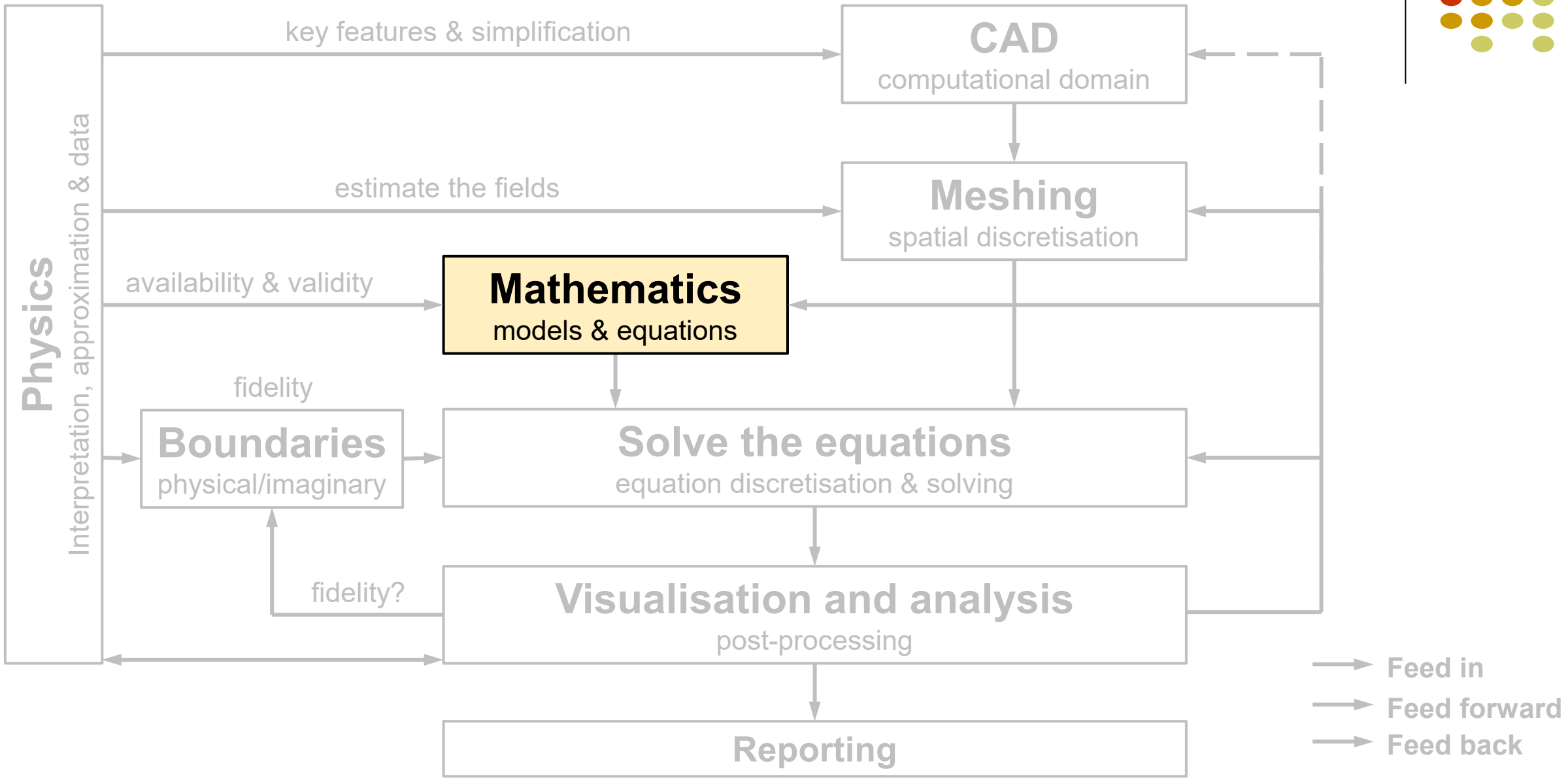
ENIGNEERING COMPUTATIONAL FLUID DYNAMICS (ECFD)

Dr Xiangdong Li

Module 4 – Turbulence

CFD workflow



This module

- What is turbulence and its roles in fluid flows
- Reynolds Averaged Navier-Stokes equations
- Turbulence models
- The eddy-viscosity models

Turbulence has never been an easy word – both in everyday life and in science

WHAT IS TURBULENCE – SCIENTIFICALLY?

Turbulence – the last challenge of classic physics

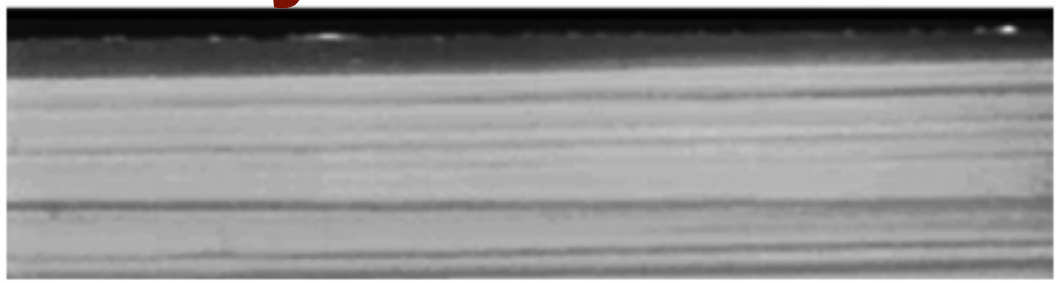
Horace Lamb in 1932:

"I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the **turbulent motion of fluids**. And about the former I am rather optimistic."

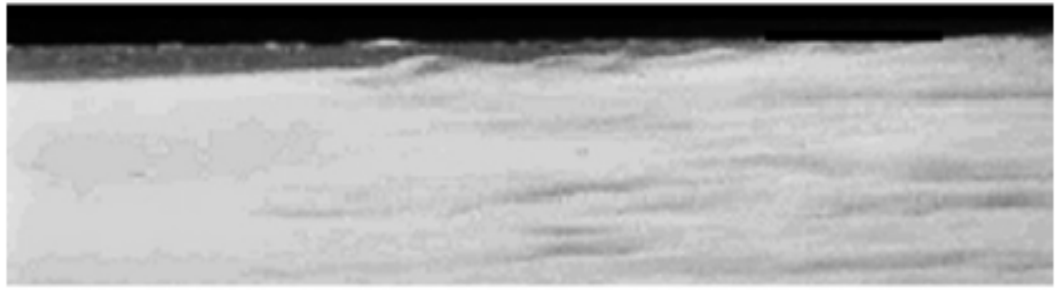
Today, turbulence remains the last unsolved problem in classical physics – including its definition.

Horace Lamb (1849 - 1934)

What you have known about

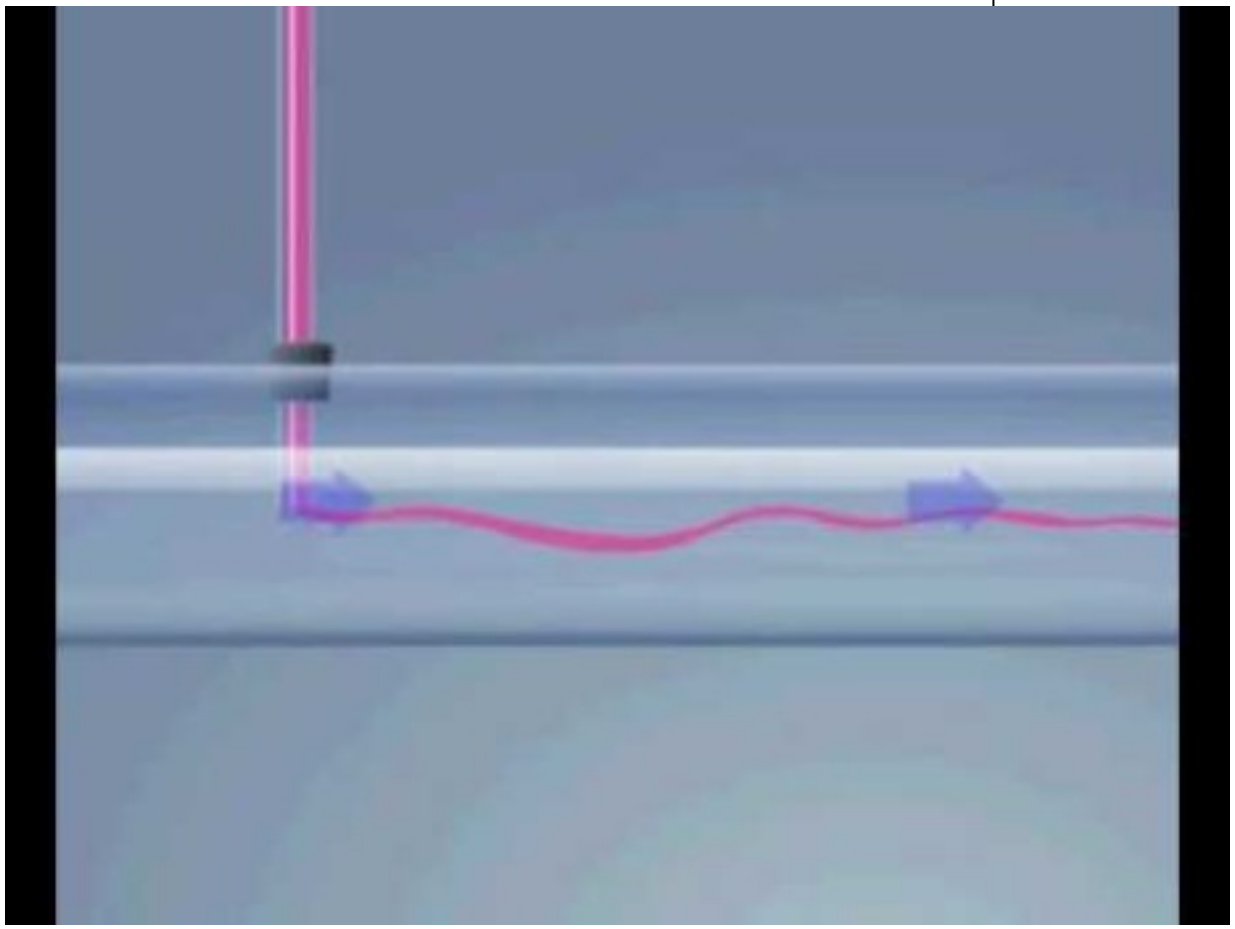


Laminar



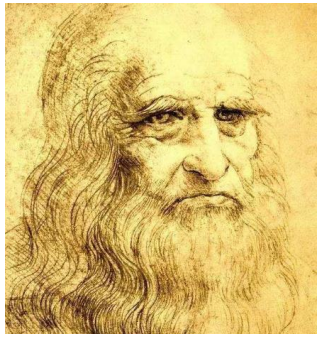
Transitional

Turbulent

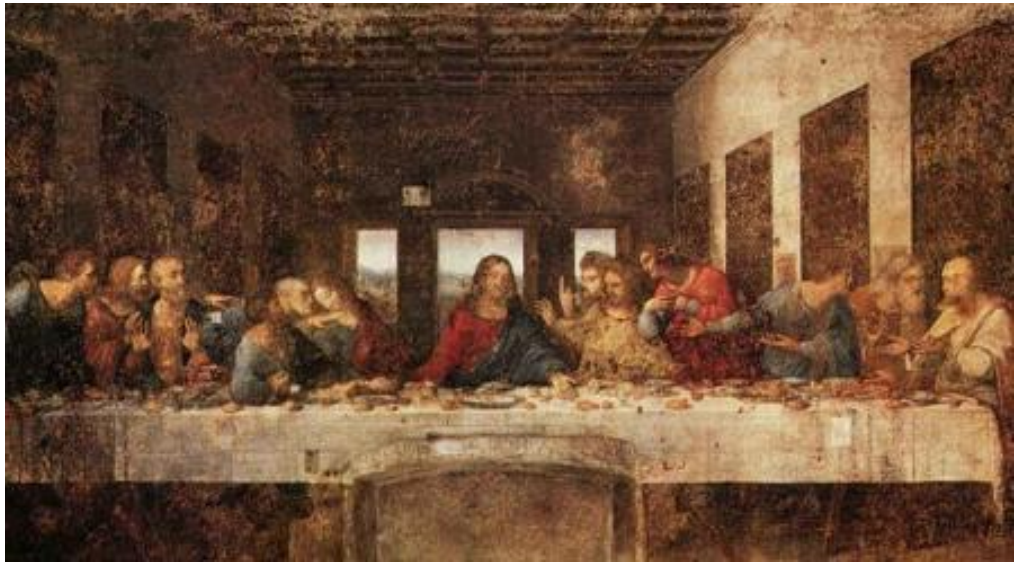


$$\mathrm{Re} = \frac{\rho U L}{\mu} = \frac{U L}{\nu}$$

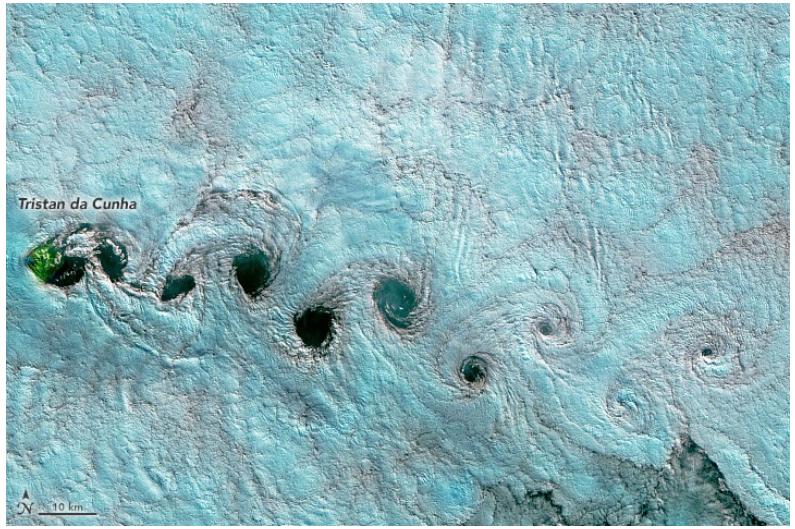
Turbulence – the earliest scientific expression, maybe?

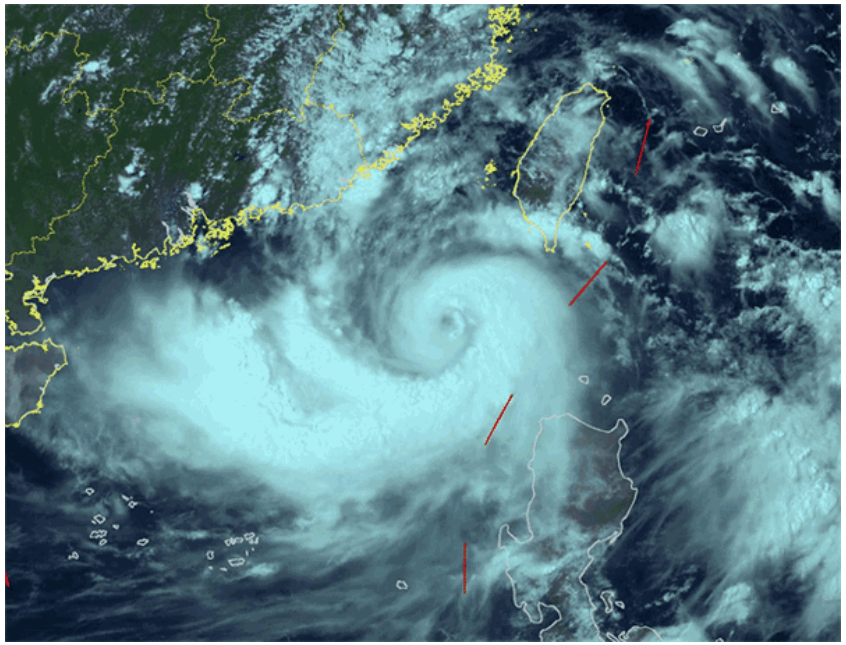


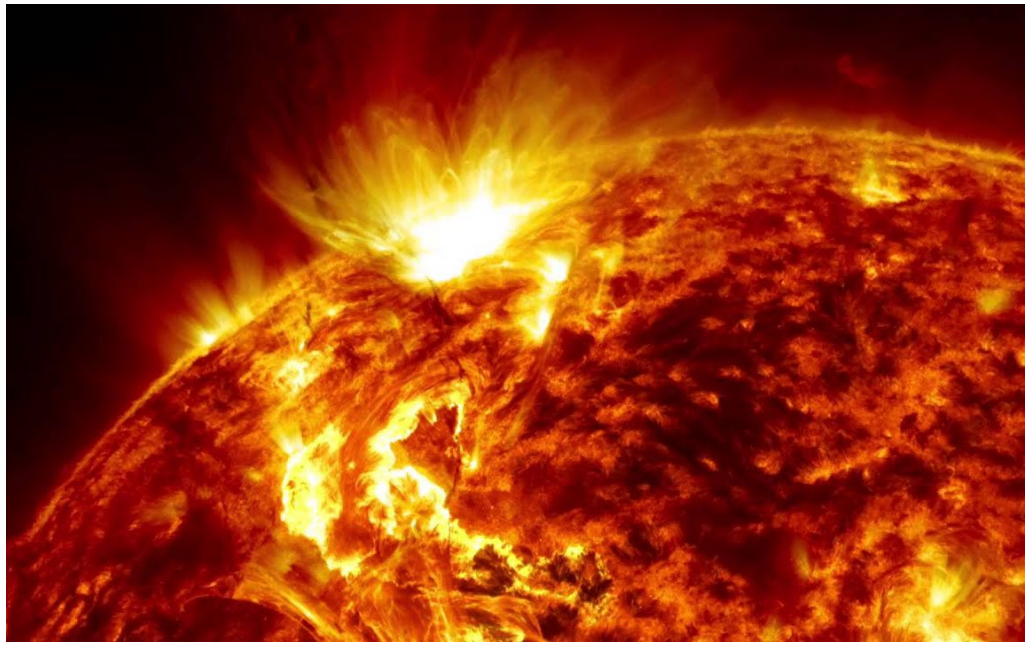
Leonardo da Vinci
1452 - 1519



Turbulence visualised







Interpretations of turbulence

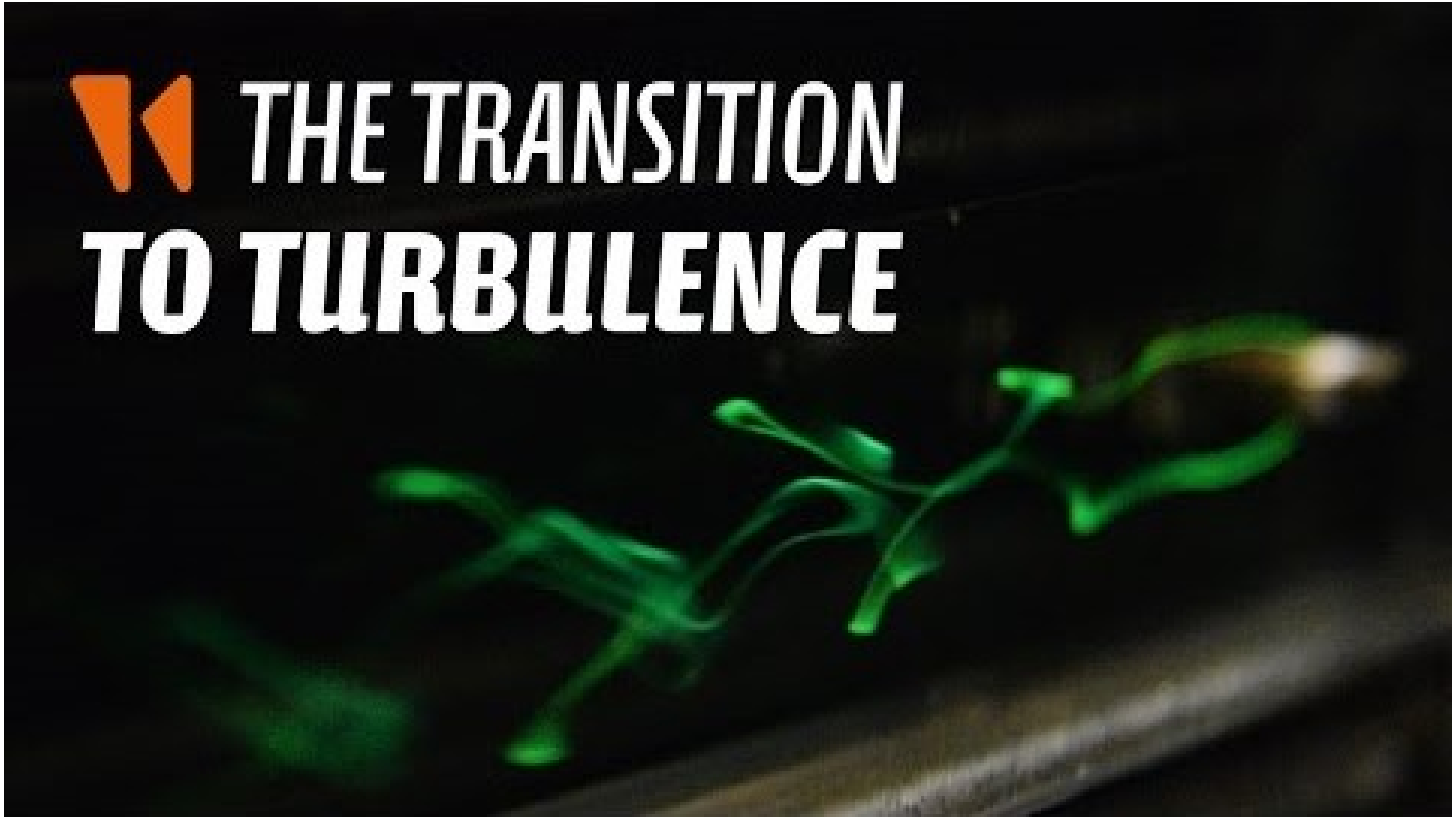
- Von Karman quoting Taylor: Turbulence is an irregular motion which in general makes its appearance in fluids, gaseous, or liquid, when they flow past solid surfaces or even neighbouring streams of the same flow past or over one another.

