ENSC 2113
Engineering Mechanics: Statics

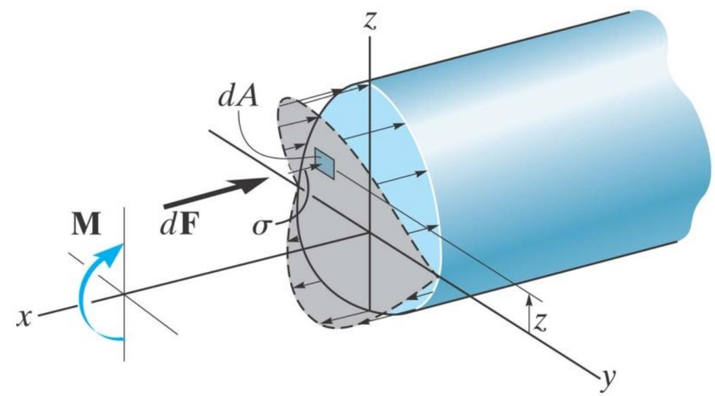
Lecture 29
Section 10.1-10.3

College of Engineering, Architecture & Technology

10.1: Moment of Inertia

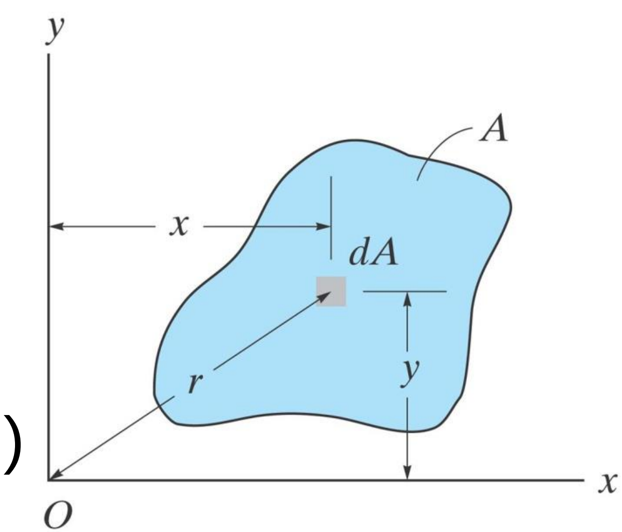
The moment of inertia of a shape is defined as *the integral of the second moment of an area*.

Moment of inertia taken about orthogonal axes:



$$I_x = \int_A y^2 \, dA$$

$$I_y = \int_A x^2 \, dA$$

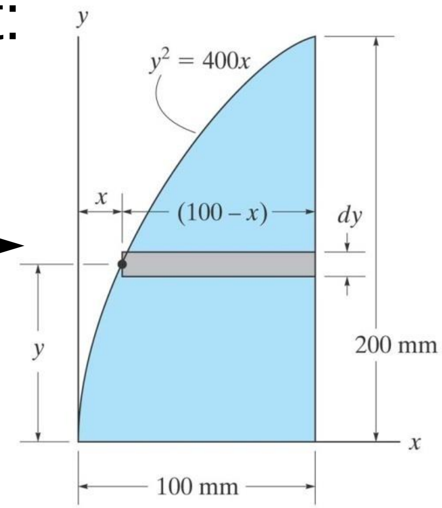


Moment of Inertia has units to the fourth power (in^4, mm^4, etc.)

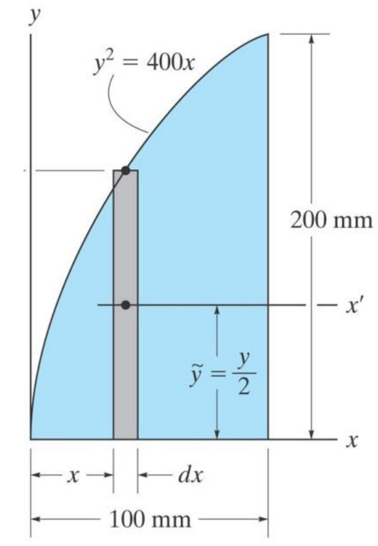
The process involves cutting a differential slice through the shape and integrating the distance squared across the area.

