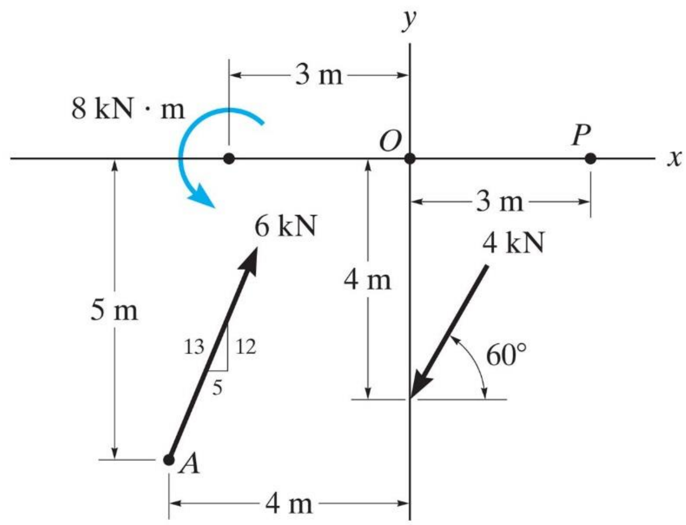
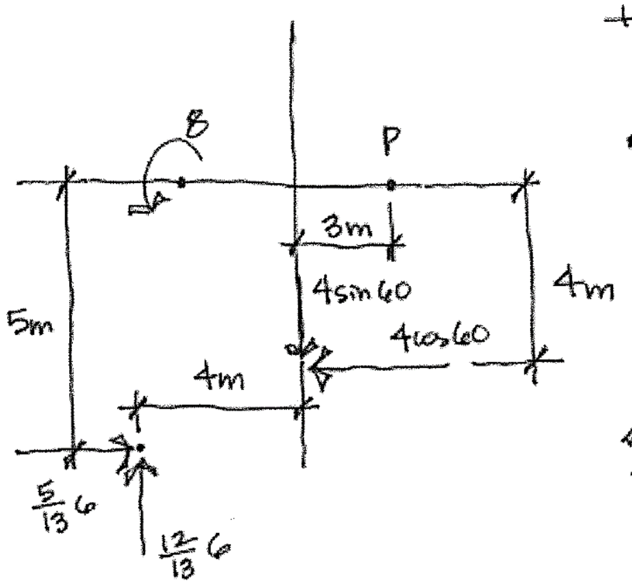


Replace the system by an equivalent force and moment at point P.

FBD:



$\xrightarrow{+} F_{RX} = \frac{5}{13}6 - 4\cos 60$

$= 0.308$

$+\uparrow F_{RY} = \frac{12}{13}6 - 4\sin 60$

$= 2.07$

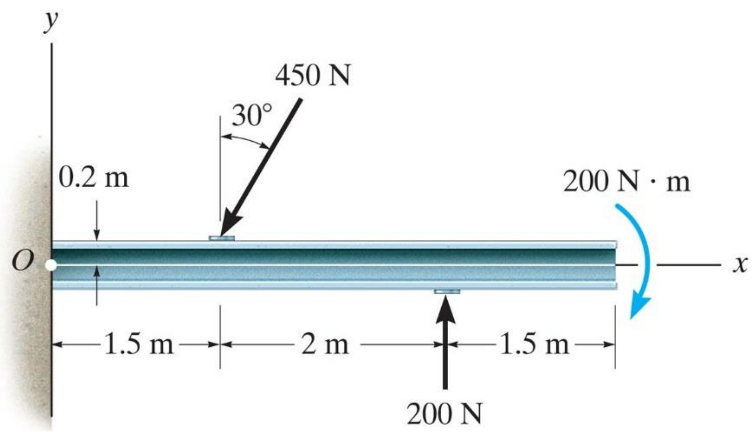
$$\boxed{F_R = \{0.308\,\hat{i} + 2.07\,\hat{j}\}\ \text{kN}}$$

