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On the classification of image features

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Abstract: While the primary purpose of edge detection schemes is to be able to produce an edge map of a given image, the ability to distinguish between different feature types is also of importance. In this paper we examine feature classification based on local energy detection and show that local energy measures are intrinsically capable of making this classification because of the use of odd and even filters. The advantage of feature classification is that it allows for the elimination of certain feature types from the edge map, thus simplifying the task of object recognition.

Key words: Edge detection, edge classification, Hilbert transform, local energy.

1. Introduction

The extraction of primitive features in an image is believed to form the first stage in the process of seeing and recognizing an object. Some of these primitive features are considered to be luminance edges, bars or lines, colour and motion. The process of recognition of the object is then based on the construction of a model for the object from the extracted primitives.

One interesting set of primitive features in an image is the class of features that are manifested due to local changes in image luminance or intensity. The profile of the luminance intensity will then decide the *type* of feature at that point. For example, some of the common feature profiles are step profiles, roof or triangular profiles, trapezoidal profiles, bars and spots (characteristic functions on line and point sets).

While the primary role of edge detectors is to produce an edge map from the original image, it is useful to be able to distinguish between the several possible feature types. If, as an extension of an edge detection algorithm, it is possible to classify the detected edges into further distinct feature types, this information can then be used as a basis for the reconstruction of the original image. Also, it is then possible to eliminate certain feature types from the edge map selectively. For instance, the penumbral portion of shadows usually appears as the ramp portion of a trapezoidal image intensity profile and therefore it is possible to eliminate this portion of a shadow from an image profile once it has been distinguished from the other features. It should be noted however that there is no easy way of distinguishing between a trapezoidal profile produced by the contours of an object and a similar profile produced by shadows; knowledge of the position of the light source, or some further information, must be used in this case.

Most currently available schemes for edge detection are based on the detection of the maxima of the first derivative or the zero crossings of the second derivative of the image intensity function. These methods have an intrinsic ability to differentiate between edge types. Canny (1983) uses different optimal operators to classify different edge types, whilst in the Marr–Hildreth (1980) technique of edge detection, an analysis of the slope of the zero crossing can be used for further feature classification.

In this paper, we examine the ability of local energy feature detection to distinguish between feature types. A brief description of these operators follows. Based on physiological evidence, a local energy feature detector (Morrone and Owens, 1987; Morrone and Burr, 1988; Owens et al., 1989) was proposed. If $F(x)$ represents the image intensity function of a one-dimensional luminance profile and $H(x)$ represents the Hilbert transform of $F(x)$, the local energy function E is defined as

$$E(x) = F^2(x) + H^2(x) \tag{1}$$

and the Argument value Arg (its derivation is provided in Section 2) as

$$\mathrm{Arg}(x) = \tan^{-1}(F(x)/H(x)) \tag{2}$$

at each point x. Extensive psychophysical work has shown that visually discernable features coincide with the maxima of this energy function.

An alternative approach to calculating E is by convolving the image with a set of spatial filters. This alternative energy function is then taken to be the sum of squares of the image convolved with a symmetric and anti-symmetric mask set. The nature of the mask set used determines the type of energy computed; for example, if the spatial filters form a quadrature pair, local energy is computed. This alternative energy function $\hat{E}$ is thus defined as

$$\hat{E}(x) = f^2(x) + h^2(x) \tag{3}$$

where $f(x)$ is the original intensity $F(x)$ convolved with a symmetric mask $S(x)$ and $h(x)$ is the original intensity $F(x)$ convolved with an anti-symmetric mask $A(x)$. The masks used in this paper have zero mean, identical L^2 norm, and are orthogonal. The maxima of $\hat{E}(x)$ then coincide with the features in the image.

Although a certain degree of feature discrimination from edge maps is possible in many edge detection algorithms, the technique of using a quadrature pair of filters lends itself to the process of feature discrimination without involving additional computation. In this paper we show that by examining the output of the odd and even masks, it is possible to obtain information about the original profile that gives rise to the feature at that point.

The format of this paper is as follows: Section 2 outlines some of the properties of local energy. Section 3 contains the classification of feature types based on local energy, whilst the analysis of edge types based on the output of the quadrature pair of filters follows in Section 4. Our conclusions are given in Section 5.

2. Properties of local energy

In order to analyse the classification of features on the basis of local energy, it is necessary to formalise the concepts of energy and phase congruency. A brief review of these concepts follows.

If, as before, $F(x)$ represents the image intensity function of a one-dimensional windowed luminance profile and $H(x)$ represents the Hilbert transform of $F(x)$, then in terms of their Fourier series expansion we may write

$$F(x) = \sum a_n \sin(n\omega x + \phi_n), \tag{4}$$

and

$$H(x) = \sum a_n \cos(n\omega x + \phi_n), \tag{5}$$

where $a_n > 0$, ϕ_n is the phase shift of the nth term and the summation is taken over the non-negative integers.

