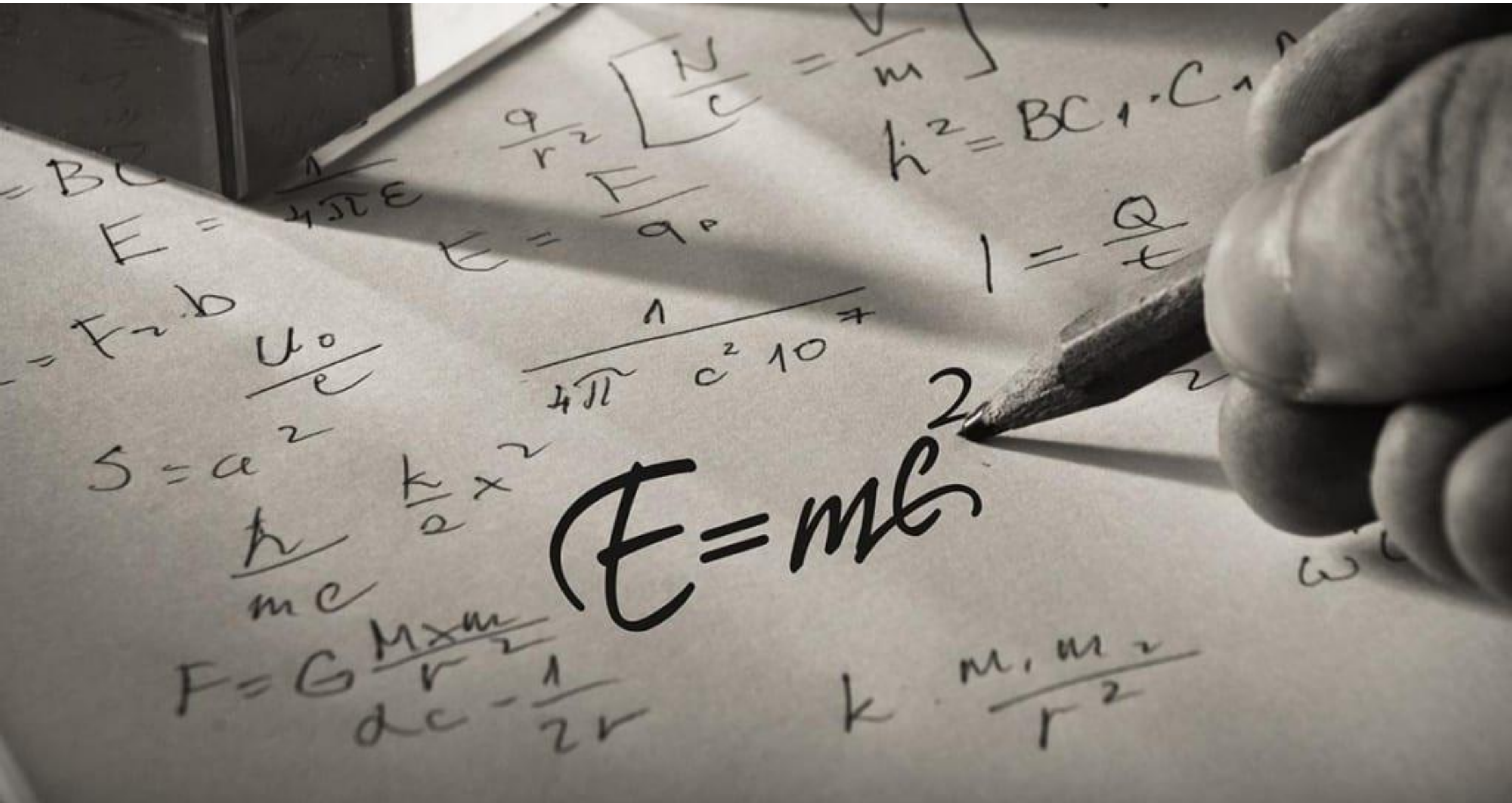
Chapter 26

The Special Theory of Relativity



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26-1 Galilean-Newtonian Relativity

Definition of an inertial reference frame:

One in which Newton's first law is valid

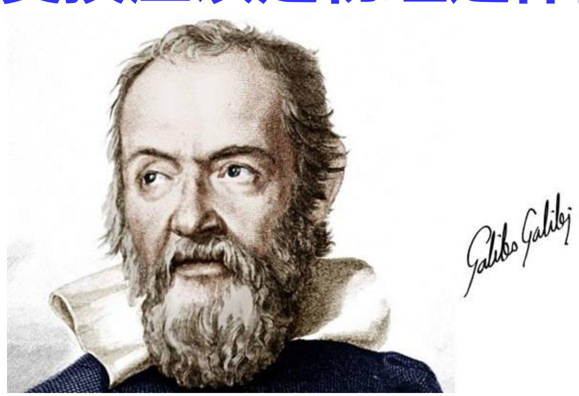
Earth is rotating and therefore not an inertial reference frame, but can treat it as one for many purposes

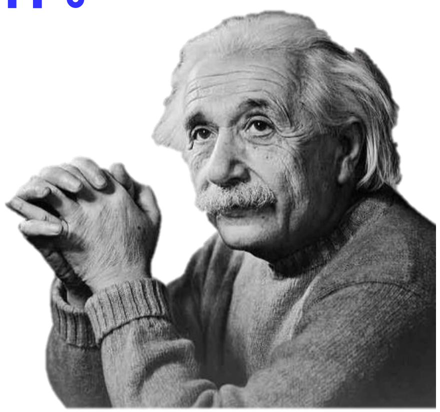
A frame moving with a constant velocity with respect to an inertial reference frame is itself inertial

An object will remain at rest or in uniform motion in a straight line unless acted upon by an external force.

物理定律不因人而异

自然规律的客观性，不应受到参考系主观选择的随意性的影响，因此物理规律(自然定律)不因人（参考系）而异，参考系变换应该是物理定律的对称操作。





All inertial frames are equivalent to the laws of mechanics—— Galilean Principle of Relativity

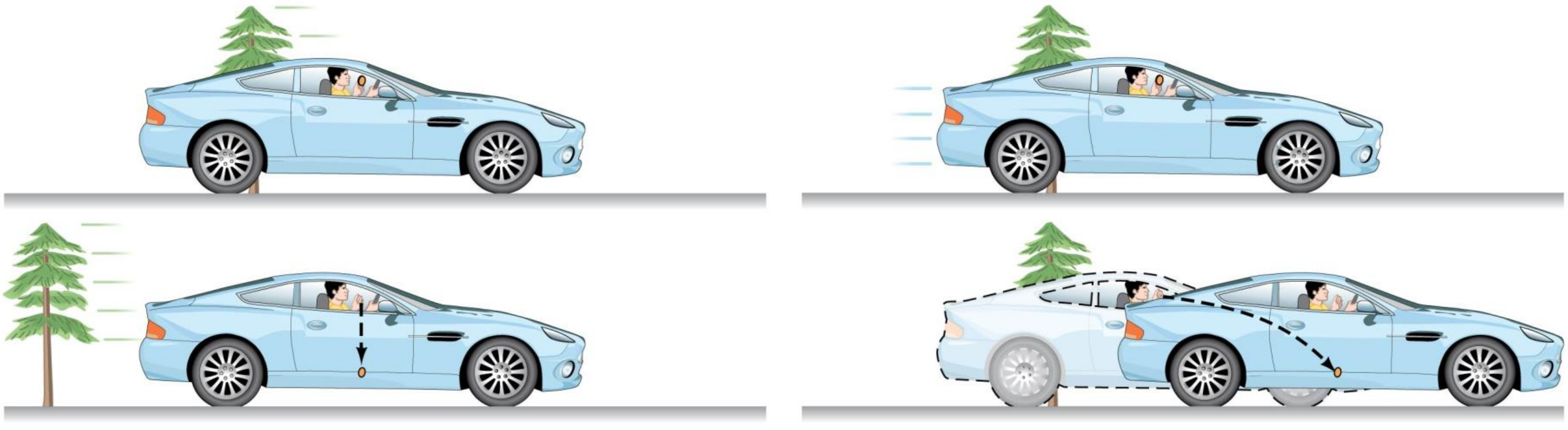
All inertial frames are equivalent to the laws of physics —— special relativity

Inertial and non-inertial frames are equivalent to the laws of physics —— general relativity

26-1 Galilean-Newtonian Relativity

Relativity principle:

The basic laws of physics are the same in all inertial reference frames.



(a)
Reference frame = car

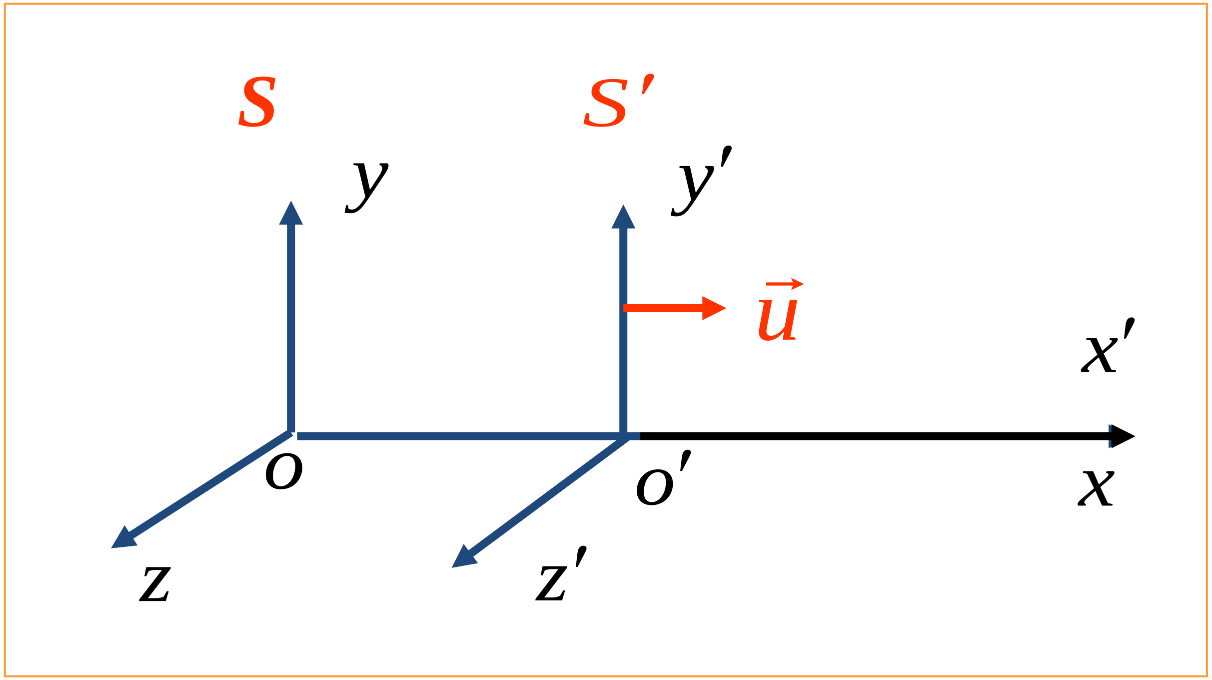
(b)
Reference frame = Earth

2、 Galilean transformation

S frame S' frame: The axes are parallel to each other

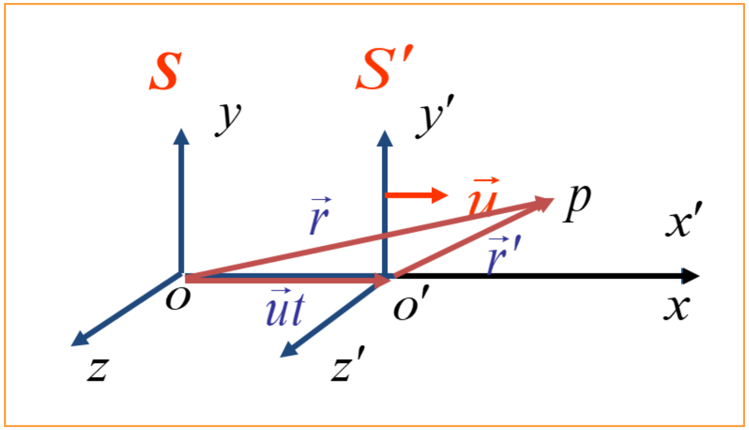
S frame S' frame: S' frame is going in the $+x$ direction at the velocity of $\boldsymbol{u}$

When O and O' overlaps, we assume $t = t' = 0$



Transformation of coordinates:

$$\vec{r}\,' = \vec{r} - \vec{u}t$$



Transformation of velocity :

$$\vec{v}\,' = \vec{v} - \vec{u}$$

Coordinate transformation in components:

$$\begin{cases} x' = x - ut \\ y' = y \\ z' = z \\ t' = t \end{cases} \qquad \begin{cases} x = x' + ut \\ y = y' \\ z = z' \\ t = t' \end{cases}$$

Forward transformation **Inverse transformation**

Velocity transformation in components：

$$\begin{cases} v_x' = v_x - u \\ v_y' = v_y \\ v_z' = v_z \end{cases} \qquad \begin{cases} v_x = v_x' + u \\ v_y = v_y' \\ v_z = v_z' \end{cases}$$

Forward transformation **Inverse transformation**

26-1 Galilean-Newtonian Relativity

This principle works well for mechanical phenomena.

However, Maxwell's equations yield the velocity of light; it is 3.0×10^8 m/s.

So, which is the reference frame in which light travels at that speed?

