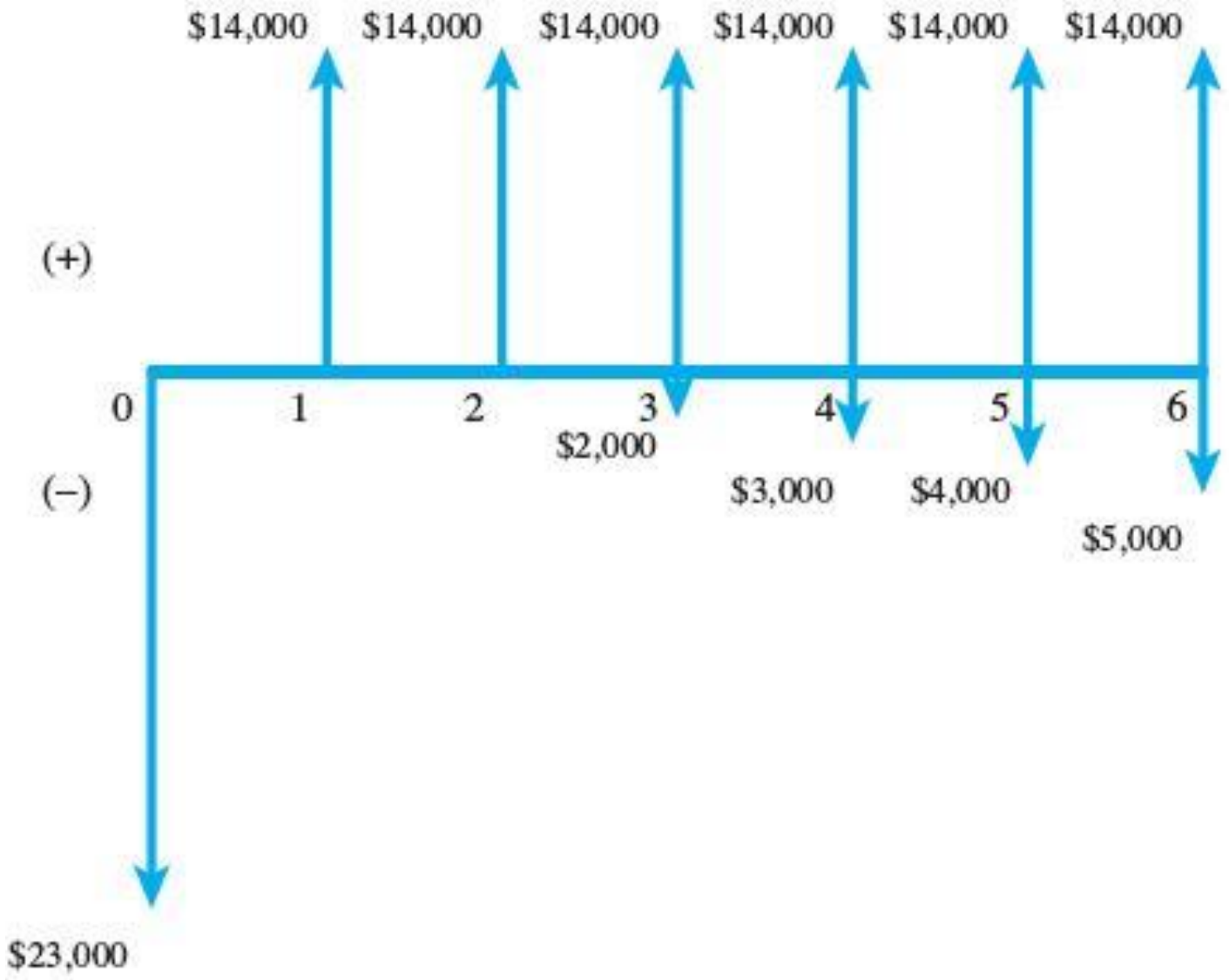
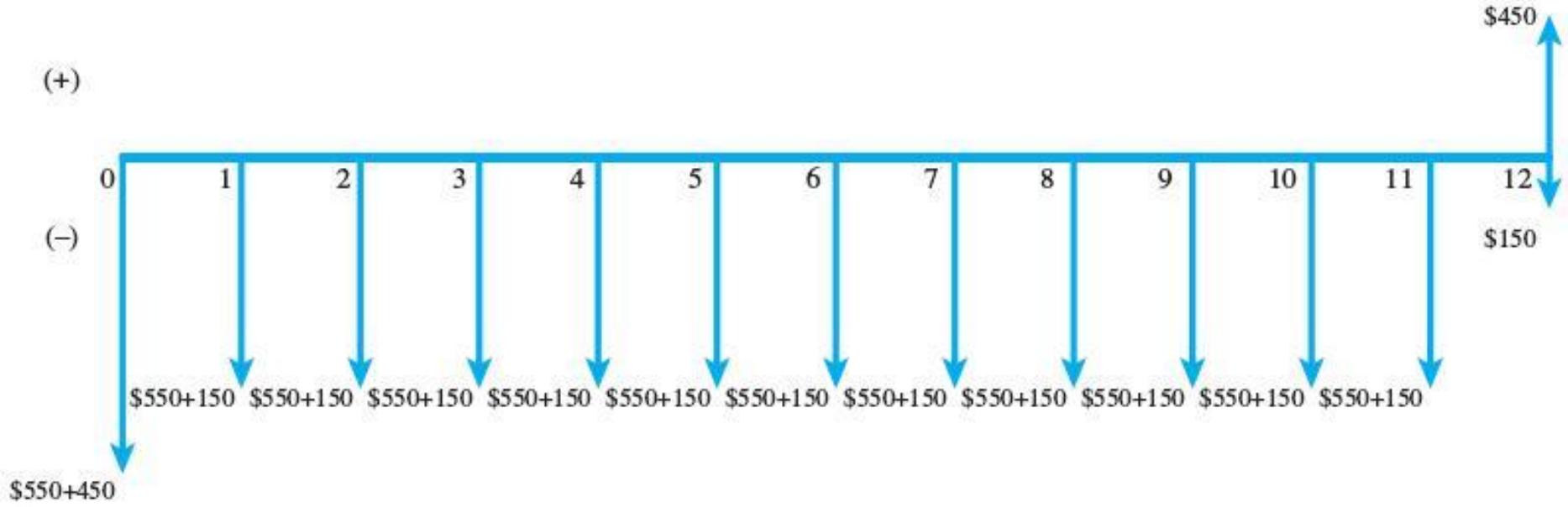
Review – Cash flow diagram

