AMATH 482/582: HOME WORK 1

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Abstract. This report addresses the problem of locating a submarine moving in Puget Sound by analyzing 3D acoustic pressure data over time. Utilizing and averaging Fourier Transform, I detect the dominant frequency generated by the submarine. A Gaussian filter is then applied to extract a denoised signal and determine a robust submarine path. The filtered data reconstruct the submarine's 3D trajectory, which is projected onto the x-y plane to visualize its motion. The report details the theoretical background, algorithm implementation, computational results, and concludes with insights into the effectiveness of the approach for submarine tracking.

1. Introduction and Overview

The ability to track submarines using signal processing is crucial in many fields. For instance, in military and surveillance applications, submarine tracking is vital for defense systems to detect and monitor threats. In this report, I aimed to track a submarine with a broad spectrum recording of acoustics pressure data. The data was obtained over 24 hours in half-hour increments. Through processing the noisy 3D data, I tried to extract useful information about the motion of the submarine. In order to do this, I used the Fast Fourier Transform as a useful tool to decompose the signal into frequency domain and determine the dominant frequency emitted by the submarine. With decomposed data, I then used Gaussian filter to denoise the signal from noise. The reconstructed signal enables the visualization of the submarine's 3D path and its projection onto the x-y plane. As a result, I find the trajectory of the submarine over time.

2. Theoretical Background

The analysis of signals in this assignment relies on mathematical frameworks including Fourier Transform and Gaussian filter. They provide the foundation for signal processing in noisy environments.

2.1. **Fourier Transform.**

The Fourier transform is used to decomposes a signal. It represents a function with sine and cosine. It is a fundamental tool since it allows us to manipulate signals in frequency domain. In our problem, I also average the transform to remove the variance of the data. For a continuous function $f(x)$, the Fourier Transform $F(k)$ is defined as:

$$F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikx} f(x) dx.$$

The $F(k)$ in the function represents the frequency-domain representation of the signal, where k is the frequency. The corresponding inverse Fourier Transform, is given by:

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ikx} F(k) dk.$$

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2.2. **Gaussian Filter.** In real-world signal processing, noise is always involved when people collect data, and often make the meaningful information obscure. Filtering techniques, such as the Gaussian filter, are employed to clean signals of interest. A Gaussian filter is defined as:

$$F(k) = \exp\left(-\tau(k-k_0)^2\right).$$

In this equation, k_0 is the center frequency (dominant frequency). The k is wavenumber, and the τ is the filter's bandwidth. The Gaussian filter effectively preserve the signal's information while eliminating noise. In this assignment, the Gaussian filter is applied in the Fourier domain to denoise the obtained signal underwater and reconstruct the submarine's trajectory.

By using these mathematical frameworks, we can process noisy 3D data effectively and extract meaningful insight about the submarine's motion.

3. Algorithm Implementation and Development

To solve the problem of locating the submarine, I used the Python packages: `numpy` for numerical computations, including Fourier Transform and Gaussian filter implementation, and `plotly` for interactive 3D and 2D visualizations of the submarine's trajectory. In order to capture the positions of submarine in three dimensions at each time point, I used for loops to iterate the data storing spatial information. Specifically, `np.fft.fftn` function and `np.fft.ifftn` are used to decompose the acoustic pressure data into its frequency components.

4. Computational Results

To locate the submarine, I first determined the dominant frequency (center frequency) emitted by the submarine.This was achieved by averaging the Fourier transform over time to reduce variance caused by noise. The center frequency corresponds to the frequency component with the greatest amplitude in the Fourier domain, which indicates the strongest signal emitted by the submarine. Identifying this frequency allows us to focus on the relevant signal and then filter out contributions from other sources by using Gaussian filter. The indices of the center frequency in the 3D frequency grid are listed in the first row of Table 1. After mapping these indices to physical frequencies, the dominant frequencies along the x-, y-, and z-directions in the Fourier domain are presented in the second row of Table 1.

39	49	20
-4.71	-7.85	3.14

Table 1. The first row is the indices of the center frequency, and the second row is dominant frequencies along the x-, y-, and z-directions. The values at the second row are round to 2 decimals.

With the dominant frequency determined, I applied a Gaussian filter to the transformed data to remove irrelevant signals. The location of the dominant frequency was tracked at each time step. Figure 1 illustrates the submarine's trajectory in the 3D spatial domain. The submarine traveled from left to right, initially went up before descending.

To provide a clearer and more direct visualization, I plotted the x-y coordinates of the submarine's path over the 24-hour period, as shown in Figure 2. This information can be utilized for monitoring the submarine's movement in the future.

