loader, lr=learnR)
    trainer.fit(epochs)
    trainer.load_best_model()
    return model, train_loader
```

In [305…

```python
def surface_plot(datadf, model, path=None, date=None, faxis=None, saxis=None,
                 train_loader=None, spot_price=None):
```

```python
# Create
d202501013 = datadf
moneyness_min = d202501013['K'].min()
moneyness_max = d202501013['K'].max()
T_min = d202501013['T'].min()
T_max = d202501013['T'].max()
# Step 1: Create meshgrid and model prediction surface
K_vals = np.linspace(moneyness_min, moneyness_max, 20)
T_vals = np.linspace(T_min, T_max, 20)
grid = np.array([[K, T] for T in T_vals for K in K_vals])  # shape (400, 2)

# Normalize the input grid using the fitted scaler
scaler_X = train_loader.dataset.scaler_X
scaler_y = train_loader.dataset.scaler_y
grid_scaled = scaler_X.transform(grid)

x_tensor = torch.tensor(grid_scaled, dtype=torch.float32)

# Model prediction (on normalized input)
model.eval()
with torch.no_grad():
    call_prices_scaled = model(x_tensor)

# Inverse-transform the output price to original scale
call_prices_np = scaler_y.inverse_transform(call_prices_scaled.cpu().numpy()).reshape(len(T_vals), len(K_vals))

# Prepare meshgrid for plotting
K_vals_grid, T_vals_grid = np.meshgrid(K_vals, T_vals)

# Step 2: Prepare scatter points from original df_cleaned
df_scatter = d202501013.copy()
scatter_x = df_scatter['K'].values
scatter_y = df_scatter['T'].values
scatter_z = df_scatter['IV'].values

# Step 3: Plot surface + scatter
fig = plt.figure(figsize=(14, 10))
ax = fig.add_subplot(111, projection='3d')

# Surface (predicted)
ax.plot_surface(K_vals_grid, T_vals_grid, call_prices_np.reshape(len(T_vals), len(K_vals)), alpha=0.7, cmap='viridis')

# Scatter (real), colored by call price
sc = ax.scatter(
    scatter_x, scatter_y, scatter_z,
    c=scatter_z,              # color by call price
```

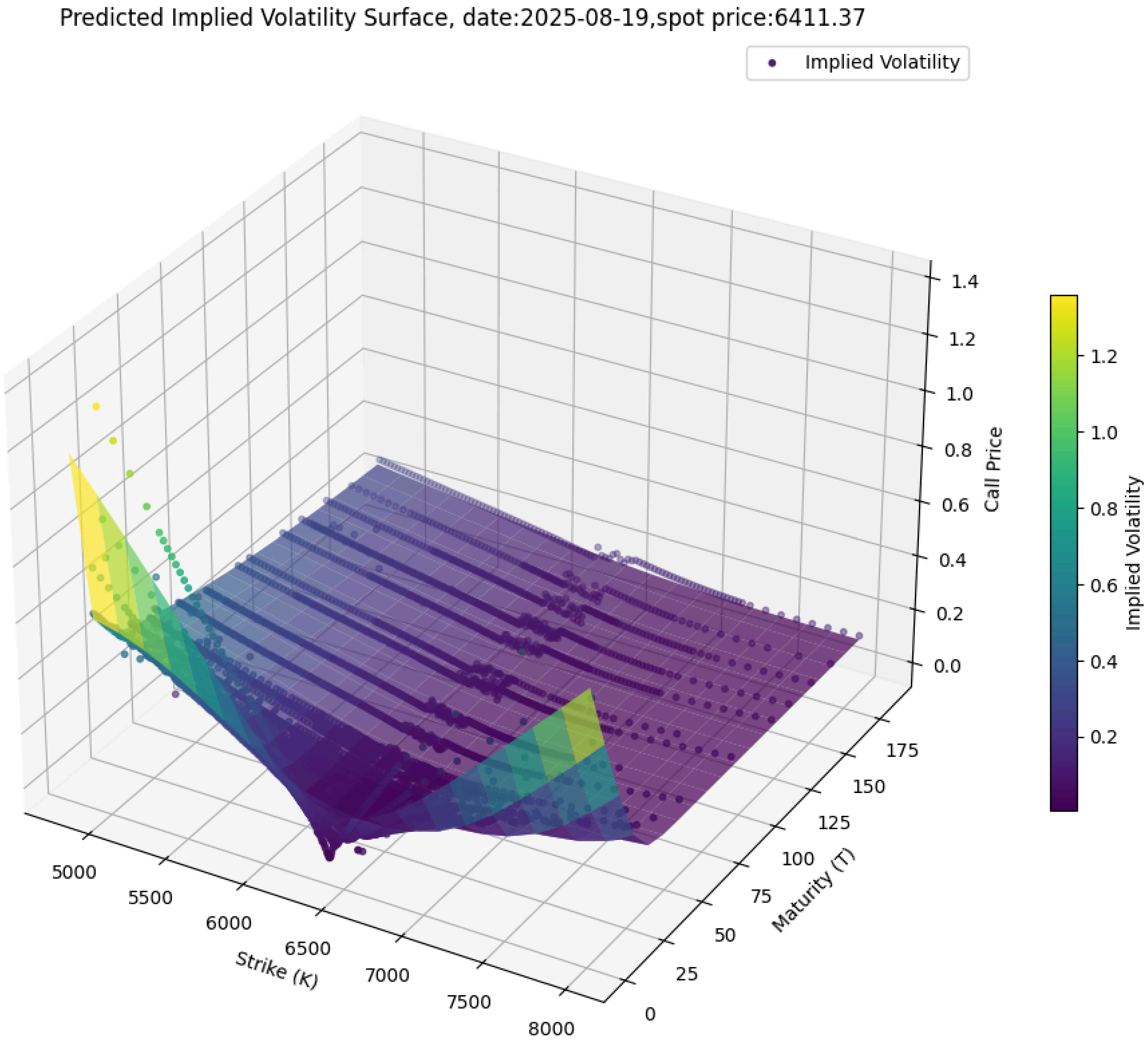
```python
        cmap='viridis',            # choose any colormap you like
        s=10,
        label='Implied Volatility'
    )

    fig.colorbar(sc, ax=ax, shrink=0.5, label='Implied Volatility')

    ax.set_xlabel("Strike (K)")
    ax.set_ylabel("Maturity (T)")
    ax.set_zlabel(f"Call Price")
    ax.set_title(f"Predicted Implied Volatility Surface, date:{date},spot price:{round(spot_price, 2)}")
    ax.legend()
    ax.view_init(elev=faxis, azim=saxis)
    if path is not None:
      plt.savefig(path + f'/{date}IVSurface.png')
    else:
      plt.show()
```

In [168…

```python
model, train_loader = full_train(blended_df)
```

In [306…

```python
surface_plot(blended_df, model, path=PATH, date=DATE, faxis=None, saxis=None,
               train_loader=train_loader, spot_price=SPOT)
```



Predicted Implied Volatility Surface, date:2025-08-19,spot price:6411.37
Implied Volatility
Call Price
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Strike (K)
5000
5500
6000
6500
7000
7500
8000
Maturity (T)
0
25
50
75
100
125
150
175
Implied Volatility
0.2
0.4
0.6
0.8
1.0
1.2

Yield Curve

In [181…

