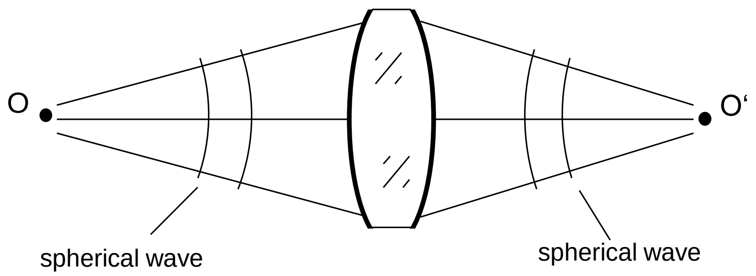
2 Modeling and Design of Lens Systems

Why is a lens so important?
It may have the property of refracting the light-rays originating from a certain point in a way that they again meet in a common point.

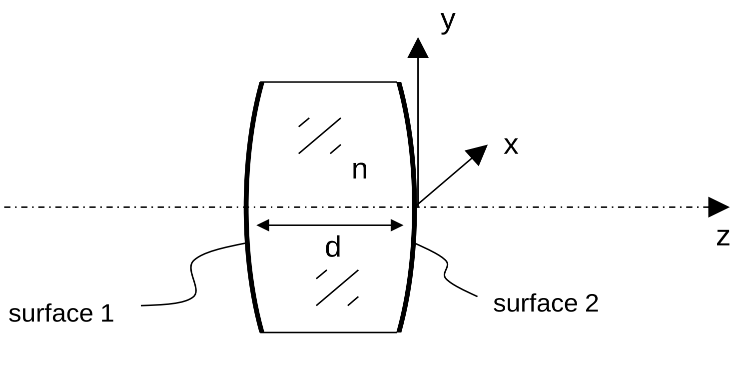
→ this property is called: **Imaging**



- sketch Point-to-Point imaging

Imaging is one of the most important applications of optical systems!
→ that’s why we will have a closer look on it in the next lectures.

What is a lens? → we already know from previous lessons.
<u>Lens:</u> a **rotational symmetric** optical element composed of a **transparent material** with refractive index n with **two spherical or conical surfaces**.



- sketch Lens

Sag formula (local coordinate system):

$$z_{1/2} = \frac{c_{1/2} \cdot r^2}{1 + \sqrt{1 - (1 + k_{1/2})c_{1/2}^2 r^2}} + a_2 r^2 + a_4 r^4 + \cdots \tag{2.1}$$

with: radius $r^2 = x^2 + y^2$

curvature $c_{1/2} = \frac{1}{R_{1/2}}$

R ... Radius of curvature

k ... conic constant → describes a conic section (Kegelschnitt)

k < -1 → hyperbola

k = -1 → parabola

-1 < k < 0 → ellipse

k = 0 → sphere

k > 0 → oblate ellipse

$1/c = R = \pm b^2/a$

$k = -\varepsilon^2 = -(1 - b^2/a^2)$ (ε ... eccentricity)

d ... lens thickness, distance of vertex points

As the exact consideration of the ray-path at a real lens may become quickly very complicated we will start with the so called **paraxial approximation**.

→ Field of **Gaussian Optics**

2.1 Paraxial Approximation / Gaussian Optics

Law of refraction:

$$n \cdot \sin\alpha = n' \cdot \sin\alpha' \qquad (2.2)$$

Paraxial approximation: α , α' small

→ $$n \cdot \alpha = n' \cdot \alpha' \qquad (2.3)$$

and $\cos \approx 1.$

If this simplification is used in:

- law of refraction
- corresponding ray angles
- equations describing optical surfaces

→ Then all equations describing the rays become linear!

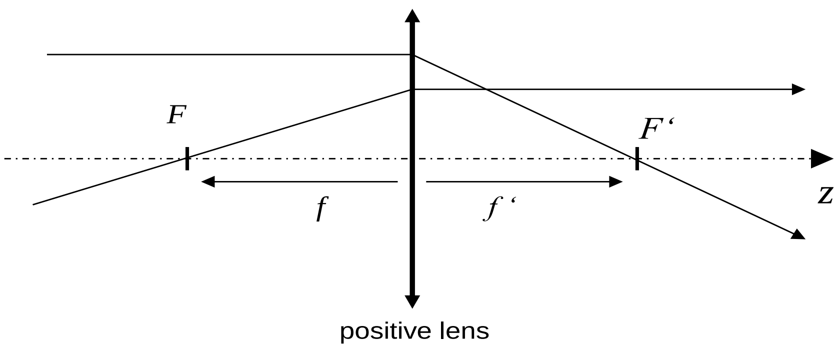
→ No aberrations of the system occur during imaging.

(as monochromatic aberrations are principally caused by the non-linearity of Snell's Law and the surface equations of order >2)

2.2 Ideal Lens

→ transforming a spherical wave into another spherical wave

Let us assume that the optical effect can be imagined as taking place in the plane of the lens.



→ sketch Positive Lens

Sign convention in optics:

Distances along the optical axis are positive if they are oriented in the sense of a vector in positive *z*-direction.

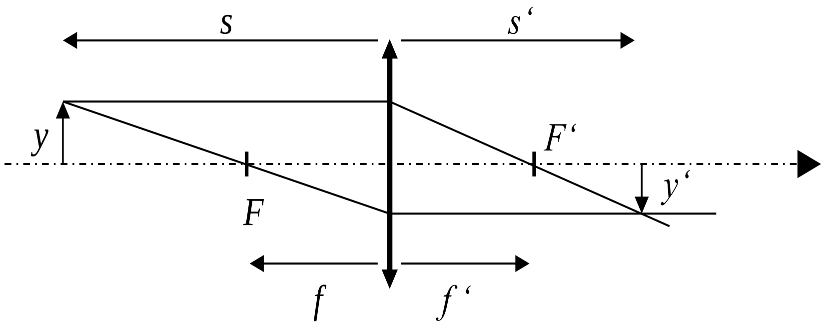
The same applies for the *x*- and *y*-coordinate.

Attention: For radii the direction is oriented from the surface towards the center of curvature!

→ in the above example: $f < 0$

$f' > 0$

Simple image formation:



→ sketch Image formation with a positive lens.

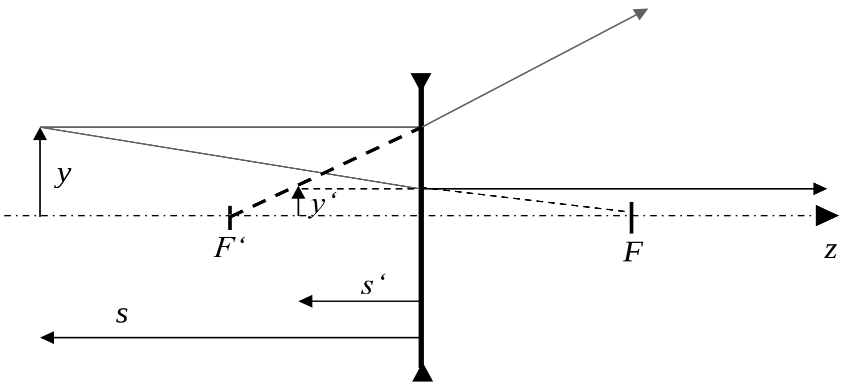
y ... object height (>0)
y' ... image height (<0)
s ... object distance
s' ... image distance (both measured from the vertex of the lens)

We introduce one additional quantity:

$$L = s - s' \tag{2.4}$$

L ... object – image distance

Image formation with a **negative lens** $f' < 0$



→ sketch Image formation with a negative lens.

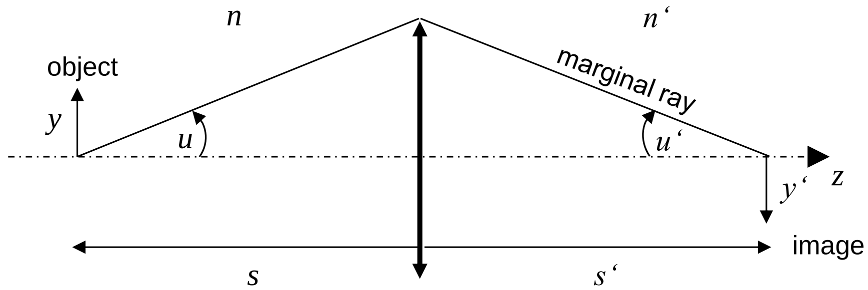
→ virtual image!

