University of Colorado **Boulder**

CSCI 3022: Intro to Data Science

Spring 2019 Tony Wong

# Lecture 2: Exploratory Data Analysis and Summary Statistics

EDA

creative adventures with your data

# Announcements and reminders

- **Canvas:** make sure you have looked over the syllabus and schedule

  https://canvas.colorado.edu/courses/24706

- **Piazza:** be on it, because no more emails, and I don't like Canvas very much!

  https://piazza.com/colorado/spring2019/csci3022/

- Get **Jupyter notebook / Anaconda Python** -- make sure you have a working install and check out the Numpy/Pandas tutorial (github/notebooks)

  https://www.anaconda.com/downloads

Quizlet 0 due Friday! (10 AM)
→ syllabus & policies, ironing stuff out

## Populations and samples

Data scientists hope to learn about some **characteristic/variable** of a **population**

But, we usually can't actually see/study the whole population → so we study a **sample**

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**Definition:** A **population** is a collection of units (people, songs, tweets, marmots)

**Definition:** A **sample** is a subset of the population

**Definition:** A **characteristic**/**variable of interest** **(VOI)** is something we want to measure for each unit.

# Populations and samples

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**Example:** S'pose the city of Boulder wants to estimate its per-household income via a phone survey. They call every 50th number on a list of Boulder phone numbers between 6 PM and 8 PM. In this case, we have:

Population: all Boulder residents

Sample: every 50th person on the phone list ... that picks up

Variable of Interest: per household income

# Populations and samples

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| | |
|---|---|
| Population: | Boulder residents |
| Sample: | every 50th person w/ phone who answers |
| Variable of Interest: | household income |

**Definition:** the **<u>sample frame</u>** is the source material or device from which sample is drawn

# Sample types

- **Simple random sample:** randomly select people from sample frame
- **Systematic sample:** order the sample frame. Choose integer *k*. Sample every *k*th unit in the sample frame.
- **Census sample:** sample literally *everyone/everything* in the population
- **Stratified sample:** if you have a heterogeneous population that can be broken up into homogeneous groups, randomly sample from each group proportionate to their prevalence in the population

Middle school: 6, 7, 8th grade (population)

Sample VOI: heights

Stratified sample: take 10% of each sub-group

| | Pop | | strat. samp. |
|---|---|---|---|
| 6th: | 100 | → | 10 |
| 7th: | 60 | → | 6 |
| 8th: | 40 | → | 4 |

# Populations and samples

Data scientists hope to learn about some **characteristic/variable** of a **population**

But, we usually can't actually see/study the whole population → so we study a **sample**

So how do we make the jump from studying a sample to drawing meaningful conclusions about the characteristic of the population?

## Populations and samples

Data scientists hope to learn about some **characteristic/variable** of a **population**

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So how do we make the jump from studying a sample to drawing meaningful conclusions about the characteristic of the population?

… **inference!**

# Exploratory data analysis (EDA)

Before we learn about **inference** though, we first need to learn how to **explore** the data

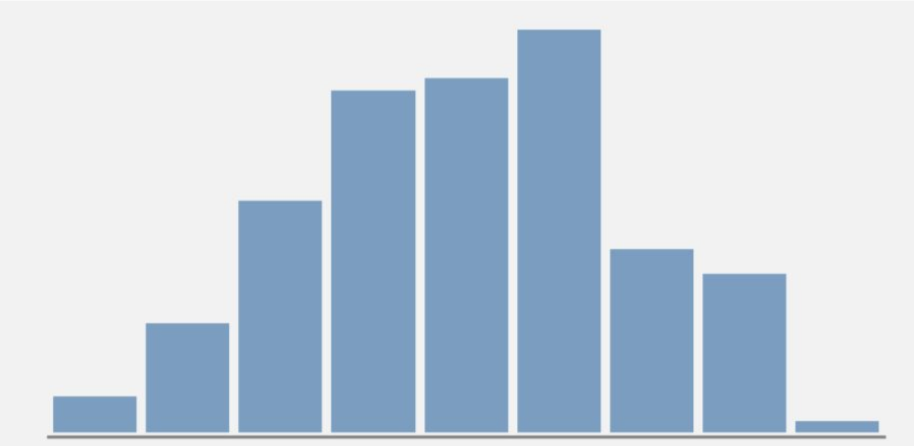
Useful for summarizing, recognizing patterns, etc. in the data

