Problem 1 Jinsong Liu 206216

(a)

The physical origin of gain narrowing is gain can shrink the amplified spectrum towards the resonance frequency if: $g(\nu_0) \cdot l = \sigma(\nu_0) \cdot n \cdot l \gg 1$.

(b)

Because we need to avoid the transition of particles from $|2\rangle$ to $|3\rangle$ during pumping.

(c)

Because the essential condition for continuous population inversion is: $S_{10} > S_{21}$.

(d)

We can reduce the influence of ASE by spatial filtering.

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Problem 2 5/5 Jinsong Liu 206216

(a)

$\because$ $g(\nu)$ has Gaussian profile

$$\therefore g(\nu) = g_0 \cdot \exp\left[-4 \cdot \ln 2 \left\{\frac{(\nu-\nu_0)}{\Delta\nu_g}\right\}^2\right]$$

g_0 is the peak value of $g(\nu)$.

$$\therefore G(\nu) = \exp[g(\nu) \cdot d] = \exp\left[g_0 \cdot \exp\left[-4 \cdot \ln 2 \left\{\frac{(\nu-\nu_0)}{\Delta\nu_g}\right\}^2\right] \cdot d\right]$$

$$\therefore G(\nu_0) = \exp[g(\nu_0) \cdot d] = \exp[g_0 \cdot d]$$

the FWHM of $G(\nu)$ is $\Delta\nu_G$

$$\therefore G(\nu_0 \pm \tfrac{1}{2}\Delta\nu_G) = \tfrac{1}{2} G(\nu_0)$$

$$G(\nu_0 \pm \tfrac{1}{2}\Delta\nu_G) = \exp\left[g_0 \cdot \exp\left[-4 \ln 2 \left\{\frac{\pm\frac{1}{2}\Delta\nu_G}{\Delta\nu_g}\right\}^2\right] \cdot d\right]$$

$$\tfrac{1}{2} G(\nu_0) = \exp\left[g_0 \cdot \exp\left[-4 \cdot \ln 2 \cdot \tfrac{1}{4} \frac{\Delta\nu_G^2}{\Delta\nu_g^2}\right] d\right]$$

$$\ln G(\nu_0) - \ln 2 = g_0 \cdot d \cdot \exp\left[-\ln 2 \left(\frac{\Delta\nu_G}{\Delta\nu_g}\right)^2\right]$$

$$\ln G(\nu_0) - \ln 2 = \ln G(\nu_0) \cdot \exp\left[-\ln 2 \left(\frac{\Delta\nu_G}{\Delta\nu_g}\right)^2\right]$$

$$1 - \frac{\ln 2}{\ln G(\nu_0)} = \exp\left[-\ln 2 \left(\frac{\Delta\nu_G}{\Delta\nu_g}\right)^2\right]$$

$$\therefore \ln\left[1-\frac{\ln 2}{\ln G(\nu_0)}\right] = -\ln 2\left(\frac{\Delta\nu_G}{\Delta\nu_g}\right)^2$$

$$\because \ln(1-x) \approx -x$$

$$\therefore -\frac{\ln 2}{\ln G(\nu_0)} = -\ln 2\left(\frac{\Delta\nu_G}{\Delta\nu_g}\right)^2$$

$$\therefore \Delta\nu_g^2 = \Delta\nu_G^2 \cdot \ln G(\nu_0)$$

$$\therefore \Delta\nu_g = \Delta\nu_G \cdot \sqrt{\ln G(\nu_0)} \qquad \Delta\nu_G = \Delta\nu_g \cdot \frac{1}{\sqrt{\ln G(\nu_0)}}$$

