Chapter 20
Magnetism

§ .1 Magnets and Magnetic Fields

§ .2 Biot-Savart Law
Gauss theorem of magnetic field

§ .3 Ampere circuital theorem

§ .4 Force on Electric Charge
Moving in a Magnetic Field

§ .5 Forces between Two Parallel Wires

Contents of Chapter 20

- Solenoids and Electromagnets
- Ampère's Law
- Torque on a Current Loop;
- Magnetic Moment（磁矩）
- Applications: Galvanometers, Motors, Loudspeakers
- Mass Spectrometer（质谱仪）
- Ferromagnetism: Domains(磁畴)Hysteresis(磁滞)

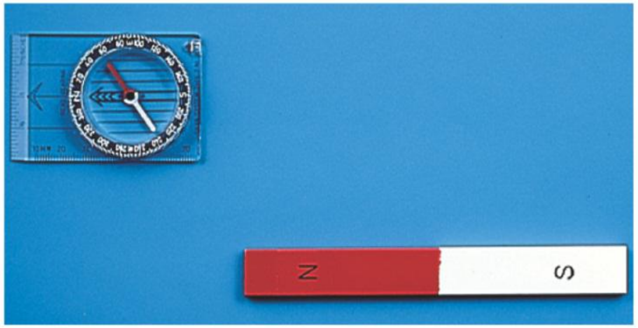
Brief history of Magnetism

1300 BC compass used to help travellers find directions in China.
800 BC Greeks observe that pieces of magnetite (Fe_3O_4) attract pieces of iron (Fe).
1269 AD Using a compass to point along field lines outside a spherical magnet

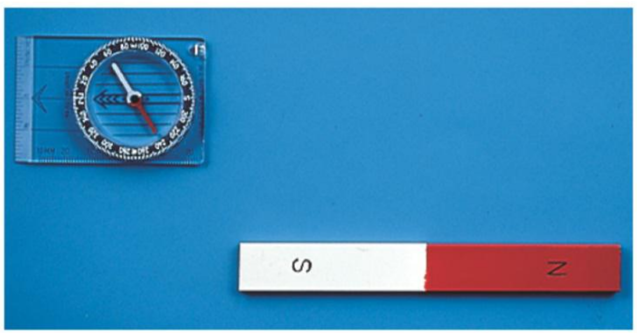
1269-1820 AD Everybody busy with wars and plagues for 550 years. No time for magnetism.

1820 AD Oersted (Denmark) observes that electric current in a wire deflects a nearby compass. Faraday(England), Henry(US), Maxwell (England) work on the physics of magnetism.
Late 1800's: motors, generators, power grids, electric lighting, electric refrigeration invented. GE, Siemens, Westinghouse, etc make these products widely available.
1889 Heaviside derives a key equation relating magnetic field B to force F on a moving charge or current.
1899 H. A. Lorentz derives the same equation, now known as the Lorentz Equation.

§ .1 Magnetic field



(a)



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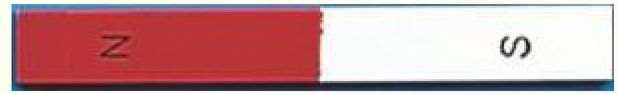


磁极

magnetic pole

Magnets have two ends—poles—called north and south.
Like poles repel; unlike poles attract.

Magnetic phenomenon

§ .1 Magnetic field

Magnetic phenomenon 磁石吸铁

Lodestone is a kind of natural magnetite ore(Fe_3O_4) Magnetism 磁性

Near a strong magnet: Magnetization 磁化

Ferromagnetic substance（铁、钴、镍及其合金）

Iron cobalt nickel

↓

暂时磁性的物体或永久磁性的物体

After magnetization

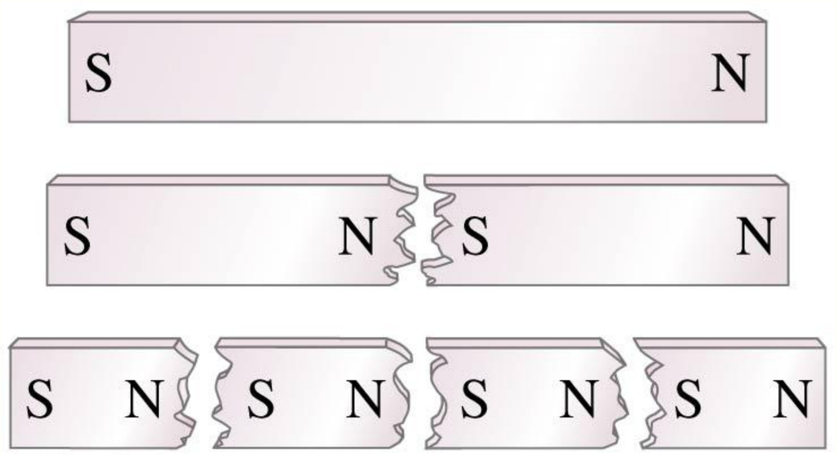
ferromagnetic substance → magnet

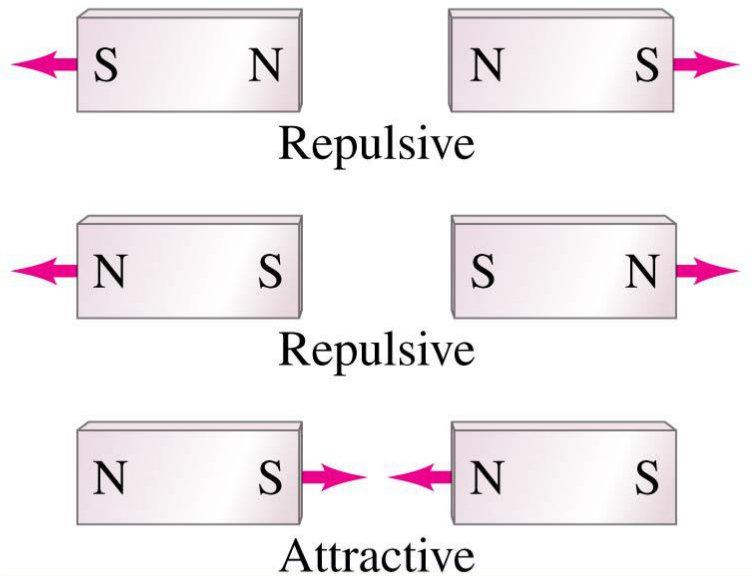
Electric coil 各种形状的具有磁性的磁铁

（载流螺线管）

20-1 Magnets and Magnetic Fields

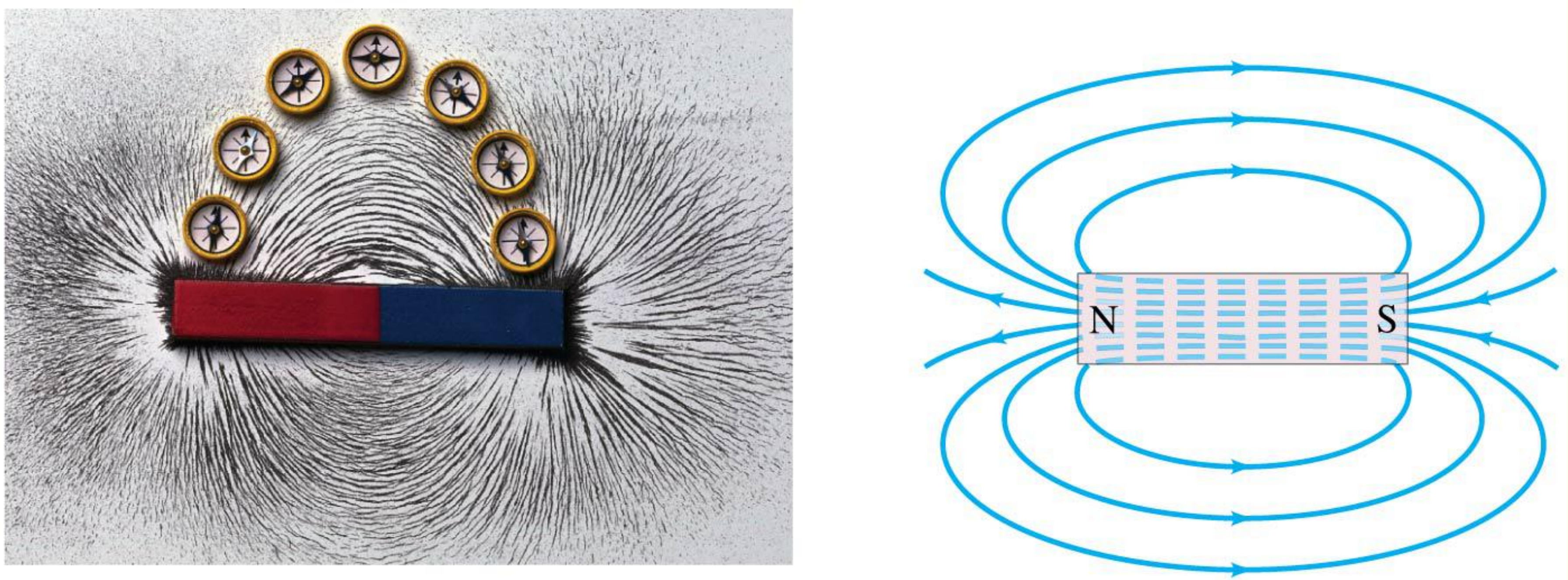
However, if you cut a magnet in half, you don't get a north pole and a south pole—you get two smaller magnets.





20-1 Magnets and Magnetic Fields

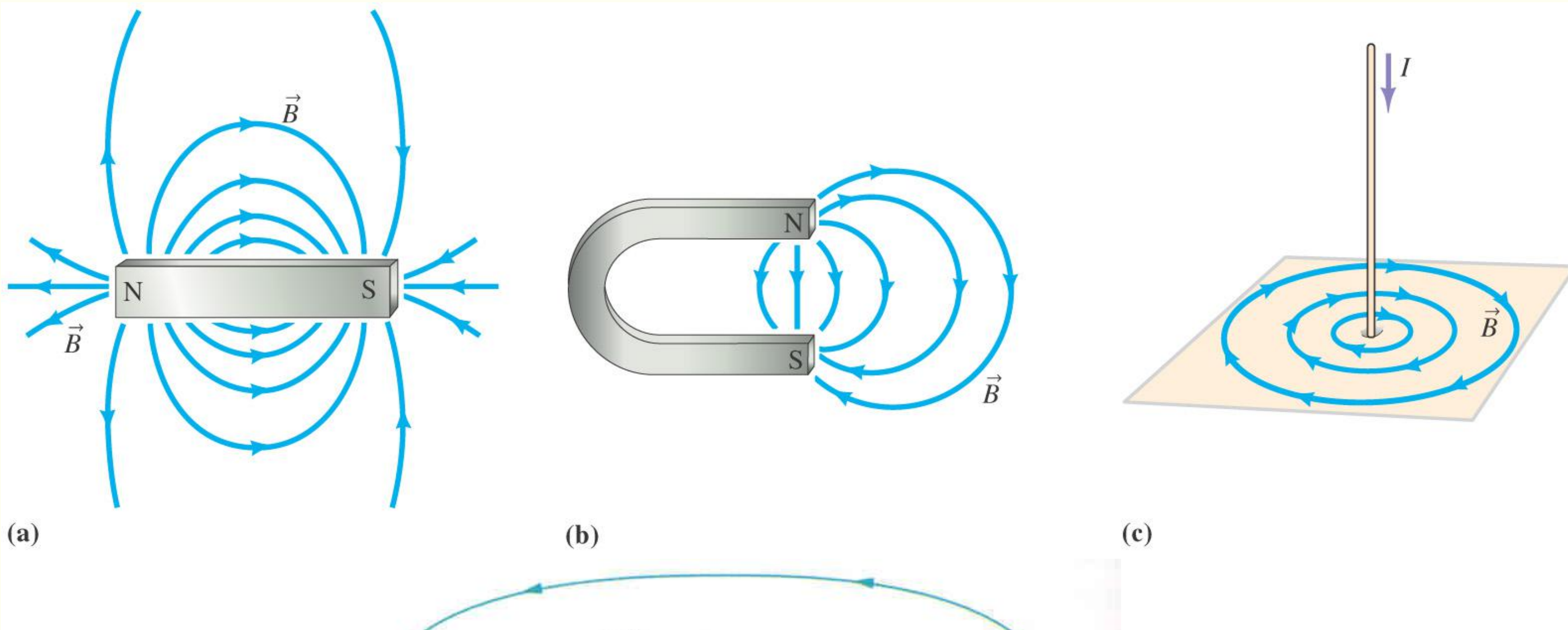
Magnetic fields can be visualized using magnetic field lines, which are always closed loops.

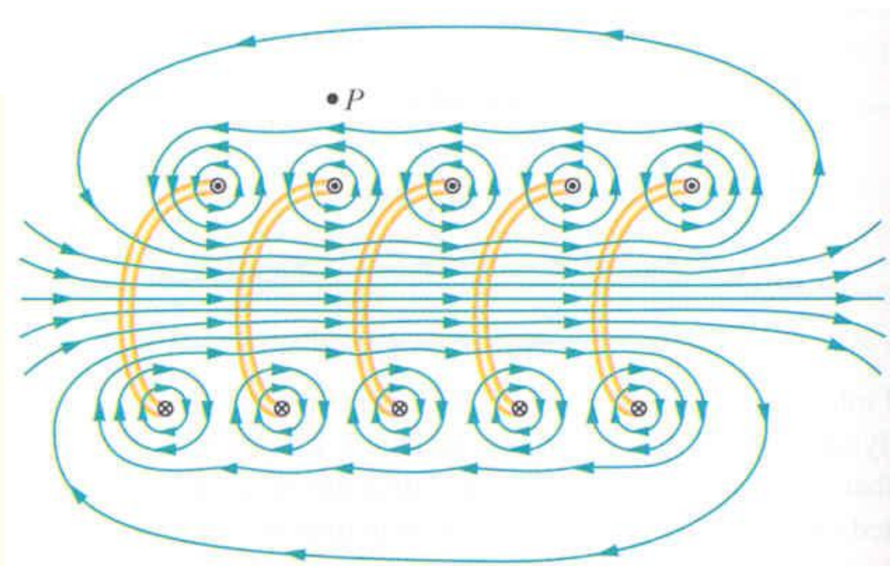


(a) (b)

Magnetic field line　　磁感应线（磁力线）

- **Magnetic field lines point from N poles to S poles**





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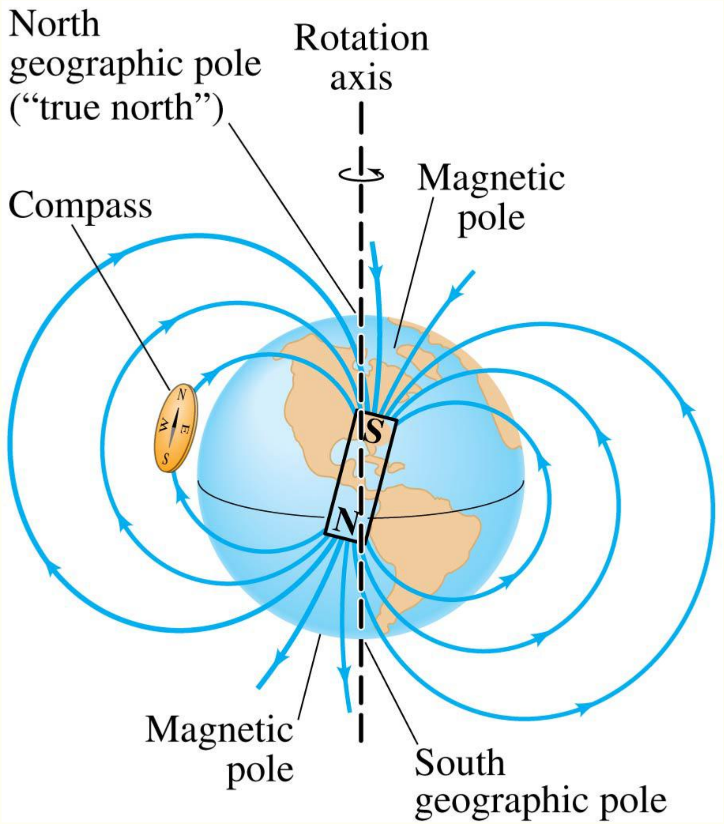
20-1 Magnets and Magnetic Fields

The Earth's magnetic field is similar to that of a bar magnet.



Note that the Earth's "North Pole" is really a south magnetic pole, as the north ends of magnets are attracted to it.

某些典型磁场的 *B*值

TABLE 28–1 • Some Magnetic Fields

Location or Source	Magnitude (T)
Interstellar space	10^{-10}
Near Earth's surface	5×10^{-5}
Refrigerator magnet for notes	10^{-2}
Bar magnet near poles	10^{-2}–10^{-1}
Near surface of Sun	10^{-2}
Large scientific magnets	2–4
Largest steady-state magnet	30
Largest pulsed field in laboratory	500–1000
Near surface of pulsar	10^{8}
Near surface of atomic nucleus	10^{12}

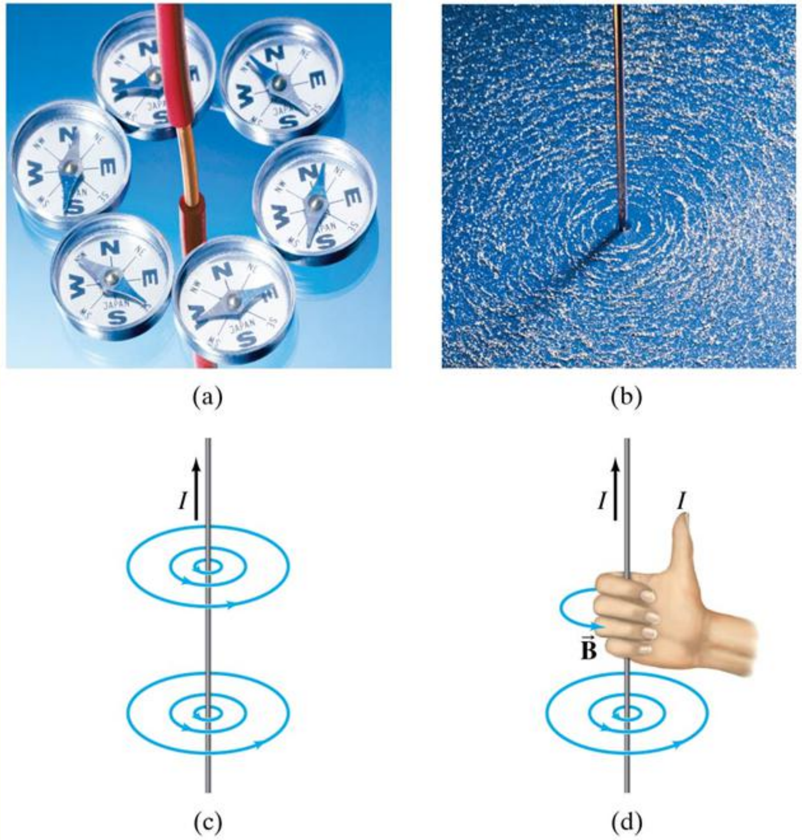
奥斯特 **Hans Christian Oersted (1777 ～ 1851)**

1820年7月21日 **1820年4月的一个晚上**

《关于电冲击对磁针影响的实验》

电流的磁效应

Experiment shows that an electric current produces a magnetic field.



