

Linear Algebra Final Exam (Form A), Fall 2019

Course Code 2475 Course Name Linear Algebra

Time allowed 120 minutes

NO.	1	2	3	4	5	6	7	8	Score
Pts									

Instructor Signature _____

Instructions:

*Please attempt every problem. You must support every solution with an appropriate amount of work and/or description. Unsupported answers may receive a score of 0. The total is **100** points. Good luck !*

1. For what values of c are the following linear system inconsistent, with unique solution, or with infinitely many solutions?

$$x_1 + 2x_2 - x_3 = c$$

$$x_1 + 3x_2 + x_3 = 1$$

$$3x_1 + 7x_2 - x_3 = 4$$

2. Find $A^{-1}C$ where $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 2 & 0 \end{bmatrix}$

3. Solve the following system of linear equations using Cramer's Rule:

$$x_1 + x_2 + x_3 = -1$$

$$2x_1 + 5x_3 = 1$$

$$4x_1 + 6x_2 = 0$$

4. Find the LU factorization of A if $A = \begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & -1 \\ 2 & 3 & -3 \end{bmatrix}$

5. Let V be the column space of A with the standard inner product, where

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ -1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}.$$

- Find a basis for V using the column space algorithm.
- Compute an orthogonal basis of V using Gram-Schmidt algorithm based on the bases found in a).
- Find the coordinate vector of $\vec{v} = (1, 3, 2)^T$ with respect to the orthogonal basis obtained from b) using Orthogonal Coordinate Theorem.
- Find an orthonormal basis of V based on the result from b).

6. Let $A = \begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, and $P = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$.

- Show that $A = PBP^{-1}$.
- Let $q(x) = 2 + x^{100}$. Find $q(A)$.

7. Consider the following expression:

$$Q(x_1, x_2, x_3) = 2x_1^2 + 2x_2^2 + 2x_3^h - 2x_1x_2^k + 2bx_1x_3 - 2x_2x_3 + lx_2^2x_3.$$

- Find values of h , k and l such that $Q(x_1, x_2, x_3)$ represents a quadratic form.
- Find the associated matrix of the quadratic form $Q(x_1, x_2, x_3)$.
- For what values of b is $Q(x_1, x_2, x_3)$ positive semidefinite?

8. Find the orthogonal canonical form of the quadratic form

$$Q(x_1, x_2) = x_1^2 + 4x_1x_2 + x_2^2.$$

In addition, give the associated coordinate transformation, canonical basis and principal axes of the given form.