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Suggestion of the DLV dimensionless number system to represent the scaled behavior of structures under impact loads

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Abstract A group of dimensionless numbers, termed density–length–velocity (DLV) system, is put forward to represent the scaled behavior of structures under impact loads. It is obtained by means of the Buckingham Π theorem with an essential basis. The distinct features of this group of dimensionless numbers are that it relates physical quantities of the impacted structures to essential basis of the density, the length and the velocity, and thus it can represent the scaled influence of material property, geometry characteristic and velocity on the behavior of structure. The newly 15 proposed dimensionless numbers reflect three advantages: (1) the intuitively clear physical significance of these dimensionless numbers, such as the ratios of force intensity, force, moment of inertia to the corresponding dynamic quantities, the Johnson's damage number D_n and Zhao's response number R_n, are naturally included; (2) the property of directly matching the dimensionless expression of response equations of dynamic problems with these dimensionless numbers through simple equation analysis; (3) the addressing ability of non-scaling problems for different materials and strain-rate-sensitive materials through adjusting impact velocity of the scaled model or adjusting density of the scaled model, as well as the VSG (initial impact velocity–dynamic flow stress–impact mass G) system. Four classical impact models are used to verify the directly matching property and the non-scaling addressing ability of the DLV system by equation analysis. The results show that the proposed dimensionless number system is simple, clear and efficient, and we suggest using it to represent the scaled behavior of structures under impact loads.

Keywords Dimensionless numbers · Structural impact · Scaling · Similarity · Johnson's damage number · Zhao's response number

1 Introduction

In order to represent the scaled behavior of structures under impact loads, Jones [1] systematically summarized previous works and attempted to describe the dynamic plastic behavior of structures through 22 dimensionless numbers based on classic mass–length–time (MLT) dimensional analysis. The structural similarity indicated the predictability that behavior of the prototype could be speculated through the scaling law, which usually linked same physical variables between the prototype and the scaled model by different scaling factors. For example, the geometric scaling factor was given as

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Table 1 Main scaling factors of structural impact in MLT system

Variable	Scaling factor	Variable	Scaling factor
Length, L	β	Stress, σ	$\beta_\sigma = 1$
Mass, M	$\beta_{\mathrm{M}} = \beta^3$	Strain, ε	$\beta_\varepsilon = 1$
Time, t	$\beta_{\mathrm{t}} = \beta$	Strain rate, $\dot{\varepsilon}$	$\beta_{\dot{\varepsilon}} = 1/\beta$
Velocity, V	$\beta_{\mathrm{V}} = 1$	Acceleration, A	$\beta_{\mathrm{A}} = 1/\beta$
Displacement, δ	$\beta_\delta = \beta$	Energy, E	$\beta_{\mathrm{E}} = \beta^3$

Table 2 Main scaling factors of structural impact in VSG system

Variable	Scaling factor	Variable	Scaling factor
Length, L	$\beta = L_{\mathrm{m}}/L_{\mathrm{p}}$	Displacement, δ	$\beta_\delta = \beta$
Density, ρ	$\beta_\rho = \rho_{\mathrm{m}}/\rho_{\mathrm{p}}$	Stress, σ	$\beta_\sigma = \beta_\rho \beta_{\mathrm{V}}^2$
Velocity, V	$\beta_{\mathrm{V}} = V_{\mathrm{m}}/V_{\mathrm{p}}$	Strain, ε	$\beta_\varepsilon = 1$
Mass, M	$\beta_{\mathrm{M}} = \beta_\rho \beta^3$	Strain rate, $\dot{\varepsilon}$	$\beta_{\dot{\varepsilon}} = \beta_{\mathrm{V}}/\beta$
Time, t	$\beta_{\mathrm{t}} = \beta/\beta_{\mathrm{V}}$	Force, F	$\beta_{\mathrm{F}} = \beta_\rho \beta^2 \beta_{\mathrm{V}}^2$
Acceleration, A	$\beta_A = \beta_{\mathrm{V}}^2/\beta$	Energy, E	$\beta_{\mathrm{E}} = \beta_\rho \beta^3 \beta_{\mathrm{V}}^2$

$$\beta = \frac{L_{\mathrm{m}}}{L_{\mathrm{p}}}, \tag{1}$$

where L was the characteristic length of structure, and the subscript m and p represented the scaled model and the prototype, respectively. When these 22 dimensionless numbers were used to relate the scaled model to the prototype, the main scaling factors of physical variable presented by this single geometric factor are listed in Table 1.

However, when the strain-rate-sensitive materials, the gravity and the fracture of structure were taken into account, the above scaling factors would become invalid [1]. And a hidden assumption of this group of scaling factors lay in the using of same material between the scaled model and the prototype; thus, the MLT system could not deal with the problem of different materials. The above two aspects of the classic MLT system limit the application to describe the scaled behavior of structures under impact loads.

Recently, Oshiro and Alves [2] proposed the VSG (V represented initial impact velocity V_0, S represented dynamic flow stress σ_{d} and G represented impact mass) dimensionless number system to represent the scaled behavior of the impacted structures, in which the dimensionless number of physical quantities is based on $V_0 - \sigma_{\mathrm{d}} - G$ dimensional analysis. The most prominent feature of this system is that it has the strong ability to address the non-scaling problems arising from strain-rate-sensitive materials through adjusting impact velocity or adjusting impact mass of the scaled model, which has been verified by Refs. [2–5]. Further study of Mazzariol et al. [6] added a new dimensionless number of structural mass to address the non-scaling problem of different materials between the prototype and the scaled model through adjusting impact velocity. In a more complete VSG system [7,8], 8 dimensionless numbers were given to express the acceleration A, the time t, the displacement δ, the strain rate $\dot{\varepsilon}$, the stress σ, the structural mass M', the force F and the energy E as follows:

$$\frac{A^3 G}{V_0^4 \sigma_{\mathrm{d}}}, \frac{t^3 \sigma_{\mathrm{d}} V_0}{G}, \frac{\delta^3 \sigma_{\mathrm{d}}}{G V_0^2}, \dot{\varepsilon}\left(\frac{G}{\sigma_{\mathrm{d}} V_0}\right)^{1/3}, \frac{\sigma}{\sigma_{\mathrm{d}}}, \frac{M'}{G}, \frac{F^3}{V_0^4 \sigma_{\mathrm{d}} G^2}, \frac{E}{G V_0^2}. \tag{2}$$

Instead of the classic MLT system, the scaling factors of main physical variables [4,6–8] for the VSG system are listed in Table 2. It could be seen that, except for the geometric scaling factor β, two more factors β_{V} and β_ρ that containing the influence of velocity and different materials were included.