Let us construct the analytic signal (Bracewell, 1965) E as

$$\begin{aligned} E(x) &= F(x) - \mathrm{j}\, H(x) \\ &= \sum a_n(\sin(n\omega x + \phi_n) - \mathrm{j}\cos(n\omega x + \phi_n)) \\ &= -\mathrm{j}(\sum a_n(\cos(n\omega x + \phi_n) \\ &\qquad + \mathrm{j}\sin(n\omega x + \phi_n))) \\ &= \mathrm{e}^{\mathrm{j}(-\pi/2 + 2k\pi)}(H(x) + \mathrm{j}\, F(x)), \end{aligned}$$

where j represents the square root of -1.

Thus

$$E(x) = H(x)\, \boldsymbol{e}_x + F(x)\, \boldsymbol{e}_y, \tag{6}$$

where $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$ represent basis vectors along the x and y axes respectively. The function E therefore, is in general an infinite vector sum, where the nth component has a length of a_n and makes an angle of $n\omega x+\phi_n$ with the positive x axis.

From equation (6) we can derive the norm of E as

$$|E(x)| = (F^2(x)+H^2(x))^{1/2}$$

and the argument as

$$\operatorname{Arg}(x) = \tan^{-1}(F(x)/H(x)).$$

Notice that the norm of $E(x)$ is the square root of the *local energy function* evaluated at x.

Since E is a vector sum of an infinite number of components, it follows that if these components are distributed with a small standard deviation about the mean phase value, the norm of the vector E will attain a local maximum. We will show mathematically that this is indeed the case and points in space where such a condition occurs are termed points of maximum phase congruency. Extensive experimentation has established that features in the image coincide with the local maxima of E, and therefore with the points of maximum phase congruency.

We define below the *Phase Congruency* function PC in order to represent the total deviation of the vector components of E from the angle the resultant makes with the positive x axis. Thus PC is defined as

$$\mathrm{PC}(x) = \max_{\theta\in[0,2\pi]} \sum (a_n \cos(n\omega x+\phi_n-\theta))/\sum a_n \quad (7)$$

where the summation is again taken over the non-negative integers. Note that the function PC weights the deviation of each component by its amplitude. Thus, the maximum value that PC can attain is 1 and this occurs when, for all n, the angle $n\omega x+\phi_n$ equals θ. The minimum value of PC is -1, which occurs when $(n\omega x+\phi_n-\theta)=\pi \pmod{2\pi}$ for all n.

The point x where $\mathrm{PC}(x)$ attains a maximum is termed a point of *maximum phase congruency*. We can justify intuitively the definition of PC by noting that when the argument of the cosine term is small, we can expand $\mathrm{PC}(x)$ using a Taylor series expansion. Retaining only the second order terms we get

$$\mathrm{PC}(x) \cong \max_{\theta\in[0,2\pi]} 1-[\textstyle\sum a_n(n\omega x+\phi_n-\theta)^2]/2\sum a_n.$$

This expression is maximised when

$$\theta=\hat{\theta}=[\textstyle\sum a_n(n\omega x+\phi_n)]/\sum a_n$$

which can be regarded as a weighted average phase. Thus

$$\mathrm{PC}(x)\cong 1-[\textstyle\sum a_n(n\omega x+\phi_n-\hat{\theta})^2]/2\sum a_n$$

and the term

$$a_n(n\omega x+\phi_n-\hat{\theta})^2$$

can be regarded as a weighted variance term, where the deviation is measured from the average phase, and the deviation is weighted by the amplitude.

Proposition 1. *A point x is a maximum of $E(x)$ iff x is a point of maximum phase congruency.*

Proof. The norm of the vector $E(x)$ can be obtained by taking the dot product of $E(x)$ with a unit vector in the same direction as $E(x)$. Therefore

$$\begin{aligned} E(x)^{1/2} &= |E(x)| \\ &= \max_{\theta\in[0,2\pi]} [\cos\theta, \sin\theta]\begin{bmatrix} \sum (a_n\cos(n\omega x+\phi_n)) \\ \sum (a_n \sin(n\omega x+\phi_n)) \end{bmatrix} \\ &= \max_{\theta\in[0,2\pi]} \textstyle\sum (a_n\cos(n\omega x+\phi_n-\theta)) \\ &= \mathrm{PC}(x)\textstyle\sum a_n. \end{aligned}$$

Therefore, $E(x)$ is a maximum if and only if $\mathrm{PC}(x)$ is a maximum.

Proposition 2. *The computation of the generalised function $\hat{E}$ with a quadrature pair of filters is equivalent to computing the magnitude of the local energy function $E(x)$.*

Proof. In our implementation of the algorithm, an alternative function $\hat{E}(x)$ is computed as

$$\hat{E}(x)=f^2(x)+h^2(x),$$

where $f(x)$ and $h(x)$ are defined from equation (3).