A. M. Ampere

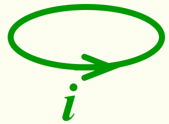
从实验上发现，一个载流螺线管在对外显示磁性和磁相互作用时，等效于一个磁棒。

An electric current-carrying coil behaves like a magnet

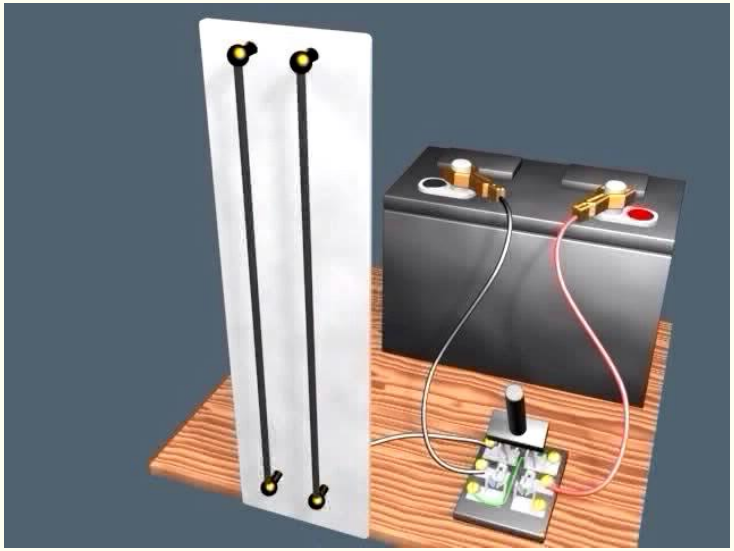
Ampere believe: The essence of magnetism is electric current. The magnetism of a substance comes from the molecular electric current in it

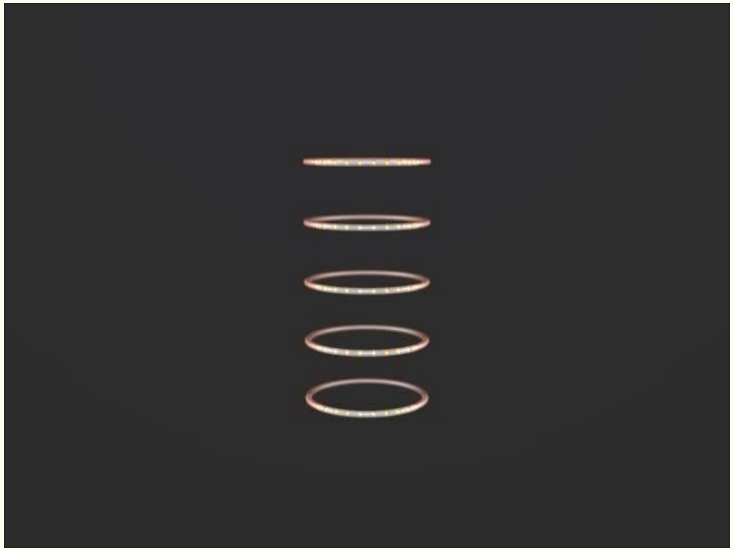
抽象和猜测

组成磁铁的每一个微粒都存在着永不停息的环形电流

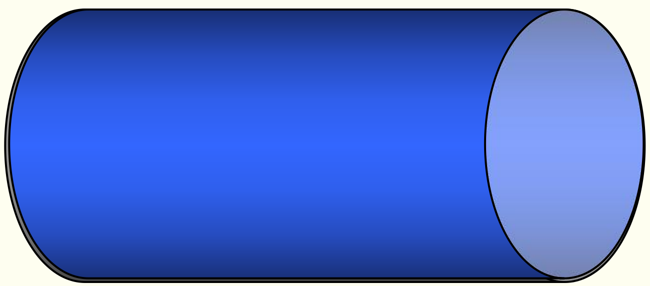


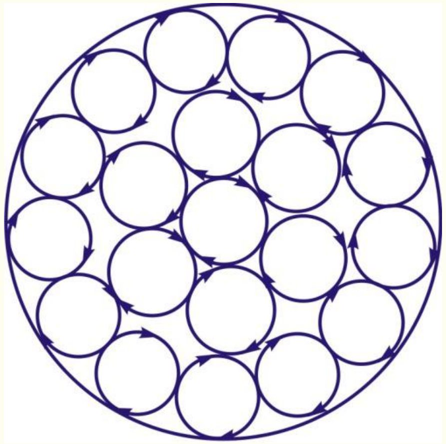
A. M. Ampere

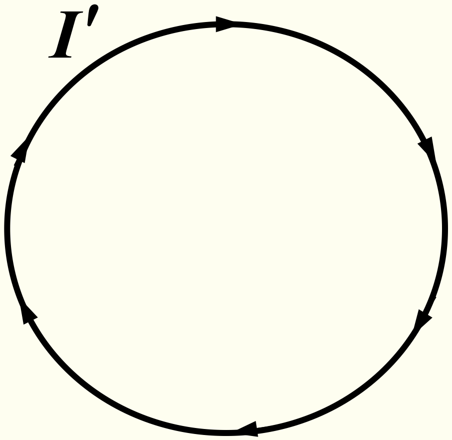




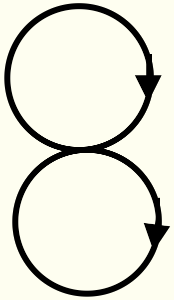
物质磁性的存在与消失，取决于其中分子电流排列的整齐与混乱。







组成类似螺线管电流的
一圈一圈的环形电流



经过磁化的铁磁性物质 → 磁铁

磁化 → 就是使其中的分子电流排列整齐

按照 Ampere提出的分子电流概念

两磁铁间的相互作用是一个磁体内的分子电流与另一个磁体内的分子电流间的相互作用。

近代 分子电流

是由原子内电子绕核的运动和电子的自旋两部分形成的

电流与电流之间的磁相互作用 磁场

Magnetic field

电流(磁铁) ⟷ 磁场 ⟷ 电流(磁铁)

运动电荷 ⟷ 磁场 ⟷ 运动电荷

§ .2 毕奥-萨伐尔定律　磁高斯定理

The Biot-Savart Law　Gauss Theorem for Magnetism

Oersted 实验以及一系列相关的实验，发现了不少前所未知的重要现象，揭示了电现象与磁现象多方面的联系，开辟了一个崭新的广阔研究领域。

一些具有重大意义的研究课题很快凝聚而成，并迅速取得了重要的成果和突破。

根据小磁针或载流导线或运动电荷在磁场中受力情况来描述磁场

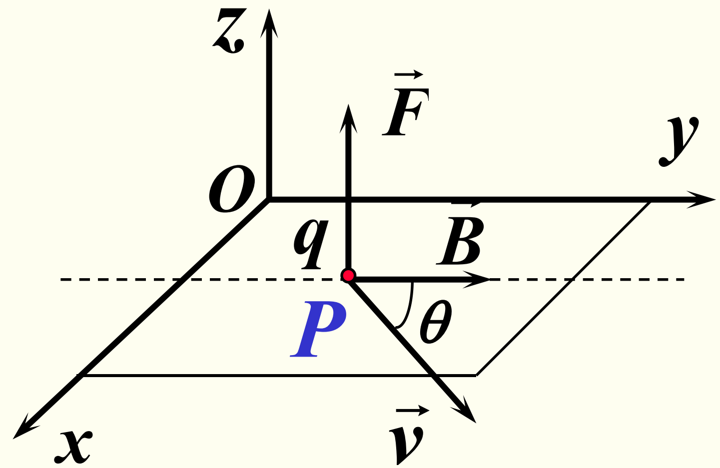
引入描述磁场的物理量

磁感应强度 $\vec{B}$

Magnetic induction

实验：小磁针N极的指向 → 磁感应强度 $\vec{B}$ 的方向

$q \rightarrow -q \quad \Rightarrow \quad \vec{F} \rightarrow -\vec{F}$



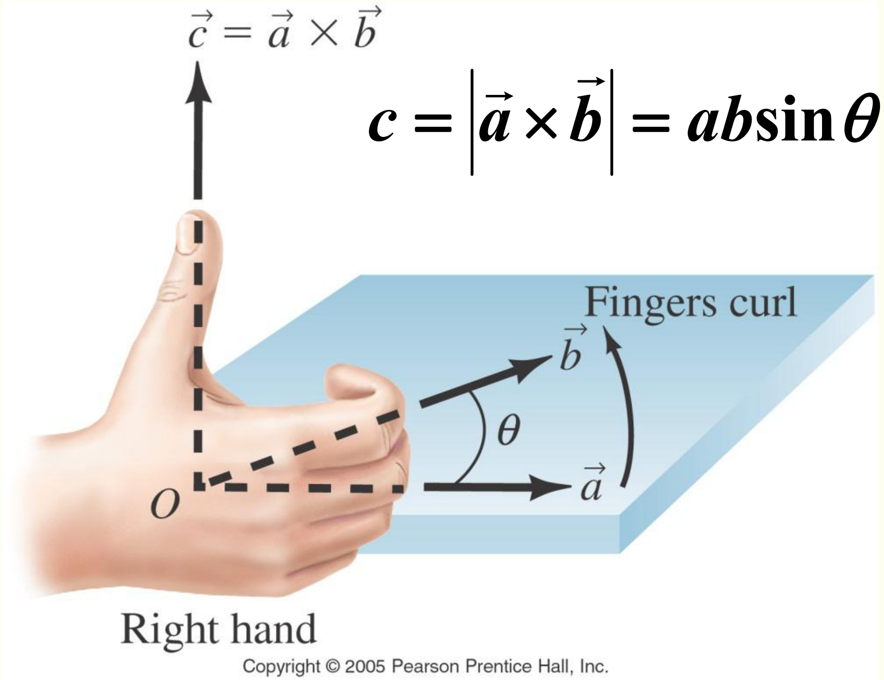
3. $F \propto qv\sin\theta$

$\dfrac{F}{qv\sin\theta}$ 与 q、v、θ 无关

Only determined by location

Definition: $B = \dfrac{F}{qv\sin\theta}$ SI 特斯拉（T）

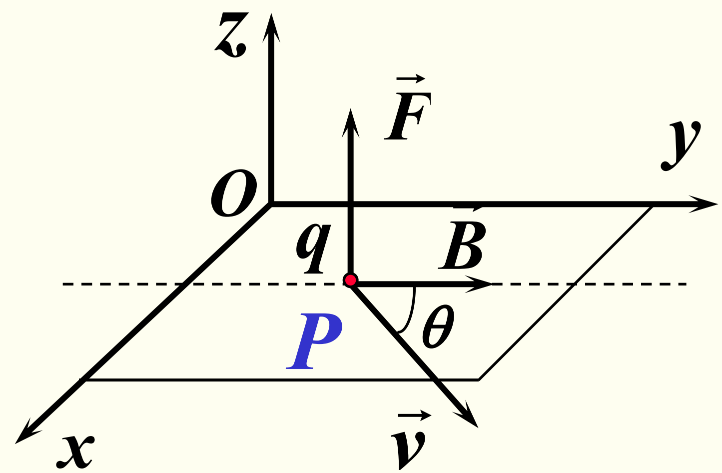
返回 退出

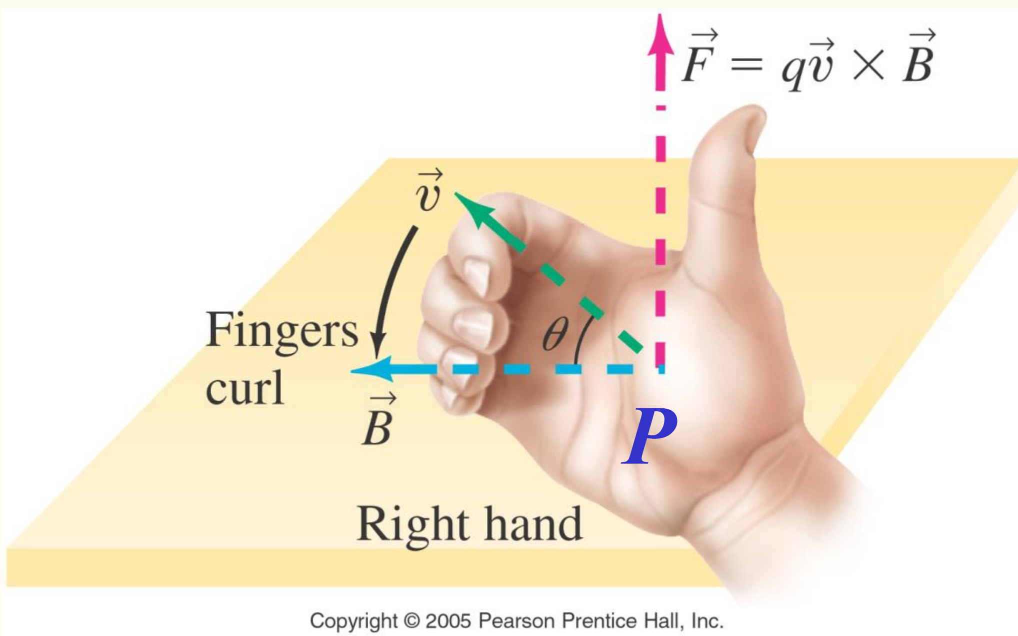


$$c = \left|\vec{a} \times \vec{b}\right| = ab\sin\theta$$

$$B = \frac{F}{qv\sin\theta}$$

$$\left|q\vec{v} \times \vec{B}\right| = Bqv\sin\theta = F$$





$$\vec{F} = q\vec{v} \times \vec{B}$$

洛仑兹力公式

Biot 和 Savart认为： Oersted 实验中磁针在电流作用下受力偏转的原因是，两个磁极分别受到电流的作用，而电流对磁极的作用力应当是构成该电流的各电流元对磁极的作用力。

Biot 和 Savart提出寻找电流元对磁极作用力的定量关系

$Id\vec{l}$ 电流元

Biot 和 Savart 通过巧妙设计的某些特殊实验的结果，经过分析（在 Laplace 的帮助下），建立了Biot-Savart定律。

$$d\vec{F} \rightarrow d\vec{B} = k\frac{Id\vec{l}\times\vec{r}}{r^3}$$

Ampere 提出了寻找电流元与电流元之间作用力所遵循的普遍规律问题。

$$d\vec{F}_{21} = k\,\frac{I_2 d\vec{l}_2 \times (I_1 d\vec{l}_1 \times \hat{r}_{12})}{r_{12}^2}$$

Ampere定律实际上包括了Biot-Savart 定律

Biot-Savart 定律 $\quad d\vec{B} = k\,\frac{I d\vec{l}\,' \quad \hat{r}}{r^2}$

$Id\vec{l}$ 电流元 infinitesimal current element

$$d\vec{B} = k\frac{Id\vec{l}\,' \quad \hat{r}}{r^2}$$

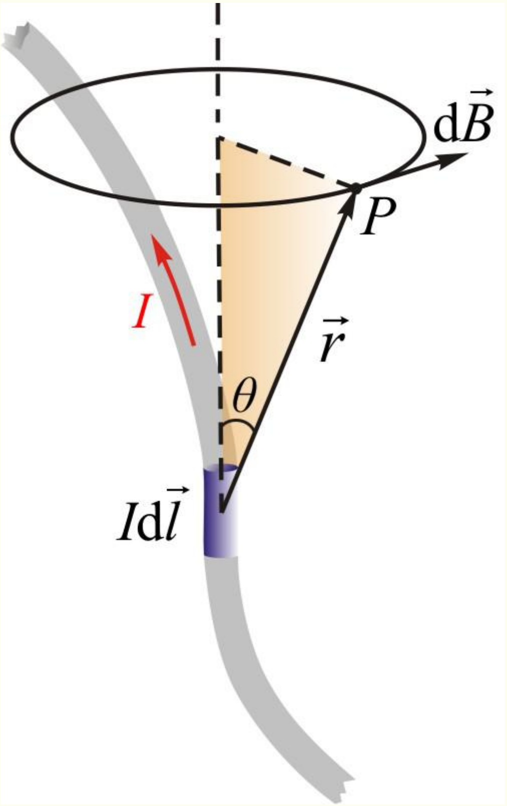
$$= \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$$

$$\mu_0 = 4\pi\times10^{-7}\,\mathrm{T\cdot m/A}$$

真空磁导率 Permeability of vacuum

magnitude $$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$

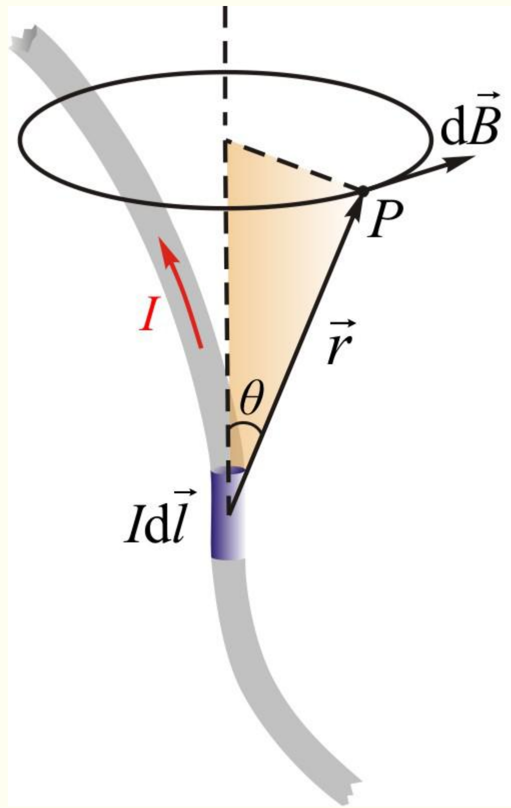
direction $$d\vec{B} \to Id\vec{l}\times\vec{r}$$



A wire that carries a current I

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \vec{r}}{r^3}$$

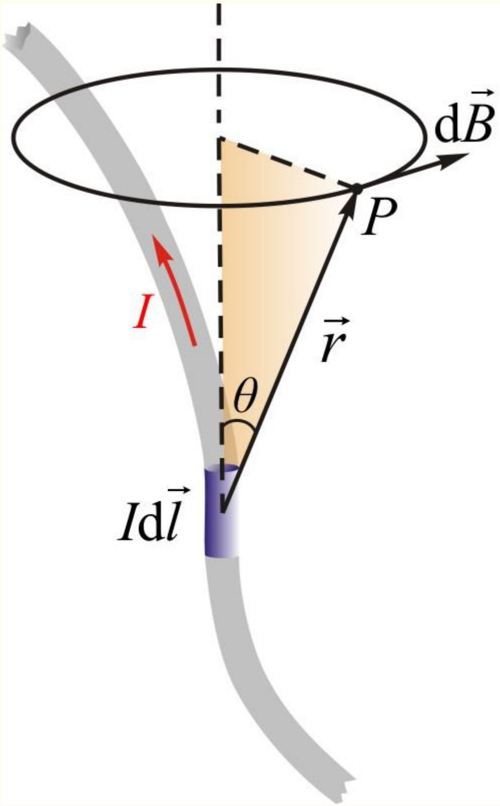
$$= \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$$



Biot-Savart Law

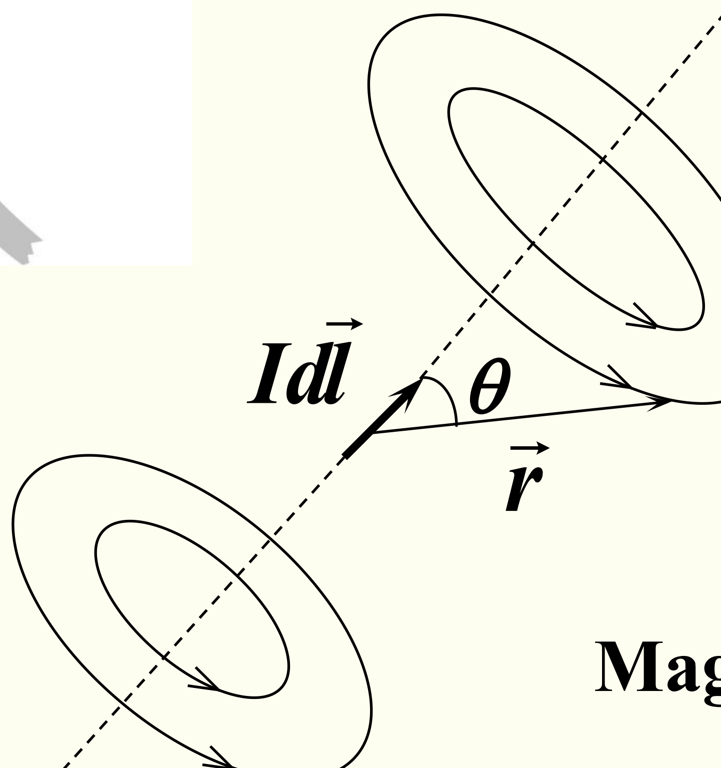
A wire that carries a current *I*

$$\vec{B} = \int d\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$$



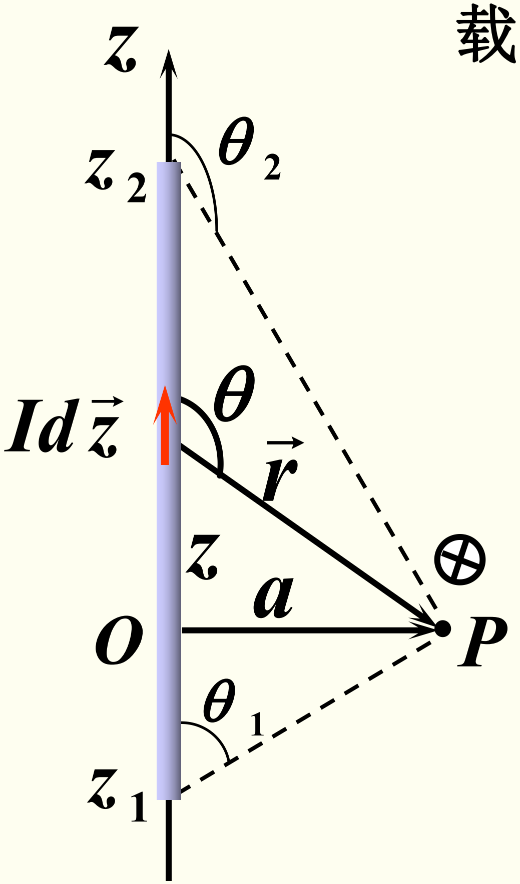
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \vec{r}}{r^3}$$

$$= \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$$



Magnetic field lines of a $Id\vec{l}$

1.Calculating the magnetic field produced by a current I in a long straight wire



载流直导线

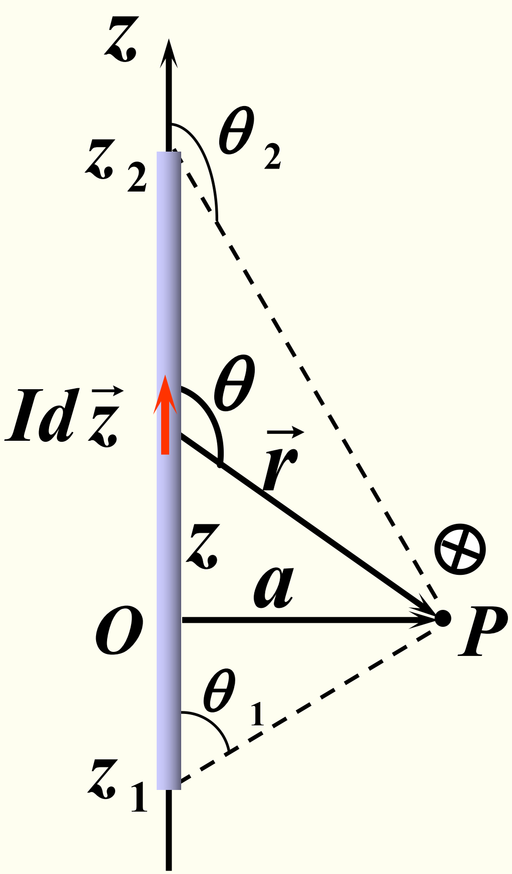
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{Idz\sin\theta}{r^2}$$

$$B = \int dB = \int \frac{\mu_0}{4\pi}\frac{Idz\sin\theta}{r^2}$$

$$z = -a\,\mathrm{ctg}\,\theta$$

$$dz = \frac{a}{\sin^2\theta}d\theta \qquad r = \frac{a}{\sin\theta}$$



$$B = \frac{\mu_0}{4\pi}\int_{\theta_1}^{\theta_2} \frac{I\sin\theta}{\frac{a^2}{\sin^2\theta}} \frac{a\,d\theta}{\sin^2\theta}$$

$$= \frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2} \sin\theta\, d\theta$$

$$= \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2)$$

infinite straight wire $\theta_1 \to 0$, $\theta_2 \to \pi$ $\Rightarrow$ $B = \dfrac{\mu_0 I}{2\pi a}$

