Class 6 Homework - Xinyi (Cynthia) Shen

Problem

Using the PDE method, price the same KO option as we had in class, but use today's option values, implied volatilities, etc.

Get Stock Price, Call and Pot Option Prices, Implied Volatilities for KO

After October 25, 2024, Friday market closes, I got a snap of the December 20, 2024 call and put option straddle for the Coca-Cola company (KO) on Yahoo! Finance.

NYSE - Nasdaq Real Time Price • USD

The Coca-Cola Company (KO)   ☆ Follow   ⤴ Compare

66.92 -0.38 (-0.56%)   **66.98** +0.06 (+0.09%)

At close: 4:00 p.m. EDT   After hours: 7:59 p.m. EDT ☾

Dec 20, 2024 ⌄   All Strike Prices ⌄   Straddle ⌄

Calls   **Puts**

Last Price	Change	% Change	Volume	Open Interest	Strike	Last Price	Change	% Change	Volume	Open Interest
-	-	-	-	-	32.50	0.01	0.00	0.00%	10	24
-	-	-	-	-	35.00	0.05	0.00	0.00%	1	1
23.09	0.00	0.00%	-	10	37.50	0.07	0.00	0.00%	1	8
31.45	0.00	0.00%	1	0	40.00	0.04	0.00	0.00%	3	5
-	-	-	-	-	42.50	0.02	0.00	0.00%	3	10
-	-	-	-	-	45.00	0.03	-0.01	-25.00%	1	78
20.40	0.00	0.00%	1	3	47.50	0.03	0.00	0.00%	4	185
20.89	0.00	0.00%	1	10	50.00	0.03	0.00	0.00%	2	671
18.75	0.00	0.00%	2	31	52.50	0.06	0.00	0.00%	2	563
12.55	0.00	0.00%	3	80	55.00	0.06	0.00	0.00%	14	920
10.23	+0.53	+5.46%	2	457	57.50	0.12	+0.01	+9.09%	2	1,834
8.30	0.00	0.00%	16	1,333	60.00	0.20	+0.01	+5.26%	90	1,623
5.00	-0.35	-6.54%	1	3,589	62.50	0.43	+0.07	+19.44%	206	1,962
2.92	-0.28	-8.75%	27	3,615	65.00	0.93	+0.11	+13.41%	253	14,581
1.32	-0.27	-16.98%	1,019	16,194	67.50	2.00	+0.20	+11.11%	142	6,866
0.51	-0.07	-12.07%	4,652	7,444	70.00	3.49	+0.10	+2.95%	14	5,301
0.16	-0.04	-20.00%	1,130	12,069	72.50	5.43	0.00	0.00%	11	1,603
0.06	-0.01	-14.29%	155	8,830	75.00	6.75	0.00	0.00%	1	310
0.05	+0.02	+66.67%	4	1,818	77.50	8.25	0.00	0.00%	8	287
0.03	0.00	0.00%	7	1,247	80.00	9.95	0.00	0.00%	1	1
0.01	0.00	0.00%	1	245	85.00	13.45	0.00	0.00%	-	0
0.02	0.00	0.00%	4	431	90.00	19.80	0.00	0.00%	2	4

At the market close of 4:00 PM EDT, KO's price was given as $66.92. We will focus on the $67.50 dollar level for the homework.

```
K = 67.5;
C_MKT = 1.32;
P_MKT = 2;
```

