Additional Questions to the Lecture

Optical Metrology and Sensing (WS 2010/2011)

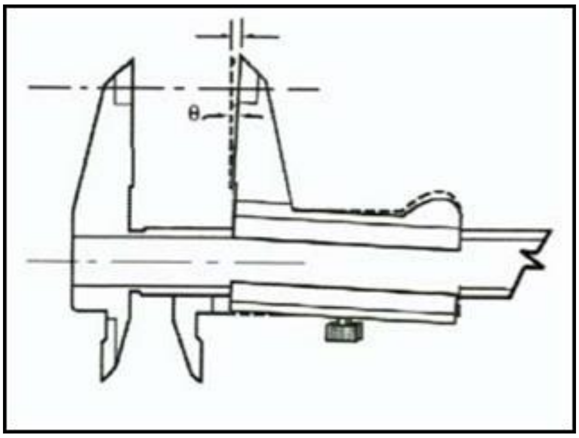
1. What is the meaning of a material measure or standard? Give some examples for it!

It is the commonly accepted measurement of physical quantities. Standards are the fundamental reference for a system of weights and measures, against which all other measuring devices are compared. For example the length in meter and mass in kilogram.

Ref.:http://en.wikipedia.org/wiki/Standard_(metrology)

2. Describe the Abbe comparator principle!

The Abbe comparator principle says that you have to measure aligning. That means that the axis of measurement/yardstick must be equal/congruent (and not only parallel) to the axis of the length (of the object under test). If the abbe comparator principle is not fulfilled you get a first order tilting error (see figure). Calipers do not fulfill the abbe comparator principle. A micrometer screw fulfills it.



$\Theta - first\ order\ tilting\ error\ at\ a\ caliper$

3. What is the meaning of the „confidence interval" of measured values?

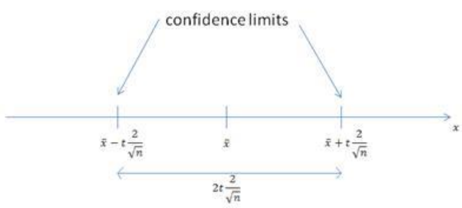
Confidence interval: $\bar{x} \pm t * \frac{s}{\sqrt{n}}$

$\bar{x}$: mean value of the measured values

s: standard deviation of the measured values

n: number of measurements

t is dependent on the so-called confidence level γ (e.g. γ =95%) and the number n and can be taken from tables

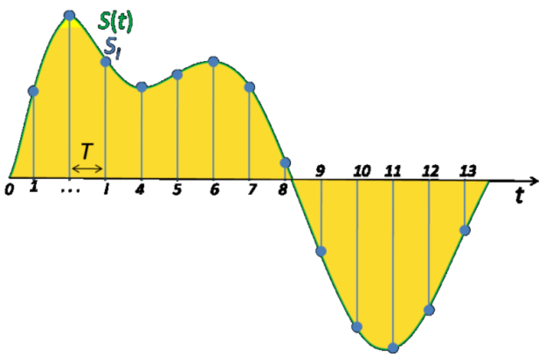


The real value/parameter, which one wants to know, lies in the confidence interval with a probability of _% where the percentage is given by γ. It is used to indicate the reliability of an estimate.
Reference: Papula, Mathematische Formelsammlung

4. ***Explain the meaning of scanning/sensing a test piece, primary and secondary standards, systematic and random errors, uncertainty of measurement!***

Sampling (scanning/sensing):

- sampling is the reduction of a continuous signal to a discrete signal (signal processing)



(Ref.: http://en.wikipedia.org/wiki/Sampling_(signal_processing))

Primary standard:

- is a standard that is accurate enough that it is not calibrated by other standards (or it's a defined standard, e.g. kg)
- they are defined via other quantities like length, mass and time
- they are used to calibrate other standards referred to as working standards

(Ref.: http://en.wikipedia.org/wiki/Primary_standard)

Secondary standards:

- are very close approximations of primary standards
- they are used as a standard of comparison, but checked against the primary standard existent elsewhere

(Ref.: http://en.wikipedia.org/wiki/Standard_(metrology))

Measurement uncertainty:

- is a parameter characterizing the dispersion of the values attributed to a measured quantity
- reflects incomplete knowledge of the quantity (probability)
- a measured value is only complete if it is accompanied by a statement of the associated uncertainty (low accuracy and precision => high measurement uncertainty)

(Ref.: http://en.wikipedia.org/wiki/Measurement_uncertainty)

Random error:

- a repeated measurement will generally provide a measured value that is different from the previous value
- random: the next measured value **can't be predicted** exactly from previous such values (**unpredictable fluctuations in reading of the measurement**)

(Ref.: http://en.wikipedia.org/wiki/Measurement_uncertainty)
(More detailed: http://en.wikipedia.org/wiki/Random_error)

Systematic error:

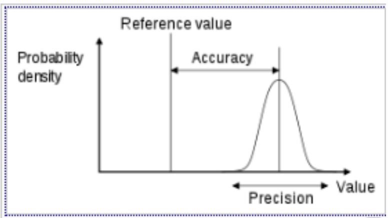
- measured value contains an **offset** (measurement bias)
- is a component of error that **remains constant** or depends in a specific manner on some other quantity (e.g. error in calibration)
- **predictable** and proportional to true value

(Ref.: http://en.wikipedia.org/wiki/Measurement_uncertainty)
(More detailed: http://en.wikipedia.org/wiki/Systematic_error)

5. Describe the difference between "accurate/correct" and "precise"!

precision: degree to obtain the same results for measuring a quantity in the same conditions

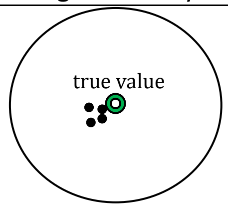
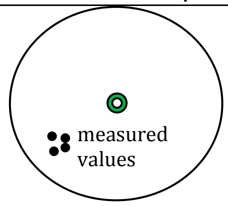
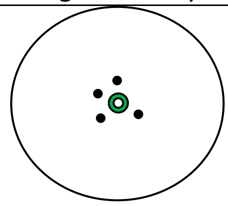
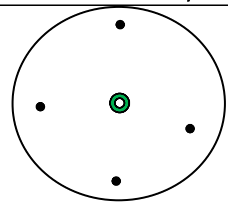
accuracy: degree of the closeness of the measured values of a quantity to the true value



Accuracy indicates proximity of measurement results to the true value, precision to the repeatability or reproducibility of the measurement

www.wikipedia.org

Example:

high precision		low precision	
high accuracy	low accuracy	high accuracy	low accuracy
			

6. Explain the measurement terms reproducibility and repeatability!

Reproducibility: It means that an experiment can be accurately reproduced, or replicated, by someone else working independently, based on the original experimental description. For a good or high reproducibility, the test results which are obtained with different operators, test apparatus, and laboratory locations should agree with the originally reported results.

Reference: http://en.wikipedia.org/wiki/Reproducibility

Repeatability: (or test-retest reliability) is the variation in measurements taken by a single person or instrument on the same item and under the same conditions. A measurement may be said to be *repeatable* when this variation is smaller than some agreed limit.

Reference: http://en.wikipedia.org/wiki/Repeatability

7. What is the meaning of the sensibility and resolution of an instrument?

Sensibility: A measure of the smallest signal the instrument can detect

Resolution: The smallest amount of input signal change that the instrument can detect reliably

Ref.: http://zone.ni.com/devzone/cda/tut/p/id/4439 (National Instruments)

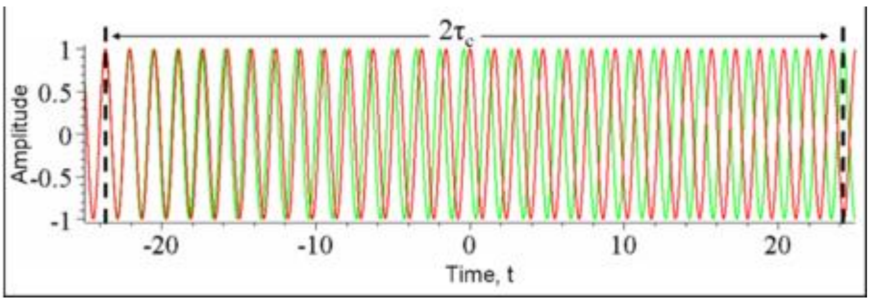
8. What has influenced the uncertainty of measurement of the primary standard of length?

The primary standard of length (1m) is the length of the path travelled by light in vacuum during a certain time interval (1/299792458 s). This means the length measurement is replaced by a time measurement. So the uncertainty of the standard meter is given by the uncertainty of time measurement, which relative uncertainty is around 10^{-14}.

9. What is the meaning of spatial and temporal coherence?
Spatial coherence means a strong correlation (fixed phase relationship) between the electric fields at different locations across the beam profile.
Temporal coherence means a strong correlation between the electric fields at one location but different times.
Reference: http://www.rp-photonics.com/coherence.html

10. How can the coherence time and the coherence length be measured?

- **Michelson interferometer** or Mach-Zehnder interferometer
- a wave is combined with a copy of itself that is delayed by time τ
- interference only visible, if the path difference of the two beams are smaller than the coherence length/time of the emitted wave trains of the atoms
- (visibility goes down)



Coherence length/time:

$$l_c = \tau_c \frac{c_0}{n}$$

(Ref.: http://en.wikipedia.org/wiki/Coherence_(physics))
(Ref.: http://de.wikipedia.org/wiki/Kohärenz_(Physik))
(or Ref.: http://www.rp-photonics.com/coherence_length.html)

From my labworks:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos 2\pi\left(\frac{r_2 - r_1}{\lambda} + \delta_2 - \delta_1\right)$$

δ … path difference (wave 1, wave 2), r … path
$\delta_2 = \delta_1$ for λ_0 is $I = I_{\max}$:

$$\frac{r_2 - r_1}{\lambda_0} = n \qquad n = 0,1,2 \ldots$$

Increasing path difference $r_2 - r_1$ => maximum for λ_0 hits minimum for λ_1 => the intensity differences due to interference are compensated

$$\frac{r_2 - r_1}{\lambda_1} = \frac{2n+1}{2}$$

Path difference equals coherence length $\boldsymbol{l_c}$:

$$\boldsymbol{r_2 - r_1} = \frac{\lambda_1 \lambda_0}{2(\lambda_0 - \lambda_1)} \approx \frac{\boldsymbol{\lambda_0^2}}{\boldsymbol{\Delta\lambda}} = \boldsymbol{l_c}$$

11. How can the spatial coherence be measured?

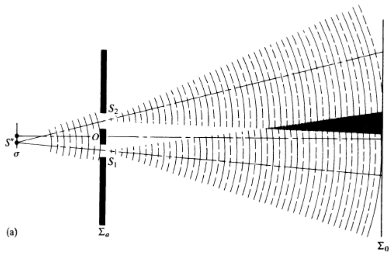
Young Interferometer:

- is the path difference ΔL between the emitted beams of the additional two sources, created by division due to the double slits, smaller than the coherence length an interference pattern could be observed on a detector
- for the measurement of the spatial coherence one has to increase the distance of the slits until the interference pattern get lost
- this results in reaching the limitation of the spatial coherence of the used lightsource
- [in formulas, where Q is a certain point on the detector:

$$I(Q) = I_1 + I_2 + 2\sqrt{I_1 I_2}|\gamma_{12}(\tau)|\cos(\Delta\varphi), \tag{11.1}$$

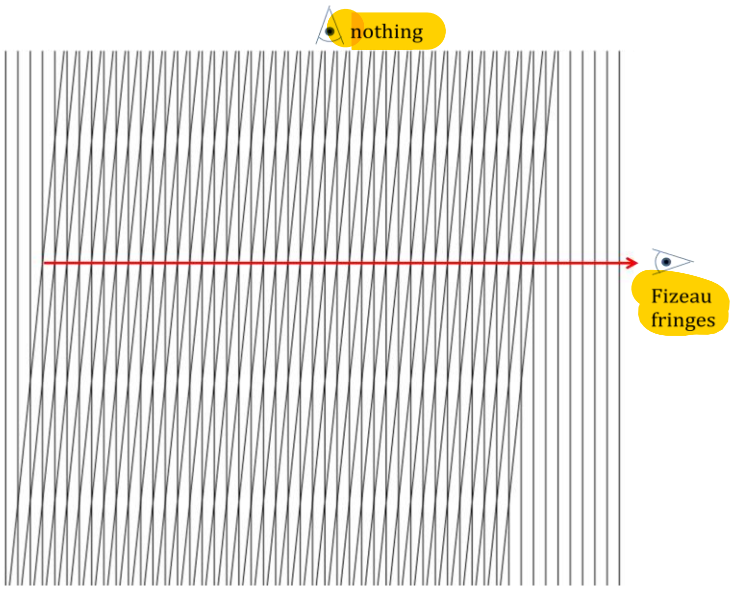
where $|\gamma_{12}|$ is complex coherence degree. If $|\gamma_{12}| = 0, \tau \geq t_c$, the interference term vanishes]

- in the figure one could see the region of spatial coherence (black), what becomes smaller if we increase the distance of slits

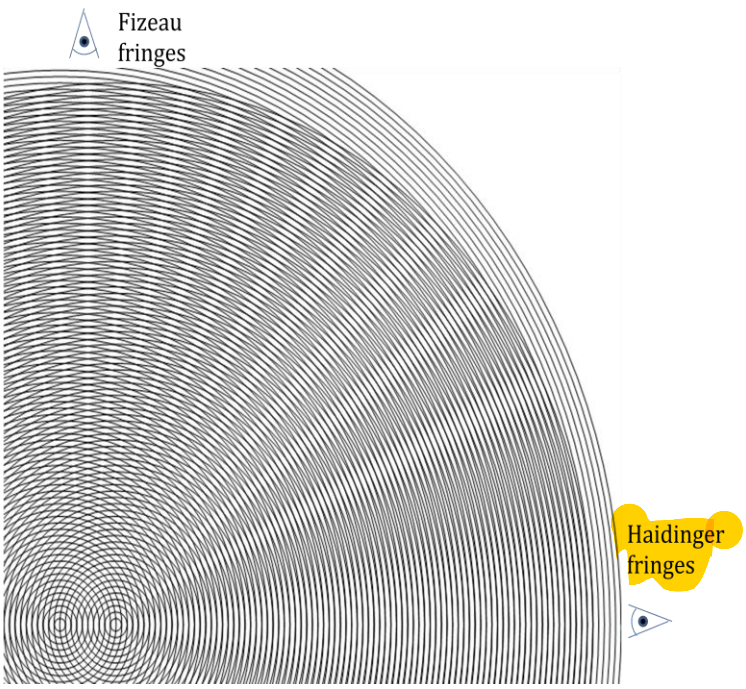


12. Which kinds of interference structures are generated if two plane waves, two spherical waves or a plane wave and a spherical wave are superposed?