Nonetheless, several main defects still existed in the VSG system: (1) Complex expression form and less physical meaning of most dimensionless numbers. (2) Difficulty in representing various impact loadings since the impact mass G was chosen as one base. (3) Denaturalization that six dimensionless numbers were expressed by the dynamic flow stress σ_{d} of material property. (4) Lack of basic physical quantities that describe structure response of impact problems such as the density, the geometrical characteristic (e.g., length, width and thickness), the strain, the angle, the angular velocity, the angular acceleration and the bending moment. These defects had been mitigated to some extent in the work of Mazzariol and Alves [9] by using VSM (velocity–dynamic flow stress–structure mass) base instead of VSG base, which further increased the number $\frac{\Psi_0^3}{V^4\sigma_\mathrm{d}M'^2}$ of plastic bending moment Ψ_0 for plate (or $\frac{\Psi_0}{V^2M'}$ for beam) and the number $\frac{\rho V^2}{\sigma_\mathrm{d}}$ of density ρ. In addition, Wei and Hu [10] proposed four numbers $\frac{L^3\rho}{G}$, $\frac{TV}{L}$, ε and $\frac{Lt^2\sigma_\mathrm{d}}{G}$ instead of VSG numbers to represent the scaled behavior of the impacted structures. The non-scaling problem arising from strain-rate-sensitive materials was addressed through adjusting the structural mass and the impact mass or impact velocity. However, these progresses cannot fundamentally overcome the defects of VSG system.

In this paper, the main objective is to propose a new group of dimensionless number system to overcome main defects of the previous MLT and the VSG dimensionless systems.

In what follows, Sect. 2 introduces our newly proposed dimensionless system including the derivation of these dimensionless numbers and the intuitive interpretation of their physical significance. Section 3 presents the scaling factors obtained by these dimensionless numbers. Section 4 verifies the features of new system through four impact models. Finally, Sect. 5 summarizes this work.

2 The DLV dimensionless numbers

In order to describe the impact behavior of structure more systematically and reasonably, we now use the Buckingham Π theorem to rederive the dimensionless system again.

The Buckingham Π theorem [11,12] postulates that if a system containing n numbers of variables X_i is expressed as a function,

$$\Phi\left(X_1, X_2, \ldots, X_n\right) = 0\,, \tag{3}$$

in which only k ($k < n$) numbers of variables are independent, the function can be reduced to a relationship about $n-k$ dimensionless numbers $Y_1, \ldots, Y_{n-k}$,

$$\Phi\left(Y_1, Y_2, \ldots, Y_{\mathrm{n-k}}\right) = 0\,, \tag{4}$$

where each Y_i is constructed from $X_1, \ldots, X_k$ by a specified form,

$$Y_\mathrm{i} = X_1^{a_1} X_2^{a_2} \ldots X_\mathrm{k}^{a_k}\,, \tag{5}$$

with the exponents $a_1, \ldots, a_k$ being the rational numbers.

Ignoring elastic effect, gravity effect, thermal effect and fracture failure, the dynamic behavior of rigid–plastic materials including strain-hardening effects and strain rate sensitivity is supposed to be mainly controlled by the following 18 interest physical variables which are density ρ, characteristic length L, velocity V, stress σ (in this paper, σ mainly presents the dynamic flow stress σ_d), force F, bending moment Ψ, time t, strain rate $\dot{\varepsilon}$, acceleration A, angular velocity $\dot{\theta}$, angular acceleration $\ddot{\theta}$, energy E, impulse I, mass M (e.g., structural mass M' and impact mass G), geometrical characteristic H' (e.g., thickness and width), displacement δ, strain ε and angle θ.

According to the Buckingham Π theorem, when the essential variables of characteristic density ρ, characteristic length L and characteristic velocity V are chosen as the base, the relationships of these 18 physical quantities can be reduced to 15 dimensionless numbers as follows:

$$\begin{aligned}
&\Pi_1 = \left[\frac{\rho V^2}{\sigma_\mathrm{d}}\right], \Pi_2 = \left[\frac{\rho L^2 V^2}{F}\right], \Pi_3 = \left[\frac{\rho L^3 V^2}{\Psi}\right], \Pi_4 = \left[\frac{tV}{L}\right], \Pi_5 = \left[\frac{\dot{\varepsilon} L}{V}\right],\\
&\Pi_6 = \left[\frac{AL}{V^2}\right], \Pi_7 = \left[\frac{\dot{\theta} L}{V}\right], \Pi_8 = \left[\frac{\ddot{\theta} L^2}{V^2}\right], \Pi_9 = \left[\frac{E}{\rho L^3 V^2}\right], \Pi_{10} = \left[\frac{I}{\rho L^3 V}\right],\\
&\Pi_{11} = \left[\frac{M}{\rho L^3}\right], \Pi_{12} = \left[\frac{H'}{L}\right], \Pi_{13} = \left[\frac{\delta}{L}\right], \Pi_{14} = [\varepsilon] \quad \text{and} \quad \Pi_{15} = [\theta]\,.
\end{aligned} \tag{6}$$

In the following context, we would explain the meaning of each number intuitively.

The number Π_1 can be interpreted as the ratio of inertia force intensity ρV^2 to the resistance ability σ_d of a material, which is well known as the damage number D_n proposed by Johnson [13] and used to measure the order of strain imposed on various impact regions of a structure.

The number Π_2 can be interpreted as the ratio of inertia force $\rho L^2 V^2$ to structural dynamic force F.

The number Π_3 can be interpreted as the ratio of inertia moment $\rho L^3 V^2$ to structural dynamic bending moment Ψ.

It should be noted that Π_2 and Π_3 can be expressed as the forms of response number R_n proposed by Zhao, which is widely used to measure the response of simple impacted structures [14–18]. For example, considering the influence of the fully plastic axial membrane force $F = \sigma_d BH$ and the fully plastic bending moment $\Psi = \sigma_d BH^2/4$ on the rectangular beam with density ρ, thickness H, width B and initial impulsive velocity $\bar{V}_0$, the number Π_2 and Π_3 can be rewritten as

$$\Pi_2 = \frac{\rho L^2 \bar{V}_0^2}{\sigma_d BH} = \frac{\rho \bar{V}_0^2}{\sigma_d} \frac{L}{H} \frac{L}{B} \tag{7}$$

and

$$\Pi_3 = \frac{\rho L^3 \bar{V}_0^2}{\sigma_d BH^2/4} = 4 \frac{\rho \bar{V}_0^2}{\sigma_d} \left(\frac{L}{H}\right)^2 \frac{L}{B}\,. \tag{8}$$

It can be seen that Π_2 is the product of the form $\left(\rho \bar{V}_0^2/\sigma_d\right) \cdot (L/H)$ and the aspect ratio L/B, while Π_3 is the product of the form $\left(\rho \bar{V}_0^2/\sigma_d\right) \cdot (L/H)^2$ and the aspect ratio L/B. The expressions of $\left(\rho \bar{V}_0^2/\sigma_d\right) \cdot (L/H)$ and $\left(\rho \bar{V}_0^2/\sigma_d\right) \cdot (L/H)^2$ are two important forms of the response number R_n expressed by the multiplicator of the damage number D_n.

