

## Review – Calculation

1. RT is about to loan his granddaughter Cynthia \$20,000 for 1 year. RT's TVOM, based upon his current investment earnings, is 8 percent. Cynthia's TVOM, based upon earnings on investments, is 12 percent.
  - a. Should they be able to successfully negotiate the terms of this loan?

Yes.

- b. If so, what range of paybacks would be mutually satisfactory? If not, how far off is each person from an agreement?

Interest rate range [8%, 12%];

payback =  $20,000 \times \text{interest rate} = [1,600, 2,400]$

TVOM=minimum attractive rate of return

When an invest yields a rate of return  $\geq$  one's TVOM, the investment is considered a good one.

RT loans money to Cynthia – as long as  $i \geq \text{TVOM}_{\text{RT}} = 8\%$ , it's profitable for her.

Cynthia borrows money from RT, and invests elsewhere – as long as  $i \leq \text{TVOM}_{\text{Cynthia}} = 12\%$ , it's profitable for her.

2. RT is about to loan his granddaughter Cynthia \$20,000 for 1 year. RT's TVOM, based upon his current investment earnings, is 12 percent, and he has no desire to loan money for a lower rate. Cynthia is currently earning 8 percent on her investments, but they are not easily available to her, and she is willing to pay up to \$2,000 interest for the 1-year loan.

a. Should they be able to successfully negotiate the terms of this loan?

No.

b. If so, what range of paybacks would be mutually satisfactory? If not, how many dollars off is each person from reaching an agreement?

$$\text{gap} = 20,000 * 12\% - 2,000 = 400$$

3. If your TVOM is 15 percent and your friend's is 20 percent, can the two of you work out mutually satisfactory terms for a 1-year, \$3,000 loan? Assume the lender has the money available and neither of you wants to go outside their acceptable TVOM range.

Clarify who is lending and the acceptable range of money paid back.

I am lending money to my friend.

Interest rate range [15%, 20%];

payback =  $3,000 \times \text{interest rate}$  = [450, 600]

## Conclusion:

When two parties A and B have different TVOMs, in order to negotiate a mutually profitable loan:

- Who should be the lender?
  - The one with lower TVOM
- Who should be the borrower?
  - The one with higher TVOM
- What is the acceptable range of interest rate (IR)?
  - The range between the two TVOMs
- Summary, The party with lower TVOM can lend money to the party with higher TVOM, and the acceptable IR range is  $[TVOM_{low}, TVOM_{high}]$

- 4.
- Barbara and Fred have decided to put in an automatic sprinkler system at their cabin. They have requested bids, and the lowest price received is \$5,500 from Water Systems Inc (WSI).
- They decide to do the job themselves and obtain a set of materials (plastic pipe, nozzles, fittings, and regulators) from an all-sales-are-final discount house for \$1,100. They begin the installation and rent a trencher at \$80 per day.
- Unfortunately, they quickly hit sandstone in many places of the yard and require a jackhammer and air compressor at another \$80 per day. They keep all the rental equipment for 5 days.
- By this time, Fred has hurt his knee, and Barbara is sick of the project. They again contact WSI, who tells them that they can use only some of the materials, reducing the cost by \$500, and only some of the trenching, reducing the cost by another \$500, bringing the total to \$4,500, finished and ready to go.

a. How much have they already spent?

$$1100 + (80 + 80) * 5 = 1900$$

b. How much will they have spent when the project is over if they accept the new offer from WSI?

$$1900 + 4500 = 6400$$

c. A different contractor, Sprinkler Systems (SS), who heard of their situation approaches Barbara and Fred and recommends a design for a sprinkler system that would require a different set of materials and a new routing of the trenches. They offer to (1) backfill all existing trenches, (2) cut new trenches with their rock-impervious Ditch Witch, and (3) install the system. Their charge is \$6,000 for a finished-and-ready-to-go project, and they correctly note that this is less than the total that will have been spent if they go with WSI. Should they go with WSI or SS? Why?

**WSI—Future cost is less ( $4500 < 6000$ ).** They only need to spend 4500 on WSI to get it done, while have to spend 6000 on SS to get it done.

\$1900 in the question a is the **suck cost**, which has already occurred no matter they choose WSI or SS.

You SHOULDN'T compare the total cost of WSI (6400) with SS (6000), and conclude that you will choose SS.

d. Of the ten principles, which one(s) is well illustrated by this problem?

2. Make investments that are economically justified

10. Past costs are irrelevant, unless they impact future costs

e. Of the systematic economic analysis technique's seven steps, which one(s) is well illustrated by this problem?

