
On the Fairness of Randomized Trials for Recommendation with Heterogeneous Demographics and Beyond

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Abstract

Observed events in recommendation are consequence of the decisions made by a policy, thus they are usually *selectively* labeled, namely the data are *Missing Not At Random* (MNAR), which often causes large bias to the estimate of true outcomes risk. A general approach to correct MNAR bias is performing small *Randomized Controlled Trials* (RCTs), where an additional *uniform policy* is employed to randomly assign items to each user. In this work, we concentrate on the fairness of RCTs under both homogeneous and heterogeneous demographics, especially analyzing the bias for the least favorable group on the latter setting. Considering RCTs' limitations, we propose a novel *Counterfactual Robust Risk Minimization* (CRRM) framework, which is totally free of expensive RCTs, and derive its theoretical generalization error bound. At last, empirical experiments are performed on synthetic tasks and real-world data sets, substantiating our method's superiority both in fairness and generalization.

1. Introduction

Conventional machine learning model, though can reach high accuracy in average, often falls short in causing high loss on minority groups at the same time. Fairness issues have been studied in various tasks, like medical treatment, criminal justice (Kearns et al., 2019; Nabi et al., 2018; Mary et al., 2019), credit loan (Liu et al., 2018), machine translation (Hashimoto et al., 2018), recommender system (Beutel et al., 2019; Singh & Joachims, 2019), etc. The reasons for improving model's fairness are not limited with respect to ensuring morality or legality, but also involving economical considerations. In (Hashimoto et al., 2018), it proves the amplification of demographic disparity in sequential ERM process. That is, the minority group which suffers

	Item 1	Item 2	Item 3
User 1	√	×	?
User 2	?	?	√
⋮			

Figure 1. Missing ratings in recommender system.

higher loss will become more likely to quit using our service, thereby providing no feedback data any longer. This negative feedback loop can cause shrinking size of users, hence reducing the income or welfare provided by our machine learning system over time.

Specifically for recommender system, there have been line of work showing that, the real-world logging policy often collects missing not at random (MNAR) (Yuan et al., 2019) data (or selective labels). For example, users tend to only reveal ratings for items they like, thus the observed users' feedback (usually described by click-through-rate, CTR) can be substantially better than those not observed yet. In Fig. 1, estimated average CTR from the observed outcome is $2/3 \approx 0.67$. However, if all unobserved outcomes are 0, the true CTR is $1/3 \approx 0.33$. This gap between the *factual* and *counterfactual* ratings, in MNAR situation, has been empirically validated by (Steck, 2013). In this work, we show that MNAR also undermines fairness of recommendation over time, by exacerbating minority group's utility.

To get unbiased estimate of true CTR without revealing all feedback, running randomized controlled trials (RCTs) (Haynes et al., 2012) has been so far the first choice, which satisfies missing-at-random (MAR) (Seaman et al., 2013) condition:

Definition 1. *The missing data are missing-at-random (MAR) if the conditional probability of observed pattern, represented by O here, satisfies*

$$P(O \mid X = x) = P(O \mid X = x') \quad \forall x, x' \in \mathcal{X} \quad (1)$$

However, the cost of RCTs is unacceptable in many scenarios, e.g. we cannot randomly assign drugs to the volunteers, or randomly display ads for users. Vast majority of existing works (Shalit et al., 2017; Rosenfeld et al., 2017; Bonner & Vasile, 2018) tend to leverage *small* RCTs \mathcal{D}_t and large

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biased logged data \mathcal{D}_c to correct the MNAR bias, through multi-task learning or domain adaptation techniques. They claim that small RCTs, though not capable of revealing each individual’s preference (Rosenfeld et al., 2017), are sufficient for reaching low-variance estimate of the *average* of all, from which we can obtain model that generalizes well to those counterfactual events. However, the conception about “small” is not well-defined yet, hence no guideline for proper number of RCTs to trade gain against cost. Besides, although RCTs are intuitively regarded as fair for each user, we argue that learning from the average from RCTs causes unfair policy, on heterogeneous demographics, i.e. users vary over populations defined by preferences.

In this work, we aim to mitigate users’ utility disparity, as well as improving global utility of all in recommendation. Our object of interest is a RCTs-free method that can achieve both two goals above. We begin from introducing *Gini index* (Gini, 1997) as a disparity measure in section 4. In section 5, we consider two heterogeneous situations, for analyzing the RCTs’s fairness, accounting for the least favorable group’s risk. Then, we propose our RCTs-free Counterfactual Robust Risk Minimization (CRRM) framework in section 6, and test its performance in section 7. The proof of all propositions and theorems can be found in appendices.

Contributions. Our main contributions are three-fold:

1. We demonstrate the unfairness of conventionally used ERM-based recommendation under MNAR data, and bridge the gap between protecting the least favorable group and ensuring fairness regarding to Gini index;
2. We first theoretically analyze the fairness of RCTs-based methods, by considering the least favorable group’s risk over multiple demographic conditions;
3. We develop a RCTs-free CRRM framework, and build its motivative error bound. Our method is proved succeeding in improving both fairness and accuracy, on both synthetic tasks and real-world data.

2. Related work

Fair machine learning. Fairness in machine learning is an emerging topic in the literature recently. The most direct principle of algorithmic fairness is to protect sensitive tags, features or labels, such as race, gender, age, etc. This formulation of fairness has been investigated in (Hardt et al., 2016; Mary et al., 2019) where the conditional independence, between the decision made by the algorithm and the variables to protect, is estimated or guaranteed. In other respects, the *Rawlsian fairness* (Rawls, 2009) is introduced to substantiate this *equal opportunity* principle in a *bandit learning* scheme (Joseph et al., 2016; Jabbari et al., 2017),

where an action is preferred only if its true quality is better. However, as far as we know, few work has ever mentioned fairness issue in MNAR scenario.

For some scenarios when the demographics are unknown, the previous works have been so far concentrating on protecting subgroups with latent demographic structure (Kearns et al., 2017), or learning a distributionally robust model for uniform performance without demographics (Duchi & Namkoong, 2018; Hashimoto et al., 2018). Although these approaches are proved to be able to protect the minority group, no evident improvement of global utility for all users is witnessed in practice.

Correcting MNAR bias. An importance sampling approach called inverse propensity score (IPS) is proposed initially to correct the sampling bias (Rosenbaum & Rubin, 1983), then followed by line of work including doubly robust (DR) (Dudík et al., 2011; Jiang & Li, 2015), weighted doubly robust (Thomas & Brunskill, 2016), joint learning doubly robust (Wang et al., 2019), self-normalized IPS (Swaminathan & Joachims, 2015; Schnabel et al., 2016), etc., for recommendation on MNAR data, by utilizing additional small RCTs. Besides, methods (Johansson et al., 2016; Bonner & Vasile, 2018) adapted from domain adaptation and representation learning techniques are used for learning from both \mathcal{D}_t and \mathcal{D}_c . However, they hardly mention their methods’ fairness and effectiveness over heterogeneous demographics.

3. Problem Setup

Let user $u \in \mathcal{U}$, and item $v \in \mathcal{V}$, then an even $x \in \mathcal{U} \times \mathcal{V}$. The outcome of an event is $y \in \mathcal{Y} = \{0, 1\}$ where

$$y = \begin{cases} 0 & \text{if the event } x \text{ was observed and clicked} \\ 1 & \text{if the event } x \text{ was observed and not clicked} \end{cases} \quad (2)$$

There are in total $|\mathcal{U}| \times |\mathcal{V}|$ possible events considering different combination of users and items, where $|\cdot|$ here means the cardinality of a set, we set $|\mathcal{U}| = M$ and $|\mathcal{V}| = N$. The true risk of predicted outcome matrix $\hat{\mathbf{Y}} \in \mathbb{R}^{M \times N}$ can be easily written out as

$$R(\hat{\mathbf{Y}}) = \frac{1}{MN} \sum_{u \in \mathcal{U}} \sum_{v \in \mathcal{V}} \delta(\hat{y}_{uv}, y_{uv}) \quad (3)$$

where y_{uv} is assumed drawn from a true outcome matrix $\mathbf{Y} \in \{0, 1\}^{M \times N}$, and $\delta(\cdot)$ is usually set as surrogate mean-squared-error, or logistic loss, for simplicity we denote it as δ_{uv} or δ_x .