The numbers Π_4 to Π_8 can be interpreted as the ratio of structural five physical quantities, time t, strain rate $\dot{\varepsilon}$, acceleration A, angular velocity $\dot{\theta}$ and angular acceleration $\ddot{\theta}$, to five characteristic quantities, L/V, V/L, V^2/L, V/L and V^2/L^2, respectively. One obvious feature is that these physical quantities relate only to two essential variables V and L.

The numbers Π_9 and Π_{10} can be interpreted as the ratio of structural energy E and impulse I to characteristic quantities $\rho L^3 V^2$ and $\rho L^3 V$, respectively.

The number Π_{11} can be interpreted as the ratio of mass M (both structural mass M' and impact mass G) to structural characteristic mass ρL^3, which means the mass of each object in an impact problem can be scaled.

The number Π_{12} can be interpreted as the ratio of various geometrical sizes H' to the characteristic length L, which indicates that geometric dimensions at different directions can be scaled in the same ratio, so that the scaled model maintains the similarity of geometrical configuration with the prototype.

The number Π_{13} can be interpreted as the ratio of displacement δ to the characteristic length L, which means that dimensionless deformation of the scaled model and those of the prototype should remain unchanged.

The numbers Π_{14} and Π_{15} can be interpreted as an invariance property of strain ε and generalized strain of angle θ for the deformed similarity. It should be noted that these two numbers should be dimensionless in nature, which in any case of scaling must be guaranteed to be constant.

From the above explanation, the advantages of DLV system are: (1) Two important numbers, Johnson's damage number D_n and Zhao's response number R_n, are directly included in the 15 dimensionless numbers, which has the ability to measure the damage and the response of impacted structures. (2) Since the density is used as one basis in DLV system instead of the impact mass in VSG system, the effect of different material densities on the scaling behavior of the structures can be directly expressed. At the same time, the various impact loadings (the initial velocity of concentrated mass V_0 or the impulsive velocity $\bar{V}_0$) would be available in DLV system. (3) These dimensionless numbers are all constructed from elementary definition of physical quantities, e.g., Π_5 in DLV system is constructed by strain rate $\dot{\varepsilon}$ with the simple form of V/L (characteristic strain rate of structures) rather than $\dot{\varepsilon}$ with the complex form of $(\sigma_d V_0/G)^{1/3}$ in VSG system. (4) 18 basic physical quantities and 15 dimensionless numbers are included in DLV system more than 11 basic physical quantities and 8 dimensionless numbers in VSG system.

Since the above derivation does not presuppose the concrete forms of any structures, these numbers are applicable to both simple structures (like beams, plates and shells) and complex structures (such as stiffened plates and stiffened shells), when they are subjected to impact loading. To observe main characters of the scaled behavior of impact problems, rigid–plastic materials with the effects of strain hardening and strain rate sensitivity are considered.

3 Scaling factors

Since the newly proposed DLV dimensionless system based on the density, the length and the velocity, three scaling factors of the density scaling factor β_ρ, the geometric scaling factor β and the velocity scaling factor $\beta_{\rm V}$ are immediately and directly used to express the factors of other physical quantities.

For perfect structural similarity, all dimensionless numbers of the scaled model must be equal to those of the prototype, which lead

$$\frac{(\Pi_1)_{\rm m}}{(\Pi_1)_{\rm p}} = \frac{\beta_\rho \beta_{\rm V}^2}{\beta_{\sigma_{\rm d}}} = 1 \rightarrow \beta_{\sigma_{\rm d}} = \beta_\rho \beta_{\rm V}^2 \,, \tag{9}$$

$$\frac{(\Pi_2)_{\rm m}}{(\Pi_2)_{\rm p}} = \frac{\beta_\rho \beta^2 \beta_{\rm V}^2}{\beta_F} = 1 \rightarrow \beta_F = \beta_\rho \beta^2 \beta_{\rm V}^2 \,, \tag{10}$$

$$\frac{(\Pi_3)_{\rm m}}{(\Pi_3)_{\rm p}} = \frac{\beta_\rho \beta^3 \beta_{\rm V}^2}{\beta_\Psi} = 1 \rightarrow \beta_\Psi = \beta_\rho \beta^3 \beta_{\rm V}^2 \,, \tag{11}$$

$$\frac{(\Pi_4)_{\rm m}}{(\Pi_4)_{\rm p}} = \frac{\beta_{\rm t} \beta_{\rm V}}{\beta} = 1 \rightarrow \beta_{\rm t} = \beta / \beta_{\rm V} \,, \tag{12}$$

$$\frac{(\Pi_5)_{\rm m}}{(\Pi_5)_{\rm p}} = \frac{\beta_{\dot\varepsilon} \beta}{\beta_{\rm V}} = 1 \rightarrow \beta_{\dot\varepsilon} = \beta_{\rm V} / \beta \,, \tag{13}$$

$$\frac{(\Pi_6)_{\rm m}}{(\Pi_6)_{\rm p}} = \frac{\beta_A \beta}{\beta_{\rm V}^2} = 1 \rightarrow \beta_A = \beta_{\rm V}^2 / \beta \,, \tag{14}$$

$$\frac{(\Pi_7)_{\rm m}}{(\Pi_7)_{\rm p}} = \frac{\beta_{\dot\theta} \beta}{\beta_{\rm V}} = 1 \rightarrow \beta_{\dot\theta} = \beta_{\rm V} / \beta \,, \tag{15}$$

$$\frac{(\Pi_8)_{\rm m}}{(\Pi_8)_{\rm p}} = \frac{\beta_{\ddot\theta} \beta^2}{\beta_{\rm V}^2} = 1 \rightarrow \beta_{\ddot\theta} = \beta_{\rm V}^2 / \beta^2 \,, \tag{16}$$

$$\frac{(\Pi_9)_{\rm m}}{(\Pi_9)_{\rm p}} = \frac{\beta_{\rm E}}{\beta_\rho \beta^3 \beta_{\rm V}^2} = 1 \rightarrow \beta_{\rm E} = \beta_\rho \beta^3 \beta_{\rm V}^2 \,, \tag{17}$$

$$\frac{(\Pi_{10})_{\rm m}}{(\Pi_{10})_{\rm p}} = \frac{\beta_I}{\beta_\rho \beta^3 \beta_{\rm V}} = 1 \rightarrow \beta_I = \beta_\rho \beta^3 \beta_{\rm V} \,, \tag{18}$$

