

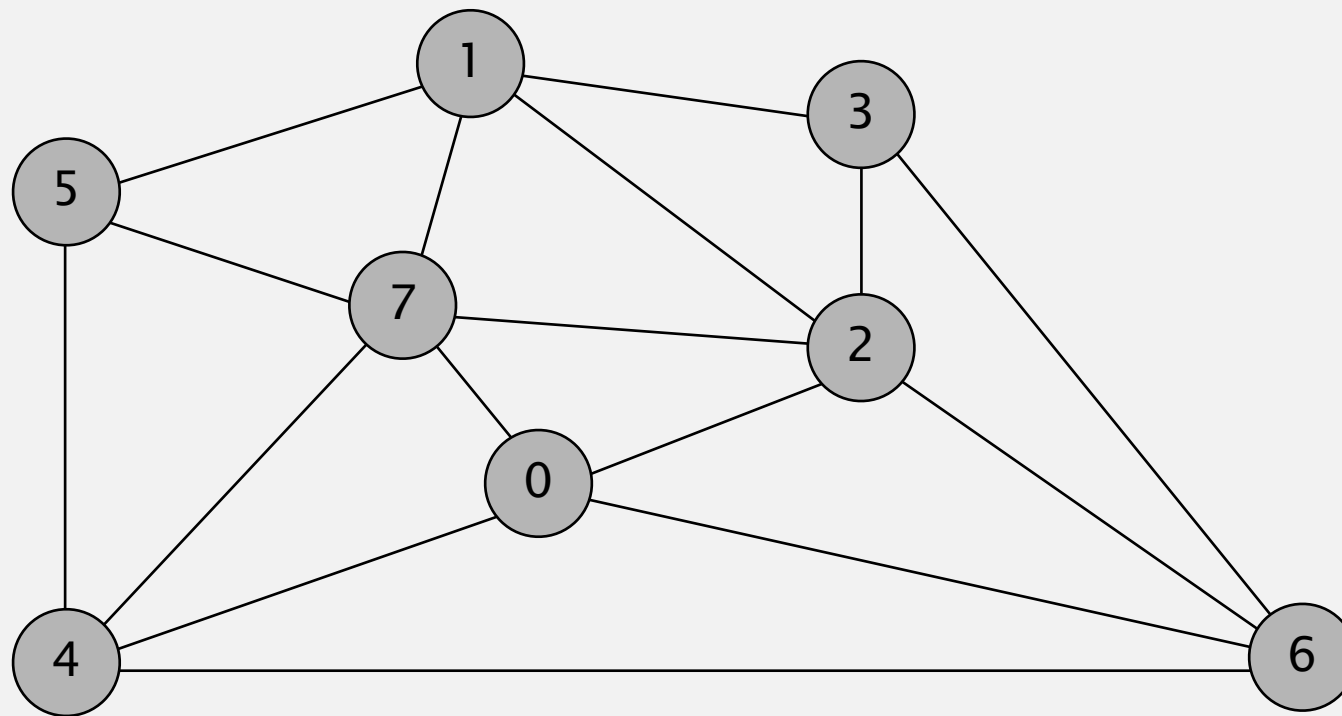
PRIM'S ALGORITHM DEMO

- ▶ Prim's algorithm
- ▶ lazy Prim
- ▶ eager Prim

- ▶ **Prim's algorithm**
- ▶ lazy implementation
- ▶ eager implementation

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

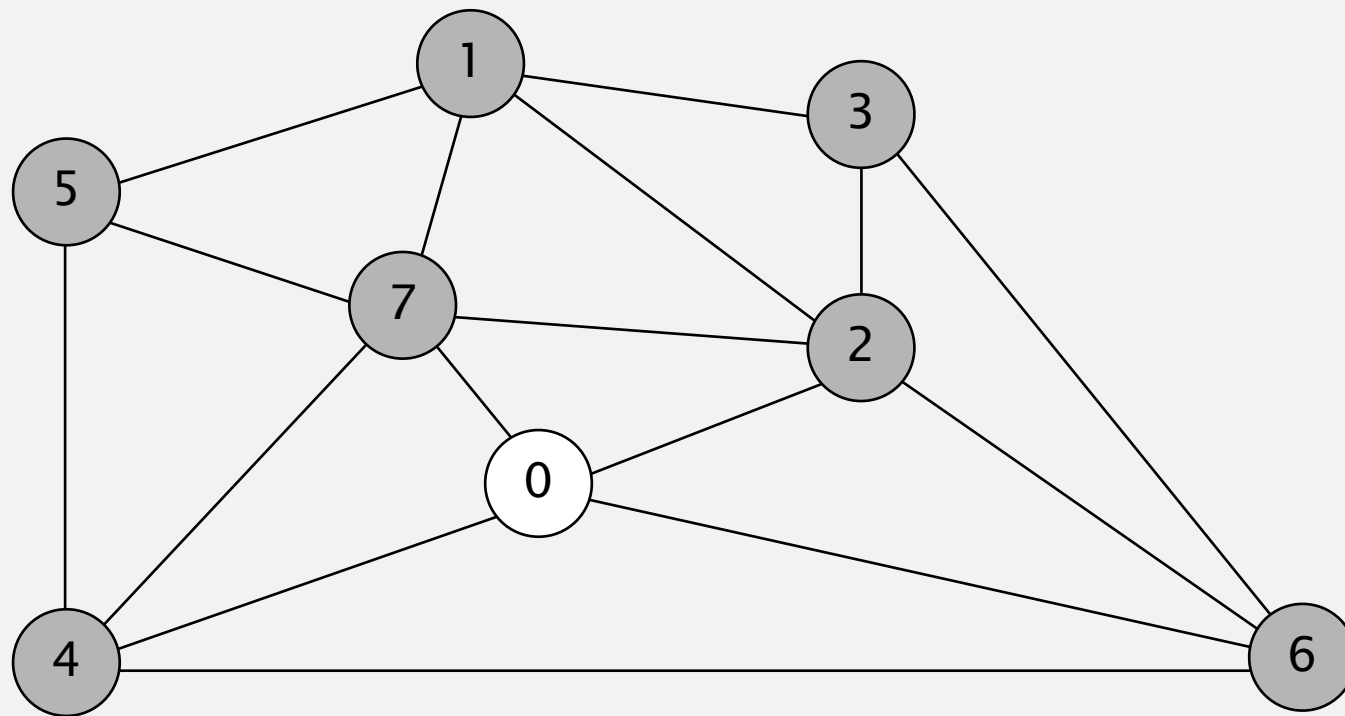


an edge-weighted graph

0-7	0.16
2-3	0.17
1-7	0.19
0-2	0.26
5-7	0.28
1-3	0.29
1-5	0.32
2-7	0.34
4-5	0.35
1-2	0.36
4-7	0.37
0-4	0.38
6-2	0.40
3-6	0.52
6-0	0.58
6-4	0.93

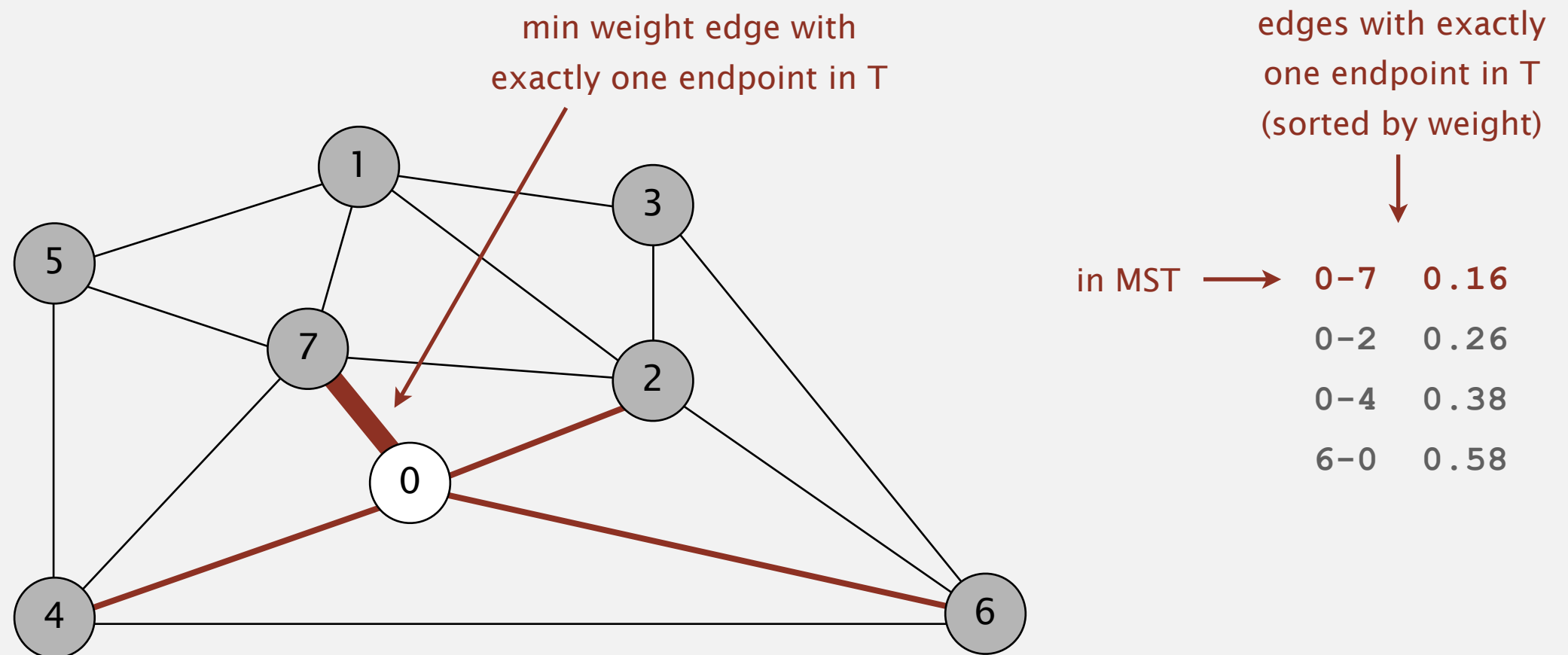
Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



Prim's algorithm

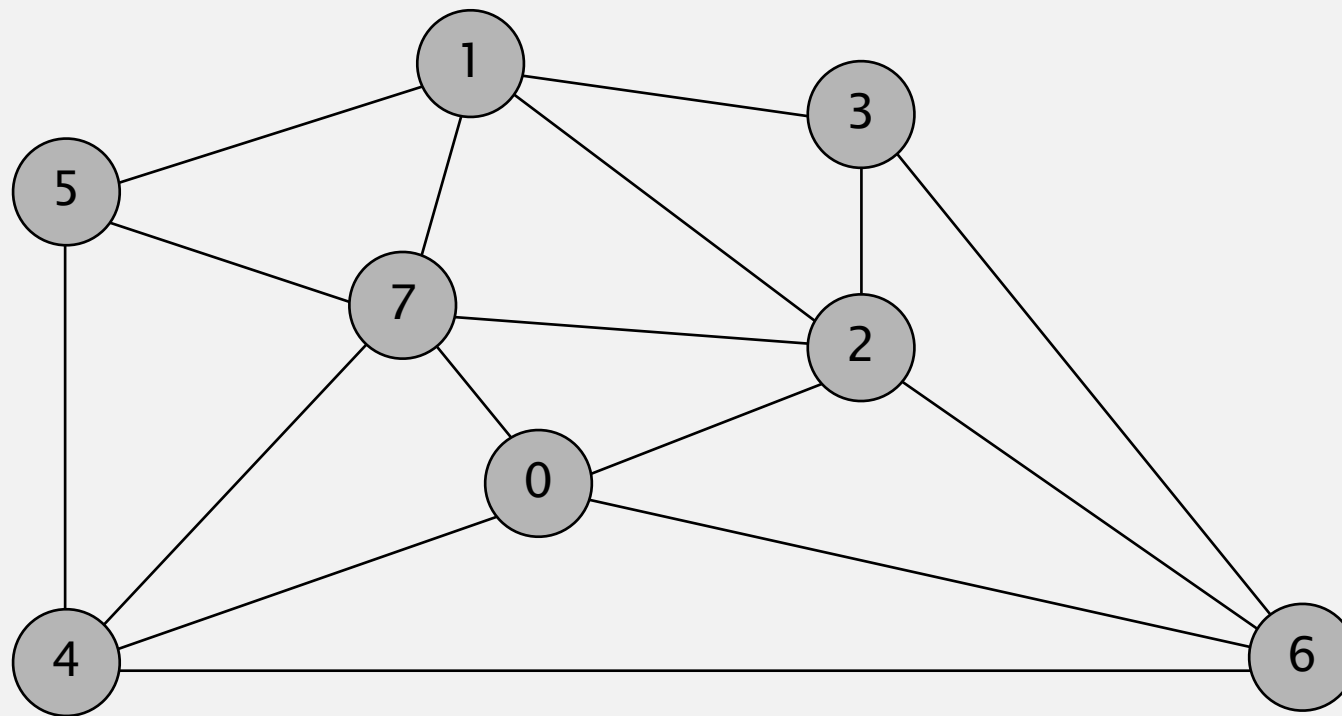
- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



- ▶ Prim's algorithm
- ▶ **lazy implementation**
- ▶ eager implementation

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

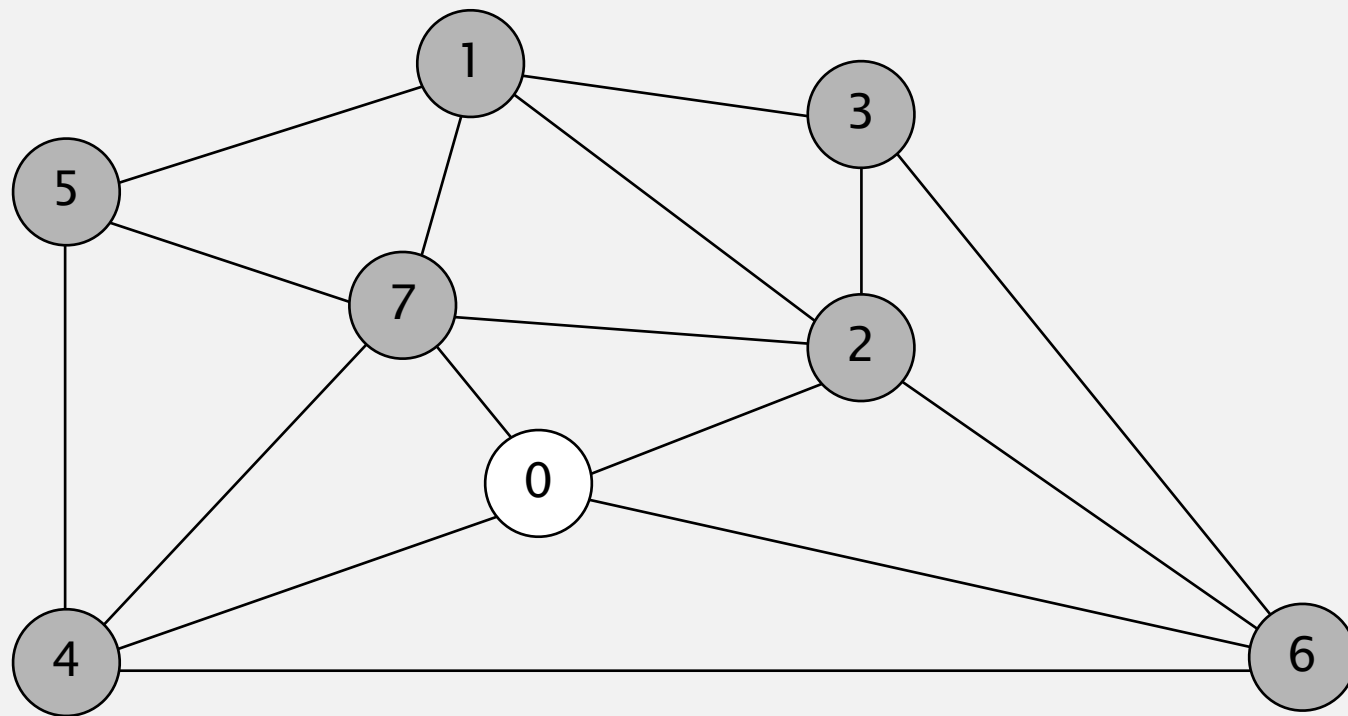


an edge-weighted graph

0-7	0.16
2-3	0.17
1-7	0.19
0-2	0.26
5-7	0.28
1-3	0.29
1-5	0.32
2-7	0.34
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1-2	0.36
4-7	0.37
0-4	0.38
6-2	0.40
3-6	0.52
6-0	0.58
6-4	0.93

Prim's algorithm

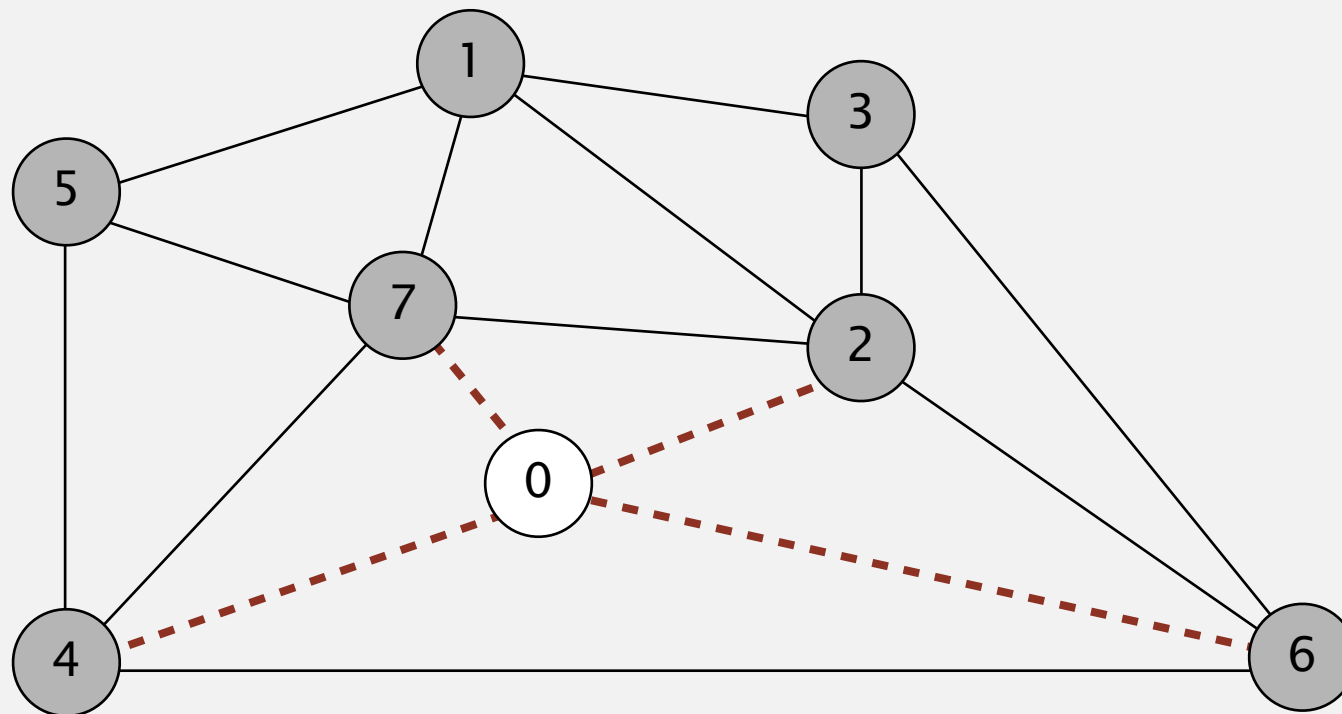
- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

add to PQ all edges incident to 0



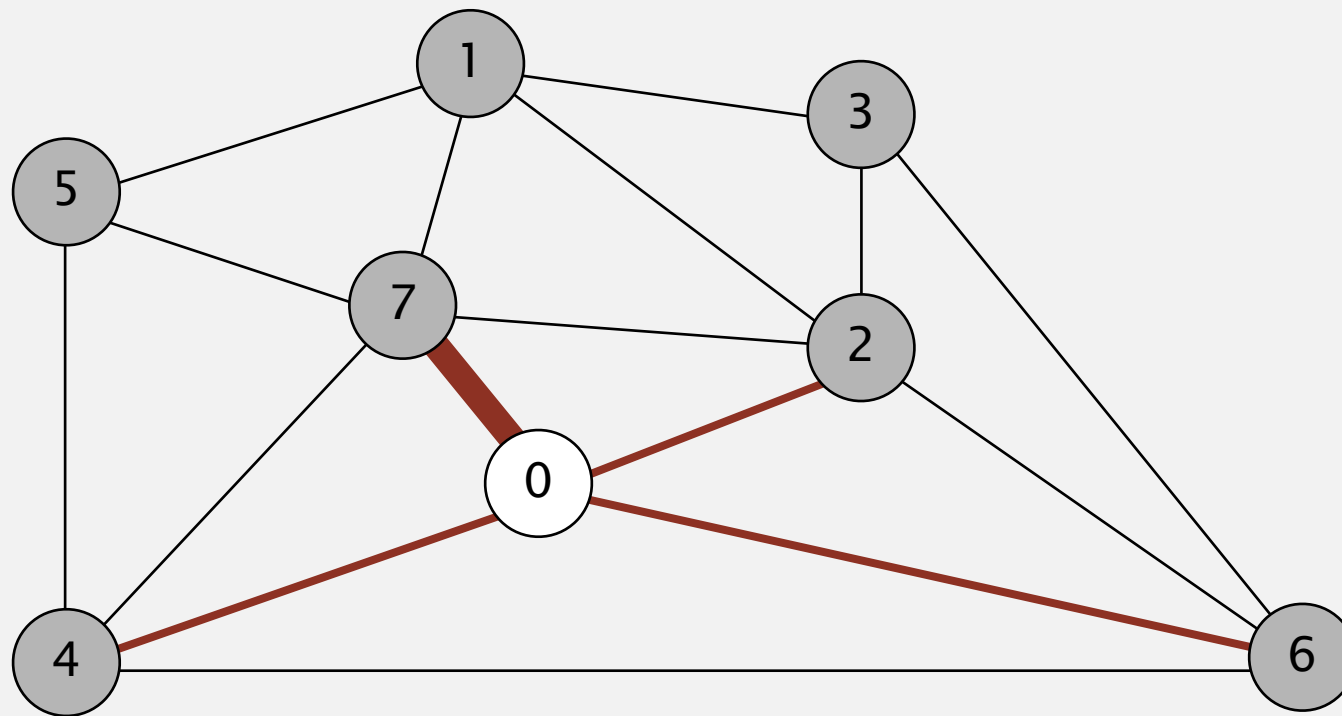
edges on PQ
(sorted by weight)

*	0-7	0.16
*	0-2	0.26
*	0-4	0.38
*	6-0	0.58

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

delete 0-7 and add to MST

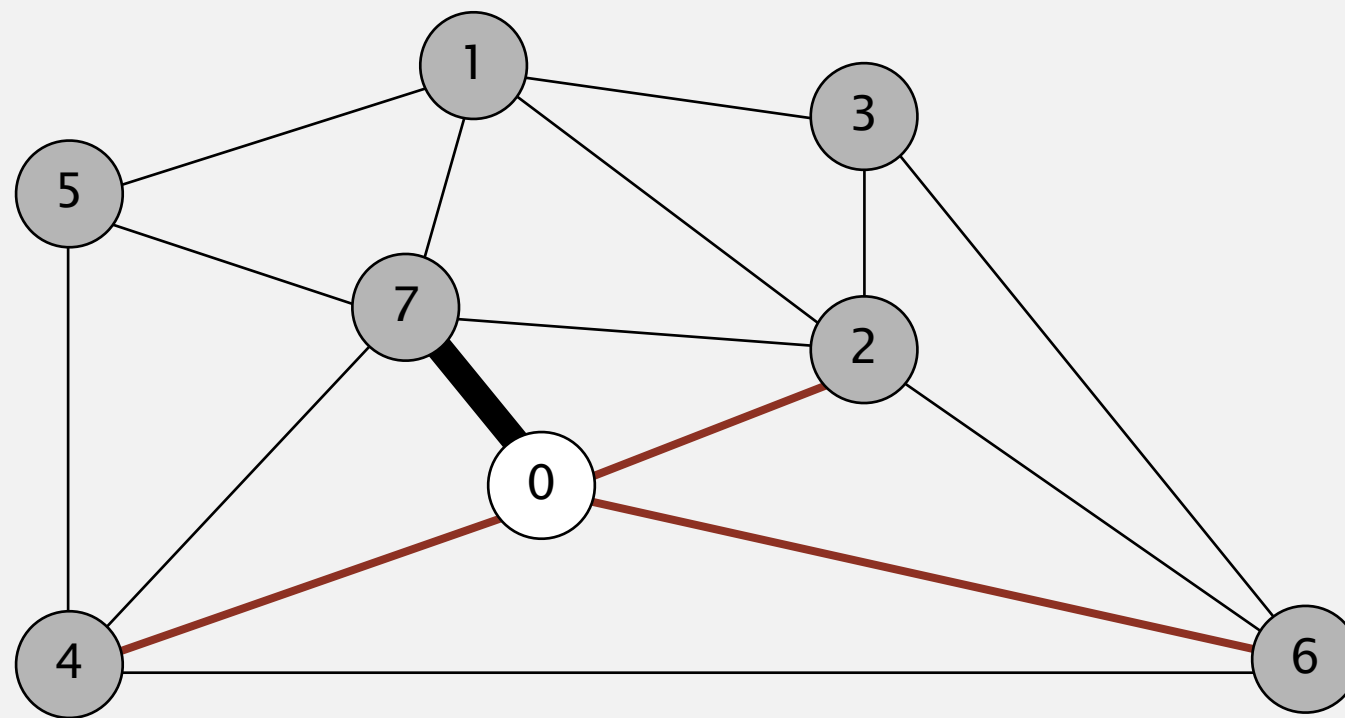


edges on PQ
(sorted by weight)

