General advice on how to learn physics:

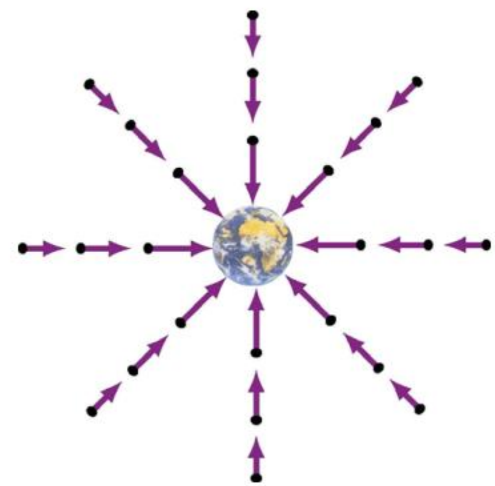
- Physics is built on basic principles ("laws") from which all results can be derived (deduced)
- Focus your attention on **understanding** the basic principles
- Practice **using** the basic principles to solve problems
- Take care to **formulate** your questions and make sure they are **answered**

Note: In general, the best way to develop your problem solving skills is to **practice** solving problems. There is less need to memorize.

As you are working problems, you may wish to construct a sheet of paper containing the basic equations you need to consult in order to solve the problems. You can take this same basic equation sheet to use during the in-class quizzes.

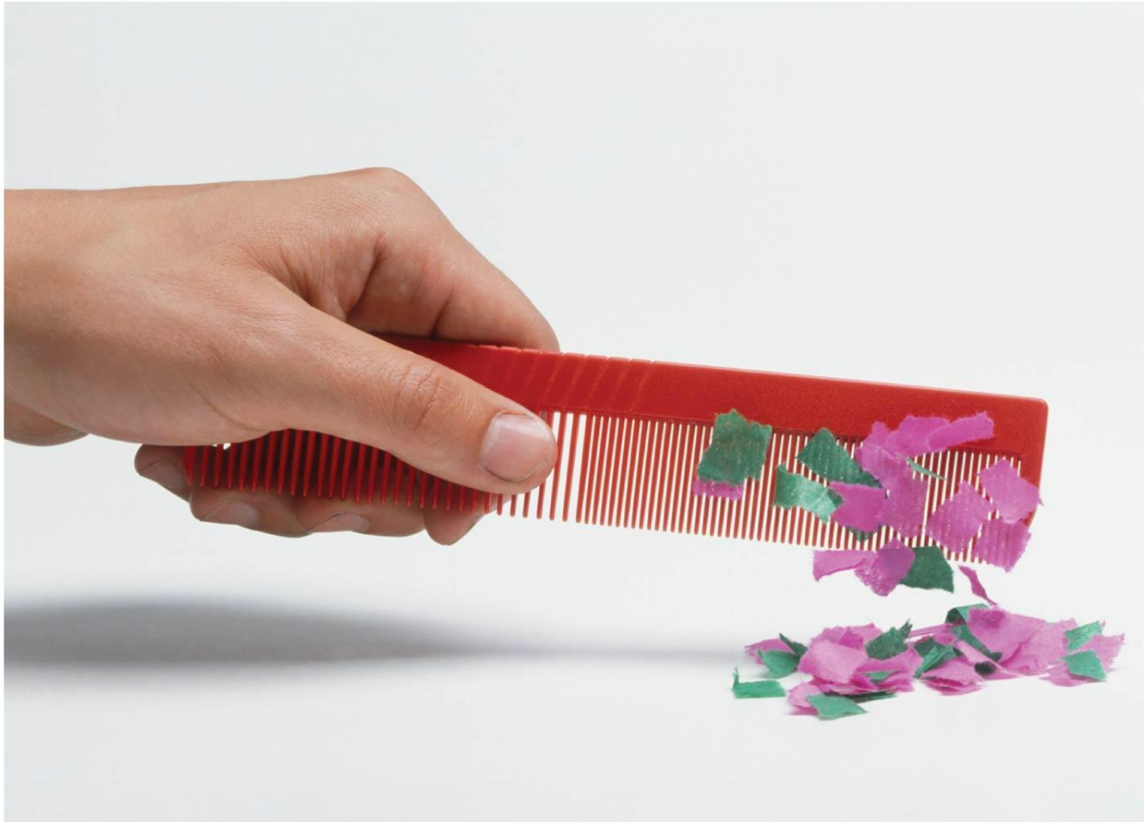
$$\vec{F}_{12} = -G\frac{m_1 m_2}{r_{12}^2}\hat{r}_{12}$$

$$\vec{\mathbf{g}} = \lim_{m\to 0}\frac{\vec{\mathbf{F}}_g}{m} = -G\frac{M}{r^2}\hat{\mathbf{r}}$$



Chapter 16

Electric Charge and Electric Field



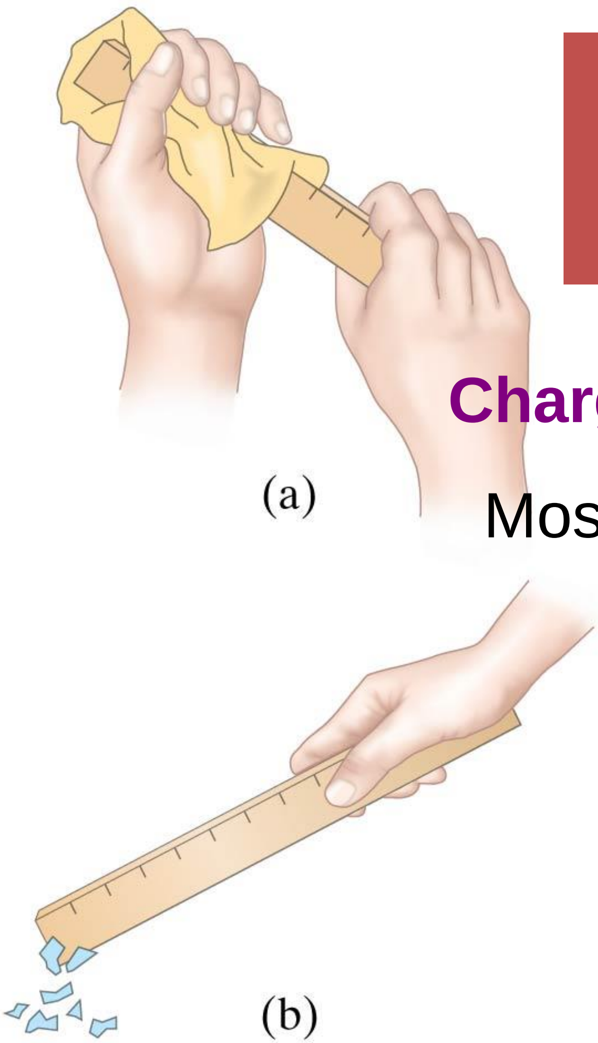
Contents of Chapter 16

- Static Electricity; Electric Charge and Its Conservation
- Electric Charge in the Atom
- Insulators and Conductors
- Induced Charge; the Electroscope
- Coulomb’s Law 库仑定律
- Solving Problems Involving Coulomb’s Law and Vectors
- The Electric Field 电场

Contents of Chapter 16

- Electric Field Lines
- Electric Fields and Conductors
- Electric Forces in Molecular Biology: DNA Structure and Replication
- Photocopy Machines and Computer Printers Use Electrostatics
- Gauss's Law

16-1 Static Electricity; Electric Charge and Its Conservation



Electrification by friction;
Electrification by contact;
Electrification by induction

Charging ahead

Most matter is made up of charged particles

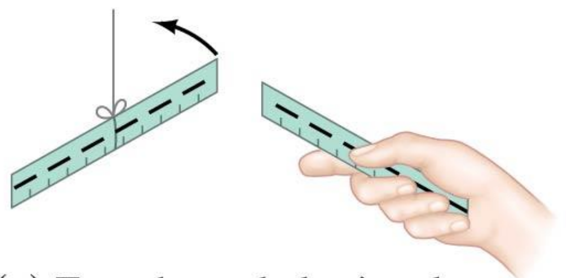
- labeled +

(proton: $q_p = 1.60217733 \times 10^{-19}$ C)

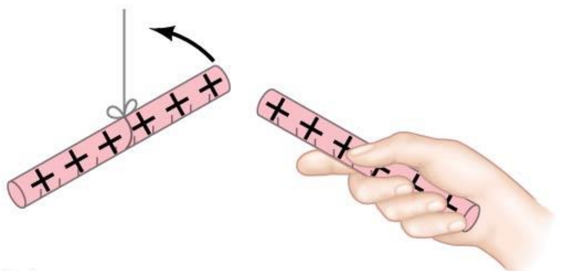
- labeled **–**

(electron: $q_e = -1.60217733 \times 10^{-19}$ C)

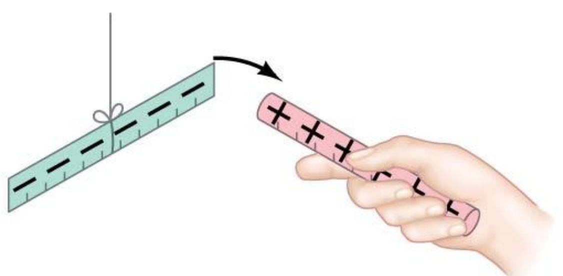
16-1 Static Electricity; Electric Charge and Its Conservation



(a) Two charged plastic rulers repel



(b) Two charged glass rods repel

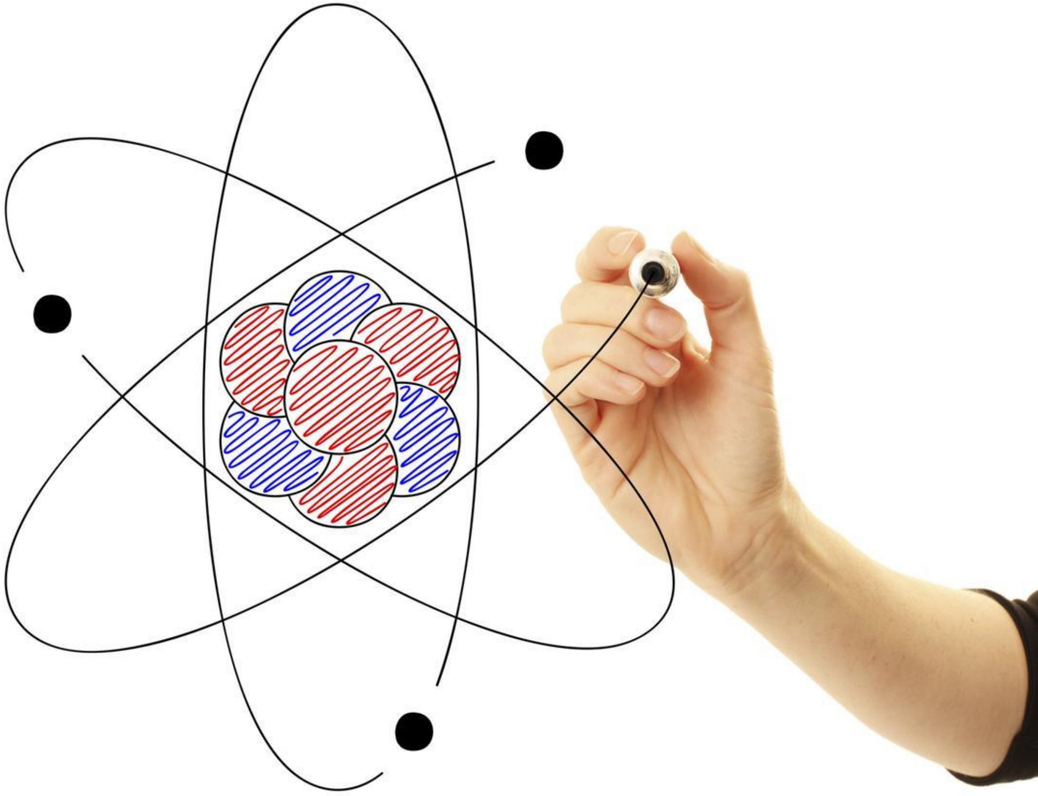


(c) Charged glass rod attracts charged plastic ruler

Charge comes in two types, positive and negative; like charges repel and opposite charges attract

16-1 Static Electricity; Electric Charge and Its Conservation

Electric charge is conserved—the arithmetic sum of the total charge cannot change in any interaction.

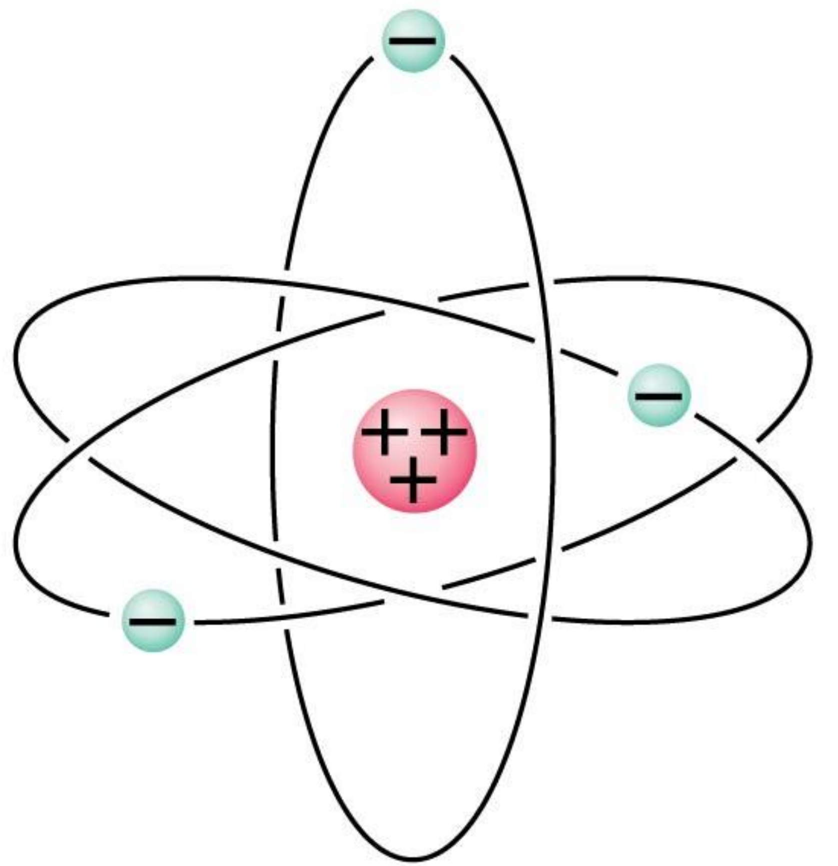


16-2 Electric Charge in the Atom

Atom:

Nucleus (small, massive, positive charge)

Electron cloud (large, very low density, negative charge)



16-2 Electric Charge in the Atom

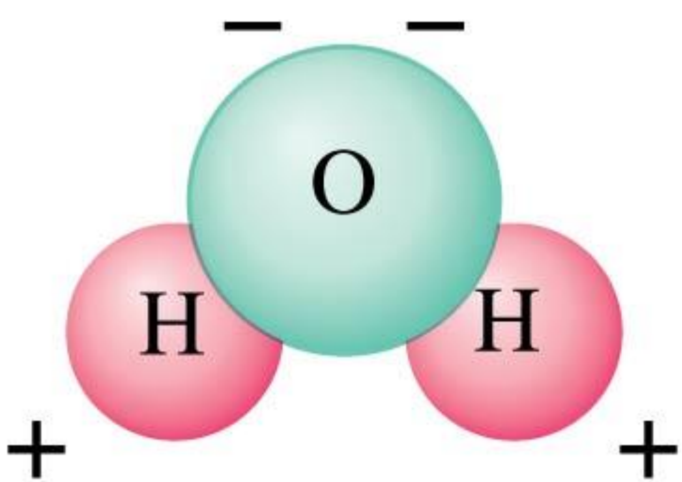
Atom is electrically neutral. 【中性】

Rubbing charges objects by moving electrons from one to the other.

(a) (b)

16-2 Electric Charge in the Atom

Polar molecule: neutral overall, but charge not evenly distributed



16-3 Insulators and Conductors

Conductor:

Charge flows freely

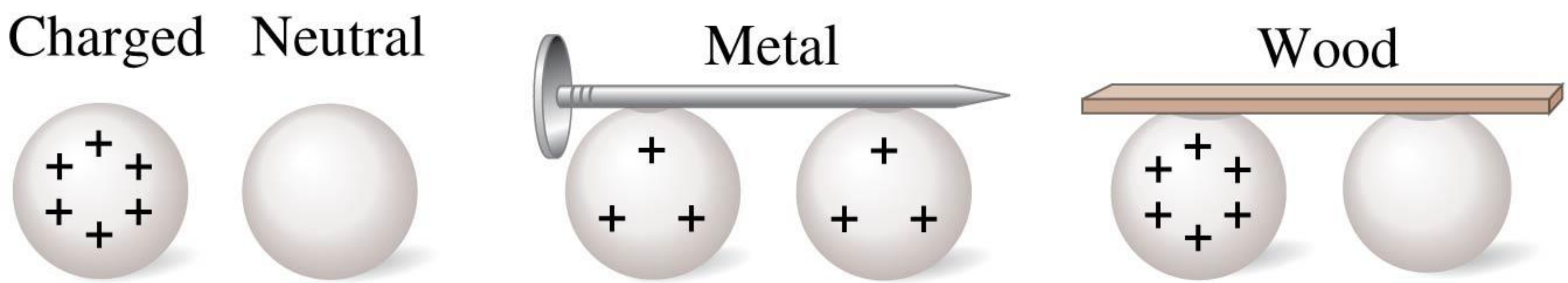
Metals

Insulator:

Almost no charge flows

Most other materials

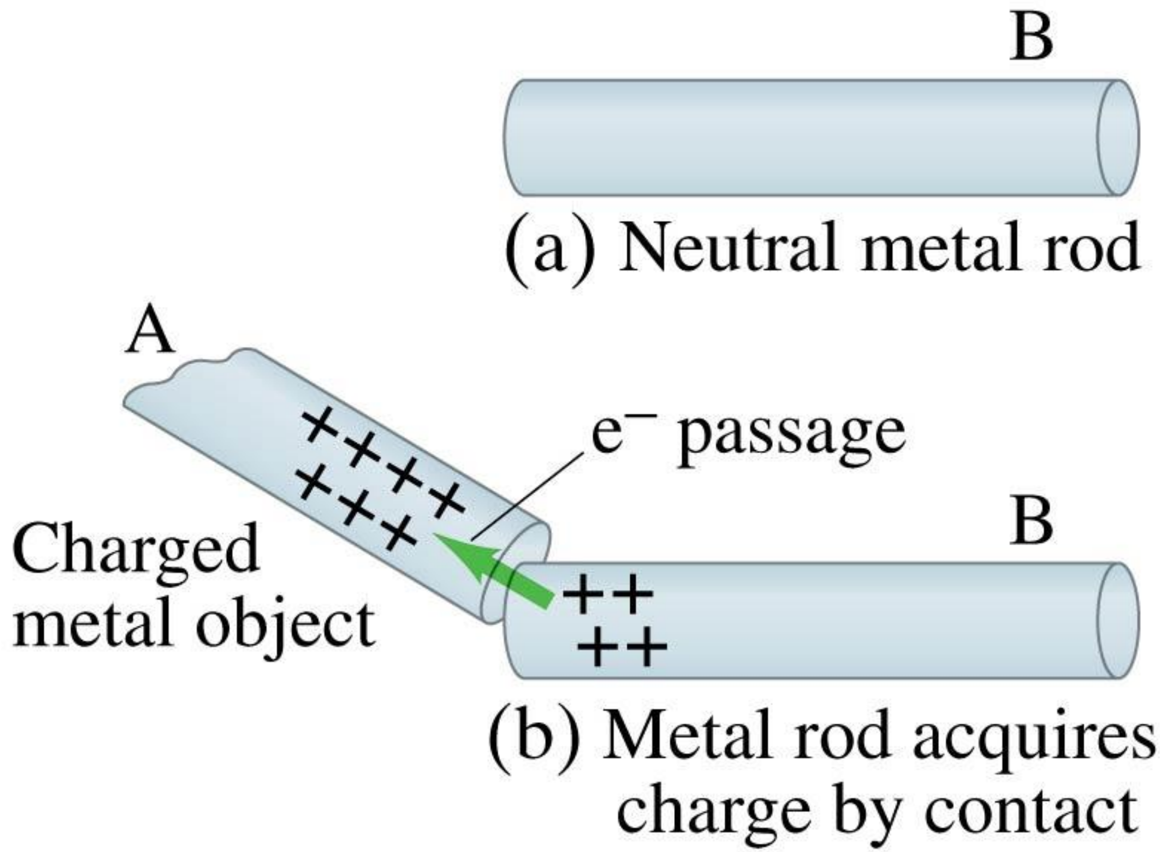
Some materials are semiconductors.



