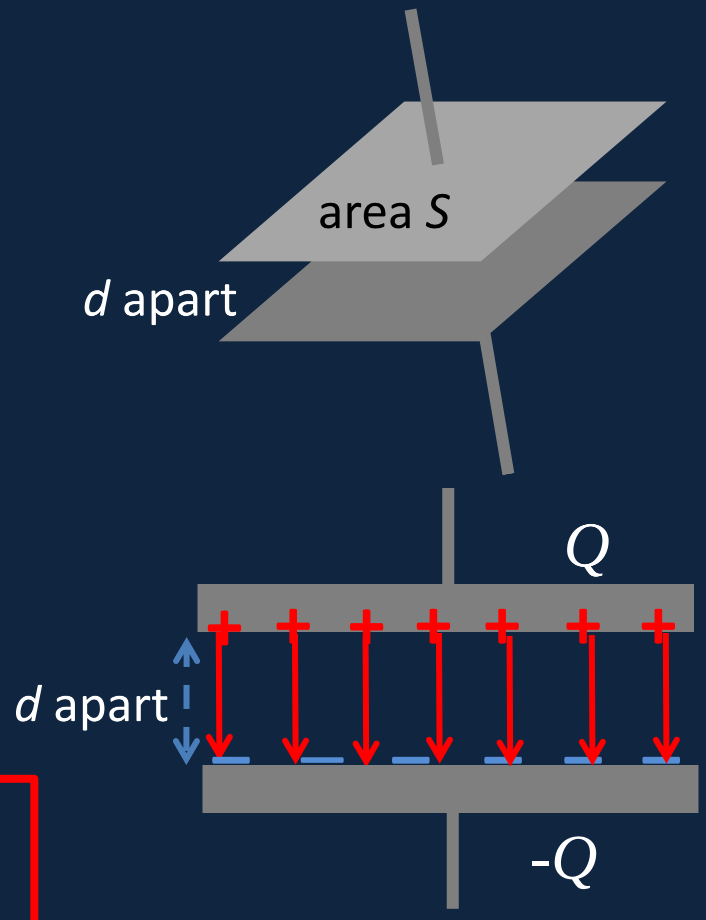
Capacitors

Parallel Plate Capacitor

- Almost all capacitors are parallel plate capacitors: two conducting plates each of area S a constant distance d apart.
- For total charge Q on the top plate and $-Q$ on the bottom, taking $d \ll \sqrt{S}$,
- $E = \sigma/\varepsilon_0 = Q/(S\varepsilon_0)$ and $V = Ed$, so

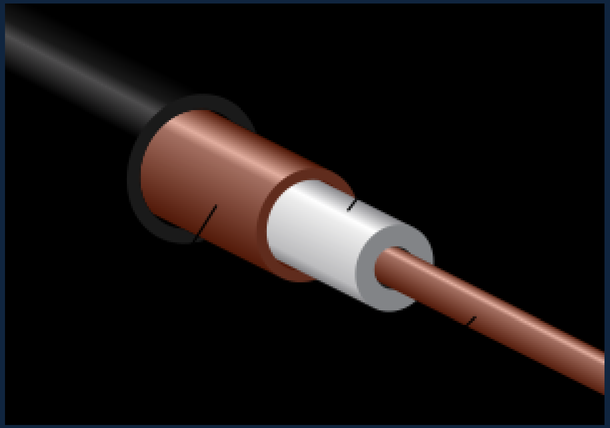
$$V = \frac{Qd}{S\varepsilon_0} = \frac{Q}{C} \text{ where } \boxed{C = \varepsilon_0 \frac{S}{d}}$$

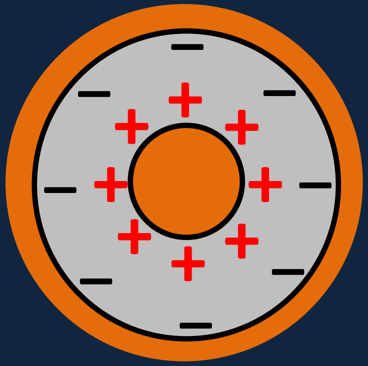


*Charge will settle on **inside** surfaces*

Cylindrical Capacitor

- A coaxial cable is a cylindrical capacitor.
- For charge density λ C/m on the inside wire (and so $-\lambda$ on the inside of the outer cylinder) the radial field $E = \lambda/(2\pi\varepsilon_0 r)$ and





$$V = \int_{R_1}^{R_2} E(r)\,dr = \frac{\lambda}{2\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{dr}{r} = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R_2}{R_1}$$

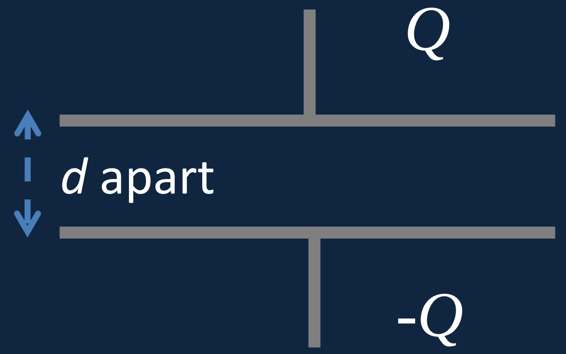
- so

$$\boxed{C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 \ell}{\ln(R_2/R_1)} \quad \text{for length } \ell}$$

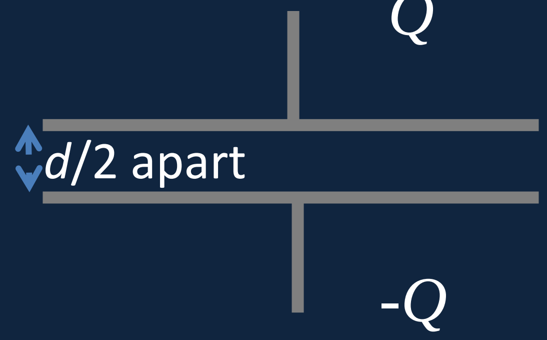
Clicker Question

- The two capacitors on the right have the same size plates, and hold the same charge Q.
- For which capacitor the voltage V between the plates is greater?

