modelling

c 1

INTRODUCTION TO OPTICAL MODELING

Uwe Zeitner

Frank Wyrowski

* login @ computer pool
  seminars start ~~of~~ in the 3rd week

Slides: www.iap.uni-jena.de → upper right corner "teaching"

Optical Modeling and Design

$\boxed{\text{Source of Light}} \longrightarrow \boxed{\text{Light transport}} \longrightarrow \boxed{\text{Detector}}$

Introduction

Modeling:

Source → optical system → $\boxed{?}$

Requires: proper models for the representation of light
proper model which represents the interaction of light & matter

Design:

Source → $\boxed{?}$ → $\boxed{\text{detector}}$

Requires: proper optimization algorithms (not trivial)

Gross "Handbook of Optical Systems"

→ Volume 1 & 2

J. Smith "Modern Optical Engineering"

Goodman "Introduction to Fourier optics"

1.1. Light representation

ght: - particle → photon

- electromagnetic wave

Maxwell's equations

└ in homogeneous & isotropic medium

⇒ wave equation:

$$\nabla^2 \vec{E} - \frac{1}{c^2} \cdot \ddot{\vec{E}} = 0 \qquad (1.1)$$

$\ddot{\vec{E}}$ second derivat to time

$\nabla^2 \vec{E}$ derivate to

s velocity of light in a medium

s electric field

$= \frac{c_0}{\sqrt{\varepsilon \cdot \mu}}$ $(\varepsilon \geq 1)$ ε = dielectric constant (permittivity) ε_{vac}

(metamaterials → ε may be $\varepsilon < 1$)

μ = magnetic permeability $\simeq 1$

in 99% optical materials ar non-mag

c_0 vacuum speed of light

$= \frac{c_0}{n}$ n - refractive index of a material

Solution of (1.1): $\vec{E}(\vec{r}, t) = \vec{E}(\vec{r}) \cdot e^{-i\omega t}$ (1.2)

$\omega = 2\pi\nu$, ν = time frequency of the oscillation

→ harmonic oscillation in time

Inserting (1.2) in (1.1) $\Rightarrow$ $\boxed{\nabla^2 \vec{E}(\vec{r}) + \frac{\omega^2}{c^2} \cdot E(\vec{r}) = 0}$ (1.3)

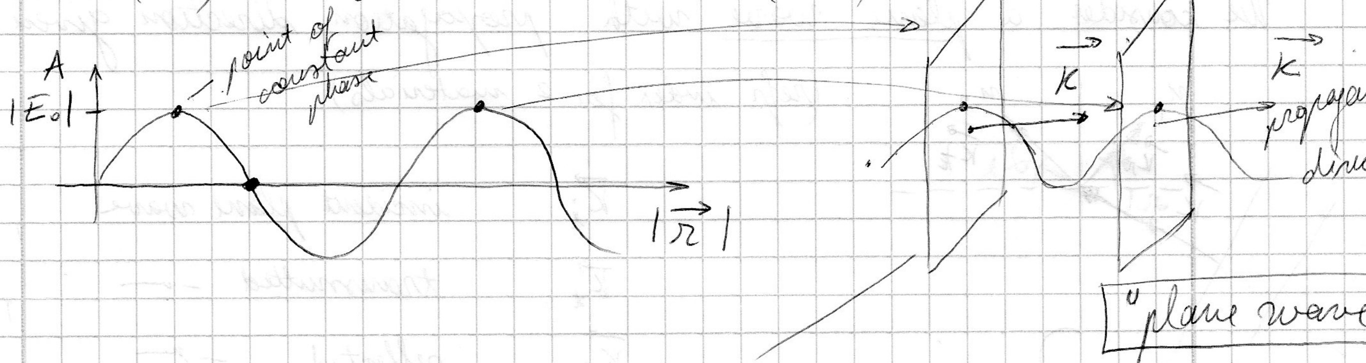
Helmholtz - Equation

$\frac{\omega^2}{c^2} = k^2 = \left(\frac{2\pi}{\lambda}\right)^2$ "modulus of the wave vector" & "wave-number"

$c = \lambda \cdot \nu$

Basic Solution of (1.3)

$\vec{E}(\vec{r}) = \vec{E}_0 \cdot e^{i \cdot \vec{k} \cdot \vec{r}}$ (1.4)



<u>wavefront</u>

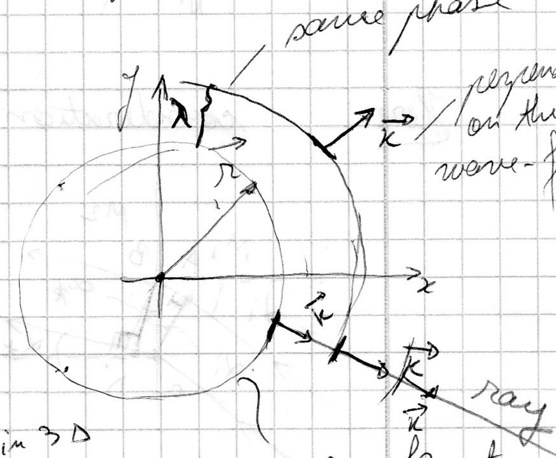
$\Rightarrow$ surface of constant phase

second solution of (1.3):

$\vec{E}(\vec{r}) = \frac{E_0}{r} \cdot e^{ikr}$

cannot go very close to the center (point & source) with this

in 2D
BUT it's SPHERICAL in 3D



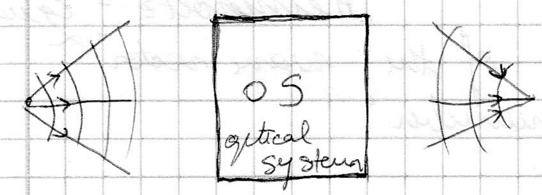
originating from a point source $\leftarrow$ $\boxed{\text{"spherical wave"}}$

$\vec{k}$ points to different directions $\Big| \Rightarrow$ "I cannot have a global $\vec{k}$, like for plane waves

<u>local $\vec{k}$</u> vectors representing the local direction of propagation

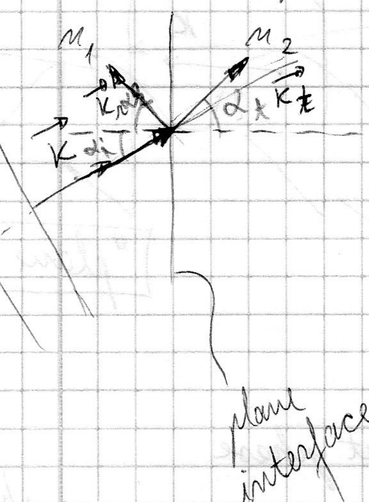
(3)

face is larger than the wavelength. Otherwise → diffraction



1.2. Interaction of light with plane interfaces

We consider a *plane wave* with propagation direction given by $\vec{k}$



(refr. index for 2 materials)

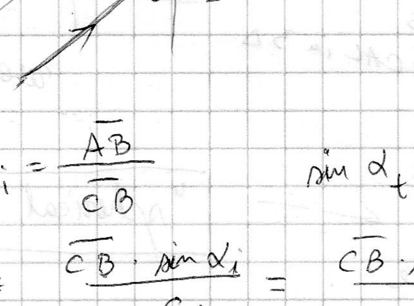
$\vec{k}_i$ incident plane wave , α_i

$\vec{k}_t$ transmitted —"— , α_t

$\vec{k}_r$ reflected —"— , α_r

relations are given by Fresnel's formu

Here: consideration of propagation speed:



same time for travelling from A to B like from C to D

$$\sin\alpha_i = \frac{\overline{AB}}{\overline{CB}} \qquad \sin\alpha_t = \frac{\overline{CD}}{\overline{CB}}$$

$$t = \frac{\overline{CB}\cdot\sin\alpha_i}{c_1} = \frac{\overline{CB}\cdot\sin\alpha_t}{c_2} \qquad c = c_0/n$$

$$\Rightarrow \boxed{n_1\cdot\sin\alpha_i = n_2\sin\alpha_t}$$

law of refraction

Snell's Law

Similar consideration for the reflected wave

$\Rightarrow$ $\boxed{\alpha_i = \alpha_r}$ law of reflection

22.10.2013
Intro to
Optical Modelling

săpt. viitoare : Wednesday $8^{15} - 9^{45}$
(I'm in group 2)

tina.weichelt@uni-jena.de

* Zemax

* Go to the POOL, log in and use "Zeemax" software

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13
27
11
~~25~~
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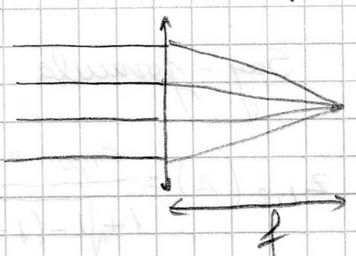
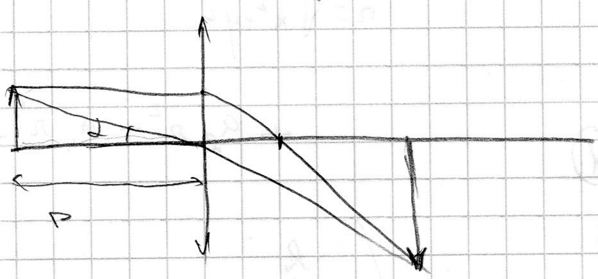




$s \tan\alpha$ = OBJECT SIZE

-3.5
-5

ZEMAX

(Gen) $\boxed{\text{Wav}}$ (λ)

Lens Data Editor

general : Aperture | Units

Layout | Spot Diagram

↳ right click
↳ choose
- Nº of Rays

* Radius
* Thickness: distance between elements
* Glass : (refraction index)

Field Data

Tools → Miscellaneous → Quick Focus

double click on Surface Type ⇒ Standard / Paraxial / ...

