Neural Language Modeling

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Evaluation of language models

Model Evaluation

- How do we know if our models are any good?
  - …and how do we know if one model is better than another?
- Well Shannon's game gives us an intuition.
  - The generated texts from the higher order models surely sound better.
    - That is, they sound more like the text the model was obtained from.
- But what does that mean? How can we make that notion operational?

Evaluating N-Gram Models

- Best evaluation for a language model
  - Put model A into an application
    - For example, a machine translation system
  - Evaluate the performance of the application with model A
  - Put model B into the application and evaluate
  - Compare performance of the application with the two models
    - **Extrinsic** evaluation

Evaluation

- Extrinsic evaluation
  - This is really time-consuming and can be hard
  - Not something you want to do unless you’re pretty sure you’ve got a good solution

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- So
  - As a intermediate evaluation, in order to run rapid experiments we evaluate N-grams with an **intrinsic** evaluation
  - An evaluation that tries to capture how good the model is intrinsically, not how much it improves performance in some larger system

Evaluation

- Standard method
  - Train parameters of our model on a training set.
  - Evaluate the model on some new data: a test set.
    - A dataset which is different from our training set, but drawn from the same source

Perplexity

- The intuition behind perplexity as a measure is the notion of surprise.
  - How surprised is the language model when it sees the test set?
    - The more surprised the model is, the lower the probability it assigned to the test set.
    - The higher the probability, the less surprised it was.

Perplexity

- Perplexity is just the probability of a test set (assigned by the language model), as normalized by the number of words:

$$
\begin{aligned}
\mathrm{PP}(W) &= P(w_1 w_2 \ldots w_N)^{-\frac{1}{N}} \\
&= \sqrt[N]{\frac{1}{P(w_1 w_2 \ldots w_N}\,)}
\end{aligned}
$$

- Chain rule:

$$
\mathrm{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i|w_1 \ldots w_{i-1})}}
$$

- For bigrams:

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- Minimizing model perplexity is the same as maximizing probability of a test set.

Lower Perplexity is Better

- Training 38 million words, test 1.5 million words, WSJ

N-gram Order	Unigram	Bigram	Trigram
Perplexity	962	170	109

Practical Issues

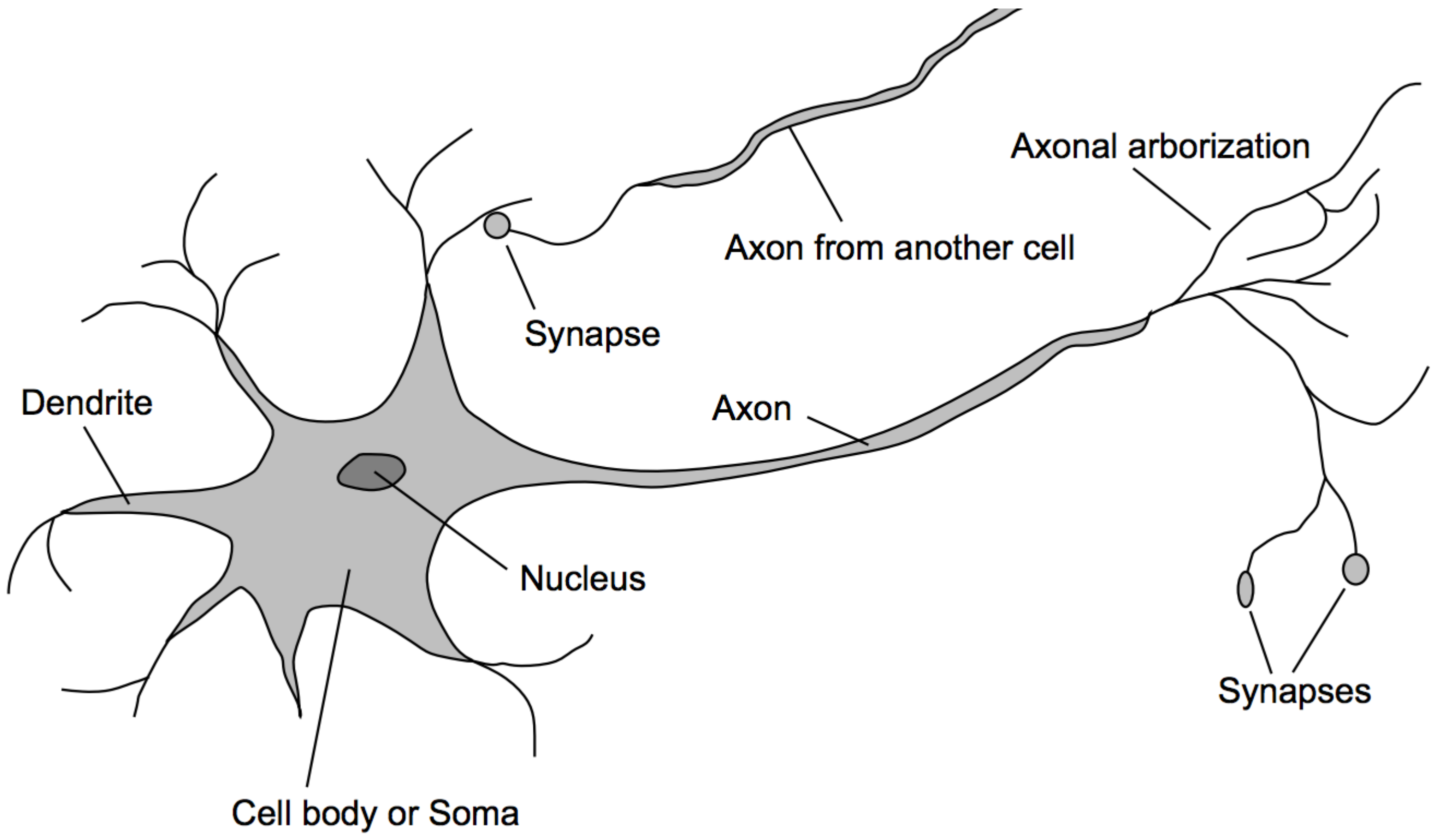
- Once we start looking at test data, we'll run into words that we haven't seen before. So our models won't work. Standard solution:
  - Given a corpus
    - Create a fixed lexicon L, of size V
      - Say from a dictionary or
      - A subset of terms from the training set
    - Any word not in L is changed to <UNK>
    - Collect counts for that as for any normal word
  - At test time
    - Use <UNK> counts for any word not seen in training

Practical Issues

- Multiplying a bunch of really small numbers < 0 is a really bad idea.
  - Underflow is likely
- So do everything in log space
  - Avoids underflow (and adding is faster than multiplying)

Neural networks

Motivation


Axonal arborization
Axon from another cell
Synapse
Dendrite
Axon
Nucleus
Synapses
Cell body or Soma

