

$$I_{\text{peak}} = \frac{I}{T}$$

## Problem Set 8: Test Exam

The only help allowed is a calculator. All the required formulas and constants are given. You can obtain a maximum of 35 exam points. Good Luck!! Please note that to calculate the points that you obtain in this problem set, the number of exam points that you get will be divided by 2, i.e. you can get a maximum of 17.5 points.

### Problem 1 (4 points)

- a) What does the acronym "Laser" stand for? Explain why it is inappropriate for describing the laser operation. (1 point)
- b) Sketch a generic laser highlighting its main components and briefly explaining their function. (2 points)
- c) Explain why a laser cannot be a system in thermodynamic equilibrium. (1 point)

### Problem 2 (8 points)

- a) In a laser in steady-state, which must be the relationship between gain and losses? Explain why. (2 points)
- b) Which is the condition that gain and losses must fulfill to enable the start of laser operation? (1 point)
- c) Why are the conditions of a) and b) different? Which mechanism has been implicitly considered? (1 point)
- d) If we have an active cylindric volume of length  $L_A$ , design a confocal resonator that uses the active medium efficiently (i.e. calculate the resonator length  $L$  and the radius of curvature of the mirrors  $R_1$  and  $R_2$ ). (4 points)

$$L = R \quad (G \cdot \left( \frac{1}{R_1} + \frac{1}{R_2} \right)) \quad f = R_1/2 \quad L = R_1/2 + R_2/2$$

### Problem 3 (6 points)

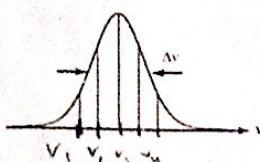
- a) Describe the difference between homogeneous and inhomogeneous broadening and name one physical origin for each. (2 points)
- b) Explain what ASE is and name two methods to reduce it. (2 points)
- c) Assume two active media with the same gain characteristics (e.g. bandwidth, spectral shape, etc.). One of these media is homogeneously broadened and the other one inhomogeneously broadened. If we want to amplify a signal much narrower than the gain bandwidth, which of these amplifiers will exhibit less ASE at the output? Why? (2 points)

### Problem 4 (2 points)

Explain what gain narrowing is and name one example when it is advantageous and another one where it is a drawback.

### Problem 5 (6 points)

We have three lasers mode locked with a saturable absorber mirror and emitting 1 Watt average power. A generic spectrum of any one of these lasers is



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# Continuity & 2D<sup>10</sup> exercise sheet

The laser parameters are as follows ( $\Delta\nu$  FWHM width,  $L_R$  resonator length):

Laser 1	$\Delta\nu = 1.7 \text{ THz}$	$L_R = 1 \text{ m}$	$75 \cdot 10^6$
Laser 2	$\Delta\nu = 1.7 \text{ THz}$	$L_R = 15 \text{ m}$	$10 \cdot 10^6$
Laser 3	$\Delta\nu = 500 \text{ GHz}$	$L_R = 3 \text{ m}$	$50 \cdot 10^6$

- a) Assuming that the active medium has an index of refraction of  $n=1$ , calculate the frequency separation between the longitudinal modes for the three lasers above. (2 points)
- b) Assuming that the spectral envelopes are Gaussian, calculate the pulse duration, repetition frequency and pulse energy of the laser outputs! (4 points)

### Problem 6 (5 points)

The Schawlow-Townes formula describes the fundamental limit for the emission bandwidth  $\Delta\nu$  of a continuous-wave (cw) laser:

$$\Delta\nu = \frac{2\pi h\nu(\delta\nu_R)^2}{P}$$

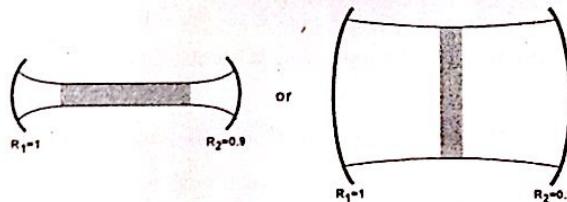
- a) Assuming that the following 2 single longitudinal mode lasers are only limited in bandwidth by the Shawlow-Townes limit, then calculate their respective coherence length. (2 points)