1. paraxial lens

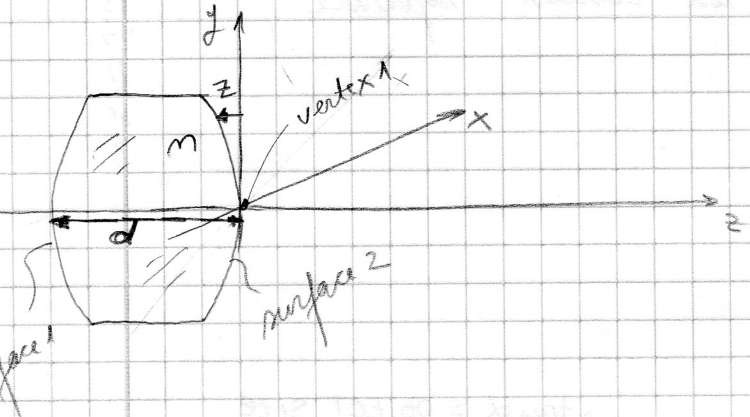
2. real lens | bi-convex

// STOP : sets the aperture:

the rays are blocked

2. MODELLING AND DESIGN OF LENS-SYSTEMS

Lens : - rotational symmetric element
- transparent material, refractive index n
- spherical or conical surfaces



Sag-formula

$$z_{1,2}(r) = \frac{c_{1,2} \cdot r^2}{1+\sqrt{1-(1+k_{1,2})\cdot c}}$$

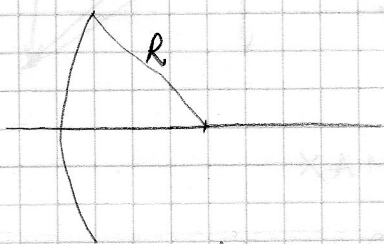
$r = \sqrt{x^2+y^2}$

$+ a_2 r^2 + a_4 r^4 + \ldots$

thickness, distance of ~~...~~ vertex points (d)

curvature $(c_{1,2}) = \frac{1}{R_{1,2}}$ d

radius of curvature (R)



conic constant (k) :

$k < -1 \Rightarrow$ hyperbola

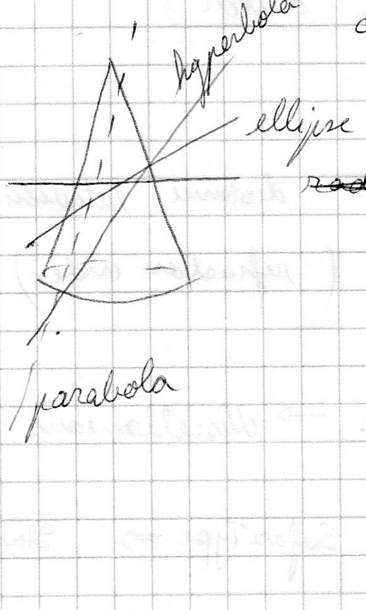
$k = -1 \Rightarrow$ parabola

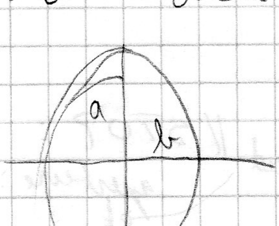
$-1 < k < 0 \Rightarrow$ ellipse

$k = 0 \Rightarrow$ sphere

$k > 0 \Rightarrow$ oblate ellipse

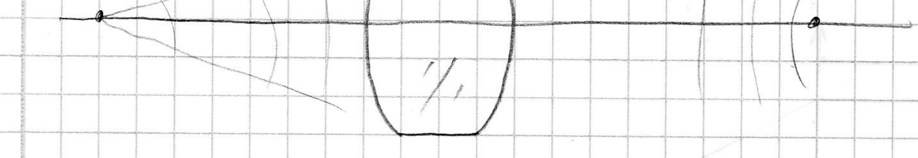
cross sections





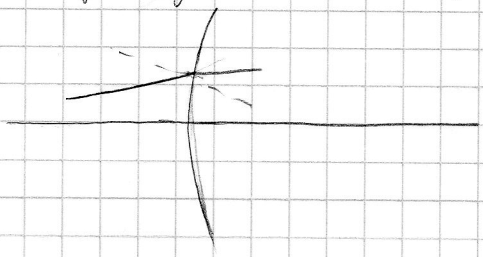
$$\frac{1}{c} = R = \pm \frac{b^2}{a}$$

$$k = -\left(1 - \frac{b^2}{\ldots}\right)$$



1. Paraxial Approximation, Gaussian Optics

→ ray angles with respect to the optical axis are small



$n \sin \alpha = n' \sin \alpha'$ (

paraxial approximation

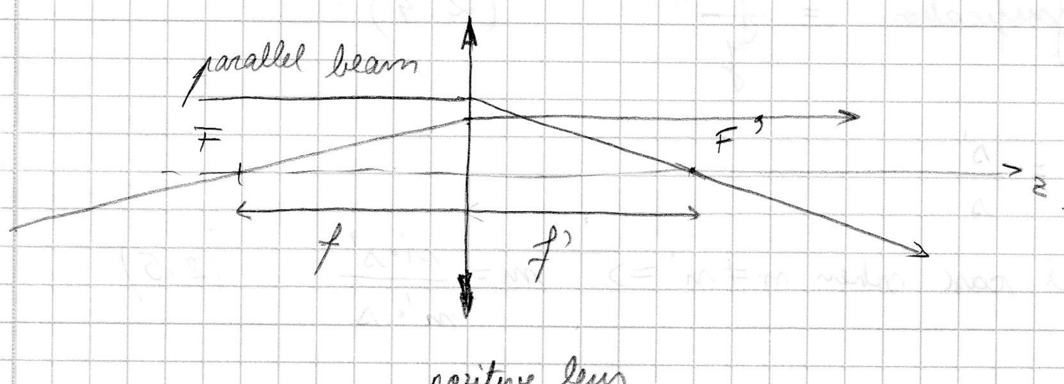
α, α' small ⇒

$\Rightarrow n \cdot \alpha = n' \cdot \alpha'$ (2.3)

$\cos \alpha \simeq 1$

2. Ideal lens

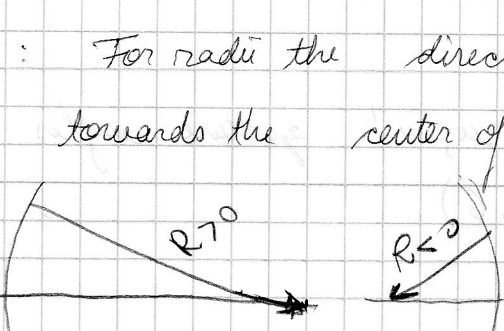
– assuming that the effect of a lens is taking place in a single p



positive lens

Sign convention in optics : distances along the optical axis a
positive if they are oriented in
sense of a vector in positive

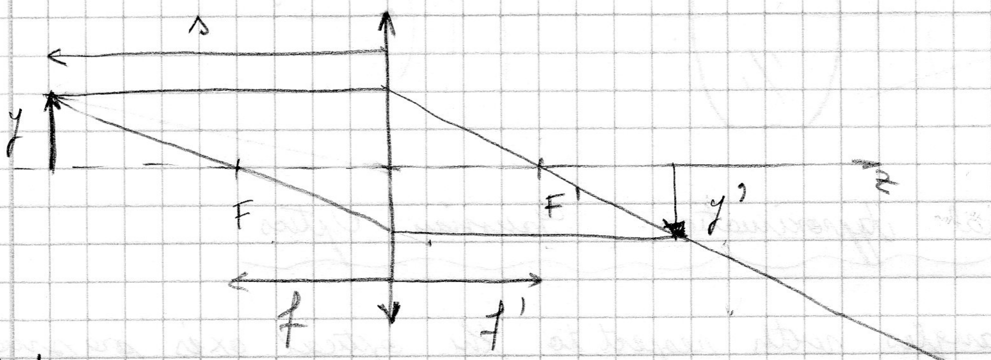
Attention : For radii the direction is oriented from the surface
towards the center of curvature !



$f < 0$

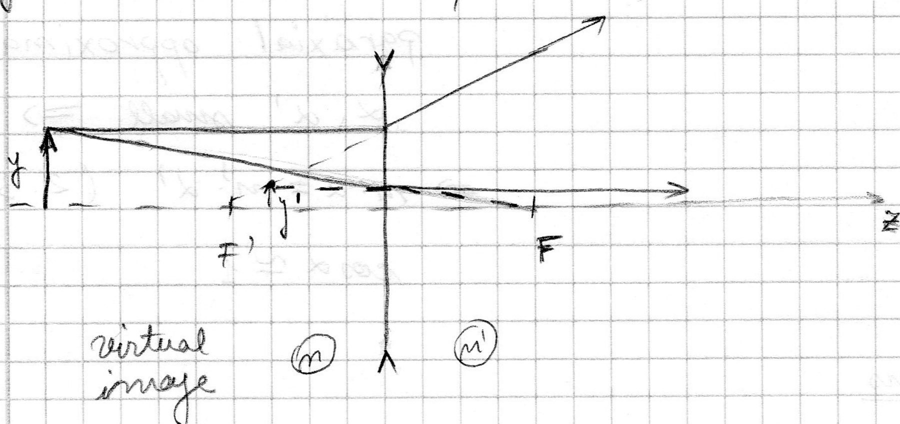
$f' > 0$

- "positive lens"



$f' > 0$

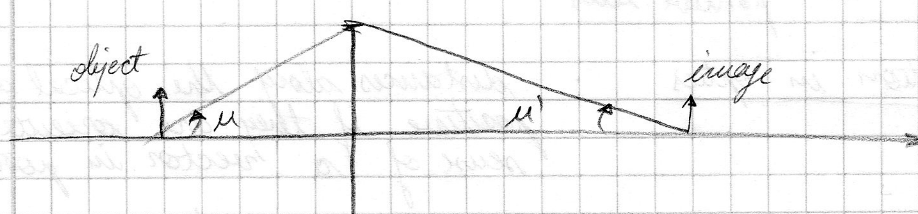
- "negative lens" $\quad f' < 0$



$$m = \text{magnification} = \frac{y'}{y} \qquad (2.4)$$

$$m = \frac{y'}{y} = \frac{s'}{s}$$

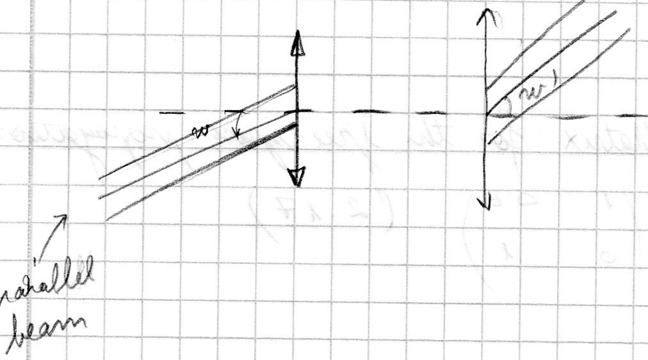
More general case when $n \neq n' \Rightarrow \quad m = \frac{n \cdot s'}{n' \cdot s} \qquad (2.5)$



$$\boxed{m = \frac{n \cdot u}{n' \cdot u'}}$$

u, u' aperture angles

(2.6)



$$\Gamma = \frac{w'}{w} \quad (2.7)$$

Lens equation

$$\frac{f'}{s'} + \frac{f}{s} = 1 \qquad (2.8)$$

in general $\frac{f'}{n'} = \frac{f}{n}$ (2.9)

