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Midterm Exam
"Fundamentals of modern optics"
WS 2016/17
to be written on December 19

Problem 1 – Maxwell's Equations $\quad 4.5 + 2.5 + 3 = 10$ points

a) Write down Maxwell's equations for the electric and magnetic field in the time domain in a material which is non-magnetizable by introducing the external sources $\rho(\mathbf{r}, t)$ and $\mathbf{j}(\mathbf{r}, t)$.

b) Derive wave equation for the electric field from these Maxwell's equations.

c) A homogeneous but dispersive medium cannot respond instantaneously when a time varying electric field is applied to it. Write down the constitutive relation between $\mathbf{D}(\mathbf{r}, \omega)$ and $\mathbf{E}(\mathbf{r}, \omega)$ in this medium and find the corresponding relation in the time domain.

Problem 2 – Poynting Vector and Normal Mode $\quad 3 + 2 + 3 + 2 = 10$ points

Consider a transverse monochromatic plane wave of frequency ω, propagating in a homogeneous isotropic medium, that is an extremely good conductor with conductivity $\sigma >> \omega\varepsilon_0$. The complex representation of the electric field has the form $\mathbf{E}(\mathbf{r}, \omega) = E_0(-\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}})e^{\mathrm{i}\beta(1+\mathrm{i})(x+y)}$, where β is a real number and $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are unit vectors in x and y-direction.

a) Specify the wave-vector $\boldsymbol{k}$ for this plane wave (with its complex amplitude and direction). What is the dispersion relation that this wave-vector satisfies? Find σ as a function of β from this dispersion relation. (You can still fully solve part b and c if you do not manage to solve part a.)

b) Find the magnetic field $\mathbf{H}(\mathbf{r}, \omega)$.

c) Write down the formula for the time-averaged Poynting vector $\langle \mathbf{S}(\mathbf{r}, \mathbf{t}) \rangle$, based on the complex representations of the electric and magnetic fields. Find $\langle \mathbf{S}(\mathbf{r}, \mathbf{t}) \rangle$ for the field given above.

d) Find the divergence of the time-averaged Poynting vector $\nabla \cdot \langle \mathbf{S}(\mathbf{r}, \mathbf{t}) \rangle$ and express it as a function of only the absolute value of the electric field $|\mathbf{E}(\mathbf{r}, \omega)|$ and conductivity σ.

Problem 3 – Beam propagation $\quad 2 + 2 + 1 + 1 = 6$ points

a) Describe an algorithm which makes use of the transfer function $H(\alpha, \beta, z)$ and is capable of calculating an optical field $u(x, y, d)$ at position $z = d$ from a field $u_0(x, y, 0)$ given at a position $z = 0$.

b) What is the explicit mathematical form of $H(\alpha, \beta, z)$ for free space? How can it be approximated for the paraxial case?

c) An initial field $u_0(x, y, 0)$ is given as the superposition of two fields

$$u_0(x, y, 0) = u_0^{(1)}(x, y, 0) + u_0^{(2)}(x, y, 0)$$

How will $u(x, y, z)$ depend on the two input fields and why?

d) Prove that the algorithm in a) corresponds to a convolution operation in real space!

Problem 4 – Propagation of Gaussian Beams $\quad 2 + 2 + 4 = 8$ points

A Gaussian beam with the Rayleigh length $z_0 = kW_0^2/2$, is propagating through a homogeneous medium.

a) What is the q-parameter for Gaussian beam propagation? How can we obtain the radius of the phase curvature and the beam width from it?

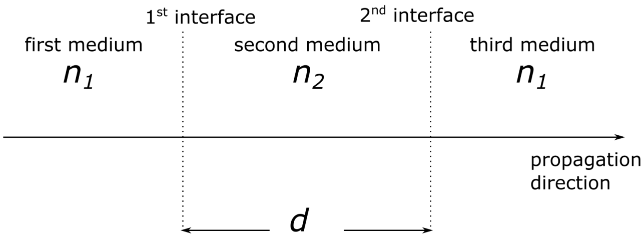
– please turn over –

Consider a Gaussian beam to propagate from a first homogeneous (refractive index n_1) through a second (n_2) into a third medium (again n_1) with $n_2 > n_1$ (see Figure). Both interfaces can be treated with the matrix method and each of them appears like a spherical interface (first interface radius $R_{int} > 0$, second interface radius $-R_{int}$). The length between the two interfaces is d.

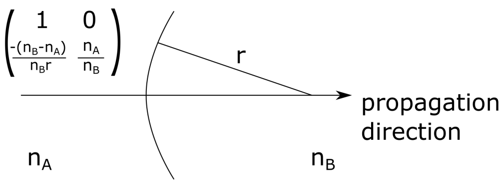
b) Assume that propagation starts directly before the first and ends directly behind the second interface. Calculate the q-parameter of the Gaussian beam after propagation through this system. The q-parameter before the first interface is q_1.

Now, consider d to be small enough to be neglected ($d = 0$). Before reaching the first interface, the beam has propagated the distance L_1 from its waist position.

c) After which distance L_2 from the second interface does the beam exhibit a waist again?



ABCD Matrix for spherical interface:



Problem 5 – Pulses $\quad 1+3+2+2=8$ points

Consider a laser source with an output power of 100 mW and a repetition rate of 100 MHz. The output of such a source is a sequence of transform-limited Gaussian pulses with central frequency ω_0. The envelope of each individual pulse in its co-moving frame is defined as $E(t') = E_0 \exp\left[-t'^2/\tau^2\right]$, where the pulse width is $\tau = 8$ ps. This pulse sequence is launched into a fiber characterized by

$$k(\omega) = k_0 + \frac{1.5}{c}(\omega - \omega_0) + \frac{D}{2}(\omega - \omega_0)^2,$$

with $D = 0.08\ \mathrm{ps}^2/\mathrm{m}$.

a) Calculate the energy of each individual pulse.

b) Find the dispersion length L_D of each individual pulse. Is the red or the blue part of the spectrum appearing earlier at the end of the fiber?

c) After a fiber length of $L_1 = 4$ km, a second type of fiber is connected to the first. It has a dispersion of $-2000\ \mathrm{fs}^2/\mathrm{m}$ and a length of L_2. How long does L_2 have to be in order to fully restore the initial pulse sequence?

d) Now, a pulse sequence with a different frequency $\omega_1 = \omega_0 + \delta\omega$ is launched into the first fiber. Suppose that the detuning $\delta\omega = 1$ THz. Find the group index n_g in that case.

Problem 6 – Fraunhofer diffraction $\quad$ 4+1=5 points

A one-dimensional optical field directly behind a specific optical element is given as

$$u_0(x,0) = \begin{cases} A\exp\left[i\Phi_0\left(\frac{x}{a}+1\right)\right] & |x| \le a \\ 0 & \text{otherwise} \end{cases}$$

a) Find the optical intensity $I(x,z)$ in the case that the paraxial approximation holds and the distance $z \gg a$. You may omit possible prefactors.

b) How large does Φ_0 have to be so that the intensity on the optical axis $x = 0$ vanishes?