

$$3. a) \quad z - z_0 = a((x - x_0)^2 + (y - y_0)^2)$$

$$\begin{aligned} z &= ax^2 - 2ax_0x + ax_0^2 + ay^2 - 2ay_0y + ay_0^2 + z_0 \\ &= a(x^2 + y^2) - 2ax_0x - 2ay_0y + ax_0^2 + ay_0^2 + z_0 \end{aligned}$$

$$\Rightarrow \quad z = m_0(x^2 + y^2) + m_1x + m_2y + m_3$$

$$m_0 = a \quad m_1 = -2ax_0 \quad m_2 = -2ay_0 \quad m_3 = ax_0^2 + ay_0^2 + z_0$$

$$\begin{array}{ccc} z & = & A \cdot m \\ \swarrow & & \downarrow \quad \downarrow \\ 475 \times 1 & & 475 \times 4 \quad 4 \times 1 \end{array}$$

$$z \rightarrow d, \text{ then } m = (A^T N^{-1} A)^{-1} (A^T N^{-1} d)$$

$$c) \quad y = \frac{x^2}{4f} = ax^2 \quad a = \frac{1}{4f}$$

calculate error bar for f : $f = \frac{1}{4a}$

$$f(a + \delta a) = f(a) + f'(a)\delta a + O(\delta a^2) \Rightarrow \delta f = f(a + \delta a) - f(a) = f'(a)\delta a = -\frac{1}{4a^2}\delta a$$

$$|\delta f| = \left| \frac{\delta a}{4a^2} \right|$$