We have shown in the proof of Proposition 1 that energy calculated as the sum of the squares of two functions that form a quadrature (or Hilbert transform) pair attains local maxima at points of maximum phase congruency. Thus, to show that the local maxima of $\hat{E}(x)$ coincide with the points of maximum phase congruency and therefore with the maxima of $E(x)$, it is necessary to establish that f and h form a Hilbert transform pair.

Let us consider the case in which the anti-symmetric and symmetric filters, A and S respectively, form a Hilbert transform pair. Then

$$A(\mathrm{j}\omega) = -\mathrm{j}\,\mathrm{sgn}(\omega)\, S((\mathrm{j}\omega)$$

where

$$\mathrm{sgn}(\omega) = 1 \quad \text{for } \omega > 0,$$
$$= -1 \quad \text{for } \omega < 0$$

and j represents the square root of -1.

Let $f(\mathrm{j}\omega)$ be the Fourier transform of $f(x)$ and $h(\mathrm{j}\omega)$ be the Fourier transform of $h(x)$. Then

$$f(x) = F(x) \circ S(x)$$

which implies

$$f(\mathrm{j}\omega) = F(\mathrm{j}\omega)\, S(\mathrm{j}\omega).$$

Similarly,

$$h(x) = F(x) \circ A(x)$$

which implies

$$\begin{aligned} h(\mathrm{j}\omega) &= F(\mathrm{j}\omega)\, A(\mathrm{j}\omega) \\ &= F(\mathrm{j}\omega)[-\mathrm{j}\,\mathrm{sgn}(\omega)\, S(\mathrm{j}\omega)] \\ &= -\mathrm{j}\,\mathrm{sgn}(\omega)\, f(\mathrm{j}\omega). \end{aligned}$$

This relation implies that $f(x)$ and $h(x)$ form a quadrature pair. By Proposition 1, $\hat{E}(x)$ will have maxima at points of maximum phase congruency. □

3. Feature classification using local energy

Specific features produce unique congruency patterns and therefore the characteristics of congruency and energy serve as a basis for feature-type signatures. Three main factors influence the feature type when a given point is a local maximum:

(1) The degree of congruency at x, which is defined as $\mathrm{PC}(x)$.

(2) The phase of $E(x)$.

(3) The slope of E in a local neighbourhood of x.

Each of these factors is detailed below.

The degree of phase congruency. The value of the phase congruency $\mathrm{PC}(x)$ determines the extent to which the components of the Fourier series expansion are in phase with the resultant at any point. For example, at a positive going step edge, all the components have zero degree phase and the function PC has the value 1. For a trapezoidal waveform, $\mathrm{PC}(x) > 1$ for all x but its local maxima occur at the points where Mach bands are perceived.

The phase of $E(x)$. Even in the case when $\mathrm{PC}(x) = 1$, it is the phase of $E(x)$ which determines the feature type. As before, $\mathrm{PC}(x) = 1$ when x is the location of a positive going step edge and in this case the phase of $E(x)$ is zero degrees. However, $\mathrm{PC}(x) = 1$ also when x is the point of a positive triangular peak; in this case all the Fourier components have ninety degrees phase at x and the phase of $E(x)$ is ninety degrees. Recall that the phase of $E(x)$ is measured by $\mathrm{Arg}(x) = \tan^{-1}(F(x)/H(x))$.

The slope of the energy function. The strength of a feature at a point x is related to the slope of the energy function in a deleted local neighbourhood of x (where a deleted neighbourhood is defined as an open set of the form $(x-\varepsilon, x) \cup (x, x+\varepsilon)$ for some $\varepsilon > 0$. Clearly, the slope of E is zero *at* the maximum.) About a fixed d.c. value, a step edge with an edge contrast of 20 grey levels will be perceived by an observer as being a stronger feature than a step edge with an edge contrast of 10 grey levels. This qualitative property is reflected in the local slope of the energy function. That is, an edge with a higher level of edge contrast will have a higher local slope of the energy function when compared with an edge of lower edge contrast.

In order to examine the effect of congruency on feature types based on the three factors outlined above, a set of synthetic images was generated. The profiles of step edges and spots (Figures 1 and 2) are based on the generation of profiles using the following Fourier expansion:

$$F(x) = \sum_n \sin((2\pi(2n+1)x/64) + \phi)/(2n+1). \quad (8)$$

The value of ϕ decides whether a step edge or a spot is generated. If ϕ is zero, the resultant profile is a step edge and if the value of ϕ is $\pi/2$ radians (90°) the profile generated is that of a spot. The summation is taken over 50 terms, and x is varied from 0 to 128.