(a) (b) (c)

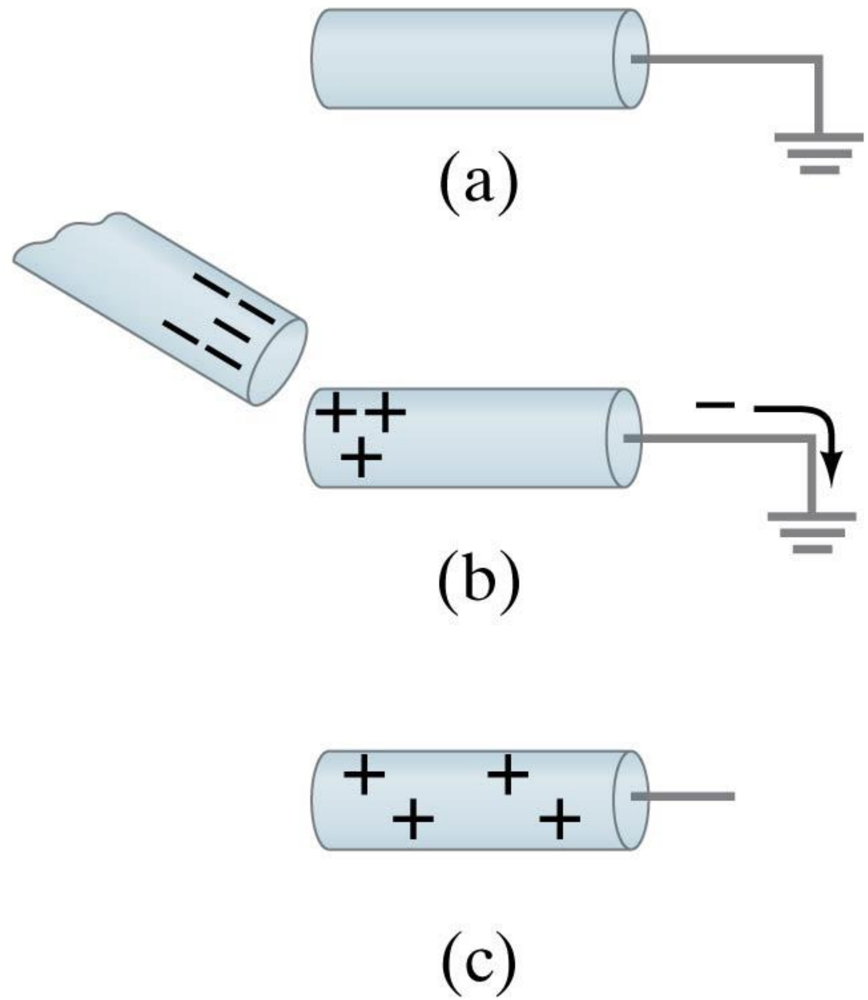
16-4 Induced Charge; the Electroscope

Metal objects can be charged by conduction:



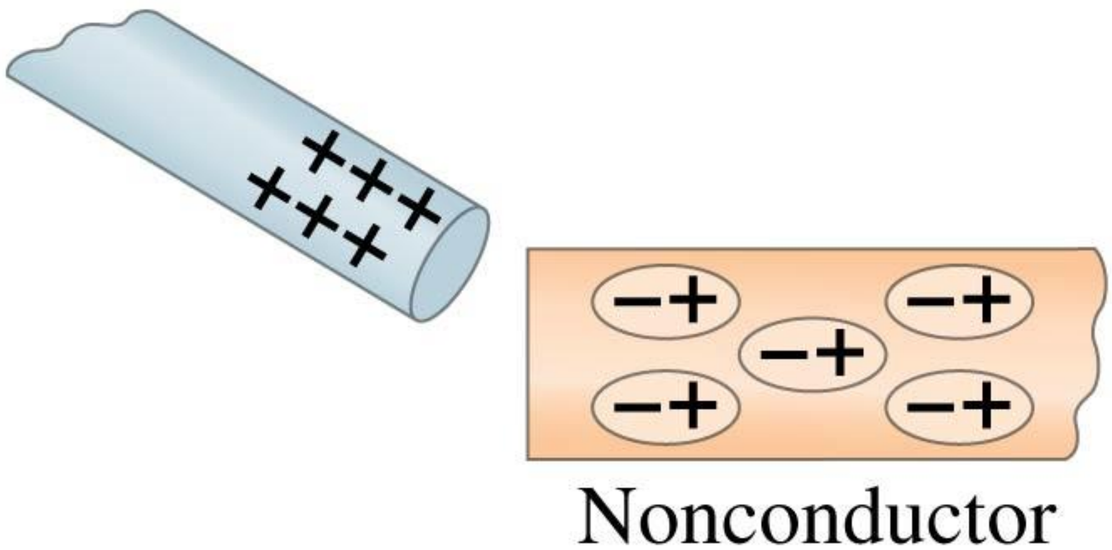
16-4 Induced Charge; the Electroscope

They can also be charged by induction:



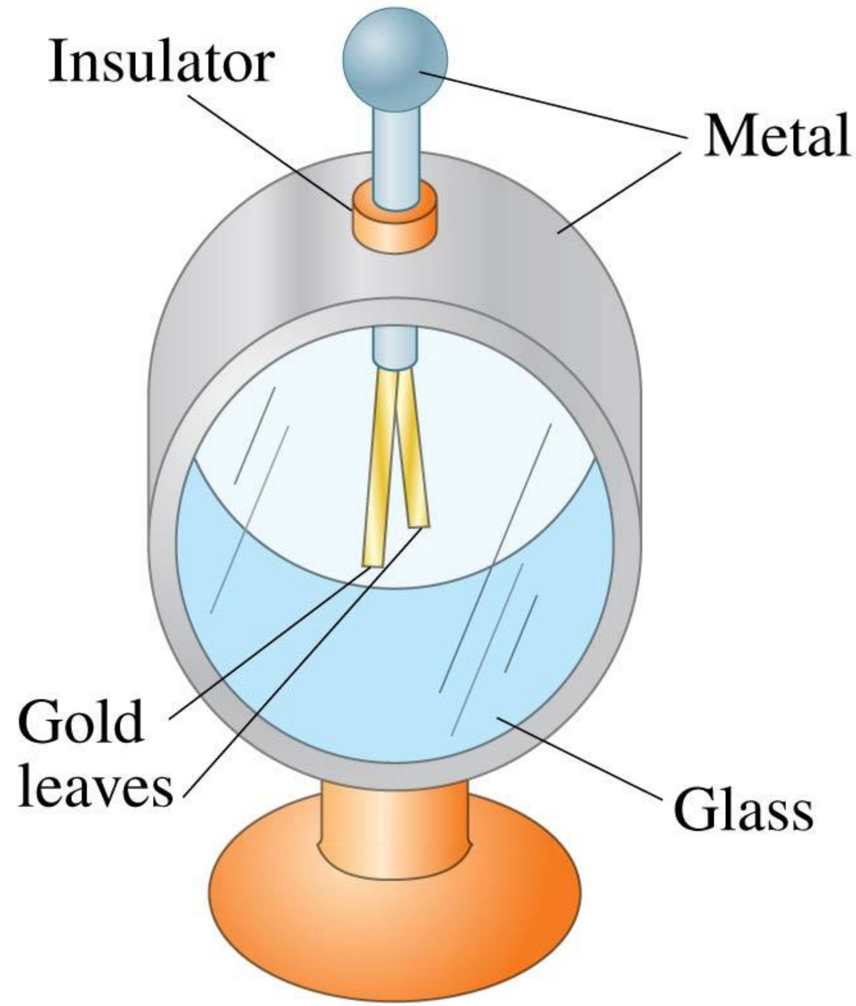
16-4 Induced Charge; the Electroscope

Nonconductors won’t become charged by conduction or induction, but will experience charge separation:



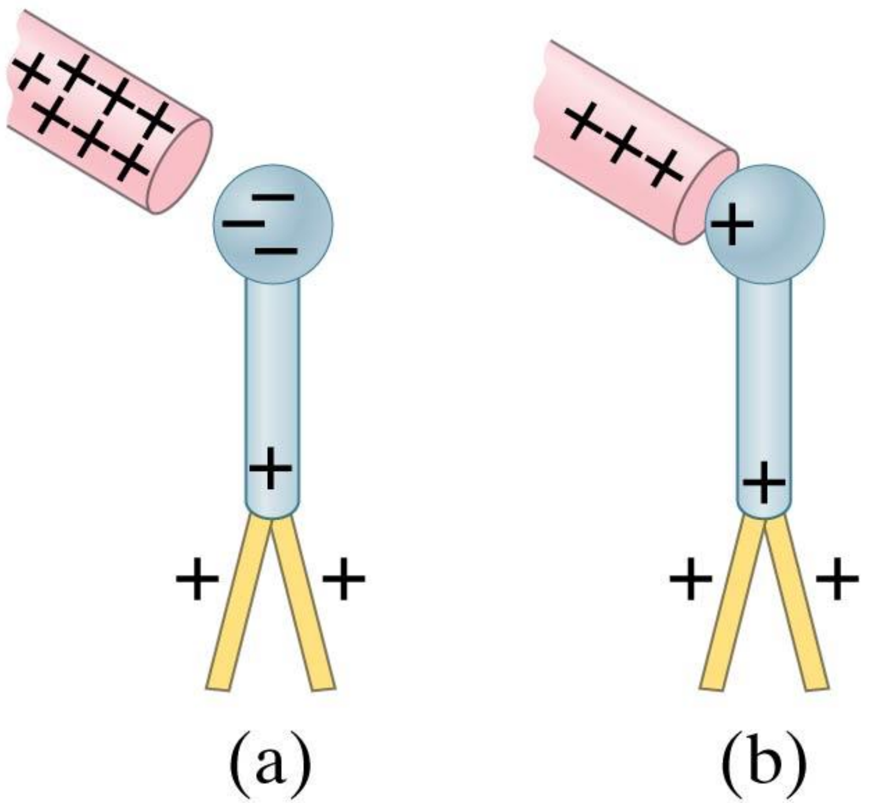
16-4 Induced Charge; the Electroscope

The electroscope can be used for detecting charge:



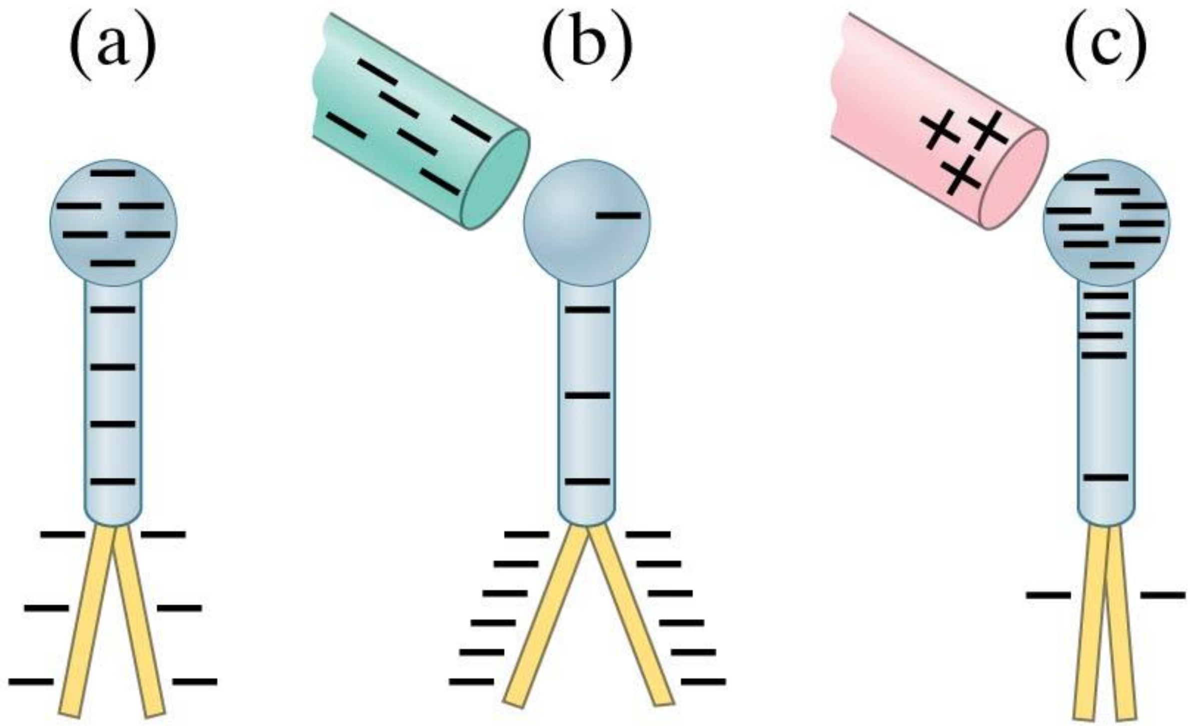
16-4 Induced Charge; the Electroscope

The electroscope can be charged either by conduction or by induction.



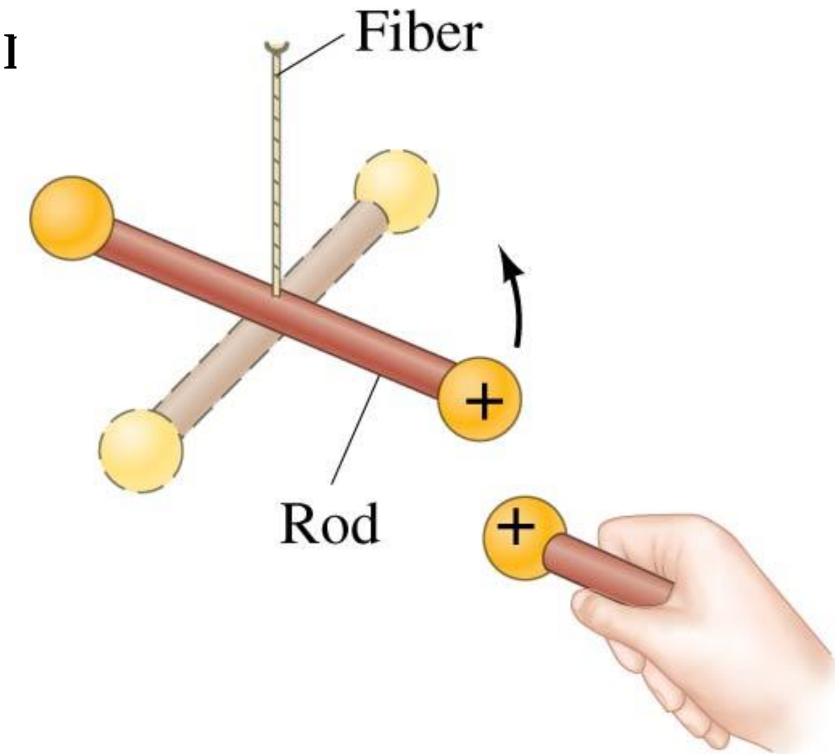
16-4 Induced Charge; the Electroscope

The charged electroscope can then be used to determine the sign of an unknown charge.



16-5 Coulomb's Law

Experiment shows that the electric force between two charges is proportional to the product of the charges and inversely proportional to the distance square between them.



§.5 库仑定律 Coulomb's Law

Coulomb是试图通过直接测量来寻找电力规律的第一人

1781 有关扭力的论文

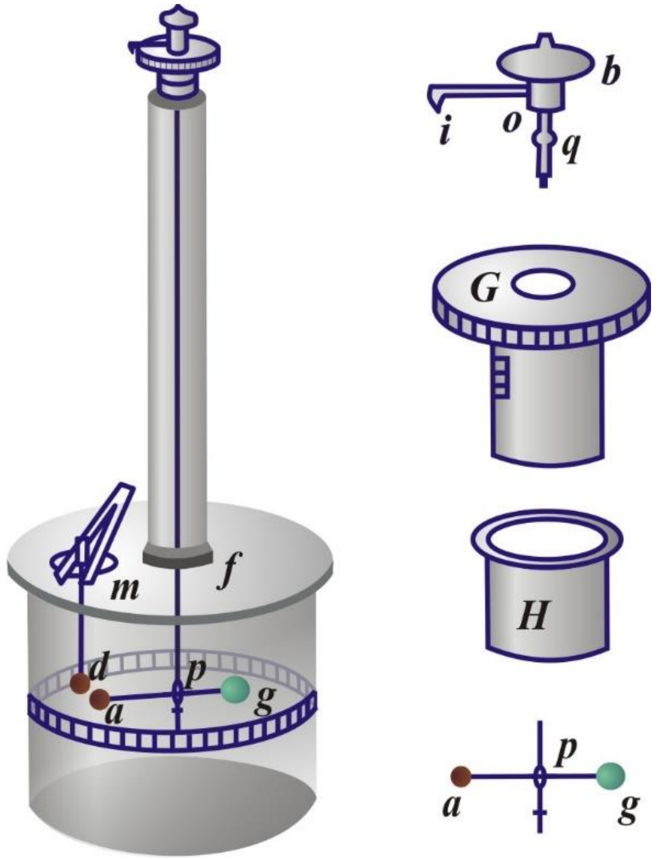
法国科学院院士

1784 在提交法国科学院的一篇论文中确定了金属丝的扭力定律

扭力正比于扭转角度

能测量小至 $\mathbf{6.48 \times 10^{-6}\,N}$ 的作用力

1785 Coulomb 扭秤实验



Coulomb 扭秤实验之前

最早提出电力平方反比律

1767 普里斯特利 (J. Priestley)

《The History and the Present State of Electricity》

1769 J. Robison首先用直接测量方法确定电力的定律

得出：两个带电小球之间的斥力 $f \propto \frac{1}{r^{2.06}}$

两个带电小球之间的引力与距离的关系比平方反比的方次要小一些

推断：正确的电力定律是平方反比律

Robison的结论比Priestley的推论晚了两年，
而且他的论文直到1801年才发表。

Coulomb 扭秤实验之前

1772-1773 卡文迪什 (H. Cavendish)

提出了精确验证电力平方反比律的方法

$$F \propto \frac{1}{r^{2\pm\delta}} \qquad \delta < 2\times 10^{-2}$$

1874 麦克斯韦 (J. C. Maxwell) 卡文迪什实验室

Cavendish-Maxwell 方法 $\delta < 5\times 10^{-5}$

1971 Williams、Faller& Hill $\delta < 2.7\times 10^{-16}$

静止的 **Resting** 点电荷 **Point charges**

$$F \propto \frac{q_1 q_2}{r^2} \qquad F = k\frac{q_1 q_2}{r^2}$$

$q_1 q_2$ 两点电荷电量的乘积 SI 库仑（C）

真空 free space $k = \dfrac{1}{4\pi\varepsilon_0} = 8.988\times10^{9}\,\mathrm{N\cdot m^{2}\cdot C^{-2}}$

Permittivity of free space $\varepsilon_0 = 8.854\times10^{-12}\,\mathrm{C^{2}\cdot N^{-1}\cdot m^{-2}}$

真空介电常量（真空电容率）

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

16-5 Coulomb's Law

Coulomb's law:

$$F = k\frac{Q_1 Q_2}{r^2}, \quad \text{[magnitudes]} \quad (16\text{-}1)$$

This equation gives the magnitude of the force.

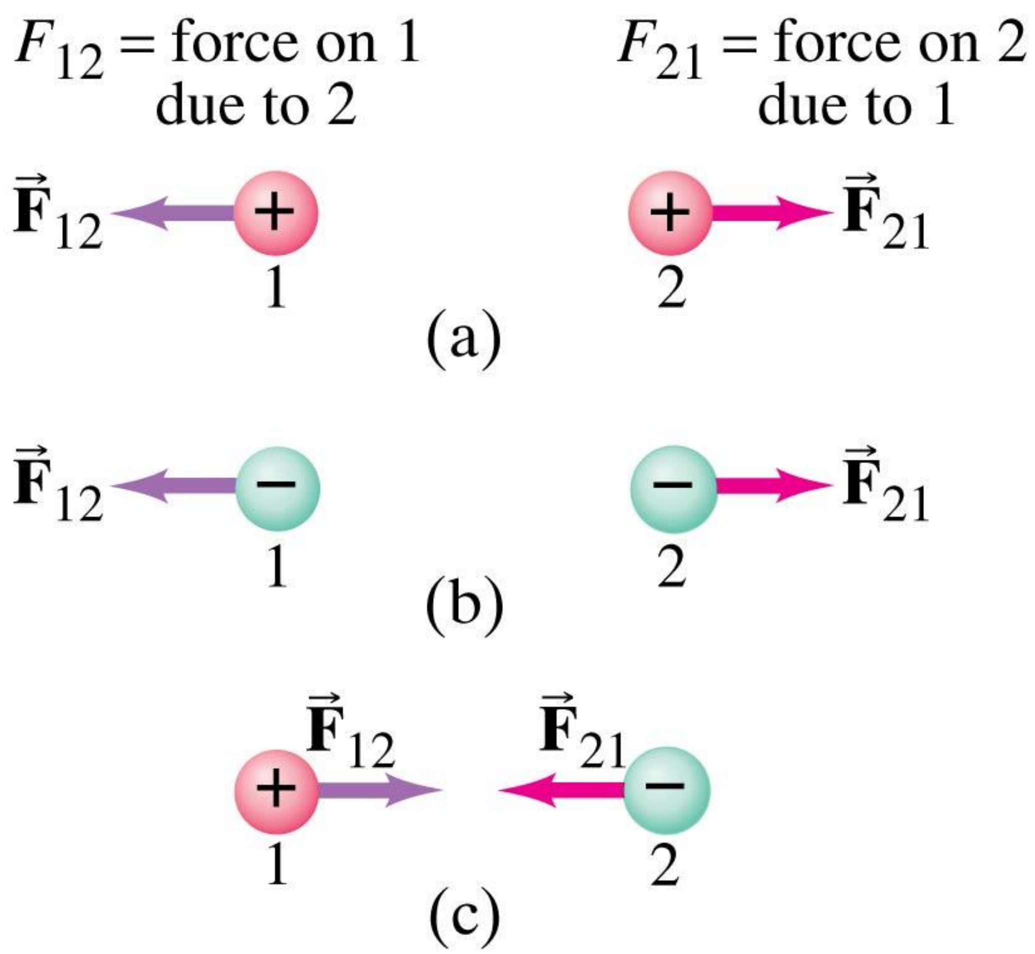
*Consider a proton (m_p=1.67x10^{-27} kg; q_p=+1.60x10^{-19} C) and an electron (m_e=9.11x10^{-31} kg; q_e=-1.60x10^{-19} C) separated by 5.29x10^{-11} m. The particles are attracted to each other by both the force of gravity and by Coulomb's law force. Which of these has the larger magnitude?

A. Gravitational force

B. Coulomb's law force

16-5 Coulomb's Law

The force is along the line connecting the charges, and is attractive if the charges are opposite, and repulsive if they are the same.



16-5 Coulomb's Law

Unit of charge: coulomb, C

The proportionality constant in Coulomb's law is then:

$$k = 8.988 \times 10^{9}\ \mathrm{N{\cdot}m^2/C^2}$$

Charges produced by rubbing are typically around a microcoulomb:

$$1\ \mu\mathrm{C} = 10^{-6}\ \mathrm{C}$$

16-5 Coulomb's Law

Charge on the electron:

$$e = 1.602 \times 10^{-19}\ \text{C}$$

Electric charge is quantized in units of the electron charge.

16-5 Coulomb's Law

The proportionality constant k can also be written in terms of ε_0, the permittivity of free space:

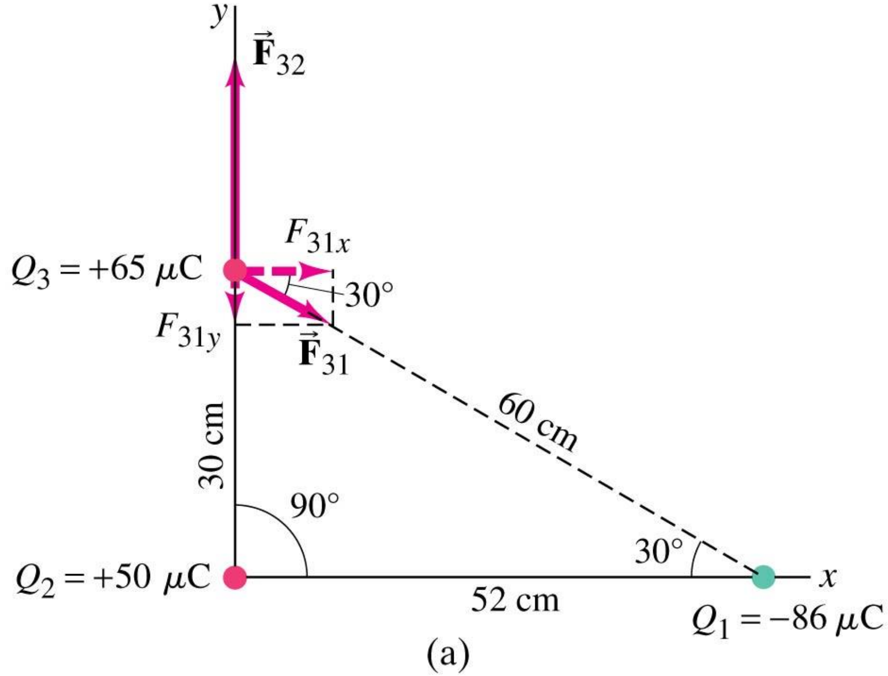
$$F = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2}, \quad (16\text{-}2)$$

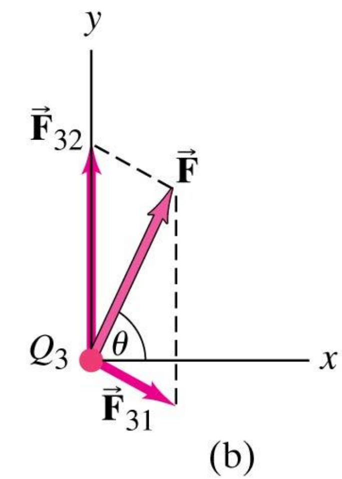
$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2}.$$

16-5 Coulomb's Law

Coulomb's law strictly applies only to point charges.