<u>For this method to work</u>, you need to cut the slice in the direction ***<u>parallel</u>*** to the axis of interest:

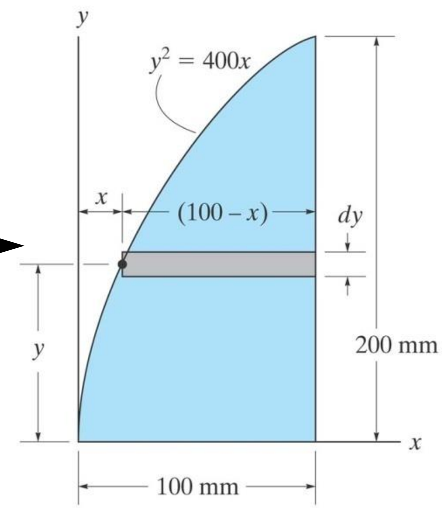
$$\boxed{I_x = \int_A y^2 \, dA}$$



$$\boxed{I_y = \int_A x^2 \, dA}$$



$$I_x = \int_A y^2\, dA$$



where,

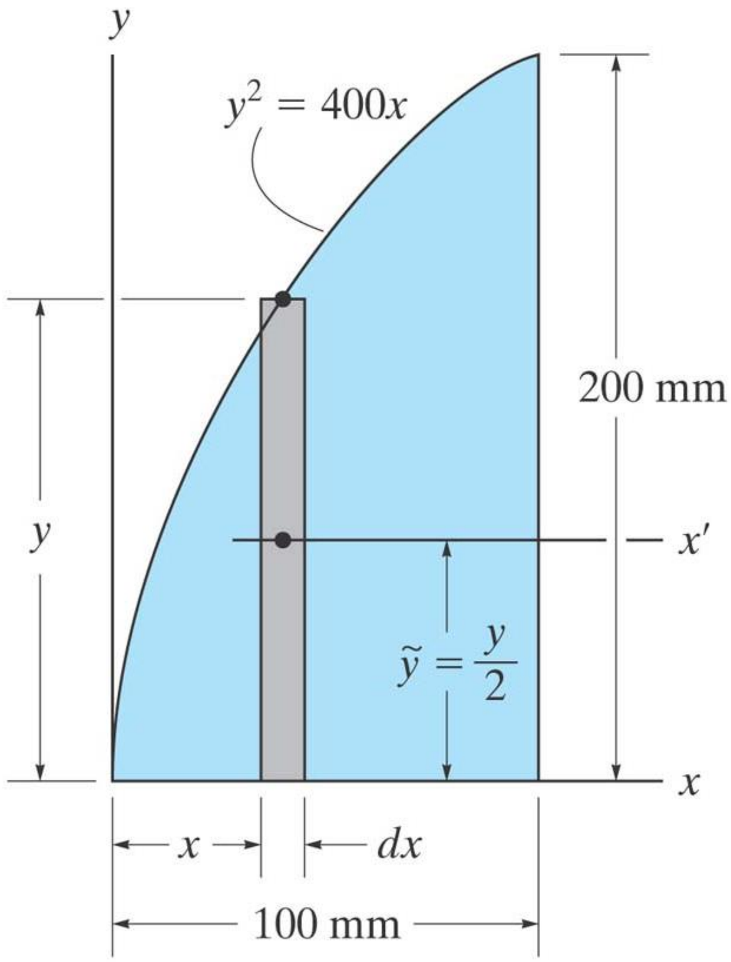
I_x or I_y = Moment of Inertia of shape about the *x* or *y* axis

x or y = Distance from *x* or *y* axis to centroid of slice

dA = Differential area of slice

If you choose to cut the differential slice perpendicular to the axis of interest, then you must use:

The Parallel-Axis Theorem



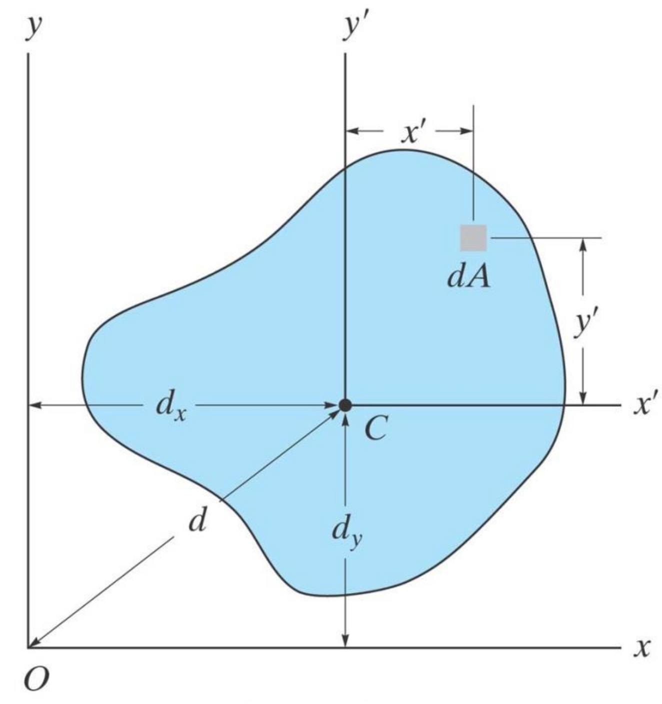
10.2: Parallel-Axis Theorem

If the *moment of inertia* is known through its centroidal axes, then it can also be found about any other axis parallel to the centroidal axis.

The eqns for this are (text eqns 10-3 & 10-4):

$$\boxed{I_x = \bar{I}_{x'} + Ad_y^2}$$

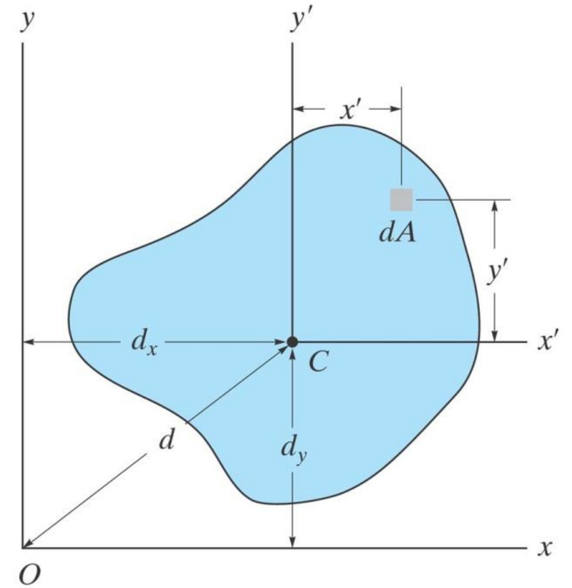
$$\boxed{I_y = \bar{I}_{y'} + Ad_x^2}$$



Definitions of the values in the eqns:

$$\boxed{I_x = \bar{I}_{x'} + Ad_y^2}$$

$$\boxed{I_y = \bar{I}_{y'} + Ad_x^2}$$



$\boldsymbol{I_x}$, $\boldsymbol{I_y}$ = *moment of inertia* about the *x* or *y* axis

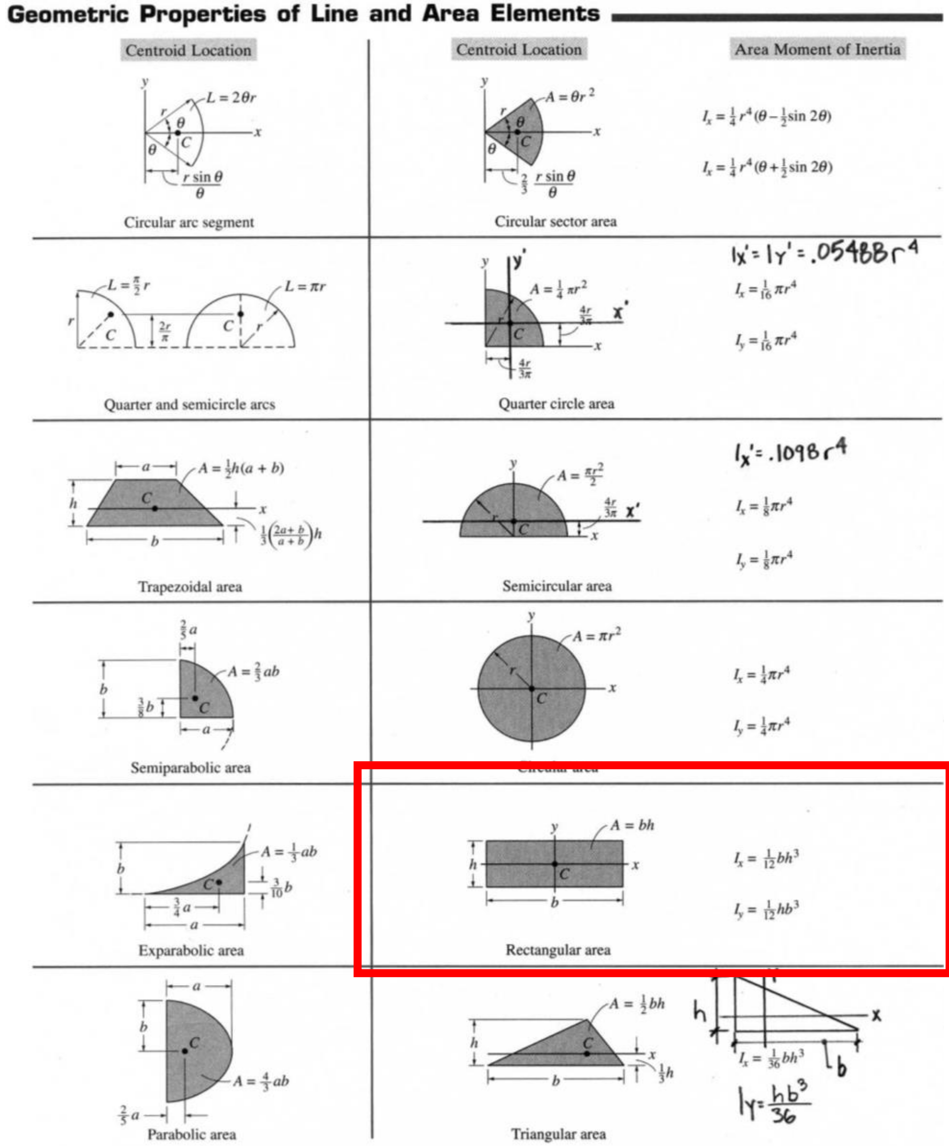
$\boldsymbol{\bar{I}_{x'}}$, $\boldsymbol{\bar{I}_{y'}}$ = *moment of inertia* about the centroid of the shape

