MMF2000 RISK MANAGEMENT ASSIGNMENT 1

Regulatory Capital and Climate Risk

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A. Balance sheet dynamics and RWA

1. Consider the toy balance sheet we discussed during the Introductory lecture on page 13.

Imagine there is a step 5, consisting of a mortgage assets of $500k defaulting, leading to a cash recovery of $300k and therefore a credit loss of $200k.

a. Show what the balance sheet looks like after this event, including the E/A ratio.

A new bank – investors provide $1 Million.

Assets	Liabilities
Assets $1	Liabilities $0
	Equity $1

E/A = 100%

Step 1: We raise $2 Million in deposits.

Assets	Liabilities
Assets $3	Liabilities $2
	Equity $1

E/A = 33.3%

Step 2: We issue $2.5 Million in mortgages.

Assets	Liabilities
Assets $2.5 mortgages $0.5 cash	Liabilities $2
	Equity $1

E/A = 33.3%

Step 3: We earn $0.2 Million after a year.

Assets	Liabilities
Assets $2.5 mortgages $0.7 cash	Liabilities $2
	Equity $1.2

E/A = 37.5%

Step 4: We return \$0.1 Million as a dividend.

Assets	Liabilities
\$2.5 mortgages \$0.6 cash	\$2
	Equity \$1.1

E/A = 35.5%

Step 5: There consists of a mortgage asset of \$0.5 Million defaulting, leading to a cash recovery of \$0.3 Million and therefore a credit loss of \$0.2 Million.

Assets	Liabilities
\$2.5 – \$0.5 = \$2 mortgages \$0.6 + \$0.3 = \$0.9 cash	\$2
	Equity \$1.1 - \$0.2 = \$0.9

E/A = \$0.9/(\$2+\$0.9) = \$0.9/\$2.9 = 31.0%

2. **Now consider a step 6 where we borrow another \$1MM and lend \$700k to a corporate borrower, use \$500k to purchase securitizations, and purchase \$200k in Cdn. government securities.**

b. **Show what the balance sheet looks like after this, including the E/A ratio.**

Step 6: We borrow another \$1 Million and lend \$0.7 Million to a corporate borrower, use \$0.5 Million to purchase securitizations, and purchase \$0.2 Million in Cdn. government securities.

Assets	Liabilities
\$2 mortgages \$0.9 + \$1 - \$0.7 - \$0.5 - \$0.2 = \$0.5 cash	\$2 + \$1 = \$3
\$0.7 corporate loan \$0.5 securitizations \$0.2 Cdn. Government securities	Equity \$0.9

E/A = \$0.9/(\$2 + \$0.5 + \$0.7 + \$0.5 + \$0.2) = \$0.9/\$3.9 = 23.1%

c. **Calculate the RWA of the final balance sheet and the corresponding RWA ratio (i.e. E/RWA.) (use the table on slide 14.)**

The RWA using the weights on slide 14 is

$$RWA = \$2 \times 50\% + \$0.5 \times 0\% + \$0.7 \times 100\% + \$0.5 \times 20\% + \$0.2 \times 0\% = \$1.8$$

The RWA ratio is

$$\frac{E}{RWA} = \frac{\$0.9}{\$1.8} = 50.0\%$$

B. RWA using OSFI scenario parameters

In this document OSFI documents how they want banks to model PD and LGD parameters in the climate scenario exercise they are planning. We shall map out in this problem how use of the climate adjusted parameters would affect RWA for a particular loan.

Looking at section 3.4.4, there are discussions regarding how to adjust the PD and the LGD to reflect climate effects. In fact, OSFI intends that they be used for provisioning (also known as ECL) which we did not discuss in much detail at all. So for the purpose of this exercise, we will assume that the same adjustments apply to the PD and LGD parameters used for RWA. I will refer to climateRWA to mean the RWA calculated from the formula I presented in the previous lecture, but using climatePD and climateLGD as inputs.

To keep things simple, assume M=1 and R=0.15 in the RWA formula I presented in the first lecture (i.e. ignore the PD dependence of R, which is important in general but is just a distraction in this context.)