Laser 1	$R_1 = 1$	$R_2 = 0.9$	$n_{Nd:YAG} = 1.82$	$L = 0.5 \text{ m}$	$P = 0.5 \text{ W}$
Laser 2	$R_1 = 0.9$	$R_2 = 0.3$	$n_{GaAs} = 3.5$	$L = 500 \mu\text{m}$	$P = 5 \text{ mW}$

- b) Explain what spatial and temporal coherence are. (2 points)
- c) Why is it important to have high spatial coherence? (1 point)

### Problem 7 (4 points)

Consider an active rectangular block used to build a laser. We use the block in two configurations:



- a) Sketch the laser output power and inversion diagrams for a general laser as a function of the pump power (assume steady state). Name two ways to increase the slope efficiency and reduce the laser threshold? (2 points)
- b) Assuming homogeneous inversion in the active block, which configuration will have a higher laser threshold? Why? (2 points)

Physical constants and units:

$$h = 6.626 \cdot 10^{-34} \text{ Js}$$

$$c = 2.998 \cdot 10^8 \text{ m/s}$$

$$e = 1.602 \cdot 10^{-19} \text{ C}$$

$$\epsilon_0 = 8.854 \cdot 10^{-12} \text{ F/m}$$

$$\tau_{ph} = -\tau_R / \ln(R_1 R_2)$$

Helpful formulas:

$$\int_{-\infty}^{\infty} \exp(-a^2 x^2) dx = \frac{\sqrt{\pi}}{a}$$

$$W_{12} = W_{21} = I_S \sigma / (h\nu)$$

$$FT(e^{-at^2}) = \sqrt{(\pi/a)} \cdot e^{-\pi^2 \nu^2 / a}$$

$$z_R^2 = g_1 g_2 (1 - g_1 g_2) \frac{L^2}{G^2}$$

$$G = g_1 + g_2 - 2g_1 g_2$$

Evaporation temperature $T_v$	1400 °C
Thermal conductivity $k$	40 W/m · K
Specific heat capacity $C_s$	0.9 kJ/kg · K
Density $\rho$	7.90 g/cm³
Absorptivity $A$	0.378

In the case of one-dimensional heat evolution with the thermal penetration depth  $l_{th} = 2\sqrt{D \cdot t}$  the temperature distribution is given by:

$$T(z, t) = \frac{A \cdot I_0}{k} l_{th} \cdot \text{i erf} \left( \frac{z}{l_{th}} \right),$$

$$\text{i erf} (x) = \frac{\exp(-x^2)}{\sqrt{\pi}} - x \cdot \text{erf} (x),$$

$$\text{erf} (x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp(-\xi^2) d\xi.$$

*Physical constants and units:*

$$h = 6.626 \cdot 10^{-34} \text{ Js}$$

$$c = 2.998 \cdot 10^8 \text{ m/s}$$

$$e = 1.602 \cdot 10^{-19} \text{ C}$$

$$\epsilon_0 = 8.854 \cdot 10^{-12} \text{ F/m}$$

$$m_e = 9.1093 \cdot 10^{-31} \text{ kg}$$

*Helpful formulas:*

$$\text{Gauss: } f(t) = e^{-4 \ln 2 \left[ \frac{t-t_o}{\Delta \tau_{FWHM}} \right]^2}$$

$$\text{Lorentz: } f(t) = \frac{(\Delta \tau_{FWHM}/2)^2}{(t-t_o)^2 + (\Delta \tau_{FWHM}/2)^2}$$

$$\int_{-\infty}^{\infty} \exp(-a^2 x^2) dx = \frac{\sqrt{\pi}}{a}$$

$$\int \frac{1}{x^2+c} dx = \frac{1}{\sqrt{c}} \arctan \left( \frac{x}{\sqrt{c}} \right)$$

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## Problem Set 7- Test Exam

You can obtain a maximum of 42 points (but the points that you obtain will be halved when calculating the amount of bonus points that you will get in the final exam). Good Luck!!

