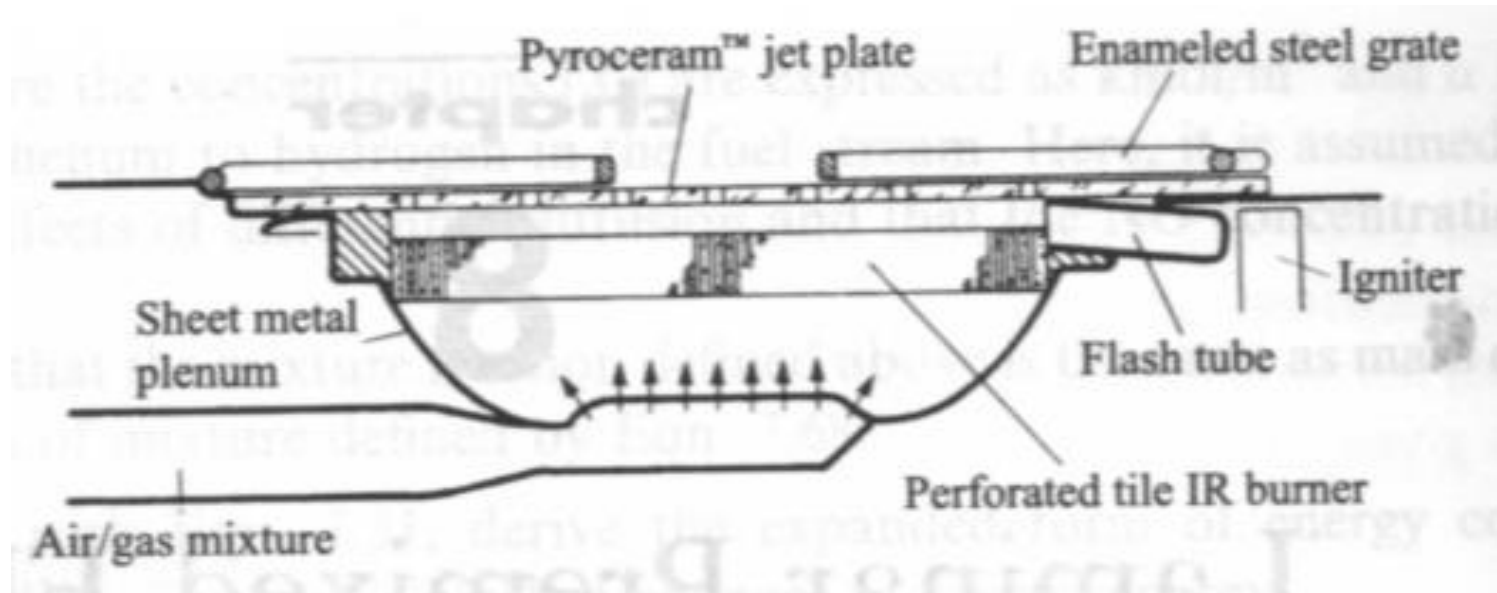


# Laminar premixed combustion

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Dr Guohong Tian  
[g.tian@surrey.ac.uk](mailto:g.tian@surrey.ac.uk)

- Laminar premixed combustion have important applications in many devices and processes.
  - Cooktop burner
  - Glass production
- Understanding laminar flame is a necessary prerequisite to the study of turbulent flames



# Physical description

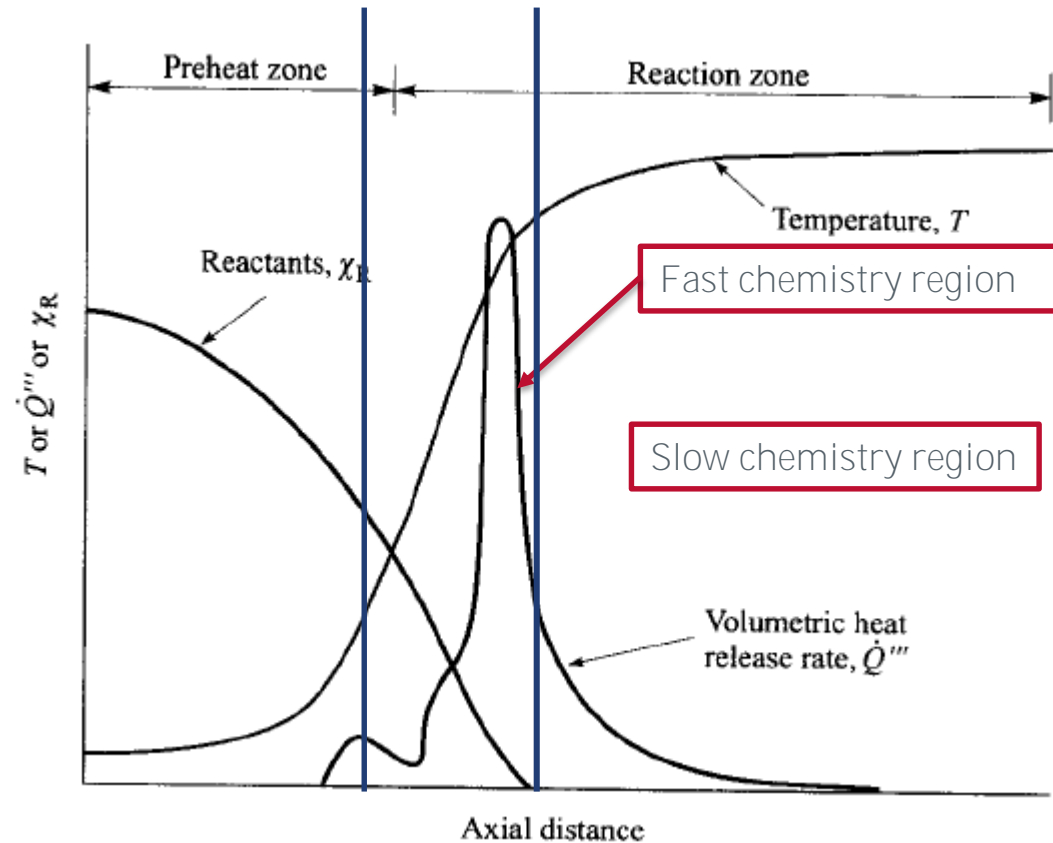
## Definition

- A **flame** is a self-sustaining propagation of a localised combustion zone at subsonic velocity
  - Localised: the flame occupies only a small portion of the combustible mixture at any one time
  - Subsonic: a discrete combustion wave that travels subsonically is termed a **deflagration**.
  - It is possible for combustion waves to propagate at supersonic velocities, called **detonation**.
- Both deflagration and detonation are different in fundamental propagation mechanisms.

# Physical description

## Principal characteristics

- The temperature profile through a flame is perhaps its most important characteristic. *how to define or distance*
- Flame would experience the *the boundary layer* unburned mixture approaching at the **flame speed,  $S_L$** 
  - Flame is equivalent to a flat flame stabilised on a burner, the reactants enter the flame with a velocity equal to the flame speed  $S_L$
  - Assume the flame is 1D and unburned gas enters the flame in a direction normal to the flame sheet



# Physical description

## Principal characteristics

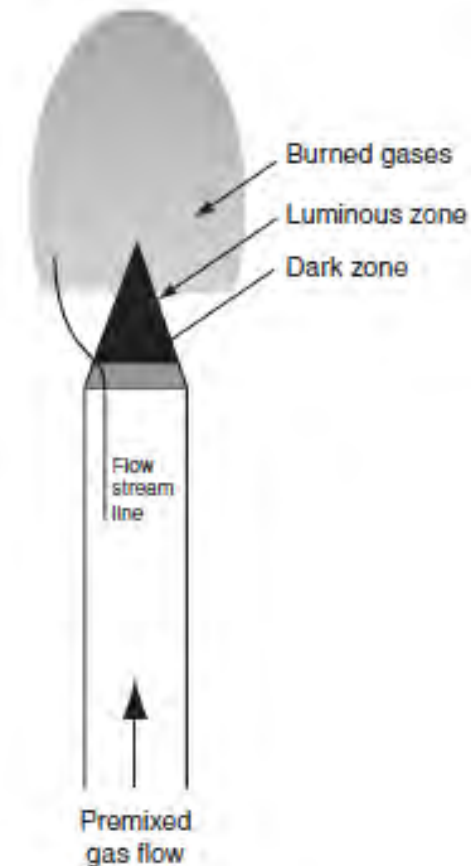
- As the flame creates hot products, the product density is less than reactant density, continuity requires the burned gas velocity is greater than the unburned gas velocity.
- Continuity:  
$$\rho_u S_L A \equiv \overset{\text{unburned}}{\rho_u v_u A} = \overset{\text{burned}}{\rho_b v_b A}$$

$\rho_u > \rho_b$   
 $v_u < v_b$
- Flame can be divided into two zones: the **preheat zone**, where little heat is released; and the **reaction zone**, where the bulk of the chemical energy is released.
  - Reaction zone can be further divided to a thin region of fast chemistry and a much wider region of slow chemistry.
  - Destruction of fuel molecule and intermediate species creation in fast-chemistry region. In this region, it is dominated by bimolecular reactions, and the region is thin (<1mm at 1 atm). Temperature gradient and concentration gradient are very high. Gradient provide driving forces do sustain flame: diffusion of heat and radical species from the reaction zone to the preheat zone.

# Physical description

## Principal characteristics

- In the slow-chemistry region, it is dominated by three body recombination reactions, which are much slower. Typical thickness is several mms at 1 atm.
- Hydrocarbon flames with excess of air
  - Fast-reaction zone appears blue, which is from excited CH radicals in the high-temperature zone.
  - Blue-green colour with air is decreased to less than stoichiometric proportions, resulted from excited  $C_2$ .
  - Bright yellow (nearly white) to dull orange colour when the flame is richer still, as a result of soot formation.

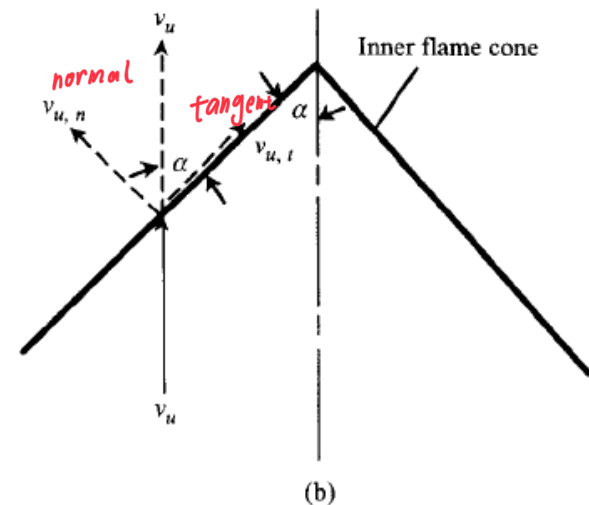
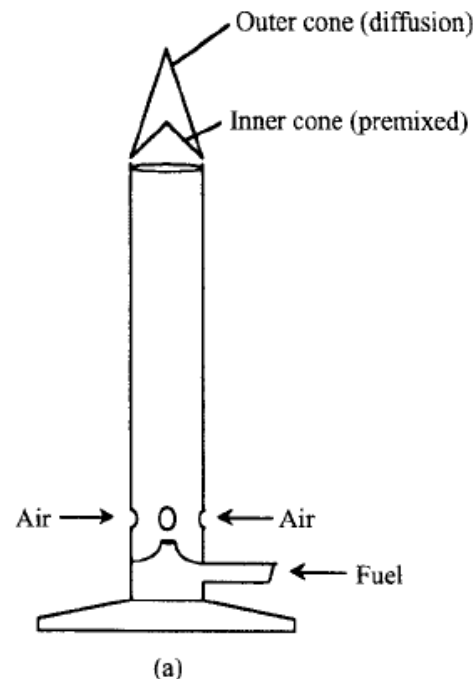