When some of outcomes are not observed, calculating Eq.(3) becomes infeasible. Such that, previous works attempt to

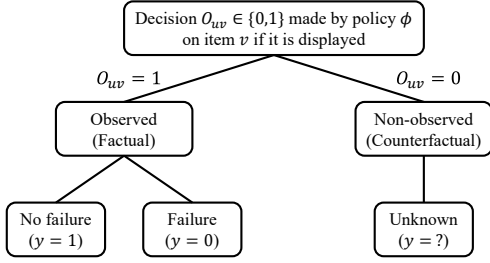


Figure 2. A demonstration of MNAR problem.

only minimize the factual empirical risk, namely ERM as

$$\hat{R}_F(\hat{Y}) = \frac{1}{|\{(u, v); O_{uv} = 1\}|} \sum_{(u, v): O_{uv} = 1} \delta_{uv} \quad (4)$$

in which random variable O_{uv} means whether a pair (u, v) is observed. The observation of outcome y_{uv} is hence constrained by the missing pattern O_{uv} , shown in Fig.2. Such that, the $\hat{R}_F(\hat{Y})$ is a biased estimate of true risk $R(\hat{Y})$:

$$\mathbb{E}_O(\hat{R}_F(\hat{Y})) \neq R(\hat{Y}) \quad (5)$$

3.1. Correcting sampling bias by propensity scores

To deal with the bias shown in Eq.(5), an unbiased inverse propensity score (IPS) estimator (Rosenbaum & Rubin, 1983) can be built as

$$\hat{R}_{IPS}(\hat{Y}) = \frac{1}{MN} \sum_{(u, v): O_{uv} = 1} \frac{\delta_{uv}}{Z_{uv}} \quad (6)$$

where Z_{uv} is the used propensity score that $\mathbb{E}_O(O_{uv}) = Z_{uv}$, hence it is easy to derive $\mathbb{E}_O(\hat{R}_{IPS}(\hat{Y})) = R(\hat{Y})$.

The propensity score Z_{uv} here, can be approximated via a simplified naive Bayes (NB) estimator (Marlin & Zemel, 2009) \hat{Z}_{uv} :

$$P(O_{uv} = 1 | Y_{uv} = y) = \frac{P(Y = y | O = 1)P(O = 1)}{P(Y = y)} \quad (7)$$

with assuming that $P(O | Y, X) = P(O | Y)$. Though the $P(Y | O)$ and $P(O)$ can be counted by MNAR data, the $P(Y = y)$ has to be estimated via MAR data, which is often achieved by taking average of n RCTs' outcomes

$$\bar{Y}_n = \frac{1}{n} \sum_{i=1}^n Y_i \quad (8)$$

Using $x = (u, v)$, the bias of IPS estimator with inaccurate propensity \hat{Z}_x is given by (Schnabel et al., 2016) that

$$R(\hat{Y}) - \mathbb{E}_O(\hat{R}_{IPS}(\hat{Y} | \hat{Z})) = \frac{1}{MN} \sum_x \delta_x \left(1 - \frac{O_x}{\hat{Z}_x}\right) \quad (9)$$

Y, \hat{Y}

U, V, X

O

u, v, x

y_x, \hat{y}_x

δ_x

$R(\hat{Y})$

$\hat{R}(\hat{Y})$

ϕ_t, ϕ_c

$\mathcal{D}_t, \mathcal{D}_c$

$\mathcal{D}_F, \mathcal{D}_{CF}$

Table 1. Main notation.

true and predicted outcome matrix
random variables: user, item and event
Bernoulli random variable: event missing
realization of U, V, X
true and predicted outcome of $x = (u, v)$
risk of predicted outcome of $x = (u, v)$
true risk of \hat{Y}
empirical risk of \hat{Y}
uniform and non-uniform policy
MAR and MNAR event sets
factual and counterfactual event sets

For notation simplicity, we omit \hat{Z} of the $\hat{R}_{IPS}(\hat{Y} | \hat{Z})$ from now on. In this work, we delve deeper into this bias, accounting for the bias of different populations. We assume that each individual u belongs to a group \mathcal{U}_l , and the group mean preference is represented by

$$\mu_l := P(Y = 1 | u \in \mathcal{U}_l) \quad l = 1, \dots, L \quad (10)$$

In this viewpoint, the \bar{Y}_n is an efficient empirical estimate of global preference for all groups. However, we question that, when demographics are heterogeneous, i.e. μ_l are different, is the \bar{Y}_n by RCTs still a fair and economical estimate? Since for ensuring fairness, we care about loss suffered by the *least favorable group*, rather than only the average over all. Besides, measuring utility disparity is also in our interests.

4. Measuring Utility Disparity by Gini Index

The conventional approach to update a policy for recommender system is repeatedly learning from the MNAR logged data, by minimizing empirical risk. In this section, we identify that the policy ϕ_c obtained by this process fails to offer fair services for users, when taking user's click-through-ratio (CTR) as their utility. We then propose Gini index for measuring this disparity, and reveal the connection between improving Gini fairness and protecting the least favorable group's utility.

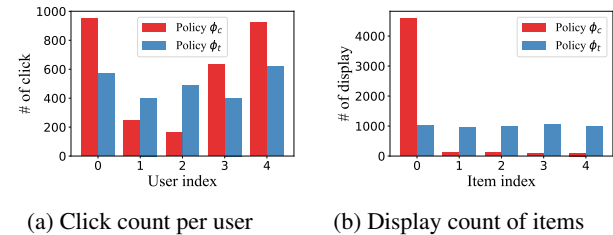


Figure 3. Analysis of simulation results of the motivating example.

4.1. Motivating example

We build a tiny system with 5 users and 5 items for simulation, and compare the user-wise and item-wise numbers

of clicks by ϕ_t and ϕ_c , shown in Fig.3. Unfair treatment towards users by ϕ_c can be witnessed in the Fig.3a. Compared with ϕ_t , though the total clicks achieved by ϕ_c is slightly higher (2916 v.s. 2484), the click rates of user #1 and #2 are much lower than their counterparts by ϕ_t . That means, the ϕ_c achieves good performance in average, at the expense of deteriorating experience of the minorities. Besides, the Fig.3b illustrates devastating diversity reduction by ϕ_c , which is also unacceptable in recommendation. Details about the implementation of simulation can be found in Appendix F.1.

4.2. Gini fairness and the minority's utility

Example above impresses the necessity to measure users' utility disparity. To this end, we introduce Gini index drawn from *Economics* to measure fairness in recommendation. The Gini index $\mathbf{GI} \in [0, 1]$, with 0 indicating perfect equality and 1 being perfect inequality, is a measure of the distribution of income across percentiles among populations.

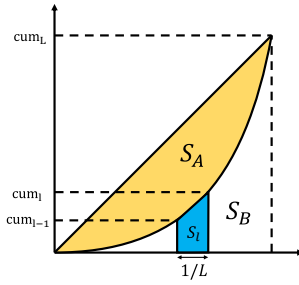


Figure 4. Computing the Gini index via the Lorenz curve.

To compute \mathbf{GI} , the *Lorenz curve* (Mankiw, 2006) plots population percentile by utility on the x-axis, and cumulative utility on the y-axis. Denote the cumulative l -th group's utility as $\mathbf{Cum}_l = \sum_{l'=1}^l \hat{\mu}_{l'}$. From Fig.4, we can compute the area under the Lorenz curve S_B by summation of all S_l

$$S_B = \sum_{l=1}^L S_l = \frac{1}{2L} \sum_{l=1}^L (\mathbf{Cum}_{l-1} + \mathbf{Cum}_l) \quad (11)$$

therefore the \mathbf{GI} can be obtained by

$$\begin{aligned} \mathbf{GI} &:= \frac{S_A + S_B - S_B}{S_A + S_B} = 1 - \frac{2}{\mathbf{Cum}_L} \times S_B \\ &= 1 - \frac{1}{L} \sum_{l=1}^L \frac{1}{\mathbf{Cum}_L} (\mathbf{Cum}_{l-1} + \mathbf{Cum}_l) \end{aligned} \quad (12)$$

We give the following Proposition 1 which identifies the connection between controlling Gini index and protecting the least favorable group.