### Problem 1 (4 Points)

- Explain what the quantum efficiency is. (1 point)
- Explain what the quantum defect is. (1 point)
- What is the biggest limitation caused by the quantum defect in a laser?. (1 point)
- Is it possible to obtain a laser using an active medium with a quantum defect of 0?. Justify your answer. (1 point)

### Problem 2 (4 Points)

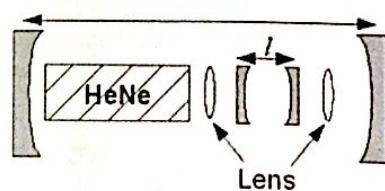
- Briefly explain the thermo-optic problems of classic solid-state lasers. (2 points)
- Which are the three main sources of thermal load in an active medium? Which of these sources is the main one? (1 point)
- Assume a 3-level system with a quantum efficiency of 99% that is pumped with a cw laser at 915nm, and that amplifies a 976nm pulsed signal with 100ps pulses at 1kHz repetition rate. When does the temperature increase the most: during the amplification of the pulses or in the time between pulses? Why? (1 point)

### Problem 3 (4 Points)

- Define the beam quality of a laser beam. (1 point)
- A diffraction-limited laser beam ( $M^2 = 1$ ) with a diameter of 10mm is focused down to a 200μm diameter spot. The wavelength of the laser beam is 1064nm. Calculate the focal length of the lens. (2 points)
- What focal length would we need if the laser beam had a  $M^2$  of 20? (1 point)

### Problem 4 (8 Points)

- Your task is to build a single longitudinal mode laser. Describe the techniques that can be employed to force single frequency operation in a laser, in which the amplification bandwidth of the active medium is broad enough to allocate many longitudinal modes. (2 points) (166, 175)
- A HeNe laser should be forced to operate in a single longitudinal mode with an etalon. The amplification bandwidth of the active medium is 2 pm and the resonator length is L=50cm. A schematic diagram of the resonator can be seen in the following picture:



Show that the transfer function of the second resonator (etalon) is given by the following function:

$$T = \frac{I_{out}}{I_{in}} = \frac{1}{1 + \frac{4 \sin^2(\delta)}{\Gamma^2}},$$

with  $\Gamma^2 = \frac{(1-R)^2}{R}$  und  $\delta = \frac{2\pi \cdot L}{\lambda}$ , where R is the (intensity) reflectivity of both mirrors. (3 points)  
*Hint : Neglect the Gouy phase-shift in your calculations.*

- c) Calculate the reflectivity  $R$  of both mirrors of the second resonator so that its finesse is 5. (1 point)  
 Hint: The finesse is approximately given by the following equation:  $F \approx \frac{\pi}{R}$ .

- d) How large should be the distance  $l$  between both mirrors, so that longitudinal modes (of the main resonator) neighbouring the mode to be selected undergo an attenuation of  $1/e$  in the etalon? Here you should assume that the resonances (longitudinal modes) of both resonators overlap exactly at one frequency (i.e. the mode that will be selected). (2 points)

**Problem 5 (5 Points)**

- a) Explain why it is not possible to obtain inversion in an optically pumped non-degenerated 2-level system. (1 point)
- b) What can a 2-level system be used for? (1 point)
- c) Can a 2-level system emit light? if yes, which are the main characteristics of this radiation? (2 points)
- d) Explain why it is possible to obtain laser radiation with 3- and 4-level systems. (1 point)

**Problem 6 (4 Points)**

- a) What limits the minimum pulse duration in a Q-switched laser? (1 point)
- b) Draw and discuss a time-dependent diagram that explains the operation principle of Q-switching. (2 points)
- c) It is known that the pulse energy of passively Q-switched lasers is nearly independent of the pump power. Explain why. (1 point)

**Problem 7 (5 Points)**

- a) What is the main characteristic of an stable resonator? (1 point)
- b) Draw the stability diagram of a passive resonator and mark the stable regions. (1 point)
- c) Draw the locus (positions) of the general confocal resonators (i.e. symmetric and asymmetric) in the stability diagram. (3 points)

