B0684
Economic Engineering Analysis

Equivalence, Loans & Bonds

Learning Objective

1. Compare the equivalence between two or more cash flow profile.
2. Analyze immediate payment and deferred payment loans, including payment amount, remaining balance, and interest and principal per payment.
3. Analyze investments in bonds and determine the purchase price, selling price, and return on such investments.
4. Calculate the worth of a cash flow profile with variable interest rates.

- Fifteen years after graduating in electrical engineering and accepting employment with Texas Instruments, Samuel Washington decides to establish a consulting business.
- Although he has invested wisely for the past 15 years, the value of his investments is only $325,000. After developing a business plan, he realizes he will need $250,000 on hand initially, plus $150,000 each successive year, to cover the expenses of an office and an assistant.
- He is unsure about how much to borrow. In talking to the loan officer of a local bank, he learns that the bank will charge him annual compound interest of 6% for a 5-year loan period or 5.5% for a 10-year loan period.
- Over the past 10 years, Samuel earned an average of 5.25 percent annually on his investments; he believes he will continue to earn at least that amount on his investment portfolio.
- If he borrows money, he can repay the loan in several ways: pay accumulated interest monthly, plus pay the principal at the end of the loan period; make equal monthly payments; make monthly payments that increase like a gradient series; make monthly payments that increase like a geometric series; or make a lump sum payment at the end of the loan period.

EQUIVALENCE

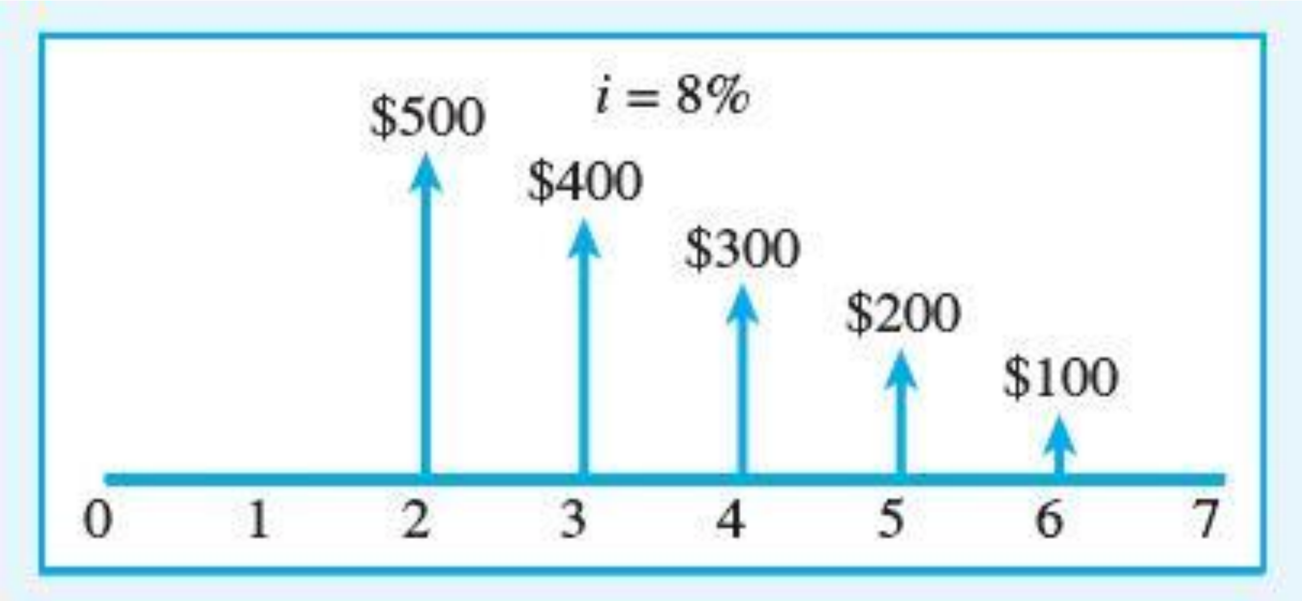
- The state of being equal in value.
- The concept is primarily applied in the comparison of two or more cash flow profiles.
- A commonly used approach to determine equivalence is to compare the present/future worth of the cash flow profiles.
- If they are equal, then the cash flow profiles are equivalent.

- Cash Flow Profile 1: Receive $1,322.50 two years from today, and the interest rate is 15%.
- Cash Flow Profile 2: Receive $1,000 today.

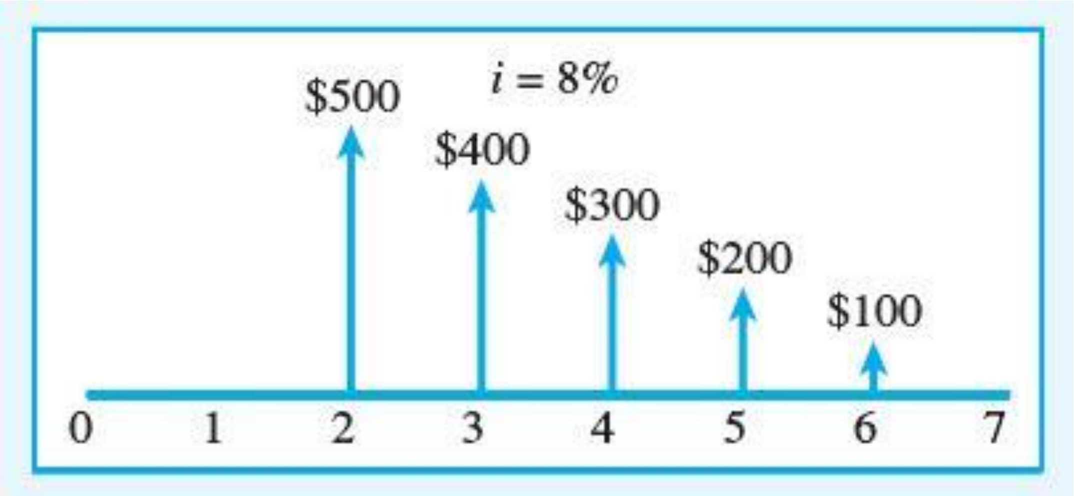
- PV1=PV(15%,2,,-1322.5)=$1,000=PV2
- The two cash flow profiles are equivalent!
- It suggest the worth of the two cash flow profiles will be the same at any particular point in time, e.g., at t_2 or t_6.

A Uniform Series Equivalency of a Gradient Series

Using an 8 percent discount rate, what uniform series over five periods, [1, 5], is equivalent to the cash flow profile given?