$n = n' \Rightarrow \boxed{\frac{1}{s'} + \frac{1}{s} = \frac{1}{f}}$ lens maker's equation (2.10)

$L = s - s'$ .. object-image distance (2.11)

|| if we need equations, we can find them in a table

2.3. ABCD - Matrix formalism

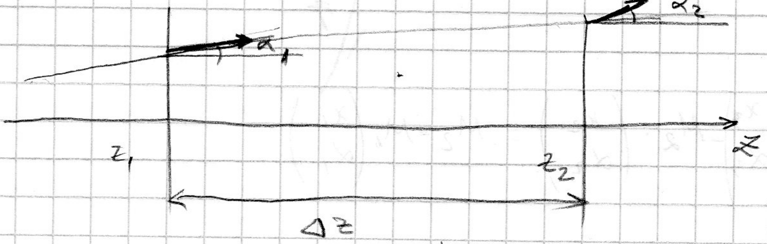
2.3.1. Derivation of the formalism

consider a free-space-propagation of a range between planes lo-

z_1 and $z_2 = z_1 + \Delta z$

a) free space propagation

(no diffraction)



ray coordinates: x, α -

in p

↳ parameters in z_2

$x_2 = x_1 + \Delta z \cdot$

(

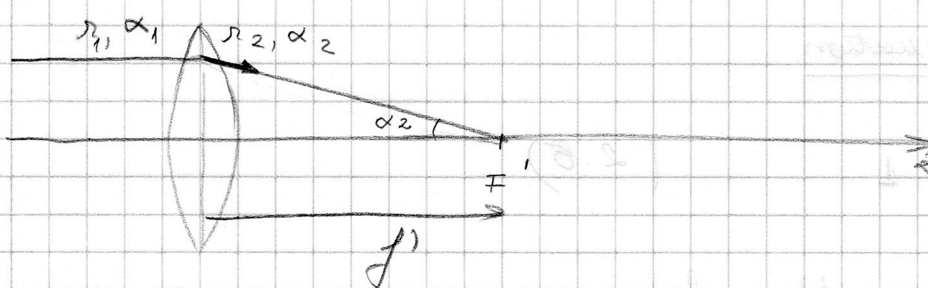
$\alpha_2 = \alpha_1$ (2.1

$$\begin{pmatrix} \dots \\ \alpha_2 \end{pmatrix} = \underbrace{\begin{pmatrix} \dots & \dots \\ 0 & 1 \end{pmatrix}}_{M_{\Delta z}} \begin{pmatrix} \dots \\ \alpha_1 \end{pmatrix} = M_{\Delta z} \begin{pmatrix} \dots \\ \alpha_1 \end{pmatrix}$$

$M_{\Delta z} \longrightarrow$ ABCD-Matrix for the free-space propagation

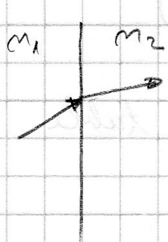
$$M_{\Delta z} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & \Delta z \\ 0 & 1 \end{pmatrix} \quad (2.17)$$

b) The thin lens:



$$\begin{cases} r_2 = r_1 \\ \alpha_2 = -\dfrac{r_1}{f} + \alpha_1 \end{cases} \qquad M_f = \begin{pmatrix} 1 & 0 \\ -\dfrac{1}{f'} & 1 \end{pmatrix} \quad (2.18)$$

c) Transition at a plane interface

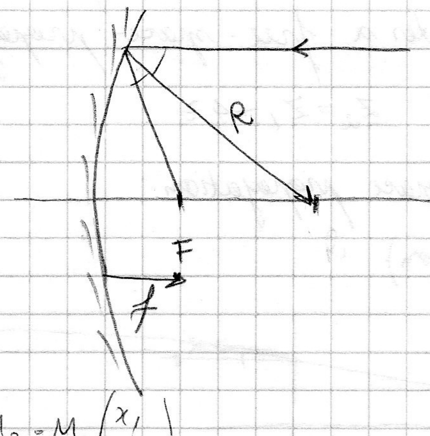


$$M_{refraction} = \begin{pmatrix} 1 & 0 \\ 0 & \dfrac{n_1}{n_2} \end{pmatrix}$$

$$\frac{\sin \alpha_1}{\sin \alpha_2} = \frac{n_2}{n_1} \qquad \alpha_2 = \alpha_1 \cdot \frac{n_1}{n_2}$$

d) Curved mirror, $R = 2f$

$$M_M = \begin{pmatrix} 1 & 0 \\ -\dfrac{2}{R} & 1 \end{pmatrix}$$



$$\begin{pmatrix} x_2 \\ \alpha_2 \end{pmatrix} = M_1 \begin{pmatrix} x_1 \\ \alpha_1 \end{pmatrix} \quad \begin{pmatrix} x_3 \\ \alpha_3 \end{pmatrix} = M_2 \cdot \begin{pmatrix} x_2 \\ \alpha_2 \end{pmatrix} = M_2 \cdot M_1 \begin{pmatrix} x_1 \\ \alpha_1 \end{pmatrix}$$

$$\begin{pmatrix} x_2 \\ \alpha_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & \Delta z \\ 0 & 1 \end{pmatrix}}_{M_{\Delta z}} \cdot \begin{pmatrix} x_1 \\ \alpha_1 \end{pmatrix} = M_{\Delta z} \cdot \begin{pmatrix} x_1 \\ \alpha_1 \end{pmatrix}$$

$M_{\Delta z} \longrightarrow$ ABCD-Matrix for the free-space propagati

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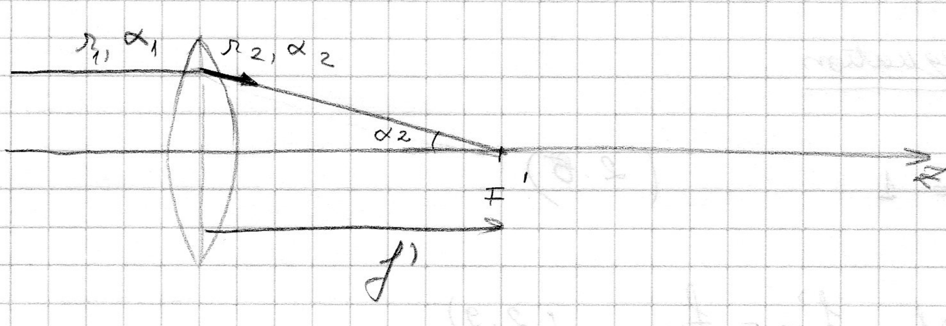

$$\begin{cases} r_2 = r_1 \\ \alpha_2 = -\dfrac{r_1}{f} + \alpha_1 \end{cases} \qquad M_f = \begin{pmatrix} 1 & 0 \\ -\dfrac{1}{f'} & 1 \end{pmatrix} \qquad (2.18)$$

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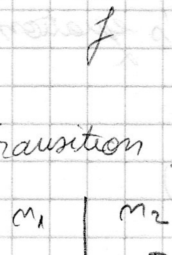

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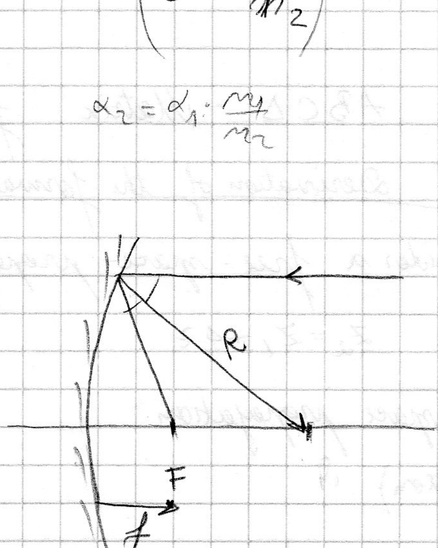
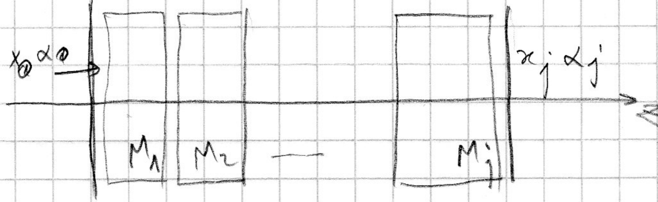
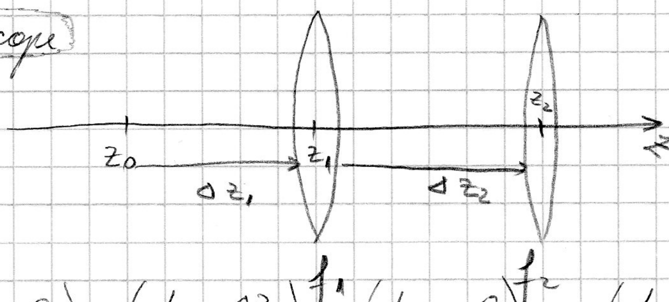




$$M_{sys} = M_j \cdot M_{j-1} \cdot \ldots \cdot M_2 \cdot M_1 \,.$$

Example: Telescope



$$M_{sys} = \underbrace{\begin{pmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{pmatrix}}_{M_4} \cdot \underbrace{\begin{pmatrix} 1 & \Delta z_2 \\ 0 & 1 \end{pmatrix}}_{M_3} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{pmatrix}}_{M_2} \cdot \underbrace{\begin{pmatrix} 1 & \Delta z_1 \\ 0 & 1 \end{pmatrix}}_{M_1} =$$

propagation in free space

$$= \begin{pmatrix} 1 - \frac{\Delta z_2}{f_1} & \Delta z_1 + \Delta z_2 - \Delta z_1 \cdot \frac{\Delta z_2}{f_1} \\ \frac{\Delta z_2}{f_1 f_2} - \frac{1}{f_1} - \frac{1}{f_2} & \left(1 - \frac{\Delta z_2}{f_2}\right)\left(1 - \frac{\Delta z_1}{f_1}\right) - \frac{\Delta z_1}{f_2} \end{pmatrix}$$