Superposition: for multiple point charges, the forces on each charge from every other charge can be calculated and then added as vectors.





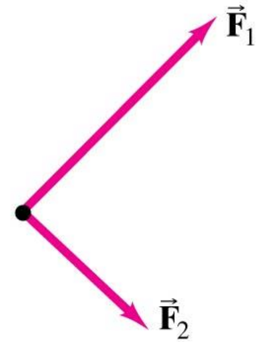
16-6 Solving Problems Involving Coulomb's Law and Vectors

The net force on a charge is the vector sum of all the forces acting on it.

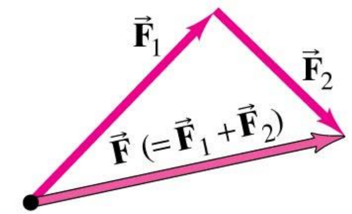
$$\vec{\mathbf{F}}_{\text{net}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \cdots .$$

16-6 Solving Problems Involving Coulomb's Law and Vectors

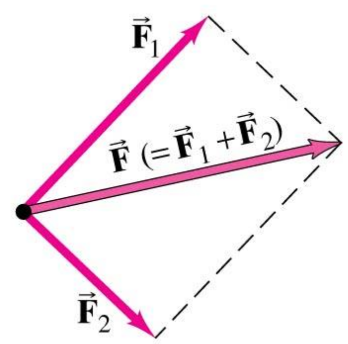
Vector addition review:



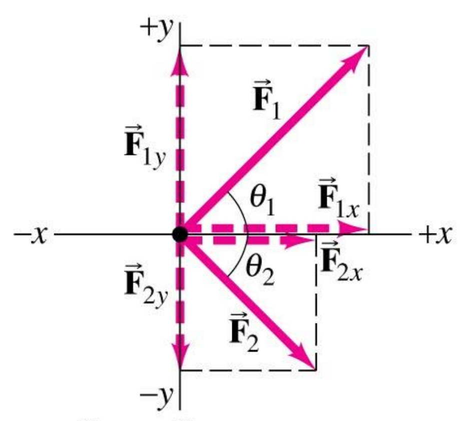
(a) Two forces acting on an object.



(b) The total, or net, force is $\vec{F} = \vec{F}_1 + \vec{F}_2$ by the tail-to-tip method of adding vectors.



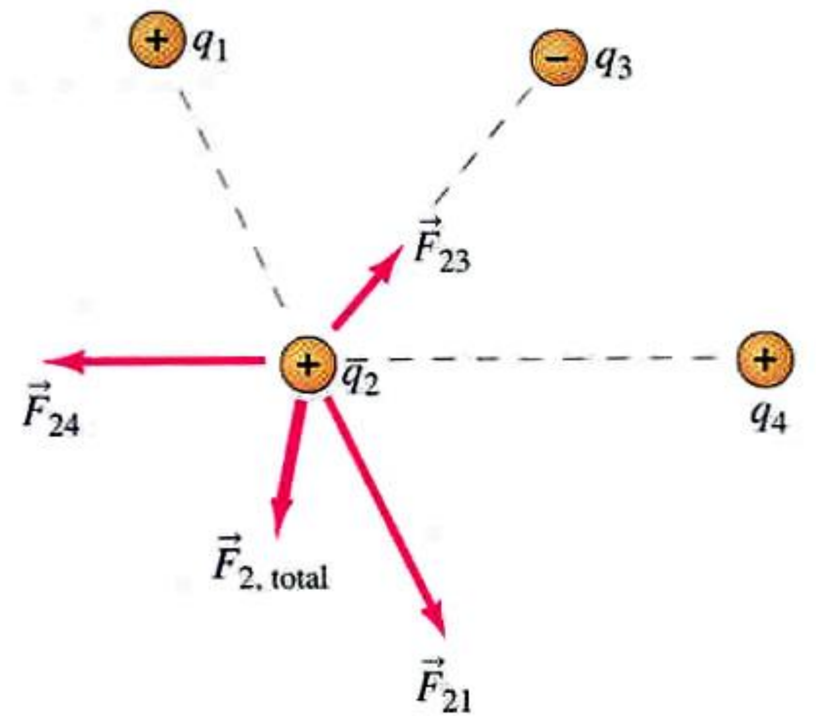
(c) $\vec{F} = \vec{F}_1 + \vec{F}_2$ by the parallelogram method.



(d) $\vec{F}_1$ and $\vec{F}_2$ resolved into their x and y components.

Forces Involving Multiple Charges

$$\vec{F}_{2,\text{total}} = \vec{F}_{21} + \vec{F}_{23} + \vec{F}_{24}$$



If there are *n* charges

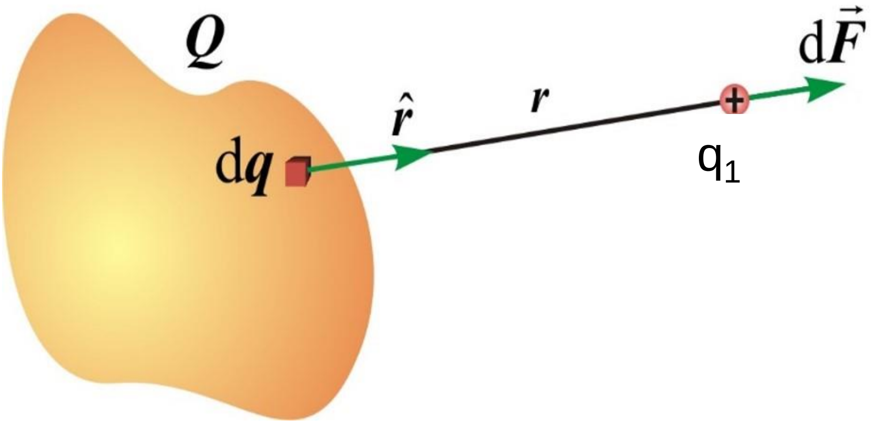
The total force $\vec{F}$ on charge *q* is the vector sum of the individual forces $\vec{F}_i$ on charge *q* due to charge q_i

$$\vec{F} = \sum_{i=1}^{n} \vec{F}_i = \sum_{i=1}^{n} \frac{q}{4\pi\varepsilon_0} \frac{q_i}{r_i^2} \hat{r}_i$$

Principle of superposition

电力叠加原理

Continuous distributions of charge



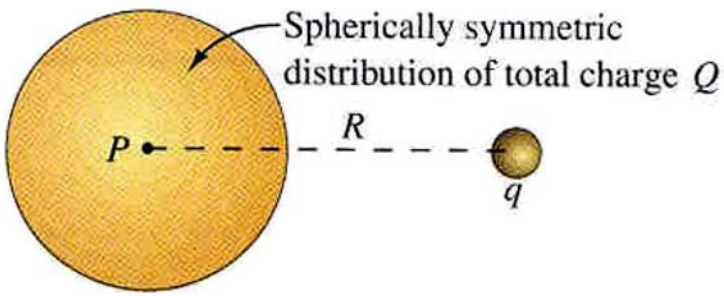
$\boldsymbol{dq}$ 电荷元

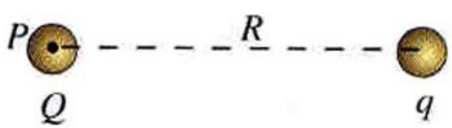
$$d\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1\, dq}{r^2}\hat{r}$$

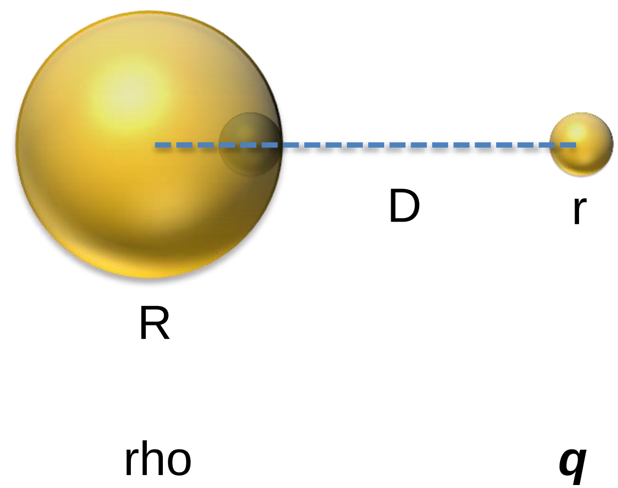
Total force on a point charge *q* due to a continuous charge distribution

$$\vec{F} = \int_{(Q)} \frac{q_1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$

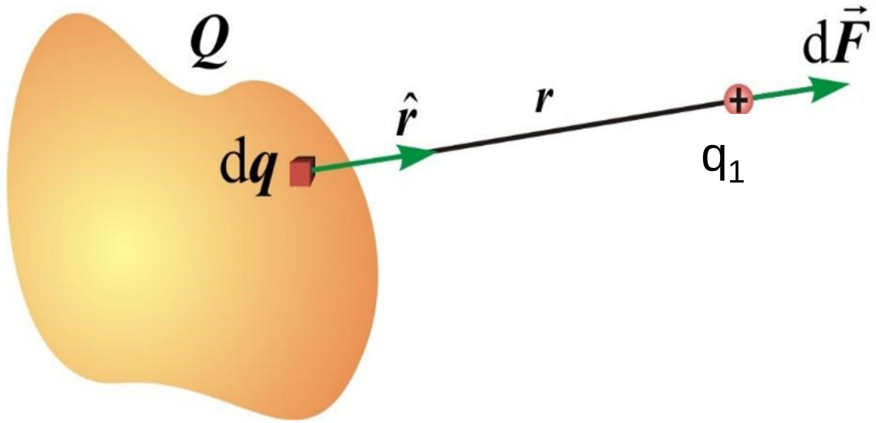
The force due to a spherically symmetric charge distribution







Continuous distributions of charge



dq 电荷元

$$d\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 dq}{r^2}\hat{r}$$

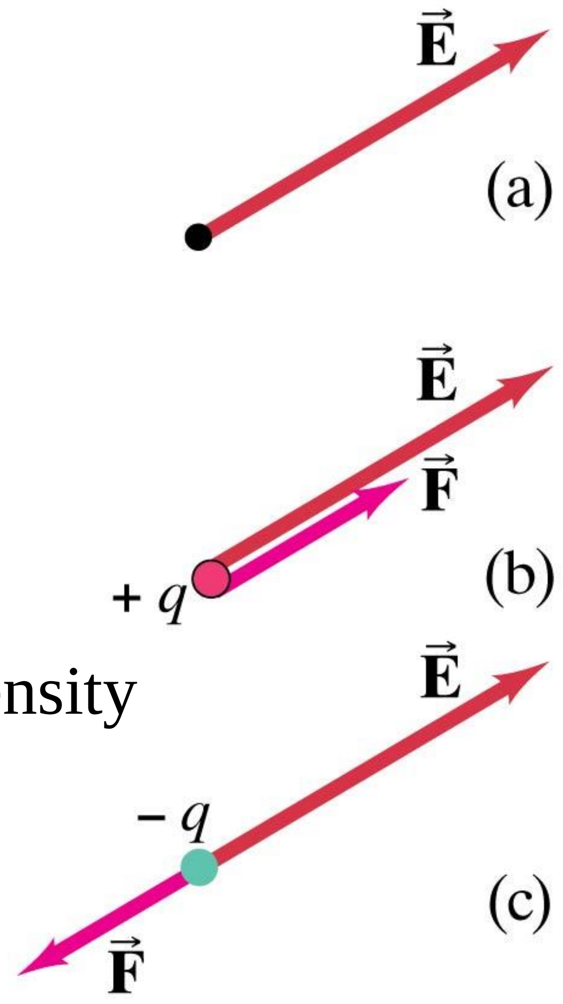
Total force on a point charge *q* due to a continuous charge distribution

$$\vec{F} = \int_{(Q)} \frac{q_1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$

16-7 The Electric Field

The electric field is the force on a small charge, divided by the charge:

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q}. \quad (16\text{-}3)$$



Electrostatic field & Electric field intensity

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

16-7 The Electric Field

For a point charge:

电荷间的相互作用是通过电场来实现的

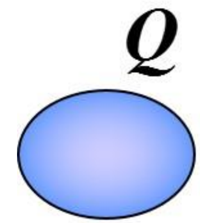
$$E = \frac{F}{q} = \frac{kqQ/r^2}{q}$$

电荷 ⟺ 电场 ⟺ 电荷

$$E = k\frac{Q}{r^2}; \qquad \text{[single point charge]} \quad (16\text{-}4a)$$

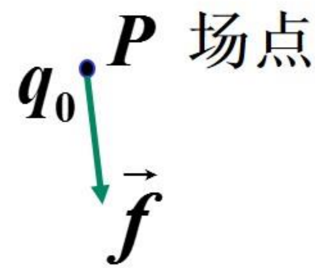
$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}. \qquad \text{[single point charge]} \quad (16\text{-}4b)$$

对电荷有作用力是电场的特征性质



试验电荷(test charge) q_0 放到场点P 处

试验电荷受力为 $\vec{f}$



试验电荷

test charge

条件 $\begin{cases} \text{电量充分地小} \\ \text{线度足够地小} \end{cases}$

实验表明：任一确定的场点 比值 $\dfrac{\vec{f}}{q_0}$ 与试验电荷无关

$$\vec{E} = \frac{\vec{f}}{q_0}$$

电场强度 场强

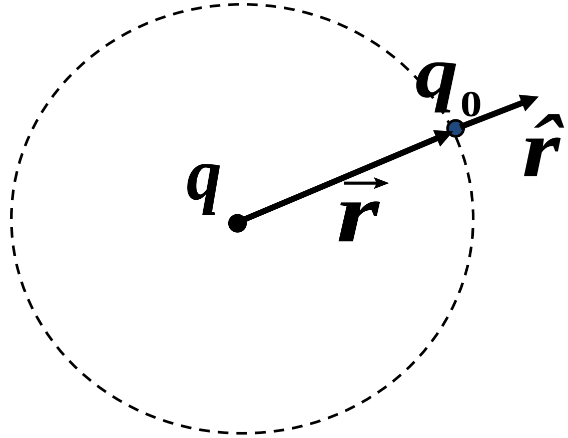
Electric field intensity

Electric field due to a point charge q

The definition of electric field $$\vec{E} = \frac{\vec{F}}{q_0}$$

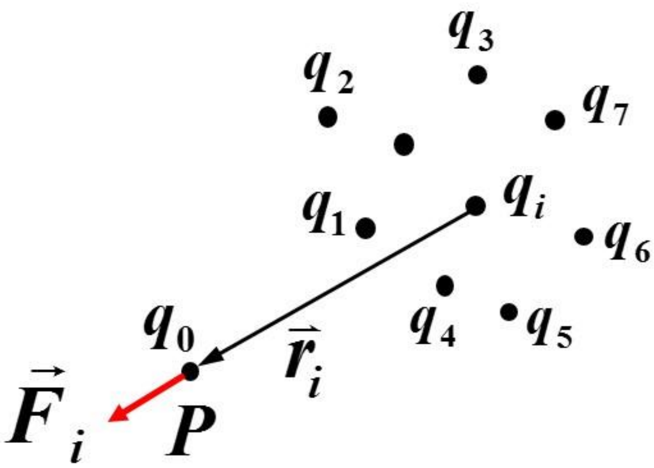
$$\vec{E} = \frac{\dfrac{1}{4\pi\varepsilon_0}\dfrac{qq_0}{r^2}\hat{r}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$$\Rightarrow \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$



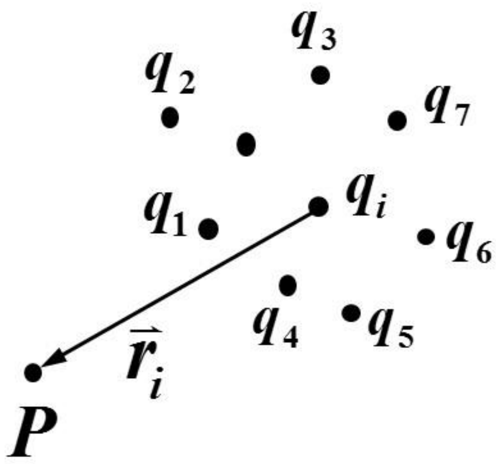
Spherical symmetry　球对称性

If there are n point charges



$$\vec{F} = \sum_{i=1}^{n} \vec{F}_i$$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{\sum_{i=1}^{n} \vec{F}_i}{q_0} = \sum_{i=1}^{n} \frac{\vec{F}_i}{q_0} = \sum_i \vec{E}_i$$



$$\vec{E} = \sum_i \vec{E}_i \qquad \vec{E} = \sum_{i=1}^{n} \frac{q_i}{4\pi\varepsilon_0 r_i^2} \hat{r}_i$$

Principle of superposition of electric field

场强叠加原理

16-7 The Electric Field

Force on a point charge in an electric field:

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}. \quad (16\text{-}5)$$

Superposition principle for electric fields:

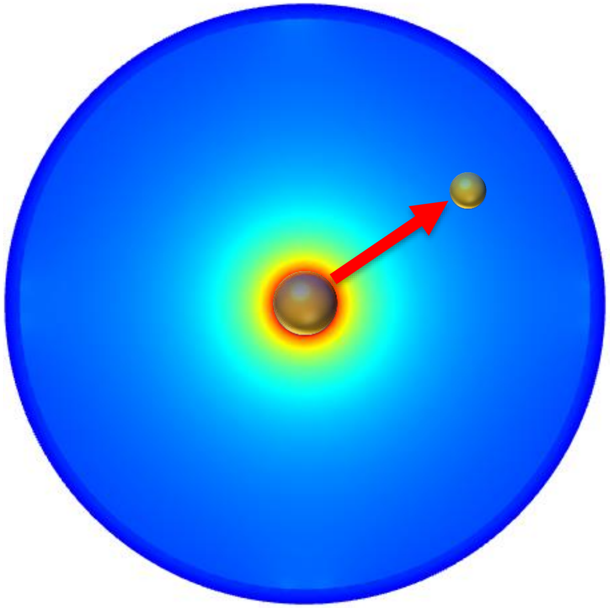
$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \cdots.$$

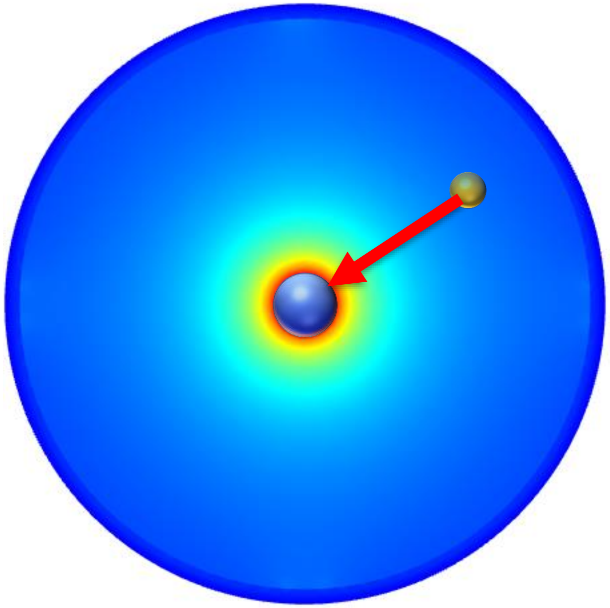
16-7 The Electric Field

Problem solving in electrostatics: electric forces and electric fields

1. Draw a diagram; show all charges, with signs, and electric fields and forces with directions
2. Calculate forces using Coulomb’s law
3. Add forces vectorially to get result

