Friedrich-Schiller-Universität Jena

Jena, Winter Semester 2015/2016

Introduction to Optical Modeling and Design

Prof. Dr. Frank Wyrowski

Jena, Germany

My Three Teams at …

- **Applied Computational Optics Group** at Friedrich Schiller University of Jena
  - R&D in optical modeling and design with emphasis on physical optics
- **Wyrowski Photonics**
  - Development of software VirtualLab Fusion

My Three Teams at …

- **Applied Computational Optics Group** at Friedrich Schiller University of Jena
  - R&D in optical modeling and design with emphasis on physical optics.
- **Wyrowski Photonics**
  - Development of software VirtualLab Fusion
- **LightTrans**
  - Distribution of VirtualLab, together with distributors worldwide
  - Service, technical support, seminars, and trainings
  - Complete optical solutions, including fabrication of micro-optics and diffractive elements (DOEs).

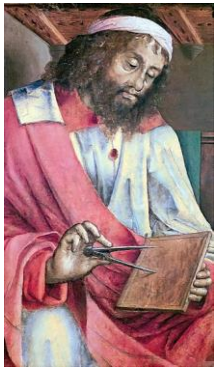
Historical Retrospective

Geometrical Optics

Light appears in form of rays.

Investigation of light propagation by geometrical rules has very long history.

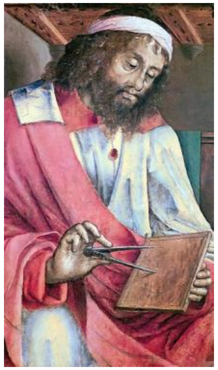
Geometrical Optics

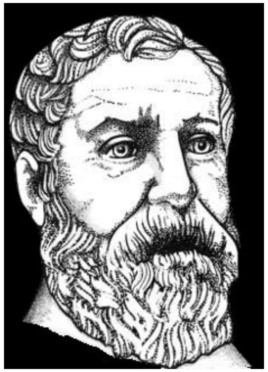


Euclid of Alexandria
(c. 325 BC – 265 BC)

Geometric considerations
of light ray propagation

Geometrical Optics





Euclid of Alexandria
(c. 325 BC – 265 BC)

Hero of Alexandria
(c. 10 – c. 70 AD)

Geometric considerations
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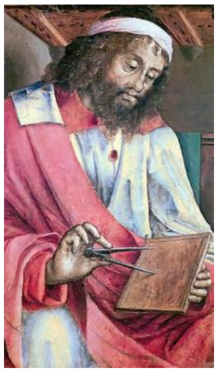
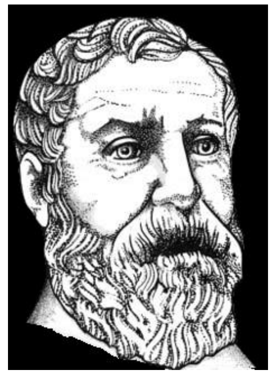
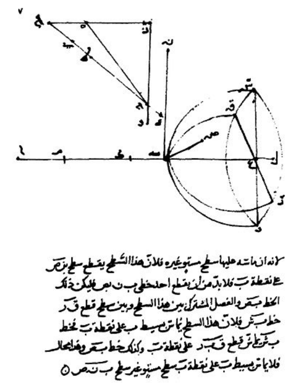
Geometrical Optics







Euclid of Alexandria
(c. 325 BC – 265 BC)

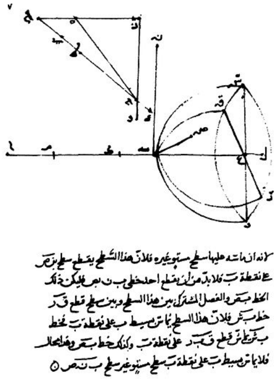
Hero of Alexandria
(c. 10 – c. 70 AD)

Ibn Sahl (ابن سهل)
(c. 940–1000)

Geometric considerations of light ray propagation

- Law of refraction
- Early theory of aspherical lenses!

Geometrical Optics



Ibn Sahl (ابن سهل)
(c. 940–1000)

Pierre de Fermat
(1601 - 1665)

1660: Fermat’s variation principle of least time: Ray path determination

Ray Optics = Geometrical Optics

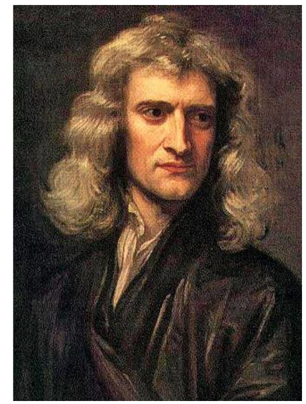
Ray optics:
- Deals with position and direction of rays
- Ray propagation is described by Fermat’s principle

Pierre de Fermat
(1601 - 1665)

Geometrical optics propagation laws
- Straight lines in homogeneous meida
- Laws of refraction and reflection

Physical reason for propagation laws?

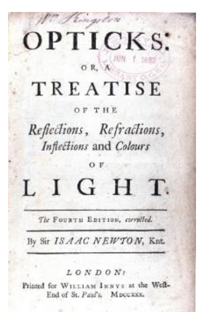
Historical Retrospective



Pierre de Fermat
(1601 - 1665)

Isaac Newton
(1642–1727)

1704



OPTICKS:
OR, A
TREATISE
OF THE
Reflections, *Refractions*,
Inflections and *Colours*
OF
LIGHT.

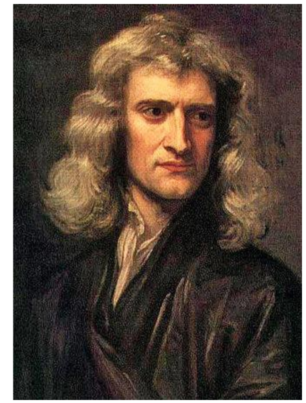
The FOURTH EDITION, *corrected*.

By Sir *ISAAC NEWTON*, Knt.

LONDON:
Printed for WILLIAM INNYS at the West-End of St. *Paul's*. MDCCXXX.

Explanation of various optical effects by a **particle** model of light.

Historical Retrospection

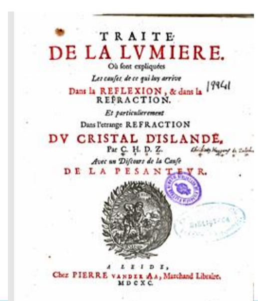


Pierre de Fermat
(1601 - 1665)

Christiaan Huygens
(1629–1695)

Isaac Newton
(1642–1727)

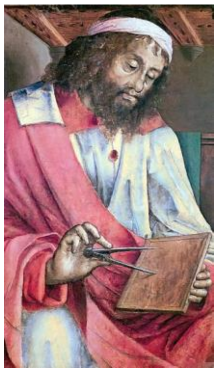
1690



TRAITE
DE LA LVMIERE.
Où sont expliquées
Les causes de ce qui luy arrive
Dans la REFLEXION, & dans la
REFRACTION.
Et particulierement
Dans l'etrange REFRACTION
DV CRISTAL DISLANDE,
Par C. H. D. Z.
Avec un Discours de la Cause
DE LA PESANTEUR.
A LEIDE,
Chez PIERRE VANDER AA, Marchand Libraire.
MDCXC.

Some first ideas on **wave model** of light.

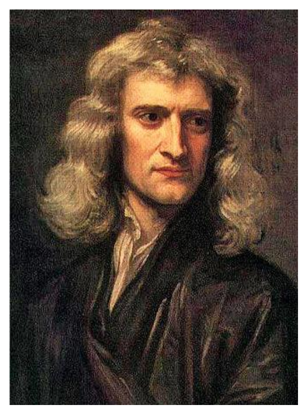
Particle versus Wave Model of Light



Wave model of light:
Rays are local normal vectors of wavefronts

Christiaan Huygens
(1629–1695)

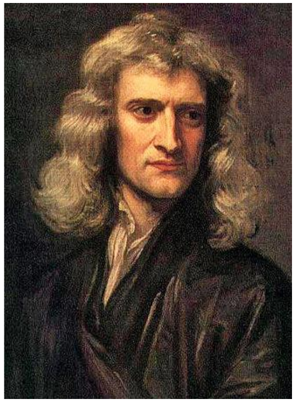
Euclid of Alexandria
(c. 325 BC – 265 BC)



Particle model of light:
Rays represent local direction of particles

Isaac Newton
(1642–1727)

Historical Retrospective

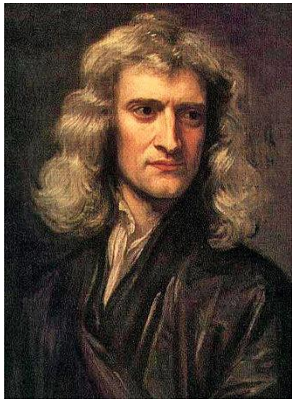


Isaac Newton
(1642–1727)

Thomas Young
(1773-1829)

1804 Young's double slit experiement: **Wave optical light model** was supported!

Historical Retrospective



Isaac Newton
(1642–1727)

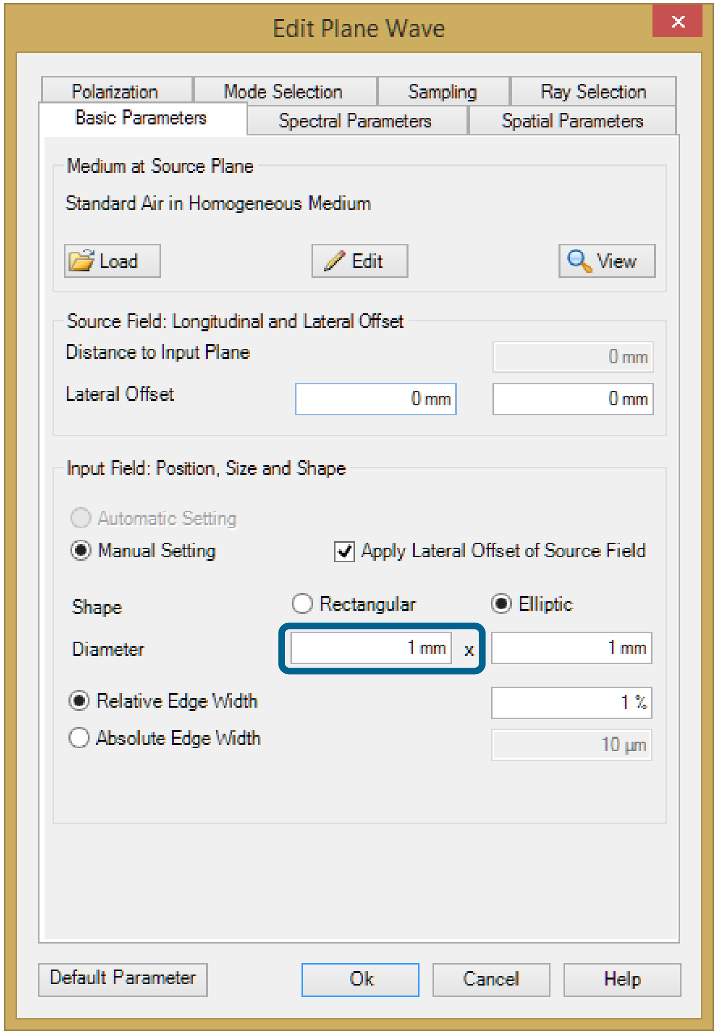
Thomas Young
(1773-1829)

Augustin-Jean Fresnel
(1788-1827)

1818

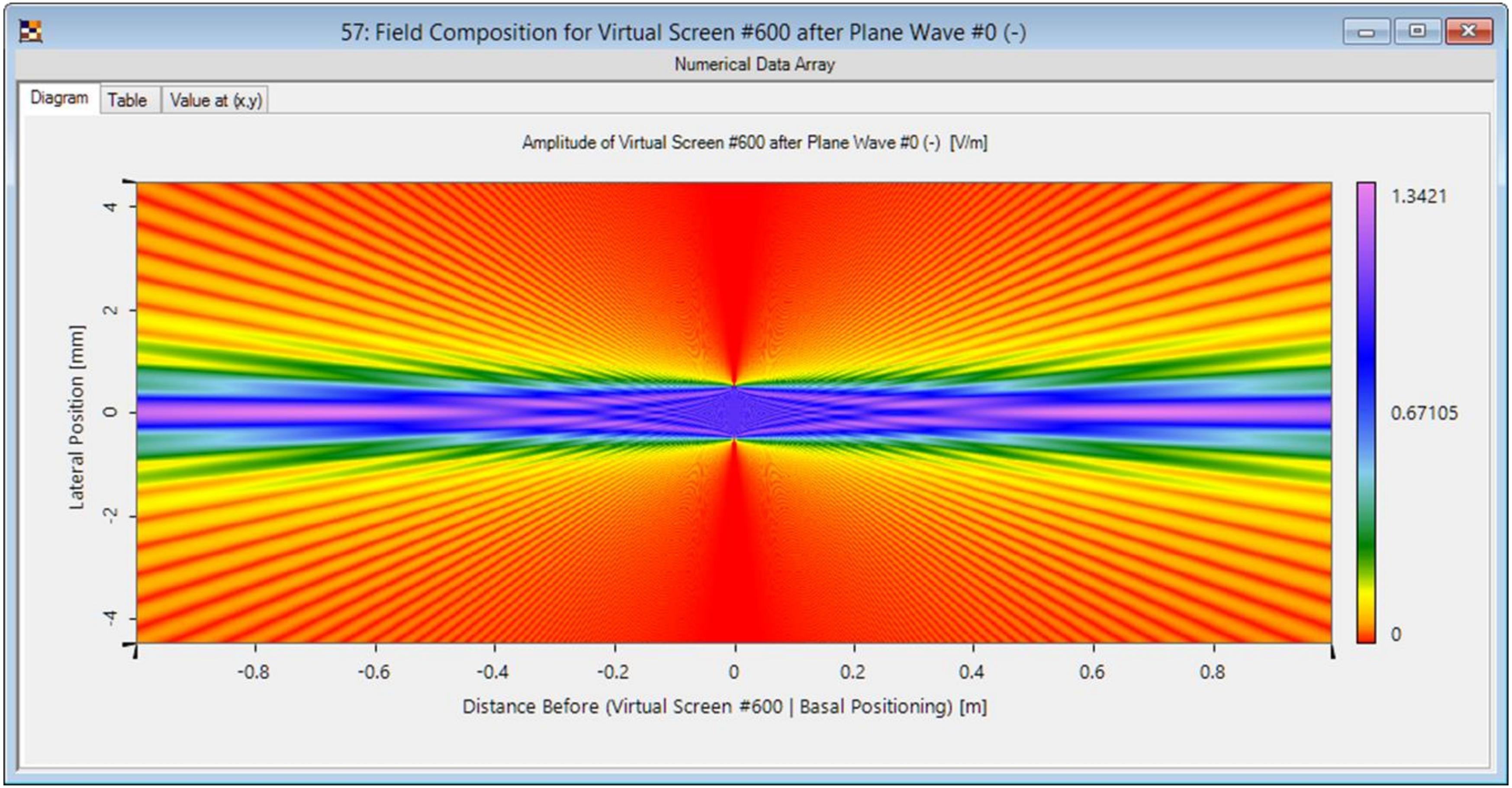
- Paraxial diffraction theory
- Poisson spot

Plane Wave: Diameter 1 mm

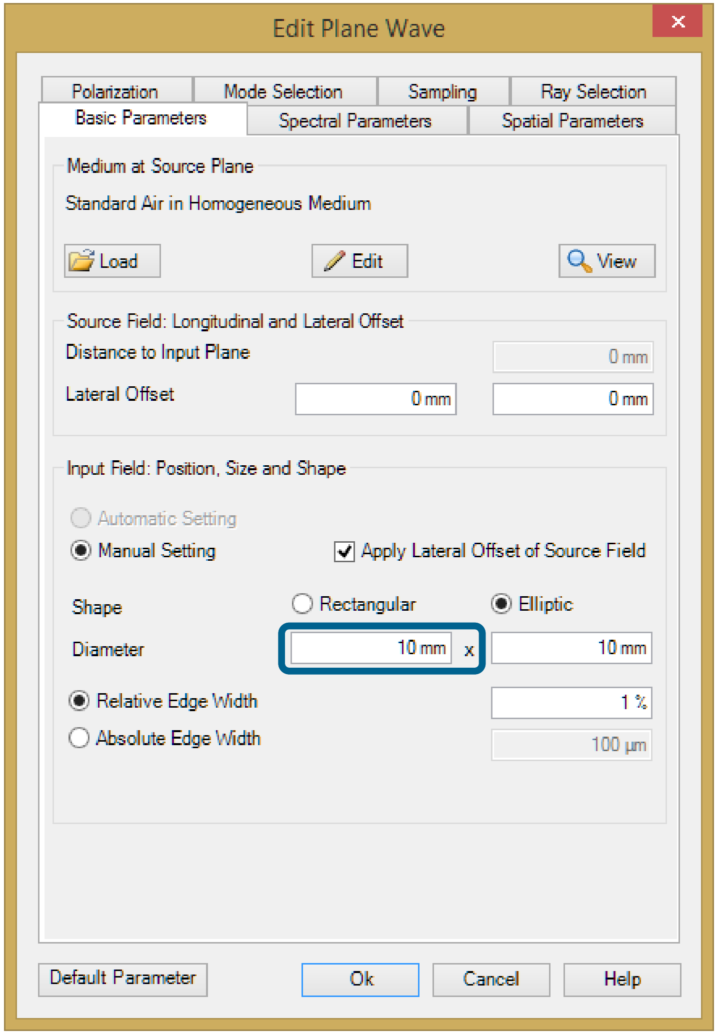


- First we consider a plane wave of 1 mm diameter.
- The edge sharpness is 10 µm.
- It is propagated from -1 m to 1 m, that is we consider a section of 2 m on the z axis.
- In all following examples the field tube is shown in logarithmic color coding.

Plane wave 1 mm diameter: z= -1, …, 1 m

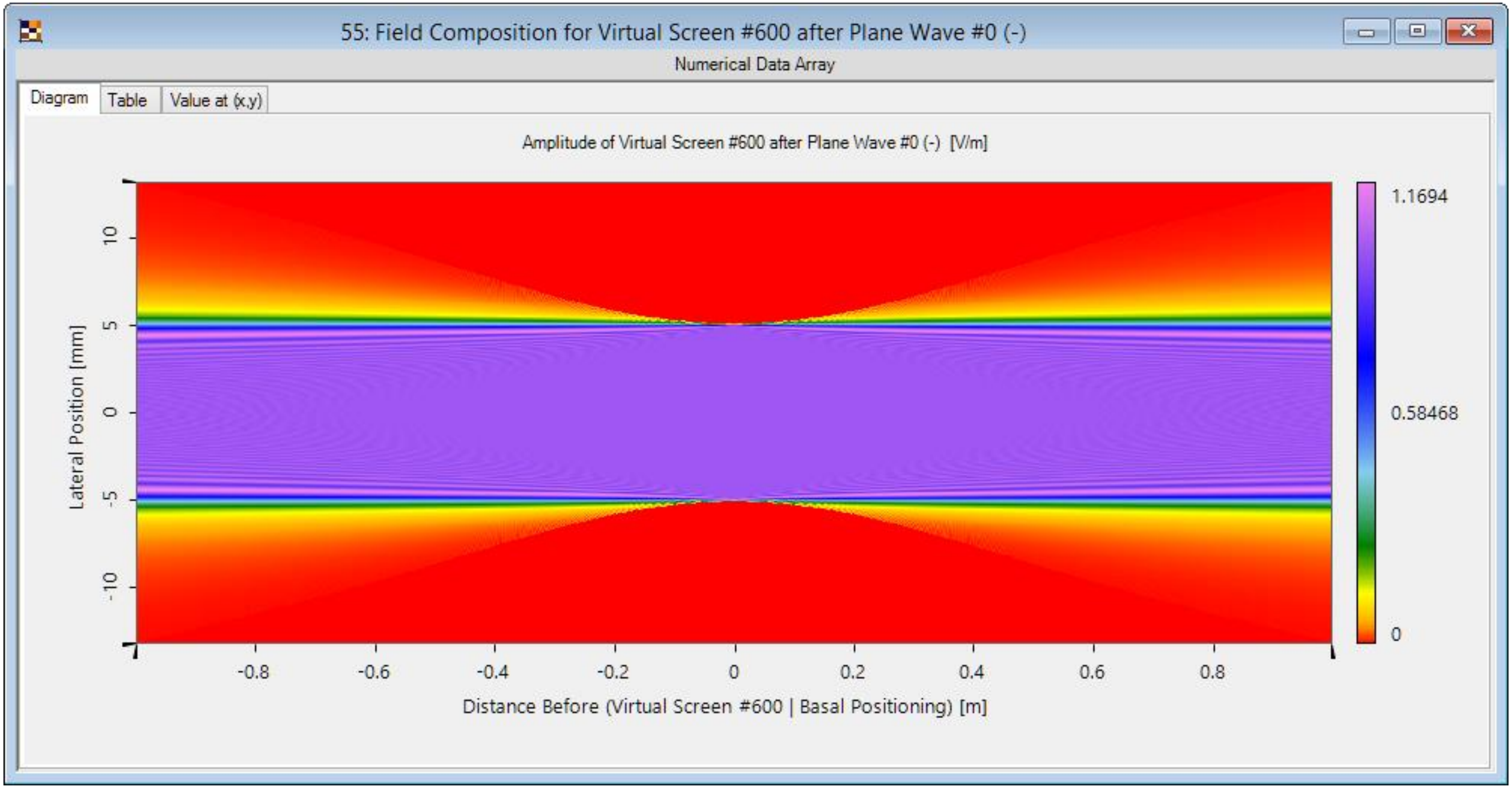


Plane Wave: Diameter 10 mm

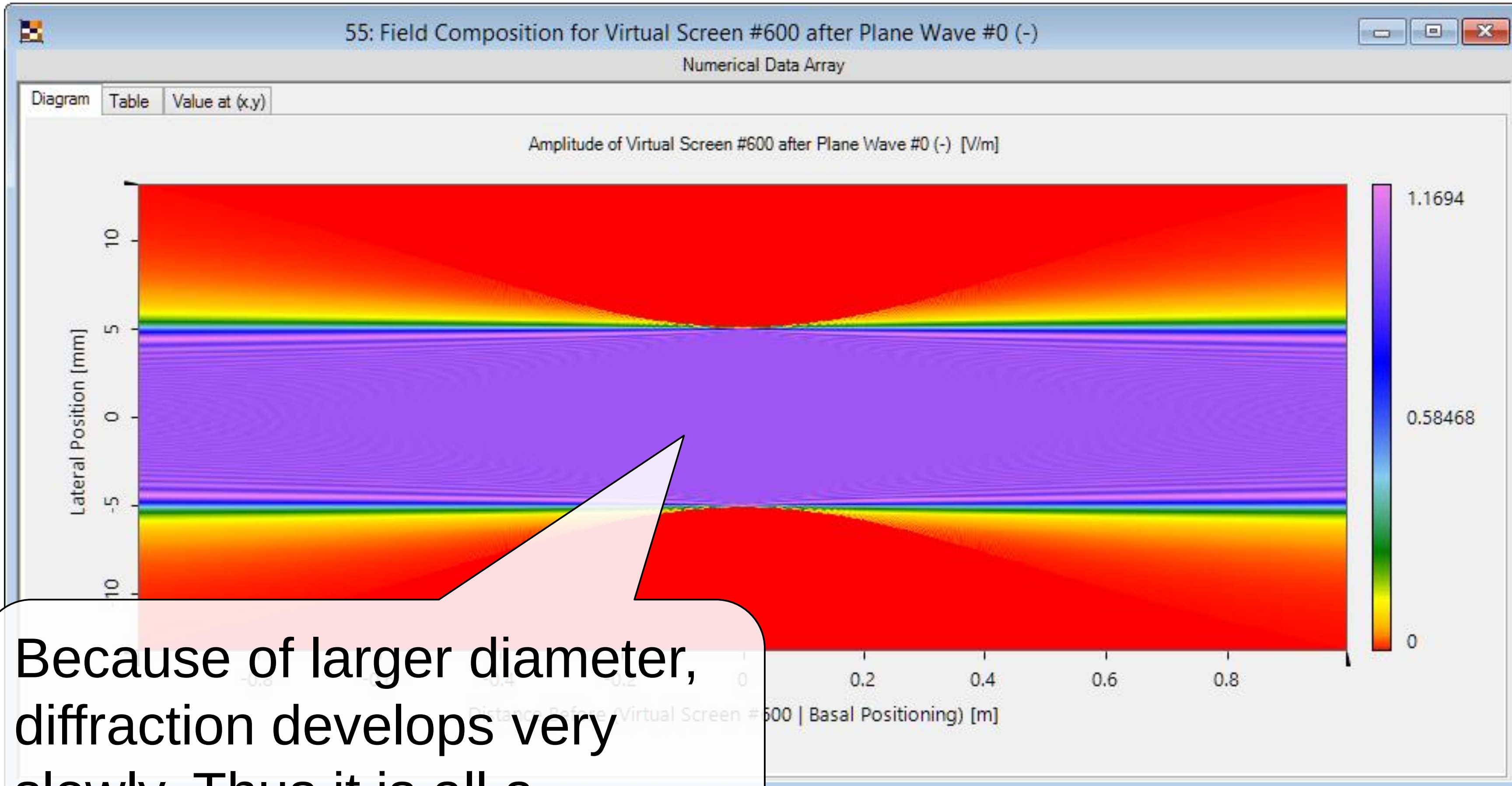


- Next we consider a plane wave of 10 mm diameter.
- The edge sharpness is 100 µm.
- It is propagated from -1 m to 1 m, that is we consider a section of 2 m on the z axis.

Plane wave 10 mm diameter: z= -1, …, 1 m

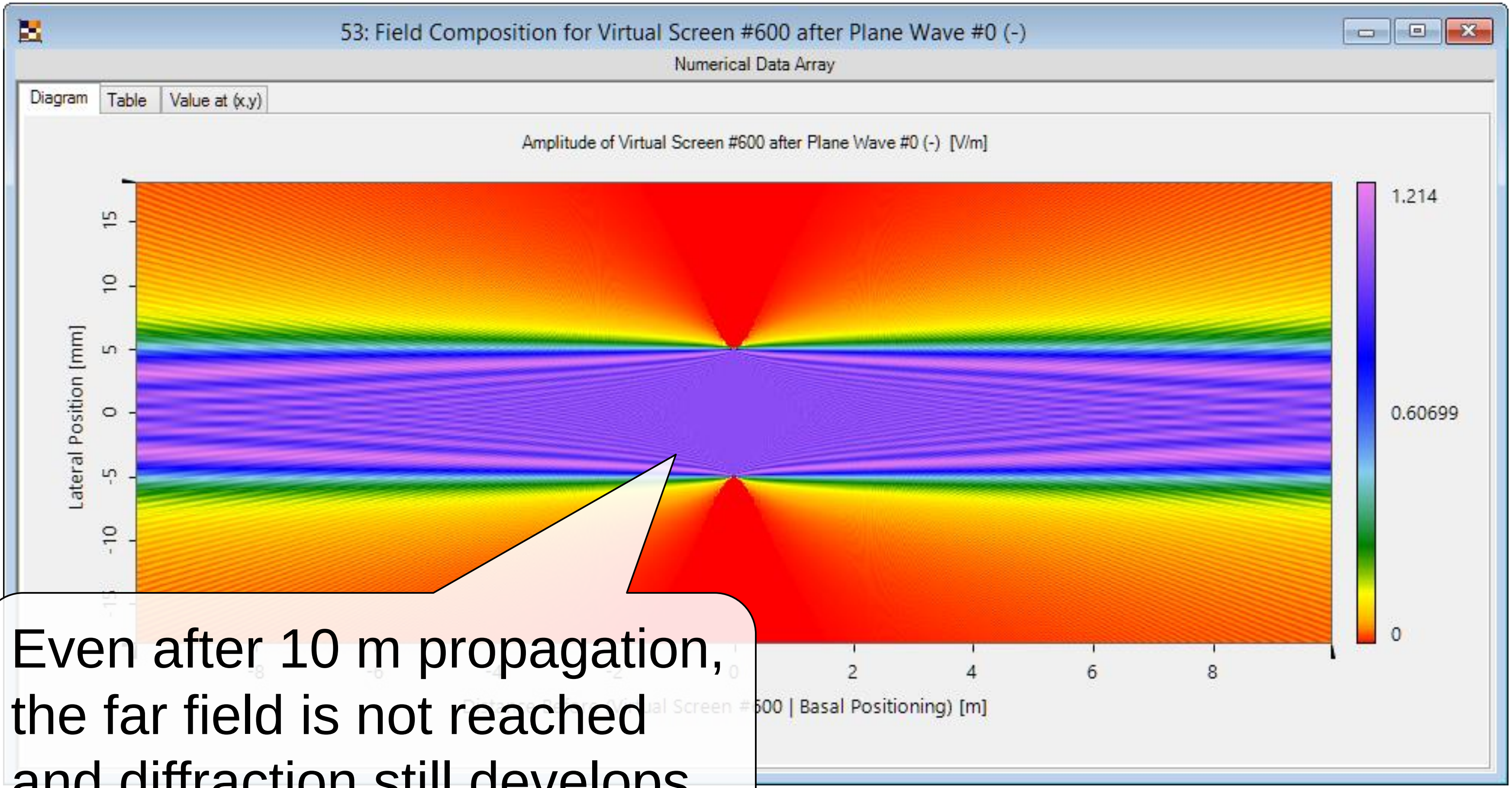


Plane wave 10 mm diameter: z= -1, …, 1 m



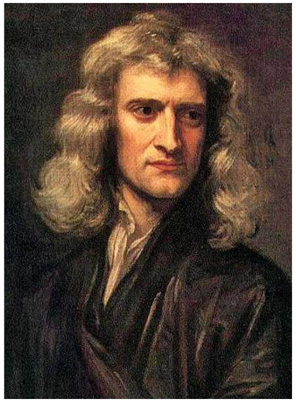
Because of larger diameter, diffraction develops very slowly. Thus it is all a diffractive region.

Plane wave 10 mm diameter: z= -10, …, 10 m



Even after 10 m propagation, the far field is not reached and diffraction still develops.

Physical Optics



Isaac Newton
(1642–1727)

Augustin-Jean Fresnel
(1788-1827)

James Clerk Maxwell
(1831-1879)

1865

Name	Integral equations	Differential equations
Gauss's law	$\oint\!\!\oint_{\partial\Omega} \mathbf{E}\cdot \mathrm{d}\mathbf{S} = \frac{1}{\varepsilon_0}\iiint_\Omega \rho\, \mathrm{d}V$	$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$
Gauss's law for magnetism	$\oint\!\!\oint_{\partial\Omega} \mathbf{B}\cdot \mathrm{d}\mathbf{S} = 0$	$\nabla\cdot\mathbf{B} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E}\cdot \mathrm{d}\boldsymbol{\ell} = -\frac{d}{dt}\iint_\Sigma \mathbf{B}\cdot \mathrm{d}\mathbf{S}$	$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$
Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B}\cdot \mathrm{d}\boldsymbol{\ell} = \mu_0\iint_\Sigma \mathbf{J}\cdot \mathrm{d}\mathbf{S} + \mu_0\varepsilon_0\frac{d}{dt}\iint_\Sigma \mathbf{E}\cdot \mathrm{d}\mathbf{S}$	$\nabla\times\mathbf{B} = \mu_0\left(\mathbf{J} + \varepsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)$

Electromagnetic field model of light was born!

