ENSC 2113
Engineering Mechanics: Statics

Lecture 30
Section 10.4

College of Engineering, Architecture & Technology

10.4: *Moment of Inertia* by Composite Bodies

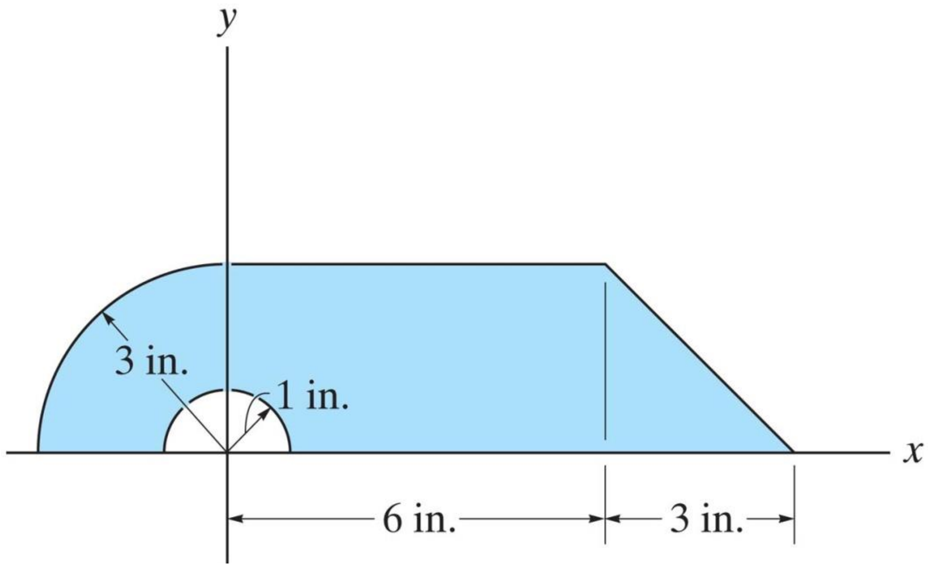
The *moment of inertia* of a shape can be found using composite bodies - A series of recognized geometric shapes that make up a section.

To determine the *Moment of Inertia* in these cases, use

The Parallel-Axis Theorem

$$I_x = \bar{I}_{x'} + Ad_y^2$$

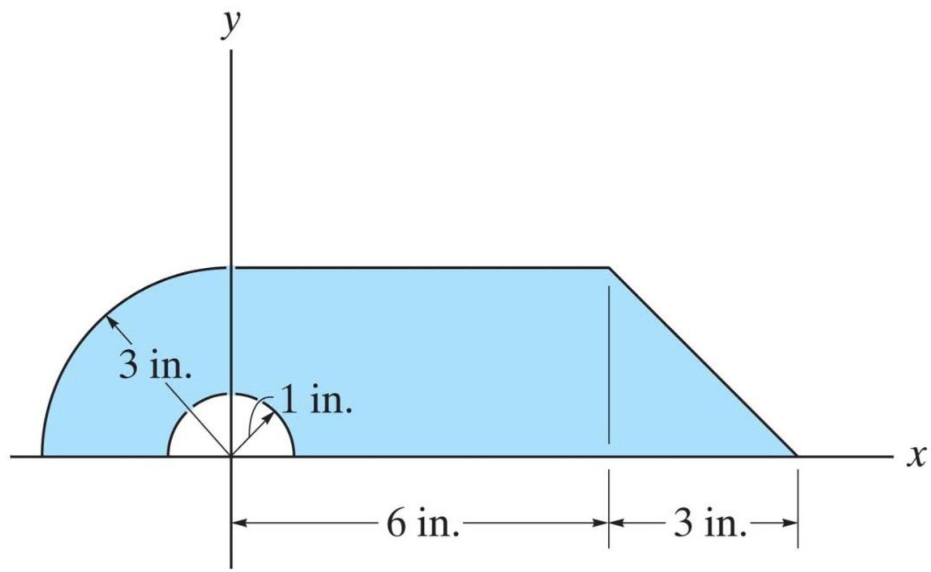
$$I_y = \bar{I}_{y'} + Ad_x^2$$



Definitions of the values in the eqns:

$$\boxed{I_x = \bar{I}_{x'} + A d_y^2}$$

$$\boxed{I_y = \bar{I}_{y'} + A d_x^2}$$



$\boldsymbol{I_x}$, $\boldsymbol{I_y}$ = *moment of inertia* about the axis of interest

