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Exam 2 Of Southwest Jiaotong University

Course name Linear Algebra and Differential Equations

Course Code FGEE001612

Form Take-Home

Semester Fall 2020

Time allowed ____________

Instructor Lukun Zheng

Instructions:

Show essential steps in your work. Answers without essential steps may receive a score of 0.

Rules for Take-home Exam

All students will have from 10:30 am to 11:59 pm on Thursday, 12/31/2020 to complete the take-home exam. The exam is **DUE AT 11:59 PM ON THURSDAY (12/31/2020) AT ZHIHUISHU. ABSOLUTELY NO EXTENSIONS.** Late submission will be severely penalized (**50% OFF**).

Please read these rules and confirm by email that you have read and understood them before you receive your exam. Failure to comply with any of the instructions below may result in **our being unable to accept or grade your exam or initiating disciplinary actions**. These instructions apply from 10:30 am to 11:59 pm on Thursday, 12/31/2020.

1. This exam is "Open Book", which means you are permitted to use your own notes from the course, the text book, and anything posted on the course website.

2. The exam must be taken completely alone. Showing it or discussing it with anyone is forbidden. No discussion of the exam or anything about the exam is allowed.

3. You may not consult with any other person regarding the exam. You may not check your exam answers with any person. You may not discuss any of the materials or concepts in this course with any other person.

4. You may not consult any external resources. This means no internet searches, materials from other classes or books or any notes you have taken in other classes etc. You may not use Baidu or any other search engines for any reason.

By signing on this form, I hereby acknowledge that I have read, understand and agree to the terms of this document relating to Exam 2.

Student Name: ____________

1. (14 pts) Determine whether the subset W is a subspace of the vector space V.

a) $V = R^{2,2}$ and $W = \left\{ \begin{bmatrix} a & b \\ b & a \end{bmatrix} \mid a, b \in R \right\}$

b) $V = C[0,1]$ and $W = \{ f(x) \in C[0,1] \mid f(1) \leq 0 \}$

2. (12 pts) Find the coordinate vector of $\vec{v}$ with respect to the following basis:

$\vec{v} = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$, basis $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ 0 & 3 \end{bmatrix}$ of the space of upper triangular 2×2 matrices.

3 (12 pts) Let $V = P_2$ and $\vec{w}_1 = x$, $\vec{w}_2 = x^2$, $\vec{v}_1 = 1 - x, \vec{v}_2 = 2 + x$, $\vec{v}_3 = 1 + x^2$. The set $\vec{v}_1, \vec{v}_2, \vec{v}_3$ is a basis of V. Determine which $\vec{v}_j's$ could be replaced by $\vec{w}_1$, and which $\vec{v}_j's$ could be replaced by both $\vec{w}_1$ and $\vec{w}_2$, while retaining the basis property.

4. (14 pts) Find bases for the row, column, and null space of each of the following matrix:

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 3 & 6 & 2 & 2 & 3 \end{bmatrix}.$$

5. (28 pts) Specify the algebraic and geometric multiplicity of each eigenvalue for the following matrix. Determine whether it is diagonalizable. If it is diagonalizable, find a matrix P such that $P^{-1}AP$ is diagonal. If it is not diagonalizable, explain why.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 6 & 2 \end{bmatrix}$$

6. (20 pts) For what values of b is the following quadratic form positive semidefinite?

$$Q(x_1, x_2, x_3) = 3x_1^2 + 2x_2^2 + 3x_3^2 + 2bx_1x_2 + 4x_1x_3 - 2x_2x_3$$

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