Friedrich Schiller University Jena
Institute of Applied Physics
Prof. Dr. Thomas Pertsch
Anastasia Romashkina, Shreyas Ramakrishna, Mostafa Abasifard, Tina (Shiu Hei Lam), Xiao Chen, Pawan Kumar

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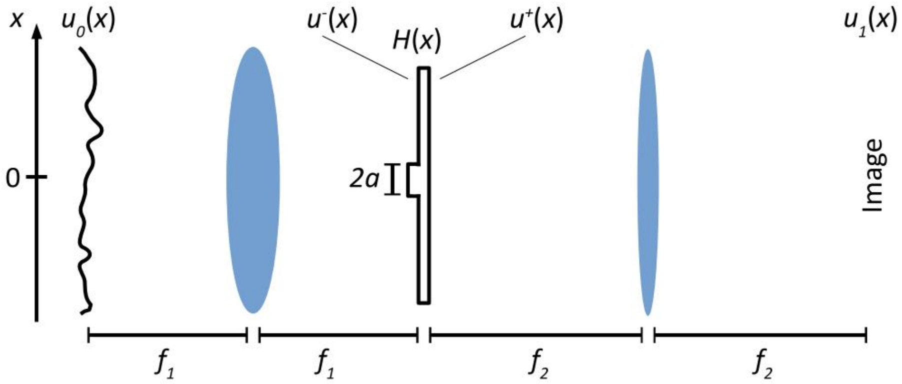
Series 10
Fundamentals of Modern Optics

to be returned on **19.01.2023**, at the beginning of the lecture

Task 1: Phase Contrast Microscopy (2+3 points)

Biological samples are often almost completely transparent. Consequently, they are very hard to see in a conventional microscope. However, these samples often have inhomogeneities of the refractive index and change the *phase* of the transmitted light. One elegant solution to make those phase profiles visible is the *phase contrast microscopy*. A simplified optical setup that implements it is shown below.



Essential is the phase plate in the centre whose effect can be described by the following transmission function

$$H(x) = \begin{cases} \exp(\mathrm{i}\varphi_0) & -a \leq x \leq a \\ 1 & \text{else} \end{cases}.$$

A part of the transmitted light is delayed compared to the rest. We will see in this task that this allows converting a phase profile into an intensity profile.

a) Derive an expression that describes how $u_1(x)$ depends on $u_0(x)$ and $H(x)$. Consider monochromatic light of wavelength λ. *Hint: Make use of the 2f-setup equation from the lecture.*

b) Calculate the image $u_1(x)$ of the initial phase-profile distribution:

$$u_0(x) = e^{\mathrm{i}A\cos(\alpha_0 x)} \approx 1 + \mathrm{i}A\cos(\alpha_0 x),$$

where $A \ll 1$ and $\alpha_0 = 2\pi/\Lambda$. This field can be thought to be caused approximately by a phase grating with a small amplitude and grating period Λ. Derive the conditions on a and φ_0 so that the setup converts the phase modulation in u_0 into an amplitude modulation in u_1.

Task 2: Prison Break (2+2+2 points)

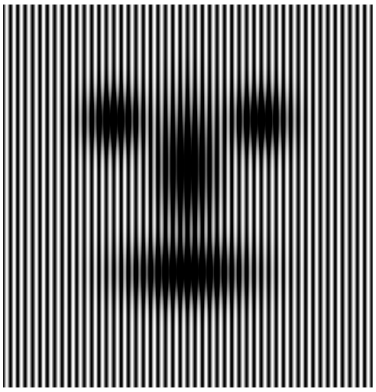


Figure 1: Hello! I am Günther! Please help me to get out of jail! I am innocent. I swear!

Your German friend Günther Gaußig was sent to prison for admitting that it is possible to overcome the resolution limit. He was locked by sheriff Peter Periodic using a grating in the x-direction which has the following form:

$$\text{PRISON} = g(x) = \frac{1}{2}\left[1 + \cos\left(\frac{2\pi}{d}x\right)\right]$$

Luckily, Günther is a very "Gaussian" person in real space, where he usually lives

$$\text{GÜNTHER} = f(x,y) = m(x,y) + n(x,y) + e_l(x,y) + e_r(x,y),$$

with

$$\text{Nose} = n(x,y) = \exp\left[-\frac{x^2}{w^2} - \frac{\left(y - \frac{R}{5}\right)^2}{l^2}\right]$$

$$\text{Mouth} = m(x,y) = \exp\left[-\frac{x^2}{l^2} - \frac{\left(y + \frac{R}{2}\right)^2}{w^2}\right]$$

$$\text{LeftEye} = e_l(x,y) = \exp\left[-\frac{\left(x - \frac{R}{2}\right)^2}{w^2} - \frac{\left(y - \frac{R}{2}\right)^2}{w^2}\right]$$

$$\text{RightEye} = e_r(x,y) = \exp\left[-\frac{\left(x + \frac{R}{2}\right)^2}{w^2} - \frac{\left(y - \frac{R}{2}\right)^2}{w^2}\right]$$

and

$$R > l > w \gg d$$

As an Abbe School of Photonics student you want to free your friend by optical means. You find Günther superimposed by the prison in the spatial domain as a starting point, i.e. the function $u_0(x,y) = f(x,y) + g(x)$ (see Figure).

a) Develop a plan to free Günther, i.e. to remove the prison, by Fourier optical filtering. Explain why it is needed the condition $R > l > w \gg d$ to be satisfied.

b) Construct and sketch an optical setup which is able to perform that task. Explain the mechanism from the initial optical field, the optics in between and the final field.

c) Why does Günther necessarily get battered (at least a little bit) by the escape from prison? How can you minimize that effect?

Hint: Notice that this task requires a physical understanding of the 4f-optical filtering. Try to be as precise as possible in your answers and avoid ambiguities. You can make sketches and write down general formulas in all questions to help the explanation.

Task 3: Jones matrix formalism (2+2 points)

The Jones formalism is a powerful technique for the treatment of polarized light. To take a look at it, let's consider a monochromatic plane wave in the vacuum of the form $\mathbf{E}(r,t) = \mathbf{E_0}\, e^{i(kz-\omega t)}$, where the electric field is polarized in the (x,y)-plane, so $\mathbf{E_0} = (E_x, E_y, 0)$. We can write it in the form of the so-called "Jones vector":

$$\mathbf{J}_{\text{in}} = \begin{pmatrix} E_x\, e^{i\varphi_x} \\ E_y\, e^{i\varphi_y} \end{pmatrix}.$$

Then light propagation through a polarizing optical element can be written as a linear transformation:

$$\mathbf{J}_{\text{out}} = \hat{\mathbf{T}} \cdot \mathbf{J}_{\text{in}}.$$

where $\hat{\mathbf{T}}$ denotes a "Jones matrix" of that element.
Recommendation: For those students who are further interested in the Jones matrix formalism we recommend to read chapter 6 of *"Fundamentals of Photonics"* by B.E.A. Saleh, M.C. Teich (*electronic copy of the book is available on Wiley online library for download*).

a) Elements of the Jones vectors and Jones matrices depend on the choice of the coordinate system. Consider, the Jones matrix of an x-polarizer is given by $\hat{\mathbf{T}} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. Now we rotate the polarizer around z axis by an angle θ. Using matrix methods, define the Jones matrix in the new coordinate system.
Hint: Make use of the rotation matrix.

b) Given is an optical element characterized by the Jones matrix $\hat{\mathbf{T}} = \frac{1}{2}\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$. Which polarization state should the light have to pass the element without change?