```
df_yield = pd.read_csv('/content/drive/My Drive/CapstoneProject/IVSurfaceWay/Data/2025-08-19yield_curve.csv')
```

In [179…

```
import numpy as np
import pandas as pd
import re

def _col_to_years(col: str) -> float:
    """
    Convert tenor column names like '1 Mo', '1.5 Month', '2 Yr', '30 Yr' to years.
    Assumes 30 days per month, 365 days per year.
    """
    s = col.strip().replace('Months', 'Month').replace('Mos', 'Mo').replace('Yrs', 'Yr')
    m = re.match(r'^(\d+(\.\d+)?)\s*(Mo|Month|Yr|Year)$', s)
    if not m:
        raise ValueError(f"Unrecognized tenor column: {col}")
    qty = float(m.group(1))
    unit = m.group(3)
    if unit in ('Mo', 'Month'):
        return (qty * 30.0) / 365.0
    else:  # 'Yr' or 'Year'
        return qty

def rate_from_yield_curve(d, df_yield, row=0, convention='cont', clamp=True):
    """
    Map days-to-maturity -> interest rate using the given yield curve.

    Parameters
    ----------
    d : int | float | array-like
        Days to maturity.
    df_yield : pd.DataFrame
        First row (by default) contains columns like '1 Mo', '1.5 Month', '2 Mo', '6 Mo', '1 Yr', ... with % yields.
    row : int
        Which row of df_yield to use (default: 0, the most recent).
    convention : {'cont','simple'}
        Return rate as continuous-compounded ('cont') or simple annualized ('simple').
    clamp : bool
        If True, clamp outside the curve to the nearest endpoint. If False, allow linear extrapolation.

    Returns
```

```
    -------
    r : float or np.ndarray
        Annual rate in decimal (e.g., 0.043 for 4.3%).
    """
    # Pick the curve row and collect tenor columns (ignore 'Date' or non-numeric cells)
    row_series = df_yield.iloc[row]
    cols = [c for c in df_yield.columns if c != 'Date']

    # Build x (years) and y (yields as decimals)
    tenors_years = []
    yields = []
    for c in cols:
        val = row_series[c]
        if pd.isna(val):
            continue
        try:
            y = float(val) / 100.0  # convert % to decimal
            t = _col_to_years(c)
            tenors_years.append(t)
            yields.append(y)
        except Exception:
            # Skip columns that aren't tenor columns
            continue

    if not tenors_years:
        raise ValueError("No valid tenor columns found in df_yield.")

    x = np.array(tenors_years, dtype=float)
    y = np.array(yields, dtype=float)

    # Sort by maturity just in case
    order = np.argsort(x)
    x = x[order]
    y = y[order]

    # Convert input days -> years
    d_arr = np.asarray(d, dtype=float)
    t_query = d_arr / 365.0

    # Interpolate (or clamp) on APR first
    if clamp:
        tq = np.clip(t_query, x[0], x[-1])
    else:
        tq = t_query

    y_interp = np.interp(tq, x, y)
```

```python
    # Return in requested compounding convention
    if convention == 'cont':
        # Convert APR to continuous rate: r_cont = ln(1 + APR)
        r = np.log1p(y_interp)
    elif convention == 'simple':
        r = y_interp
    else:
        raise ValueError("convention must be 'cont' or 'simple'")

    # Preserve scalar if input was scalar
    if np.isscalar(d):
        return float(r)
    return r
```

In [182…

```python
df_yield.head(1)
```

Out[182…

	Date	1 Mo	1.5 Month	2 Mo	3 Mo	4 Mo	6 Mo	1 Yr	2 Yr	3 Yr	5 Yr	7 Yr	10 Yr	20 Yr	30 Yr
0	08/19/2025	4.46	4.4	4.35	4.3	4.21	4.11	3.91	3.75	3.7	3.82	4.04	4.3	4.89	4.9

In [195…

```python
# Using your df_yield (most recent row at index 0)
r_45d = rate_from_yield_curve(90, df_yield, convention='cont')    # 45 days
r_45d
```

Out[195…

```
0.04210117601863539
```

Recovered call price

In [196…

```python
# def a function that gives the predict unscaled output while input is a non scaled input
def model_prediction(model_used=None, K=6000, T=100, train_loader=None):
    model_used.eval()
    scaler_X = train_loader.dataset.scaler_X
    scaler_y = train_loader.dataset.scaler_y
    with torch.no_grad():
        x_tensor = torch.tensor([[K, T]], dtype=torch.float32)
        x_transformed = scaler_X.transform(x_tensor.numpy())
        x_tensor = torch.tensor(x_transformed, dtype=torch.float32)
        y_pred_scaled = model_used(x_tensor)
        y_pred_np = scaler_y.inverse_transform(y_pred_scaled.cpu().numpy()).reshape(1, 1)
    return y_pred_np
```

In [217…

```
import numpy as np
from scipy.stats import norm

# def r_from_CPP(c, p, T, S, K):
#     # Returns the risk free rate used in black scholes for the current option pair
#     T = T / 365
#     # # Avoid division by zero or log of non-positive values
#     # if T <= 0 or K <= 0 or (p + S - c) / K <= 0:
#     #   return np.nan
#     r = (np.log((p+S-c)/K))/(-T)
#     return r

def black_scholes_call(S, K, T, r, sigma):
    """
    Black–Scholes European call price (no dividends).

    Parameters
    ----------
    S : float
        Spot price
    K : float
        Strike price
    T : float
        Time to maturity in DAYS
    r : float
        Annualized continuously compounded risk-free rate (e.g., 0.04 for 4%)
    sigma : float
        Annualized volatility
    """
    # Convert days -> years
    T = T / 365.0

    if T <= 0 or sigma <= 1e-9:  # handle expiry and tiny vol
        return max(S - K, 0.0)

    d1 = (np.log(S / K) + (r + 0.5 * sigma**2) * T) / (sigma * np.sqrt(T))
    d2 = d1 - sigma * np.sqrt(T)
    result = S * norm.cdf(d1) - K * np.exp(-r * T) * norm.cdf(d2)

    return result[0, 0]
```

In [322…

```
def single_price_prediction(K=None, T=None, model_used=None, train_loader=None, yield_data=df_yield, spot_price=SPOT):
    r_t = rate_from_yield_curve(T, yield_data, convention='cont')
    IV = model_prediction(model_used=model_used, K=K, T=T, train_loader=train_loader)
    p = black_scholes_call(S=spot_price, K=K, T=T, r=r_t, sigma=IV)
```

```python
    return p

def recoverd_call_price(model_used=None, delta_percent=0.01,
                    llim=-12, rlim=13, start=7, end=185, freq=7,
                    train_loader=None, spot_price=None, path=None, date=None):

    K_range = spot_price * (1 + np.arange(llim, rlim) * delta_percent)  # 25 strikes from −12% to +12%
    T_range = np.arange(start, end, freq)  # weekly: 0 to 350

    # Build grid
    results = []

    for K in K_range:
        for T in T_range:
            r_t = rate_from_yield_curve(T, df_yield, convention='cont')
            IV = model_prediction(model_used=model_used, K=K, T=T, train_loader=train_loader)
            p = black_scholes_call(S=spot_price, K=K, T=T, r=r_t, sigma=IV)
            results.append({
                'Strike': round(K, 0),
                'T_days': T,
                'p_K_T': p
            })