5. Summary and Conclusions

By applying the Fourier Transform and Gaussian filter, I identified the dominant frequency emitted by the submarine and obtained denoised data, enabling the accurate tracking of the submarine's

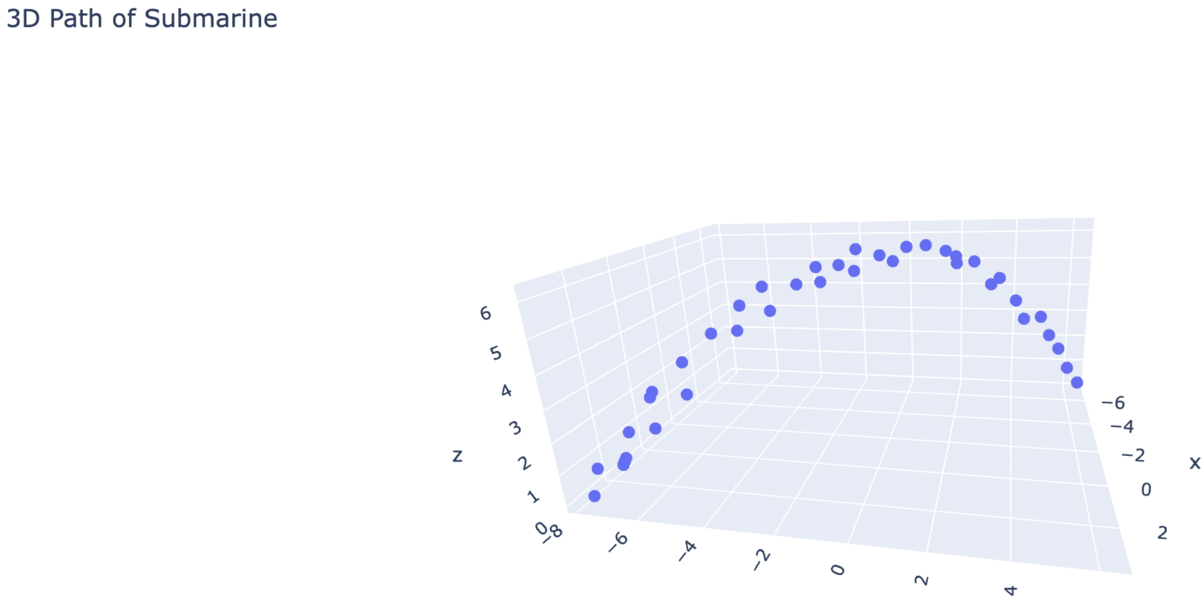

FIGURE 1. This figure shows the 3D path of the submarine. The beginning point is at the most left point in the plot. The submarine is traveling from left to right along the route. The axies represent three directions in space.

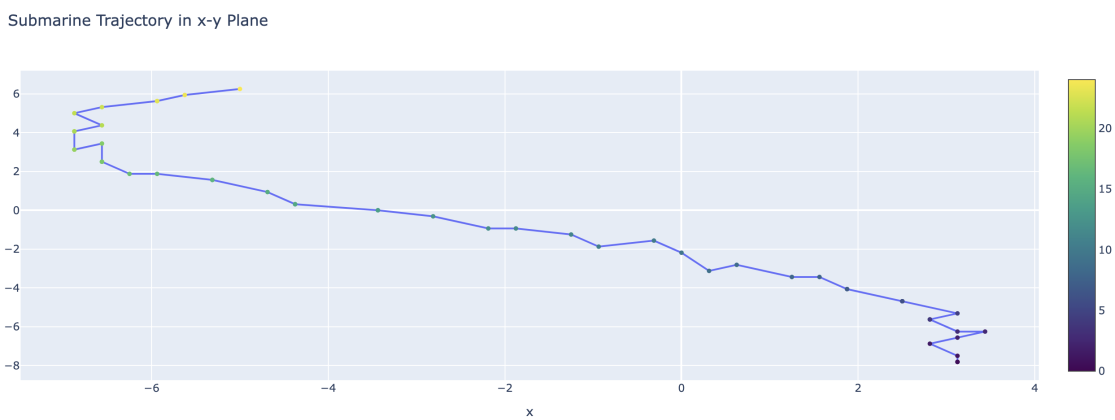

FIGURE 2. This plot represents the submarine's 2D trajectory in the x-y plane. The submarine traveled from the right side of the plot to the left. The color bar on the right indicates time progression, with darker points representing earlier time steps and lighter points corresponding to later ones.

trajectory. The results highlight the effectiveness of the Fourier Transform and the Gaussian filter's capability to clean signal by filtering irrelevant noise.

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References

[1] J. N. Kutz. *Data-driven modeling & scientific computation: methods for complex systems & big data*. OUP Oxford, 2013.