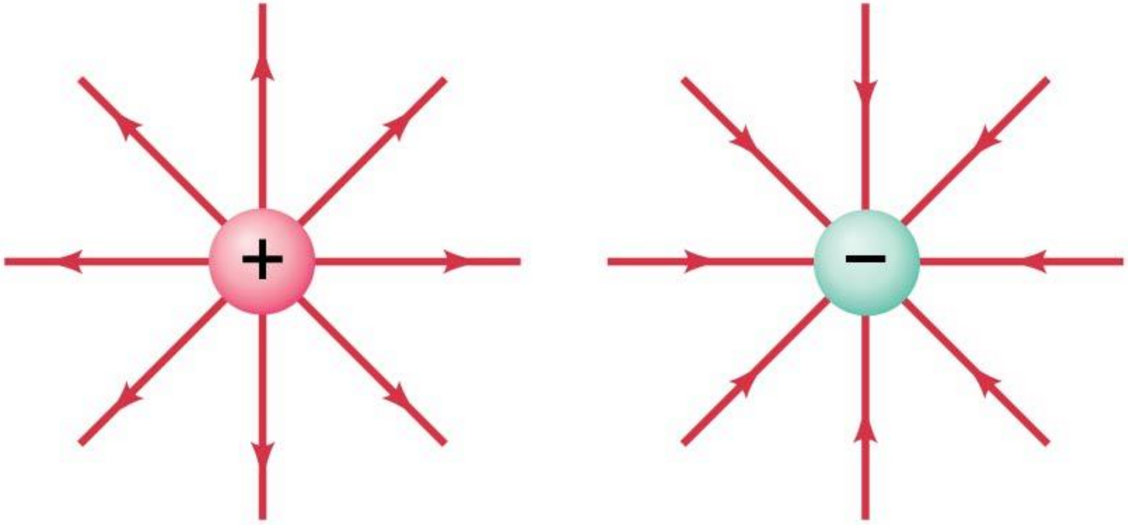
16-7 The Electric Field





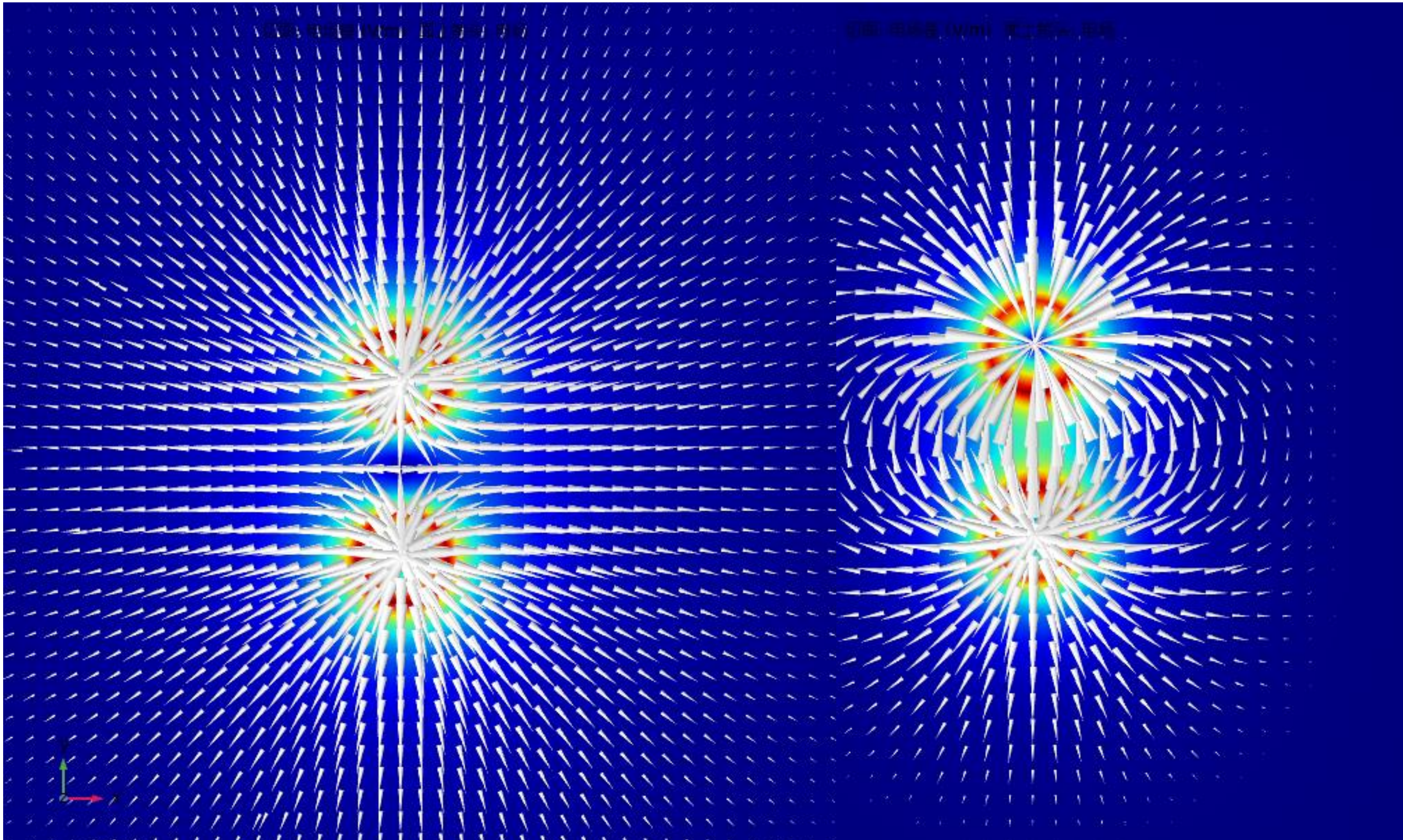
16-8 Electric Field Lines

The electric field can be represented by field lines. These lines start on a positive charge and end on a negative charge.



(a) (b)

电场线 Electric Field Lines



where the field lines are far apart, E is small

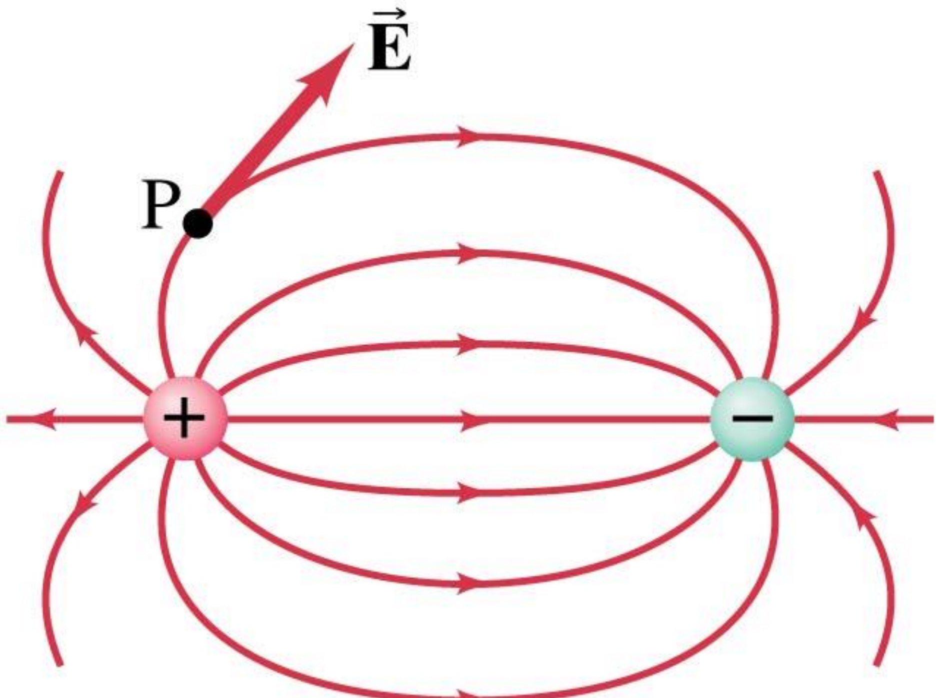
16-8 Electric Field Lines

The number of field lines starting (ending) on a positive (negative) charge is proportional to the magnitude of the charge.

The electric field is stronger where the field lines are closer together.

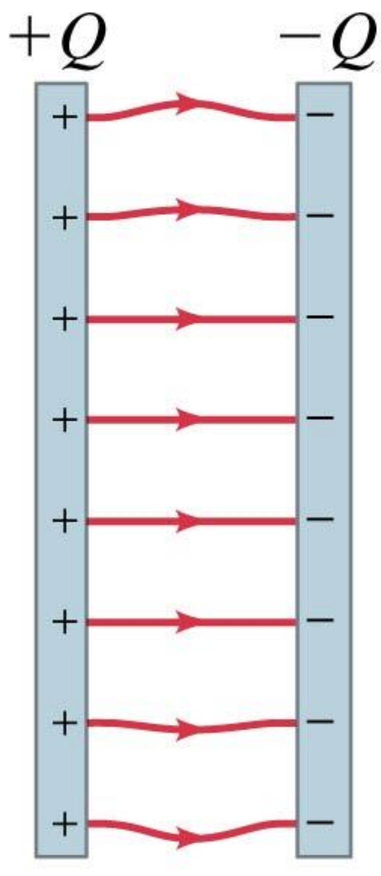
16-8 Electric Field Lines

Electric dipole: two equal charges, opposite in sign:



(a)

16-8 Electric Field Lines



(d)

The electric field between two closely spaced, oppositely charged parallel plates is constant.

16-8 Electric Field Lines

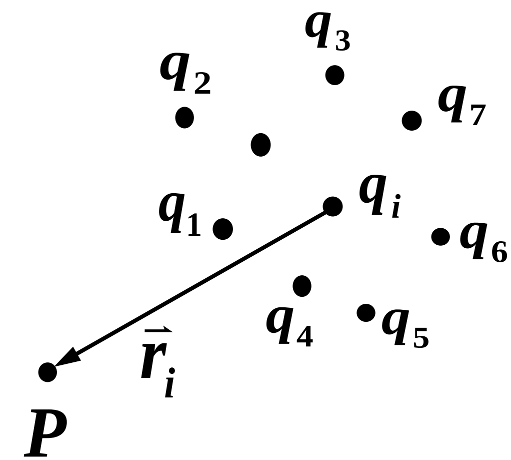
Summary of field lines:

1. Field lines indicate the direction of the field; the field is tangent to the line.
2. The magnitude of the field is proportional to the density of the lines.
3. Field lines start on positive charges and end on negative charges; the number is proportional to the magnitude of the charge.

§ .9 电场强度的计算

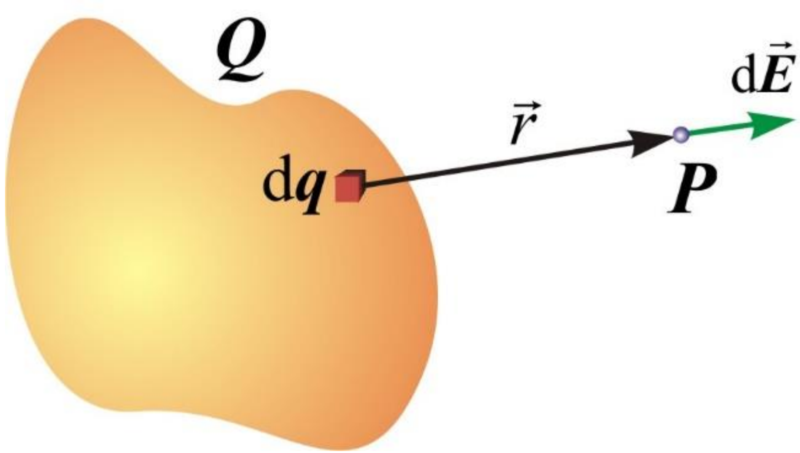
Calculation of Electric field intensity

$$\vec{E}=\sum_{i=1}^{n}\frac{q_i}{4\pi\varepsilon_0 r_i^2}\hat{r}_i$$



Continuous distributions of charge

tiny charge element 电荷元 dq

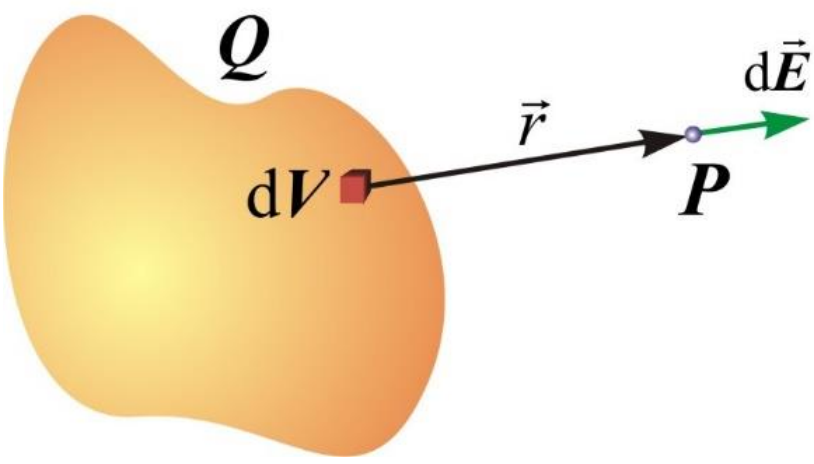


$$d\vec{E}=\frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$

$$\vec{E}=\int_{(Q)}d\vec{E}=\int_{(Q)}\frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$

The electric field at point *P* due to the charged object is

$$\vec{E} = \int_{(Q)} d\vec{E} = \int_{(Q)} \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$



Volume (three dimensions)

$$\rho = \frac{dq}{dV}$$

volume charge density

体电荷密度

$$dq = \rho dV$$

Surface (two dimensions)

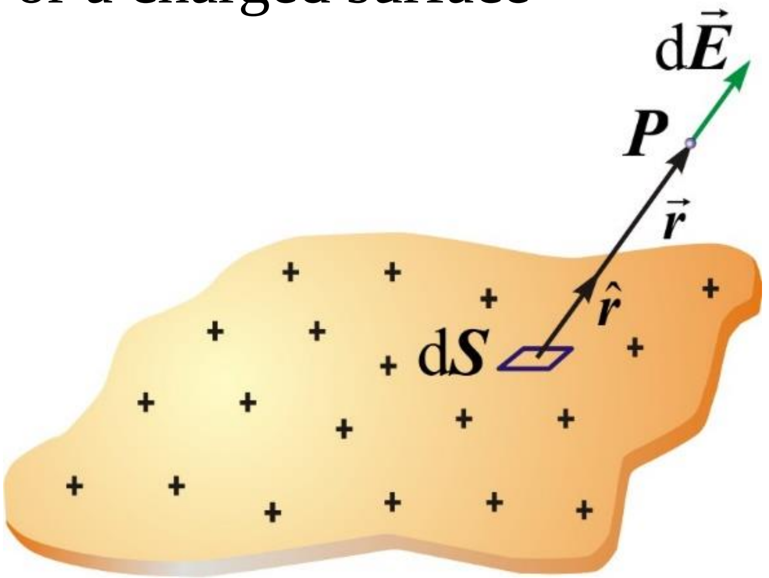
$$\sigma = \frac{\mathrm{d}q}{\mathrm{d}S}$$

surface charge density

面电荷密度

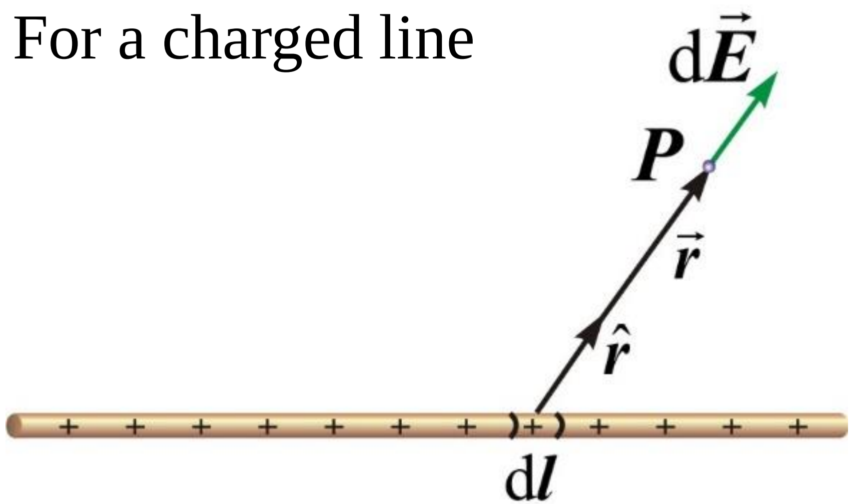
$$dq = \sigma \mathrm{d}S$$

For a charged surface



Line segment (one dimension)

For a charged line



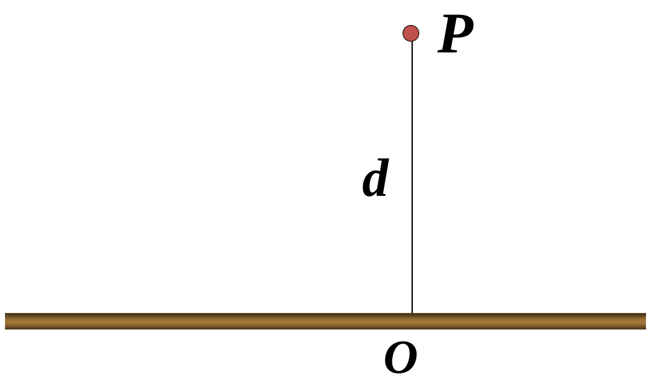
$$\lambda = \frac{\mathrm{d}q}{\mathrm{d}l}$$

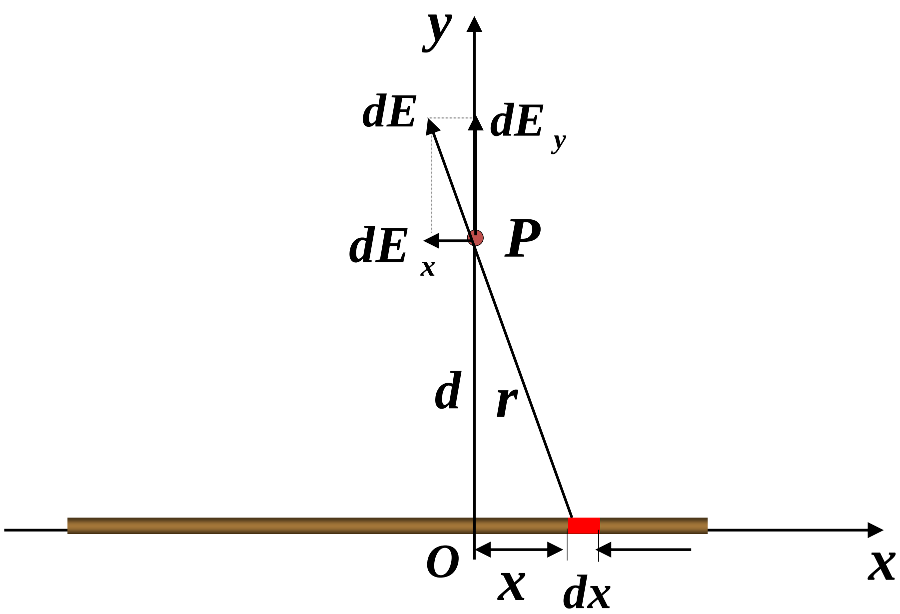
linear charge density

线电荷密度

$$dq = \lambda \mathrm{d}l$$

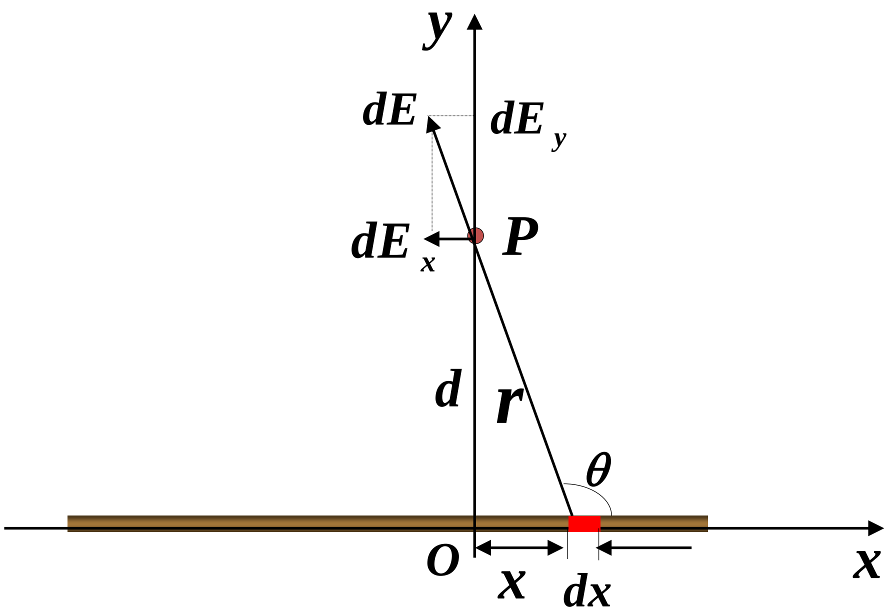
A straight insulating rod of length L carries a uniform linear charge density λ. Determine the electric field at point P, a perpendicular distance d from the rod.





$$dq = \lambda dx$$

$$dE = \frac{dq}{4\pi\varepsilon_0 r^2}$$



$$dE = \frac{dq}{4\pi\varepsilon_0 r^2}$$

$$r = \frac{d}{\sin\theta}$$

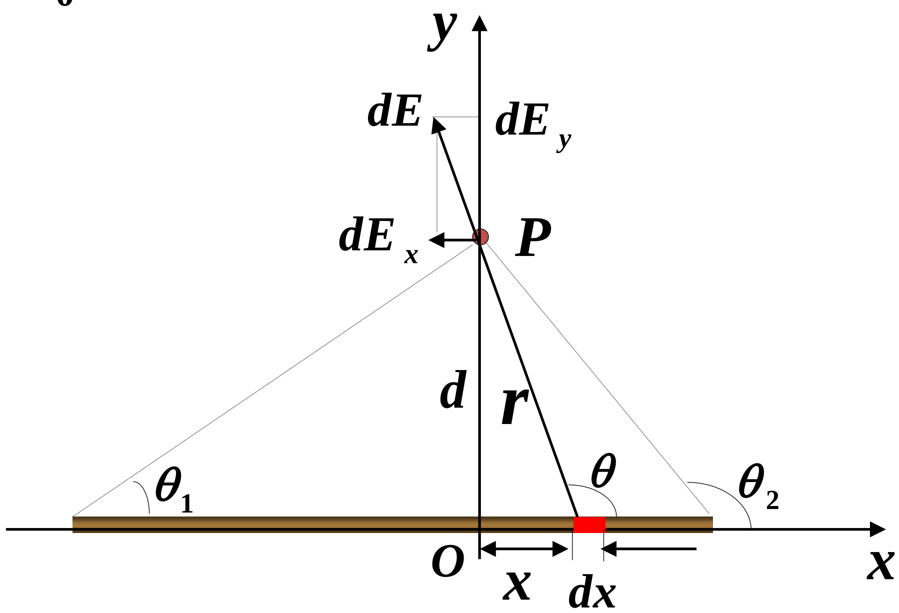
$$dE_x = dE\cos\theta$$

$$x = -d\cot\theta \qquad dx = \frac{d}{\sin^2\theta}d\theta$$

$$dE_y = dE\sin\theta$$

$$E_x = \int dE_x = \int \frac{\lambda dx}{4\pi\varepsilon_0 r^2}\cos\theta \qquad E_y = \int dE_y = \int \frac{\lambda dx}{4\pi\varepsilon_0 r^2}\sin\theta$$

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 d}\int \cos\theta d\theta \qquad E_y = \frac{\lambda}{4\pi\varepsilon_0 d}\int \sin\theta d\theta$$



$$E_x = \frac{\lambda}{4\pi\varepsilon_0 d}\int_{\theta_1}^{\theta_2} \cos\theta d\theta$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 d}\int_{\theta_1}^{\theta_2} \sin\theta d\theta$$

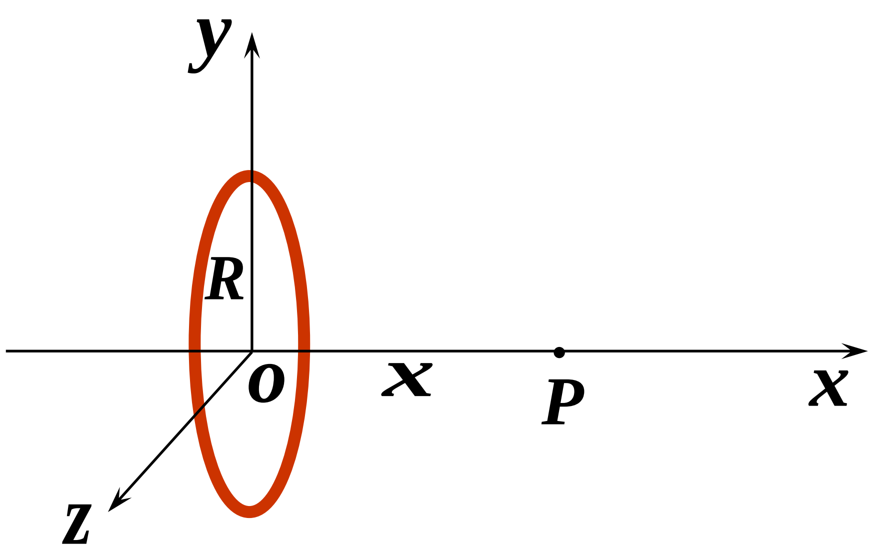
$$E_x = \frac{\lambda}{4\pi\varepsilon_0 d}(\sin\theta_2 - \sin\theta_1)$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 d}(\cos\theta_1 - \cos\theta_2) \qquad \vec{E} = E_x\vec{i} + E_y\vec{j}$$

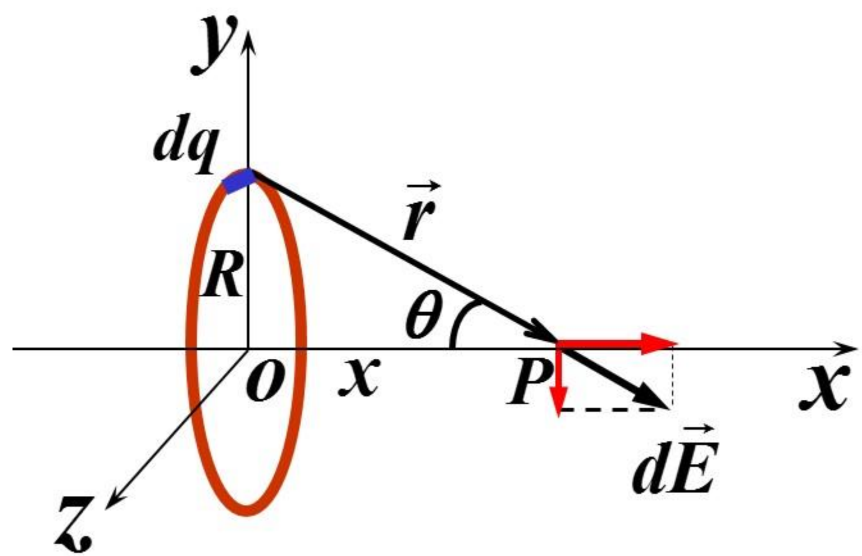
Discuss: infinitely long rod $d<<L$ $\theta_1 = 0$ $\theta_2 = \pi$

$$E_x = 0 \qquad E_y = \frac{\lambda}{2\pi\varepsilon_0 d} \qquad \vec{E} = \frac{\lambda}{2\pi\varepsilon_0 d}\vec{j}$$

Find the electric field at point P located on the axis of uniformly charged ring of total charge Q . The radius of the ring is R, and the point P is located a distance x from the center of the ring.