There are two main types of data exploration: **numerical** and **graphical**

# Numerical summaries

The calculation and interpretation of certain summarizing numbers can help us gain a better understanding of the data

These sample numerical summaries are called **sample statistics**



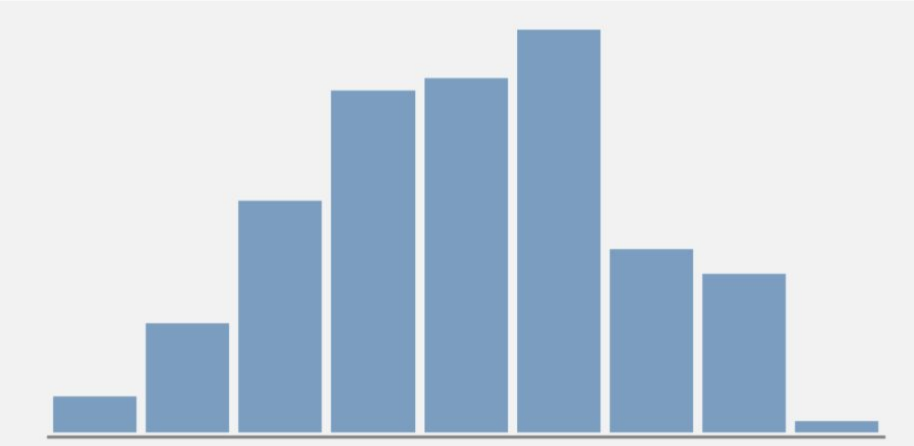
# Measures of centrality

Summarizing the “center” -- or better yet -- “**central tendency**” of the sample data is a popular and important characteristic of a set of numbers

**Goal:** capture something about the “typical” unit in the sample with respect to the VOI

3 main measures:

1) Mean
2) Median
3) Mode



center?

## Sample mean

$n$ = # data pts // Python: n = len(x)

**Definition:** For a given set of numbers $x_1, x_2, \ldots, x_n$, the **sample mean** is given by:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{n}\left[x_1 + x_2 + \ldots + x_n\right]$$

Also called the **arithmetic average**

**Example:** Compute the sample mean of the data 2, 4, 3, 5, 6, 4 ← $n=6$

$$\bar{x} = \frac{1}{6}\left[\underbrace{2+4}_{6}+\underbrace{3+5}_{8}+\underbrace{6+4}_{10}\right] = \frac{24}{6} = \boxed{4}$$

## Sample mean

**Definition:** For a given set of numbers $x_1, x_2, \ldots, x_n$, the **sample mean** is given by:

$$\bar{x} = \frac{1}{n}\sum_{k=1}^{n} x_k$$

Also called the **arithmetic average**

**Example:** Compute the sample mean of the data 2, 4, 3, 5, 6, 4

$$\begin{aligned}\bar{x} &= \frac{1}{6}(2+4+3+5+6+4) \\ &= \frac{1}{6}\cdot 24 \\ &= 4\end{aligned}$$

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$$\bar{x} = \frac{1}{n}\sum_{k=1}^{n} x_k$$

Also called the **arithmetic average**

**Advantages:** easy & fast to calculate ; widely used

**Disadvantages:** outliers can make interpretation misleading

"Extreme" data points

# Sample mean

**Definition:** For a given set of numbers $x_1$, $x_2$, … , $x_n$, the **sample mean** is given by:

$$\bar{x} = \frac{1}{n}\sum_{k=1}^{n} x_k$$

Also called the **arithmetic average**

**Advantages:** easy to calculate

**Disadvantages:** outliers can make interpretation misleading

## Sample median

**Definition:** For a given set of numbers the **sample median** is the “middle” value when the observations are ordered from smallest to largest.

half data are above $\tilde{x}$ & half are below

1 3 4 (4)

**Calculation:**

- Order the *n* observations from smallest to largest
- Include multiple instances of repeated values

- If *n* is odd, then $\tilde{x} = \left(\frac{n+1}{2}\right)^{th}$ ordered value

$\tilde{x} = 3$

- If *n* is even, then $\tilde{x} =$ average of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ ordered values

$\tilde{x} = \frac{1}{2}[3+4] = 3.5$

## Sample median

**Definition:** For a given set of numbers the **sample median** is the "middle" value when the observations are ordered from smallest to largest.

