Problem set 4

HW 6.1

1. For this TM. it first checks if the number of a is less than or equal to 2. (case 1) If the # of a is 1, which is a^{2^0}. so we accept it. If the # of a is 2. which is a^{2^1} which is accepted. If the number of a is 3. which is not the power of 2 so we reject it (case 2) For input have more than 3 a, in each pass from right to left. we mark the first unmark a with '/' In each pass from left to right. we mark the first unmark a with '\' If in any case. we mark a "/" and can't mark a "\", we reject the input. (The length of input is odd since odd length can't be the power of 2 expect 1 a, which is already covered as special case.

After we mark all the inputs, we cut the input to half where symbol '/' and the symbol a with '\' meets. For example,

⊢ a a a a a a t ␣ ␣ ␣ ... ⊢ à à à á á á t ␣ ␣ ␣ ...

⊢ a a a a a a á t ␣ ␣ ␣ ... after all the input characters been marked

⊢ à a a a a á t ␣ ␣ ␣ ...

Then we cut the input to half. then delete the right half and unmark all the new input

⊢ à à à | á á á t ␣ ␣ ␣ ... ⊢ a a a t ␣ ␣ ␣ ...

After mark and cut the input to half. we delet all the right part with table '/'. Then unmark the new input. By keep looping all the steps, it will eventually goes either case 1 or case 2. If it goes case 1. the TM will accpet it. if it goes case 2. the TM will reject it.

2. Input $x \in \{0,1\}^*$ with no leading 0s.

$x \in L(1(1|10)^* | 0)$

For this TM, it first checks the rightmost of the tape. If the right most is 0, then change it to 1. ((case 1). If the rightmost is a 1, then move 1 step to the left, keep doing this step until it finds a 0 on the tape, convert it to 1. Then convert every elements on the right of the current location to 0. ((case 2)
If we go through elements on the tape and still can't find any 0. Start from the second element on the left, convert all 1s to 0. Then add an 0 at the end of rightmost of the tape. (case 3) If there is only one 0 one the tape, just change the 0 to 1 (case 4).

Case 1.

original ⊢ 101010 ⊔⊔⊔...

add 1, start from the rightmost, convert to 1

⊢ 10101 ⊔⊔⊔...

Case 2.

Original ⊢ 10011 ⊔⊔⊔...

Since rightmost is not 0, move one step to the left

⊢ 10011 ⊔⊔⊔...

keep looping until find a 0, convert it to 1

⊢ 10011 ⊔⊔⊔...

⊢ 10111 ⊔⊔⊔

Convert everything on the right to 0.

⊢ 10100 ⊔⊔⊔...

Case 3.

⊢ 1111 ⊔⊔⊔...

After go through all the input, still can't find a 0.
start from second element, convert all the element to 1.

⊢ 10000 ⊔⊔⊔...

add an 0 on the rightmost.

⊢ 100000 ⊔⊔⊔...

Case 4

If only one 0, convert it to 1.

⊢ 0 ⊔⊔⊔...

⊢ 1 ⊔⊔⊔...

After doing all these step, check the converted input is on the tape, if it is, let the TM halt it, if it is not on the tape, reject it.

3 Since A, B are r.e., so they are both accepted by the TM. Let M_A and M_B be the TM that accpet the input A, B.

Union: For $A \cup B$. we create a new TM, simulate both M_A M_B, simultaneously on input x. The new TM will accpet if either M_A and M_B accpet the input. Since it is an union, then r.e. is closed under union.

Intersect: For $A \cap B$, We create a new TM, simulate both M_A M_B on input x. The new TM will accpect if both M_A M_B accpet the input x. Since it is an intersection, the re set will only be closed if both M_A and M_B accept it.

Concatenation: r.e. set is closed.

Homomorphisms: re set is not closed.

re v : re set is closed