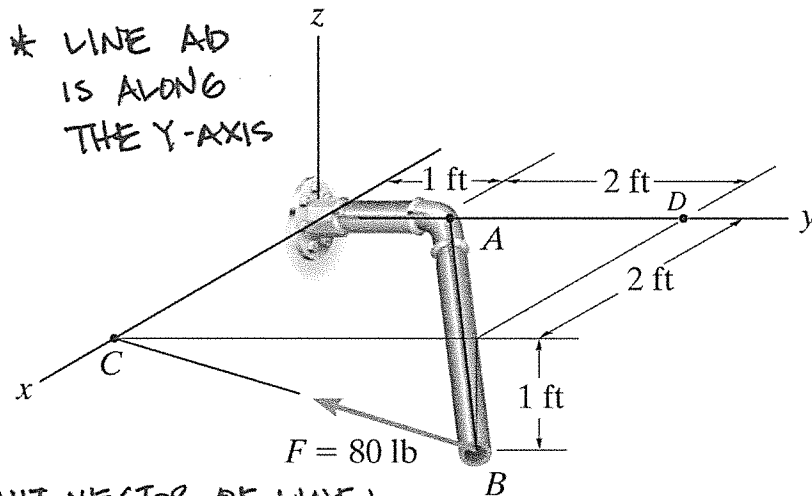


ENSC 2113 – SPRING 2018 – FINAL EXAM

EACH PROBLEM IS WORTH 25 PTS. BOX YOUR ANSWERS AND PROVIDE PROPER UNITS, WHERE APPLICABLE. CALCULATIONS AND FREE BODY DIAGRAMS MUST BE SHOWN THAT SUPPORT THE ANSWER TO RECEIVE CREDIT.

1. A force with a magnitude of 80 lb follows along a line from B to C. Determine the moment created by the force about a line defined from A to D. Express the result as a Cartesian Vector.



COORDINATES:

$$\begin{aligned} A & (0, 1, 0) \\ B & (2, 3, -1) \\ C & (2, 0, 0) \\ D & (0, 3, 0) \end{aligned}$$

POSITION VECTORS:

$$\begin{aligned} \vec{r}_{AC} &= \{2\hat{i} - 1\hat{j} + 0\hat{k}\} \\ \vec{r}_{DC} &= \{2\hat{i} - 3\hat{j} + 0\hat{k}\} \\ \vec{r}_{AB} &= \{2\hat{i} + 2\hat{j} - 1\hat{k}\} \\ \vec{r}_{DB} &= \{2\hat{i} + 0\hat{j} - 1\hat{k}\} \\ \vec{r}_{OC} &= \{2\hat{i} + 0\hat{j} + 0\hat{k}\} \\ \vec{r}_{OB} &= \{2\hat{i} + 3\hat{j} - 1\hat{k}\} \end{aligned}$$

UNIT VECTOR OF LINE:

$$\vec{u}_{AD} = \{0\hat{i} + 1\hat{j} + 0\hat{k}\}$$

FORCE VECTOR:

$$\vec{r}_{BC} = \{0\hat{i} - 3\hat{j} + 1\hat{k}\}$$

$$|\vec{r}_{BC}| = \sqrt{0^2 + (-3)^2 + (1)^2} = \sqrt{10}$$

$$\vec{u}_{BC} = \left\{0\hat{i} - \frac{3}{\sqrt{10}}\hat{j} + \frac{1}{\sqrt{10}}\hat{k}\right\}$$

$$\vec{F}_{BC} = |F| \vec{u}_{BC}$$

$$= 80 \left\{0\hat{i} - \frac{3}{\sqrt{10}}\hat{j} + \frac{1}{\sqrt{10}}\hat{k}\right\}$$

$$= \{0\hat{i} - 75.9\hat{j} + 25.3\hat{k}\} \text{ lb}$$

MOMENT ABOUT A LINE:

$$|M| = (\vec{r} \times \vec{F}) \cdot \vec{u}$$

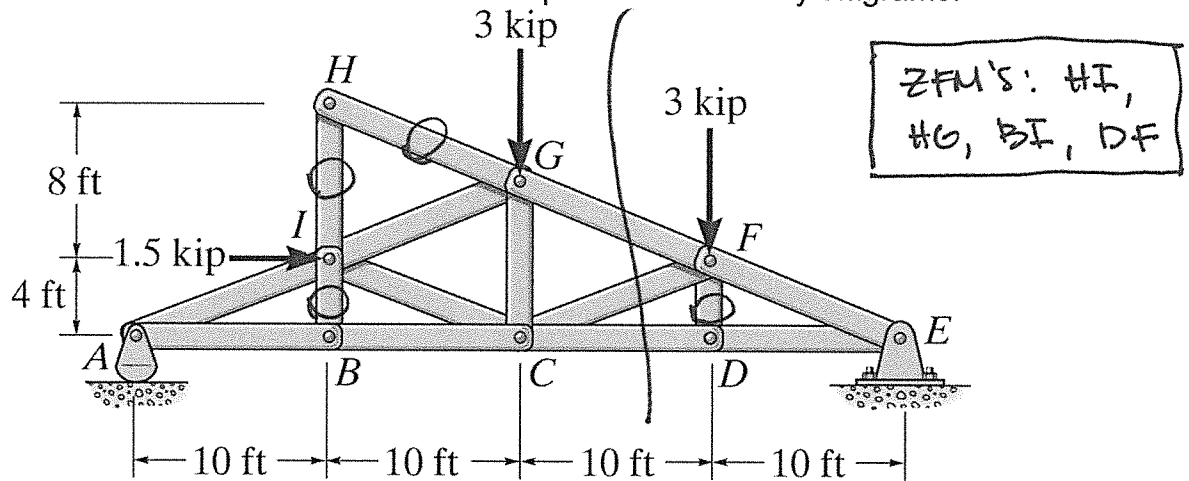
$$= \begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 0 \\ 0 & -75.9 & 25.3 \end{vmatrix}$$

$$\begin{aligned} &= [(0)(25.3) - (0)(-75.9)](0) \\ &\quad - [(2)(25.3) - (0)(0)](1) \\ &\quad + [(2)(-75.9) - (0)(0)](0) \end{aligned}$$

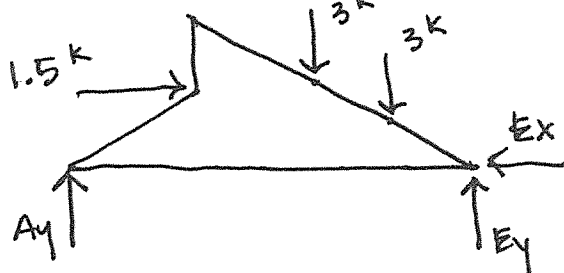
$$= -50.6$$

$$\vec{M} = -50.6 \{0\hat{i} + 1\hat{j} + 0\hat{k}\} = \boxed{\{50.6\hat{j}\} \text{ LB-FT}}$$

2. Determine the force in members GF, CF, and CD by the method of sections and the force in member FE by method of joints and indicate if the members are in tension or compression. The truss is supported with a rocker at A and a pin at E. List all zero-force members and draw all pertinent free-body diagrams.



OVERALL FBD:



$$+\circlearrowleft \sum M_A = 0 = -1.5(4) - 3(20) - 3(30) + E_y(40)$$

$$E_y = 3.9 \text{ k} \uparrow$$

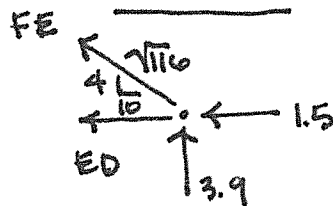
$$+\uparrow \sum F_y = 0 = A_y - 3 - 3 + E_y$$

$$A_y = 2.1 \text{ k} \uparrow$$

$$+\rightarrow \sum F_x = 0 = 1.5 - E_x$$

$$E_x = 1.5 \text{ k} \leftarrow$$

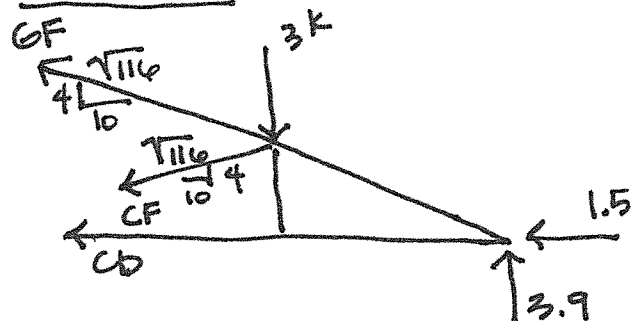
JOINT E:



$$+\uparrow \sum F_y = 0 = 3.9 + \frac{4}{\sqrt{116}} FE$$

$$FE = 10.5 \text{ k (c)}$$

CUT FBD:



$$+\circlearrowleft \sum M_F = 0 = -CD(4) + 3.9(10) - 1.5(4)$$

$$CD = 8.25 \text{ k (T)}$$

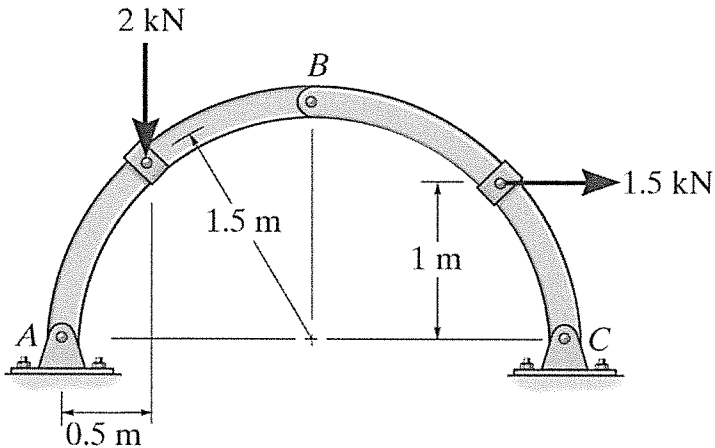
$$+\circlearrowleft \sum M_C = 0 = \frac{10}{\sqrt{116}} GF(4) + \frac{4}{\sqrt{116}} GF(10) - 3(10) + 3.9(20)$$

$$GF = 6.46 \text{ k (c)}$$

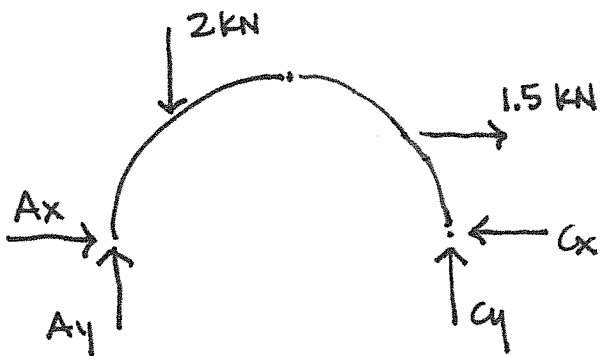
$$+\uparrow \sum F_y = 0 = \frac{4}{\sqrt{116}} (-6.46) - 3 + 3.9 - \frac{4}{\sqrt{116}} CF$$

$$CF = 4.04 \text{ k (c)}$$

3. Given the two-member frame, made up of members AB and BC, calculate the external support reactions for the pins at A and C. Member AB is pinned to member BC with an internal pin at point B. Draw all pertinent free-body diagrams and indicate direction with arrows in your answer. The locations of the applied forces are not pins.



OVERALL FBD:



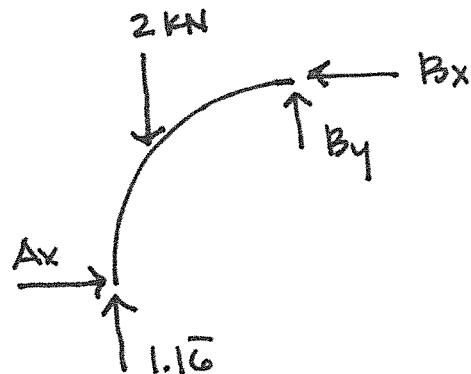
$$+\circlearrowleft \sum M_A = 0 = -2(0.5) - 1.5(1) + C_y(3)$$

$$\boxed{C_y = 0.8\bar{3} \text{ kN } \uparrow}$$

$$\uparrow \sum F_y = 0 = A_y - 2 + 0.8\bar{3}$$

$$\boxed{A_y = 1.1\bar{6} \text{ kN } \uparrow}$$

MEMBER AB:



$$+\circlearrowleft \sum M_B = 0 = 2(1) - 1.1\bar{6}(1.5) + A_x(1.5)$$

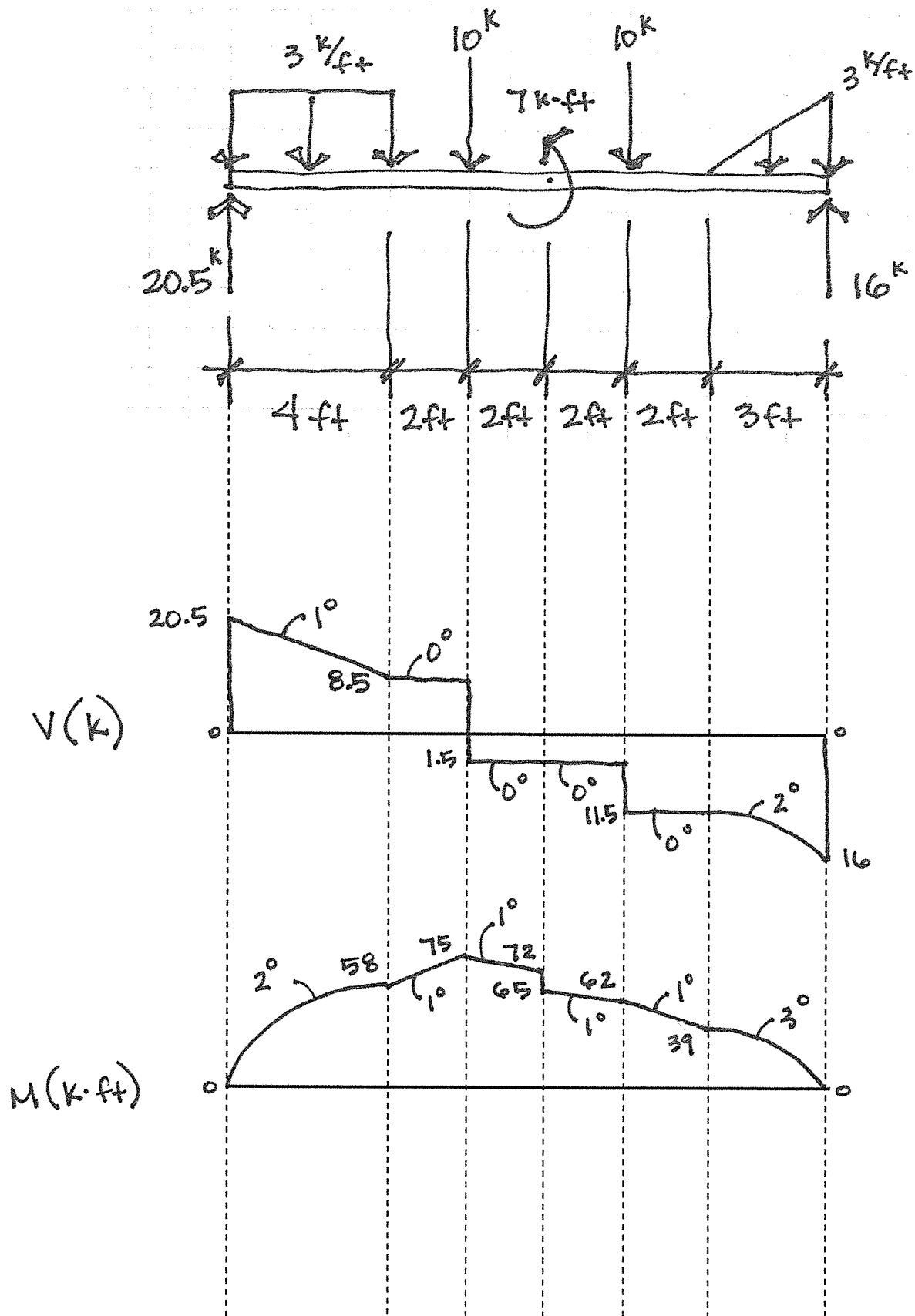
$$\boxed{A_x = 0.1\bar{6} \text{ kN } \leftarrow}$$

FROM OVERALL:

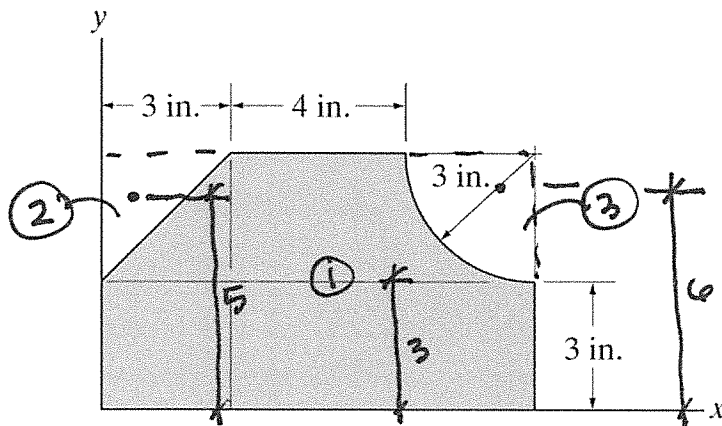
$$\rightarrow \sum F_x = 0 = -0.1\bar{6} + 1.5 - C_x$$

