Probability Theory

POISSON DISTRIBUTION Probability $R(t)$ component will not fail during $(0,t)$

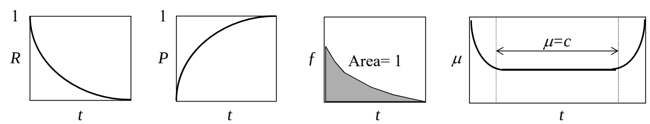
This is called the reliability:

$R(t) = e^{-\mu t}$ $\quad P(t) = 1 - R(t) = 1 - e^{-\mu t}$

- **R** Reliability (no units)
- **μ** Average failure rate (time^{-1})
- **P** Failure probability (no units)
- **f(t)** Failure density (time^{-1})

$$\text{Mean Time Between Failures (MTBF)} = E(t) = \int_0^\infty t f(t)\,dt = \frac{1}{\mu}$$

Bathtub curve



Probability Theory

Example: A device is found to fail once every 2 years. What is the failure rate, the failure probability and the reliability at the end of 1 year, and the MTBF?

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Answer:

The failure rate, μ, is given by: $\mu = 1/2 \text{ years } = 0.5 \text{ yr}^{-1}$

The reliability is given by Equation:

$$R(t) = e^{-\mu t} = \exp\left[-\left(0.5\,\text{yr}^{-1}\right)\left(1\,\text{yr}\right)\right] = 0.607$$

The failure probability is given by Equation:

$$P(t) = 1 - R(t) = 1 - 0.607 = 0.393$$

The Mean Time Between Failure:

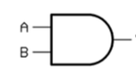
$$MTBF = \frac{1}{\mu} = \frac{1}{0.5\text{ yr}} = 2\text{ years}$$

Interaction

COMPONENTS IN PARALLEL: Both components must fail

$$P = \prod_{i=1}^{n} P_i$$

n = the total number of components
P_i = the failure probability of each component



AND

$$R = 1 - \prod_{i=1}^{n} (1 - R_i)$$

R_i = the reliability of an individual process component.

Interaction

COMPONENTS IN SERIES: Either component fails

$$R = \prod_{i=1}^{n} R_i$$

OR

$$P = 1 - \prod_{i=1}^{n} (1 - P_i)$$

$$P\,(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A})\,P(\text{B})$$

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Failure Rate Data for Various Selected Process Components

Instrument	Faults/Year
Controller	0.29
Control valve	0.60
Flow measurement (fluids)	1.14
Flow measurement (solids)	3.75
Flow switch	1.12
Gas–liquid chromatograph	30.6
Hand valve	0.13
Indicator lamp	0.044
Level measurement (liquids)	1.70
Level measurement (solids)	6.86
Oxygen analyzer	5.65
pH meter	5.88
Pressure measurement	1.41
Pressure relief valve	0.022
Pressure switch	0.14
Solenoid valve	0.42
Stepper motor	0.044
Strip chart recorder	0.22
Thermocouple temperature measurement	0.52
Thermometer temperature measurement	0.027
Valve positioner	0.44

Basic Fact: The more complex the device the higher the failure rate!

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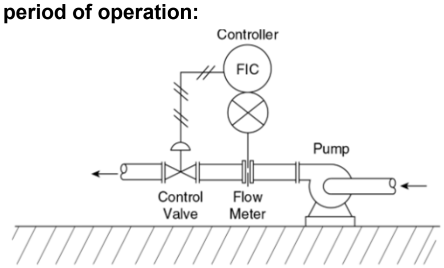
Example: Compute the overall <u>failure rate</u>, the <u>unreliability</u>, and the <u>MTBF</u> of the following flow control loop. Assume a 1 year period of operation:



We have 3 components: the control valve, the controller and the DP cell. These components are related in series, i.e. if any one component fails the entire flow control loop fails.

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Look up the failure rates for these three components from the Table. Then compute the reliability and failure probability for each component for a 1 year time period.

Component	Failure Rate μ (faults/yr)	Reliability $R = e^{-\mu t}$	Failure Probability $P = 1 - R$
Control valve	0.60	0.55	0.45
Controller	0.29	0.75	0.25
DP cell	1.41	0.24	0.76

The overall reliability for components in series is given:

$$R = \prod_{i=1}^{3} R_i = (0.55)(0.75)(0.24) = 0.10$$

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The failure probability is then given:

$$P = 1 - R = 1 - 0.1 = 0.90 \,/\, \text{year}$$

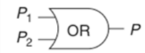
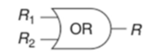
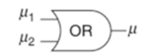
The overall failure rate is computed from the definition of the reliability:

$$R = 0.10 = e^{-\mu}$$

$$\mu = -\ln(0.10) = 2.30 \text{ failures/year}$$

The MTBF is given by Equation (12-5):

$$MTBF = \frac{1}{\mu} = \frac{1}{2.30\,/\,\text{yr}} = 0.43 \text{ yr}$$

Failure Probability	Reliability	Failure Rate
		
$P = 1 - (1 - P_1)(1 - P_2)$ $P = 1 - \prod_{i=1}^{n} (1 - P_i)$	$R = R_1 R_2$ $R = \prod_{i=1}^{n} R_i$	$\mu = \mu_1 + \mu_2$ $\mu = \sum_{i=1}^{n} \mu_i$
Series link of components:	The failure of either component adds to the total system failure.	
		
$P = P_1 P_2$ $P = \prod_{i=1}^{n} P_i$	$R = 1 - (1 - R_1)(1 - R_2)$ $R = 1 - \prod_{i=1}^{n} (1 - R_i)$	$\mu = (-\text{in } R)/t$
Parallel link of components:	The failure of the system requires the failure of both components. Note that there is no convenient way to combine the failure rate.	