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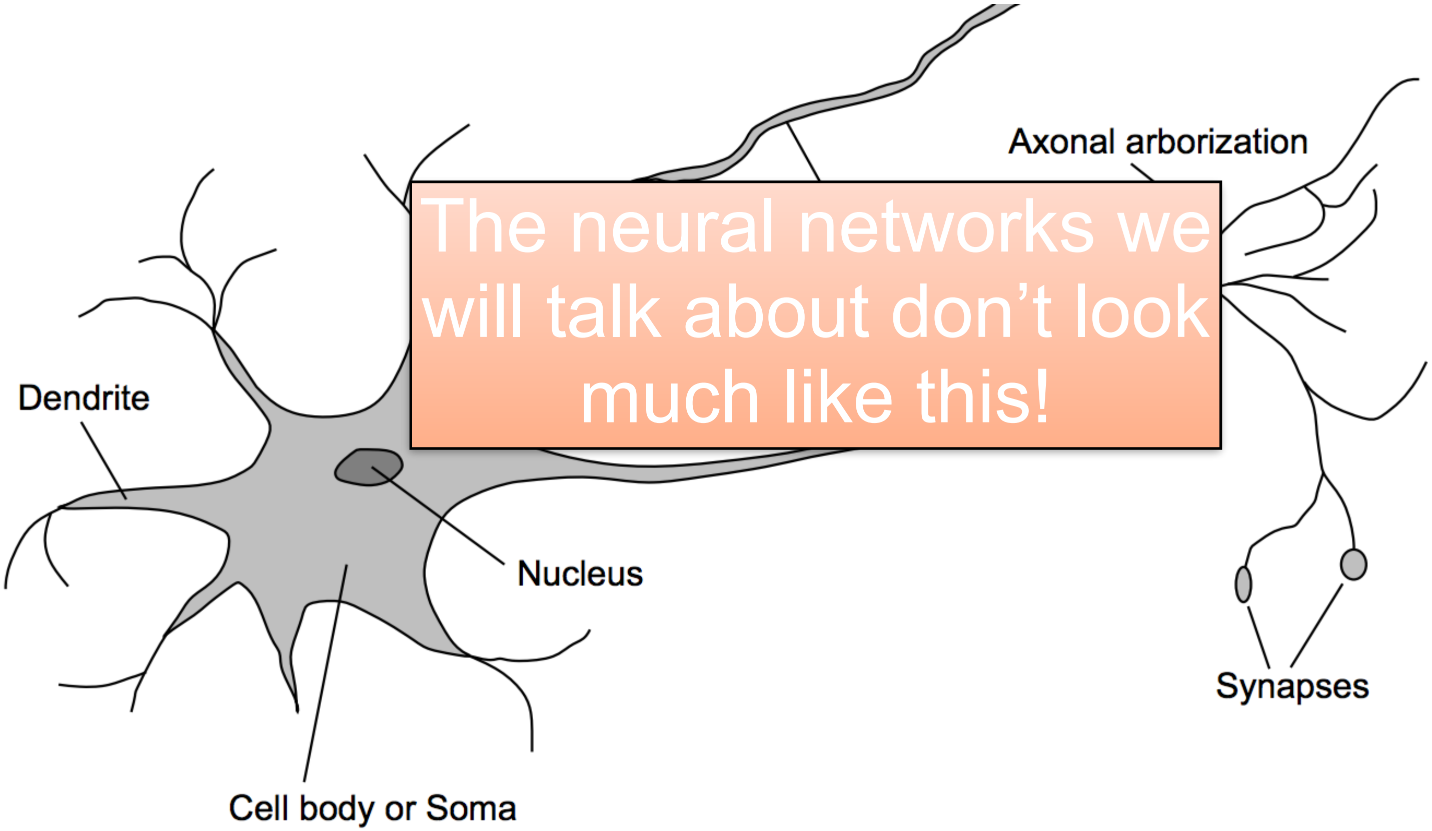

The neural networks we will talk about don't look much like this!

Overview

- Advantage: no feature engineering needed.
- Neural networks detect on their own what parts of the raw input are important.
- They are really good at that!

Applications (Some Examples)

- Sequence classification
  - Sentiment analysis
  - Natural language inference

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  - Part-of-speech tagging
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  - Natural language inference
- Sequence labeling
  - Part-of-speech tagging
  - Named entity recognition
- Sequence generation
  - Language modeling
  - Machine translation

Perceptron

- Single neural unit
- Binary output
- Linear classifier

$$z = b + \Sigma_0^i w_i z_i$$

$$z > 0 \rightarrow y = 1$$
$$z \leq 0 \rightarrow y = 0$$

Perceptron - Example

$$z = b + \Sigma_0^i w_i x_i$$

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- Assume
  - weights w=[2, 4, 8, 3], bias b = 0
  - Inputs x=[0, 0.2, 1, 0]
- What is the output?

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- What is the output?
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  - y = 1

Perceptrons for AND, OR, and XOR

- 2 binary inputs (1 or 0; TRUE or FALSE)
- Which parameters (w and b) give you
  - the AND function?
  - the OR function?
  - the XOR function?

AND

$$z = b + \Sigma_0^i w_i z_i \qquad \begin{array}{l} z > 0 \rightarrow y = 1 \\ z \leq 0 \rightarrow y = 0 \end{array}$$

- 2 binary inputs (1 or 0; TRUE or FALSE)
- We want:
  - [0, 0] -> 0
  - [0, 1] -> 0
  - [1, 0] -> 0
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- 2 binary inputs (1 or 0; TRUE or FALSE)
- We want:
  - [0, 0] -> 0 $\quad b \leq 0$
  - [0, 1] -> 0 $\quad b + w_2 \leq 0$
  - [1, 0] -> 0 $\quad b + w_1 \leq 0$
  - [1, 1] -> 1 $\quad b + w_1 + w_2 > 0$
- Possible answer: w = [1, 1]; b = -1

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$$1 \geq w_1 > 1 - w_2 \geq 0$$

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Set $w_1 = 1.$

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Set $w_2 = 1.$

$$1 \geq w_1 > 1 - w_2 \geq 0$$

In-class Exercise

- 2 binary inputs (1 or 0; TRUE or FALSE)
- Find parameters for
  - the OR function
  - the XOR function

OR			XOR		
x1	x2	y	x1	x2	y
0	0	0	0	0	0
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	1	0

XOR Function

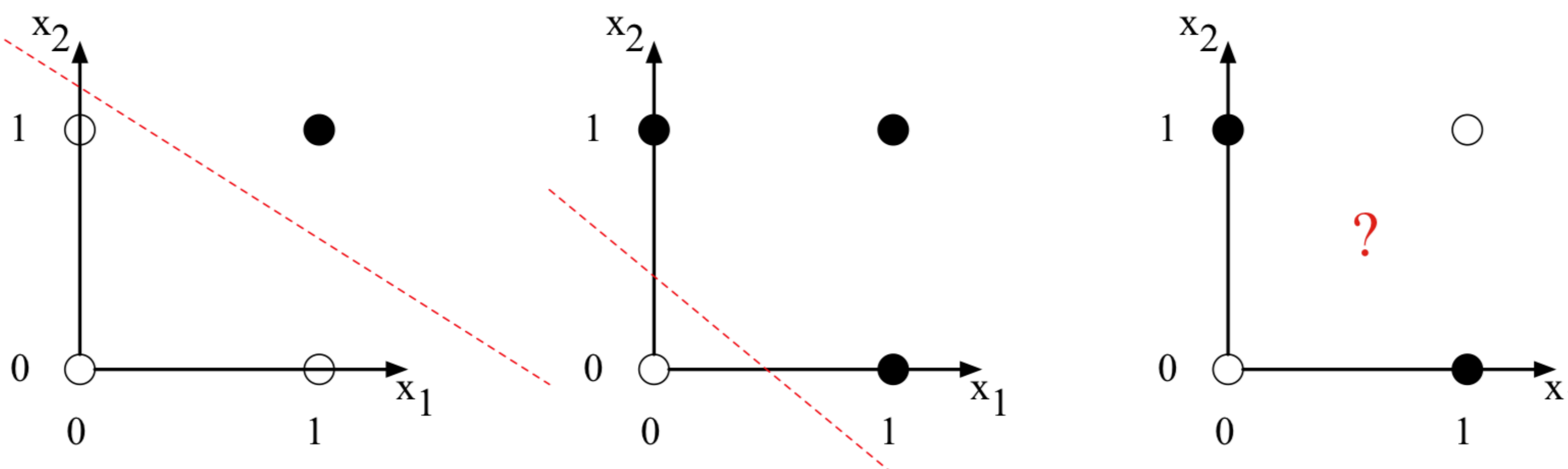
- The perceptron cannot compute XOR
  - The perceptron is a linear classifier.
  - Decision boundary is a line.