A:



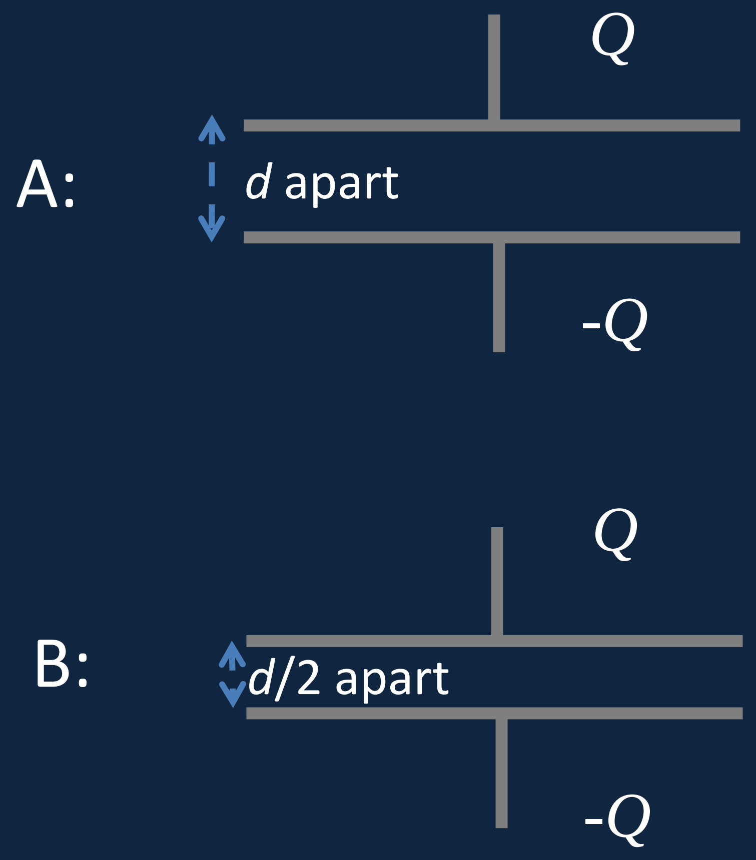
B:



C: they have the same V

Clicker Answer

- The two capacitors on the right have the same size plates, and hold the same charge Q.
- The same charge density means the same strength electric field between the plates: so to move a charge between plates for A takes twice the work—twice the voltage.



C: they have the same V

Clicker Question

- If the voltage applied to a parallel plate capacitor is doubled, what happens to the capacitance C?

A. It’s doubled

B. It’s halved

C. It doesn’t change

Clicker Answer

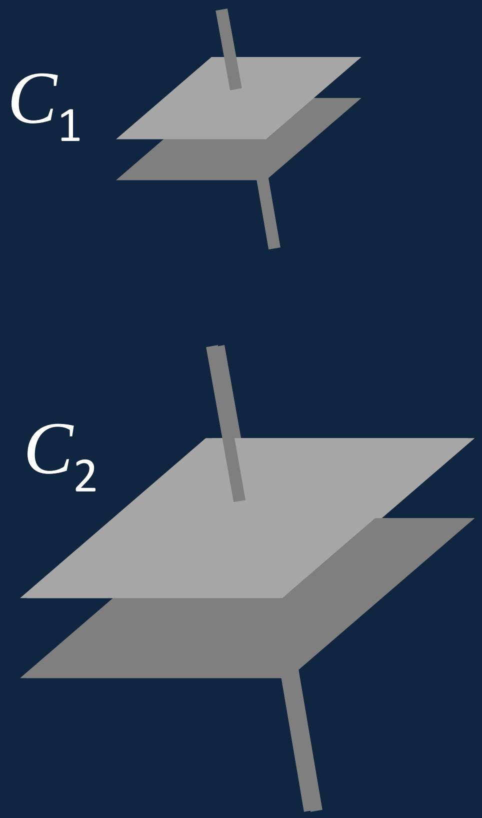
- If the voltage applied to a parallel plate capacitor is doubled, what happens to the capacitance C?

<u>It doesn't change</u>: capacitance depends on plate area and plate separation, NOT on the charge stored.

Don't use formulas before thinking, however briefly!

Clicker Question

- Capacitor C_2 is a scale model of capacitor C_1, with all linear dimensions up by a factor of 2.
- What is the ratio of capacitances?



A. C_2 is 8 times C_1

B. C_2 is 4 times C_1

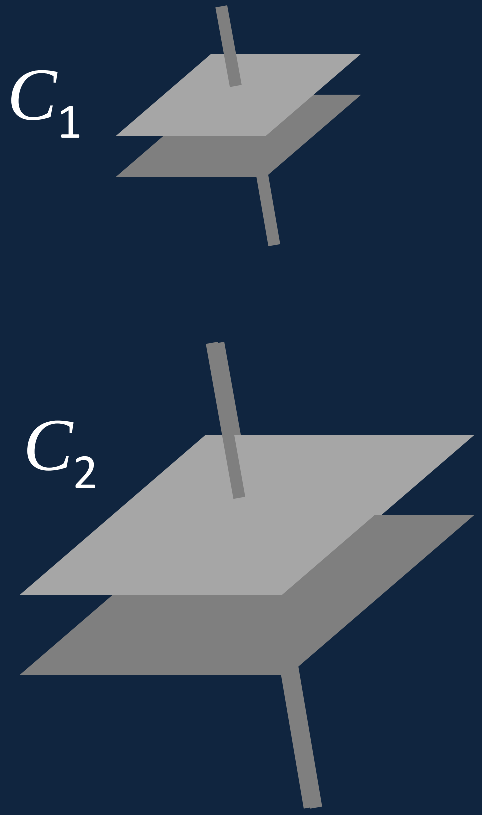
C. C_2 is twice C_1

Clicker Answer

- CapacitorC_2 is a scale model of capacitor C_1, with all linear dimensions up by a factor of 2.
- What is the ratio of capacitances?

C_2 is twice C_1:

Area is up by a factor of 4, but doubling of separation distance halves capacitance.



Clicker Question

- A parallel plate capacitor, consisting simply of two metal plates held parallel, is charged with a battery to a voltage V, then the battery is disconnected.
- The plates are now physically pulled further apart : what happens to the voltage?

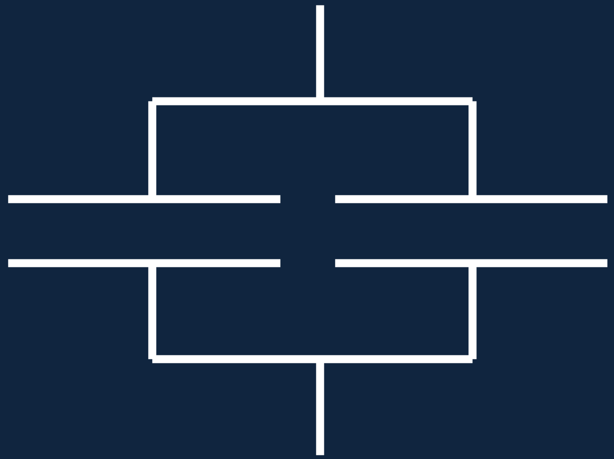
A. It stays the same
B. It decreases
C. It increases

Clicker Answer

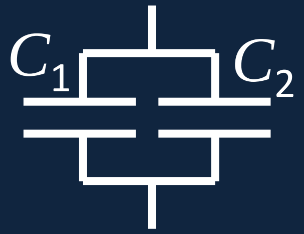
- A parallel plate capacitor, consisting simply of two metal plates held parallel, is charged with a battery to a voltage V, then the battery is disconnected.
- The plates are now physically pulled further apart : what happens to the voltage?
- It increases: the charge on the plates cannot change, so the electric field stays the same, and voltage is field strength x distance.