13. A laser cutting machine is purchased today for $23,000. There are no maintenance costs for the next two years. Maintenance at the end of year 3 is expected to be $2,000, with each subsequent year's maintenance costs exceeding the previous year's by $1,000. A revenue of $14,000 per year is expected. The planning horizon is 6 years. Draw the cash flow diagram.


$14,000
$14,000
$14,000
$14,000
$14,000
$14,000
(+)
0
1
2
3
4
5
6
$2,000
(–)
$3,000
$4,000
$5,000
$23,000

Review

14. You rent an apartment for $550 per month, payable at the beginning of the month. An initial deposit of $450 is required. Utilities are an additional $150 per month payable at the end of the month. The deposit is refundable at the time you move out, assuming a clean apartment in good condition. Draw a monthly cash flow diagram, assuming you keep the apartment for 12 full months.


$450
(+)
0
1
2
3
4
5
6
7
8
9
10
11
12
(–)
$150
$550+150
$550+150
$550+150
$550+150
$550+150
$550+150
$550+150
$550+150
$550+150
$550+150
$550+150
$550+450

Review– Calculation

15. How much money would have to be deposited today to accumulate:

a. $10,000 after 6 years if the investment earns 5%/year compounded annually?

b. $6,500 after 4 years if the investment earns 8%/year compounded annually?

c. $3,400 after 12 years if the investment earns 6%/year compounded annually?

d. $13,500 after 5 years if the investment earns 10%/year compounded annually?

a. P=PV(0.05,6,,-10000)=7462.15

b. P=PV(0.08,4,,-6500)=4777.69

c. P=PV(0.06,12,,-3400)=1689.70

d. P=PV(0.1,5,,-13500)=8382.44

Review

16. The cash flow profile for an investment is given below and the interest rate is 6.5% compounded annually. Find the present worth and future worth of this cash flow series.

End of Year	Net Cash Flow	End of Year	Net Cash Flow
0	$0	4	−$300
1	−$500	5	$500
2	$200	6	−$200
3	$400	7	$100

P=100*NPV(0.065,-5,2,4,-3,5,-2,1)=97.02

F=FV(0.065,7,,-97.02)=150.77

Review

17. Jason has been making equal annual payments of $7,500 to repay a college loan. He wishes to pay off the loan immediately after having made an annual payment. He has eight payments remaining. With an annual compound interest rate of 6%, how much should Jason pay?

P=PV(0.06,8,-7500)=46573.45

Review

18. On Juan's 26th birthday, he deposited $7,500 in a retirement account. Each year thereafter he deposited $1,000 more than the previous year. Determine how much was in the account immediately after his 35th birthday deposit if :

a. The account earned annual compound interest of 5%.