$$w_1 x_1 + w_2 x_2 + b = 0$$

$$\rightarrow x_2 = -\frac{w_1}{w_2} x_1 - b$$

XOR Function

- XOR is not linearly separable



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J&M, Ch. 7

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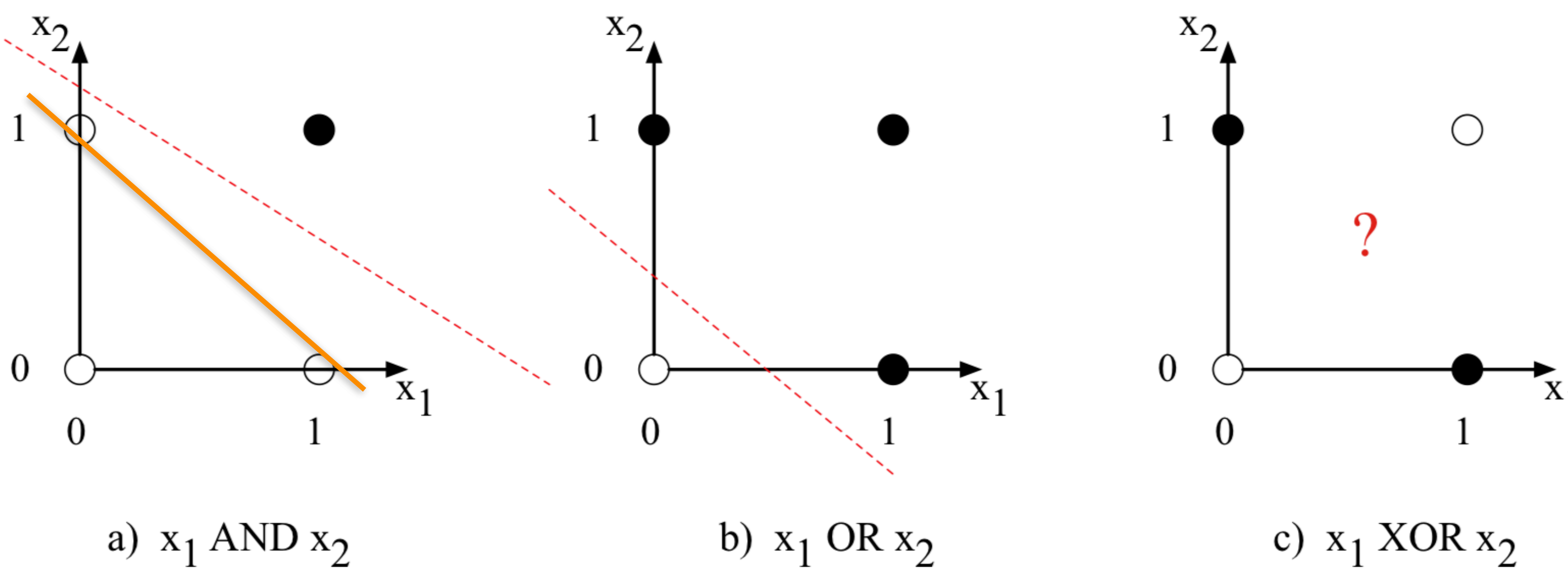

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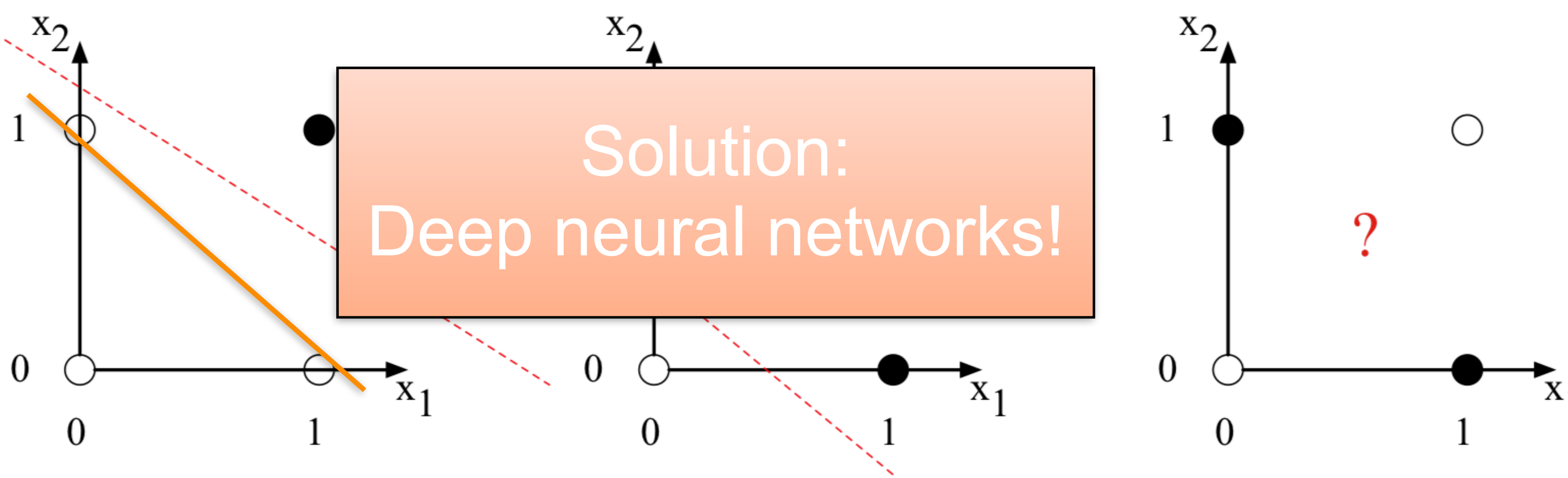


Solution:
Deep neural networks!

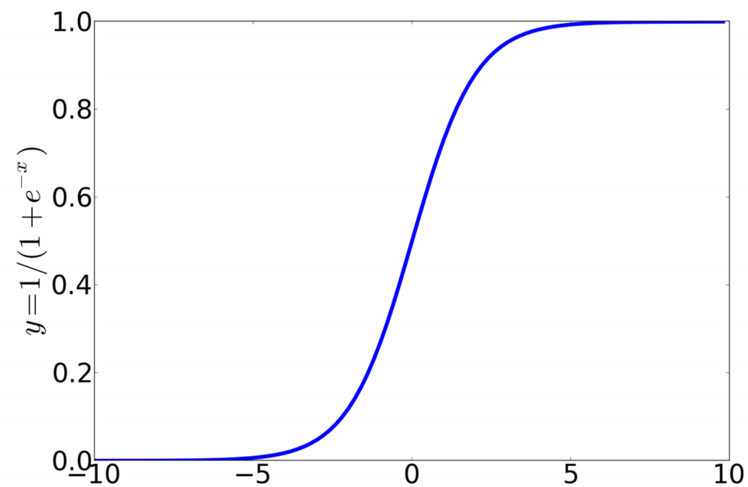
a) x_1 AND x_2 b) x_1 OR x_2 c) x_1 XOR x_2

J&M, Ch. 7

Non-linear output functions

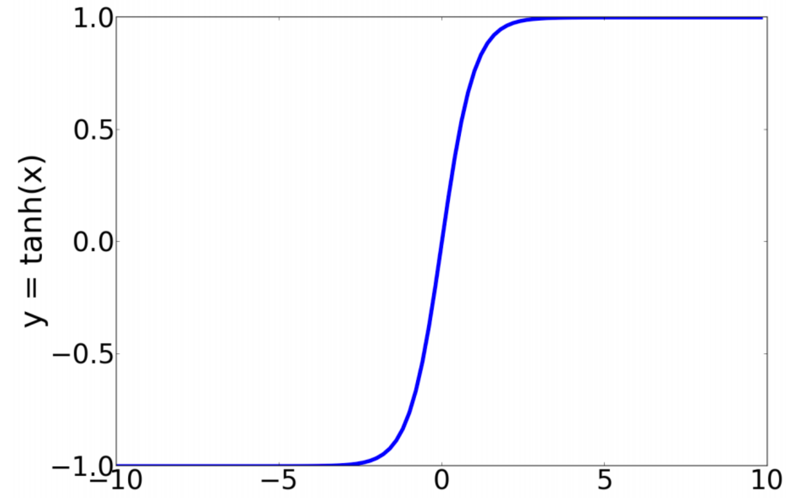
Sigmoid Function

- Maps output to a range between 0 and 1
  - Useful for outliers!
- Can represent probabilities.
- Is differentiable.
  - Handy for learning!



