Fundamentals of Modern Optics WS 2012/13
Midterm Exam

to be written December 10, 8:15 – 9.45 a.m.

Problem 1 – Maxwell's equations

4 + 3 + 2 = 12 points

a) In the time domain, write down Maxwell's equations for optics in a general form and name the different fields and parameters.

b) In the frequency domain, derive the wave equation for the electric field in a non-magnetizable, isotropic, dispersive and homogeneous medium without sources of charges or current.

c) Show that the continuity equation for charges and currents reads as

$$\nabla \cdot \mathbf{j}(\mathbf{r},t) + \frac{\partial \rho(\mathbf{r},t)}{\partial t} = 0.$$

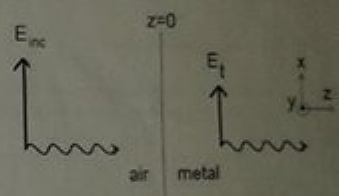
Derive and explain the integral form of this relation.

Problem 2 – Normal modes

8 + 4 = 12 points

A semi-infinite block of metal is illuminated perpendicularly with a plane wave from air. The electric field of the incoming field in air is

$$\mathbf{E}_{inc} = E_{inc} e^{ik_0 z - i\omega t}\,\hat{\mathbf{x}}$$



The electric field of the transmitted plane wave inside the metal reads as

$$\mathbf{E}_t = \frac{2E_{inc}}{1+(n+i\kappa)} e^{i(n+i\kappa)k_0 z - i\omega t}\,\hat{\mathbf{x}},$$

where $n + i\kappa = \sqrt{\epsilon' + i\epsilon''}$ and $k_0 = \omega/c$.

a) Calculate the magnetic field of the incoming and transmitted waves using Maxwell's equation. Afterwards, calculate the time averaged Poynting vector of the incoming wave $\langle \mathbf{S}_{inc} \rangle$ and transmitted wave $\langle \mathbf{S}_t \rangle$.

b) The reflectivity of this metal surface under normal incidence reads as

$$R = \left| \frac{n + i\kappa - 1}{n + i\kappa + 1} \right|^2.$$

Discuss R for the two cases:

I) Lossless metal: $\epsilon' < 0$, $\epsilon'' = 0$.

II) Low loss metal: $\epsilon' < 0$, $\epsilon'' > 0$, $\epsilon'' \ll |\epsilon'|$.

$n + m = \sqrt{\varepsilon'}$

Problem 3 – Beam propagation

3 + 3 + 4 = 12 points

Given is the field directly behind a one dimensional amplitude mask

$$u_0(x, z_0) = A\left[1 + \cos\left(\frac{2\pi}{G}x\right)\right].$$

The field is propagating through vacuum.

a) Calculate the spatial frequency spectrum $U_0(\alpha, z_0)$.

b) Calculate the field $u(x, z)$ for all $z > z_0$ without approximation.

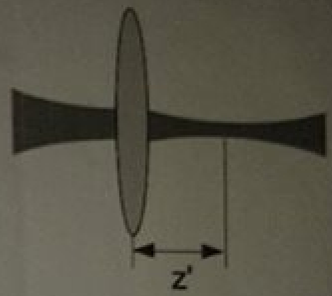
c) The field will reproduce itself except for a constant phase factor $e^{i\varphi}$ after a certain propagation length z_T (Talbot effect). Calculate z_T as a function of G and the wavelength λ.

please turn over

Problem 4 – Gaussian beams

6 + 4 = 10 points

A focusing lens of focal length f is placed in the beam waist of a Gaussian beam of Rayleigh range $z_0 = \pi W_0^2/\lambda$, where W_0 is the waist radius of the incoming Gaussian beam.



a) Show that the beam waist of the transformed beam is positioned at a distance:

$$z' = \frac{f}{1 + (f/z_0)^2},$$

and the corresponding waist radius is:

$$W_0' = \frac{W_0}{\sqrt{1 + (z_0/f)^2}},$$

Hint: Free space propagation is simply described by $q' = q_0 + z$.

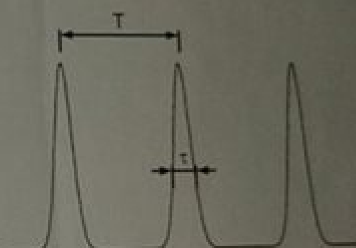
b) A collimated laser beam of wavelength $\lambda_0 = 633\,\text{nm}$ has a waist radius of $100\,\mu\text{m}$ and is focused by a $f = 25\,\text{mm}$ lens. Calculate the waist radius after the lens. Argue that the paraxial approximation is valid.

Problem 5 – Pulse propagation

3 + 3 = 6 points

A train of transform-limited Gaussian pulses with central frequency ω_0, where the envelope of each pulse is defined as

$$E(t) = E_0 \exp\left[-\frac{(t - t_0)^2}{\tau^2}\right]$$



is launched into a fiber. The pulse width is given by $\tau = 1\,\text{ps}$ and the separation T between consecutive pulses is $T = \sqrt{101}\,\text{ps} \approx 10.05\,\text{ps}$. The fiber is characterized by a dependence of the wavenumber k such that

$$k(\omega) = k_0 + (1.5/c)(\omega - \omega_0) + \frac{10^{-1}\,\text{ps}^2/\text{m}}{2}(\omega - \omega_0)^2.$$

a) Find the dispersion length $L_D = \tau^2 \left[\frac{\partial^2 k(\omega)}{\partial \omega^2}\right]^{-1}_{\omega=\omega_0}$ of each individual pulse. Is the red or the blue part of the pulse spectrum appearing earlier at the end of the fiber?

b) Find the interaction length L_{Int} after which the dispersion-broadened pulses start to overlap considerably. At L_{Int} each pulse is supposed to have a pulse width that is equal to the separation T between neighboring pulses.

$z = (z_0)\frac{1}{\tau}\sqrt{T^2 - \tau^2}$

$L_{in} =$