1 2 3 4 5 6 7 8 9 10

**Example:** Calculate the sample median of the data ~~36~~, ~~15~~, ~~39~~, ~~41~~, ~~40~~, ~~42~~, ~~47~~, ~~49~~, ~~7~~, ~~6~~

1.) sort em: $6, 7, 15, 36, \boxed{39, 40}, 41, 42, 47, 49$

1 2 3 4 5 6

2.) $n = 10$, so $\tilde{x}$ = avg. of $5^{th}$ & $6^{th}$ data pts: $\tilde{x} = \frac{1}{2}(39 + 40)$

$$\boxed{\tilde{x} = 39.5}$$

LaTeX: $\tilde{x}$ = \tilde{x}

## Sample median

**Definition:** For a given set of numbers the **sample median** is the “middle” value when the observations are ordered from smallest to largest.

**Example:** Calculate the sample median of the data 36, 15, 39, 41, 40, 42, 47, 49, 7, 6

**Solution:** $n = 10$ is even so it’s the average of the middle 2 numbers when sorted:

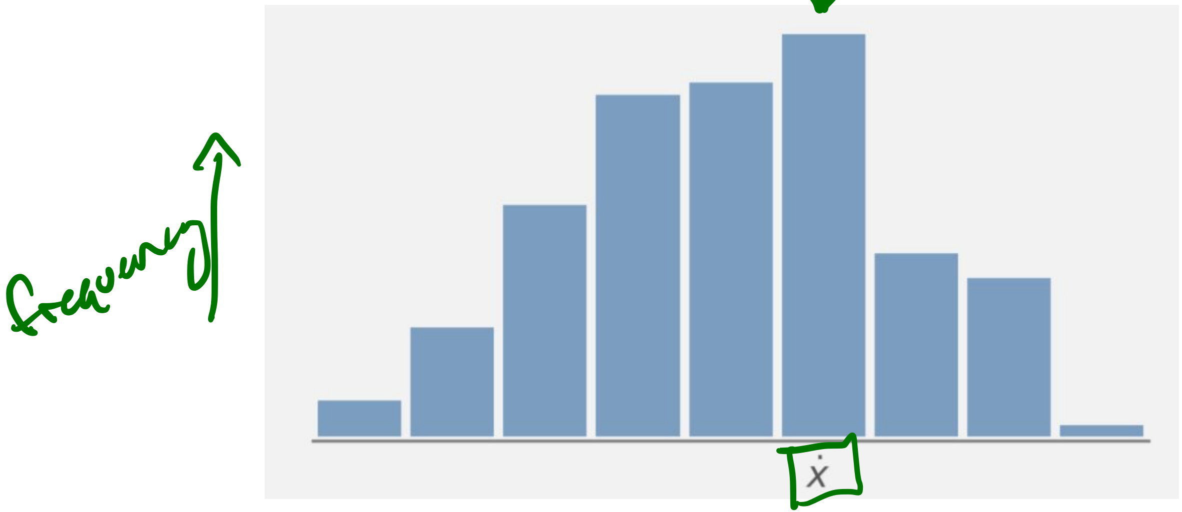
6, 7, 15, 36, **39, 40**, 41, 42, 47, 49