Capacitors in Parallel

- Let's look first at hooking up two identical parallel plate capacitors in parallel: that means the wires from the two top plates are joined, similarly at the bottom, so effectively they become one capacitor.
- What is its capacitance? From the picture, the combined capacitor has twice the area of plates, the same distance apart.
- We see that $\boxed{C = C_1 + C_2}$



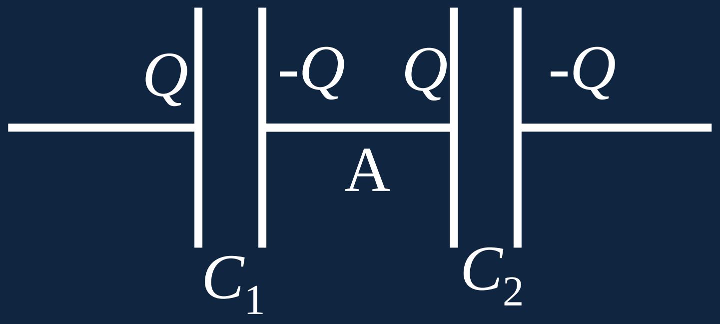
Capacitors in Parallel



- If two capacitors C_1, C_2 are wired together as shown they have the same voltage V between plates.
- Hence they hold charges $Q_1 = C_1V$, $Q_2 = C_2V$, for total charge $Q = Q_1 + Q_2 = (C_1 + C_2)V = CV$.
- So capacitors in parallel just add:

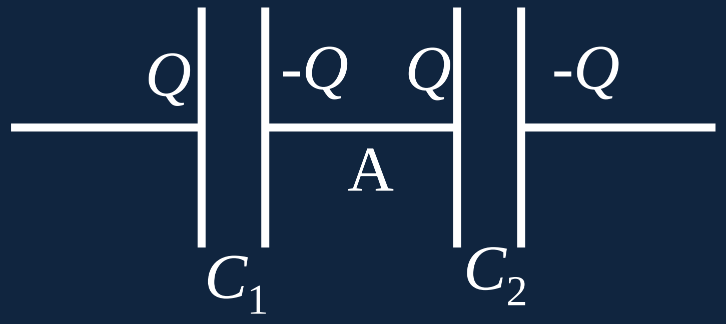
$$\boxed{C = C_1 + C_2 + C_3 + \ldots}$$

Capacitors in Series



- Regarding the above as a single capacitor, the important thing to realize is that in adding charge via the outside end wires, no charge is added to the central section labeled A: it's isolated by the gaps between the plates.
- Charge Q on the outside plate of C_1 will attract $-Q$ to the other plate, this has to come from C_2, as shown.
- Series capacitors all hold the same charge.

Capacitors in Series



- <u>Series capacitors all hold the same charge.</u>
- The voltage drop V_1 across C_1 is $V_1 = Q/C_1$.
- The voltage drop across C_2 is $V_2 = Q/C_2$.
- Denoting the total capacitance of the two taken together as C, then the total voltage drop is $V = Q/C$.
- But $V = V_1 + V_2$, so $Q/C = Q/C_1 + Q/C_2$,

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$

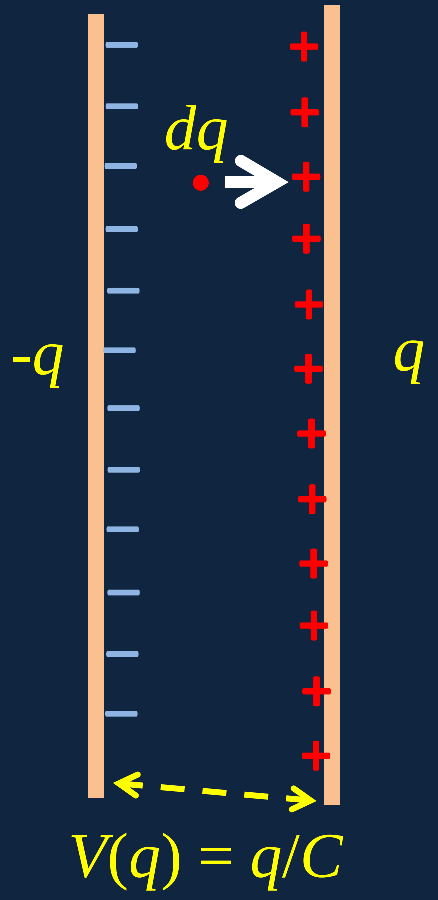
It Takes Work to Charge a Capacitor

- Suppose a sphere capacitor C already contains charge q. Then it's at a potential $V(q) = q/C$.
- To bring a further little charge dq from far away, against the repulsive force of the charge already there, takes work $V(q)dq$.
- So, to deliver a total charge Q to the capacitor, one bit dq at a time, takes total work:

$$W = \int_0^Q V(q)\,dq = \int_0^Q \frac{q}{C}\,dq = \frac{Q^2}{2C}$$

Work Done in Charging a *Parallel Plate* Capacitor

- The math is identical to charging a sphere by bringing up little charges from far away (see the last slide) but for the parallel plate capacitor we only have to bring charge across from one plate to another: the work is still $V(q)dq$ for each dq.
- A capacitor is actually charged, of course, by using a battery to pump charge from one plate to the other via an outside wire, but the route taken doesn't affect the gain in potential energy of the charge transferred.



Energy Stored in a Capacitor

- The work needed to place charge in a capacitor is stored as electrostatic potential energy in the capacitor:

$$\boxed{U = \frac{Q^2}{2C} = \tfrac{1}{2} C V^2 = \tfrac{1}{2} Q V}$$

- We proved the first, the others come from $V = Q/C$.

Clicker Question

- How much work does it take to put charge Q on to a 2mF capacitor compared with putting the same charge on to a 1mF capacitor?

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV$$

A. Twice as much
B. The same
C. Half as much

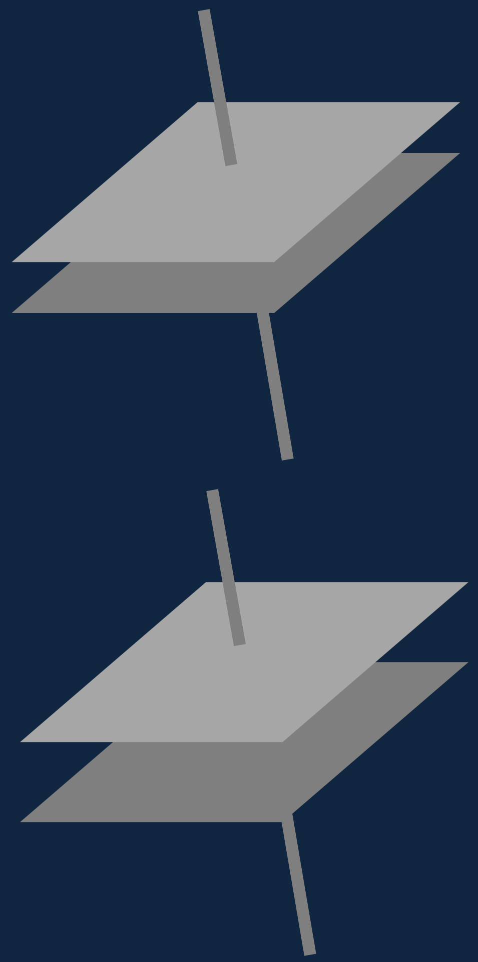
Clicker Answer

- How much work does it take to put charge Q on to a 2mF capacitor compared with putting the same charge on to a 1mF capacitor?