# Physical description

## Typical laboratory flames

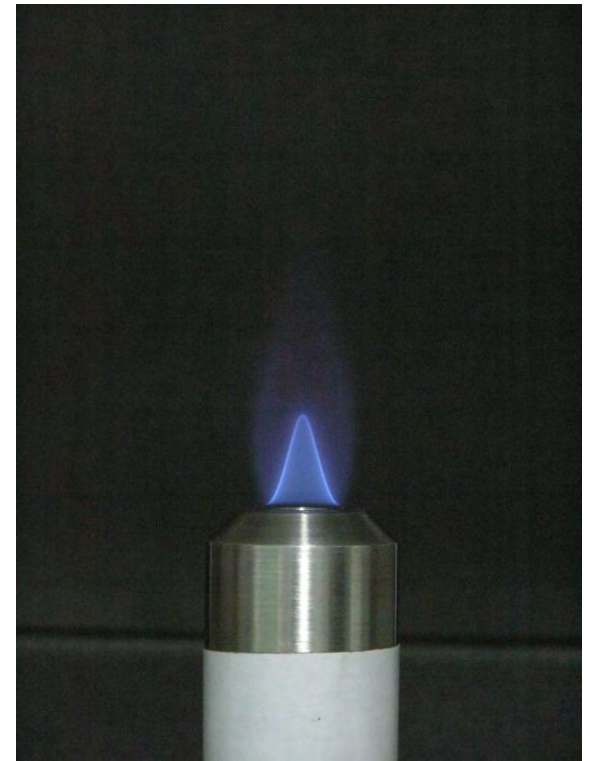
- Bunsen burner
  - A jet of fuel at the base induces a flow of air through the variable area port, and the air and fuel mix as they flow up through the tube
  - Typical Bunsen burner flame is a dual flame: a fuel-rich premixed inner flame surrounded by a diffusion flame. Outer flame results when CO and H products from the rich inner flame.
  - Shape of flame is determined by the combined effects of the velocity profile and heat losses to the tube wall.
  - The flame speed equal the speed of the normal component of unburned gas at each location

$$S_L = v_u \sin \alpha$$



# Typical laboratory flames

## Bunsen burner

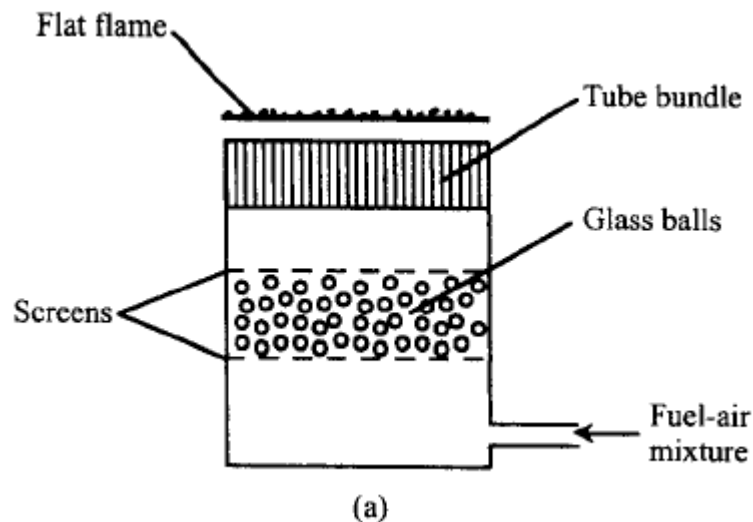




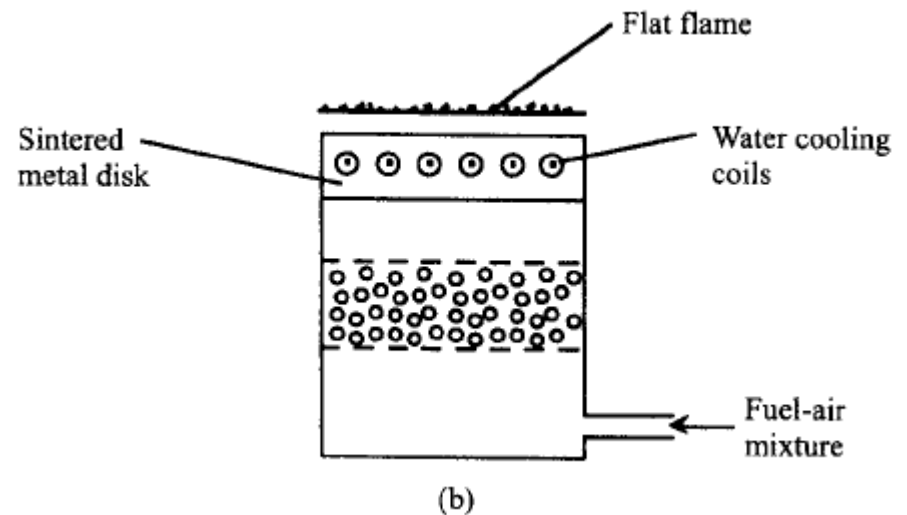
# Typical laboratory flames

## One-dimensional flat flames

- In the adiabatic burner, a flame is stabilised over a bundle of small tubes through which the fuel-air mixture passes laminarily. The flame is stable in a narrow range of conditions.
- The non-adiabatic burner utilises a water cooled face that allows heat to be extracted from the flame, which decreases the flame speed, allowing flames to be stabilised over a relatively wide range of flow conditions.



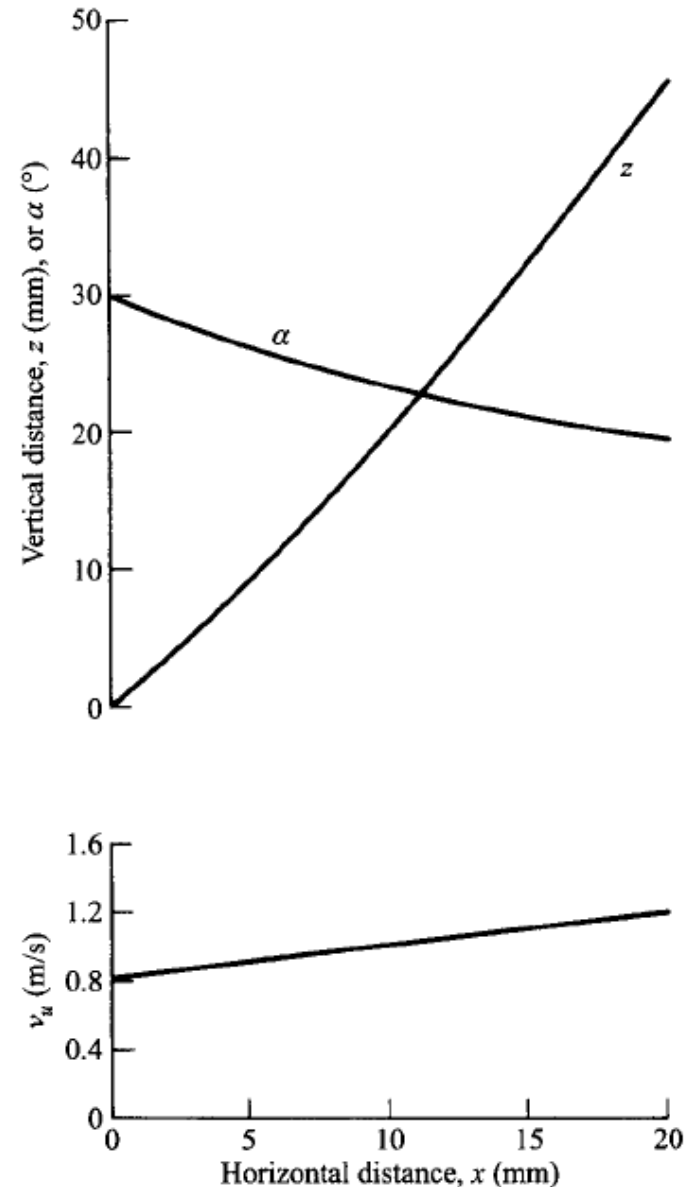
Adiabatic burner



Non-adiabatic burner

## Example

A premixed laminar flame is stabilised in a one-dimensional gas flow where the vertical velocity of the unburned mixture,  $v_u$ , varies linearly with the horizontal coordinate,  $x$ , as shown in the figure. Determine the flame shape and the distribution of the local angle of the flame surface from vertical. Assume the flame speed is independent of position and equal to  $0.4 \text{ m/s}$ , a nominal value for a stoichiometric methane-air flame.



From the figure, we see that the local angle,  $\alpha$ , which the flame sheet makes with a vertical plane is

$$\alpha = \sin^{-1}(S_L / v_u)$$

From the figure last page,

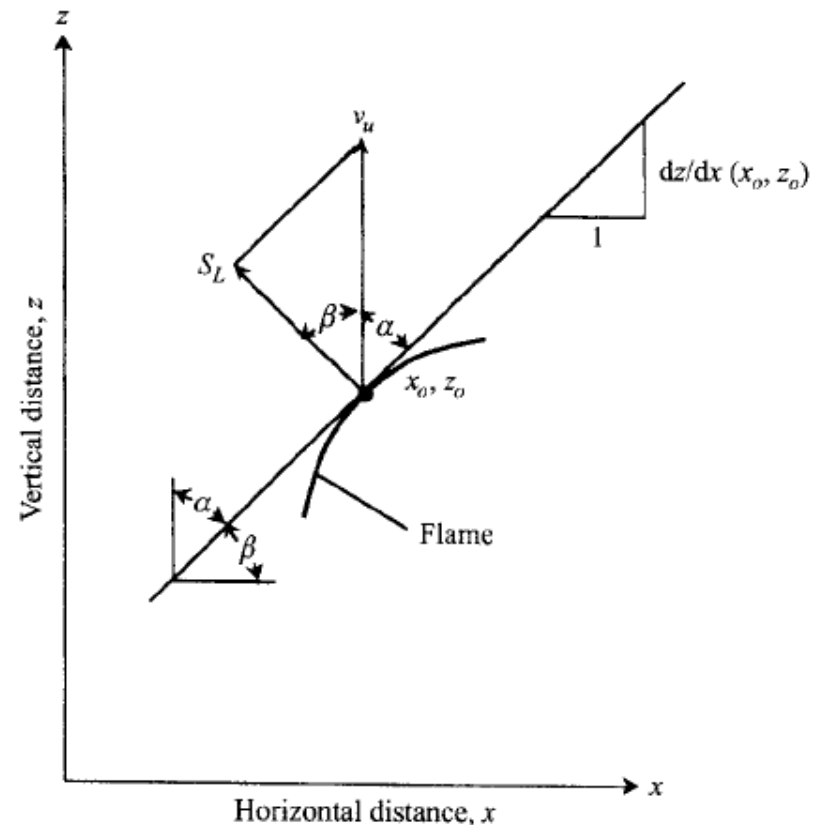
$$v_u = 800 + \frac{1200-800}{20} x \text{ (mm)}$$

So

$$\alpha = \sin^{-1}\left(\frac{400}{800+20x}\right)$$

It has values ranging from  $30^\circ$  at  $x=0$  to  $19.5^\circ$  at  $x=20$  mm, which is shown in the top part of the figure last slide.

To calculate the flame position, we first obtain an expression for the local slope of the flame sheet  $\frac{dz}{dx}$  in the x-z plane, and integrate this expression with respect to x to find  $z(x)$



$$\frac{dz}{dx} = \tan \beta = \left( \frac{v_u^2(x) - S_L^2}{S_L^2} \right)^{1/2}$$

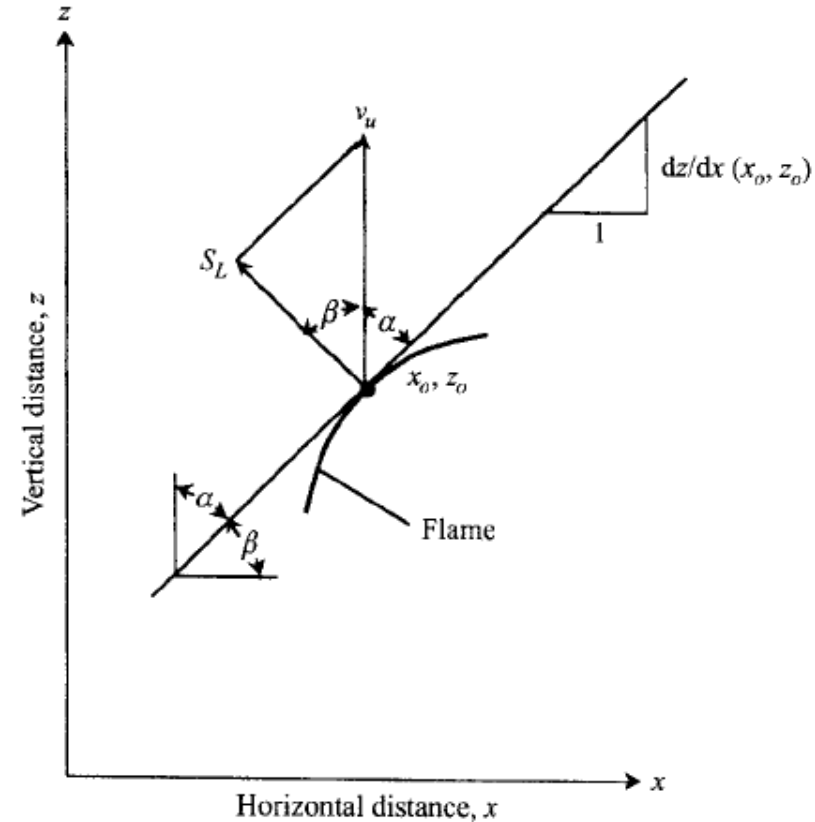
Which, for  $v_u \equiv A + Bx$ , becomes

$$\frac{dz}{dx} = \left[ \left( \frac{A}{S_L} + \frac{Bx}{S_L} \right)^2 - 1 \right]^{1/2}$$

Integrate the above with  $\frac{A}{S_L} = 2$  and  $\frac{B}{S_L} = 0.05$  yields

$$\begin{aligned} z(x) &= \int_0^x \frac{dz}{dx} dx = \int_0^x [(2 + 0.05x)^2 - 1]^{1/2} dx \\ &= (x^2 + 80x + 1200)^{\frac{1}{2}} \left( \frac{x}{40} + 1 \right) \\ &\quad - \ln \left[ (x^2 + 80x + 1200)^{\frac{1}{2}} + (x + 40) \right] - 20\sqrt{3} \\ &\quad + 10 \ln(20\sqrt{3} + 40) \end{aligned}$$

The flame position  $z(x)$  is plotted in the figure two slides ago.



# Simplified analysis

- Based on Spalding's theory.
- Objective is to find a simple analytical expression for the laminar flame speed.
- Analysis couples principles include
  - Heat transfer
  - Mass transfer
  - Chemical kinetics
  - Thermodynamics
- Assumptions
  1. One dimensional , constant-area, steady flow
  2. Kinetic and potential energies, viscous shear work, thermal radiation neglected.
  3. Small pressure difference across the flame neglected constant pressure
  4. Diffusion of heat and mass governed by Fourier's and Fick's laws respectively; binary diffusion assumed.

(tbc)

(continue...)

5. The Lewis number,  $Le$ , which expresses the ratio of thermal diffusivity to mass diffusivity, is unity ( $=1$ ). This has the result that  $\frac{k}{c_p} = \rho D$ , greatly simplifies the energy equation

$$Le \equiv \frac{\alpha}{D} = \frac{k}{\rho c_p D}$$

6. Mixture specific heat depends neither on temperature nor on the mixture composition. (individual species specific heats are all equal and constant)
7. Fuel and oxidiser form products in a single step exothermic reaction.
8. Oxidiser is present in stoichiometric or excess proportions; thus the fuel is completely consumed at the flame.

