22-2 Production of Electromagnetic Waves

麦克斯韦（J.C.Maxwell）

1865年预言了电磁波的存在，
指出光是一种电磁波

电磁波在真空中的传播速度

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3 \times 10^8 \mathrm{m \cdot s^{-1}}$$

真空中的光速

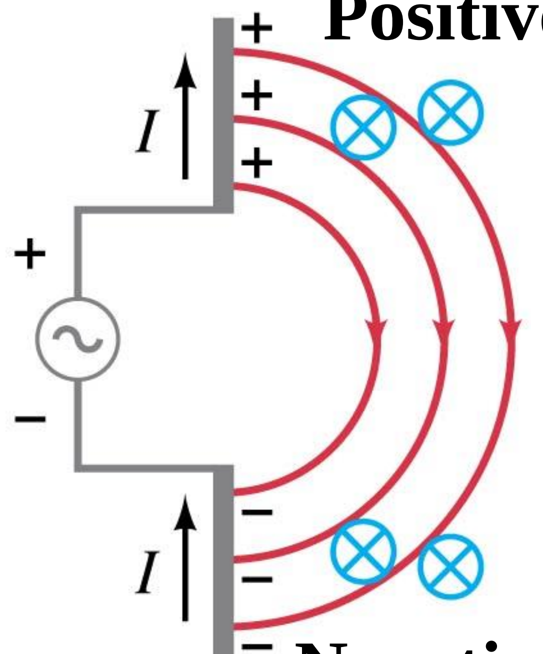
Henirich Hertz

1888年　赫兹（ Henirich Hertz)

第一个用实验证实电磁波的存在

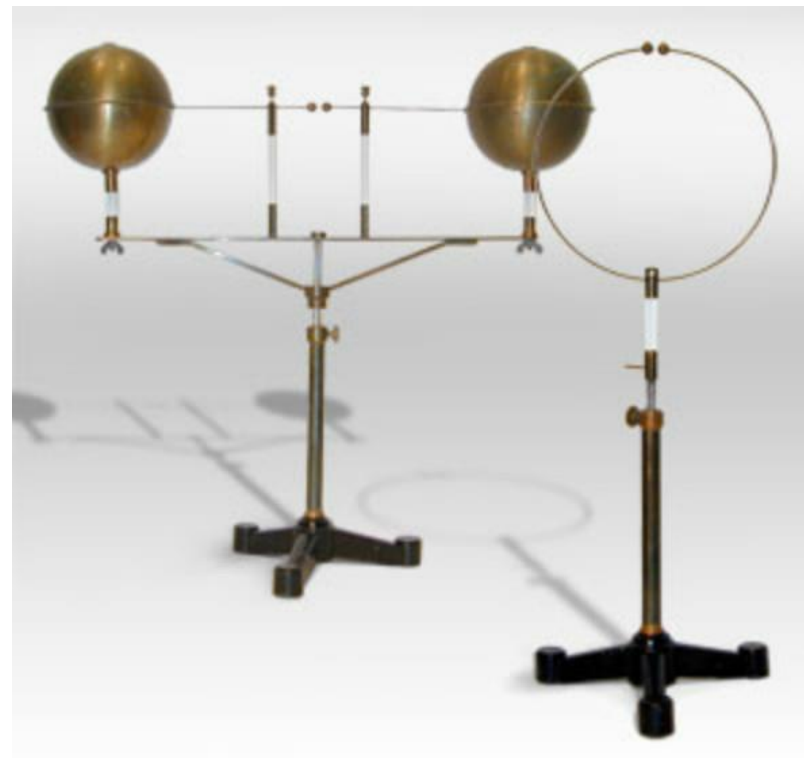
Oscillating charges will produce electromagnetic waves:

Positive electrode



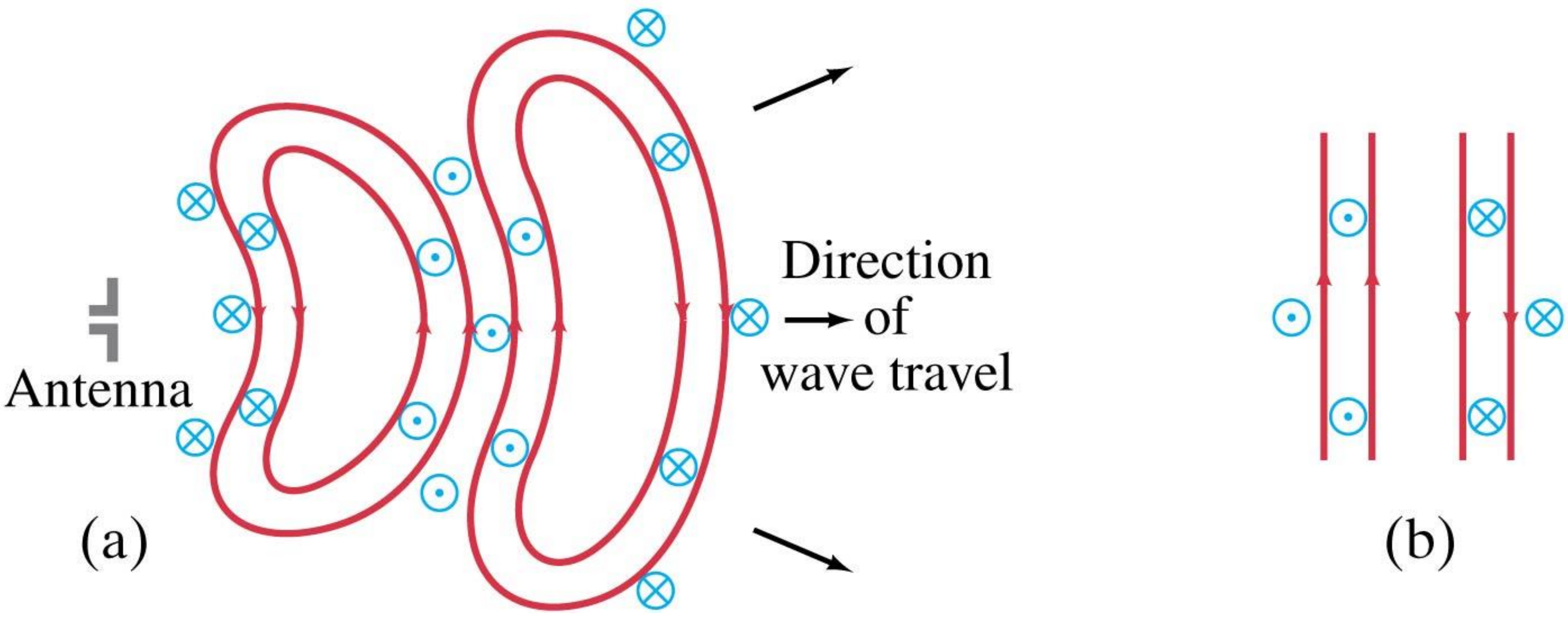
Negative electrode

(a)



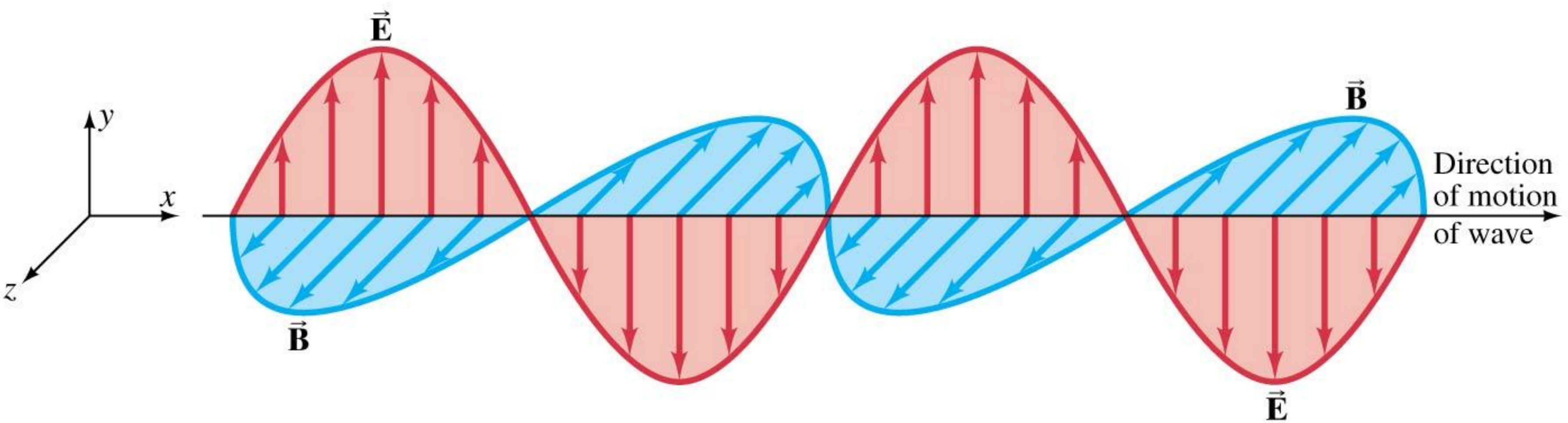
22-2 Production of Electromagnetic Waves

Far from the source, the waves are plane waves:



22-2 Production of Electromagnetic Waves

The electric and magnetic waves are perpendicular to each other, and to the direction of propagation.



22-2 Production of Electromagnetic Waves

When Maxwell calculated the speed of propagation of electromagnetic waves, he found:

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}.$$

Using the known values of ε_0 and μ_0 gives
$c = 3.00 \times 10^8$ m/s. (22-3)

This is the speed of light in a vacuum.

22-3 Light as an Electromagnetic Wave and the Electromagnetic Spectrum

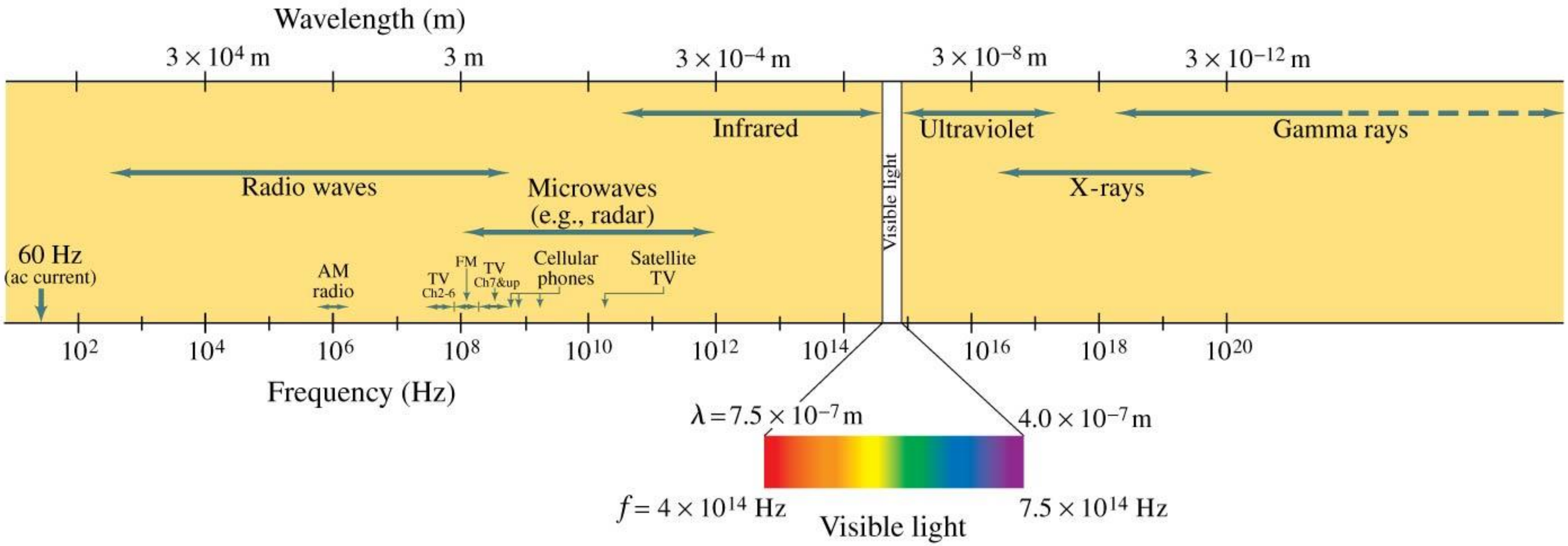
Light was known to be a wave. The production and measurement of electromagnetic waves of other frequencies confirmed that light was an electromagnetic wave as well.

The frequency of an electromagnetic wave is related to its wavelength:

$$c = \lambda f, \qquad (22\text{-}4)$$

22-3 Light as an Electromagnetic Wave and the Electromagnetic Spectrum

Electromagnetic waves can have any wavelength; we have given different names to different parts of the electromagnetic spectrum.



§.8 电磁波的辐射和传播

1.电磁场的能量和能流

电磁场能量是按一定方式分布于电磁场内的

由于电磁场在运动着，电磁场能量不是固定地分布于空间中，而是随着电磁场的运动在空间中传播。

能量密度 $$w=\frac{1}{2}(\vec{E}\cdot\vec{D}+\vec{B}\cdot\vec{H})$$

能流密度 $$\vec{S}=\vec{E}\times\vec{H}$$

坡印廷矢量

2.平面电磁波　Plane electromagnetic wave

$$\vec{E}(x,y,z,t)=E_0\cos\omega(t-\frac{x}{v})\hat{e}_y=E_0\cos(\omega t-kx)\hat{e}_y$$

$$\vec{B}(x,y,z,t)=B_0\cos\omega(t-\frac{x}{v})\hat{e}_z=B_0\cos(\omega t-kx)\hat{e}_z$$

平面电磁波的主要性质

CAI

1) $\vec{E}$和$\vec{B}$都与传播方向垂直，横波　$\vec{k}$

2) $\vec{E}$和$\vec{B}$互相垂直，且 $\vec{E}\times\vec{B}$沿着电磁波的传播方向。

3) $\vec{E}$和$\vec{B}$同相位

4) $\vec{E}$和$\vec{B}$的振幅比为 $\frac{|\vec{E}|}{|\vec{B}|}=v$　$(\sqrt{\varepsilon}E=\sqrt{\mu}H)$

5) 推迟因子 $\frac{x}{v}$　t 时刻　$t-\frac{x}{v}$ 时刻

$$v=\frac{1}{\sqrt{\mu\varepsilon}}=\frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}}$$

In vacuum $c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}$

$$=\frac{c}{\sqrt{\mu_r\varepsilon_r}}=\frac{c}{n}$$

$n=\sqrt{\mu_r\varepsilon_r}$ 折射率 refractive index

$$w=\frac{1}{2}(\varepsilon E^2+\mu H^2)=\varepsilon E^2=\mu H^2$$

$$\sqrt{\varepsilon}E=\sqrt{\mu}H$$

$$\vec{S}=\vec{E}\times\vec{H}=EH\hat{k}=\sqrt{\frac{\varepsilon}{\mu}}E^2\hat{k}=v\varepsilon E^2\hat{k}=vw\hat{k}$$

电磁波强度

$$E^2=\vec{E}\cdot\vec{E}=E_0^2\cos^2\omega(t-\frac{x}{v})$$

$$I=\langle S\rangle=\langle\sqrt{\frac{\varepsilon}{\mu}}E^2\rangle=\frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}E_0^2=\langle w\rangle v$$

Chapter 24
The Wave Nature of Light

preface

A brief historical review

In the second half of the seventeenth century

牛顿：**Corpuscular theory of light**

惠更斯：**Wave theory of light**

Early 19th century

托马斯•杨： Double slit interference experiment

菲涅尔： Wave theory of light （**1815—1826**）

Mid-19th century

麦克斯韦： Electromagnetic wave

Late 19th century early 20th century

- 爱因斯坦： The photon hypothesis; Quantum of light （**1905**）

Optics

1. Geometric optics $\left(\frac{\lambda}{a} \to 0\right)$

以光的直线传播特性为基础研究光学仪器的成像理论 Rectilinear propagation

2. Wave optics $\left(\frac{\lambda}{a} \sim 1\right)$

以**Maxwell** 的电磁波理论为基础的 **Huygens-Fresnel** 理论

研究光的传播规律——干涉、衍射、偏振性质

3. Quantum optics $\left(\frac{\lambda}{a} >> 1\right)$

以量子力学为基础——研究光与物质的相互作用

§ 24.1 Light source

发射辐射的发光体　反射辐射的物体（ The secondary light source ）

特性：大小、强度、颜色

点光源　线度小到可以忽略的光源

1. Luminescence mechanism of light source

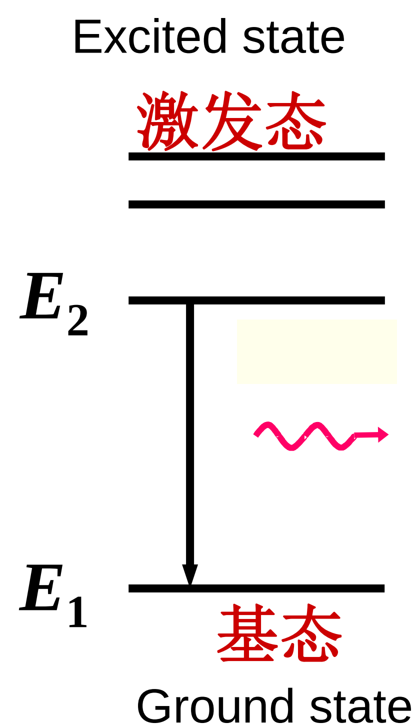
Energy level

The atoms emitting

Lifetime of excited state $10^{-11} \sim 10^{-8}$ s

Spontaneous radiation $\nu = \dfrac{\Delta E}{h}$

➢一定频率，长度有限的光波列

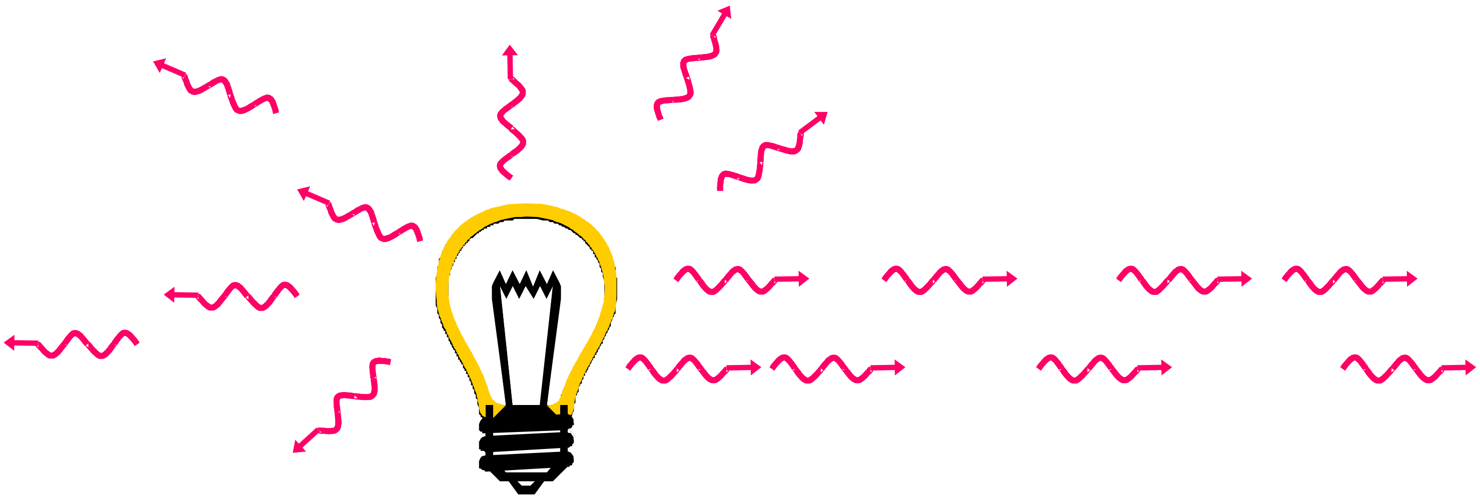


Molecules, condensed matter luminescence

Quasi-continuous

A train of light waves of continuous frequency

大量原子和分子持续、随机地发射的光波列



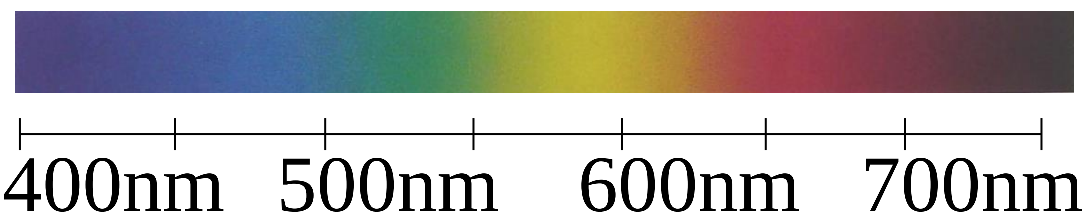
2、Monochromatic radiation & Polychromatic radiation

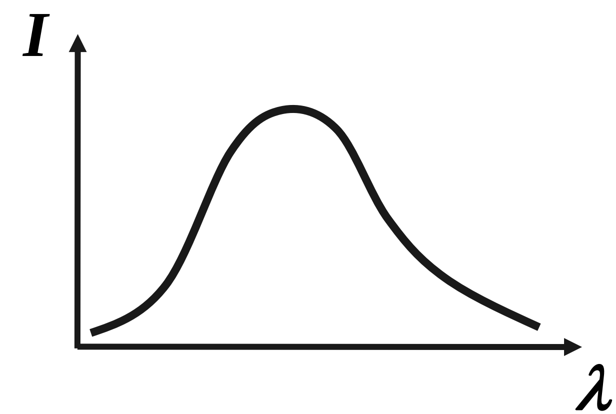
Thermal radiation source 白炽灯、弧光灯、太阳

Non-thermal radiation source 水银灯、日光灯

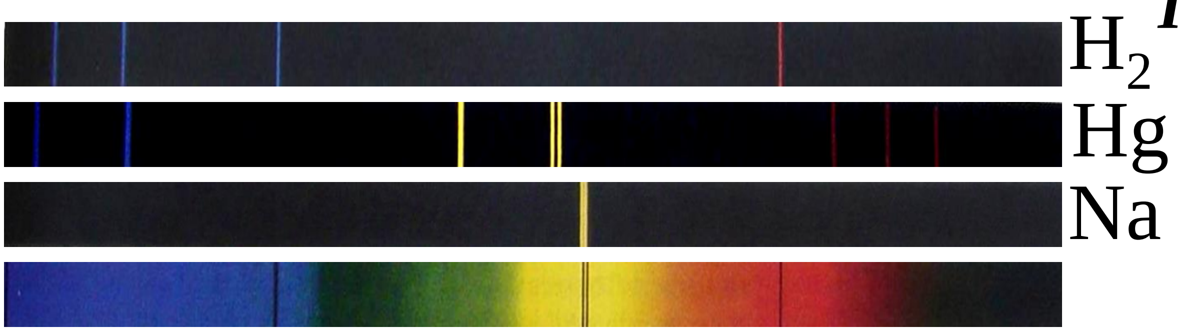
Spectrum光谱 光的强度按频率（或波长）的分布

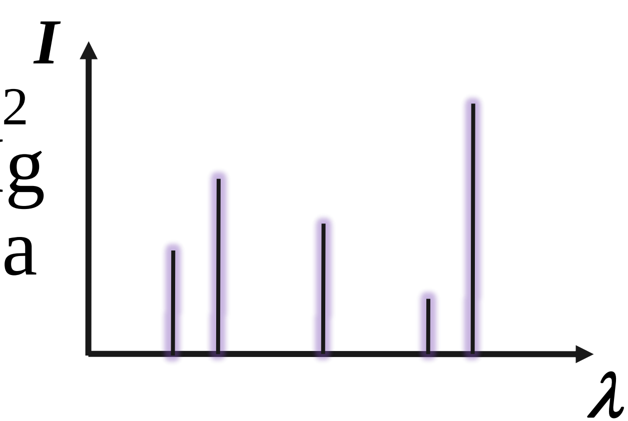
连续光谱





线光谱

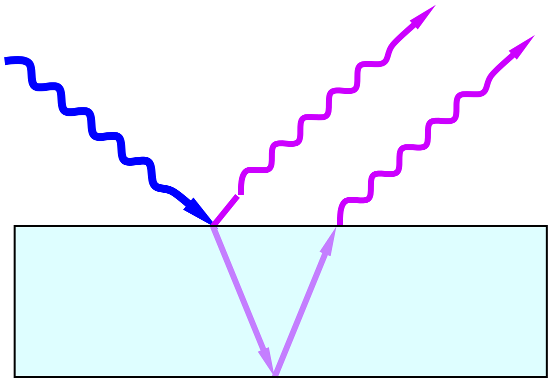




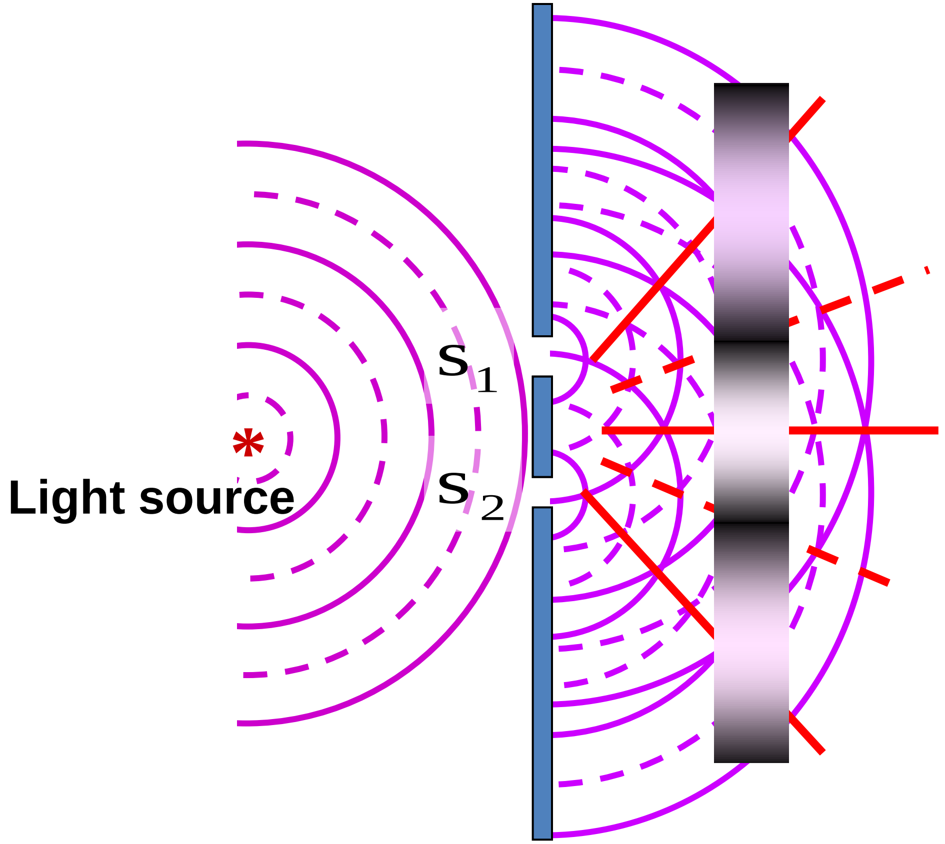
太阳吸收光谱

2 The generation of coherent light

Division of amplitude



Division of wavefront



Contents of Chapter 24

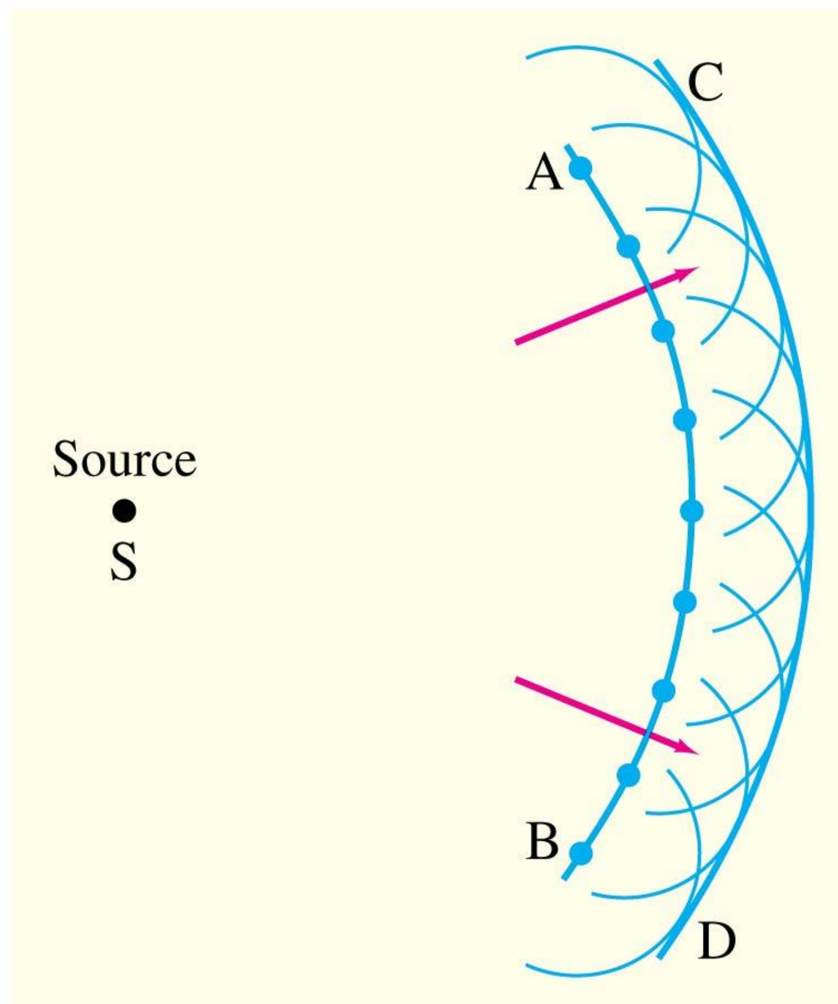
- **Waves Versus Particles; Huygens' Principle and Diffraction**
- **Huygens' Principle and the Law of Refraction**
- **Interference—Young's Double-Slit Experiment**
- **The Visible Spectrum and Dispersion**
- **Diffraction by a Single Slit or Disk**
- **Diffraction Grating**

Contents of Chapter 24

- The Spectrometer and Spectroscopy
- Interference in Thin Films
- Michelson Interferometer
- Polarization
- Liquid Crystal Displays (LCD)
- Scattering of Light by the Atmosphere

24-1 Waves Versus Particles; Huygens' Principle and Diffraction

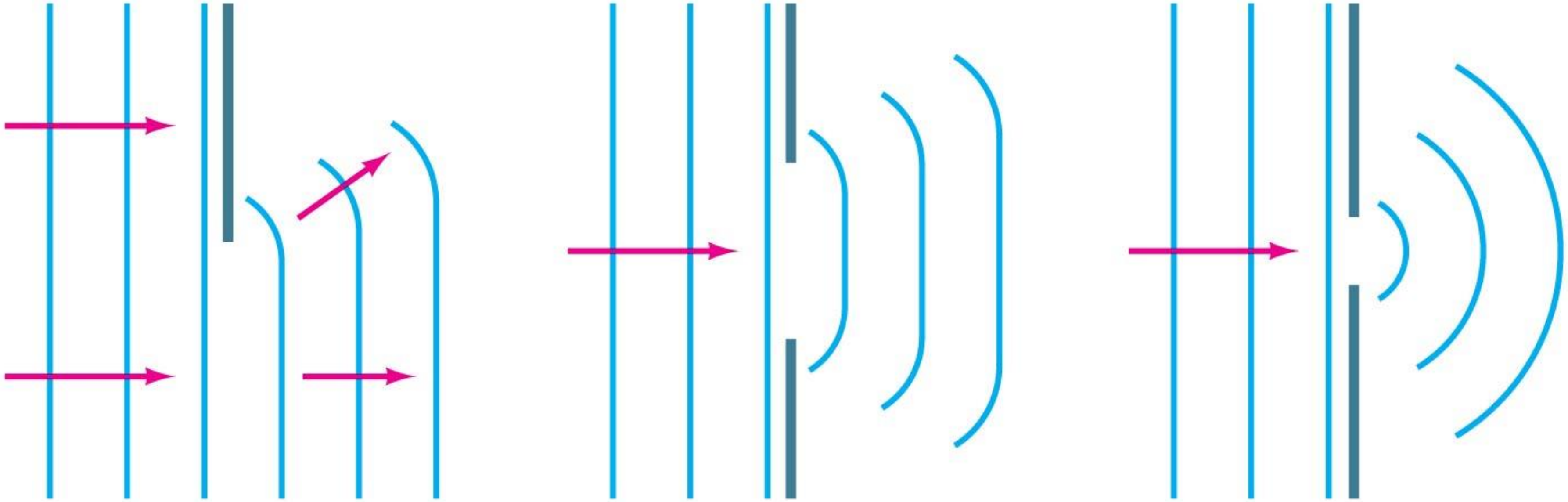
Huygens' principle: Every point on a wave front acts as a point source; the wavefront as it develops is tangent to their envelope



24-1 Waves Versus Particles; Huygens' Principle and Diffraction

Rectilinear propagation VS Diffraction

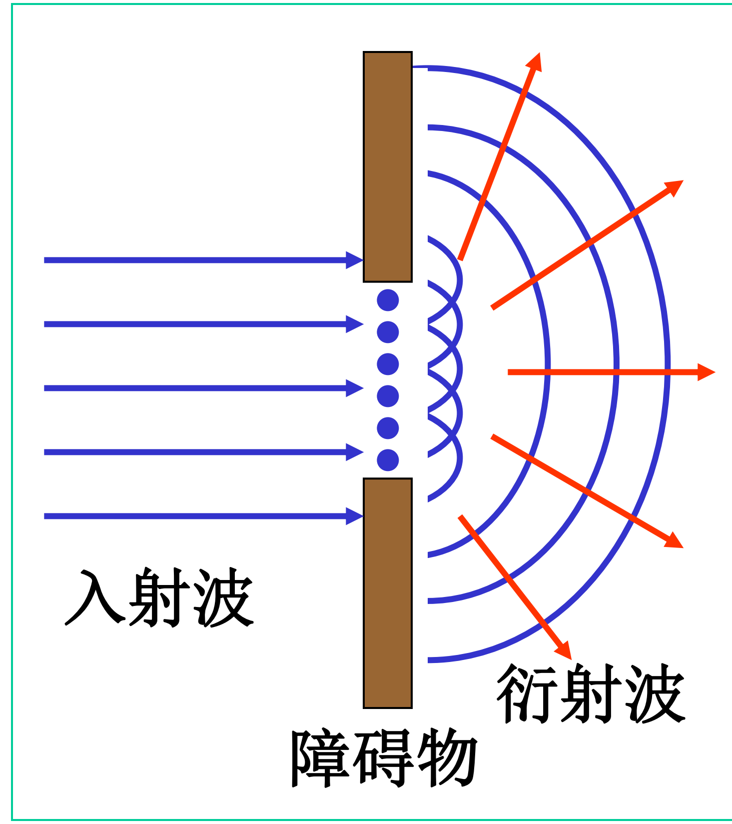
Huygens' principle is consistent with diffraction:



(a) (b) (c)

Huygens' principle

Huygens' principle: Every point on a wave front acts as a point source; the wavefront as it develops is tangent to their envelope

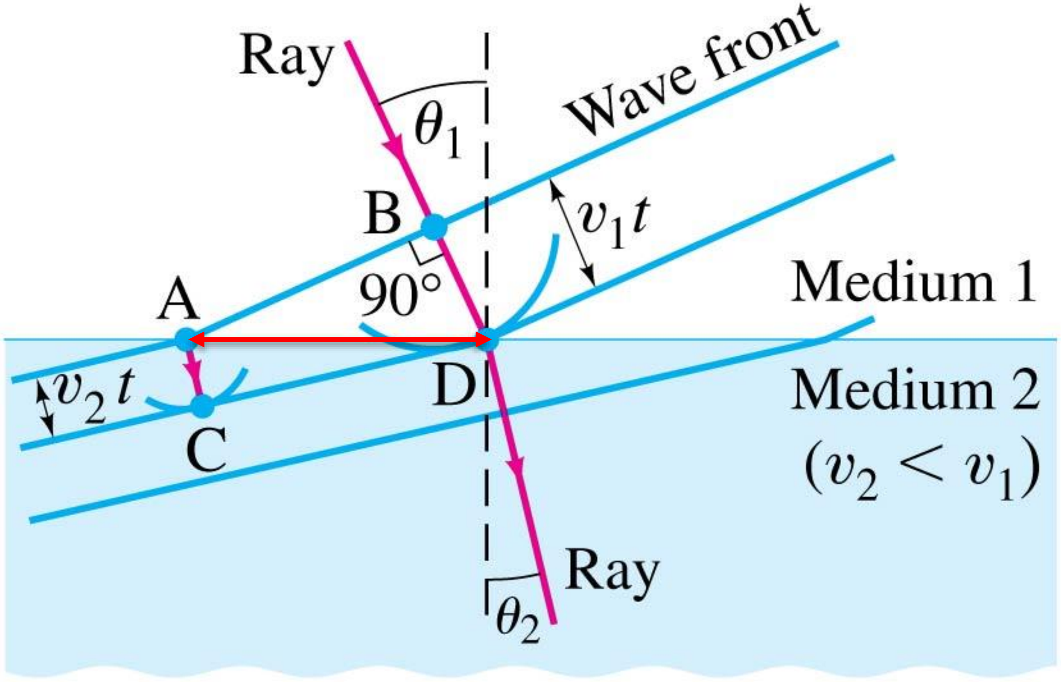


成功：可解释衍射成因，用几何法作出新的波面，推导反射、折射定律，解释晶体中的双折射 。

不足：不能定量说明衍射波的强度分布

可能出现倒退波。

24-2 Huygens' Principle and the Law of Refraction



$$v = \frac{C}{n}$$

$$BD = ADsin\theta_1$$

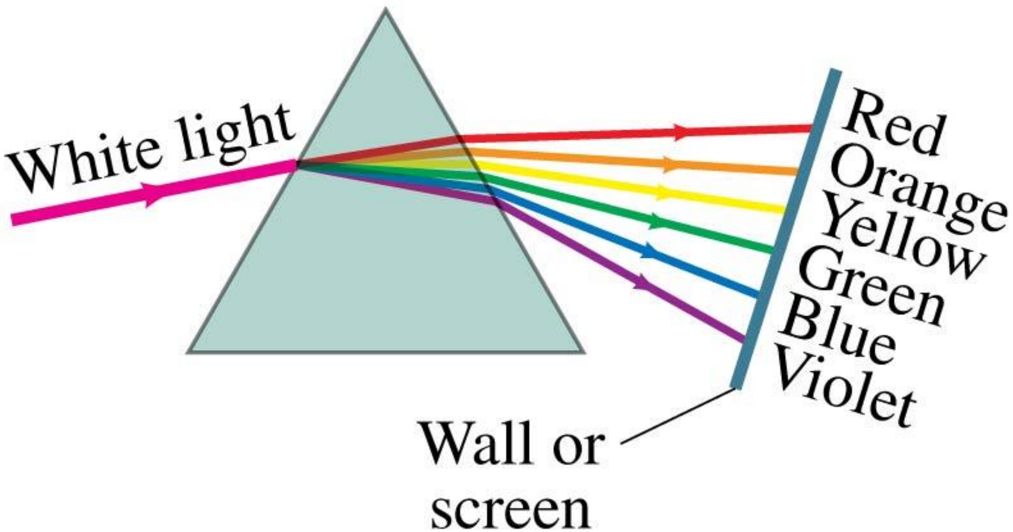
$$AC = ADsin\theta_2$$

$$BD = v_1 t = \frac{C}{n_1} t$$

$$AC = v_2 t = \frac{C}{n_2} t$$

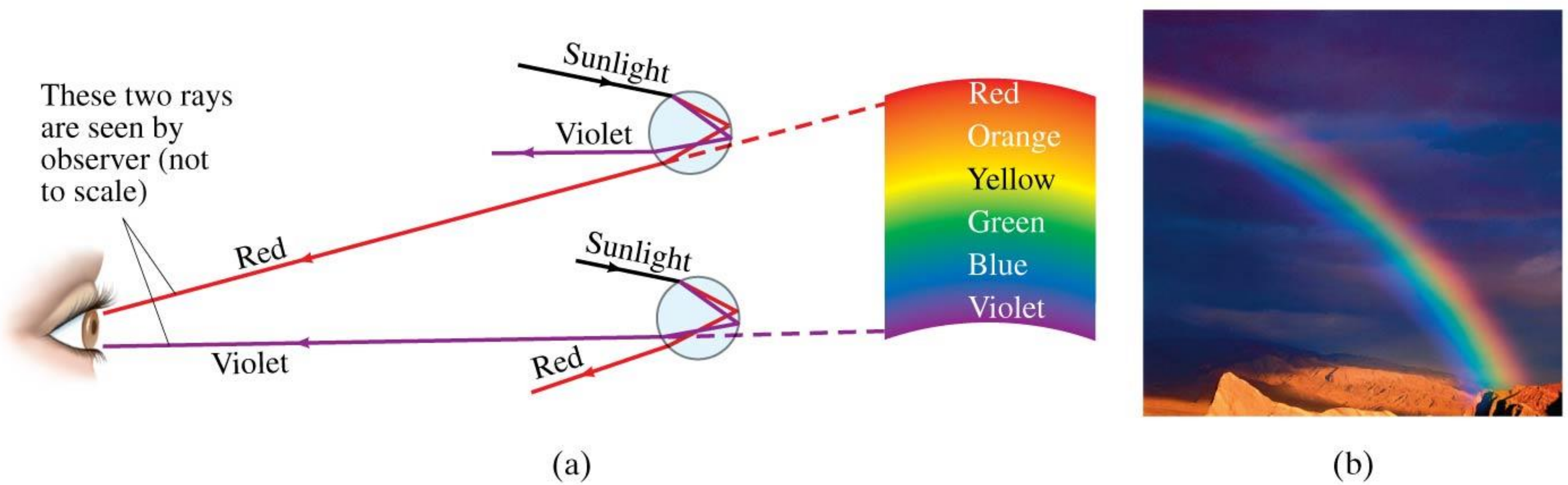
24-4 The Visible Spectrum and Dispersion

This variation in refractive index is why a prism will split visible light into a rainbow of colors.



24-4 The Visible Spectrum and Dispersion

Atmospheric rainbows are created by dispersion in tiny drops of water.



24-2 Huygens' Principle and the Law of Refraction

Huygens' principle can also explain the law of refraction.

As the wavelets propagate from each point, they propagate more slowly in the medium of higher index of refraction.

This leads to a bend in the wavefront and therefore in the ray.

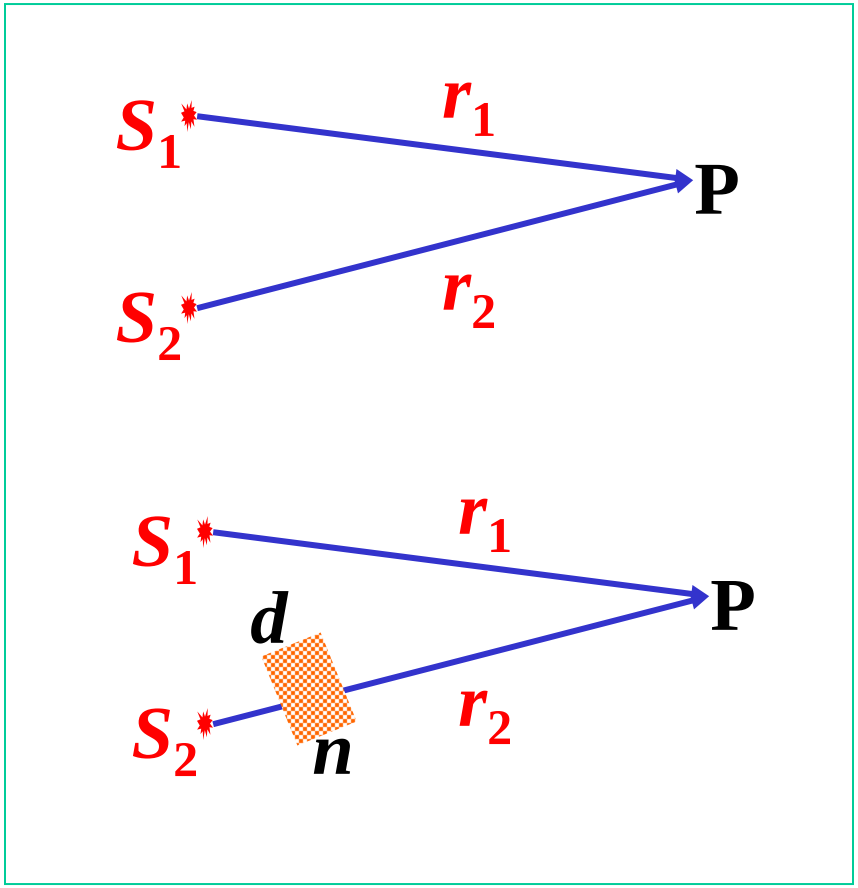
24-2 Huygens' Principle and the Law of Refraction

The frequency of the light does not change, but the wavelength does as it travels into a new medium.

$$\frac{\lambda_2}{\lambda_1} = \frac{v_2 t}{v_1 t} = \frac{v_2}{v_1} = \frac{n_1}{n_2}$$

$$\lambda_n = \frac{\lambda}{n} \qquad (24\text{-}1)$$

二、Optical path Optical path difference



$$\Delta\varphi = \varphi_2 - \varphi_1 - 2\pi\frac{r_2 - r_1}{\lambda}$$

when $\phi_2 = \phi_1$:

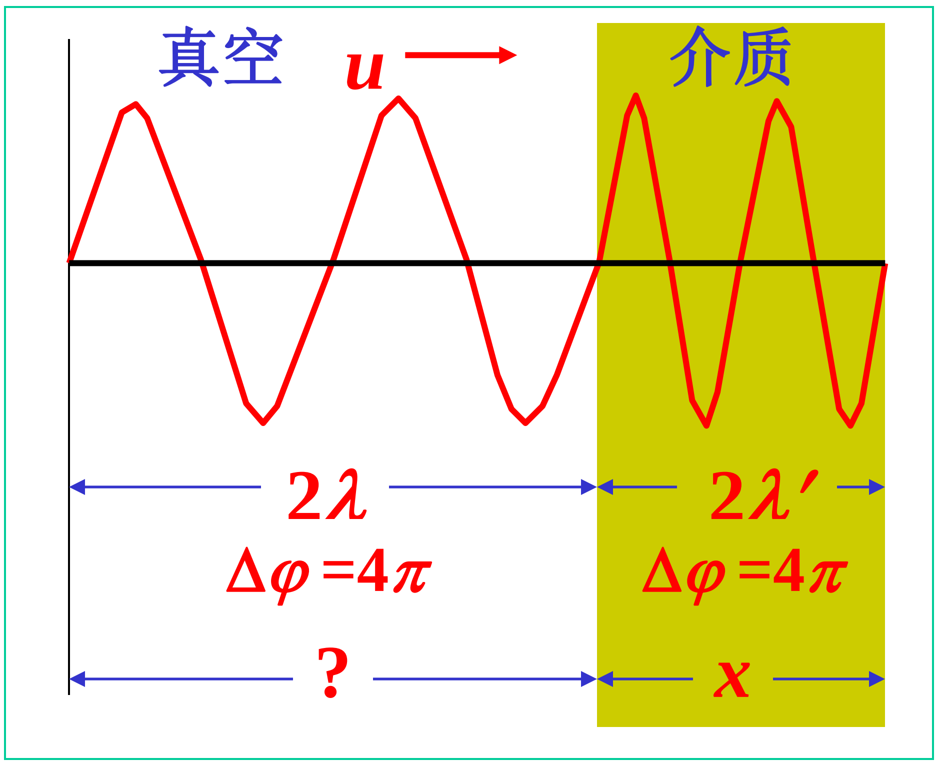
$$\Delta\varphi = 2\pi\frac{r_1 - r_2}{\lambda}$$

$$\Delta\varphi = 2\pi\frac{r_1}{\lambda} - 2\pi\left(\frac{r_2 - d}{\lambda} + \frac{d}{\lambda'}\right)$$

How to simplify ?

