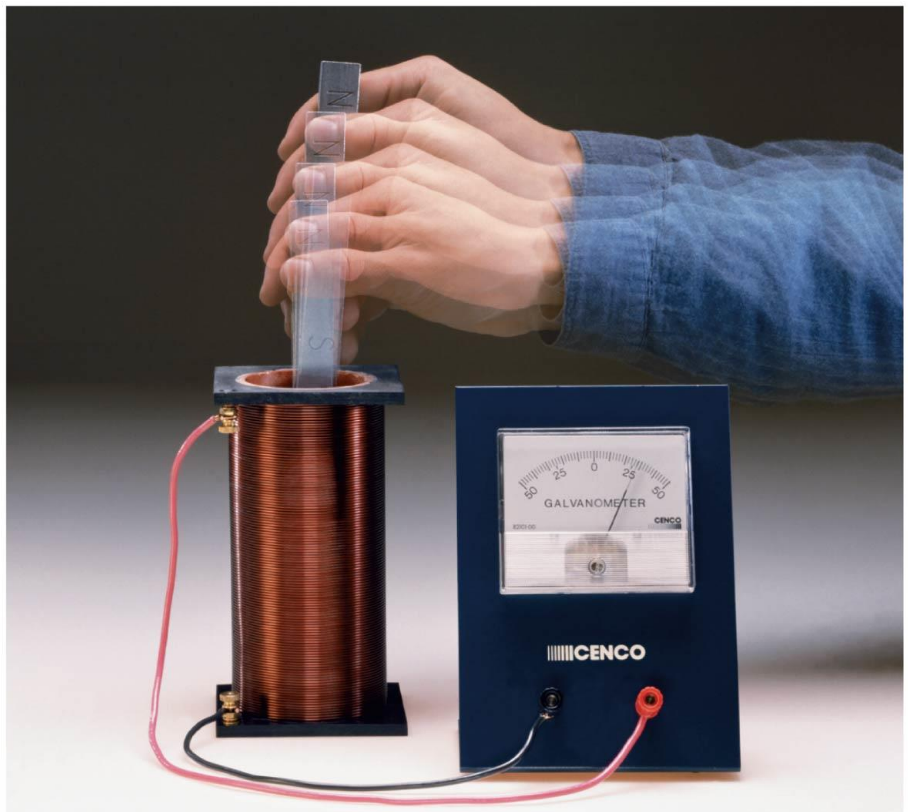
Chapter 21
Electromagnetic Induction and Faraday's Law



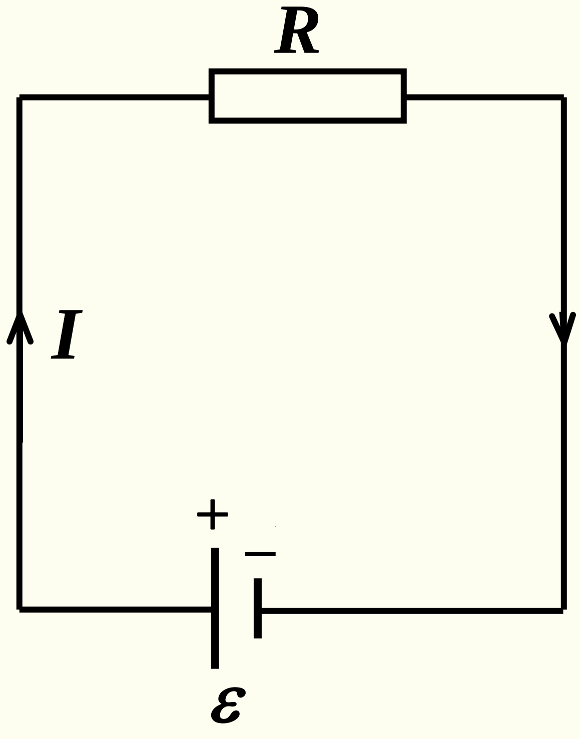
Contents of Chapter 21

- Induced EMF Electromotive force   EMF(电动势）
- Faraday’s Law of Induction; Lenz’s Law
- EMF Induced in a Moving Conductor
- Changing Magnetic Flux Produces an Electric Field
- Electric Generators

The change of the magnetic flux through any surface bounded by a loop causes a induction current in that loop. 感应电流

感应电流的出现表明存在着某种电动势

electromotive force emf



电动势 $\varepsilon = \int_{-}^{+} \frac{\vec{F}_k}{q} \cdot d\vec{l} = \int_{-}^{+} \vec{E}_k \cdot d\vec{l}$

$$\varepsilon = \oint_l \vec{E}_k \cdot d\vec{l}$$

科拉顿 Jean-Daniel Colladon

1823年 电流计↔小磁针的偏转

在相同条件下，不同金属导体回路中产生的感应电流与导体的导电能力成正比。

感应电动势 **induction emf .**

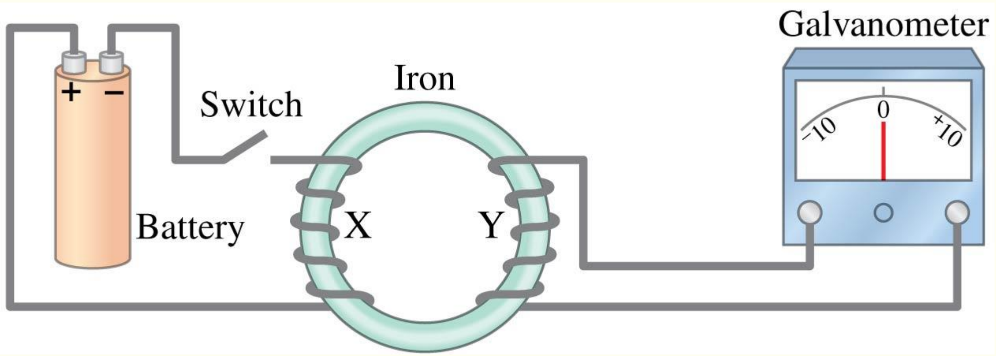
The induction emf leads to an induction current

§ .1 法拉第电磁感应定律

Faraday's Discovery and the Law of Induction

Faraday's law has far-reaching technological applications. It lies behind our entire system of electrical power generation and plays a role in most of the electronic devices we use.

Almost 200 years ago, Faraday looked for evidence that a magnetic field would induce an electric current with this apparatus



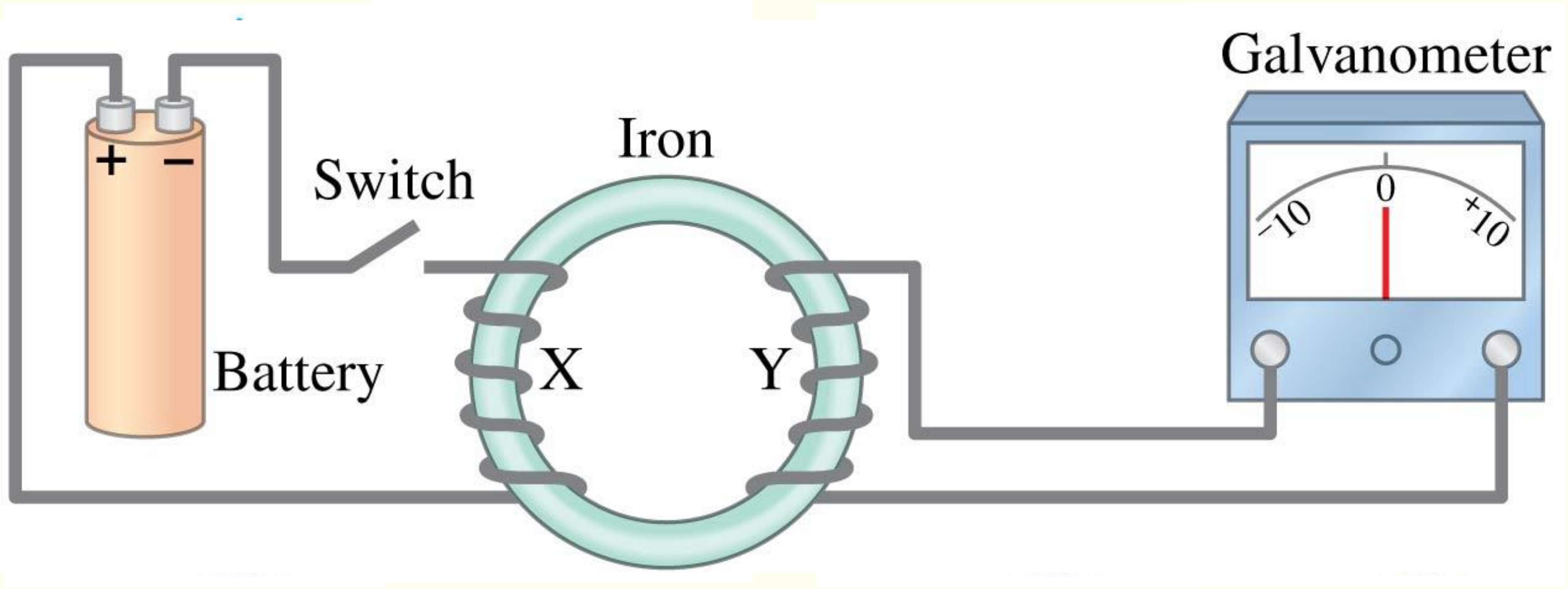
Four Experiments

He found no evidence when the current through the left-hand loop was steady, but did see a current induced in the right-hand loop when the switch was turned on or off.

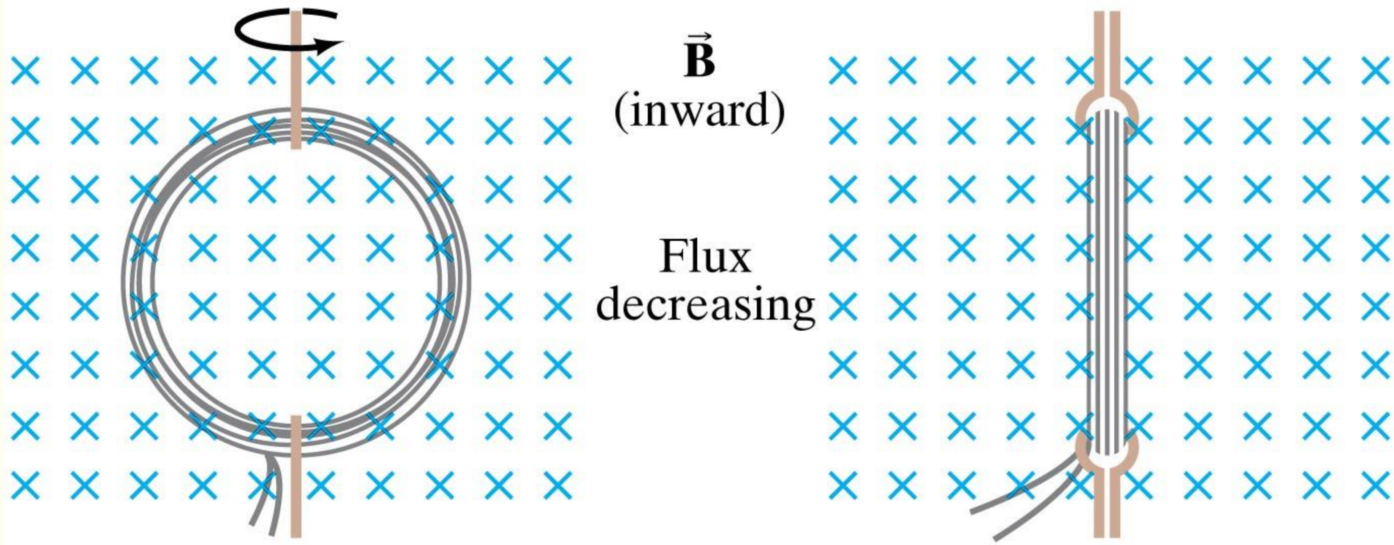
When The rod is moving

⑴**relative motion**

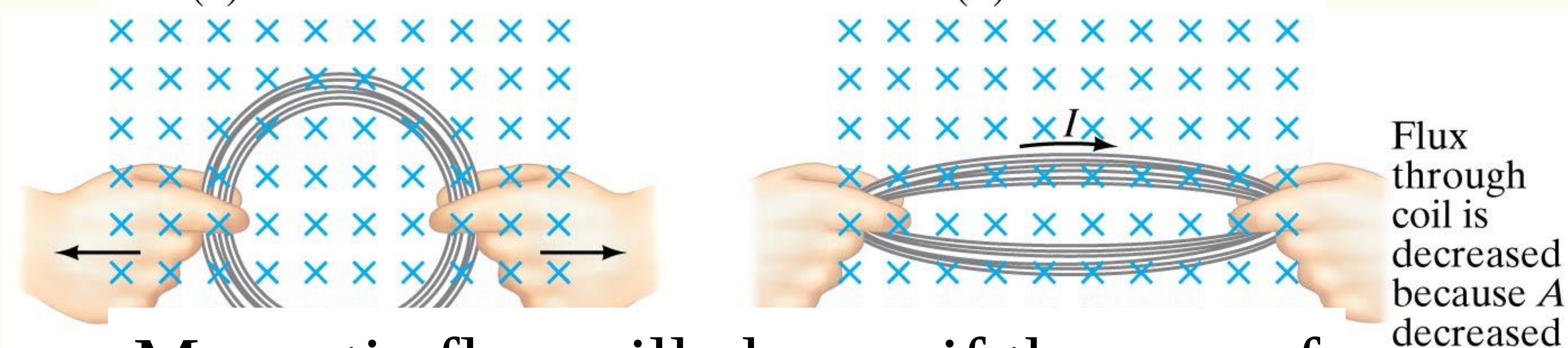
⑵**varying of field**



Magnetic flux will change if the angle between the loop and the field changes:



(a) Maximum flux (b) Zero flux

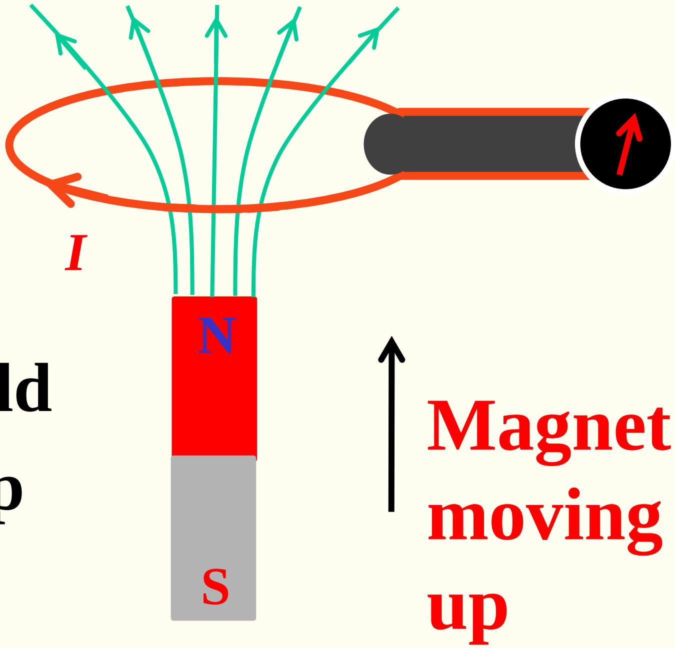


Magnetic flux will change if the area of the loop changes:

磁场发生变化 changing the magnetic field

运动不能产生EMF？

Is this allege completely right？



uniform magnetic field

Unchanging the magnetic field

Changing the area of the loop

穿过回路的磁通量发生了变化

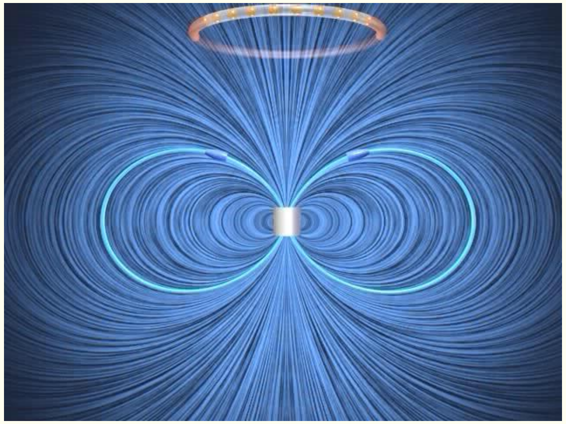
change the **magnetic flux** through the loop

Suppose a loop enclosing an area S is placed in a **magnetic field** $\vec{B}$. Then the magnetic flux through the loop is

$$\Phi_B = \iint_S \vec{B} \cdot d\vec{S}$$

•Lenz's Law

The magnitude of the emf ε induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time.



$$\varepsilon \propto \frac{\mathrm{d}\Phi_B}{\mathrm{d}t} \Rightarrow \varepsilon = -k\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$

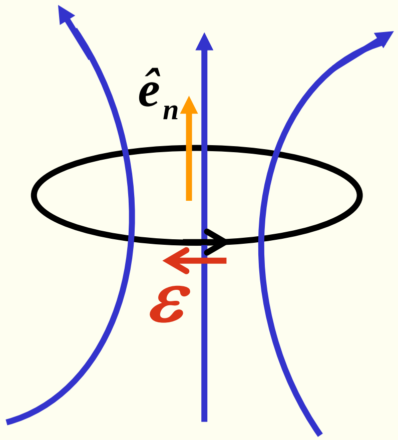
in SI $$\varepsilon = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$

$$\varepsilon = \oint_L \vec{E}_k \cdot d\vec{l}$$

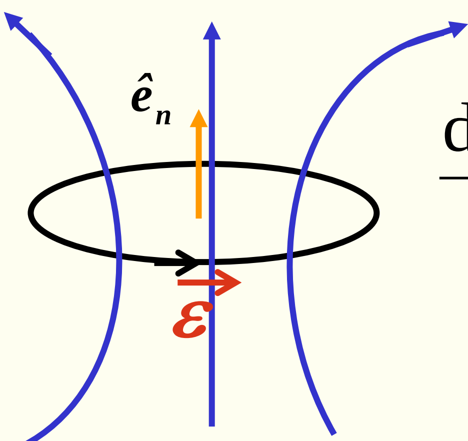
$$\oint_L \vec{E}_k \cdot d\vec{l} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \vec{B} \cdot d\vec{S}$$

$$\Phi_B = \iint_S \vec{B} \cdot d\vec{S}$$

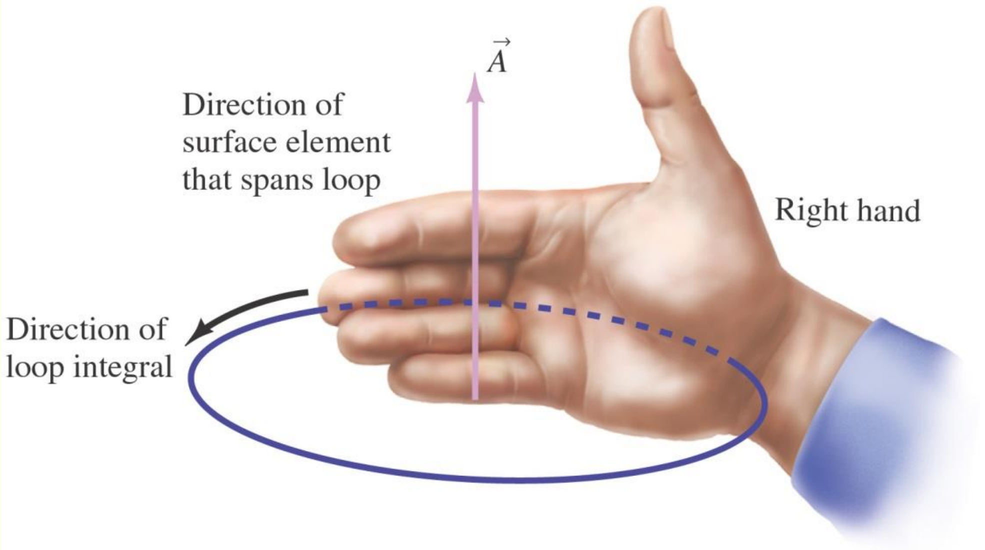
$$\oint_L \vec{E}_k \cdot d\vec{l} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \vec{B}\cdot d\vec{S}$$



$$\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} > 0$$

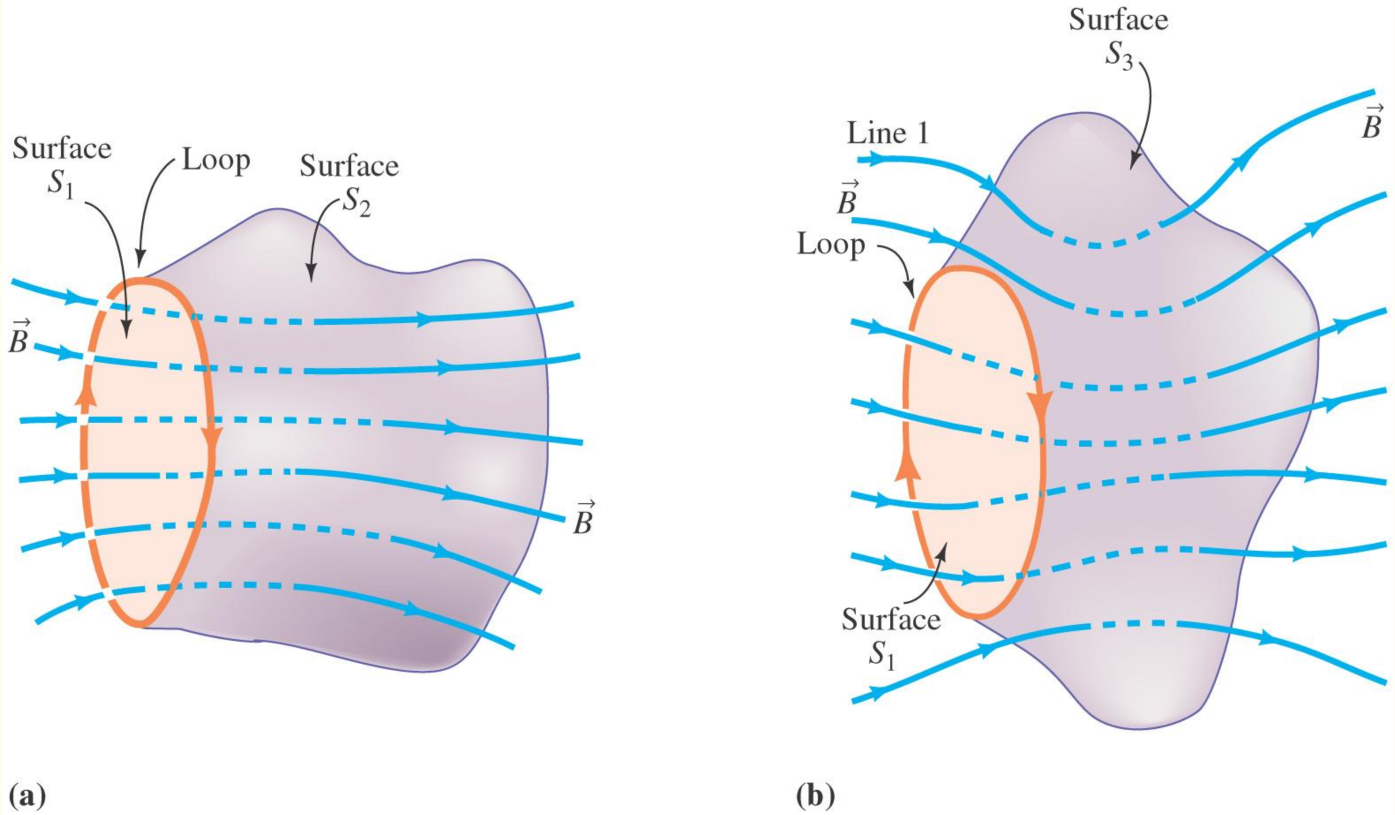


$$\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} < 0$$



Right-hand Rule for a Loop

$$\oint_L \vec{E}_k \cdot d\vec{l} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \vec{B} \cdot d\vec{S}$$



The same number of field lines pass through any two surfaces bounded by the same closed loop.

Loop → ***N*** **Turns**

When the flux through the loop changes, the induced electromotive force is generated in each turn of the coil。

$$\Phi_1,\ \Phi_2,\ \Phi_3,\ldots,\ \Phi_i,\ldots,\ \Phi_N \qquad \varepsilon=\sum_{i=1}^{N}\left(-\frac{d\Phi_i}{dt}\right)$$

$$\varepsilon=-\sum_{i=1}^{N}\frac{d\Phi_i}{dt}=-\frac{d\sum_{i=1}^{N}\Phi_i}{dt}=-\frac{d(N\Phi_i)}{dt}=-\frac{d\Psi}{dt}$$

If the magnetic flux through each turn is the same

$$\Psi=\sum_{i=1}^{N}\Phi_i=N\Phi_i$$

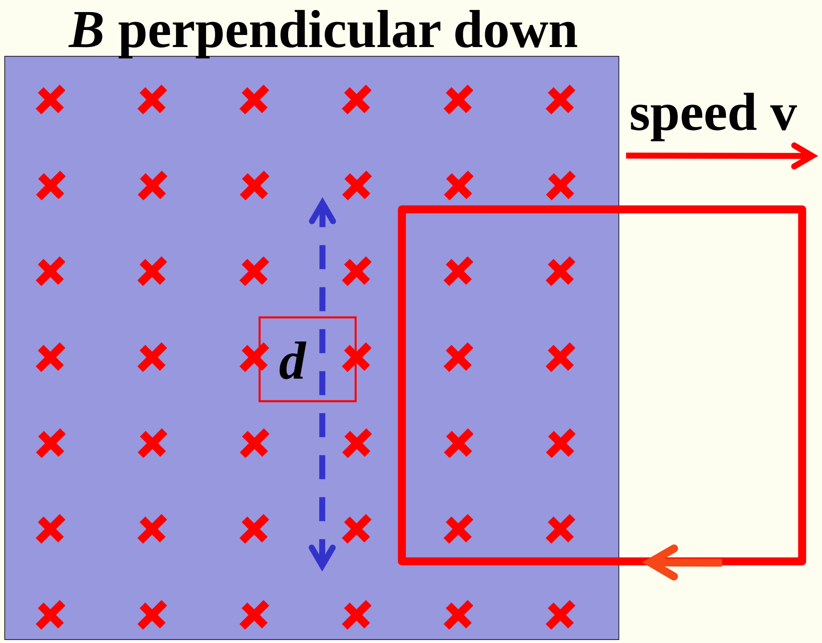
磁链 magnetic flux linkage

Diamagnetic Levitation…

- Diamagnetism is usually weak, but a strong enough field (in this case 16 Tesla) can levitate ordinary material, for example a frog, making an appearance here inside a solenoid (in Holland).
- 