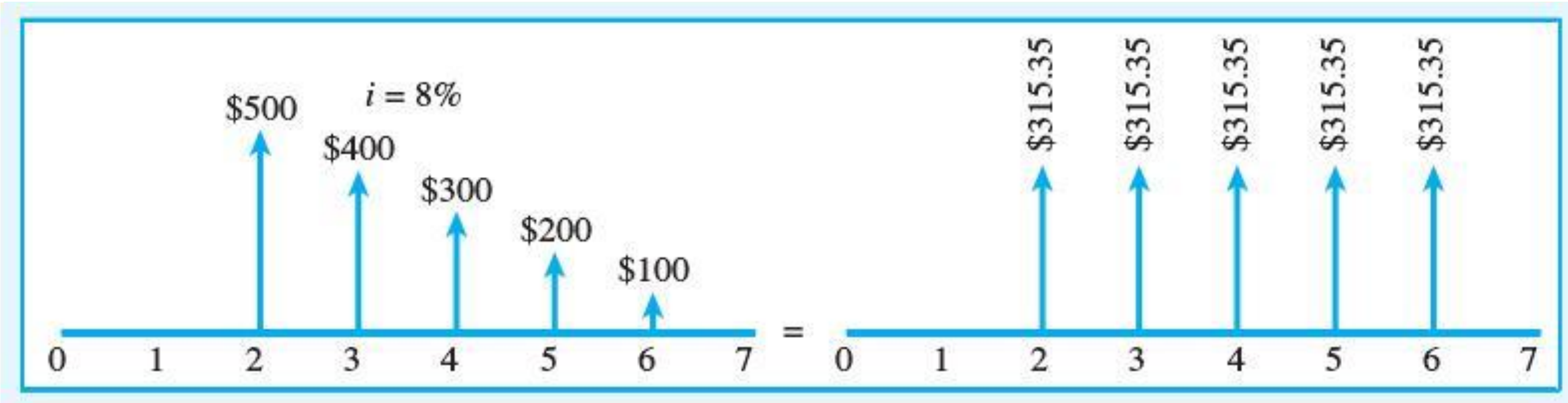


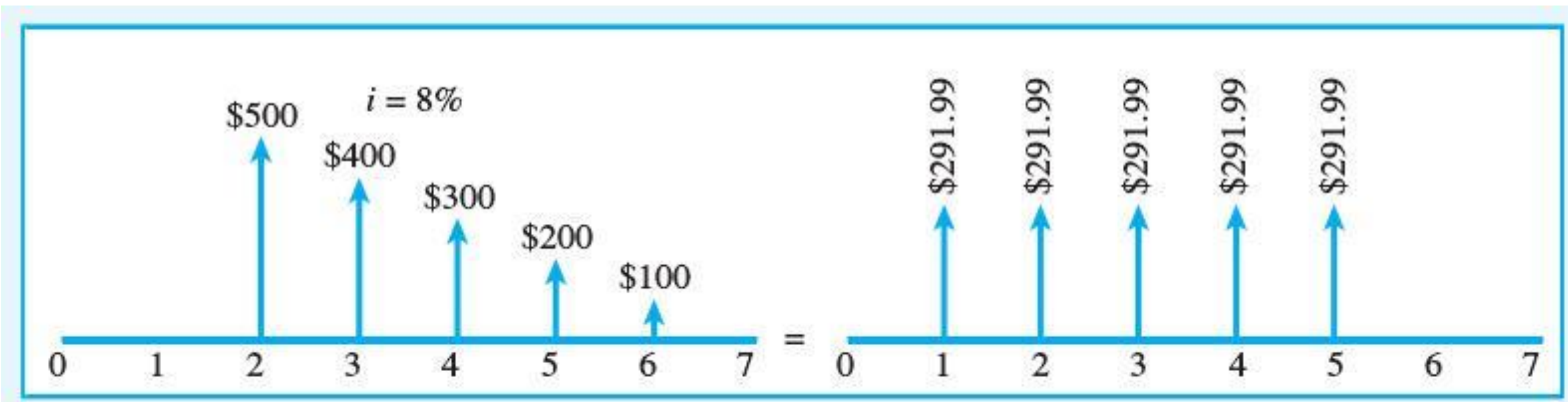
Draw CFD!! Pay attention to the time period!



Solution 1:

- P1=100*NPV(0.08,5,4,3,2,1)=1259.1125; P1 occurs at t_1.
- A=PMT(0.08,5,-1259.1125)=315.35; P1 occurs at t_1, and this equivalent uniform series occur at period [2,6], which is one time period after t_1!
- The question is to find the equivalent uniform series at period [1,5], thus discount A backward one time period:
- A'=315.35/(1+8%)=291.99


i = 8%
$500
$400
$300
$200
$100
0 1 2 3 4 5 6 7
=
$315.35
$315.35
$315.35
$315.35
$315.35
0 1 2 3 4 5 6 7

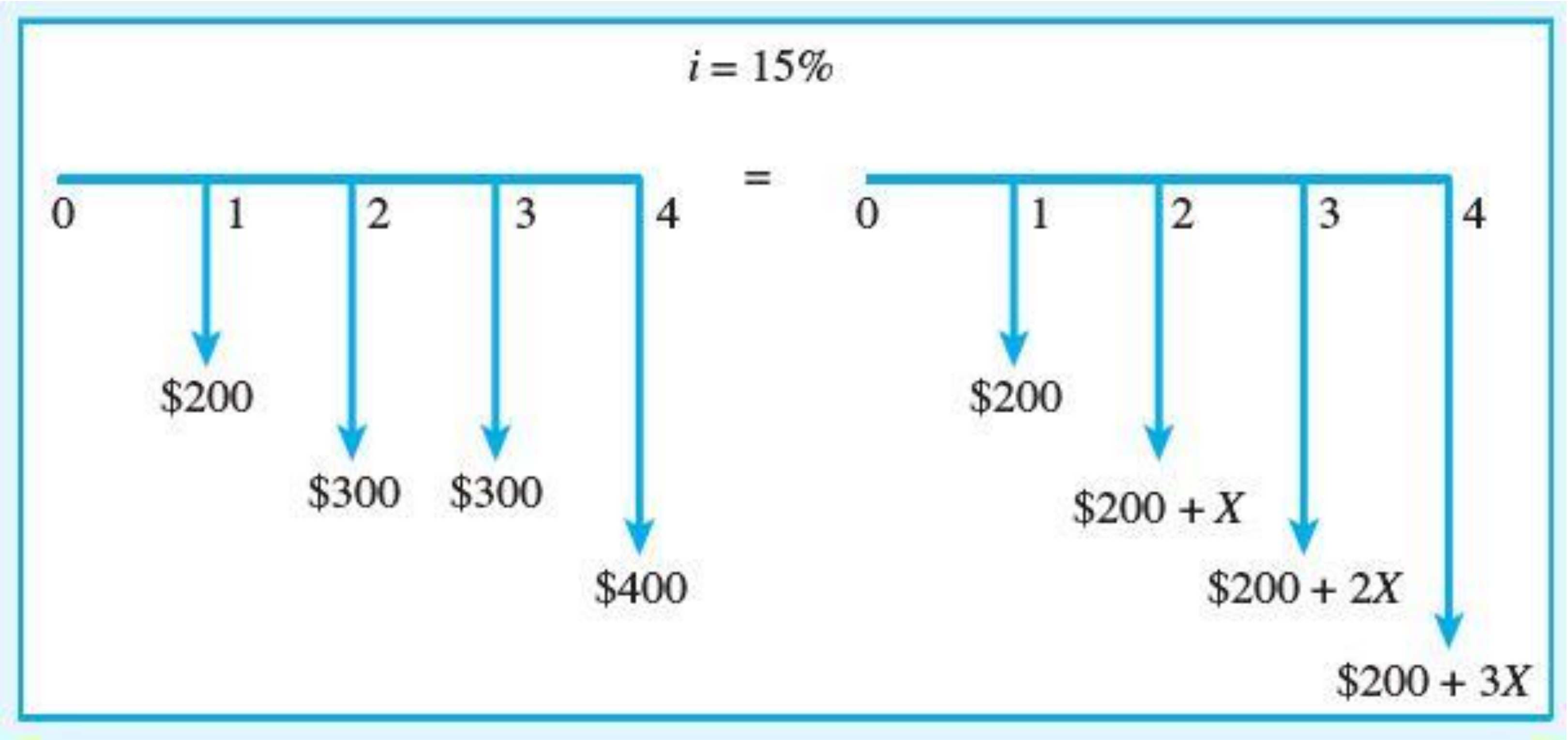

i = 8%
$500
$400
$300
$200
$100
0 1 2 3 4 5 6 7
=
$291.99
$291.99
$291.99
$291.99
$291.99
0 1 2 3 4 5 6 7

Solution 2:

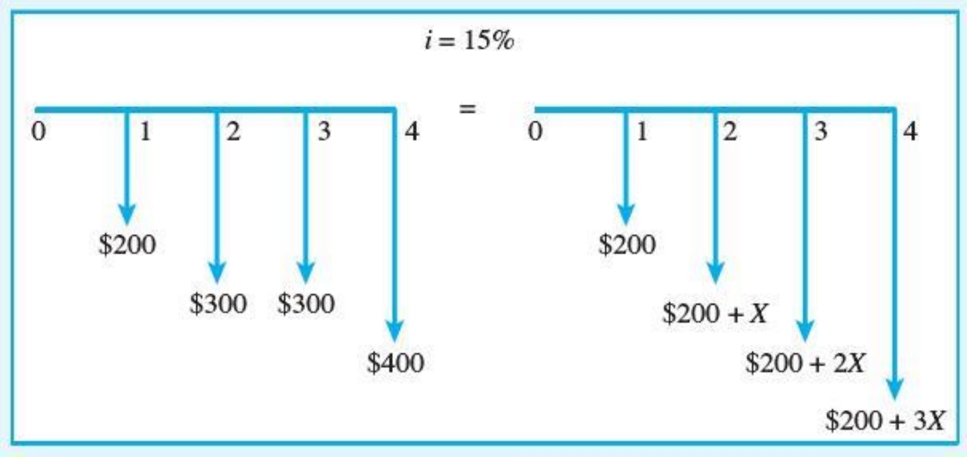
- P1=100*NPV(0.08,5,4,3,2,1)=1259.1125; P1 occurs at t_1.
- Discount P1 to t_0, P0=PV(0.08,1,,-1259.1125)=1165.84
- Then find the equivalent uniform series at period [1,5], thus A=PMT(0.08,5,-1165.84)=291.99

Determining an Equivalent Gradient Step

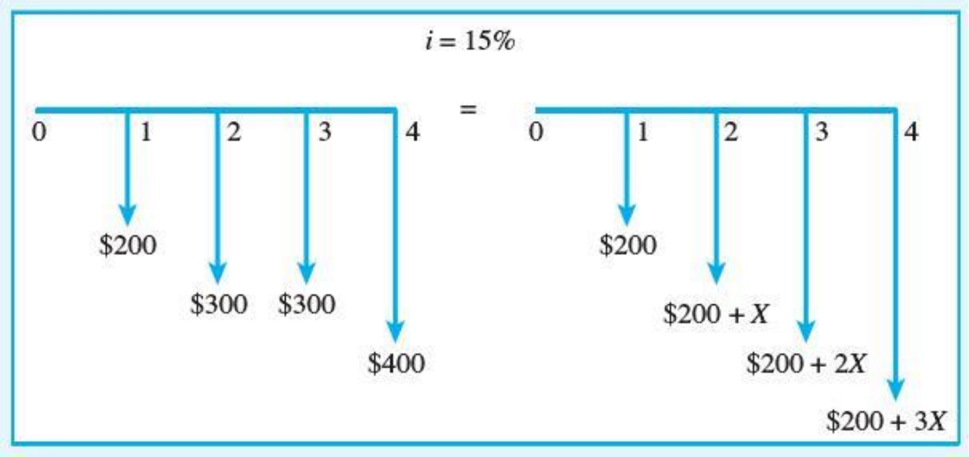
Determine the value of X that makes the two cash flow profiles equivalent using a TVOM of 15 percent.



