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Problem set 3

HW 5.2

$$S \to aSb \mid bSa \mid SS \mid \varepsilon$$

Proof by induction

Base case: let $S \to \varepsilon$. since it has 0 a's and b's it holds the original claim.

Inductive hypothesis: Assume $S \xrightarrow[G]{n} \beta \in \Sigma^*$ number of a in β = number of b in β

Inductive step: According to inductive hypothesis $S \xrightarrow[G]{n} \beta$ $\beta \in \Sigma^*$

where $\beta = \beta_1 S \beta_2$, for $S \xrightarrow[G]{n+1}$

According to IH $D(x) := \#b(x) - \#a(x) = 0$

Since $S \to aSb \mid bSa \mid SS \mid \varepsilon$

$$D(xy) = \#b(xy) - \#a(xy)$$
$$= (\#b(x) + \#b(y)) - (\#a(x) + \#a(y))$$
$$= (\#b(x) - \#a(x)) + (\#b(y) - \#a(y))$$
$$= D(x) + D(y)$$

$$\beta = \beta_1 aSb \beta_2$$
$$= \beta_1 bSa \beta_2$$
$$= \beta_1 SS \beta_2$$
$$= \beta_1 \beta_2$$

According to IH, $D(\beta) = 0$,

For $D(\beta_1 S \beta_2) = D(\beta_1) + D(S) + D(\beta_2)$ Since S is non-terminal

$$= D(\beta_1) + D(\beta_2) = 0$$

Thus, $S \xrightarrow[G]{n+1}$ contains the same numbers of a and b.

therefore for CFG, $S \to aSb \mid bSa \mid SS \mid \varepsilon$, it contains the same numbers of a and b. $\square$

ME 73.

a) $A = \{x \mid rev\ x = \overline{x}\}$

$S \to 0S1 \mid 1S0 \mid \varepsilon$

b) Chomsky Normal Form

Remove NULL Production

$S \to 0S1 \mid 1S0 \mid 01 \mid 10$

add nonterminals A, B and replace these productions with

$S \to ASB \mid BSA \mid AB \mid BA$ $\quad A \to 0 \quad B \to 1$

add nonterminal C, D replace and replace $S \to ASB \mid BSA$ with

$S \to AC \mid BD \mid AB \mid BA \quad A \to 0 \quad B \to 1 \quad C \to SB \quad D \to SA$

Griebach Normal Form.

Base on above after remove NULL Production and replace the production

$S \to ASB \mid BSA \mid AB \mid BA$ replace S with A_1

$A \to 0$ A with A_2

$B \to 1$ B with A_3

we get

$A_1 \to A_2A_1A_3 \mid A_3A_1A_2 \mid A_2A_3 \mid A_3A_2$

$A_2 \to 0$

$A_3 \to 1$

Since $A_2 \to 0 \quad A_3 \to 1$

$A_1 \to 0A_1A_3 \mid 1A_1A_2 \mid 0A_3 \mid 1A_2$

$A_2 \to 0$

$A_3 \to 1$

put back the original terminal symbol

$S \to 0SB \mid 1SA \mid 0B \mid 1A$

$A \to 0$

$B \to 1$

3.

a $\{a^k b^l a^m b^n \quad k=m \text{ or } l=n\}$

Since $k=m$, or $l=n$

$$a^m b^l a^m b^n \mid a^k b^n a^m b^n$$

which is same

$$a^{2m} b^l b^n \mid a^k b^{2n} a^m$$

The language is context free because according to the stack of PDA, the machine could still recognize the number of a and b it put into the stack, so after pop out the stack, (because $k=m$ or $l=n$) the stack is still empty.

Suppose $D = \{a^k b^l a^m b^n \quad k=m \text{ or } l=n\}$ is regular, then $D \cap a^* b^*$ would also be regular, $D \cap L(a^* b^*) = \{a^n b^n \mid n \geq 0\}$, but $a^n b^n$ is not regular, therefore $\{a^k b^l a^m b^n \quad k=m \text{ or } l=n\}$ is not regular.

b. $\{a^k b^l a^m b^n \quad k=m \text{ and } l=n\}$

replace k and l

$$a^m b^n a^m b^n = a^{2m} b^{2n}$$

The language is not context free because according to PDA, after pushing k number of a and l number of b, then we could not recognize how many b we need to pop out the stack. Therefore, the stack is not empty.

Since $\{a^k b^l a^m b^n \quad k=m \text{ and } l=n\}$ is not context free, it is not regular.

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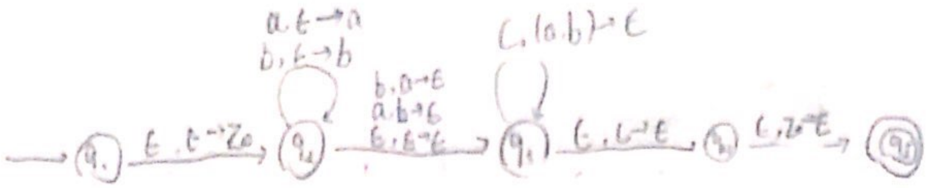
4 $\{a^n b^m c^k \quad k = m+n\}$



b)