ENIGNEERING COMPUTATIONAL FLUID DYNAMICS (ECFD)

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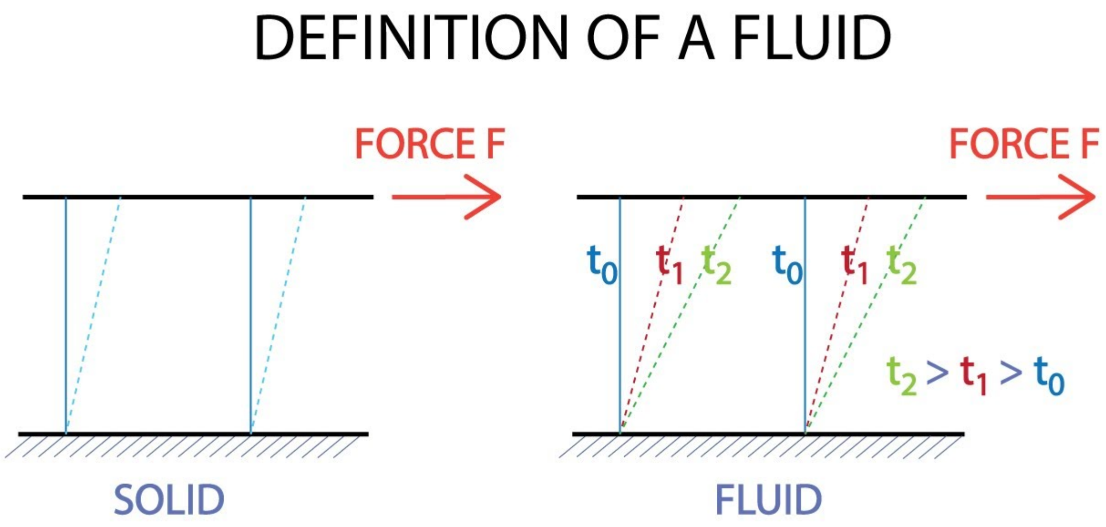
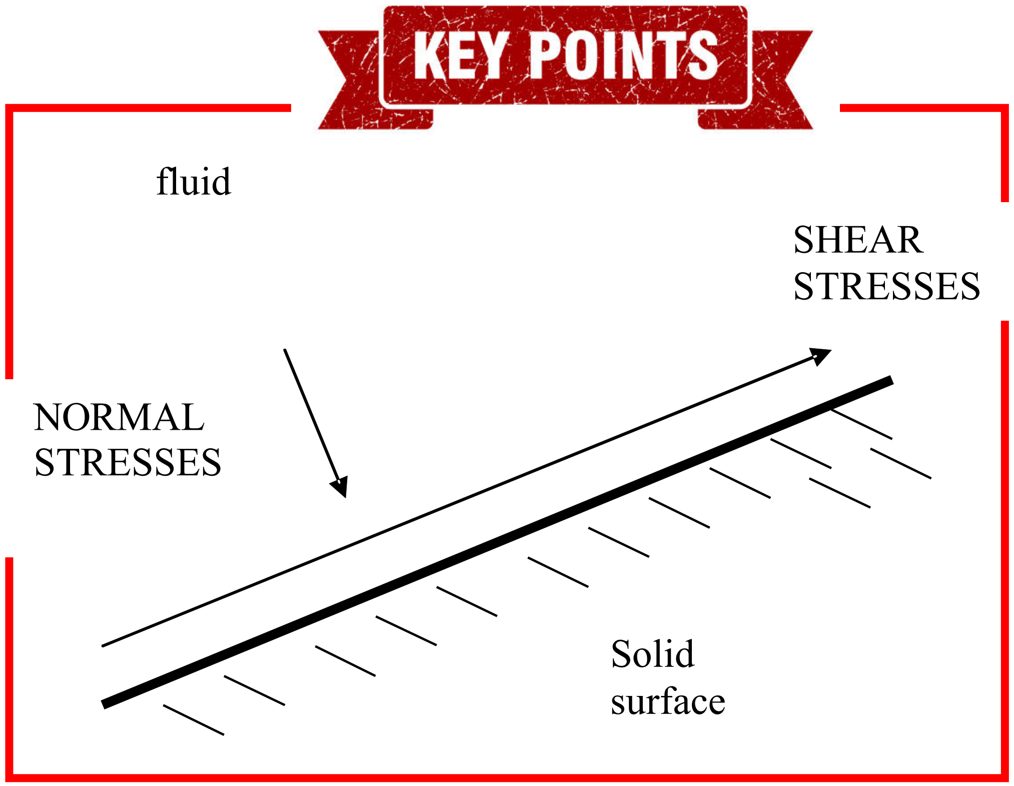
Some fundamental knowledge

FLUID MECHANICS FUNDAMENTALS

Fluid

A substance which **deforms continuously** when acted by a **shear stress of any magnitude**.