- <u>Half as much</u>: the 2mF capacitor is only at half the voltage of the 1mF capacitor if they have the same charge q, so bringing up extra charge dq takes only half the work.

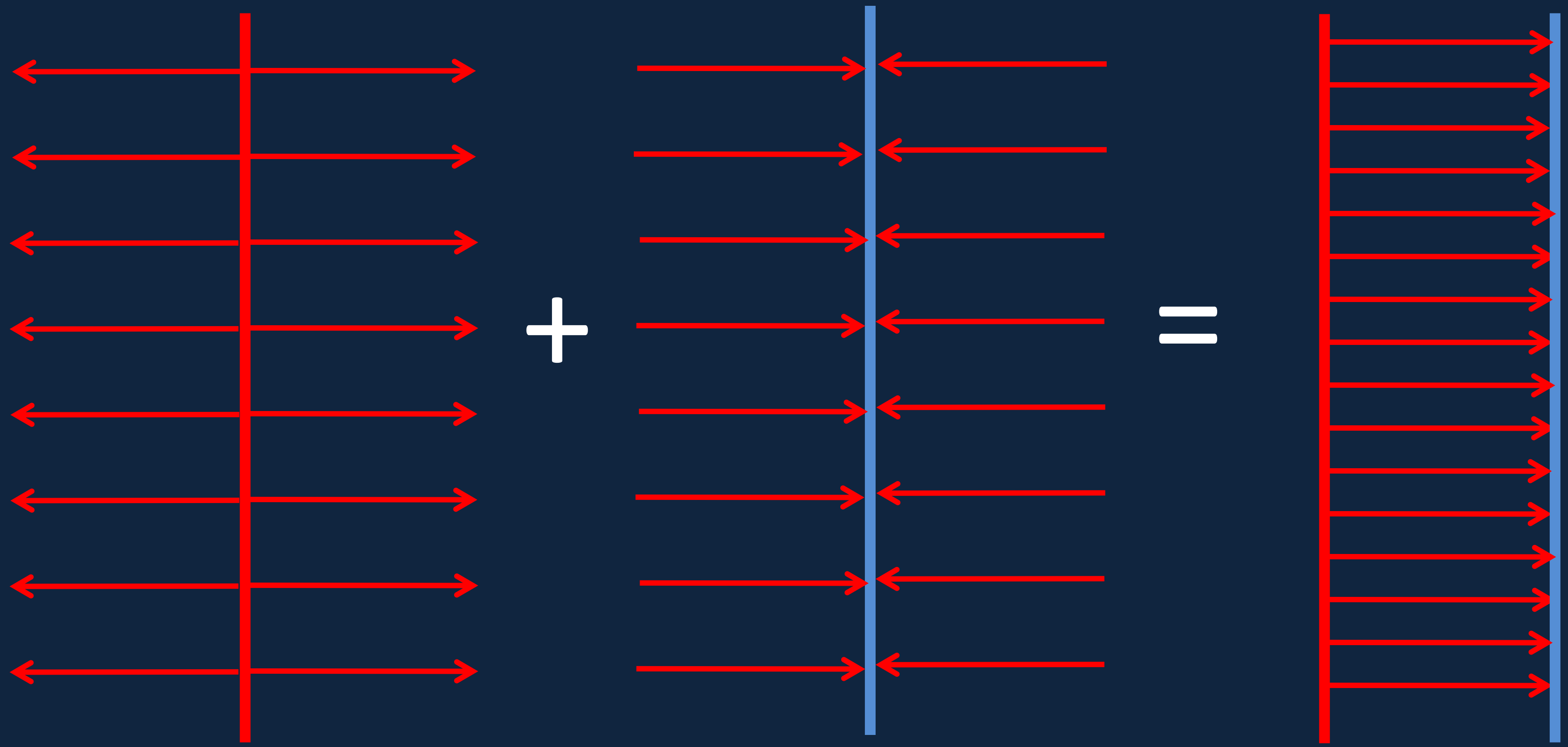
Pulling the Plates Apart

- Suppose we have charge Q on a parallel plate capacitor having area S and plate separation d.
- We now pull the plates to a greater distance apart, say $2d$.
- Assume first that the capacitor is disconnected from the battery, so no charge can flow.
- Since the plates are oppositely charged, it takes work to pull them apart.



Reminder from lecture 3

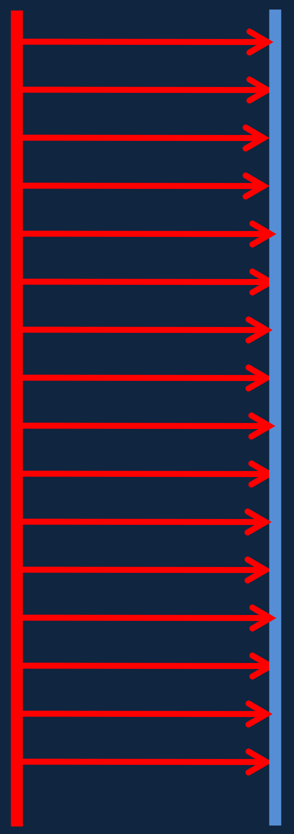
Field for Two Oppositely Charged Planes



Superpose the field lines from the negatively charged plate on the parallel positively charged one, and you'll see the total field is double in the space between the plates, but exactly <u>zero</u> outside the plates.

Working to Pull the Plates Apart

- From the last slide, the electric field $E = \sigma/\varepsilon_0$ between the plates is $\sigma/2\varepsilon_0$ from the top plate and $\sigma/2\varepsilon_0$ from the bottom plate.
- Therefore, in finding the work done against the electric field in moving the top plate, charge Q, we can only count the field from the bottom plate—a charge can't do work moving in its own field!