思路： Try to convert the path distance of light in a medium into the path distance of light in a vacuum，

Equivalent principle： **Equivalent in causing phase changes of light waves.**



介质中 x 距离内波数： $\dfrac{x}{\lambda'}$

真空中同样波数占据的距离

$$\frac{x}{\lambda'}\cdot\lambda=\frac{x\cdot c\cdot T}{u\cdot T}=x\cdot\frac{c}{u}=x\cdot n$$

Refractive index

Conclusion:

Optical path: $\boldsymbol{x = n x_n}$

Optical path difference: difference of equivalent vacuum path

$$\Delta = n_1 r_1 - n_2 r_2$$

$$\Delta\varphi = \varphi_2 - \varphi_1 + 2\pi \frac{\Delta}{\lambda}$$

Δ → Optical path difference

λ → Vacuum wavelength

$$\Delta\varphi = 2\pi \frac{r_1}{\lambda} - 2\pi\left(\frac{r_2 - d}{\lambda} + \frac{d}{\lambda'}\right)$$

$$\Delta\varphi = \varphi_2 - \varphi_1 + 2\pi\frac{\Delta}{\lambda}$$

Δ → Optical path difference

λ → Optical wave length

When $\Delta\varphi = \begin{cases} 2k\pi & \textbf{constructive} \sim \textbf{bright} \\ (2k+1)\pi & \textbf{destructive} \sim \textbf{dark} \end{cases}$

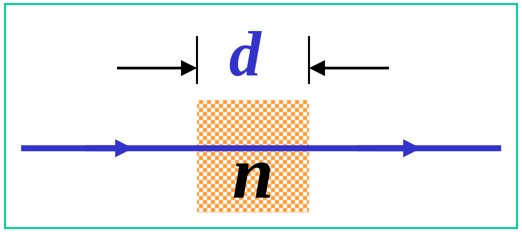
$k = 0, \pm 1, \pm 2 \cdots$

When $\varphi_1 = \varphi_2$ $\Delta\varphi = 2\pi\frac{\Delta}{\lambda}$

When $\Delta = \begin{cases} k\lambda & \textbf{Bright} \\ (2k+1)\frac{\lambda}{2} & \textbf{Dark} \end{cases}$ $k = 0, \pm 1, \pm 2 \cdots$

Common situation：

① 真空中加入厚 d 的介质、增加 $(n-1)d$ 光程



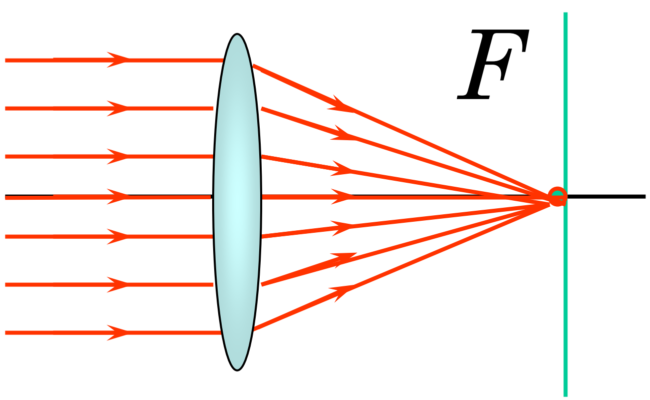
$$nd - d = (n-1)d$$

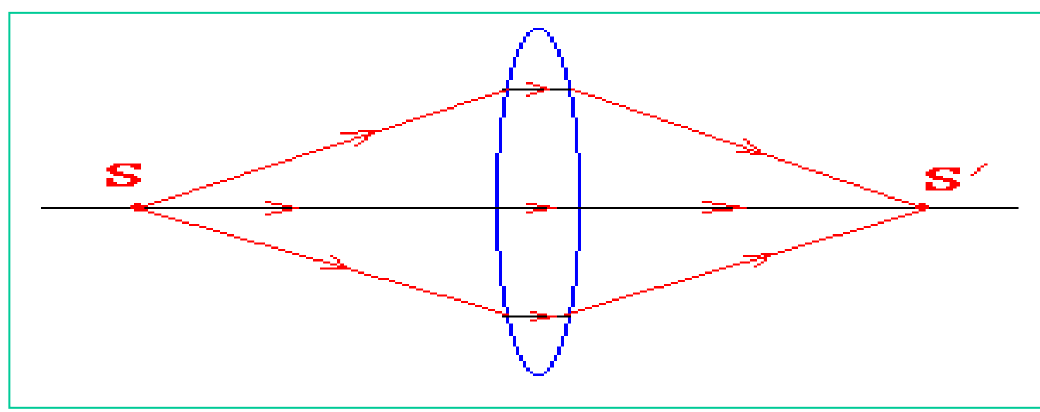
②光由光疏介质射到光密介质界面上反射时附加 $\lambda/2$ 光程差

折射率n较小　　n较大　　（半波损失）

Optically thinner medium　Optically denser medium

③薄透镜不引起附加光程差(物点与象点间各光线等光程)

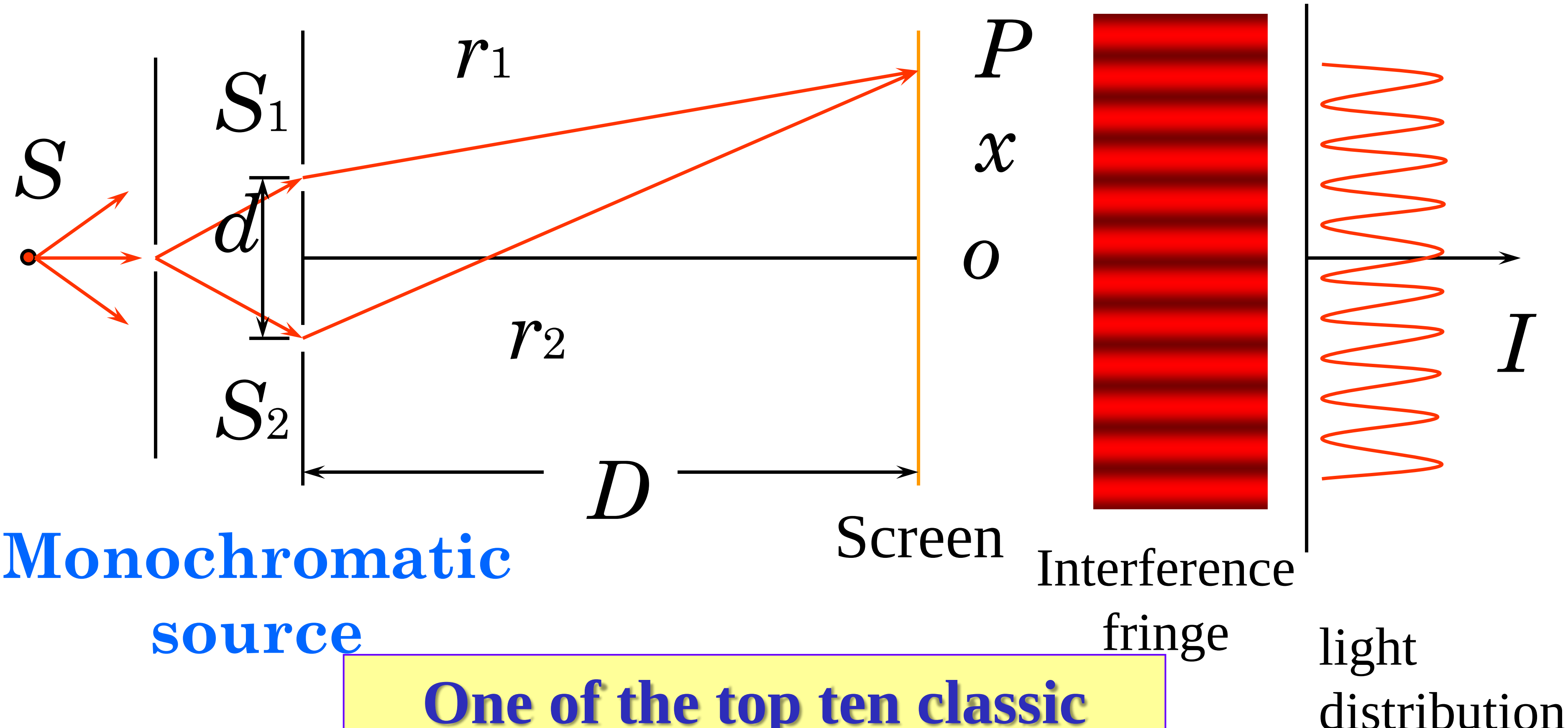




3、 Double-Slit interference

1、 Young's Double-Slit Experiment

① Instrument：

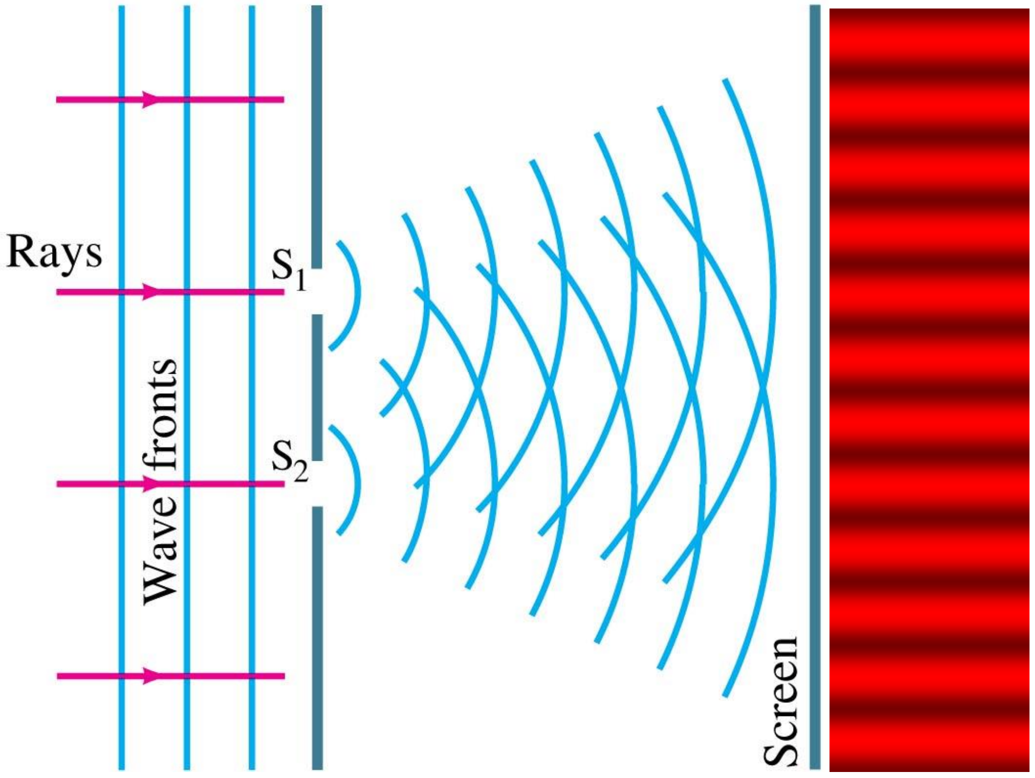


Monochromatic source

One of the top ten classic experiments in the world

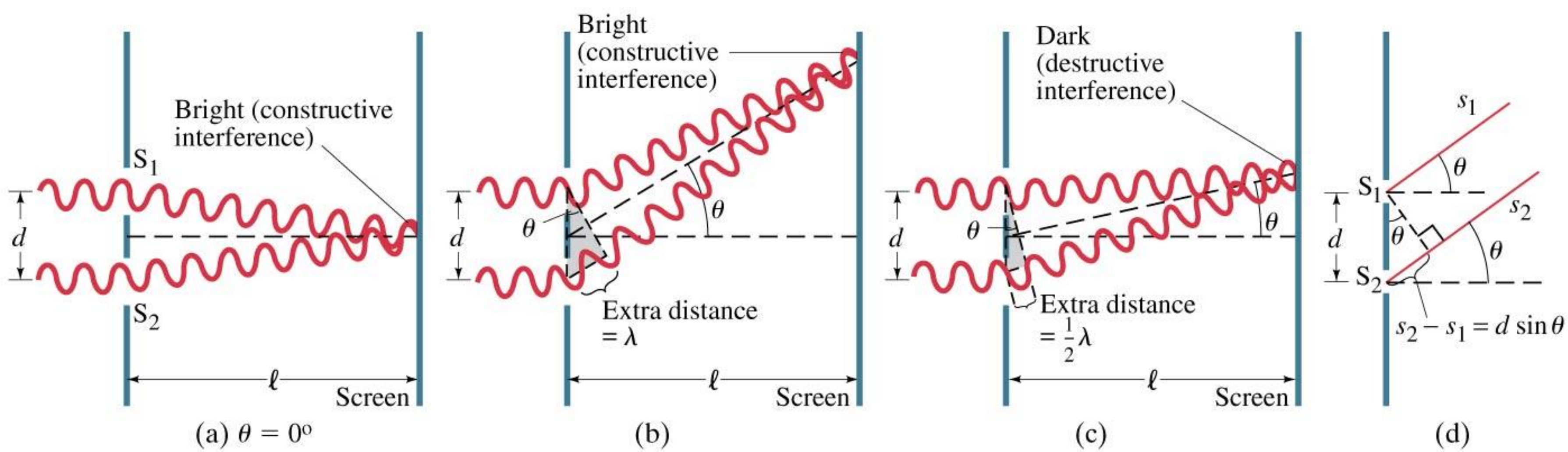
Huygens' principle :

Huygens' principle: Every point on a wave front acts as a point source; the wavefront as it develops is tangent to their envelope

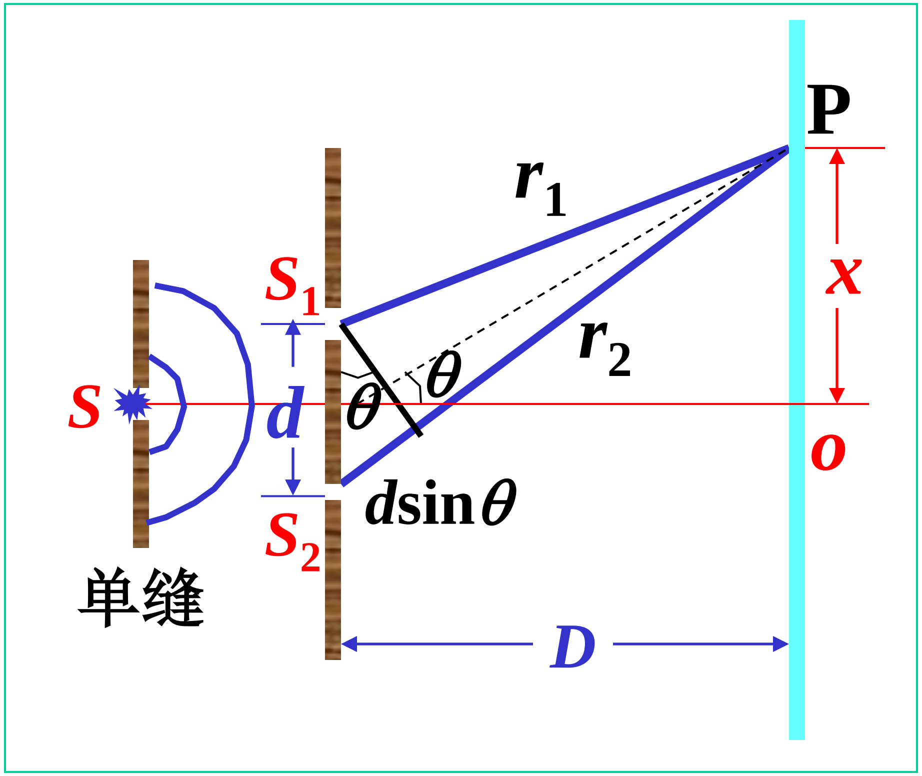


24-3 Interference—Young's Double-Slit Experiment

The interference occurs because each point on the screen is not the same distance from both slits. Depending on the path length difference, the wave can interfere constructively (bright spot) or destructively (dark spot).



② The position of interference bright &dark fringe



For $\varphi1 = \varphi2$

$$\Delta = r_2 - r_1 \approx d\sin\theta \approx d\frac{x}{D}$$

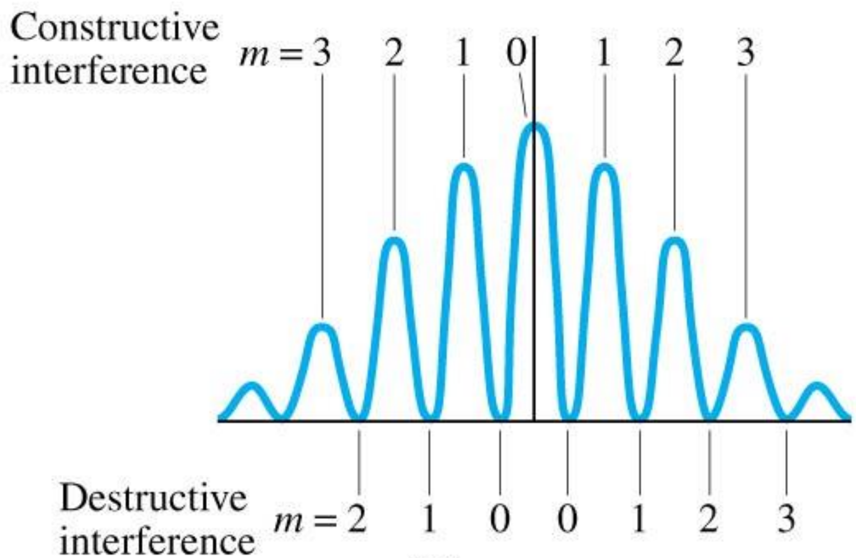
$$\Delta = \begin{cases} \pm k\lambda & \textbf{Bright} \quad k = 0,1,2,\cdots \\ \pm(2k-1)\dfrac{\lambda}{2} & \textbf{Dark} \quad k = 1,2,\cdots \end{cases}$$

$$x = \begin{cases} \pm\dfrac{kD}{d}\lambda & \textbf{Bright} \quad k = 0,1,2,\cdots \\ \pm(2k-1)\dfrac{D}{d}\cdot\dfrac{\lambda}{2} & \textbf{Dark} \quad k = 1,2,\cdots \end{cases}$$

k **fringe order**

24-3 Interference—Young's Double-Slit Experiment

Between the maxima and the minima, the amplitude varies smoothly.



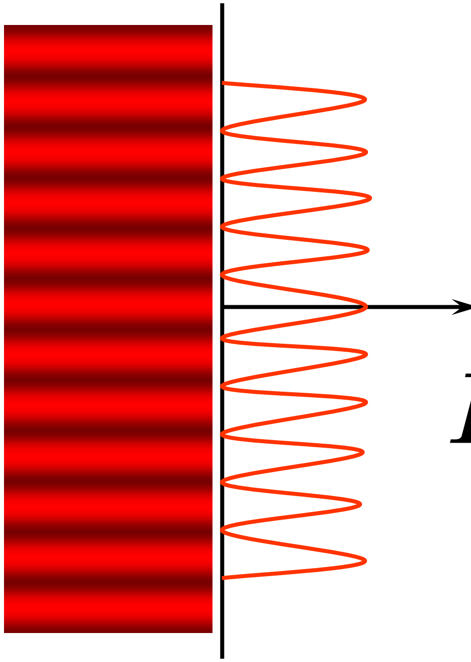
(b)

③ Fringe characteristics

Pattern： 平行于缝的等亮度、等间距、明暗相间条纹

Fringe intensity： $I_{max} = 4I_1 \quad I_{min} = 0$

$$x = \begin{cases} \pm \dfrac{kD}{d}\lambda & \textbf{bright} \quad k = 0,1,2,\cdots \\ \pm(2k-1)\dfrac{D}{d}\cdot\dfrac{\lambda}{2} & \textbf{dark} \quad k = 1,2,\cdots \end{cases}$$



Fringe width： $\Delta x = \dfrac{D}{d}\lambda$

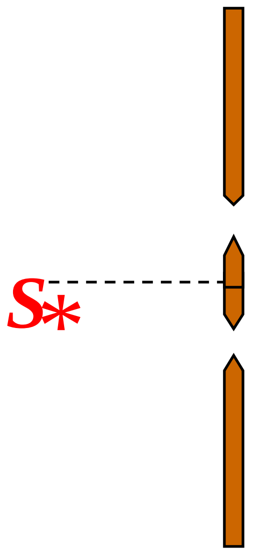
λ ***constant*** **:** $\Delta x \propto D \qquad \Delta x \propto \dfrac{1}{d}$

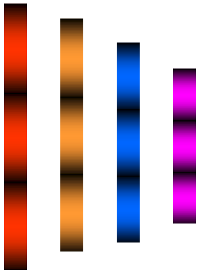
d、D ***constants*** **:** $\Delta x \propto \lambda \quad \Delta x_{红} > \Delta x_{紫}$

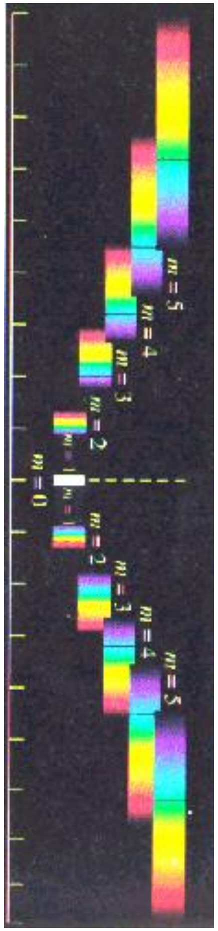
白光照射双缝:

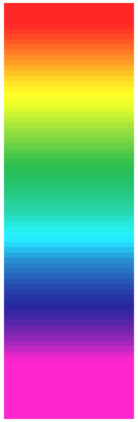
零级明纹：白色　其余明纹：彩色光谱（内紫外红）

高级次重叠





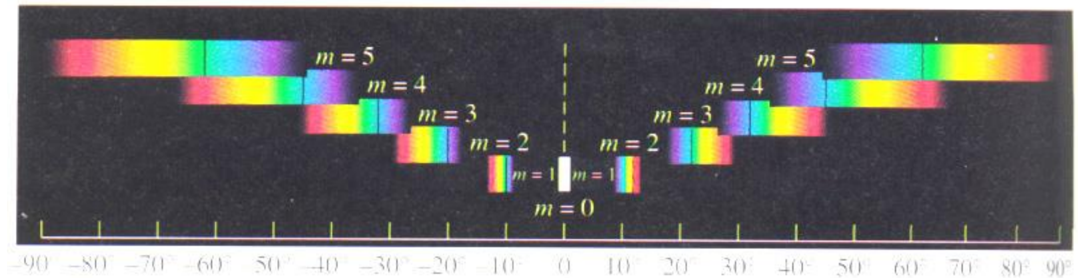




例：

用白光光源进行双缝干涉实验，求清晰可辩光谱的级次

$\lambda: 4000 \sim 7000\ \mathring{A}$



零级　　一级　　二级　　三级

最先重叠：　某级红光和高一级紫光 Δ相同

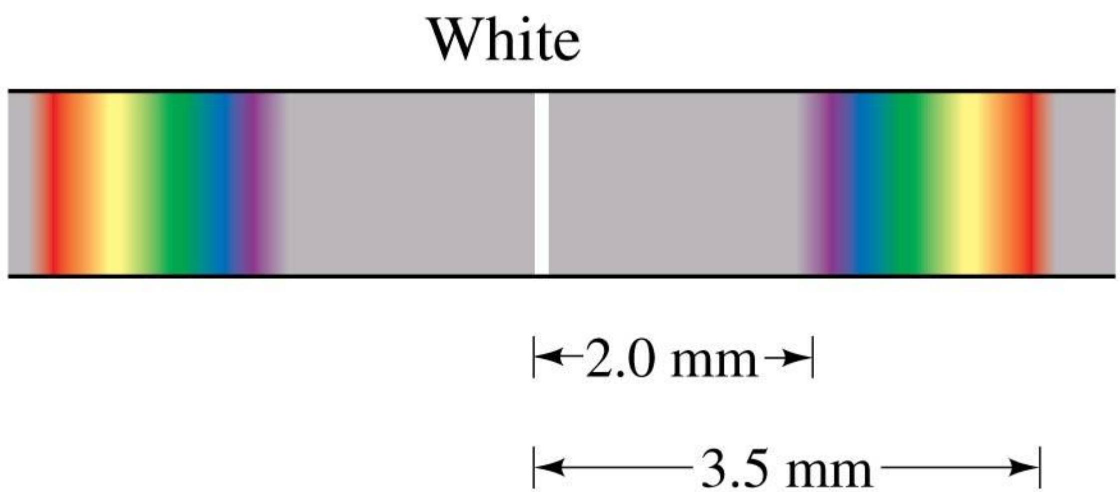
$$\boxed{\Delta = k\lambda_{红} = (k+1)\ \lambda_{紫}}$$

$$k = \frac{\lambda_{紫}}{\lambda_{红} - \lambda_{紫}} = \frac{4000}{7000 - 4000} \approx 1 \cdot 3$$

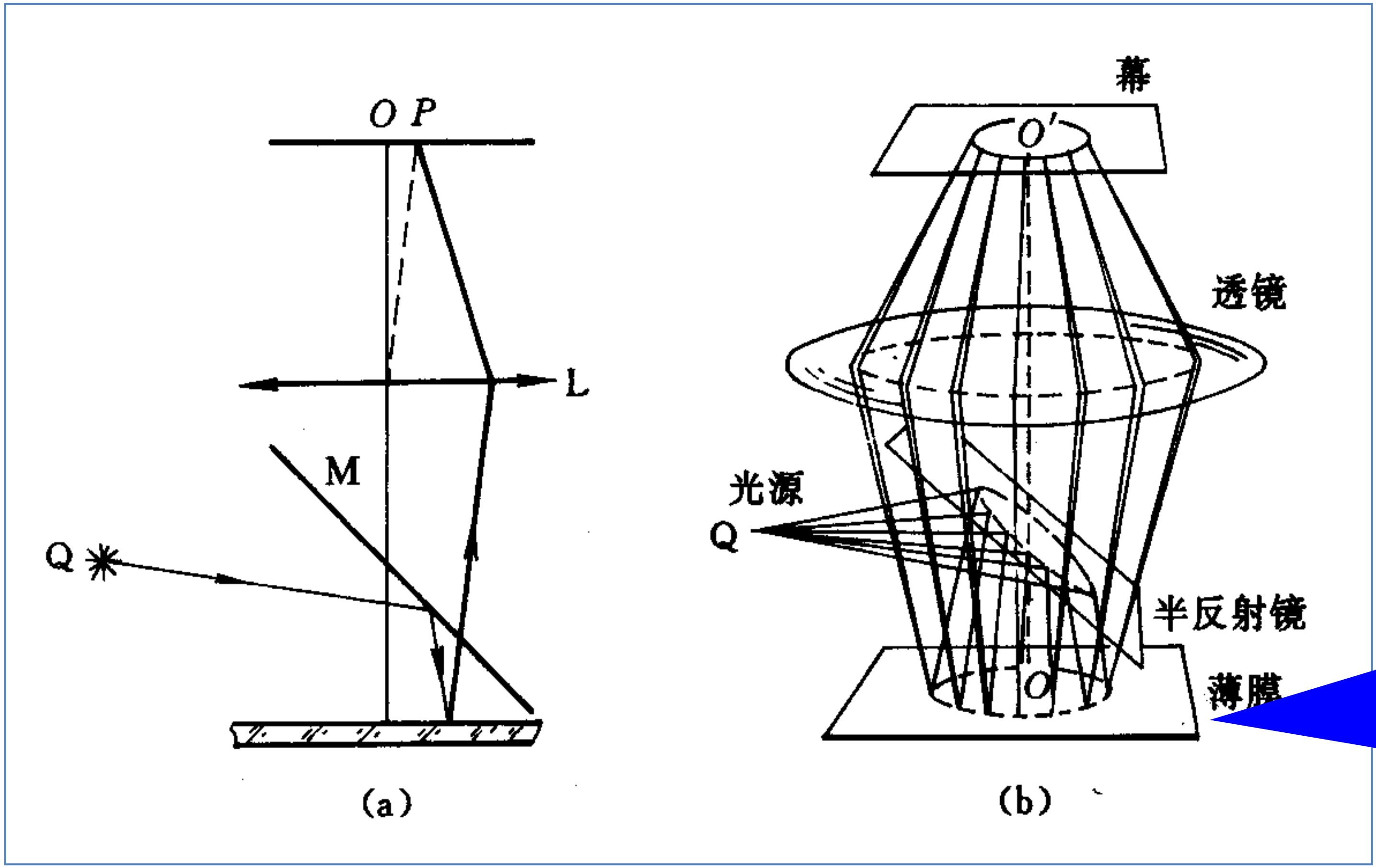
未重叠的清晰光谱只有一级。

24-3 Interference—Young's Double-Slit Experiment

Since the position of the maxima (except the central one) depends on wavelength, the first- and higher-order fringes contain a spectrum of colors.

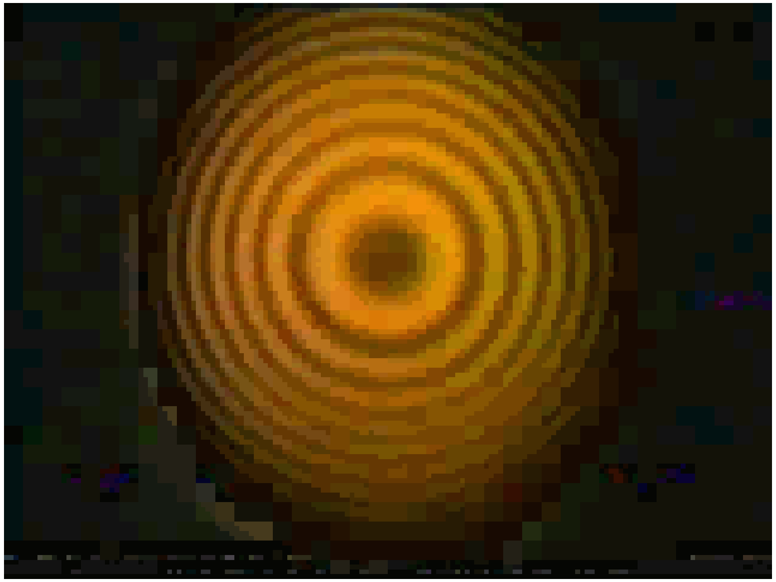


2、Equal inclination interference



表面相互平行的平面薄膜

等倾干涉图样

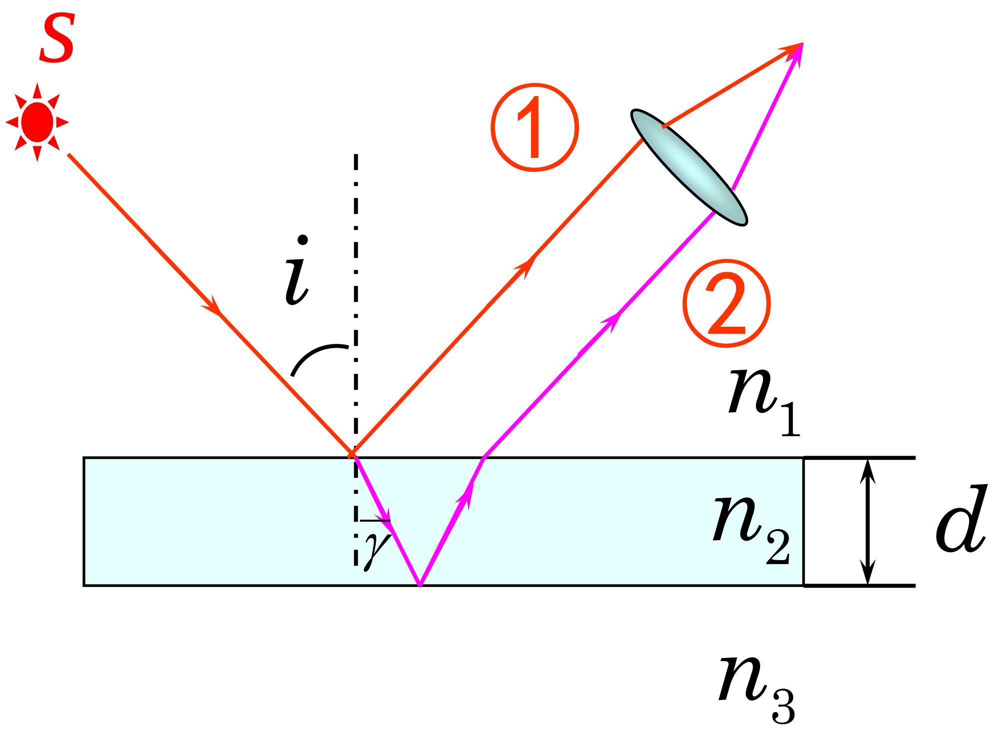


干涉条纹为一系列内疏外密的同心圆环

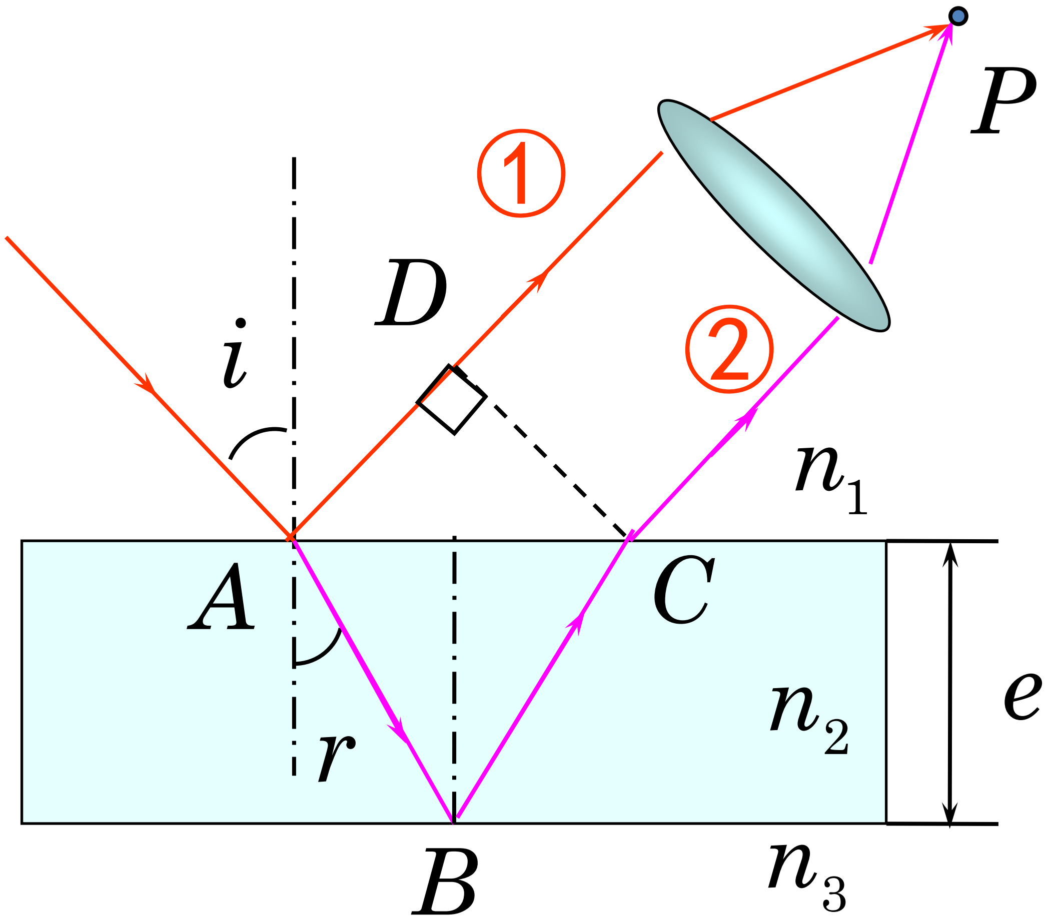
The interference fringe is a series of concentric rings

1、Constructive interference conditions

Monochromatic light enters a medium of refractive index n_2 from a medium of refractive index n_1 at the incident Angle i ,



Monochromatic light enters a medium of refractive index n_2 from a medium of refractive index n_1 at the incident Angle i ,



$$\Delta' = n_2(\overline{AB} + \overline{BC}) - n_1\overline{AD}$$

$$n_1 \sin i = n_2 \sin r$$

$$\Delta' = 2e\sqrt{n_2^2 - n_1^2 \sin^2 i}$$

Consider the half-wave loss:

$n_1 < n_2 < n_3$

$n_1 > n_2 > n_3$

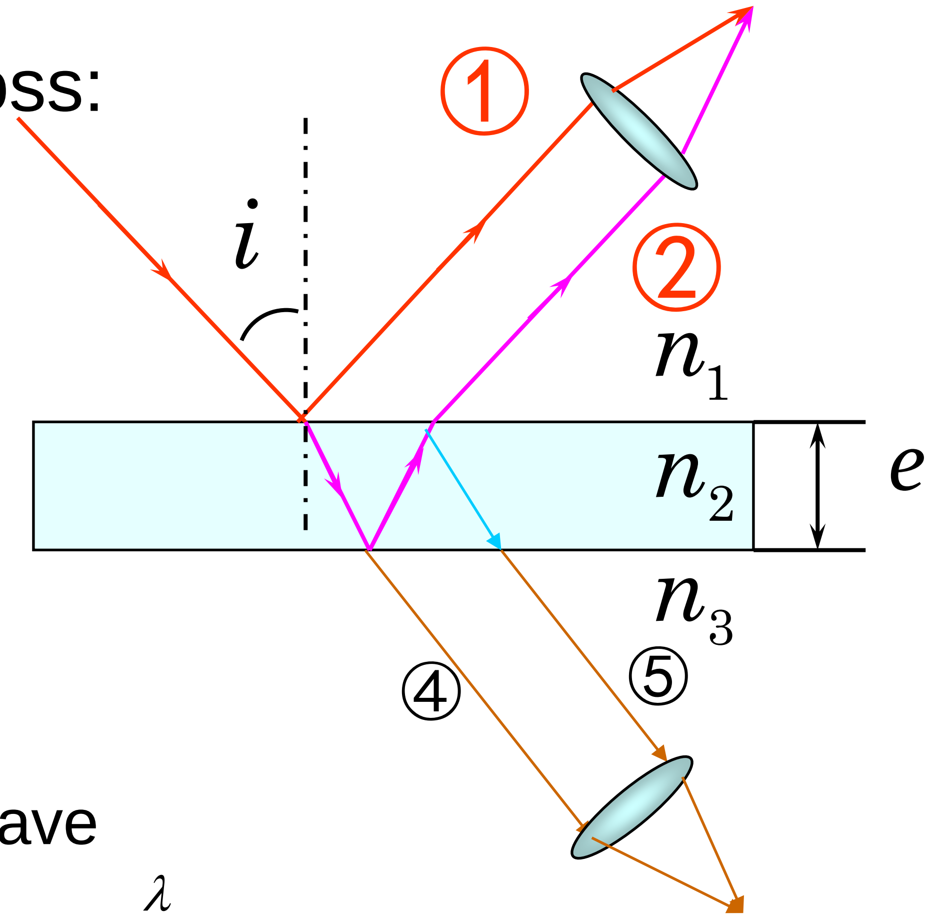
Optical path difference $\frac{\lambda}{2}$ is canceled

$n_1 < n_2 > n_3$

$n_1 > n_2 < n_3$

The half-wave loss introduces $\frac{\lambda}{2}$

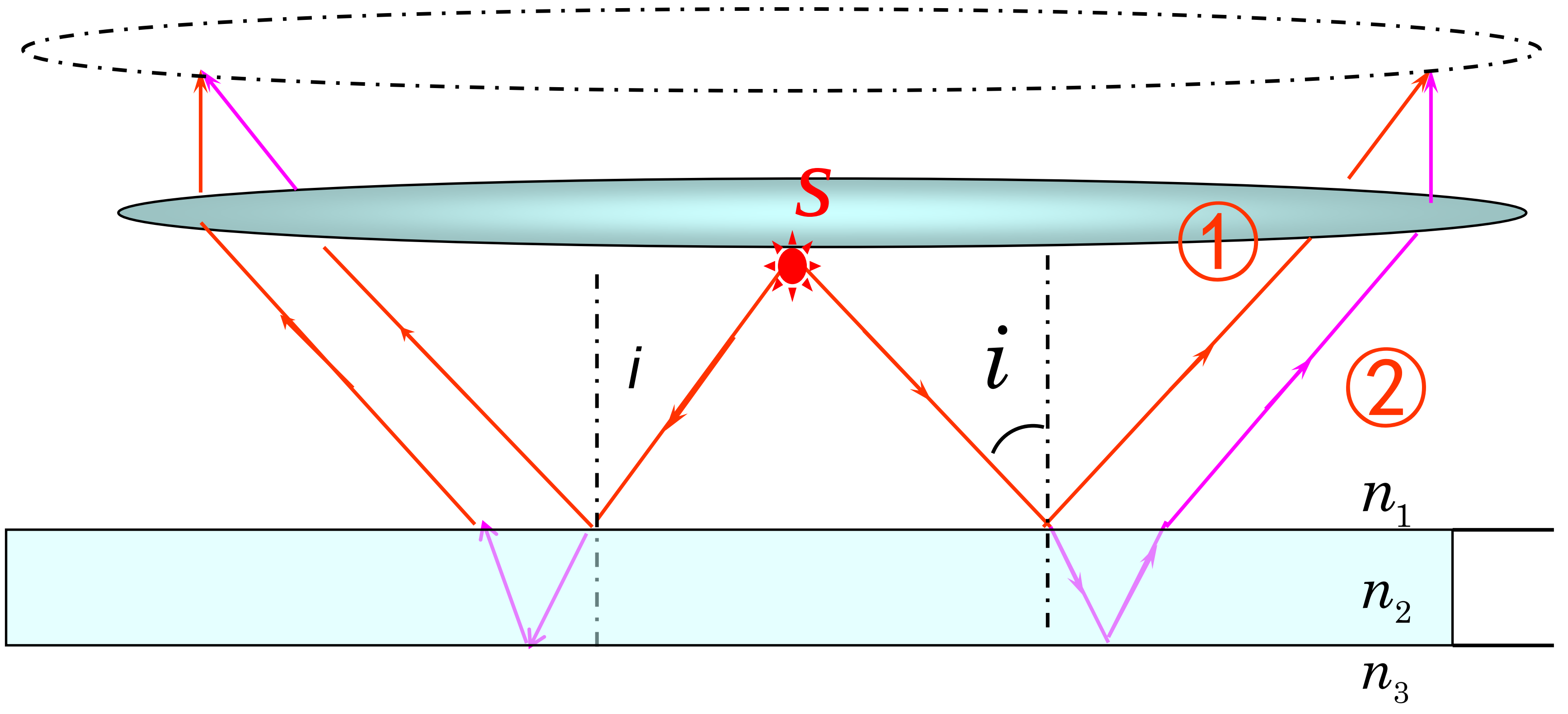
$$\Delta = 2e\sqrt{n_2^2 - n_1^2 \sin^2 i} + \boxed{\frac{\lambda}{2}}$$



折射光半波损失的分析与反射光情况正好相反!!!

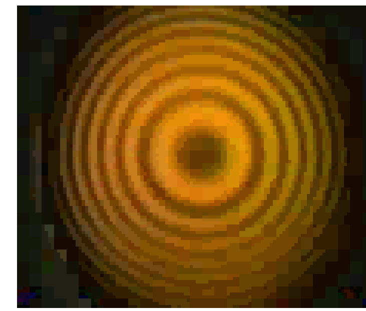
$$\Delta = 2e\sqrt{n_2^2 - n_1^2 \sin^2 i} + \boxed{\frac{\lambda}{2}} \begin{cases} \pm k\lambda & \text{C}\quad k=0,1,.. \\ \pm(2k-1)\dfrac{\lambda}{2} & \text{D}\quad k=1,2\ldots \end{cases}$$

Rays with the same incident Angle are located on the same fringe, which is a series of concentric rings



2、The pattern of fringe

$$\Delta = 2e\sqrt{n_2^2 - n_1^2 \sin^2 i} + \frac{\lambda}{2} = \begin{cases} \pm k\lambda & \text{C} \\ \pm (2k-1)\dfrac{\lambda}{2} & \text{D} \end{cases}$$

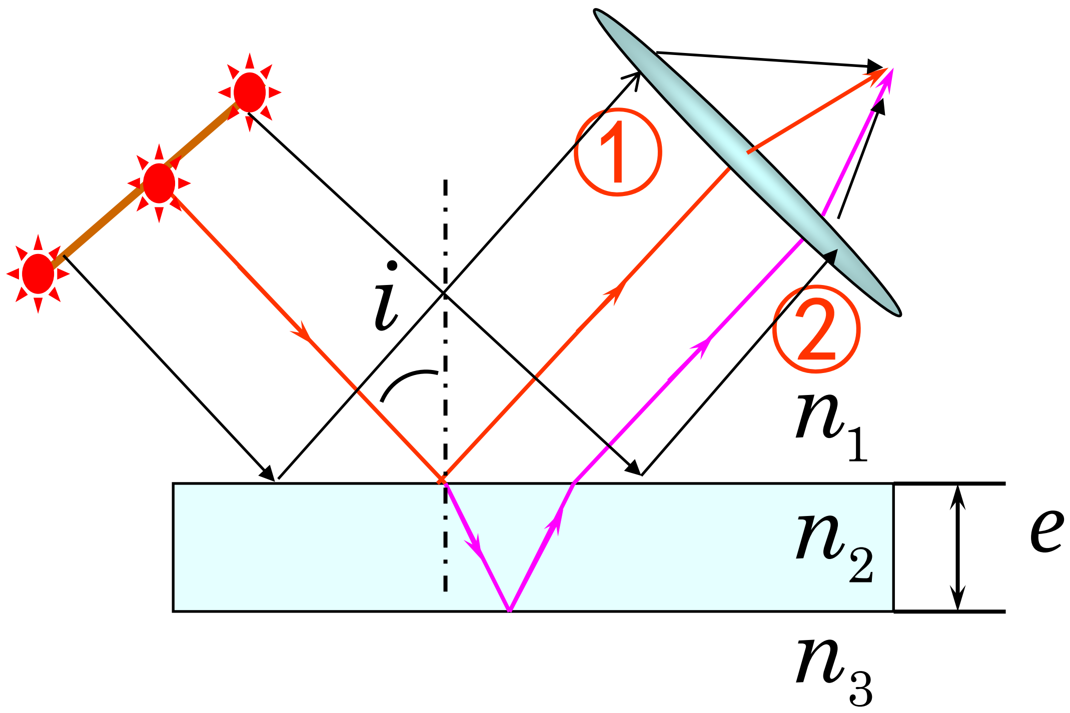


The central fringe is the widest and corresponds to zero incident Angle

The fringe shows the pattern that changes from dense to thin at edge to center

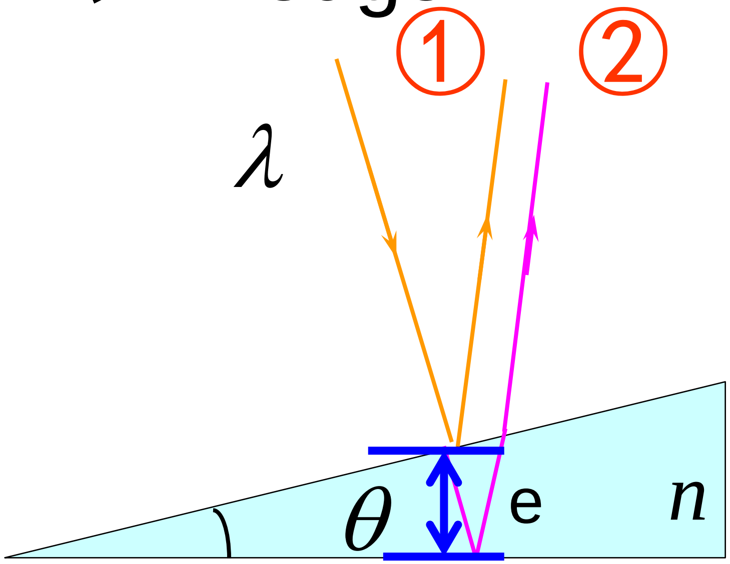
$$\Delta = 2e\sqrt{n_2^2 - n_1^2 \sin^2 i} + \frac{\lambda}{2} = k\lambda \qquad \left|\mathrm{d}k / \mathrm{d}i\right| \propto \sin\gamma \propto \sin i$$

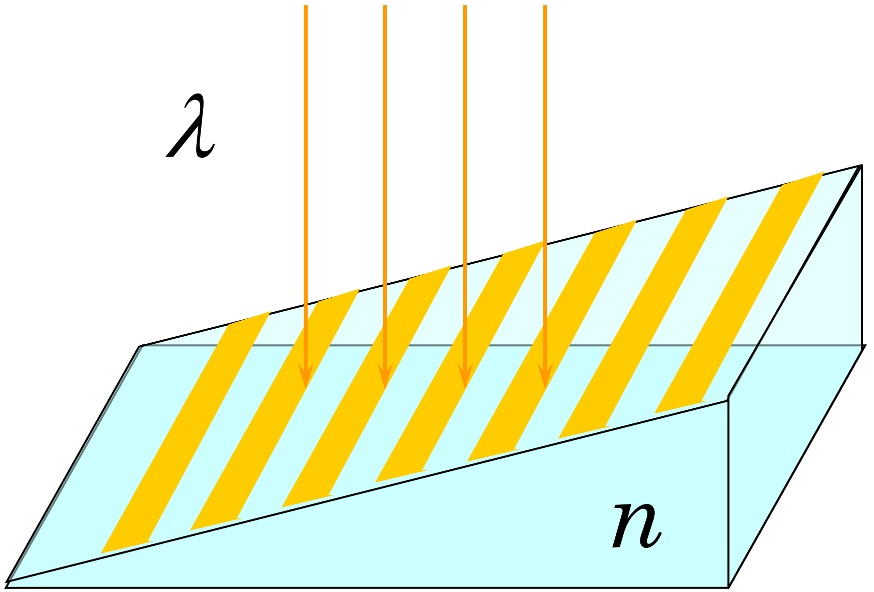
The size of the light source has no effect on the distribution of interference fringe, only on the brightness of fringe.



3. Equal thickness interference

1、 Wedge





For vertical incidence, with small θ, beams 1 and 2 actually coincide, and their optical path difference is approximately:

$$\Delta = 2e\sqrt{n_2^2 - n_1^2 \sin^2 i} + \boxed{\frac{\lambda}{2}} = 2ne + \boxed{\frac{\lambda}{2}} = \begin{cases} k\lambda & \text{C} \quad k = 1、2\cdots \\ (2k+1)\dfrac{\lambda}{2} & \text{D} \quad k = 0、1、2\cdots \end{cases}$$

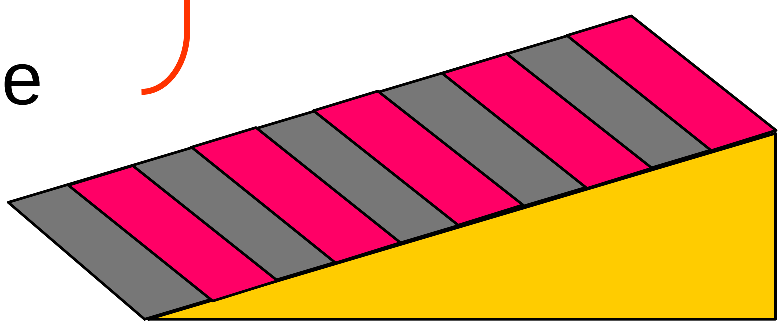
$$\Delta = 2ne + \frac{\lambda}{2} = \begin{cases} k\lambda & \text{C} \quad k = 1、2\cdots \\ (2k+1)\dfrac{\lambda}{2} & \text{D} \quad k = 0、1、2\cdots \end{cases}$$

The same interference fringe corresponds to the same thickness of the film

The different thickness of the film corresponds to different interference fringes

The fringes are the same shape as the thickness of the film

Equal thickness interference



$$\Delta = 2ne + \frac{\lambda}{2} = \begin{cases} k\lambda & k = 1、2\cdots \\ (2k+1)\dfrac{\lambda}{2} & k = 0、1、2\cdots \end{cases}$$

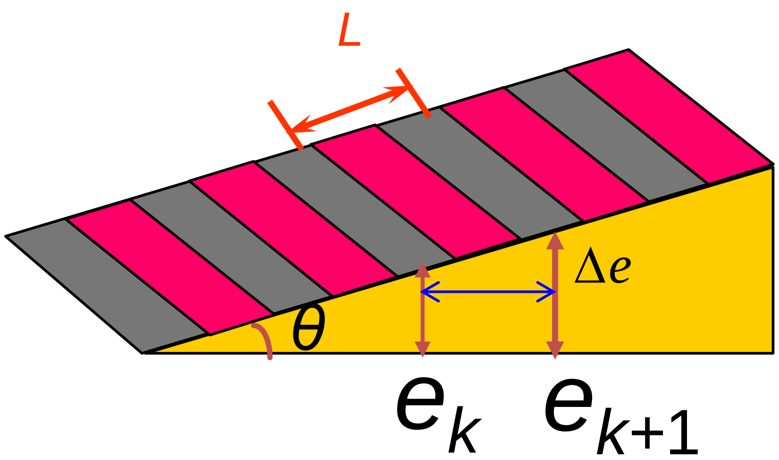
C

D

When $e = 0$ $\quad \Delta = \dfrac{\lambda}{2}$

Adjacent bright (dark) fringes corresponding to the film thickness difference:

$$\Delta e = \frac{\lambda}{2n}$$



Fringe width (spacing between two adjacent dark lines)

$$L = \frac{\Delta e}{\sin\theta} = \frac{\lambda}{2n\sin\theta} \approx \frac{\lambda}{2n\theta}$$

Fringe width: the distance between two adjacent dark lines or the distance between two adjacent bright lines

$$L = \frac{\Delta e}{\sin\theta} = \frac{\lambda}{2n\sin\theta} \approx \frac{\lambda}{2n\theta}$$

Discussion:

1、 What factors can affect the fringes?