Calls

In The Money

Contract Name	Last Trade Date (EDT)	Strike	Last Price	Bid	Ask	Change	% Change	Volume	Open Interest	Implied Volatility
KO241220C00037500	2024-04-22 4:21 p.m.	37.5	23.09	23.60	27.55	0.00	0.00%	-	10	0.00%
KO241220C00040000	2024-09-12 7:14 p.m.	40	31.45	28.10	31.00	0.00	0.00%	1	0	140.72%
KO241220C00047500	2024-10-23 3:01 p.m.	47.5	20.40	19.55	20.00	0.00	0.00%	1	3	57.57%
KO241220C00050000	2024-10-16 4:47 p.m.	50	20.89	16.10	17.75	0.00	0.00%	1	10	63.04%
KO241220C00052500	2024-09-12 2:16 p.m.	52.5	18.75	17.35	18.25	0.00	0.00%	2	31	94.97%
KO241220C00055000	2024-10-24 5:23 p.m.	55	12.55	12.10	12.25	0.00	0.00%	3	80	36.28%
KO241220C00057500	2024-10-25 3:28 p.m.	57.5	10.23	9.65	10.00	+0.53	+5.46%	2	457	35.08%
KO241220C00060000	2024-10-23 4:05 p.m.	60	8.30	7.25	7.35	0.00	0.00%	16	1,333	25.29%
KO241220C00062500	2024-10-24 6:29 p.m.	62.5	5.00	4.90	5.00	-0.35	-6.54%	1	3,589	20.63%
KO241220C00065000	2024-10-25 7:39 p.m.	65	2.92	2.88	2.94	-0.28	-8.75%	27	3,615	17.68%
KO241220C00067500	2024-10-25 7:55 p.m.	67.5	1.32	1.34	1.40	-0.27	-16.98%	1,019	16,194	15.94%
KO241220C00070000	2024-10-25 7:56 p.m.	70	0.51	0.49	0.51	-0.07	-12.07%	4,652	7,444	14.97%
KO241220C00072500	2024-10-25 7:44 p.m.	72.5	0.16	0.15	0.18	-0.04	-20.00%	1,130	12,069	15.38%
KO241220C00075000	2024-10-25 5:15 p.m.	75	0.06	0.05	0.07	-0.01	-14.29%	155	8,830	16.41%
KO241220C00077500	2024-10-25 7:55 p.m.	77.5	0.05	0.02	0.05	+0.02	+66.67%	4	1,818	19.04%
KO241220C00080000	2024-10-24 5:41 p.m.	80	0.03	0.01	0.08	0.00	0.00%	7	1,247	24.22%
KO241220C00085000	2024-10-24 4:29 p.m.	85	0.01	0.00	0.05	0.00	0.00%	1	245	28.71%
KO241220C00090000	2024-10-01 7:29 p.m.	90	0.02	0.00	0.15	0.00	0.00%	4	431	40.63%

The implied volatility for the $67.5 strike call was 15.94%

```
sigma67p5Call = 0.1594;
```

Puts

Contract Name	Last Trade Date (EDT)	Strike	Last Price	Bid	Ask	Change	% Change	Volume	Open Interest	Implied Volatility
KO241220P00032500	2024-08-29 7:04 p.m.	32.5	0.01	0.00	0.16	0.00	0.00%	10	24	84.77%
KO241220P00035000	2024-04-04 7:49 p.m.	35	0.05	0.00	0.16	0.00	0.00%	1	1	76.76%
KO241220P00037500	2024-04-05 2:08 p.m.	37.5	0.07	0.00	0.17	0.00	0.00%	1	8	69.92%
KO241220P00040000	2024-05-31 4:36 p.m.	40	0.04	0.01	0.19	0.00	0.00%	3	5	64.45%
KO241220P00042500	2024-07-30 7:03 p.m.	42.5	0.02	0.00	0.76	0.00	0.00%	3	10	73.73%
KO241220P00045000	2024-10-25 4:32 p.m.	45	0.03	0.00	0.05	-0.01	-25.00%	1	78	46.29%
KO241220P00047500	2024-10-11 3:25 p.m.	47.5	0.03	0.00	0.22	0.00	0.00%	4	185	52.05%
KO241220P00050000	2024-10-16 7:14 p.m.	50	0.03	0.01	0.23	0.00	0.00%	2	671	45.90%
KO241220P00052500	2024-10-23 2:45 p.m.	52.5	0.06	0.02	0.07	0.00	0.00%	2	563	31.64%
KO241220P00055000	2024-10-23 5:17 p.m.	55	0.06	0.04	0.14	0.00	0.00%	14	920	29.98%
KO241220P00057500	2024-10-25 1:33 p.m.	57.5	0.12	0.08	0.31	+0.01	+9.09%	2	1,834	29.35%
KO241220P00060000	2024-10-25 7:55 p.m.	60	0.20	0.18	0.21	+0.01	+5.26%	90	1,623	20.66%
KO241220P00062500	2024-10-25 7:44 p.m.	62.5	0.43	0.38	0.42	+0.07	+19.44%	206	1,962	18.26%
KO241220P00065000	2024-10-25 7:49 p.m.	65	0.93	0.90	0.93	+0.11	+13.41%	253	14,581	16.72%
KO241220P00067500	2024-10-25 7:55 p.m.	67.5	2.00	1.91	1.97	+0.20	+11.11%	142	6,866	15.85%
KO241220P00070000	2024-10-25 7:07 p.m.	70	3.49	3.30	3.70	+0.10	+2.95%	14	5,301	16.31%
KO241220P00072500	2024-10-24 7:59 p.m.	72.5	5.43	4.80	5.90	0.00	0.00%	11	1,603	18.12%
KO241220P00075000	2024-10-23 6:03 p.m.	75	6.75	8.15	8.30	0.00	0.00%	1	310	21.00%
KO241220P00077500	2024-10-10 3:37 p.m.	77.5	8.25	8.80	10.75	0.00	0.00%	8	287	23.98%
KO241220P00080000	2024-10-21 1:55 p.m.	80	9.95	12.20	13.25	0.00	0.00%	1	1	27.88%
KO241220P00085000	2024-09-19 1:41 p.m.	85	13.45	14.50	14.70	0.00	0.00%	-	0	0.00%
KO241220P00090000	2024-10-17 2:09 p.m.	90	19.80	23.05	24.10	0.00	0.00%	2	4	51.66%

