

Wave Aberrations and Zernike Coefficients

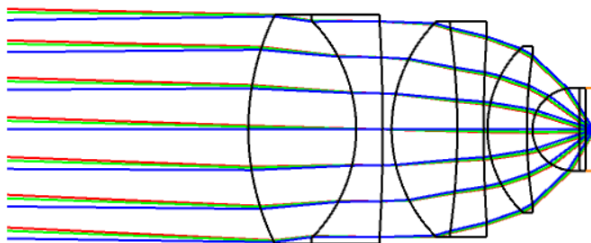
Load the system data from Moodle. It is a microscopic lens with high NA = 1.28 from the book of Laikin.

- Show the rms wave aberrations as a function of the defocussing . Discuss the results
- Show the rms wave aberration as a function of the field for all wavelengths. Is the system diffraction limited ?
- Calculate the Zernike coefficients for the primary wavelength on axis and for the maximum field size. What kind of aberration limits the performance in the field ?
- Calculate the Zernikes on axis behind the first three components and in the image. What can be seen for the changes and the compensation effects in the spherical aberration coefficients ?

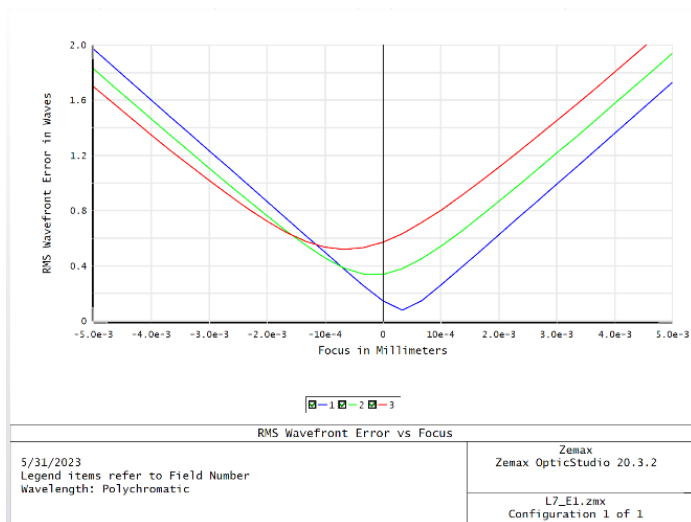
Solution:

The system data are as follows:

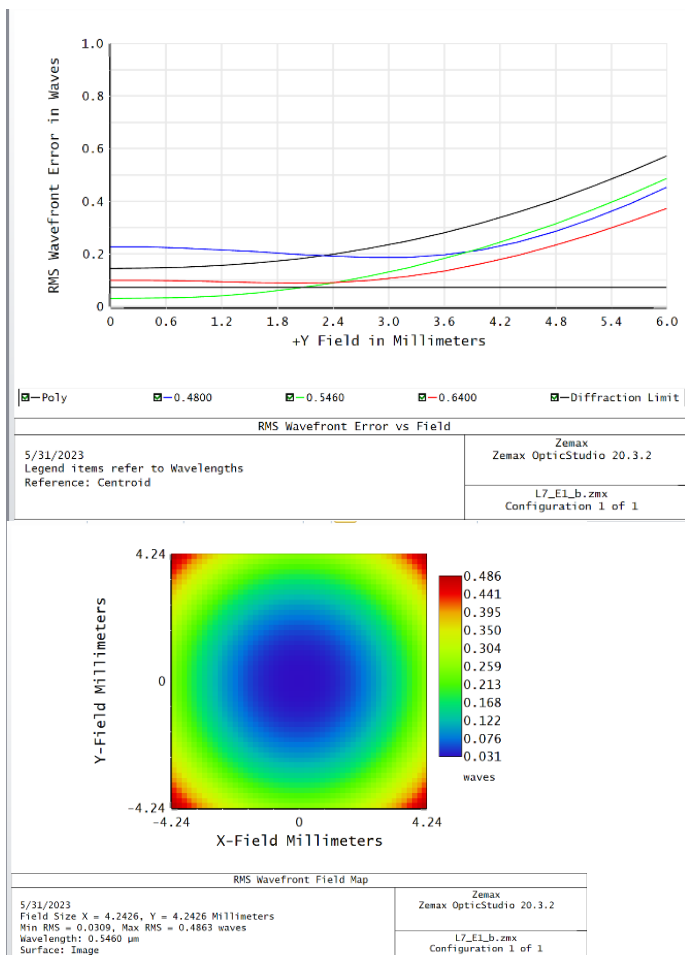
	Surface Type		Comment	Radius	Thickness	Material	Coating	Clear Semi-Dia
0	OBJECT	Standard ▾		Infinity	167.867320			6.000000
1		Standard ▾		Infinity	5.000000			2.477504
2		Standard ▾		5.341620	2.232660	CAF2		2.361041
3		Standard ▾		-3.180080	0.548640	F2		2.239519
4	STOP	Standard ▾		-38.305740	0.185420			2.220908
5		Standard ▾		3.126740	1.358900	CAF2		2.217043
6		Standard ▾		-15.306040	0.546100	F2		2.136609
7		Standard ▾		30.728920	0.078740			1.964774
8		Standard ▾		2.372360	0.815340	SK14		1.713994
9		Standard ▾		10.200640	0.115734			1.596243
10		Standard ▾		0.853440	0.970280	BK7		0.852155
11		Standard ▾		Infinity	0.128972	V TYPEA		0.571722
12		Standard ▾		Infinity	0.177800	N-K5		0.350250
13	IMAGE	Standard ▾		Infinity	-	WATER		0.063182



- The Rms of the wave aberration as a function of the defocus is shown here for the 3 field points. It is seen, that the performance is decreasing for larger field points. Especially, also a shift in best image position is seen, which results from the lack of field flattening.



b) If the wave aberrations are plotted as a function of the field, it is seen, that the system is only diffraction limited in the green inside a field circle with radius 1.8 mm. The same can be seen in the field distribution of the rms for the primary wavelength.



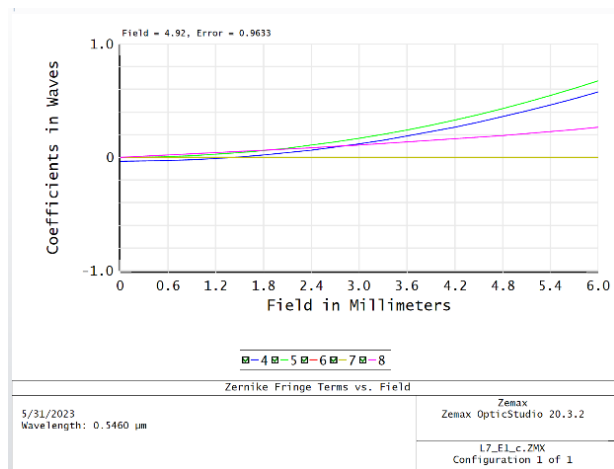
c) The Zernikes on axis and in the field also demonstrate the change of correction over the field size. In the field, the Marechal approximation for the Strehl ratio completely breaks down.

Surface	: Image	Surface	: Image
Field	: 0.0000 mm	Field	: 6.0000 mm
Wavelength	: 0.5460 μm	Wavelength	: 0.5460 μm
Peak to Valley (to chief)	: 0.59580369 waves	Peak to Valley (to chief)	: 4.37741709 waves
Peak to Valley (to centroid)	: 0.59580369 waves	Peak to Valley (to centroid)	: 2.78519800 waves
RMS (to chief)	: 0.09103532 waves	RMS (to chief)	: 1.07370619 waves
RMS (to centroid)	: 0.09103532 waves	RMS (to centroid)	: 0.47819760 waves
Variance	: 0.00828743 waves squared	Variance	: 0.22867294 waves squared
Strehl Ratio (Est)	: 0.72095786	Strehl Ratio (Est)	: 0.00000000
RMS fit error	: 0.00045823 waves	RMS fit error	: 0.00050678 waves
Maximum fit error	: 0.00229943 waves	Maximum fit error	: 0.00328283 waves

