
Models for Crowd Movement and Egress Simulation

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Summary. This paper discusses basic findings on crowd movement and their application to simulation models. This includes empirical and experimental results concerning group behavior, pedestrian motion, and emergency egress. Next to a literature review, we will present own empirical investigations on walking speed distribution and the dependency of walking speed on group size.

The second part of the paper relates these findings to modelling and simulation of crowd movement. This comprises the representation of behavior, calibration and verification and the connection to many particle systems.

Finally, we will present an extension of the current microscopic theory (basically underlying all the “individual” models used for real world applications). This includes route-choice behavior and links microscopic and macroscopic behavior.

Keywords: crowd movement, simulation, egress, evacuation, cellular automata, pedestrian dynamics

1 Introduction: What is it Good for?

This paper describes fundamental properties of models for crowd movement. The real-world systems investigated can – as a crude but justified approximation – be mapped onto many particle systems far from equilibrium. The evacuation from, e.g. a building, is then equivalent to a transportation process. Of course, this mapping can only be done by simplifying the influences present in social systems and neglecting most of them, especially the psychological ones. An example for such a simplifications is the description of space as a grid of (usually quadratic cells), as it is done in the software package *PedGo*. The specific details of the PedGo model are described in another article in these proceedings [21].

In general, the aforementioned model assumptions lead to several questions. As a motivation, we will start with the most fundamental ones:

1. Why simulate crowd movement and evacuations?
2. And: How?

The answer to the first question is illustrated in fig. 1. We assume that the necessity for doing research on (empirically and theoretically) crowd movement and emergency egress is unquestioned. Then, the experimental investigation of crowd movement and especially emergency egress are limited by practical, ethical, financial, and logical constraints.

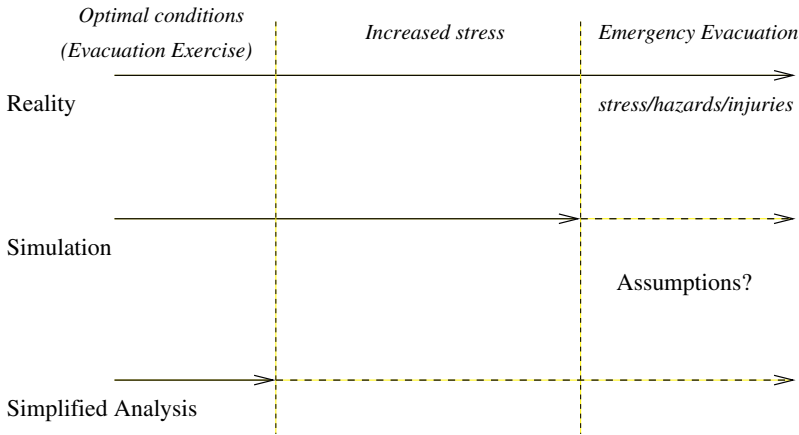


Fig. 1. A simplified analysis can basically cover the same range as an evacuation exercise. Aggravating circumstances like hazards can be included via extrapolation of the results obtained. A simulation, however, allows to include those influences via the adaption of the parameters, i.e., the extrapolation is made on the input and leads to an output different from the optimal case.

Furthermore, the scenarios we have in mind usually comprise large populations and structures, e.g. subway stations, football stadiums, passenger ships, etc.

At this point it is also worthwhile to clarify the meaning of the term crowd. Figure 2 sheds light especially on the distinction between gatherings and mobs.⁴ When the term crowd or mass is used in this paper (as defined in the figure), only the former are addressed.

This paper focusses mainly on the theoretical aspects of crowd dynamics modelling. We will refer to our other paper in this book [21] when it comes to

⁴ We would also like to note that the terms “mob” and “panic” are at least controversial [12, 33].

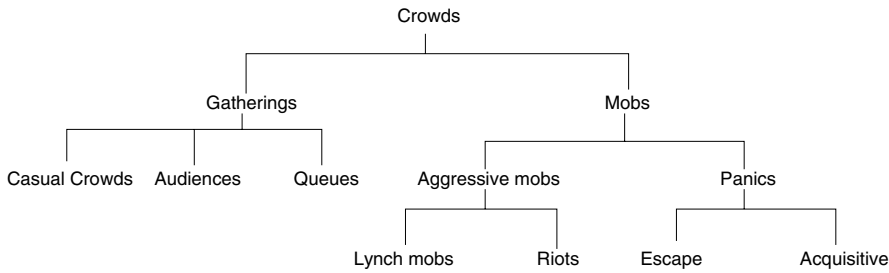


Fig. 2. Classification of crowds: [5]. Crowds are large groups that occupy a single location and share a common focus. One should probably also add passengers to the left branch.

specific model details of the cellular automaton model. First, however, let's have a look at empirical findings.