Scientists searched for variations in the speed of light depending on the direction of the ray, but found none.

1. The measurement of time

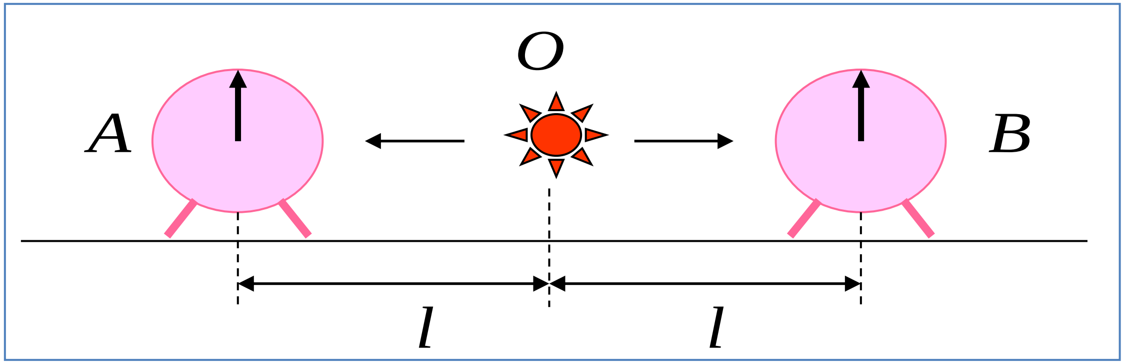
The measurement of time：clock "钟"

Any periodic process can be used to measure time. For example, the rotation or revolution of a planet; Simple pendulum. Crystal vibration; Molecular and atomic level transition radiation……

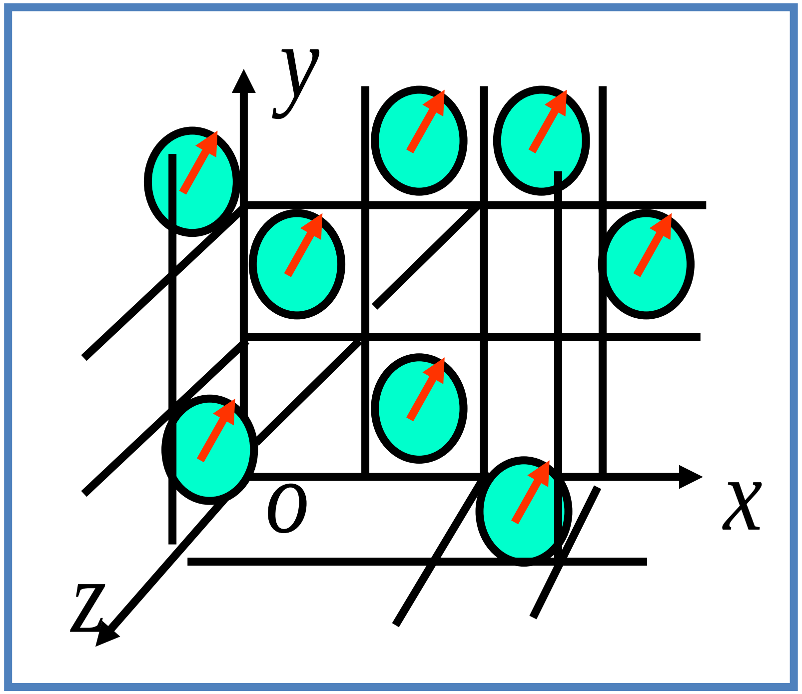
SI the international system of units： Second"秒"

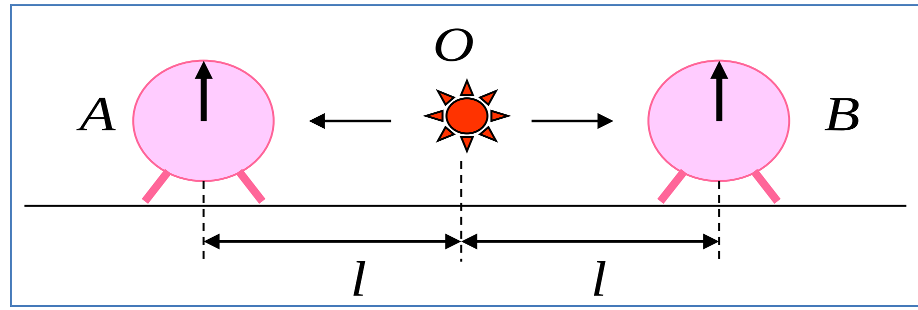
与铯133原子基态两个超精细能级之间跃迁相对应的辐射周期的9,192,631,700倍(精确度10^{-12}~10^{-13})

The calibration of clock:



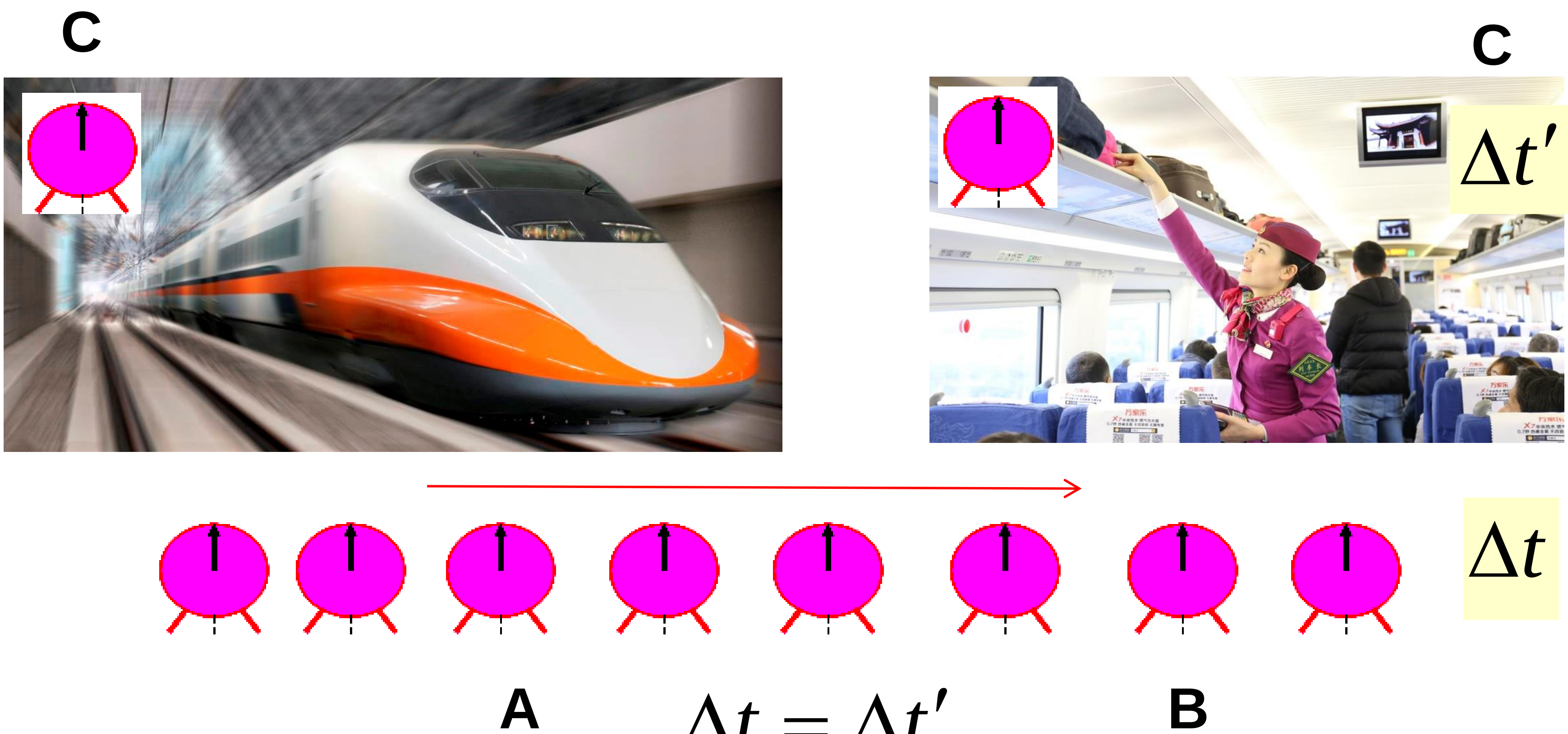
Establish uniform time coordinates at different points in the inertial system：





The calibration of clock

当某一事件发生时，我们通过该事件发生地的时钟读出该事件发生的时刻；当发生两个事件时，由两事件发生地的钟读数差得出两事件发生的时间间隔。



$$\Delta t = \Delta t'$$

Galilean-Newtonian Relativity

Measuring the time of the same event or the time interval between two events in different inertial frames yields the same result。

2. The measurement of space

To characterize the extensibility of matter and its motion

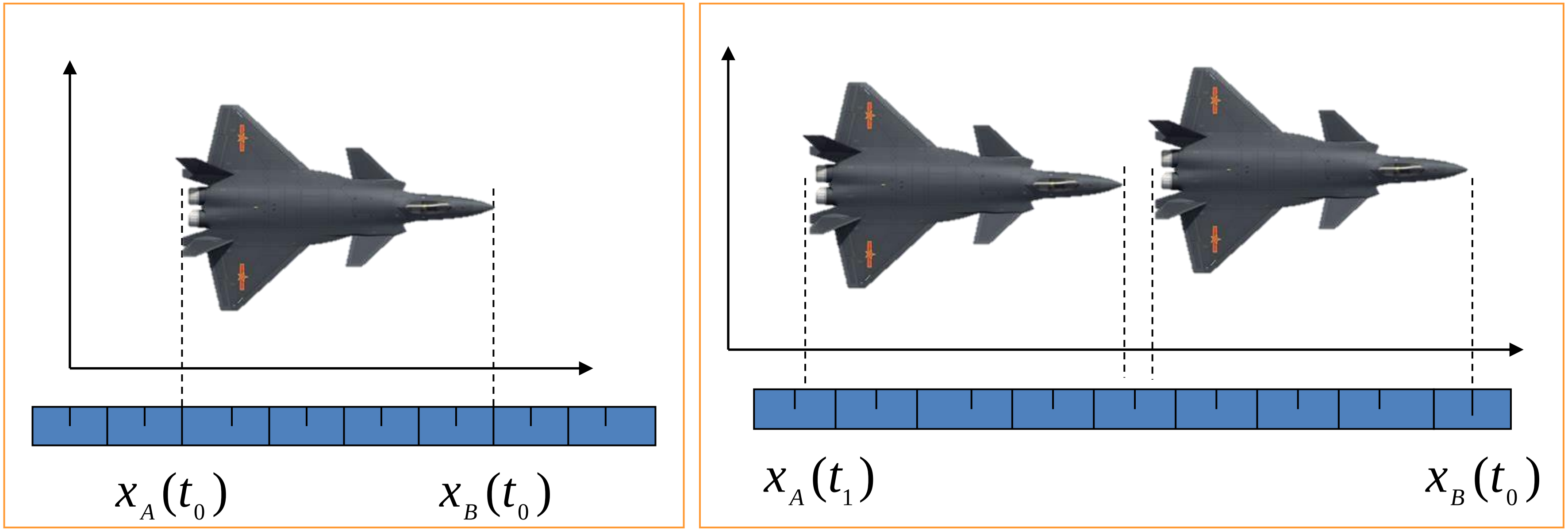
用以表征物质及其运动的广延性

The measurement of space：The rigid rod

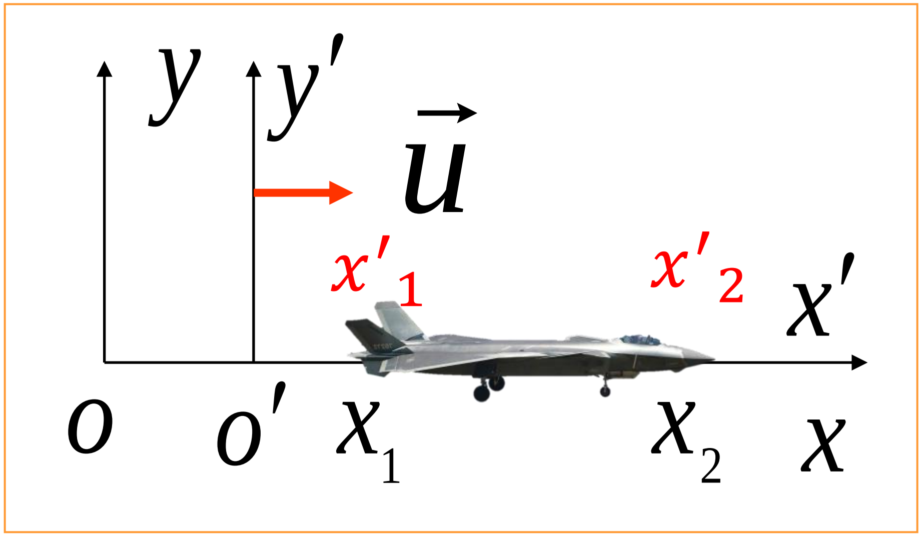
SI International system of unit: meter

光在真空中 $(299792458)^{-1}$ **秒的时间间隔内传播的距离。**

Notice： **The length is always measured with a ruler that is stationary relative to the reference frame. Coordinates at both ends must be recorded as the object moves.**



Measuring scales **at the same time** actually shows that time and space are intertwined