<u>Magnification:</u>

$$m = \frac{y'}{y} \tag{2.5}$$

With aperture angles u, u'

$$m = \frac{n \cdot \sin u}{n' \cdot \sin u'} \approx \frac{n \cdot u}{n' \cdot u'} \tag{2.6}$$



→ Sketch: Magnification with aperture angles

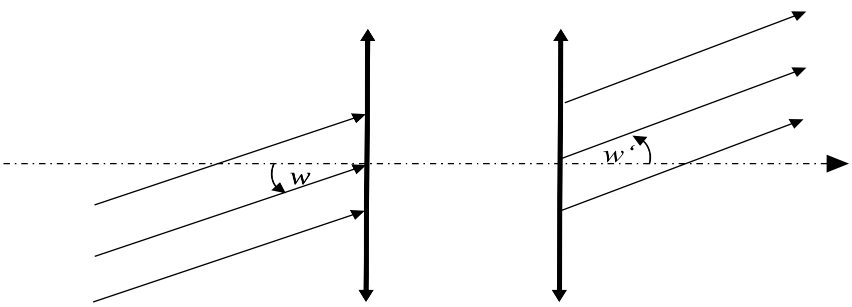
$$m = \frac{n \cdot s'}{n' \cdot s} \tag{2.7}$$

Special case: object and image are at infinity (afocal system) → Telescope

→ y, y' are not defined

→ definition of the angular magnification by the chief ray angles w, w'.

$$\Gamma = \frac{w'}{w} \tag{2.8}$$



→ Sketch: Angular magnification

<u>Lens Equation:</u>

From the above sketches the following equation can easily be derived combining the focal length and object / image distances:

$$\frac{f'}{s'} + \frac{f}{s} = 1 \tag{2.9}$$

→ classical image equation

In general

$$\frac{f'}{n'} = -\frac{f}{n} \tag{2.10}$$

holds.

With $n=n'$ Eq. (2.9) becomes

$$\frac{1}{s'} - \frac{1}{s} = \frac{1}{f'} \tag{2.11}$$

"lens makers formula"

→

$$s' = \frac{s \cdot f'}{s + f'} \tag{2.12}$$

$$m = \frac{f' - s'}{f'} \tag{2.13}$$

→ With these quantities a set of 30 equations for the calculation of one of the other interesting parameters can be derived.

→ see Slide **(S)**

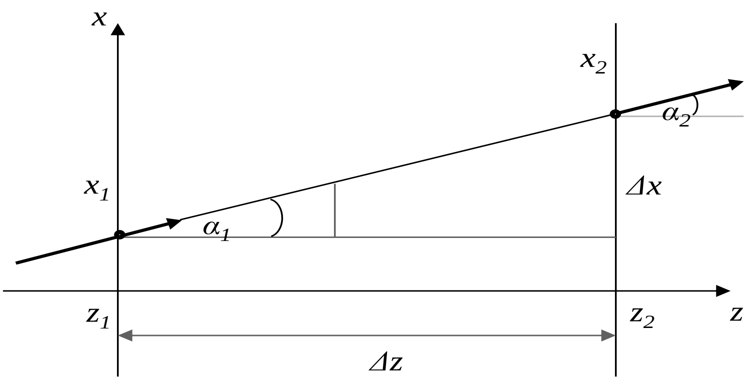
2.3 ABCD-Matrix Formalism

- This formalism is based on a geometrical optics consideration of field propagation.
- Thus, in a first attempt it seems to be restricted to the cases where the ray-model is valid and diffraction effects are not included.
- However, we will see that the ABCD-matrices which are the core of this formalism are much more powerful and can be used also for studying (paraxial) diffraction phenomena.
- First of all the formalism is a simple method for the treatment of complex optical systems

2.3.1 Derivation of the Formalism

Let's consider the free-space propagation of a ray between planes z_1 and $z_2=z_1+\Delta z$.

a) Free-Space Propagation:



→ Sketch: Ray between two planes

Parameters of the ray: x … ray-coordinate
α … ray-angle → given in plane z_1

Calculating the ray parameters in plane z_2 :

$$x_2 = x_1 + \Delta z \cdot \tan\alpha_1 \tag{2.14}$$

$$\alpha_2 = \alpha_1 \tag{2.15}$$

Paraxial approximation: tan α ≈ α → replace in Eq. (2.15)

Note: It is not really necessary to use the paraxial approximation. All following equations are valid as well if one is using the ray-slope tan α = x'= dx/dz instead of α alone in Eq. (2.15).

We can rewrite Eqs. (2.14) and (2.15) as one equation for the vector (x α)

$$\begin{pmatrix} x_2 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} 1 & \Delta z \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ \alpha_1 \end{pmatrix} = M_{\Delta z} \cdot \begin{pmatrix} x_1 \\ \alpha_1 \end{pmatrix} \tag{2.16}$$

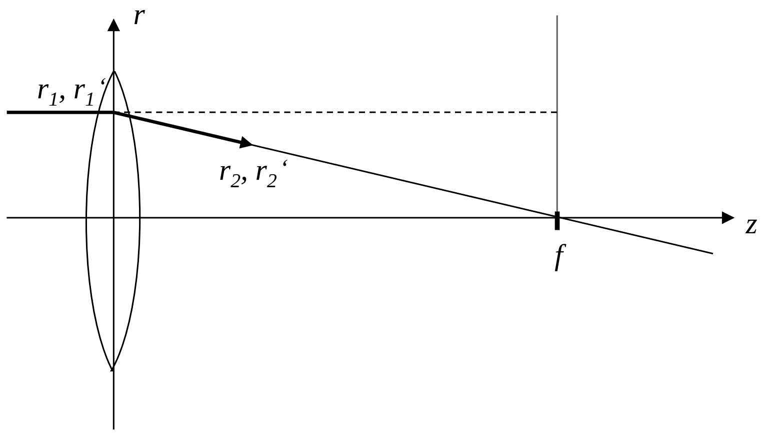
→ ABCD-matrix for free-space propagation within a homogeneous medium.

$$M_{\Delta z} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & \Delta z \\ 0 & 1 \end{pmatrix} \tag{2.17}$$

<u>In general:</u> One can find such a matrix for almost every optical element, but typically then A = f(x)
B = f(x) … are functions of the coordinate x (or r).

There are a number of (important) optical elements for which the matrix elements are constants and do not depend on the coordinate x!
→ "well behaved" elements

b) <u>The Thin Lens:</u>



→ Sketch: Thin Lens for ABCD

We see $r_2 = r_1$

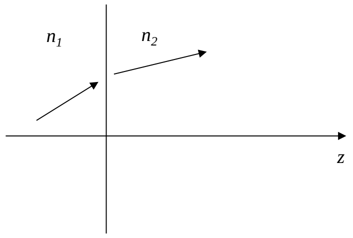
$\alpha_2 = -r_1/f + \alpha_1$ (paraxial)

$$M_{\mathrm{f}} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \tag{2.18}$$

→ components are not a function of r !

<u>Other Examples:</u>

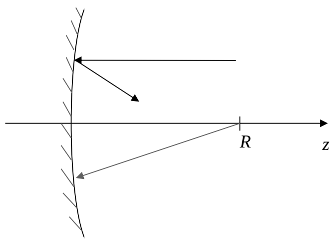
c) transition at a plane interface $n_1 \rightarrow n_2$



$$M_{\mathrm{ref}} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix} \tag{2.19}$$

→Sketch: refraction at an interface

d) Curved mirror, radius $R = 2f$



$$M_{\mathrm{M}} = \begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix} \tag{2.20}$$

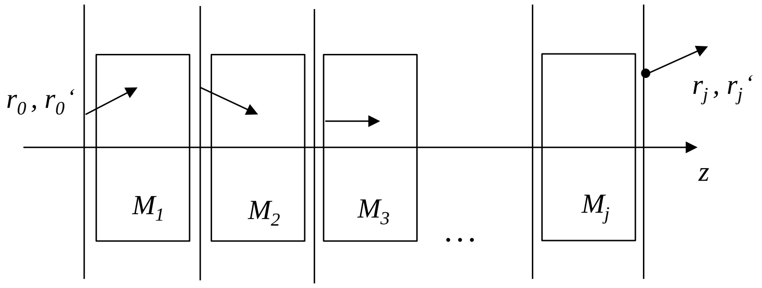
→ Sketch: curved mirror

e) Magnification change:

$$M_{\mathrm{m}} = \begin{pmatrix} m & 0 \\ 0 & \frac{1}{m} \end{pmatrix}$$

A number of other matrices for other elements can be found in the literature.