$$\frac{(\Pi_{11})_{\rm m}}{(\Pi_{11})_{\rm p}} = \frac{\beta_{\rm M}}{\beta_\rho \beta^3} = 1 \rightarrow \beta_{\rm M} = \beta_\rho \beta^3 \,, \tag{19}$$

$$\frac{(\Pi_{12})_{\rm m}}{(\Pi_{12})_{\rm p}} = \frac{\beta_{H'}}{\beta} = 1 \rightarrow \beta_{H'} = \beta \,, \tag{20}$$

$$\frac{(\Pi_{13})_{\rm m}}{(\Pi_{13})_{\rm p}} = \frac{\beta_\delta}{\beta} = 1 \rightarrow \beta_\delta = \beta \,, \tag{21}$$

$$\frac{(\Pi_{14})_{\rm m}}{(\Pi_{14})_{\rm p}} = \beta_\varepsilon = 1 \,, \tag{22}$$

and

$$\frac{(\Pi_{15})_{\rm m}}{(\Pi_{15})_{\rm p}} = \beta_\theta = 1 \,. \tag{23}$$

Equations (9)–(23) show that all physical quantities can be directly expressed by the three basic scaling factors β_ρ, β and $\beta_{\rm V}$. The main scaling factors of physical variable for the DLV system are listed in Table 3. Since the expressions of scaling factor in Eqs. (9), (10), (12), (13), (14), (17), (19), (21) and (22) are completely identical with those in Table 2, the DLV system would have the same ability with the VSG to deal with the non-scaling problems for different materials and strain-rate-sensitive materials. However, the derivation procedure of these factors in DLV system is very simple without any intermediate steps, while the same procedure in VSG system resorts to some extra relations. For example, the factor $\beta_{\sigma_{\rm d}} = \beta_{\rm V}^2$ can be obtained through the

Table 3 Main scaling factors of structural impact in DLV system

Variable	Scaling factor	Variable	Scaling factor
Density, ρ	$\beta_\rho = \rho_\mathrm{m}/\rho_\mathrm{p}$	Angular velocity, $\dot{\theta}$	$\beta_{\dot{\theta}} = \beta_\mathrm{V}/\beta$
Characteristic length, L	$\beta = L_\mathrm{m}/L_\mathrm{p}$	Angular acceleration, $\ddot{\theta}$	$\beta_{\ddot{\theta}} = \beta_\mathrm{V}^2/\beta^2$
Velocity, V	$\beta_\mathrm{V} = V_\mathrm{m}/V_\mathrm{p}$	Energy, E	$\beta_\mathrm{E} = \beta_\rho \beta^3 \beta_\mathrm{V}^2$
Stress, σ	$\beta_\sigma = \beta_\rho \beta_\mathrm{V}^2$	Impulse, I	$\beta_I = \beta_\rho \beta^3 \beta_\mathrm{V}$
Force, F	$\beta_F = \beta_\rho \beta^2 \beta_\mathrm{V}^2$	Mass, M	$\beta_\mathrm{M} = \beta_\rho \beta^3$
Bending moment, Ψ	$\beta_\Psi = \beta_\rho \beta^3 \beta_\mathrm{V}^2$	Geometrical characteristic, H'	$\beta_{H'} = \beta$
Time, t	$\beta_\mathrm{t} = \beta/\beta_\mathrm{V}$	Displacement, δ	$\beta_\delta = \beta$
Strain rate, $\dot{\varepsilon}$	$\beta_{\dot{\varepsilon}} = \beta_\mathrm{V}/\beta$	Strain, ε	$\beta_\varepsilon = 1$
Acceleration, A	$\beta_A = \beta_\mathrm{V}^2/\beta$	Angle, θ	$\beta_\theta = 1$

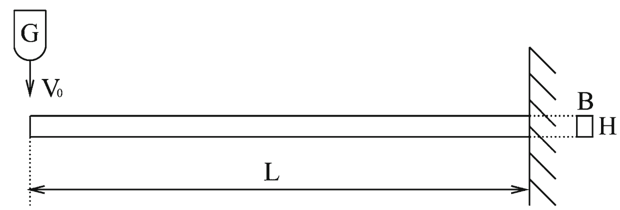

Fig. 1 A cantilever beam subject to impact mass at the free end

direct derivation of Eq. (9) in DLV system when considering the case using same material (i.e., $\beta_\rho = 1$), while in VSG system the derivation would be performed through the dimensionless number $\frac{\delta^3\sigma_\mathrm{d}}{GV_0^2}$ with the procedure of $\left(\delta^3\sigma_\mathrm{d}/GV_0^2\right)_\mathrm{m} / \left(\delta^3\sigma_\mathrm{d}/GV_0^2\right)_\mathrm{p} = \beta_\delta^3 \beta_{\sigma_\mathrm{d}}/\beta_G \beta_\mathrm{V}^2 = \beta^3 \beta_{\sigma_\mathrm{d}}/\beta^3 \beta_\mathrm{V}^2 = 1$ which resorts to two extra scaling relations $\beta_\delta = \beta$ and $\beta_G = \beta^3$ [2].

4 Verification

In this section, four simple typical beam models subjected to mass impact or impulsive velocity are chosen to verify the features of the DLV dimensionless system in directly matching the dimensionless expression of response equations and their corresponding scaling analysis.

4.1 Impact of a mass on a cantilever beam

The first structure we studied is a cantilever with length of L, width of B and height of H, and it is struck at the free end by a concentrated mass G with an initial impact velocity V_0, as shown in Fig.1.

4.1.1 Response equations

Parkes [19,20] carried out the model-based theoretical research on a perfectly plastic material. The response function of the final displacement W_f at the free end was given as

$$W_\mathrm{f} = \frac{\rho' V_0^2 L^2 \gamma^2}{3\varphi_0}\left[\frac{1}{1+2\gamma} + 2\ln\left(1 + \frac{1}{2\gamma}\right)\right], \quad (24)$$

where $\rho' = \rho BH$ is the material density per unit length, $\varphi_0 = \sigma_\mathrm{d} BH^2/4$ is the fully plastic bending moment and $\gamma = G/\rho' L$ is the mass ratio of the concentrated mass to the structure mass of the cantilever. The final rotation angle at the root is given as

$$\theta_{\mathrm{f}}=\frac{\rho' L V_0^2}{6\varphi_0}(1+3\gamma)\left(1+\frac{1}{2\gamma}\right)^{-2}. \tag{25}$$

The response equation of final time is given as

$$T_{\mathrm{f}}=\frac{G V_0 L}{\varphi_0}. \tag{26}$$

4.1.2 Dimensionless expression

Firstly, we rewrite the response equation of Eq. (24) to a dimensionless form:

$$\frac{W_{\mathrm{f}}}{L}=\frac{4}{3}\frac{\rho V_0^2}{\sigma_{\mathrm{d}}}\frac{L}{H}\gamma^2\left[\frac{1}{1+2\gamma}+2\ln\left(1+\frac{1}{2\gamma}\right)\right]. \tag{27}$$

If we regard the term of $\rho V_0^2/\sigma_{\mathrm{d}}$ as Π_1, the term of H/L as Π_{12}, the term of $\gamma=G/\rho LBH$ as Π_{11} and the term of W_{f}/L as Π_{13}, Eq. (27) reflects a functional relationship among the dimensionless numbers Π_1, Π_{12}, Π_{11} and Π_{13}.

