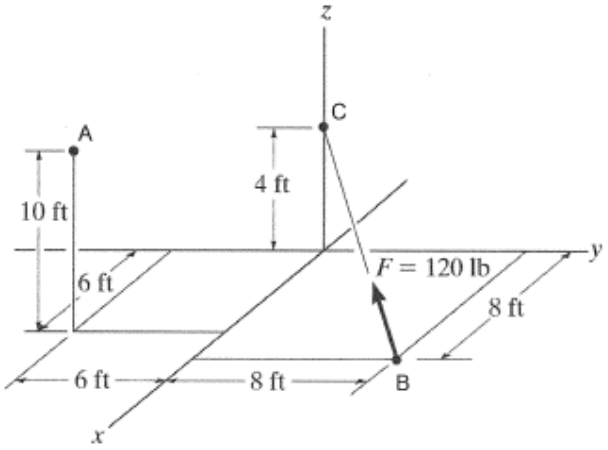
Name: _____ **CWID #:**_____ **Section Number:**_____

ENSC 2113 – SPRING 2016 – FINAL EXAM

EACH PROBLEM IS WORTH 25 PTS. BOX YOUR ANSWERS AND PROVIDE PROPER UNITS, WHERE APPLICABLE. CALCULATIONS AND FREE BODY DIAGRAMS MUST BE SHOWN THAT SUPPORT THE ANSWER TO RECEIVE CREDIT.

1) A force follows along a line from B to C. Determine the moment created by the force about point A. Express the result as a Cartesian vector.



COORDINATES:

$A(6, -6, 10)$

$B(8, 8, 0)$

$C(0, 0, 4)$

POSITION VECTORS:

$\bar{r}_{AC} = \{-6\hat{i} + 6\hat{j} - 6\hat{k}\}$

$\bar{r}_{AB} = \{2\hat{i} + 14\hat{j} - 10\hat{k}\}$

FORCE IN CARTESIAN FORM:

$\bar{F}_{BC} = |F|\,\bar{u}_{BC}$

$\bar{r}_{BC} = \{-8\hat{i} - 8\hat{j} + 4\hat{k}\}$

$|r_{BC}| = \sqrt{(-8)^2 + (-8)^2 + (4)^2}$

$= 12$

$\bar{F}_{BC} = 120\left\{-\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}\right\}$

$\bar{F}_{BC} = \{-80\hat{i} - 80\hat{j} + 40\hat{k}\}$

MOMENT:

$\bar{M} = \bar{r} \times \bar{F}$

$$\bar{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 6 & -6 \\ -80 & -80 & +40 \end{vmatrix}$$

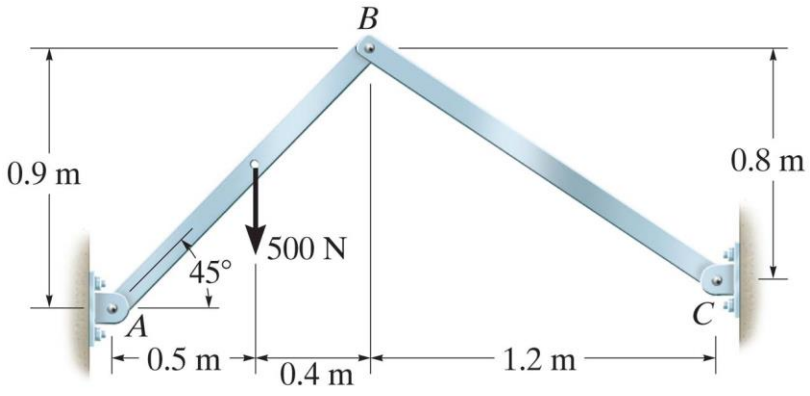
$\bar{M} = [6(40) - (-6)(-80)]\,\hat{i}$

$- [(-6)(40) - (-6)(-80)]\,\hat{j}$

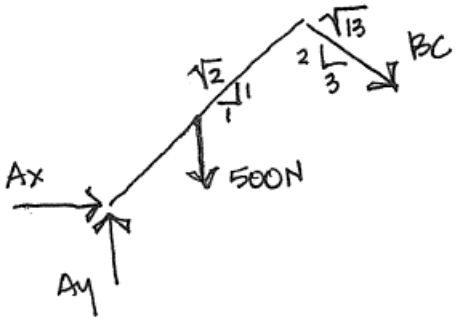
$+ [(-6)(-80) - (6)(-80)]\,\hat{k}$

$$\boxed{\bar{M} = \{-240\hat{i} + 720\hat{j} + 960\hat{k}\}\ \text{LB-FT}}$$

2) Determine the external support reactions for the pins at A and C. Member AB is pinned to member BC with an internal pin at point B. NOTE: A and C are not aligned vertically. Draw all pertinent free-body diagrams and indicate direction with arrows in your answer.



