CS520 - INTRO TO ARTIF INTEL

Final: Question 4 - GoatDiscoveryBot

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You are a GoatDiscoveryBot. Your job is to discover goats. You know there is a goat hidden in one of three locations, A, B, or C.

a) Logically, how could you model this information? Probabilistically, how could you model this information?Hint: Consider the events In A, In B, In C.

Logically, I model the information with the model more similar to the Minesweeper. More specifically, in this question, I use A+B+C=1 to describe that there is only one goat hidden in three locations. And if I was told that there is no goat in the B, so I set B=0 and A+C=1 which means there is only one goat hidden between A and C. And under this occasion, I can enumerate all the possible situations to count the cases of Goat hidden in A or B or C to determine my last choice. For example, if I know A+C=1, there are two occasions that the Goat hides in A or the Goat hides in B. So, this information will lead me randomly choose a location between A and C.

Probabilistically, I will use a Bayesian formula to calculate the probability of Goat hiding in A or B or C., And it will depend on what CBMHBot told me together with my previous selection. Then, my choice will be based on the probability of the Goat hiding in such three locations.

At this point, you want to search a location for the goat. How can you determine which location to select?

b) Under the logical formulation, how can you compare the value/results of actions 'Select A', 'Select B', 'Select C'? Is there an obvious choice of best action?

There is no extra information about Goat being A or B or C. So, under the logical formulation, A+B+C=1, there are three possible occasions: 1. A=0, B=0, C=1; 2. A=0, B=1, C=0; 3.A=1,B=0,C=0. So, the chance of goat hiding in A or B or C are the same as ⅓. So, there is no obvious choice of choosing A or B or C.

c) **Under the probabilistic formulation, how can you compare the value/results of actions 'Select A', 'Select B', 'Select C'? Is there an obvious choice of best action?**

Under the probabilistic formulation, according to Full probability formula, $P(A) + P(B) + P(C) = 1.$ Because there is no extra information about Goat being A or B or C, so $P(A) = P(B) = P(c) = 1/3$. Also, there is no obvious choice of choosing A or B or C.

Suppose, for argument's sake, you select location A. Before you search location A, you consult your friend, CouldBe- MoreHelpfulBot (who knows where the goat is, but will only tell you where the goat isn't). CBMHBot will look at the two locations you didn't pick, and name one of them that does not have the goat. CBMHBot tells you the goat is not in location B. Given this new information:

d) Update your logical formulation to reflect this new information.

For the logical formulation, when I was told that there is no Goat hiding in location B, I set B=0. Therefore, A+C=1. And under this occasion, I can enumerate all the possible situations to count the cases of Goat hidden in A or B or C to determine my last choice. For example, if I know A+C=1, there are two occasions that the Goat hides in A or the Goat hides in B which represents as A=1, C=0 or A=0, C=1. So, this information will lead me randomly choose a location between A and C.

e) Update your probabilistic formulation to reflect this new information. Hint: The CBMHBot's decision to tell you the goat is not in B depended both on which location you selected, and where the goat actually is.

In this question, with the information that Goat doesn't hide in B, I can update the probability of Goat being in A or B or C which will be represented by $P(A)\ or\ P(B) or\ P(c)$.

First, assuming that I previously select A and CBMHBot tells me there is no Goat hiding in B, we an update $P(A)\ or\ P(B) or\ P(c)$ to the equations below.

$$P(A) \ = P(Goat\ in\ A|select\ A, told\ Goat\ not\ in\ B),$$

$$P(B) \ = P(Goat\ in\ A|select\ A, told\ Goat\ not\ in\ B),$$

$$P(C) \ = P(Goat\ in\ A|select\ A, told\ Goat\ not\ in\ B).$$

And according to the Bayesian formula，we can convert the problems to below equations,

$$P(Goat\ in\ A|select\ A, told\ Goat\ not\ in\ B) = \frac{P(selectA,\ \ told\ Goat\ not\ in\ B|Goat\ in\ A)P(Goat\ in\ A)}{P(select\ A,\ told\ Goat\ not\ in\ B)},$$

$$P(Goat\ in\ B|select\ A, told\ Goat\ not\ in\ B) = \frac{P(selectA,\ \ told\ Goat\ not\ in\ B|Goat\ in\ B)P(Goat\ in\ B)}{P(select\ A,\ told\ Goat\ not\ in\ B)}$$

$$P(Goat\ in\ C|select\ A, told\ Goat\ not\ in\ B) = \frac{P(selectA,\ \ told\ Goat\ not\ in\ B|Goat\ in\ C)P(Goat\ in\ C)}{P(select\ A,\ told\ Goat\ not\ in\ B)}.$$