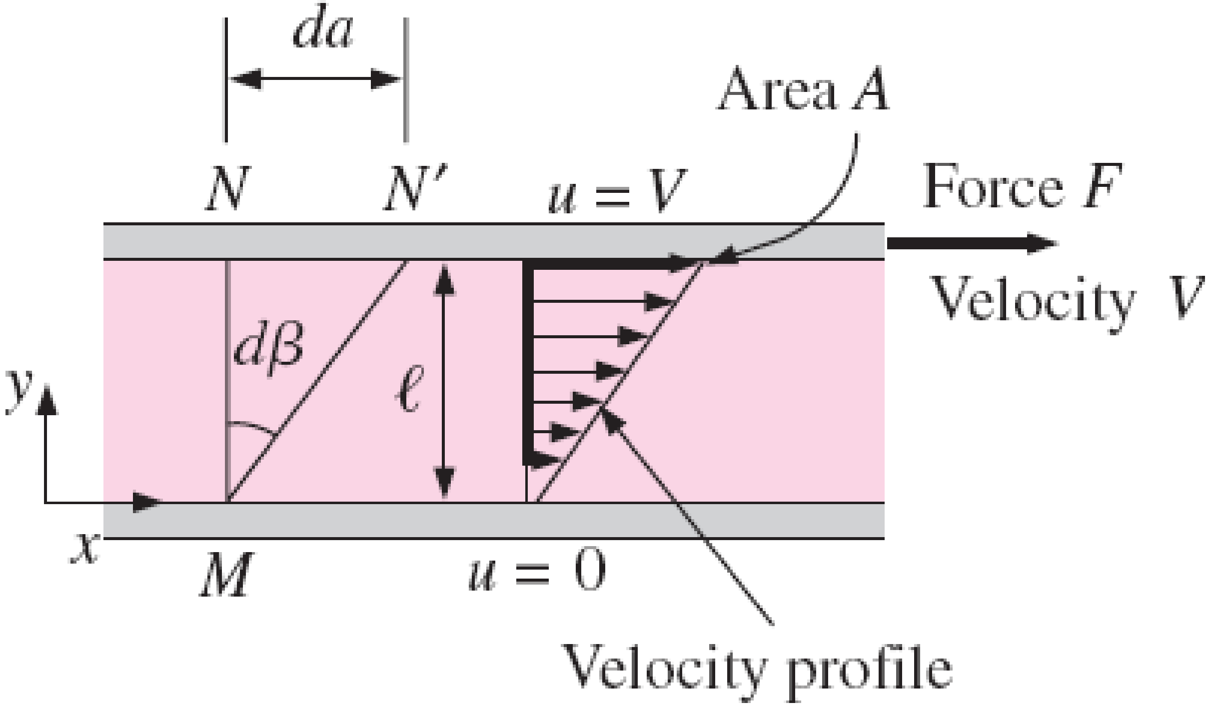


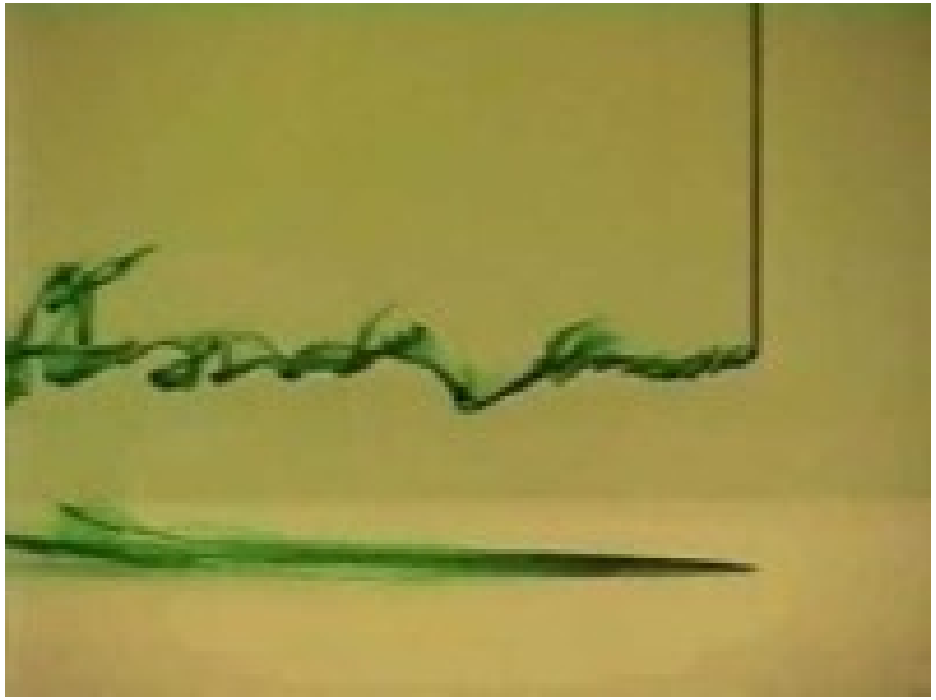


The non-slip condition

KEY POINTS

- When fluid contacts a solid surface, it stick to the surface and there is no slip of the fluid relative to the surface **– zero relative velocity**.
- If the surface is fixed, the fluid immediately adjacent to the surface must have zero velocity - for a high-speed stream moving past a fixed boundary, this means a high rate of shearing of the fluid in the boundary layer.

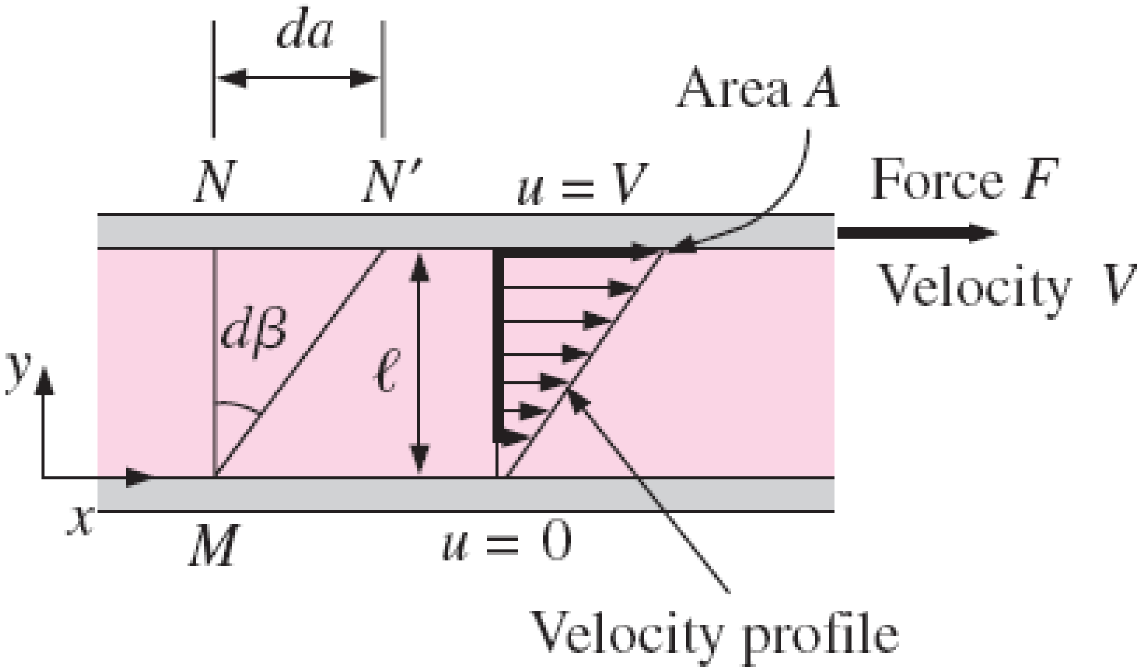




Viscosity

Viscosity and shear stress

KEY POINTS



Shear force

$$F = \tau A$$

$$\tau \propto \frac{d(d\beta)}{dt} \qquad \text{or} \qquad \tau \propto \frac{du}{dy}$$

shear strain rate

Shear stress

$$\tau = \mu \frac{du}{dy} \qquad (\mathrm{N/m^2})$$

Dynamic viscosity

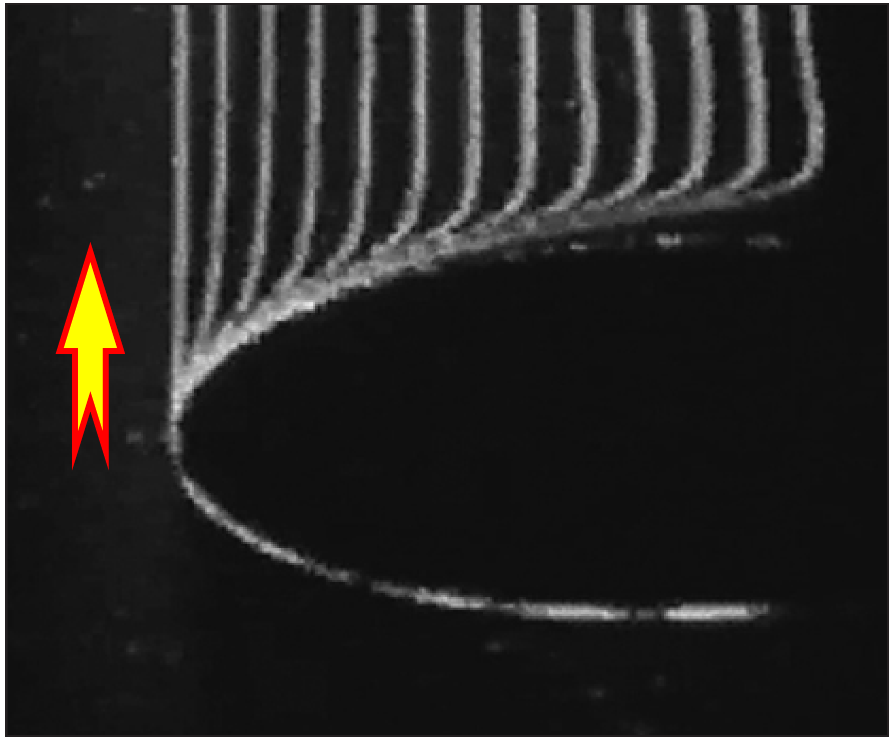
$$\left[\frac{\mathrm{kg}}{\mathrm{m \cdot s}} = \frac{\mathrm{N \cdot s}}{\mathrm{m^2}} = \mathrm{Pa \cdot s}\right]$$