FBD AB:



$+\circlearrowleft \Sigma M_A = 0 = -500(0.5) - \frac{2}{\sqrt{13}}BC(0.9) - \frac{3}{\sqrt{13}}BC(0.9)$

$250 = -1.25\,BC$

$BC = -200$

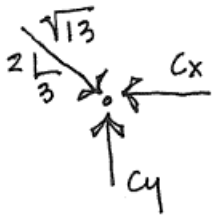
$\uparrow \Sigma F_y = 0 = A_y - 500 - \frac{2}{\sqrt{13}}(-200)$

$\boxed{A_y = 389\ N \uparrow}$

$\rightarrow \Sigma F_x = 0 = A_x + \frac{3}{\sqrt{13}}(-200)$

$\boxed{A_x = 166.4\ N \rightarrow}$

FBD @ C:



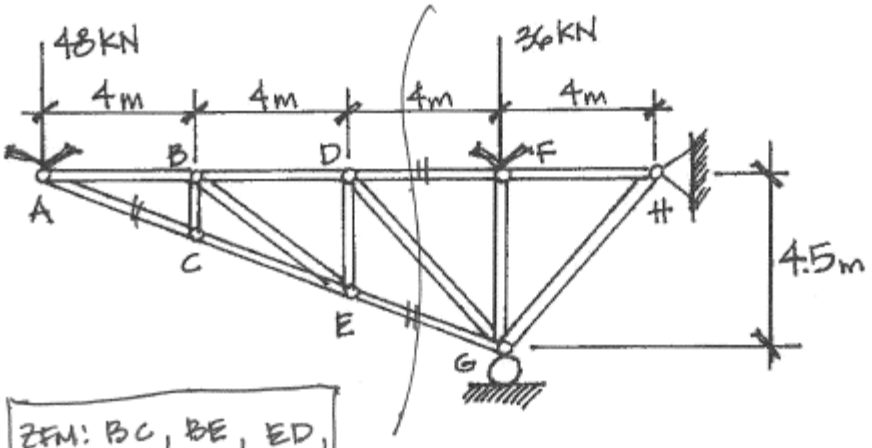
$\uparrow \Sigma F_y = 0 = \frac{-2}{\sqrt{13}}(200) + C_y$

$\boxed{C_y = 111\ N \uparrow}$

$\rightarrow \Sigma F_x = 0 = \frac{3}{\sqrt{13}}(200) - C_x$

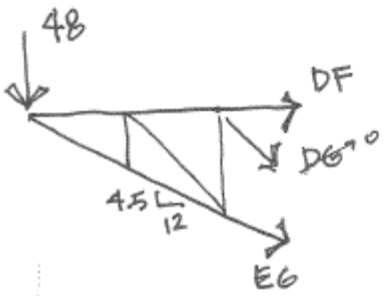
$\boxed{C_x = 166.4\ N \leftarrow}$

3) Determine the axial force in truss members DF, and EG by the method of sections and the force in member AC by method of joints. Indicate if the member is in tension or compression. List all zero force members and draw all pertinent free-body diagrams. Point H is a pin and point G is a roller.



