B0684
Economic Engineering Analysis

Equivalence, Loans & Bonds

Learning Objective

1. Compare the equivalence between two or more cash flow profile.
2. Analyze immediate payment and deferred payment loans, including payment amount, remaining balance, and interest and principal per payment.
3. Analyze investments in bonds and determine the purchase price, selling price, and return on such investments.
4. Calculate the worth of a cash flow profile with variable interest rates.

- Fifteen years after graduating in electrical engineering and accepting employment with Texas Instruments, Samuel Washington decides to establish a consulting business.
- Although he has invested wisely for the past 15 years, the value of his investments is only $325,000. After developing a business plan, he realizes he will need $250,000 on hand initially, plus $150,000 each successive year, to cover the expenses of an office and an assistant.
- He is unsure about how much to borrow. In talking to the loan officer of a local bank, he learns that the bank will charge him annual compound interest of 6% for a 5-year loan period or 5.5% for a 10-year loan period.
- Over the past 10 years, Samuel earned an average of 5.25 percent annually on his investments; he believes he will continue to earn at least that amount on his investment portfolio.
- If he borrows money, he can repay the loan in several ways: pay accumulated interest monthly, plus pay the principal at the end of the loan period; make equal monthly payments; make monthly payments that increase like a gradient series; make monthly payments that increase like a geometric series; or make a lump sum payment at the end of the loan period.

EQUIVALENCE

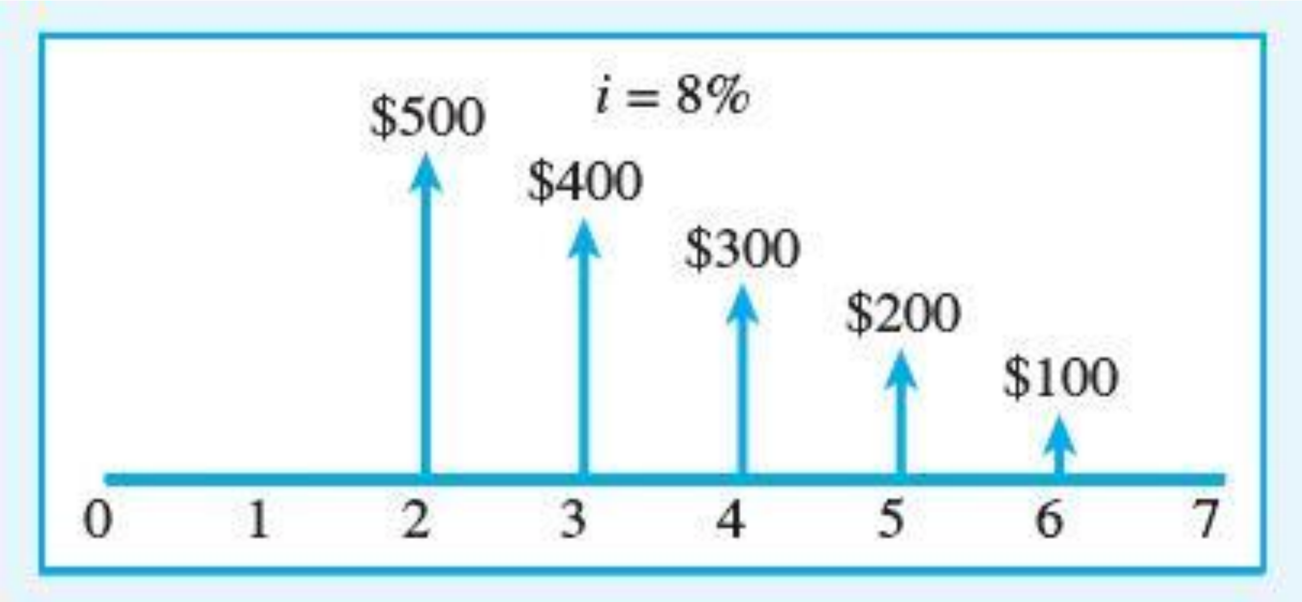
- The state of being equal in value.
- The concept is primarily applied in the comparison of two or more cash flow profiles.
- A commonly used approach to determine equivalence is to compare the present/future worth of the cash flow profiles.
- If they are equal, then the cash flow profiles are equivalent.

- Cash Flow Profile 1: Receive $1,322.50 two years from today, and the interest rate is 15%.
- Cash Flow Profile 2: Receive $1,000 today.

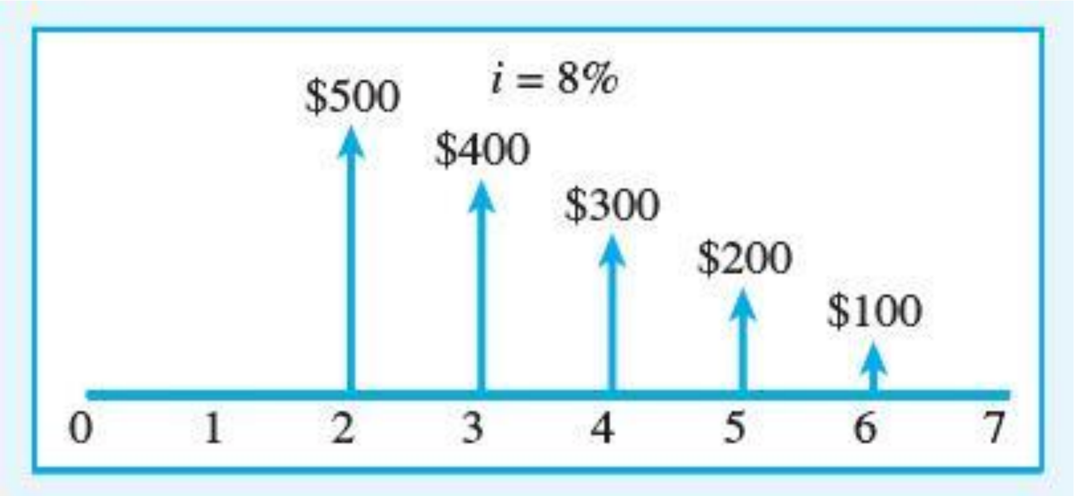
- PV1=PV(15%,2,,-1322.5)=$1,000=PV2
- The two cash flow profiles are equivalent!
- It suggest the worth of the two cash flow profiles will be the same at any particular point in time, e.g., at t_2 or t_6.