- Solution 1: breaking down the cash flow on the right into a uniform series A=200 at [1,4], and a gradient series {X, 2X, 3X} at [2,4], calculate PV at t_0
- P=100*NPV(0.15,2,3,3,4)=826.71,
- $P_{uniform}$=PV(0.15,4,-200)=571.00,
- $P_{gradient}$=P-$P_{uniform}$=255.71,
- As the gradient series occurs at [2,4], PV should occur one time period before at t_1, thus move $P_{gradient}$ forward one time period.
- P'=$P_{gradient}$*(1+0.15)=294.07
- X*NPV(0.15,1,2,3)=294.07
- X*4.35=294.07
- X=67.53



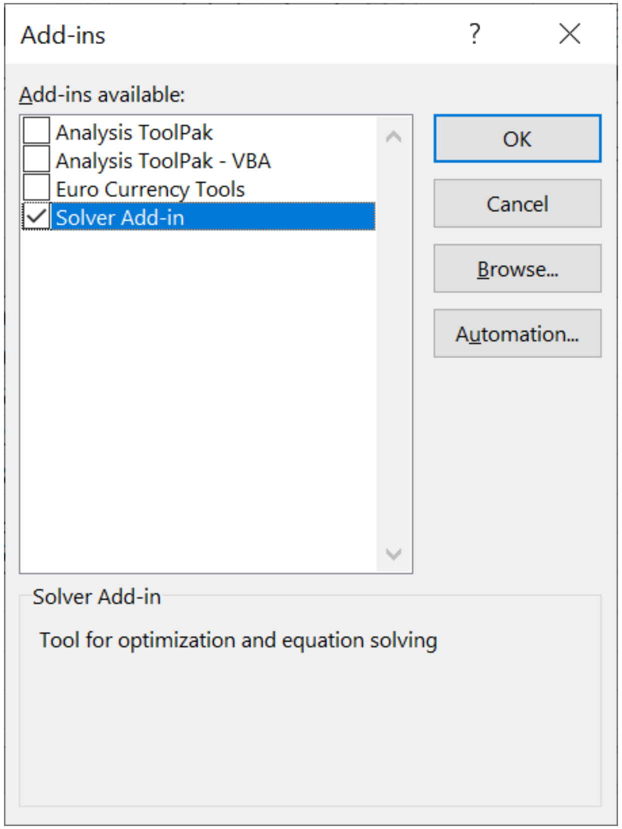
- Solution 2: all cash flows minus 200, calculate PV at t_1
- 100*NPV(0.15,1,1,2)=X*NPV(0.15,1,2,3),
- 100*2.94=X*4.35
- X=67.59



- **Solution 3: using the Excel Solver Tool**

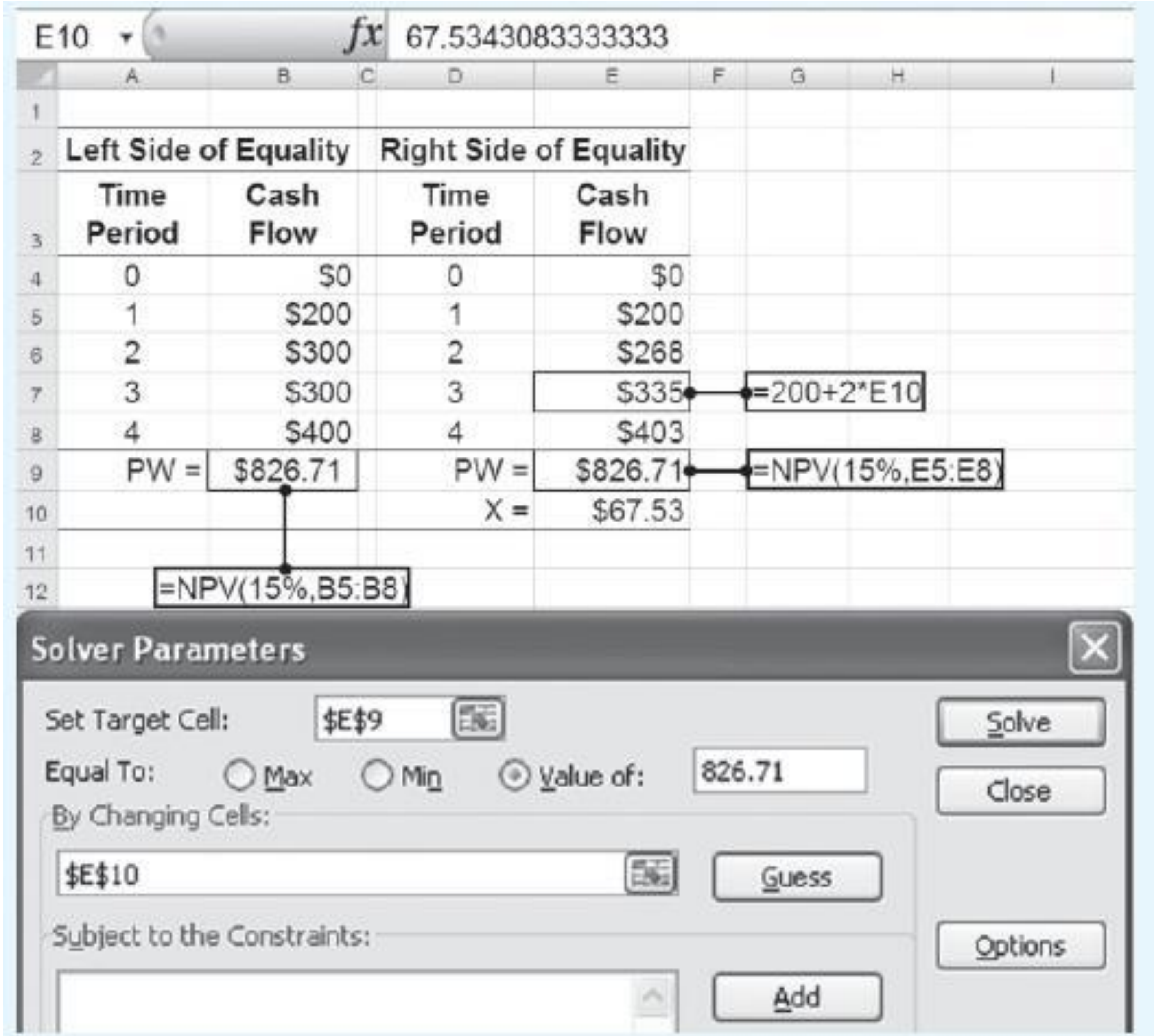
First add on the Solver tool:

- In **Excel** 2010 and later, go to File > Options. ...
- Click **Add**-Ins, and then in the Manage box, select **Excel Add**-ins.
- Click Go.
- In the **Add**-Ins available box, select the **Solver Add**-in check box, and then click OK. ...
- After you load the **Solver Add**-in, the **Solver** command is available on the Data tab.



Alternatively, search for “Solver” in the search tool bar of Excel.