- Hinze: Turbulent fluid motion is an irregular condition of flow in which the various quantitates show a random variation with time and space coordinates, so that the statistical average can be discerned.

- Bradshaw added that: Turbulence has a wide range of scales.

- …

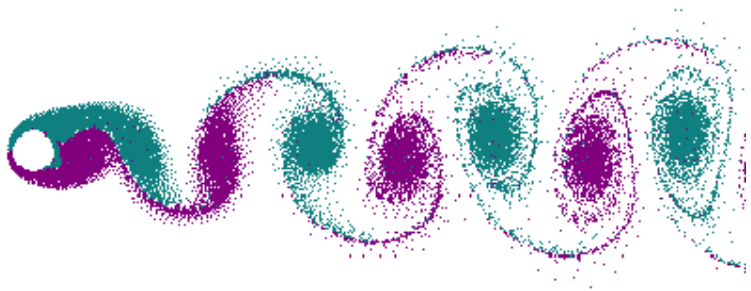
How does turbulence occur?

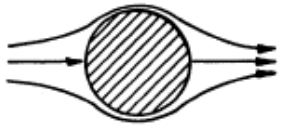


Vortexes/vortices

Eddies

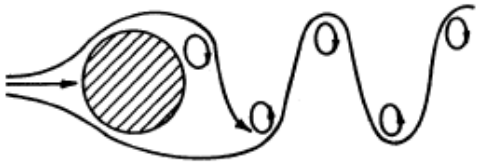
von Karman vortex street





Re < 5 REGIME OF UNSEPARATED FLOW.

5 TO 15 ≤ Re < 40 A FIXED PAIR OF FÖPPL VORTICES IN THE WAKE.

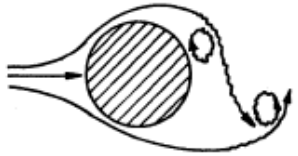


40 ≤ Re < 90 AND 90 ≤ Re < 150

TWO REGIMES IN WHICH VORTEX STREET IS LAMINAR:

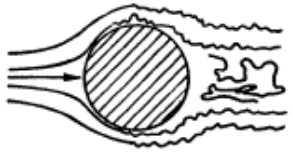
PERIODICITY GOVERNED IN LOW Re RANGE BY WAKE INSTABILITY

PERIODICITY GOVERNED IN HIGH Re RANGE BY VORTEX SHEDDING.



150 ≤ Re < 300 TRANSITION RANGE TO TURBULENCE IN VORTEX.

300 ≤ Re ≲ 3×10^5 VORTEX STREET IS FULLY TURBULENT.

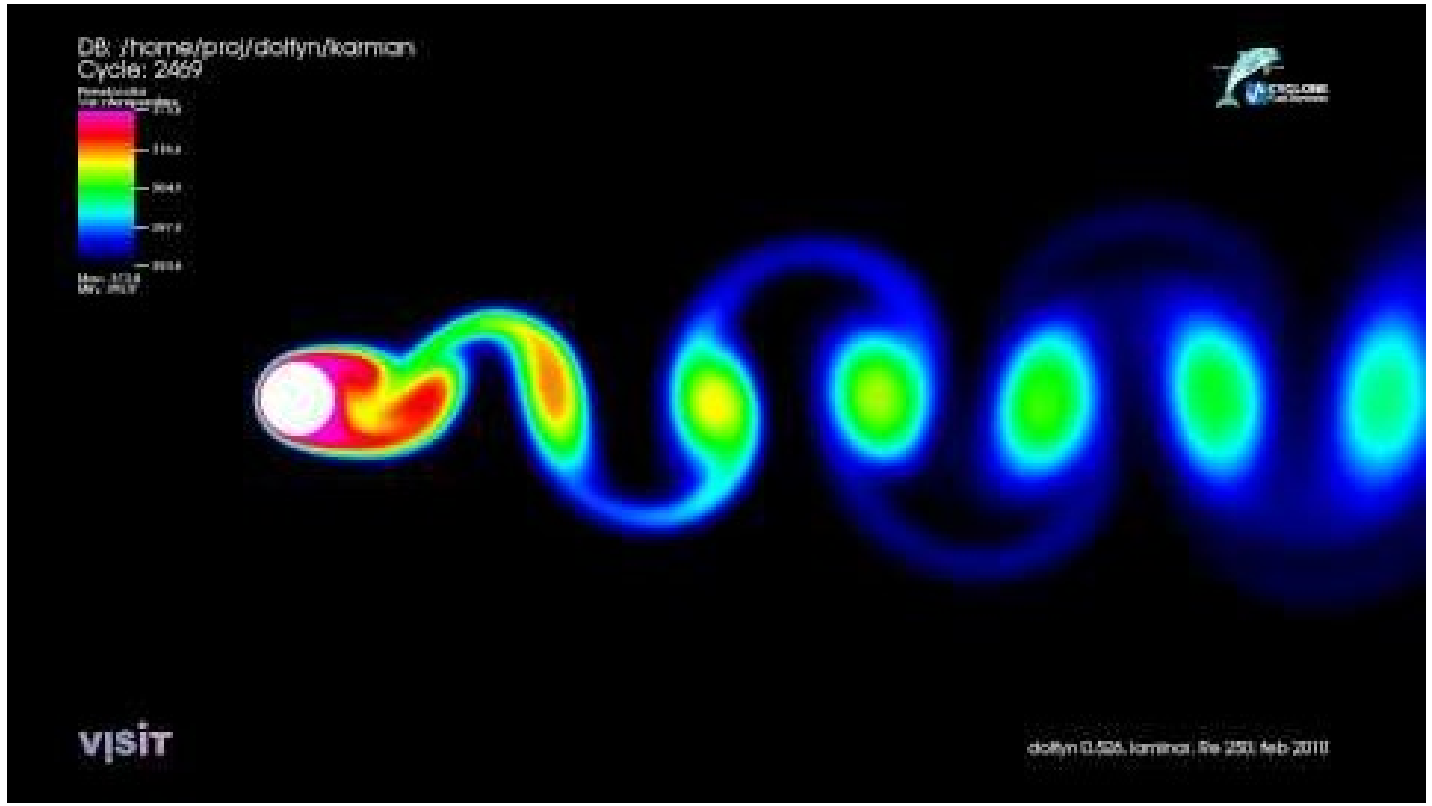


$3 \times 10^5 \lesssim Re < 3.5 \times 10^6$

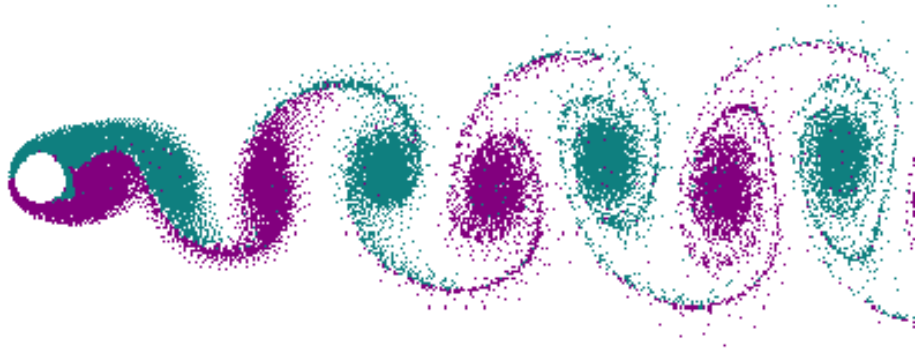
LAMINAR BOUNDARY LAYER HAS UNDERGONE TURBULENT TRANSITION. THE WAKE IS NARROWER AND DISORGANIZED. NO VORTEX STREET IS APPARENT.

$3.5 \times 10^6 \leq Re < \infty$ (?)

RE-ESTABLISHMENT OF THE TURBULENT VORTEX STREET THAT WAS EVIDENT IN $300 \leq Re \lesssim 3 \times 10^5$. THIS TIME THE BOUNDARY LAYER IS TURBULENT AND THE WAKE IS THINNER.



How does turbulence occur?

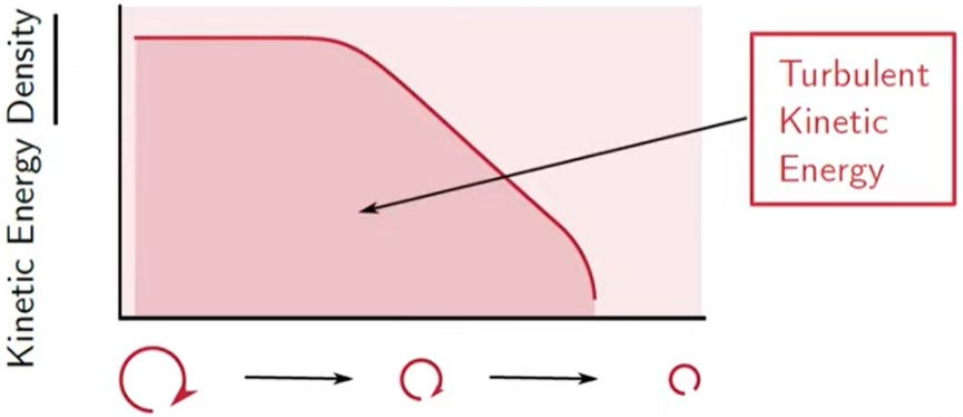


- Turbulence develops as an **instability of laminar flow**
- Vortex stretching is one of the main mechanisms that onsets turbulence from laminar instability
  - Larger eddies randomly stretch vortex elements that compromise smaller eddies and cascades energy to them
  - Vortex stretching does not occur in 2D, so turbulence must be 3D

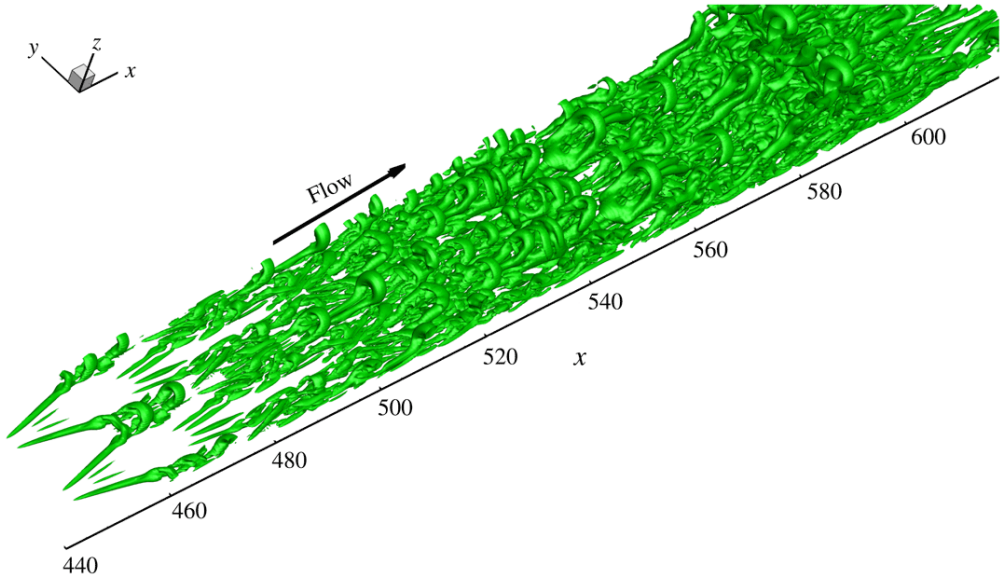
Defining features of turbulence – eddies

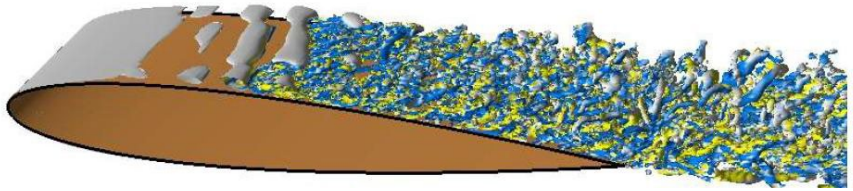
- Turbulence is characterised by **random 3D eddies** with many scales in a continuous spectrum
- Large eddies carry smaller eddies, the eddies overlap and form a cascade process
- Most of the energy within the flow is at the large eddies
- Large eddies are responsible for enhanced mixing, diffusivities and stresses
- Smaller eddies dissipate most energy through viscous friction, the rate of dissipation is proportional to that received from larger eddies

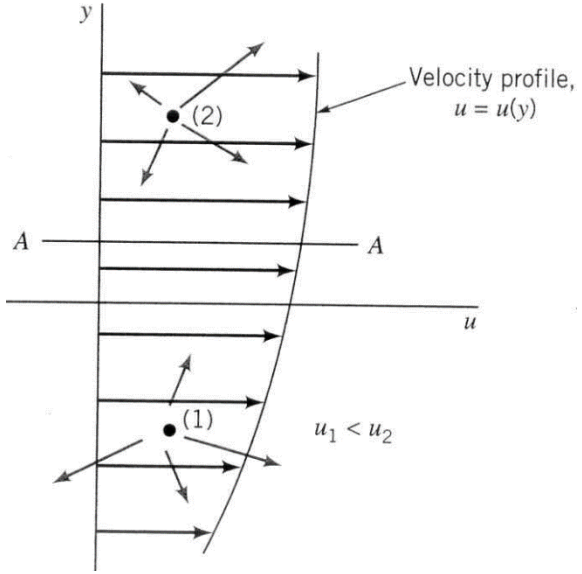
Turbulence cascade

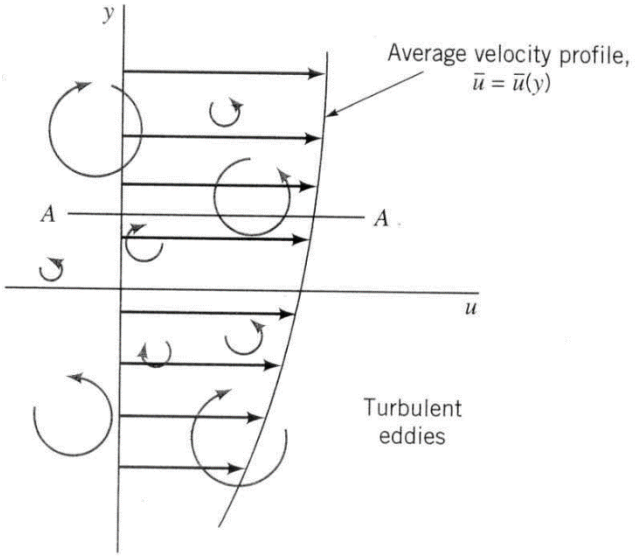


Turbulence eddies



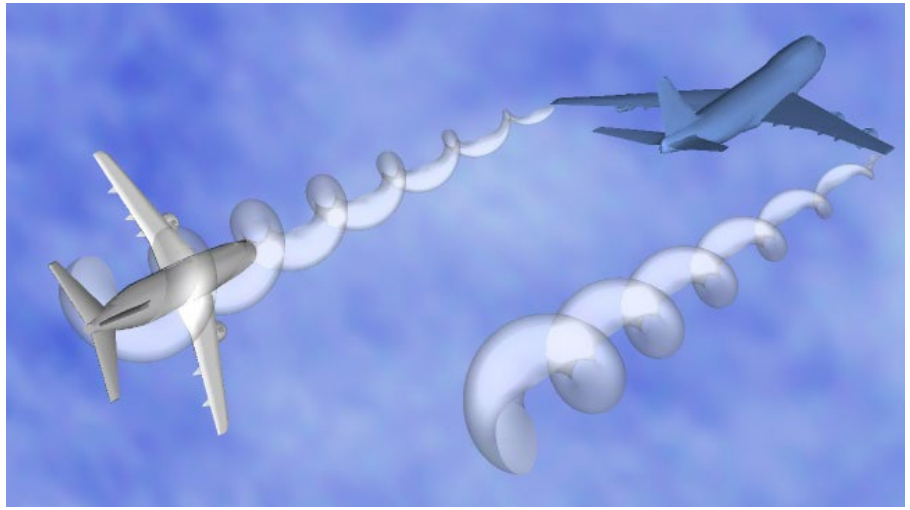






- Frictional drag
- Flow mixing rate
- Energy dispersion
- …..

Turbulence eddies



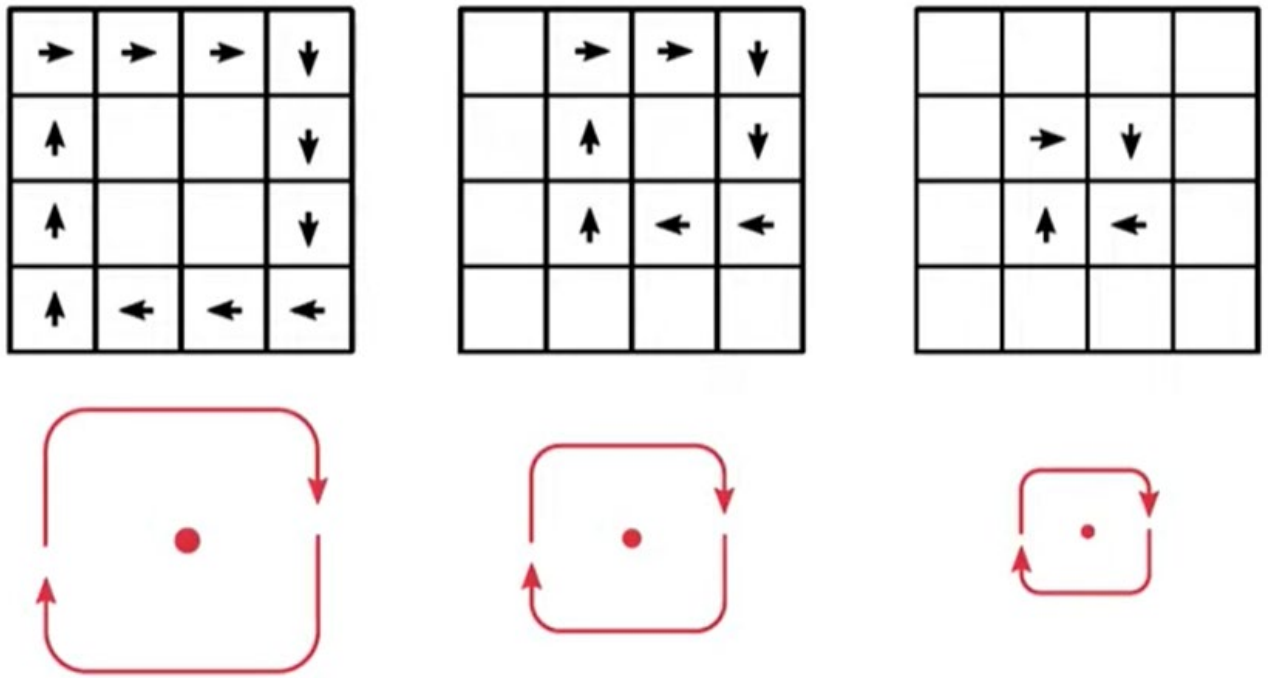
https://youtu.be/FNqXf6t7e-w

MATHEMATICAL DESCRIPTION OF TURBULENCE

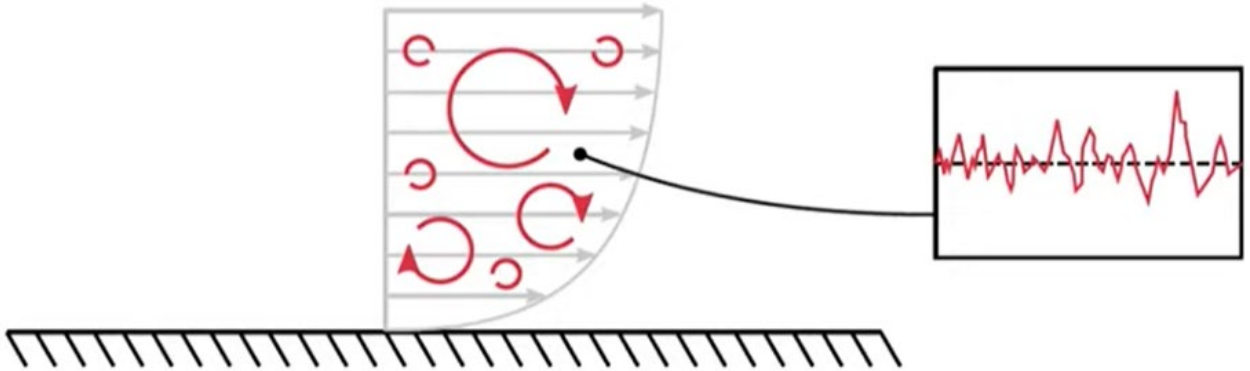
Mathematical description of turbulence

- **Hypothesis**: Turbulence is a continuum phenomenon, even the smallest scales are far larger than molecular scales – the N-S equations remains valid for the physics in a turbulent flow
- Turbulence is characterised by random eddies that almost mandates the use of **statistical methods** for analysis
- Most popular statistical method was introduced by Reynolds (1895)
- However, statistical methods prevent exploring details