*why for control volume, mass is conserved*

## Mass conservation

$$\frac{d(\rho v_x)}{dx} = 0$$

Or

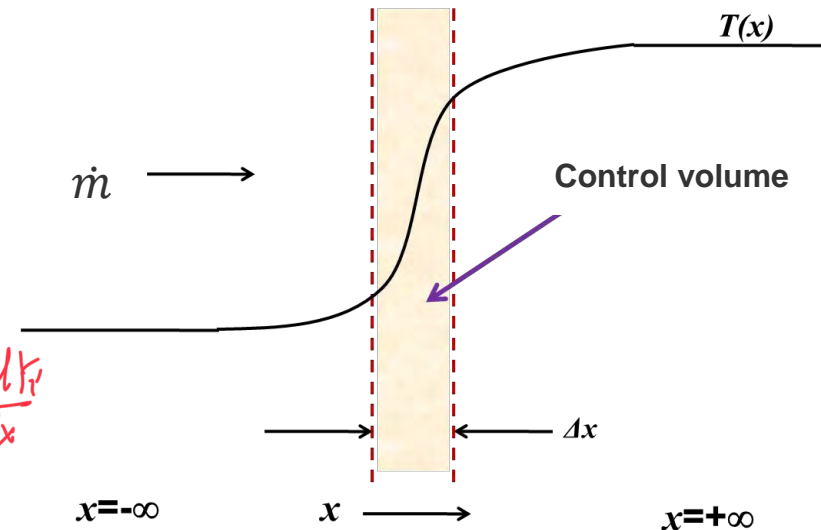
$$\dot{m}'' = \rho v_x = \text{constant}$$

## Species conservation

$$\frac{d\dot{m}_i''}{dx} = \dot{m}_i''' \quad \text{with } \dot{m}_i'' = \dot{m}'' Y_i - \rho D \frac{dY_i}{dx}$$

With the application of Fick's law

$$\frac{d \left[ \dot{m}'' Y_i - \rho D \frac{dY_i}{dx} \right]}{dx} = \dot{m}_i'''$$



Where  $\dot{m}_i'''$  is the mass production rate of species i per unit volume ( $\text{kg} \cdot \text{s}^{-1} \cdot \text{m}^{-3}$ )

If we have the overall reaction as



Thus

$$\dot{m}_F''' = \frac{1}{\nu} \dot{m}_{Ox}''' = -\frac{1}{\nu + 1} \dot{m}_{Pr}'''$$

# Conservation Laws

Thus the species conservation equation becomes for each species:

*Fuel*

$$\dot{m}'' \frac{dY_F}{dx} - \frac{d \left( \rho D \frac{dY_F}{dx} \right)}{dx} = \dot{m}_F'''$$

*Oxidiser*

$$\dot{m}'' \frac{dY_{Ox}}{dx} - \frac{d \left( \rho D \frac{dY_{Ox}}{dx} \right)}{dx} = \dot{m}_{Ox}'''$$

*Products*

$$\dot{m}'' \frac{dY_{Pr}}{dx} - \frac{d \left( \rho D \frac{dY_{Pr}}{dx} \right)}{dx} = \dot{m}_{Pr}'''$$

**Energy conservation**

$$\dot{m}'' c_p \frac{dT}{dx} - \frac{d}{dx} \left[ (\rho D c_p) \frac{dT}{dx} \right] = - \sum h_{f,i}^0 \dot{m}_i'''$$

change of T      heat diffusion      enthalpy change  
heat release

Or

$$- \sum h_{f,i}^0 \dot{m}_i''' = - \dot{m}_F''' \Delta h_c$$

Where  $\Delta h_c$  is the heat of combustion of the fuel,  $\Delta h_c \equiv h_{f,F}^0 + \nu h_{f,Ox}^0 - (\nu - 1) h_{f,Pr}^0$

Rearrange, replacing  $\rho D c_p$  with  $k$  by Lewis number = 1 approximation

$$\dot{m}'' \frac{dT}{dx} - \frac{1}{c_p} \frac{d \left( k \frac{dT}{dx} \right)}{dx} = - \frac{\dot{m}_F''' \Delta h_c}{c_p}$$



Objective: find laminar flame speed, related simply to the mass flux,  $\dot{m}''$ , by

$$\dot{m}'' = \rho_u S_L$$

- Boundary conditions

- Far upstream of the flame

$$T(x \rightarrow -\infty) = T_u$$

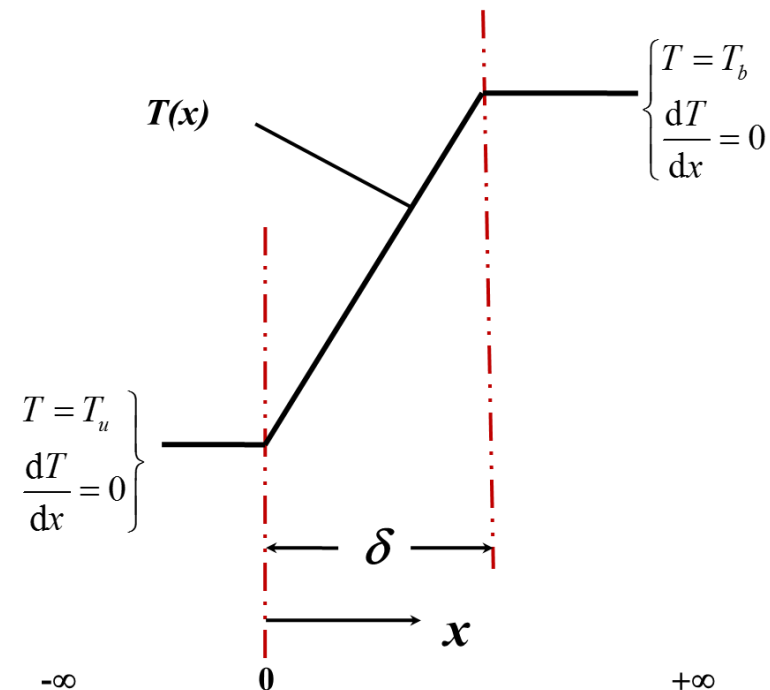
$$\frac{dT}{dx}(x \rightarrow -\infty) = 0$$

- Far downstream of the flame

$$T(x \rightarrow +\infty) = T_b$$

$$\frac{dT}{dx}(x \rightarrow +\infty) = 0$$

- For simplicity, we assume a simple linear temperature profile from  $T_u$  to  $T_b$  over the small distance  $\delta$ , which is defined as the flame thickness



# Solution

Objective: find laminar flame speed, related simply to the mass flux,  $\dot{m}''$ , by

$$\dot{m}'' = \rho_u S_L$$

- Integrating the above second-order ordinary differential equation, over  $x$ , applying the boundary conditions,

$$\dot{m}''[T]_{T=T_u}^{T=T_b} - \frac{k}{c_p} \left[ \frac{dT}{dx} \right]_{\frac{dT}{dx}=0}^{\frac{dT}{dx}=0} = -\frac{\Delta h_c}{c_p} \int_{-\infty}^{\infty} \dot{m}_F''' dx$$

By evaluating the limits, simplifies to

$$\dot{m}''(T_b - T_u) = -\frac{\Delta h_c}{c_p} \int_{-\infty}^{\infty} \dot{m}_F''' dx$$

If we change the limits on the reaction rate integral (right hand side) from space to temperature, since  $\dot{m}_F'''$  is only non-zero over the region  $\delta$

$$\frac{dT}{dx} = \frac{T_b - T_u}{\delta} \quad \text{or} \quad dx = \frac{\delta}{T_b - T_u} dT$$

The last equation becomes

$$\dot{m}''(T_b - T_u) = -\frac{\Delta h_c}{c_p} \frac{\delta}{T_b - T_u} \int_{-\infty}^{\infty} \dot{m}_F''' dT$$

With recognition of the definition of the average reaction rate

$$\overline{\dot{m}_F'''} = \frac{1}{T_b - T_u} \int_{-\infty}^{\infty} \dot{m}_F''' dT \quad \text{mean value theorem for definite integrals}$$

We obtain the simple result that

$$\dot{m}''(T_b - T_u) = -\frac{\Delta h_c}{c_p} \delta \overline{\dot{m}_F'''} \quad \text{mean value theorem for definite integrals}$$

The above equation is a simple equation involving the two unknowns  $\dot{m}''$  and  $\delta$ . Therefore we need another equation to complete the solution. Now we repeat the procedure before but integrating from  $x = -\infty$  to  $x = \delta/2$ . because the reaction zone of a flame is in the high temperature region, we assume that  $\dot{m}_F'''$  is zero in the interval  $-\infty < x \leq \delta/2$ . At  $x = \delta/2$ , we have  $T = \frac{T_b + T_u}{2}$ , and

$$\frac{dT}{dx} = \frac{(T_b - T_u)}{\delta}$$

So solving previous equations with modified limits,

$$\dot{m}'' \frac{dT}{dx} - \frac{1}{c_p} \frac{d(k \frac{dT}{dx})}{dx} = -\frac{\dot{m}_F''' \Delta h_c}{c_p} \quad \frac{\dot{m}'' \delta}{2} - \frac{k}{c_p} = 0$$

And we yield

$$\dot{m}'' = \left[ 2 \frac{k}{c_p^2 (T_b - T_u)} \bar{\dot{m}}_F''' \right]^{\frac{1}{2}}$$

assumption:  
 $\dot{m}_F''' = 0$

And

$$\delta = \frac{2k}{c_p \dot{m}''}$$

Applying the definition of flame speed,  $S_L \equiv \dot{m}'' / \rho_u$ , and thermal diffusivity,  $\alpha \equiv k / \rho_u c_p$ , and recognising that  $\Delta h_c = (\nu + 1) c_p (T_b - T_u)$ , we have the final results:

product

$$S_L = \left[ -2\alpha(\nu + 1) \frac{\bar{\dot{m}}_F'''}{\rho_u} \right]^{\frac{1}{2}}$$

$$\delta = \left[ -\frac{2\rho_u \alpha}{(\nu + 1) \bar{\dot{m}}_F'''} \right]^{\frac{1}{2}}$$

Or

$$\delta = 2\alpha / S_L$$

Estimate the laminar flame speed of a stoichiometric propane-air ( $C_3H_8$ ) mixture using the simplified-theory result. Use the global one-step reaction mechanism to estimate the mean reaction rate.

$$\frac{d[C_xH_y]}{dt} = -A \exp\left(-\frac{E_a}{R_0T}\right) [C_xH_y]^m [O_2]^n$$

Fuel	Pre-exponential Factor, $A^a$	Activation Temperature, $E_a/R_u$ (K)	$m$	$n$
CH <sub>4</sub>	$1.3 \cdot 10^8$	24,358 <sup>b</sup>	-0.3	1.3
CH <sub>4</sub>	$8.3 \cdot 10^5$	15,098 <sup>c</sup>	-0.3	1.3
C <sub>2</sub> H <sub>6</sub>	$1.1 \cdot 10^{12}$	15,098	0.1	1.65
C <sub>3</sub> H <sub>8</sub>	$8.6 \cdot 10^{11}$	15,098	0.1	1.65
C <sub>4</sub> H <sub>10</sub>	$7.4 \cdot 10^{11}$	15,098	0.15	1.6
C <sub>5</sub> H <sub>12</sub>	$6.4 \cdot 10^{11}$	15,098	0.25	1.5
C <sub>6</sub> H <sub>14</sub>	$5.7 \cdot 10^{11}$	15,098	0.25	1.5
C <sub>7</sub> H <sub>16</sub>	$5.1 \cdot 10^{11}$	15,098	0.25	1.5

To evaluate the laminar flame speed, we use

$$S_L = \left[ -2\alpha(\nu + 1) \frac{\bar{m}_F'''}{\rho_u} \right]^{\frac{1}{2}}$$

Mainly to evaluate  $\alpha$  and  $\bar{m}_F'''$ .

