Fundamentals of Modern Optics

series 4

10.11.2014

to be returned on 17.11.2014, at the beginning of the lecture

Problem 1 - Poynting Vector for Plane Waves (2+2+2+2 points)

Given is a monochromatic, plane wave of frequency ω with a linear polarization along the **x**-direction and a **k**-vector which is pointing in **z**-direction. The wave is propagating through a linear, isotropic, homogeneous, non-magnetic medium with complex permittivity $\varepsilon(\omega) = \varepsilon' + i\varepsilon''$. At the plane $z = 0$ the electric field has a modulus of E_0.

a) Find the dispersion relation $\omega = \omega(\mathbf{k})$ and identify the refractive index n as a function of ε.

b) Calculate both the $\mathbf{E}(\mathbf{r})$ and $\mathbf{H}(\mathbf{r})$-fields as functions of z for all $z > 0$. Remember that the complex fields for a monochromatic wave are connected to the real time varying fields via the relation $\mathbf{E}_r(\mathbf{r}, t) = \frac{1}{2}\left[\mathbf{E}(\mathbf{r})\, e^{-i\omega t} + \mathbf{E}(\mathbf{r})^*\, e^{i\omega t}\right]$.

c) Calculate the time averaged Poynting vector $\langle \mathbf{S}(\mathbf{r}, t)\rangle = \frac{1}{2}\mathrm{Re}[\mathbf{E}(\mathbf{r}) \times \mathbf{H}(\mathbf{r})^*]$ and its divergence $\nabla \cdot \langle \mathbf{S}\rangle$.

d) Identify ranges of ε which correspond to the cases: (i) propagating waves without loss, (ii) propagating waves with loss, (ii) nonpropagating waves without loss.

Problem 2 - Poynting Vector for a Surface Guided Wave (2*+2+2+2*+2+2 points)

We have learned in the previous exercise series about the separation of EM modes in TE and TM for the cases that we have invariance of the structure in one direction. Let us investigate one such structure with invariance in y direction: A metal-to-air-interface at $x = 0$ with $\varepsilon(x < 0) = \varepsilon_M < 0$ and $\varepsilon(x > 0) = 1$. It turns out that this geometry supports a TM surface guided mode (propagating in the **z**-direction and confined to the interface along the **x**-direction from both sides) with frequency ω with the following structure of the magnetic field: (can not propagating in x-direction)

$$\mathbf{H} = \begin{bmatrix} 0 \\ H_y \\ 0 \end{bmatrix}; \quad H_y = H_0 e^{ik_z z} \begin{cases} \exp\{-\sqrt{k_z^2 - \frac{\omega^2}{c^2}}\, x\} & , x > 0 \\ \exp\{+\sqrt{k_z^2 - \frac{\omega^2}{c^2}\varepsilon_M}\, x\} & , x < 0 \end{cases}$$

with $k_z > 0$.

a*) Show that if we want to have such a guided/confined wave with the above mentioned characteristics, H_y would indeed have the form given above. (*Hint*: The eigenmode of each of this half spaces are still plane waves of the form $H_y = e^{i(k_x x + k_z z)}$, where the wave numbers satisfy the dispersion relations of their corresponding media. The key to have a mode that coexists in both media is that both waves in metal and air should have the same k_z (satisfying the boundary condition for tangential magnetic field across the interface).)

b) Use Maxwell's equations to find the expression for the electric field.

c) Make a drawing of both fields and their behaviours close to the metal-air interface.

d*) Use the continuity of E_z at the interface to show that

$$-\varepsilon_M \sqrt{k_z^2 - \frac{\omega^2}{c^2}} = \sqrt{k_z^2 - \frac{\omega^2}{c^2}\varepsilon_M}$$

From this relation it can be seen that having $\varepsilon_M < 0$ is essential for having a confined wave in the x direction. But is it enough for having it propagating in the z direction? Find k_z from this equation and find the condition on ε_M for having a real valued k_z.

e) Calculate the Poynting vector as a function of distance x from the metal-air interface. Make a schematic drawing of it.

f) Calculate the net flow of energy per unit length in the y-direction by integrating the Poynting vector from e) over $x \in [-\infty, \infty]$. In which direction does the net energy flow?

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Problem 1 - Poynting Vector for Plane Waves

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a) according to the conditions: $\vec{E}(\vec{r},t) = E_0 e^{i(kz-\omega t)} \vec{e}_x$

the Maxwell's equations:
$$\begin{cases} \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \\ \nabla\cdot\vec{E} = 0 \\ \nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t} \\ \nabla\cdot\vec{B} = 0 \end{cases}$$

$$\Rightarrow \nabla\times\nabla\times\vec{E} = -\mu_0\varepsilon_0\varepsilon\frac{\partial^2}{\partial t^2}\vec{E}(\vec{r},t)$$

$$\Rightarrow -k^2\vec{E}(\vec{r},t) = -\mu_0\varepsilon_0\varepsilon\frac{\partial^2}{\partial t^2}\vec{E}(\vec{r},t) = -\mu_0\varepsilon_0\varepsilon\,\omega^2\vec{E}(\vec{r},t)$$

$\Rightarrow k^2 = \omega^2\mu_0\varepsilon_0\varepsilon = \frac{\omega^2}{c^2}\varepsilon(\omega)$ ($c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ speed of light in vaccum)

if $\varepsilon(\omega) \neq 0$ (while $\varepsilon(\omega_L) = 0$ - case of longitudinal waves)

we have transversal wave $\vec{E}(\vec{r},t) \perp \vec{k}$

So $k^2 = \frac{\omega^2}{c^2}\varepsilon(\omega)$ and $k(\omega) = \frac{\omega}{c}[n(\omega) + i\kappa(\omega)]$

$$\Rightarrow \begin{cases} k^2 = \frac{\omega^2}{c^2}[n^2(\omega) - \kappa^2(\omega) + 2i n(\omega)\cdot\kappa(\omega)] \\ k^2 = \frac{\omega^2}{c^2}\varepsilon(\omega) = \frac{\omega^2}{c^2}[\varepsilon'(\omega) + i\varepsilon''(\omega)] \end{cases}$$

$$\Rightarrow \begin{cases} \varepsilon'(\omega) = n^2(\omega) - \kappa^2(\omega) \\ \varepsilon''(\omega) = 2n(\omega)\kappa(\omega) \end{cases}$$

solve this equations, we can get

$$n = \sqrt{\frac{\varepsilon' + \sqrt{(\varepsilon')^2 + (\varepsilon'')^2}}{2}}$$

$$\kappa(\omega) = \sqrt{\frac{\sqrt{(\varepsilon')^2 + (\varepsilon'')^2} - \varepsilon'}{2}}$$

b) $\vec{E}(\vec{r},t)=\frac{1}{2}[\vec{E}(\vec{r})e^{-i\omega t}+E(\vec{r})^{*}e^{i\omega t}]$