Eddies and mesh

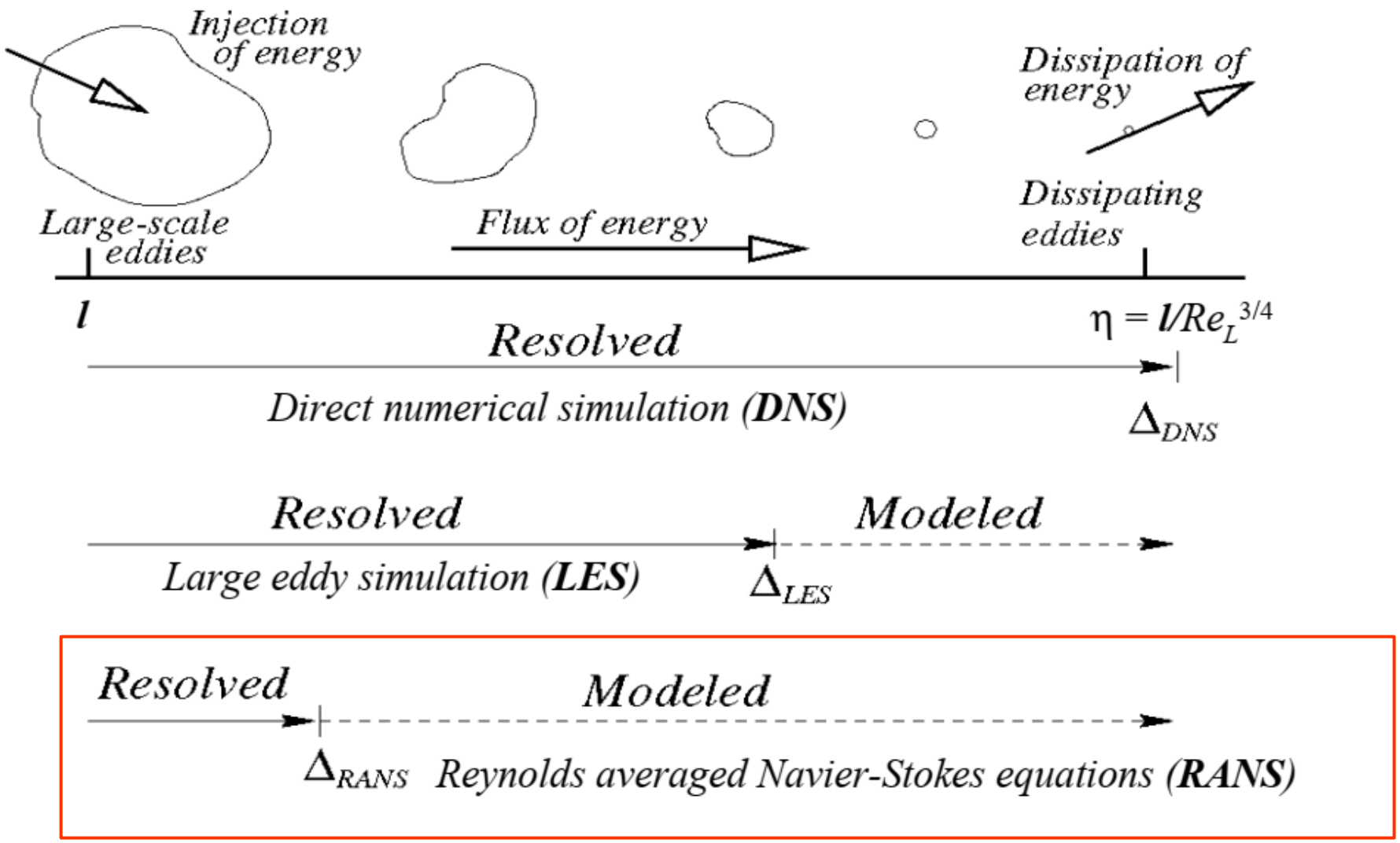


- In 2D, you need at least 4 mesh cells to capture an eddy
- Turbulence is instantaneous

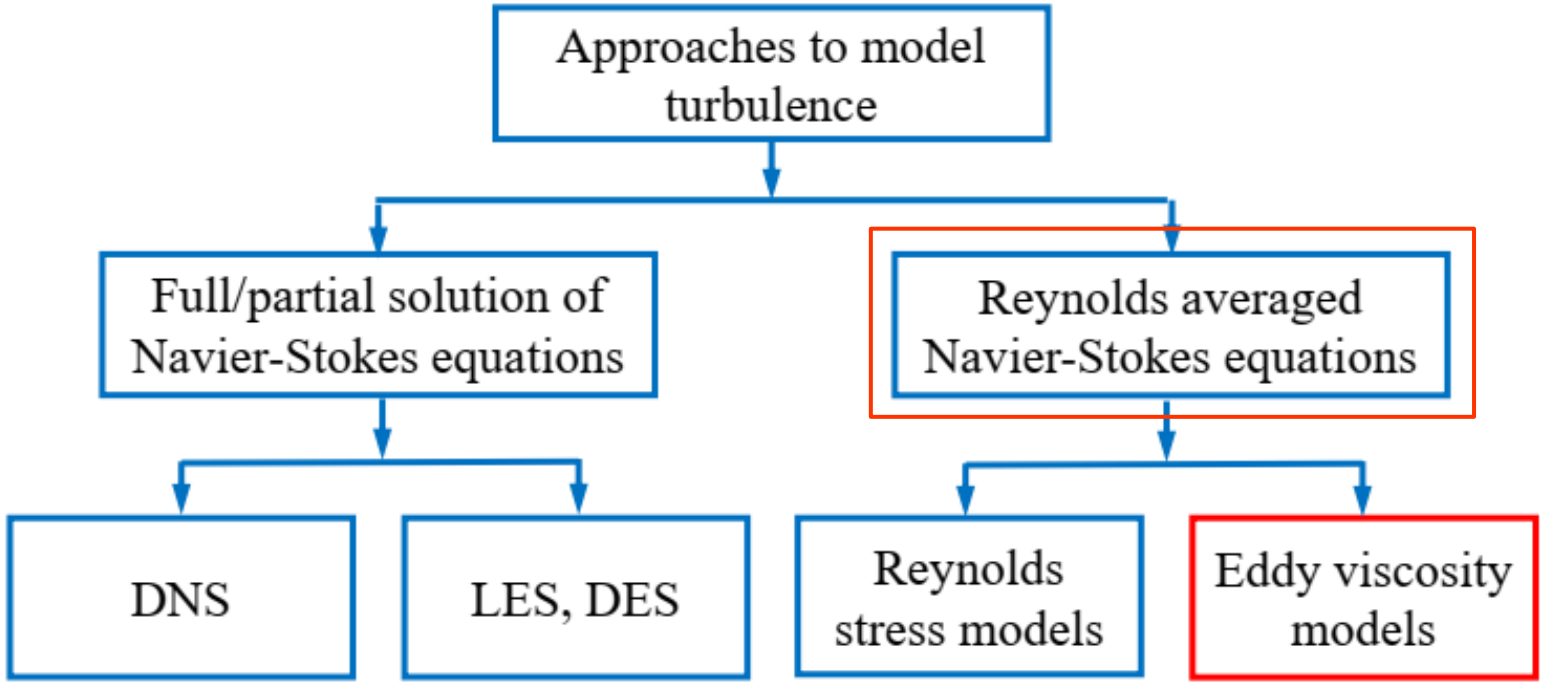


- Statistical average could be a cost-efficient way

Approaches to turbulence modelling



Approaches to turbulence modelling

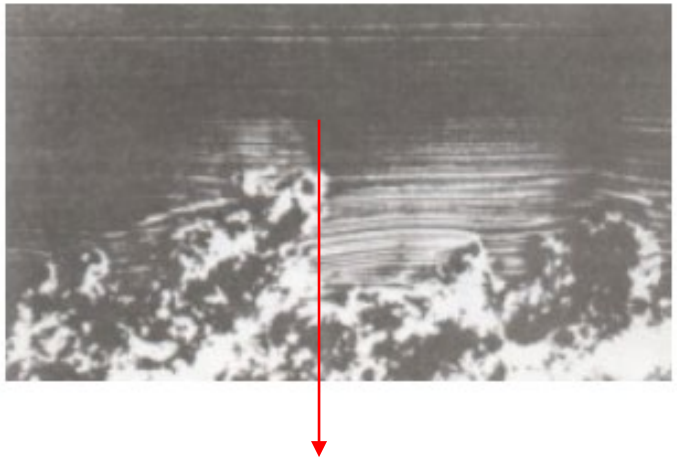


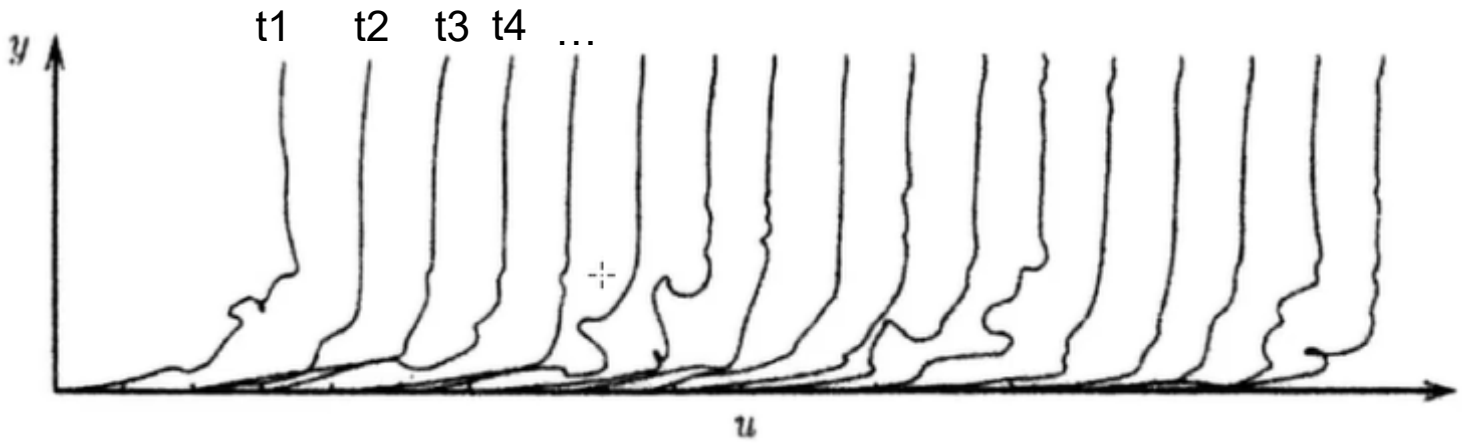
Vortexes are random and can be large or small.
--- Consider the mesh

REYNOLDS AVERAGED NAVIER-STOKES (RANS) EQUATIONS

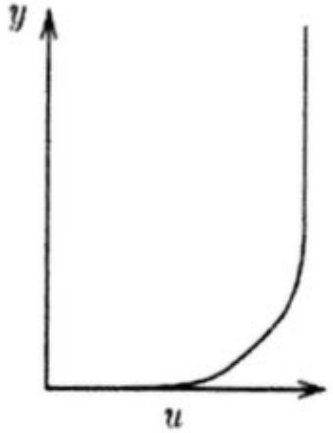
Reynolds (statistical) average

Consider a near-wall region in a “steady flow”





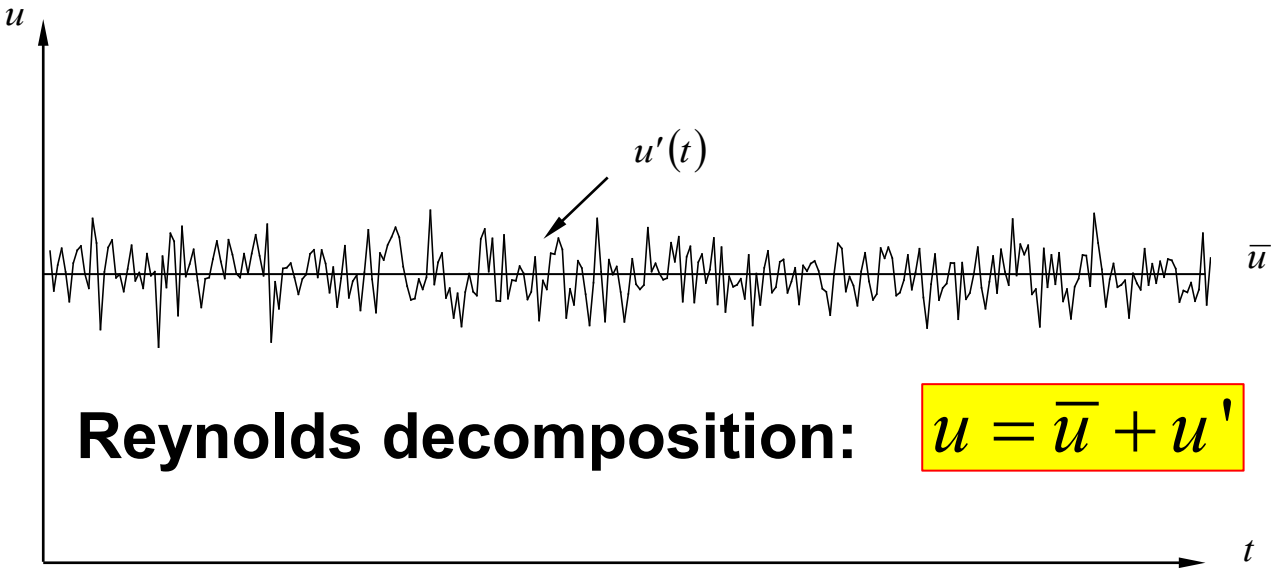
All fluctuations



Mean BL profile

How to take into account the fluctuations?

Turbulent velocity fluctuations



Reynolds decomposition: $u = \bar{u} + u'$

$$\bar{u} = \frac{1}{T}\int_{t_0}^{t_0+T} u(x,y,z,t)dt$$

$$\overline{u'} = \frac{1}{T}\int_{t_0}^{t_0+T} \left(u - u'\right)dt = 0$$

$$\overline{u'^2} = \frac{1}{T}\int_{t_0}^{t_0+T} (u')^2 dt > 0$$

Turbulent kinetic energy: $k = \frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$

Turbulence intensity:

$$\frac{\textit{root-mean-square fluctuation}}{\textit{mean velocity}} = \frac{u'_{rms}}{\bar{u}} = \frac{\sqrt{2k/3}}{\bar{u}}$$

Fluctuation carries kinetic energy!

Reynolds Averaged Navier-Stokes (RANS)

$$\tau_{xx} = \frac{2}{3}\nabla\vec{U} + 2\left(\bar{u}+u'\right)\frac{\partial\left(\bar{u}+u'\right)}{\partial x} \qquad \tau_{yx} = \mu\left(\frac{\partial\left(\bar{v}+v'\right)}{\partial x} + \frac{\partial\left(\bar{u}+u'\right)}{\partial y}\right) \qquad \tau_{zx} = \mu\left(\frac{\partial\left(\bar{w}+w'\right)}{\partial x} + \frac{\partial\left(\bar{u}+u'\right)}{\partial z}\right)$$

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\boxed{\rho uu}) + \frac{\partial}{\partial y}(\boxed{\rho vu}) + \frac{\partial}{\partial z}(\boxed{\rho wu}) = -\frac{\partial p}{\partial x} + \frac{\partial \boxed{\tau_{xx}}}{\partial x} + \frac{\partial \boxed{\tau_{yx}}}{\partial y} + \frac{\partial \boxed{\tau_{zx}}}{\partial z} + body\ forces$$

$$\overline{uu} = \overline{\left(\bar{u}+u'\right)\left(\bar{u}+u'\right)} = \bar{u}^2 + \overline{u'^2}$$

$$\overline{uv} = \overline{\left(\bar{u}+u'\right)\left(\bar{v}+v'\right)} = \overline{uv} + \overline{u'v'}$$

$$\overline{uw} = \overline{\left(\bar{u}+u'\right)\left(\bar{w}+w'\right)} == \overline{uw} + \overline{u'w'}$$

Reynolds Averaged Navier-Stokes (RANS)

Laminar momentum equation

$$\frac{\partial}{\partial t}\left(\rho\vec{U}\right)+\nabla\cdot\left(\rho\vec{U}\vec{U}\right)=-\nabla p+\nabla\cdot\left(\mu\left(\nabla\vec{U}+\left(\nabla\vec{U}\right)^{T}\right)\right)-\nabla\left(\frac{2}{3}\mu\left(\nabla\cdot\vec{U}\right)\right)+S_M$$

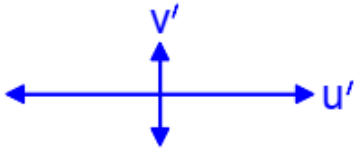
Turbulent momentum equation

the Reynolds stress

$$\frac{\partial}{\partial t}\left(\rho\vec{U}\right)+\nabla\cdot\left(\rho\vec{U}\vec{U}\right)=-\nabla p+\nabla\cdot\left(\mu\left(\nabla\vec{U}+\left(\nabla\vec{U}\right)^{T}\right)\right)-\nabla\left(\frac{2}{3}\mu\left(\nabla\cdot\vec{U}\right)\right)-\boxed{\nabla\cdot\left(\overline{\rho\vec{U}'\vec{U}'}\right)}+S_M$$

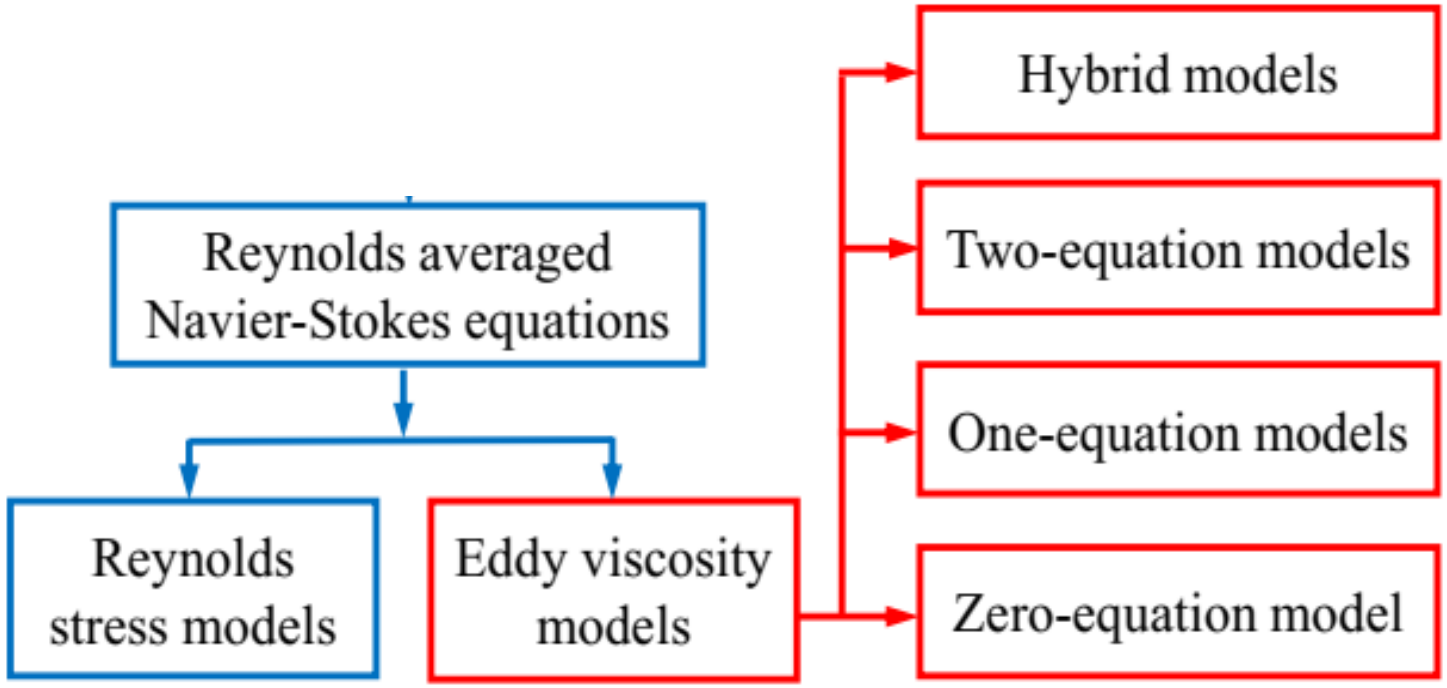
Normal stresses: $\overline{u'^2}, \overline{v'^2}, \overline{w'^2}$

Shear stresses: $\overline{v'w'}, \overline{w'u'}, \overline{u'v'}$



The key to the RANS model is solving the Reynolds stresses

Solving the Reynolds stress



Eddy-viscosity (Boussinesq) hypothesis

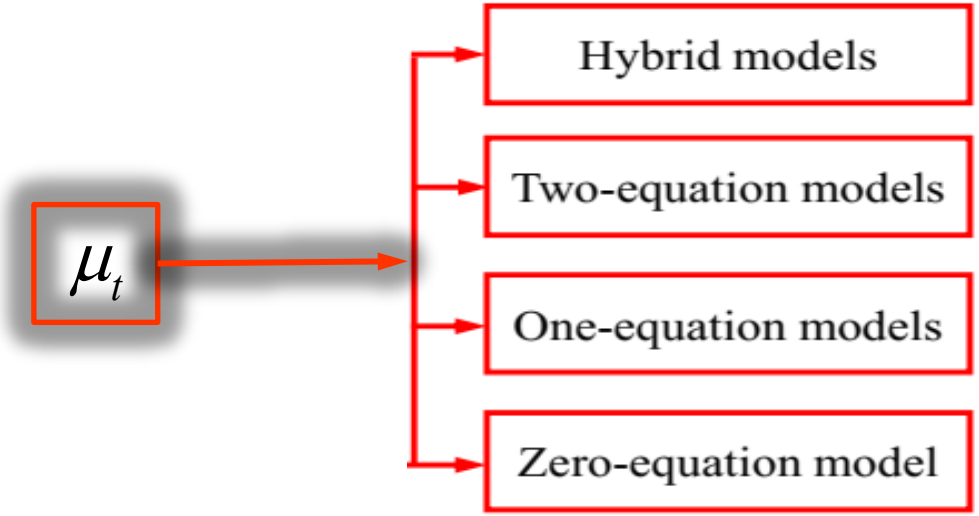
The simplest method - Assuming that the turbulent stress can be modelled in a similar way to the viscous stress.

$$-\overline{\rho \vec{U}'\vec{U}'} = \mu_t \left(\nabla \vec{U} + \left(\nabla \vec{U} \right)^T \right) - \frac{2}{3}\rho k I - \frac{2}{3}\left(\nabla \cdot \vec{U} \right) I$$