Galilean transformation:

$$x'_1 = x_1 - ut_1$$

$$x'_2 = x_2 - ut_2$$

The length of the ruler

$$\Delta x = x'_2 - x'_1 = x_2 - x_1 - u(t_2 - t_1)$$

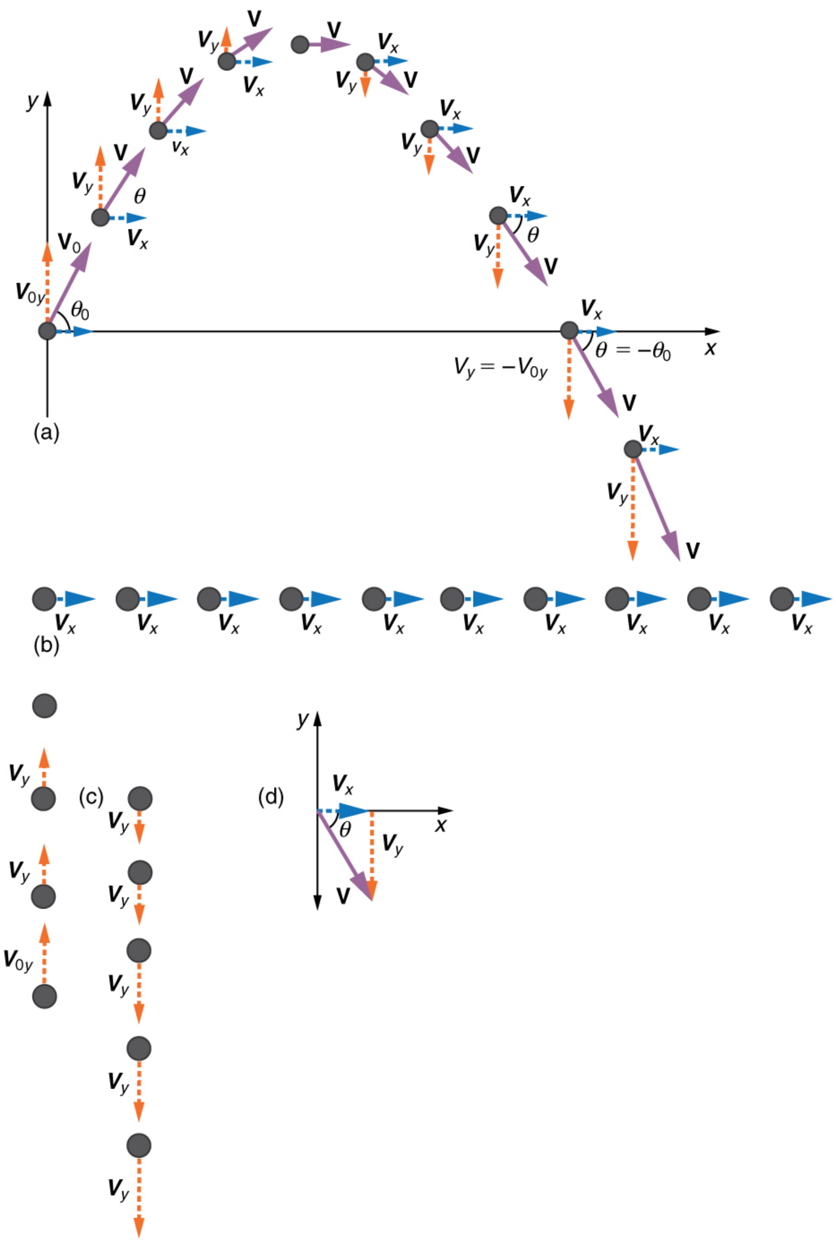
$$\because t_1 = t_2 \qquad \therefore \quad x_2 - x_1 = x'_2 - x'_1$$

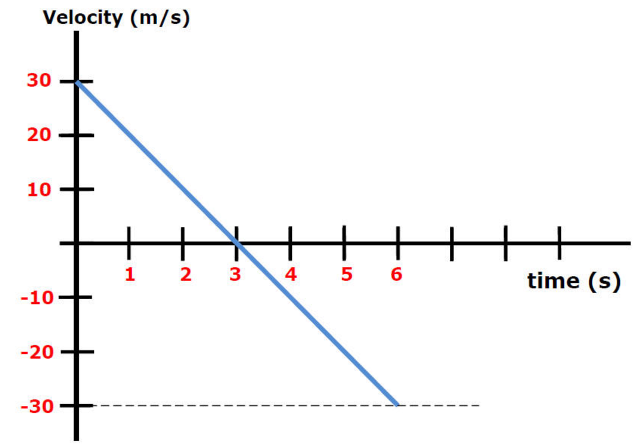
so:

尺的长度与其运动状态无关；空间测量与惯性系的选择无关。

Galilean-Newtonian Relativity

The relationship of space and time





Space time coordinate

Describe motion in term of events:

When and where what happens

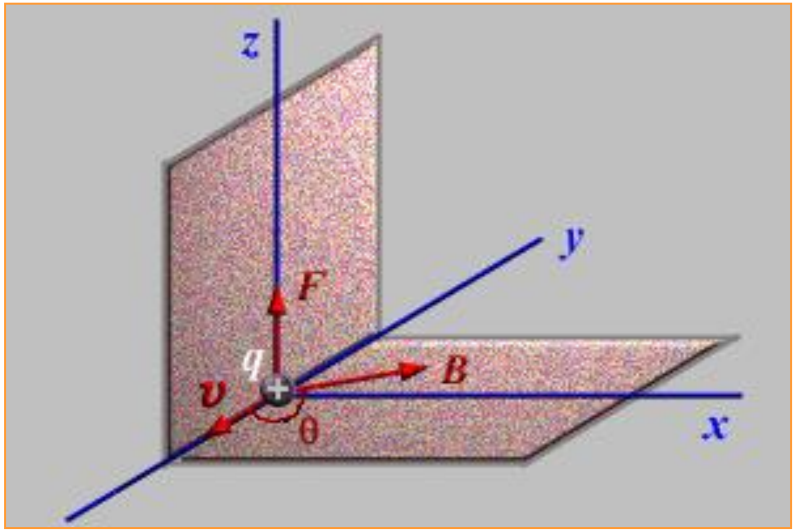
It is an order of event

5、The problem of Galilean transformation

Force on charged particles：

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

电场力　　洛仑兹力



Lorentz force： $F = qvB\sin\theta$

Perpendicular to the $\vec{B}$, $\vec{v}$ determined plane

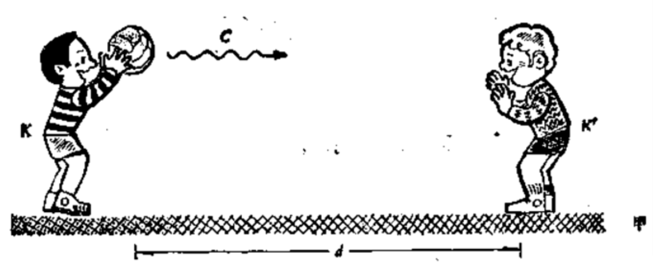
因速度 $\vec{v}$ 与参考系有关，所以经伽利略变换后洛仑兹力将发生变化，经典电磁定律不具有伽利略变换的不变性。

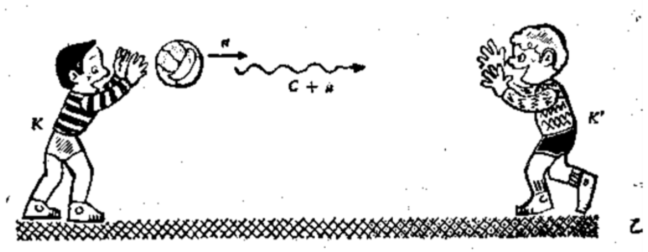
All forces associated with velocities do not have the invariance of the Galilean transformation!

5、The problem of Galilean transformation

$$\boxed{\vec{v} = \vec{v}' + \vec{u}}$$

The speed of light does not satisfy the law of velocity transformation





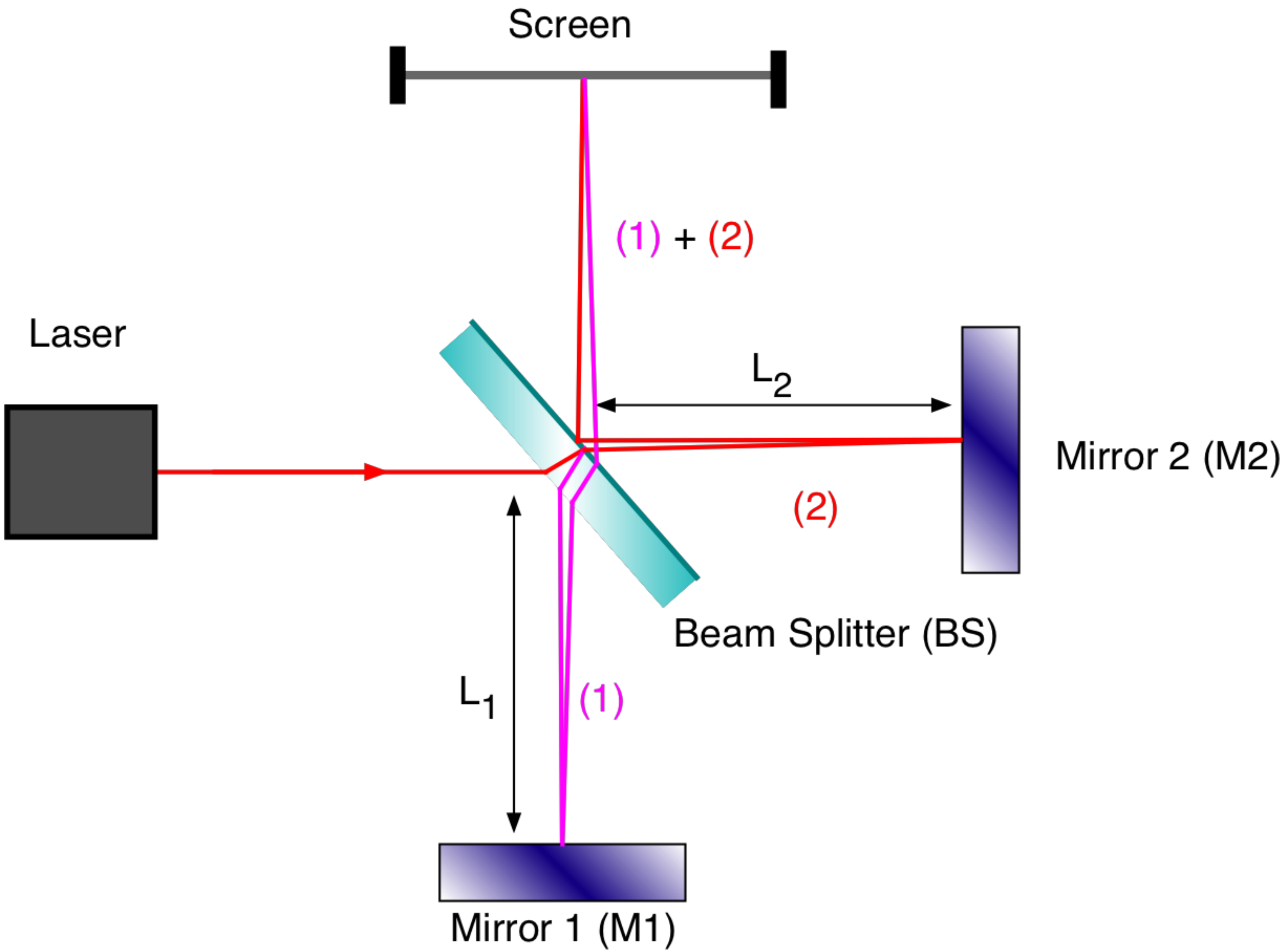
$$\Delta t = \frac{d}{c}$$

Less than

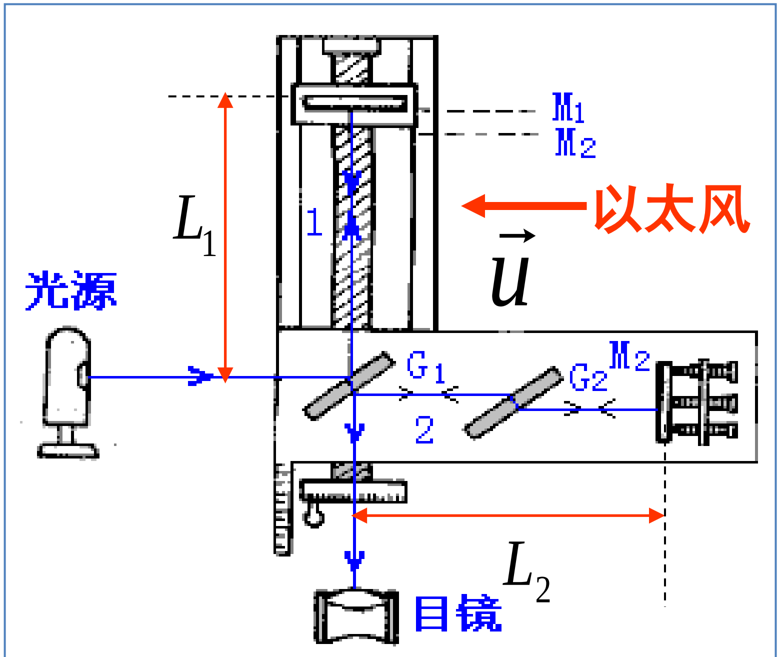
$$\Delta t' = \frac{d}{c+u}$$

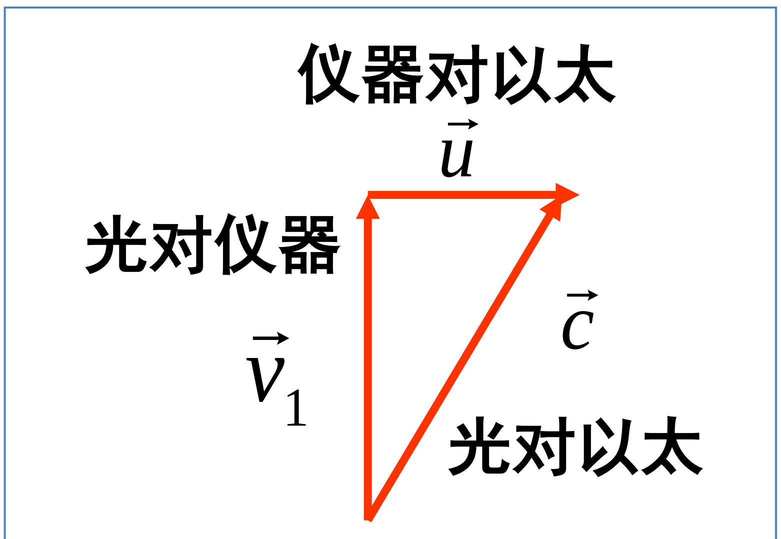
把速度合成公式应用到光传播问题上得到的一个混乱结果，它使我们先看到后发生的事，后看到先发生故事。然而，这种颠倒先后的怪现象谁也没有看到过。这就证明，光速并不满足速度合成公式,

