

Name (with capital letters):

Matriculation number:

| Only to be used by the corrector! |   |   |   |   |   |   |          |      |
|-----------------------------------|---|---|---|---|---|---|----------|------|
| 1                                 | 2 | 3 | 4 | 5 | 6 | 7 | $\Sigma$ | mark |
|                                   |   |   |   |   |   |   |          |      |

**Mid-Term-Exam Structure of Matter Winter Term 2016 / 2017 (19.02.2016)**

**90 minutes time**

**Please note:** Please write your name and your matriculation number on each sheet! Please write your solution on the sheets provided and use only other sheets when necessary! Notes made with pencils or with colors others than blue or black will not be accepted!

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**Task 1**

**6 Points**

**Scales and Dimensions**

- Please determine the value for the de Broglie wavelength of a resting car ( $\vec{v} = 0$ ).
- Give an estimate of the size of an atom in meters!
- Give a typical wavelength of IR, visible and UV radiation?
- Order the colors or spectral ranges with respect to the wavelength. Start with the shortest wavelength: yellow, red, blue, green, IR, UV, X-ray
- Which of the colors or spectral ranges mentioned in d) will create the fastest electrons via the photoelectric effect and why?
- A proton and a rocket travel at  $|\vec{v}| = 1000 \text{ m/s}$ . Which of them has the shorter de Broglie wavelength and why?

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## Task 2

Parabolic potential

Consider a particle, which is restricted to a onedimensional potential

$$V(x) = \frac{m\omega_0^2}{2} x^2$$

The particle shall be in a superposition composed of the ground state and the second excited state. The wavefunction at  $t=0$  can be written as

$$\Psi(x, t=0) = c_0 \psi_0(x) + c_2 \psi_2(x),$$

where  $c_0$  and  $c_2$  are arbitrary constants and  $\psi_0(x)$  and  $\psi_2(x)$  are the eigenfunctions corresponding to the ground state and the second excited state, respectively.

Determine the shortest time  $T_0$ , after which the probability density recovers in the sense that

$$|\Psi(x, t=0)|^2 = |\Psi(x, t=T_0)|^2 \text{ holds.}$$

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**Task 3**

**8 Points**

Energy loss

We study a monochromatic field with frequency  $\omega$  propagating in a homogenous and isotropic medium with negligible magnetization and a dielectric constant  $\epsilon(\omega)$ .

- a) Write down Maxwell's Equations for this case.
- b) Calculate the divergence of the time averaged Poyntingvector  $\langle \vec{s} \rangle = \frac{1}{2\mu_0} \text{Re}(\vec{E} \times \vec{B}^*)$  by using

Maxwell's equations. Express it as a function of the absolute value square of the interacting fields, the frequency and the dielectric constant!

- c) Which part of the complex dielectric constant is zero in a lossless medium?

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**Task 4**

**6 Points**

Lossy material

The polarization of a strongly damped material follows an evolution equation in the time domain like

$$\left[ \gamma \frac{\partial}{\partial t} + \omega_0^2 \right] \tilde{P}_{\text{real}}(t) = \frac{e^2 N}{m} \tilde{E}_{\text{real}}(t)$$

where the second derivative is neglected as friction induced forces are much larger than the contribution of inertia.

- Determine the frequency dependent dielectric constant  $\epsilon(\omega)$  of this material!
- Draw the real and imaginary parts of that dielectric constant in a principal sketch!

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### Task 5

3 Points

Balmer series

The Balmer series of the emission spectrum of excited hydrogen atoms results from electron transitions towards the  $n=2$  state. Calculate the three lowest energies of the emitted photons in eV!

Important constants

mass of the electron:  $m_{\text{Elektron}} = 9,11 \cdot 10^{-31} \text{ kg}$ , Planck's constant  $h = 6,63 \cdot 10^{-34} \text{ Js}$ , elementary charge:  $e = 1,602 \cdot 10^{-19} \text{ As}$ , velocity of light in vacuum  $c = 3 \cdot 10^8 \text{ m/s}$ , dielectric constant of the vacuum  $\epsilon_0 = 8,85 \cdot 10^{-12} \text{ As/Vm}$ , Rydberg energy:  $E_R = 13,6 \text{ eV}$

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**Task 6**

**4 Po**

Show by using the commutator relation of angular momentum operators  
 $[\hat{L}_i, \hat{L}_j] = i\hbar \sum_k \hat{\epsilon}_{ijk} \hat{L}_k$  where  $\hat{\epsilon}_{ijk}$  is the completely antisymmetric tensor,  
that the momentum operators  $\hat{L}_y$  and  $\hat{\mathbf{L}}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$  commute!

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**Task 7**

Relativistic electron

**6 Points**

An electron is accelerated reaching a velocity of  $0.95\ c$ , with  $c$  being the velocity of light. Which voltage is required to achieve that velocity? Please use relativistic calculus!

Important constants

mass of the electron:  $m_{\text{Elektron}} = 9,11 \cdot 10^{-31} \text{ kg}$ , Planck's constant  $h = 6,63 \cdot 10^{-34} \text{ Js}$ , elementary charge:  $e = 1,602 \cdot 10^{-19} \text{ As}$ , velocity of light in vacuum  $c = 3 \cdot 10^8 \text{ m/s}$ , dielectric constant of the vacuum  $\epsilon_0 = 8,85 \cdot 10^{-12} \text{ As/Vm}$ , Rydberg energy:  $E_R = 13,6 \text{ eV}$