Proposition 1 (Least favorable group's utility and Gini index). *Assume that $\hat{\mu}_1 \leq \hat{\mu}_2 \leq \dots \leq \hat{\mu}_L$, controlling the worst-case $\hat{\mu}_1$ to protect the minority group can improve fairness,*

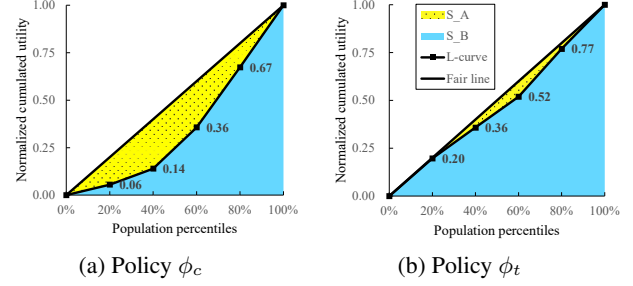


Figure 5. The *Lorenz curve* of cumulative utility μ obtained by policy ϕ_c and ϕ_t , e.g. in (a), the point (40%, 0.14) means the tail two users obtain 14% of total utility; and in (b), the (40%, 0.36) means 36% of total utility. The value of Gini index is defined as the area of S_A , therefore the ϕ_t is much more fair than ϕ_c .

by virtue of controlling the supremum of the \mathbf{GI} as

$$0 \leq \mathbf{GI} \leq 1 - \frac{1}{L} - \frac{L-1}{\mathbf{Cum}_L} \hat{\mu}_1 \quad (13)$$

if and only if $\hat{\mu}_1 = \hat{\mu}_2 = \dots = \hat{\mu}_L$ the equality holds.

Based on above analysis, picking the least favorable group's average utility can be a surrogate metric for fairness, since the \mathbf{GI} 's upper bound is restricted by $\hat{\mu}_1$. By the Lorenz curve of the motivating example in Fig.5, the ϕ_t has much less Gini index than ϕ_c , and the minimum utility earned by ϕ_t is larger than ϕ_c as well.

Another concern is that the *global utility* \mathbf{gU} , namely the utility enjoyed by all users, is also important for evaluating a policy's effectiveness:

$$\mathbf{gU} := \frac{1}{L} \mathbf{Cum}_L = \frac{1}{L} \sum_{l=1}^L \hat{\mu}_l \quad (14)$$

because it decides the total income of the system. Then comes two principles for developing a recommender system:

- Reduce Gini index \mathbf{GI} (Eq. (12)) for fairness
- Improve global utility \mathbf{gU} (Eq. (14)) for effectiveness

We will use the above two metrics for comparison in the experiment section.

5. Fairness of RCTs Considering Heterogeneous Users Demographics

Recall that our goal is to control the utility disparity of users, which can be realized by controlling the least favorable group's utility $\hat{\mu}_1$, as above mentioned in Eq.(13). We proceed in two steps. First, we show how the empirical mean \bar{Y}_n influences the minority's risk by IPS estimator. Then, we analyze this risk regarding to several demographics assumptions, i.e. homogeneous and heterogeneous. As

a result, we delve deeper into the limitations in fairness of randomized trials, specifically when the \bar{Y}_n by RCTs is used as a trustworthy knowledge for debiasing MNAR data in the literature.

5.1. Risk of the least favorable group

Let $\mu_1 \leq \mu_2 \leq \dots \leq \mu_L$, then the least favorable group is \mathcal{U}_1 , and the event set related to \mathcal{U}_1 is $\mathcal{X}_1 = \mathcal{U}_1 \times \mathcal{V}$. Bias of IPS estimator on \mathcal{X}_1 , similar to Eq.(9), can be derived as

$$\begin{aligned} \text{Bias}(\hat{R}_{IPS}(\hat{Y} | \mathcal{X}_1)) &= R(\hat{Y} | \mathcal{X}_1) - \mathbb{E}_O(\hat{R}_{IPS}(\hat{Y} | \mathcal{X}_1)) \\ &= \frac{1}{|\mathcal{X}_1|} \sum_x \delta_x \left(1 - \frac{O_x}{\hat{Z}_x}\right) \end{aligned} \quad (15)$$

Proposition 2 (Bias of the least favorable group's risk by IPS). *The bias of IPS risk suffered by the least favorable group \mathcal{U}_1 , has a lower bound as*

$$\text{Bias}(\hat{R}_{IPS}(\hat{Y} | \mathcal{X}_1)) \geq \left(1 - \frac{\bar{Y}_n}{\mu_1}\right) \frac{1}{|\mathcal{X}_1|} \sum_x \delta_x \quad (16)$$

Roughly speaking, the above inequality demonstrates that the \bar{Y}_n 's property matters. In particular, even under uniform policy, the bias for each group would be dependent on the preference demographics, as shown next.

5.2. Homogeneity: same mean and same variance

Intuitively, when randomly assigning items for users, under MAR condition, the \bar{Y}_n is a fair and efficient estimate for debiasing. However, we argue that this only holds under *homogeneous* and *weak heterogeneous* demographics.

Assumption 1 (Homogeneous demographics.). *Users' preference are same mean and same variance, namely the $Y_i \sim \mathcal{F}(\mu, \sigma^2) \forall i$.*

With above assumption, and by law of large numbers, it is easy to get the consistency of \bar{Y}_n , i.e. $\bar{Y}_n - \mu \xrightarrow{p} 0$. Recall the inequality in Eq.(16), the right term approaches to zero at this situation because $\mu_1 = \mu$, which holds for other groups as well. That is the reason why in our motivating example, uniform ϕ_t reaches good fairness. However, these homogeneity hardly happen in practice. A real-world recommender system often serves hundred million of users, such that it is reasonable to assume that the demographics of users are highly *heterogeneous*. We discuss it by two cases next, i.e. weak and strong heterogeneity.

5.3. Weak heterogeneity: with same mean but different variance

We begin our analysis from a simple case without loss of generality, in which users are from two distributions:

$\mathcal{F}(\mu_0, \sigma_0^2)$ and $\mathcal{F}(\mu_1, \sigma_1^2)$, with probability $p \in [0, 1]$ from the first and $1 - p$ from the second one. Formally, this is a common case in *Robust Statistics* (Maronna et al., 2019) called *contaminated distribution*, as

$$Y_i \sim p\mathcal{F}(\mu_0, \sigma_0^2) + (1 - p)\mathcal{F}(\mu_1, \sigma_1^2) \quad (17)$$

For simplicity, we assume that both are Gaussian distribution (we will extend it to more general sub-Gaussian family later). Based on Eq.(8) and Eq.(17), it is easy to derive that

$$\begin{aligned} \mathbb{E}(\bar{Y}_n) &= p\mu_0 + (1 - p)\mu_1 \\ \text{Var}(\bar{Y}_n) &= \frac{p\sigma_0^2 + (1 - p)\sigma_1^2}{n} \end{aligned} \quad (18)$$

We first consider the setting that groups' preferences are same mean but different variance (SMDV), namely $\mu_0 = \mu_1 = \mu$, and $\sigma_0 < \sigma_1$. We argue that only if variance is controllable, the RCTs under SMDV demographics lead to asymptotically normal and fair \bar{Y}_n :

Theorem 1 (Asymptotic Normality under SMDV). *Let $s_n^2 = \sum_{i=1}^n (p\sigma_0^2 + (1 - p)\sigma_1^2)$, under SMDV assumption, if*

$$\lim_{n \rightarrow \infty} \frac{1}{s_n^2} \sigma_1^2 \rightarrow 0 \quad (19)$$

then the empirical mean \bar{Y}_n satisfies asymptotic normality as

$$\frac{n}{s_n} (\bar{Y}_n - \mu) \xrightarrow{d} \mathcal{N}(0, 1) \quad (20)$$

Similar as in homogeneous demographics, with weak heterogeneity and mild condition on variance, the \bar{Y}_n also leads to fair result.

5.4. Strong heterogeneity: with different mean and different variance

Although the above analysis proves that RCTs do lead to fair estimator in theory, the fundamental challenge faced is that, no matter for homogeneity or weak heterogeneity, the *same mean* assumption seems to be too strong. This argument urges us to explore more general setting, that is, when the demographics are different mean and different variance (DMDV), how well the \bar{Y}_n performs, in terms of its bias and fairness. Accordingly, we continue to L DMDV groups with sub-Gaussian assumption next.

