

Problem 1

Jinsong Liu

(a)

$$P_\nu = \frac{I_\nu}{C} \quad \phi_\nu = \frac{I_\nu}{h\nu}$$

For monochromatic light:

$$W_i = \int_0^\infty \phi_\nu \cdot \sigma_\nu \delta(\nu - \nu_0) d\nu = \phi_{\nu_0} \cdot \sigma_{\nu_0}$$

(b)

$\because \sigma_\nu$ is much narrower than $P(\nu)/h\nu$

$$W_i = \int_0^\infty \frac{P_\nu C}{h\nu} \cdot \sigma_\nu d\nu$$

$$= \frac{P(\nu_{21}) C}{h\nu} \cdot \int_0^\infty \sigma_\nu d\nu$$

$$= \frac{P(\nu_{21}) C}{h\nu} \cdot \sigma$$

$$\therefore \sigma = \int_0^\infty \sigma_\nu d\nu = \frac{B_{21} \cdot h\nu_{21}}{C}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \quad B_{12} = B_{21} \quad A_{21} = 1/Z_2$$

$$\therefore \sigma = \frac{\lambda^2}{8\pi Z_2}$$

$$\therefore W_i = B_i \cdot P(\nu)$$

$$\therefore B_i = \frac{\lambda^3}{8\pi h Z_2}$$

$$\therefore W_i = \frac{P(\nu_{21}) C}{h\nu} \cdot \frac{\lambda^3}{8\pi Z_2}$$

Problem 2

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(a)

the wave number = $\frac{1}{\lambda}$ [cm⁻¹] suppose $k = \frac{1}{\lambda}$

∴ the emission wavelength for all laser transitions :

$$\lambda_{ij} = \frac{1}{k_i - k_j} \Leftarrow (E = h\nu = hc/\lambda, h\nu_{ij} = E_i - E_j)$$

$$k_i = 10930 \text{ cm}^{-1}, 10624 \text{ cm}^{-1}, 10327 \text{ cm}^{-1}$$

$$k_j = 785 \text{ cm}^{-1}, 612 \text{ cm}^{-1}, 565 \text{ cm}^{-1}, 0 \text{ cm}^{-1}$$

∴ λ_{ij} has 12 values.

k_i [cm ⁻¹]	k_j [cm ⁻¹]	λ_{ij} [nm]	k_i [cm ⁻¹]	k_j [cm ⁻¹]	λ_{ij} [nm]
10930	0	914.9	10327	0	968.3
10930	565	964.8	10327	565	1024.4
10930	612	969.2	10327	612	1029.3
10930	785	985.7	10327	785	1048.0
10624	0	941.3	10327		
10624	565	994.1			
10624	612	998.8			
10624	785	1016.4			

(b)

Because of the Stark-effect, inhomogeneous broadening occurs in the cross-sections. The spontaneous emission of different excited particles also contributes to the broadening.

(c)

Because the transition energy doesn't transfer into the heat at the zero-phonon line.

(d)

For a four-level system :

the maximum value of absorption occurs at 941.3 nm.
 $(k_i = 10624 \text{ cm}^{-1}, k_j = 0 \text{ cm}^{-1})$

Meanwhile, the maximum value of emission occurs at 1029.3 nm $(k_i = 10327 \text{ cm}^{-1}, k_j = 612 \text{ cm}^{-1})$

In summary, the four-level system is:

10624 cm^{-1}

10327 cm^{-1}

612 cm^{-1}

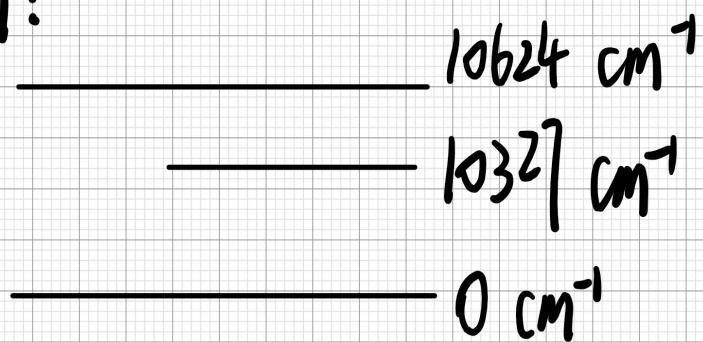
0 cm^{-1}

(c) For a three-level system:

the maximum value of absorption occurs at 941.3 nm
($k_i = 10624 \text{ cm}^{-1}$, $k_j = 0 \text{ cm}^{-1}$)

the emission between $k_i = 10327 \text{ cm}^{-1}$ and 0 cm^{-1} is the zero-phonon line.