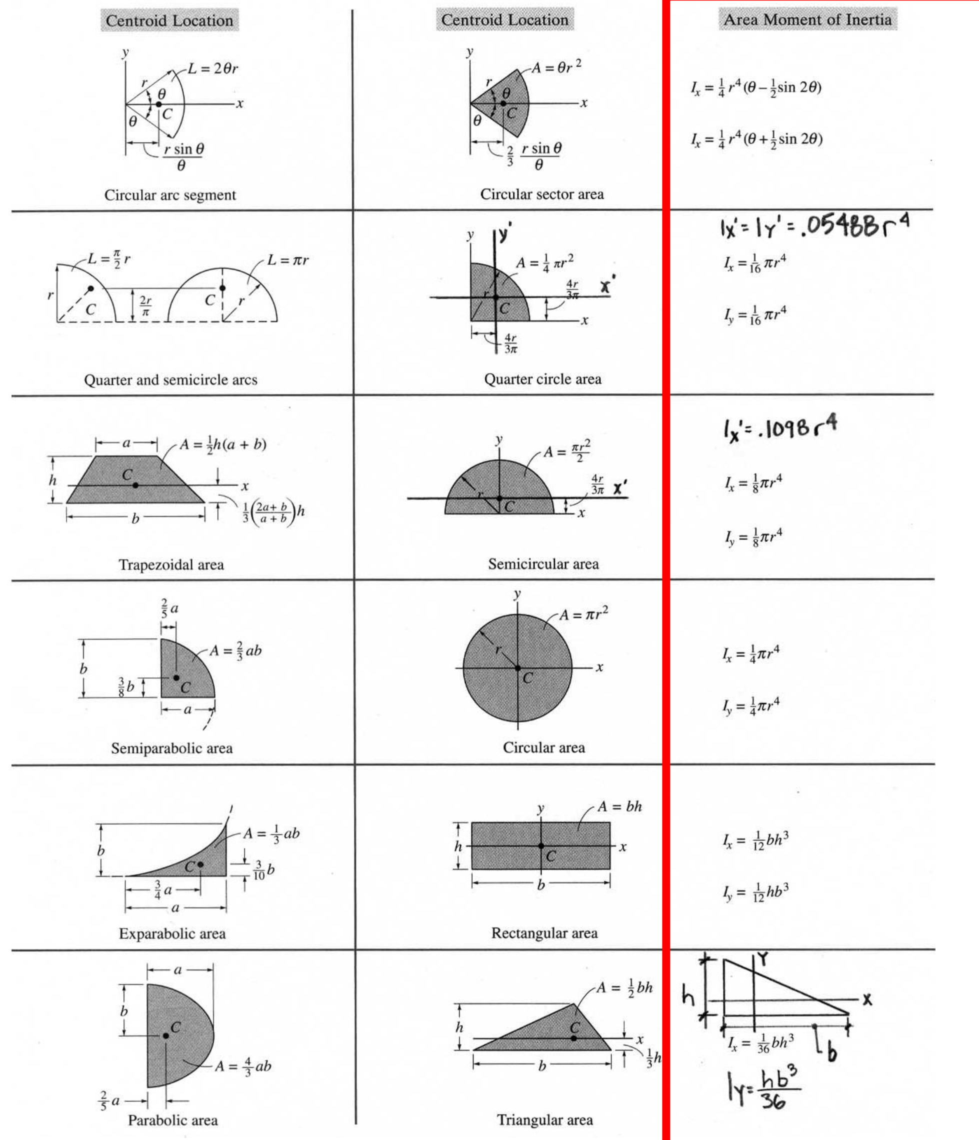
$\boldsymbol{\bar{I}_{x'}}$, $\boldsymbol{\bar{I}_{y'}}$ = *moment of inertia* about the centroid of the shape

