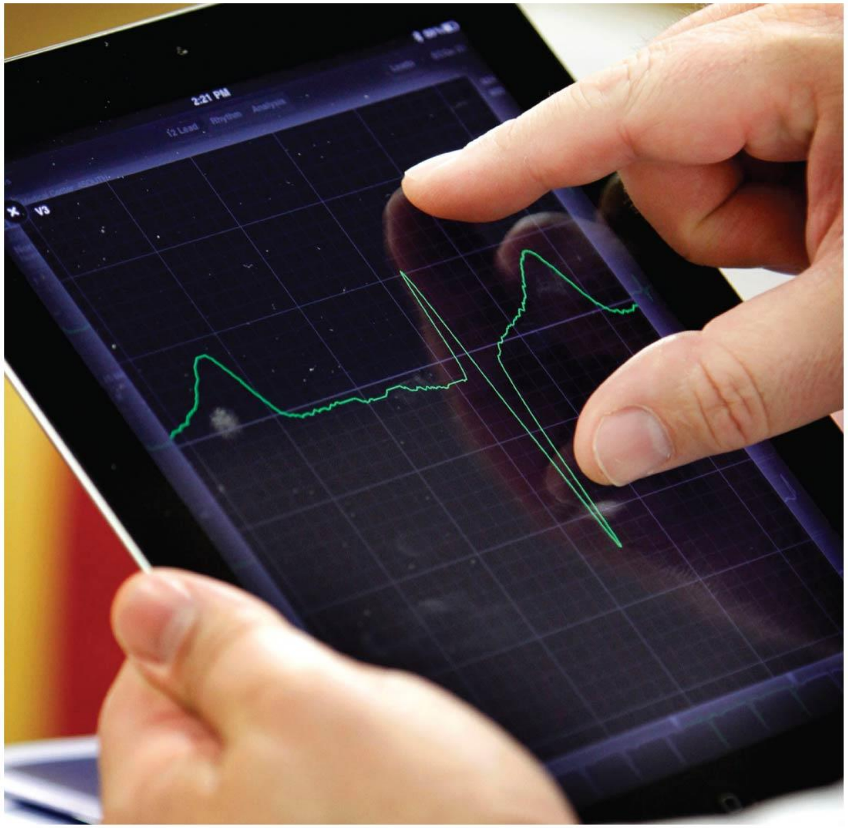
Chapter 17

Electric Potential



§ .6 静电场中的导体

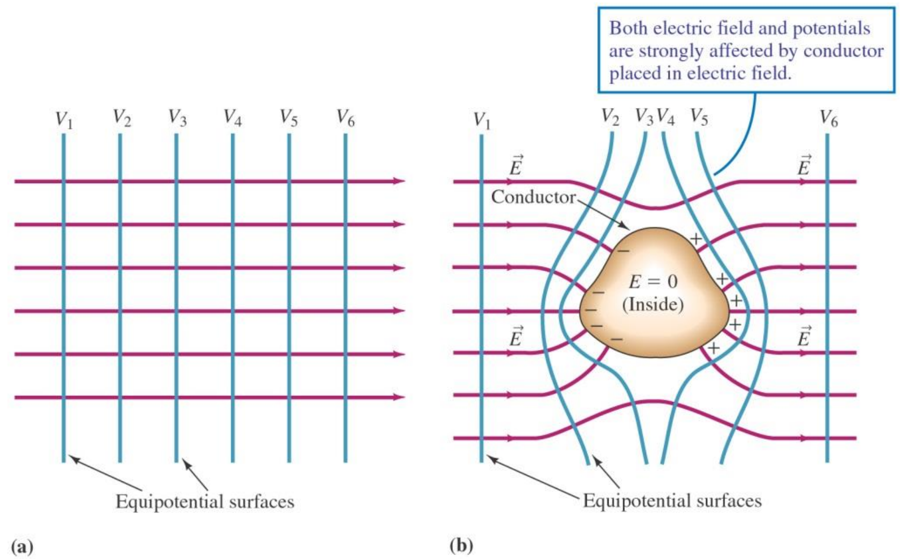
金属　　晶体点阵（晶格）　　自由电子　　自由电荷

crystal lattice

当导体上的电荷分布不再随时间变化时，电场分布不随时间变化

导体达到了静电平衡状态　In electrostatic equilibrium

静电感应现象 Electrostatic Induction



均匀导体的静电平衡条件：

导体内部场强处处为零

- 导体是等势体，导体表面为等势面。

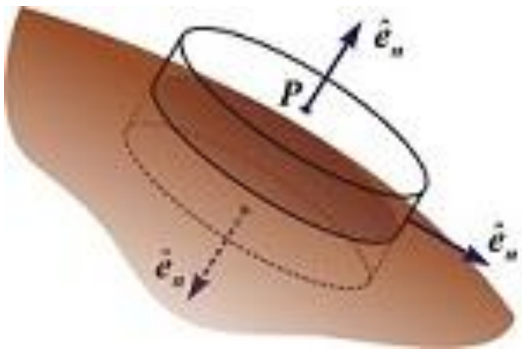
Equipotential body　　Isopotential surface

达到静电平衡时导体上的电荷分布 Electrostatic equilibrium

1. 导体内部处处没有未抵消的净电荷（即体电荷密度 $\rho = 0$），电荷只分布在导体的表面。 Net charge

2. 导体表面外侧附近空间的电场强度大小与该处导体表面的面电荷密度有如下关系：

$$E = \frac{\sigma}{\varepsilon_0}$$



证明：

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{q_{net}}{\varepsilon_0} = \frac{\sigma \Delta S}{\varepsilon_0}$$

$$\oint_S \vec{E} \cdot d\vec{S} = \int_{(\text{上底})} \vec{E} \cdot d\vec{S} + \int_{(\text{下底})} \vec{E} \cdot d\vec{S} + \int_{(\text{侧面})} \vec{E} \cdot d\vec{S} = \int_{(\text{上底})} \vec{E} \cdot d\vec{S} = E \cdot \Delta S$$

3. 在一个 孤立导体的表面上，面电荷密度的大小与其表面的曲率有关：

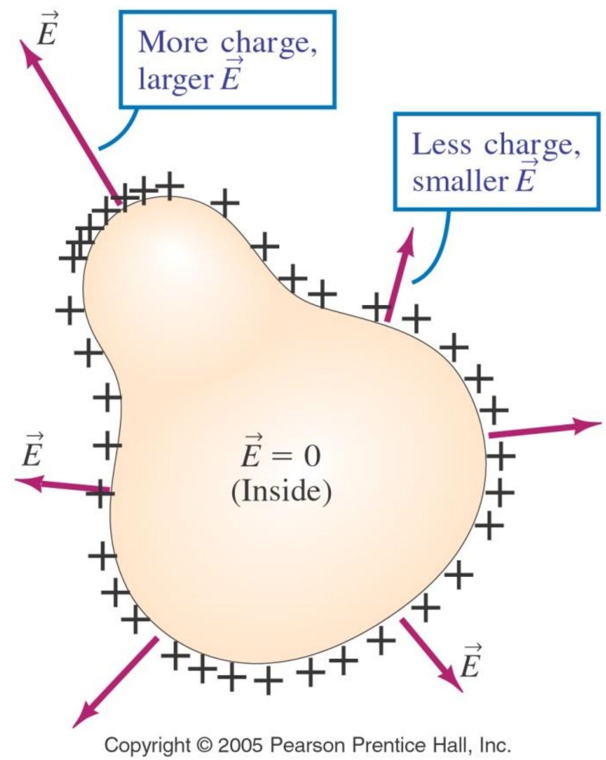
Curvature

表面曲率大处的面电荷密度大，表面曲率小处的面电荷密度小，凹面处更小。

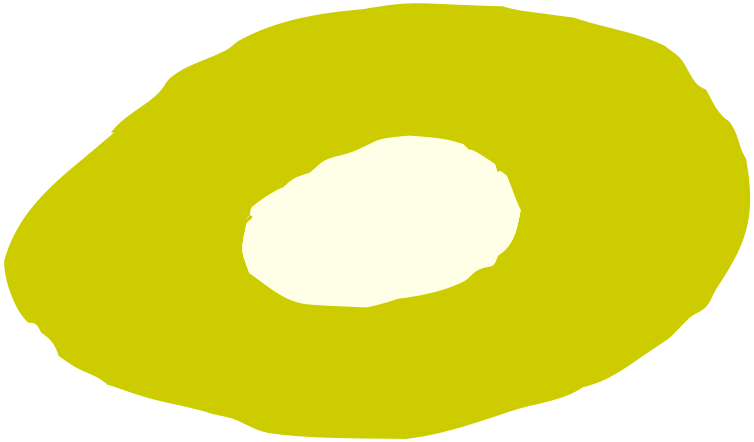
曲率的倒数就是曲率半径

$$V=\frac{q_1}{4\pi\varepsilon_0 R_1}=\frac{q_2}{4\pi\varepsilon_0 R_2}$$

$$\frac{q_1}{q_2}=\frac{R_1}{R_2}=\frac{\sigma_1 R_1^2}{\sigma_2 R_2^2} \qquad \frac{\sigma_1}{\sigma_2}=\frac{R_2}{R_1}$$

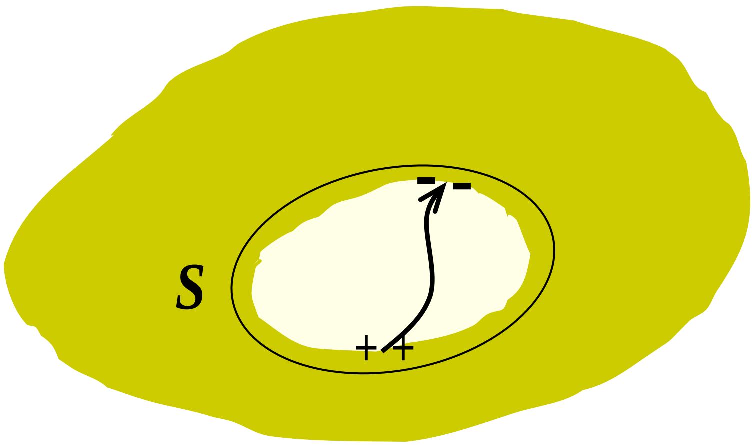


•4. If there is an empty cavity inside the conductor, there is no other Electricity body in the cavity



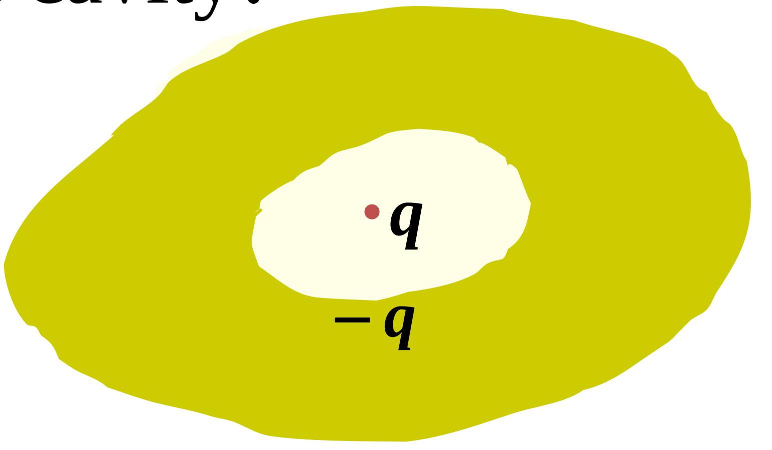
- There is no charge at all on the surface of the cavity the charge is only distributed outside the conductor

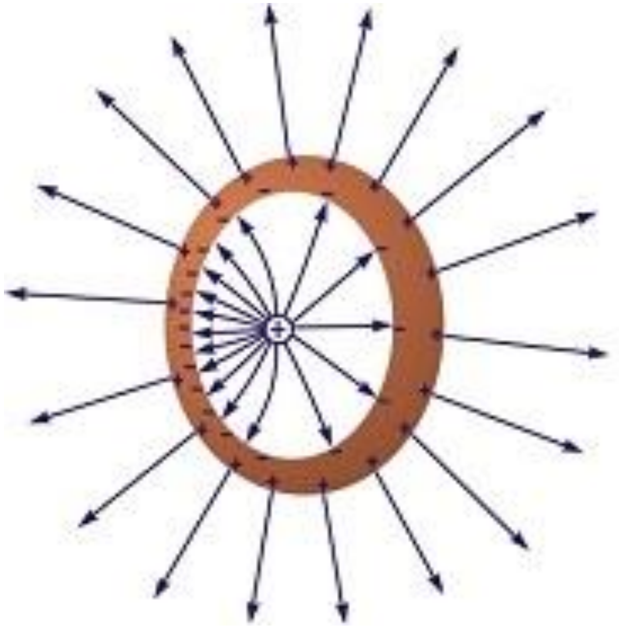
- There's no electric field in the cavity