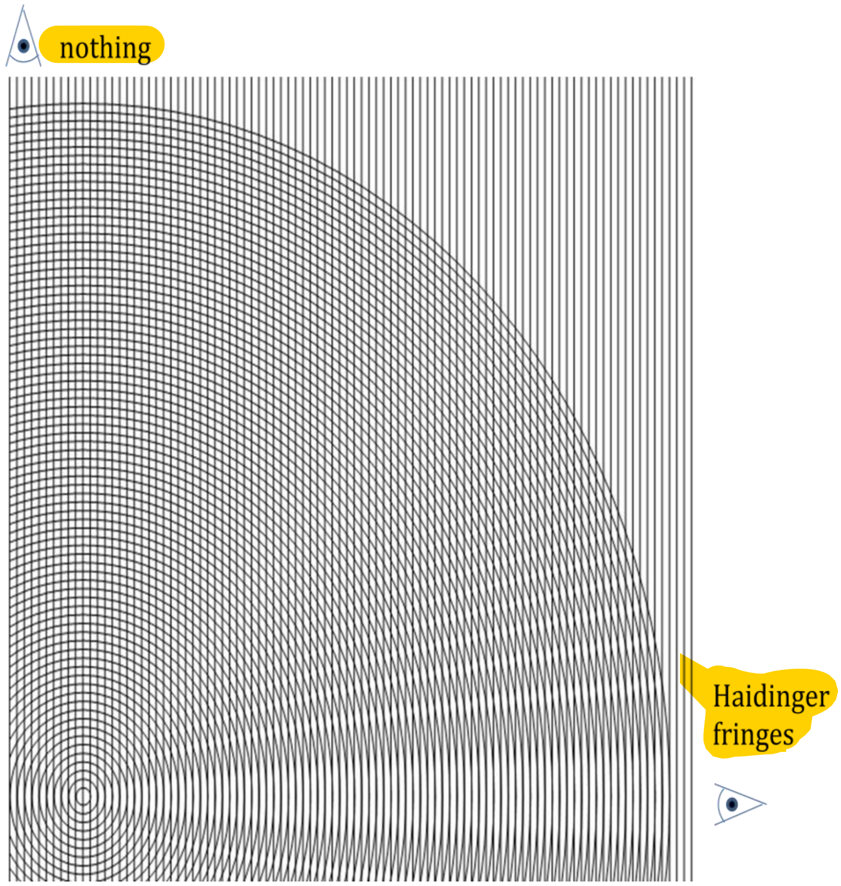
- plane-plane:



- spherical-spherical:



- plane-spherical:



13. What is the visibility of fringes and how can it be determined?

Measurement of the contrast of the interference phenomenon is the visibility of the interference fringes, defined by:

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

If the two optical fields are ideally monochromatic (consist of only single wavelength) point sources then the predicted visibility will be:

$$V_{(ideal)} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$$

Where I_1 and I_2indicates the intensity of the respective wave. Any dissimilarity between the optical fields will decrease the visibility from the ideal. In this sense, the visibility is a measure of the coherence between two optical fields. This can be observed in Michelson Interferometer by creating phase difference.

Ref.: http://en.wikipedia.org/wiki/Interferometric_visibility

14. What does the degree of coherence describe and how does it influence the law of

interference?

Complex degree of coherence γ is the normalized correlation of electric fields:

$$\gamma_{12(\tau)} = \frac{\Gamma_{12(\tau)}}{\sqrt{\Gamma_{11(0)}\Gamma_{22(0)}}} = \frac{\langle E_{1(t+\tau)}E^*_{2(t)}\rangle}{\sqrt{I_1 I_2}} = |\gamma_{12(\tau)}|e^{i\phi_{12(\tau)}} \qquad \text{(for two beam interference)}$$

where the degree of coherence $|\gamma_{12(\tau)}|$ is a quantity for the coherence of the light signals.

For $|\gamma_{12(\tau)}| = 1$ → ideal coherent light

For $|\gamma_{12(\tau)}| = 0$ → ideal incoherent light

For $0 < |\gamma_{12(\tau)}| < 1$ → partial coherent light

The general law of interference for stationary fields:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}|\gamma_{12(\tau)}|\cos(\phi_{12(\tau)} - \Delta\varphi)$$

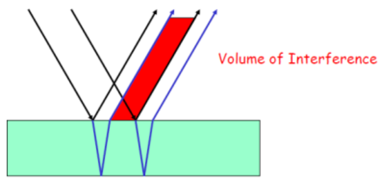
so that the amplitude $|\gamma_{12(\tau)}|$ of the complex degree of coherence influences the intensity of the interference pattern and the phase $\phi_{12(\tau)}$ of the complex degree of coherence influences the lateral position of interference maxima and minima.

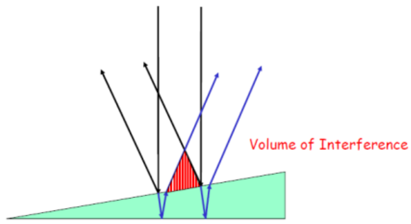
For $I_1 = I_2$ the visibility V for two beam interference is given:

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = |\gamma_{12(\tau)}|$$

Reference: Hecht – Optik pp. 917-921.

15. What is the meaning of localized interference structures and when do they appear?





Non-localized fringes: when interference fringes can be found in a (infinite) large volume. Above figure on right hand side: fringes need to be imaged (lens) in order to analyze them (e.g. count them).

Localized fringes: when interference fringes can only be found in a very small volume, e.g. directly at the surface of the wedge in the figure on the right side.

16. How can white-light interference patterns be generated with a Michelson interferometer?

- Superposition of continuously varying wave lengths (spectrum) only creates interference pattern **within the coherence length**
- Need light source with very **short coherence length** **($\Delta t\, \Delta\nu \approx 1$)** and a **compensation plate** to compensate **dispersion**
- Interference pattern of high contrast only available if **path difference of the two arms are smaller than coherence length** **(BS-M1 = BS-M2)**
- high quality of the surfaces
- Coherence length of white light (λ center wavelength, Δλ band width):

$$l_c = \frac{\lambda^2}{\Delta\lambda}$$

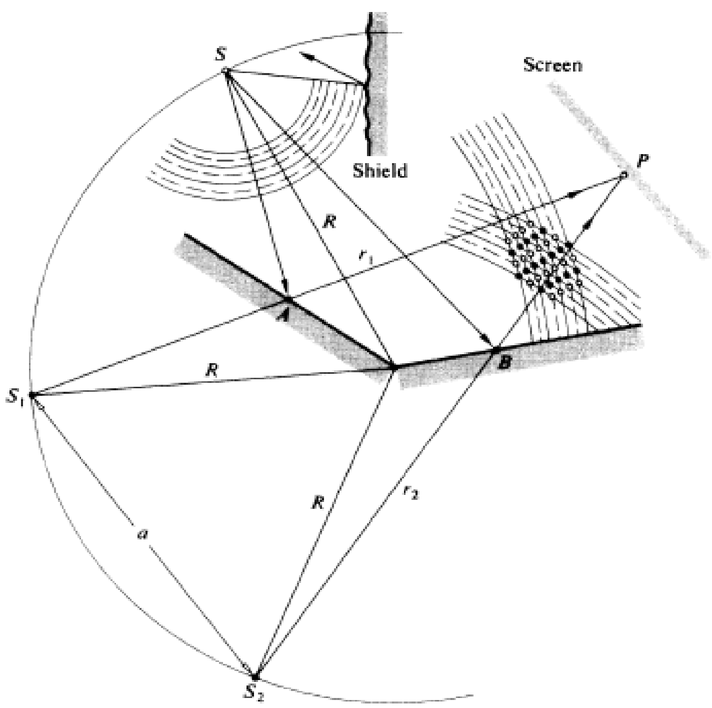
(Ref.: Script: 3.7 White-Light Interferometry)
(Ref.: http://de.wikipedia.org/wiki/Interferenz_(Physik)#Wei.C3.9Flichtinterferenz)
(Ref.: http://books.google.de/books?id=N8huAnJjJbwC&pg=PA401&lpg=PA401&dq=michelson+wei%C3%9Flicht&source=bl&ots=k6aUOFh0rQ&sig=92TGIE-YJL2UfB2POle737F7VNA&hl=de&ei=E2YTTe2mDsftsgaBk9nYDA&sa=X&oi=book_result&ct=result&resnum=9&ved=0CDIQ6AEwCA#v=onepage&q=michelson%20wei%C3%9Flicht&f=false)

17. Give Examples for interferometers with division of amplitudes and wavefronts respectively?

Division of amplitudes	Division of Wavefronts
• Michelson-IF • Mach-Zehnder IF • Sagnac IF • Pohl IF	• YOUNG IF • Fresnel Prism and Mirror • Lloyd's Mirror • Rayleigh-IF

18. How can interference structures be generated with Fresnel`s mirror?

Setup:



Description:

Wavefront originates from slit S. Can not directly reach screen because of shield.

One part of the wavefront gets reflected from mirror A, another part from mirror B (splitting of wavefront).

Both parts interfere and give interference pattern on screen. For point P it looks like that two wavefronts come from the virtual sources S1 and S2. S1 and S2 have the same distance to P. Remark: Angle between mirrors has to be small.

=> **Interference Pattern**: Straight periodic lines (Fizeau fringes).

Reference: E. Hecht, „Optics“

19. Explain the measurement of the complex degree of coherence with the Young interferometer!

Conditions:

- Coherent sources
- Phase of one source is known

The general interference law for partially coherent light is defined as
$I(\tau) = I_1 + I_2 + 2\sqrt{I_1 I_2}\ Re\{\gamma_{12}(\tau)\}$

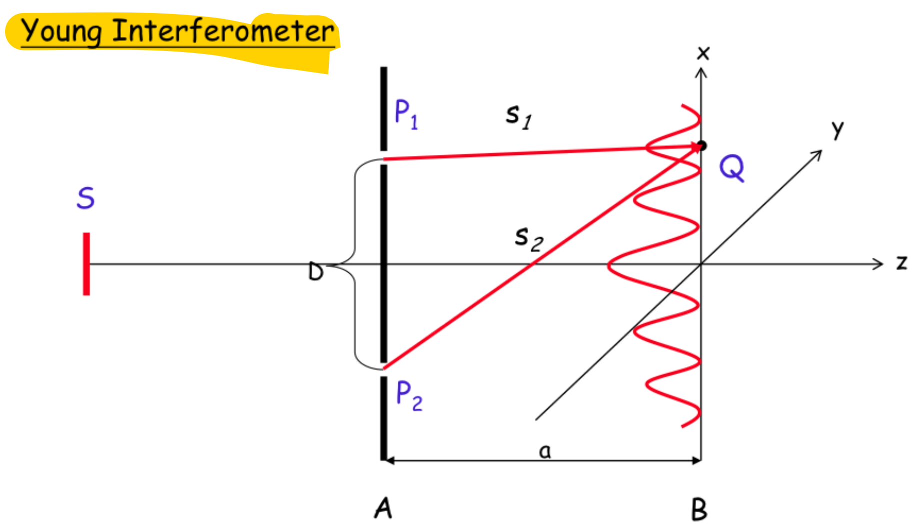
The complex degree of coherence is $\gamma_{12}(\tau)$ and can be expressed as:

$$\gamma_{12}(\tau) = |\gamma_{12}(\tau)| e^{i\phi_{12}(\tau)}$$

Where the phase angle $\phi_{12}(\tau) = \phi_1(\tau) - \phi_2(\tau)$ is related to the phase of the fields at each slit P_1 and P_2. By measuring the intensity of each field, and the varying the distance of the pinholes we can observe the shift position of maxima and therefore obtain the complex degree of coherence.

Example:

$\boldsymbol{\|\gamma_{12}(\tau)\|}$	**Degree of coherence**
1	Two completely coherent waves
0	Two completely incoherent waves
$0<\|\gamma_{12}(\tau)\|<1$	Partial coherence



Ref.: Demtröder – Laser Spectroscopy, 2.8.4 The Coherence Function and the Degree of Coherence

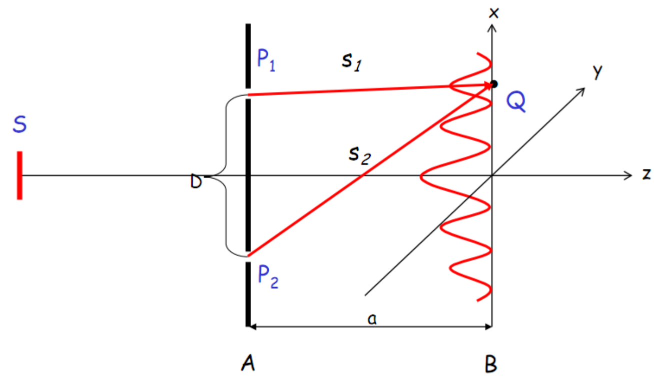
Re.: Lecture Notes, 2.2.1. Young Interferometer

20. Calculate the shift of the positions of interference maxima in a Young interferometer for a given phase of the degree of coherence!

The general law of interference is:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\left|\gamma_{12(\tau)}\right| \cos\left(\phi_{12(\tau)} - \Delta\varphi\right)$$

with $\Delta\phi = \frac{2\pi}{\lambda}(S_2 - S_1)$.



Young-IF

For D<<a,x,y and a>>x,y we can write $\Delta\phi \approx \frac{2\pi}{\lambda}\frac{xD}{a}$.

Getting the position of interference maxima it must be fulfilled $\cos\left(\phi_{12(\tau)} - \Delta\varphi\right) = 1$

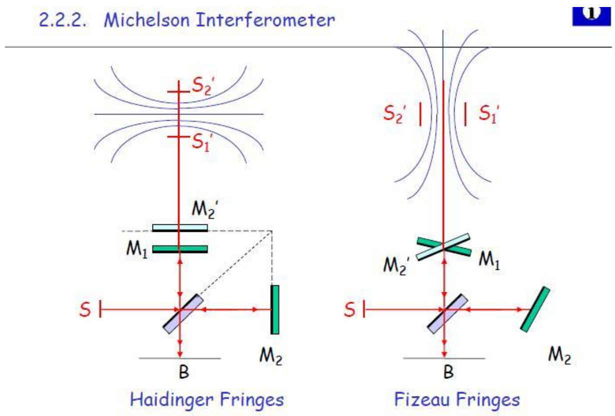
So we get $\phi_{12(\tau)} = \Delta\varphi \approx \frac{2\pi}{\lambda}\frac{xD}{a}$.

The shift of the position Δx then is:

$\Delta x \approx \frac{\lambda a \phi_{12(\tau)}}{2\pi D}$.

21. Describe the realization of Haidinger fringes and Fizeau fringes with a Michelson interferometer!

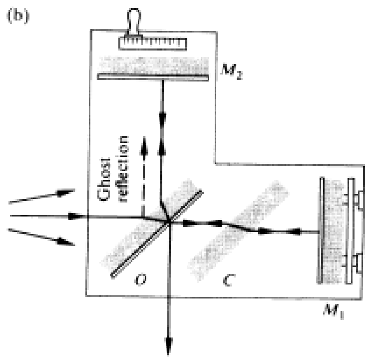
Haidinger fringes pattern is formed when two mirrors are parallel to each other forming an interference plate which eventually generate fringes with localization at infinity. The fringes are co-centric (?? do they mean concentric??) in this case and have equal inclination.