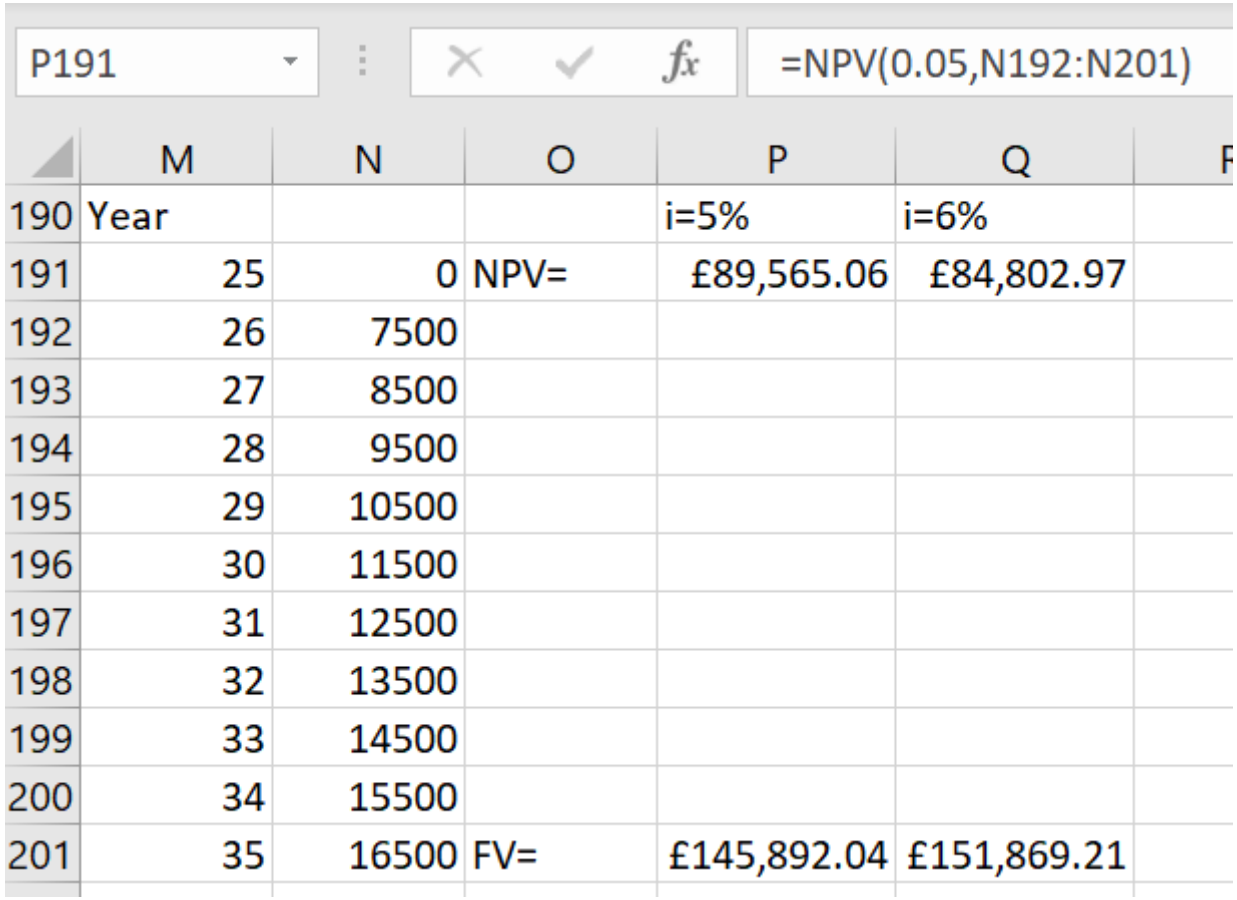
b. The account earned annual compound interest of 6%.

a. P=100*NPV(0.05,75,85,95,105,115,125,135,145,155,165)=89565.06

F=FV(0.05,10,,-89565.06)=145892.04

b. P=100*NPV(0.06,75,85,95,105,115,125,135,145,155,165)=84802.97

F=FV(0.06,10,,-84802.97)=151869.21



	M	N	O	P	Q
190	Year			i=5%	i=6%
191	25	0	NPV=	£89,565.06	£84,802.97
192	26	7500			
193	27	8500			
194	28	9500			
195	29	10500			
196	30	11500			
197	31	12500			
198	32	13500			
199	33	14500			
200	34	15500			
201	35	16500	FV=	£145,892.04	£151,869.21

Review

19. Suppose you make 30 annual investments in a fund that pays 5% compounded annually. If your first deposit is $7,500 and each successive deposit is 5% greater than the preceding deposit, how much will be in the fund immediately after the 30th deposit?

C2=C1*1.05,……, C30=C29*1.05

P=NPV(0.05,C1:C30)=214285.71

F=FV(0.05,30,,-214285.71)=926130.51

7500	$214,285.71
7875	
8268.75	
8682.188	
9116.297	
9572.112	
10050.72	
10553.25	
11080.92	
11634.96	
12216.71	
12827.55	
13468.92	
14142.37	
14849.49	
15591.96	
16371.56	
17190.14	
18049.64	
18952.13	
19899.73	
20894.72	
21939.46	
23036.43	
24188.25	
25397.66	
26667.55	
28000.92	
29400.97	
30871.02	$926,130.51

Review

20. A refrigerator sold for $500. The store financed the refrigerator by charging 0.5% monthly interest on the unpaid balance. If the refrigerator is paid for with 30 equal end-of-month payments:

a. What will be the size of the monthly payments?

b. If the first payment is not made until one year after the purchase, what will be the size of the monthly payments?