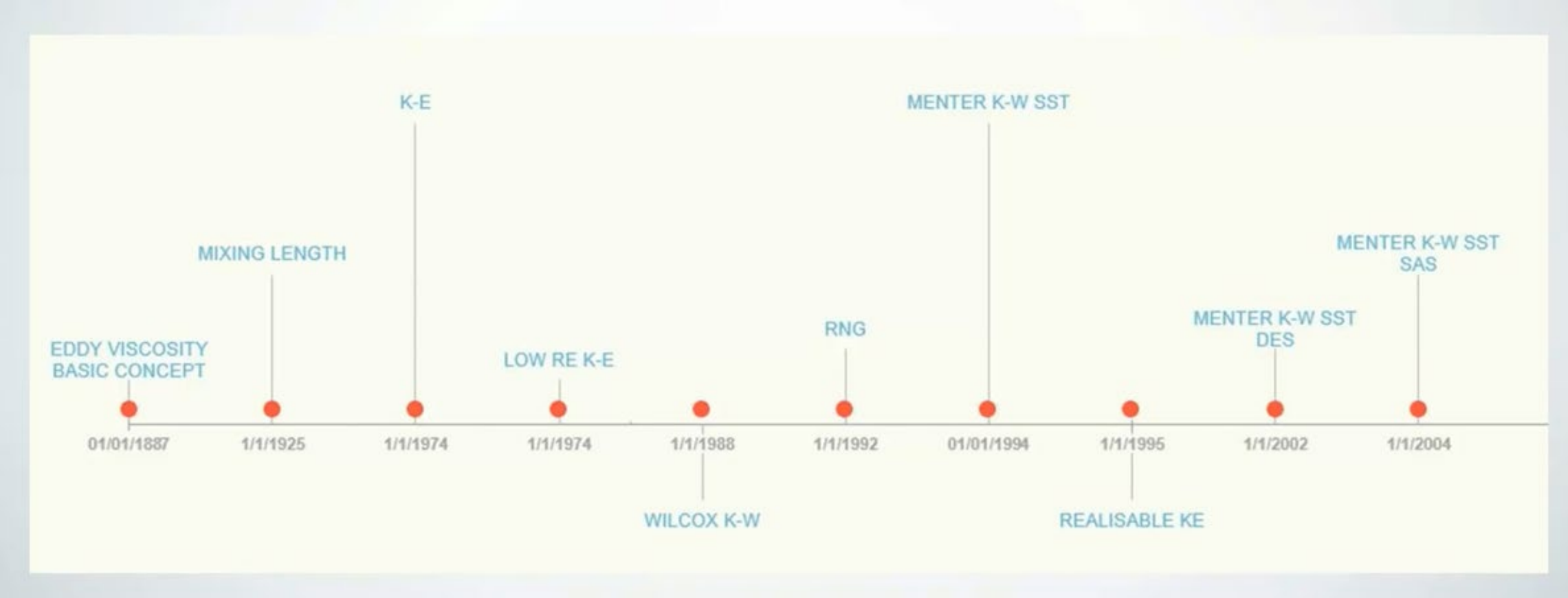
Reynolds stress

Mean velocity gradients

This is called the **eddy-viscosity model**



Eddy-viscosity turbulence models – Timeline



Zero-Equation model – the mixing length theory

$$\mu_t = \rho k^{1/2} l_m \quad \text{or} \quad \mu_t = \rho l_m^2 \left| \frac{\partial U}{\partial y} \right|$$

$$l_m = \kappa y \qquad \kappa = 0.41$$

The wall blocks the maximum size of the eddies

One-Equation model – eddy viscosity transport equation

$$\mu_t = \rho \tilde{\nu}_t$$

$$\frac{\partial \rho \tilde{\nu}_t}{\partial t} + \frac{\partial \rho U_j \tilde{\nu}_t}{\partial x_j} = c_1 D_1 \rho \tilde{\nu}_t S - c_2 \rho \left(\frac{\tilde{\nu}_t}{L_{vK}} \right) S_{1e} + \left(\mu + \frac{\rho \tilde{\nu}_t}{\sigma_t} \right) \frac{\partial \tilde{\nu}}{\partial x_j}$$

Two-equation models

- In the two-equation models, the turbulence viscosity is modelled as the production of a turbulent velocity and length scale.

- Both the velocity and length scales are solved using separate transport equations.

- The turbulent velocity scale is computed from the turbulent kinetic energy while the turbulence length scale is estimated from two property parameters of the turbulence fields.

- Depending on the parameters selected to calculate the length scale, there are the k-ε and k-ω group models.

The k-epsilon (k-ε) model

$$\mu_t = C_\mu \frac{\rho \kappa}{\varepsilon} \qquad C_\mu = 0.09$$

Turbulent kinetic energy *k*

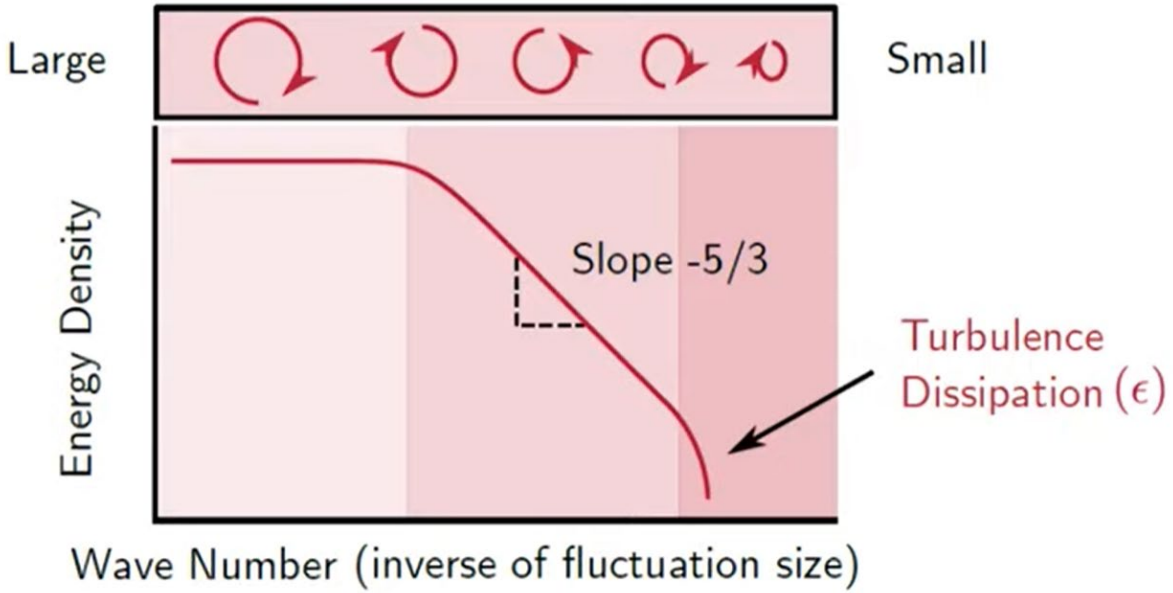
$$k = \frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$$

Turbulence dissipation rate *ε*

$$\varepsilon = \upsilon_t \overline{(\frac{\partial u_i'}{\partial x_i})(\frac{\partial u_i'}{\partial x_j})}$$

$$l_m = \frac{C_\mu k^{3/2}}{\varepsilon}$$

- Turbulence dissipation rate ε is the rate (m^2/s^3) that turbulent kinetic energy is converted into thermal energy by the viscous effects



- We can expect ε is high near wall and in shear layers

The standard k-ε model

$$\frac{\partial}{\partial t}(\rho k)+\nabla\cdot(\rho\vec{U}k)=\nabla\cdot\left(\left(\mu+\frac{\mu}{\sigma_k}\right)\nabla k\right)+P_k+P_b-\boxed{\rho\varepsilon}+S_k$$

$$\frac{\partial}{\partial t}(\rho\varepsilon)+\nabla\cdot(\rho\vec{U}\varepsilon)=\nabla\cdot\left(\left(\mu+\frac{\mu}{\sigma_\varepsilon}\right)\nabla\varepsilon\right)+C_1\frac{\varepsilon}{k}(P_k+C_3P_b)-C_2\frac{\rho\varepsilon^2}{k}+S_\varepsilon$$

- P_k is the production due to mean velocity shear
- P_b is the production due to buoyancy
- S_k is a user-defined source

$C_\mu=0.09$ $\sigma_k=1.0$ $\sigma_\varepsilon=1.3$

C_1 C_2 C_3 vary between models

Model	σ_k	σ_ϵ	C_1	C_2	C_μ
Jones & Launder (1972)	1.0	1.3	**1.55**	**2.0**	0.09
Launder & Spalding (1974)	1.0	1.3	1.44	1.92	0.09
Launder & Sharma (1974)	1.0	1.3	1.44	1.92	0.09

The Low-Re k-ε model – the dumping functions

- Model coefficients C_1, C_2 and C_μ are dumped using dumping functions in the viscous sublayer $y^+<5$.

$$f_1 = 1 \qquad f_2 = 1 - 0.3\exp\left(-\mathrm{Re}_T^2\right) \qquad f_\mu = \exp\left(-\frac{-3.4}{\left(1+\left(\mathrm{Re}_T/50\right)\right)^2}\right)$$

- Dumping functions are not very accurate in the presence of adverse pressure gradients

The k-ε model – pros and cons

- Pros
  - Robust for a range of models (wall bounded and internal flows)
  - Long history of industry benchmark
  - Mesh friendly (3- 5 inflation layers, y+ < 350)
  - Reasonably successful

Aerofoil Seperation

Diffuser

- Cons
  - Inadequate for strong curvature/swirl flows
  - Strong buoyancy effects
  - Eddy viscosity is determined from a single turbulence length scale

The k-omega (k-ω) model

- ω is the specific turbulence dissipation rate (unit 1/s)

$$\omega = \frac{\varepsilon}{C_\mu k^2} \qquad C_\mu = 0.09$$

- Both ε and ω describe the dissipation rate of turbulent kinetic energy
- We can easily convert between ε and ω using the above equation

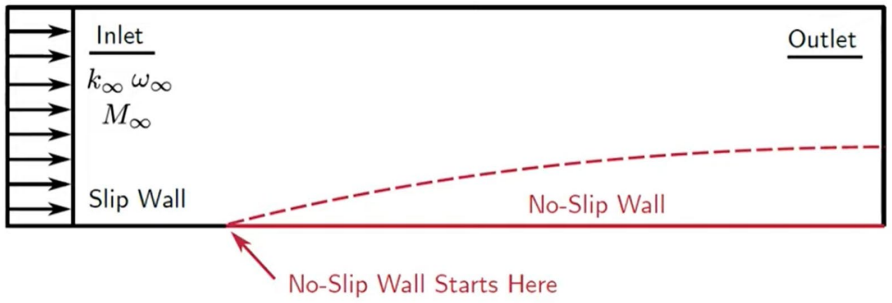
$$\frac{\partial}{\partial t}(\rho k) + \nabla \cdot (\rho \vec{U} k) = \nabla \cdot \left(\left(\mu + \frac{\mu}{\sigma_k} \right) \nabla k \right) + P_k + P_b - \rho\varepsilon + S_k$$

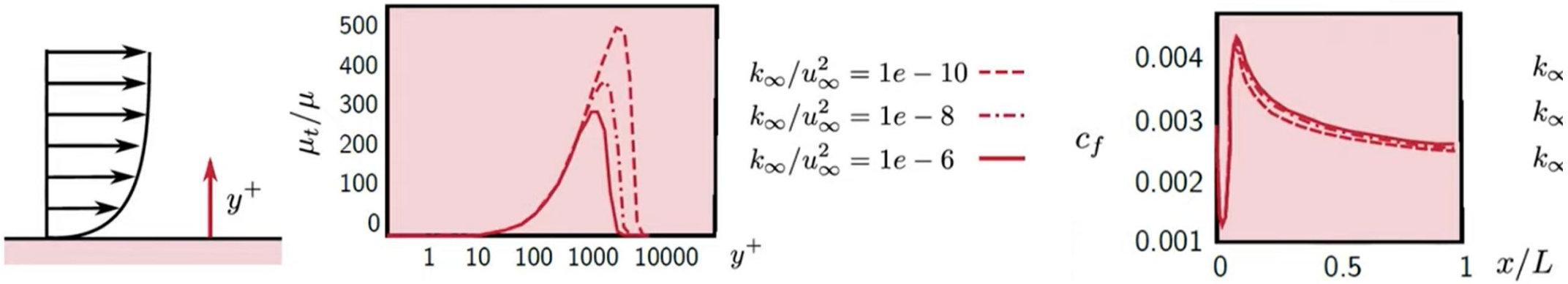
$$\frac{\partial}{\partial t}(\rho \omega) + \nabla \cdot (\rho \vec{U} \omega) = \nabla \cdot \left(\left(\mu + \frac{\mu}{\sigma_k} \right) \nabla \omega \right) + C_1 \frac{\gamma}{\nu_t} P_k - \beta \rho \omega^2 + S_\omega$$

- k-ω model has different empirical coefficients

The k-ω model – pros and cons

- k-ω model does not need damping functions
- Accurate for near-wall region, even with adverse pressure gradients
- Dependent on the freestream turbulence conditions

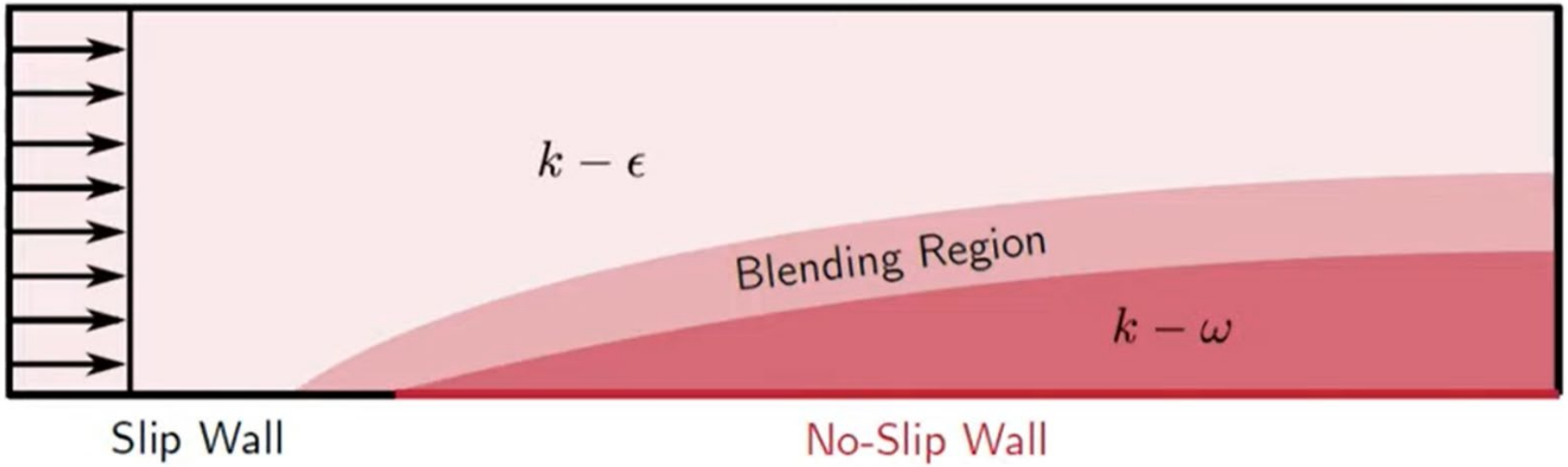




- **Small** changes in k_∞ lead to **large** changes in the turbulent viscosity μ_t!

Blending the k-ε and k-ω models

- k-ε dumping functions are unreliable
- k-ω model is sensitive to freestream turbulence conditions
- Sometimes shear stress is over-predicted
- Any solution?



Blending the k-ε and k-ω models

❖ Substitute $\varepsilon = \omega C_\mu k^2$ to the ε equation

$$\frac{\partial}{\partial t}(\rho\omega) + \nabla\cdot(\rho\vec{U}\omega) = \nabla\cdot\left(\left(\mu + \frac{\mu}{\sigma_k}\right)\nabla\omega\right) + \frac{\gamma}{\nu_t}P_k - \beta\rho\omega^2 + \boxed{2\frac{\rho\sigma_{\omega 2}}{\omega}\nabla k : \nabla\omega}$$

Additional term

❖ The ω equation

$$\frac{\partial}{\partial t}(\rho\omega) + \nabla\cdot(\rho\vec{U}\omega) = \nabla\cdot\left(\left(\mu + \frac{\mu}{\sigma_k}\right)\nabla\omega\right) + \frac{\gamma}{\nu_t}P_k - \beta\rho\omega^2$$

❖ Use a function (1-F1) to blend k-ε and k-ω models

$$2(1-F1)\frac{\rho\sigma_{\omega 2}}{\omega}\nabla k : \nabla\omega$$

- F1 = 1, k-ε model
- F1 = 0, k-ω model

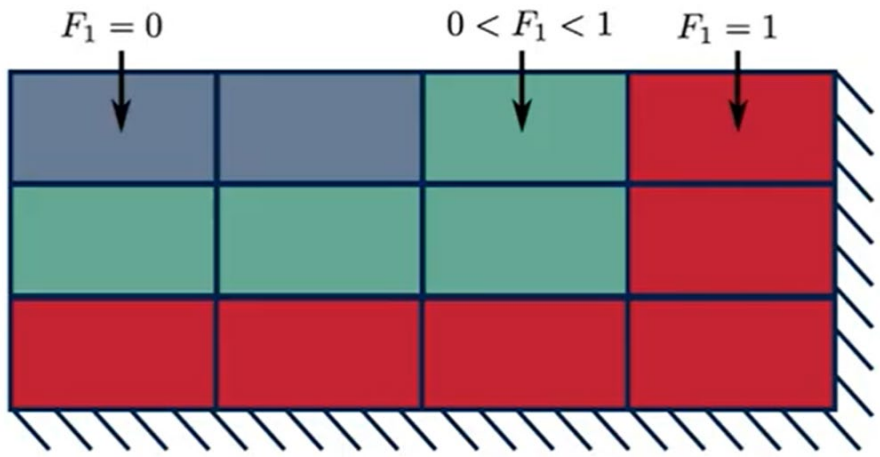
Blending the k-ε and k-ω models

$$\frac{\partial}{\partial t}(\rho k)+\nabla\cdot\left(\rho\vec{U}k\right)=\nabla\cdot\left(\left(\mu+\frac{\mu}{\sigma_k}\right)\nabla k\right)+P_k+P_b-\rho\varepsilon+S_k$$

$$\frac{\partial}{\partial t}(\rho\omega)+\nabla\cdot\left(\rho\vec{U}\omega\right)=\nabla\cdot\left(\left(\mu+\frac{\mu}{\sigma_k}\right)\nabla\omega\right)+\frac{\gamma}{\nu_t}P_k-\beta\rho\omega^2$$

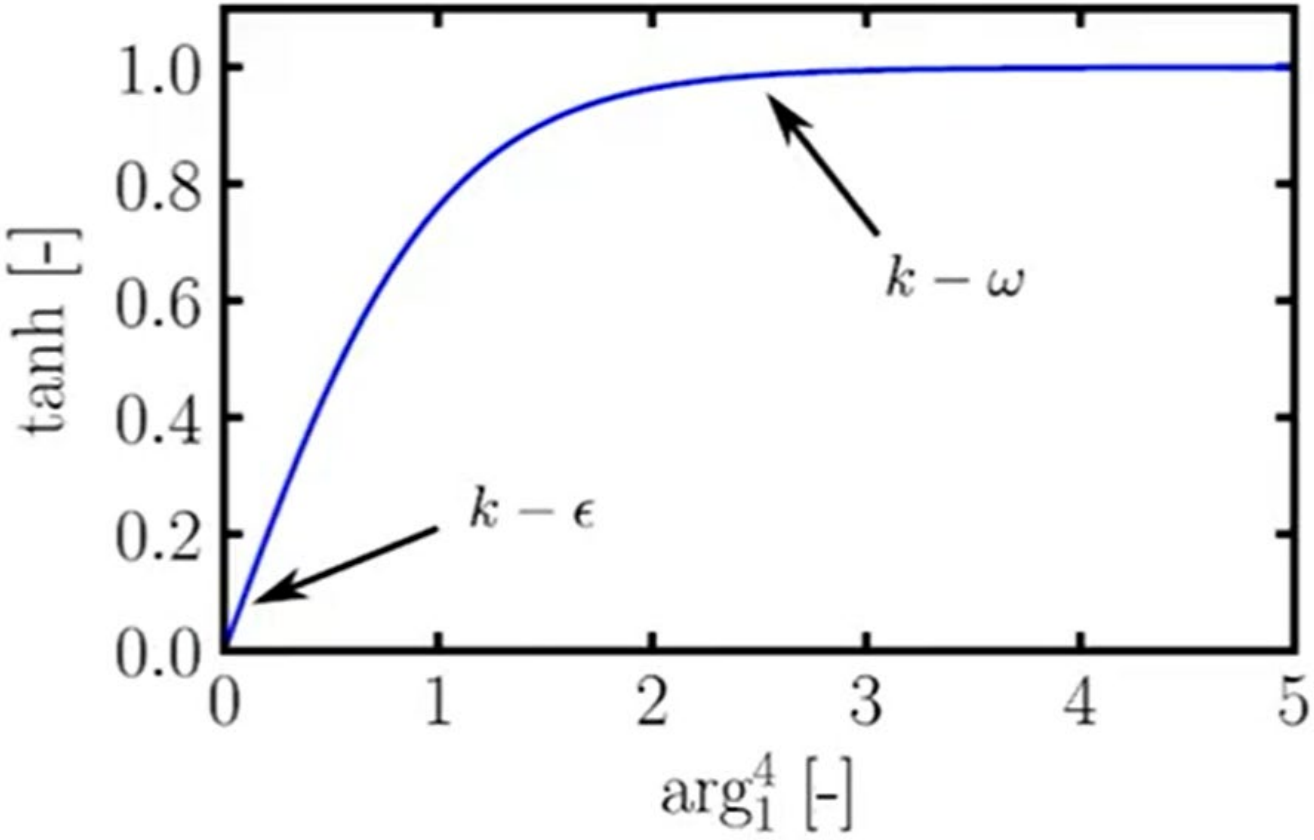
$$+2(1-F1)\frac{\rho\sigma_{\omega 2}}{\omega}\nabla k:\nabla\omega$$

$$\nabla k:\nabla\omega=\frac{\partial k}{\partial x}\frac{\partial\omega}{\partial x}+\frac{\partial k}{\partial y}\frac{\partial\omega}{\partial y}+\frac{\partial k}{\partial z}\frac{\partial\omega}{\partial z}$$



The k-ω BSL (baseline) model

$$F1 = \tanh\left(\arg_1^4\right)$$



The k-ω SST (shear stress transport) model

- In the k-ω SST model, a viscosity limiter is further included

Original $\mu_t = \dfrac{\rho k}{\omega}$ SST scheme $\mu_t = \dfrac{a_1 \rho k}{\max\left(a_1 \omega, S F_2\right)}$ $F_2 = \tanh\left(\arg_2^2\right)$

- F_2 is another blending function similar to F_1.
- The viscosity is limited when S or F_2 is large.
- Better agreement with experimental measurements

The k-ω model – pros and cons

- Pros
  - Robust across a range of models (flow separation, adverse pressure)
  - Accurate for high Prandtl number wall heat flux prediction

- Cons
  - Can require a very fine boundary layer mesh (10-15 mesh enhancement layers)

Effects of turbulence modelling

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Evaluation of the eddy viscosity turbulence models for the simulation of convection–radiation coupled heat transfer in indoor environment

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Indoor environment

ABSTRACT

This paper presents a numerical evaluation of the eddy viscosity turbulence models (the zero-eq one-equation, k-ε group and k-ω group models) in terms of CFD modelling of convection-radiatio pled heat transfer in indoor environment. The results show that all the models except the zero-e model have acceptable accuracies to predict the bulk air temperature. However, the zero-equati one-equation models seriously overpredict the convective heat transfer coefficient at the heater s while the k-ε group models severely underpredict it, ultimately resulting in the incorrect pred of the thermal buoyancy flows and contaminant dispersion. On the contrary, the k-ω group mod form best to calculate the convection-radiation heat split, which is critical to the successful predic the near-wall flow field and distribution patterns of the contaminants. This study also demonstra importance of simultaneously considering the convective and radiative heat transfer mechanism evaluating the turbulence models for indoor environment simulations.

