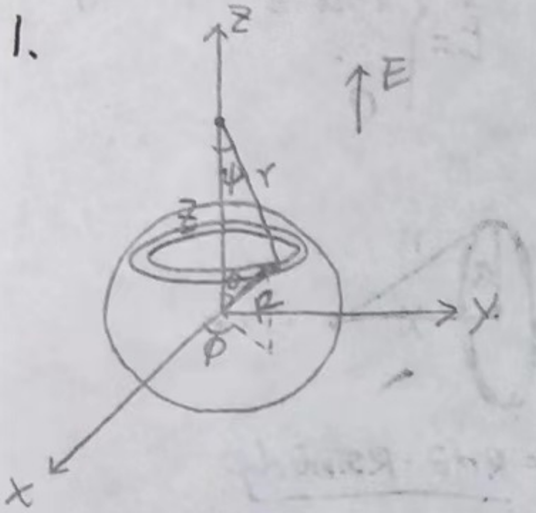
PHYS 512 Problem Set 2

1.



$\vec{E}$ in the z direction

$$\begin{cases} dq = \sigma da = \sigma R^2 \sin\theta \, d\theta \, d\phi \\ r = (R^2 + z^2 - 2Rz\cos\theta)^{\frac{1}{2}} \\ \cos\psi = \dfrac{z - R\cos\theta}{r} \end{cases}$$

$$E_z = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r^2} \cos\psi$$

$$= \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma R^2 \sin\theta \, d\theta \, d\phi \, (z - R\cos\theta)}{(R^2 + z^2 - 2Rz\cos\theta)^{\frac{3}{2}}}$$

total amount of electricity

$$\sigma = \frac{Q}{4\pi R^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot 2\pi R^2 \cdot \frac{Q}{4\pi R^2} \int_0^{\pi} \frac{(z - R\cos\theta) \, d\theta}{(R^2 + z^2 - 2Rz\cos\theta)^{\frac{3}{2}}}$$

$u = \cos\theta$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{2} \int_{-1}^{1} \frac{(z - Ru) \, du}{(R^2 + z^2 - 2Rzu)^{\frac{3}{2}}}$$