Theorem 2 (Tail bound under strong heterogeneity). *Given the $Y_i \sim \sum_{l=1}^L p_l \mathcal{F}(\mu_l)$, where $\mathcal{F}(\mu_l) \in [a_l, b_l]$, then*

$$\bar{Y}_n \leq \sum_{l=1}^L p_l \mu_l + \sum_{l=1}^L p_l (b_l - a_l) \sqrt{\frac{1}{2n} \log \frac{1}{\eta}} \quad (21)$$

with probability $1 - \eta$. Specifically when the sub-Gaussian parameters are all same, i.e. $b_l - a_l = 1, \forall l$, we have

$$\bar{Y}_n \leq \sum_{l=1}^L p_l \mu_l + \sqrt{\frac{1}{2n} \log \frac{1}{\eta}} \quad (22)$$

Moreover, For the bias of IPS estimator on the least favorable group, we have

$$\begin{aligned} & \text{Bias}(\hat{R}_{IPS}(\hat{Y} \mid \mathcal{X}_1)) \\ & \geq \sum_x \frac{\delta_x}{|\mathcal{X}_1|} - \frac{1}{\mu_1} \left(\sum_l p_l \mu_l + \sqrt{\frac{1}{2n} \log \frac{1}{\eta}} \right) \sum_x \frac{\delta_x}{|\mathcal{X}_1|} \end{aligned} \quad (23)$$

Above bound identifies that inaccurate propensities obtained by NB estimator, with \bar{Y}_n from RCTs as the outcome prior, contributes to the gap between the average and on \mathcal{X}_1 . Though the $\hat{R}_{IPS}(\hat{Y})$ is unbiased, its part on \mathcal{X}_1 is worse than other \mathcal{X}_l , since the μ_l matters for the lower bound of $\text{Bias}(\hat{R}_{IPS}(\hat{Y} \mid \mathcal{X}_l))$. For other multi-task learning or domain adaptation methods, we argue that this challenge appears as well, because small RCTs are only competent for average estimation, rather than representative for all groups.

This challenge inspires us to develop a machinery, which can correct MNAR bias without RCTs, as well as achieving better fairness for recommendation.

6. Beyond Randomized Trials

For ensuring fairness on known demographics, the disparity metrics are used by (?) to measure the outcome disparity conditioned by predefined group:

$$\text{KL}(\mathbb{P}_{\hat{Y}|X \in \mathcal{X}_1} \parallel \mathbb{P}_{\hat{Y}|X \notin \mathcal{X}_1}) \quad (24)$$

such that more fair model can be obtained by minimizing it. However, demographics of user's preference in recommendation are usually unknown, thus defining groups and optimizing the Eq.(24) directly is difficult.

To deal with unknown demographics, distributionally robust optimization (DRO) technique is employed for controlling the least favorable group's risk (Hashimoto et al., 2018), by minimizing the upper bound of the risk defining on an uncertainty distributions set. Some concerns of it are (1) the importance sampling scheme introduces additional variance in optimization; (2) the result is sensitive to the distance metric for defining the uncertainty set and (3) the DRO often degenerates to ERM (Hu et al., 2018) in many scenarios. In this section, we propose a more easy-to-use and effective counterfactual robust risk minimization framework to reduce utility disparity coping with unknown demographics. We first present the definition of objective function and algorithm of the framework, then give its theoretical generalization error bound.

6.1. Counterfactual robust risk minimization

One would expect that disparity could be controlled on the least favorable group \mathcal{X}_1 , by constructing a synthetic set \mathcal{X}'

Algorithm 1 Mini-batch SGD learning for CRRM

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1: Input: Event sets  $\mathcal{D}_F, \mathcal{D}_{CF}$ ; Batch size  $B_0, B_1$ ; Power iteration step  $J$ ; Epoch  $T$ ; Hyper parameters  $\lambda, \alpha, \xi, \epsilon, \omega$ .
2: Initialize model parameters  $\Theta^{(0)} = \{\Theta_U^{(0)}, \Theta_V^{(0)}\}$ 
3: for  $t = 1$  to  $T$  do
4:   /* Build the empirical risk term
5:    $\{\mathbf{x}_1, y_1\} \leftarrow \text{SampleMiniBatch}(\mathcal{D}_F, B_1; \Theta^{(t)})$ 
6:    $\hat{y}_F \leftarrow \text{Forward}(\mathbf{x}_1; \Theta^{(t)})$ 
7:    $\hat{R}_1 \leftarrow \frac{1}{B_1} \sum_i \delta(y_1^{(i)}, \hat{y}_1^{(i)}) + \lambda \Omega(\Theta^{(t)})$ 
8:   /* Build the counterfactual robust risk term
9:    $\mathbf{x}_0 \leftarrow \text{SampleMiniBatch}(\mathcal{D}_{CF} \cup \mathcal{D}_F, B_0; \Theta^{(t)})$ 
10:   $\hat{y}_0 \leftarrow \text{Forward}(\mathbf{x}_0; \Theta^{(t)})$ 
11:  Randomly generate an unit vector  $\mathbf{r}^{(0)}$ 
12:  for  $j = 1$  to  $J$  do
13:    /* Power iteration
14:     $\hat{y}_0' \leftarrow \text{Forward}(\mathbf{x}_0 + \xi \mathbf{r}^{(j)}; \Theta^{(t)})$ 
15:     $\mathbf{g} \leftarrow \nabla_{\mathbf{r}} \text{KL}(\hat{y}_0 \parallel \hat{y}_0')$ 
16:     $\mathbf{r}^{(j+1)} \leftarrow \epsilon \frac{\mathbf{g}}{\|\mathbf{g}\|_2}$ 
17:  end for
18:   $\hat{y}_0' \leftarrow \text{Forward}(\mathbf{x}_0 + \mathbf{r}^{(J)}; \Theta^{(t)})$ 
19:   $\hat{R}_0 \leftarrow \frac{1}{B_0} \text{KL}(\hat{y}_0 \parallel \hat{y}_0')$ 
20:  /* Learning from the joint CRR
21:   $\Theta^{(t+1)} \leftarrow \Theta^{(t)} - \omega \nabla_{\Theta}(\hat{R}_1 + \alpha \hat{R}_0)$ 
22: end for
    