As you see, the impact of climate change is all mediated by a single parameter which OSFI calls climateAdd-on$_i$ which I will shorten to the variable name c, for brevity. (I am not a fan of using words as variables when doing math, but that is how things are in this world.) OSFI points to a placeholder table which they haven't yet filled in for hypothetical c values.

d. Show that when c=0, both PD and LGD are unaffected: that climatePD = PD and climateLGD = LGD.

When $\text{climateAdd-on} = c = 0$, climatePD becomes

$$\begin{aligned}
\text{climatePD} &= \frac{1}{1+e^{-\text{logit(PD)}}} \\
&= \frac{1}{1+e^{-ln\frac{\text{PD}}{1-\text{PD}}}} \\
&= \frac{1}{1+e^{ln\frac{1-\text{PD}}{\text{PD}}}} \\
&= \frac{1}{1+\frac{1-\text{PD}}{\text{PD}}} \\
&= \frac{1}{\frac{\text{PD}+1-\text{PD}}{\text{PD}}} \\
&= \text{PD}
\end{aligned}$$

Since when $c = 0$, $\text{climatePD} = \text{PD}$, then climateLGD becomes

$$\begin{aligned}
\text{climateLGD} &= \frac{\phi[\phi^{-1}(PD) - \phi^{-1}(\text{PD}) + \phi^{-1}(\text{PD}\times\text{LGD})]}{\text{PD}} \\
&= \frac{\phi\left(\phi^{-1}(\text{PD}\times\text{LGD})\right)}{\text{PD}} \\
&= \frac{\text{PD}\times\text{LGD}}{\text{PD}} \\
&= \text{LGD}
\end{aligned}$$

Therefore, we have proved that when $c = 0$, climatePD = PD, climateLGD = LGD.

e. Consider a hypothetical loan with PD=1%, LGD=40% and EAD=$1 million. Calculate its base RWA using the formula I presented in the first lecture but as modified per my instruction above (M=1 and R=0.15). Next plot the ratio of climateRWA to base RWA as a function of c for c in the range from 0 to 2.

Since $PD = 1\%$, we have

$$b = (0.11852 - 0.05478 \times \ln PD)^2 = (0.11852 - 0.05478 \times \ln 1\%)^2 = 0.1375$$

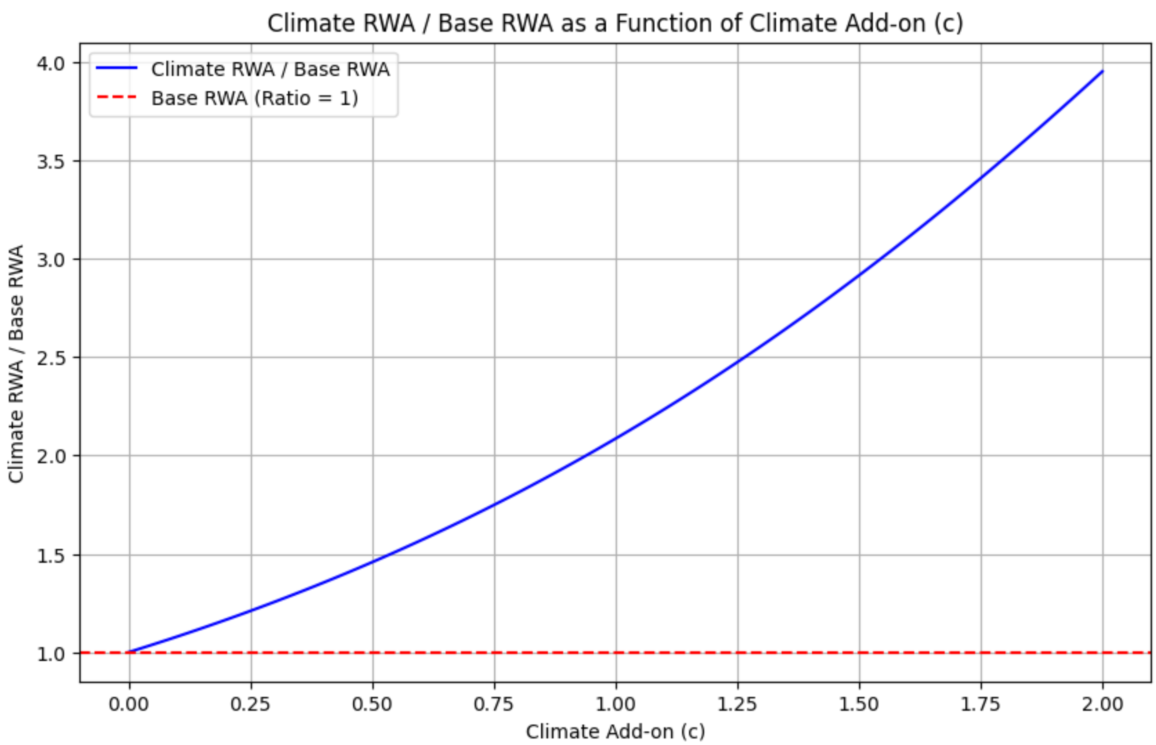
Substituting $PD = 1\%$, $LGD = 40\%$, $M = 1$, and $R = 0.15$, the capital requirement is

