Fire Dynamics
Fire plume I

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Objective

- Understanding the fire plume structure
- Understanding the definition of flame height

Fire plume structure

- Fire plume structure

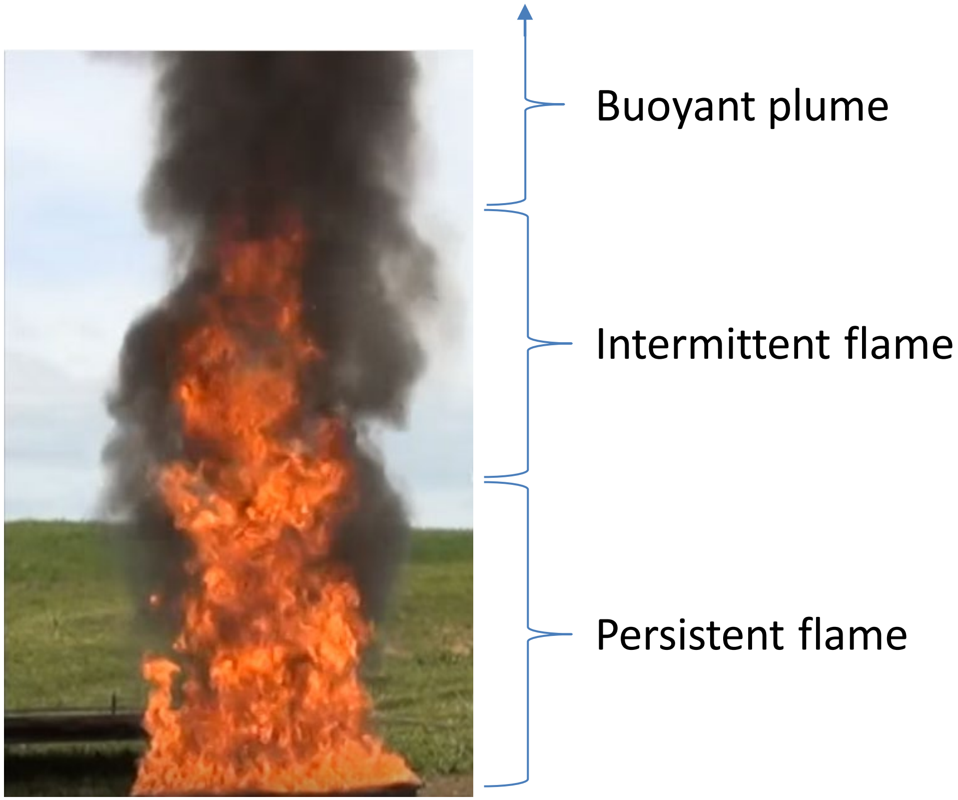
Far field or buoyant plume

Flame height (50% intermittency)

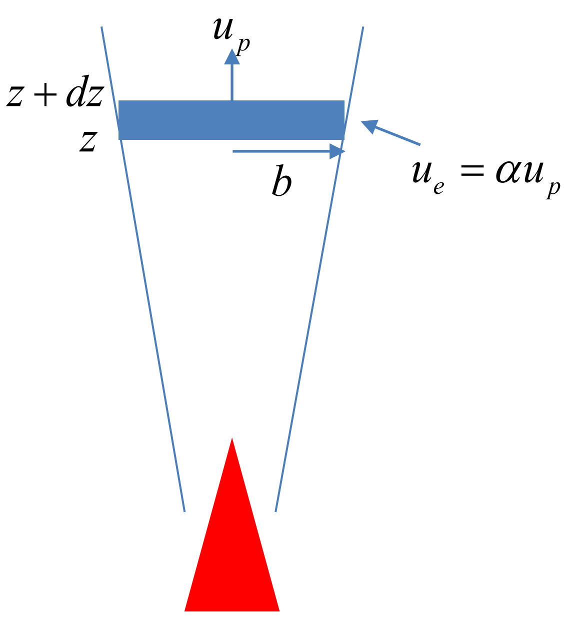
Near field or combusting plume

Fire plume structure

- Fire plume structure



Ideal fire plume



Mass conservation:

- For entrained air,

$$\dot{m}_e = \dot{m}_e(z+dz) - \dot{m}_e(z)$$

$$= \rho_a \dot{V} = \rho_a A u_e = \rho_a (2\pi b)(dz)(\alpha u_p)$$

$$\frac{\dot{m}_e(z+dz) - \dot{m}_e(z)}{dz} = \frac{d\dot{m}_e}{dz} = \rho_a (2\pi b)(\alpha u_p)$$

- For plume,

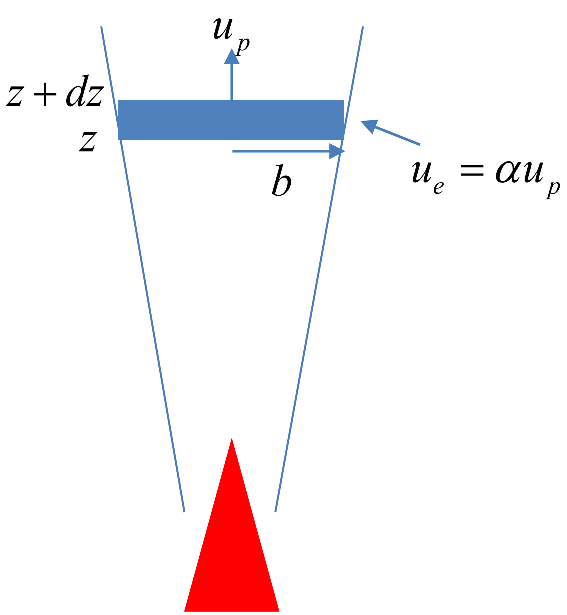
$$\dot{m}_p = \rho_p \dot{V} = \rho_p A u_p = \rho_p \pi b^2 u_p \approx \rho_a \pi b^2 u_p$$

- Equating these two equations,

$$\frac{d\dot{m}_p}{dz} = \frac{d}{dz}\left(\rho_a \pi b^2 u_p\right) = \rho_a (2\pi b)(\alpha u_p)$$

$$\frac{d}{dz}\left(b^2 u_p\right) = (2b)(\alpha u_p)$$

Ideal fire plume



Momentum conservation: $\frac{d}{dt}(m_p u_p) = \sum \mathrm{F}$

$$\frac{d}{dt}(m_p u_p) = \frac{dm_p}{dt}\mathrm{u}_p + m_p \frac{du_p}{dt} \approx \dot{m}_p u_p$$

$$F = PA = (\rho g h)A = (\rho_\infty - \rho_p)g(dz)(\pi b^2)$$

Equating,

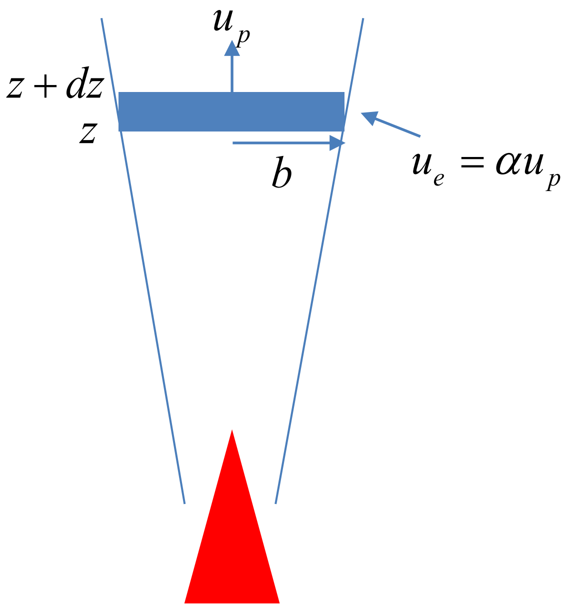
$$\frac{\dot{m}_p u_p(z+dz) - \dot{m}_p u_p(z)}{dz} = \frac{d(\dot{m}_p u_p)}{dz} = (\rho_\infty - \rho_p)g(\pi b^2)$$

$$\frac{d}{dz}(\dot{m}_p u_p) = \frac{d}{dz}(\rho_\infty(\pi b^2)u_p{}^2) = (\rho_\infty - \rho_p)g(\pi b^2)$$