Conclusion

Fermat's principle

Ray or
geometrical optics

Maxwell's equations

Physical or
electromagnetic optics

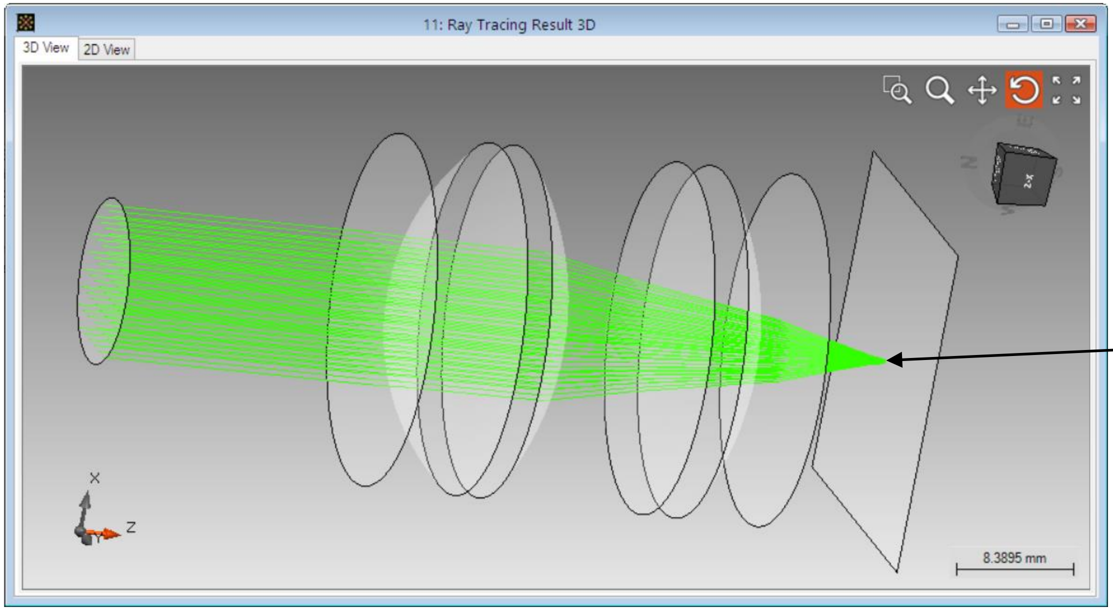
Optical Design

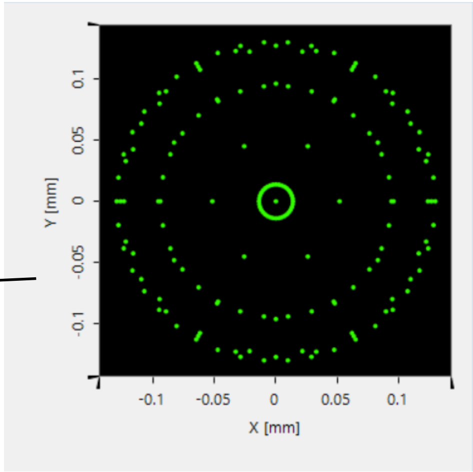
Light appears
in form of rays.

Design of optical instruments was based on geometrical optics and that has not changed a lot since then!

Geometrical Optics and Ray Tracing

- Geometrical optics is the basic approach in optical design.
- Ray Tracing, the geometrical-optics based modeling algorithm, is the most often used tool in optical design.





Geometrical Optics and Ray Tracing

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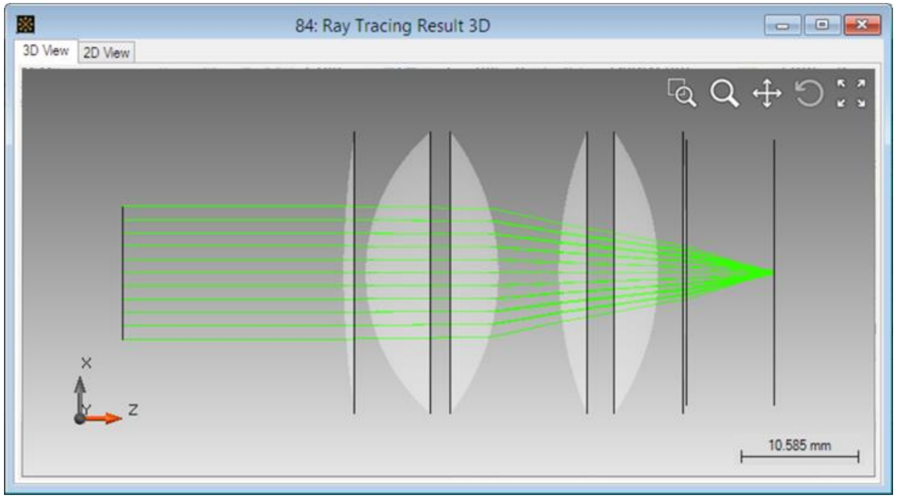
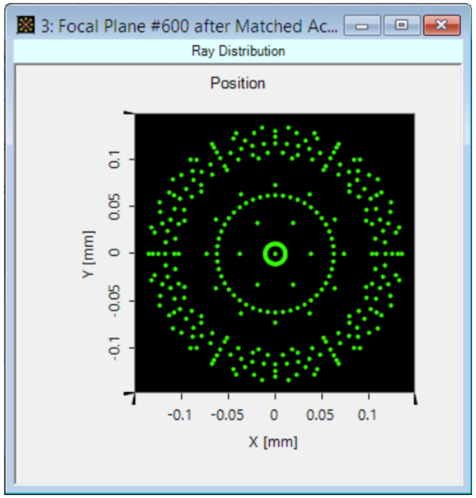
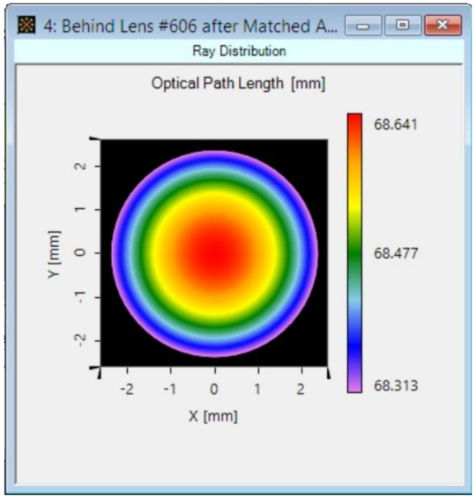
VirtualLab Fusion includes a full ray tracing engine for modeling and optimization.

Ray Tracing in VirtualLab Fusion







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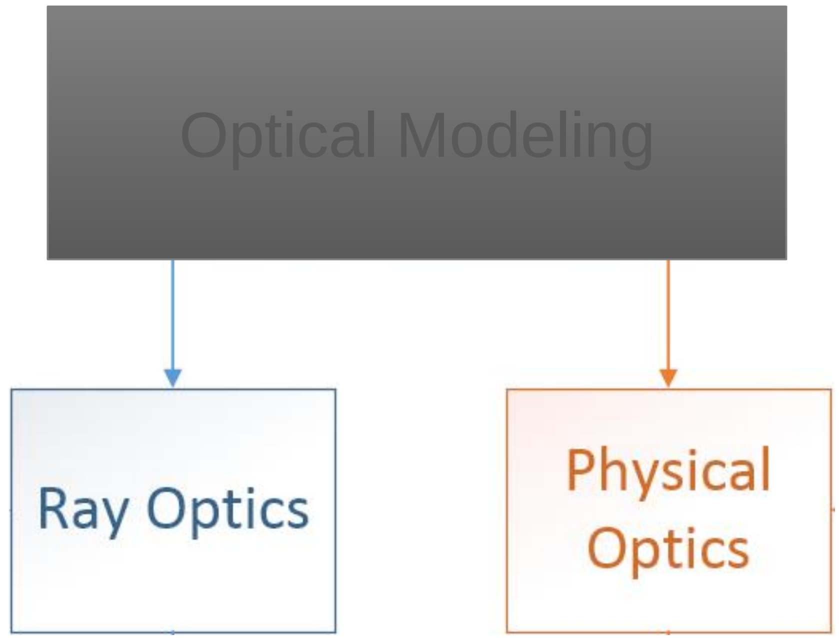
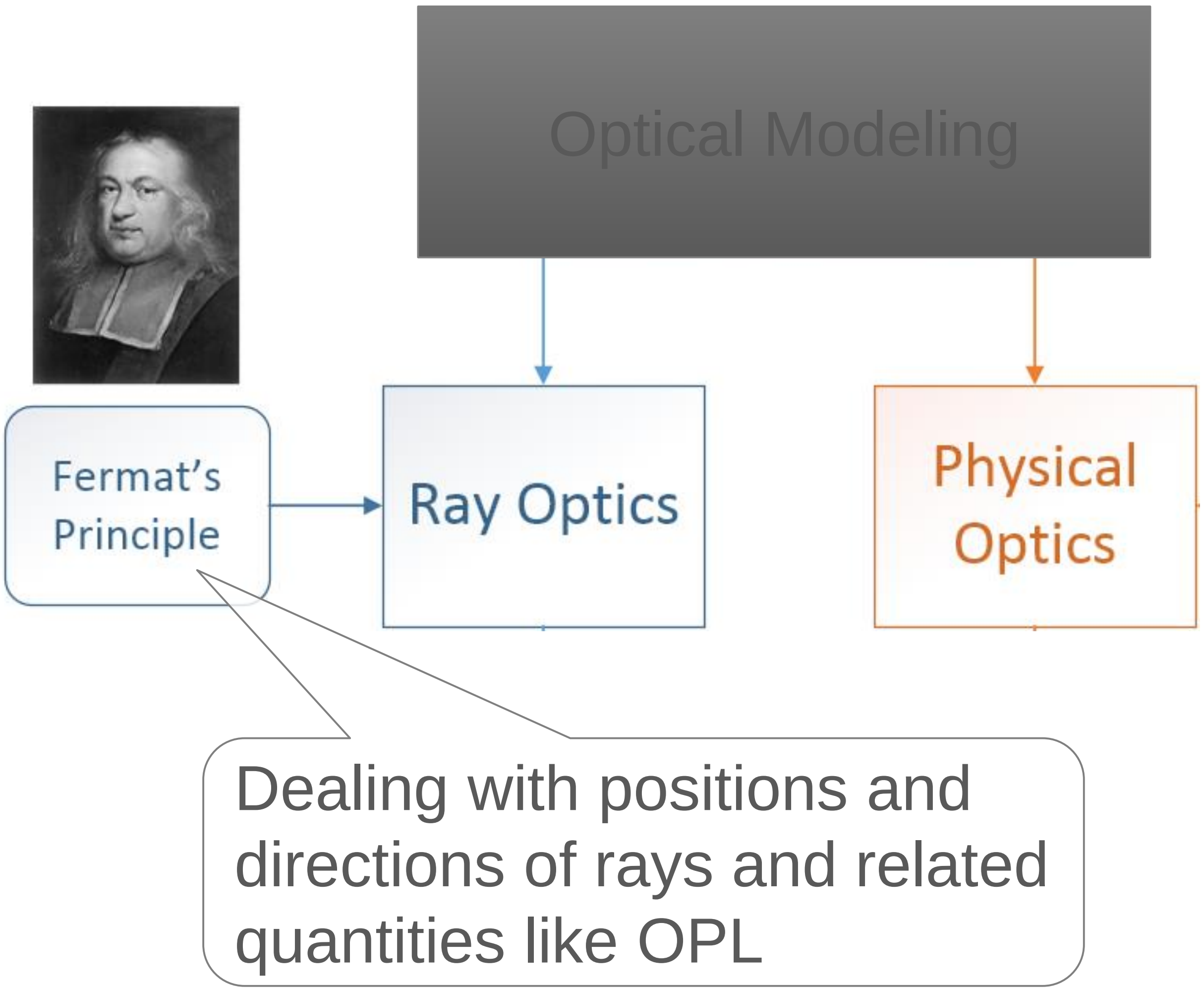
Ray and Field Tracing

Optical Modeling

Ray and Field Tracing



Ray and Field Tracing



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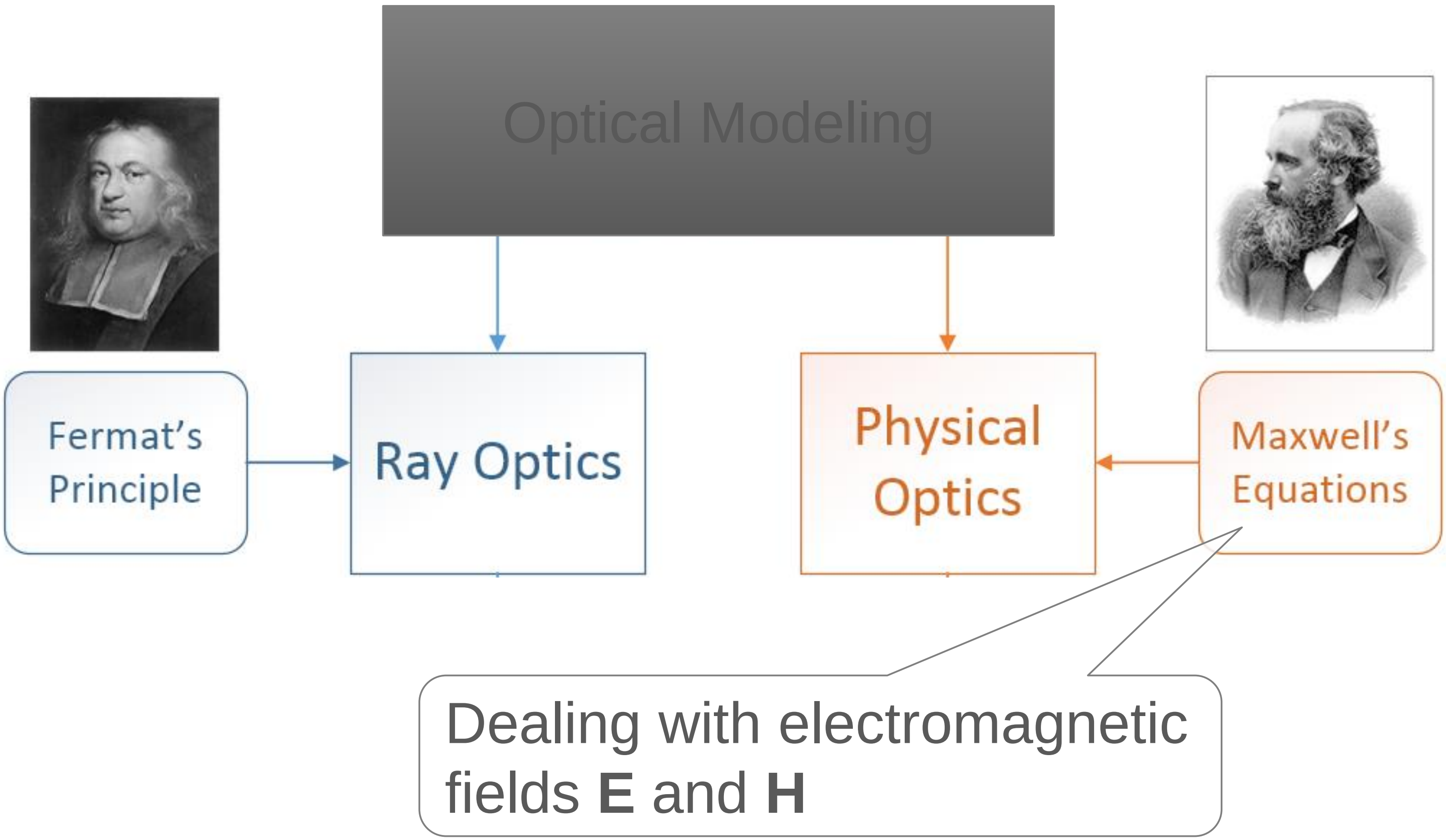

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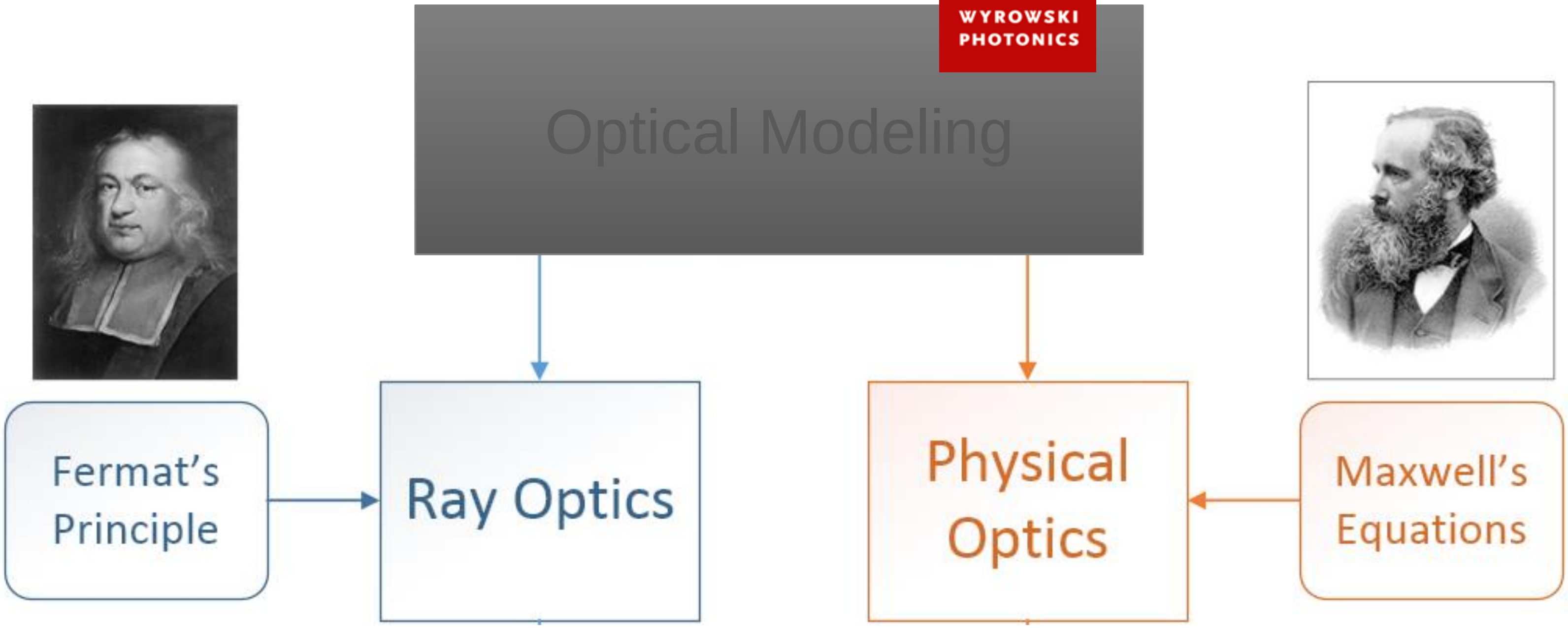

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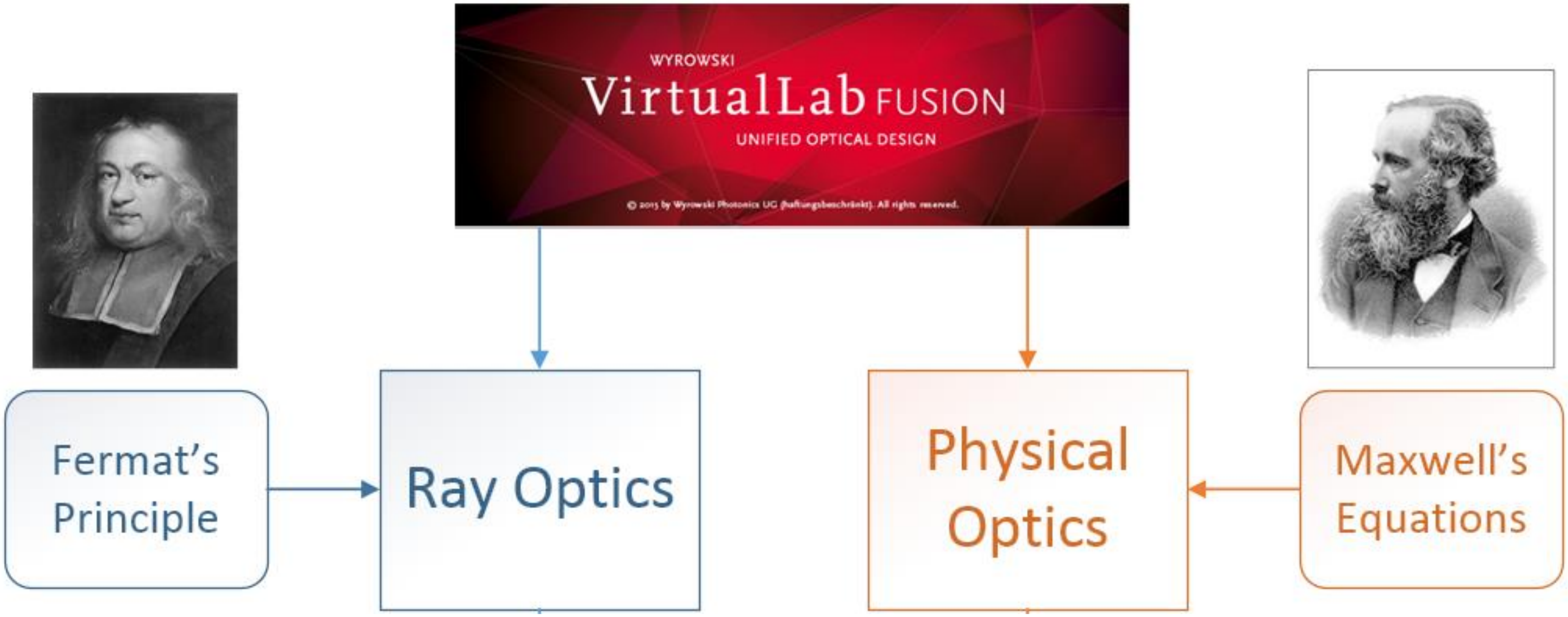

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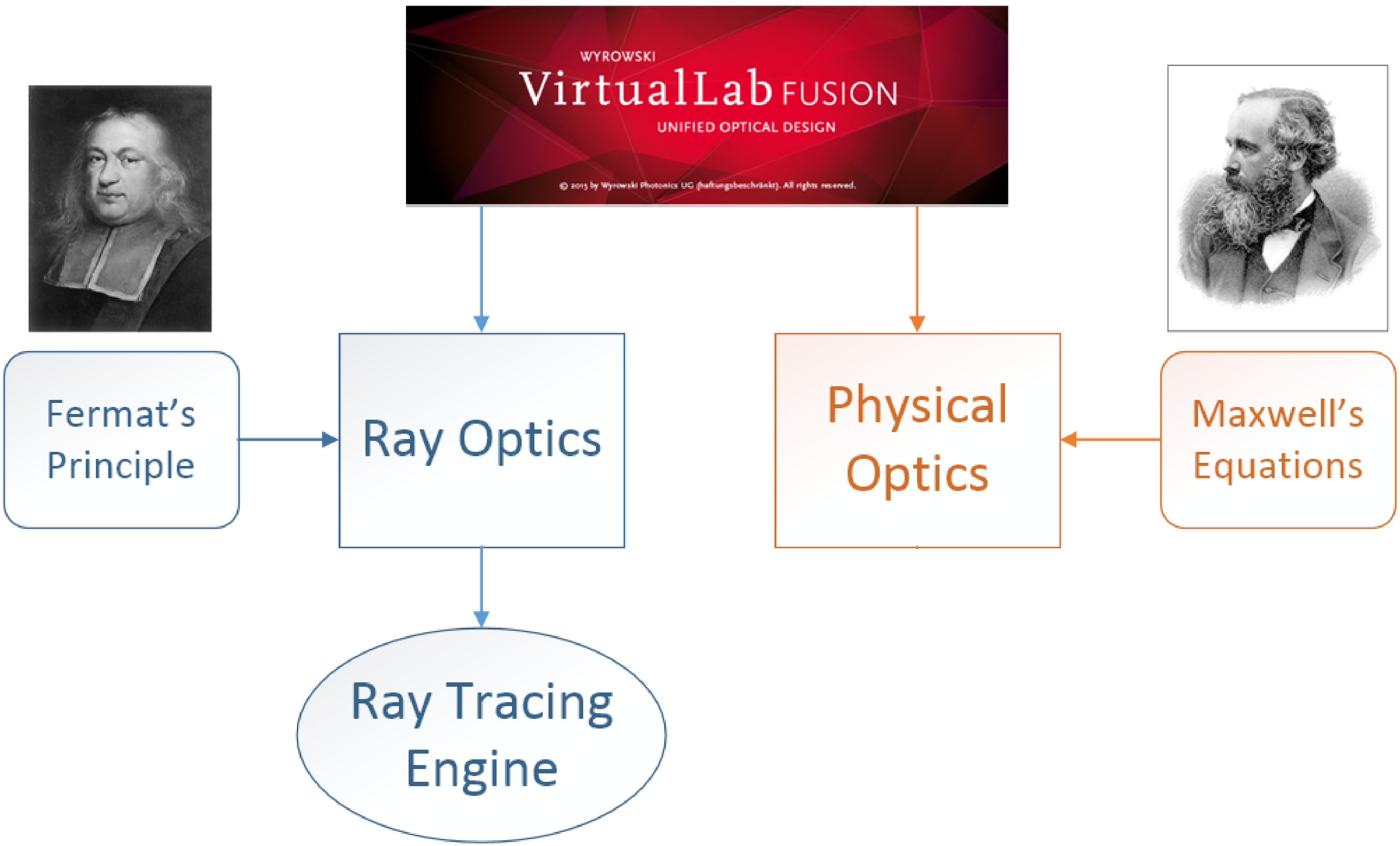

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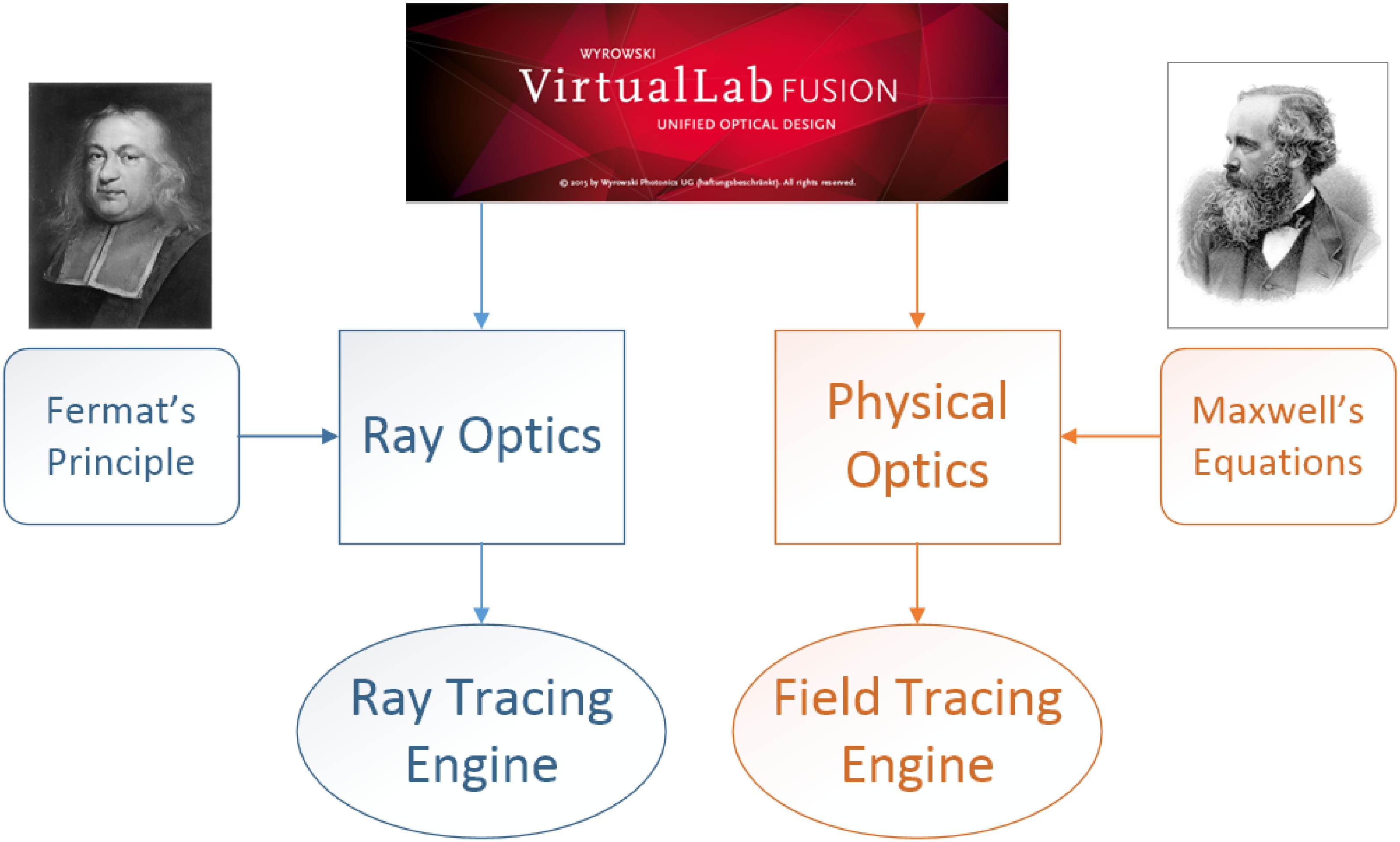
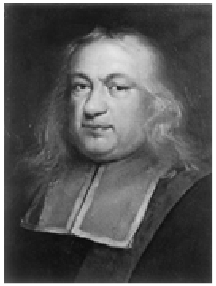
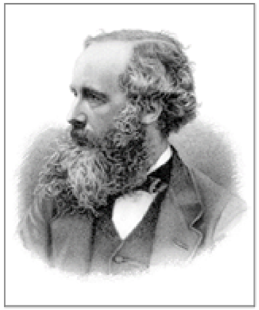


Ray and Field Tracing



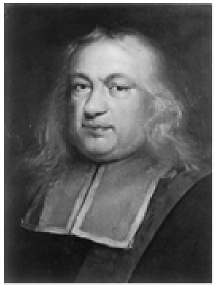


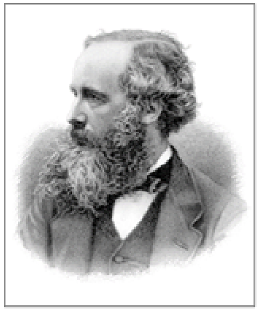
Fermat's Principle → Ray Optics → Ray Tracing Engine

