

1)

4)

Solution:

$$\vec{k} \cdot \vec{r} = \text{const}$$

2) When  $\epsilon(\omega) \neq 0$

$$\text{div } \vec{E}(\vec{r}, \omega) = i\vec{k} \cdot \vec{E}(\vec{r}, \omega) = 0$$

$$\therefore \vec{k} \perp \vec{E}(\vec{r}, \omega)$$

$$3) \because \epsilon_0 \epsilon(\omega) \text{div } \vec{E}(\vec{r}, \omega) = 0$$

$$\epsilon_0, \epsilon(\omega) \neq 0$$

$$\therefore \text{div } \vec{E}(\vec{r}, \omega) = 0$$

$$\text{div } \vec{E}(\vec{r}, \omega) = i\vec{k} \cdot \vec{E}(\vec{r}, \omega)$$

$$\therefore \vec{k} \perp \vec{E}(\vec{r}, \omega) \text{ is required}$$