

# ENSC 2113

## Engineering Mechanics: Statics

Lecture 29  
Section 10.1-10.3



College of Engineering, Architecture & Technology

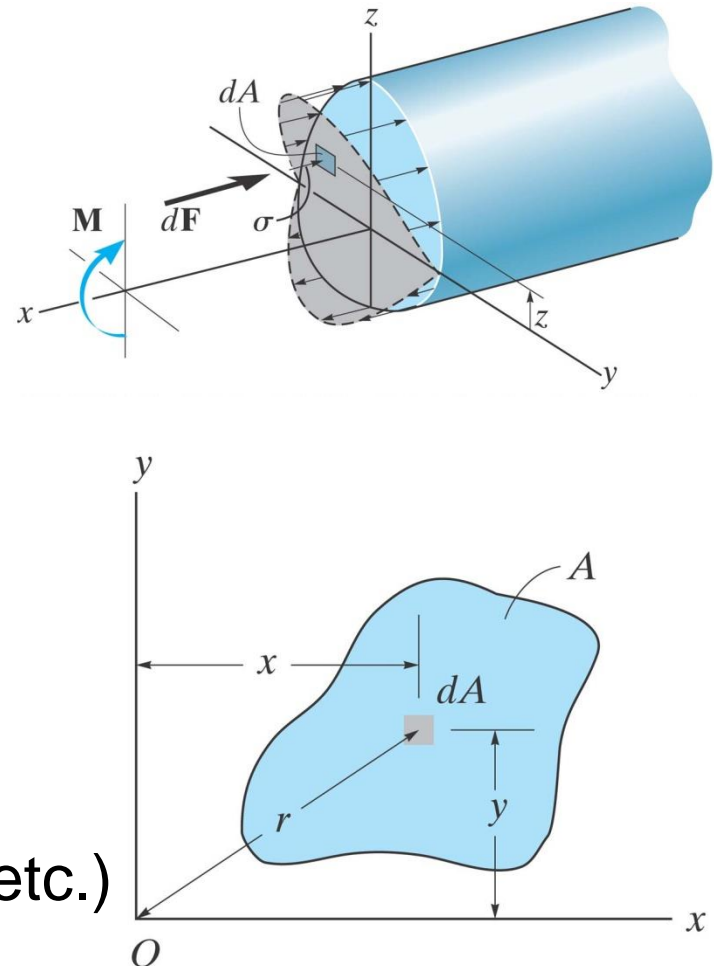
## 10.1: Moment of Inertia

The moment of inertia of a shape is defined as *the integral of the second moment of an area*.

Moment of inertia taken about orthogonal axes:

$$I_x = \int_A y^2 dA$$
$$I_y = \int_A x^2 dA$$

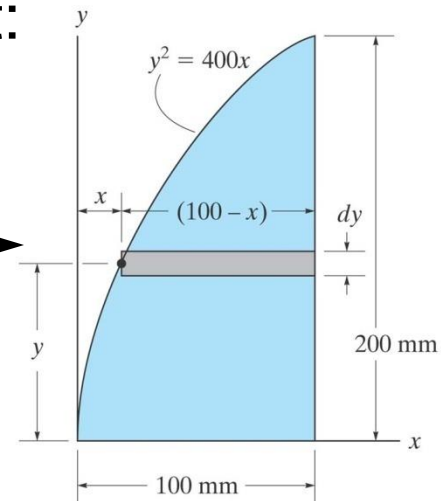
Moment of Inertia has units to the fourth power (in<sup>4</sup>, mm<sup>4</sup>, etc.)



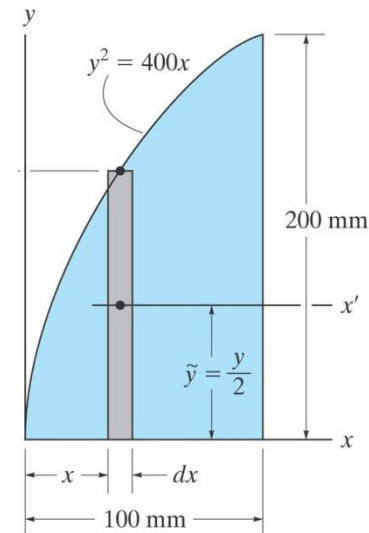
The process involves cutting a differential slice through the shape and integrating the distance squared across the area.

