Exam 'Introduction to Nanooptics', WS 2015/2016

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Problem 1: Plasmonic nanoantennas: Directional scattering by a pair of electric dipoles (13 points)

Consider a y-polarized monochromatic plane wave ($\omega_{\mathrm{inc}} = 2\pi \cdot 375\,\mathrm{THz}$) incident along z-direction, which is scattered from two plasmonic nanoparticles p_1 and p_2 (indicated as arrows in Fig. 1 (a)) in vacuum with particle plasmon resonances near (but not necessarily centered at) ω_{inc} and with intrinsic phases $\phi_{1,\mathrm{int}}$ and $\phi_{2,\mathrm{int}}$, respectively. Neglect near-field coupling. The nanoparticles are located at a distance L with respect two each other. Focus on the 1D problem of light scattered in opposite directions along the x-axis.

(a) The optical intensity at position x is given by the sum of the contributions from the two scatterers I_1 and I_2 and an interference term:

$$I(x) = I_1(x) + I_2(x) + 2\sqrt{I_1(x)I_2(x)}cos(\phi_2(x) - \phi_1(x)). \tag{1}$$

Formulate the (infinite set of) conditions for constructive interference in the positive x-direction and for destructive interference in the negative x-direction as a function of in intrinsic phase difference $\Delta\phi_{\mathrm{int}} = \phi_{2,\mathrm{int}} - \phi_{1,\mathrm{int}}$ and the distance L. Hint: for each scatterer the phase $\phi_j(x), j = \{1,2\}$ at position x consists of the intrinsic phase and the phase related to the propagation away from the scatterer. (4 points)

(b) Calculate the minimal positive distance between the two dipoles required to fulfill both conditions simultaneously. What is the corresponding difference in the intrinsic phase shifts $\Delta\phi_{\mathrm{int}}$ of the two dipoles? (4 points)

(c) The described system can be implemented by two short plasmonic nanowires with their long axis oriented in y-direction. Which one of the nanowires has to be longer and which one has to be shorter and why? Can the value calculated in (b) be achieved? What is the maximal value for the intrinsic phase difference that can be achieved in this picture, and why is it desirable to work at smaller values? (3 points)

(d) How would near-field coupling qualitatively change the resonance frequencies of the two nanowires in the considered geometry assuming equal resonance frequencies? How does that change if the scatterers are displaced along y-direction instead as indicated in Fig. 1 (b)? (2 points)