$$\begin{aligned} K &= \left[LGD \times N\left(\frac{N^{-1}(PD) + \sqrt{R} \times N^{-1}(0.999)}{\sqrt{1-R}}\right) - (LGD \times PD)\right] \times \left(\frac{1 + (M - 2.5) \times b}{1 - 1.5 \times b}\right) \\ &= \left[40\% \times N\left(\frac{N^{-1}(1\%) + \sqrt{0.15} \times N^{-1}(0.999)}{\sqrt{1-0.15}}\right) - (40\% \times 1\%)\right] \times \left(\frac{1 + (1 - 2.5) \times 0.1375}{1 - 1.5 \times 0.1375}\right) \\ &= 0.0401 \end{aligned}$$

Then we calculate the base RWA:

$$RWA = 12.5 \times EAD \times K = 12.5 \times \$1{,}000{,}000 \times 0.0401 = \$501{,}323.78$$

The plot of ratio of climateRWA and base RWA as a function of c for c in the range from 0 to 2 is



f. Provide some commentary regarding the ability of OSFI's parameterisation to affect RWA.

OSFI's parameterisation lies in adjusting credit risk parameters like PD and LGD by adding "climateAdd-on" parameter, i.e., c, to adjust PD to "climatePD" and hence "climateLGD".

Since for each scenario narrative and year, the climate PD add-ons will vary across underlying exposure characteristics, considering regional sector, industry sector, credit quality bucket, to calculate climate transition adjusted PDs. For example, in Section 3.2.1, climate PD add-ons are under three climate transition scenarios: 1) below 2℃ immediate, 2) below 2℃ delayed, and 3) net-zero 2050. This shows that OSFI's framework has different parameterisations of c under various climate transition scenarios. Therefore, incoporating this climate adjustment parameter ensures climateRWA reflects climate-related risks more accurately, highlighting the potential impacts of climate policies, technological advancements, and shifts in consumer preferences.

Higher values of c lead to increased PD and LGD values, which intern inflates the capital requirements (K) via the Basel RWA formula. The climate adjustment reflects the increased likelihood of default and thus higher loss severities under adverse climate scenarios, ensuring that OSFI capital buffer is adequate to absorb risks brought by changing climate conditions.

According to the plot of ratio of climateRWA to base RWA, we also note that the ratio of climateRWA to base RWA is a non-linear upwarding curve and is higher than the ratio of base RWA to base RWA. For small values of c, the climateRWA is only slightly higher than the base RWA, but as c grows, the ratio rises exponentially. The plot suggests that the ability of OSFI's parameterisation to affect RWA is great since climateRWA is very sensitive to the climate PD add-on parameter. OSFI's parameterisation allows regulators to dynamically scale RWA in response to varying levels of climate-related risk exposure, suggesting that it can be fine-tuned to enforce stringent capital requirements for institutions exposed to higher climate risks.