The implied volatility for the $67.5 strike put was 15.85%.

```
sigma67p5Put = 0.1585;
```

SOFR Rate:

Rate	Latest	Date
30 - Day Average SOFR	▼ 4.85648	25-October-2024
90 - Day Average SOFR	▼ 5.17199	25-October-2024
180 - Day Average SOFR	▼ 5.30090	25-October-2024

```
% Given SOFR 180-day average
SOFR_180_day = 0.053009; % 5.3009% as a decimal
```

```
% Annualize the rate with semi-annual compounding
r = (1 + SOFR_180_day / 2)^2 - 1
```

```
r =
0.0537
```

Dividends & Splits

Forward Annual Dividend Rate [4]	1.94
Forward Annual Dividend Yield [4]	2.90%
Trailing Annual Dividend Rate [3]	1.91
Trailing Annual Dividend Yield [3]	2.85%
5 Year Average Dividend Yield [4]	3.02
Payout Ratio [4]	79.46%
Dividend Date [3]	2024-12-16
Ex-Dividend Date [4]	2024-11-29
Last Split Factor [2]	2:1
Last Split Date [3]	2012-08-13

The trailing annual dividend yield is 2.85%.

```
q = 0.0285;
b = r - q
```

```
b =
0.0252
```

For December 20, 2024 we calculate T in years

```
To = datenum(2024,10,25);
TExpiration = datenum(2024,12,20);
TDays = (TExpiration - To); % in days
T = (TExpiration - To) / 365 % in years
```

```
T =
0.1534
```

Download Historical Stock Data for KO

```
% Download data for Coke
startDate = '2019-10-25';
endDate = '2024-10-26';
KO_data = py.yfinance.download('KO', pyargs('start', startDate, 'end', endDate,
'interval', '1d'));
```

```
[*********************100%***********************]  1 of 1 completed
```

```
TTKO = table(KO_data);
TTKO = renamevars(TTKO, ["Adj Close"], ["Price"]);
```

4

```matlab
TTKO = removevars(TTKO,["Open","High","Low","Close","Volume"])
```

TTKO = 1259×2 table

	Date	Price
1	25-Oct-2019	46.0380
2	28-Oct-2019	45.8839
3	29-Oct-2019	45.7468
4	30-Oct-2019	46.2008
5	31-Oct-2019	46.6205
6	01-Nov-2019	46.1665
7	04-Nov-2019	45.5155
8	05-Nov-2019	44.8989
9	06-Nov-2019	45.2243
10	07-Nov-2019	44.7875
11	08-Nov-2019	44.7190
12	11-Nov-2019	44.4021
13	12-Nov-2019	44.2907
14	13-Nov-2019	44.8903

⋮

```matlab
% What is the historical volatility of the stock over different time frames
tHistorical = datenum(TTKO.Date);
SHistorical = TTKO.Price;
So = SHistorical(end)
```