0-7	0.16
0-2	0.26
0-4	0.38
6-0	0.58

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



edges on PQ
(sorted by weight)

0-2	0.26
0-4	0.38
6-0	0.58

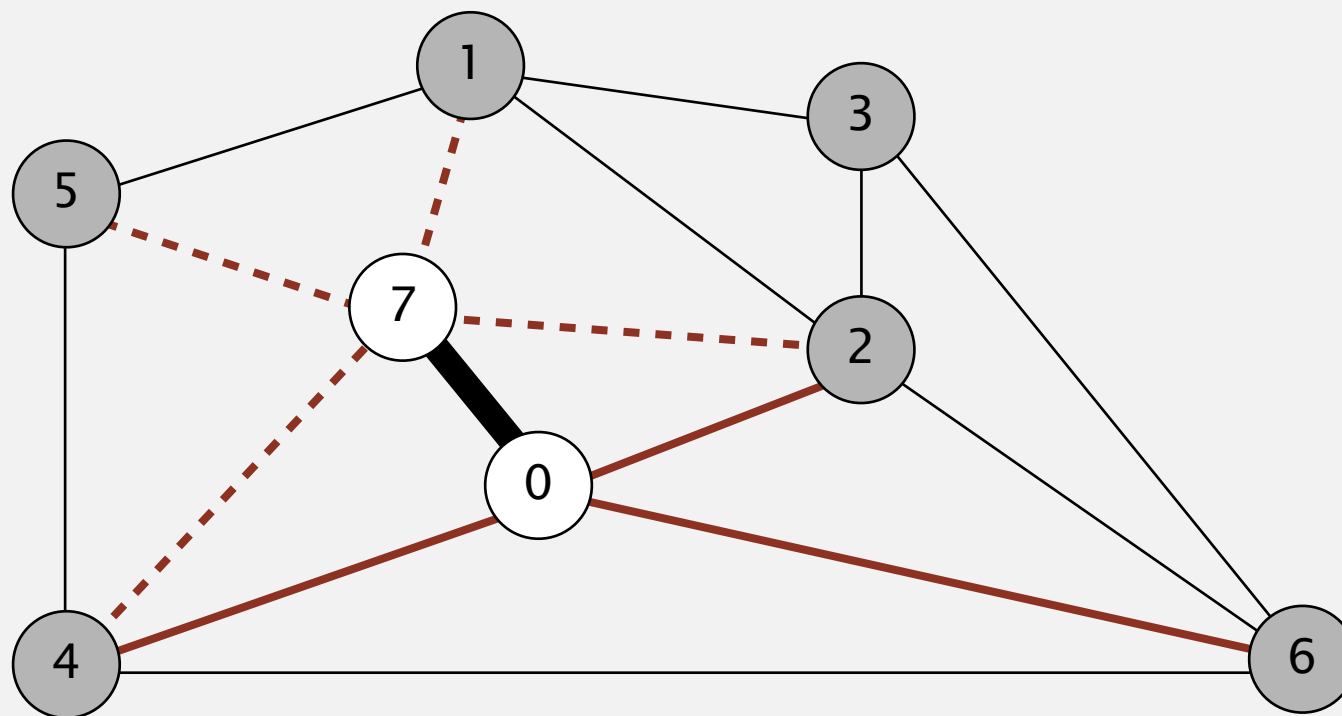
MST edges

0-7

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

add to PQ all edges incident to 7



MST edges

0-7

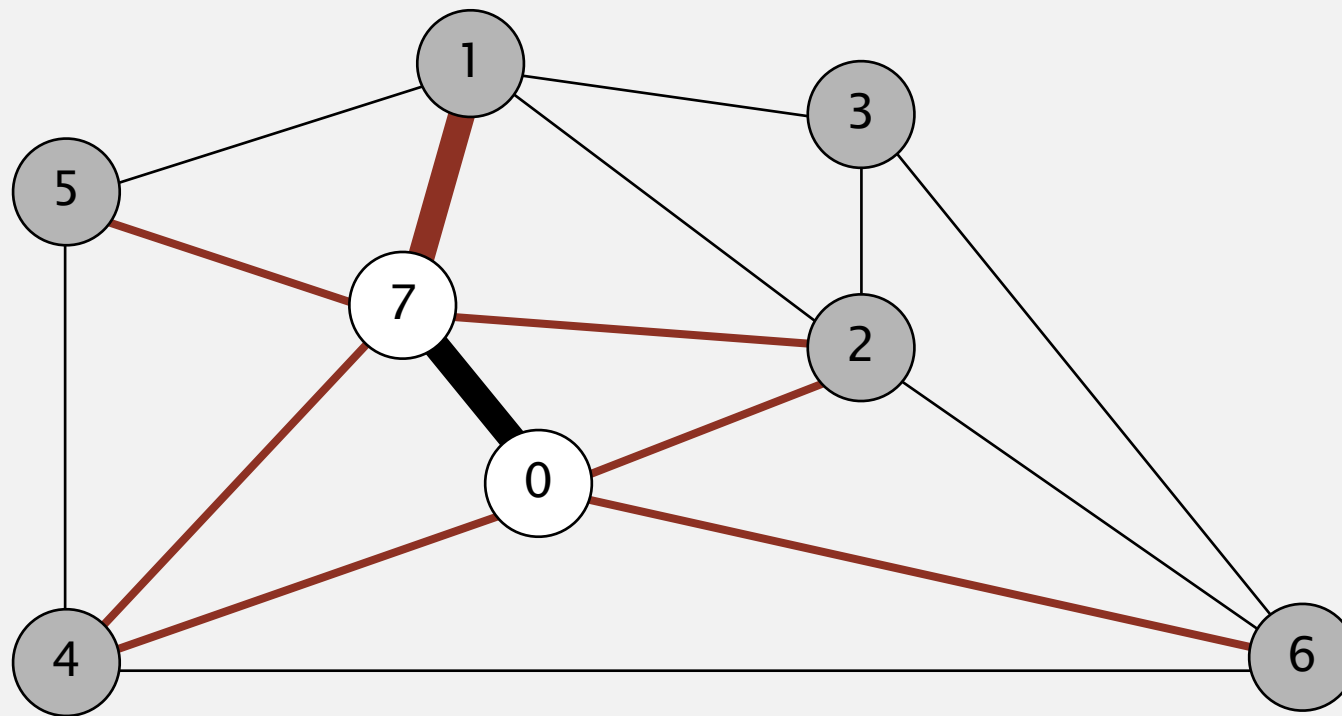
edges on PQ
(sorted by weight)

*	1-7	0.19
	0-2	0.26
*	5-7	0.28
*	2-7	0.34
*	4-7	0.37
	0-4	0.38
	6-0	0.58

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

delete 1-7 and add to MST



MST edges

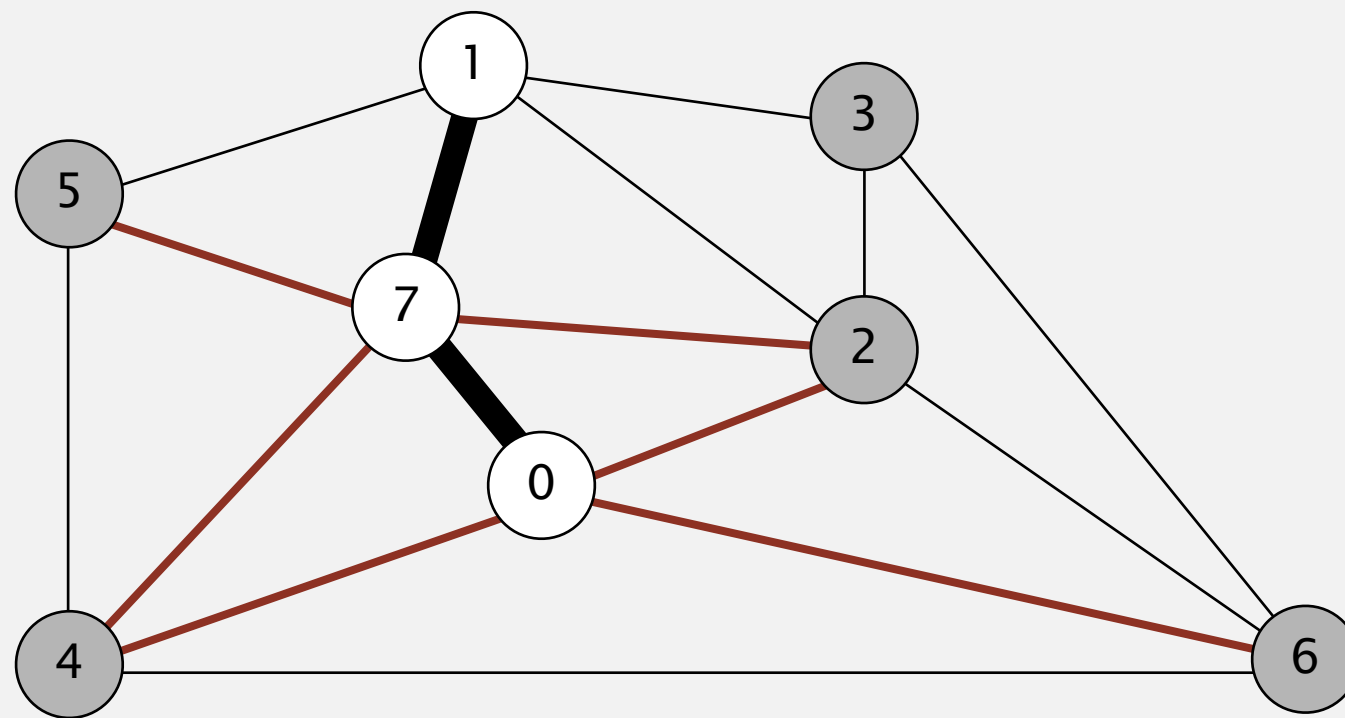
0-7

edges on PQ
(sorted by weight)

1-7	0.19
0-2	0.26
5-7	0.28
2-7	0.34
4-7	0.37
0-4	0.38
6-0	0.58

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



MST edges

0-7 1-7

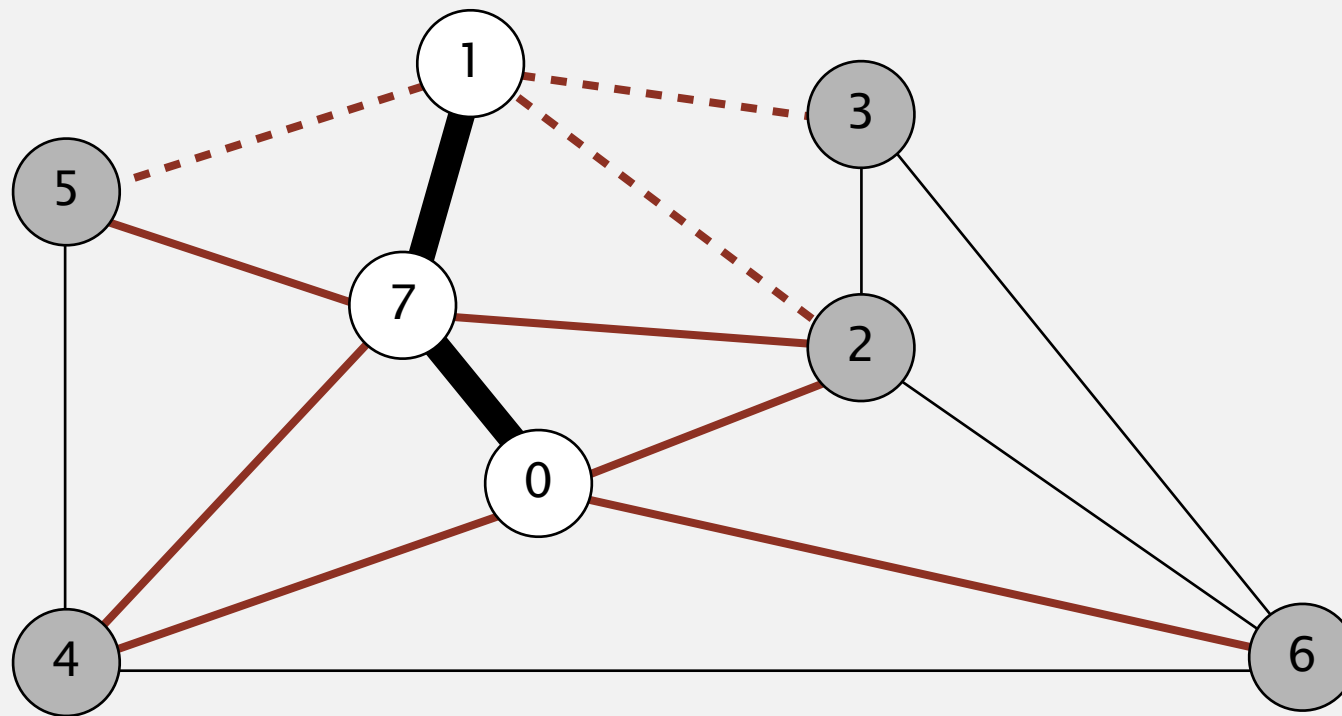
edges on PQ
(sorted by weight)

0-2	0.26
5-7	0.28
2-7	0.34
4-7	0.37
0-4	0.38
6-0	0.58

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

add to PQ all edges incident to 1



MST edges

0-7 1-7

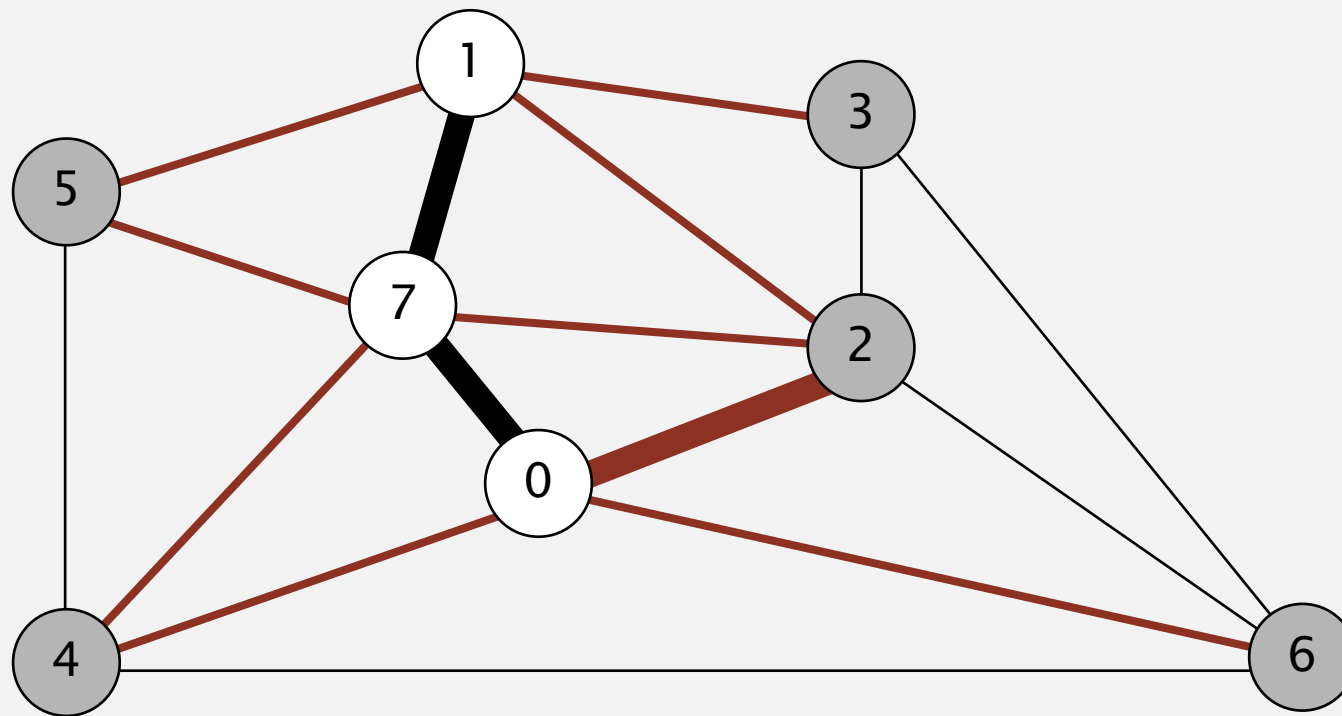
edges on PQ
(sorted by weight)

0-2	0.26
5-7	0.28
* 1-3	0.29
* 1-5	0.32
2-7	0.34
* 1-2	0.36
4-7	0.37
0-4	0.38
6-0	0.58

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

delete edge 0-2 and add to MST



MST edges

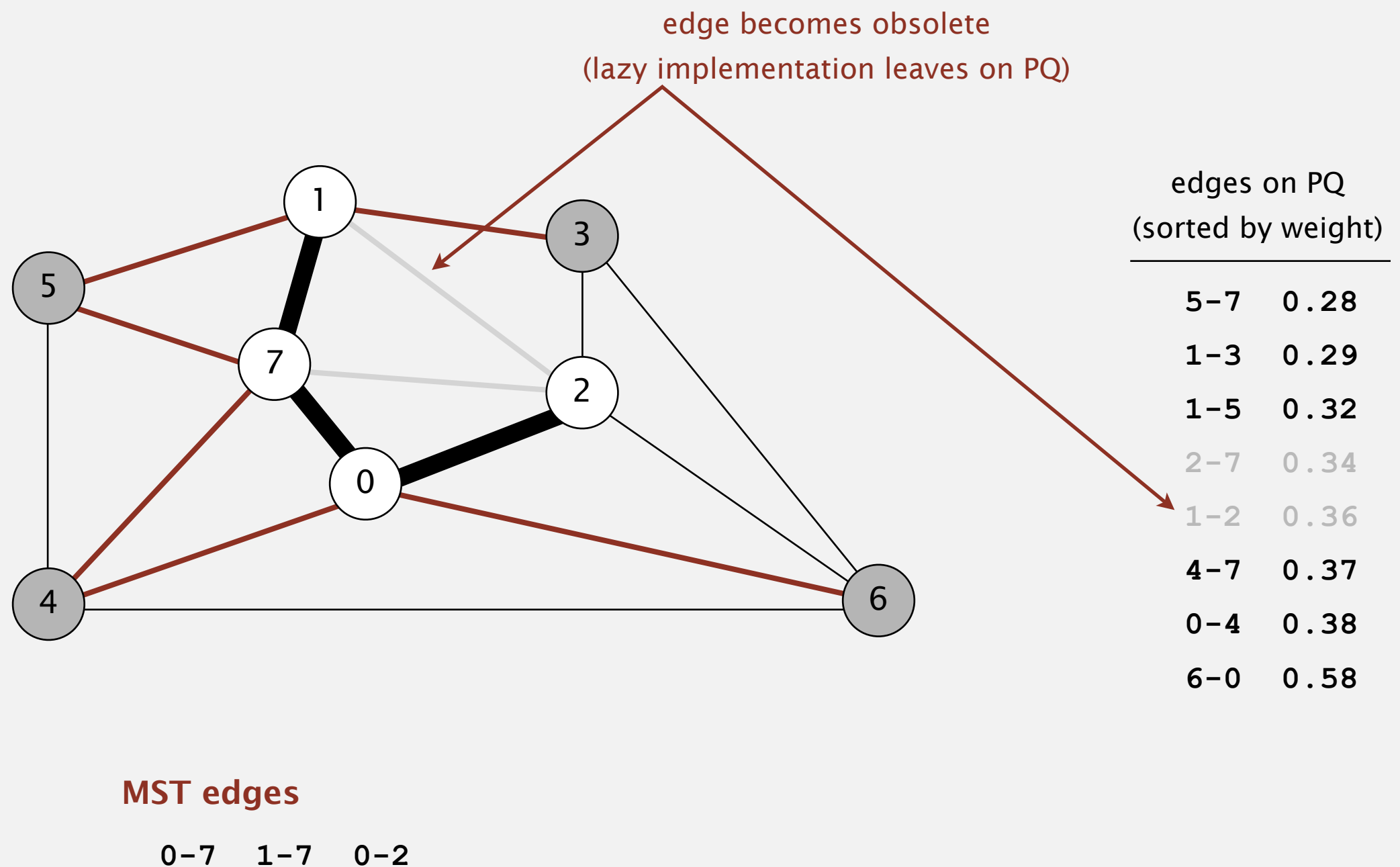
0-7 1-7

edges on PQ
(sorted by weight)