2、 The top surface of the wedge moves up parallelly. How do the fringes change ?

3、How do fringes change when you gently press the wedge?

4、There is a groove on the underside of the wedge. What is the shape of the interference fringe?

$$L = \frac{\Delta e}{\sin\theta} = \frac{\lambda}{2n\sin\theta} \approx \frac{\lambda}{2n\theta}$$

(1) Any change on n、 λ、 θ will affect the fringe

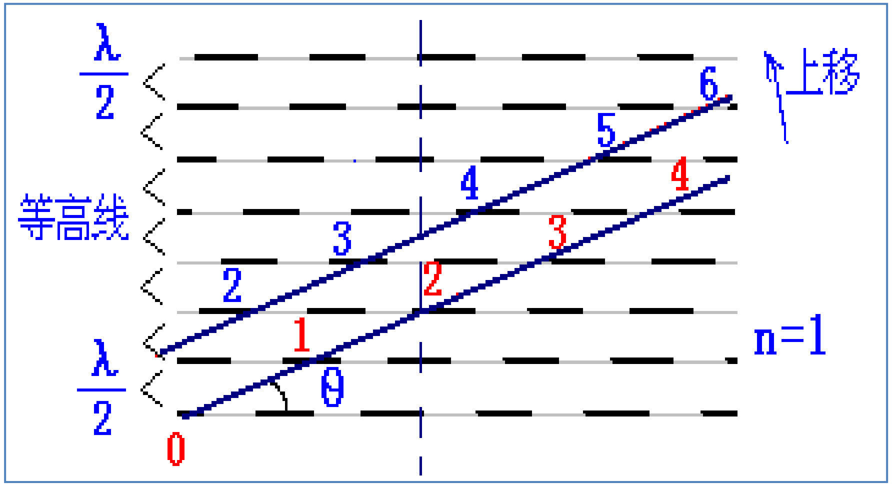
n、λ, $\theta\uparrow$ $L\downarrow$ **Angle increasing cause fringe denser**

n、θ, $\lambda\uparrow$ $L\uparrow$ $L_{red} > L_{purple}$ **Color fringes appear when white light is incident**

λ、θ, $n\uparrow$ $L\downarrow$ **Air wedge filled with water cause fringe denser**

Invariant

(2) The top surface of the wedge moves up parallelly. How do the fringes change ?

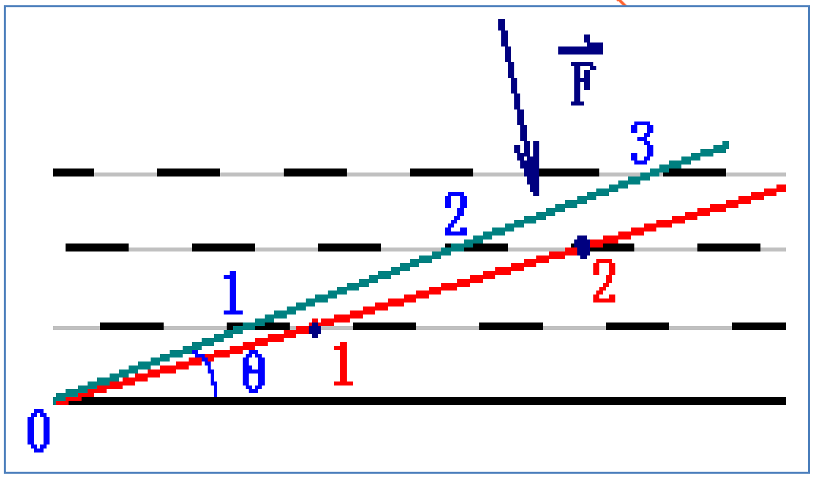


θ invariant

fringe width unchanged

The fringe moves left (towards the edge)

(3) How do fringes change when you gently press the wedge?

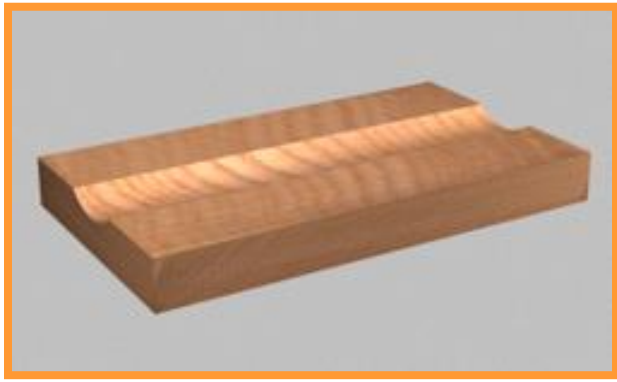


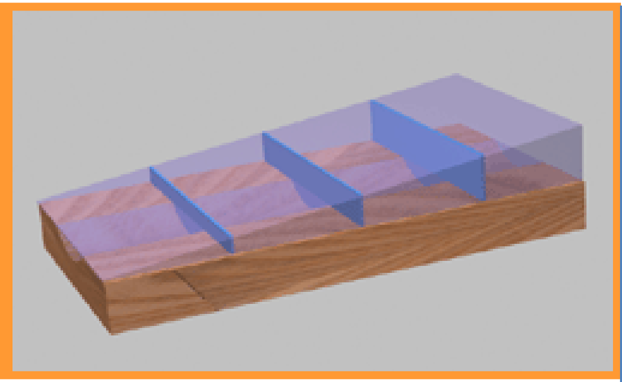
θ smaller, fringe width increase

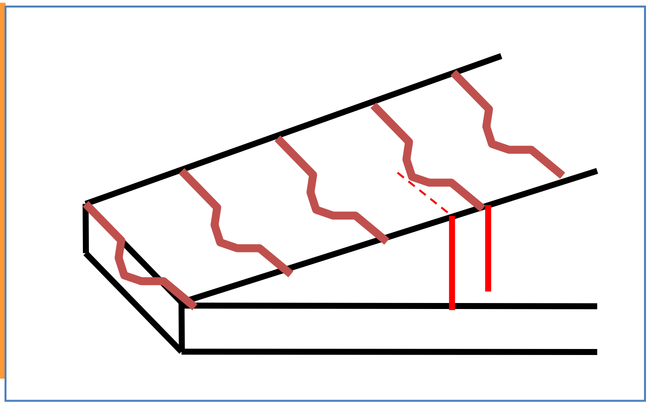
The fringe moves to the right (away from the edge)

$$L \approx \frac{\lambda}{2n\theta}$$

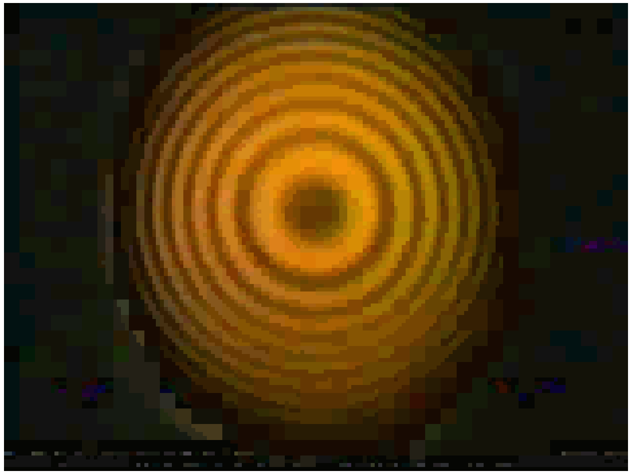
(4) There is a groove on the underside of the wedge.
What is the shape of the interference fringe ?



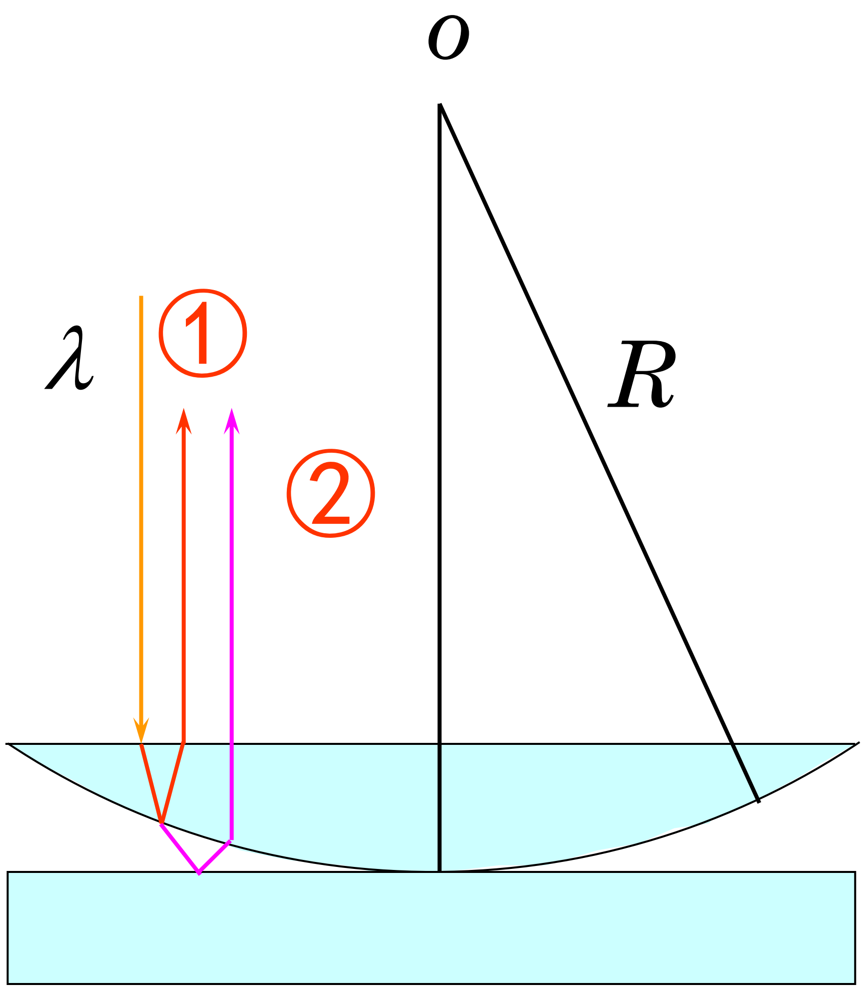




3、Equal thickness interference



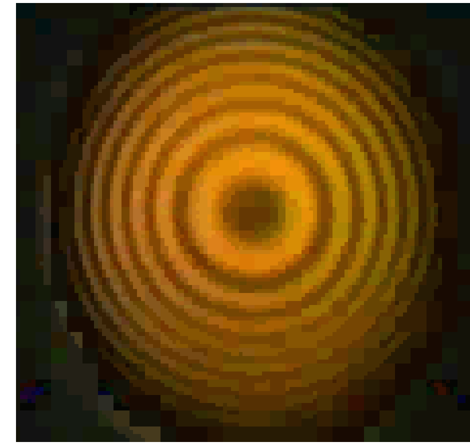
2、Newton rings



$$\Delta = 2ne + \frac{\lambda}{2}$$

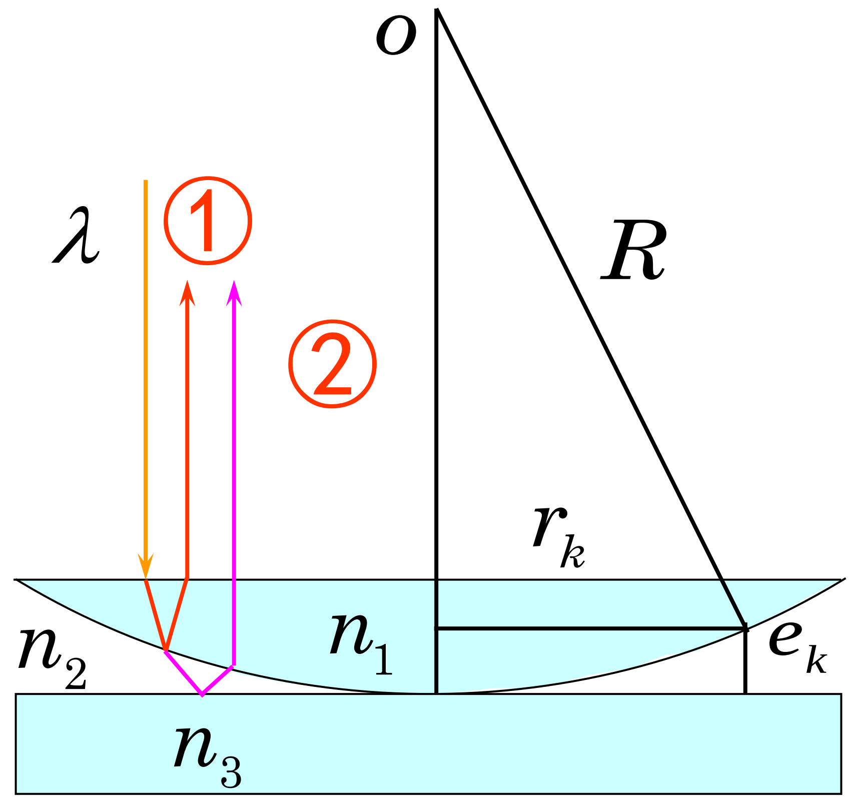
$$\begin{cases} k\lambda & \text{C} \quad k=1,2\ldots \\ (2k+1)\dfrac{\lambda}{2} & \text{D} \quad k=0,1,2\ldots \end{cases}$$

Concentric rings of light and dark centered on contact points



At center $e=0 \quad \Delta=\frac{\lambda}{2}$ Dark fringe

中央级次高，边缘级次低



•Fringe radius:

$$R^2=r_k^{\ 2}+(R-e_k)^2$$

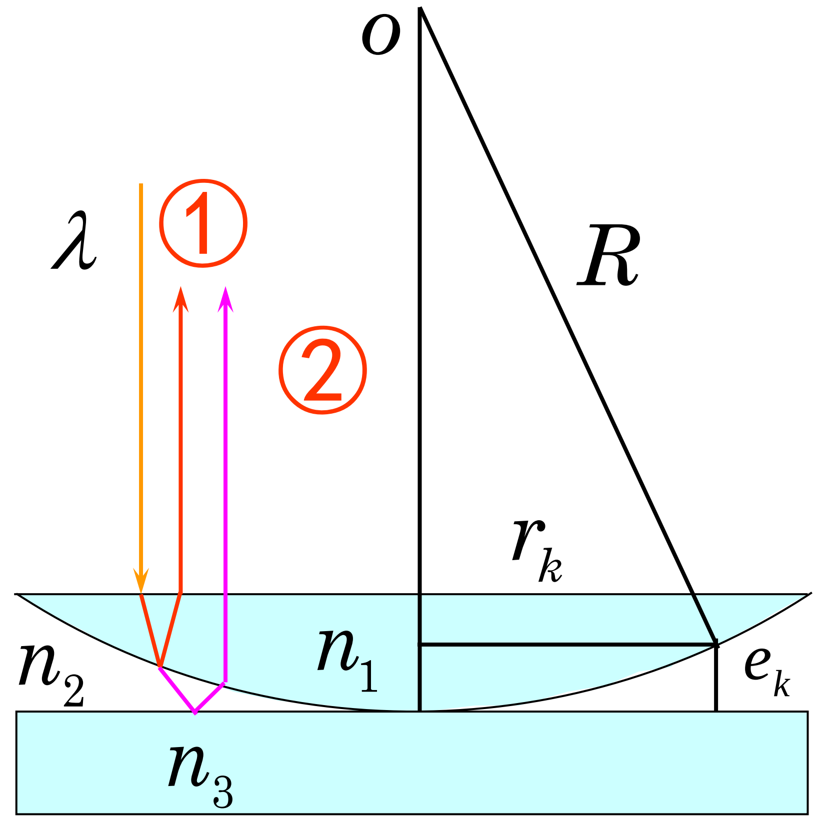
$$=r_k^{\ 2}+R^2-2Re_k+e_k^{\ 2}$$

$$e_k \ll R \qquad e_k=\frac{r_k^{\ 2}}{2R}$$

•Fringe radius：

$$e_k = \frac{r_k^2}{2R} \qquad r_k = (2Re_k)^{\frac{1}{2}}$$

$$\Delta = 2ne_k + \frac{\lambda}{2} = \begin{cases} k\lambda & \text{C} \\ (2k+1)\dfrac{\lambda}{2} & \text{D} \end{cases}$$



C $r_k = \sqrt{(k-1/2)R\lambda/n} \qquad (k = 1,2\cdots)$

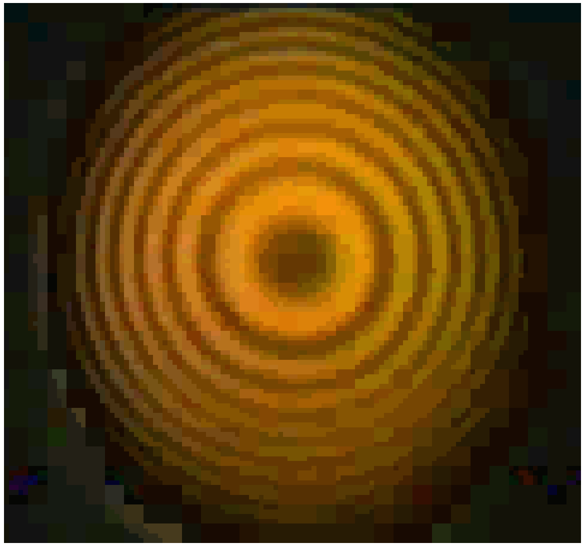
D $r_k = \sqrt{kR\lambda/n} \qquad (k = 0,1,2\cdots)$

$$r_k = \sqrt{(k-1/2)R\lambda/n} \qquad r_k = \sqrt{kR\lambda/n}$$

$$r \propto \sqrt{k}$$

$$\frac{dk}{dr} \propto r$$

The fringes are thin inside and dense outside

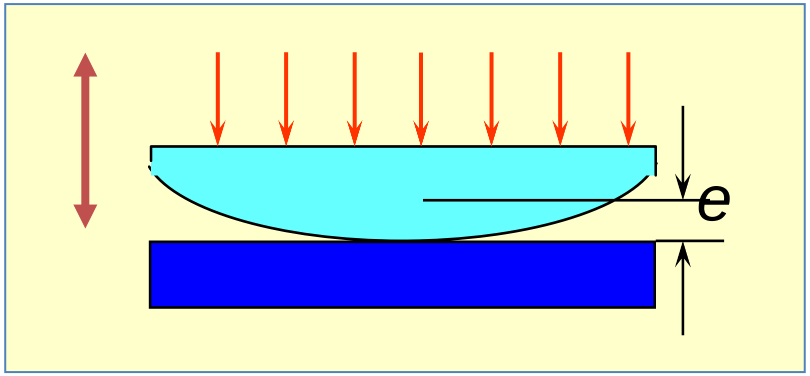


$$r \propto \sqrt{\lambda}$$

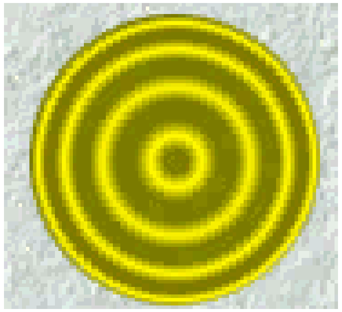
Colored rings appear under white light

Discussion：

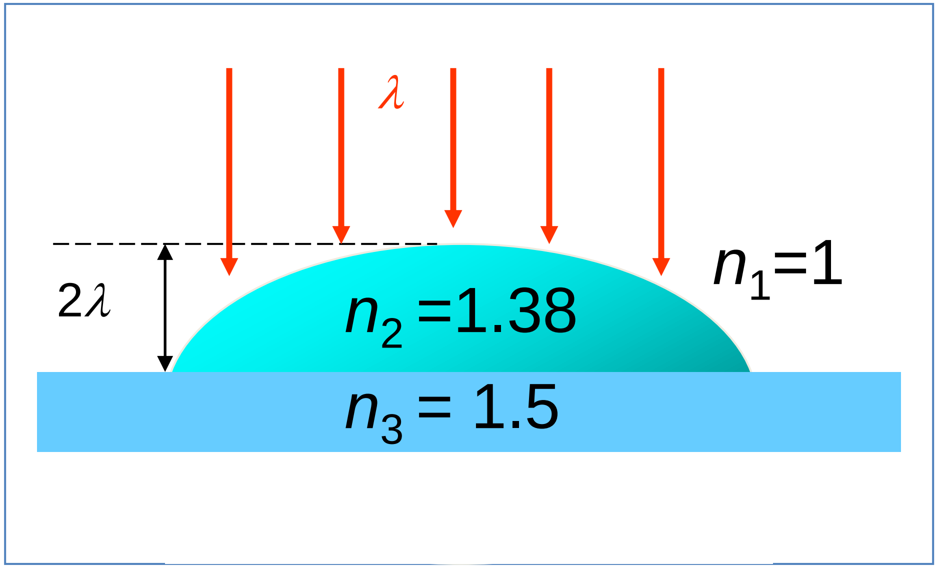
1、How will the fringes change when the flat convex lens moves up (down)?

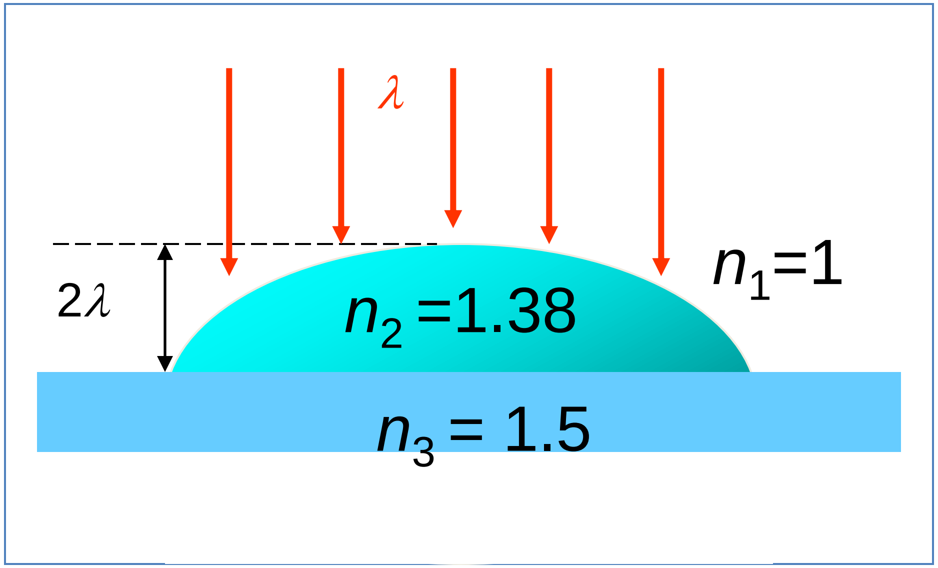


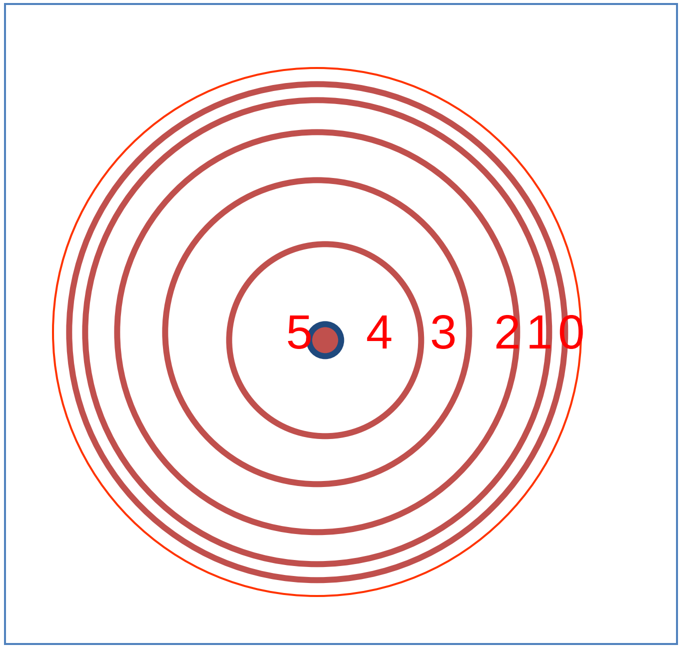
The operation will cause the fringes to contract inward or expand outward



2、What does the fringe behavior when a flat convex lens (shown on the right) is used in a Newton's rings experiment?





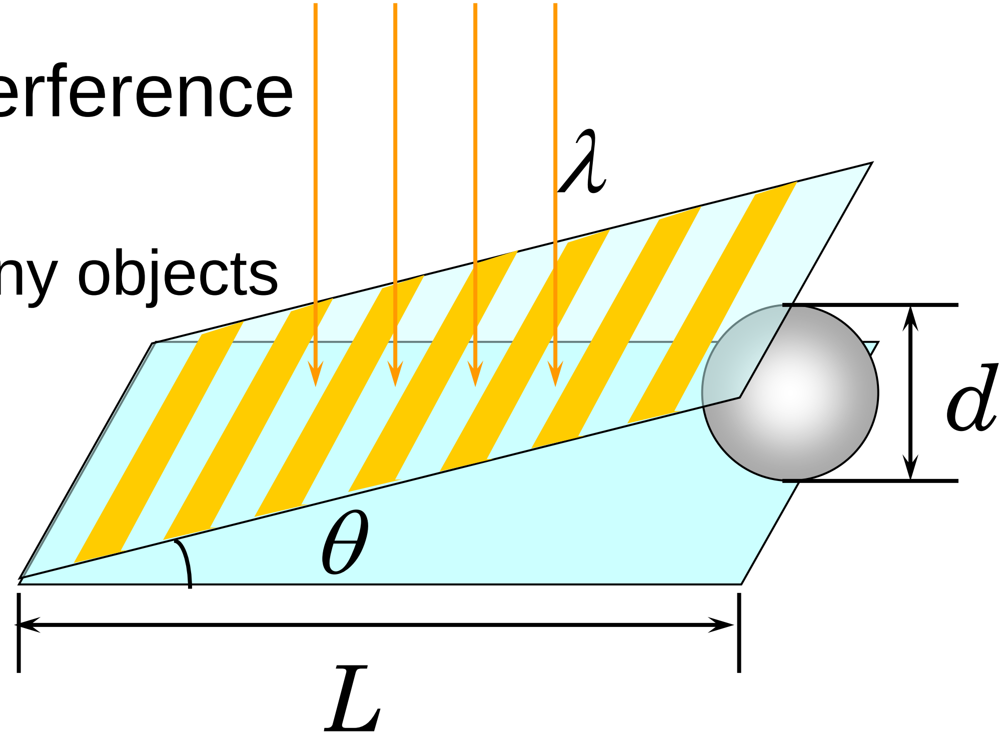


Δ中有无$\lambda/2$项?

$$\Delta = 2n_2 e \begin{cases} \text{Edge } e = 0 \quad \Delta = 0 \quad \text{C} \quad k = 0 \\ \text{Center } e = 2\lambda \quad \Delta = 4n_2\lambda \approx 5 \cdot 5\lambda \quad \text{D} \quad k = 5 \end{cases}$$

等厚线：圆环，条纹为内疏外密同心圆，共6条暗纹。

Application of thin film interference

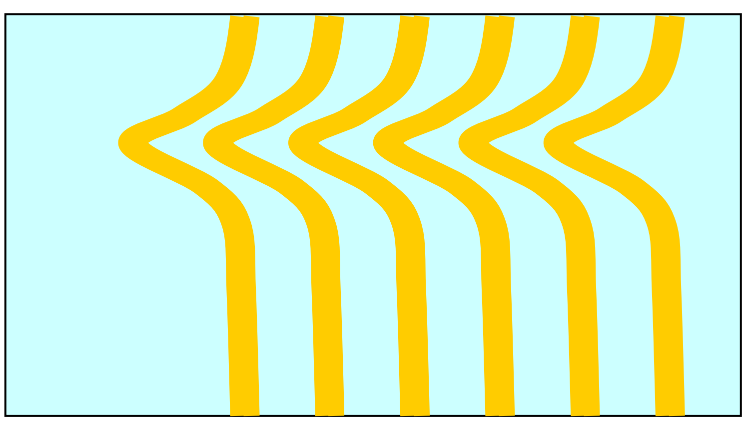


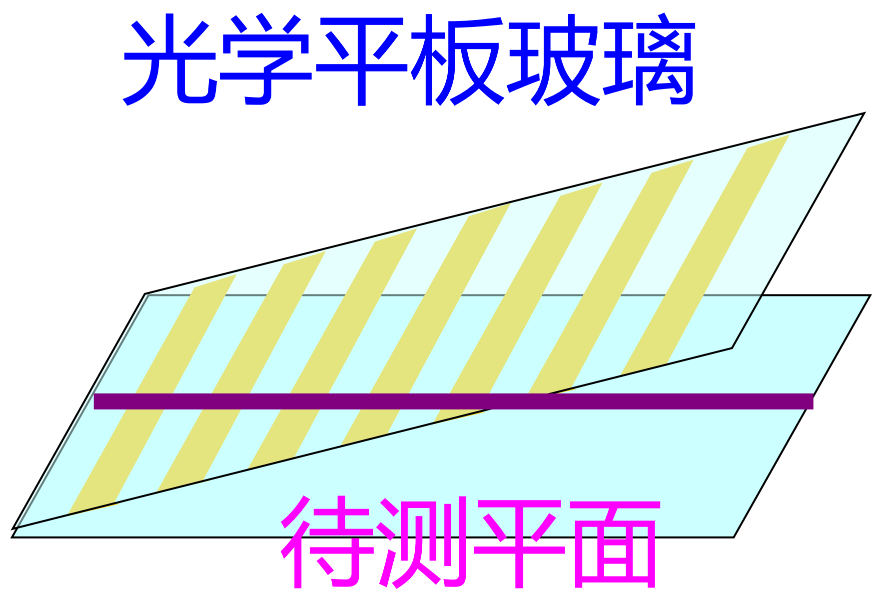
1. Measure the thickness of tiny objects

$$l = \frac{\lambda}{2n\sin\theta} \approx \frac{\lambda}{2n\theta} = \frac{\lambda}{2n}\frac{L}{d}$$

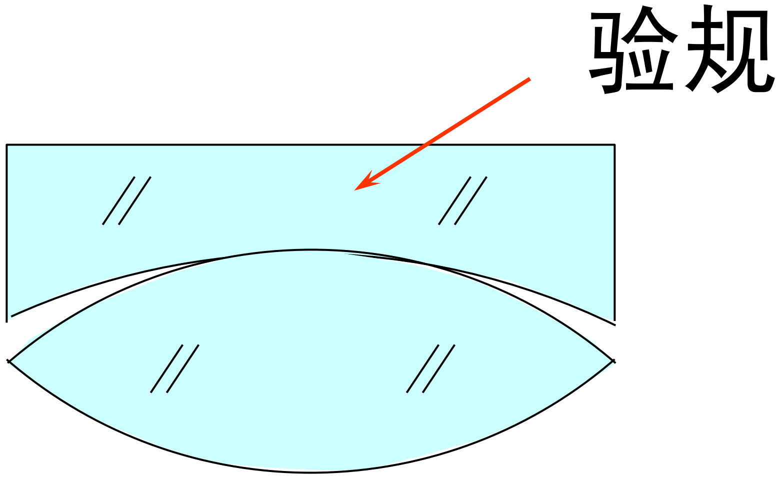
$$d = \frac{\lambda}{2n}\frac{L}{l}$$

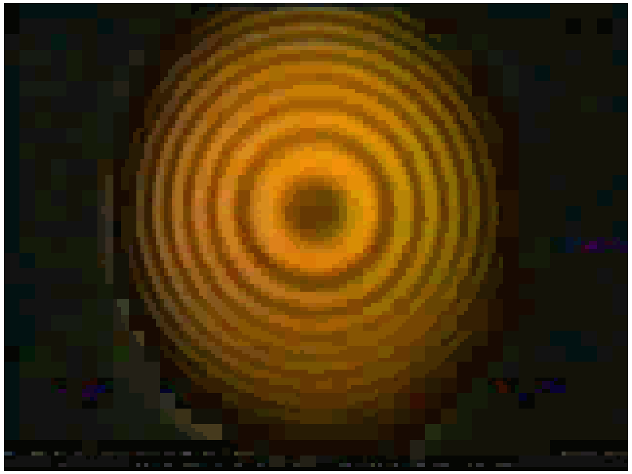
2. Measure the flatness of the plate under test





3、Check the surface curvature of optical lens





4、Measure the wavelength of an unknown monochromatic parallel light

$$r_k = \sqrt{kR\lambda / n} \qquad r_m^2 - r_k^2 = \frac{mR\lambda - kR\lambda}{n}$$

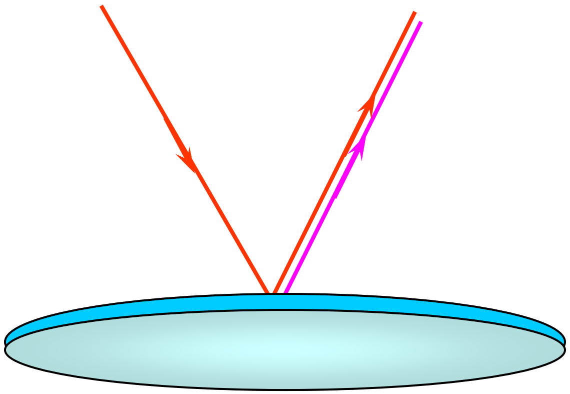
$$\boxed{\lambda = \frac{(r_m^2 - r_k^2)n}{(m-k)R}}$$

5、Anti-reflection film or anti-return film is made by film interference

Antireflection film

增透膜是使膜上下两表面的反射光满足减弱条件。

光学镜头为减少反射光，通常要镀增透膜。



MgF2 薄膜

$$\Delta = 2e\sqrt{n_2^2 - \sin^2 i} + \frac{\lambda}{2} = (2k+1)\frac{\lambda}{2}$$

$$(k = 0,1,2\cdots)$$

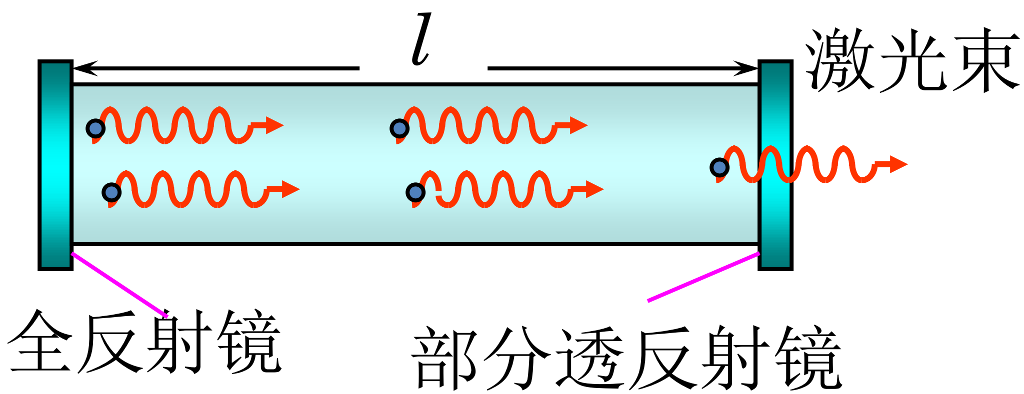
D

对人眼和感光底片最敏感的黄绿光 反射最小，使得蓝紫光反射最强，所以平常看照像机镜头都成蓝紫色。

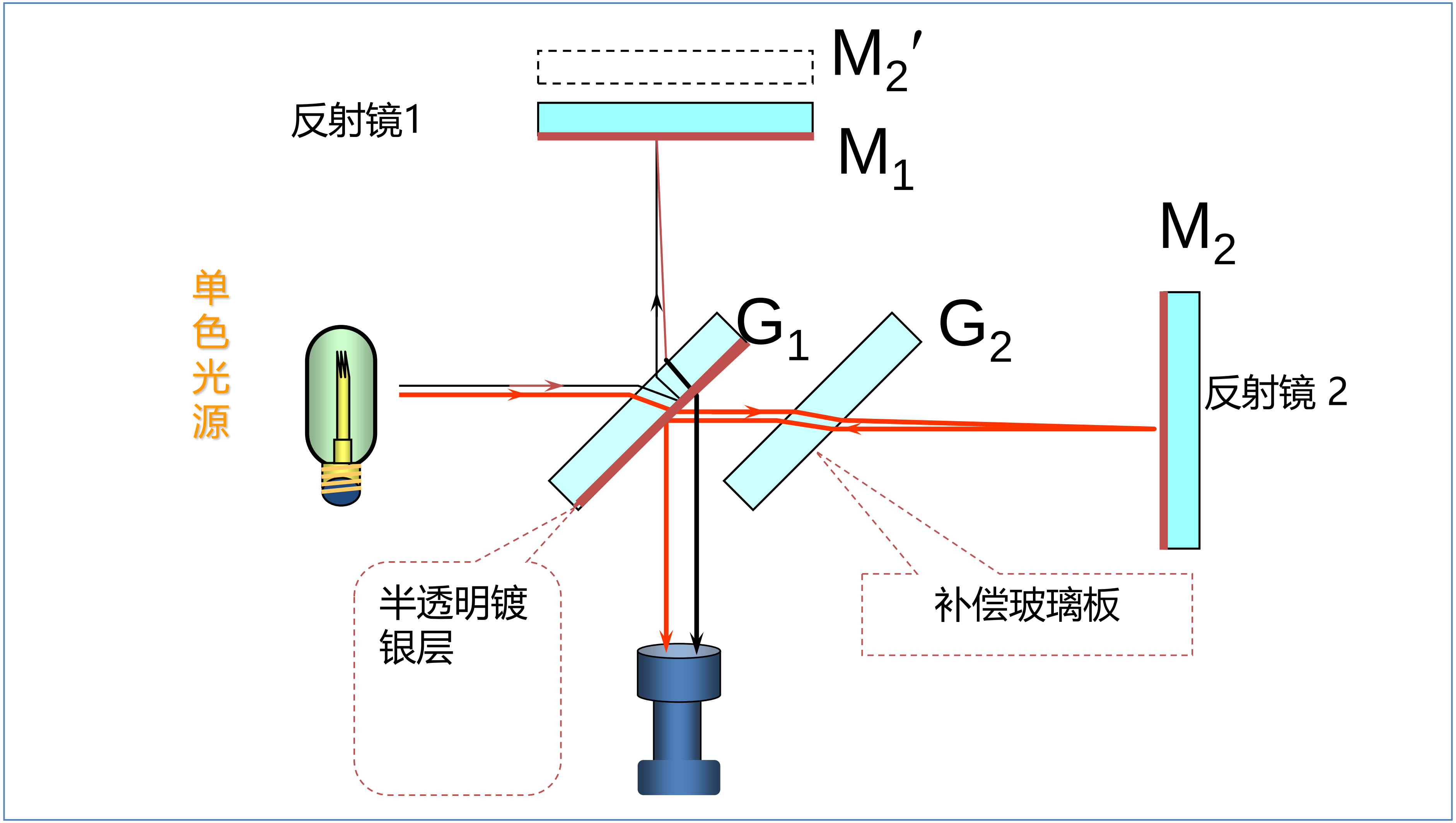
Anti-return film

减少透光量，增加反射光，使膜上下两表面的反射光满足加强条件。

例如：激光器谐振腔反射镜采用优质增反膜介质薄膜层已达15层，其反射率99.9%。



6、 Michelson Interferometer



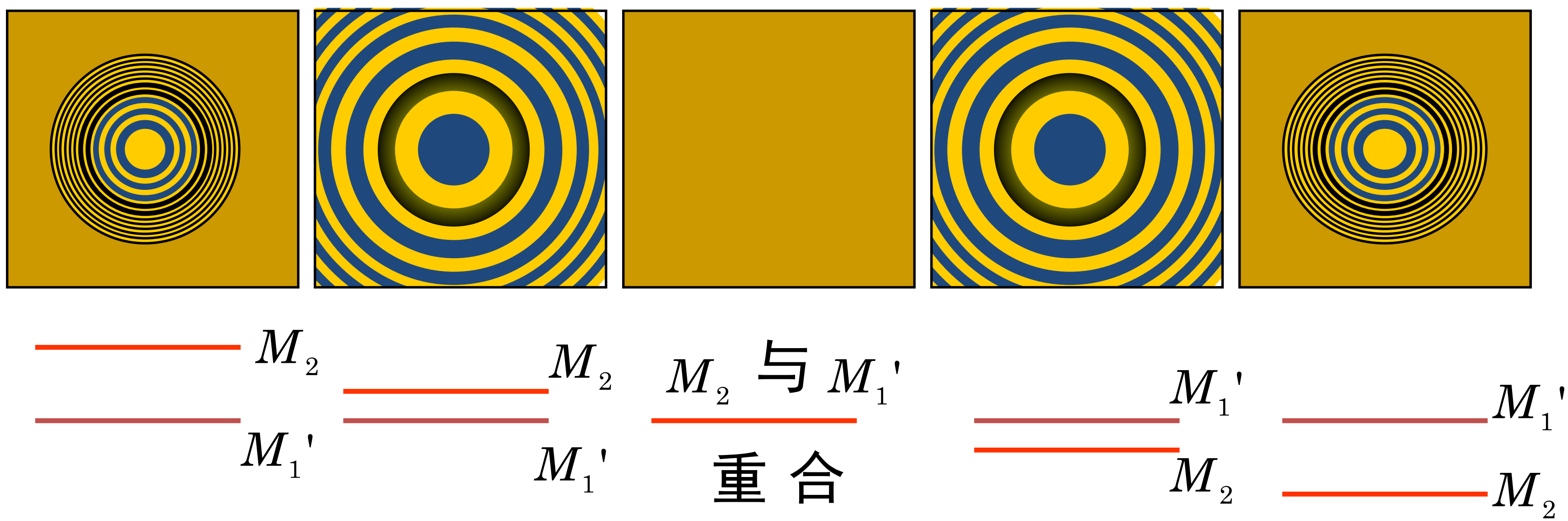
当 $M_2 \perp M_1$ 时， $M_2 \parallel M_1'$，所观察到的是等倾干涉条纹，即相同倾角下光程差相同。

M_2 、M_1'之间距离变大时，圆形干涉条纹向外扩张，干涉条纹变密。

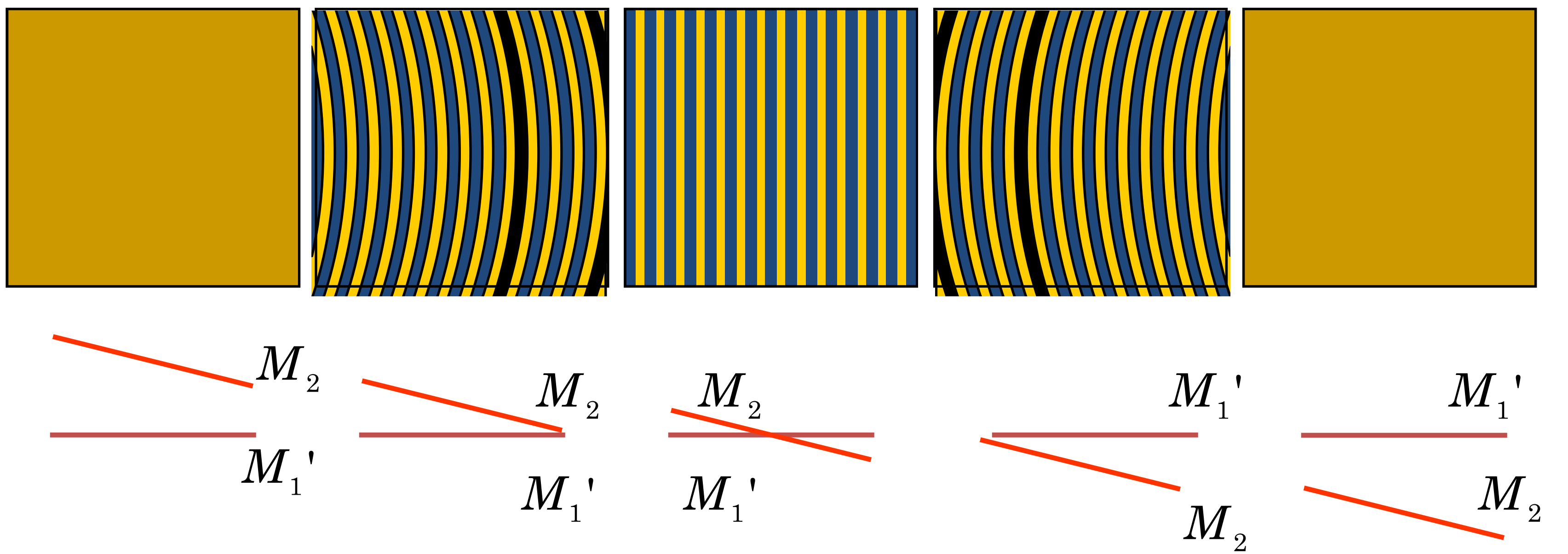
M_2 与M_1不垂直时时，形成劈尖，干涉条纹为等厚干涉条纹。

$$d = N \cdot \frac{\lambda}{2}$$

等倾干涉条纹

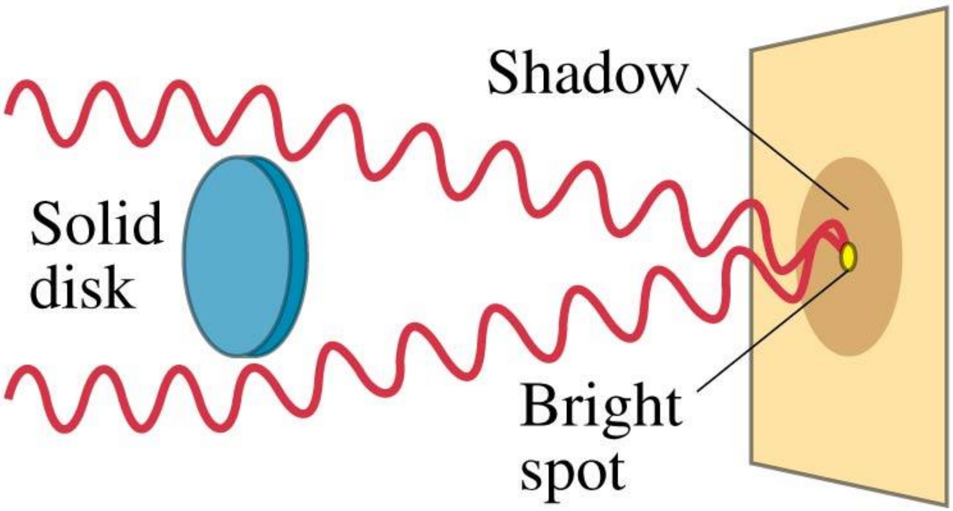


等厚干涉条纹



24-5 Diffraction by a Single Slit or Disk

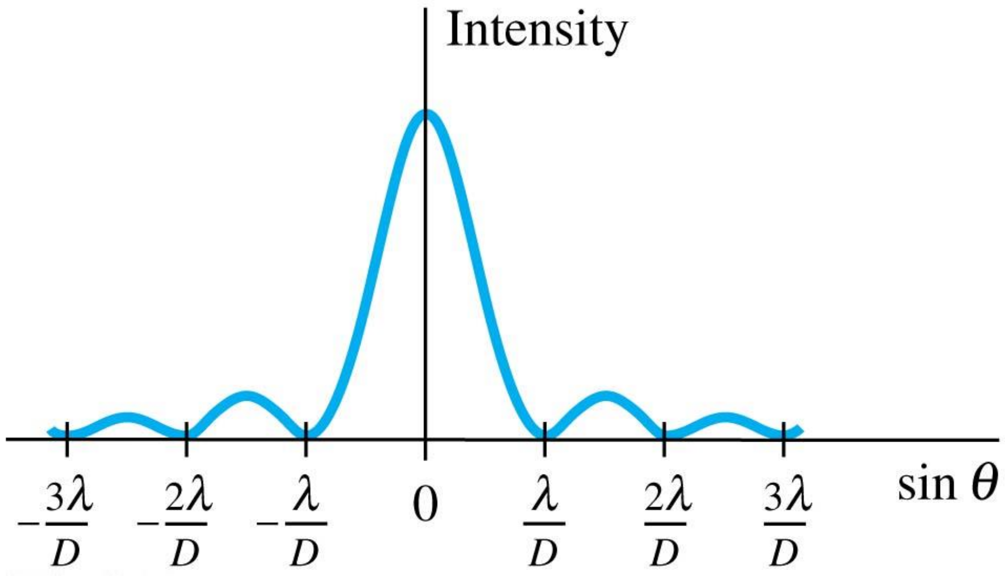
Light will also diffract around a single slit or obstacle.



24-5 Diffraction by a Single Slit or Disk

The resulting pattern of light and dark stripes is called a diffraction pattern.