∴ the three-level system:



(f) For an inverse three-level system:

Problem 3

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(a)

Gain saturation occurs when the increasing intensity leads to the decrease of the gain. The physical origin is the absorption particles (cross-sections) are saturable.

(b) The differential equation is:

$$\frac{dI(z)}{dz} = \frac{g_0}{1 + \frac{I(z)}{I_{\text{sat}}}} I(z) = g_{\text{sat}}(z) I(z)$$

$$\therefore \frac{dI(z)}{I(z)} \left(1 + \frac{I(z)}{I_{\text{sat}}} \right) = g_0 dz$$

$$\frac{dI(z)}{I(z)} + \frac{dI(z)}{I_{\text{sat}}} = g_0 dz$$

Assume that $I(z=0) = I_{\text{in}}$ $I(z=L) = I_{\text{out}}$

$$\int_{I_{\text{in}}}^{I_{\text{out}}} \frac{1}{I(z)} + \frac{1}{I_{\text{sat}}} dI(z) = \int_{z=0}^{z=L} g_0 \cdot dz$$

$$\ln(I_{\text{out}}) - \ln(I_{\text{in}}) + \frac{I_{\text{out}} - I_{\text{in}}}{I_{\text{sat}}} = g_0 \cdot L$$

$$\ln\left(\frac{I_{out}}{I_{in}}\right) + \frac{I_{out} - I_{in}}{I_{sat}} = g_0 \cdot L$$

$$\exp\left[\ln\left(\frac{I_{out}}{I_{in}}\right) + \frac{I_{out} - I_{in}}{I_{sat}}\right] = \exp(g_0 \cdot L)$$

$$\frac{I_{out}}{I_{in}} \cdot \exp\left(\frac{I_{out} - I_{in}}{I_{sat}}\right) = \exp(g_0 \cdot L)$$

$$\therefore G \cdot \exp\left(\frac{I_{out} - I_{in}}{I_{sat}}\right) = G_0$$

$$G = G_0 \cdot \exp\left(-\frac{I_{out} - I_{in}}{I_{sat}}\right)$$

(c)

$$L_{dB} = 10 \lg \frac{I_{out}}{I_{in}}$$

$$L_1 = 10 \text{ dB}$$

$$L_2 = 9 \text{ dB}$$

$$I_{in} = 1 \text{ W/cm}^2$$

$$I'_{in} = 2 \text{ W/cm}^2$$

$$I_{out} = 10 \text{ W/cm}^2$$

$$I'_{out} = 15.9 \text{ W/cm}^2$$

$$G_1 = G_0 e^{-\frac{I_{out} - I_{in}}{I_{sat}}}$$

$$G_2 = G_0 e^{-\frac{I'_{out} - I'_{in}}{I_{sat}}}$$

$$G_1 = 10$$

$$G_2 = 7.95$$

$$\therefore I_{sat} \approx 21.4 \text{ W/m}^2 \quad G_0 \approx 15.22$$

$$(d) I_{\text{extr1}} = 10 - 1 \text{ W/m}^2 = 9 \text{ W/m}^2$$

$$I_{\text{extr2}} = 15.9 - 2 \text{ W/m}^2 = 13.9 \text{ W/m}^2$$

When I_{in} is 2 W/m^2 , the extracted intensity is bigger.

$\because I_{\text{in}} < I_{\text{saturation}}$

\therefore the extracted intensity increase with the increasing input intensity.

$$(e) \frac{dI(z)}{dz} = g(z)I(z) - \alpha I(z) = \left[\frac{g_0}{1 + \frac{I(z)}{I_{\text{sat}}}} - \alpha \right] I(z)$$

(f)