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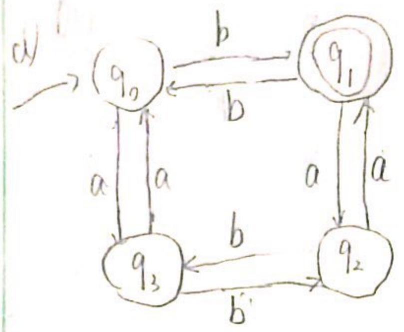
HW 2.

3.1 c) $(b^*ab^*ab^*)^* + a^*ba^*(ba^*ba^*)^*$

d)



DFA for even number of a and odd number of b

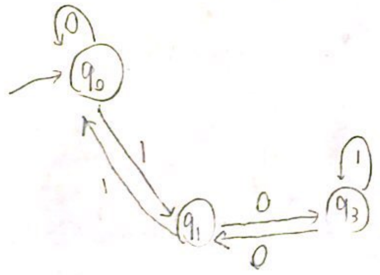
b, aab, baa, baaaa, abaaa, aabaa, aaaba, aaaab
bbbaa, bbaba, bbaab.

$$\alpha_{q_0q_0}^{\{q_0,q_1,q_2,q_3\}} = \alpha_{q_0q_0}^{\{q_0,q_1\}} + \alpha_{q_0q_1}^{\{q_0,q_1\}}\left(\alpha_{q_1q_1}^{\{q_0,q_1\}}\right)^* \alpha_{q_1q_0}^{\{q_0,q_1\}}$$

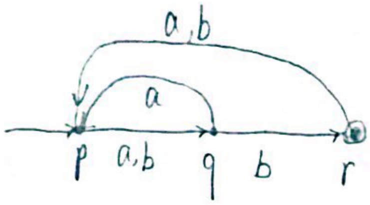
$$= ((aa+ba(bb)^*ba) + (b+ab(bb)^*a) + (a(bb)^*a)^*$$

$$(b+a(bb)^*ba))^* (b+ab(bb)^*a)(a(bb)^*a)^*$$

3.2 $(0+1(01^*0)^*1)^*$



ME 15



$$\alpha_{pr}^{\{p,q,r\}} = \alpha_{pr}^{\{p,r\}} + \alpha_{pq}^{\{p,r\}} \left(\alpha_{qq}^{\{p,r\}}\right)^* \alpha_{qr}^{\{p,r\}}$$

$\alpha_{pr}^{\{p,r\}} = \phi$ no direct path from p to r.

$$\alpha_{pq}^{\{p,r\}} = a+b$$

$$\alpha_{qq}^{\{p,r\}} = \varepsilon + a(a+b) + b(a+b)(a+b)$$

$$\alpha_{qr}^{\{p,r\}} = b$$

$$\alpha_{pr}^{\{p,q,r\}} = (a+b)\left(\varepsilon + a(a+b) + b(a+b)(a+b)\right)^* b$$

HW 4

1.b) $\{x \in \{a,b,c\}^* \mid x$ is palindrome, i.e., $x = rev(x)\}$

pumping lemma:

Let the string be a^*a^*

suppose we pick $x = a^k$ $y = a^k$ $z = \varepsilon$

then $xyz = a^k a^k$ which satisfie the requirement

$|y| = k$

Then, for u, v, w, which $y = uvw$ and $v \notin \varepsilon$.

suppose the length of u, v, w is j, m, n with $k = j+m+n$ and $m > 0$. suppose $i = 2$

$xuv^2wz = a^k a^j a^m a^m a^n$

$= a^k a^{j+2m+n}$

$= a^k a^{k+m}$

Since $m > 0$, $k \neq k+m$. therefore for $x \in \{a, b, c\}^*$ and x is palindrome, x is not regular □