Maxwell's Equations → Physical Optics → Field Tracing Engine

Powerful basis of optical design

Ray and Field Tracing





Fermat's Principle

Ray Optics

Ray Tracing Engine

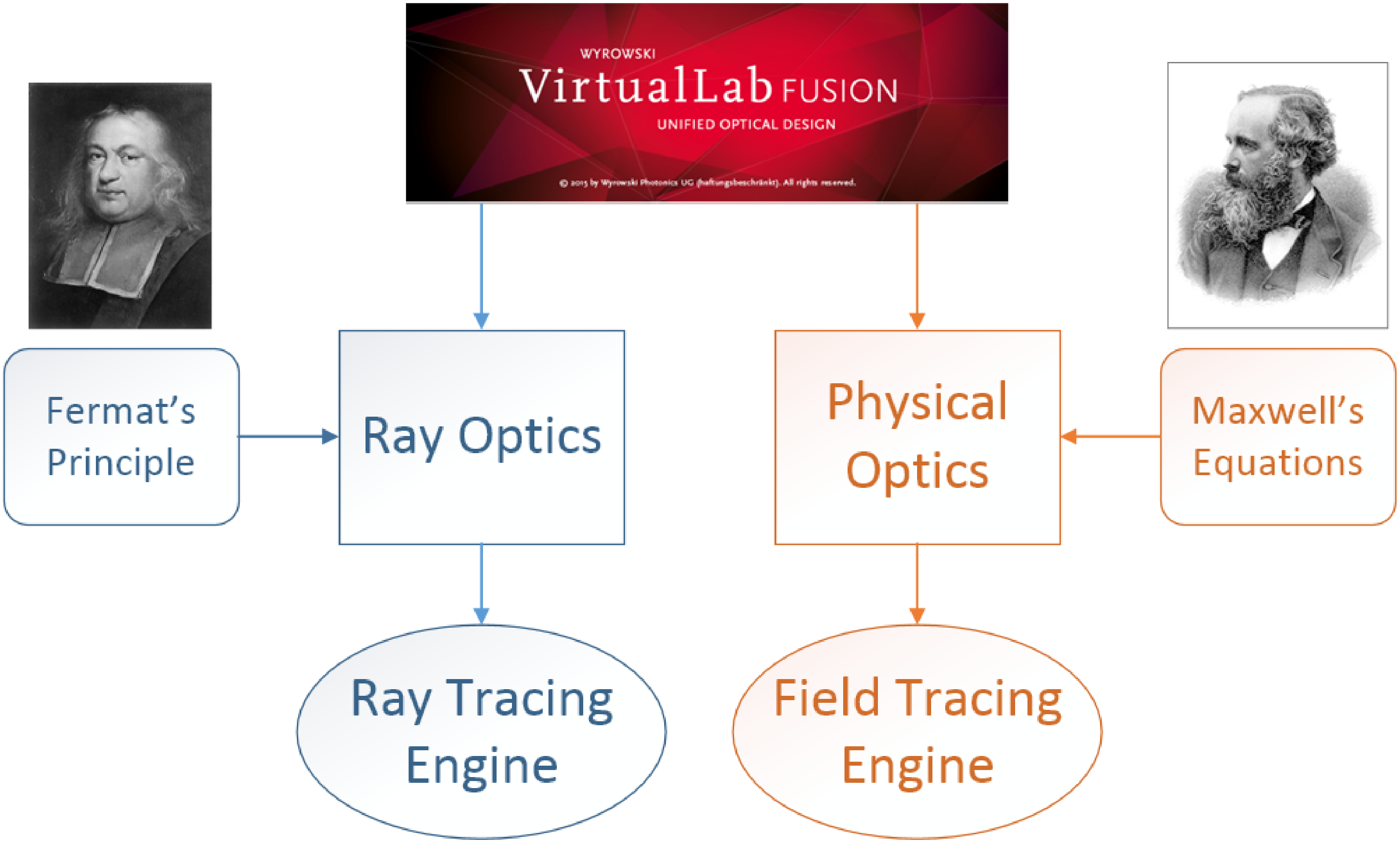
Physical Optics

Maxwell's Equations

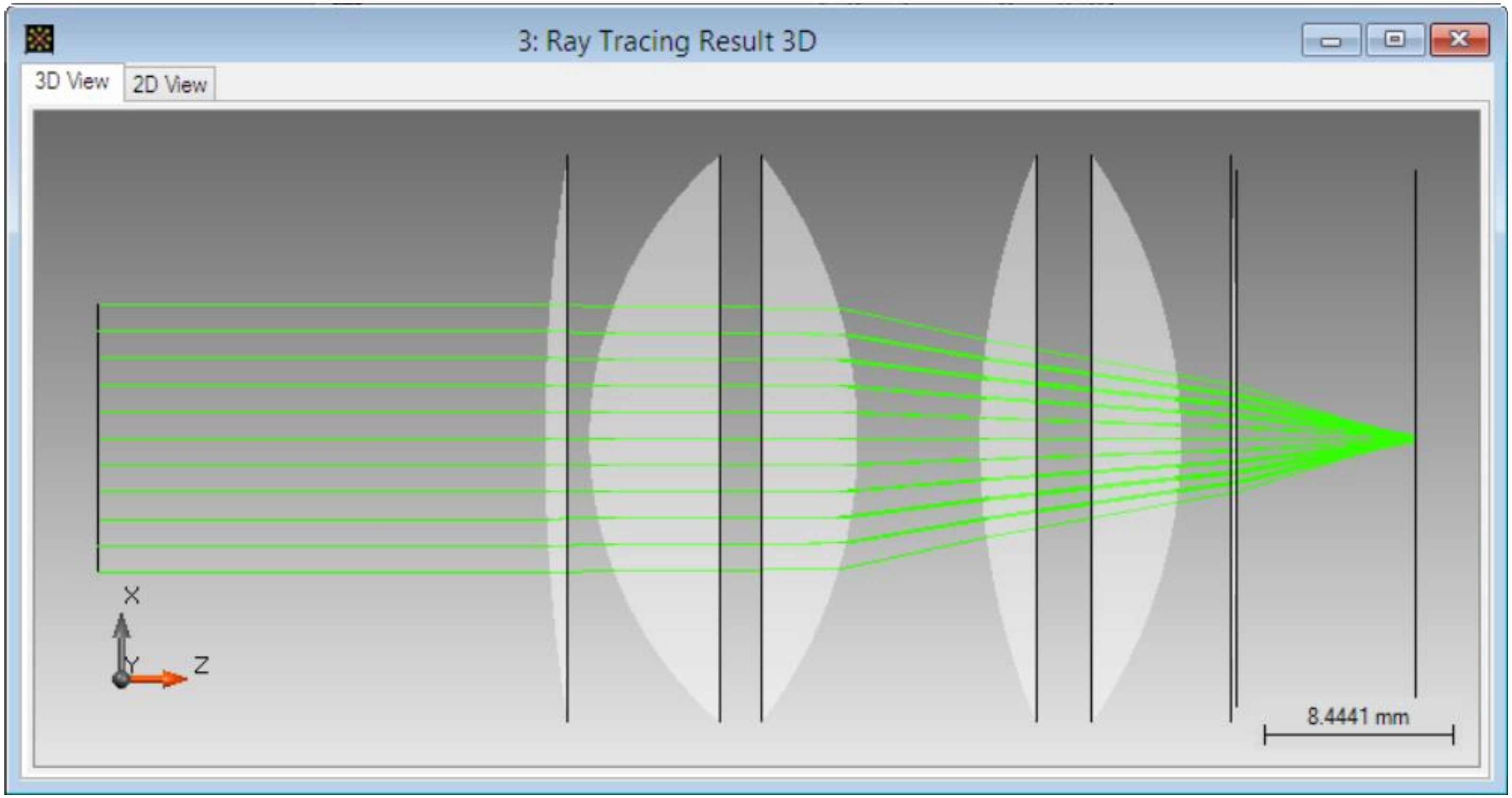
Field Tracing Engine

Going beyond the limitations of ray tracing

Ray and Field Tracing

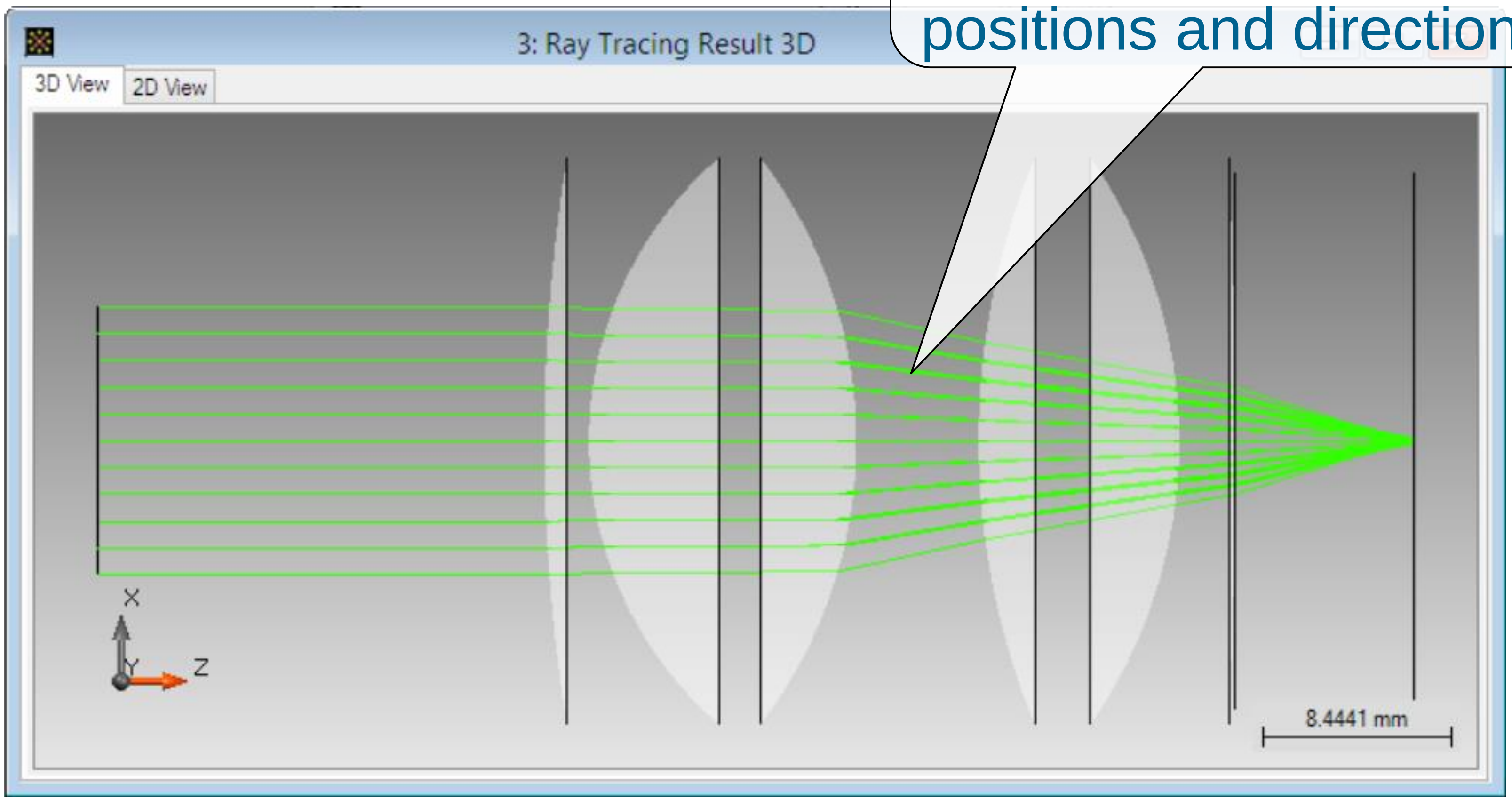


Ray Tracing Concept and Limitations

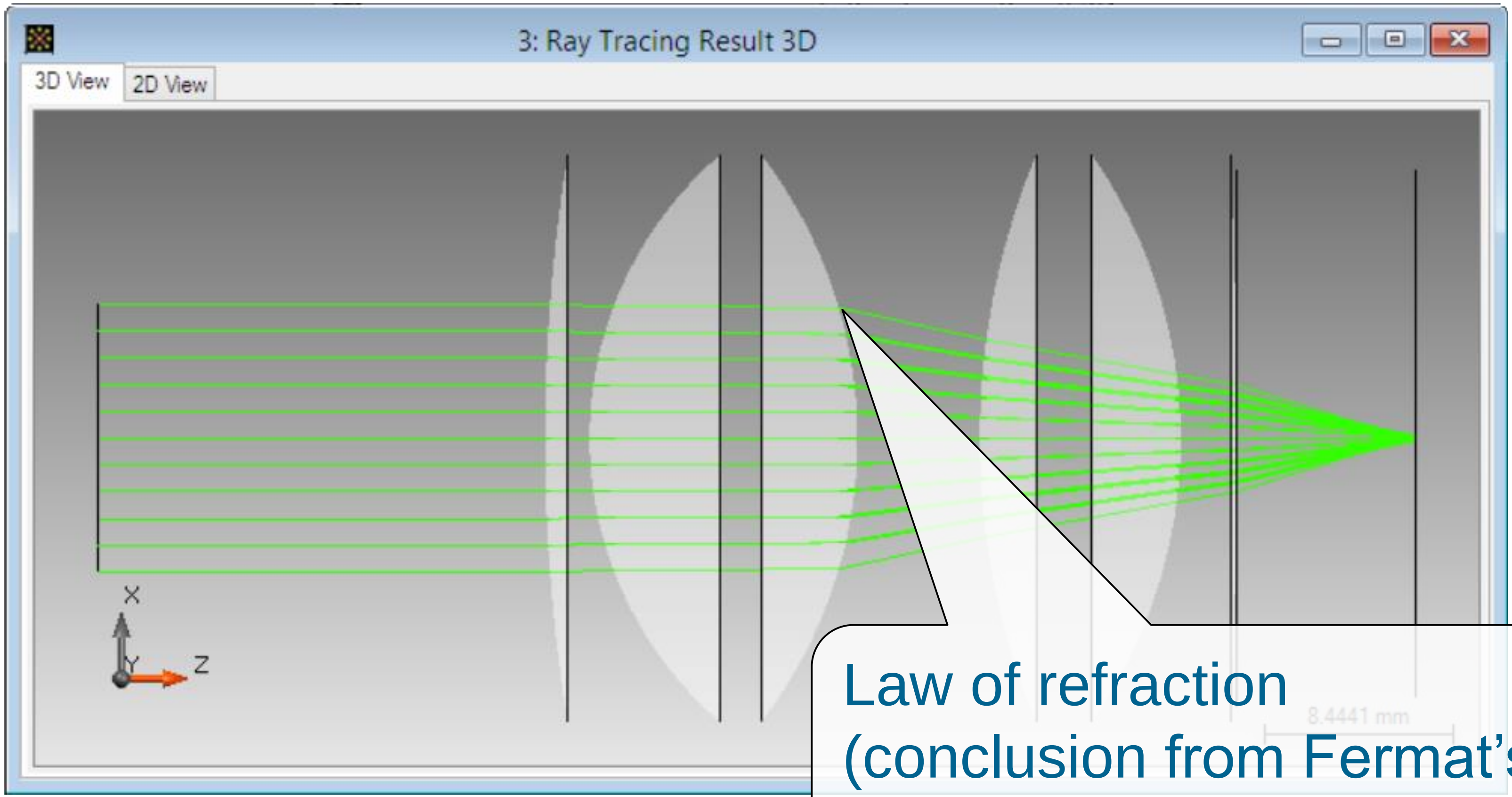


Ray Tracing Concept and Limitations

Rays are traced by determining their positions and directions.

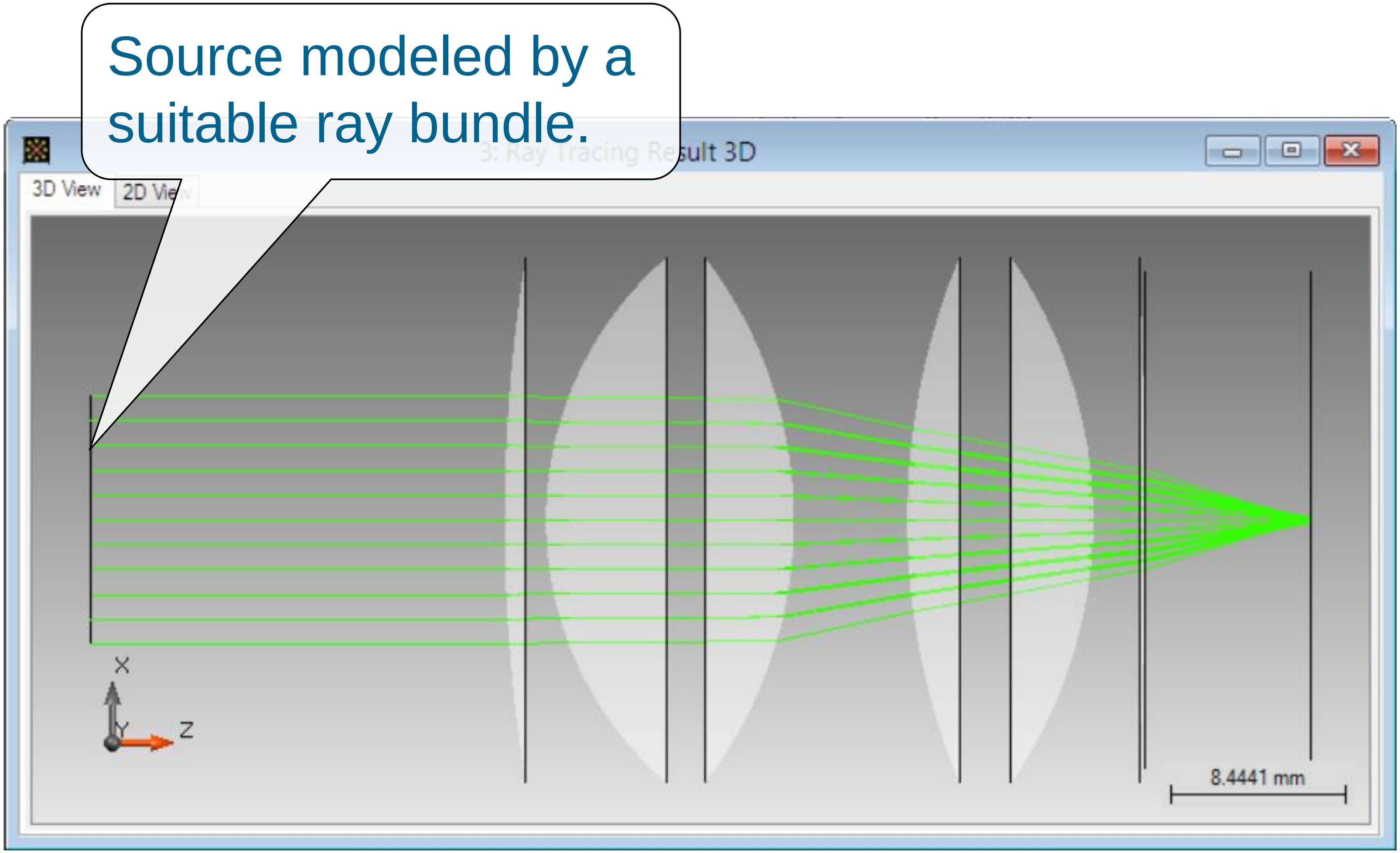


Ray Tracing Concept and Limitations

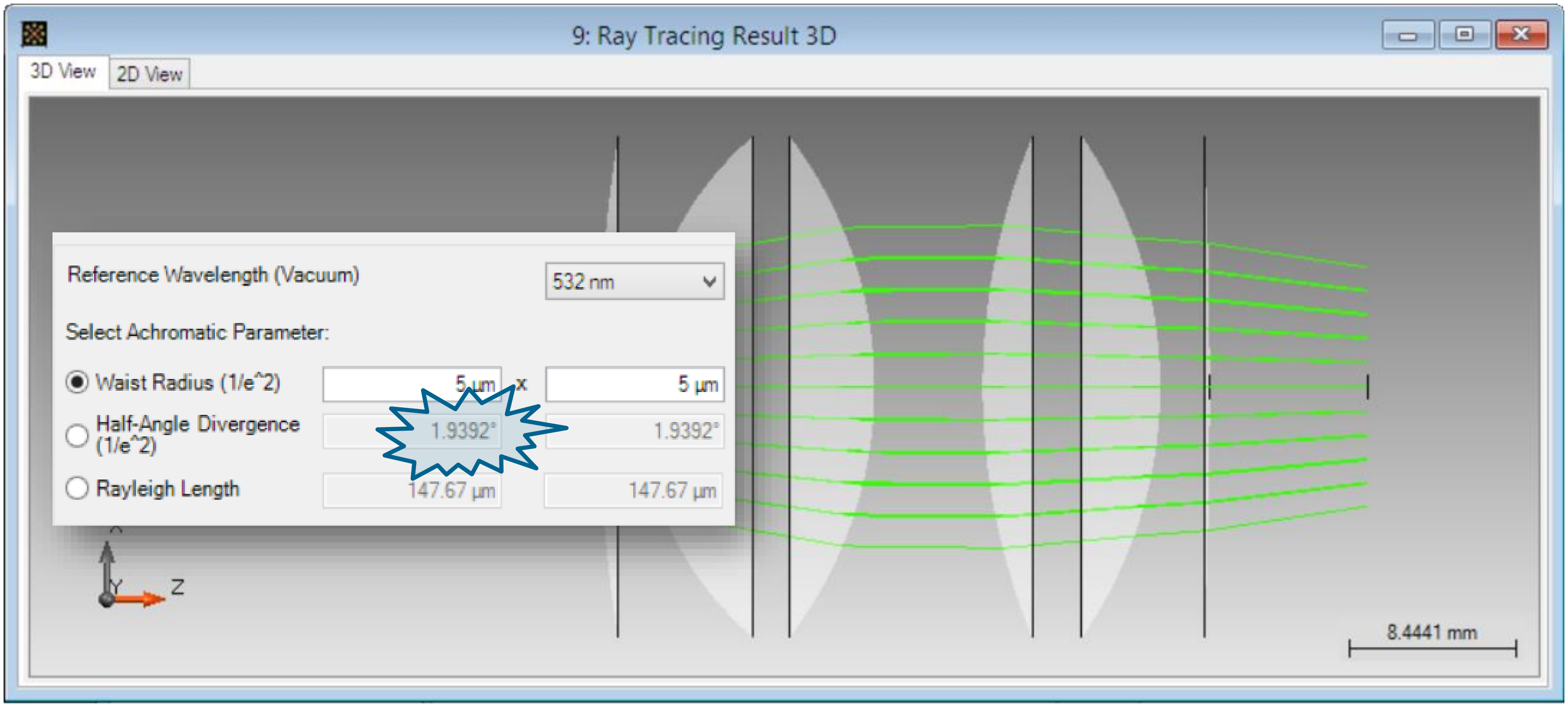


Law of refraction (conclusion from Fermat's principle) is applied per ray.

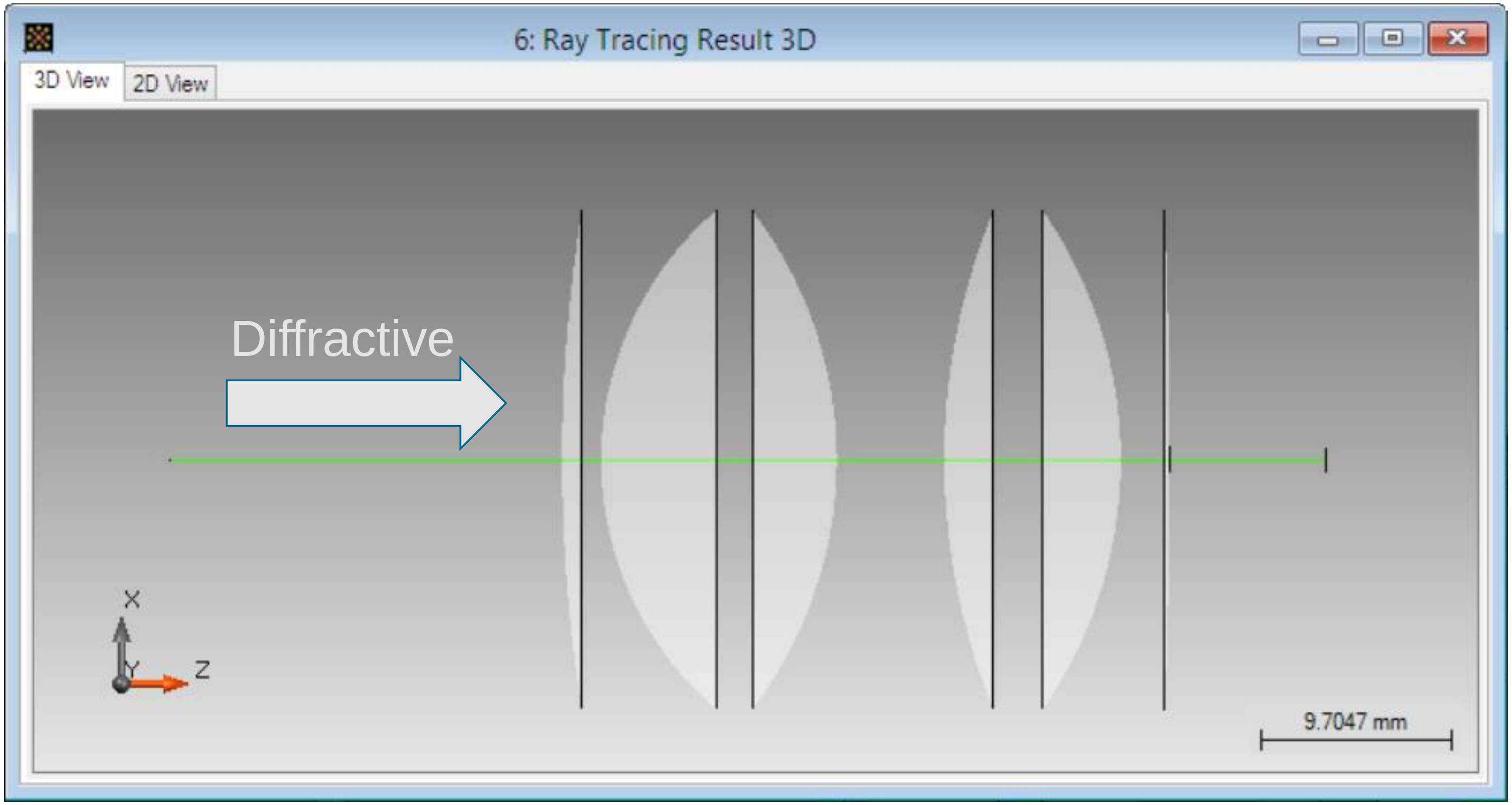
Ray Tracing Concept and Limitations



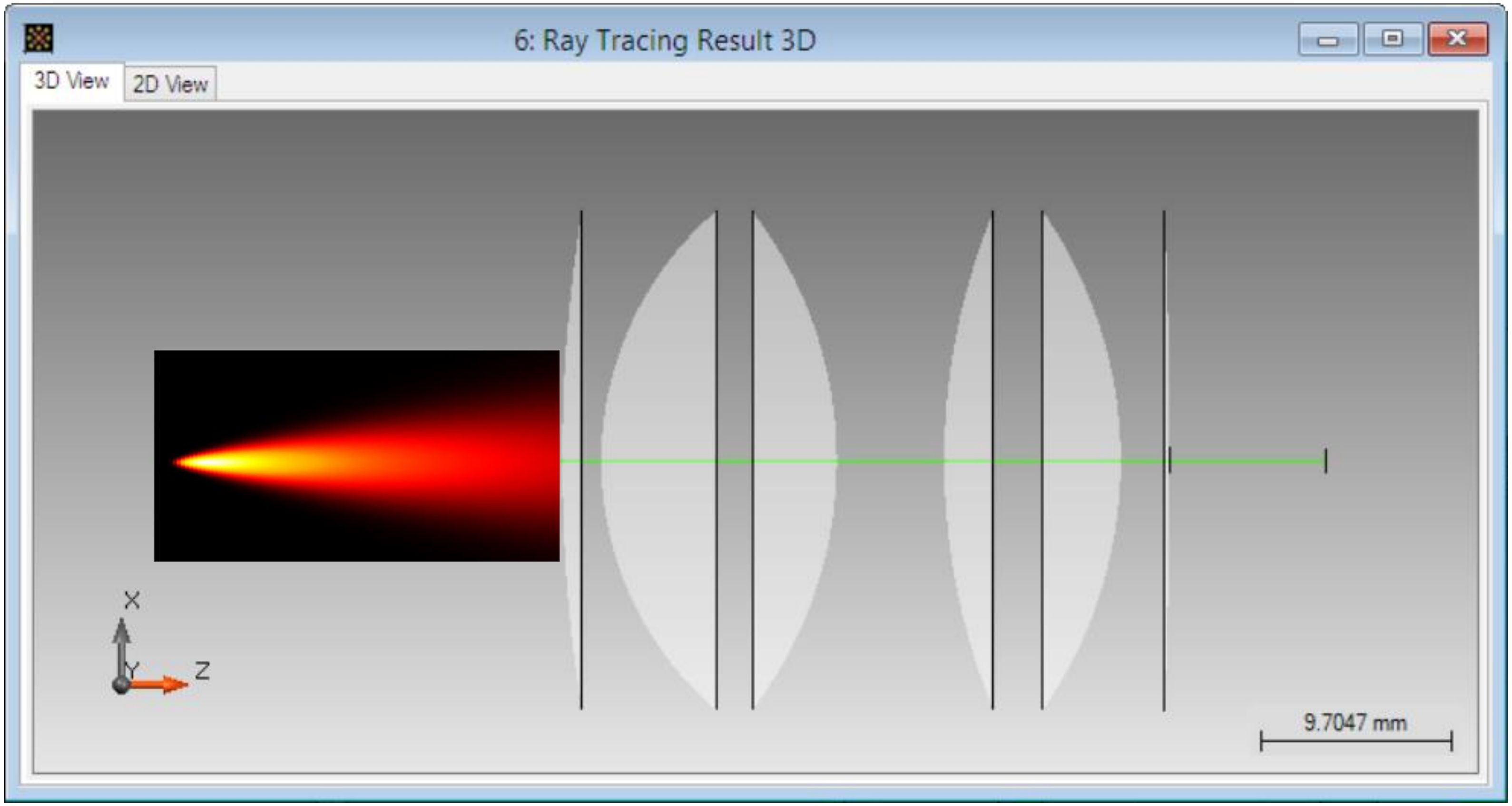
Ray Tracing Limitations: Gaussian Source



