Structure of matter: Homework to exercise 4

Electrical and optical properties of continuous media

Due on November 2nd 2023 at noon

Please indicate your name on the solution sheets and send it to your seminar leader!

1. Multiple-choice test: Please tick the box(es) with the correct answer(s)!
   (correctly ticked box: +1/2 point; wrongly ticked box: -1/2 point)

The wavenumber ν = 5000cm^{-1} corresponds to a wavelength $V = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{V} = \frac{1}{5000}\,cm = 2\times10^{-6}\,m = 2\mu m$	5000nm	
	2000nm	✓
	2μm	✓
The refractive index of a material is	dimensionless	✓
	given in s^{-1}	
	given in cm^{-1}	

2. True or wrong? Make your decision!

   (2 points): 1 point per correct decision, 0 points per wrong or no decision

Assertion	true	wrong
In linear optics, electromagnetic energy dissipation occurs when the imaginary part of the dielectric function is larger than zero	✓	
All dielectrics have negative refractive indices.		✓

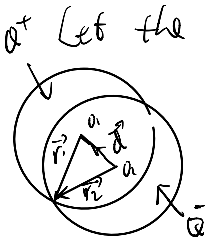
3. Let a material have the absorption coefficient α. Which path must be travelled by the electromagnetic wave in order to reduce its intensity down to 10%? (2 points)
4. Find an expression for the electric field inside a homogeneously polarized dielectric sphere located in vacuum!
   Note: The task is easily solved when regarding the single polarized sphere as a superposition of two homogeneously charged spheres with slightly shifted central points. (6 points)
5. Find an expression for the static polarizability of a spherical particle located in vacuum with radius R, built from a dielectric material with the static dielectric constant ε_{stat}. Also, consider the case of a metal sphere, formally replacing ε_{stat} by $\varepsilon \to -\infty$. Basing on the expression for the static polarizability of the metal sphere, estimate the polarizability of a fictive atom, assuming the latter as a sphere with a radius equal to 0.05nm. (6 points)

3. $I = I_0 e^{-\alpha z}$ $I = 10\% I_0 \Rightarrow \frac{1}{10} I_0 = I_0 e^{-\alpha z} \Rightarrow e^{-\alpha z} = \frac{1}{10} \Rightarrow \alpha z = \ln 10 \Rightarrow z = \frac{\ln 10}{\alpha}$

4. $\vec{E} = \frac{\vec{F}}{q}$ $\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq\vec{r}}{r^3} \Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\vec{r}}{r^3}$ $Q = \frac{4}{3}\pi r^3 \rho$

$\Rightarrow E = \frac{\rho\vec{r}}{3\varepsilon_0}$ ρ is charge volume density

Let the distance of the slightly shifted central points is $\vec{d}$, and sphere O_1 is full of positive charges and sphere O_2 is full of negative charges



Then: The $\vec{E}_1 = \frac{\rho\vec{r}_1}{3\varepsilon_0}$ $\vec{E}_2 = -\frac{\rho\vec{r}_2}{3\varepsilon_0}$ $\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho\vec{r}_1}{3\varepsilon_0} + \left(-\frac{\rho\vec{r}_2}{3\varepsilon_0}\right) = \frac{\rho}{3\varepsilon_0}(\vec{r}_1 - \vec{r}_2) = \frac{-\rho\vec{d}}{3\varepsilon_0} = -\frac{\vec{P}}{3\varepsilon_0}$

$\vec{j} = q N_e V_D$ $\qquad m = 63.5u = \frac{63.5}{N_A} g = 63.5 \times 1.66 \times 10^{-24} g$

$I = \pi r^2 \vec{j} = \pi r^2 q N_e V_D$ $\qquad N_e = \frac{\rho}{m} = \frac{8.93\ g/cm^3}{63.5 \times 1.66 \times 10^{-24} g}$

6. $\Rightarrow V_D = \frac{I}{\pi r^2 q N_e}$ $\qquad \Rightarrow V_D = \frac{1A \cdot 63.5 \times 1.66 \times 10^{-24} g}{3.14 \times 0.0815^2 cm^2 \cdot 1.602 \times 10^{-19} A \cdot s \cdot 8.93 g \cdot cm^{-3}} = 353.28 \times 10^{-5}\ cm/s$

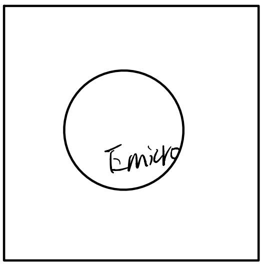
$= 3.5328 \times 10^{-5}\ m/s$

$\approx 0.127\ m/h$

$V_D < V_{snail} = 3 m/h$

Assume a current I = 1A flowing through a copper wire with a diameter d of 1.63mm. Estimate the drift velocity of the electrons, assuming that there is approximately 1 free electron per copper atom, a mass density of ρ=8.93gcm^{-3}, and a mass number of copper of 63.5. Note that the Roman snail in the figure moves with a velocity of approximately 3 meters per hour. Is the drift velocity of the conduction electron higher or smaller than the propagation velocity of the Roman snail?(6 points)