Reference: 2nd year students

22. What is the role of the compensation plate in the Michelson interferometer?

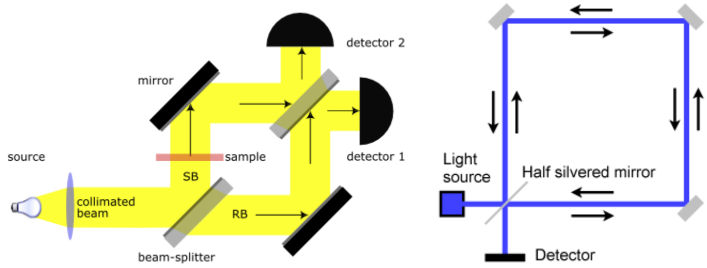
- **Equals the aberrations of the two arms**
- **Ensure that the optical path in each arm is equivalent**
- Because **one arm passes 3 times and the other 1 time the material** of the beam splitter plate (not for cube) => with compensation plate are both same times
- **Dispersion compansated** of the beamsplitter => optical path is a f(λ)
- for quantitative work without compensator plate only with quasimonochromatic source
- Compensator negates the effect of dispersion => source with broad bandwidth will generate fringes



(Ref.: Hecht (International Edition), Optics, 4 Ed., p. 407-408)
(Ref.: Hecht, Optik, 4. Aufl., S. 658-659)

23. Are their compensation plates required for Mach-Zehnder or Sagnac IF too?

If this two IF are very exactly justified there is in general no additional compensation plate needed, but only if the optical path in the BSers of both beams is equal. This is caused by traveling the BS(ers) of the two beams in the same amount, different to Michelson, where a compensation plate is needed. Imagine the case in the Mach-Zehnder IF that the BSers have a different thickness d1 and d2, so you need a compensation plate with thickness $|d1 - d2|$.



Mach-Zehnder-IF (left) and Sagnac-IF (right)

24. How can wavelengths and path differences be measured with a Michelson interferometer?

The Michelson Interferometer can be used to make extremely accurate length measurements. As the moveable mirror is displaced by $\lambda_0/2$, each fringe will move to the position previously occupied by an adjacent fringe. One need only count the number of fringes N, or portions thereof, that have moved past a reference point to determine the distance traveled by the mirror Δd, that is: $\Delta d = N(\lambda_0/2)$.

If one want to measure the wavelength one can use the same technique, but in this case one has to know the travelled distance Δd of course to determine an unknown wavelength:

$\lambda_0 = (2\ \Delta d)/N$

Reference: E. Hecht, „Optics“

25. What is the meaning of a traceability measurement like it was performed out by Michelson with a cadmium lamp and the primary (secondary) standard?

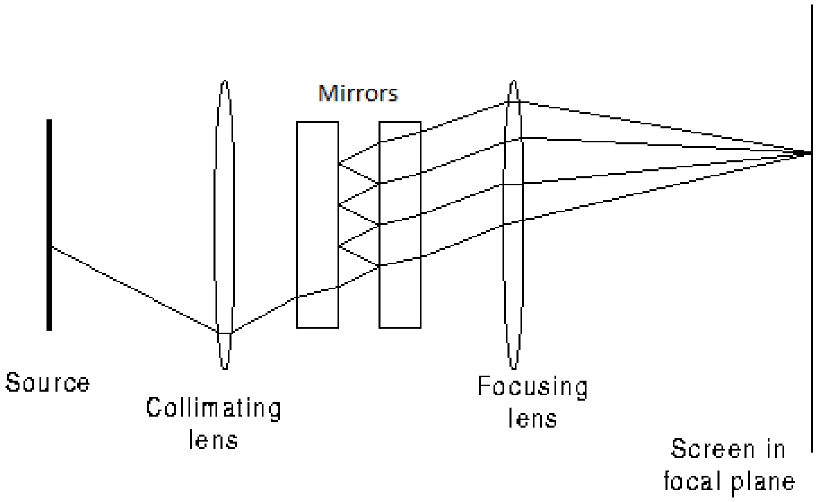
Traceability measurement (RückverfolgbarkeiteinerMessung):
Is used to refer to an unbroken chain of comparisons relating an instrument's measurements to a known standard (example: meter, kilogram...)

Ref.: http://en.wikipedia.org/wiki/Traceability

Michelson related the wavelength of Cd with the standard of a meter by comparison with etalons (varying their position) in the Michelson interferometer. The result was $1\,m =$ 1,553,163.5+/- some uncertainty.

Ref.:https://docs.google.com/Doc?docid=0AVru3kJAsa2DZDR4enN6Y180Z2N4Mjk2ZzY&hl=en_GB

26. Explain the generation of interference fringes for a Fabry-Perot interferometer!



The interference fringes in a Fabry-Perot interferometer are created by multiple-beam-interference. Multiple-beam interference is created by two mirrors for example. Each incident light ray is reflected between the mirrors several times. At every reflection point a part of intensity is transmitted. This transmitted light rays are focused by a lens to a single point (for equal inclination) on the screen and interfere with each other. The relative phase of each transmitted light ray to another transmitted light ray depends on the optical path d between the mirrors, the angle of light incidence and the light wavelength. Each point on the screen is only illuminated by only one inclination of incident rays from the source. Because of radial symmetry of the lenses and the mirrors we get haidinger fringes. Important note: incident light rays with equal inclination from DIFFERENT lateral positions (S1 and S2) from the source are focused to one point on the screen but don't interfere to each other because normally they are ideal incoherent to each other.

27. What is the meaning of finesse for multiple-beam interferometers?

The Finesse is defined as distance between the fringe maxima (=2π) divided by their half-width ε (FWHM-width): $\mathcal{F} = \frac{2\pi}{\varepsilon}$. With F, coefficient of finesse: $F = \frac{4\pi}{(1-R)^2}$ is follows:

Finesse $\mathcal{F} = \frac{2\pi}{\varepsilon} = \frac{\pi\sqrt{R}}{1-R}$

The Finesse represents the sharpness of the fringes and is dependent on the reflectivity R. Finesse is a factor given to quantity the performance of Fabry Perot interferometer.

Reference:
http://www.optics.arizona.edu/jcwyant/Optics505%282000%29/ChapterNotes/Chapter07/multiplebeaminterference.pdf

28. Derive the Airy-formulas of the reflected and transmitted intensities for multiple beam interferences given in the lecture!

Multible beam interference in general:

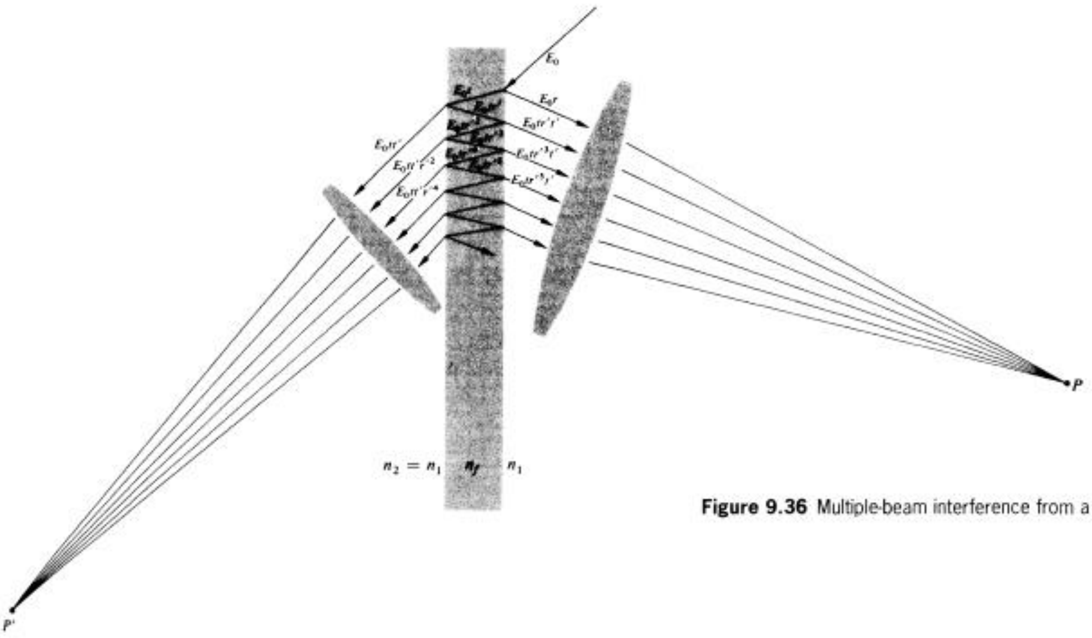


Figure 9.36 Multiple-beam interference from a parallel film.

Let $n_1 = n_2$, thereby avoiding the need to introduce different reflection and transmission coefficients at each surface. The optical fields at point *P* are given by

$$\vec{E}_{1r} = E_0 r e^{i\omega t}$$

$$\vec{E}_{2r} = E_0 t r' t' e^{i(\omega t - \delta)}$$

$$\vec{E}_{3r} = E_0 t r'^3 t' e^{i(\omega t - 2\delta)}$$

$$\dots$$

$$\vec{E}_{Nr} = E_0 t r'^{(2N-3)} t' e^{i(\omega t - (N-1)\delta)}$$

where $E_0 e^{i\omega t}$ is the incident wave.

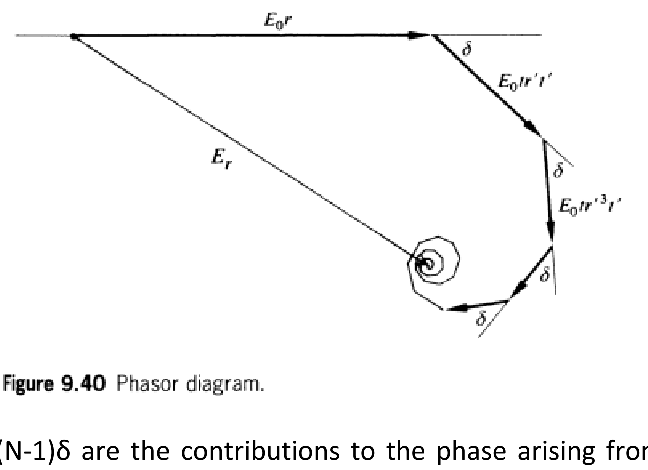


Figure 9.40 Phasor diagram.

The terms δ, 2δ,…, (N-1)δ are the contributions to the phase arising from an optical path length difference between adjacent rays, $\delta = k_0 \Lambda$ where $\Lambda = 2 n_f d \cos \Theta_r$ is the difference in optical path length. There is an additional phase contribution arising from the optical distance traversed in reaching point *P*, but this is common to each ray and has been omitted. The relative phase shift undergone by the first ray as aresult of the reflection is embodied in the quantity *r'*. The resultant *reflected scalar wave* is then

$$\vec{E}_r = \vec{E}_{1r} + \vec{E}_{2r} + \cdots + \vec{E}_{Nr}$$

or substitued

$$\vec{E}_r = E_0 r e^{i\omega t} + E_0 t r' t' e^{i(\omega t - \delta)} + \cdots + E_0 t r'^{(2N-3)} t' e^{i(\omega t-(N-1)\delta)}$$

This can be written as

$$\vec{E}_r = E_0 e^{i\omega t}\left\{r + r' t t' e^{-i\delta}\left[1 + \left(r'^2 e^{-i\delta}\right) + (r'^2 e^{-i\delta})^2 + \cdots + (r'^2 e^{-i\delta})^{N-2}\right]\right\}$$

If $\left|r'^2 e^{-i\delta}\right| < 1$, and if the number of terms in the series approaches infinity, the series converges. The resultant wave becomes

$$\vec{E}_r = E_0 e^{i\omega t}\left[r + \frac{r' t t' e^{-i\delta}}{1 - r'^2 e^{-i\delta}}\right]$$

In case of zero absorption, no energy being taken out of the waves, we can use the relation $r = -r'$ and $tt' = 1 - r^2$ to rewrite

$$\vec{E}_r = E_0 e^{i\omega t}\left[\frac{r(1 - e^{-i\delta})}{1 - r^2 e^{-i\delta}}\right]$$

The reflected flux density at *P* is then $I_r = \vec{E}_r \vec{E}_r^{\,*}/2$, that is,

$$I_r = \frac{E_0^2 r^2 (1 - e^{-i\delta})(1 - e^{+i\delta})}{2(1 - r^2 e^{-i\delta})(1 - r^2 e^{+i\delta})}$$

which can be transformed into (eq. (1))

$$I_r = I_i \frac{2r^2(1 - \cos\delta)}{(1 + r^4) - 2r^2\cos\delta}$$

The symbol $I_i = E_0^2/2$ represents the incident flux density, since, of course, E_0 was the amplitude of the incident wave. Similarly, the amplitudes of transmitted waves given by

$$\vec{E}_{1t} = E_0 t t' e^{i\omega t}$$

$$\vec{E}_{2t} = E_0 t t' r'^2 e^{i(\omega t - \delta)}$$

$$\vec{E}_{3t} = E_0 t t' r'^4 e^{i(\omega t - 2\delta)}$$

$$\cdots$$

$$\vec{E}_{Nt} = E_0 t t' r'^{2(N-1)} e^{i(\omega t - (N-1)\delta)}$$

can be added to yield

$$\vec{E}_t = E_0 e^{i\omega t}\left[\frac{tt'}{1 - r^2 e^{-i\delta}}\right]$$

(Because we are interested in the irradiance, a common factor of $e^{-i\delta/2}$, arising from the transmission through the thin film, was omitted. It contributes to the fact that there is a phase difference of $\pi/2$ between the reflected and transmitted waves, but that is of no concern here.) Multipliing by its complex conjugate yields the irradiance of the transmitted beam (eq. (2))

$$I_t = \frac{I_i (tt')^2}{(1+r^4) - 2r^2 \cos\delta}$$

Using the trigonometric identity $\cos\delta = 1 - 2\sin^2(\delta/2)$, it becomes

$$I_r = I_i \frac{[2r/1-r^2]^2 \sin^2(\delta/2)}{1 + [2r/1-r^2]^2 \sin^2(\delta/2)}$$

and

$$I_t = I_i \frac{1}{1 + [2r/1-r^2]^2 \sin^2(\delta/2)}$$

where energy is not absorbed, that is, $tt' + r^2 = 1$.

(Ref.: Hecht (International Edition), Optics, 4 Ed., p. 417-419)
(Ref.: Hecht, Optik, 4. Aufl., S. 673-675)

Or like in the script: Using the trigonometric identity $\cos\delta = 1 - 2\sin^2(\delta/2)$ and $R = r^2 = r'^2$ as well as the binomial formula, the eq. (1) becomes

$$I_r = \frac{I_i 4R \sin^2(\delta/2)}{(1-R)^2 + 4R\sin^2(\delta/2)}$$

Additionly using $T = tt'$, the eq. (2) becomes

$$I_t = \frac{I_i T^2}{(1-R)^2 + 4R\sin^2(\delta/2)}$$

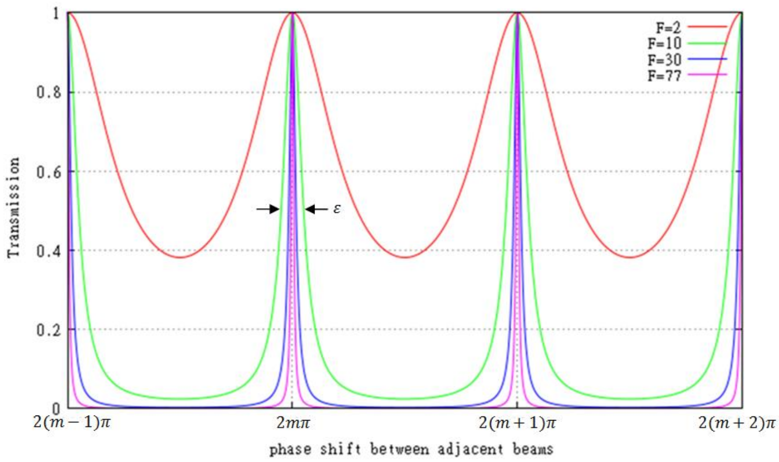
29. Explain the instrumental function of a Fabry-Perot interferometer!