A Uniform Series Equivalency of a Gradient Series

Using an 8 percent discount rate, what uniform series over five periods, [1, 5], is equivalent to the cash flow profile given?

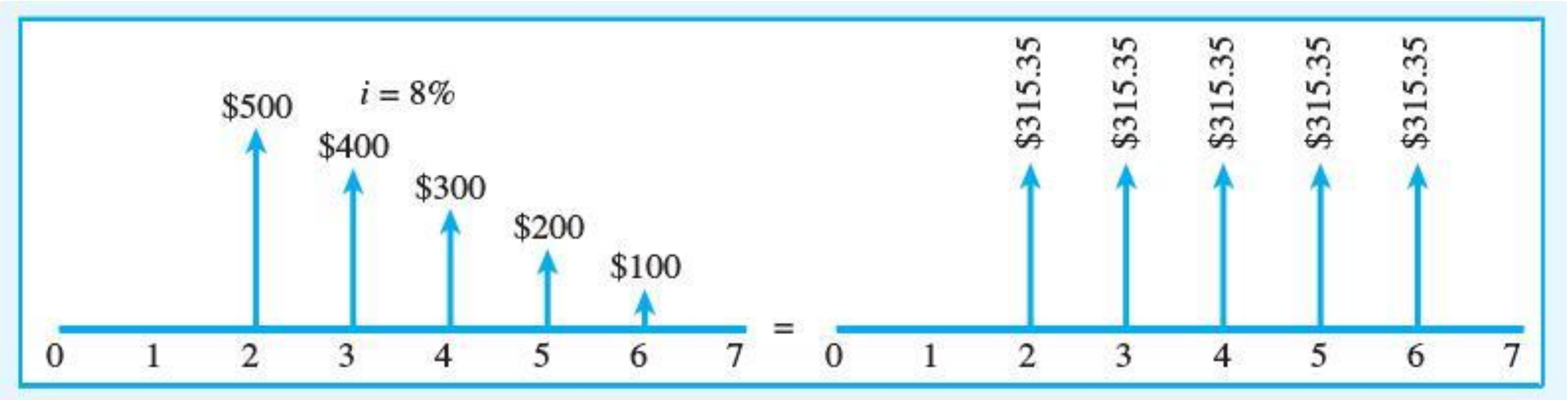


Draw CFD!! Pay attention to the time period!

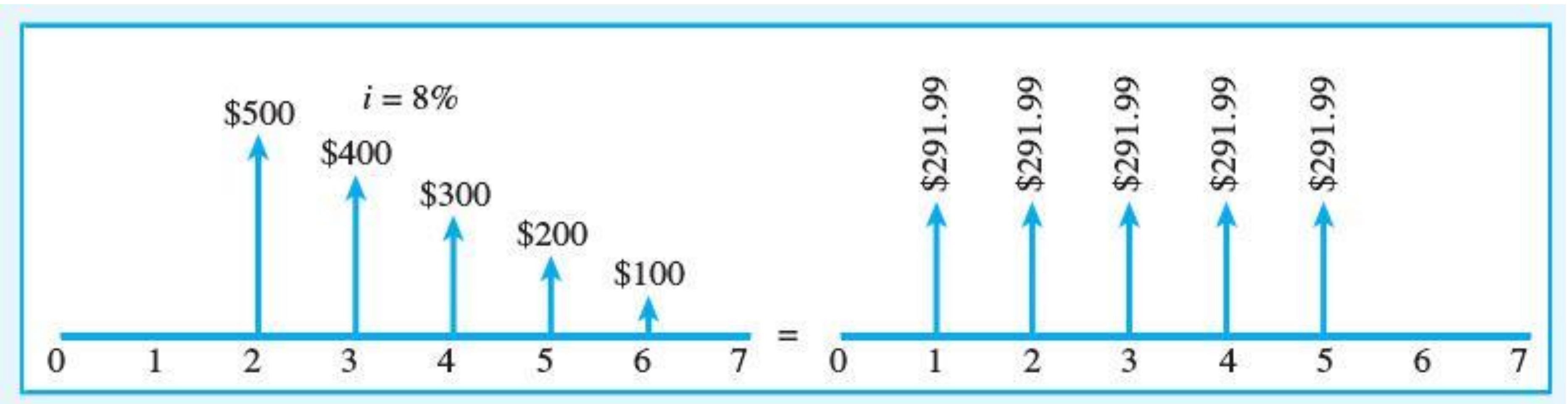


Solution 1:

- P1=100*NPV(0.08,5,4,3,2,1)=1259.1125; P1 occurs at t_1.
- A=PMT(0.08,5,-1259.1125)=315.35; P1 occurs at t_1, and this equivalent uniform series occur at period [2,6], which is one time period after t_1!
- The question is to find the equivalent uniform series at period [1,5], thus discount A backward one time period:
- A'=315.35/(1+8%)=291.99



$500
$400
$300
$200
$100
i = 8%
$315.35
$315.35
$315.35
$315.35
$315.35
=
0
1
2
3
4
5
6
7



