Supplementary Material for "Integrating Event-Triggered Control and Deep Reinforcement Learning for Brain-Computer Shared Control"

I. Part A: Derivation of the WMR linear time-varying discrete model (Related to Section II. B *WMR Kinematic Model*)

By defining the system state vector as $z = [x, y, v, \phi]^{\mathrm{T}}$ and the control input vector as $u = [a, \delta]^{\mathrm{T}}$, we consider a continuous-time nonlinear kinematic model of a wheeled mobile robot (WMR). To facilitate controller design, this model is linearized around an equilibrium point $(\bar{z}, \bar{u})$ and discretized using the forward Euler method with a sampling time of dt. The resulting linear time-varying discrete model is expressed as:

$$z_{k+1} = Az_k + Bu_k + C$$

where x and y denote the position coordinates, v represents the linear velocity, and ϕ is the heading angle. The inputs are the acceleration a and the steering angle δ.

a) Vehicle Model Linearization: The original nonlinear vehicle model is given by:

$$\begin{aligned}
\dot{x} &= v\cos(\phi), \\
\dot{y} &= v\sin(\phi), \\
\dot{v} &= a, \\
\dot{\phi} &= \frac{v\tan(\delta)}{L},
\end{aligned}$$

where L is the wheelbase of the vehicle. This can be written compactly as:

$$\dot{z} = f(z, u) = A'z + B'u,$$

where the Jacobian matrices A' and B' are computed at the linearization point $(\bar{z}, \bar{u})$.

$$A' = \begin{bmatrix} 0 & 0 & \cos(\bar{\phi}) & -\bar{v}\sin(\bar{\phi}) \\ 0 & 0 & \sin(\bar{\phi}) & \bar{v}\cos(\bar{\phi}) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\tan(\bar{\delta})}{L} & 0 \end{bmatrix}$$

$$B' = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & \frac{\bar{v}}{L\cos^2(\bar{\delta})} \end{bmatrix}$$

Applying forward Euler discretization yields:

$$\begin{aligned}
z_{k+1} &= z_k + f(z_k, u_k)dt \\
&= z_k + (f(\bar{z}, \bar{u}) + A'z_k + B'u_k - A'\bar{z} - B'\bar{u})dt \\
&= (I + dtA')z_k + (dtB')u_k + (f(\bar{z}, \bar{u}) - A'\bar{z} - B'\bar{u})dt
\end{aligned}$$

Thus, the discrete-time model becomes:

$$z_{k+1} = Az_k + Bu_k + C$$

where:

$$A = I + dtA' = \begin{bmatrix} 1 & 0 & \cos(\bar{\phi})dt & -\bar{v}\sin(\bar{\phi})dt \\ 0 & 1 & \sin(\bar{\phi})dt & \bar{v}\cos(\bar{\phi})dt \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{\tan(\bar{\delta})}{L}dt & 1 \end{bmatrix}$$

$$B = dtB' = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ dt & 0 \\ 0 & \frac{\bar{v}}{L\cos^2(\bar{\delta})}dt \end{bmatrix}$$

$$C = (f(\bar{z}, \bar{u}) - A'\bar{z} - B'\bar{u})dt = \begin{bmatrix} \bar{v}\sin(\bar{\phi})\bar{\phi}dt \\ -\bar{v}\cos(\bar{\phi})\bar{\phi}dt \\ 0 \\ -\frac{\bar{v}\bar{\delta}}{L\cos^2(\bar{\delta})}dt \end{bmatrix}$$