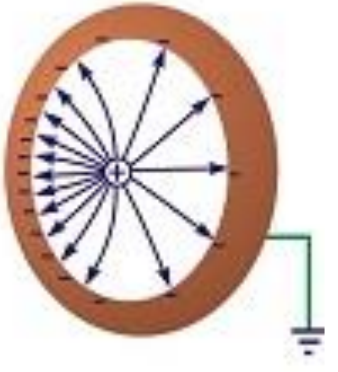


5. If there is an empty cavity in the conductor, and there are other charges in the cavity：

- 空腔内表面上电荷的电量与腔内电荷的电量的代数和为零







金属壳内、壳外的电场被金属壳完全隔开，互不影响。

静电屏蔽

Electrostatic screening

Electrostatic shielding

给定所讨论空间的电荷分布　　　**叠加原理**

原则上，空间各点的场强和电势都可确定

带电体是导体　　　**无体电荷分布**　　　**有面电荷分布**

实际上，确定导体表面上的面电荷分布是极其困难的

容易确定的是每个导体上的总电量或电势

给定静电场中各带电导体的几何形状和相对位置

- **给定每个导体的总电量**
- **给定每个导体的电势**

边界条件

Boundary conditions

空间各点的场强是是否是唯一确定的？

静电场边值问题的唯一性定理 Uniqueness theorem

给定静电场中各导体的几何形状和相对位置，已知该空间内自由电荷的分布，如果再给定每个导体上的总电量，则空间各点的场强是唯一确定的。

给定所讨论空间内的自由电荷分布，给定该空间的边值条件，该空间的电场分布是唯一的。

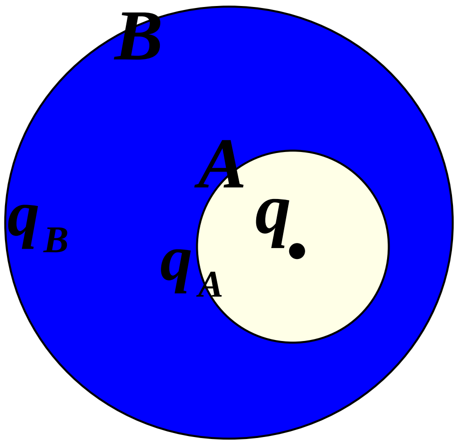
边界条件

Neutral metal ball　　　　Acentric sphere cavity

有一个中性金属球，内部有一个偏心的球状空腔，在空腔中心有一个点电荷q

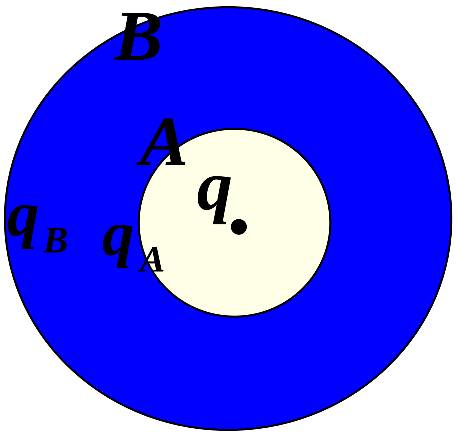
空腔表面A和金属球表面B上的电荷分布?

腔内和球外的场强分布?



$$q_A = -q \qquad q_B = q$$

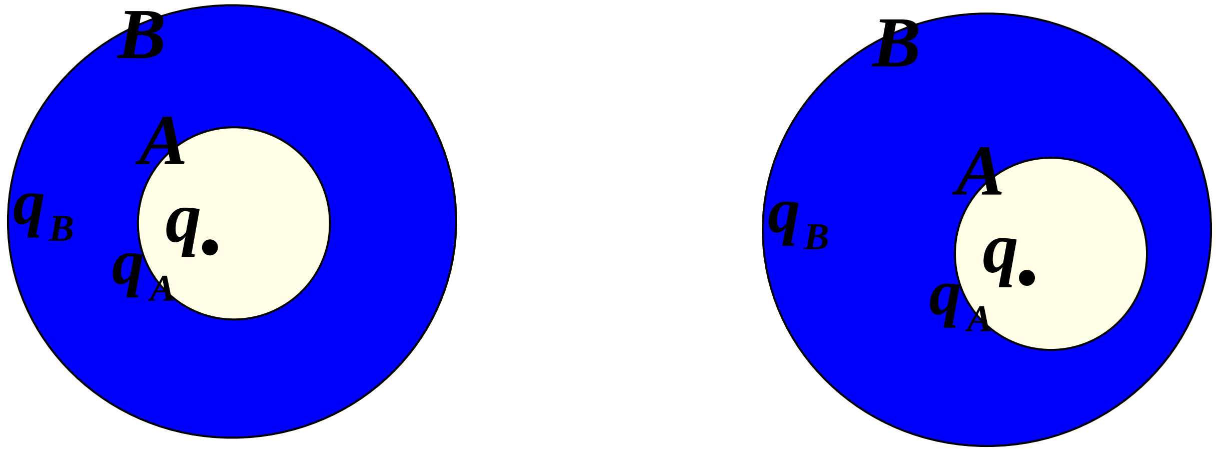
q_A 均匀分布，q_B 均匀分布



如果点电荷 q 不在空腔中心

$q_A = -q$

$q_B = q$

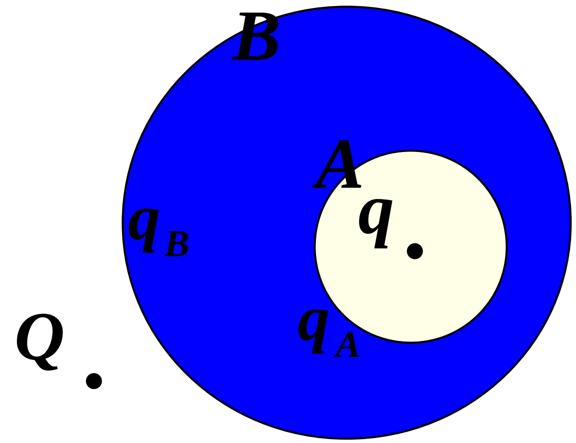


q_A 非均匀分布，q_B 均匀分布

无论 q 在空腔中如何移动，无论空腔在金属导体内如何移动，q_B 的分布都不变，总是均匀的。

B面以外空间的场强分布不变，完全取决于 q_B 的分布，丝毫不受空腔的位置以及 q的位置影响。

中性金属球外有一点电荷Q，空腔内点电荷q 在球面A 的球心。



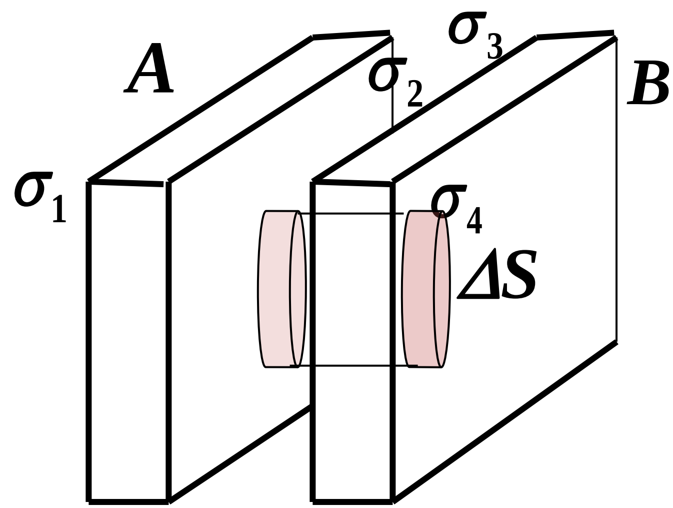
$\boldsymbol{q_A}$ 均匀分布，$\boldsymbol{q_B}$ 非均匀分布

不论 Q 如何移动，不论 Q 的数值如何变化，它只对 $\boldsymbol{q_B}$ 的分布有影响，而对空腔中的场强以及 $\boldsymbol{q_A}$ 的分布毫无影响。

A、B为平行放置的两个均匀带电的很大的导体平板，A 板带电荷为 Q_A，B 板带电荷为 Q_B 。

在静电平衡下，两导体板的四个表面上的面电荷密度 σ_1、σ_2、σ_3 和 σ_4 之间的关系?

作圆柱形高斯面



$$\oint_S \vec{E}\cdot d\vec{S} = \frac{(\sigma_2+\sigma_3)}{\varepsilon_0}\Delta S$$

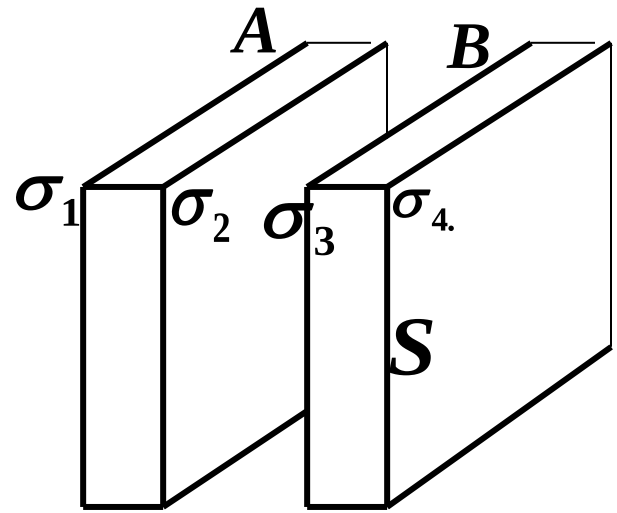
$$\oint_S \vec{E}\cdot d\vec{S} = 0 \qquad \sigma_2 = -\sigma_3$$

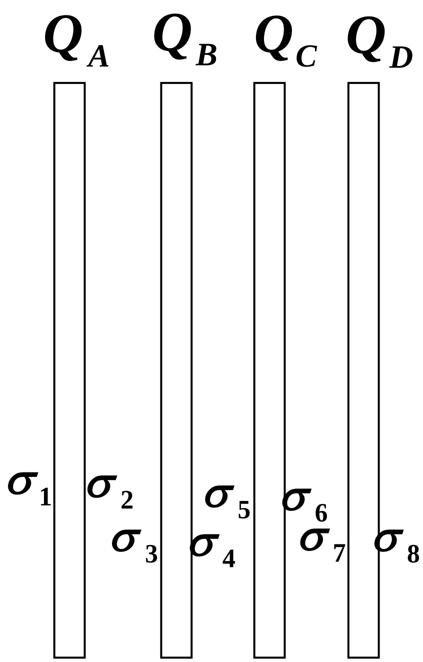
导体板A内的总场强为零

$$E = \frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} - \frac{\sigma_3}{2\varepsilon_0} - \frac{\sigma_4}{2\varepsilon_0} = 0 \qquad \therefore \sigma_1 = \sigma_4$$

$$\sigma_2 = -\sigma_3$$

$$\sigma_1 = \sigma_4$$





$$\sigma_1 = \sigma_8$$

$$\sigma_2 = -\sigma_7$$

$$\sigma_3 = -\sigma_6$$

$$\sigma_4 = -\sigma_5$$

$$\sigma_1 = \sigma_8$$

$$\sigma_2 = -\sigma_3$$

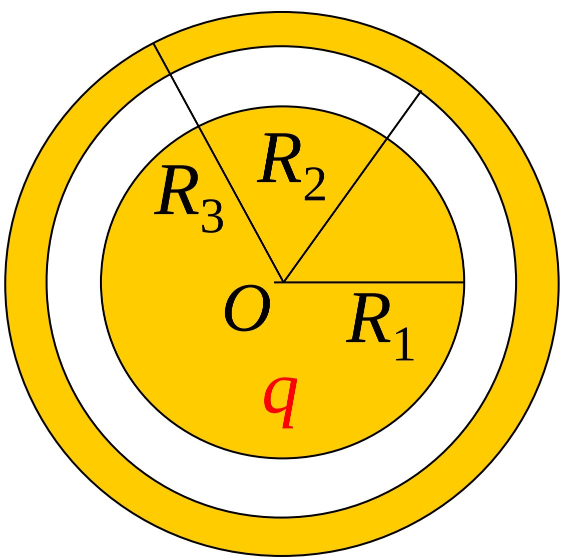
$$\sigma_4 = -\sigma_5$$

$$\sigma_6 = -\sigma_7$$

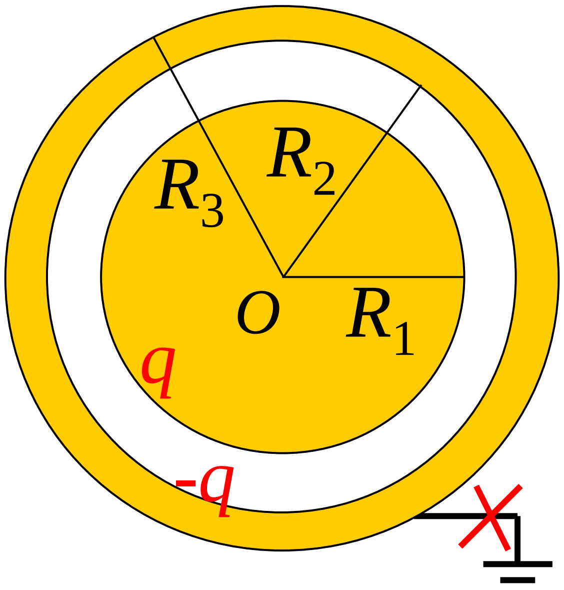
电量为 q（$q>0$）、半径为 R_1 的导体球，球外同心地放置一个不带电的金属球壳，球壳内、外半径分别为 R_2、R_3。

(1) 把外球壳接地后，再与地绝缘。电势分布？

(2) 再把内球接地，内球的电量和外球壳的电势？



（1）把外球壳接地后，再与地绝缘。电势分布?