2.3.2. General Properties of ABCD-Matrices

1) mathematical property

Determinant M

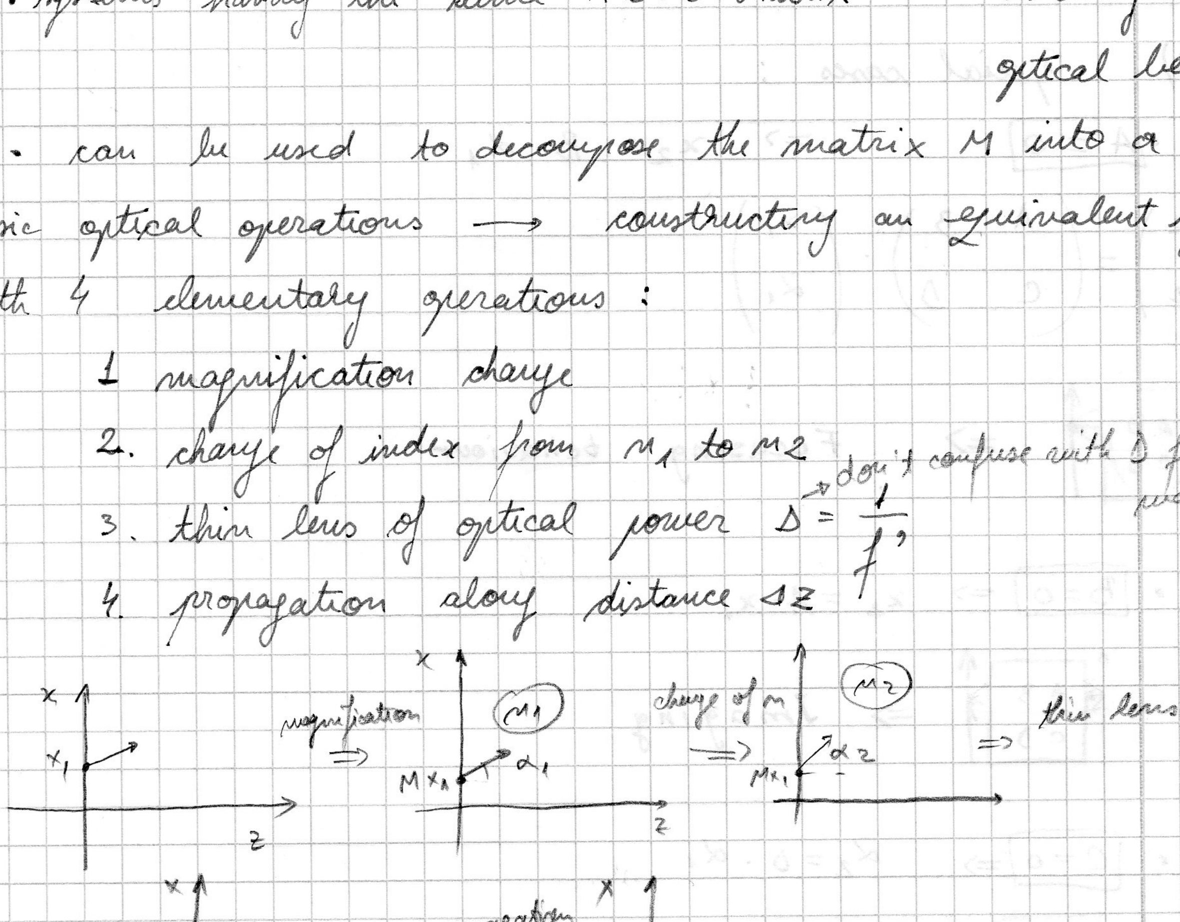
$|M| = AD - BC = \frac{n_1}{n_2}$ → only 3 independent components of M

2) Equivalent optical system

- systems having the same ABCD-Matrix → showing the same optical behaviour

- can be used to decompose the matrix M into a set of basic optical operations → constructing an equivalent system with 4 elementary operations:

1. magnification change
2. change of index from n_1 to n_2
3. thin lens of optical power $\Delta = \frac{1}{f'}$ ← don't confuse with D from matrix!
4. propagation along distance Δz



$$M_{eq} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & \Delta z \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\frac{1}{f' n_2} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix} \begin{pmatrix} M & 0 \\ 0 & \frac{1}{M} \end{pmatrix}$$

↑ lens is embedded in a medium of index n_2.

⇒ 4 variables: $\boxed{\frac{n_1}{n_2} = AD - BC}$ $\boxed{M = \frac{AD - BC}{D}}$

$\boxed{\frac{1}{f' n_2} = -\frac{C D}{AD - BC} = -\frac{C}{M}}$ $\boxed{\Delta z = \frac{B}{D}}$

* these operations can also be applied on arbitrary fields $\vec{E}$.
Considerations are not restricted to rays.

Diffraction integral (like Fresnel diffraction formula) in combination with ABCD matrix $\Rightarrow$ Collins Integral

If $D = 0$ $\Rightarrow$ another arrangement of the above sequence solves the singularity.

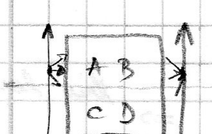
3) Special cases:

- $\boxed{A = 0}$ $\Rightarrow$ $x_2 = B \cdot \alpha_1$

$$\begin{pmatrix} x_2 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ \alpha_1 \end{pmatrix}$$

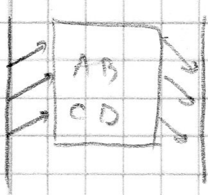
$\Rightarrow$ Focusing behaviour

- $\boxed{B = 0}$ $\Rightarrow$ $x_2 = A \cdot x_1$



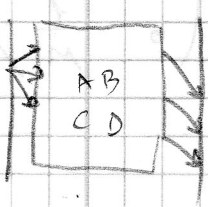
$\Rightarrow$ Imaging

- $\boxed{C = 0}$ $\Rightarrow$ $\alpha_2 = D \cdot \alpha_1$



$\Rightarrow$ Deflection

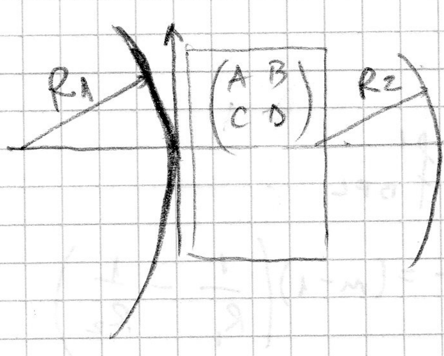
- $\boxed{D = 0}$ $\Rightarrow$ $\alpha_2 = C \cdot x_1$



Collimation

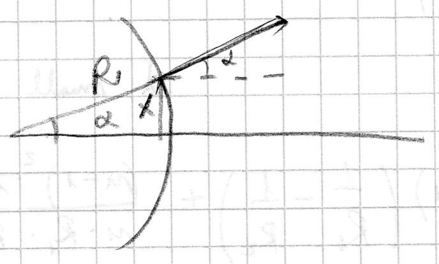
4) Transformation of a spherical wave

– usual waves in imaging systems !



R_1 : radius of a spherical wave entering the system

Question : $R_2 = ?$



small angles :

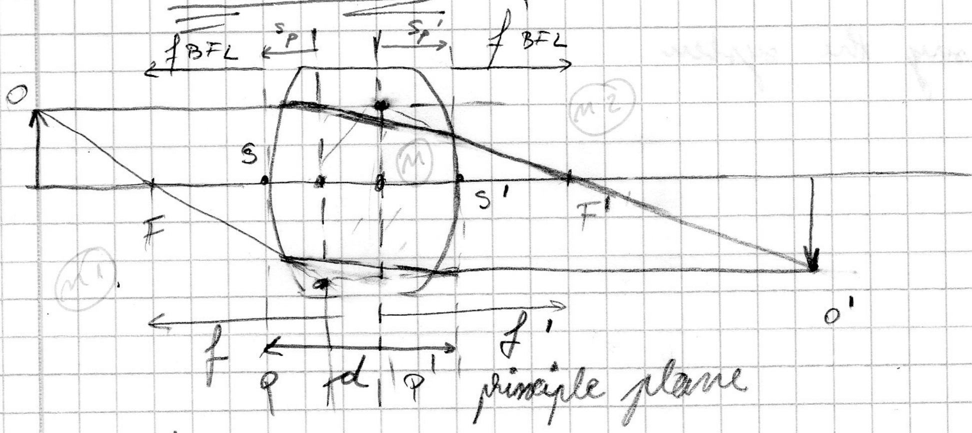
$\tan\alpha \simeq \alpha = \dfrac{x}{R} \Rightarrow \begin{array}{l} x_1 = R_1\,\alpha_1 \\ x_2 = R_2\,\alpha_2 \end{array}$

- put into the definition of the ABCD matrix (2.16)

$$\Rightarrow \quad R_2 = \frac{A\cdot R_1 + B}{C\cdot R_1 + D}$$

Example : lens f' $\Rightarrow \dfrac{1}{R_2} = \dfrac{1}{R_1} - \dfrac{1}{f'}$

2.4. Real Lenses



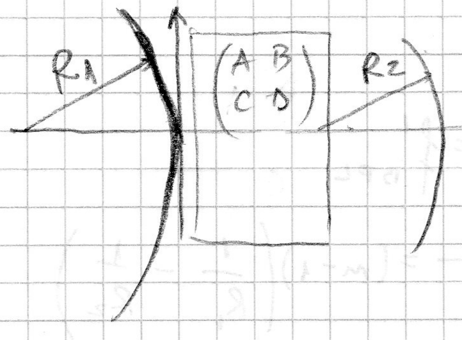
P, P' = principle planes

f_{BFL}, f'_{BFL} = back focal length

refractive power $\quad \phi = -\dfrac{n_1}{f} = \dfrac{n_2}{f'}$

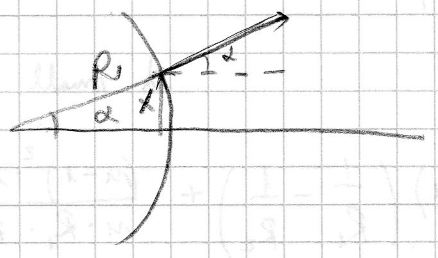
4) Transformation of a spherical wave

– usual waves in imaging systems!