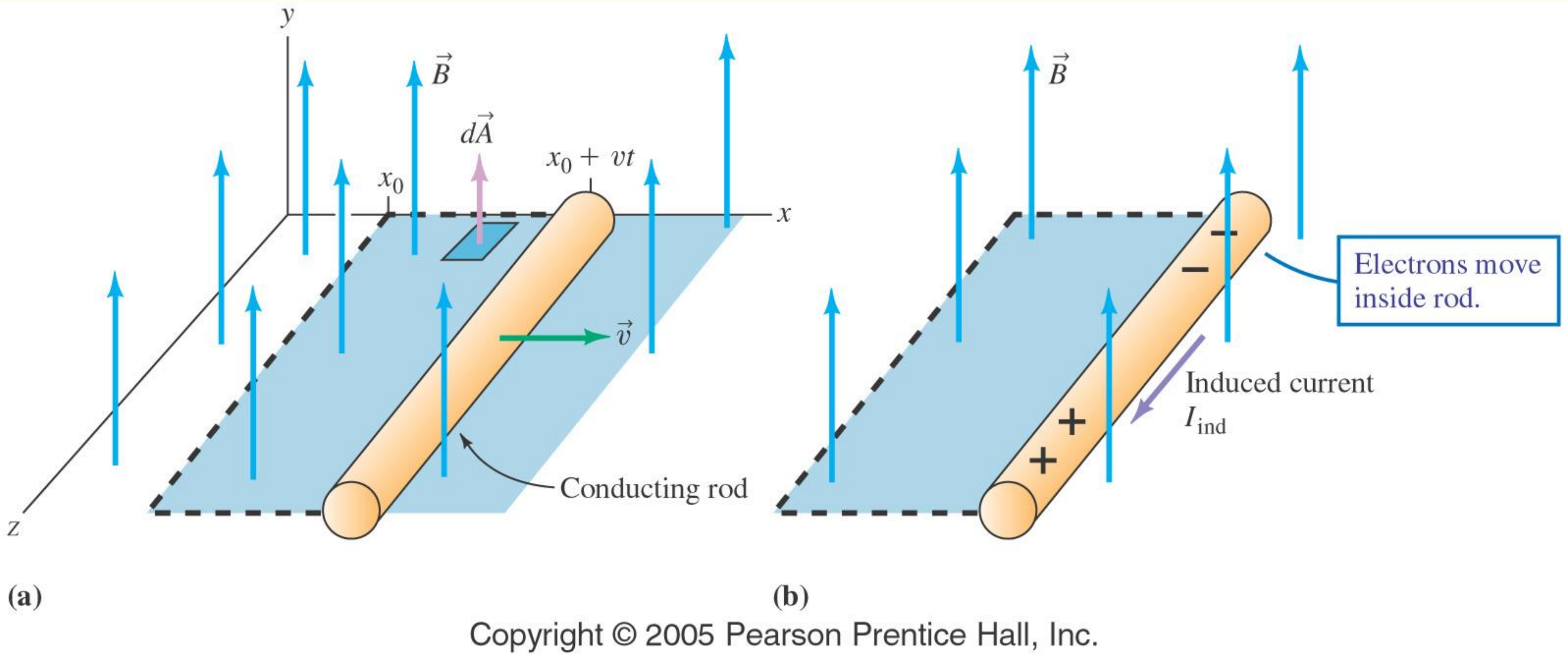
Pulling a Loop out of a Field

- A square loop of side *d* is moving at speed *v* out of a region of uniform field *B*.
- The induced emf is
  $$E = -d\Phi_B/dt = Bvd$$
- What about direction? The downward flux through the loop is decreasing, the loop will try to oppose this by making more downward flux (Lenz's law).

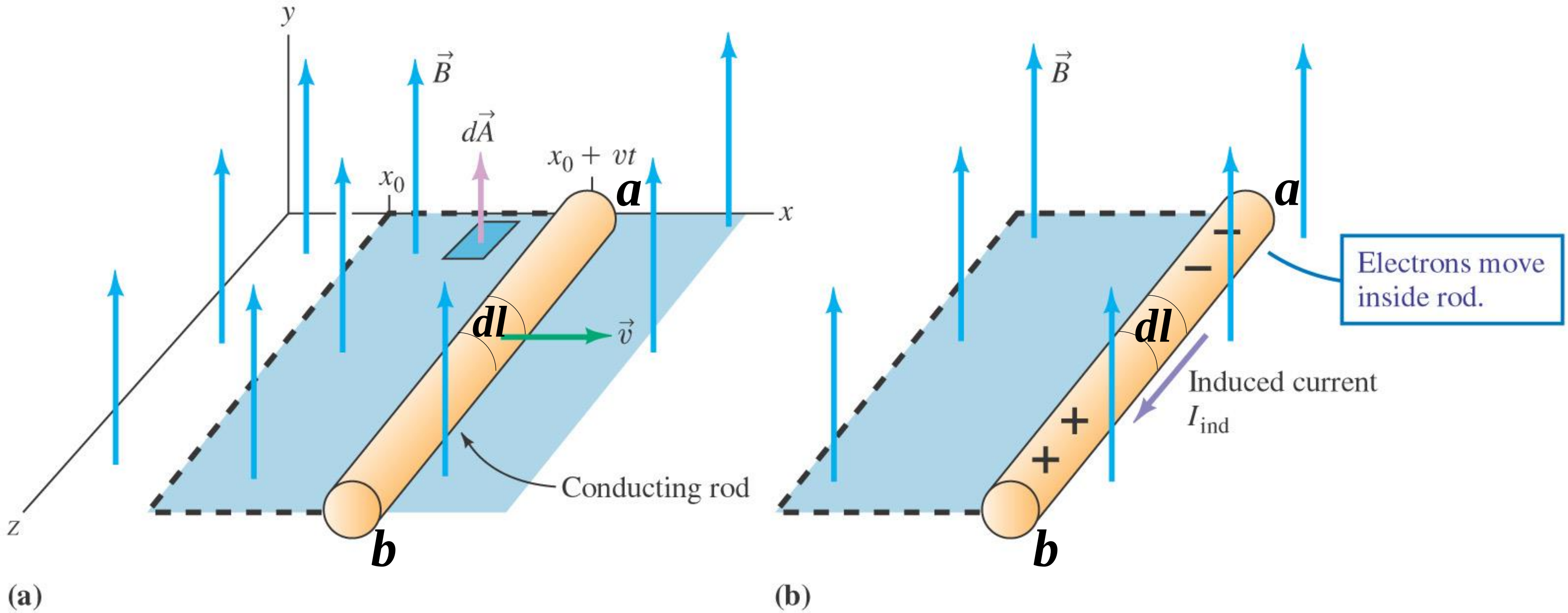


In time *dt* the loop will move distance *vdt*, so the area of lost magnetic flux will be *vdt*x*d*.

§ .2 动生电动势　Motional emf



- **Created by a conductor moving in a magnetic field**
- **"Loop" is imaginary but real emf is induced in conductor**



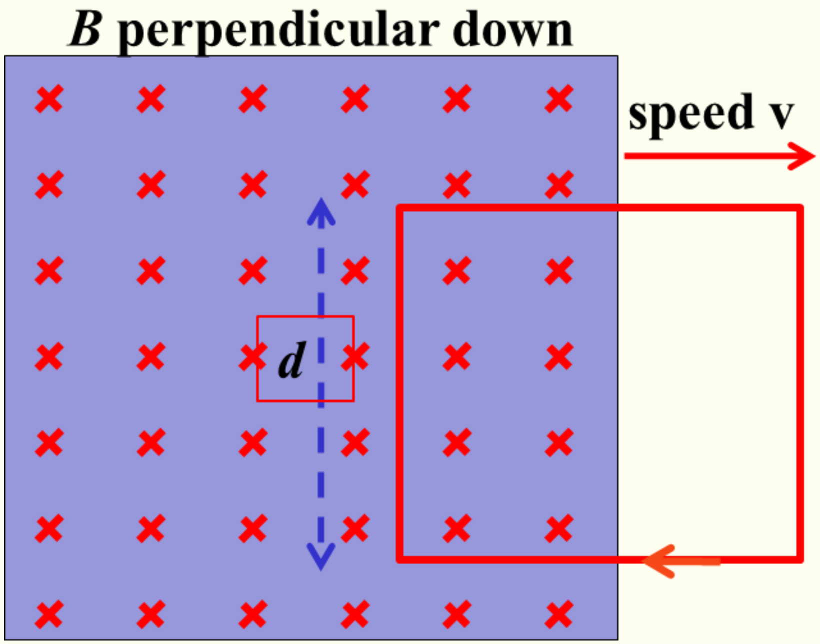
$$\Phi_B = Blvt \qquad \frac{d\Phi_B}{dt} = Blv$$

Motional emf $\quad \varepsilon = -\dfrac{d\Phi_B}{dt} = -Blv$

$$\int_a^b \vec{E}_k \cdot d\vec{l} = \int_a^b \frac{q\vec{v} \times \vec{B}}{q} \cdot d\vec{l} = \int_a^b vBdl = Blv$$

Pulling a Loop out of a Field II

- There's another way to see this induced emf!
- A charge q in the left hand side wire is moving at v along with the wire through the field B, so will feel a force $q\vec{v} \times \vec{B}$ upwards. This is equivalent to an electric field, which acting the length of the side gives a potential difference vBd: this *is* the induced emf.

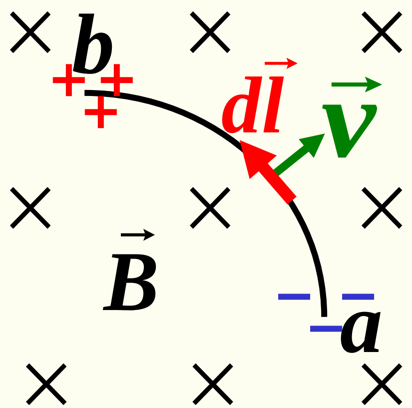


U=vBd

A magnetic field of magnitude *B*

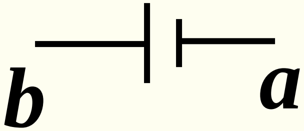
$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\vec{E}_k = \frac{\vec{F}}{q} = \vec{v} \times \vec{B} \qquad q > 0$$



$$\vec{E}_k \cdot d\vec{l} = (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

$$\varepsilon = \int_a^b \vec{E}_k \cdot d\vec{l} = \int_a^b (\vec{v} \times \vec{B}) \cdot d\vec{l}$$



$$\varepsilon = \int_a^b (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

对于导体回路 L $\qquad \varepsilon = \oint_L (\vec{v} \times \vec{B}) \cdot d\vec{l}$

$$\oint_L \vec{E}_k \cdot d\vec{l} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \vec{B}\cdot d\vec{S}$$

（1）在恒定磁场$\vec{B}$中，因导体运动产生的感应电动势

动生电动势　　**Motional emf**

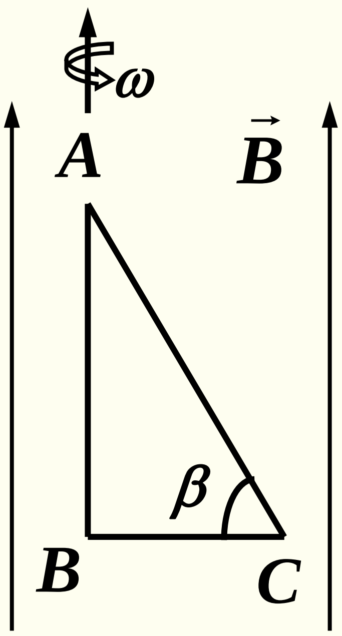
（2）导体不动，因磁场 $\vec{B}$ 变化产生的感应电动势

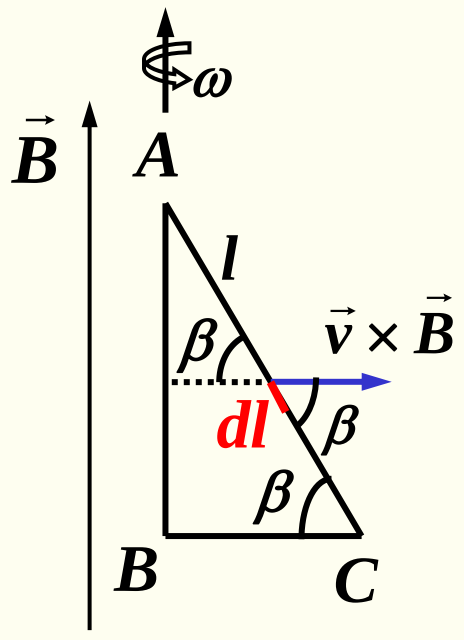
感生电动势　　**Induced emf**

磁场 $\vec{B}$ 变化，导体也有运动

动生电动势 + 感生电动势

由金属导体制成的直角三角形框架，**BC** 边 的长度为 a，**BC** 边与斜边之间的夹角为 β 。放在磁感应强度为 $\vec{B}$ 的均匀磁场中。此框架以 **AB** 边为轴以匀角速度 ω 转动，在图示位置时整个框架里产生的感应电动势？