$=\frac{1}{2}E_0\vec{e_x}\cdot(e^{ikz-i\omega t}+e^{-ikz+i\omega t})$

$=\frac{1}{2}E_0\vec{e_x}\cdot[e^{-\frac{\omega}{c}Kz}\cdot e^{i(\frac{\omega}{c}nz-\omega t)}+e^{-\frac{\omega}{c}Kz}\cdot e^{-i(\frac{\omega}{c}nz-\omega t)}]$

$=E_0\vec{e_x}\cdot e^{-\frac{\omega}{c}Kz}\cdot\cos(\frac{\omega}{c}nz-\omega t)$

here $n=n(\omega)$, $K=K(\omega)$ and $k=k'+ik''=\frac{\omega}{c}(n+iK)$

$\Rightarrow\begin{cases}k'(\omega)=\frac{\omega}{c}\cdot n\\ k''(\omega)=\frac{\omega}{c}\cdot K\end{cases}$

therefore, $\vec{E}(\vec{r},t)=E_0\vec{e_x}\cdot e^{-k''z}\cos(k'z-\omega t)$

In Fourier domain, complex form of $\vec{E}$ field,

$\vec{E}=E_0e^{ikz}\delta(\omega'-\omega)\vec{e_0}$

$\nabla\times\vec{E}(\vec{r},\omega)=i\mu_0\omega\vec{H}(\vec{r},\omega)$

$\Rightarrow E_0\delta(\omega'-\omega)\cdot\nabla\times(e^{ikz}\vec{e_x})=i\mu_0\omega\vec{H}(\vec{r},\omega)$

since $\nabla\times(e^{ikz}\vec{e_x})=ike^{ikz}\vec{e_y}$

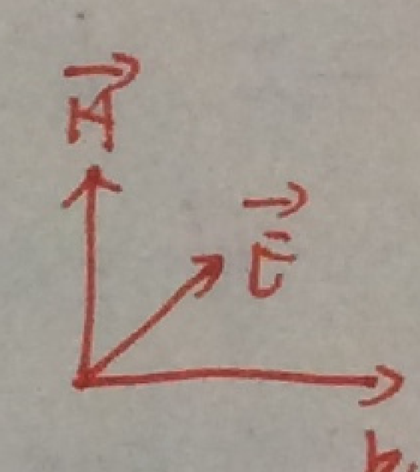
another ways:

$\nabla\times\vec{E}=\nabla\times(\vec{E_0}e^{i\vec{k}\vec{r}})$

$=\nabla e^{i\vec{k}\vec{r}}\times\vec{E_0}$

$=i(\vec{k}\times\vec{E_0})e^{i\vec{k}\cdot\vec{r}}$

$=i\omega\mu_0\vec{H}$



$\Rightarrow\vec{H}(\vec{r},\omega)=\frac{k}{\mu_0\omega}E_0\delta(\omega'-\omega)e^{ikz}\vec{e_y}=\sqrt{\frac{\varepsilon_0}{\mu_0}}\cdot\sqrt{\varepsilon(\omega)}\cdot E_0\delta(\omega'-\omega)e^{ikz}\vec{e_y}$

In time domain

$\vec{H}(\vec{r},t)=\int_{-\infty}^{\infty}\vec{H}e^{i\omega' t}d\omega'=\sqrt{\frac{\varepsilon_0}{\mu_0}}\sqrt{\varepsilon(\omega)}\cdot E_0e^{ikz}e^{-i\omega t}\cdot\vec{e_y}=\sqrt{\frac{\varepsilon_0}{\mu_0}}\sqrt{\varepsilon(\omega)}\cdot E_0\cdot e^{-k''z}\cdot e^{ik'z}\cdot e^{-i\omega t}\vec{e_y}$

lets define $\vec{E}(\vec{r})$ and $\vec{H}(\vec{r})$ as follows

$\vec{E}(\vec{r},t)=\vec{E}(\vec{r})e^{-i\omega t}$

$\vec{H}(\vec{r},t)=\vec{H}(\vec{r})e^{-i\omega t}$

$\Rightarrow\begin{cases}\vec{E}(\vec{r})=E_0\cdot e^{-k''z}\cdot e^{ik'z}\cdot\vec{e_x}\\ \vec{H}(\vec{r})=\sqrt{\frac{\varepsilon_0}{\mu_0}}\cdot\sqrt{\varepsilon(\omega)}\cdot E_0e^{-k''z}e^{ik'z}\cdot\vec{e_y}\end{cases}$ ✓ ✓

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c) $\vec{E}(\vec{r}) \times \vec{H}(\vec{r})^* = \sqrt{\frac{\varepsilon_0 \varepsilon(\omega)}{\mu_0}}^{\,*} |E_0|^2 e^{-2k''z} \vec{e}_z$

$\Rightarrow \langle \vec{S}(\vec{r},t) \rangle = \frac{1}{2} \mathrm{Re}[\vec{E}(\vec{r}) \times H(\vec{r})^*] = \frac{1}{2}|E_0|^2 \cdot \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot e^{-2k''z} \cdot n(\omega) \vec{e}_z$ ✓

$\nabla \cdot \langle \vec{S} \rangle = \frac{1}{2}|E_0|^2 \cdot \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot n(\omega) \frac{\partial}{\partial z}(e^{-2k''z}) = -2k'' \langle \vec{S} \rangle = -k''|E_0|^2 \cdot \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot n(\omega) e^{-2k''z}$ ✓ 2/2

energy loss due to medium absorption.

d) (i) propagation waves without loss

$\Rightarrow k'' = 0 = \frac{\omega}{c}\kappa(\omega)$

$\boxed{\begin{pmatrix} \langle S \rangle \neq 0 \\ \nabla \cdot \langle S \rangle = 0 \end{pmatrix} \text{adding method}}$ $\Rightarrow \begin{matrix} n \neq 0 \\ \varepsilon'' = 0 \end{matrix} \Rightarrow \varepsilon' > 0 \ \varepsilon'' = 0$

$\begin{cases} \varepsilon' = n^2 - \kappa^2 \\ \varepsilon'' = 2n\kappa \end{cases} \Rightarrow \begin{cases} \varepsilon' = n^2 > 0 \\ \varepsilon'' = 0 \end{cases}$ ✓

(ii) propagation waves with loss

$\boxed{\begin{pmatrix} \langle S \rangle \neq 0 \\ \nabla \cdot \langle S \rangle < 0 \end{pmatrix} \Rightarrow \begin{matrix} n \neq 0 \\ \varepsilon'' > 0 \end{matrix} \begin{cases} \varepsilon' > 0, \varepsilon'' > 0 & \text{loss in dielectric} \\ \varepsilon' < 0 \ \varepsilon'' > 0 & \text{loss in metal} \end{cases}}$