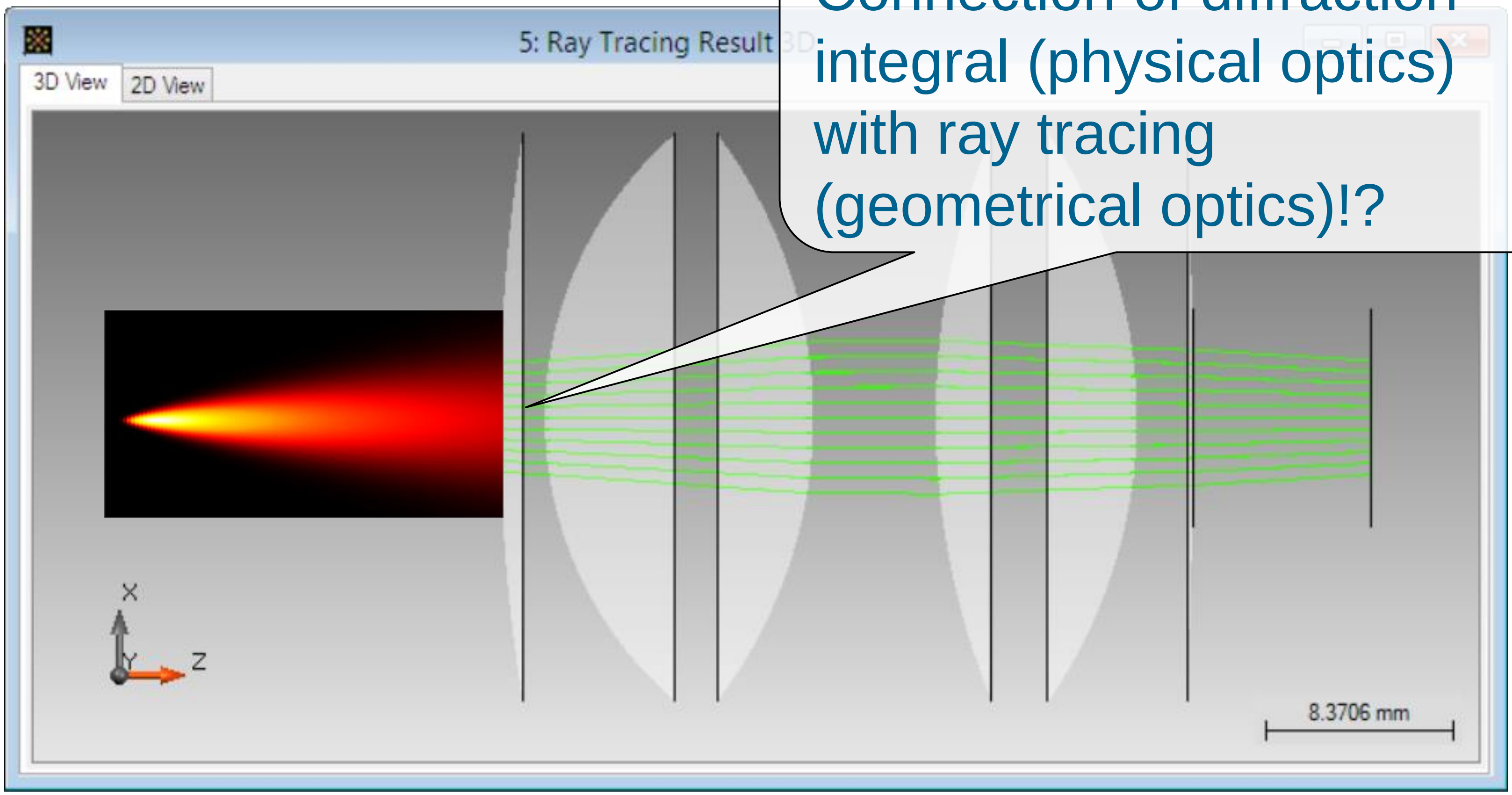
Ray Tracing Limitations: Gaussian Source



Ray Tracing Limitations: Gaussian Source

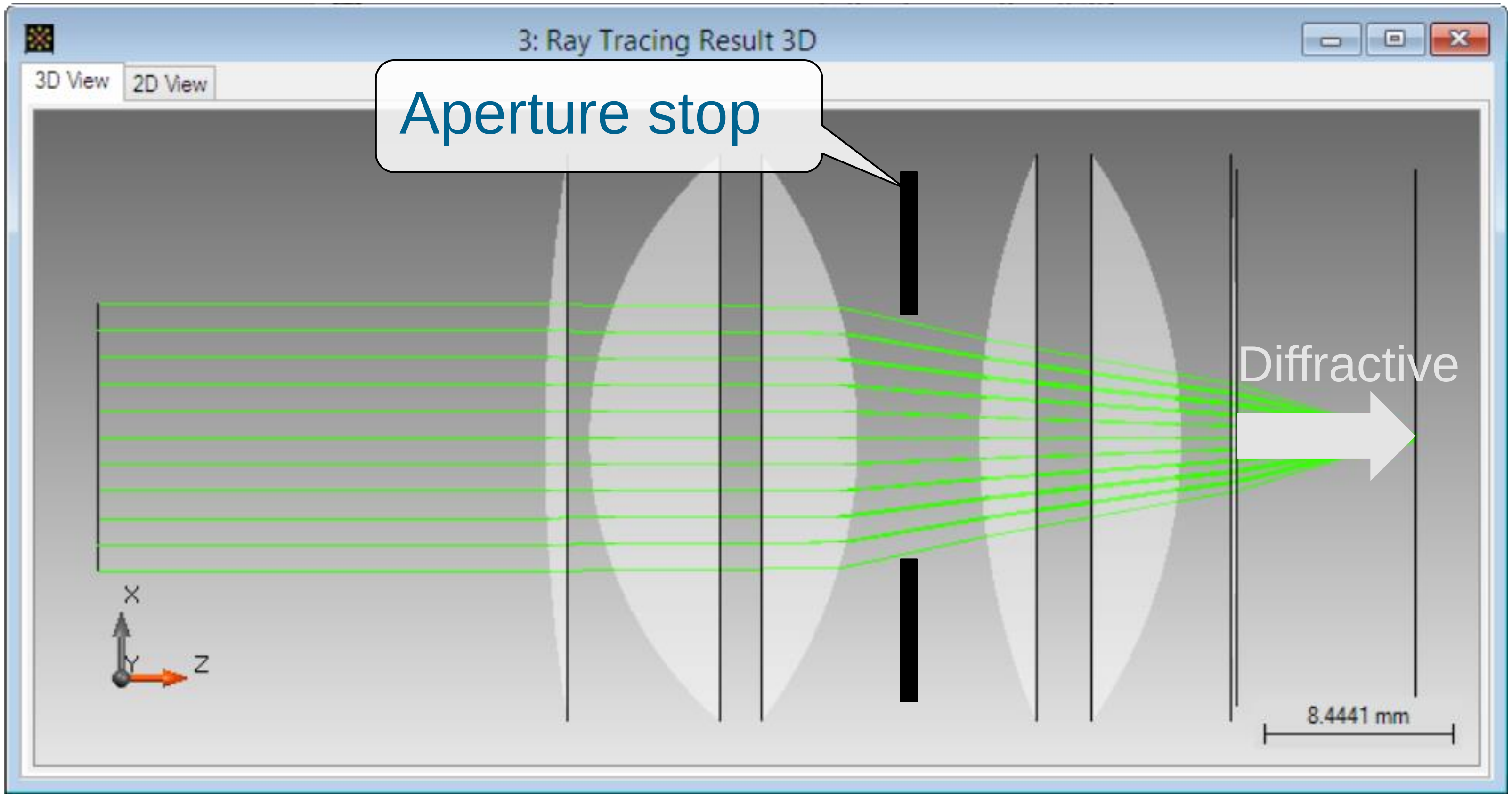


Ray Tracing Limitations: Gaussian Source

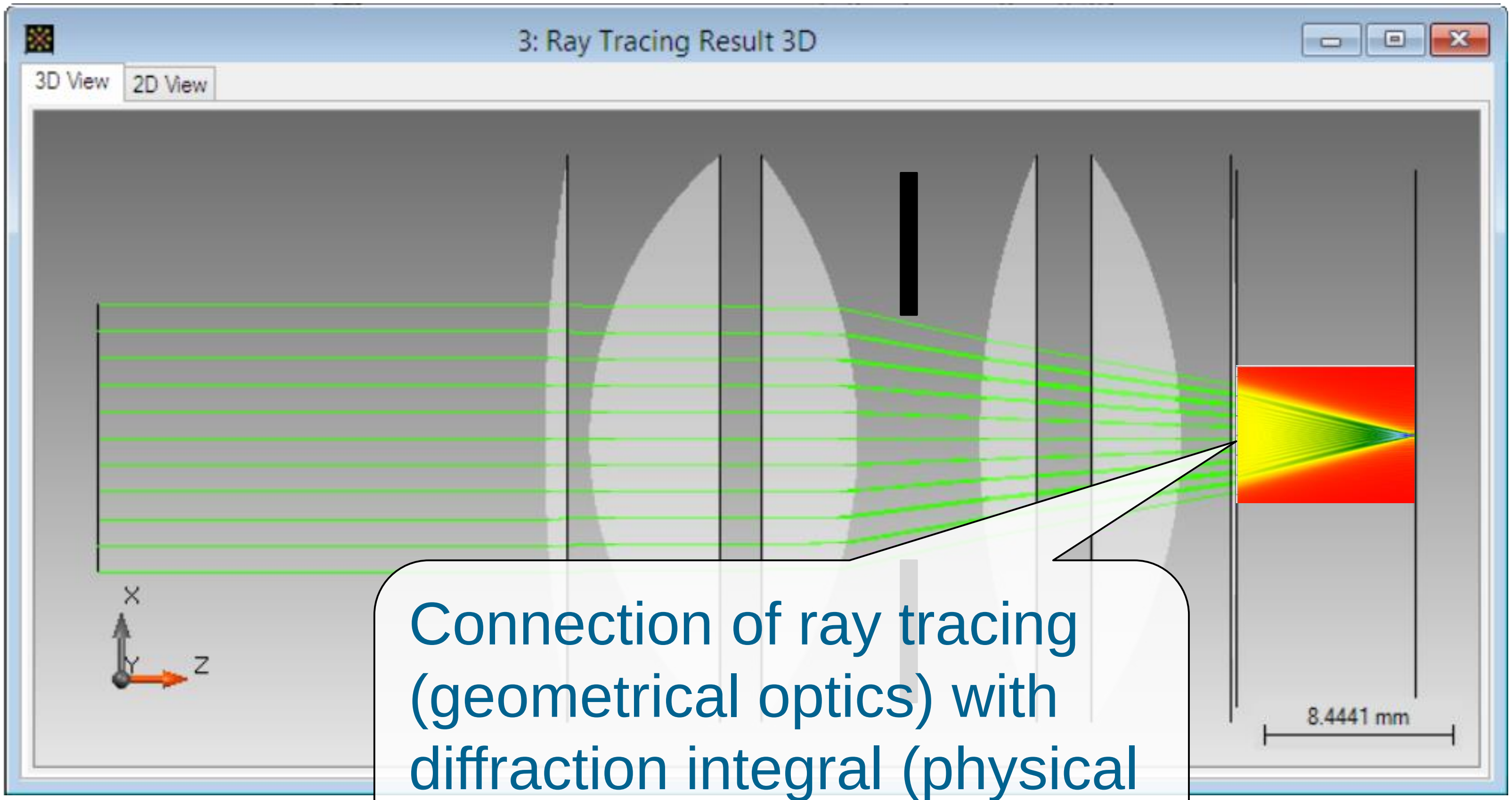


Connection of diffraction integral (physical optics) with ray tracing (geometrical optics)!?

Ray Tracing Limitations: Focusing

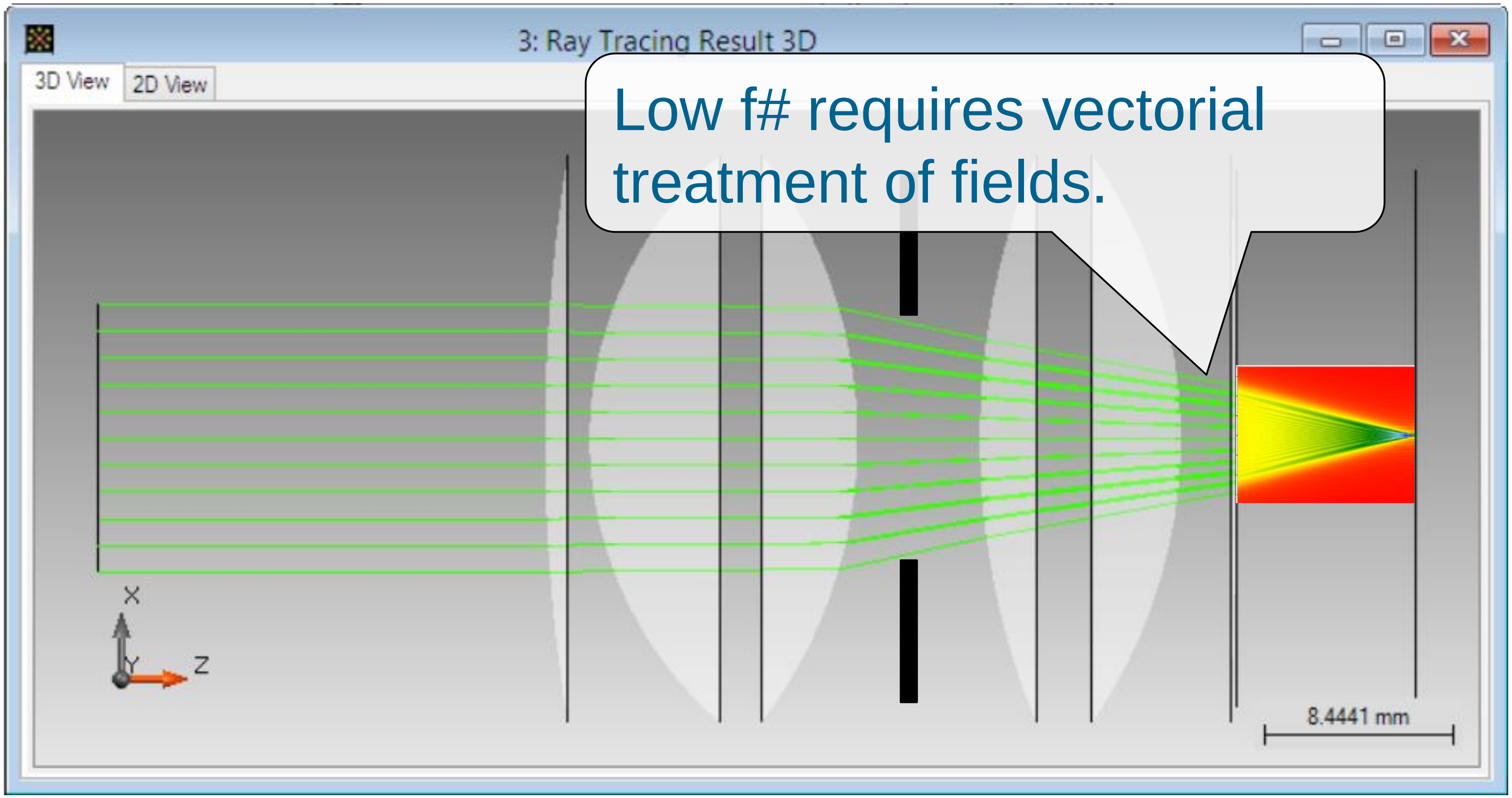


Ray Tracing Limitations: Focusing



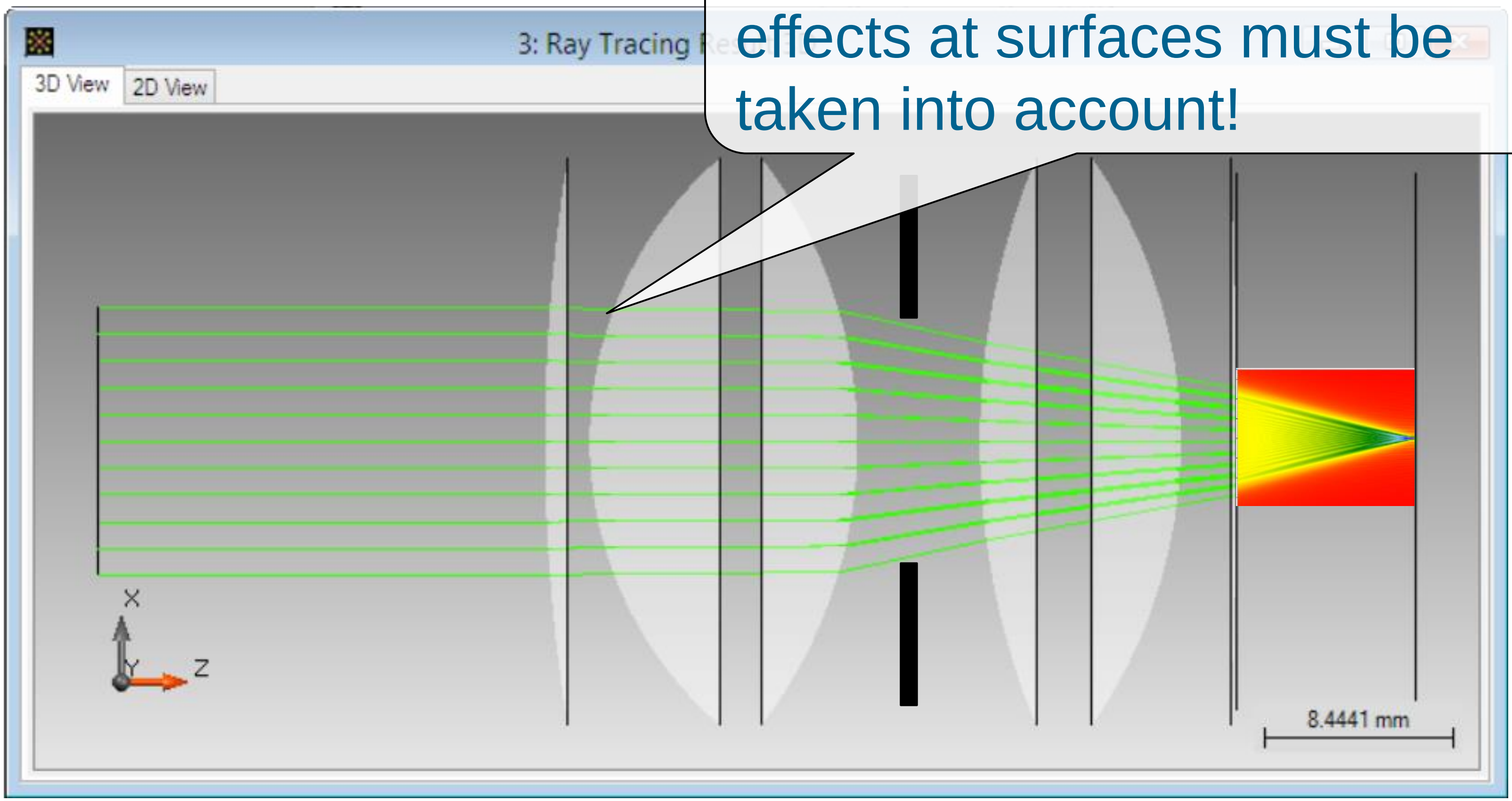
Connection of ray tracing (geometrical optics) with diffraction integral (physical optics)!?

Ray Tracing Limitations: Vectorial Modeling



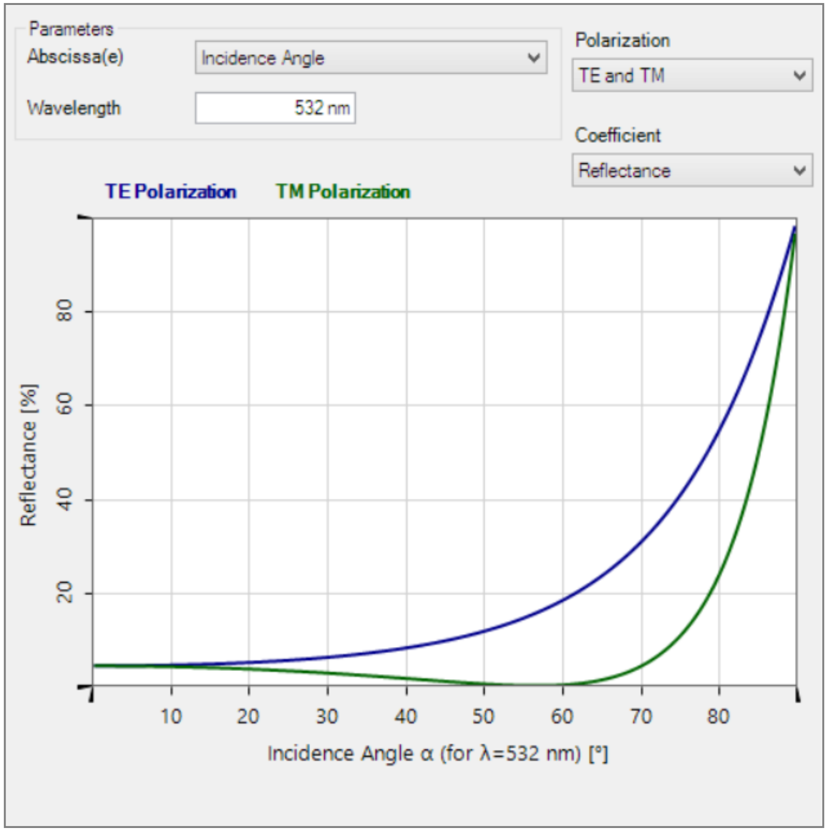
Ray Tracing Limitations: Vectorial Modeling

Energy losses and polarization effects at surfaces must be taken into account!

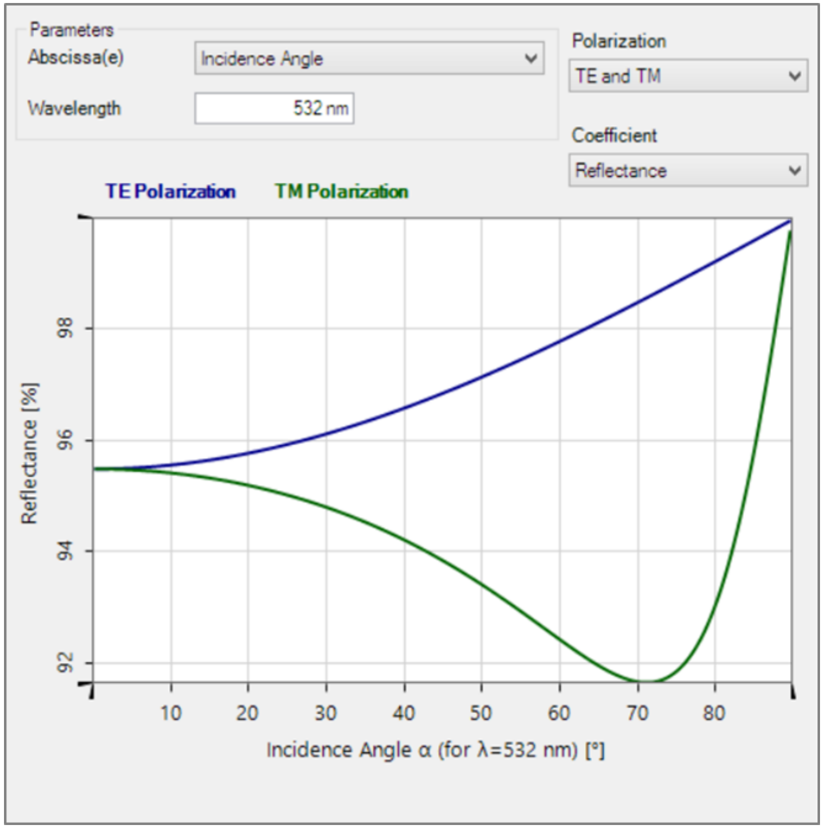


Fresnel Effect: Reflectance for TE/TM

Air vs. BK7 glass

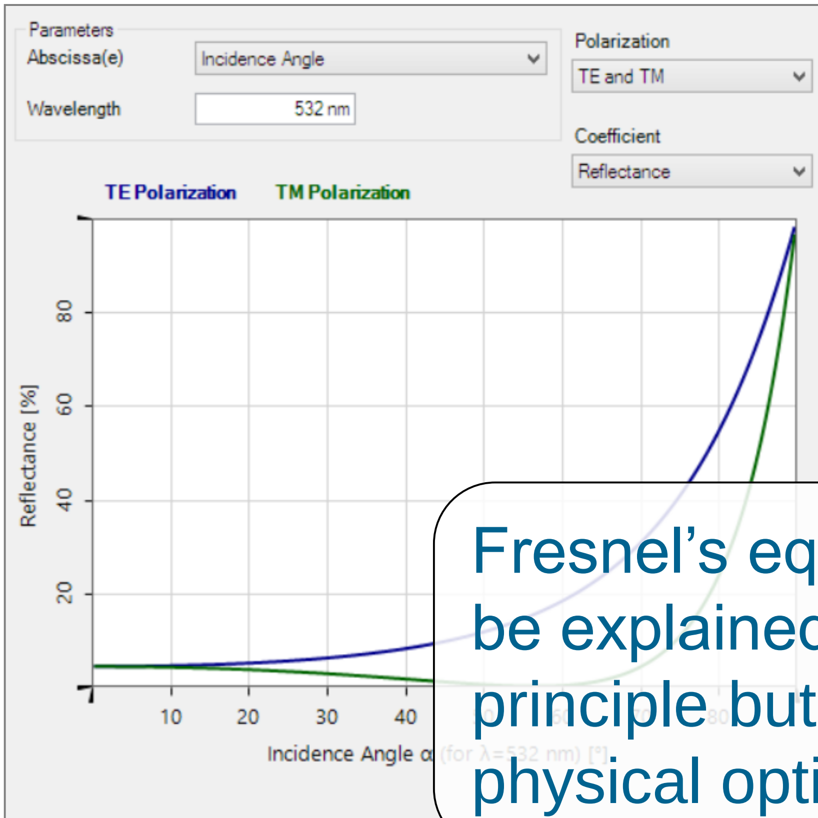


Air vs. silver

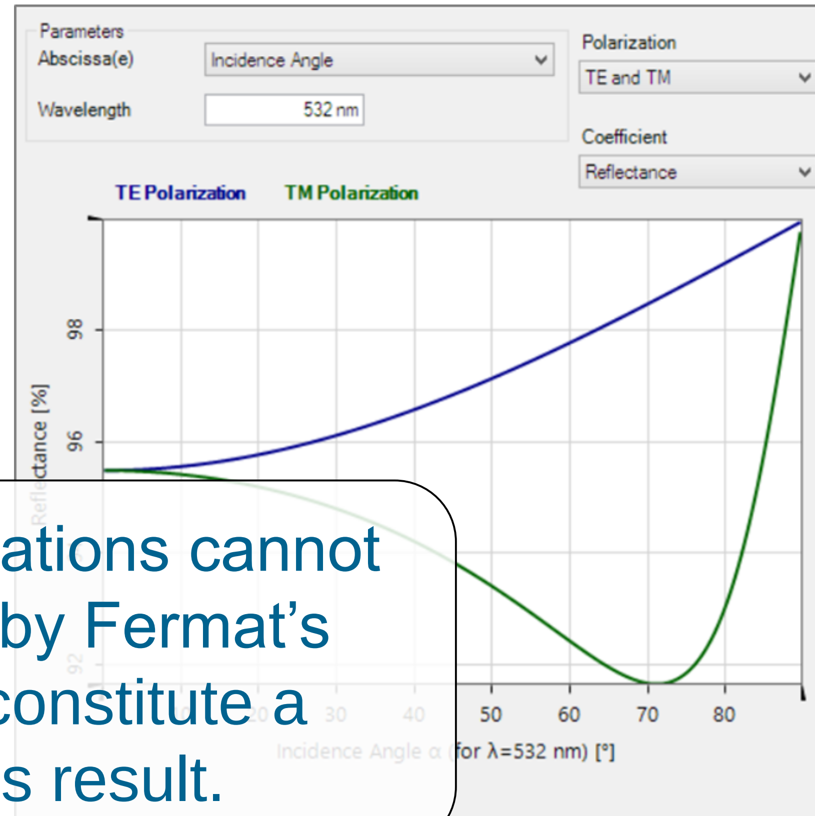


Fresnel Effect: Reflectance for TE/TM

Air vs. BK7 glass

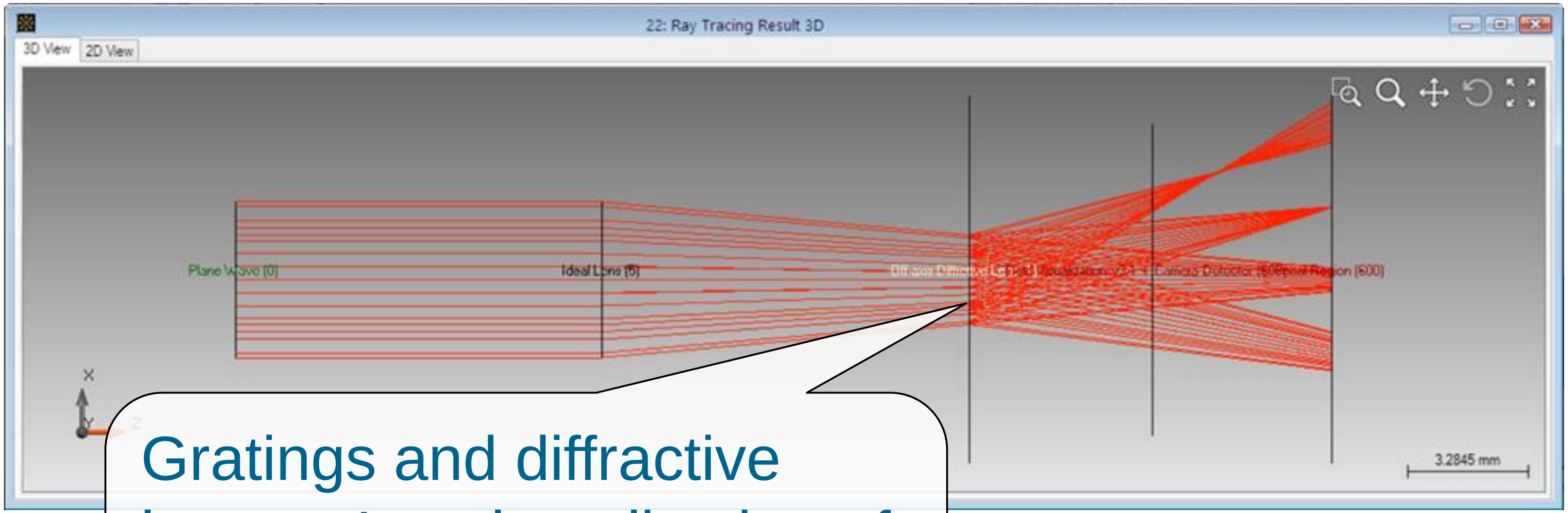


Air vs. silver



Fresnel's equations cannot be explained by Fermat's principle but constitute a physical optics result.