2 Investigation of Crowd Movement

Researchers from many disciplines – mostly traffic and civil engineers – have investigated pedestrian and crowd movement for planning and design purposes. [3, 4, 6, 7, 29, 30, 37, 39]

The following figure 3 shows a rough classification of empirical and experimental situations.

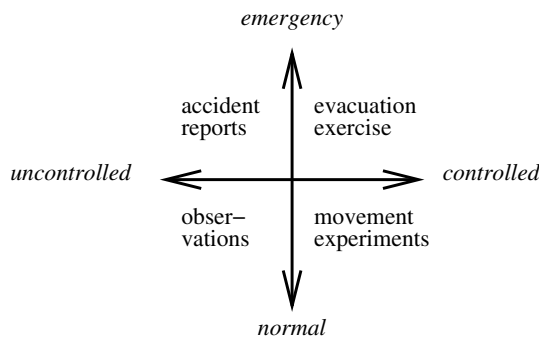


Fig. 3. Empirical data (including experiments) can be roughly classified according to controlled/uncontrolled and emergency/normal situations. Of course, there are other important criteria, like validity, reliability and objectivity. This becomes especially important for the uncontrolled situations, where those criteria can usually be questioned due to the lack of an operational definition of the situation. Whether an evacuation exercise can be called controlled depends largely on the number of repetitions.

Empirical in this context corresponds to observations (uncontrolled) and experimental to controlled situations.

2.1 Empirical Investigations

The following table 1 summarizes some of the major sources for data on pedestrian (single persons) and crowd (many persons) movement. In [14] the distribution for the directed walking speed (x -component) is Maxwellian and for its absolute value (velocity) – according to Maxwell’s theory for kinetic gases – Gaussian.

This summary is restricted to quantitative results and therefore mainly comprises data on walking speed, interpersonal distances and flow-density-relations.

Figure 4 shows the curve of a flow-density-relation [38]. It was obtained by averaging over several results for walk- and passageways. Such a curve is especially useful for model calibration [21]: The same curve can be obtained by simulating a long and narrow hallway for all accessible densities (0 to ρ_{\max} , where ρ_{\max} is the maximum density the model allows) [18, 23]. !

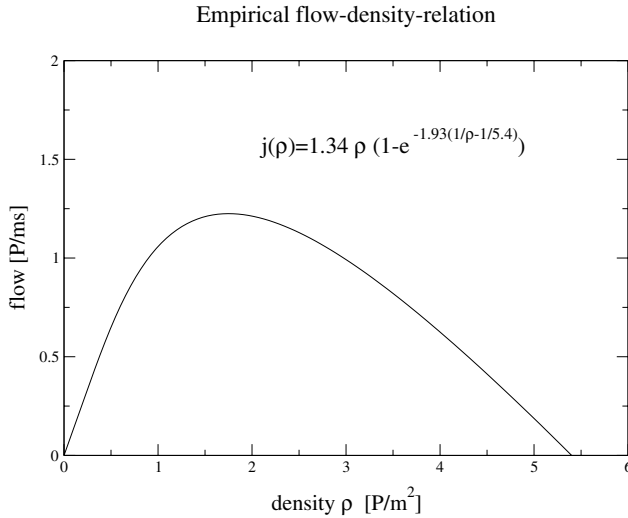


Fig. 4. Flow density relation for pedestrian movement. The analytical expression is shown at the top of the figure. The curve is a fit to empirical data [38].

For the case of stairs, the flow depends on the slope, of course. However, it does not seem to decrease significantly compared to flat terrain for the scenario depicted in fig. 5. This is probably due to the fact that stairs are equipped with handrails. However, the walking speed might well decrease (note that $j = \rho \cdot \langle v \rangle$) [2].

Table 1. Summary of the empirical data found in the literature. The results are described in more detail in the text. FWHM is short for Full width at half maximum (θ). If no explicit formula or type of distribution is given θ is used to characterize the width of the distribution (for the probability density it holds $f(\mu \pm \theta) = 1/2 f(\mu)$). Additional reviews for the walking speed on stairs can be found in [6] and for the flow on stairs and and surface level in [9].