$\begin{cases} n > 0 \\ \kappa > 0 \end{cases} \Rightarrow$ two cases $\begin{cases} n > \kappa, \ \varepsilon' > 0, \varepsilon'' > 0 & \text{propagation dominates} \\ & \text{weakly damped quasi-homogeneous waves} \\ n < \kappa, \ \varepsilon' < 0, \varepsilon'' > 0 & \text{damping dominates} \\ & \text{strongly damped quasi-homogeneous waves.} \end{cases}$ ✓

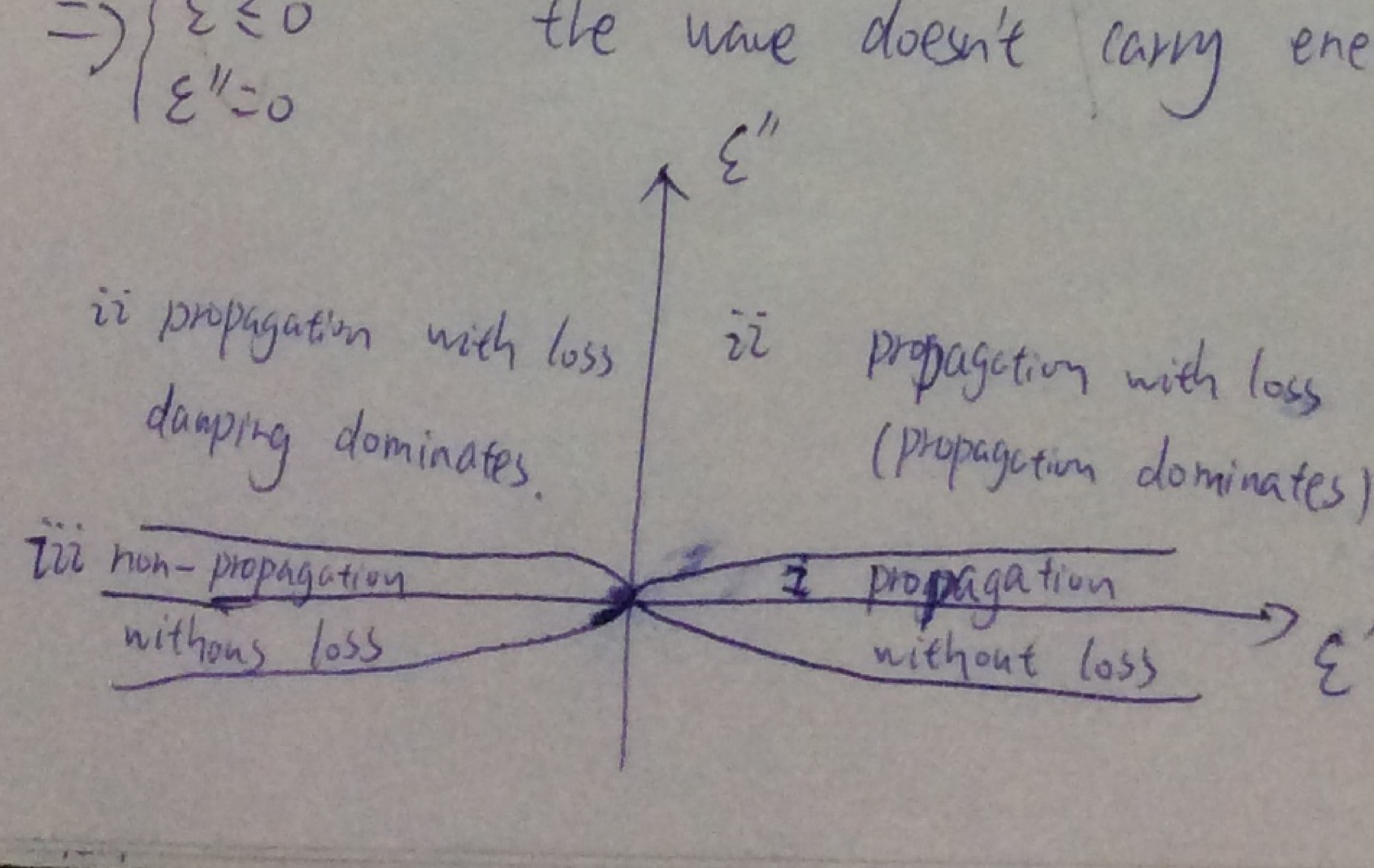
(iii) nonpropagating waves withous loss

$\boxed{\begin{pmatrix} \langle S \rangle = 0 \Rightarrow n = 0 \\ \nabla \cdot \langle S \rangle = 0 \Rightarrow \varepsilon'' = 0 \end{pmatrix} \Rightarrow \varepsilon' < 0 \ \varepsilon'' = 0}$

$k' = \frac{\omega}{c} n = 0 \Rightarrow \nabla \cdot \langle \vec{S} \rangle = 0$

even we have non-zero k'' $\Rightarrow \kappa \geq 0$

$\Rightarrow \begin{cases} \varepsilon' \leq 0 \\ \varepsilon'' = 0 \end{cases}$ the wave doesn't carry energy and doesn't lose it



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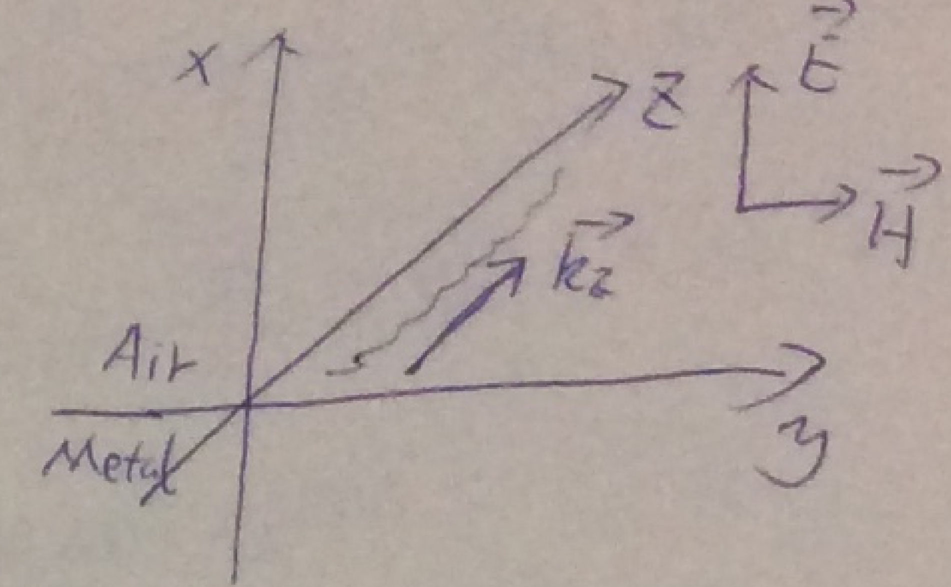
Problem 2 Poynting Vector for a Surface Guided Wave

a) a) $H_y = \begin{cases} H_1 e^{i(k_{1x}x + k_{1z}z)} & x>0 \\ H_2 e^{i(k_{2x}x + k_{2z}z)} & x<0 \end{cases}$



for boundry condition, we have $k_{1z} = k_{2z} = k_z$

and $H_y|_{x=0^-} = H_y|_{x=0^+} \Rightarrow H_1 e^{ik_z \cdot z} = H_2 e^{ik_z \cdot z} \Rightarrow H_1 = H_2 = H_0$