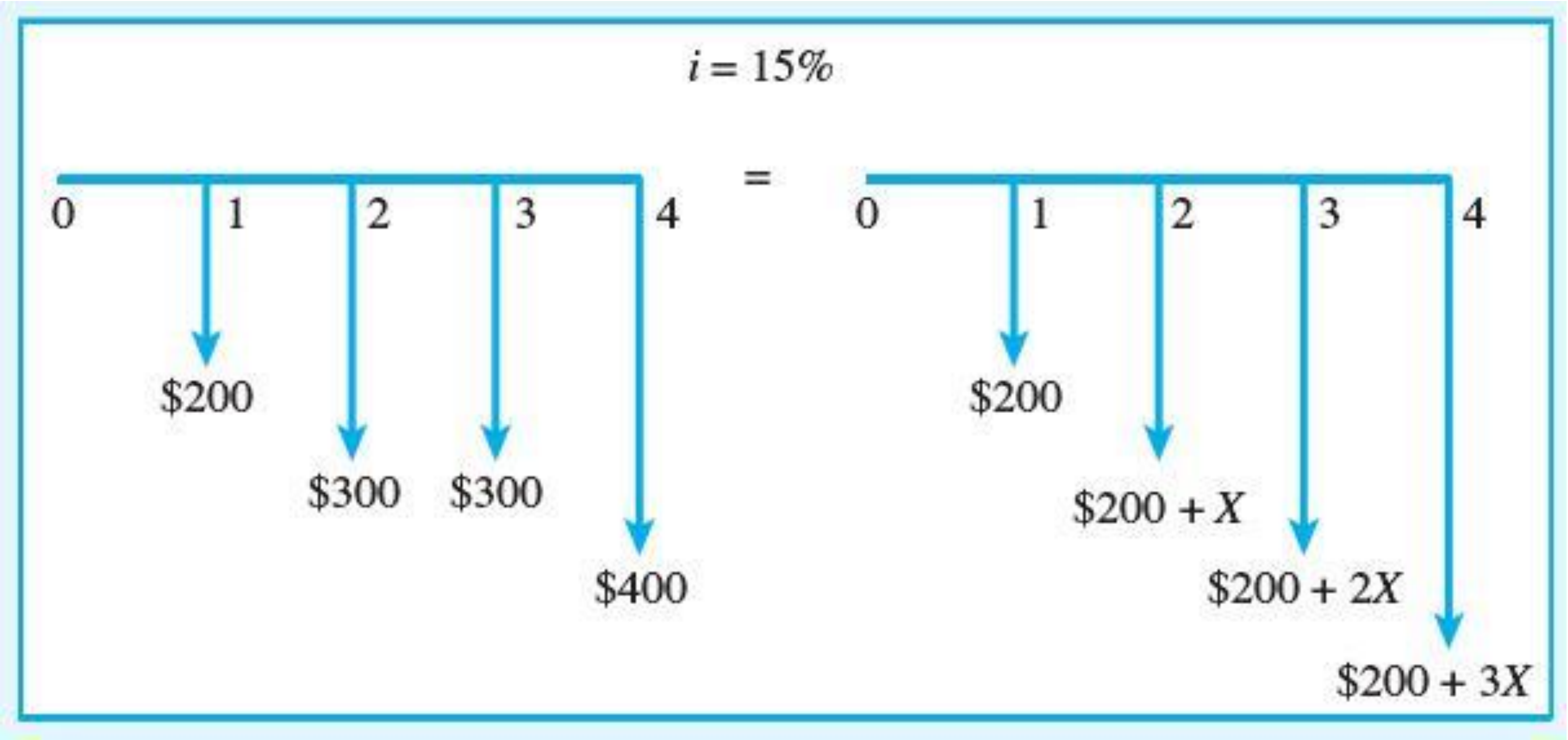
$500
$400
$300
$200
$100
i = 8%
$291.99
$291.99
$291.99
$291.99
$291.99
=
0
1
2
3
4
5
6
7

Solution 2:

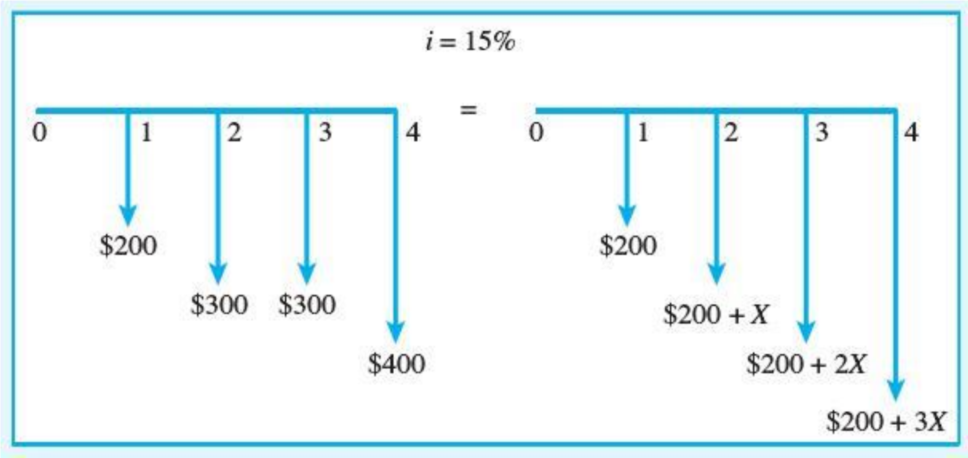
- P1=100*NPV(0.08,5,4,3,2,1)=1259.1125; P1 occurs at t_1.
- Discount P1 to t_0, P0=PV(0.08,1,,-1259.1125)=1165.84
- Then find the equivalent uniform series at period [1,5], thus A=PMT(0.08,5,-1165.84)=291.99