The images of the triangular (Figure 3) and trapezoidal (Figure 4) profiles are based on the following Fourier expansion:

$$F(x) = 64/(\pi^2 t) \sum_n \sin(\pi t(2n+1)) \times \sin(2\pi(2n+1)x/64)/(2n+1)^2 \quad (9)$$

Again the summation is taken over 50 terms, and x is varied from 0 to 128. The factor t determines the kind of feature produced: t equals 0.5 in the case of a triangular profile and t equals 0.25 for a trapezoidal profile.

Case 1: *Step edge*

Figure 1 shows the profile generated from the expression in (8) with ϕ set to zero. Note that the energy peaks at the point of the step edge. The *slope* of this function is related to the *size* or contrast of the step edge. The magnitude of the slope of the energy function is greater for the higher contrast step as shown in Figure 1e. The phase congruency is 1 in both cases indicating that *all* the components of the Fourier expansion are in phase with the resultant which makes an angle of 0 degrees with the positive x axis. A negative going step will produce results that are identical except that the resultant will make an angle of 180 degrees with the x axis.

Case 2: *Spots*

Figure 2 is generated from the expression in (8) with the value of ϕ as $\pi/2$ radians (90°). This profile is perceived as a spot. The degree of phase congruency is 1 and the maximum phase congruency occurs when the resultant makes an angle of 90° with the positive x axis. Once again a negative going spot would differ in that the resultant makes an angle of 270° with the positive x axis.

Case 3: *Roof and ramp edges*

Figure 3 shows a profile that is perceived as a roof edge, while Figure 4 shows a trapezoidal profile that causes a human observer to see Mach bands. At the point of the roof edge PC is 1 and the resultant of a positive going triangular waveform makes an angle of 90°. While these factors are similar to those produced by the profile of a spot analysed in Case 2, the local slope of the energy function is smaller in the case of the roof profile than in the case of a spot. The similarity between the roof profile and the spot profile is not surprising, as a spot can be considered as a roof profile with a very sharp gradient. The difference, therefore, should be reflected as a difference in the strength of the feature as is demonstrated in these results.

As indicated in Figure 4e, the function PC has a maximum value of 0.89 at the point where the plateau meets the ramp. Also, for a positive going trapezoid given by equation (9) with t as 0.25, the resultant makes an angle of 44.2 degrees with the x axis. If the value of t is changed, the trapezoidal profile produced will also change and so will its properties. For example, if $t = 0.35$, the resultant makes an angle of 62.74 degrees with the x axis and PC is 0.96 at the point of the feature.

As introduced in Section 2, the local energy function is computed as the sum of squares of the image and its Hilbert transform. It then follows that by analysing the behaviour of the original profile F, and its Hilbert transform H, it is possible to uniquely identify the edge type at that point. Figure 5 shows the behaviour of F and H at typical feature types that include positive and negative step edges, spots, roof edges and trapezoidal profiles.

From Figure 5 we can draw the following conclusions:

(a) A local maximum of the energy function is coincident with a perceived feature in the image profile. (The local minima do not coincide with any known features that humans perceive). Once a feature has been identified as a local maximum of the energy function, it is possible to identify the profile that is responsible for that feature type by examining the values of F and H at that point.

(b) Local energy is therefore a sound and robust mechanism for identifying feature points in an image. The use of two functions F and H provides valuable information that serves as a signature of the feature type in the original luminance profile.

4. Classification based on the output of odd and even filters

While the study of feature classification using the concepts of congruency and local energy is of theoretical interest, for a real image a decomposi-

