1.

a)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p | q | r |  |  |
| T | T | T | T | T |
| T | T | F | T | F |
| T | F | T | T | T |
| T | F | F | T | F |
| F | T | F | T | F |
| F | T | T | T | T |
| F | F | T | F | T |
| F | F | F | F | T |

b)

2.

a) my statement:

I believe the statement:

Proof: Let m,n , assume 7|(m-5), that

Also, assume 7|(n-2) that is

Let =7+2+5+1

Then 7=49+14+35+7

=(7+5)(7+2)-3

=mn-3

b)converse:

I disbelieve the converse:

Proof: Take m=10,n=1,

But that is

Hence, the converse is false.

3.

a)proof:

Let The pigeonhole principle says that:

This is equal to

We can define the question as a function f:AB , domain A is the set of people at the party, range B is the set of the number of people each person shook hands with.

Since when a people shook hand with |A|-1 people, there will be no people have not shake hand with anybody. Vice versa. Hence the range B may be 0~|A|-2 or 1~|A|-1, so |B||A|-1, hence |B|<|A|.

With the pigeonhole principle above, we can prove that that is there are at least 2 people who shake hands with the same number of other people.

4.

a)Proof:

From the couse note we have this statement : and the proof of this statement is:

Let n . Assume that n is prime. We need to prove that n > 1

and that Atomic(n) are true.

For the first part, the definition of prime tells us immediately that n > 1.

For the second part, we want to prove that ,Let and assume that n a and n b. We want to prove

that n ab.

We’ll first prove that there exist , , n + ab = 1. By Claim

1 and the assumption that n is prime, there exist , ,,  such

that n + a = 1 and n + b = 1. Let  =n + a + b

and = .

Then we can multiply the first two equations to obtain:

(n + a)(n + b) = 1

+ an + bn + ab = 1

(n + a + b)n + ab = 1

n + ab = 1

So then there exist , , n+ab = 1. Then using Claim 2 (and

again the assumption that n is prime), we can conclude that n ab.

And the claim1： which can be proved by the claim3 and claim 6 in the Tutorial4 worksheet

Claim 2: which can be proved by the claim 6 in the Tutorial4 worsheet.

Gcd(a,p)=1 and Prime(p) means ( same as the claim2 above)

And we also know that  for n is belong to T ( T={1,...,p-1})

With the statement. Hence (an) must be one of 1,....,p-1

In all,

b)Proof:

reduction ad absurdum:

assume that there are two distinct numbers  and in T, that (a(a)

we also know that p| a(a and p| a(a

so we can get the statement that p| a- )

since , and they are distinct so |- |- ) cannot be divisible by p, we also know that a is not divisible by p, With the statement- ). And this is contradict to the conclusion we assumed. So the statement we assumed is not true . And we can get the result that If and are distinct numbers in T, then (a (a)

c)Proof:

Let The pigeonhole principle says that:

since if and are distinct numbers in T, then (a (a)（claim b）that meet OneToOne(f)

So |T|

since (claim a) so|T|

s

d)Proof:

since For finite sets A and B if A B then |B|=|B\A| + |A| , both and T are finite sets and (claim1) so |T|=|T\| +|

since (claim c)we can get that |T\|=0 that means T\= hence we can conclude that =T

e)Proof:

Since i = 1~P-1 so i is all the element of T. So  is the product of all elements in and is the product of all elements in T.

We also know that =T (claim d) , so the the product of all elements in is equal to the product of all elements in T.

Hence we can prove that

f)Proof:

As a consequence of Example 2.18, if for (mod p).

We can find that  since .

We also know that

So (mod p)

So p|(

That is p|123....(p-1)-123...(p-1)]

That is p| [123....(p-1)]]

Since none of {1,2,3...(p-1)} can be divisible by p, must can be divisible by p.

As an extension of Example 2.14, that for any k>1, if prime ppp, then p so p since p

So must can be divisible by p means 1(mod p)

Since 1 is the smallest integer grater than zero we can find that

g)Proof:

Since a is an arbitrary natural number that is not divisible by 5 that means gcd(a,5)=1

And we all know that 5 is a prime number since it can only be divisible by 1 or 5 itself and 5>1

So we can get the conclusion that (claim f) that is 1(mod 5)

As a consequence of Example 2.18, if for (mod p). so (mod 5) that is (mod 5)

Since 1 is the smallest integer grater than zero we can find that

5.

a)Proof:

Let k, take n=2+(k+2)!

We can write n,n+1,...,n+k as n+a(a

So n+a

Since a ,so (2+a), and they are both interger.

So we can write n+a=(2+a)[123(1+a)(3+a)(4+a)...(k+2)+1]

So n+a can be divisible by (2+a) which does not equal to 1 or n+a for (2+a)>1 since a and n>2 since n=2+(k+2)! and (k+2)!>1 for k

So we can say n+a is composite.

Hence for any k there is some nsuch that n,n+1，...n+k are composite.

b)Proof:

Let n>0 and n

Take a look at n!+1 ,

If n!+1 is a prime number then p=n!+1 since n<n!+1<n!+2 (n>0) the statement is proved.

If n!+1 is not a prime number then n!+1 must can be writed as a(a,b,c...is prime number) and a,b,c,...>n for the reason below:

We assume that there exist a number p{a,b,c...} and p then p|n! and p|n!+1

So p|n!+1-n! that is p|1 and this is impossible for p is a prime so p>1. So what we assumed is false and

Since n!+2>n!+1= a(a,b,c...is prime number) so

In all and this meet the statement.

Hence , For any positive natural number n there exists a prime p with n < p < n!+2.