Determining an Equivalent Gradient Step

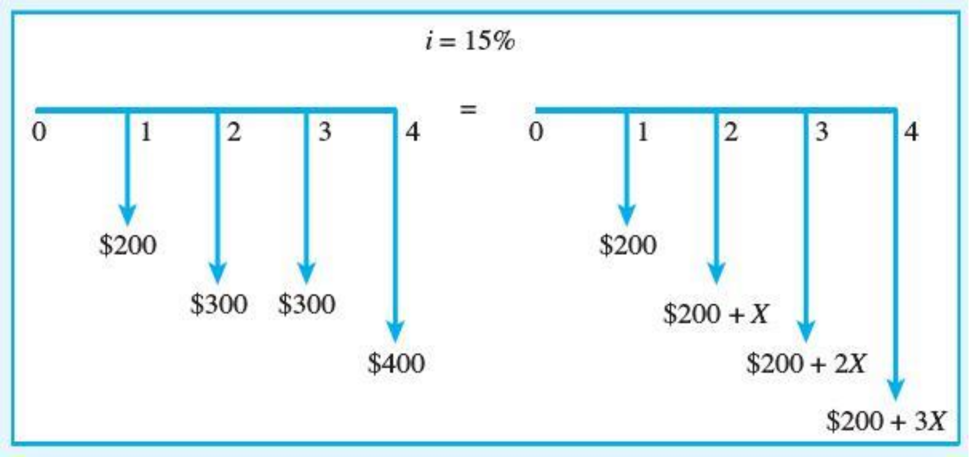
Determine the value of X that makes the two cash flow profiles equivalent using a TVOM of 15 percent.



- Solution 1: breaking down the cash flow on the right into a uniform series A=200 at [1,4], and a gradient series {X, 2X, 3X} at [2,4], calculate PV at t_0
- P=100*NPV(0.15,2,3,3,4)=826.71,
- $P_{uniform}$=PV(0.15,4,-200)=571.00,
- $P_{gradient}$=P-$P_{uniform}$=255.71,
- As the gradient series occurs at [2,4], PV should occur one time period before at t_1, thus move $P_{gradient}$ forward one time period.
- P'=$P_{gradient}$*(1+0.15)=294.07
- X*NPV(0.15,1,2,3)=294.07
- X*4.35=294.07
- X=67.53



- Solution 2: all cash flows minus 200, calculate PV at t_1
- 100*NPV(0.15,1,1,2)=X*NPV(0.15,1,2,3),
- 100*2.94=X*4.35
- X=67.59



- **Solution 3: using the Excel Solver Tool**

  First add on the Solver tool:

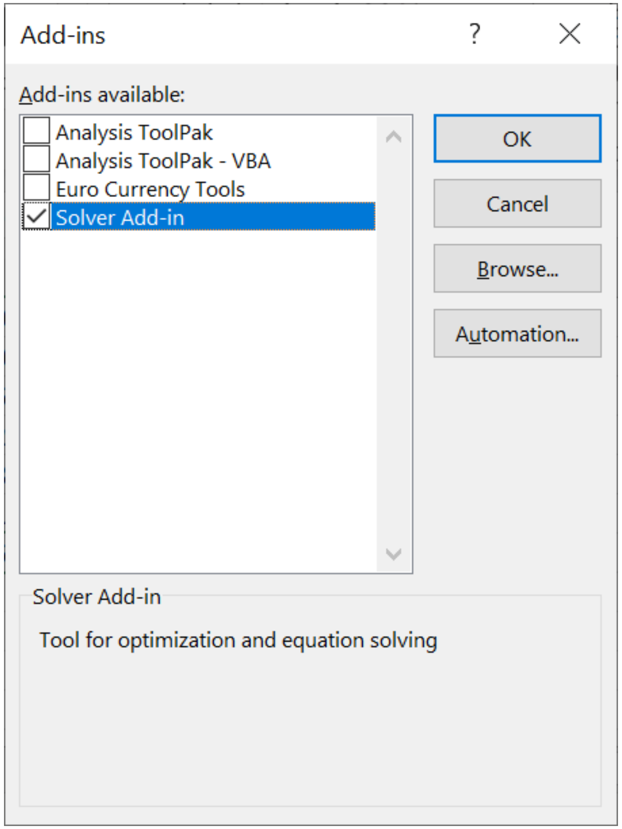
  – In **Excel** 2010 and later, go to File > Options. ...

  – Click **Add**-Ins, and then in the Manage box, select **Excel Add**-ins.

  – Click Go.

