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Problem 1 3/3

(a) 1/1

$$F_L(\nu) = \frac{C_1}{(\nu-\nu_{21})^2+(B_1\Delta\nu)^2}$$

when $\nu = \nu_{21}$, $F_L(\nu)$ equals to the maximum value.

suppose when $\nu=\nu'$, $F_L(\nu') = \frac{1}{2}F_L(\nu_{21})$

$\nu' - \nu_{21} = \pm\frac{1}{2}\Delta\nu$

$$\therefore \frac{\frac{C_1}{\frac{1}{4}\Delta\nu^2+(B_1\Delta\nu)^2}}{\frac{C_1}{(B_1\Delta\nu)^2}} = \frac{1}{2}$$

$$2B_1^2 = \frac{1}{4}+B_1^2$$

$$B_1 = \frac{1}{2}$$ ✓

$$F_G(\nu) = C_2\exp\left[-B_2\frac{(\nu-\nu_{21})^2}{\Delta\nu^2}\right]$$

when $\nu=\nu_{21}$, $F_G(\nu)$ equals to the maximum value

suppose when $\nu=\nu'$, $F_G(\nu') = \frac{1}{2}F_G(\nu_{21})$

$\nu' - \nu_{21} = \pm\frac{1}{2}\Delta\nu$

$$\therefore \frac{C_2\exp\left[-B_2\frac{\frac{1}{4}\Delta\nu^2}{\Delta\nu^2}\right]}{C_2} = \frac{1}{2}$$

$$\therefore \exp\left[-\frac{B_2}{4}\right] = \frac{1}{2}$$

$$-\frac{B_2}{4} = -\ln 2$$

$$B_2 = 4\ln 2$$ ✓

(b) 1/1

$$\because F_L(\nu_{21}) = \frac{C_1}{(B_1\Delta\nu)^2} = 1$$

$$\because B_1 = \frac{1}{2} \quad \therefore C_1 = \frac{\Delta\nu^2}{4}$$ ✓

$$\because F_G(\nu_{21}) = C_2 = 1 \quad \therefore C_2 = 1$$ ✓

(c) 1/1

$$\int_{-\infty}^{\infty} F_L(\nu)\,d\nu = 1$$

$$\int_{-\infty}^{\infty}\frac{C_1}{(\nu-\nu_{21})^2+(B_1\Delta\nu)^2}\,d\nu = 1$$

$$\int_{-\infty}^{\infty}\frac{C_1}{(\nu-\nu_{21})^2+(\frac{\Delta\nu}{2})^2}\,d\nu \qquad (\nu-\nu_{21})\cdot\frac{2}{\Delta\nu} = t$$

$$= C_1\int_{-\infty}^{\infty}\left(\frac{2}{\Delta\nu}\right)^2\frac{1}{t^2+1}\,dt\,\frac{\Delta\nu}{2}$$

$$= C_1\cdot\frac{2}{\Delta\nu}\arctan\left[(\nu-\nu_{21})\cdot\frac{2}{\Delta\nu}\right]\Bigg|_{-\infty}^{\infty}$$

$$= C_1\cdot\frac{2}{\Delta\nu}\cdot\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = 1$$

$$\therefore C_1 = \frac{\Delta\nu}{2\pi}$$ ✓

$$\int_{-\infty}^{\infty} F_G(\nu)\,d\nu = 1$$

$$\int_{-\infty}^{\infty} C_2\exp\left[-B_2\frac{(\nu-\nu_{21})^2}{\Delta\nu}\right]d\nu = 1$$

$$\int_{-\infty}^{\infty} C_2\exp\left[-4\ln 2\cdot\frac{(\nu-\nu_{21})^2}{\Delta\nu^2}\right]d\nu$$

$$= C_2\sqrt{2\pi}\frac{\Delta\nu}{\sqrt{8\ln 2}}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\cdot\frac{\Delta\nu}{\sqrt{8\ln 2}}}\exp\left(-\frac{(\nu-\nu_{21})^2}{2(\frac{\Delta\nu}{\sqrt{8\ln 2}})^2}\right)d\nu$$

$$\therefore C_2 = \frac{2}{\Delta\nu}\sqrt{\frac{\ln 2}{\pi}}$$ ✓

Problem 2 Jinsong Liu 206216

3.5/4

(a)

suppose the radius of the circular aperture is a

$x^2+y^2=r^2$

$$T=\frac{P_a}{P_\infty}=\frac{\int_0^a\int_0^{2\pi}\frac{2}{\pi W^2}\exp(-2\frac{r^2}{W^2})\,2\pi r\,dr\,d\theta}{\int_0^\infty\int_0^{2\pi}\frac{2}{\pi W^2}\exp(-2\frac{r^2}{W^2})\,2\pi r\,dr\,d\theta}=1-\exp(-\frac{2a^2}{W^2})=0.99$$

1/1

$\therefore\ a\approx 1.52W$

d ?