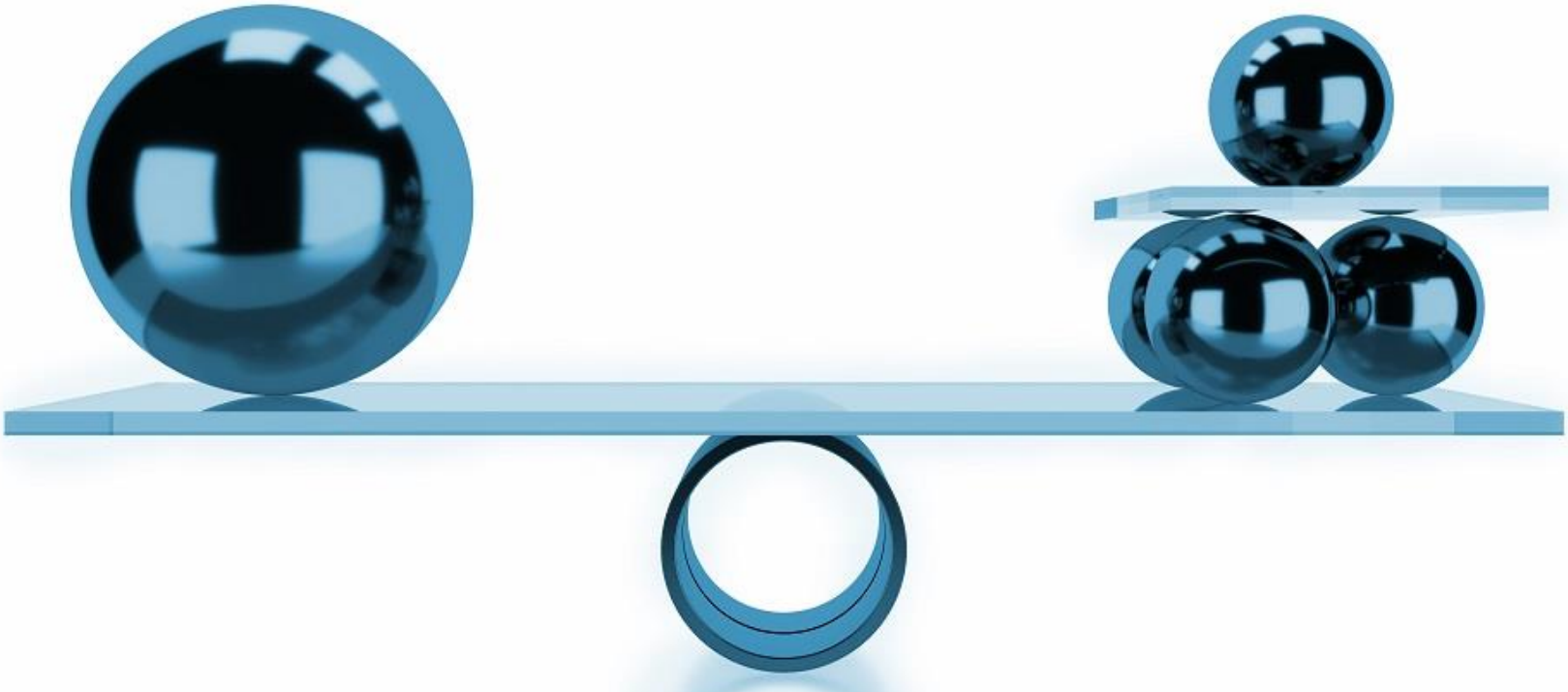
a. A=PMT(0.005,30,-500)=17.99

b. F=FV(0.005,12,,-500)=530.84

A=PMT(0.005,30,-530.84)=19.10

B0684
Economic Engineering Analysis

Equivalence, Loans & Bonds



Learning Objective

1. Compare the equivalence between two or more cash flow profile.
2. Analyze immediate payment and deferred payment loans, including payment amount, remaining balance, and interest and principal per payment.
3. Analyze investments in bonds and determine the purchase price, selling price, and return on such investments.
4. Calculate the worth of a cash flow profile with variable interest rates.

- Fifteen years after graduating in electrical engineering and accepting employment with Texas Instruments, Samuel Washington decides to establish a consulting business.
- Although he has invested wisely for the past 15 years, the value of his investments is only $325,000. After developing a business plan, he realizes he will need $250,000 on hand initially, plus $150,000 each successive year, to cover the expenses of an office and an assistant.
- He is unsure about how much to borrow. In talking to the loan officer of a local bank, he learns that the bank will charge him annual compound interest of 6% for a 5-year loan period or 5.5% for a 10-year loan period.
- Over the past 10 years, Samuel earned an average of 5.25 percent annually on his investments; he believes he will continue to earn at least that amount on his investment portfolio.
- If he borrows money, he can repay the loan in several ways: pay accumulated interest monthly, plus pay the principal at the end of the loan period; make equal monthly payments; make monthly payments that increase like a gradient series; make monthly payments that increase like a geometric series; or make a lump sum payment at the end of the loan period.