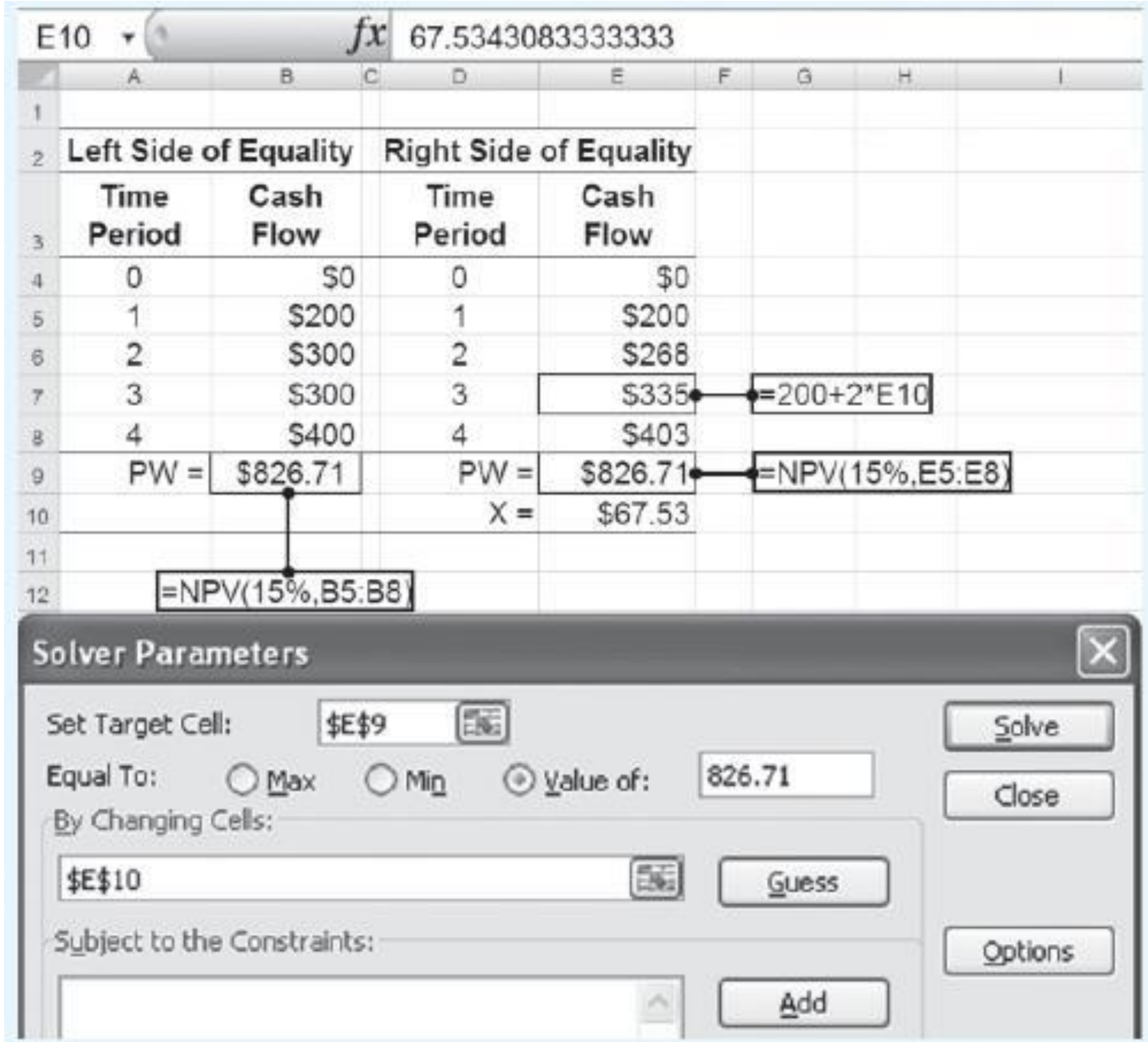
  – In the **Add**-Ins available box, select the **Solver Add**-in check box, and then click OK. ...

  – After you load the **Solver Add**-in, the **Solver** command is available on the Data tab.



Alternatively, search for "Solver" in the search tool bar of Excel.

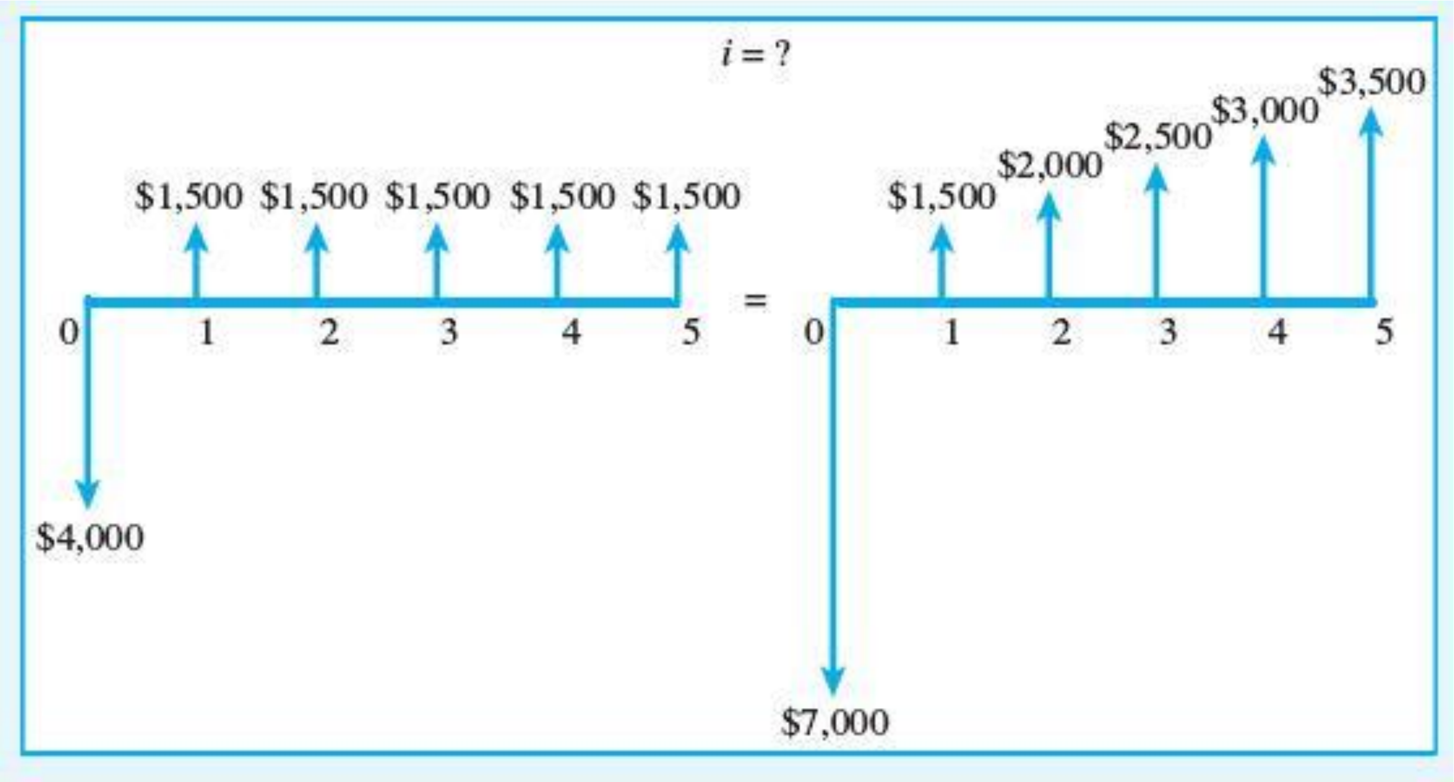
- Let the value of E10 as X to be solved.
- Input the left cash flow.
- Find PV at B9
- Input the right cash flow. For the value of E6, E7, E8, use E10 to substitute X.
- Find PV at E9.
- As E9=B9, open solver, set as the following:
- Set target cell: E9
- Equal to: Value of 826.71
- By changing cells: E10.
- Click Solve, click OK
- X will be returned in E10.