**Problem 8 (6 Points)**

- a) Why is it not possible to generate ultra-short laser pulses with Q-switched lasers? (1 point)
- b) What is the basic idea behing mode-locking? (2 points)
- c) A Nd:YAG mode-locked laser delivers Gaussian pulses ( $P(t) = P_{peak} \cdot e^{-4 \cdot \ln 2 \cdot \frac{t^2}{\tau_{FWHM}^2}}$ ) with 20nJ pulse energy, a repetition rate of 1 MHz and a FWHM pulse duration of 1 ps.
- Calculate the average power emitted by this laser. (1 point)  $\Rightarrow E_T \cdot P_{av} = E_p \times \text{Repetit.}$
  - The time-bandwidth product ( $TBWP$ ) of a Gaussian pulse is 0.44 ( $TBWP = \tau_{FWHM} \cdot \Delta\nu_{FWHM}$ ). Calculate the spectral bandwidth of the mode-locked pulses. Give this bandwidth in Hz and in nm. (1 point)
  - How large is the spectral separation of the longitudinal modes of this laser? (1 point)  $\Delta\nu_{LHM} = \frac{c}{D\lambda_{FWHM}}$

**Problem 9: Extra problem (2 Points)**

In order to get single longitudinal mode operation, an etalon of thickness  $d$  can be placed inside of the cavity. The etalon should be tilted at an angle  $\theta_0$  with respect to the cavity axis to tune the wavelength of its resonances. Calculate the general expression for the transmissivity of an etalon (consider that the etalon is made of a material with index of refraction  $n$  and that the reflectivities of its coated surfaces are  $R_e$ ).

## Exam

Only a calculator is allowed as help. All the required formulas and constants are given. You can obtain a maximum of 35 points. Good Luck!!

**Problem 1** (4 points)

Sketch a "normal" 3- and 4-level system and mark the fast nonradiative decays and metastable levels. How many atoms need to be in the upper laser level of each system to obtain inversion?

**Problem 2** (4 points)

A saturable absorber can be described by a 2-level system.

- Write the rate equation of a 2 level system for the inversion  $n$  as a function of the total ion density ( $N_{tot}$ ) and of the signal intensity ( $I_S$ ). (2 points)
- The laser process starts when the signal intensity reaches the so-called saturation intensity  $I_{sat}$  (Intensity at which the steady state value of the inversion  $n = -N_{tot}/2$ ). Calculate an expression for  $I_{sat}$ . (1 point)
- Name two types of lasers that can be built using a saturable absorber. (1 points)

**Problem 3** (6 points)

- Describe the difference between homogeneous and inhomogeneous broadening and name one physical origin for each. (2 points)
- Explain what ASE is and name two methods to reduce it. (2 points)
- Assume two active media with the same gain characteristics (e.g. bandwidth, spectral shape, etc.). One of these media is homogeneously broadened and the other one inhomogeneously broadened. If we want to amplify a signal much narrower than the gain bandwidth, which of these amplifiers will exhibit less ASE at the output? Why? (2 points)

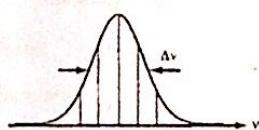
**Problem 4** (2 points) *(Rpu photonics)*

Explain what gain narrowing is and name one example when it is advantageous and another one where it is a drawback.

*longitudinal mode mode locking*

**Problem 5** (6 points)

We have the following three mode locked lasers emitting 1 Watt average power. A generic spectrum of any one of these lasers is



The laser parameters are as follows ( $\Delta\nu$  FWHM width,  $L_R$  resonator length):

Laser 1	$\Delta\nu = 1.7 \text{ THz}$	$L_R = 1 \text{ m}$
Laser 2	$\Delta\nu = 1.7 \text{ THz}$	$L_R = 15 \text{ m}$
Laser 3	$\Delta\nu = 500 \text{ GHz}$	$L_R = 3 \text{ m}$

- Assuming that the active medium has an index of refraction of  $n=1$ , calculate the frequency separation between the longitudinal modes for the three lasers above. (2 points)
- Assuming that the spectral envelopes are Gaussian, calculate the pulse duration, repetition frequency and pulse energy of the laser outputs! (4 points)

## Exam

Only a calculator is allowed as help. You can obtain a maximum of 40 points. Good Luck!!