Working to Pull the Plates Apart

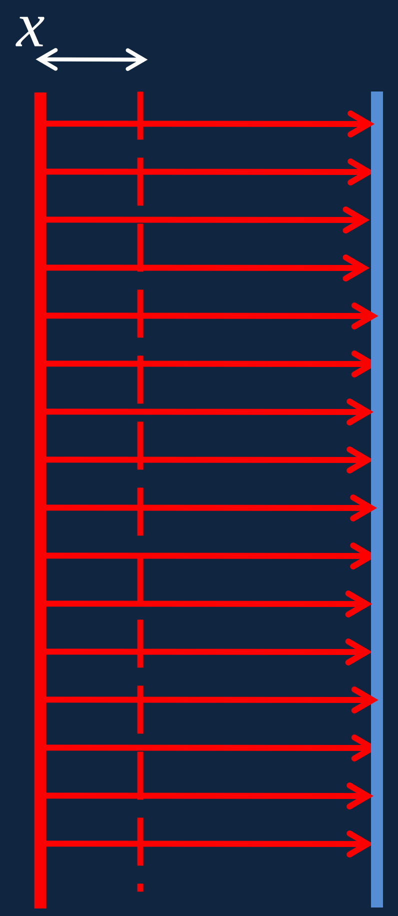
- Moving the top plate, charge $Q = S\sigma$, a distance x outwards in the electric field $\sigma/2\varepsilon_0$ from the bottom plate takes work

$$W = Fx = Q\frac{\sigma}{2\varepsilon_0}x = \frac{Q^2}{2\varepsilon_0 S}x = \tfrac{1}{2}Q^2\frac{x}{\varepsilon_0 S}$$

- Now initially $\dfrac{1}{C} = \dfrac{d}{\varepsilon_0 S}$, finally $\dfrac{1}{C} = \dfrac{d+x}{\varepsilon_0 S}$

so work done = capacitor energy change:

$$W = \frac{Q^2}{2C_{\text{final}}} - \frac{Q^2}{2C_{\text{initial}}}$$

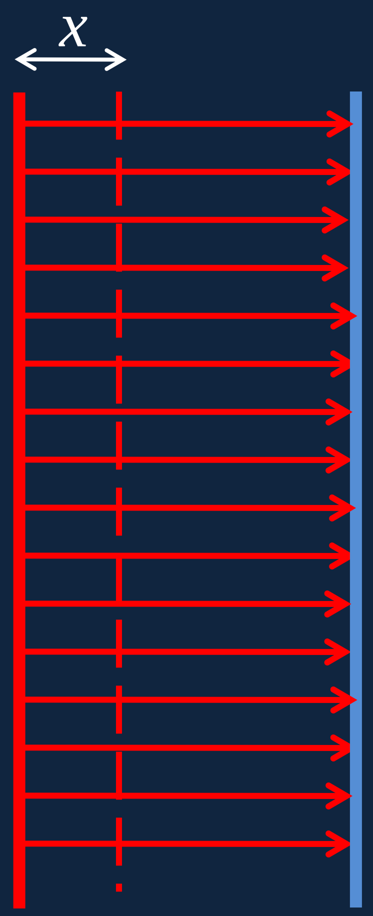


Working to Pull the Plates Apart

- Moving the top plate, charge Q, a distance x outwards created an extra volume $\Delta V = Sx$ between the plates, filled with the constant electric field E, and took work:

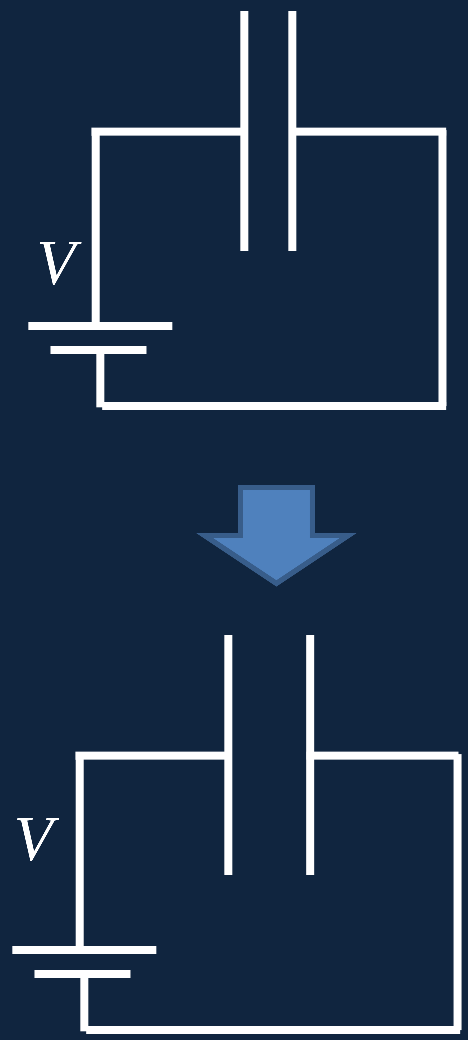
$$W = \frac{Q\sigma}{2\varepsilon_0}x = \frac{\sigma^2 Sx}{2\varepsilon_0} = \tfrac{1}{2}\varepsilon_0 E^2 \Delta V$$

- **The electric field itself is the store of energy**: and this is true in general, for varying as well as constant fields, the energy density in an electric field is $\underline{\tfrac{1}{2}\varepsilon_0 E^2}$.



Clicker Question

- Suppose the parallel plates are pulled apart from separation d to $2d$, the plates having a constant potential difference V from a battery.
- What happens to the electric field strength between the plates?



A. It's doubled

B. It's halved

C. It's constant

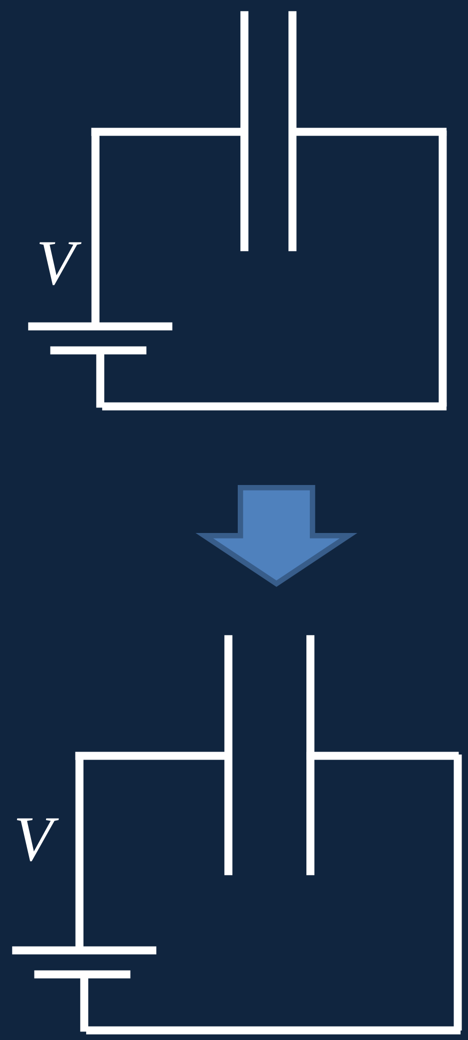
Clicker Answer

- Suppose the parallel plates are pulled apart from separation d to $2d$, the plates having a constant potential difference V from a battery.
- What happens to the electric field strength between the plates?