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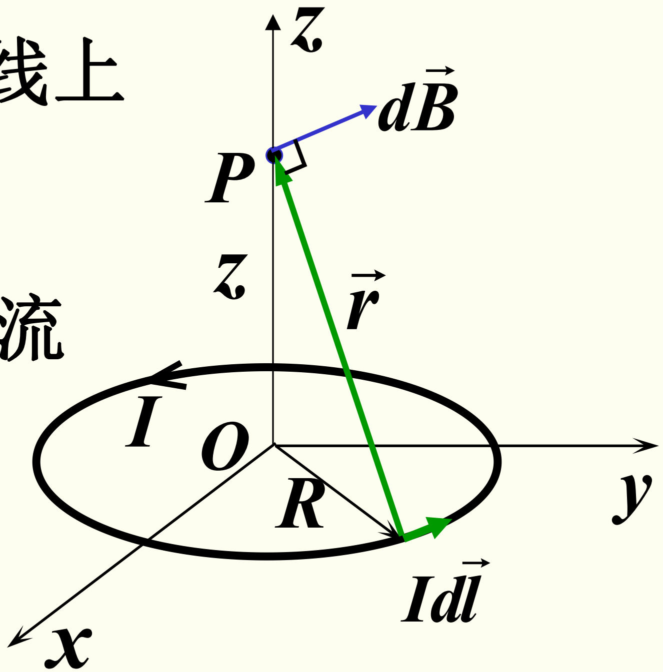
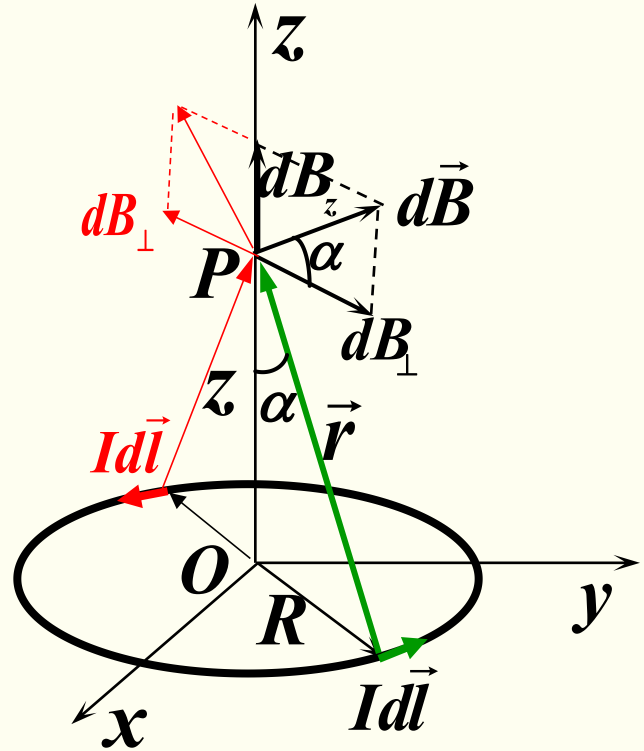
2. Magnetic Field Due to a Current in a Circular of Wire

轴线上

圆电流





$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\vec{r}}{r^3}$$

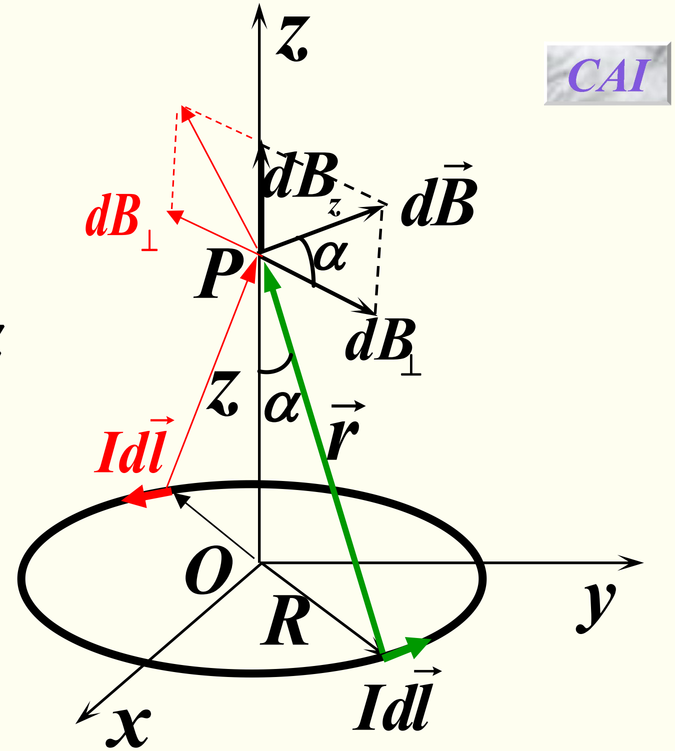
$$dB = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}\sin 90^0$$

$$dB_z = dB\sin\alpha = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}\sin\alpha$$

$$B=\int dB_z=\frac{\mu_0 I}{4\pi}\int\frac{dl\sin\alpha}{r^2}$$

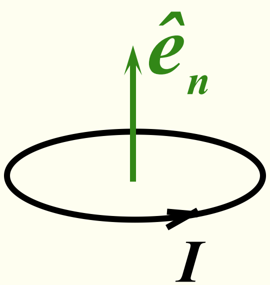
$$=\frac{\mu_0 I}{4\pi r^2}\sin\alpha\int_0^{2\pi R}dl=\frac{\mu_0 IR}{2r^2}\sin\alpha$$

$$=\frac{\mu_0 R^2 I}{2(R^2+z^2)^{3/2}}$$



$z>>R$ $$B\approx\frac{\mu_0 R^2 I}{2z^3}$$

$$\vec{B}=\frac{\mu_0}{4\pi}\frac{2\vec{m}}{z^3}$$

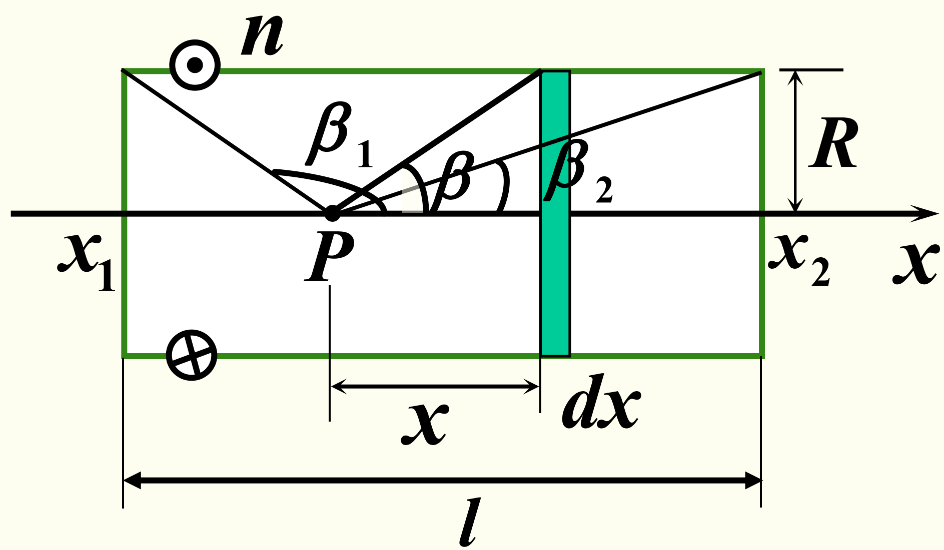


$S\to 0$ 磁偶极子 magnetic dipole

$\vec{m}=SI\hat{e}_n$ 磁偶极矩 (磁矩) magnetic dipole moment

3. Magnetic Field at any point along the center axis of a Solenoid 载流螺线管内的磁场（轴线上）

$$dB_p = \frac{\mu_0 R^2 Indx}{2(R^2 + x^2)^{3/2}}$$



$$B_p = \int_{x_1}^{x_2} \frac{\mu_0 R^2 Indx}{2(R^2 + x^2)^{3/2}}$$

场点P为坐标原点

$$= -\frac{\mu_0 nI}{2}\int_{\beta_1}^{\beta_2} \frac{R^3 \csc^2 \beta d\beta}{R^3 \csc^3 \beta}$$

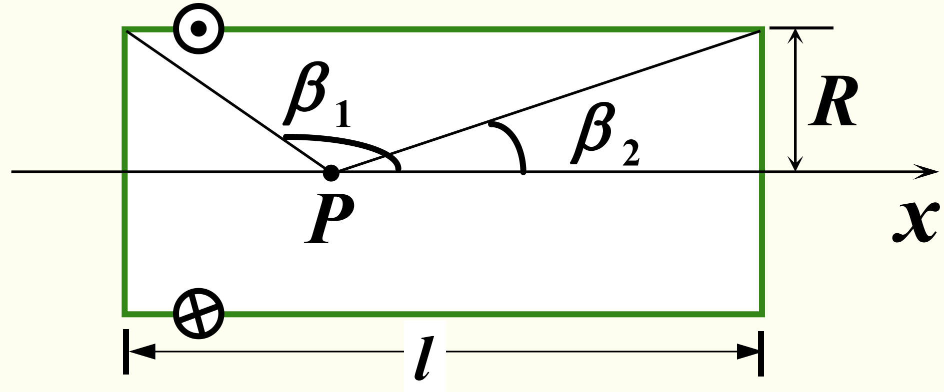
$$x = R\cot\beta$$

$$dx = -R\csc^2\beta d\beta$$

$$= \frac{\mu_0 nI}{2}(\cos\beta_2 - \cos\beta_1)$$

$$R^2 + x^2 = R^2 \csc^2 \beta$$

$$B_p = \frac{\mu_0 n I}{2}(\cos\beta_2 - \cos\beta_1)$$



1. infinite

$l >> R \quad \beta_1 \to \pi \quad \beta_2 \to 0$

$$B = \mu_0 n I$$

2. semi-infinite

left end of the solenoid $\quad \beta_1 = \frac{\pi}{2} \quad \beta_2 \to 0$

$$B = \frac{1}{2}\mu_0 n I$$

CAI

Gauss theorem for electricity

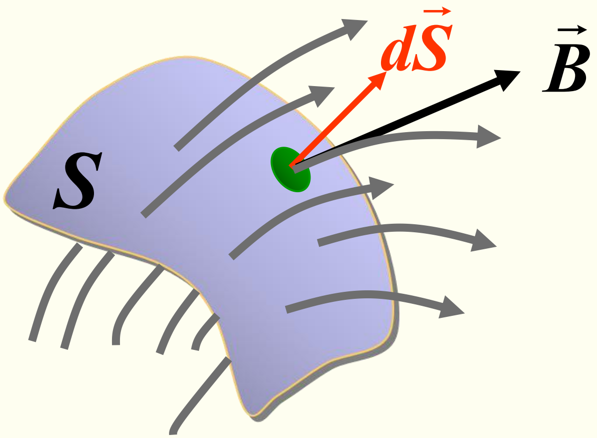
$$\oint_S \vec{E}\cdot \mathrm{d}\vec{S} = \frac{q}{\varepsilon_0} \qquad \oint_S \vec{B}\cdot \mathrm{d}\vec{S} = ?$$

$\vec{B}$ S $\mathrm{d}S$ $\mathrm{d}\vec{S}$

磁通量

$$\mathrm{d}\Phi_m = \vec{B}\cdot \mathrm{d}\vec{S}$$

Magnetic flux



$$\mathrm{d}\Phi_m = B\cos\theta \mathrm{d}S = B\mathrm{d}S_{\perp}$$

$$\Phi_m = \int_S \vec{B}\cdot \mathrm{d}\vec{S}$$

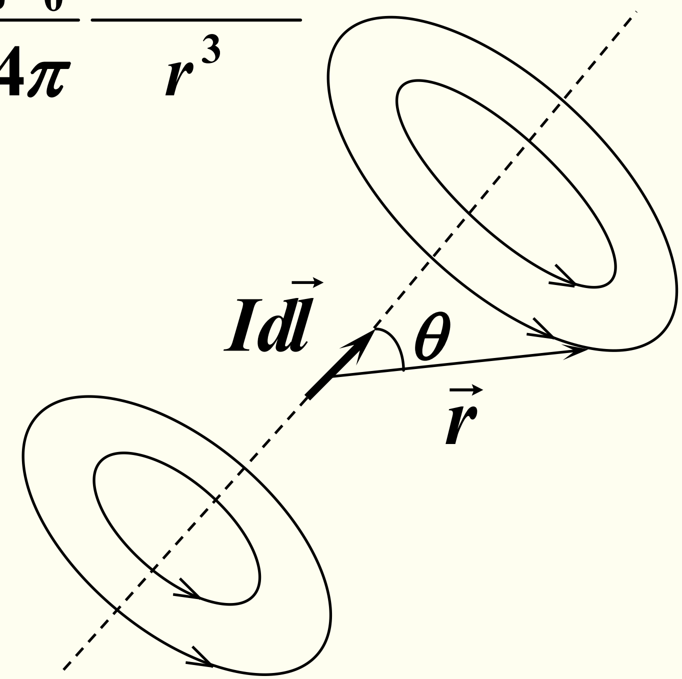
SI $\mathrm{T}\cdot \mathrm{m}^2 = \mathrm{wb}$

$$\oint_S \vec{B} \cdot d\vec{S} = ?$$

Molecular current hypothesis

Any current always consists of many current elements

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \vec{r}}{r^3}$$



$Id\vec{l}_1$、$Id\vec{l}_2$、$Id\vec{l}_3$、$\cdots$

$d\Phi_1$、$d\Phi_2$、$d\Phi_3$、$\cdots$

0、 0、 0、$\cdots$

$$\Phi_m = d\Phi_1 + d\Phi_2 + d\Phi_3 + \cdots = 0$$

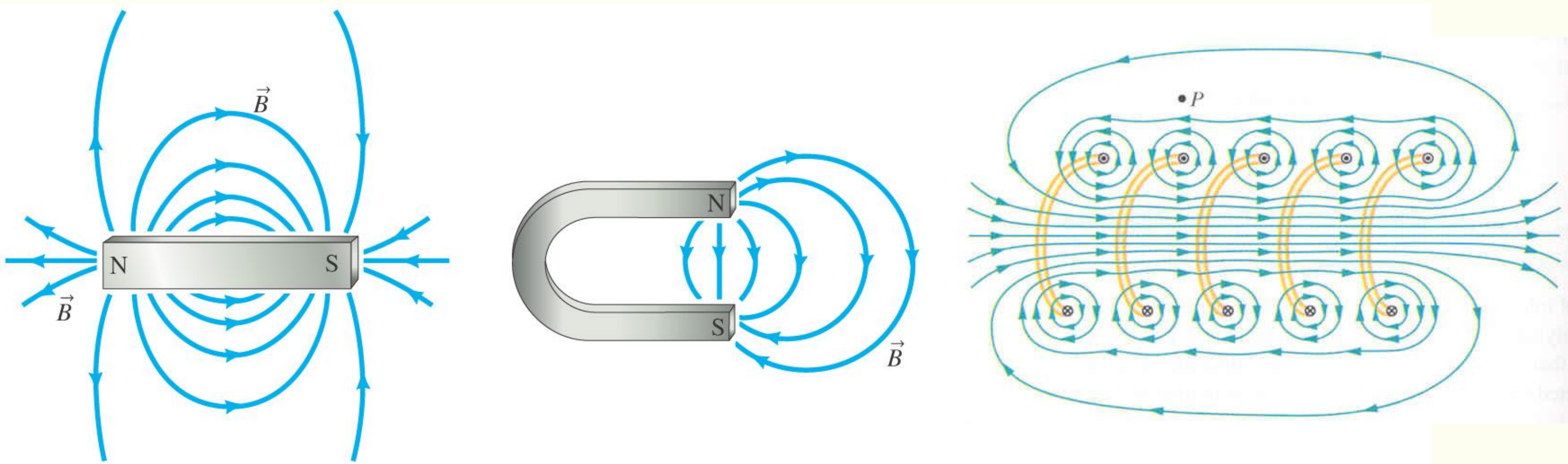
$$\Phi_m = \oint_S \vec{B} \cdot d\vec{S} = 0$$

$$\oint_S \vec{B} \cdot d\vec{S} = 0$$

Gauss Theorem for Magnetism

磁高斯定理

The passive field 无源场



(a) (b) (c)

1931年　狄拉克（P. A. M. Dirac）　磁单极

20世纪60年代　美国阿波罗计划　登月飞行

先后从月球运回许多月球岩石

$10^{16}\,\mathrm{GeV}/c^2$

1975年　加州大学　Price

装有探测宇宙射线仪器的气球

距地面 40km 的高空

记录到一条电离性很强的粒子留下的径迹

斯坦福大学　Cobrera　　超导磁单极探测器

1982年2月14日

磁通量突然有很大变化

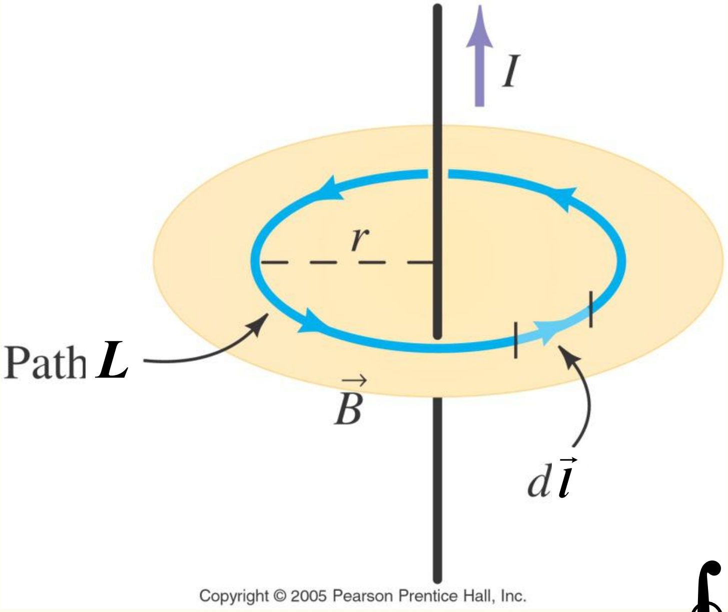
与一个磁单极引起的变化相当　　……

§ .3 Ampère Circuital Theorem 安培环路定理

Electrostatic field $\oint_L \vec{E}\cdot d\vec{l} = 0$

Magnetic field $\oint_L \vec{B}\cdot d\vec{l} = ?$

Andre Marie Ampère (1775-1836)



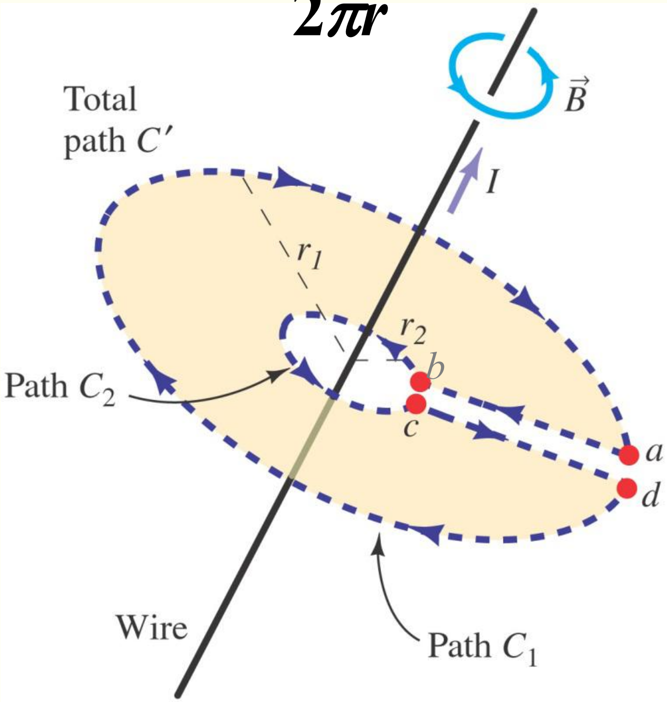
$$B = \frac{\mu_0 I}{2\pi r}$$

$$\oint_L \vec{B}\cdot d\vec{l} = \oint_L B\cdot dl = B\oint_L dl$$

$$= \frac{\mu_0 I}{2\pi r} 2\pi r = \mu_0 I$$

$$\oint_L \vec{B}\cdot d\vec{l} = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$



$$\int_{ab} \vec{B} \cdot d\vec{l} = 0 \qquad \int_{cd} \vec{B} \cdot d\vec{l} = 0$$

$$\oint_{C'} \vec{B} \cdot d\vec{l} = \int_{C_1} \vec{B} \cdot d\vec{l} + \int_{C_2} \vec{B} \cdot d\vec{l}$$

$$= -\int_{C_1} B \cdot dl + \int_{C_2} B \cdot dl$$

$$= -B_1(2\pi r_1) + B_2(2\pi r_2)$$

$$= -\frac{\mu_0 I}{2\pi r_1}(2\pi r_1) + \frac{\mu_0 I}{2\pi r_2}(2\pi r_2)$$

$$= 0$$

$$\oint_{C'} \vec{B} \cdot d\vec{l} = 0$$

$$\oint_L \vec{B} \cdot d\vec{l} = \mu_0 \sum_i I_i$$

Ampère Circuital Theorem

$$\oint_L \vec{B}\cdot d\vec{l} = \mu_0 \sum_i I_i$$ **Ampère Circuital Theorem**

