CSC236 Fall 2018

Assignment#1: induction

1.

(a) Yes.The proof is the following.

Proof:

Assume P(234), that is: every bipartite graph on 234 vertices has no more than  edges.

I will use this to prove P(235), that is: every bipartite graph on 235 vertices has no more than edges.

Let G = (V, E) be a bipartite graph on 235 vertices

Let , be such that

Thus

Assume  （without loss of generality）

Then

Let be an arbitrary vertex in

Let be the number of edges incident with

Since every edge incident with must have the other endpoint in , and

Then

Taking away from G results in a new bipartite graph

Thus （Since takes away k also takes away the x edges that are incident with k.）

And , which means is a bipartite graph on 234 vertices

Also,

(since )

(since P(234))

(since )

(b)

No, I cannot directly use P(235) to prove P(236)

Explanation:

Assume P(235), that is: every bipartite graph on 235 vertices has no more than edges.

Since the number of edges must be an integer, we can say that: every bipartite graph on 235 vertices has no more than edges.

I will use this to prove P(236), that is: every bipartite graph on 236 vertices has no more than edges.

Let G = (V, E) be a bipartite graph on 236 vertices

Let , be such that

Thus

Assume （without loss of generality）

Then

Let be an arbitrary vertex in

Let be the number of edges incident with

Since every edge incident with must have the other endpoint in , and

Then

Taking away from G results in a new bipartite graph

Thus （Since takes away k also takes away the x edges that are incident with k.）

And , which means is a bipartite graph on 235 vertices

Also,

(since )

By P(235) and the fact that the number of edges must be an integer, we know: every bipartite graph on 235 vertices has no more than edges.

Then:

(since )

(since )

From the above proof we can say that P(236) holds, by assuming P(235) and using the fact that the number of edges must be an integer.

So, I’m actually using the assumption that: every bipartite graph on 235 vertices has no more than edges, not P(235). That is P(235) is not a necessary part of my proof.

Therefore, I cannot prove P(236) by directly using P(235).

(c)

Proof:

P(n): Every bipartite graph on n vertices has no more than edges.

Wants to show:

Define H(n): Every bipartite graph on n vertices has no more than edges.

Since

Then

I will prove that by simple induction.

Base case:

Every bipartite graph on 0 vertices has 0 edges, which is no more than edges.

This verifies H(0).

Inductive step:

Let

Assume H(n), that is: every bipartite graph on n vertices has no more than edges.

I will show that H(n+1) follows, that is: every bipartite graph on (n+1) vertices has no more than edges.

Let G = (V, E) be a bipartite graph on (n+1) vertices.

Let , be such that

Thus

Assume （without loss of generality）

Then

Let be an arbitrary vertex in

Let be the number of edges incident with

Since every edge incident with must have the other endpoint in , and

Then

Taking away from G results in a new bipartite graph

Thus （Since takes away k also takes away the x edges that are incident with k.）

And , which means is a bipartite graph on n vertices

Also,

Case 1: (n+1) is odd

When (n+1) is odd,

Thus,

Then,

(since H(n))

(since )

(since is an integer, )

(since )

Case 2: (n+1) is even

When (n+1) is even,

Thus,

Also, when (n+1) is even, n is odd

Then, by H(n):

Since n is odd, n+1 and n-1 are all even

Let such that

Then

Thus,

Since , then

Thus,

Then,

Then,

(since )

(since is an integer, )

So, I’ve proven H(n+1)

In all, I’ve proven that ,

that is, (since )

2.(a)Yes, I can prove P(29) by assuming P(3), the proof is the following:

Proof:

* Assume P(3) that is f(3) is a multiple of 4, so
* I am going to use this to prove P(29) that is f(29) is a multiple of 4. So we are going to show that .
* Let Since 29>0 by the question we know that:

f(29)=

=

=

=

=

=

So f(29) is a multiple of 4. So we can prove P(29) by assuming P(3).

(b)No, I cannot prove P(29) directly by P(4), the proof is the following:

Proof:

* Assume P(4) that is f(4) is a multiple of 4.

