Linear Algebra Homework #4
Fall 2020

Name: Due: Monday, Dec. 7th, 2020

**Instructions:**

*Please include essential steps in your solution. For most of the problems, answers without essential steps may receive a score of 0.*

1. Convert the following $3 \times 3$ elementary operations to matrix form and convert matrices to elementary operation form.

$$(a)\ E_{23}(3);\quad (b)\ E_3(2);\quad (c)\begin{bmatrix}1 & 3 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix};\quad (d)\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ -a & 0 & 0\end{bmatrix}.$$

2. Compute the reduced row echelon form of the following matrix and express each form as a product of elementary matrices and the original matrix.

$$\begin{bmatrix}1 & 1 & 1\\ 0 & 0 & 0\\ 1 & 2 & 2\end{bmatrix}$$

3. Identify a complete list of simple structure descriptions that apply to the following matrices. (e.g., for the matrix $\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{bmatrix}$: the complete list is: diagonal matrix; upper triangular matrix; lower triangular matrix; triangular matrix; tridiagonal matrix.)

$$(a)\ \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 3\\ 0 & 0 & 0\end{bmatrix};\quad (b)\begin{bmatrix}2 & 0 & 0 & 0\\ 1 & 2 & 0 & 0\\ 0 & 0 & 1 & 0\\ 1 & 0 & 1 & 1\end{bmatrix};\quad (c)\ \begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$$

4. Express the matrix $\begin{bmatrix}1 & 2 & 1\\ 0 & 0 & 0\\ 2 & 4 & 2\end{bmatrix}$ as an outer product of two vectors.

5. Compute the transpose and conjugate transpose of the following matrices and determine which are symmetric or Hermitian.

$$(a)\ \begin{bmatrix}1+i & 1\\ 1 & 1-i\end{bmatrix};\ (b)\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix};\quad (c)\ \begin{bmatrix}1 & i\\ -i & 1\end{bmatrix};\quad (d)\ \begin{bmatrix}1 & 1 & 3\\ 1 & 0 & 0\\ 3 & 0 & 2\end{bmatrix}$$

6. A square matrix A is called normal if $A^*A = AA^*$. Determine if $\begin{bmatrix} 1 & i \\ 1 & 2+i \end{bmatrix}$ is normal.

7. Find the inverse or show that it does not exist.

$$(a)\ \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}; (b)\ \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}; (c)\ \begin{bmatrix} 2 & -2 & 1 \\ 0 & 2 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

8. Solve the following system by matrix inversion.

$$\begin{aligned} x_1 + 6x_2 - x_3 &= 4 \\ x_1 + \ x_2 \qquad &= 0 \\ 2x_2 \qquad &= 1 \end{aligned}$$

9. Solve $AX = C$ for $X$, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & 1 \\ 2 & 5 & -6 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & -1 & 1 & 1 \\ 2 & 0 & -6 & 0 \end{bmatrix}$.

10. Determine for what values of $k$ the following matrix is invertible and find the inverse in that case.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ k & 0 & 1 \end{bmatrix}.$$