0-2	0.26
5-7	0.28
1-3	0.29
1-5	0.32
2-7	0.34
1-2	0.36
4-7	0.37
0-4	0.38
6-0	0.58

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

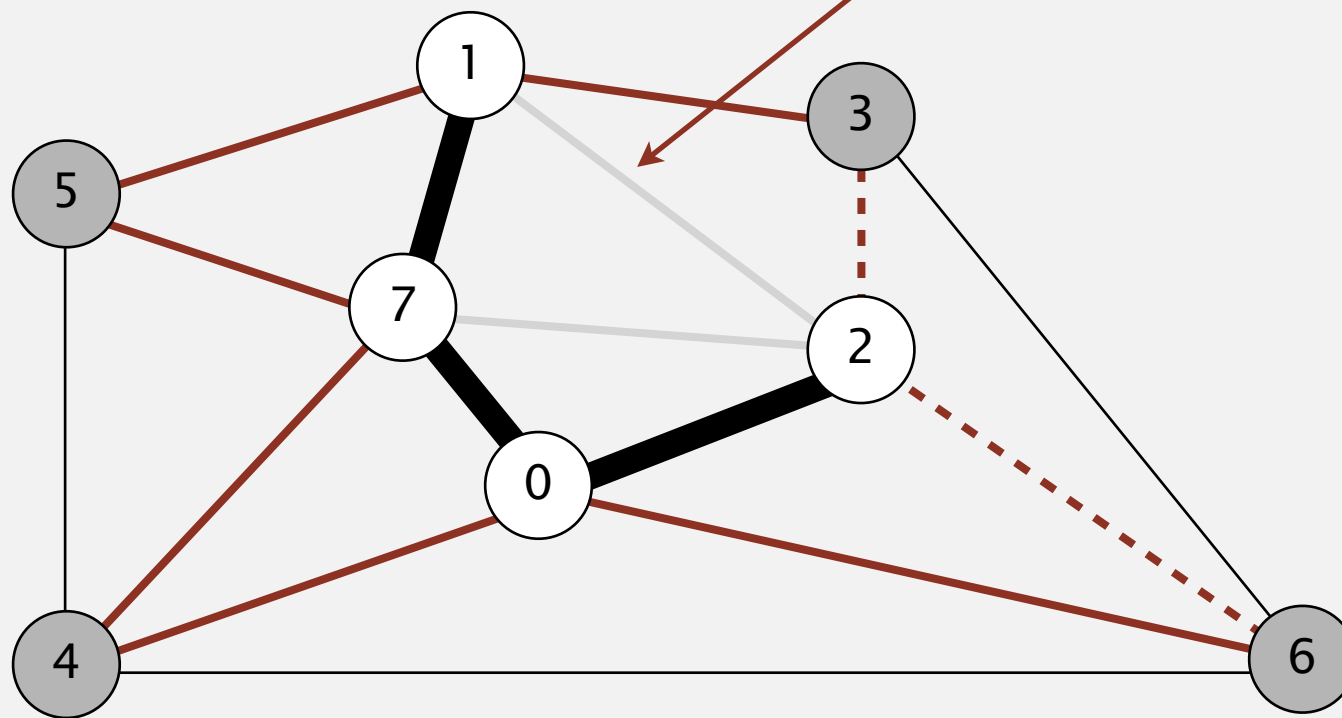


Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

add to PQ all edges incident to 2

no need to add edge 1-2 or 2-7
because it's already obsolete



MST edges

0-7 1-7 0-2

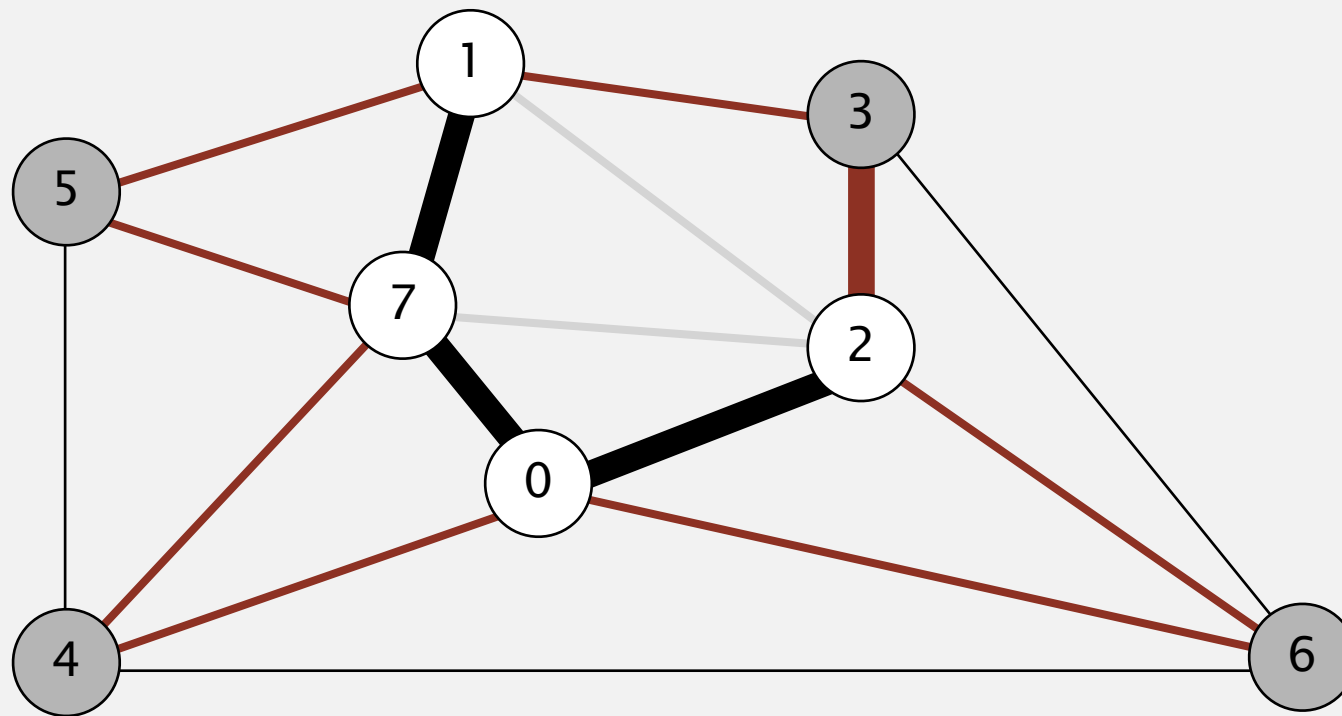
edges on PQ
(sorted by weight)

*	2-3	0.17
	5-7	0.28
	1-3	0.29
	1-5	0.32
	2-7	0.34
	1-2	0.36
	4-7	0.37
	0-4	0.38
*	6-2	0.40
	6-0	0.58

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

delete 2-3 and add to MST



MST edges

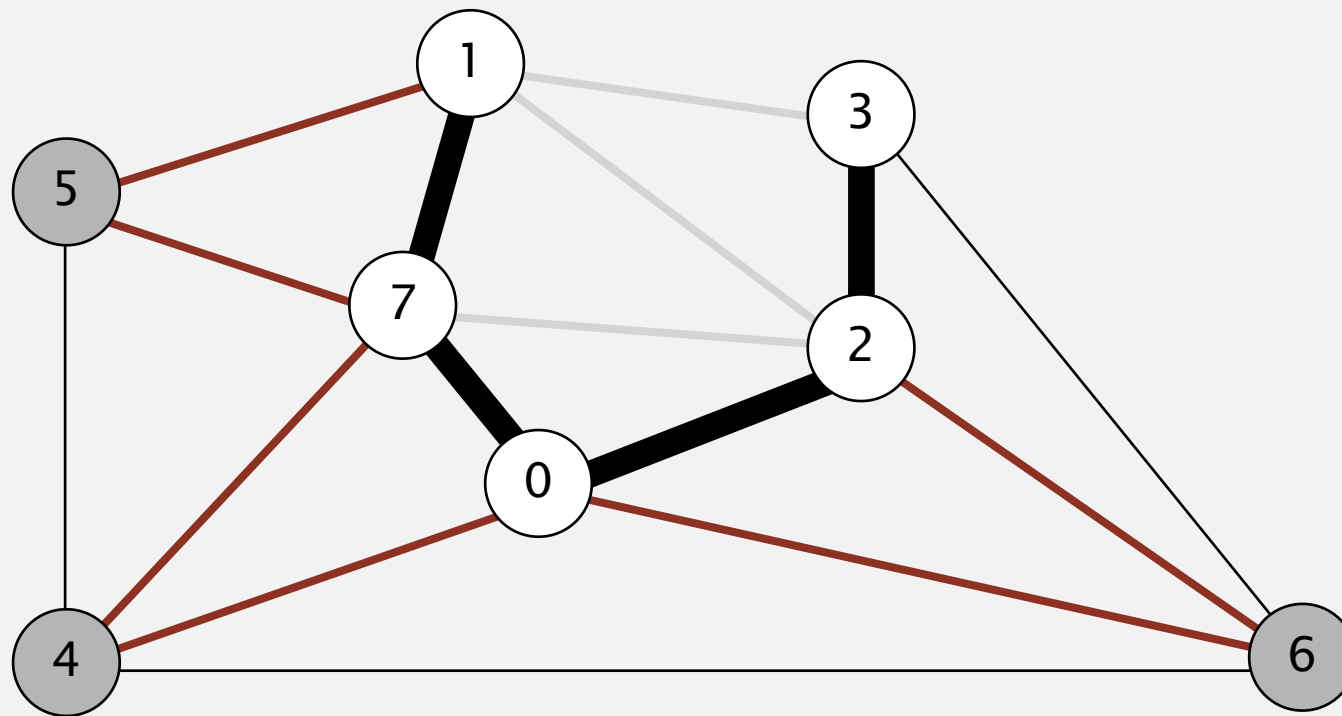
0-7 1-7 0-2

edges on PQ
(sorted by weight)

*	2-3	0.17
	5-7	0.28
	1-3	0.29
	1-5	0.32
	2-7	0.34
	1-2	0.36
	4-7	0.37
	0-4	0.38
*	6-2	0.40
	6-0	0.58

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



MST edges

0-7 1-7 0-2 2-3

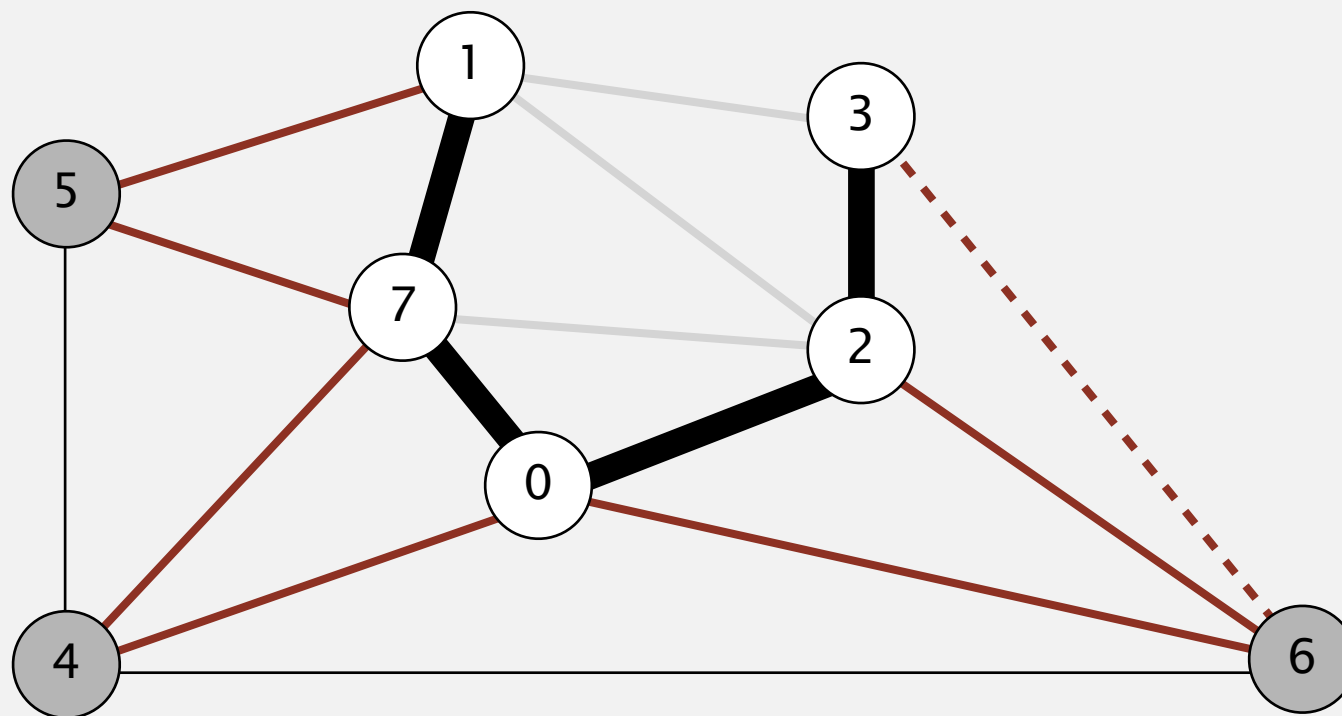
edges on PQ
(sorted by weight)

5-7	0.28
1-3	0.29
1-5	0.32
2-7	0.34
1-2	0.36
4-7	0.37
0-4	0.38
6-2	0.40
6-0	0.58

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

add to PQ all edges incident to 3



MST edges

0-7 1-7 0-2 2-3

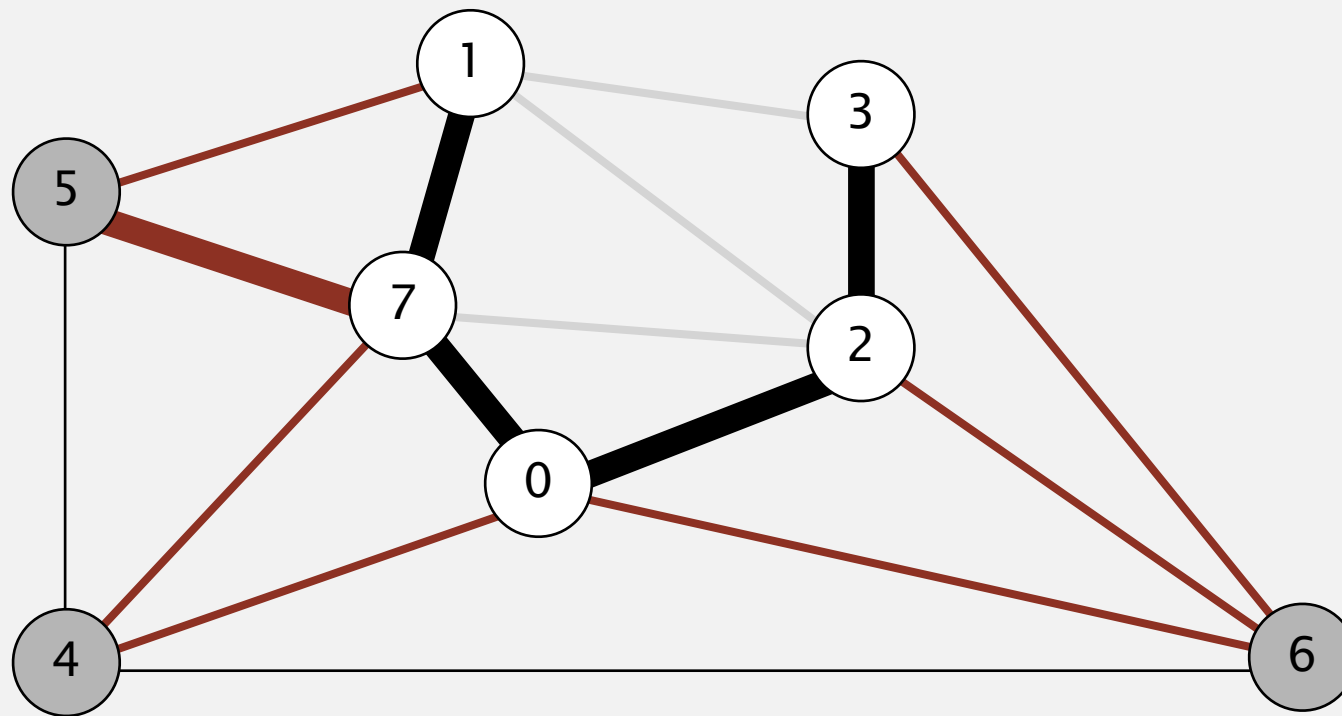
edges on PQ
(sorted by weight)

5-7	0.28
1-3	0.29
1-5	0.32
2-7	0.34
1-2	0.36
4-7	0.37
0-4	0.38
6-2	0.40
* 3-6	0.52
6-0	0.58

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

delete 5-7 and add to MST



MST edges

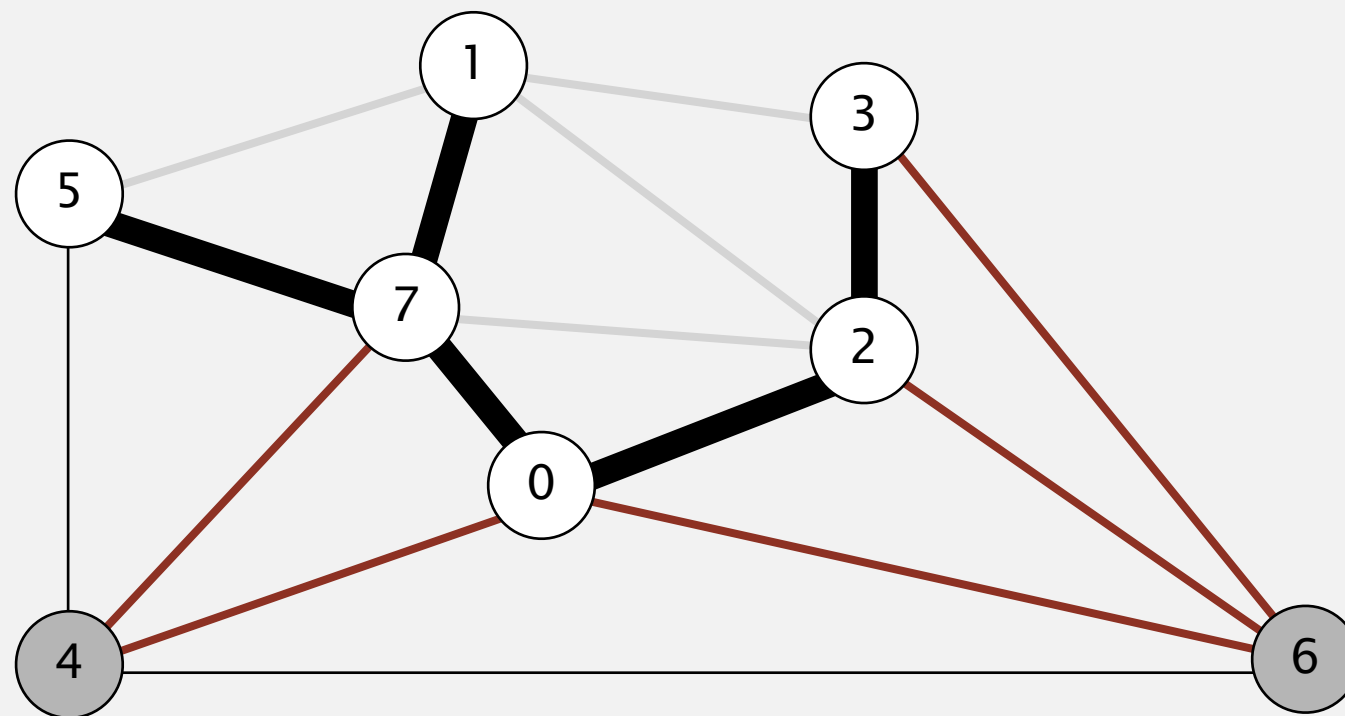
0-7 1-7 0-2 2-3

edges on PQ
(sorted by weight)

5-7	0.28
1-3	0.29
1-5	0.32
2-7	0.34
1-2	0.36
4-7	0.37
0-4	0.38
6-2	0.40
3-6	0.52
6-0	0.58

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



MST edges

0-7 1-7 0-2 2-3 5-7

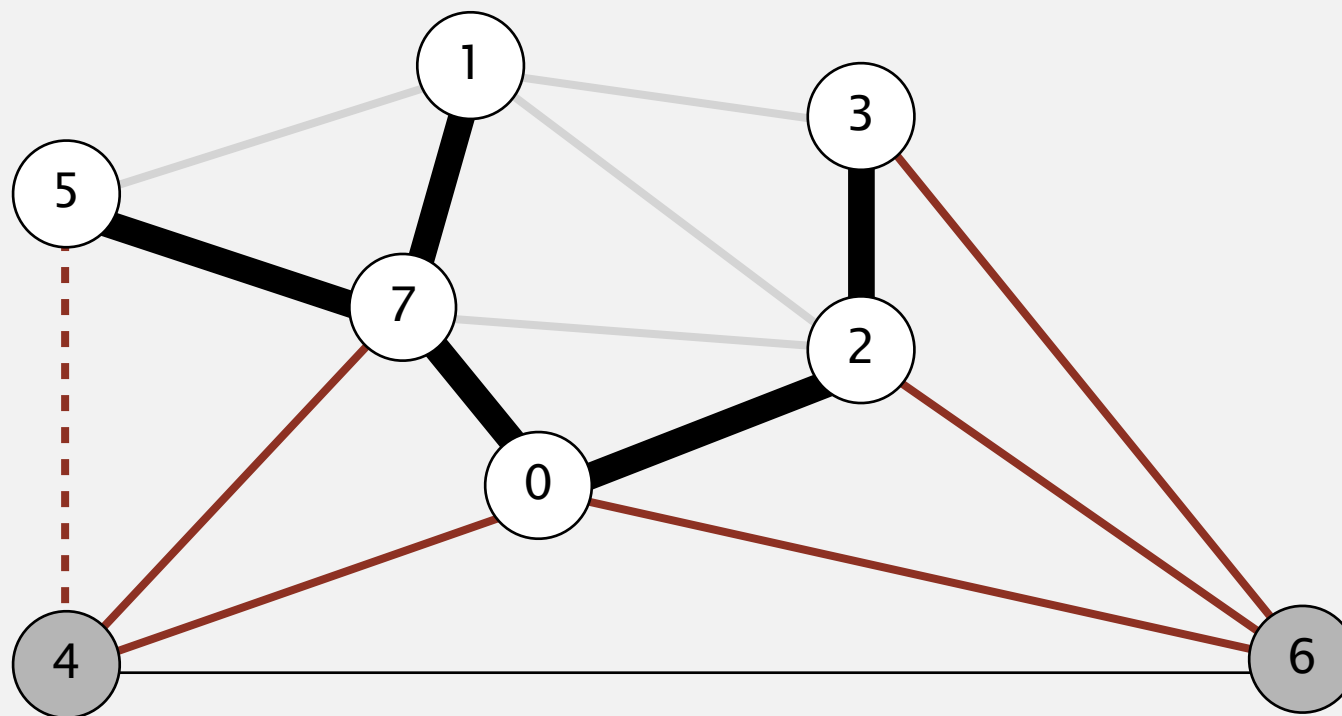
edges on PQ
(sorted by weight)

1-3	0.29
1-5	0.32
2-7	0.34
1-2	0.36
4-7	0.37
0-4	0.38
6-2	0.40
3-6	0.52
6-0	0.58

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

add to PQ all edges incident to 5



MST edges

0-7 1-7 0-2 2-3 5-7

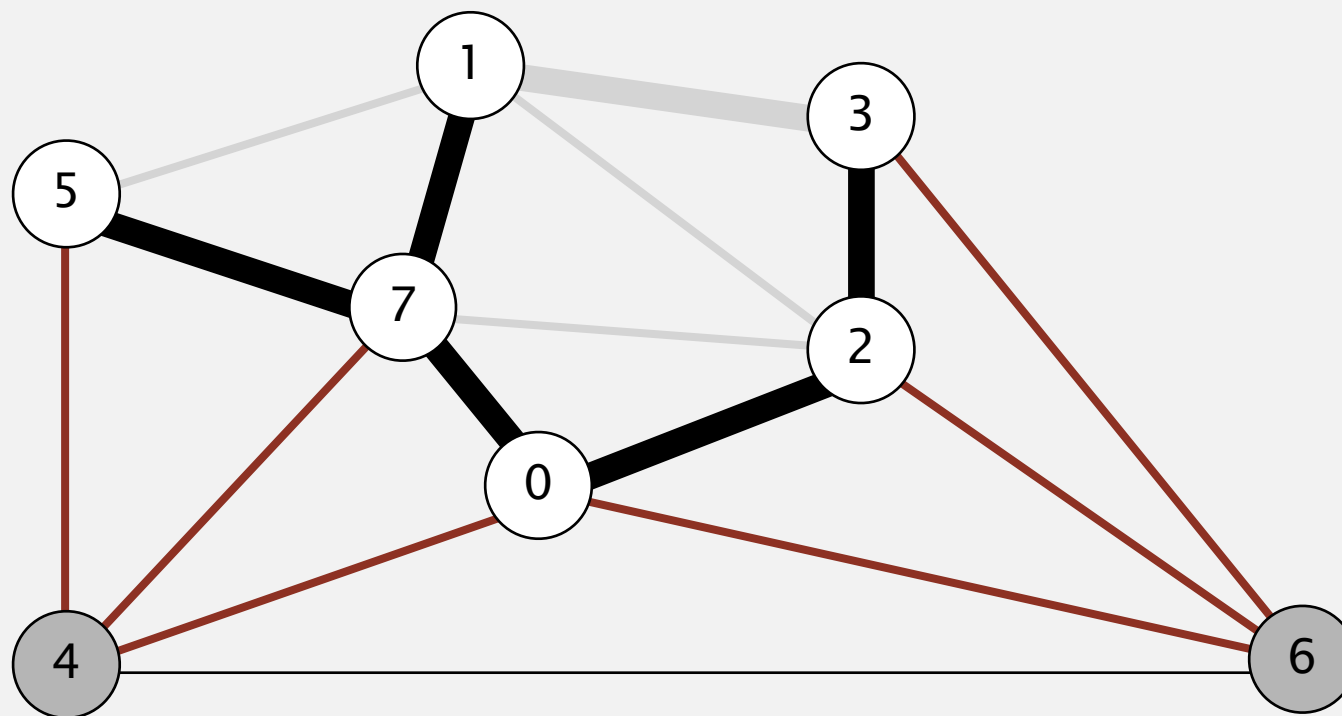
edges on PQ
(sorted by weight)

1-3	0.29
1-5	0.32
2-7	0.34
* 4-5	0.35
1-2	0.36
4-7	0.37
0-4	0.38
6-2	0.40
3-6	0.52
6-0	0.58