L 在磁场中任取的一闭合线，规定一个绕行方向

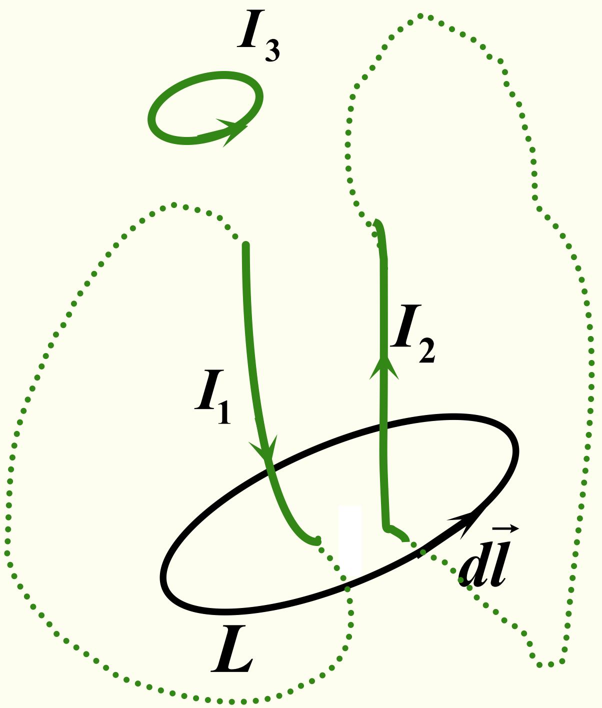
$\mathrm{d}\vec{l}$ L 上的任一线元 **infinitesimal length**

The integral is taken around that closed path L

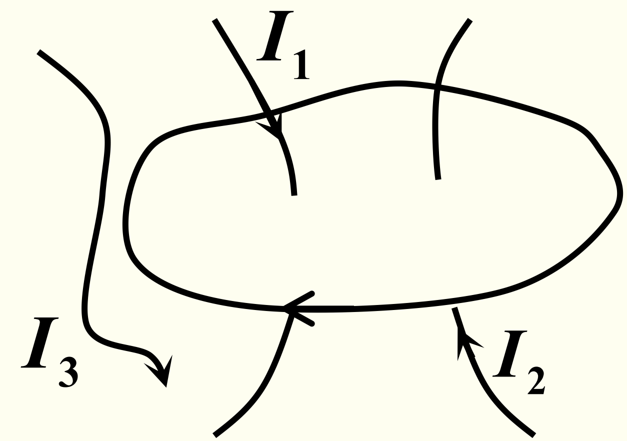
$\sum_i I_i$ **The total current enclosed by any closed path L**

电流的符号规定：

Curl your right hand around the loop L, with the fingers pointing in the direction of integration. A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.



$$\oint_L \vec{B}\cdot d\vec{l} = \mu_0(-I_1 + I_2)$$



$$\oint_L \vec{B}\cdot d\vec{l} = \mu_0(I_1 - I_2)$$

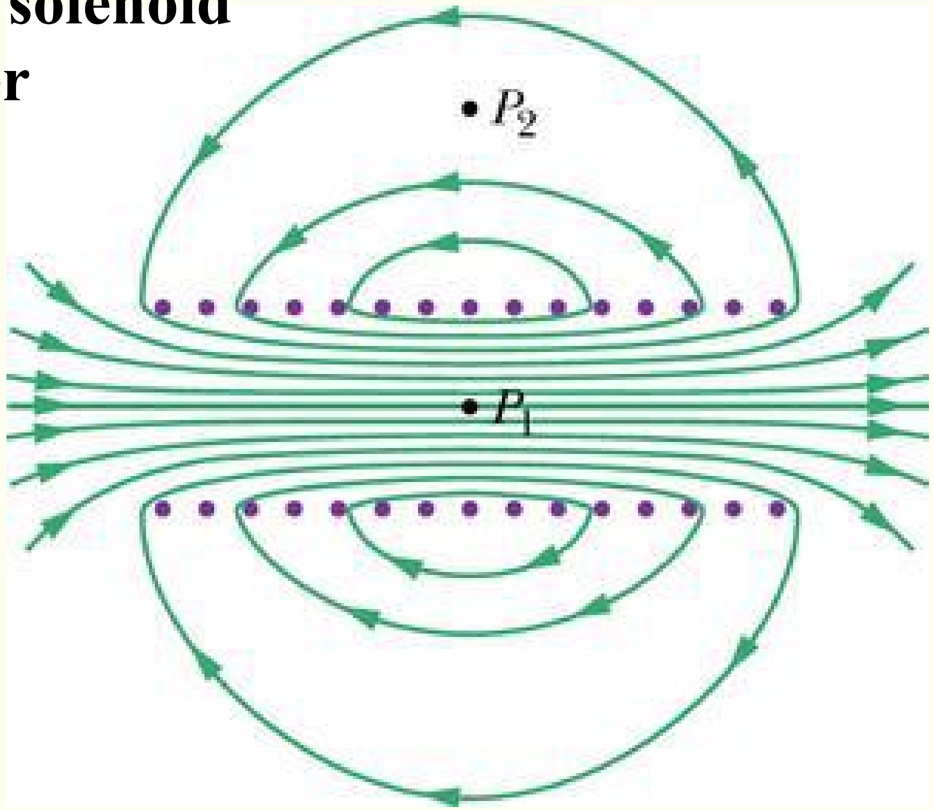
Rotational field 涡旋场

$\vec{B}$ 所有电流在空间产生磁场的叠加

环流仅与穿过回路L所围曲面的电流有关

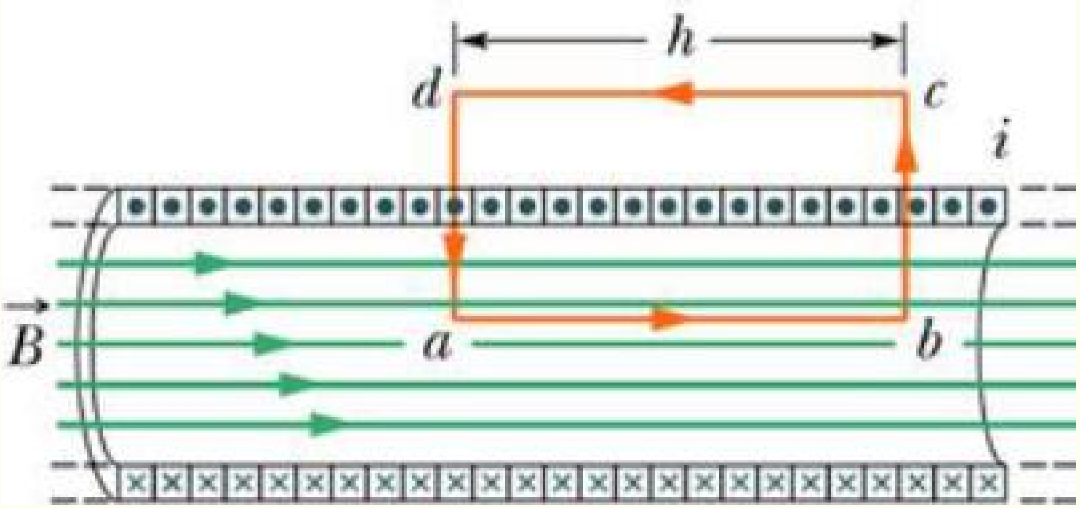
1. 长直载流螺线管内的磁感应强度 Solenoid

We assume that the length of the solenoid is much greater than the diameter



Magnetic field lines for a real solenoid of finite length.

The field is strong and uniform at interior points such as P_1 but relatively weak at external points such as P_2.



$$\oint_L \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}\cdot d\vec{l} + \int_b^c \vec{B}\cdot d\vec{l} + \int_c^d \vec{B}\cdot d\vec{l} + \int_d^a \vec{B}\cdot d\vec{l}$$

$$\int_b^c \vec{B}\cdot d\vec{l} = \int_d^a \vec{B}\cdot d\vec{l} = 0$$

$$\int_c^d \vec{B}\cdot d\vec{l} = 0$$

$$\oint_L \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}\cdot d\vec{l} = B\cdot h$$

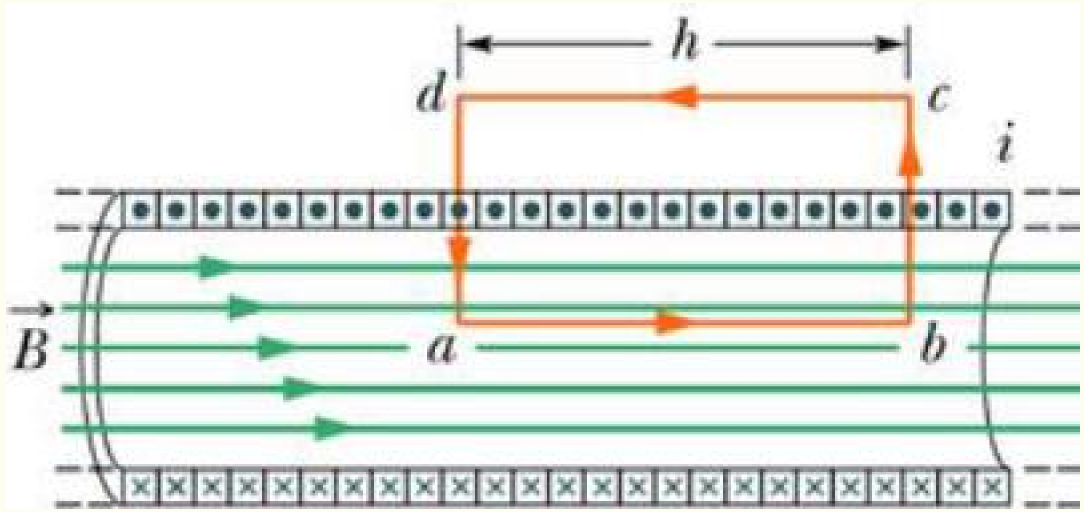
The rectangular loop encloses the current:

$$\sum I_i = i\cdot n\cdot h$$

$$\oint_L \vec{B}\cdot d\vec{l} = \mu_o \sum I_i$$

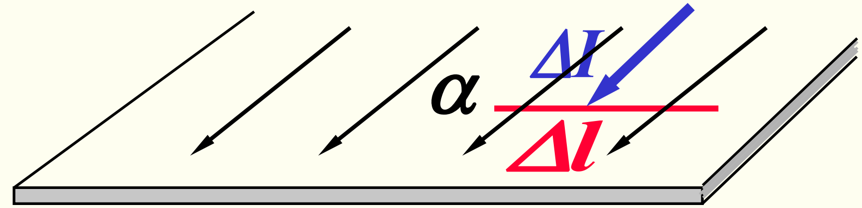
$$B\cdot h = \mu_o i\cdot n\cdot h$$

$$B = \mu_o n i$$



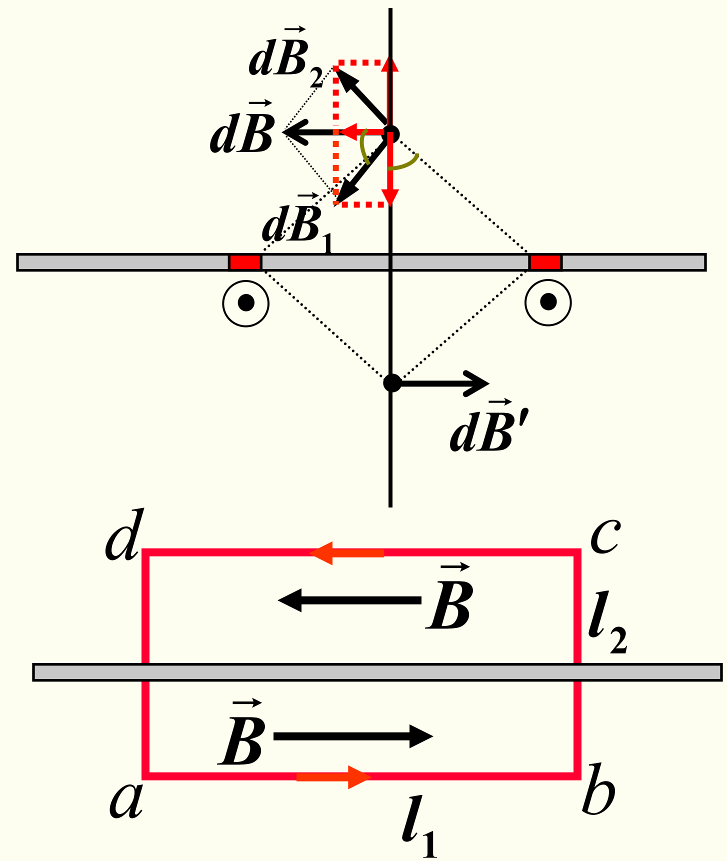
2. 无限大平面电流的磁场

An infinite slab carries a uniform current



α 面电流密度 $\alpha = \dfrac{\Delta I}{\Delta l}$

Surface current density



$$\oint_L \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}\cdot d\vec{l} + \int_b^c \vec{B}\cdot d\vec{l} + \int_c^d \vec{B}\cdot d\vec{l} + \int_d^a \vec{B}\cdot d\vec{l}$$

$$= \int_a^b \vec{B}\cdot d\vec{l} + \int_c^d \vec{B}\cdot d\vec{l}$$

$$= Bl_1 + Bl_1$$

$$Bl_1 + Bl_1 = \mu_0 \alpha l_1 \qquad B = \frac{\mu_0 \alpha}{2}$$

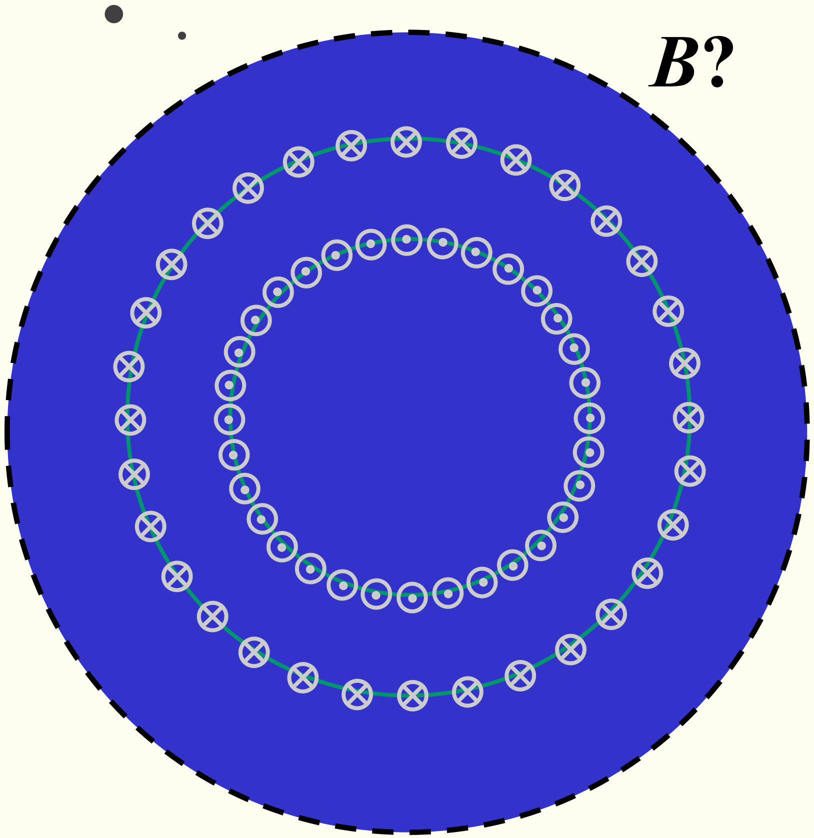
Clicker Question

- The field inside / outside a toroidal solenoid:

A. Decreases as $1/r$ with distance from the center of the solenoid.

B. Decreases as $1/r^2$.

C. Is zero.



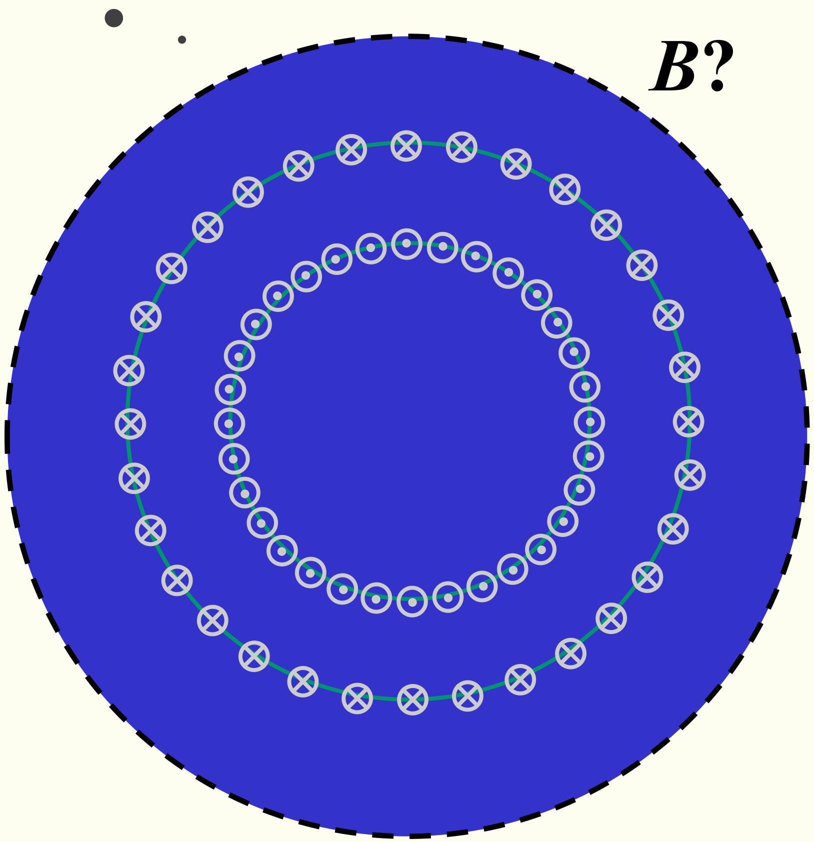
Clicker Answer

- The field outside a toroidal solenoid:

A. Decreases as $1/r$ with distance from the center of the solenoid.

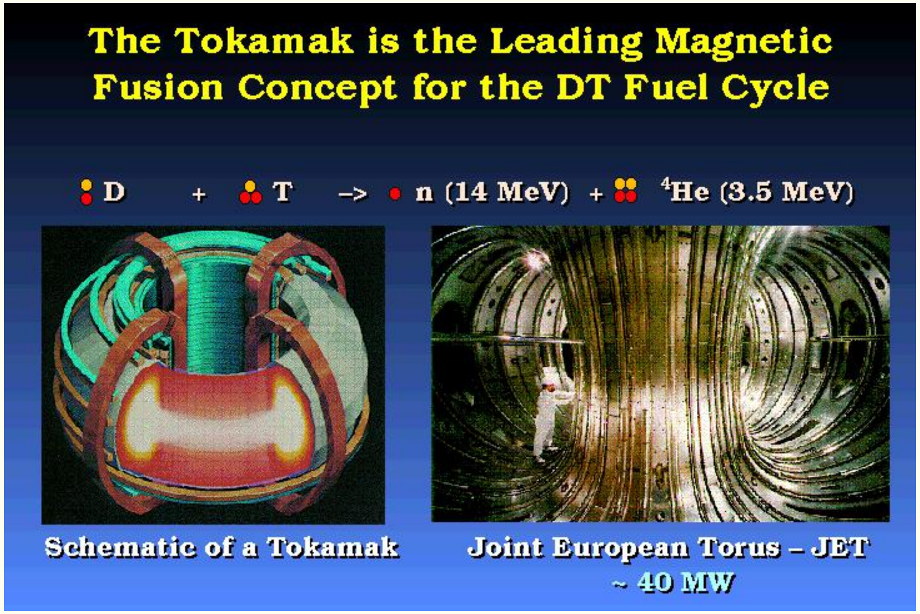
B. Decreases as $1/r^2$.

