Name (with capital letters):
Matriculation number:

Only to be used by the corrector!								
1	2	3	4.1	4.2	5.1	5.2	Σ	mark

Exam Structure of Matter Winter Term 2015 / 2016 (19.02.2016)

90 minutes time

Please note: Please write your name and your matriculation number on each sheet! Please write your solution on the sheets provided and use only other sheets when necessary! Notes made with pencils or with colors others than blue or black will not be accepted!

Task 1 **3 Points**

Imagine that a beam of unpolarized light propagates in x-direction. A subwavelength spherical particle interacts with the beam and scatters light into the y-direction.

a) Into which direction may the E field of the light of the beam point?

b) What is the main orientation of the dipole which is excited by the beam and scatters light into the y-direction?

c) How is the light polarized, which is scattered into the y-direction ?

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Task 2 **6 Points**

The spectral energy density of so-called black-body radiation has the form

$$u_\omega(\omega) = \frac{\hbar\omega^3}{\pi^2 c^3} \frac{1}{\exp(\hbar\omega/k_B T) - 1}$$

a) Show that its maximum growths linearly with temperature!

b) Show that the total energy density of thermal radiation is proportional to T^4 !

c) Show that $u_\omega(\omega)$ growths for small frequencies quadratically with ω!

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Task 3 **4 Points**

Show by using the commutator relation of angular momentum operators

$\left[\hat{L}_i, \hat{L}_j\right] = i\hbar \sum_k \hat{\varepsilon}_{ijk} \hat{L}_k$ where $\hat{\varepsilon}_{ijk}$ is the completely antisymmetric tensor,

that the momentum operators $\hat{L}_x$ and $\hat{\mathbf{L}}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$

a) commute and

b) possess a joint eigenfunction system!

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You should solve either task 4.1 or task 4.2. If you solve both only the points of the task where you have achieved the best result will be counted!

Task 4.1 **6 Points**

Derive from Maxwell's equations for monochromatic optical fields in homogenous nonmagnetic media

a) that the time averaged Poyntingvector $\langle \vec{s} \rangle = \frac{1}{2\mu_0} \mathrm{Re}\left(\vec{E} \times \vec{B}^*\right)$ obeys the relation

$$\mathrm{div}\langle \vec{s} \rangle = +\frac{\omega}{2} \mathrm{Re}\left(i\vec{E}^* \vec{D}\right) \quad !$$

b) Express the energy loss of the electromagnetic field in terms of the dielectric constant!

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Task 4.2 **6 Points**

A material shows two sharp absorption lines: One in the infrared and one in the ultraviolet. The imaginary part of the dielectric constant of the material can therefore for $\omega > 0$ be approximated by two δ functions as

$$\mathrm{Im}\left[\varepsilon(\omega)\right] = A\,\delta(\omega - \omega_{IR}) + B\,\delta(\omega - \omega_{UV}) \text{ with } \omega_{IR} < \omega_{UV}$$

a) Determine $\mathrm{Im}\left[\varepsilon(\omega)\right]$ for $\omega < 0$!

b) Determine $\mathrm{Re}\left[\varepsilon(\omega)\right]$!

c) Draw $\mathrm{Re}\left[\varepsilon(\omega)\right]$ in the visible spectral range, i.e. for $\omega_{IR} < \omega < \omega_{UV}$ and determine the sign of dispersion (normal or anomalous)!

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You should solve either task 5.1 or task 5.2. If you solve both only the points of the task where you have achieved the best result will be counted!

Task 5a **6 Points**

An electron with a kinetic energy E_{kin} comes from $-\infty$ and hits straight a wall approximated by a δ potential $V(x) = V_0 \delta(x)$.

a) Determine the principle shape of the electron wavefunction at $\pm\infty$!

b) What is the continuity condition at $x = 0$?

c) Calculate the probability that the electron passes the wall as a function of the potential V_0 !

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Task 5b **6 Points**

An electron in the ground state of a hydrogen atom is lifted into a superposition of several excited states ψ_{nlm} as $\psi(t=0)=\frac{1}{\sqrt{5}}\left[\psi_{210}+2\psi_{310}\right]$ by a spectrally broad optical excitation.

a) How and was the optical pulse polarized?
b) How big is the probability to detect the atom in the n=3 state?
c) What is the mean energy (expectation value of the energy) of the new state?
d) Calculate the minimum time which it takes the wavefunction to recover to its initial shape at $t=0$!