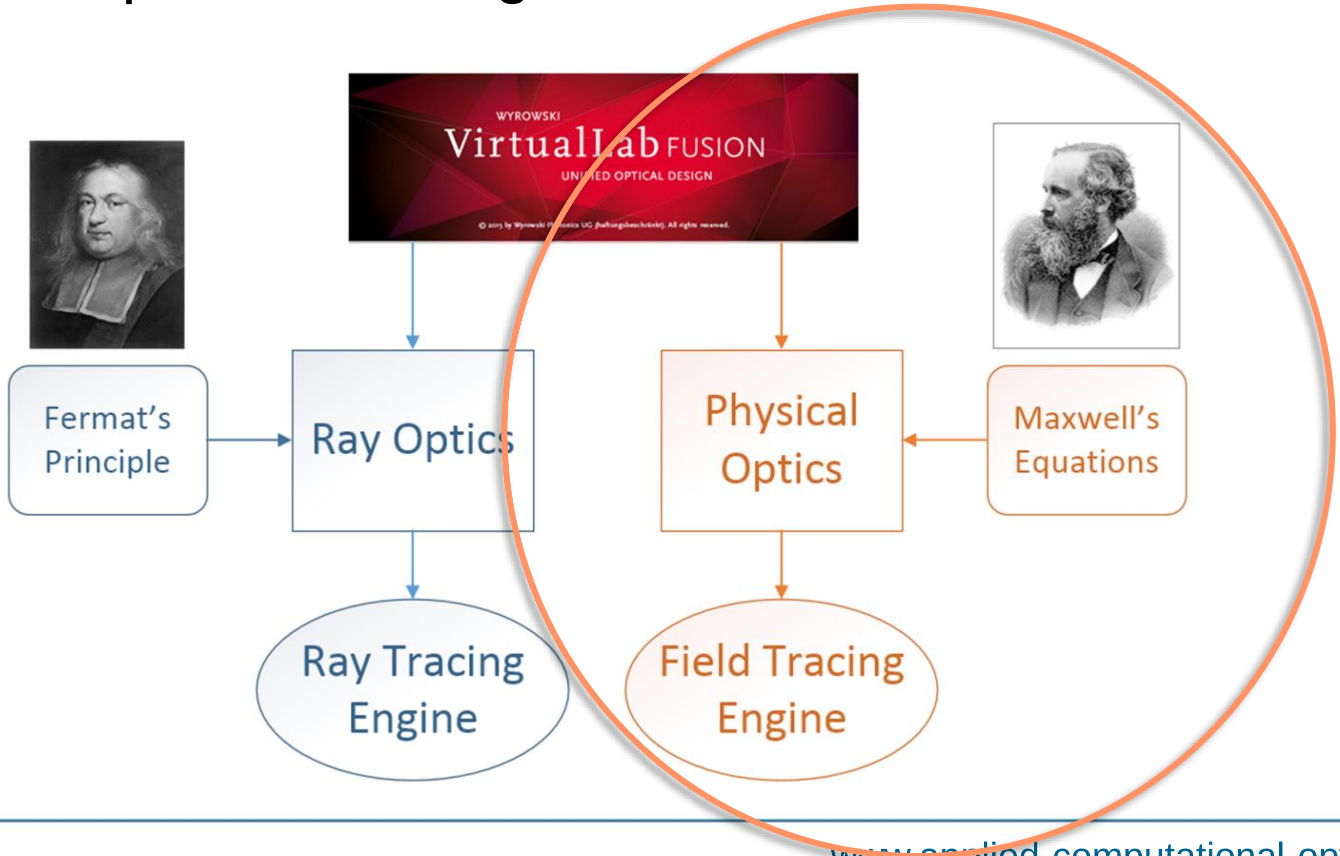
Ray Tracing Limitations: Gratings



Gratings and diffractive lenses: Local application of vectorial grating response by grating theory.

Physical Optics Modeling

- Ray tracing is far too be restricted for ultrafast optics modeling.
- Physical optics modeling is demanded!

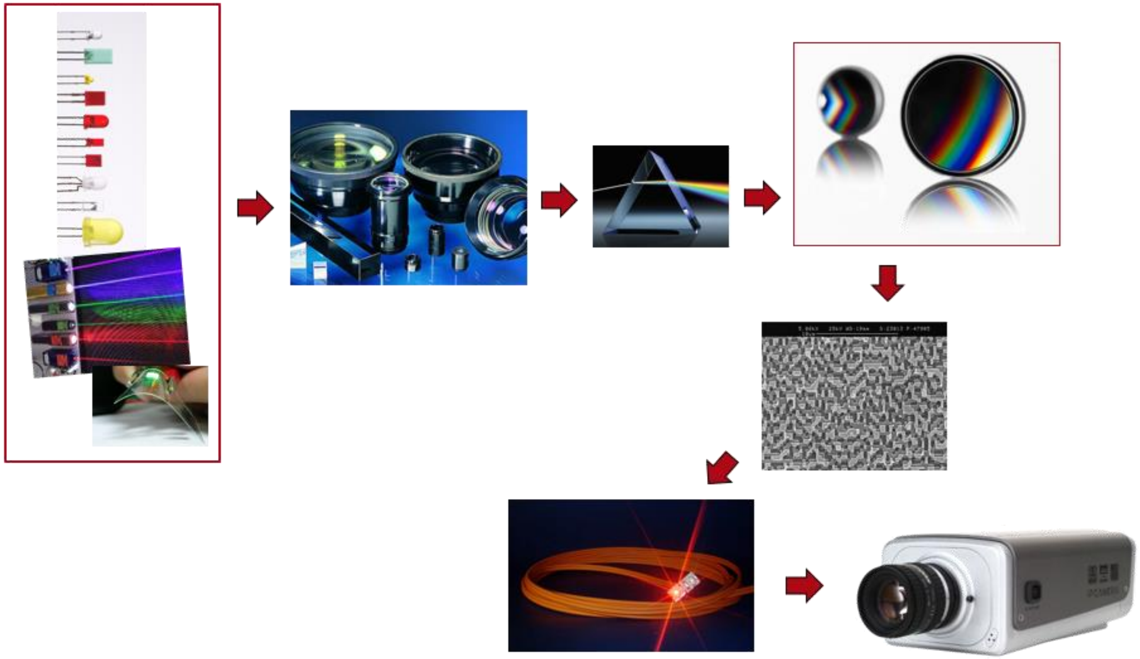


Modeling by Field Tracing

Solve Maxwell’s equations!

But …

- We all know, **universal and rigorous** Maxwell Solver like FDTD and FEM, are typically not practical beyond nanooptics and special effects simulation.



But …

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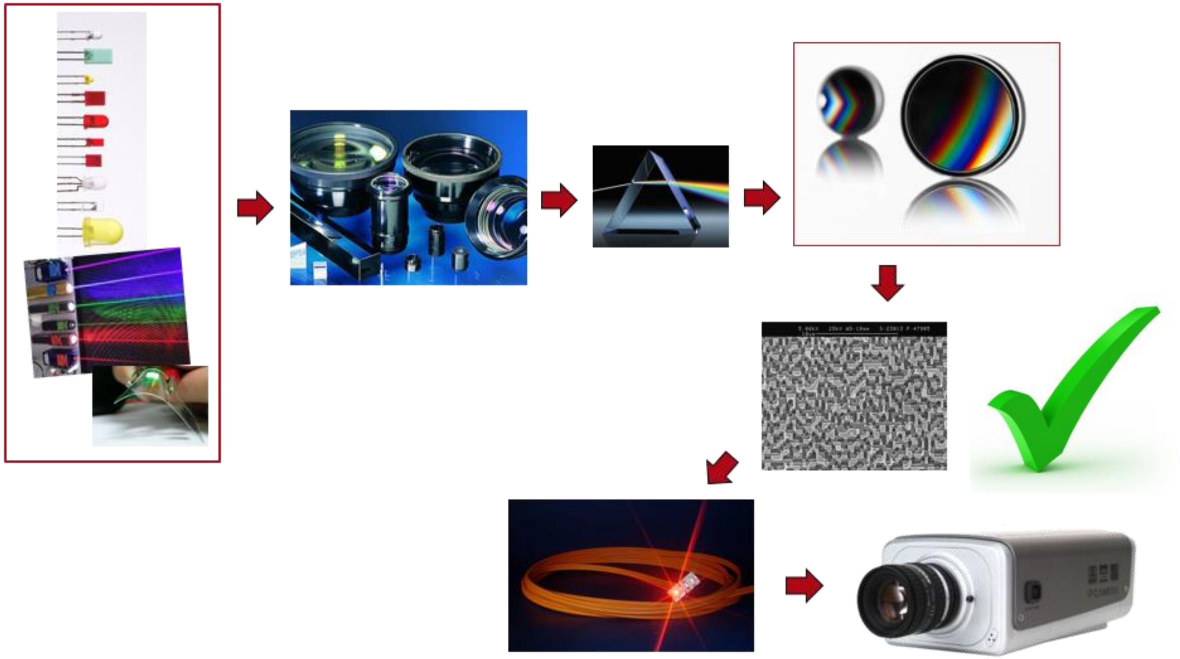

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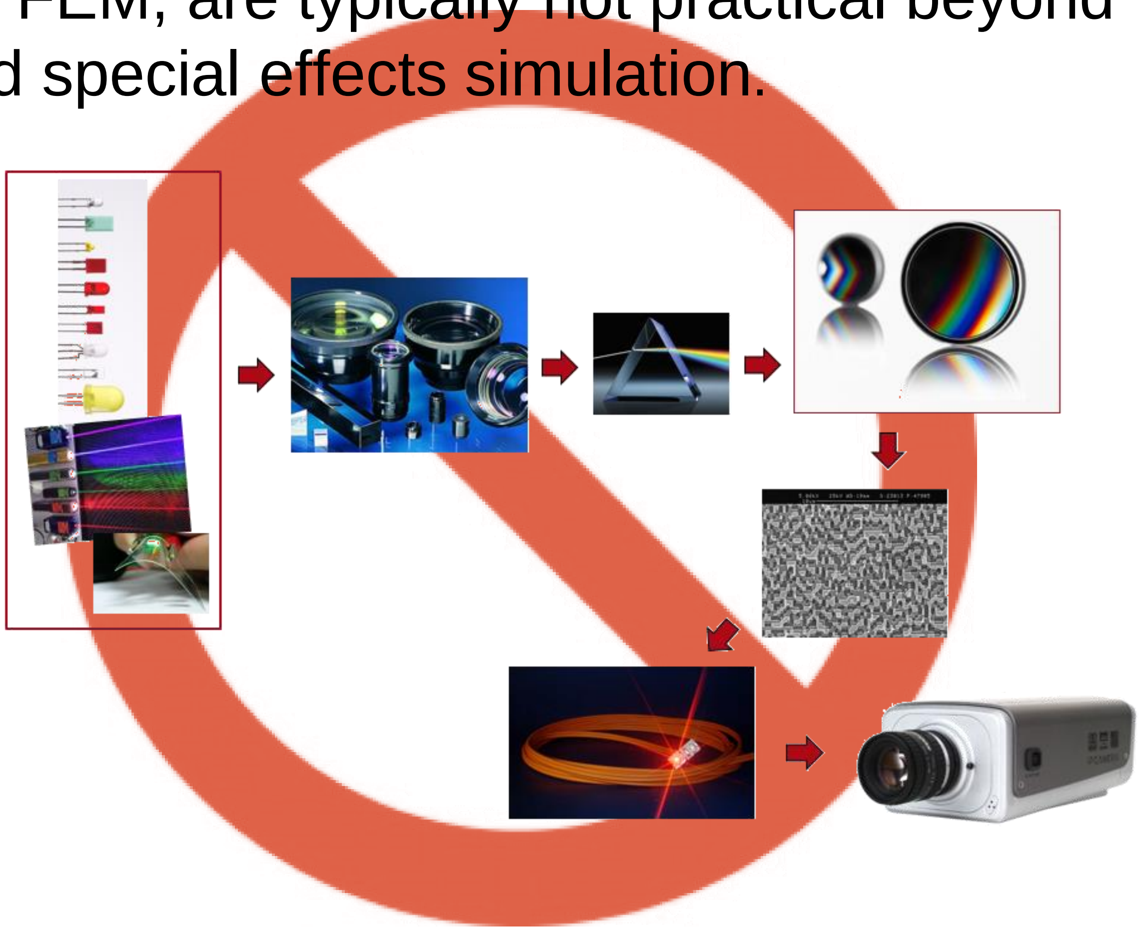
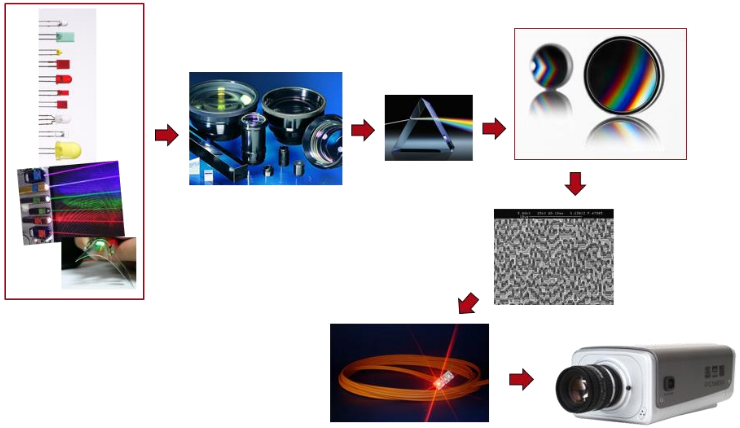


But …

- We all know, **universal and rigorous** Maxwell Solver like FDTD and FEM, are typically not practical beyond nanooptics and special effects simulation.

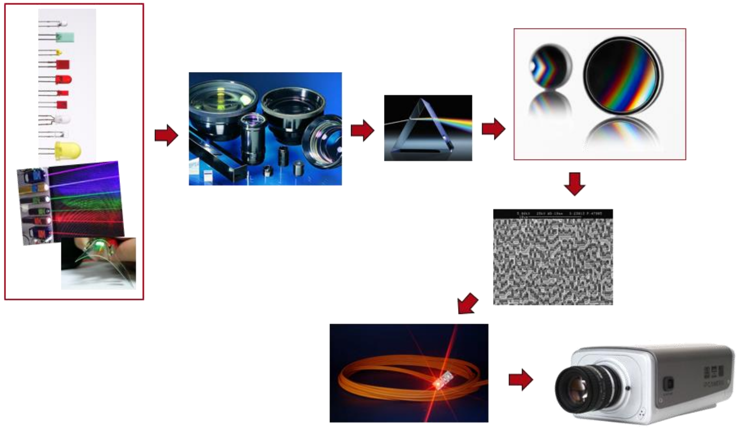
How to solve this basic problem?



Specialized and/or Approximated Solver

- Instead of one universal and rigorous solver we must combine **specialized** and/or **approximated** solver!

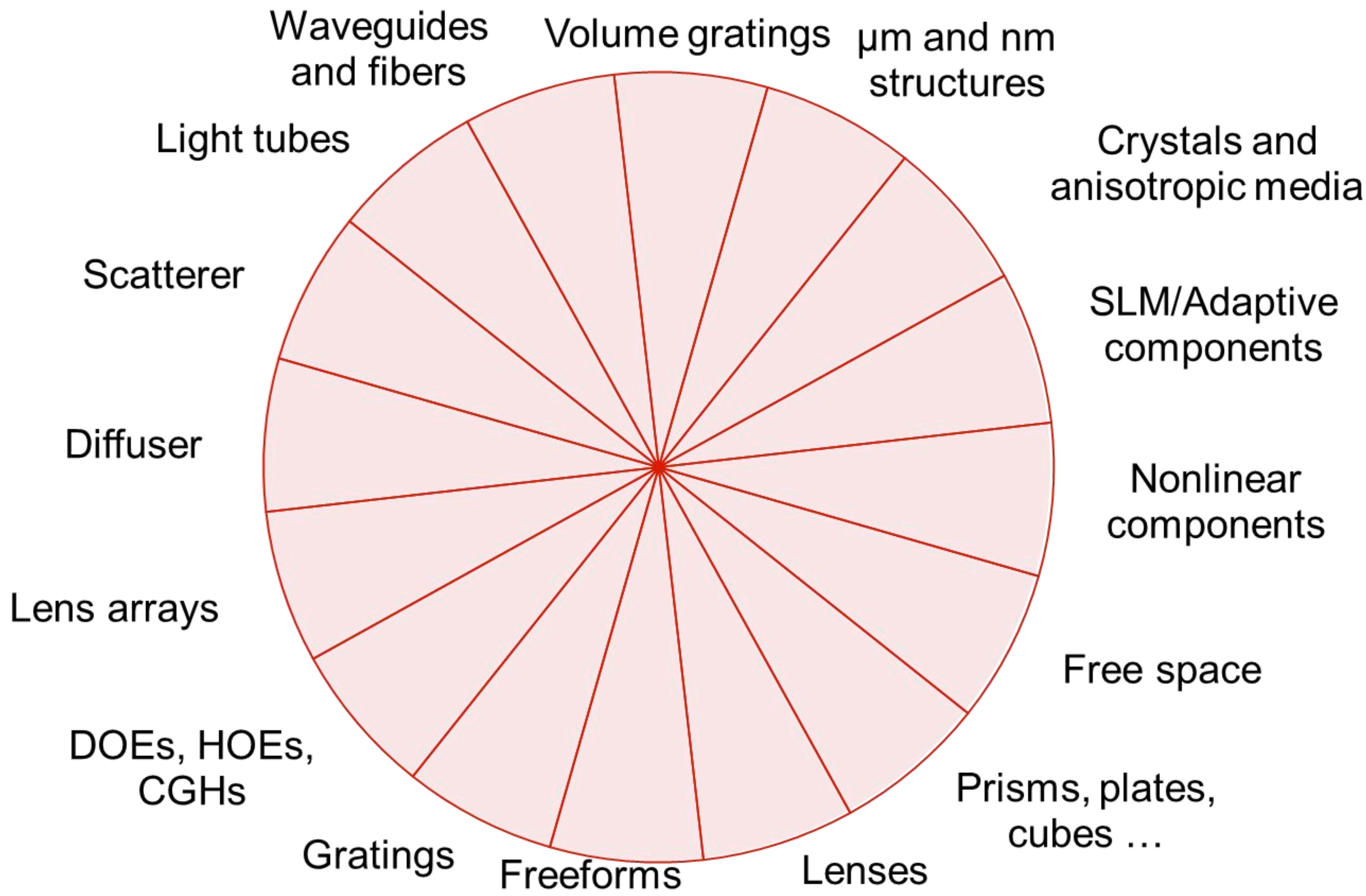
Unified Field Tracing



Unified Field Tracing

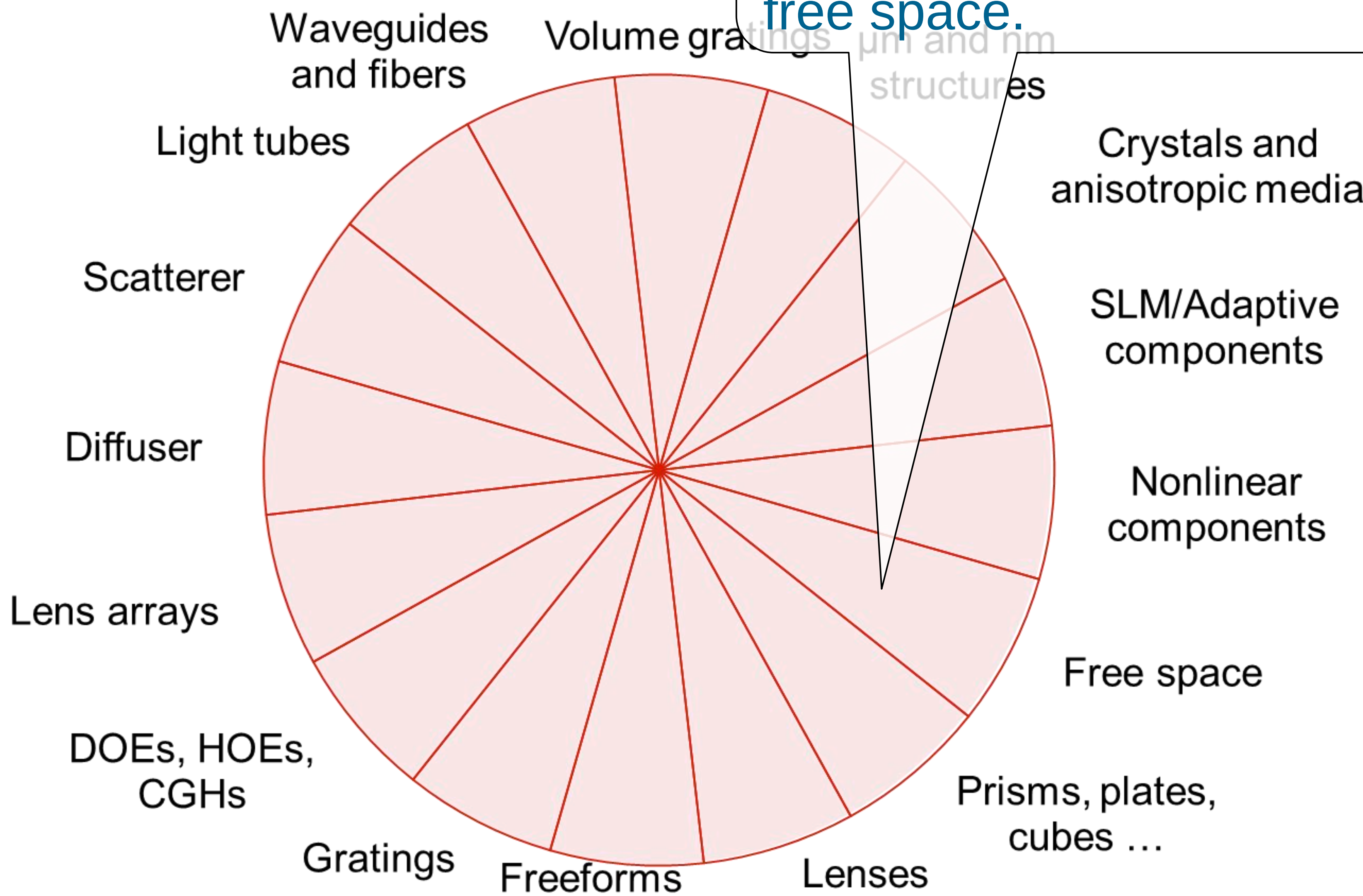
Solve Maxwell’s equations!

Unified Field Tracing



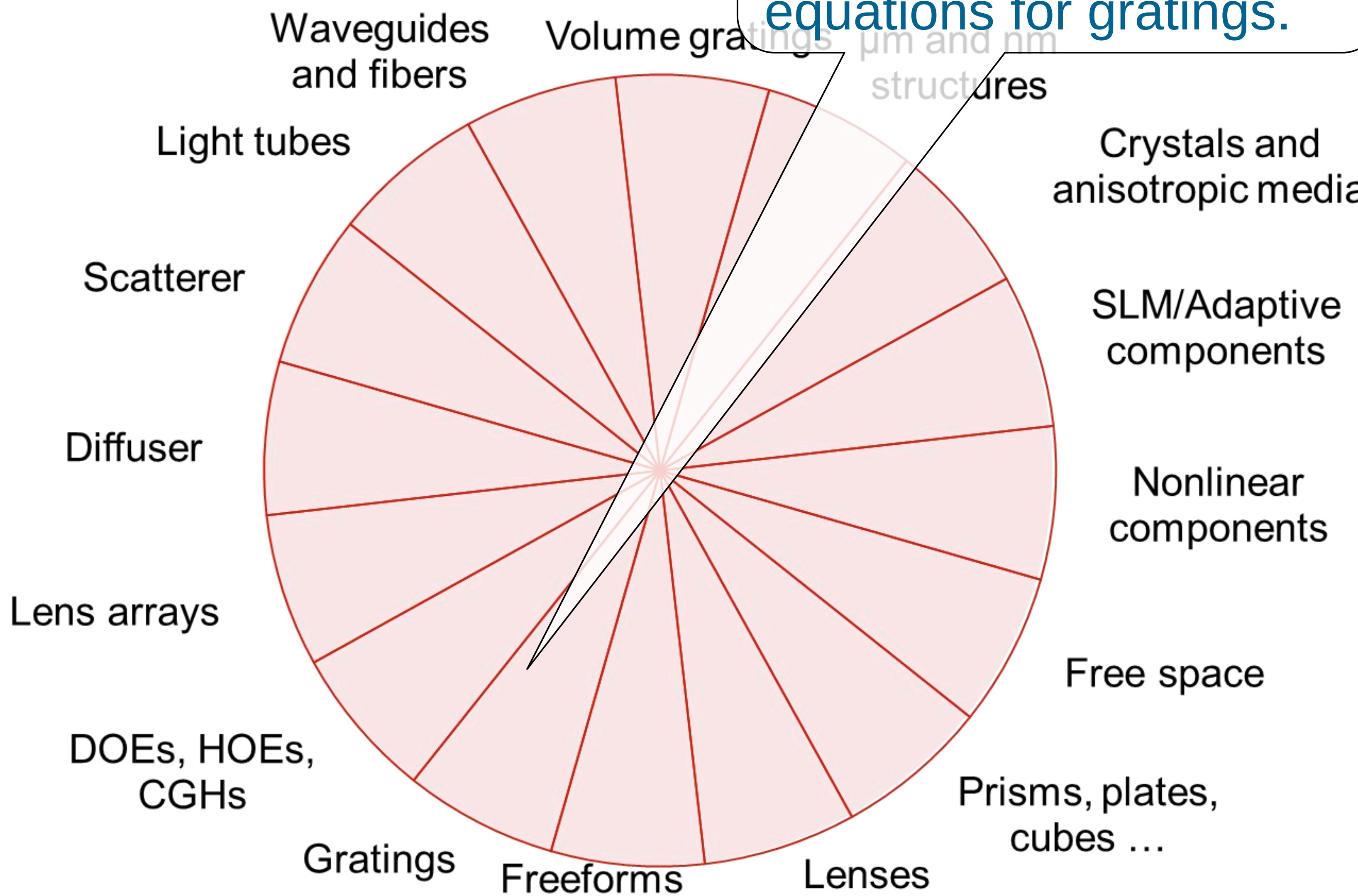
Unified Field Tracing

Special methods to solve Maxwell's equations in free space.



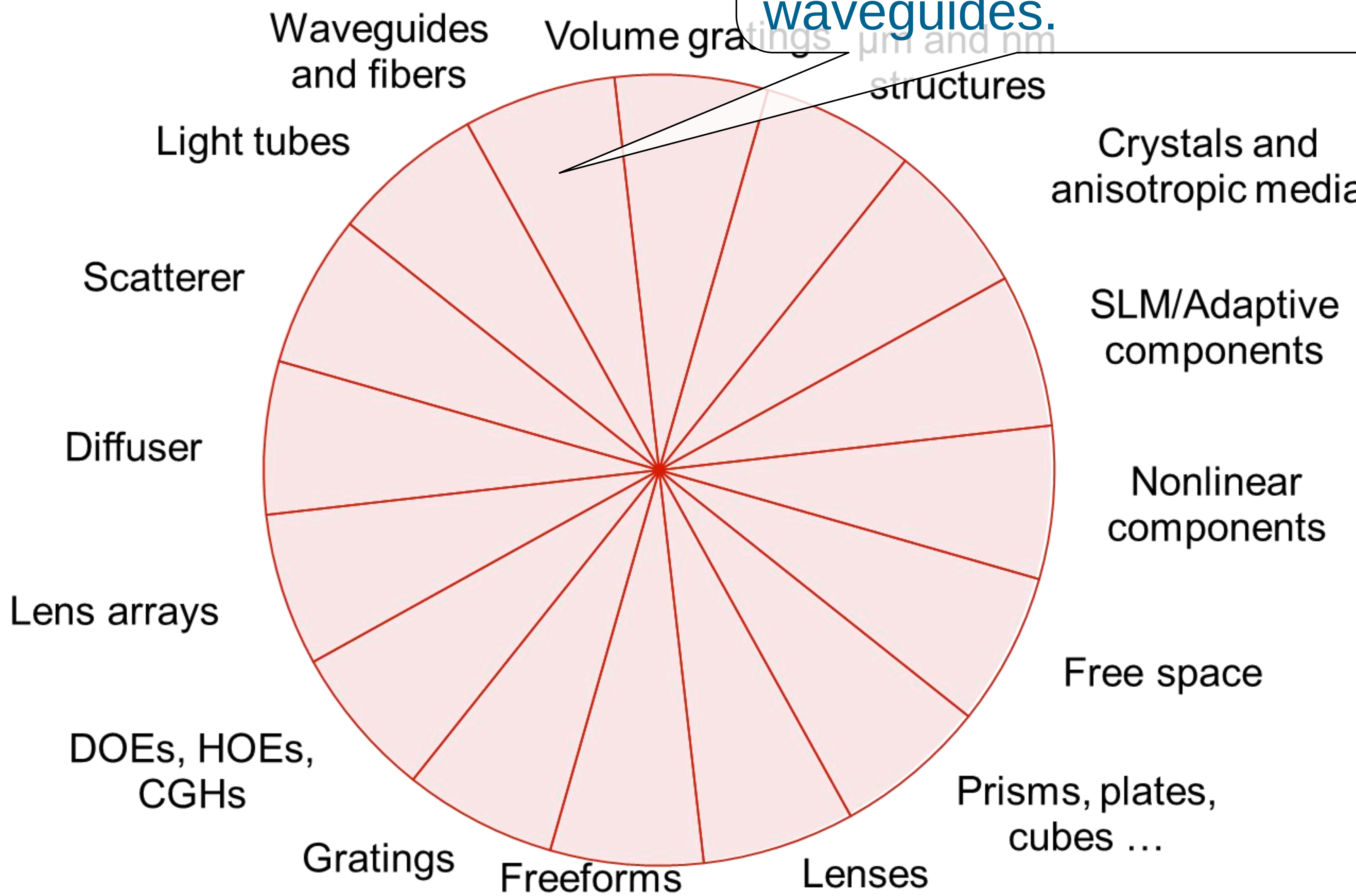
Unified Field Tracing

Special methods to solve Maxwell's equations for gratings.