A. It’s doubled

B. It’s halved

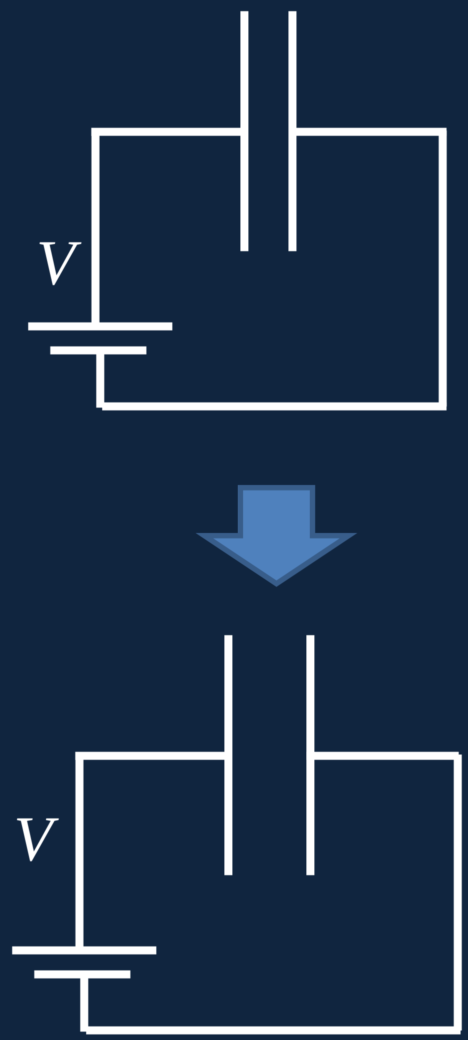
same voltage, double distance: V/m.



Clicker Question

- Suppose the parallel plates are pulled apart from separation d to $2d$, the plates having a constant potential difference V from a battery.
- What happens to the total energy stored in the capacitor?

A. It's doubled
B. It's halved
C. It's constant
D. It increases by a factor of 4
E. It decreases by a factor of 4



Clicker Answer

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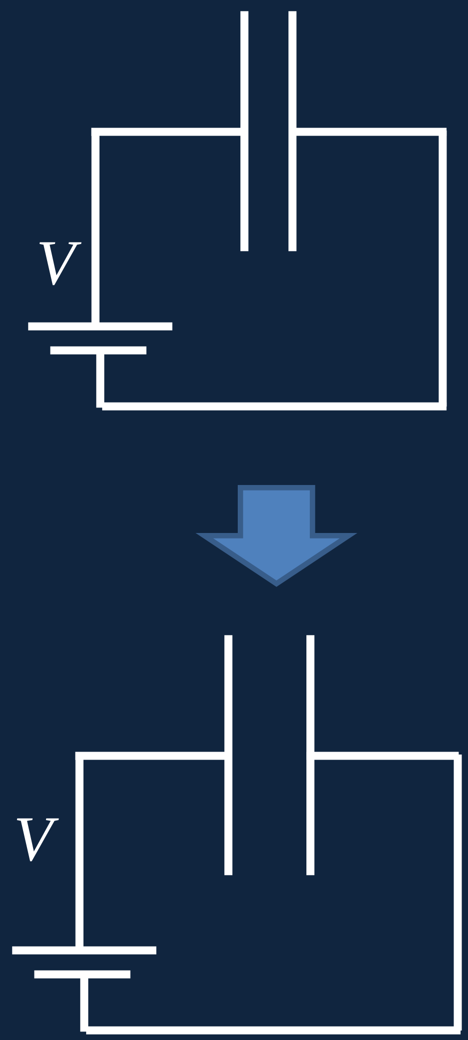
- What happens to the total energy stored in the capacitor?

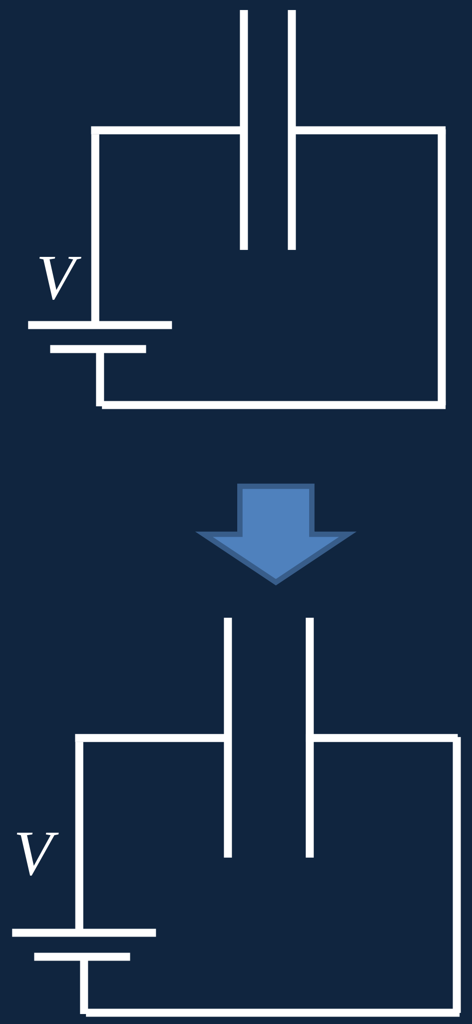


A. It’s doubled
B. It’s halved ← $U = \frac{1}{2}CV^2$
C. It’s constant
D. It increases by a factor of 4
E. It decreases by a factor of 4

Puzzle…

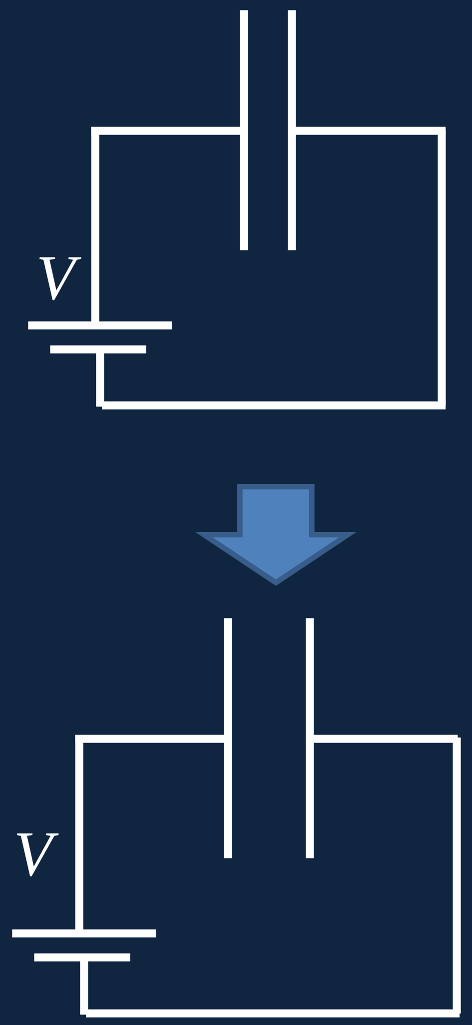
- Pulling the plates apart takes external work, since the plates are oppositely charged and attract each other.
- Yet after pulling them to double the initial separation, there is *less energy* stored in the capacitor than before!