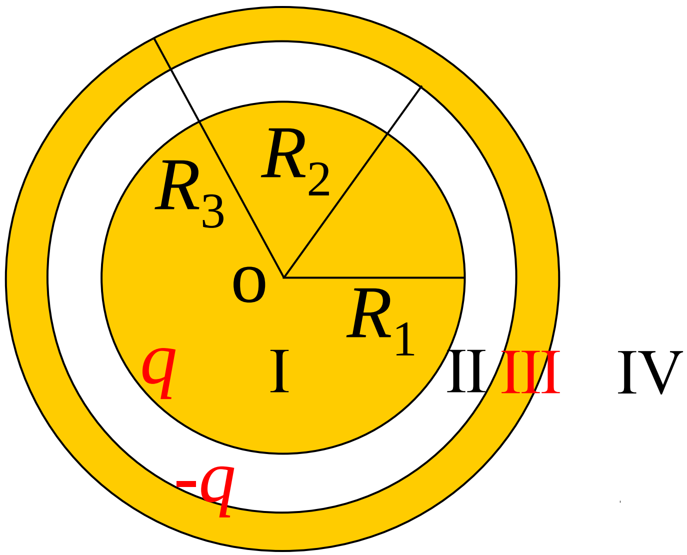
内球的 q 均匀地

分布在它的外表面

外球壳

内表面均匀地分布 $-q$

外表面无电荷



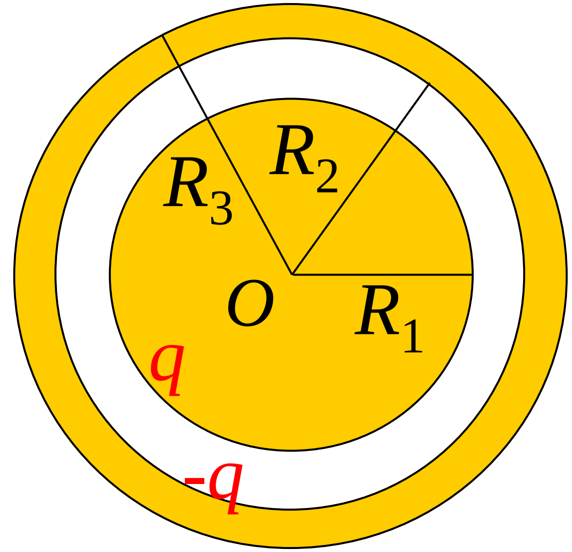
Ⅰ区： $V_1 = \dfrac{q}{4\pi\varepsilon_0 R_1} + \dfrac{-q}{4\pi\varepsilon_0 R_2}$

Ⅱ区： $V_2 = \dfrac{q}{4\pi\varepsilon_0 r} + \dfrac{-q}{4\pi\varepsilon_0 R_2}$

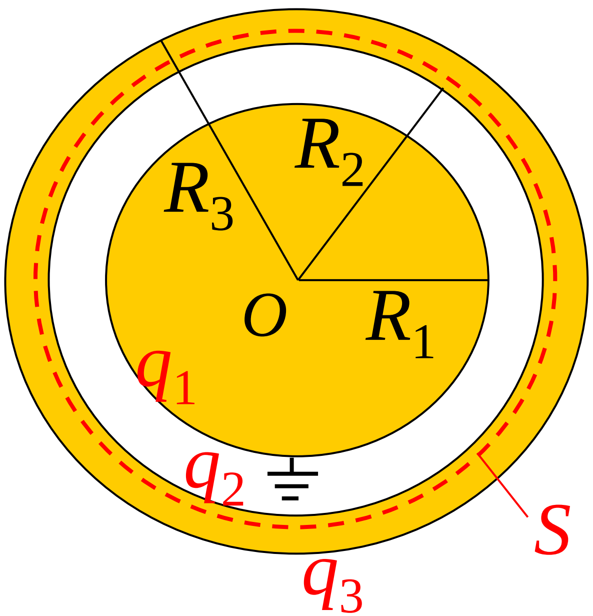
Ⅲ区： $V_3 = \dfrac{q}{4\pi\varepsilon_0 r} + \dfrac{-q}{4\pi\varepsilon_0 r} = 0$

Ⅳ区： $V_4 = \dfrac{q}{4\pi\varepsilon_0 r} + \dfrac{-q}{4\pi\varepsilon_0 r} = 0$

（2）再把内球接地，内球的电量和外球壳的电势？



假设接地后，内球外表面的电量为q_1，外球壳的内、外表面的电量为q_2、q_3



在外球壳上作高斯面 S

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{q_1 + q_2}{\varepsilon_0} = 0$$

$$q_1 + q_2 = 0 \cdots (1)$$

由电荷守恒

$$q_2 + q_3 = -q \cdots (2)$$

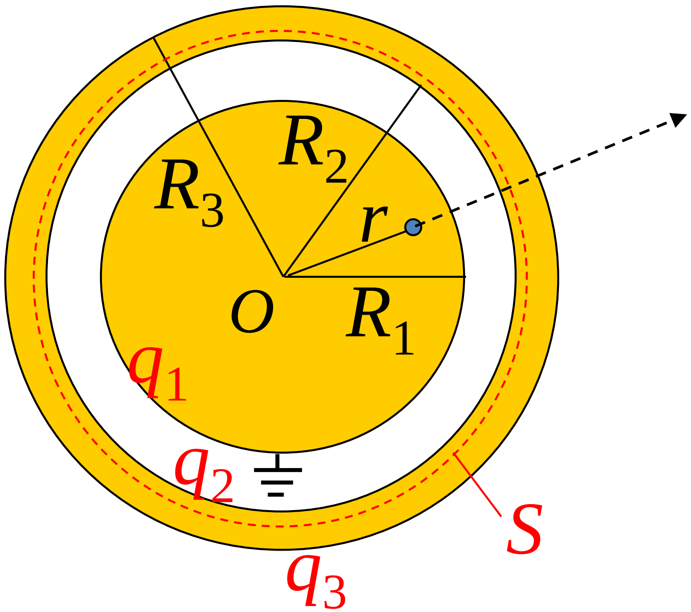
还缺一个方程

返回 退出

内球接地 -----电势为零

$$V_1=\int_r^{\infty}\vec{E}\cdot d\vec{r}=\int_r^{R_1}\vec{E}\cdot d\vec{r}+\int_{R_1}^{R_2}\vec{E}\cdot d\vec{r}+\int_{R_2}^{R_3}\vec{E}\cdot d\vec{r}+\int_{R_3}^{\infty}\vec{E}\cdot d\vec{r}$$

$$=0+\int_{R_1}^{R_2}\frac{q_1}{4\pi\varepsilon_0 r^2}dr+0+\int_{R_3}^{\infty}\frac{q_3}{4\pi\varepsilon_0 r^2}dr$$



$$=\frac{q_1}{4\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+\frac{q_3}{4\pi\varepsilon_0 R_3}\overset{\text{接地}}{==}0$$

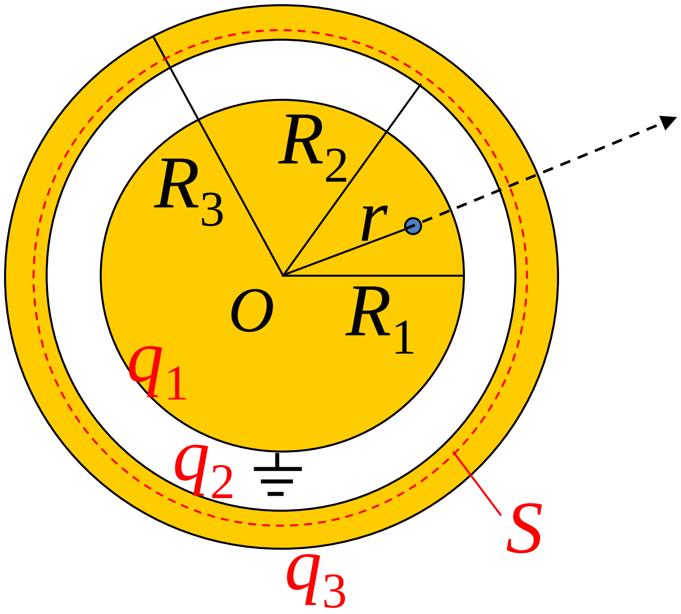
$$q_1\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+\frac{q_3}{R_3}=0\cdots(3)$$

(1)、(2)、(3) 联立，即可得到 q_1、 q_2、 q_3

$$q_1 = \frac{R_1 R_2}{R_2 R_3 - R_1 R_3 + R_1 R_2} q$$

$$R_2 > R_1 \Longrightarrow q_1 > 0 \qquad q_1 < q$$

$$q_2 = -q_1 \qquad q_2 < 0$$



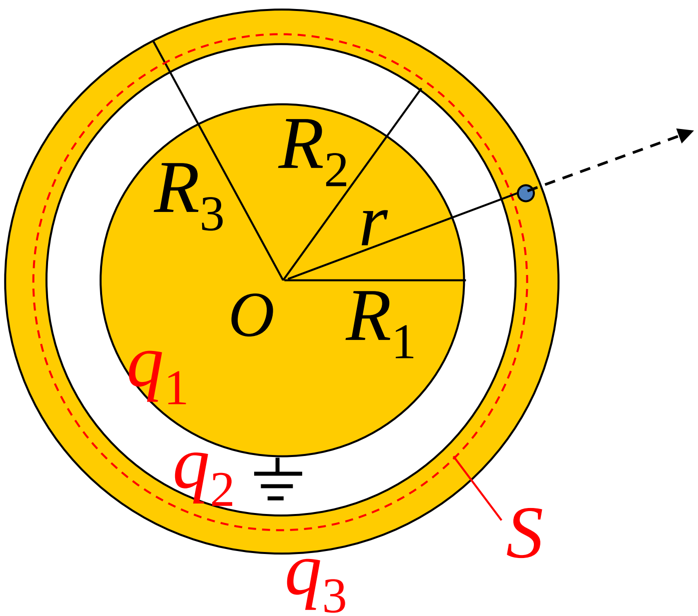
$$q_3 = \frac{(R_1 - R_2) R_3}{R_2 R_3 - R_1 R_3 + R_1 R_2} q$$

$$q_3 < 0$$

返回 退出

下面计算外球壳的电势

$$V_{外壳}=\int_{R_3}^{\infty}\vec{E}\cdot d\vec{r}=\int_{R_3}^{\infty}\frac{q_1+q_2+q_3}{4\pi\varepsilon_0 r^2}dr$$



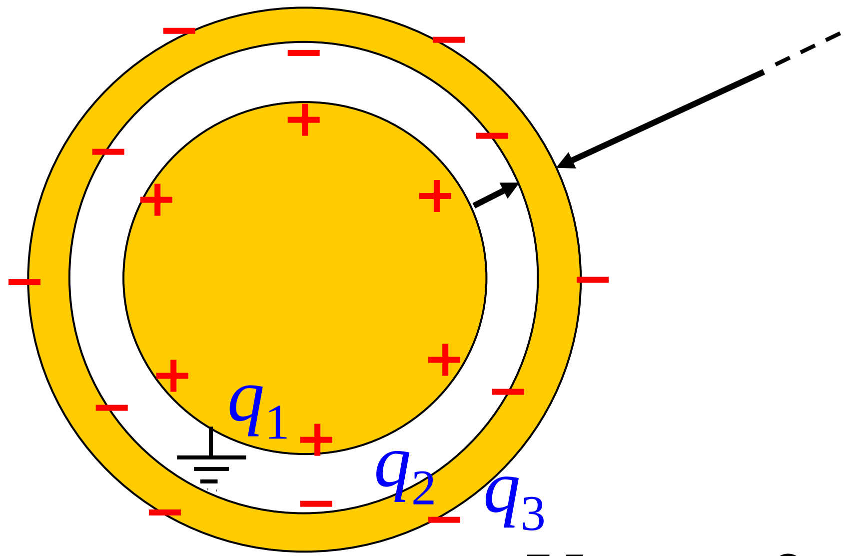
$$=\int_{R_3}^{\infty}\frac{q_3}{4\pi\varepsilon_0 r^2}\mathrm{d}r=\frac{q_3}{4\pi\varepsilon_0 R_3}$$

$$=\frac{(R_1-R_2)}{4\pi\varepsilon_0(R_2R_3-R_1R_3+R_1R_2)}q$$

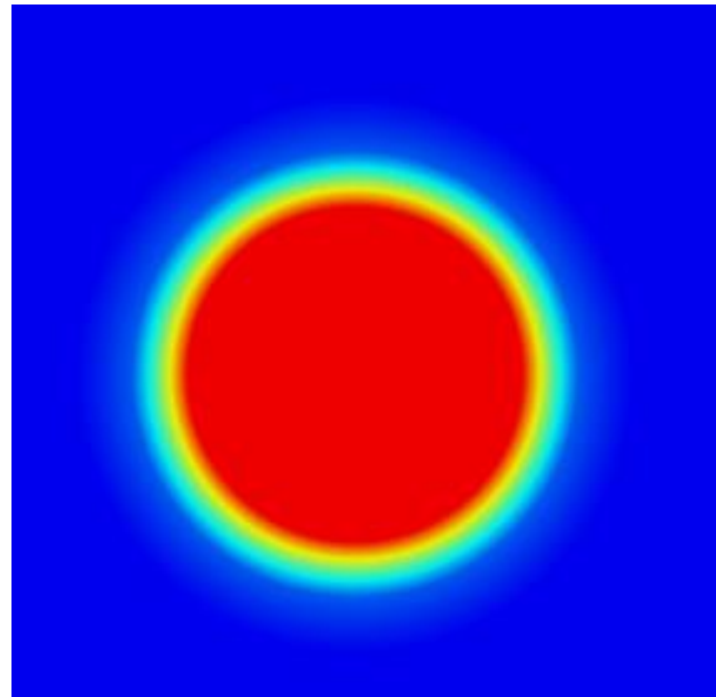
$$V_{外壳}<0$$

从电场线看，外球壳的电势确实是负的。

$q_1 > 0 \qquad q_2 < 0 \qquad q_3 < 0$



$V_{外壳} < 0$



接地的内球有电场线指向外球壳内表面；

必有从无穷远来的电场线指向外球壳外表面。

Capacitance and capacitor 电容和电容器

For isolated conductor：There's some sort of proportional relationship between its potential and the amount of charge it's carrying. So the concept of Capacitance is introduced 。

$$q = CV$$

$$C = \frac{q}{V}$$ 孤立导体的电容 **SI （F）**

When there are other conductors or charged bodies nearby:

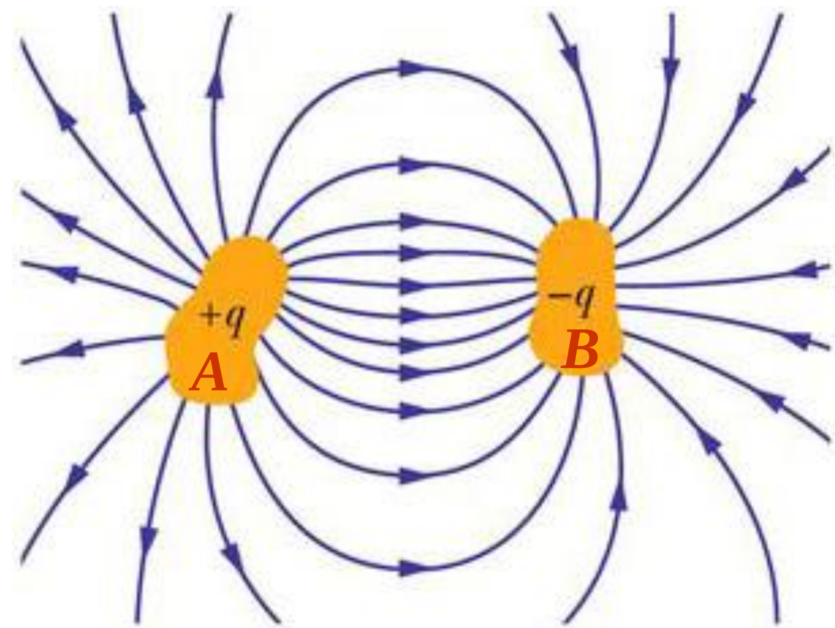
The electric potential V
与本身所带的电荷有关
与周围其他导体和带电体都有关

一般情况下，非孤立导体的电量与其电势不成正比

Two conductors, isolated electrically from each other and from their surroundings.

When the conductors are charged, the conductors have equal but opposite charges of magnitude q.

$$\boldsymbol{q \propto V_A - V_B}$$



$$\boldsymbol{q = C(V_A - V_B)}$$

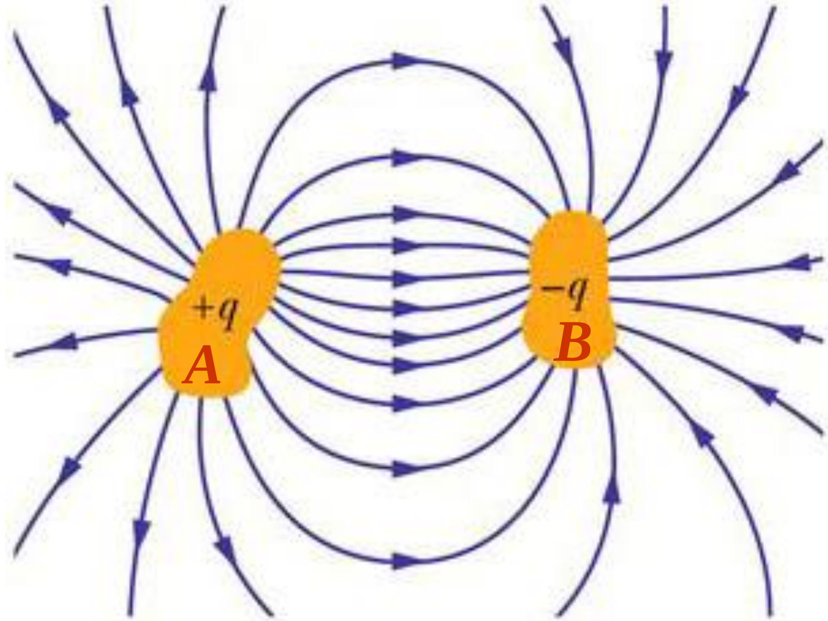
capacitance 电容

$$\boldsymbol{C = \frac{q}{V_A - V_B}}$$

1. 附近有其他带电体或导体时

⇩

两导体间的电势差与电量的正比关系受影响



2. 附近有其他带电体或导体时

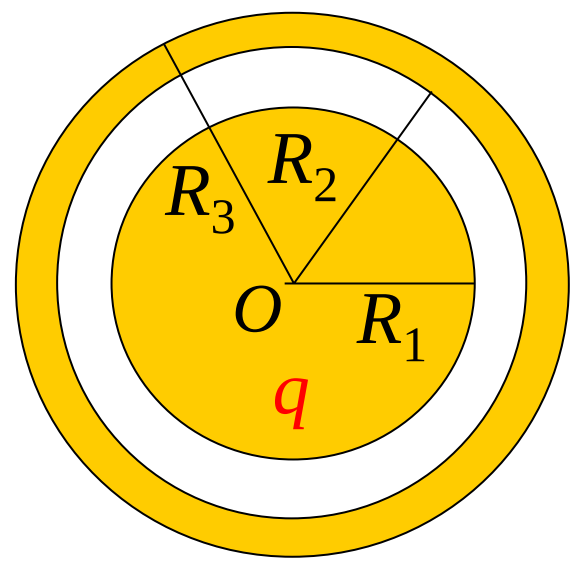
⇩

两导体间的电势差与电量的正比关系不受影响

$$C = \frac{q}{V_A - V_B}$$

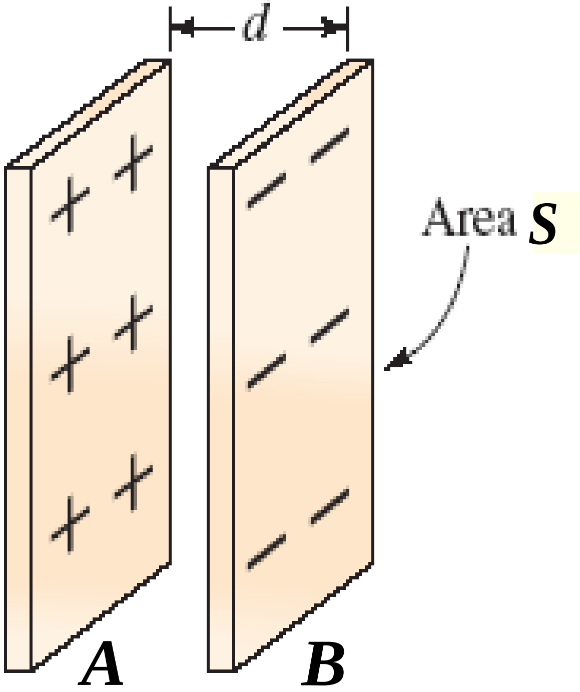
capacitor

电容器



几种形状的电容器及其电容

1. 平板电容器 Parallel-plate capacitor



$$d << \sqrt{S}$$ 无限大

设A板带$+q$　　B板带$-q$

$$\sigma = \frac{q}{S} \qquad E = \frac{\sigma}{\varepsilon_0}$$

$$V_A - V_B = \int_A^B \vec{E} \cdot d\vec{l} = Ed = \frac{\sigma d}{\varepsilon_0}$$

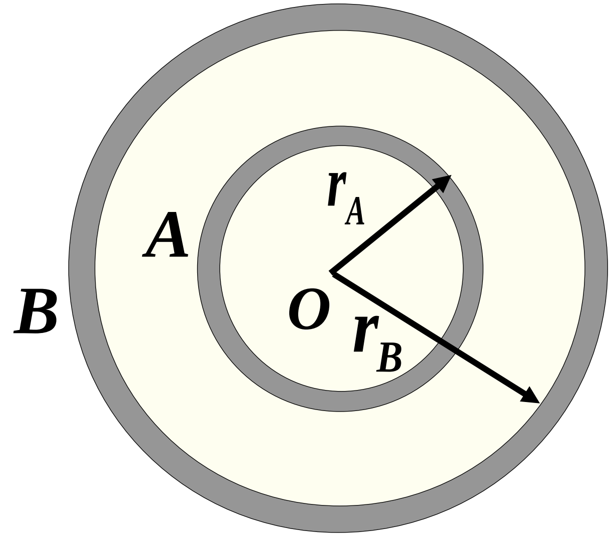
$$C = \frac{q}{V_A - V_B} = \frac{\varepsilon_0 S}{d}$$

2. 球形电容器 Spherical capacitor

两个同心导体球壳 $\boldsymbol{A}$ 和 $\boldsymbol{B}$

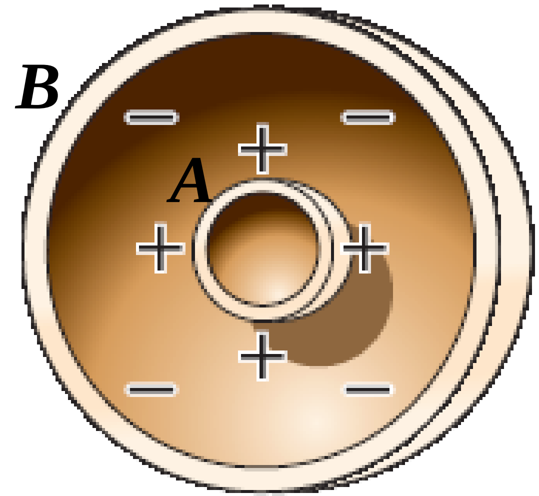
半径分别为 $\boldsymbol{r_A}$ 和 $\boldsymbol{r_B}$

带电量分别为 $+q$ 和 $-q$



$$V_A - V_B = \int_A^B \vec{E}\cdot d\vec{l}$$

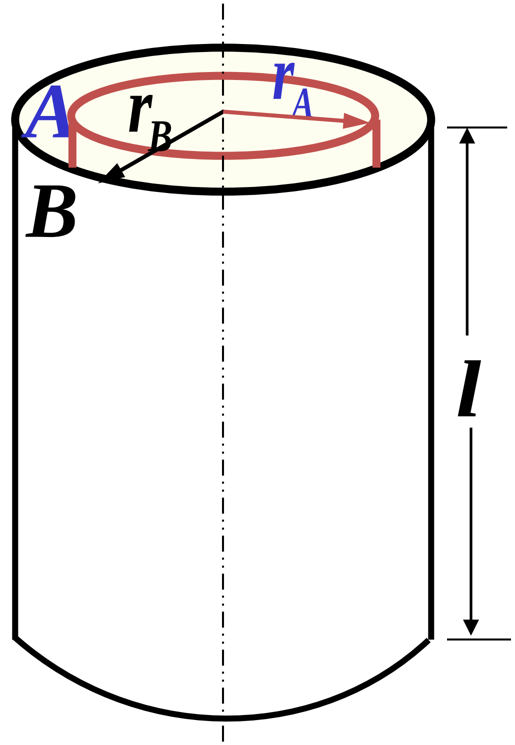
$r_A < r_B$



$$= \int_{r_A}^{r_B} \frac{q}{4\pi\varepsilon_0 r^2} dr = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

$$C = \frac{q}{V_A - V_B} = 4\pi\varepsilon_0\left(\frac{r_A r_B}{r_B - r_A}\right)\frac{q}{q}$$

3. 圆柱形电容器



两个长直同轴导体圆筒 ***A*** 和 ***B***

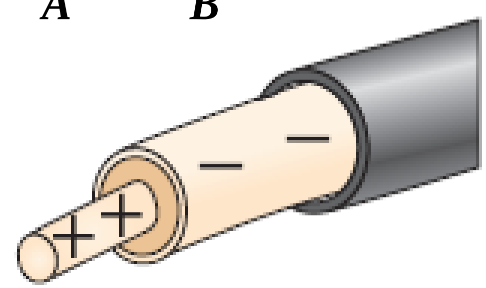
$$r_A < r_B << l \qquad \lambda = \frac{q}{l}$$

$$E \cdot 2r\pi \cdot l = \lambda \cdot l \cdot \frac{1}{\varepsilon_0}$$

$$V_A - V_B = \int_A^B \vec{E} \cdot d\vec{l} \qquad E = \frac{\lambda}{2r\pi\varepsilon_0}$$

$$= \int_{r_A}^{r_B} \frac{\lambda}{2\pi\varepsilon_0 r} dr = \frac{\lambda}{2\pi\varepsilon_0} \ln\frac{r_B}{r_A}$$

$$r_A < r_B$$



$$C = \frac{q}{V_A - V_B} = \frac{\lambda l}{V_A - V_B} = \frac{2\pi\varepsilon_0 l}{\ln\dfrac{r_B}{r_A}}$$

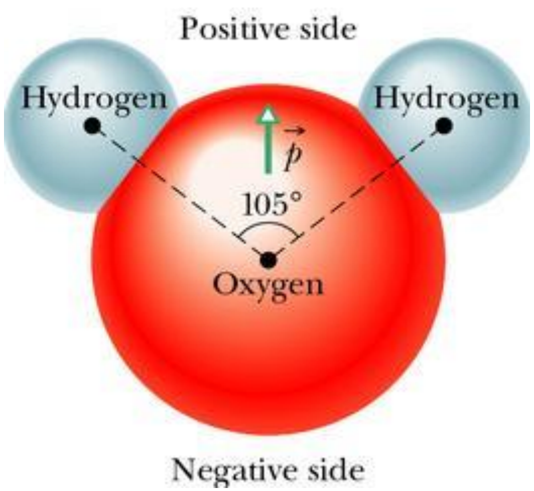
§ .7 Dielectrics in Electric Fields

电介质(Dielectrics) 就是绝缘体(insulator)

电介质由中性分子组成

Many molecules such as water have *permanent* electric dipole moments.