Environment	Type of data	Main result
Walkways	frequency distr.	$\mu = 1.34 \text{ m/s}$, $\sigma = 0.26 \text{ m/s}$ [38]
Urban	frequency distr.	$\mu = 1.19 \text{ m/s}$, FWHM=0.21 m/s [7]
Campus	frequency distr.	$\mu = 1.53 \text{ m/s}$ [14]
Zebra crossing	frequency distr.	$\mu = 1.44 \text{ m/s}$ [14]
Walkways	flow vs. density	$\rho_{\max} = 5.4 \text{ P/m}^2$ [38]
Urban	speed vs. density	$j(\rho)$, $\rho \sim 1 \text{ P/m}^2$ [7]
Commuters	walking speed	$v(\text{age})$ [2]
School yard	walking speed	sexual differences [15]
Aircraft mockup	egress time	critical exit width [27]
Ships	behavior	panic is very rare [12]
Stair Mockup	upstairs/downstairs	$d_{\text{gap}} \geq 0.25 \text{ m}$, v_{\uparrow} , v_{\downarrow} [6]
Overviews		
Walkways/Urban		
		[7, 37, 38]
Buildings		
		[29, 30]

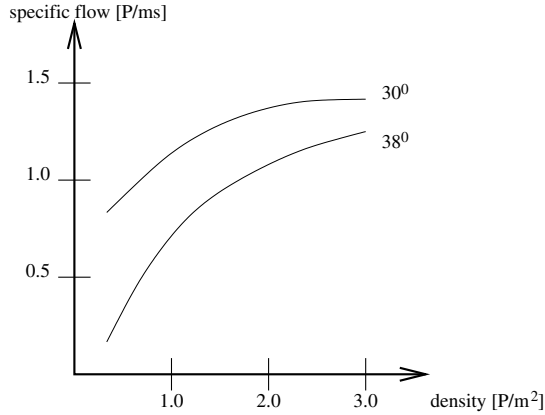


Fig. 5. Flow density relation for pedestrian movement on stairs. The dependence on the slope is most prominent for low densities, where steeper stairs perform worst. The experiments have been carried out in a Dutch football stadium. Additionally, the influence of the motivational level on the flow has been investigated [9].

We have also carried out studies on person flow at the world exhibition (EXPO) 2000 in Hannover, Germany. The frequency distribution for the walking speed on a (flat) pedestrian bridge is shown in fig. 6.

A normal distribution has been fit to the data using the mean and standard deviation of the empirical distribution as well as adapted values.⁵ The data and the fitted curves are shown in fig. 6. The mean value obtained was $\mu = \langle v_x \rangle = 1.30$ m/s and the standard deviation $\sigma = 0.21$ m/s. The third moment of the distribution $E(x^3 - \langle x^3 \rangle)$ is 0.41 (m/s)³. This shows that the distribution is not symmetric but slightly skewed towards the origin. The parameters for the second fit-curve shown in fig. 6 are $\mu = 1.28$ m/s and $\sigma = 0.2$ m/s.

A second aspect of this investigation is the dependence of the walking speed on the group size. Several persons were identified as a group if the distance between at least two of them was not larger than about 1 m, they walked at the same speed, and in the same ‘formation’, i.e., they actually formed a social group.

Table 2 shows the decrease of the walking speed with increasing group size. It is interesting to note that groups larger than 6 persons were not observed. Of course the statistics for the larger groups are less reliable since they rarely occurred.

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⁵ This differs from [14], where a Maxwell-Boltzmann distribution was used.

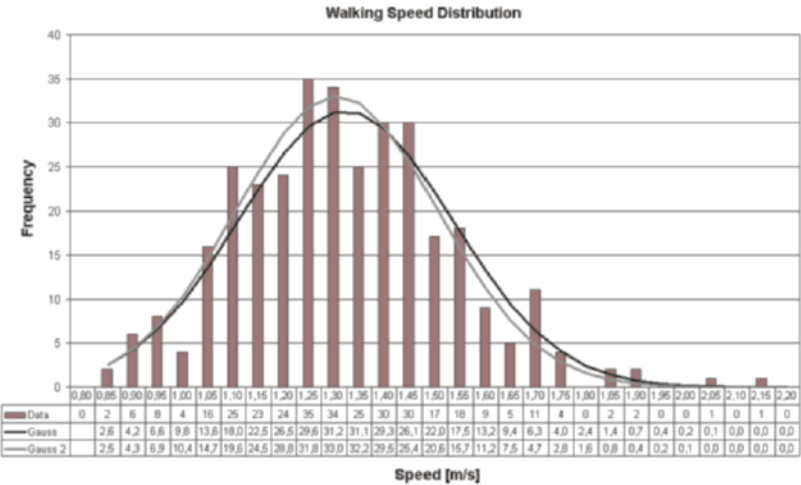


Fig. 6. Shown is the walking speed distribution for the pedestrian bridge at the World Exhibition (Expo) 2000 in Hannover. The length that was walked by the pedestrians was 15.5 m. A Gaussian distribution is fitted to the results with μ and σ obtained either from the sample or using adapted values where the medians of the fitted and empirical curves are closer to each other. The data is shown in the table below the horizontal axis. Groups are represented by one data point (cf. table 2).