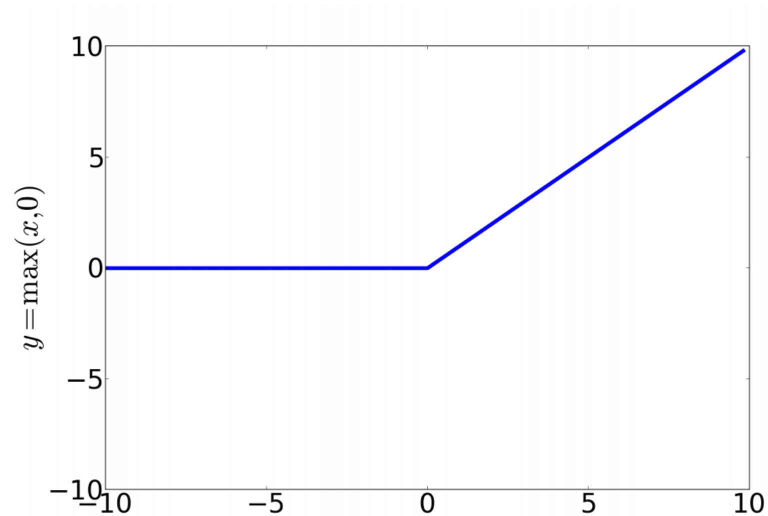
Tanh Function

- More commonly used as an activation function
- In practice often better results than sigmoid



ReLU Function

- Simplest activation function
- Most commonly used
- Nearly linear



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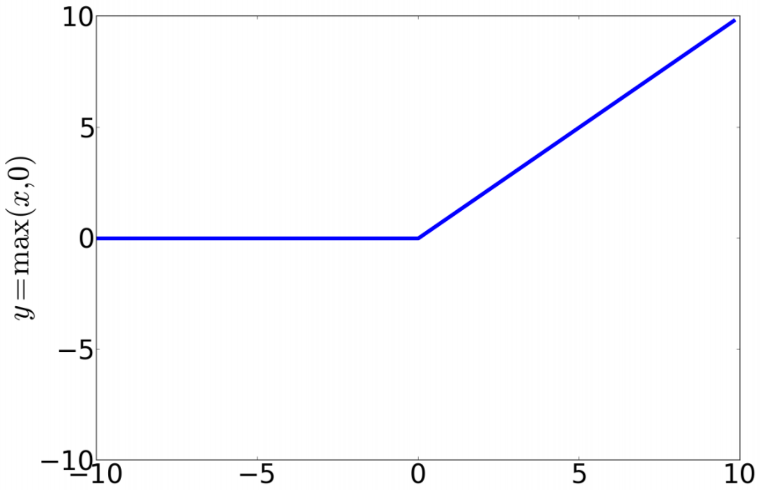

If in doubt, explore different options using your development set!

Feedforward networks

Feedforward Networks

- Also called multi-layer perceptrons.
- Multiple layers of perceptrons with non-linear functions after each layer.
- Can represent more complex decision boundaries.
    - (They can compute XOR!)

Feedforward Networks

$$k = b + \Sigma_0^i w_i z_i$$

$$l = \text{tanh}(k)$$

$$m = b + \Sigma_0^i w_i l_i$$

$$y = \text{sigmoid}(m)$$

Feedforward Networks

$$k = b + \Sigma_0^i w_i z_i$$

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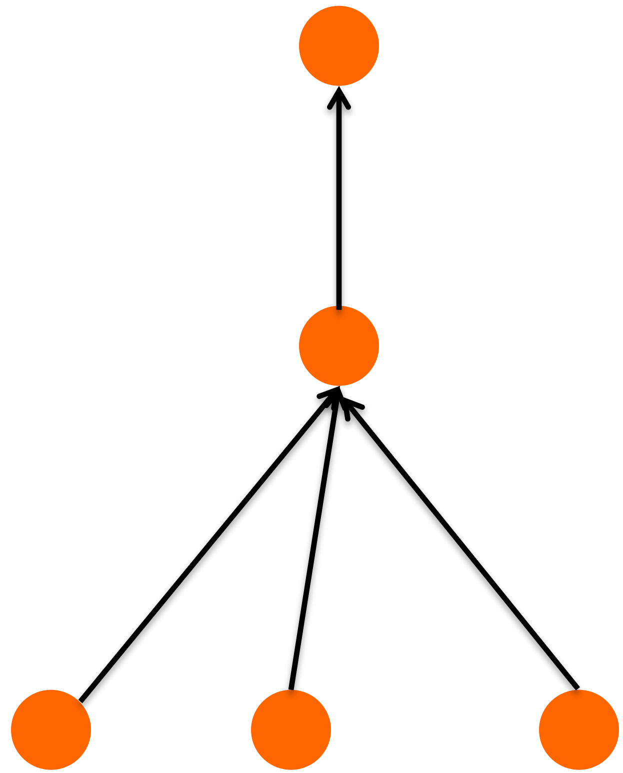

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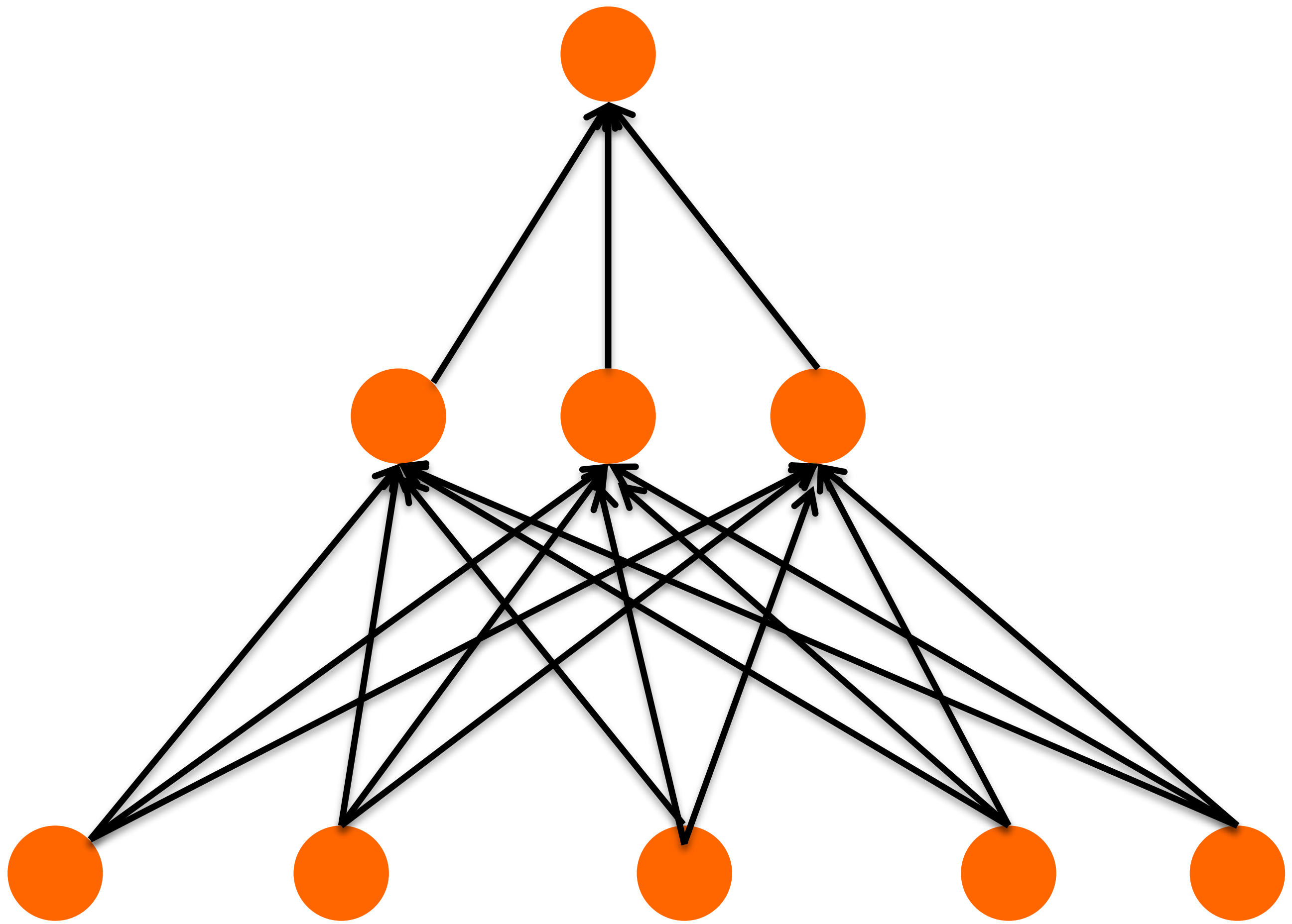