```

from

$$\max_{\mathcal{X}'} \text{KL}(\mathbb{P}_{\hat{Y}|X \in \mathcal{X}'} \parallel \mathbb{P}_{\hat{Y}|X \in \mathcal{X}}) \quad (25)$$

where the \mathcal{X}' is the group that causes the steepest disparity increase. By optimizing on it, our model is encouraged to maintain uniform performance among groups. We formalize this intuition to a tractable optimization regime, by presenting a robust risk that

$$\tilde{\delta}_x := \max_{\|\mathbf{x}' - \mathbf{x}\| \leq \xi} \text{KL}(f(\mathbf{x}; \Theta) \parallel f(\mathbf{x}'; \Theta)) \quad (26)$$

where the $f : \mathcal{X} \rightarrow \mathbb{R}$ is a predictor parametrized by Θ . In this work, we pick the matrix factorization (MF) (Koren et al., 2009), hence the input \mathbf{x} is an embedding vector looked up from the parameter Θ , and the worst-case input \mathbf{x}' is restricted in a ball, with radius ξ around the original input \mathbf{x} . The intuition behind is that for each event $x \notin \mathcal{X}_1$, we find its counterfactual counterpart $x' \in \mathcal{X}_1$, then try to minimize the disparity between them. The objective function for our CRRM framework is defined by

$$\hat{R}_{CRR}(\hat{Y}) = \frac{1}{MN} \sum_x \mathbb{I}\{O_x = 1\} \delta_x + \tilde{\delta}_x \quad (27)$$

For observed events, we try to minimize both the robust risk $\tilde{\delta}_x$ and empirical risk δ_x ; for the unobserved, as δ_x is intractable, only $\tilde{\delta}_x$ is applied. The empirical risk can be defined as any surrogate loss for supervised learning, e.g. mean-squared-error. For the robust risk, it is tractable by a power iteration (Miyato et al., 2018) approach, albeit it

was used for improving local distributional smoothness for semi-supervised learning.

Algorithm 1 details our CRRM framework, where trainable embeddings for users and items are $\Theta_U \in \mathbb{R}^{M \times d}$ and $\Theta_V \in \mathbb{R}^{N \times d}$. The input hyper parameters include: regularization term λ , counterfactual risk weight α , robust radius ξ , perturbation radius ϵ , learning rate ω , power iteration step J and total learning epoch T , albeit few of them need being tuned in practice. Their setups are discussed in experiments section.

6.2. Bounding the risk over counterfactuals

As aforementioned, the CRRM tries to balance the predictor’s performance between a synthetic \mathcal{X}_1 and $\mathcal{X} \oplus \mathcal{X}_1$, by penalizing samples with small factual risk but large robust risk. We formally explain it by deriving the CRRM’s theoretical generalization error bound next.

Definition 2 (CRRM for Recommendation). *Given a hypothesis space \mathcal{H} of predictions as \hat{Y} , and CRRM’s objective function $\hat{R}(\hat{Y})$ from Eq.(27), the algorithm selects $\hat{Y} \in \mathcal{H}$ that optimizes:*

$$\hat{Y} = \arg \min_{Y' \in \mathcal{H}} \hat{R}_{CRR}(Y') \quad (28)$$

To illustrate the validity and principle of CRRM approach, we state the following bound, where only finite hypothesis set \mathcal{H} is considered for the sake of conciseness.

Theorem 3 (Generalization Error Bound of CRRM). *For any finite hypothesis space of predictions $\mathcal{H} = \{\hat{Y}_1, \dots, \hat{Y}_{|\mathcal{H}|}\}$, and empirical loss δ_x is bounded $0 \leq \delta_x \leq \Delta$, $\forall x$, the true risk $R(\hat{Y})$ of our CRR minimizer \hat{Y} from \mathcal{H} , is bounded with probability $1 - \eta$ by:*

$$\begin{aligned} R(\hat{Y}) &\leq \hat{R}_{CRR}(\hat{Y}) + \frac{1}{MN} \sum_x |\delta_x - \tilde{\delta}_x| \\ &\quad + \frac{\Delta}{MN} \sqrt{\frac{\log(2|\mathcal{H}|/\eta)}{2}} \sqrt{\sum_x \left(1 + \frac{1}{\beta_x}\right)^2} \end{aligned} \quad (29)$$

where $\beta_x := \delta_x / \tilde{\delta}_x$ and we assume $\beta_x > 0 \forall x$.

This generalization error bound is constructive, as it motivates a general principle for designing fair and effective estimator for learning from MNAR data, with the proposed CRRM. In particular, it not only emphasizes on minimizing the gap between δ_x^1 and δ_x^0 for reducing bias, but also points out the large variance incurred by small factual risk. As the bound shows, the bound can become loose due to small δ_x and relatively large $\tilde{\delta}_x$, such that the fairness controller $|\delta_x - \tilde{\delta}_x|$ even contributes to reducing variance, hence pick the hypothesis with more tight bound on the true risk $R(Y)$.

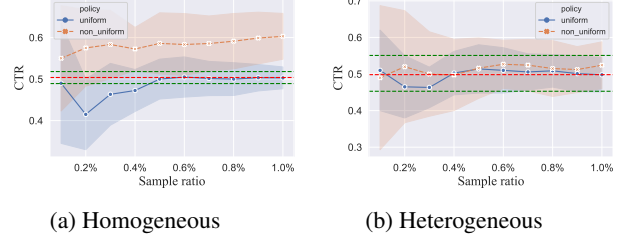


Figure 6. Estimate of \bar{Y}_n under two different setting of demographics, left: homogeneous, right: heterogeneous. Green dot lines are the maximum and minimum of μ_l among all groups, and red one is the average of all.

Table 2. mU and gU vary under data sparsity.

Sparsity	0.1%		0.5%		1%	
Metric	mU	gU	mU	gU	mU	gU
LR	0.27	0.53	0.04	0.47	0.05	0.51
MF	0.32	0.50	0.29	0.52	0.20	0.52
IPS	0.39	0.53	0.34	0.52	0.27	0.52
SNIPS	0.40	0.54	0.32	0.50	0.30	0.51
CRRM	0.47	0.60	0.36	0.60	0.23	0.57

7. Experiments

In this section, we perform experiments to explore the performance of proposed CRRM, in terms of both global utility and Gini fairness. Furthermore, additional experiments on real-world data sets (Yahoo¹ and Coat) are conducted to compare CRRM with other methods, according to the commonly used measurements, e.g. MSE, AUC. We here consider logistic regression (LR), matrix factorization (MF-naive), inverse propensity score based MF (MF-IPS) (Schnabel et al., 2016), self-normalized IPS (MF-SNIPS) (Swaminathan & Joachims, 2015) and MF incorporated in our CRRM framework (MF-CRRM).

7.1. Synthetic task

The experimental setup is similar as the motivating example done in Section 4.1, but we supplement the heterogeneous demographics setting, i.e. each user’s average preference is different, and extend the scale to $M = 10$, $N = 1000$. For experiments on the non-uniform ϕ_c , each time ϕ_c decides the top 1 item for each user, and collects the users’ feedback. Then it learns from the logged data, and continues to give next round of top 1 recommendation.

How does demographic heterogeneity influence \bar{Y}_n ’s property by RCTs and ERM-based policy? We want to compare the performance of \bar{Y}_n with ϕ_t and ϕ_c , with varying heterogeneity, in the synthetic task. The results are shown in Fig.6, where the x-axis is the observed events ratio. With homogeneous demographics in Fig.6a, the \bar{Y}_n obtained by ERM policy is much higher than the global average, and