R_1: radius of a spherical wave entering the system

Question : $R_2 = ?$



small angles:

$\tan\alpha \simeq \alpha = \frac{x}{R} \Rightarrow x_1 = R_1\,\alpha_1$

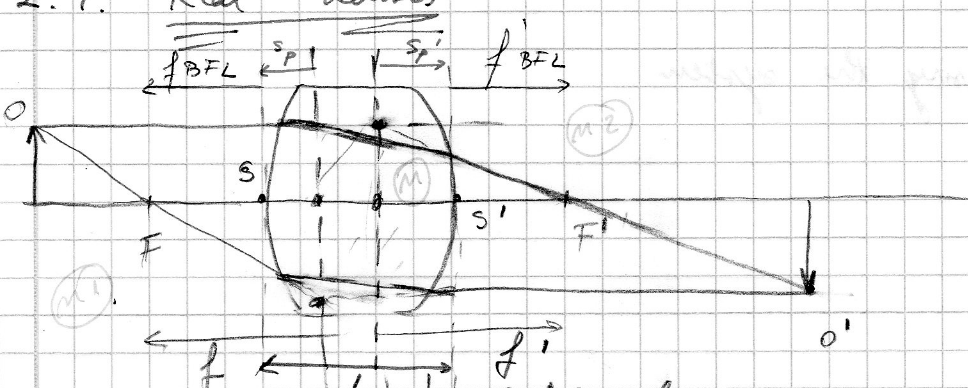
$x_2 = R_2\,\alpha_2$

– put into the definition of the ABCD matrix (2.16)

$$\Rightarrow R_2 = \frac{A \cdot R_1 + B}{C \cdot R_1 + D}$$

Example: lens f' $\Rightarrow \frac{1}{R_2} = \frac{1}{R_1} - \frac{1}{f'}$

2.4. Real Lenses



P, P' = principle planes

f_{BFL}, f'_{BFL} = back focal length

refractive power $\phi = -\frac{n_1}{f} = \frac{n_2}{f'}$

"thin lens" : radii of curvature of lens surface is large compared to d : $|C_{1/2} \cdot d| \ll 1$

curvature

, the principle planes coincide and $f' = f'_{BFL}$

If $n_1 = n_2 = 1$ (lens in air) $\Rightarrow \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

d small

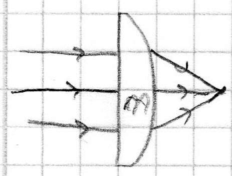
d not negligible $\quad \phi = \frac{1}{f'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{(n-1)^2 \cdot d}{n \cdot R_1 \cdot R_2}$

Position of principle planes :

$$s_p = \frac{R_1 \cdot d}{(n-1)d + n(R_2 - R_1)}$$

$$s_{p'} = - \frac{R_2 \cdot d}{(n-1)d + n(R_1 - R_2)}$$

this was analysing the system



$n = f(\lambda)$

2.5. Optical Materials

- Glasses
- Crystals
- Plastics
- Liquids
- Gases
- Glues or Cements

Here we restrict to isotropic materials
dielectric materials

$$\tilde{n}(\lambda) = \underbrace{n(\lambda)}_{\text{refraction}} + i \cdot \underbrace{K(\lambda)}_{\text{absorption}}$$ complex refractive ind

These parameters are given at normal conditions: $T = 293\,K$
$p = 1013\,m$

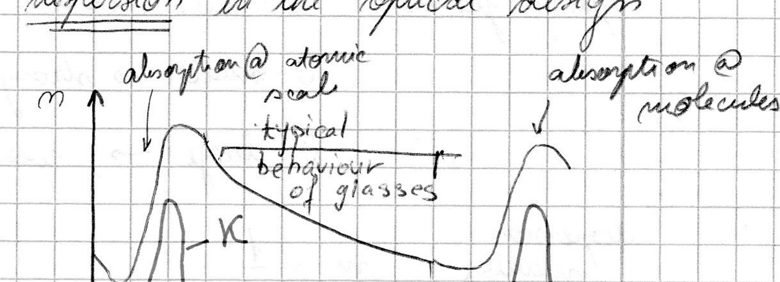
$\Rightarrow \frac{dn}{dT}$ thermooptical coefficient

2.5.1. Treatment of dispersion in the optical design

dispersion $n = f(\lambda)$

$\frac{dn}{d\lambda} < 0$ NORMAL DISPERSION

(for all transparent materials)



$\frac{1}{f} = (n-1) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$; $\lambda \searrow \Rightarrow n \nearrow \Rightarrow f \searrow$

$f_2 > f_1$ because $\lambda_1 < \lambda_2$

$d\lambda$

In optical design, dispersion is characterized by 3 wavelengths:

[pr]imary wavelength λ_0

[t]wo secondary colors λ_1, λ_2

VIS : 1^{st} choice for λ_0: $\lambda_e = 546.1$ nm, $\lambda_{F'} = 480$ nm, $\lambda_{C'} = 643.8$ nm — typical choice for Microscopy ν_e

2^{nd} choice for λ_0: $\lambda_d = 587.6$ nm, $\lambda_F = 486.1$ nm, $\lambda_C = 656.3$ nm — for Photogra[phy] application ν_d

wavelength are red-shifted because ... are more se[nsitive] to longer λ.

Characterisation of the dispersion by a single number

Abbe number : $\nu = \dfrac{n_{\lambda_0} - 1}{n_{\lambda_1} - n_{\lambda_2}}$

vacuum: $n = 1 \simeq$ air

optical glasses : $\nu = 20 \dots 120$

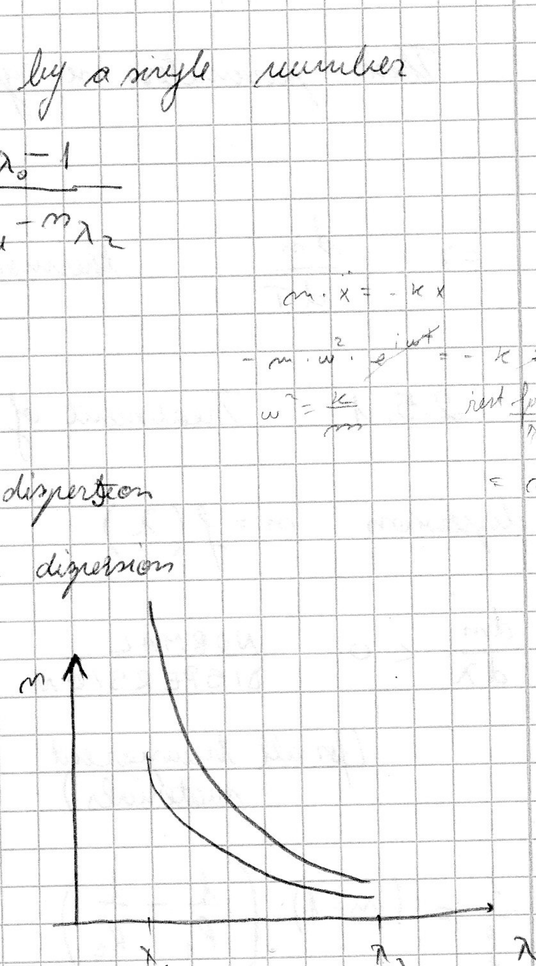
ν small $\rightarrow$ strong dispersion

ν large $\rightarrow$ weak dispersion

dispersion number : $n_e = \dfrac{1}{\nu_e}$

Typical behaviour :

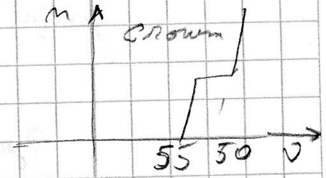
$n \uparrow \Rightarrow \dfrac{dn}{d\lambda} \uparrow$



Historical distinction:

- crown glass : $\nu > 55$, $n < 1.6$
  $\nu > 50$, $n > 1.6$

- flint glass : $\nu < 55$, $n < 1.6$
  $\nu < 50$, $n > 1.6$



Fit equations for $n(\lambda)$

Sellmeier 1: $$n(\lambda) = \sqrt{\frac{K_1 \cdot \lambda^2}{\lambda^2 - L_1} + \frac{K_2 \lambda^2}{\lambda^2 - L_2} + \frac{K_3 \lambda^3}{\lambda^2 - L_3} + 1}$$

K_3 - fitting parameters

Sellmeier 2: $$n(\lambda) = \sqrt{A + \frac{B_1 \lambda^2}{\lambda^2 - \lambda_1} + \frac{B_2 \lambda^2}{\lambda^2 - \lambda_2} + 1}$$

λ_1 here $\neq \lambda_1$ up

others: Schott - Formula

Herzberger - Formula

2.5.2. Design of an achromatic lens

- calculating a <u>doublet</u> lens with reduced chromatic aberation

$$\Phi = \frac{1}{f'} = \underset{n(\lambda)}{(n-1)} \underbrace{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}_{\text{geometrical parameters} \neq f(\lambda)}$$

$$\Phi = (n-1) \cdot A$$

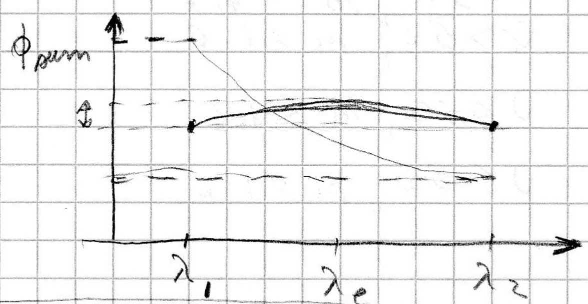
() neglecting the distance between them: $\Phi_{sum} = \Phi_1 + \Phi_2$

(check "=" with matrix)

$$\Phi_{sum} = (n_1 - 1) A_1 + (n_2 - 1) A_2$$

central wavelength λ_e $$\begin{cases} \Phi_1 = (n_{1e} - 1) A_1 \\ \Phi_2 = (n_{2e} - 1) A_2 \end{cases}$$

general $\phi_{sum}(\lambda) = [n_1(\lambda) - 1] A_1 + [n_2(\lambda) - 1] A_2$ (*)



$$\boxed{\phi_{sum}(\lambda_{F'}) \overset{!}{=} \phi_{sum}(\lambda_{C'})} \quad (**)$$ same refractive power for 2 wavelengths

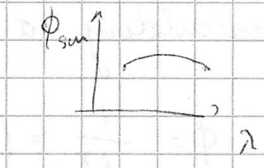
Inserting (*) into (**) and replace the A_1 & A_2 with $\bar{\phi}_1$ and $\bar{\phi}_2$, respectively

$$\Rightarrow \underbrace{\frac{n_{1F'} - n_{1C'}}{n_{1e} - 1}}_{1/\nu_{e_1}} \cdot \bar{\phi}_1 = \underbrace{\frac{n_{2F'} - n_{2C'}}{n_{2e} - 1}}_{1/\nu_{e_2}} \cdot \bar{\phi}_2$$

$$\Rightarrow \boxed{\frac{\bar{\Phi}_1}{\nu_1} + \frac{\bar{\Phi}_2}{\nu_2} = 0}$$

Condition of achromacy

$\Rightarrow$



$\cancel{\bar{\Phi}_1 = \frac{\bar{\Phi}_2}{\nu_2}}$

ens: $\bar{\Phi}_1 = \frac{\nu_1}{\nu_1 - \nu_2} \cdot \phi_{sum}$

lens: $\bar{\Phi}_2 = \frac{\nu_2}{\nu_2 - \nu_1} \cdot \phi_{sum}$

$\nu_1 \neq \nu_2$! otherwise the equations are not solvable

$\Rightarrow$ need - different materials with distinct ν_1 and ν_2.