$$\mu = \rho \nu$$

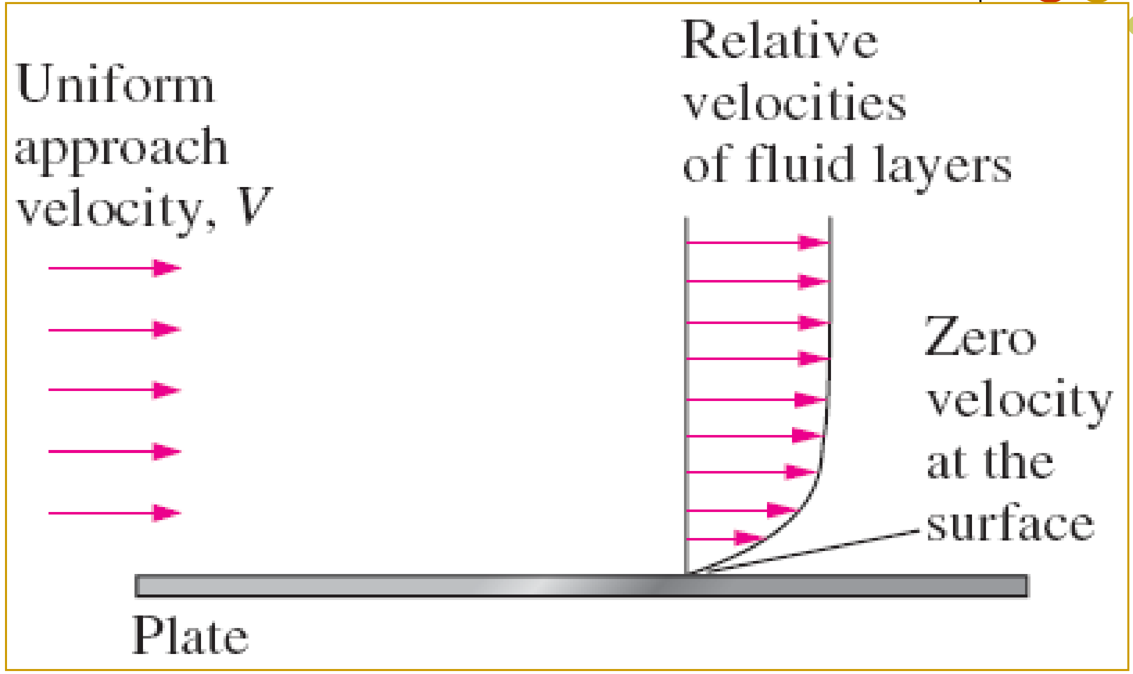
Kinematic viscosity

$$\left[\frac{\mathrm{m^2}}{\mathrm{s}}\right]$$

Boundary layer



The development of a velocity profile due to the no-slip condition as a fluid flows upwards over a blunt nose.



A fluid flowing over a stationary surface comes to a complete stop at the surface because of the no-slip condition.

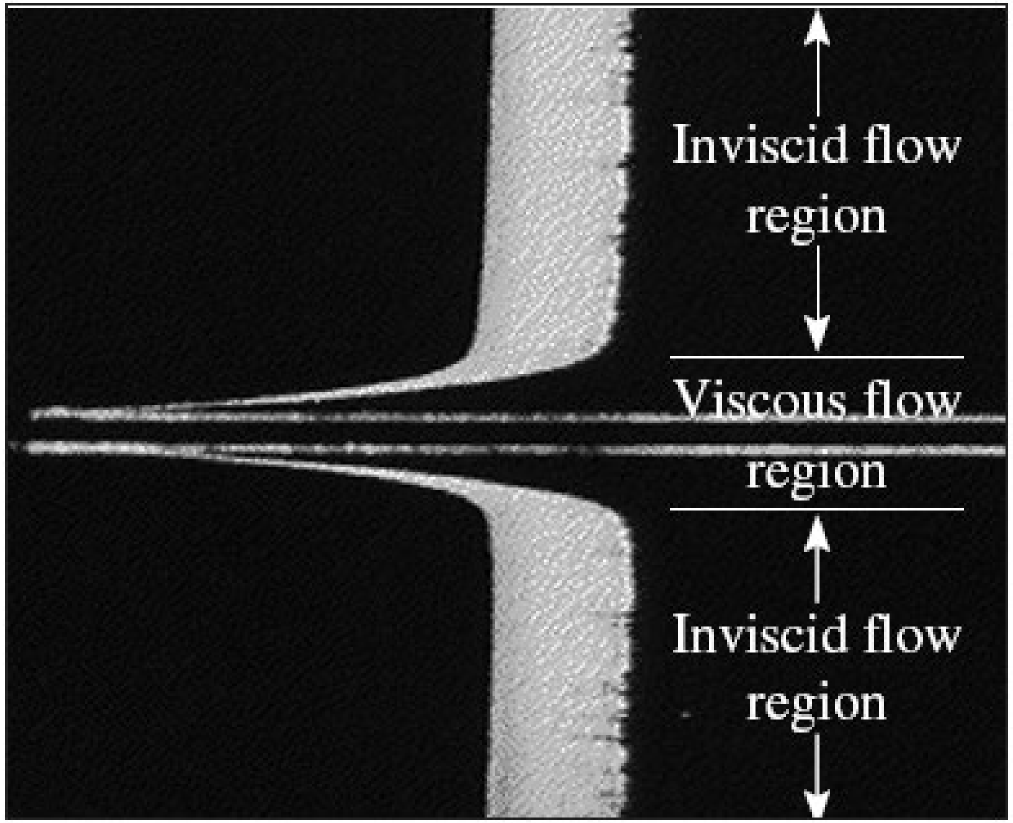
Boundary layer: The flow region adjacent to the wall in which the velocity gradients are significant. (99% mainstream velocity?)