    # Convert to DataFrame and pivot
    df_state_price = pd.DataFrame(results)
    pivot_table = df_state_price.pivot(index='Strike', columns='T_days', values='p_K_T')
    # replace any neagtive to 0
    pivot_table[pivot_table < 0] = 0
    if path is not None:
      pivot_table.to_excel(path + f'/{date}RecovedCall.xlsx')
    return pivot_table
```

In [317…

```python
recoved_call_df = recoverd_call_price(model_used=model, delta_percent=0.015,
                    llim=-12, rlim=13, start=7, end=185, freq=7,
                    train_loader=train_loader, spot_price=SPOT, path=PATH, date=DATE)
```

In [318…

```python
#recoved_call_df.to_excel('recoved_call.xlsx')
recoved_call_df.head(3)
```

Out[318…

T_days	7	14	21	28	35	42	49	56	63	70	...
Strike											
5257.0	1158.725615	1163.506436	1168.571034	1174.125918	1180.009079	1186.058562	1192.156321	1198.190965	1204.131016	1209.980537	...
5353.0	1062.695190	1067.613710	1072.838291	1078.611164	1084.767700	1091.129692	1097.561223	1103.937093	1110.217478	1116.401708	...
5450.0	966.654438	971.699080	977.089904	983.100326	989.559616	996.270574	1003.077773	1009.840838	1016.510158	1023.078851	...

3 rows × 26 columns

Surface plot function for K T

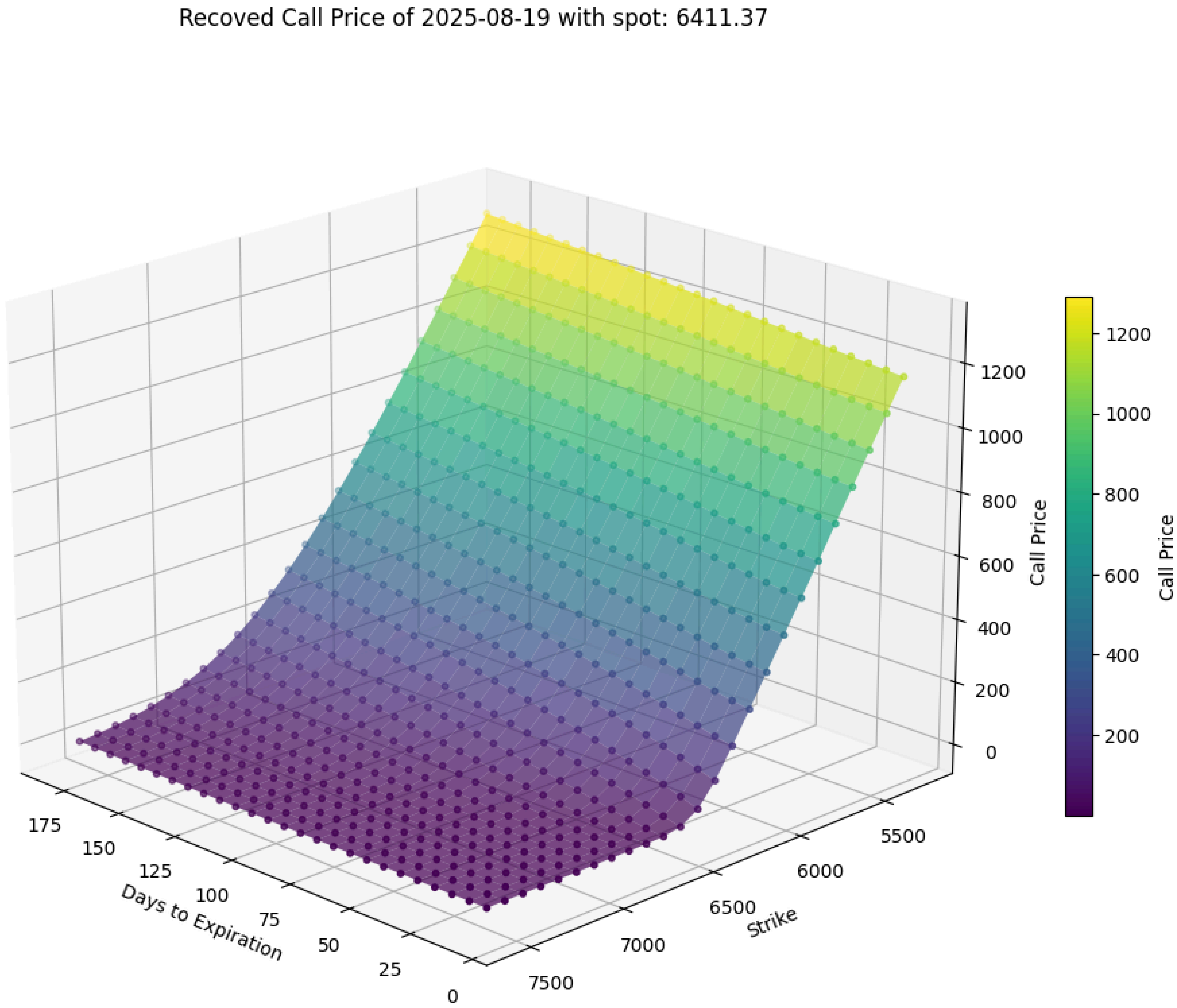
In [359…

```
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
import numpy as np
import pandas as pd
from datetime import datetime


def plot_3d_scatter(dataset, size=(14, 10), elev=None, azim=None,
                    path=None, date=None, spot=SPOT, title='Recoved Call Price',
                    x_label='Days to Expiration', y_label='Strike', z_label='Call Price',
                    file_name='RecovedCall'):
  # Ensure columns (dates) are sorted
  panel_cleaned = dataset.sort_index(axis=1)

  # Get the column and index values as 1D arrays
  X_vals_1d = panel_cleaned.columns.values
  Y_vals_1d = panel_cleaned.index.values
  Z_vals = panel_cleaned.values

  # Create 2D meshgrid for X and Y values
  X_vals, Y_vals = np.meshgrid(X_vals_1d, Y_vals_1d)

  # Reshape Z_vals to match the meshgrid shape
  z_reshaped = Z_vals.reshape(len(Y_vals_1d), len(X_vals_1d))

  # Plot
  fig = plt.figure(figsize=(14, 10))
  ax = fig.add_subplot(111, projection='3d')

  # Surface (predicted)
```

```python
    ax.plot_surface(X_vals, Y_vals, z_reshaped, alpha=0.7, cmap='viridis')

     # Create X, Y, Z values for scatter plot
    X_scatter = []
    Y_scatter = []
    Z_scatter = []

    for i, strike in enumerate(panel_cleaned.index):
        for j, exp_date in enumerate(panel_cleaned.columns):
            last_price = panel_cleaned.loc[strike, exp_date]
            if pd.notna(last_price):
                X_scatter.append(exp_date)
                Y_scatter.append(strike)
                Z_scatter.append(last_price)
     # Scatter (real), colored by call price


    scatter = ax.scatter(X_scatter, Y_scatter, Z_scatter, c=Z_scatter, cmap='viridis', s=10)


    ax.set_xlabel(x_label)
    ax.set_ylabel(y_label)
    ax.set_zlabel(z_label)
    ax.set_title(f'{title} of {date} with spot: {round(spot, 2)}')

    fig.colorbar(scatter, ax=ax, shrink=0.5, label=z_label)


    ax.view_init(elev, azim)

    if path is not None:
      plt.savefig(path + f'/{date}{file_name}.png')
    plt.show()
```

In [313…