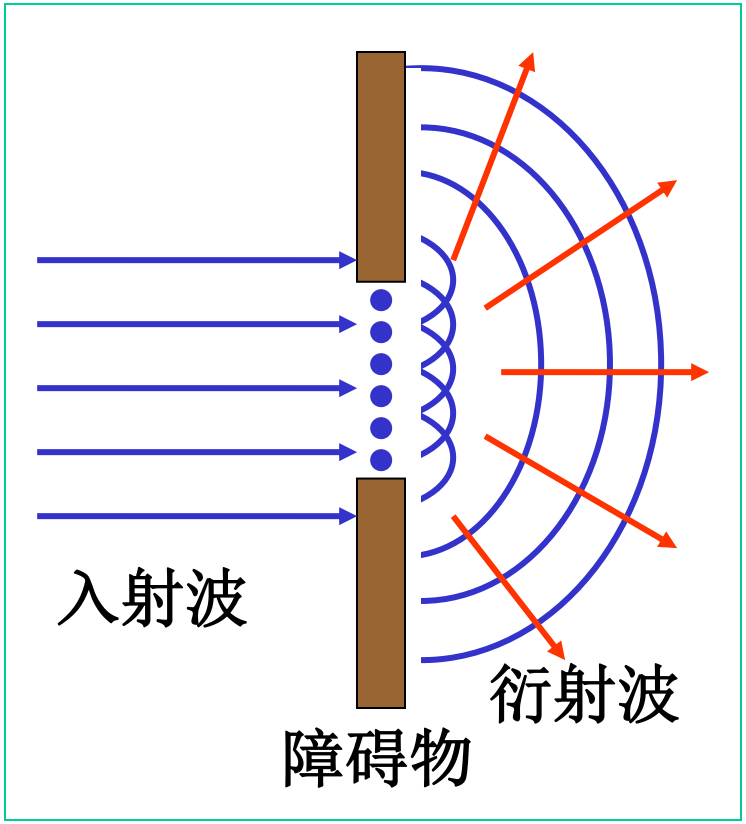
This pattern arises because different points along a slit create wavelets that interfere with each other just as a double slit would.



3. Huygens-Fresnel principle

1、Huygens’ principle

Huygens’ principle: Every point on a wave front acts as a point source; the wavefront as it develops is tangent to their envelope



成功：可解释衍射成因，用几何法作出新的波面，推导反射、折射定律，解释晶体中的双折射 。

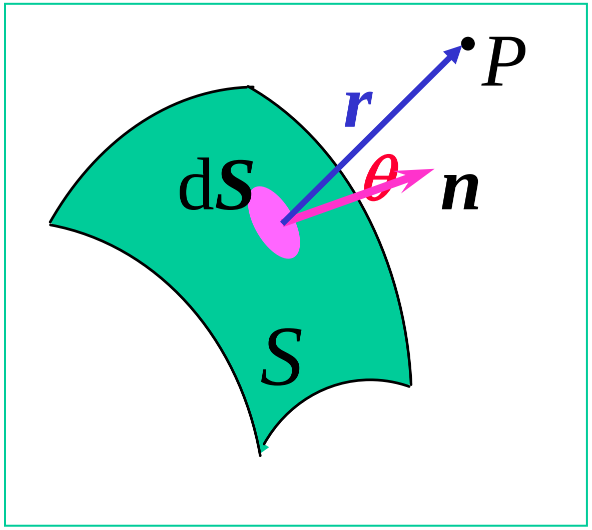
不足：不能定量说明衍射波的强度分布

可能出现倒退波。

2、Huygens-Fresnel principle

① **The amplitude and phase of wavelet are described quantitatively**

Each surface element on a wavefront is a source of wavelet



各子波初相相同：φ_0

子波在P点相位：$\omega t+\varphi_0-2\pi\dfrac{r}{\lambda}$

子波在P点振幅：

$$A\propto\frac{1}{r};\qquad A\propto\frac{1}{2}f(\theta)\mathrm{d}s$$

Inclination factor :

$$f(\theta)=\frac{1}{2}(1+\cos\theta)=\begin{cases}1 & (\theta=0)\\ 1/2 & (\theta=\pi/2)\\ 0 & (\theta=\pi)\end{cases}$$

wavelet : $$\mathrm{d}\psi=\frac{c}{2r}(1+\cos\theta)\cdot\cos(\omega t+\varphi_0-2\pi\frac{r}{\lambda})\cdot\mathrm{d}s$$

② The vibration of any point in space is the result of the coherent superposition of all wavelet at that point

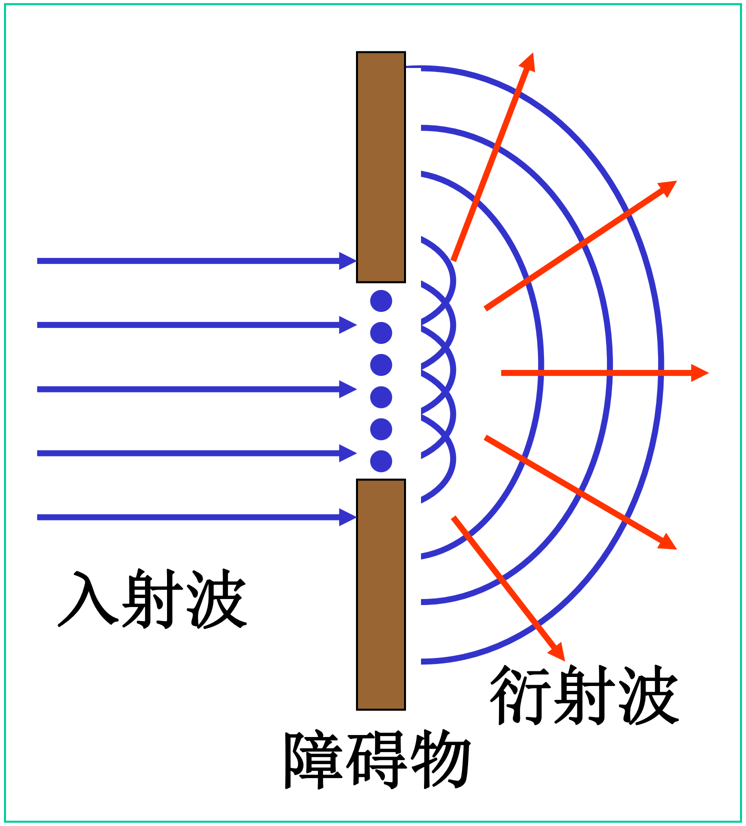
合振动: $$\psi=\int\mathrm{d}\psi$$

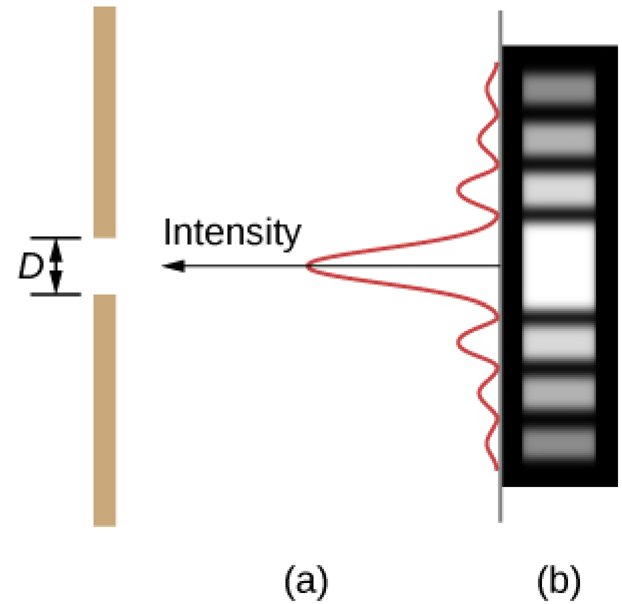
衍射本质:子波的相干叠加

- 有限个分立相干波叠加 —— 干涉
- 无限多个连续分布子波源相干叠加 —— 衍射

The Huygens-Fresnel principle indicates the superposition of all the waves on the same surface at a given observation point。

当波面完全不遮蔽时，所有次波在任何观察点叠加的结果乃形成光的直线传播。如果波面不完整，叠加时少了这些部分次波的参加，便发生了有明暗条纹花样的衍射现象。至于衍射现象是否显著，则和障碍物的线度及观察的距离有关。





Because of the diffraction of waves, ocean waves entering through an opening in a breakwater can spread throughout the bay. (credit: modification of map data from Google Earth)

Classification of diffraction:

Fresnel diffraction：

有限 距离　　有限 距离

波源 —— 障碍物 —— 屏

（或二者之一有限远）

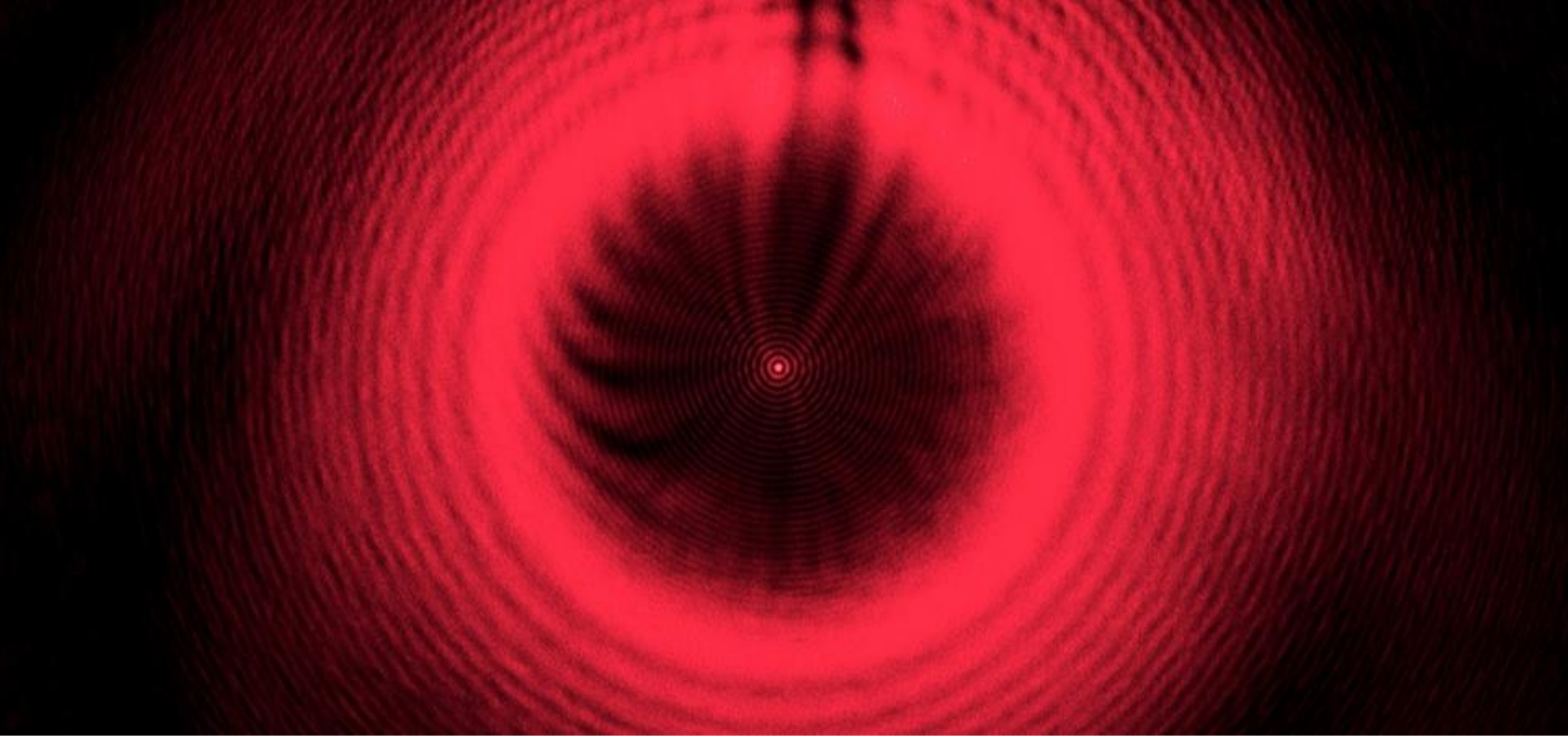
Fraunhofer diffraction：

波源 —— 障碍物 —— 屏

无限远　　无限远

L_1　　L_2

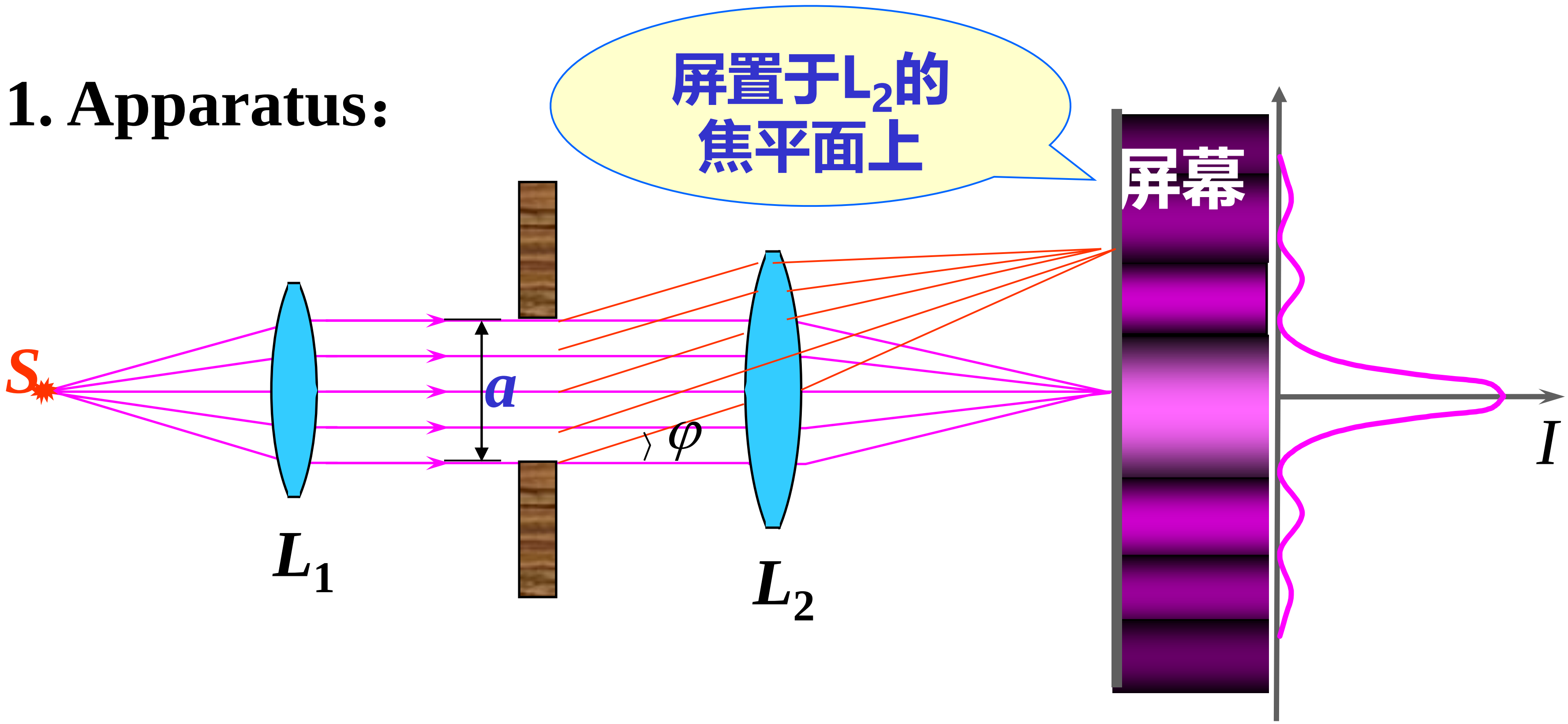
即平行光衍射



Figure： A steel ball bearing illuminated by a laser does not cast a sharp, circular shadow. Instead, a series of diffraction fringes and a central bright spot are observed. Known as Poisson’s spot, the effect was first predicted by Augustin-Jean Fresnel (1788–1827) Based on principles of ray optics, Siméon-Denis Poisson (1781–1840) argued against Fresnel’s prediction.

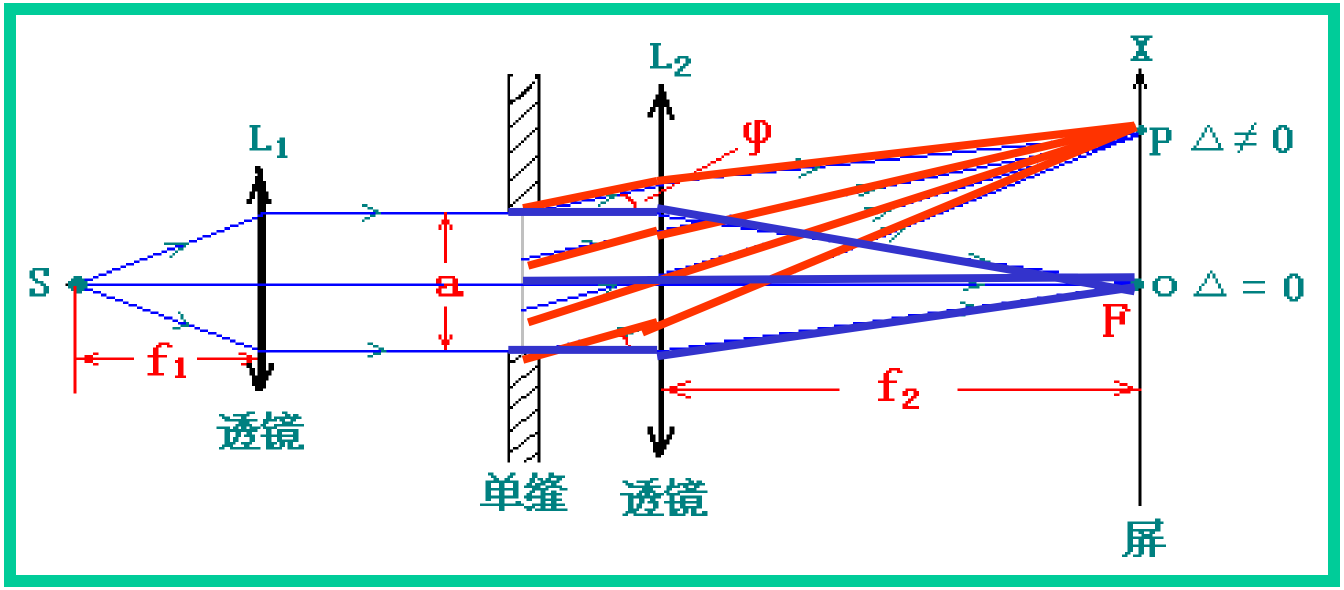
4、 Single slit Fraunhofer diffraction

1. Apparatus：



Slit width *a*: 其上每一点均为子波源，发出球面波

Diffraction angle *φ*: 衍射光线与波面法线夹角



Screen is placed on the focal plane of L2

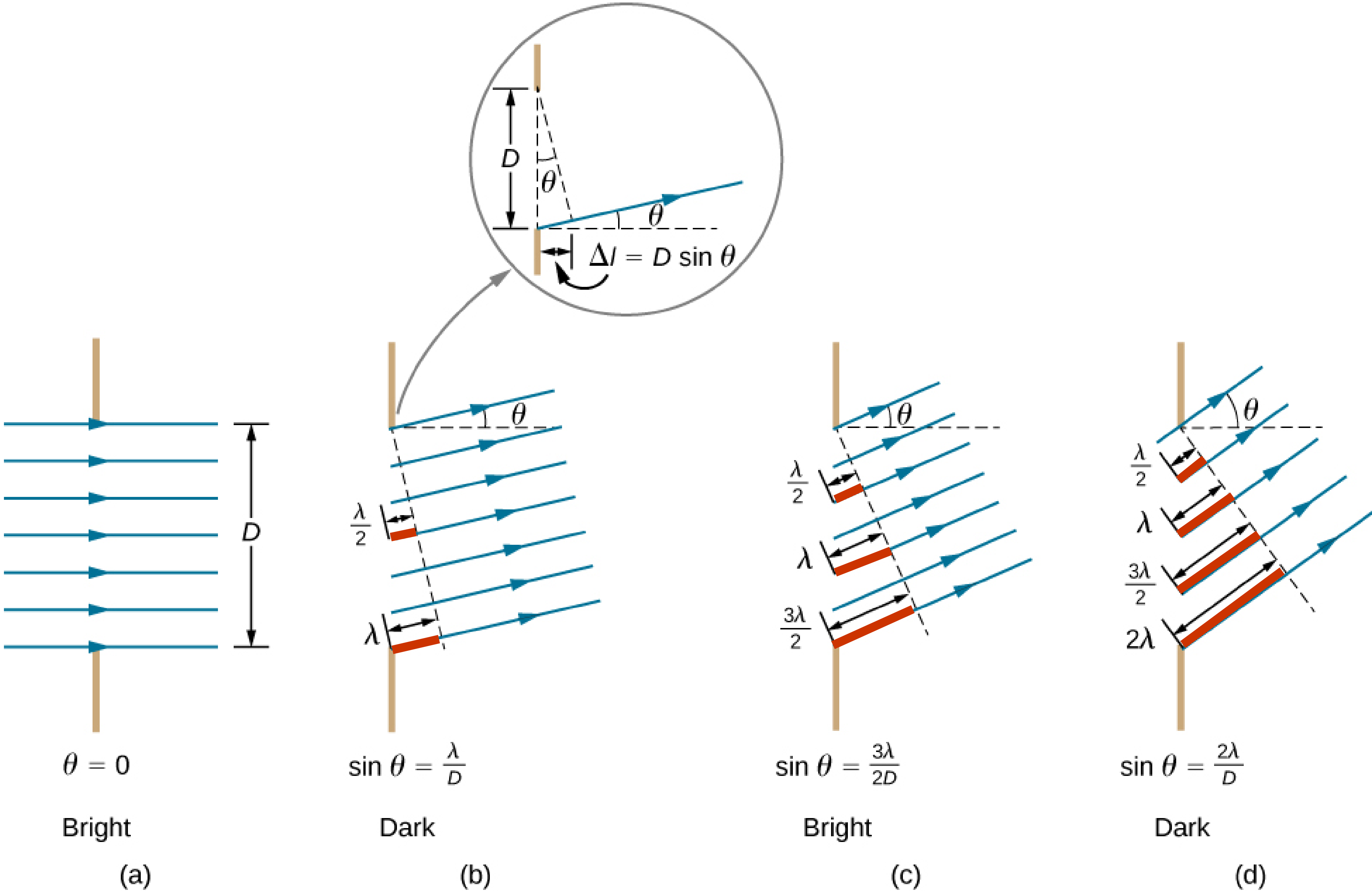
$\varphi = 0$ 衍射光线汇集于L_2焦点F

$\Delta = 0$ 中央明纹中心

$\varphi \neq 0$ 衍射光线汇集于L_2焦平面上某点P

$\Delta \neq 0$ P处光强可由菲涅耳公式计算

$$\psi = \int \mathrm{d}\psi$$



D
θ
Δl = D sin θ
λ/2
λ
3λ/2
2λ
θ = 0
Bright
sin θ = λ/D
Dark
sin θ = 3λ/2D
Bright
sin θ = 2λ/D
Dark
(a)
(b)
(c)
(d)

2. Fresnel zone plate method (semi-quantitative method)

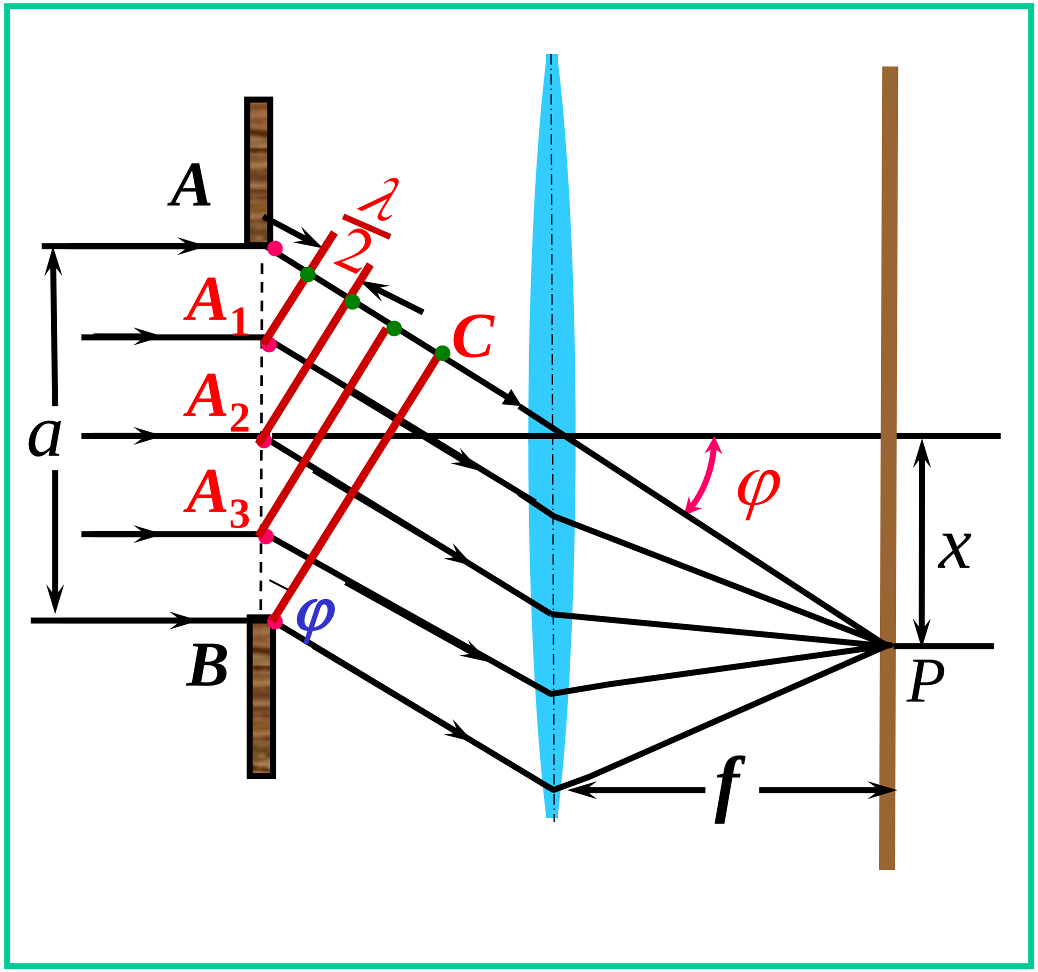
Diffraction Angle*is* ϕ
Maximum difference of light path:

$$\Delta = AC = a\sin\varphi$$

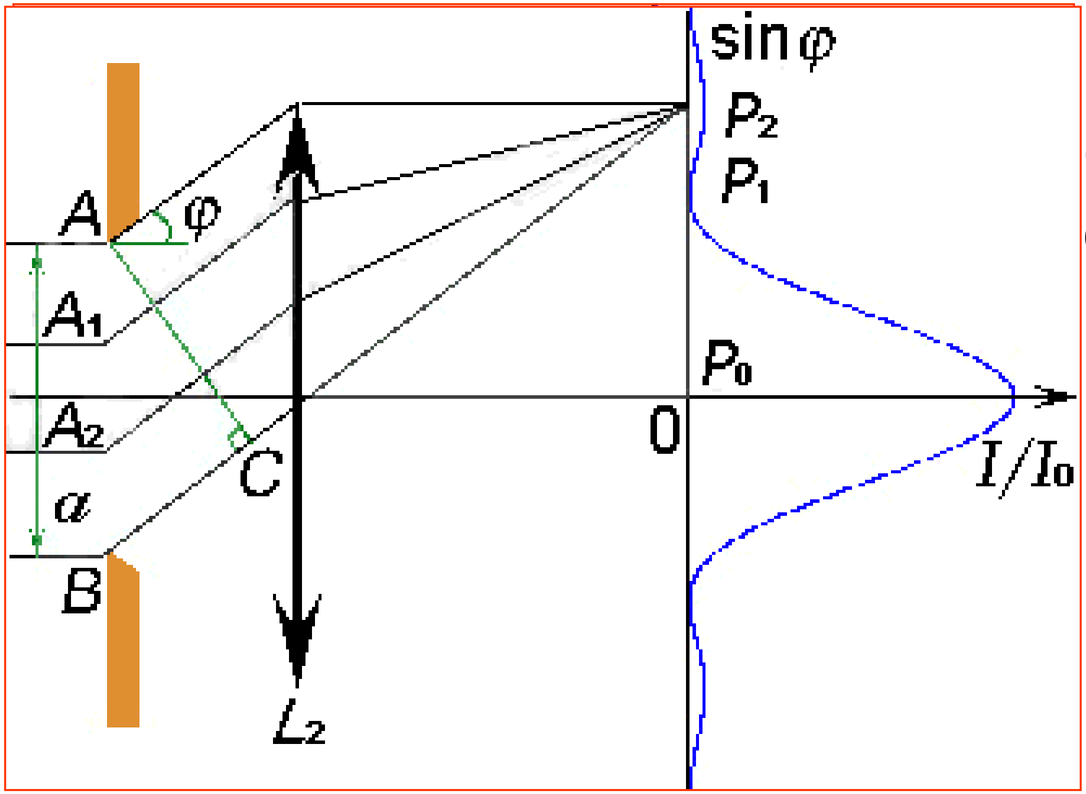
用$\frac{\lambda}{2}$去分Δ，设$\Delta = n\cdot\frac{\lambda}{2}$

对应的单缝*a*被分为

***n*个半波带**



For a certain slit width, the number of half wave zone is determined by the diffraction Angle .



$\varphi = 0 \quad \boldsymbol{n = 0}$

Corresponding to the center of the central bright fringe

n the even number：

相邻两半波带中对应光线

$$\Delta = \frac{\lambda}{2}, \quad \Delta\varphi = \pi$$

Because of the cancellation，

Dark fringe

n the odd number :

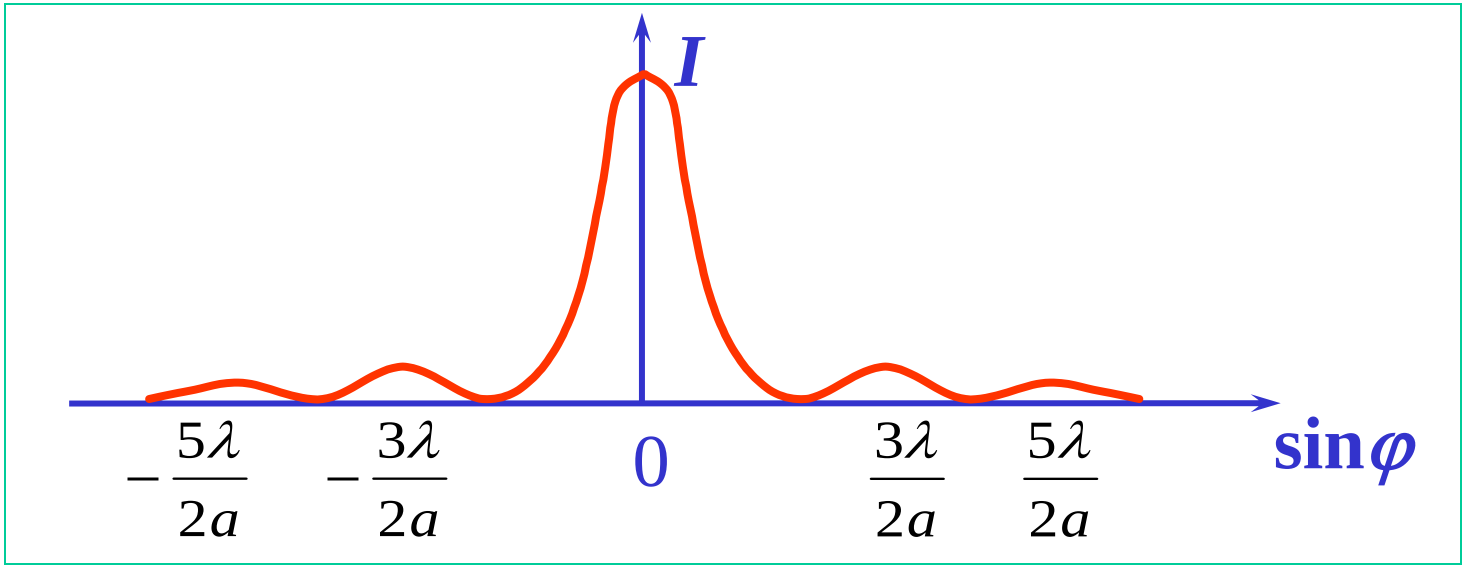
The left half wave zone leads to bright fringe

n non-integer: **Corresponding to the non - bright, dark center of the rest of the position**

*Interference condition:

$$\Delta = a\sin\varphi = \begin{cases} 0 & \text{The central bright fringe} \\ \pm(2k+1)\dfrac{\lambda}{2} & \text{bright} \\ \pm k\lambda & \text{dark} \end{cases}$$

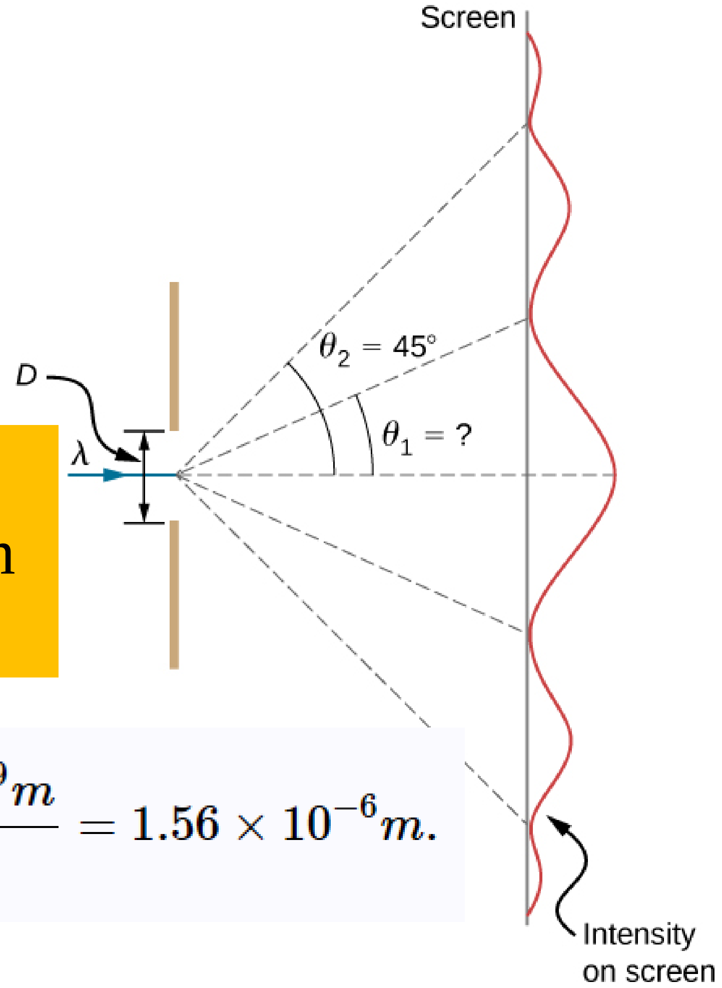
$k = 1、2、3\cdots$ 注意: $k \neq 0$



Example:

Visible light of wavelength 550 nm falls on a single slit and produces its second diffraction minimum at an angle of 45.0° to the incident direction of the light, as in Figure

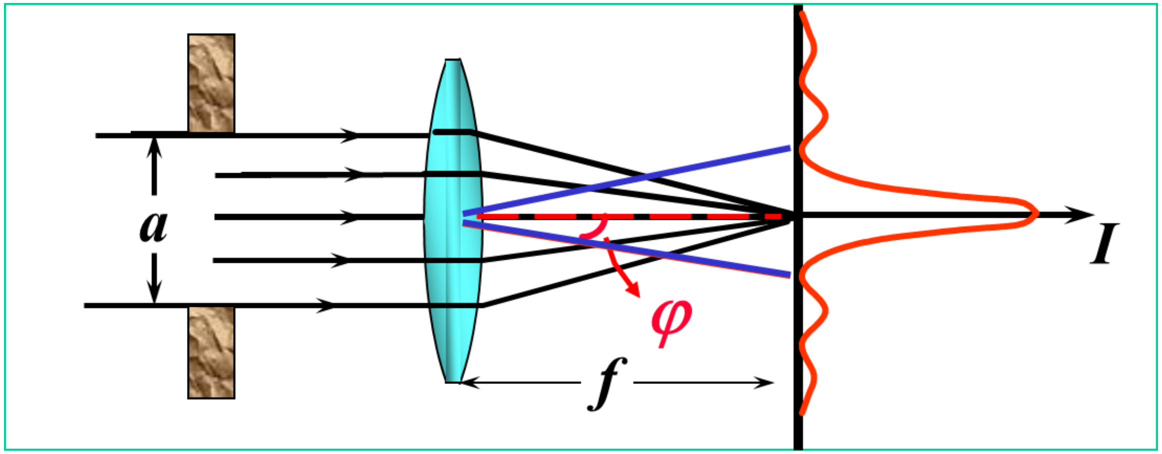
What is the width of the slit?
At what angle is the first minimum produced



$$D = \frac{m\lambda}{\sin\,\theta_2} = \frac{2(550\,nm)}{sin\,45.0^{\circ}} = \frac{1100\times10^{-9}m}{0.707} = 1.56\times10^{-6}m.$$

*Fringe angle width

$$\sin\varphi \approx \varphi = \begin{cases} 0 & \text{The central bright fringe} \\ \pm k\dfrac{\lambda}{a} & \text{dark} \quad k = 1、2、\cdots \\ \pm(2k+1)\dfrac{\lambda}{2a} & \text{bright} \end{cases}$$



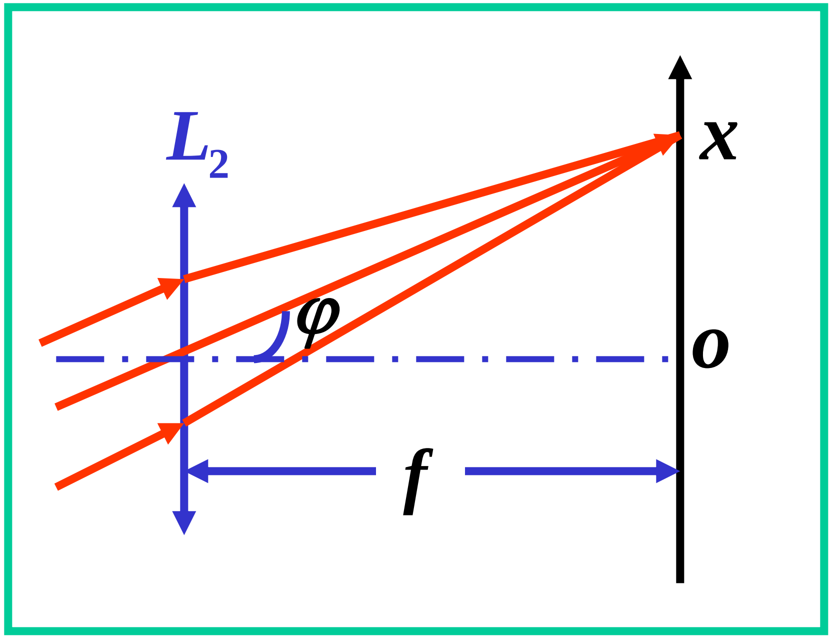
The central bright fringe

$$\Delta\varphi = \frac{2\lambda}{a}$$

The left bright fringes

$$\Delta\varphi = \frac{\lambda}{a}$$

*Calculate the width of the diffraction fringe:



$$x = f\,\mathrm{tg}\varphi$$

$$\Delta x = f(\mathrm{tg}\varphi_2 - \mathrm{tg}\varphi_1)$$

$$\Delta x = f(\varphi_2 - \varphi_1) = f \cdot \Delta\varphi$$

$\lambda = a \cdot \sin\varphi$
$\approx a \cdot \varphi$

中央明纹 $\Delta x = \dfrac{2\lambda}{a} \cdot f$　　其余明纹 $\Delta x = \dfrac{\lambda}{a} \cdot f$

Discussion

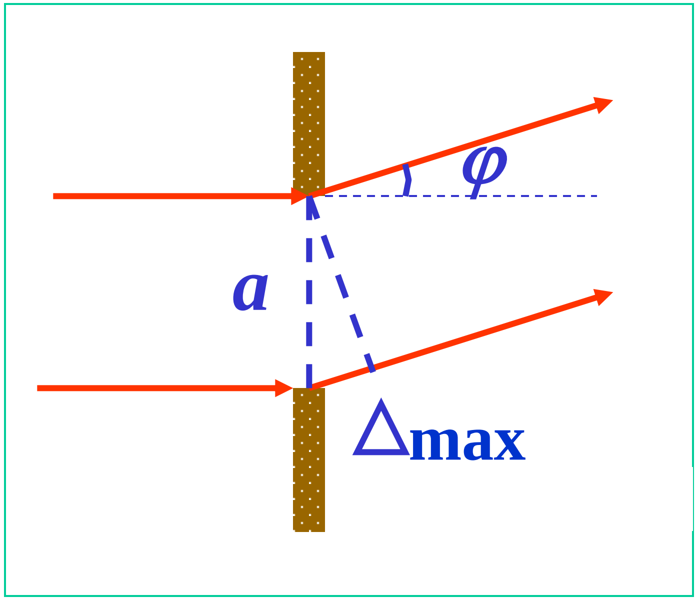
Double slit interference

$$\Delta = \begin{cases} \pm k\lambda & \text{C} \\ \pm(2k+1)\dfrac{\lambda}{2} & \text{D} \end{cases} \qquad k = 0、1、2、\cdots$$

Single slit diffraction

$$\Delta = \begin{cases} \pm(2k+1)\dfrac{\lambda}{2} & \text{C} \\ \pm k\lambda & \text{D} \end{cases} \qquad k = 1、2、\cdots$$

Whether the bright and dark fringe conditions contradict to each other?



Are the fringes evenly distributed and why?

Discussion

Fresnel zone plate method :

中央明纹中心:

全部光线干涉相长

一级明纹中心:

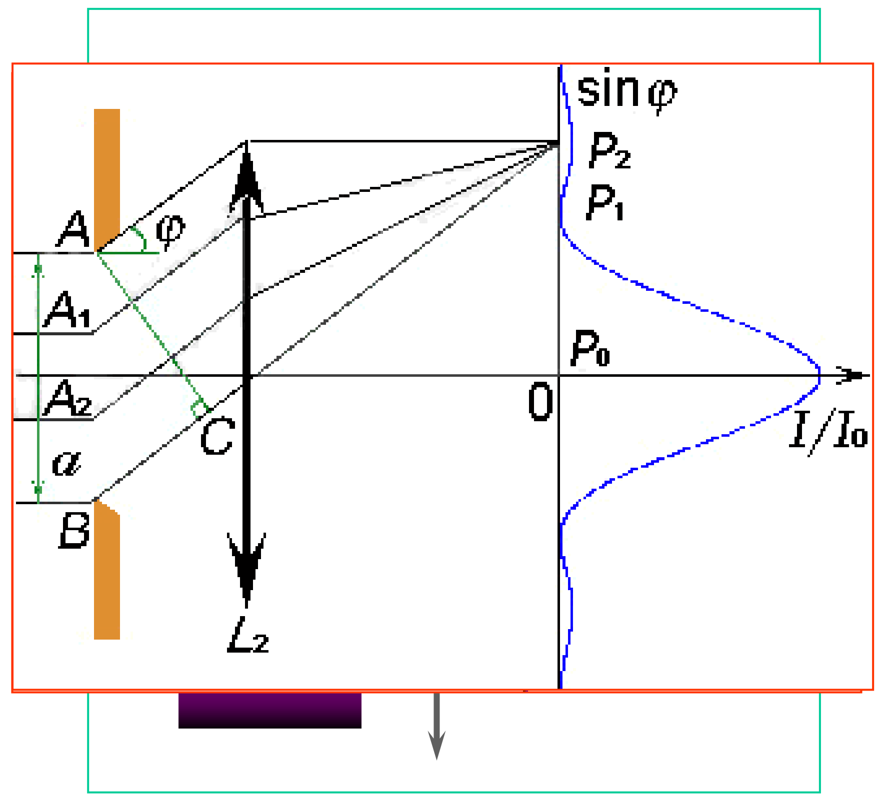
$\frac{1}{3}$部分光线干涉相长

二级明纹中心:

$\frac{1}{5}$部分光线干涉相长

……

中央明纹集中大部分能量，明条纹级次越高亮度越弱。



讨论条纹随λ、a的变化

Discussion

中央明纹 $\Delta x = \dfrac{2\lambda}{a} \cdot f$　　其余明纹 $\Delta x = \dfrac{\lambda}{a} \cdot f$

λ一定
- $a\downarrow$　$\Delta\varphi\uparrow$　衍射显著　$a\downarrow\downarrow$ 光强太弱
- $a\uparrow$　$\Delta\varphi\downarrow$　衍射不明显　$a\uparrow\uparrow$　直线传播

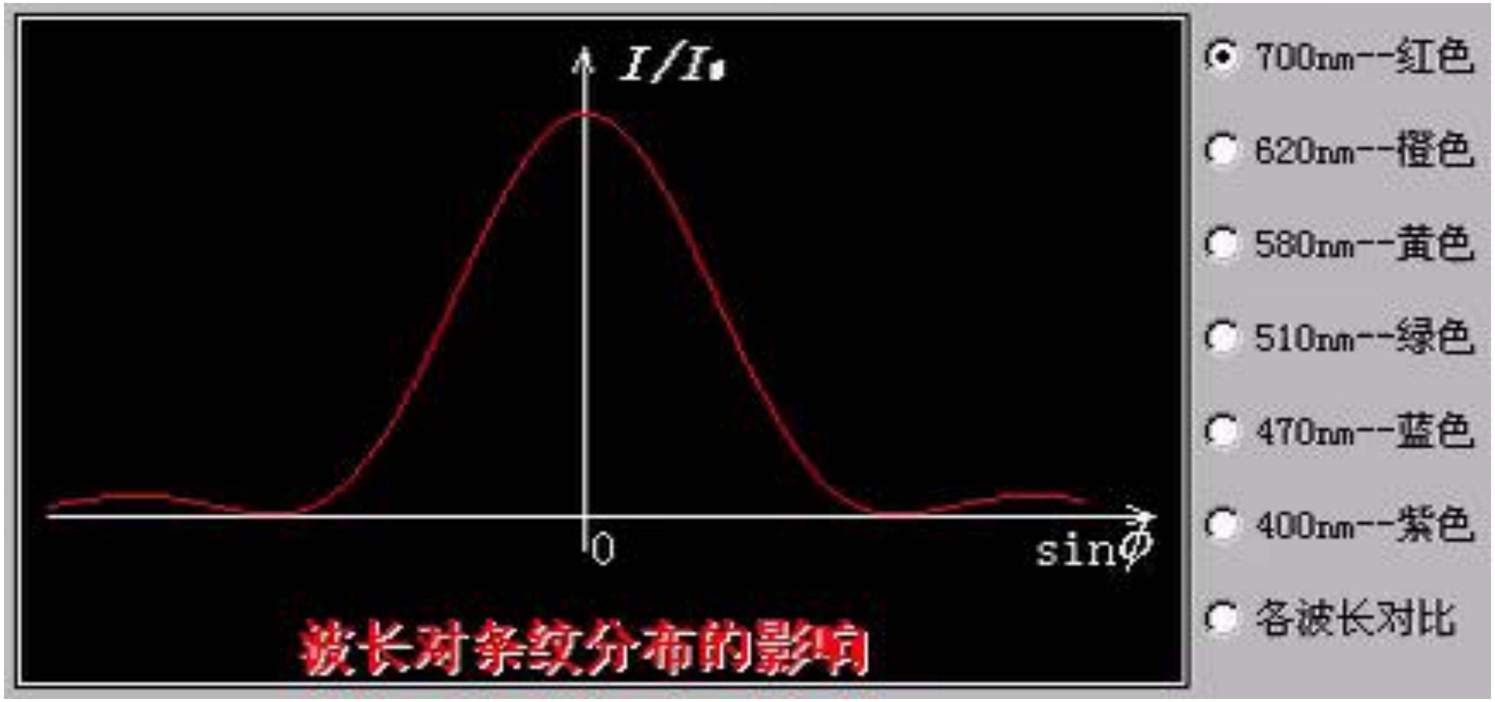
a一定
- $\lambda\uparrow$　$\Delta\varphi\uparrow$