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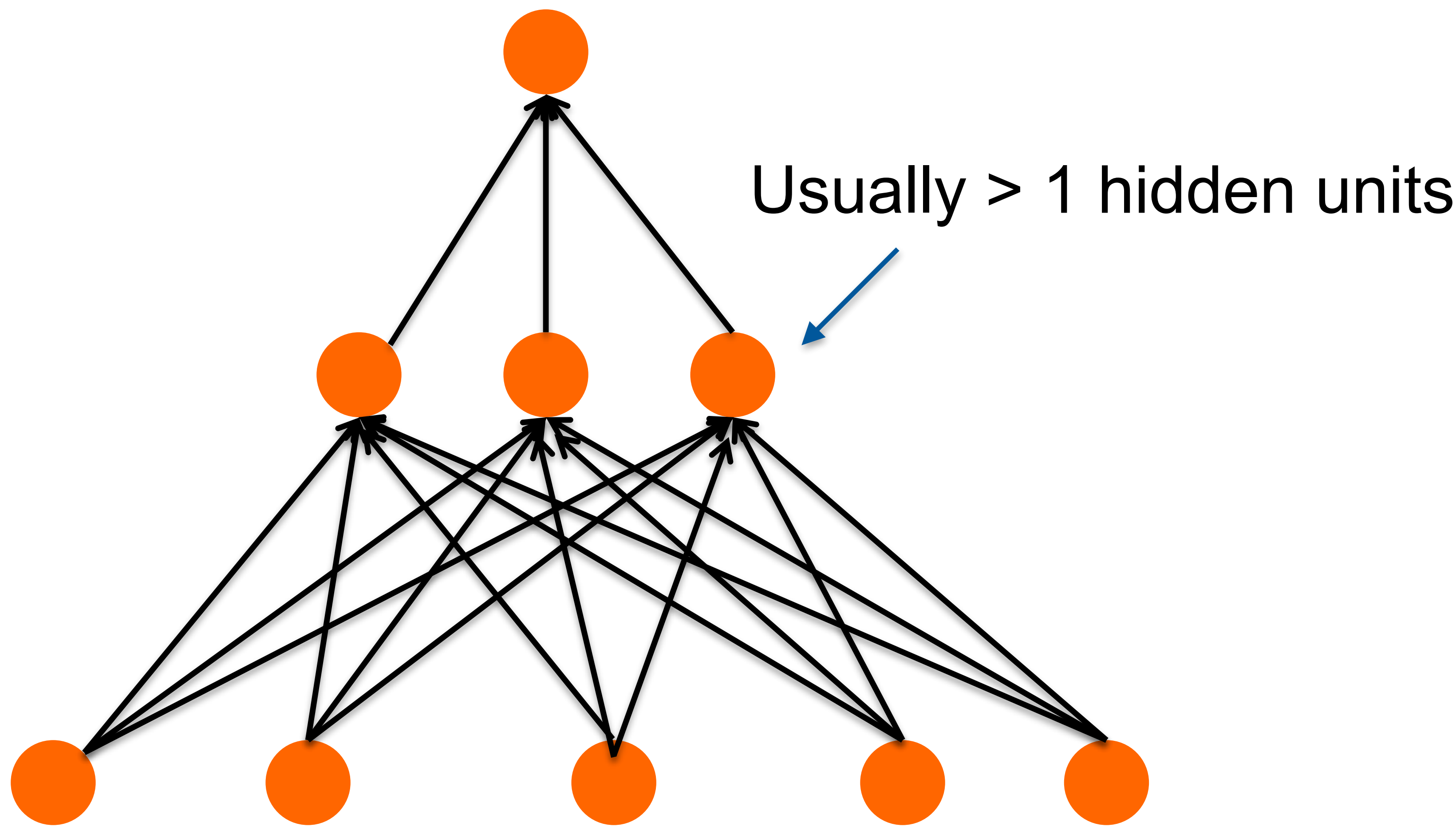

Feedforward Network - Example

- Weights w_1=[2, 4, 8]
- Bias b_1 = 0
- Weights w_2 = [3]
- Bias b_2 = 1
- Inputs x=[0, 0.2, 1]
- What is the output?

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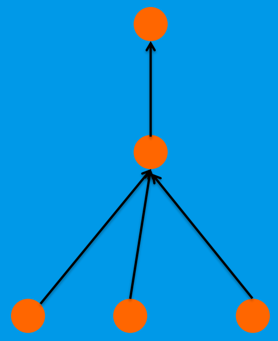

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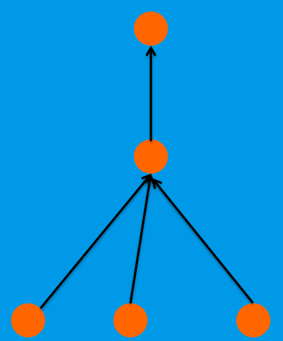

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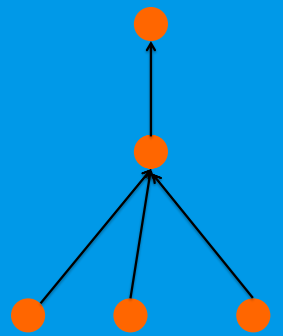

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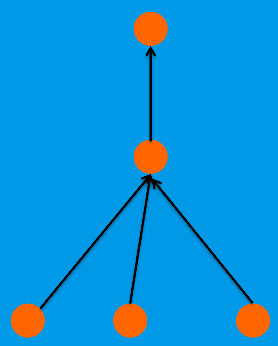

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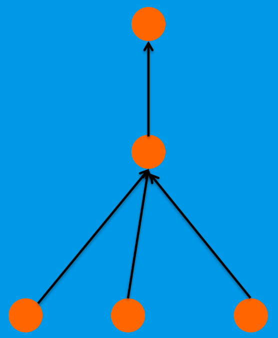

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  - m = 1 + 3*1 = 4
  - y = sigmoid(4) = 0.98

Multi-Class Outputs

- What if you have more than two output classes?
  - Add more output units (one for each class)
  - Use a softmax layer:

$$\text{softmax}(z_i) = \frac{e^{z_i}}{\sum_{j=1}^{k} e^{z_j}} \quad 1 \le i \le D$$

Softmax

$$\text{softmax}(z_i) \;=\; \frac{e^{z_i}}{\sum_{j=1}^{k} e^{z_j}} \quad 1 \leq i \leq D$$

- Use for one-of-many predictions
- Gives you a probability distribution
  - Values are all between 0 and 1
  - Values add up to 1

Softmax

$$\text{softmax}(z_i) = \frac{e^{z_i}}{\sum_{j=1}^{k} e^{z_j}} \quad 1 \leq i \leq D$$

- Example:
  - 3 classes
  - z = [130, 34, 91]

Softmax

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- Example:
  - 3 classes
  - z = [130, 34, 91]

- $softmax(z_i) = \dfrac{e^{130}}{e^{130} + e^{34} + e^{91}}$

Neural language models

Neural Language Models

- Back to language modeling: Calculating the probability of the next word in a sequence given some history.

$$P(w_t|w_1^{t-1}) \approx P(w_t|w_{t-N+1}^{t-1})$$

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- How do we represent our input?
- Option 1: one-hot vectors
    - Potentially really large (depending on the vocabulary size)
    - No information
- Option 2: word embeddings
    - Dense vectors, so not too many weight parameters needed
    - Pretrained embeddings already contain useful information!