Z 1	-0.16594574	:	1
Z 2	0.00000000	:	(p) * COS (A)
Z 3	0.00000000	:	(p) * SIN (A)
Z 4	-0.02585636	:	(2p^2 - 1)
Z 5	0.00000000	:	(p^2) * COS (2A)
Z 6	0.00000000	:	(p^2) * SIN (2A)
Z 7	0.00000000	:	(3p^2 - 2) p * COS (A)
Z 8	0.00000000	:	(3p^2 - 2) p * SIN (A)
Z 9	0.12910795	:	(6p^4 - 6p^2 + 1)
Z 10	0.00000000	:	(p^3) * COS (3A)
Z 11	0.00000000	:	(p^3) * SIN (3A)
Z 12	0.00000000	:	(4p^2 - 3) p^2 * COS (2A)
Z 13	0.00000000	:	(4p^2 - 3) p^2 * SIN (2A)
Z 14	0.00000000	:	(10p^4 - 12p^2 + 3) p * COS (A)
Z 15	0.00000000	:	(10p^4 - 12p^2 + 3) p * SIN (A)
Z 16	0.12252524	:	(20p^6 - 30p^4 + 12p^2 - 1)
Z 17	-0.00001223	:	(p^4) * COS (4A)
Z 18	0.00000000	:	(p^4) * SIN (4A)
Z 19	0.00000000	:	(5p^2 - 4) p^3 * COS (3A)
Z 20	0.00000000	:	(5p^2 - 4) p^3 * SIN (3A)
Z 21	0.00000000	:	(15p^4 - 20p^2 + 6) p^2 * COS (2A)
Z 22	0.00000000	:	(15p^4 - 20p^2 + 6) p^2 * SIN (2A)
Z 23	0.00000000	:	(35p^6 - 60p^4 + 30p^2 - 4) p * COS (A)
Z 24	0.00000000	:	(35p^6 - 60p^4 + 30p^2 - 4) p * SIN (A)
Z 25	0.17460700	:	(70p^8 - 140p^6 + 90p^4 - 20p^2 + 1)
Z 26	0.00000000	:	(p^5) * COS (5A)
Z 27	0.00000000	:	(p^5) * SIN (5A)
Z 28	-0.00001120	:	(6p^2 - 5) p^4 * COS (4A)
Z 29	0.00000000	:	(6p^2 - 5) p^4 * SIN (4A)
Z 30	0.00000000	:	(21p^4 - 30p^2 + 10) p^3 * COS (3A)
Z 31	0.00000000	:	(21p^4 - 30p^2 + 10) p^3 * SIN (3A)
Z 32	0.00000000	:	(56p^6 - 105p^4 + 60p^2 - 10) p^2 * COS (2A)
Z 33	0.00000000	:	(56p^6 - 105p^4 + 60p^2 - 10) p^2 * SIN (2A)
Z 34	0.00000000	:	(126 p^8 - 280p^6 + 210p^4 - 60p^2 + 5) p * COS (A)
Z 35	0.00000000	:	(126 p^8 - 280p^6 + 210p^4 - 60p^2 + 5) p * SIN (A)
Z 36	0.05037324	:	(252p^10 - 630p^8 + 560p^6 - 210p^4 + 30p^2 - 1)
Z 37	0.01091366	:	(924p^12 - 2772p^10 + 3150p^8 - 1680p^6 + 420p^4 - 42p^2 + 1)