¹<https://webscope.sandbox.yahoo.com/>

Table 3. Test MSE and AUC on real-world data sets.

	COAT		YAHOO	
	MSE	AUC	MSE	AUC
MF-naive	0.656	0.681	0.340	0.687
MF-IPS	0.714	0.687	0.332	0.666
MF-SNIPS	0.605	0.684	0.325	0.667
MF-CRRM	0.277	0.717	0.216	0.713

Table 4. Test GI and gU on real-world data sets.

	COAT		YAHOO	
	GI	gU	GI	gU
MF-naive	0.351	0.486	0.549	0.274
MF-IPS	0.333	0.508	0.558	0.264
MF-SNIPS	0.341	0.501	0.559	0.264
MF-CRRM	0.316	0.538	0.538	0.290

ϕ_t converges to global average with high confidence. However, with strongly heterogeneous demographics defined in section 5.4, the Fig.6b displays high variance of the ϕ_t . Unlike in homogeneous setting, the blue band converges much slower with sampling ratio increasing. Besides, the orange line (ϕ_c) also has deteriorating performance compared with before. The result demonstrates that heterogeneity of demographics can cause failure of RCTs-based IPS because of high variance incurred by \bar{Y}_n , and also do harm to ERM-based ϕ_c in practice.

Does CRRM improve fairness and generalization? Results on strong demographic heterogeneity are reported in Table 2, where \mathbf{mU} is the minimum of user’s enjoyed utility. We test our CRRM method against LR, naive MF and IPS-based MF, and significant superiority of our method is witnessed. The MF-CRRM substantially perform better on both fairness and effectiveness in general, as it gains the best \mathbf{mU} and \mathbf{gU} almost under each sparsity setting. By contrast, MF-IPS with propensity scores learned from MAR data with 10% of number of MANR data, does not win over naive MF, due to high variance caused by \bar{Y}_n under heterogeneous demographics.

7.2. Real-world data sets

Further experiments conducted on two real-world data sets, which both collected a small test set where users are provided randomly displayed items for labeling. Following previous works’ setting, we train all models on MNAR training set \mathcal{D}_c , and test them on MAR test set. More details about experimental setup are in appendix F.2.

Yahoo ! R3 Data Set. This is a user-song rating data set (Marlin & Zemel, 2009), where MNAR training set has over 300K ratings for songs that were selected by users. The test set contains ratings by 5400 users who were asked to rate

10 randomly selected songs.

Coat Shopping Data Set. This data set was collected and used in (Schnabel et al., 2016), which contains 290 users and 300 items. Each user self-selected 24 items, and additional 16 items were randomly drawn for them to rate.

Results. Results are reported by Table 3 and Table 4, where our CRRM substantially wins over other baselines on both data sets, either in effectiveness metric \mathbf{gU} , MSE, AUC or in fairness metric \mathbf{GI} . Unlike other IPS-based methods, CRRM has much lower cost since it does not need additional RCTs. It contributes to less variance via controlling both robust risk and factual risk, thus facilitating generalization without RCTs, as presented in Theorem 3. It should be noted that on YAHOO data set, for computation efficiency, our CRRM only sampling 0.1% from $\mathcal{D}_F \cup \mathcal{D}_{CF}$ for optimizing on the \hat{R}_0 in each epoch, which sheds light on our CRRM’s promising prospects in real-world recommender system.

8. Discussion & Conclusion

In this work, we focus on mitigating MNAR bias and unfair policy, which are ubiquitous in recommendation. Previous works tend to leverage empirical outcome mean \bar{Y}_n from small RCTs for correcting MNAR bias, e.g. IPS family. However, our study on the bias of these RCTs-based IPS estimator, with introducing more general different mean demographics, identifies the risk estimator with \bar{Y}_n sacrifices the least favorable group for ensuring average performance.

We therefore propose an easy-to-use and RCTs-free counterfactual robust risk minimization framework, in order to circumvent the cost of RCTs, and mitigate the unfairness from previous works. The key insight of our CRRM is to balance between robust and factual risk, thus both reducing bias and facilitating fairness, explained by the generalization bound in our Theorem 3. Moreover, by random sampling when alternative events set is large, our algorithm only costs a little additional computation, thus being scalable to large recommender systems.

We conjecture that the cutting edge sampling approaches (Wang et al., 2020), rather than randomly sampling from all events, can be plugged in our framework for further improvement. Besides, other existing models for recommendation, e.g. deep factorization machine (Guo et al., 2017), graph neural networks (Fan et al., 2019), can be retrofit with our CRRM for even better results.

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Beyond Randomized Trials: Appendix

A. Proof of Proposition 1

Proposition 1 (Least favorable group's utility and Gini index). *Assume that $\hat{\mu}_1 \leq \hat{\mu}_2 \leq \dots \leq \hat{\mu}_L$, controlling the worst-case $\hat{\mu}_1$ to protect the minority group can improve fairness, by virtue of controlling the supremum of the **GI** as*

$$0 \leq \mathbf{GI} \leq 1 - \frac{1}{L} - \frac{L-1}{\mathbf{Cum}_L} \hat{\mu}_1 \quad (\text{A.1})$$

if and only if $\hat{\mu}_1 = \hat{\mu}_2 = \dots = \hat{\mu}_L$ the equality holds.

Proof.

$$\mathbf{GI} = 1 - \frac{1}{L \times \mathbf{Cum}_L} \sum_{l=1}^L \left(\sum_{l'=1}^{l-1} \hat{\mu}_{l'} + \sum_{l'=1}^l \hat{\mu}_{l'} \right) \quad (\text{A.2})$$

$$= 1 - \frac{1}{L \times \mathbf{Cum}_L} \left(2 \sum_{l=1}^L \sum_{l'=1}^{l-1} \hat{\mu}_{l'} + \sum_{l=1}^L \hat{\mu}_l \right) \quad (\text{A.3})$$

$$= 1 - \frac{1}{L} - \frac{2}{L \times \mathbf{Cum}_L} \times \sum_{l=1}^L \sum_{l'=1}^{l-1} \hat{\mu}_{l'} \quad (\text{A.4})$$

$$\leq 1 - \frac{1}{L} - \frac{L-1}{\mathbf{Cum}_L} \hat{\mu}_1 \quad (\text{A.5})$$

where the last inequality holds because

$$\sum_{l=1}^L \sum_{l'=1}^{l-1} \hat{\mu}_{l'} \geq \sum_{l=1}^L (l-1) \hat{\mu}_1 = \frac{L(L-1)}{2} \hat{\mu}_1 \quad (\text{A.6})$$

□

B. Proof of Proposition 2

Proposition 2 (Bias of the least favorable group's risk). *The bias of IPS risk suffered by the least favorable group \mathcal{U}_1 , has a lower bound as*

$$\mathbf{Bias}(\hat{R}_{IPS}(\hat{Y} \mid \mathcal{X}_1)) \geq \left(1 - \frac{\bar{Y}_n}{\mu_1} \right) \frac{1}{|\mathcal{X}_1|} \sum_x \delta_x \quad (\text{B.1})$$