Then according to the reality, $P(Goat\ in\ A) = P(Goat\ in\ B) = P(Goat\ in\ C) = 1/3.$

Assuming the Goat hidding in A, CBMHBot can randomly tell you there is no Goat in B or there is no Goat in C. Therefore, $P(selectA,\ \ told\ Goat\ not\ in\ B|Goat\ in\ A) = 1/2$.
Assuming the Goat hidding in B, CBMHBot can't tell you there is no Goat in B, because it can't lie to us. Therefore, $P(selectA,\ \ told\ Goat\ not\ in\ B|Goat\ in\ B) = 0.$.
Assuming the Goat hidding in C, CBMHBot can only tell you there is no Goat in B. Therefore, $P(selectA,\ \ told\ Goat\ not\ in\ B|Goat\ in\ C) = 1$.

Then, we calculate $P(select\ A,\ told\ Goat\ not\ in\ B)$. According to Full probability formula

$$P(select\ A,\ told\ Goat\ not\ in\ B) = P(selectA,\ \ told\ Goat\ not\ in\ B|Goat\ in\ A)P(Goat\ in\ A) +$$

$$P(selectA,\ \ told\ Goat\ not\ in\ B|Goat\ in\ B)P(Goat\ in\ B) +$$

$$P(selectA,\ \ told\ Goat\ not\ in\ B|Goat\ in\ C)P(Goat\ in\ C) = 1/2\ * 1/3 + 0 * 1/3 + 1 * 1/3 = 1/2$$

Finally, we can calculate the probability of Goat being in A or B or C as $P(A)\ or\ P(B) or\ P(c)$.

$$P(A) = P(Goat\ in\ A | select\ A, told\ Goat\ not\ in\ B) = \frac{(1/2)*(1/3)}{1/2} = 1/3$$
$$P(B) = P(Goat\ in\ A | select\ A, told\ Goat\ not\ in\ B) = \frac{0*(1/3)}{1/2} = 0$$
$$P(C) = P(Goat\ in\ A | select\ A, told\ Goat\ not\ in\ B) = \frac{1*(1/3)}{1/2} = 2/3.$$

At this point, you want to re-assess your earlier decision of which action to take as you now have more information than you did previously.

f) Under the logical formulation, how can you compare the value/results of actions 'Re-Select A', 'Re-Select B', 'Re-Select C'? Is there an obvious choice of best action?

For the logical formulation, when I was told that no Goat are hiding in location B, I set B=0. Therefore, A+C=1. And under this occasion, I can enumerate all the possible situations to count the cases of Goat hidden in A or B or C to determine my last choice. For example, if I know A+C=1, there are two occasions that the Goat hides in A or the Goat hides in B which represents as A=1, C=0 or A=0, C=1. Because I want to find the Goat, I will rationally make a choice more likely to discover the Goat. So, I won't Re-select B., And there is both occasion of the Goat being in A or C. Based on that I will randomly choose a location between A and C to discover. So, there is also no apparent choice between Re-select A or Re-select B under the logical formulation.

g) Under the probabilistic formulation, how can you compare the value/results of actions 'Re-Select A', 'Re-Select B', 'Re-Select C'? Is there an obvious choice of best action?

Because I want to find the Goat, I will rationally make the choice more likely to discover the Goat. So, I should compare the probability of the Goat being in A or B or C as $P(A)\ or\ P(B) or\ P(c)$ to detemine which location to Re-select.

And according to e), $P(A) = 1/3, P(B) = 0, P(C) = 2/3,$ so we will Re-select C, for the Goat is more likely being in C.

Did CouldBeMoreHelpfulBot provide anything of actual value?

h) Under the logical formulation, having initially selected location A, should you stick with location A or change? Justify your choice.

Under the logical formulation, there is no obvious choice between Re-select A or Re-select C. So, rational stick or change will both be Okay. However, for myself, I won't change, because I always believe my First Sense.

i) Under the probabilistic formulation, having initially selected location A, should you stick with location A or change? Justify your choice.

Under the probabilistic formulation, I will change to Re-select C, because the Goat is more likely hiding in location C. I want to discover the Goat so that I will turn to Re-select C.

j) Who is more successful in their mission, LogicalGoatDiscoveryBot or ProbabilisticGoatDiscoveryBot? Justify.

I think ProbabilisticGoatDiscoveryBot is more successful because ProbabilisticGoatDiscoveryBot can tell us the probability of The Goat being in each position, which will lead us to make an optimal selection. However, for the LogicalGoatDiscoveryBot, it only helps me find out all the possible occasions of the Goat being in each location, which will not lead us to make an optimal selection.