A = Area of the shape

$\boldsymbol{d_x}$, $\boldsymbol{d_y}$ = distance from centroid of shape to axis of interest

Moment of Inertia about the centroid of a shape, $\bar{I}_{x'}$, $\bar{I}_{y'}$:

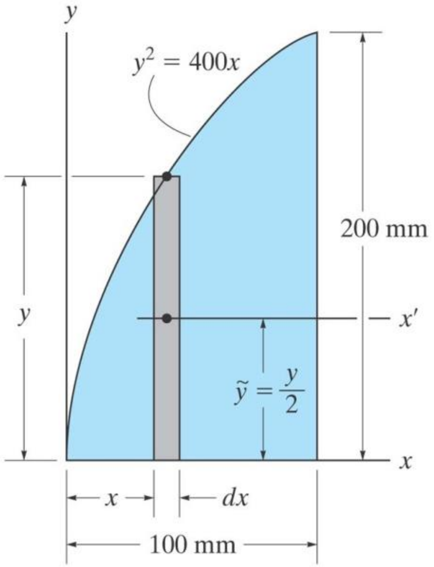
Refer to the inside cover of the back of your text for eqns ...



For use in *Parallel Axis Theorem*

The eqn using the *Parallel-Axis* theorem becomes:

$$I_x = \int dI_x$$



$$\boxed{I_x = \bar{I}_{x'} + Ad_y^2}$$

where,

$$dI_x = d\bar{I}_{x'} + dA\tilde{y}^2$$

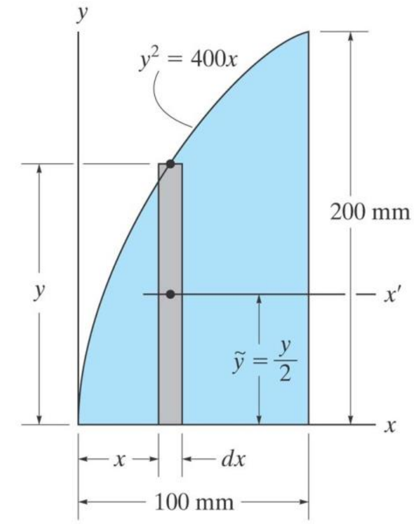
$d\bar{I}_{x'}$ = *moment of inertia of differential slice about its centroid*

dA = *area of differential slice*

$\tilde{y}$ = *distance from centroid of slice to axis of interest*

Procedures for determining *Moment of Inertia*:

1. Choose a differential slice to use. Its orientation will affect the integration equation used:
   a) If differential slice is taken parallel to the axis, use the *basic* eqn.
   b) If the slice is taken perpendicular to the axis, use the *Parallel-Axis Theorem* eqn.



2. Define slice size & moment arm to use. Draw these on the sketch for reference.
3. Perform the integrations & apply eqns previously derived. *Integrate in the direction perpendicular to the slice.*

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