C. Is zero.



A surface spanning the dotted circle will have zero total current penetrating—as

Toroids


The Tokamak is the Leading Magnetic Fusion Concept for the DT Fuel Cycle
D + T –> n (14 MeV) + 4He (3.5 MeV)
Schematic of a Tokamak
Joint European Torus – JET
~ 40 MW

§ .4 带电粒子在磁场中的运动

The Motion of a Charged Particle in a Magnetic Field

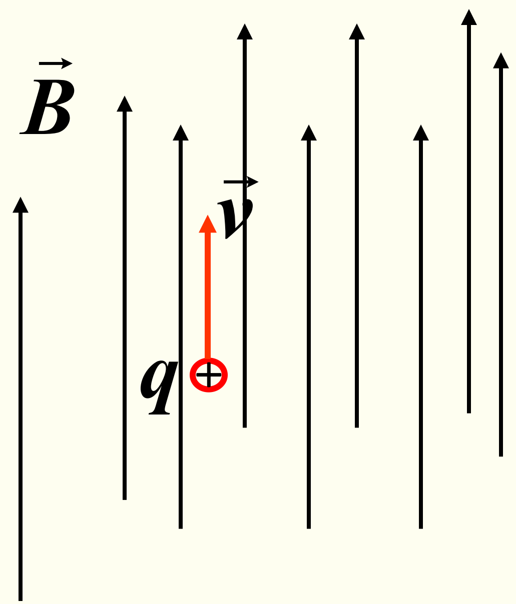
(1) A charged particle moves parallel to a constant magnetic field

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$F = qvB\sin\theta$$

$$\theta = 0$$

$$F = 0$$



匀速直线运动

uniform straight line motion

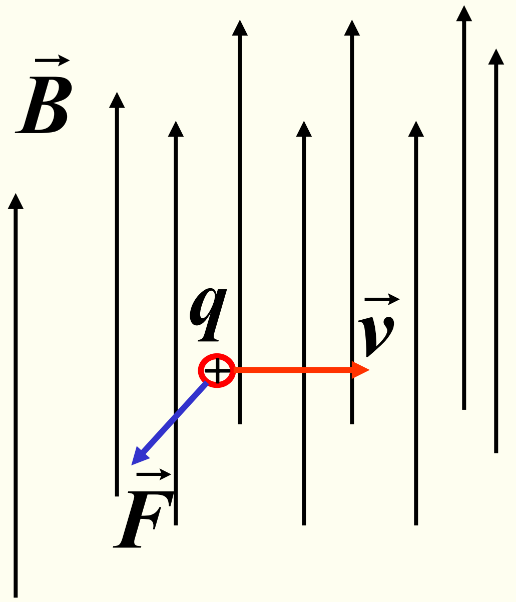
(2) A charged particle moves perpendicularly to a constant magnetic field

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$F = qvB\sin\theta = qvB$$

匀速圆周运动

uniform circular motion



$$qvB = m\frac{v^2}{R}$$

$$R = \frac{mv}{qB}$$

$$R = \frac{mv}{qB}$$

uniform circular motion

周期
The period

$$T = \frac{2\pi R}{v} = \frac{2\pi m}{qB}$$

频率
The frequency

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

$$\frac{q}{m}$$

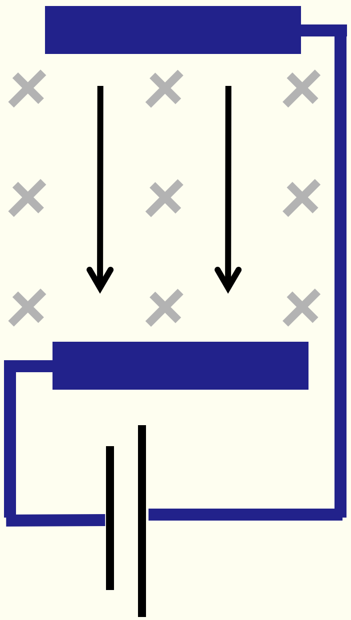
荷质比
charge-to-mass ratio

The period and frequency are independent of the speed.

All particles with the same charge-to-mass ratio take the same time T (the period) to complete one round trip.

Magnetohydrodynamic Drive

- Seawater conducts electricity: the idea behind the Red October silent drive was that an electric current through seawater.



(3)Initial velocity is not perpendicular to magnetic field

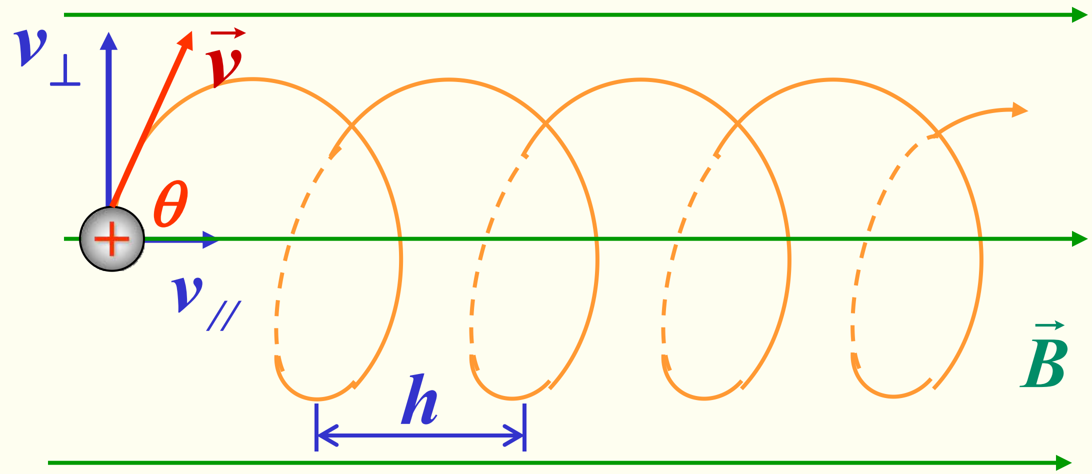
The particle will move in a helical path about the direction of the field vector 螺旋运动

$$v_{//} = v\cos\theta$$

uniform straight line motion

$$v_{\perp} = v\sin\theta$$

uniform circular motion

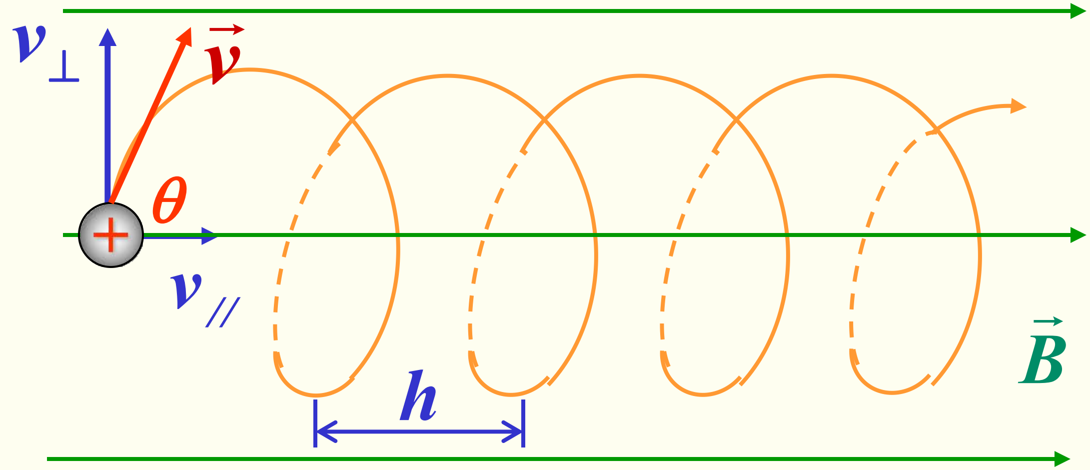


pitch of the helix 螺距 **the distance between adjacent turns**

The velocity vector $\vec{v}$ of such a particle resolved into two components, one parallel to $\vec{B}$ and one perpendicular to it.

move in a helical path

螺旋运动

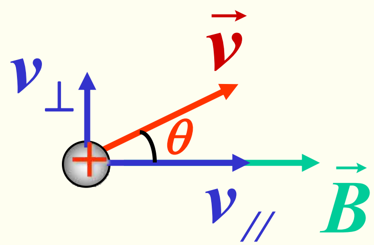


$$T = \frac{2\pi m}{qB}$$

pitch of the helix 螺距

$$h = v_{//} T = \frac{2\pi m}{qB} v\cos\theta$$

A narrow stream of charged particles is emitted from a point in the magnetic field



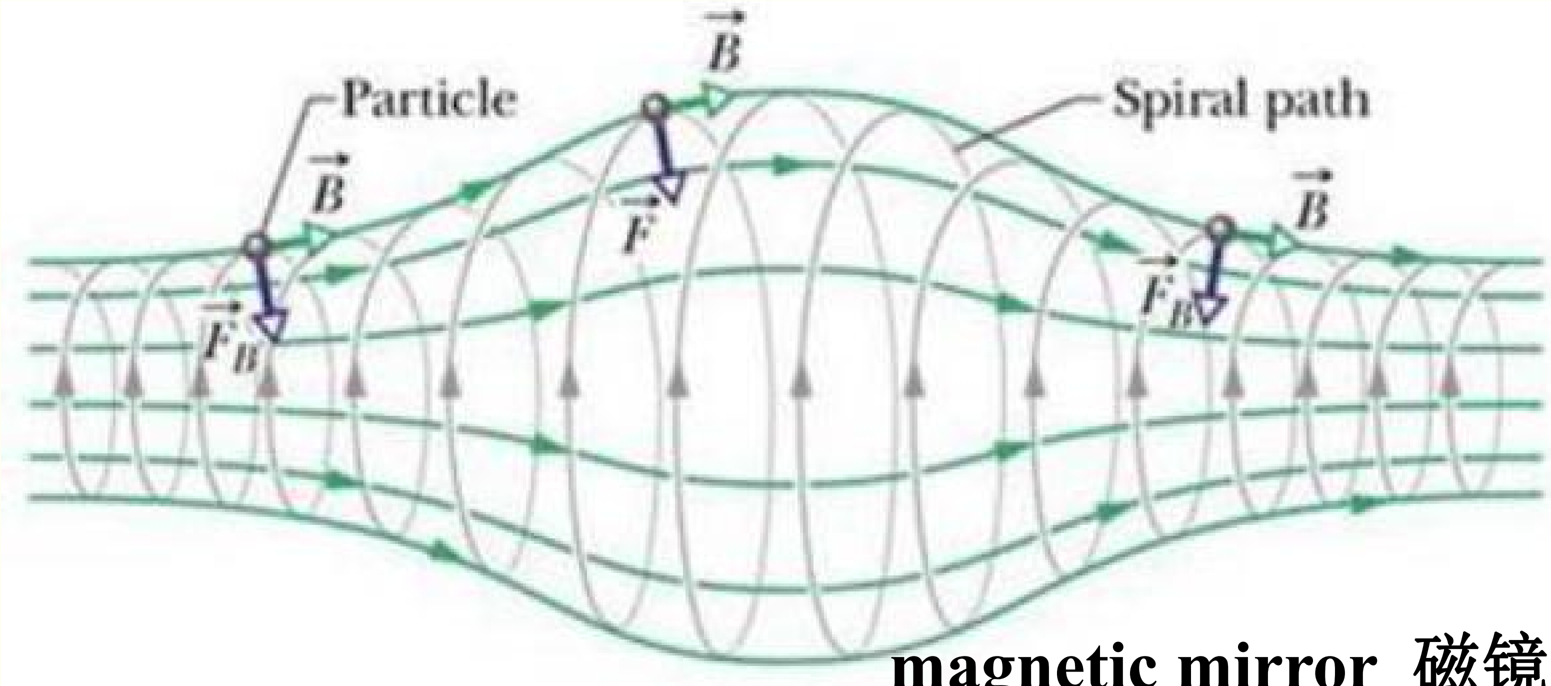
$$v_{//} = v\cos\theta \approx v$$

$$v_\perp = v\sin\theta \approx v\theta$$

$$h = \frac{2\pi m}{qB} v_{//} = \frac{2\pi m}{qB} v$$

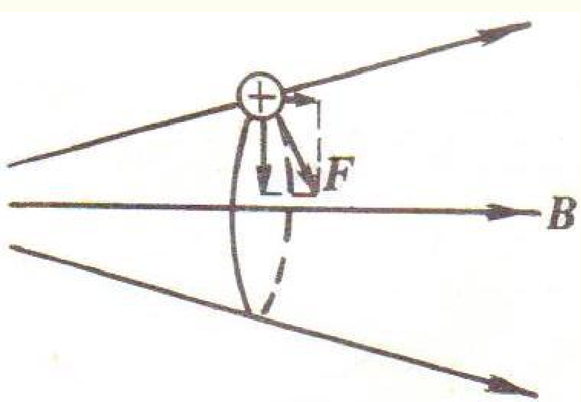
CAI

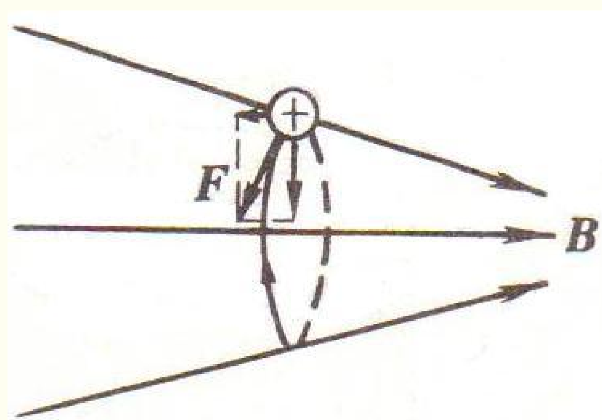
Magnetic focusing 磁聚焦



magnetic mirror 磁镜

The particle can become trapped, spiraling back and forth between the strong field regions at either end.

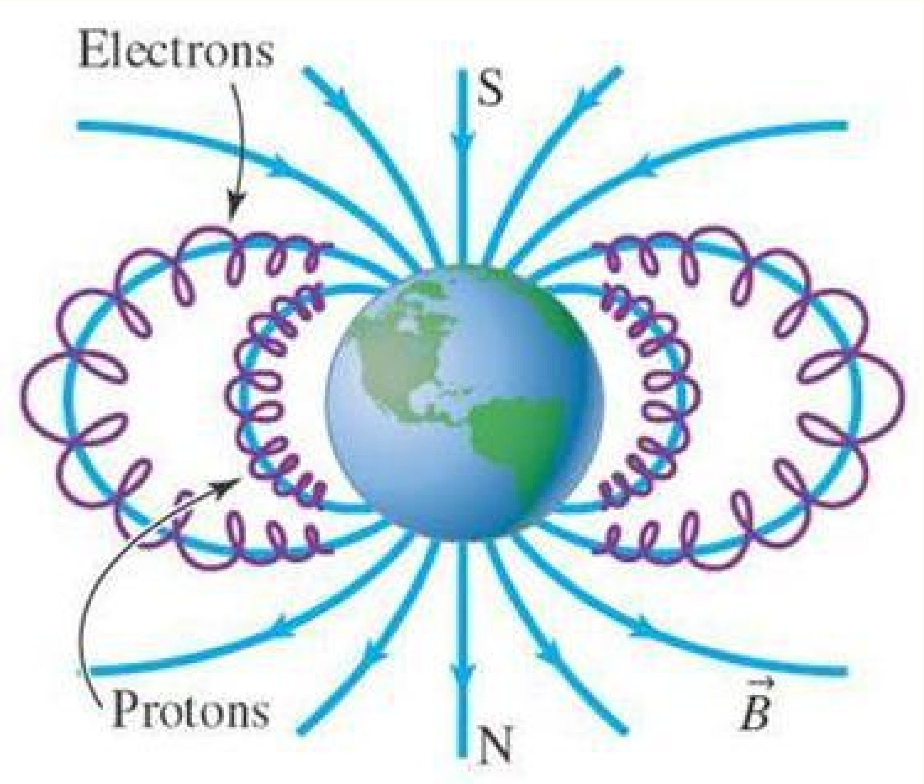




The magnetic force vectors at the left and right sides have a component pointing toward the the center of the figure.

范艾仑（Van Allen）辐射带

- Van Allen belt



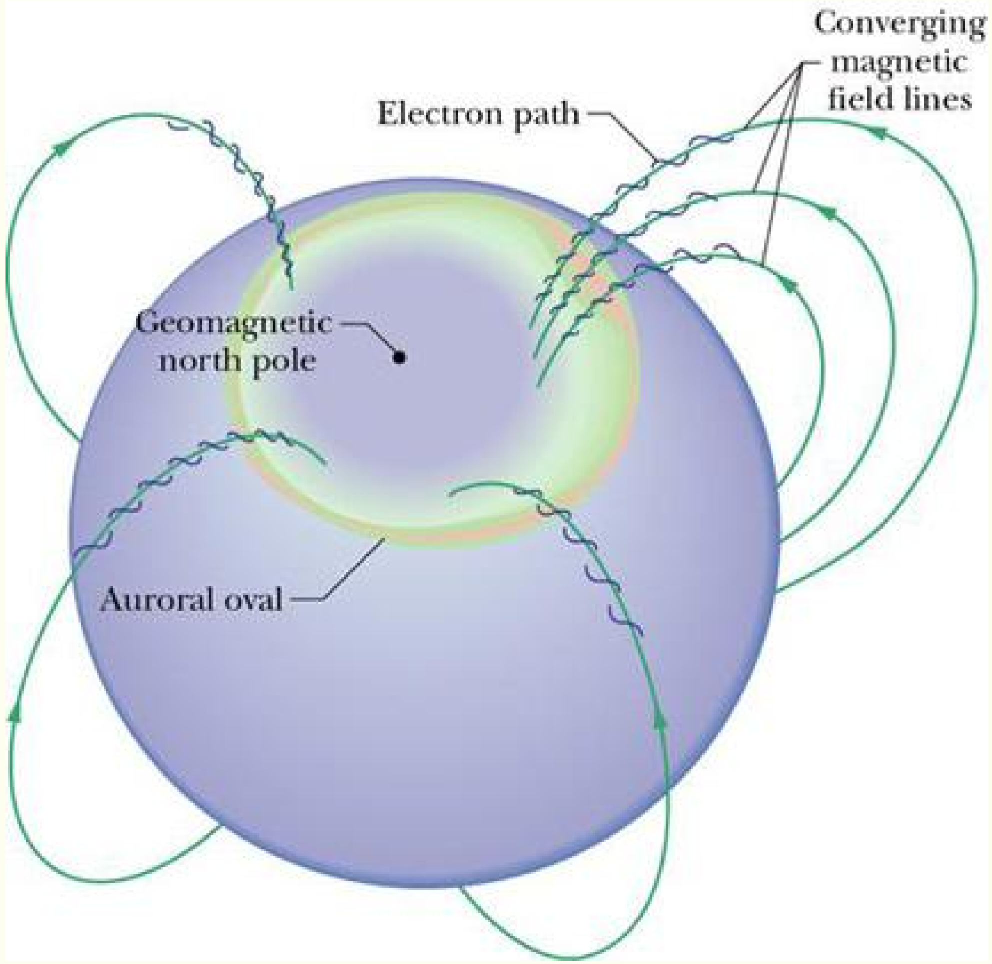
1958年

探索者1号卫星

赤道上空3000km

赤道上空15000km

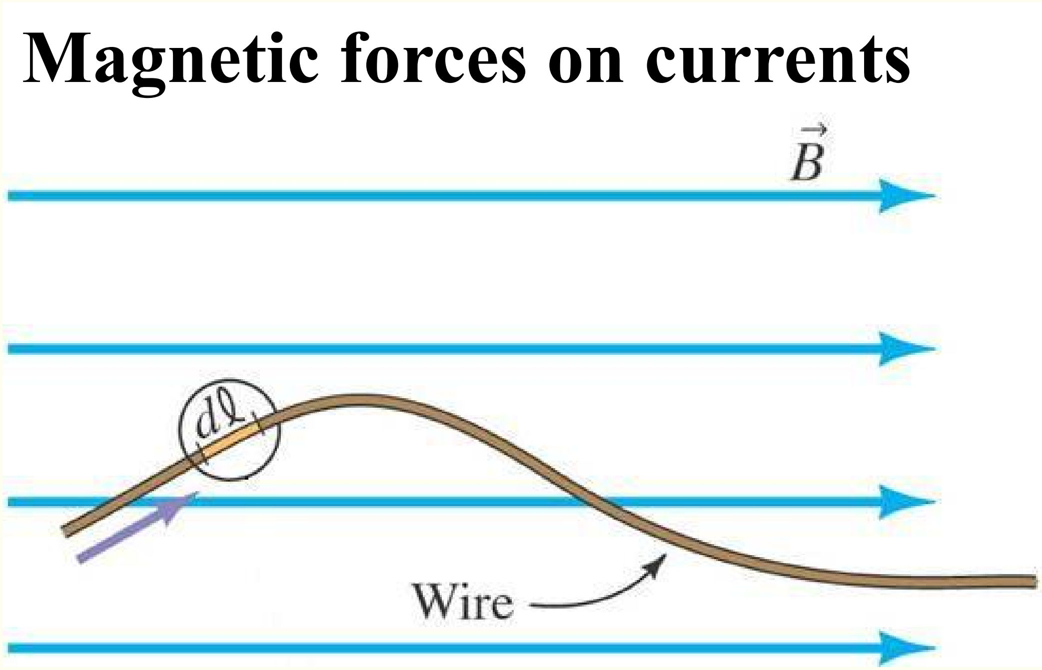
Cosmic rays, solar wind



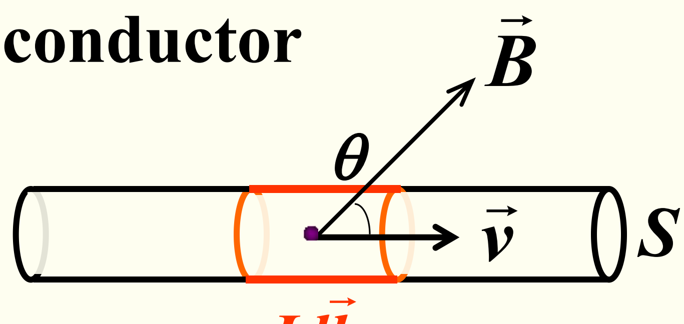
The auroral oval surrounding Earth's geomagnetic north pole. Magnetic field lines converge toward that pole. Electrons moving toward Earth are "caught by" and spiral around these field lines, entering the terrestrial atmosphere at high latitudes and producing aurora within the oval.