### Problem 1 (4 points)

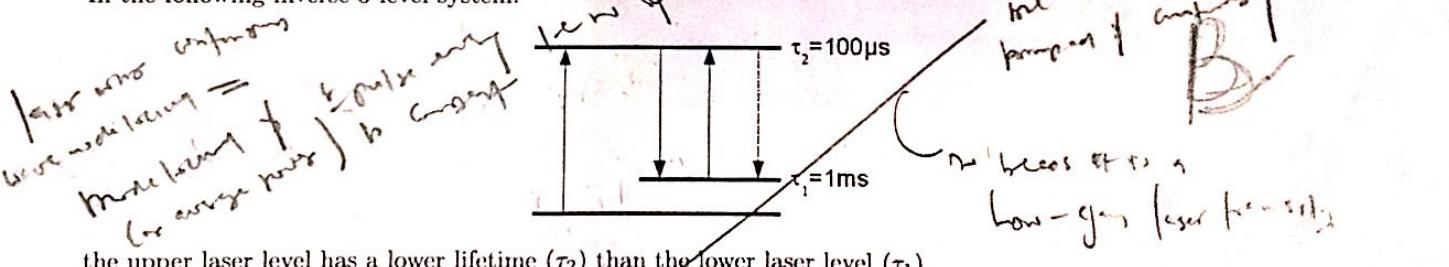
- a) Explain the difference between homogeneous and inhomogeneous broadening. Name an example for each. (2 points)
- ✓ b) If the upper laser level lifetime of a generic active medium is  $\tau_2$ , is it possible to amplify pulses shorter than  $\tau_2$ ? Justify your answer. (1 point)
- ✓ c) Explain what saturation is. (1 point)

### Problem 2 (8 points)

- $\phi_{\text{G}}$  =  $\frac{\pi}{2} - \frac{\lambda L}{2\pi n}$  additional phase shift occurs in  $\propto \text{prop}^2$
- a) Explain what the Gouy-phase shift is. (1 point)
- b) Which is the consequence of the Gouy-phase shift in a plan-plan resonator? (1 point)
- c) Which is the consequence of the Gouy-phase shift in a confocal resonator? (1 point)
- d) Draw the stability diagram for passive laser resonators marking the regions of stability. (1 point)
- e) Calculate the trajectories (i.e. different positions) described in the stability diagram by a general resonator as its length is varied. Draw two exemplary trajectories. (3 points)
- f) According to the results obtained in d), which condition has to be satisfied so that a resonator is always unstable independently of its length  $L$  (with  $L > 0$ )? (1 point)

### Problem 3 (5 points)

In the following inverse 3-level system:

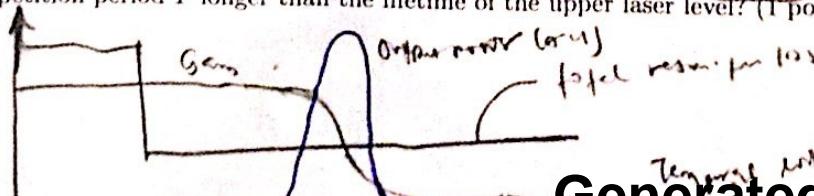


the upper laser level has a lower lifetime ( $\tau_2$ ) than the lower laser level ( $\tau_1$ ).