Z 1	0.61332979	:	1
Z 2	0.00000000	:	(p) * COS (A)
Z 3	1.92417391	:	(p) * SIN (A)
Z 4	0.56265128	:	(2p^2 - 1)
Z 5	0.67929048	:	(p^2) * COS (2A)
Z 6	0.00000000	:	(p^2) * SIN (2A)
Z 7	0.00000000	:	(3p^2 - 2) p * COS (A)
Z 8	0.25054032	:	(3p^2 - 2) p * SIN (A)
Z 9	-0.04459339	:	(6p^4 - 6p^2 + 1)
Z 10	0.00000000	:	(p^3) * COS (3A)
Z 11	-0.04221987	:	(p^3) * SIN (3A)
Z 12	0.23084224	:	(4p^2 - 3) p^2 * COS (2A)
Z 13	0.00000000	:	(4p^2 - 3) p^2 * SIN (2A)
Z 14	0.00000000	:	(10p^4 - 12p^2 + 3) p * COS (A)
Z 15	-0.55294531	:	(10p^4 - 12p^2 + 3) p * SIN (A)
Z 16	0.12078918	:	(20p^6 - 30p^4 + 12p^2 - 1)
Z 17	0.00016941	:	(p^4) * COS (4A)
Z 18	0.00000000	:	(p^4) * SIN (4A)
Z 19	0.00000000	:	(5p^2 - 4) p^3 * COS (3A)
Z 20	0.00180260	:	(5p^2 - 4) p^3 * SIN (3A)
Z 21	0.00639322	:	(15p^4 - 20p^2 + 6) p^2 * COS (2A)
Z 22	0.00000000	:	(15p^4 - 20p^2 + 6) p^2 * SIN (2A)
Z 23	0.00000000	:	(35p^6 - 60p^4 + 30p^2 - 4) p * COS (A)
Z 24	-0.07231759	:	(35p^6 - 60p^4 + 30p^2 - 4) p * SIN (A)
Z 25	0.17579142	:	(70p^8 - 140p^6 + 90p^4 - 20p^2 + 1)
Z 26	0.00000000	:	(p^5) * COS (5A)
Z 27	-0.0000555	:	(p^5) * SIN (5A)
Z 28	0.00057060	:	(6p^2 - 5) p^4 * COS (4A)
Z 29	0.00000000	:	(6p^2 - 5) p^4 * SIN (4A)
Z 30	0.00000000	:	(21p^4 - 30p^2 + 10) p^3 * COS (3A)
Z 31	0.00263237	:	(21p^4 - 30p^2 + 10) p^3 * SIN (3A)
Z 32	-0.00152163	:	(56p^6 - 105p^4 + 60p^2 - 10) p^2 * COS (2A)
Z 33	0.00000000	:	(56p^6 - 105p^4 + 60p^2 - 10) p^2 * SIN (2A)
Z 34	0.00000000	:	(126 p^8 - 280p^6 + 210p^4 - 60p^2 + 5) p * COS (A)
Z 35	-0.01184470	:	(126 p^8 - 280p^6 + 210p^4 - 60p^2 + 5) p * SIN (A)
Z 36	0.05030055	:	(252p^10 - 630p^8 + 560p^6 - 210p^4 + 30p^2 - 1)
Z 37	0.01079007	:	(924p^12 - 2772p^10 + 3150p^8 - 1680p^6 + 420p^4 - 42p^2 + 1)

From the variation of the Zernikes over the field size it is seen, that a large tilt takes place. If only the primary aberrations are considered, Coma (c8), astigmatism (c5) and field curvature (c4) are approximately of the same size.