$\circlearrowleft \Sigma M_P = 8 + \frac{5}{13}(6)(5) - \frac{12}{13}(6)(\;)$

$+ 4\sin 60(3) - 4\cos 60(4)$

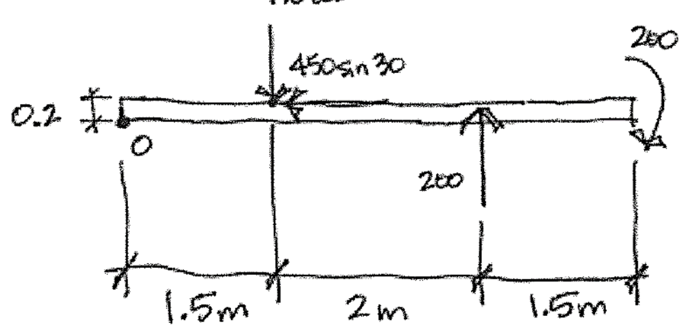
$= -16.83$

$$\boxed{M_P = 16.83\ \text{kN-m} \circlearrowright}$$



Replace the loading with an equivalent force and specify where this force intersects the centerline of the beam, measured from point O.

FBD:



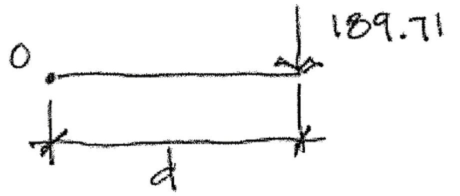
$\xrightarrow{+} F_{Rx} = -450\sin 30 = -225$

$+\uparrow F_{Ry} = -450\cos 30 + 200 = -189.71$

$\boxed{F_R = \{-225i - 189.71j\}\,N}$

$+\circlearrowleft \Sigma M_O = -450\cos 30(1.5) + 200(3.5) - 200 + 450\sin 30(0.2)$

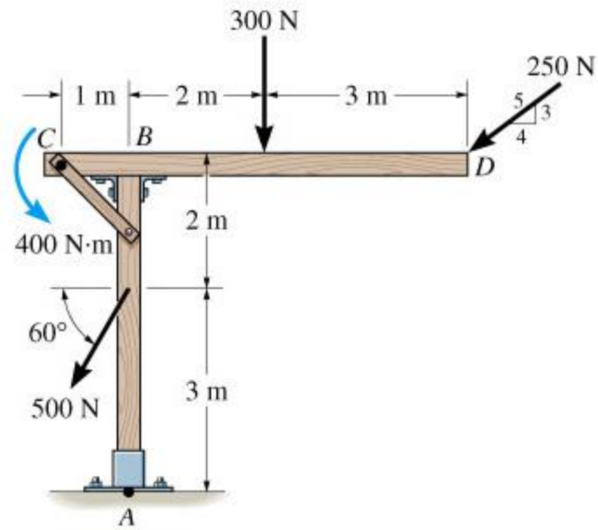
$= -39.57$



$M = Fd$

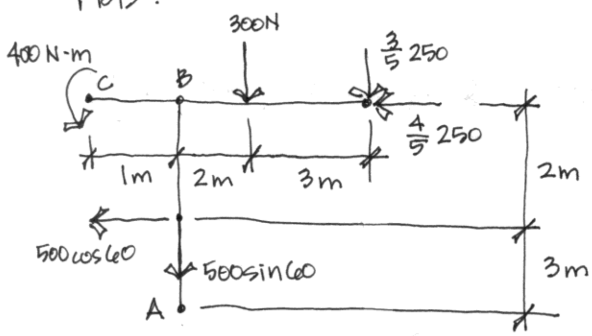
$-39.57 = -189.71\,d$

$\boxed{d = 0.21\,m}$



Replace the loading on the frame with a resultant force and specify where its line of action intersects member CD, measured from C.

