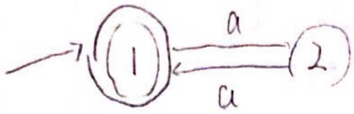
Collaborator Qiuyang Wang

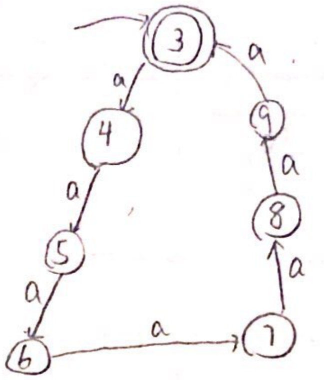
HW 1.1

b) the set of string $\{a\}^*$ divisible by 2 or 7.

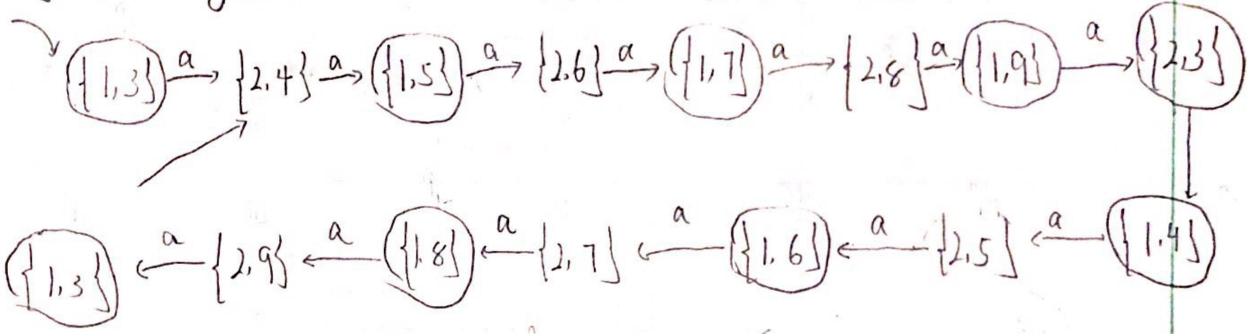
Divisible by 2



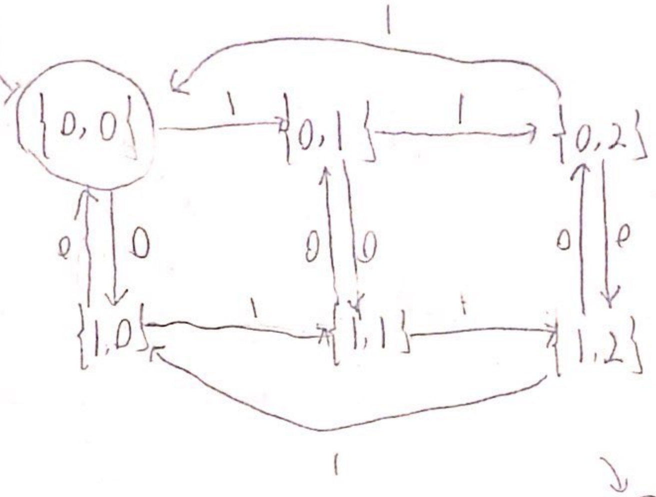
Divisible by 7



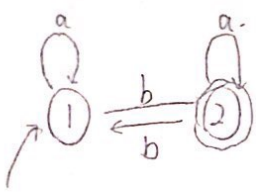
putting those together

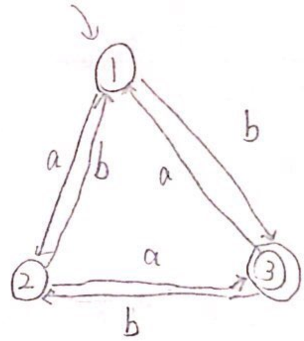


c) In order for x to be even, x mod 2 have to be equal to 0 with 2 cases {0, 1}
In order for x to be mutiple of 3, x mod 3 have to be equal to 0, with 3 cases {0, 1, 2}

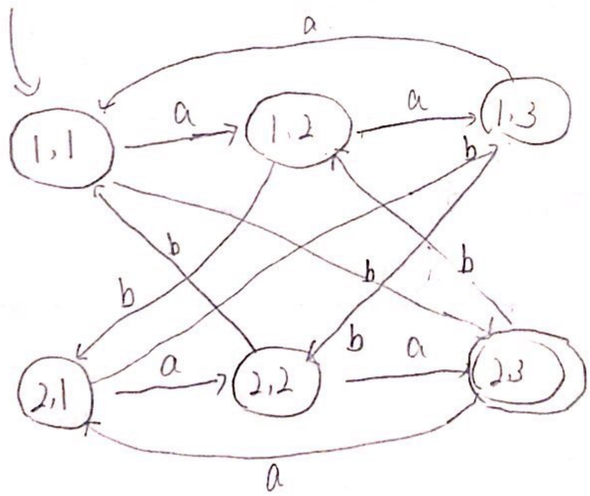


2.