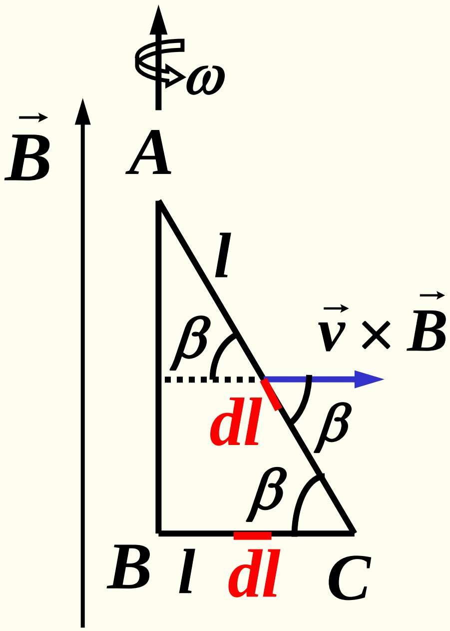


$$\vec{v}=\vec{\omega}\times\vec{r}$$

$$\varepsilon_{AC}=\int_A^C(\vec{v}\times\vec{B})\cdot d\vec{l}$$

$$=\int_A^C vBdl\cos\beta$$

$$=\int_A^C \omega(l\cos\beta)Bdl\cos\beta$$



$$= \int_A^C \omega(l\cos\beta)Bdl\cos\beta$$

$$= \omega B\cos^2\beta\int_A^C ldl$$

$$= \omega B\cos^2\beta\,\frac{1}{2}\,\overline{AC}^2$$

$$\varepsilon_{BC} = \int_B^C (\vec{v}\times\vec{B})\cdot d\vec{l}$$

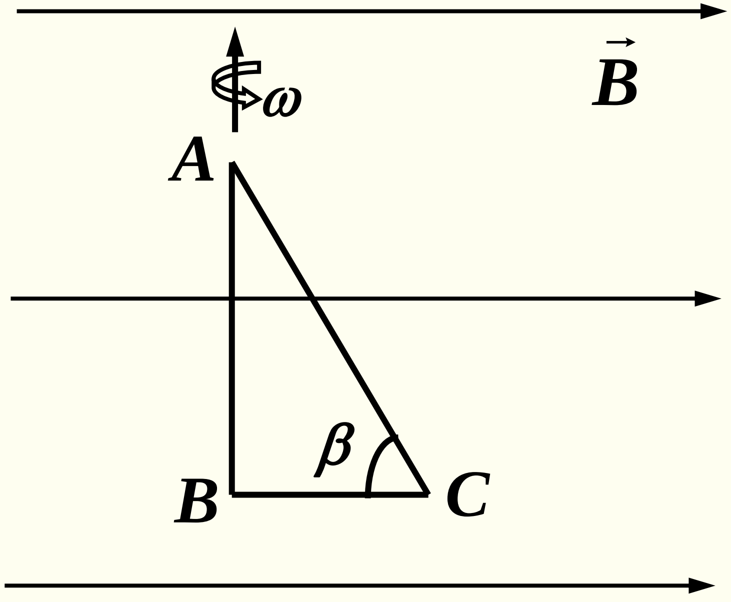
$$= \frac{1}{2}\omega B\cos^2\beta\left(\frac{a}{\cos\beta}\right)^2$$

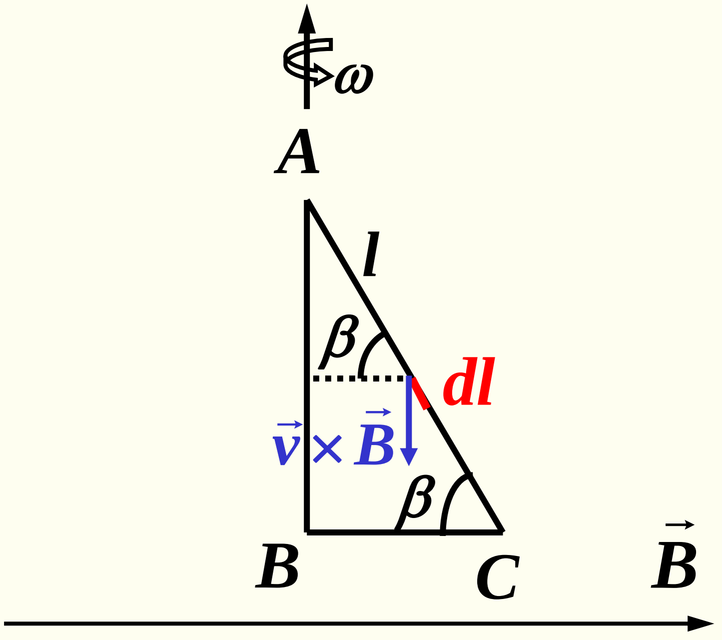
$$= \int_B^C vBdl$$

$$\varepsilon_{AC} = \frac{1}{2}\omega Ba^2 \qquad \varepsilon_{CA} = -\frac{1}{2}\omega Ba^2$$

$$= \int_B^C \omega lBdl = \omega B\int_B^C ldl = \frac{1}{2}\omega Ba^2$$

$$\varepsilon = \varepsilon_{BC} + \varepsilon_{CA} = 0$$

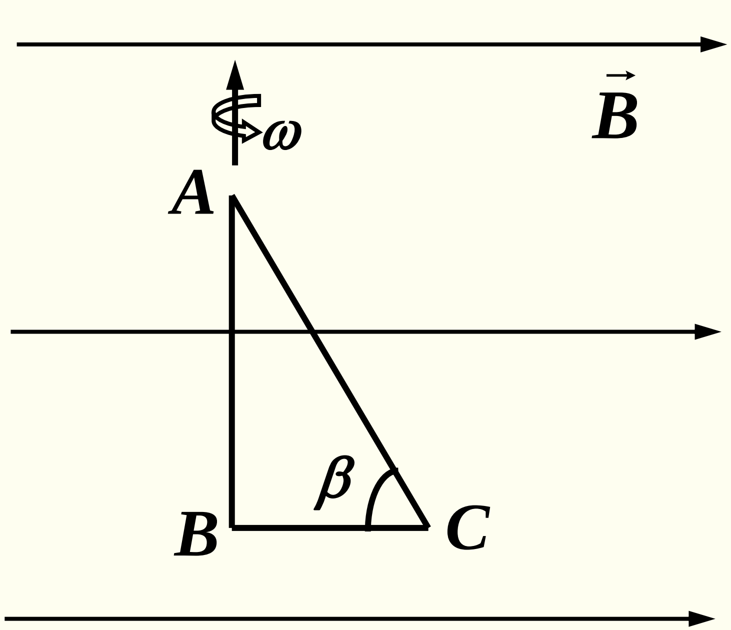


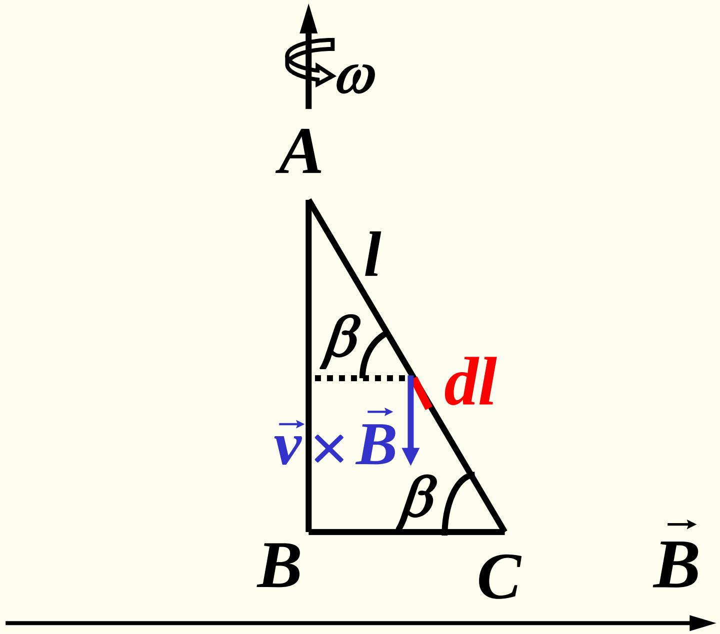


$$\varepsilon_{AC} = \int_A^C (\vec{v}\times\vec{B})\cdot d\vec{l} = \int_A^C vBdl\sin\beta = \int_A^C \omega(l\cos\beta)Bdl\sin\beta$$

$$= \omega B\cos\beta\sin\beta\int_A^C ldl = \omega B\cos\beta\sin\beta\frac{1}{2}\overline{AC}^2$$

$$= \frac{1}{2}\omega B\cos\beta\sin\beta\left(\frac{a}{\cos\beta}\right)^2 = \frac{1}{2}\omega Ba^2\tan\beta$$





$$\varepsilon = \varepsilon_{AC} = \frac{1}{2}\omega B a^2 \tan\beta$$

$$\Phi = \vec{B}\cdot\vec{S} = BS\cos\alpha$$

$$\varepsilon = -\frac{d\Phi}{dt} = -BS\frac{d(\cos\alpha)}{dt} = BS\sin\alpha\frac{d\alpha}{dt}$$

$$= B\frac{1}{2}a(a\tan\beta)\sin\alpha\,\omega = \frac{1}{2}\omega B a^2\tan\beta\sin\alpha = \frac{1}{2}\omega B a^2\tan\beta$$

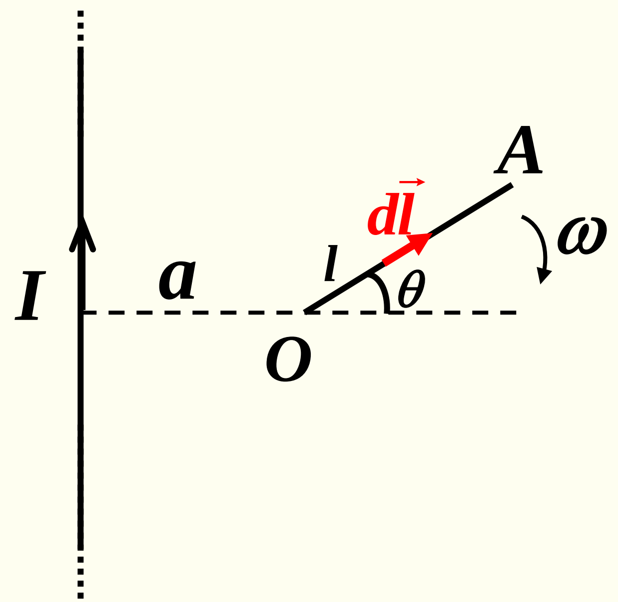
1. Constant current I, a, $OA = b$, ω, θ. $\varepsilon = ?$

$$B = \frac{\mu_0 I}{2\pi\left(a + l\cos\theta\right)}$$

$$d\varepsilon = (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

$$= vBdl$$

$$= l\omega \frac{\mu_0 I}{2\pi\left(a + l\cos\theta\right)} dl$$



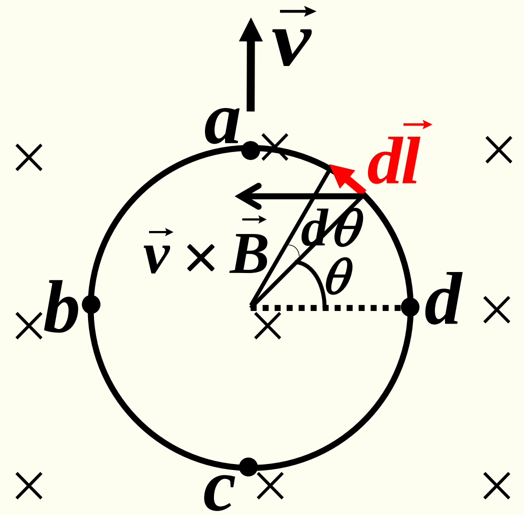
$$\varepsilon = \int_0^b d\varepsilon = \frac{\mu_0 \omega I}{2\pi\cos\theta}\left(b - \frac{a}{\cos\theta}\ln\frac{a + b\cos\theta}{a}\right)$$

2. Uniform magnetic field $\vec{B}$, the loop of radius R, moves with speed $\vec{v}$

(1) The distribution of *emf*

(2) $\vec{E}_k$ of a and c

(3) *emf* between b and d



(1) $d\varepsilon = (\vec{v}\times\vec{B})\cdot d\vec{l}$

$$= vB\sin\theta dl \quad = vB\sin\theta R d\theta$$

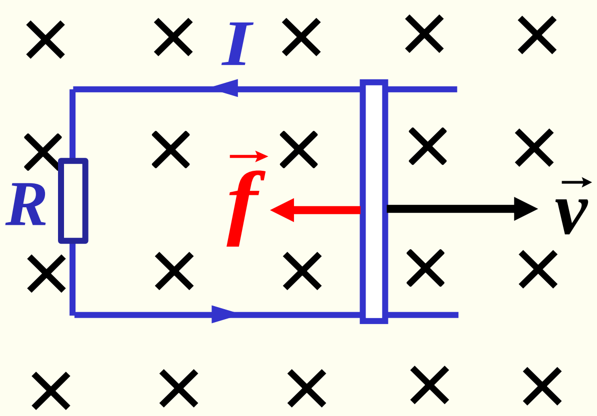
(2) $\vec{E}_k = \vec{v}\times\vec{B}$ $\qquad E_{ka} = E_{kc} = vB$ $\qquad$ 方向：水平向左

(3) $\varepsilon = \int_d^b (\vec{v}\times\vec{B})\cdot d\vec{l} \quad = \int_0^\pi vBR\sin\theta d\theta \quad = 2vBR$

返回 退出

Forces and Energy in Motional emf

均匀磁场 $\vec{B}$ L m $\vec{v}_0$



$$\varepsilon = \oint_L (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

$$= 0 + 0 + 0 + \int_L (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

$$= \int_L vBdl = vBL$$

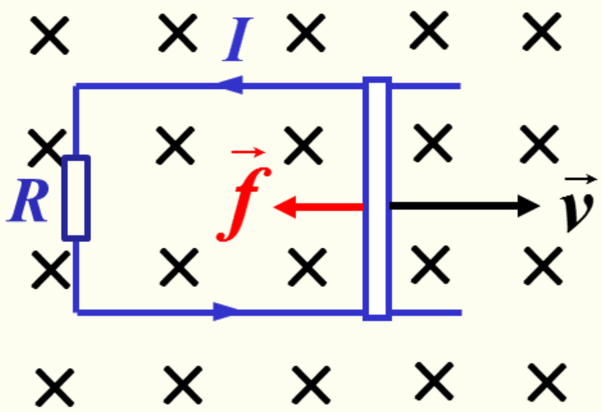
$$I = \frac{\varepsilon}{R} = \frac{BLv}{R}$$