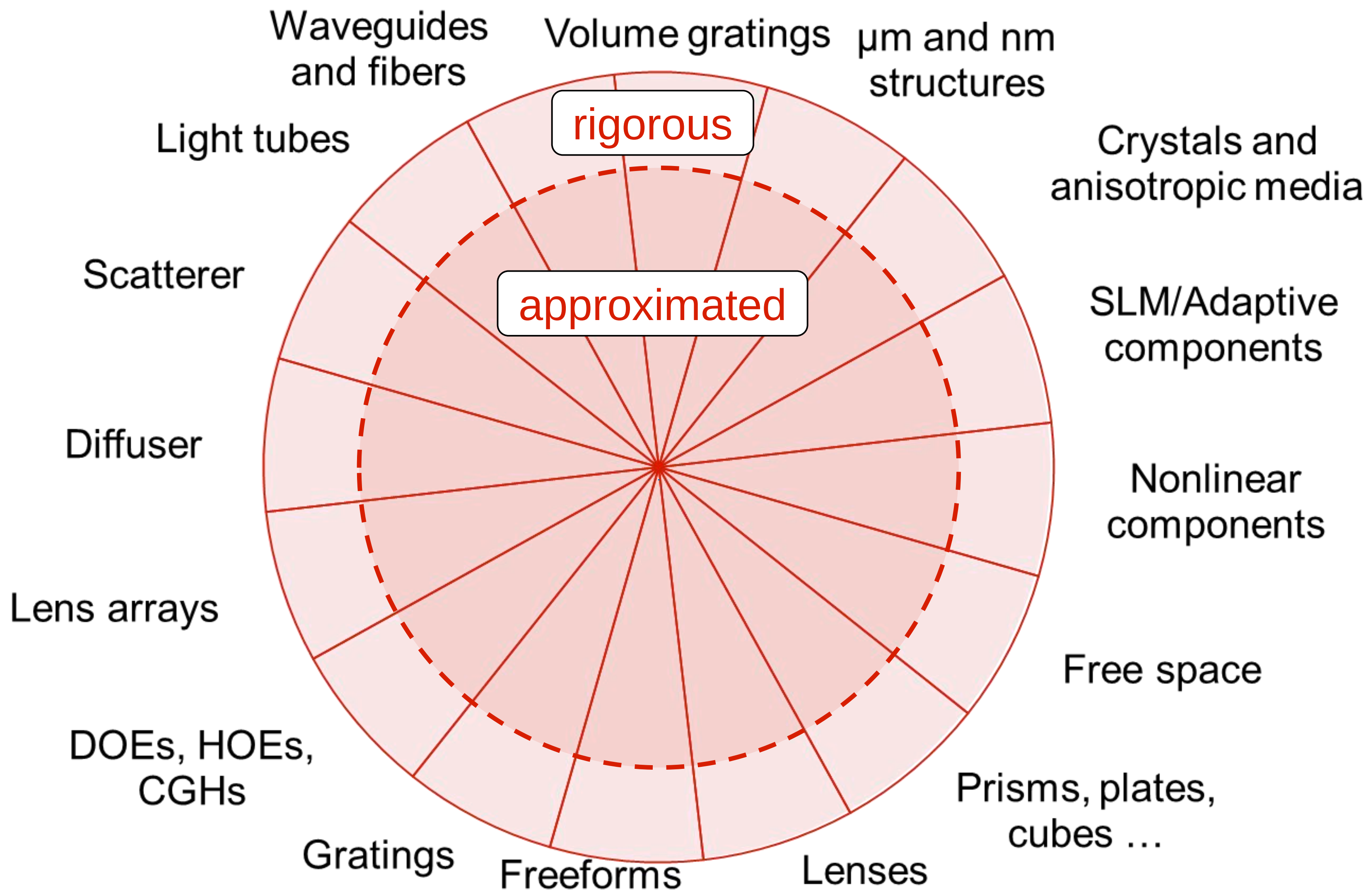


Unified Field Tracing

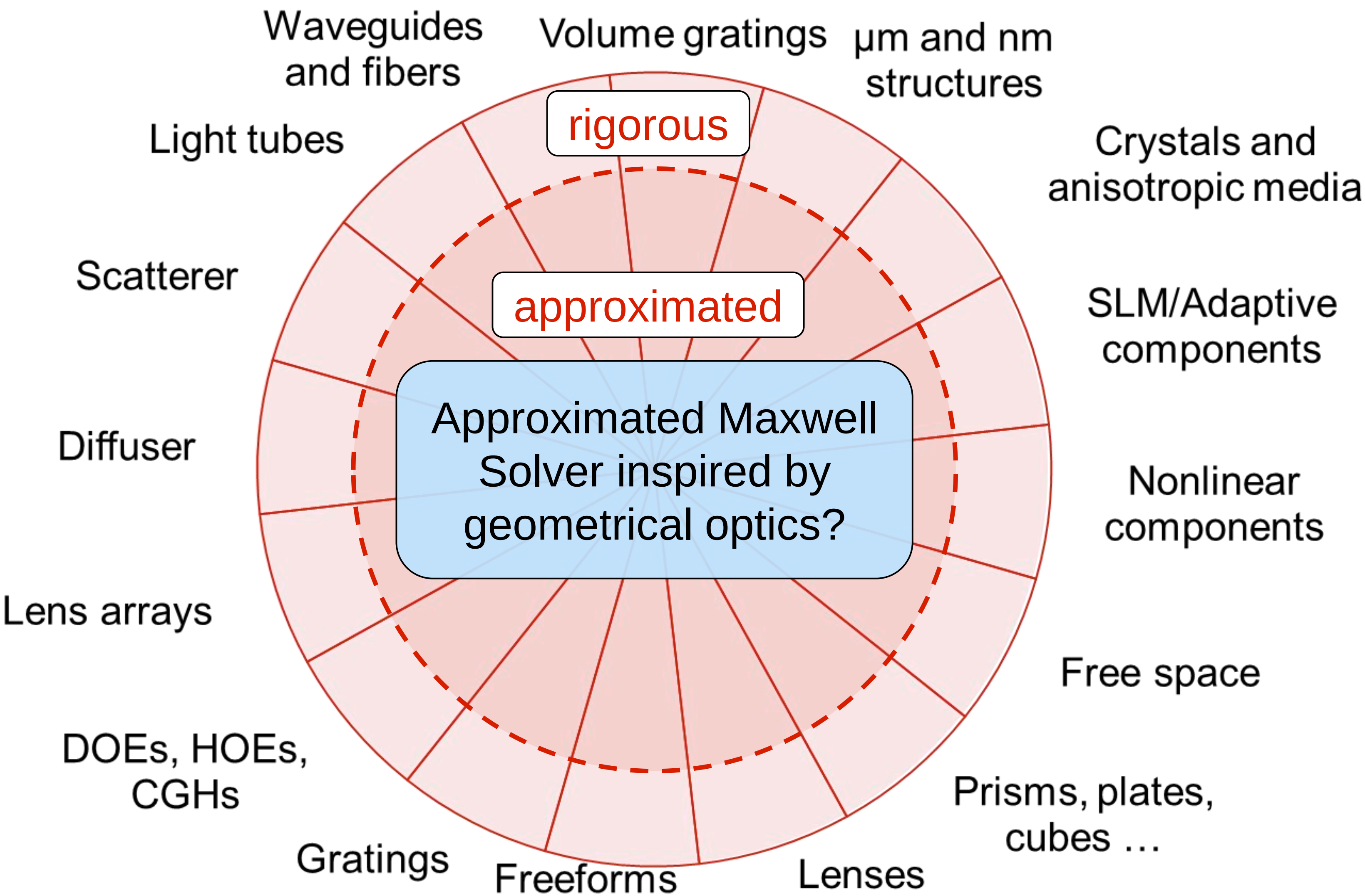
Special methods to solve Maxwell's equations for waveguides.



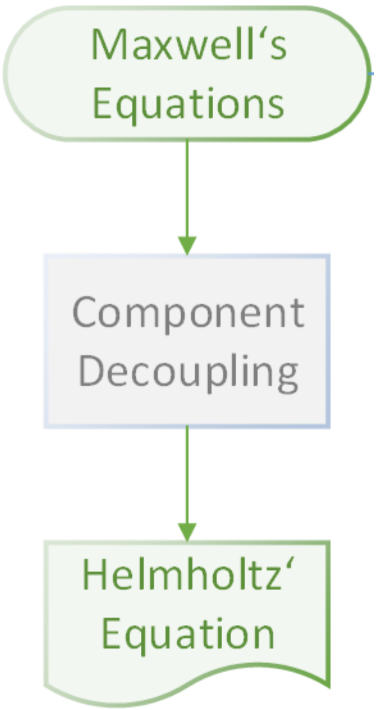
Unified Field Tracing



Geometrically Inspired Maxwell Solver

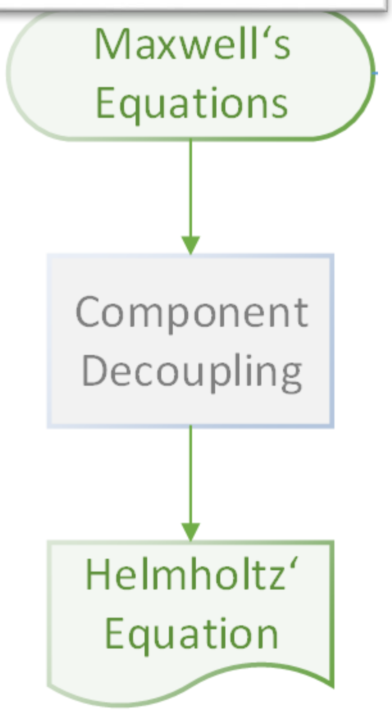


Overview Field Equations (Homogemeous Media)



Overview Field Equations

$$\begin{aligned}
\nabla \times \boldsymbol{E}(\boldsymbol{r}) &= \mathrm{i}\omega\mu_0 \boldsymbol{H}(\boldsymbol{r})\,, \\
\nabla \times \boldsymbol{H}(\boldsymbol{r}) &= -\mathrm{i}\omega\epsilon_0\check{\epsilon}_{\mathrm{r}}(\omega)\boldsymbol{E}(\boldsymbol{r}) \\
\nabla \cdot \boldsymbol{E}(\boldsymbol{r}) &= 0\,, \\
\nabla \cdot \boldsymbol{H}(\boldsymbol{r}) &= 0
\end{aligned}$$



Overview Field Equations

$$
\begin{aligned}
\nabla \times \boldsymbol{E}(\boldsymbol{r}) &= \mathrm{i}\omega\mu_0 \boldsymbol{H}(\boldsymbol{r})\,, \\
\nabla \times \boldsymbol{H}(\boldsymbol{r}) &= -\mathrm{i}\omega\epsilon_0 \check{\epsilon}_\mathrm{r}(\omega) \boldsymbol{E}(\boldsymbol{r}) \\
\nabla \cdot \boldsymbol{E}(\boldsymbol{r}) &= 0\,, \\
\nabla \cdot \boldsymbol{H}(\boldsymbol{r}) &= 0
\end{aligned}
$$

Maxwell's Equations

↓

Component Decoupling

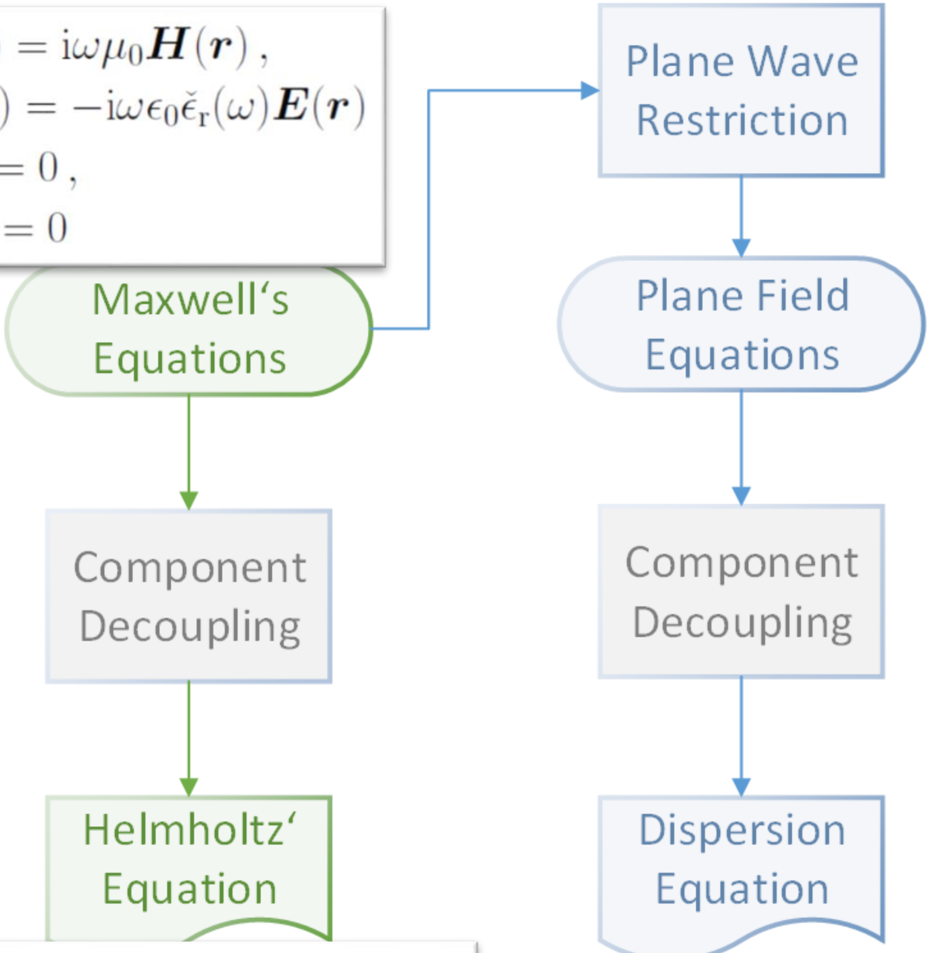
↓

Helmholtz' Equation

$$\nabla^2 \boldsymbol{E}(\boldsymbol{r}, \omega) + k_0^2 \check{\epsilon}_\mathrm{r}(\omega) \boldsymbol{E}(\boldsymbol{r}, \omega) = 0$$

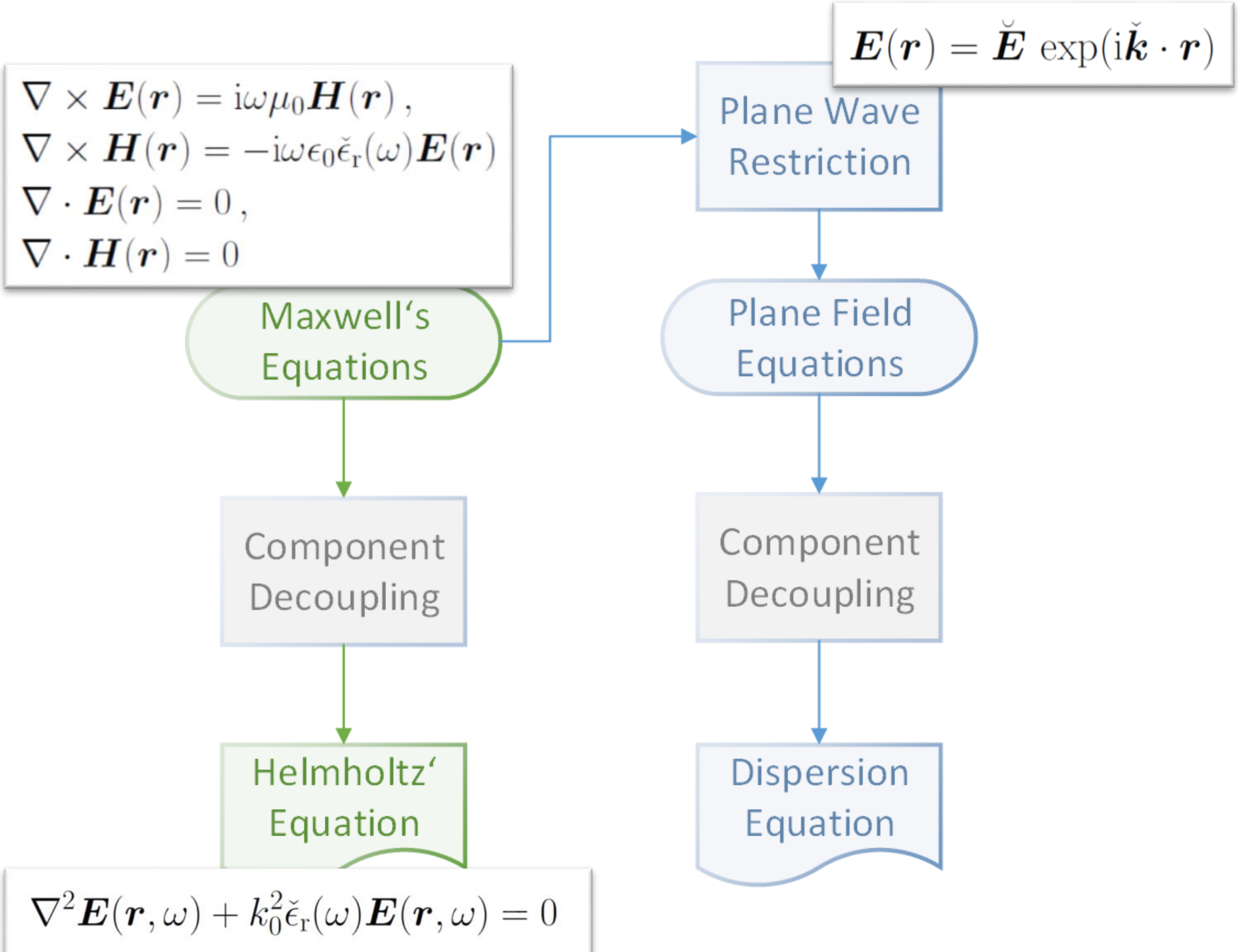
Overview Field Equations

$$\nabla \times \boldsymbol{E}(\boldsymbol{r}) = \mathrm{i}\omega\mu_0 \boldsymbol{H}(\boldsymbol{r})\,,$$
$$\nabla \times \boldsymbol{H}(\boldsymbol{r}) = -\mathrm{i}\omega\epsilon_0\check{\epsilon}_\mathrm{r}(\omega)\boldsymbol{E}(\boldsymbol{r})$$
$$\nabla \cdot \boldsymbol{E}(\boldsymbol{r}) = 0\,,$$
$$\nabla \cdot \boldsymbol{H}(\boldsymbol{r}) = 0$$



$$\nabla^2 \boldsymbol{E}(\boldsymbol{r},\omega) + k_0^2\check{\epsilon}_\mathrm{r}(\omega)\boldsymbol{E}(\boldsymbol{r},\omega) = 0$$

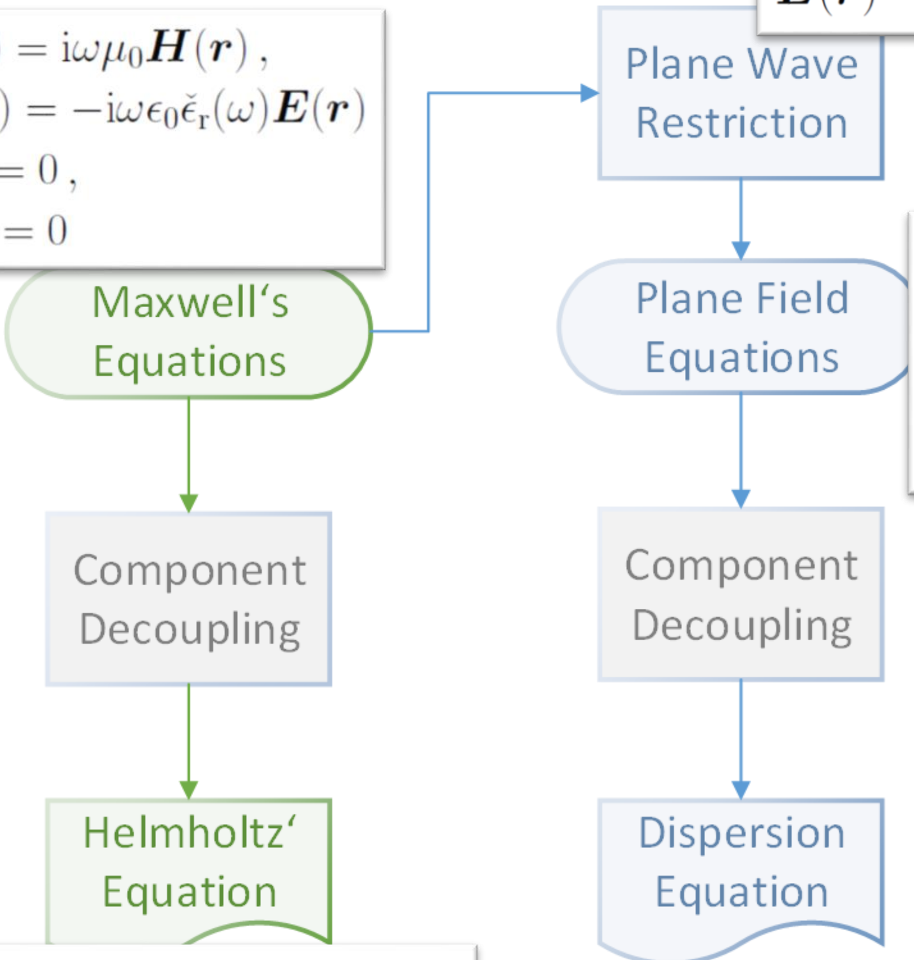
Overview Field Equations



Overview Field Equations

$$
\begin{aligned}
\nabla \times \boldsymbol{E}(\boldsymbol{r}) &= \mathrm{i}\omega\mu_0 \boldsymbol{H}(\boldsymbol{r})\,, \\
\nabla \times \boldsymbol{H}(\boldsymbol{r}) &= -\mathrm{i}\omega\epsilon_0 \breve{\epsilon}_{\mathrm{r}}(\omega)\boldsymbol{E}(\boldsymbol{r}) \\
\nabla \cdot \boldsymbol{E}(\boldsymbol{r}) &= 0\,, \\
\nabla \cdot \boldsymbol{H}(\boldsymbol{r}) &= 0
\end{aligned}
$$

$$\boldsymbol{E}(\boldsymbol{r}) = \breve{\boldsymbol{E}}\,\exp(\mathrm{i}\breve{\boldsymbol{k}} \cdot \boldsymbol{r})$$



$$
\begin{aligned}
\breve{\boldsymbol{k}} \times \breve{\boldsymbol{E}} &= \omega\mu_0 \breve{\boldsymbol{H}} \\
\breve{\boldsymbol{k}} \times \breve{\boldsymbol{H}} &= -\omega\epsilon_0 \breve{\epsilon}_{\mathrm{r}}(\omega)\breve{\boldsymbol{E}} \\
\breve{\boldsymbol{k}} \cdot \breve{\boldsymbol{E}} &= 0 \\
\breve{\boldsymbol{k}} \cdot \breve{\boldsymbol{H}} &= 0
\end{aligned}
$$

$$\nabla^2 \boldsymbol{E}(\boldsymbol{r},\omega) + k_0^2 \breve{\epsilon}_{\mathrm{r}}(\omega)\boldsymbol{E}(\boldsymbol{r},\omega) = 0$$

Overview Field Equations

$$\nabla \times \boldsymbol{E}(\boldsymbol{r}) = \mathrm{i}\omega\mu_0\boldsymbol{H}(\boldsymbol{r})\,,$$
$$\nabla \times \boldsymbol{H}(\boldsymbol{r}) = -\mathrm{i}\omega\epsilon_0\breve{\epsilon}_\mathrm{r}(\omega)\boldsymbol{E}(\boldsymbol{r})$$
$$\nabla \cdot \boldsymbol{E}(\boldsymbol{r}) = 0\,,$$
$$\nabla \cdot \boldsymbol{H}(\boldsymbol{r}) = 0$$

Maxwell's Equations

Component Decoupling

Helmholtz' Equation

$$\nabla^2\boldsymbol{E}(\boldsymbol{r},\omega) + k_0^2\breve{\epsilon}_\mathrm{r}(\omega)\boldsymbol{E}(\boldsymbol{r},\omega) = 0$$

$$\boldsymbol{E}(\boldsymbol{r}) = \breve{\boldsymbol{E}}\,\exp(\mathrm{i}\breve{\boldsymbol{k}}\cdot\boldsymbol{r})$$

Plane Wave Restriction

Plane Field Equations

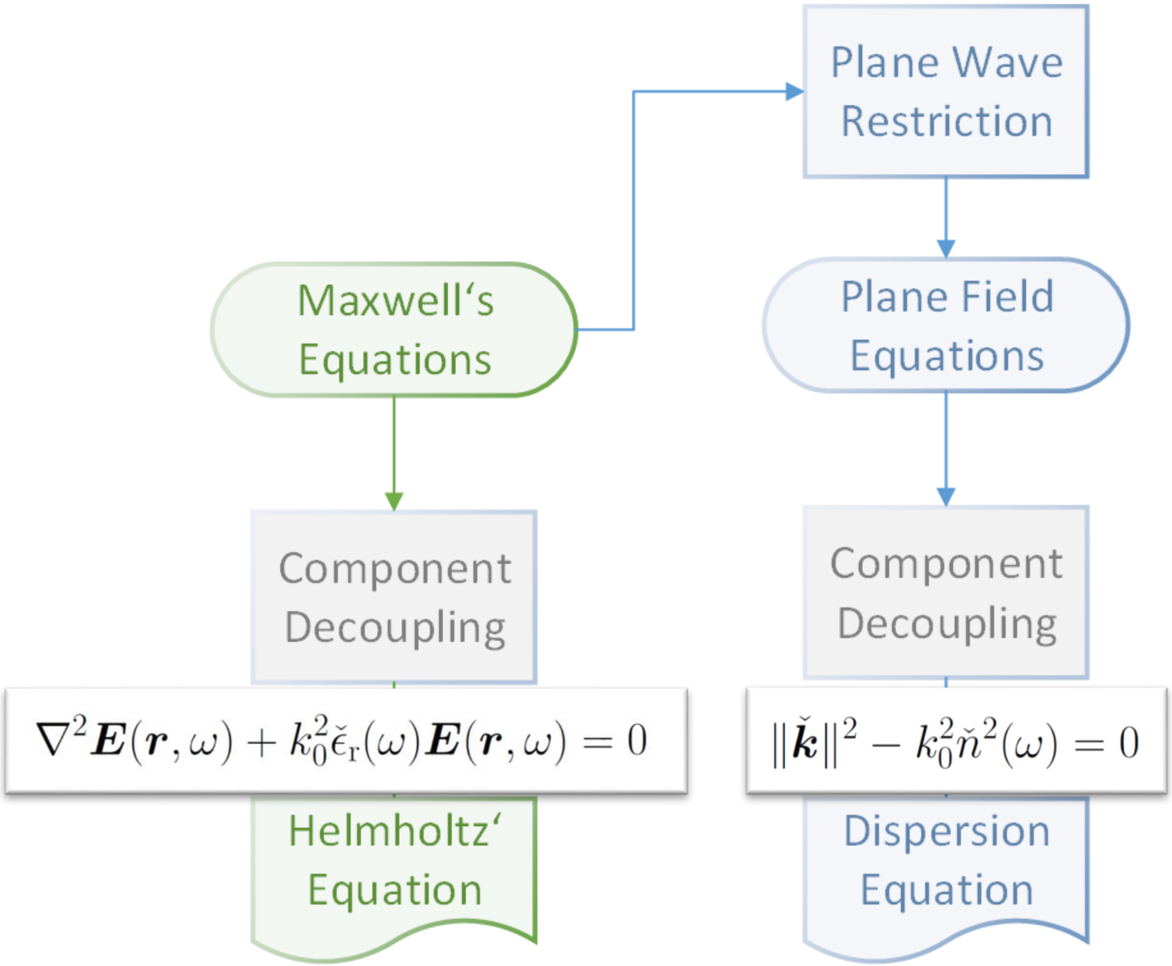
$$\breve{\boldsymbol{k}} \times \breve{\boldsymbol{E}} = \omega\mu_0\breve{\boldsymbol{H}}$$
$$\breve{\boldsymbol{k}} \times \breve{\boldsymbol{H}} = -\omega\epsilon_0\breve{\epsilon}_\mathrm{r}(\omega)\breve{\boldsymbol{E}}$$
$$\breve{\boldsymbol{k}} \cdot \breve{\boldsymbol{E}} = 0$$
$$\breve{\boldsymbol{k}} \cdot \breve{\boldsymbol{H}} = 0$$