$\rightarrow$ **39.5**

## Sample mode

**Definition:** The **sample mode** is the value that occurs the most often in the sample.

highest frequency data points : occurs most often

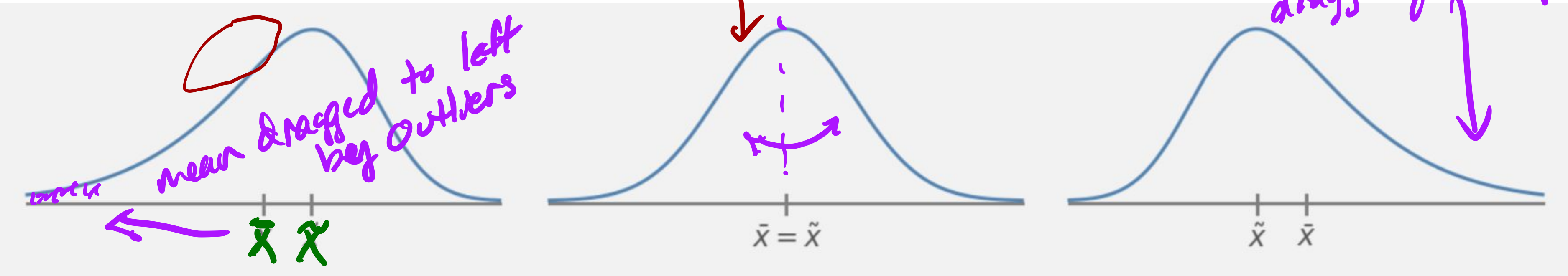


# Mean vs median

The population mean and median will generally not be equal.

If the population distribution is positively or negatively skewed …

extreme high data pts are dragging mean up




negative skew (left-skew)

$\bar{x} < \tilde{x}$

symmetric

"roughly"

positive skew (right skew)

$\bar{x} > \tilde{x}$

left side has fewer values → left side got screwed!

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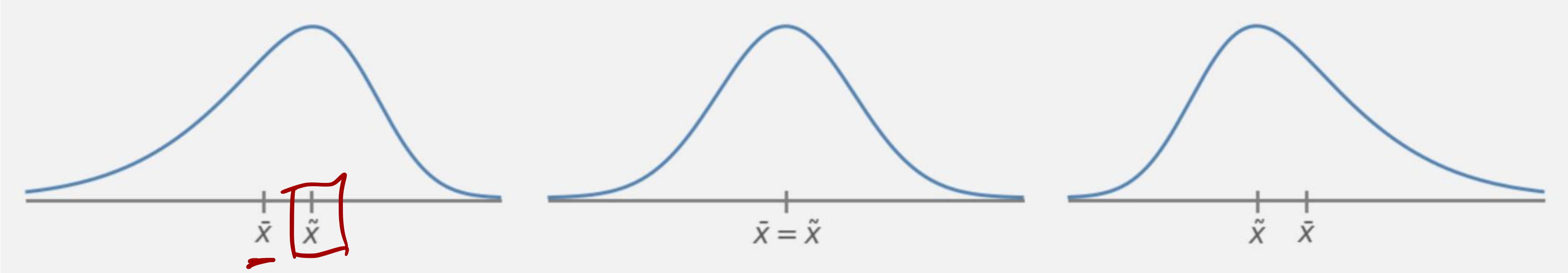
If the population distribution is positively or negatively skewed …



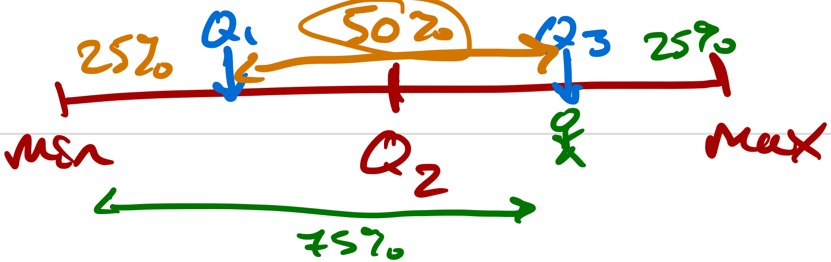

negative skew
(left-skew)

symmetric

positive skew
(right skew)

→ Which measure of central tendency is most important? → depends on what you're trying to measure; $\bar{x}$ is going to be influenced by outliers

# Other sample measures




**Quartiles:** Divide the data into 4 equal parts

- Lower quartile ($Q_1$ or $P_{25}$) splits the lowest 25% of the data from the other 75%
- Middle quartile ($Q_2$ or $P_{50}$) splits the data in half (i.e., the **median**)
- Upper quartile ($Q_3$ or $P_{75}$) splits the highest 25% of the data from the lowest 75%

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Different sources use different conventions for including or not the $\tilde{x}$ in $Q_1$ & $Q_3$ calculation. (MIPS book does include $\tilde{x}$ in both halves; we'll use that convention)

**Computation:**

1) Use the median to divide the ordered data set into 2 halves
   - If *n* is odd, include the median in both halves
   - If *n* is even, split the data exactly in half
2) The lower quartile is the median of the lower half