Proof. It is easy to derive that

$$O_x = P(O_+ \mid y, x) = \frac{P(y \mid O_+, x) P(O_+ \mid x)}{P(y \mid x)} \quad (\text{B.2})$$

$$\hat{Z}_x = \hat{P}(O_+ \mid y) = \frac{\hat{P}(y \mid O_+) \hat{P}(O_+)}{\hat{P}(y)} \quad (\text{B.3})$$

Such that, we concentrate on O_x/\hat{Z}_x in the Eq.(15):

$$\frac{O_x}{\hat{Z}_x} = \frac{P(y | O_+, x)}{\hat{P}(y | O_+)} \times \frac{P(O_+ | x)}{\hat{P}(O_+)} \times \frac{\hat{P}(y)}{P(y | x)} \quad (\text{B.4})$$

$$\leq \frac{\hat{P}(y)}{P(y | x)} \quad (\text{B.5})$$

$$= \frac{1}{\mu_1} \hat{P}(y) \quad (\text{B.6})$$

The second inequality Eq.(B.5) comes from the fact that the least favorable group \mathcal{U}_1 's true average preference must be less than the empirical average. Combining the result above and Eq.(15) yields the result. \square

C. Proof of Theorem 1

Definition C.1 (Triangular Array (TA)). *A triangular array of random variables is like*

$$\begin{array}{ccccc} X_{11} & & & & \\ X_{21} & X_{22} & & & \\ X_{31} & X_{32} & X_{33} & & \\ \vdots & & & & \end{array} \quad (\text{C.1})$$

For each row n , the random variables $X_{n1} \dots X_{nn}$ are assumed independent, with $\mathbb{E}(X_{nk}) = \mu_{nk}$, and $\text{Var}(X_{nk}) < \infty$. We denote this triangular array as $\text{TA}(X_{n,k})$ henceforth.

Definition C.2 (Lindeberg's condition (Zabell, 1995)). *For random variables Y_1, Y_2, \dots, Y_n , we have $s_n^2 = \sum_{i=1}^n \sigma_i^2$ where the variance σ_i^2 of Y_i is finite $\forall i$. We define a triangular array as $\text{TA}(\frac{1}{s_n}(Y_i - \mu))$, which satisfies Lindeberg's condition if for*

$$\Lambda_n(\epsilon) = \sum_{i=1}^n \mathbb{E} \left(\left(\frac{Y_i}{s_n} \right)^2 \mathbb{I}\{|Y_i| \geq s_n \epsilon\} \right) \quad (\text{C.2})$$

we have

$$\lim_{n \rightarrow \infty} \Lambda_n(\epsilon) = 0 \quad (\text{C.3})$$

Lemma C.1 (Lindeberg's condition under SMDV). *Given $Y_i \sim p\mathcal{F}(\mu, \sigma_0^2) + (1-p)\mathcal{F}(\mu, \sigma_1^2)$, and $\sigma_1 > \sigma_0 \geq 1$, if the variance is controllable, namely*

$$\lim_{n \rightarrow \infty} \frac{1}{s_n^2} \sigma_1^2 \rightarrow 0 \quad (\text{C.4})$$

then the $\text{TA}(\frac{1}{s_n}(Y_i - \mu))$ satisfies Lindeberg's condition.

Proof. Assume without loss of generality that $\mu = 0$, and let Y a random variable distributed like every Y_i , then for $\epsilon > 0$,

$$\Lambda_n(\epsilon) = \sum_{i=1}^n \mathbb{E} \left(\left(\frac{Y_i}{s_n} \right)^2 \mathbb{I}\{|Y_i| \geq s_n \epsilon\} \right) \quad (\text{C.5})$$

$$= \frac{p}{s_n^2} \sum_{i=1}^n \mathbb{E} \left(Y^2 \mathbb{I}\{|Y| \geq \frac{s_n \epsilon}{\sigma_0}\} \right) + \frac{1-p}{s_n^2} \sum_{i=1}^n \mathbb{E} \left(Y^2 \mathbb{I}\{|Y| \geq \frac{s_n \epsilon}{\sigma_1}\} \right) \quad (\text{C.6})$$

$$\leq \frac{n}{s_n^2} \mathbb{E} \left(Y^2 \mathbb{I}\{|Y| \geq \epsilon \sqrt{\frac{s_n^2}{\sigma_0^2}}\} \right) + \frac{n}{s_n^2} \mathbb{E} \left(Y^2 \mathbb{I}\{|Y| \geq \epsilon \sqrt{\frac{s_n^2}{\sigma_1^2}}\} \right) \quad (\text{C.7})$$

$$\leq \mathbb{E} \left(Y^2 \mathbb{I}\{|Y| \geq \epsilon \sqrt{\frac{s_n^2}{\sigma_0^2}}\} \right) + \mathbb{E} \left(Y^2 \mathbb{I}\{|Y| \geq \epsilon \sqrt{\frac{s_n^2}{\sigma_1^2}}\} \right) \quad (\text{C.8})$$

in which the second inequality holds because it is easy to derive that $s_n^2 \geq n$. When $n \rightarrow \infty$, the last two terms converge to zero, because of the condition set in Eq.(C.4). \square

Theorem 1 (Asymptotic Normality under SMDV). *Let $s_n^2 = \sum_{i=1}^n (p\sigma_0^2 + (1-p)\sigma_1^2)$, under SMDV assumption, if*

$$\lim_{n \rightarrow \infty} \frac{1}{s_n^2} \sigma_1^2 \rightarrow 0 \quad (\text{C.9})$$

then the empirical mean \bar{Y}_n satisfies asymptotic normality as

$$\frac{n}{s_n} (\bar{Y}_n - \mu) \xrightarrow{d} \mathcal{N}(0, 1) \quad (\text{C.10})$$

Proof. It is easy to derive that the summation over the n -th row of $\text{TA}(\frac{1}{s_n}(Y_i - \mu))$ is $\frac{n}{s_n}(\bar{Y}_n - \mu)$. Based on the condition set by Eq.(C.9), applying Lemma C.1 yields the result that $\text{TA}(\frac{1}{s_n}(Y_i - \mu))$ satisfies Lindeberg's condition, namely

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \mathbb{E} \left(\left(\frac{Y_i}{s_n} \right)^2 \mathbb{I}\{|Y_i| \geq s_n \epsilon\} \right) = 0 \quad (\text{C.11})$$

From the Lindeberg-Feller Central Limit Theorem (CLT) (Feller, 2008), the summation over the each row of $\text{TA}(\frac{1}{s_n}(Y_i - \mu))$ converges to a zero-mean unit variance Gaussian distribution, namely

$$\sum_{i=1}^n \left(\frac{1}{s_n} (Y_i - \mu) \right) = \frac{n}{s_n} (\bar{Y}_n - \mu) \xrightarrow{d} \mathcal{N}(0, 1) \quad (\text{C.12})$$

□

D. Proof of Theorem 2

First, we introduce Hoeffding's lemma on bounded random variables (Hoeffding, 1994) without proof.

Lemma D.1 (Hoeffding's lemma (Hoeffding, 1994)). *if $a \leq X \leq b$ with probability 1, and $\mathbb{E}(X) = 0$, then X is sub-Gaussian with parameter $\frac{(b-a)^2}{4}$.*

Theorem 2 (Tail bound under strong heterogeneity). *Given the $Y_i \sim \sum_{l=1}^L p_l \mathcal{F}(\mu_l)$, where $\mathcal{F}(\mu_l) \in [a_l, b_l]$, then*

$$\bar{Y}_n \leq \sum_{l=1}^L p_l \mu_l + \sum_{l=1}^L p_l (b_l - a_l) \sqrt{\frac{1}{2n} \log \frac{1}{\eta}} \quad (\text{D.1})$$

with probability $1 - \eta$. Specifically when the sub-Gaussian parameters are all same, i.e. $b_l - a_l = 1, \forall l$, we have

$$\bar{Y}_n \leq \sum_{l=1}^L p_l \mu_l + \sqrt{\frac{1}{2n} \log \frac{1}{\eta}} \quad (\text{D.2})$$

Moreover, For the bias of IPS estimator on the least favorable group, we have

$$\begin{aligned} & \text{Bias}(\hat{R}_{IPS}(\hat{Y} \mid \mathcal{X}_1)) \\ & \geq \sum_x \frac{\delta_x}{|\mathcal{X}_1|} - \frac{1}{\mu_1} \left(\sum_l p_l \mu_l + \sqrt{\frac{1}{2n} \log \frac{1}{\eta}} \right) \sum_x \frac{\delta_x}{|\mathcal{X}_1|} \end{aligned} \quad (\text{D.3})$$

Proof. From the definition of Y_i and \bar{Y}_n , it is easy to derive

$$\mathbb{E}(\bar{Y}_n) = \mathbb{E}(Y_i) = \sum_{l=1}^L p_l \mu_l \quad (\text{D.4})$$

And the interval of Y_i is

$$Y_i \in \left[\underbrace{\sum_{l=1}^L p_l a_l}_{\tilde{a}_i}, \underbrace{\sum_{l=1}^L p_l b_l}_{\tilde{b}_i} \right] \quad (\text{D.5})$$

Applying Hoeffding's inequality for Y_n yields

$$P(\bar{Y}_n - \mathbb{E}(\bar{Y}_n) \geq \epsilon) \leq \exp\left(\frac{-2n^2\epsilon^2}{\sum_{i=1}^n (\tilde{b}_i - \tilde{a}_i)^2}\right) = \eta \quad (\text{D.6})$$