```python
plot_3d_scatter(recoved_call_df, size=(14, 10), elev=20, azim=135, path=PATH, date=DATE, spot=SPOT)
```


Recoved Call Price of 2025-08-19 with spot: 6411.37
Days to Expiration
175
150
125
100
75
50
25
0
Strike
5500
6000
6500
7000
7500
Call Price
0
200
400
600
800
1000
1200
Call Price
200
400
600
800
1000
1200

Second Order Derivative

In [337…

```
# Define the seond order derivative function
def second_order_derivative(model_used=None, K=None, T=None, delta_change=0.001,
                            train_loader=None, yield_data=df_yield, spot_price=SPOT):
    delta_K = K * delta_change

    C_K_PdeltaK = single_price_prediction(K+delta_K, T, model_used=model_used, train_loader=train_loader, yield_data=yield_data
    C_K_MdeltaK = single_price_prediction(K-delta_K, T, model_used=model_used, train_loader=train_loader, yield_data=yield_data
    C_K_T = single_price_prediction(K, T, model_used=model_used, train_loader=train_loader, yield_data=yield_data, spot_price=s

    result = (C_K_PdeltaK - 2 * C_K_T + C_K_MdeltaK) / (delta_K ** 2)
    return result.item()
```

In [342…

```
second_order_derivative(model_used=model, K=6500, T=7, delta_change=0.005,
                            train_loader=train_loader, yield_data=df_yield, spot_price=SPOT)
```

Out[342…

```
0.002934795853812485
```

Raw State price

In [348…

```
def raw_state_price(model_used=None, delta_percent=0.015,
                    llim=-12, rlim=13, start=7, end=185, freq=7,
                    train_loader=None, spot_price=None, path=None, date=None):

    K_range = spot_price * (1 + np.arange(llim, rlim) * delta_percent)  # 25 strikes from -12% to +12%
    T_range = np.arange(start, end, freq)  # weekly: 0 to 350

    # Build grid
    results = []

    for K in K_range:
        for T in T_range:
            p = second_order_derivative(K=K, T=T, model_used=model_used, train_loader=train_loader)
            results.append({
                'Strike': round(K, 0),
                'T_days': T,
                'p_K_T': p
            })
```

```
    # Convert to DataFrame and pivot
    df_state_price = pd.DataFrame(results)
    pivot_table = df_state_price.pivot(index='Strike', columns='T_days', values='p_K_T')
    # check negative
    if pivot_table.min().min() < 0:
      print('Negative found')

    # replace any neagtive to 0
    pivot_table[pivot_table < 0] = 0
    if path is not None:
      pivot_table.to_excel(path + f'/{date}RawStatePrice.xlsx')
    return pivot_table
```

In [350…

```
raw_state_df = raw_state_price(model_used=model, train_loader=train_loader, spot_price=SPOT, path=PATH, date=DATE)
raw_state_df.head(3)
```

```
Negative found
```

Out[350…

T_days	7	14	21	28	35	42	49	56	63	70	...	119	126	133	14(
Strike															
5257.0	0.0	0.0	0.0	0.000000e+00	0.000002	0.000006	0.000009	0.000013	0.000018	0.000022	...	0.000050	0.000054	0.000057	0.00006(
5353.0	0.0	0.0	0.0	3.087192e-07	0.000003	0.000007	0.000012	0.000017	0.000022	0.000027	...	0.000061	0.000065	0.000069	0.00007:
5450.0	0.0	0.0	0.0	2.095512e-06	0.000006	0.000012	0.000017	0.000023	0.000030	0.000036	...	0.000076	0.000080	0.000084	0.00008ε

3 rows × 26 columns

Normalized State Price

In [351…

```
def build_discount_df(yield_data=df_yield, n_weeks=52):
    weeks = np.arange(1, n_weeks)
    t_years = weeks / 52
    days = weeks * 7
    r_t = rate_from_yield_curve(days, yield_data, convention='cont')
    D_t = np.exp(-r_t * t_years)

    df_discount = pd.DataFrame({
        'Week': weeks,
        'T (Years)': np.round(t_years, 6),
        'Interpolated Yield': np.round(r_t * 100, 6),
```

```python
        'Discount Factor': np.round(D_t, 6),
        'Days': weeks * 7
    })

    return df_discount
```

In [354…

```python
build_discount_df().head(3)
```

Out[354…

	Week	T (Years)	Interpolated Yield	Discount Factor	Days
0	1	0.019231	4.363404	0.999161	7
1	2	0.038462	4.363404	0.998323	14
2	3	0.057692	4.363404	0.997486	21

In [355…

```python
def state_price_matrix(df_discount=None, pivot_table=None, path=None, date=None):
  # Ensure column headers in pivot_table are integers (days)
  pivot_table.columns = pivot_table.columns.astype(int)

  # Build mapping from day to discount factor
  discount_map = dict(zip(df_discount['Days'], df_discount['Discount Factor']))

  # Normalize each column
  normalized_table = pivot_table.copy()

  for col in normalized_table.columns:
      if col in discount_map:
          col_sum = normalized_table[col].sum()
          if col_sum > 0:
              normalized_table[col] *= discount_map[col] / col_sum
  if path is not None:
      normalized_table.to_excel(path + f'/{date}(Normalized)StatePrice.xlsx')