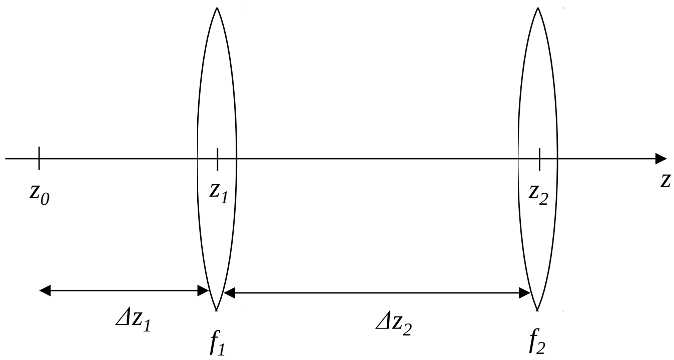
These matrices are descriptions for elementary units of complex optical systems.
The overall ABCD-matrix of a complex system can be found by multiplication (non-commutative) of the matrices of the single elements the system is composed of.



→ Sketch: System composed of multiple matrices.

$$M_{\text{sys}} = M_j \cdot \ldots \cdot M_3 \cdot M_2 \cdot M_1 \tag{2.21}$$

Example System: Microscope



→ Sketch: Telescope

$$\begin{aligned} M_{\text{sys}} &= \begin{pmatrix} 1 & 0 \\ -\dfrac{1}{f_2} & 1 \end{pmatrix} \begin{pmatrix} 1 & \Delta z_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\dfrac{1}{f_1} & 1 \end{pmatrix} \begin{pmatrix} 1 & \Delta z_1 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 - \dfrac{\Delta z_2}{f_1} & \Delta z_1 + \Delta z_2 - \dfrac{\Delta z_1 \Delta z_2}{f_1} \\ \dfrac{\Delta z_2}{f_1 f_2} - \dfrac{1}{f_1} - \dfrac{1}{f_2} & \left(1 - \dfrac{\Delta z_2}{f_2}\right)\left(1 - \dfrac{\Delta z_1}{f_1}\right) - \dfrac{\Delta z_1}{f_2} \end{pmatrix} \end{aligned} \tag{2.22}$$

2.3.2 General Properties of ABCD-Matrices

1) Mathematical property:
   Determinant of M:

$$|M| = AD - BC = \frac{n_1}{n_2} \tag{2.23}$$

n_1 … refractive index in start-region

n_2 … refractive index in end-region

→ A simple rule for checking the matrix!

→ Only **3** independent variables possible.

2) Equivalent optical System:
Systems having the same ABCD-matrix!
→ showing the same optical behavior

We can use this to decompose a given matrix into a "fixed" series of basic operations
→ constructing an <u>equivalent optical system:</u>

<u>Four elementary operations</u> for the equivalent ABCD-matrix:

- magnification change m
- change of index $n_1 \rightarrow n_2$
- thin lens of optical power $\Phi = 1/f$
- propagation of distance Δz

→ equivalent matrix:

$$M_{eq} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & \Delta z \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f \cdot n_2} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix} \begin{pmatrix} m & 0 \\ 0 & \frac{1}{m} \end{pmatrix} \tag{2.24}$$

(lens embedded in a medium of refr. index n_2)

→ 4 free variables describing the operations in terms of the given ABCD-matrix:

$$\frac{n_1}{n_2} = AD - BC \tag{2.25}$$

$$m = \frac{AD - BC}{D} \tag{2.26}$$

$$\frac{1}{f \cdot n_2} = -\frac{CD}{AD - BC} = -\frac{C}{m} \tag{2.27}$$

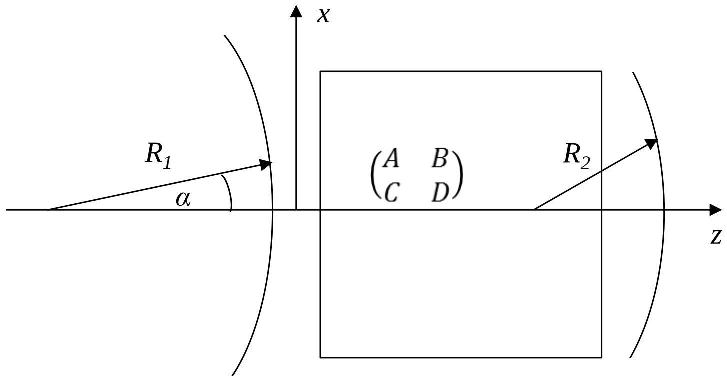
$$\Delta z = \frac{B}{D} \tag{2.28}$$

→ these operations can easily be applied also on arbitrary fields $\vec{E}$
→ not only a ray-based consideration!

Thus, if one can find an ABCD-matrix (coordinate independent components) for a given optical system, it is easy to propagate a field through the system!
Another use of this property is the Collins-Integral.

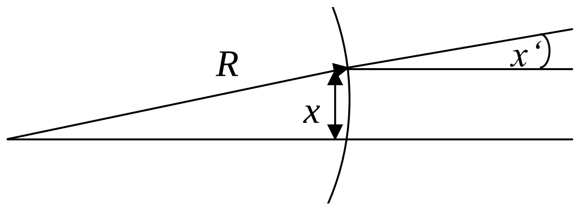
Note: If D=0 an other arrangement of the above sequence of four operations is useful
→ propagation to the exact focus point.

3) Transformation of a spherical wave:
Consider the illumination of the system with a spherical wave with radius of curvature R_1



→ Sketch: Spherical wave and ABCD-system

<u>Question:</u>



→ Sketch: Ray-angle and R

small angles: $\tan \alpha \approx \alpha \approx x/R$

→ $x_1 = R_1 \cdot \alpha_1$

$x_2 = R_2 \cdot \alpha_2$

Together with the definition of the ABCD-matrix from Eq. (2.16) and a bit algebra we get

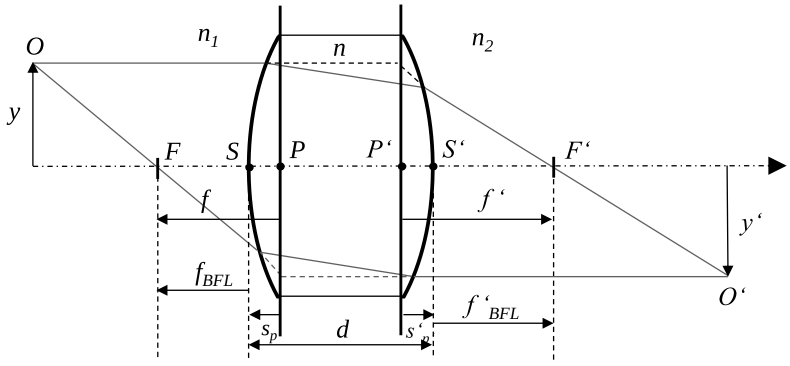
$$R_2 = \frac{AR_1 + B}{CR_1 + D} \tag{2.29}$$

Example: lens with focal length f

→ $\frac{1}{R_2} = \frac{1}{R_1} - \frac{1}{f}$

2.4 Real Lens

Again: let's sketch a lens



→ Sketch: Real lens and quantities

→ Refraction at 2 surfaces. How to define the focal length?
→ Introduction of the principle planes:

Incident ray parallel to axis intersects with the refracted ray in the <u>principle plane P'</u>
(and vice versa for <u>principle plane P</u>)

→ "Back focal length" f'_{BFL}

Refractive power of the lens is given by

$$\Phi = -\frac{n_1}{f} = \frac{n_2}{f'} \tag{2.30}$$

A lens is called "thin" if the radii of curvature of the lens are relatively large compared with the thickness.