- combination of a positive and a negative lens

$\nu_1 - \nu_2 = (-1) \cdot (\nu_2 - \nu_1)$; $\nu_1, \nu_2 > 0$

Example

BK7 glass : $n_e = 1.518$

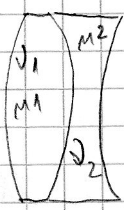
$\nu_e = 63.9$

SF6 $n_e = 1.805$

$\nu_e = 25.4$

$f' \overset{!}{=} 200$ mm (! = define) $\Rightarrow$ $f_1' = 120.5$ mm

$f_2' = -303.1$ mm



$$\frac{1}{f'} = (n-1) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Assume : • $R_1 = -R_2$ for lens 1 $f = 200$ mm

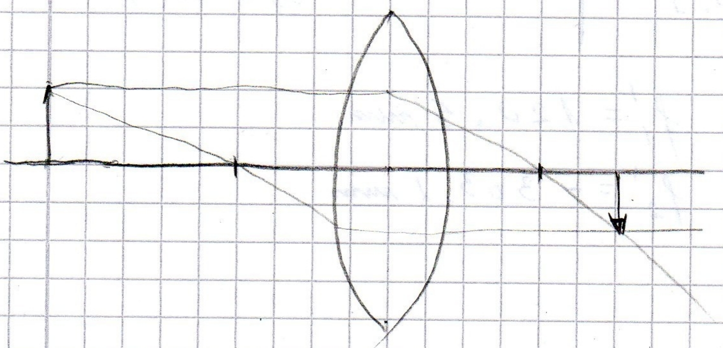
$\Leftrightarrow$ $R_1 = 124.8$ mm
$R_2 = -124.8$ mm

• for lens 2 we have no air gap $\Rightarrow$

$R_1 = -124.8$ mm

$R_2 = +255.5$ mm

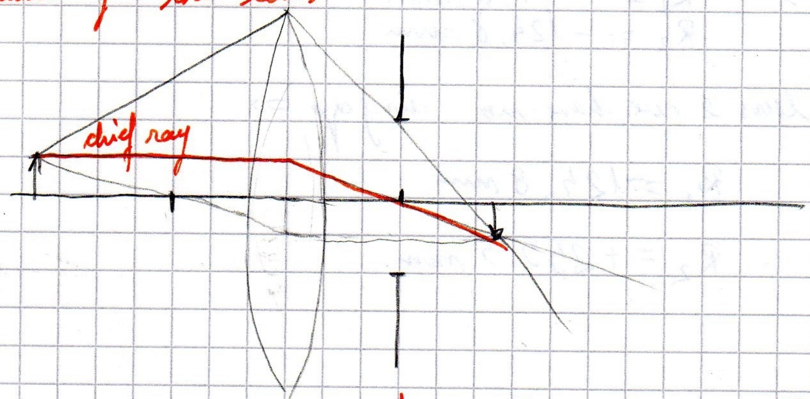
6. Imaging Systems



Essential element of each imaging system : the stop/aperture

- stop (as the limiting aperture) can be located at **any** position in the system

* One special position : $\boxed{\text{stop}}$ is located at the back focal plane of the lens



↳ the ray cone on the object side is symmetric to the ray parallel to the optical axis.

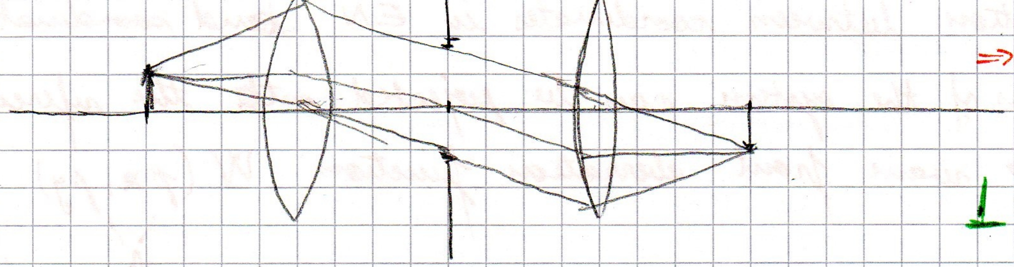
⇓

object-side **telecentric**

$\boxed{\text{Chief ray}}$: the ray that goes through the center of the stop

\+ reverse arrangement : stop in the front focal-plane

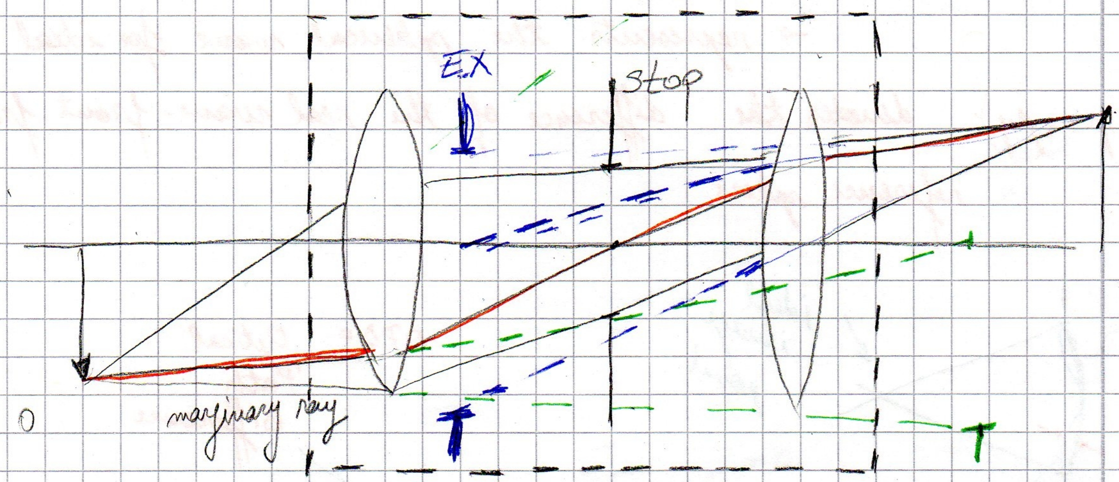
→ image side telecentric system



=> both-side telecentric

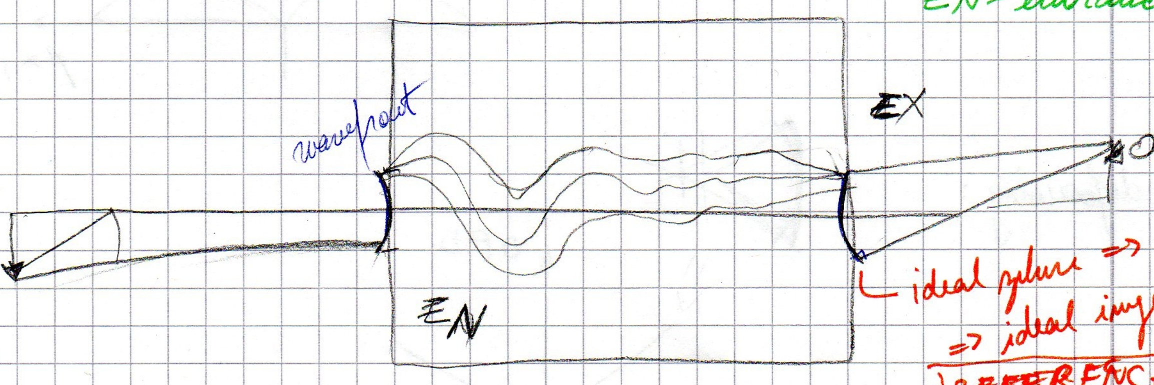
EN

• general two lens systems



EX - exit pupil

EN - entrance pupil



└ ideal sphere =>
=> ideal image point
=> REFERENCE SPHERE

the concept of EN and EX combines the computational elegance of the ray-tracing based propagation with the power of a wave optical analysis of imaging systems

wavefront: equal phase !

EX and EN are images of the same limiting aperture of the system.

→ strict relation between coordinates in EN ~~and~~ and coordinates in EX

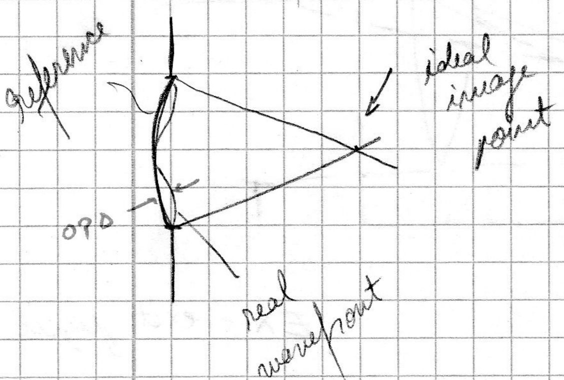
→ aberrations of the system can be projected onto the reference sphere in EX → wave front aberration function $W(p_x, p_y)$

pupil coordinates

Reference sphere : located in EX

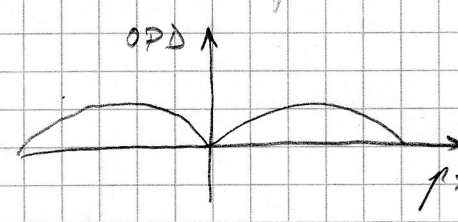
→ represents the spherical wave for ideal imaging

$W(p_x, p_y)$ denotes the difference of the real wave-front from the reference sphere

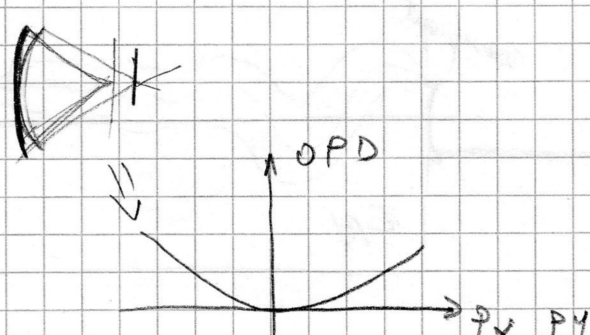


OPD = Optical Path Difference

to the chief ray



* defocusing :