   $Q_1$
3) The upper quartile is the median of the upper half

   $Q_3$

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1 2 3 4 5 6 7 8 9 10 11

**Example:** Compute the quartiles of the data 6, 7, 15, 36, 39, 40, 41, 42, 43, 47, 49

$n = 11$ is odd, so $\tilde{x} = \boxed{Q_2 = 40}$

$Q_1$: take lower half of data, including $\tilde{x}$: $Q_1 = \frac{1}{2}[15 + 36] = \boxed{25.5}$

$Q_3 = \frac{1}{2}(42 + 43) = \boxed{42.5}$

# Other sample measures

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**Example:** Compute the quartiles of the data 6, 7, 15, 36, 39, 40, 41, 42, 43, 47, 49

**Solution:**

1) Data are already sorted
2) Compute median → $n$=11 is odd, so middle value is median, $Q_2 = 40$
3) Compute $Q_1$ and $Q_3$ from first and second halves of data:

    $Q_1$ = median of first half (6, 7, 15, 36, 39, 40) = (15+36)/2 = 25.5

    $Q_3$ = median of second half (40, 41, 42, 43, 47, 49) = (42+43)/2 = 42.5

# Other sample measures

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**Percentiles:**

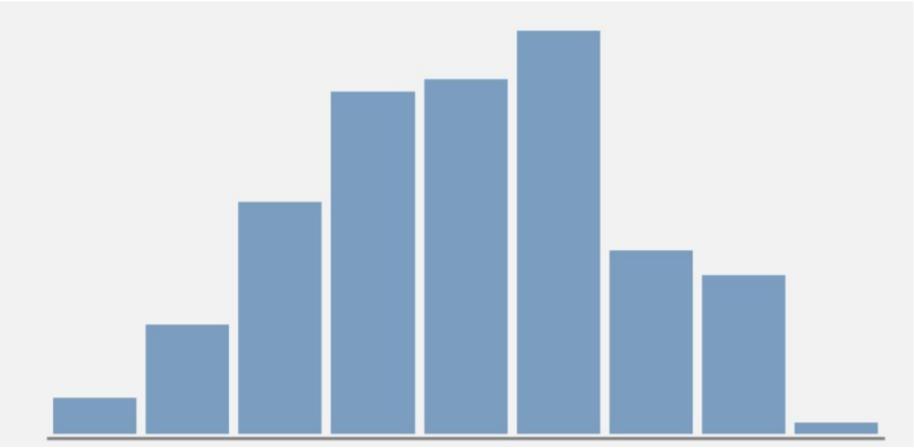
- Generalization of quartiles
- $Q_1$ is the 25th percentile, $P_{25}$
- Can also calculate general percentiles:

  e.g., the 16th percentile ($P_{16}$) splits off the lower 16% of the data.

# Variability

So far, we have learned about measuring the **central tendency** of data

But what about the **spread**?



wide?
spread?