Secondly, we rewrite the final rotation angle equation of Eq. (25) to a dimensionless form:

$$\theta_{\mathrm{f}}=\frac{2}{3}\frac{\rho V_0^2}{\sigma_{\mathrm{d}}}\frac{L}{H}(1+3\gamma)\left(1+\frac{1}{2\gamma}\right)^{-2}. \tag{28}$$

If we regard the term of $\rho V_0^2/\sigma_{\mathrm{d}}$ as Π_1, the term of H/L as Π_{12}, the term of $\gamma=G/\rho LBH$ as Π_{11} and the term of θ_{f} as Π_{15}, Eq. (28) reflects a functional relationship among the dimensionless numbers Π_1, Π_{12}, Π_{11} and Π_{15}.

Thirdly, we rewrite the final time equation of Eq. (26) to a dimensionless form:

$$\frac{T_{\mathrm{f}} V_0}{L}=4\frac{\rho V_0^2}{\sigma_{\mathrm{d}}}\frac{L}{H}\gamma. \tag{29}$$

If we regard the term of $\rho V_0^2/\sigma_{\mathrm{d}}$ as Π_1, the term of H/L as Π_{12}, the term of $\gamma=G/\rho LBH$ as Π_{11} and the term of $T_{\mathrm{f}} V_0/L$ as Π_4, Eq. (29) reflects a functional relationship among the dimensionless numbers Π_1, Π_{12}, Π_{11} and Π_4.

The above analysis shows that the numbers $\Pi_1=\rho V_0^2/\sigma_{\mathrm{d}}$, $\Pi_{12}=H/L$ and $\Pi_{11}=G/\rho LBH$ govern the final deformation response $\Pi_{13}=W_{\mathrm{f}}/L$, the rotation angle response $\Pi_{15}=\theta_{\mathrm{f}}$ and the final time response $\Pi_4=T_{\mathrm{f}} V_0/L$.

From the equation analysis of the cantilever model, the DLV dimensionless number system shows its property to directly match dimensionless response equation of Eqs. (27), (28) and (29). However, in VSG system the term of W_{f} in Eq. (24) would be rewritten to the form of $W_{\mathrm{f}}/\left(G V_0^2/\sigma_{\mathrm{d}}\right)^{1/3}$, which is a little complicated, unnatural and difficult to be understood. The main reason for the different rewritten forms is the lacking of essential physical quantities of density and length in the VSG basis.

4.1.3 Scaling analysis

For the scaled model and the prototype, Eq. (27) can be written as

$$\frac{(W_{\mathrm{f}})_{\mathrm{m}}}{L_{\mathrm{m}}}=\frac{4}{3}\frac{\rho_{\mathrm{m}}(V_0)_{\mathrm{m}}^2}{(\sigma_{\mathrm{d}})_{\mathrm{m}}}\frac{L_{\mathrm{m}}}{H_{\mathrm{m}}}\gamma_{\mathrm{m}}^2\left[\frac{1}{1+2\gamma_{\mathrm{m}}}+2\ln\left(1+\frac{1}{2\gamma_{\mathrm{m}}}\right)\right] \tag{30}$$

and

$$\frac{(W_{\mathrm{f}})_{\mathrm{p}}}{L_{\mathrm{p}}}=\frac{4}{3}\frac{\rho_{\mathrm{p}}(V_0)_{\mathrm{p}}^2}{(\sigma_{\mathrm{d}})_{\mathrm{p}}}\frac{L_{\mathrm{p}}}{H_{\mathrm{p}}}\gamma_{\mathrm{p}}^2\left[\frac{1}{1+2\gamma_{\mathrm{p}}}+2\ln\left(1+\frac{1}{2\gamma_{\mathrm{p}}}\right)\right], \tag{31}$$

respectively.

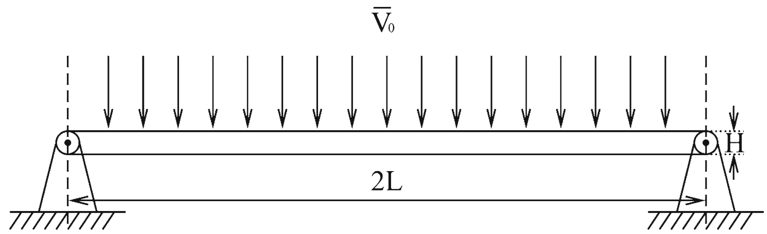

Fig. 2 A simply supported beam subject to impulsive velocity

Substituting $\beta_\rho = \rho_m/\rho_p$, $\beta = L_m/L_p = B_m/B_p = H_m/H_p$, $\beta_V = (V_0)_m / (V_0)_p$, $\beta_\gamma = \gamma_m/\gamma_p = (G_m/\rho_m L_m B_m H_m) / \left(G_p/\rho_p L_p B_p H_p\right) = \beta_G/\beta_\rho\beta^3$, $\beta_{\sigma_d} = (\sigma_d)_m / (\sigma_d)_p$ and $\beta_{W_f} = (W_f)_m / (W_f)_p$ into Eq. (30), it becomes

$$\begin{aligned} \frac{\beta_{W_f}}{\beta}\frac{(W_f)_p}{L_p} = \frac{\beta_\rho\beta_V^2}{\beta_{\sigma_d}}\left(\frac{\beta_G}{\beta_\rho\beta^3}\right)^2\left[\frac{4}{3}\frac{\rho_p(V_0)_p^2}{(\sigma_d)_p}\frac{L_p}{H_p}\gamma_p^2\right]\times \\ \left[\frac{1}{1+2\gamma_p\frac{\beta_G}{\beta_\rho\beta^3}} + 2\ln\left(1+\frac{1}{2\gamma_p\frac{\beta_G}{\beta_\rho\beta^3}}\right)\right]. \end{aligned} \tag{32}$$

The combination of Eqs. (31) and (32) gives the scaling relations $\beta_G/\beta_\rho\beta^3 = 1$, $\beta_\rho\beta_V^2/\beta_{\sigma_d} = 1$ and $\beta_{W_f}/\beta = 1$ for Eq. (27), which are in full accord with Eqs. (19), (9) and (21), respectively.

