

Problem 1

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(a)

The physical origin of gain narrowing is gain can shrink the amplified spectrum towards the resonance frequency if : $g(\nu_0) \cdot l = \sigma(\nu_0) \cdot n \cdot l \gg 1$.

(b)

Because we need to avoid the transition of particles from $|2\rangle$ to $|3\rangle$ during pumping.

(c)

Because the essential condition for continuous population inversion is : $S_{10} > S_{21}$.

(d)

We can reduce the influence of ASE by spatial filtering.

Problem 2

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(a)

$\therefore g(v)$ has Gaussian profile

$$\therefore g(v) = g_0 \cdot \exp \left[-4 \cdot \ln 2 \cdot \frac{(v-v_0)^2}{\Delta v_g} \right]$$

g_0 is the peak value of $g(v)$.

$$\therefore G(v) = \exp [g(v) \cdot d] = \exp [g_0 \cdot \exp \left[-4 \cdot \ln 2 \cdot \frac{(v-v_0)^2}{\Delta v_g} \right] \cdot d]$$

$$\therefore G(v_0) = \exp [g(v_0) \cdot d] = \exp [g_0 \cdot d]$$

the FWHM of $G(v)$ is Δv_g

$$\therefore G(v_0 \pm \frac{1}{2} \Delta v_g) = \frac{1}{2} G(v_0)$$

$$G(v_0 \pm \frac{1}{2} \Delta v_g) = \exp \left[g_0 \cdot \exp \left[-4 \ln 2 \cdot \frac{\left(\pm \frac{1}{2} \Delta v_g \right)^2}{\Delta v_g} \right] \cdot d \right]$$

$$\frac{1}{2} G(v_0) = \exp \left[g_0 \cdot \exp \left[-4 \cdot \ln 2 \cdot \frac{1}{4} \frac{\Delta v_g^2}{\Delta v_g} \right] d \right]$$

$$\ln G(v_0) - \ln 2 = g_0 \cdot d \cdot \exp \left[-\ln 2 \left(\frac{\Delta v_g}{\Delta v_g} \right)^2 \right]$$

$$\ln G(v_0) - \ln 2 = \ln G(v_0) \cdot \exp \left[-\ln 2 \left(\frac{\Delta v_g}{\Delta v_g} \right)^2 \right]$$

$$1 - \frac{\ln 2}{\ln G(v)} = \exp \left[-\ln 2 \left(\frac{\Delta v_g}{\Delta v_g} \right)^2 \right]$$

$$\ln \left[1 - \frac{\ln 2}{\ln G(V_0)} \right] = -\ln 2 \left(\frac{\Delta k_g}{\Delta V_g} \right)^2$$

$$\because \ln(1-x) \approx -x$$

$$\therefore -\frac{\ln 2}{\ln G(V_0)} = -\ln 2 \left(\frac{\Delta k_g}{\Delta V_g} \right)^2$$

$$\therefore \Delta V_g^2 = \Delta k_g^2 \cdot \ln G(V_0)$$

$$\therefore \Delta V_g = \Delta k_g \cdot \sqrt{\ln G(V_0)} \quad \Delta k_g = \Delta V_g \cdot \frac{1}{\sqrt{\ln G(V_0)}}$$

(b)

$$G(v) = \exp[g(v) \cdot d] = \exp[\sigma(v) \cdot n \cdot d]$$

$$\therefore \sigma(v) = a - \sigma_0 \left[\frac{v - v_0}{\Delta V_g} \right]^2 = \exp \left[(a - \sigma_0 \left[\frac{v - v_0}{\Delta V_g} \right]^2) \cdot n \cdot d \right] \textcircled{1}$$

is valid for $|v| < v_0 + \Delta V \cdot \sqrt{\frac{a}{\sigma_0}}$

From equation ①, we can get:

$$G(v_0) = \exp[a \cdot n \cdot d]$$

$$G(v_0 \pm \frac{1}{2} \Delta V_g) = \exp \left[n \cdot d \cdot \left(a - \sigma_0 \left[\frac{\pm \frac{1}{2} \Delta V_g}{\Delta V_g} \right]^2 \right) \right]$$

$$G(v_0 \pm \frac{1}{2} \Delta V_g) = \frac{1}{2} G(v_0)$$

$$\frac{1}{2} \exp(a \cdot n \cdot d) = \exp\left[n \cdot d \left(a - \sigma_0 \left[\frac{\Delta V_G}{2\Delta k_B}\right]^2\right)\right]$$

$$a \cdot n \cdot d - \ln 2 = a \cdot n \cdot d - \sigma_0 \cdot n \cdot d \left[\frac{\Delta V_G}{2\Delta k_B}\right]^2$$

$$\ln 2 = \sigma_0 \cdot n \cdot d \left[\frac{\Delta V_G}{2\Delta k_B}\right]^2$$

$$\therefore \frac{\ln 2}{\sigma_0 \cdot n \cdot d} (2\Delta k_B)^2 = \Delta V_G^2$$

$$\therefore \Delta V_G = 2\Delta k_B \cdot \sqrt{\frac{\ln 2}{\sigma_0 \cdot n \cdot d}}$$

(C)

$$G(v) = \exp[g(v) \cdot d] = \exp[\sigma_v \cdot n \cdot d]$$

$$\therefore \sigma(v) = \sigma_0 \left[1 - \left(\frac{v - v_0}{\Delta V_g}\right)^2\right]$$

$$\therefore G(v) = \exp\left[\sigma_0 \cdot n \cdot d \cdot \left[1 - \left(\frac{v - v_0}{\Delta V_g}\right)^2\right]\right]$$