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

delete 1-3 and discard obsolete edge



MST edges

0-7 1-7 0-2 2-3 5-7

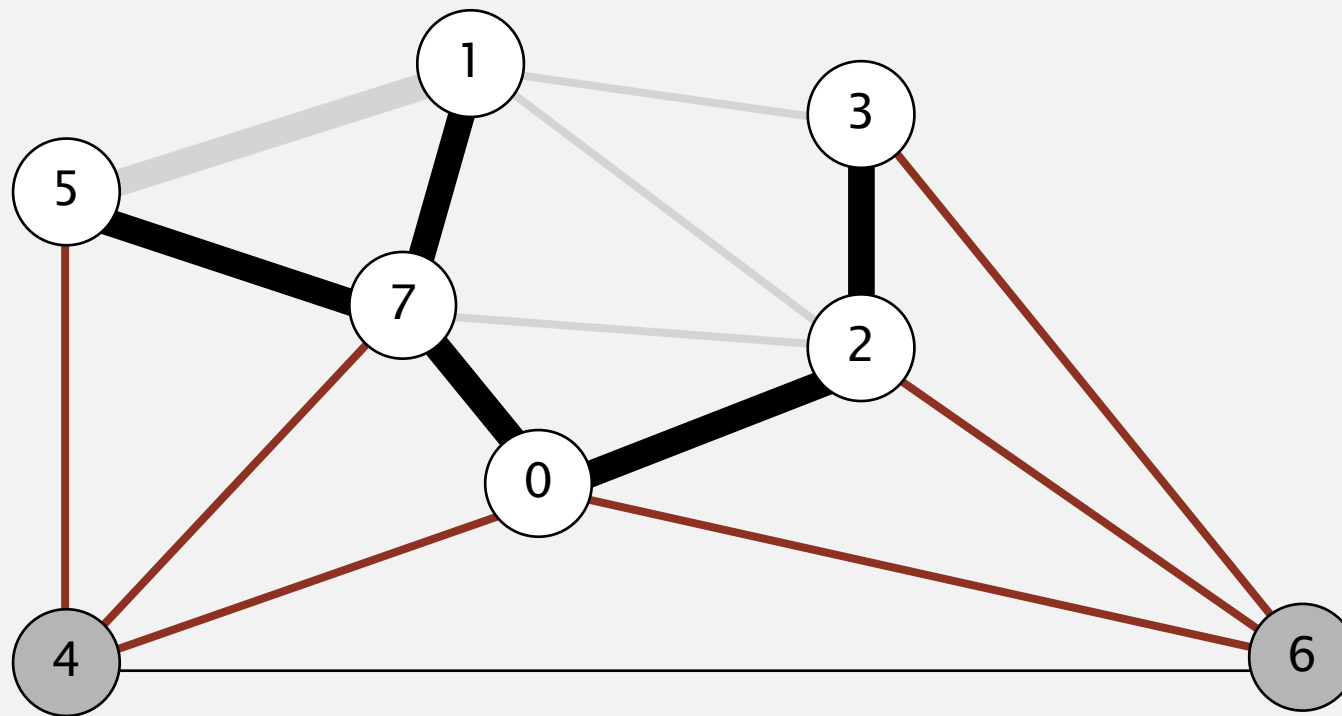
edges on PQ
(sorted by weight)

1-3	0.29
1-5	0.32
2-7	0.34
4-5	0.35
1-2	0.36
4-7	0.37
0-4	0.38
6-2	0.40
3-6	0.52
6-0	0.58

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

delete 1-5 and discard obsolete edge



MST edges

0-7 1-7 0-2 2-3 5-7

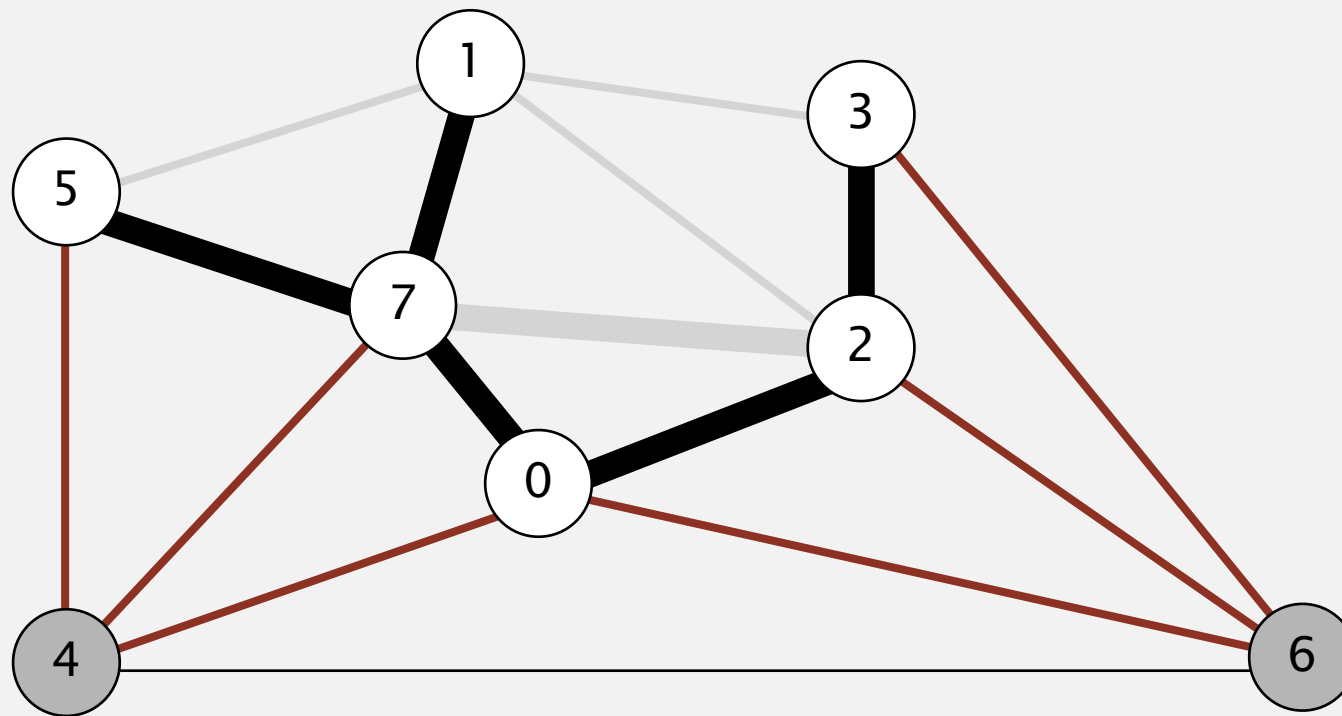
edges on PQ
(sorted by weight)

1-5	0.32
2-7	0.34
4-5	0.35
1-2	0.36
4-7	0.37
0-4	0.38
6-2	0.40
3-6	0.52
6-0	0.58

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

delete 2-7 and discard obsolete edge



edges on PQ
(sorted by weight)

2-7	0.34
4-5	0.35
1-2	0.36
4-7	0.37
0-4	0.38
6-2	0.40
3-6	0.52
6-0	0.58

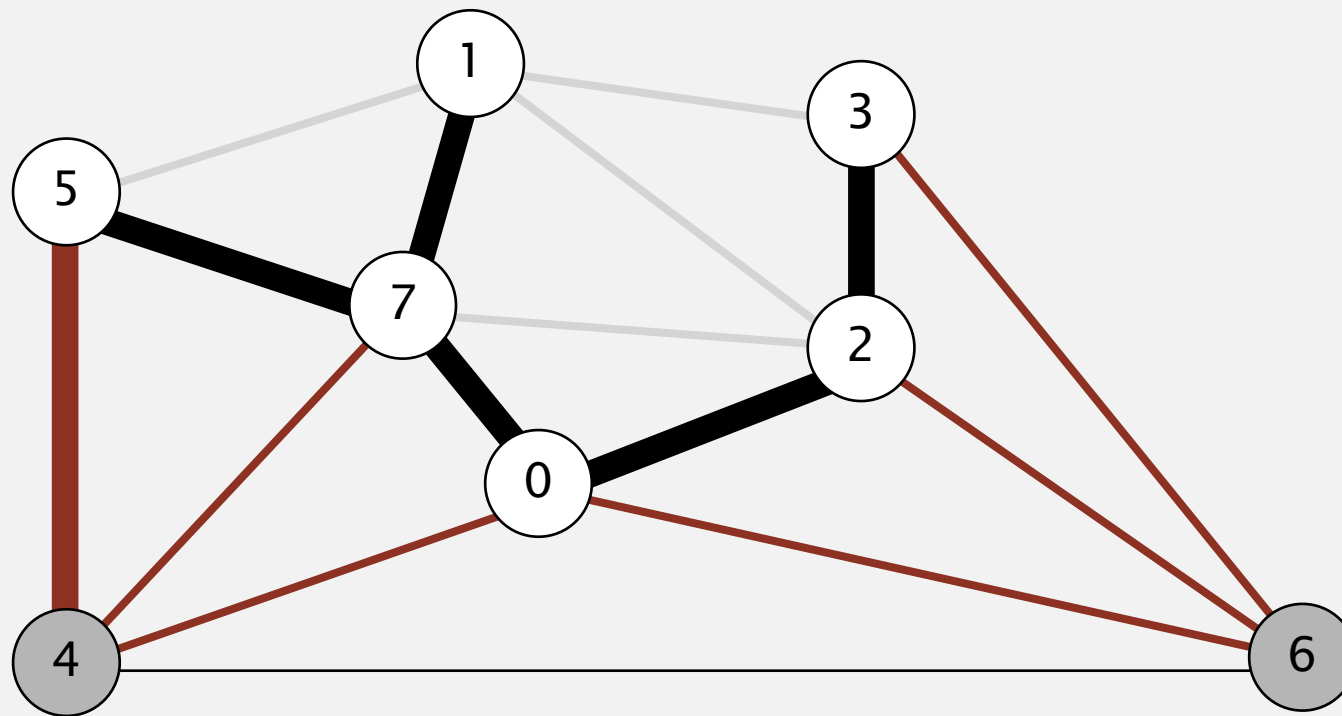
MST edges

0-7 1-7 0-2 2-3 5-7

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

delete 4-5 and add to MST



edges on PQ
(sorted by weight)

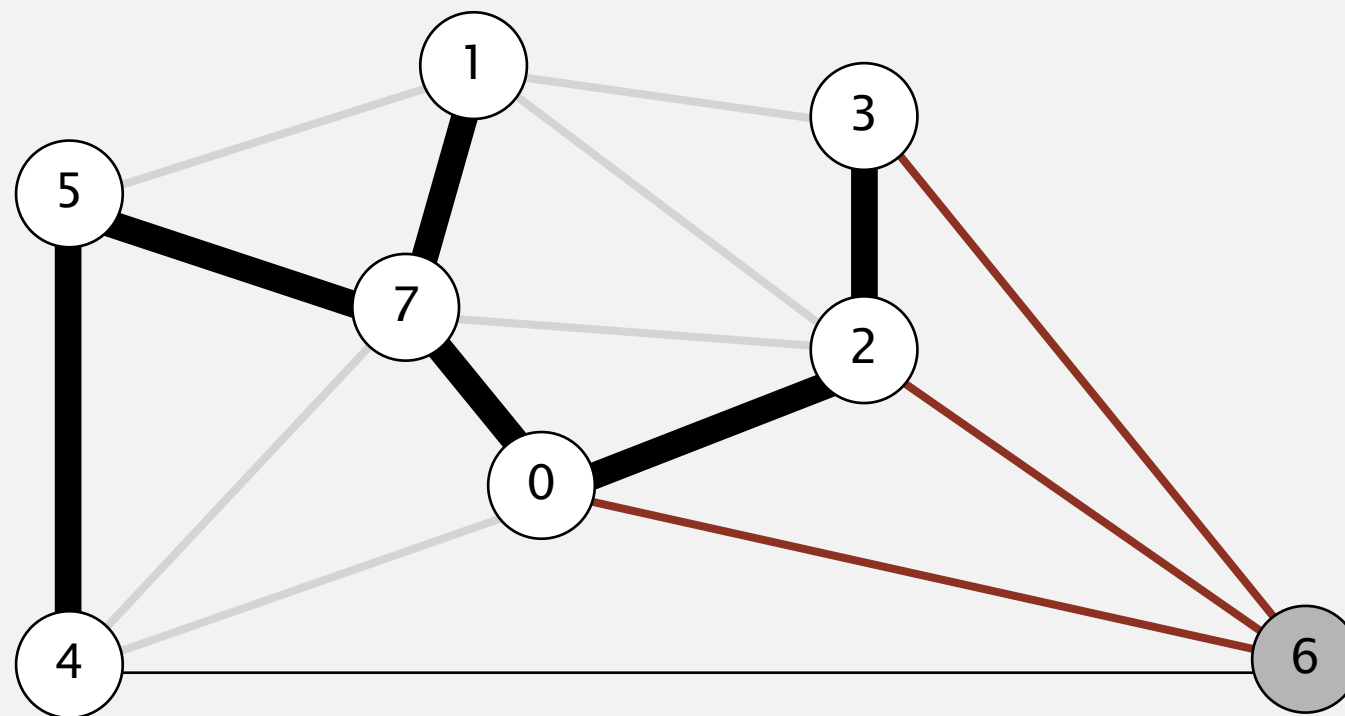
4-5	0.35
1-2	0.36
4-7	0.37
0-4	0.38
6-2	0.40
3-6	0.52
6-0	0.58

MST edges

0-7 1-7 0-2 2-3 5-7

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



edges on PQ
(sorted by weight)

1-2	0.36
4-7	0.37
0-4	0.38
6-2	0.40
3-6	0.52
6-0	0.58

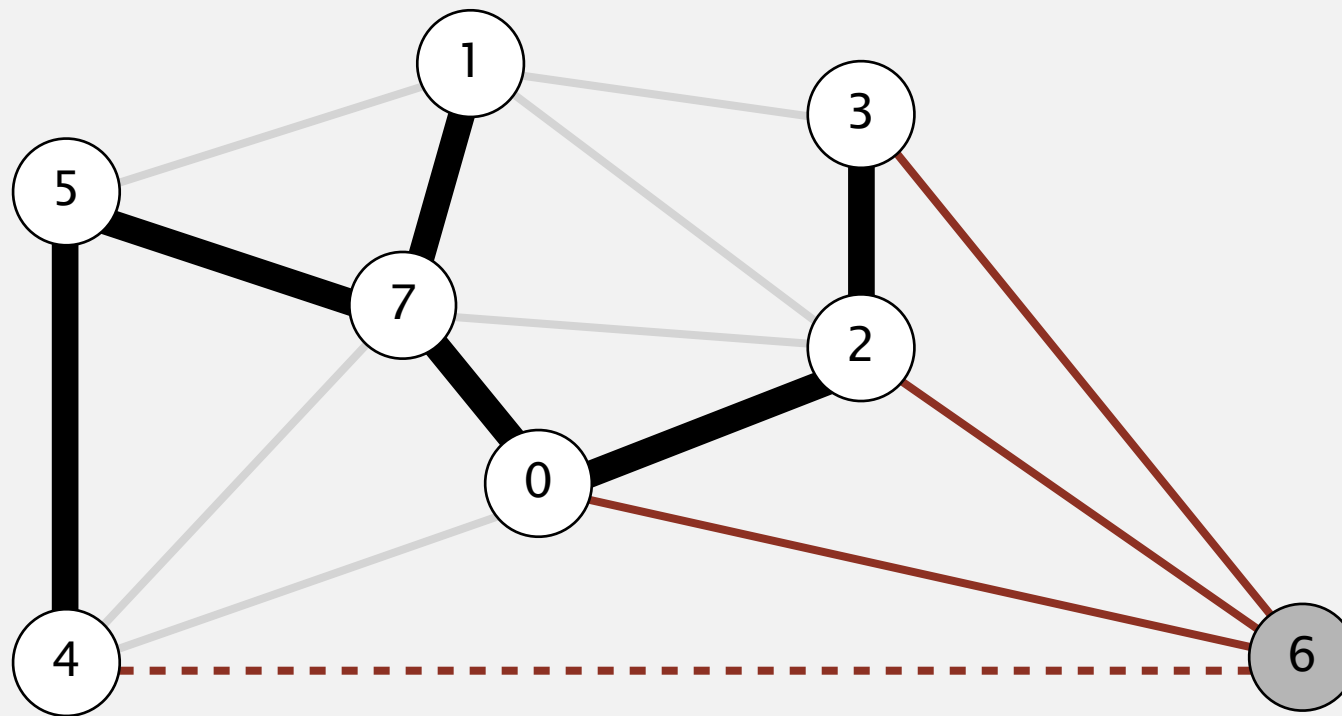
MST edges

0-7 1-7 0-2 2-3 5-7 4-5

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

add to PQ all edges incident to 4



edges on PQ
(sorted by weight)

1-2	0.36
4-7	0.37
0-4	0.38
6-2	0.40
3-6	0.52
6-0	0.58
* 6-4	0.93

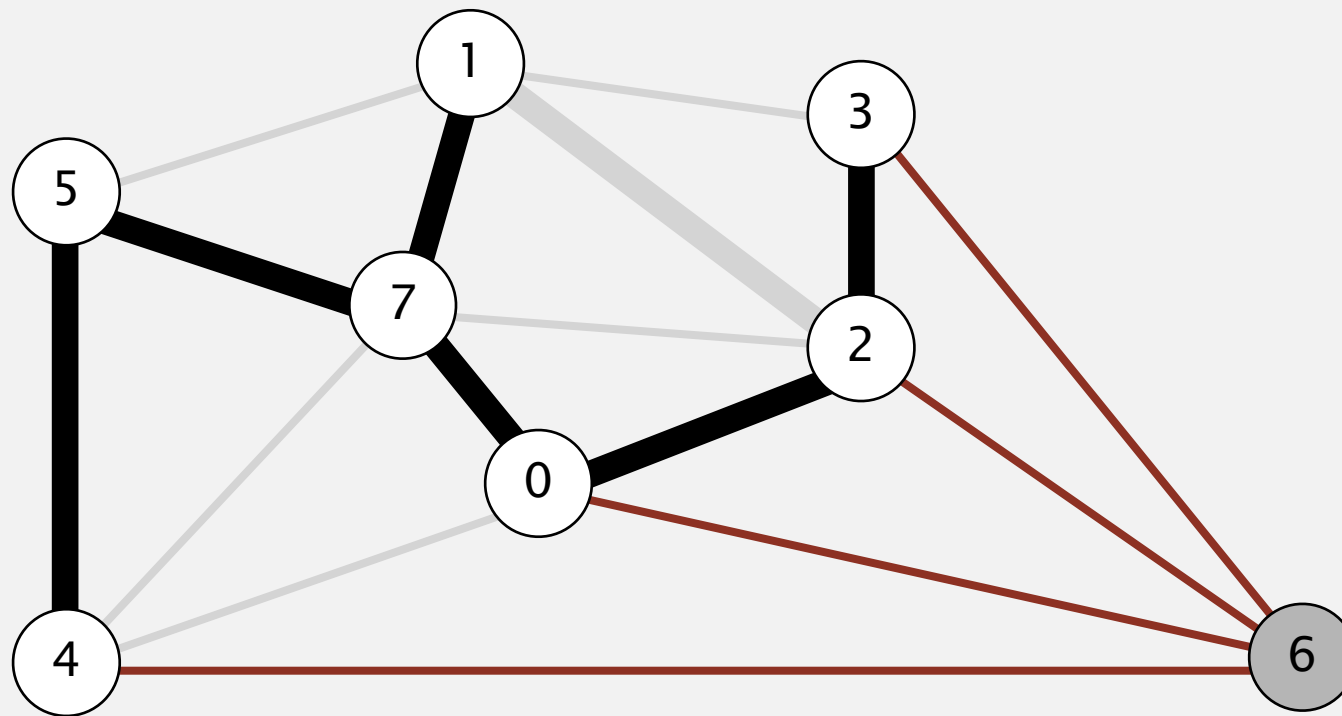
MST edges

0-7 1-7 0-2 2-3 5-7 4-5

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

delete 1-2 and discard obsolete edge



edges on PQ
(sorted by weight)

1-2	0.36
4-7	0.37
0-4	0.38
6-2	0.40
3-6	0.52
6-0	0.58
6-4	0.93

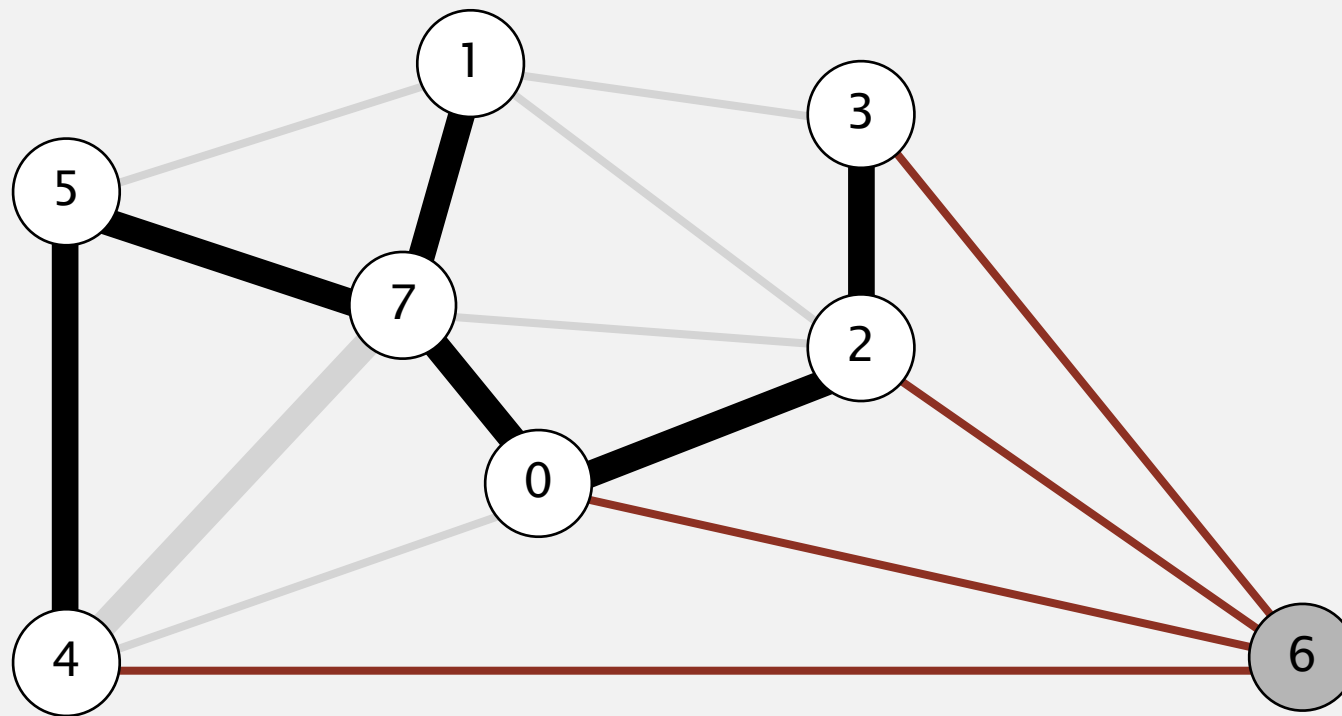
MST edges

0-7 1-7 0-2 2-3 5-7 4-5

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

delete 4-7 and discard obsolete edge



edges on PQ
(sorted by weight)