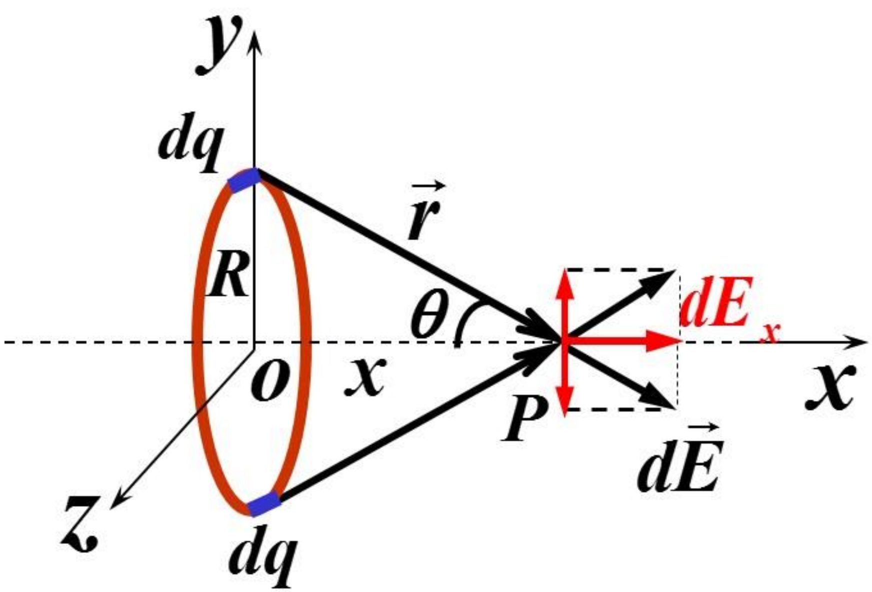


$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$



Find the electric field at point P located on the axis of uniformly charged ring of total charge Q . The radius of the ring is R, and the point P is located a distance x from the center of the ring.

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$



$$dE_x = dE\cos\theta$$

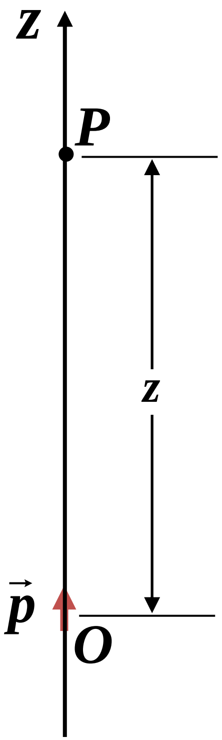
$$E_x = \int \frac{dq}{4\pi\varepsilon_0 r^2}\cos\theta$$

$$= \frac{xQ}{4\pi\varepsilon_0\left(x^2 + R^2\right)^{3/2}}$$

$$\vec{E} = E_x\vec{i}$$

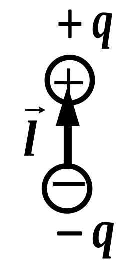
The Electric Field Due to an Electric Dipole 电偶极子

Two charged particles of magnitude q but of opposite sign, separated by a distance l $(l << r)$ we call this configuration an electric dipole.



electric dipole moment

电偶极矩（电矩） $\vec{p}$

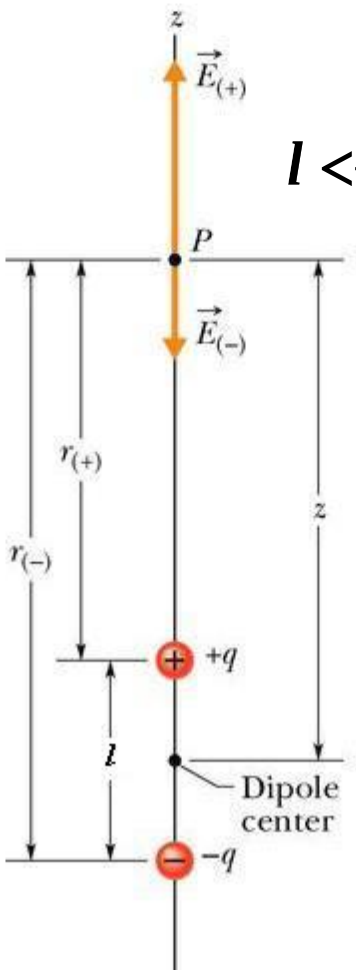


$$\vec{p} = q\vec{l} \qquad l \to 0 \ (l << r)$$

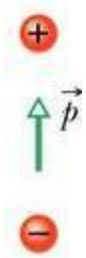
Let us find the electric field due to the dipole at a point P, a distance z from the electric dipole.

The Electric Field Due to an Electric Dipole 电偶极子

Two charged particles of magnitude q but of opposite sign, separated by a distance l $(l << r)$ we call this configuration an electric dipole.



$l << z$



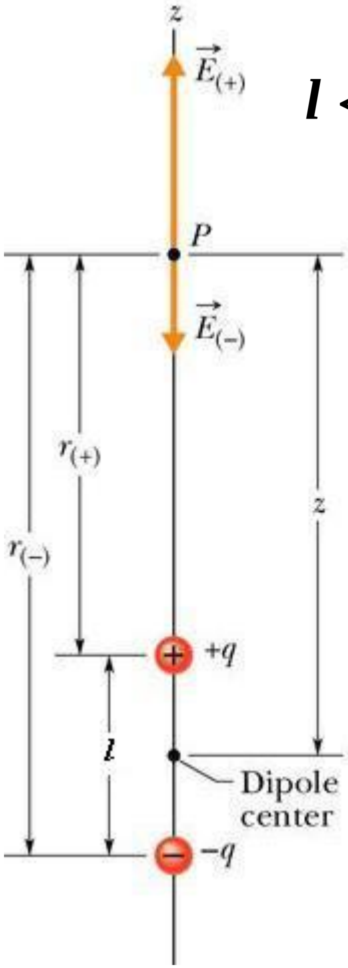
electric dipole moment $\vec{p}$

电偶极矩（电矩）

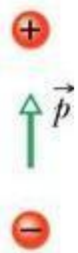
$$\vec{p} = q\vec{l} \qquad l \to 0 \quad (l << r)$$

$$E = E_+ + E_-$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{r_{(-)}^2}$$



$l \ll z$



$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(z-\frac{l}{2})^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(z+\frac{l}{2})^2}$$

$$= \frac{q}{4\pi\varepsilon_0 z^2}\left[(1-\frac{l}{2z})^{-2} - (1+\frac{l}{2z})^{-2}\right]$$

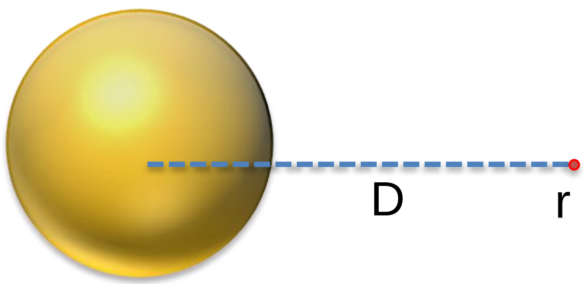
$$= \frac{q}{4\pi\varepsilon_0 z^2}\left[(1+\frac{l}{z}) - (1-\frac{l}{z})\right]$$

$$= \frac{q}{4\pi\varepsilon_0 z^2}\frac{2l}{z} = \frac{1}{2\pi\varepsilon_0}\frac{ql}{z^3}$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}$$

$$\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\vec{p}}{z^3}$$

Quizzes

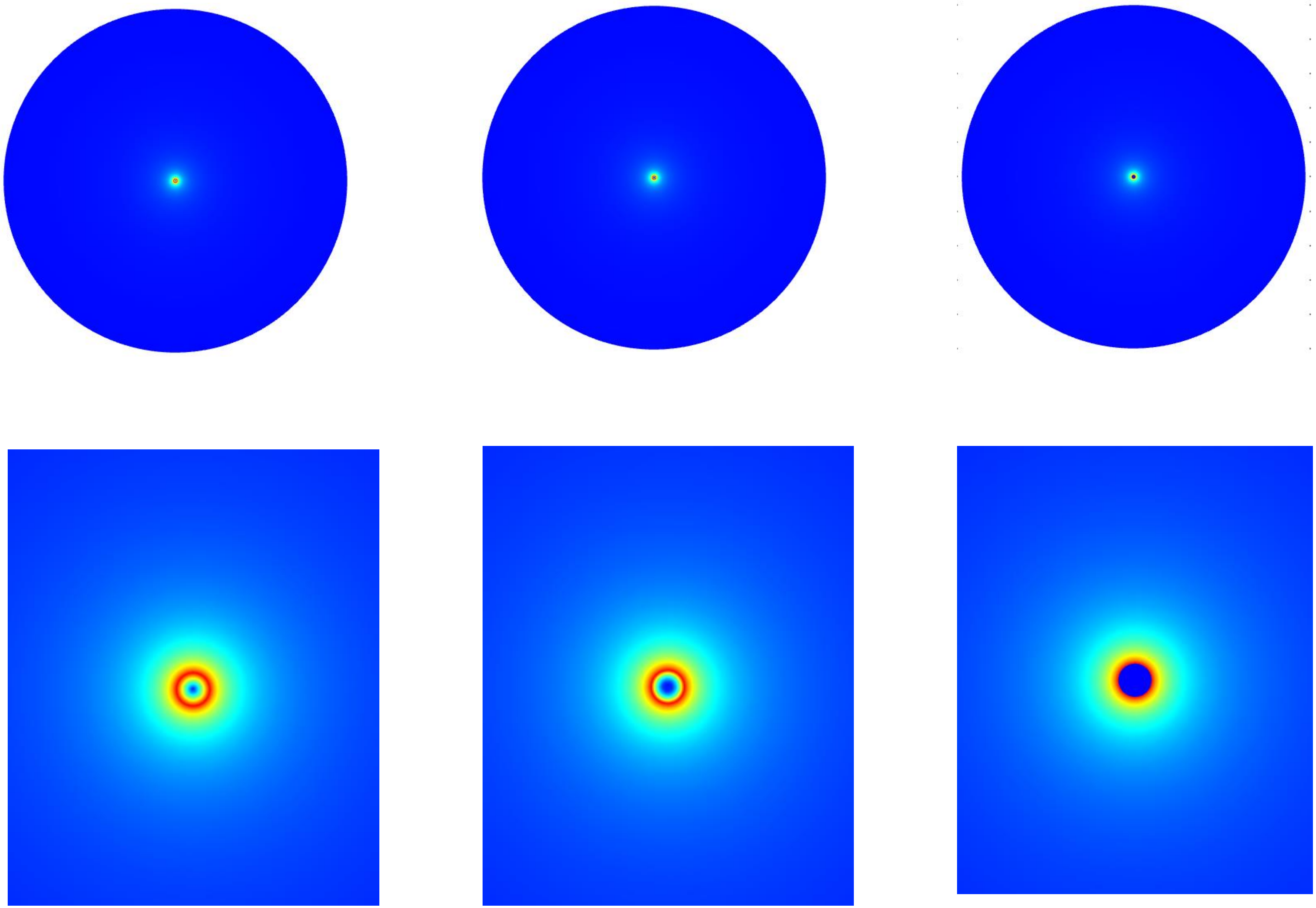


R=0.01[m]

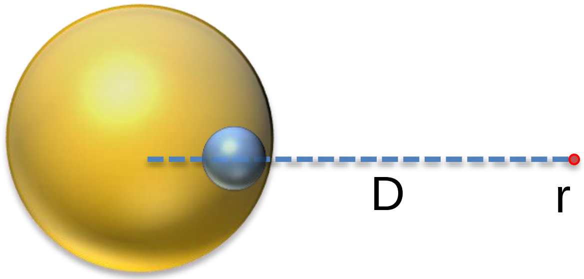
Rho=r[m]

Rho=$\frac{3}{40}\,[C/m^3]$

$$\delta = \frac{1}{400}\,[C/m^2]$$



Quizzes



R=0.01[m]

Compensation method

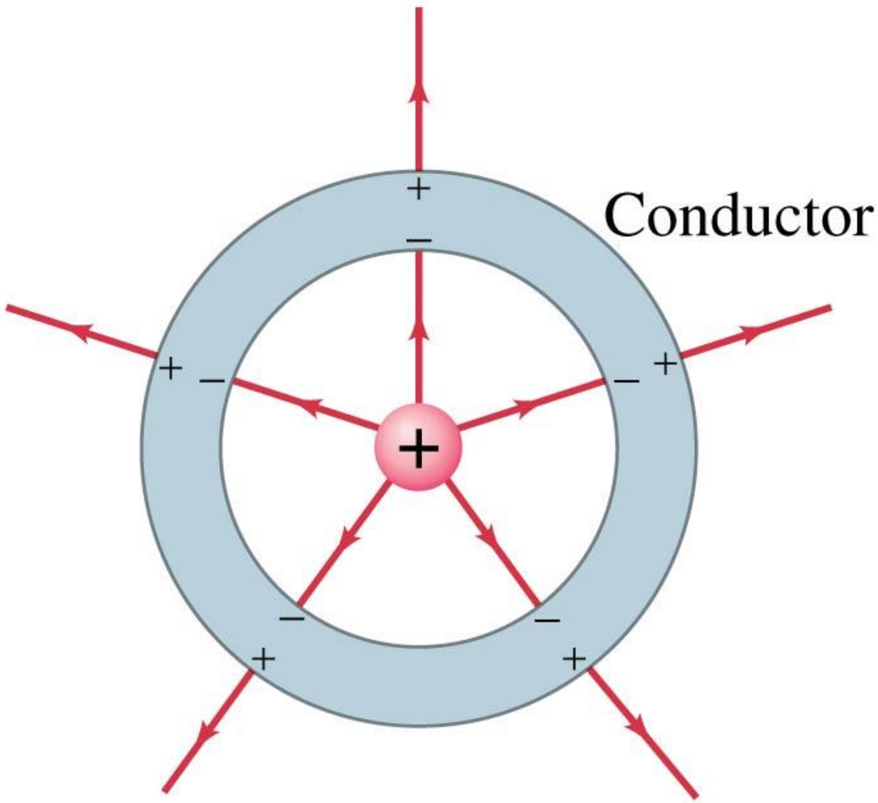
Rho=r[m]

$\boxed{\text{Rho}=\frac{3}{40}\,[C/m^3]}$

$$\delta = \frac{1}{400}\,[C/m^2]$$

16-9 Electric Fields and Conductors

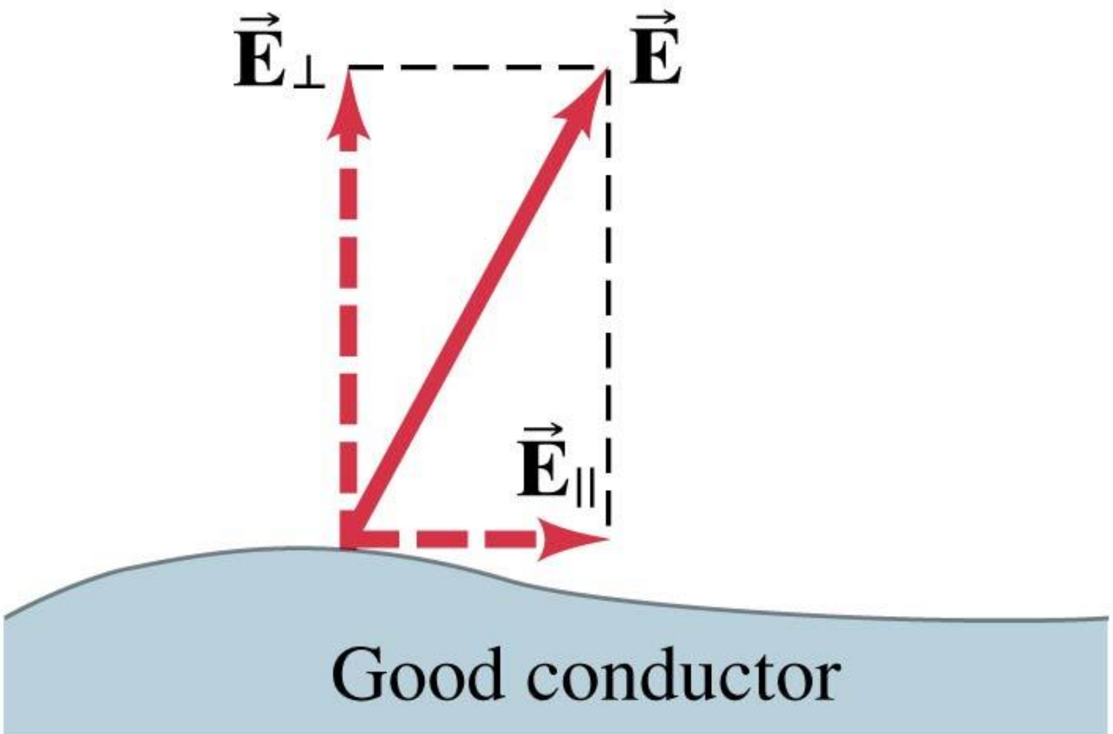
The static electric field inside a conductor is zero—if it were not, the charges would move.



The net charge on a conductor is on its surface.

16-9 Electric Fields and Conductors

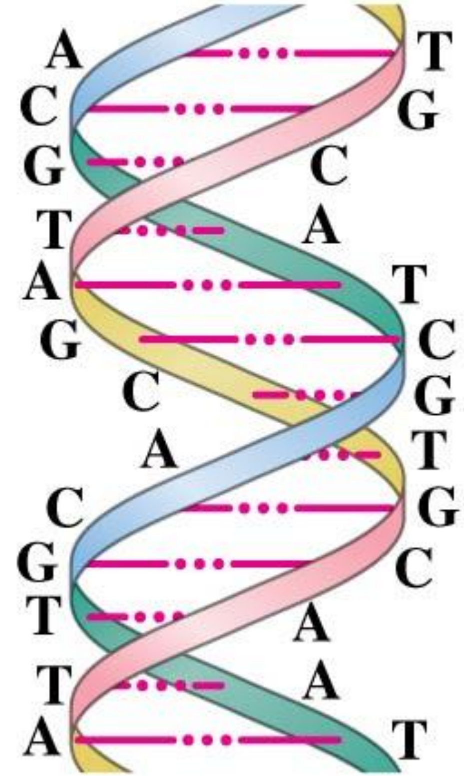
The electric field is perpendicular to the surface of a conductor—again, if it were not, charges would move.



16-10 Electric Forces in Molecular Biology: DNA Structure and Replication

Molecular biology is the study of the structure and functioning of the living cell at the molecular level.