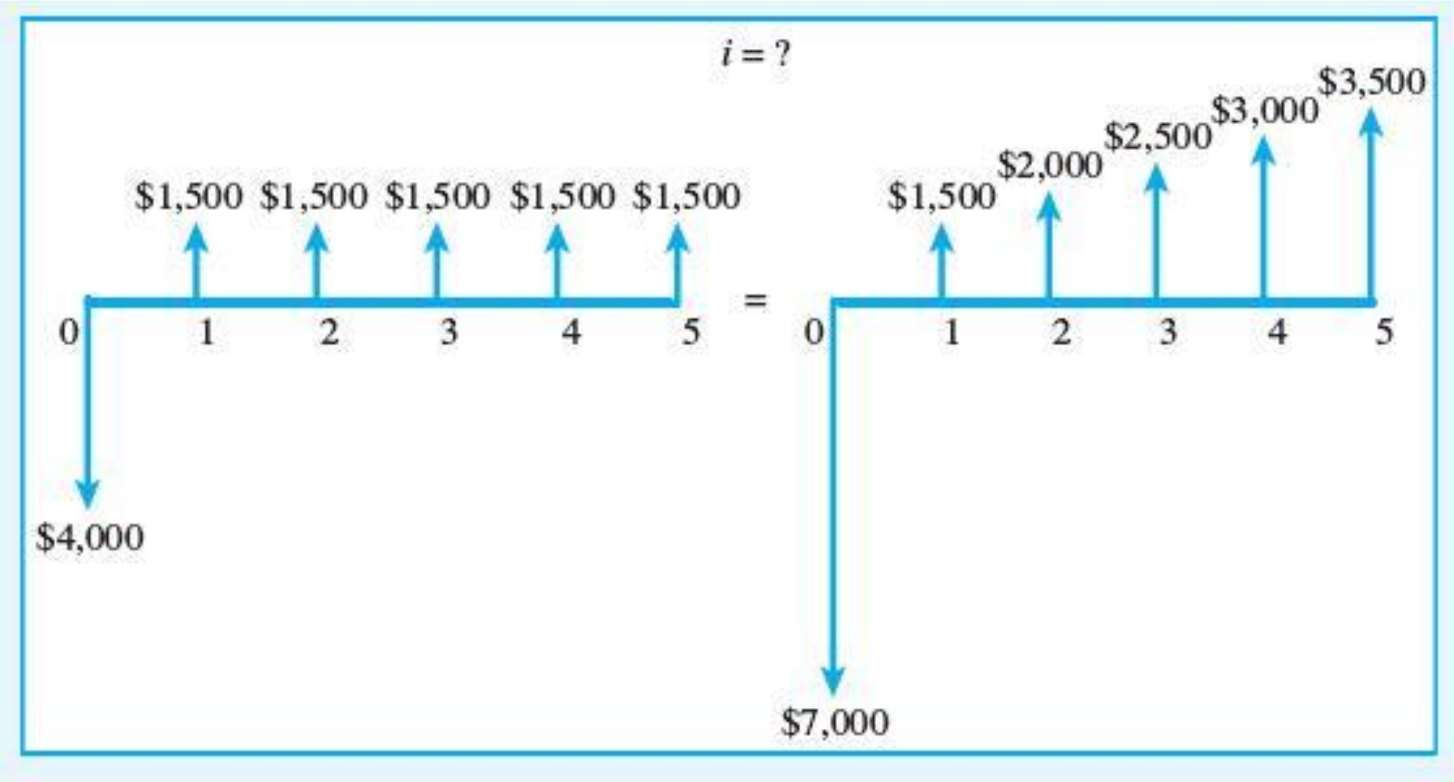
- Let the value of E10 as X to be solved.
- Input the left cash flow.
- Find PV at B9
- Input the right cash flow. For the value of E6, E7, E8, use E10 to substitute X.
- Find PV at E9.
- As E9=B9, open solver, set as the following:
- Set target cell: E9
- Equal to: Value of 826.71
- By changing cells: E10.
- Click Solve, click OK
- X will be returned in E10.



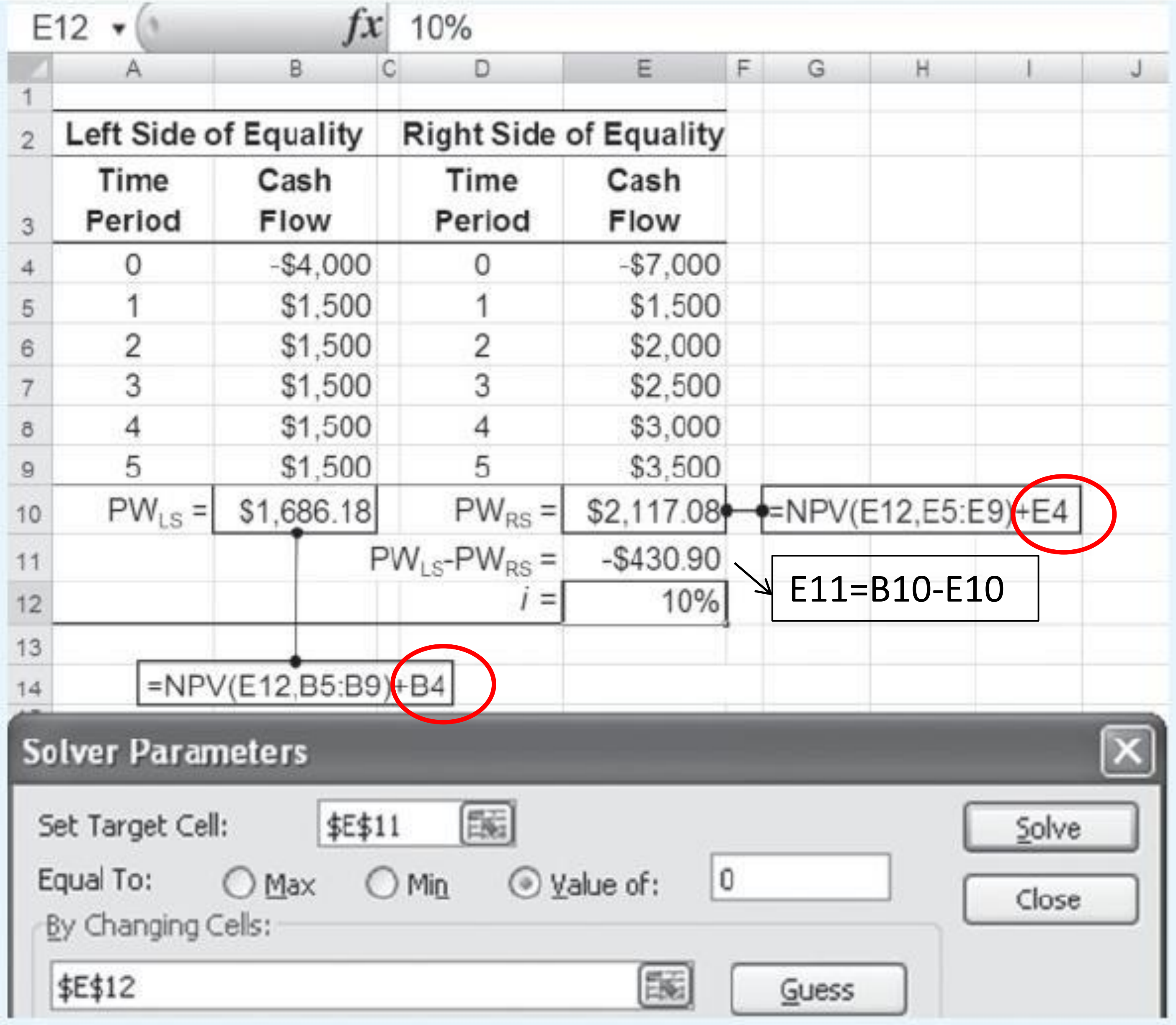
	K	L	M	N	O	P
226	t	left cashflow		right cashflow		
227	0	0		0		
228	1	200		200		
229	2	300		267.534	<-"=200+N233"	
230	3	300		335.069	<-"=200+2*N233"	
231	4	400		402.603	<-"=200+3*N233"	
232	NPV=	£826.71	<-to the value of	£826.71	<-Set objective	
233			X=	67.5343	<-by changing cell	

Determining an Equivalent Interest Rate

For what interest rate are the two cash flow profiles equivalent?



Solution: using the Solver Tool



E11 f_x =B10-E10

	A	B	C	D	E
1					
2	**Left Side of Equality**			**Right Side of Equality**	
3	**Time Period**	**Cash Flow**		**Time Period**	**Cash Flow**
4	0	-$4,000		0	-$7,000
5	1	$1,500		1	$1,500
6	2	$1,500		2	$2,000
7	3	$1,500		3	$2,500
8	4	$1,500		4	$3,000
9	5	$1,500		5	$3,500
10	PW_{LS} =	$1,166.04		PW_{RS} =	$1,166.04
11				PW_{LS}-PW_{RS} =	$0.00
12				i =	13.8677%

LOANS

- When you have a loan, the (equal sized) payment is repaid every period as a uniform series.
- Some proportion of the payments are paid for the interest (interest payment) and the other are paid for the principal (principal/equity payment).
- **The first thing paid in repaying a loan is interest.**
  - Your payments are first paid for interest.
  - When interest reduces to 0, your payments start to be paid for principal.

Purchasing a Car

Sara Beth wants to purchase a used car in excellent condition. She has decided on a car with low mileage that will cost $20,000. After considering several alternatives, she identified a local lending source that will charge her an interest rate of 6 percent per annum compounded monthly for a 48-month loan:

(a) What will be the size of her monthly payments?

(b) What will be the remaining balance on her loan immediately after making her 24th payment?

(c) If she chooses to pay off the loan at the time of her 36th payment, how much must she pay?

(d) What portion of her 12th payment is interest?

(e) What portion of her 12th payment is an equity payment?

a. i_{per}=6%/12=0.5%/month

A=PMT(0.5%,48,-20000)=$469.70

b. P24=PV(0.5%,24,-469.70)=$10,597.79

c. The payment on the 36th month=the sum of the rest 12 month payments + the payment at the 36th month

P36=PV(0.5%,12,-469.70)+469.70=5457.41+469.70=$5,927.11

d.

- The Excel IPMT function determines the amount of a periodic payment that is interest.
- Parameters in order are: interest rate, period for which the payment occurs, number of periodic payments, present worth, future worth, and type.
- For conventional loans, the future amount and type parameters are not needed.
- I_{12}=IPMT(0.5%,12,48,-20000)=$79.15

e.