Issues

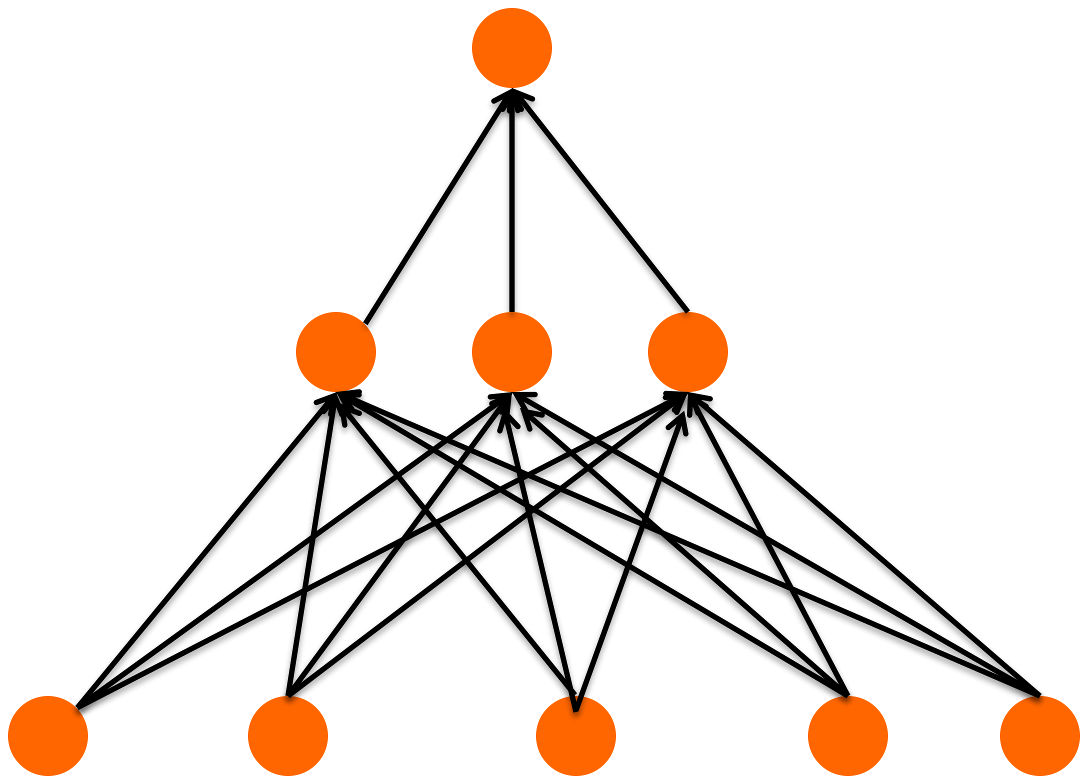
- How many possible classes are there?

Issues

- How many possible classes are there?
- |V|: one for each word in the vocabulary
    - Training takes forever.
    - So does forward inference.
    - In most applications we don’t need the full distribution, just the probability of a small candidate set.
    - Only use the most frequent words!

On the Number of Parameters

- How many parameters does the following network contain?



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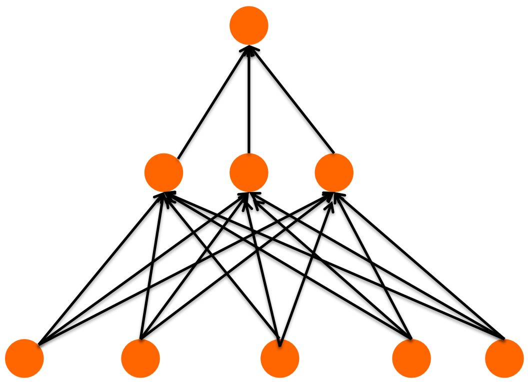
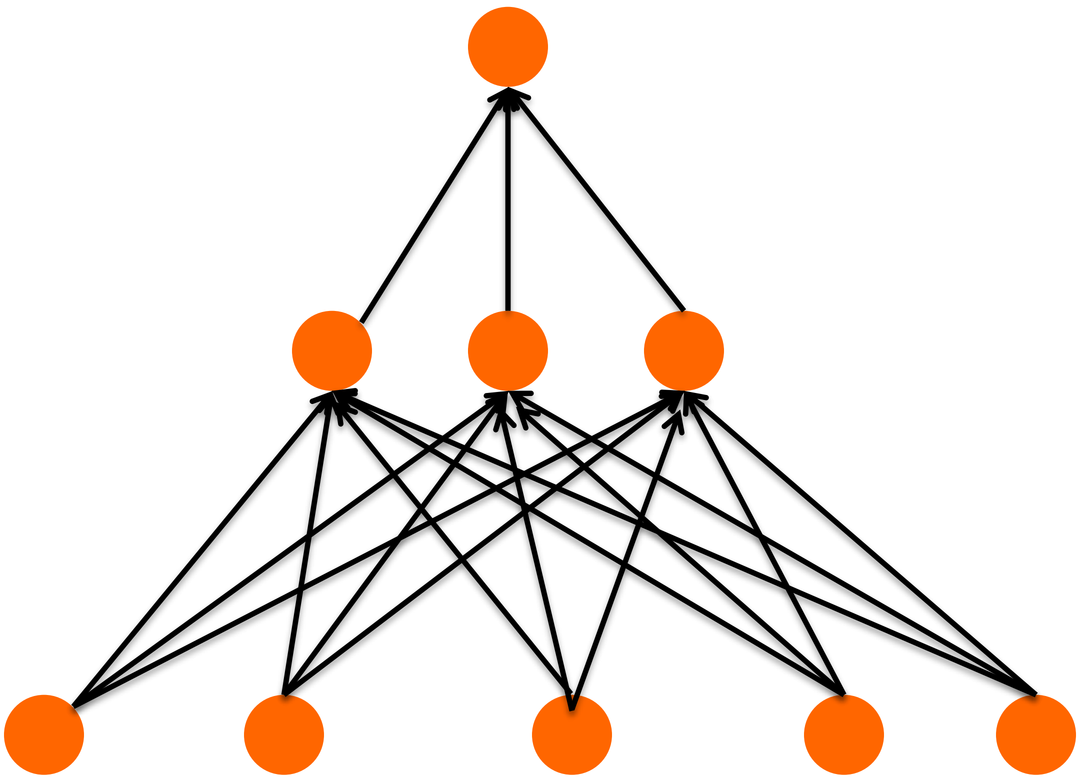