	K	L	M	N	O	P
226	t	left cashflow		right cashflow		
227	0	0		0		
228	1	200		200		
229	2	300		267.534	<-"=200+N233"	
230	3	300		335.069	<-"=200+2*N233"	
231	4	400		402.603	<-"=200+3*N233"	
232	NPV=	£826.71	<-to the value of	£826.71	<-Set objective	
233			X=	67.5343	<-by changing cell	

Determining an Equivalent Interest Rate

For what interest rate are the two cash flow profiles equivalent?



Solution: using the Solver Tool

E12 fx 10%

	A	B	C	D	E
1					
2	Left Side of Equality			Right Side of Equality	
3	Time Period	Cash Flow		Time Period	Cash Flow
4	0	-$4,000		0	-$7,000
5	1	$1,500		1	$1,500
6	2	$1,500		2	$2,000
7	3	$1,500		3	$2,500
8	4	$1,500		4	$3,000
9	5	$1,500		5	$3,500
10	PW_{LS} =	$1,686.18		PW_{RS} =	$2,117.08
11				$PW_{LS}-PW_{RS}$ =	-$430.90
12				i =	10%

=NPV(E12,E5:E9)+E4

E11=B10-E10

=NPV(E12,B5:B9)+B4

Solver Parameters

Set Target Cell: E11

Equal To: Max Min Value of: 0

By Changing Cells:

E12

Solve

Close

Guess

E11 fx =B10-E10

	A	B	C	D	E
1					
2	Left Side of Equality			Right Side of Equality	
3	Time Period	Cash Flow		Time Period	Cash Flow
4	0	-$4,000		0	-$7,000
5	1	$1,500		1	$1,500
6	2	$1,500		2	$2,000
7	3	$1,500		3	$2,500
8	4	$1,500		4	$3,000
9	5	$1,500		5	$3,500
10	PW_{LS} =	$1,166.04		PW_{RS} =	$1,166.04
11				PW_{LS}-PW_{RS} =	$0.00
12				i =	13.8677%

LOANS

- When you have a loan, the (equal sized) payment is repaid every period as a uniform series.
- Some proportion of the payments are paid for the interest (interest payment) and the other are paid for the principal (principal/equity payment).
- **The first thing paid in repaying a loan is interest.**
  - Your payments are first paid for interest.
  - When interest reduces to 0, your payments start to be paid for principal.

Purchasing a Car

Sara Beth wants to purchase a used car in excellent condition. She has decided on a car with low mileage that will cost $20,000. After considering several alternatives, she identified a local lending source that will charge her an interest rate of 6 percent per annum compounded monthly for a 48-month loan:

(a) What will be the size of her monthly payments?

(b) What will be the remaining balance on her loan immediately after making her 24th payment?

(c) If she chooses to pay off the loan at the time of her 36th payment, how much must she pay?

(d) What portion of her 12th payment is interest?

(e) What portion of her 12th payment is an equity payment?

a. i_{per}=6%/12=0.5%/month

A=PMT(0.5%,48,-20000)=$469.70

b. P24=PV(0.5%,24,-469.70)=$10,597.79

c. The payment on the 36th month=the sum of the rest 12 month payments + the payment at the 36th month

P36=PV(0.5%,12,-469.70)+469.70=5457.41+469.70=$5,927.11

d.

- The Excel IPMT function determines the amount of a periodic payment that is interest.
- Parameters in order are: interest rate, period for which the payment occurs, number of periodic payments, present worth, future worth, and type.
- For conventional loans, the future amount and type parameters are not needed.
- I_{12}=IPMT(0.5%,12,48,-20000)=$79.15

e.

- The Excel PPMT function determines the amount of a periodic payment that reduces the unpaid principal on a loan.
- It has the same parameters as IPMT.
- P_{12}= PPMT(0.5%,12,48,-20000)=$390.55

- When asked to find IPMT and PPMT, always find IPMT first! (The interest is always paid back first)
- Find period equal size payment PMT (PMT=IPMT+PPMT).
- If IPMT<PMT, then interest payment=IPMT; PPMT=PMT-IPMT
- If IPMT>=PMT, then interest payment=PMT; PPMT=0
- To conclude, interest payment=MIN(IPMT, PMT)