固有电偶极矩



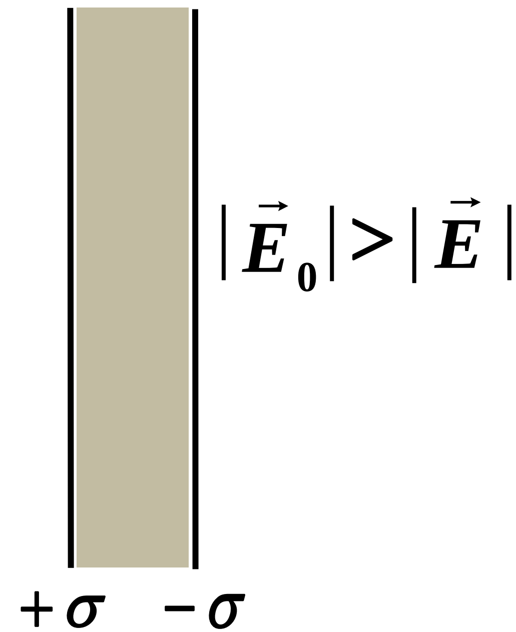
A molecule of H_2O

负电荷中心

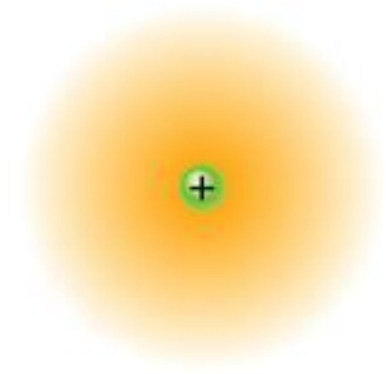
正电荷中心

有极分子

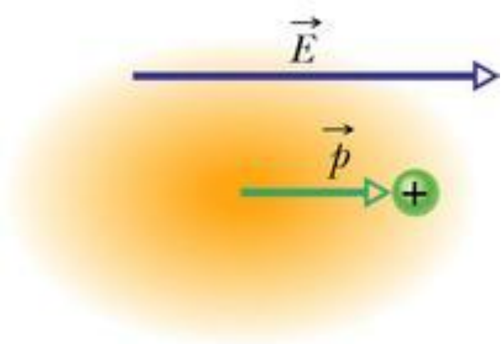
Polar molecule



In other molecules, the centers of the positive and negative charges coincide and thus no dipole moment is set up.



(a)



(b)

Nonpolar molecule

无极分子

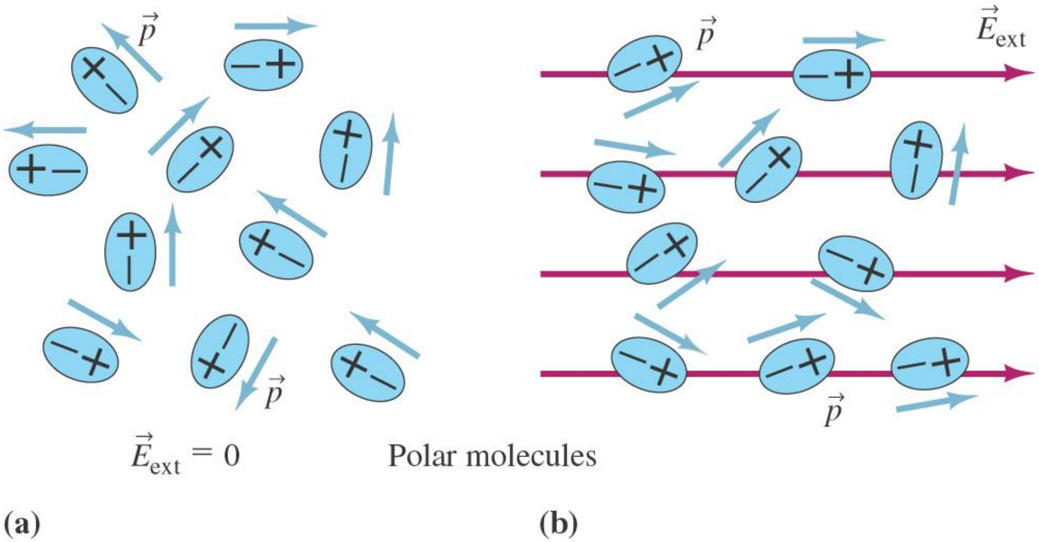
If a nonpolar molecule is placed in an external electric field $\vec{E}$

The field distorts the electron orbits and separates the centers of positive and negative charge

An induced dipole moment $\vec{p}$ appears.

感生电偶极矩

External electric field

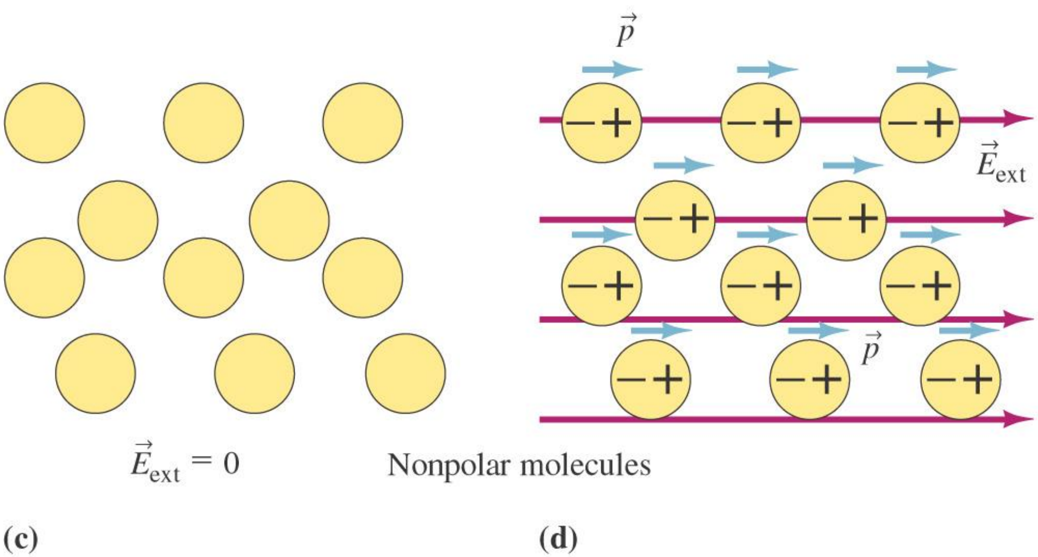


Polar molecule
有极分子

体积元　宏观小、微观大

Orientation dielectric
取向极化

$$\sum_i \vec{p}_i = 0 \rightarrow \sum_i \vec{p}_i \neq 0$$

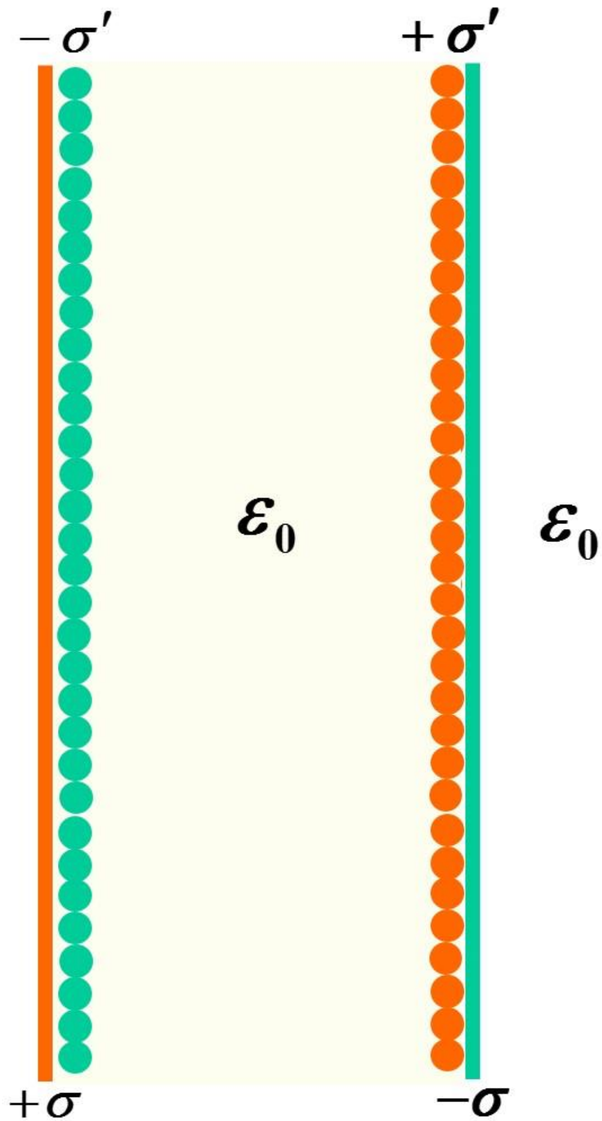


Nonpolar molecule
无极分子

体积元　宏观小、微观大

Displacement dielectric
位移极化

Generally，these two kind of polarization have same effect for macroscope description of dielectric.