FBD:



$F_{Rx} = -500\cos 60 - \frac{4}{5}(250)$

$= -450$ OR $\boxed{450\text{ N} \leftarrow}$

$F_{Ry} = -500\sin 60 - 300 - \frac{3}{5}(250)$

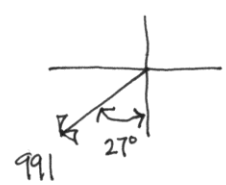
$= -883.01$ OR $\boxed{883.01\text{ N} \downarrow}$

$$\boxed{F_R = -450\,i - 883.01\,j \text{ N}}$$

$F_R = \sqrt{(-450)^2 + (-883.01)^2} = 991.1$ N

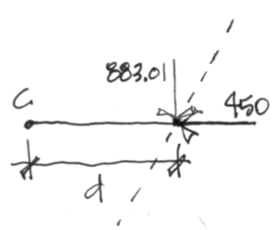
$\circlearrowleft \Sigma M_C = 400 - 500\cos 60(2) - 300(3)$
$- \frac{3}{5}(250)(6) - 500\sin 60(1)$

$M_C = -2333 = 2333$ N-m $\circlearrowright$



$\theta = \tan^{-1}\left(\frac{450}{883.01}\right)$

$\theta = 27°$

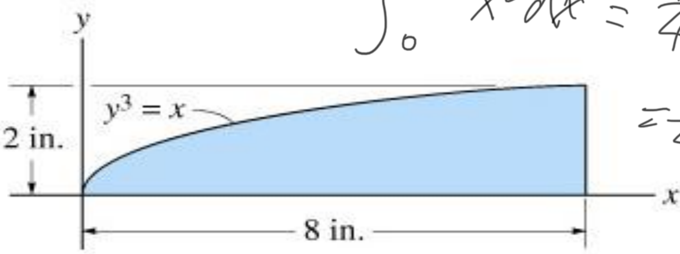


$M = Fd$

$-2333 = -883.01d$

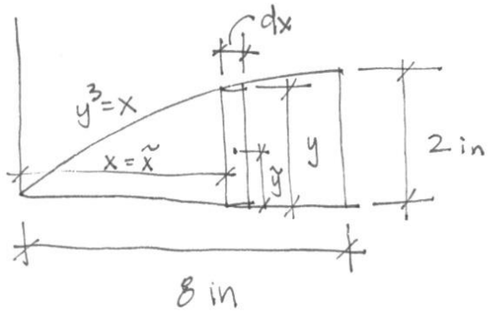
$$\boxed{d = 2.64\text{m}}$$

$$\boxed{F_R = 991\text{ N @ } 117° \text{ CW FROM POS X}}$$



$$\int_0^8 x^{1/3}\,dx = \frac{3}{4} x^{4/3} \Big|_0^8$$
$$= \frac{3}{4} \cdot 8^{4/3}$$
$$= 12$$

Find x bar & y bar of the shaded area.



$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA}$$

$\tilde{x} = x$

$\tilde{y} = \frac{1}{2} y$

$dA = y\,dx$

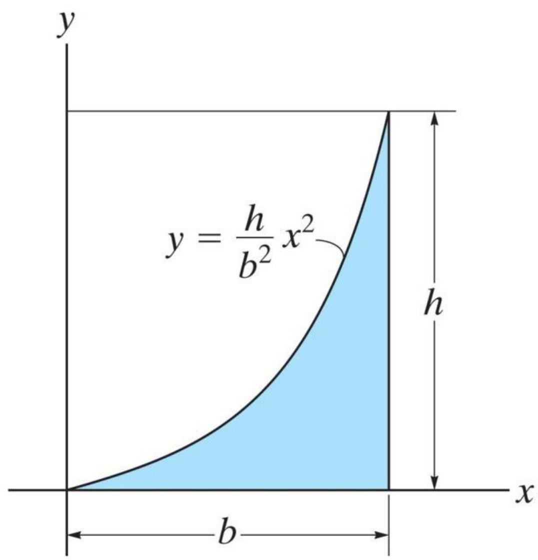
$y^3 = x \Rightarrow y = x^{1/3}$

$$\bar{x} = \frac{\int_0^8 x\,(x^{1/3})\,dx}{\int_0^8 x^{1/3}\,dx} = \frac{\int_0^8 x^{4/3}\,dx}{\int_0^8 x^{1/3}\,dx}$$

