Last Time:

- How to land a space ship

Today:

- How to walk

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* History

  - First legged robots built in ~ 1960s
  - "Serious" research started in ~ 1980s

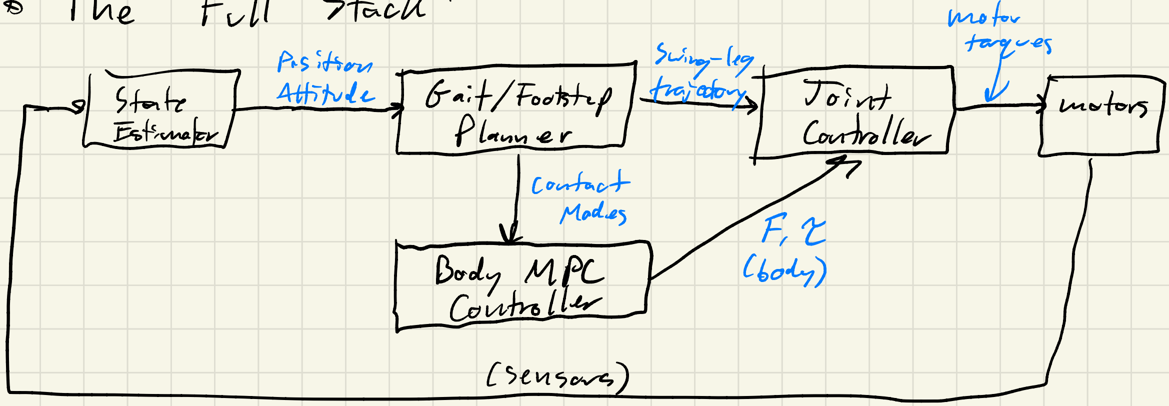
  - Different Approaches:

    Honda/Waseda: Humanoids. Control based on industrial manipulator ideas.

    Raibert/CMU/MIT: Hoppers. Control focused on floating-base dynamics + balance.

  - Past ~10 years: lot of work on mechanical design (direct-drive motors etc.) and MPC for quadrupeds (Cheetah, ANYmal, Spot) and humanoids (Boston Dynamics), RL on quadrupeds.

The "Full Stack"



- Sensors: Joint encoders, IMU, Contact force, vision/GPS
- State Estimator: Some kind of EKF with lots of tricks for reasoning about contact + foot slip.
- Gait/Footstep Planner: Plans placement + timing, swing-leg trajectories based on pre-specified gait + desired body velocity.
- MPC Controller: Treats the robot as a single rigid body and assumes contact modes given by gait planner to compute contact forces.
- Joint controller: Generates motor torque commands to produce desired force + torque from MPC + desired swing-leg trajectory from gait planner.

* Legged Robot Dynamics:

$$x = \begin{bmatrix} q \\ v \end{bmatrix}, \quad q = \begin{bmatrix} \text{body position} \\ \text{body attitude} \\ \text{joint angles} \end{bmatrix}, \quad v = \dot{q}$$

$$\underbrace{M(q)}_{\text{Mass Matrix}} \dot{v} + \underbrace{C(q,v)}_{\text{Coriolis + Gravity}} = \underbrace{B(q)}_{\text{input Jacobian}} u + \underbrace{J(q)}_{\text{contact Jacobian}}^T \overbrace{f}^{\text{Contact Forces}}$$

$$\underbrace{\phi(q)}_{\text{signed-distance function}} \geq 0 \quad \Big\} \text{ Interpenetration Constraint}$$

$$\| \underbrace{f^{2:3}}_{\text{tangential force}} \| \leq \mu \overbrace{f^1}^{\text{normal force}} \quad \Big\} \text{ Friction Cone}$$

- Very messy for online control
- State dimension is large (~ 36). TrajOpt scales like n^3
- Highly nonlinear + non smooth

* Single Rigid Body / "Centroidal" Dynamics

- Assumptions:

  1) Leg mass/inertia << body (~10%)

  2) Leg actuators are very fast compared to body motion.

=> Use a lumped single-rigid-body model for the whole robot:

$$m\dot{v} = -mg + \sum_i f_i$$ ← sum of contact forces

$$J\dot{\omega} + \omega \times J\omega = \sum_i r_i \times f_i$$

r_i ← foot position in body frame

$$\underbrace{\dot{r} = v \quad , \quad \dot{q} = \tfrac{1}{2} q * \omega}_{\text{kinematics}}$$

- With a few more assumptions we can get a linear model:

  3) body angular velocities are small

  4) body pitch + roll are small

  5) Foot positions track reference (almost) exactly

$\Downarrow$

$$m\dot{v} = -mg + \sum_i f_i$$

$$J\dot{\omega} = \sum_i \hat{R}_i f_i$$

$\hat{r}_i$: cross-product matrix using reference footsteps

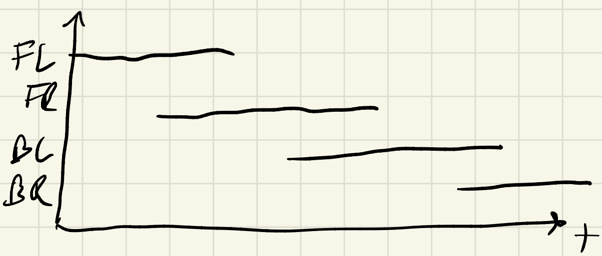
$$\dot{r} = v$$

$$\dot{\theta} = \omega$$

θ = roll, pitch, yaw angles

$$\underbrace{\begin{bmatrix} \dot{r} \\ \dot{\theta} \\ \dot{v} \\ \dot{\omega} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} r \\ \theta \\ v \\ \omega \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} & 0 & & \\ I & I & I & I \\ R_1 & R_2 & R_3 & R_4 \end{bmatrix}}_{B} \underbrace{f}_{4}$$

* Foot step/gait planner
  - Specify foot contact mode timing



- Where to put the feet down?
  - Standard technique: "Raibert heuristic"

$$r_{foot} = r_{hip} + \frac{\Delta t}{2} V_{body}$$

Δt: gait period

* MPC Controller

  - Track desired body motion

  - Horizon 1 ~ 2 gait periods

  - With linearized dynamics this is convex:

$$\min_{\substack{x_{1:N} \\ u_{1:N-1}}} \sum \frac{1}{2}(x-\bar{x})^T Q (x-\bar{x}) + \frac{1}{2}(u-\bar{u})^T R (u-\bar{u})$$

$$\text{s.t. } x_{n+1} = A x_n + B u_n$$

$$\| u_n^{2:3} \|_2 \le \mu u_n^1$$

tangential   normal

- Friction cone is usually linearized to make this a QP.

- Difficult to enforce torque limits