- The Excel PPMT function determines the amount of a periodic payment that reduces the unpaid principal on a loan.
- It has the same parameters as IPMT.
- P_{12}= PPMT(0.5%,12,48,-20000)=$390.55

- When asked to find IPMT and PPMT, always find IPMT first! (The interest is always paid back first)
- Find period equal size payment PMT (PMT=IPMT+PPMT).
- If IPMT<PMT, then interest payment=IPMT; PPMT=PMT-IPMT
- If IPMT>=PMT, then interest payment=PMT; PPMT=0
- To conclude, interest payment=MIN(IPMT, PMT)

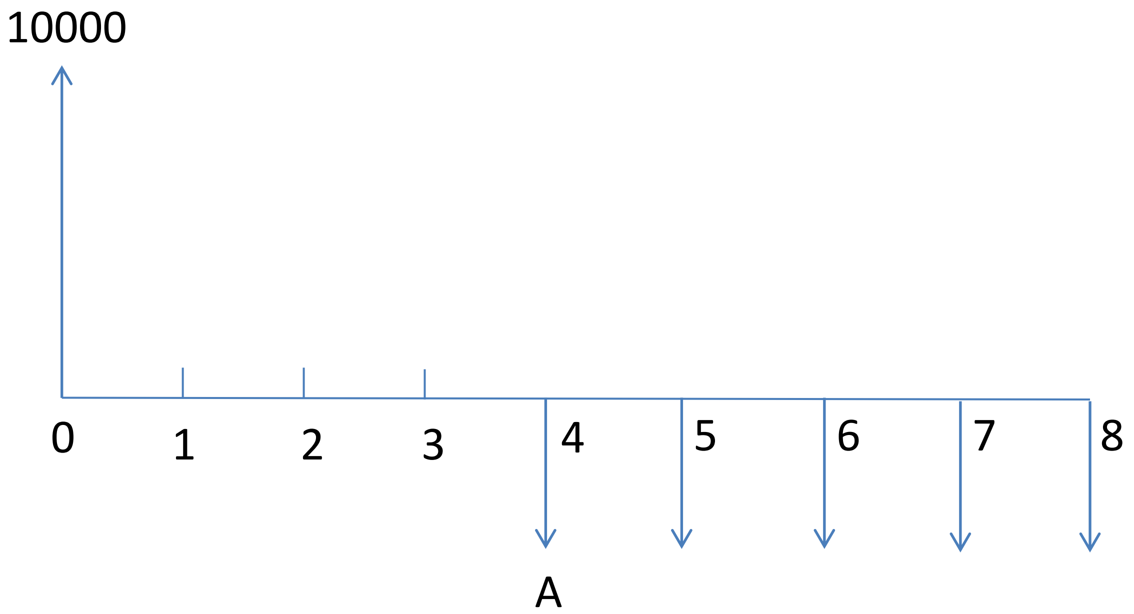
Interest and Equity Payments in Deferred conditions*

The owner of a small business borrows $10,000 at 15 percent annual compound interest. Five equal annual payments will be made to repay the loan, but the first will not occur until 4 years after receipt of the principal amount.

How much of each payment will be paid for the interest and principal?

*optional content, not required

Solution: Draw CFD!



conpound P to t_3, P3=FV(15%,3,,-10000)=15208.75

A=PMT(15%,5,-15208.75)=4537.01

Recall that your payments are paid for interest first!

Year	Unpaid Balance Before Payment (UB)	Interest During Year (Int)	Unpaid Interest Before Payment (UIB)	Amount Owed (AO)	Loan Payment (A_d)	Interest Payment ($IPmt$)	Principal Payment ($PPmt$)	Unpaid Interest After Payment (UIA)	Unpaid Balance After Payment (UBA)
1	$10,000.00	$1,500.00	$1,500.00	$11,500.00	$0.00	$0.00	$0.00	$1,500.00	$11,500.00
2	$11,500.00	$1,725.00	$3,225.00	$13,225.00	$0.00	$0.00	$0.00	$3,225.00	$13,225.00
3	$13,225.00	$1,983.75	$5,208.75	$15,208.75	$0.00	$0.00	$0.00	$5,208.75	$15,208.75
4	$15,208.75	$2,281.31	$7,490.06	$17,490.06	$4,537.01	$4,537.01	$0.00	$2,953.06	$12,953.06
5	$12,953.06	$1,942.96	$4,896.01	$14,896.01	$4,537.01	$4,537.01	$0.00	$359.01	$10,359.01
6	$10,359.01	$1,553.85	$1,912.86	$11,912.86	$4,537.01	$1,912.86	$2,624.15	$0.00	$7,375.85
7	$7,375.85	$1,106.38	$1,106.38	$8,482.23	$4,537.01	$1,106.38	$3,430.63	$0.00	$3,945.22
8	$3,945.22	$591.78	$591.78	$4,537.01	$4,537.01	$591.78	$3,945.22	$0.00	$0.00
	$UB_t = UBA_{t-1}$	$Int_t = IR_t \times UB_t$	$UIB_t = Int_t + UIA_{t-1}$	$AO_t = UB_t + Int_t$	A_{dt}	$IPm_t = \min(UIB_t; A_{dt})$	$PPmt_t = A_{dt} - IPmt_t$	$UIA_t = UIB_t - IPmt_t$	$UBA_t = AO_t - A_{dt}$

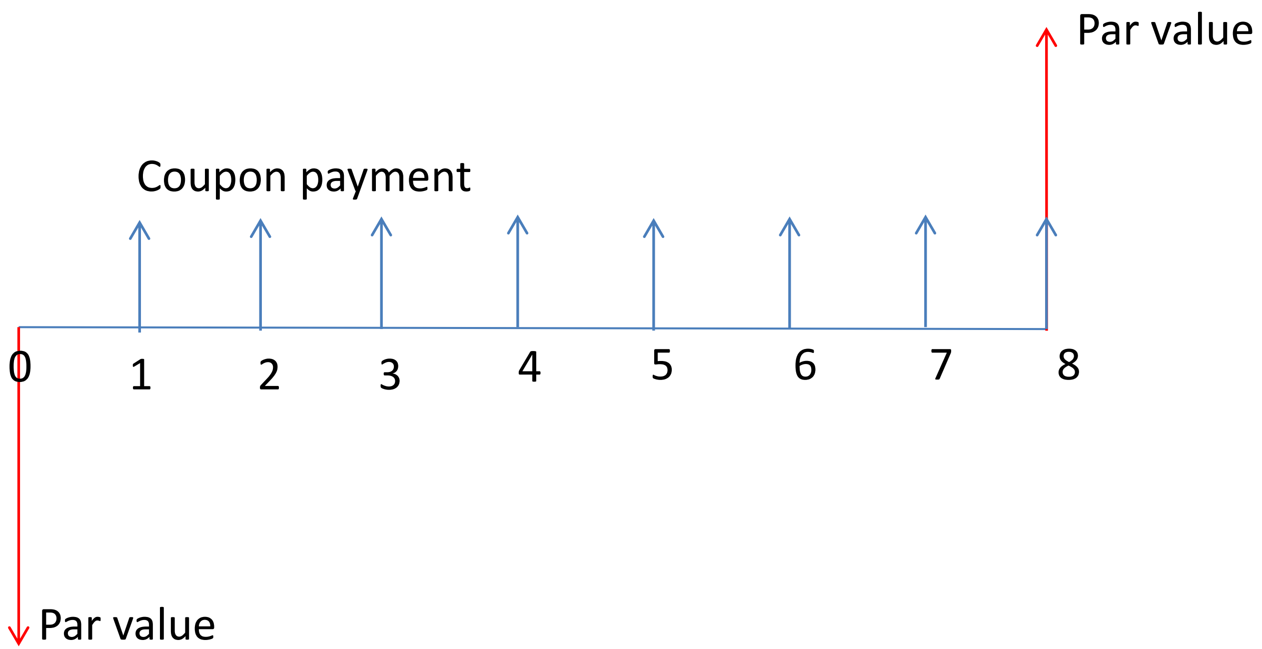
What are the total interest and principal paid when the loan is closing?

- $P3_{interest}$=NPV(0.15,4537.01,4537.01,1912.86,1106.38,591.78)=9560.39
- $FV_{interest}$=FV(0.15,5,,-9560.39)=19229.36
- $P5_{principal}$=NPV(0.15,2624.15,3430.63,3945.22)=7469.96
- $FV_{principal}$=FV(0.15,3,,-7469.96)=11360.88
- *You pay more for the interest than for the principal!!*

BOND

- A bond is a long-term note issued by the borrower (a corporation or governmental agency) to the lender, typically for the purpose of financing a large project.
- The stated value on the individual bond is the face/par value.
- However the price you pay for the bond may differ from its face value.
- The issuing unit is obligated to redeem the bond *at par value* at maturity.
- The issuing unit is obligated to pay a bond rate on the face value between the date of issuance and the date of redemption.
- The interest payment per period is coupon payment.
- Coupon payment=face value*bond rate

CFD of a bond



The buying price or selling price of a bond may be different from its par value!

Determining the Selling Price for a Bond

On January 1, 2011, Austin plans to pay $1,050 for a $1,000, 12 percent semiannual bond. He will keep the bond for 3 years, receive six coupon payments, and then sell it. How much should he sell the bond for in order to receive a yield of 10 percent compounded semiannually?