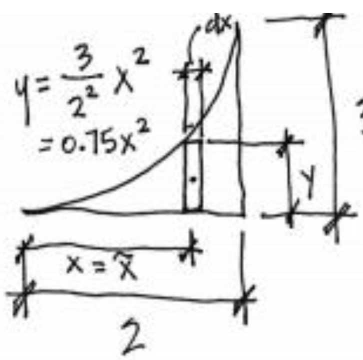
$$= \frac{\frac{3x^{7/3}}{7}\Big|_0^8}{\frac{3x^{4/3}}{4}\Big|_0^8} = \frac{54.86}{12} = \boxed{4.57\text{ in}}$$

$$\bar{y} = \frac{\int_0^8 \frac{1}{2}(x^{1/3})(x^{1/3})\,dx}{\int_0^8 x^{1/3}\,dx} = \frac{\int_0^8 \frac{1}{2} x^{2/3}\,dx}{\int_0^8 x^{1/3}\,dx}$$

$$= \frac{\frac{1}{2}\left(\frac{3x^{5/3}}{5}\right)\Big|_0^8}{\frac{3x^{4/3}}{4}\Big|_0^8} = \frac{9.6}{12} = \boxed{0.8\text{ in}}$$



Calculate x bar of the shaded area. Take b=2 inches and h=3 inches.



$$x = \tilde{x}$$

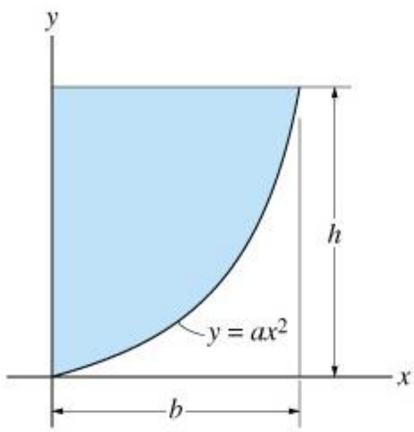
$$dA = y\,dx$$

$$= 0.75x^2\,dx$$

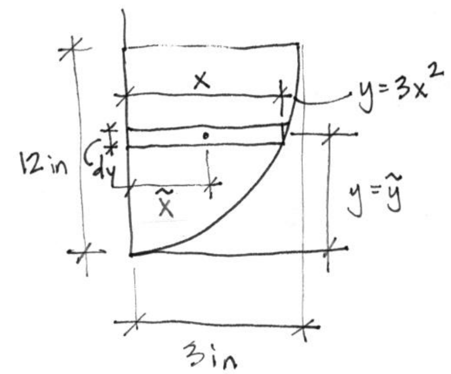
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA}$$

$$\bar{x} = \frac{\int_0^2 x(0.75x^2)dx}{\int_0^2 (0.75x^2)dx} = \frac{\int_0^2 0.75x^3dx}{\int_0^2 0.75x^2dx}$$

$$= \frac{\left.\frac{0.75x^4}{4}\right|_0^2}{\left.\frac{0.75x^3}{3}\right|_0^2} = \frac{3}{2} = \boxed{1.5 = \bar{x}}$$



Find x bar of the shaded area. Take a=3 mm, b=2 mm, and h=12 mm.



$x = \left(\frac{y}{3}\right)^{1/2}$

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA}$$

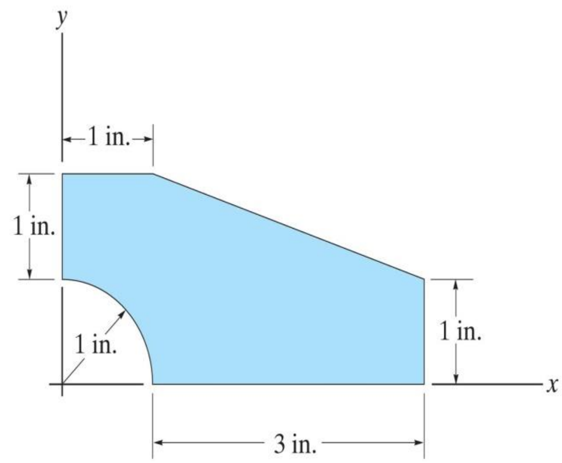
$$\tilde{x} = \frac{x}{2} = \frac{1}{2}\left(\frac{y}{3}\right)^{1/2} = 0.289\, y^{1/2}$$

