Cirrhosis Patient Survival Prediction

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Background

Cirrhosis Overview

Related research

Study Objective

Methods

Dataset Description

The dataset used in this analysis originates from a study on primary biliary cirrhosis (PBC) conducted at the Mayo Clinic. It contains data for 276 patients, each characterized by 20 variables reflecting demographic, clinical, and laboratory features. Demographic variables include age and sex. Clinical features include the presence or absence of ascites, hepatomegaly, spiders, and edema, as well as the histologic stage of the disease. Laboratory markers, such as bilirubin, albumin, copper, alsekaline phosphatase (alk_phos), SGOT, triglycerides, platelets, and prothrombin time, provide insight into liver function and disease severity. Outcome measures include the number of days from registration to death, liver transplantation, or censoring, and patient status. For the purpose of analysis, censored cases (C: censored, CL: censored due to liver transplantation) were grouped together as a single category (C), while death (D) remained as a separate outcome.

Missing data were handled by removing rows with incomplete values. Out of the original dataset, 142 entries were removed due to missing information, resulting in a final dataset of 276 complete cases. This approach ensures the integrity of statistical analysis by avoiding imputation biases. By excluding records with missing data, the analysis avoids biases introduced by imputation but acknowledges the trade-off between sample size and data quality.

As shown in Table 1, the baseline characteristics of patients reveal significant differences across survival outcomes. Patients who died had the shortest survival time, highest bilirubin, alkaline phosphatase, and SGOT levels, as well as the most advanced disease stage (50% in stage 4). In contrast, younger patients were more likely to undergo liver transplantation, with a mean age of 40.7 years compared to 53.4 years in the death group. Clinical features such as ascites, hepatomegaly, and edema were more prevalent among those who died or underwent transplantation, indicating disease severity. Additionally, lower albumin levels and prolonged prothrombin time in the death group highlight impaired liver function as a key prognostic factor.

Table 1: Baseline Characteristics

Characteristic	Censored $N = 147^1$	Censored due to liver tx $N = 18^1$	Death $N = 111^{1}$
N_days	2,391.8 / 2,224.0 (984.3)	$1,511.6 \ / \ 1,368.0 \ (754.4)$	1,508.5 / 1,191.0 (1,110.4)
Drug			
D-penicillamine	70 (48%)	9 (50%)	57 (51%)
Placebo	77 (52%)	9 (50%)	54 (49%)
Age	48.3 / 48.0 (10.3)	40.7 / 40.5 (6.0)	$53.4 \ / \ 53.0 \ (10.0)$
Sex			
Female	137~(93%)	15 (83%)	90 (81%)
Male	10 (6.8%)	3 (17%)	21 (19%)
Ascites	1 (0.7%)	0 (0%)	18 (16%)
Hepatomegaly	55 (37%)	12 (67%)	75 (68%)
Spiders	29 (20%)	5 (28%)	46 (41%)
Edema	8 (5.4%)	2 (11%)	32 (29%)

Table 1: Baseline Characteristics

Characteristic	Censored $N = 147^1$	Censored due to liver $tx N = 18^1$	Death $N = 111^{1}$
Bilirubin	1.6 / 0.9 (1.8)	$3.2 \ / \ 3.3 \ (2.0)$	5.7 / 3.3 (6.2)
Cholesterol	$326.9 \ / \ 293.0 \ (168.1)$	$439.5 \; / \; 343.5 \; (335.5)$	418.9 / 344.0 (277.9)
Albumin	$3.6 \ / \ 3.6 \ (0.3)$	$3.6 \ / \ 3.6 \ (0.4)$	3.4 / 3.4 (0.5)
Copper	$68.1 \ / \ 52.0 \ (58.7)$	$123.3 \ / \ 101.0 \ (102.9)$	140.3 / 121.0 (100.9)
Alk_phos	1,501.1 / 1,120.0 (1,376.8)	$1,\!509.7\ /\ 1,\!253.5\ (854.4)$	2,731.8 / 1,794.0 (2,765.3)
SGOT	110.2 / 97.0 (54.4)	$130.2 \ / \ 123.5 \ (38.0)$	$141.5 \ / \ 134.9 \ (57.7)$
Tryglicerides	111.1 / 103.0 (47.8)	133.9 / 124.0 (70.5)	141.8 / 124.0 (79.3)
Platelets	$267.0 \ / \ 265.0 \ (86.4)$	294.8 / 297.5 (79.9)	249.5 / 236.0 (102.1)
Prothrombin	$10.4 \ / \ 10.2 \ (0.9)$	$10.4 \ / \ 10.2 \ (0.6)$	11.2 / 11.0 (1.0)
Stage			
1	11 (7.5%)	0 (0%)	1 (0.9%)
2	42 (29%)	3 (17%)	14 (13%)
3	62 (42%)	8 (44%)	41 (37%)
4	32 (22%)	7 (39%)	55 (50%)

 $^{^{1}}$ Mean / Median (SD); n (%)

Survival Analysis

Kaplan-Meier Estimates

The Kaplan-Meier estimator was used to estimate survival probabilities. This method provides a non-parametric estimation of the survival function S(t), defined as the probability of survival beyond time t. For a set of ordered survival times t_1, t_2, \ldots, t_k , the Kaplan-Meier survival estimate is computed as:

$$\hat{S}(t) = \prod_{t_i \le t} \left(1 - \frac{d_i}{n_i} \right)$$

where:

- d_i is the number of events (deaths or censoring) at time t_i ,
- n_i is the number of individuals at risk just before time t_i .

The Kaplan-Meier survival curves were plotted for overall survival, as well as stratified by key variables such as treatment type (D-penicillamine vs. placebo), presence of edema, and histological stage of disease. Confidence intervals were calculated for survival probabilities to assess the precision of estimates.

Log-Rank Test

The log-rank test was employed to compare survival distributions between groups, such as those defined by drug treatment, edema status, and disease stage. The log-rank test statistic is computed as:

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

where:

- O_i is the observed number of events in group i,
- E_i is the expected number of events in group i, under the null hypothesis.

This statistic follows a chi-squared distribution with degrees of freedom equal to the number of groups minus one. P-values were calculated to assess the significance of differences in survival.

Cox PH

(Feature selection, model assumptions, model evaluation)

We used a cox proportional hazards model to assess the effect of the covariates on the hazard of developing cirrhosis. The assumption of the model is that hazard ratios between groups is constant over time. In addition, the effects of the covariates on the hazard are assumed to be proportional. To determine if the Cox PH model violated the proportional hazard assumptions we used the cox.zph() function in R.

After the proportional hazards assumption is met, interaction terms between model covariates are considered using the likelihood ratio test. Suppose there are p covariates in the model. First, each one of the $\frac{p(p-1)}{2}$ interaction terms is added to the model to obtain $\frac{p(p-1)}{2}$ p value, and the interaction term with the lowest p value is added to the model. Then, the rest of the interaction terms are added to the new model one by one to obtain the p value. The process is repeated until p > 0.05 holds for all the likelihood ratio tests. The model with the added interaction terms is the final model.

With the final model, model evaluation is conducted. Deviance residuals and Cox-Snell residuals are used to evaluate model fit. For models with good fit, deviance residuals should be randomly distributed around 0, and for KM survival estimates using Cox-Snell residuals as the pseudo survival time, log(-log(S(t))) should be approximately equal to log(t). Influence diagnostics with LD option is used to identify influential individuals. It evaluates how much the log-likelihood would change if the i^{th} person was removed from the sample. After identifying outliers and influential individuals, these subjects are removed from the sample and the model is re-fit to see if the results would change.

Results

Descriptive Statistics (EDA)

Figure 1 provides insights into the distributions of continuous variables through boxplots, revealing heterogeneity in liver disease severity. Variables such as bilirubin, alkaline phosphatase, SGOT, and prothrombin exhibit highly skewed distributions with significant outliers, reflecting the heterogeneity in liver disease severity among patients. These patterns highlight the diversity in clinical markers and their potential implications for survival outcomes.