KEY POINTS

Viscous and non-viscous flows

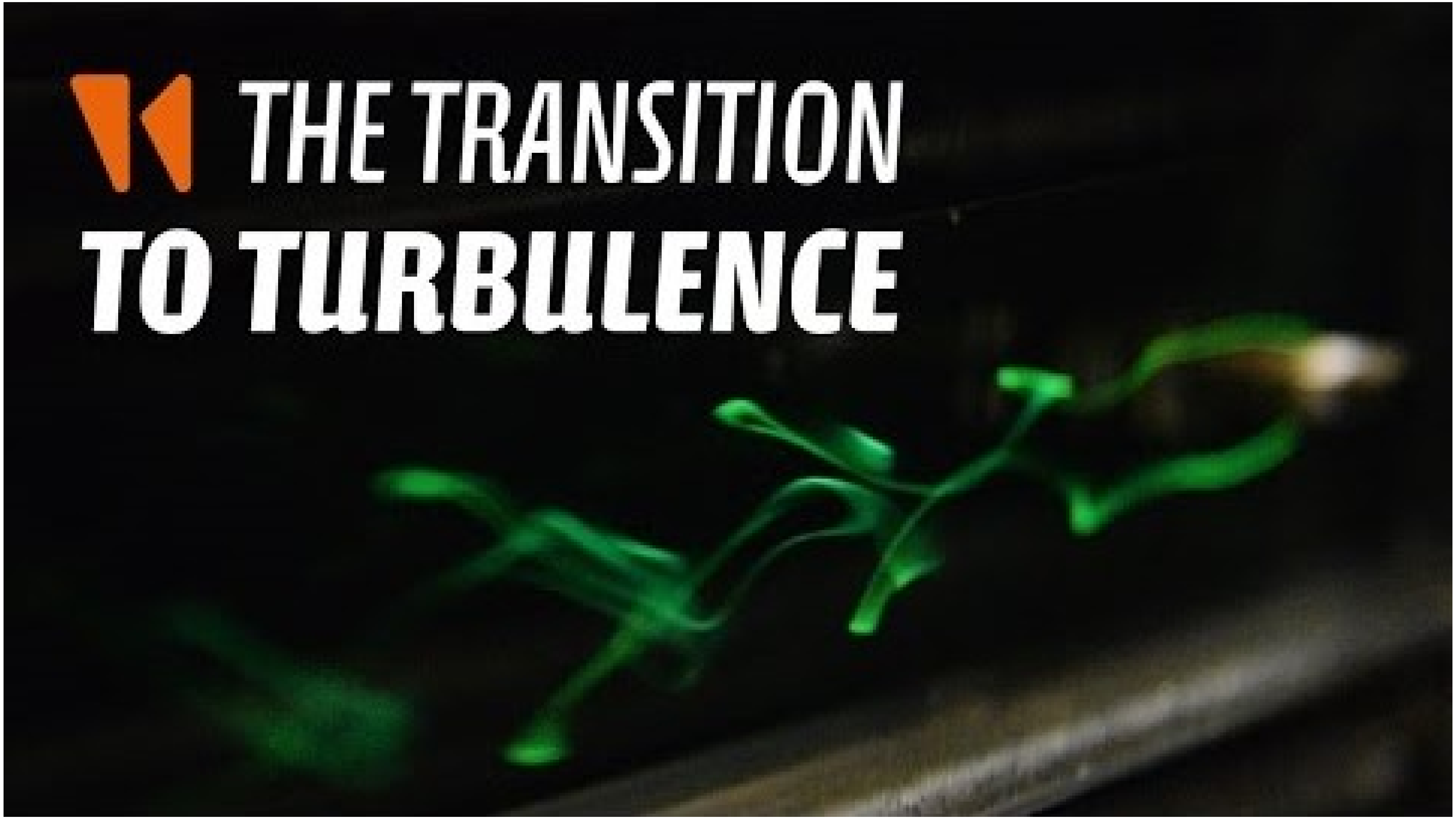
KEY POINTS

- **Viscous flows** - Fluids are very viscous or they are subject to high shearing rates, viscosity influences the flow behaviour and must be included in analysis.
- **Inviscid flows** - Fluids with low viscosity or not subject to high shearing rates are often not influenced much by viscous effects, which are therefore sometimes ignored.



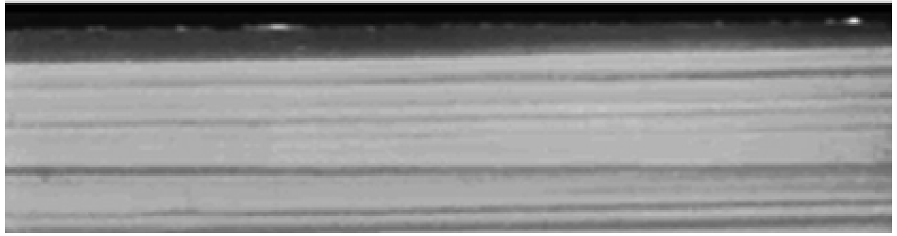
$$\tau = \mu \frac{du}{dy} \qquad (\mathrm{N/m^2})$$

Laminar and turbulent flows



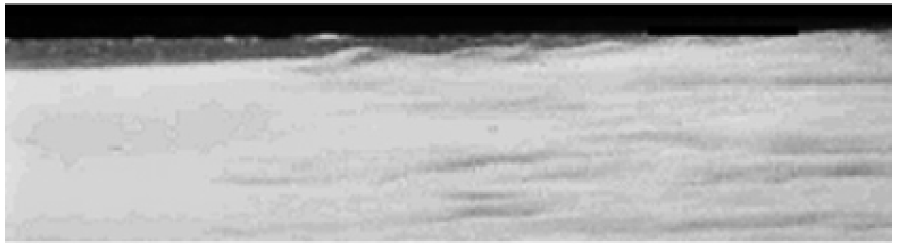
Laminar and turbulent flows

- **Laminar flow:** The highly ordered fluid motion characterized by smooth layers of fluid.



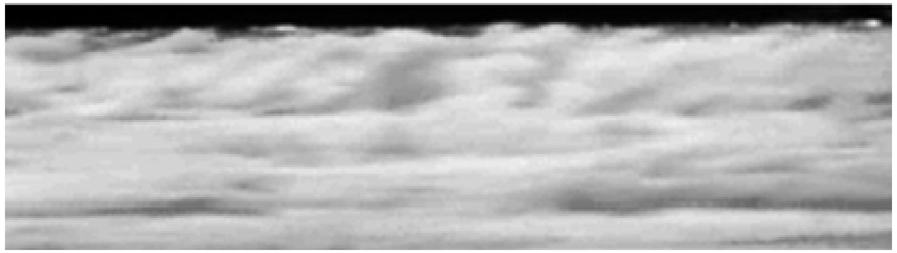
Laminar

- **Transitional flow:** A flow that alternates between being laminar and turbulent.



Transitional

- **Turbulent flow:** The highly disordered fluid motion that is characterized by velocity fluctuations.



Turbulent

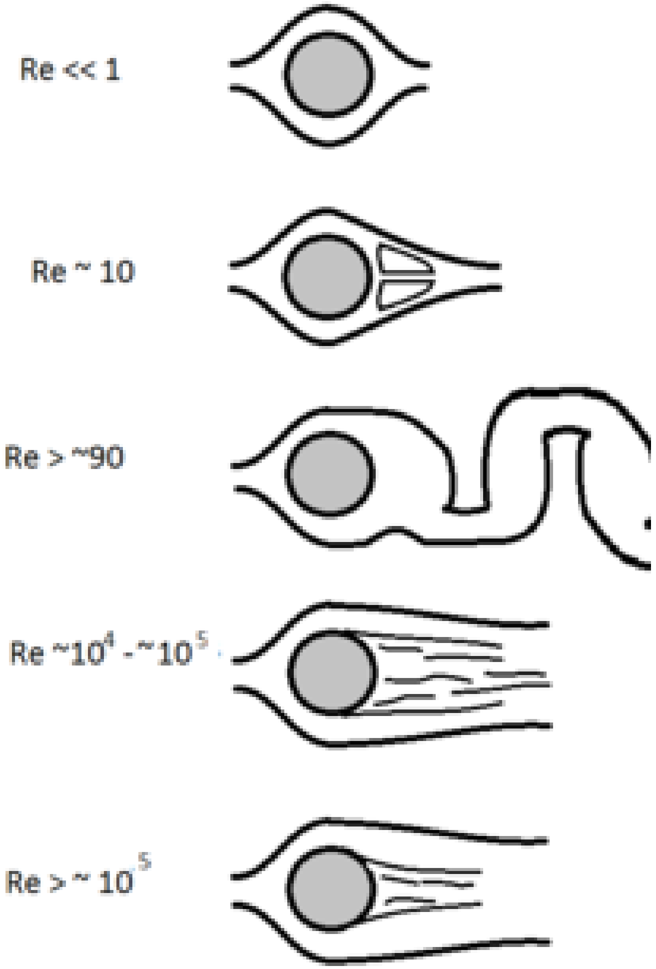
The Reynolds number

KEY POINTS

$$Re = \frac{uL}{\nu} = \frac{u\rho L}{\mu}$$

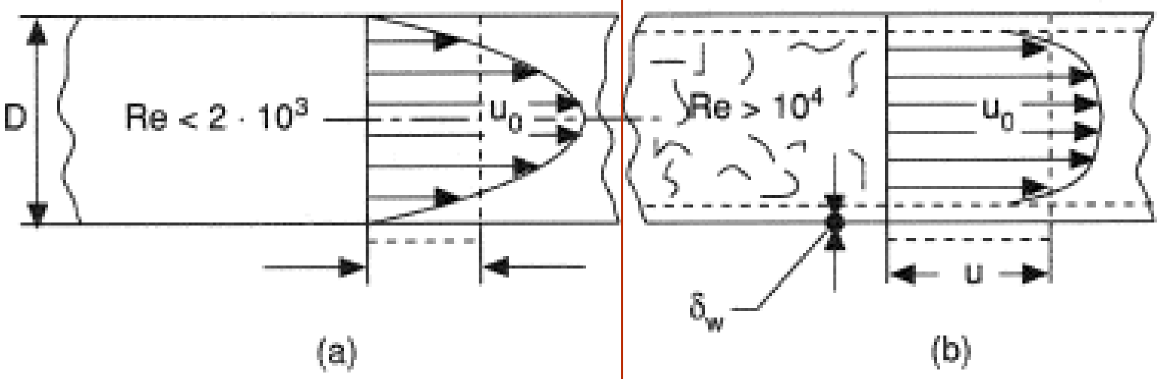
The Reynolds number is the ratio of inertial force to viscous force within a fluid which is subjected to relative internal movement due to different fluid velocities.