<u>"Thin Lens":</u>

$$|c_{1/2} \cdot d| \ll 1 \tag{2.31}$$

Then the principle planes coincide
and $\qquad f' = f'_{BFL}$

If $\qquad n_1 = n_2 = 1 \qquad$ (lens in air)

$$\frac{1}{f'} = (n-1) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{2.32}$$

→ A simple formula for making a first rough estimation of the required $R_{1/2}$ for a given focal length f'.

more general (in air): if d is not negligible

$$\Phi = \frac{1}{f'} = (n-1) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{(n-1)^2 \cdot d}{n \cdot R_1 \cdot R_2} \tag{2.33}$$

Position of the principle planes w.r.t. the vertices:

$$s_P = -\frac{R_1 \cdot d}{(n-1)\cdot d + n\cdot(R_2 - R_1)}$$
$$s'_{P'} = -\frac{R_2 \cdot d}{(n-1)\cdot d + n\cdot(R_2 - R_1)} \qquad \textbf{(2.34)}$$

We have now the most necessary information required if we like to make
- a first rough estimation of important system parameters
- do a rough sketch of the ray path at a real lens
- or (most important) understand the image formation and the relevant parameters given in connection with a lens in a catalogue or in a lens-makers shop.

Doing more in a pure analytical way would require a huge amount of trigonometry and algebra and would not really bring us much further.
Therefore, we will have a more detailed view onto the important effects by using the optical design program ZEMAX instead.

→ **Program Experiments:** **(P)**

a) short introduction of the program philosophy

b) a spherical lens (bi- / plano-convex) EPD: 50mm
   → spherical aberrations R_1= - R_2=100
   → distortion diagram, spot diagram d=20

c) lens shape for ideal on-axis focusing
   → aspherical lens
   → behavior if illuminated off-axis

d) illumination with multiple wavelengths
   → chromatic aberrations

2.5 Optical Materials

We will restrict ourselves here to (isotropic) dielectric materials → transparent materials which interact with the light.
In general optical materials can be:
- Glasses
- Crystals
- Plastics
- Liquids
- Gases
- Glues and Cements

For the optical design the most important parameter is the refractive index $n(\lambda)$.
Sometimes also the absorption α is of interest → not here
In data sheets of glasses, the values for $n(\lambda)$ are typically given relative to air at normal conditions:

T = 293K p = 1013mbar

Other (sometimes) interesting properties are

dn / dT ...	thermooptic coefficient
...	thermal expansion

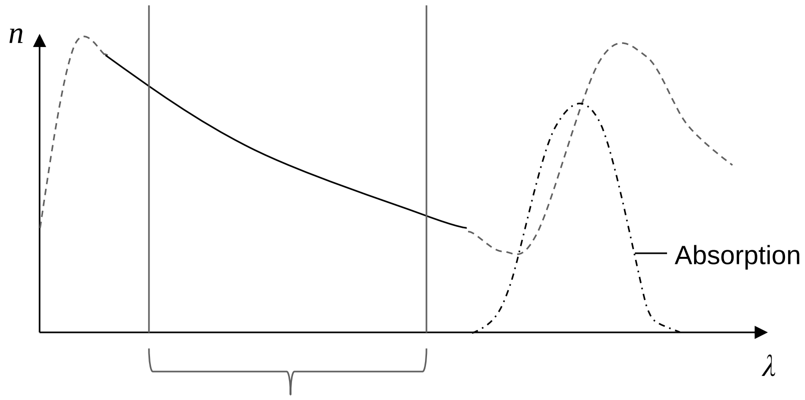
2.5.1 Treatment of dispersion in the optical design

<u>Dispersion:</u> dependence of $n = f(\lambda)$

For transparent materials / transmissive spectral region

$$\frac{dn}{d\lambda} < 0 \qquad (2.35)$$

→ normal dispersion



→ Sketch: typical dispersion curve

In the optical design the dispersion is often characterized by only three wavelengths:

- primary / main wavelength λ_o → in the center of the used spectral region
- two secondary colors λ_1, λ_2

VIS:	I)	λ_e = 546.07 nm	
		$\lambda_{F'}$ = 480.0 nm	Microscopy
		$\lambda_{C'}$ = 643.8 nm	
	II)	λ_d = 587.56 nm	
		λ_F = 486.1 nm	Photography
		λ_C = 656.3 nm	

However, there is a larger number of other wavelengths used, depending on the particular application.

→ see Slide (spectroscopic wavelengths) **(S)**

Characterization of the dispersion by a single number:

$$\nu = \frac{n_{\lambda_o} - 1}{n_{\lambda_1} - n_{\lambda_2}} \tag{2.36}$$

→ Abbe - number

or in particular

$$\nu_e = \frac{n_e - 1}{n_{F'} - n_{C'}} \quad ; \quad \nu_d = \frac{n_d - 1}{n_F - n_C} \tag{2.37}$$

Optical glasses:		ν_e = 20 ... 120
	ν small	→ large dispersion
	ν large	→ small dispersion

Other coefficient: dispersion number

$$m_e = \frac{1}{\nu_e} \tag{2.38}$$

One can now visualize the dispersion characteristics of different types of optical glasses by the so called glass-diagram.
This diagram shows each glass as a position in a diagram of the refractive index n and the Abbe-number.

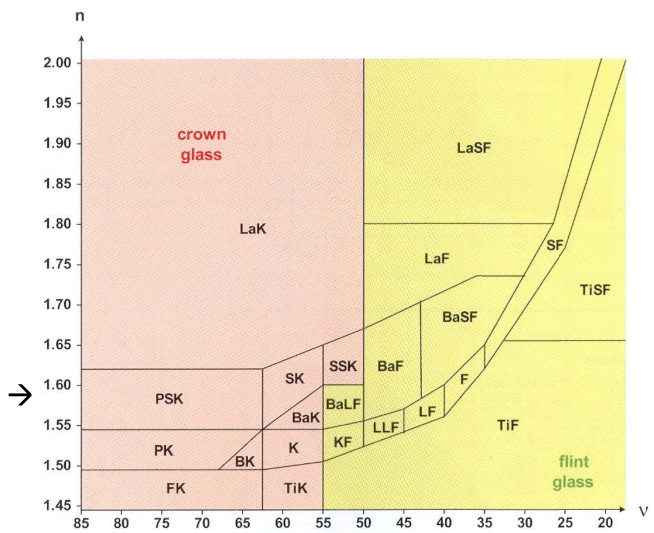


Figure 4-28: Glass diagram of Schott. Division into ranges of glass families.

→ see Slide (Abbe diagram) **(S)**

Rough classification:

n < 1.6	$\nu_e > 55$	crown glasses
	$\nu_e < 55$	flint glasses
n > 1.6	$\nu_e > 50$	crown glasses
	$\nu_e < 50$	flint glasses

→ historical distinction

→ see Slide (Crown / Flint glasses) **(S)**

2.5.2 Design of an achromatic lens

→ example of using different materials in an optical design

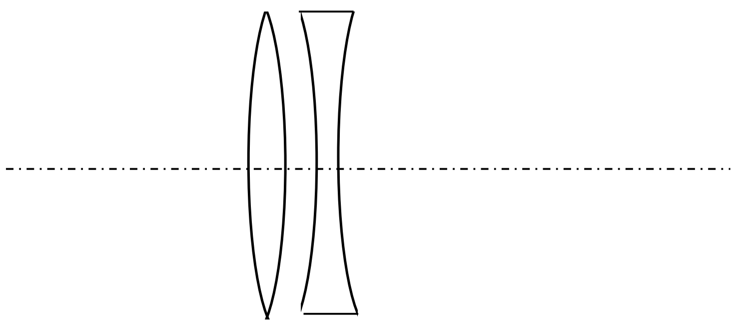
We will now try to calculate a lens / doublet, which is showing a considerable less chromatic aberration.
First, we recall the lens formula (thin lenses) Eq. (2.32)

$$\Phi = \frac{1}{f'} = (n-1) \cdot \underbrace{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}_{\substack{\text{- only geometrical parameters} \\ \text{- do not change with } \lambda}}$$