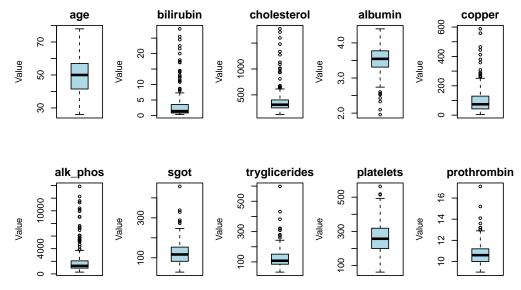


Figure 1: Boxplots for Continuous Variables

As shown in the Figure 2, the majority of patients are female, lack ascites, and are evenly distributed regarding hepatomegaly. Most are in stages 2 and 3 of the disease, with a notable proportion in stage 4, indicating disease progression. Drug distribution is balanced between D-penicillamine and placebo groups, supporting comparability in treatment outcomes.

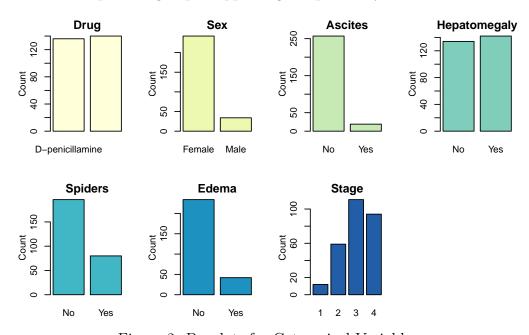


Figure 2: Barplots for Categorical Variables

Finally, Figure 3 presents the correlation matrix, which highlights strong positive associations

between bilirubin, alkaline phosphatase, and SGOT, emphasizing their relationship with liver dysfunction. In contrast, albumin and platelet counts negatively correlate with disease stage, indicating their decline as the disease advances. These relationships highlight key biomarkers of cirrhosis progression.

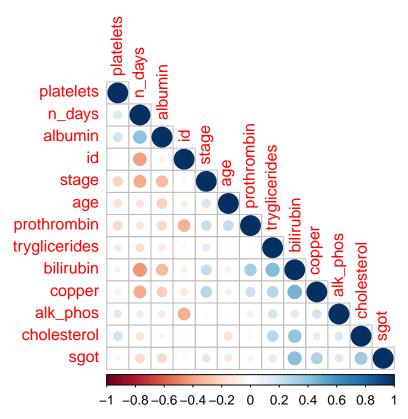


Figure 3: Correlation Matrix

KM and Log-Rank Test

Table 2: Kaplan-Meier Survival Summary in Years

Time Interval (Years)	At Risk	Events	Censored	Survival Probability	Lower CI	Upper CI
(0, 1)	276	19	0	0.93	0.90	0.96
[1, 2)	257	10	1	0.89	0.86	0.93
[2, 3)	246	22	12	0.81	0.77	0.86
[3, 4)	212	14	29	0.76	0.71	0.81
[4, 5)	169	9	24	0.71	0.66	0.77
[5, 6)	136	6	18	0.68	0.62	0.74
[6, 7)	112	9	23	0.62	0.55	0.69
[7, 8)	80	6	15	0.57	0.50	0.64

[8, 9) [9, 10)	59 41	5 6	13 8	$0.51 \\ 0.42$	$0.43 \\ 0.34$	$0.60 \\ 0.53$
[10, 11)	27	3	7	0.37	0.28	$0.48 \\ 0.45$
[11, 12)	17	2	10	0.31	0.21	

The Kaplan-Meier survival analysis provides valuable insights into overall survival probabilities over time. The survival curve shows a consistent decline in survival probability Figure 4 (a), with a median survival probability of 9 years and a 75% survival probability of 4 years. Key survival probabilities at yearly intervals are summarized in Table 2.

Table 3: Log-Rank Test Results

Chi-Squared	d Statistic	Degrees of Freedom	P-Value
Drug	0.4049	1	0.5246
Edema	53.0933	1	< 0.0001
Stage	44.6499	3	< 0.0001

A comparison of survival probabilities between the two treatment groups—D-penicillamine and placebo—demonstrated no statistically significant differences. The log-rank test produced a p-value of 0.5246, far above the significance threshold of 0.05. The Kaplan-Meier curves for these groups overlap substantially, indicating that D-penicillamine does not significantly improve survival outcomes compared to placebo (Table 3, Figure 4 (b)).

Edema shows as a critical factor influencing survival, as highlighted by the log-rank test (p-value < 0.0001). Patients without edema exhibit significantly better survival probabilities than those with edema (Table 3, Figure 4 (c)). The stark contrast in survival curves underscores edema as a key predictor of survival and its importance in risk stratification.

The histologic stage of the disease also plays a significant role in survival outcomes. The log-rank test for stage groups yielded a highly significant p-value (< 0.0001), indicating marked differences in survival curves across stages (Table 3, Figure 4 (d)). Patients in advanced stages (3 and 4) have substantially lower survival probabilities compared to those

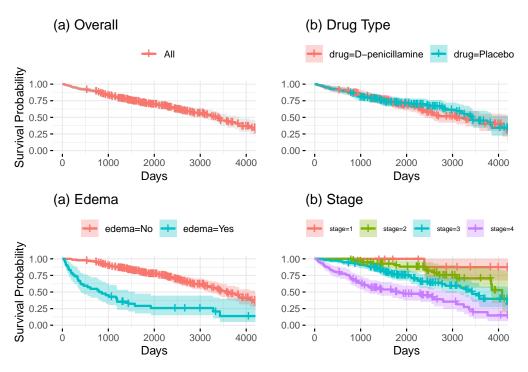


Figure 4: Kaplan-Meier Survival Curve

in earlier stages (1 and 2).

Cox Model

Feature Selection

Table 4: Model Comparison

Model	Log_Lik	AIC	Kept_Variable
Forward	-467.8089	969.6179	drug, age, sex, ascites, hepatomegaly,
			spiders, edema, bilirubin, cholesterol,
			albumin, copper, alk_phos, sgot,
			tryglicerides, platelets, prothrombin
Forward	-469.1367	968.2734	drug:bilirubin, drug:stage,
with			drug:albumin, drug:copper,
Interaction			${\it drug:} {\it prothrombin, drug:} {\it age, drug:} {\it sgot,}$
			drug
Backward	-469.5553	957.1105	age, edema, bilirubin, albumin, copper,
			sgot, prothrombin, stage, drug
Backward	-463.5469	957.0937	drug, age, edema, bilirubin, cholesterol,
with			albumin, copper, sgot, tryglicerides,
Interaction			platelets, prothrombin, stage,
			drug:sgot, drug:tryglicerides,
			drug:platelets
Stepwise	-469.5553	957.1105	age, edema, bilirubin, albumin, copper,
			sgot, prothrombin, stage, drug
Stepwise	-463.5469	957.0937	drug, age, edema, bilirubin, cholesterol,
with			albumin, copper, sgot, tryglicerides,
Interaction			platelets, prothrombin, stage,
			drug:sgot, drug:tryglicerides,
			drug:platelets
LASSO	-469.5552	959.1105	age, ascites, edema, bilirubin, albumin,
			copper, sgot, prothrombin, stage, drug

Collett's	-473.7799	963.5599	drug, age, bilirubin, albumin, copper,
Model			sgot,,drug:stage
Selection			

Forward, backward, and stepwise selection were used to identify relevant predictors, while LASSO was applied to incorporate shrinkage and reduce overfitting. Collett's approach was included to enhance robustness in model selection. Forward, backward, and stepwise selection were conducted in R using the stepAIC() function from the MASS package for forward, backward, and stepwise selection, which performs model selection based on the Akaike Information Criterion (AIC). The LASSO approach was implemented using the glmnet package, and Collett's model selection approach was performed manually following the guidelines outlines:

- 1. Fit a univariate model for each covariate, and identify the predictors significant at $p_1 = 0.20$.
- 2. Fit a multivariate model with all significant variable. Perform backward selection.
- 3. Starting with final step (2) model, consider each of the non-significant variables from step (1) using forward selection, with significance level p3, say 0.10
- 4. Do final pruning of main-effects model (omit variables that are non-significant, add any that are significant), using stepwise regression with significance level p4. At this stage, you may also consider adding interactions between any of the main effects currently in the model, under the hierarchical principle.