ΣFM: BC, BE, ED, DG

FBD OF CUT:



$+\circlearrowleft \Sigma M_G = 0 = 48(12) - DF(4.5)$

$DF = 128\ kN\ (T)$

$+\uparrow \Sigma F_y = 0 = -48 - \frac{4.5}{\sqrt{164.25}} EG$

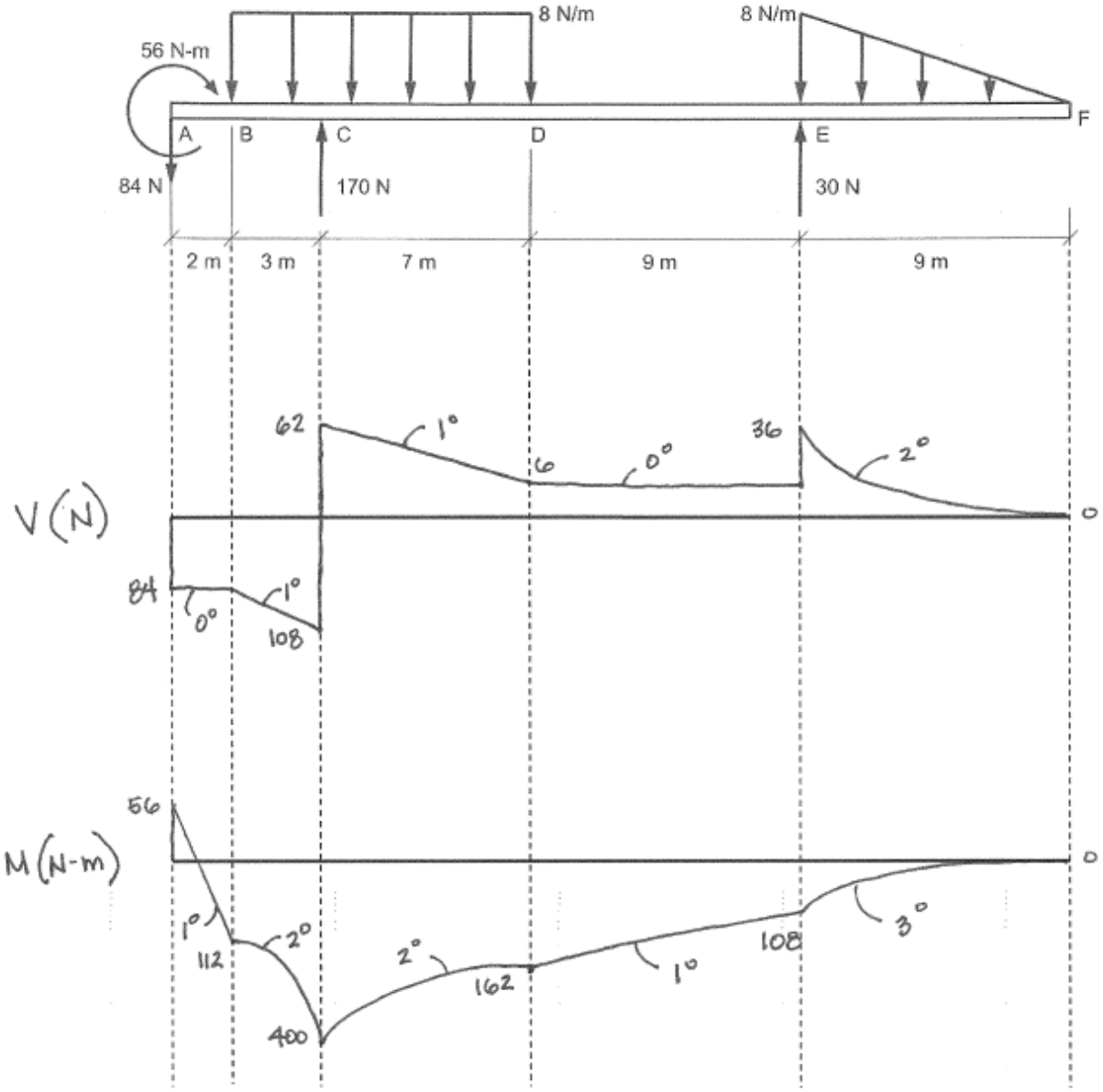
$EG = 136.7\ kN\ (C)$

JOINT FBD:

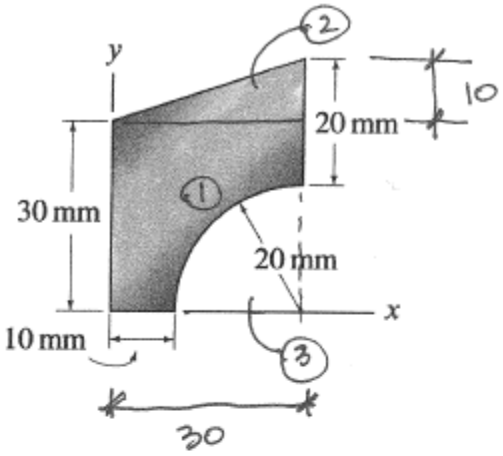
$+\uparrow \Sigma F_y = 0 = -48 - \frac{4.5}{\sqrt{164.25}} AC$

$AC = 136.7\ kN\ (C)$

4) Draw the shear and bending moment diagrams for the loading condition below. Label the diagrams appropriately (units, order of slope, etc.).