伽利略变换与高速运动物体的实验结果不符。



Screen
(1) + (2)
Laser
L_2
Mirror 2 (M2)
(2)
Beam Splitter (BS)
L_1
(1)
Mirror 1 (M1)





由伽利略变换

$/\!/\vec{u}$方向，光对仪器的速率 为：

$$v_2 = c - u$$

$$v_2' = c + u$$

$\perp\vec{u}$方向，光对仪器的速率 为：

$$v_1 = \sqrt{c^2 - u^2}$$

光线1、2相遇，出现干涉条纹，将装置转动90度，干涉条纹应移动（预计0.37条）。反复实验，“零结果”

26-2 Postulates of the Special Theory of Relativity

1. The laws of physics have the same form in all inertial reference frames.

2. Light propagates through empty space with speed *c* independent of the speed of source or observer.

This solves the problem—the speed of light is in fact the same in all inertial reference frames.

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Problem—the speed of light

$$\varepsilon_0\mu_0 \frac{\partial E_y}{\partial t} = -\frac{\partial B_z}{\partial x} \xrightarrow[\frac{\partial}{\partial t}]{} \varepsilon_0\mu_0 \frac{\partial^2 E_y}{\partial t^2} = -\frac{\partial^2 B_z}{\partial t \partial x}$$

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x}$$

$$\frac{\partial^2 B_z}{\partial t \partial x} = \frac{\partial^2 B_z}{\partial x \partial t} = \frac{\partial}{\partial x}\left(\frac{\partial B_z}{\partial t}\right) = -\frac{\partial}{\partial x}\left(\frac{\partial E_y}{\partial x}\right)$$

$$\varepsilon_0\mu_0 \frac{\partial^2 E_y}{\partial t^2} = \frac{\partial^2 E_y}{\partial x^2}$$

Similarly, we can obtain $\varepsilon_0\mu_0 \frac{\partial^2 B_z}{\partial t^2} = \frac{\partial^2 B_z}{\partial x^2}$

Remember the wave equation? (lecture 5) $\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$

This is a wave with speed

$$v = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

This value is essentially identical to the speed of light measured by Foucault in 1860! (3×10^8 m/s) Maxwell identified light as an electromagnetic wave.

26-3 The locations of same events

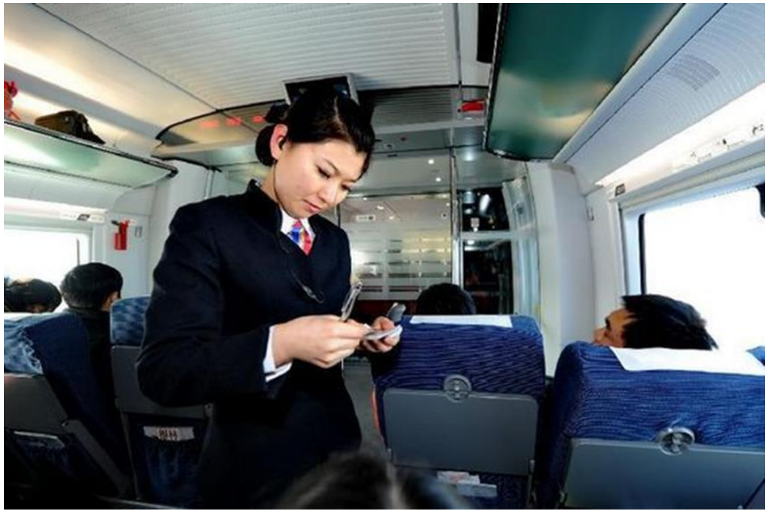
Different events happened
at same location

Different events happened
at different locations

26-3 Simultaneity

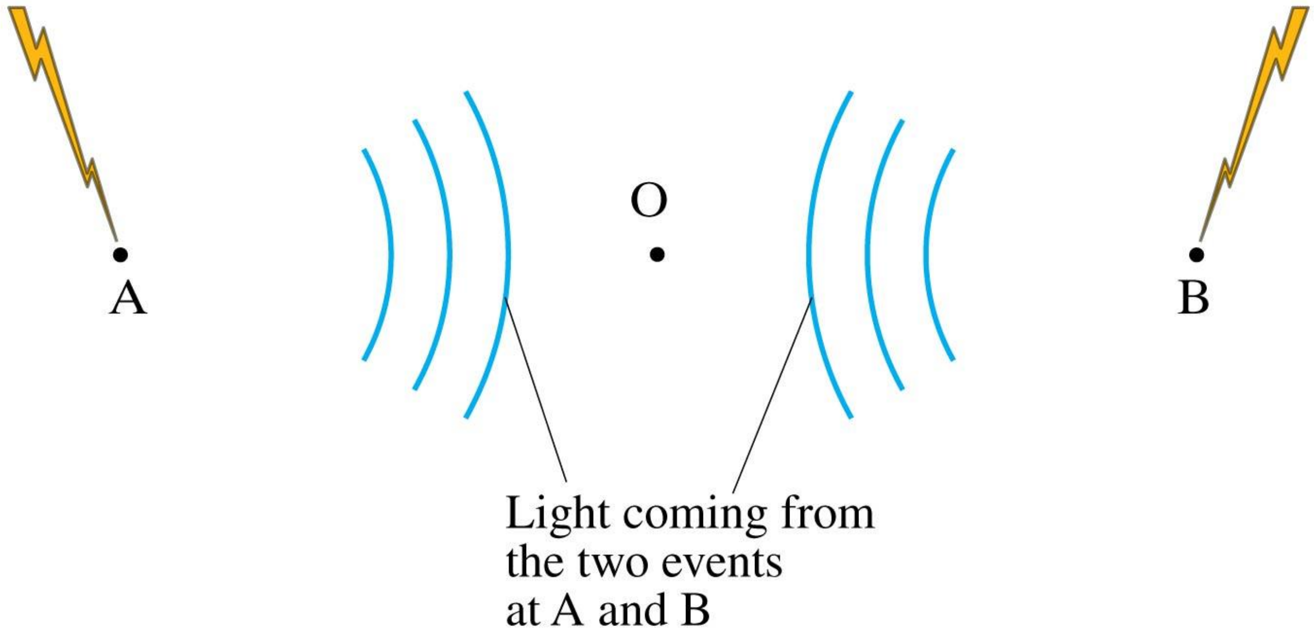
One of the implications of relativity theory is that time is not absolute. Distant observers do not necessarily agree on time intervals between events, or on whether they are simultaneous or not.

In relativity, an “event” is defined as occurring at a specific place and time.



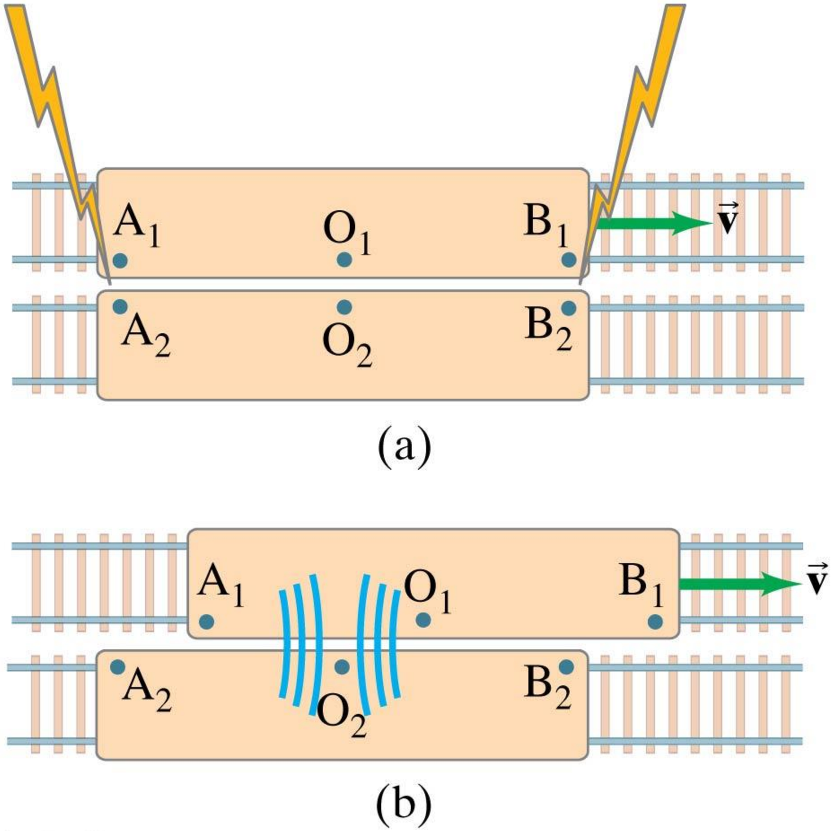
26-3 Simultaneity

Thought experiment: Lightning strikes at two separate places. One observer believes the events are simultaneous—the light has taken the same time to reach her—but another, moving with respect to the first, does not.



26-3 Simultaneity

Here, it is clear that if one observer sees the events as simultaneous, the other cannot, given that the speed of light is the same for each.



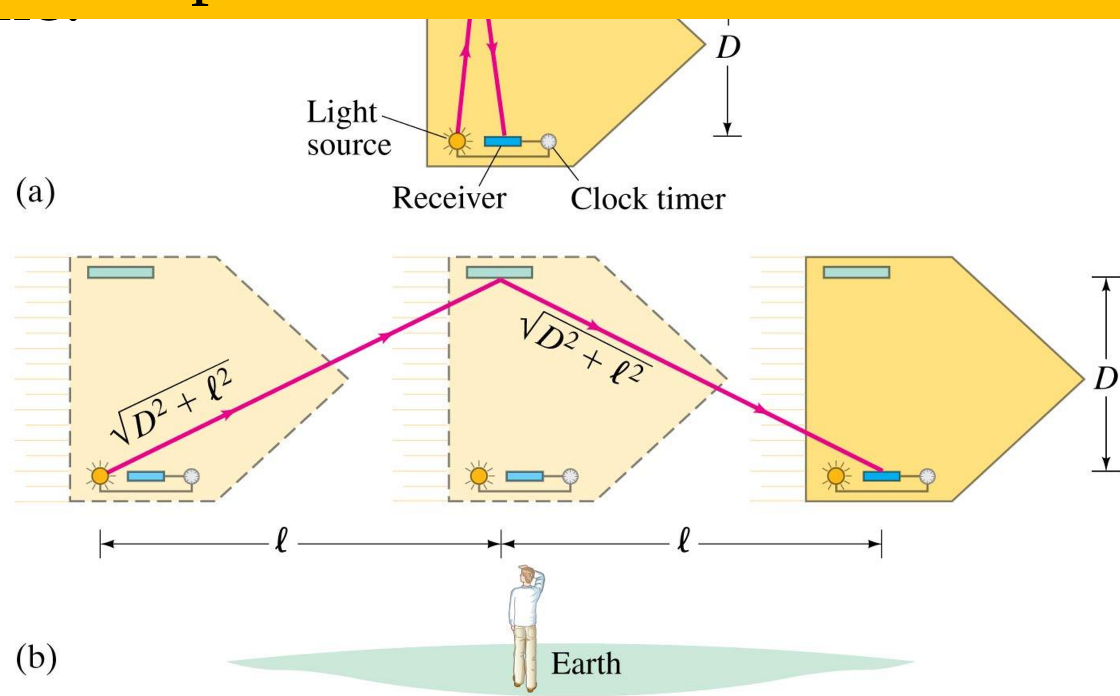
(a)

(b)

26-4 Time Dilation and the Twin Paradox

1. The laws of physics have the same form in all inertial reference frames.

2. Light propagates through empty space with speed c independent of the speed of source or observer.



26-4 Time Dilation and the Twin Paradox

Calculating the difference between clock "ticks," we find that the interval in the moving frame is related to the interval in the clock's rest frame:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}. \quad (26\text{-}1a)$$

The time interval between two events in the same place measured by a clock at rest relative to the place where the event occurred is proper time

相对于事件发生地静止的钟所测量的两个同地事件的时间间隔为原时

26-4 Time Dilation and the Twin Paradox

The factor multiplying t_0 occurs so often in relativity that it is given its own symbol, γ.