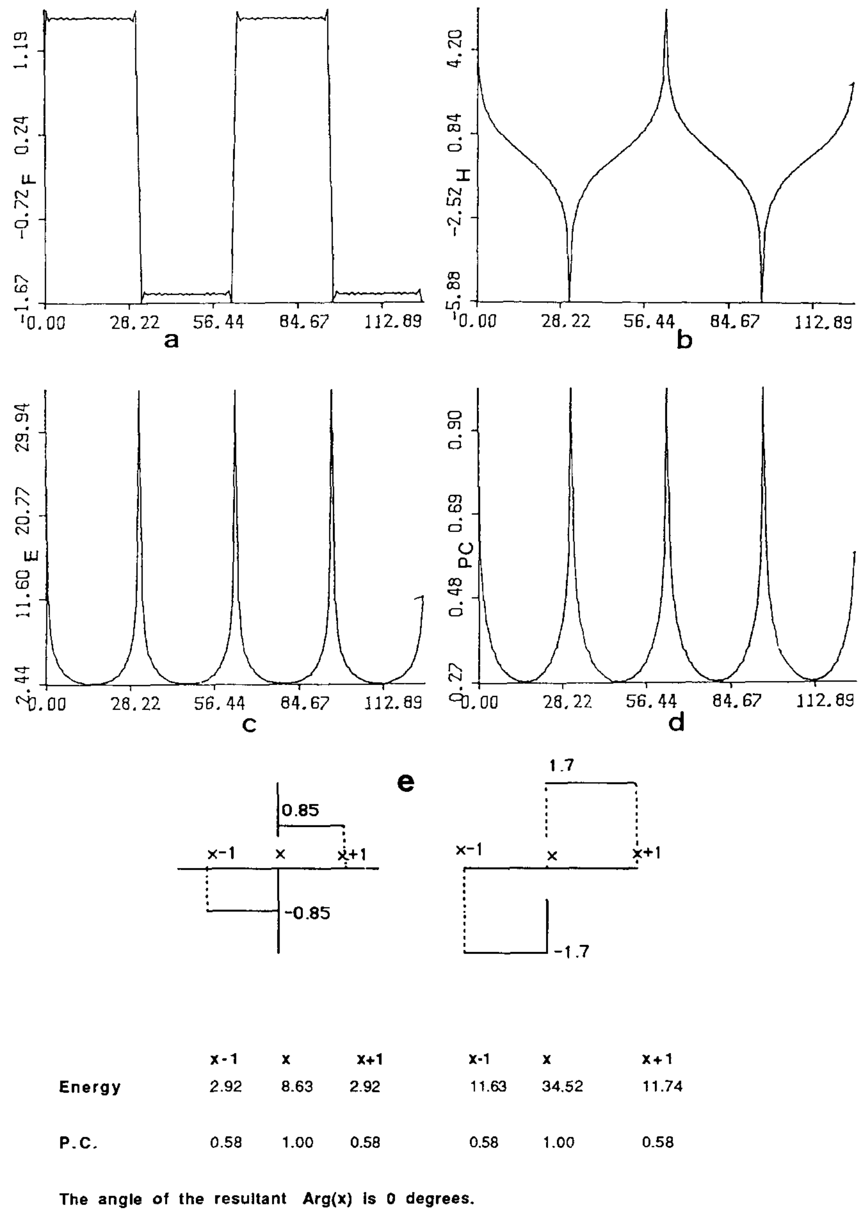


Figure 1. (a) Fourier series approximation to a step edge. (b) The Hilbert transform of (a). (c) The local energy function E. (d) The phase congruency function PC. (e) Two step edges of varying contrast examined at the feature point and one spatial point on either side. Notice that the slope of the energy function is greater for the edge with higher contrast.

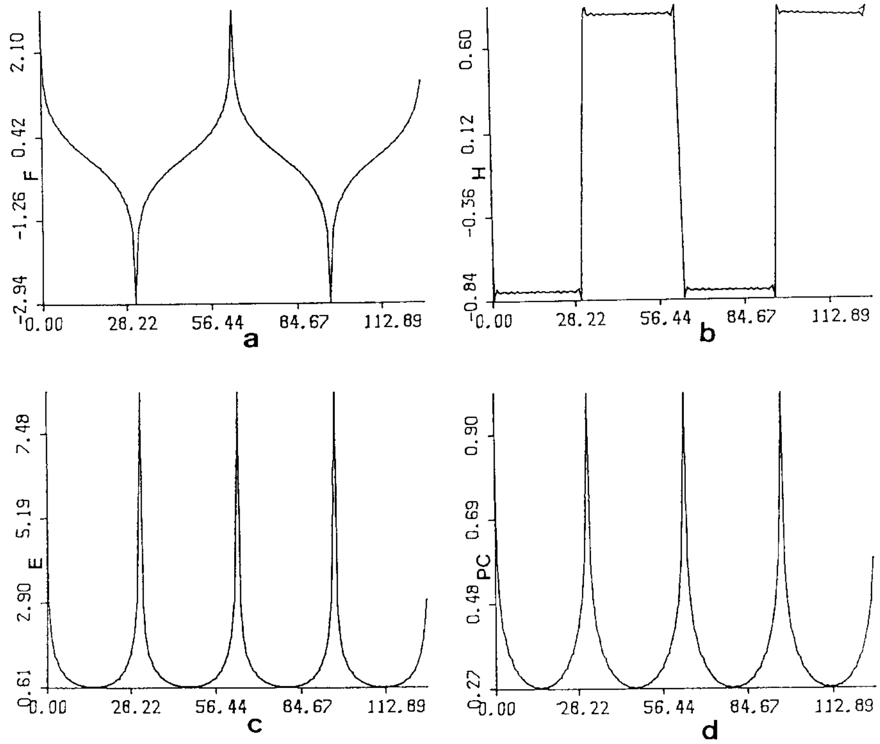


Figure 2. (a) Fourier series approximation to a spot. (b) The Hilbert transform of (a). (c) The local energy function E. (d) The phase congruency function PC.