Table 2. Walking speed vs. group size for the pedestrian bridge. The speed is obtained by dividing the distance of 7 m by the travel time, i.e., $\langle v_x \rangle$, if the ‘direction’ of the bridge is denoted x .

Group size	Number of groups	Mean Velocity
1	95	1.38
2	149	1.28
3	59	1.24
4	17	1.24
5	10	1.22
6	2	1.10
700		1.30

Table 2 shows the decrease of the walking speed with increasing group size. It is interesting to note that groups larger than 6 persons were not observed. Of course the statistics for the larger groups are less reliable since they rarely occurred.

Nevertheless, this information could be useful when integrating the influence of group size into a theory for crowd movmeent. At the moment there is – to our knowledge – no model that incorporates the influence of groups on the dynamics of crowd movement. (i.e., walking speed of groups vs. group

size). This could – as a first approximation – be done by reducing the walking speed according to the group size. Of course, this would also require knowledge about the division of the population into groups and the distribution of group sizes.

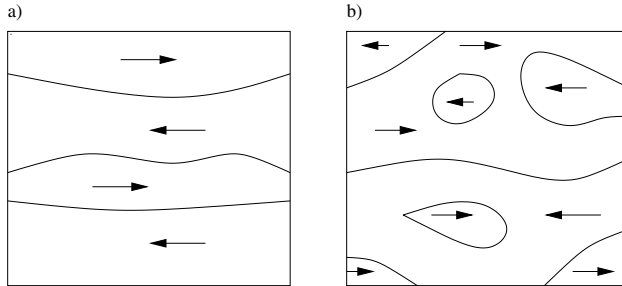


Fig. 7. A typical phenomenon in pedestrian movement is the formation of lanes (a) and clusters (b) with uniform movement direction. [41] has introduced a band index, which is basically the ratio of pedestrians in lanes to the overall number. For a) this would be nearly 1. High band indices have only been observed for large numbers of pedestrians (100 and above).

Finally, we would like to mention two phenomena that can be observed in moving crowds: lane formation [41] (cf. fig. 7 and clogging [28]. For a more detailed overview and discussion, please refer to [20].

2.2 Experiments

Hitherto, we have described empirical results (uncontrolled situations, cf. fig. 3. Another important source of knowledge are ewxperiments, of course. However, they are often expensive and complicated to carry out. A full scale trial for the evacuation of a Ro-Ro passenger ferry, e.g., cost about GBP 30,000 in 1999 [40].

An experiment on the evacuation of passenger aircraft is described in [27]. One of the major results, namely the existence of a minimal critical exit width, could quantitatively be reproduced in simulations! [19]. The empirical data are shown in fig. 8.

2.3 Accident Reports

Finally, data about behavior in a real emergency can usually only be obtained via accident reports. Sometimes, there are also CCTV (closed circuit television) recordings available.

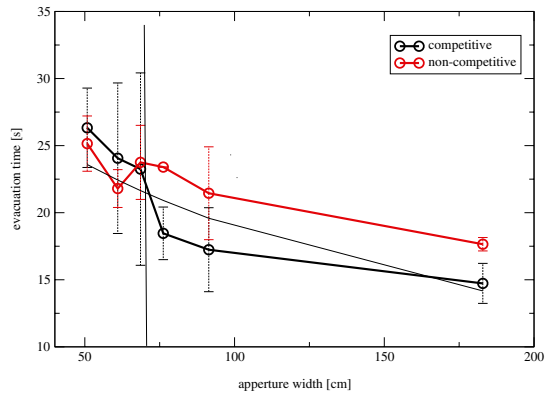


Fig. 8. The evacuation time increases with decreasing aperture width. For non-competitive situations, this decrease is rather smooth. However, if there is competition, at a certain aperture width ($w_c \approx 70$ cm) the increase in the egress time is quite drastic and the performance is worse than in the case of non-competition [27].

The awareness and response (pre-movement) times for complex buildings were investigated in [31]. The collapse of the World Trade Center in 2001 is subject of an intensive investigation by NIST [10].