4-7	0.37
0-4	0.38
6-2	0.40
3-6	0.52
6-0	0.58
6-4	0.93

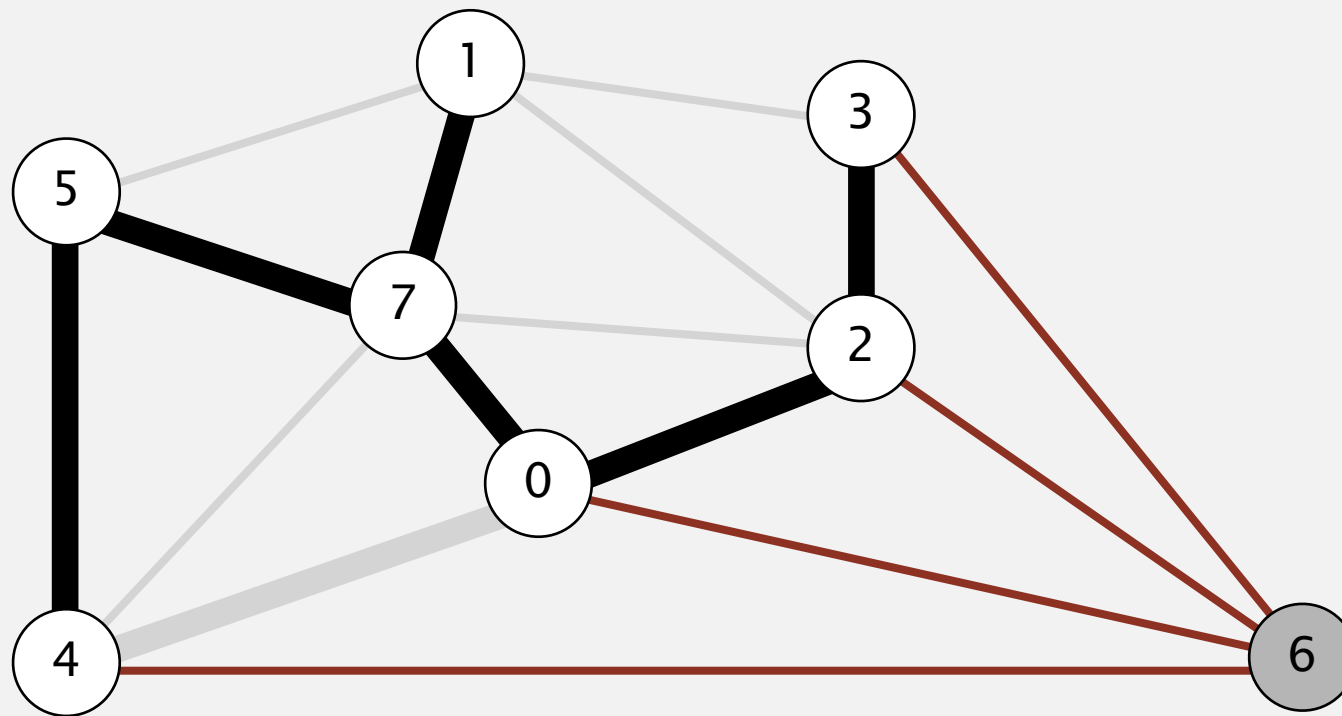
MST edges

0-7 1-7 0-2 2-3 5-7 4-5

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

delete 0-4 and discard obsolete edge



edges on PQ
(sorted by weight)

0-4	0.38
6-2	0.40
3-6	0.52
6-0	0.58
6-4	0.93

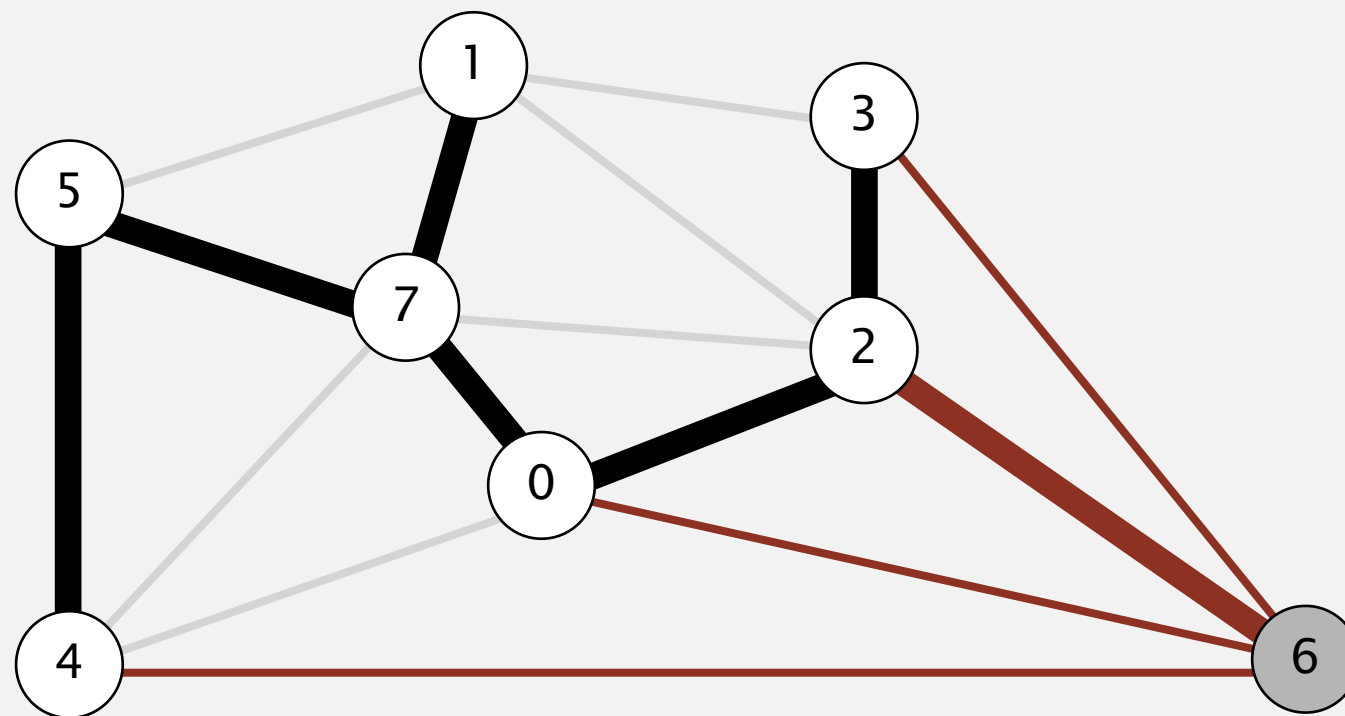
MST edges

0-7 1-7 0-2 2-3 5-7 4-5

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

delete 6-2 and add to MST



edges on PQ
(sorted by weight)

6-2	0.40
3-6	0.52
6-0	0.58
6-4	0.93

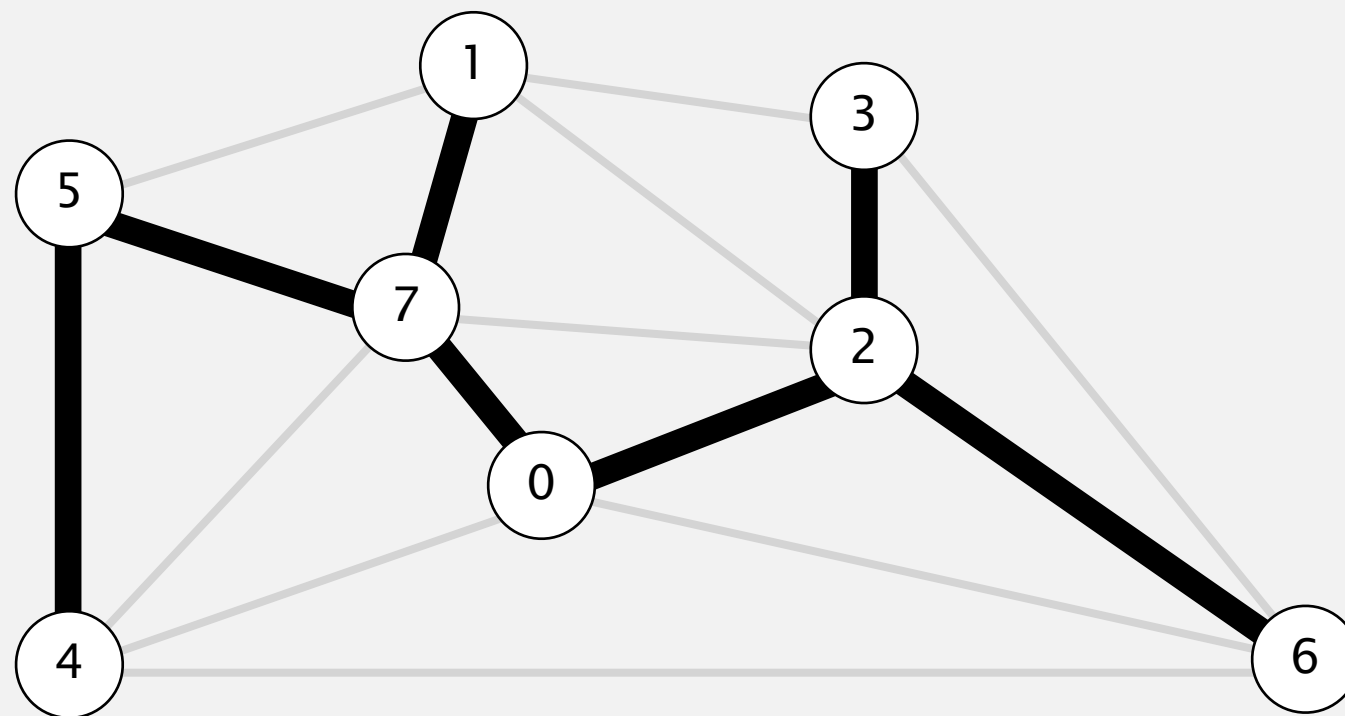
MST edges

0-7 1-7 0-2 2-3 5-7 4-5

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

delete 6-2 and add to MST



edges on PQ
(sorted by weight)

3-6	0.52
6-0	0.58
6-4	0.93

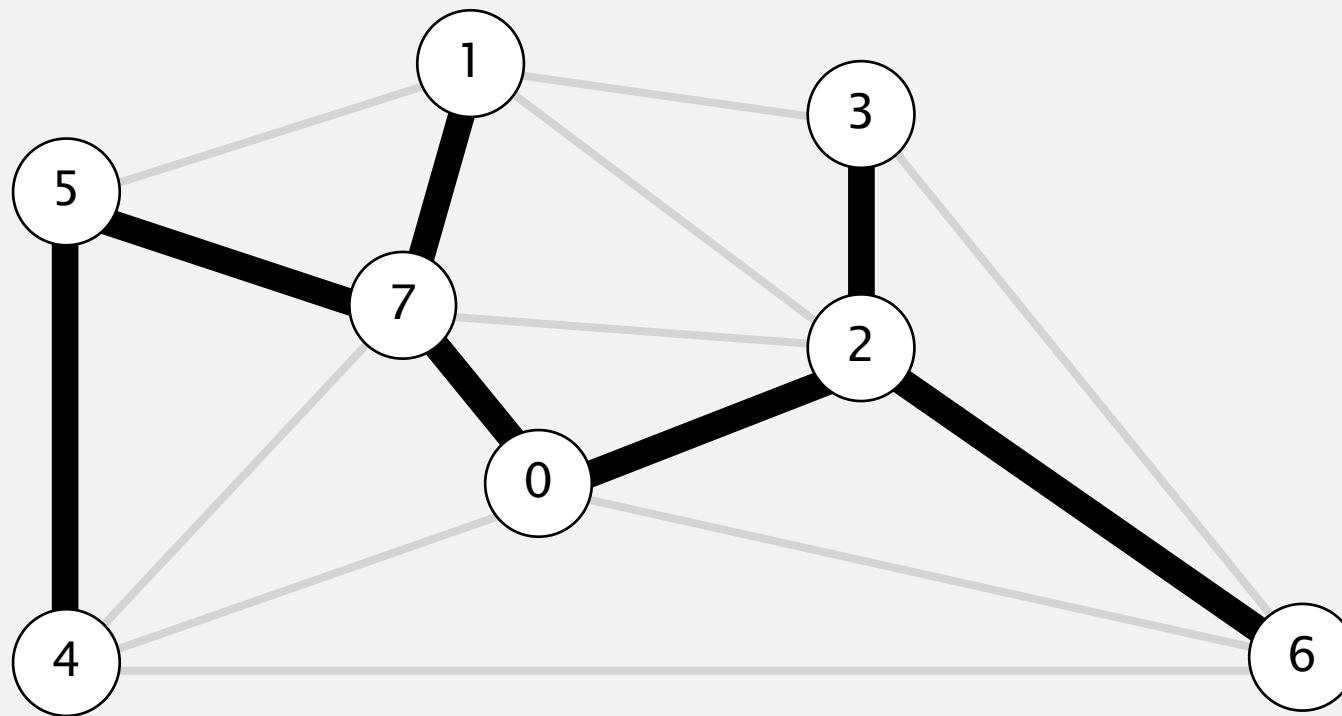
MST edges

0-7 1-7 0-2 2-3 5-7 4-5 6-2

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

stop since $V-1$ edges



edges on PQ
(sorted by weight)

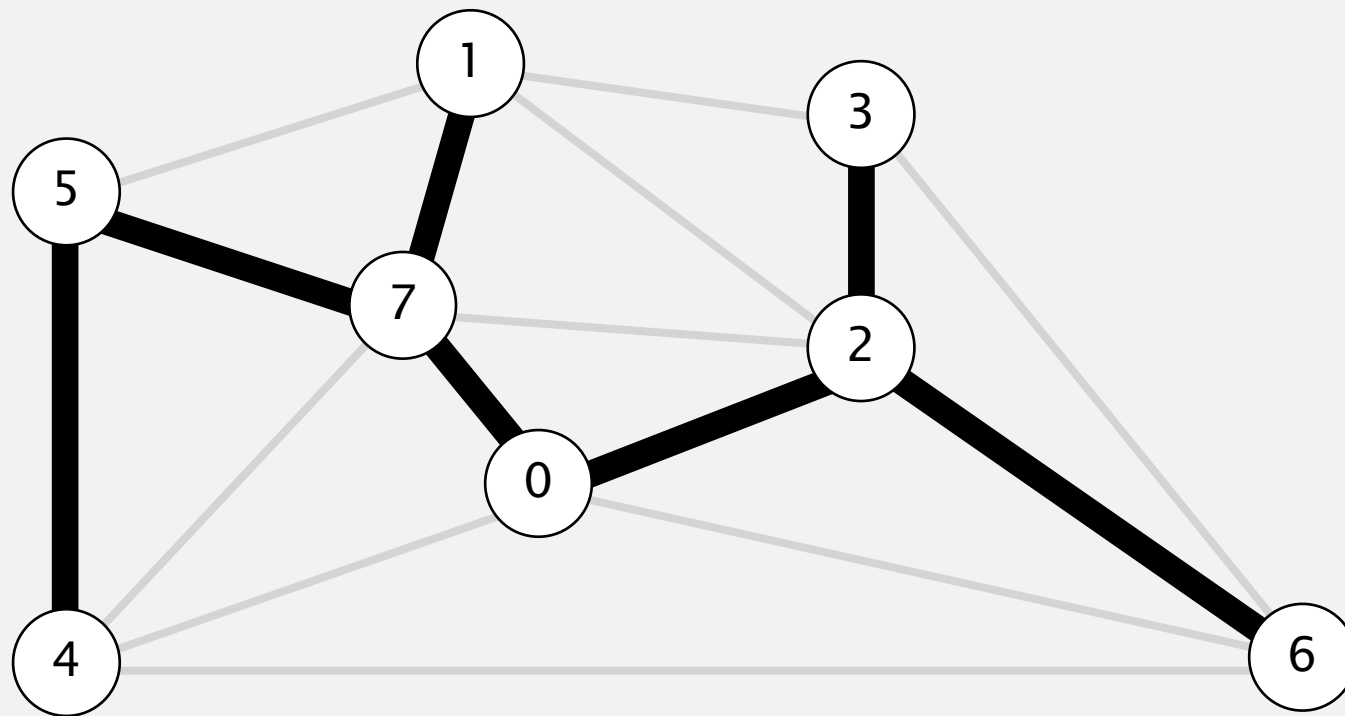
3-6	0.52
6-0	0.58
6-4	0.93

MST edges

0-7 1-7 0-2 2-3 5-7 4-5 6-2

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



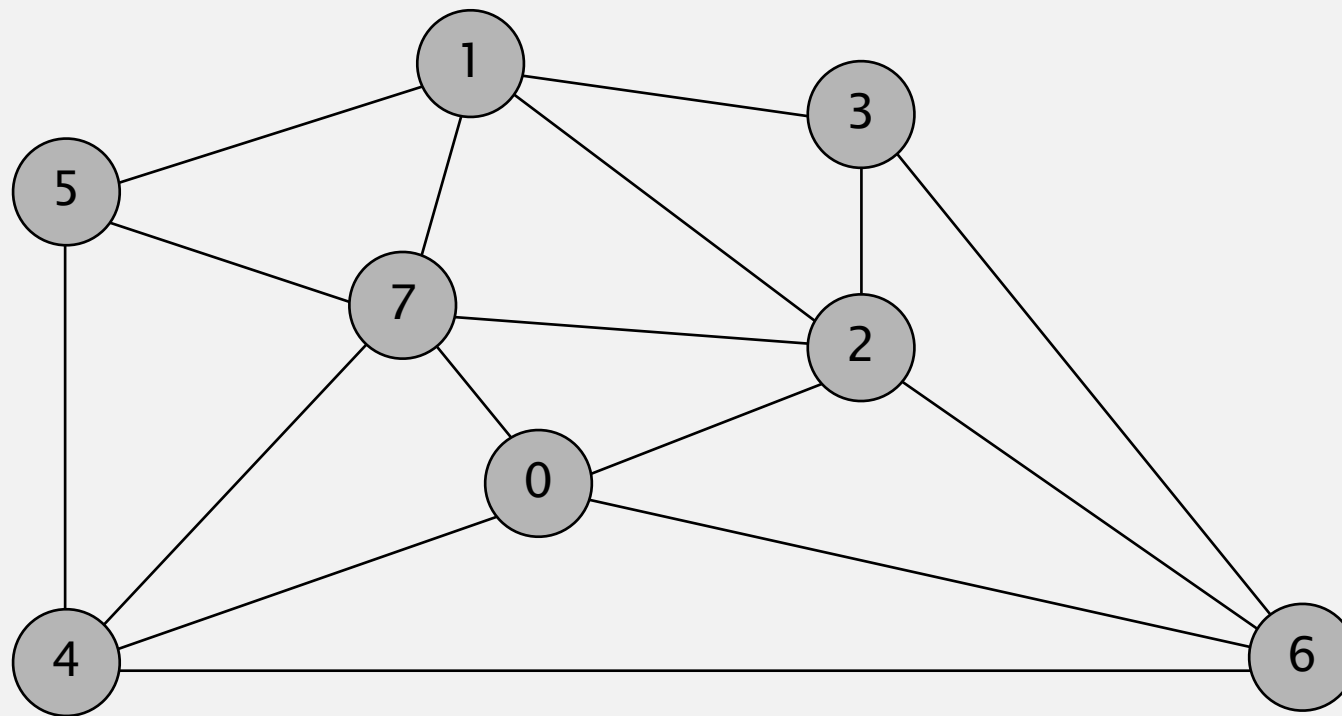
MST edges

0-7 1-7 0-2 2-3 5-7 4-5 6-2

- ▶ Prim's algorithm
- ▶ lazy implementation
- ▶ **eager implementation**

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

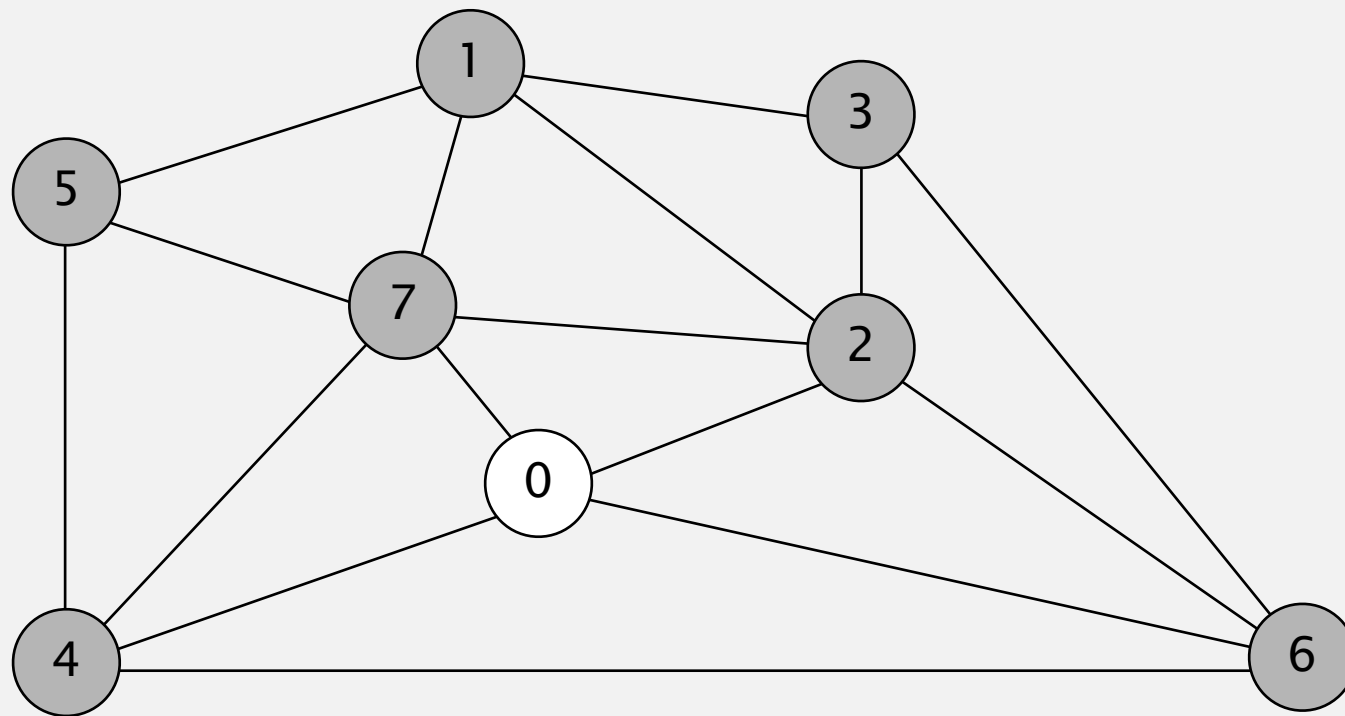


an edge-weighted graph

0-7	0.16
2-3	0.17
1-7	0.19
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5-7	0.28
1-3	0.29
1-5	0.32
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4-5	0.35
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4-7	0.37
0-4	0.38
6-2	0.40
3-6	0.52
6-0	0.58
6-4	0.93