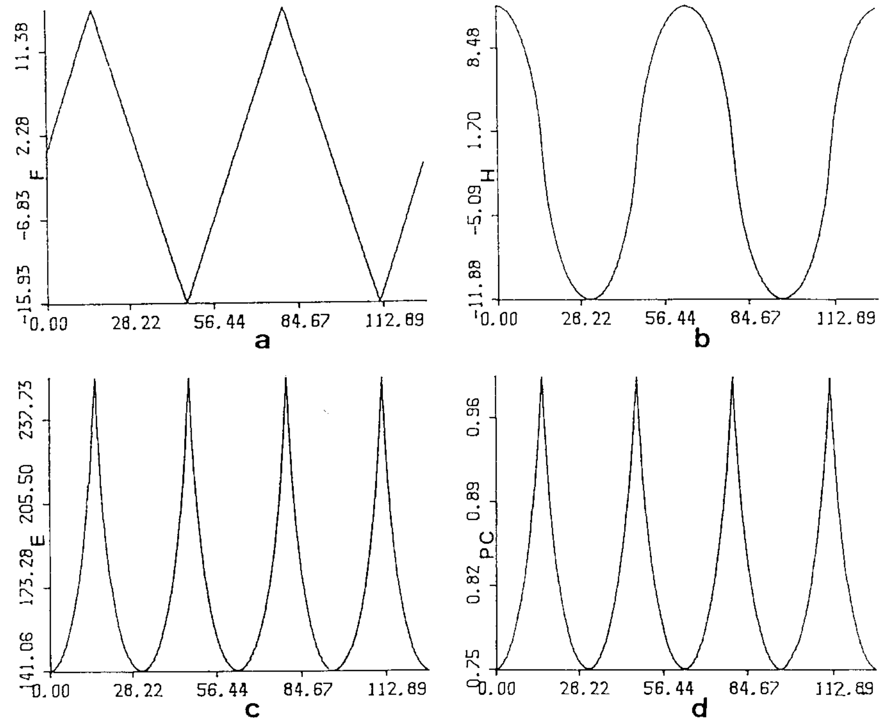
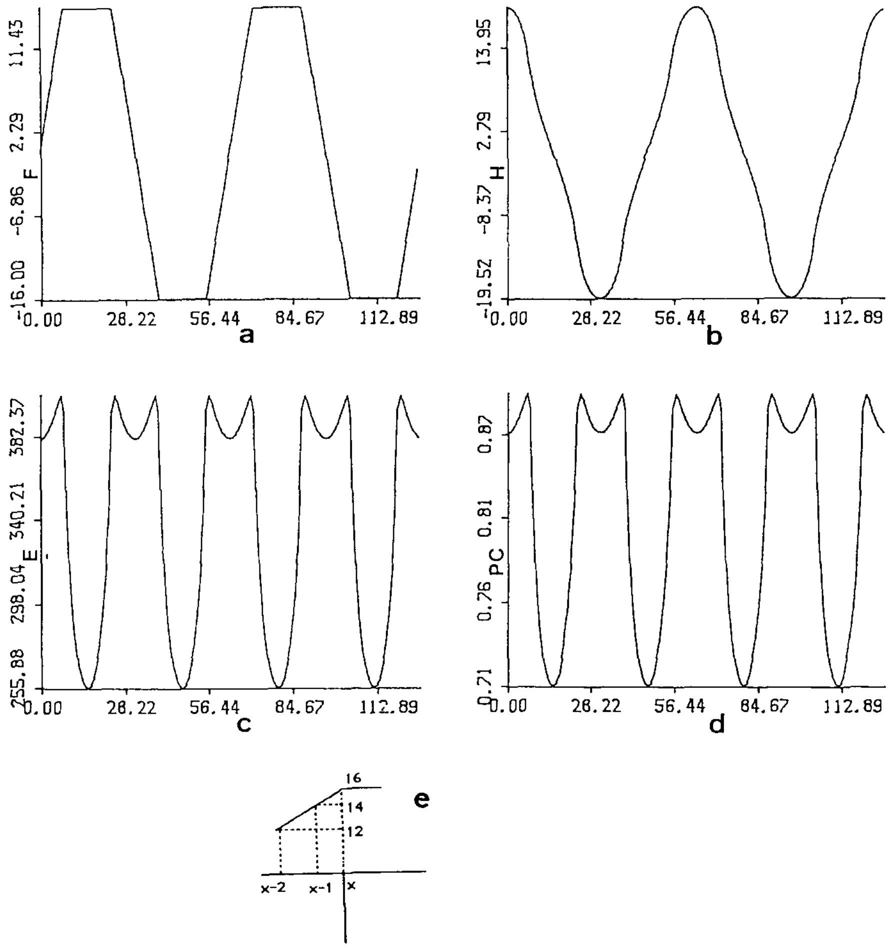


Figure 3. (a) Fourier series approximation to a roof edge. (b) The Hilbert transform of (a). (c) The local energy function E. (d) The phase congruency function PC.

tion of feature types on these principles is difficult as the process may involve complex spectral decompositions. However, the computation of energy with the use of odd and even filters naturally lends itself to feature type classification by using the information generated by the convolution of the image with an odd and even filter. Venkatesh and Owens (1988) have proposed a set of masks for the computation of energy. The three point masks given in the legend to Figure 6 are an example of an orthogonal pair of masks. Figure 6 shows the output of the odd and even filters when the three point masks are applied to a set of images.