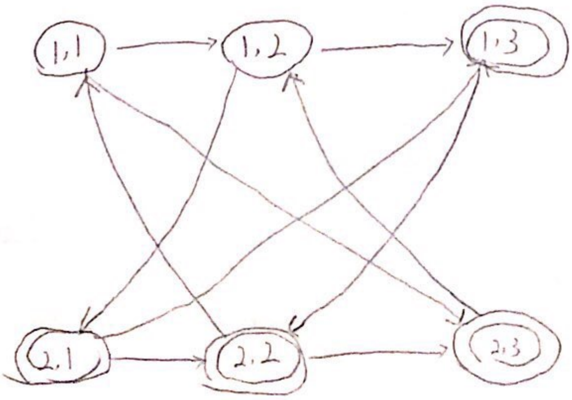




a)



b)



H 1.3

Prove $\hat{\delta}(q, xy) = \hat{\delta}(\hat{\delta}(q,x), y)$ $x, y \in \Sigma^*$ and $q \in Q$

Proof by induction:

By the claim $\hat{\delta}(q, xy) = \hat{\delta}(\hat{\delta}(q,x), y)$ for any $x, y \in \Sigma^*$ and $q \in Q$

Base case: let $y = \varepsilon$, therefore $x\varepsilon = x$ and $\hat{\delta}(q,x) = \hat{\delta}(\hat{\delta}(q,x), \varepsilon)$

which RHS $= \hat{\delta}(q,x)$

Inductive hypothesis: assume the claim is true

Inductive step: then for $\hat{\delta}(q, xyz)$

$$\hat{\delta}(q, xyz) = \hat{\delta}(\hat{\delta}(q, xy), z)$$

By the claim

$$= \hat{\delta}(\hat{\delta}(\hat{\delta}(q,x), y), z)$$

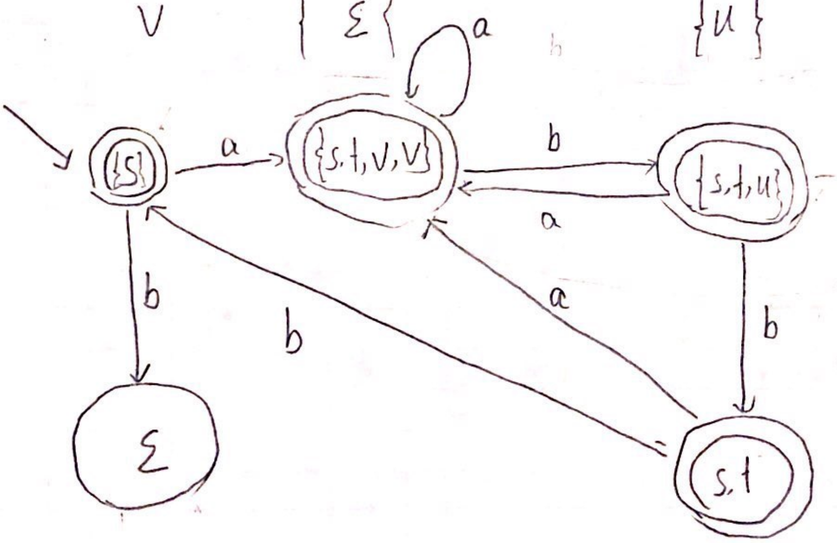
$$= \hat{\delta}(\hat{\delta}(q,x), yz)$$

Therefore, $\hat{\delta}(q, xy) = \hat{\delta}(\hat{\delta}(q,x), y)$

4. a) abbbb

b)

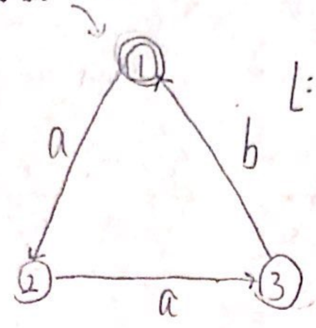
NFA state	possible state with a	possible state with b.
s	{s, t, u, v}.	{ε}
t	{ε}	{s}
u	{ε}	{t}
v	{ε}	{u}



5 $rev\{a, ab, aab, aaab\} = \{a, ba, baa, baaa\}$

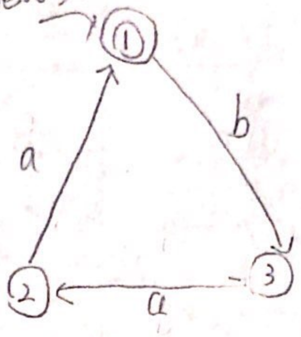
Prove for $A \in \Sigma^*$ if A is regular, rev A also regular

origional DFA



L: aab

reversed DFA



L: baa

for regular DFA $M = (Q, \Sigma, \delta, s, F)$

for rev(A) $M^R = (Q^R \Sigma, \delta', s', F')$

Proof: $Q^R = Q \cup \{s'\}$ which indicates adding a new start state.

Also, because the origional DFA is reversed, therefore $F' = \{s\}$

In addition, $\delta^{-1}(q, a)$ is all the possible states that have transition to q with input a, therefore by adding a new starting state

Proof by induction $\delta(q_0', \epsilon) = F$ if $q = q_0'$ and $a = \epsilon$

Inductive step: $\delta'(q, a) = \delta^{-1}(q, a)$ if $q \neq q_0'$ and $a \neq \epsilon$

δ' indicates runing all pass nondeterministically on M with M's transcation function. Therefore it reverse the direction arrow.

For all the other case, it could not find the path, and could not provide a validate state.

Therefore, for any input $L_0 L_1 L_2 \cdots L_n$, $q_0 q_1 \cdots q_n$ is an acpect computation, then for $L_n L_{n-1} \cdots L_1$, $q_n q_{n-1} \cdots q_0$ is also an accpect computation, because $\delta(q_n, L_{n+1}) = q_{n+1}$ if and only if $q_n \in \delta'(q_{n+1}, w_{41})$, thus there exists an NFA the reconize M^R, so revA is regular