$$\boxed{C_x = 1.3\bar{3} \text{ kN } \leftarrow}$$

4. Draw the shear and bending moment diagrams for the loading condition below. Label all diagrams appropriately.



5. Using tabular form, calculate the moment of inertia about the centroidal x axis, x' , for the object below. The coordinates for the overall centroid of the shape are (4.83 in, 2.56 in).



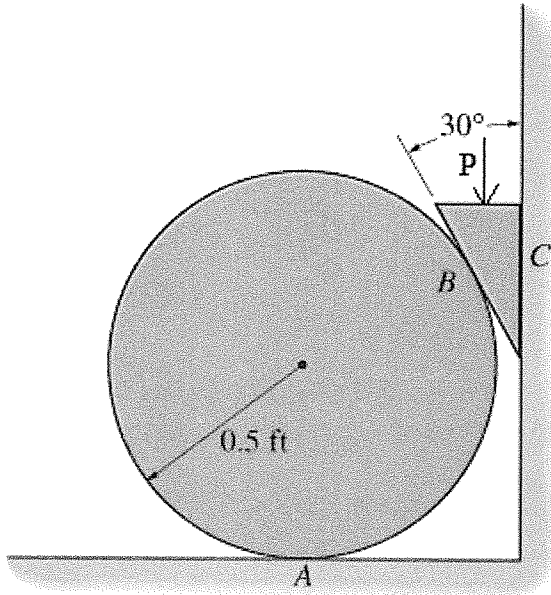
$$I_x = \bar{I}_x' + A(dy)^2$$

$$6 - \frac{4r}{3\pi} = 4.73$$

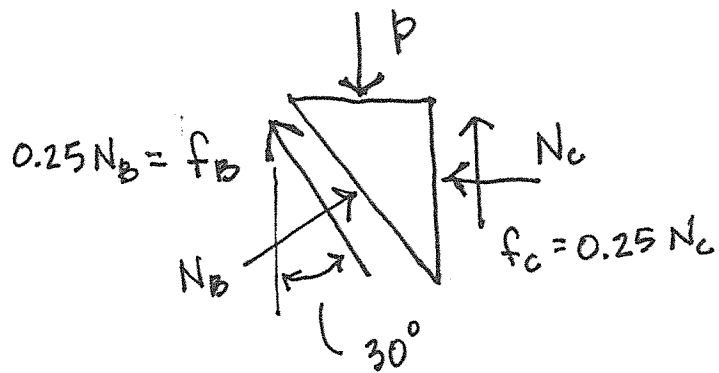
SHAPE	\bar{I}_x'	AREA	dy	$A(dy)^2$
①	$\frac{bh^3}{12} = 180$	$bh = 60$	$2.56 - 3$ -0.44	11.62
② VOID	$-\frac{bh^3}{36} = -2.25$	$\frac{1}{2}bh = 4.5$	$2.56 - 5$ -2.44	-26.79
③ VOID	$-0.05488r^4$ $= -4.45$	$-\frac{\pi r^2}{4}$ $= -7.07$	$2.56 - 4.73$ -2.17	-33.29
	173.3			-48.46

$$I_x = 173.3 - 48.46 = \boxed{124.84 \text{ in}^4}$$

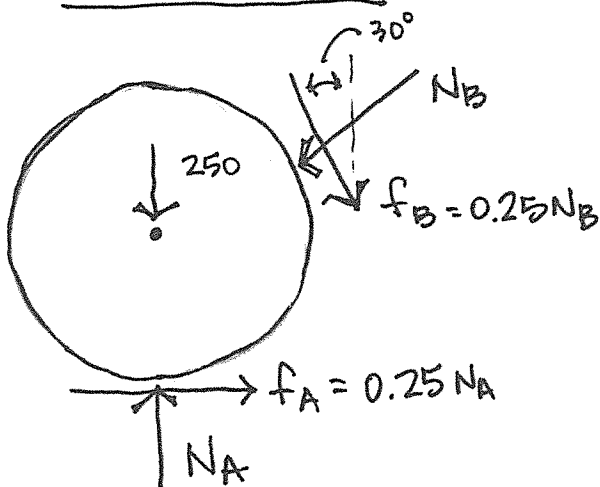
- 3) Determine the force P required for impending motion. The cylinder has a weight of 250 lb and the static coefficient of friction at all surfaces is 0.25. Neglect the size and weight of the wedge. Draw all pertinent free-body diagrams.