$\therefore k^2 = k_x^2 + k_z^2 = \frac{w^2}{c^2}\varepsilon = \begin{cases} \frac{w^2}{c^2} & x>0 \\ \frac{w^2}{c^2}\varepsilon_m & x<0 \end{cases}$ ✓

for metal ($x<0$), $k_x^2 = \frac{w^2}{c^2}\varepsilon_m - k_z^2 < 0 \Rightarrow k_x = \pm i\sqrt{k_z^2 - \frac{w^2}{c^2}\varepsilon_m}$ k_x is a Imaginary number

$H_y = H_1 e^{i(k_{1x}x + k_{1z}z)} = H_0 \cdot e^{ik_z z} \cdot e^{\pm\sqrt{k_z^2 - \frac{w^2}{c^2}\varepsilon_m}\cdot x}$ $x<0$

in x-direction, for confined wave, we have

$H_y = H_0 e^{ik_z \cdot z} \cdot e^{\sqrt{k_z^2 - \frac{w^2}{c^2}\varepsilon_m}\cdot x}$ $x<0$ $e^{\sqrt{k_z^2 - \frac{w^2}{c^2}\varepsilon_m}\cdot x} \to 0$ for $x \to -\infty$

for air ($x>0$) $k_x^2 = \frac{w^2}{c^2} - k_z^2 \Rightarrow k_x = \sqrt{\frac{w^2}{c^2} - k_z^2} = \pm i\sqrt{k_z^2 - \frac{w^2}{c^2}}$ k_x might be a real number or Imaginary number

$H_y = H_2 e^{i(k_{2x}\cdot x + k_{2z}\cdot z)} = H_0 \cdot e^{ik_z \cdot z} \cdot e^{\pm\sqrt{k_z^2 - \frac{w^2}{c^2}}\cdot x}$ $x>0$

~~In x-direction~~, for confined wave, we have

$H_y = H_0 \cdot e^{ik_z \cdot z} e^{-\sqrt{k_z^2 - \frac{w^2}{c^2}}\cdot x}$ $x>0$ $e^{-\sqrt{k_z^2 - \frac{w^2}{c^2}}\cdot x} \to 0$ for $x \to +\infty$

So, $H_y = H_0 \cdot e^{ik_z} \begin{cases} e^{-\sqrt{k_z^2 - \frac{w^2}{c^2}}\cdot x} & x>0 \\ e^{+\sqrt{k_z^2 - \frac{w^2}{c^2}\varepsilon_m}\cdot x} & x<0 \end{cases}$

✓

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b) For a metal, there is no current inside, for the Maxwell's equation, we have

a)

$$\vec{E}(\vec{r},\omega) = \frac{i}{\varepsilon\varepsilon_0\omega}\nabla\times\vec{H}(\vec{r},\omega)$$

$$\vec{H}(\vec{r},\omega) = H_0\, e^{i(k_z\cdot z + k_x x)}\cdot\vec{e_y}$$

$$\vec{E}(\vec{r},\omega) = E_0(\omega)\cdot\vec{E}(\vec{r})$$

$$\nabla\times\vec{H} = H_0(\omega)\cdot\nabla\times[e^{i(k_z\cdot z + k_x\cdot x)}\vec{e_y}]$$

$$= H_0(\omega)\cdot[\vec{e_z}\cdot ik_x\cdot e^{i(k_z\cdot z + k_x x)} - \vec{e_x}\cdot ik_z\cdot e^{i(k_z\cdot z + k_x\cdot x)}]$$

$$\Rightarrow \vec{E}(\vec{r},\omega) = \frac{H_0(\omega)}{\varepsilon\cdot\varepsilon_0\omega}\begin{pmatrix} k_z\cdot e^{i(k_z\cdot z + k_x\cdot x)} \\ 0 \\ -k_x\, e^{i(k_z\cdot z + k_x\cdot x)}\end{pmatrix}$$

$$\therefore E_x = \frac{H_0(\omega)}{\varepsilon_0\omega}\cdot k_z\cdot e^{ik_z z}\begin{cases} e^{-\sqrt{k_z^2-\frac{\omega^2}{c^2}}\cdot x} & x>0 \\ \frac{e^{\sqrt{k_z^2-\frac{\omega^2}{c^2}\varepsilon_m}\cdot x}}{\varepsilon_m} & x<0\end{cases}$$

$$E_z = \frac{H_0(\omega)}{\varepsilon_0\omega}\cdot e^{ik_z z}\begin{cases} -i\sqrt{k_z^2-\frac{\omega^2}{c^2}}\, e^{-\sqrt{k_z^2-\frac{\omega^2}{c^2}}\cdot x} & x>0 \\ \frac{i\sqrt{k_z^2-\frac{\omega^2}{c^2}\varepsilon_m}}{\varepsilon_m}\, e^{\sqrt{k_z^2-\frac{\omega^2}{c^2}\varepsilon_m}\cdot x} & x<0\end{cases}$$