\because the input signal has a Gaussian spectrum

$$\therefore I_{in}(v) = I_0 \cdot \exp\left[-4\ln 2 \left(\frac{v - v_0}{\Delta V_{in}}\right)^2\right]$$

$$I_{out}(v) = I_{in}(v) \cdot G(v)$$

$$= I_0 \exp\left\{\sigma_0 \cdot n \cdot d \left[1 - \left(\frac{v - v_0}{\Delta V_g}\right)^2\right] - 4\ln 2 \left(\frac{v - v_0}{\Delta V_{in}}\right)^2\right\}$$

$$\because C = \lambda \cdot V \quad V = \frac{C}{\lambda} \quad \ln V = \ln C - \ln \lambda$$

$$\frac{1}{V} dV = -\frac{1}{\lambda} d\lambda$$

$$\therefore \left| \frac{\Delta V}{V} \right| = \left| \frac{\Delta \lambda}{\lambda} \right|$$

$$\because \lambda = 1 \mu\text{m} \quad \begin{cases} \Delta \lambda_{\text{in}} = 4.5 \text{ nm} \\ \Delta \lambda_{\text{out}} = 2 \text{ nm} \end{cases} \quad \therefore \Delta V_{\text{in}} = 1.35 \times 10^2 \text{ Hz} \\ \therefore \Delta V_{\text{out}} = 6 \times 10^{11} \text{ Hz}$$

$$I_{\text{out}}(V_0) = I_0 \exp(\sigma_0 \cdot n \cdot d)$$

$$I_{\text{out}}(V \pm \frac{1}{2} \Delta V_{\text{out}}) = I_0 \exp \left\{ \sigma_0 \cdot n \cdot d \cdot \left(1 - \frac{\Delta V_{\text{out}}^2}{4 \Delta V_g^2} \right) - 4 \ln 2 \cdot \frac{\Delta V_{\text{out}}^2}{4 \Delta V_m^2} \right\}$$

$$\frac{1}{2} I_{\text{out}}(V_0) = I_{\text{out}}(V \pm \frac{1}{2} \Delta V_{\text{out}})$$

$$\therefore \frac{1}{2} I_0 \exp(\sigma_0 \cdot n \cdot d) = I_0 \exp \left\{ \sigma_0 \cdot n \cdot d \cdot \left(1 - \frac{\Delta V_{\text{out}}^2}{4 \Delta V_g^2} \right) - 4 \ln 2 \cdot \frac{\Delta V_{\text{out}}^2}{4 \Delta V_m^2} \right\}$$

$$\sigma_0 \cdot n \cdot d - \ln 2 = \sigma_0 \cdot n \cdot d \left(1 - \frac{\Delta V_{\text{out}}^2}{4 \Delta V_g^2} \right) - 4 \ln 2 \cdot \frac{\Delta V_{\text{out}}^2}{4 \Delta V_m^2}$$

$$-\ln 2 = -\sigma_0 \cdot n \cdot d \frac{\Delta V_{\text{out}}^2}{4 \Delta V_g^2} - 4 \ln 2 \cdot \frac{\Delta V_{\text{out}}^2}{4 \Delta V_m^2}$$

$$\frac{4 \ln 2}{\Delta V_{\text{out}}^2} = \sigma_0 \cdot n \cdot d \frac{1}{\Delta V_g^2} + 4 \cdot \ln 2 \frac{1}{\Delta V_m^2}$$

$$\sigma_0 \cdot n \cdot d \frac{1}{\Delta V_g^2} = 4 \ln 2 \frac{1}{\Delta V_{\text{out}}^2} - 4 \ln 2 \frac{1}{\Delta V_m^2}$$

$$\therefore d = \frac{\Delta V_g^2}{\sigma_0 \cdot n} \cdot 4 \ln 2 \left(\frac{1}{\Delta V_{out}^2} - \frac{1}{\Delta V_{in}^2} \right)$$

$$\therefore d \approx 0.206 \text{ m}$$

(d)

$$\therefore G(\nu) = \exp \left[\sigma_0 \cdot n \cdot d \left[1 - \left(\frac{\nu - \nu_0}{\Delta V_g} \right)^2 \right] \right]$$

\therefore the maximum value of $G(\nu)$ is :

$$G(\nu_0) = \exp (\sigma_0 \cdot n \cdot d) \approx 482.992$$

Problem 3

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(a)

Ne atoms have 5 energy-levels, the life time of 3P, 4S, 4P and 5S is much shorter than 3S. So it's really difficult to realize population inversion.

(b)

Mission: ① The laser process in a He-Ne laser starts with collision of electrons from the electrical discharge with the helium atoms, which excites helium from the ground state to the 2^3S , and 2^1S , metastable excited states.
② Collision of the excited helium atoms with the ground-state neon atoms results in transfer of energy to the neon atoms. He atoms can transfer energy to 5S and 4S of Ne atoms.

(c)

Because the laser starts from the collision of the electrons and He atoms. The more the He atoms exist, the higher pump efficiency is.

(d)

From energy conservation , the extra energy comes from the kinetic energy of He. And the excess energy becomes the kinetic energy of Ne.

(e)

∴ It can increase the probability of collisions of He atoms with Ne atoms.

(f)

From the manual of He-Ne laser experiment , we know that normally a He-Ne laser is working at 632. 81 nm with a very narrow gain bandwidth of a few GHz , which is dominated by Doppler broadening .