Therefore,

$$\frac{d}{dz}(b^2 u_p{}^2) = \frac{(\rho_\infty - \rho_p)}{\rho_\infty} g b^2$$

Ideal fire plume



Energy conservation:

$$\frac{c_p\dot{m}_pT_p(z+dz)-c_p\dot{m}_pT_p(z)}{dz}=\frac{d(c_p\dot{m}_pT_p)}{dz}$$

$$=\frac{d(c_p(\rho_\infty(\pi b^2)u_p)T_p)}{dz}$$

Using mass conservation $\dot{m}_p \approx \rho_a \pi b^2 u_p$,

Therefore, $\dfrac{d(\dot{m}_pu_p)}{dz}=\dfrac{d(\rho_a\pi b^2u_p{}^2)}{dz}$

$$\frac{d(c_p\rho_\infty\pi b^2u_pT_p)}{dz}=c_p\left(\rho_\infty(2\pi b)(\alpha u_p)\right)T_\infty$$

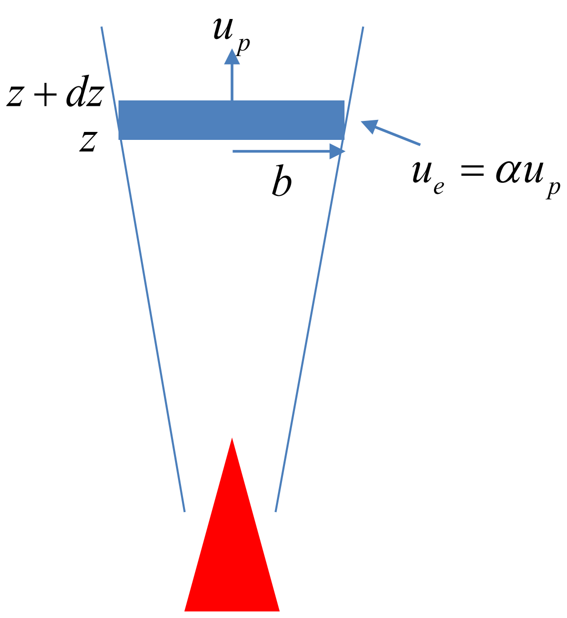
Rewriting this,

$$\frac{d}{dz}(c_p\rho_\infty\pi b^2u_p(T_p-T_\infty))=0$$

Integrating over z,

$$c_p\rho_\infty\pi b^2u_p(T_p-T_\infty)=\dot{Q}_c=\dot{Q}(1-X_r)$$

Ideal fire plume



Mass conservation:

$$\frac{d}{dz}\left(b^2 u_p\right) = (2b)(\alpha u_p)$$

Momentum conservation:

$$\frac{d}{dz}(b^2 u_p{}^2) = \frac{(\rho_\infty - \rho_p)}{\rho_\infty} g b^2$$

Energy conservation:

$$c_p \rho_\infty \pi b^2 u_p (T_p - T_\infty) = \dot{Q}_c$$

From, $$\frac{(\rho_\infty - \rho_p)}{\rho_\infty} = 1 - \frac{T_\infty}{T_p} = \frac{T_p - T_\infty}{T_p} = \frac{T_p - T_\infty}{T_\infty}\left(\frac{T_\infty}{T_p}\right) \approx \frac{T_p - T_\infty}{T_\infty}$$

$$\frac{d}{dz}(b^2 u_p{}^2) = \frac{(\rho_\infty - \rho_p)}{\rho_\infty} g b^2 = \frac{\dot{Q}_c g}{c_p \rho_\infty \pi b^2 u_p T_\infty}$$