If we assume the  $T_b = T_a = 2260 \text{ K}$  (adiabatic flame temperature from chap 3), and  $T_u = 300 \text{ K}$ , the mean temperature of the reaction is

$$\bar{T} = \frac{1}{2} \left( \frac{1}{2} (T_b + T_u) + T_b \right) = 1770 \text{ K}$$

Where we also assume in the second half of flame thickness we have reaction. Assume no fuel or oxygen left in burned gas, mean concentration is

$$\bar{Y}_F = \frac{1}{2} (Y_{F,u} + 0) = \frac{0.061015 \cdot \frac{1}{6+1}}{2} = 0.0301$$

And

$$\bar{Y}_{O_2} = \frac{1}{2} \left( \frac{2 \times 0.21}{28} + 0 \right) = 0.1095$$

Where the A/F of a stoichiometric propane-air mixture is 15.625 ( $= \nu$ ), so and the mass fraction of oxygen is 0.233

# Solution

The reaction rate, given by

$$\dot{\omega}_F \equiv \frac{d[C_3H_8]}{dt} = -k_G [C_3H_8]^{0.1} [O_2]^{1.65}$$

$8.6 \times 10^{11} \left( \frac{g \text{ mol}}{m^3} \right)^{0.75}$   
 $\frac{1}{5} = 4.836 \times 10^9 \left( \frac{kmol}{m^3} \right)^{-0.75}$

With  $k_G = 4.836 \cdot 10^9 \exp\left(-\frac{15098}{T}\right) [=] \left(\frac{kmol}{m^3}\right)^{-0.75} \frac{1}{s}$  (given)

Can be transformed to

$$\bar{\dot{\omega}}_F = -k_G(\bar{T}) \bar{\rho}^{1.75} \left[ \frac{\bar{Y}_F}{\bar{M}_F} \right]^{0.1} \left[ \frac{\bar{Y}_{O_2}}{\bar{M}_{O_2}} \right]^{1.65}$$

Can we use our selected mean values

$$k_G = 4.836 \cdot 10^9 \exp\left(-\frac{15098}{1770}\right) = 9.55 \cdot 10^5 \left(\frac{kmol}{m^3}\right)^{-0.75} \frac{1}{s}$$

$$\bar{\rho} = \frac{p}{\left(\frac{R_o}{\bar{M}}\right) \bar{T}} = \frac{101325}{\frac{8315}{29} 1770} = 0.1997 \text{ kg/m}^3$$

$$\dot{\omega}_F = -9.55 \cdot 10^5 (0.1997)^{1.75} \left(\frac{0.0301}{44}\right)^{0.1} \left(\frac{0.1095}{32}\right)^{1.65} = -2.439 \frac{kmol}{s \text{ m}^3}$$

$$\bar{m}_F''' = \bar{\omega}_F M_F = -2.439(44) = -107.3 \frac{kg}{s m^3}$$

The thermal diffusivity is defined as

$$\alpha = \frac{k(\bar{T})}{\rho_u c_p(\bar{T})}$$

Now we use the mean temperature across the whole flame thickness since conduction occurs over this interval,

$$\bar{T} = \frac{1}{2} (T_b + T_u) = 1280 K$$

And

$$\alpha = \frac{k(\bar{T})}{\rho_u c_p(\bar{T})} = \frac{0.0809 \overset{\text{book}}{\bar{T}}}{1.16(1186) \overset{\bar{T}}{\bar{T}}} = 5.89 \cdot 10^{-5} m^2/s$$

Where air properties were used to evaluate  $k$ ,  $\rho$  and  $c_p$ . Then

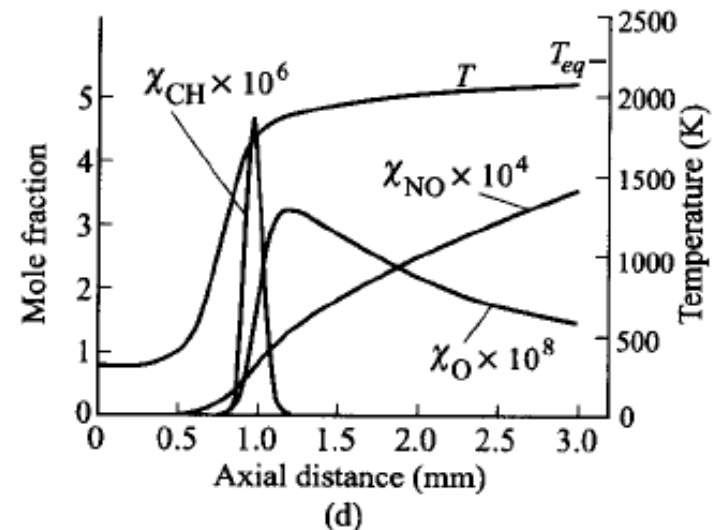
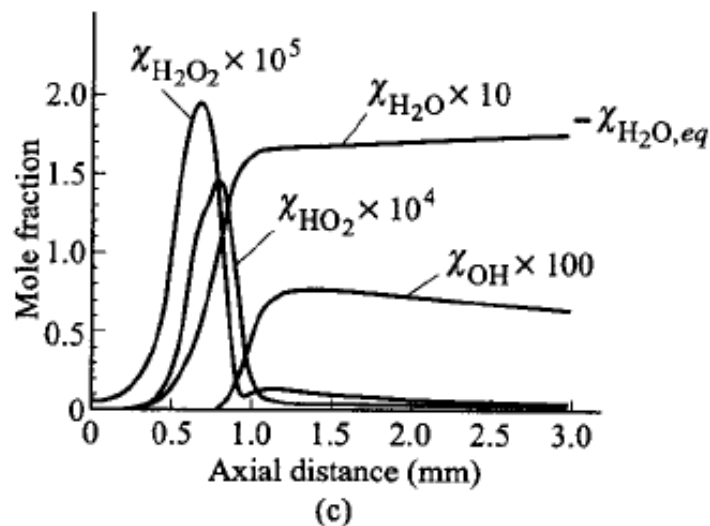
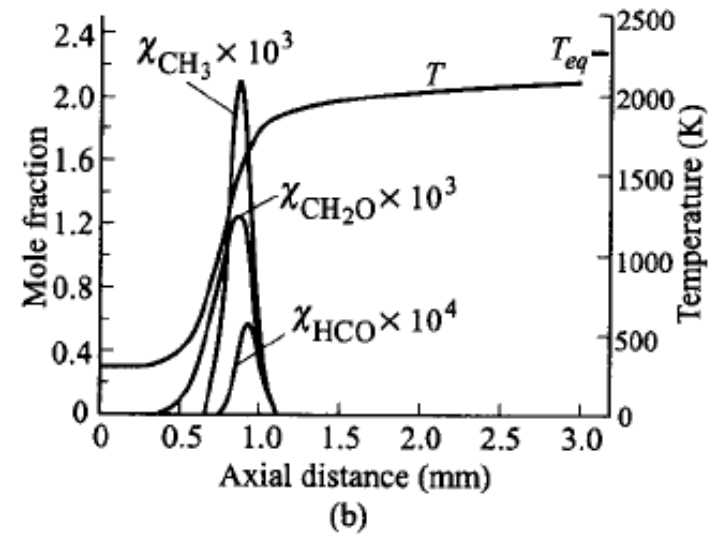
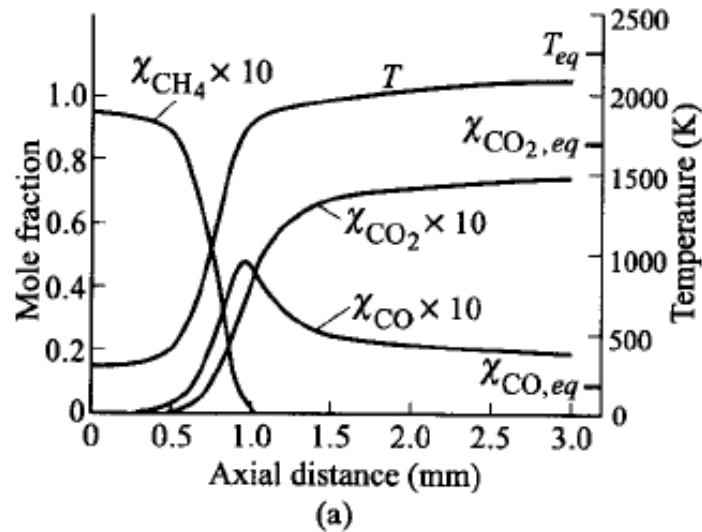
$$S_L = \left[ \frac{-2 (5.89 \cdot 10^{-5})(15.625 + 1)(-107.3)}{1.16} \right]^{\frac{1}{2}} = 0.425 m/s$$

Comments: the measured value of  $S_L$  for this mixture is 38.9 cm/s. rigorous theory with detailed kinetics can be used to obtain more accurate results.

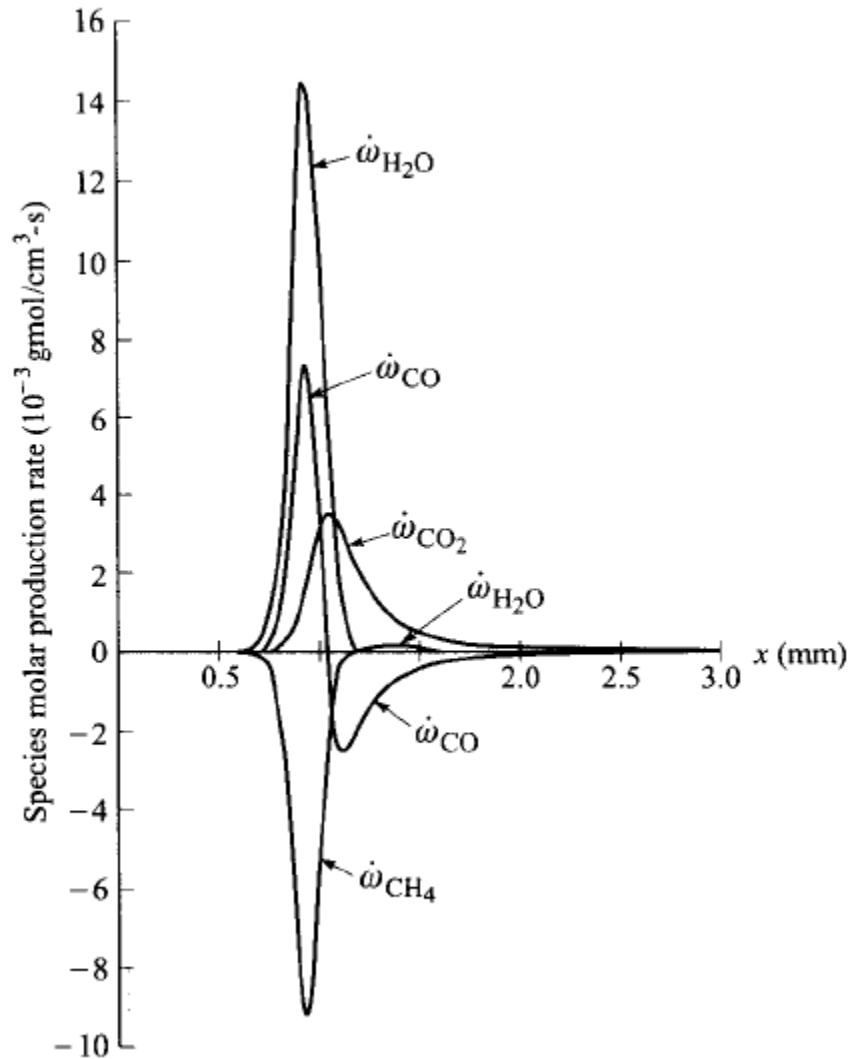


# Structure of CH<sub>4</sub>-air flame

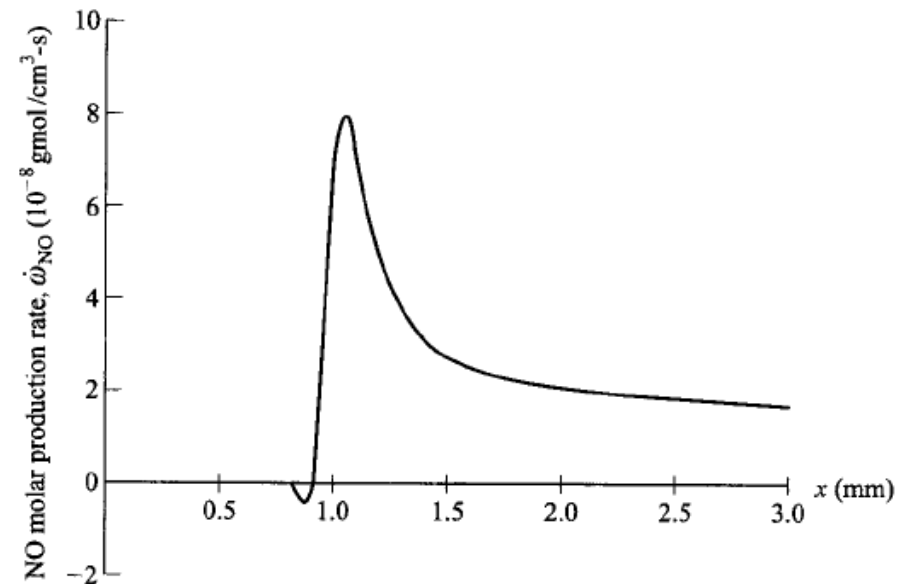
Laminar stoichiometric CH<sub>4</sub>-air premixed flame



# Structure of CH<sub>4</sub>-air flame



Volumetric production rate profile



Nitric oxide molar production rate profile

# Factors influencing flame velocity and thickness

## Temperature

Laminar flame speed has a strong temperature dependence.