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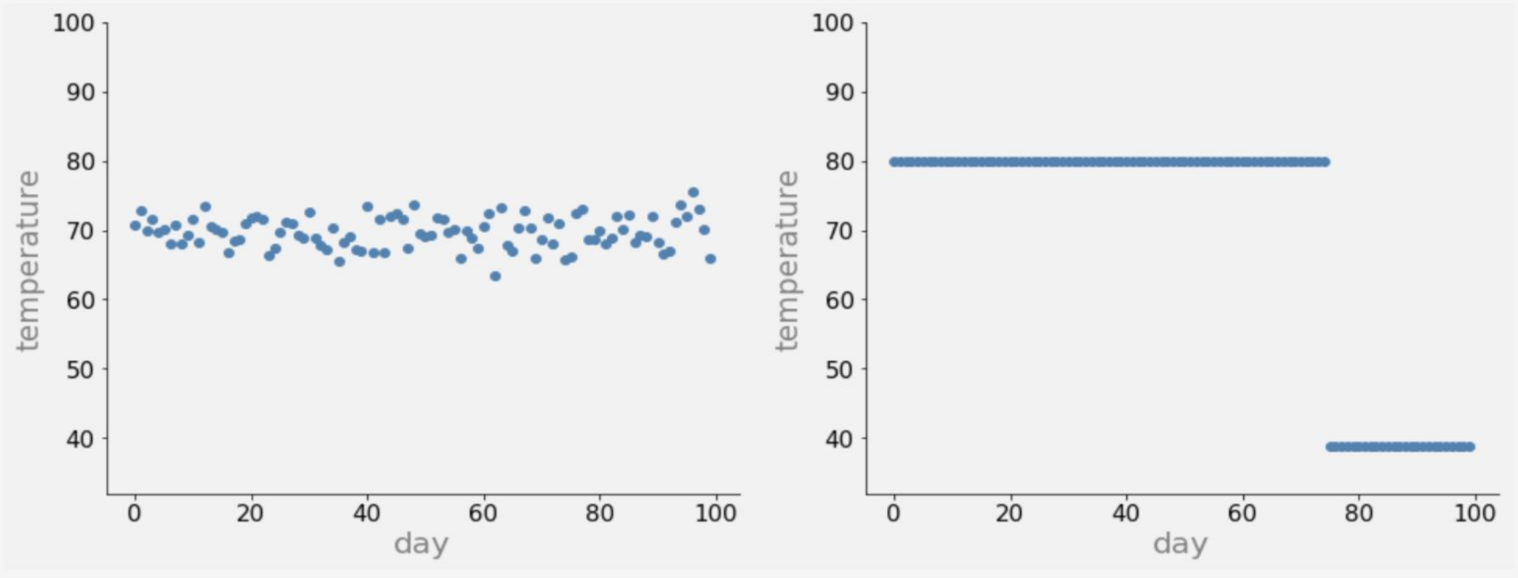
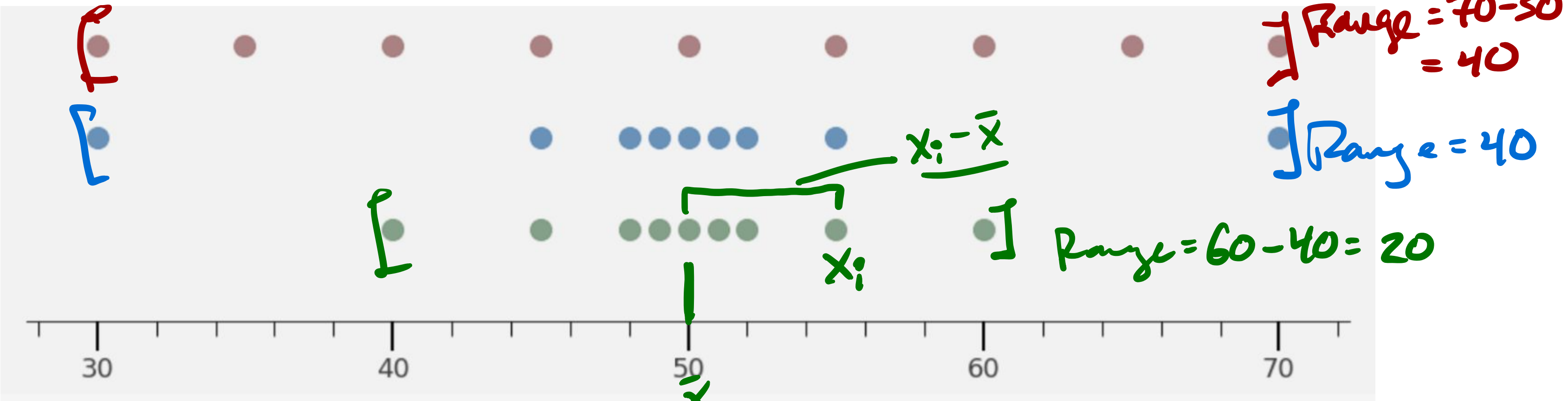


# Variability

The simplest measure of variability is the **range**

**Definition:** The **range** of a sample is the difference between the max and min values



samples with identical measures of centrality but different variability

## Variability

What if we combined the deviations into a single quantity by finding the average deviation?

A more robust measure of variation takes into account deviations from the mean

$$x_1 - \bar{x}, x_2 - \bar{x}, \ldots, x_n - \bar{x}$$

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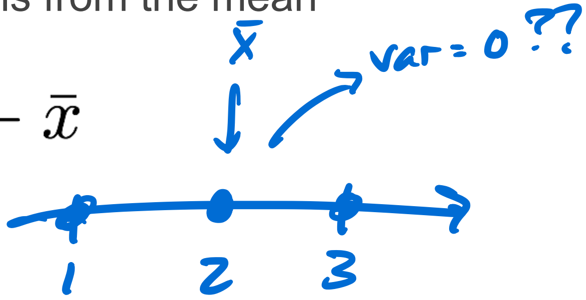



So… what do we do with these things?