Interaction term between drug and other variables were included to assess the possibility that the effect of drug on the outcome might differ across levels of other variables. The variable drug was forcibly included in all models as it serves as the primary focus of the analysis.

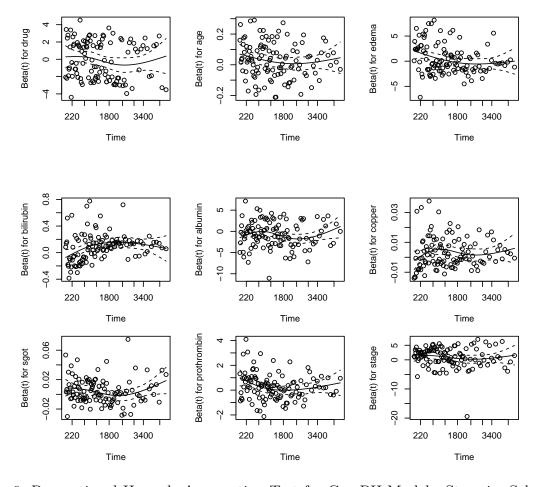
Final model selection was guided by AIC as the primary selection criterion. Among the models, backward and stepwise selection (AIC: 957.1105) and backward/stepwise selection with the interaction term (AIC: 957.0937) produced the lowest AIC values. Although the model with the interaction term had a slightly lower AIC (957.0937) compared to the simpler model (AIC: 957.1105), the difference was not substantial. The LASSO approach included additional

Table 5: Multivariate Cox Proportional Hazards Analysis - Stepwise Selection Model

Characteristic	$\mathbf{H}\mathbf{R}^{1}$	95% CI ¹	p-value
drug			
D-penicillamine			
Placebo	0.94	0.63 to 1.40	0.75
age	1.03	1.01 to 1.05	0.004
edema			
No			
Yes	1.47	0.88 to 2.47	0.14
bilirubin	1.09	1.05 to 1.13	< 0.001
albumin	0.47	0.28 to 0.82	0.007
copper	1.00	1.00 to 1.00	0.002
sgot	1.00	1.00 to 1.01	0.015
prothrombin	1.33	1.07 to 1.64	0.010
stage			
1			
2	3.88	0.47 to 32.1	0.21
3	5.29	0.68 to 41.1	0.11
4	8.02	1.04 to 61.8	0.046

¹HR = Hazard Ratio, CI = Confidence Interval

variables such as ascites and spiders in the model. In contrast, Collett's Model Selection retained far fewer variables but resulted in a significantly higher AIC of 965.2379. Based on the principle of parsimony, the simpler model was preferred to enhance interpretability and reduce the risk of overfitting. However, the final model selection will be revisited if further assessment, following the evaluation of the proportional hazards (PH) assumption, suggests a different approach is warranted.



 ${\it Table 6: Proportional\ Hazards\ Assumption\ Test\ for\ Cox\ PH\ Model\ -\ Stepwise\ Selection}$

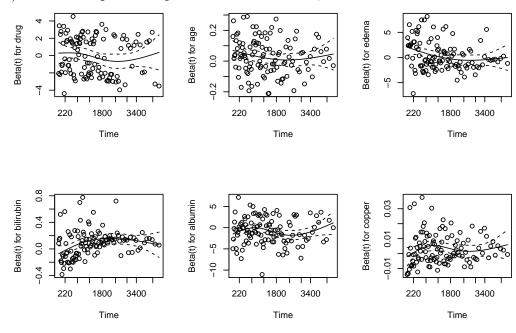
	chisq	df	p
drug	0.1600772	1	0.6890854
age	2.6909476	1	0.1009198
edema	6.1134319	1	0.0134158
bilirubin	8.3071868	1	0.0039489
albumin	0.6258766	1	0.4288719
copper	0.1021024	1	0.7493211
sgot	1.3384725	1	0.2473035
prothrombin	5.0189196	1	0.0250718
stage	4.5185052	3	0.2106456
GLOBAL	24.6087203	11	0.0103973

Table 7: Multivariate Cox Proportional Hazards Analysis - Stratification of Edema Model

Characteristic	$\mathbf{H}\mathbf{R}^{1}$	$\mathbf{95\%} \mathbf{CI}^{\scriptscriptstyle 1}$	p-value
drug			
D-penicillamine			
Placebo	0.88	0.59 to 1.31	0.53
age	1.03	1.01 to 1.05	0.005
bilirubin	1.08	1.04 to 1.13	< 0.001
albumin	0.50	0.29 to 0.86	0.011
copper	1.00	1.00 to 1.00	0.004
sgot	1.00	1.00 to 1.01	0.033
prothrombin	1.36	1.09 to 1.70	0.006
stage			
1			
2	4.64	0.55 to 39.4	0.16
3	6.55	0.83 to 52.0	0.075
4	9.43	1.20 to 74.3	0.033

¹HR = Hazard Ratio, CI = Confidence Interval

We can see from Table $\ref{Table points}$ that edema(p=0.013), bilirubin (p=0.003), and prothrombin (p=0.025) violate the ph assumptions. To reduce bias, stratification was conducted.



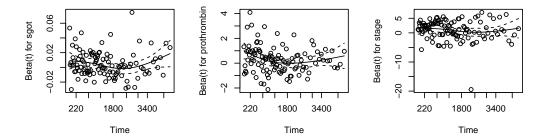


Table 8: Proportional Hazards Assumption Test COX PH Model Stratified for Edema

	chisq	df	р
drug	1.4344763	1	0.2310353
age	1.8937409	1	0.1687806
bilirubin	11.5075689	1	0.0006931
albumin	0.0049180	1	0.9440915
copper	0.3914633	1	0.5315312
sgot	1.0484233	1	0.3058705
prothrombin	2.6117691	1	0.1060734
stage	3.0850010	3	0.3787045
GLOBAL	18.9514038	10	0.0408843

In Table ??, after the straification of edema, bilirubin (p=0.0006) is the only variable violating PH assumptions. A time interactin is added to bilirubin to make the model appropriate.

Now, interaction terms are considered between the covariates in the model. During the first iteration, five interaction terms, including interaction for Copper with age, Albumin, SGOT, Prothrombin, and diease stage are selected using criteria p < 0.05, and the interaction between Albumin and Copper is added to the model as it has the lowest p value. During the second iteration, none of the interaction terms gets selected, and the iteration ends. The final model can then be specified as:

$$log(HR) = \beta_1 I(drug=D-penicillamine) + \beta_2 age + \beta_3 billirubin + \beta_4 albumin + \beta_5 copper$$
$$+ \beta_6 sgot + \beta_7 prothrombin + \beta_8 I(stage=2) + \beta_9 I(stage=3) + \beta_{10} I(stage=4)$$
$$+ \beta_{11} billirubin : n_days + \beta_{12} albumin : copper$$

Table 9 shows the model estimates. It can be concluded that, holding other covariates constant:

- The primary variable of interest, drug, has a negative yet insignificant impact on survival. Other covariates that impose a negative significant impact include age, Bilirubin, SGOT, Prothrombin, Stage 4 (compared with Stage 1), and interaction between Albumin and Copper. Albumin, Copper, and interaction between Bilirubin and number of days instead have a protective significant impact.
- The hazard for PBC patients treated with D-penicillamine is 1.2720 times that of PBC patients treated with Placebo.
- The hazard ratio for PBC patients with one year increase in age is 1.0343.
- The hazard ratio for PBC patients with 1 mg/dl increase in Bilirubin is 1.2798.
- The hazard ratio for PBC patients with 1 gm/dl increase in Albumin is 0.2316.
- The hazard ratio for PBC patients with 1 ug/day increase in Copper is 0.9779.
- The hazard ratio for PBC patients with 1U/ml in SGOT is 1.0065.
- The hazard ratio for PBC patients with 1s increase in Prothrombin is 1.3257.
- The hazard for PBC patients at Stage 2, 3, and, 4 is 3.7034, 5.4604, and 8.0139 times that of PBC patients at Stage 1.
- For the same level of Bilirubin, a unit increase in survive time results in -0.0002Bilirubin change in the effect of Bilirubin on log hazard ratio for PBC patients.
- For the same level of Albumin, a unit increase in Copper results in 0.0076Albumin change in the effect of Albumin on log hazard ratio for PBC patients.