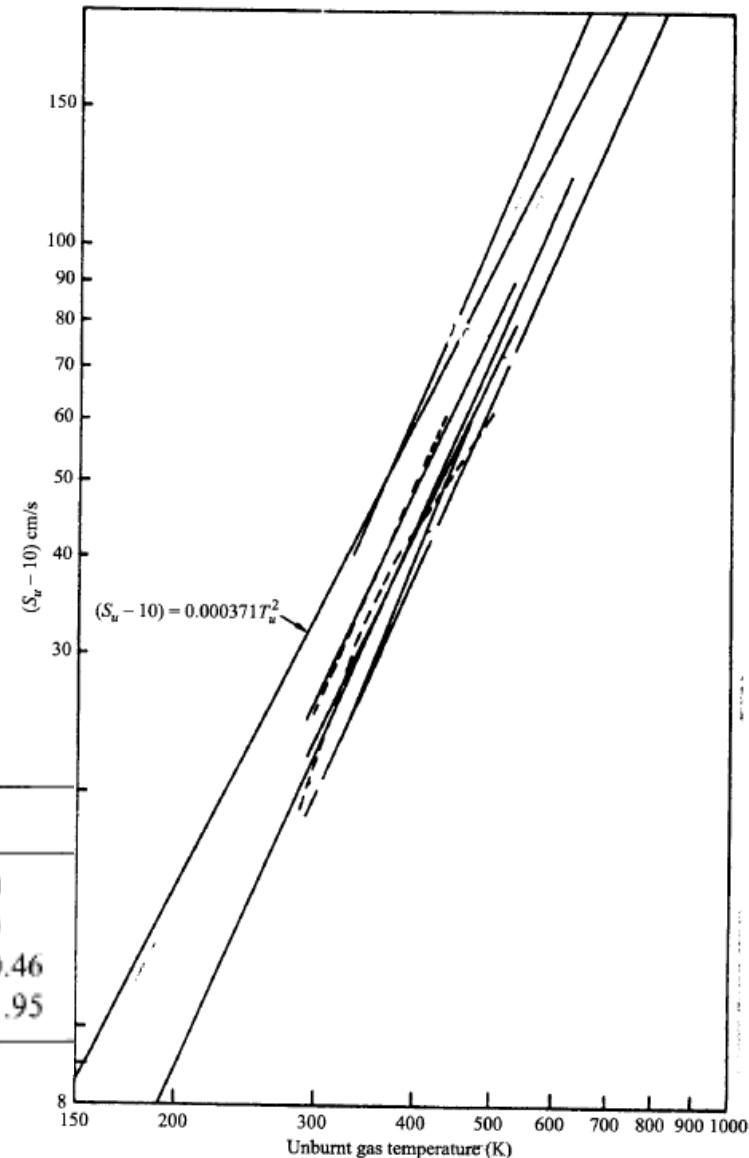
For stoichiometric CH<sub>4</sub>-air flames

$$S_L \left( \frac{\text{cm}}{\text{s}} \right) = 10 + 3.71 \cdot 10^{-4} [\underbrace{T_u}_{T_u} (K)]^2$$

In the table we have 3 cases of different unburned temperature for CH<sub>4</sub>-air

Case	A	B	C
$T_u$ (K)	300	600	300
$T_b$ (K)	2,000	2,300	1,700
$S_L/S_{L,A}$	1	3.64	0.46
$\delta/\delta_A$	1	0.65	1.95

*flame thickness*



Stoichiometric CH<sub>4</sub>-Air flame at 1 atm

# Factors influencing flame velocity and thickness

## Pressure

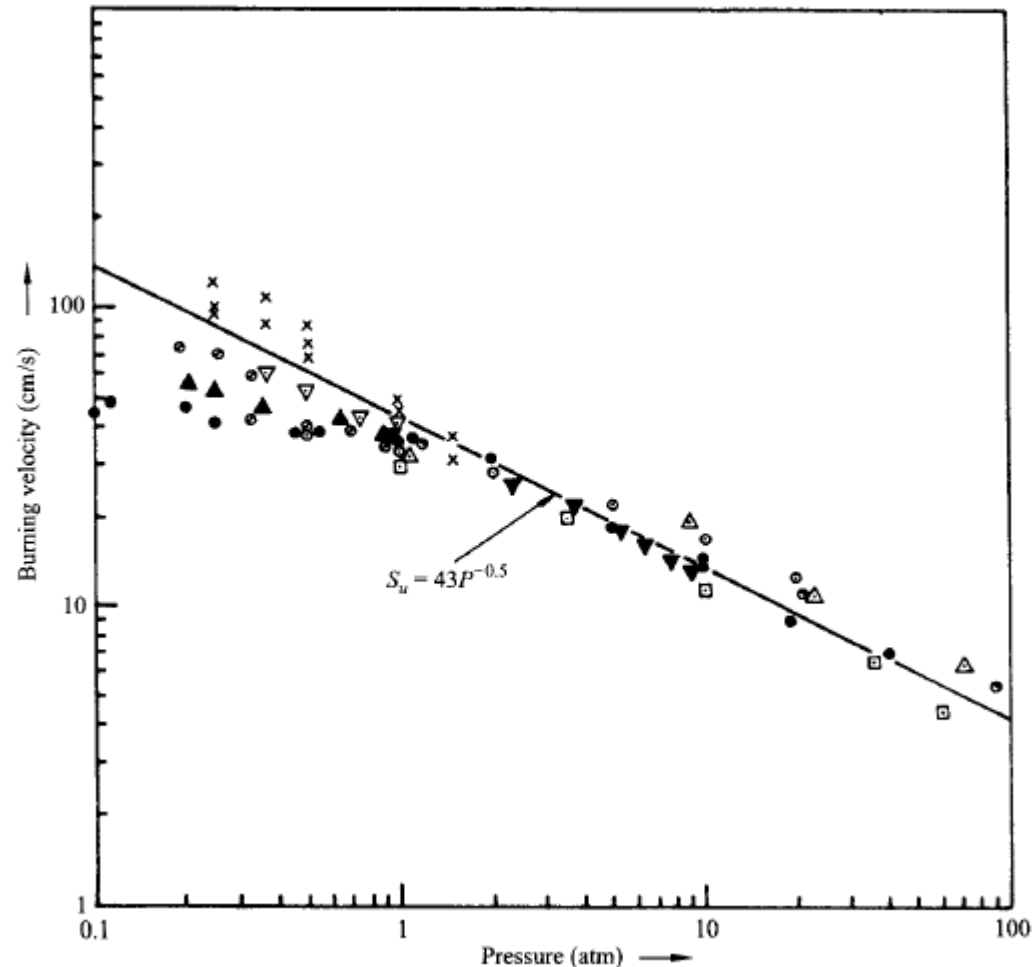
Flame speed in general have a negative dependence of pressure.

For CH<sub>4</sub>-air flames and  $p > 5$  atm, empirical equation

$$S_L \left( \frac{\text{cm}}{\text{s}} \right) = 43 [p(\text{atm})]^{-0.5}$$

Can be used to estimate flame speed.

*weaker than T*



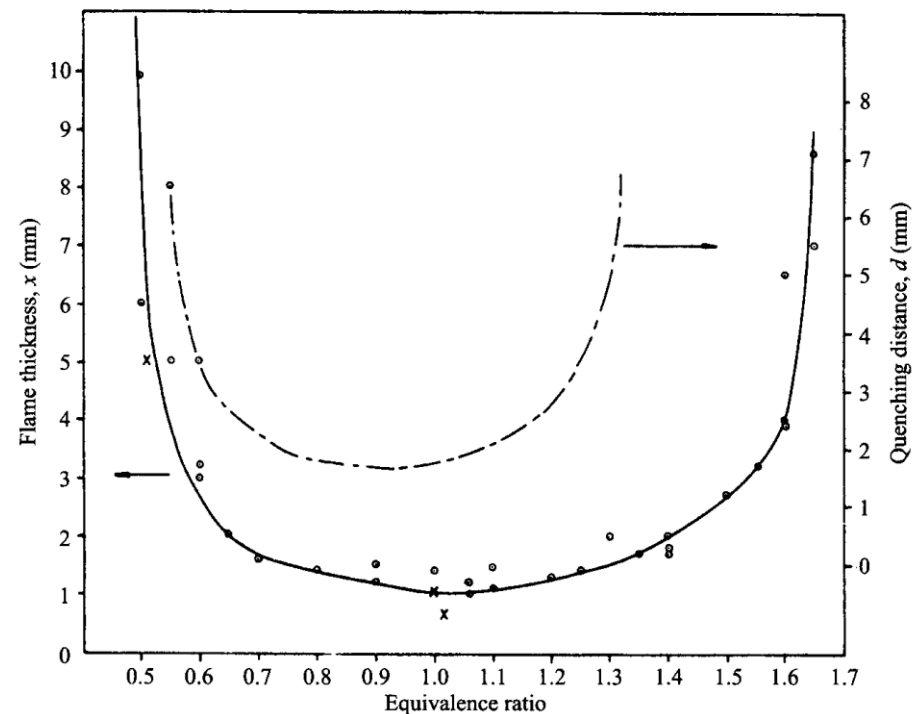
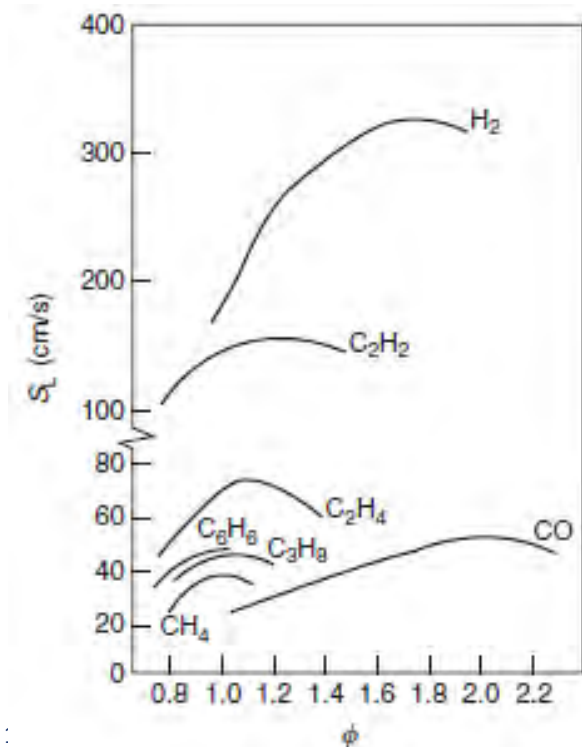
# Factors influencing flame velocity and thickness

## Equivalence ratio

The primary effect of equivalence ratio on flame speed and thickness is a result of the flame temperature.

The flame speeds are at their maximum values at a slight rich mixture and fall off either side.

Flame thickness shows the reverse trend, having a minimum near stoichiometric.

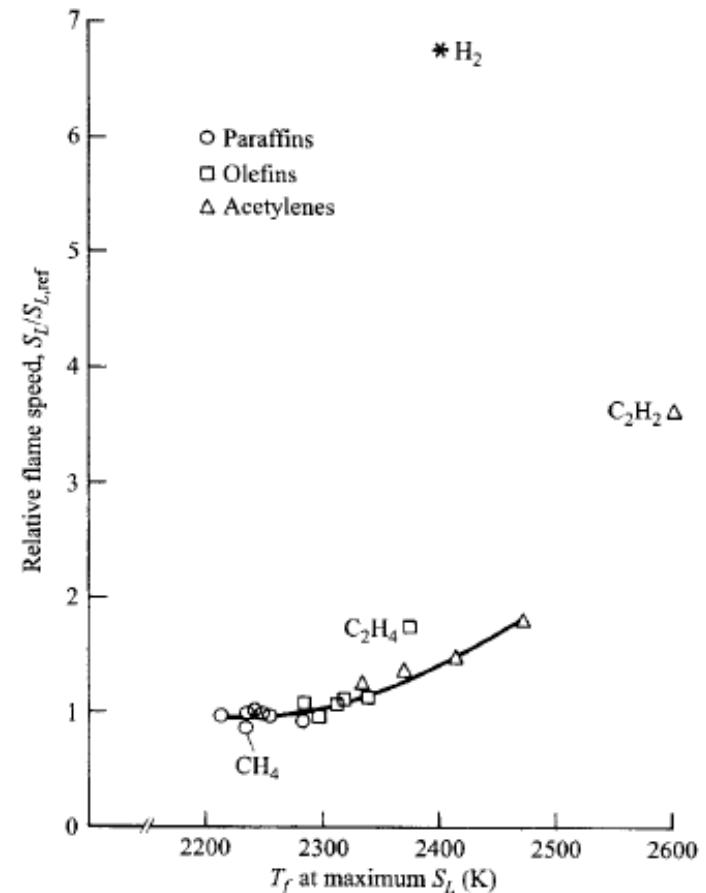
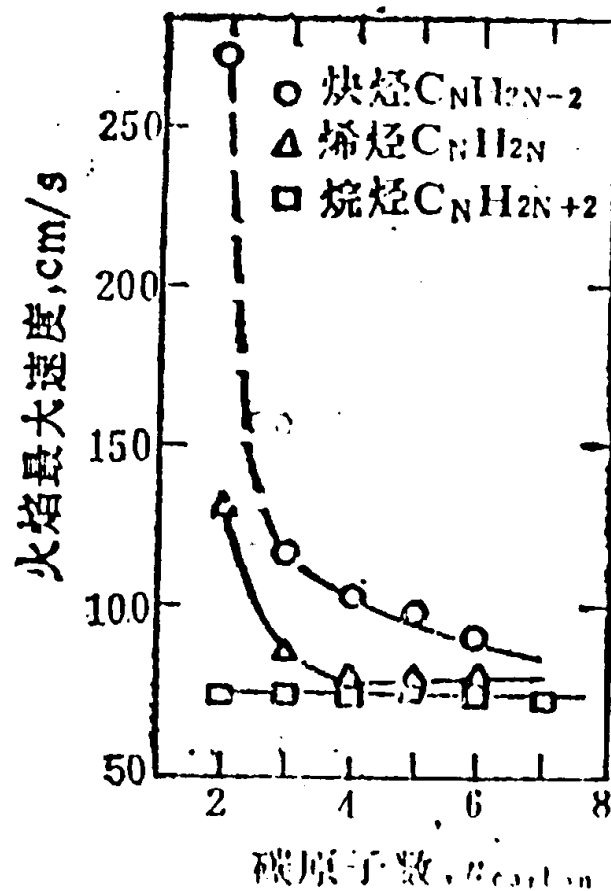


# Factors influencing flame velocity and thickness

## Fuel types

For paraffins, the maximum flame speed is independent of carbon numbers

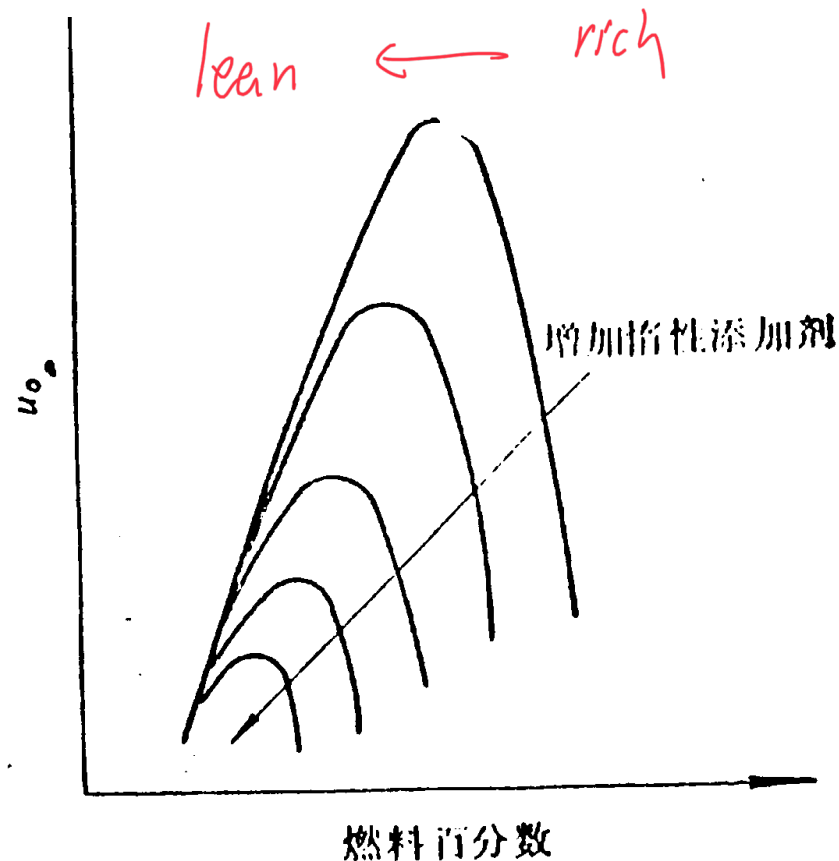
For olefins and acetylenes (unsaturated bonds) smaller carbon number has higher flame speed.



