FUNDAMENTALS OF MODERN OPTICS
Series 9

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Problem 1.

P1 3
P2 0

a) $t(x) = \sum_{l=0}^{N-1} \tilde{F}(x - lD)$

$$\Rightarrow T(\alpha) = \mathcal{F}\{t(x)\} \underset{\uparrow}{=} \sum_{l=0}^{N-1} e^{-i\alpha l D} \tilde{F}(\alpha) = \tilde{F}(\alpha) \sum_{l=0}^{N-1} e^{-i\alpha l D}$$

Fourier Shifting theorem

Notice that $\sum_{l=0}^{N-1} e^{-i\alpha l D}$ is a geometric series ~~[illegible]~~

$$\Rightarrow \sum_{l=0}^{N-1} e^{-i\alpha l D} = \frac{1 - e^{-i\alpha D(N\cancel{-1}\cancel{+1})}}{1 - e^{-i\alpha D}} = \frac{e^{-i\alpha D\frac{N}{2}}\left(e^{+i\alpha\frac{DN}{2}} - e^{-i\alpha D\frac{N}{2}}\right)}{e^{-i\alpha\frac{D}{2}}\left(e^{+i\alpha\frac{D}{2}} - e^{-i\alpha\frac{D}{2}}\right)}$$

$$= e^{-i\alpha D\frac{N}{2} + i\alpha\frac{D}{2}} \frac{\cancel{(2i)} \sin\left(\frac{\alpha D N}{2}\right)}{\cancel{(2i)} \sin\left(\alpha\frac{D}{2}\right)}$$

$$= e^{+i\frac{\alpha D}{2}(-N+1)} \frac{\sin\left(\frac{\alpha N D}{2}\right)}{\sin\left(\frac{\alpha D}{2}\right)}$$

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$$\therefore T(\alpha) = \tilde{F}(\alpha) \exp\left[i\frac{\alpha D}{2}(1-N)\right] \frac{\sin\left(\frac{\alpha N D}{2}\right)}{\sin\left(\frac{\alpha D}{2}\right)}$$