Collaborator: Qiuyang Wang

Problem Set 6.

1. m crimes k associates

n stoplight p officers

Since there are m crimes with k associates. Therefore, to stop all the crimes, we need to catch at least one associates in each crimes with limited to p officers (so that we can't assign 1 officer to 1 stoplight). However, we could not decide if assign police at certain stoplights would stop all the crimes, but we could solve this by try all possible assign way.

For example.

$$\begin{array}{ll} \text{stop light} & \left[\begin{matrix} 5 & 7 & 6 & 4 \\ 3 & 2 & 3 & 3 \end{matrix}\right] \\ \text{crime associate} & \end{array}$$

Base on the graph. we could stop all the crime by put officer at stoplight 5 and 7. but we could not figure that out without try all the possible way assign officer.

Therefore, we could have this algorithm

```
count=0
for each associate in the given solution S
    remove the crime from the crime set
    count=count+1
    if count = m (m is numbe of crimes)
    then
        the given solution is correct
    else
        the given solution is wrong
```

This algorithm explains this problem can be solve in polynomial time. which is in class of NP.

To prove this problem is NP-Hard We reduct from this problem to vertex cover problem (which is NP-complete. so is also NP-Hard)

Consider this problem in a graph (G) and a non-negative integer k we need to check if we could stop all crimes in p officers where k=P

For each associates at all the stoplights. If one of the associate is caught by officers, then the crime have been stopped. Similar with vertex cover problem. the officers need to find a way to stop all the crimes By caught each associates, the number of crimes caught increase by 1. Keep catching associates until all crimes have been covered by number of crimes n. Therefore, all crimes have been stops.

After all crimes have stoped, compare the number of officers used with p values, we would have if it is possible to stop all crimes with p officers (if number of officers used $\leq p$, it is possible to stop all crimes using p officers)

Therefore, we can say that by reducing to vertex cover problem, we could decide if p officer could stops all the crimes Therefore, the problem is NP-Hard. Since it is also NP (proved above), so it is in the class of NP-Complete.

2.

Since $k \le 2$, so there are at most 2 associates in each stop light.

Therefore, we could use this algorithm:

count = 0

for catch each associates that in each stop lights (associates ≤ 2) ①

remove the crime from the crime set. ②

count = count + 1 ③

if count != m (number of crimes)

repeat step 1, 2, 3 until count = m

which means all the crimes has been stoped.

Since we can't decide the optimal way to place officer without try all the possible placement but we can verify if all the crimes have been stopped by compare if count equal to m (crime number). The time complexity for this algorithm is $O(k^n)$ where $k \le 2$. Therefore, it is still linear time solvable. which is in Group of NP.

To show this problem is NP-Hard, we can reduce this problem to Vertex Cover problem (Since Vertex Cover problem is NP-Complete which is also NP-Hard). Let m be the number of crimes, p be the number of officers. For officers at each stoplights if the officers catch 1 or 2 associates, the crime that the associate related will be removed from the crime sets. Same with the vertex cover problem, when there is no crimes in the crime set. We say all the crimes have been stoped. Then by camparing the officer use with P, we get if it is possible to stop all the crim by p officers.

Therefore, this question is NP-Hard. Since it is also in class of NP. This problem is in class of NP-Complete.

Since crime occupy in a contigous section of road, we could find a stoplight 'a' that the crime associates "started" and by removing crimes from the crime set. When there are no crimes in the crime set, so we stop all the crimes.

We could use this algrothm

count = 0

For catch each associates in stop light

count = count + 1

go be stop light futher, check if count = number of crimes.

repeat these step until count = number of crimes

Base on the algorithm, we could decide when we find the "start" spot 'a' by comparing count with number of crimes. At certain point, we could find the "stare" stoplight. Since the time complexity is $O(n^b)$ where b is a constant. Therefore, this problem is in class of P.

3. n artifacts

fit into 7 cases

each cases fit upto W kg

Base on the description of the problem, the person need to pack all the artifacts, so he need to optimize the way he put artifacts to each cases. which he need use dynamic programming to put in largest amount of artifact. Therefore, we can use bin-pack problem strategy to solve this problem

Algorithm: sort all the artifacts from smallest to largest.

start from heaviest artifact

put in that artifact to the case that has largest space remaining

if it can fit the artifact chosen

place it into the case

else

go to another case that has second largest space remaing

repeat this step until there is no more artifacts or cannot place more artifacts

if no more artifact

the 7 cases can fit all artifact

else

the person should quit.

Base on the algorithm above, the run time is $O(7^n)$

Also, because we can verify if one artifact can fit the size of the case by adding up all the artifacts that already in the case, and subtract by the size of the case, see if the value is greater than the put in artifact's weight. Therefore, it is linear time solveable. Therefore it is in group of NP

扫描全能王 创建

4. $n \times m$ grid of street light

type of A or B

	1	2	3	4
row 1	~~A~~		~~B~~	~~A~~
row 2		A	A	B
row 3	B	~~B~~		

shut off

1. A
2. B
3. B
4. A

Since the city has $n \times m$ grid of street light, but we can only shut down type A or B in one column, and we have the position and grid of the light. We can check if there is light on in each row by shuting off all the type A/B light For example, in a 3×4 grid (the graph above), but shuttling off A, B, B, A. row 1 would not have one light on. Keep doing this step (try possible ways shutting off light) We would get a way that have 1 light on. Base on the algorithm, it is linear time solvable. the time complixity is $O(mn)$. which is in group of NP.

To show the problem is NP-Hard. we reduce this problem with 3SAT where $3SAT \leq_p$ the problem (because 3SAT is NPC, so it is NP-Hard).

For each street light, it has type $\bar{x}_i$ or x_i. let $x_1, x_2 \cdots x_i$ be the variable represents the street lights. Then, for each clause, as long as there is one x_i equal to true, then the whole clause will be true. If for one clause, all the x_i, x_{i+1}, x_{i+2} are false, then the whole clause is false. Thefore, by using the algorithm

$$\Phi = (x_1 \vee x_2 \vee x_3) \wedge (x_4 \vee x_4 \vee x_5) \wedge \cdots \wedge (x_{i-2}, x_{i-1}, x_i) \quad (3SAT)$$

If the value of Φ is true, which means every row has at least on light on.
If Φ is false, then there is at least one row doesn't have a light

Therefore, we reduce the problem to 3SAT problem. Since 3SAT is NP-Hard. This problem is also NP-Hard. Since this problem is also in group of NP It is NP-Complete.