Model evaluation is then conducted on this final model. Figures 5 and 6 show the deviance residuals distribution and the KM estimates using Cox-Snell residuals as pseudo survival time respectively. As there is no obvious trend in the deviance plots and the line is close to the reference line in Cox-Snell plot, it can be concluded that the model is a relatively good fit.

For influence diagnostics, the individuals that provide the 5 largest absolute differences for the LD option are selected. After removing the outliers (identified by a deviance residual larger than 3, 2 are selected) and the 5 influential individuals, the model is re-fit. Table 10 compares the model estimates for the two models. There are subtle differences between model

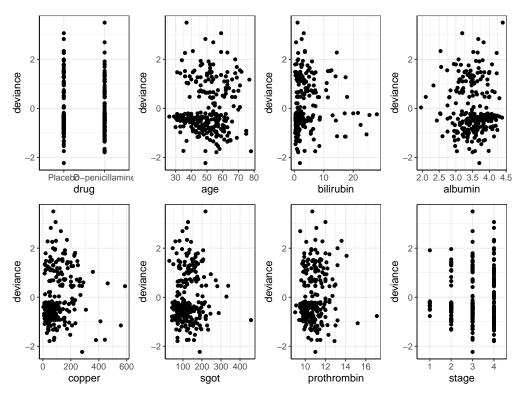


Figure 5: Deviance Residuals Scatterplot for Individual Variable

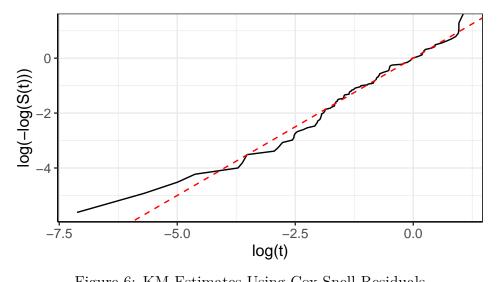


Figure 6: KM Estimates Using Cox-Snell Residuals

Table 9: Final Model Hazard Ratio Estimates

Characteristic	$\mathbf{H}\mathbf{R}^{1}$	95% CI 1	p-value
drug			
drug.L	1.2720	0.9451 to 1.7118	0.1124
age	1.0343	1.0119 to 1.0572	0.0025
bilirubin	1.2798	1.1927 to 1.3731	0.0000
albumin	0.2316	0.1089 to 0.4927	0.0001
copper	0.9779	0.9641 to 0.9919	0.0020
sgot	1.0065	1.0026 to 1.0104	0.0010
prothrombin	1.3257	1.0521 to 1.6704	0.0168
stage			
1			
2	3.7034	0.4513 to 30.392	0.2228
3	5.4604	0.6981 to 42.713	0.1058
4	8.0139	1.0305 to 62.324	0.0467
bilirubin * n_days	0.9998	0.9997 to 0.9999	0.0000
albumin * copper	1.0076	1.0034 to 1.0119	0.0004

 $^{^{}T}$ HR = Hazard Ratio, CI = Confidence Interval

estimates, but the direction of impact stays the same for all the variables, thus resulting in similar conclusions.

Table 10: Model Parameter Estimates Comparison

	Original Model			New Model			
	Estimate	Hazard Ratio	p value	Estimate	Hazard Ratio	p value	
drug.L	0.2406	1.2720	0.1124	0.2907	1.3374	0.0662	
age	0.0337	1.0343	0.0025	0.0303	1.0308	0.0098	
bilirubin	0.2467	1.2798	0.0000	0.3033	1.3543	0.0000	
albumin	-1.4627	0.2316	0.0001	-1.5639	0.2093	0.0001	
copper	-0.0224	0.9779	0.0020	-0.0226	0.9777	0.0021	
sgot	0.0065	1.0065	0.0010	0.0063	1.0063	0.0024	
prothrombin	0.2819	1.3257	0.0168	0.3428	1.4089	0.0063	
stage2	1.3093	3.7034	0.2228	1.4232	4.1505	0.1921	
stage3	1.6975	5.4604	0.1058	1.7935	6.0104	0.0930	

stage4	2.0812	8.0139	0.0467	2.1404	8.5030	0.0440
bilirubin:n_days	-0.0002	0.9998	0.0000	-0.0002	0.9998	0.0000
albumin:copper	0.0076	1.0076	0.0004	0.0078	1.0078	0.0003

Discussion

The analysis demonstrates that D-penicillamine is inefficient in improving survival outcomes for patients with primary biliary cirrhosis (PBC). The hazard ratio for those treated with the drug is 1.2720 compared to placebo, indicating no survival benefit and a potential negative effect. This finding suggests the need to reconsider its use and focus on alternative therapies that may offer greater efficacy.

Kaplan-Meier analysis further confirmed the inefficacy of D-penicillamine, as survival probabilities between the drug and placebo groups showed no statistically significant difference. The overlapping survival curves reinforce the finding that D-penicillamine does not confer a survival advantage and may require reevaluation as a treatment option. Additionally, Kaplan-Meier analysis highlighted the importance of edema, as patients without edema showed significantly better survival probabilities, making it a critical factor in risk stratification.

Age and disease stage emerged as critical determinants of survival, underscoring the importance of early detection and stage-specific care. The hazard of mortality increases significantly with each advancing stage, with Stage 4 patients facing an eightfold higher risk compared to Stage 1. Kaplan-Meier survival curves demonstrated markedly reduced survival probabilities in advanced stages, with a sharp decline observed in stages 3 and 4. Early interventions to halt disease progression are vital, as is tailoring treatment strategies to the patient's disease stage.

Liver function indicators such as Bilirubin, Copper, SGOT, Prothrombin, and Albumin are vital in survival outcomes. Elevated Bilirubin, SGOT, and Prothrombin levels are associated with higher hazards, reflecting liver damage and dysfunction. Conversely, higher albumin levels provide a strong protective effect, emphasizing the importance of maintaining good

nutritional and synthetic liver function. The modest protective impact of Copper is proved by previous studies where Copper deficiency is identified as a risk factor for mortality in advanced liver disease (Yu et al. 2019). These findings highlight the necessity of monitoring liver function and metabolic health closely to identify high-risk patients and address reversible factors.

The analysis also revealed key interactions affecting survival. A negative Albumin-Copper interaction was observed, indicating a synergistic detrimental effect. This underscores the complexity of metabolic interactions in liver disease and the need to address deficiencies or toxicities in a balanced manner. Furthermore, continued investigation of other potential interactions is needed to uncover personalized treatment strategies.

To improve patient outcomes, it is crucial to monitor high-risk patients routinely, focusing on those with poor liver function indicators or advanced disease stages. Additionally, conducting biological investigations into the protective role of copper and its interactions with other variables could provide valuable insights. These efforts can pave the way for personalized therapies that address the unique risk profiles of individual patients and enhance survival outcomes.

There were several limitations within the project including missing data, imbalanced data, and a high censoring rate. Regarding missing data, 147 observation had missing values, which may require the application of imputations techniques to address potential bias. Furthermore, the data imbalance was attributed to the distribution of sex, where we had about 80-90% female participants and the right-skwedness of bilirubin. Lastly, with more than 50% of data being censored, high censoring data was a major limitation on the survival analysis and the robustness of the results.

Conclusion

References

Yu, L., I. W. Liou, S. W. Biggins, M. Yeh, F. Jalikis, L. N. Chan, and J. Burkhead. 2019.
"Copper Deficiency in Liver Diseases: A Case Series and Pathophysiological Considerations."
Hepatology Communications 3 (8): 1159–65. https://doi.org/10.1002/hep4.1393.