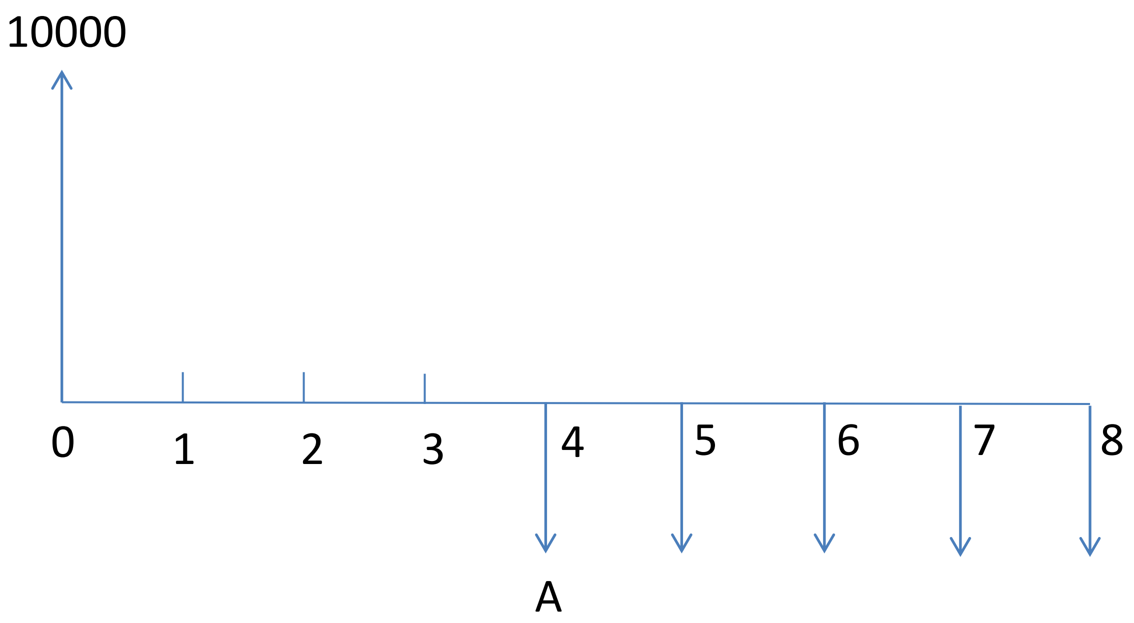
Interest and Equity Payments in Deferred conditions*

The owner of a small business borrows $10,000 at 15 percent annual compound interest. Five equal annual payments will be made to repay the loan, but the first will not occur until 4 years after receipt of the principal amount.

How much of each payment will be paid for the interest and principal?

*optional content, not required

Solution: Draw CFD!



conpound P to t_3, P3=FV(15%,3,,-10000)=15208.75

A=PMT(15%,5,-15208.75)=4537.01

Recall that your payments are paid for interest first!

Year	Unpaid Balance Before Payment (UB)	Interest During Year (Int)	Unpaid Interest Before Payment (UIB)	Amount Owed (AO)	Loan Payment (A_d)	Interest Payment ($IPmt$)	Principal Payment ($PPmt$)	Unpaid Interest After Payment (UIA)	Unpaid Balance After Payment (UBA)
1	$10,000.00	$1,500.00	$1,500.00	$11,500.00	$0.00	$0.00	$0.00	$1,500.00	$11,500.00
2	$11,500.00	$1,725.00	$3,225.00	$13,225.00	$0.00	$0.00	$0.00	$3,225.00	$13,225.00
3	$13,225.00	$1,983.75	$5,208.75	$15,208.75	$0.00	$0.00	$0.00	$5,208.75	$15,208.75
4	$15,208.75	$2,281.31	$7,490.06	$17,490.06	$4,537.01	$4,537.01	$0.00	$2,953.06	$12,953.06
5	$12,953.06	$1,942.96	$4,896.01	$14,896.01	$4,537.01	$4,537.01	$0.00	$359.01	$10,359.01
6	$10,359.01	$1,553.85	$1,912.86	$11,912.86	$4,537.01	$1,912.86	$2,624.15	$0.00	$7,375.85
7	$7,375.85	$1,106.38	$1,106.38	$8,482.23	$4,537.01	$1,106.38	$3,430.63	$0.00	$3,945.22
8	$3,945.22	$591.78	$591.78	$4,537.01	$4,537.01	$591.78	$3,945.22	$0.00	$0.00
	$UB_t = UBA_{t-1}$	$Int_t = IR_t \times UB_t$	$UIB_t = Int_t + UIA_{t-1}$	$AO_t = UB_t + Int_t$	A_{dt}	$IPm_t = \min(UIB_t; A_{dt})$	$PPmt_t = A_{dt} - IPmt_t$	$UIA_t = UIB_t - IPmt_t$	$UBA_t = AO_t - A_{dt}$