Since 3>0 , so according to the question f(3)=

=

Since 4>0, so according to the question f(4)=

=

Hence we can conclude that f(3)=f(4) (since =)

We also know from assumption that f(4) is a multiple of 4, so f(3) is also a multiple of 4. (This is actually P(3)), So

* I am going to use this to prove P(29) that is f(29) is a multiple of 4. So we are going to show that .
* Let Since 29>0 by the question we know that:

f(29)=

=

=

=

=

=

So f(29) is a multiple of 4. So P(29) is proved to be true.

From the above proof we can conclude that the value of f(29) actually depends on the value of f(3) rather than f(4). That is P(29) actually depends on P(3) rather than P(4). So although we can indirectly prove P(29) by assuming P(4), P(4) is not a necessary part of our proof for P(29). Hence I cannot prove P(29) directly by P(4).

(c)WTS:

* I am going to prove this by complete induction.
* Inductive step: Let n be a natural number greater than 0. Assume P(1) and...P(n-1). I will show that P(n) follows, that is f(n) is a multiple of 4.

There are two cases to consider: n<3 and .

1. Base case: : So . So P(n) follows in this case.
2. Case :

* Since , , hence .
* Since it is clear in calculus that . Since , we know that .

In all, . Also we know by definition that is an integer. So we may use the assumption P( （since we assumed P(1) and...P(n-1) before）, in other words, is a multiple of 4. Let k be an integer such that .

So #since n>0

= #since

=

So P(n) follows in this case（since with k be an integer, will also be an integer, so we can conclude that f(n) is a multiple of 4）

So P(n) follows in both possible cases.

In all we’ve proven that

3.

Proof by contradiction:

Assume, for the sake of contradiction, the negation of what we are proving, that is:

Define

By our assumption is non-empty, so by the Principle of Well-Ordering it has a smallest element.

Let be the smallest element of

Let such that

Then:

(since and 5 is a prime number)

(by clue for A1 Q3)

Let

Then:

(divide through by 5)

(by clue for A1 Q3)

Let

Then:

(divide through by 5)

(since and 5 is a prime number)

(by clue for A1 Q3)

Let

Then:

(divide through by 5)

----><---- contradiction!

, but is the smallest element of

Since assuming that leads to a contradiction, the assumption is false.

4.

(a）WTS:

Proof: Define P(t):. So we will prove that I will prove this by structural induction.

* Basis: Let t be “\*”, since there is no “(“ in “\*”, hence left\_count(t) = 0.

Since there is no “(“ and no “)” in “\*”, so the left surplus for all prefixes in “\*” is 0 (since 0-0=0), hence max\_left\_surplus(t) = 0

Since , hence . Hence P(“\*”) holds.

* Inductive step: Let . Assume P() and P(). We will show that P((, that is .

**=**

*(since according to the structure of and left\_count() function return the number of “(“)*

*(by P() and P())*

*(since*

*hence*

*and )*

**=**

**=**

**=**

*(since)*

So P(( follows.

In all we’ve proven that

(b)WTS:

Proof: Define P(t): . So we will prove that I will prove this by structural induction.

* Basis: Let t be “\*”, since there is no “(“ and no “)” in “\*”, hence there is no “((“ and no “))” in “\*”. Hence according to the definition of double\_count function Hence P(“\*”) holds.
* Inductive step: Let . Assume P() and P(). We will show that P((, that is .

I will show that in 4 cases.

1. Case 1,

* Then (
* Since ,hence . That is

So P(( follows in this case.

1. Case 2,

* Then (
* . Since there is another ”)” in the right hand side of and we know that since there must be a “)” in the rightmost Hence there is an additional “))” in rather than .
* =, since we add a bracket around and , and

Hence

= +1 (#by P() and )

=

=

(#since =

So P(( follows in this case.

1. Case 3,

* Then (
* . Since there is another ”(” in the left hand side of and we know that since there must be a “(” in the leftmost Hence there is an additional “((” in rather than .
* =, since we add a bracket around and , and

Hence

= +1 (#by P() and )

=

=

(#since =

So P(( follows in this case.

1. Case 4,

* Then (
* . Since there is another ”(” in the left hand side of and we know that since there must be a “(” in the leftmost Hence there is an additional “((” in . Also, since there is another ”)” in the right hand side of and we know that since there must be a “)” in the rightmost Hence there is an additional “))” in .
* =, since we add a bracket around and , there is an additional “(“ in .

Hence

= (#by P() and )

=

=

(#since =

So P(( follows in this case.

In all, P(( follows.

In all we’ve proven that