5) Using tabular form, calculate the moment of inertia about the centroidal x-axis, x', for the object below. The centroid of the shape is (13.24 mm, 21.52 mm).



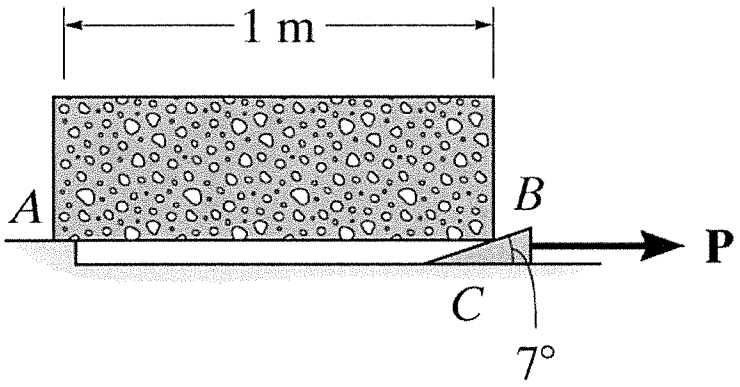
$$I_x = \bar{I}_x' + A(d_y)^2$$

SHAPE	$\bar{I}_x'$	A	d_y	$A(d_y)^2$
(1)	$\frac{bh^3}{12}$ 67,500	900	21.52 − 15 = 6.52	38,259.36
(2)	$\frac{bh^3}{36}$ 833.33	150	21.52 − 33.33 = −11.81	20933.2
(3) VOID	$-0.05488r^4$ = −8780.8	$-\frac{\pi r^2}{4}$ − 314.16	$21.52 - \frac{4r}{3\pi}$ = 13.03	−53352.6
	59552.5			5839.98

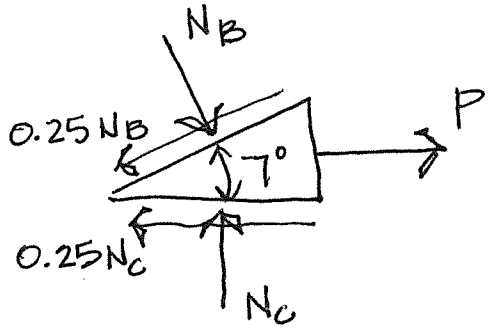
$$I_x = 59552.5 + 5839.98$$

$$\boxed{I_x = 65392.5 \text{ mm}^4}$$

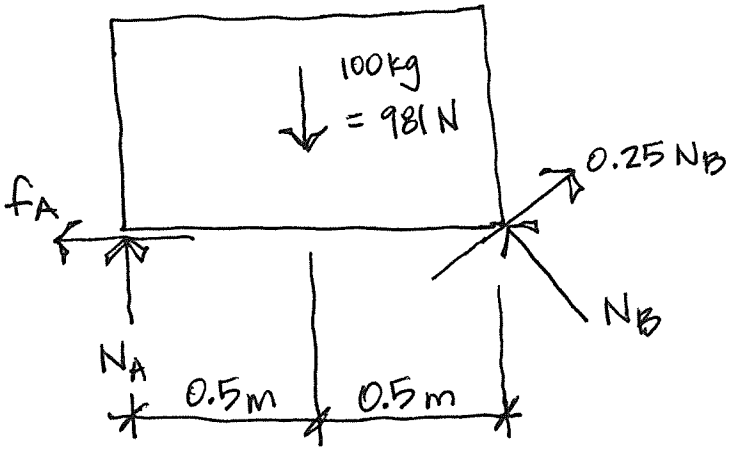
3) Determine the force required to move the wedge to the right as the 100 kg block remains in equilibrium. The static coefficient of friction between the block and the wedge and the wedge and the floor is 0.25. Neglect the size and weight of the wedge. Draw all pertinent free-body diagrams.