  return normalized_table
```

In [357…

```python
NormState = state_price_matrix(df_discount=build_discount_df(), pivot_table=raw_state_df, path=PATH, date=DATE)
NormState
```

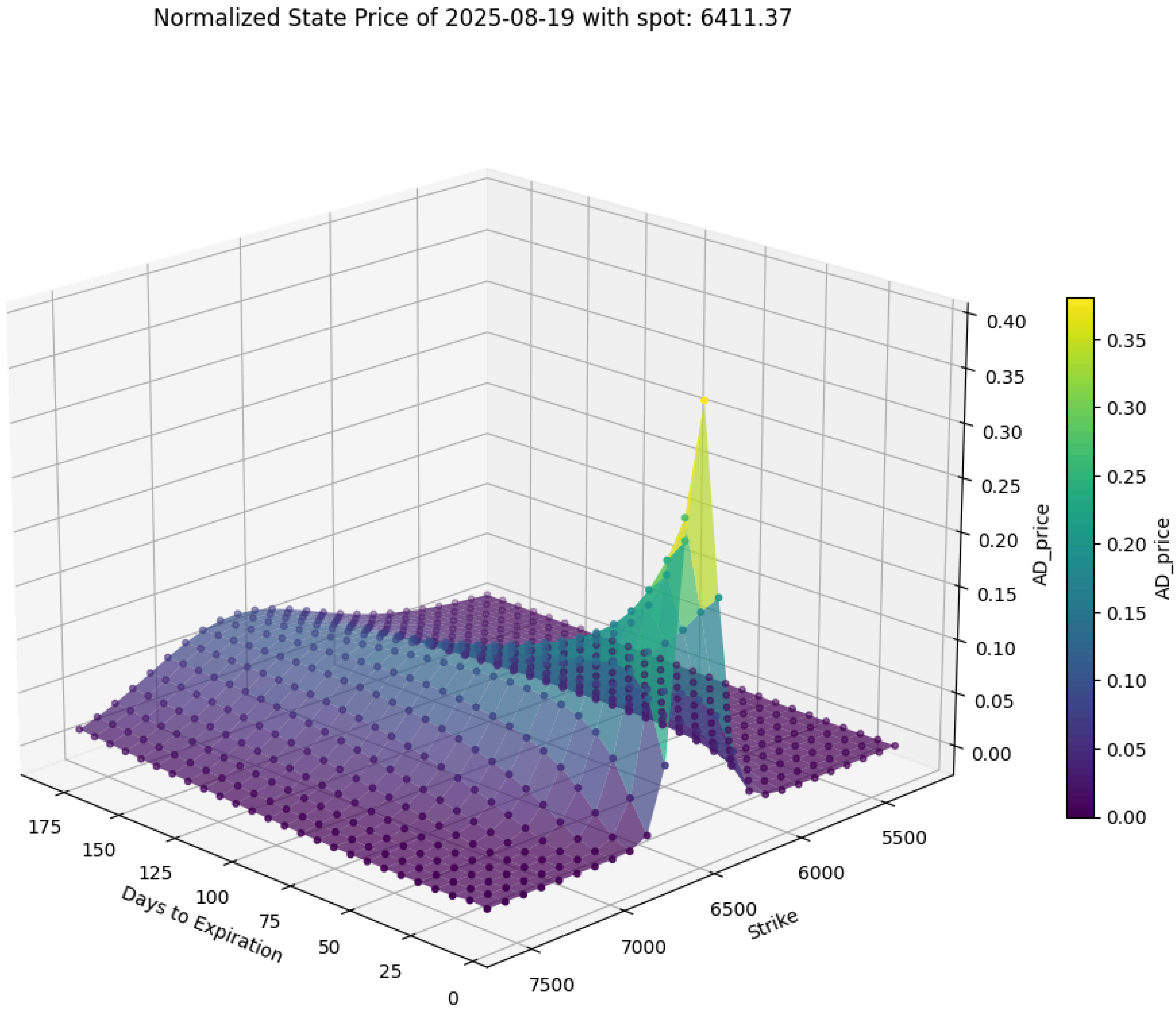
Out[357…

T_days	7	14	21	28	35	42	49	56	63	70	...	119	126	
Strike														
5257.0	0.000000	0.000000	0.000000	0.000000	0.000233	0.000533	0.000889	0.001285	0.001709	0.002145	...	0.005032	0.005383	0.005
5353.0	0.000000	0.000000	0.000000	0.000030	0.000333	0.000713	0.001152	0.001635	0.002156	0.002686	...	0.006135	0.006549	0.006
5450.0	0.000000	0.000000	0.000000	0.000202	0.000623	0.001120	0.001674	0.002272	0.002902	0.003535	...	0.007594	0.008062	0.008
5546.0	0.000000	0.000000	0.000126	0.000633	0.001253	0.001933	0.002644	0.003385	0.004136	0.004890	...	0.009521	0.010056	0.010
5642.0	0.000000	0.000053	0.000648	0.001513	0.002476	0.003434	0.004372	0.005282	0.006172	0.007041	...	0.012128	0.012689	0.013
5738.0	0.000011	0.000537	0.001689	0.003202	0.004719	0.006093	0.007309	0.008410	0.009425	0.010375	...	0.015602	0.016151	0.016
5834.0	0.000319	0.001532	0.003816	0.006459	0.008818	0.010732	0.012259	0.013514	0.014579	0.015511	...	0.020211	0.020686	0.02
5931.0	0.001036	0.003898	0.008472	0.012913	0.016326	0.018751	0.020449	0.021664	0.022566	0.023266	...	0.026263	0.026552	0.026
6027.0	0.003289	0.010463	0.019204	0.025793	0.029901	0.032288	0.033591	0.034246	0.034513	0.034565	...	0.034005	0.033938	0.033
6123.0	0.012661	0.029898	0.043474	0.050356	0.053152	0.053862	0.053502	0.052606	0.051449	0.050196	...	0.043561	0.042972	0.042
6219.0	0.055309	0.082187	0.091341	0.091217	0.088174	0.084389	0.080516	0.076779	0.073258	0.069986	...	0.054741	0.053421	0.052
6315.0	0.201183	0.182682	0.161075	0.143369	0.129943	0.119468	0.110916	0.103666	0.097358	0.091781	...	0.066820	0.064673	0.062
6411.0	0.381034	0.272651	0.216597	0.183692	0.162379	0.147211	0.135535	0.126012	0.117905	0.110814	...	0.078407	0.075511	0.072
6508.0	0.262688	0.240574	0.208597	0.184189	0.166782	0.153821	0.143578	0.135043	0.127626	0.120989	...	0.087532	0.084229	0.081
6604.0	0.067035	0.121676	0.141077	0.142621	0.139282	0.134939	0.130550	0.126297	0.122178	0.118146	...	0.092114	0.089007	0.08
6700.0	0.009351	0.037493	0.068042	0.086004	0.095158	0.099823	0.102167	0.103189	0.103370	0.102957	...	0.090736	0.088473	0.086
6796.0	0.001203	0.008027	0.024282	0.041217	0.054024	0.063104	0.069605	0.074360	0.077894	0.080525	...	0.083351	0.082391	0.081
6892.0	0.000233	0.001433	0.006742	0.016080	0.025954	0.034637	0.041892	0.047904	0.052920	0.057141	...	0.071415	0.071836	0.07
6988.0	0.000133	0.000267	0.001553	0.005233	0.010728	0.016750	0.022580	0.027958	0.032835	0.037245	...	0.057266	0.058770	0.059
7085.0	0.000202	0.000080	0.000323	0.001458	0.003869	0.007220	0.011022	0.014946	0.018816	0.022551	...	0.043251	0.045350	0.047
7181.0	0.000354	0.000091	0.000069	0.000359	0.001233	0.002798	0.004912	0.007378	0.010041	0.012795	...	0.031005	0.033235	0.035
7277.0	0.000547	0.000245	0.000020	0.000082	0.000352	0.000982	0.002010	0.003382	0.005019	0.006845	...	0.021264	0.023304	0.025
7373.0	0.000733	0.000659	0.000019	0.000018	0.000091	0.000314	0.000759	0.001445	0.002358	0.003466	...	0.014051	0.015745	0.017
7469.0	0.000875	0.001433	0.000063	0.000005	0.000022	0.000092	0.000265	0.000577	0.001044	0.001665	...	0.008995	0.010309	0.01
7565.0	0.000964	0.002443	0.000256	0.000003	0.000005	0.000025	0.000086	0.000216	0.000437	0.000760	...	0.005600	0.006570	0.00

25 rows × 26 columns

In [369…

```
plot_3d_scatter(NormState, size=(14, 10), elev=20, azim=135,
                path=PATH, date=DATE, spot=SPOT, title='Normalized State Price', x_label='Days to Expiration',
                y_label='Strike', z_label='AD_price', file_name='(Normalized)StatePrice')
```


Normalized State Price of 2025-08-19 with spot: 6411.37
Days to Expiration
175
150
125
100
75
50
25
0
Strike
5500
6000
6500
7000
7500
AD_price
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
AD_price
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35

State Price Transition Matrix

In [374…

```python
import numpy as np
import pandas as pd
from scipy.optimize import nnls, lsq_linear

def fit_state_price_transition_columnwise(Pt_matrix, ridge=0.0, scale=False):
    """
    Column-wise NNLS fit of P from p^{t+1} = p^{t} P.