EQUIVALENCE

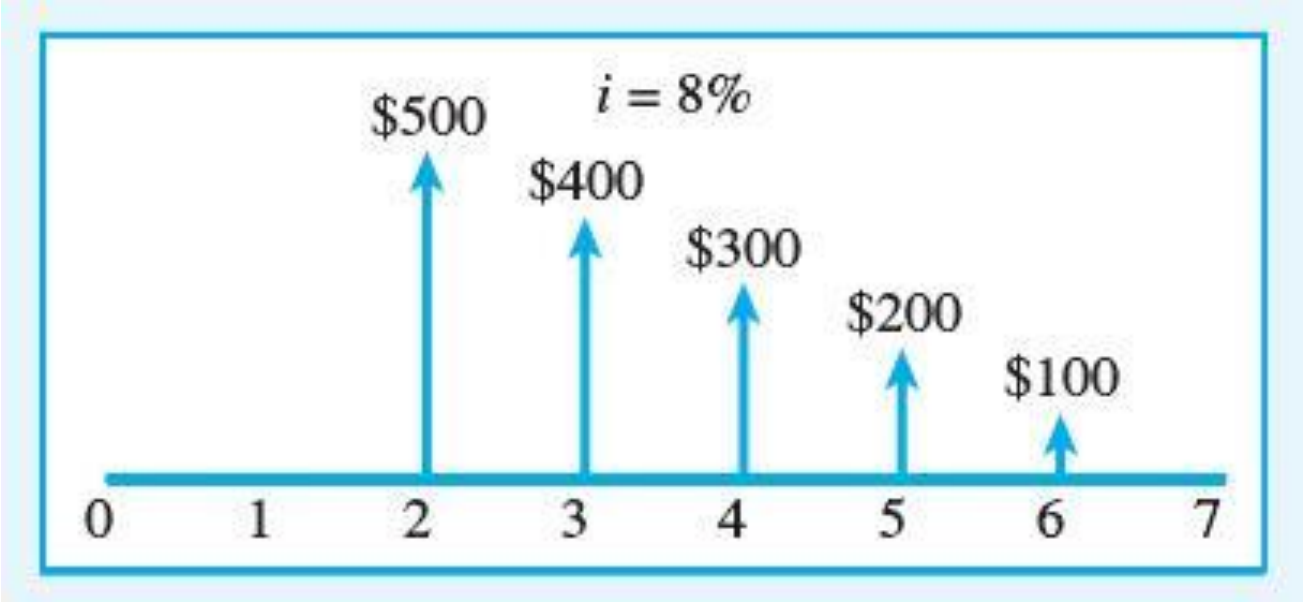
- The state of being equal in value.
- The concept is primarily applied in the comparison of two or more cash flow profiles.
- A commonly used approach to determine equivalence is to compare the present/future worth of the cash flow profiles.
- If they are equal, then the cash flow profiles are equivalent.

- Cash Flow Profile 1: Receive $1,322.50 two years from today, and the interest rate is 15%.
- Cash Flow Profile 2: Receive $1,000 today.

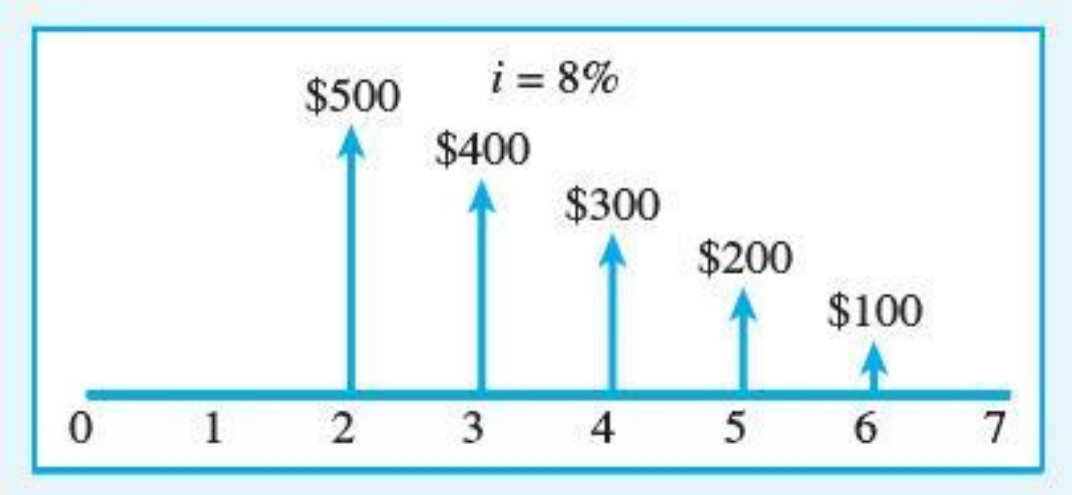
- PV1=PV(15%,2,,-1322.5)=$1,000=PV2
- The two cash flow profiles are equivalent!
- It suggest the worth of the two cash flow profiles will be the same at any particular point in time, e.g., at t_2 or t_6.

A Uniform Series Equivalency of a Gradient Series

Using an 8 percent discount rate, what uniform series over five periods, [1, 5], is equivalent to the cash flow profile given?