- bond rate=12%/2=6%, n=6,
- coupon payment=face value*bond rate=1000*6%=60
- i_{per}=r/m=10%/2=5%
- Fbuy=Fpayment+Fsell
- Fbuy=FV(0.05,6,,-1050)=1407.10
- Fpayment=FV(0.05,6,-60)=408.11
- Fsell=Fbuy-Fpayment=998.99

- Bond rate (6%) is only used for calculating coupon payment. In other cases please use interest rate (5%).

Determining the Purchase Price for a Bond

Emma plans to purchase a $1,000, 12 percent semiannual bond, hold it for 3 years, receive six coupon payments, and redeem it at par value. What is the maximum amount she should pay for the bond if she wants to earn at least 14 percent compounded semiannually on her investment?

- bond rate=12%/2=6%, n=6,
- coupon payment=1000*6%=60
- i_{per}=r/m=14%/2=7%
- Psell=PV(0.07,6,,-1000)=666.34
- Ppayment=PV(0.07,6,-60)=285.99
- Pbuy=Psell+Ppayment=952.33

Determining the Rate of Return for a Bond Investment

Charlotte purchased a $1,000, 12 percent quarterly bond for $1,020, kept it for 3 years, received 12 coupon payments, and sold it for $950. What was her quarterly interest rate on her bond investment? What was her effective annual rate of return?

- bond rate=12%/4=3%, n=4*3=12
- Coupon payment=1000*3%=30
- Using Excel Solver Tool
- Fpayment=FV(i,12,-30)
- Fbuy=FV(i,12,,-1020)
- Fsell=950
- Fbuy-Fpayment=Fsell
- i_{per}=0.02442,
- Using Excel EFFECT function
- r=0.02442*4=0.09768
- i_{eff}=EFFECT(0.09768,4)=0.10132

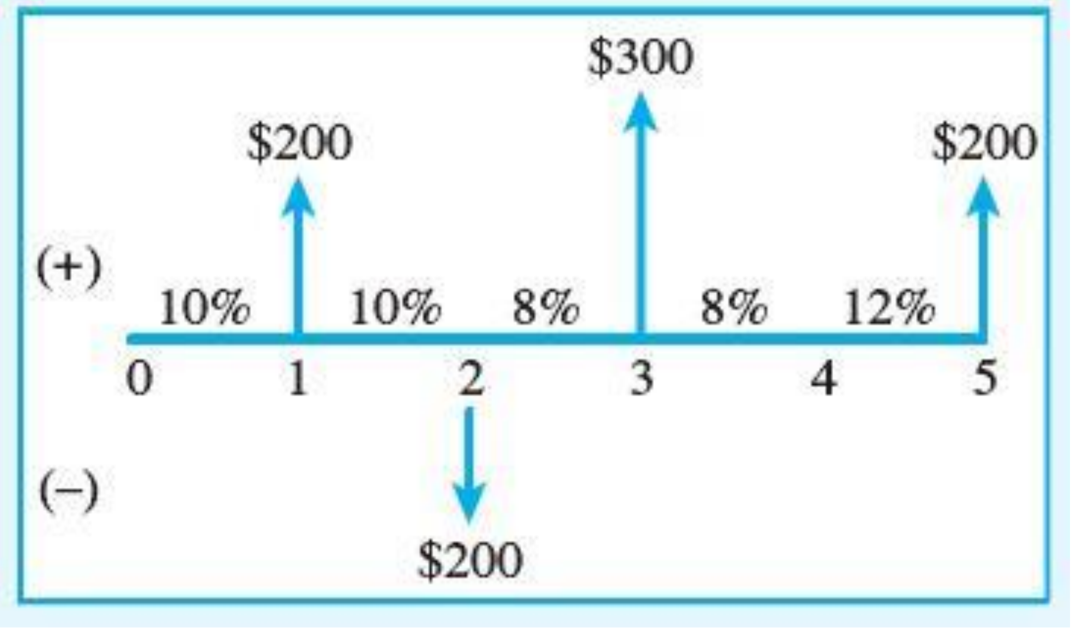
- Alternatively, using Excel RATE function
- Parameters in order are: number of periods (nper), equal size payment (pmt), pv, fv
- i_{per}=RATE(12,30,-1020,950)=2.442%
  - This is from the bond buyer's perspective: bond payment (pmt) and selling price (fv) are positive cashflows, while buying price (pv) is a negative cashflow.
- Or, i_{per}=RATE(12,-30,1020,-950)=2.442%
  - This is from the bond issuer's perspective: bond payment (pmt) and redemption price (fv) are negative cashflows, while selling price (pv) is a positive cashflow.

VARIABLE INTEREST RATES

- So far, the interest rates are fixed in the planning horizon.
- It’s NOT likely the case, if the time period extends over several years.
- In such cases, variable interest rates apply.
- $F=P(1+i_1)(1+i_2)\ldots(1+i_{n-1})(1+i_n)$
- $P=F/[(1+i_1)(1+i_2)\ldots(1+i_{n-1})(1+i_n)]$

Example

Consider the CFD given with the appropriate interest rates indicated. Determine the present worth, future worth, and uniform series equivalents for the cash flow series.



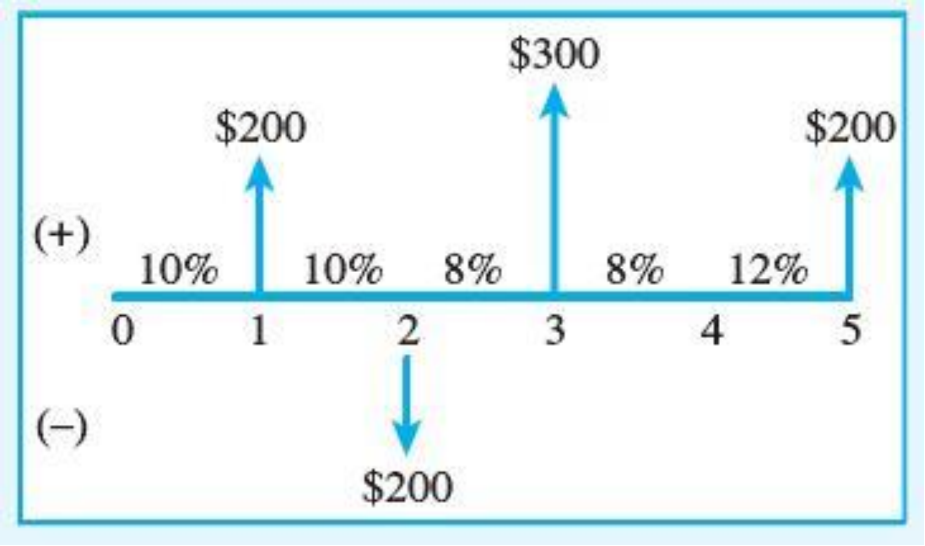
Solution 1:

$F=200(1+10\%)(1+8\%)(1+8\%)(1+12\%)-200(1+8\%)(1+8\%)(1+12\%)+300(1+8\%)(1+12\%)+200=589.01$

$P= 200(1+10\%)^{-1}-200(1+10\%)^{-1}(1+10\%)^{-1}+300(1+8\%)^{-1}(1+10\%)^{-1}(1+10\%)^{-1}+200(1+12\%)^{-1}(1+8\%)^{-1}(1+8\%)^{-1}(1+10\%)^{-1}(1+10\%)^{-1}=372.63$

i=RATE(5,,-372.63,589.01)=9.5895%

A=PMT(9.5895%,5,-372.63)=97.27



Solution 2:

Using Excel FVSCHEDULE function to calculate further value.

Parameters in order are: principal, schedule.

schedule is the variable interest rate series, schedule={$i_1, i_2, \ldots, i_n$}

It is applied to single cash flows.

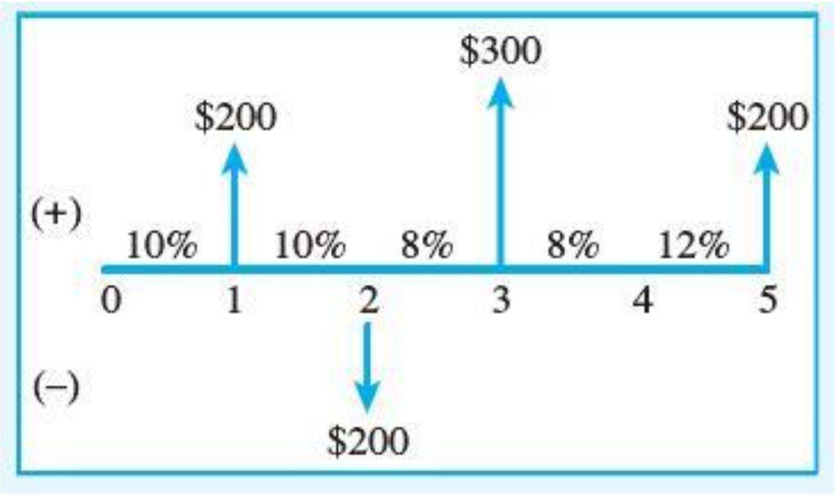
F=FVSCHEDULE(200,{0.1,0.08,0.08,0.12})

-FVSCHEDULE(200,{0.08,0.08,0.12})

+FVSCHEDULE(300,{0.08,0.12})

+200

=$589.01



ANNUAL PERCENTAGE RATE

Loan lenders are not required to provide the effective annual interest rate to borrowers.