5.



$$d = \varepsilon_0 \beta_{stat} \vec{E}_{micro} \qquad N d = P = N \varepsilon_0 \beta_{stat} \vec{E}_{micro}$$

$$E_{micro} = E - E_{sphere} = E + \frac{P}{3\varepsilon_0}$$

$$\Rightarrow P = N \varepsilon_0 \beta_{stat} E + \frac{N \varepsilon_0 \beta_{stat} P}{3\varepsilon_0}$$

$$\Rightarrow \left(1 - \frac{N\beta_{stat}}{3}\right) P = N \varepsilon_0 \beta_{stat} E \qquad P = \varepsilon_0 (\varepsilon_{stat} - 1) E$$

$$(\varepsilon_{stat} - 1) - \frac{1}{3} N \beta_{stat} (\varepsilon_{stat} - 1) = N \beta_{stat}$$

$$\Rightarrow N \beta_{stat} \left(1 + \frac{1}{3}\varepsilon_{stat} - \frac{1}{3}\right) = \varepsilon_{stat} - 1$$

$$\Rightarrow \beta_{stat} = \frac{3(\varepsilon_{stat} - 1)}{N(2 + \varepsilon_{stat})}$$

Metal sphere : $\beta_{stat} = \frac{3}{N}$ $\qquad N = \frac{n}{V}$ $\quad$ n is the number of free e^-

fictive atom $\Rightarrow \beta_{stat} = \frac{3}{n} V = \frac{4\pi R^3}{n} = \frac{1.57}{n} \times 10^{-30} m$

a fictive atom $\Rightarrow n = 1 \Rightarrow \beta = 1.57 \times 10^{-30}$

Oriental Polarization

$Re(\varepsilon) = 1 + \frac{\chi_{stat}}{1+w^2\tau^2}$: → Debye's formula

$Im(\varepsilon) = \frac{-1}{\pi} VP \int_{-\infty}^{\infty} \frac{[Re\,\varepsilon(w_2) - 1]}{w_2 - w} dw$ → kramers kronig relation

↓ Cauchy's principle function

$w_2 = \xi \quad Re\,\varepsilon(w) - 1 = \frac{\chi_{stat}}{1+w^2\tau^2} \quad Im\,\varepsilon(w) = -\frac{\chi_{stat}}{\pi} VP \int_{-\infty}^{\infty} \frac{d\xi}{(\xi - w)(1+\xi^2\tau^2)}$

$x = \tau\xi \Rightarrow Im\,\varepsilon(w) = -\frac{\chi_{stat}}{\pi} VP \int_{-\infty}^{\infty} \frac{dx}{(x - w\tau)(1+x^2)} = -\frac{\chi_{stat}}{\pi} \lim_{A\to\infty} \lim_{B\to 0} \left[\int_{-A}^{w(-B)} \frac{dx}{(x-w\tau)(1+x^2)} + \int_{w\tau+B}^{A} \frac{dx}{(x-w\tau)(1+x^2)} \right]$

$\int \frac{dx}{(x+b)(x^2+a^2)} = \frac{1}{a^2+b^2} \left[\ln|x+b| - \frac{1}{2}\ln|x^2+a^2| + \frac{b}{a}\tan^{-1}\left(\frac{x}{a}\right) \right]$

$\Rightarrow Im\,\varepsilon(w) = \underbrace{-\frac{\chi_{stat}\,w\tau}{\pi(1+w^2\tau^2)} \lim_{A\to\infty} [\tan^{-1}(-A) - \tan^{-1}(A)]}_{①} - \underbrace{\frac{\chi_{stat}}{\pi(1+w^2\tau^2)} \lim_{B\to 0} \left[\frac{1}{2}\ln \overbrace{\left| \frac{(w\tau+B)^2+1}{(w\tau-B)^2+1} \right|}^{\to 0} + w\tau \underbrace{[\tan^{-1}(w\tau+B) - \tan^{-1}(w\tau-B)]}_{0} \right]}_{B\to 0 \Rightarrow 0}$

$\lim_{A\to\infty} \tan^{-1}(A) = \frac{\pi}{2} \quad \tan^{-1}(-A) = -\tan^{-1}(A)$

① $Im\,\varepsilon(w) = -\frac{\chi_{stat}\,w\tau}{\pi(1+w^2\tau^2)}(-\pi) = \frac{\chi_{stat}\,w\tau}{(1+w^2\tau^2)}$

$V_{ph} = \frac{w}{k} \quad V_g = \frac{dw}{dk}$

$R = \left| \frac{n_1 - n_2}{n_1 + n_2} \right|^2 = \frac{V_2 - V_1}{V_1 + V_2}$