  白光照射，中央白色，其余明纹形成内紫外红光谱，高级次重叠
- $\lambda\downarrow$　$\Delta\varphi\downarrow$

  浸入液体中、条纹变密

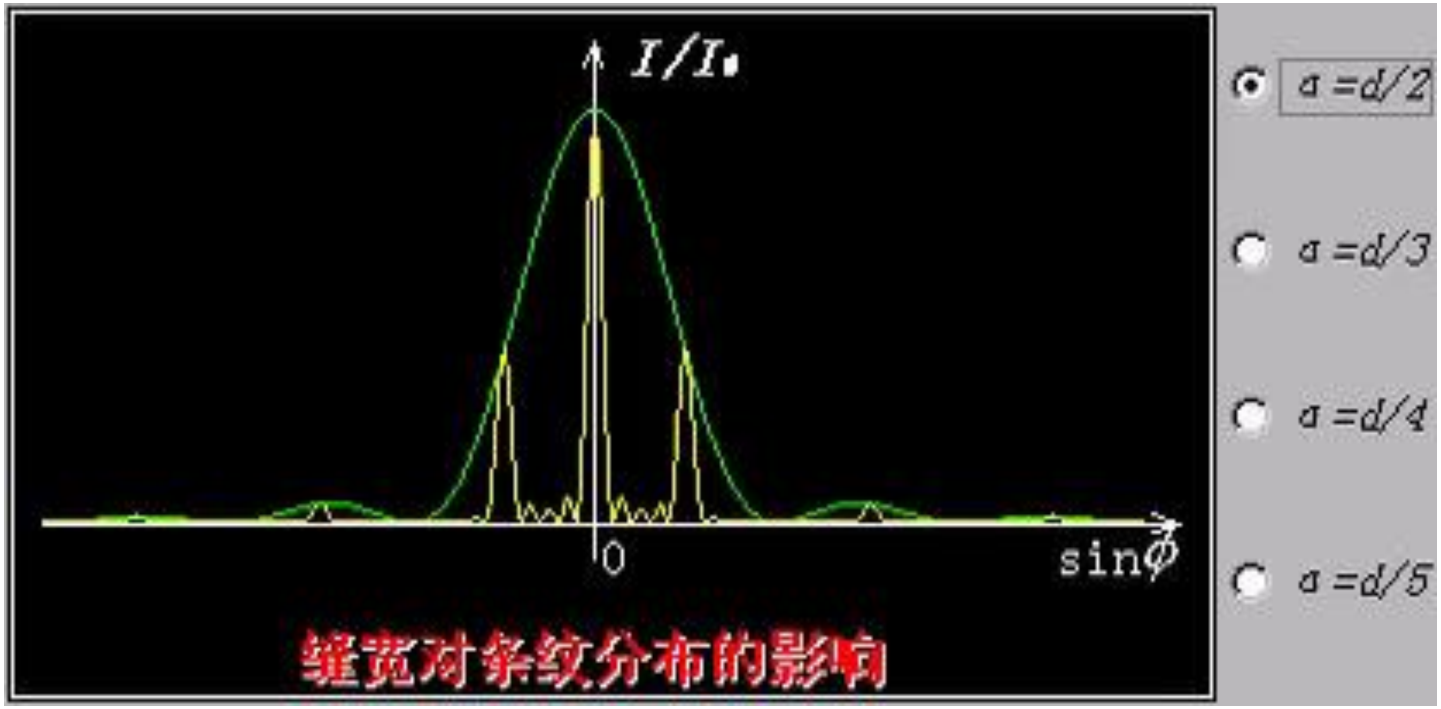
Discussion

- The influence of wavelength on diffraction pattern



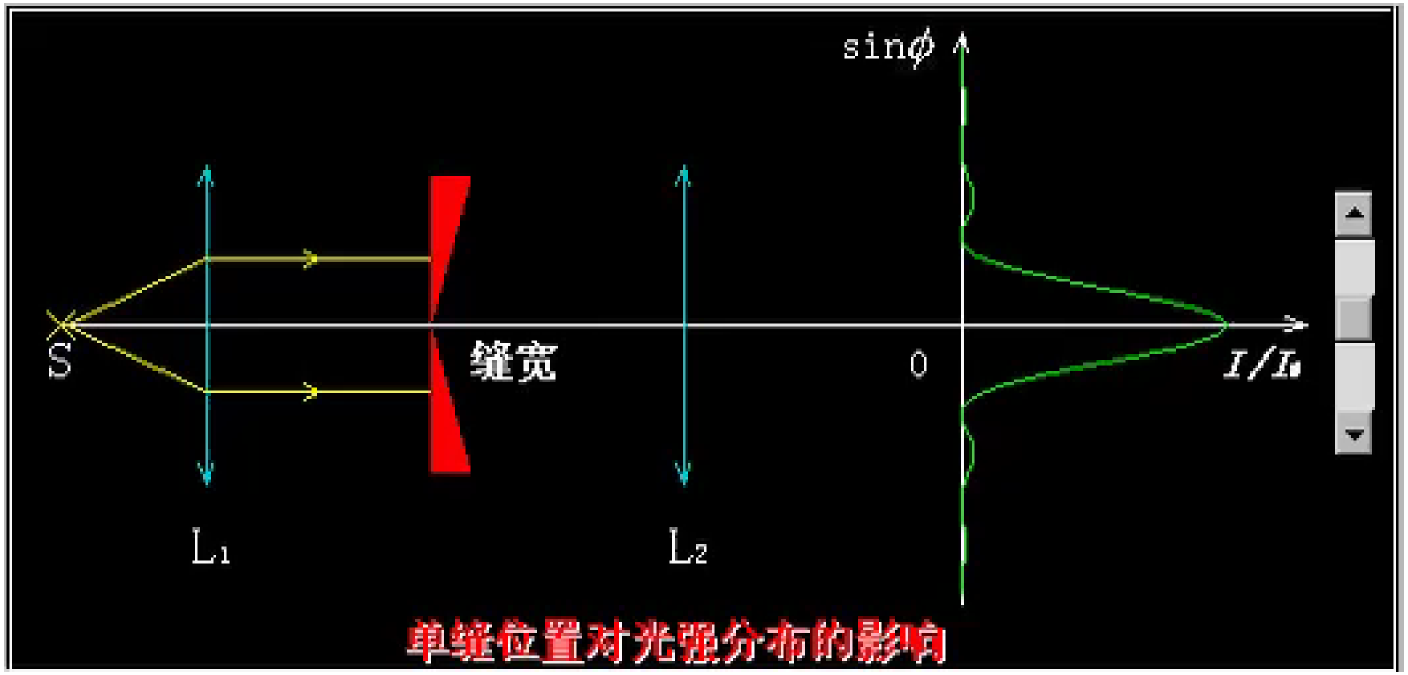
Discussion

- The influence of slit width on diffraction pattern



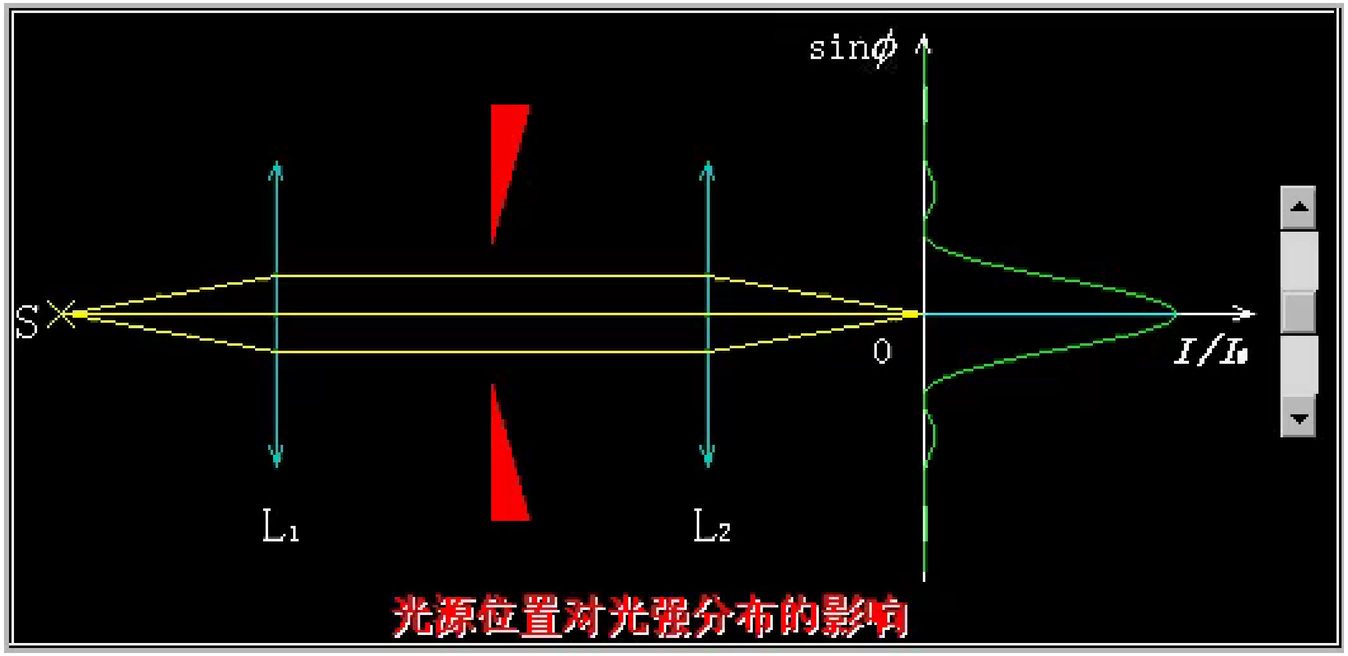
•**The influence of slit location on diffraction pattern**

Discussion

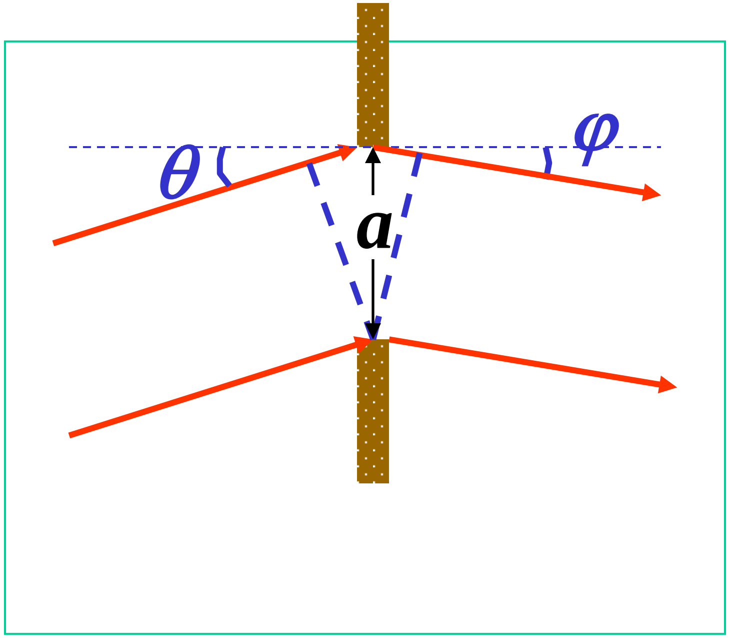


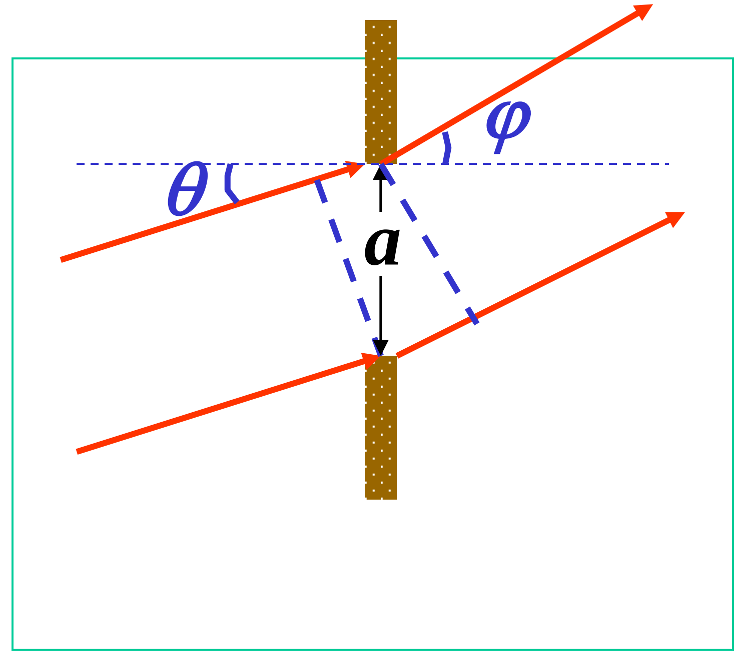
Discussion

- The influence of light source location on diffraction pattern



Discussion





$\Delta = a\sin\theta + a\sin\varphi$ $\qquad$ $\Delta = a\sin\theta - a\sin\varphi$

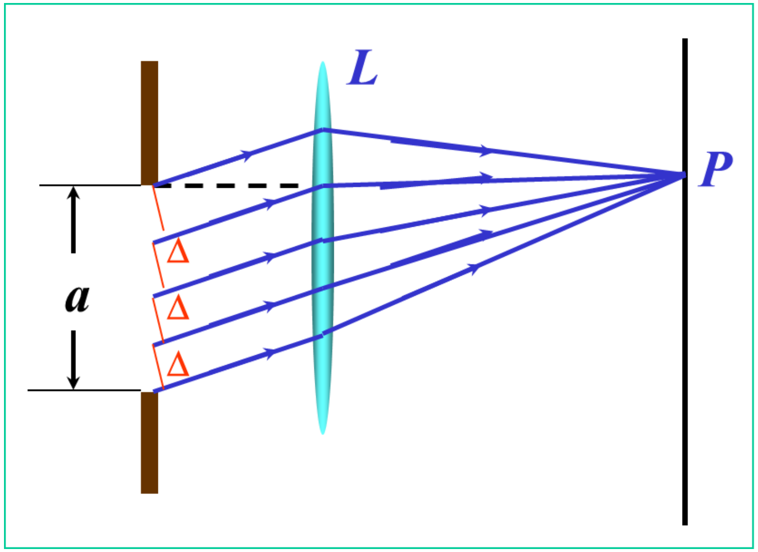
If parallel light is not vertically incident, the formula of path difference and the condition of bright and dark fringe are obtained

$$\Delta = a\sin\theta \pm a\sin\varphi = \begin{cases} 0 & \text{中央明纹中心} \\ \pm k\lambda & \text{暗} \\ \pm(2k+1)\dfrac{\lambda}{2} & \text{明} \end{cases} \qquad k = 1、2、3\cdots$$

3. Amplitude vector superposition (quantify)

Divide a into N equal widths $\left(\frac{a}{N}\right)$ Zone，

The internal energy is concentrated in the light rays shown in the figure



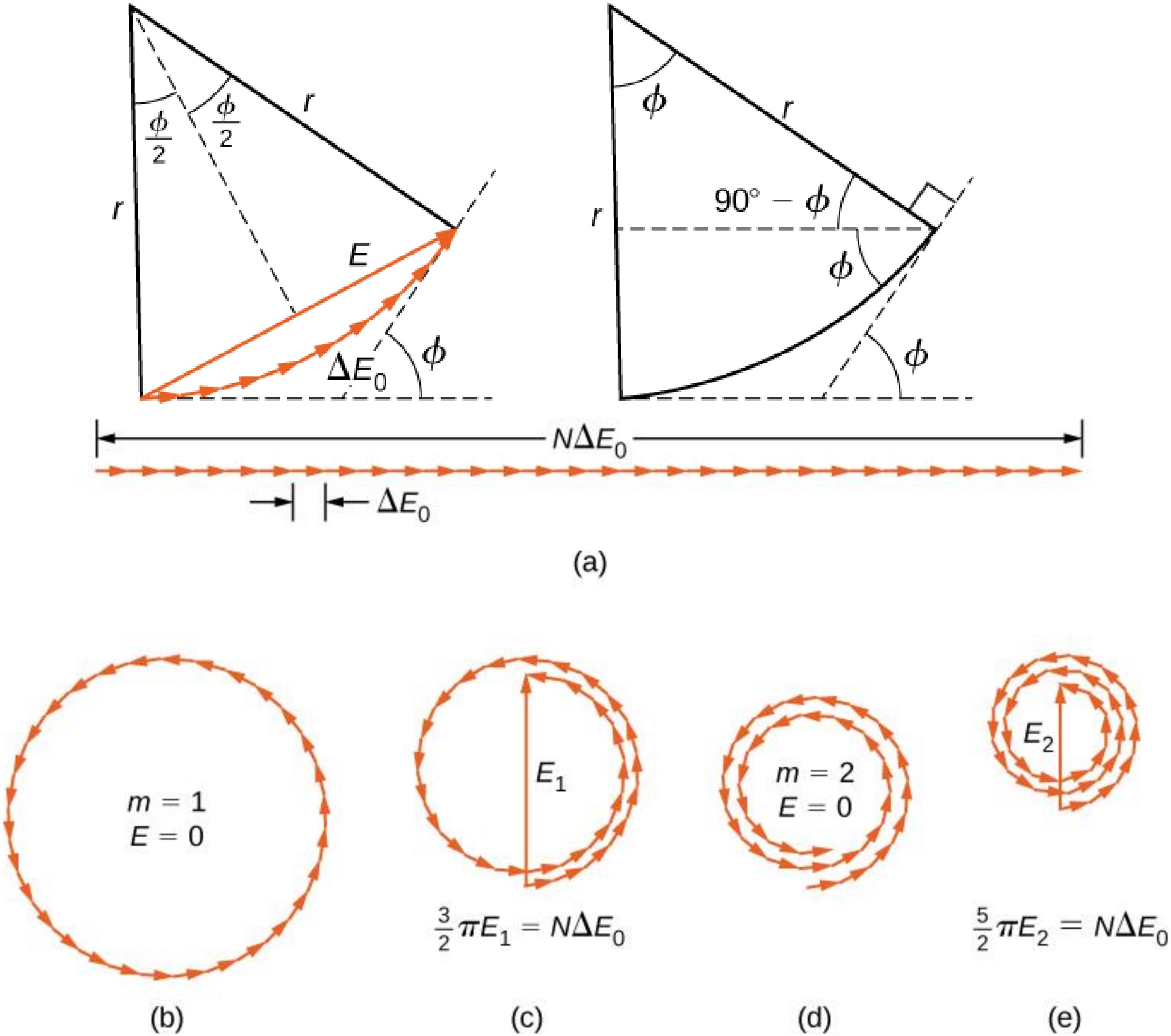
The path difference between two adjacent rays

$$\Delta = \frac{a}{N}\sin\varphi \ (不一定为\frac{\lambda}{2})$$

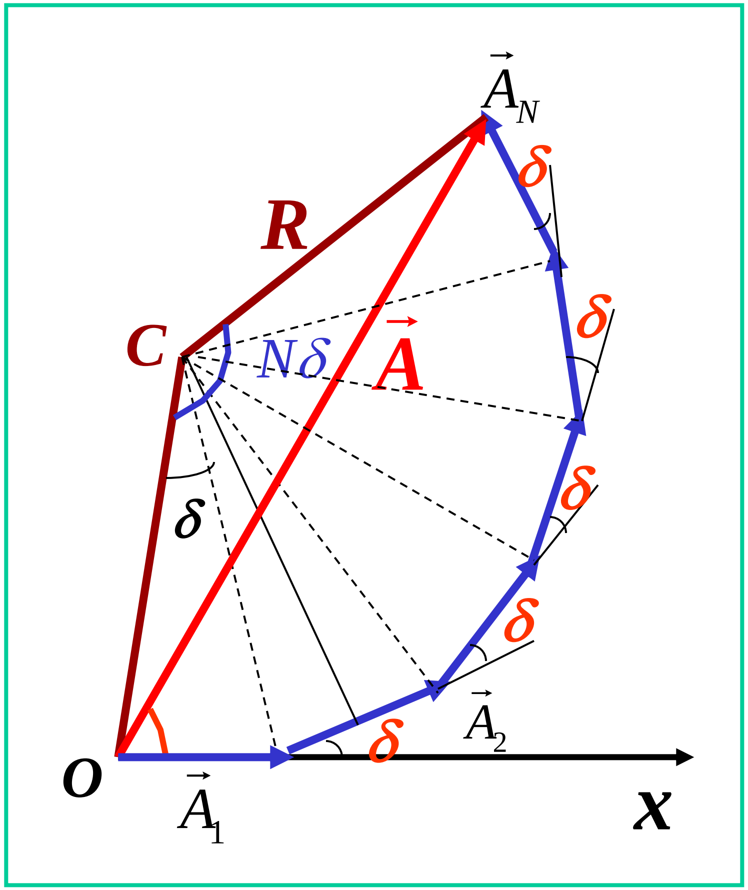
The phase difference between two adjacent rays

$$\delta = 2\pi\frac{\Delta}{\lambda} = \frac{2\pi}{\lambda}\cdot\frac{a}{N}\sin\varphi$$

Equal amplitude of each light rays $A_1 = A_2 = \cdots = A_N$



r
ϕ/2
ϕ/2
r
E
ΔE0
ϕ
ϕ
r
90° − ϕ
r
ϕ
ϕ
NΔE0
ΔE0
(a)
m = 1
E = 0
E1
3/2 πE1 = NΔE0
m = 2
E = 0
E2
5/2 πE2 = NΔE0
(b)
(c)
(d)
(e)



$$A_1 = 2R\sin\frac{\delta}{2}$$

$$A = 2R\sin\frac{N\delta}{2}$$

$$A = A_1\frac{\sin\frac{N\delta}{2}}{\sin\delta/2} \approx A_1\frac{\sin\frac{N\delta}{2}}{\delta/2}$$

$$= NA_1\frac{\sin\frac{N\delta}{2}}{N\delta/2}$$

$$\boxed{\alpha = \frac{N\delta}{2} = \frac{\pi a\sin\varphi}{\lambda}}$$

$$A_0 = NA_1$$

$$\alpha = \frac{N\delta}{2} = \frac{\pi a \sin\varphi}{\lambda}$$

$$A_0 = NA_1$$

This is the amplitude at the center of the central bright fringe

so $$A = A_0 \frac{\sin\alpha}{\alpha} \qquad I = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2$$

The central bright fringe intensity

式中$I_0 = (NA_1)^2$为中央明纹光强

$$\text{When}\ \frac{\P\, I}{\P\, \alpha} = 0$$

when $\dfrac{\partial I}{\partial \alpha} = 0$ The extremum is derived $\tan\alpha = \alpha \quad \sin\alpha = 0$

$$\alpha = \frac{N\delta}{2} = \frac{\pi a \sin\varphi}{\lambda}$$

（1）Primary maximum

$\varphi = 0$， $\alpha = 0$ $\quad I = I_0$ 中央明纹光强

（2）Secondary maximum $\tan\alpha = \alpha$

$$\alpha = \pm 1.43\pi, \quad \pm 2.46\pi, \quad \pm 3.47\pi \cdots\cdots$$

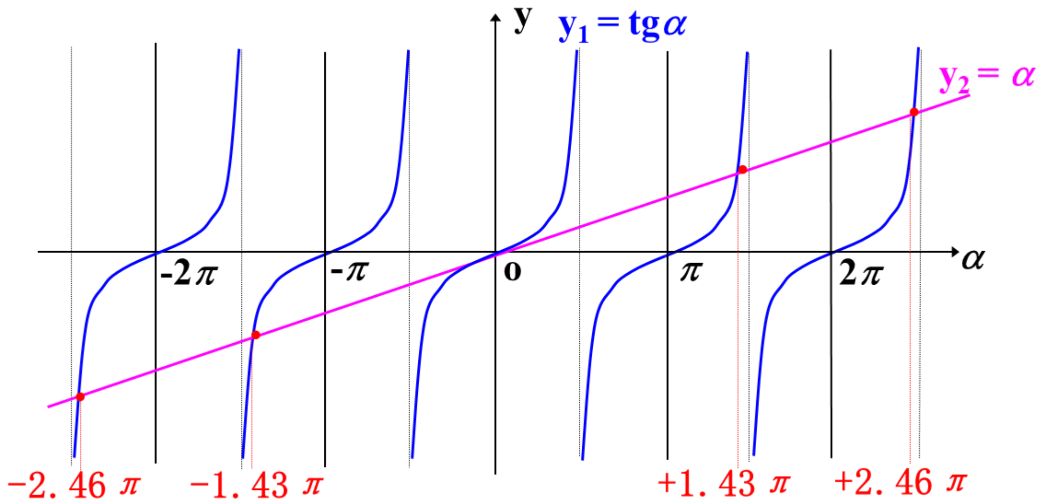
$$a\sin\varphi = \pm 1.43\lambda, \quad \pm 2.46\lambda, \quad \pm 3.47\lambda \cdots\cdots$$

（3）Minimum

$\alpha = k\pi \quad k = \pm 1, \pm 2 \cdots\cdots \quad I = 0$ 暗纹中心

$$\downarrow$$

$$a\sin\varphi = k\lambda$$



so: $\alpha = \pm 1.43\pi, \pm 2.46\pi, \pm 3.47\pi, \cdots$

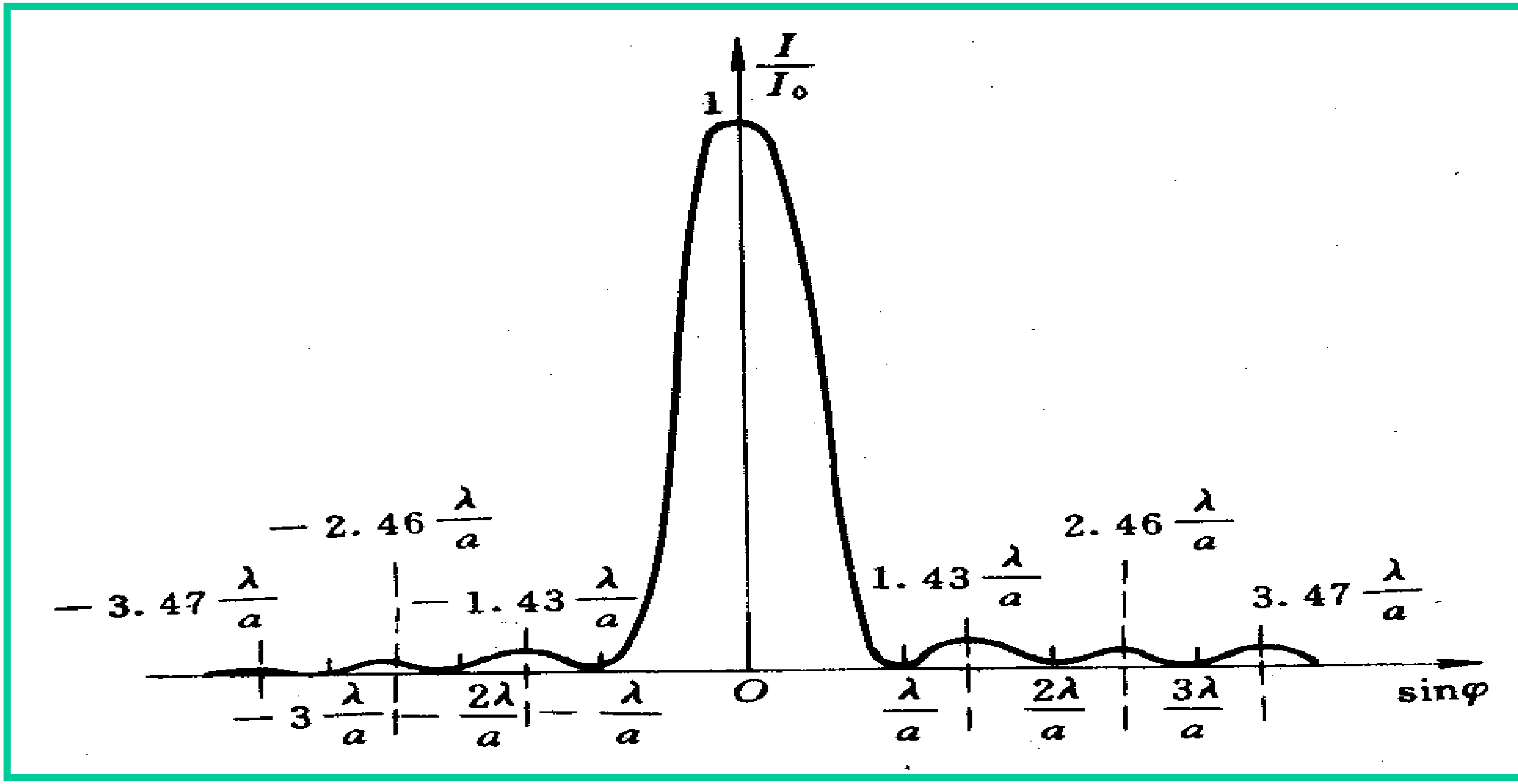
Correspondingly :

$$a \sin\varphi = \pm 1.43\lambda, \pm 2.46\lambda, \pm 3.47\lambda, \cdots$$

(4)Intensity : **for different secondary maximum**

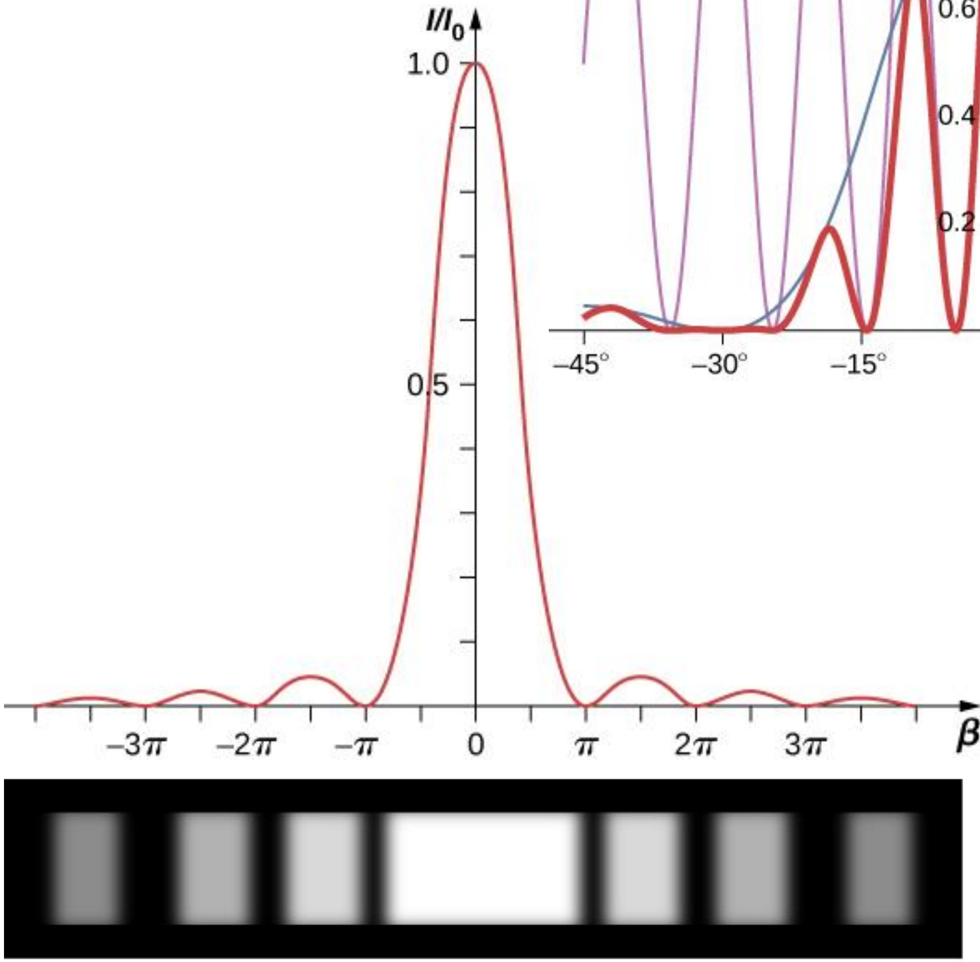
0.0472*I*$_0$, 0.0165*I*$_0$, 0.0083*I*$_0$, ...

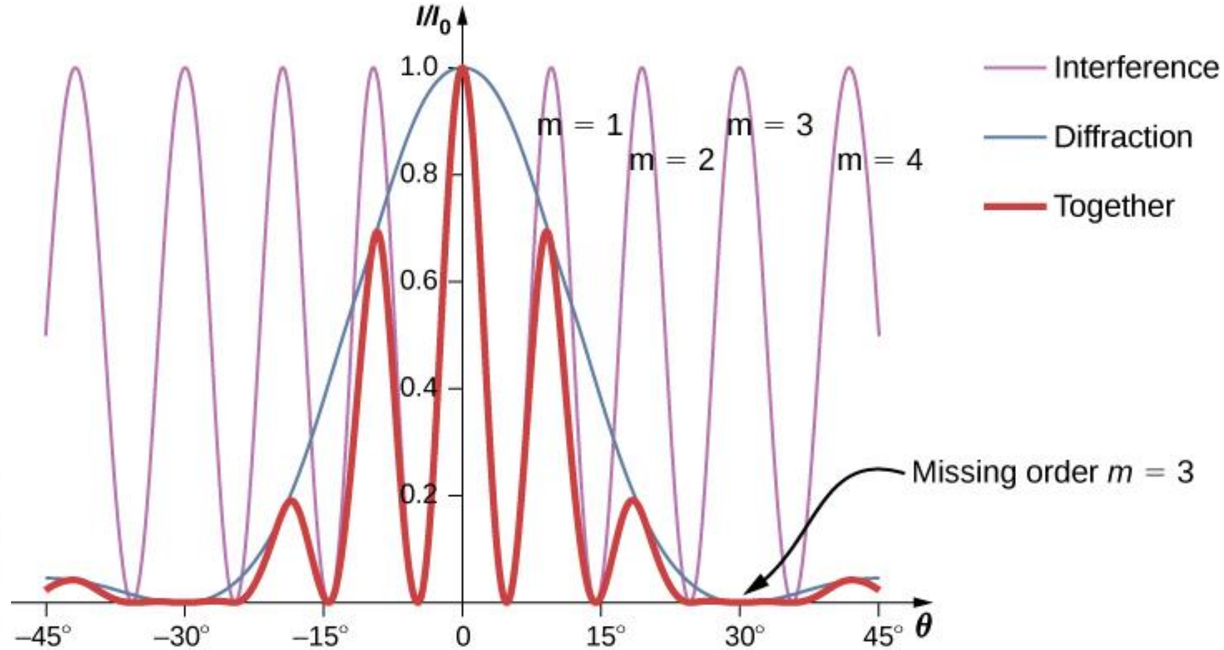
$\therefore \quad I_{sec} << I_{prim}$



Bright fringe: $\sin\varphi = 0, \pm 1.43\dfrac{\lambda}{a}, \pm 2.46\dfrac{\lambda}{a}, \cdots$

Dark fringe: $\sin\varphi = \dfrac{\lambda}{a}, \dfrac{2\lambda}{a}, \dfrac{3\lambda}{a}, \cdots$



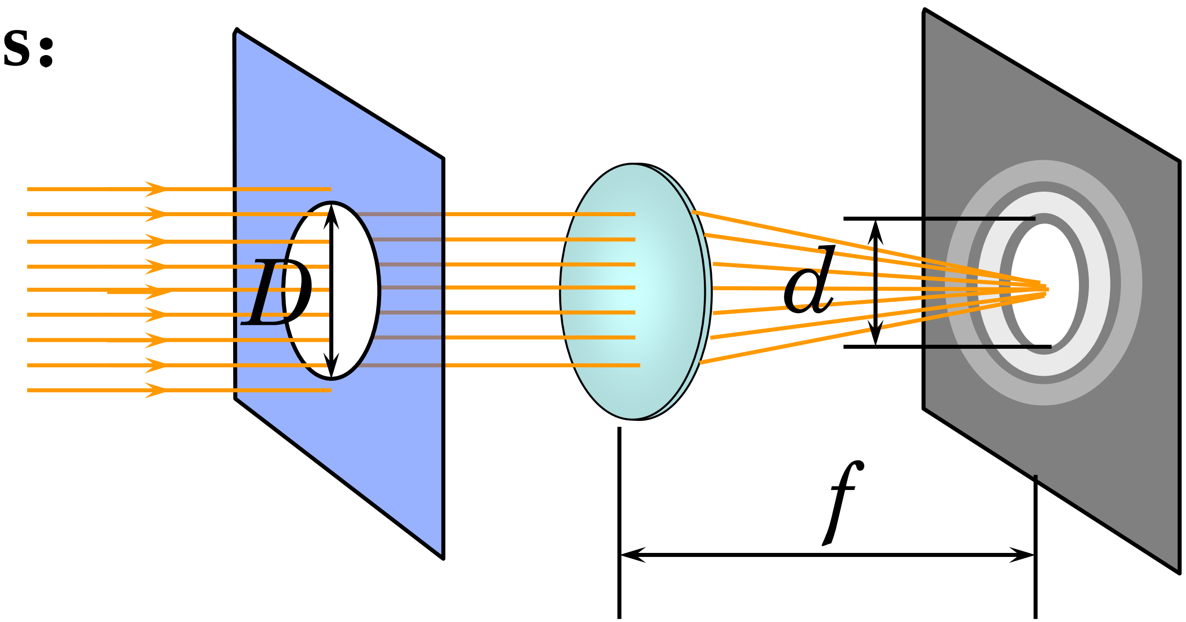


Double-Slit Diffraction

§ 24.3 diffraction of light

1、Fraunhofer diffraction for circle aperture

1. Apparatus:



2. Fringes:

Concentric rings of bright and dark

The primary fringe:
Airy Disc

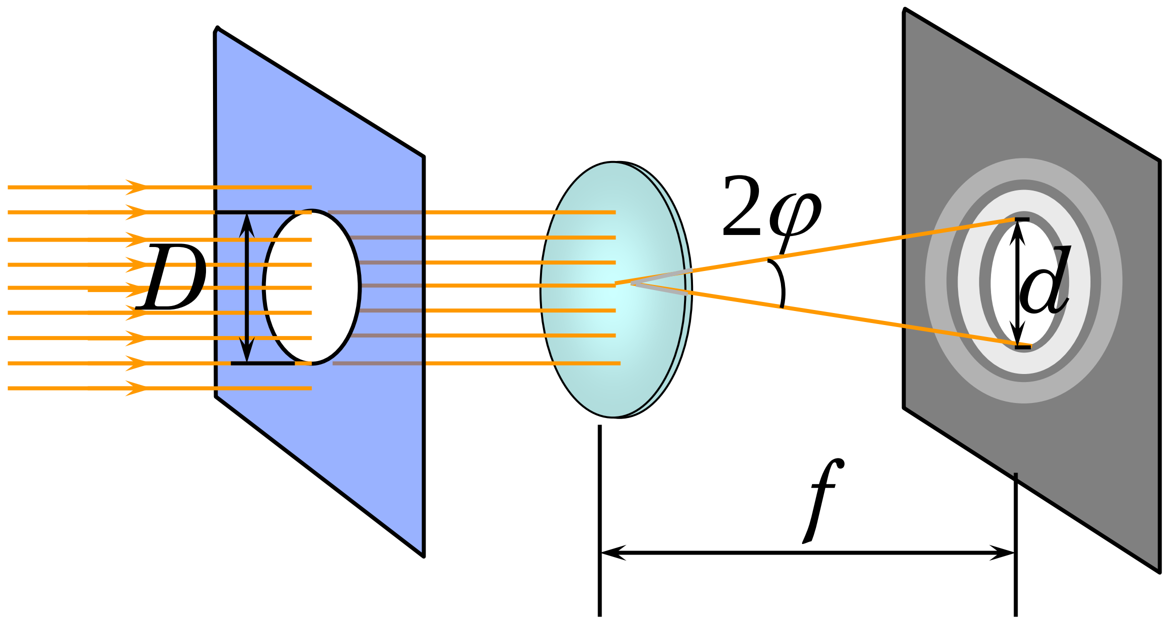
集中大部分能量

角宽度约为其余明纹2倍

半角宽度： $1.22\dfrac{\lambda}{D}$

Half of the central Angle of the airy disc is deduced theoretically:

$$\varphi = \frac{d}{2f} = 1.22\frac{\lambda}{D}$$



With the increase of D , φ decreases， and the diffraction phenomenon is less conspicuous 。

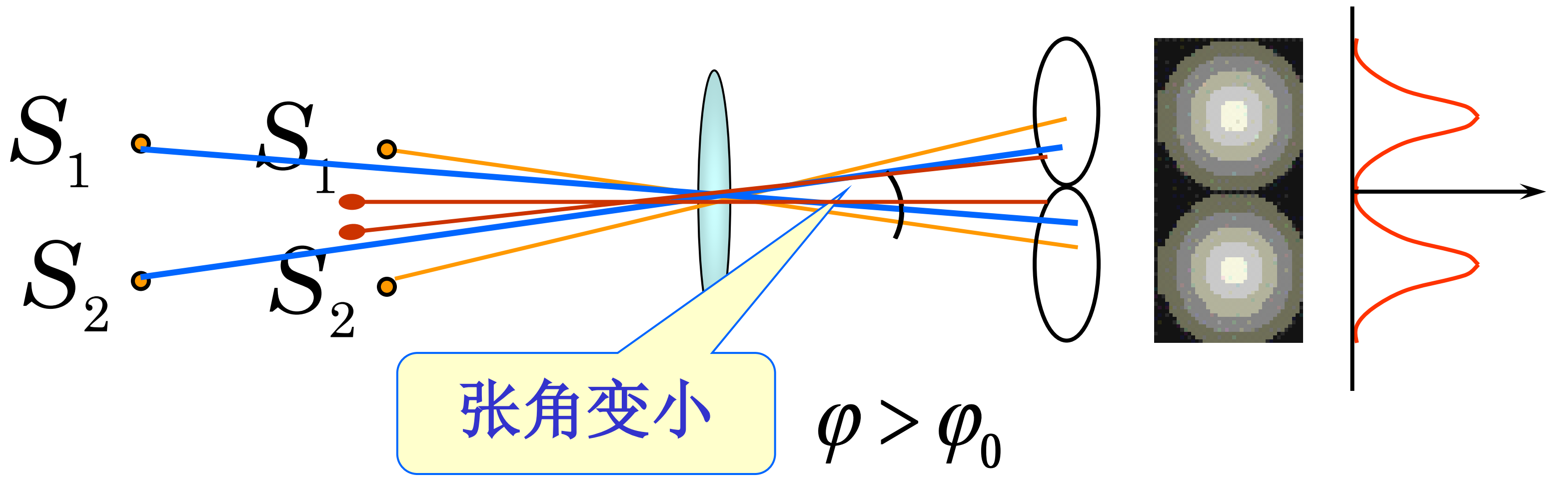
3、 Resolution of optical instrument

Objective ~ Round hole

The image ~ diffraction pattern of an point source

(1). Rayleigh criterion

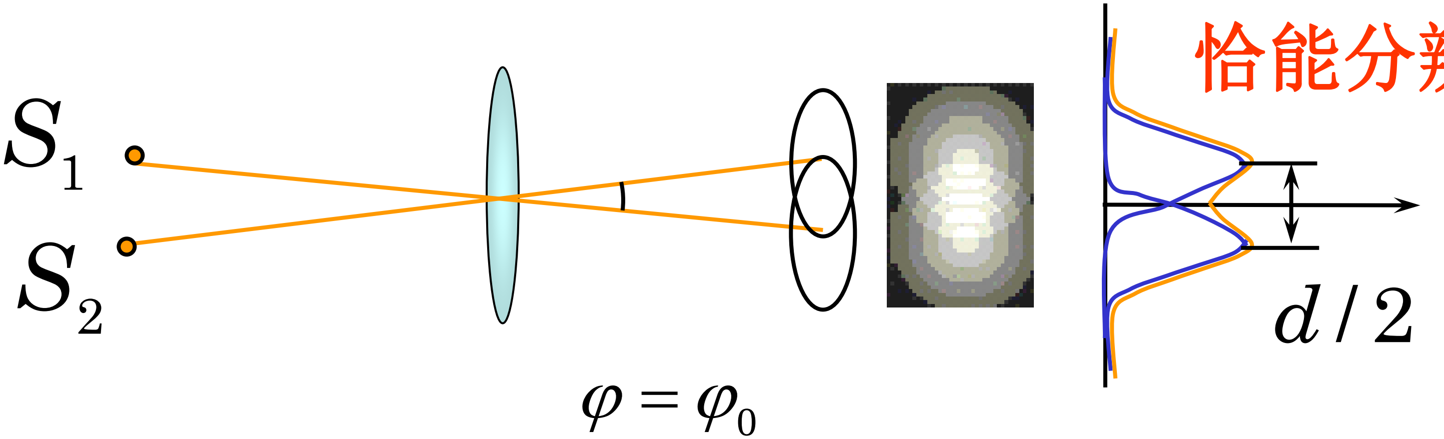
·Two point sources are far apart and can be distinguished.