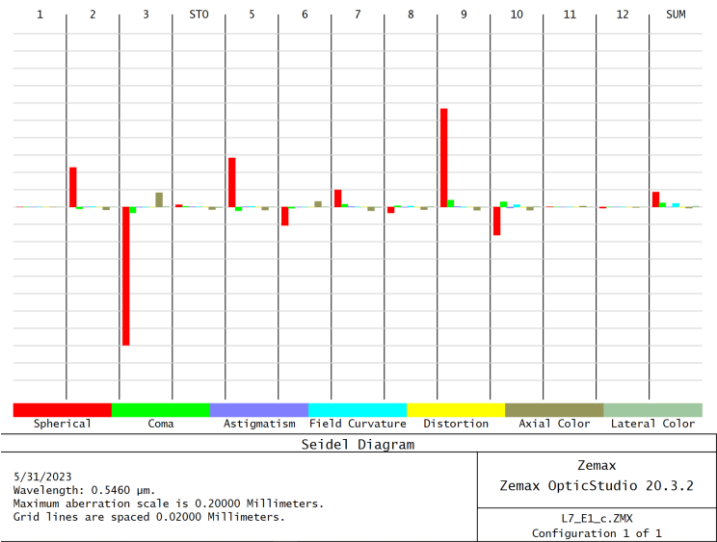
d) If the Zernikes are calculated for the surface indices 4, 7 and 9 and the image, we get the following data:

Z 1	-16.44721392	Z 1	-9.33855398	Z 1	5.38758877	Z 1	-0.16594574	:	1
Z 2	0.00000000	Z 2	0.00000000	Z 2	0.00000000	Z 2	0.00000000	:	(p) * COS (A)
Z 3	0.00000000	Z 3	0.00000000	Z 3	0.00000000	Z 3	0.00000000	:	(p) * SIN (A)
Z 4	-26.12095807	Z 4	-15.10166076	Z 4	8.31509634	Z 4	-0.02585636	:	(2p^2 - 1)
Z 5	0.00000000	Z 5	0.00000000	Z 5	0.00000000	Z 5	0.00000000	:	(p^2) * COS (2A)
Z 6	0.00000000	Z 6	0.00000000	Z 6	0.00000000	Z 6	0.00000000	:	(p^2) * SIN (2A)
Z 7	0.00000000	Z 7	0.00000000	Z 7	0.00000000	Z 7	0.00000000	:	(3p^2 - 2) p * COS (A)
Z 8	0.00000000	Z 8	0.00000000	Z 8	0.00000000	Z 8	0.00000000	:	(3p^2 - 2) p * SIN (A)
Z 9	-10.74084716	Z 9	-6.56724253	Z 9	3.29615815	Z 9	0.12910795	:	(6p^4 - 6p^2 + 1)

It is seen, that the contributions of the various components compensate the effect of the c9. The contributions are approximately (subtracting the absolute values):

-10.74 (surfaces 2,3,4) , +4.18 (surfaces 5,6,7) , +9.86 (surfaces 8,9) , -3.16 (surfaces 10,11,12)

If the Seidel contributions are inspected, qualitatively the same result occurs. It is seen in the Zernike table, that also a considerable contribution of 5th order spherical aberrations should be compensated.



Aplanatic lens

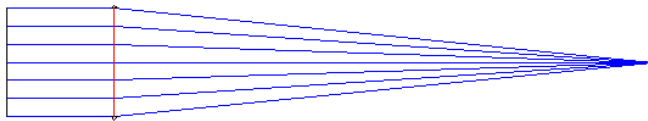
Consider a collimated incoming beam with wavelength 500 nm and diameter 10 mm. This bundle should be focussed by a perfect lens of focal length $f = 50$ mm.

- Place an aplanatic-concentric lens shortly behind the ideal lens with the material SF57. What is the resulting numerical aperture in the image space ? Show at least two different methods to find the best image position.
- Show that the spherical aberration of this setup is exactly zero for all orders.
- Aplanatic means, that the linear coma vanishes and the imaging is free of coma for a small but finite field size. Show this property by using a small field of 2° for the current system. What is the largest present aberration ?