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \quad (26\text{-}2)$$

We can then write:

$$\Delta t = \gamma \, \Delta t_0 \quad (26\text{-}1b)$$

在所有关于时间的测量中原时最短

Proper time is the shortest in all measurements of time

26-4 Time Dilation and the Twin Paradox

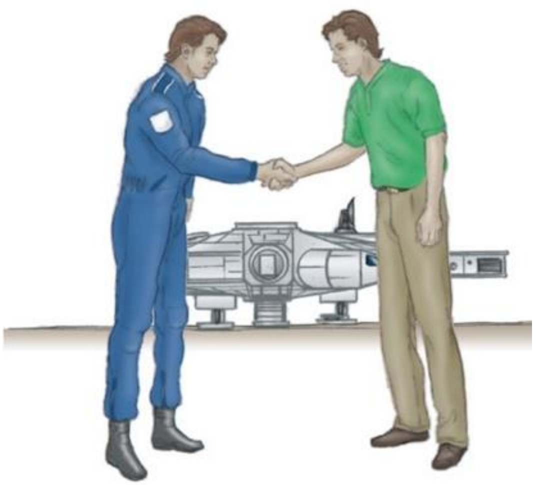
To clarify:

- The time interval in the frame where two events occur in the same place is t_0.
- The time interval in a frame moving with respect to the first one is Δt.

Twin Paradox

It has been proposed that space travel could take advantage of time dilation—if an astronaut's speed is close enough to the speed of light, a trip of 100 light-years could appear to the astronaut as having been much shorter.

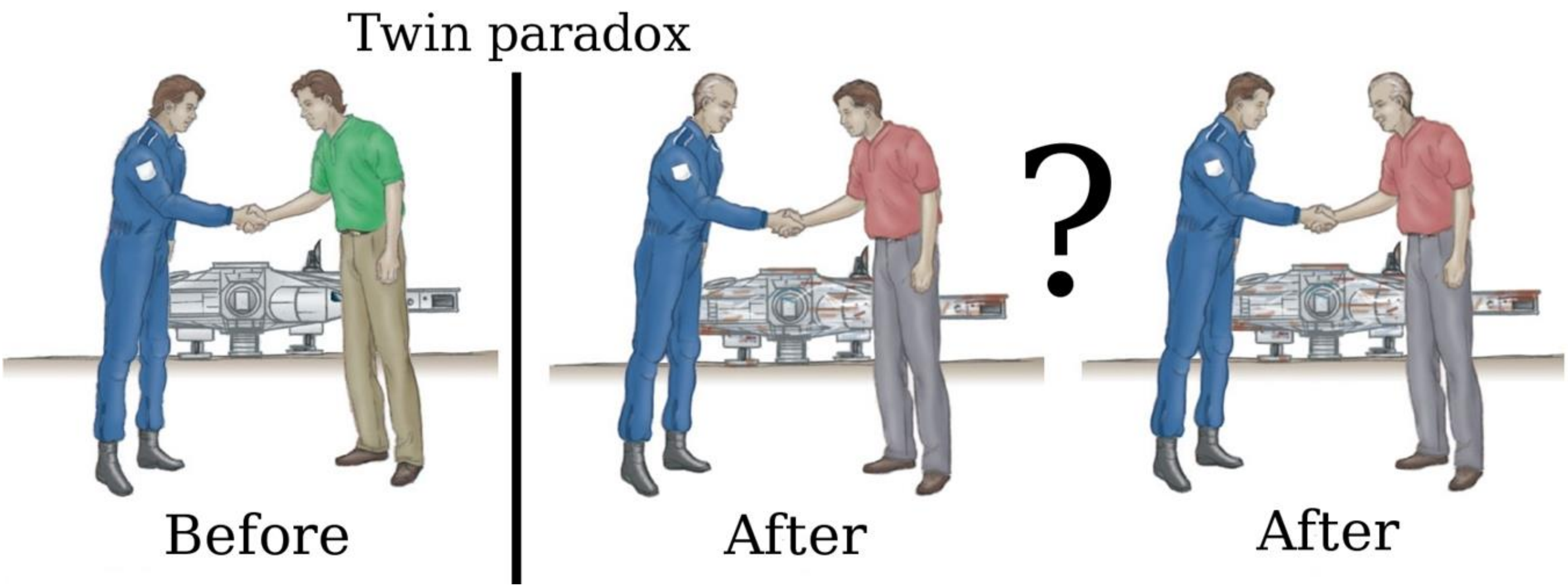
The astronaut would return to Earth after being away for a few years, and would find that hundreds of years had passed on Earth.



Before

After

26-4 Time Dilation and the Twin Paradox



26-4 Time Dilation and the Twin Paradox

This brings up the twin paradox—if any inertial frame is just as good as any other, why doesn't the astronaut age faster than the Earth traveling away from him?

The solution to the paradox is that the astronaut's reference frame has not been continuously inertial—he turns around at some point and comes back. It is impossible to do this without accelerating.

26-4 Time Dilation and the Twin Paradox

TABLE 26–1 Values of γ

v	γ
0	1.00000 …
$0.01c$	1.00005
$0.10c$	1.005
$0.50c$	1.15
$0.90c$	2.3
$0.99c$	7.1

断章

卞之琳

你站在桥上看风景，
看风景的人在楼上看你。
明月装饰了你的窗子，
你装饰了别人的梦。

3. Lorentz transformation

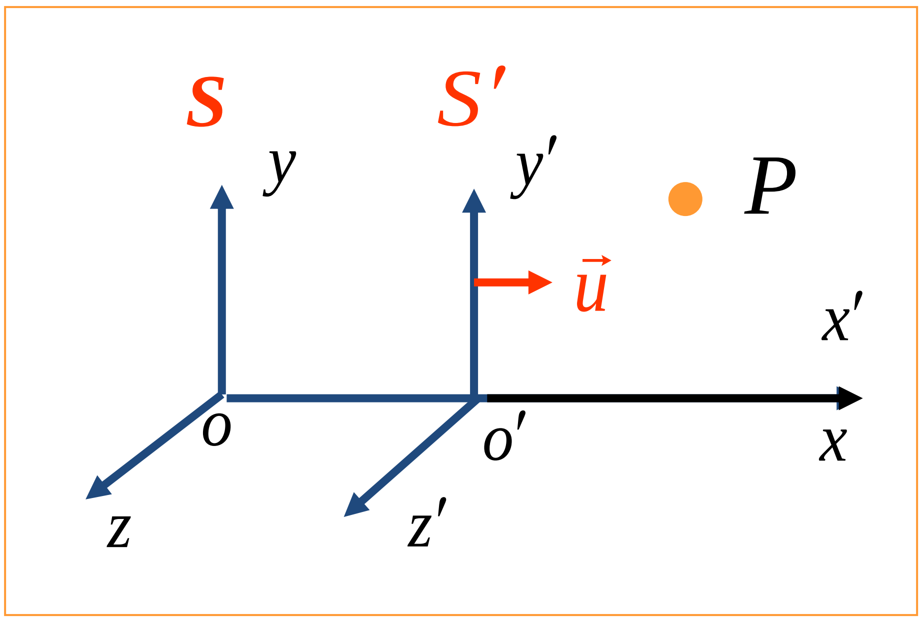
(1) transformation of coordinates

s $\quad P(x, y, z, t)$

s' $\quad P(x', y', z', t')$

seeking →

For the same objective event P, the relationship between the the corresponding coordinate values of two inertial frames。



When $t = t' = 0$ ，

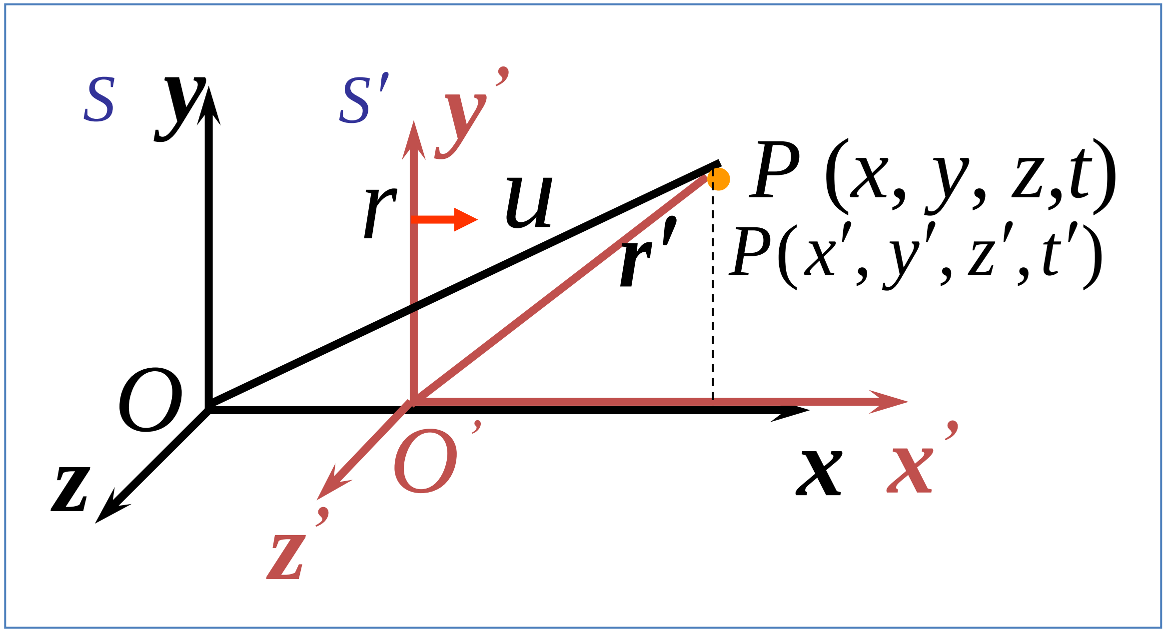
optical signal

is emitted from $o(o')$

And arrived at P ：

$S: \quad P(x, y, z, t)$

$S': \quad P(x', y', z', t')$



All the speed of light in S and S' in a vacuum is c

$$r = \sqrt{x^2 + y^2 + z^2} = ct$$

$$r' = \sqrt{x'^2 + y'^2 + z'^2} = ct'$$

$$x^2 + y^2 + z^2 - c^2t^2 = 0$$

$$x'^2 + y'^2 + z'^2 - c^2t'^2 = 0$$

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = x^2 + y^2 + z^2 - c^2 t^2 = 0$$

S, S'系只在x方向有相对运动

$$y = y' \quad ; \quad z = z'$$

$$\therefore \quad x'^2 - c^2 t'^2 = x^2 - c^2 t^2$$

显然，伽利略变换不满 足上式。

Coordinate transformations satisfy linear relationships in X direction：

$$\left.\begin{aligned} x' &= k(x - ut) \\ x &= k'(x' + ut') \end{aligned}\right\} k = k' = 1 \Big/ \sqrt{1 - u^2/c^2}$$

Assume $\beta = \dfrac{u}{c}$ $\qquad \gamma = \dfrac{1}{\sqrt{1-\beta^2}}$ **So:**

Forward transformation | **Inverse transformation**

$$\boxed{\begin{aligned} x' &= \gamma(x - ut) \\ y' &= y \\ z' &= z \\ t' &= \gamma\left(t - \frac{u}{c^2}x\right) \end{aligned}} \qquad \boxed{\begin{aligned} x &= \gamma(x' + ut') \\ y &= y' \\ z &= z' \\ t &= \gamma\left(t' + \frac{u}{c^2}x'\right) \end{aligned}}$$

注意: $\gamma = 1 \Big/ \sqrt{1 - u^2/c^2} \geq 1$

(2) speed transformation

设S系: $\vec{v}(v_x, v_y, v_z)$ S'系: $\vec{v}'(v_x', v_y', v_z')$

According to the velocity definition: $v_x' = \dfrac{\mathrm{d}x'}{\mathrm{d}t'}$ $v_x = \dfrac{\mathrm{d}x}{\mathrm{d}t}$

$\because \quad x' = \gamma(x - ut) \qquad \therefore \quad \mathrm{d}x' = \gamma(\mathrm{d}x - u\,\mathrm{d}t)$

$\because \quad t' = \gamma(t - \dfrac{u}{c^2}x) \qquad \therefore \quad \mathrm{d}t' = \gamma(\mathrm{d}t - \dfrac{u}{c^2}\mathrm{d}x)$

$$\therefore \quad v_x' = \frac{\mathrm{d}x'}{\mathrm{d}t'} = \frac{\mathrm{d}x - u\,\mathrm{d}t}{\mathrm{d}t - u\,\mathrm{d}x/c^2} = \frac{v_x - u}{1 - uv_x/c^2}$$

同理可得: v_y', v_z'

Lorentz space-time transformation:

$$\begin{cases} x' = \dfrac{x - ut}{\sqrt{1 - \dfrac{u^2}{c^2}}} \\ y' = y \\ z' = z \\ t' = \dfrac{t - \dfrac{u}{c^2}x}{\sqrt{1 - \dfrac{u^2}{c^2}}} \end{cases}$$

$$\begin{cases} x = \dfrac{x' + ut'}{\sqrt{1 - \dfrac{u^2}{c^2}}} \\ y = y' \\ z = z' \\ t = \dfrac{t' + \dfrac{u}{c^2}x'}{\sqrt{1 - \dfrac{u^2}{c^2}}} \end{cases}$$

Forward transformation **Inverse transformation**

速度变换公式

正变换：

$$\begin{cases} v'_x = \dfrac{v_x - u}{1 - uv_x/c^2} \\ v'_y = \dfrac{v_y}{\gamma(1 - uv_x/c^2)} \\ v'_z = \dfrac{v_z}{\gamma(1 - uv_x/c^2)} \end{cases}$$

逆变换：

$$\begin{cases} v_x = \dfrac{v'_x + u}{1 + uv'_x/c^2} \\ v_y = \dfrac{v'_y}{\gamma(1 + uv'_x/c^2)} \\ v_z = \dfrac{v'_z}{\gamma(1 + uv'_x/c^2)} \end{cases}$$

Some notes on Lorentz transformation：

1. Lorentz transformation satisfies the correspondence principle. At low speed, Lorentz transform is reduced to Galilean transformation。

$$u << c \quad \frac{u}{c} \to 0 \quad \gamma \to 1$$

$$\left.\begin{aligned} x' &= \gamma(x - ut) \\ y' &= y \\ z' &= z \\ t' &= \gamma\left(t - \frac{u}{c^2}x\right) \end{aligned}\right\} \quad \begin{aligned} x' &= x - ut \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}$$

洛仑兹变换 ⟶ 伽利略变换，满足对应原理

2. Lorentz transformation represents the transformation relation of space-time coordinates of the same physical event in different inertial frames

3. Clocks and rulers in different inertial frames must be at rest relative to the reference frame. Thus, the difference in the space-time measurement results of each inertial system is reflected in the difference in the motion state of the consolidated clock and ruler of these inertial systems.