    Pt_matrix: array (m, T) with columns p^t (state-price vectors).
    ridge:     nonnegative Tikhonov weight.
    scale:     True -> row-scale X for conditioning.
    """
    Pt = np.asarray(Pt_matrix, float)
    if (Pt < 0).any():
        Pt = np.maximum(Pt, 0.0)  # clip tiny negatives
    m, T = Pt.shape
    if T < 2:
        raise ValueError("Need at least two tenors (T >= 2).")

    # X shape: (T-1, m); Y shape: (T-1, m)
    X = Pt[:, :-1].T
    Y = Pt[:, 1:].T

    # Optional scaling
    if scale:
        col_scale = np.median(np.abs(X), axis=0)
        col_scale[col_scale == 0] = 1.0
        Xs = X / col_scale
    else:
        col_scale = np.ones(m)
        Xs = X

    # Optional ridge via augmentation
    if ridge > 0.0:
        R = np.sqrt(ridge) * np.eye(m)
        X_aug = np.vstack([Xs, R])
    else:
        X_aug = Xs

    P = np.zeros((m, m))
```

```python
    for j in range(m):
        y = Y[:, j]
        if ridge > 0.0:
            y = np.concatenate([y, np.zeros(m)])

        coef, resnorm = nnls(X_aug, y)
        coef = coef / col_scale  # undo scaling
        P[:, j] = coef

    return P



def Transition_Matrix(spot_price, df_rename, path=None, date=None, ridge=0.000001):
    """

    """
    df_rename = df_rename.reset_index()
    df_rename['Strike'] = np.round(df_rename['Strike'] / spot_price, 2)
    df_rename = df_rename.rename(columns={'Strike': 'Moneyness'})

    maturity_cols = df_rename.columns[1:25]  # pick your tenors
    df_filtered = df_rename[['Moneyness'] + list(maturity_cols)].copy()

    # STATE PRICES per tenor, on same moneyness grid
    Pt_matrix = df_filtered.iloc[:, 1:].to_numpy(dtype=float)
    moneyness_states = df_filtered['Moneyness'].to_numpy()

    # Clean tiny negatives from numerical extraction
    Pt_matrix = np.maximum(Pt_matrix, 0.0)

    # 1) Estimate P (no row-sum=1 constraint)
    P_hat = fit_state_price_transition_columnwise(Pt_matrix, ridge=ridge, scale=False)


    P_df = pd.DataFrame(P_hat, index=moneyness_states, columns=moneyness_states)
    P_df = P_df.sort_index(axis=0).sort_index(axis=1)

    if path is not None:
        P_df.to_excel(f"{path}/{date}TransitionMatrix.xlsx")