# Factors influencing flame velocity and thickness

## Inert gas

With addition of inert gas, flame velocity will reduce and the maximum flame velocity point will shift towards the leaner direction.

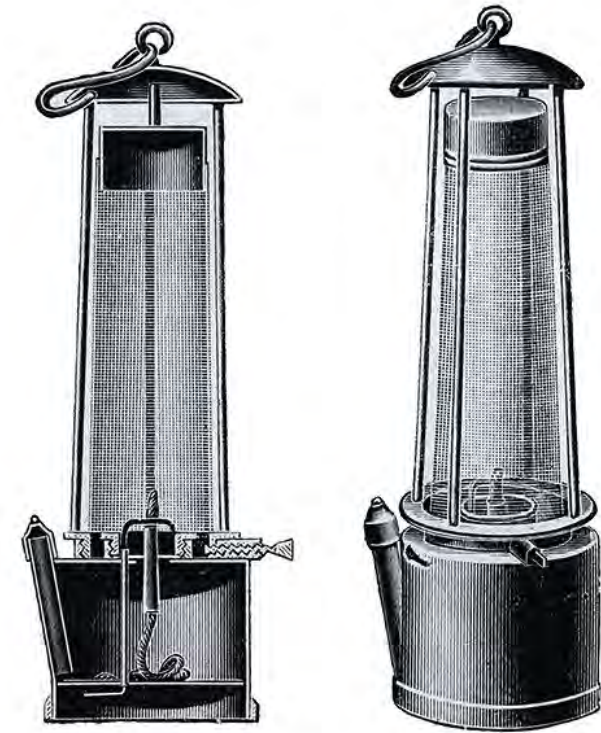


# Quenching, flammability and ignition

- There are many ways for flames to extinguish.
  - When passing through narrow passageways
  - Addition of diluents such as water (thermal effect) or chemical suppressants such as halogens (chemical kinetics).
  - Blowing the flame away from the reactants



Sir Humphry Davy



The Davy lamp



## Quenching by a cold wall

- Flames extinguish upon entering a sufficiently small passageway. If the passageway is not too small, flames will propagate through it.
- The critical diameter of a circular tube where a flame extinguishes, is called the quenching distance.
- **Ignition and quenching criteria** (by Williams)
  - Criterion I – ignition will only occur if enough energy is added to the gas to heat a slab about as thick as a steadily propagating laminar flame to the adiabatic flame temperature.
  - Criterion II – the rate of liberation of heat by chemical reactions inside the slab must approximately balance the rate of heat loss from the slab by thermal conduction.

# Simplified quenching analysis

Consider a flame that just enter a slot formed by two plane-parallel plates. Our objective is to determine the distance  $d$ , the quenching distance. From the second criterion, the energy released from combustion must equal to the conductive heat losses.

$$\dot{Q}'''V = \dot{Q}_{cond,tot}$$

Where the volumetric heat release rate  $\dot{Q}'''$  is related to  $\bar{m}_F'''$

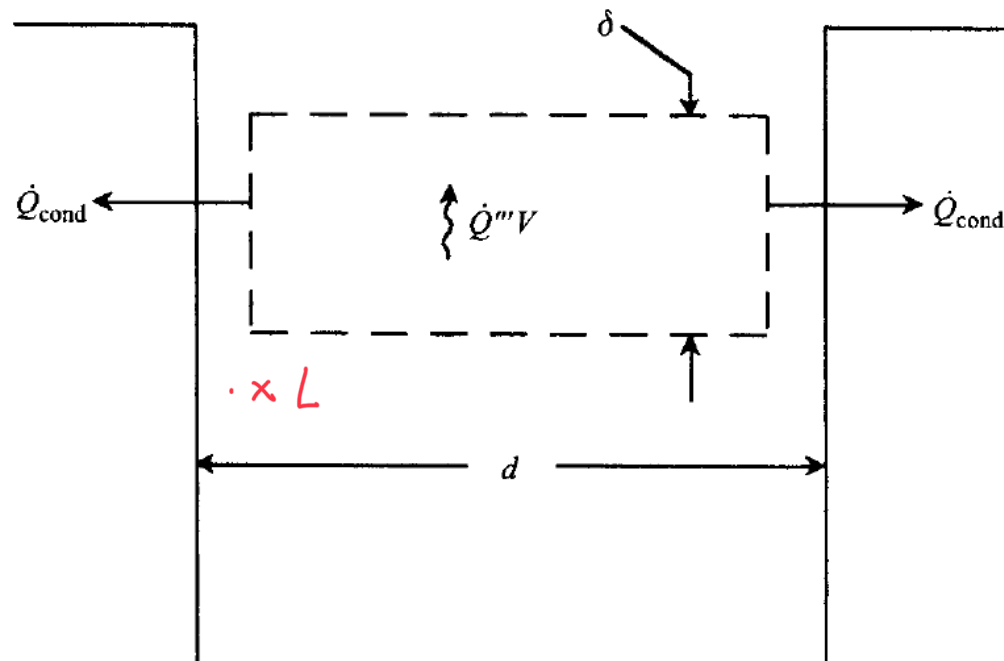
$$\dot{Q}''' = -\bar{m}_F''' \Delta h_c$$

We also consider the thickness of the slab of gas has been taken to be  $\delta$ , the adiabatic laminar flame thickness.

The heat loss by Fourier's law

$$\dot{Q}_{cond} = -kA \left. \frac{dT}{dx} \right|_{\text{in gas at wall}}$$

Where both  $k$  and temperature gradient are evaluated in the gas at the wall. Area  $A$  is  $2\delta L$ , where  $L$  is the slot width (perpendicular to the page). Factor 2 is due to the flame has contact both sides.



## Simplified quenching analysis

The temperature gradient is assumed a linear distribution from the centreline plane at  $T_b$  to wall at  $T_w$ . The  $dT/dx$  is likely much greater than this, we introduce an arbitrary constant  $b$ , so we have

$$\left| \frac{dT}{dx} \right| \equiv b \frac{T_b - T_w}{d} = \frac{T_b - T_w}{d/b}$$

And substitute this to previous equation, we have the criteria as

$$(-\bar{m}_F''' \Delta h_c)(\delta dL) = k(2\delta L) \frac{T_b - T_w}{d/b}$$

Rearrange

$$d^2 = \frac{2kb(T_b - T_w)}{-\bar{m}_F''' \Delta h_c}$$

Assume  $T_w = T_u$ , use the relationships between  $\bar{m}_F'''$  and  $S_L$ , and relating  $\Delta h_c = (\nu + 1)c_p(T_b - T_u)$

$$d = 2\sqrt{b}\alpha/S_L$$

Or

$$d = 2\sqrt{b}\delta$$

This result is in agreement with measured values for methane.

# Quenching distance

Flammability, quenching distance and minimum ignition energy

Fuel	Flammability Limits			Quenching Distance		Minimum Ignition Energy	
	$\Phi_{\min}$ (Lean or Lower Limit)	$\Phi_{\max}$ (Rich or Upper Limit)	Stoichiometric Mass Air-Fuel Ratio	For $\Phi = 1$ (mm)	Absolute Minimum (mm)	For $\Phi = 1$ ( $10^{-5}$ J)	Absolute Minimum ( $10^{-5}$ J)
Acetylene, $C_2H_2$	0.19 <sup>b</sup>	$\infty^b$	13.3	2.3	—	3	—
Carbon monoxide, CO	0.34	6.76	2.46	—	—	—	—
<i>n</i> -Decane, $C_{10}H_{22}$	0.36	3.92	15.0	2.1 <sup>c</sup>	—	—	—
Ethane, $C_2H_6$	0.50	2.72	16.0	2.3	1.8	42	24
Ethylene, $C_2H_4$	0.41	> 6.1	14.8	1.3	—	9.6	—
Hydrogen, $H_2$	0.14 <sup>b</sup>	2.54 <sup>b</sup>	34.5	0.64	0.61	2.0	1.8
Methane, $CH_4$	0.46	1.64	17.2	2.5	2.0	33	29
Methanol, $CH_3OH$	0.48	4.08	6.46	1.8	1.5	21.5	14
<i>n</i> -Octane, $C_8H_{18}$	0.51	4.25	15.1	—	—	—	—
Propane, $C_3H_8$	0.51	2.83	15.6	2.0	1.8	30.5	26

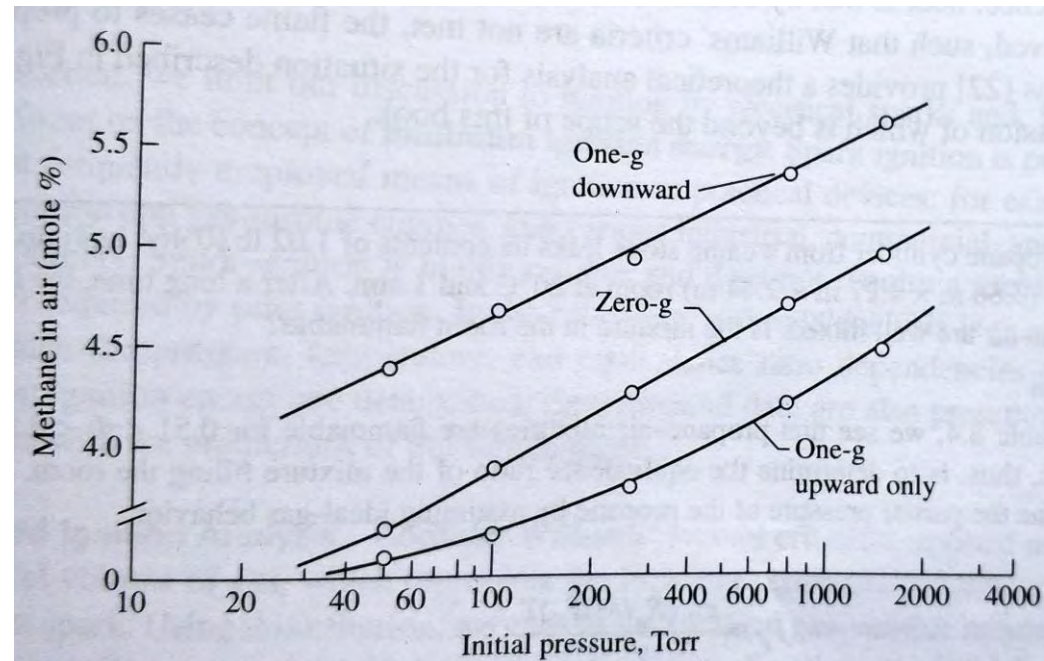
<sup>a</sup>SOURCE: Data from Ref. [21] unless otherwise noted.

<sup>b</sup>Zabetakis [U.S. Bureau of Mines, Bulletin 627, 1965].

<sup>c</sup>Charniak [25].

# Flammability limits

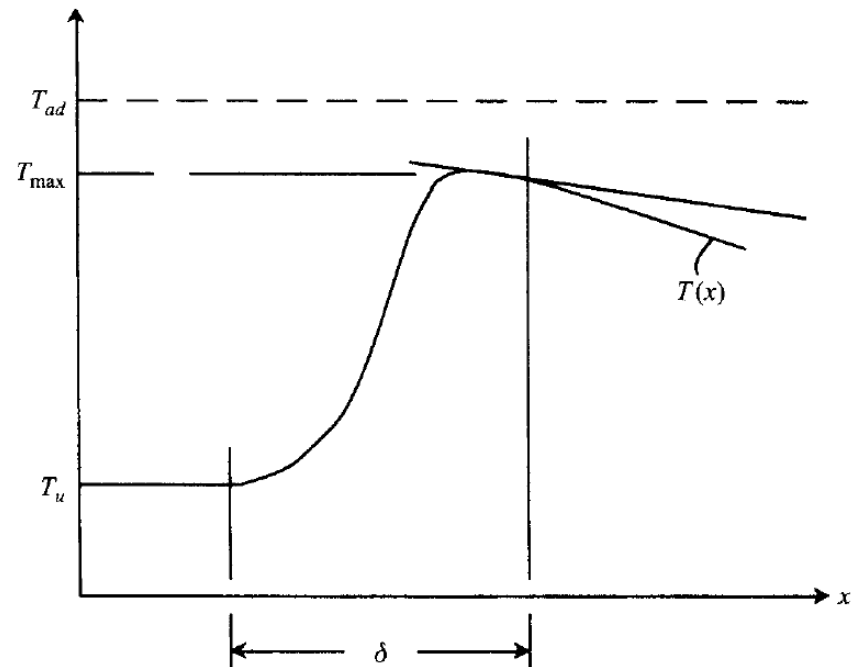
- Experiments show that a flame will only propagate with a range of mixture strengths between the so-called lower and upper limits of flammability.
- The **lower limit** is the leanest mixture ( $\Phi < 1$ ) that will allow steady flame propagation.
- The **upper limit** is the richest mixture ( $\Phi > 1$ ) that allow steady propagation.
- Flammability limits are often quoted as percent fuel by volume in the mixture.
- The effect of pressure on the lower flammability limit is relatively weak.