The magnetic forces on the conductor act to slow down the conductor

$$\vec{f} = \int I d\vec{l} \times \vec{B}$$

$$f = ILB = \frac{B^2 L^2 v}{R}$$

Force tends to slow the conductor down (drag force)



$$f = \frac{B^2 L^2 v}{R}$$

时间常数 $\tau = \dfrac{mR}{B^2 L^2}$

$$-\frac{B^2 L^2 v}{R} = m\frac{\mathrm{d}v}{\mathrm{d}t}$$

$$\frac{\mathrm{d}v}{v} = -\frac{B^2 L^2}{mR}\mathrm{d}t = -\frac{\mathrm{d}t}{\tau}$$

$$\int_{v_0}^{v}\frac{\mathrm{d}v}{v} = -\int_0^t\frac{\mathrm{d}t}{\tau}$$

$$v = v_0 e^{-\frac{t}{\tau}}$$

$$v = v_0 e^{-\frac{t}{\tau}} \qquad \tau = \frac{mR}{B^2 L^2}$$

杆以初速度 $\vec{v}_0$ 向右运动

The initial kinetic energy

$$\frac{1}{2} m v_0^2$$

Joule heat consumed

$$W = \int_0^\infty I^2 R \mathrm{d}t = \frac{B^2 L^2}{R} \int_0^\infty v^2 \mathrm{d}t$$

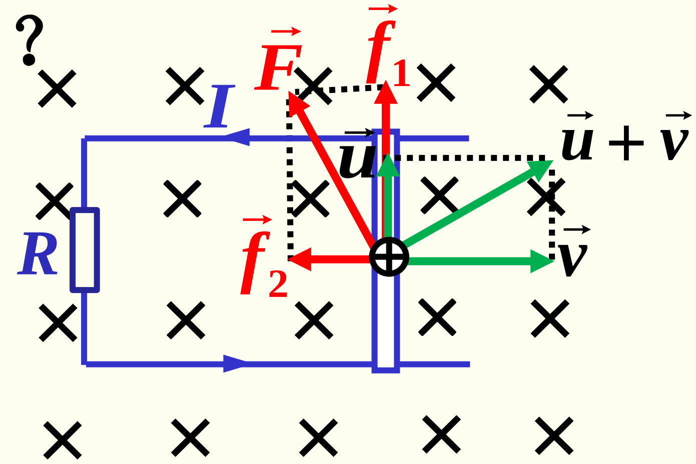
$$= \frac{B^2 L^2 v_0^2}{R} \int_0^\infty e^{-\frac{2t}{\tau}} \mathrm{d}t$$

$$= \frac{B^2 L^2 v_0^2}{2R} \tau$$

$$= \frac{1}{2} m v_0^2$$

杆的初始动能 $\frac{1}{2} m v_0^2$ 全部转化为回路中感应电流释放的焦耳热

Does Lorentz force apply work？



The total lorentz force on a charged particle

$$\vec{F} = q(\vec{v}+\vec{u})\times\vec{B}$$

杆以速度 $\vec{v}$ 向右运动时

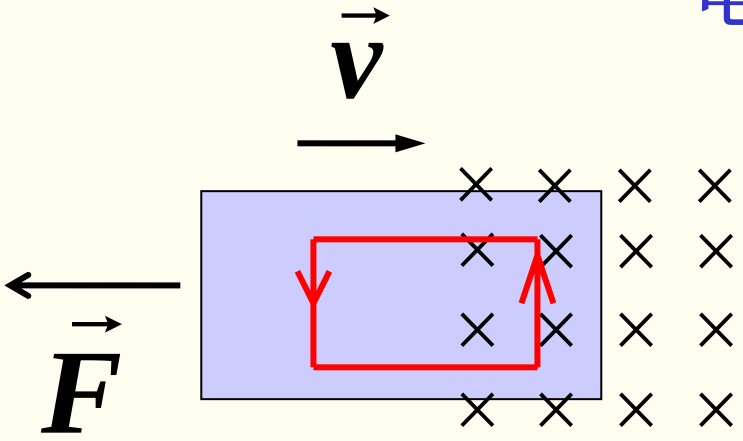
$$= q\vec{v}\times\vec{B} + q\vec{u}\times\vec{B} = \vec{f}_1 + \vec{f}_2$$

$$\vec{f}_1\cdot\vec{u} = q(\vec{v}\times\vec{B})\cdot\vec{u} = q(\vec{B}\times\vec{u})\cdot\vec{v} = -q(\vec{u}\times\vec{B})\cdot\vec{v}$$

$$= -\vec{f}_2\cdot\vec{v}$$

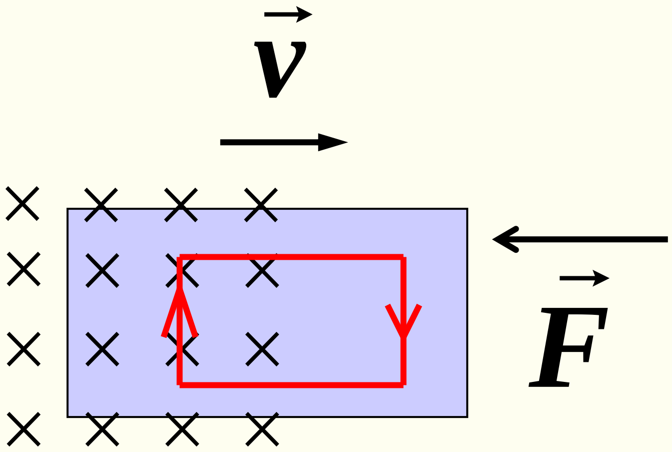
外力克服洛伦兹力的一个分力 $\vec{f}_2$ 所做的功率通过另一个分力 $\vec{f}_1$ 做正功全部转化为感应电动势提供的功率

电磁阻尼



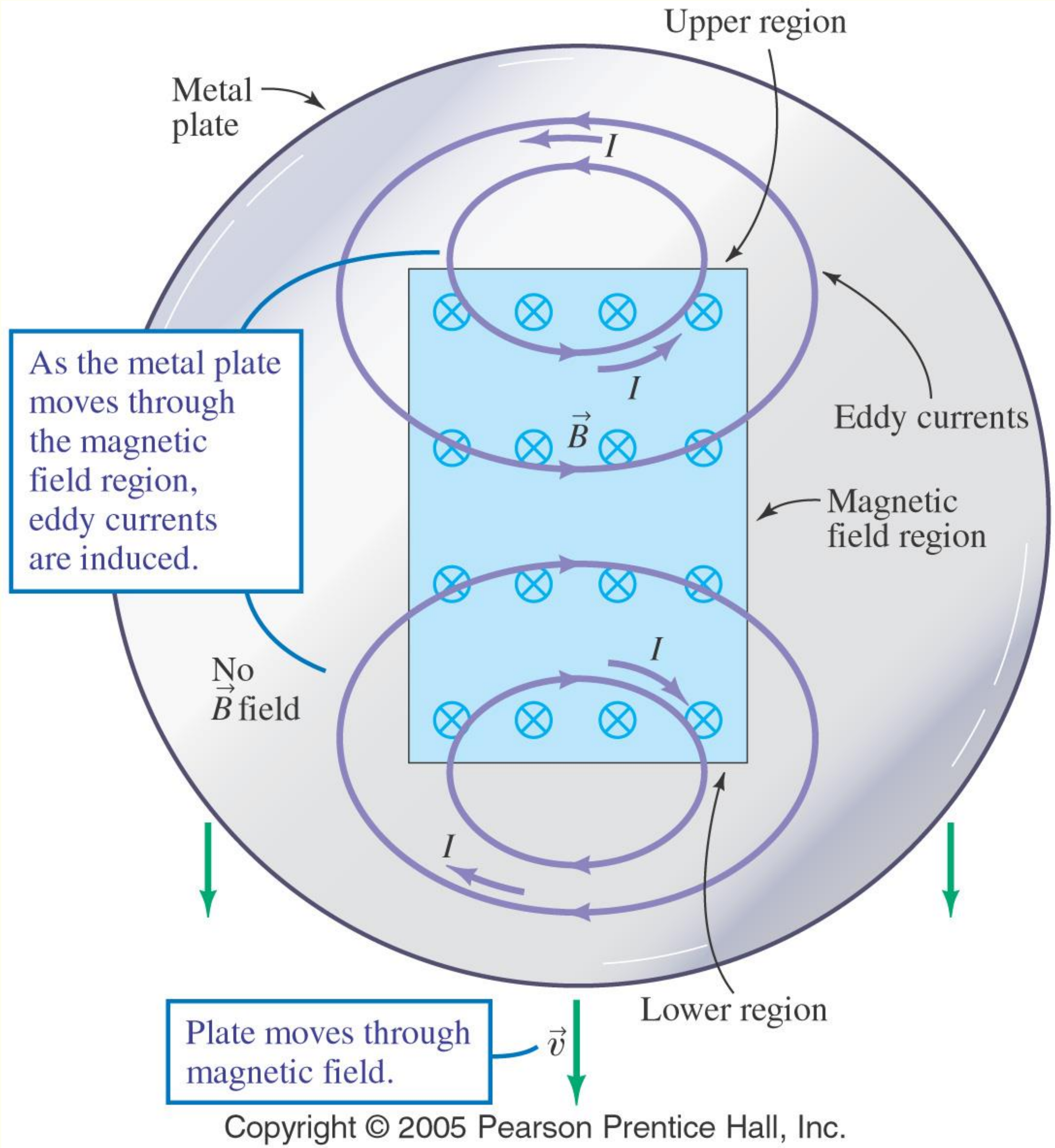
Eddy Currents

涡电流

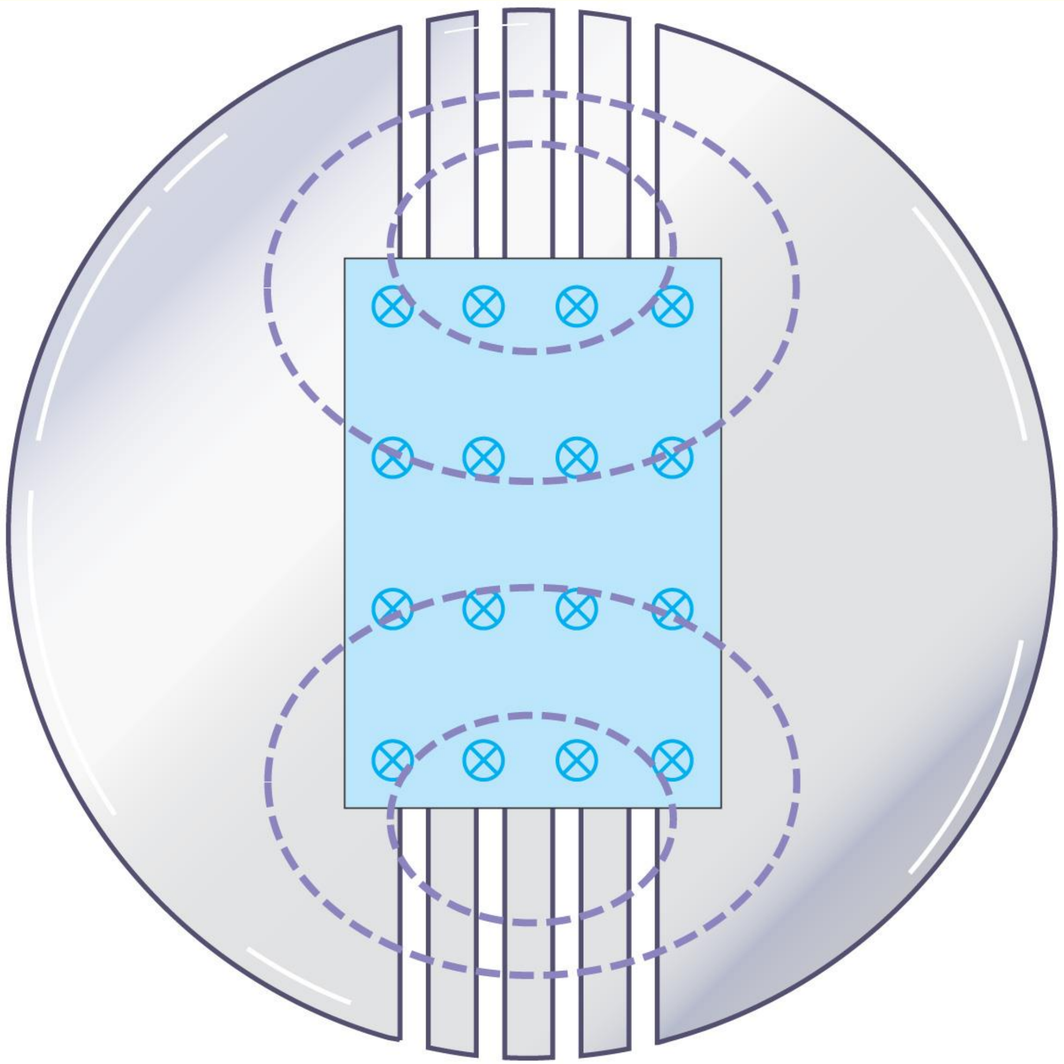


Eddy currents: produced in conductor moving in magnetic field

- **Can make a very effective brake**
- **If you don't want a brake, eddy currents can be foiled by cutting holes (slots) in conductor**



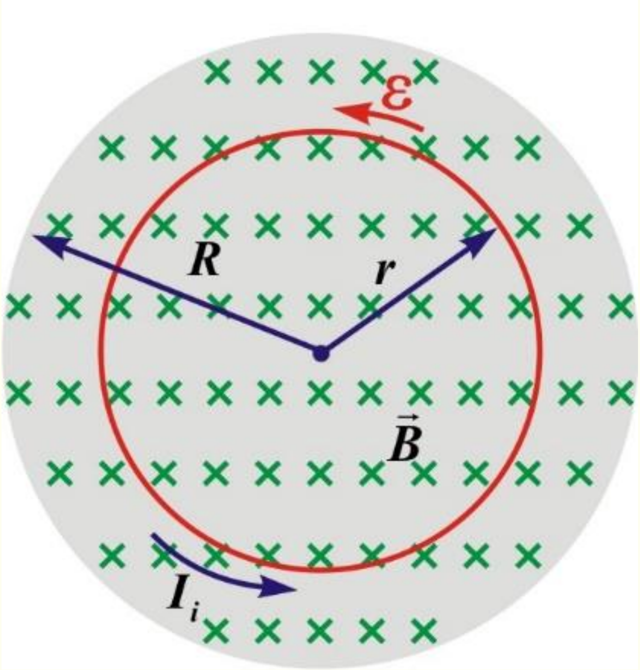
Eddy Currents



To inhibit the development of eddy currents in the moving metal plate, slots can be cut in the plate

