Quiz 05: Normal modes and their propagation

The normal mode solutions of Maxwell's equations in homogeneous media with $\varepsilon(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega)$ in the frequency domain are plan waves:

$$\overline{\mathbf{E}}(\mathbf{r},\omega) = \overline{\mathbf{E}}_0(\mathbf{k}(\omega),\omega)\exp\left(\mathbf{i}\mathbf{k}(\omega)\mathbf{r}\right) \text{ with } \mathbf{k}(\omega) = \mathbf{k}'(\omega) + \mathbf{i}\mathbf{k}'(\omega).$$

1) Write down the dispersion relation between $\mathbf{k}(\omega)$ and $\varepsilon(\omega)$. [1 point]
2) Derive the set of equations which connect $\mathbf{k}'(\omega)$ and $\mathbf{k}''(\omega)$ to $\varepsilon'(\omega)$ and $\varepsilon''(\omega)$. [2 points]
3) Assuming $\mathbf{k}'(\omega) + \mathbf{i}\mathbf{k}''(\omega) = \hat{\mathbf{k}}\frac{\omega}{c}(n + i\kappa)$, derive the equations connecting n and κ to $\varepsilon'(\omega)$ and $\varepsilon''(\omega)$. [2 points]
4) Write down the equation, which defines the amplitudes of excited plane waves $U_0(\alpha,\beta)$ depending on the field distribution in the excitation plane $u_0(x,y)$ in scalar approximation. [3 points]
5) How does the amplitude $U(\alpha,\beta;z)$ of an excited plane wave depend on the propagation coordinate z? [2 points]

You have 10 minutes!

Make sure that you indicate your name and seminar group on your answer sheet and send your answer to teaching.ngo@uni-jena.de at 09:15am.