MMF2000 Assignment 1 Climate Risk Code

November 21, 2024

1 Part (e) Python Code

[5]:
```
import numpy as np
from scipy.stats import norm

# Constants
PD = 0.01  # Probability of Default (1%)
LGD = 0.4  # Loss Given Default (40%)
EAD = 1_000_000  # Exposure at Default ($1 million)
M = 1  # Maturity (1 year)
R = 0.15  # Correlation factor

# Step 1: Calculate b
b = (0.11852 - 0.05478 * np.log(PD)) ** 2

# Step 2: Calculate K
N_inv_PD = norm.ppf(PD)  # Inverse CDF (N^-1) for PD
N_inv_999 = norm.ppf(0.999)  # Inverse CDF (N^-1) for 99.9%
K = (
    LGD * norm.cdf((N_inv_PD + np.sqrt(R) * N_inv_999) / np.sqrt(1 - R))
    - LGD * PD
) * ((1 + (M - 2.5) * b) / (1 - 1.5 * b))

# Step 3: Calculate RWA
RWA = 12.5 * EAD * K

# Output the results
print(f"b: {b:.4f}")
print(f"K: {K:.4f}")
print(f"RWA: ${RWA:,.2f}")
```

```
b: 0.1375
K: 0.0401
RWA: $501,323.78
```

[2]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm
```

```python
# Given parameters
PD = 0.01  # Base probability of default (1%)
LGD = 0.4  # Loss given default (40%)
EAD = 1e6  # Exposure at default ($1 million)
M = 1  # Maturity
R = 0.15  # Correlation
c_values = np.linspace(0, 2, 100)  # Climate add-on range

# Helper functions
def calculate_logit(p):
    return np.log(p / (1 - p))

def calculate_pd_from_logit(logit):
    return 1 / (1 + np.exp(-logit))

def calculate_b(pd):
    return (0.11852 - 0.05478 * np.log(pd))**2

def calculate_climate_lgd(climate_pd, base_pd, base_lgd):
    term_1 = norm.ppf(climate_pd)  # N^{-1}(climatePD)
    term_2 = norm.ppf(base_pd)  # N^{-1}(PD)
    term_3 = norm.ppf(base_pd * base_lgd)  # N^{-1}(PD * LGD)
    adjusted_term = term_1 - term_2 + term_3
    climate_lgd = norm.cdf(adjusted_term) / climate_pd
    return climate_lgd

def calculate_k(pd, lgd, r, m):
    b = calculate_b(pd)
    z_99 = norm.ppf(0.999)
    term_1 = norm.ppf(pd) + np.sqrt(r) * z_99
    k_base = (lgd * norm.cdf(term_1 / np.sqrt(1 - r))) - (lgd * pd)
    adjustment = (1 + (m - 2.5) * b) / (1 - 1.5 * b)
    return k_base * adjustment

# Base RWA calculation
b_base = calculate_b(PD)
K_base = calculate_k(PD, LGD, R, M)
base_RWA = 12.5 * EAD * K_base

# Climate RWA calculation
climate_RWA_ratios = []
for c in c_values:
    climate_logit_pd = calculate_logit(PD) + c
    climate_PD = calculate_pd_from_logit(climate_logit_pd)
    climate_LGD = calculate_climate_lgd(climate_PD, PD, LGD)
    K_climate = calculate_k(climate_PD, climate_LGD, R, M)
```

```python
    climate_RWA = 12.5 * EAD * K_climate
    climate_RWA_ratios.append(climate_RWA / base_RWA)

# Plotting
plt.figure(figsize=(10, 6))
plt.plot(c_values, climate_RWA_ratios, label="Climate RWA / Base RWA",
 ↪color="blue")
plt.axhline(1, color="red", linestyle="--", label="Base RWA (Ratio = 1)")
plt.title("Climate RWA / Base RWA as a Function of Climate Add-on (c)")
plt.xlabel("Climate Add-on (c)")
plt.ylabel("Climate RWA / Base RWA")
plt.legend()
plt.grid()
plt.show()
```