Instead, they are required to provide the annual percentage rate (APR).

There are multiple methods of how APR is calculated.

APR might be a nominal annual interest rate, an effective annual interest rate, or something quite different.

Review – Multiple Choice Question (MCQ) – Only one correct answer!

1. What single sum of money at t = 4 is equivalent to receiving \$5,000 at t = 1, \$6,000 at t = 2, \$7,000 at t = 3, and \$8,000 at t = 4 if money is compounded at a rate of 8% per time period?

a. \$28,857 c. \$30,892

b. \$26,000 d. \$33,363

P=1000*NPV(0.08,5,6,7,8)=21210.73

F4=FV(0.08,4,,-21210.73)=28856.96

a.

Review

2. You borrow $5,000 at 10% per year and will pay off the loan in 3 equal annual payments starting one year after the loan is made. The end-of-year payments are $2,010.57. Which of the following is true for your payment at the end of year 2?

a. Interest is $500.00 and principal is $1,510.57.

b. Interest is $450.00 and principal is $1,560.57.

c. Interest is $348.94 and principal is $1,661.63.

d. Interest is $182.78 and principal is $1,827.79.

Pi=IPMT(0.1,2,3,-5000)=348.94

Pp=A-Pi=1661.63

c.

Review

3. You borrow $10,000 at 15% per year and will pay off the loan in three equal annual payments with the first occurring at the end of the fourth year after the loan is made. The three equal annual payments will be $6,661.08. Which of the following is true for your first payment at the end of year 4?

a. Interest = $6,661.08; principal = $0.00

b. Interest = $2,281.31; principal = $4,379.77

c. Interest = $1,500.00; principal = $5,161.08

d. Interest = $0.00; principal = $6,661.08

P4=FV(0.15,4,,-10000)=17490.06

unpaid interest I4=P4-10000=7490.06>A=6661.08

As interest should be paid FIRST, all of the first payment will be used to pay for interest

a.

Review

4. Consider a cash flow and interest profile as shown:

	Year 0	Year 1	Year 2	Year 3
Cash Flow at End of Year	−$1,000	$3,000	$2,000	$1,000
Interest Rate During Year	NA	6%	8%	10%

The worth at the end of Year 3 of these cash flows is:

a. $5,000.00 c. $5,994.56

b. $5,504.72 d. $5,440.00

F=FVSCHEDULE(-1000,{0.06,0.08,0.1})
+FVSCHEDULE(3000,{0.08,0.1})+2000*(1+0.1)+1000=5504.72

b.

Review

5. A $200,000 bond having a bond rate of 8% payable annually is purchased for $190,500 and kept for 6 years, at which time it is sold. How much should it sell for in order to yield a 7% effective annual return on the investment?

a. $168,000 c. $174,000

b. $171,000 d. $177,000

coupon payment=200000*8%=16000

Fbuy=FV(0.07,6,,-190500)=285889.13

Fpayment=FV(0.07,6,-16000)=114452.65

Fsell=Fbuy-Fpayment=171436.48

b.

Review

6. \$150,000 is deposited in a fund that pays 5% annual compound interest for 2 years, 3% annual compound interest for 2 years, and 4% annual compound interest for 2 years. If uniform annual withdrawals occur over the 6-year period, what will be the magnitude of the annual withdrawals?

a. \$27,689.63 c. \$28,804.50

b. \$28,614.29 d. \$29,552.62

FV=FVSCHEDULE(150000,{0.05,0.05,0.03,0.03,0.04,0.04})=189762.76

i=RATE(6,,-150000,189762.16)=4%

A=PMT(0.04,6,-150000)=28614.29

b.

Review

7. A house is to be purchased for $180,000 with a 10% down payment (the first payment to secure the contract), the rest will be home loan and mortgage. There are no “points” or other closing charges associated with the loan. A conventional 30-year loan is used at 7.5% compounded monthly. The interest portion of the first monthly payment will be what?

a. $1,012.50 c. $120.23

b. $682.73 d. The answer cannot be determined without more information.

- PV=180000*90%=162000, n=12*30=360, i_{per}=7.5%/12=0.625%
- A=PMT(0.625%,360,-162000)=1132.73
- i1=162000*0.625%=1012.50<A
- iPMT=i1
- or iPMT=IPMT(0.625%,1,360,-162000)=1012.50

a.

Review– Calculation

8. Quarterly deposits of $1,000 are made at t = 1, 2, 3, 4, 5, 6, and 7. Then, withdrawals of size A are made at t = 12, 13, 14, and 15. If the fund pays interest at a quarterly compounding rate of 4%, what value of A will deplete the fund with the fourth withdrawal?

Solution:

F7=FV(0.04,7,-1000)=7898.29

F11=FV(0.04,4,,-7898.29)=9239.89

A=PMT(0.04,4,-9239.89)=2545.50

Review

9. Kinnunen Company wishes to give its customers three options on payments for office equipment when the initial purchase price is over a certain amount. For example, the following three payment plans are options on a typical purchase, and Kinnunen wants to be sure they are equivalent at a their TVOM of 14%. Determine the values of Q and R.

End of Year	Option 1	Option 2	Option 3
0	\$0	\$0	\$0
1	1,800	Q	R
2	1,800	$2Q$	$(1.1)R$
3	1,800	$3Q$	$(1.1)^2R$
4	1,800	$4Q$	$(1.1)^3R$
5	1,800	$5Q$	$(1.1)^4R$

Solution: Using Excel Solver Tool

0	0		0
1800	656.9648144		1511.339074
1800	1313.929629		1662.472982
1800	1970.894443		1828.72028
1800	2627.859258		2011.592308
1800	3284.824072		2212.751539
	$6,179.55		$6,179.55
$11,898.19	$11,898.19		$11,898.19
Q=	656.9648144	R=	1511.339074

Review

10. Consider the following three cash flow series:

Determine the values of X and Y so that you are indifferent between all three cash flows if your TVOM is 11% per year compounded yearly.

End of Year	Cash Flow Series A	Cash Flow Series B	Cash Flow Series C
0	\$3.0X	\$1,000	2Y
1	2.5X	1,500	2Y
2	2.0X	2,000	2Y
3	1.5X	2,500	Y
4	1.0X	3,000	Y
5	−1,000	−2,500	Y

Solution: Using Excel Solver Tool

	2394.361913		1000	1699.455404
	1995.301594		1500	1699.455404
	1596.241276		2000	1699.455404
	1197.180957		2500	849.7277019
	798.1206378		3000	849.7277019
	-1000		-2500	849.7277019
	$6,295.14		$6,295.14	$6,295.14
X=	798.1206378		Y=	849.7277019

Review

11. In order to buy a car, you borrow $25,000 from a friend at 12%/year compounded monthly for 4 years.

a. How much are the monthly payments?

b. How much interest is in the 23rd payment?

c. What is the remaining balance after the 37th payment?

d. Three and one-half years after borrowing the money, you decide to pay off the loan. You have not yet made the payment due at that time. What is the payoff amount for the loan?

Solution:

- a. i_{per}=12%/12=1%, A=PMT(0.01,48,-25000)=658.35
- b. i23=IPMT(0.01,23,48,-25000)=150.07
- c. P37=PV(0.01,11,-658.35)=6825.53
- d. P42=PV(0.01,6,-658.35)=3815.45
  P=P42+A=3815.45+658.35=4473.80

Review

12. $25,000 is borrowed at an annual compound rate of 8%. The loan is repaid with 5 annual payments, each of which is $500 greater than the previous payment.

a. How much of the 2nd payment will be a principal payment?

b. How much of the last payment will be an interest payment?

c. When the loan is closing after the 5 payments, how much in total is paid for interest? How much is paid for principal?

Solution:

- First, calculate the cash flows, using Excel Solver tool.
- Second, draw a table of cash flow profiles for each year.
- Third, calculate NPV of the interest and principal, and then calculate FV.
- Answers highlighted in yellow.

	K	L	M	N	O	P	Q
276	pmt1	5338.17557					
277	pmt2	5838.17557					
278	pmt3	6338.17557					
279	pmt4	6838.17557					
280	pmt5	7338.17557					
281	NPV=	$25,000.00	25000				
282	X=	5338.17557					
283							
284	balance before pmt	interest this year	total owned	payment this year	iPMT	pPMT	balance after pmt
285	25000.00000	2000.00000	27000.00000	5338.17557	2000.00000	3338.17557	21661.82443
286	21661.82443	1732.94595	23394.77038	5838.17557	1732.94595	4105.22962	17556.59482
287	17556.59482	1404.52759	18961.12240	6338.17557	1404.52759	4933.64798	12622.94683
288	12622.94683	1009.83575	13632.78258	6838.17557	1009.83575	5828.33982	6794.60701
289	6794.60701	543.56856	7338.17557	7338.17557	543.56856	6794.60701	0.00000
290	bbp.t=bam.(t-1)	ity.t=bbp.t*8%	to.t=bbp.t+ity.t	pty.t=pmt.t	ipmt.t=ity.t	ppmt.t=pty.t-ipmt.t	bap.t=to.t-pty.t
291							
292				NPV=	¥5,564.74	¥19,435.26	25000
293				FV=	¥8,176.42	¥28,556.78	¥36,733.20

Review

13. Five 15-year bonds each having a face value of $1,000 and a coupon rate of 6% per 6 months payable semiannually were purchased for $7,000 8 years ago, and the 16th coupon payment was just made. What can they be sold for now to a buyer if that buyer's desired return is 4% per 6 months?

