CSC165H1: Problem Set 2

Due October 25 2017 before 10pm

1.(a) WTS:

Let

* since, so
* To show is same to show or .Since we want to show.

Take d=n+1 since

Since (n+1)(n+2)=, so (

Hence we have proven

We have proven as needed.

(b) WTS:

Let

* since, so
* To show is same to show or .Since we want to show.

Take d=n+1 since

Since (n+1)(n+5)=, so (

Hence we have proven

We have proven as needed.

2.(a)WTS:

Construct a set A=

* Since , and A is not empty since it has element .(
* Since A= A is a finite set of real numbers since there are finite positive natural numbers which is smaller than

In all A is a non-empty, finite set of real numbers.

Since the fact that any non-empty, finite set of real numbers has a minimum element, A has a minimum element.

Take m be this minimum element. Since , so

Let . I am going to prove in two cases:

* , then (since ,

thus since m is the minimum element of A.

* ,

since , (since m is the minimum element of A)

we know

so

We have proven as needed.

(b)WTS: ,where m is introduced in 2(a).

Let

to show is to show

* Since and so
* Since , let , be that value.

Take x=k,y=k, since so k k

ax+by=ak+bk=k(=km

we have proven as needed.

(c)WTS: , ,where m is introduced in 2(a).

We will prove this by contradiction.

Assume ,

Since , so . thus according to the Quotient-Remainder Theorem, there exist such that c=qm+r and 0 also since , so r, so 0<r<m.

Since, ,, let be that value.

Since c=qm+r so hence r=( so ( also since 0<r and so . Hence , therefore r (since m is the minimum element of l). So we get a contradiction with 0<r<m

Hence we have proven , as needed.

(d)WTS: ,where a,b is introduced in the question and m is introduced in 2(a).

To show is same to show

Since , and they are not both 0. We can prove the statement in two cases:

For a:

* a=0: take k=0,km=0=a, so
* a. Take x=1,y=0,(they are both integers) so ax+by=a, hence

according to 2(c) , we know that , hence .

For b:

* b=0: take k=0,km=0=b, so
* b. Take x=0,y=1,(they are both integers) so ax+by=b, hence

according to 2(c) , we know that , hence .

Hence we have proven as needed.

(e)WTS: ,where a,b is introduced in the question and m is introduced in 2(a).

Let ,

For n=0 :since is false, is true.

For nwe assume, that is , let be that value.

We want to show

Since ,, let x,y be that value.

Take k=, since so

kn==ax+by=m.

Hence we have proven as needed.

(f)WTS: ,where a,b is introduced in the question and m is introduced in 2(a).

We have proven in 2(d)

We have proven in 2(e)

Since and so

Hence

We have proven as needed, so m is the greatest common divisor of a and b.

(g)WTS: ,where a,b is introduced in the question and m is introduced in 2(a).

Let , we assume

Since m=1 and , so , let x,y be that value.

Since a|bc, so, let be that value.

We want to prove , that is

Take k=, since

Since c,x,y,, so , hence k

ka=

Hence we have proven as needed

3.WTS:

I will prove this by contradiction.

Assume that this statement is false, i.e., that there are finite numbers of P. Let k be

the number of elements of P, and letbe the elements.(), so

Our statement Q will be “for all n, n is prime and if and only if n is one

of { }

Define the number p=4()+3 ,hence (since is an integer). Also since p is even bigger than . Therefore p must not be a prime. So p is a composite number since p is not a prime and p is bigger than 1.

* I am going to prove: by contradiction

Let , we assume that is , let k be that value.

If n|p that is , let a be that value, so p=na=4ak

So 4()+3=4ak

Hence 4()=-3

Hence = and that is impossible since must be an integer, so we get a contradiction.

Hence we have proven as needed.

* I am going to prove: by contradiction

Let , we assume that is , let k be that value.

If n|p that is , let a be that value, so p=na=4ka+2a

So 4()+3=4ak+2a

Hence 2()=-3

Hence = and that is impossible since must be an integer, so we get a contradiction.

Hence we have proven as needed.

* I am going to prove:

That is to prove p is divisable by one of since for all n, n is prime and if and only if n is one of { }.

For : this is impossible for otherwise 4() is divisible by 3 while and are primes that is not equal to 3.

For , this is also impossible for otherwise one of would divide P-4()= 3, while all of is bigger than 3.

Hence we have proven as needed.

Since so

And

And

And p is not a prime

And the fact that any integer greater than 1 is a product of prime

So p must be a product of prime (t)

According to the [modular multiplication] that says that the product of 2 or more numbers (mod m) is congruent to the product of numbers congruent to them so

But p=4()+3 ,hence , so we get a contradiction

So what we assumed at first is false

Hence we have proven as needed

4.(a)WTS:

Take , since so

Let we assume , we want to show , that is 2n+1650

2n+1650<250n+250000 (

+ (

Hence we have shown as needed

(b) WTS:

Let

Take =max, since, so, so

Let , we assume , we want to prove that is an+b

Since =max and this implies:

* n 4a, so since

hence 0.25

* n, so since

hence 0.25

In all 0.25an+b ,that is an+b

Hence we have shown as needed