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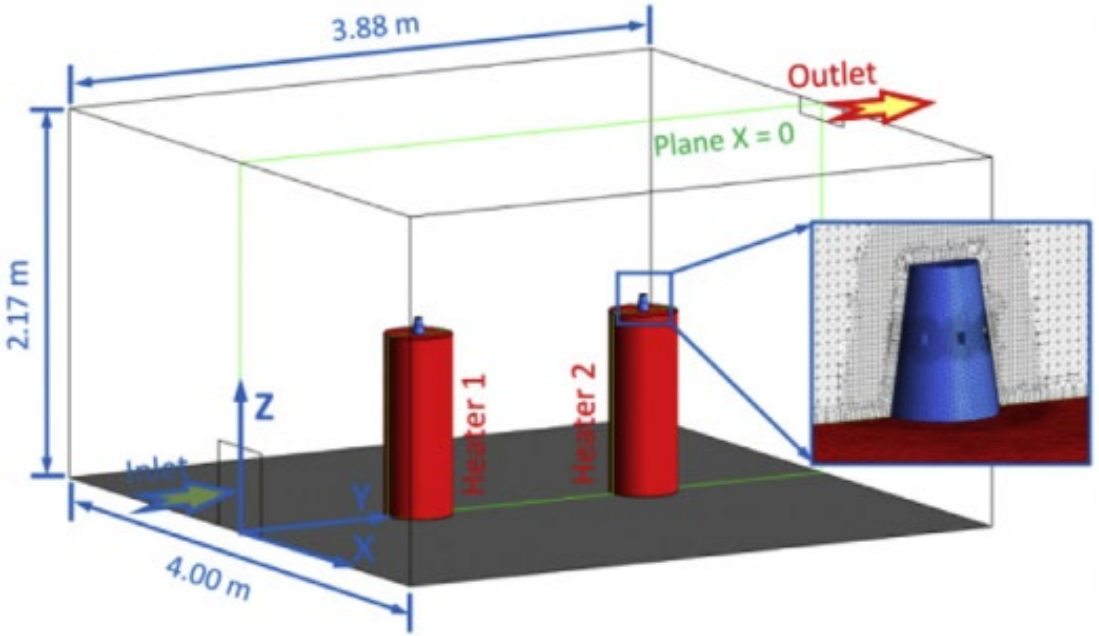

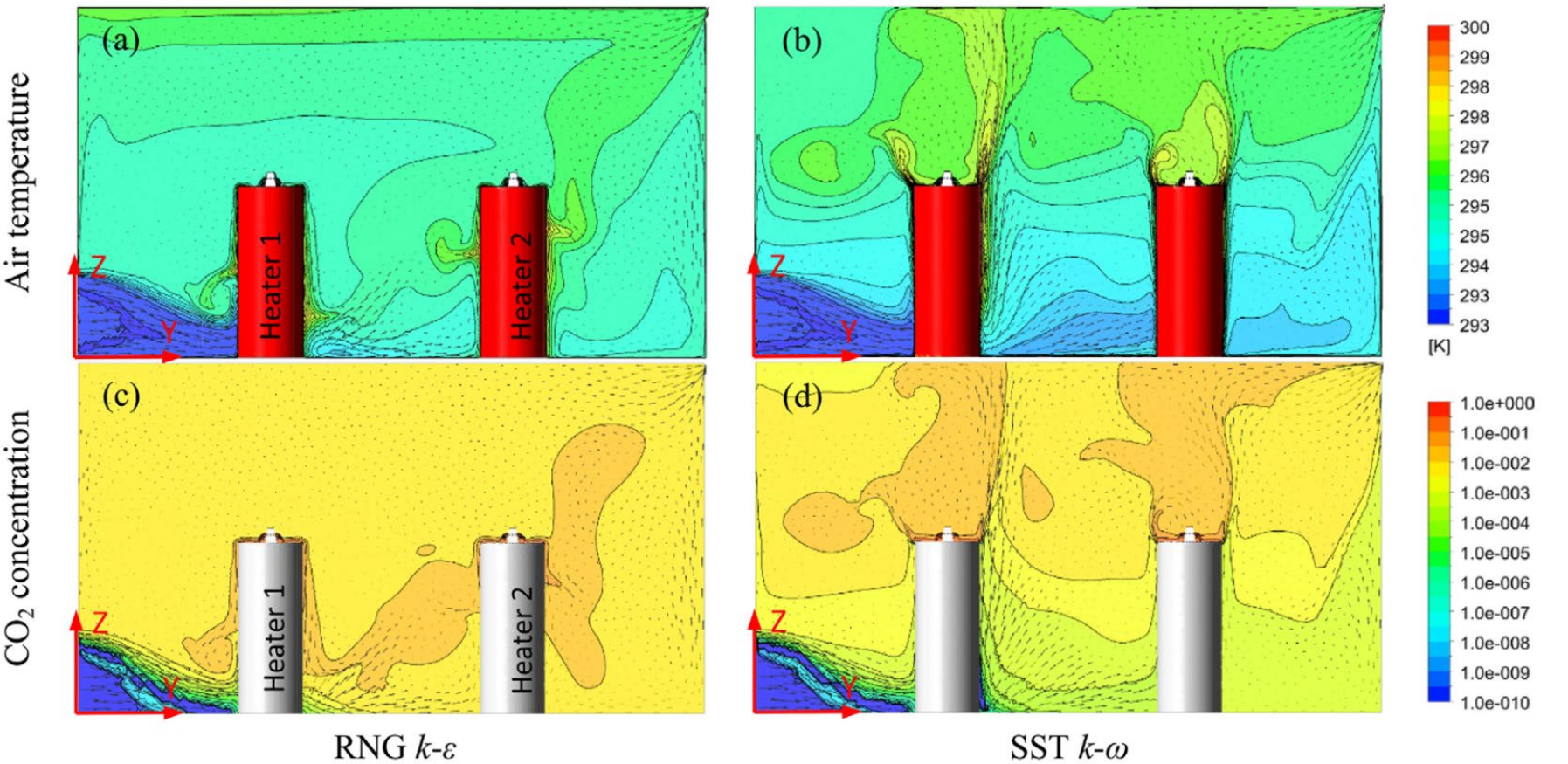
Air temperature
CO_2 concentration
(a)
(b)
(c)
(d)
Heater 1
Heater 2
Z
Y
300
299
298
298
297
296
295
295
294
293
293
[K]
1.0e+000
1.0e-001
1.0e-002
1.0e-003
1.0e-004
1.0e-005
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1.0e-008
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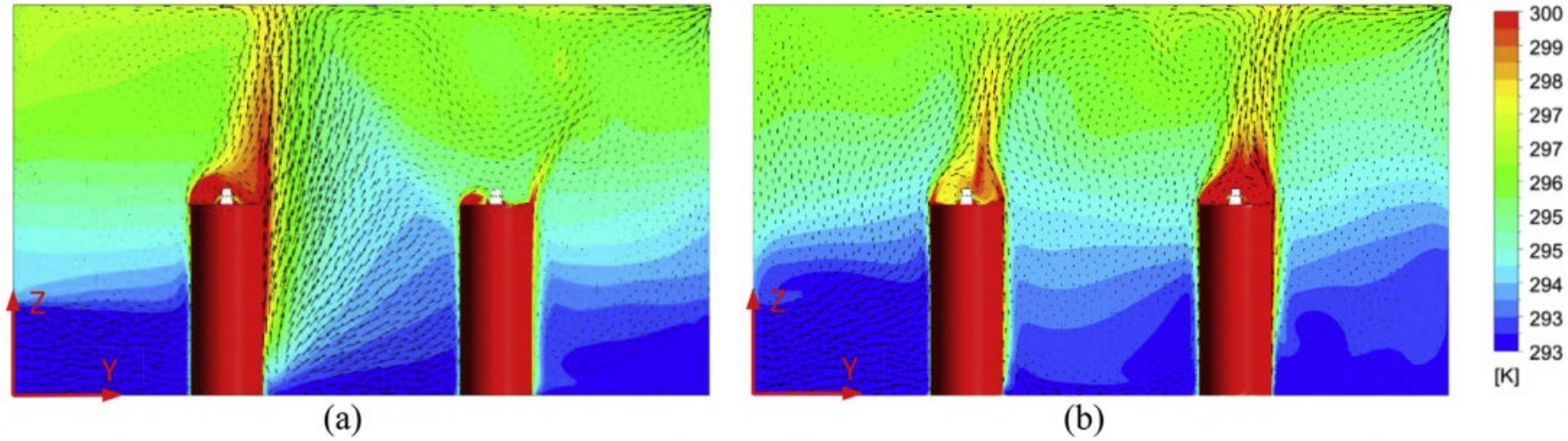
1.0e-010
RNG k-ε
SST k-ω


Z
Y
300
299
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296
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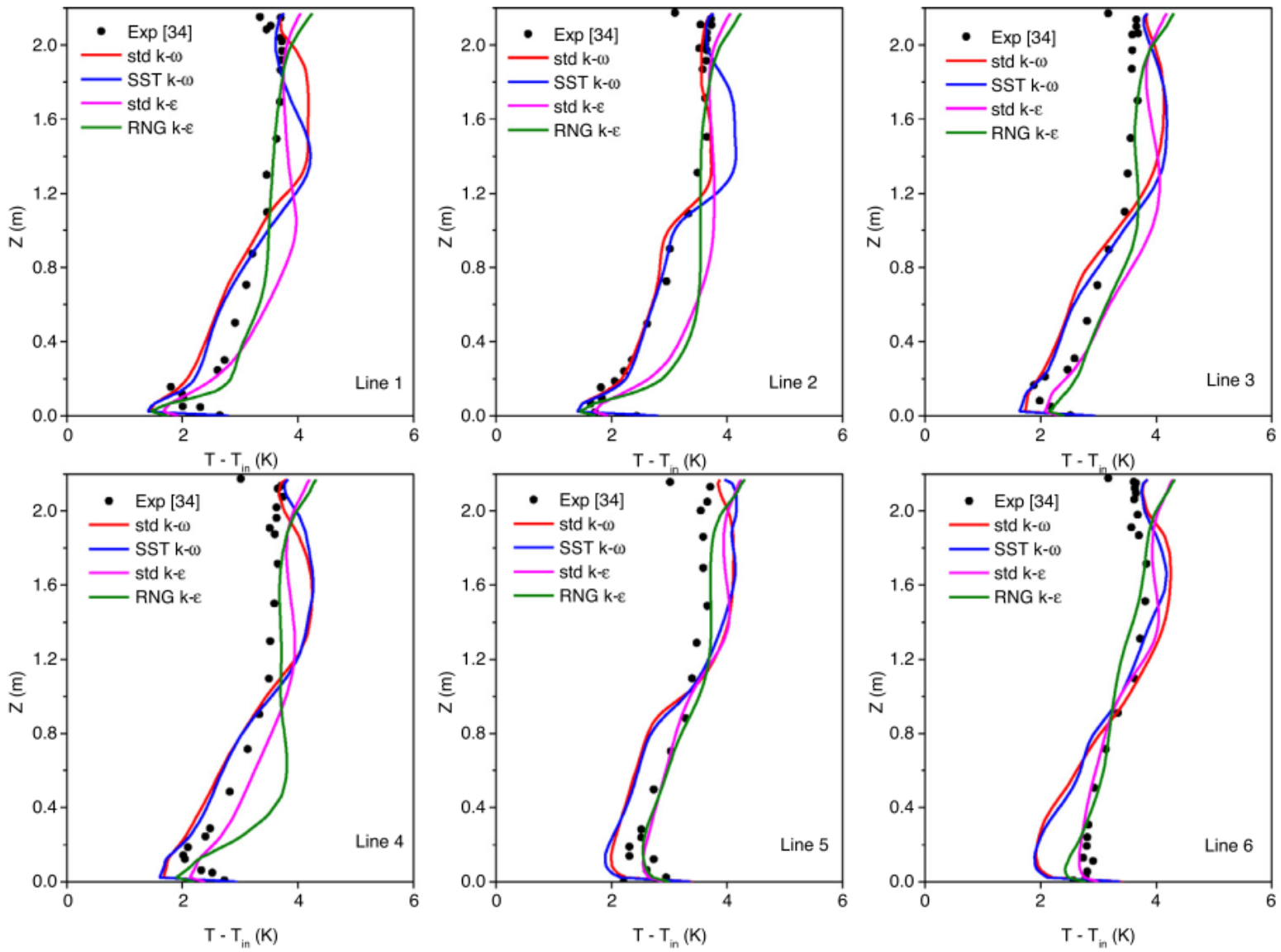
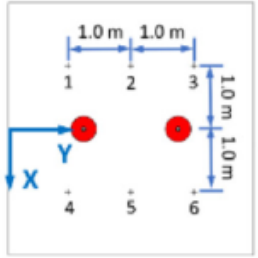
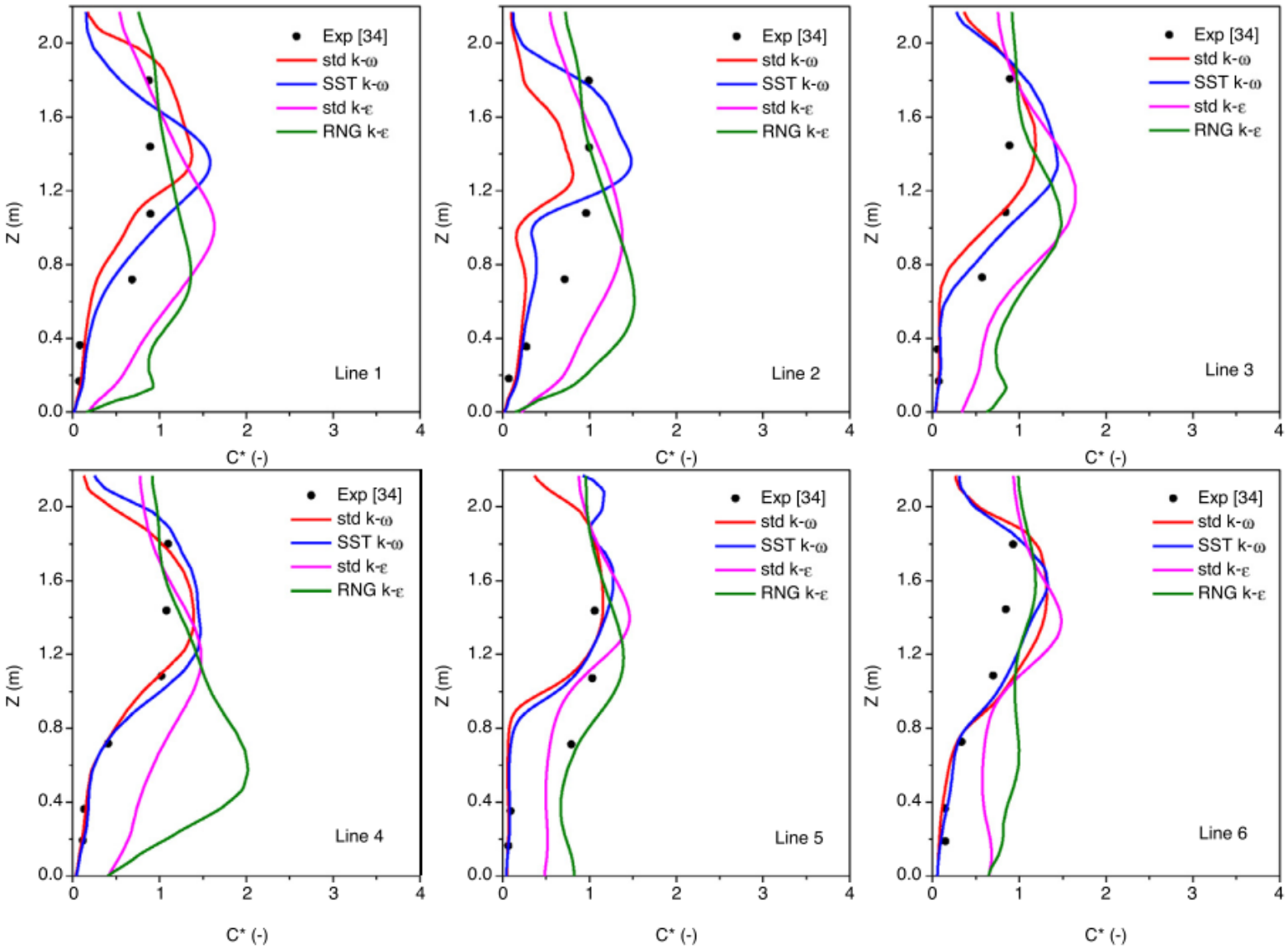
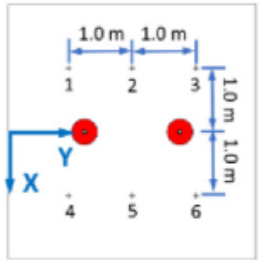
294
293
293
[K]
(a)
(b)


Exp [34]
std k-ω
SST k-ω
std k-ε
RNG k-ε
Line 1
Line 2
Line 3
Line 4
Line 5
Line 6
Z (m)
T - T_in (K)


1.0 m
1.0 m
1.0 m
1.0 m
1
2
3
4
5
6
Y
X

(a)


Exp [34]
std k-ω
SST k-ω
std k-ε
RNG k-ε
Line 1
Line 2
Line 3
Line 4
Line 5
Line 6
Z (m)
C* (-)


1.0 m
1.0 m
1.0 m
1.0 m
1
2
3
4
5
6
X
Y

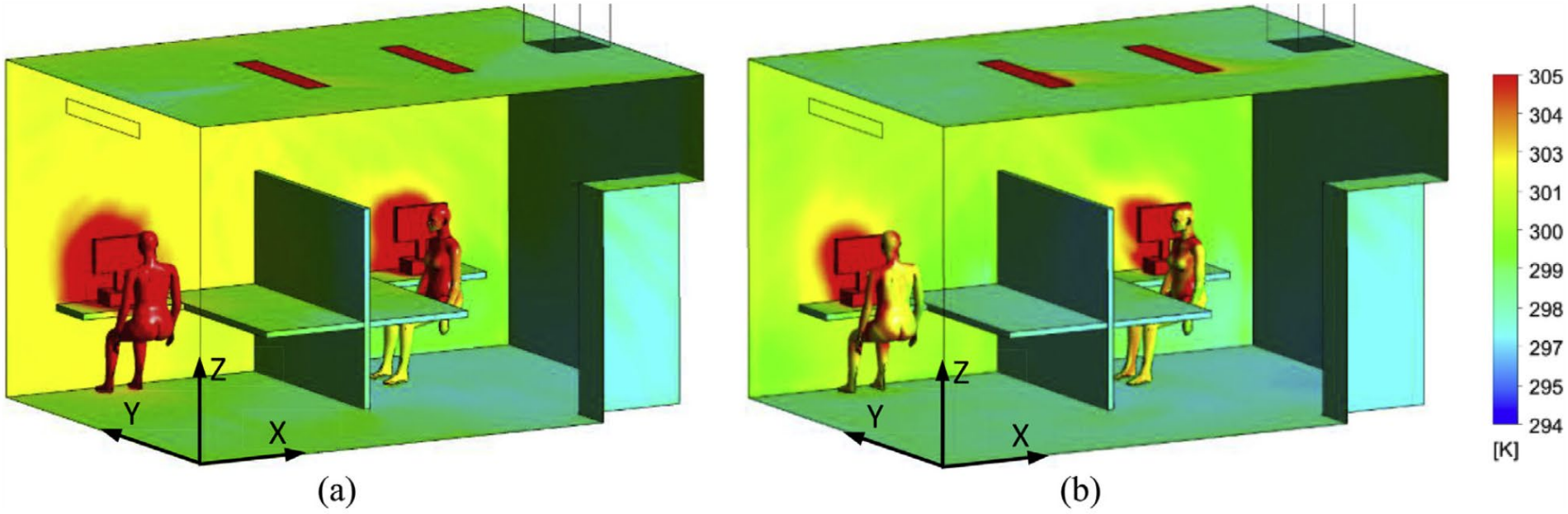


Fig. 11. The surface temperature predicted with different turbulence models, (a) RNG k-ε, (b) SST k-ω.