(b)

$$G(\nu) = \exp[g(\nu)\cdot d] = \exp[\sigma(\nu)\cdot n\cdot d]$$

$$= \exp\left[\left(a-\sigma_0\left[\frac{\nu-\nu_0}{\Delta\nu_0}\right]^2\right)\cdot n\cdot d\right] \quad ①$$

$$\because \sigma(\nu) = a - \sigma_0\left[\frac{\nu-\nu_0}{\Delta\nu_0}\right]^2$$

is valid for $|\nu| < \nu_0 + \Delta\nu\cdot\sqrt{\frac{a}{\sigma_0}}$

From equation ①, we can get:

$$G(\nu_0) = \exp[a\cdot n\cdot d]$$

$$G(\nu_0 \pm \tfrac{1}{2}\Delta\nu_G) = \exp\left[n\cdot d\cdot\left(a-\sigma_0\left[\frac{\pm\frac{1}{2}\Delta\nu_G}{\Delta\nu_0}\right]^2\right)\right]$$

$$G(\nu_0 \pm \tfrac{1}{2}\Delta\nu_G) = \tfrac{1}{2}G(\nu_0)$$

$$\frac{1}{2}\exp(a\cdot n\cdot d)=\exp\left[n\cdot d\left(a-\sigma_0\left[\frac{\Delta\nu_G}{2\Delta\nu_0}\right]^2\right)\right]$$

$$a\cdot n\cdot d-\ln 2 = a\cdot n\cdot d-\sigma_0\cdot n\cdot d\left[\frac{\Delta\nu_G}{2\Delta\nu_0}\right]^2$$

$$\ln 2 = \sigma_0\cdot n\cdot d\left[\frac{\Delta\nu_G}{2\Delta\nu_0}\right]^2$$

$$\therefore \frac{\ln 2}{\sigma_0\cdot n\cdot d}(2\Delta\nu_0)^2=\Delta\nu_G^2$$

$$\therefore \Delta\nu_G = 2\Delta\nu_0\cdot\sqrt{\frac{\ln 2}{\sigma_0\cdot n\cdot d}}$$

(c)

$$G(\nu)=\exp[g(\nu)\cdot d]=\exp[\sigma_\nu\cdot n\cdot d]$$

$$\because \sigma(\nu)=\sigma_0\left[1-\left(\frac{\nu-\nu_0}{\Delta\nu_g}\right)^2\right]$$

$$\therefore G(\nu)=\exp\left[\sigma_0\cdot n\cdot d\cdot\left[1-\left(\frac{\nu-\nu_0}{\Delta\nu_g}\right)^2\right]\right]$$

$\because$ the input signal has a Gaussian spectrum

$$\therefore I_{in}(\nu)=I_0\cdot\exp\left[-4\ln 2\left\{\frac{\nu-\nu_0}{\Delta\nu_{in}}\right\}^2\right]$$

$$I_{out}(\nu)=I_{in}(\nu)\cdot G(\nu)$$

$$=I_0\exp\left\{\sigma_0\cdot n\cdot d\left[1-\left(\frac{\nu-\nu_0}{\Delta\nu_g}\right)^2\right]-4\ln 2\left(\frac{\nu-\nu_0}{\Delta\nu_{in}}\right)^2\right\}$$

$\because\ c = \lambda \cdot \nu \qquad \nu = \frac{c}{\lambda} \qquad \ln\nu = \ln c - \ln\lambda$

$$\frac{1}{\nu} d\nu = -\frac{1}{\lambda} d\lambda$$

$$\therefore \left|\frac{\Delta\nu}{\nu}\right| = \left|\frac{\Delta\lambda}{\lambda}\right|$$

$\because\ \lambda = 1\mu m \quad \begin{cases} \Delta\lambda_{in} = 4.5\,nm \\ \Delta\lambda_{out} = 2\,nm \end{cases} \quad \begin{matrix} \therefore\ \Delta\nu_{in} = 1.35\times 10^{12}\ Hz \\ \therefore\ \Delta\nu_{out} = 6\times 10^{11}\ Hz \end{matrix}$