→
$$\Phi = (n-1) \cdot A \tag{2.39}$$

For a combination of two lenses:

$$\Phi_{\mathrm{sum}} = \Phi_1 + \Phi_2 = (n_1 - 1) \cdot A_1 + (n_2 - 1) \cdot A_2 \tag{2.40}$$



→ Sketch: Doublet

Central wavelength: λ_e

$$\begin{aligned} \bar{\Phi}_1 &= (n_{1e} - 1) \cdot A_1 \\ \bar{\Phi}_2 &= (n_{2e} - 1) \cdot A_2 \end{aligned} \tag{2.41}$$

$$\Phi_{\mathrm{sum}}(\lambda) = [n_1(\lambda) - 1] \cdot A_1 + [n_2(\lambda) - 1] \cdot A_2 \tag{2.42}$$

Design goal: <u>same refractive power</u> for 2 wavelengths near the border of the spectral region:

$$\Phi_{\mathrm{sum}}(\lambda_{F'})\ ! = \Phi_{\mathrm{sum}}(\lambda_{C'}) \tag{2.43}$$

With Eq. (2.42) we get for the doublet the condition:

$$(n_{1F'} - 1) \cdot A_1 + (n_{2F'} - 1) \cdot A_2\ ! = \ (n_{1C'} - 1) \cdot A_1 + (n_{2C'} - 1) \cdot A_2 \tag{2.44}$$

Replacing A_1 and A_2 with the average refractive power from Eq. (2.41) and rearranging the terms gives

$$\underbrace{\frac{n_{1F'} - n_{1C'}}{n_{1e} - 1} \bar{\Phi}_1}_{\frac{1}{\nu_1}\bar{\Phi}_1} = - \underbrace{\frac{n_{2F'} - n_{2C'}}{n_{2e} - 1} \bar{\Phi}_2}_{\frac{1}{\nu_2}\bar{\Phi}_2} \tag{2.45}$$

$$\Rightarrow \qquad \frac{\overline{\Phi}_1}{\nu_1} + \frac{\overline{\Phi}_2}{\nu_2} = 0 \qquad (2.46)$$

Condition of achromasie

Together with Eq. (2.40) we have now a system of two linear equations for calculating the refractive powers of $\overline{\Phi}_1$ and $\overline{\Phi}_2$ out of Φ_{sum} , ν_1 and ν_2:

$$\overline{\Phi}_1 = \frac{\nu_1}{\nu_1 - \nu_2}\Phi_{sum}$$
$$\overline{\Phi}_2 = \frac{\nu_2}{\nu_2 - \nu_1}\Phi_{sum} \qquad (2.47)$$

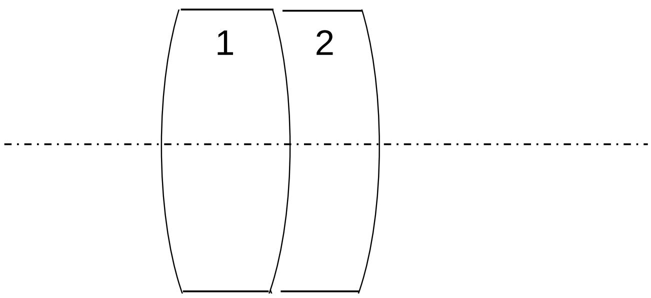
→ An achromatic doublet is always composed of a positive and a negative lens.

Example: We need two glasses with considerably distinct Abbe-numbers
→ out of the glass diagram **(S)**

eg. BK7 n_e = 1.518
ν_e = 63.9
SF6 n_e = 1.805
ν_e = 25.4

The lens combination should have a focal length of f' = 200 mm

→ with Eq. (2.47): f_1' = 120.5 mm f_2' = -303.1 mm

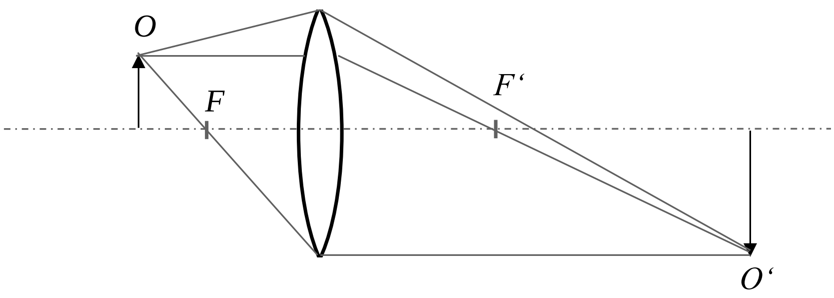


Lens 1: assume a symmetric positive lens with $R_1 = -R_2$
→ R_1 = 124.8 mm
Lens 2: R_1 = - 124.8 mm
R_2 = - 255.5 mm

→ **Program Experiment:** **(P)**
EPD: 50 mm
d_1= 10 mm
d_2= 5 mm → d_3= 194.9 mm

2.6 Imaging Systems

We already know the basic imaging set-up consisting of a single lens:

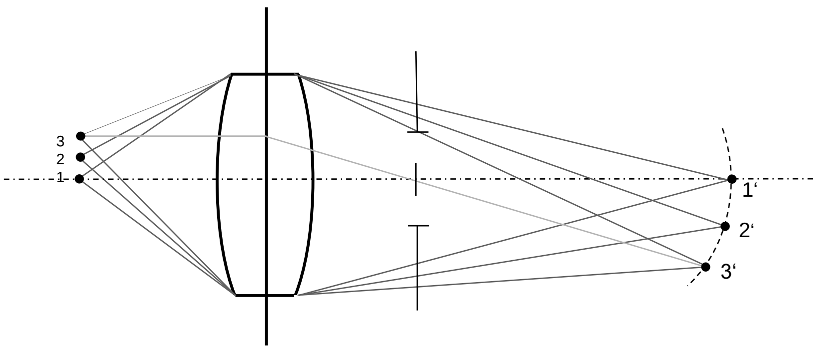


→ Sketch: Simple 1-lens imaging

- magnified image → Microscope
- inverse arrangement: de-magnified image → Camera

However, real imaging is more complex than considering only one object point:

→ **Program Experiment:** set-up in Zemax **(P)**
- different field points



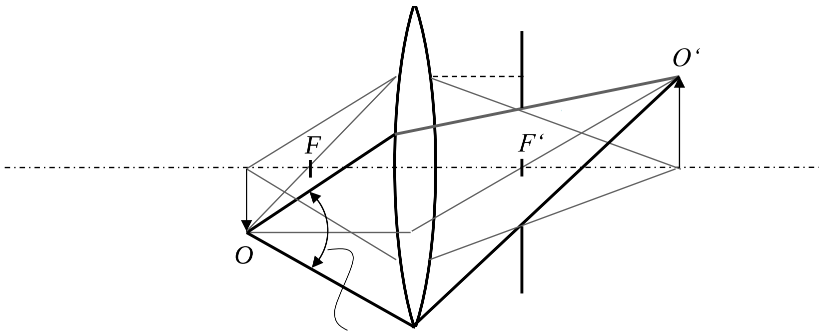
→ Sketch: Image with different field points -> field curvature

Field curvature is a special aberration which always occurs if only a single lens is used for the imaging.
→ We will consider this later together with other aberrations.

Before we will stick to another important element of each imaging system which has a very important influence on the image quality: the stop / aperture / diaphragm

In the above image the ray-cones are only limited by the aperture of the lens.
One can of curse also add a separate stop which limits the ray–cone.

One special example: we locate the stop in the back focal plane of the imaging lens.



symmetrical cone around the ray parallel to the optical axis!