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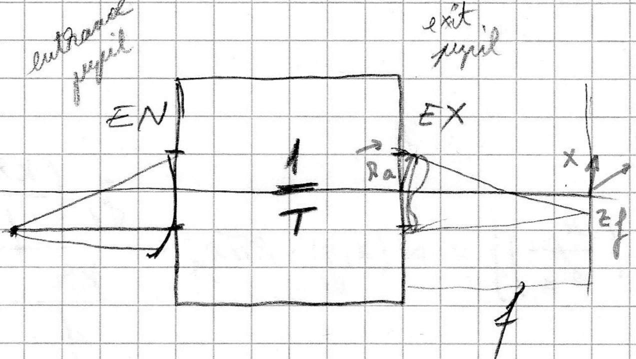
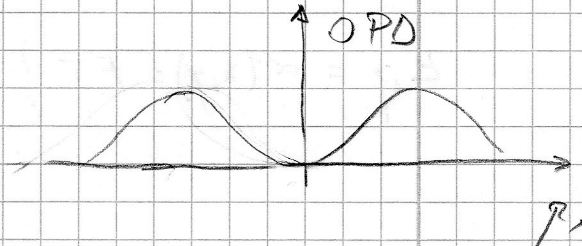
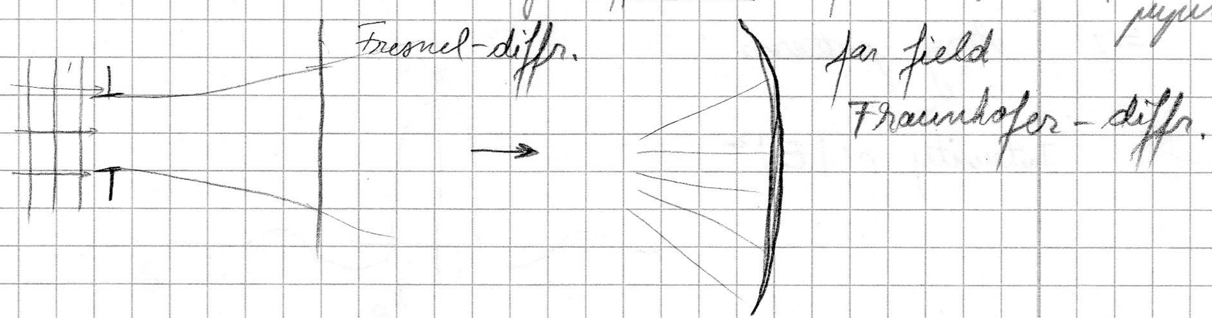
Optical Modeling e 6





$\Rightarrow$ Resolution is determined by <u>diffraction</u> of the aperture / exit pupil

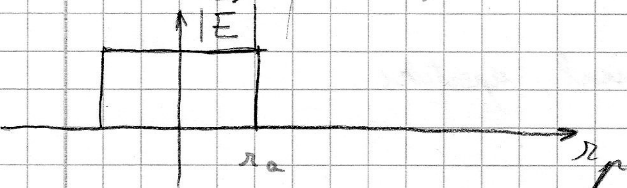


OPD defined on a sphere $\longrightarrow$ wave propagation into the focal plane can be computed by the Fraunhofer diffraction integral

$$\vec{E}(\omega, x, y, z_f) = \underbrace{\frac{\omega}{i 2\pi c f} \cdot e^{ikf} \cdot e^{ik\frac{x^2+y^2}{2f}}}_{\alpha(x,y)} \cdot \underbrace{\iint \vec{E}(\underbrace{\omega, \rho_x, \rho_y, z_p}_{\text{pupil coordinates}}) \, d\rho_x d\rho_y}_{FT\{\vec{E}(\omega, \rho_x, \rho_y, z_p)\}}$$

$\rho_x \; \rho_y$

$\vec{E}$ on the reference sphere in EX



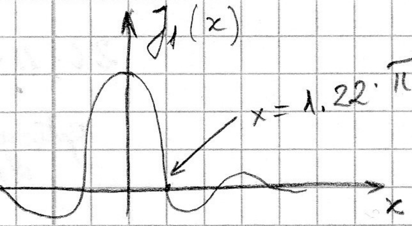
$\arg(\vec{E}) \longrightarrow$ OPD

ideal case : OPD = 0

$$E_i(r_p, z_p) = \text{circ}\left(\frac{r_p}{r_a}\right) \qquad \text{circ}(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| \geq 1 \end{cases}$$

$$FT\left\{\text{circ}\left(\frac{r_p}{r_a}\right)\right\} = 2\pi r_a^2 \cdot \frac{J_1\left(\frac{kr}{f} \cdot r_a\right)}{\frac{kr}{f} \cdot r_a}$$

J_1 = Bessel function of first kind

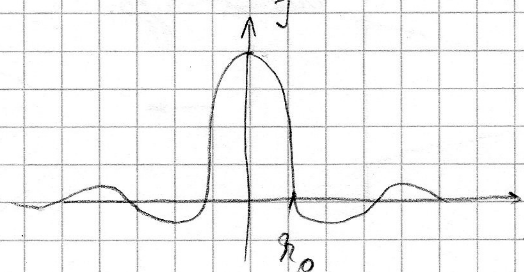


$$E_{if} = \alpha(x,y)\cdot FT\left\{ circ\left(\frac{r_p}{r_a}\right)\right\} = \alpha(x,y)\cdot 2\pi r_a^2 \cdot \frac{J_1\left(\frac{kr}{f}\cdot r_a\right)}{\frac{kr}{f}\cdot r_a}$$

$$r = \sqrt{x^2+y^2}$$

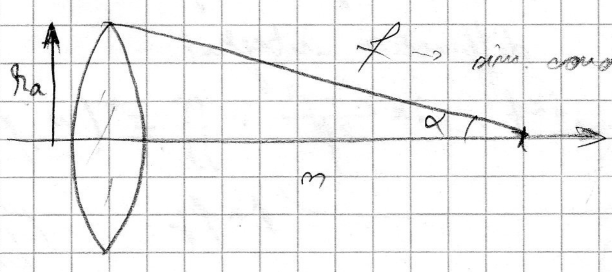
Airy pattern

Intensity $\sim |E|^2$



Radius of the central peak:

$$\boxed{r_0 = 0.61\cdot\frac{\lambda f}{r_a}}$$



$f \rightarrow$ sin condition

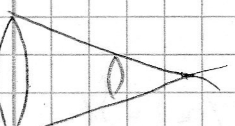
$$\sin\alpha = \frac{r_a}{f} \qquad \lambda = \frac{\lambda_0}{n}$$

$$\boxed{r_0 = 0.61\cdot\frac{\lambda_0}{n\cdot\sin\alpha}}$$

ABBE'S RESOLUTION FORMULA

$$\boxed{NA = n\cdot\sin\alpha}$$

nummerical aperture

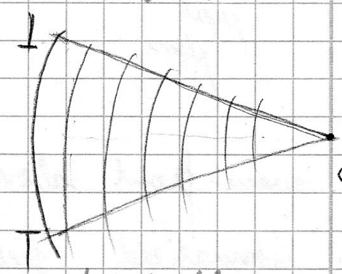


$\Rightarrow$ size of lens is not important

$$FT\{\vec{E}_p\}$$

Ideal imaging : – 1:1 relation between object and image points
– ~~diff~~ equal lateral object distances will result in equal lateral image distances

"diffraction limited imaging"

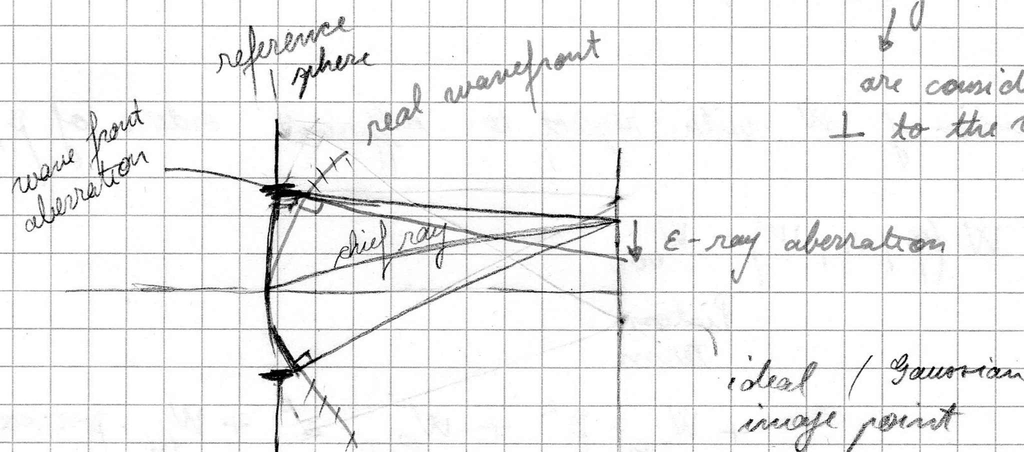


– real lens systems : typically we don't have perfect spherical waves

wave-front deformations are described by OPD

↳ can also consider ray aberrations

↓ are considered to be ⊥ to the wavefront

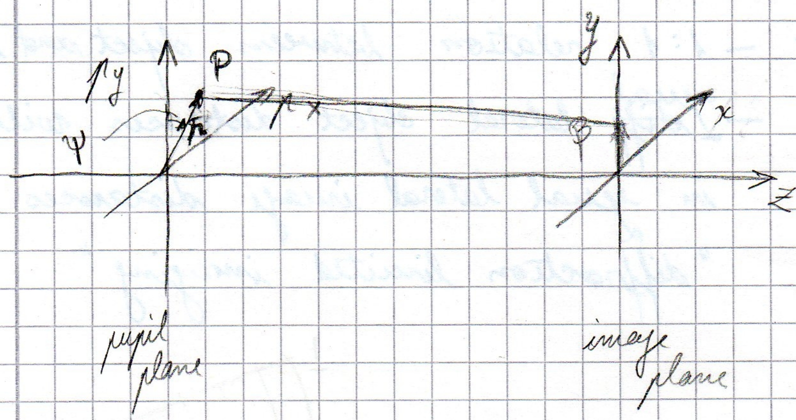


$$\varepsilon_x \sim \frac{\partial W}{\partial x} \quad ; \quad \varepsilon_y \sim \frac{\partial W}{\partial y}$$

lens design ≡ aberration balancing as one cannot avoid aberrations completely in a real system.