其它超光速的现象：

(1) 粒子在媒质中的传播速度可能超过媒质中的光速。在这种情况下会发生辐射，称为切仑科夫效应。

(2) 第三观察者观察两个物体的相对速度是可以超过光速的。

(3) 物质波的波速可以超过光速。

例.观察者甲、乙，分别静止在惯性系 S，S' 中，S' 相对 S 以 u 运动， S' 中一个固定光源发出一束光与 u 同向

(1)乙测得光速为 c .
(2)甲测得光速为 $c+u$;
(3)甲测得光速为 $c-u$ ；
(4)甲测得光相对于乙的速度为 $c-u$。

正确的答案是:

(A) (1),(2),(3); (B) (1),(4)
(C) (2),(3); (D) (1),(3),(4)

[B]

26-5 Length Contraction

Forward transformation

$$
\boxed{
\begin{aligned}
x' &= \gamma\left(x - ut\right)\\
y' &= y\\
z' &= z\\
t' &= \gamma\left(t - \frac{u}{c^2}x\right)
\end{aligned}
}
$$

Inverse transformation

$$
\boxed{
\begin{aligned}
x &= \gamma\left(x' + ut'\right)\\
y &= y'\\
z &= z'\\
t &= \gamma\left(t' + \frac{u}{c^2}x'\right)
\end{aligned}
}
$$

注意:

$$\gamma = 1\Big/\sqrt{1-u^2/c^2} \geq 1$$

26-5 Length Contraction

If time intervals are different in different reference frames, lengths must be different as well. Length contraction is given by:

$$\ell = \ell_0\sqrt{1 - v^2/c^2} \qquad (26\text{-}3a)$$

or

$$\ell = \frac{\ell_0}{\gamma}. \qquad (26\text{-}3b)$$

Length contraction occurs only along the direction of motion.

§ 8.3 Special theory of relativity time space

The concept of time and space refers to the understanding of the physical properties of time and space 。

Transpositional consideration

Before Einstein, people’s view of time and space was an absolute view of time and space represented by Newton. Where simultaneity, measurement of time， time intervals, measurement of space， space intervals, etc., are absolute. In relativity, however, everything becomes relative。

The relativity of ‘simultaneity’

“凡是时间在里面起作用的一切判断，总是关于同时事件的判断” ——爱因斯坦

Question:
Are simultaneous events in one inertial reference frame simultaneous in another inertial reference frame?

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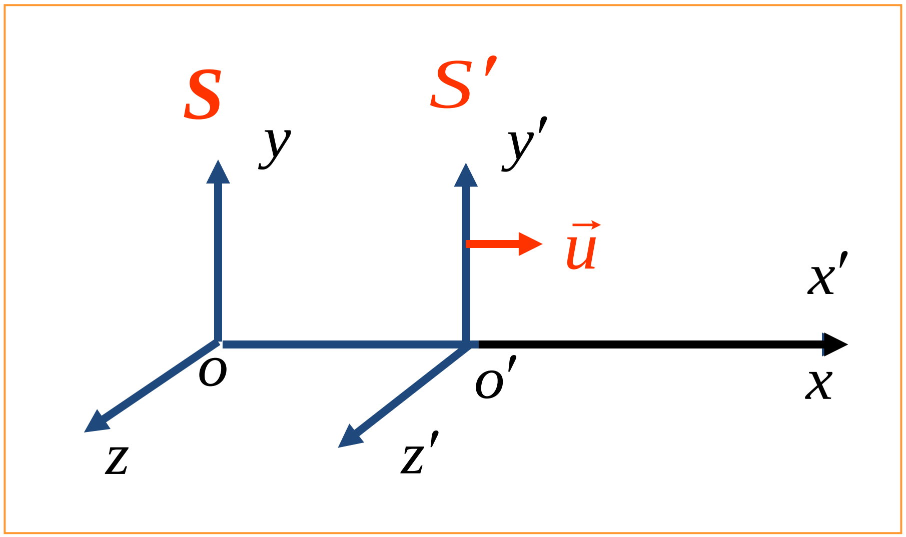
Are simultaneous events in one inertial reference frame simultaneous in another inertial reference frame?



	Event1	Event2
S	$x_{1,}\ t_1$	$x_{2,}\ t_2$
S'	$x'_{1,}\ t'_1$	$x'_{2,}\ t'_2$

Lorentz transformation：

$$t'_1 = \gamma\left(t_1 - \frac{u}{c^2}x_1\right) \quad ; \quad t'_2 = \gamma\left(t_2 - \frac{u}{c^2}x_2\right)$$

$$\Delta t' = t'_2 - t'_1 = \gamma\left[\left(t_2 - t_1\right) - \frac{u}{c^2}\left(x_2 - x_1\right)\right] = \gamma\left(\Delta t - \frac{u}{c^2}\Delta x\right)$$

$$\Delta t' = t_2' - t_1' = \gamma\left[\left(t_2 - t_1\right) - \frac{u}{c^2}\left(x_2 - x_1\right)\right] = \gamma\left(\Delta t - \frac{u}{c^2}\Delta x\right)$$

***S* frame two simultaneous events , $\Delta t = 0$**

s' frame $\left\{ \begin{array}{l} \textbf{When}: \Delta x = 0 \quad \Delta t' = 0 \text{，两事件同时发生。} \\ \textbf{When}: \Delta x \neq 0 \quad \Delta t' \neq 0 \text{，两事件不同时发生。} \end{array} \right.$

即：一个惯性系中的同时、同地事件，在其它惯性系中必为同时事件；一个惯性系中的同时、异地事件，在其它惯性系 中必为不同时事件。

结论： 同时性概念是因参考系而异的，在一个惯性系中认为同时发生的两个事件，在另一惯性系中看来，不一定同时发生。同时性具有相对性。

Discussion: Timing sequence and causality of two events

In *s* frame $\Delta t = t_2 - t_1 > 0$ **Event 1 happens earlier**

Thus in s' **frame: there are two situations**

$$\Delta t' = \gamma(\Delta t - \frac{u}{c^2}\Delta x) > 0$$

$$\Delta t > \frac{u}{c^2}\Delta x$$

$$\frac{\Delta x}{\Delta t} < \frac{c^2}{u}$$

时序不变

$$\Delta t' = \gamma(\Delta t - \frac{u}{c^2}\Delta x) \le 0$$

$$\Delta t \le \frac{u}{c^2}\Delta x$$

$$\frac{\Delta x}{\Delta t} \ge \frac{c^2}{u}$$

时序变化

即在 S' 系中观测，事件1有可能比事件2先发生、同时发生、或后发生，时序有可能倒置。

Does it contradict causality?

因果律就是保证关联事件先后次序的绝对性

可以用讯号联系的事件称为关联事件，因果关系的事件一定是关联事件。

$$\boldsymbol{v} = \frac{x_2 - x_1}{t_2 - t_1} \qquad u < c \longrightarrow \frac{\Delta x}{\Delta t} < \frac{c^2}{u}$$

有因果关联的事件时序不变，无因果关联的事件才可能发生时序变化。

Example 1：The relativity of time series

谁先动手?

列车时速0.6c，A，B相距10m

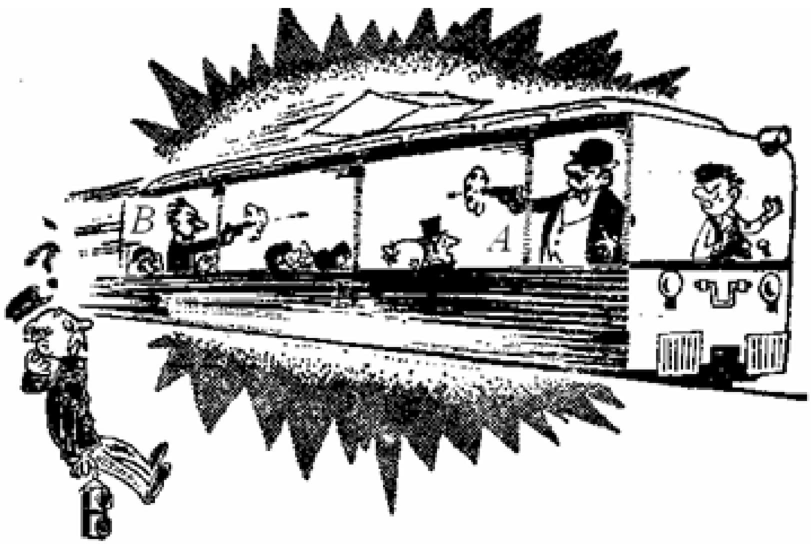
站台系：A先于B12.5纳秒开枪

列车系：B先于A10纳秒开枪

这个具体事件中、谁先谁后是有相对性的！

例1：时序的相对性与因果律关系

A和B并不满足这个必要条件(在十几个纳秒时间内，光信号走不到$10m$远)，所以，A和B开枪动作并不是关联事件。所以其先后是相对的。



$$\frac{\Delta x}{\Delta t}=\frac{10}{10\times10^{-9}}>c$$

例2 事件1：某天小孩甲在北京出生 → 飞机由北京起飞；

事件2：第二天小孩乙在巴黎出生 → 飞机抵达巴黎；

事件1和事件2无因果关联。 → 事件1和事件2可能有因果关联，时序不变。

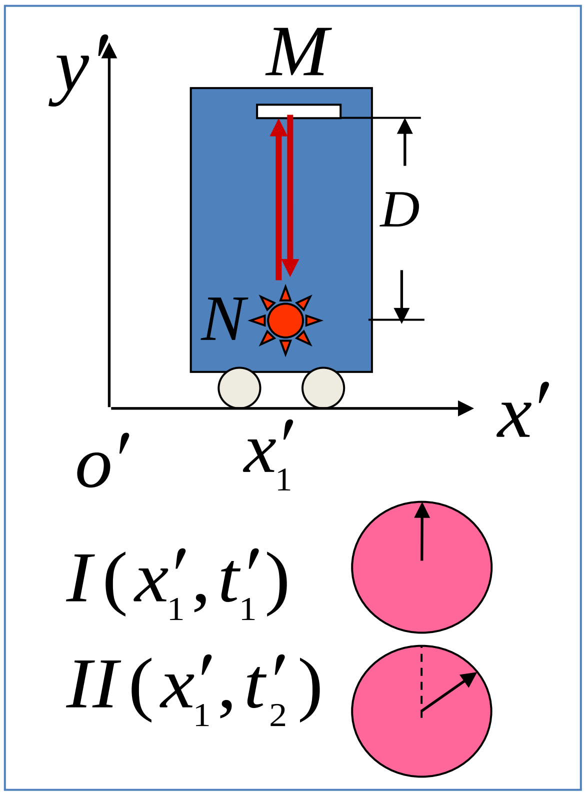
例3 事件1：某天小孩甲在北京出生；

事件2：0.03秒后小孩乙在巴黎出生；

$$\frac{\Delta x}{\Delta t} \approx \frac{11000}{0.03} \approx 3.67\times 10^{5}\,\mathrm{km}\cdot\mathrm{s}^{-1} > c$$

事件1和事件2无因果关联，，可能在某个飞船上的观察者看来，巴黎小孩先出生。

Relativity of time measurement (time dilation)