Appendix

Code

```
knitr::opts chunk$set(echo = FALSE, message = F, warning = F, out.width = "80%", fig.ali
options(knitr.kable.NA = '')
library(tidyverse)
library(RColorBrewer)
library(corrplot)
library(gtsummary)
library(flextable)
library(stringr)
library(survival)
library(survminer)
library(kableExtra)
library(ggplot2)
library(ggpubr)
library(gridExtra)
library(MASS)
library(glmnet)
write_matex <- function(x) {</pre>
  begin <- "$$\\begin{bmatrix}"</pre>
  end <- "\\end{bmatrix}$$"</pre>
    apply(x, 1, function(x) {
      paste(
        paste(x, collapse = "&"),
        "////"
      )
    })
  writeLines(c(begin, X, end))
}
theme_set(
  theme bw()+
  theme(
    plot.title = element_text(size = 16, hjust = 0.5),
    axis.title.x = element_text(size = 12),
    axis.title.y = element_text(size = 12),
    axis.text = element_text(size = 10),
    axis.line = element_line(color = "black", size = 0.5),
  )
cirrhosis <- read_csv("data/cirrhosis.csv")|>
```

```
janitor::clean_names() |>
 mutate(age = round(age / 365),
         sex = if_else(sex == "M", "Male", "Female"),
         ascites = if_else(ascites == "N", "No", "Yes"),
         hepatomegaly = if_else(hepatomegaly == "N", "No", "Yes"),
         spiders = if_else(spiders == "N", "No", "Yes"),
         edema = if_else(edema == "N", "No", "Yes"))|>
 na.omit()
theme_gtsummary_journal(journal = "nejm")
cirrhosis_df <- cirrhosis |>
 mutate(
    status = case_when(
      status == "C" ~ "Censored",
      status == "CL" ~ "Censored due to liver tx",
      status == "D" ~ "Death",
      TRUE ~ status))
table_1 <- cirrhosis_df |>
 dplyr::select(-id) |>
 tbl_summary(
   by = status,
    statistic = list(
      all_continuous() ~ "{mean} / {median} ({sd})",
     all_categorical() ~ ^{n} ({p}%)"
    ),
    digits = all_continuous() ~ 1,
   missing = "no",
   label = list(
   n_days ~ "N_days",
    drug ~ "Drug",
    age ~ "Age",
    sex ~ "Sex",
    ascites ~ "Ascites",
   hepatomegaly ~ "Hepatomegaly",
    spiders ~ "Spiders",
    edema ~ "Edema",
    bilirubin ~ "Bilirubin",
    cholesterol ~ "Cholesterol",
    albumin ~ "Albumin",
    copper ~ "Copper",
    alk_phos ~ "Alk_phos",
    sgot ~ "SGOT",
    tryglicerides ~ "Tryglicerides",
```

```
platelets ~ "Platelets",
   prothrombin ~ "Prothrombin",
    stage ~ "Stage"
 )) |>
 modify_caption("Baseline Characteristics") |>
 as_flex_table() |>
 set_table_properties(width = 0.8, layout = "autofit") # Set width to 80%
table 1
conti vars = cirrhosis |>
 dplyr::select(age, bilirubin, cholesterol, albumin, copper, alk_phos, sgot, trygliceri
# Boxplot for all continuous variables
par(mfrow = c(2, 5), oma = c(2, 2, 3, 1), mar = c(4, 4, 2, 1))
conti names <- names(conti vars)</pre>
p1 <- for (i in seq_along(conti_names)) {</pre>
 boxplot(conti_vars[[conti_names[i]]],
          main = conti_names[i],
          ylab = "Value",
          col = "lightblue",
          outline = TRUE) # Show outliers
cate vars = cirrhosis |>
 dplyr::select(drug, sex, ascites, hepatomegaly, spiders, edema, stage)
par(mfrow = c(2, 4), #2 rows, 5 columns)
   oma = c(2, 2, 3, 1), # Outer margins
    mar = c(4, 4, 2, 1), \# Inner margins for individual plots
                        # Margins for axis labels and titles
   mgp = c(2, 1, 0)
colors <- c(brewer.pal(9, "YlGnBu"), "darkblue")</pre>
barplot(table(cate_vars$drug), main = "Drug", ylab = "Count",col = colors[1])
barplot(table(cate_vars$sex), main = "Sex", ylab = "Count",col = colors[2])
barplot(table(cate_vars$ascites), main = "Ascites", ylab = "Count", col = colors[3])
barplot(table(cate_vars$hepatomegaly), main = "Hepatomegaly", ylab = "Count", col = colo
barplot(table(cate_vars$spiders), main = "Spiders", ylab = "Count", col = colors[5])
barplot(table(cate_vars$edema), main = "Edema", ylab = "Count", col = colors[6])
barplot(table(cate_vars$stage), main = "Stage", ylab = "Count", col = colors[7])
numeric_cirr <- cirrhosis |>
 select_if(is.numeric)
cor matrix <- cor(numeric cirr, use = "complete.obs")</pre>
```

```
corrplot(cor matrix, method = "circle", type = "lower", order = "hclust")
cirrhosis <- read_csv("../data/cirrhosis.csv")|>
  janitor::clean_names() |>
  mutate(age = round(age / 365),
         sex = if_else(sex == "M", "Male", "Female"),
         ascites = if_else(ascites == "N", "No", "Yes"),
         hepatomegaly = if_else(hepatomegaly == "N", "No", "Yes"),
         spiders = if_else(spiders == "N", "No", "Yes"),
         edema = if_else(edema == "N", "No", "Yes")) |>
  drop_na()
cirrhosis$event <- ifelse(cirrhosis$status == "D", 1, 0)</pre>
surv object <- Surv(time = cirrhosis$n days, event = cirrhosis$event)</pre>
km_fit <- survfit(surv_object ~ 1, data = cirrhosis)</pre>
plot_all = ggsurvplot(km_fit, conf.int = TRUE,
           title = "(a) Overall",
           xlab = "Days", ylab = "Survival Probability",
           legend.title = "",
           ggtheme = theme_minimal())
km_fit_drug <- survfit(surv_object ~ drug, data = cirrhosis)</pre>
plot drug = ggsurvplot(km fit drug, conf.int = TRUE,
           title = "(b) Drug Type",
           xlab = "Days", ylab = " ",
           legend.title = "",
           ggtheme = theme_minimal() )
# Fit survival curves by edema
km fit edema <- survfit(surv object ~ edema, data = cirrhosis)</pre>
plot_edema <- ggsurvplot(</pre>
 km fit edema, conf.int = TRUE,
 title = "(a) Edema",
  xlab = "Days", ylab = "Survival Probability",
 legend.title = "",
  ggtheme = theme_minimal()
)
# Fit survival curves by stage
```

```
km fit stage <- survfit(surv object ~ stage, data = cirrhosis)</pre>
plot stage <- ggsurvplot(</pre>
  km_fit_stage, conf.int = TRUE,
  title = "(b) Stage",
  xlab = "Days", ylab = " ",
  legend.title = "",
  ggtheme = theme_minimal() +
    theme(legend.text = element_text(size = 6)) # Adjust legend text size
)
# Arrange the plots side by side with adjusted spacing
grid.arrange(
  plot_all$plot,
 plot_drug$plot,
 plot edema$plot,
 plot_stage$plot,
 ncol = 2.
 widths = c(1, 1) # Equal sizing for both plots
)
max_time <- max(cirrhosis$n_days, na.rm = TRUE)</pre>
max years <- floor(max time / 365)</pre>
yearly times \leftarrow seq(0, max years * 365, by = 365)
km_summary_yearly <- summary(km_fit, times = yearly_times)</pre>
# Create the data frame from the KM summary
surv yearly table <- data.frame(</pre>
  years = yearly_times / 365,
  n risk = km summary yearly$n.risk,
 n event = km summary yearly$n.event,
  n_censor = km_summary_yearly$n.censor,
  survival = km_summary_yearly$surv,
 lower ci = km summary yearly$lower,
 upper ci = km summary yearly$upper
)
# If time=0 row does not exist, add it
if (!any(yearly_times == 0)) {
  surv_yearly_table <- rbind(</pre>
    data.frame(
      years = 0,
      n_risk = km_fit$n.risk[1],
```

```
n = 0,
      n censor = 0,
      survival = 1,
      lower ci = 1,
      upper_ci = 1
    ),
    surv_yearly_table
  )
}
surv_yearly_table <- surv_yearly_table[order(surv_yearly_table$years), ]</pre>
interval_labels <- sapply(2:nrow(surv_yearly_table), function(i) {</pre>
  paste0("[", surv_yearly_table$years[i-1], ", ", surv_yearly_table$years[i], ")")