Average them? $\quad$ variability $= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})$

# Variability

What if we combined the deviations into a single quantity by finding the average deviation?

A more robust measure of variation takes into account deviations from the mean

$$x_1 - \bar{x}, x_2 - \bar{x}, \ldots, x_n - \bar{x}$$

So… what do we do with these things?

… add them?

$$\frac{1}{n}\left[(x_1 - \bar{x}) + (x_2 - \bar{x}) + \ldots + (x_n - \bar{x})\right]$$

# Variability

What if we combined the deviations into a single quantity by finding the average deviation?

A more robust measure of variation takes into account deviations from the mean

$$x_1 - \bar{x}, x_2 - \bar{x}, \ldots, x_n - \bar{x}$$

So… what do we do with these things?

… add them?

$$\frac{1}{n}\left[(x_1 - \bar{x}) + (x_2 - \bar{x}) + \ldots + (x_n - \bar{x})\right]$$

… square, ***then*** add them?

$$\frac{1}{n}\left[(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2\right]$$

# Variability

**Definition:** The **sample variance**, denoted by $s^2$, is given by

units: degrees$^2$

$$s^2 = \frac{1}{n-1}\sum_{k=1}^{n}(x_k - \bar{x})^2$$

we'll talk about degrees of freedom

**Definition:** The **sample standard deviation**, denoted by $s$, is given by the (positive) square root of the variance:

$$s = \sqrt{s^2}$$

LOOK HERE!

WARM-UP PROBLEM!

**Example:** Compute the SD of the data:

2, 4, 3, 5, 6, 4

**NB:**

- The variance and SD are both **nonnegative** ($\geq 0$)
- The units for SD are the same as for the data

# Variability

**Example:** Compute the SD of the data: 2, 4, 3, 5, 6, 4

## Variability

**Example:** Compute the SD of the data: 2, 4, 3, 5, 6, 4

**Solution:**

1) Need $\bar{x}$ … = (2+4+3+5+6+4) / 6 = 24 / 6 = 4

2) Calculate $s^2$ …

$$s^2 = \frac{1}{6-1}\left[(2-4)^2 + (4-4)^2 + (3-4)^2 + (5-4)^2 + (6-4)^2(4-4)^2\right]$$
$$= \frac{1}{5}\left[4+0+1+1+4+0\right]$$
$$= \frac{1}{5}\cdot 10 = 2$$