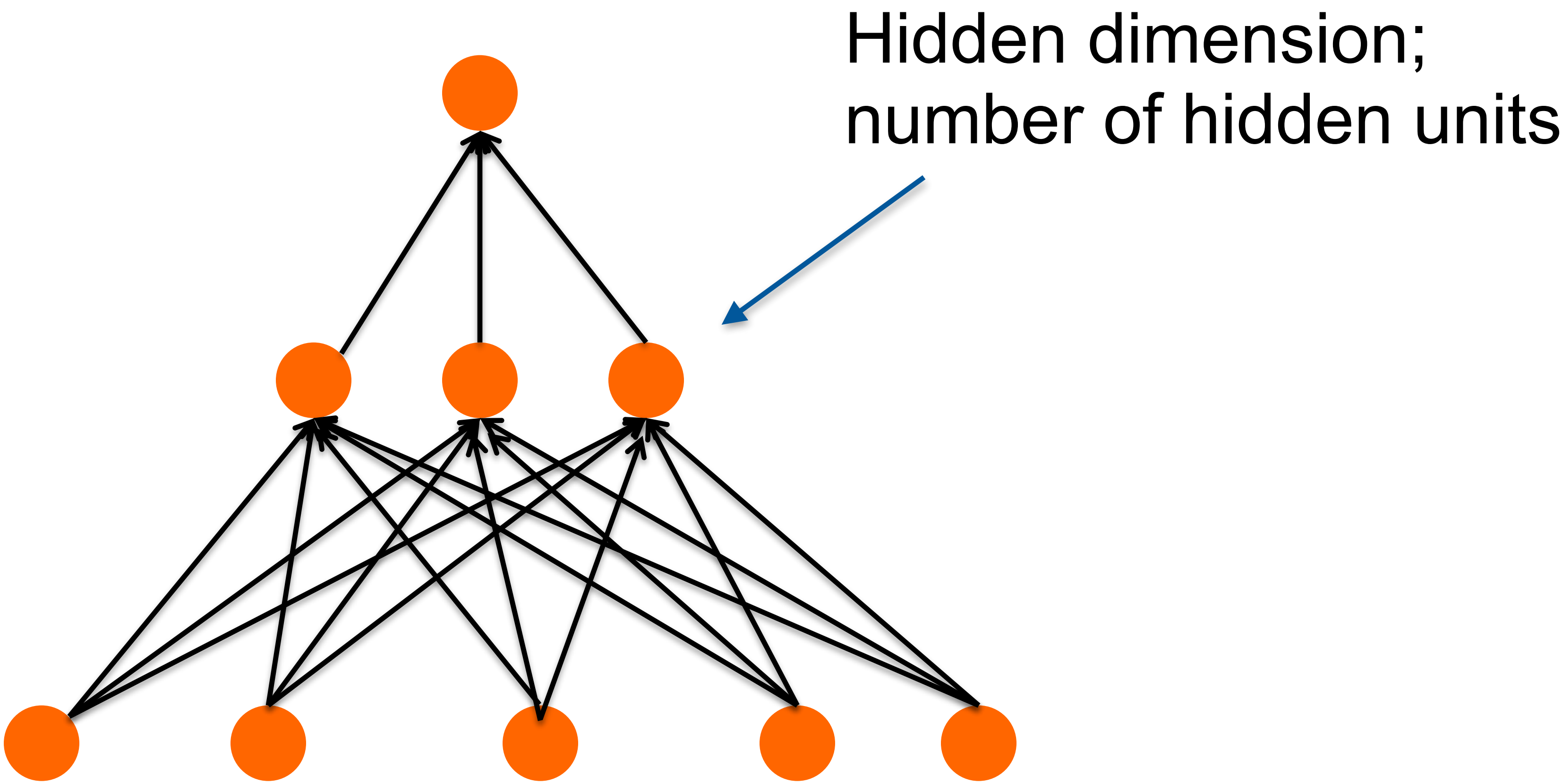
- One for each connection.
- One bias for each neuron.
- 5*3 + 3 for the first layer: 18
- 3*1 + 1 for the second layer: 4

- Total of 22 parameters. This is a tiny network!

On Names



On Names



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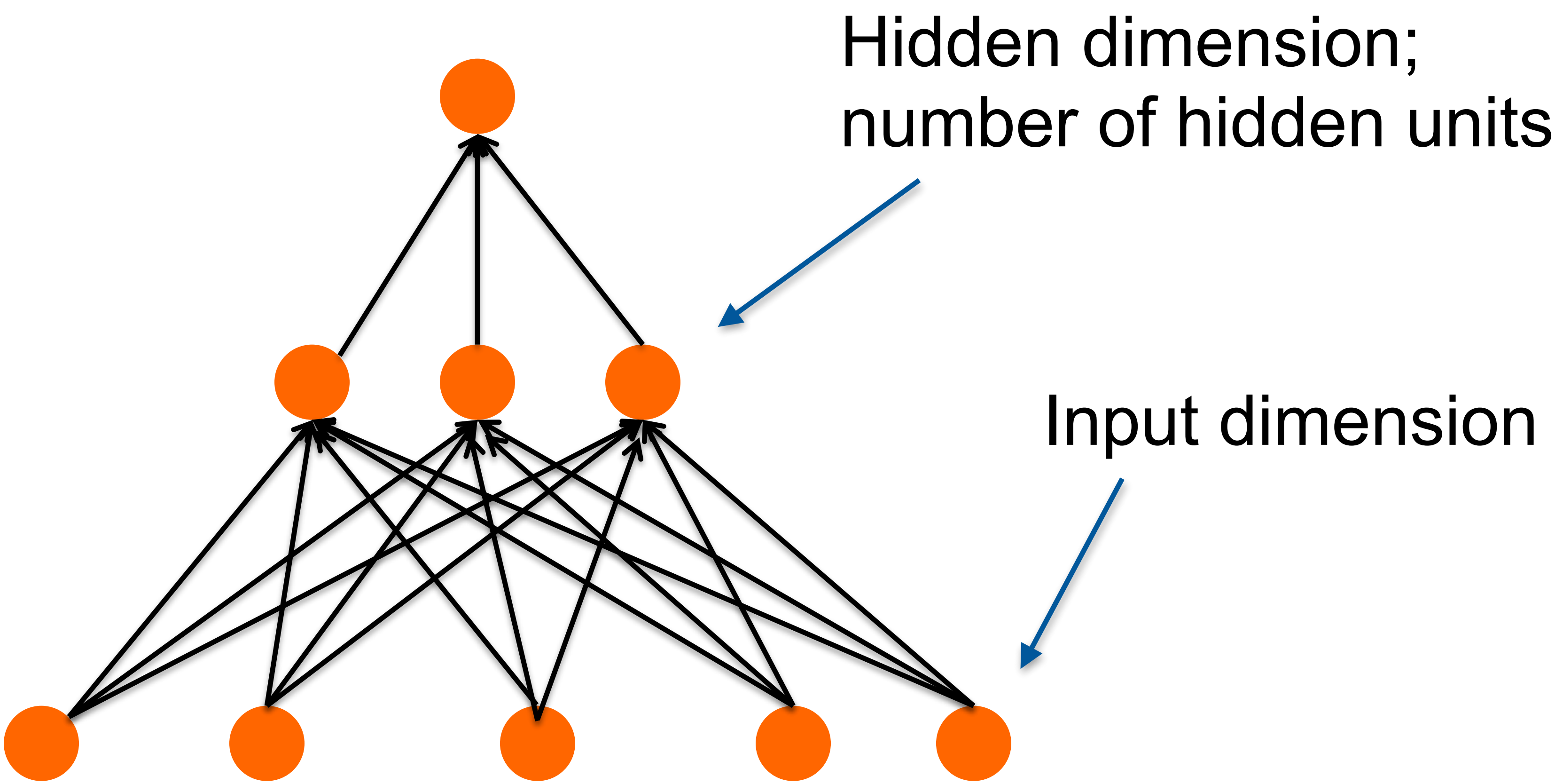