A = Area of the shape

$\boldsymbol{d_x}$, $\boldsymbol{d_y}$ = distance from centroid of shape to axis of interest

Moment of Inertia about the centroid of a shape, $\bar{I}_{x'}$, $\bar{I}_{y'}$:

Refer to the inside cover of the back of your text for eqns.



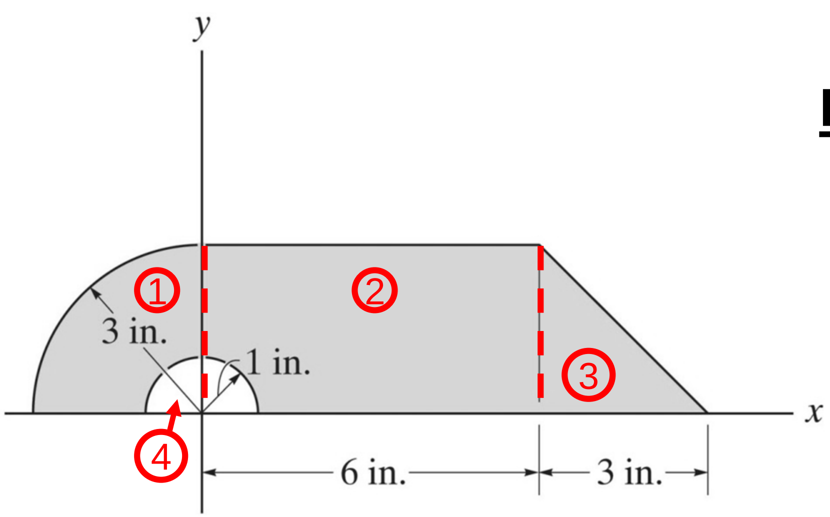
Geometric Properties of Line and Area Elements

Centroid Location	Centroid Location	Area Moment of Inertia
$L = 2\theta r$; $\frac{r\sin\theta}{\theta}$ Circular arc segment	$A = \theta r^2$; $\frac{2}{3}\frac{r\sin\theta}{\theta}$ Circular sector area	$I_x = \frac{1}{4}r^4(\theta - \frac{1}{2}\sin 2\theta)$ $I_x = \frac{1}{4}r^4(\theta + \frac{1}{2}\sin 2\theta)$
$L = \frac{\pi}{2}r$; $L = \pi r$; $\frac{2r}{\pi}$ Quarter and semicircle arcs	$A = \frac{1}{4}\pi r^2$; $\frac{4r}{3\pi}$; y'; x' Quarter circle area	$I_{x'} = I_{y'} = .05488 r^4$ $I_x = \frac{1}{16}\pi r^4$ $I_y = \frac{1}{16}\pi r^4$
$A = \frac{1}{2}h(a+b)$; $\frac{1}{3}\left(\frac{2a+b}{a+b}\right)h$ Trapezoidal area	$A = \frac{\pi r^2}{2}$; $\frac{4r}{3\pi}$; x' Semicircular area	$I_{x'} = .1098 r^4$ $I_x = \frac{1}{8}\pi r^4$ $I_y = \frac{1}{8}\pi r^4$
$A = \frac{2}{3}ab$; $\frac{2}{5}a$; $\frac{3}{8}b$ Semiparabolic area	$A = \pi r^2$ Circular area	$I_x = \frac{1}{4}\pi r^4$ $I_y = \frac{1}{4}\pi r^4$
$A = \frac{1}{3}ab$; $\frac{3}{4}a$; $\frac{3}{10}b$ Exparabolic area	$A = bh$ Rectangular area	$I_x = \frac{1}{12}bh^3$ $I_y = \frac{1}{12}hb^3$
$A = \frac{4}{3}ab$; $\frac{2}{5}a$ Parabolic area	$A = \frac{1}{2}bh$; $\frac{1}{3}h$ Triangular area	$I_x = \frac{1}{36}bh^3$ $I_Y = \frac{hb^3}{36}$