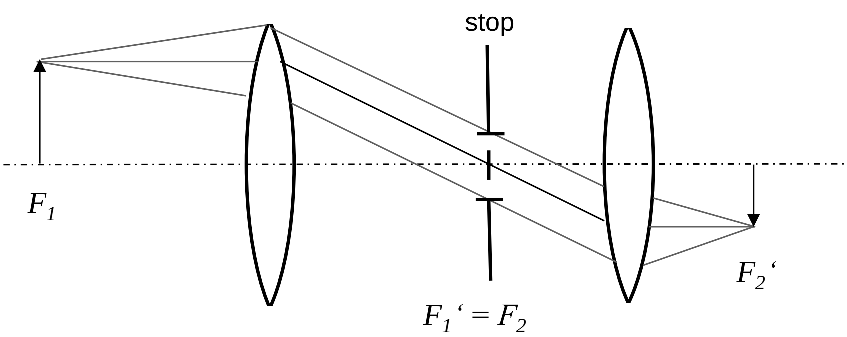
→ Sketch: Imaging with a stop at F'

→ symmetrical cone around the ray parallel to the optical axis!
→ this is true for each object point!
→ Property: object side telecentric

This property of the optical system is useful e.g. for measurement tasks:
if the image plane is fixed at the position O' a change of the object distance would only lead to a defocus but the lateral magnification of the image remains constant!

Placing the stop in the front focal plane F would give an image side telecentric system.

If one uses a 4*f*-set-up for the imaging and places the stop at the common intermediate focus the system becomes both side telecentric.

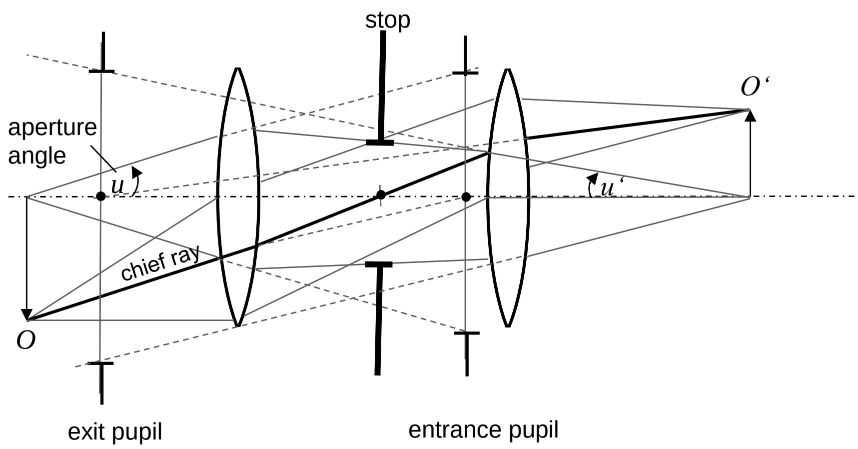


→ Sketch: 4f-set-up with intermediate stop

→ good for systems with constant magnification in case of changing image and object distances

We can see that the aperture can have an important influence on the imaging properties of the system. But we did not yet see all!
The impact is much stronger as the stop may also have a huge impact on the aberrations(!) and other important parameters like the resolution.
Therefore, we will introduce some general terms connected with the aperture.

Let's consider a "general" two-lens imaging system.



→ Sketch: general two-lens imaging system with stop

chief ray: ray from an object point which goes through the center of the stop

With the help of the chief ray one can construct the position where the image of the stop will be located as seen either from the object or the image position.

→ simply extend the chief ray to the point where it crosses the optical axis

entrance pupil: stop, as seen from the object **(EN)**
exit pupil: stop, as seen from the image **(EX)**

→ both are images of the same limiting aperture !

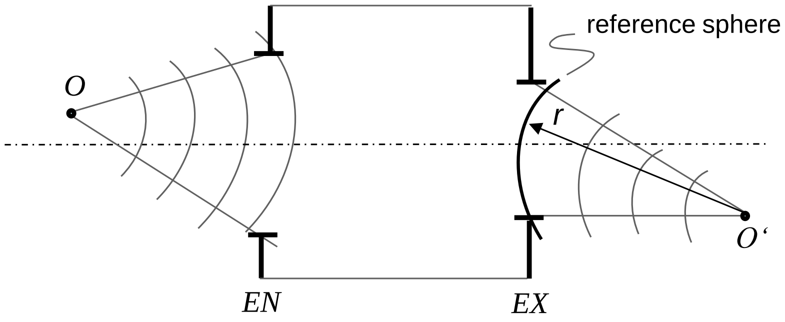
Attention: Except for a 1-lens "system" the size and shape of the pupil depend on the field point. On-axis field points typically have a different pupil shape than field points near the border of the FOV

→ **Program Example:** Zemax: "Cooke 40 degree field_Stop Size" **(P)**

→ this fact needs to be considered in calculations like
- transmission
- resolution (-limit)
- MTF, etc.

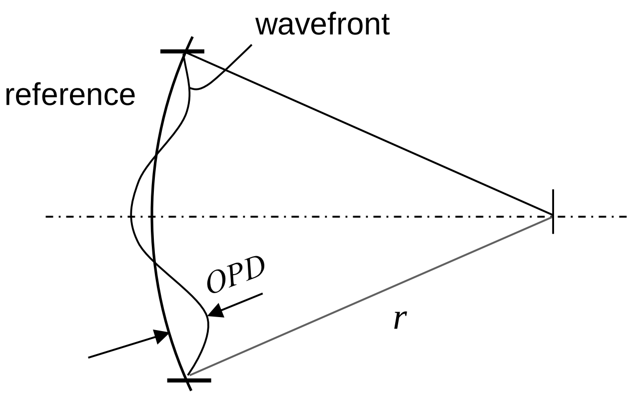
What is the main feature of the entrance – exit pupil concept?
It provides an easy way to connect the computational elegance of the ray-based propagation through (arbitrarily) complex imaging systems with a wave optical analysis of the imaging properties of the system!

We know: Imaging:
transformation of a spherical wave from an object point into an other spherical wave generating the corresponding image point



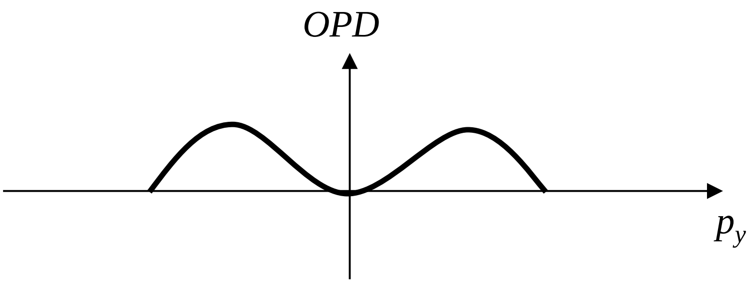
→ Sketch: Spherical wave imaging with EN, EX → reference sphere

Reference sphere: → located in the exit pupil
→ represents the spherical wave for ideal imaging, center: ideal image point
→ lateral extension gives resolution limit



→ Sketch: Reference sphere in exit pupil

Real wave-front: can be calculated by tracing rays from the EN to the EX and plot the actual optical path length for each pupil position
(without the chief ray path length)



→ Sketch: OPD-plot

OPD: gives the <u>difference</u> between the ideal <u>reference sphere</u> and the <u>real wave-front</u>
→ represents the aberrations introduced by the optical system

The field in the focus of the optics can not be calculated by ray-tracing!
→ wave-optical propagation methods need to be applied.

As we defined the OPD on a sphere (not a plane!) in the exit pupil the propagation into the focus can easily computed by

$$\vec{E}(\omega; x, y, z_f) = \frac{\omega}{\mathrm{i}2\pi c f} \mathrm{e}^{\mathrm{i}kf} \cdot \mathrm{e}^{\mathrm{i}k\frac{\sqrt{x^2+y^2}}{2f}} \cdot \mathrm{FT}\{\vec{E}(\omega; p_x, p_y, z_{EX})\} \tag{2.48}$$

f … focal length (distance from EX to z_f)

→ Fourier transformation (evaluated at $\left(\frac{2\pi}{\lambda f}p_x, \frac{2\pi}{\lambda f}p_y\right)$) + spherical phase factor

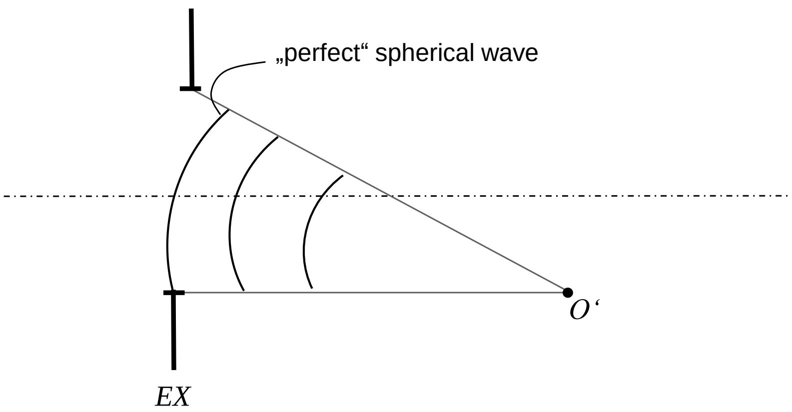
→ also valid for high NA optics (only very weak approximations ← small off-axis distance from ideal focal point)

→ discussion of resolution, introduction of NA

2.7 Aberrations

Aberrations are image imperfections.