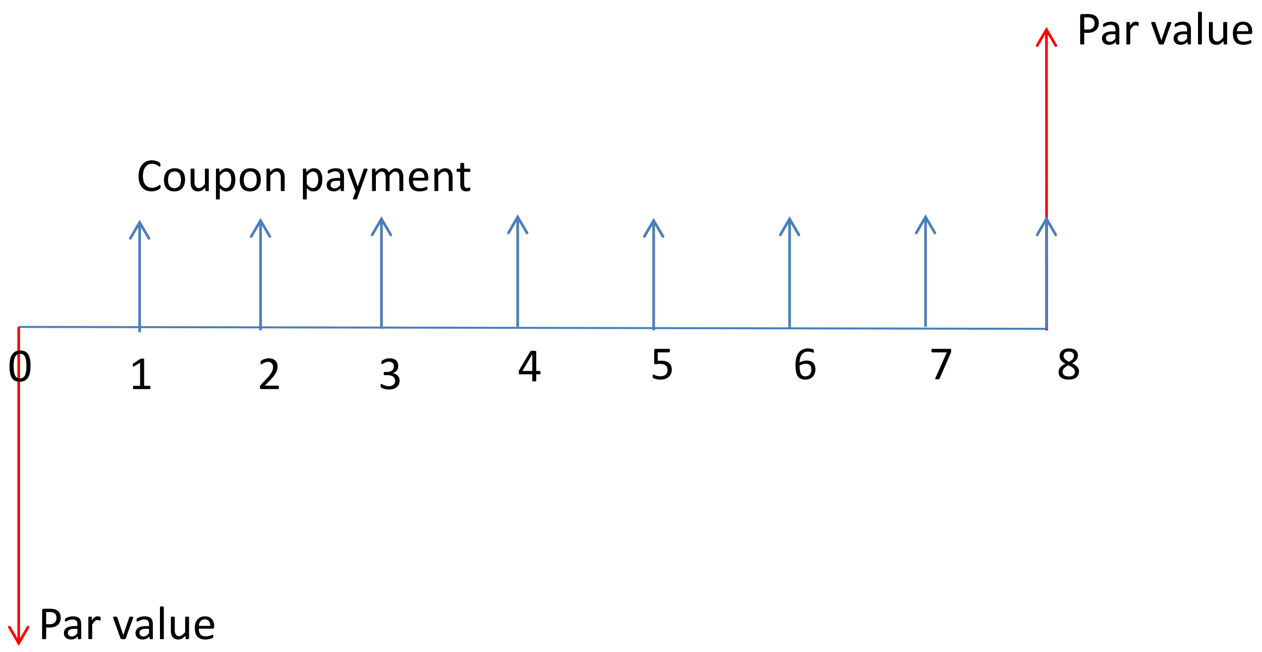
What are the total interest and principal paid when the loan is closing?

- $P3_{interest}$=NPV(0.15,4537.01,4537.01,1912.86,1106.38,591.78)=9560.39
- $FV_{interest}$=FV(0.15,5,,-9560.39)=19229.36
- $P5_{principal}$=NPV(0.15,2624.15,3430.63,3945.22)=7469.96
- $FV_{principal}$=FV(0.15,3,,-7469.96)=11360.88
- *You pay more for the interest than for the principal!!*

BOND

- A bond is a long-term note issued by the borrower (a corporation or governmental agency) to the lender, typically for the purpose of financing a large project.
- The stated value on the individual bond is the face/par value.
- However the price you pay for the bond may differ from its face value.
- The issuing unit is obligated to redeem the bond *at par value* at maturity.
- The issuing unit is obligated to pay a bond rate on the face value between the date of issuance and the date of redemption.
- The interest payment per period is coupon payment.
- Coupon payment=face value*bond rate

CFD of a bond



The buying price or selling price of a bond may be different from its par value!

Determining the Selling Price for a Bond

On January 1, 2011, Austin plans to pay $1,050 for a $1,000, 12 percent semiannual bond. He will keep the bond for 3 years, receive six coupon payments, and then sell it. How much should he sell the bond for in order to receive a yield of 10 percent compounded semiannually?

- bond rate=12%/2=6%, n=6,
- coupon payment=face value*bond rate=1000*6%=60
- i_{per}=r/m=10%/2=5%
- Fbuy=Fpayment+Fsell
- Fbuy=FV(0.05,6,,-1050)=1407.10
- Fpayment=FV(0.05,6,-60)=408.11
- Fsell=Fbuy-Fpayment=998.99

- Bond rate (6%) is only used for calculating coupon payment. In other cases please use interest rate (5%).

Determining the Purchase Price for a Bond

Emma plans to purchase a $1,000, 12 percent semiannual bond, hold it for 3 years, receive six coupon payments, and redeem it at par value. What is the maximum amount she should pay for the bond if she wants to earn at least 14 percent compounded semiannually on her investment?

- bond rate=12%/2=6%, n=6,
- coupon payment=1000*6%=60
- i_{per}=r/m=14%/2=7%
- Psell=PV(0.07,6,,-1000)=666.34
- Ppayment=PV(0.07,6,-60)=285.99
- Pbuy=Psell+Ppayment=952.33

Determining the Rate of Return for a Bond Investment

Charlotte purchased a $1,000, 12 percent quarterly bond for $1,020, kept it for 3 years, received 12 coupon payments, and sold it for $950. What was her quarterly interest rate on her bond investment? What was her effective annual rate of return?

- bond rate=12%/4=3%, n=4*3=12
- Coupon payment=1000*3%=30
- Using Excel Solver Tool
- Fpayment=FV(i,12,-30)
- Fbuy=FV(i,12,,-1020)
- Fsell=950
- Fbuy-Fpayment=Fsell
- i_{per}=0.02442,
- Using Excel EFFECT function
- r=0.02442*4=0.09768
- i_{eff}=EFFECT(0.09768,4)=0.10132

- Alternatively, using Excel RATE function
- Parameters in order are: number of periods (nper), equal size payment (pmt), pv, fv
- i_{per}=RATE(12,30,-1020,950)=2.442%
  - This is from the bond buyer's perspective: bond payment (pmt) and selling price (fv) are positive cashflows, while buying price (pv) is a negative cashflow.
- Or, i_{per}=RATE(12,-30,1020,-950)=2.442%
  - This is from the bond issuer's perspective: bond payment (pmt) and redemption price (fv) are negative cashflows, while selling price (pv) is a positive cashflow.

VARIABLE INTEREST RATES

- So far, the interest rates are fixed in the planning horizon.
- It's NOT likely the case, if the time period extends over several years.
- In such cases, variable interest rates apply.
- $F=P(1+i_1)(1+i_2)\ .\ .\ .\ (1+i_{n-1})(1+i_n)$
- $P=F/[(1+i_1)(1+i_2)\ .\ .\ .\ (1+i_{n-1})(1+i_n)]$

Example

Consider the CFD given with the appropriate interest rates indicated. Determine the present worth, future worth, and uniform series equivalents for the cash flow series.