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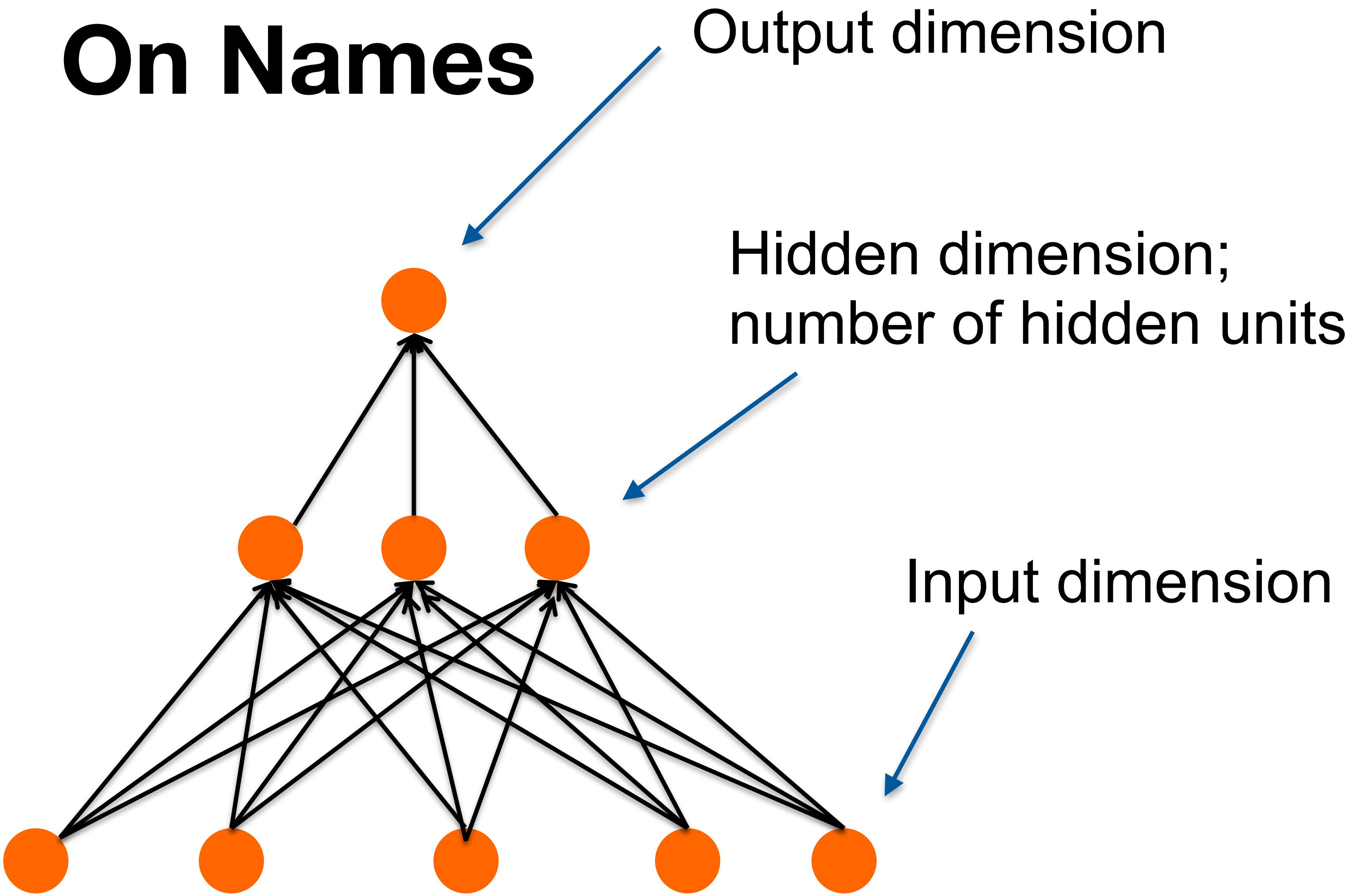
Output dimension
Hidden dimension;
number of hidden units
Input dimension

Back to Neural Language Models

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  - Plus <s> and </s>!
- How many dimensions do our (co-occurrence) word embeddings have?
- If we want to make a neural bigram language model, how many dimensions does our input have?

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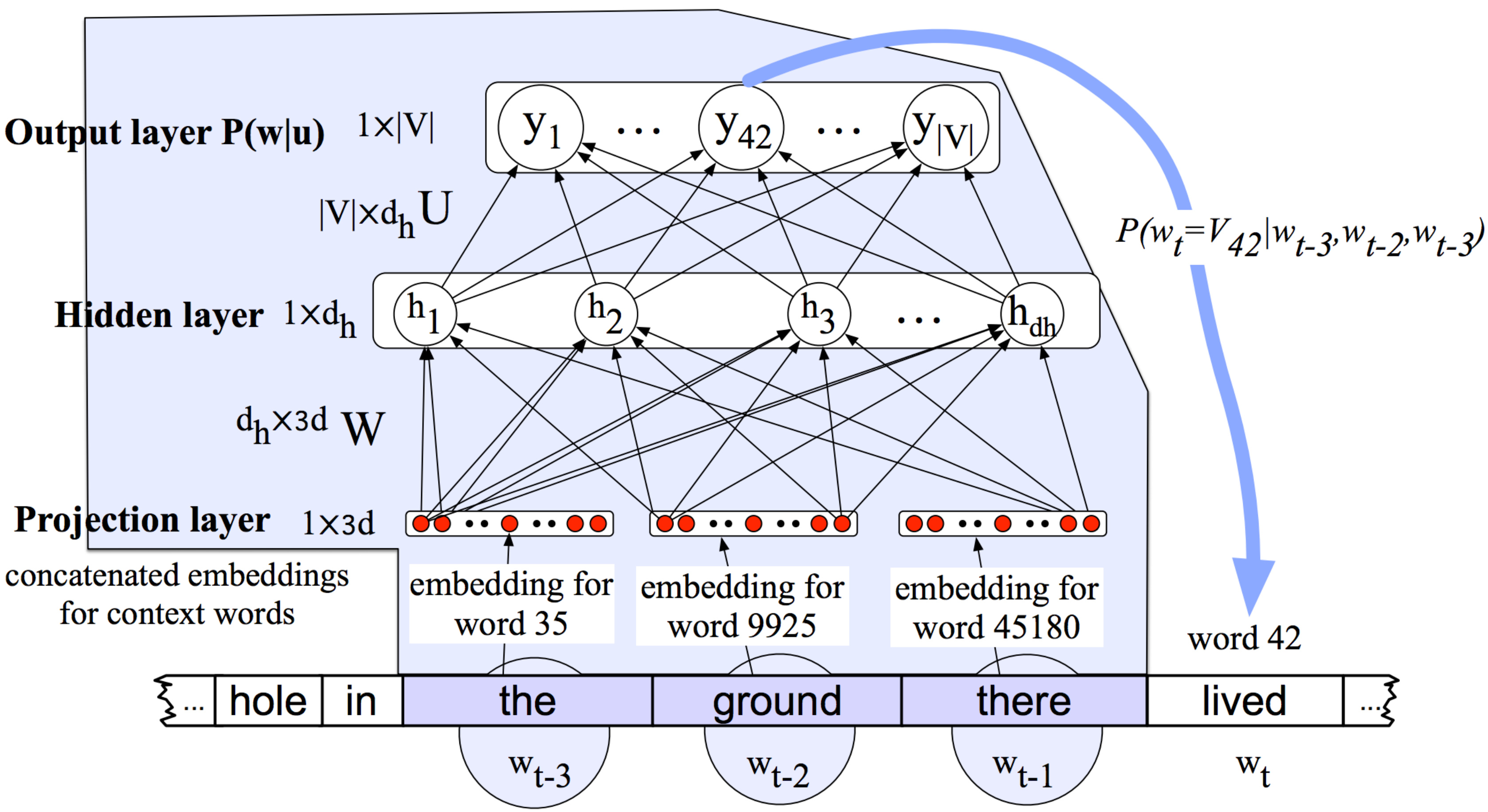
Back to Neural Language Models



Open Question

- Where do we get the network's weights from?
  - Will be discussed in some weeks!

Wrapping up

- Discussed today:
  - Evaluation of language models
  - Perceptrons
  - Feedforward networks
  - Neural language models based on feedforward networks
- Next Monday: Sentiment analysis: Feature-based methods