CH<sub>4</sub>-air mixture, note 5% cor. to  $\Phi=0.476$

# Flammability limits

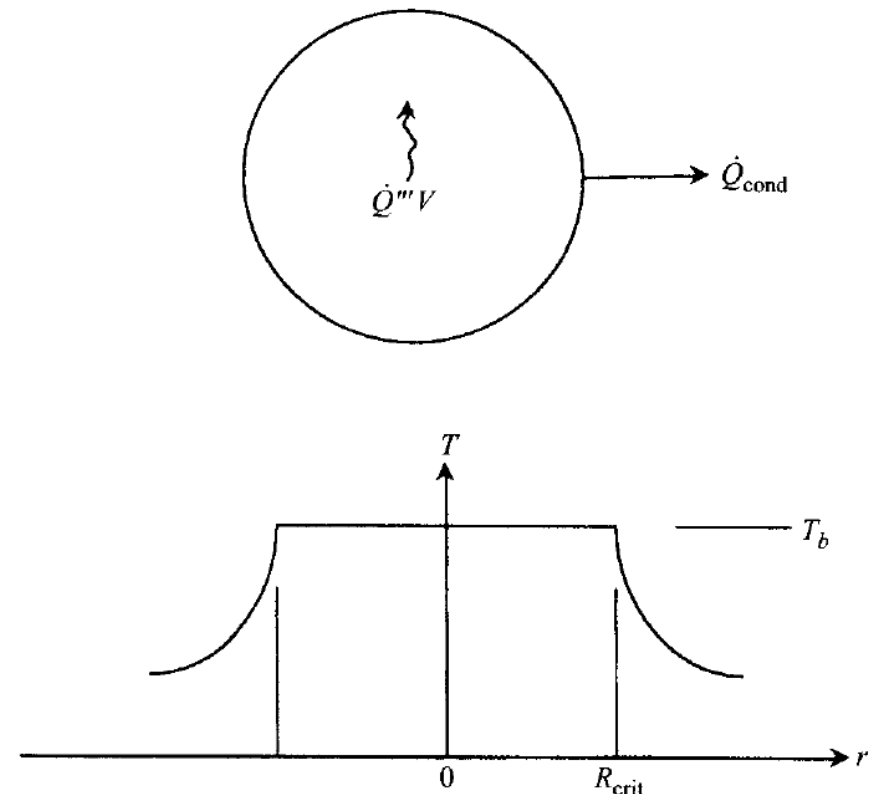
- Flammability can be defined as a physiochemical property of a fuel-air mixture, the actual limits are related to heat losses from the system.
- Even if conduction losses are minimal, radiation losses can be significant.
- The figure illustrate an instantaneous axial temperature profile along the centreline of a tube in which a flame is propagating. Due to the radiation from the high temperature products, there is a negative temperature gradient at the rear of the flame zone.
- When sufficient heat is lost, Williams' criteria are not met, the flame ceases to propagate.



# Ignition

## Simplified analysis

- We focus our discussion on ignition by electrical sparks, and the concept of **minimum ignition energy**.
- Apply Williams' criterion II to a spherical volume of gas. We define a critical gas volume radius below which a flame will not propagate.
- The minimum ignition energy is supplied by the spark and the energy is required to heat the critical gas volume from its initial temperature to the flame temperature.
- The objective is to determine the minimum ignition energy  $E_{ign}$ .



To determine the critical radius,  $R_{crit}$ , we equate the heat released by reaction to the rate of heat lost to the cold gas by conduction

$$\dot{Q}'''V = \dot{Q}_{cond}$$

And

$$-\bar{m}_F''' \Delta h_c \frac{4\pi R_{crit}^3}{3} = -k4\pi R_{crit}^2 \left. \frac{dT}{dr} \right|_{R_{crit}}$$

The temperature gradient at the sphere boundary can be evaluated as

$$\left. \frac{dT}{dr} \right|_{R_{crit}} = -\frac{T_b - T_u}{R_{crit}}$$

So we have

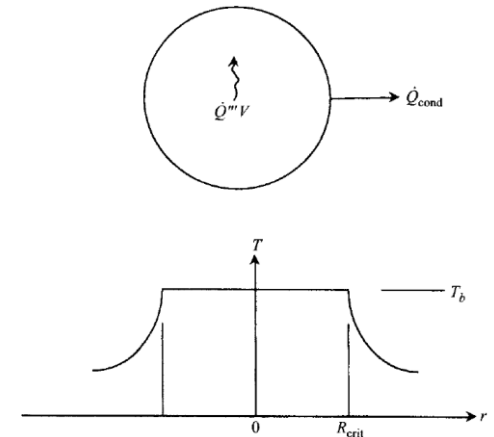
$$R_{crit}^2 = \frac{3k(T_b - T_u)}{-\bar{m}_F''' \Delta h_c}$$

In the form with regards to  $S_L$  and  $\delta$

$$R_{crit} = \frac{\sqrt{6}\alpha}{S_L}$$

Or

$$R_{crit} = \frac{\sqrt{6}\delta}{2}$$





Due to the approximations we have made, we can consider the critical radius roughly equal to or a few times larger than the laminar flame thickness. In contrast, the quenching distance is many times larger than the flame thickness. Now we determine the minimum ignition energy. We simply assume the energy added by the spark heats the critical volume to the burned gas temperature.

$$E_{ign} = m_{crit} c_p (T_b - T_u)$$

$$R_c \approx \frac{\sqrt{\delta}}{2}$$

Where the mass within the critical volume is  $\rho_b 4\pi R_{crit}^3 / 3$

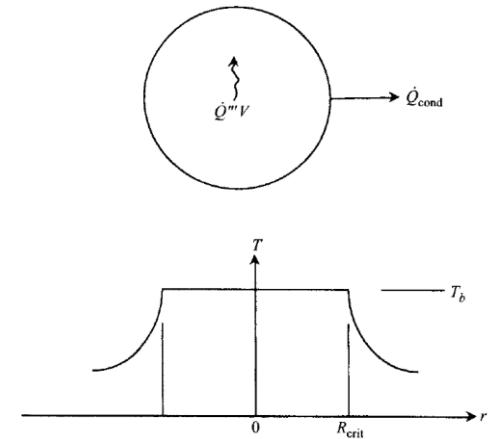
So we have

$$E_{ign} = 61.6 \rho_b c_p (T_b - T_u) (\alpha / S_L)^3$$

Eliminating  $\rho_b$  using the ideal gas equation

$$E_{ign} = 61.6 p \left( \frac{c_p}{R_b} \right) \left( \frac{T_b - T_u}{T_b} \right) (\alpha / S_L)^3$$

Where  $R_b = R_0 / M_b$



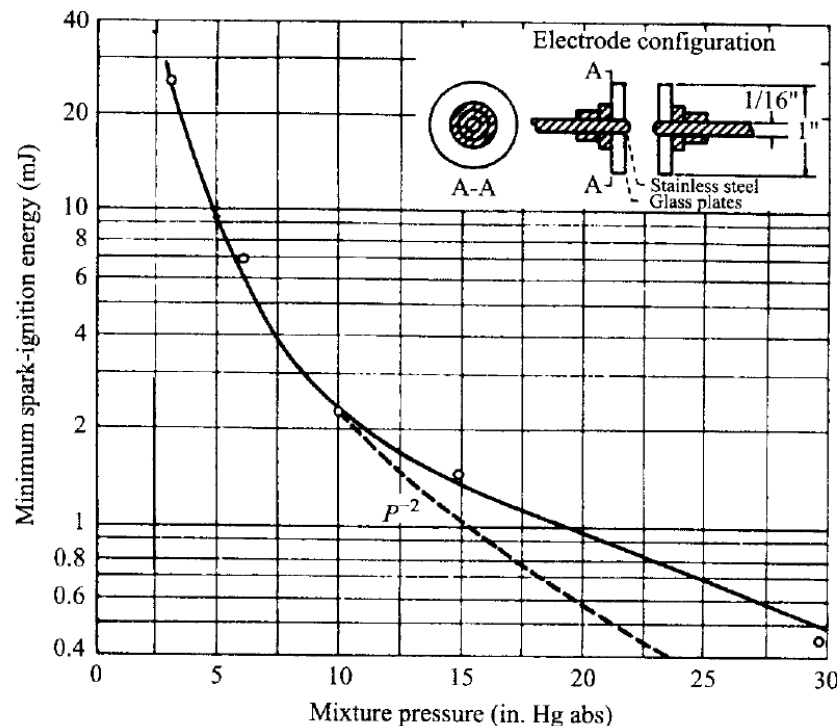
# Ignition

## Pressure dependency

From the previous equation, minimum ignition energy is directly affected by pressure and indirectly affected by thermal diffusivity  $\alpha$  and flame speed  $S_L$  both are influenced by pressure. The combined effect is

$$E_{ign} \propto p^{-2}$$

This agrees very well with experimental results.



# Ignition

Temperature dependency

In general increasing initial temperature results in decreased ignition energies.

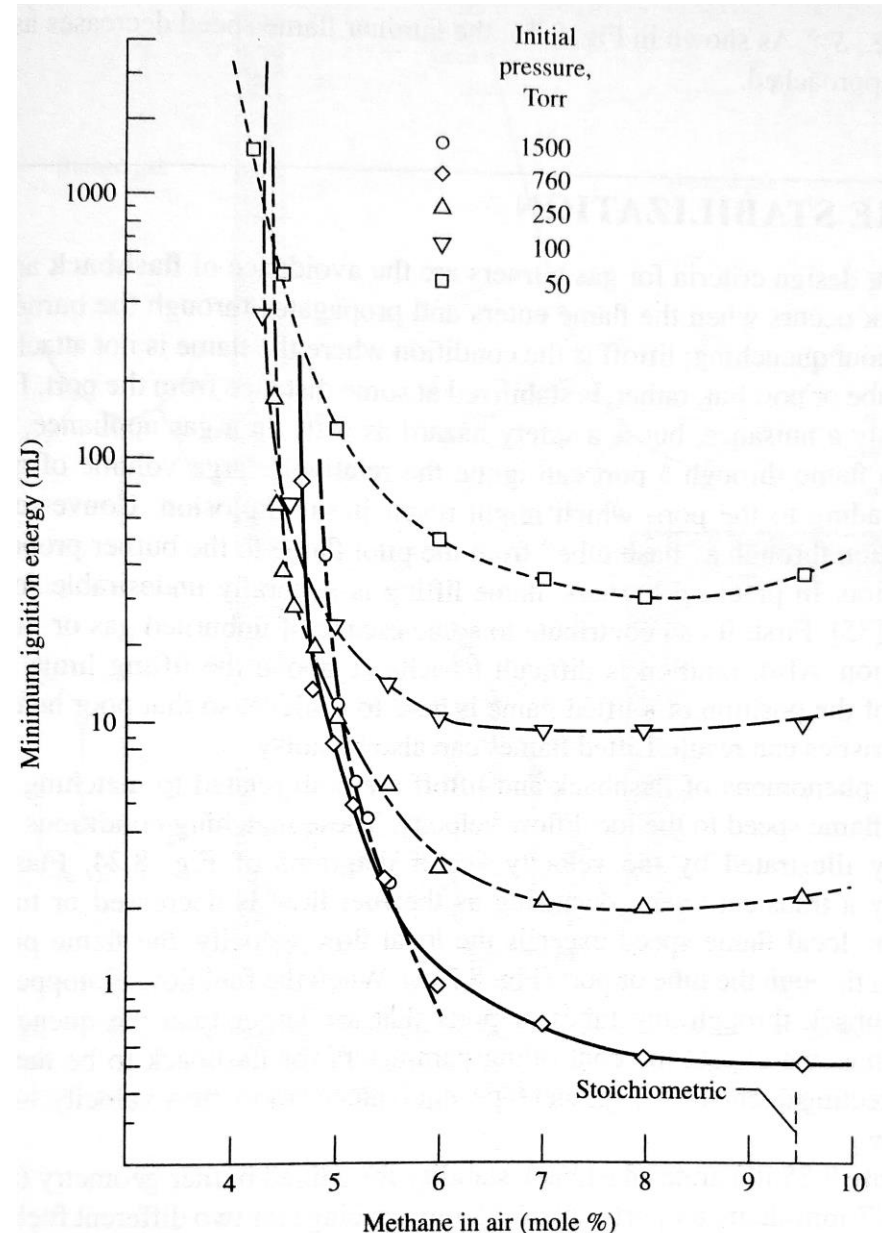
Fuel	Initial Temperature (K)	$E_{\text{ign}}$ (mJ) <sup>a</sup>
<i>n</i> -Heptane	298	14.5
	373	6.7
	444	3.2
Isooctane	298	27.0
	373	11.0
	444	4.8
<i>n</i> -Pentane	243	45.0
	253	14.5
	298	7.8
	373	4.2
	444	2.3
Propane	233	11.7
	243	9.7
	253	8.4
	298	5.5
	331	4.2
	356	3.6
	373	3.5
	477	1.4

