Linear Algebra Homework #8
Fall 2020

Name: Due: Monday, Dec. 28th, 2020

Instructions:

Please include essential steps in your solution. For most of the problems, answers without essential steps may receive a score of 0.

1. Find eigensystems for the following matrices. Specify the algebraic and geometric multiplicity of each eigenvalue and identify any defective matrices.

$$(a) \quad \begin{bmatrix} 1+i & 3 \\ 0 & i \end{bmatrix}. \qquad (b) \begin{bmatrix} 2 & 1 & -1 & -2 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

2. Let A be a square matrix and $f(x)$ an arbitrary polynomial. Show that if λ is an eigenvalue of A, then $f(\lambda)$ is an eigenvalue of $f(A)$. Verify this fact with the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ and $f(x) = 3 + 2x + x^2$. That is, find each eigenvalue λ of A and then verify that $f(\lambda)$ is an eigenvalue of $f(A)$.

3. Determine if the matrices in Problem 1 are diagonalizable. If so, find a matrix P such that $P^{-1}AP$ is diagonal. If not, explain why.

4. Show that the matrices $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & 2 \\ 0 & 3 & 0 \\ 0 & -1 & 2 \end{bmatrix}$ are similar as follows: find diagonalizing matrices P,Q for A,B, respectively, that yield identical diagonal matrices, set $S = PQ^{-1}$, and confirm that $S^{-1}AS = B$.