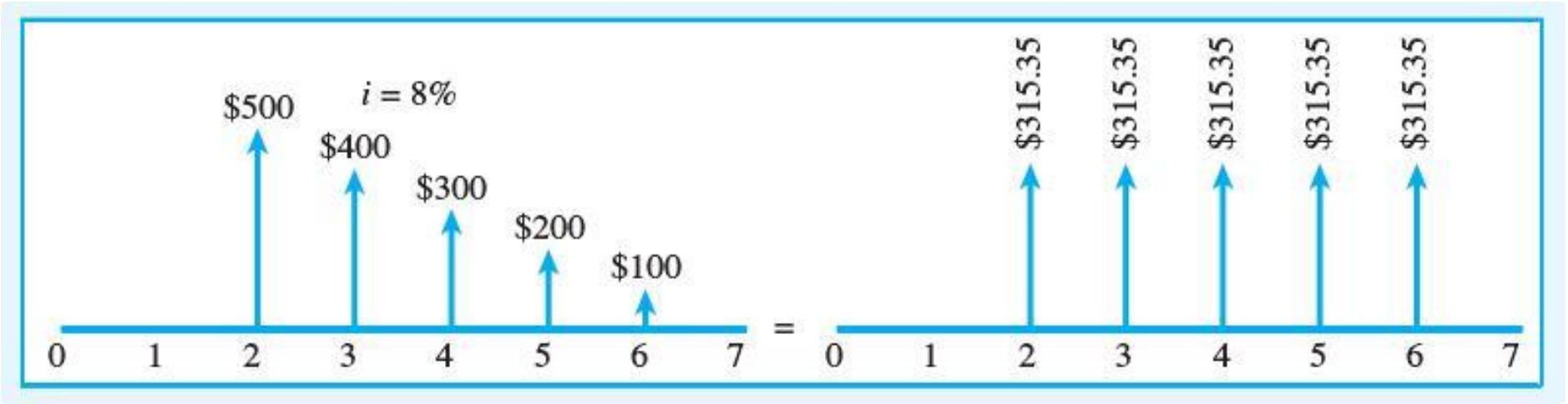


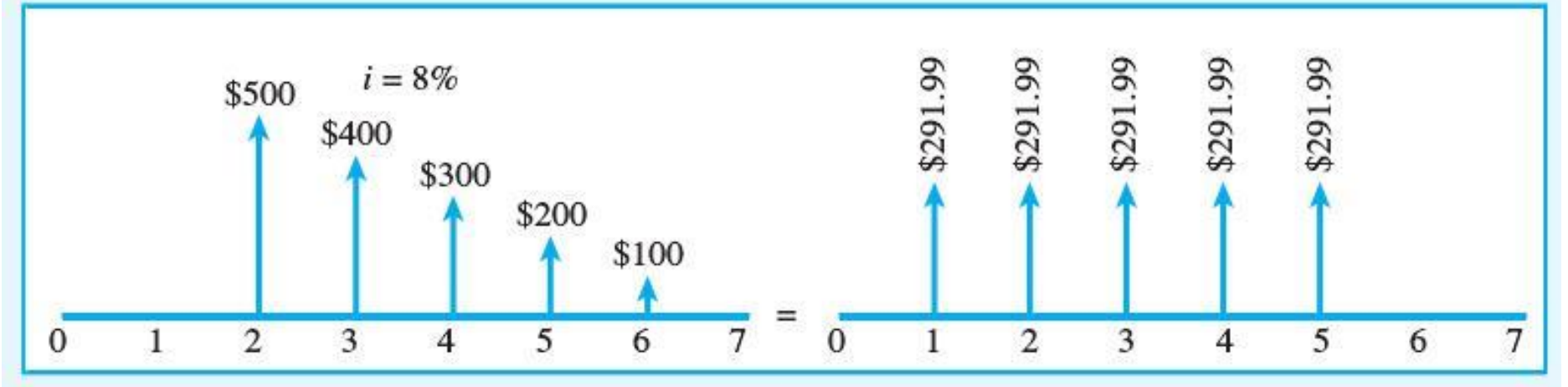
Draw CFD!! Pay attention to the time period!



Solution 1:

- P1=100*NPV(0.08,5,4,3,2,1)=1259.1125; P1 occurs at t_1.
- A=PMT(0.08,5,-1259.1125)=315.35; P1 occurs at t_1, and this equivalent uniform series occur at period [2,6], which is one time period after t_1!
- The question is to find the equivalent uniform series at period [1,5], thus discount A backward one time period:
- A'=315.35/(1+8%)=291.99


$500
$400
$300
$200
$100
i = 8%
0 1 2 3 4 5 6 7
=
$315.35
$315.35
$315.35
$315.35
$315.35
0 1 2 3 4 5 6 7