Think about the effects of L and μ?



Laminar and turbulent flows

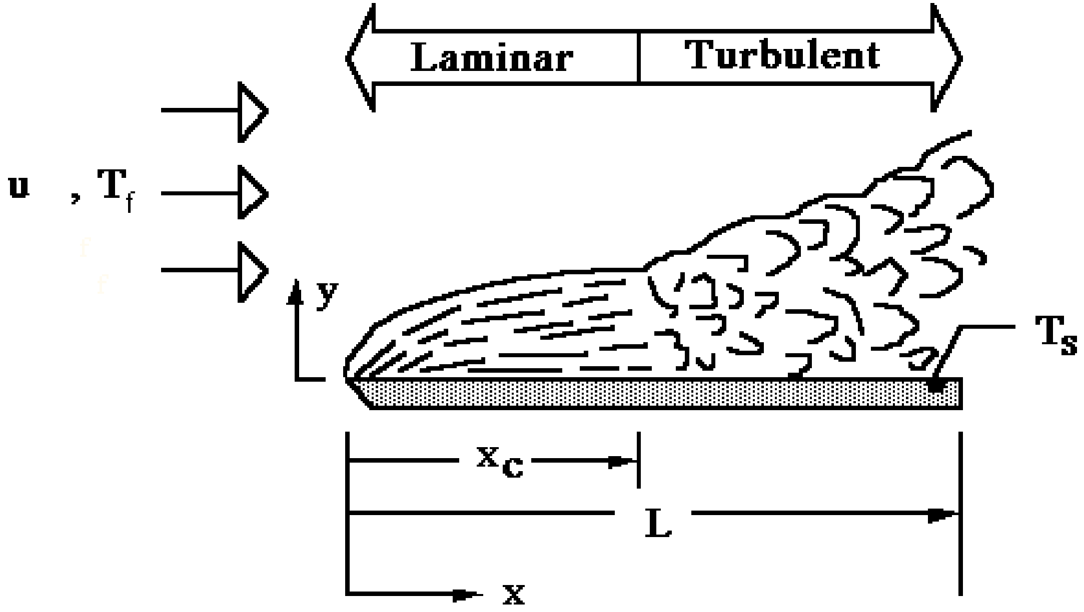
Fully developed pipe flow



$Re_D < 2300$ | $Re_D > 2900$

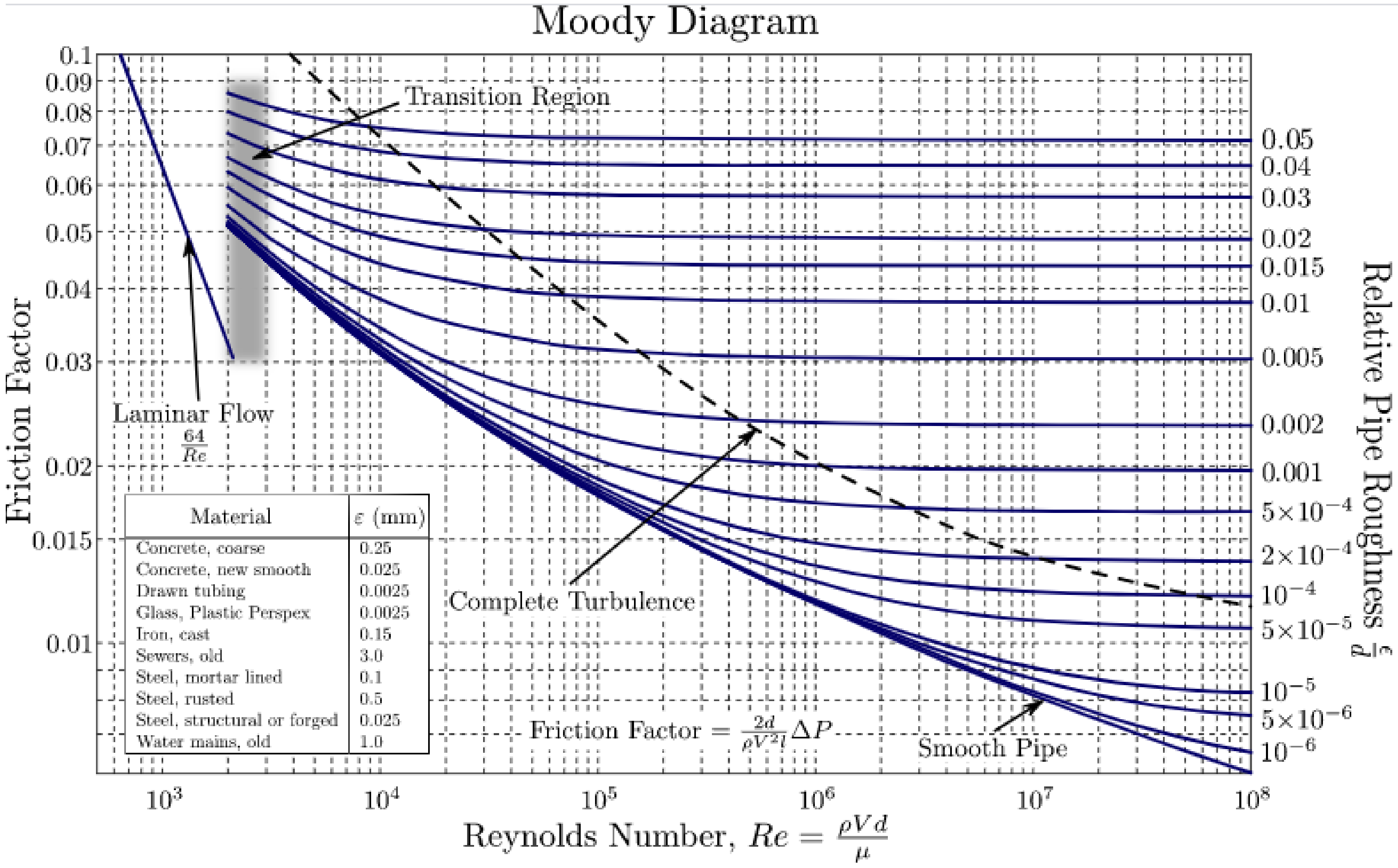
$Re_D < 2000$ | $Re_D > 10000$

Flow over a flat plate

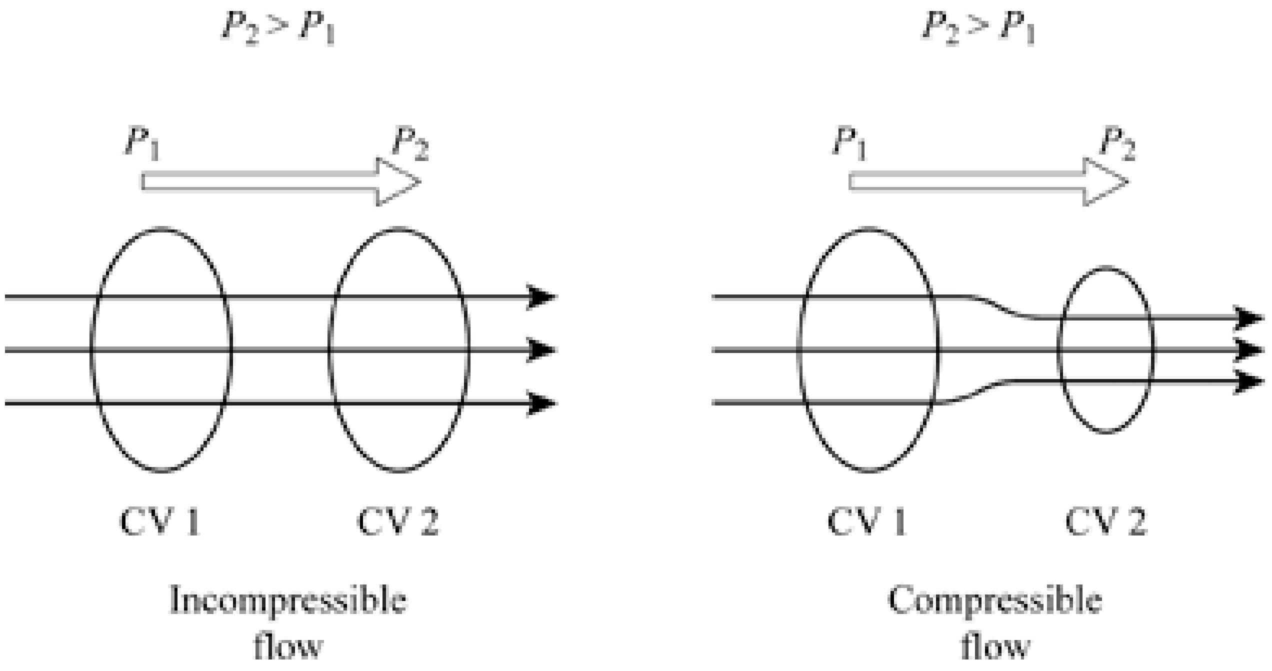


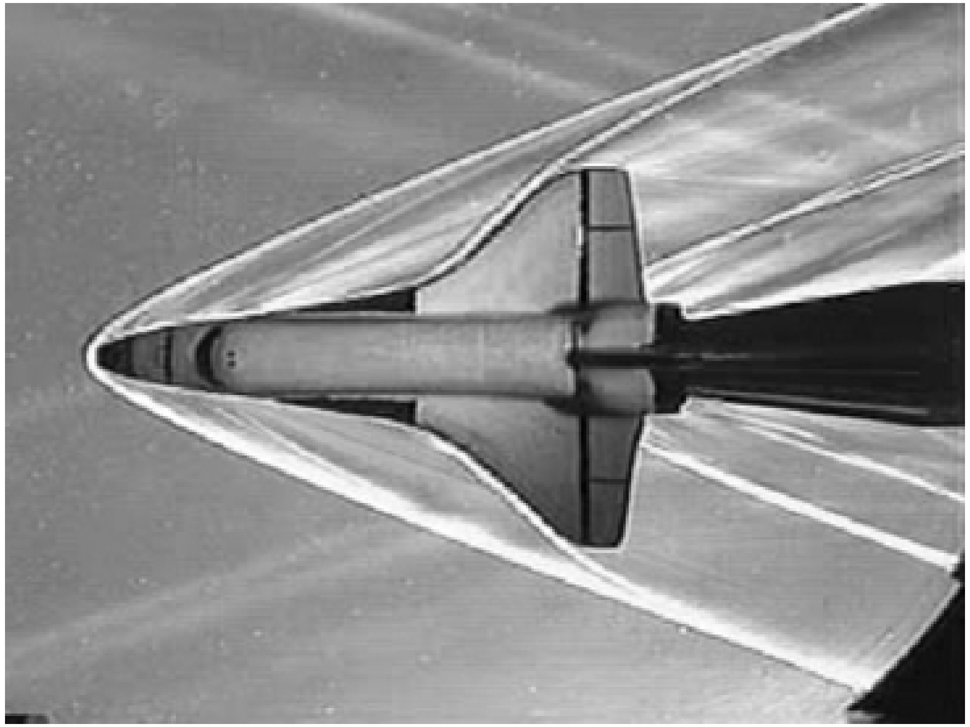
$$Re_x = 5 \times 10^5$$

The Reynolds number



Compressible and incompressible flows

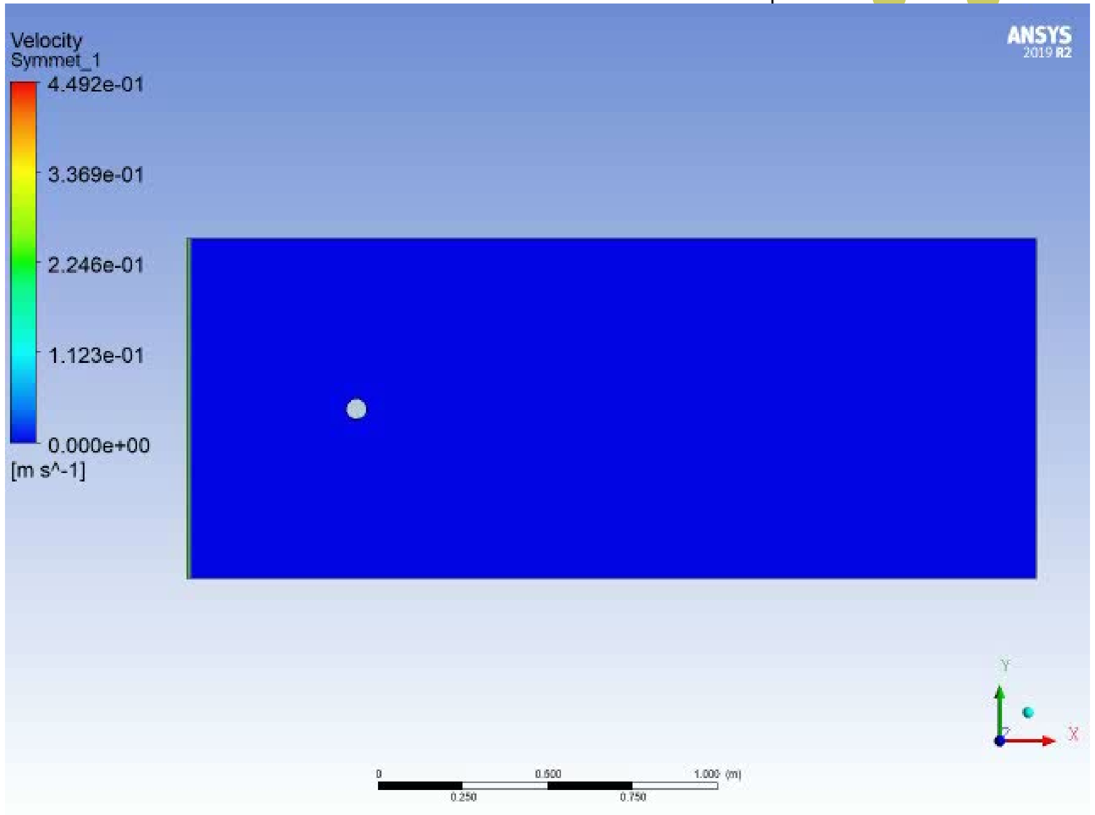




- The **density – pressure** relationship
- Flows with near constant density are usually modelled as **incompressible**.
- When density variation is significant the flow is termed as **compressible.**

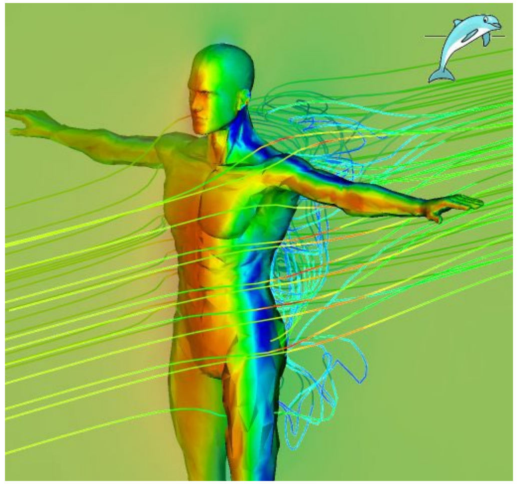
Steady and unsteady flows

- The term **steady** implies *no change with time* – **Only ideal conditions**.
- The opposite of steady is **unsteady**.
- Many devices operate for long periods of time under the same conditions, and they are classified as **steady-flow devices**.