Prim's algorithm

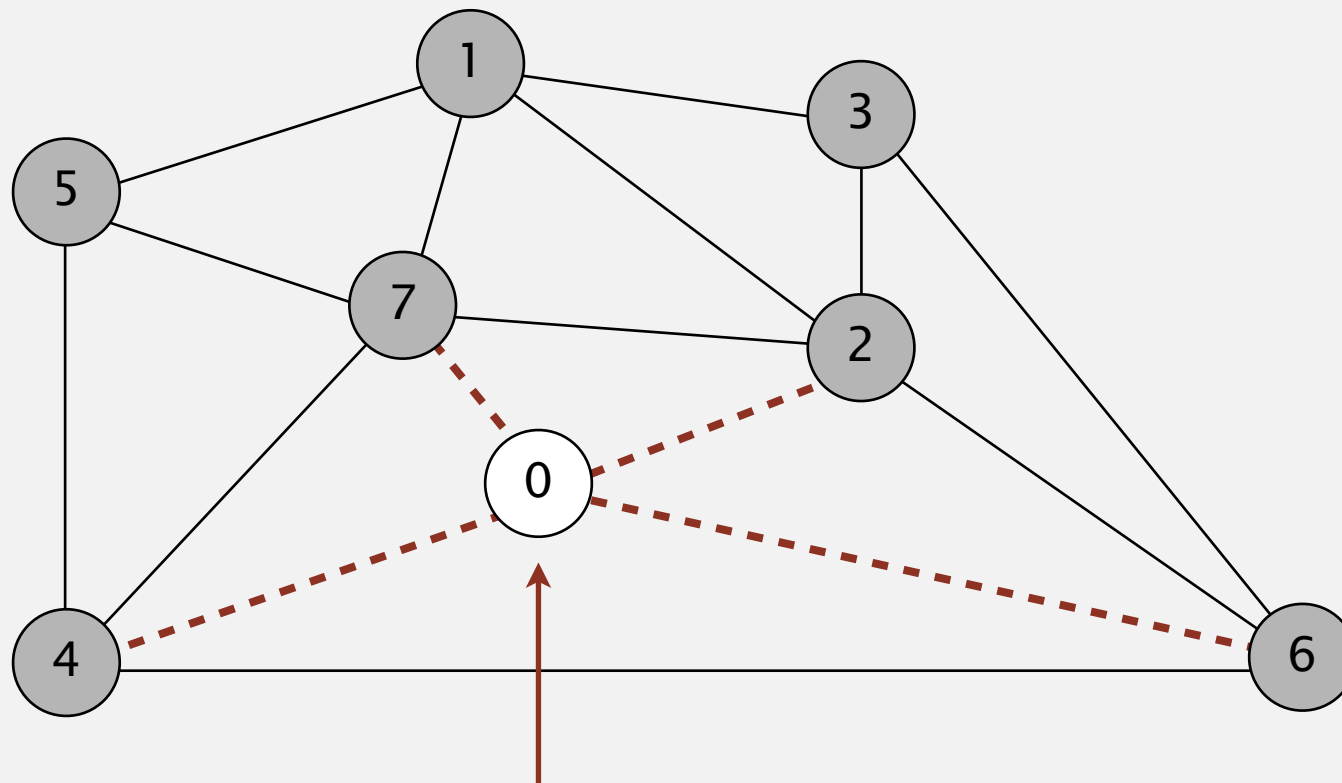
- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



v	edgeTo[]	distTo[]
→ 0	-	-

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



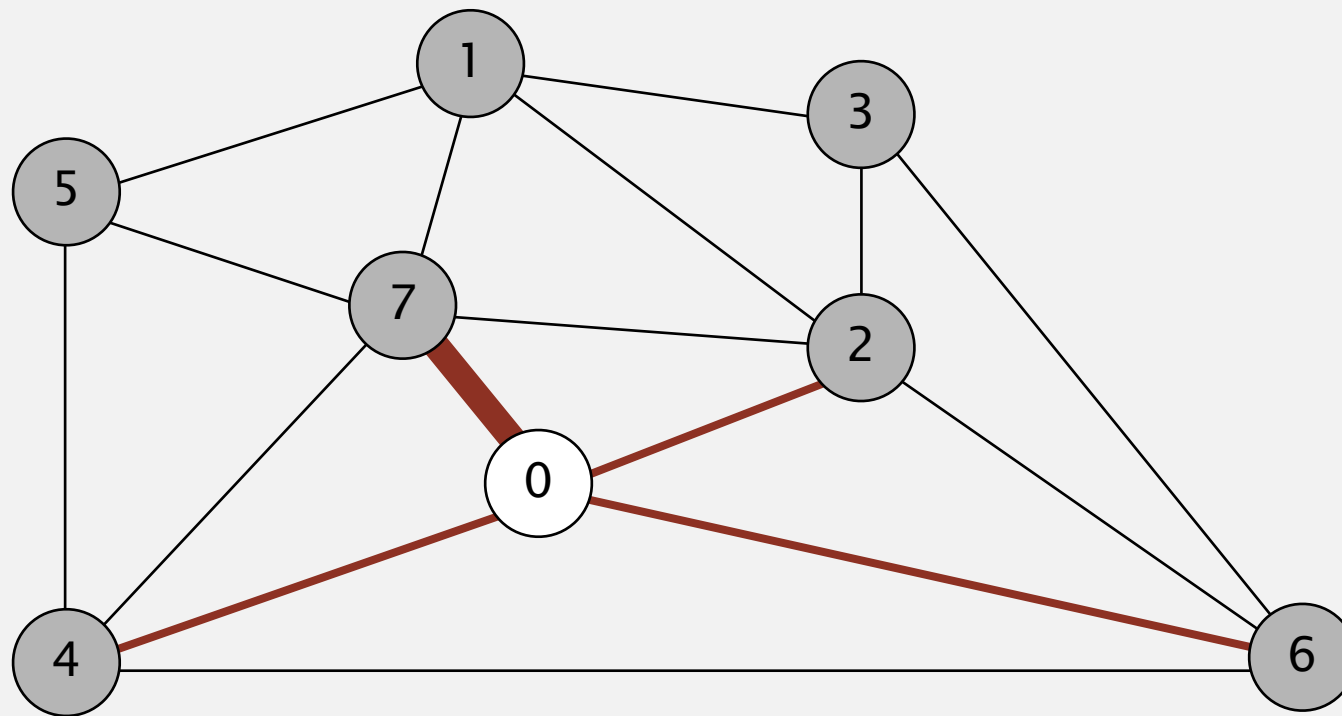
add vertices 7, 2, 4, and 6 to PQ

v	edgeTo[]	distTo[]
→ 0	-	-
7	0-7	0.16
2	0-2	0.26
4	0-4	0.38
6	6-0	0.58

vertices on PQ
(sorted by weight)

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

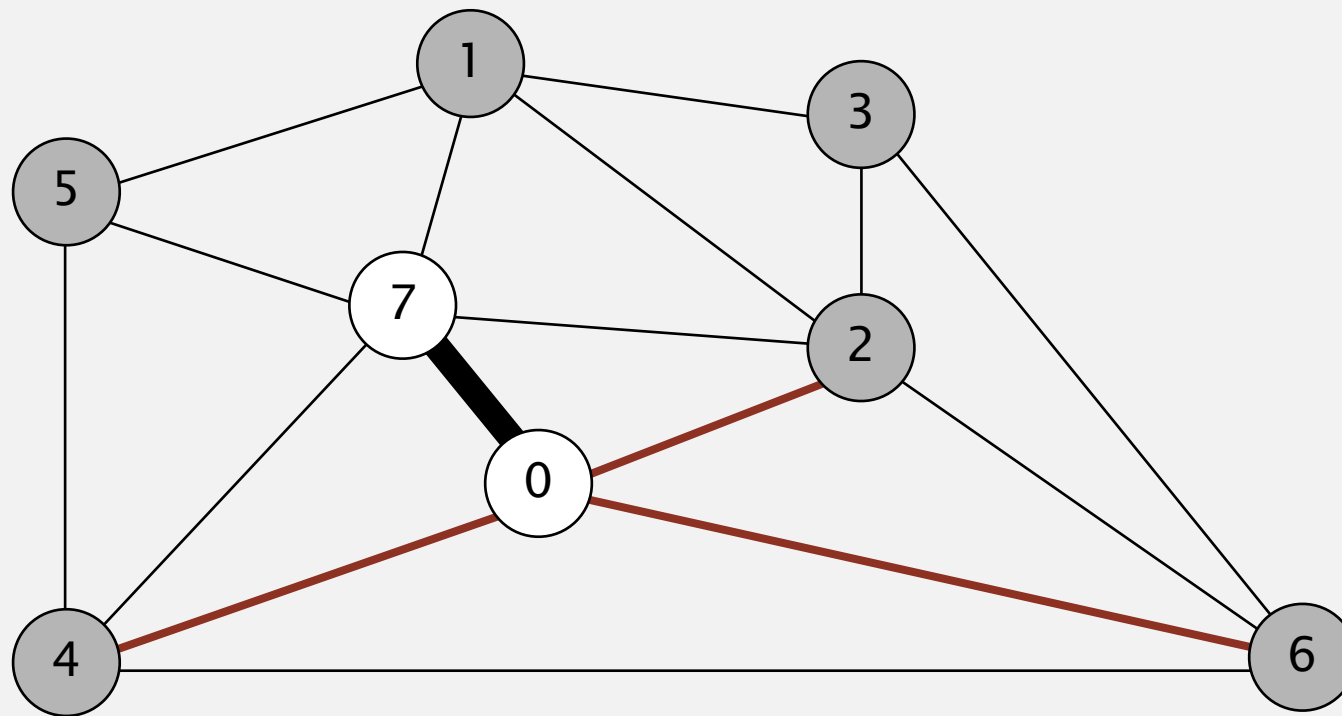


v	edgeTo[]	distTo[]
0	-	-
→ 7	0-7	0.16
2	0-2	0.26
4	0-4	0.38
6	6-0	0.58

vertices on PQ
(sorted by weight)

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



MST edges

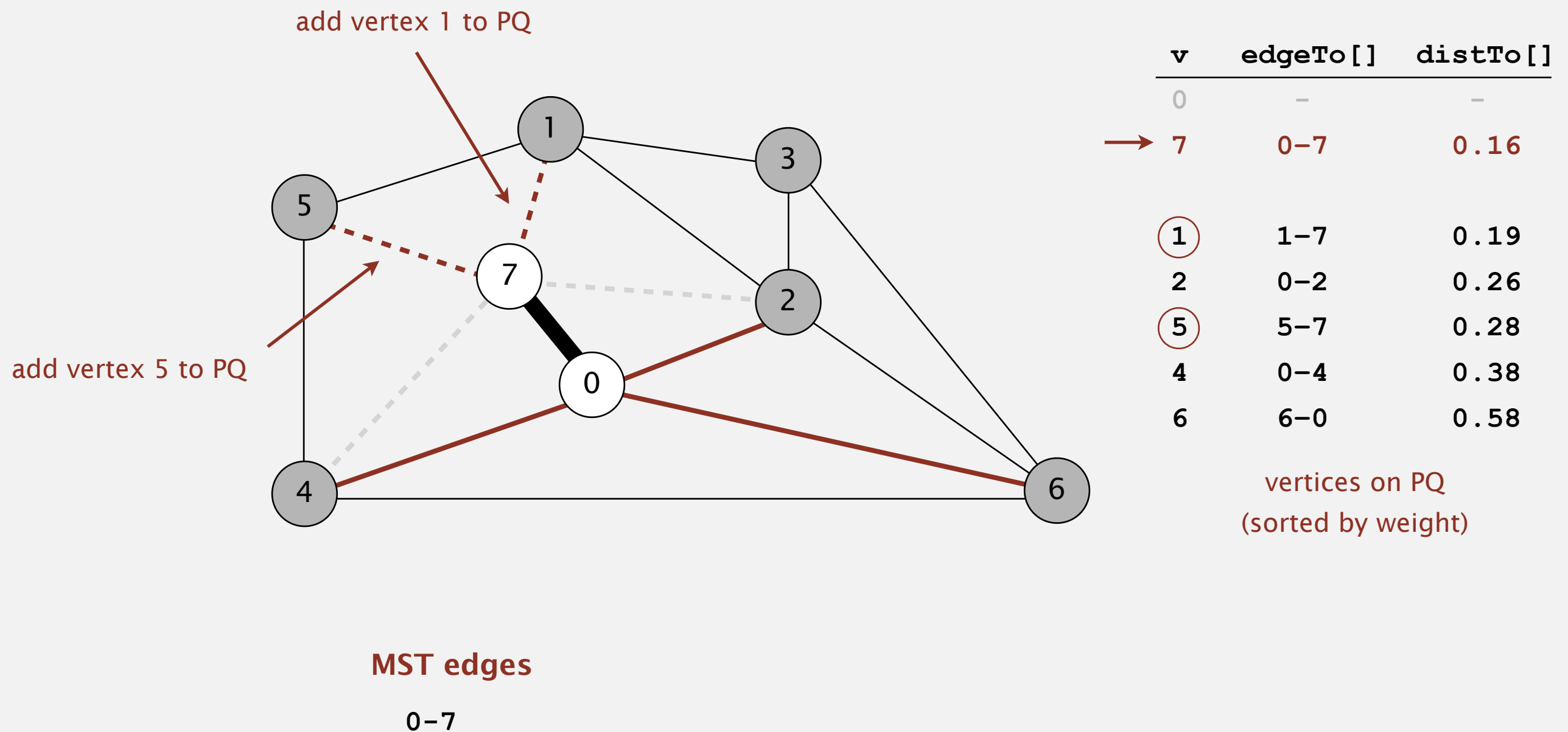
0-7

v	edgeTo[]	distTo[]
0	-	-
→ 7	0-7	0.16
2	0-2	0.26
4	0-4	0.38
6	6-0	0.58

vertices on PQ
(sorted by weight)

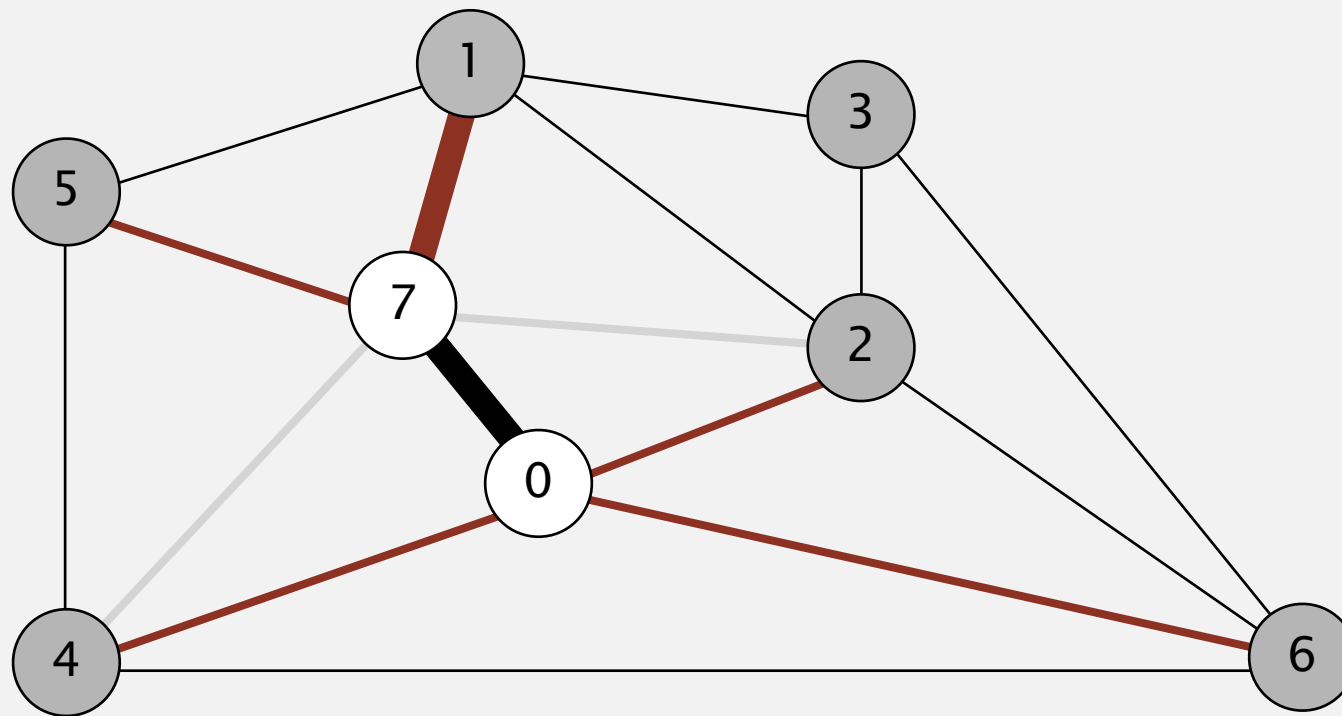
Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



MST edges

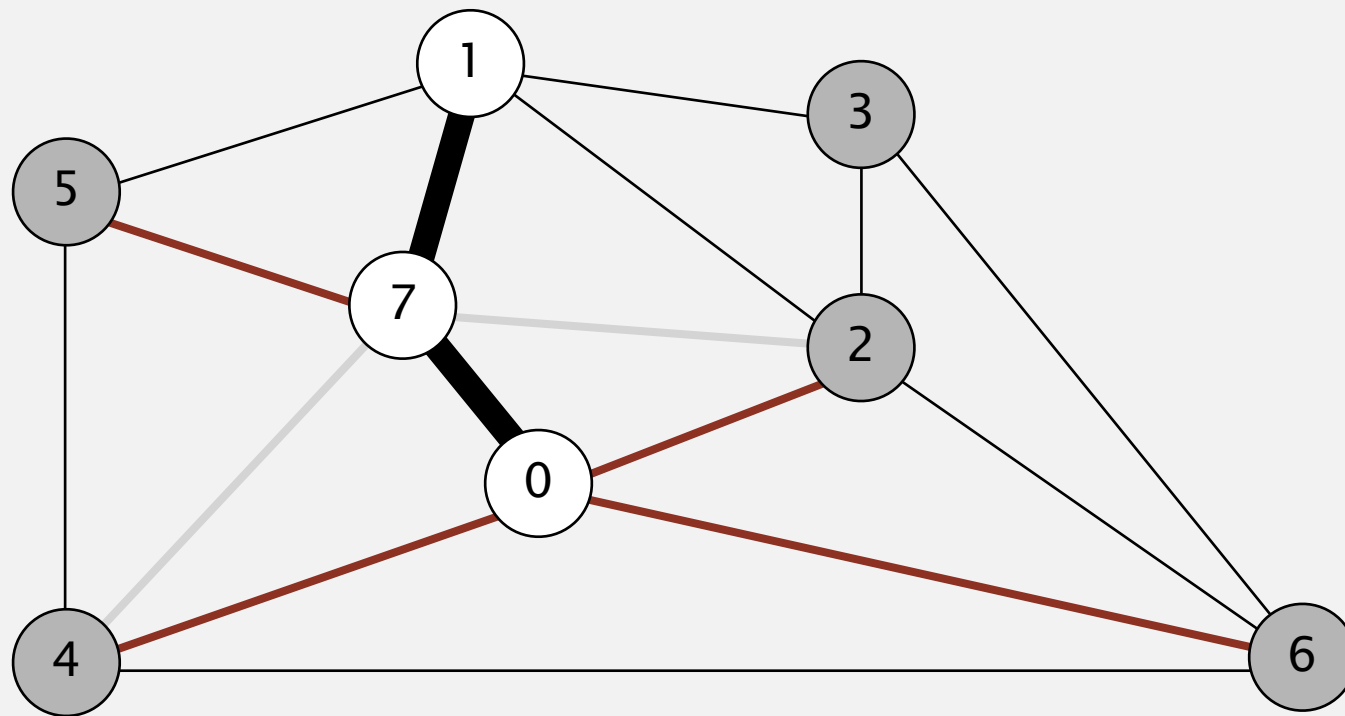
0-7 1-7

v	edgeTo[]	distTo[]
0	-	-
7	0-7	0.16
→ 1	1-7	0.19
2	0-2	0.26
5	5-7	0.28
4	0-4	0.38
6	6-0	0.58

vertices on PQ
(sorted by weight)

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



MST edges

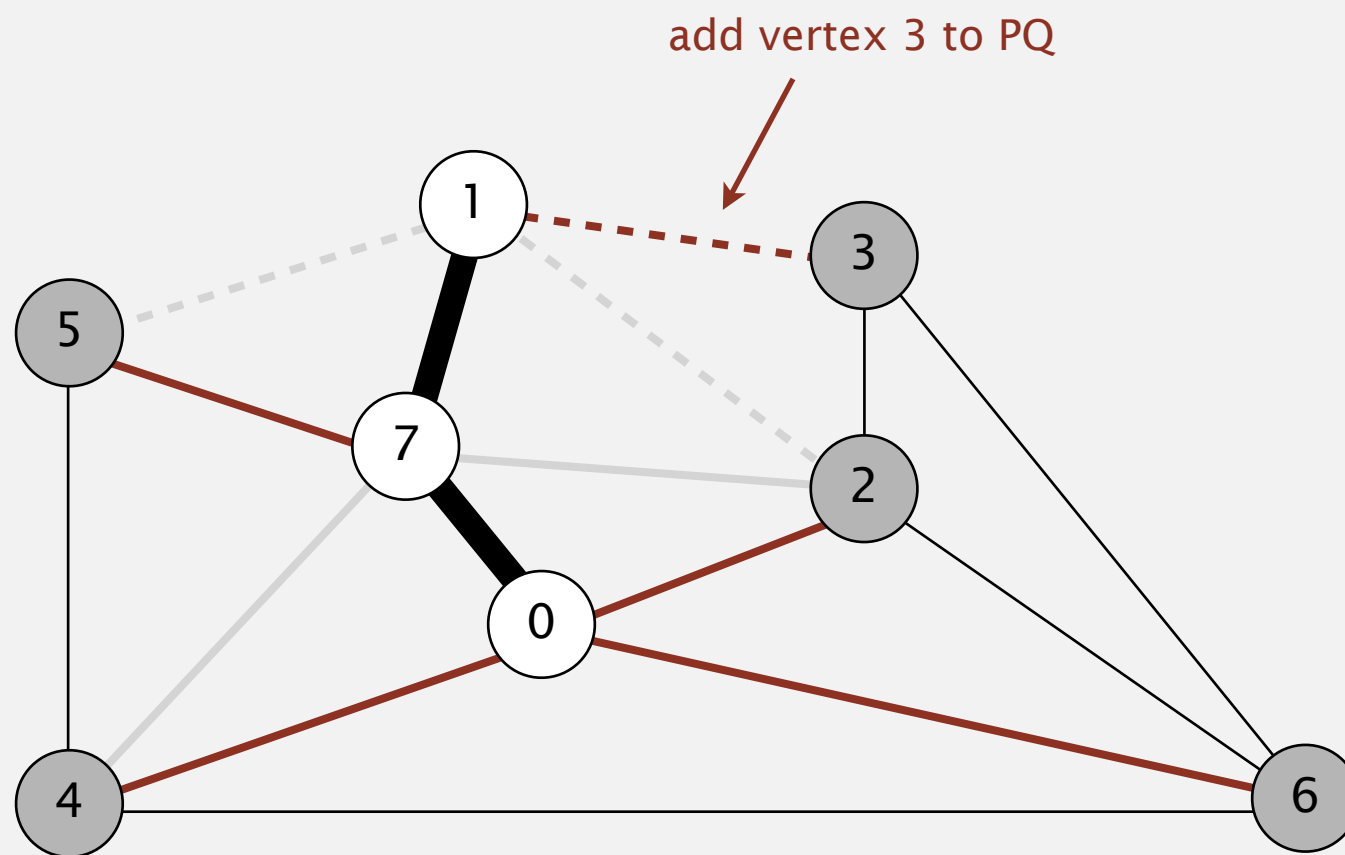
0-7 1-7

v	edgeTo[]	distTo[]
0	-	-
7	0-7	0.16
→ 1	1-7	0.19
2	0-2	0.26
5	5-7	0.28
4	0-4	0.38
6	6-0	0.58

vertices on PQ
(sorted by weight)

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



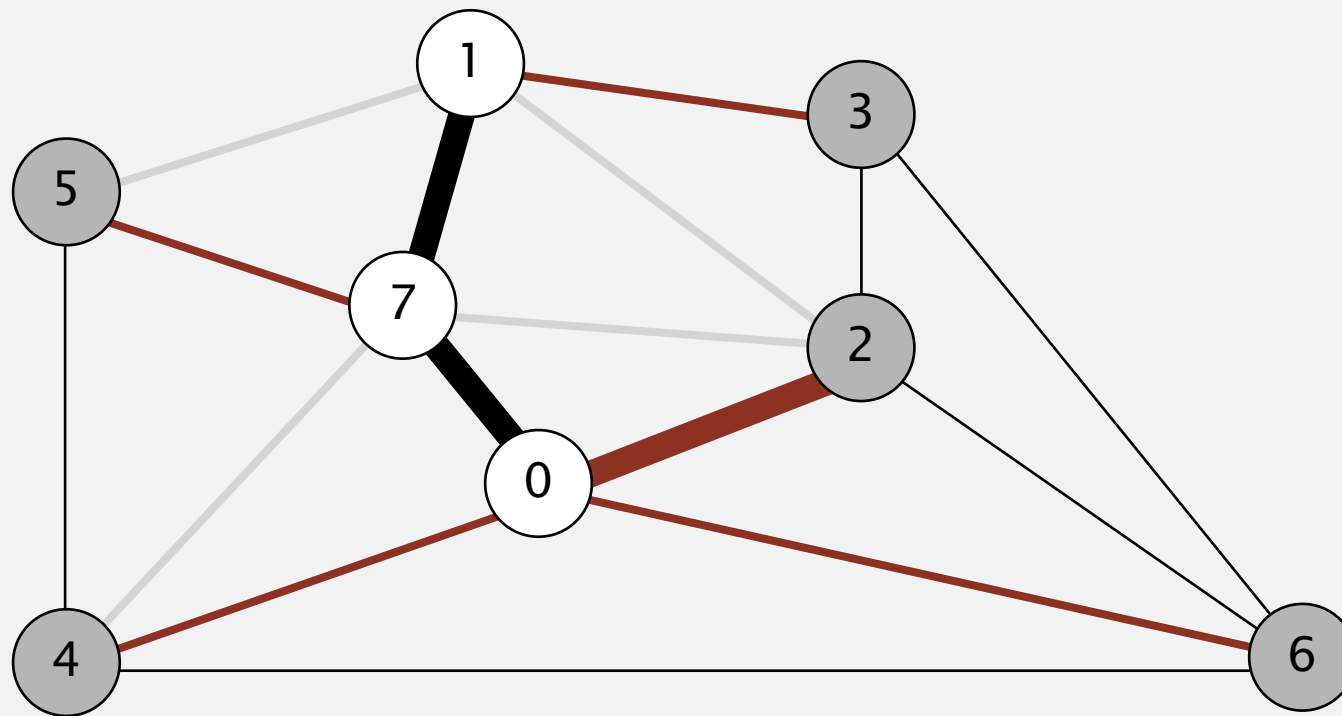
v	edgeTo[]	distTo[]
0	-	-
7	0-7	0.16
→ 1	1-7	0.19
2	0-2	0.26
5	5-7	0.28
3	1-3	0.29
4	0-4	0.38
6	6-0	0.58