The analysis of Eqs. (30)–(32) shows that the scaling procedure based on DLV numbers is simple and intuitive. Further, we can obtain the scaling relations for an impact model more directly through writing the DLV numbers into their corresponding scaling factors.

In the same way, the scaling relations for Eqs. (28) and (29) can be obtained, which lead to other two scaling relations $\beta_{\theta_f} = 1$ and $\beta_T\beta_V/\beta = 1$, respectively.

When we conduct a scaling testing for this cantilever, the input parameters consisting of three aspects of the material properties (including material density ρ and dynamic flow stress σ_d), the geometry size (including the length L, the width B and the height H) and the external loads (including impact mass G and its initial velocity V_0) could be considered. It is obvious that when the scaling relations of these input parameters satisfy $\beta = \beta_L$, $\beta_B = \beta$, $\beta_H = \beta$, $\beta_G = \beta_\rho\beta^3$ and $\beta_V = \sqrt{\beta_{\sigma_d}/\beta_\rho}$, the final responses of the beam should be completely scaled by the relations of $\beta_{W_f} = \beta$, $\beta_{\theta_f} = 1$ and $\beta_T = \beta/\beta_V$.

The above scaling procedure actually adjusts the impact velocity of scaled model by the factor $\beta_V = \sqrt{\beta_{\sigma_d}/\beta_\rho}$. However, when the impact velocity of scaled model does not satisfy the factor, we can use the method of adjusting density by the factor $\beta_\rho = \beta_{\sigma_d}/\beta_V^2$ to conduct the scaling testing. In this case, the velocity factor β_V is arbitrary and the density factor β_ρ can be determined by β_V and β_{σ_d}. Then, the scaling procedure becomes: if the scaling relations of input parameters satisfy $\beta = \beta_L$, $\beta_B = \beta$, $\beta_H = \beta$, $\beta_\rho = \beta_{\sigma_d}/\beta_V^2$ and $\beta_G = \beta_\rho\beta^3 = \beta_{\sigma_d}\beta^3/\beta_V^2$, the final responses of the beam should be completely scaled by the relations of $\beta_{W_f} = \beta$, $\beta_{\theta_f} = 1$ and $\beta_T = \beta/\beta_V$. The factor $\beta_G = \beta_\rho\beta^3 = \beta_{\sigma_d}\beta^3/\beta_V^2$ means the impact mass also needs to be adjusted according to the factor $\beta_\rho = \beta_{\sigma_d}/\beta_V^2$. It is obvious that the adjusting density includes the adjustment of structural mass and impact mass.

As can be seen from the above scaling analysis, the factors β_ρ, β and β_V are three basic scaling factors to control the scaling procedure in essence, which further proves that the DLV base could be the most essential base. In addition, compared with the previous methods adjusting impact velocity in Refs. [2–4,6], adjusting impact mass in Ref. [5] or adjusting the structural mass and the impact mass or impact velocity in Ref. [10], the presented scaling analysis further expands the scope by simultaneously adjusting impact velocity and density.

4.2 Simply supported beam subjected to impulsive loading

The second structure to be verified is a simply supported rectangular cross-section beam with a length of $2L$, height of H and initial impulsive velocity $\bar{V}_0$ on the entire span, as shown in Fig.2.

4.2.1 Response equations

Zhao [21] carried out the model-based theoretical research on a perfectly plastic material and taking into account the influence of finite displacements. The response function of the final maximum dimensionless displacement W_f/H at the mid-span is given as

$$\frac{W_\mathrm{f}}{H}=\frac{1}{4}\left\{\left[\frac{1}{2}+8\frac{\rho\bar{V}_0^2L^2}{\sigma_\mathrm{d}H^2}+\frac{1}{2}\left(1+\frac{32}{3}\frac{\rho\bar{V}_0^2L^2}{\sigma_\mathrm{d}H^2}\right)^{1/2}\right]^{1/2}-1\right\}. \tag{33}$$

And the response equation of time is given as

$$T=\frac{H}{8\bar{V}_0}\left[\left(1+\frac{32}{3}\frac{\rho\bar{V}_0^2L^2}{\sigma_\mathrm{d}H^2}\right)^{1/2}-1\right]. \tag{34}$$

4.2.2 Dimensionless expression and scaling analysis

Firstly, we rewrite the response equation of Eq. (33) to a dimensionless form:

$$\frac{W_\mathrm{f}}{L}=\frac{1}{4}\frac{H}{L}\left\{\left[\frac{1}{2}+8\frac{\rho\bar{V}_0^2}{\sigma_\mathrm{d}}\left(\frac{L}{H}\right)^2+\frac{1}{2}\left(1+\frac{32}{3}\frac{\rho\bar{V}_0^2}{\sigma_\mathrm{d}}\left(\frac{L}{H}\right)^2\right)^{1/2}\right]^{1/2}-1\right\}. \tag{35}$$

Similar to Eq. (27), if we regard the term of $\rho\bar{V}_0^2/\sigma_\mathrm{d}$ as Π_1, the term of H/L as Π_{12} and the term of W_f/L as Π_{13}, Eq. (35) reflects a functional relationship among the dimensionless numbers Π_1, Π_{12} and Π_{13}.

Secondly, we rewrite the time equation of Eq. (34) to a dimensionless form:

$$\frac{T\bar{V}_0}{L}=\frac{1}{8}\frac{H}{L}\left\{\left[1+\frac{32}{3}\frac{\rho\bar{V}_0^2}{\sigma_\mathrm{d}}\left(\frac{L}{H}\right)^2\right]^{1/2}-1\right\}. \tag{36}$$

Similar to Eq. (29), if we regard the term of $\rho\bar{V}_0^2/\sigma_\mathrm{d}$ as Π_1, the term of H/L as Π_{12} and the term of $T\bar{V}_0/L$ as Π_4, Eq. (36) reflects a functional relationship between the dimensionless numbers Π_1, Π_{12} and Π_4.

When we conduct a scaling testing for this simply supported beam with input parameters of three aspects as the first cantilever model, the final responses should be completely scaled by the relations of $\beta_{W_\mathrm{f}}=\beta$ and $\beta_T=\beta/\beta_\mathrm{V}$ if the relations of input parameters satisfy (1) $\beta=\beta_L$, $\beta_H=\beta$ and $\beta_\mathrm{V}=\sqrt{\beta_{\sigma_\mathrm{d}}/\beta_\rho}$ or (2) $\beta=\beta_L$, $\beta_H=\beta$ and $\beta_\rho=\beta_{\sigma_\mathrm{d}}/\beta_\mathrm{V}^2$. The differences between these two models are the input parameters and its scaling relations to the impact mass G.