Ideal fire plume

$$\frac{d}{dz}\left(b^2 u_p\right) = (2b)(\alpha u_p) \text{ and } \frac{d}{dz}(b^2 u_p{}^2) = \frac{(\rho_\infty - \rho_p)}{\rho_\infty} g b^2 = \frac{\dot{Q}_c g}{c_p \rho_\infty \pi b^2 u_p T_\infty}$$

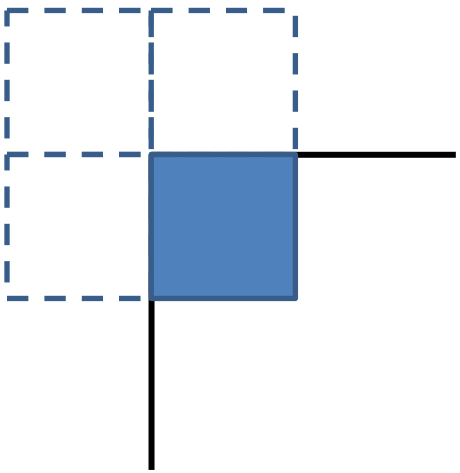
Assuming $b = C_1 z^m$ and $u_p = C_2 z^n$

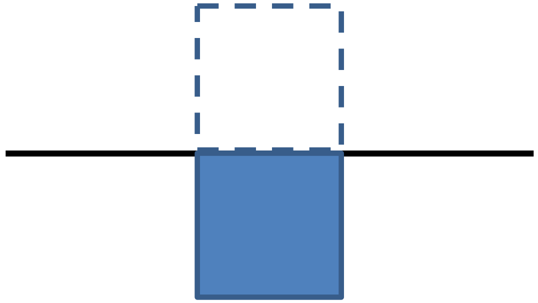
$$u_p = \left(\frac{25}{48\alpha^2} \frac{\dot{Q}_c g}{\pi c_p T_\infty \rho_\infty}\right)^{1/3} z^{-1/3} = 1.94\left(\frac{g}{\rho_\infty c_p T_\infty}\right)^{1/3} \dot{Q}_c{}^{1/3} z^{-1/3} \text{ with } \alpha \approx 0.15$$

$$\dot{m}_p = 0.2\left(\frac{\rho_\infty{}^2 g}{c_p T_\infty}\right)^{1/3} \dot{Q}_c{}^{1/3} z^{5/3}$$

$$\Delta T = 5.0\left(\frac{T_\infty}{g c_p{}^2 \rho_\infty{}^2}\right)^{1/3} \dot{Q}_c{}^{2/3} z^{-5/3}$$

Plume in the corner and against a wall





Using Zukoski plume ($\dot{m}_p = 0.071\dot{Q}^{1/3} z^{5/3}$),

– In the corner,

$$\dot{m}_{p,corner} \approx \frac{1}{4}\left(0.071(4\dot{Q})^{1/3} z^{5/3}\right)$$