1. Identify the investment alternatives

4. Estimate the cash flows

5. Compare the alternatives

7. Select the preferred investment

**B0684**

# **Economic Engineering Analysis**

## **Time Value of Money**





# Learning Objective

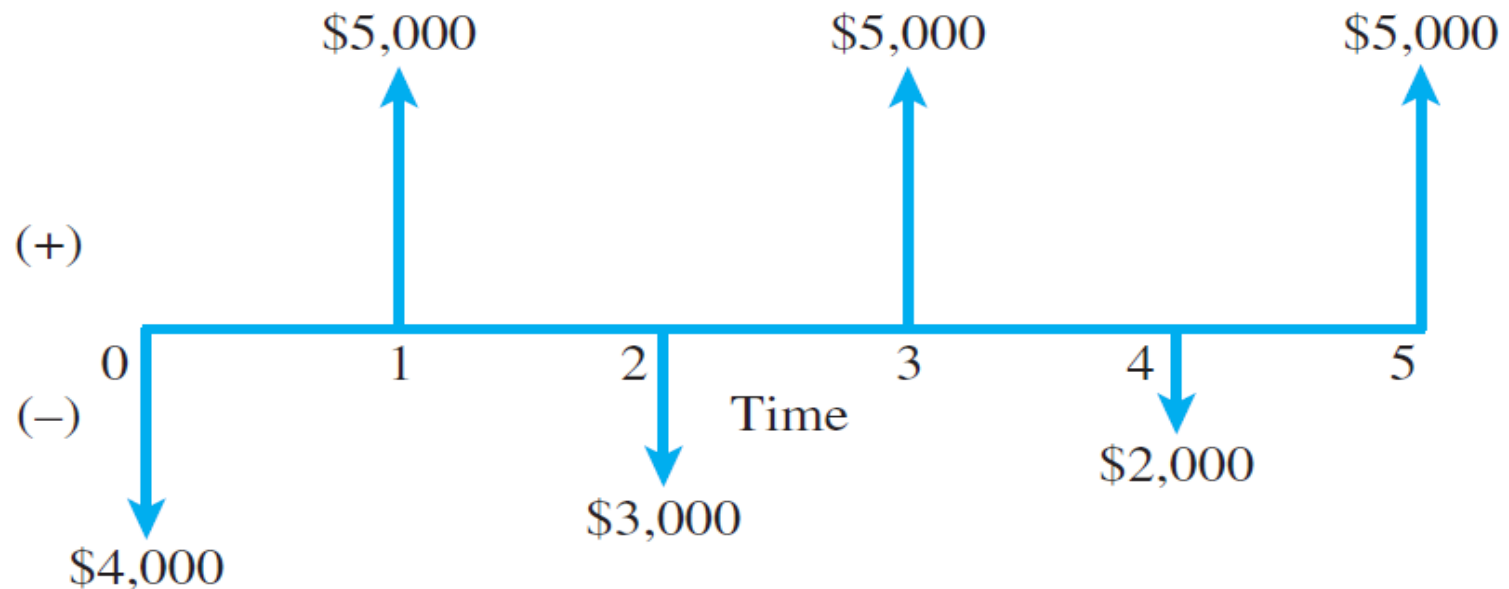
- Cash Flow Diagram (CFD)
- Single cash flow calculation
  - Future worth
  - Present worth
- Multiple cash flow calculation
  - Irregular cash flow
  - Uniform series
  - Gradient series
  - Geometric series
- Compounding frequency
  - Period interest rate
  - Effective annual interest rate

This chapter is **very important!!**

It serves as a foundation for the remainder of the module.