4.3 Clamped beam subject to impulsive velocity

The third structure to be verified is a clamped rectangular cross-section beam with a length of $2L$, height of H and initial impulsive velocity $\bar{V}_0$ on the entire span, as shown in Fig.3.

4.3.1 Response equations

Jones [1] carried out the model-based theoretical research on a perfectly plastic material and taking into account the influence of finite displacements. The response function of the final maximum dimensionless displacement W_f/H at the mid-span is given as

$$\frac{W_\mathrm{f}}{H}=\frac{1}{2}\left[\left(1+\frac{3\rho\bar{V}_0^2L^2}{\sigma_\mathrm{d}H^2}\right)^{1/2}-1\right]. \tag{37}$$

And the average strain rate equation is given as

$$\dot{\varepsilon}=\frac{\bar{V}_0W_\mathrm{f}}{3\sqrt{2}L^2}. \tag{38}$$

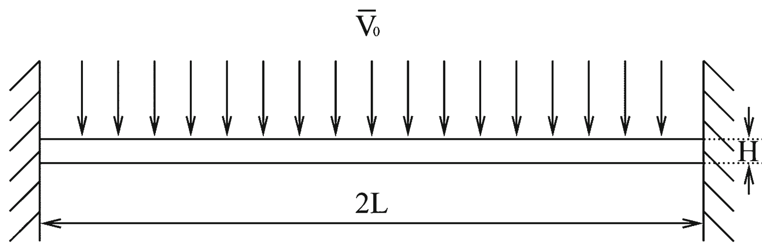

Fig. 3 A clamped beam subject to impulsive velocity

4.3.2 Dimensionless expression and scaling analysis

Firstly, we rewrite the response equation of Eq. (37) to a dimensionless form:

$$\frac{W_\mathrm{f}}{L} = \frac{1}{2}\frac{H}{L}\left[\left(1 + 3\frac{\rho \bar{V}_0^2}{\sigma_\mathrm{d}}\left(\frac{L}{H}\right)^2\right)^{1/2} - 1\right]. \tag{39}$$

Similar functional relationship among the dimensionless numbers Π_1, Π_{12} and Π_{13} can be found for the impulsive velocity loading.

Secondly, we rewrite the strain rate equation of Eq. (38) to a dimensionless form:

$$\frac{\dot{\varepsilon} L}{\bar{V}_0} = \frac{1}{3\sqrt{2}}\frac{W_\mathrm{f}}{L}. \tag{40}$$

And substituting Eq. (39) into Eq. (40), it becomes

$$\frac{\dot{\varepsilon} L}{\bar{V}_0} = \frac{1}{6\sqrt{2}}\frac{H}{L}\left[\left(1 + 3\frac{\rho \bar{V}_0^2}{\sigma_\mathrm{d}}\left(\frac{L}{H}\right)^2\right)^{1/2} - 1\right]. \tag{41}$$

If we regard the term of $\rho \bar{V}_0^2/\sigma_\mathrm{d}$ as Π_1, the term of H/L as Π_{12} and the term of $\dot{\varepsilon} L/\bar{V}_0$ as Π_5, Eq. (41) reflects a functional relationship among the dimensionless numbers Π_1, Π_{12} and Π_5. The strain rate-related dimensionless number Π_5 appears.

The above analysis shows that the numbers $\Pi_1 = \rho \bar{V}_0^2/\sigma_\mathrm{d}$ and $\Pi_{12} = H/L$ govern the final deformation response $\Pi_{13} = W_\mathrm{f}/L$ and the strain rate response $\Pi_5 = \dot{\varepsilon} L/\bar{V}_0$. When we conduct a scaling testing for this clamped beam, if the relations of input parameters satisfy (1) $\beta = \beta_L$, $\beta_H = \beta$ and $\beta_\mathrm{V} = \sqrt{\beta_{\sigma_\mathrm{d}}/\beta_\rho}$ or (2) $\beta = \beta_L, \beta_H = \beta$ and $\beta_\rho = \beta_{\sigma_\mathrm{d}}/\beta_\mathrm{V}^2$, the final responses should be completely scaled by the relations of $\beta_{W_\mathrm{f}} = \beta$ and $\beta_{\dot{\varepsilon}} = \beta_\mathrm{V}/\beta$. For strain-rate-sensitive materials materials, $\beta_{\sigma_\mathrm{d}} = (\sigma_\mathrm{d})_\mathrm{m} / (\sigma_\mathrm{d})_\mathrm{p} = f_\mathrm{m}(\dot{\varepsilon}_\mathrm{m})/f_\mathrm{p}(\dot{\varepsilon}_\mathrm{p}) = f_\mathrm{m}\left(\dot{\varepsilon}_\mathrm{p}\frac{\beta_\mathrm{V}}{\beta}\right)/f_\mathrm{p}(\dot{\varepsilon}_\mathrm{p})$ by substituting factor $\beta_{\dot{\varepsilon}} = \beta_\mathrm{V}/\beta$ into material constitutive equation $\sigma_\mathrm{d} = f(\dot{\varepsilon})$.

It is also good to find that the input and output relations of this clamped beam for the strain-rate-sensitive material were verified by the numerical calculation in Refs. [2,4], which used the method of adjusting impact velocity of scaled model by the factor $\beta_\mathrm{V} = \sqrt{\beta_{\sigma_\mathrm{d}}/\beta_\rho}$. The presented scaling analysis further uses the method of adjusting density of scale model by the factor $\beta_\rho = \beta_{\sigma_\mathrm{d}}/\beta_\mathrm{V}^2$.

4.4 Clamped beam struck at mid-span

The fourth structure we studied is a rectangular cross section beam with a length of $2L$, width of B and height of H, and it is struck at mid-span by a concentrated mass G with an initial impact velocity V_0, as shown in Fig. 4.