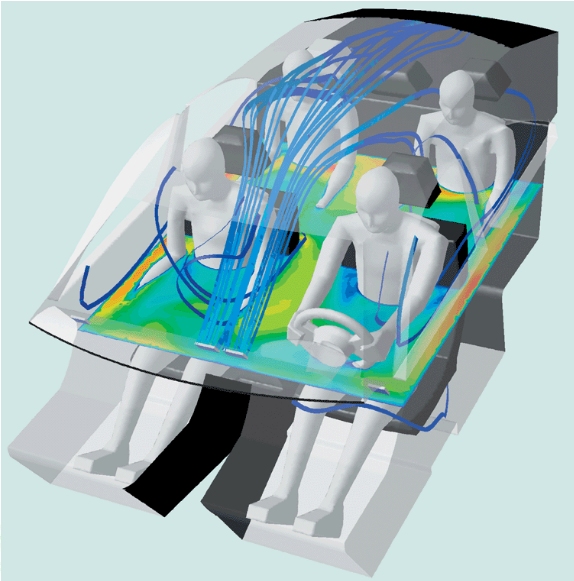
1D, 2D and 3D flows

- **CAUTION: All actual flows are 3D**
- The properties of the simplest flows may vary with only one dimension and are analysed as **one-dimensional**
- Some flows vary in **two-dimensions**
- More complex flows vary in all directions and must be modelled as **three-dimensional**

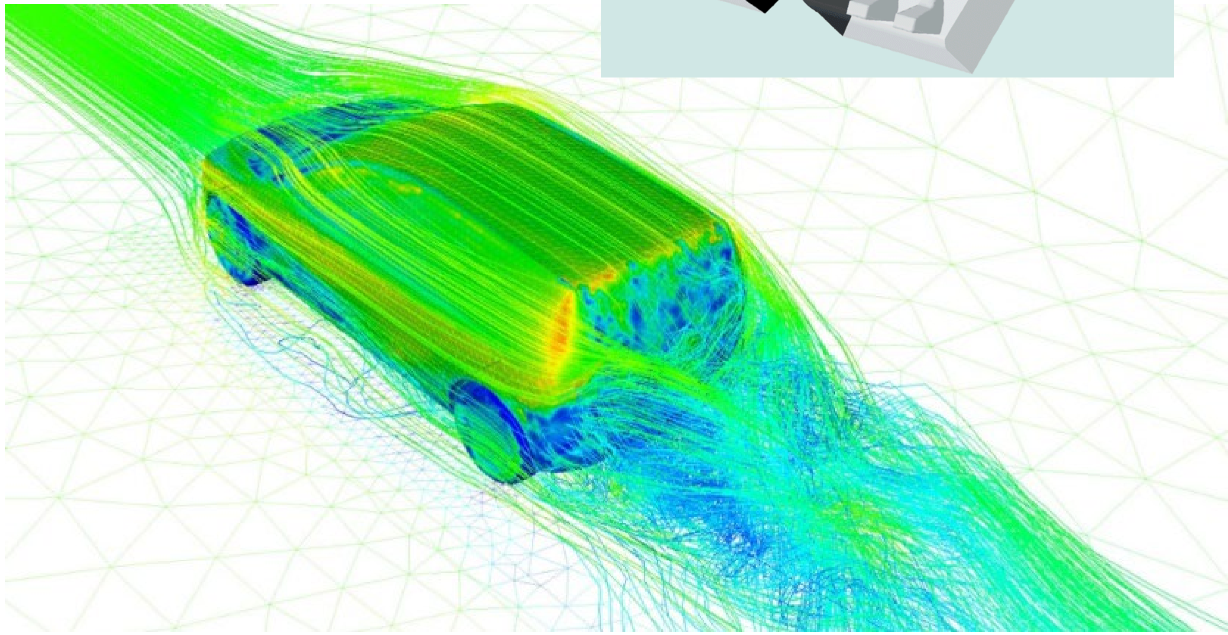


Internal and external flows

- Flows that are contained in a pipe/duct or other enclosing surfaces are analysed as **Internal flows**.



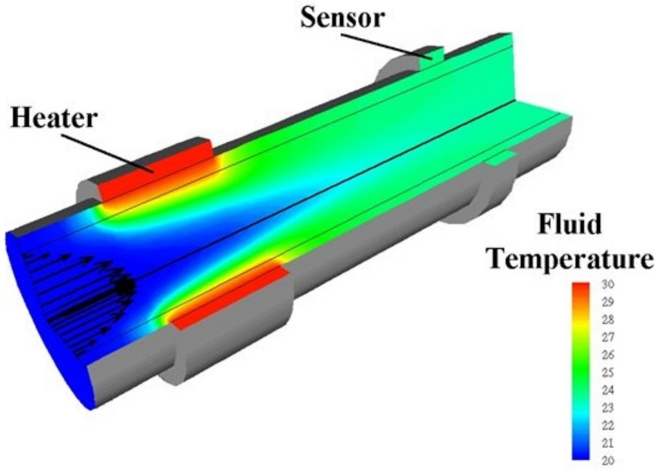
- Flows that pass over surfaces are modelled as **External Flows**.



Isothermal and thermal flows

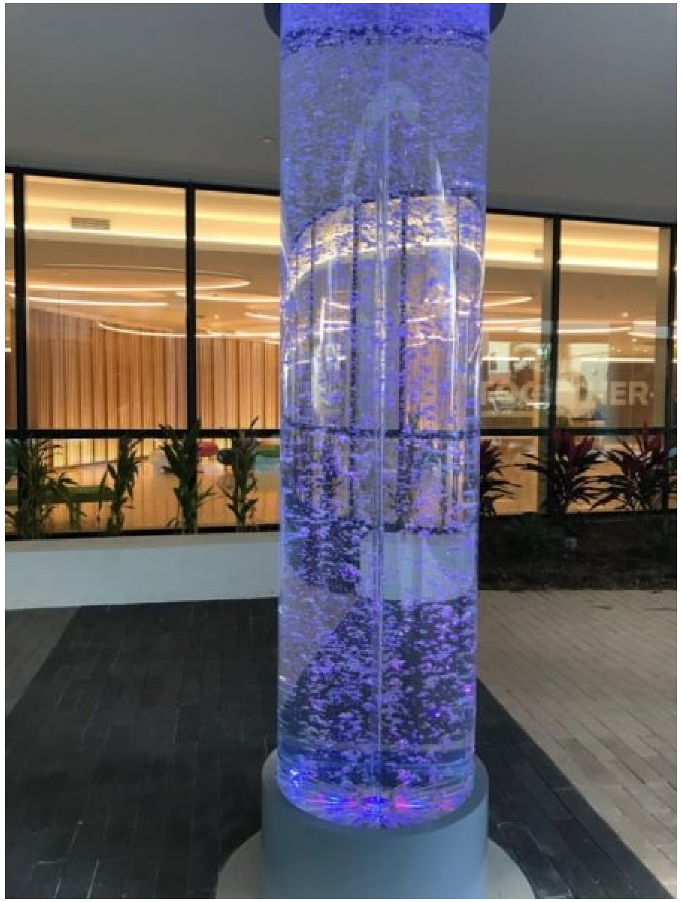
- **Isothermal flows**: Flows that have a constant temperature – **without heat transfer**.