Ideal imaging: All image points are generated by a perfect spherical wave.
Then the resolution of the image is only determined by the angular extend / NA of this wave.
→ <u>"diffraction limited imaging"</u>



→ Sketch: "perfect" spherical wave

In real lens systems the wave-front deviates from this condition.

<u>**Lens design = aberration balancing**</u>

Design goals:

- high resolution

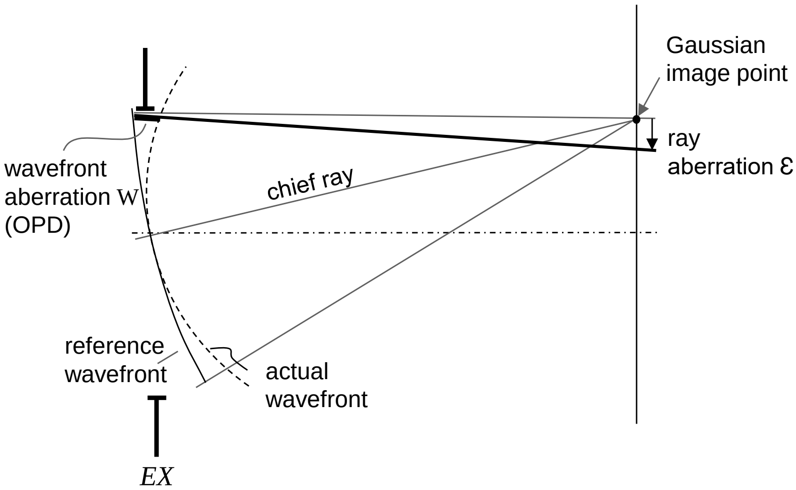
- high image contrast
- homogenous illumination
- similarity to object

Chromatic aberrations → we already know from previous discussion
→ caused by different focusing powers for different wavelengths, "chromatic defocusing"
→ balancing with different lens materials

<u>Here:</u> monochromatic aberrations
→ caused by the higher terms in the description of the law of refraction and the surface profiles (deviation from the paraxial region)

<u>Visualization of aberrations:</u>
→ "wave-front deformation", <u>OPD</u>
→ can also be considered as <u>ray aberrations</u> → ray stays perpendicular on the wave-front
→ rays do not meet in a single point but spread out.
→ ray fan plot & spot diagram

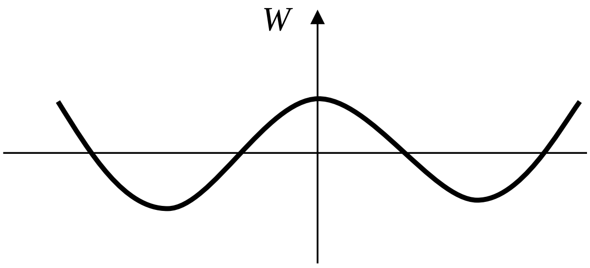
Let us consider again the wave-front in the exit pupil:



→ Sketch: Wave-front in exit pupil, wave-front/ray aberration

$$\varepsilon_y \sim \frac{\partial W}{\partial y}\,;\, \varepsilon_x \sim \frac{\partial W}{\partial x} \qquad (2.49)$$

Diagram:



→ Sketch: wave-front aberration function

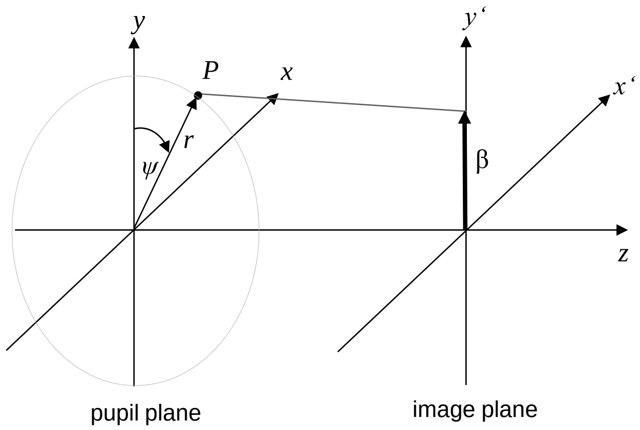
W ... wave-front aberration function

$W = W(\beta, r, \psi)$

β ... normalized field height

r ... normalized pupil height (of point P)

ψ ... azimuth of point P



→ Sketch: pupil plane / image plane

Expansion of W with respect to orders of β, r, $\cos\psi$:

$$W(\beta, r, \psi) = \underbrace{W_{000}}_{\text{Piston Error}}$$

$$+ \underbrace{W_{200} \cdot \beta^2}_{\text{Piston error}} + \underbrace{W_{020} \cdot r^2}_{\text{Defocus}} + \underbrace{W_{111} \cdot \beta \cdot r \cdot \cos\psi}_{\text{Lateral Magnification Error}}$$

$$+ \underbrace{W_{400} \cdot \beta^4}_{\text{Piston Error}} + \underbrace{W_{040} \cdot r^4}_{\text{SA}} + \underbrace{W_{131} \cdot \beta \cdot r^3 \cdot \cos\psi}_{\text{Coma}} + \underbrace{W_{222} \cdot \beta^2 \cdot r^2 \cdot \cos^2\psi}_{\text{Astigmatism}} \qquad (2.50)$$

$$+ \underbrace{W_{220} \cdot \beta^2 \cdot r^2}_{\text{Field Curvature}} + \underbrace{W_{311} \cdot \beta^3 \cdot r \cdot \cos\psi}_{\text{Distortion}}$$

+ ... aberrations of higher order

→ **5 classical Seidel Aberrations**

Philipp Ludwig Ritter von Seidel (1821 – 1896)

First order aberrations: - Defocus **(S)**

- Lateral Magnification Error

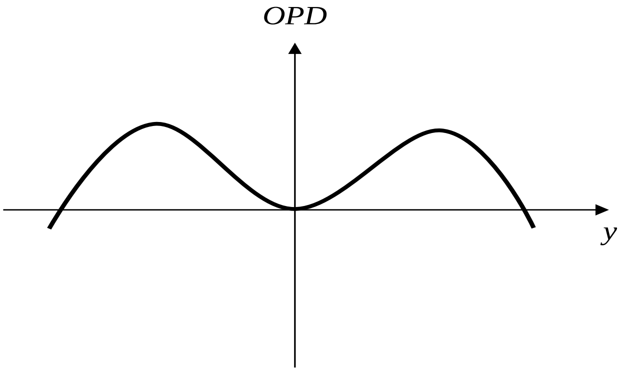
2.7.1 Spherical aberration $\sim r^4$

→ Occurs when rays passing the aperture far from the optical axis have focal lengths different from the paraxial rays

→ $f = f(r)$

→ Slide: spherical aberration **(S)**

Spherical aberration in the OPD-diagram:



→ Sketch: OPD for spherical aberrations

Getting rid of spherical aberrations:

- balancing with defocus → **(S)**
- bending the lens → **(S)**
- splitting the lens
- increasing the refractive index
- aspheric lens