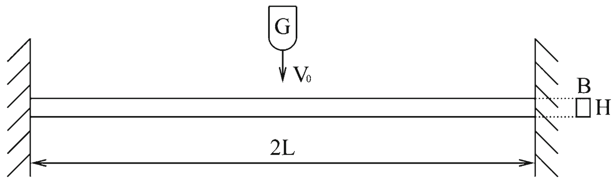

Fig. 4 A clamped beam subject to impact mass

4.4.1 Response equations

Liu and Jones [22] carried out the beam struck at any point of the span on a perfectly plastic material and taking into account the influence of the finite displacements. For the case of a large mass G (relative to small mass of the beam) at mid-span, the final maximum dimensionless displacement W_{f}/H at the loading point is expressed as

$$\frac{W_{\mathrm{f}}}{H}=\frac{1}{2}\left[\left(1+\frac{2GV_0^2L}{BH^3\sigma_{\mathrm{d}}}\right)^{1/2}-1\right]. \tag{42}$$

As for true equivalent strain, the expression at any point on the span was derived by Alves and Jones [23]. And it can be expressed at the mid-span as follows:

$$\varepsilon_{eq}=\begin{cases}\sqrt{\left(\frac{3W_H}{\gamma'^2}\right)^2+\left(\frac{4k'}{\sqrt{3}}\frac{W_H}{\gamma'}\right)^2}, & \text{for } W_{\mathrm{f}}/H\le 1\\ \sqrt{\left\{\frac{1}{2}\left[\left(\frac{W_H}{\gamma'}\right)^2+\frac{5}{\gamma'^2}\right]\right\}^2+\left(\frac{4k'}{\sqrt{3}}\frac{W_H}{\gamma'}\right)^2}, & \text{for } W_{\mathrm{f}}/H>1\end{cases}, \tag{43}$$

where $\gamma'=L/H$, $W_H=W_{\mathrm{f}}/H$ and k' is a dimensionless constant that accounts the influence of transverse shear and is assumed to be same for different materials.

The expression of the equivalent strain rate was adopted in Refs. [4,23], and it can be expressed at mid-span as follows:

$$\dot{\varepsilon}_{eq}=\begin{cases}\frac{V_0}{L}\left[\frac{9}{2}\left(\frac{H}{L}\right)^2+\frac{8k'^2}{3}\right]^{1/2}, & \text{for } W_{\mathrm{f}}/H\le 1\\ \frac{V_0}{L}\left[\frac{1}{2}\left(\frac{W_{\mathrm{f}}}{L}\right)^2+\frac{8k'^2}{3}\right]^{1/2}, & \text{for } W_{\mathrm{f}}/H>1\end{cases}. \tag{44}$$

4.4.2 Dimensionless expression and scaling analysis

It can be learned from the above three models, for the concentrated mass impact loading, the beam response equations of deformation, strain and strain rate would reflect functional relations between numbers Π_{13}, Π_{14}, Π_5 and numbers Π_1, Π_{11}, Π_{12} respectively, which could be verified easily by the same equation analysis for Eqs. (42)–(44). For the scaling relations, if input parameters satisfy (1) $\beta=\beta_L$, $\beta_B=\beta$, $\beta_H=\beta$, $\beta_G=\beta_\rho\beta^3$ and $\beta_{\mathrm{V}}=\sqrt{\beta_{\sigma_{\mathrm{d}}}/\beta_\rho}$ or (2) $\beta=\beta_L$, $\beta_B=\beta$, $\beta_H=\beta$, $\beta_\rho=\beta_{\sigma_{\mathrm{d}}}/\beta_{\mathrm{V}}^2$ and $\beta_G=\beta_\rho\beta^3=\beta_{\sigma_{\mathrm{d}}}\beta^3/\beta_{\mathrm{V}}^2$, the final responses of the beam should be completely scaled with the relation of $\beta_{W_{\mathrm{f}}}=\beta$, $\beta_\varepsilon=1$ and $\beta_{\dot{\varepsilon}}=\beta_{\mathrm{V}}/\beta$. For strain-rate-sensitive materials and strain-hardening materials, $\beta_{\sigma_{\mathrm{d}}}=(\sigma_{\mathrm{d}})_{\mathrm{m}}/(\sigma_{\mathrm{d}})_{\mathrm{p}}=f_{\mathrm{m}}(\varepsilon_{\mathrm{m}},\dot{\varepsilon}_{\mathrm{m}})/f_{\mathrm{p}}(\varepsilon_{\mathrm{p}},\dot{\varepsilon}_{\mathrm{p}})=f_{\mathrm{m}}\left(\varepsilon_{\mathrm{p}},\dot{\varepsilon}_{\mathrm{p}}\frac{\beta_{\mathrm{V}}}{\beta}\right)/f_{\mathrm{p}}(\varepsilon_{\mathrm{p}},\dot{\varepsilon}_{\mathrm{p}})$ by substituting the factors $\beta_\varepsilon=1$ and $\beta_{\dot{\varepsilon}}=\beta_{\mathrm{V}}/\beta$ into material constitutive equation $\sigma_{\mathrm{d}}=f(\varepsilon,\dot{\varepsilon})$. It is also good to find that the input and output relations of this beam for considering strain-rate-sensitive material have been verified by numerical calculation in Ref. [4], which used the method of adjusting impact velocity of the scaled model by the factor $\beta_{\mathrm{V}}=\sqrt{\beta_{\sigma_{\mathrm{d}}}/\beta_\rho}$. This model in addition proves the applicable relations for simultaneously considering the effect of material strain hardening by the equation analysis. And the method of adjusting density of scaled model is further used by the factor $\beta_\rho=\beta_{\sigma_{\mathrm{d}}}/\beta_{\mathrm{V}}^2$ in the presented scaling analysis.

From the above four examples, the structure dynamic responses of deformation, angle, time, strain and strain rate of different impact problems all could be presented directly to dimensionless expression by the newly proposed dimensionless number system for different loadings of concentrated mass impact and impulsive velocity. At the same time, when we conduct a scaling testing, the scaling relations of input and output parameters can be simultaneously obtained by writing these dimensionless numbers to the form of scaling factors. In addition, the DLV base provides three essential scaling factors, i.e., the density factor β_ρ, the geometric factor β and the velocity factor β_V. When the geometric configuration is scaled by the geometric factor β, the density factor β_ρ and the velocity factor β_V can be adjusted simultaneously to address the non-scaling problems for different materials and strain-rate-sensitive materials.

5 Conclusions

A new group of dimensionless numbers termed DLV system is suggested in this paper in order to represent the scaled behavior of impacted structures for rigid–plastic materials with strain-hardening effects and strain rate sensitivity. It is obtained by means of Buckingham Π theorem with density–length–velocity as the essential basis and is verified by four classical impacted models with equation analysis. Compared with the previous dimensionless system, the results have shown the advancements of this group of numbers in clear physical significance, naturally including the well-known damage number and response number, the directly matching dimensionless expression of response equations, the addressing ability of the non-scalability through adjusting impact velocity of the scaled model or adjusting density of the scaled model, etc. At the same time, the numbers in DLV system have been proved to be very important in dimensionless expression and scaling analysis for the structure dynamic response. Because of its simple, clear and efficient properties, this newly proposed DLV (density–length–velocity) dimensionless number system is suggested to be an alternative system to represent the scaled behavior of structures under impact loads.

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