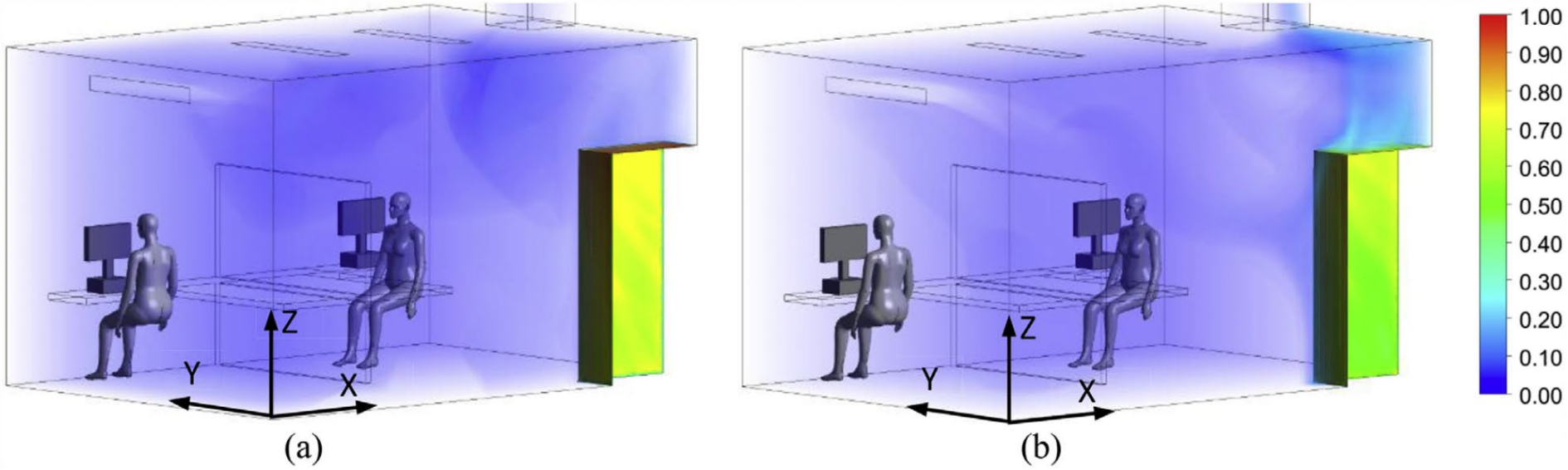


Fig. 13. The volumetric distribution of formaldehyde concentration ($C^* = C$ / C_{max}), (a) RNG k-ε, (b) SST k-ω.

Use of turbulence models

Basic Level

Standard k-ε

RNG k-ε

k-ω

Low-Re k-ε

Realisable k-ε

k-ω BSL

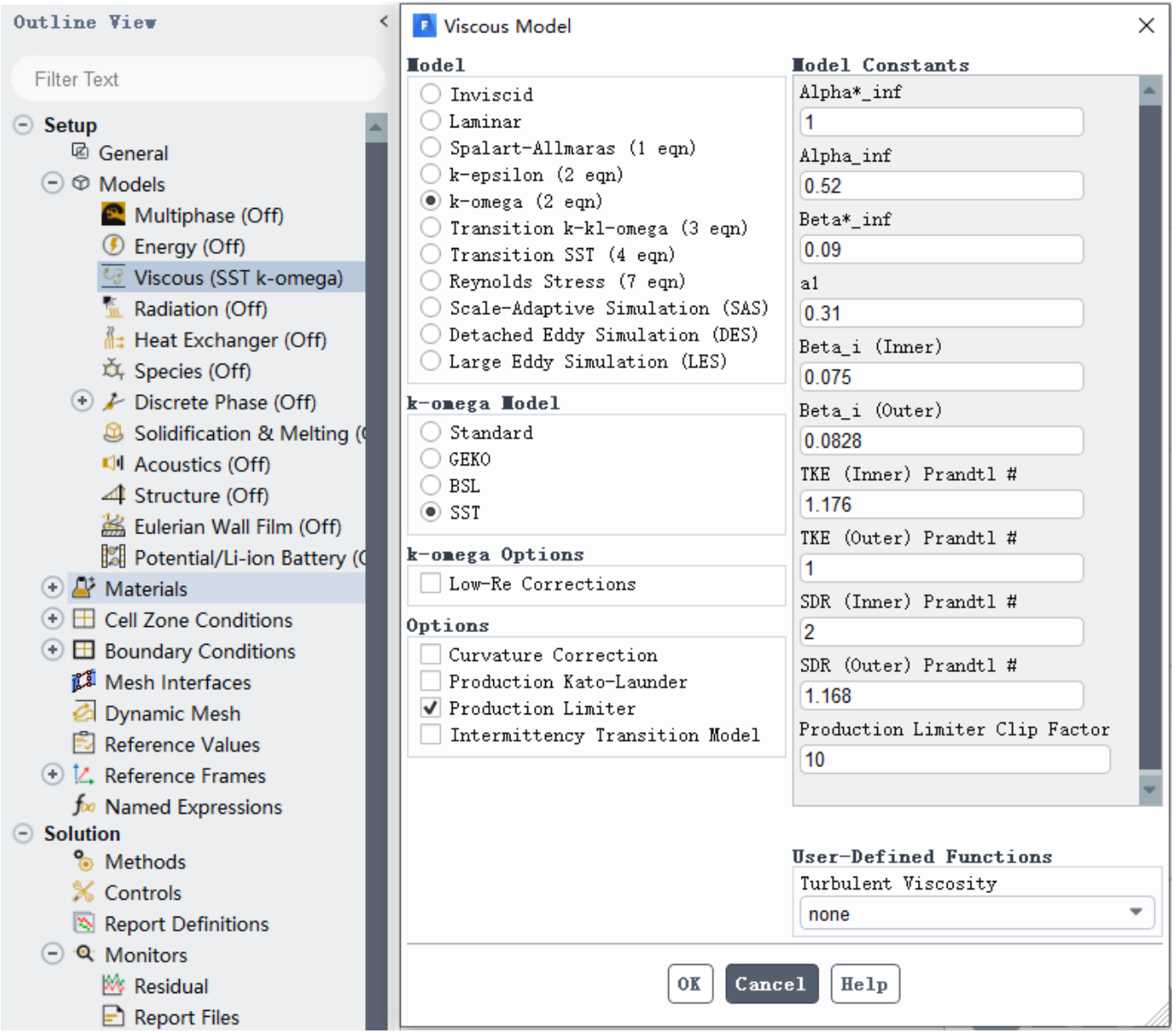
Advanced Level

k-ω SST

- Everywhere else
- Initial value

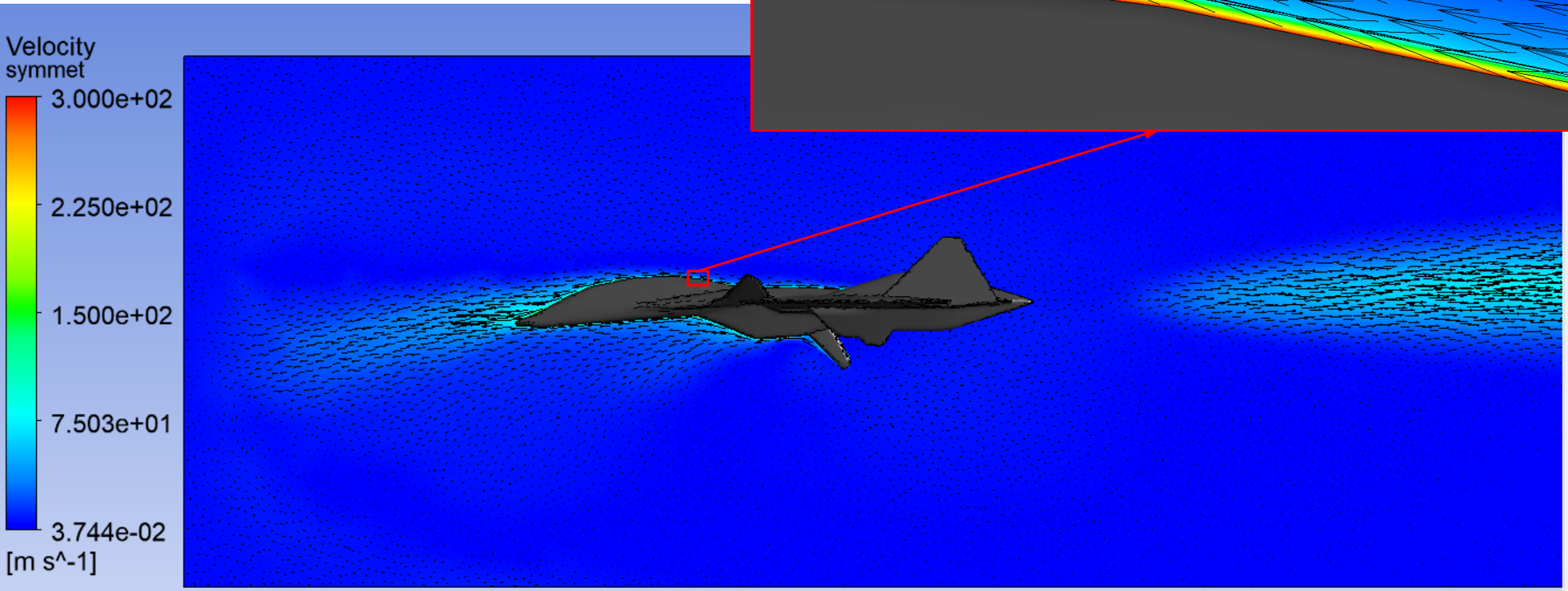
- External aero
- L-T Transient
- Significant wake

Turbulence models in Fluent

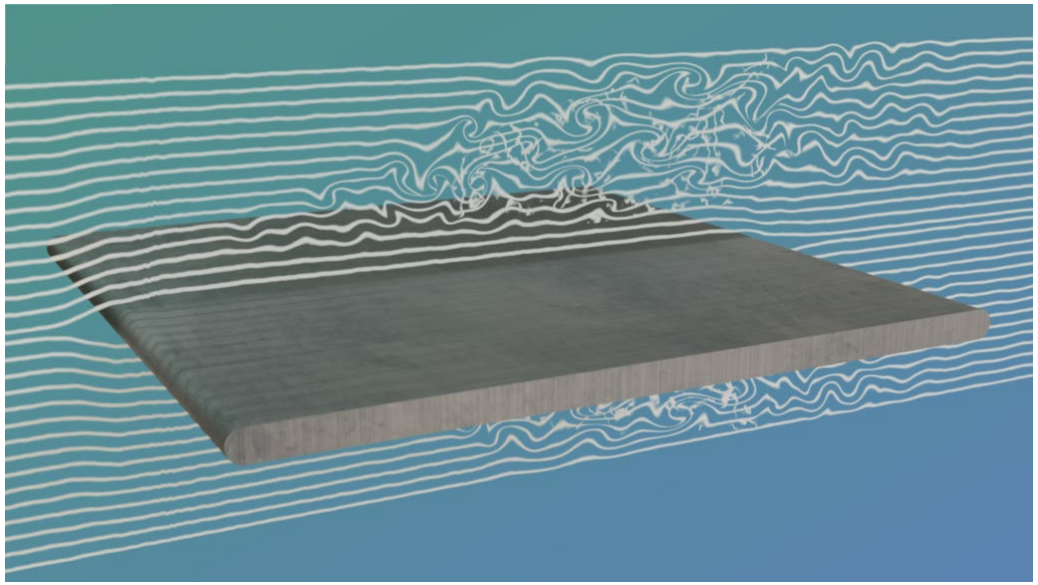


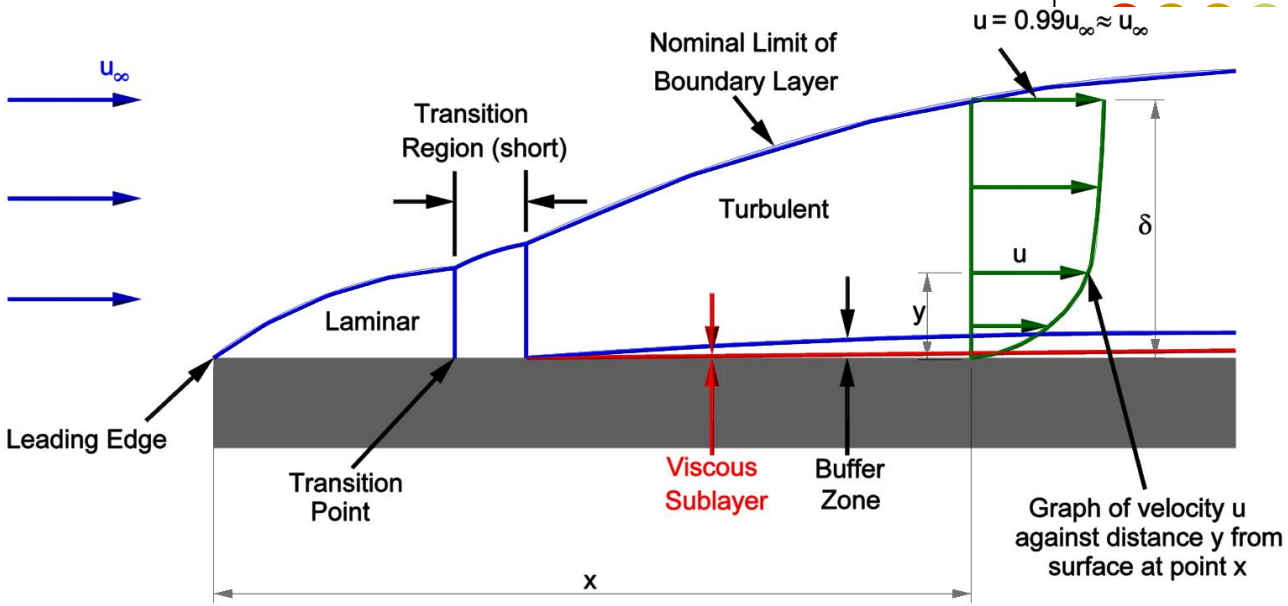
BOUNDARY LARYER

Boundary layer



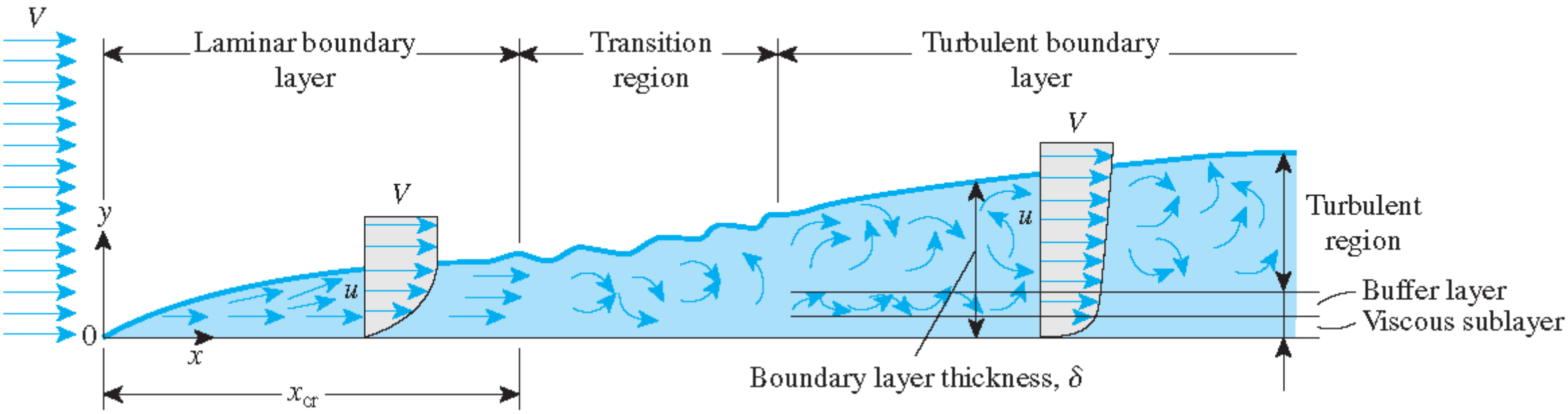
Boundary layer





- Boundary layer is the thin layer on solid surface in which viscous effects are important
- Fluid velocity increases from zero to the free stream velocity
- The velocity component normal to the surface is much smaller than the parallel velocity v << u (velocity gradient is much larger)
- The boundary thickness is defined as the distance away from the surface where the velocity reaches **99%** of the free-stream velocity.

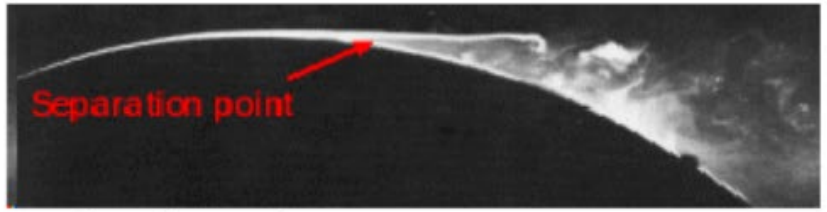
Structures of boundary layer



https://physics.stackexchange.com/questions/281281/about-the-definition-of-boundary-layer

- A developing process: Laminar boundary layer → Transition region → Turbulence boundary layer
- Laminar boundary layer: a relatively short region
- Transition region: generally short, but containing too many uncertainties
- Turbulent boundary layer: our focus

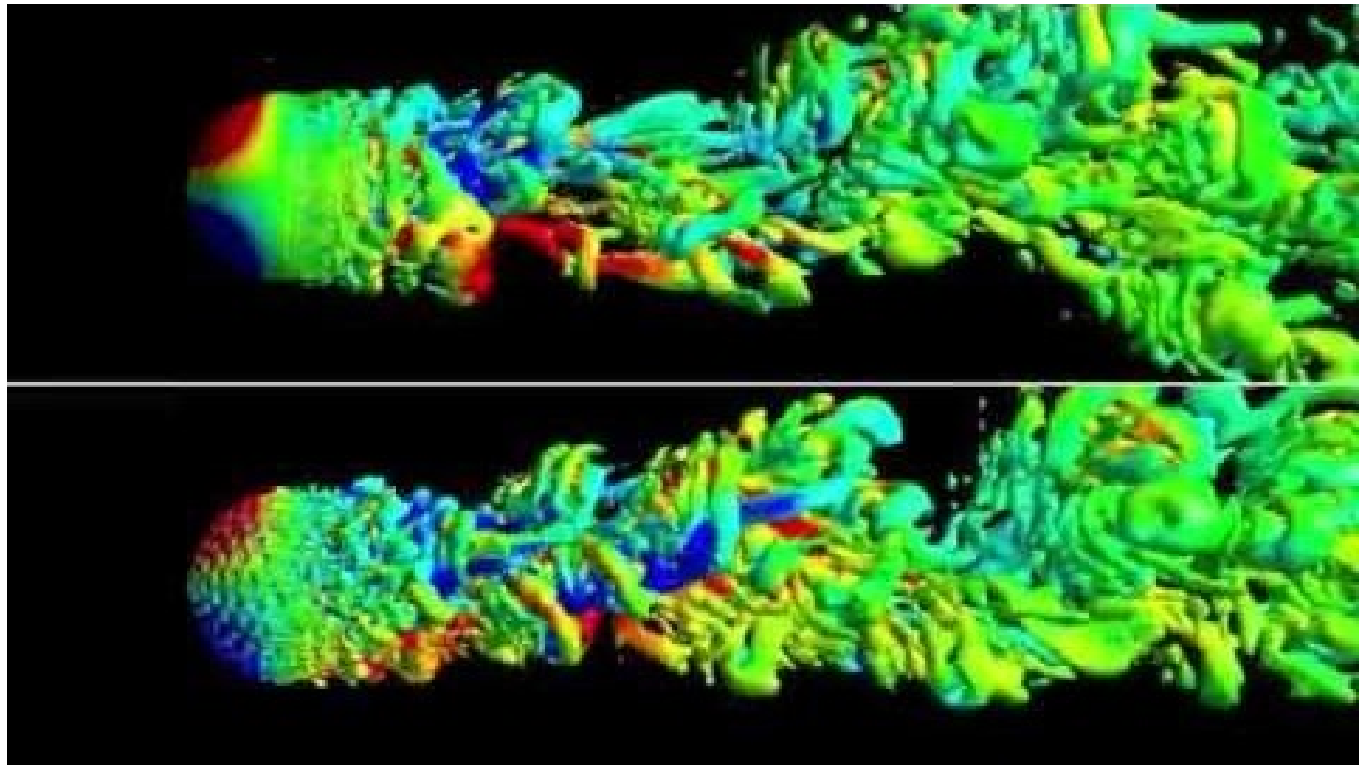
Turbulent vs laminar boundary layer



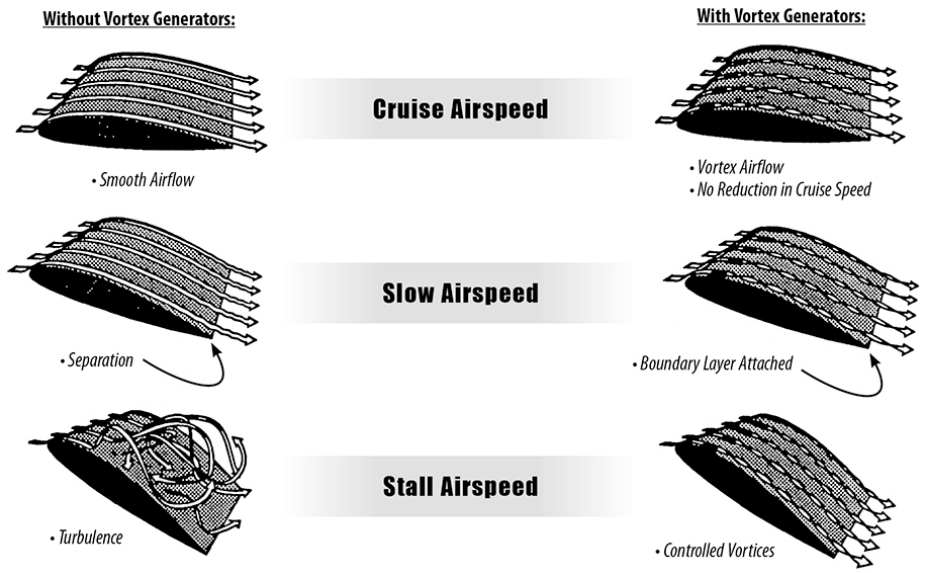
Laminar Separation

Turbulent Separation

- Turbulent boundary layer more resistant to flow separation
- Increased streamwise momentum in near-wall region due to the enhanced momentum transport by turbulence from the mainstream (due to the Reynolds stress)
- Allows flow to resist to adverse pressure gradient