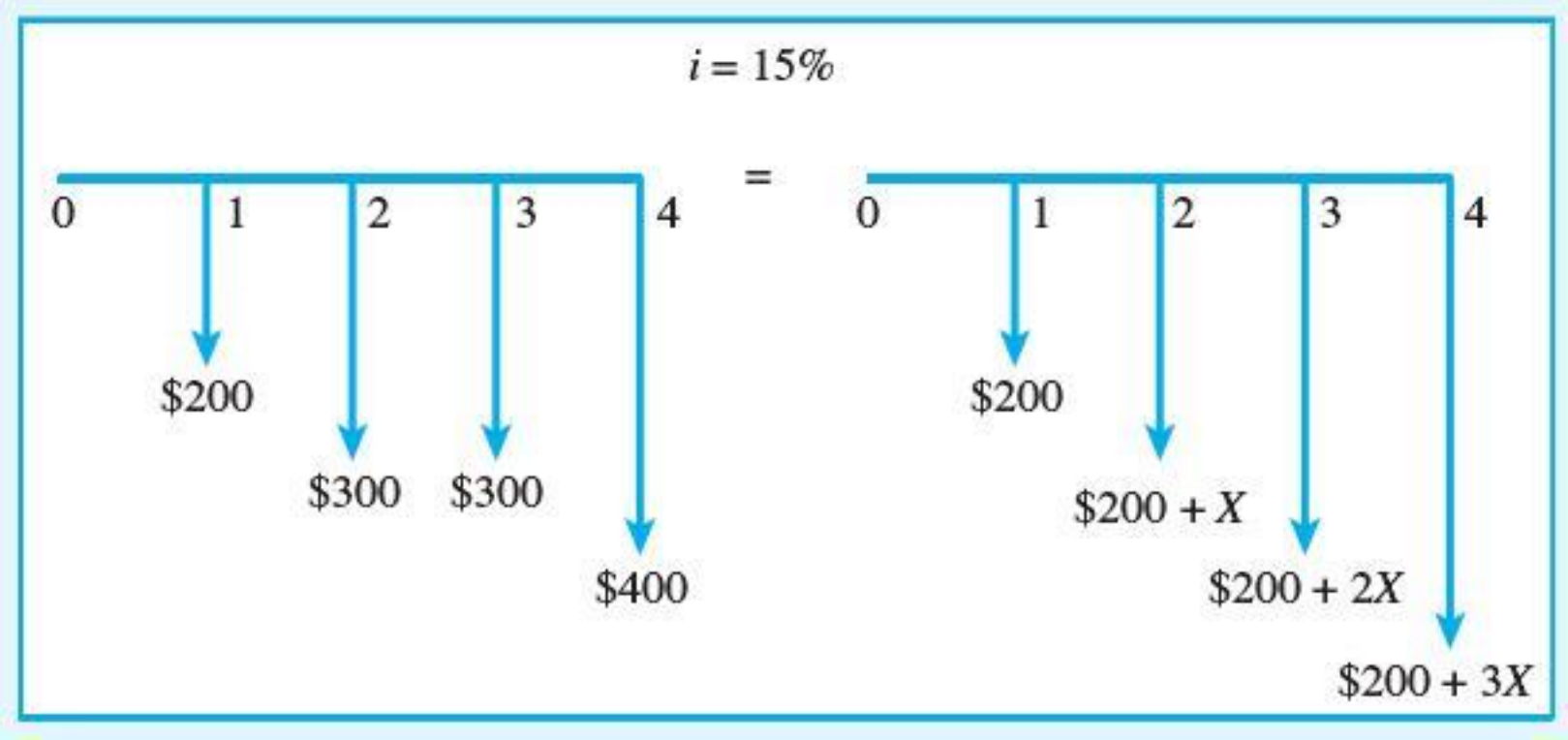

$500
$400
$300
$200
$100
i = 8%
0 1 2 3 4 5 6 7
=
$291.99
$291.99
$291.99
$291.99
$291.99
0 1 2 3 4 5 6 7

Solution 2:

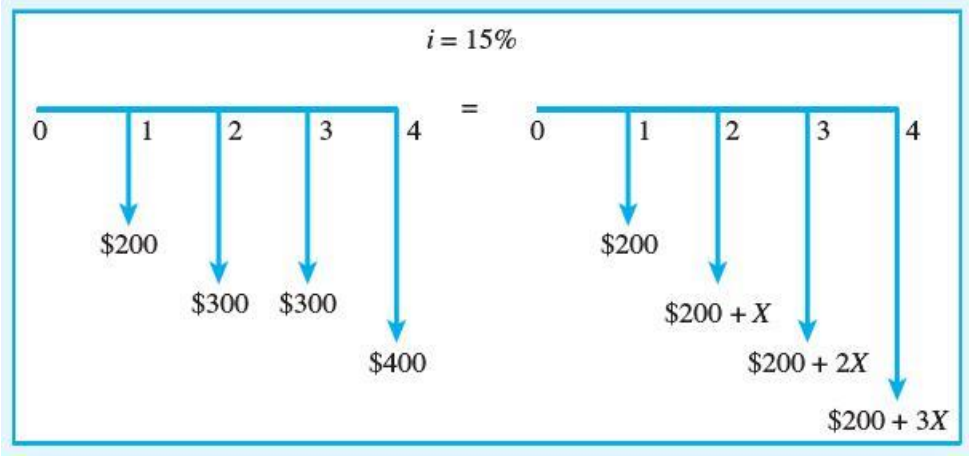
- P1=100*NPV(0.08,5,4,3,2,1)=1259.1125; P1 occurs at t_1.
- Discount P1 to t_0, P0=PV(0.08,1,,-1259.1125)=1165.84
- Then find the equivalent uniform series at period [1,5], thus A=PMT(0.08,5,-1165.84)=291.99