MST edges

0-7 1-7

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



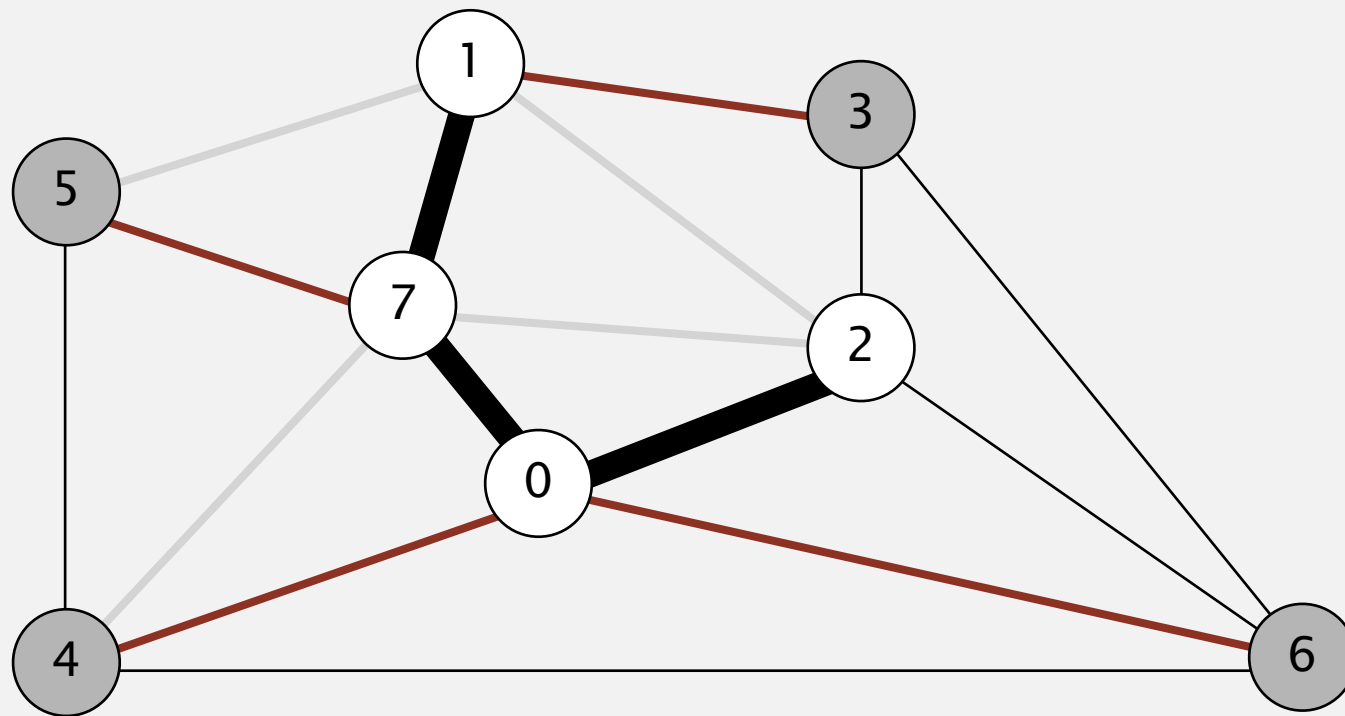
v	edgeTo[]	distTo[]
0	-	-
7	0-7	0.16
1	1-7	0.19
→ 2	0-2	0.26
5	5-7	0.28
3	1-3	0.29
4	0-4	0.38
6	6-0	0.58

MST edges

0-7 1-7

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



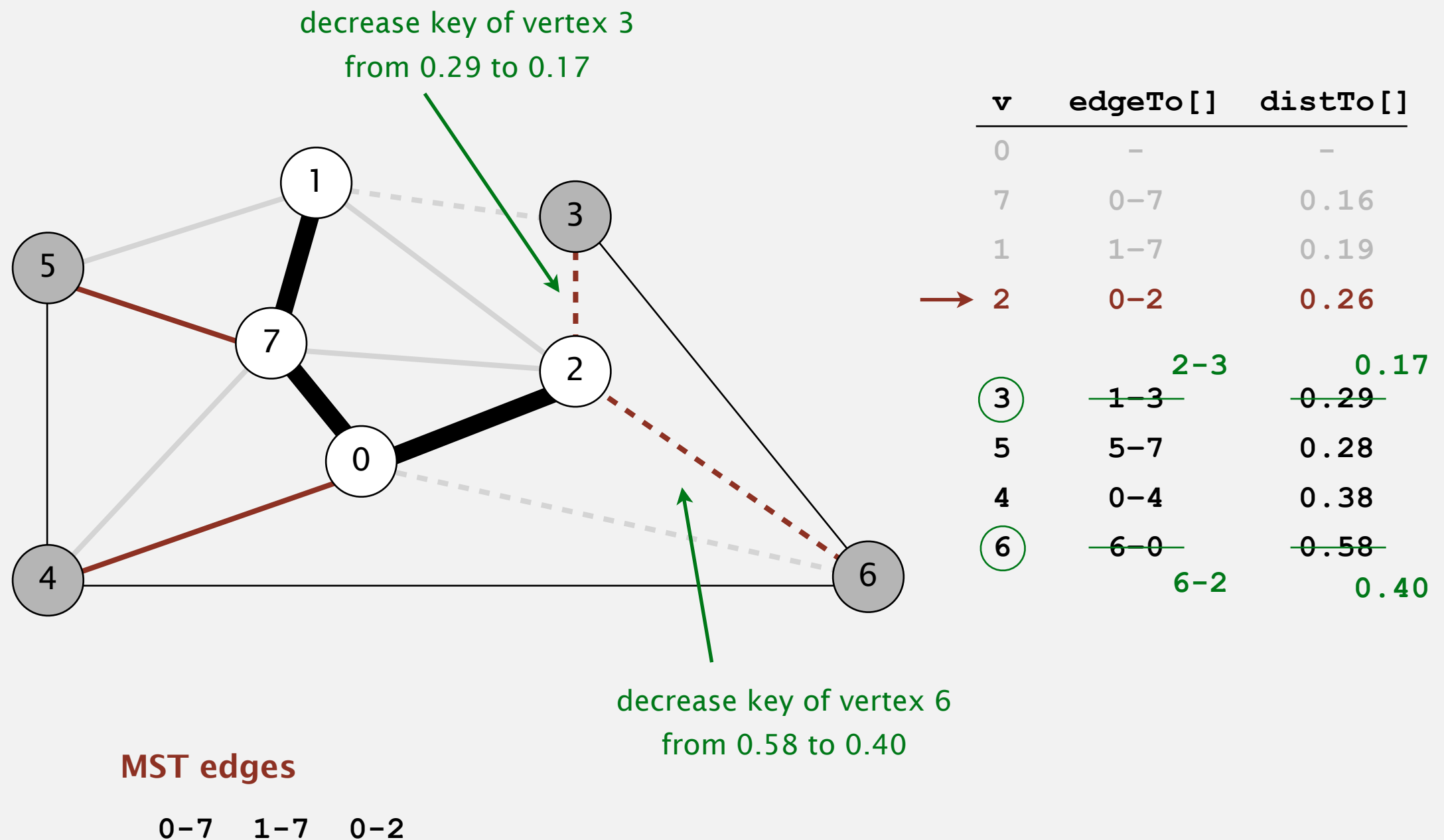
v	edgeTo[]	distTo[]
0	-	-
7	0-7	0.16
1	1-7	0.19
→ 2	0-2	0.26
5	5-7	0.28
3	1-3	0.29
4	0-4	0.38
6	6-0	0.58

MST edges

0-7 1-7 0-2

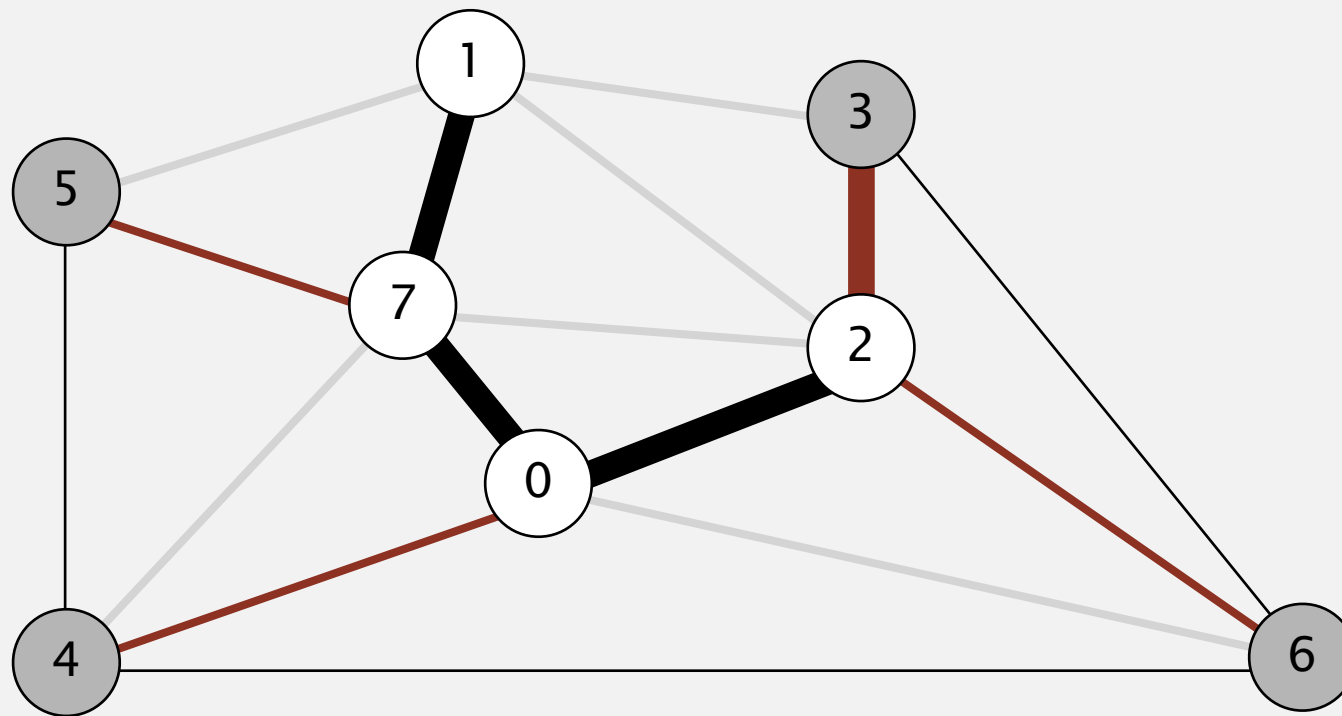
Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



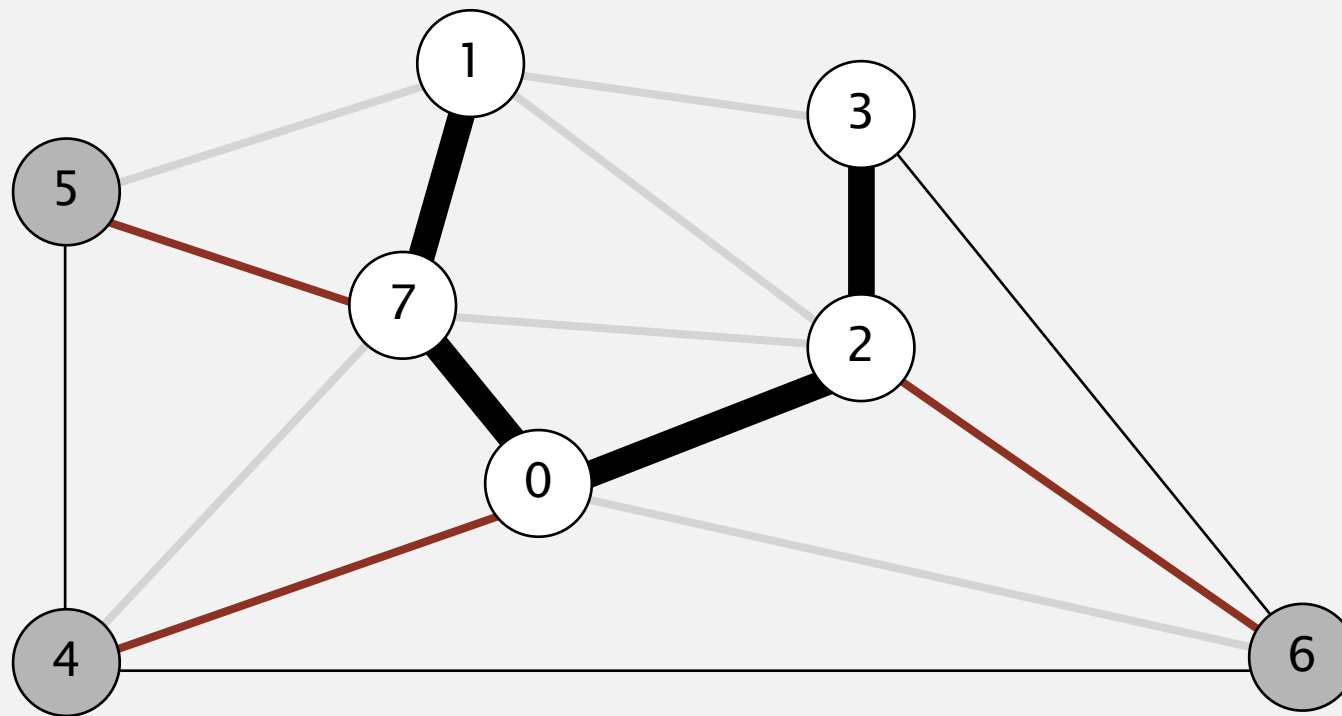
v	edgeTo[]	distTo[]
0	-	-
7	0-7	0.16
1	1-7	0.19
2	0-2	0.26
→ 3	2-3	0.17
5	5-7	0.28
4	0-4	0.38
6	6-2	0.40

MST edges

0-7 1-7 0-2 2-3

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



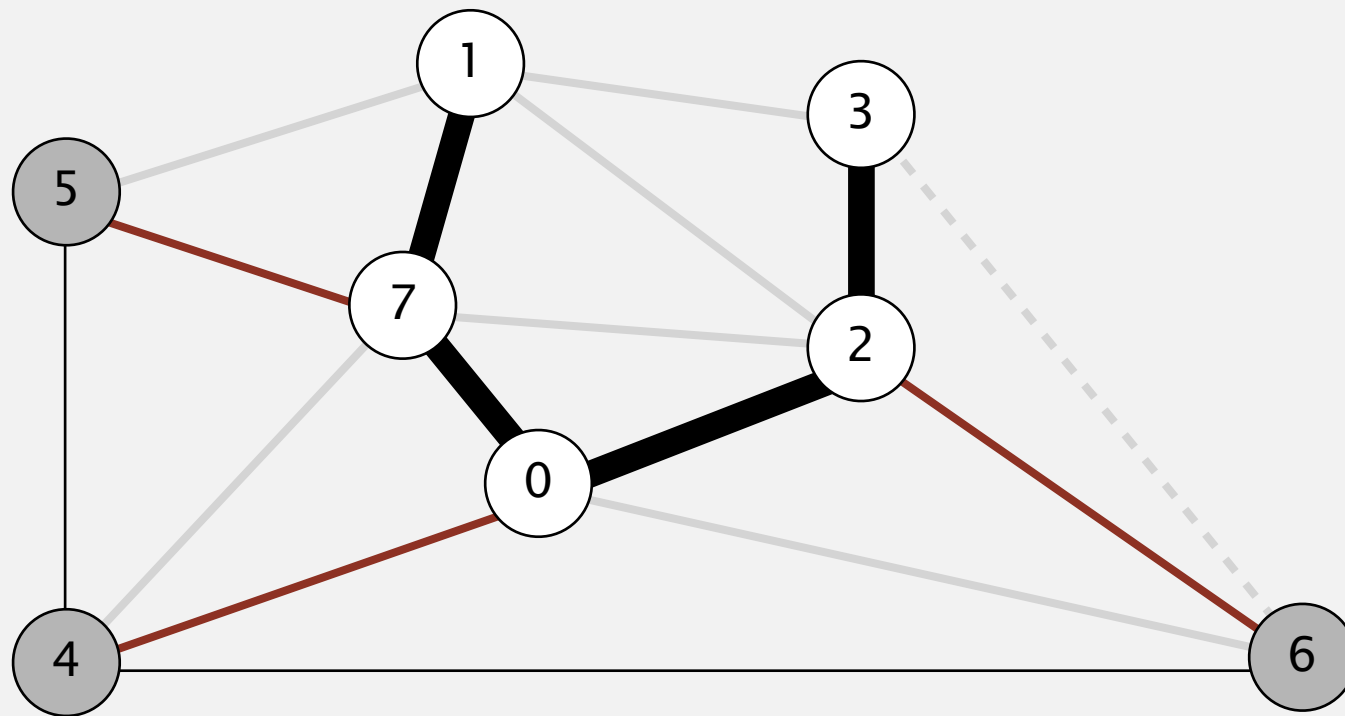
v	edgeTo[]	distTo[]
0	-	-
7	0-7	0.16
1	1-7	0.19
2	0-2	0.26
→ 3	2-3	0.17
5	5-7	0.28
4	0-4	0.38
6	6-2	0.40

MST edges

0-7 1-7 0-2 2-3

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



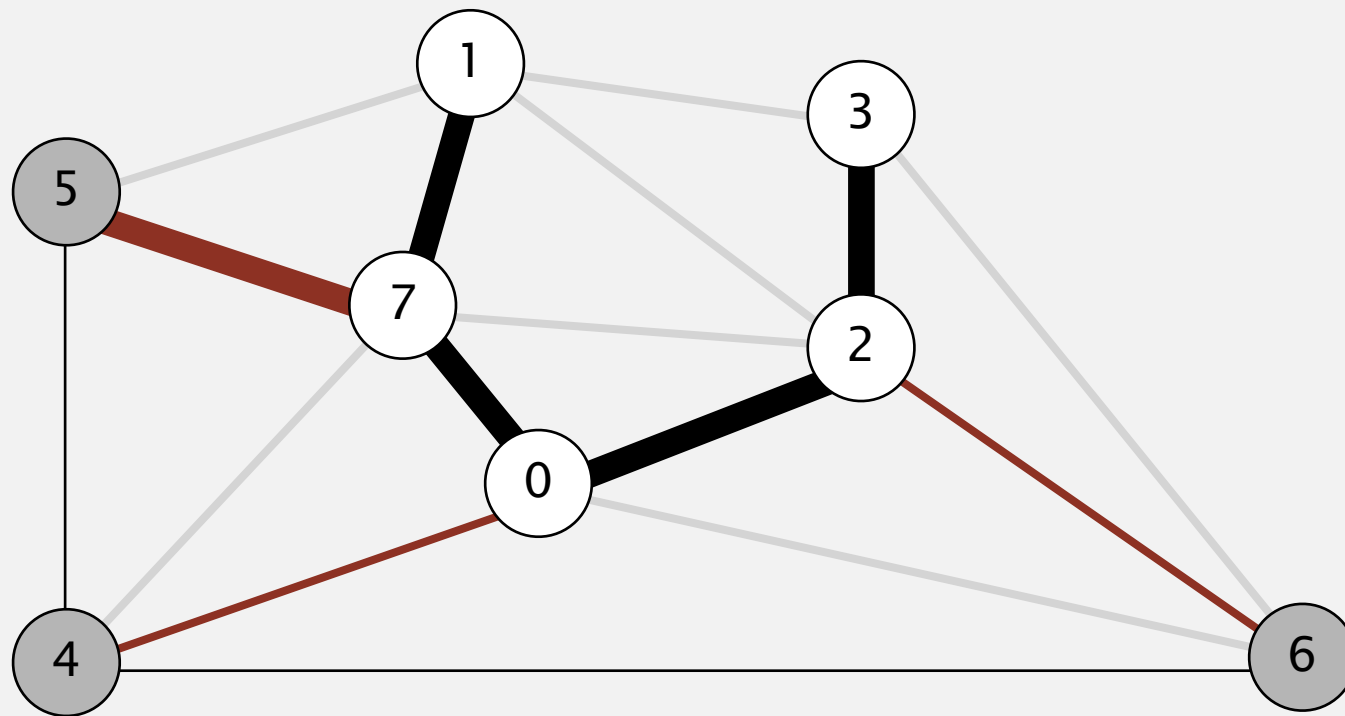
v	edgeTo[]	distTo[]
0	-	-
7	0-7	0.16
1	1-7	0.19
2	0-2	0.26
→ 3	2-3	0.17
5	5-7	0.28
4	0-4	0.38
6	6-2	0.40

MST edges

0-7 1-7 0-2 2-3

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



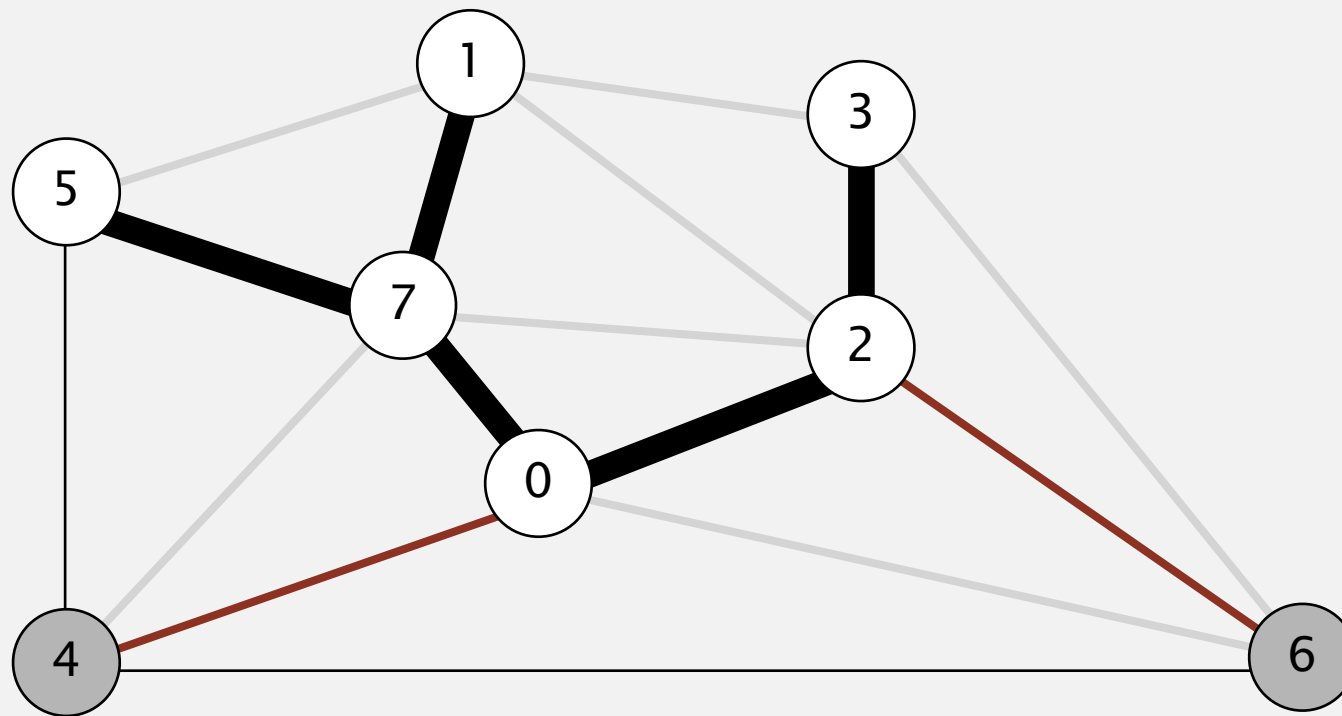
v	edgeTo[]	distTo[]
0	-	-
7	0-7	0.16
1	1-7	0.19
2	0-2	0.26
3	2-3	0.17
→ 5	5-7	0.28
4	0-4	0.38
6	6-2	0.40