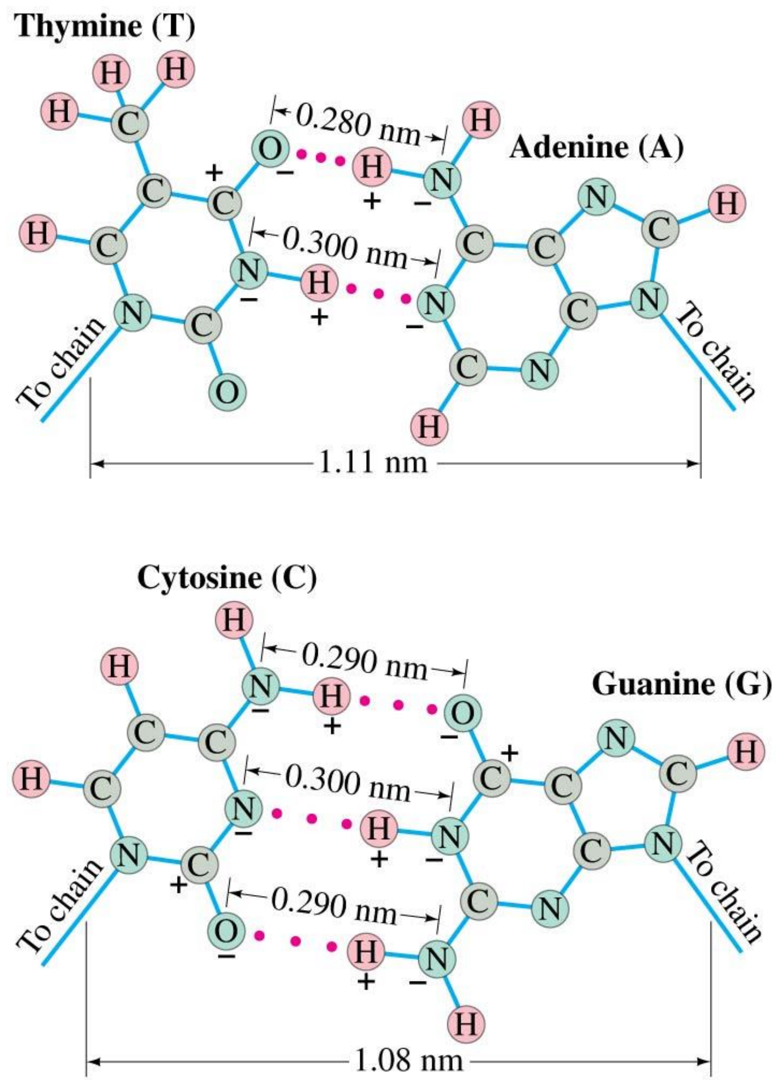
The DNA molecule is a double helix:



(a)

16-10 Electric Forces in Molecular Biology: DNA Structure and Replication

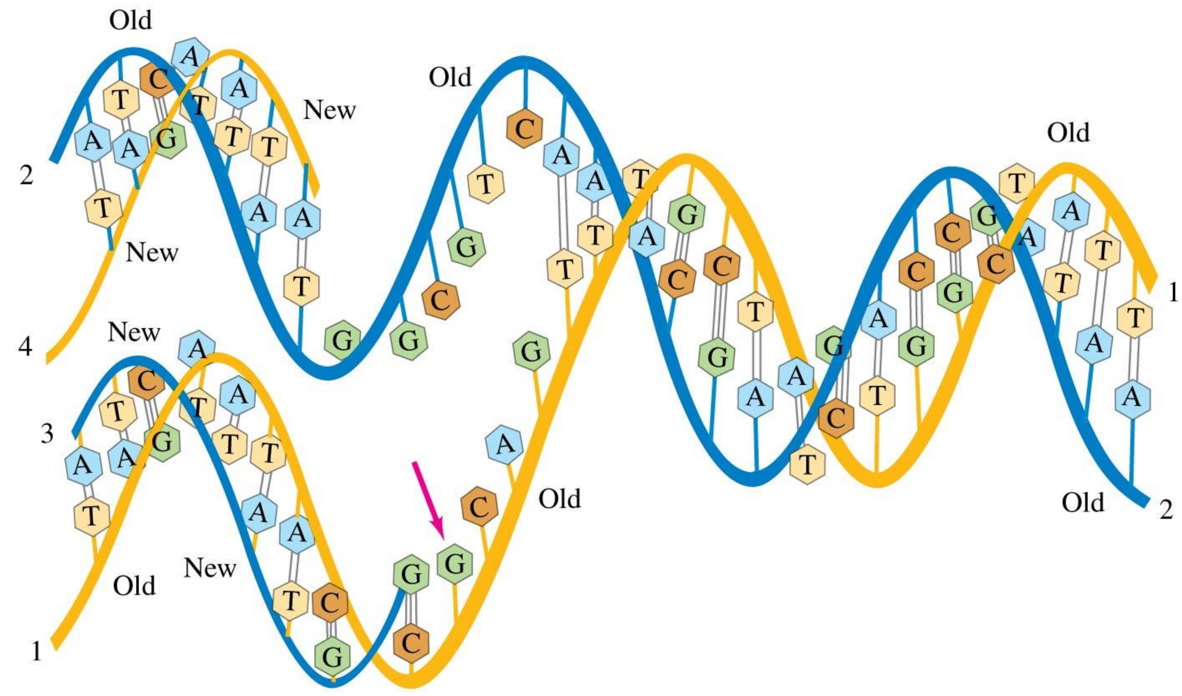
The A-T and G-C nucleotide bases attract each other through electrostatic forces.



(b)

16-10 Electric Forces in Molecular Biology: DNA Structure and Replication

Replication: DNA is in a "soup" of A, C, G, and T in the cell. During random collisions, A and T will be attracted to each other, as will G and C; other combinations will not.

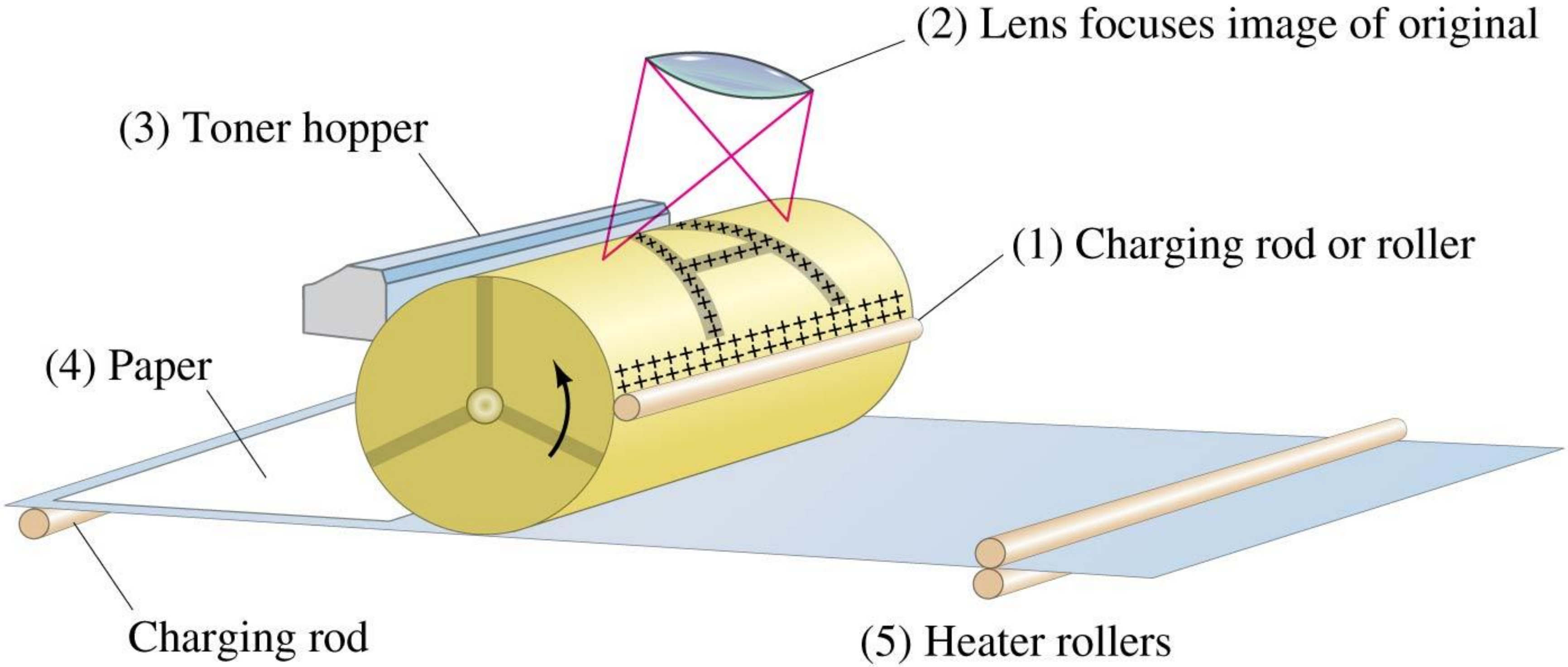


16-11 Photocopy Machines and Computer Printers Use Electrostatics

Photocopy machine:

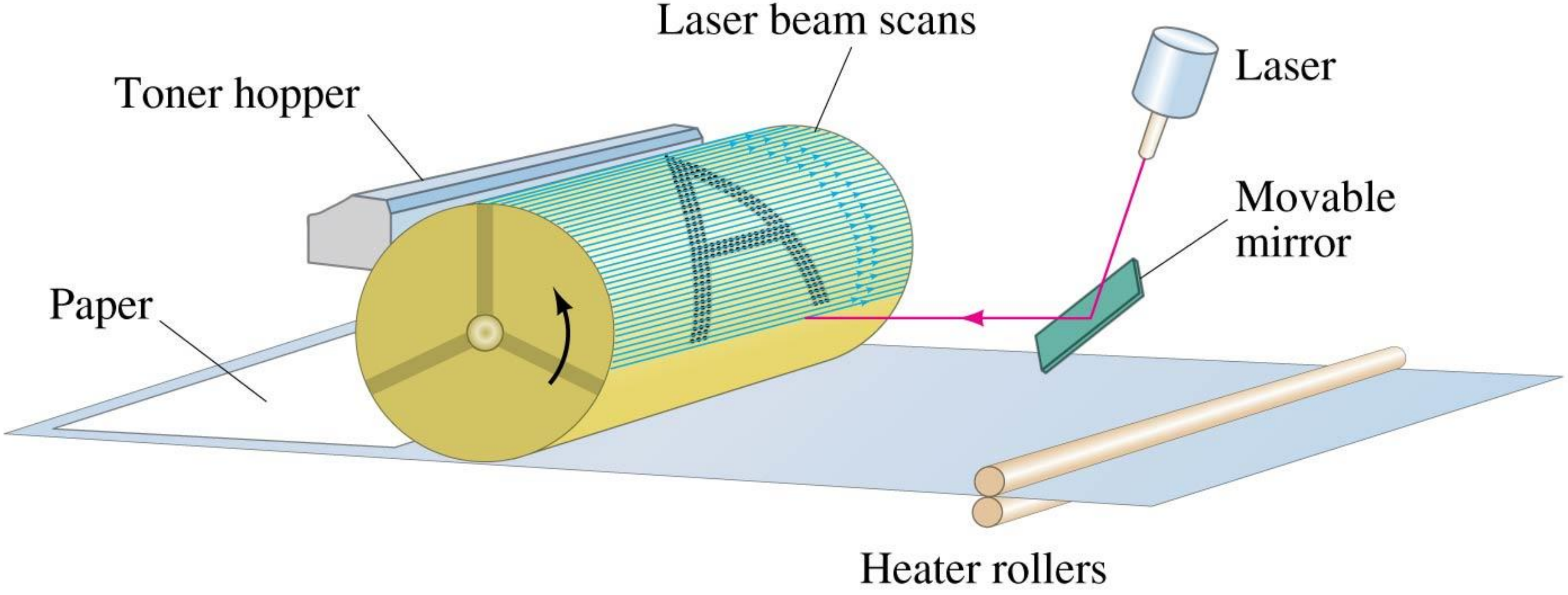
- drum is charged positively
- image is focused on drum
- only black areas stay charged and therefore attract toner particles
- image is transferred to paper and sealed by heat

16-11 Photocopy Machines and Computer Printers Use Electrostatics

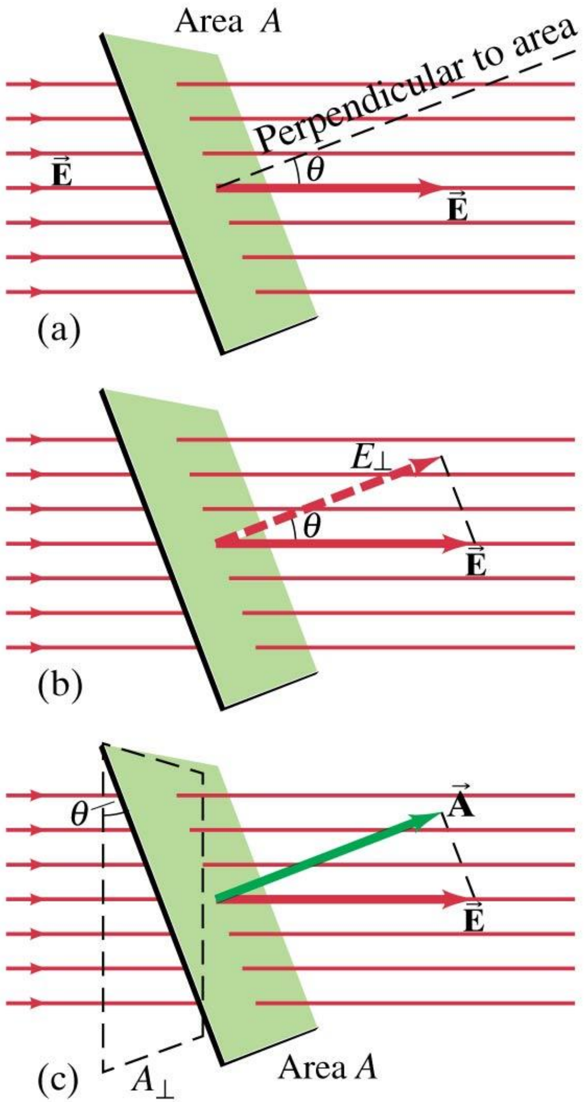


16-11 Photocopy Machines and Computer Printers Use Electrostatics

Laser printer is similar, except a computer controls the laser intensity to form the image on the drum



16-12 Gauss's Law



Electric flux:

$$\Phi_E = E_\perp A = EA_\perp = EA\cos\theta, \quad (16\text{-}7)$$

Electric flux through an area is proportional to the total number of field lines crossing the area.

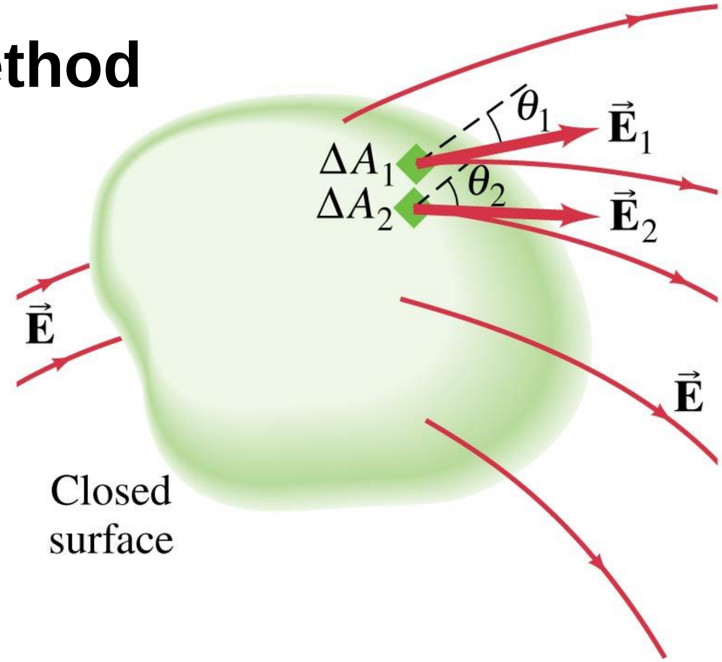
16-12 Gauss's Law

Flux through a closed surface:

$$\Phi_E = E_1\,\Delta A_1 \cos\theta_1 + E_2\,\Delta A_2 \cos\theta_2 + \cdots$$

$$= \sum E\,\Delta A \cos\theta = \sum E_\perp\,\Delta A,$$

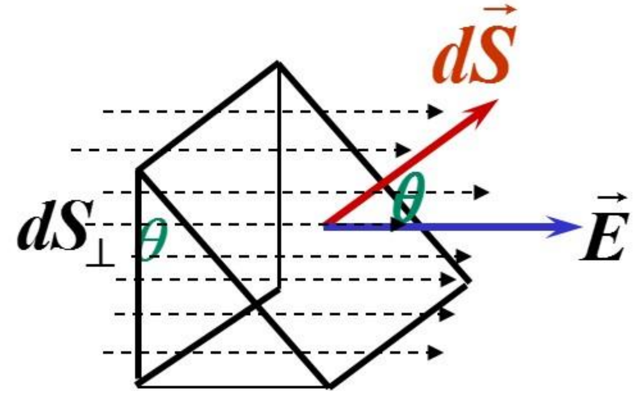
Infinitesimal method



The electric flux crossing the infinitesimal area dS

通过面积元dS 的电通量

$$d\Phi = \vec{E} \cdot d\vec{S}$$

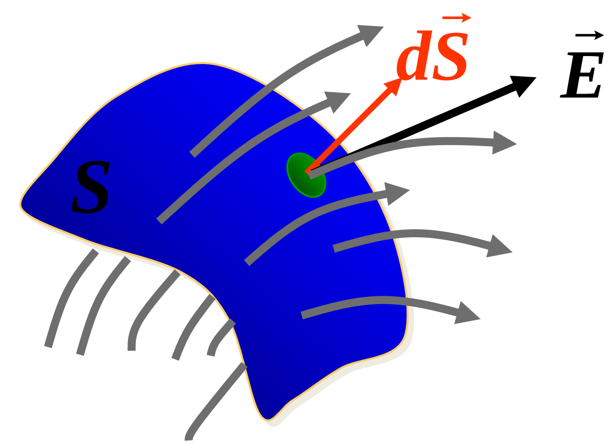


Infinitesimal method

The electric flux crossing a surface S

通过任意曲面 S 的电通量

We divide the surface S into tiny elements, each of infinitesimal area is dS

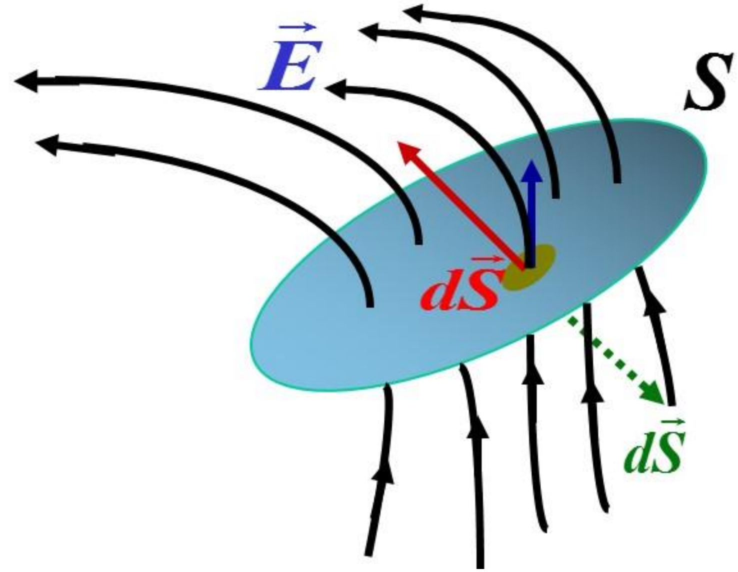


$$d\Phi = \vec{E} \cdot d\vec{S}$$

$$\Phi = \int_S d\Phi = \int_S \vec{E} \cdot d\vec{S}$$

In a nonuniform electric field $\vec{E}$

$$d\Phi = \vec{E} \cdot d\vec{S}$$



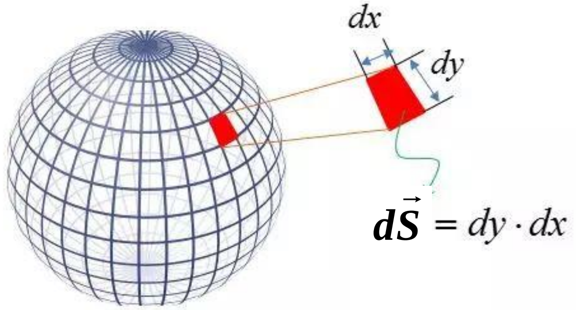
$$\vec{E} \cdot d\vec{S} > 0$$

$$\vec{E} \cdot d\vec{S} < 0$$

The electric flux through a closed surface

通过闭合曲面的电通量

$$\Phi = \oint_S \vec{E} \cdot d\vec{S}$$



The area vector $d\vec{S}$ at any point is perpendicular to the surface and directed outward from the interior.

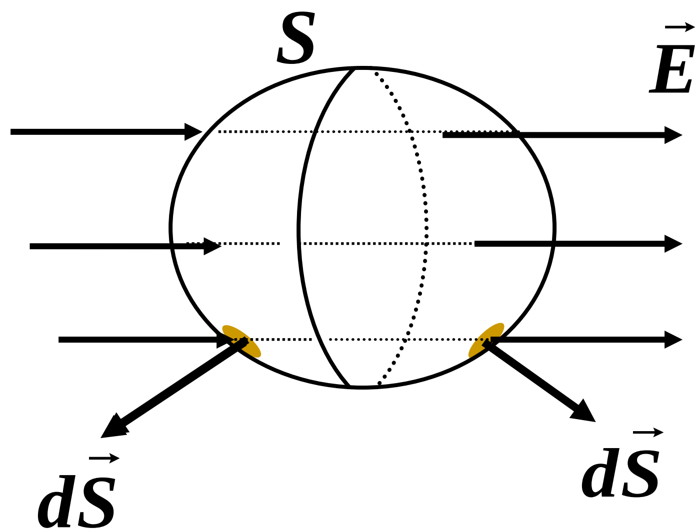
$$\Phi = \oint_S \vec{E} \cdot d\vec{S}$$

Field lines leave the surface

$$\vec{E} \cdot d\vec{S} > 0$$

Field lines enter the surface

$$\vec{E} \cdot d\vec{S} < 0$$



$$\Phi = \oint_S \vec{E} \cdot d\vec{S} = 0$$

在静电场中，通过任意一个闭合曲面的电通量等于该闭合曲面所包围的电荷电量的代数和的 $1/\varepsilon_0$ 倍，与闭合面外的电荷无关。

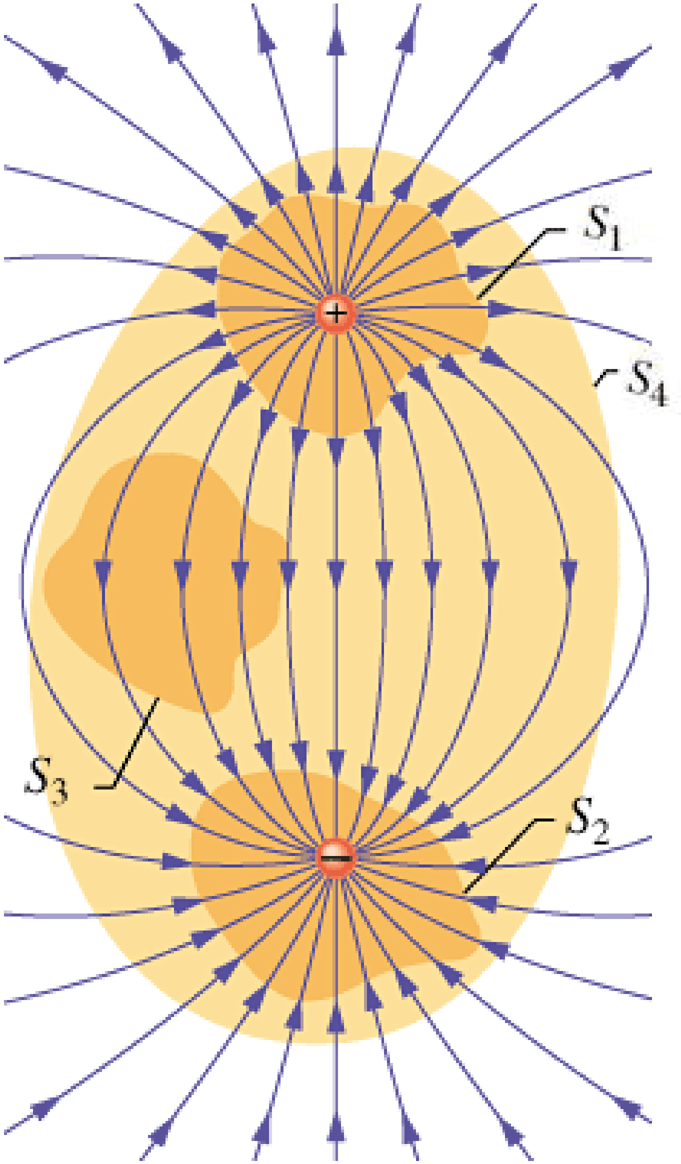
$$\oint_S \vec{E} \cdot d\vec{S} = \frac{q_{net}}{\varepsilon_0}$$

高斯定理

$S \rightarrow$ Gaussian surface　高斯面

The quantity $\vec{E}$ on the left side of the equation is the electric field resulting from *all* charges, both those inside and those outside the Gaussian surface.