Solution:

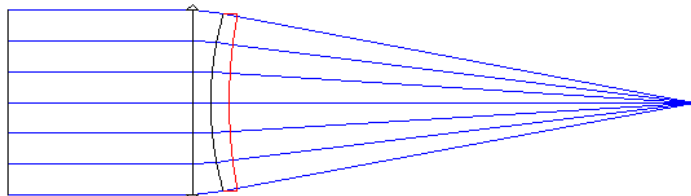
The initial focussing lens is established as follows:

		Comment	Radius	Thickness	Glass	Semi-Diameter	Conic	Par 0 (unused)	Par 1 (unused)
OBJ	Standard		Infinity	Infinity		0.0000000	0.0000000		
1	Standard		Infinity	10.0000000		5.0000000	0.0000000		
STO	Paraxial			0.0000000		5.0000000			50.0000000
3	Standard		Infinity	50.0000000		5.0000000	0.0000000		
IMA	Standard		Infinity	-		0.0000000	0.0000000		



- A lens with thickness 1 mm is placed 1 mm behind the lens. The first surface is made aplanatic by a solve, the second surface is made concentric by choosing the solve 'marginal ray normal' to force the marginal ray to be concentric.

Surf:	Type	Comment	Radius	Thickness	Glass	Semi-Diameter	Conic	Par 0 (unused)	Par 1 (unused)
OBJ	Standard		Infinity	Infinity		0.0000000	0.0000000		
1	Standard		Infinity	10.0000000		5.0000000	0.0000000		
STO	Paraxial			0.0000000		5.0000000			50.0000000
3	Standard		Infinity	1.0000000		5.0000000	0.0000000		
4	Standard		17.0884803 A	1.0000000	SF57	4.8303111	0.0000000		
5	Standard		25.2392874 N	25.2392874		4.6898661	0.0000000		
IMA	Standard		Infinity	-		1.776E-015	0.0000000		



If a single ray trace is performed, we get the direction cosine of the marginal ray to be 0.1858. It can also be seen, that the marginal ray is concentric at the surface 5.

3: Ray Trace					
Update Settings Print Window					
Ray Trace Data					
File : C:\Gross\Lectures - Gross\Optical Design with Zemax\Aplanatic lens.ZMX					
Title:					
Date : 13.11.2012					
Units : Millimeters					
Wavelength : 0.500000 μm					
Coordinates : Local					
Direction cosines are after refraction or reflection from the surface or object.					
Angles are in degrees.					
Normalized X Field Coord (Hx) : 0.0000000000					
Normalized Y Field Coord (Hy) : 0.0000000000					
Normalized X Pupil Coord (Px) : 0.0000000000					
Normalized Y Pupil Coord (Py) : 1.0000000000					
Real Ray Trace Data:					
Surf	X-coordinate	Y-coordinate	Z-coordinate	X-cosine	Y-cosine
OBJ	Infinity	Infinity	Infinity	0.0000000000	0.0000000000
1	0.0000000000E+000	5.0000000000E+000	0.0000000000E+000	0.0000000000	0.0000000000
2	0.0000000000E+000	5.0000000000E+000	0.0000000000E+000	0.0000000000	-0.0995037190
3	0.0000000000E+000	5.0000000000E+000	0.0000000000E+000	0.0000000000	-0.0995037190
4	0.0000000000E+000	4.8303110655E+000	6.9688934549E-001	0.0000000000	-0.1858161068
5	0.0000000000E+000	4.6898661252E+000	4.3955384906E-001	0.0000000000	-0.1858161068
6	0.0000000000E+000	1.7763568394E-015	0.0000000000E+000	0.0000000000	-0.1858161068
Paraxial Ray Trace Data:					

The best image position can be obtained by

1. Quick focus option
2. Solve at the last surface with marginal ray height 0
3. Pick up on the last (concentric) surface radius
4. Optimizing the last thickness as a variable with minimal spot size

b) If the Zernike polynomials are calculated, they are exactly zero for all orders.