surv yearly intervals <- surv yearly table[-1, ] # Remove the first row if needed
surv_yearly_intervals$interval <- interval_labels</pre>
surv_yearly_intervals$n_risk[1] <- surv_yearly_table$n_risk[1]</pre>
for (i in 2:nrow(surv yearly intervals)) {
  surv_yearly_intervals$n_risk[i] <- surv_yearly_intervals$n_risk[i-1] -</pre>
                                        surv yearly intervals$n event[i-1] -
                                        surv yearly intervals$n censor[i-1]
}
surv_table = surv_yearly_intervals %>%
  rownames_to_column() %>% # Convert any existing row names to a column
  dplyr::select(-rowname) %>%
                                           # Remove the converted row names column
  dplyr::select(interval, n_risk, n_event, n_censor, survival, lower_ci, upper_ci) |>
  as.data.frame()
kable(
  surv_table,
  caption = "Kaplan-Meier Survival Summary in Years",
      col.names = c("Time Interval (Years)", "At Risk", "Events", "Censored", "Survival
  digits = 2,
  booktabs = TRUE
) %>%
  kable_styling(full_width = FALSE)
log_rank_test <- survdiff(surv_object ~ drug, data = cirrhosis)</pre>
log rank results drug <- data.frame(</pre>
```

```
Group = "Drug",
  Statistic = log rank test$chisq,
  Degrees_of_Freedom = 1,
  P Value = log rank test$pvalue
)
log_rank_results_drug[, -1] <- log_rank_results_drug[, -1] %>%
  mutate(across(where(is.numeric), ~ round(., 4)))
# Log-rank test for edema
log rank test edema <- survdiff(surv object ~ edema, data = cirrhosis)</pre>
log_rank_results_edema <- data.frame(</pre>
  Group = "Edema",
  Statistic = log rank test edema$chisq,
  Degrees_of_Freedom = 1,
  P Value = ifelse(log rank test edema$pvalue < 0.0001, "<0.0001", log rank test edema$pvalue
)
# Log-rank test for stage
log_rank_test_stage <- survdiff(surv_object ~ stage, data = cirrhosis)</pre>
log_rank_results_stage <- data.frame(</pre>
  Group = "Stage",
  Statistic = log_rank_test_stage$chisq,
  Degrees of Freedom = 3,
  P_Value = ifelse(log_rank_test_stage$pvalue < 0.0001, "<0.0001", log_rank_test_stage$pvalue
log_rank_results_combined <- rbind(log_rank_results_drug,</pre>
                                    log_rank_results_edema,
                                    log_rank_results_stage)
kable(
  log rank results combined[, -1], # Exclude the "Group" column for main table
  digits = 4,
  col.names = c("Chi-Squared Statistic", "Degrees of Freedom", "P-Value"),
  caption = "Log-Rank Test Results"
) %>%
  pack_rows("Drug", 1, 1) |>
  pack_rows("Edema", 2, 2) |>
  pack_rows("Stage", 3, 3)
main effects terms = cirrhosis |>
  dplyr::select(-matches("^(id|n_days|status|event)")) |>
  colnames()
```

```
initial models = map(main effects terms, \setminus(x)
  coxph(as.formula(paste0("surv_object ~", x)), data = cirrhosis)
)
# Extract significant variables (p < 0.20)
initial_significant_vars = map(initial_models, \(x)
  summary(x)$coefficients[, "Pr(>|z|)"] < 0.20</pre>
) |> unlist()
significant_var_names = main_effects_terms[initial_significant_vars]
# Step 2: Fit multivariate model and perform backward selection
initial_terms = significant_var_names
formula_str = paste("surv_object ~ drug +", paste(initial_terms, collapse = " + "))
initial model = coxph(as.formula(formula str), data = cirrhosis, model = TRUE)
# Extract the exact data used in the initial model
cirrhosis_clean = cirrhosis[complete.cases(cirrhosis[, c("n_days", "status", "drug", "evaluation")
# Start backward selection
current_model = initial_model
current_terms = initial_terms
while (TRUE) {
  terms_to_test = setdiff(current_terms, "drug")
  if (length(terms_to_test) == 0) break
  best_model = current_model
  term_to_remove = NULL
  for (term in terms_to_test) {
    reduced terms = setdiff(current terms, term)
    formula_str = paste("surv_object ~ drug +", paste(reduced_terms, collapse = " + "))
    reduced model = coxph(as.formula(formula str), data = cirrhosis clean)
    lrt = anova(current_model, reduced_model, test = "LRT")
    lrt_p_value = lrt[2, "Pr(>|Chi|)"]
    if (lrt_p_value > 0.10) {
      best_model = reduced_model
      term_to_remove = term
      break
    }
```

```
if (is.null(term to remove)) break
 current_model = best_model
 current terms = setdiff(current terms, term to remove)
}
backward_model = current_model
# Step 3: Forward selection of previously non-significant variables
current_model = backward_model
excluded vars = setdiff(significant var names, attr(terms(current model), "term.labels")
for (var in excluded vars) {
 current_terms = attr(terms(current_model), "term.labels")
 formula terms = unique(c(current terms, var))
 formula str = paste("surv object ~", paste(union("drug", formula terms), collapse = "
 extended_model = coxph(as.formula(formula_str), data = cirrhosis_clean)
 lrt = anova(current model, extended model, test = "LRT")
 lrt_p_value = lrt[2, "Pr(>|Chi|)"]
 if (lrt_p_value <= 0.10) {</pre>
   current model = extended model
 }
forward model = current model
# Step 4: Final stepwise selection (backward + forward)
current model = forward model
while (TRUE) {
 current_terms = attr(terms(current_model), "term.labels")
 if (length(setdiff(current terms, "drug")) == 0) break
 best_model = current_model
 term_to_remove = NULL
 term_to_add = NULL
 for (term in setdiff(current_terms, "drug")) {
    remaining_terms = setdiff(current_terms, term)
    formula_str = paste("surv_object ~", paste(union("drug", remaining_terms), collapse
    reduced model = coxph(as.formula(formula str), data = cirrhosis clean)
```

```
lrt = anova(current model, reduced model, test = "LRT")
    lrt p value = lrt[2, "Pr(>|Chi|)"]
    if (lrt_p_value > 0.10) {
     best model = reduced model
     term_to_remove = term
     break
   }
 }
 excluded_vars = setdiff(significant_var_names, current_terms)
 for (var in excluded vars) {
    formula terms = union(current terms, var)
    formula_str = paste("surv_object ~", paste(union("drug", formula_terms), collapse =
    extended_model = coxph(as.formula(formula_str), data = cirrhosis_clean)
    lrt = anova(current_model, extended_model, test = "LRT")
    lrt_p_value = lrt[2, "Pr(>|Chi|)"]
    if (lrt p value <= 0.10) {
     best model = extended model
     term_to_add = var
     break
   }
 }
 if (is.null(term_to_remove) && is.null(term_to_add)) break
 current model = best model
}
# Interaction terms
main_effects = setdiff(attr(terms(current_model), "term.labels"), "drug")
for (var in main_effects) {
  interaction_term = paste("drug:", var)
 interaction_formula = paste(deparse(current_model$formula), "+", interaction_term)
 interaction_model = coxph(as.formula(interaction_formula), data = cirrhosis_clean)
 lrt = anova(current model, interaction model, test = "LRT")
 lrt_p_value = lrt[2, "Pr(>|Chi|)"]
 if (lrt_p_value <= 0.10) {</pre>
    current_model = interaction_model
```

```
}
}
colletts_model = current_model
forward model = stepAIC(
  coxph(
    Surv(n_days, event) ~ .,
    data = cirrhosis |>
      dplyr::select(-status) |>
      dplyr::select(-id) |> na.omit()
  ),
  direction = "forward",
  trace = F
)
# Forward already has drug
forward_model_interact = stepAIC(