therefore

$$\bar{Y}_n - \mathbb{E}(\bar{Y}_n) \leq \sum_{l=1}^L p_l(b_l - a_l) \sqrt{\frac{1}{2n} \log \frac{1}{\eta}} \quad (\text{D.7})$$

□

E. Proof of Theorem 3

Lemma E.1 (Bias of Counterfactual Robust Risk). *Let δ_x the empirical risk, and $\tilde{\delta}_x$ the robust risk, for bounded $\delta_x \leq \Delta$, we have*

$$\mathbf{Bias}(\hat{R}_{CRR}(\hat{\mathbf{Y}})) \leq \frac{1}{MN} \sum_x |\delta_x - \tilde{\delta}_x| \quad (\text{E.1})$$

Proof.

$$\mathbf{Bias}(\hat{R}_{CRR}(\hat{\mathbf{Y}})) = R(\hat{\mathbf{Y}}) - \mathbb{E}_O(\hat{R}_{CRR}(\hat{\mathbf{Y}})) \quad (\text{E.2})$$

$$= \frac{1}{MN} \sum_x \delta_x - \mathbb{E}_O \left(\sum_x \frac{1}{MN} (\mathbb{I}\{O_x = 1\} \delta_x + \tilde{\delta}_x) \right) \quad (\text{E.3})$$

$$= \frac{1}{MN} \sum_x (\delta_x - (O_x \delta_x + \tilde{\delta}_x)) \quad (\text{E.4})$$

$$= \frac{1}{MN} \sum_x ((1 - O_x) \delta_x - \tilde{\delta}_x) \quad (\text{E.5})$$

$$\leq \frac{1}{MN} \sum_x |\delta_x - \tilde{\delta}_x| \quad (\text{E.6})$$

because $O_x \in [0, 1]$. □

Theorem 3 (Generalization Error Bound of CRRM). *For any finite hypothesis space of predictions $\mathcal{H} = \{\hat{\mathbf{Y}}_1, \dots, \hat{\mathbf{Y}}_{|\mathcal{H}|}\}$, and empirical loss δ_x is bounded $\delta_x \leq \Delta$, $\forall x$, the true risk $R(\hat{\mathbf{Y}})$ of our CRR minimizer $\hat{\mathbf{Y}}$ from \mathcal{H} , is bounded with probability $1 - \eta$ by:*

$$\begin{aligned} R(\hat{\mathbf{Y}}) &\leq \hat{R}_{CRR}(\hat{\mathbf{Y}}) + \frac{1}{MN} \sum_x |\delta_x - \tilde{\delta}_x| \\ &\quad + \frac{\Delta}{MN} \sqrt{\frac{\log(2|\mathcal{H}|/\eta)}{2}} \sqrt{\sum_x \left(1 + \frac{1}{\beta_x}\right)^2} \end{aligned} \quad (\text{E.7})$$

where $\beta_x := \delta_x / \tilde{\delta}_x$ and we assume $\beta_x > 0 \forall x$.

Proof. For notation simplicity, we omit the subscript *CRR* next.

$$R(\hat{\mathbf{Y}}) = R(\hat{\mathbf{Y}}) - \mathbb{E}_O(\hat{R}(\hat{\mathbf{Y}})) + \mathbb{E}_O(\hat{R}(\hat{\mathbf{Y}})) \quad (\text{E.8})$$

$$= \mathbf{Bias}(\hat{R}(\hat{\mathbf{Y}})) + \mathbb{E}_O(\hat{R}(\hat{\mathbf{Y}})) \quad (\text{E.9})$$

$$\leq \frac{1}{MN} \sum_x |\delta_x - \tilde{\delta}_x| + \mathbb{E}_O(\hat{R}(\hat{\mathbf{Y}})) \quad (\text{E.10})$$

which follows from Lemma E.1. Considering Hoeffding inequality and union bounds, we are now going to bound the following

$$\mathbb{P} \left(|\hat{R}(\hat{\mathbf{Y}}) - \mathbb{E}_O \hat{R}(\hat{\mathbf{Y}})| \leq \epsilon \right) \geq 1 - \eta \quad (\text{E.11})$$

$$\Leftrightarrow \mathbb{P} \left(\max_{\hat{\mathbf{Y}}_h} |\hat{R}(\hat{\mathbf{Y}}_h) - \mathbb{E}_O \hat{R}(\hat{\mathbf{Y}}_h)| \leq \epsilon \right) \geq 1 - \eta \quad (\text{E.12})$$

$$\Leftrightarrow \mathbb{P} \left(\bigcup_{\hat{\mathbf{Y}}_h} |\hat{R}(\hat{\mathbf{Y}}_h) - \mathbb{E}_O \hat{R}(\hat{\mathbf{Y}}_h)| \geq \epsilon \right) < \eta \quad (\text{E.13})$$

$$\Leftrightarrow \sum_{h=1}^{|\mathcal{H}|} \mathbb{P} \left(|\hat{R}(\hat{\mathbf{Y}}_h) - \mathbb{E}_O \hat{R}(\hat{\mathbf{Y}}_h)| \geq \epsilon \right) < \eta \quad (\text{E.14})$$

$$\Leftrightarrow |\mathcal{H}| \times 2 \exp \left(\frac{-2\epsilon^2 M^2 N^2}{\sum_x \delta_x^2 (1 + \frac{1}{\beta_x})^2} \right) < \eta \quad (\text{E.15})$$

solving for ϵ yields the bound that

$$|\hat{R}(\hat{\mathbf{Y}}) - \mathbb{E}_O \hat{R}(\hat{\mathbf{Y}})| \leq \frac{1}{MN} \sqrt{\frac{\log(2|\mathcal{H}|/2)}{2}} \sqrt{\sum_x \delta_x^2 (1 + \frac{1}{\beta_x})^2} \quad (\text{E.16})$$

As we know that $\delta_x \leq \Delta$, so it is easy to derive the final bound as

$$\mathbb{E}_O \hat{R}(\hat{\mathbf{Y}}) \leq \hat{R}(\hat{\mathbf{Y}}) + \frac{\Delta}{MN} \sqrt{\frac{\log(2|\mathcal{H}|/\eta)}{2}} \sqrt{\sum_x (1 + \frac{1}{\beta_x})^2} \quad (\text{E.17})$$

Substitute $\mathbb{E}_O \hat{R}(\hat{\mathbf{Y}})$ in Eq.(E.10) yields the final result. \square

F. Experimental setup

F.1. Simulation Tasks

For motivating example, we build a 5×5 matrix as the users' preference over each item, where each element is generated uniformly within $[0, 1]$, so they are homogeneous. For simulating user's behavior, we introduce an additional latent variable uniformly drawn from $[0, 1]$, such that for each time an event x appears, the click happens when the latent variable is smaller than y . We adopt a bandit feedback setting here, that is, the policy ϕ_c firstly ranks the Top-1 predicted reward item for each user, collects the feedbacks as click or not click; then we do policy learning via ERM from these logged feedbacks. For controlled experiment, we perform the same setting for an uniform policy ϕ_t as well.

After 100 epochs of logging feedbacks and learning policy, we can calculate the empirical outcome distribution from the observed logged data, e.g. $\hat{P}_1^c(Y | X)$ from \mathcal{D}_c and $\hat{P}_1^t(Y | X)$ from \mathcal{D}_t . The Fig.F.1 demonstrates the $\hat{P}_1^c(Y | X)$, generated by the non-uniform policy ϕ_c , compared with the true $P(Y | X)$. Significant bias of the estimated $\hat{P}_1^c(Y | X)$ is depicted in Fig.F.2b, which is caused by inaccurately modeling on the observed events, with non-uniform policy and assuming that $\hat{P}_1^c(Y | X) = P(Y | X)$. From Fig.3b we observe that the ϕ_c causes devastating reduction of *diversity* to the system, all items except for item #0 are underrepresented, because they are relatively less interested by the users in average. The insufficient exploration over items causes large bias of estimated marginal distribution over items, shown in Fig.F.2a, i.e. bias of $\hat{P}_1^c(Y | V = 0)$ is near zero, while others' are large.

F.2. Real-world data

The original data set contains ratings from 1 to 5, we binarize it by setting 3 as threshold. For fair comparison, we set same hyper-parameters for all used MF-based methods: regularization term $\lambda = 10^{-3}$, learning rate $\omega = 0.05$, and embedding size $d = 4$. Specifically for our CRRM, we maintain $\epsilon = 0.1$, $\xi = 1$, $\alpha = 1$, and $J = 1$ all the time. For the IPS methods, propensity scores are learned by naive Bayes estimator from 5% of MAR test set.

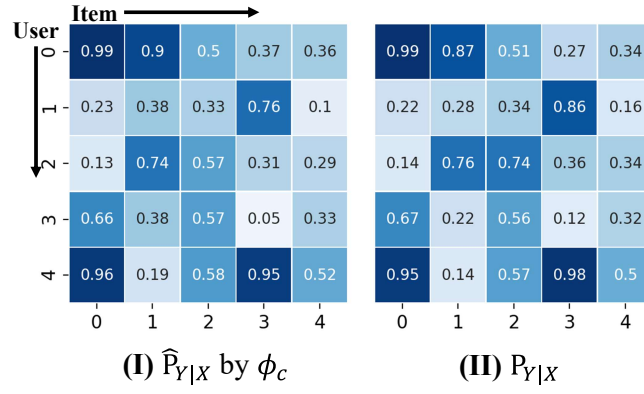


Figure F.1. Comparison between the generated preference and empirically estimated preference by ϕ_c .

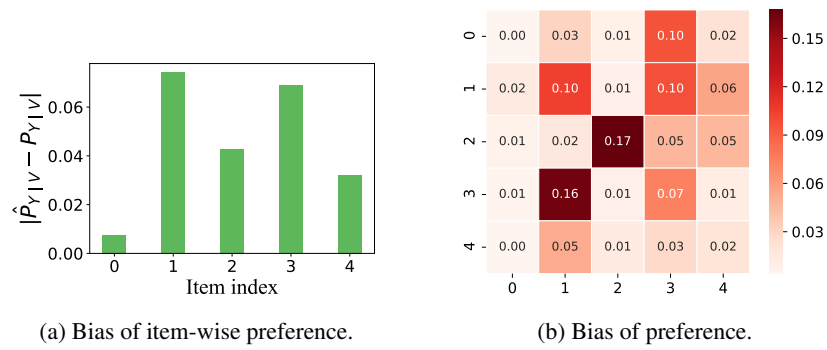


Figure F.2. Analysis of simulation results of the motivating example.