

PHYS 512 PS5

5) a) $1 + e^{-\frac{2\pi i k}{N}} + e^{-\frac{4\pi i k}{N}} + e^{-\frac{6\pi i k}{N}} + \dots + e^{-\frac{2\pi i k(N-1)}{N}}$ ← geometric series

$$= \frac{1 - e^{-2\pi i k N}}{1 - e^{-2\pi i k}}$$

$$\Rightarrow \sum_{x=0}^{N-1} \exp(-2\pi i k x/N) = \frac{1 - \exp(-2\pi i k)}{1 - \exp(-2\pi i k/N)}$$

b) as $k \rightarrow 0$ $\exp(-2\pi i k x/N) \rightarrow 1$ thus $\sum_{x=0}^{N-1} \exp(-2\pi i k x/N) \rightarrow N$ ~~$\frac{\exp(-2\pi i k)}{1 - \exp(-2\pi i k/N)}$~~

when $k \neq mN$ ($m \in \mathbb{Z}$) $\frac{1 - \exp(-2\pi i k)}{1 - \exp(-2\pi i k/N)} = 0$ thus $\sum_{x=0}^{N-1} \exp(-2\pi i k x/N) = 0$ when $k \neq mN$ ($m \in \mathbb{Z}$)

c) choose $k = \frac{3}{2}$ DFT: $X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{j2\pi}{N} \cdot \frac{3}{2} n}$