$$\vec{E}(\vec{r},\omega) = E_x\vec{e_x} + E_z\vec{e_z}$$

✓

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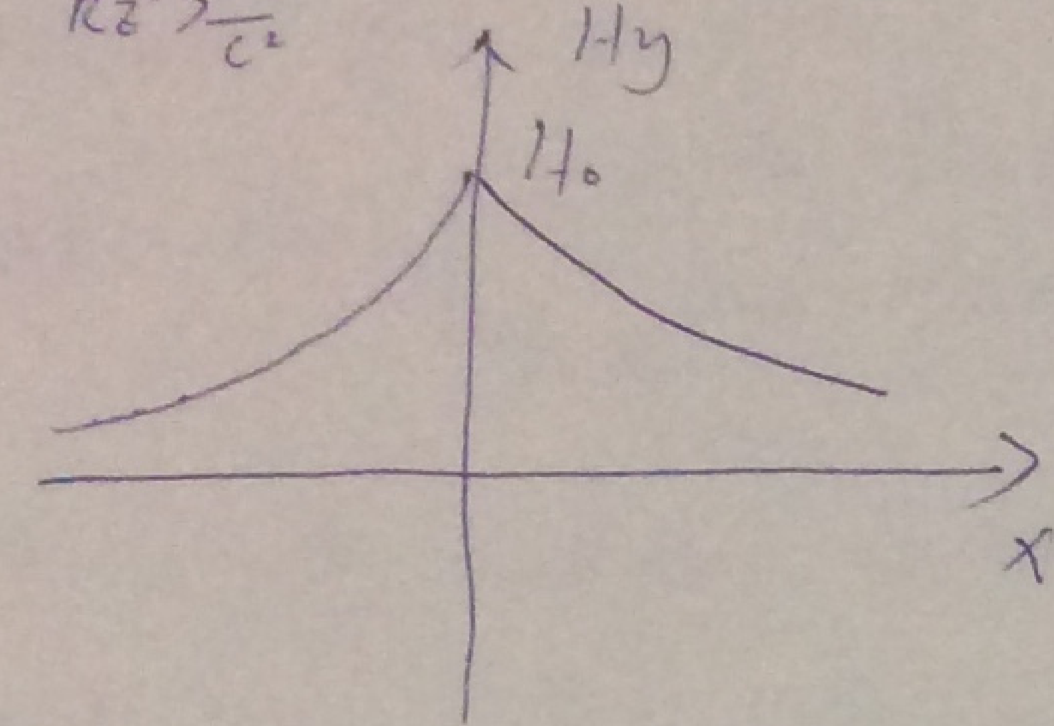
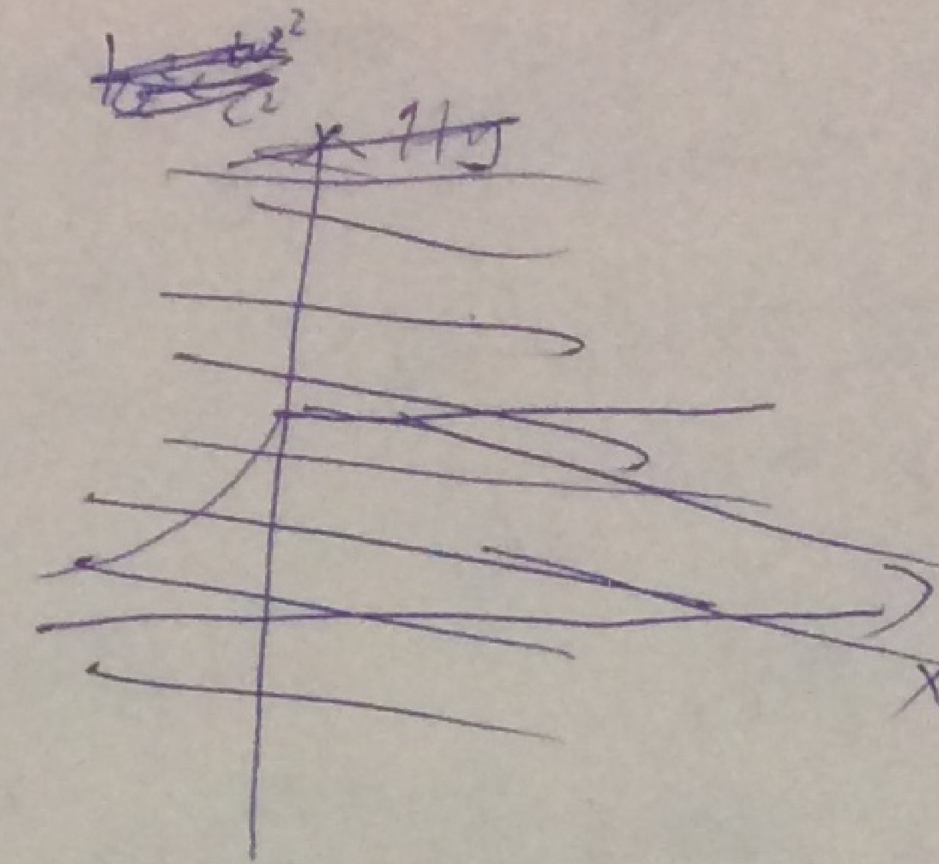
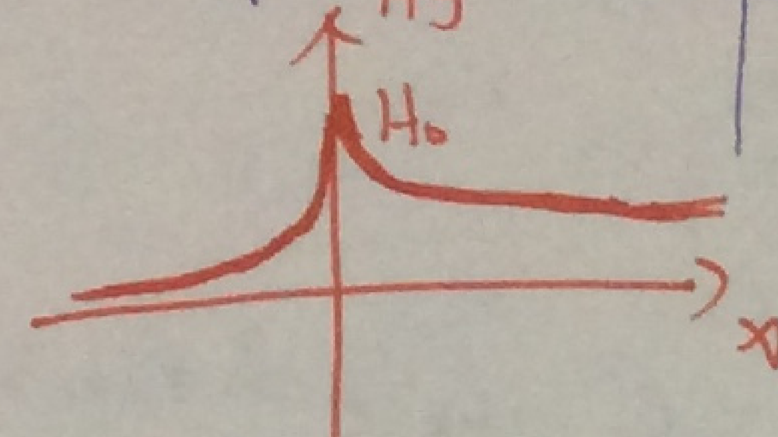
c) H_y, E_x, E_z at $z=0$