- the instrumental function of a FPI is

$$W(\sigma) = \frac{\left(1 - \frac{A}{1-R}\right)^2}{1 + \frac{4R}{(1-R)^2} \sin^2(\pi n d \sigma)} \tag{29.1}$$

- this function is the Transmission function for the FPI to describe the changing of the spectral density (spectrum)
- it depends on the absorption A, reflection R and on the refraction index, the thickness d of the FPI and the wavenumber (property of the beam)
- in dependence of the reflectivity (also seen on the finesse) the instrumental function shows that only certain wavelengths (producing certain phase differences) are transmitted through the FPI

- therefore look on the figure below, which describes the transmittance of the FPI (Airy-fct.!)



30. Which rule do surface imperfections of mirrors play in the Fabry-Perot interferometer?

Surface imperfections like a rough surface or even scratches scatter the light and create rays with different phase due to the differnet optical path of the scattered rays. These rays interfere as well with the other and result in smeared out resonance maxima. So the finesse decreases. The quality of optical surfaces in terms of its mean roughness or mean height deviation Δh is measured in fraction of the wavelength, $\Delta h = \lambda/n$. The functional dependence of the finesse is $\boldsymbol{F} \approx n/2$.

31. What is the meaning of effective finesse?

It is the finesse after taking into account absorption/fluctuations in the cavity, thesurface imperfectionsand misalignment.

Ref.:https://docs.google.com/Doc?docid=0AVru3kJAsa2DZDR4enN6Y180Z2N4Mjk2ZzY&hl=en_GB

32. How is the free spectral range of a Fabry-Perot interferometer defined? Is there a difference with regard to the grating interferometer?

The free spectral range is the largest wavelength range for a given order that does not overlap the same range in an adjacent order. If the $(m+1)^{th}$ order of λ and $(m)^{th}$ order of $(\lambda + \Delta\lambda)$ lie at the same position/phase difference φ we get:

$$\Delta\lambda = \frac{\lambda}{m}$$

For a grating-IF (using the grating equation):

$$\Delta\lambda = \frac{\lambda^2}{g(\sin\theta_m - \sin\Theta_i)}$$

For Fabry-Perot-IF:

$$\Delta\lambda = \frac{\lambda_0^2}{2nl\cos\theta + \lambda_0} \approx \frac{\lambda_0^2}{2nl\cos\theta}$$

where n is the refractive index of the medium between the mirrors, l the distance between the mirrors, $\lambda_0$the central wavelength and θ the angle of incidence.

Normally:

$$\Delta\lambda_{\text{grating IF}} \gg \Delta\lambda_{\text{Fabry-Perot\_IF}}$$

33. Which basic principles should be observed in interferometric wavefront analysis to get usable experimental data?

- In principle, 3 independent intensity measurements required, so that 3 unknowns can be calculated (see formula below).

  $$I(r) = I_0(r)\{1 + V(r)cos\varphi(r)\}$$

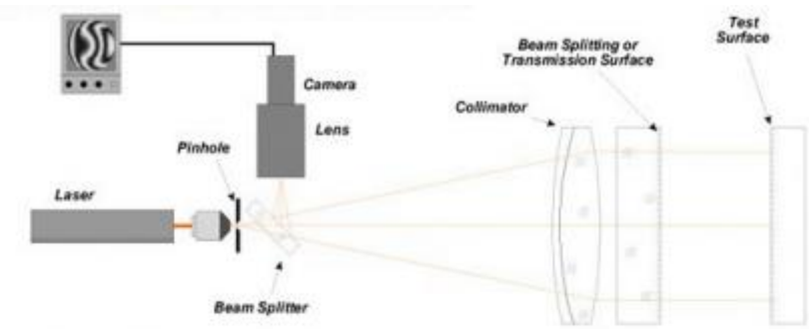
  Whereas $I_0(r)$, $V(r)$, and $\varphi(r)$ are unknown.
- Nyquist criterion (sampling theorem): to resolve fringe requires more than 1 detector per fringe ?!?

Reference: see 3.4 in lecture notes

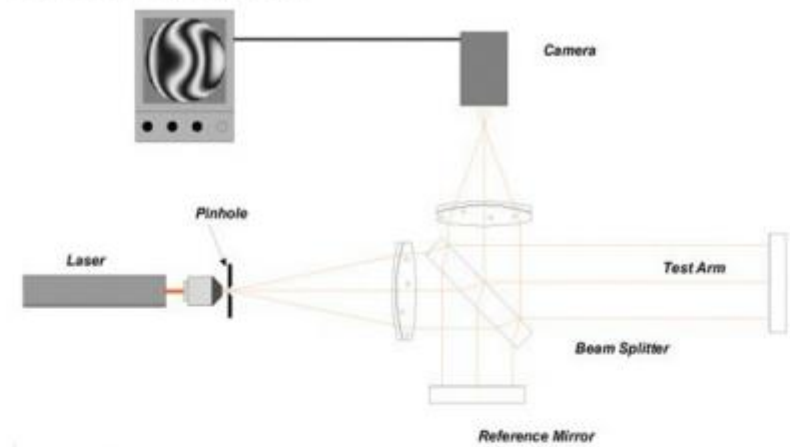
- Minimum surface irregularitions;

Reference: 2nd year students

34. Discuss the advantages and disadvantages of Fizeau and Twyman-Green interferometers!



Laser Fizeau Interferometer



Twyman Green Interferometer

Fizeau Interferometer:

+ **Common path (one arm) => very stable (insensitive adjustment)**
+ Collimated light leads to fringes of equal thickness
+ Flatness of surfaces could be tested, parallelism, also concave and convex surfaces against a flat surface
    + Standard method for evaluating the quality of optics
+ Very simple arrangement (could be small; beamsplitter not 45°, acts only to get beam on screen; very simple and robust)

-+ Surface of collimating lens must be of good optical quality

- A problem, commonly encountered in flatness testing of transparent plates, is unwanted interference effects (**ghost fringes**) arising from reflections from the back surface. (This problem can be overcome by using a multimode laser.)
- **Reference and test surface distance is very close => risk of damage**

Twyman-Green Interferometer:

(Similar test geometries can also be implemented with a Fizeau interferometer.)

+ Using plane waves and a lens, which permits all the light from the aperture to enter the eye so that the entire field can be seen
+ A continuous laser => convenience of long path length differences and short photographic exposure times *(for all interferometers)*
+ These tend to minimize unwanted vibration effects
+ Collimated light leads to fringes of equal thickness
+ The two optical paths can be made nearly equal => interference fringes with good visibility
+ **Can use both arms in different ways (different wavefronts)**

- **Two arms** (differently affected by mechanical shocks or temperature fluctuations)

(Ref.: Script)
(Ref.: Hecht (International Edition), Optics, 4 Ed., p. 434)
(Ref.: Hecht, Optik, 4. Aufl., S. 700-701)
(Ref.: http://books.google.de/books?id=sWbGSSQ6fPYC&pg=PA67#v=onepage&q&f=false)
(Ref.: http://books.google.de/books?id=EGdMO3rfVj4C&pg=PA119#v=onepage&q&f=false)
(Ref.: http://books.google.de/books?id=mHsU4mPToB0C&pg=PA52#v=onepage&q&f=false)
(Ref.: http://de.wikipedia.org/wiki/Fizeau-Interferometer)

35. Discuss the advantages and disadvantages of Shearing interferometers!

Advantages

- wavefront-sensor with the accuracy of an interferometer, because the wavefront difference is created by interferometry
- very simple and compact setup → stable adjustment caused by only one interferometer arm (lateral Shearing)
- precise measuring of the wavefront of an incoming beam (often used to focusing other optical system because the high sensitivity of defocused wavefronts)
- variable shear influences the sensitivity of the instrument → the higher the shear the smaller the changes of wavefronts are visible
  → caused by the interference pattern at the overlap region

- using grating shearing with rotating gratings with respect to one another, the first diffraction orders produced by the two gratings move across one another to give two lateral shear interferograms having shear in orthogonal directions → measurement of wavefronts having much aberrations
- with radial shearing same results like Twymann-Green IF but more simple setup

Disadvantages

- one has to be taken two measurements with orthogonal shear because the wavefront information is only describable for one shear (x- **or** y-direction) → lateral shearing

$$\Delta W(x,y) = W(\boldsymbol{x}-\boldsymbol{s},y) - W(\boldsymbol{x}+\boldsymbol{s},y) \tag{35.1}$$

  → this could be avoided by using grating or radial shearing interferometers
- very high quality required for the plate or wedge to measure the focusing of an incoming beam → the surfaces of the shear plate have to be plan parallel and should not cause any wavefront imperfections (lateral shearing)
- no reference wavefront needed
-

Lit.: Yoshizawa T. Handbook of optical Metrology; http://www.optics.arizona.edu/jcwyant/optics513/ChapterNotes/Chapter08/shearinginterferometryForInternet.pdf!!!

36. What is the difference between null tests and non-null tests?

Null Test	Non- Null Test
if we have an ideal object under test, the output would be straight, parallel lines, e.g. Fizeau fringes One can use: -conventional null optics -Holographic null optics -CGH	if we have an ideal object under test, the output would be no straight, no parallel lines. Methods which are used: -Lateral/Radial shear interferometry -Two-wavelength holography/Interferometry

- Aspheric surfaces are difficult to measure accurately.

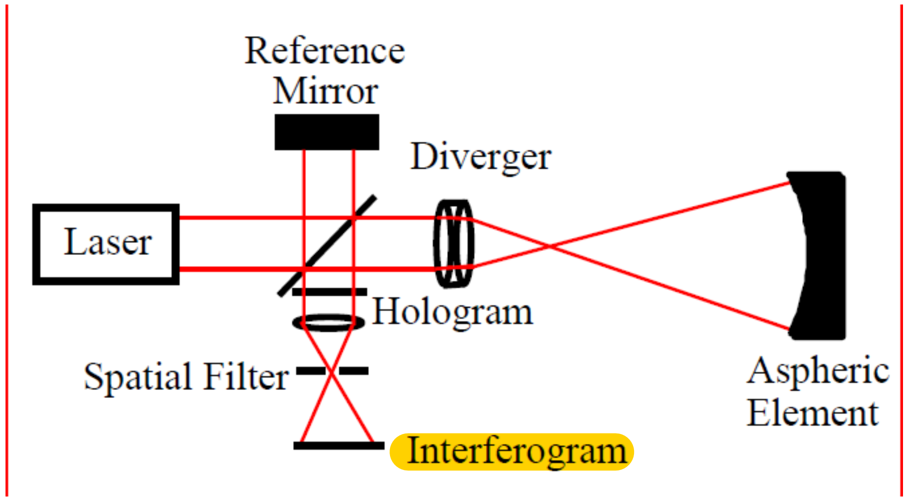
- Common method: construct an optical system (null lens, null mirror) which converts the

wavefront, produced by the aspheric surface under test, into a spherical or plane wavefront

which is then interferometrically compared with a known reference wavefront.

- Problem: Null optics are expensive

=> Solution: CGH or combination of “simple null optic” + CGH



Quelle: Script + Tafelbild zu Kapitel 3.3

More information about "Special Interferometric Tests for Aspherical Surfaces" can be found here:

http://www.optics.arizona.edu/jcwyant/optics513/ChapterNotes/Chapter09/chapter9NotesForInternet-Part A.pdf

37. Describe the Fourier transform method for the static evaluation of interference fringes!

We know:

$$I = I_0 + \{1 + v\cos\phi\} = a + \frac{1}{2}b(e^{i\phi} + e^{-i\phi})$$

We can rewrite the expression to:

$$I = a + c + c^* \; while \; c = \frac{1}{2}be^{i\phi}$$

Fourier Transform:

$$\mathcal{F}\{I\} = A + C + C^*$$

For an ideal case we get 3-separated information, whereby A does not provide any information about the phase, but C does and is symmetric to C*.

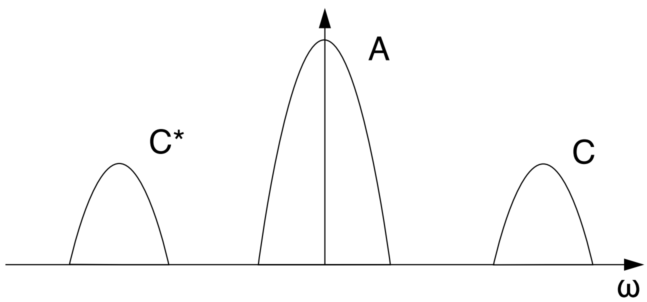


Figure 1 Signal from a Fourier Transformation method for extracting phase information

Ref.: Lecture notes

38. What are the advantages of carrier-frequency techniques?

- You only need 1 interferogram → more suitable for dynamic event analysis.

- No specific components are needed to perform carrier-frequency techniques

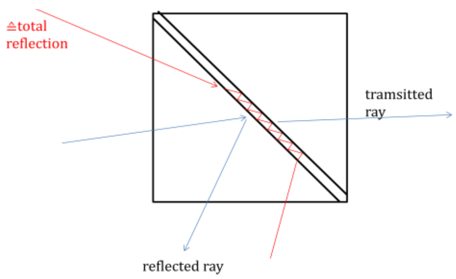
- Easy calibration

- No moving parts

→ Ref: M. Takeda, Hideki Ina and S. Kobayashi: Fourier-transform method of fringe-pattern analysis for computer-based topography and interferometry, J. Opt. Soc. Am. Vol .72, No.1 (January 1982)

Other References: http://adsabs.harvard.edu/abs/2004SPIE.5458..287K

39. How can be Herschel fringes realized (lecture experiment)?



- Possible: from two-beam to multiple-beam interference
- Dark fringes in transmission are bright fringes in reflection. Superposition leads to homogeneous light.
- Color in transmission has the corresponding complementary color in reflection. Superposition leads to white light.