Determining an Equivalent Gradient Step

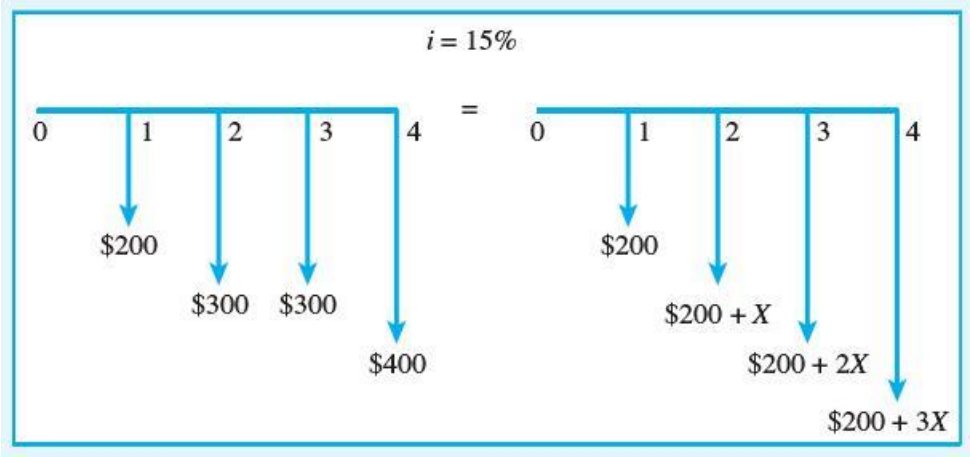
Determine the value of X that makes the two cash flow profiles equivalent using a TVOM of 15 percent.



- Solution 1: breaking down the cash flow on the right into a uniform series A=200 at [1,4], and a gradient series {X, 2X, 3X} at [2,4], calculate PV at t_0
- P=100*NPV(0.15,2,3,3,4)=826.71,
- $P_{uniform}$=PV(0.15,4,-200)=571.00,
- $P_{gradient}$=P-$P_{uniform}$=255.71,
- As the gradient series occurs at [2,4], PV should occur one time period before at t_1, thus move $P_{gradient}$ forward one time period.
- P'=$P_{gradient}$*(1+0.15)=294.07
- X*NPV(0.15,1,2,3)=294.07
- X*4.35=294.07
- X=67.53



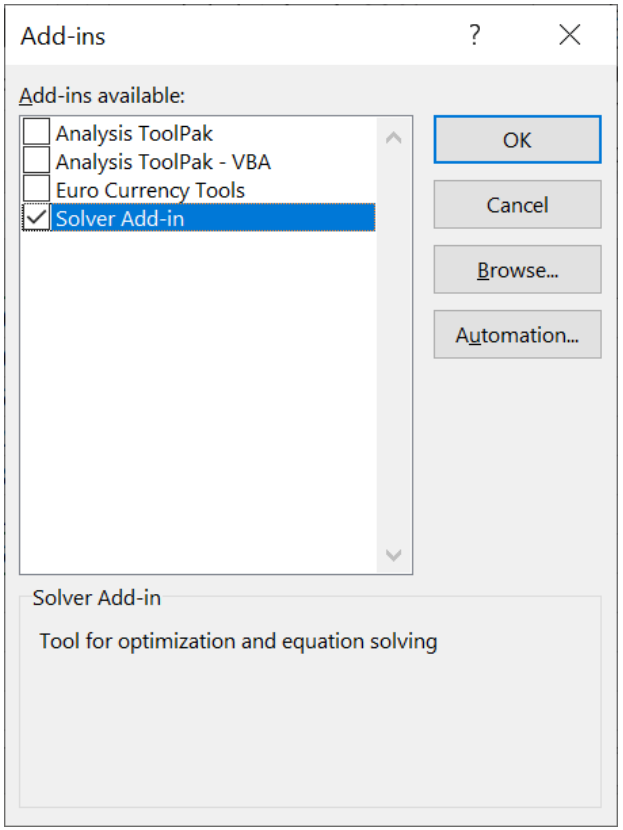
- Solution 2: all cash flows minus 200, calculate PV at t_1
- 100*NPV(0.15,1,1,2)=X*NPV(0.15,1,2,3),
- 100*2.94=X*4.35
- X=67.59



- **Solution 3: using the Excel Solver Tool**

  First add on the Solver tool:

  – In **Excel** 2010 and later, go to File > Options. ...
  – Click **Add**-Ins, and then in the Manage box, select **Excel Add**-ins.
  – Click Go.
  – In the **Add**-Ins available box, select the **Solver Add**-in check box, and then click OK. ...
  – After you load the **Solver Add**-in, the **Solver** command is available on the Data tab.



Alternatively, search for “Solver” in the search tool bar of Excel.