  coxph(
    Surv(n days, event) ~ 1,
    data = cirrhosis |>
      dplyr::select(-status) |>
      dplyr::select(-id) |> na.omit()
  scope = list(lower = ~ 1, upper = as.formula(paste(
    "~ · +",
    paste0(
      "drug:",
      cirrhosis |> dplyr::select(-id, -n days, -status, -drug, -event) |> colnames(),
      collapse = " + "
    )
  ))),
  tract = F,
  direction = "forward"
)
forward_model_interact = coxph(
  as.formula(
    paste(
      "Surv(n days, event) ~",
      forward_model_interact$formula[3],
      "+ drug"
    )
  ),
```

```
data = cirrhosis |>
    dplyr::select(-status) |>
   dplyr::select(-id) |> na.omit()
backward model = stepAIC(
 coxph(
   Surv(n_days, event) ~ .,
    data = cirrhosis |>
      dplyr::select(-status) |>
      dplyr::select(-id) |> na.omit()
 ),
 direction = "backward",
 trace = F
)
backward model = coxph(
 as.formula(
   paste("Surv(n_days, event) ~", backward_model$formula[3], "+ drug")
 ),
 data = cirrhosis |>
   dplyr::select(-status) |>
   dplyr::select(-id) |> na.omit()
)
backward_model_interact = stepAIC(
 coxph(
    as.formula(paste(
      "Surv(n_days, event) ~ . +",
     paste0(
        "drug:",
        cirrhosis |> dplyr::select(-id, -n_days, -status, -drug, -event) |> colnames(),
        collapse = " + "
     )
    )),
    data = cirrhosis |>
      dplyr::select(-status) |>
      dplyr::select(-id) |> na.omit()
 ),
 direction = "backward",
 trace = F
stepwise_model = stepAIC(
 coxph(
   Surv(n_days, event) ~ .,
```

```
data = cirrhosis |>
      dplyr::select(-status) |>
      dplyr::select(-id) |> na.omit()
 ),
 direction = "both",
 trace = F
stepwise_model = coxph(
 as.formula(
   paste("Surv(n_days, event) ~", stepwise_model$formula[3], "+ drug")
 ),
 data = cirrhosis |>
   dplyr::select(-status) |>
   dplyr::select(-id) |> na.omit()
)
stepwise_model_interact = stepAIC(
 coxph(
    as.formula(paste(
      "Surv(n days, event) ~ . +",
     paste0(
        "drug:",
        cirrhosis |> dplyr::select(-id, -n days, -status, -drug, -event) |> colnames(),
        collapse = " + "
      )
    )),
   data = cirrhosis |>
      dplyr::select(-status) |>
     dplyr::select(-id) |> na.omit()
 ),
 direction = "both",
 trace = F
x = model.matrix(Surv(n_days, event) ~ . - id, cirrhosis |> na.omit() |> dplyr::select(
y = Surv(cirrhosis |>
           na.omit() |>
          pull(n_days),
         cirrhosis |>
           na.omit() |>
           pull(event))
cv_model = cv.glmnet(x, y, family = "cox", alpha = 1)
best_lambda = cv_model$lambda.min
selected_coefficients = coef(cv_model, s = best_lambda)
```

```
selected var name = rownames(selected coefficients)[which(selected coefficients != 0)]
selected_var_name = map(selected_var_name, str_remove_all, "Yes|Placebo") |> unlist()
lasso_model = coxph(as.formula(paste0("surv_object ~ ", paste(selected_var_name, collap."))
                               dplyr::select(-id) |> na.omit())
lasso_model = coxph(as.formula()
  paste(
    "surv_object ~",
    lasso_model$formula[3],
    "+ drug"
), data = cirrhosis |>
  dplyr::select(-status) |>
  dplyr::select(-id) |> na.omit())
model_comparison = data.frame(
  Model = c(
    "Forward",
    "Forward with Interaction",
    "Backward",
    "Backward with Interaction",
    "Stepwise",
    "Stepwise with Interaction",
    "LASSO",
    "Collett's Model Selection"
  ),
  Log_Lik = c(
    logLik(forward_model),
    logLik(forward_model_interact),
    logLik(backward_model),
    logLik(backward_model_interact),
    logLik(stepwise_model),
    logLik(stepwise_model_interact),
    logLik(lasso_model),
    logLik(colletts_model)
  ),
  AIC = c(
    AIC(forward_model),
    AIC(forward_model_interact),
    AIC(backward model),
    AIC(backward_model_interact),
    AIC(stepwise_model),
    AIC(stepwise_model_interact),
    AIC(lasso_model),
    AIC(colletts_model)
```

```
),
 Kept Variable = c(
    str_replace_all(paste(forward_model$formula[[3]][2]), " \\+", ","),
    str_replace_all(paste(forward model interact$formula[3]), " \\+", ","),
    str_replace_all(paste(formula(backward model)[3]), " \\+", ","),
    str_replace_all(paste(formula(backward model interact)[3]), " \\+", ","),
    str_replace_all(paste(formula(stepwise model)[3]), " \\+", ","),
    str_replace_all(paste(formula(stepwise_model_interact)[3]), " \\+", ","),
    str_replace_all(paste(formula(lasso model)[3]), " \\+", ","),
    str_replace_all(paste(formula(colletts model)[3]), " \\+", ",")
 )
)
model comparison |>
 knitr::kable(caption = "Model Comparison") |>
 kable_styling(full width = TRUE) |>
 column_spec(1, width = "2cm") |>
 column_spec(2, width = "2cm") |>
 column_spec(3, width = "2cm") |>
  column_spec(4, width = "7cm")
cirrhosis = read csv("data/cirrhosis.csv") |>
  janitor::clean_names() |>
 mutate(age = round(age / 365),
         sex = if_else(sex == "M", "Male", "Female"),
        ascites = if_else(ascites == "N", "No", "Yes"),
        hepatomegaly = if_else(hepatomegaly == "N", "No", "Yes"),
         spiders = if_else(spiders == "N", "No", "Yes"),
         edema = if_else(edema == "N", "No", "Yes")) |>
 na.omit()
cirrhosis = cirrhosis |>
 mutate(
    status = case when(
      status == "D" ~ 1, # Event of interest (death)
      status == "C" | status == "CL" ~ 0, # Censored data
      TRUE ~ as.numeric(status)),
      stage = factor(stage) # Convert 'stage' to a factor
)
# cox model based on stepwise selection varibales above (ixta)
cox model a = coxph(Surv(n days, status) ~ drug + age + edema +
                          bilirubin + albumin + copper + sgot +
                          prothrombin + stage,
                    id=id,
                   data = cirrhosis)
```

```
# Summarize the results
cox summary a = tbl_regression(cox model a, exponentiate = TRUE) |>
  modify_caption("Multivariate Cox Proportional Hazards Analysis - Stepwise Selection Mo
cox summary a
ph_assumption_a = cox.zph(cox_model_a)
par(mfrow = c(2, 3))
plot(ph_assumption_a)
ph_assumption_df = as.data.frame(ph_assumption_a$table)
knitr::kable(ph_assumption_df, caption = "Proportional Hazards Assumption Test for Cox F
cox model edema strat = coxph(Surv(n days, status) ~ drug + age + strata(edema) +
                                bilirubin + albumin + copper + sgot +
                                prothrombin + stage,
                                id = id,
                                data = cirrhosis)
# Summarize the results
cox_summary_b = tbl_regression(cox_model_edema_strat, exponentiate = TRUE) |>
  modify_caption("Multivariate Cox Proportional Hazards Analysis - Stratification of Ed
cox summary b
ph_assumption_b = cox.zph(cox_model_edema_strat)
par(mfrow = c(2, 3))
plot(ph_assumption_a)
ph_assumption_edema_strat = cox.zph(cox_model_edema_strat)
ph assumption edema = as.data.frame(ph assumption edema strat$table)
knitr::kable(ph_assumption_edema, caption = "Proportional Hazards Assumption Test COX PH
cirrhosis = read_csv("../data/cirrhosis.csv")|>
  janitor::clean_names() |>
  mutate(age = round(age / 365),
         sex = if_else(sex == "M", "Male", "Female"),