Simplified model

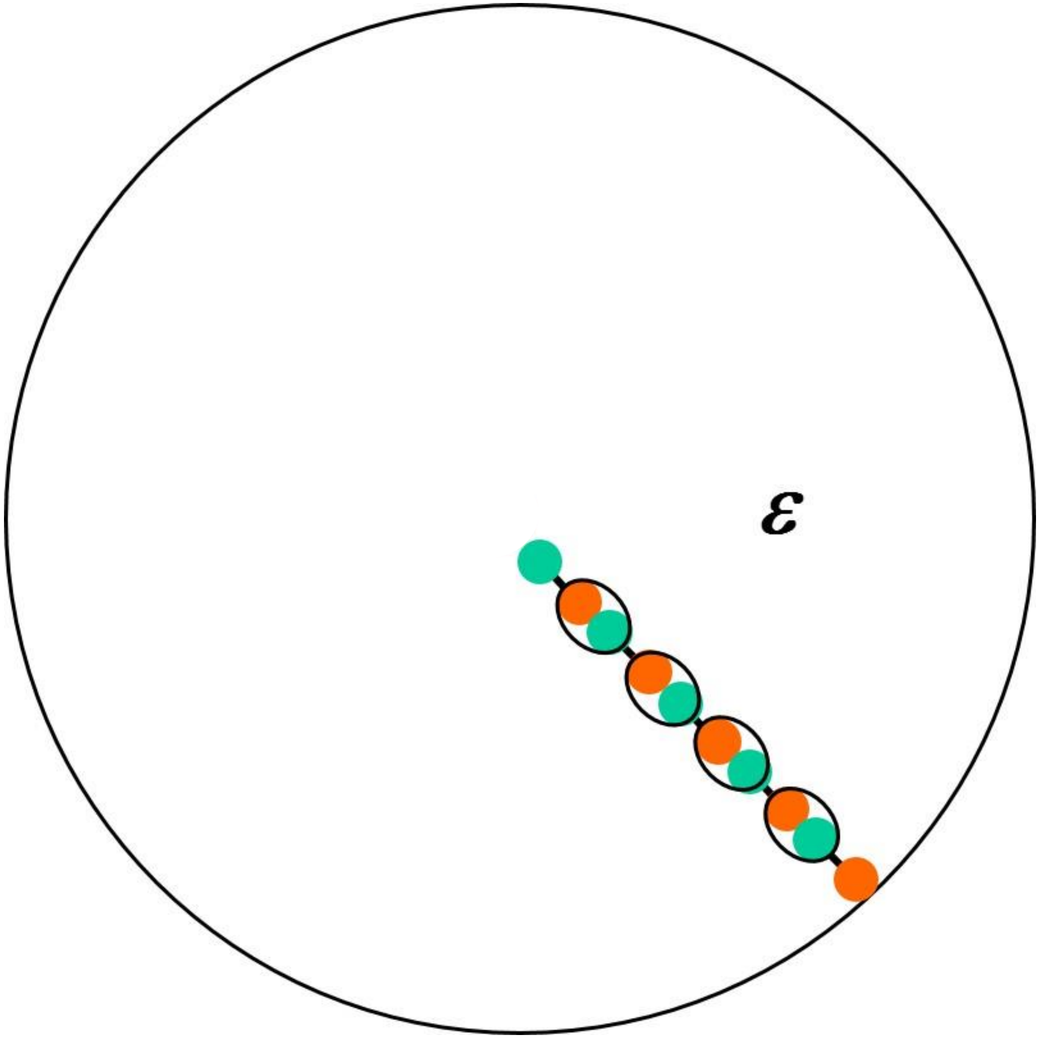
束缚电荷

Bound charge

极化电荷

Polarization charge

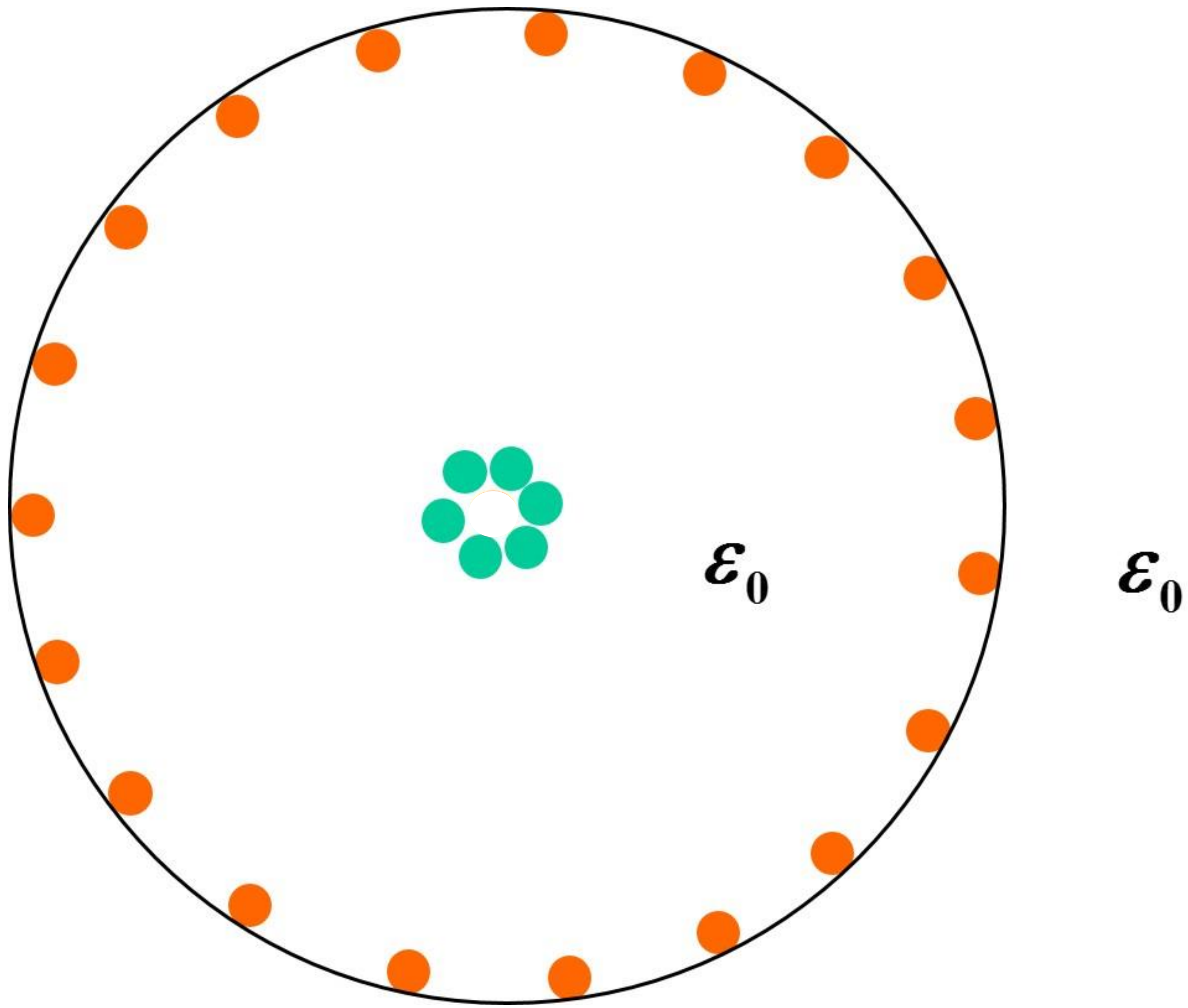
Generally，these two kind of polarization have same effect for macroscope description of dielectric.



Simplified model

ε_0

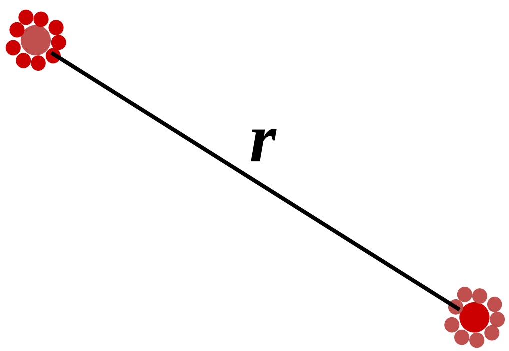
Bound charge 束缚电荷 Polarization charge 极化电荷



返回 退出

- After homogeneous dielectric polarization, polarized charge only appears at the nearby of free charge and dielectric interface.
- After heterogeneous polarization, polarized charges generally appear in the interior of the media.
- Infinitely isotropic linearly homogeneous medium

极化电荷的作用相当于减少自由点电荷的电量



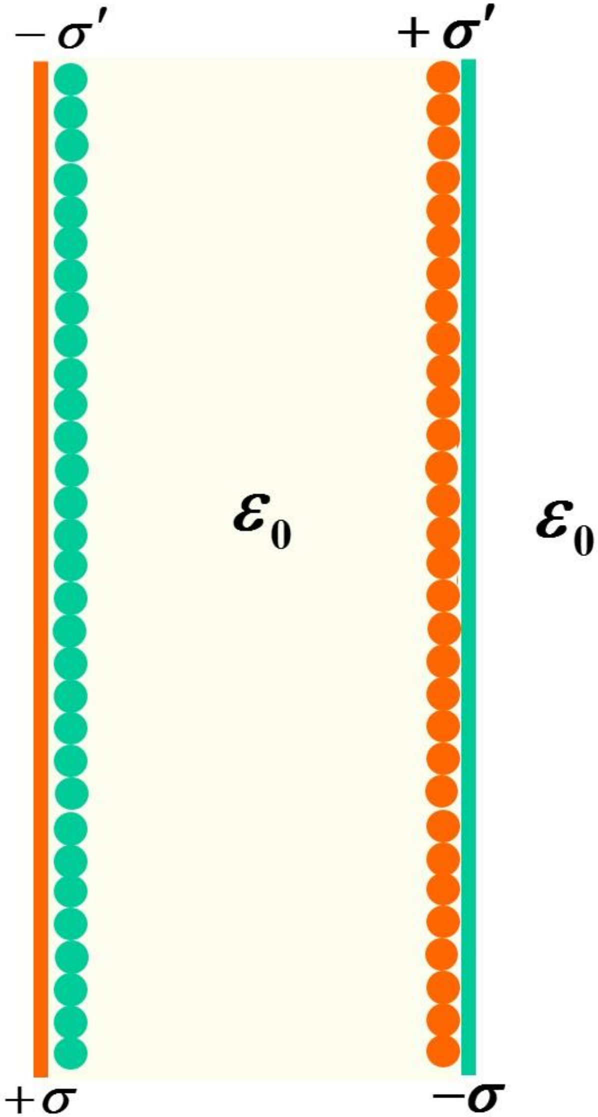
介质中两点电荷之间的相互作用力要比真空中时小

$$\vec{f}=\frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{r^3}\vec{r}=\frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^3}\vec{r}$$

$$\vec{E}=\frac{1}{4\pi\varepsilon}\frac{q}{r^3}\vec{r}$$

真空　　　无限大各向同性均匀介质

$$\varepsilon_0 \longrightarrow \varepsilon$$



$$C = \frac{q}{V_A - V_B} = \frac{\varepsilon_0 S}{d}$$

Bound charge 束缚电荷 Polarization charge 极化电荷

In the macroscope, polarized charges appear in the dielectric

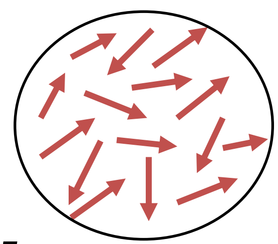
In the Polarization of Dielectric

使分子电偶极子有一定的取向并增大其电矩的过程

极化强度

Electric polarization density

$$\vec{P} = \frac{\sum_i \vec{p}_i}{\Delta V}$$



ΔV

宏观小 微观大

Isotropy linearity dielectric 各向同性线性电介质

$$\vec{P} = \chi_e \varepsilon_0 \vec{E}$$ （Experiment）

$$\chi_e = \varepsilon_r - 1$$

χ_e Polarizability 电介质的极化率

ε_r Relative permittivity

相对电容率（电介质的相对介电常量）

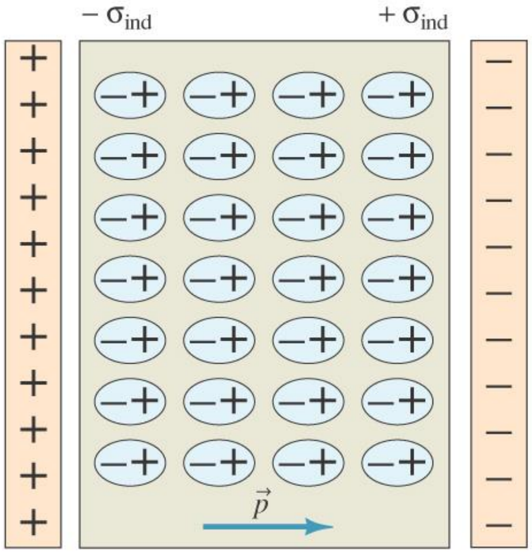
$$\varepsilon = \varepsilon_r \varepsilon_0$$

Permittivity

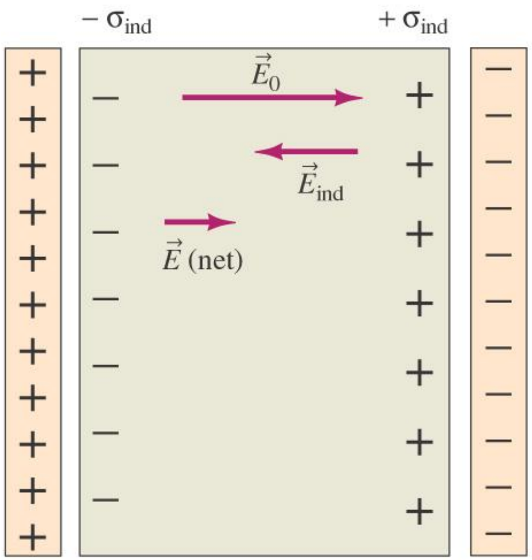
电容率（电介质的介电常量）

Anisotropy linearity dielectric 各向异性线性电介质

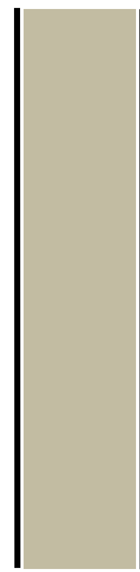
Tensor 张量



(a)



(b)

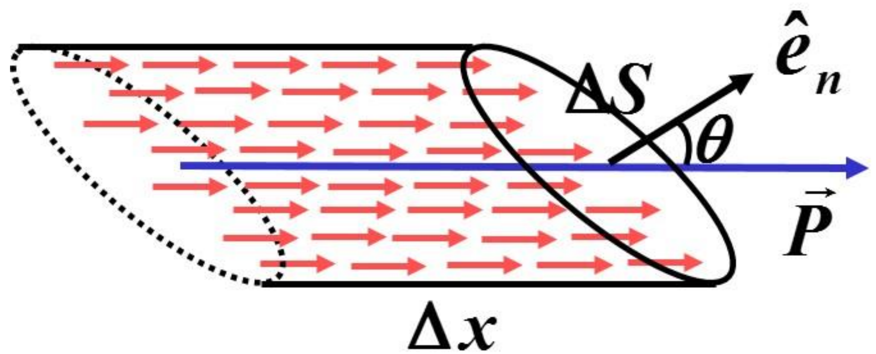


$$|\vec{E}_0| > |\vec{E}|$$

The relationship between the surface density of polarized charge and the polarization intensity