$k_z^2 > \frac{\omega^2}{c^2}$







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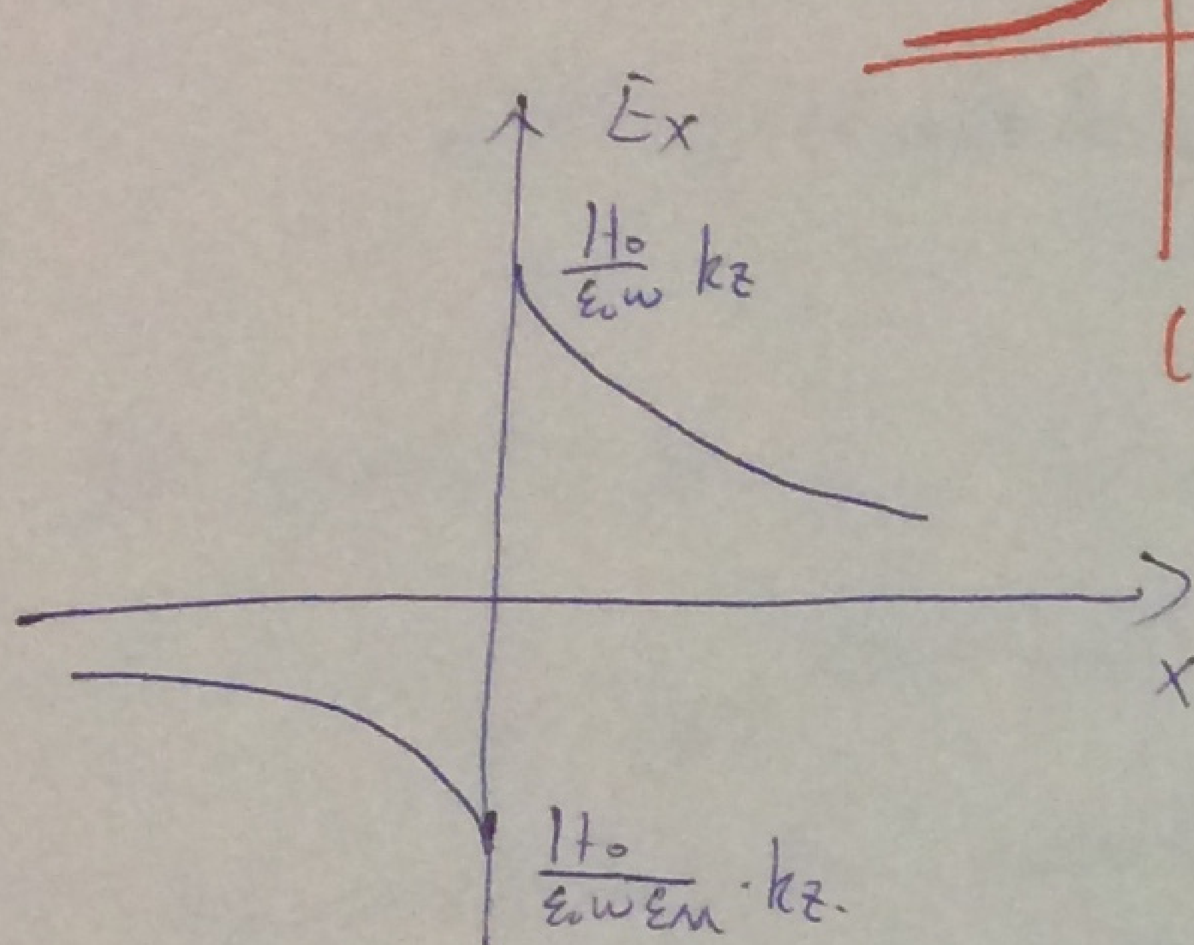