§ .3 感生电动势　涡旋电场

$$\varepsilon = \oint_L \vec{E}_k \cdot d\vec{l} = -\frac{d\Phi_m}{dt}$$



$$\frac{\partial \vec{B}}{\partial t} > 0$$

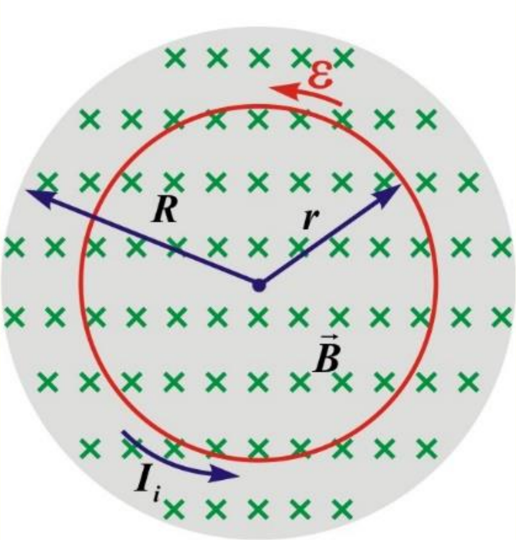
动生电动势

Motional emf $\varepsilon = \oint_L \vec{E}_k \cdot d\vec{l} = \oint_L (\vec{v} \times \vec{B}) \cdot d\vec{l}$

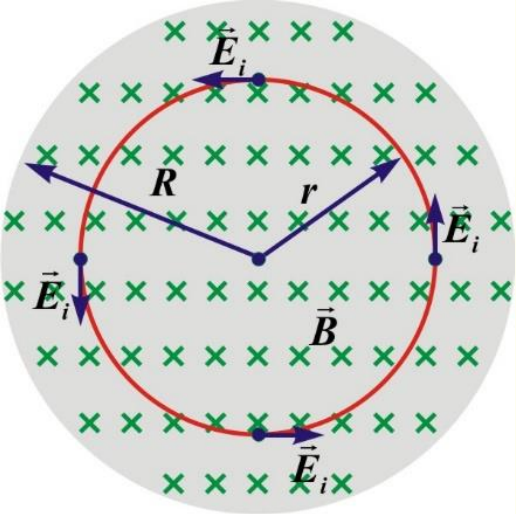
感生电动势

非静电力

Induced emf $\varepsilon = \oint_L \vec{E}_k \cdot d\vec{l}$



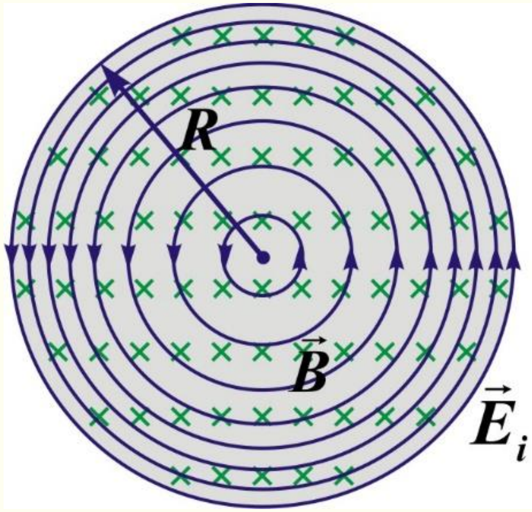
$$\frac{\partial \vec{B}}{\partial t} > 0$$

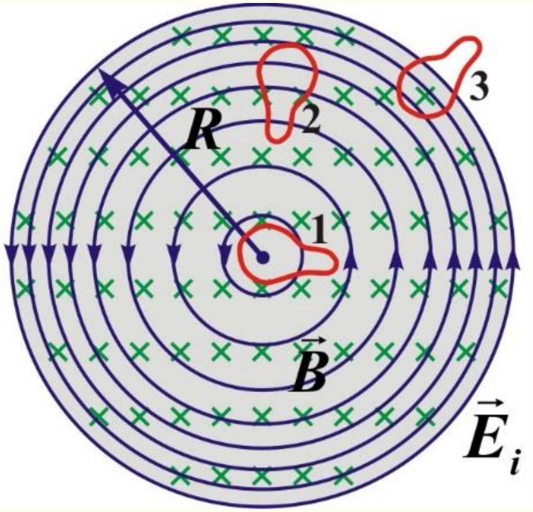


1861年

麦克斯韦

J.C.Maxwell





涡旋电场

vortex electric field

感应电场

induced electric field

A changing magnetic field produces an electric field

涡旋电场与静电场

electrostatic fields are produced by static charges

vortex electric fields are produced by changing magnetic field

electrostatic field 发散状

vortex electric field 涡旋状

either field will exert a force $q\vec{E}$ on a charge q

$$\varepsilon = \oint_L \vec{E}_k \cdot d\vec{l} = \oint_L \vec{E}_i \cdot d\vec{l}$$

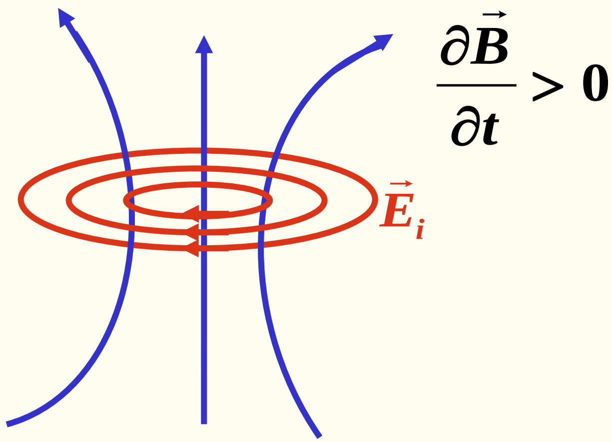
$\vec{E}_i$ 涡旋电场的场强

$$\oint_L \vec{E}_i \cdot d\vec{l} = -\frac{d\Phi_m}{dt}$$

$$\oint_L \vec{E}_i \cdot d\vec{l} = -\frac{d\Phi_m}{dt}$$

$$\frac{d\Phi_m}{dt} = \frac{d}{dt}\iint_S \vec{B}\cdot d\vec{S} = \iint_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$$

$$\oint_L \vec{E}_i \cdot d\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$$



涡旋电场

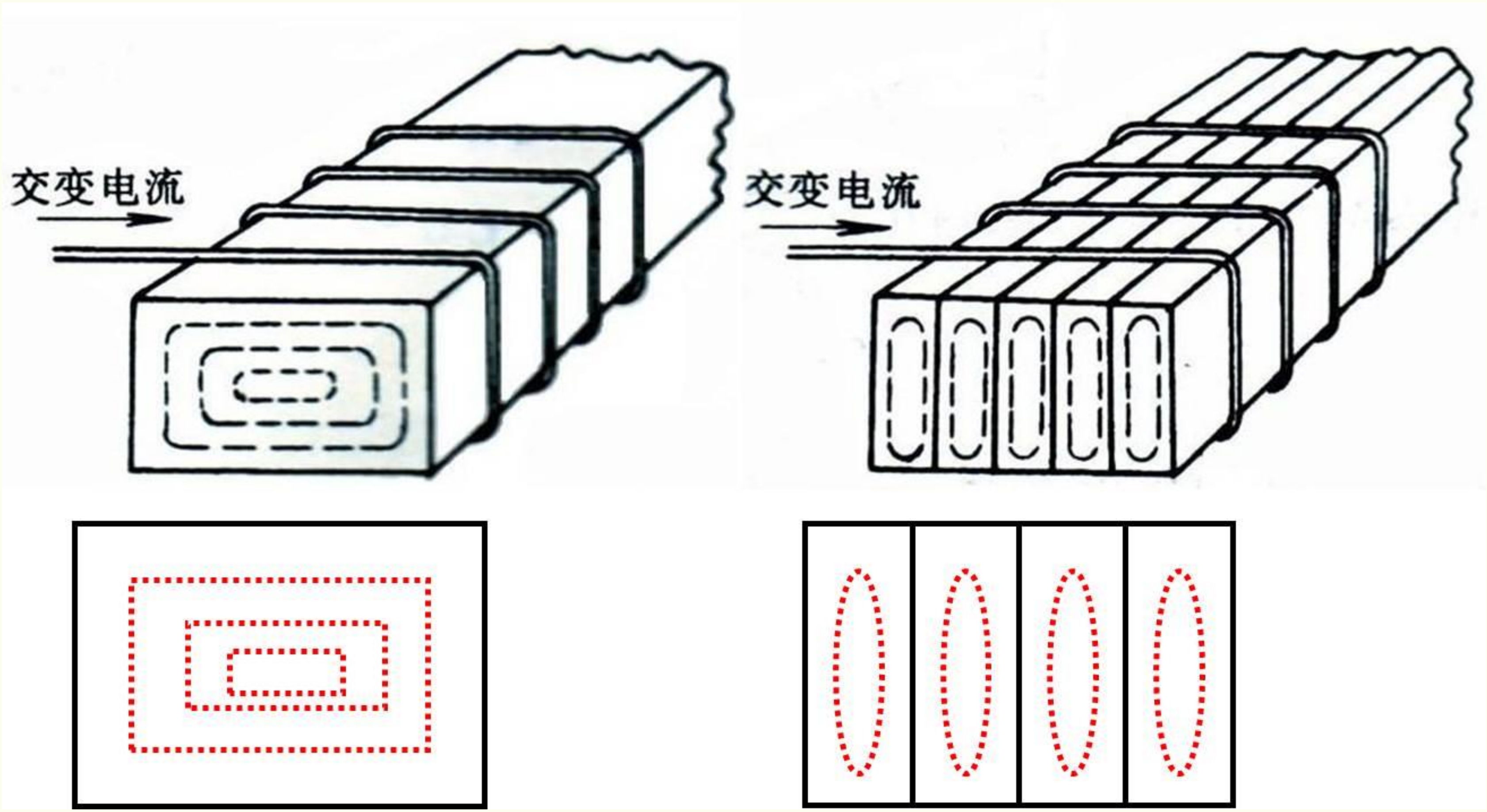
vortex electric field

左旋

left-hand rule

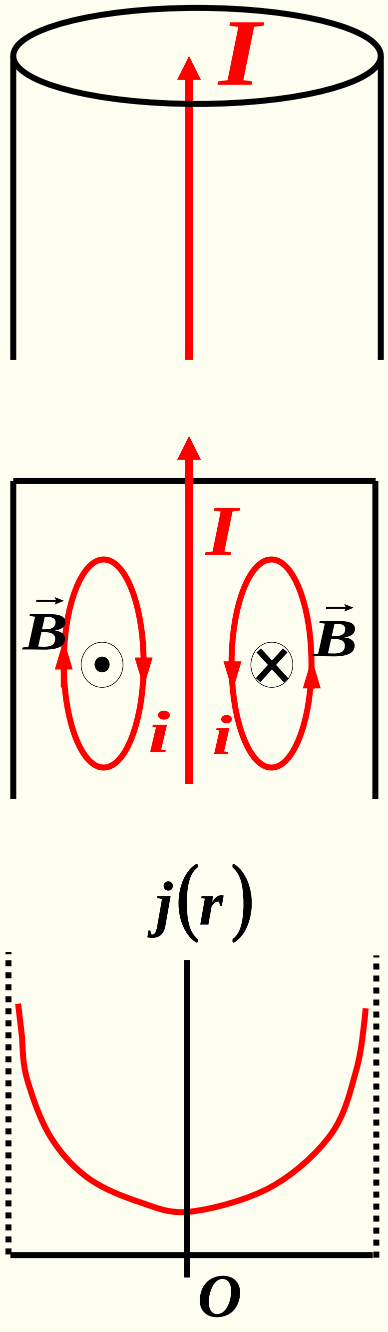
麦克斯韦电磁场理论的基本假设之一

Induction eddy current (感生涡电流)



减小涡电流的方法：　高电阻率的材料 (硅钢、铁氧体)

