Linear Algebra Homework #6
Fall 2020

Name: Due: Monday, Dec. 21st, 2020

Instructions:

Please include essential steps in your solution. For most of the problems, answers without essential steps may receive a score of 0.

1. Let U and V be subspaces of W. Use the subspace test to prove the following.
   (a) The set intersection $U \cap V$ is a subspace of W.
   (b) The sum of the spaces, $U + V = \{\vec{u} + \vec{v} | \vec{u} \in U and \vec{v} \in V\}$, is a subspace of W.
   (c) The set union $U \cup V$ is not a subspace of W unless one of U or V is contained in the other.
2. Show that $span \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\} = span \left\{ \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \right\}$.
3. Let $\vec{u} = (2, -1, 1)$, $\vec{v} = (0, 1, 1)$, and $\vec{w} = (2, 1, 3)$. Show that $span \{\vec{u} + \vec{w}, \vec{v} - \vec{w}\} \subset span \{\vec{u}, \vec{v}, \vec{w}\}$ and determine whether or not these spans are actually equal.
4. Determine if the following sets are linearly independent in $V = R^4$.
   a) $[1,\ 1,\ 1,\ 1]^T,\ [1,\ 0,\ 1,\ 0]^T,\ [1,\ 0,\ 1,\ 0]^T$.
   b) $[0,\ 1,\ -1,\ 2]^T,\ [0,\ 1,\ 3,\ 4]^T,\ [0,\ 2,\ 2,\ 6]^T$.
   c) $[1,\ -1,\ 0,\ 1]^T,\ [-2,\ 2,\ 1,\ 1]^T$.
5. Find the coordinate vector of $\vec{v}$ with respect to the following bases:
   (a) $\vec{v} = (0, 1, 2)$, basis $(2, 0, 1), (-1, 1, 0), (0, 1, 1)$ of R^3.
   (b) $\vec{v} = 2 + x^2$, basis $1 + x,\ x + x^2,\ 1 - x$ of P_2.