Component Decoupling

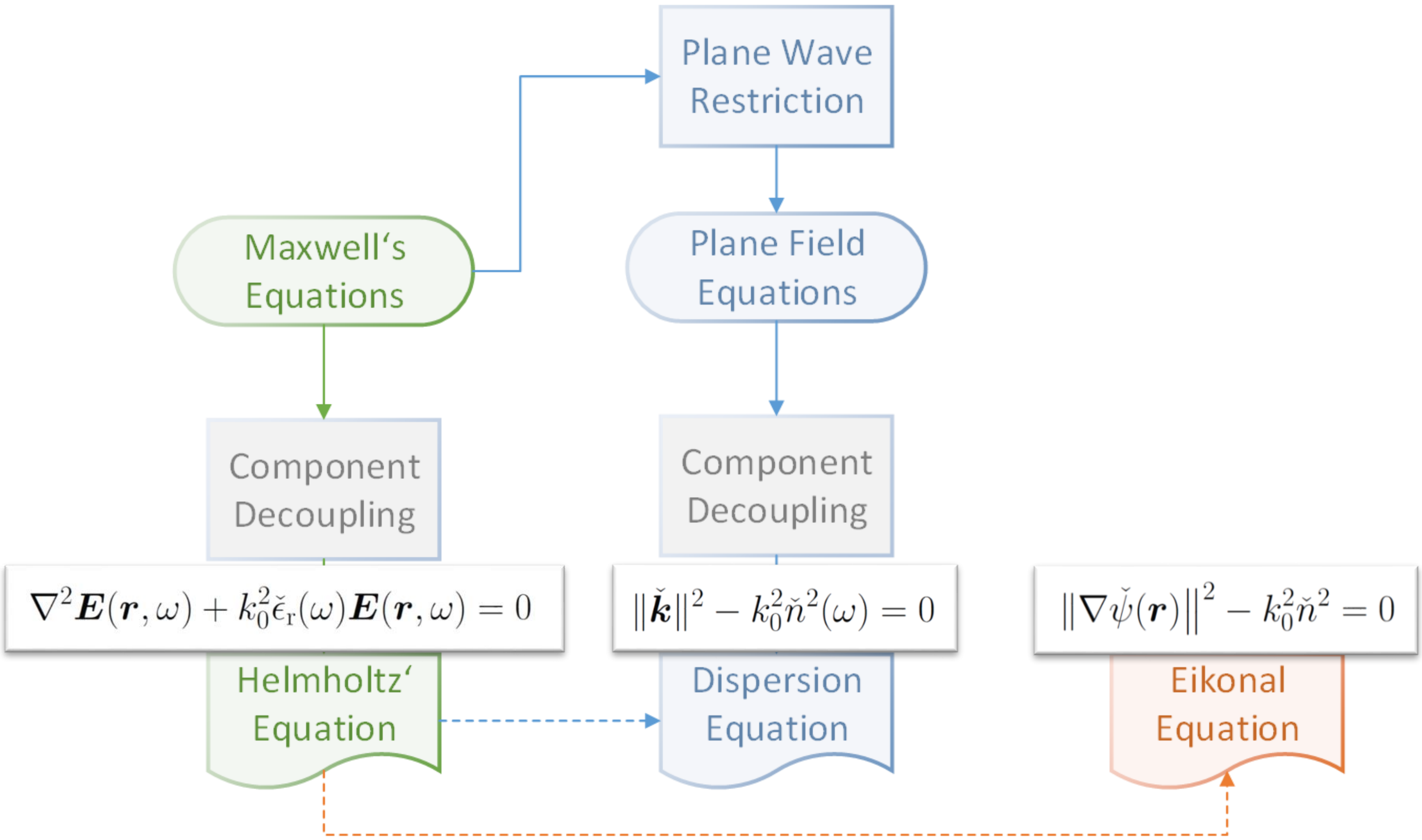
Dispersion Equation

$$\|\breve{\boldsymbol{k}}\|^2 - k_0^2\breve{n}^2(\omega) = 0$$

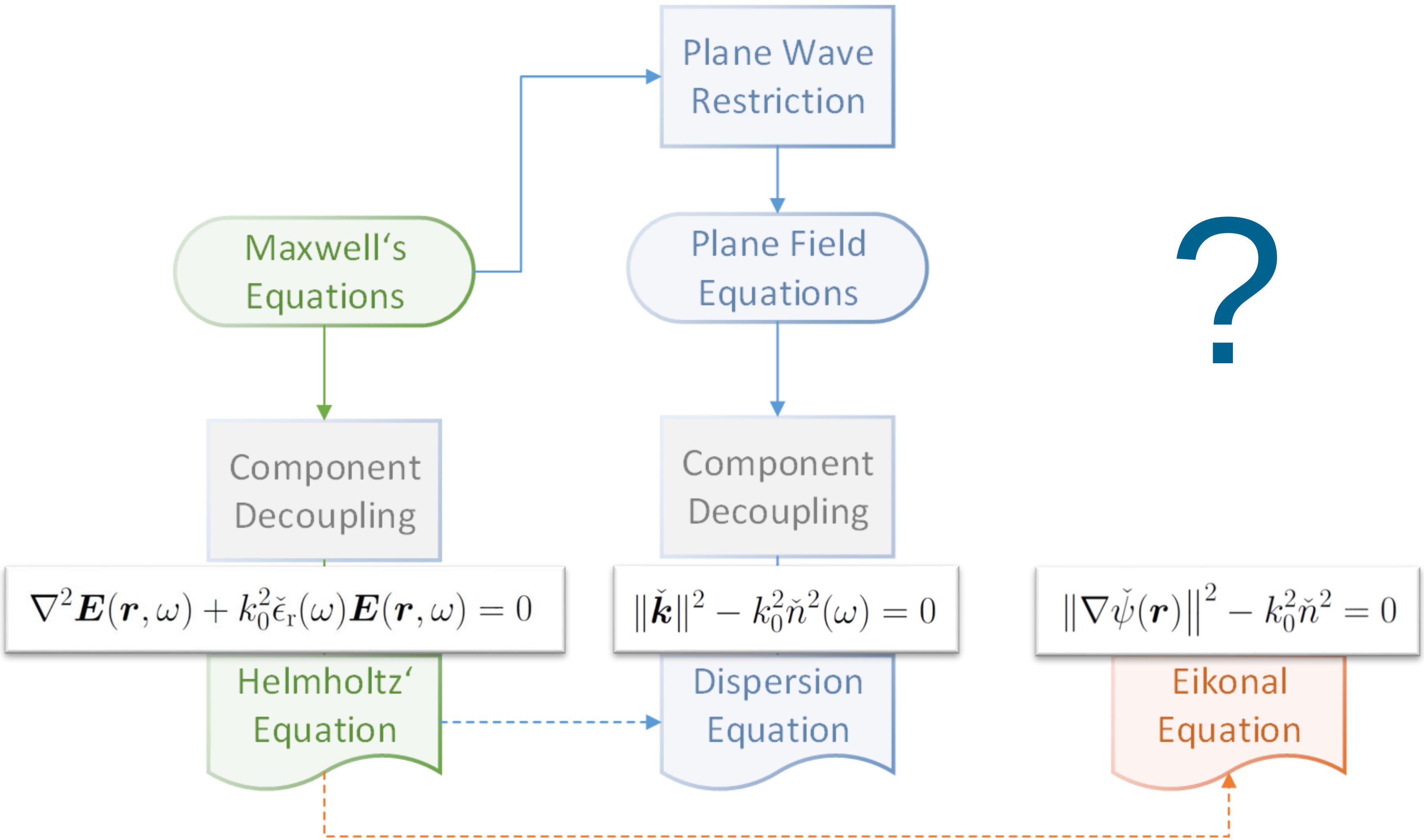
Overview Field Equations



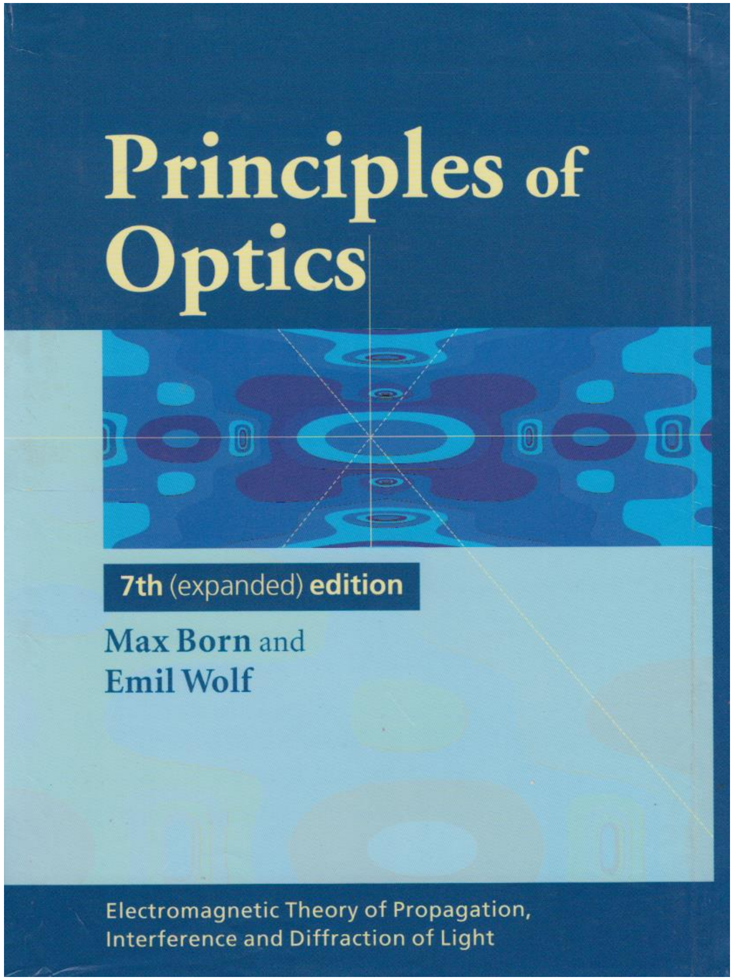
Overview Field Equations

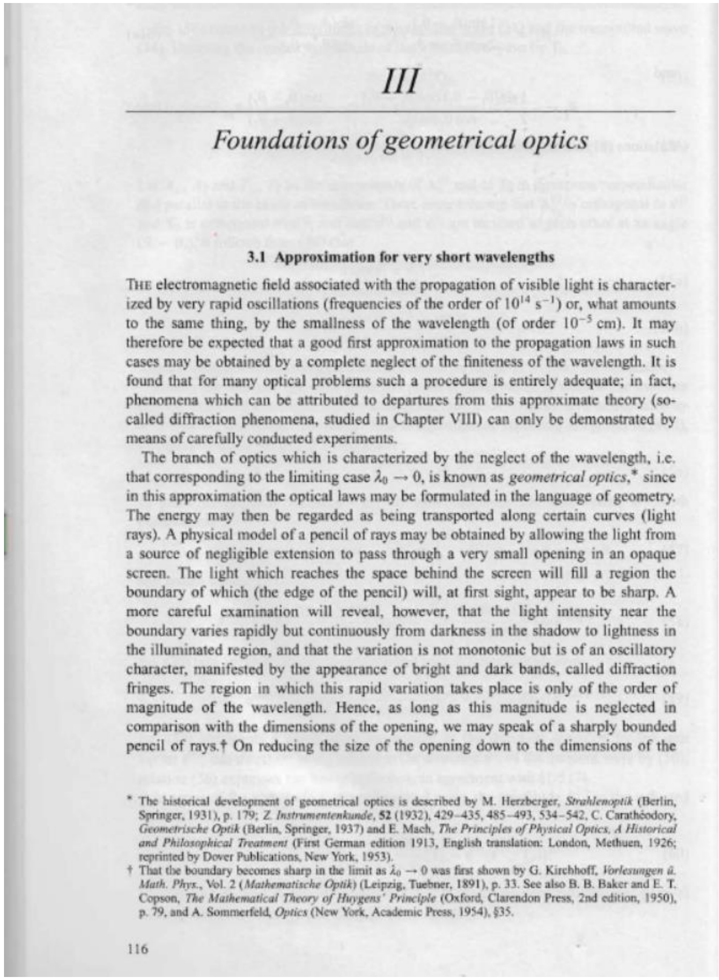


Overview Field Equations



Geometrical Optics of Electromagnetic Fields





III

Foundations of geometrical optics

3.1 Approximation for very short wavelengths

THE electromagnetic field associated with the propagation of visible light is characterized by very rapid oscillations (frequencies of the order of $10^{14}\ s^{-1}$) or, what amounts to the same thing, by the smallness of the wavelength (of order 10^{-5} cm). It may therefore be expected that a good first approximation to the propagation laws in such cases may be obtained by a complete neglect of the finiteness of the wavelength. It is found that for many optical problems such a procedure is entirely adequate; in fact, phenomena which can be attributed to departures from this approximate theory (so-called diffraction phenomena, studied in Chapter VIII) can only be demonstrated by means of carefully conducted experiments.

The branch of optics which is characterized by the neglect of the wavelength, i.e. that corresponding to the limiting case $\lambda_0 \rightarrow 0$, is known as *geometrical optics*,* since in this approximation the optical laws may be formulated in the language of geometry. The energy may then be regarded as being transported along certain curves (light rays). A physical model of a pencil of rays may be obtained by allowing the light from a source of negligible extension to pass through a very small opening in an opaque screen. The light which reaches the space behind the screen will fill a region the boundary of which (the edge of the pencil) will, at first sight, appear to be sharp. A more careful examination will reveal, however, that the light intensity near the boundary varies rapidly but continuously from darkness in the shadow to lightness in the illuminated region, and that the variation is not monotonic but is of an oscillatory character, manifested by the appearance of bright and dark bands, called diffraction fringes. The region in which this rapid variation takes place is only of the order of magnitude of the wavelength. Hence, as long as this magnitude is neglected in comparison with the dimensions of the opening, we may speak of a sharply bounded pencil of rays.† On reducing the size of the opening down to the dimensions of the

* The historical development of geometrical optics is described by M. Herzberger, *Strahlenoptik* (Berlin, Springer, 1931), p. 179; *Z. Instrumentenkunde*, **52** (1932), 429–435, 485–493, 534–542, C. Carathéodory, *Geometrische Optik* (Berlin, Springer, 1937) and E. Mach, *The Principles of Physical Optics, A Historical and Philosophical Treatment* (First German edition 1913, English translation: London, Methuen, 1926; reprinted by Dover Publications, New York, 1953).

† That the boundary becomes sharp in the limit as $\lambda_0 \rightarrow 0$ was first shown by G. Kirchhoff, *Vorlesungen ü. Math. Phys.*, Vol. 2 (*Mathematische Optik*) (Leipzig, Tuebner, 1891), p. 33. See also B. B. Baker and E. T. Copson, *The Mathematical Theory of Huygens' Principle* (Oxford, Clarendon Press, 2nd edition, 1950), p. 79, and A. Sommerfeld, *Optics* (New York, Academic Press, 1954), §35.

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Geometrical Optics of Electromagnetic Fields

3.1 Approximation for very short wavelengths 125

But by (38), (15) and (25),

$$d\tau = \frac{d\mathcal{S}}{(\mathrm{grad}\,\mathcal{S})^2} = \frac{1}{n^2}\,d\mathcal{S} = \frac{1}{n}\,ds, \tag{39}$$

so that we finally obtain the following expressions for the ratio of the intensities at any two points of a ray:

$$\frac{I_2}{I_1} = \frac{n_2}{n_1}\,e^{-\int_{\mathcal{S}_1}^{\mathcal{S}_2} \frac{\nabla^2\mathcal{S}}{n^2}\,d\mathcal{S}} = \frac{n_2}{n_1}\,e^{-\int_{s_1}^{s_2} \frac{\nabla^2\mathcal{S}}{n}\,ds}, \tag{40}$$

the integrals being taken along the ray.*

3.1.3 Propagation of the amplitude vectors

We have seen that, when the wavelength is sufficiently small, the transport of energy may be represented by means of a simple hydrodynamical model which may be completely described in terms of the real scalar function $\mathcal{S}$, this function being a solution of the eikonal equation (15). According to traditional terminology, one understands by geometrical optics this approximate picture of energy propagation, using the concept of rays and wave-fronts. In other words polarization properties are excluded. The reason for this restriction is undoubtedly due to the fact that the simple laws of geometrical optics concerning rays and wave-fronts were known from experiments long before the electromagnetic theory of light was established. It is, however, possible, and from our point of view quite natural, to extend the meaning of geometrical optics to embrace also certain geometrical laws relating to the propagation of the 'amplitude vectors' **e** and **h**. These laws may be easily deduced from the wave equations (16)–(17).

Since $\mathcal{S}$ satisfies the eikonal equation, it follows that $\boldsymbol{K} = 0$, and we see that when k_0 is sufficiently large (λ_0 small enough), only the $\boldsymbol{L}$-terms need to be retained in (16) and (17). Hence, in the present approximation, the amplitude vectors and the eikonal are connected by the relations $\boldsymbol{L} = 0$. If we use again the operator $\partial/\partial\tau$ introduced by (38), the equations $\boldsymbol{L} = 0$ become

$$\frac{\partial \mathbf{e}}{\partial\tau} + \frac{1}{2}\left(\nabla^2\mathcal{S} - \frac{\partial \ln\mu}{\partial\tau}\right)\mathbf{e} + (\mathbf{e}\cdot\mathrm{grad}\ln n)\,\mathrm{grad}\,\mathcal{S} = 0, \tag{41}$$

$$\frac{\partial \mathbf{h}}{\partial\tau} + \frac{1}{2}\left(\nabla^2\mathcal{S} - \frac{\partial \ln\varepsilon}{\partial\tau}\right)\mathbf{h} + (\mathbf{h}\cdot\mathrm{grad}\ln n)\,\mathrm{grad}\,\mathcal{S} = 0. \tag{42}$$

These are the required *transport equations* for the variation of **e** and **h** along each ray. The implications of these equations can best be understood by examining separately the variation of the magnitude and of the direction of these vectors.

* It has been shown by M. Kline, *Comm. Pure and Appl. Maths.*, **14** (1961), 473 that the intensity ratio (40) may be expressed in terms of an integral which involves the principal radii of curvature of the associated wavefronts. Kline's formula is a natural generalization, to inhomogeneous media, of the formula (34). See also M. Kline and I. W. Kay, *ibid*, 184.

page 125

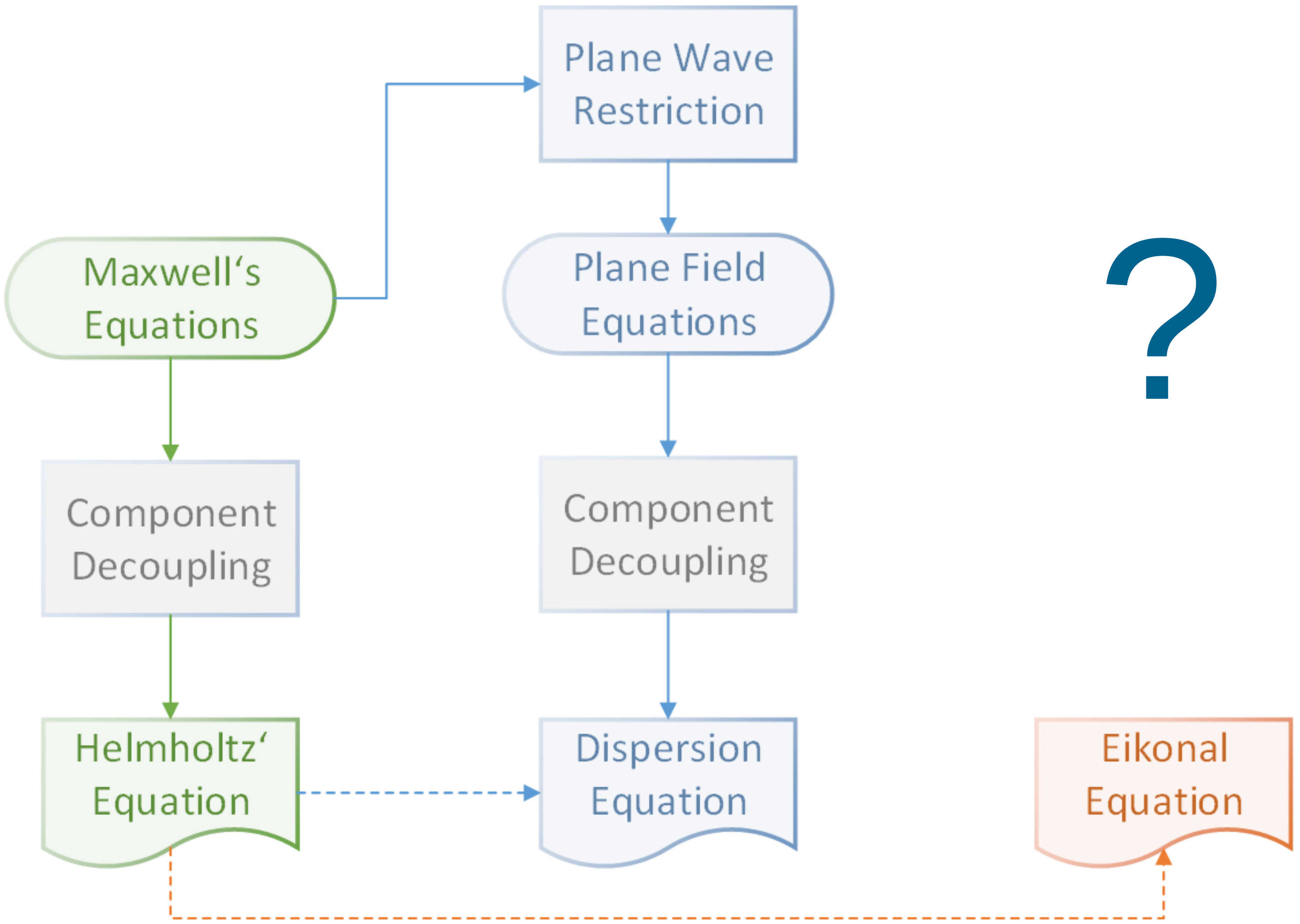
"According to traditional terminology, one understands by geometrical optics this approximate picture of energy propagation, using the concept of rays and wave-fronts. In other words polarization properties are excluded. The reason for this restriction is undoubtedly due to the fact that the simple laws of geometrical optics concerning rays and wave-fronts were known from experiments long before the electromagnetic theory of light was established. **It is, however, possible, and from our point of view quite natural, to extend the meaning of geometrical optics to embrace also certain geometrical laws relating to the propagation of the 'amplitude vectors' E and H."**

Geometric Field Equations

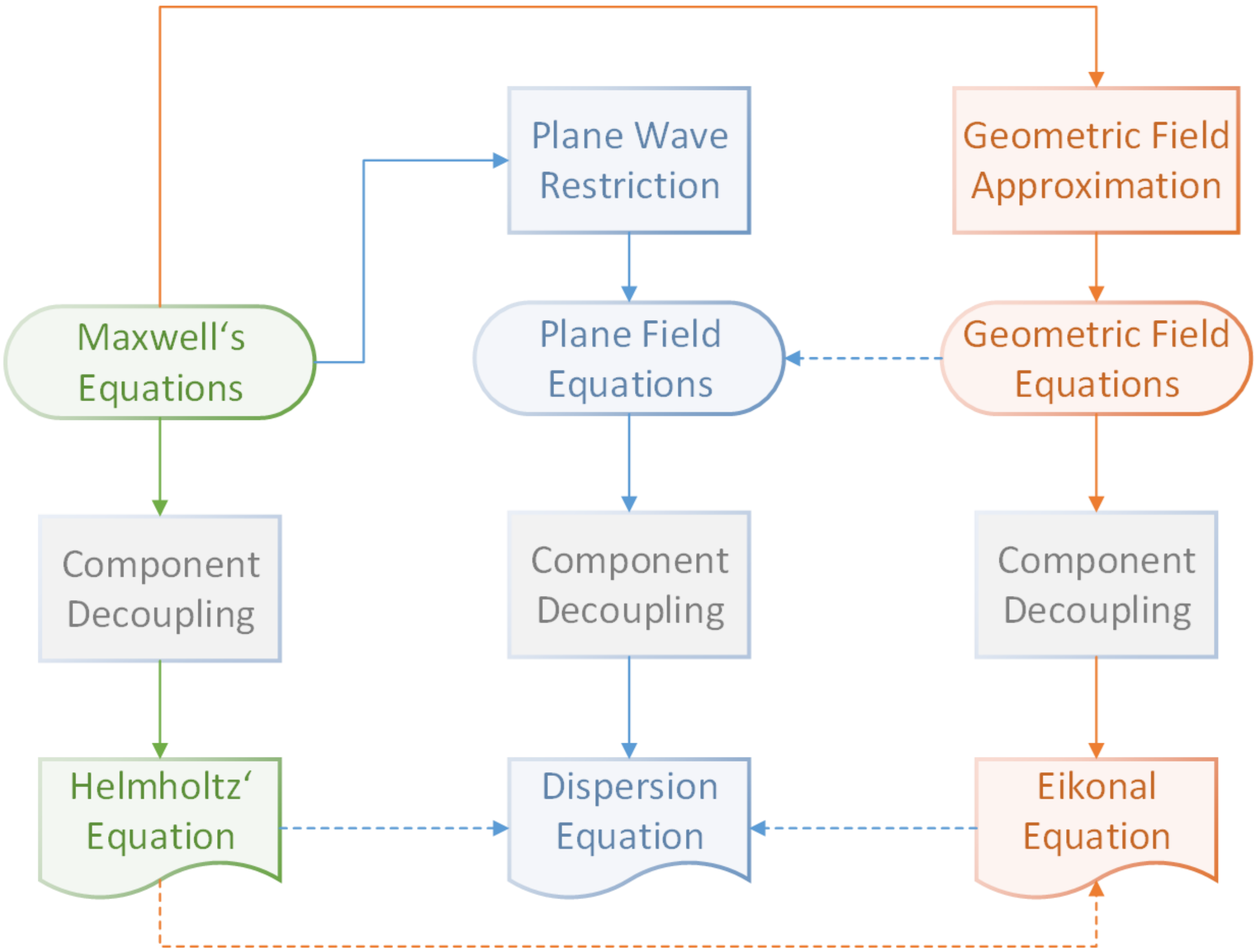
We follow Max Born's and
Emil Wolf's point of view!

Development and implementation of
algorithms to solve Maxwell's equations
in its **geometric field approximation**!

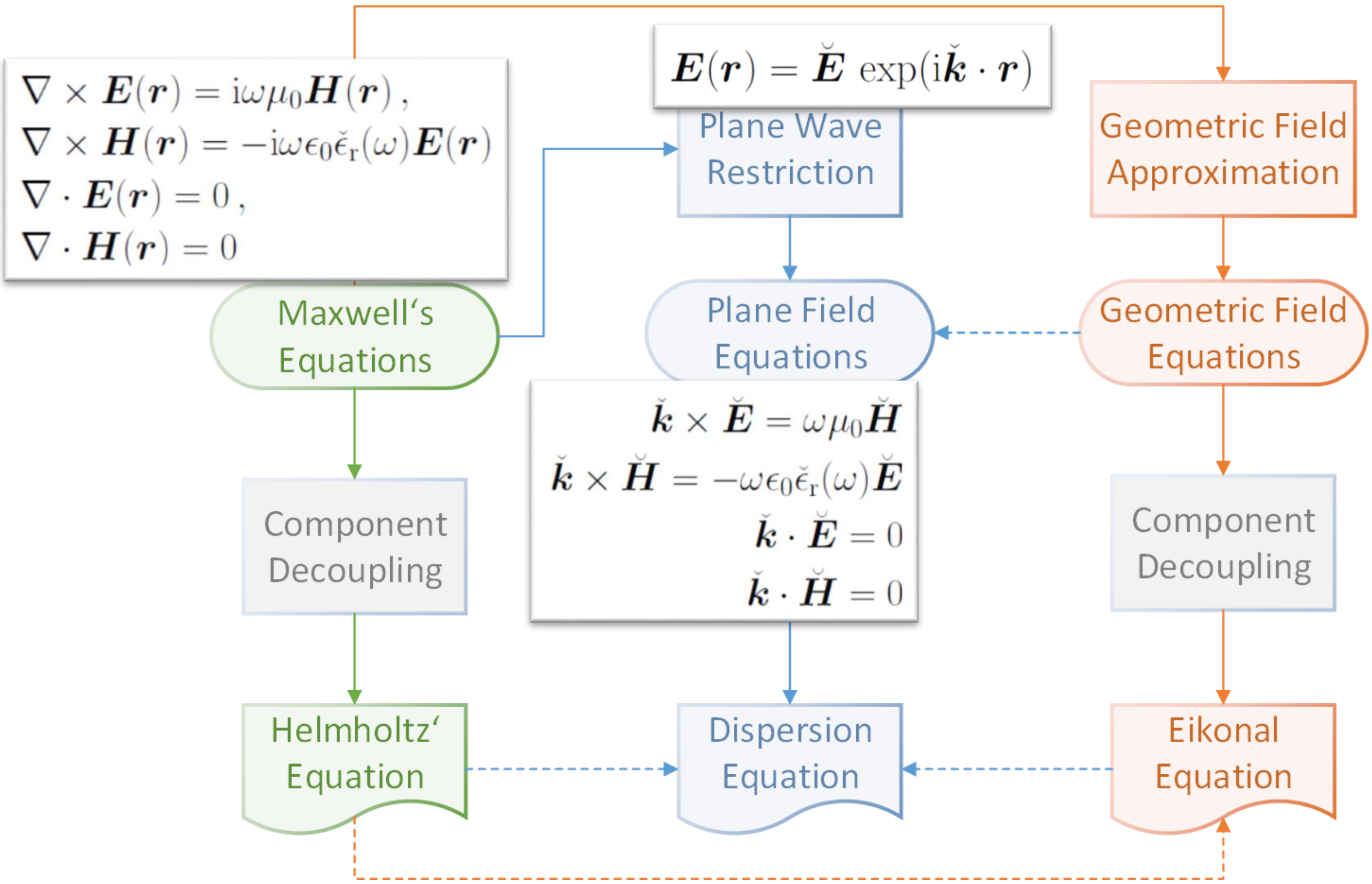
Overview Field Equations



Overview Field Equations

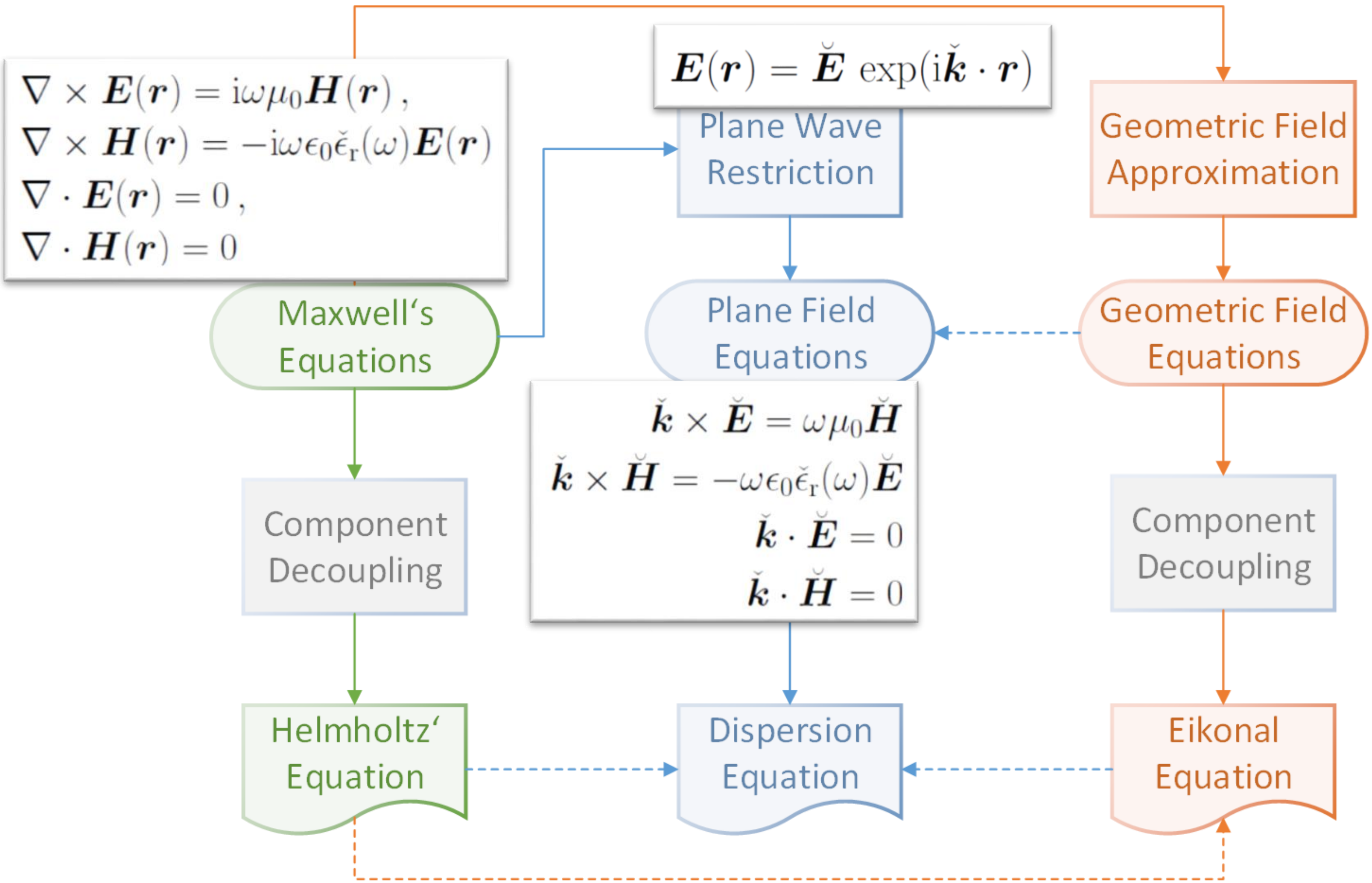


Overview Field Equations



Overview Field Equations

$$\boldsymbol{E}(\boldsymbol{r}) = \tilde{\boldsymbol{E}}(\boldsymbol{r})\exp\big(\mathrm{i}\breve{\psi}(\boldsymbol{r})\big) \text{ with}$$
$$\tilde{E}_\ell(\boldsymbol{r}) = \|\tilde{E}_\ell(\boldsymbol{r})\|\exp\big(\mathrm{i}\tilde{\gamma}_\ell(\boldsymbol{r})\big) \text{ and}$$
$$\breve{\psi}(\boldsymbol{r}) = \psi(\boldsymbol{r}) + \mathrm{i}\psi'(\boldsymbol{r})$$



Overview Field Equations

$$\boldsymbol{E}(\boldsymbol{r}) = \tilde{\boldsymbol{E}}(\boldsymbol{r})\exp\big(\mathrm{i}\breve{\psi}(\boldsymbol{r})\big) \text{ with}$$
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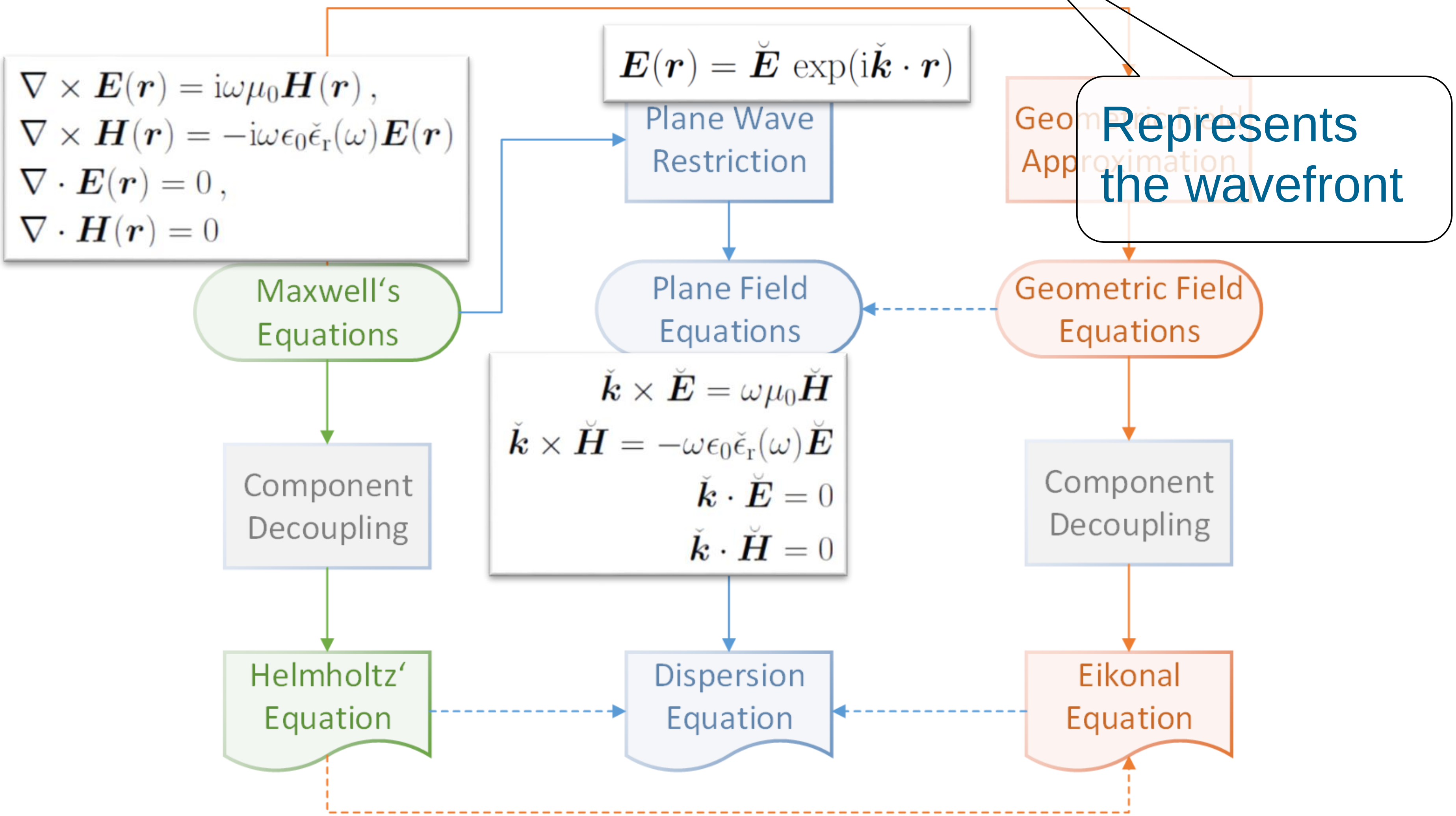