Molecule ➡ Electric dipole $\vec{p} = q\vec{l}$

Oblique cylindrical volume element



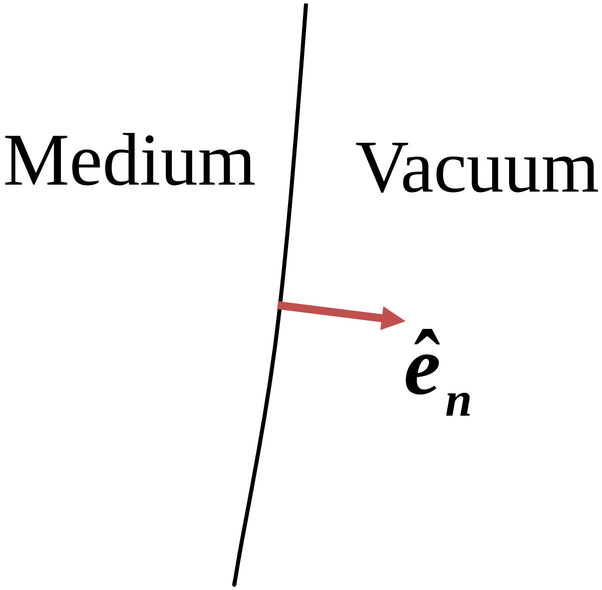
$$\Delta V = \Delta x \cos\theta \cdot \Delta S \qquad -q' = -\sigma' \Delta S \qquad +q' = +\sigma' \Delta S$$

$$\left| \sum_i \vec{p}_i \right| = P\Delta V \qquad (\sigma' \Delta S)\Delta x = P\Delta V = P\Delta x \cos\theta \cdot \Delta S$$

$$\left| \sum_i \vec{p}_i \right| = (\sigma' \Delta S)\Delta x$$

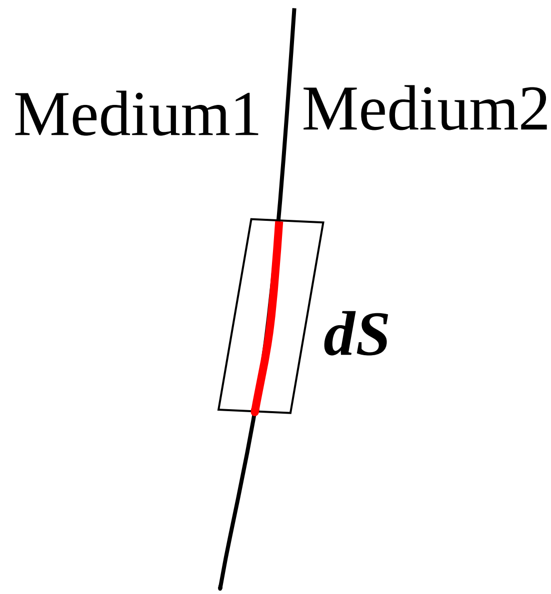
$$\sigma' = P\cos\theta$$

$$\sigma' = \vec{P} \cdot \hat{e}_n$$



$$\sigma' = \vec{P} \cdot \hat{e}_n$$

The surface polarization charge on the interface between two media

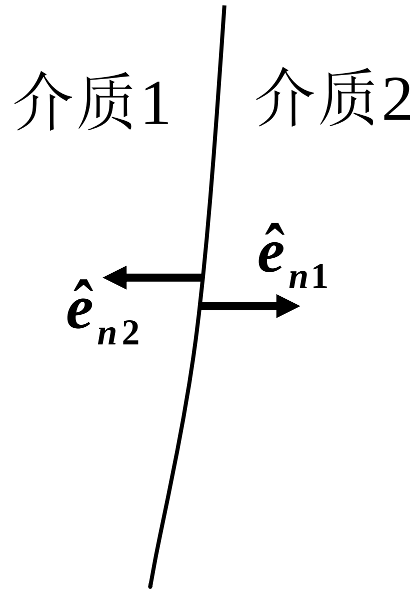


Take a certain thickness of thin layer on both sides of the interface

在此薄层内出现的极化电荷与 dS 之比称为分界面上的极化电荷面密度 Surface density of Polarized charge

For medium 1 $\vec{P}_1 \cdot \hat{e}_{n1}$

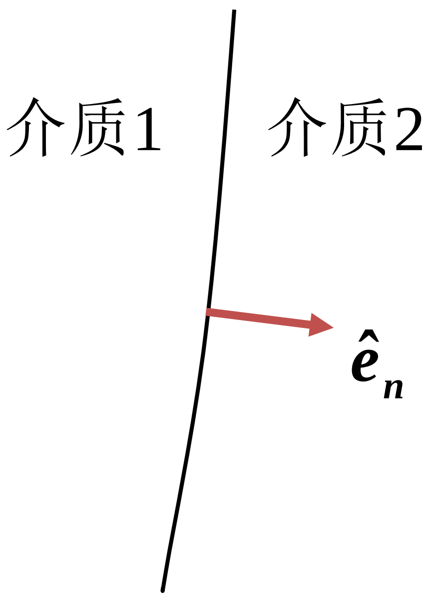
For medium 2 $\vec{P}_2 \cdot \hat{e}_{n2}$



$$\vec{P}_1 \cdot \hat{e}_{n1} + \vec{P}_2 \cdot \hat{e}_{n2} = \sigma'$$

$$\hat{e}_{n1} = -\hat{e}_{n2}$$

$$\vec{P}_1 \cdot \hat{e}_{n1} - \vec{P}_2 \cdot \hat{e}_{n1} = \sigma'$$



$$\vec{P}_1 \cdot \hat{e}_n - \vec{P}_2 \cdot \hat{e}_n = \sigma'$$

Electrostatic field in medium

q_0 Free charge $\quad q'$ Polarization charge

$$\vec{E} = \vec{E}_0 + \vec{E}'$$

$$\oint_L \vec{E}_0 \cdot d\vec{l} = 0 \qquad \oint_L \vec{E}' \cdot d\vec{l} = 0$$

- Circuital Theorem of Electrostatic Field $\quad \oint_L \vec{E} \cdot d\vec{l} = 0$

- Gauss's law

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{\sum_i q_i + \sum_i q_i'}{\varepsilon_0}$$

介质中任意闭合曲面 S 内的极化电荷代数和 $\sum_i q_i'$

当电偶极子的正电荷穿出闭合曲面 S 时，同一电偶极子的负电荷就处于 S 内

闭合曲面 S 的任意面积元 dS 上的极化电荷 $\sigma' dS$

对应的 S 内的极化电荷 $dq' = -\sigma' dS$

闭合曲面 S 内的极化电荷

$$\sum_i q_i' = -\oint_S \sigma' dS = -\oint_S \vec{P} \cdot \hat{e}_n dS = -\oint_S \vec{P} \cdot d\vec{S}$$

$$\sum_i q_i' = -\oint_S \vec{P} \cdot d\vec{S}$$

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{\sum_i q_i - \oint_S \vec{P} \cdot d\vec{S}}{\varepsilon_0}$$

$$\oint_S (\varepsilon_0 \vec{E} + \vec{P}) \cdot d\vec{S} = \sum_i q_i$$

Auxiliary quantity ➡ electric displacement vector $\vec{D}$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

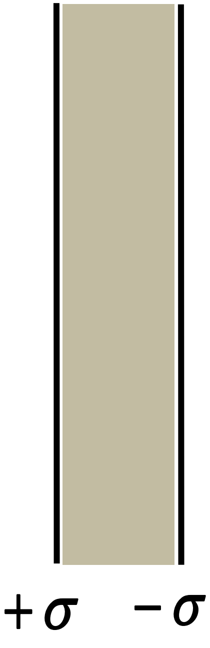
$$\oint_S \vec{D} \cdot d\vec{S} = \sum_i q_i$$

Gauss's law in the presence of medium

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \vec{E} + \chi_e \varepsilon_0 \vec{E} = (1 + \chi_e) \varepsilon_0 \vec{E} = \varepsilon_r \varepsilon_0 \vec{E}$$

$$\vec{D} = \varepsilon \vec{E}$$ Isotropic linear dielectric

充满相对介电常量为 ε_r 的均匀各向同性线性电介质



$|\vec{E}_0| > |\vec{E}|$

均匀极化　表面出现极化电荷

介质内部的电场由自由电荷和极化电荷共同产生

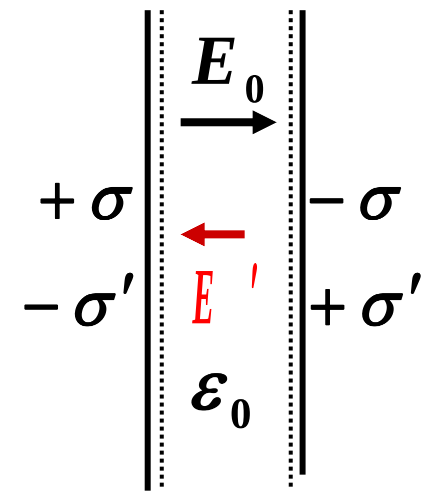
$\pm\sigma$　　$E_0 = \dfrac{\sigma}{\varepsilon_0}$　　$\pm\sigma'$　　$E' = \dfrac{\sigma'}{\varepsilon_0}$

$$E = E_0 - E' = \frac{\sigma}{\varepsilon_0} - \frac{\sigma'}{\varepsilon_o}$$

$$\sigma' = \vec{P}\cdot\hat{e}_n = P_n$$

$$= \varepsilon_0(\varepsilon_r - 1)E$$

$$\vec{P} = \varepsilon_0(\varepsilon_r - 1)\vec{E}$$



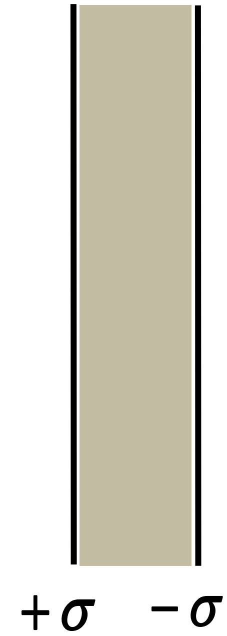
联立

$$E = \frac{\sigma}{\varepsilon_0\varepsilon_r} = \frac{E_0}{\varepsilon_r}$$　　$$\vec{E} = \frac{\vec{E}_0}{\varepsilon_r}$$

充满相对介电常量为 ε_r 的均匀各向同性线性电介质

均匀极化　表面出现极化电荷

介质内部的电场由自由电荷和极化电荷共同产生



$$|\vec{E}_0| > |\vec{E}|$$

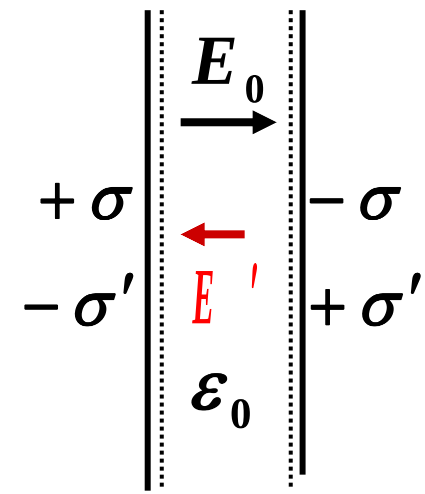
$\pm\sigma$　$E_0 = \dfrac{\sigma}{\varepsilon_0}$　$\pm\sigma'$　$E' = \dfrac{\sigma'}{\varepsilon_0}$

$$E = E_0 - E' = \frac{\sigma}{\varepsilon_0} - \frac{\sigma'}{\varepsilon_o}$$

$$\sigma' = \vec{P}\cdot\hat{e}_n = P_n$$

$$= \varepsilon_0(\varepsilon_r - 1)E$$

$$\vec{P} = \varepsilon_0(\varepsilon_r - 1)\vec{E}$$

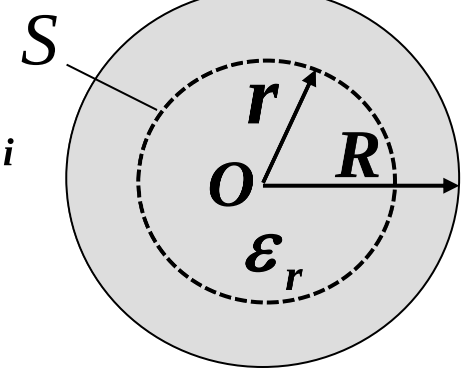


联立 $\Longrightarrow$ $E = \dfrac{\sigma}{\varepsilon_0\varepsilon_r} = \dfrac{E_0}{\varepsilon_r}$　$\vec{E} = \dfrac{\vec{E}_0}{\varepsilon_r}$

有一各向同性均匀介质球中均匀分布着体密度为 ρ_0 的自由电荷，介质球半径为 R，相对介电常量为 ε_r，球心电势及极化电荷分布？

The electric field distribution is spherically symmetric

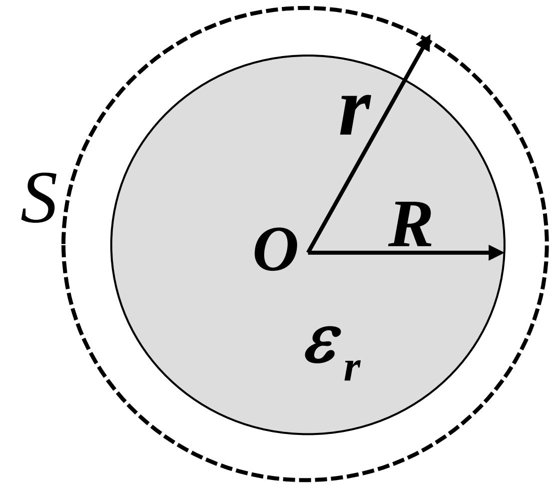
$$\oint_S \vec{D}\cdot d\vec{S} = \sum_i q_i$$