**For this method to work**, you need to cut the slice in the direction **parallel** to the axis of interest:

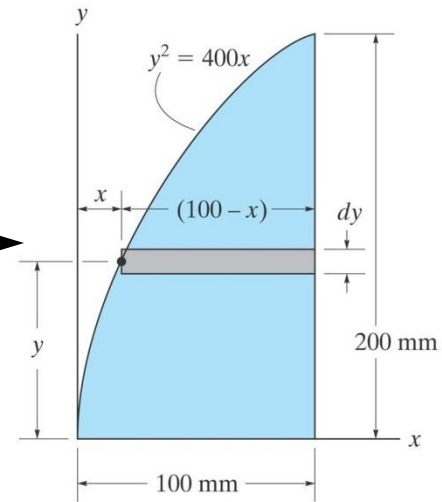
$$I_x = \int_A y^2 dA$$



$$I_y = \int_A x^2 dA$$



$$I_x = \int_A y^2 dA$$



where,

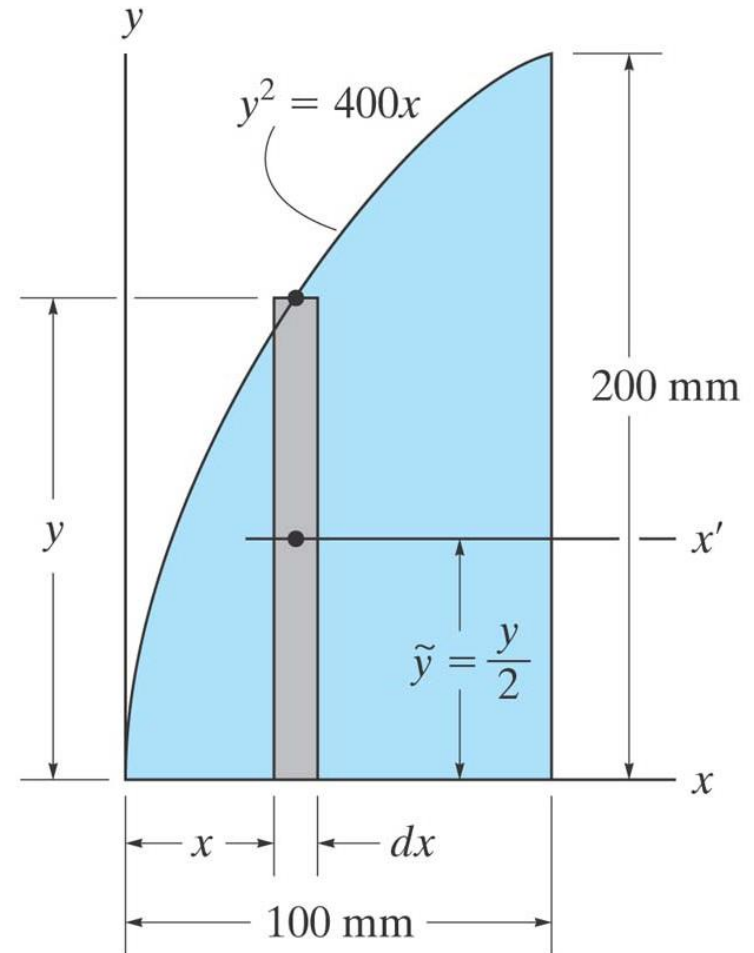
$I_x$  or  $I_y$  = Moment of Inertia of shape about the  $x$  or  $y$  axis

$x$  or  $y$  = Distance from  $x$  or  $y$  axis to centroid of slice

$dA$  = Differential area of slice

If you choose to cut the differential slice perpendicular to the axis of interest, then you must use:

## ***The Parallel-Axis Theorem***



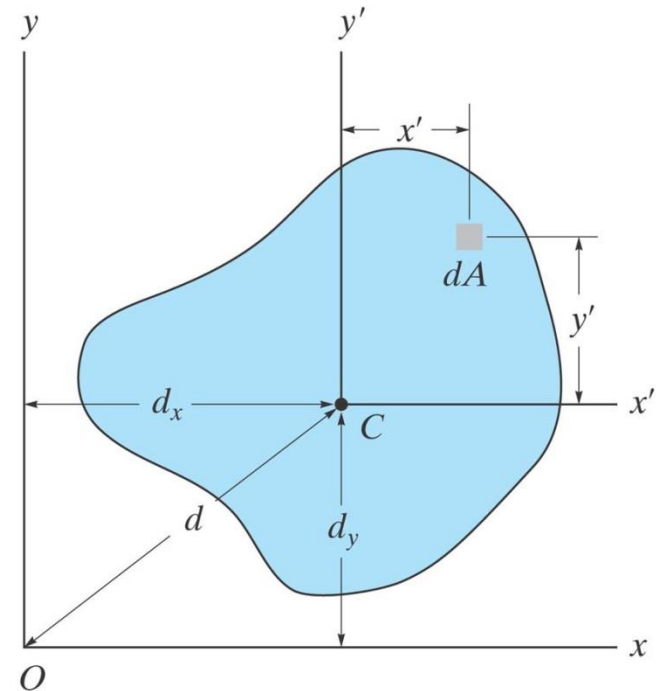
## 10.2: Parallel-Axis Theorem

If the *moment of inertia* is known through its centroidal axes, then it can also be found about any other axis parallel to the centroidal axis.

The eqns for this are (text eqns 10-3 & 10-4):

$$I_x = \bar{I}_{x'} + Ad_y^2$$

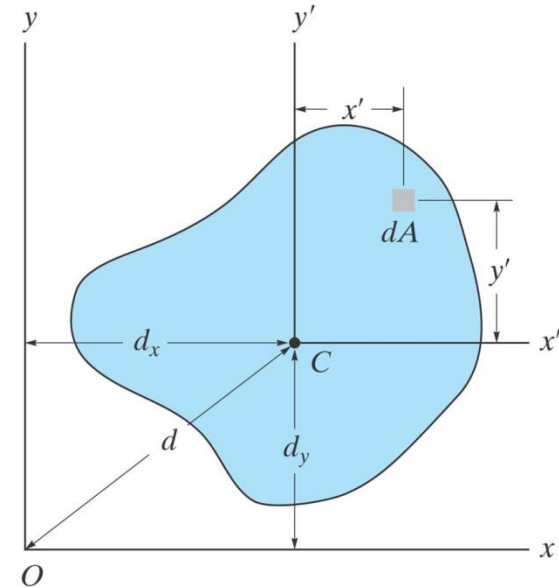
$$I_y = \bar{I}_{y'} + Ad_x^2$$



Definitions of the values in the eqns:

$$I_x = \bar{I}_{x'} + Ad_y^2$$

$$I_y = \bar{I}_{y'} + Ad_x^2$$



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$I_x, I_y$  = *moment of inertia* about the x or y axis

$\bar{I}_{x'}, \bar{I}_{y'}$  = *moment of inertia* about the centroid of the shape

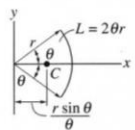
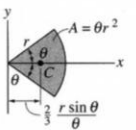
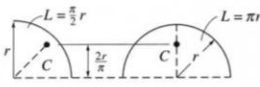
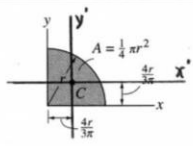
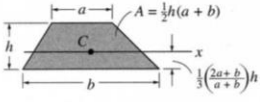
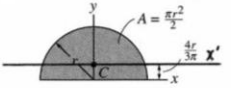
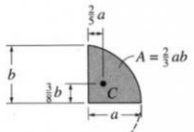
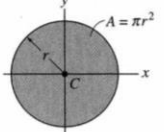
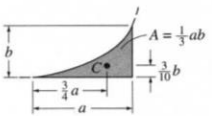
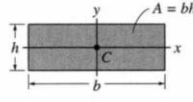
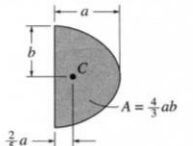
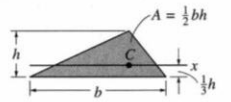
**A** = Area of the shape

$d_x, d_y$  = distance from centroid of shape to axis of interest

Moment of Inertia about the centroid of a shape,  $\bar{I}_x$ ,  $\bar{I}_y$ :

Refer to the inside cover of the back of your text for eqns ...