2: Zernike Fringe Coefficients		
Update Settings Print Window		
Wavelength : 0.5000 μm		
Peak to Valley (to chief) : 0.00000001 waves		
Peak to Valley (to centroid) : 0.00000001 waves		
RMS (to chief) : 0.00000000 waves		
RMS (to centroid) : 0.00000000 waves		
Variance : 0.00000000 waves		
Strehl Ratio (Est) : 1.00000000		
RMS fit error : 0.00000000 waves		
Maximum fit error : 0.00000001 waves		
Z 1	0.00000000	: 1
Z 2	0.00000000	: (p) * COS (A)
Z 3	0.00000000	: (p) * SIN (A)
Z 4	0.00000000	: (2p ² - 1)
Z 5	0.00000000	: (p ²) * COS (2A)
Z 6	0.00000000	: (p ²) * SIN (2A)
Z 7	0.00000000	: (3p ² - 2) p * COS (A)
Z 8	0.00000000	: (3p ² - 2) p * SIN (A)
Z 9	0.00000000	: (6p ⁴ - 6p ² + 1)
Z 10	0.00000000	: (p ³) * COS (3A)
Z 11	0.00000000	: (p ³) * SIN (3A)
Z 12	0.00000000	: (4p ² -3) p ² * COS (2A)
Z 13	0.00000000	: (4p ² -3) p ² * SIN (2A)
Z 14	0.00000000	: (10p ⁴ - 12p ² + 3) p * COS
Z 15	0.00000000	: (10p ⁴ - 12p ² + 3) p * SIN
Z 16	0.00000000	: (20p ⁶ - 30p ⁴ + 12p ² - 1)
Z 17	0.00000000	: (p ⁴) * COS (4A)
Z 18	0.00000000	: (p ⁴) * SIN (4A)
Z 19	0.00000000	: (5p ² - 4) p ³ * COS (3A)
Z 20	0.00000000	: (5p ² - 4) p ³ * SIN (3A)
Z 21	0.00000000	: (15p ⁴ - 20p ² + 6) p ² * COS
Z 22	0.00000000	: (15p ⁴ - 20p ² + 6) p ² * SIN
Z 23	0.00000000	: (35p ⁶ - 60p ⁴ + 30p ² - 4)
Z 24	0.00000000	: (35p ⁶ - 60p ⁴ + 30p ² - 4)
Z 25	0.00000000	: (70p ⁸ - 140p ⁶ + 90p ⁴ - 2)
Z 26	0.00000000	: (p ⁵) * COS (5A)
Z 27	0.00000000	: (p ⁵) * SIN (5A)
Z 28	0.00000000	: (6p ² - 5) p ⁴ * COS (4A)
Z 29	0.00000000	: (6p ² - 5) p ⁴ * SIN (4A)
Z 30	0.00000000	: (21p ⁴ - 30p ² + 10) p ³ * COS
Z 31	0.00000000	: (21p ⁴ - 30p ² + 10) p ³ * SIN
Z 32	0.00000000	: (56p ⁶ - 105p ⁴ + 60p ² - 1)
Z 33	0.00000000	: (56p ⁶ - 105p ⁴ + 60p ² - 1)
Z 34	0.00000000	: (126 p ⁸ - 280p ⁶ + 210p ⁴ - 35p ² + 3)
Z 35	0.00000000	: (126 p ⁸ - 280p ⁶ + 210p ⁴ - 35p ² + 3)
Z 36	0.00000000	: (252p ¹⁰ - 630p ⁸ + 560p ⁶ - 210p ⁴ + 35p ² - 3)