a. Initial pressure = 1atm

# Ignition

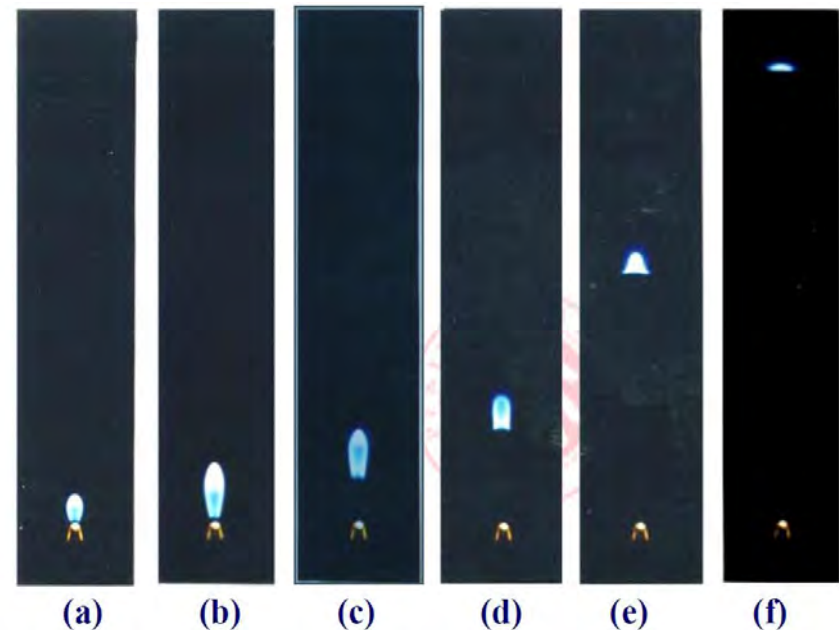
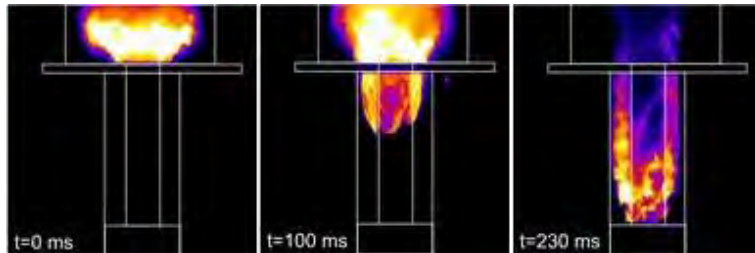
## Equivalence ratio dependency

- At sufficiently lean equivalence ratio, the minimum ignition energy increases.
- Near the lean flammability limit, ignition energies increase more than an order of magnitude from their values at stoichiometric conditions



# Flame stability

- Important design criteria for gas burners are the avoidance of **flashback** and **liftoff**.
- Flashback occurs when the flame enters and propagates through the burner tube or port without quenching (safety hazard). *back to tube*
- Liftoff is the condition where the flame is not attached to the burner tube or port is stabilised at some distance from the port. This can leave unburned gas or incomplete combustion, and ignition is difficult to achieve above the lifting limit.

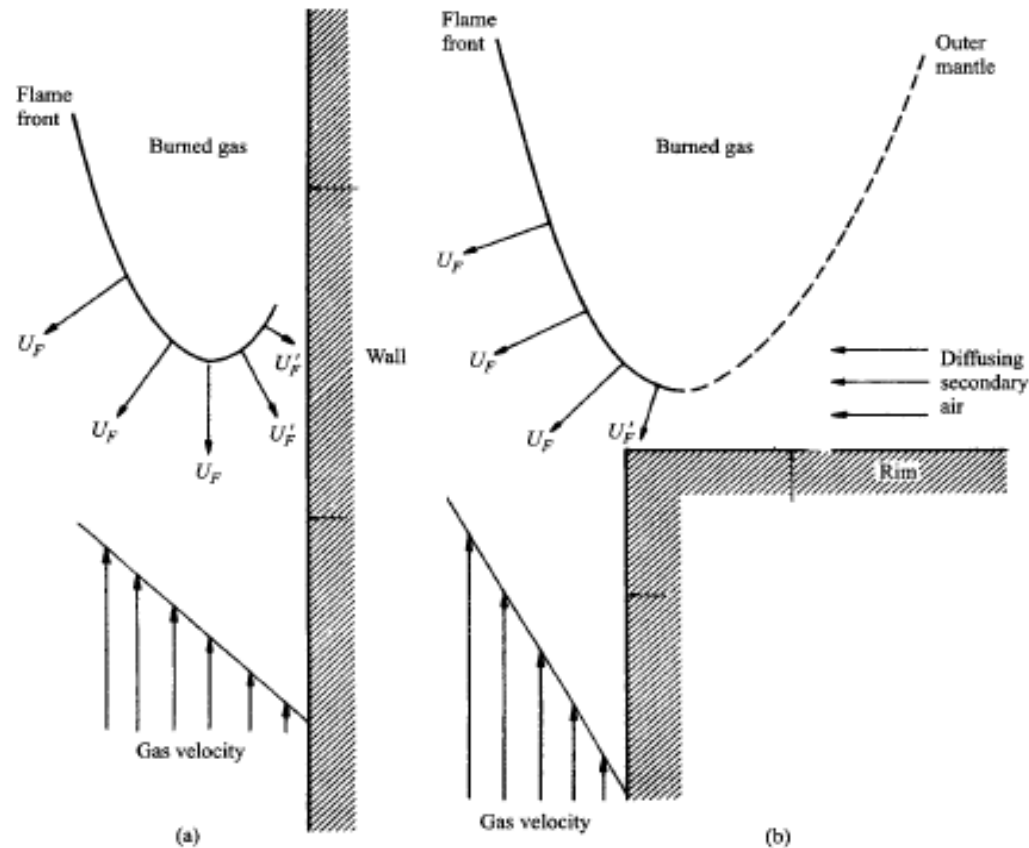


$d = 0.195 \text{ mm}$ ; Propane

Flow rate: (a)  $Q=0.195$  (b) 17.3 (c) 18.3 (d) 20.3 (e) 22.3 (f) 26.4 ml/min 5

Both flashback and liftoff phenomena are related to matching the local laminar flame speed to the local flow velocity.

- Flashback is generally a transient event, when fuel flow is decreased or turned off.
- When the local flame speed exceeds the local flow velocity, the flame propagates upstream.
- When the fuel flow is stopped, flames flashback through tubes or ports are larger than the quenching distance.



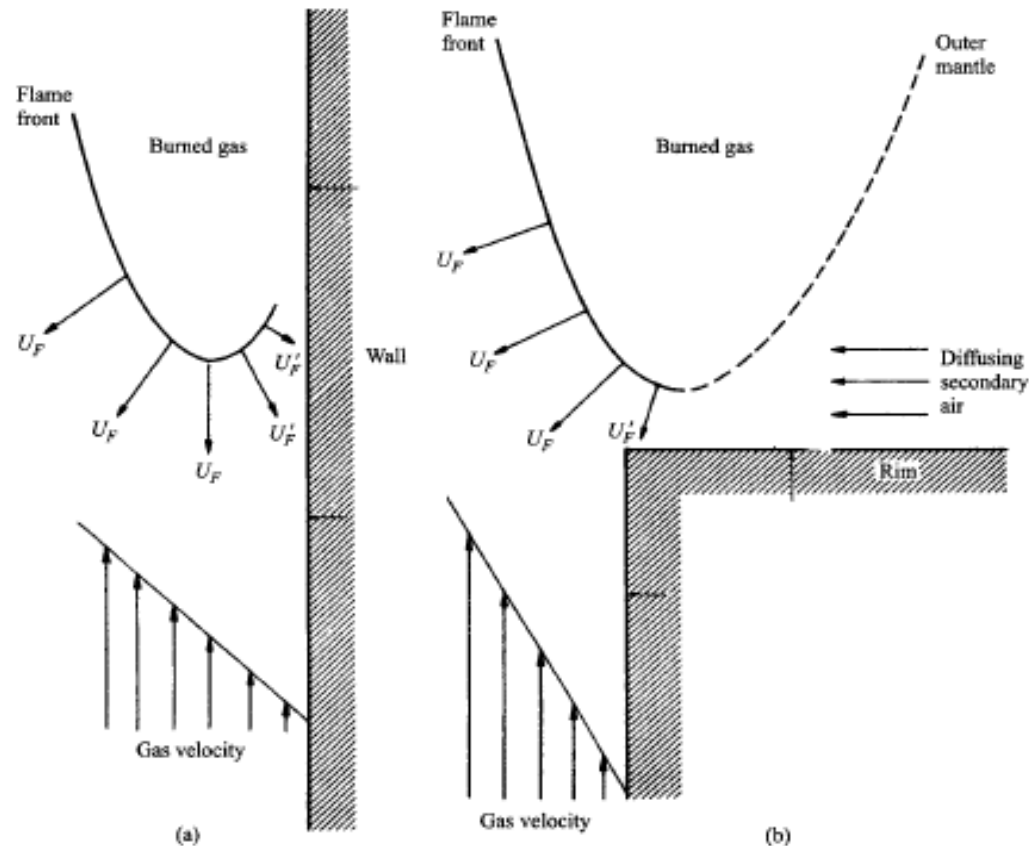
Both flashback and liftoff phenomena are related to matching the local laminar flame speed to the local flow velocity.

- Flame lifting depends on local flame and flow near the edges of the burner port.

- When velocity increases, the cone angle decreases because

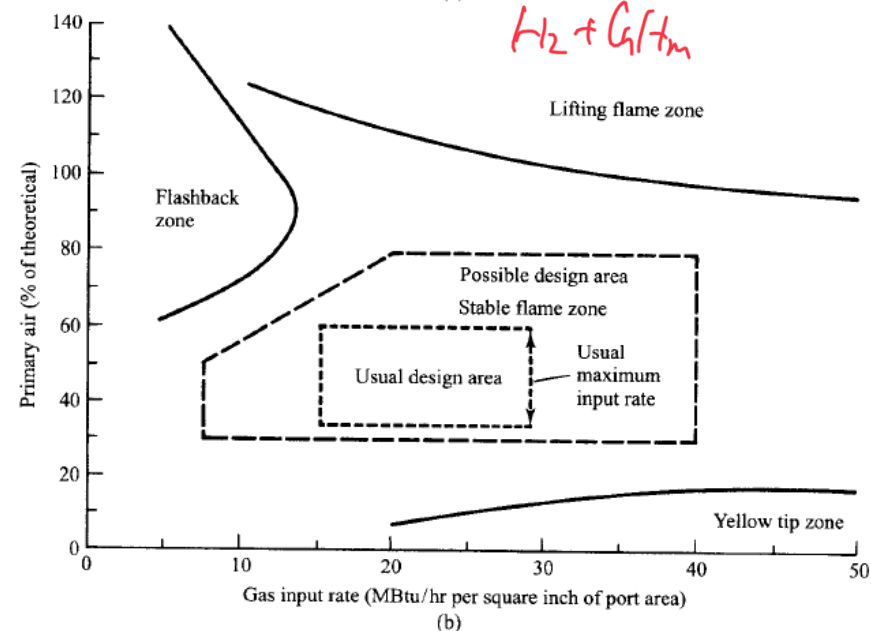
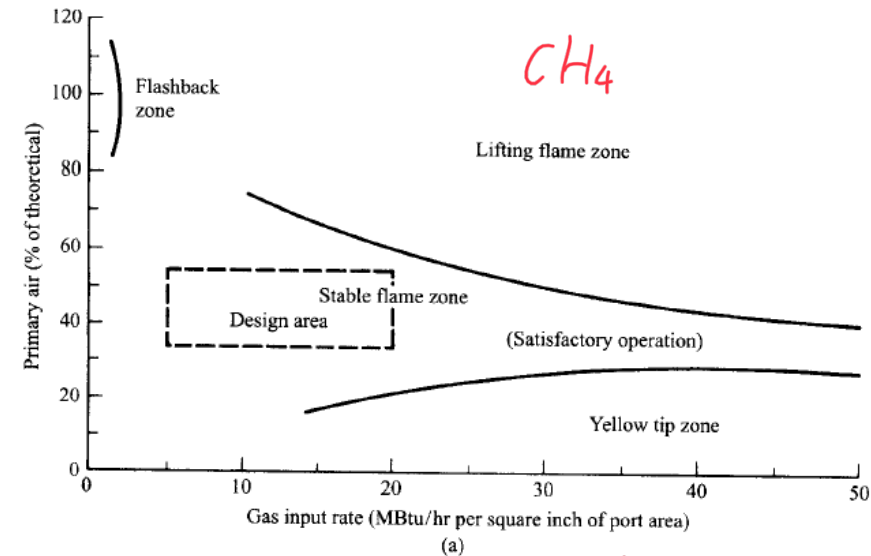
$$\alpha = \sin^{-1}\left(\frac{S_L}{v_u}\right)$$

- When further increases in flow velocity, a critical velocity is reached where the flame edge jumps to a downstream position far from the burner lip.
- Increasing the velocity beyond the liftoff value results in increasing the liftoff distance until the flame blows off.



# Flame stability

- This figure illustrates flashback stability for a fixed burner geometry for natural gas and another gas contained hydrogen.
- Flashback occurs in the region to the left of the flashback zone, flashback-free occurs to the right where velocities are higher.
- Slightly rich stoichiometries provide the least tolerance to flashback because flame speeds are the highest with slightly rich mixture.
- Flashback stability of natural gas is much greater than the second gas because hydrogen has very high flame speed.
- Lift-free operation as a function of stoichiometry and flow velocity.
- Natural gas is more prone to lifting due to the lower flame speed and less stability.





- Flame speed  $S_L$
- Flame thickness  $\delta$
- Quenching distance
- Flammability limits
- Minimum ignition energy
- Pressure and temperature dependencies of flame speed and velocity and  $S_L \propto (\alpha \bar{m}_F''' / \rho_u)^{1/2}$