aurora

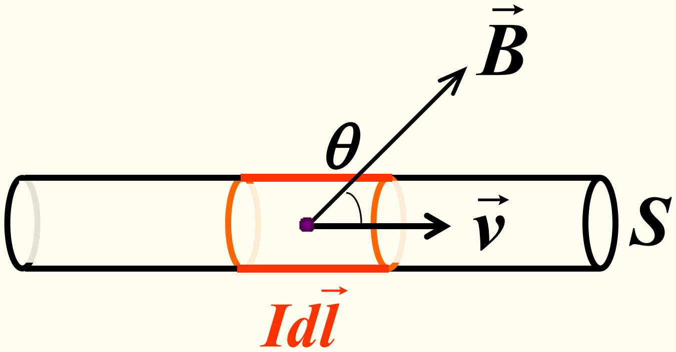
§ .5 The effect of magnetic field on current carrying wire



The effect of magnetic field on current carrying conductor is actually the macroscopic manifestation of Lorentz force on a large number of free carriers in the current carrying conductor



a charge carrier: $\vec{f} = q\vec{v} \times \vec{B}$



a charge carrier: $\vec{f} = q\vec{v} \times \vec{B}$

The value of the magnetic field $\vec{B}$ is uniform in the range of the current element $Id\vec{l}$ ⇨ $nSdl$ n Carrier number density

$$d\vec{F} = nSdl(q\vec{v} \times \vec{B}) = nSv(qd\vec{l} \times \vec{B})$$

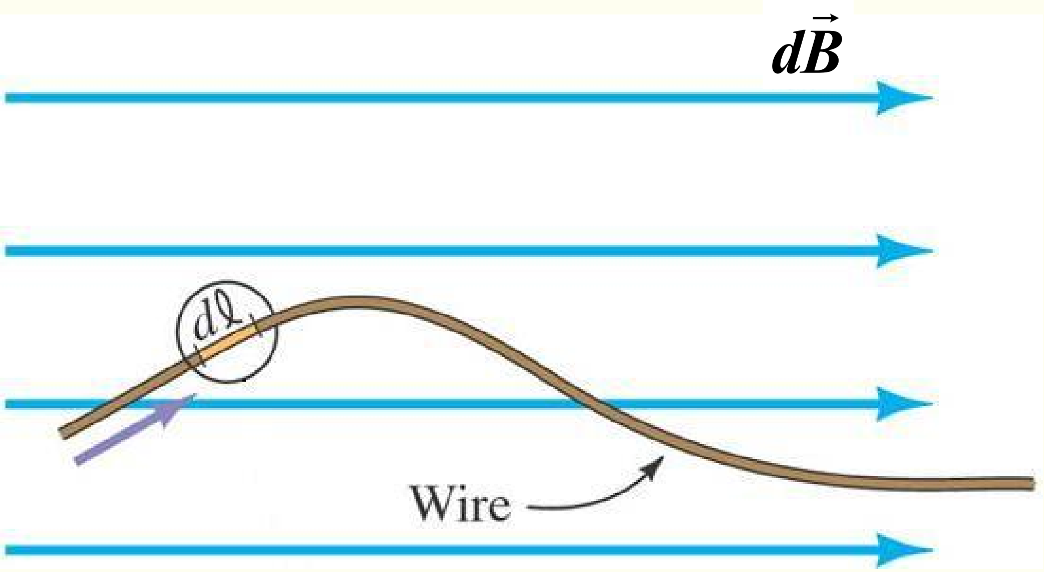
$$I = qnvS$$

$$d\vec{F} = I(d\vec{l} \times \vec{B}) = Id\vec{l} \times \vec{B}$$

$$d\vec{F} = Id\vec{l} \times \vec{B}$$

安培力
Ampère force

$$d\vec{F} = Id\vec{l} \times \vec{B}$$ **Ampère force 安培力**



Magnetic force on finite wire with current

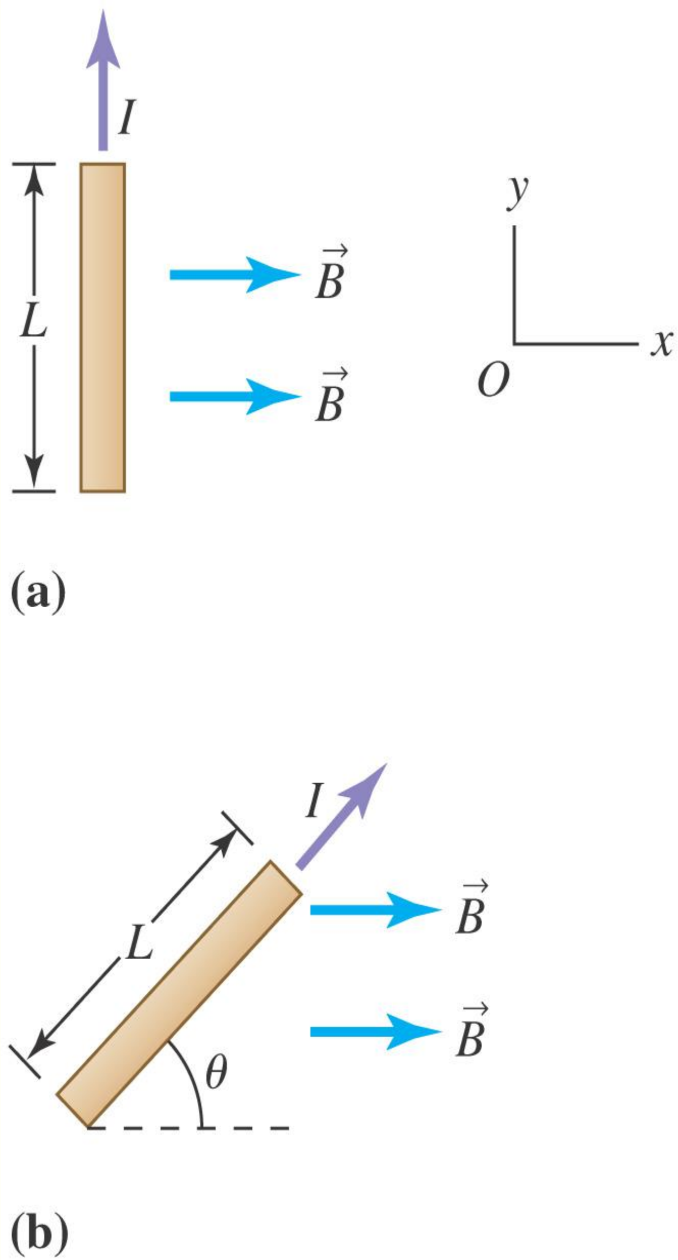
$$\vec{F} = \int_L Id\vec{l} \times \vec{B}$$

$$Id\vec{l} \to I_2 d\vec{l}_2$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I_1 d\vec{l}_1 \times \hat{r}_1}{r_1^2}$$

$$d\vec{F}_{21} = I_2 d\vec{l}_2 \times \frac{\mu_0}{4\pi} \frac{I_1 d\vec{l}_1 \times \hat{r}_{21}}{r_{21}^2}$$

$$d\vec{F}_{21} = \frac{\mu_0}{4\pi} \frac{I_2 d\vec{l}_2 \times (I_1 d\vec{l}_1 \times \hat{r}_{21})}{r_{21}^2}$$ **Ampère Law**



Finite wire with current

$$d\vec{F} = I d\vec{l} \times \vec{B}$$

$$d\vec{F} = I(dl\hat{j}) \times (B\hat{i})$$

$$= I(dl)B(\hat{j} \times \hat{i})$$

$$= I(dl)B(-\hat{k})$$

$$\vec{F} = I\int_L dlB(-\hat{k}) = IB(-\hat{k})\int_L dl$$

$$\vec{F} = -IBL\hat{k}$$

$$d\vec{F} = I(dl)B\sin\theta(-\hat{k})$$

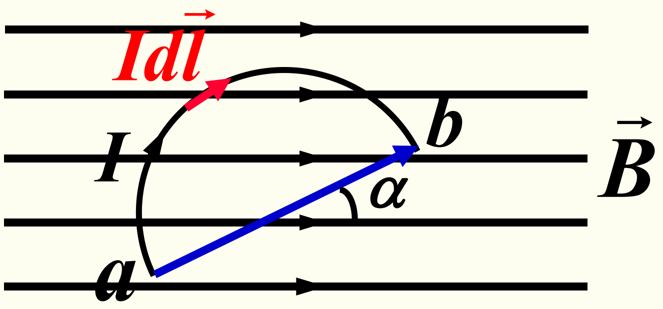
$$\vec{F} = -IBL\sin\theta\hat{k}$$

A half-circle wire of radius R carries a current I. The wire is placed in a uniform magnetic field. The angle between the path from one end a to another end b of the wire and the magnetic field is $\alpha=30^{\circ}$. What is the magnetic force on the wire?

infinitesimal wire with current $Id\vec{l}$

$$\overline{ab} = 2R$$

$$\vec{F} = \int_{ab} Id\vec{l} \times \vec{B}$$



uniform magnetic field

$$= I\left[\int_{ab} d\vec{l}\right] \times \vec{B} = I\overrightarrow{ab} \times \vec{B}$$

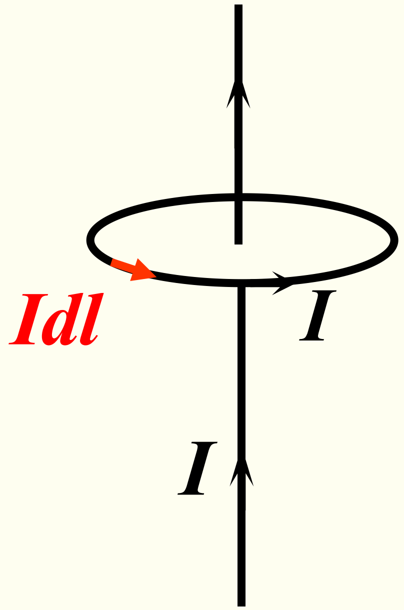
direction: $\otimes\ \vec{F}$

$$\left|\vec{F}\right| = I\left|\overrightarrow{ab}\right| B \sin\alpha = IBR$$

A long straight wire passes through the center of circular flat loop vertically, both wires carrying current *I*. what is the interactive force?

$$d\vec{F} = Id\vec{l} \times \vec{B}$$

infinitesimal wire with current $Id\vec{l}$



$$\vec{F} = \oint_L Id\vec{l} \times \vec{B}$$

$$\left| Id\vec{l} \times \vec{B} \right| = 0$$

$$\vec{F} = 0$$

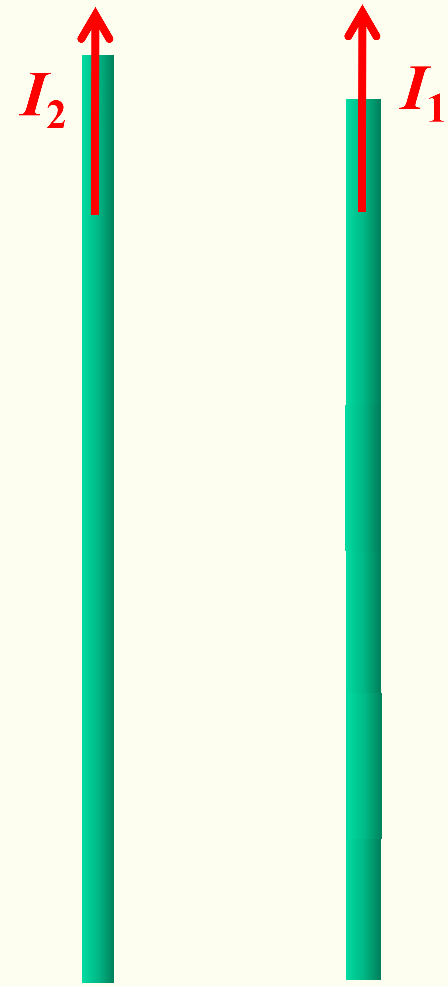
Clicker Question

- Currents flow in the same direction in parallel wires.
- Do the wires

A. Repel each other?

B. Attract each other?

C. Neither attract nor repel?



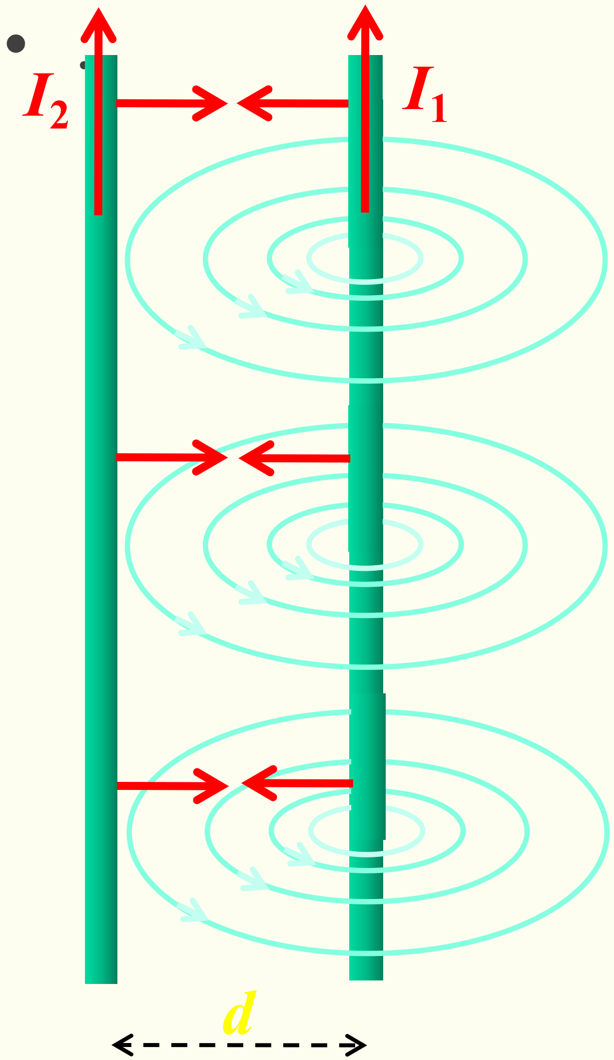
Force Between Parallel Wires

- The field from current I_1 is $B = \frac{\mu_0}{2\pi}\frac{I_1}{r}$, circling the wire, and the current I_2 will feel a force $I_2 \vec{\ell} \times \vec{B}$ per length ℓ, so the force per meter on wire 2 is

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

towards wire 1, and wire 1 will feel the opposite force.

- Like currents attract.



Definition of the Ampère and Coulomb

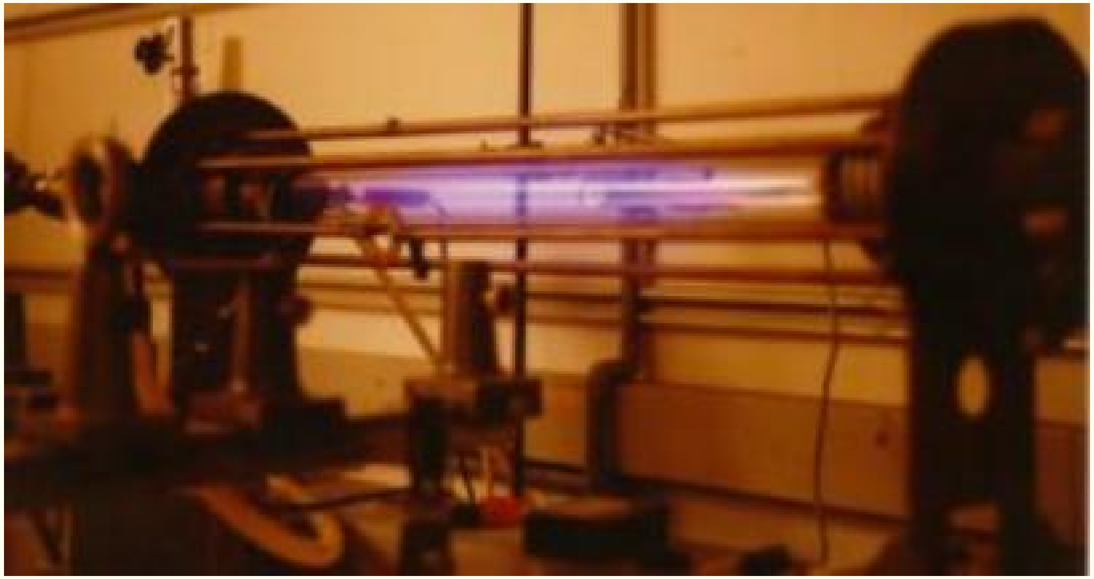
- In the formula for the attraction between long parallel wires carrying steady currents

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

the constant μ_0 has <u>precisely</u> the value $4\pi \times 10^{-7}$.

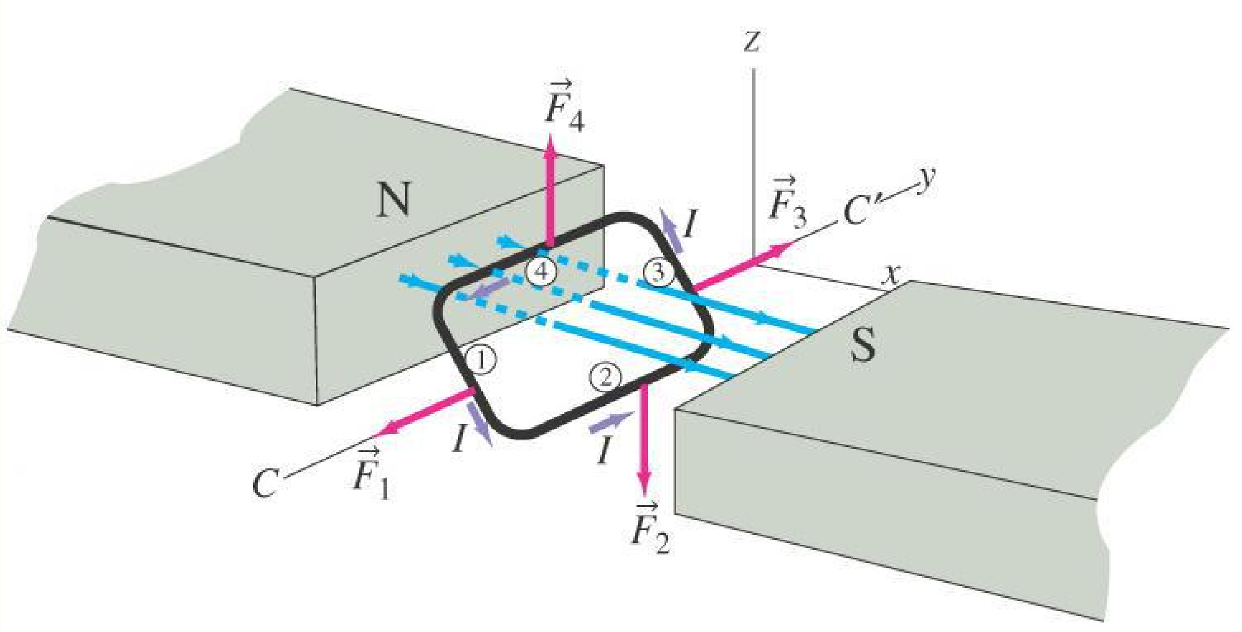
- Fixing μ_0 <u>defines</u> the unit of current, the ampère, as that current which in a long wire one meter away from an equal current feels a force of $\mu_0/2\pi$ N/m—and 1 amp = 1 coulomb/sec.

Like Currents Attracting



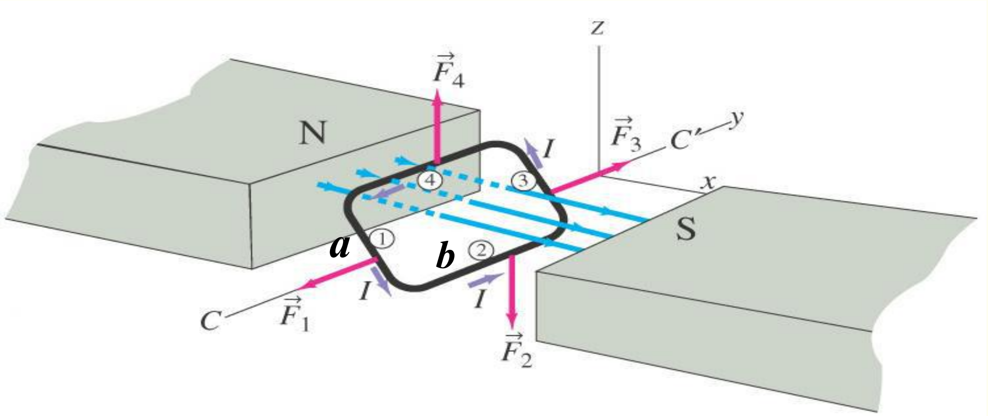
- The picture on the left is of a copper pipe used as a lightning conductor—after it conducted. The parallel currents all attracted each other.
- On the right, an intense current is sent through a plasma—the self compression generates intense heat. The hope is to induce thermonuclear fusion.