The equation hold only when the net charge is located in a vacuum



Surface S_1

The electric field is outward for all points on this surface.

Surface S_2

The electric field is inward for all points on this surface.

Surface S_3

This surface encloses no charge.

Surface S_4

This surface encloses no *net* charge.

16-12 Gauss's Law

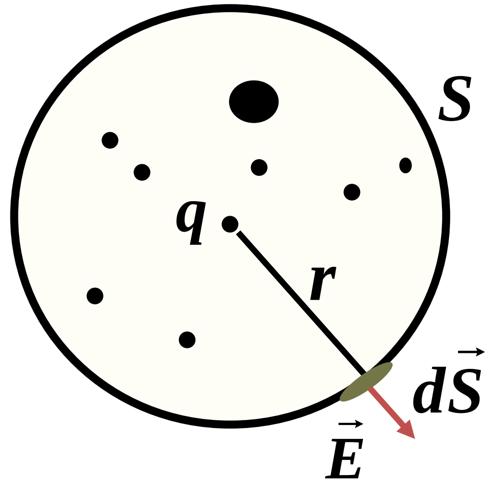
The net number of field lines through the surface is proportional to the charge enclosed, and also to the flux, giving Gauss's law:

$$\sum_{\substack{\text{closed}\\\text{surface}}} E_\perp \, \Delta A = \frac{Q_{\text{encl}}}{\epsilon_0}, \qquad (16\text{-}9)$$

This can be used to find the electric field in situations with a high degree of symmetry.

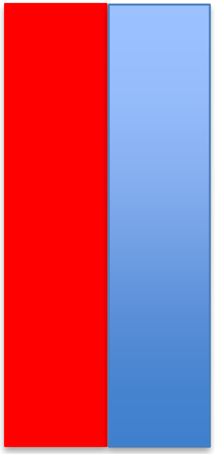
Gauss' law is also a powerful tool for the determination of electric fields in situations where there is a high degree of symmetry.

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{q_{net}}{\varepsilon_0}$$



$$\oint_S E dS = \frac{q_{net}}{\varepsilon_0}$$

$$E\oint_S dS = \frac{q_{net}}{\varepsilon_0} \qquad E = \frac{q_{net}}{\varepsilon_0 \oint_S dS} \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

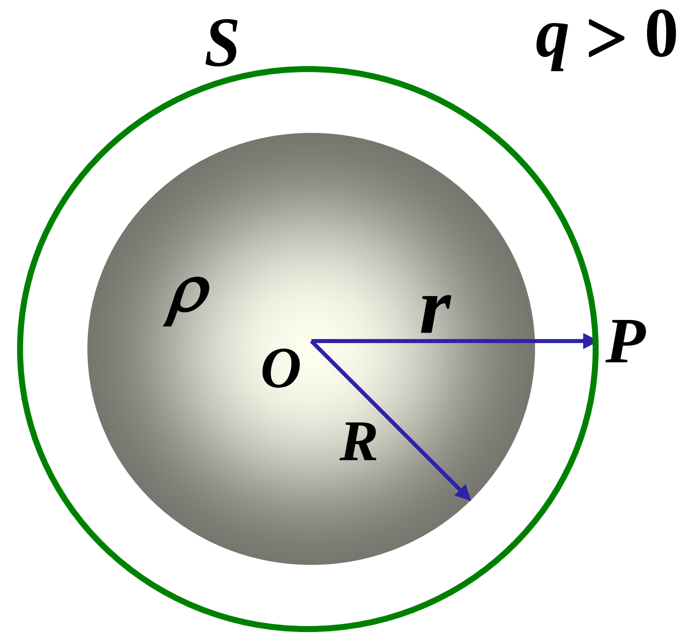


$$\rho = 0, \qquad (x < -L, x > L)$$

$$\rho = -ax, \qquad (-L < x < L)$$

均匀带电球体.

Find the electric field outside and inside a solid, nonconducting sphere of radius R that contains a total charge q uniformly distributed throughout its volume.



Spherical symmetry　球对称性

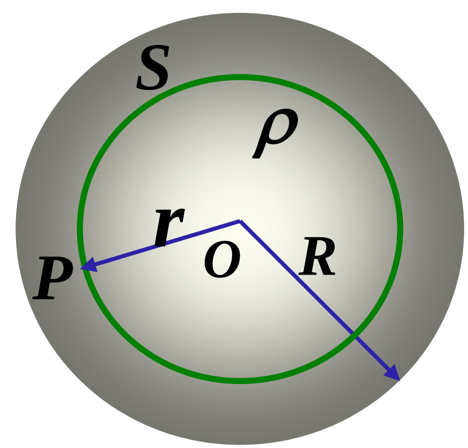
1） $r > R$　$\oint_S \vec{E} \cdot d\vec{S} = \dfrac{q}{\varepsilon_o}$

$$\oint_S \vec{E} \cdot d\vec{S} = \oint_S E\,dS = E \oint_S dS = \frac{q}{\varepsilon_o}$$

$4\pi r^2$

$$E = \frac{q}{4\pi\varepsilon_o r^2}$$

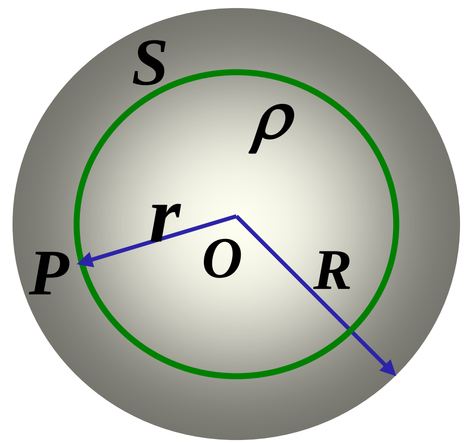
$$\vec{E} = \frac{q}{4\pi\varepsilon_o r^2}\hat{r} \quad (r \geq R)$$



2） $r < R$ $\oint_S \vec{E} \cdot d\vec{S} = \frac{\sum q_i}{\varepsilon_o}$

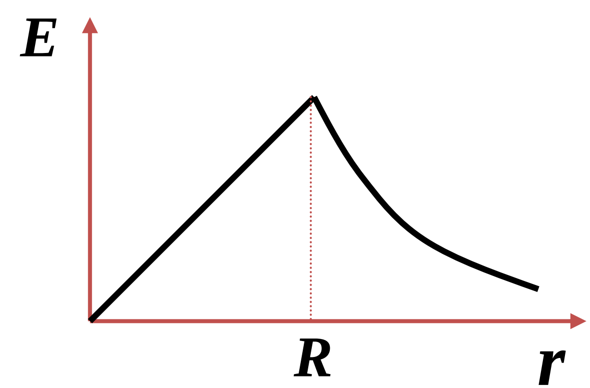
$$\rho = \frac{q}{\frac{4}{3}\pi R^3}$$

$$\oint_S E dS = E \oint_S dS = \frac{\rho \cdot \frac{4}{3}\pi r^3}{\varepsilon_o}$$

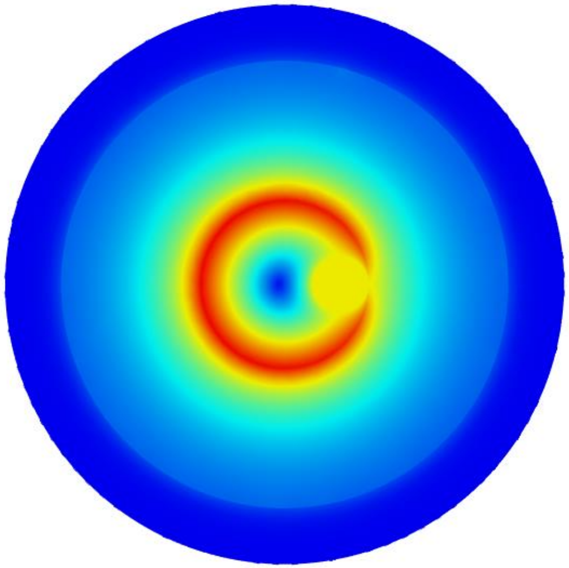


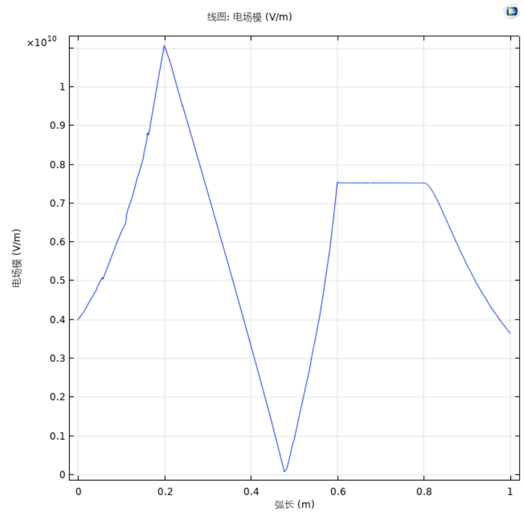
$$E \cdot 4\pi r^2 = \frac{qr^3}{\varepsilon_o R^3}$$

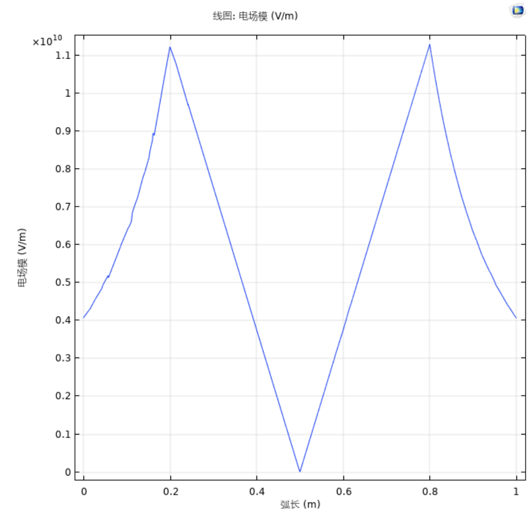
$$E = \frac{qr}{4\pi\varepsilon_o R^3}$$

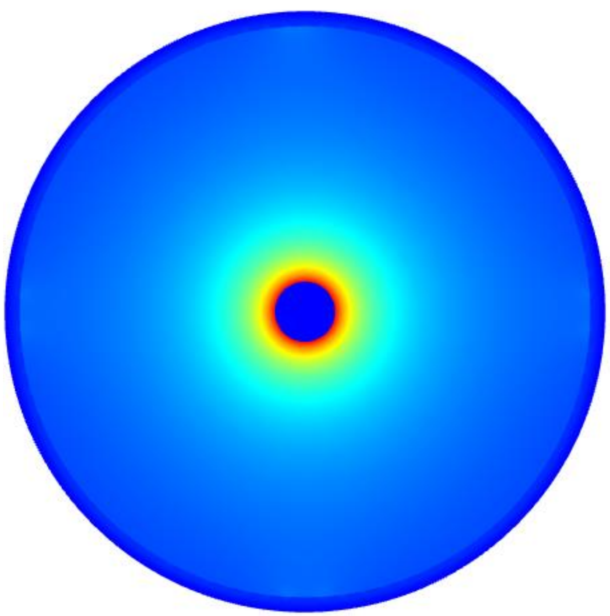


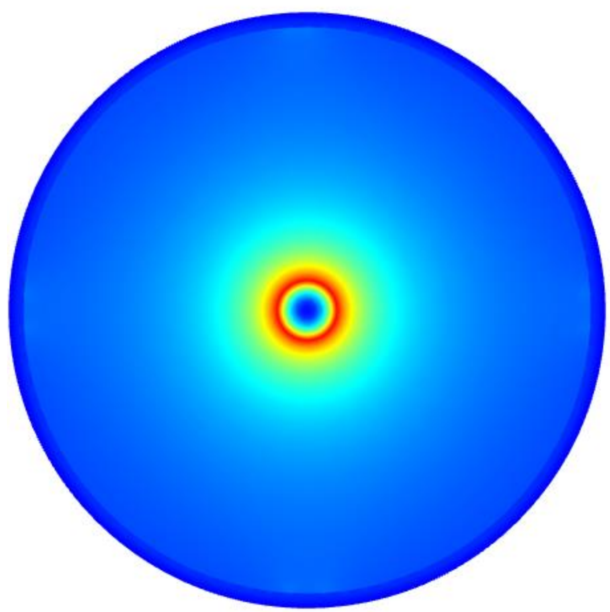
$$\vec{E} = \frac{qr}{4\pi\varepsilon_o R^3}\hat{r} \qquad (r < R)$$

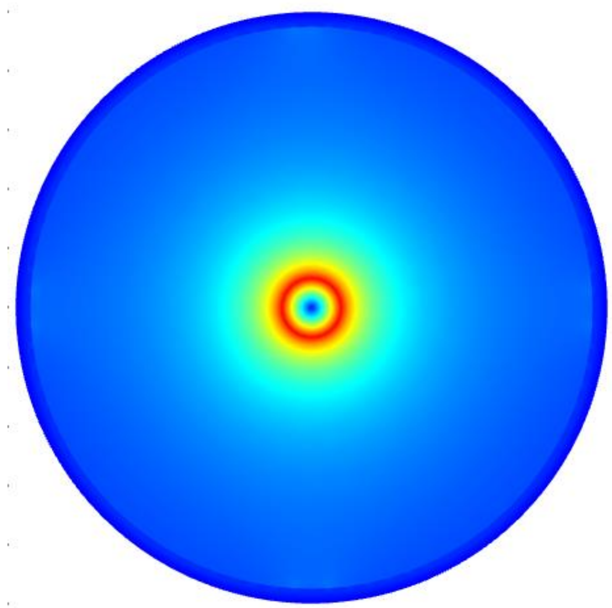



线图: 电场模 (V/m)
×10^10
电场模 (V/m)
弧长 (m)


线图: 电场模 (V/m)
×10^10
电场模 (V/m)
弧长 (m)



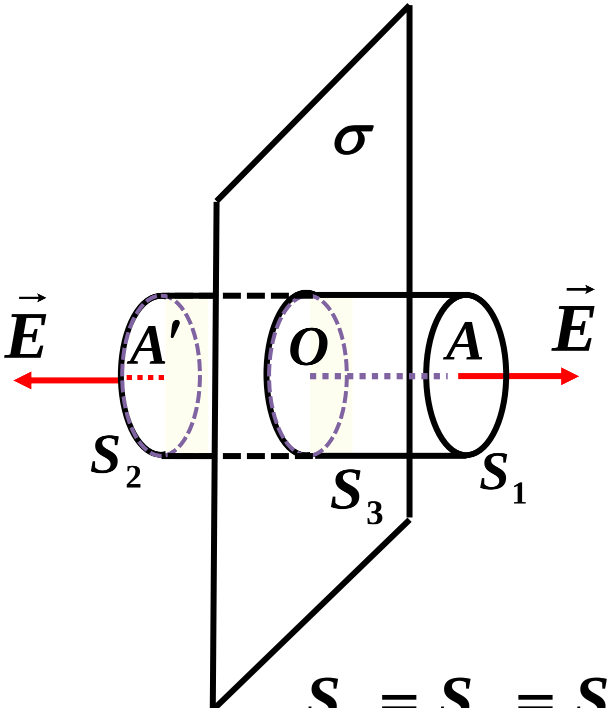




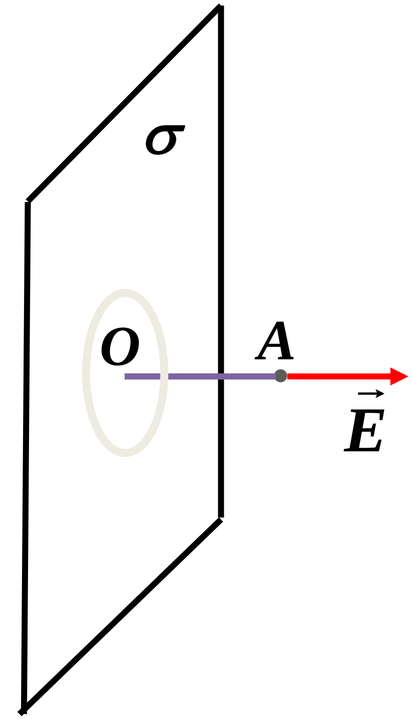
无限大均匀带电平面.

Find the electric field outside an infinite, nonconducting plane of charge with uniform charge density $\boldsymbol{\sigma}$.

$$\oint_S \vec{E}\cdot d\vec{S} = \frac{\sigma S}{\varepsilon_0}$$



$$S_1 = S_2 = S$$



$$ES + ES = \frac{\sigma S}{\varepsilon_0}$$

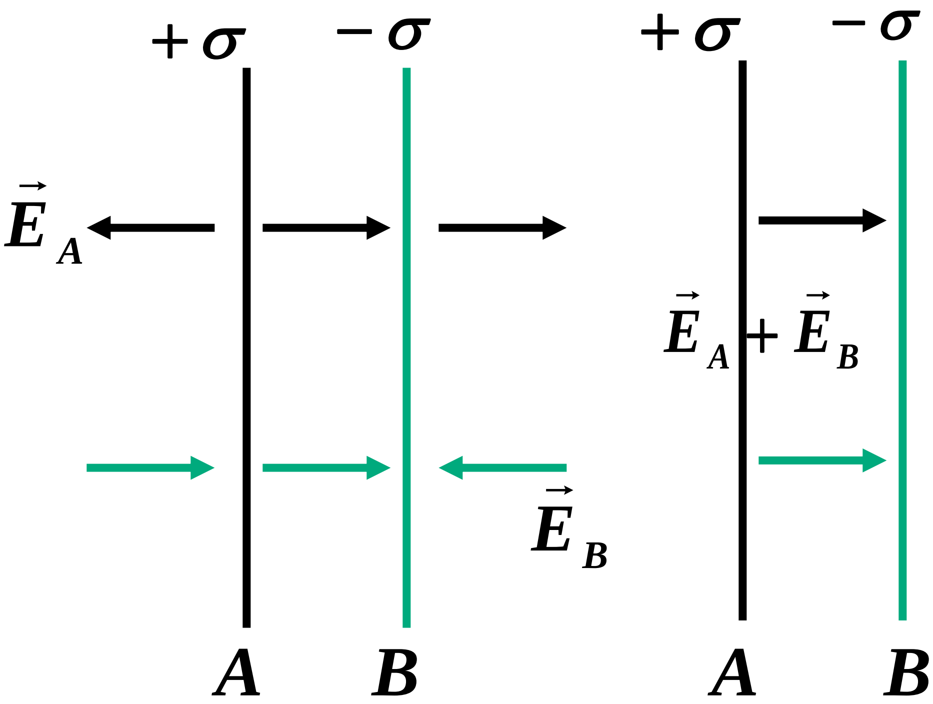
$$E = \frac{\sigma}{2\varepsilon_0}$$

$$\int_{S_1} \vec{E}\cdot d\vec{S} + \int_{S_2} \vec{E}\cdot d\vec{S} + \underbrace{\int_{S_3} \vec{E}\cdot d\vec{S}}_{0} = \frac{\sigma S}{\varepsilon_0}$$

电场是由两块相互平行的无限大均匀带电平面产生的，两平面的面电荷密度分别为 $+\sigma$ 和 $-\sigma$ 。

$$E_A = \frac{\sigma}{2\varepsilon_0}$$

$$E_B = \frac{\sigma}{2\varepsilon_0}$$



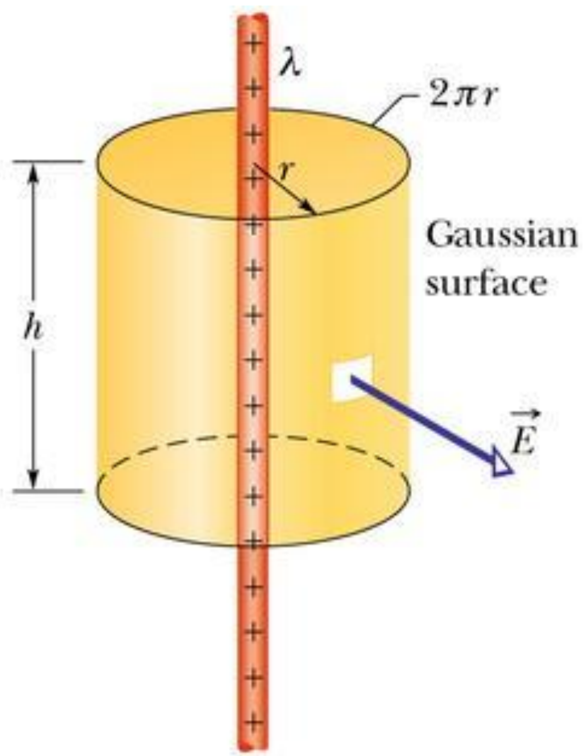
The electric field at any point between the plates

$$E = E_A + E_B = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

The electric field to the left and right of the plates

$$E = E_A - E_B = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

An infinitely long cylindrical plastic rod with a uniform positive linear charge density λ. Let us find an expression for the magnitude of the electric field $\vec{E}$ at a distance r from the axis of the rod.



$$\oint_S \vec{E} \cdot d\vec{S} = \frac{q_{net}}{\varepsilon_0}$$

$$\int_{\text{上面}} \vec{E} \cdot d\vec{S} + \int_{\text{下面}} \vec{E} \cdot d\vec{S} + \int_{\text{侧面}} \vec{E} \cdot d\vec{S} = \frac{\lambda h}{\varepsilon_0}$$

$$\int_{\text{侧面}} E dS = \frac{\lambda h}{\varepsilon_0} \qquad E(2\pi r h) = \frac{\lambda h}{\varepsilon_0}$$