φ_0 The Angle of the Airy disc against the lens

When two points of light approaching

• 两爱里斑中心距离为爱里斑的半径时，或者第一个象的爱里斑中心与第二个象的的爱里斑边缘重合时恰能分辨----瑞利判据。

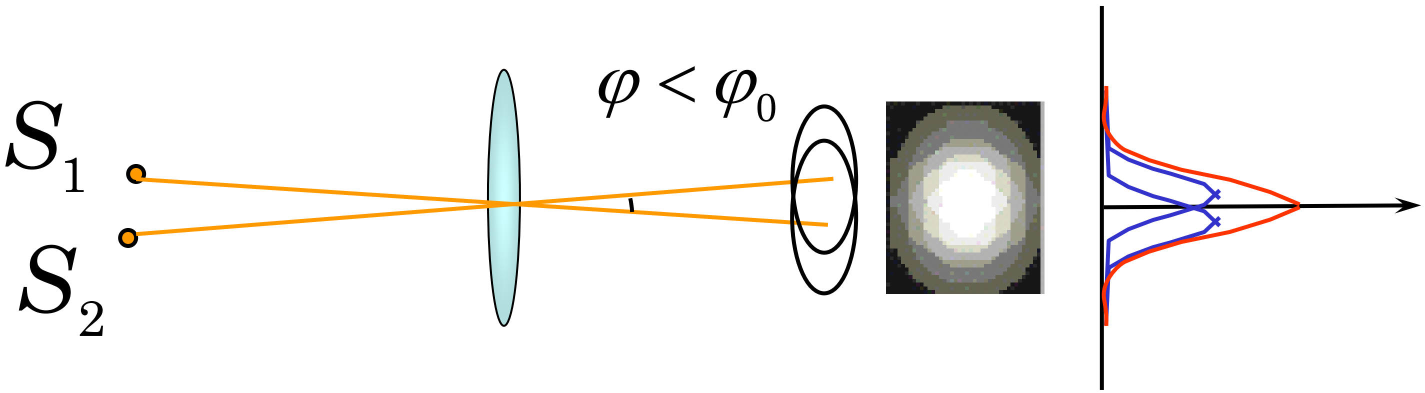


At this time, the light intensity of the overlap of the two airy disc is the80%of maximum center intensity of airy disc

φ_0 is called the minimum resolution Angle

Two point sources of light continue to approach

$\varphi < \varphi_0$ **Unable to distinguish**

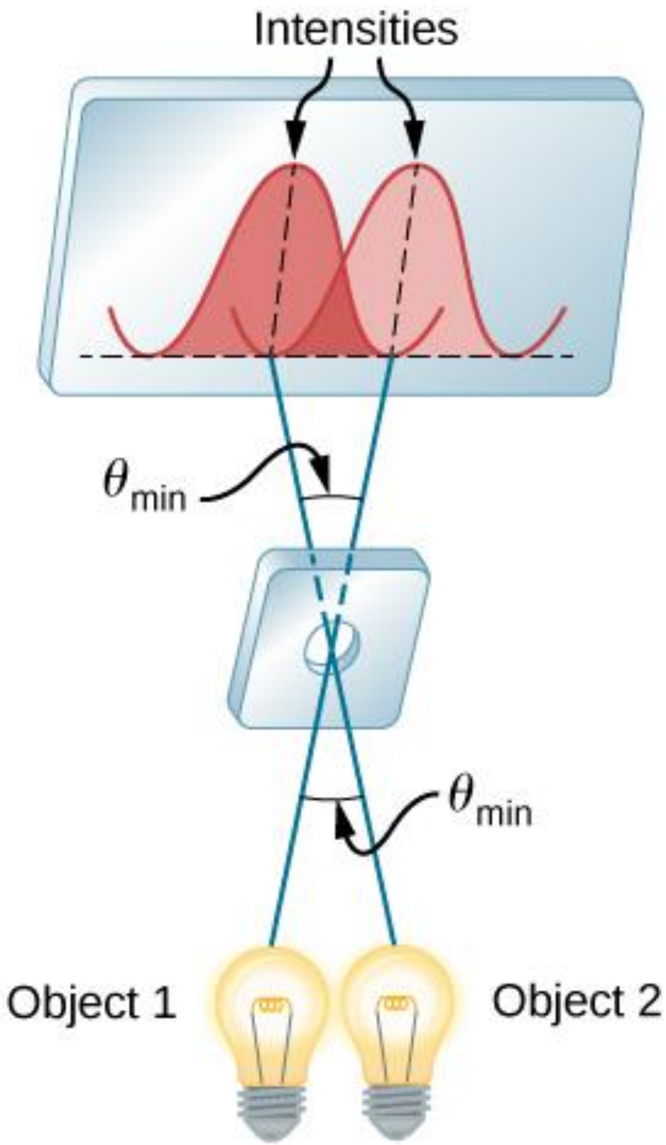


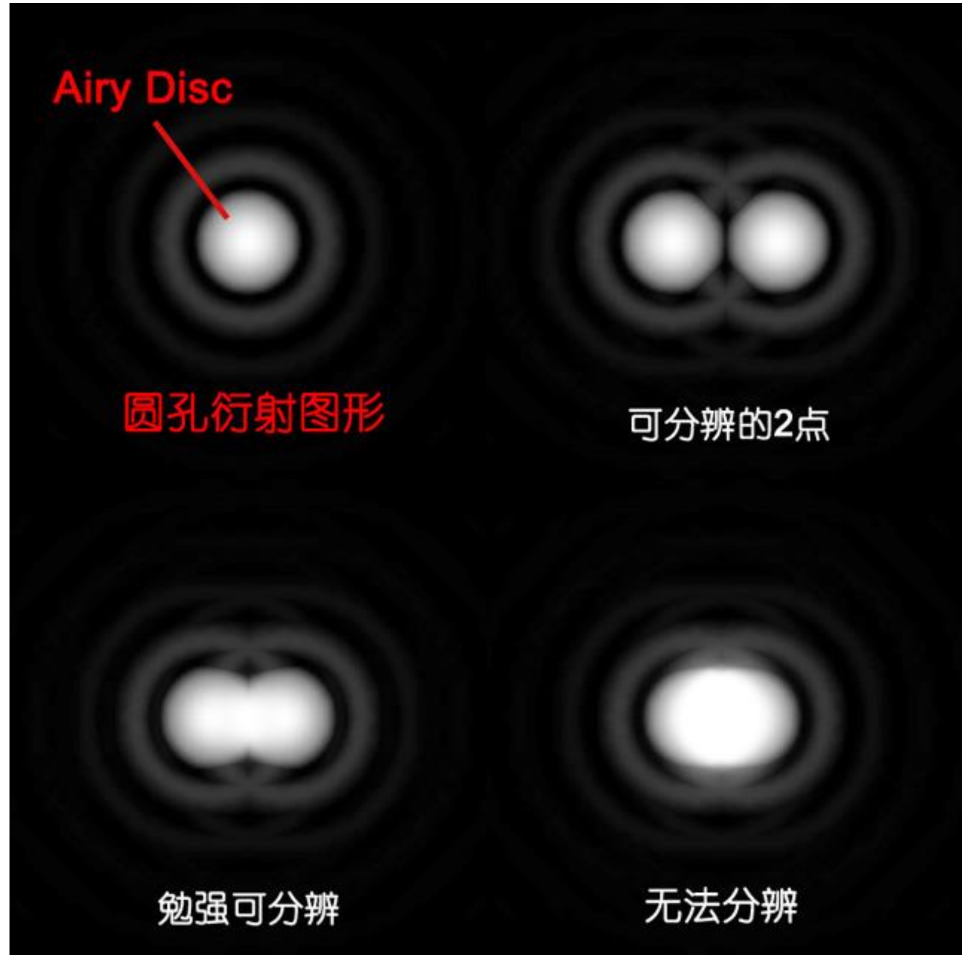
3. Resolution of optical instrument

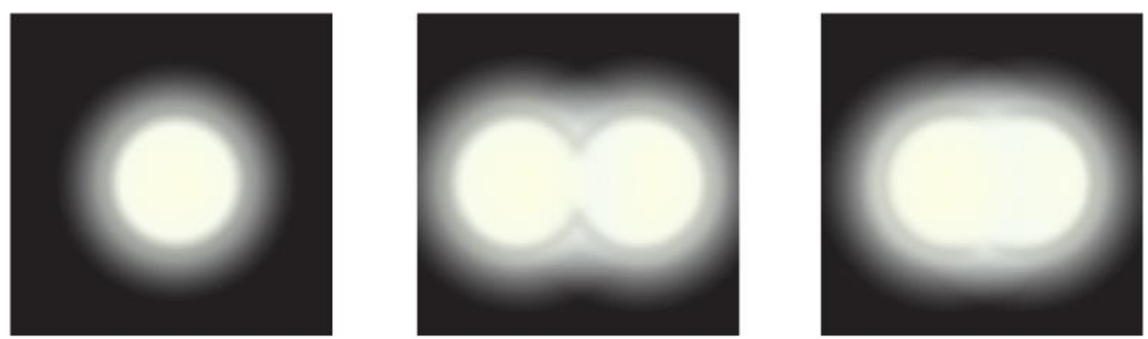
Reciprocal of the minimum resolution Angle

$$R = \frac{1}{\phi_0} = \frac{1}{1\times22} \times \frac{D}{\lambda}$$

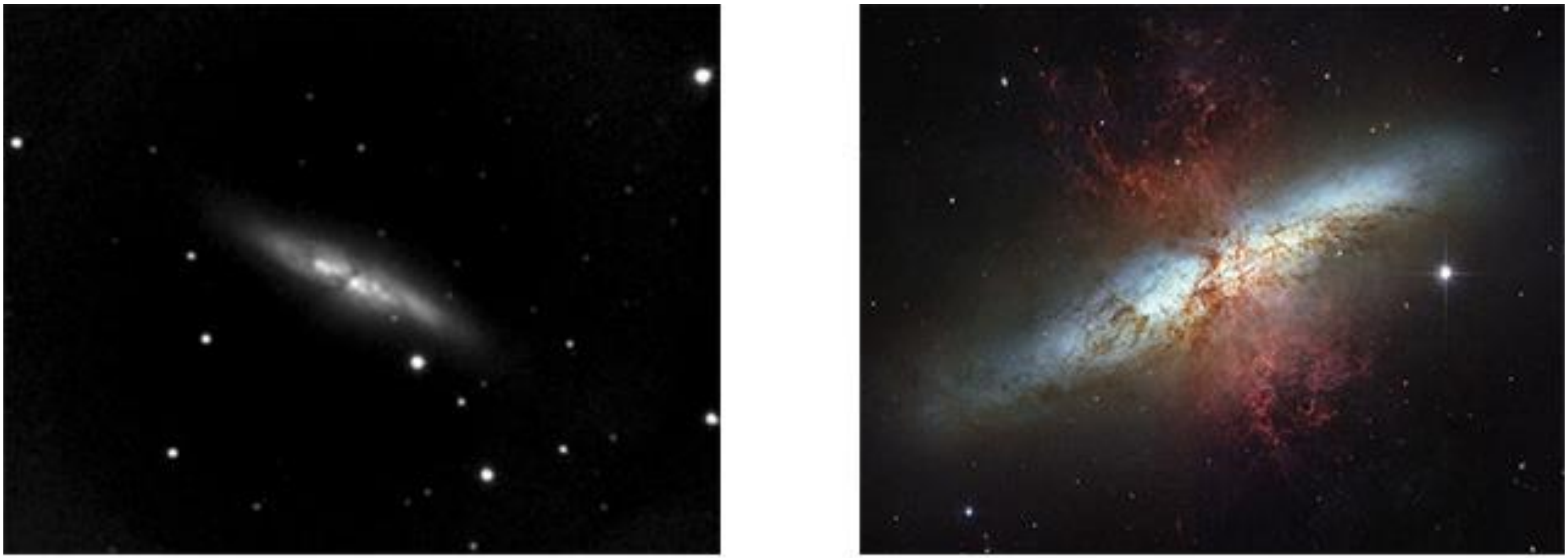
光学仪器的最小分辨角越小，分辨率就越高。


Intensities
θ_{min}
θ_{min}
Object 1
Object 2


Airy Disc
圆孔衍射图形
可分辨的2点
勉强可分辨
无法分辨



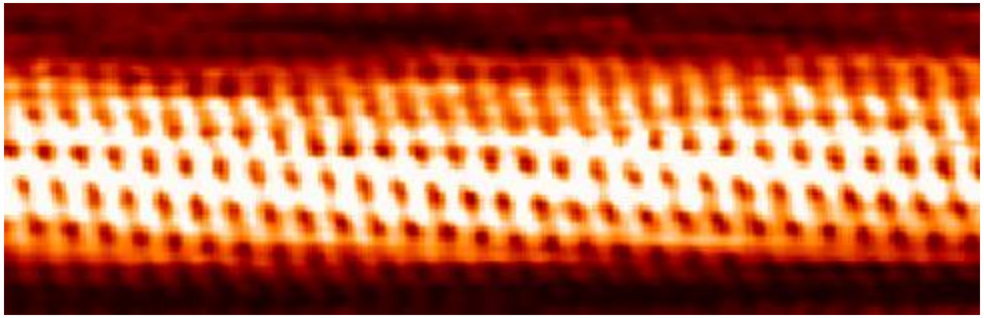
(a) (b) (c)



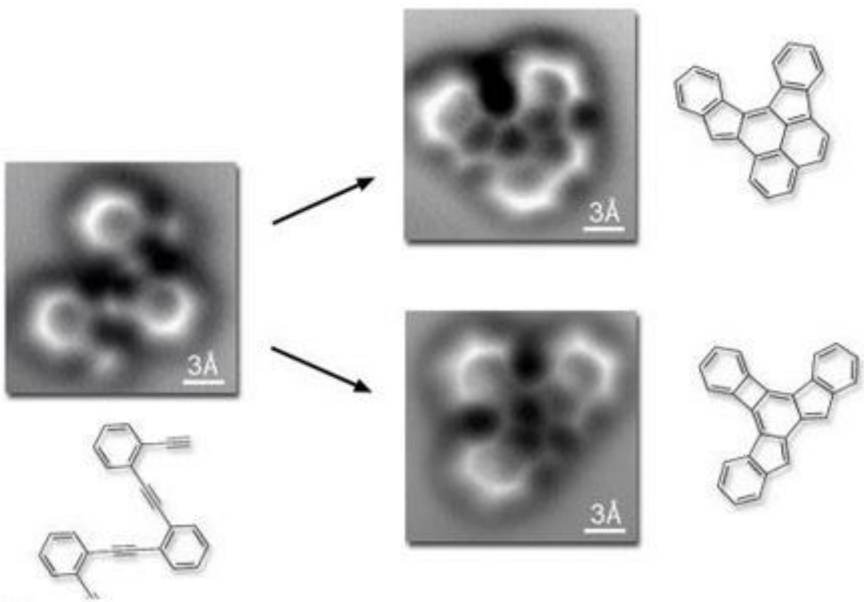
(a) (b)

These two photographs of the M82 Galaxy give an idea of the observable detail using (a) a ground-based telescope and (b) the Hubble Space Telescope.

Five-hundred-meter Aperture Spherical Telescope



controlled conditions.



Electron scanning tunneling microscope

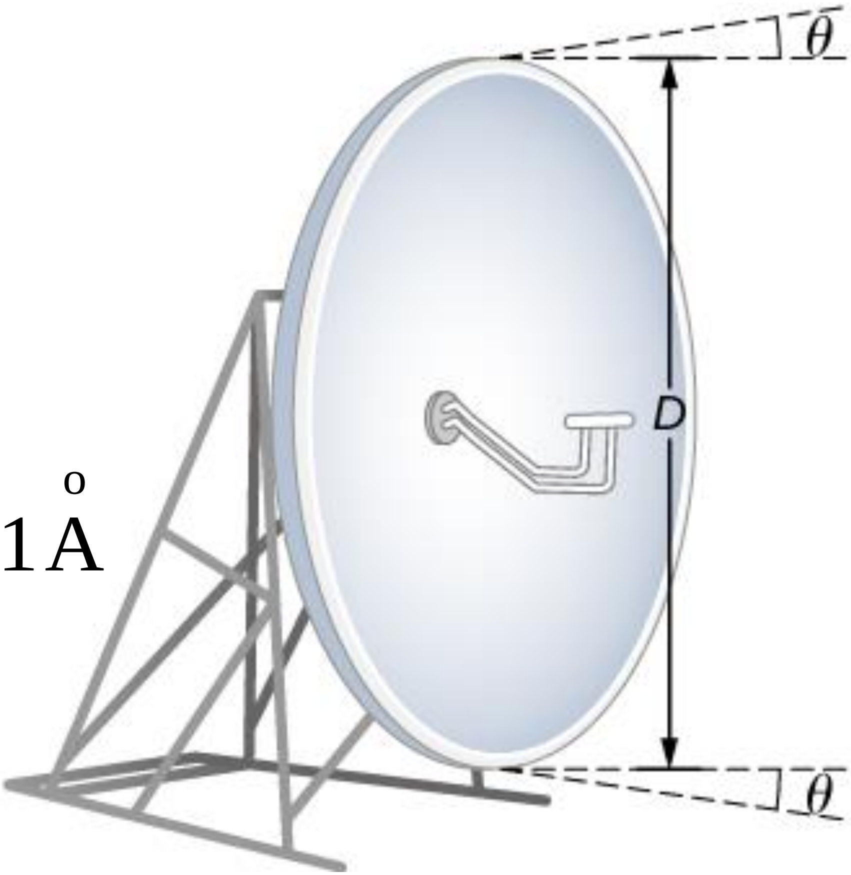
- 采用波长较短的光，也可提高分辨率。

电子显微镜用加速的电子束代替光束，其波长约 0.1nm，用它来观察分子结构。

光学显微镜 **0.2 μm**

电子显微镜 **1** Å

扫描遂道显微镜 0·01Å



▲ 4. Grating Fraunhofer diffraction

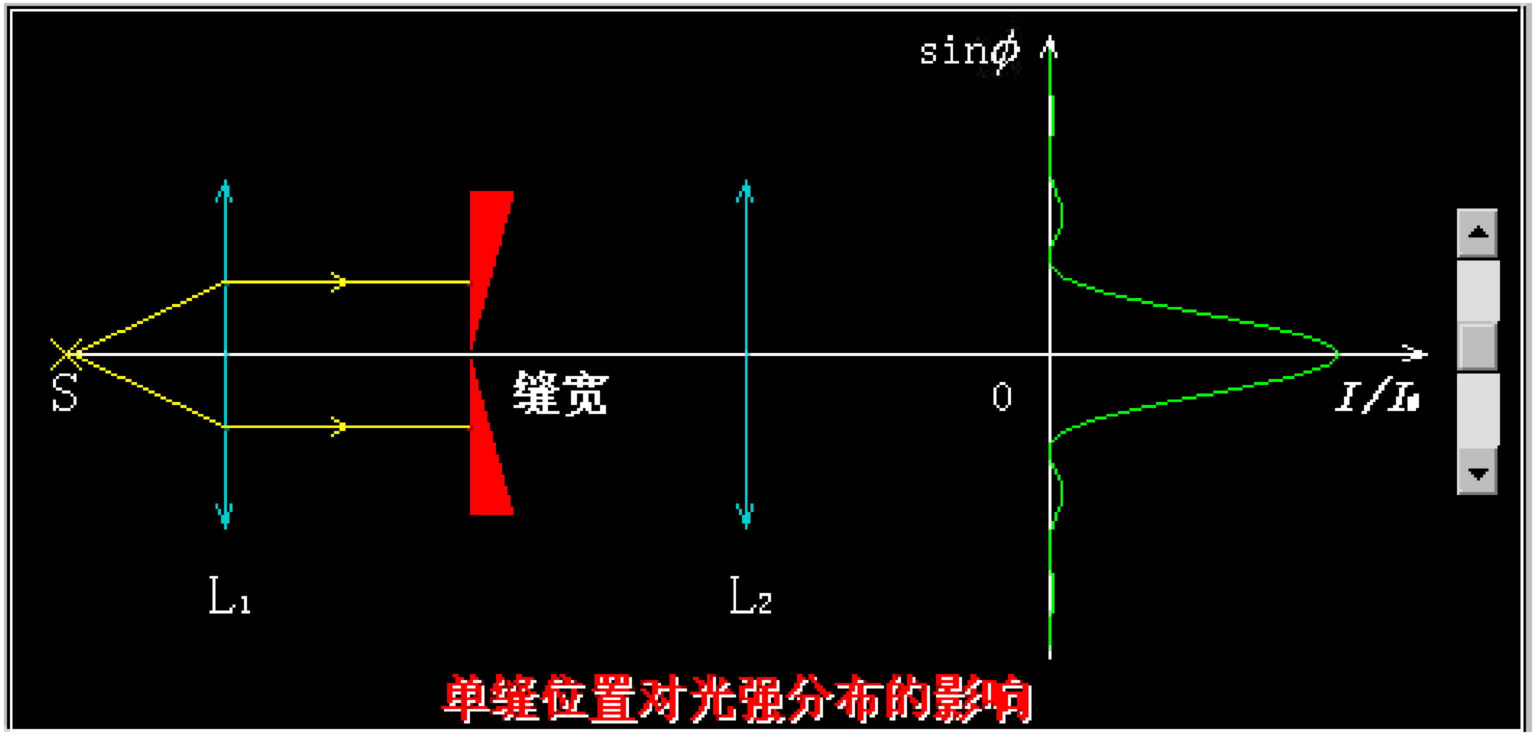
single slit diffraction:

$$a\downarrow : \Delta\varphi\uparrow ; I\downarrow$$

Discussion:

Whether the influence of N single slit diffraction is consistent with each other?

• Effect of single slit position on diffraction pattern



▲ 4. Grating Fraunhofer diffraction

Single slit diffraction:

$$a\downarrow:\Delta\varphi\uparrow;I\downarrow$$

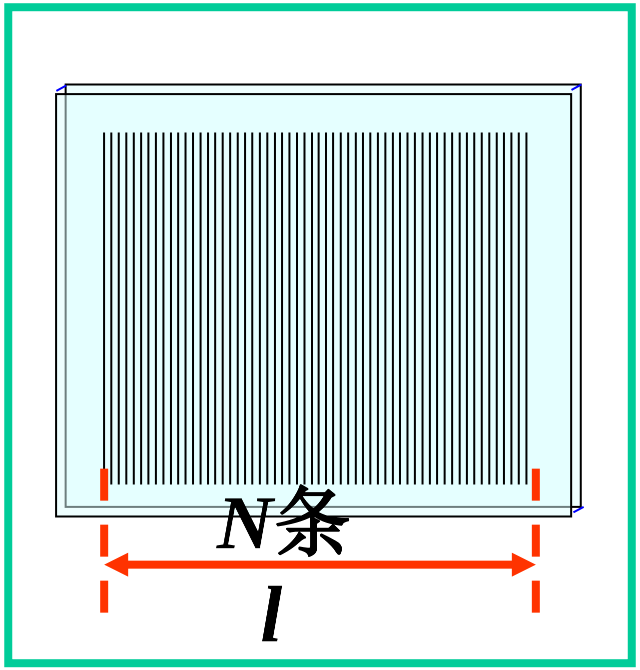
讨论 ***Whether the influence of N single slit diffraction is consistent with each other?***

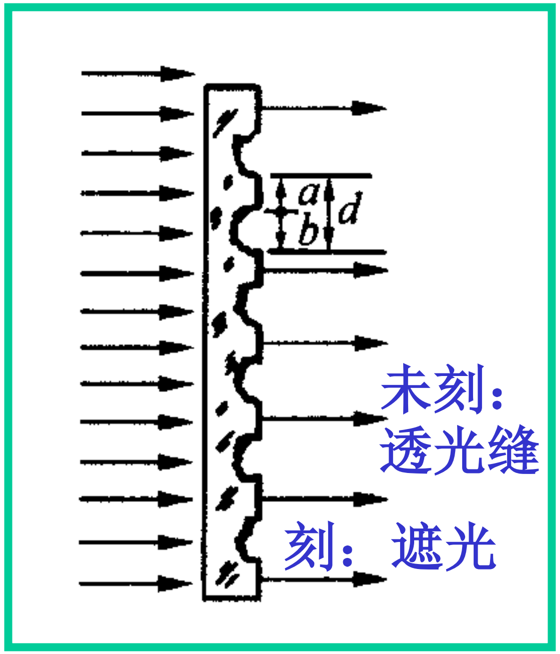
Solution: **A series of parallel single slits are used**

Grating : **An optical element consisting of parallel, equal-width, and equal-spacing slits (or reflective surfaces)**

Transmission grating: 刻痕玻璃

Make a series of parallel, equidistant scratches on the glass, where the cut is opaque.





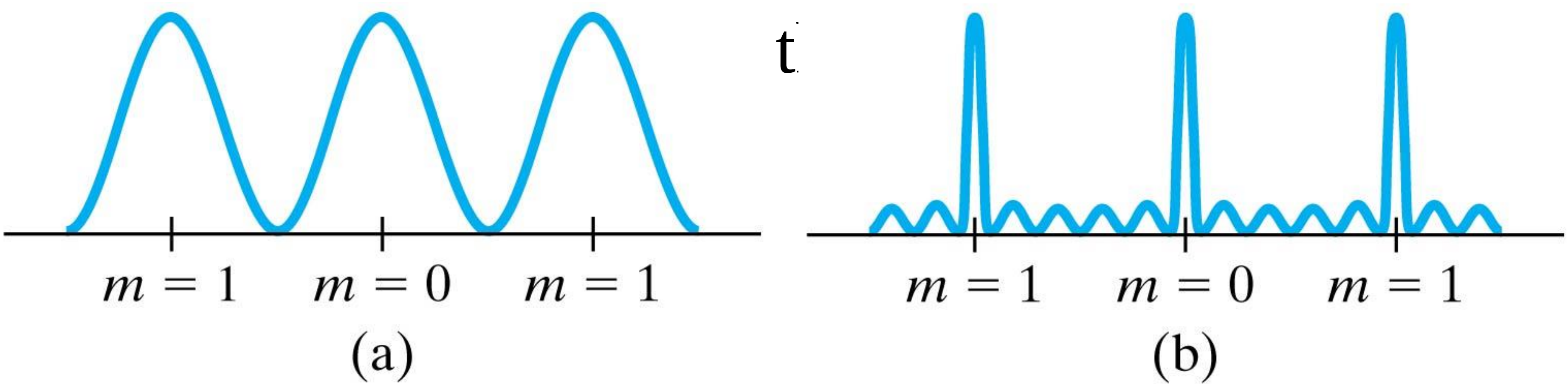
Grating constant:

$$\boxed{d = a + b = \frac{l}{N}}$$

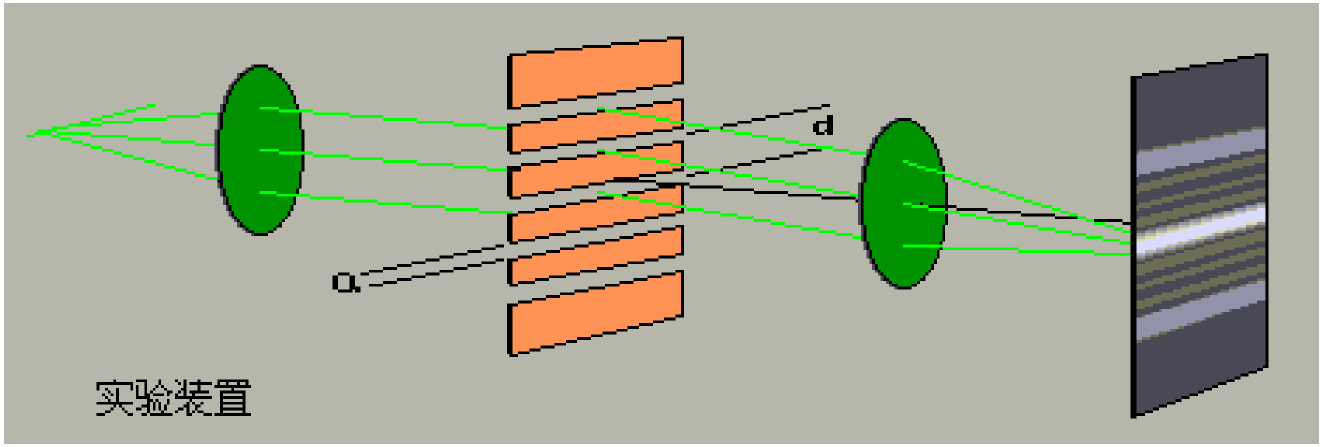
$(10^{-3} \sim 10^{-4}\,\mathrm{cm})$

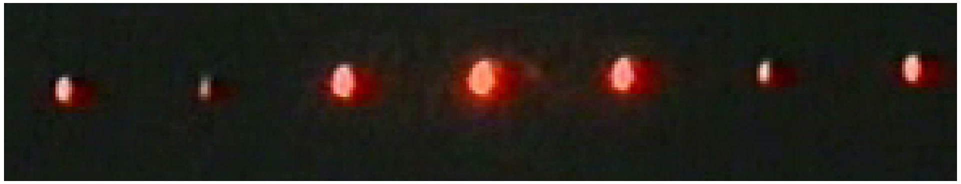
24-6 Diffraction Grating

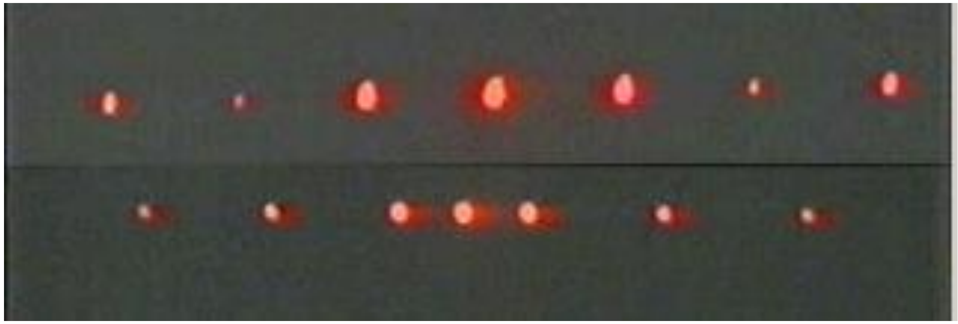
A diffraction grating consists of a large number of equally spaced narrow slits or lines. A transmission grating has slits, while a reflection grating has lines that reflect light.



1、Apparatus:

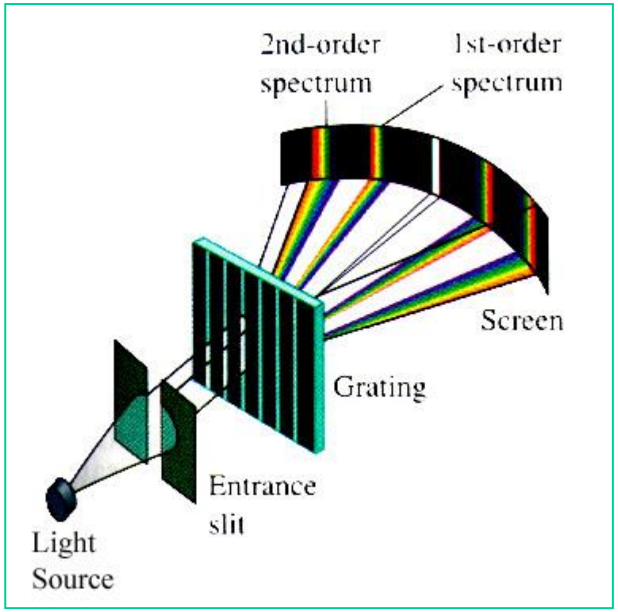






Diffraction pattern characteristics：

Bright narrow fringes of varying intensity



复色光入射时，每一种波长的光都会产生细窄明亮的条纹，因此常把作为一种分光装置，用于形成光谱。

2、Diffraction intensity analysis

Strategy:

①First, the slit width is not taken into account, and the intensity of each slit is concentrated on a first-line light source

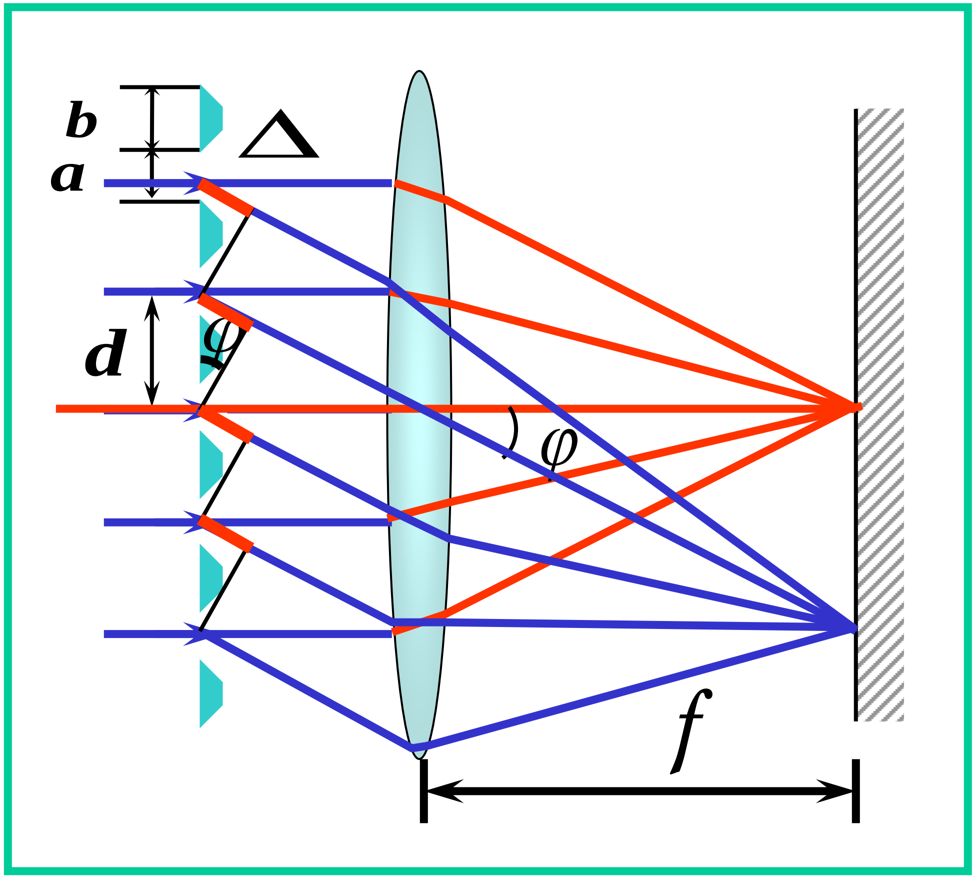
The interference of N light sources is discussed

② Taken slit width into account : the effect of N single slit diffraction

1. *N* -slit interference

$$\Delta = (a+b)\sin\varphi$$

$$= d\sin\varphi$$



$$\delta = \frac{2\pi d\sin\varphi}{\lambda}$$

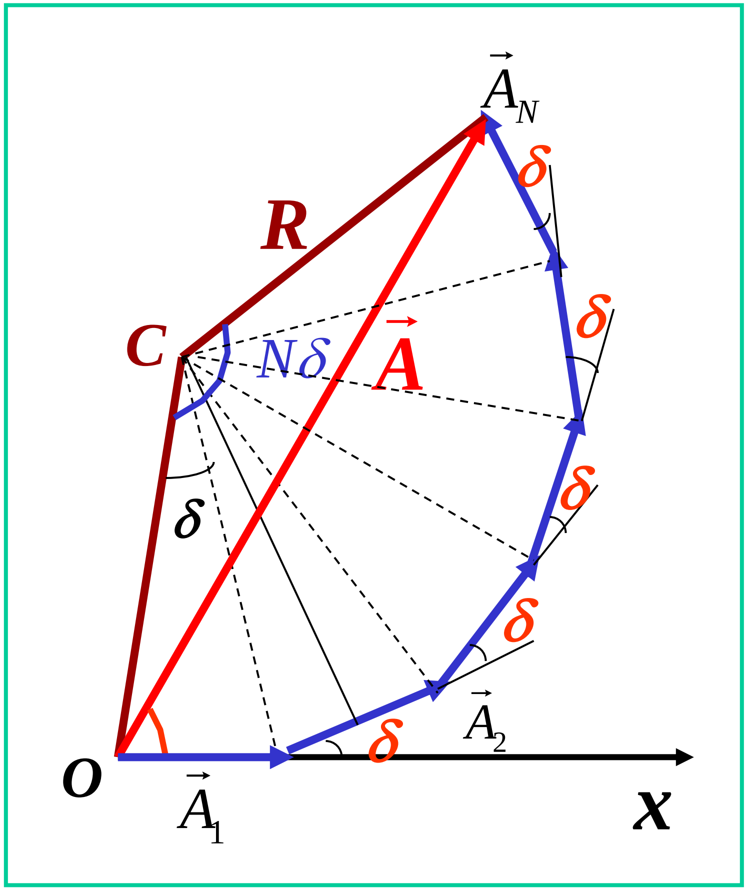
$$\beta = \frac{\delta}{2} = \frac{\pi d\sin\varphi}{\lambda}$$

1. *N* -slit interference

$$\Delta = (a+b)\sin\varphi$$
$$= d\sin\varphi$$

$$\delta = \frac{2\pi d\sin\varphi}{\lambda}$$

$$\beta = \frac{\delta}{2} = \frac{\pi d\sin\varphi}{\lambda}$$



$$A = A_1 \cdot \frac{\sin\frac{N\delta}{2}}{\sin\frac{\delta}{2}} = A_1 \cdot \frac{\sin N\beta}{\sin\beta}$$

Intensity: $I = I_1\left(\frac{\sin N\beta}{\sin\beta}\right)^2$

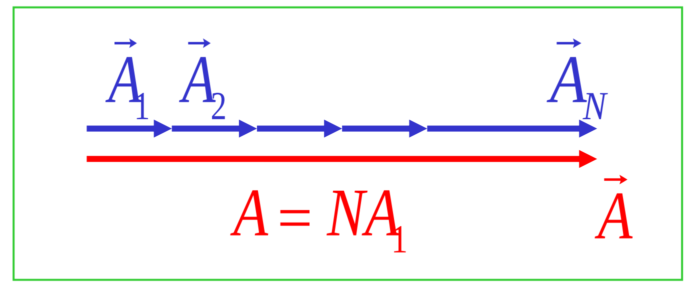
令 $\dfrac{\partial I}{\partial \beta}=0$ 得极值位置 $\beta=\dfrac{\delta}{2}=\dfrac{\pi d\sin\varphi}{\lambda}$ $I=I_1\left(\dfrac{\sin N\beta}{\sin\beta}\right)^2$

$\sin N\beta=0$ $\tan N\beta=N\tan\beta$

$$\beta=\frac{\delta}{2}=\frac{\pi d\sin\varphi}{\lambda}$$

(1) Primary fringes:

$\beta=k\pi \quad k=0,\pm1,\pm2\cdots\cdots$



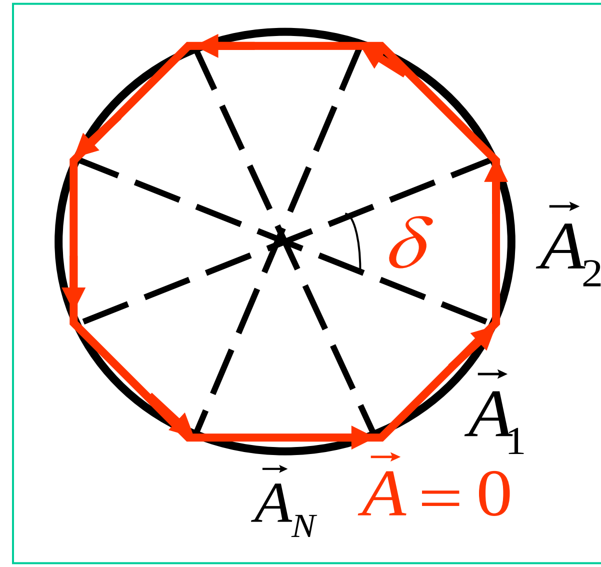
$$\boxed{d\sin\varphi=\pm k\lambda\ (k=0,1,2,..)}$$

Grating formula

$I=N^2I_1$

光栅公式决定了衍射主极大的位置，衍射主极大位置只由 **d** 决定，与N无关

$$\sin N\beta = 0 \quad \beta = \frac{\delta}{2} = \frac{\pi d \sin\varphi}{\lambda}$$



(2) Minimal value　　暗纹中心

$$\beta = k'\pi / N \qquad k' \neq kN \qquad I = 0$$

$$\sin\varphi = \frac{k'}{N}\cdot\frac{\lambda}{d} \qquad \sin\varphi = k\frac{\lambda}{d} \qquad k = 0, \pm 1, \pm 2 \cdots\cdots$$

$k = 0$　　　**1**　　　**2**

$$k' \neq 0,\ 1,\ 2, \cdots N-1,\ \neq N,\ N+1,\ N+2, \cdots 2N-1,\ \neq 2N,\ 2N+1, \cdots$$

两个主极大之间有N-1个最小值　　$I = I_1\left(\frac{\sin N\beta}{\sin\beta}\right)^2$

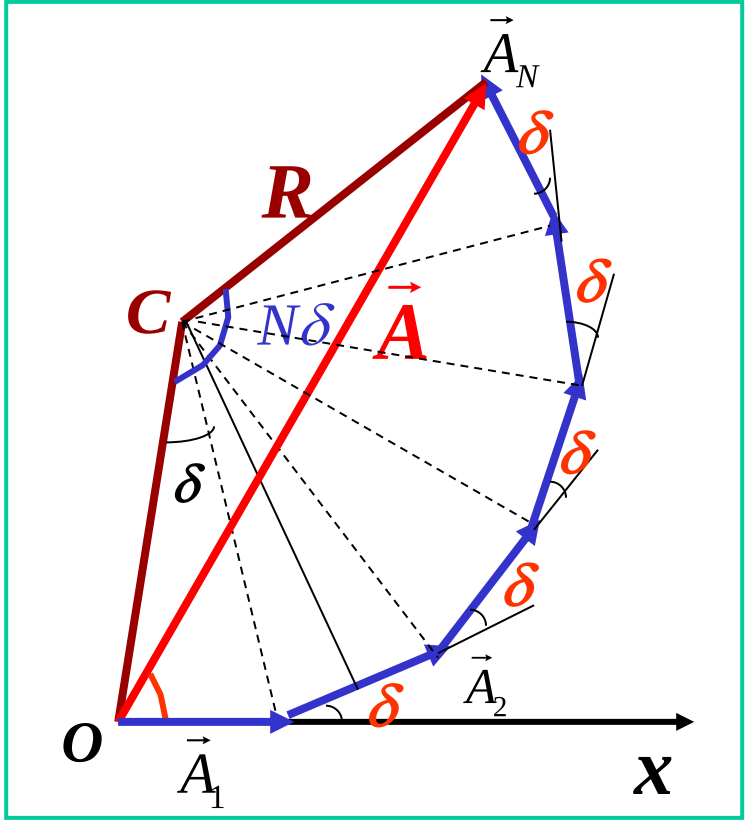
$$\tan N\beta = N\tan\beta$$

一般情况 $I = I_1\left(\dfrac{\sin N\beta}{\sin\beta}\right)^2$

(3) Secondary maximum

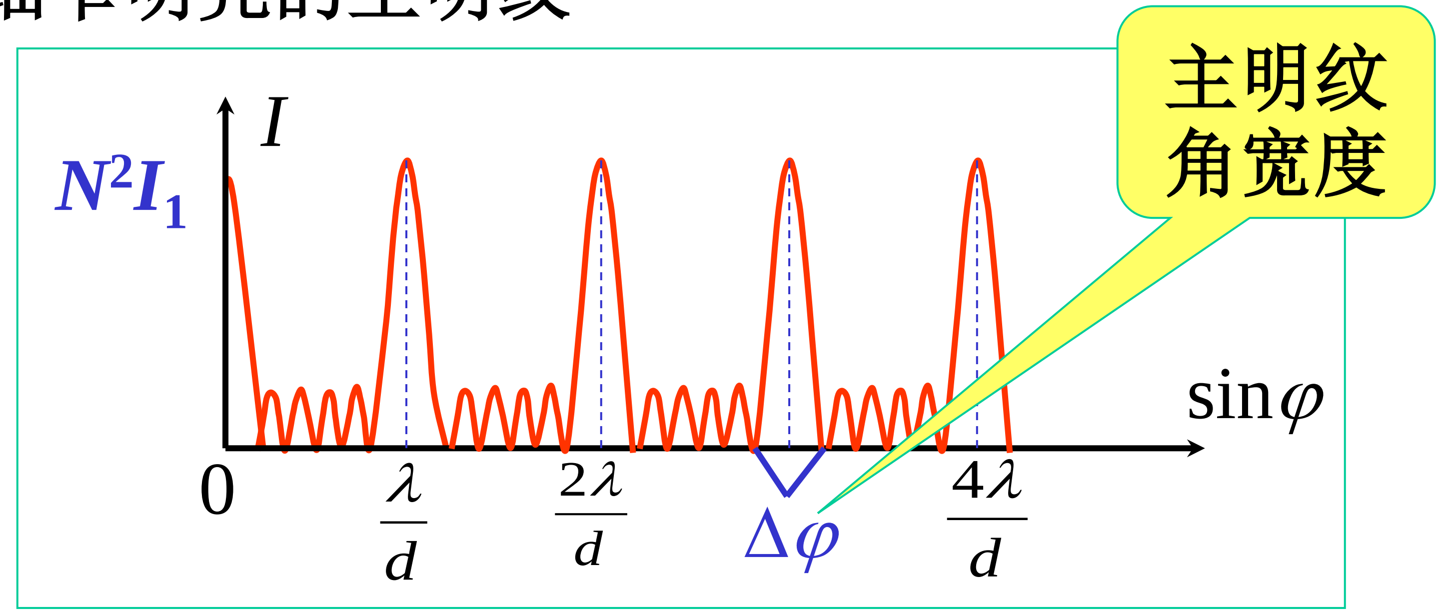
两个主极大之间的N-1个最小值，就由N-2个次极大隔开

次极大中心对应的光强最大值不超过主最大的1/23，所以两主极大值之间是宽大的弱暗背景



多缝干射光强分布图

暗区（***N*-1条暗纹，*N*-2条次级大**）背景上出现细窄明亮的主明纹



Location: $\sin\varphi = k\dfrac{\lambda}{d}$ **Highest order:** $\therefore k_{\mathrm{m}} < \dfrac{d}{\lambda}$

主明纹角宽度

K级主明纹的角宽度：在*kN*-1和*kN*+1两条暗纹之间，

对应 $\Delta k' = 2$

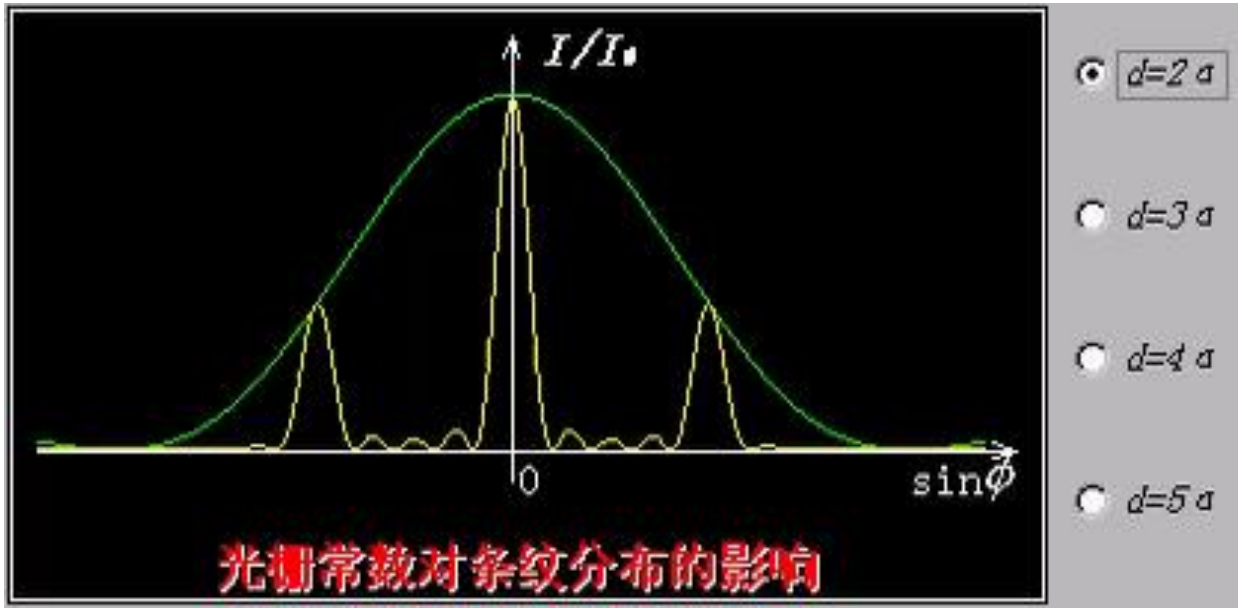
由暗纹条件 $\beta = k'\pi / N$ $\quad \beta = \dfrac{\delta}{2} = \dfrac{\pi d \sin\varphi}{\lambda}$

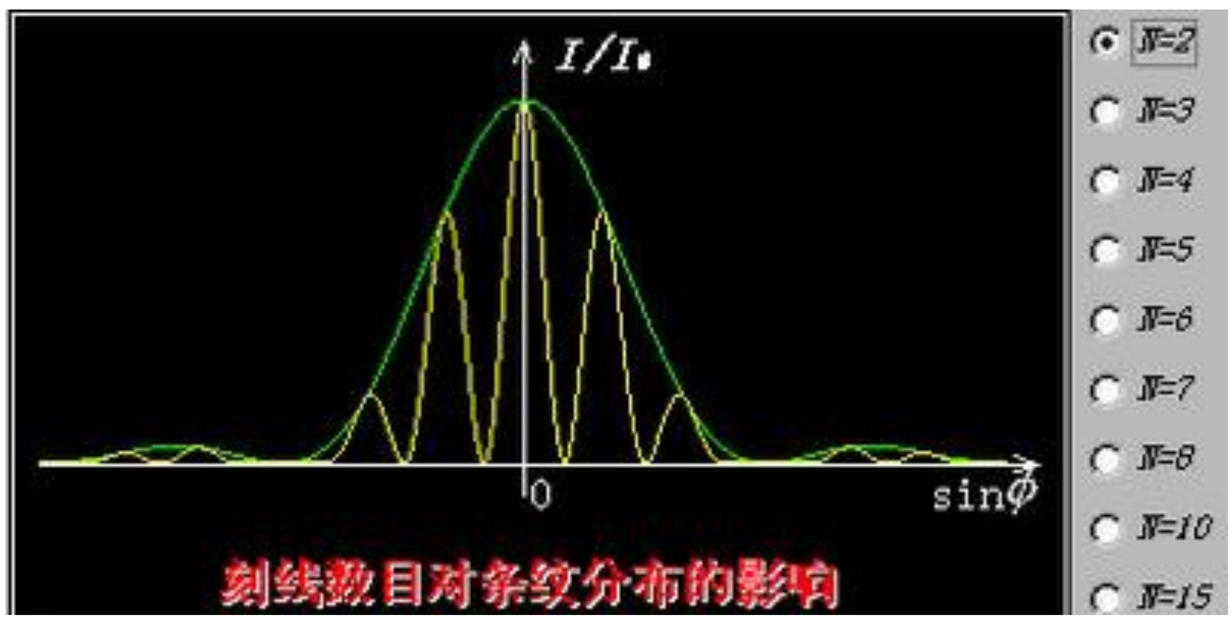
$$\sin\varphi = \frac{k'}{N} \cdot \frac{\lambda}{d} \qquad \cos\varphi \cdot \Delta\varphi = \frac{\lambda}{Nd} \cdot \Delta k'$$

$$\Delta k' = 2 \qquad \Delta\varphi = \frac{2\lambda}{Nd\cos\varphi} \approx \frac{2\lambda}{Nd}$$

(用于低级次 $\varphi \to 0, \cos\varphi \to 1$)

$N\uparrow$ 主明纹越细窄明亮，光 栅分辨本领越高


I/I₀
0
sinφ
d=2 a
d=3 a
d=4 a
d=5 a
光栅常数对条纹分布的影响


I/I₀
0
sinφ
N=2
N=3
N=4
N=5
N=6
N=7
N=8
N=10
N=15
刻线数目对条纹分布的影响

Grating resolution:

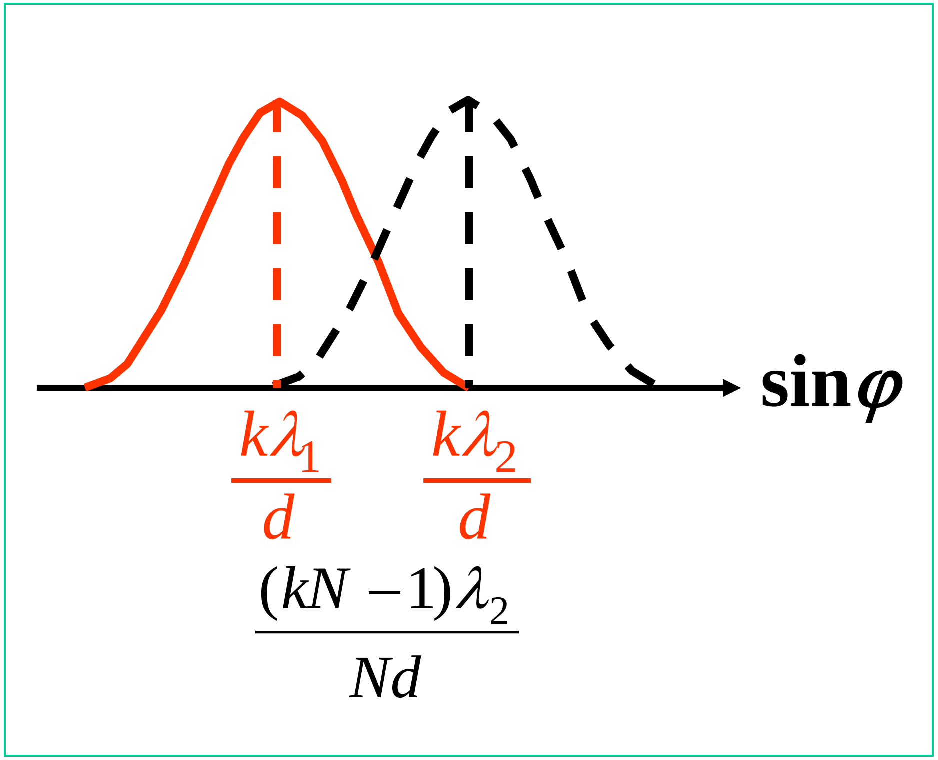
由瑞利判据，λ_1，λ_2光第k 级主明纹恰能分辨条件：

λ_1的主极大在λ_2相邻最近的暗纹处

$$\begin{cases} d\sin\varphi = k\lambda_1 \\ d\sin\varphi = \dfrac{kN-1}{N}\cdot\lambda_2 \end{cases}$$

$$= \left(k - \frac{1}{N}\right)\cdot\lambda_2$$

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda_2-\lambda_1}{\lambda_2} = \frac{1}{kN}$$

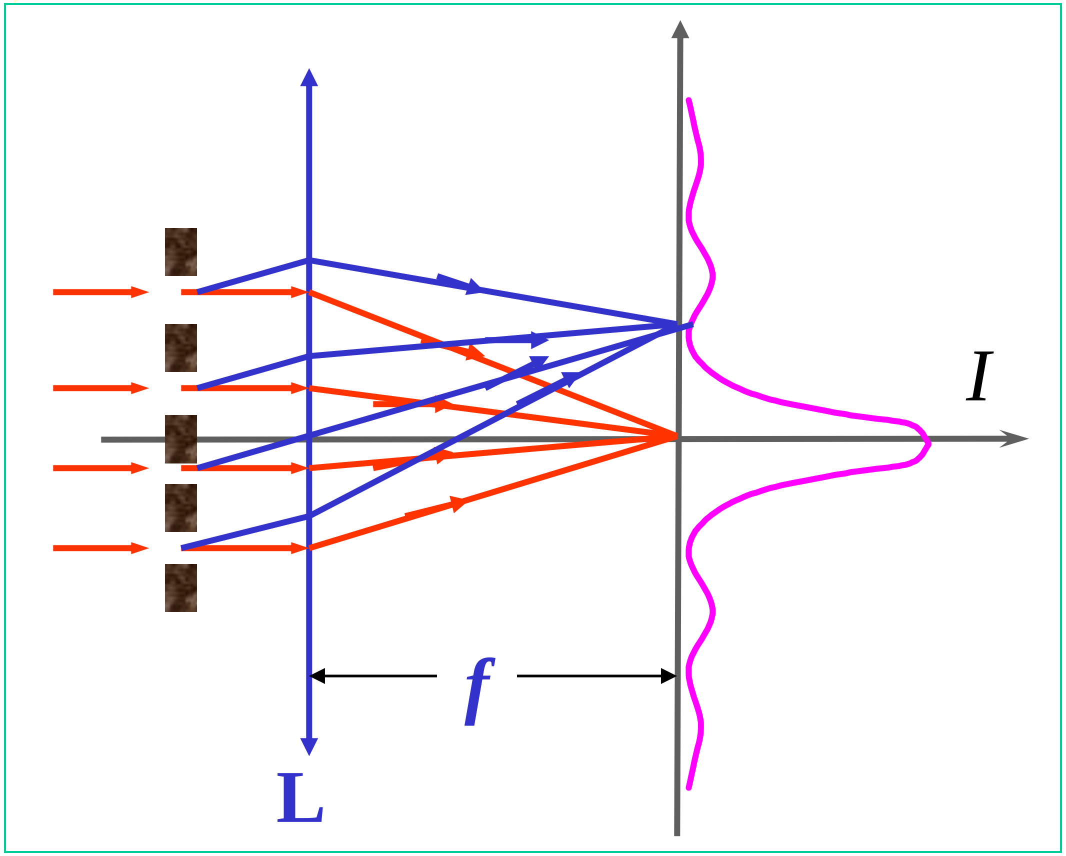


分辨本领: $R = \dfrac{\lambda}{\Delta\lambda} = kN$ 　 $R \propto N$,与d无关

2. *The influence of N single slit diffraction overlap*

讨论

N 个单缝衍射的影响彼此是否一致？



每条缝的单缝衍射条纹彼此重合

讨论　考虑缝宽后　会带来什么影响？

缝宽：　* 分为偶数个半波带：

缝内光线自身干涉相消　　$I_1 = 0$

即使缝间干涉相长　　$\boldsymbol{I = N^2 I_1 = 0}$

该主明纹不出现——缺级 order missing

* 分为奇数个半波带：

缝内光线部分干涉相消，条纹级次越高，光强越弱；

N 缝叠加后，光栅主明纹光强非均匀分布——亮度调制 Intensity modulation

（1）Intensity modulation:

light distribution: $$I = I_1\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

$$I_1 = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2; \quad \alpha = \frac{\pi a\sin\varphi}{\lambda}$$ **（single slit diffraction）**

$$\therefore \quad I = I_0\underbrace{\left(\frac{\sin\alpha}{\alpha}\right)^2}_{\text{单缝衍射因子}} \cdot \underbrace{\left(\frac{\sin N\beta}{\sin\beta}\right)^2}_{\text{多}(N)\text{缝干涉因子}}$$

