Linear Algebra Homework #3
Fall 2020

Name: Due: Wednesday, Dec. 2nd, 2020

Instructions:

Please include essential steps in your solution. For most of the problems, answers without essential steps may receive a score of 0.

1. Let $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 \\ 1 & -2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \end{bmatrix}$, and compute the following when possible

$$(a)\ A + 3B; \quad (b)\ 2C - 3(A - 2C).$$

2. Write the following vectors as a linear combination of constant vectors with scalar coefficients x, y, z, or w.

$$(a)\ \begin{bmatrix} 3x + y \\ x + y + z \end{bmatrix} \quad (b)\ \begin{bmatrix} 3x + 2y - w \\ w - z \\ x + y - 2w \end{bmatrix}.$$

3. Find scalars a, b, c, d such that

$$\begin{bmatrix} d & 2a \\ 2d & a \end{bmatrix} = \begin{bmatrix} a - b & b + c \\ a + b & c - b + 1 \end{bmatrix}$$

4. Express the matrix $D = \begin{bmatrix} 3 & 3 \\ 1 & -3 \end{bmatrix}$ as a linear combination of the matrices $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix}$.

5. For each pair of matrices A,B, calculate the product AB or indicate that the product is undefined.

$$(a)\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 3 & -2 & 0 \\ -2 & 5 & 8 \end{bmatrix}; \quad (b)\ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 & 0 \\ -2 & 8 \end{bmatrix}$$

6. Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$ and $X = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$. Find the coefficient matrix of the linear system $AX - XA = I_2$ in the variables x, y, z, w.

7. Let $A = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$, $f(x) = 1 + x + x^2$, $g(x) = 1 - x$, and $h(x) = 1 - x^3$. Verify that $f(A)g(A) = h(A)$.

8. Given that $A = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$ and $AB = \begin{bmatrix} 5 & -2 & 3 \\ -1 & 1 & -6 \end{bmatrix}$ for a suitable matrix B, find the third column of B.

9. Balance the following chemical equation:

$$C_8H_{18} + O_2 \to CO_2 + H_2O.$$