Figure 7 illustrates the capacity of the energy feature detector to distinguish between different feature types. We have used the method of classifying features on the output of the odd and even masks, and the local energy operator has been applied using the three point masks. Figure 7a illustrates a disc corrupted by bright spots. The output of the feature detector is shown in Figure 7b. If the output of the energy feature detector is controlled



	x-2	x-1	x
Energy	399.25	403.39	394.96
P.C.	0.88	.89	0.88

Angle of the resultant Arg(x) : 44.2 degrees.

Figure 4. (a) Fourier series approximation to a trapezoidal edge. (b) The Hilbert transform of (a). (c) The local energy function E. (d) The phase congruency function PC. (e) Examination of the feature point and two spatial points. Notice that the energy function peaks at the point $x-1$. This can be attributed to digitization effects.

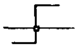
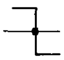
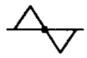
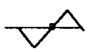
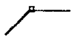
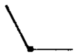
Feature	F	H	Energy	Arg (degrees)
	Z.C.	Max	Max	0
	Z.C.	Min	Max	180
	Min	Z.C.	Max	270
	Max	Z.C.	Max	90
	Z.C.	Min	Min	-
	Z.C.	Max	Min	-
	Max	--	Max	44.2
	Max	--	Max	135.8
	Min	--	Max	-135.8
	Min	--	Max	-44.2
	--	Z.C.	Min	90
	--	Z.C	Min	-90

Figure 5. Analysis of the behaviour of an image profile (F) and its Hilbert transform (H) at a point of maximum energy. (Trapezoidal profile is generated with $t=0.25$ and roof profile is generated with $t=0.5$. See Section 3 for details. Z.C. Stands for zero crossing.)

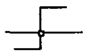
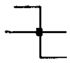
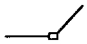
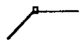
Feature	Output of even filter	Output of odd filter	Energy
	Z.C.	Max	Max
	Z.C.	Min	Max
	Min	Z.C.	Max
	Max	Z.C.	Max
	Min	Min	Max
	Min	Max	Max
	Max	Max	Max
	Max	Min	Max

Figure 6. An analysis of the behaviour of the output which results from convolving an image profile F with a pair of odd and even filters. For the above results, the three point masks were used. (See text for explanation.) The filters were: $S=(-0.40825, +0.81650, -0.40825)$, $A=(-0.70711, 0.0, +0.70711)$.

to allow for the detection of step edges only, the bright spots are eliminated and the output produced is shown in Figure 7c. In a similar fashion, Figure 7d shows a positive step edge enclosed by receding trapezoidal profiles. The output of the feature detector is shown in Figure 7e. Again, if the output of the feature detector is restricted to the detection of step edges only, the result indicates the elimination of the features corresponding to Mach bands, as is illustrated in Figure 7f.

In summary, a maximum of the energy function is coincident with a visually identifiable feature in the image. The specific feature types we have considered in our study are the positive and negative going step edges, spots or bars, roof edges and trapezoidal edges. In all cases, the behaviour of the odd and even filters at the point of the feature is unique, being either a local maximum, a local minimum or a local zero crossing. The eight possible combinations of the behaviour pairs categorise the type of feature observed. Note that the pair (zero crossing, zero crossing) is impossible to obtain by the nature of the definition of E.

5. Conclusion

While the primary purpose of edge detection schemes is to be able to produce an edge map of a given image, the ability to distinguish between