- a) Is it possible to obtain continuous-wave (CW) laser operation in the system depicted above? why? (2 points)
- b) Is it possible to obtain pulsed laser operation? If so, how would you realize it? and which kind of laser would it be? (3 points)

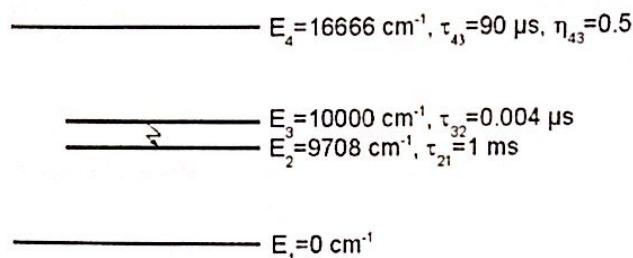
### Problem 4 (5 points)

- a) Why is it not possible to obtain ultra-short pulses with Q-switched lasers? (1 point)
- ✓ b) Explain active Q-switching using a diagram of the temporal evolution of the losses, the photon density and the inversion. (2 points)
- c) In the diagram depicted in b), when is the maximum photon density reached? Justify your answer. (1 point)
- ✓ d) Why is it advisable to pulse the pump power in an actively Q-switched laser that is operated with a pulse repetition period  $T$  longer than the lifetime of the upper laser level? (1 point)



**Problem 5** (6 points)

Assume the following energy diagram of an active medium:



where  $\tau_{21}$  represents the lifetime of the transition between levels 2 and 1,  $\tau_{32}$  represents the lifetime of the transition between levels 3 and 2,  $\tau_{43}$  represents the lifetime of the transition between levels 4 and 3, and  $\eta_{43}$  is the probability of non-radiative transition between levels 4 and 3 (i.e. percentage of ions that decay to level 3 without emitting a photon).

- a) Calculate all the emission/absorption wavelengths of the radiative transitions of the system. (1 point)
- b) Explain what quantum efficiency is. (1 point)
- c) Explain what quantum defect is. (1 point)
- ✓ d) Using the energy diagram above as a 3-level system and pumping in level 4, what would you expect from the slope efficiency and thermo-optical problems of such a laser? Justify your answer. (1 point)
- ✗ e) What would you do to significantly reduce the quantum-defect heating of this laser without changing the wavelengths of either pump or signal (i.e. pumping in level 4 and emitting from level 2)? Justify your answer. (2 points)

- **Problem 6** (5 points)

An ultra-short pulse mode-locked laser is built with a 1m long resonator and a roundtrip loss of 40%. The inhomogeneously broadened active medium fills the whole resonator. It has a refractive index of 1.5, a maximum gain of  $G_o^{dB} = 20dB$ , and a small-signal gain bandwidth of 10nm (FWHM) with a central wavelength of 1 $\mu\text{m}$ . The gain profile in frequency domain is:

$$G(f) = 1 + G_o \cdot e^{-4\ln 2 \frac{(f-f_c)^2}{\Delta f_{FWHM}^2}}$$

The laser can deliver an average output power of 2W.

- a) Calculate the number of longitudinal modes supported by this laser configuration. (2 points)
- b) Assuming that the spectral bandwidth calculated in a) corresponds to the  $1/e^2$  bandwidth of the emitted Gaussian pulses, demonstrate that the transform-limited pulse duration of the output pulses is 218fs. (1 point)
- c) Estimate the output peak power of the pulses emitted by this laser. (1 point)
- d) An etalon is placed into the laser beam to filter out a single longitudinal mode of this mode-locked laser. Describe the properties of the filtered laser radiation. (1 point)

**Problem 7** (4 points)

Name 4 methods of Doppler-free spectroscopy and explain one of them.

**Problem 8** (3 points)

- a) Give a typical value of the optical penetration depth for metals. (1 point)
- b) At which wavelength do you expect significant changes of this value? (1 point)
- c) Aluminium has a diffusivity  $D = 8 \cdot 10^{-5} \text{ m}^2/\text{s}$ . Calculate the pulse duration at which the optical penetration depth equals the thermal penetration length. (1 point)

Final question = set question

Exam

0.5%

0.2%

No help other than a calculator is allowed in the exam. All the constants and formulae required to solve the exam can be found on the second page. A maximum of 27 points can be achieved. Good luck!!

use reiteration time  
•  $\Delta t = \frac{c}{2n}$

Problem 1 (6 Points)

A mode-locked laser consists of an hemispheric resonator with an effective resonator length of  $1.5 \text{ m}$ . Pulses with a FWHM duration  $t_p = 1 \text{ ps}$  at an average power  $P_{av}$  of  $1 \text{ W}$  are obtained from this laser. The central emission wavelength is  $\lambda_c = 1.064 \text{ nm}$ .