Closure properties

proof by contradiction: Suppose $D = \{x \in \{a,b,c\}^* \mid x$ is palindrome$\}$ is regular

then $D \cap a^*a^*$ would also be regular

$D \cap L(a^*a^*) = \{a^n a^n \mid n \geq 0\}$

but $\{a^n a^n \mid n \geq 0\}$ is not regular (given in the write up), therefore, for $\{x \in \{a,b,c\}^* \mid x$ is palindrome$\}$ is not regular □

d. pumping lemma

Let the string be ( )

suppose we pick $x = (^k$ $y =)^k$ $z = \varepsilon$

then $xyz = (^k)^k$ which satisfie the requirement

$|y| = k$

Then, for u, v, w which $y = uvw$ and $v \neq \varepsilon$

Suppose the length of u, v, w is j, m, n with $k = j+m+n$ and $m > 0$

suppose $i = 2$

$$(^k u v^2 w = (^k)^j)^m)^m)^n$$

$$= (^k)^{j+2m+n}$$

$$= (^k)^{k+m}$$

Since $m > 0$, $k \neq k+m$, therefore the set PAREN of balanced strings of parentheses is not regular. □

ME 37.

d) $\{a^{p-1} \mid p \text{ is prime}\}$

pumping lemma

$x = a^p \quad y = a^{-1} \quad z = \varepsilon$

$xyz = a^{p-1}$

for $i=2$

$xyz = a^{p-1} \cdot a$

$= a^p$

which is not a prime, thus $\{a^{p-1} \mid p$ is prime is not regular$\}$

e) $D = \{xcx \mid x \in (a,b)^*\}$

Suppose D is regular, the $D \cap a^*b^*$ is regular

$D \cap L(a^*b^*) = \{a^n c b^n \mid n \geq 0\}$

which is not regular (given in the write up)

therefore D is not regular.

f) $\{xcy \mid x, y \in (a,b)^*\}$

Since regular language is closed under concatenation

xcy is concatention of $\{x \mid x \in (a,b)^*\}$, therefore

it is regular.

g) $\{a^n b^{n+481} \mid n \geq 0\}$

$x = a^k \quad y = b^{k+481} \quad z = \varepsilon \quad xyz = a^k b^{k+481}$ and $|y| = k$

For $y = uvw$ and $v \neq \varepsilon$, length of u, v, w is j, m, n

with $k = j+m+n$ and $m > 0$, suppose $i=2$

$xuv^2wz = a^k a^j a^m a^m a^n$

$= a^k a^{k+481+m}$

Since $m \neq 0$ thus $a^k a^{k+481+m} \notin a^n b^{n+481}$

therefore $a^n b^{n+481}$ is not regular.

l) $D = \{a^n b^m \mid n - m \le 481\}$

suppose $x = \varepsilon$ $y = a^k$ $z = b^k$ $xyz = a^k b^k$ $k - k = 0$ and $|y| = k$ which satisfie the requirement.

For u, v, w which $y = uvw$ and $v \notin \varepsilon$

suppose $i = 500$

$$|uv^{500}w| = 500 + k$$

where $k - k + 500 > 481$ which dose not satisfie $n - m \le 481$

therefore D is not regular

m) $D = \{$syntactically correct python program$\}$

For example, we could chose arr = [a, a]

This is saying $D = \{x \in (a, b)^* \mid x \text{ is palindrome}\}$

which we have already prove in HW.4.1, it is not regular.

HW 4.3

a) 1, 2, 3F, 4F, 5, 6 are accessible, 7, 8 are unaccessible

because start from 1, we cannot get 7 and 8.

b)

1					
X	2				
X	X	3			
X	X	_	4		
X	_	X	X	5	
_	X	X	X	X	6.

Base on the graph, the equivalent classes are $\{1, 6\}$, $\{2, 5\}$, $\{3, 4\}$

$1 \approx 6$ $2 \approx 5$ $3 \approx 4$

c) Since 3 and 4 are final state. the automaton obtained by collapsing equivalent is

	a	b
{1,6}	{1,6}	{3,4}
{2,5}	{3,4}	{1,6}
{3,4}	{3,4}	{2,5}