Linear Algebra Homework #2
Fall 2020

Name: Due: Monday, Nov 30th, 2020

Instructions:

Please include essential steps in your solution. For most of the problems, answers without essential steps may receive a score of 0.

1. Circle leading entries and determine which of the following matrices are in echelon form or reduced echelon form.

(a) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ (f) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (g) $\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ (h) $[1\ 3]$

2. Use Gaussian elimination to find the general solution to the system. Show the elementary operations you use.

$$\begin{aligned} x_1 + x_2 &= 1 \\ 2x_1 + 2x_2 + x_3 &= 1 \\ 2x_1 + 2x_2 &= 2 \end{aligned}$$

3. Show that the following nonlinear systems become linear if we view the unknowns as 1/x, 1/y, and 1/z rather than x, y, and z. Use this to find the solution sets of the nonlinear systems. (You must also account for the possibilities that one of x, y, z is zero.)

$$\begin{aligned} 2x - y + 3xy &= 0 \\ 4x + 2y - xy &= 0 \end{aligned}$$

4. Find the rank of the augmented and coefficient matrix of the following linear systems and the solution sets to the following systems.

(a)
$$\begin{aligned} x_1 + x_2 + x_3 - x_4 &= 2 \\ 2x_1 + x_2 - 2x_4 &= 1 \\ 2x_1 + 2x_2 + 2x_3 - 2x_4 &= 4 \end{aligned}$$

(b)
$$\begin{aligned} x_3 + x_4 &= 0 \\ -2x_1 - 4x_2 &= 0 \\ 3x_1 + 6x_2 - x_3 + x_4 &= 0 \end{aligned}$$

5. For what values of c are the following systems inconsistent, with unique solution, or with infinitely many solutions?

$$\begin{aligned} cx_1 + x_2 + x_3 &= 2 \\ x_1 + cx_2 + x_3 &= 2 \\ x_1 + x_2 + cx_3 &= 2 \end{aligned}$$