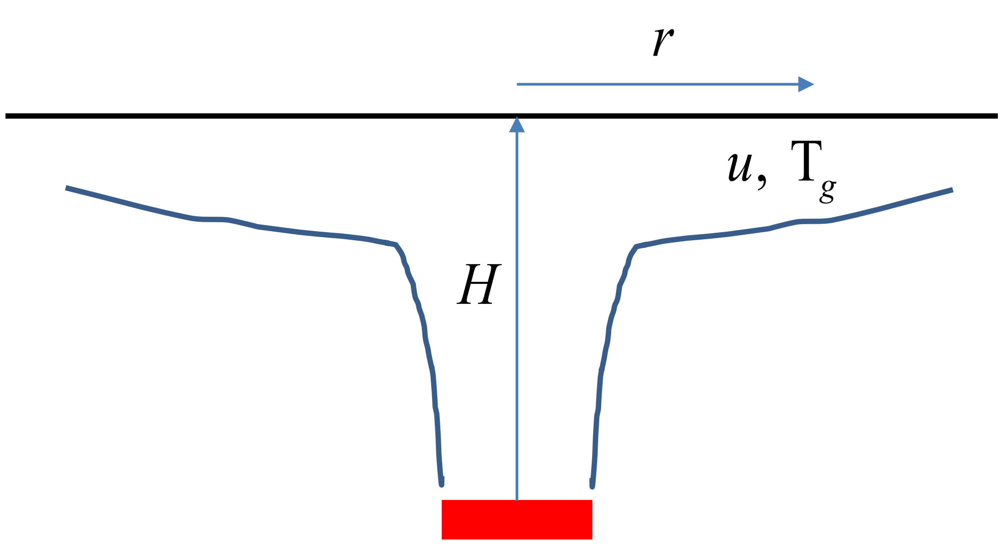
$$\approx 0.028\dot{Q}^{1/3} z^{5/3}$$

– Against the wall,

$$\dot{m}_{p,wall} \approx \frac{1}{2}\left(0.071(2\dot{Q})^{1/3} z^{5/3}\right)$$

$$\approx 0.045\dot{Q}^{1/3} z^{5/3}$$

S-S ceiling jet correlation



$T_g\ [°C], u\ [m/s], \dot{Q}\ [kW], r, and\ H\ [m]$

For r/H ≤ 0.18,

$$T_g - T_\infty = \frac{16.9\dot{Q}^{2/3}}{H^{5/3}}$$

For r/H > 0.18,

$$T_g - T_\infty = \frac{5.38(\dot{Q}/r)^{2/3}}{H}$$

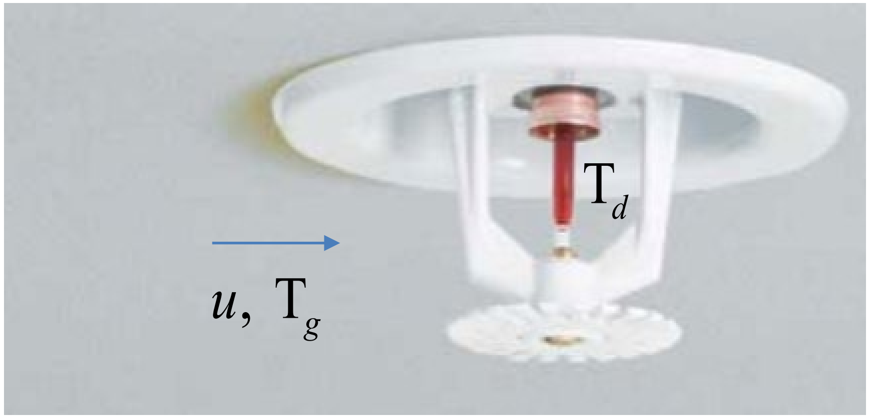
For r/H > 0.15,

$$u = \frac{0.20\dot{Q}^{1/3}H^{1/2}}{r^{5/6}}$$

For r/H ≤ 0.15,

$$u = 0.95\left(\frac{\dot{Q}}{H}\right)^{1/3}$$

Sprinkler activation time calc.



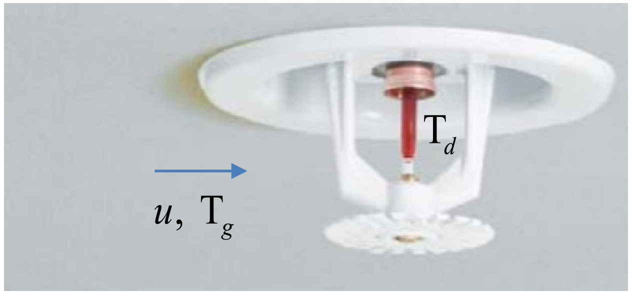
$$\frac{c_{p,d} m_d}{h_c A_d} = \tau$$

$$\Rightarrow h_c = \frac{c_{p,d} m_d}{\tau A_d}$$

$$h_c A_d (T_g - T_d) = c_{p,d} m_d \frac{dT_d}{dt}$$

$$\Rightarrow \frac{dT_d}{dt} = \frac{h_c A_d}{c_{p,d} m_d}(T_g - T_d) = \frac{(T_g - T_d)}{\left(\dfrac{c_{p,d} m_d}{h_c A_d}\right)} = \frac{(T_g - T_d)}{\tau}$$