So =
66.9200

```matlab
rKO = diff(SHistorical) ./ SHistorical(1:end-1);

sigmaHist1YEAR = std(rKO(end-252+1:end)) * sqrt(252);
sigmaHist3YEAR = std(rKO(end-2*252+1:end)) * sqrt(252);
sigmaHist5YEAR = std(rKO(1:end)) * sqrt(252);

% Results and Output
figure()
set(gcf,'Position',[0,0,500,300])

barh([sigmaHist1YEAR, sigmaHist3YEAR, sigmaHist5YEAR, sigma67p5Call, sigma67p5Put])

xlabel('Annual Volatility')
set(gca,'YTickLabels', {'Historical 1Y', ...
                        'Historical 3Y', ...
                        'Historical 5Y',...
                        'Implied $67.5 Call', ...
```
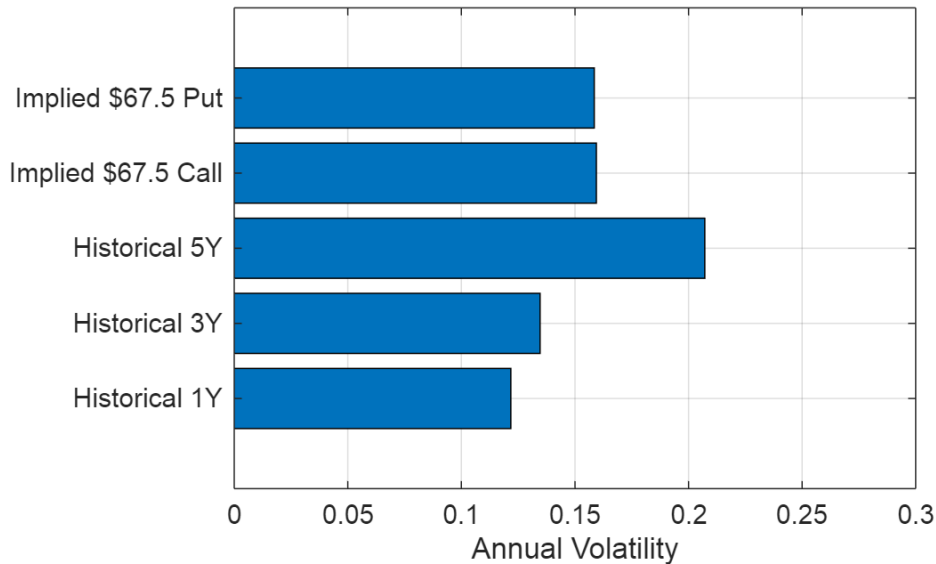
```
                              'Implied $67.5 Put'})
xlim([0 0.3])
box on
grid on
```


Use PDE to Price KO American Call Option as of December 20, 2024

(1) Construct Grid of Prices, Times, etc.

```
% The Grid
SMin = K/2;
SMax = K*2;

XMin = log(SMin/So);
XMax = log(SMax/So);
NX = 300;

% The 1+return vector
XGridPDE = linspace(XMin, XMax, NX)';
dX = (XMax - XMin) / NX;

% The Stock Price vector
SGridPDE = So * exp(XGridPDE);

% The Time vector
TMin = 0;
TMax = T;
NT = 1000;

dT = (TMax - TMin) / NT;
TGridPDE = linspace(TMin, TMax, NT);
```

(2) Define Option Value V(S,t)

```
% Option Value
IntrinsicVPDE = max(SGridPDE - K, 0);
VPDE = NaN(NX, NT);
```

(3) Define Binary Conditions

```
% Boundary Conditions
% at expiration
VPDE(:,end) = max(SGridPDE - K, 0);

% at the boundary SMax
VPDE(end,:) = max(SGridPDE(end) - K, 0);

% at the boundary SMin
VPDE(1,:) = max(SGridPDE(1) - K, 0);
```

(4) Define Key Calculation Terms

```
% Key Calculations
pu = (sigma67p5Call^2 * dT) / (2 * dX^2) + (b - sigma67p5Call^2/2) * dT / (2*dX);
pm = 1 - sigma67p5Call^2 * dT / (dX^2);
pd = (sigma67p5Call^2 * dT) / (2 * dX^2) - (b - sigma67p5Call^2/2) * dT / (2*dX);
```

(5) Backward PDE Algorithm

```
% Adjust the PDE loop for American option
for n = (NT-1):-1:1
    for j = 2:NX-1
        VPDE(j,n) = max(IntrinsicVPDE(j), (pu * VPDE(j+1,n+1) + pm * VPDE(j,n+1) +
pd * VPDE(j-1,n+1)) / (1 + r*dT));
    end
end
ICurrentPrice = find(SGridPDE >= So, 1, 'first');
C_PDE_AMER = VPDE(ICurrentPrice:ICurrentPrice);
formatted_price = sprintf('American Call Option Price using PDE: $%.2f',
C_PDE_AMER);
disp(formatted_price);
```

```
American Call Option Price using PDE: $1.57
```