Figure 9 shows the basic strategies of occupants in case of an emergency in complex buildings.

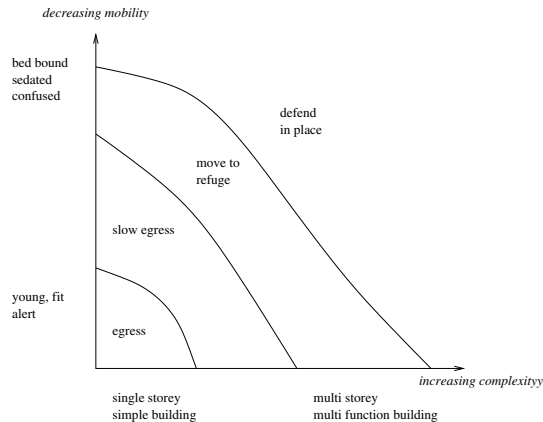


Fig. 9. Egress vs refuge in building evacuation: The more complex a building is and the less the mobility the more difficult is the egress from a building. This leads to a distinction between four different strategies: egress, slow egress, move to refuge, and defend in place [1].

3 Simulation of Crowd Movement

3.1 Basics

The method of choice to extent the results to scenarios not accessible (due to the aforementioned constraints) via observations and experiments are simulations.

On the other hand, looking at fig. 10 it is clear, that one cannot model crowd movement based on first principles (like it is possible for, e.g., Molecular Dynamics simulations).

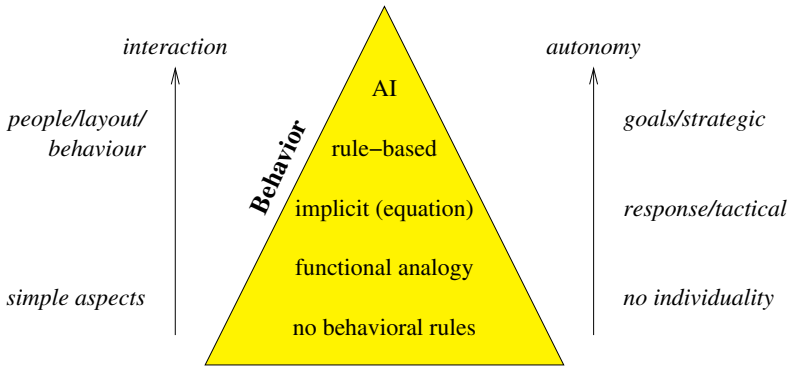


Fig. 10. The representation of behavior can range from its neglecting to artificial intelligence (AI), which aims at even modeling the decision making process [17]. However, the more complex approaches are not completely different from the basic ones but include additional features. Therefore, the autonomy and interaction increase first the top but are not completely absent and completely present in one case or the other. These qualities are rather a question of interpretation than of direct representation in a set of rules.

Once it is clear, that a model for crowd movement is phenomenological anyway, there seems to be no reason why not to choose the most simple approach one can come up with. However: As simple as possible but no simpler. This means – looking at fig. 10 – starting at the bottom and then adding complexity when necessary. The corresponding scenarios range from a single room or hallway to measure flow-density-relations (corresponding to “no individuality” in fig. 10), via structures with one floor and few exits (platforms, small buildings), to very complex scenarios (large arenas, cruise ships, airports, etc.) corresponding to “strategic” in fig. 10.

We will first use grid-based models as an example to specify some of the thoughts previously outlined.

3.2 Model Details – Grid Based Models

The scenarios considered in an investigation of pedestrian dynamics can be classified according to several criteria. These criteria are summarized in fig. 11.

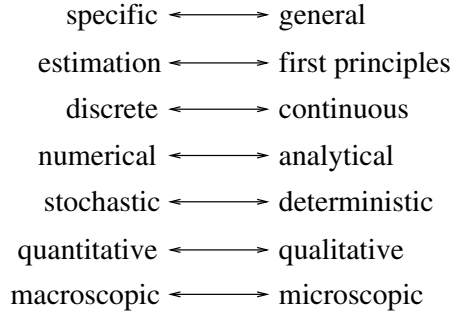


Fig. 11. Modeling criteria that can be used for classifying different theories and models [8]. The major choices for models and simulations of crowd movement are discrete vs. continuous and stochastic vs. deterministic.

According to these modelling criteria, a grid-based model (or cellular automaton, cf. table. 3 is microscopic, general, quantitative, and usually stochastic.