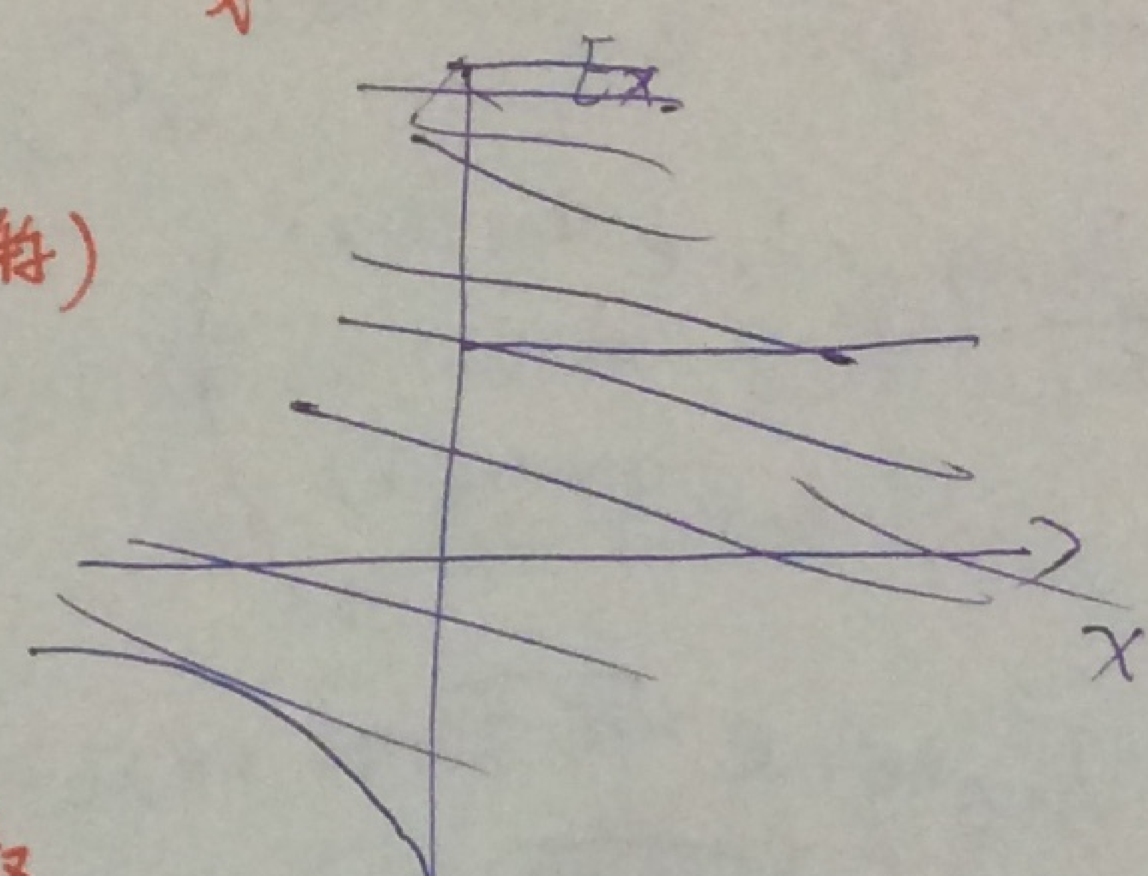

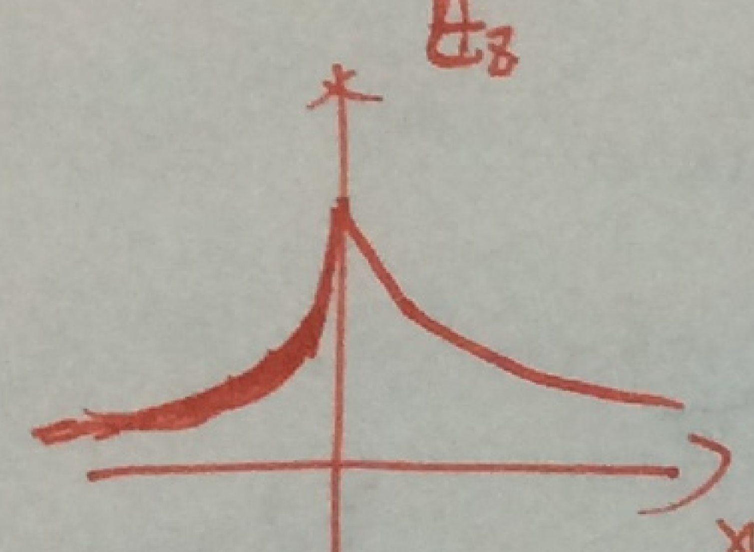

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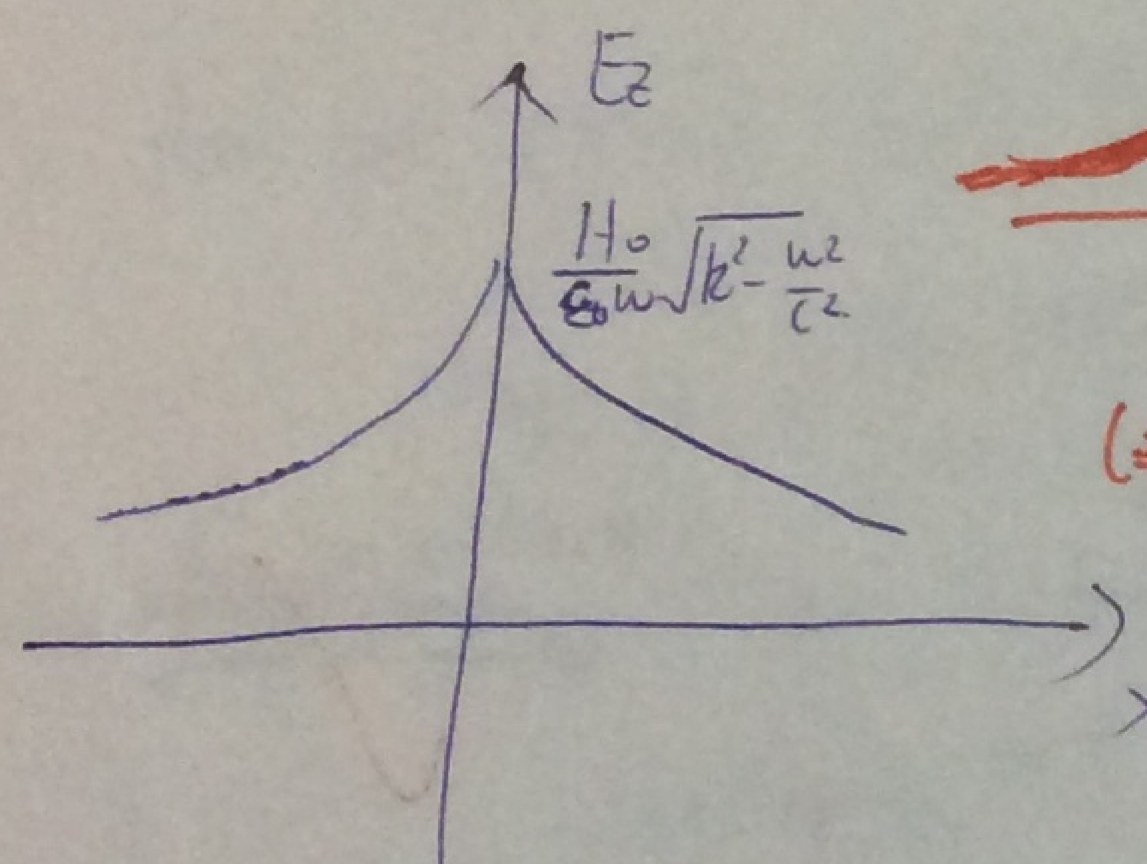

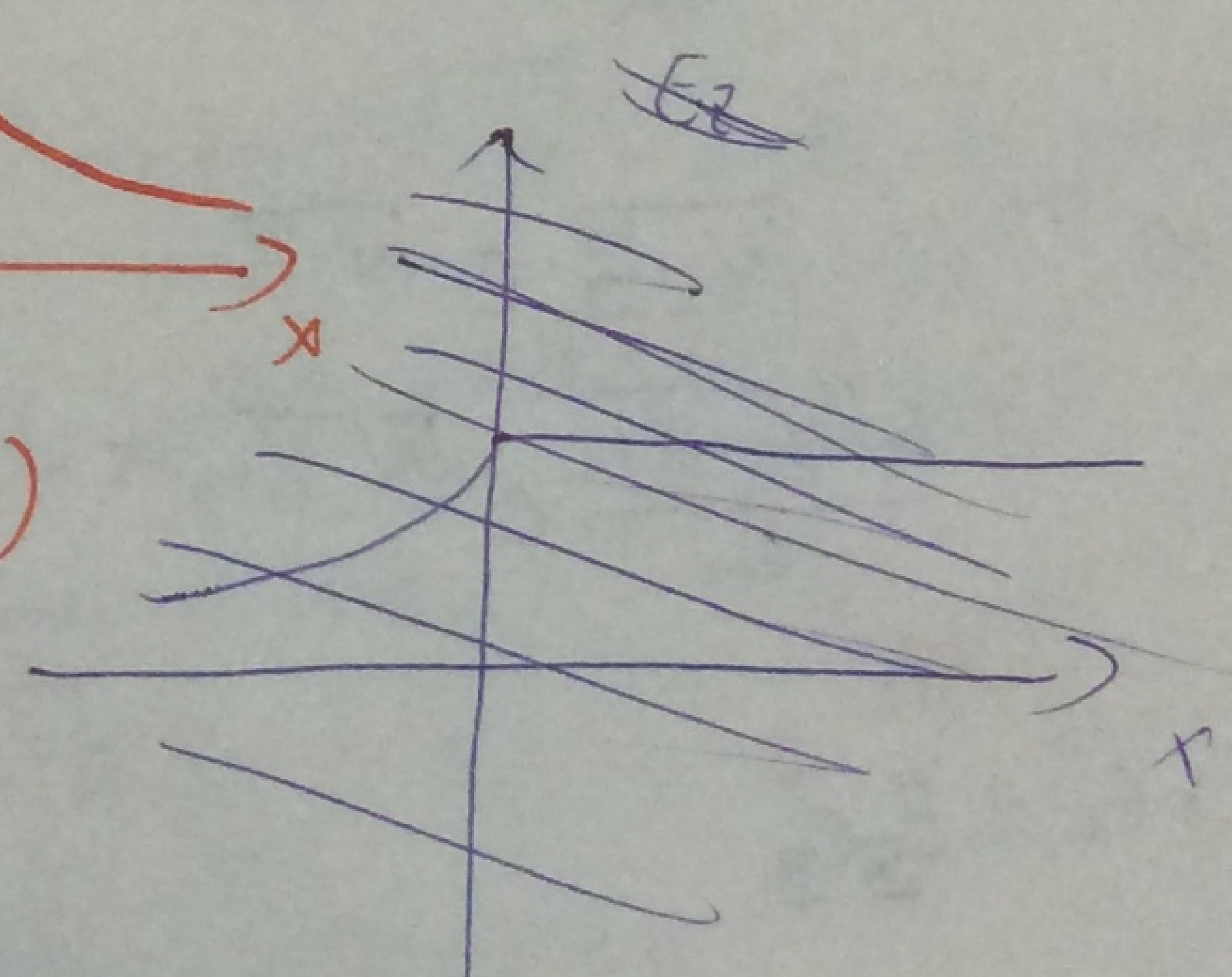

-- you should have emphasized the asymmetry of H_y and ~~非~~ E_z more clearly

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d) As ~~fas~~ as E_z satisfy continuity condition:

$$E_z|_{z=0^-} = E_z|_{z=0^+}$$

$$\Rightarrow -i\sqrt{k_z^2-\frac{\omega^2}{c^2}} = \frac{i}{\varepsilon_m}\sqrt{k_z^2-\frac{\omega^2}{c^2}\varepsilon_m} \Rightarrow -\varepsilon_m\sqrt{k_z^2-\frac{\omega^2}{c^2}} = \sqrt{k_z^2-\frac{\omega^2}{c^2}\varepsilon_m}$$

for a confined wave, $k_z^2-\frac{\omega^2}{c^2} > 0$

we have $\sqrt{k_z^2-\frac{\omega^2}{c^2}\varepsilon_m} > 0$, $\sqrt{k_z^2-\frac{\omega^2}{c^2}} > 0$

to have k_z propagating in z-direction, we must have k_z a real number or complex number, $\varepsilon_m < 0$ for metal

So $\varepsilon_m^2\left(k_z^2-\frac{\omega^2}{c^2}\right) = k_z^2-\frac{\omega^2}{c^2}\varepsilon_m$

$$k_z^2 = \frac{\frac{\omega^2}{c^2}(\varepsilon_m^2-\varepsilon_m)}{\varepsilon_m^2-1} = \frac{\varepsilon_m\cdot\frac{\omega^2}{c^2}}{\varepsilon_m+1}$$

if $\varepsilon_m < 0$ and $k_z^2 > 0 \Rightarrow \varepsilon_m + 1 < 0$

$\Rightarrow \varepsilon_m < -1$ ✓

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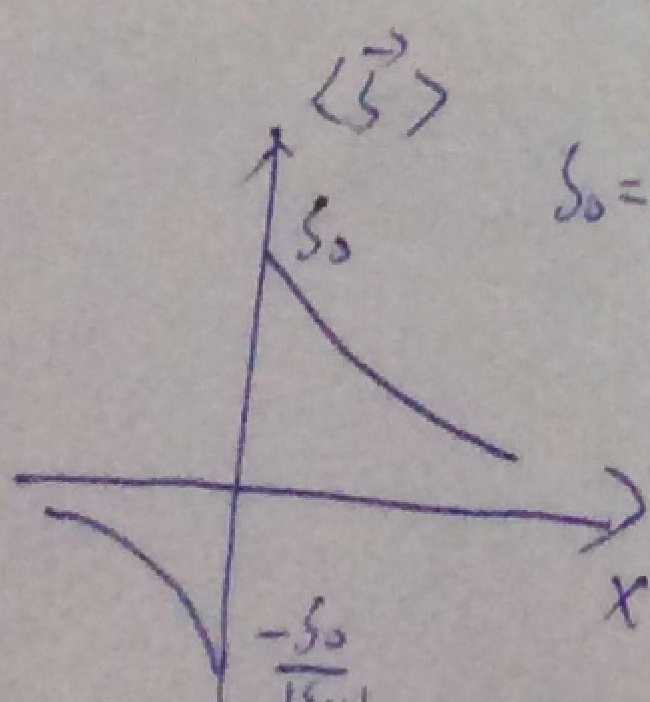
e) $\langle\vec{S}\rangle = \frac{1}{2}\mathrm{Re}[\vec{E}(\vec{r}) \times \vec{H}^*(\vec{r})]$

$$= \frac{1}{2}\mathrm{Re}\left[\frac{H_0}{\varepsilon\varepsilon_0\omega}\left(k_z e^{-k''x} e^{ik_z z}\vec{e}_x - k_x e^{-k''x} e^{ik_z z}\vec{e}_z\right) \times H_0^*\left(e^{-k''x} e^{-ik_z z}\vec{e}_y\right)\right]$$

$$= \frac{|H_0|^2 e^{-2k''x}}{2\varepsilon\varepsilon_0\omega}\mathrm{Re}(k_z\vec{e}_z + k_x\vec{e}_x) = k_z\vec{e}_z\cdot\frac{|H_0|^2 e^{-2k''x}}{2\varepsilon\varepsilon_0\omega} \quad (\text{here } k'' = ik_x)$$

So, $\langle\vec{S}\rangle = k_z\vec{e}_z\frac{|H_0|^2}{2\varepsilon_0\omega}\cdot\begin{cases} e^{-2\left(\sqrt{k_z^2-\frac{\omega^2}{c^2}}\cdot x\right)} & x>0 \ ✓ \\ \dfrac{e^{+2\left(\sqrt{k_z^2-\frac{\omega^2}{c^2}\varepsilon_m}\cdot x\right)}}{\varepsilon_m} & x<0 \ ✓ \end{cases}$



$S_0 = \frac{|H_0|^2}{2\varepsilon_0\omega}$

In metal, $\langle\vec{S}\rangle$ has oppsite direction to $\vec{k_z}$. It means that energy flow is guided oppositely to wave propagation. ✓ ✓

2/2

f) $\Phi = \iint_{0\,-\infty}^{l\;\infty} \langle \vec{S} \rangle \, dy \, dx = \int_{-\infty}^{\infty} \langle \vec{S} \rangle dx = k_z \vec{e_z} \frac{|H_0|^2}{2\varepsilon_0 \omega} \cdot \Big(\int_0^{\infty} e^{-2\sqrt{k_z^2 - \frac{\omega^2}{c^2}} \cdot x} dx$

$+ \int_{-\infty}^{0} \frac{1}{\varepsilon_m} e^{+2\sqrt{k_z^2 - \frac{\omega^2}{c^2}\varepsilon_m} \cdot x} dx \Big)$

$= k_z \vec{e_z} \frac{|H_0|^2}{2\varepsilon_0 \omega} \left(\frac{1}{2\sqrt{k_z^2 - \frac{\omega^2}{c^2}}} + \frac{1}{\varepsilon_m} \frac{1}{2\sqrt{k_z^2 - \frac{\omega^2}{c^2}\varepsilon_m}} \right)$

$= \frac{k_z \cdot \vec{e_z} \cdot |H_0|^2}{4\varepsilon_0 \omega \sqrt{k_z^2 - \frac{\omega^2}{c^2}}} \cdot \left(1 - \frac{1}{\varepsilon_m^2}\right) \quad \left(-\varepsilon_m \cdot \sqrt{k_z^2 - \frac{\omega^2}{c^2}} = \sqrt{k_z^2 - \frac{\omega^2}{c^2}\varepsilon_m} \right)$

for $\varepsilon_m < -1$, the new energy flow Φ goes along the wave propagation direction $\vec{e_z}$ ✓

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