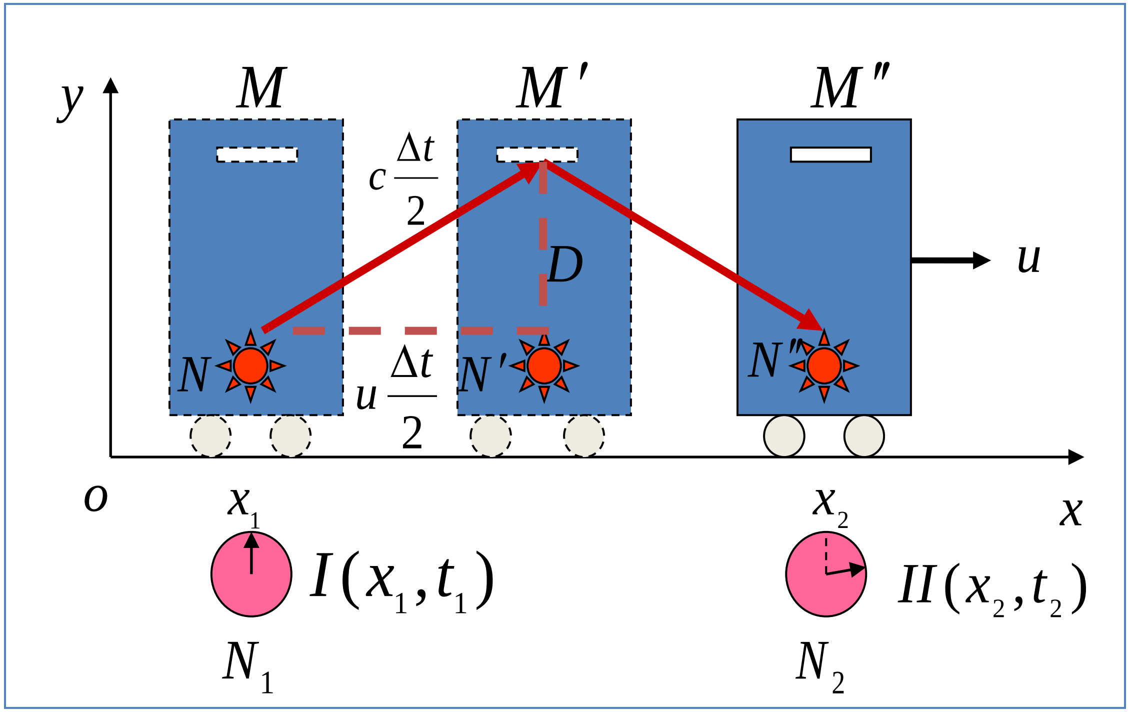
$$\Delta t' = t'_2 - t'_1 = \frac{2D}{c}$$

Measured with a stationary clock relative to where the event occurred （the reference frame）

The time interval between two events in the same place — proper time

↓

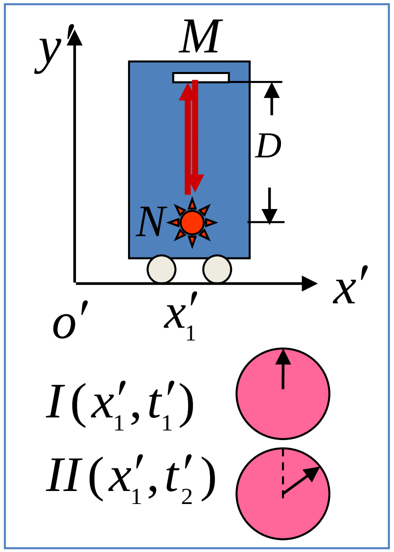
（Intrinsic time）

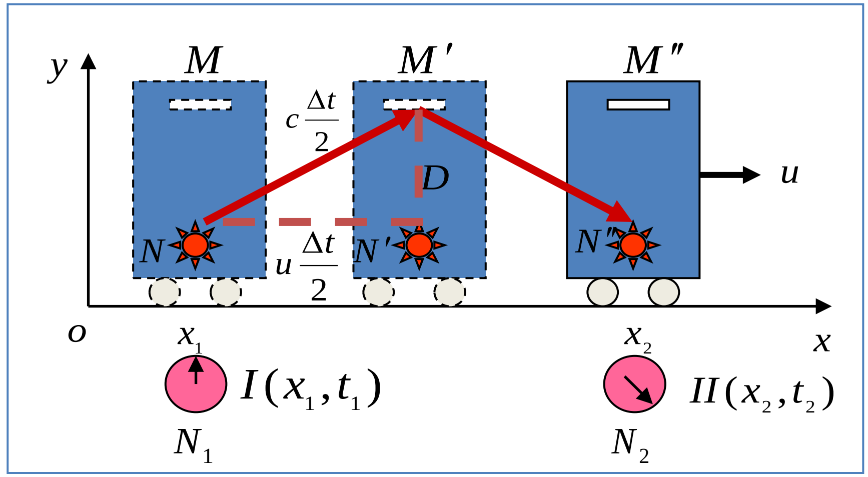


The two events are remote events, and two clocks are needed to measure the time interval:
non-proper time

$$\left(c\frac{\Delta t}{2}\right)^2 = D^2 + \left(u\frac{\Delta t}{2}\right)^2$$

$$\Delta t = \frac{2D}{c}\frac{1}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{\Delta t'}{\sqrt{1-\left(\frac{u}{c}\right)^2}} = \gamma\Delta t' > \Delta t'$$





$$\Delta t = \frac{\Delta t'}{\sqrt{1-\left(\frac{u}{c}\right)^2}} = \gamma\,\Delta t' > \Delta t'$$

non-proper time **proper time**

在 S 系中用 N_1、N_2 钟测量 S' 系中 N 钟所测得的原时 $\Delta t'$，将获得一个放大了的时间间隔 Δt ——时间膨胀

在 s 系中看来，相对它运动的 S' 系内的钟走慢了。

——动钟变慢

Discussion: what if the signal system is stationary relative to the platform?

$$\Delta t = \frac{\Delta t'}{\sqrt{1-\left(\frac{u}{c}\right)^2}} = \gamma\,\Delta t' > \Delta t'$$

non-proper time (Δt)   proper time ($\Delta t'$)

Conclusion:

the measurement of time interval is relative and related to the choice of inertial system;

- Of all time measurements, the proper time is the shortest （时间膨胀）。
- The observer in each reference frame will assume that the clock moving relative to him is slower than his own clock（动钟变慢）

原时：在相对事件发生地静止的参考系中，用同一个钟测定的两个同地事件之间的时间间隔 τ_0

若在相对事件发生地运动的参考系中，该两事件必为异地事件，需用两只钟测出其时间间隔 τ ，则：

$$\tau = \gamma \tau_0$$

静系中同地事件的时间间隔为原时，

动系中异地事件的时间间隔非原时。

Time expansion can be obtained directly from lorentz transformation：

$$\underset{\text{非原时}}{\Delta t'} = \gamma(\underset{\text{原时}}{\Delta t} - \frac{u}{c^2}\underset{0}{\Delta x})$$

$$\underset{\text{非原时}}{\Delta t} = \gamma(\underset{\text{原时}}{\Delta t'} + \frac{u}{c^2}\underset{0}{\Delta x'})$$

$\boxed{\text{在一切时间测量中，原时最短！}}$

Experimental verification：

1) μ decay

宇宙射线和大气相互作用时能产生π介子衰变，在大气上层放出μ子。这些μ子的速度约为 0.998*c*，如果在实验室中测得静止μ子的寿命为 $2.2\times10^{-6}\text{s}$，试问，在 8000 m 高空由π介子衰变放出的μ子能否飞到地面？

解： 按照经典理论，μ子飞行的距离为

$$s = u\tau = 0.998\times3\times10^{8}\times2.2\times10^{-6} = 658.7\text{m}$$

显然，μ子不能飞到地面。 <<8000 m

按照相对论理论，应该如何计算？

按照相对论理论，地面参考系测得的 μ 子的寿命应为：

$$\Delta t = \gamma \Delta t' = \gamma \tau$$

在地面参考系看来，μ 子的飞行距离为

$$s = u\Delta t = \gamma u \tau = \frac{0.998 \times 3 \times 10^{8} \times 2.2 \times 10^{-6}}{\sqrt{1 - 0.998^{2}}}$$

$$= 10420\,\mathrm{m} > 8000\,\mathrm{m}$$

显然，μ 子可以飞到地面。

测量结果：到达地面的 μ 子流为 $500\ \mathrm{m}^{-2} \cdot \mathrm{s}^{-1}$

验证了相对论时间膨胀效应。

有一类青蛙，寿命为10天。一宇宙飞船以0.866C相对地面飞行，飞船上载有这类青蛙。

地面上的观察者认为飞船上的青蛙寿命是多少？

飞船上的观察者认为地面上的青蛙寿命是多少？

20天

这就产生了一个有趣的问题，哪里的青蛙先死？

当我们问“究竟哪里的青蛙先死时？”，我们没有说出来的埋藏在心底的信念仍然是，有一个普适的真实时间。但是请记住，并没有任何单一而普适的时间。相反的只是每个个体观察者的时间。

例. 半人马座α星是距离太阳系最近的恒星，它距离地球4.3×10^{16}m，设有一宇宙飞船自地球飞到半人马α星，若宇宙飞船相对地球的速度为0.999c，按地球上的时钟计算要用多少年时间？如以飞船上的时钟计算，所需时间又为多少年？

Consider: which time is the Proper time?

Earth system: non-proper time;

spacecraft system : proper time

Solution: For the clock on the earth :

$$\Delta t=\frac{s}{v}=\frac{4.3\times 10^{16}}{0.999\times 3\times 10^{8}\times 365\times 24\times 3600}=4.55Y$$

For the clock on the spacecraft:

$$\tau = \gamma^{-1}\Delta t = \sqrt{1-0.999^2} \times 4.55 = 0.203\,Y$$

正是时间膨胀效应使得在人的有生之年进行星际航行成为可能。

实验验证：

2) 飞机载铯原子钟环球航行

1971年： 地球赤道地面钟： A

地球赤道上空约一万米处钟

向东飞行： B

向西飞行： C

A，B，C 对太阳参考系均向东： $v_B > v_A > v_C$

结果：钟 B 慢于 A 慢于 C

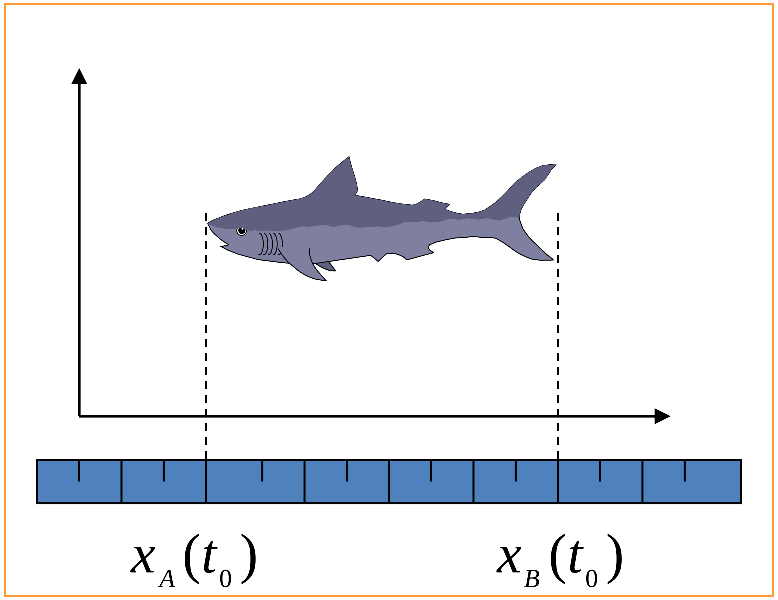
$B \xrightarrow[59\text{ns}]{\text{慢于}} A \xrightarrow[273\text{ns}]{\text{慢于}} C$

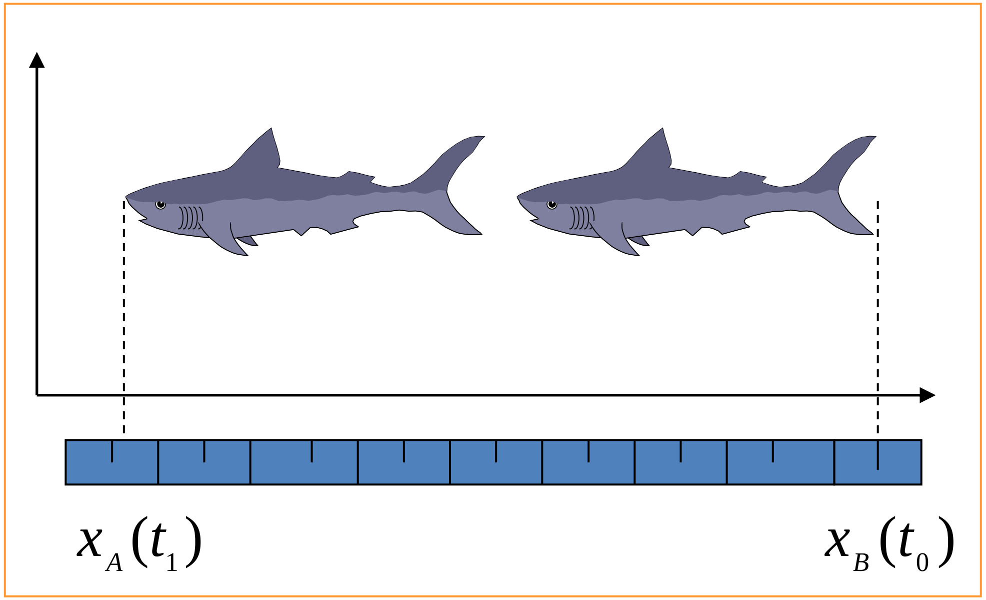
验证了相对论时间膨胀效应。

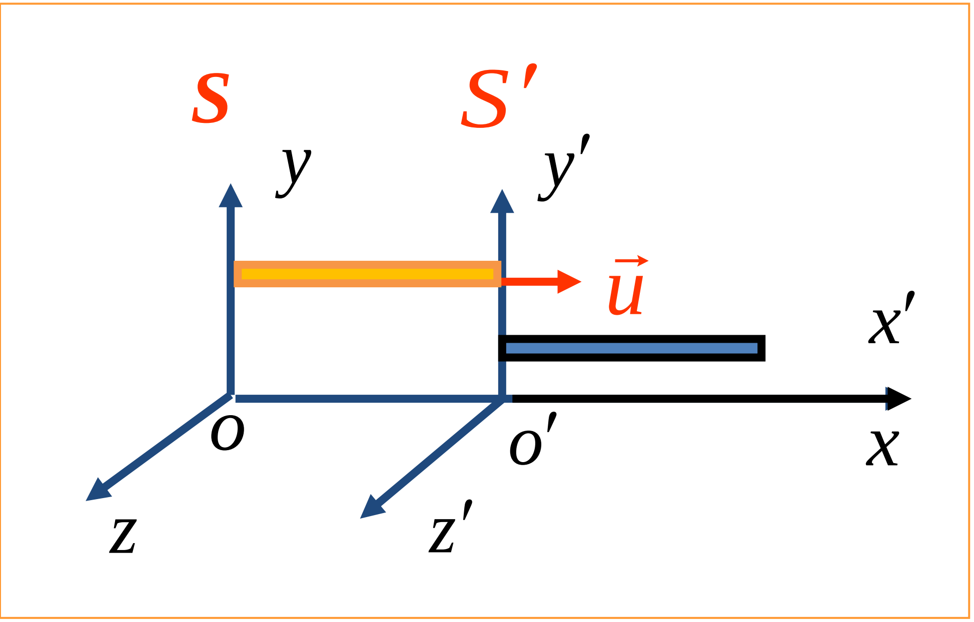
Relativity of spatial measurement

Attention：

Both ends must be recorded as the object moves at same time。







设尺相对于 S' 系静止

测量其两端坐标：

	Event1	Event2
S	$x_{1},\ t_1$	$x_{2},\ t_2$
S'	$x'_{1},\ t'_1$	$x'_{2},\ t'_2$

The length measured in a frame of reference at rest with respect to an object——proper length