- What about conservation of energy?



Puzzle Answer

- What about conservation of energy?
- The capacitance goes down, the voltage is constant, so charge flows from the capacitance into the battery. ($Q = CV$)
- And, it flows the *wrong way*—against the battery's potential, so this takes work. The battery is being charged, it's storing energy.



Field Energy for a Charged Sphere

- For a charged spherical conductor of radius R:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q\hat{r}}{r^2} \quad \text{and} \quad V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}.$$

- The energy stored in the electric field is

$$U = \int \tfrac{1}{2}\varepsilon_0 E^2 dv = \tfrac{1}{2}\varepsilon_0 \int_R^\infty \left(\frac{Q}{4\pi\varepsilon_0 r^2}\right)^2 4\pi r^2 dr = \frac{Q^2}{8\pi\varepsilon_0 R}$$

and this is just $\tfrac{1}{2}QV$, so the capacitor's energy *is* in the electric field.

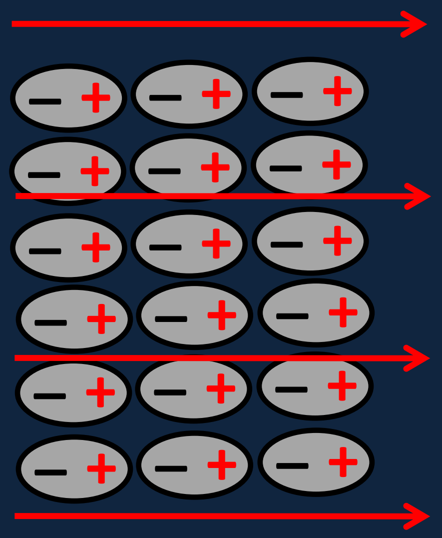
How Big is an Electron?

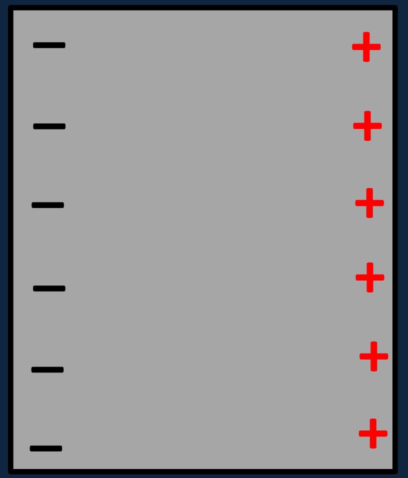
- We've just seen that a charge Q on a sphere of radius R has electric field energy

$$U = \frac{Q^2}{8\pi\varepsilon_0 R}$$

- This means that since the electron has a charge, if the inverse square law holds up at smaller and smaller distances, it can't be infinitely small!
- A lower limit on its size is given by assuming its mass comes entirely from this electrostatic energy, using $U = E = mc^2$.
- This gives R about 10^{-15}m: called the classical radius of the electron. In fact, at this R, Coulomb's law breaks down—and we need quantum mechanics.

Dielectrics

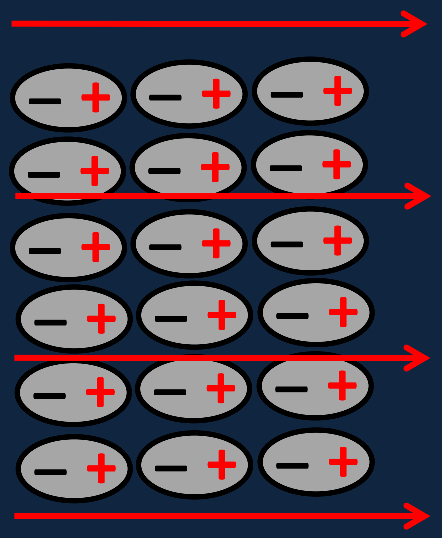


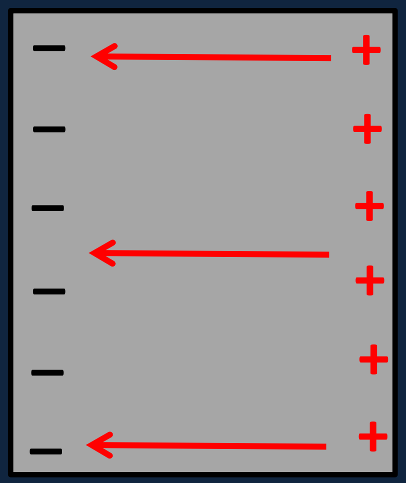


- If a nonconducting material is placed in an electric field, the electrons will still move a little, remaining within their home molecules or atoms, which will therefore become polar.
- The overall effect of this polarization is to generate a layer of positive charge on the right and negative charge on the left.

Dielectrics

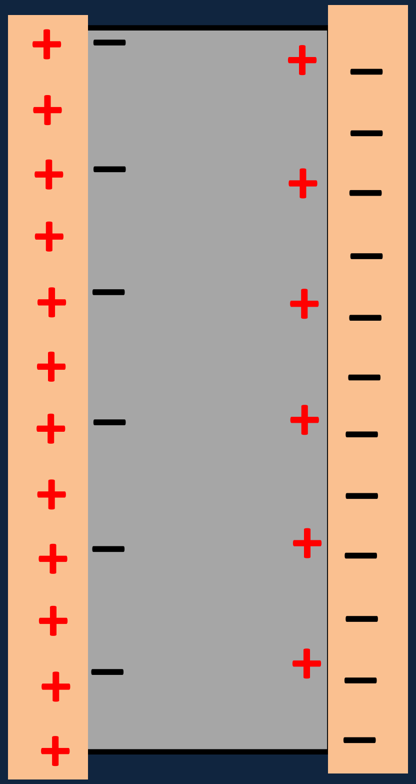
- These “layers of surface excess charge” created by the polarization generate an electric field opposing the external field.
- However, unlike a conductor, this field cannot be strong enough to give zero field inside, because then the polarization would all go away.





Dielectric in a Capacitor

- If a dielectric material is placed between the parallel plates of a capacitor, the effect of the dielectric produced "surface layers of charge" is to partially cancel the charge on the plates as seen from inside the capacitor.
- Therefore, the dielectric will reduce the electric field strength, and therefore the voltage between the plates for given Q capacitor charge.



The Dielectric Constant K

- It is found experimentally that putting dielectric material between the plates of a capacitor reduces the magnitude of the electric field by a constant K that varies with the material used.
- This means that it takes more plate charge to give the same voltage: in other words, <u>the capacitance increases by a factor K</u>.

