Linear Algebra Homework #5
Fall 2020

Name: Due: Wednesday, Dec. 16th, 2020

Instructions:

Please include essential steps in your solution. For most of the problems, answers without essential steps may receive a score of 0.

1. determine whether the given set and operations define a vector space. If not, indicate which laws fail.
   (a) $V = \left\{ \begin{bmatrix} a & b \\ 0 & a+b \end{bmatrix} : \textit{ where } a,\ b \in R \right\}$ with standard matrix addition and scalar multiplication.
   (b) $V = \left\{ \begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix} : \textit{ where } a \in R \right\}$ with standard matrix addition and scalar multiplication.
   (c) V consists of all quadratic polynomial functions $f(x) = ax^2 + bx + c, a \neq 0$ with the standard function addition and scalar multiplication.
2. Determine which of the these formulas for $T:\ R^3 \to R^2$ is a linear operator. If so, write the operator as a matrix multiplication. Here $\vec{x} = (x, y, z)$ and $T(x)$ follows
   (a) $(x,\ x + 2y - 4z)$.
   (b) $(x + y, xy)$.
3. Let $V = C[0, 1]$ and define an operator $T : V \to V$ by the following formulas for $T(f)$ as a function of the variable x. Which of these operators is linear? If so, is the range $Range(V)$ of the operator equal to V?
   (a) $f(1)x^2$
   (b) $\int_0^x f(s)ds$.
4. Use the definition of vector space to prove the vector law of arithmetic (2): $c\vec{0} = \vec{0}$.
5. Determine whether the subset W is a subspace of the vector space V
   (a) $V = R^3$ and $W = \{(a, b, a - b + 1) | a, b \in R\}$.

(b) $V = R^3$ and $W = \{(a, b, c)|2a - b + c = 0\}$

(c) $V = R^{2,2}$ and W is the set of all matrices $A = \begin{bmatrix} a & b \\ -b & c \end{bmatrix}$, for some scalars $a,\ b,\ c$.

(d) $V = R^{n,n}$ and W is the set of all invertible matrices in V.