$$L' = x'_2 - x'_1$$ 两端坐标不一定同时测量。

在 s 系中测尺的长度，两端坐标一定要同时测量。

$$L = x_B - x_A$$ **——non-proper length**

静系： $\Delta t'$不一定为零，$\Delta x'$为原长

动系： Δt 一定为零，Δx非原长

Derived from Lorentz transformation：

$$\Delta x' = \gamma(\Delta x - u\Delta t) \qquad \Delta x' = \gamma \Delta x > \Delta x$$

原长　　观测长度（非原长）　　0

若尺相对于 S 系静止

$$\Delta x = \gamma(\Delta x' + u\Delta t') \qquad \Delta x = \gamma \Delta x' > \Delta x'$$

原长　　观测长度（非原长）　　0

原长最长！

Conclusions：

The measurement of the space interval is relative, and the length of the object is related to the selection of the inertial system;

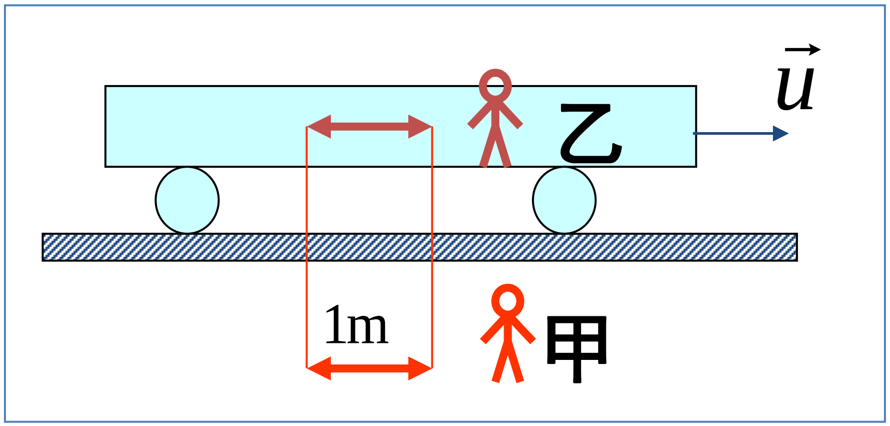
The proper length is the longest of all length measurements;

在其它惯性系中测量相对其运动的尺，总得到比原长小的结果 —— 动尺缩短。

注意:

- 尺缩效应只在相对运动方向上发生;
- 尺缩效应是高速运动物体的测量形象，不是视觉形象。

例：一列高速火车以速率 u 驶过车站，站台上的观察者甲观察到固定于站台、相距 1m 的两激光打标机车厢上同时打出两个光斑，求车厢上的观察者乙测出两个光斑间的距离为多少？



Consider:

Which is proper length?

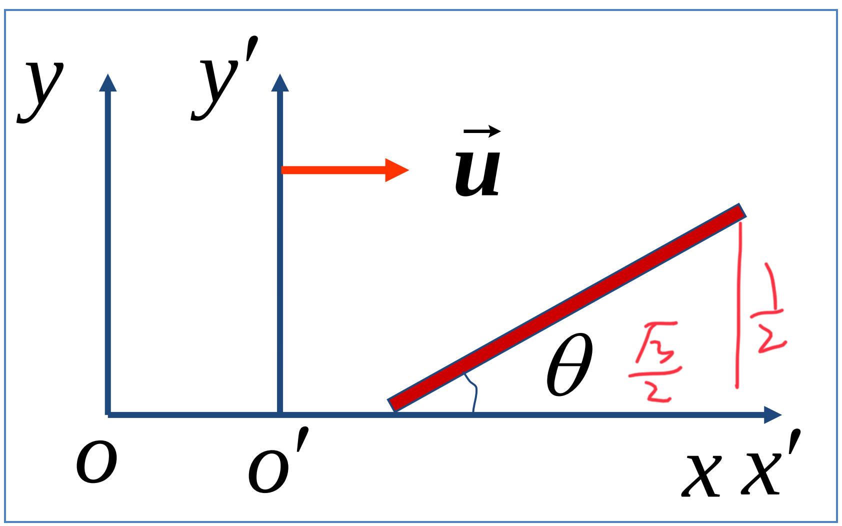
站台系：动系，两端同时测 $\Delta s = 1\text{m}$ 非原长

车厢系：静系，$\Delta s' = ?$ 为原长

$$\Delta s' = \gamma \Delta s = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} > 1(\text{m})$$

考题：

一根米尺静止放置在s'系中，与 $o'x'$ 轴成 30° 角，如果在 S 系中测得米尺与 ox 轴成 45° 角，那么，s'系相对于S 系的运动速度 u为多大？ S 系中测得米尺的长度是多少？



$$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$1-\frac{u^2}{c^2} = \frac{1}{3}$$

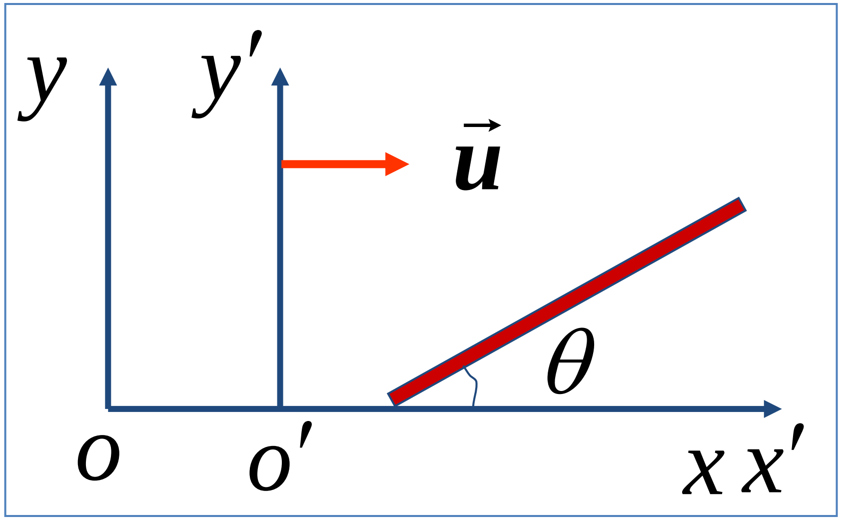
$$\frac{u^2}{c^2} = \frac{2}{3}$$

$$u = \frac{\sqrt{6}}{3}c$$

解： 由题意可知

$$\operatorname{tg}30^\circ = \frac{\Delta y'}{\Delta x'}$$

$$\operatorname{tg}45^\circ = \frac{\Delta y}{\Delta x}$$



由 $\Delta y = \Delta y'$ 得 $\dfrac{\Delta x}{\Delta x'} = \dfrac{\operatorname{tg}30^\circ}{\operatorname{tg}45^\circ}$

根据相对论“尺缩”效应，有

$$\Delta x = \Delta x'\sqrt{1-\left(\frac{u}{c}\right)^2}$$

即 $$\frac{\Delta x}{\Delta x'} = \sqrt{1-\left(\frac{u}{c}\right)^2}$$

于是

$$\sqrt{1-\frac{u^2}{c^2}}=\frac{\mathrm{tg}\,30^\circ}{\mathrm{tg}\,45^\circ}$$

$$\sqrt{1-\frac{u^2}{c^2}}=\frac{\sqrt{3}}{2}$$

$$\frac{u^2}{c^2}=\frac{1}{4}$$

米尺还有什么发生变化?

得

$$u=\sqrt{\frac{2}{3}}c=0.816c$$

由于

$$\Delta y=\Delta y'$$

所以

$$L'\sin 30^\circ=L\sin 45^\circ$$

米尺长度

$$L=\frac{\sin 30^\circ}{\sin 45^\circ}L'=\frac{\sqrt{2}}{2}\times 1=0.707\mathrm{m}$$

静系中同地事件的时间间隔为原时，（Proper time）

相对于参考系静止的事件发生地同时测量的空间间隔为原长，（Proper length）

1.宇宙飞船相对于地面以速度 v 作匀速直线飞行，某一时刻飞船头部的宇航员向飞船尾部发出一个光讯号，经过 Δt （飞船上的钟）时间后，被尾部的接收器收到，则由此可知飞船的固有长度为

(A) $c \cdot \Delta t$

(B) $v \cdot \Delta t$

(C) $c \cdot \Delta t \cdot \sqrt{1-(v/c)^2}$

(D) $\dfrac{c \cdot \Delta t}{\sqrt{1-(v/c)^2}}$

[A]

6.相对于地球的速度为 v 的一飞船，要到离地球为 5 光年的星球上去。若飞船的宇航员测得该旅程的时间为 3 年，则 v 应为：

(A) $c/2$　　(B) $3c/5$

(C) $9c/10$　　(D) $4c/5$

[D]

7.坐标轴相互平行的两惯性系 S、S'，S 相对沿 ox 轴正方向以 v 匀速运动，在 S' 中有一根静止的刚性尺，测得它与 ox' 轴成 $30°$ 角，与 ox 轴成 $45°$ 角，则 v 应为：

(A) $2c/3$　　(B) $c/3$

(C) $(2/3)^{1/2}c$　　(D) $(1/3)^{1/3}c$

[C]

8.观察者甲、乙，分别静止在惯性系 S、S' 中，S' 相对 S 以 u 运动，S' 中一个固定光源发出一束光与 u 同向

(1)乙测得光速为 c .

(2)甲测得光速为 $c+u$;

(3)甲测得光速为 $c-u$ ；

(4)甲测得光相对于乙的速度为 $c-u$。

正确的答案是:

(A) (1),(2),(3);　　(B) (1),(4)

(C) (2),(3);　　(D) (1),(3),(4)

[B]

由此得 $$t'_1 - t'_2 = \frac{v(x_2 - x_1)/c^2}{\sqrt{1-(v/c)^2}}$$

$$= 5.77 \times 10^{-6}\,\mathrm{s}$$

26-11 The Impact of Special Relativity

The predictions of special relativity have been tested thoroughly, and verified to great accuracy.

The correspondence principle says that a more general theory must agree with a more restricted theory where their realms of validity overlap. This is why the effects of special relativity are not obvious in everyday life.

Summary of Chapter 26

- Inertial reference frame: one in which Newton's first law holds
- Principles of relativity: the laws of physics are the same in all inertial reference frames; the speed of light in vacuum is constant regardless of speed of source or observer
- Time dilation:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}.$$

$$\Delta t = \gamma \, \Delta t_0$$

Summary of Chapter 26

- Length contraction:

$$\boxed{\ell = \ell_0\sqrt{1 - v^2/c^2}}$$

$$\boxed{\ell = \frac{\ell_0}{\gamma}.}$$

- Gamma:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}.$$

Summary of Chapter 26

- Relativistic momentum:

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}} = \gamma mv.$$

- Mass-energy relation:

$$E = mc^2.$$

Summary of Chapter 26

- Kinetic energy:

$$\text{KE} = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2.$$

$$\text{KE} = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2.$$

- Total energy:

$$E = \text{KE} + mc^2.$$

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}}.$$

Summary of Chapter 26

- Relationship between energy and momentum:

$$E^2 = \frac{m^2c^2c^2}{1 - v^2/c^2} = \frac{m^2c^2(c^2 - v^2 + v^2)}{1 - v^2/c^2} = \frac{m^2c^2v^2}{1 - v^2/c^2} + \frac{m^2c^2(c^2 - v^2)}{1 - v^2/c^2}$$

$$= p^2c^2 + \frac{m^2c^4(1 - v^2/c^2)}{1 - v^2/c^2}$$

or

$$E^2 = p^2c^2 + m^2c^4.$$