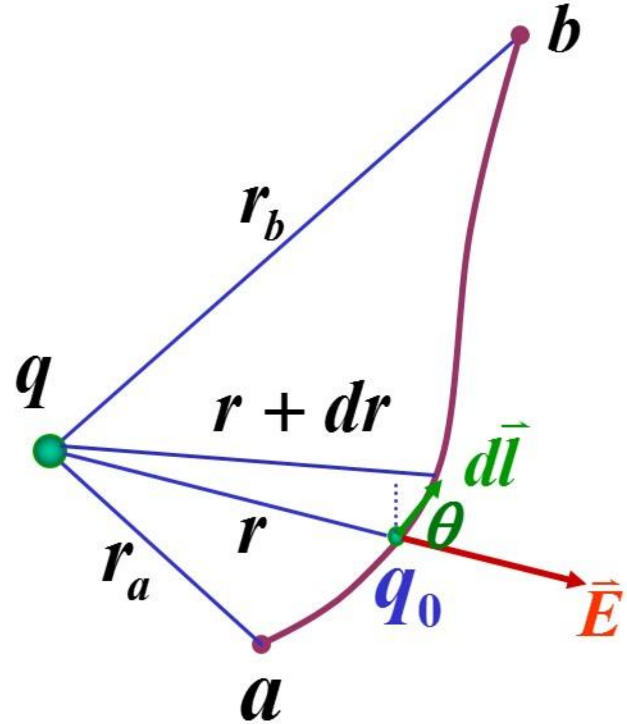
柱对称性

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

§.5 静电场的环路定理　电势

Ampere circuital theorem

1. 点电荷的静电场



$$dA = q_o \vec{E} \cdot d\vec{l} = q_o E \cos\theta \, dl$$

$$E = \frac{q}{4\pi\varepsilon_o r^2} \qquad \cos\theta \, dl = dr$$

$$dA = \frac{q_o q}{4\pi\varepsilon_o r^2} dr$$

$$A = \int_a^b dA = \int_{r_a}^{r_b} \frac{q_0 q}{4\pi\varepsilon_0 r^2} dr = \frac{q_0 q}{4\pi\varepsilon_0} \left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

$$A=\oint_L q_0\vec{E}\cdot d\vec{l}=0$$

2. 点电荷系的静电场

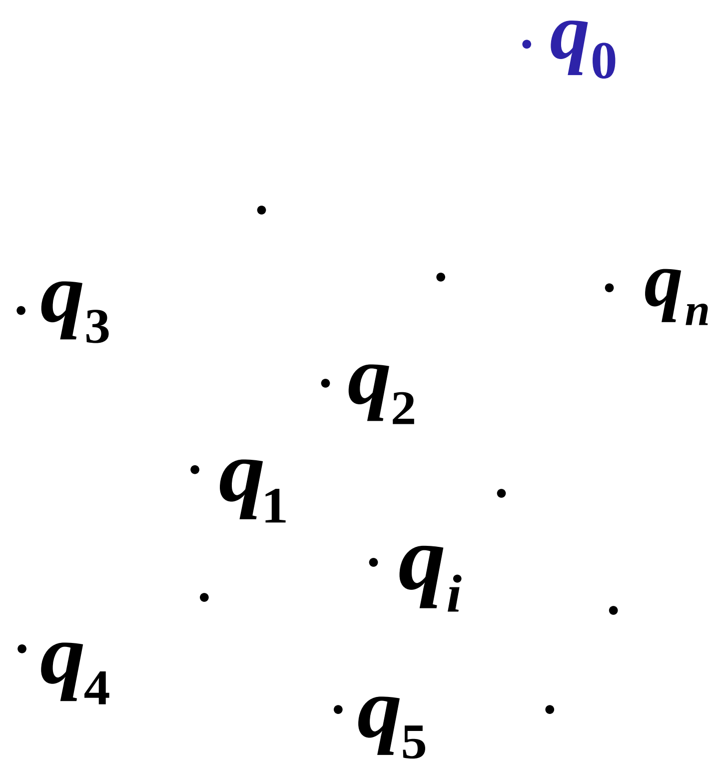
There are *n* point charges

$$A=\oint_L q_0\vec{E}\cdot d\vec{l} \qquad \vec{E}=\sum_{i=1}^{n}\vec{E}_i$$

$$=\oint_L q_0\sum_{i=1}^{n}\vec{E}_i\cdot d\vec{l}$$

$$=\sum_{i=1}^{n}\oint_L q_0\vec{E}_i\cdot d\vec{l}=0$$

$$\oint_L q_0\vec{E}\cdot d\vec{l}=0$$



3. 连续分布电荷的静电场

连续分布电荷是由大量电荷元 ***dq***（可视为点电荷）组成的

$$\oint_L q_0 \vec{E} \cdot d\vec{l} = 0$$

Conclusion： When the test charge moves in the electrostatic field, the work done by the electric field force depends only on the position of the starting and ending points of the test charge, and has nothing to do with the path

静电场力是保守力 Conservative force

4. 静电场的环路定理

$$\oint_L q_0 \vec{E} \cdot d\vec{l} = 0$$

The characteristic of conservative force in doing work

$$q_0 \neq 0 \quad \Rightarrow \quad \oint_L \vec{E} \cdot d\vec{l} = 0$$

静电场的环路定理

Irrotational field 无旋场

Conservative field 静电场是保守场

在静电场中，电场强度沿任意闭合路径的环流为零。

5. 电势能

可以引进电势能的概念

Electrostatic field force is conservative force → Electric potential energy

The work done by conservative force equals to the decrease in electric potential energy

$$A=\int_{\vec{r}_a}^{\vec{r}_b} q_0\vec{E}\cdot d\vec{l} = -[\mathrm{U}(\vec{r}_b)-\mathrm{U}(\vec{r}_a)]$$

规定b点的电势能为零 $\mathrm{U}(\vec{r}_b)=0$

a点的电势能 $\mathrm{U}(\vec{r}_a)=\int_{\vec{r}_a}^{\vec{r}_b} q_0\vec{E}\cdot d\vec{l}$

$$U(\vec{r}_a) = \int_{\vec{r}_a}^{\vec{r}_b} q_0 \vec{E} \cdot d\vec{l}$$

试验电荷 $\boldsymbol{q_0}$ 在空间某处的电势能在数值上就等于将 $\boldsymbol{q_0}$ 从该处移至电势能的零点处电场力所作的功

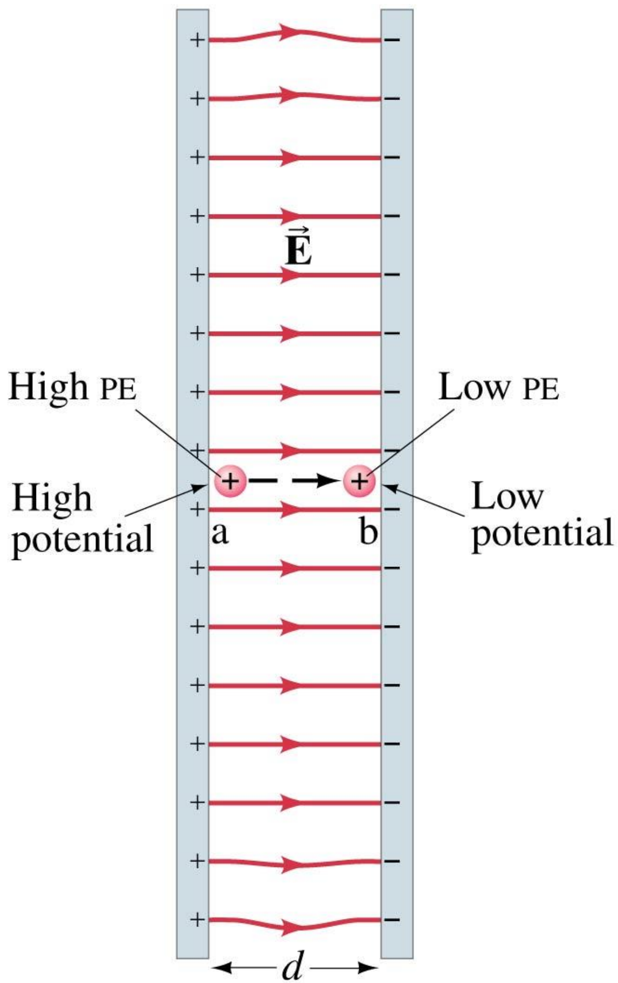
电势能的零点可以任意选取

Usually, when the source charge **is a finite charged body**, the zero potential energy point is selected at infinity

$[\boldsymbol{V = 0 \;\; at\, r = \infty}]$

However, when the source charge **is a infinite charged body**, the zero potential energy point is selected at some special point.

17.1 Electric Potential Energy and Potential Difference



The electrostatic force is conservative—potential energy can be defined

Change in electric potential energy is negative of work done by electric force:

$$\mathrm{PE_b} - \mathrm{PE_a} = -qEd \quad (17\text{-}1)$$

17.1 Electric Potential Energy and Potential Difference

Electric potential is defined as potential energy per unit charge; analogous to definition of electric field as force per unit charge:

$$V_{\mathrm{a}} = \frac{\mathrm{PE_a}}{q}. \quad (17\text{-}2a)$$

Unit of electric potential: the volt (V).

$$1\ \mathrm{V} = 1\ \mathrm{J/C}.$$

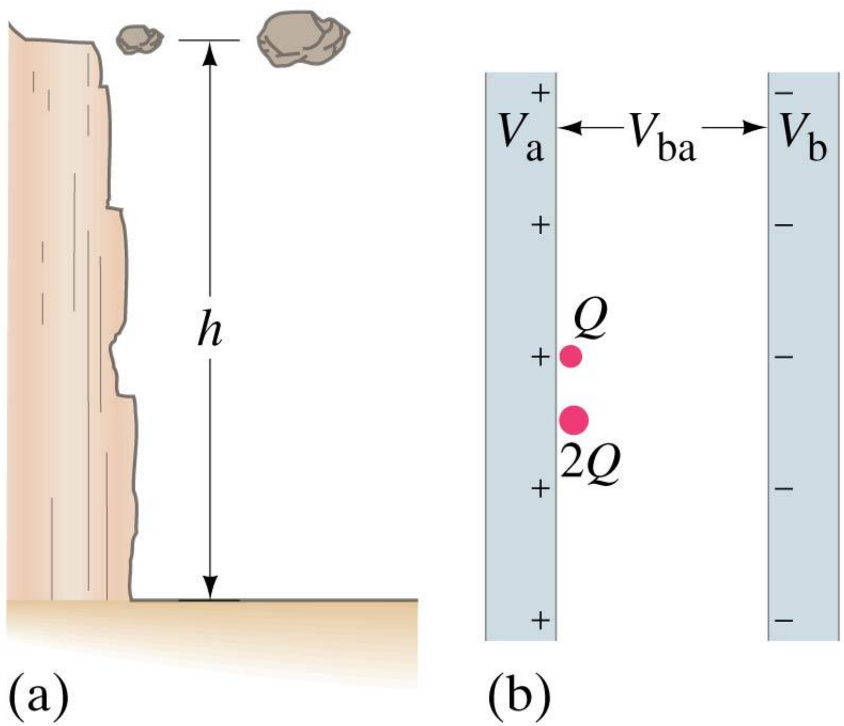
17.1 Electric Potential Energy and Potential Difference

Only changes in potential can be measured, allowing free assignment of $V = 0$.

$$V_{\mathrm{ba}} = V_{\mathrm{b}} - V_{\mathrm{a}} = \frac{\mathrm{PE_b} - \mathrm{PE_a}}{q} = -\frac{W_{\mathrm{ba}}}{q}. \quad \text{(17-2b)}$$

17.1 Electric Potential Energy and Potential Difference

Analogy between gravitational and electrical potential energy. Just as the more massive rock has more potential energy, so does the larger charge:



17.2 Relation between Electric Potential and Electric Field

Work is charge multiplied by potential difference:

$$W = -q(V_{\mathrm{b}} - V_{\mathrm{a}}) = -qV_{\mathrm{ba}}.$$

Work is also force multiplied by distance:

$$W = Fd = qEd,$$

17.2 Relation between Electric Potential and Electric Field

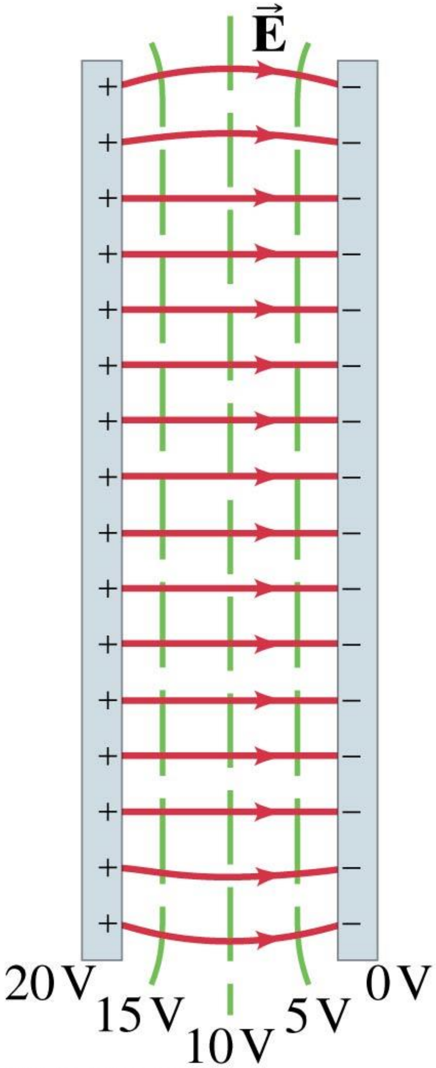
Solving for the field,

$$E = -\frac{V_{\mathrm{ba}}}{d}. \qquad [\text{uniform } \vec{\mathbf{E}}] \qquad (17\text{-}4b)$$

In general, the electric field in a given direction at any point in space is equal to the rate at which the electric potential decreases over distance in that direction.

Gradient

17.3 Equipotential Lines and Surfaces



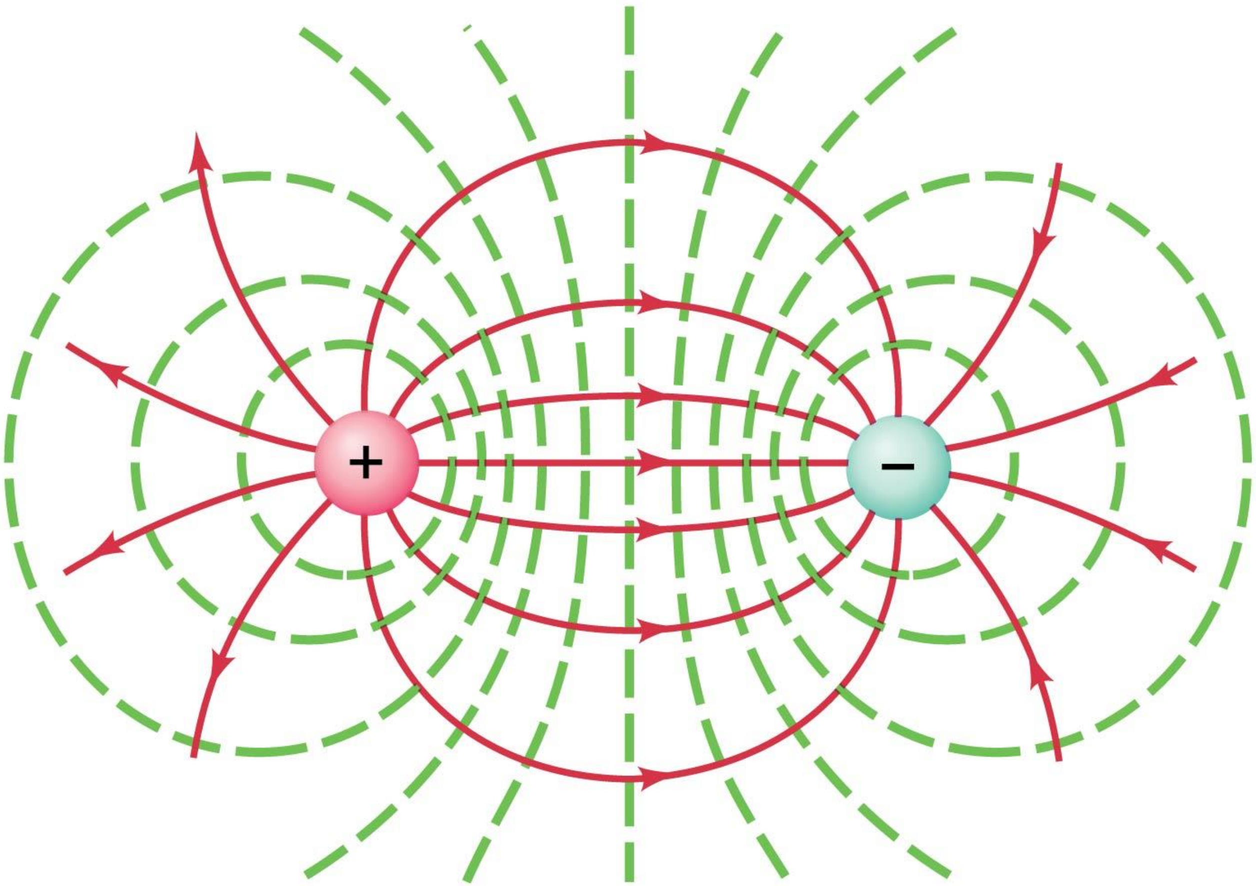
An equipotential is a line or surface over which the potential is constant.

Electric field lines are perpendicular to equipotentials.

The surface of a conductor is an equipotential.

17.3 Equipotential Lines and Surfaces

Equipotential lines of an electric dipole:



17.4 The Electron Volt, a Unit of Energy

One electron volt (eV) is the energy gained by an electron moving through a potential difference of one volt.

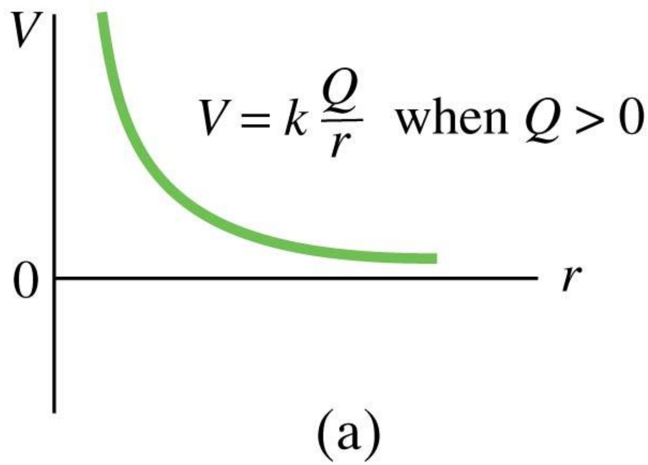
$$1 \text{ eV} = 1.6022 \times 10^{-19} \approx 1.60 \times 10^{-19} \text{ J.}$$

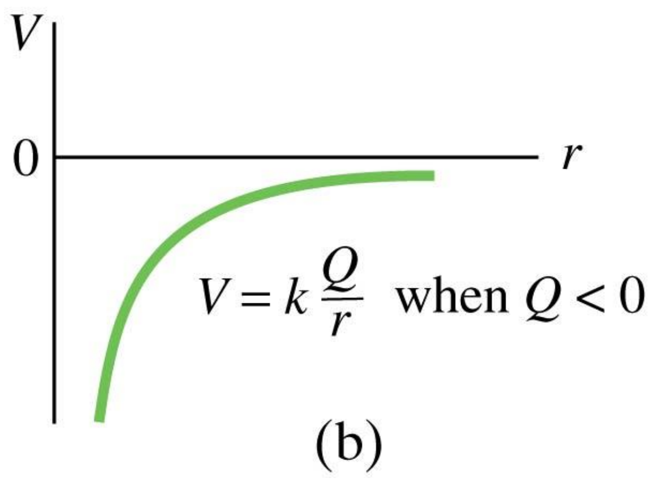
17.5 Electric Potential Due to Point Charges

The electric potential due to a point charge can be derived using calculus.

$$
\begin{aligned}
V &= k\frac{Q}{r} \\
&= \frac{1}{4\pi\epsilon_0}\frac{Q}{r},
\end{aligned}
\qquad (17\text{-}5)
$$

17.5 Electric Potential Due to Point Charges





These plots show the potential due to (a) positive and (b) negative charge.

17.5 Electric Potential Due to Point Charges

Using potentials instead of fields can make solving problems much easier—potential is a scalar quantity, whereas the field is a vector.

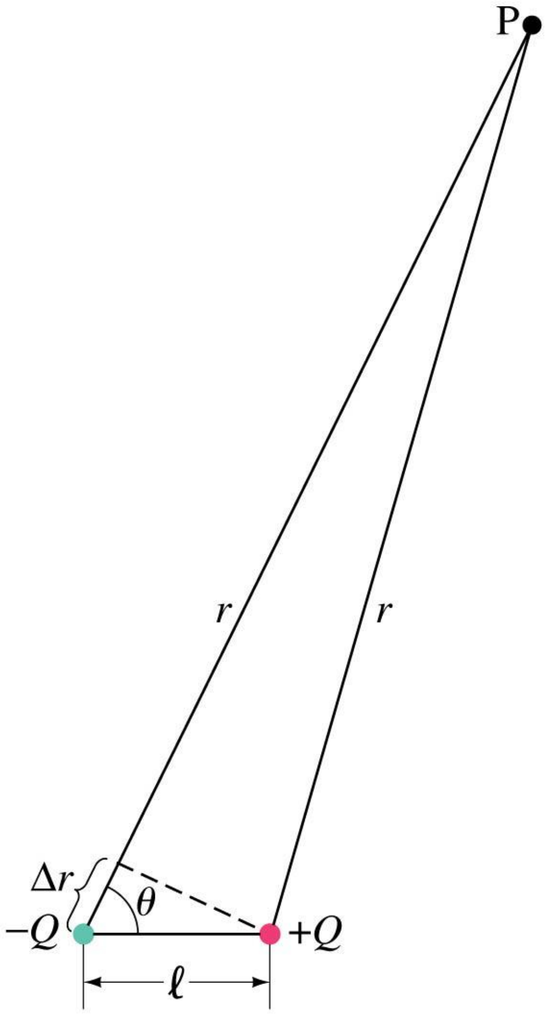
17.6 Potential Due to Electric Dipole; Dipole Moment

The potential due to an electric dipole is just the sum of the potentials due to each charge, and can be calculated exactly.

17.6 Potential Due to Electric Dipole; Dipole Moment

Approximation for potential far from dipole:

$$V \approx \frac{kQ\ell\cos\theta}{r^2}. \quad (17\text{-}6a)$$



17.6 Potential Due to Electric Dipole; Dipole Moment

Or, defining the dipole moment $p = Ql$,

$$V \approx \frac{kp\cos\theta}{r^2}. \quad (17\text{-}6b)$$