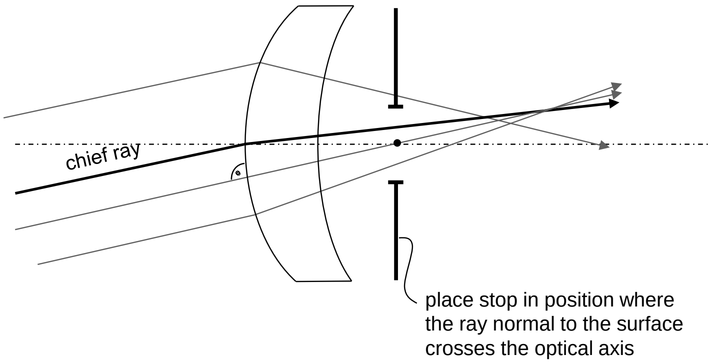
2.7.2 Coma $\sim \beta \cdot r^3 \cdot \cos\psi$

→ Occurs for ray bundles whose chief ray is not symmetric with respect to the optical axis. "non-symmetry error"

→ Slide: coma **(S)**

→ different bending of rays in upper and lower part of the ray bundle
→ stronger bending results in larger influence of higher order terms in the law of refraction

Getting rid of coma:

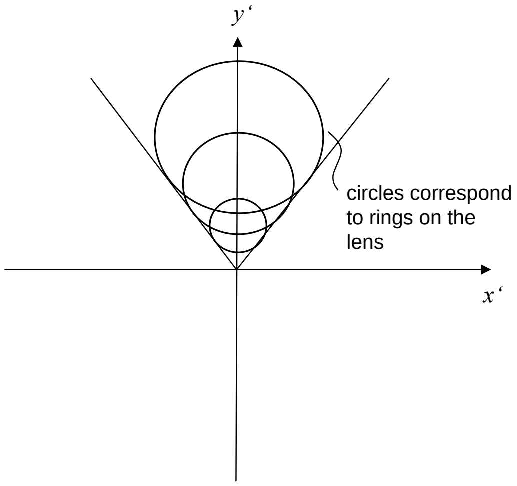


→ Sketch: tilted ray-bundle through lens

→ place the stop in the position where the ray which is normal to the surface crosses the optical axis.

→ move stop
→ make the ray bundle symmetric to this surface-perpendicular ray

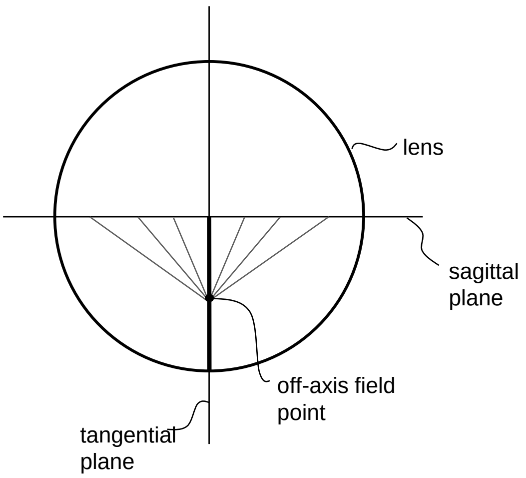
Coma figure:



→ Sketch: Coma figure

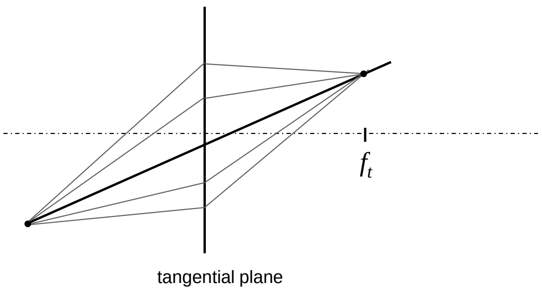
2.7.3 Astigmatism $\sim \beta^2 \cdot r^2 \cdot \cos^2 \psi$

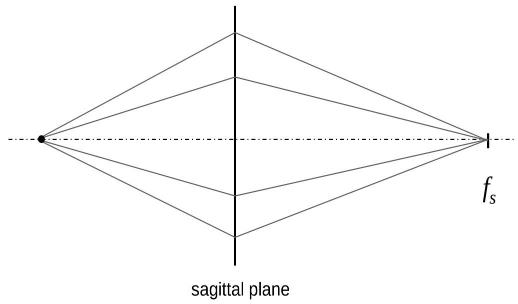
→ Occurs for off-axis field points
→ Ray bundle passes the lens asymmetrical and the sagittal and tangential rays do experience different radii of curvature of the lens surface



→ Sketch: View along the optical axis

side view: top view:





→ Sketch: side view / top view

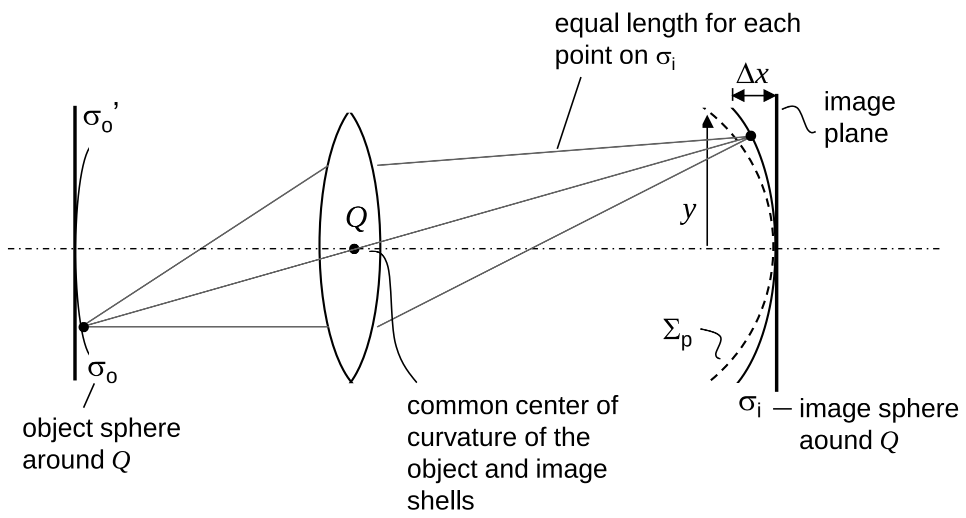
$$f_t < f_s$$

→ Slide: Astigmatism **(S)**

2.7.4 Field Curvature $\sim \beta^2 \cdot r^2$

→ Occurs as the natural image surface is spherical, not planar
"Petzval curvature" Hungarian Mathematician Josef Max Petzval (1807 – 1891)

A spherical object is imaged by a spherical lens onto a spherical surface:



→ Sketch: Field curvature

If we now flatten σ_o to σ_o' the corresponding image points move <u>towards</u> the lens → onto the so called <u>Petzval surface</u> Σ_p

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(*) Distance of image point from image plane:

$$\Delta x = \frac{y}{2} \cdot \underbrace{\sum_{j=1}^{m} \frac{1}{n_j \cdot f_j}}_{\text{Petzval-sum}}$$

n_j , f_j ... focal length and refractive index of lens j in a system

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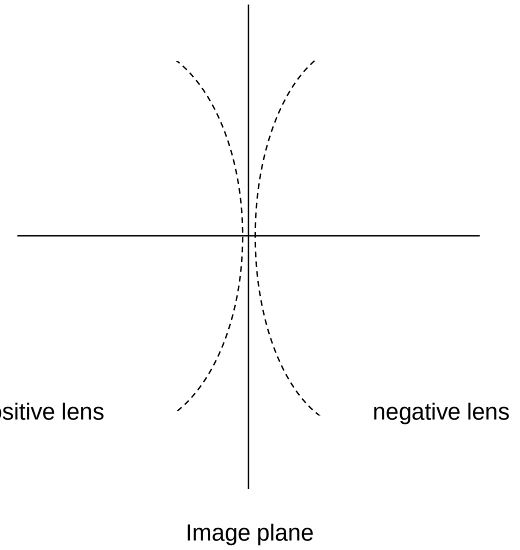
For a positive lens: negative radius of the Petzval surface
negative lens: positive radius ----"----



→ Sketch: Petzval curvature for pos./neg. lens

→ A proper combination of positive and negative lenses in a system can be used to compensate the field curvature.
Example: Cooke Triplet

2.7.5 Distortion $\sim \beta^3 \cdot r \cdot \cos\psi$

→ Deformation of the image scale due to different transversal magnifications for each field point.

<u>Reason:</u> spherical aberration of the chief ray

→ Slide: Distortion **(S)**

→ Distortion is a function of the stop position!