    return P_df
```

In [396…

```python
TransState = Transition_Matrix(spot_price=SPOT, df_rename=NormState,
                               path=PATH, date=DATE, ridge=0.000001)
TransState
```

Out[396…

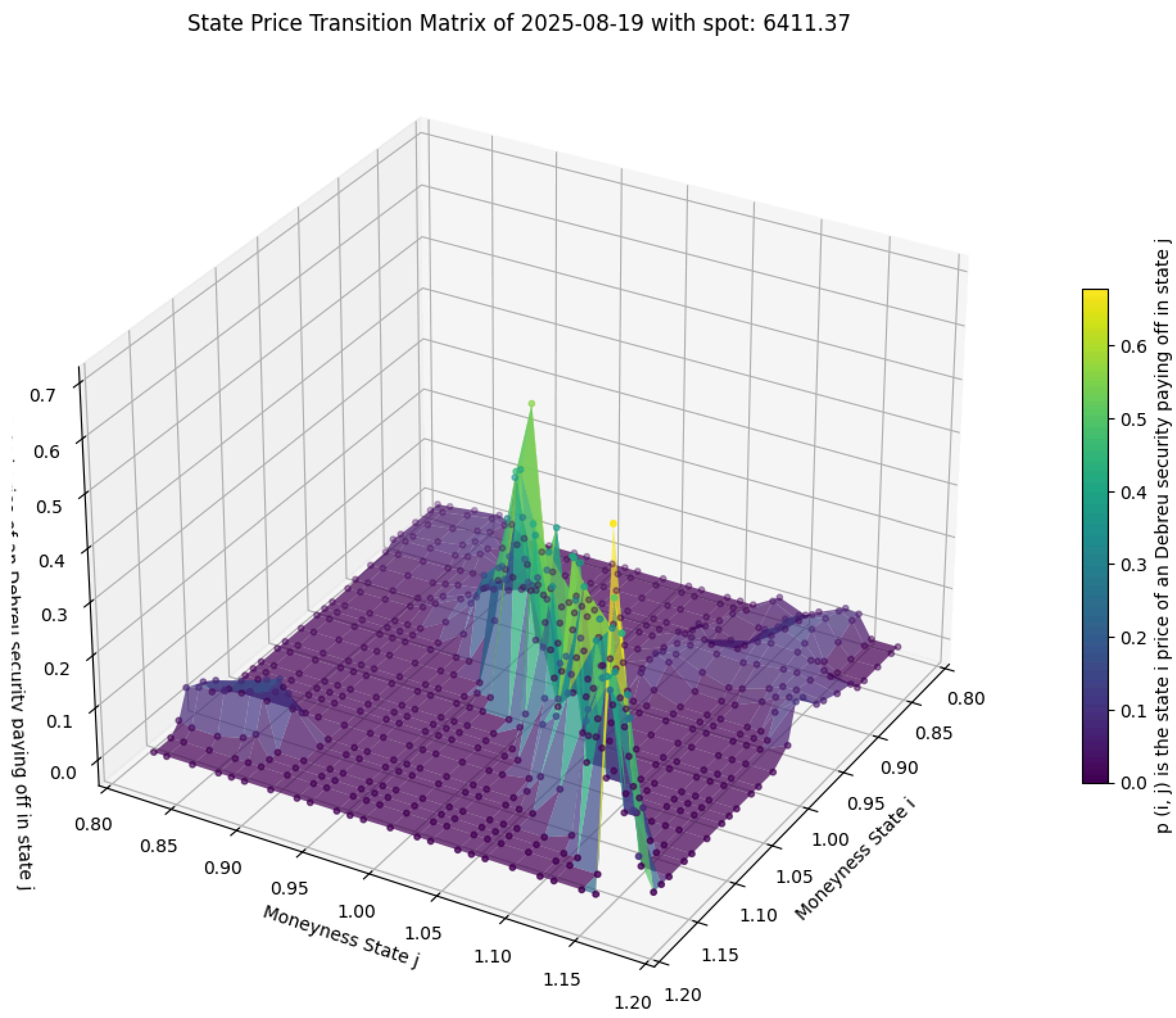
	0.82	0.83	0.85	0.87	0.88	0.89	0.91	0.93	0.94	0.96	...	1.05	1.06	1.0
0.82	0.017732	0.018272	0.018731	0.020616	0.023043	0.010430	0.009049	0.006302	0.001160	0.000000	...	0.000000	0.000000	0.00000
0.83	0.020459	0.021269	0.021628	0.023406	0.026042	0.012175	0.010210	0.006885	0.001488	0.000000	...	0.000000	0.000000	0.00000
0.85	0.026877	0.029229	0.028398	0.027449	0.027059	0.013543	0.010476	0.006245	0.001754	0.000000	...	0.000000	0.000000	0.00000
0.87	0.031859	0.037543	0.038137	0.035484	0.027342	0.014730	0.011052	0.006432	0.003026	0.000000	...	0.000000	0.000000	0.00000
0.88	0.020906	0.032100	0.043463	0.049543	0.032122	0.017214	0.015971	0.013565	0.008893	0.000000	...	0.000000	0.000000	0.00000
0.89	0.000000	0.000000	0.018589	0.042644	0.044937	0.018968	0.021234	0.024720	0.023311	0.000000	...	0.000000	0.000000	0.00000
0.91	0.000000	0.000000	0.000000	0.000000	0.028912	0.019819	0.029633	0.043061	0.047399	0.027977	...	0.000000	0.000000	0.00000
0.93	0.000000	0.000000	0.000000	0.000000	0.001301	0.021002	0.040597	0.069045	0.085473	0.079959	...	0.000000	0.000000	0.00000
0.94	0.000000	0.000000	0.000000	0.000000	0.000000	0.022271	0.050209	0.094952	0.131189	0.142422	...	0.000000	0.000000	0.00000
0.96	0.000000	0.000000	0.000000	0.000000	0.000000	0.004263	0.045470	0.098743	0.162710	0.193780	...	0.000000	0.000000	0.00000
0.97	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.005892	0.033231	0.134222	0.275453	...	0.000000	0.000000	0.00000
0.98	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.045877	...	0.000000	0.000000	0.00000
1.00	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000109	0.000000	0.000000	0.000000	...	0.000000	0.000000	0.00000
1.02	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	...	0.028087	0.000000	0.00000
1.03	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.001265	0.033670	...	0.420233	0.075392	0.00000
1.05	0.000000	0.000000	0.000000	0.000000	0.006131	0.023068	0.018552	0.021430	0.016999	0.000000	...	0.174060	0.331811	0.13466
1.06	0.000000	0.000000	0.000000	0.015620	0.033430	0.019471	0.034261	0.045137	0.054350	0.043111	...	0.380033	0.208546	0.14157
1.07	0.000000	0.002173	0.021452	0.016432	0.000000	0.022681	0.038020	0.047164	0.044892	0.017263	...	0.000000	0.391056	0.41585
1.09	0.029857	0.035386	0.008893	0.000000	0.006853	0.037282	0.036324	0.027078	0.003333	0.000000	...	0.000000	0.000000	0.31447
1.11	0.009608	0.010974	0.021477	0.034570	0.060119	0.049079	0.034769	0.012655	0.000000	0.000000	...	0.000000	0.000000	0.00000
1.12	0.023520	0.031363	0.055897	0.075149	0.073151	0.054009	0.039119	0.018309	0.000000	0.000000	...	0.000000	0.000000	0.00000
1.14	0.045882	0.053793	0.064822	0.070090	0.055876	0.051699	0.045247	0.034277	0.022077	0.000000	...	0.000000	0.000000	0.00000
1.15	0.042084	0.042527	0.026029	0.014641	0.028057	0.043398	0.046212	0.046045	0.039026	0.000000	...	0.000000	0.000000	0.00000
1.16	0.000000	0.000000	0.000000	0.000000	0.006141	0.031617	0.039320	0.047216	0.046717	0.044833	...	0.000000	0.000000	0.00000
1.18	0.000000	0.000000	0.000000	0.000000	0.000000	0.020135	0.028733	0.041953	0.048598	0.088973	...	0.000000	0.000000	0.00000

25 rows × 25 columns

Visualization

In [420…

```
plot_3d_scatter(TransState, size=(14, 10), elev=30, azim=30,
                path=PATH, date=DATE, spot=SPOT, title='State Price Transition Matrix', x_label='Moneyness State i',
                y_label='Moneyness State j', z_label='p (i, j) is the state i price of an Debreu security paying off in state j
```



State Price Transition Matrix of 2025-08-19 with spot: 6411.37
p (i, j) is the state i price of an Debreu security paying off in state j
Moneyness State j
Moneyness State i
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.80
0.85
0.90
0.95
1.00
1.05
1.10
1.15
1.20

Natural probabiliy

In [404…

```
import numpy as np
from numpy.linalg import eig
def is_irreducible(df: pd.DataFrame) -> bool:
    if not isinstance(df, pd.DataFrame):
        raise TypeError("Input must be a pandas DataFrame.")

    if df.shape[0] != df.shape[1]:
        raise ValueError("Input matrix must be square.")

    if not (df.values >= 0).all():
        raise ValueError("Matrix entries must be non-negative.")

    # Convert to boolean adjacency matrix: True if entry > 0
    adj_matrix = (df.values > 0).astype(bool)
    n = adj_matrix.shape[0]

    # Reachability matrix: initially only direct connections
    reachability = adj_matrix.copy()

    # Warshall's algorithm to compute transitive closure
    for k in range(n):
        for i in range(n):
            for j in range(n):
                reachability[i, j] = reachability[i, j] or (
                    reachability[i, k] and reachability[k, j]
                )

    # Ensure diagonal is reachable (self-reachability)
    np.fill_diagonal(reachability, True)

    # Matrix is irreducible if every state can reach every other
    return np.all(reachability)

def ross_D_from_P(P, tol=1e-10, gap_tol=1e-8, imag_tol=1e-12):
    P = np.asarray(P, dtype=float)

    # 1) Eigen-decomposition (right eigenpairs)
    vals, vecs = eig(P)

    # 2) Identify (numerically) real eigenvalues and pick the Perron root among them
```

```python
    real_mask = np.isclose(vals.imag, 0.0, atol=imag_tol)
    if not np.any(real_mask):
        raise ValueError("No (numerically) real eigenvalues found.")

    real_idx = np.where(real_mask)[0]                # indices into original arrays
    vals_real = vals[real_idx].real                  # real eigenvalues as reals
    j = int(np.argmax(vals_real))                    # index within the real subset
    i = int(real_idx[j])                             # map back to original index
    delta = float(vals_real[j])

    # 3) Uniqueness: spectral gap among real eigenvalues
    if vals_real.size > 1:
        others = np.delete(vals_real, j)
        gap = delta - np.max(others)
        if gap <= gap_tol:
            raise ValueError(
                f"Perron root not unique or gap too small (gap={gap:.3e}). "
                "Check irreducibility / symmetry of P."
            )

    # 4) Perron right eigenvector (corresponding to the original index i)
    z = vecs[:, i]
    if np.max(np.abs(z.imag)) > np.sqrt(imag_tol):
        # Allow tiny numerical noise but flag if it's not tiny
        raise ValueError("Dominant eigenvector has significant imaginary part; check P.")
    z = z.real

    # 5) (Optional) Enforce positivity if PF assumptions hold
    # if np.all(z <= 0): z = -z
    # if (z <= 0).any(): raise ValueError("Perron vector not strictly positive; P may be reducible or invalid.")