Magnetic force on current loops



CAI

A rectangular loop of wire, carrying a current and free to rotate about a fixed axis, is placed in a magnetic field. Magnetic forces on the wire produce a torque that rotates it.



$$F_1 = F_3 = BIa\sin\theta$$

$$F_2 = F_4 = BIb$$

$$\vec{F} = 0$$

The moment of force:

CAI

$$M = \frac{a}{2}F_2 b\sin\phi + \frac{a}{2}F_4 b\sin\phi$$

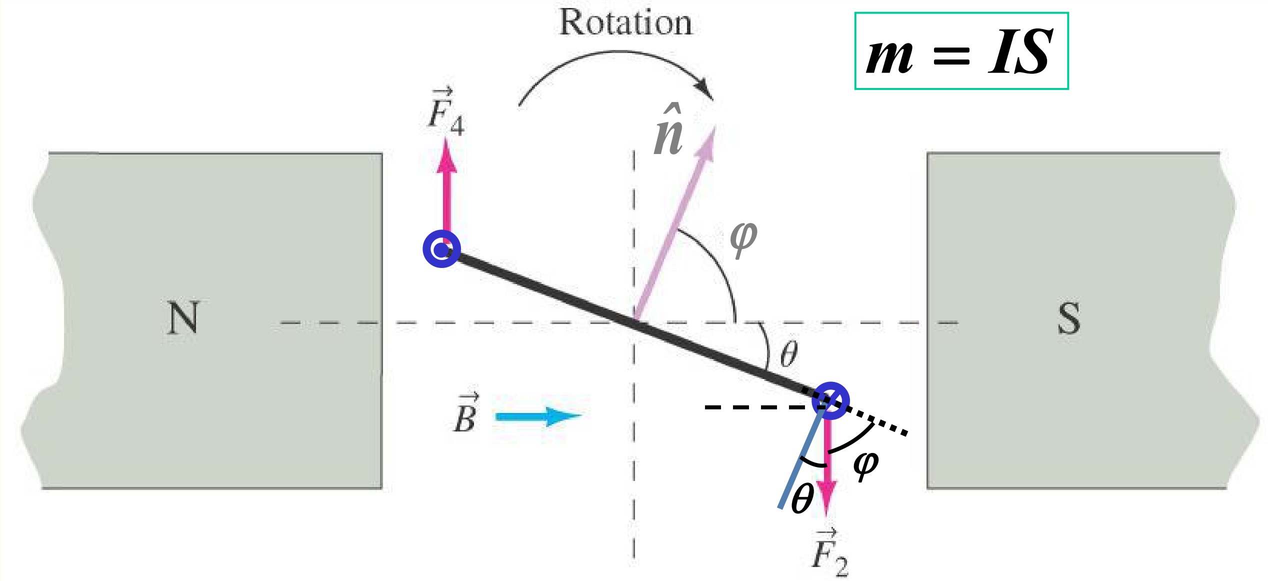
$$= BIba\sin\varphi = BIS\sin\varphi = mB\sin\varphi$$

N turns

$$M = NBIS\sin\varphi$$

$$= mB\sin\varphi$$

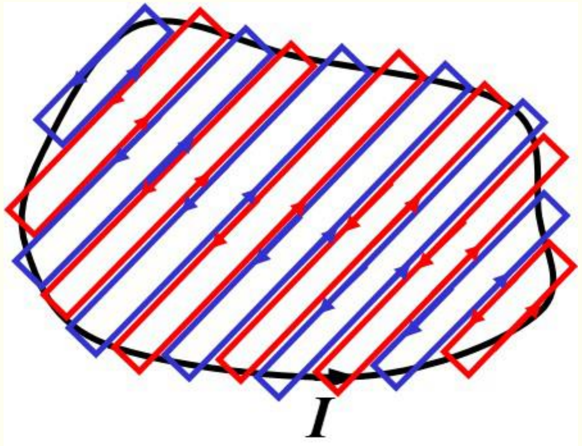
$$m = NIS$$



$$\boxed{m = IS}$$

$$\vec{M} = \vec{m} \times \vec{B}$$

An arbitrary shape flat loop of wire , carrying current *I* through uniform magnetic field.



$$\vec{F}=0$$

Infinitesimal rectangular loops of wire, carrying current *I*

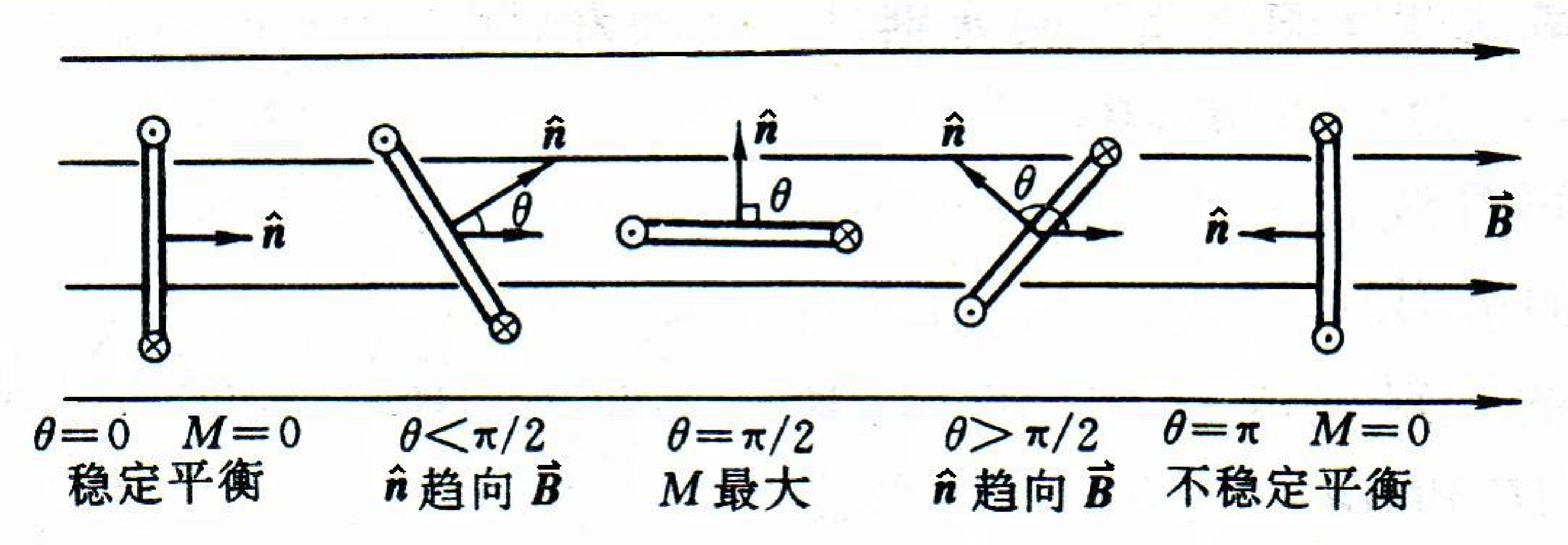
$$d\vec{m}=IdS\hat{n} \qquad d\vec{M}=d\vec{m}\times\vec{B}$$

$$\vec{M}=\int d\vec{M}=\int(d\vec{m}\times\vec{B})$$

$$=\int(IdS\hat{n}\times\vec{B})=I(\int dS)\,\hat{n}\times\vec{B}$$

$$=IS\hat{n}\times\vec{B}=\vec{m}\times\vec{B}$$

$$\vec{M}=\vec{m}\times\vec{B}$$

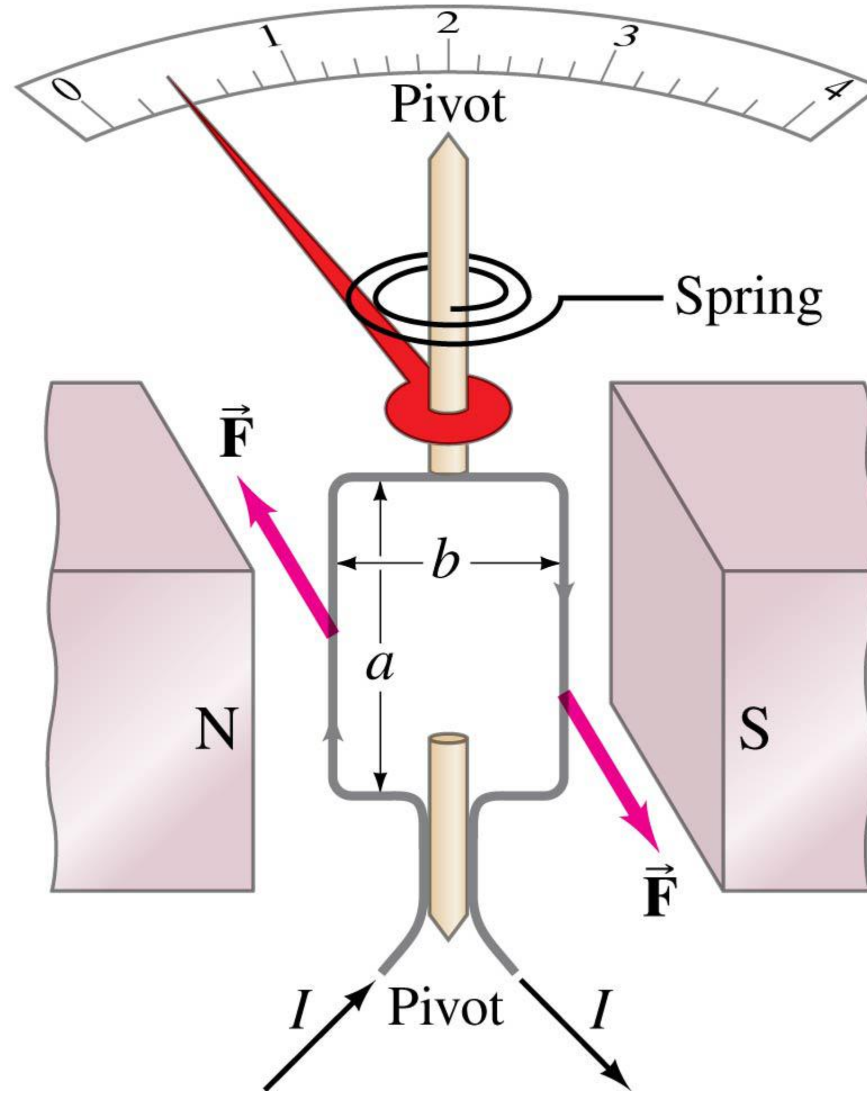


$\hat{n}$
θ
$\vec{B}$
$\theta=0 \quad M=0$
稳定平衡
$\theta<\pi/2$
$\hat{n}$ 趋向 $\vec{B}$
$\theta=\pi/2$
M 最大
$\theta>\pi/2$
$\hat{n}$ 趋向 $\vec{B}$
$\theta=\pi \quad M=0$
不稳定平衡

CAI

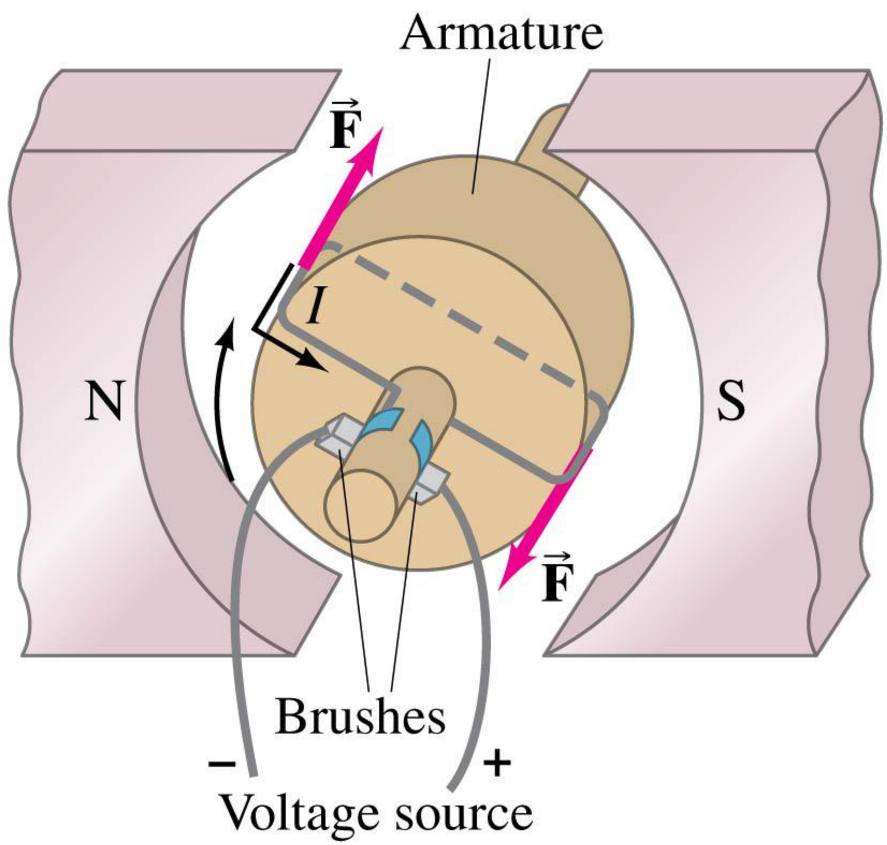
20-10 Applications: Galvanometers, Motors, Loudspeakers

A galvanometer takes advantage of the torque on a current loop to measure current.



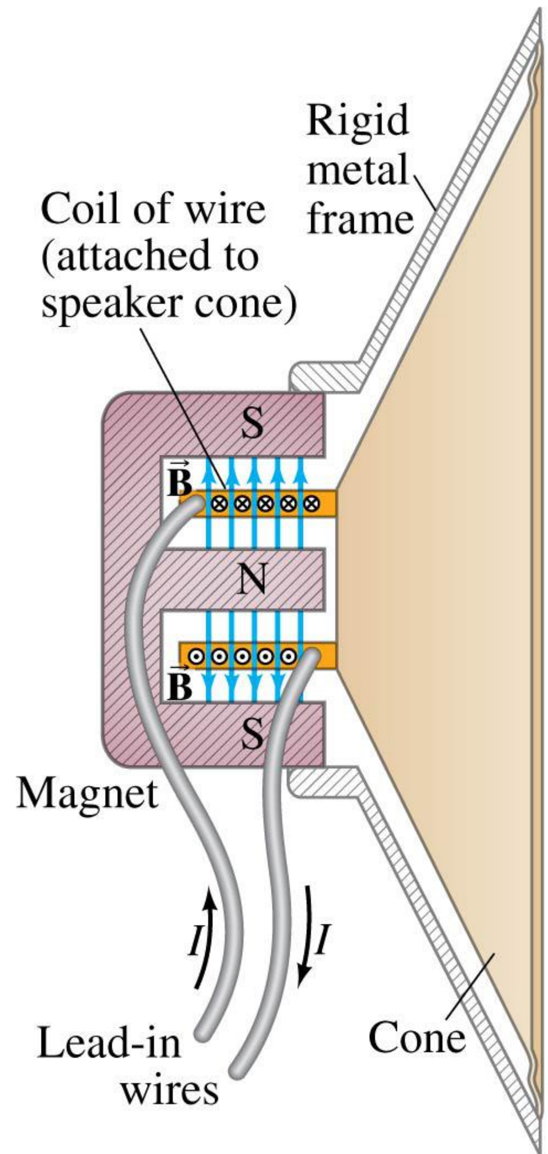
20-10 Applications: Galvanometers, Motors, Loudspeakers

An electric motor also takes advantage of the torque on a current loop, to change electrical energy to mechanical energy.



20-10 Applications: Galvanometers, Motors, Loudspeakers

Loudspeakers use the principle that a magnet exerts a force on a current-carrying wire to convert electrical signals into mechanical vibrations, producing sound.



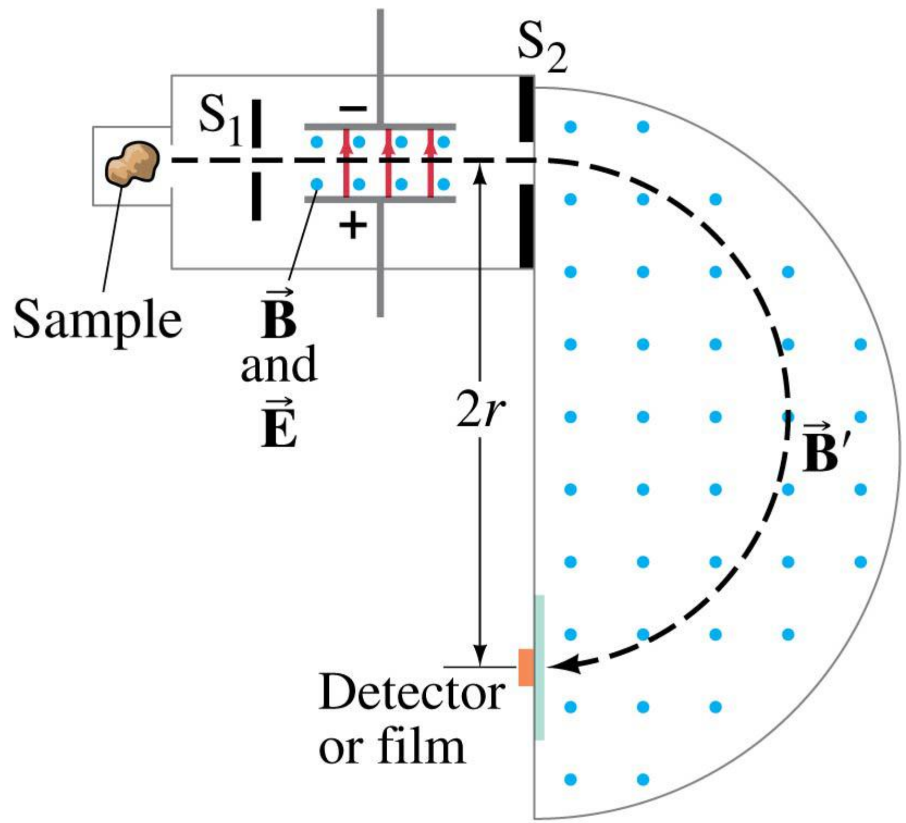
20-11 Mass Spectrometer

A mass spectrometer measures the masses of atoms. If a charged particle is moving through perpendicular electric and magnetic fields, there is a particular speed at which it will not be deflected:

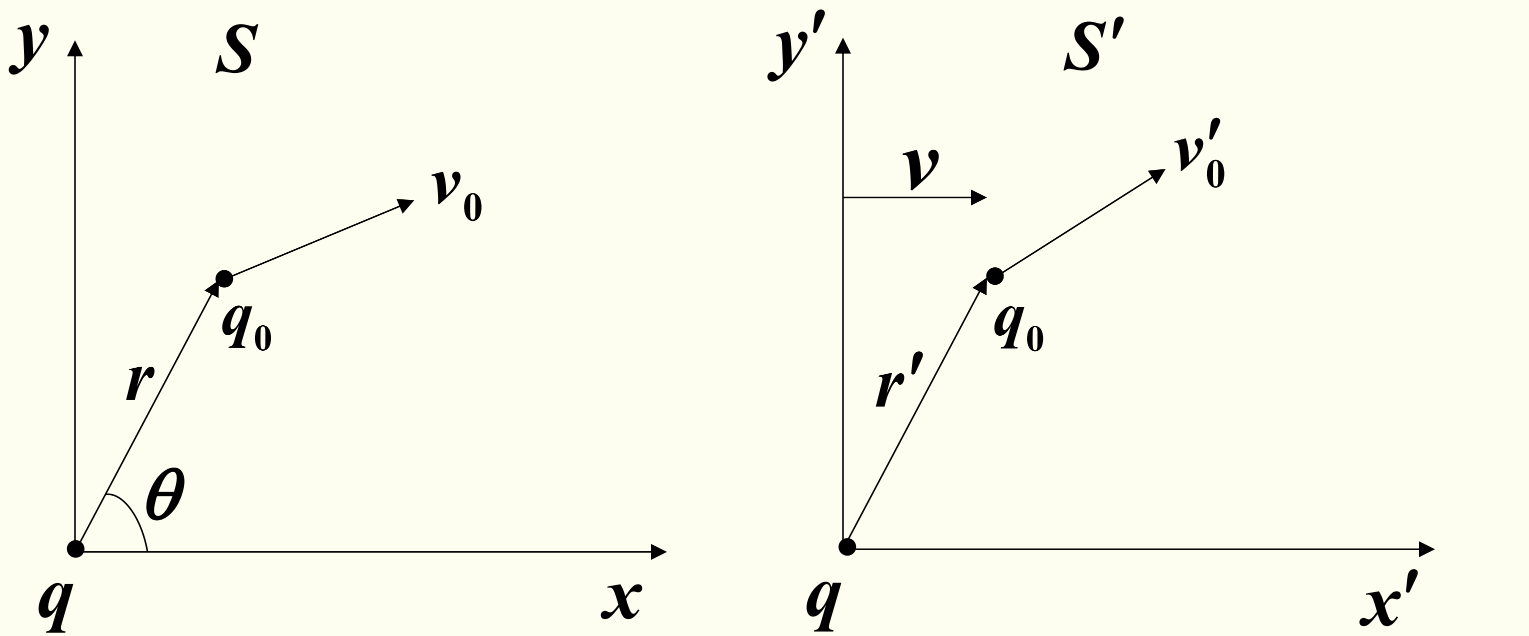
$$v = \frac{E}{B}.$$

20-11 Mass Spectrometer

All the atoms reaching the second magnetic field will have the same speed; their radius of curvature will depend on their mass.