WEDGE FBD:



BLOCK FBD:



BLOCK FBD:

$+\circlearrowleft \Sigma M_A = 0 = -981(0.5) + N_B \cos 7(1) + 0.25 N_B \sin 7(1)$

$490.5 = 0.918 N_B$

$N_B = 479.5\ N$

WEDGE FBD:

$+\uparrow \Sigma F_y = 0 = N_C - N_B \cos 7 - 0.25 N_B \sin 7$

$N_C = (0.25)479.5 \sin 7 + 479.5 \cos 7$

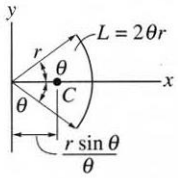
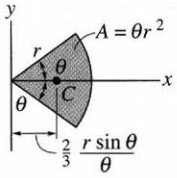
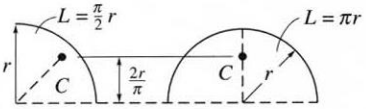
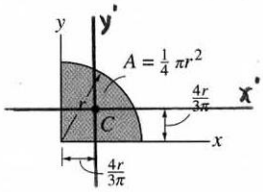
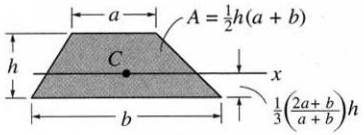
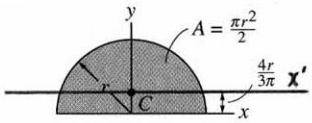
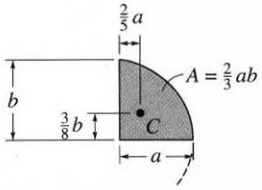
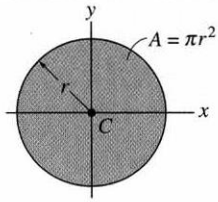
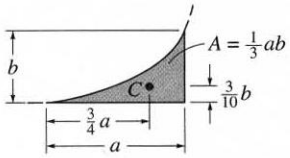
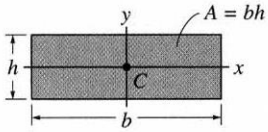
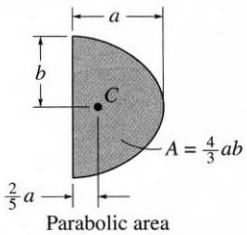
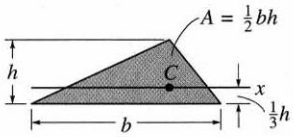
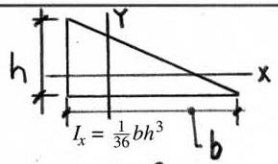
$N_C = 490.5\ N$

$+\rightarrow \Sigma F_x = 0 = -0.25 N_B \cos 7 + N_B \sin 7 - 0.25 N_C + P$

$P = (0.25)479.5 \cos 7 - 479.5 \sin 7 + (0.25)490.5$

$\boxed{P = 183.2\ N}$

Geometric Properties of Line and Area Elements

Centroid Location	Centroid Location	Area Moment of Inertia
 Circular arc segment	 Circular sector area	$I_x = \frac{1}{4}r^4(\theta - \frac{1}{2}\sin 2\theta)$ $I_x = \frac{1}{4}r^4(\theta + \frac{1}{2}\sin 2\theta)$
 Quarter and semicircle arcs	 Quarter circle area	$I_{x'} = I_{y'} = .0548Br^4$ $I_x = \frac{1}{16}\pi r^4$ $I_y = \frac{1}{16}\pi r^4$
 Trapezoidal area	 Semicircular area	$I_{x'} = .1098r^4$ $I_x = \frac{1}{8}\pi r^4$ $I_y = \frac{1}{8}\pi r^4$
 Semiparabolic area	 Circular area	$I_x = \frac{1}{4}\pi r^4$ $I_y = \frac{1}{4}\pi r^4$
 Exparabolic area	 Rectangular area	$I_x = \frac{1}{12}bh^3$ $I_y = \frac{1}{12}hb^3$
 Parabolic area	 Triangular area	 $I_x = \frac{1}{36}bh^3$ $I_Y = \frac{hb^3}{36}$