MST edges

0-7 1-7 0-2 2-3

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



v	edgeTo[]	distTo[]
0	-	-
7	0-7	0.16
1	1-7	0.19
2	0-2	0.26
3	2-3	0.17
→ 5	5-7	0.28
4	0-4	0.38
6	6-2	0.40

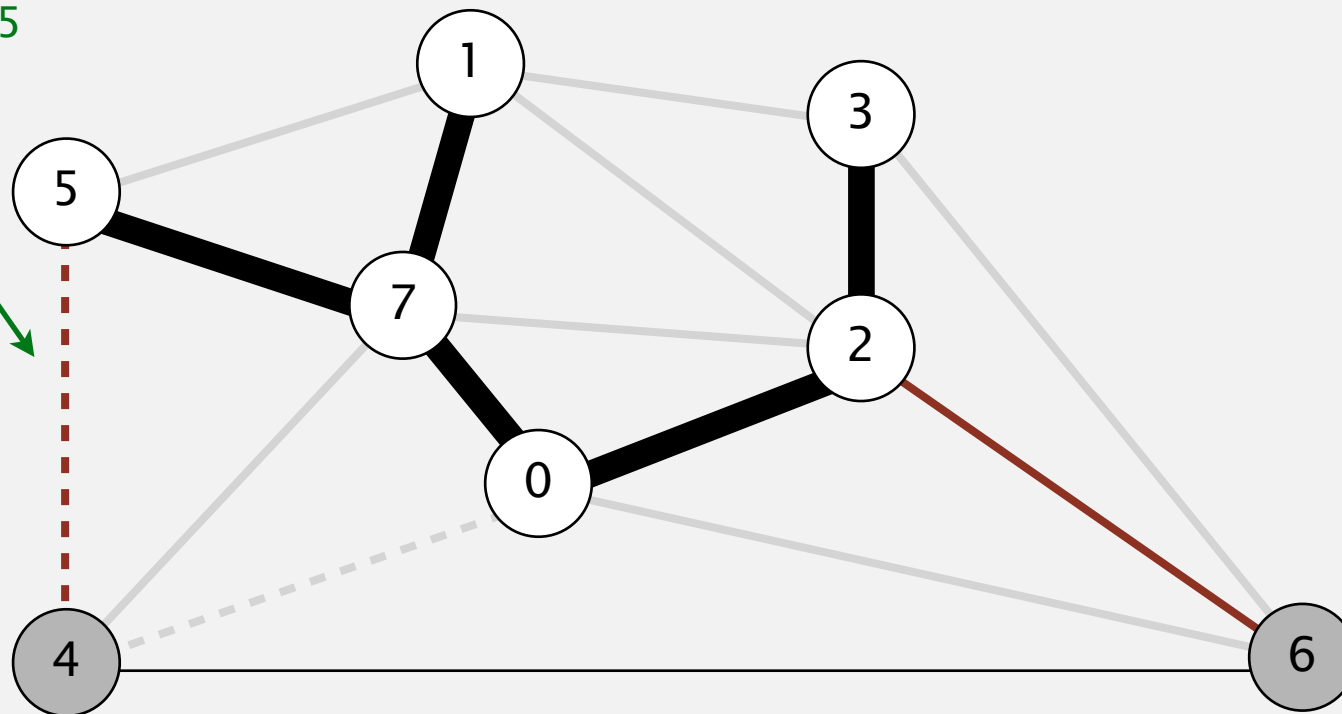
MST edges

0-7 1-7 0-2 2-3 5-7

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .

decrease key of 4
from 0.38 to 0.35



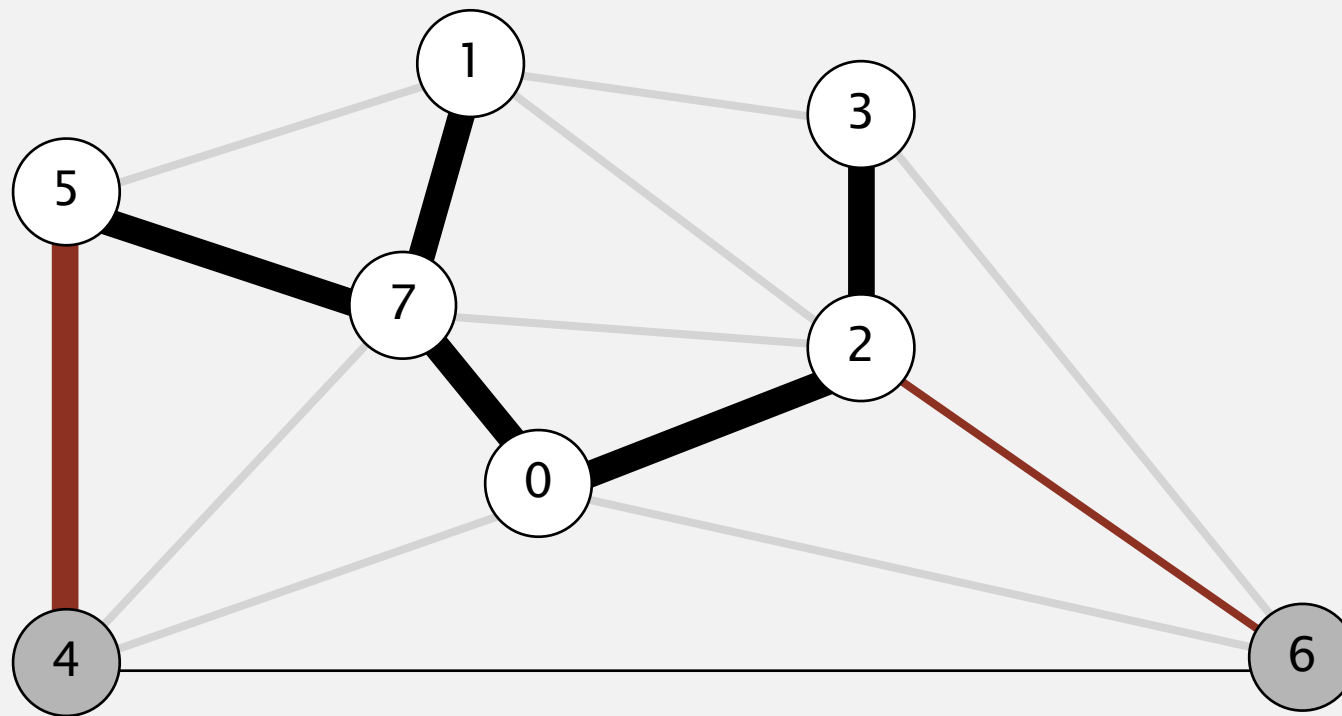
MST edges

0-7 1-7 0-2 2-3 5-7

v	edgeTo[]	distTo[]
0	-	-
7	0-7	0.16
1	1-7	0.19
2	0-2	0.26
3	2-3	0.17
→ 5	5-7	0.28
4	0-4 ⁴⁻⁵	0.38 ^{0.35}
6	6-2	0.40

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



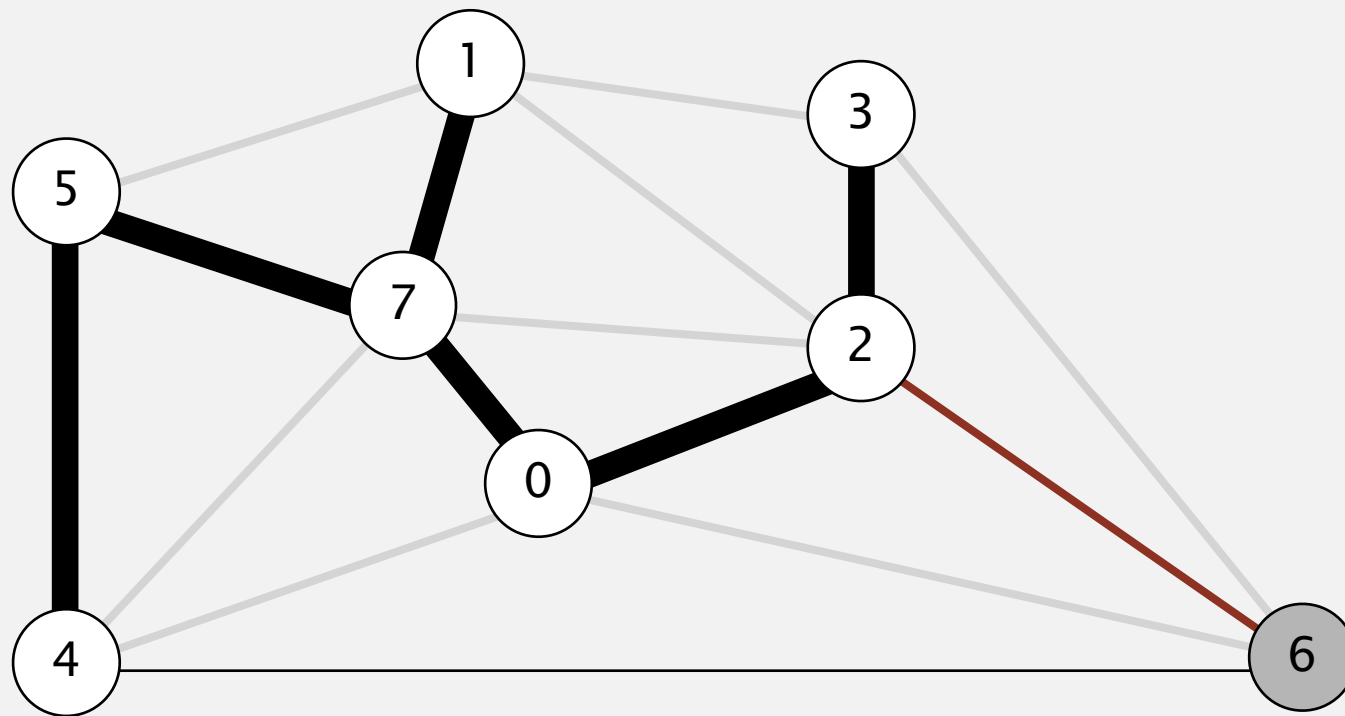
v	edgeTo[]	distTo[]
0	-	-
7	0-7	0.16
1	1-7	0.19
2	0-2	0.26
3	2-3	0.17
5	5-7	0.28
→ 4	4-5	0.35
6	6-2	0.40

MST edges

0-7 1-7 0-2 2-3 5-7

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



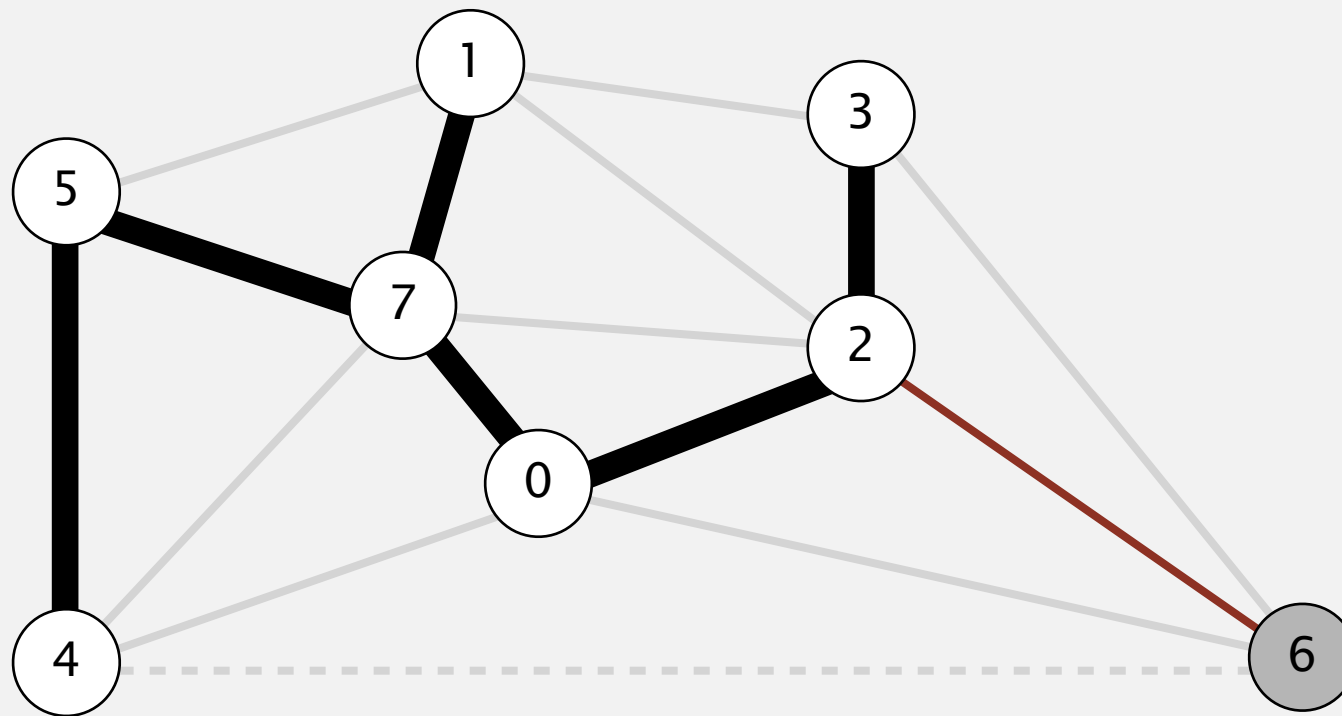
v	edgeTo[]	distTo[]
0	-	-
7	0-7	0.16
1	1-7	0.19
2	0-2	0.26
3	2-3	0.17
5	5-7	0.28
→ 4	4-5	0.35
6	6-2	0.40

MST edges

0-7 1-7 0-2 2-3 5-7 4-5

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



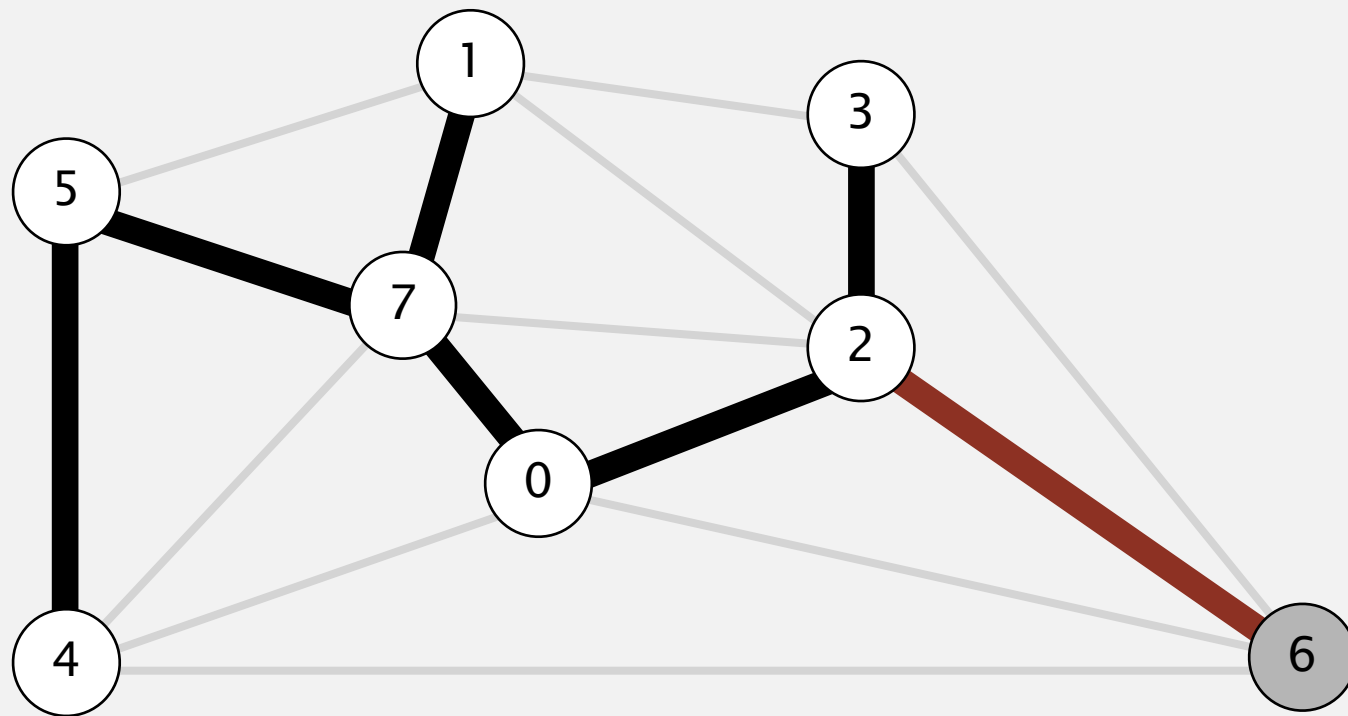
v	edgeTo[]	distTo[]
0	-	-
7	0-7	0.16
1	1-7	0.19
2	0-2	0.26
3	2-3	0.17
5	5-7	0.28
→ 4	4-5	0.35
6	6-2	0.40

MST edges

0-7 1-7 0-2 2-3 5-7 4-5

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



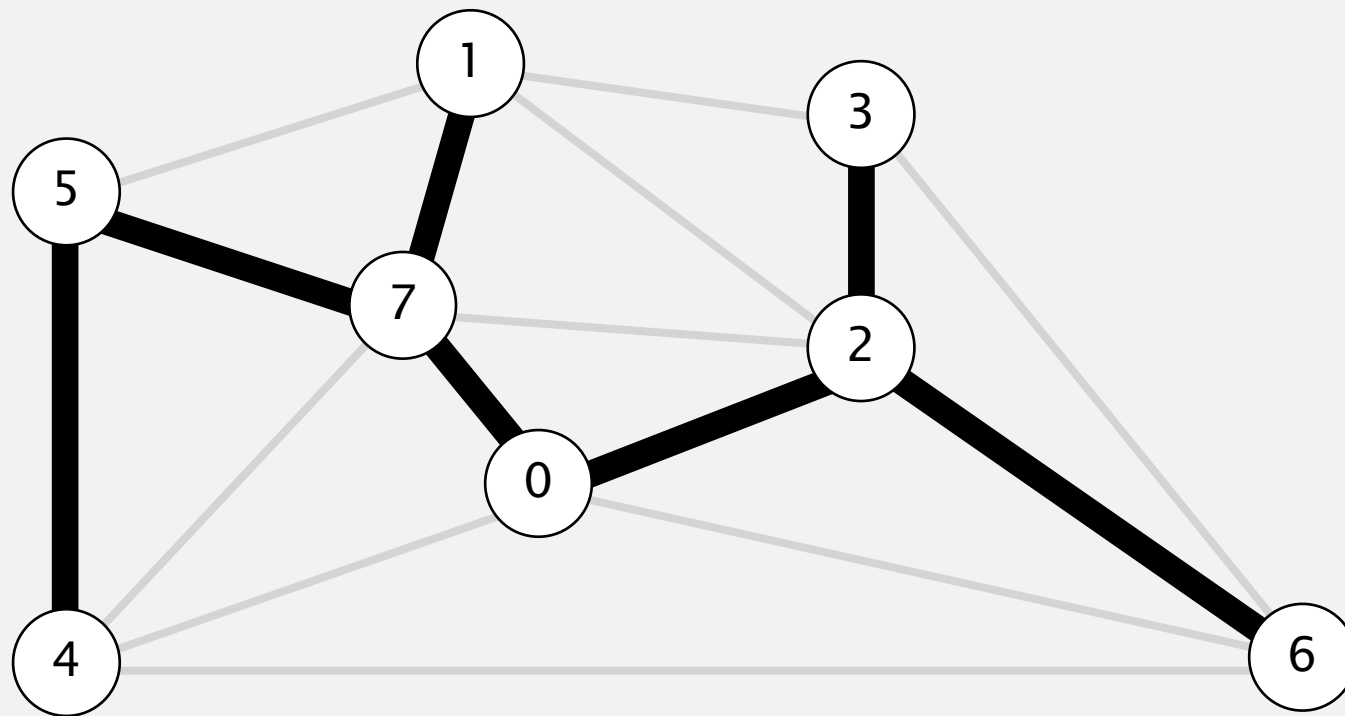
v	edgeTo[]	distTo[]
0	-	-
7	0-7	0.16
1	1-7	0.19
2	0-2	0.26
3	2-3	0.17
5	5-7	0.28
4	4-5	0.35
6	6-2	0.40

MST edges

0-7 1-7 0-2 2-3 5-7 4-5

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



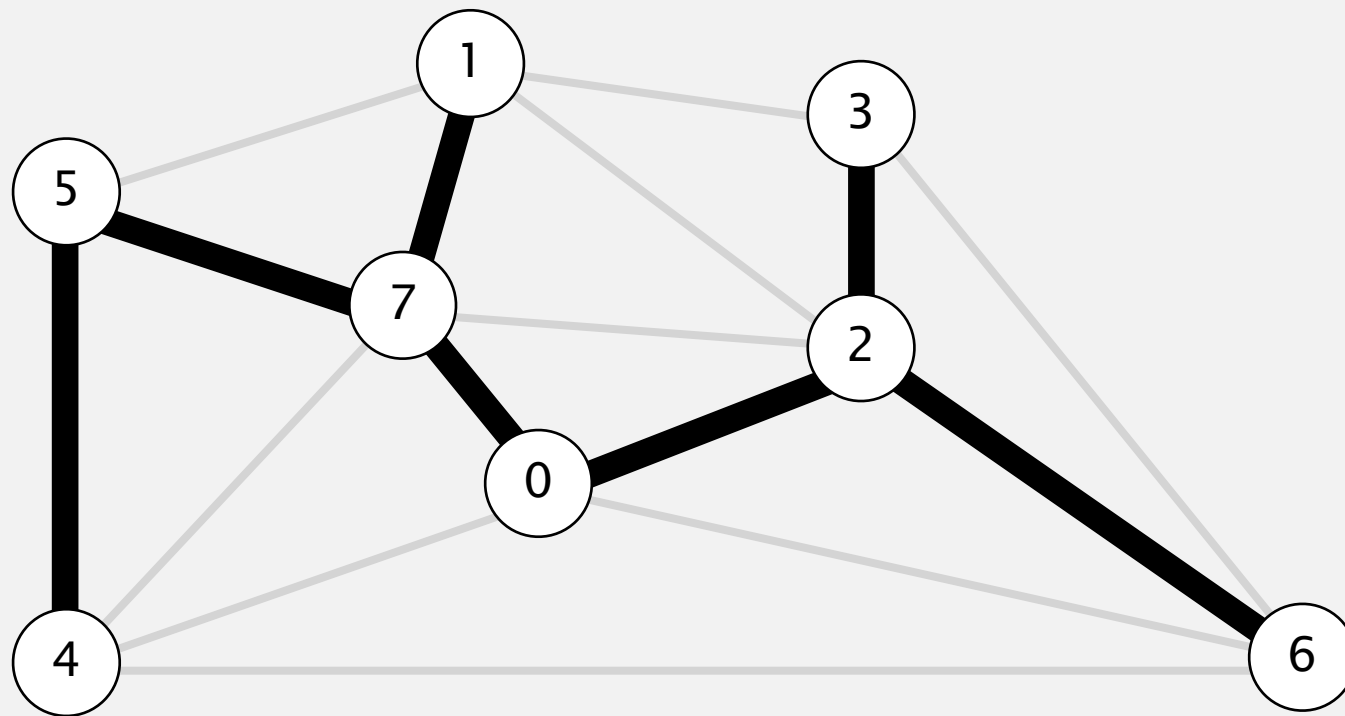
v	edgeTo[]	distTo[]
0	-	-
7	0-7	0.16
1	1-7	0.19
2	0-2	0.26
3	2-3	0.17
5	5-7	0.28
4	4-5	0.35
→ 6	6-2	0.40

MST edges

0-7 1-7 0-2 2-3 5-7 4-5 6-2

Prim's algorithm

- Start with vertex 0 and greedily grow tree T .
- At each step, add to T the min weight edge with exactly one endpoint in T .



MST edges

0-7 1-7 0-2 2-3 5-7 4-5 6-2

v	edgeTo[]	distTo[]
0	-	-
7	0-7	0.16
1	1-7	0.19
2	0-2	0.26
3	2-3	0.17
5	5-7	0.28
4	4-5	0.35
6	6-2	0.40