40. How do phase shift methods work?

<u>Phase-Shifting:</u>

1) Modulate phase (moving mirror, diffraction grating, Bragg Cell, tilted glass plate, rotating half-wave plate)
2) Record several frames (min. 3)
3) Calculate OPD (optical path difference)

<u>Advantages of Phase-Shifting Interferometry:</u>

- High measurement accuracy
- Rapid measurement
- Good results with low contrast fringes
- Results independent of intensity variations across pupil

(Ref.:
http://www.optics.arizona.edu/jcwyant/Optics505(2000)/ChapterNotes/Chapter09/phaseshiftinginterferometry.pdf)

Phase-Measurement Algorithms:

- intensity $I(x,y)$ of an interferogram at a point (x,y):

$$I(x,y) = I_m(x,y) + I_a(x,y)\cos[\Phi(x,y)]$$

- 3 unknowns in the equation (I_m, I_a, Φ) => **3 equations needed**
- recording a series of intensity distributions with a uniform change of phase

$$I_1(x,y) = I_m(x,y) + I_a(x,y)\cos[\Phi(x,y) - \delta]$$

$$I_2(x,y) = I_m(x,y) + I_a(x,y)\cos[\Phi(x,y)]$$

$$I_3(x,y) = I_m(x,y) + I_a(x,y)\cos[\Phi(x,y) + \delta]$$

- I_1, I_2, I_3 are the intensity distributions recorded with phase change of $-\delta$, 0, $+\delta$
- phase $\Phi(x,y)$ can be determined:

$$\Phi(x,y) = \arctan\left[\frac{1-\cos(\delta)}{\sin(\delta)}\frac{I_1(x,y) - I_3(x,y)}{2I_2(x,y) - I_1(x,y) - I_3(x,y)}\right]$$

- When $\delta = \frac{2}{3}\pi$ it becomes

$$\Phi(x,y) = \arctan\left[\sqrt{3}\frac{I_1(x,y) - I_3(x,y)}{2I_2(x,y) - I_1(x,y) - I_3(x,y)}\right]$$

- more accurate **four phase-shifted images**:

$$I_1(x,y) = I_m(x,y) + I_a(x,y)\cos[\Phi(x,y)]$$

$$I_2(x,y) = I_m(x,y) + I_a(x,y)\cos[\Phi(x,y) + \frac{1}{2}\pi]$$

$$I_3(x,y) = I_m(x,y) + I_a(x,y)\cos[\Phi(x,y) + \pi]$$

$$I_4(x,y) = I_m(x,y) + I_a(x,y)\cos[\Phi(x,y) + \frac{3}{2}\pi]$$

- phase:

$$\Phi(x,y) = \arctan\left[\frac{I_4(x,y) - I_2(x,y)}{I_1(x,y) - I_3(x,y)}\right]$$

- **five images** (*Hariharan*), to minimize the cases of denominators with zero or near zero values:

$$I_1(x,y) = I_m(x,y) + I_a(x,y)\cos[\Phi(x,y) - 2\delta]$$

$$I_2(x,y) = I_m(x,y) + I_a(x,y)\cos[\Phi(x,y) - \delta]$$

$$I_3(x,y) = I_m(x,y) + I_a(x,y)\cos[\Phi(x,y)]$$
$$I_4(x,y) = I_m(x,y) + I_a(x,y)\cos[\Phi(x,y) + \delta]$$
$$I_5(x,y) = I_m(x,y) + I_a(x,y)\cos[\Phi(x,y) + 2\delta]$$

- phase:

$$\Phi(x,y) = \arctan\left[\frac{1-\cos(\delta)}{\sin(\delta)} \frac{I_2(x,y) - I_4(x,y)}{2I_3(x,y) - I_1(x,y) - I_5(x,y)}\right]$$

- If $\delta = \frac{1}{2}\pi$, then

$$\Phi(x,y) = \arctan\left[\frac{I_2(x,y) - I_4(x,y)}{2I_3(x,y) - I_1(x,y) - I_5(x,y)}\right]$$

- ***Carré* method**, where the phase shift amount is also treated as an unknown, uses **four phase-shifted images** as

$$I_1(x,y) = I_m(x,y) + I_a(x,y)\cos[\Phi(x,y) - \frac{3}{2}\delta]$$
$$I_2(x,y) = I_m(x,y) + I_a(x,y)\cos[\Phi(x,y) - \frac{1}{2}\delta]$$
$$I_3(x,y) = I_m(x,y) + I_a(x,y)\cos[\Phi(x,y) + \frac{1}{2}\delta]$$
$$I_4(x,y) = I_m(x,y) + I_a(x,y)\cos[\Phi(x,y) + \frac{3}{2}\delta]$$

- Assuming the phase shift is linear and does not change during the measurements, the phase at each point is determined as

$$\Phi(x,y) = \arctan\left[\frac{\sqrt{[(I_1 - I_4) + (I_2 - I_3)][3(I_2 - I_3) - (I_1 - I_4)]}}{(I_2 + I_3) - (I_1 + I_4)}\right]$$

- advantage of *Carré*: requires no accurate calibration of the phase shifting mechanism

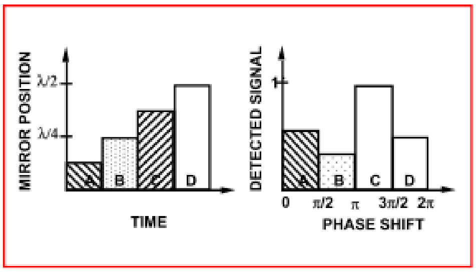
(Ref.: http://www.opticist.org/node/36)

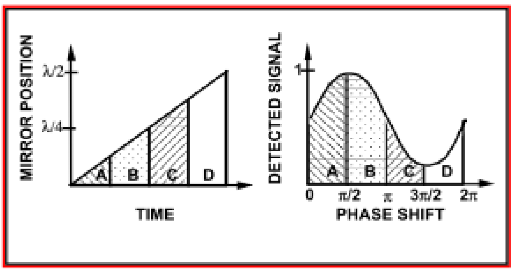
Removing Phase Ambiguities:

- Arctan Mod 2π (Mod 1 wave)
- Require adjacent pixels less than π difference (1/2 wave OPD)
- Trace path
- When phase jumps by > π
  - Add or subtract N2π
  - Adjust so < π

Phase-Stepping Phase Measurement:

Integrated-Bucket Phase Measurement:



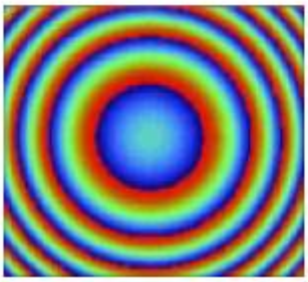


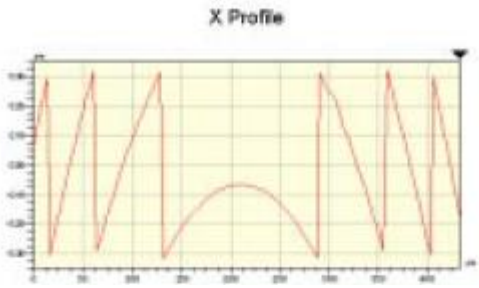
Integration:

$$I_i = \frac{I_0}{\Delta}\int_{\alpha_i-\Delta/2}^{\alpha_i+\Delta/2} \{1 + V\cos[\Phi + \alpha_i(t)]\}\, \mathrm{d}\alpha(t)$$

Before integration:

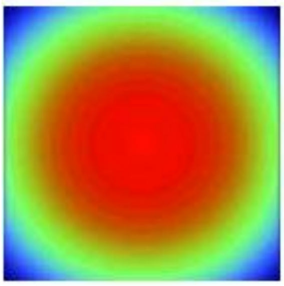
2 π Phase Steps





After integration:

Phase Steps Removed



(Ref.: http://www.optics.arizona.edu/jcwyant/Optics505(2000)/ChapterNotes/Chapter09/phaseshiftinginterferometry.pdf)

41. What are the advantages and disadvantages of phase step and integrating bucket methods?

	advantages	disadvantages
Phase step method	• high accuracy (measuring more than 4)	• steps have to be equidistant

both	• simple setup (Twymann-Green IF) • measuring the phase in every point of the image possible	• at least 4 measurements are needed • only for small varying surfaces • very low exposure time for detectors needed
Integrated bucket method	• automatic measurement possible without knowing the distance for phase 2pi • faster than phase step	• decrease contrast (visibility)

42. Which methods of phase shifting can be used?

-Phase stepping

- *Carré* method

-Integrated bucket (continous phase changes)

43. ***How does a heterodyne interferometer work and what is the reason for its very high accuracy of measurement?***

In heterodyne interferometer the beams of each arm gets modulated with a different frequency ($\Delta\omega \leq 100\ MHz$, small frequency shift), mostly done by acoustooptical modulator.

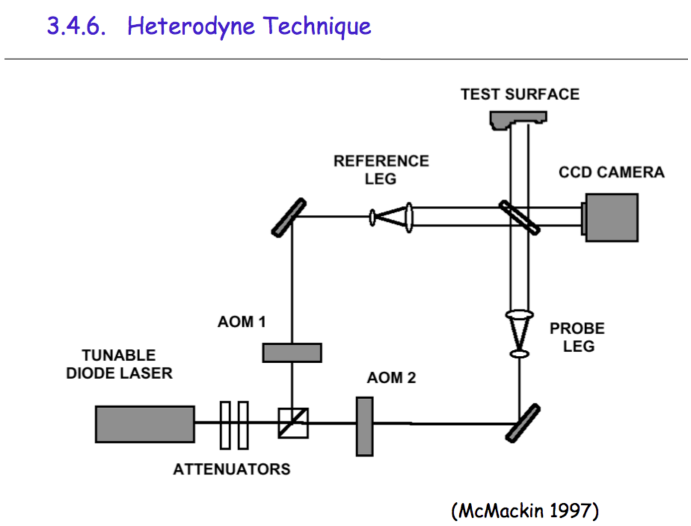


Figure 2Schematic drawing of the heterodyne interferometer

Ref.: Lecture notes

This gives

$$I(\mathrm{t}) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(2\pi(v_1 - v_2)t + (\varphi_1 - \varphi_2)$$

The output contains an ac component at the difference frequency $(v_1 - v_2)$ whose phase is $(\varphi_1 - \varphi_2)$. The phase difference between the interfering light waves can then be obtained from measurement of the ac component with respect to a reference signal. This gives a very stable subdivision of the light wavelength with high resolution (nm range) on the basic of a phase measurement.

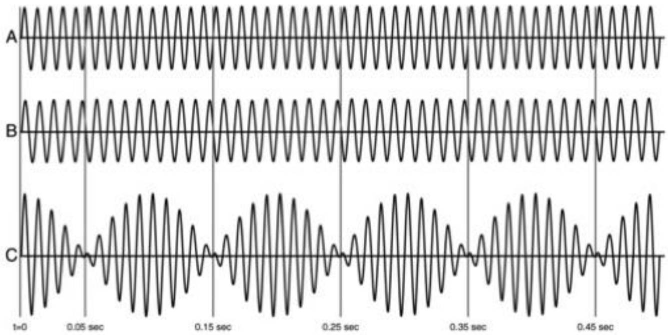


Figure 3 A and B are signals with a small frequency shift, C is the superimposed signal -> beat frequency

44. Describe the calibration of phase shifters using the ellipse fitting technique!

The measured fringe field distribution for a two beam interferometer can be written as:

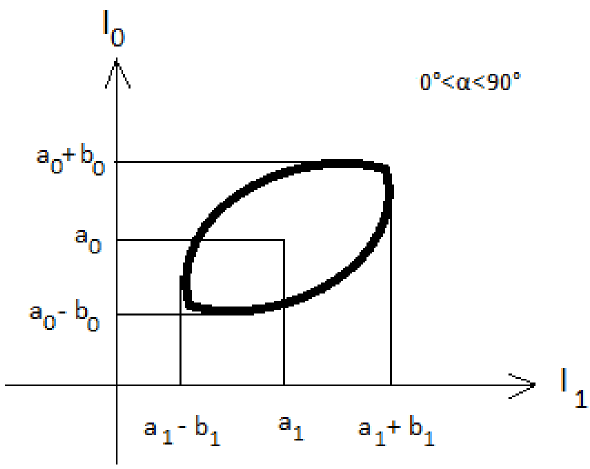
$$I_0 = a_0 + b_0 \cos(\phi)$$

a – average intensity of the fringe field, ϕ - phase to be measured, b- amplitude of intensity modulation

If we now add an unknown phase step α to it (using the phase shifter) the intensity distribution changes:

$$I_1 = a_1 + b_1 \cos(\phi + \alpha)$$

We can plot a Lissajous figure (fit an ellipse) for 0°<α<90° :



We can calculate the unknown phase step α using:

$$\frac{(I_0 - a_0)^2}{b_0} + \frac{2\cos\alpha\,(I_0 - a_0)(I_1 - a_1)}{b_1 b_0} + \frac{(I_1 - a_1)^2}{b_1} = \sin^2\alpha$$

Ref: Farrell, Player: Phase step measurement and variable step algorithms in phase-shifting interferometry. Meas. Sci. Technol. 3 953 (Inet: http://iopscience.iop.org/0957-0233/3/10/003/pdf/0957-0233_3_10_003.pdf)

45. What is the meaning of interferometry with effective or synthetic wavelengths?

If one wants to measure a rough surface without measuring the irregularities of the surface, one needs a bigger wavelength → reduced accuracy. This can either can achieved by using another laser source or by the following techniques:

Effective wavelength:
Goal: measure shape of rough surface by using interferometry with reduced accuracy. Therefore, either the wavelength or the angle of incidence need to be increased ("grazing incidence") -> to increase the effective wavelength:

$$\frac{\lambda}{cos\alpha} = \Lambda_{eff} \quad \text{(see lecture notes)}$$

Synthetic wavelength:
In a two wavelength interferometry setup, the process of using two wavelengths, which are close to each other creates a virtual or synthetic wavelength Λ, which is much larger than the individual wavelengths (λ_1, λ_2) and is given by:

$$\Lambda = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$$

Where Λ is something like the beat freguency, which is used for measurement.

(Schwebungsprinzip)
(siehe Vorlesungsunterlagen)

46. How does a grazing-incidence interferometer work and for which applications can it be used?

- With conventional interferometers it is not possible to obtain interference fringes with rough surfaces (no reflections with visible light at normal incidence)
- One way to solve that problem: using longer wavelengths => infrared interferometers (CO_2)
- Or simpler: using visible light incident obliquely on the surface at an angle at which it is specularly reflected
- Rough surfaces appear smooth when viewed at a grazing angle (speckle noise reduced, because the effective wavelength is increased by a factor)

$$\lambda_{\text{eff}} = \frac{\lambda}{\cos\alpha}$$

(Ref.: http://www.zygo.com/library/papers/AO_39_10_1527.pdf)

(Ref.: http://books.google.de/books?id=sWbGSSQ6fPYC&pg=PA77&lpg=PA77&dq=grazing-incidence+interferometer&source=bl&ots=zVAgeW9oLo&sig=rV7VI-

IGgUicNIzoyKky837RJvk&hl=de&ei=wTEbTa6dCY6MswbH_736DQ&sa=X&oi=book_result&ct=result&resnum=1&ved=0CCcQ6AEwADgK#v=onepage&q=grazing-incidence%20interferometer&f=false)

(Ref.: http://mpl.mpg.de/mpf/php/abteilung1/files/jabe/anrep_99_move.pdf)

Setup:

- beam splitting and beam combining with two identical DOE (diffractive optical elements)
- expanded laser beam hits the first DOE => split into several diffraction orders => one of the first orders plays the role of the probe beam for the surface under test
- after reflection the wave is diffracted at the second DOE and recombined with the undiffracted zero order wave to form the interference pattern

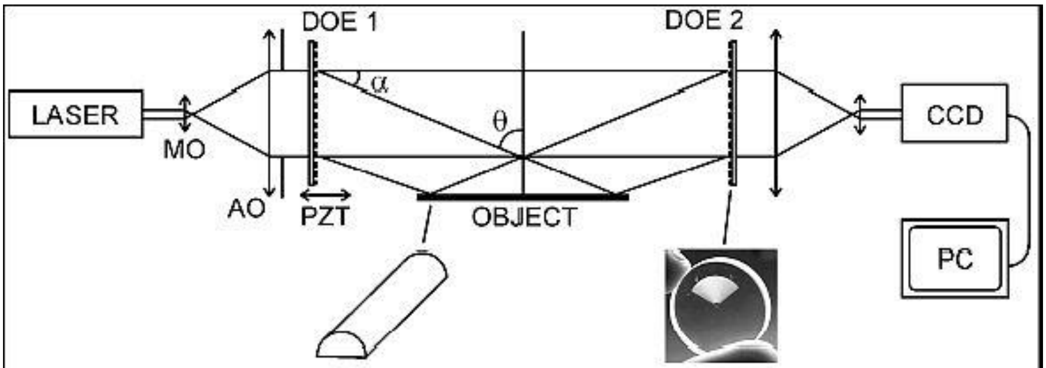


Fig 1: Scheme of the grazing incidence interferometer for testing rod shaped objects (cylindrical lenses are a special case). The He-Ne-laser light is expanded by means of a microscope objective (MO) and an astronomical achromat (AO). The DOE 1,2 perform the beam shaping and splitting. The interference pattern is detected by a CCD-array. The first DOE can be moved axially to provide the necessary phase variations for phase shifting interferometry (PSI).