Eqns to use

<u>Procedures for determining *Moment of Inertia*</u>:

1. Divide shape into geometric parts & indicate perpendicular distance from the centroid of each part to axis of interest.
2. Determine *moment of inertia* for each part at its centroidal axis – ***Use equation on inside of back cover of text***.
3. Determine the area of each of the composite parts.
4. Set up a tabular form to keep track of the values.
5. Sum appropriate values to obtain the *Moment of Inertia*.



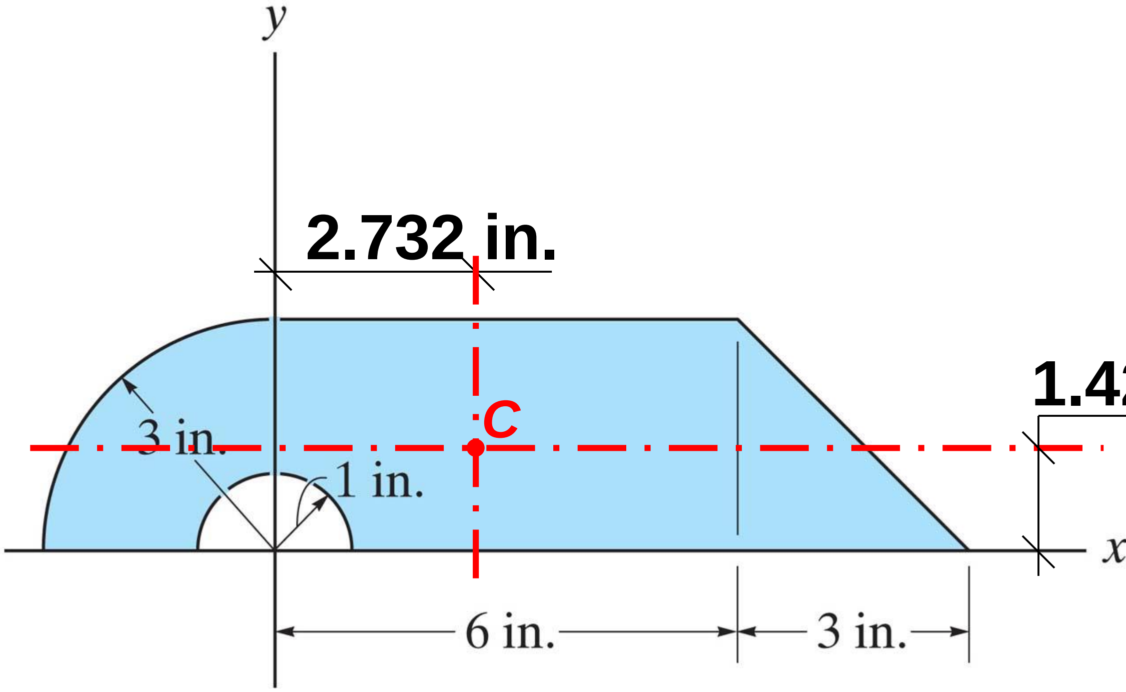
<u>Break shape into 4 elements:</u>

- Quarter circle
- Rectangle
- Triangle
- Semi-circle (Void)

Location of Axis of Interest:

The *moment of inertia* can be found using the *Parallel-axis Theorem* at **any** axis that is parallel to a centroidal axis.

Often, the *moment of inertia* is found about the centroidal axis of a shape:



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