Bonus: You initially select location A. Suppose that you know CouldBeMoreHelpfulBot is biased, in the following way: if CBMHBot has a choice between telling you B and C, then CBMHBot tells you B with probability p, and C with probability 1−p. CBMHBot tells you that the goat is not in location B. What is the utility of sticking with your initial selection? What is the utility of switching to C? What is the rational choice, and does it depend on p?

In this question, with the information that Goat doesn't hide in B, I can update the probability of Goat being in A or B or C which will be represented by $P(A)\ or\ P(B) or\ P(c)$.

First, assuming that I previously select A and CBMHBot tells me there is no Goat hiding in B, we can update $P(A)\ or\ P(B) or\ P(c)$ to the equations below.

$P(A)\ = P(Goat\ in\ A | select\ A, told\ Goat\ not\ in\ B),$

$P(B)\ = P(Goat\ in\ A | select\ A, told\ Goat\ not\ in\ B),$

$P(C)\ = P(Goat\ in\ A | select\ A, told\ Goat\ not\ in\ B).$

And according to the Bayesian formula， we can convert the problems to below equations,

$P(Goat\ in\ A | select\ A, told\ Goat\ not\ in\ B) = \frac{P(selectA,\ \ told\ Goat\ not\ in\ B | Goat\ in\ A) P(Goat\ in\ A)}{P(select\ A,\ told\ Goat\ not\ in\ B)},$

$P(Goat\ in\ B | select\ A, told\ Goat\ not\ in\ B) = \frac{P(selectA,\ \ told\ Goat\ not\ in\ B | Goat\ in\ B) P(Goat\ in\ B)}{P(select\ A,\ told\ Goat\ not\ in\ B)}$

$P(Goat\ in\ C | select\ A, told\ Goat\ not\ in\ B) = \frac{P(selectA,\ \ told\ Goat\ not\ in\ B | Goat\ in\ C) P(Goat\ in\ C)}{P(select\ A,\ told\ Goat\ not\ in\ B)}.$

Then according to the the reality, $P(Goat\ in\ A) = P(Goat\ in\ B) = P(Goat\ in\ C) = 1/3.$

Assuming the Goat hiding in A, CBMHBot can tell you there is no Goat in B with probability as p or there is no Goat in C with probability as $1 - p$.

Therefore, $P(selectA,\ \ told\ Goat\ not\ in\ B | Goat\ in\ A) = p$.

Assuming the Goat hidding in B, CBMHBot can't tell you there is no Goat in B, because it can't lie to us. Therefore, $P(selectA,\ \ told\ Goat\ not\ in\ B | Goat\ in\ B) = 0.$.

Assuming the Goat hidding in C, CBMHBot can only tell you there is no Goat in B. Therefore, $P(selectA,\ \ told\ Goat\ not\ in\ B | Goat\ in\ C) = 1$.

Then, we calculate $P(select\ A,\ told\ Goat\ not\ in\ B).$ According to Full probability formula

$P(select\ A,\ told\ Goat\ not\ in\ B) = P(selectA,\ \ told\ Goat\ not\ in\ B | Goat\ in\ A) P(Goat\ in\ A) +$

$P(selectA,\ \ told\ Goat\ not\ in\ B|Goat\ in\ B)P(Goat\ in\ B) +$
$P(selectA,\ \ told\ Goat\ not\ in\ B|Goat\ in\ C)P(Goat\ in\ C) = p\ * 1/3 + 0 * 1/3 + 1 * 1/3 = (1 + p)/3$

Finally, we can calculate the probability of Goat being in A or B or C as $P(A)\ or\ P(B) or\ P(c)$.
$P(A)\ = P(Goat\ in\ A|select\ A, told\ Goat\ not\ in\ B) = \frac{p*(1/3)}{(1+p)/3} = p/(p + 1)$
$P(B)\ = P(Goat\ in\ A|select\ A, told\ Goat\ not\ in\ B) = \frac{0 * (1/3)}{(1+p)/3} = 0$
$P(C)\ = P(Goat\ in\ A|select\ A, told\ Goat\ not\ in\ B) = \frac{1 * (1/3)}{(1+p)/3} = 1/(p + 1).$
So the probability of Goat being in A or B or C depends on the p itself. Because $p/(p + 1) \leqslant 1/(p + 1)$, which means if p is not 1, the probability of Goat being in C is biggest, we need to change our selection to C. If p is 1, we can either switch to C or stick the selection of A, because $P(A)\ = P(c) = 1/2.$.
Therefore, swithching to C or sticking to A depends on p itself.