(Ref.: http://www.google.de/imgres?imgurl=http://www.ndt.net/article/v07n04/lamprecht/fig1.gif&imgrefurl=http://www.ndt.net/article/v07n04/lamprecht/lamprecht.htm&usg=__vbLV3DSS1C5gXsZXcSB8xWbcFPQ=&h=205&w=584&sz=11&hl=de&start=9&zoom=1&um=1&itbs=1&tbnid=iQYk0eMeEjxd9M:&tbnh=47&tbnw=135&prev=/images%3Fq%3Dgrazing-incidence%2Binterferometer%26um%3D1%26hl%3Dde%26client%3Dopera%26sa%3DN%26rls%3Dde%26channel%3Dsuggest%26tbs%3Disch:1%26prmd%3Divns)

(Additional Ref.: http://www.grahamoptical.com/graze.html)

47. What is the meaning of optical phase conjugation?

- phase conjugation means that the wavefront and the direction of an electromagnetic wave (E_i) is inversed to their propagation if it is conjugated (E_c) (t.e. by a PCM)

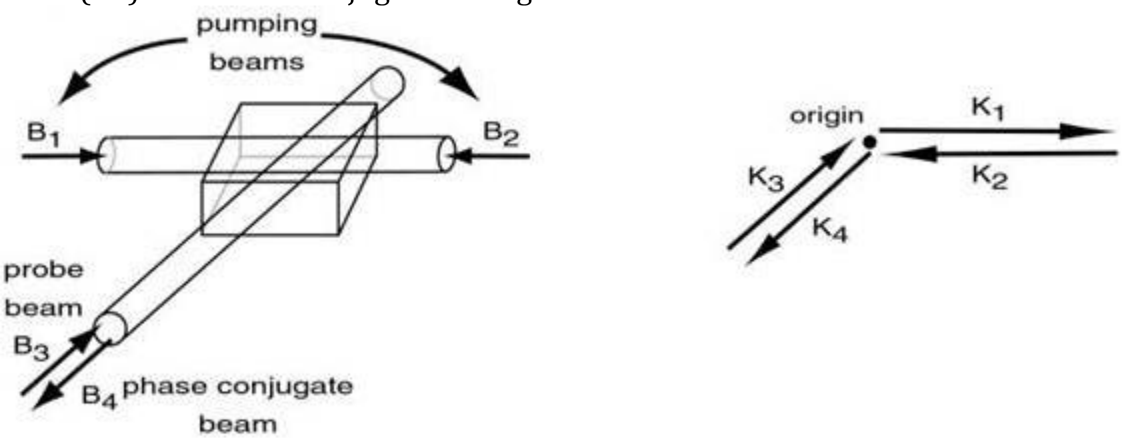
$$\boldsymbol{E}_i(\boldsymbol{r},t) = \Re\{\boldsymbol{A}(x,y)e^{i(wt-\boldsymbol{kr})}e^{i\Phi(xy)}\}$$

$$\boldsymbol{E}_c(\boldsymbol{r},t) = \Re\{\boldsymbol{A}^*(x,y)e^{i(wt+\boldsymbol{kr})}e^{-i\Phi(xy)}\}$$

- therefore the consequence is that a wavefront reflected by a PCM propagates back to their origin with exact the same form before (you look on PCM you will see your points of retina – but intensity is to low)

- only the direction of the wavefront is changed (inverse) – this can be reached by conjugate the phase term → phase conjugation!

48. How can phaseconjugated waves be generated?

- Phase conjugated waves can be generated with the help of a non-linear optical medium.
- A technique which is often used to generate phase conjugated waves is the so called "degenerate four-*wave* mixing":
- Two beams (B1,B2) interfere to form some type of grating (e.g. intensity grating) and a third beam (B3) scatters off this grating, generating the fourth beam (B4) named the conjugated or signal beam.
- 

- (Please click link to see more detailed explanation (page 81-83)).
- http://scholar.lib.vt.edu/theses/available/etd-022299-083514/unrestricted/CHAPTER3.PDF

49. Which are the advantages of phase-conjugate mirrors in interferometry?

Advantages of surface measurement:

1. No high-corrected elements in the reference arm
2. No high-corrected collimation optics, no plane referencemirrors
3. No compensation plate in the test arm and no high-symmetrical beam splitter
4. Simple adjustment (self-adjustment of the reference arm)
5. Limitation of the aperture by only the beam splitter

Advantages of wavefront measurement:

1. Measurement of the absolute phase distribution without any reference wave(self-referencing)
2. Double sensitivity of phase measurement
3. Fringe contrast ≠ f(A(x,y))
4. Compensation of aberrations in the reference arm
5. Simple interpretation of the interferogram (conjugated shear)
6. Only one plane surface of high optical quality is required

Ref.: Lecture notes, 3.6.2. Two-Beam Interferometers with PCM

50. Describe the advantages of a Fizeau interferometer with a PCM!

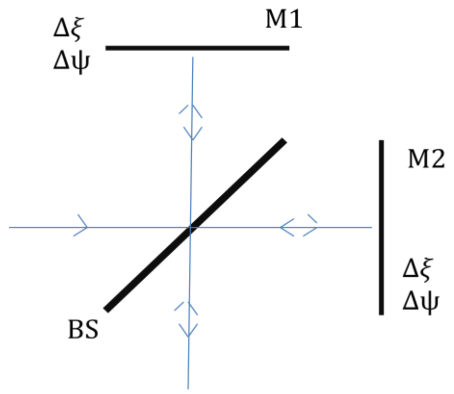
- measurement of the absolute phase distribution without any reference wave (self-referencing)
- Double sensitivity of phase-measurement
- Fringe contrast is independent to unwanted irradiance variations A(x,y) arising from nonuniform light reflection or transmission by a test object. → Ref: M. Takeda....
- compensation of aberrations in the reference arm
- simple interpretation of the interferogram (conjugated shear)
- only one plane surface of high optical quality is required

51. How is it possible to measure reciprocal and non-reciprocal effects with interferometers?

-reciprocal effect: effect on incident beam is independent from propagation direction and is therefore the doubled effect is obtained when propagating through a reciprocal medium in forward and backward direction.

- non-reciprocal effect: dependent from propagation direction of an incident beam the properties of this beam changes; therefore the effect is compensated when propagating twice (forward and backward) through a non-reciprocal medium (examples: Faraday effect, Sagnac effect)

- measurement of reciprocal and non-reciprocal effects is possible in a Michelson interferometer with one PCM



	M1	M2	$\lvert\Delta\Phi\rvert$	effect
Conventional MIF	2Δψ 0	2Δψ 0	0 0	reciprocal non-reciprocal
MIF with one PCM (M2)	2Δψ 0	0 2Δξ	$\lvert 2\Delta\psi\rvert$ $\lvert 2\Delta\xi\rvert$	reciprocal non-reciprocal

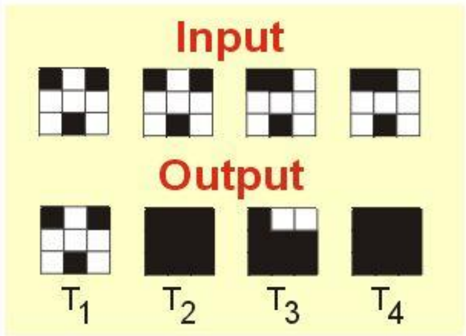
- depending on either a reciprocal or a non-reciprocal effect the phase difference in the interferometer with the PCM is either doubled (reciprocal effect) or compensated

52. What is a novelty filter and how can it be realized?

- are devices whose output consists of only the changing part of the input *(Novelty – Neuigkeit)*

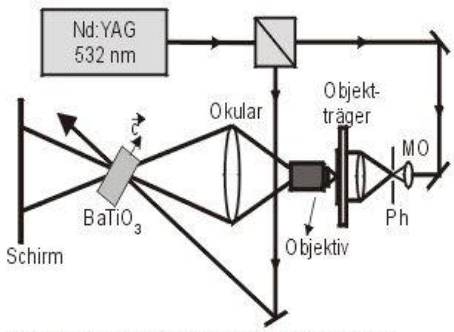
  (Ref.: Bass M. (ed.) Handbook Of Optics. Volume 4, 12.32)

- (temporal highpass filter)
- shows novelties in an input pattern with respect to previously lerned patterns



Die obere Reihe zeigt den Eingang des Neuigkeitsfilters zu verschiedenen aufeinanderfolgenden Zeitpunkten und die untere Reihe den jeweils dazugehörigen Ausgang. Von T1 zu T2 haben das 3. und 4. Pixel ihre Werte ausgetauscht. Da beide Änderungen neu sind erscheinen beide Pixel am Ausgang hell.

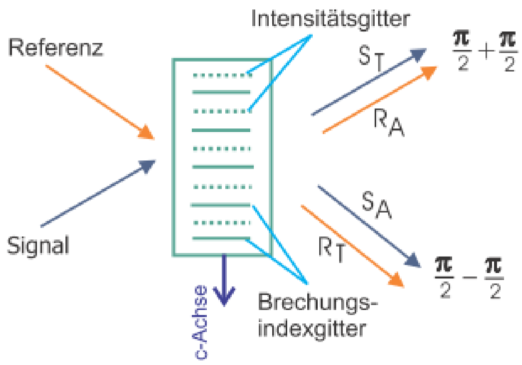
- photorefractive crystal ($BaTiO_3$)
- response time of the crystal could be in the second range
- **Principle 1:** two-beam coupling
- two beams are incident



Aufbau eines neuigkeitsfilternden Mikroskops.

(Ref.: http://pap093.uni-muenster.de/?id=n_research_biophot_noveltyfilter&lang=de)

- energy transfer between signal beam and reference beam; signal beam is attenuated and reference beam is gained; change in signal beam destroys stationary condition (beam coupling needs some time) and will be transmitted (more complicated setup)



- **Principle 2:** Beamfanning
- only one beam is incident
- amplifying of self-induced scattered light due to crystal defects or roughness of the surface (easy setup)
- again only changes of signal beam could be measured due to response time of the crystal

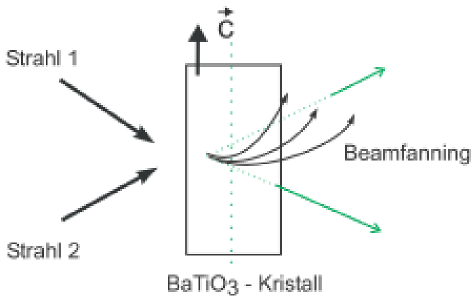


Abbildung 3.10: Schematische Darstellung des Effekts des Beamfannings in einem photorefraktiven Kristall. Strahlen, die in eine Richtung gestreut werden, wo Zweiwellenmischungseffekt stattfindet, werden verstärkt. Die gerichtete Verstärkung verläuft in Richtung der **c**-Achse des Kristalls.

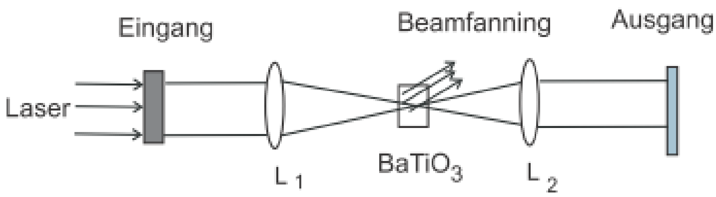


Abbildung 3.11: Konfiguration für einen Neuigkeitsfilter durch den Effekt des Beamfannings. L: Linse.

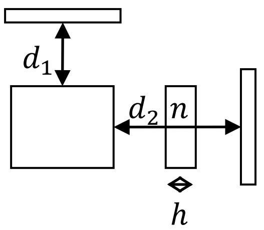
(Ref.: Karaboué C., Ein photorefraktiver Neuigkeitsfilter in der Sichtprüfung technischer Objekte, Dissertation, S. 23-27, 32
http://deposit.ddb.de/cgi-bin/dokserv?idn=971628467&dok_var=d1&dok_ext=pdf&filename=971628467.pdf)

53. How does a white-light interferometer work and for which application is it well suited?

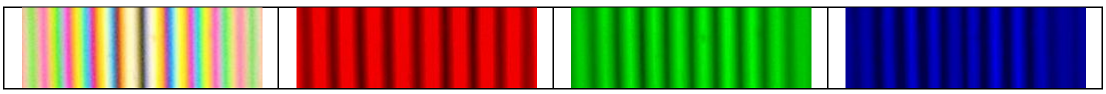
- interferometer with a white-light source (broad optical bandwidth), which has a short l_c (small temporal coherence) and **high** spatial coherence
- typically the setup is realized by a MIF
- therefore the distances of the object/(ref)mirror to beam splitter has to be in micrometer range, to create interference pattern on the detector
  →(path difference between object- and referencewave) < l_c
- the detector records the signal either in time domain (photodetector) or in frequency domain (spectrometer)
- the interference pattern is in visible, IR and UV possible – more wavelengths, more information about the path differences between them → more information about the topology of an surface!
- **important:** to compensate the dispersion effects, one has to use a compensation plate with the following setting:

  with # of fringes $N = \frac{\Delta d}{\lambda}$ and the condition the dispersion vanishes $\frac{dN}{d\lambda} = 0$ and

  $\Delta d = 2d_1 - 2(d_2 + nh)$ we get the following condition for the setup with compensation plate:



$$d_1 - \left\{d_2 + h\left(n - \lambda\frac{dn}{d\lambda}\right)\right\} = 0$$



www.wikipedia.org (probably without a compensation plate!)

Applications

- Analysis the **Topology** of a surface of an object under test by using time domain detection
- Measurement of the **chromatic dispersion** of an object (frequency domain)
- the wave are also transmitted in the object and reflected/scattered on surfaces with different optical behavior (reflectance) – this could be used for **optical coherence tomography**
  - depth resolution few mm (depends on the losses) – time domain OCT
  - in time domain the refmirror has to be moved for scanning the depth (tomography)
  - in frequency domain all reflections are recorded in one measurement - fourier domain OCT
  - widely used in ophthalmology for scanning the retina (layers)

- OCR (optical coherence radar) is a white-light interferometer with using speckle patterns (container of depth info) to analyze rough surface structures of an object under test

OCM (optical coherence microscopy)

54. Why are wavefront aberrations determined in the plane of the exit pupil?

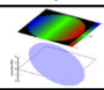
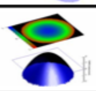
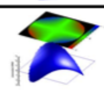
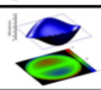
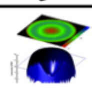
To be able to compare measurement results, since the exit pupil is well defined and can be constructed for every optical system.

The aberrations are described by the optical path difference (OPD) of the measured wavefront to the ideal spherical wavefront in the exit pupil.