    # 6) Normalize (avoid dividing by ~0: pick first reasonably large entry)
    # anchor_idx = np.argmax(np.abs(z))
    # if np.abs(z[anchor_idx]) < tol:
    #     raise ZeroDivisionError("All components of z are ~0; cannot normalize.")
    # z = z / z[anchor_idx]

    # 7) Build D with d_ii = 1 / z_i
    if (np.abs(z) < tol).any():
        raise ZeroDivisionError("A component of z is ~0; cannot form D = diag(1/z).")
    D = np.diag(1.0 / z)

    return D, delta, z


def natural_probability(AD, path=None, date=None):
```

```python
    if np.linalg.matrix_rank(AD) == len(AD):
      print(is_irreducible(AD))

    # Calculate e
    e = np.ones((25,1))

    # Calcualte eigenvalues and eigen vectors
    D, delta, z = ross_D_from_P(AD)

    # Changes data types from complex to real given nature of np.linalg functions
    # if np.all(z.values.imag ==0):
    #     z = z.astype(float)
    #     delta = delta.astype(float)

    # Creation of Pricing Kernel
    phi = (1/z) * delta

    # Creation of Natural probabilities
    F = pd.DataFrame(np.empty((25, 25)) * np.nan)
    for i in range(len(AD)):
        for j in range(len(AD)):
            F.iloc[i,j] = (1/delta)*(z[j]/z[i])*AD.iloc[i,j]
    # AD_inv = np.linalg.inv(AD)
    # D_inv = np.linalg.inv(D)
    # F = (1.0 / delta) * (D * AD * D_inv)
    F.index = AD.index
    F.columns = AD.columns

    if path is not None:
        F.to_excel(path + f'/{date}NaturalProbability.xlsx')
        # phi.to_excel(path + f'/{date}PricingKernal.xlsx')
    # print(D)
    # print(F)
    return F, phi, delta
```

In [399…

```python
# Check irreducibility
is_irreducible(TransState)
```

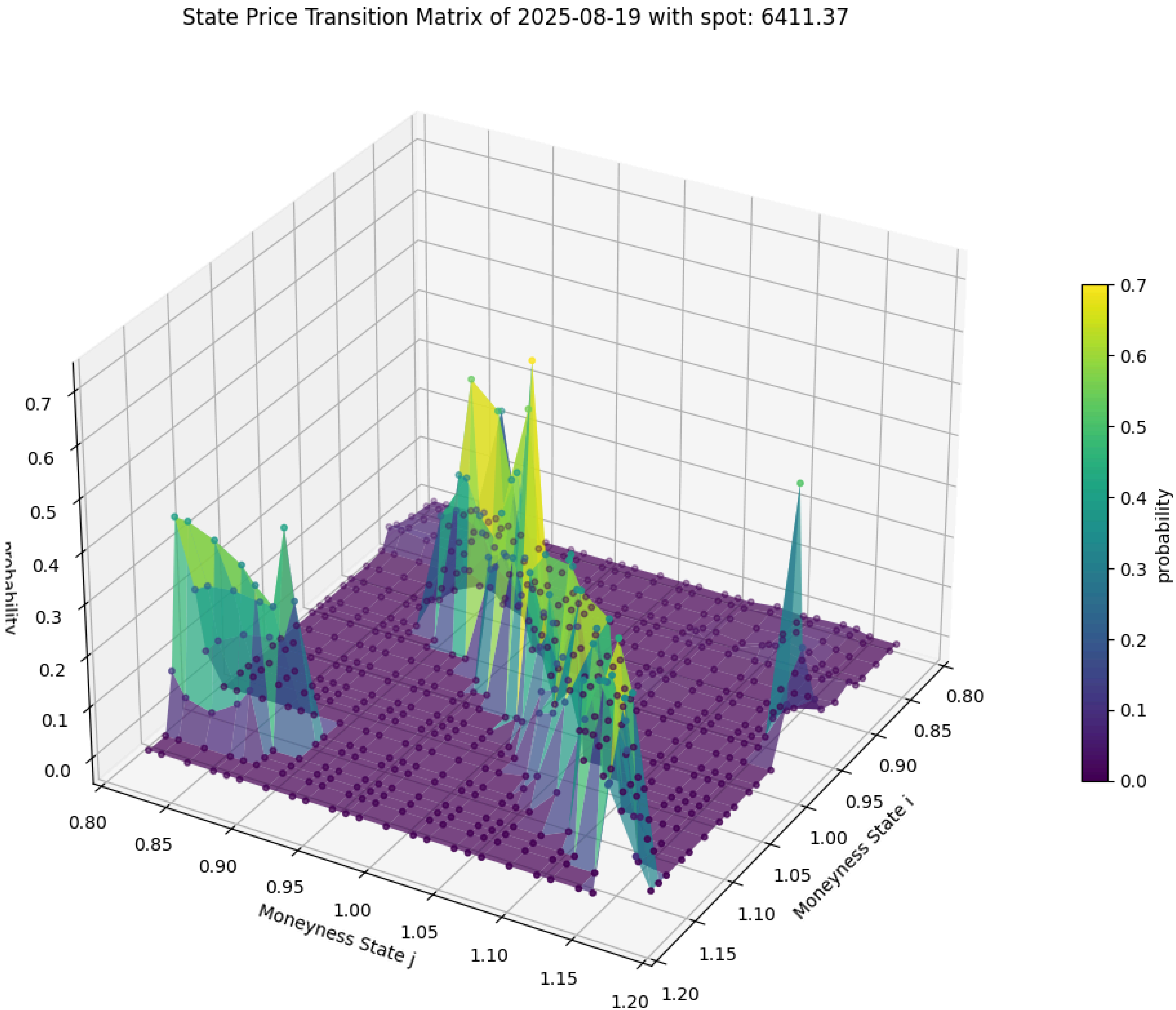
Out[399…

```
np.True_
```

In [419…

```python
NaturalMatrix, phi, delta= natural_probability(TransState, path=PATH, date=DATE)

plot_3d_scatter(NaturalMatrix, size=(14, 10), elev=30, azim=30,
                path=PATH, date=DATE, spot=SPOT, title='State Price Transition Matrix', x_label='Moneyness State i',
                y_label='Moneyness State j', z_label='probability', file_name='NaturalProbabilitySurface')
```

```
True
```



State Price Transition Matrix of 2025-08-19 with spot: 6411.37
probability
Moneyness State j
Moneyness State i
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.80
0.85
0.90
0.95
1.00
1.05
1.10
1.15
1.20

In [407…

```python
# plot phi and sav
plt.figure()
plt.plot(phi)
plt.title("Pricing Kernal")
plt.xlabel("State")
plt.ylabel("Scaled Kernel")
plt.savefig(PATH + f'/{DATE}PricingKernal.png')
plt.show()
plt.close()
```