(6) Results and Graphs

```
figure()
set(gcf,'Position',[0,0,1000,500])

subplot(1,2,1)
plot(SGridPDE, VPDE(:,1), 'r-', SGridPDE, VPDE(:,round(NT/2)), 'g-', SGridPDE,
VPDE(:,NT), 'b-')

xlabel('Stock Price [$]')
ylabel('Call Option Value [$]')
title('American Call Option Using PDE Explicit Method')
```
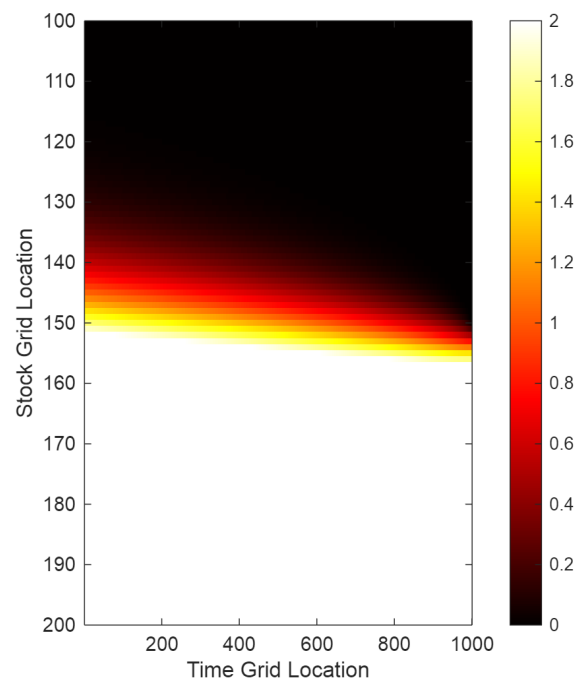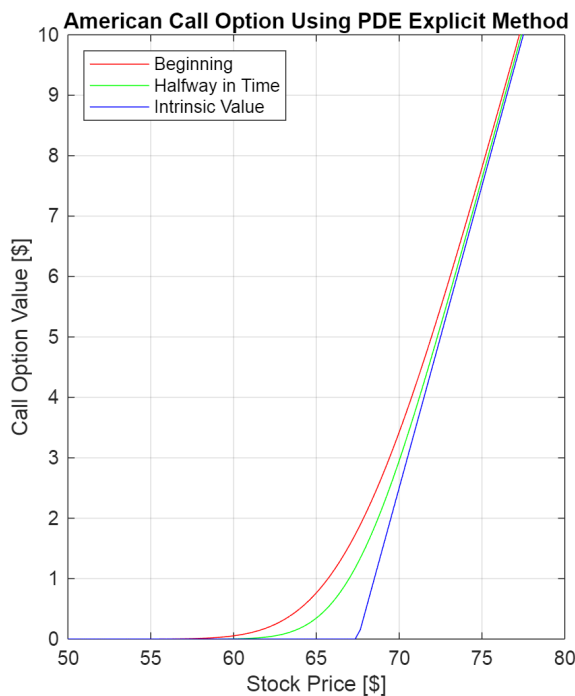
```matlab
legend({'Beginning','Halfway in Time', 'Intrinsic Value'}, 'location', 'northwest')

grid on
xlim([50 80])
ylim([0 10])

subplot(1,2,2)
%imagesc(SGridPDE, TGridPDE, VPDE)
imagesc(VPDE)

xlabel('Time Grid Location')
ylabel('Stock Grid Location')
colormap(hot)
ylim([100,200])
%xlim([50,100])
caxis([0,2])
colorbar
```



```matlab
% Update on call option price by various methods
figure()

set(gcf,'Position',[0,0,800,500])

bar([C_MKT, C_PDE_AMER])
title('Comparison of Various Numerical Methods for Option Pricing')
ylabel('Call Price [$]')
set(gca, 'XTickLabels', {'Market', 'AmerPDE'})

box on
```

```
grid on
```


Comparison of Various Numerical Methods for Option Pricing