sample variance

3) $s = \sqrt{s^2} = \sqrt{2}$

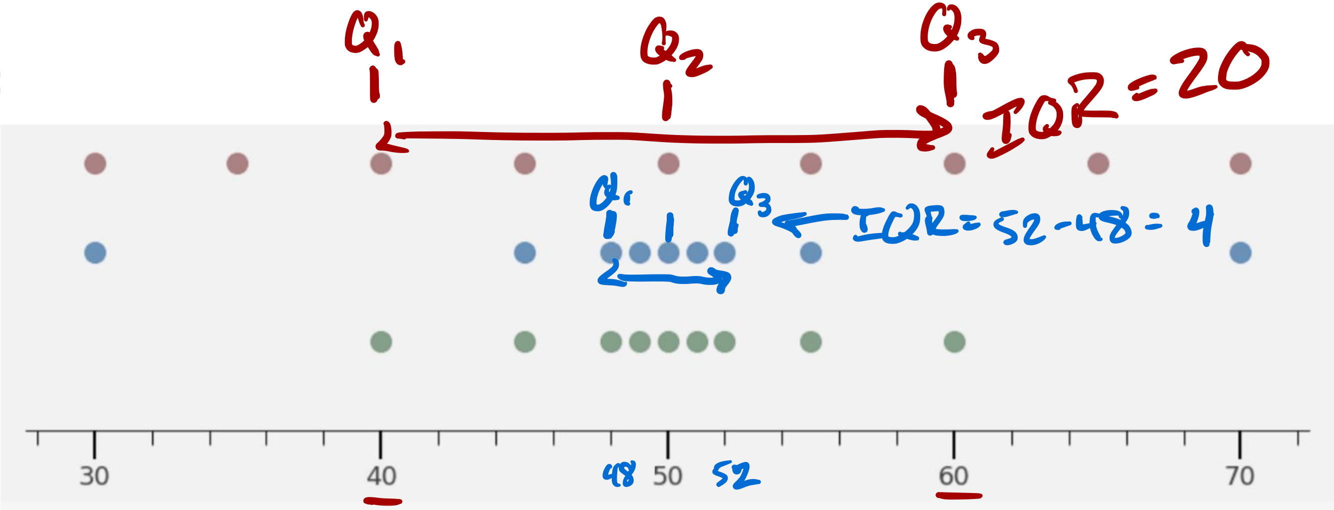
# Interquartile range

**Definition:** The **interquartile range** is defined to be the difference between the upper and lower quartiles:

$$\mathrm{IQR} = Q_3 - Q_1$$

→ IQR gives the spread of 50% of the data

**Examples:**



## Interquartile range

**Example:** Compute the IQR of the data 6, 7, 15, 36, 39, 40, 41, 42, 43, 47, 49

$\tilde{x} = Q_2 = 40$

$Q_1 = 25.5$

$Q_3 = 42.5$

$IQR = Q_3 - Q_1 = 42.5 - 25.5 = \boxed{17}$

# Interquartile range

**Example:** Compute the IQR of the data 6, 7, 15, 36, 39, 40, 41, 42, 43, 47, 49

**Solution:**

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2) Compute median → $n$=11 is odd, so middle value is median, $Q_2$ = 40
3) Compute $Q_1$ and $Q_3$ from first and second halves of data:

   $Q_1$ = median of first half (6, 7, 15, 36, 39, 40) = (15+36)/2 = 25.5

   $Q_3$ = median of second half (40, 41, 42, 43, 47, 49) = (42+43)/2 = 42.5

4) IQR = $Q_3$ - $Q_1$ = 42.5 - 25.5 = 17

# Tukey 5-number summary

John Tukey, father of modern EDA, advocated summarizing data sets with 5 values:

1) Min value ✓
2) Lower quartile ✓ $Q_1$
3) Median ✓
4) Upper quartile ✓ $Q_3$
5) Max value ✓

$Q_2$

1 2 3 4 5 6 7 8 9 10 11

**Example:** Find the 5-number summary of the data 6, 7, 15, 36, 39, 40, 41, 42, 43, 47, 49

Min = 6

$Q_1$ 25.5

$Q_2$ = 40

$Q_3$ = 42.5

Max = 49

# Tukey 5-number summary

John Tukey, father of modern EDA, advocated summarizing data sets with 5 values:

1) Min value
2) Lower quartile
3) Median
4) Upper quartile
5) Max value

**Example:** Find the 5-number summary of the data 6, 7, 15, 36, 39, 40, 41, 42, 43, 47, 49

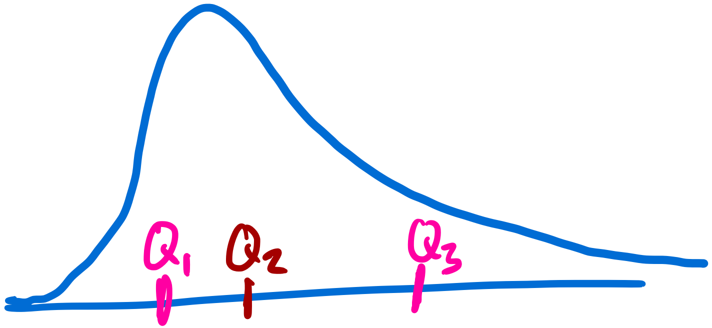
**Advantages:**

- gives the center of the data
- gives the spread of the data (range in IQR)
- gives and idea of skewness

## Tukey 5-number summary

**Advantages:**

- gives the center of the data
- gives the spread of the data (range in IQR)
- gives and idea of skewness
    - E.g., if $Q_2$ is closer to $Q_1$ than to $Q_3$, then you know the median is "leaning left" (so, distribution is right-skewed)



## Next time…

- We'll see how to visualize this!
  (histograms and box-whisker plots)