Table 3. Definition of a cellular automaton. The assumption of a regular lattice and a uniform neighborhood is in accordance with complex geometries, since the set of states S also contains information about whether a cell is accessible or not (i.e., a wall cell, w in table 4).

Definition	Description
\mathcal{L}	consists of a regular discrete lattice of cells
$t \rightarrow t + 1$	evolution takes place in discrete time steps
S	set of ‘finite’ states
$f : S^n \rightarrow S$	each cell evolves according to the same rule (transition function) which depends only on the state of the cell, and a finite number of neighboring cells
$\mathcal{N} : \forall c \in \mathcal{N}, \forall r \in \mathcal{L} : r + c \in \mathcal{L}$	the neighborhood relation is local and uniform

For the ease of the reader, table 4 summarizes the characteristics of the *PedGo* model [21, 24–26, 36]. Of course, the criteria shown in table 3 apply also.

Table 4. Assumptions for the model and the empirical correlate. The symbols are explained in the text. The parameters can vary and the values given are typical ones.

empirical	model
orientation at exit signs	$V(r) \sim d(r, r_{\text{exit}})$
$\rho_{\text{max}} = 6.25\text{P/m}^2$	$a = 0.4\text{ m}$
$v_{\text{max}}^{\text{emp}} \approx 2\text{ m/s} \wedge \Delta t \approx 1\text{ s}$	$v_{\text{max}}^{\text{mod}} = 5$
stopping due to orientation	$p_{\text{dec}} = 0 \dots 0.1$
deviations from the optimal direction	$p_{\text{sway}} = 0 \dots 0.03$
walls are ‘black’ cells	$w = 1$
hard core exclusion	$o \in \{0, 1\}$

3.3 Other Models

We have already mentioned (in the introduction) that crowd movement can be mapped onto transportation processes. In this context, the connection between transportation processes and spin models might well become of interest in the future also for such phenomena like lane formation, shock waves, or jamming. Especially, the connection between person flows and granular materials research could be fruitful. This connection is briefly outlined in fig. 12.

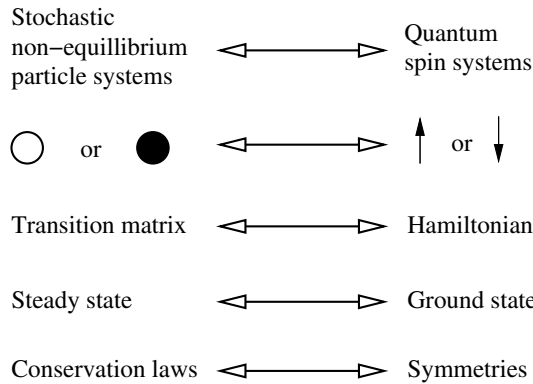


Fig. 12. Correspondence between stochastic non-equilibrium particle systems and quantum many body systems. For single-species exclusion processes the mapping can be to spin systems, as indicated in the second row [35].

3.4 Calibration

Concerning the calibration of the basic properties of crowd movement, there are three major issues:

- Maximum density
- Walking speed
- Time scale (decision making or reaction time) ⁶

For the case of grid-based models, these issues have been investigated by varying cell size and walking speed in [18].

3.5 Full Scale Trials – Verification

Finally, real world scenarios comprise many influences, which usually cannot be separated into isolated factors. Therefore, it is also necessary to reproduce the results of full scale trials in a simulation, where these factors influence each other and might lead to phenomena not present in less complex situations. An illustrative example is counterflow, which will – due to the lack of space – not be elaborated at this point, however. The interested reader might refer to [16, 32].

3.6 The Micro-Macro Link

Microscopic models might not be able to cover all aspects of crowd movement, even when neglecting most of the psychological and social influences (cf. section 1). The most prominent example in this context is presumably route-choice, which is a strategic (cf. fig. 10) and non-local (cf. fig. 3, bottom and fig. 13) task. Figure 13 (right) exemplifies an elegant solution of the route-choice problem within the microscopic framework: A graph superstructure is defined to represent the topology of the floorplan. Then, standard graph theoretical algorithms (e.g., Dijkstra's) can be applied to find the shortest path using the appropriate weights. These weights are set to represent the scenario under consideration (e.g., distance, familiarity, availability, crew influence, etc.).

For more and general information on network-flow approaches to the evacuation problem, please refer to [11].

⁶ Please note that we used the terms awareness and response time in section 2.3 to quantify the pre-movement phase, which is in accordance with the generally accepted terminology [17].