55. How can wavefront aberrations be described by Zernike polynomials?

The wave aberrations can be expressed as a weighted sum of Zernike polynomials:

$$W(\rho,\theta) = \sum_{n}^{k} \sum_{m=-n}^{n} W_n^m Z_n^m(\rho,\theta)$$

Meaning	$Z_{n,m}(\rho,\alpha)$	Graph
Tilt	$Z_{11} = \rho \cos\alpha$	
Defocus (Sphere)	$Z_{20} = 2\rho^2 - 1$	
Astigmatism	$Z_{22} = \rho^2 \cos 2\alpha$	
Coma	$Z_{31} = (3\rho^3 - 2\rho)\cos\alpha$	
Sph. Aberration	$Z_{40} = 6\rho^4 - 6\rho^2 + 1$	

Ref.: Lecture notes, 4.1. Introduction

56. Which aberrations are called Seidel aberrations?

Zernike polynomials	Aberration Name
$Z_{40}=f(p^4)$	Spherical Aberration
$Z_{31}=f(p^3, \cos\alpha)$	Coma
$Z_{22}=f(p^2, \cos 2\alpha)$	Astigmatism
$Z=f(p^2)$	Field Curvature
$Z=f(p,\cos\alpha)$	Distortion

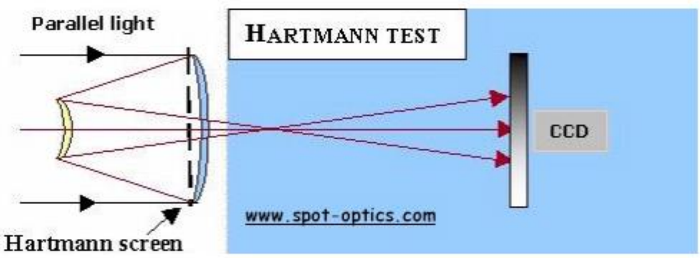
LERNHILFE: erste Zahl der Zernikepolynome steht für Ordnung von p und zweite Zahl für Faktor vor α in cos-Term.

57. Give examples for indirect and direct wavefront sensors?

Direct	Indirect
Radial Shearing	Foucault knife edge
Point Diffraction Interferometer	Pyramid sensor
	Wire test
	Lateral shearing
	Grating shearing
	Hartmann sensor
	Shack-Hartmann sensor

Reference: Script

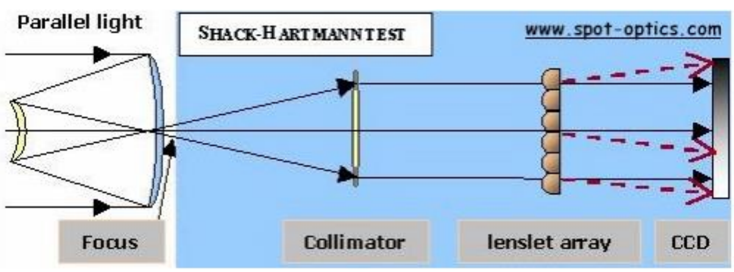
58. Which are the advantages of Hartmann-Shack sensors compared with Hartmann sensors?



Hartmann

- (convergent beam) a screen with pinholes is placed before the element being tested
- spots are recorded on a CCD before or after (as in the figure) the focus
- the positions of these spots are mapped back onto the optical element, and the difference between the positions of these spots and the holes of the screen are used to compute the aberrations

Shack-Hartmann:



- lenslet array is placed in the transferred pupil
- light from the optical system being tested comes to a focus and is made parallel by the collimator (collimated beam), which also images the pupil on the array
- displacements due to the aberrations with respect to the ideal wavefront are used to compute the aberrations (in terms of Zernike polynomials) and the wavefront

Advantages of Hartmann-Shack sensor compared with Hartmann sensor:

- the system is easier to calibrate
- is of much higher precision
- a lenslet array leads to higher sensitivity
- recommended for high deviated wavefronts
- focusing makes the light energy density of the spot much higher, Hartmann blocks much photons
- to increase the measurement accuracy the focal length must be increased, but which decreases the dynamic range (largest measurable tilting)
- Hartmann test was to slow to allow active correction

(Ref.: http://www.spot-optics.com/downloads/pdf/hartmann_sh.pdf)
(Ref.: http://de.wikipedia.org/wiki/Hartmann-Shack-Sensor)
(Ref.: http://de.wikipedia.org/wiki/Hartmann-Shack-Sensor)
(Ref.: Script from Mundus)

59. Which are the main fields of application of wavefront sensors?

Astronomy

- Alignment of telescopes
- increase the image quality acquired with telescopes by analyzing the wavefront and reducing the aberrations with adaptive optics

Medicine

- Ophthalmology – detection of aberrations of the eye; lens testing

Optics

- Alignment of optical systems
- Testing optical elements (LCD panels, lens, beam expanders …)
- Quality of lasers

Military

- Radar measurements
- correct detection of objects from military interest (missile defence)

Biology

- microscopy

60. What is the meaning of adaptive optics?

Adaptive optics is a technique to improve the quality of imaging optical systems.
The Basic idea is to compensate wavefront aberrations.
Example Astronomy: Distorted wavefronts from stars by fluctuating atmosphere.
System consists of:
- Wavefront sensor which measures aberrated wavefronts

- Computer which calculated the OPD, the corresponding aberration and the correction
  profile
- Deformable optic (e.g. mirror) which can be shaped in a way to correct the aberration,
  based on the correction profile given by the Pc.

It is an iterative process.

61. What limits the resolution of earth telescopes?

Limits:

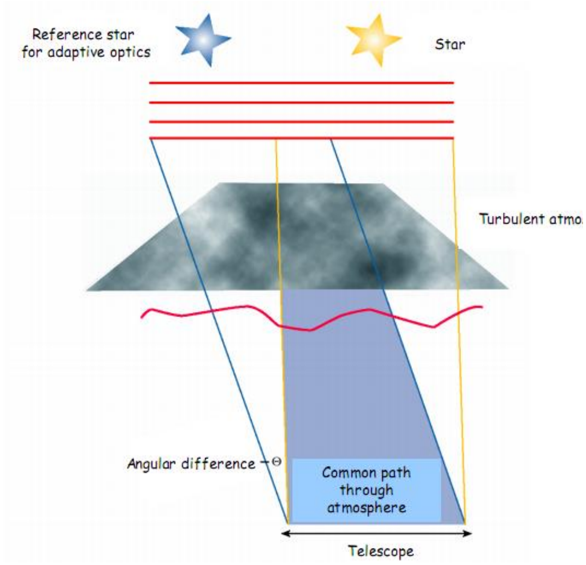
- Numerical aperture
- Turbulence in atmosphere

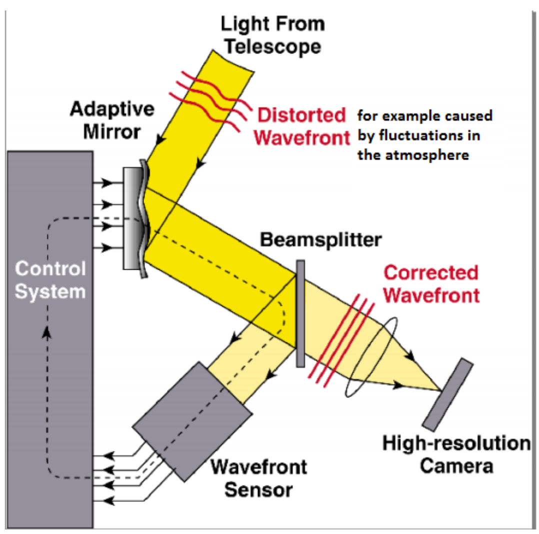
In General the numerical aperture limits the resolution, the better the angular resolution ($sin\theta = \frac{1.22\,\lambda}{D}$, $D$is the aperture) and light-gathering power are. However for large ground-based telescopes, the resolution is limited by turbulence in the atmosphere. This limit can be overcome by placing the telescopes above the atmosphere, e.g., on the summits of high mountains, on balloon and high-flying airplanes, or in space. Resolution limits can also be overcome by adaptive optics, speckle imaging or lucky imaging for ground-based telescopes.

Ref.: http://en.wikipedia.org/wiki/Optical_telescope

62. How does a telescope with adaptive optics works?

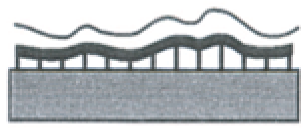
A reference star near the object of interest is chosen (angular distance), so the wavefront coming from this reference include aberrations that are shared with the object. Then, the aberrations related to reference is corrected using an adaptive optics system between the telescope and the camera. After correcting the wavefront the object of interest can be observed almost aberration free.





63. How can adaptive mirrors be realized?

Adaptive mirrors in general consist of a reflecting membrane and attached to this, several actuators. These actuators can be moved separately, such that the membrane is locally deformed. This effect is used to shape the membrane inverse to the incoming wavefront and thus, for a known aberrated wavefront shape, such deformable mirrors can be used to correct or reshape the incoming wavefront.
Example: Piezo-electric mirror



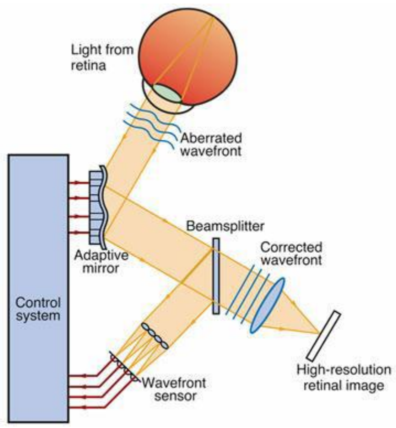
Piezo-electric mirror

64. What is the meaning of guide stars and how can they be generated?

- artificial star created for use in astronomical adaptive optics imaging (since bright stars are not available in all parts of the sky)
- shining a laser into the atmosphere
- two main types (often pulsed): sodium and Rayleigh beacon *(Leuchte)* guide stars
- Sodium beacons are using a laser (589.2 nm) to energize a layer of sodium atoms which is naturally present in the mesosphere (altitude 90 km); sodium atoms re-emit the laser light, producing a glowing artificial star (yellow like street lights)
- Rayleigh beacons rely on the scattering of light by the molecules which make up the lower atmosphere

(Ref.: http://en.wikipedia.org/wiki/Laser_guide_star)

65. How can adaptive optics be used in ophthalmology?



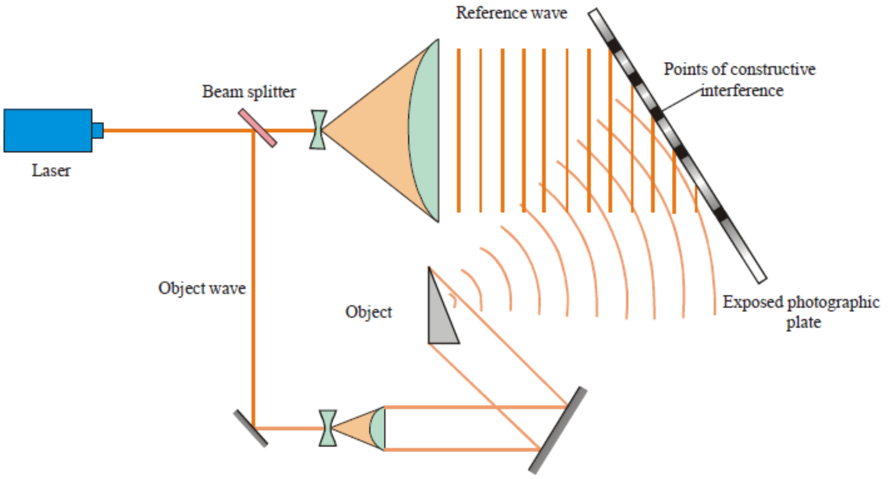
High quality image of the retina is required for the diagnose of diseases. The imaging system of the **eye** (cornea, crystalline, humors) **introduces aberrations** when retina is scanned with shown setup. Identifying those aberrations and compensating them in the optical path between camera and retina **increases** the **contrast and resolution** of the retina image formed in the camera. A **wavefront sensor measures the distortions** of the light reflected by the retina, the control system send such information to the **adaptive mirror** to **compensate** the distorted wavefront coming from the eye and then image formed in the camera is **distortion free**.

66. Explain the principle of holographic recording and reconstruction of wavefronts!

Recording:

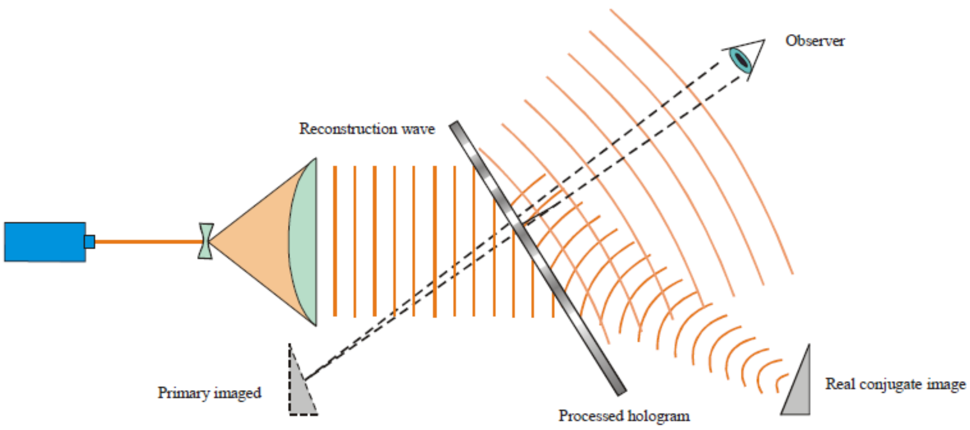
-Laser with sufficient coherence length (spatially as well as temporal)

-Object with rough surface, so that light is scattered
-Superposition of (scattered) object wave and reference wave
-Record interference pattern in e.g. holographic plate (medium with linear response)



Reconstruction:

-Illuminate hologram (developed holographic plate) just with reference wave
-Hologram act like grating and reference wave gets refracted into a primary imaginary and a
 real conjugate image.



67. Which physical and technical conditions have to be fulfilled for holography?

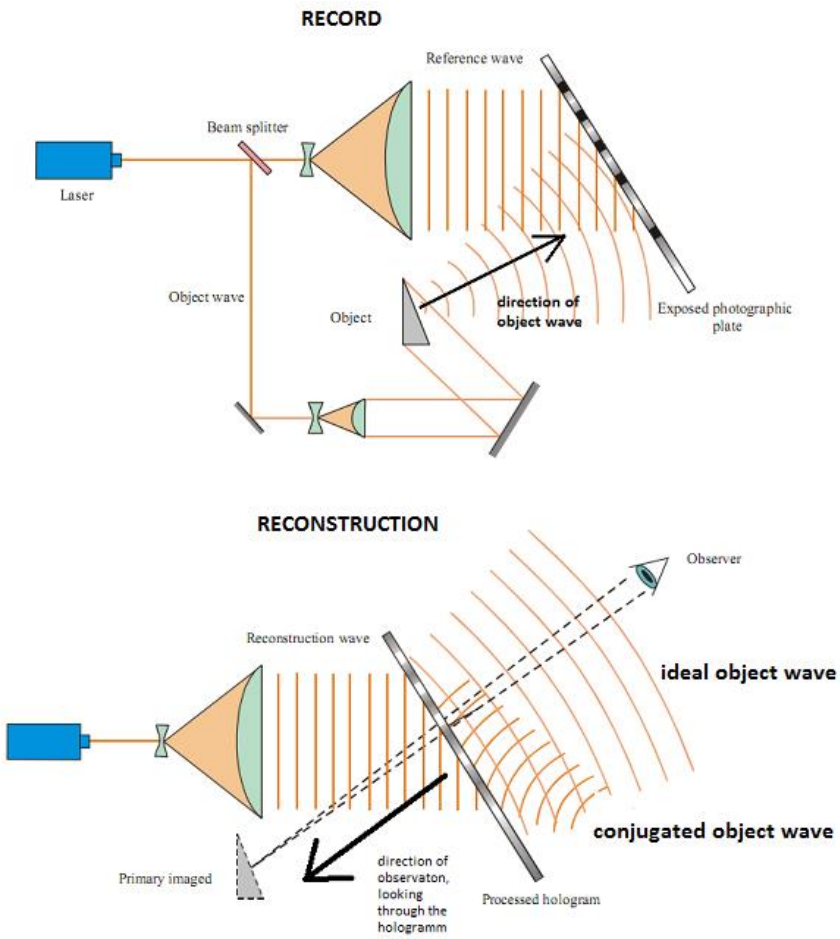
Conditions:

- Temporally and spatially coherent light source
- The recording and constructing wave should be the same
- Linear response of the recording material to intensity
- Rough surface of the object

Ref.: Lecture notes

68. Under which conditions can a virtual image be reconstructed?

In general you have to separate the 0. diffraction order wave, the conjugated object wave and the ideal object wave lateral. You can do that if you use an off-axis illumination of the hologram. To see a virtual image in the reconstruction process, the direction of observation through the hologram must be antiparallel to the direction of the object wave in the record process.