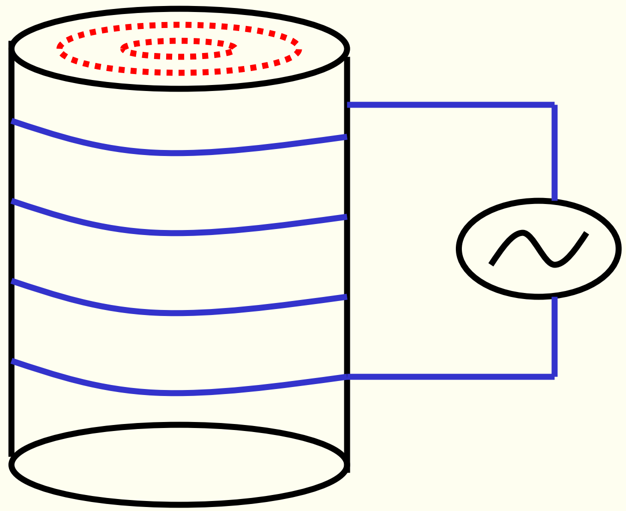
使用叠合起来的彼此绝缘的铁片　铁片的绝缘层与涡流垂直



趋肤效应　skin effect

Effective section $S\downarrow \Rightarrow R\uparrow$

有效截面



高频感应电炉

在坩埚的外缘绕有线圈

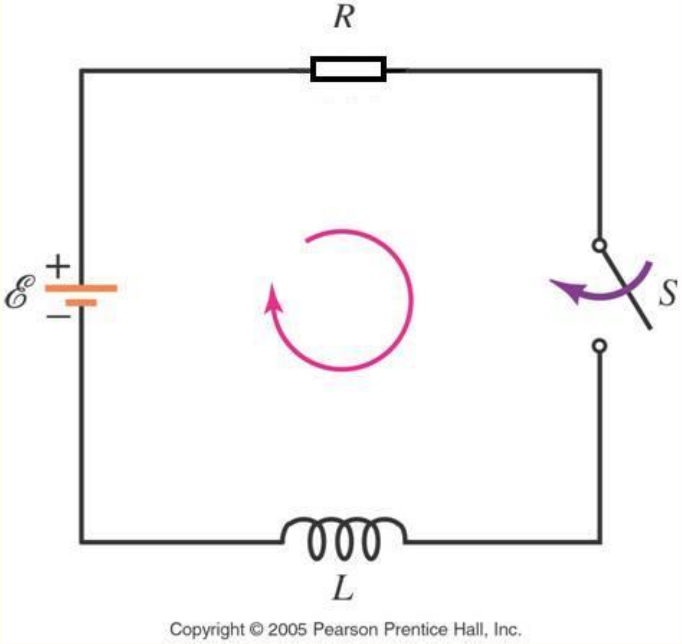
大功率交变电源

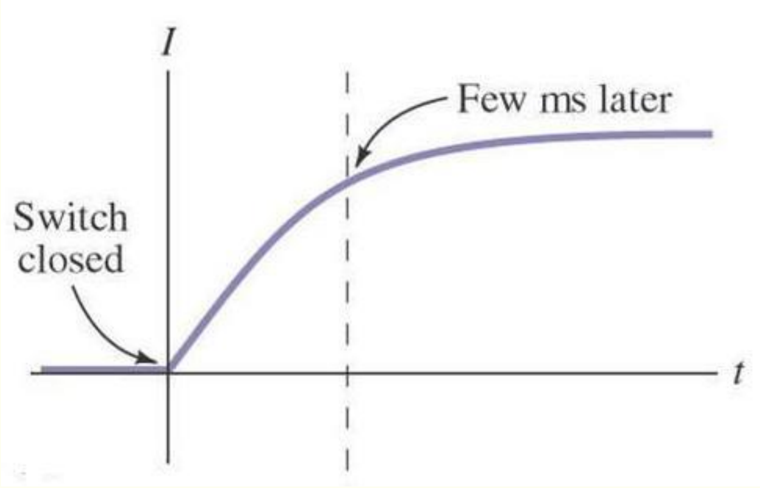
高频交变电流

高频感应加热淬火

§ .4 自感和互感

Self-Inductance and Mutual Inductance





亨利 J. Henry 1829

self-induced emf

自感电动势 $$\varepsilon_L = -\frac{d\Phi}{dt}$$

$$\Phi = \iint_S \vec{B} \cdot d\vec{S} \propto I$$

$$\Phi = LI$$

L 自感系数

L self-inductance coefficient

自感 **Self-Inductance**

$$\Phi = LI$$ SI 亨利 (H)

L It depends on the shape and size of the loop and the surrounding magnetic medium

$$\varepsilon_L = -\frac{d\Phi}{dt} = -L\frac{dI}{dt} - I\frac{dL}{dt}$$

If L stays the same $\varepsilon_L = -L\frac{dI}{dt}$

If we establish a current *I* in a coil of *N* turns, the current produces a magnetic flux through the coil.

磁链 magnetic flux linkage

$$\Psi = \Phi_1 + \Phi_2 + \ldots + \Phi_N$$

If the coil is tightly wound (*closely packed*), so that the same magnetic flux passes through all the turns.

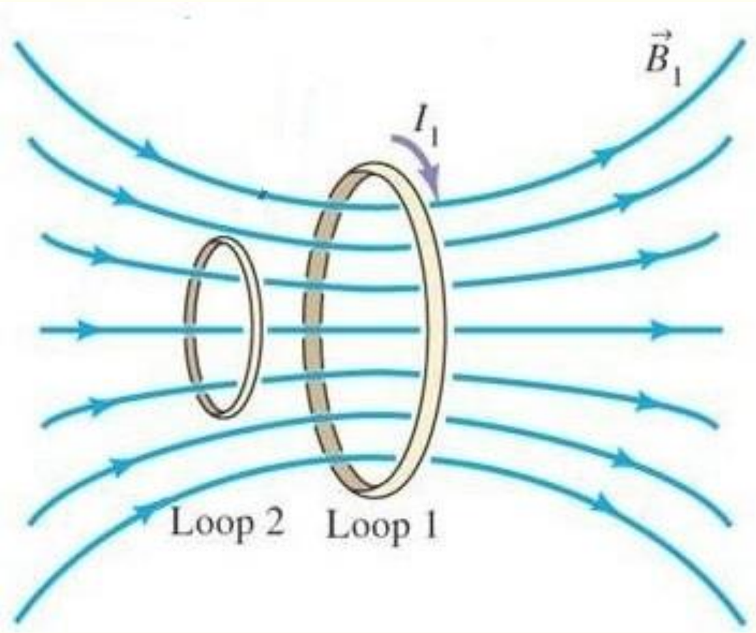
$$\Psi = N\Phi$$

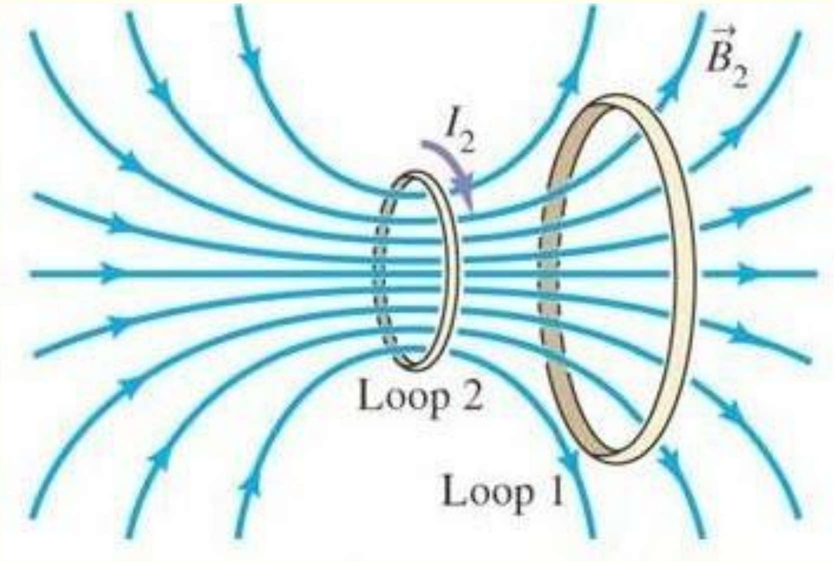
$$\Psi = LI$$

L self-inductance coefficient

$$\varepsilon_L = -\frac{d\Psi}{dt} = -L\frac{dI}{dt}$$

Mutual Inductance 互感





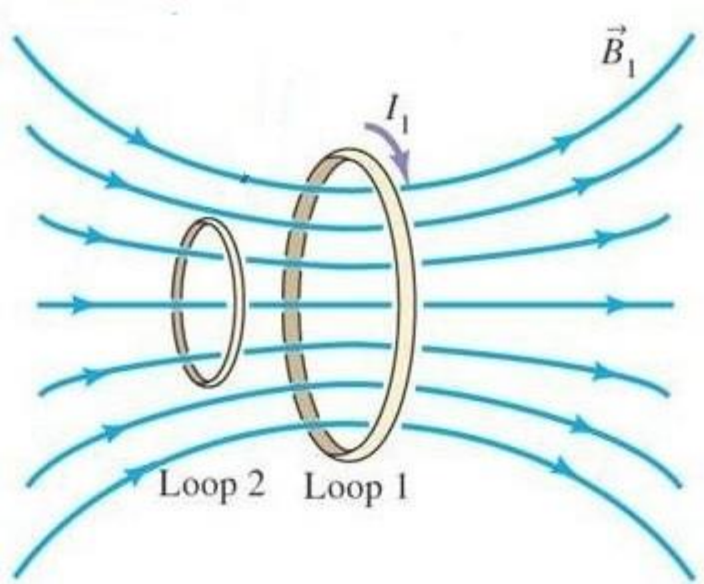
$$\Phi_{21} = M_{21} I_1$$

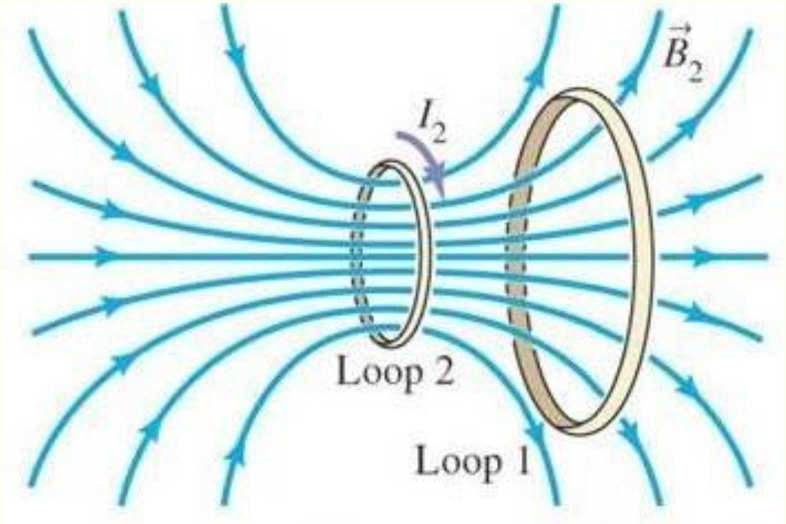
$$\Phi_{12} = M_{12} I_2$$

$$I_1 \rightarrow \Phi_{21} \rightarrow \varepsilon_{21} = -\frac{d\Phi_{21}}{dt}$$

$$I_2 \rightarrow \Phi_{12} \rightarrow \varepsilon_{12} = -\frac{d\Phi_{12}}{dt}$$

互感电动势





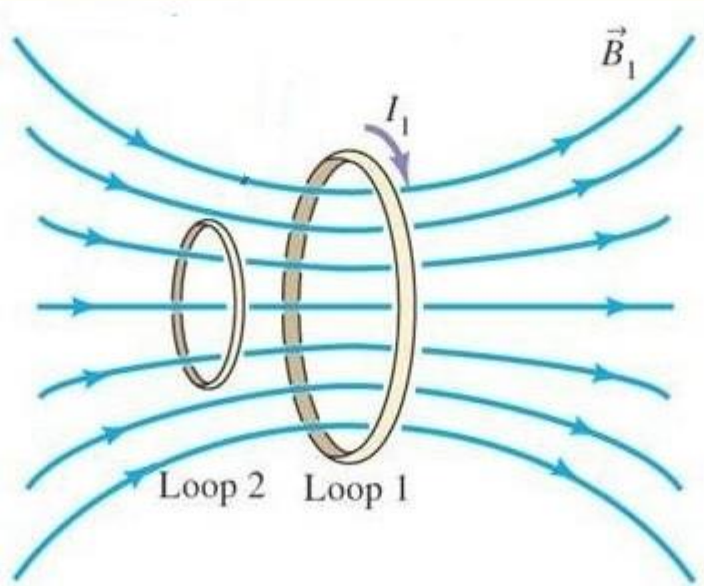
$$\Phi_{21} = M_{21} I_1 \qquad \Phi_{12} = M_{12} I_2$$

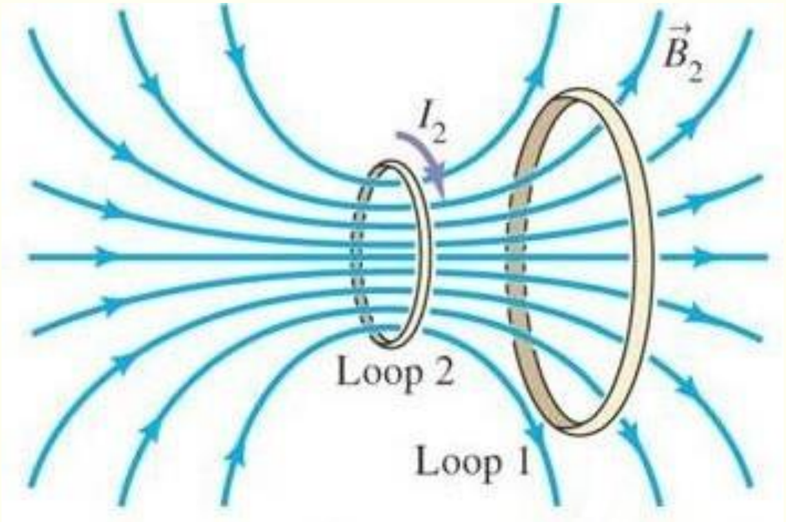
$$M_{12} = M_{21} = M$$

互感系数

互感 **Mutual inductance**

$$\Phi_{21} = M I_1 \qquad \Phi_{12} = M I_2$$





$$\Phi_{21} = MI_1 \qquad \Phi_{12} = MI_2$$

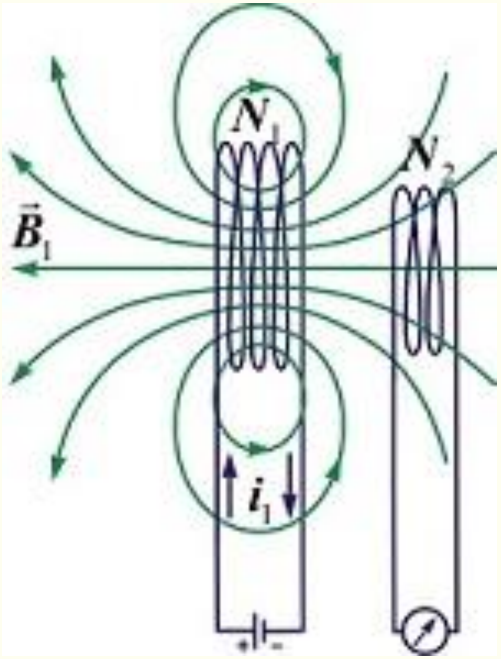
如果 M 保持不变

$$\varepsilon_{21} = -\frac{d\Phi_{21}}{dt} = -M\frac{dI_1}{dt} - I_1\frac{dM}{dt}$$