电磁场的相对论性变换

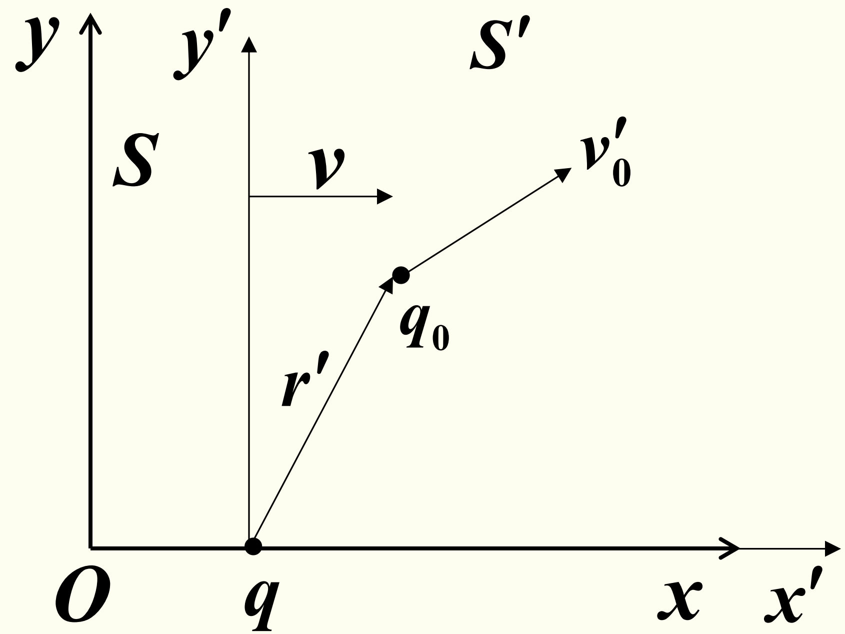


在 S' 中，有一点电荷q 在坐标原点 O' 处，并与S'系保持相对静止，另有一点电荷q_0在 S' 系中以速度$\vec{v}'_0$运动

在S系中，运动电荷q_0所受的作用力?

在S'系中计算点电荷q所产生的静电场$\vec{E}'$

$$\vec{E}' = \frac{q\vec{r}'}{4\pi\varepsilon_0 r'^3}$$



q_0在S'系中所受的电场力

$$\vec{F}' = q_0\vec{E}' = q_0\frac{q\vec{r}'}{4\pi\varepsilon_0 r'^3}$$

q_0在S系中所受的力

$$\vec{F} = q_0\frac{q\vec{r}}{4\pi\varepsilon_0 r^3}\cdot\frac{1-\dfrac{v^2}{c^2}}{(1-\dfrac{v^2}{c^2}\sin^2\theta)^{3/2}} + q_0\vec{v}_0\times\frac{q\vec{v}\times\vec{r}}{4\pi\varepsilon_0 c^2 r^3}\frac{1-\dfrac{v^2}{c^2}}{(1-\dfrac{v^2}{c^2}\sin^2\theta)^{3/2}}$$

$$= q_0\vec{E} + q_0\vec{v}_0\times\vec{B}$$

q_0在 S 系中所受的力

$$\vec{F}=q_0\frac{q\vec{r}}{4\pi\varepsilon_0 r^3}\cdot\frac{1-\frac{v^2}{c^2}}{(1-\frac{v^2}{c^2}\sin^2\theta)^{3/2}}+q_0\vec{v}_0\times\frac{q\vec{v}\times\vec{r}}{4\pi\varepsilon_0 c^2 r^3}\frac{1-\frac{v^2}{c^2}}{(1-\frac{v^2}{c^2}\sin^2\theta)^{3/2}}$$

$$=q_0\vec{E}+q_0\vec{v}_0\times\vec{B}$$

$$\vec{E}=\frac{q\vec{\mathrm{r}}}{4\pi\varepsilon_0 r^3}\cdot\frac{1-\frac{v^2}{c^2}}{(1-\frac{v^2}{c^2}\sin^2\theta)^{3/2}}$$

$$\vec{B}=\frac{q\vec{v}\times\vec{r}}{4\pi\varepsilon_0 c^2 r^3}\frac{1-\frac{v^2}{c^2}}{(1-\frac{v^2}{c^2}\sin^2\theta)^{3/2}}$$

$$\vec{E} = \frac{q\vec{r}}{4\pi\varepsilon_0 r^3} \cdot \frac{1-\frac{v^2}{c^2}}{(1-\frac{v^2}{c^2}\sin^2\theta)^{3/2}}$$

（1）运动电荷q 的电场 $\vec{E}$

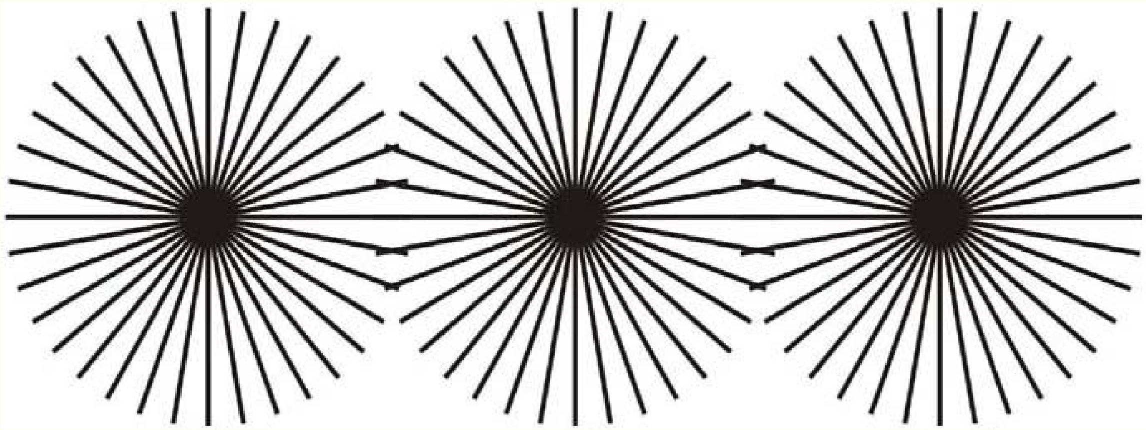
$$\frac{1-\frac{v^2}{c^2}}{(1-\frac{v^2}{c^2}\sin^2\theta)^{3/2}} \to 1 \qquad \vec{E} = \frac{q\vec{r}}{4\pi\varepsilon_0 r^3}$$

If v is small，i.e. $\frac{v}{c} \ll 1$

The field has the same form as in electrostatics, which is spherically symmetric. In this approximation the field is the Coulomb field centered at the moving position of q.

$$\vec{E} = \frac{q\vec{r}}{4\pi\varepsilon_0 r^3} \cdot \frac{1-\dfrac{v^2}{c^2}}{\left(1-\dfrac{v^2}{c^2}\sin^2\theta\right)^{3/2}}$$

$$\frac{v}{c} << 1$$



Coulomb field

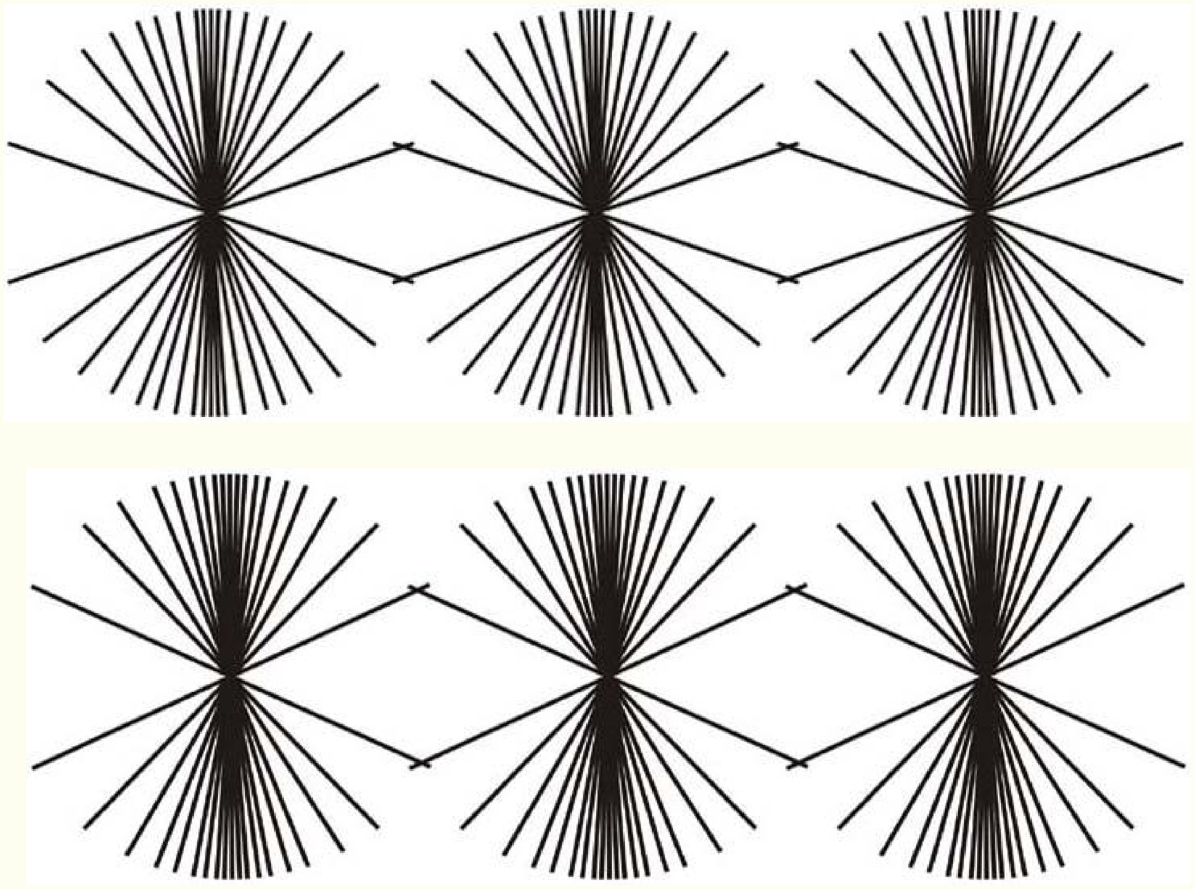
$$\vec{E} = \frac{q\vec{r}}{4\pi\varepsilon_0 r^3}$$

If v is large， approaching c， then the electric field is quite different from the Coulomb field.

In the direction of motion, $\theta = 0$, the magnitude of the field is decreased by the factor $(1-\dfrac{v^2}{c^2})$

In the direction perpendicularly to the direction of motion, $\theta = \dfrac{\pi}{2}$, the magnitude of the field is increased by the factor $\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$.

$$\vec{E} = \frac{q\vec{\mathbf{r}}}{4\pi\varepsilon_0 r^3}\cdot\frac{1-\dfrac{v^2}{c^2}}{\left(1-\dfrac{v^2}{c^2}\sin^2\theta\right)^{3/2}}$$



$v \to c$

(2) 运动电荷q 的磁场$\vec{B}$

使一探测电荷 q_0 静止于电场和磁场均存在的空间 P点，测量它的受力

$$\vec{F} = q_0\vec{E} + q_0\vec{v}_0 \times \vec{B}$$

此力为电场力 $\vec{F}_e$

再使 q_0 以某一速度 $\vec{v}_0$ 通过P 点，测出力 $\vec{F}$

合力

则磁场力 $\vec{F}_m = \vec{F} - \vec{F}_e$

$$\vec{F} = q_0\vec{E} + q_0\vec{v}_0 \times \vec{B}$$

改变 $\vec{v}_0$ 方向测量磁场力，P 点处有一个特殊方向，当 $\vec{v}_0$ 沿此方向通过 P点时，不管其速率多大，测得的磁场力恒为零，则磁场$\vec{B}$ 就在此方向上。

当在垂直此方向通过P 点时，磁场力有最大值 F_{max}，于是，我们定义磁感应强度 $\vec{B}$ 的大小为

$$B = \frac{F_{max}}{q_0 v_0}$$

$$B = \frac{F}{q_0 v_0 \sin\theta}$$

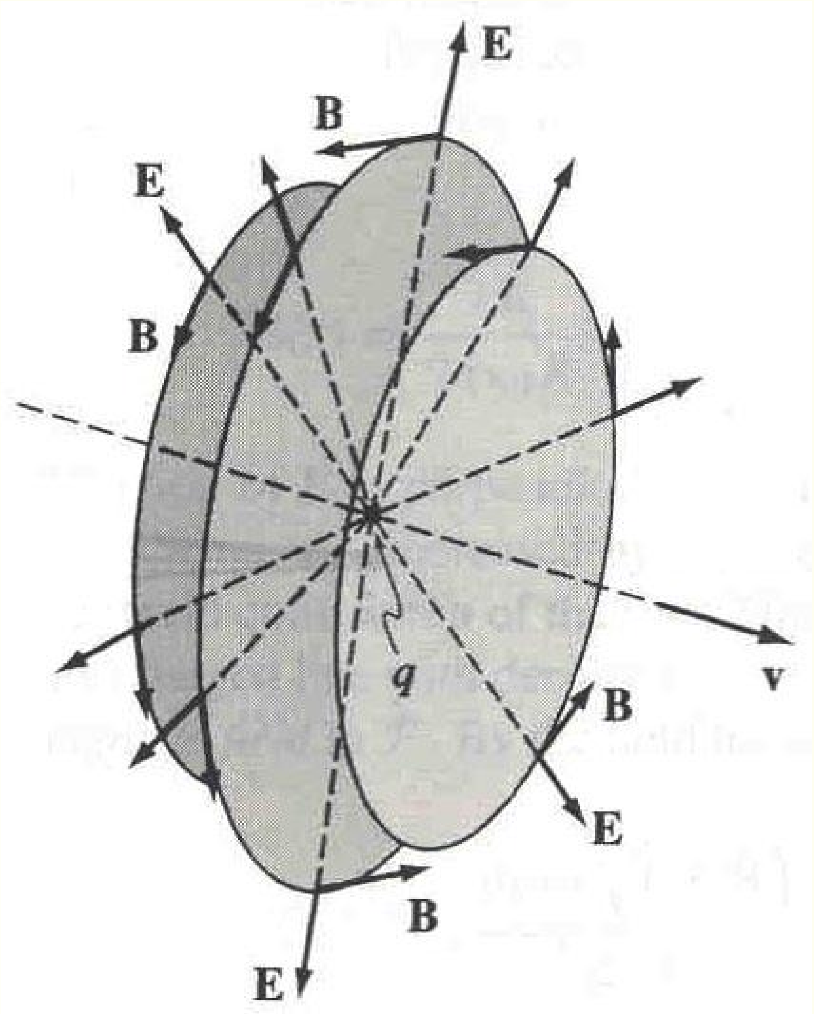
$$\vec{B}=\frac{q\vec{v}\times\vec{r}}{4\pi\varepsilon_0 c^2 r^3}\frac{1-\frac{v^2}{c^2}}{(1-\frac{v^2}{c^2}\sin^2\theta)^{3/2}}$$

If the speed of charge *v* is small

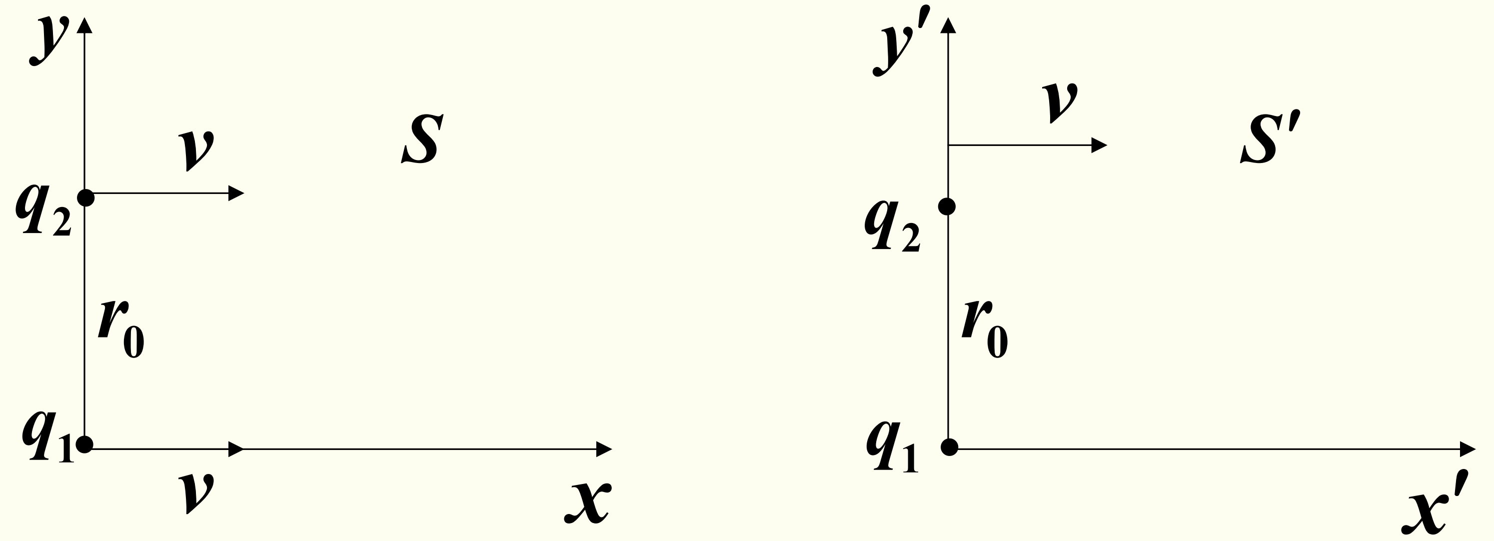
$$\frac{v^2}{c^2}<<1$$

$$\vec{B}=\frac{q\vec{v}\times\vec{r}}{4\pi\varepsilon_0 c^2 r^3}$$

磁感应线是以电荷运动轨迹为轴的一组同心圆



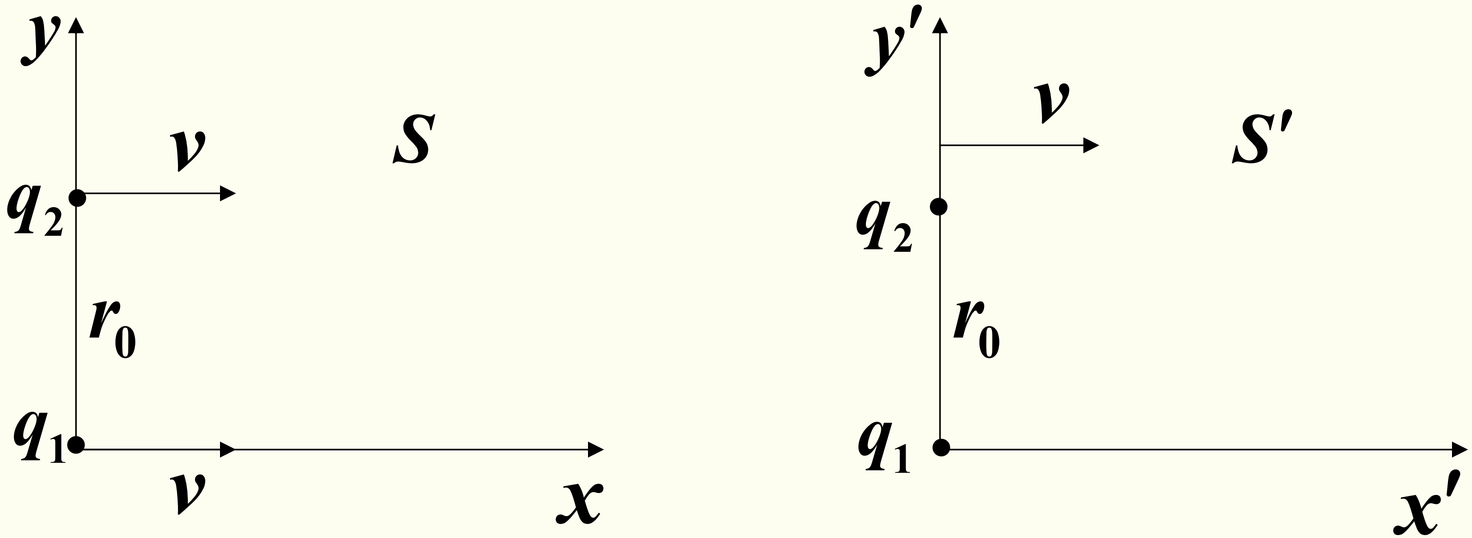
（3）磁场力是对电场力的一种相对论修正



在 S' 中，q_1、q_2 相对于参考系静止

仅观测到静电场力 $F'_e = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r_0^{\ 2}}$

（3）磁场力是对电场力的一种相对论修正



在 S 中，$\theta=\pi/2$

电场力 $F_e=\dfrac{q_1q_2}{4\pi\varepsilon_0 r_0^{\,2}}\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$

磁场力 $F_m=\dfrac{q_1q_2v^2}{4\pi\varepsilon_0 c^2 r_0^{\,2}}\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$

返回 退出

作用在电荷q_1 或 q_2 上的合力为 $\vec{F} = \vec{F}_e + \vec{F}_m$

$$F = F_e - F_m = \frac{q_1 q_2}{4\pi\varepsilon_0 r_0^{2}} \cdot \sqrt{1 - \frac{v^2}{c^2}}$$

两个场力的比值

$$\frac{F_m}{F_e} = \left(\frac{v}{c}\right)^2$$

在高速运动中，$\frac{v}{c} \to 1$ ，此时磁场力与电场力有相同数量级，磁场力将起到与电场力相当的作用

在两根平行通电导线之间

在通电导线中，导线中电子的漂移速度极小，大约为$10^{-4}\mathrm{ms}^{-1}$数量级 $(v/c)^2 \approx 10^{-25}$

$$\frac{F_m}{F_e} = (\frac{v}{c})^2 \approx 10^{-25}$$

如果存在电场力，磁场力肯定可忽略

通电导线能保持严格的电中性，通电导体之间电场力消失的程度远较 10^{-25} 小。

使磁场力保留下来，成为通电导体之间相互作用的主要项。