- **Thermal/non-isothermal flows**: Flows with varying temperatures – **with heat transfer**.



Single-phase and multi-phase flows

- **Single-phase flows**: Flows that only include ONE phase of matter
- **Multiphase-phase flows**: Flows that include MORE THAN ONE phase of matter.



- Nitrogen/oxygen flow?
- Water/sand flow?
- Water/oil flow?

Complex flows

- Complex flows refer to those involving complex physical/chemical processes, examples include
  - Multicomponent flows
  - Combustion – flows of your discipline interest!
  - Reacting flows
  - Multiphase flows with multiple flow regimes
  - Multiphase flows with heat/mass transfer
  - …

Simplest flows to analyse

- Steady
- Incompressible
- Inviscid
- One-dimensional
- Isothermal
- Single-phase

Bernoulli Equation

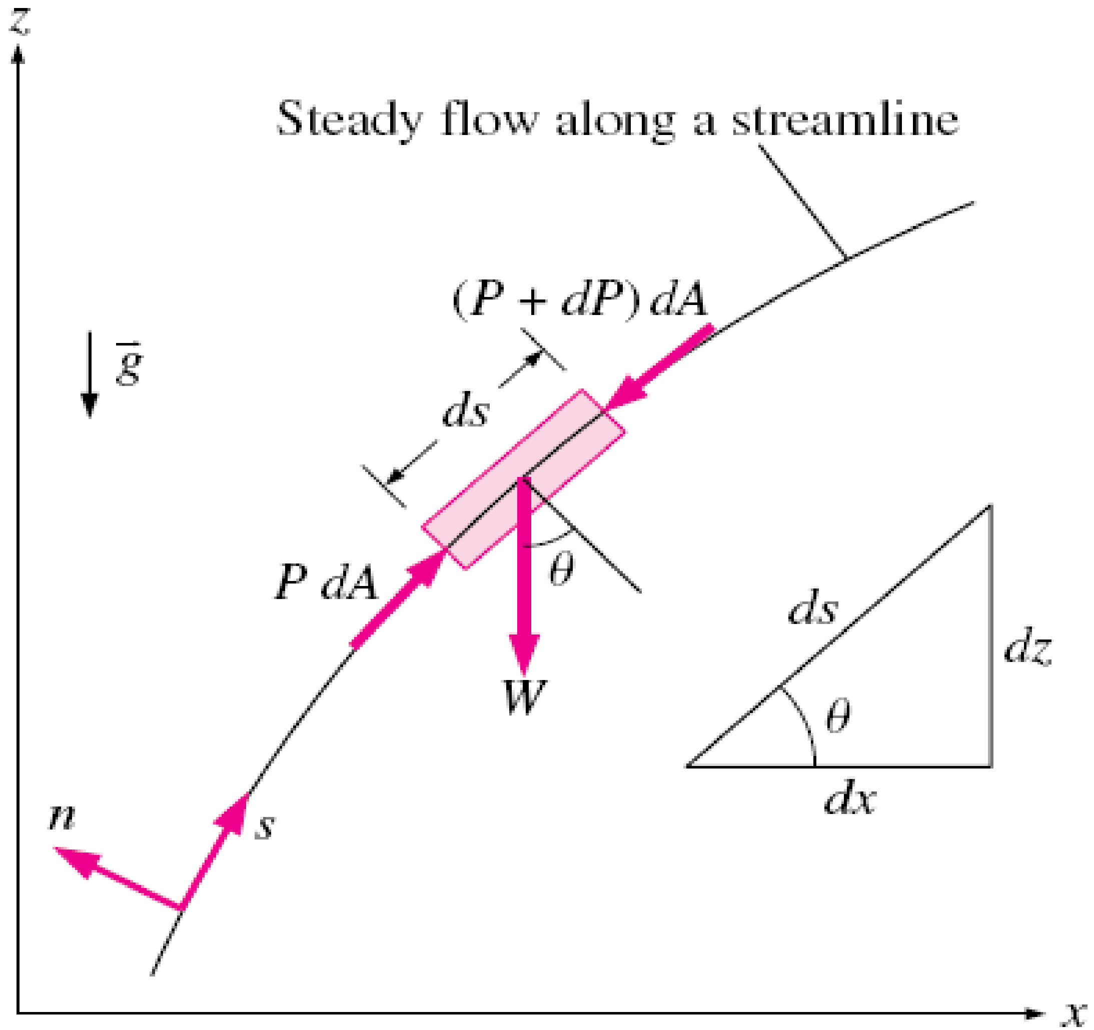
Hardest flows to analyse

- Non-steady
- Compressible
- Viscous
- Three-dimensional
- Thermal
- Multi-phase
- Complex flows

Computational Fluid Dynamics (CFD)

BASIC APPROACHES

Bernoulli equation



Steady, incompressible and inviscid flow:

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a streamline)}$$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

MATHEMATICS FUNDAMENTALS - Divergence and curl

The total derivative

KEY POINTS

Any variable φ in the flow field can be expressed as a function of time and its spatial coordinates in the space $\varphi = f(x, y, z, t)$

The total derivative of φ is expressed as

$$d\phi \equiv \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy + \frac{\partial \phi}{\partial z}dz + \frac{\partial \phi}{\partial t}dt$$

In fluid dynamics, we are more interested in the time changing rate of φ

$$\frac{d\phi}{dt} = \frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial x}\frac{dx}{dt} + \frac{\partial \phi}{\partial y}\frac{dy}{dt} + \frac{\partial \phi}{\partial z}\frac{dz}{dt}$$

$$= \frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial x}u + \frac{\partial \phi}{\partial y}v + \frac{\partial \phi}{\partial z}w$$

The Del operator

KEY POINTS

$$\nabla \equiv \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

It can be used to find

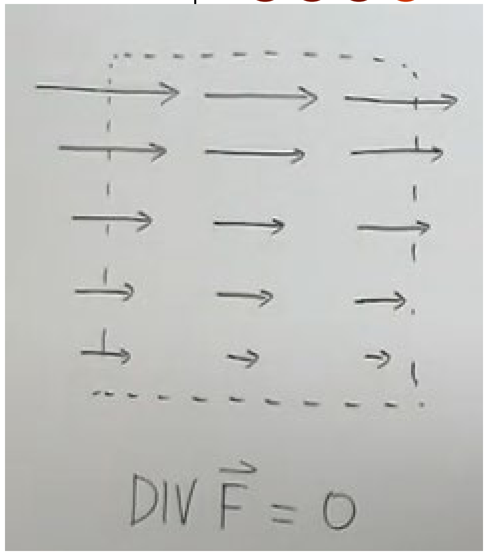
- The gradient of scalar, resulting in a vector --- Del φ $\nabla\varphi$
- The divergence of a vector, resulting in a scalar --- Del dot φ $\nabla\cdot\varphi$
- The curl of a vector, resulting in a vector --- Del cross φ $\nabla\times\varphi$

Gradient

$$grad\,\varphi = \nabla\varphi \equiv \vec{i}\,\frac{\partial\varphi}{\partial x} + \vec{j}\,\frac{\partial\varphi}{\partial y} + \vec{k}\,\frac{\partial\varphi}{\partial z}$$

- The magnitude of gradient presents the slope along the tangent to the surface
- The direction of gradient points to the greatest rate of increase

Divergence



$$div\vec{\varphi} = \nabla \cdot \vec{\varphi} \equiv \frac{\partial \varphi}{\partial x} + \frac{\partial \varphi}{\partial y} + \frac{\partial \varphi}{\partial z}$$

- The net gain or loss of φ anywhere in the field