Use of turbulent layer



Use of turbulent layer

Quantify the turbulent boundary layer

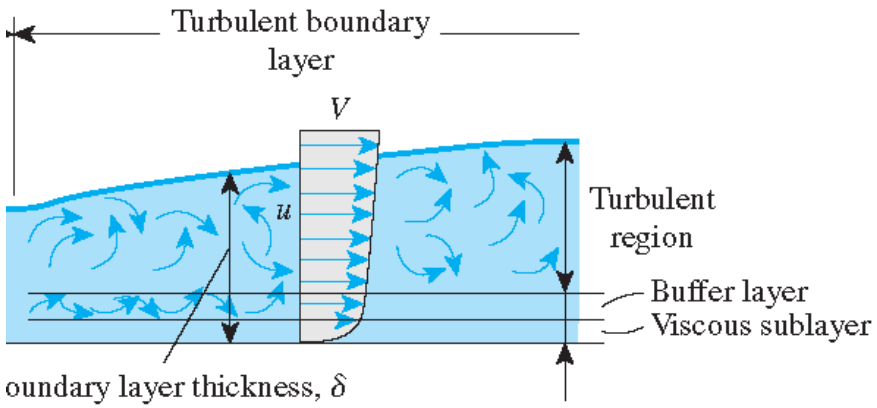
- Let's first define two dimensionless quantities

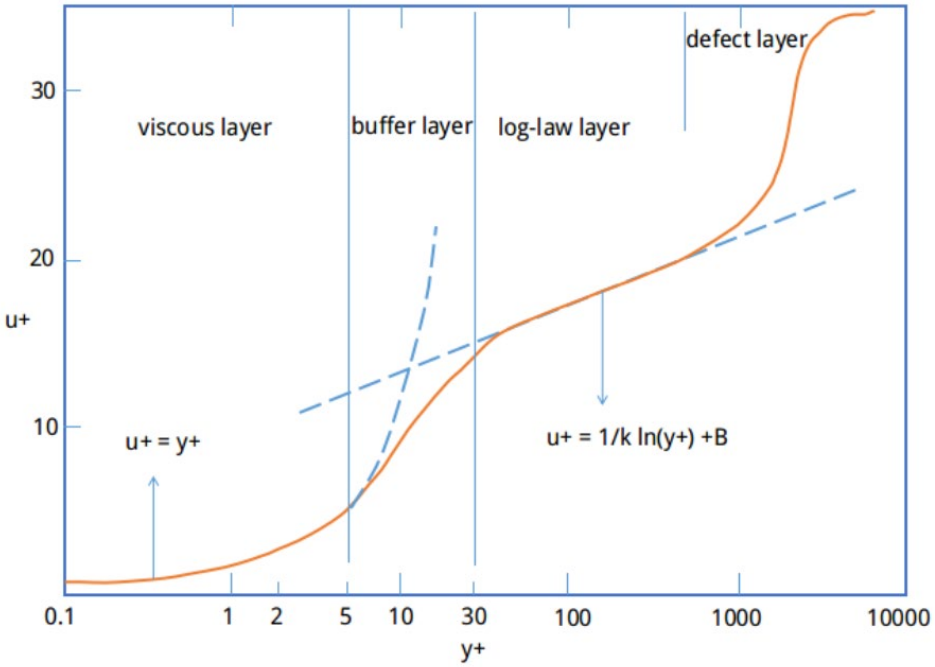
$$y^+ = \frac{y u_\tau}{\nu} \qquad U^+ = \frac{U}{u_\tau}$$

- u_τ is a reference velocity called frictional velocity

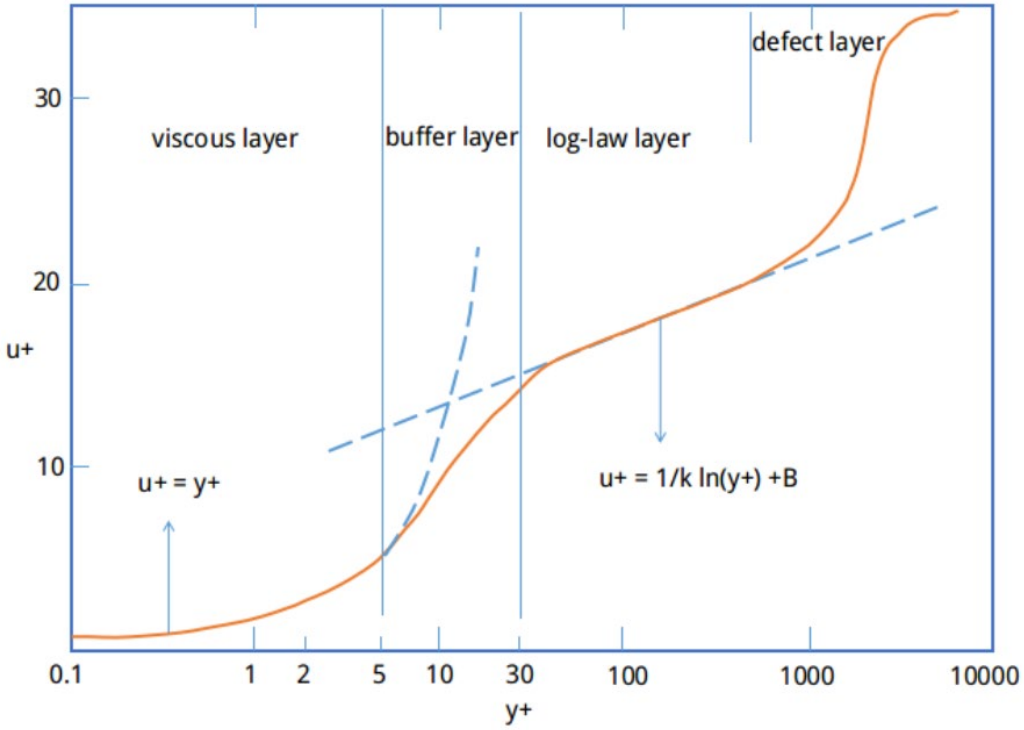
$$u_\tau = \sqrt{\tau_w / \rho}$$

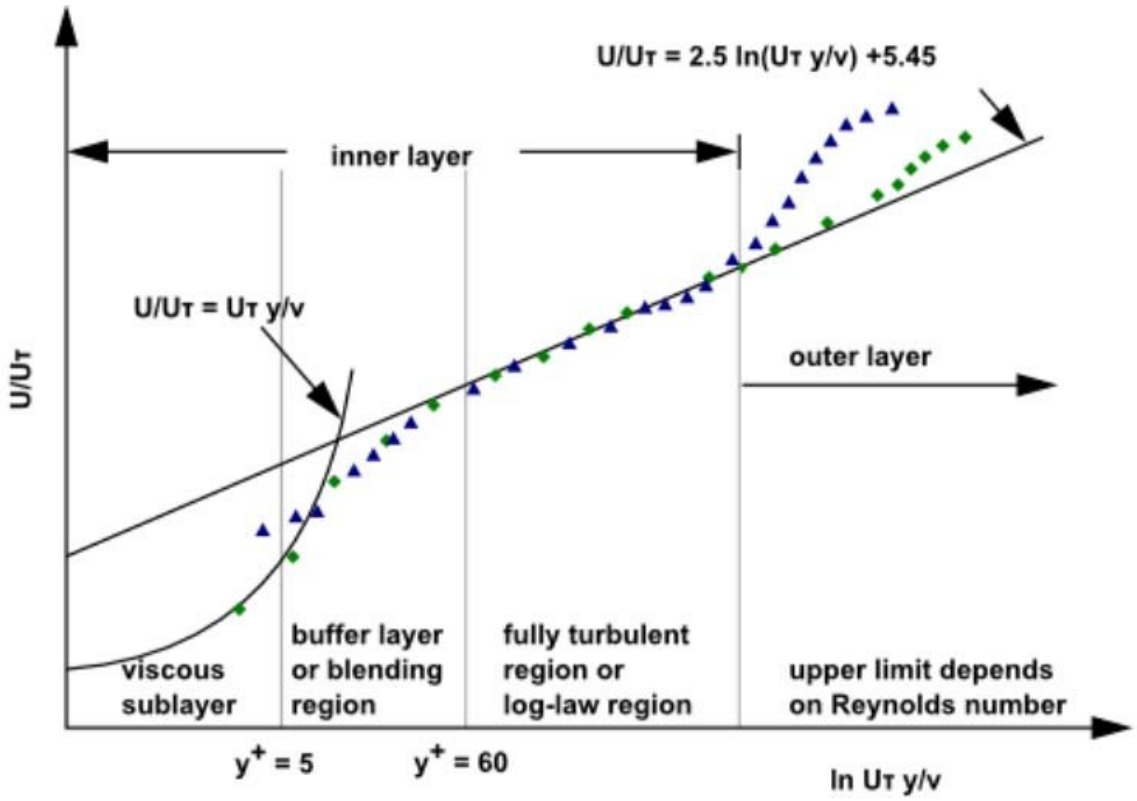
$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_w$$





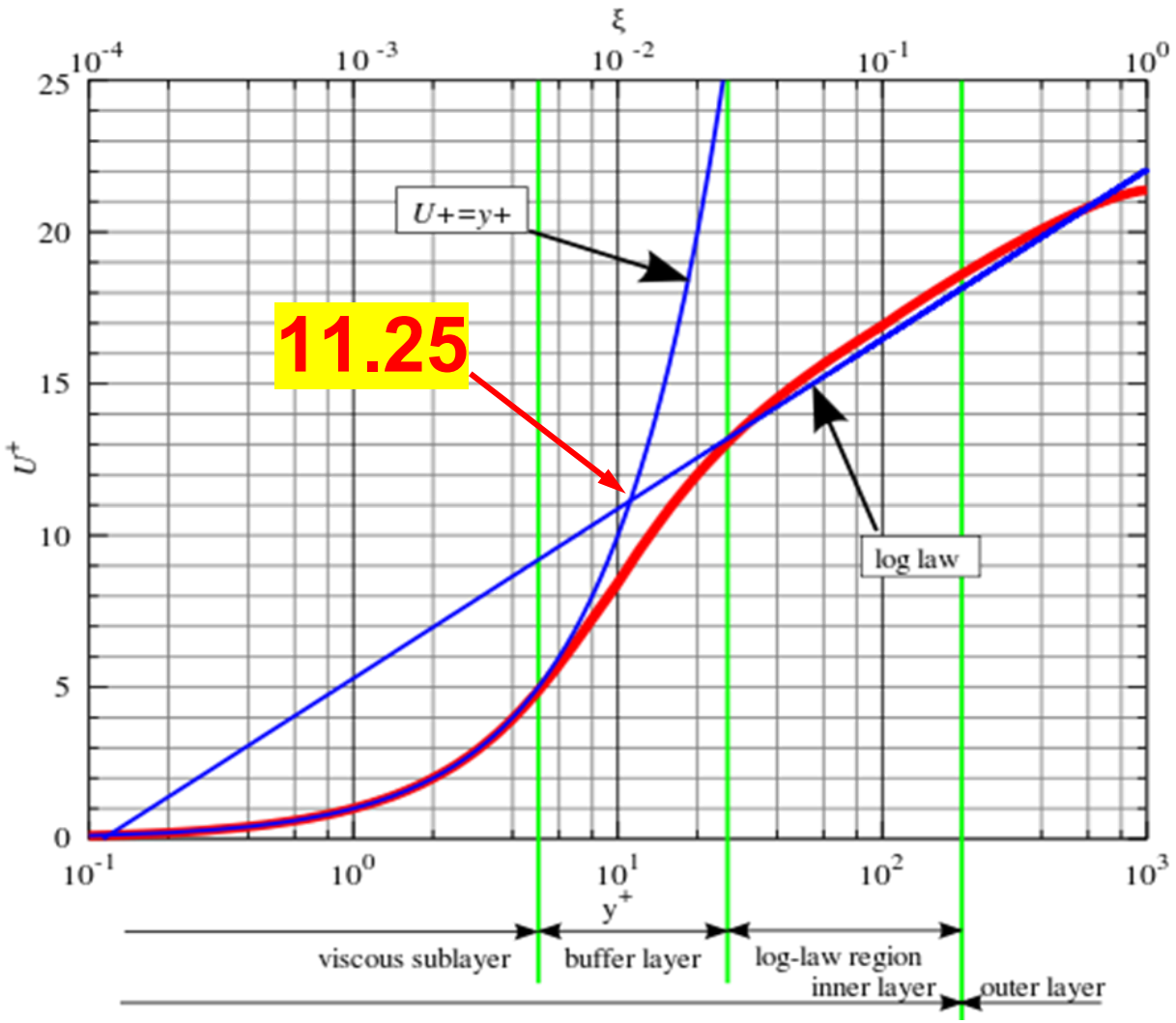
Quantify the turbulent boundary layer





No standardized division!!!

Wall functions – the standard wall function

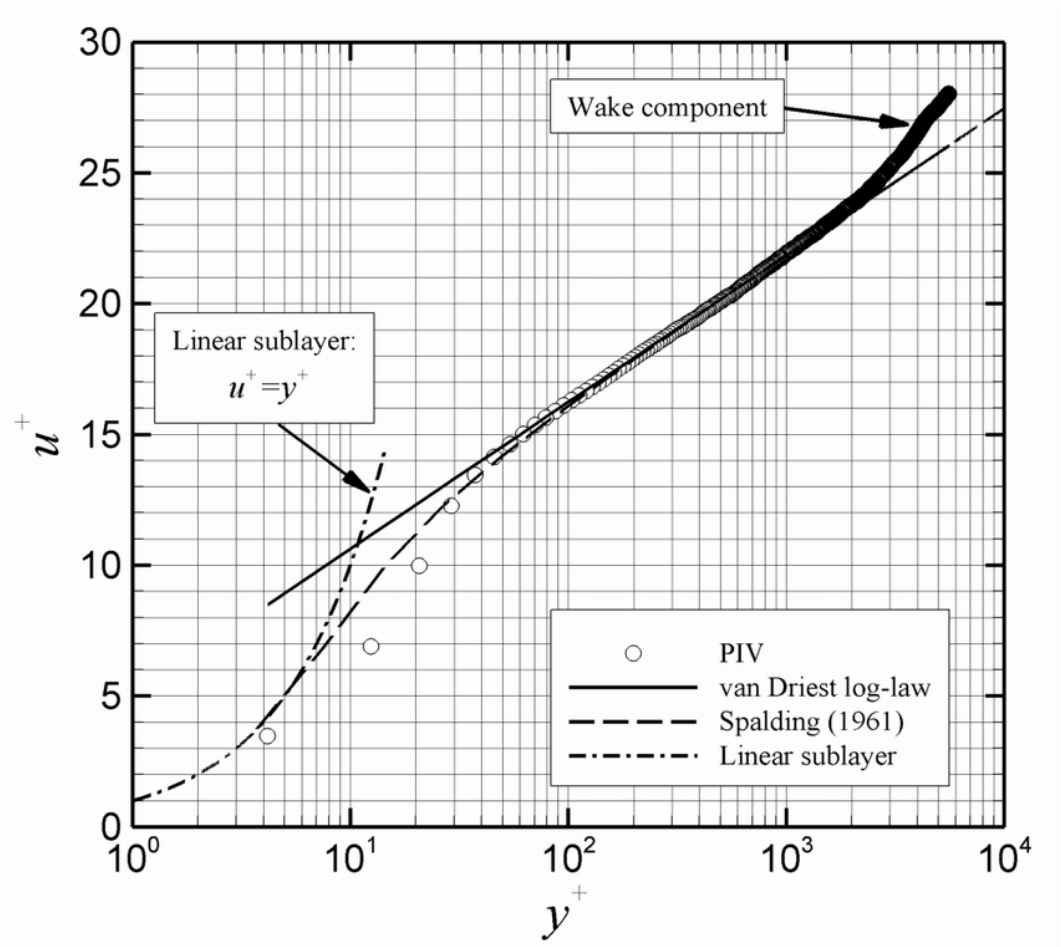


$$U^+ = \begin{cases} y^+ & y^+ < 11.25 \\ \dfrac{1}{\kappa}\log\left(Ey^+\right) & 200 > y^+ > 11.25 \end{cases}$$

$$\kappa = 0.4187 \quad E = 9.793$$

Wall functions – the blending wall function

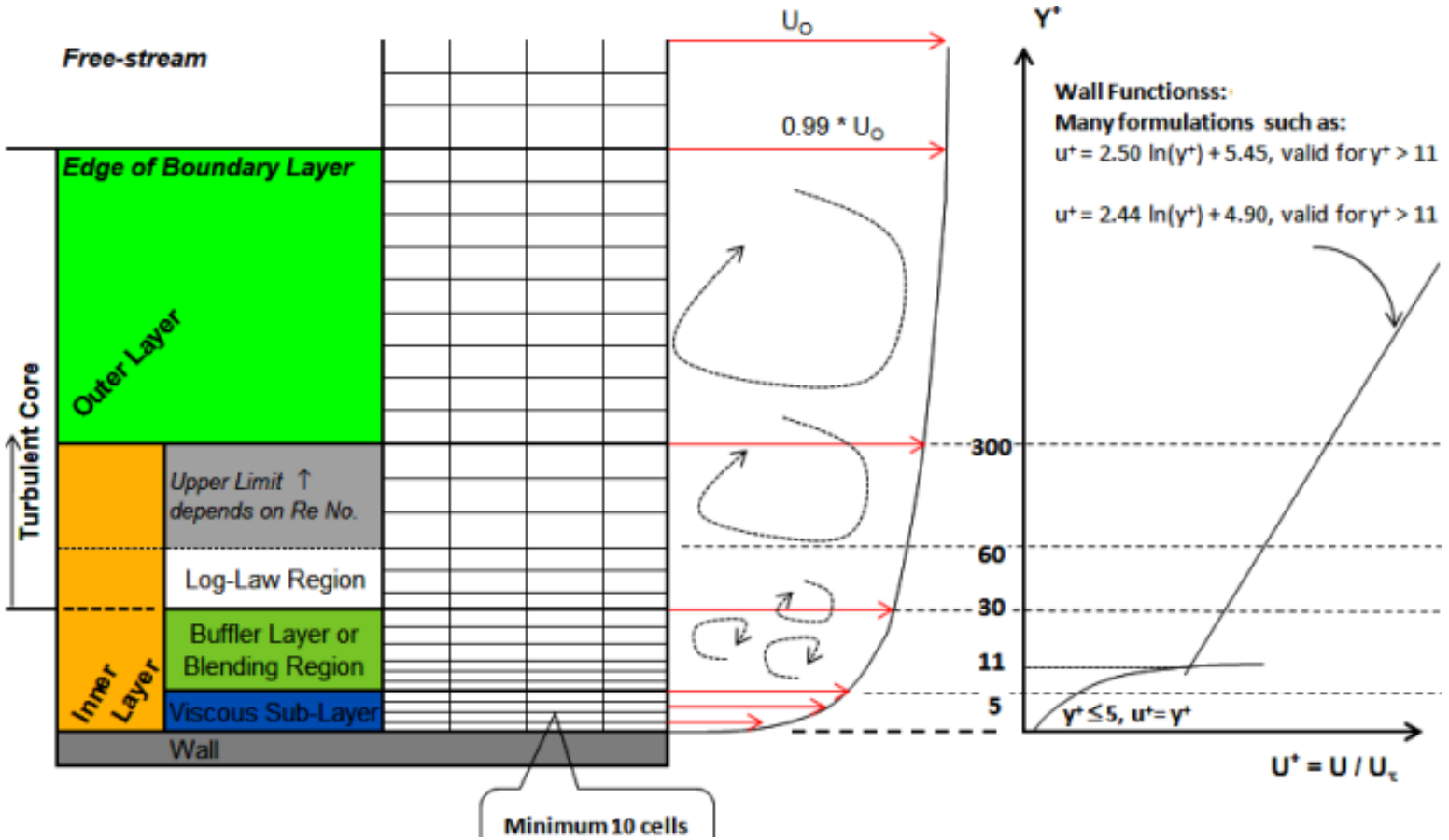
$$y^+ = U^+ + 0.1108\left(e^{0.4U^+} - 1 - 0.4U^+ - \frac{1}{2}\left(0.4U^+\right)^2 - \frac{1}{6}\left(0.4U^+\right)^3\right)$$



Why important – think about your mesh

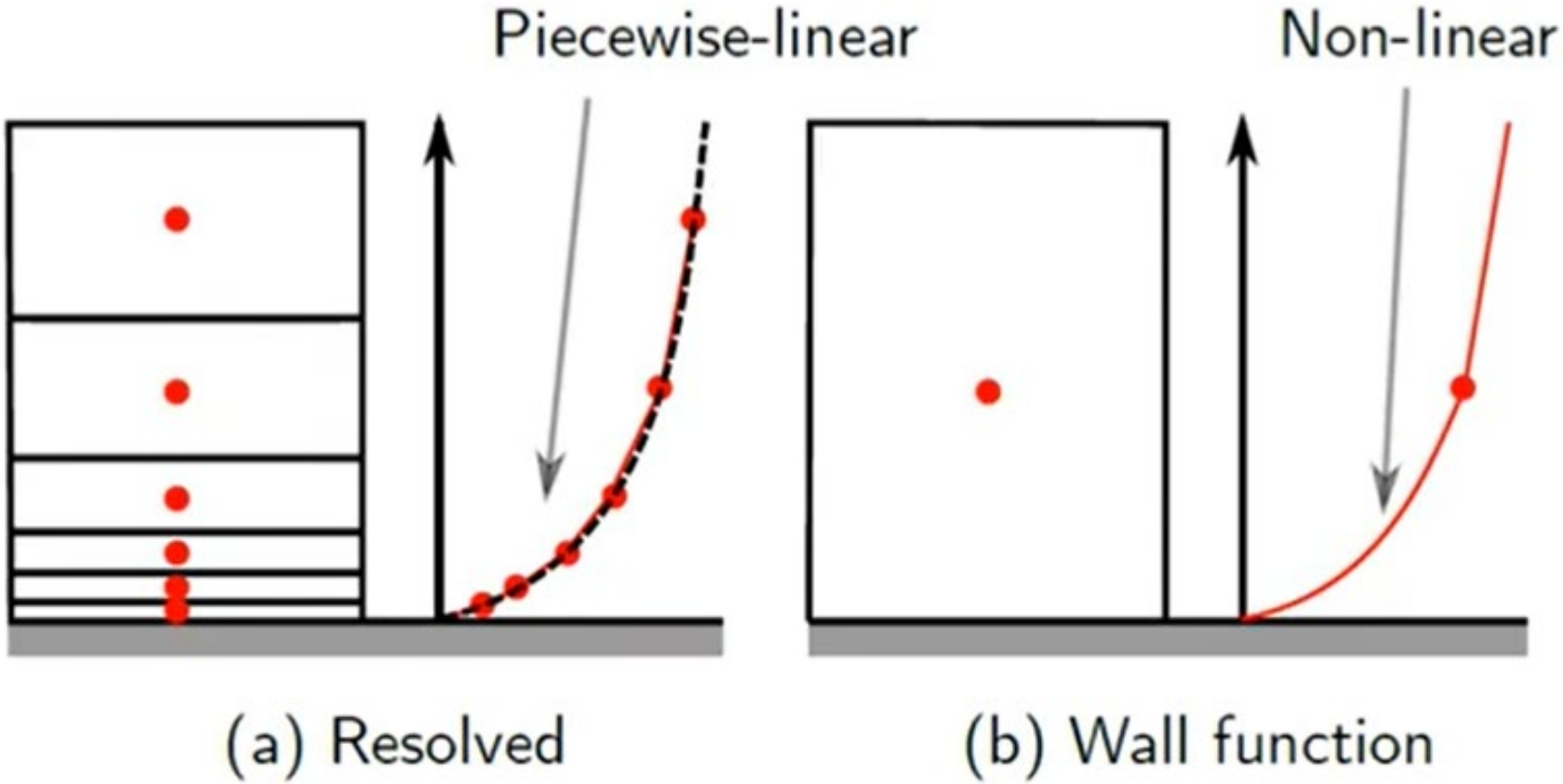
Some research suggests you need at least 10 layers of mesh elements in the viscous sublayer