# CASH FLOW DIAGRAMS

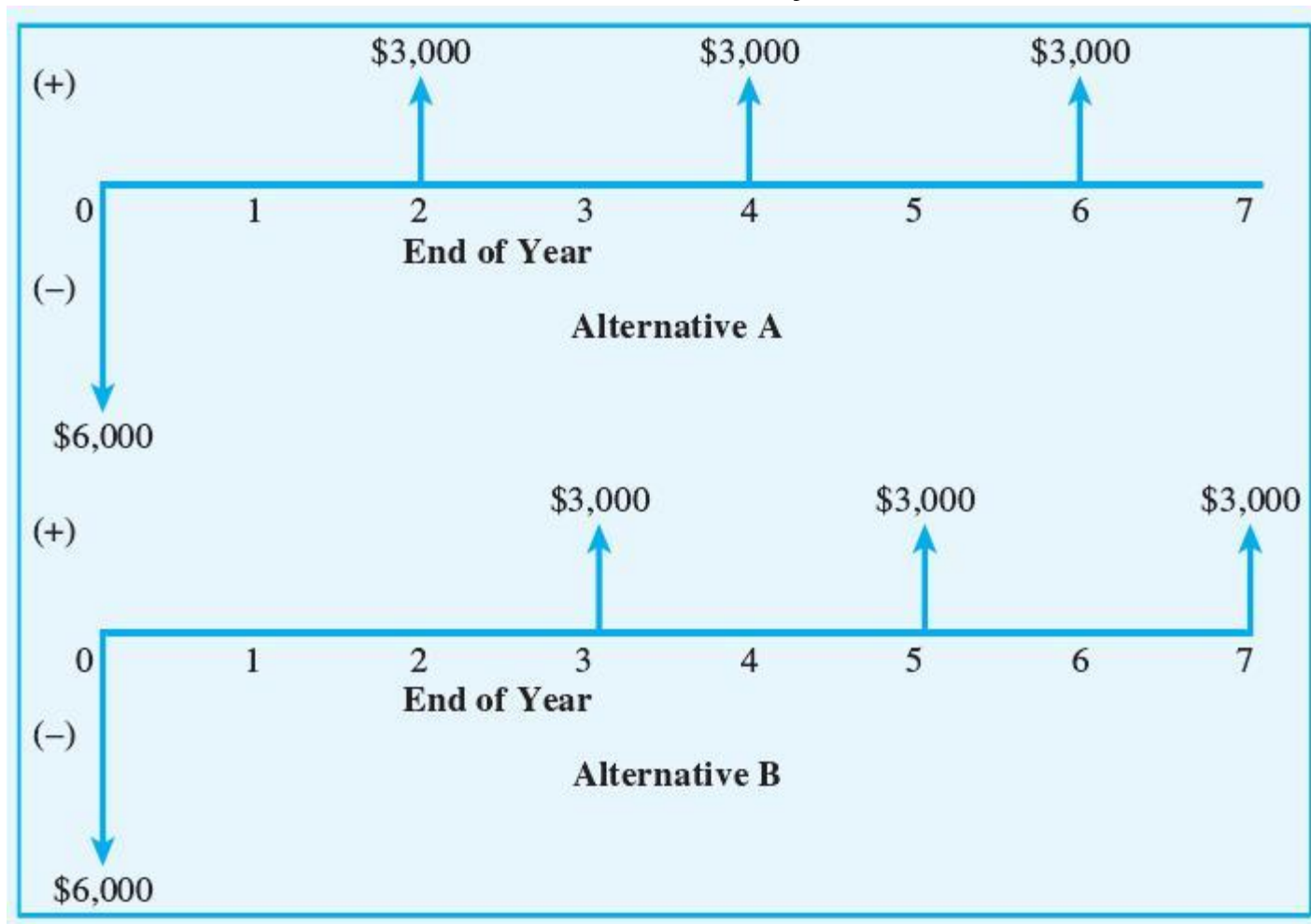
- A diagram depicting the **magnitude** and **timing** of cash **flowing in** and **out** of the investment alternative.



**FIGURE 2.1** A Cash Flow Diagram (CFD)

- A horizontal line as a time scale
- Vertical arrows indicating cash flows
  - An upward arrow indicates a cash inflow or positive-valued cash flow (receipts)
  - A downward arrow indicates a cash outflow or negative-valued cash flow (expenditures)
- The lengths of the arrows suggest the magnitudes of cash flows (but not precisely)

Which investment alternative will you choose?



Recall TVOM, when receiving a given sum of money, we prefer to receive it **sooner!**

**Alternative A is preferred** to Alternative B

- End-of-period cash flows, end-of-year cash flows, end-of-period compounding, are assumed unless otherwise noted
- The end of period  $t$  is the beginning of period  $t+1$
- The role of CFDs:
  - Clearly communicate cash flows (than words do)
  - Identify cash flow patterns (e.g., a uniform/gradient/geometric series)

# SINGLE CASH FLOWS

The simplest scenario: there is only one cash flow in the planning horizon.

Compound interest is used in almost all business and lending situations:

Interest should be charged (or earned) against both the principal and accumulated interest to date.

# Future Worth

$$F_n = F_{n-1}(1+i)$$

$F_0 = P$ , where  $P$  is the **present value** of a single sum of money;  $F_n$  is the accumulated value of  $P$  over  $n$  years;  $i$  – interest rate

$$I_n = \sum_{t=1}^n iF_{t-1}$$

$I_n$  – the accumulated (total) interest over  $n$  years



## *Illustration*

Suppose you loan \$10,000 for 1 year to an individual who agrees to pay you interest at a compound rate of 10 percent/year. At the end of 1 year, the individual asks to extend the loan period an additional year. The borrower repeats the process several more times. Five years after loaning the person the \$10,000, how much would the individual owe you?

Year	Unpaid Balance at the Beginning of the Year	Annual Interest	Payment	Unpaid Balance at the End of the Year
1	\$10,000.00	\$1,000.00	\$0.00	\$11,000.00
2	\$11,000.00	\$1,100.00	\$0.00	\$12,100.00
3	\$12,100.00	\$1,210.00	\$0.00	\$13,310.00
4	\$13,310.00	\$1,331.00	\$0.00	\$14,641.00
5	\$14,641.00	\$1,464.10	\$16,105.10	\$0.00

$$F=P(1+i)^n$$

P = present worth.

F = future worth. F occurs n periods after P.

i = the interest rate, expressed as a decimal or percentage.

n = the number of interest periods.

- $(1+i)^n$  is referred to as *the single sum, future worth factor*.
- It is denoted  $(F|P\ i\%,\ n)$ , and reads “**the F, given P factor at i% for n periods**” Thus,

$$F=P(F|P\ i\%,\ n)$$