         ascites = if_else(ascites == "N", "No", "Yes"),
         hepatomegaly = if_else(hepatomegaly == "N", "No", "Yes"),
         spiders = if_else(spiders == "N", "No", "Yes"),
         edema = if_else(edema == "N", "No", "Yes"),
         stage = factor(stage),
         drug = factor(drug, levels = c("Placebo", "D-penicillamine"), order = T)) |>
  na.omit()
cirrhosis = cirrhosis |>
  mutate(
    status = case_when(
      status == "D" ~ 1, # Event of interest (death)
      status == "C" | status == "CL" ~ 0, # Censored data
      TRUE ~ as.numeric(status)))
# Interaction between Convariates
cox_init = coxph(Surv(n_days, status) ~ drug + age + strata(edema) +
```

```
bilirubin + albumin + copper + sgot +
                                prothrombin + stage + bilirubin : n days,
                         id = id,
                        data = cirrhosis |> na.omit())
variables = c("drug", "age", "albumin", "copper", "sgot",
              "prothrombin", "stage")
vars df = tibble()
for(var in variables[1 : (length(variables) - 1)])
  left vars = variables[(which(variables == var) + 1) : length(variables)]
  for(var2 in left vars)
    cox_fit = coxph(Surv(n_days, status) ~ drug + age + strata(edema) +
                                bilirubin + albumin + copper + sgot +
                                prothrombin + stage + bilirubin : n days +
                      eval(parse(text = var2)) : eval(parse(text = var)),
                         id = id.
                        data = cirrhosis |> na.omit())
     # aic vec= c(aic vec, AIC(model four))
     chisq stat=-2 * (logLik(cox init)-logLik(cox fit))
     p_val = 1 - pchisq(chisq_stat,
                        attr(logLik(cox fit), "df") -
                          attr(logLik(cox init), "df"))
     if(p val < 0.05)
       vars df = vars_df |> rbind(c(round(p_val, 4), var, var2))
 }
}
colnames(vars df) = c("p value", "variable1", "variable2")
# vars_df />
   mutate(interaction = paste0(variable1, " * ", variable2)) |>
    select(interaction, p value) />
    knitr::kable(col.names = c("Interaction Term", "P Value"),
      caption = "Siginificant Interaction term")
# We first add the albumin*copper term into the model and evaluate again.
cox fit2 = coxph(Surv(n days, status) ~ drug + age + strata(edema) +
                   bilirubin + albumin + copper + sgot + prothrombin + stage +
                   bilirubin : n days + albumin * copper,
                 id = id, data = cirrhosis)
vars df = tibble()
```

```
for(var in variables[1 : (length(variables) - 1)])
 left_vars = variables[(which(variables == var) + 1) : length(variables)]
 for(var2 in left vars)
    cox_fit = coxph(Surv(n_days, status) ~ drug + age + strata(edema) +
                      bilirubin + albumin + copper + sgot + prothrombin +
                      stage + bilirubin : n_days + albumin * copper +
                      eval(parse(text = var2)) : eval(parse(text = var)),
                         id = id,
                        data = cirrhosis)
     # aic_vec= c(aic_vec, AIC(model_four))
     chisq stat=-2 * (logLik(cox fit2)-logLik(cox fit))
     p_val = 1 - pchisq(chisq_stat,
                        attr(logLik(cox_fit), "df") -
                          attr(logLik(cox fit2), "df"))
     if(p val < 0.05)
      vars_df = vars_df |> rbind(c(round(p_val, 4), var, var2))
 }
}
# This is our final model.
cox final = cox fit2
# summary(cox final)$coefficient %>% .[, c(1, 2, 5)] >
    data.frame() |> mutate(significance = c("", "**", "***", "***", "**", "**", "**",
                                             "", "", "*", "***", "***")) />
#
    knitr::kable(col.names = c("", "Estimate", "Hazard Ratio", "p value", "Siq."),
#
                 digits = 4, caption = "Final Model Parameter Results")
cox_final |> tbl_regression(
 exponentiate = T,
 estimate_fun = purrr::partial(style_ratio, digits = 4),
 pvalue_fun = purrr::partial(style_sigfig, digits = 4)) |>
 modify_caption("Final Model Hazard Ratio Estimates")
deviance_res = residuals(cox_final, type = "deviance", var = stage)
dev_drug = cirrhosis |>
 mutate(deviance = deviance res) |>
 ggplot(aes(x = drug, y = deviance)) +
 geom_point()
dev age = cirrhosis |>
 mutate(deviance = deviance_res) |>
 ggplot(aes(x = age, y = deviance)) +
```

```
geom_point()
dev bili = cirrhosis |>
  mutate(deviance = deviance res) |>
  ggplot(aes(x = bilirubin, y = deviance)) +
  geom_point()
dev albu = cirrhosis |>
  mutate(deviance = deviance res) |>
  ggplot(aes(x = albumin, y = deviance)) +
  geom_point()
dev_copper = cirrhosis |>
  mutate(deviance = deviance_res) |>
  ggplot(aes(x = copper, y = deviance)) +
  geom_point()
dev_sgot = cirrhosis |>
  mutate(deviance = deviance res) |>
  ggplot(aes(x = sgot, y = deviance)) +
  geom_point()
dev proth = cirrhosis |>
  mutate(deviance = deviance res) |>
  ggplot(aes(x = prothrombin, y = deviance)) +
  geom_point()
dev_stage = cirrhosis |>
  mutate(deviance = deviance res) |>
  ggplot(aes(x = stage, y = deviance)) +
  geom_point()
ggarrange(dev_drug, dev_age, dev_bili, dev_albu, dev_copper,
          dev_sgot, dev_proth, dev_stage, ncol = 4, nrow = 2)
# plot(deviance_res, ylab = "Deviance Residuals", xlab = "Index",
       main = "Deviance Residuals Scatterplot")
# abline(h = c(-3, 3), col = "red", lty = 2) # Flag large residuals
# which(deviance res > 3)
coxsnell_res = - (predict(cox_final, type = "survival") |> log())
# hist(coxsnell_res, main = "Cox-Snell Residuals Histogram", freq = F, breaks = 15)
\# curve(exp(-x), add = T, col = "red")
# plot(coxsnell res, ylab = "Cox-Snell Residuals", xlab = "Index",
       main = "Cox-Snell Residuals Scatterplot")
km fit = cirrhosis |> mutate(pseudo time = coxsnell res) |>
  survfit(Surv(pseudo_time, status) ~ 1, id = id, data = )
km_summary = summary(km_fit)
tibble(
  t = km_summary$time,
  survival = km_summary$surv
```

```
) |>
 mutate(y = log(- log(survival))) |>
 ggplot(aes(x = log(t), y = y)) +
 geom_line() +
 geom_abline(intercept = 0, slope = 1, color = "red", lty = 2) +
 labs(y = "log(-log(S(t)))", title = "")
ld res = c()
for(i in 1 : nrow(cirrhosis))
 dat = cirrhosis |> slice(- i)
 model_ld = coxph(Surv(n_days, status) ~ drug + age + strata(edema) +
                   bilirubin + albumin + copper + sgot + prothrombin + stage +
                   bilirubin : n days + albumin * copper,
                 id = id, data = dat)
 ld_res = c(ld_res, 2 * abs(logLik(model_ld) - logLik(cox_final)))
cox after = cirrhosis |>
 slice(c(- 77, - 143, - 82, - 100, - 108, - 129, - 210)) |>
 coxph(Surv(n_days, status) ~ drug + age + strata(edema) +
                   bilirubin + albumin + copper + sgot + prothrombin + stage +
                   bilirubin : n days + albumin * copper,
                 id = id, data = )
summary(cox final)$coefficient %>% .[, c(1, 2, 5)] |>
 cbind(summary(cox after)$coefficient %>% .[, c(1, 2, 5)]) |>
 knitr::kable(col.names = c(" ", rep(c("Estimate", "Hazard Ratio", "p value"), 2)),
               digits = 4, caption = "Model Parameter Estimates Comparison") |>
 add_header_above(header = c(" " = 1, "Original Model" = 3, "New Model" = 3))
```