Sprinkler activation time calc.



$$Nu = \frac{h_c D}{k} = C\,\mathrm{Re}_D^{\ 1/2}\,\mathrm{Pr}^{1/3}$$

$$= C\left(\frac{\rho_g u D}{\mu}\right)^{1/2}\mathrm{Pr}^{1/3}$$

$$\Rightarrow h_c = C\frac{k}{D^{0.5}}\left(\frac{\rho_g}{\mu}\right)^{1/2}\mathrm{Pr}^{1/3}\,u^{1/2}$$

$$\Rightarrow h_c = C\frac{k}{D^{0.5}}\left(\frac{\rho_g}{\mu}\right)^{1/2}\mathrm{Pr}^{1/3}\,u^{1/2}$$

Therefore, for a given detector,

$$\frac{c_{p,d}m_d}{\tau A_d} = C\frac{k}{D^{0.5}}\left(\frac{\rho_g}{\mu}\right)^{1/2}\mathrm{Pr}^{1/3}\,u^{1/2}$$

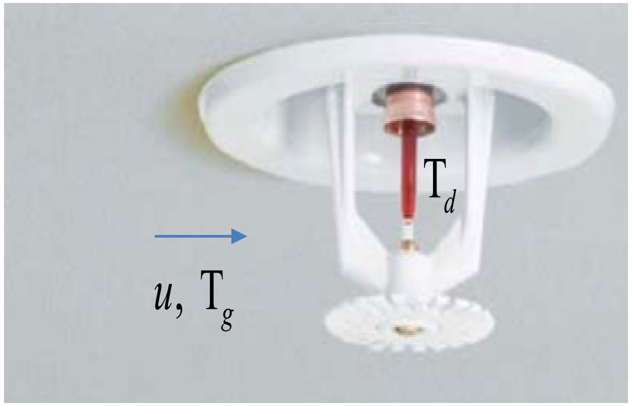
$$\Rightarrow \tau u^{1/2} = \frac{c_{p,d}m_d D^{0.5}}{A_d C k\,\mathrm{Pr}^{1/3}}\left(\frac{\mu}{\rho_g}\right)^{1/2} \approx const.$$

In a lab environment (plunge test) with a specific velocity, u

$$\tau u^{1/2} = \frac{c_{p,d}m_d D^{0.5}}{A_d C k\,\mathrm{Pr}^{1/3}}\left(\frac{\mu}{\rho_g}\right)^{1/2} \approx const = \tau_o u_o^{\ 1/2}$$

$$= RTI$$

Sprinkler activation time calc.



$$\frac{dT_d}{dt} = \frac{(T_g - T_d)}{\tau} = \frac{u^{1/2}(T_g - T_d)}{RTI}$$

$$\Rightarrow T_d - T_a = (T_g - T_a)\left[1 - \exp\left(\frac{-t_r u^{0.5}}{RTI}\right)\right]$$

or

$$\Rightarrow t_r = \frac{RTI}{u^{0.5}} \ln\left(\frac{T_g - T_a}{T_g - T_d}\right)$$

Example

How long does it take for a sprinkler head (RTI = 55 $m^{0.5}s^{0.5}$) to activate if it were 2.4 m away from the center of a 0.5 m^2 kerosene pool fire in a 3.3 m high compartment? Ambient Temp. = 20 ^oC. Activation Temperature of the sprinkler = 57 ^oC.