- Calculate the pulse energy  $E_p$  and peak power  $P_{peak}$  assuming that the emitted pulses are square. (1 Point)
- Calculate  $P_{peak}$  assuming Gaussian pulses  $P(t) = P_{peak} e^{-4 \ln 2 \left( \frac{t}{t_p} \right)}$ . (1 Point)
- How can the pulse energy be doubled without doubling the pump power? What should be modified in the resonator design? (1 Point)
- Calculate the spectral bandwidth of the Gaussian pulses in nm (time-bandwidth product: 0.441). (1 Point)
- Explain briefly the mode-locking technique to create ultrashort pulses. Point out differences between active and passive mode-locking. (2 Points)

Problem 2 (3 Points)

- Draw the stability diagram with the resonator parameters  $g_i$ . Include in the diagram the points for the following resonators ( $R_n$  = radius of curvature of the mirror n,  $l_R$  = length of the resonator) (2 Points):
  - (I) Plane-Parallel ( $R_1 = R_2 = \infty, l_R < \infty$ ), (II) Hemispheric ( $R_1 = l_R, R_2 = \infty$ ),
  - (III) Confocal ( $R_1 = R_2 = l_R$ ), (IV) Concentric ( $R_1 = R_2 = l_R/2$ )
- Which mechanism is at work, from the wave-optics point of view, for out-coupling in an unstable resonator? (1 Point)

Problem 3 (2 Points)

The rate equations of a two-level system are given by:

$$\frac{\partial N_2}{\partial t} = -\frac{\partial N_1}{\partial t} = -\frac{N_2}{\tau} + \frac{\sigma I}{h\nu} (N_1 - N_2)$$

where  $N_1$  and  $N_2$  are the populations of the lower and upper level respectively,  $\tau$  is the average lifetime in the upper level,  $\sigma$  is the cross-section,  $\nu$  is the transition frequency and  $I$  is the intensity of the incoming radiation.

- Identify the different terms in the rate equations with their related physical processes. (1 Point)
- Demonstrate with a short calculation that it is not possible with this two-level system to obtain a continuous emitting (cw) laser. (1 Point)

Problem 4 (2 Points)

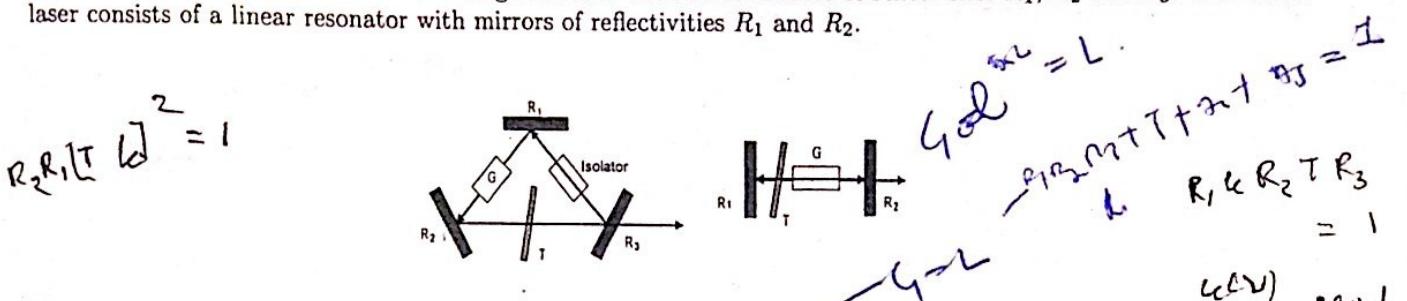
- Why is it possible, under the same conditions, to drill ten times smaller holes with a Nd:YAG laser ( $M^2 = 1$ ) than with a CO<sub>2</sub> laser ( $M^2 = 100$ )? (1 Point)
- If the thickness of the materials to be drilled doesn't play a role, how is it possible to reduce the size of the holes drilled with the CO<sub>2</sub> laser so that they are equal to those drilled with the Nd:YAG laser? (1 Point)