**Geometric Properties of Line and Area Elements**

| Centroid Location  | Centroid Location  | Area Moment of Inertia   |
|--|--|--|
|  <p>Circular arc segment</p>        |  <p>Circular sector area</p> | $I_x = \frac{1}{4} r^4 (\theta - \frac{1}{2} \sin 2\theta)$<br>$I_y = \frac{1}{4} r^4 (\theta + \frac{1}{2} \sin 2\theta)$ |
|  <p>Quarter and semicircle arcs</p> |  <p>Quarter circle area</p>  | $I_x' = I_y' = .05488 r^4$<br>$I_x = \frac{1}{16} \pi r^4$<br>$I_y = \frac{1}{16} \pi r^4$                                 |
|  <p>Trapezoidal area</p>            |  <p>Semicircular area</p>    | $I_x' = .1098 r^4$<br>$I_x = \frac{1}{8} \pi r^4$<br>$I_y = \frac{1}{8} \pi r^4$   |
|  <p>Semiparabolic area</p>          |  <p>Circular area</p>        | $I_x = \frac{1}{4} \pi r^4$<br>$I_y = \frac{1}{4} \pi r^4$   |
|  <p>Exparabolic area</p>          |  <p>Rectangular area</p>   | $I_x = \frac{1}{12} b h^3$<br>$I_y = \frac{1}{12} h b^3$   |
|  <p>Parabolic area</p>            |  <p>Triangular area</p>    | $I_x = \frac{1}{36} b h^3$<br>$I_y = \frac{1}{36} h b^3$   |

For use in  
*Parallel Axis  
Theorem*



The eqn using the *Parallel-Axis* theorem becomes:

$$I_x = \int dI_x$$

where,

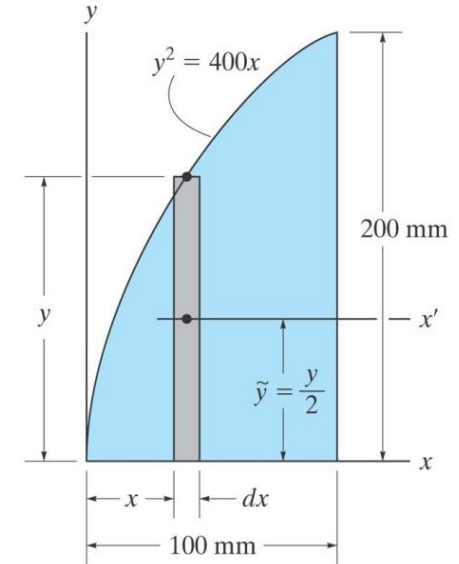
$$I_x = \bar{I}_{x'} + Ad_y^2$$

$$dI_x = d\bar{I}_{x'} + dA\tilde{y}^2$$

$d\bar{I}_{x'}$  = moment of inertia of differential slice about its centroid

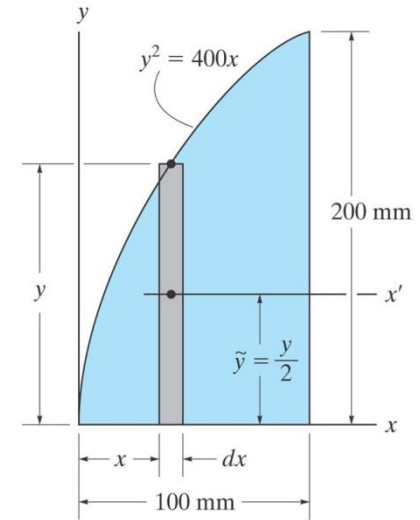
$dA$  = area of differential slice

$\tilde{y}$  = distance from centroid of slice to axis of interest



## Procedures for determining *Moment of Inertia*:

1. Choose a differential slice to use. Its orientation will affect the integration equation used:
  - a) If differential slice is taken parallel to the axis, use the *basic* eqn.
  - b) If the slice is taken perpendicular to the axis, use the *Parallel-Axis Theorem* eqn.
2. Define slice size & moment arm to use. Draw these on the sketch for reference.
3. Perform the integrations & apply eqns previously derived. *Integrate in the direction perpendicular to the slice.*



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