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Problem Set 5

HW 9.

4 r.e. set is recursive iff $\exists$ an enumeration machine that enumerates in increasing order.

Proof: Suppose set A is recursive iff there exist a machine E such that $L(E)=A$ and E would prints string in an increasing order.
Since set A is recursive, there exist a total machine M such that for all strings in lexicographic order and run M on that string If M accepts it, print that string. Since M is a total machine, it always has a membership relationship with E such that $L(M)=A=L(E)$, therefore the accepted string will be enumerate in finate time (will halt), so that r.e set is recursive.

HW 9

1. TM M and state q of M. M enter state q on some input?

Proof by contradiction: Suppose the statement is decideable. Then for the TM M have state q on input x We need to decide if M will halt on input x. So, we construct a new TM M' that contain state q On any input x M' would only halt if x enter the state q

Structure of the algorithm

```
M(x)
if M'(x) enters state q
    while (false)
else
    return
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Since we assume the problem is decideable, then wheather M' will accept / reject will also be decideable. But since halting problem is undecideable (definition of halting problem) Then there is a contradiction Therefore the statement is undecideable.

3. $L = \{ M \mid M$ halts on all inputs of length less than $3434 \}$

For the TM M on input x, we construct from a given TM M and string x a TM M' that accept input of length less than 3434 iff M halts on x (If M halt on x, M' accept input y).

Then if M takes input that length less than 3434 Then it halt. so that it is HP. Therefore HP is r.e. complete

$\sim L = \{ M \mid M$ halt on $|y| \geq 3434 \}$ and $HP = \{ M\#x \mid M$ halts on $x \}$

$M\#x \in HP \Longleftrightarrow \sigma(M\#x) \in L$

M halts on $x \Longleftrightarrow \sigma(M\#x)$ halts on input $y \in |y| < 3434$. new machine

M halts on $x \Longleftrightarrow \sigma(M\#x)$

M loops on $x \Longleftrightarrow \underbrace{\sigma(M\#x)}_{= M'}$ loops on some input y such that $|y| < 3434$

$M'(y)$
  if $|y| < 3434$
  $M(x)$
else:
  return.

Therefore $\sim L(M') = \begin{cases} \Sigma^* & \text{if } M' \text{ loops on } x \\ \phi & \text{if } M' \text{ halt on } x \end{cases}$

Then for all r.e. sets R, $R \leq_m L$ and L is r.e.

therefore $\sim L$ is not r.e. and L is r.e. complete

4 a) $L = \{ M\#x \mid$ on input x, M never writes the symbol $a \in \Sigma$ on the tape $\}$

It is co-r.e because for TM M, M must halt on input x. however it is undecidable whether it accept/reject (halting problem is decadeable. Therefore it is not re. For $\sim L = \{ M\#x$, M always write the symbol $a \in \Sigma$ on the tape$\}$. if the machine M write on tape, then $\sim L$ is in finite time. which yields that $\sim L$ is re. Therefore, since L is not re and $\sim L$ is re The statement is co.re.

b) $L = \{ M\#N\#x \mid M$ halts on x but N loops on $x \}$

It is Neither re or co-re because

For the TM M, since M halts on x. for input x M is r.e. And since N always loops on x, which means it could not create an output which means on input x and TM N, it is not r.e. Therefore it is Neither re or core.