69. Which methods of holographic interferometry (HI) do you know and what are they used for?

- Real-time HI: to observe the changing of the object's state while change is taking place (in real time)
- Double exposure HI: used to compare 2 different states of an object, which is changing
- Pulse HI: used for very fast changes → pulse captures one state of the changing
- Time averaged HI: usually used for harmonic oscillations

70. Which conditions have objects to fulfill for holographic recording?

- **Rough objects** => high light scattering
- You will not see the optical objects in the hologram (e.g. lenses, mirrors)

(Ref.: Script)

71. Which are the advantages and disadvantages of holographic interferometry compared with conventional interferometry?

advantages	disadvantages
• with Holographic interferometry (HI) one cold record the whole **thermal or concentration field** of an object • HI is cheaper than the classical interferometer (no high requests optical components quality) • with HI one could obtain a **three-dimensional display of the object** (**3D displacement vector**) • simple wavefronts for reference wave are not needed anymore (but have to be reproducible) • **applicable for more rough surfaces** • larger objects can be measured with lower expend • measurement of movements and vibrations with **more accuracy** • wavefront of object could be interfered with the wavefront of the same object on another time (**comparison with Master**)	• **no information included about the distance in space of the object** • **the evaluation of the holographic interference patterns are more difficult** • **photo plates could only used once**

72. Which effects lead to interference fringes in double exposure techniques?

In general: we observe interference if the object state changed compared to the reference state!
So the effects which change the object state lead to interference fringes.
Examples: -Any kind of force which is acting on the object
-Simply a displacement of the object

73. Which advantages and disadvantages of real-time holographic interferometry do you know?

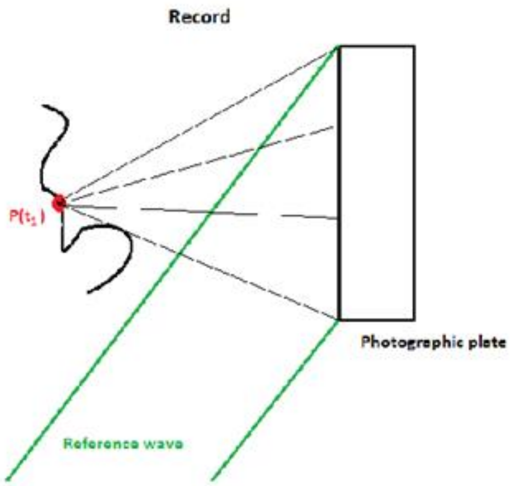
Advantage	Disadvantage
• Observing an object and the dynamic processes of changingsequentially • Continues recording by camera • Method applies to opaque and transparent object	• Develop the hologram in its original place or exact setting into original position • Recording materials decide about the quality • Dimensional changes during their

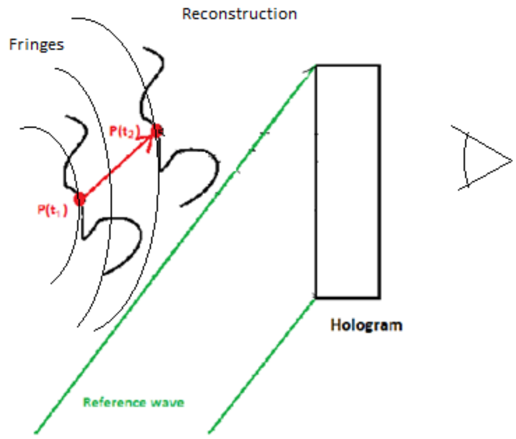
chemical processing and drying of recording material

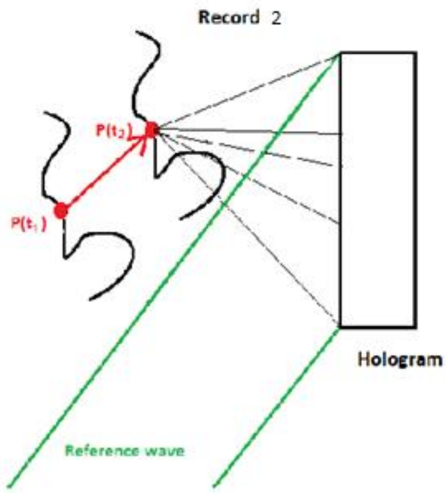
Ref.: Lecture notes 5.2.1. Real-Time Holographic Interferometry

74. Explain the concept of homologous points.

A homologous point is the same point P of a rough object at different times. If there is for example a one dimensional translation displacement of the object it can be fully specified by the knowledge of the displacement of 1 point P. You can use double exposure technique to calculate the displacement of point P. 1. Step: Record $P(t_1)$, 2. Step: second record with $P(t_2)$. 3. Reconstruction. 4. Counting fringes between $P(t_1)$ and $P(t_2)$ →Calculation of displacement of P in time t_2-t_1. If there is a displacement with more than one degree of freedom you need more homologous points to specify the displacement of the object.







75. Why holography is well-suited for rough objects in contrast to classic interferometry?

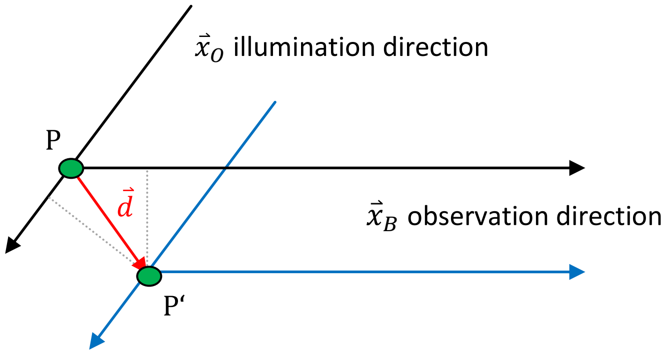
- In holographic interferometry (HI), fringe density is lower for rough objects
- In HI, one needs to know where a certain point of the surface of the object is to be found when the object surface is changed → to determine displacement vector (with a mirror for instance, we can't determine between different points) →"Homologous points"

76. What is the sensitivity vector?

Evaluation of holograms:

- Whole body movement (P -> P' homologous points)

$$\Delta\phi = \frac{2\pi}{\lambda}(\vec{x}_B - \vec{x}_O)\vec{d}$$



- $\vec{d}$ is the displacement vector (displacement of object)
- $\vec{x}_B - \vec{x}_O$ is the sensitivity vector
- Gives the scaling
- Determines the accuracy of measurements

77. How can holographic interferograms be quantitatively evaluated?

The basis of quantitative evaluation of the interference pattern is the movements of fringes over the surface while varying the observation directions of the hologram. Therefore two principles are possible the Dynamic and the Static Evaluation.

<u>Dynamic Evaluation</u>

- you count the number of moving fringes in a fixed point P while moving the observation direction in µm range
- these are related to a phase difference $\Delta\varphi = 2\pi N$ and this difference is related to a certain displacement (vectors)
- so you have more unknown about the direction → evaluation of holograms more than once necessary
- another possibility is to move or deform the object, therefore see figure 15.7 and determine the translation vector $\vec{L}$ → $\Delta\varphi = (\vec{k}_1 - \vec{k}_2)\vec{L}$; sensitivity vector

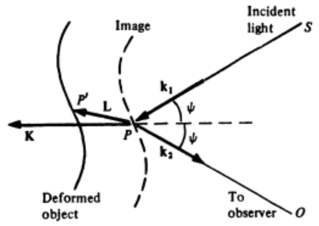
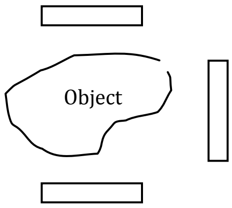


Fig. 15.7. Calculation of the phase difference in the interference pattern.

Static Evaluation

- Observation of one point P and a certain known fringe (0'th order)
- now compare 3 orthogonally arranged holograms or 3 incoherent double exposure holograms and count the changing of fringes by observing P relative to the known fringe (0'th order)



78. How can the shape of objects be measured by holographic means?

79. Which types of contour lines can be generated, if the wavelength or the refractive index is changed?

The change of the refractive index or the wavelength can create elliptical contour lines.

80. How can the 3D shape of an object be measured with the two-source method?

You have to perform a double exposure technique with 2 light sources with a wavelength difference $\Delta\lambda$. For a fixed position of the object, the hologram and the light source we get for the fringe number N(P):

$$N(P) = \left(\frac{\Delta\lambda}{\lambda}\right)\left(r_Q + r_B - r_R\right),$$

Where r_Q is the distance between the object point P and the light source, r_B is the distance between the object point P and the hologram. r_R is the distance between the reference R and the hologram (fixed for both recordings and reconstruction). Using triangulation we are able to determine the object point P in space.

81. What means digital holography and what are its advantages and disadvantages compared with conventional holography?

Digital holography: camera (electronic detector) as storage medium (CCD, CMOS,...)

advantages	disadvantages
Reconstruction of hologram possible with computer (→ close to real time)	Limited/bad resolution due to pixels of camera
Double exposure etc. possible as well	Quality of reconstructed holograms bad due to bad resolution
Calculation/ reconstruction is easy and fast	Only small angles possible → only small objects or objects far away (otherwise, density of fringes too high)
Editing possible, e.g. filtering	

82. Why is the pixel size of the camera the main resolution limiting factor of digital holography?

- Resolution is limited by both, Nyquist sampling theorem and resolution limit after Abbe

Nyquist sampling theorem:

- reduction of a continuous signal to a discrete signal *(task 4)*
- Maximum number of sampling points equals the number of pixels

Resolution limit after Abbe *(Abbesche Auflösungsgrenze)*:

Die maximale Auflösung ist der minimale Abstand zwischen zwei unterscheidbaren Strukturen $d_{\min}$. In der Mikroskopie ist die Größe des Fokus durch Beugung begrenzt und proportional zur Wellenlänge λ des verwendeten Lichtes sowie umgekehrt proportional zur numerischen Apertur:

$$d_{\min} = \frac{\lambda}{2\,NA} = \frac{\lambda}{2\,n \sin\theta}$$

(allgemein für rechteckige und runde Aperturen:
Rechteckfunktion => FT => Sinc-Funktion)

(NA numerische Apertur, θ halber objektseitiger Öffnungswinkels (Akzeptanzwinkel), Brechungsindex n)

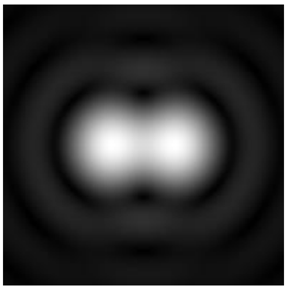
(Ref.: http://de.wikipedia.org/wiki/Numerische_Apertur)

Nur Info:

Rayleigh-Kriterium:

Das Rayleigh-Kriterium definiert, wann zwei Lichtquellen als aufgelöst betrachtet werden können.
Bei einem Mikroskop spricht man von der Abbeschen Auflösungsgrenze, die durch die numerische Apertur NA und die Wellenlänge λ bestimmt wird. Hier wird normalerweise die Auflösung über den kleinsten Abstand zweier (Punkt-)Objekte beschrieben. Nach Rayleigh sind zwei (Punkt-)Objekte mit dem Abstand $d_{\min}$ gerade dann noch auflösbar, wenn das Beugungsscheibchen des ersten Objekts auf das erste Minimum des Beugungsscheibchens des zweiten Objekts fällt:

$$d_{\min} = \frac{1.22\,\lambda}{2\,NA} = \frac{0.61\,\lambda}{n\sin\theta}$$

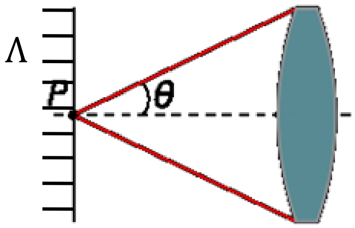


(nur für runde Aperturen:
Kreisfunktion => FT => BesselSinc-Funktion)

- For a CCD the inverse Abbe limit is also:

$$f = \frac{2}{\lambda}\sin\frac{\alpha}{2} = \frac{1}{\Lambda}$$

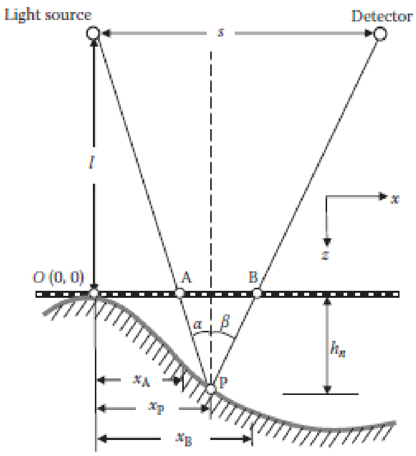
(Spatial frequency f, wavelength λ, opening angle α, pixel pitch Λ)



- the Pixel size Λ is limited (µm), therefore only small angles are possible, which means that only objects can be resolved, which are far away
- Hence: Problems to record big objects (also too big amount of data)

83. How do shadow Moiré and projection Moiré work?

Shadow Moire



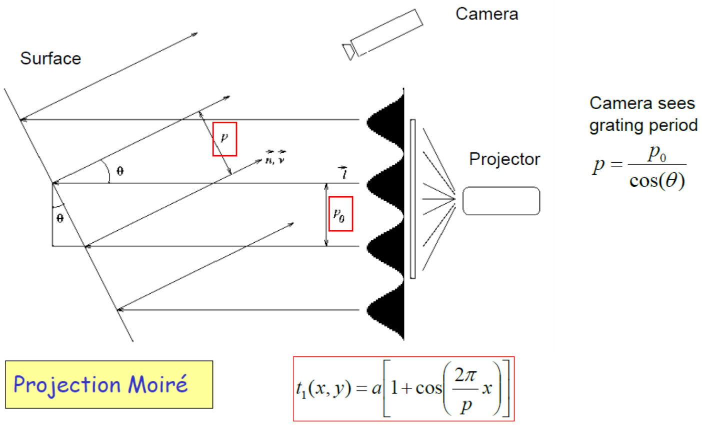
The moire pattern observed from the detector is the result of superposition between the grating elements contained in OB (**reference grating**) and the elements contained in OP (**objected grating**), which is the shadow of elements contained in OA of the reference grating

→ Analyzing of topological shape of the object (sense of moiré pattern)

$$h_n = \frac{np}{tan\alpha + tan\beta}$$

where n is the # of grating lines between A and B (n=6 in pic.) and p is the grating period

Projection Moire



→ you detect the projected grating period on the shape of an object -> contours

(there are different methods for projection moiré: either with a reference grating or not (pic), phase or frequency shift method -> Ref. Yoshizawa T. Handbook of optical Metrology)