Problem 5 (3 Points)

- Name one homogeneous and one inhomogeneous spectral broadening mechanism. (1 Point)
- Explain the mechanisms of spectral and spatial hole-burning. (2 Points)

**Problem 6 (3 Points)**

Two different lasers comprise each an etalon with transmissivity  $T$  and an active medium with gain  $G$  (single-pass gain). One of these lasers consists of a ring-resonator with three mirrors of reflectivities  $R_1$ ,  $R_2$  and  $R_3$ . The other laser consists of a linear resonator with mirrors of reflectivities  $R_1$  and  $R_2$ .



- a) Write the equilibrium condition of each resonator for continuous wave operation. (2 Points)  
 b) Why is the ring-resonator more useful for single longitudinal mode operation? (1 Point)

**Problem 7 (2 Points)**

An spherical target for nuclear fusion with  $d = 1 \text{ mm}$  diameter is simultaneously irradiated with 192 laser pulses coming from all directions. Each pulse has an energy  $E_p = 35 \text{ kJ}$ . The synchronization of the pulses leads to an effective irradiation time of 3 ns. As a simplification assume a beam diameter  $d$  and an homogeneous irradiation of the target surface, which has a 100% reflectivity.

- a) Calculate the light pressure induced at the target surface (1 Point)  
 b) Which value has the effective parameter  $\gamma$  if the target surface has 90% absorption and 10% reflexion? (1 Point)

**Problem 8 (6 Points)**

An atomic beam of Na atoms (atomic weight 23 u) with an initial speed  $v = 750 \text{ m/s}$  is directed against a dye-laser beam of fixed wavelength  $\lambda = 589 \text{ nm}$  and FWHM bandwidth  $\Delta\nu = 1 \text{ MHz}$  (both beams are exactly counterpropagating). The lifetime of the excited Na atoms is  $\tau = 16 \text{ ns}$ . The laser intensity is so high that the Na atoms are, in average, excited every 16 ns. The excited atoms emit isotropically in all directions. The Na atoms will be slowed down through light pressure.

- a) It is assumed that the laser wavelength is perfectly tuned to the Na atom absorption line at the initial speed  $v$ . Sketch this initial situation showing the spectral profiles of the laser emission and the absorption line of the Na atoms. Sketch also the spectral shift of the Na atom absorption line due to the Doppler effect. For simplicity assume a Gauss profile both for the laser emission and for the Na atom absorption line. Additionally, assume for the atomic transition that the spectral bandwidth (FWHM) is  $\Delta\nu = 1/(2\pi\tau)$ . (2 Points)  
 b) In the process of slowing down, the Na atoms run out of resonance with the laser emission. This results in a decreasing overlap of the spectral profile curves. Let's assume that the minimum overlap, that still results in some slowing down, coincides with the situation in which the points at half maximum height for both spectral profiles are situated at the same frequency (wavelength). Which atomic line shift is necessary to yield this coincidence? (1 Point)  
 c) At which speed do the Na atoms move in case b)? (1 Point)  
 d) How much time since the beginning of the slowing down process is elapsed until the situation in case b) is achieved? (1 Point)  
 e) How long do the Na atoms travel during the slowing down process until case b) is achieved? (1 Point)

constants/units:

$$h = 6,626 \cdot 10^{-34} \text{ Js}$$

$$c = 2,998 \cdot 10^8 \text{ m/s}$$

$$k_B = 1,381 \cdot 10^{-23} \text{ J/K}$$

$$1 \text{ u} = 1,661 \cdot 10^{-27} \text{ kg}$$

formulae:

$$\int_{-\infty}^{\infty} \exp(-a^2 x^2) dx = \frac{\sqrt{\pi}}{a}$$