$r < R$

$$\oint_S \vec{D}\cdot d\vec{S} = \oint_S D dS = D4\pi r^2$$

$$\sum_i q_i = \frac{4\pi r^3}{3}\rho_0 \qquad D = \frac{\rho_0 r}{3}$$



$r > R$

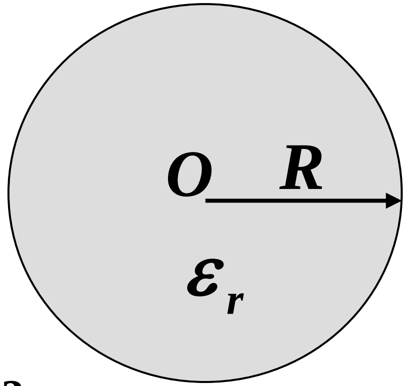
$$\oint_S \vec{D}\cdot d\vec{S} = \oint_S D dS = D4\pi r^2$$

$$\sum_i q_i = \frac{4\pi R^3}{3}\rho_0 \qquad D = \frac{\rho_0 R^3}{3r^2}$$

$$r < R, \quad D = \frac{\rho_0 r}{3} \qquad E = \frac{\rho_0 r}{3\varepsilon_0 \varepsilon_r}$$

$$\vec{D} = \varepsilon \vec{E}$$

$$r > R, \quad D = \frac{\rho_0 R^3}{3r^2} \qquad E = \frac{\rho_0 R^3}{3\varepsilon_0 r^2}$$



Potential at the center of sphere

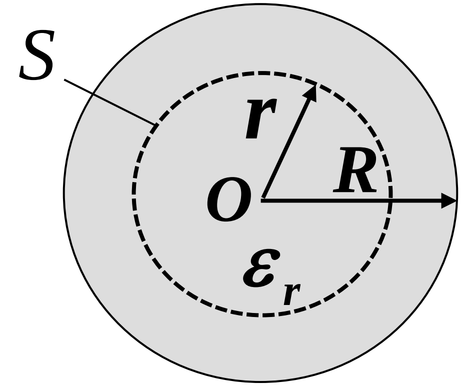
$$V_O = \int_O^\infty \vec{E} \cdot d\vec{l}$$

$$= \int_O^R \frac{\rho_0 r}{3\varepsilon_0 \varepsilon_r} dr + \int_R^\infty \frac{\rho_0 R^3}{3\varepsilon_0 r^2} dr$$

$$V_O = \frac{\rho_0 R^2}{6\varepsilon_0 \varepsilon_r}(1 + 2\varepsilon_r)$$

The distribution of Polarization charge

$r < R$ Polarized charge in the region（球内极化电荷）



$$\vec{P} = \varepsilon_0(\varepsilon_r - 1)\vec{E} = \frac{\rho_0 r(\varepsilon_r - 1)}{3\varepsilon_r}\hat{r}$$

对半径为 r 的同心球面 S，内有极化电荷

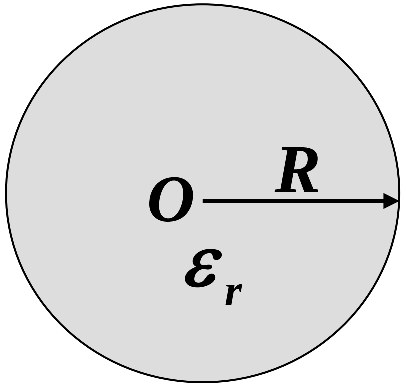
$$q' = -\oint_S \vec{P}\cdot d\vec{S} = -\oint_S P dS = -P\cdot 4\pi r^2 = -\frac{\varepsilon_r - 1}{\varepsilon_r}\rho_0 \frac{4\pi r^3}{3}$$

$q' \propto V$ $\Rightarrow$ A uniformly polarized sphere

$$\Rightarrow \rho' = -\frac{\varepsilon_r - 1}{\varepsilon_r}\rho_0$$

Polarized charge on the surface of the sphere

$$\vec{P} = \frac{\rho_0 r(\varepsilon_r - 1)}{3\varepsilon_r}\hat{r}$$



$$\sigma' = \vec{P}\cdot\hat{e}_n\Big|_{r=R} = P\Big|_{r=R} = \frac{\rho_0 R(\varepsilon_r - 1)}{3\varepsilon_r}$$

Uniformly polarized charge distribution on spherical surface

Summary:

(1) 球内部极化电荷与球表面极化电荷的总电量应当为零

$q_V' + q_S'$ 均匀带电球体 (极化电荷) + 均匀带电球面 (极化电荷)

$$= -\frac{\varepsilon_r - 1}{\varepsilon_r}\rho_0\frac{4\pi R^3}{3} + \frac{\rho_0 R(\varepsilon_r - 1)}{3\varepsilon_r}\cdot 4\pi R^2 = 0$$

介质是电中性的

(2) 从最后得到的电荷分布来检验场强对不对？

均匀带电球体 + 均匀带电球面

即：(自由电荷、极化电荷)+ (极化电荷)

在球内的场强，均匀带电球面没有贡献，只是均匀带电球体的贡献

$$\rho' = -\frac{\varepsilon_r - 1}{\varepsilon_r}\rho_0 \qquad \rho^* = \rho_0 + \rho' = \frac{\rho_0}{\varepsilon_r}$$

$$\vec{E}_{r<R} = \frac{\rho^* \vec{r}}{3\varepsilon_0} = \frac{\rho_0}{\varepsilon_r}\frac{\vec{r}}{3\varepsilon_0} = \frac{\rho_0 \vec{r}}{3\varepsilon_0\varepsilon_r} = \frac{\vec{E}_0}{\varepsilon_r}$$

这和前面得到的一样：

$$r < R \qquad D = \frac{\rho_0 r}{3} \qquad E = \frac{\rho_0 r}{3\varepsilon_0\varepsilon_r}$$

在球外的场强，极化电荷的均匀带电球面和极化电荷的均匀带电球体总的没有贡献，只有自由电荷的均匀带电球体有贡献。

$$\vec{E}_{r>R} = \vec{E}_0 = \frac{1}{4\pi\varepsilon_0 r^2}\cdot\rho_0\frac{4\pi R^3}{3}\hat{r} = \frac{\rho_0 R^3}{3\varepsilon_0 r^2}\hat{r}$$

这和前面得到的也一样：

$$r > R \qquad D = \frac{\rho_0 R^3}{3r^2} \qquad E = \frac{\rho_0 R^3}{3\varepsilon_0 r^2}$$

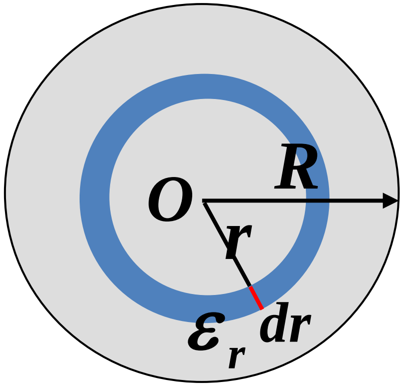
(3) 用电势叠加计算电势 V_O 来检验

$$V_O=\int_{q^*} dV^* + V_S \quad = \int_O^R \frac{\rho^* 4\pi r^2 dr}{4\pi\varepsilon_0 r}+\frac{\sigma' 4\pi R^2}{4\pi\varepsilon_0 R}$$

带电球面的电势

带电球体各壳层的电势叠加

$$=\int_O^R \frac{\rho^* r}{\varepsilon_0}dr+\frac{\sigma' R}{\varepsilon_0} \quad = \frac{\rho^*}{\varepsilon_0}\frac{1}{2}R^2+\frac{\sigma' R}{\varepsilon_0}$$



将 $\rho^*=\dfrac{\rho_0}{\varepsilon_r}$ 及 $\sigma'=\dfrac{\rho_0 R(\varepsilon_r-1)}{3\varepsilon_r}$

代入可得 $V_O=\dfrac{\rho_0 R^2}{6\varepsilon_0\varepsilon_r}(1+2\varepsilon_r)$ 与前面结果一样

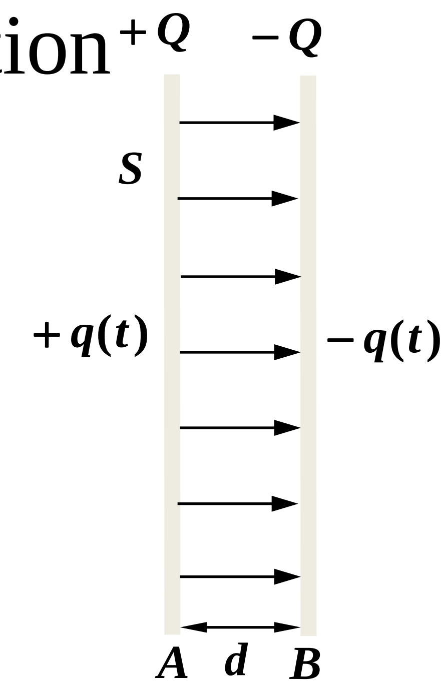
Electrostatic energy

The energy of a charged system

Work must be done by external forces during the setup of field distribution。

Charge plane-parallel capacitor

Energy transformation

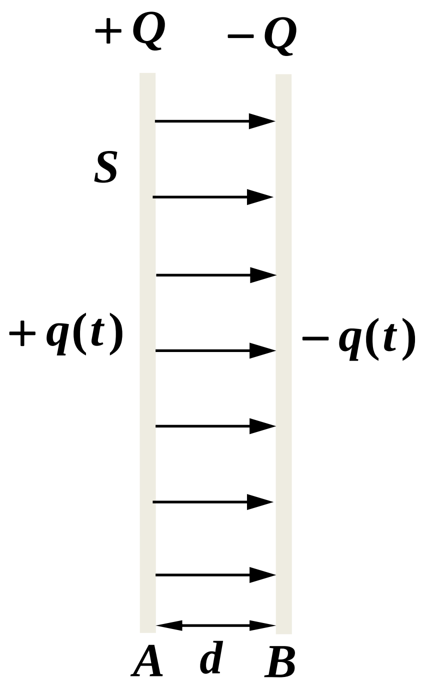


设充电时，在电源的作用下把正的电荷元 $\mathrm{d}q$ 不断地从 B板上拉下来，再推到 A板上去。

设在 t 时刻，电容器两极板分别带电荷 $\pm q(t)$

这时两极板间的电势差 $V(t)=\dfrac{q(t)}{C}$

若继续从 B板迁移电荷元 $\mathbf{d}q$ 到 A板，则电源必须做功



$$dA = V(t)dq = \frac{q(t)}{C}dq$$

从开始极板上无电荷直到极板上带电量为 Q 时，电源所做的功为

$$A = \int dA = \int_0^Q \frac{q(t)}{C}dq = \frac{Q^2}{2C}$$

极板上带电量为 Q 时两极板间的电势差为 U

$$A = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

$$C = \frac{Q}{V}$$

返回 退出

If the dielectric loss is negligible, the amount of work done by the source is equal to the electrostatic energy of the capacitor

$$W = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

The energy of a charged system

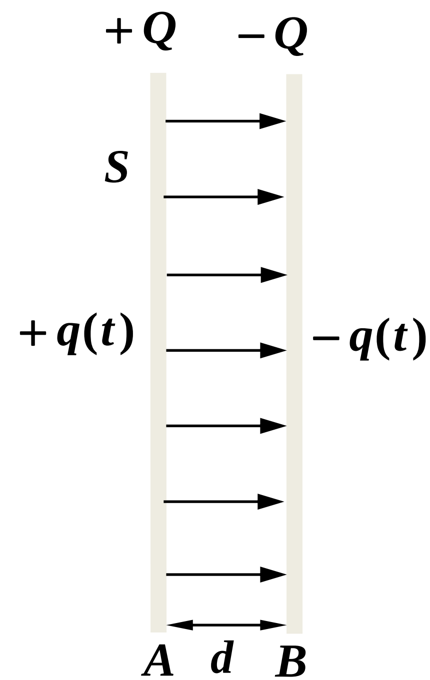
储藏在电场中的能量

电场携带的能量

电场的能量

When you charge a capacitor, you have an electric field in the capacitor. During charging, the power supply works on the capacitor, causing it to consume other forms of energy and convert them into energy stored in the electric field

$$W = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$



$$U = Ed \qquad Q = \sigma S = \varepsilon ES$$

$$W = \frac{1}{2}QV = \frac{1}{2}\varepsilon E^2 Sd = \frac{1}{2}DEV_{体积}$$

The energy density of an electrostatic field

$$w = \frac{1}{2}DE = \frac{1}{2}\vec{D}\cdot\vec{E}$$

$$W = \int_V w dV = \frac{1}{2}\int_V \vec{D}\cdot\vec{E} dV$$

Isotropic linear medium