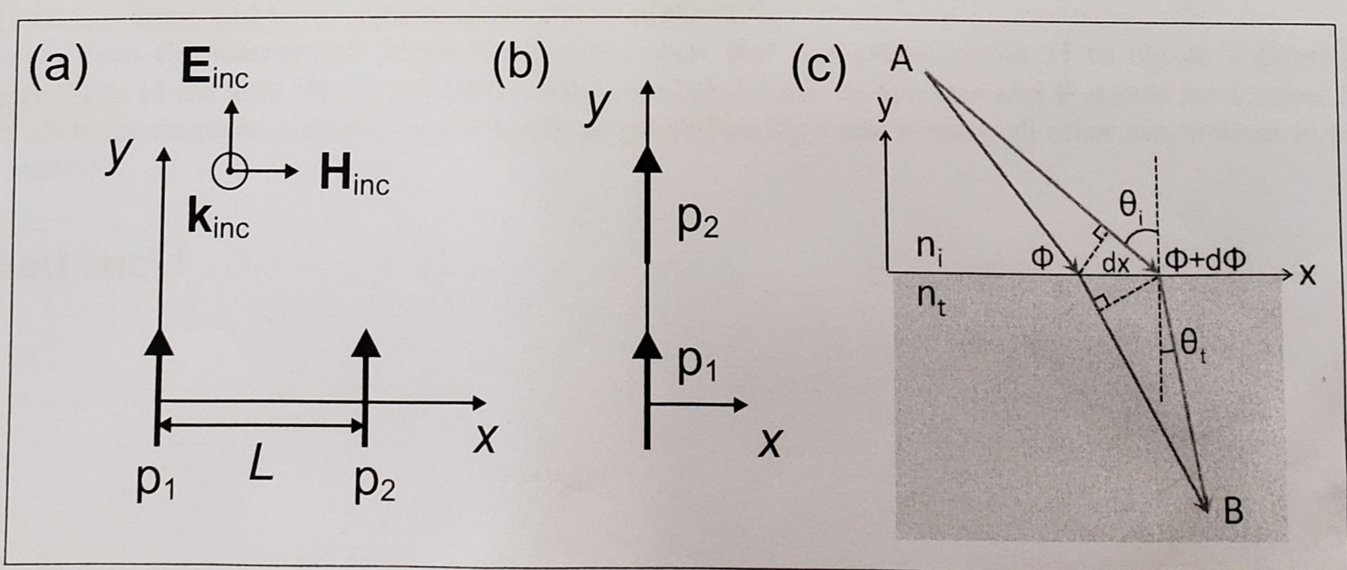

Figure 1: (a) Sketch of the arrangement of the two scatterers in problem 1(a)-(d). (b) Changed arragement considered in problem 1(e). (c) Sketch for the derivation of the generalized Snell's law for problem 2 (g). Taken from Science 334, 333-337 (2011).

Problem 2: Refraction by metamaterials and metasurfaces (16 points)

Assume a planar, infinitely extended interface (located at $y = 0$ and parallel to the (x, z)-plane) between air and a medium with an electrical permittivity $\epsilon = -1$ and a magnetic permeability $\mu = -1$.

(a) What is the reflectivity of the interface? (1 point)

(b) Consider an s-polarized plane wave incident from air onto the interface at an angle θ_i from the surface normal. The plane of incidence is in (x, y)-direction. Formulate the continuity conditions for the components of the electric field ($\mathbf{E}$) and the magnetic inductance field ($\mathbf{B}$) at the interface. (2 points)

(c) Making use of the continuity conditions, draw the $\mathbf{E}$- and $\mathbf{B}$-field vectors in both half spaces. (2 points)

(d) How do the wave vector $\mathbf{k}$ and the Poynting vector $\mathbf{S}$ connect to the field vectors? Add them to the drawing in both half spaces. (2 points)

(e) What is the angle of refraction θ_t resulting from the graphical construction using the sign conventions of the usual Snell's law? What is the refractive index that can be assigned to the $(\epsilon = -1,\ \mu = -1)$-medium in Snell's law? (1 point)

(f) Assume now that the medium has an electrical permittivity $\epsilon = -2$ and magnetic permeability $\mu = -2$. What is now the reflectivity of the interface? What is the angle of refraction for an incidence angle of 30 degrees? (1 point)

(g) Consider now an interface between air and a medium with $\epsilon = 4$ and $\mu = 1$. Assume that the interface is a metasurface that imprints a position dependent phase Φ onto the transmitted plane wave. The imprinted phase changes linearly as a function of the x-coordinate as $\frac{d\Phi}{dx} = -\frac{\pi}{600\,\mathrm{nm}}$. To derive a generalization of Snell's law for this case consider two paths from A to B (see Fig. 1 (c)) which are infinitesimally close to the actual light path, and use the stationary phase condition. What is the angle of refraction, if light with a wavelength $\lambda_{\mathrm{inc}} = 800\,\mathrm{nm}$ impinges onto this interface at an incident angle of 30 degrees? (5 points)

(h) Say in one sentence what would happen for $\frac{d\Phi}{dz} \neq 0$. (1 point)

(i) Name two functionalities other than beam deflection that can be achieved by imprinting a position dependent phase onto an incident plane wave. (1 points)

Problem 3: Photonic crystals: Photonic eigenvalue problems for magnetic materials (10 points)

In the lecture, we have eliminated the electric field $\mathbf{E}$ from the macroscopic Maxwell equations (no free charges, no free currents) to obtain a Hermitian eigenvalue problem (also called the master equation) in the magnetic field $\mathbf{H}$ for a system consisting of mixed dielectric media, all of which are linear, macroscopic, isotropic, transparent, and nondispersive, and have magnetic permeability $\mu(\mathbf{r}) = \mathbf{1}$.

Starting from the macroscopic Maxwell equations, show that one can eliminate $\mathbf{H}$ to obtain a generalized eigenproblem of the form $\hat{A}\mathbf{F}(\mathbf{r}) = \frac{\omega^2}{c^2}\hat{B}\mathbf{F}(\mathbf{r})$, where the hat denotes an operator and $\mathbf{F}$ stands for a vector field, if we allow for magnetic materials $\mu(\mathbf{r}) \neq 1$ with μ real and positive, and keeping all other assumptions in place. (10 points)

Good luck!