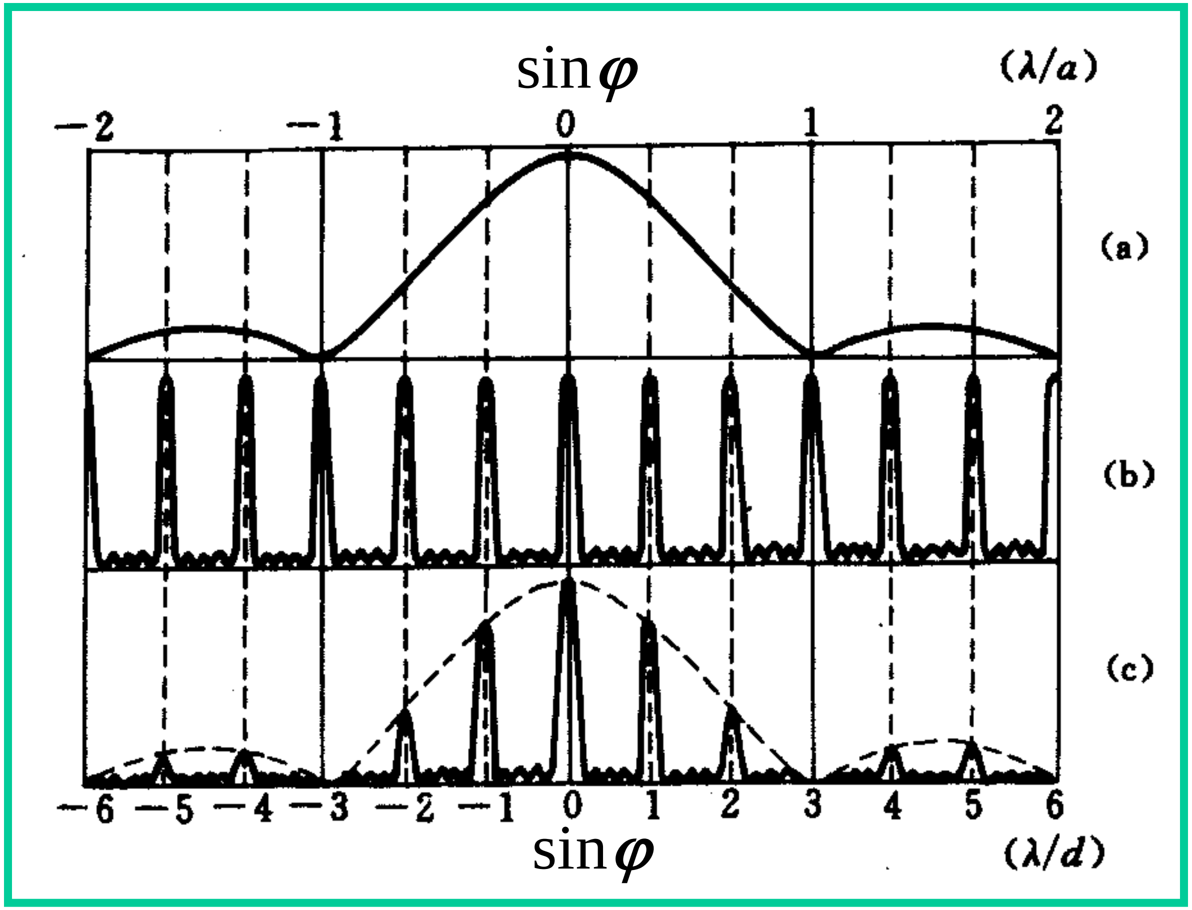
$$\alpha = \frac{\pi a\sin\varphi}{\lambda} \qquad \beta = \frac{\pi d\sin\varphi}{\lambda}$$

a: **Slit width** d: **Grating constant *a*+*b***

φ: **Diffraction angle**

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 \cdot \left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

同一缝中的子波相干影响亮度分布



$k\uparrow, I\downarrow$

（2） Order missing conditions：

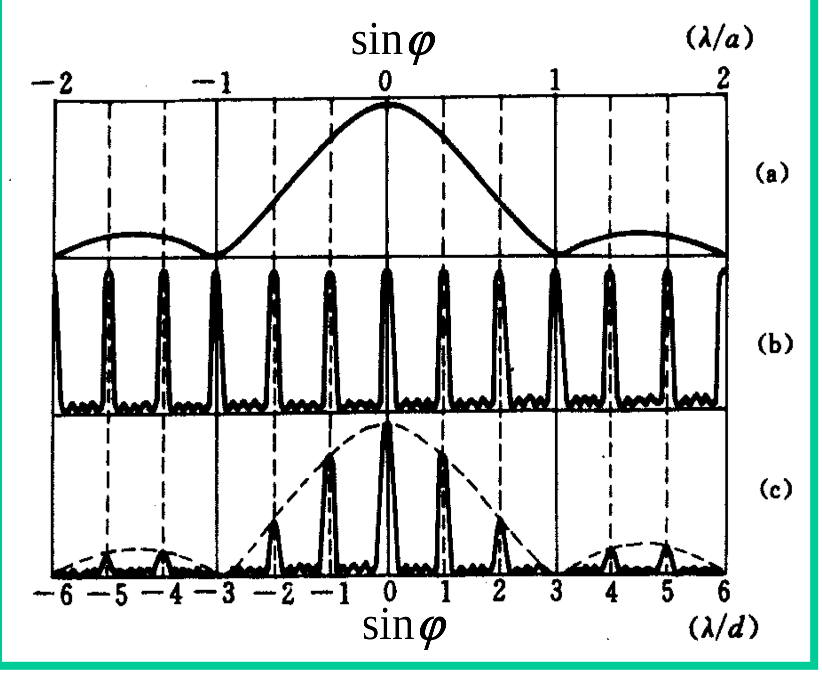
Primary fringe: $d\sin\varphi = (a+b)\sin\varphi = k\lambda \quad (k = 0,\pm1,\pm2\cdots)$

Dark fringe (single slit): $a\sin\varphi = k'\lambda \quad (k' = \pm1,\pm2\cdots)$

If conditions are satisfied at the same time, the k-th bright fringe disappeared。

When $\dfrac{d}{a} = \dfrac{a+b}{a} = \dfrac{k}{k'}$ （为整数比）

Order missing: $k = \dfrac{d}{a}\cdot k'(k' = \pm1,\pm2\cdots)$



（3）单缝衍射中央明纹区主极大条数

$$2(\frac{d}{a}_{\text{进整}})+1-2 = 2(\frac{d}{a}_{\text{进整}})-1$$

小结

光栅衍射是N缝干涉和N个单缝衍射的总效果

光强分布

$$I = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2 \cdot \left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

式中：

$$\alpha = \frac{\pi a \sin\varphi}{\lambda} \qquad \beta = \frac{\pi d \sin\varphi}{\lambda}$$

I_0：零级主明纹光强

（1） 细窄明亮的主明纹

位置：$d \sin\varphi = k\lambda \qquad k = (0, \pm 1, \cdots)$ ——光栅公式

缺级：

$$a \sin\varphi = k'\lambda$$

$$k = \frac{d}{a} k' \qquad (k' = \pm 1, \pm 2 \cdots)$$

角宽度： $\Delta\varphi = \dfrac{2\lambda}{Nd\cos\varphi}$

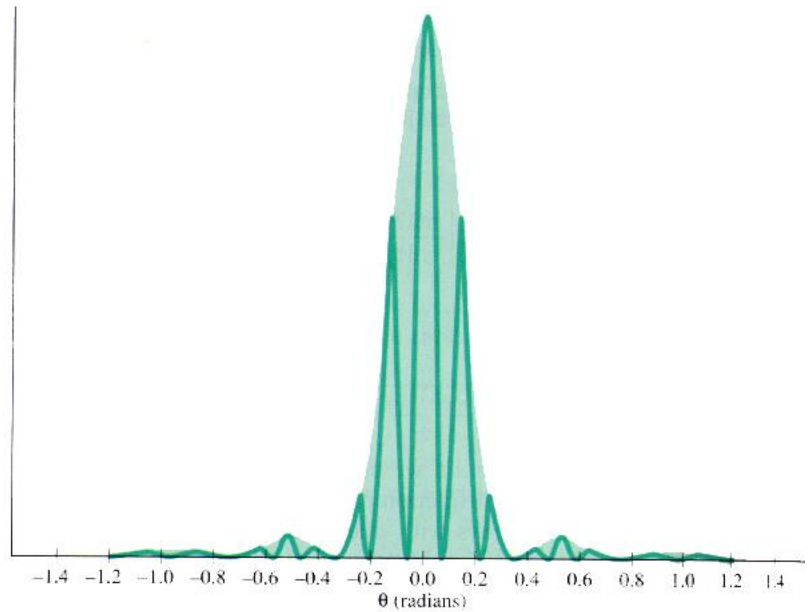
最高级次： $k_m < \dfrac{d}{\lambda}$

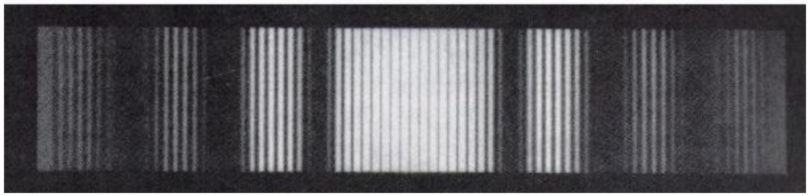
单缝中央明纹区主明纹条数：

$$2(\frac{d}{a}_{进整}) - 1$$

（2）相邻主明纹间较宽暗区

（N-1条暗纹，N-2条次极大）

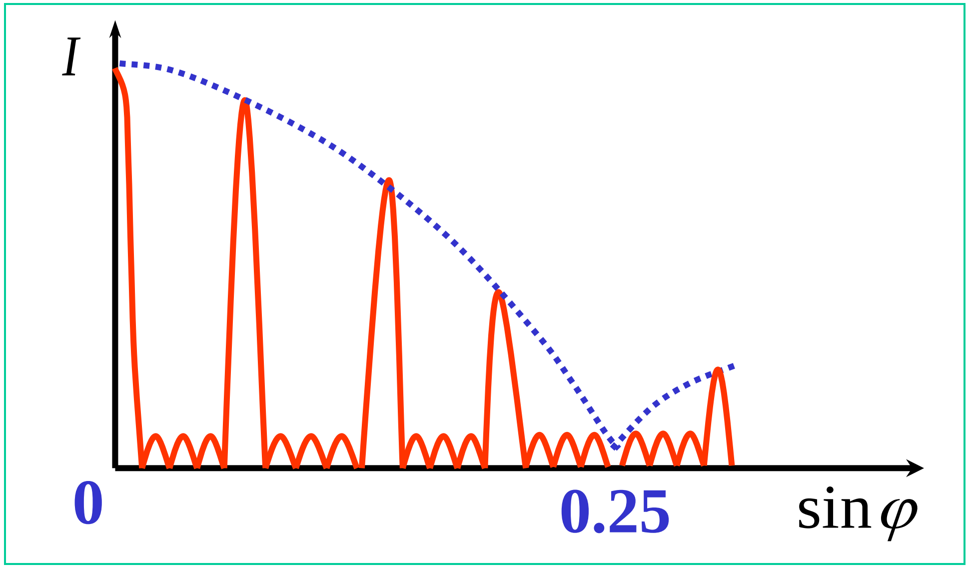




例： 入射光λ =500nm, 由图中衍射光强分布确定

① 缝数N=? ② 缝宽 a =? ③光栅常数 $d=a+b=?$

解： ① $N=5$



② $a\sin\varphi = k'\lambda$

$k'=1 \qquad \sin\varphi = 0\cdot 25$

$$a=\frac{5000}{0\cdot 25}=2\times 10^{4}\,\text{Å}$$

③ $d\sin\varphi = k\lambda$

$k=4 \qquad \sin\varphi = 0\cdot 25$

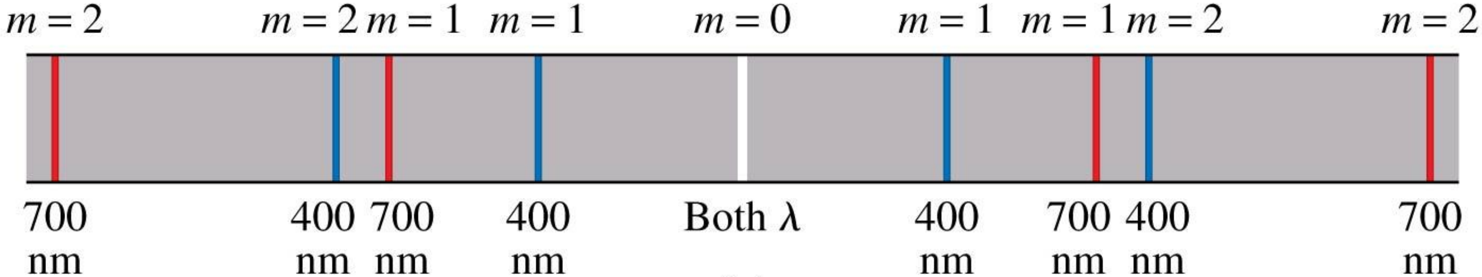
$$d=\frac{4\times 5000}{0\cdot 25}=8\times 10^{4}\,\text{Å}$$

或由缺级 $\dfrac{d}{a}=4 \qquad d=4a=8\times 10^{4}\,\text{Å}$

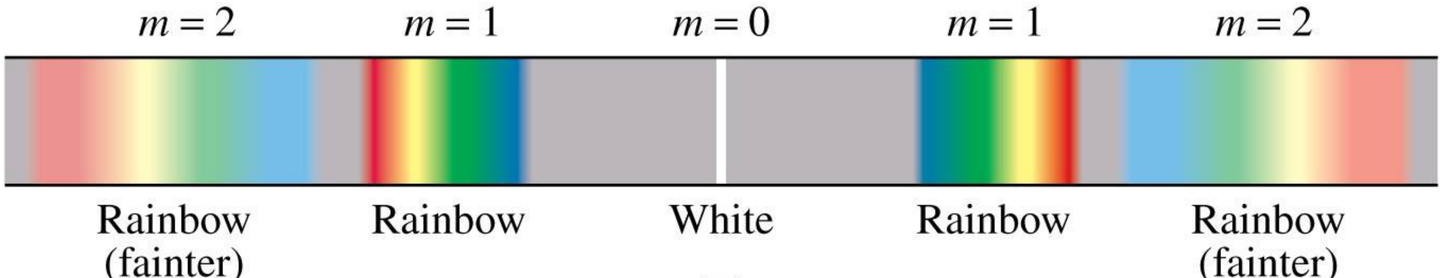
24-6 Diffraction Grating

The maxima of the diffraction pattern are defined by

$$\sin\theta = \frac{m\lambda}{d}, \quad m = 0, 1, 2, \cdots \qquad (24\text{-}4)$$



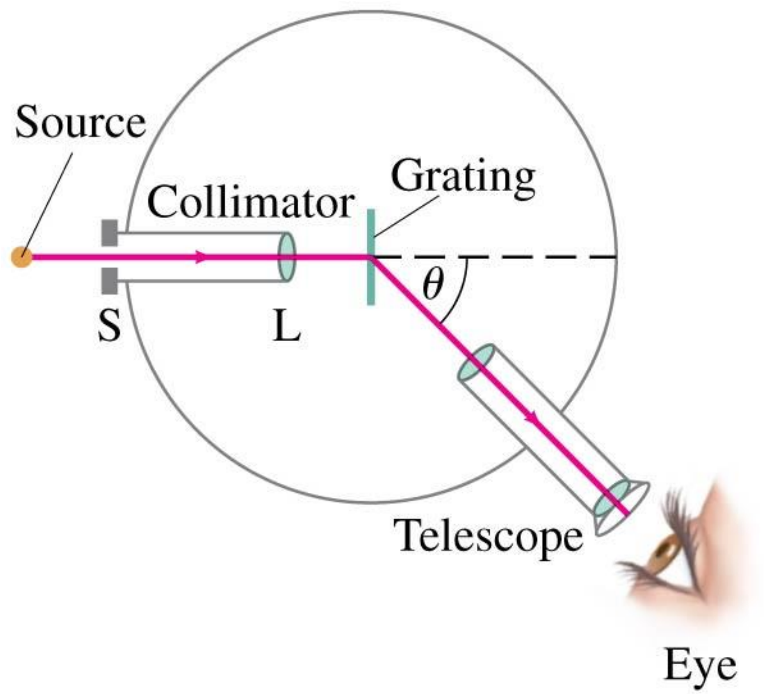
(a)



(b)

24-7 The Spectrometer and Spectroscopy

A spectrometer makes accurate measurements of wavelengths using a diffraction grating or prism.



24-7 The Spectrometer and Spectroscopy

The wavelength can be determined to high accuracy by measuring the angle at which the light is diffracted.

$$\lambda = \frac{d}{m} \sin\theta$$

Atoms and molecules can be identified when they are in a thin gas through their characteristic emission lines.

四、晶格衍射（X光衍射）

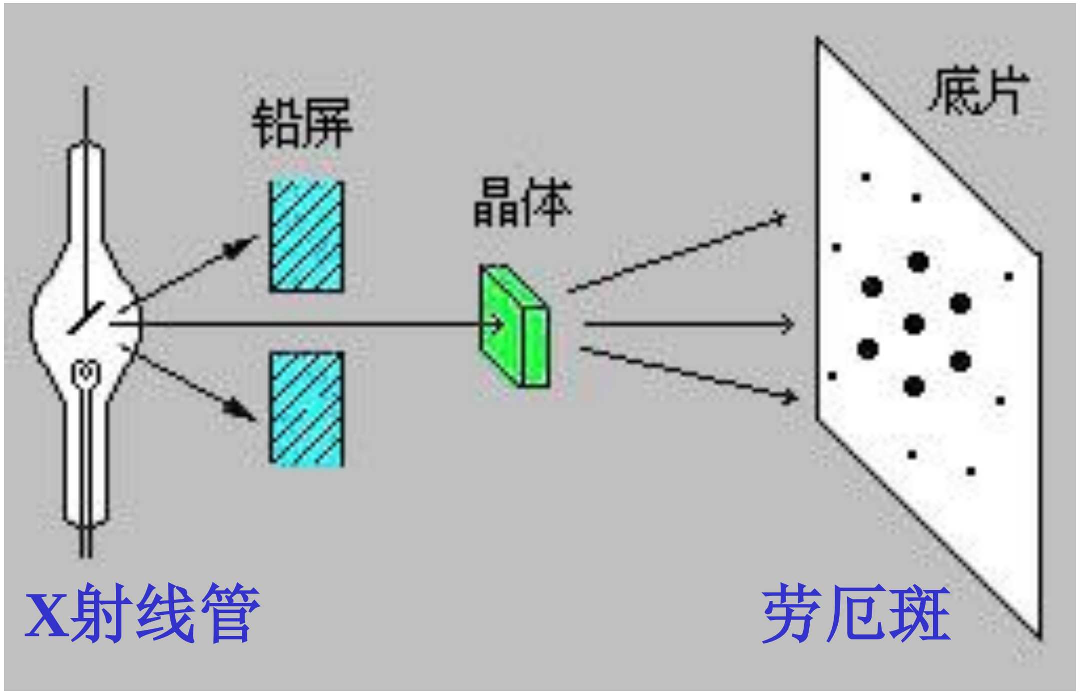
X射线：1895年，德国，伦琴在阴极射线实验中发现。
特点：不带电，穿透本领强，能使底片感光，是以前所未知的，所以称X射线(又称伦琴射线)。

Mrs. Roentgen looked at the picture and said, with fear ” It looks like the hand of the devil。”

In order to study its fluctuation,

To find the corresponding grating

1912年德国慕尼黑大学的实验物理学教授冯·劳厄用晶体中的衍射拍摄出X射线衍射照片。由于晶体的晶格常数约10nm，与 X 射线波长接近，衍射现象明显。



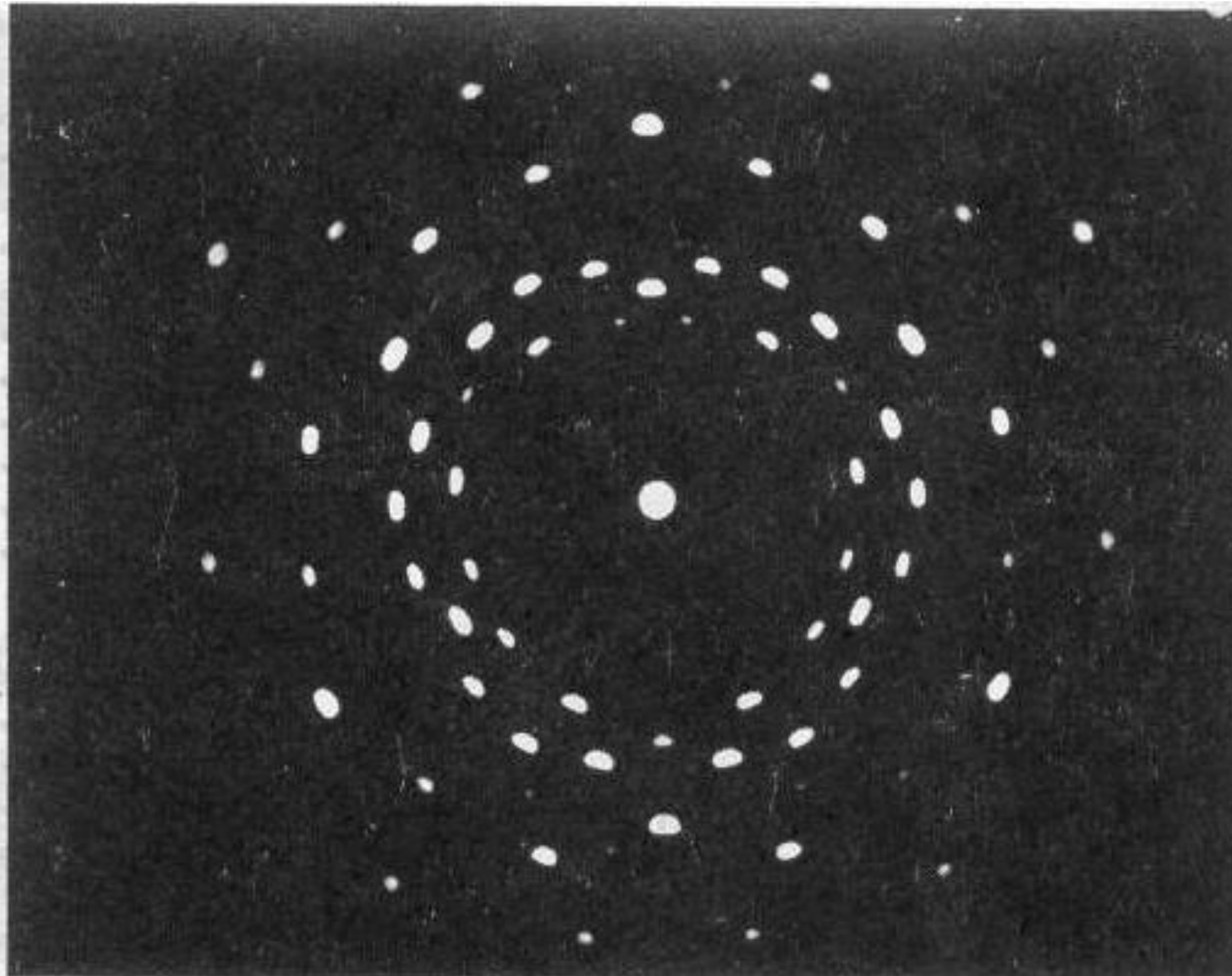
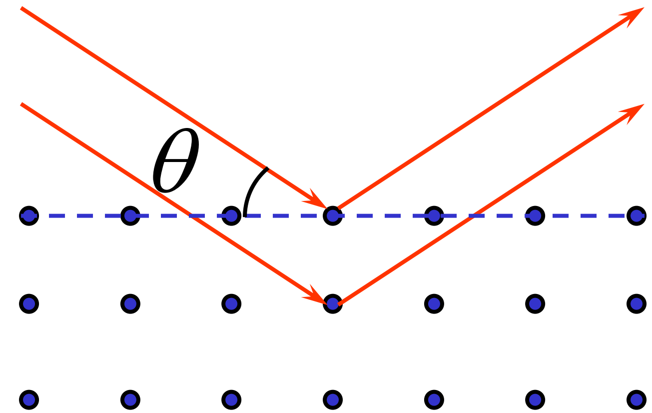


Fig. X-ray diffraction pattern for quartz (SiO_2).

Bragg equation:

1913年英国的布拉格父子，提出了另一种精确研究X射线的方法，并作出了精确的定量计算。由于父子二人在X射线研究晶体结构方面作出了巨大贡献，于1915年共获诺贝尔物理学奖。

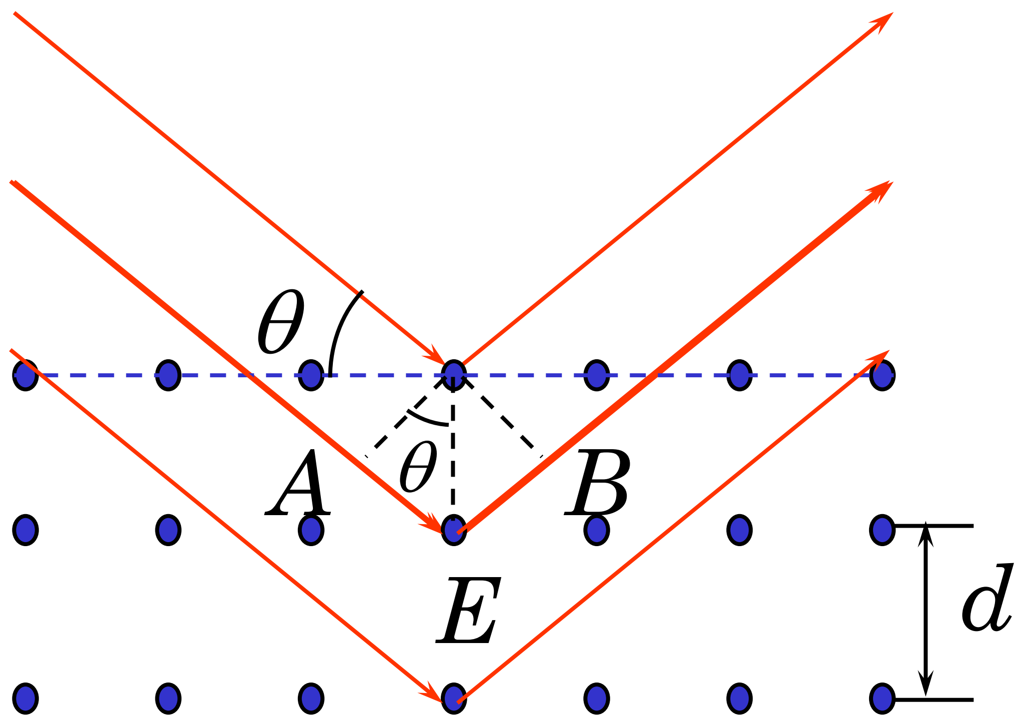


Crystals are composed of layers of atoms that are parallel to each other, and the position of the atoms can be regarded as the source of the secondary wavelet

掠射角θ：　X射线射到晶面时与晶面夹角。

晶格常数：　d

X射线经两晶面反射产生干涉，两束光的光程差为：



$$\Delta = \overline{AE} + \overline{EB}$$

$$= 2d\sin\theta$$

布拉格公式

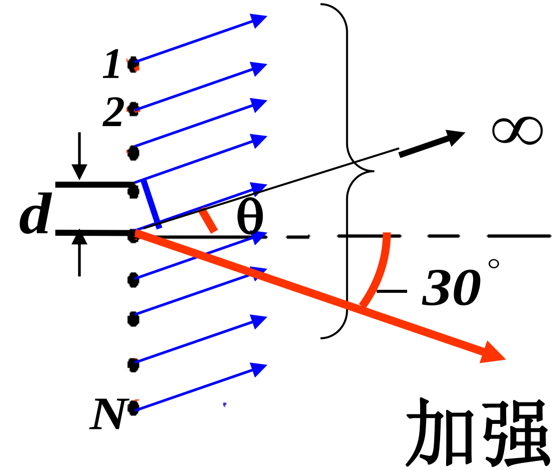
$$\boxed{\Delta = 2d\sin\theta = k\lambda \quad (k = 0,1,2\cdots) \quad \text{加强}}$$

例2. N 根天线位相依次落后 $\pi/2$，天线间距 $d=\frac{\lambda}{2}$

求： θ 为何值时该方向辐射加强？

解： 可视为N个缝的光栅

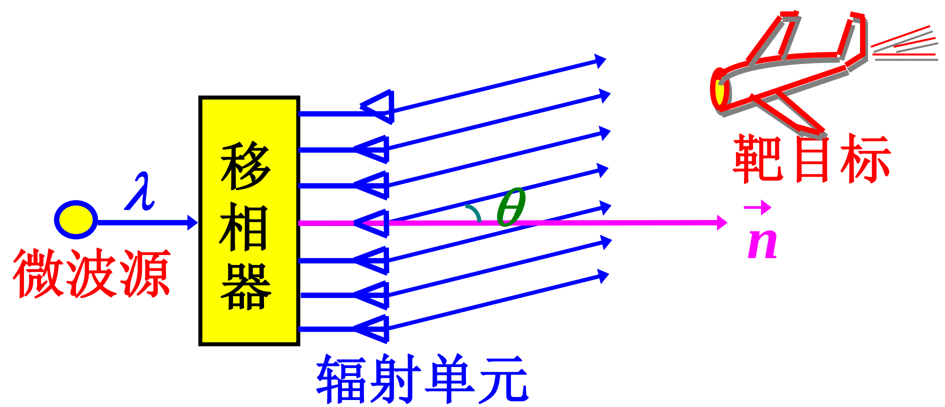
$$\boxed{d\sin\theta+\underline{\frac{\lambda}{4}}=k\lambda}$$



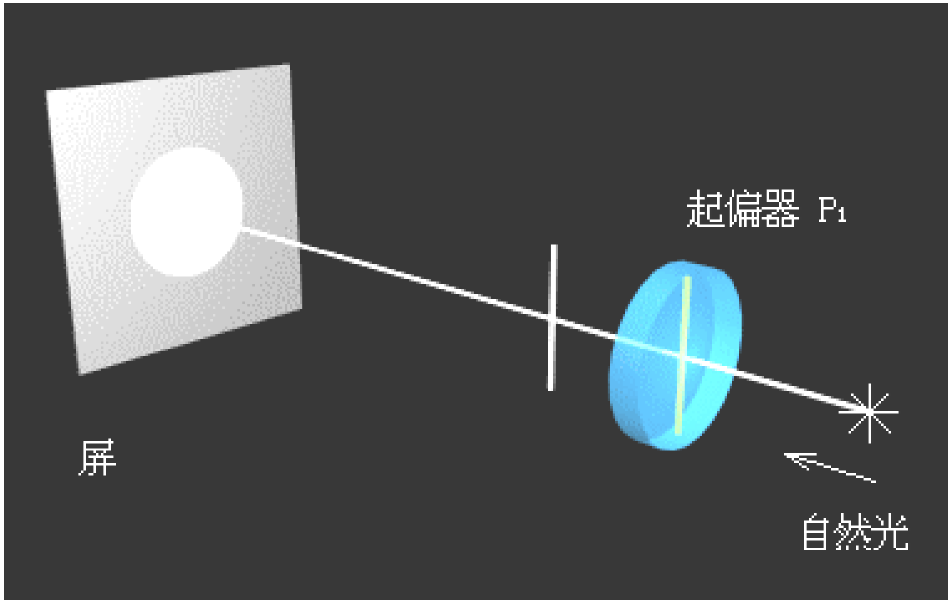
$$\frac{\lambda}{2}\sin\theta+\frac{\lambda}{4}=k\lambda \longrightarrow \sin\theta=2\left(k-\frac{1}{4}\right)$$

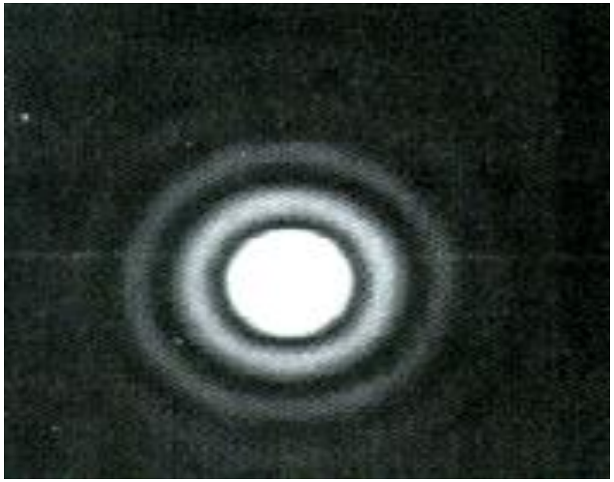
$$\begin{cases} k=0 & \sin\theta=-\frac{1}{2} \longrightarrow \underline{\theta=-30^\circ} \\ k\geq 1 & \text{无解} \end{cases}$$

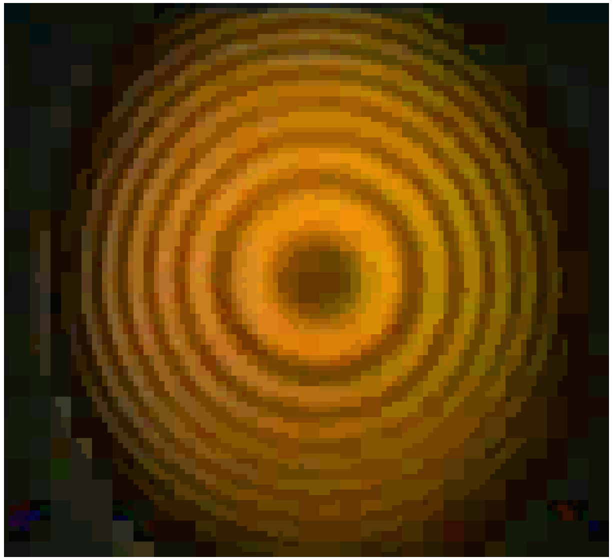
一维阵列的相控阵雷达



第十四章　波动光学



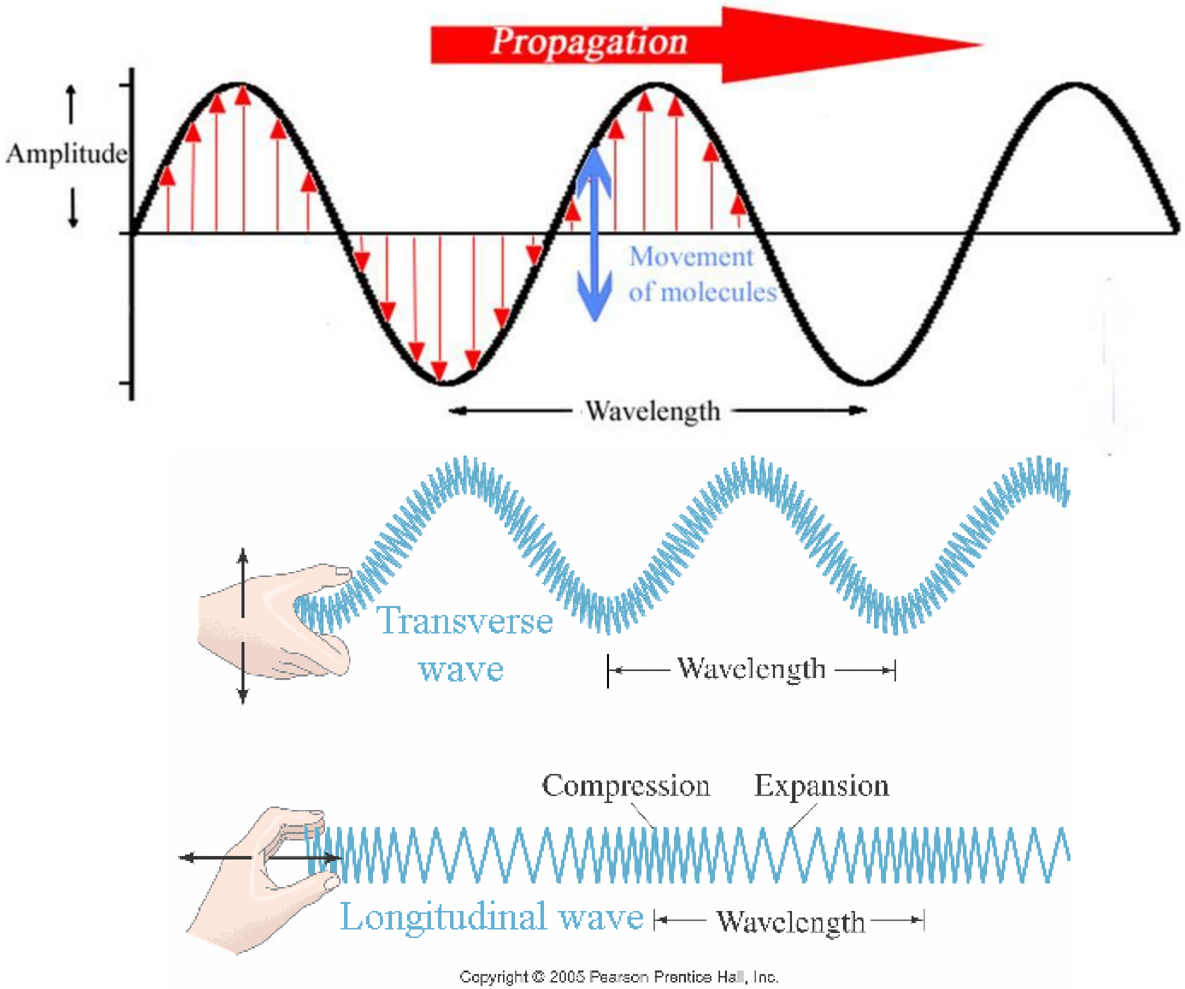


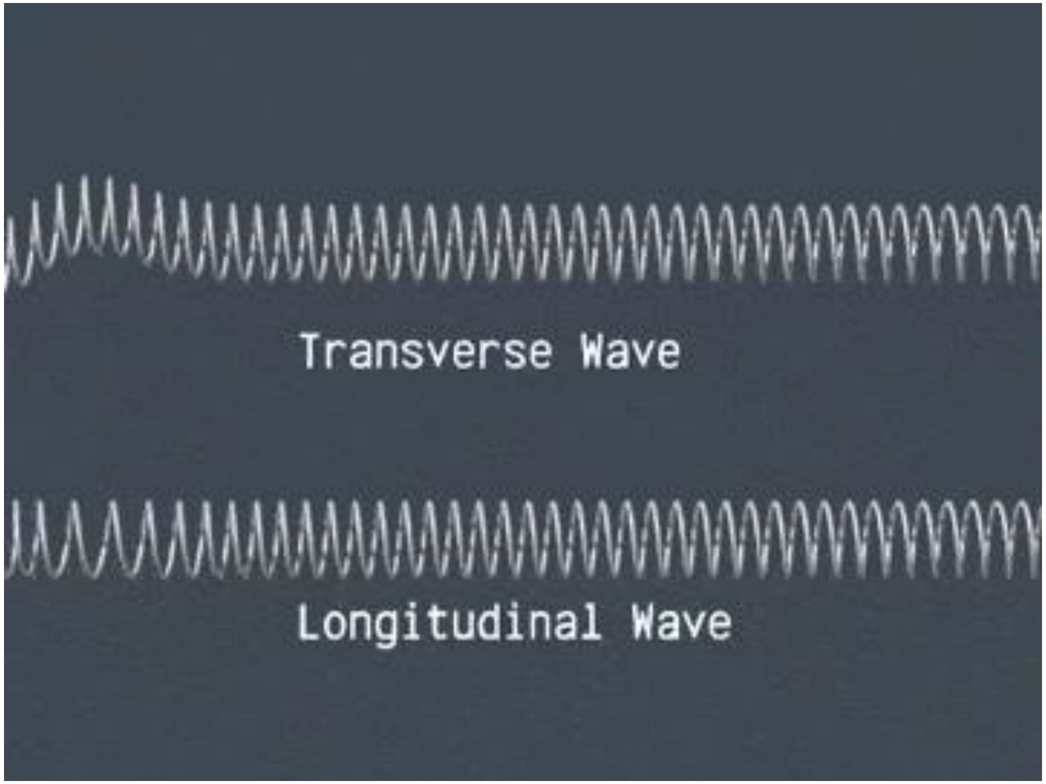


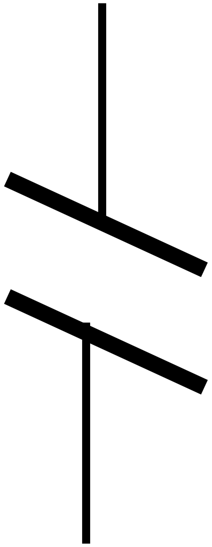
波的叠加原理　惠更斯-菲涅耳原理

§ 24.10 Polarization

一、Polarization states of light



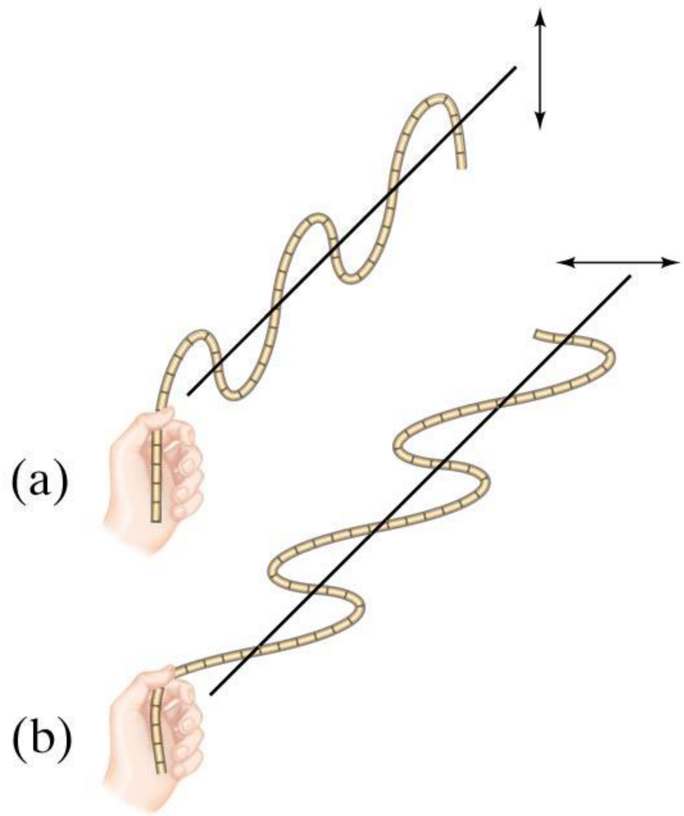




polarization： **The asymmetry of the direction of wave vibration with respect to the direction of propagation**

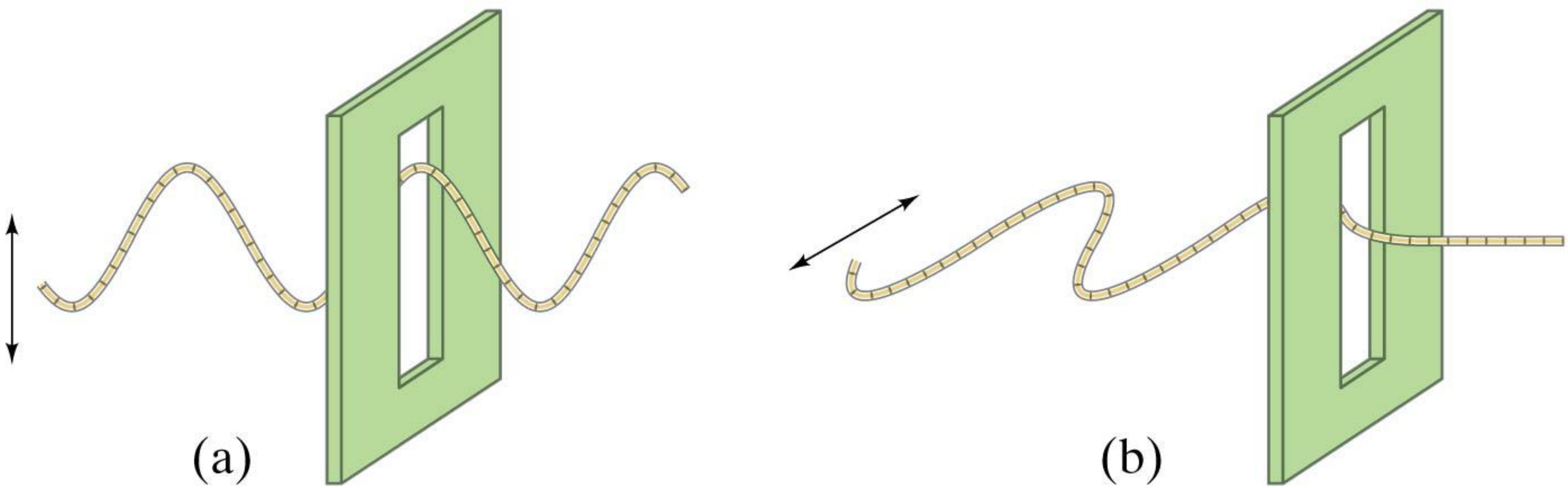
It is the most obvious sign of the difference between transverse wave and longitudinal wave

（1）Theory and experiment have proved that light is electromagnetic wave, so it has polarization。

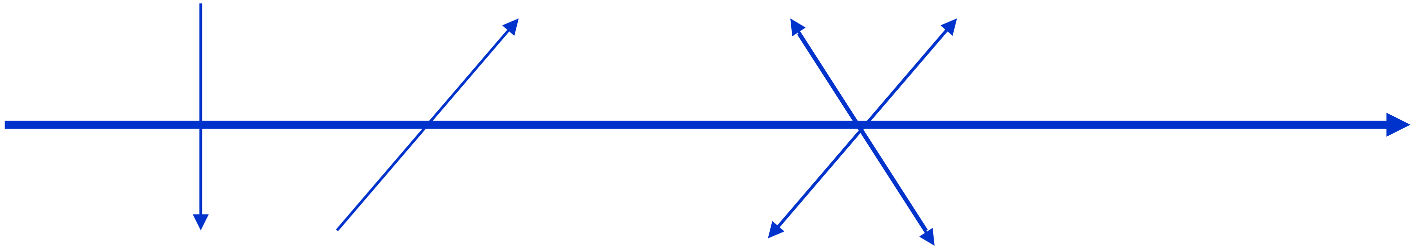


24-10 Polarization

Light is polarized when its electric fields oscillate in a single plane, rather than in any direction perpendicular to the direction of propagation.