WEDGE FBD:



CYLINDER FBD:



FROM CYLINDER FBD:

$$\rightarrow \sum F_x = 0 = 0.25 N_A + 0.25 N_B \sin 30^\circ - N_B \cos 30^\circ$$

$$N_A = 2.96 N_B$$

$$\uparrow \sum F_y = 0 = -250 + 2.96 N_B - 0.25 N_B \cos 30^\circ - N_B \sin 30^\circ$$

$$250 = 2.243 N_B$$

$$N_B = 111.4 \text{ lb}$$

FROM WEDGE:

$$\rightarrow \sum F_x = 0 = -0.25 N_B \sin 30^\circ + N_B \cos 30^\circ - N_C$$

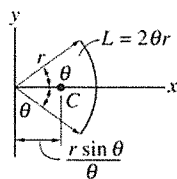
$$N_C = 82.6 \text{ lb}$$

$$\uparrow \sum F_y = 0 = 0.25 N_B \cos 30^\circ + N_B \sin 30^\circ - P + 0.25 N_C$$

$$\boxed{P = 100.5 \text{ lb}}$$

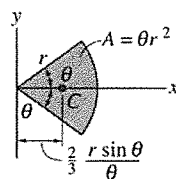
Geometric Properties of Line and Area Elements

Centroid Location



Circular arc segment

Centroid Location

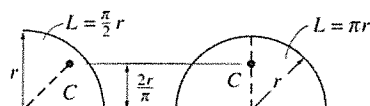


Circular sector area

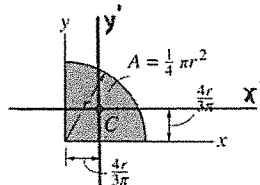
Area Moment of Inertia

$$I_x = \frac{1}{4} r^4 (\theta - \frac{1}{2} \sin 2\theta)$$

$$I_y = \frac{1}{4} r^4 (\theta + \frac{1}{2} \sin 2\theta)$$



Quarter and semicircle arcs

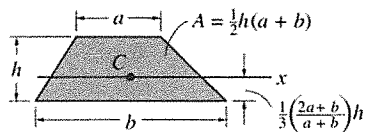


Quarter circle area

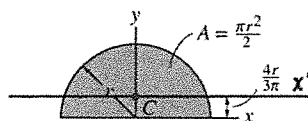
$$I_x' = I_y' = .05488 r^4$$

$$I_x = \frac{1}{16} \pi r^4$$

$$I_y = \frac{1}{16} \pi r^4$$



Trapezoidal area

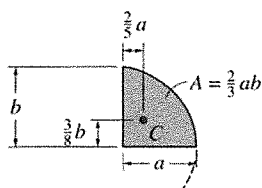


Semicircular area

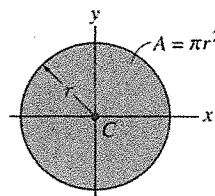
$$I_x' = .1098 r^4$$

$$I_x = \frac{1}{8} \pi r^4$$

$$I_y = \frac{1}{8} \pi r^4$$



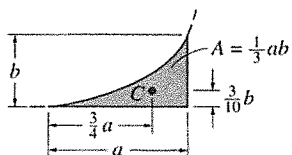
Semiparabolic area



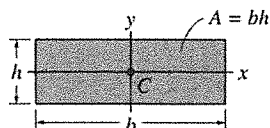
Circular area

$$I_x = \frac{1}{4} \pi r^4$$

$$I_y = \frac{1}{4} \pi r^4$$



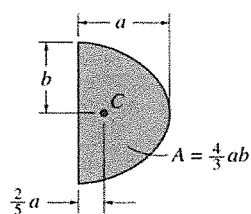
Exparabolic area



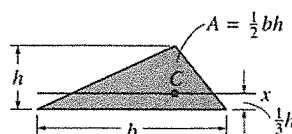
Rectangular area

$$I_x = \frac{1}{12} b h^3$$

$$I_y = \frac{1}{12} h b^3$$



Parabolic area



Triangular area