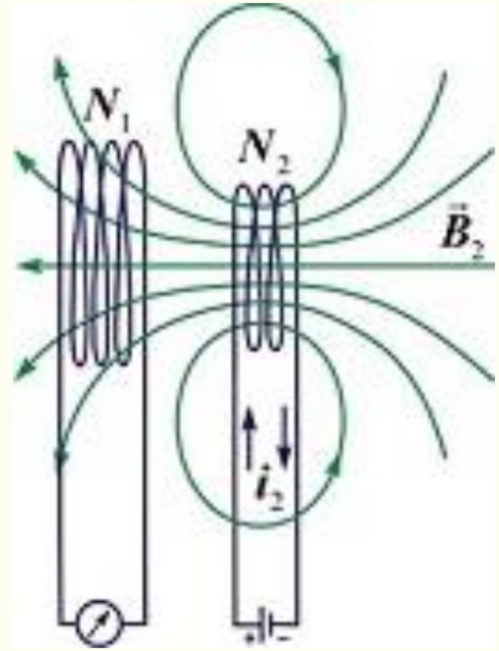
$$\varepsilon_{21} = -M\frac{dI_1}{dt}$$

$$\varepsilon_{12} = -\frac{d\Phi_{12}}{dt} = -M\frac{dI_2}{dt} - I_2\frac{dM}{dt}$$

$$\varepsilon_{12} = -M\frac{dI_2}{dt}$$



线圈



A **magnetic** **flux Φ_{21} (the flux through coil 2 associated with the current in coil 1) links the N_2 turns of coil 2.**

$$\Psi_{21} = N_2\Phi_{21} \qquad \Psi_{21} = MI_1 \qquad \varepsilon_{21} = -M\frac{dI_1}{dt}$$

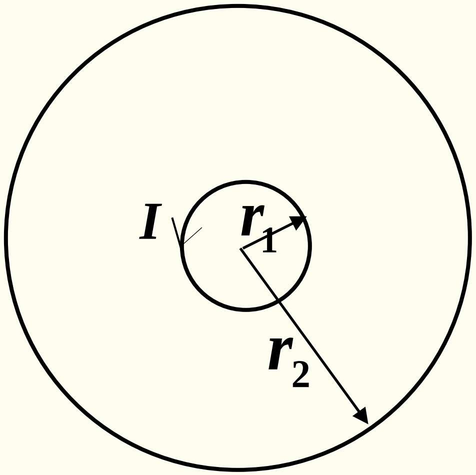
$$\Psi_{12} = N_1\Phi_{12} \qquad \Psi_{12} = MI_2 \qquad \varepsilon_{12} = -M\frac{dI_2}{dt}$$

two circular loops, the smaller (radius r_1) being concentric with the larger (radius r_2) and in the same plane. $r_1 << r_2$

$I = kt$, k is a positive constant. Find the emf induced in the larger loop due to the change of magnetic field.

$$\Phi_{21} = MI_1 \qquad \Phi_{12} = MI_2 \qquad \varepsilon_{21} = -M\frac{dI}{dt}$$

$$M = \frac{\Phi_{12}}{I_2} = \frac{B_1 \pi r_1^{\ 2}}{I_2} \qquad B_1 = \frac{\mu_0 I_2}{2r_2}$$



$$\Rightarrow M = \frac{\mu_0 \pi r_1^{\ 2}}{2r_2}$$

$$\varepsilon_{21} = -M\frac{dI}{dt} = -\frac{\mu_0 \pi r_1^{\ 2}}{2r_2} \cdot k$$

1. I, x_0, a, b

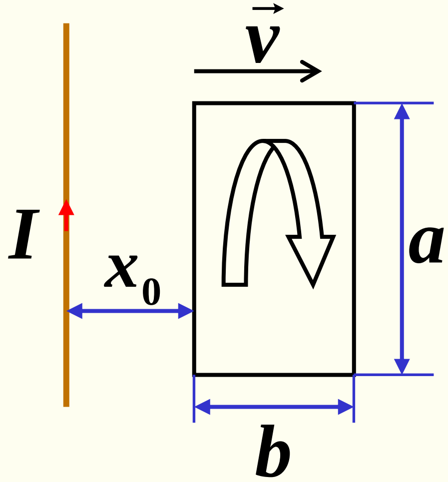
(1) the loop is moving to the right with speed v at position x_0 $\qquad \varepsilon_1 = ?$

(2) the loop is rest at x_0 , $\quad \dfrac{dI}{dt} > 0 \qquad \varepsilon_2 = ?$

(3) the loop is moving to the right with speed v at position x_0, $\quad \dfrac{dI}{dt} > 0 \qquad \varepsilon_3 = ?$

(1) $$\varepsilon = \oint_L (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

clockwise

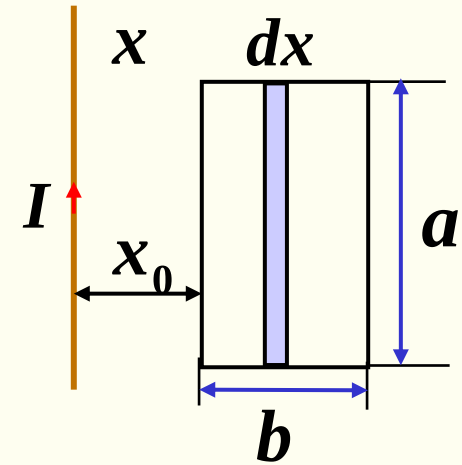


$$\varepsilon_1 = \int_{l_1} vBdl - \int_{l_2} vBdl$$

$$= Bav\Big|_{x_0} - Bav\Big|_{x_0+b} = \frac{\mu_0 Iav}{2\pi}\left(\frac{1}{x_0} - \frac{1}{x_0+b}\right)$$

(2)

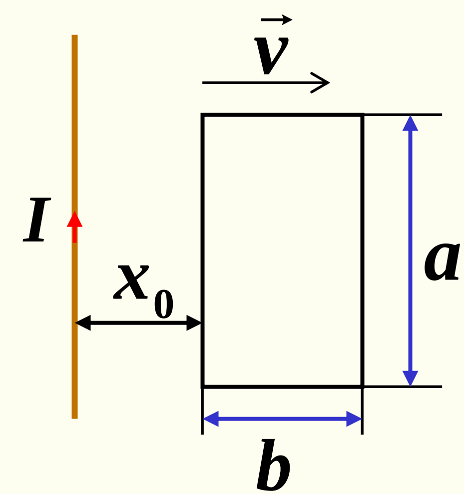
$$d\Phi = \frac{\mu_0 I}{2\pi x} a dx$$



$$\Phi = \int_{x_0}^{x_0+b} \frac{\mu_0 I}{2\pi x} a dx = \frac{\mu_0 I a}{2\pi} \ln \frac{x_0 + b}{x_0}$$

$$\varepsilon_2 = -\frac{d\Phi}{dt} = -\frac{\mu_0 a \frac{dI}{dt}}{2\pi} \ln \frac{x_0 + b}{x_0}$$

(3)

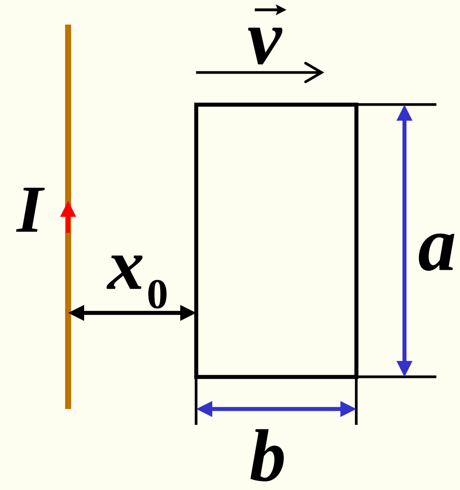


$$\varepsilon_3 = -\frac{d\Phi(I, x_0)}{dt}$$

$$= -\frac{\mu_0 a \frac{dI}{dt}}{2\pi} \ln \frac{x_0 + b}{x_0} + \frac{\mu_0 I a v}{2\pi} \left(\frac{1}{x_0} - \frac{1}{x_0 + b} \right) = \varepsilon_2 + \varepsilon_1$$

or

$$\Phi = \frac{\mu_0 I a}{2\pi} \ln \frac{x_0 + b}{x_0}$$



$$\Phi = MI$$

$$M = \frac{\Phi}{I} = \frac{\mu_0 a}{2\pi} \ln \frac{x_0 + b}{x_0}$$

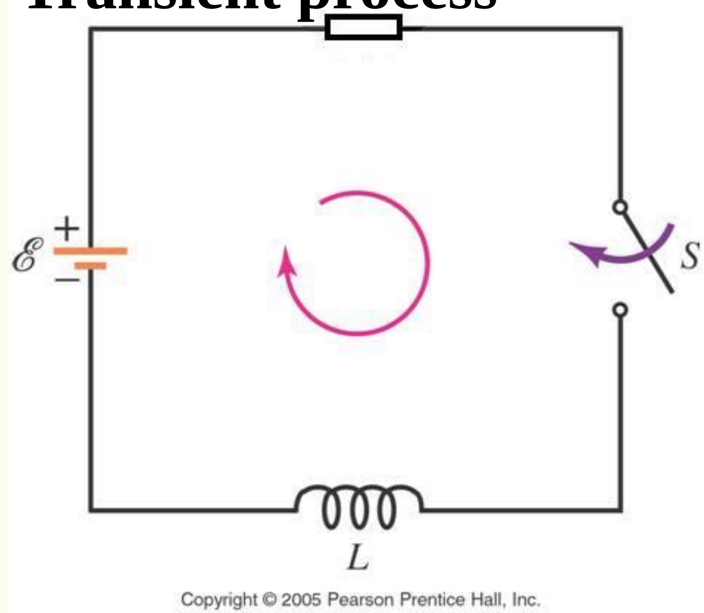
$$\varepsilon_3 = -M\frac{dI}{dt} - I\frac{dM}{dt}$$

$$= -\frac{\mu_0 a}{2\pi} \ln \frac{x_0 + b}{x_0} \frac{dI}{dt} + \frac{\mu_0 I a v}{2\pi} \left(\frac{1}{x_0} - \frac{1}{x_0 + b} \right)$$

$$= \varepsilon_2 + \varepsilon_1$$

§ .5 磁场能量

暂态过程 Transient process



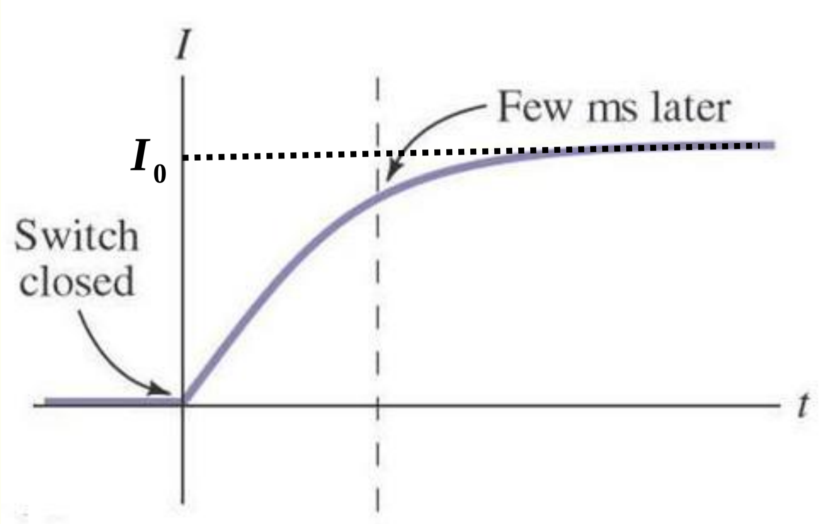
线圈与电源接通时，电流 $0 \to I_0$

$$\varepsilon + \varepsilon_L = IR$$

$$\varepsilon I dt = -\varepsilon_L I dt + I^2 R dt$$

$$\varepsilon_L = -L\frac{dI}{dt}$$

$$\varepsilon I dt = LIdI + I^2 R dt$$



当 $t = 0$ 时 $\quad I = 0$

当电流达到稳定值 I_0 时

$$W_m = \int_0^{I_0} LIdI = \frac{1}{2}LI_0^2$$

$$W_m = \frac{1}{2} L I^2$$ **Magnetic energy stored in a field**

Long straight solenoid

Non-ferromagnetic media $$B = \mu n I = \mu \frac{N}{l} I$$

$$\Psi = NBS = \mu \left(\frac{N}{l} \right)^2 ISl = \mu n^2 IV$$

$$\Psi = LI$$

$$L = \mu n^2 V$$

$$W_m = \frac{1}{2} L I^2 = \frac{1}{2} \mu n^2 V \frac{B^2}{\mu^2 n^2} = \frac{1}{2} BHV$$

Magnetic energy density

$$w_m = \frac{1}{2} \vec{B} \cdot \vec{H}$$

$$w_m = \frac{1}{2} BH$$

$$W_m = \iiint_V w_m dV$$

Isotropic linear medium