(b)

$$I_p=P_p\cdot\frac{2}{\pi W^2}$$

$$=\frac{2}{\pi\times(5\times10^{-6})^2}\times10^{15}\ W/m^2$$

$$=2.55\times10^{25}\ W/m^2$$

1/1

$\because\ I=\frac{1}{2}c\varepsilon_0E^2$

$$E=\sqrt{\frac{2I}{c\varepsilon_0}}=\sqrt{\frac{2\times2.55\times10^{25}}{3\times10^8\times8.854\times10^{-12}}}\ V/m=1.39\times10^{14}\ V/m$$

(c)

$$E_{pulse} = \int_{-\infty}^{\infty} P(t)\, dt = \int_{-\infty}^{\infty} P_p \exp\left[-4\ln 2 \left(\frac{t}{\tau}\right)^2\right] dt$$

$$= 25 \times \sqrt{\frac{\pi}{4\ln 2}}\ J$$

$$\approx 25 \times 1.064\ J = 26.6\ J$$

1/1

$$P_{avg} = E_{pulse} \cdot f = 26.6\ W$$

(d)

We can use a photodiode to measure the average power, then we can calculate the peak power

0.5/1

Problem 3

(a)

$$\frac{dN_2}{dt} = -A_{21} \cdot N_2 - B_{21}\,\rho(\nu)\,N_2 + B_{12}\,\rho(\nu)\,N_1$$

1/1

(b)

$\because$ thermal equilibrium and that the atoms are enclosed in a conducting cavity

$$\therefore \quad \cancel{dN_2} \quad \frac{dN_2}{dt} = 0 \qquad \frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-\frac{h\nu}{k_B T}} \quad ①$$

From Planck's formula, we can get

$$\rho(\nu) = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{h\nu/k_B T} - 1} \quad ②$$

2/2

$$\therefore \quad A_{21} \cdot N_2 = -B_{21}\,\rho(\nu) N_2 + B_{12}\,\rho(\nu)\,N_1 \quad ③$$

$$\therefore \quad A_{21} \cdot N_2 = N_2 \frac{8\pi h\nu^3}{c^3} \left[e^{\frac{h\nu}{k_B T}} - 1\right]^{-1} \cdot \left[B_{12} \frac{g_1}{g_2} e^{\frac{h\nu}{k_B T}} - B_{21}\right]$$

(c)

From the equation ③ $A_{21} \cdot N_2 = -B_{21}\,\rho(\nu) N_2 + B_{12}\,\rho(\nu)\,N_1$

and the equation ① $\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-\frac{h\nu}{k_B T}}$

we can get

$$\rho(\nu) = \frac{A_{21}/B_{21}}{\frac{g_1}{g_2}\frac{B_{12}}{B_{21}} e^{h\nu/k_B T} - 1} \quad ④$$

2/2

compare to the equation ②

we can get

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \qquad A_{21} = \frac{8\pi h\nu^3}{c^3} B_{21}$$

$$\frac{g_1}{g_2}\frac{B_{12}}{B_{21}} = 1 \qquad B_{21} = \frac{g_1}{g_2} B_{12}$$

Problem 3

(d)

$$B_{21} = \frac{g_1}{g_2} B_{12}$$

physical meaning: For the same degeneracy factors ($g_1 = g_2$), the ~~prob~~ probability of ~~..~~ stimulated emission and absorption are ~~..~~ equal. The ratio of B_{21} and B_{12} depends on the degeneration of two levels.

$$A_{21} = \frac{8\pi h\nu^3}{c^3} B_{21}$$

physical meaning: the number of modes per unit volume at frequency ν is $n(\nu) = \frac{8\pi\nu^2}{c^3}$

$$\therefore \frac{A_{21}}{n(\nu)} = B_{21} h\nu$$

which means the probability of spontaneous emission in each mode is equal to the proability of stimulated emission induced by a photon.

For a 2-level system with $g_2 = 2$ and $g_1 = 1$ when $T \to \infty$ $\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-\frac{h\nu}{k_B T}} = \frac{g_2}{g_1} = 2$

$\therefore$ inversion is possible

$$\because \frac{\left.\frac{dN_2}{dt}\right|_{induced}}{\left.\frac{dN_1}{dt}\right|_{absorption}} = \frac{B_{21}}{B_{12}} \cdot \frac{N_2}{N_1} = \frac{g_1}{g_2} \frac{g_2}{g_1} e^{-\frac{h\nu}{k_B T}} = e^{-\frac{h\nu}{k_B T}} < 1$$

$$\therefore \left.\frac{dN_2}{dt}\right|_{induced} < \left.\frac{dN_1}{dt}\right|_{absorption}$$

$\therefore$ gain is impossible.

2/2