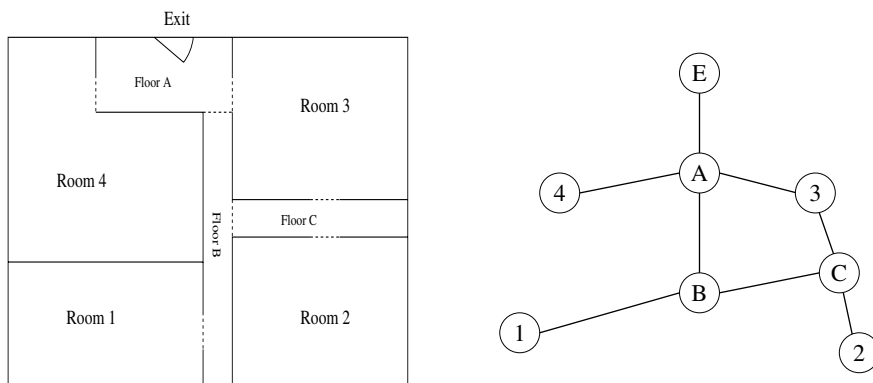


Fig. 13. A simple geometry and its graphical representation.

4 Summary and Conclusions

In this paper, the basic properties of a specific grid based model for simulating the dynamics of crowd movement was described. The focus was mainly on egress simulation. However, the model is applicable also to non-emergency situations.

When comparing the discrete space (and time) model to other similar and especially continuous models, it becomes clear that the differences are subtle. It depends mainly on the application one has in mind, what the preferable model or model variant is. However, I am aware of the fact, that this statement is controversial and other researchers argue, that models for pedestrian dynamics can be universal in the sense that there is no adaption of the parameters necessary for different scenarios.

Acknowledgements

We would like to thank Michael Schreckenberg and the members of his research team (*Physics of Transport and Traffic, Duisburg, Germany*) for many fruitful discussions and their extensive support in general. My (HK's) special thanks go to Andreas Schadschneider and Ansgar Kirchner (formerly) at *Cologne University, Germany* and Katsuhiro Nishinari *Ryukoku University, Shiga, Japan*).

References

1. J. Abrahams. Fire escape in difficult circumstances. In P. Stollard and L. Johnson, editors, *Design against fire*, London, New York, 1994. SFPE.

2. K. Ando, H. Ota, and T. Oki. Forecasting the flow of people. *Railway Research Review*, 45:8–14, 1988. (in Japanese).
3. D. Canter, editor. *Fires and Human Behaviour*. David Fulton Publishers, London, 2nd edition, 1990.
4. P. DiNenno, editor. *SFPE Handbook of Fire Protection Engineering*. National Fire Protection Association, Washington, 2nd edition, 1995.
5. D. R. Forsyth. *Group Dynamics*. Wadsworth Publishing, Belmont, CA, 3 edition, 1999.
6. H. Frantzich. Study of movement on stairs during evacuation using video analysing techniques. Technical report, Department of Fire Safety Engineering, Lund Institute of Technology, Lund University, 1996.
7. J. Fruin. *Pedestrian Planning and Design*. Metropolitan Association of Urban Designers and Environmental Planners, New York, 1971.
8. N. Gershenfeld. *The Nature of Mathematical Modelling*. Cambridge University Press, Cambridge, 1999.
9. E. Graat, C. Midden, and P. Bockholts. Complex evacuation; effects of motivation level and slope of stairs on emergency egress time in a sports stadium. *Safety Science*, 31:127–141, 1999.
10. W. Grosshandler, S. Sunder, and J. Snell. Building and fire safety investigation of the world trade center disaster. In E. Galea, editor, *Pedestrian and Evacuation Dynamics 2003*, pages 279–281, London, 2003. University of Greenwich, CMS press. <http://wtc.nist.gov>.
11. H. Hamacher and S. Tjandra. Mathematical modelling of evacuation problems – a state of the art. In Schreckenberg and Sharma [32], pages 227–266.
12. J. Harbst and F. Madsen. The behaviour of passengers in a critical situation on board a passenger vessel or ferry. Technical report, Danish Investment Foundation, Copenhagen, 1996.
13. D. Helbing, L. Buszna, and T. Werner. Self-organized pedestrian crowd dynamics and design solutions. <http://www.helbing.org>, Dec 2003.
14. L. Henderson. The statistics of crowd fluids. *Nature*, 229:381–383, 1971.
15. L. Henderson. Sexual differences in human crowd motion. *Nature*, 240:353–355, 1972.
16. IMO, London. *Interim Guidelines for Evacuation Analyses for New and Existing Passenger Ships*, 2002. MSC/Circ. 1033.
17. ISO. Fire safety engineering. Technical Recommendation TR 13387, International Organization for Standardization, Geneva, 1999.
18. A. Kirchner, H. Klüpfel, K. Nishinari, A. Schadschneider, and M. Schreckenberg. Discretization effects and influence of walking speed in cellular automata models for pedestrian dynamics. *In preparation for Journal of Statistical Mechanics: Theory and Experiment (JSTAT)*, 2004.
19. A. Kirchner, K. Nishinari, and A. Schadschneider. Friction effects and clogging in a cellular automaton model for pedestrian dynamics. *Phys. Rev. E*, 67:056122, 2003. cond-mat/0209383.
20. H. Klüpfel. *A Cellular Automaton Model for Crowd Movement and Egress Simulation*. PhD thesis, University Duisburg–Essen, 2003. <http://www.ub.uni-duisburg.de/ETD-db/theses/available/duett-08012003-092540>.
21. H. Klüpfel and T. Meyer-König. Simulation of the evacuation of a football stadium. In Schreckenberg and Sharma [32].