c) If a field of 2° is introduced and the Zernike coefficients are calculated for the field point in the image and behind the 4th surface (the aplanatic), we get the following picture:

```

Surface          : 4
Field            : 2.0000 (deg)
Wavelength       : 0.5000  $\mu$ m
Peak to Valley (to chief) : 0.42436528 waves
Peak to Valley (to centroid) : 0.41949035 waves
RMS (to chief)   : 0.11961177 waves
RMS (to centroid) : 0.11958708 waves
Variance         : 0.01430107 waves squared
Strehl Ratio (Est) : 0.56859687

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```

RMS fit error      : 0.00000000 waves
Maximum fit error  : 0.00000001 waves

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```

Z 1 0.20794872 : 1
Z 2 0.00000000 : (p) * COS (A)
Z 3 0.00490450 : (p) * SIN (A)
Z 4 0.20837470 : (2p^2 - 1)
Z 5 -0.00021696 : (p^2) * COS (2A)
Z 6 0.00000000 : (p^2) * SIN (2A)
Z 7 0.00000000 : (3p^2 - 2) p * COS (A)
Z 8 0.00246706 : (3p^2 - 2) p * SIN (A)
Z 9 0.00042809 : (6p^4 - 6p^2 + 1)
Z 10 0.00000000 : (p^3) * COS (3A)
Z 11 -0.00000102 : (p^3) * SIN (3A)
Z 12 -0.00001191 : (4p^2-3) p^2 * COS (2A)
Z 13 0.00000000 : (4p^2-3) p^2 * SIN (2A)
Z 14 0.00000000 : (10p^4 - 12p^2 + 3) p * COS (A)
Z 15 0.00000997 : (10p^4 - 12p^2 + 3) p * SIN (A)
Z 16 0.00000211 : (20p^6 - 30p^4 + 12p^2 - 1)
Z 17 0.00000000 : (p^4) * COS (4A)
Z 18 0.00000000 : (p^4) * SIN (4A)
Z 19 0.00000000 : (5p^2 - 4) p^3 * COS (3A)
Z 20 -0.00000006 : (5p^2 - 4) p^3 * SIN (3A)

```

```

Surface          : Image
Field            : 2.0000 (deg)
Wavelength       : 0.5000  $\mu$ m
Peak to Valley (to chief) : 0.98906306 waves
Peak to Valley (to centroid) : 0.97834634 waves
RMS (to chief)   : 0.22880861 waves
RMS (to centroid) : 0.22874623 waves
Variance         : 0.05232484 waves squared
Strehl Ratio (Est) : 0.12672931

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```

RMS fit error      : 0.00000000 waves
Maximum fit error  : 0.00000001 waves

```

```

Z 1 0.34701665 : 1
Z 2 0.00000000 : (p) * COS (A)
Z 3 0.01078681 : (p) * SIN (A)
Z 4 0.34699634 : (2p^2 - 1)
Z 5 -0.27812765 : (p^2) * COS (2A)
Z 6 0.00000000 : (p^2) * SIN (2A)
Z 7 0.00000000 : (3p^2 - 2) p * COS (A)
Z 8 0.00539378 : (3p^2 - 2) p * SIN (A)
Z 9 -0.00001686 : (6p^4 - 6p^2 + 1)
Z 10 0.00000000 : (p^3) * COS (3A)
Z 11 -0.00140829 : (p^3) * SIN (3A)
Z 12 0.00065732 : (4p^2-3) p^2 * COS (2A)
Z 13 0.00000000 : (4p^2-3) p^2 * SIN (2A)
Z 14 0.00000000 : (10p^4 - 12p^2 + 3) p * COS (A)
Z 15 0.00000041 : (10p^4 - 12p^2 + 3) p * SIN (A)
Z 16 0.00000345 : (20p^6 - 30p^4 + 12p^2 - 1)
Z 17 0.00000683 : (p^4) * COS (4A)
Z 18 0.00000000 : (p^4) * SIN (4A)
Z 19 0.00000000 : (5p^2 - 4) p^3 * COS (3A)
Z 20 0.00000466 : (5p^2 - 4) p^3 * SIN (3A)

```

In the image, defocus (which is here in field the field curvature) and astigmatism are the dominating aberrations. Directly behind the aplanatic surface, only defocus has a considerable amount. This shows, that the concentric surface limits the system performance by astigmatism and field curvature. The change in the coma of the aplanatic surface is extremely small.