TABLE 24–1
Dielectric Constants (at 20°C)

Material	Dielectric constant K	Dielectric strength (V/m)
Vacuum	1.0000	
Air (1 atm)	1.0006	3×10^6
Paraffin	2.2	10×10^6
Polystyrene	2.6	24×10^6
Vinyl (plastic)	2–4	50×10^6
Paper	3.7	15×10^6
Quartz	4.3	8×10^6
Oil	4	12×10^6
Glass, Pyrex	5	14×10^6
Porcelain	6–8	5×10^6
Mica	7	150×10^6
Water (liquid)	80	
Strontium titanate	300	8×10^6

Energy Storage in a Dielectric

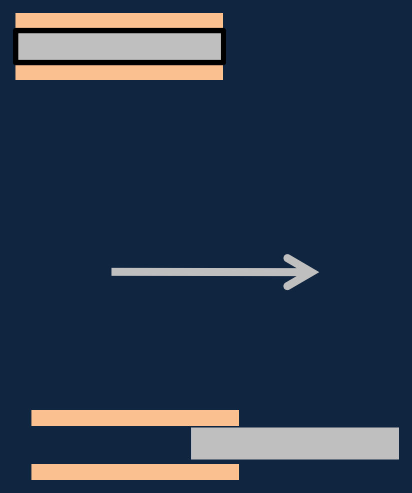
- Inserting dielectric between the plates of a capacitor increases the capacitance from C_0 to KC_0.
- This means that the energy stored at voltage V goes from $\frac{1}{2}C_0V^2$ to $\frac{1}{2}KC_0V^2$: yet inside the capacitor, the electric field has the same strength, V/d, as before. Where is the extra energy stored?
- In the dielectric: the stretched molecules store energy like little springs, so the total energy density of a field in a dielectric is

$$\boxed{u = \frac{1}{2}K\varepsilon_0 E^2 = \frac{1}{2}\varepsilon E^2}$$

Note: ε is called the permittivity of the material

Clicker Question

- An isolated (no battery connection) parallel plate capacitor has charges $+Q$, $-Q$ on its plates, and dielectric ($K = 3$) between them.
- The dielectric is now removed, without disturbing the charge on the plates.
- The capacitor's energy has:



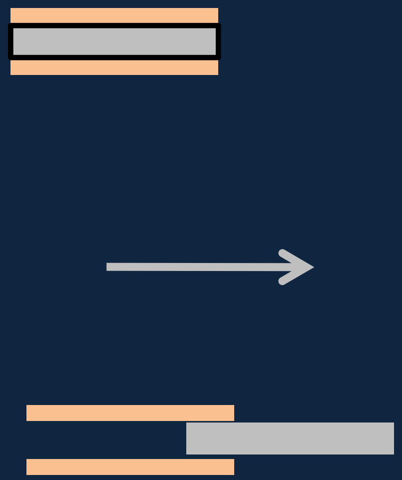
A. increased.

B. decreased.

C. stayed the same.

Clicker Answer

- An isolated (no battery connection) parallel plate capacitor has charges $+Q$, $-Q$ on its plates, and dielectric (K = 3) between them.
- The dielectric is now removed, without disturbing the charge on the plates.
- The capacitor's energy has:



A. Increased.

$U = Q^2/2C$, Q is constant, C decreases.

(The charge on the dielectric surface attracts that on the plates, so it takes work to separate them.)

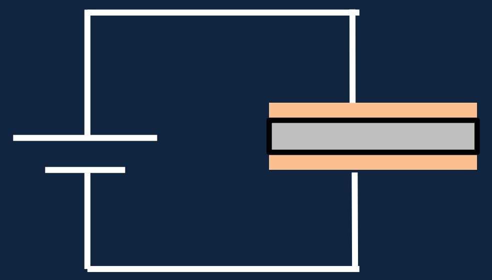
Clicker Question

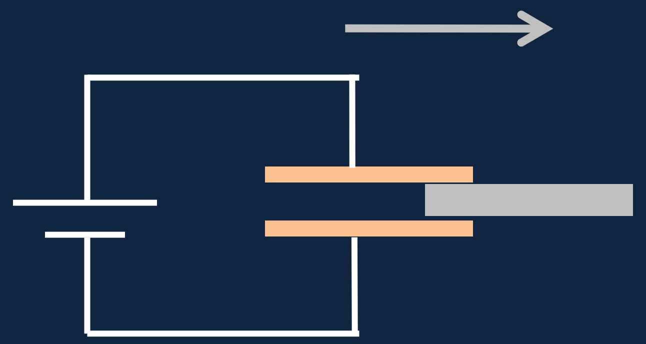
- A parallel plate capacitor has its plates connected to a 100V battery, and dielectric (K = 3) between them.
- The dielectric is now removed, while keeping the 100V battery connection.
- The capacitor's energy has:

A. increased.

B. decreased.

C. stayed the same.



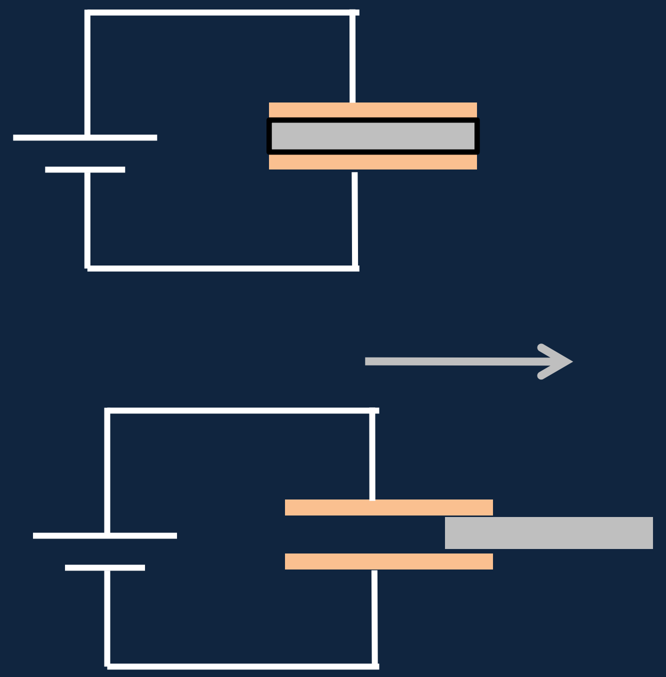


Clicker Answer

- A parallel plate capacitor has its plates connected to a 100V battery, and dielectric (K = 3) between them.
- The dielectric is now removed, while keeping the 100V battery connection.
- The capacitor's energy has:

B. Decreased.

- $U = \frac{1}{2}CV^2$, V is constant, C decreases.



It still took work—but now you're charging the battery!

Capacitor Driven Bus

- Bus energy is stored in a capacitor (about 1kF).
- Recharges in two minutes at stops every two miles. (Those overhead wires are only at recharging station).
- Recharges much faster than batteries—but only 10% storage capacity/kg currently.