22. H. Klüpfel, T. Meyer-König, and M. Schreckenberg. Microscopic modelling of pedestrian motion – comparison of simulation results with an evacuation exercise in a primary school. In M. Fukui, Y. Sugiyama, M. Schreckenberg, and D. Wolf, editors, *TGF '03*, Berlin, 2003. Springer.
23. H. Klüpfel, T. Meyer-König, J. Wahle, and M. Schreckenberg. Microscopic simulation of evacuation processes on passenger ships. In S. Bandini and T. Worsch, editors, *ACRI 2000*, pages 63–71, London, 2000. Springer.
24. T. Meyer-König. Mikroskopische Simulation von Evakuierungsprozessen. *VDI Technische Überwachung*, 43(9):10–13, September 2002. available for download at www.traffgo.com.
25. T. Meyer-König, H. Klüpfel, A. Keßel, and M. Schreckenberg. Simulating mustering and evacuation processes onboard passenger vessels: Model and applications. In *The 2nd International Symposium on Human Factors On Board (ISHFOB)*, 2001. CD-Rom.
26. T. Meyer-König, H. Klüpfel, and M. Schreckenberg. Assessment and analysis of evacuation processes on passenger ships by microscopic simulation. In Schreckenberg and Sharma [32], pages 297–302.
27. H. Muir. Effects of motivation and cabin configuration on emergency aircraft evacuation behavior and rates of egress. *Intern. J. Aviat. Psych.*, 6(1):57–77, 1996.
28. K. Müller. Die Evakuierung von Personen aus Gebäuden – nach wie vor ein nationales und internationales Problem. *vfd-Zeitschrift*, 3:131, 1999.
29. J. Pauls. Movement of people. In DiNenno [4], pages 3–263—3–285.
30. W. Predtetschenski and A. Milinski. *Personenströme in Gebäuden – Berechnungsmethoden für die Modellierung*. Müller, Köln-Braunsfeld, 1971.
31. G. Proulx. Evacuation time and movement in apartment buildings. *Fire Safety Journal*, 24:229–246, 1995.
32. M. Schreckenberg and S. Sharma, editors. *Pedestrian and Evacuation Dynamics*, Berlin, 2002. Springer.
33. J. Sime. The concept of panic. In Canter [3], chapter 5, pages 63–82.
34. R. Smith and J. Dickie, editors. *Engineering for Crowd Safety*. Elsevier, Amsterdam, 1993.
35. R. Stinchcombe. Stochastic non-equilibrium systems. *Advances in Physics*, 50(5):431–496, 2001.
36. TraffGo GmbH, Duisburg. *PedGo Users' Manual*, 2004. www.traffgo.de.
37. Transportation Research Board, Washington, D.C. *Highway Capacity Manual*, 1994.
38. U. Weidmann. Transporttechnik der Fußgänger. Schriftenreihe des IVT 90, ETH Zürich, 1992. (in German).
39. U. Weidmann. Grundlagen zur Berechnung der Fahrgastwechselzeit. Schriftenreihe des IVT 106, ETH Zürich, Juni 1995.
40. A. Wood. *Validating Ferry Evacuation Standards*, 1997. Available from MCA, 105 Commercial Road, Southampton.
41. K. Yamori. Going with the flow: Micro-macro dynamics in the macrobehavioral patterns of pedestrian crowds. *Psychological Review*, 105(3):530–557, 2001.