Design goals : – high resolution
– high image contrast
– homogeneous illumination
– similarity between image and object

Here: monochromatic aberrations



W ... wave-front aberration function in the pupil plane

β ... normalized field ~~height~~

r ... normalized pupil height of point P

Ψ ... azimuth of P

$$W = W(\beta, r, \Psi)$$

Expansion of W with respect to different orders of β, r, cos Ψ

$$W(\beta, r, \Psi) = \underbrace{W_{000}}_{\text{Piston term}}$$

$$+ W_{200} \cdot \beta^2 + \underbrace{W_{020} \cdot r^2}_{\text{DEFOCUS}} + \underbrace{W_{111} \cdot \beta \cdot r \cdot \cos \Psi}_{\text{LATERAL MAGNIFICATION ORDER}}$$

the odd ones are 0

$$+ W_{400}\, \beta^4 + \underbrace{W_{040}\, r^4}_{\text{SPHERICAL ABERRATION}} + \underbrace{W_{131}\, \beta r^3 \cos \Psi}_{\text{COMA}} + \underbrace{W_{222}\, \beta^2 r^2 \cos^2 \Psi}_{\text{ASTIGMATISM}}$$

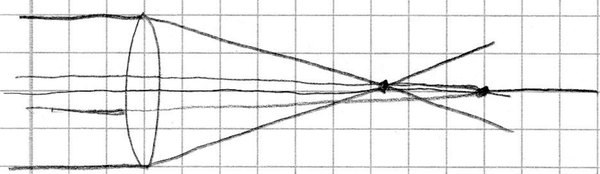
$$+ \underbrace{W_{220} \cdot \beta^2 \cdot r^2}_{\text{FIELD CURVATURE}} + \underbrace{W_{311}\, \beta^3 \cdot r \cdot \cos \Psi}_{\text{DISTORTION}}$$

+ higher other terms

=> 5 classical Seidel-Aberrations

3·12·2013
Opt. Model
c 7

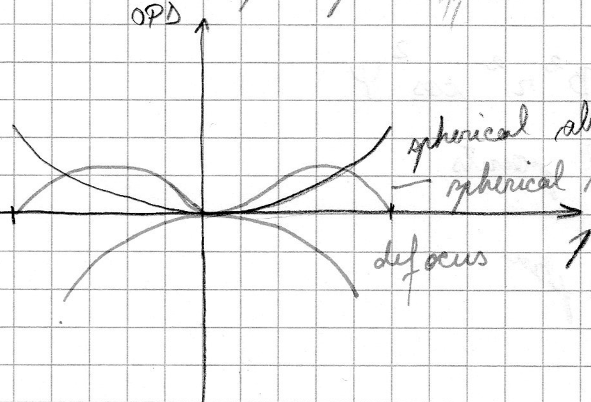
2.7.1. Spherical Aberration $\sim r^4$



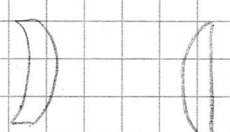
* to avoid spherical aber. we should use a hiperboloid lens

cause for sph. ab. : $f' = f(r)$

different focal length for different ray rail in the pupil plane



Lens equation : $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$



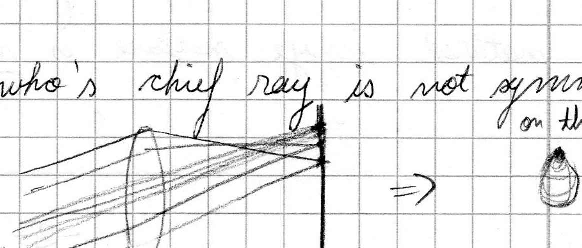
shape factor : $c = \frac{R_2 + R_1}{R_2 - R_1}$

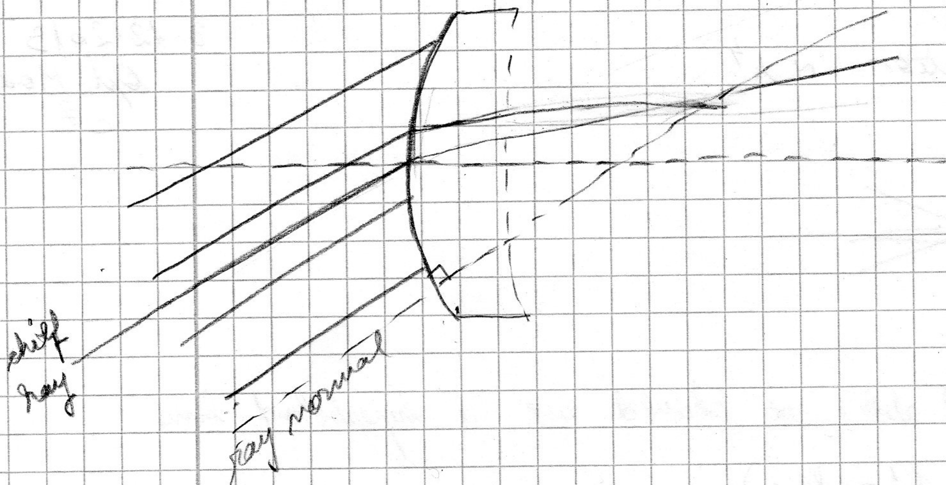
Minimize sph. aber. :  - more surfaces => more degrees of freedom

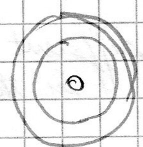
(1) non-liniarity of reflexion law

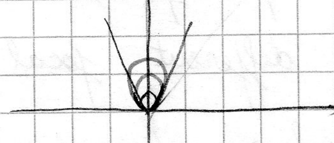
2.7.2. Coma $\sim \beta \cdot r^3 \cos \Psi$

- "non-symmetry error"
- occurs for ray bundles who's chief ray is not symmetric to the optical axis.





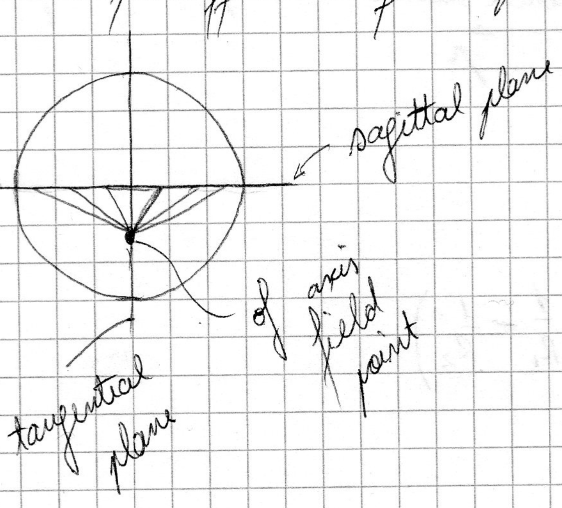


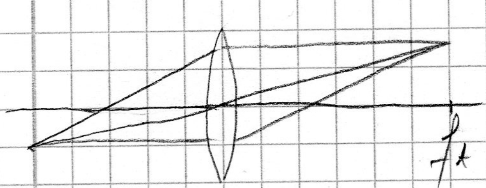


2.7.3. Astigmatism $\sim \beta^2 r^2 \cos^2 \Psi$

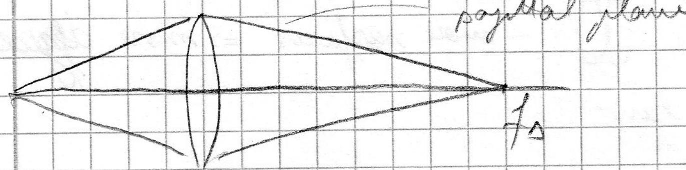
- occurs for off-axis field points

-



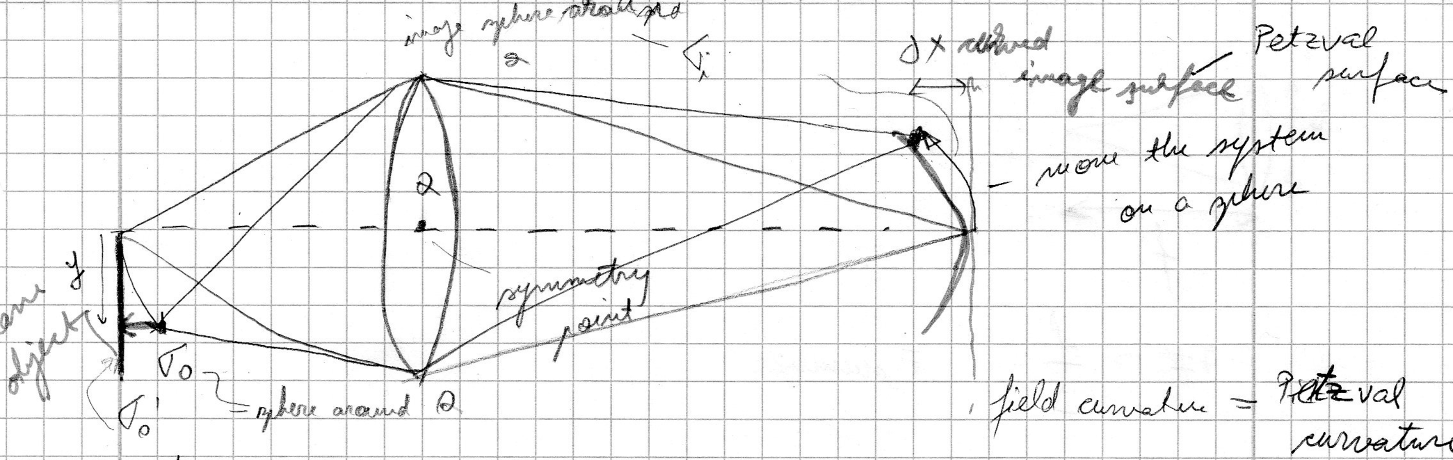


top view



2.7.4. Field curvature $\sim \beta^2 \cdot r^2$

- natural image surface is <u>not</u> a plane



$$\frac{1}{f'} = \frac{1}{s} - \frac{1}{s'}$$

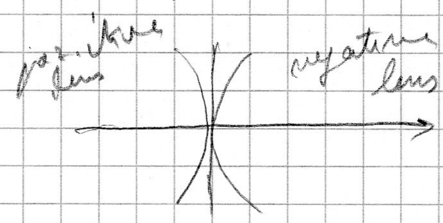
Josef Max Petzval (1807–1891)

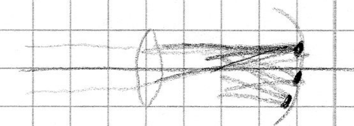
Distance of the image point from the image plane:

$$\Delta x = \frac{y}{2} \cdot \underbrace{\sum_{j=1}^{m} \frac{1}{n_j' f_j'}}_{\text{Petzval sum}}$$

m = number of lenses in the optical systems

For a positive lens we have a negative radius of the Petzval surface and for neg. lens, → positive



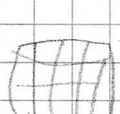


→ to improve: field flattener

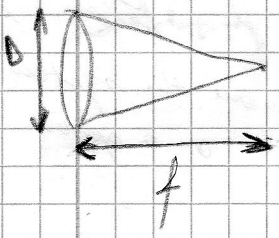
The Cook triplet

2.7.5. Distortion $\sim \rho^3 \cdot r \cdot \cos\Psi$

– distorsion of image scale due to different transversal magnifications for each field point.

positive dist:  barrel

neg. dist: pincushion



$$F\# = \frac{f}{D}$$ F number