Is there a better way?



http://www.cfdyna.com/CFDHT/turbulenceCFD.html

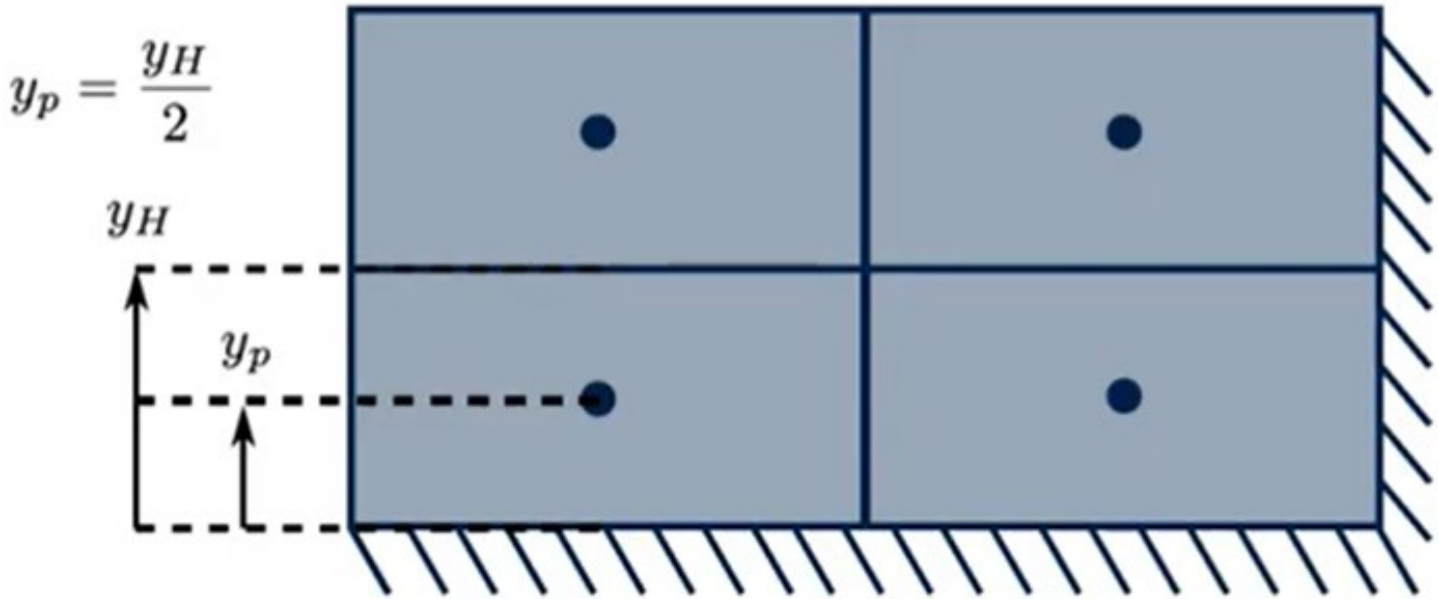
Why important – think about your mesh



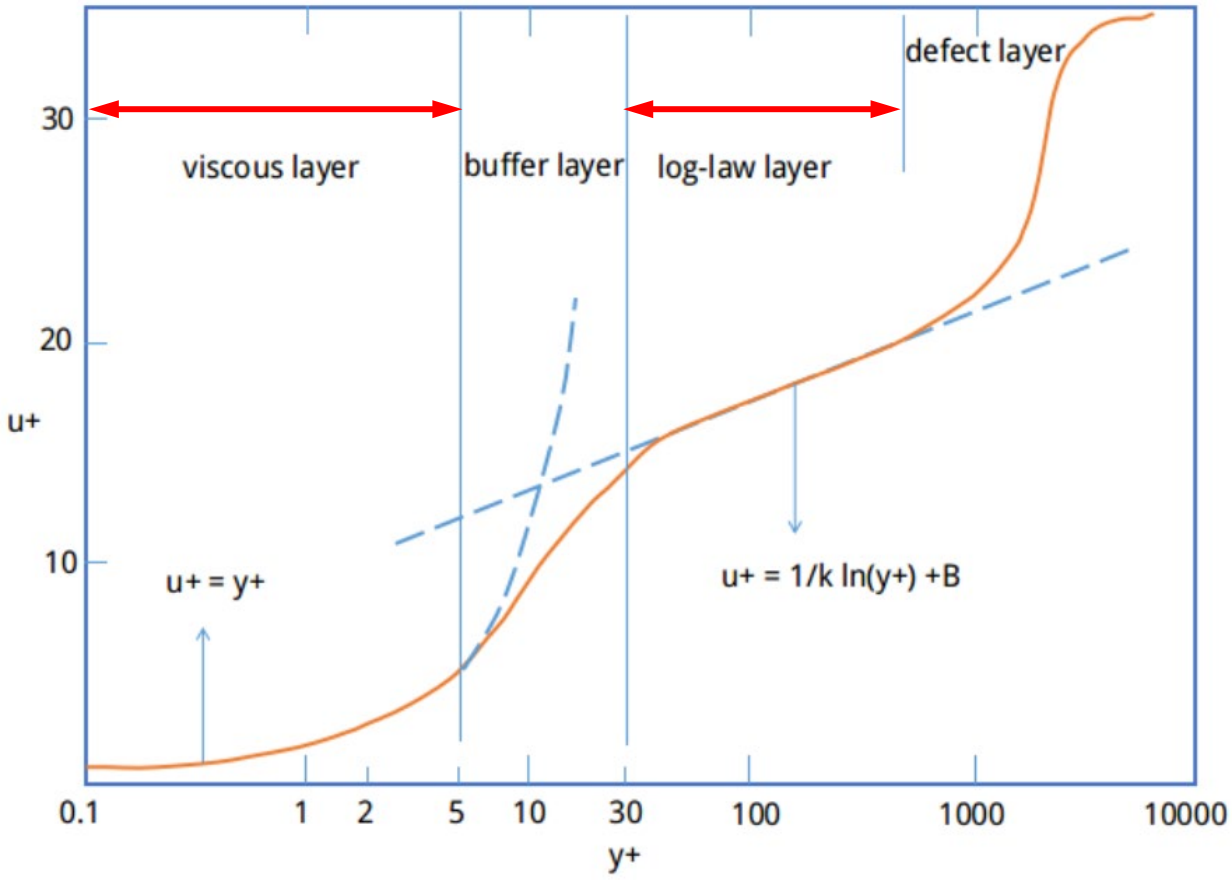
Velocity in the next-wall mesh is calculated using a wall

Only look at the first mesh layer

$$y^+ = \frac{y_p u_\tau}{\nu}$$



Desired wall function regions – generally



- Wall functions are only valid in a certain y+ range
- y+ too high, the first node is above the boundary layer, and wall function applied to mainstream flow
- y+ too low, the first node is in the laminar sublayer

y+ in different turbulence models

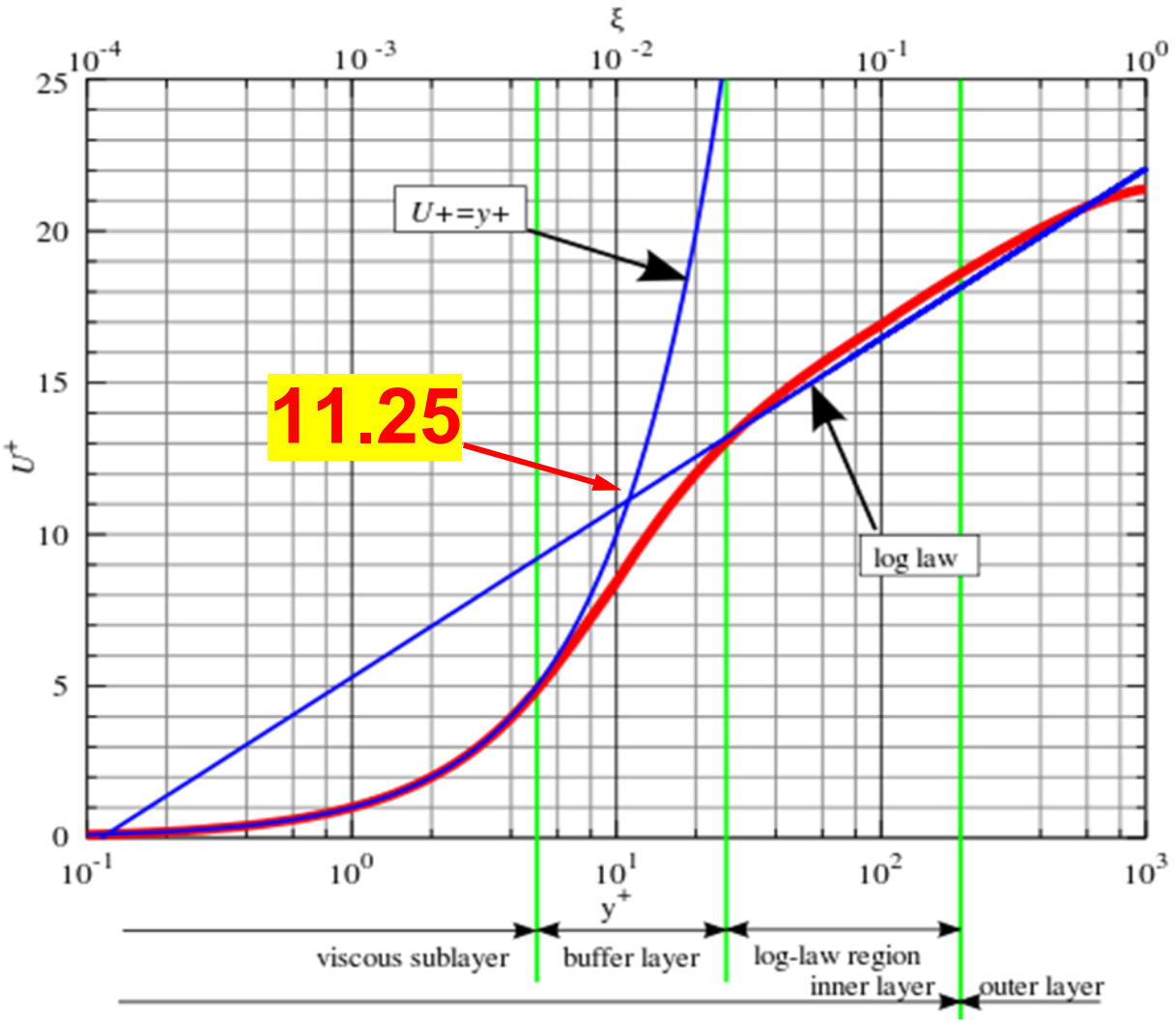
k-ε models

- Standard: 35 < y+ < 350
  - Not modelling laminar sublayer
  - Not accurate for high-Prandtl number wall heat flux
- Low-Re: No wall function
- Scalable wall function: y+ < 350
  - Not sensitive to mesh refinement – great for mesh adaption studies
  - Increased performance in transition flows

k-ω SST

- y+ can be up to 100
- Robust between 10 < y+ < 100
- Ideally, y+ < 2 with a target of 0.3

In Ansys Fluent, the target is



$$y^+ < 5 \qquad U^+ = y^+$$

$$30 < y^+ < 200 \qquad U^+ = \frac{1}{\kappa}\log\left(Ey^+\right)$$

$$\kappa = 0.4187$$

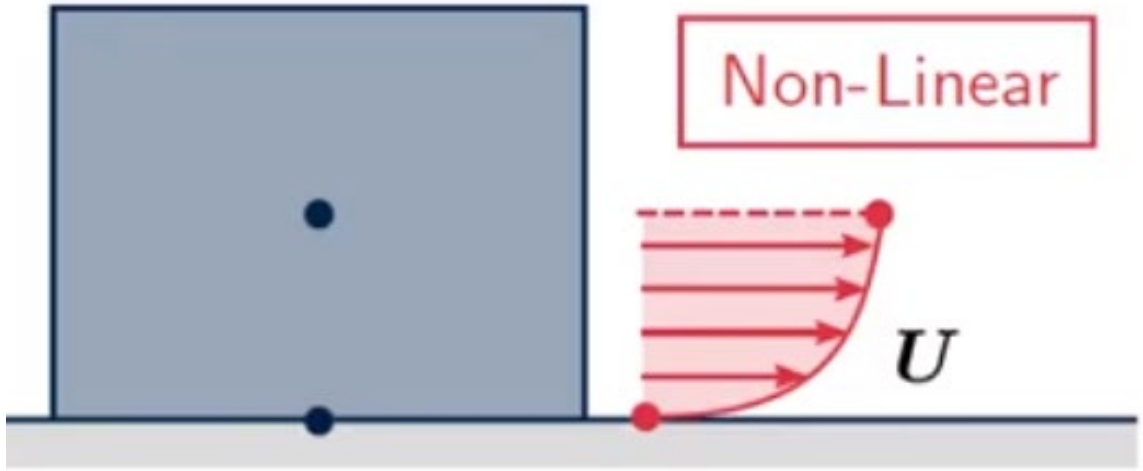
$$E = 9.793$$

The curves intersect at y+ = 11.25

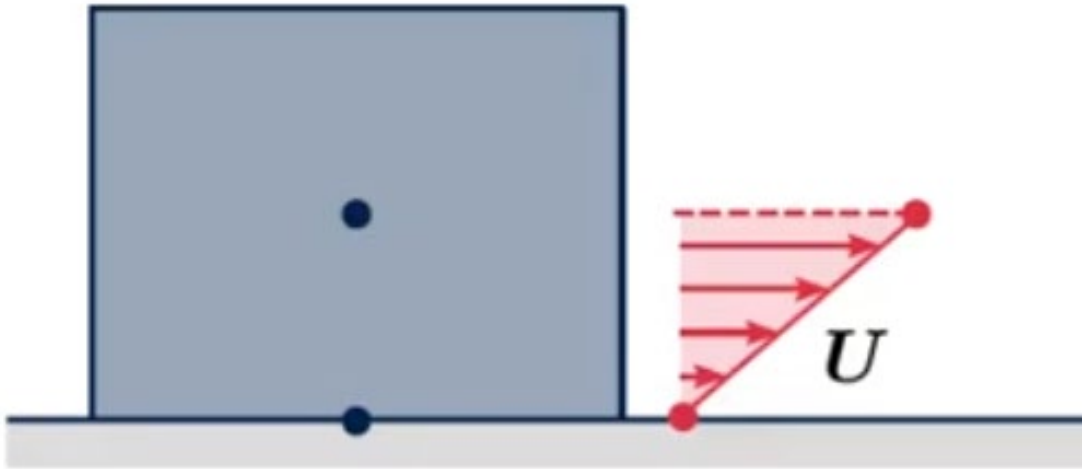
How does y+ affect results?

Wall Shear Stress = Viscosity × Velocity Gradient at Wall

Physically $$\tau_w = \rho \nu \frac{\partial u}{\partial y}\Big|_w$$

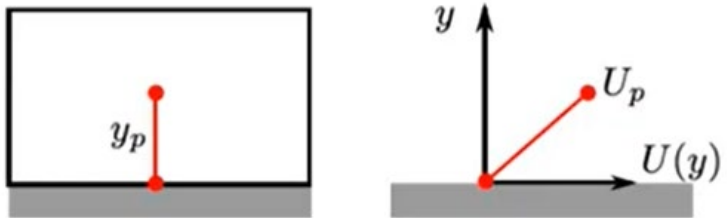


In CFD code $$\tau_w = \rho \boxed{\nu_w} \frac{U_p}{y_p}$$



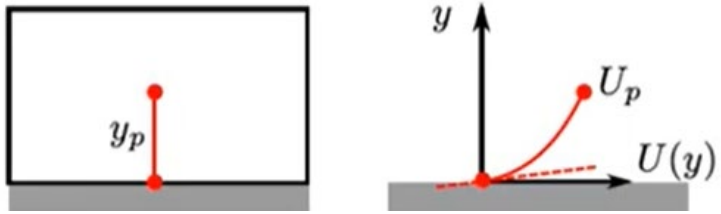
How does y+ affect results?

$$\tau_w = \mu_w \frac{U_p}{y_p} = \rho \nu_w \frac{U_p}{y_p}$$



y+ locates in the linear layer

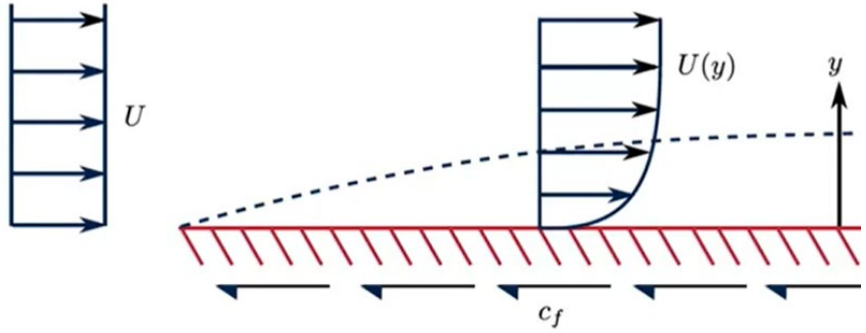
$$\tau_w = \nu_w \frac{\rho \nu_w U_p}{\frac{1}{\kappa}\log\left(Ey^+\right)}$$



y+ locates in the log layer

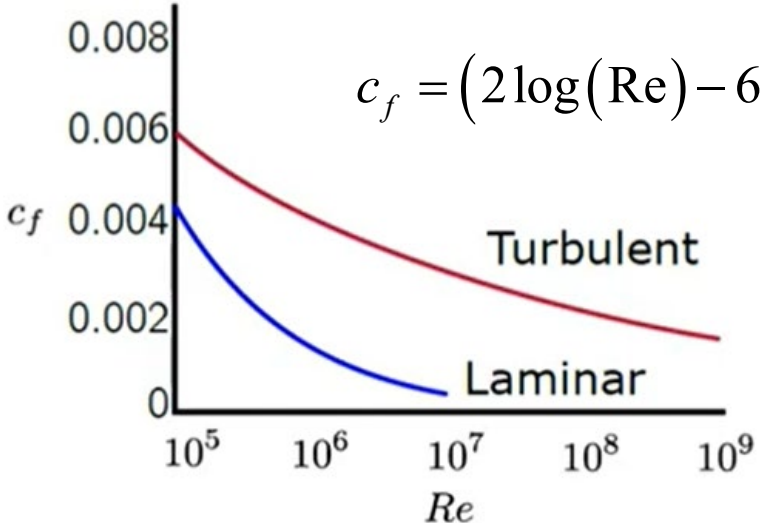
$$\nu_w = \nu + \nu_t \qquad \nu_t = \begin{cases} 0, & y^+ < 11.25 \\ \nu\left(\dfrac{y^+}{\frac{1}{\kappa}\log\left(Ey^+\right)} - 1\right), & y^+ > 11.25 \end{cases}$$

Initial guess of wall-adjacent cell height



- Estimate Re $\quad \mathrm{Re} = \dfrac{\rho U L}{\mu}$

- Calculate the skin friction coefficient



$$c_f = \left(2\log(\mathrm{Re}) - 6.5\right)^{-2.3}$$

- Calculate the wall shear stress

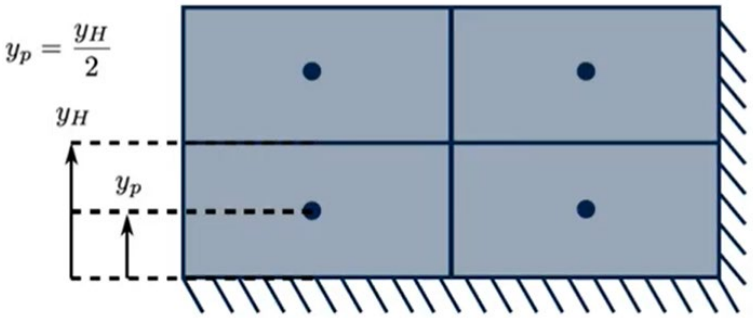
$$\tau_w = \frac{c_f}{2}\rho U^2$$

- Calculate the friction velocity

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

- Here is your initial guess

$$y_p = \frac{y^+ \mu}{\rho u_\tau}$$



Useful y+ calculators

ATTN: These calculators only give you a rough estimation, you still need to iterate to find the best mesh size.

- https://cfdyna.com/CFDHT/Y_Plus.html
- https://www.cfd-online.com/Tools/yplus.php
- http://ichrome.com/blogs/archives/421

Q&A TIME