$$I_{out}(\nu_0) = I_0 \exp(\sigma_0 \cdot n \cdot d)$$

$$I_{out}(\nu \pm \tfrac{1}{2}\Delta\nu_{out}) = I_0 \exp\left\{\sigma_0 \cdot n \cdot d \cdot \left(1 - \frac{\Delta\nu_{out}^2}{4\Delta\nu_g^2}\right) - 4\ln 2 \cdot \frac{\Delta\nu_{out}^2}{4\Delta\nu_{in}^2}\right\}$$

$$\frac{1}{2} I_{out}(\nu_0) = I_{out}(\nu \pm \tfrac{1}{2}\Delta\nu_{out})$$

$$\therefore\ \frac{1}{2} I_0 \exp(\sigma_0 \cdot n \cdot d) = I_0 \exp\left\{\sigma_0 \cdot n \cdot d \cdot \left(1 - \frac{\Delta\nu_{out}^2}{4\Delta\nu_g^2}\right) - 4\ln 2 \cdot \frac{\Delta\nu_{out}^2}{4\Delta\nu_{in}^2}\right\}$$

$$\sigma_0 \cdot n \cdot d - \ln 2 = \sigma_0 \cdot n \cdot d \left(1 - \frac{\Delta\nu_{out}^2}{4\Delta\nu_g^2}\right) - 4\ln 2 \cdot \frac{\Delta\nu_{out}^2}{4\Delta\nu_{in}^2}$$

$$-\ln 2 = -\sigma_0 \cdot n \cdot d \frac{\Delta\nu_{out}^2}{4\Delta\nu_g^2} - 4\ln 2 \cdot \frac{\Delta\nu_{out}^2}{4\Delta\nu_{in}^2}$$

$$\frac{4\ln 2}{\Delta\nu_{out}^2} = \sigma_0 \cdot n \cdot d \frac{1}{\Delta\nu_g^2} + 4 \cdot \ln 2 \frac{1}{\Delta\nu_{in}^2}$$

$$\sigma_0 \cdot n \cdot d \frac{1}{\Delta\nu_g^2} = 4\ln 2 \frac{1}{\Delta\nu_{out}^2} - 4\ln 2 \frac{1}{\Delta\nu_{in}^2}$$

$$\therefore\ d = \frac{\Delta\nu_g^2}{\sigma_0 n} \cdot 4\ln 2 \left(\frac{1}{\Delta\nu_{out}^2} - \frac{1}{\Delta\nu_{in}^2} \right)$$

$$\therefore\ d \approx 0.206\,m$$

(d)

$$\because\ G(\nu) = \exp\left[\sigma_0 \cdot n d \left[1 - \left(\frac{\nu - \nu_0}{\Delta\nu_g} \right)^2 \right] \right]$$

$\therefore$ the maximum value of $G(\nu)$ is :

$$G(\nu_0) = \exp(\sigma_0 \cdot n \cdot d) \approx 482.992$$

Problem 3 4.5/8 Jinsong Liu 206216

(a)

Ne atoms have 5 energy-levels. the life time of 3p, 4s, 4p and 5s is much shorter than 3s. So it's really difficult to realize population inversion. 1/2

(b)

Mission: ① The laser process in a He-Ne laser starts with collision of electrons from the electrical discharge with the helium atoms, which excites helium from the ground state to the 2^3S_1 and 2^1S_0 metastable excited states.

② Collision of the excited helium atoms with the ground-state neon atoms results in transfer of energy to the neon atoms. He atoms can transfer energy to 5s and 4s of Ne atoms.

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(c)

Because the laser starts from the collision of the electrons and He atoms. The more the He atoms exist, the higher pump efficiency is.

0

(d)

From energy conservation, the extra energy comes from the kinetic energy of He. And the excess energy becomes the kinetic energy of Ne.

1

(e)

∵ It can increase the proabability of collisions of He atoms with Ne atoms.

the tube, not the He atom

0

(f)

From the manual of He-Ne laser experiment, we know that normally a He-Ne laser is working at 632.816nm with a very narrow gain bandwidth of a few GHz, which is dominated by Doppler broadening.

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