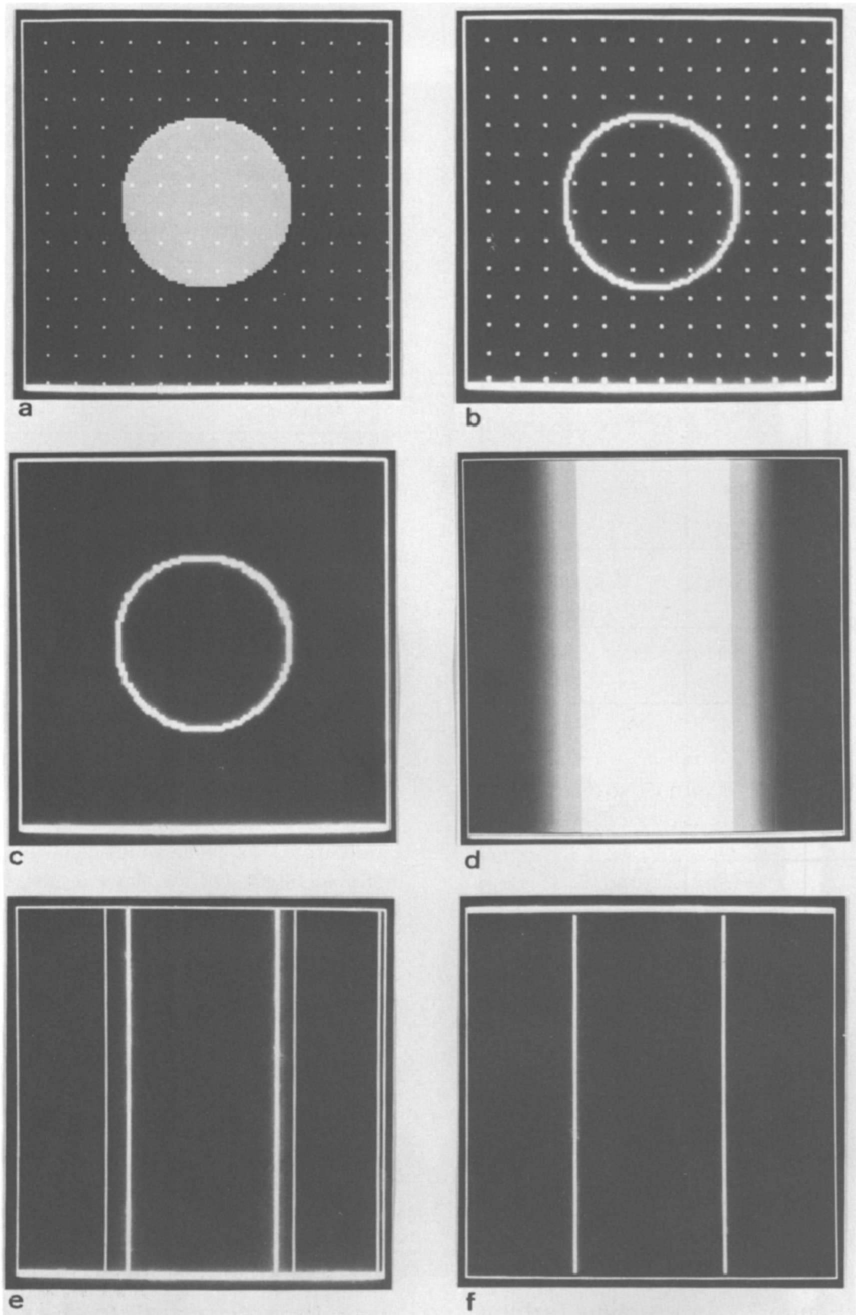


Figure 7. (a) Image of a disc corrupted with dots. (b) Output of the energy feature detector when applied to (a). (c) Output of the energy detector when output features are restricted to step edges. (d) Image of step edge encased by trapezoidal profiles. (e) Output of the energy feature detector when applied to (d). (f) Output of the energy detector when output features are restricted to step edges.

feature types is also of importance. It is computationally advantageous if the edge detection process allows this distinction to be made without the application of additional transformations.

In this paper we have examined feature classification based on energy feature detectors and have shown that energy measures are intrinsically capable of making this classification because of the use of a quadrature pair of functions in the calculation of energy. There are three distinct factors that contribute to a feature and therefore to the signature of the feature type. If a uni-dimensional profile is considered as a Fourier expansion in terms of sine components, then these factors are:

(1) The degree of congruency of these terms.

(2) The phase of the energy function at the point where the terms attain a local maximum in phase congruency.

(3) The local slope of the energy function at the local maximum.

Whilst the slope of the energy function is instrumental in deciding the edge strength, the phase at the point of congruency and the degree of congruency are instrumental in deciding the feature type.

The local energy, calculated as the sum of the squares of the image and its Hilbert transform, allows for feature type distinction by examining the joint behaviour of the image and its Hilbert transform at a point. It is shown that this behavioural pair is sufficient to discriminate between positive and negative step edges, spots and bars, roof and trapezoidal edges. In a similar fashion, if the energy is computed as the sum of squares of the image convolved with an odd and even mask, the outputs of these convolutions characterise the feature type.

In conclusion, we have analysed the factors that are instrumental in deciding the feature type. Although our study has been limited to one dimensional profiles, this is not considered to be a restriction. In the algorithm we have devised for 2D images the energy function is calculated as the sum of the horizontal and vertical energy: horizontal energy is computed by applying 1D filters tuned to vertical edges and vertical energy is computed analogously. If we try to reconstruct the original image from the information obtained from the output of the odd and even filters at relevant edge points, it is possible to recover the image to within a scale factor. The information lost in each row is the original dc component, and therefore the relative shading information between rows is lost. However, an interesting point that arises is that, even though one is attempting to recover the image from the components of the horizontal energy alone, this is sufficient to reconstruct features in the original image such as corners. Thus, although corners and intersections are essentially 2D features (Noble, 1988), a one-dimensional analysis along two orthogonal axes is sufficient to identify corners. Current work has established that it is possible to differentiate between different corner types with the method formulated in this paper. The results will appear in a forthcoming publication.

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