Solution:

- n=(15-8)*2=14 periods
- i_{per}=4%
- coupon pmt=1000*6%=60
- Ppmt=PV(0.04,14,-60)=633.79
- Psell=PV(0.04,14,,-1000)=577.48
- Pbuy=Ppmt+Psell=1211.26
- nPbuy=5*Pbuy=6056.31

Review

14. You wish to purchase a $1,000 bond from a friend who needs the money. There are 7 years remaining until the bond matures, and interest payments are quarterly. You decide to offer $750.08 for the bond because you want to earn exactly 16% per year compounded quarterly on the investment. What is the annual bond rate of interest?

Solution:

- i_{per}=16%/4=4%, n=7*4=28
- Fbuy=FV(0.04,28,,-750.08)=2249.27
- let coupon rate=i, coupon pmt=1000*i
- Fpmt=FV(0.04,28,-1000*i)
- Fsell=1000=Fbuy-Fpmt
- Using Excel Solver tool, i=2.5%, i_{annual}=i*4=10%

K302 =FV(0.04,28,-1000*K305)

	I	J	K
301		Fbuy=	$2,249.27
302		Fpmt=	$1,249.27
303		Fbuy-Fpmt=	$1,000.00
304		F sell =	1000
305		i=	0.025001557
306		iannual=	0.100006229

Review

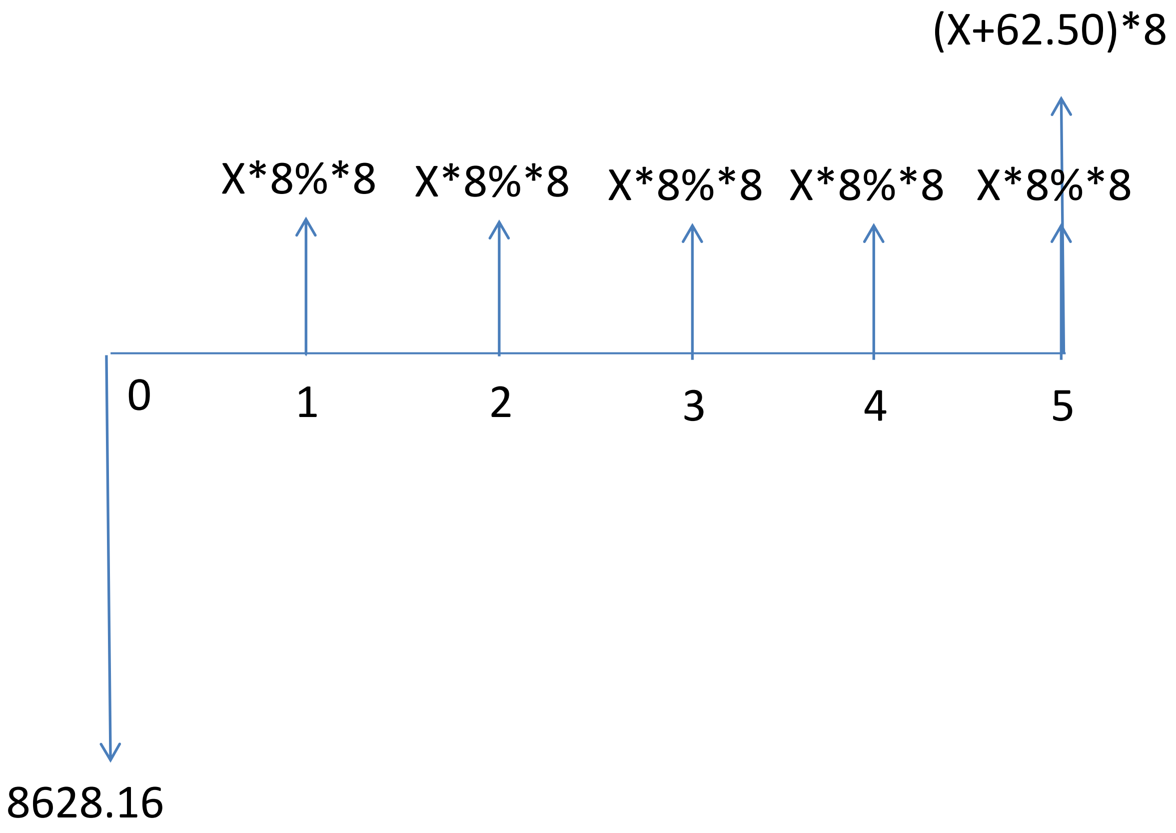
15. Eight bonds were purchased for $8,628.16. They were kept for 5 years and coupon payments were received at the end of each of the 5 years. Immediately following receipt of the 5th coupon payment, the owner sells each bond for $62.50 more than its par value. The bond coupon rate is 8%, and the owner's money yields a 10% annual return.

a. Draw a clear, completely labeled cash flow diagram of the entire bond transaction using dollar amounts where they are known and using $X to represent the face value of the bond.

b. Determine the face value of each bond.

Solution:

a.



b.

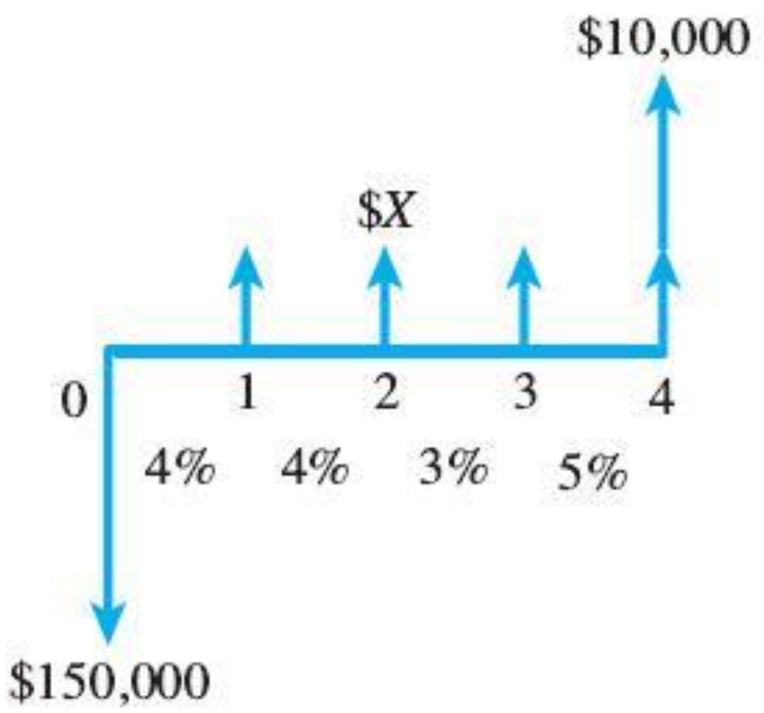
- Fbond=FV(0.1,5,,-8628.16)=13895.74
- Fpmt=FV(0.1,5,-K313*8%*8)
- Fsell=(X+62.5)*8
- Fbuy-Fpmt-Fsell=0
- Using Excel Solver tool, X=1125

K302 | f_x | =FV(0.04, 28, -1000*K305)

	I	J	K
309		Fbuy=	¥13,895.74
310		Fpmt=	¥4,395.69
311		Fsell=	9500.044318
312		Fbuy-Fpmt-Fsell=0=	-$0.00
313		X=	1125.00554

Review

16. Based on the interest rates and cash flows shown in the cash flow diagram, determine the value of $X.



Solution:

present worth method,

- $150000-X/(1+4\%)-X/(1+4\%)^2-X/[(1+4\%)^2*(1+3\%)]-(X+10000)/[(1+4\%)^2*(1+3\%)*(1+5\%)]=0$
- $150000-0.9615X-0.9246X-0.8976X-0.8549X-8549=0$
- $3.6386*X=141451$
- $X=38875.12$

or future worth method,

- $150000*(1+4\%)^2*(1+3\%)*(1+5\%)-X*(1+4\%)*(1+3\%)*(1+5\%)-X*(1+3\%)*(1+5\%)-X*(1+5\%)-(X+10000)=0$
- $150000*1.1698-1.1248X-1.0815X-1.05X-X-10000=0$
- $4.2563*X=165470$
- $X=38876.49$

Final

- MCQ: 20 points. 2 points*10
- Short answers (1 CFD drawing): 30 points. 10 points*3
- Calculation: 50 points. 10 points*5

- You can only open an empty Excel file!!
- If you are found open other file/page/PPT, it's cheating in the exam and your result is 0!!