$$dA = x\, dy = \left(\frac{y}{3}\right)^{1/2} = 0.577 y^{1/2} dy$$

$$\bar{x} = \frac{\int_0^{12} 0.289\, y^{1/2}\left(0.577\, y^{1/2}\right) dy}{\int_0^{12} 0.577\, y^{1/2}\, dy}$$

$$\bar{x} = \frac{\int_0^{12} 0.167\, y\, dy}{\int_0^{12} 0.577\, y^{1/2}\, dy} = \frac{0.167\, \frac{y^2}{2}\Big|_0^{12}}{\frac{0.577\, y^{1\frac{1}{2}}}{1.5}\Big|_0^{12}}$$

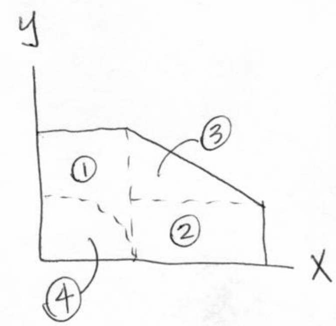
$$\bar{x} = \frac{12.024}{15.99} = \boxed{0.752}$$



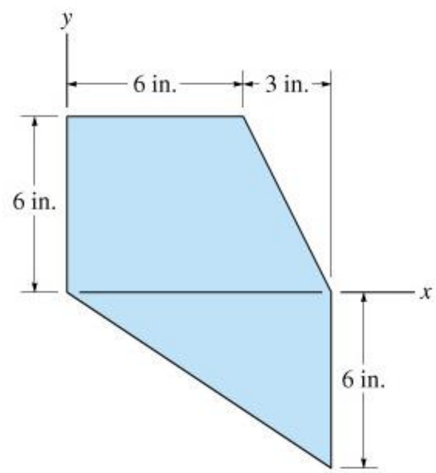
Locate the centroid (x bar, y bar) of the shaded area.

$$\bar{X} = \frac{\Sigma A\tilde{x}}{\Sigma A} \qquad \bar{y} = \frac{\Sigma A\tilde{y}}{\Sigma A}$$

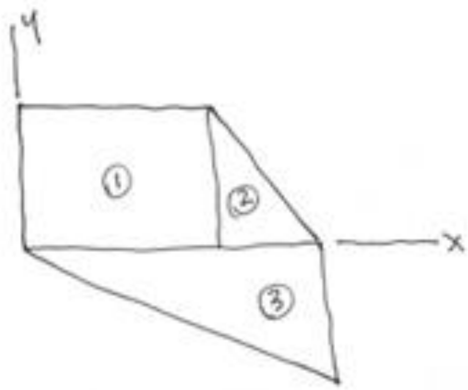
Shape	Area	$\tilde{x}$	$A\tilde{x}$	$\tilde{y}$	$A\tilde{y}$
1	$1(2) = 2$	0.5	1	1	2
2	$1(3) = 3$	2.5	7.5	0.5	1.5
3 VOID	$-\frac{1}{4}\pi r^2 = -0.785$	$4r/3\pi = 0.424$	-0.33	$4r/3\pi$ 0.424	-0.333
4	$\frac{1}{2}(3)(1) = 1.5$	2	3	$1.3\bar{3}$	2
Σ	5.71		11.17		5.17



$$\bar{X} = \frac{11.17}{5.71} = \boxed{1.96\text{ in}} \qquad \bar{y} = \frac{5.17}{5.71} = \boxed{0.91\text{ in}}$$



Locate the centroid of the area from both the ***x*** and ***y*** axes.



SHAPE	AREA	$\tilde{x}$	$\tilde{y}$	$A\tilde{x}$	$A\tilde{y}$
1	6(6) 36	3	3	108	108
2	½(3)(6) 9	7	2	63	18
3	½(9)(6) 27	6	-2	162	-54
	72			333	72

$$\bar{y} = \frac{\Sigma A\tilde{y}}{\Sigma A} = \frac{72}{72} = 1.0 \text{ in}$$

$$\bar{x} = \frac{\Sigma A\tilde{x}}{\Sigma A} = \frac{333}{72} = 4.625 \text{in}$$