光的横波性只表明电矢量与光的传播方向垂直，在与传播方向垂直的平面内还可能有各式各样的状态。



（2）. polarization state of light

- Light
  - Natural light
  - Polarized light
    - Completely polarized light
      - Linearly polarized lights
      - Elliptically polarized light
      - Circularly polarized light
    - Partial polarized light

1. Natural light

普通光源发光

每个光波列：横波 — 偏振

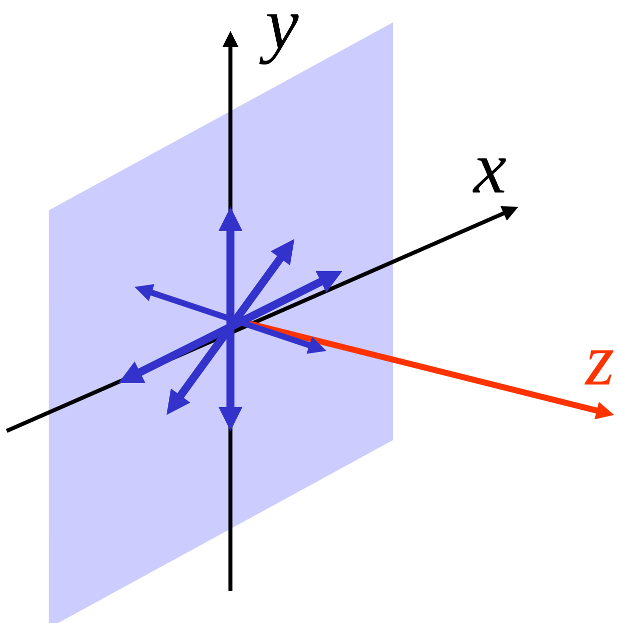
原子发光的独立性和随机性：

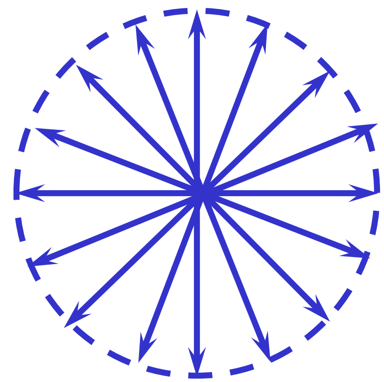
一段观测时间内电矢量统计平均值既有空间分布的均匀性，又有时间分布的均匀性。

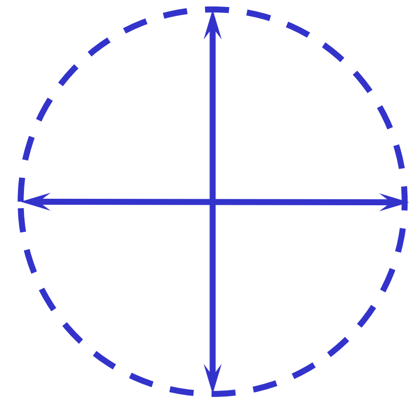
光振动各向振幅大小相同

↓

光矢量对传播方向均匀对称分布 — 非偏振



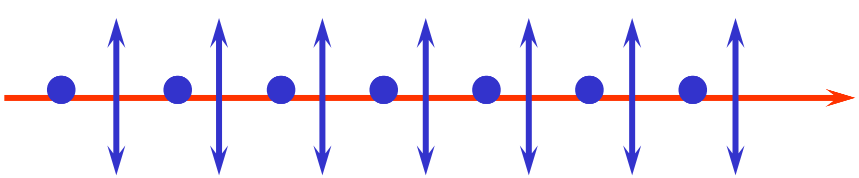


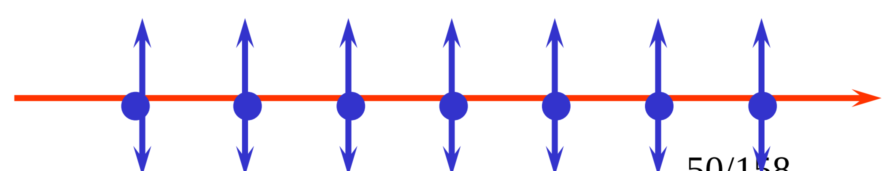


$$A_x = \sum a_{ix} \qquad A_y = \sum a_{iy} \qquad I_y = I_x = \frac{1}{2} I$$

The superposition of a pair of mutually perpendicular, equally oscillating, independent plane polarized light

无固定相位差,非相干叠加

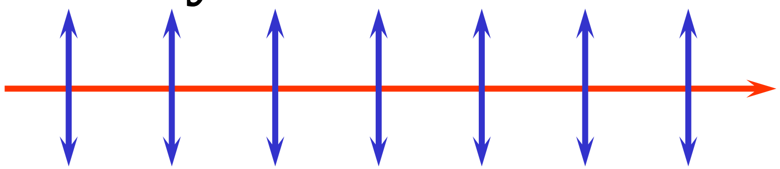


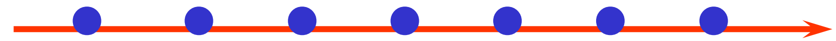


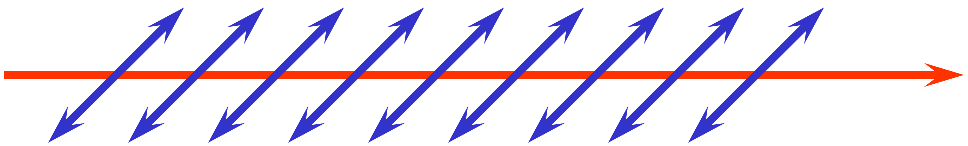
2. Linearly polarized lights

The vibration of light has only one definite direction

只有一个振动面



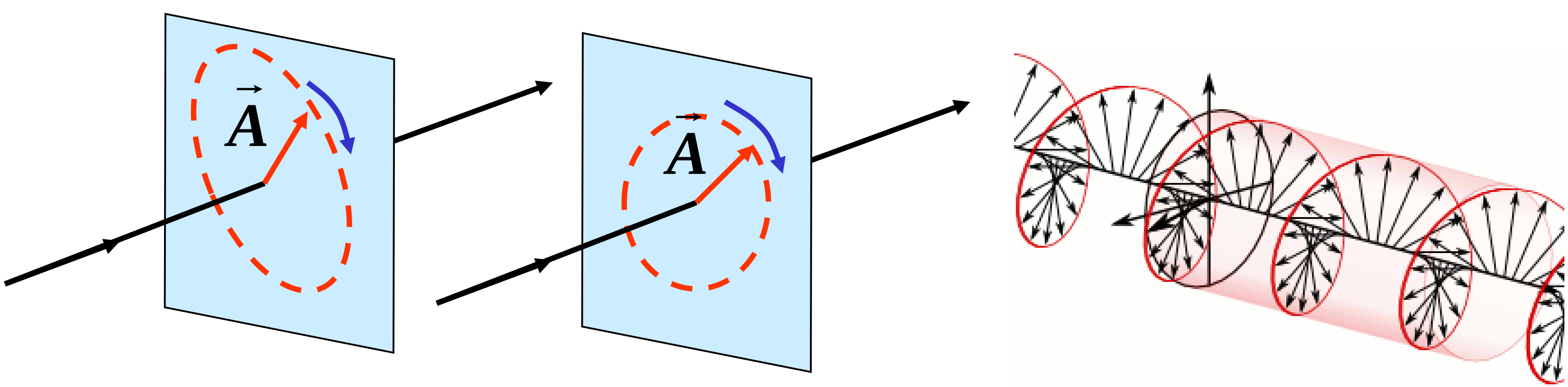




3. Elliptically polarized light

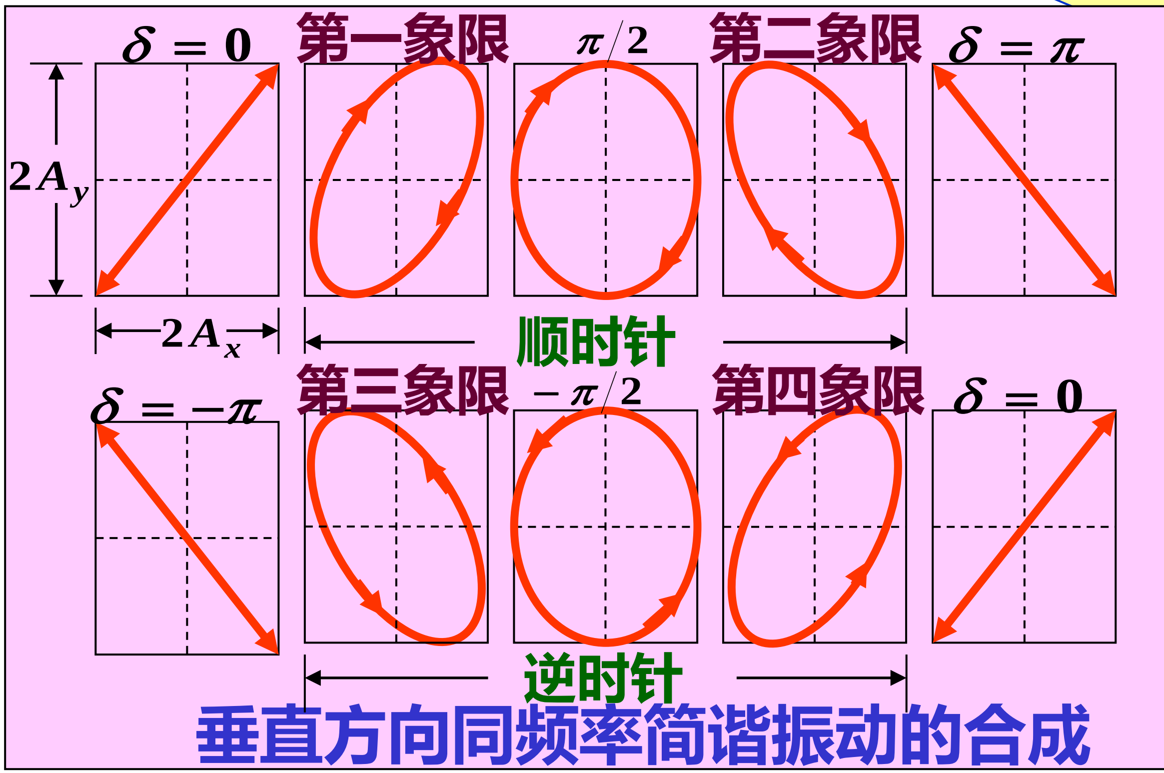
4. Circularly polarized light

光矢量旋转，不只一个振动面其端点轨迹为截面是 椭圆 圆 的螺旋线



椭圆偏振光和圆偏振光

分解为两个互相垂直、频率相同、有确定相位差的光振动

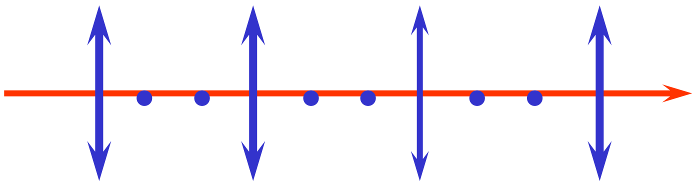


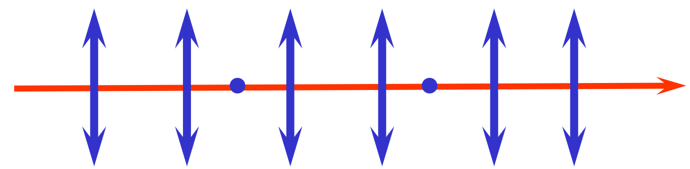
规定：迎着光的传播方向看，光矢量顺时针旋转为**右旋偏振光**（圆或椭圆）；光矢量逆时针旋转为**左旋偏振光**（圆或椭圆）。

5. Partial polarized light

自然光+线偏振光

光振动在某方向上占优势





• Method and rules of polarization

获得线偏振光

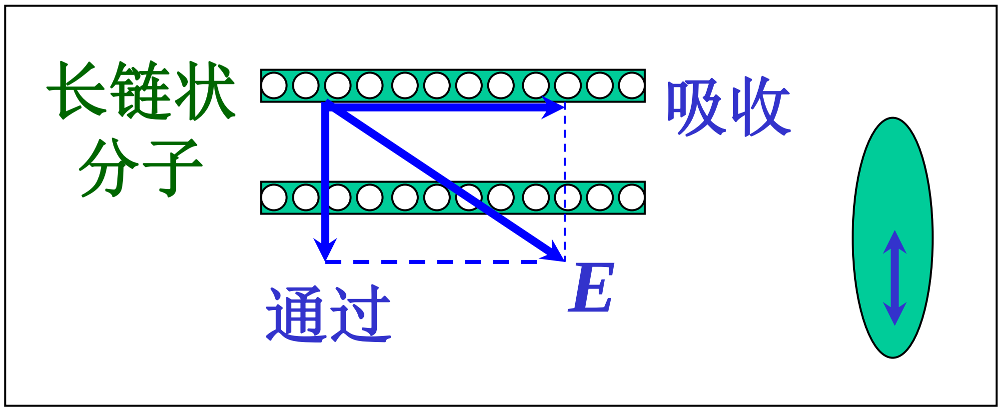
Methods:

- **Using the reflection and refraction of light on the interface of two media**
- **Using the propagation of light in anisotropic media**
  - **Polarizer**
  - **Birefringence**

二、Polarization　Malus law

①原理：晶体的二相色性：只让某一方向振动的光通过，而吸收其它方向的光振动

Polarization direction

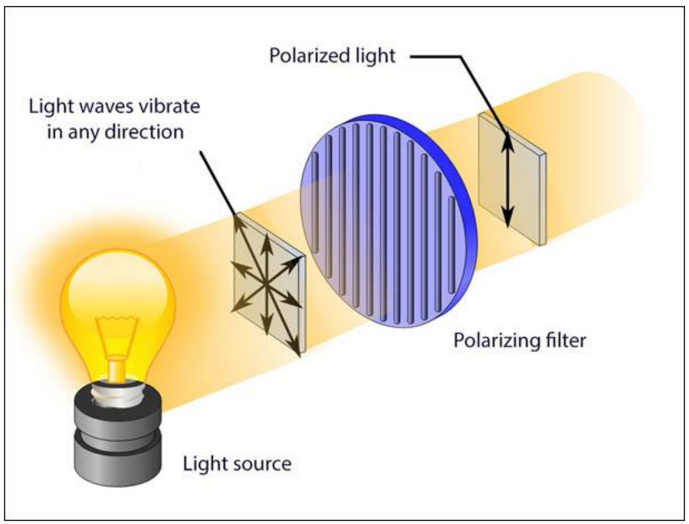


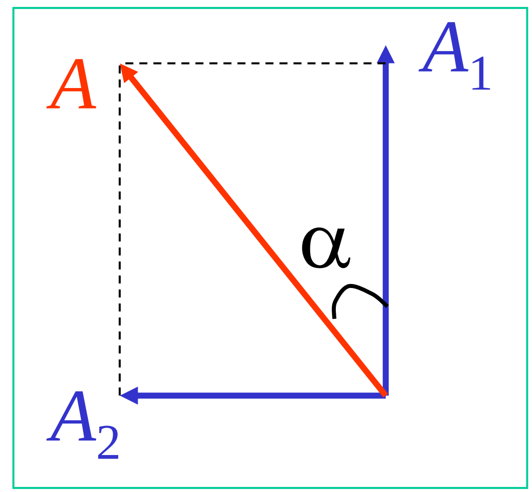
②效果：A linearly polarized light whose vibration direction is the same as the polarization direction is obtained

Natural light incidence $I_0 \xrightarrow{\text{polarizer}}$ $\boldsymbol{I = \frac{1}{2} I_0}$

Linearly polarized light incidence $I_0 \xrightarrow{\text{polarizer}}$ $\boldsymbol{I = I_0 \cos^2 \alpha}$

α：**The Angle between the direction of light vibration and the polarization direction of the polarizer**





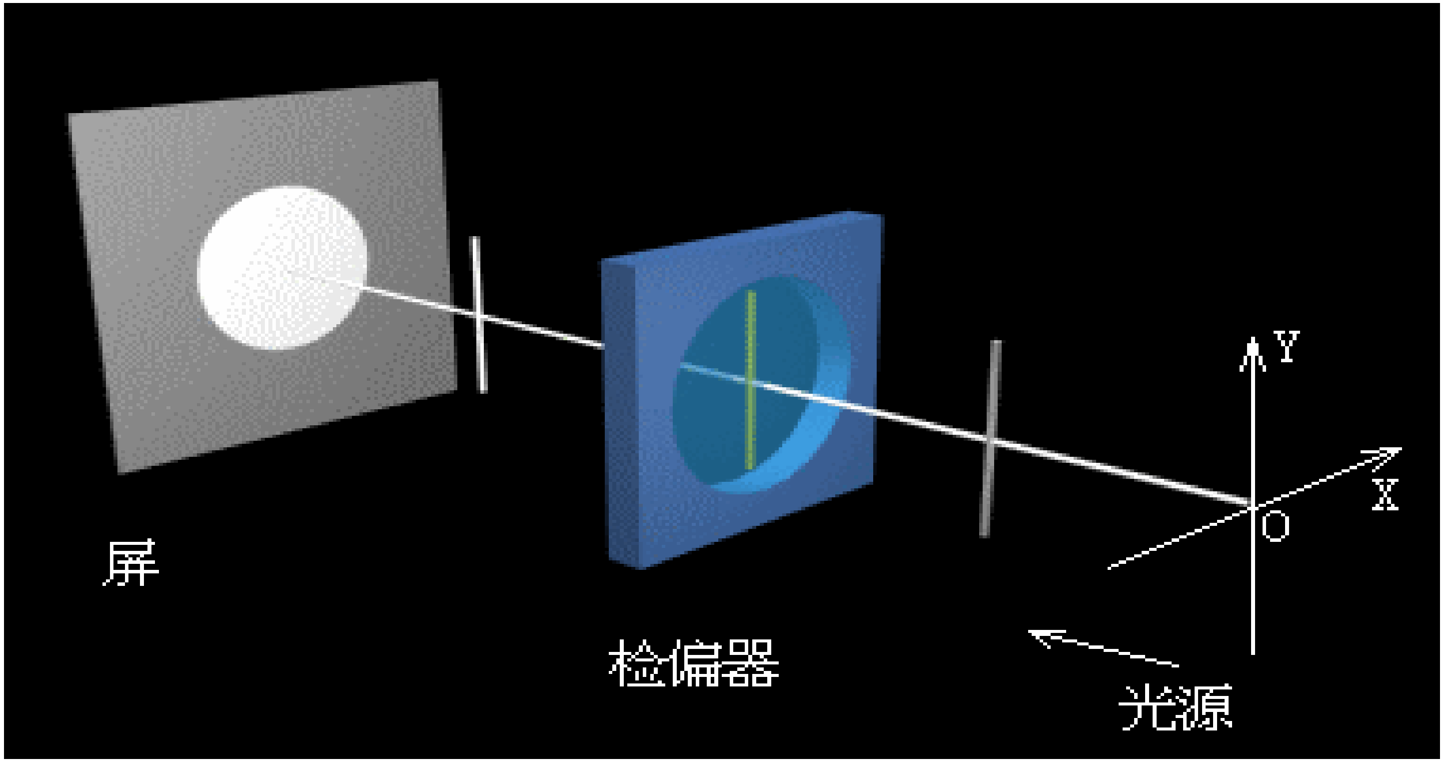
$$A_1 = A\cos\alpha$$

Intensity variation: **Malus law:** $\dfrac{I}{I_0} = \dfrac{A_1^2}{A^2} = \cos^2\alpha$

部分偏振光入射：自然光与线偏振光叠加

24-10 Polarization

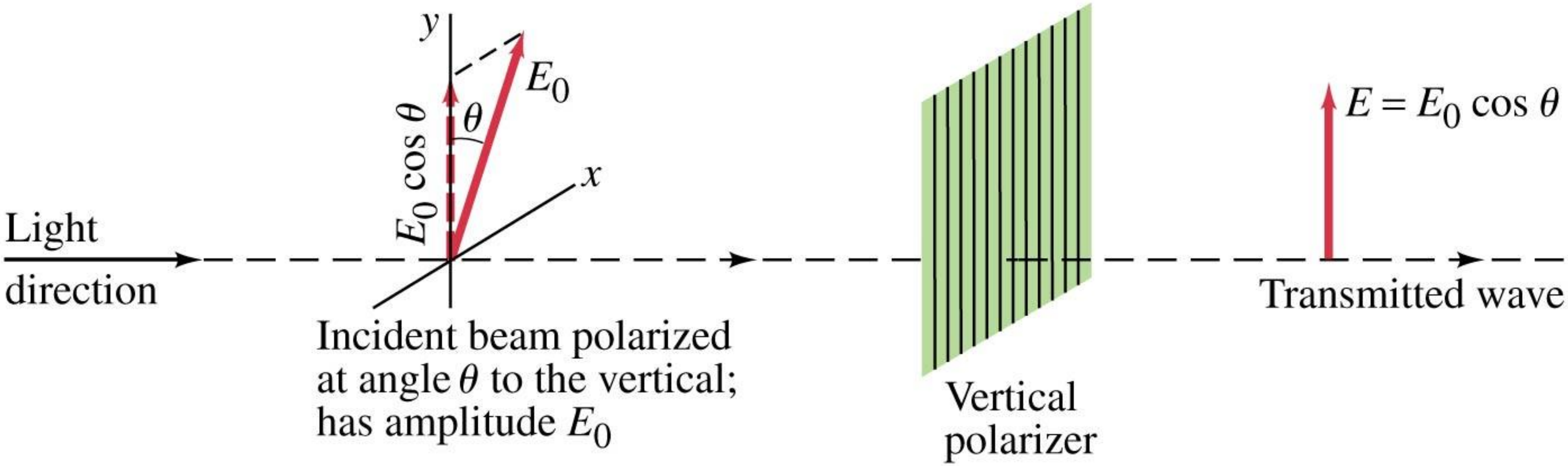
Polarized light will not be transmitted through a polarized film whose polarization direction is perpendicular to the polarization direction.



24-10 Polarization

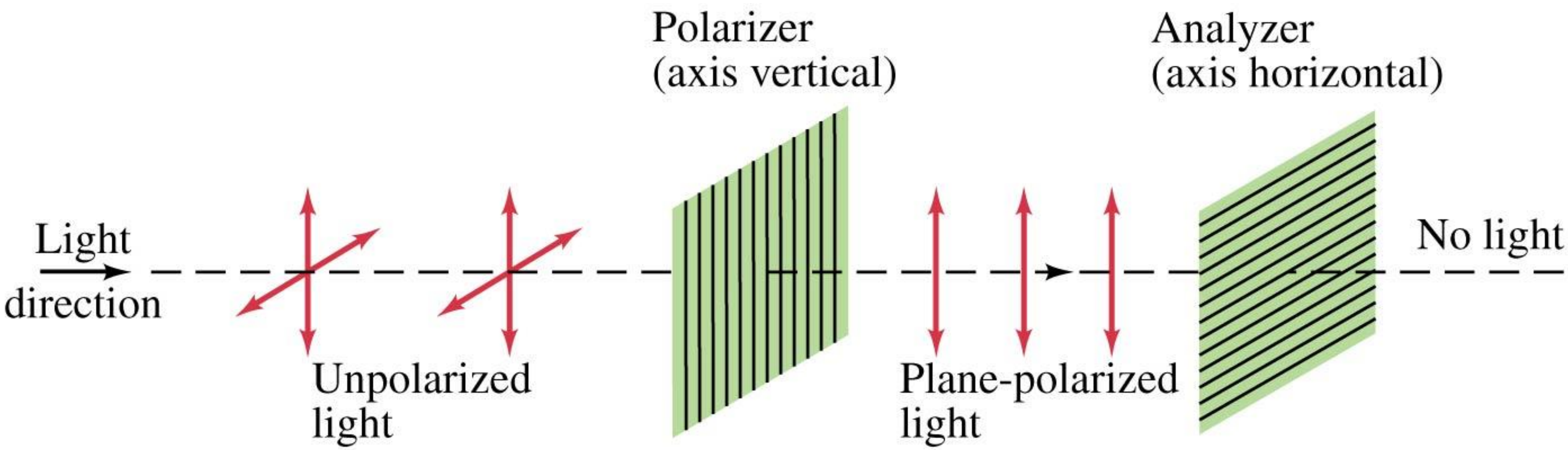
When light passes through a polarizer, only the component parallel to the polarization axis is transmitted. If the incoming light is plane-polarized, the outgoing intensity is:

$$I = I_0 \cos^2 \theta \qquad (24\text{-}5)$$



24-10 Polarization

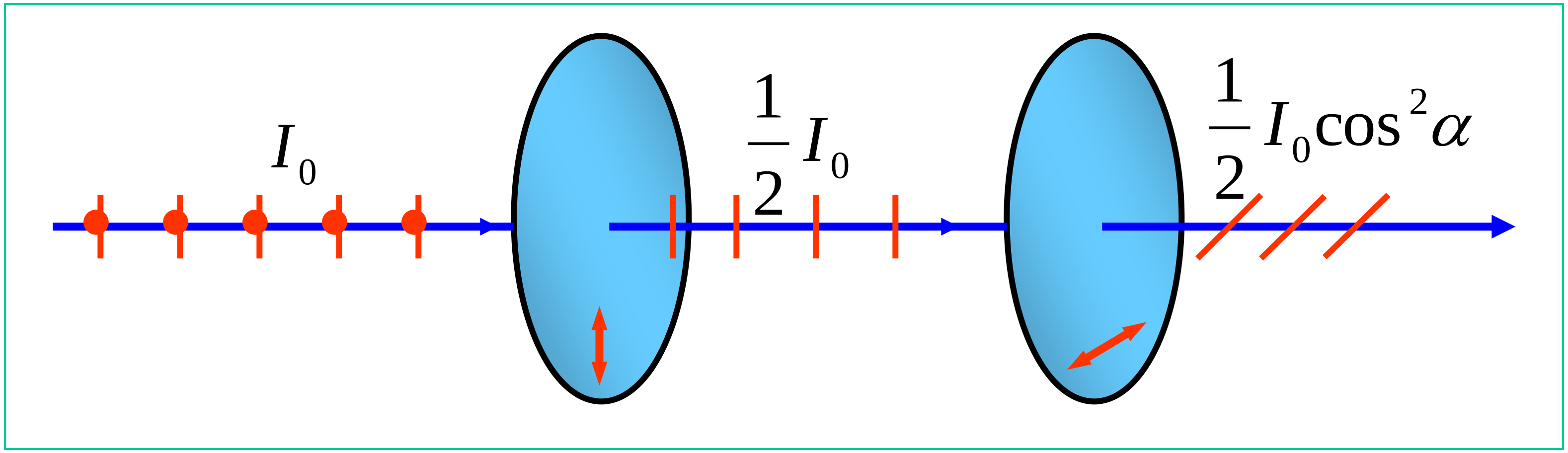
This means that if initially unpolarized light passes through crossed polarizers, no light will get through the second one.



练习

1) 一束光强为I_0的自然光通过两个偏振化方向成60°的偏振片后，光强为

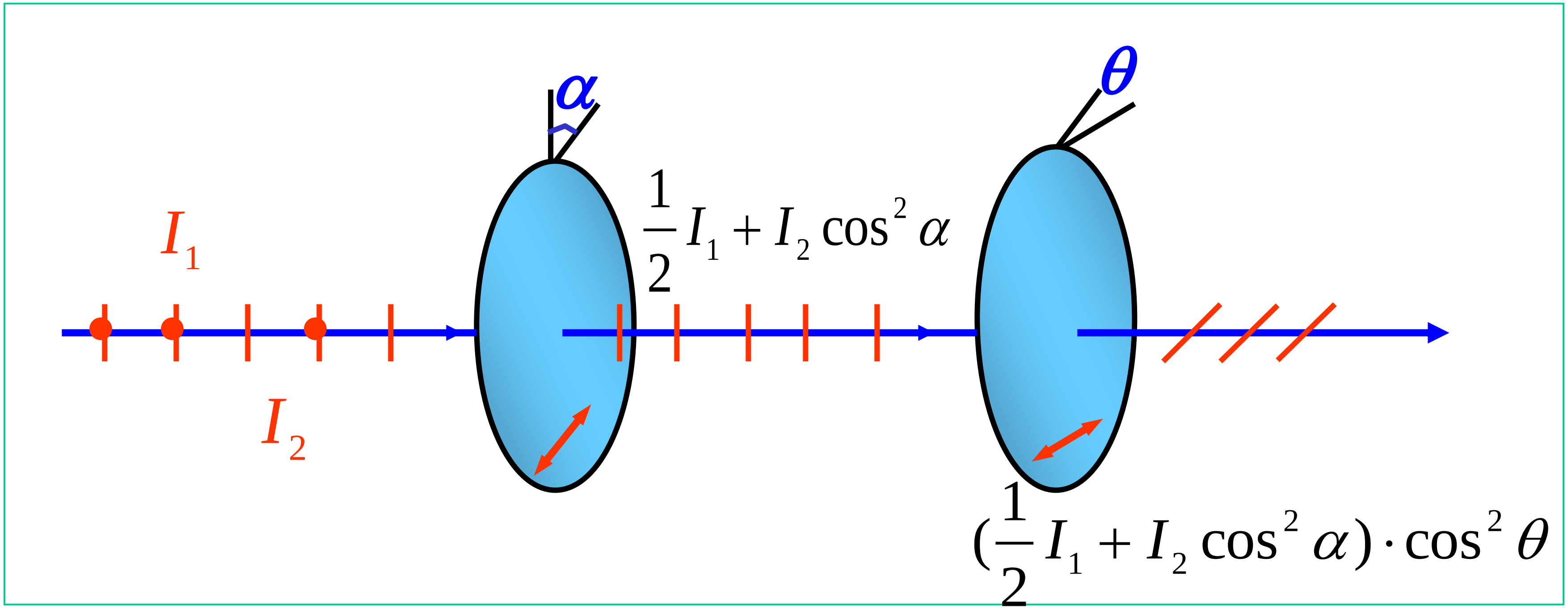
① $\frac{1}{2}I_0$　② $\frac{1}{4}I_0$　③ $\frac{1}{8}I_0$　④ $\frac{1}{16}I_0$



$$\frac{1}{2}I_0\cos^2\alpha = \frac{1}{2}I_0\cdot\cos^2 60^\circ = \frac{1}{8}I_0$$

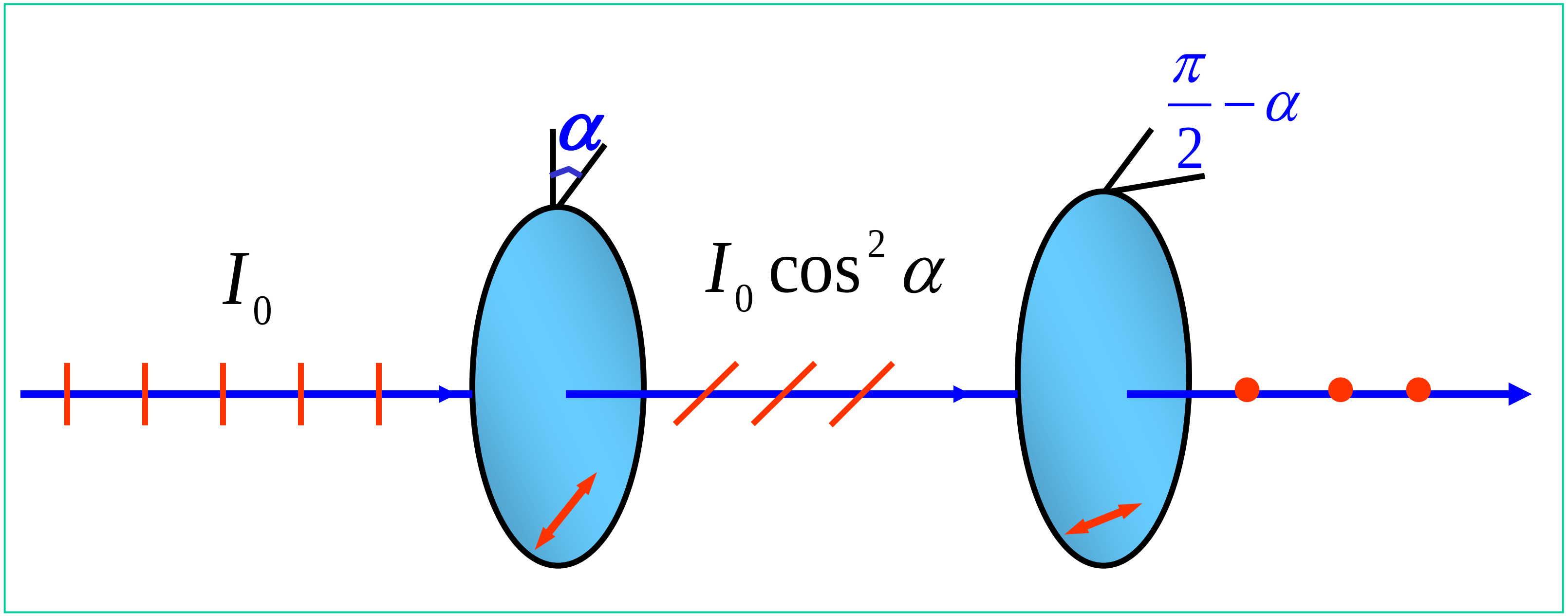
2）一束部分偏振光可视为由强度 I_1 的自然光和强度 I_2 的线偏振光组成，

让它连续通过偏振片 P_1、P_2，求出射光强。



3) 要让一束线偏振光的振动方向旋转90°至少要几块偏振片？如何放置？

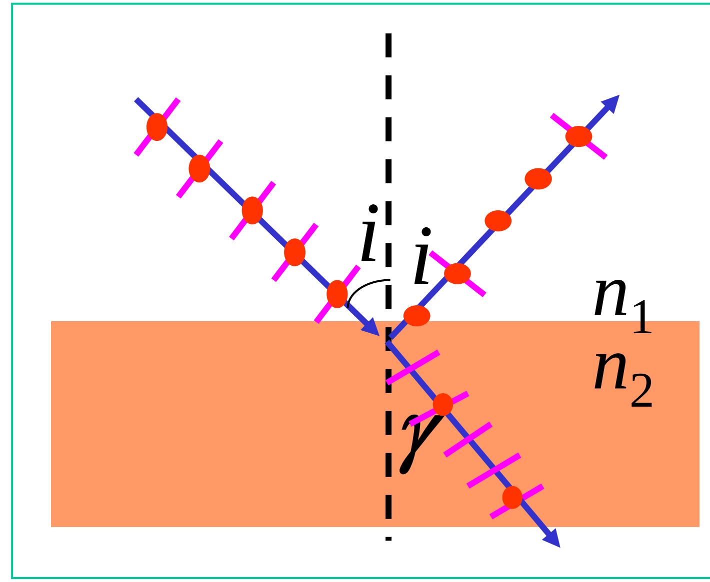
至少两块偏振片，如图放置：



$$I_0\cos^2\alpha\cdot\cos^2(\frac{\pi}{2}-\alpha) = I_0\cos^2\alpha\cdot\sin^2\alpha = \frac{1}{4}I_0\sin^2 2\alpha$$

当 $\alpha=45^\circ$ 时，出射光强最大：$\frac{1}{4}I_0$

三. Polarization of reflection and refraction Brewster's theory



Fresnel formula:

$$\frac{A'_{p1}}{A_{p1}}=\frac{tg(i-\gamma)}{tg(i+\gamma)} \qquad \frac{A'_{s1}}{A_{s1}}=-\frac{\sin(i-\gamma)}{\sin(i+\gamma)}$$

$$\frac{A_{p2}}{A_{p1}}=\frac{2\sin\gamma\cos i}{\sin(i+\gamma)\cos(i-\gamma)} \qquad \frac{A_{s2}}{A_{s1}}=\frac{2\sin\gamma\cos i}{\sin(i+\gamma)}$$

$$\frac{A'_{p1}}{A'_{s1}}=\frac{A_{p1}}{A_{s1}}\frac{\cos(i+\gamma)}{\cos(i-\gamma)} \qquad \frac{A'_{p1}}{A'_{s1}}<\frac{A_{p1}}{A_{s1}}$$

$$\frac{A_{p2}}{A_{s2}}=\frac{A_{p1}}{A_{s1}}\frac{1}{\cos(i-\gamma)} \qquad \frac{A_{p2}}{A_{s2}}>\frac{A_{p1}}{A_{s1}}$$

自然光入射一般情况下得部分偏振光

$\left\{\begin{array}{l}\text{反射光 } /\!/ < \perp \\ \text{折射光 } /\!/ > \perp\end{array}\right.$

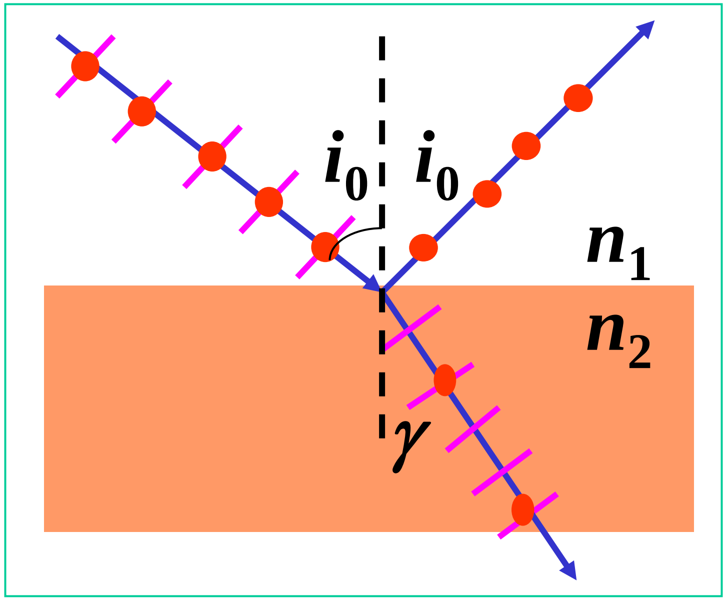
1. Polarization of reflection and refraction

$$\frac{A'_{p1}}{A'_{s1}} = \frac{A_{p1}}{A_{s1}} \frac{\cos(i+\gamma)}{\cos(i-\gamma)}$$

$$\frac{A_{p2}}{A_{s2}} = \frac{A_{p1}}{A_{s1}} \frac{1}{\cos(i-\gamma)}$$

$$i_0 + \gamma = \frac{\pi}{2}$$

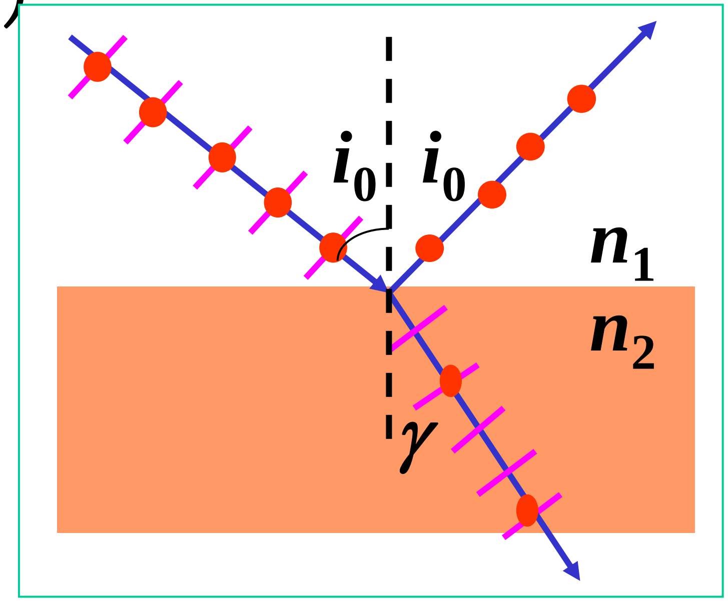
The reflected ray is perpendicular to the refracted ray



反射光不含平行分量，只含垂直分量，成为线偏振光

折射光既含平行分量，又含垂直分量，为部分偏振光，平行分量占优。

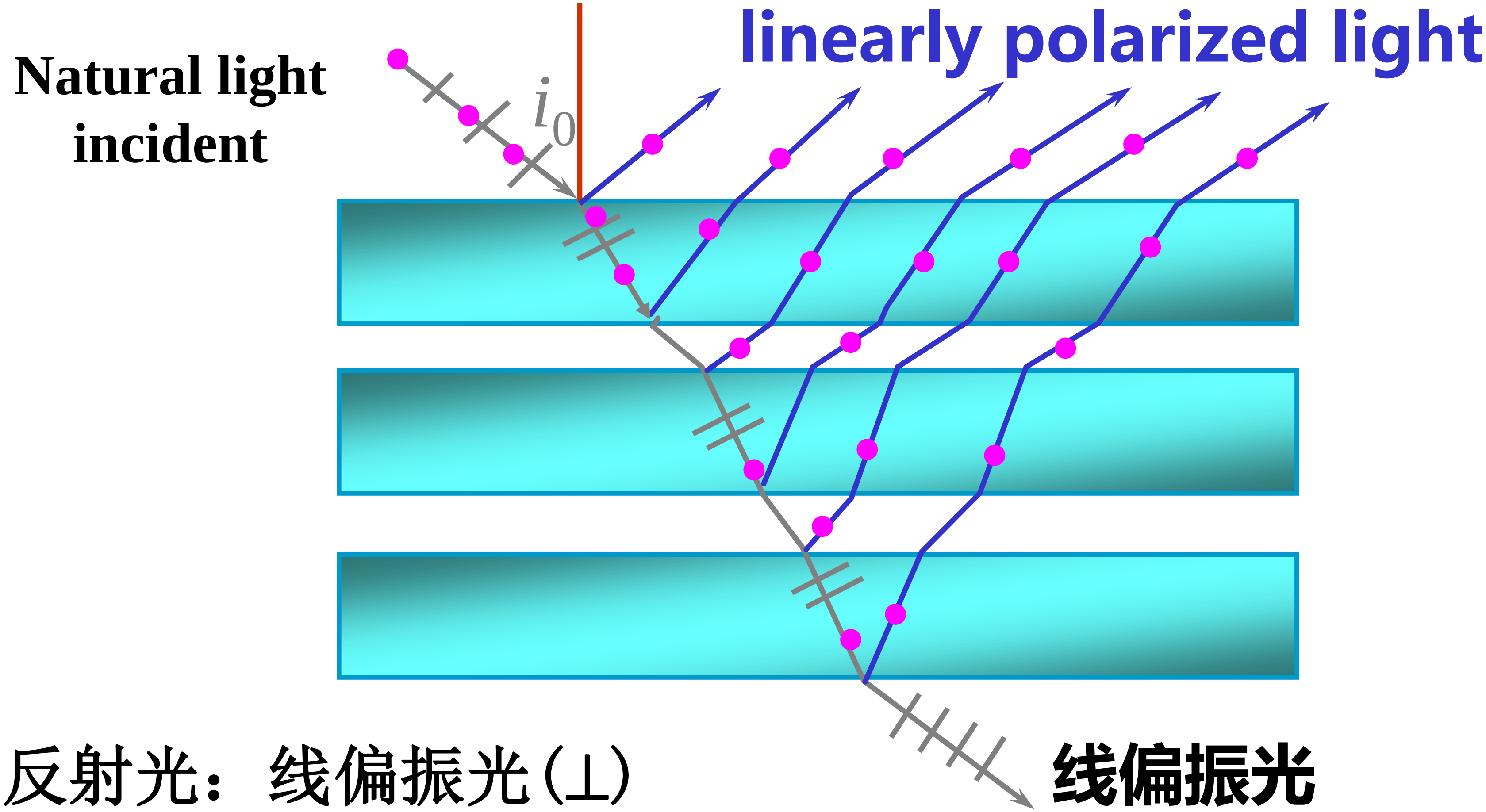
i_0 Brewster angle（起偏振角）



$$\left.\begin{aligned} \frac{\sin i_0}{\sin \gamma} = \frac{n_2}{n_1} \\ i_0 + \gamma = \frac{\pi}{2} \end{aligned}\right\} \tan i_0 = \frac{n_2}{n_1}$$

Brewster law

When light incidents at $i = i_0$ into glass stack:



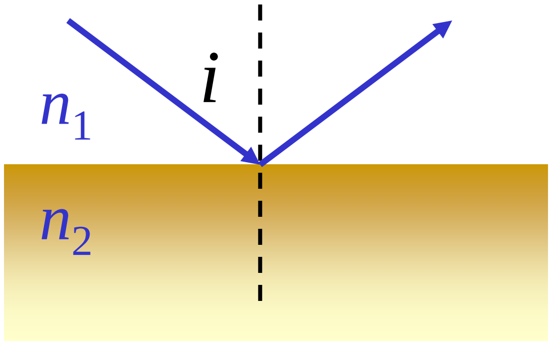
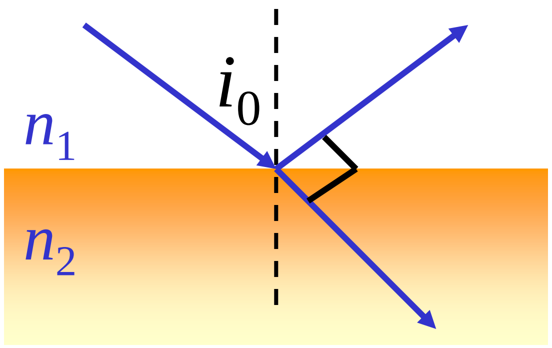
反射光：线偏振光(⊥)

折射光：近似线偏振光(∥)

光强可达 $\frac{1}{2}I$

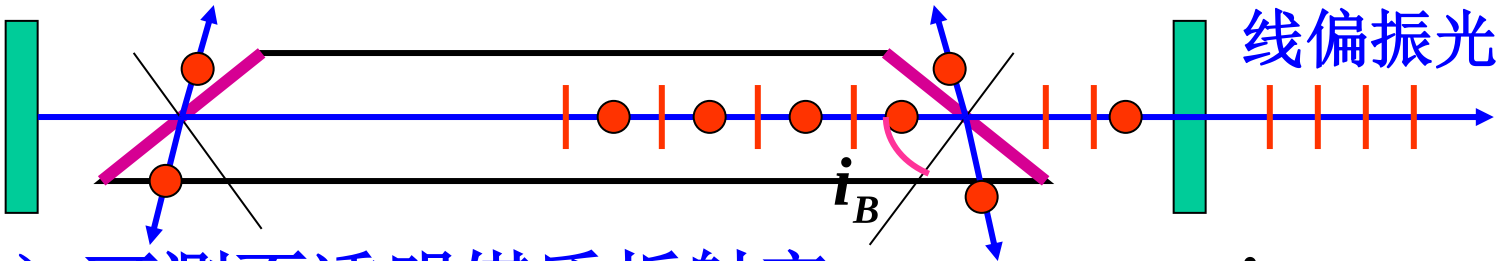
（垂直分量一次次被反射掉）

试比较起偏角与全反射临界角

	条件	关系式	现　象
全反射	光密→光疏 $i \geq i_0$	$\sin i_0 = \dfrac{n_2}{n_1}$	 无折射线
起偏振	光密↔光疏 $i = i_0$	$\tan i_0 = \dfrac{n_2}{n_1}$	 折射线与反射线垂直

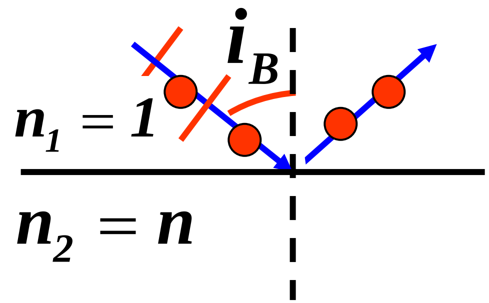
应用：（1）可由反射获得线偏振光（玻璃片就是起偏器）

例如激光器中的布儒斯特窗

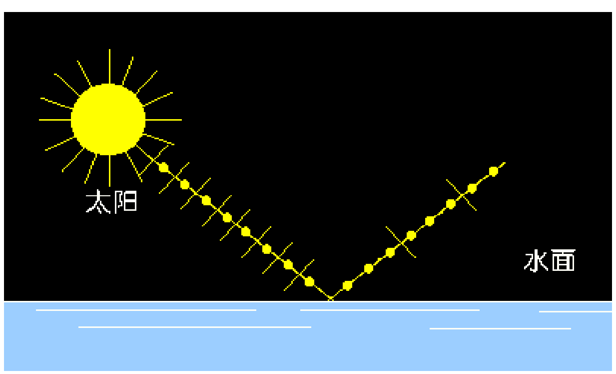


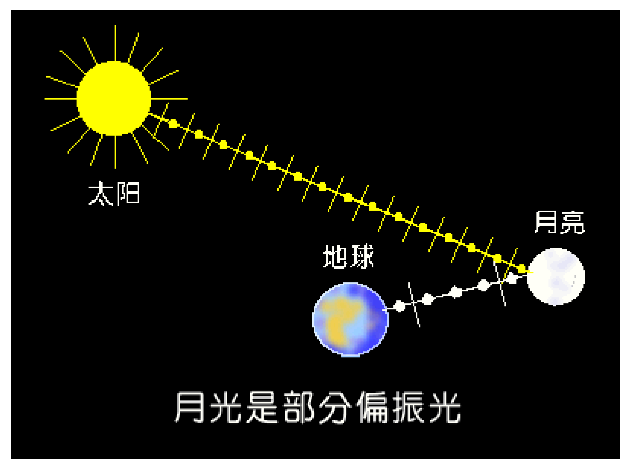
（2）可测不透明媒质折射率

$$tg\, i_B = n$$



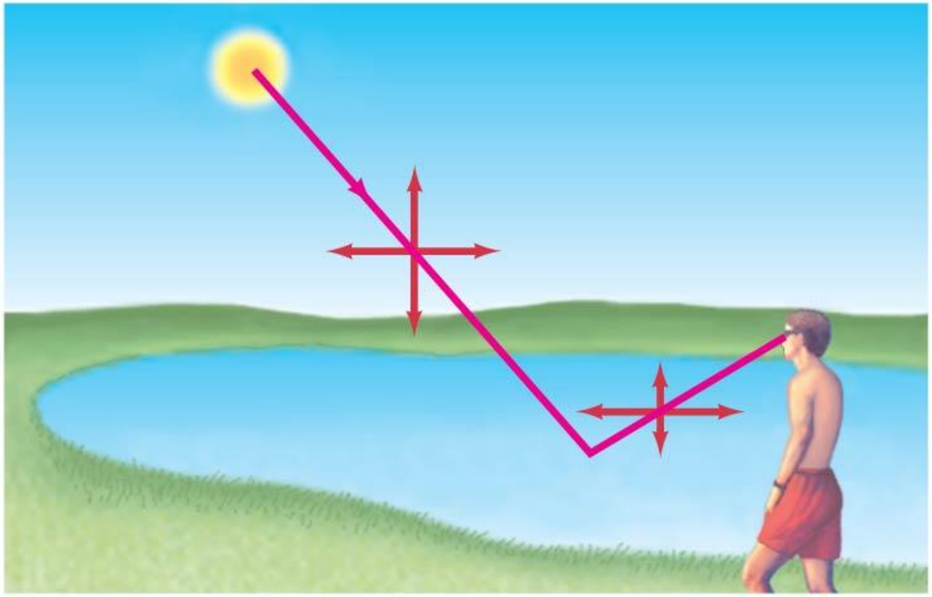
（3）反射光是部分偏振光，利用偏振片可消去大部分反射光（如镜头前加偏振片、偏光望远镜等）。





24-10 Polarization

Light is also partially polarized after reflecting from a nonmetallic surface. At a special angle, called the polarizing angle or Brewster's angle, the polarization is 100%.



$$\tan \theta_p = \frac{n_2}{n_1} \qquad (24\text{-}6a)$$

24-11 Liquid Crystal Displays (LCD)

Liquid crystals are unpolarized in the absence of an external voltage, and will easily transmit light. When an external voltage is applied, the crystals become polarized and no longer transmit; they appear dark.