Overview Field Equations

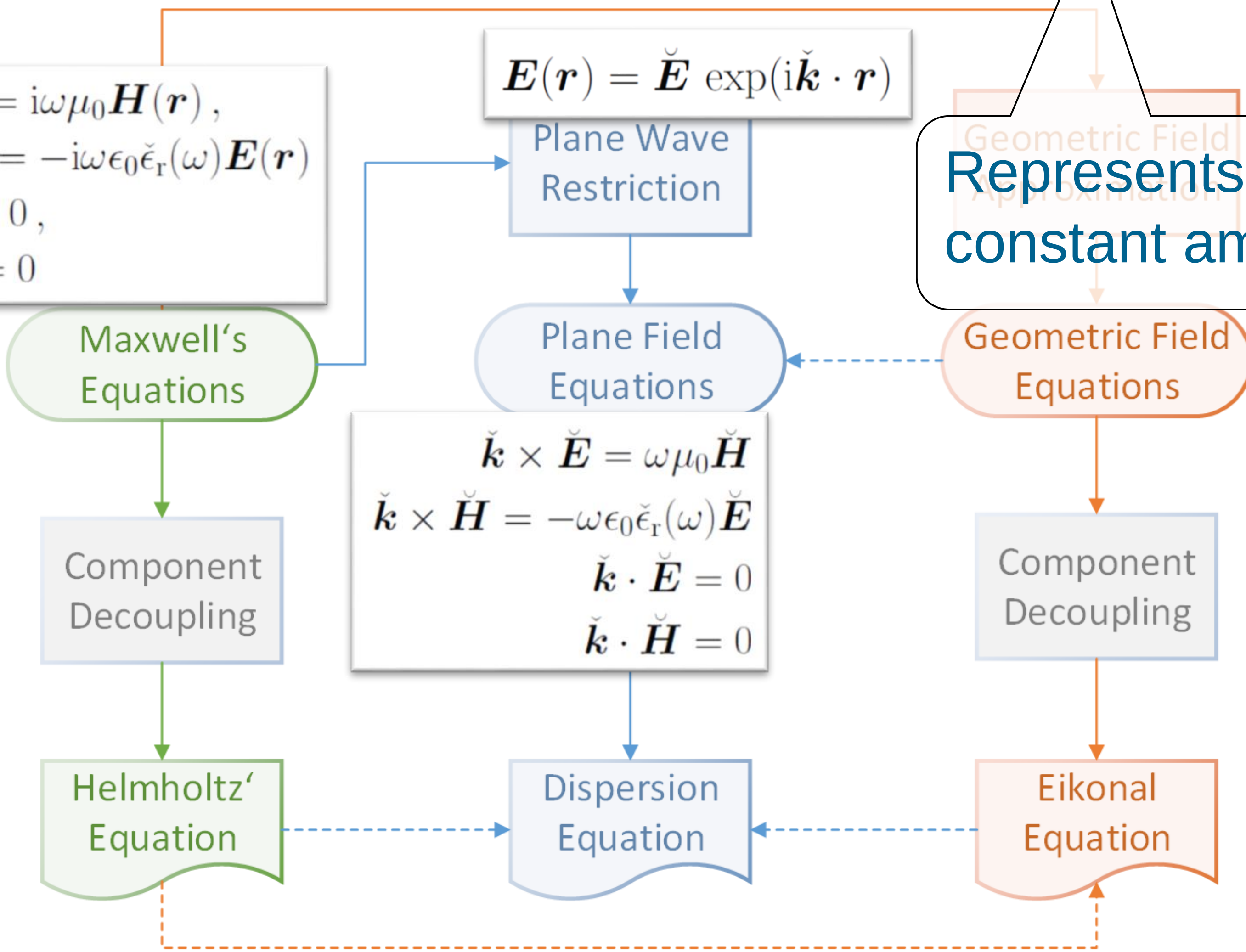
$$\boldsymbol{E}(\boldsymbol{r}) = \tilde{E}(\boldsymbol{r})\exp\left(\mathrm{i}\breve{\psi}(\boldsymbol{r})\right) \text{ with}$$
$$\tilde{E}_\ell(\boldsymbol{r}) = \|\tilde{E}_\ell(\boldsymbol{r})\|\exp\left(\mathrm{i}\tilde{\gamma}_\ell(\boldsymbol{r})\right) \text{ and}$$
$$\breve{\psi}(\boldsymbol{r}) = \psi(\boldsymbol{r}) + \mathrm{i}\psi'(\boldsymbol{r})$$

Represents front of constant amplitude

$$\nabla \times \boldsymbol{E}(\boldsymbol{r}) = \mathrm{i}\omega\mu_0\boldsymbol{H}(\boldsymbol{r})\,,$$
$$\nabla \times \boldsymbol{H}(\boldsymbol{r}) = -\mathrm{i}\omega\epsilon_0\breve{\epsilon}_\mathrm{r}(\omega)\boldsymbol{E}(\boldsymbol{r})$$
$$\nabla \cdot \boldsymbol{E}(\boldsymbol{r}) = 0\,,$$
$$\nabla \cdot \boldsymbol{H}(\boldsymbol{r}) = 0$$

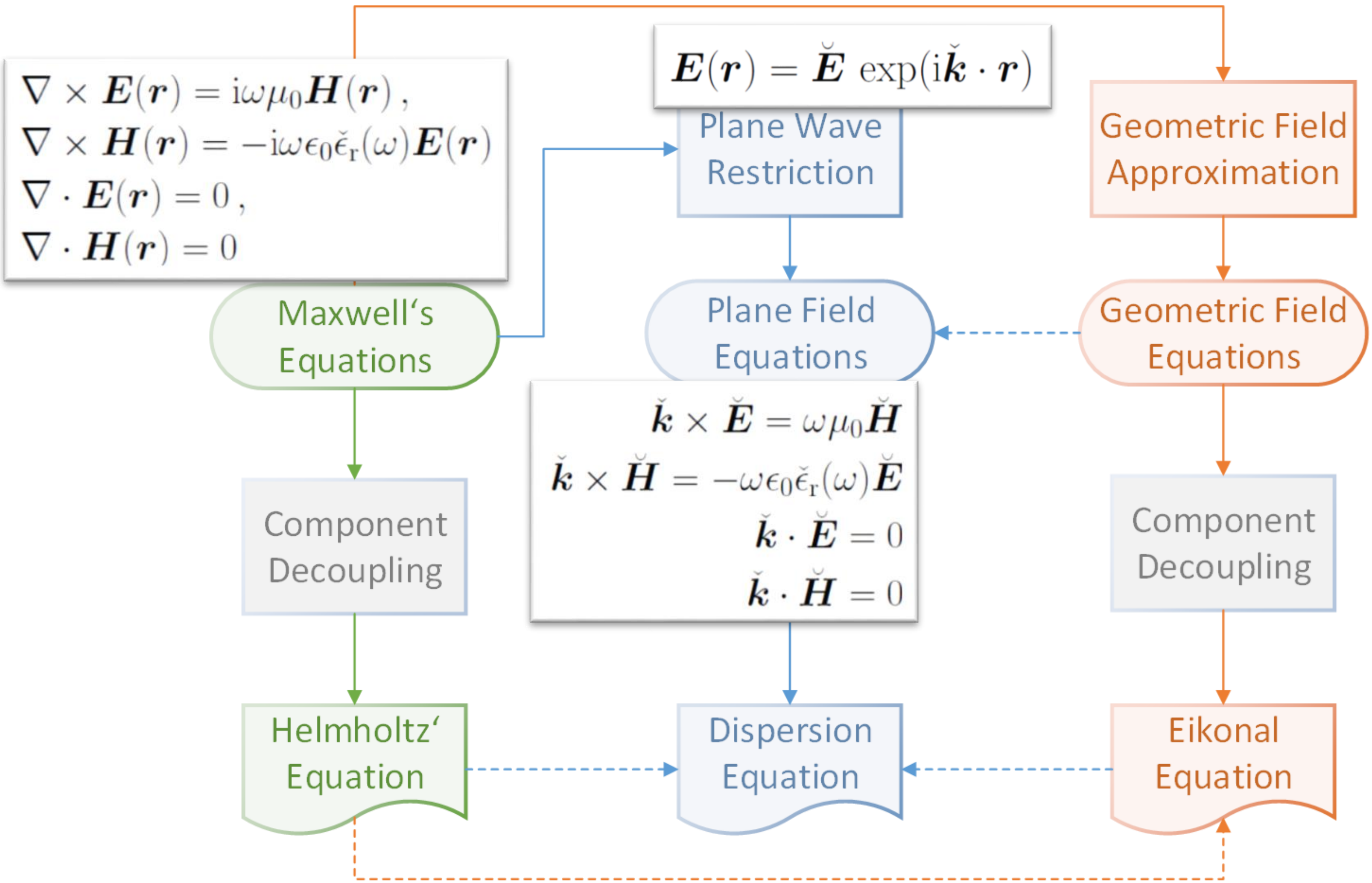
$$\boldsymbol{E}(\boldsymbol{r}) = \breve{\boldsymbol{E}}\exp(\mathrm{i}\breve{\boldsymbol{k}}\cdot\boldsymbol{r})$$

$$\breve{\boldsymbol{k}} \times \breve{\boldsymbol{E}} = \omega\mu_0\breve{\boldsymbol{H}}$$
$$\breve{\boldsymbol{k}} \times \breve{\boldsymbol{H}} = -\omega\epsilon_0\breve{\epsilon}_\mathrm{r}(\omega)\breve{\boldsymbol{E}}$$
$$\breve{\boldsymbol{k}} \cdot \breve{\boldsymbol{E}} = 0$$
$$\breve{\boldsymbol{k}} \cdot \breve{\boldsymbol{H}} = 0$$



Overview Field Equations

$$\boldsymbol{E}(\boldsymbol{r}) = \tilde{\boldsymbol{E}}(\boldsymbol{r}) \exp\big(\mathrm{i}\breve{\psi}(\boldsymbol{r})\big) \text{ with}$$
$$\tilde{E}_\ell(\boldsymbol{r}) = \|\tilde{E}_\ell(\boldsymbol{r})\| \exp\big(\mathrm{i}\tilde{\gamma}_\ell(\boldsymbol{r})\big) \text{ and}$$
$$\breve{\psi}(\boldsymbol{r}) = \psi(\boldsymbol{r}) + \mathrm{i}\psi'(\boldsymbol{r})$$

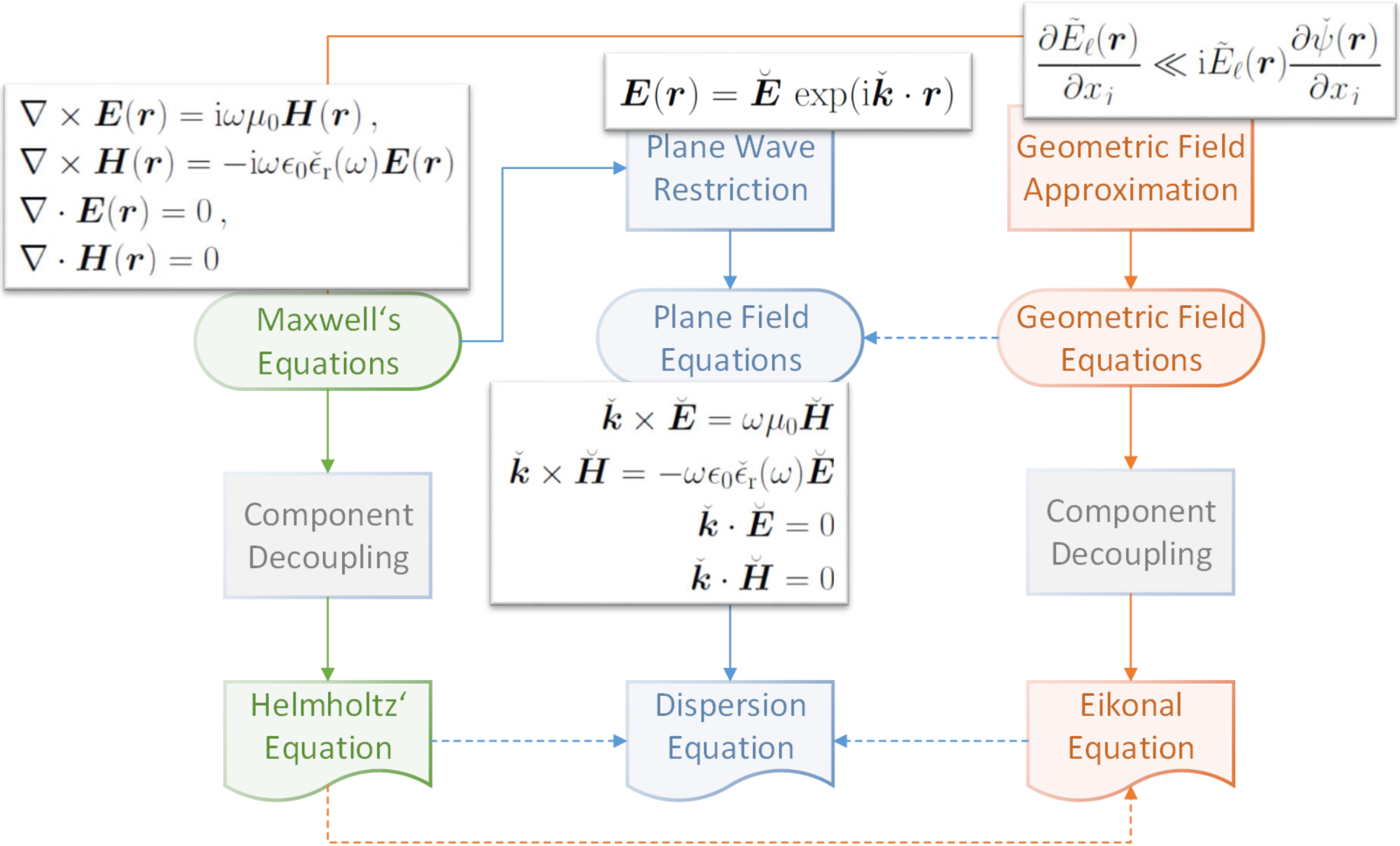


Overview Field Equations

$\boldsymbol{E}(\boldsymbol{r}) = \tilde{\boldsymbol{E}}(\boldsymbol{r}) \exp\left(\mathrm{i}\breve{\psi}(\boldsymbol{r})\right)$ with

$\tilde{E}_\ell(\boldsymbol{r}) = \|\tilde{E}_\ell(\boldsymbol{r})\| \exp\left(\mathrm{i}\tilde{\gamma}_\ell(\boldsymbol{r})\right)$ and

$\breve{\psi}(\boldsymbol{r}) = \psi(\boldsymbol{r}) + \mathrm{i}\psi'(\boldsymbol{r})$



Overview Field Equations

$\boldsymbol{E}(\boldsymbol{r}) = \tilde{\boldsymbol{E}}(\boldsymbol{r}) \exp(\mathrm{i}\breve{\psi}(\boldsymbol{r}))$ with
$\tilde{E}_\ell(\boldsymbol{r}) = \|\tilde{E}_\ell(\boldsymbol{r})\| \exp(\mathrm{i}\tilde{\gamma}_\ell(\boldsymbol{r}))$ and
$\breve{\psi}(\boldsymbol{r}) = \psi(\boldsymbol{r}) + \mathrm{i}\psi'(\boldsymbol{r})$

$$\frac{\partial \tilde{E}_\ell(\boldsymbol{r})}{\partial x_j} \ll \mathrm{i}\tilde{E}_\ell(\boldsymbol{r}) \frac{\partial \breve{\psi}(\boldsymbol{r})}{\partial x_j}$$

$$\boldsymbol{E}(\boldsymbol{r}) = \breve{\boldsymbol{E}} \exp(\mathrm{i}\breve{\boldsymbol{k}} \cdot \boldsymbol{r})$$

$$\begin{aligned}
\nabla \times \boldsymbol{E}(\boldsymbol{r}) &= \mathrm{i}\omega\mu_0 \boldsymbol{H}(\boldsymbol{r}), \\
\nabla \times \boldsymbol{H}(\boldsymbol{r}) &= -\mathrm{i}\omega\epsilon_0\breve{\epsilon}_\mathrm{r}(\omega) \boldsymbol{E}(\boldsymbol{r}) \\
\nabla \cdot \boldsymbol{E}(\boldsymbol{r}) &= 0, \\
\nabla \cdot \boldsymbol{H}(\boldsymbol{r}) &= 0
\end{aligned}$$

Plane Wave Restriction

Geometric Field Approximation

Maxwell‘s Equations

Plane Field Equations

Geometric Field Equations

$$\begin{aligned}
\breve{\boldsymbol{k}} \times \breve{\boldsymbol{E}} &= \omega\mu_0 \breve{\boldsymbol{H}} \\
\breve{\boldsymbol{k}} \times \breve{\boldsymbol{H}} &= -\omega\epsilon_0\breve{\epsilon}_\mathrm{r}(\omega) \breve{\boldsymbol{E}} \\
\breve{\boldsymbol{k}} \cdot \breve{\boldsymbol{E}} &= 0 \\
\breve{\boldsymbol{k}} \cdot \breve{\boldsymbol{H}} &= 0
\end{aligned}$$

$$\begin{aligned}
\nabla\breve{\psi}(\boldsymbol{r}) \times \tilde{\boldsymbol{E}}(\boldsymbol{r}) &= \omega\mu_0 \tilde{\boldsymbol{H}}(\boldsymbol{r}), \\
\nabla\breve{\psi}(\boldsymbol{r}) \times \tilde{\boldsymbol{H}}(\boldsymbol{r}) &= -\omega\epsilon_0\breve{\epsilon}_\mathrm{r}(\omega) \tilde{\boldsymbol{E}}(\boldsymbol{r}), \\
\nabla\breve{\psi}(\boldsymbol{r}) \cdot \tilde{\boldsymbol{E}}(\boldsymbol{r}) &= 0, \\
\nabla\breve{\psi}(\boldsymbol{r}) \cdot \tilde{\boldsymbol{H}}(\boldsymbol{r}) &= 0.
\end{aligned}$$

Component Decoupling

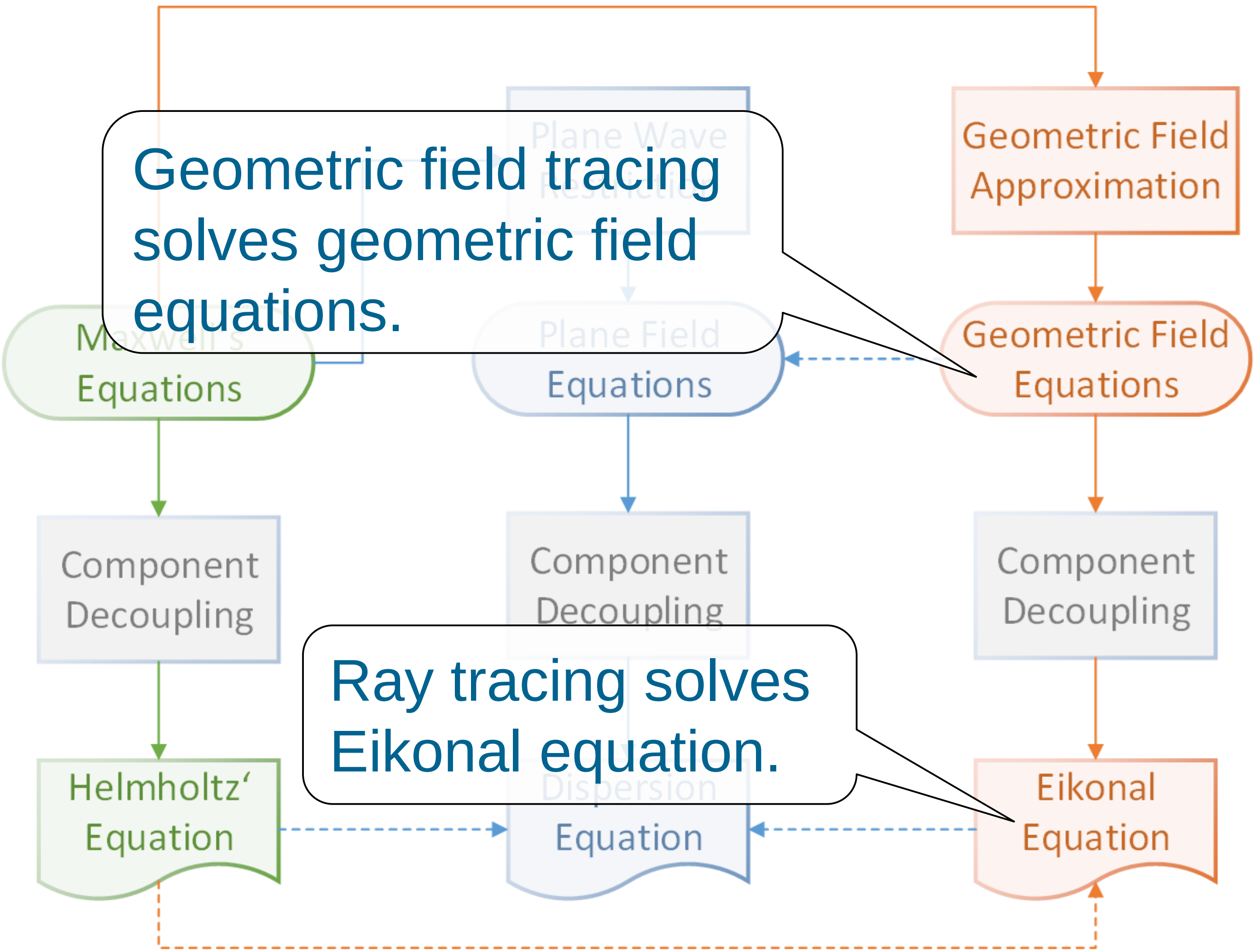
Helmholtz‘ Equation

Dispersion Equation

Eikonal Equation

Local plane wave equations. Includes plane wave equations.

Overview Field Equations



Geometric field tracing solves geometric field equations.

Ray tracing solves Eikonal equation.

Geometric Field Tracing is mainly …

… Ray Tracing with Smart Rays

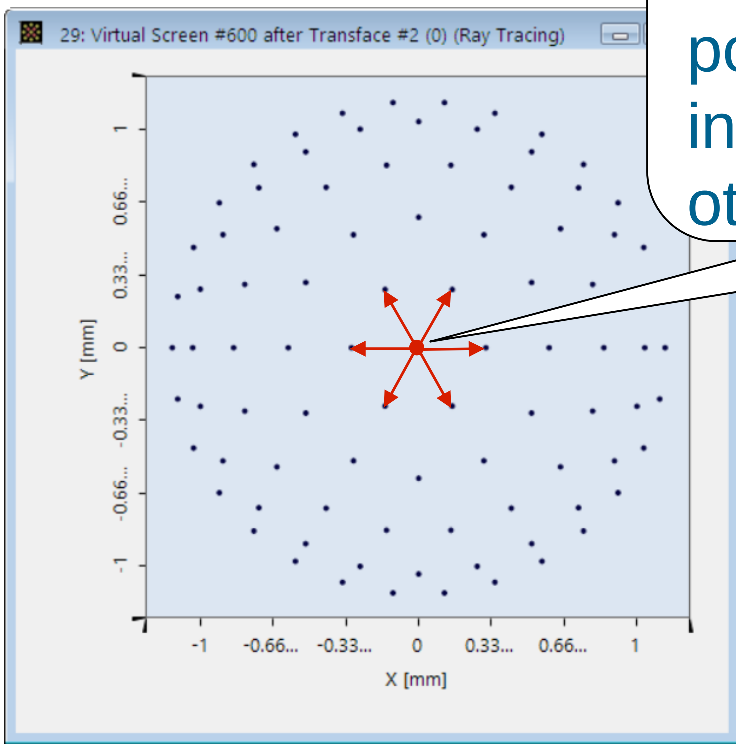
Smart rays know …

- … any electromagnetic field information at its position
- … its neighbors with respect to the wavefront

… Ray Tracing with Smart Rays

Smart rays know …

- … any electromagnetic field information at its position
- … its neighbors with re



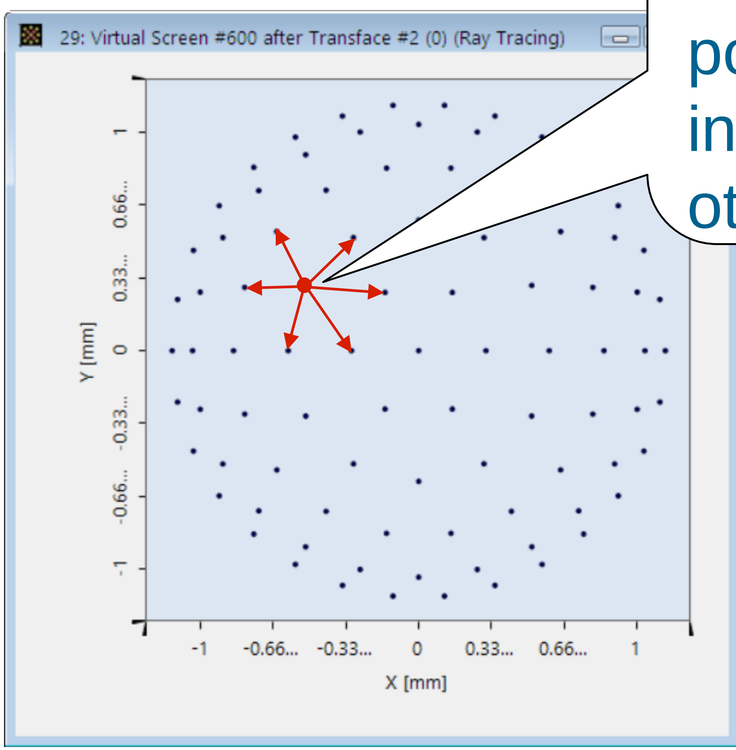
Smart rays come with information about spatial position, ray direction, and indexes for relative position to other rays.

… Ray Tracing with Smart Rays

Smart rays know …

- … any electromagnetic field information at its position
- … its neighbors with re

Smart rays come with information about spatial position, ray direction, and indexes for relative position to other rays.

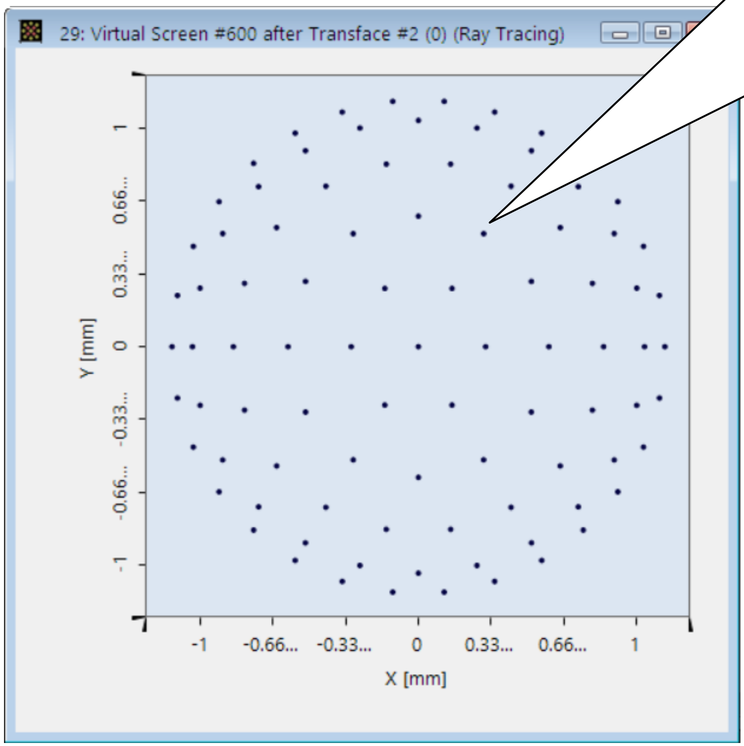


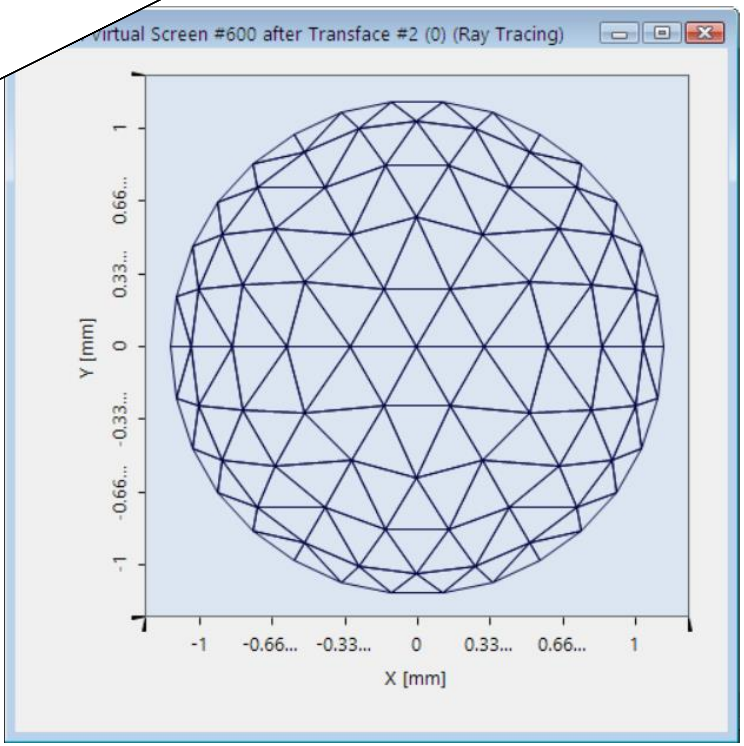
… Ray Tracing with Smart Rays

Smart rays know …

- … any electromagne
- … its neighbors with

Ray position can be interpreted as nodes of a mesh which can be obtained by triangularization on demand.





… Ray Tracing with Smart Rays

Smart rays know …

- … any electromagnetic field information at its position
- … its neighbors with respect to the wavefront

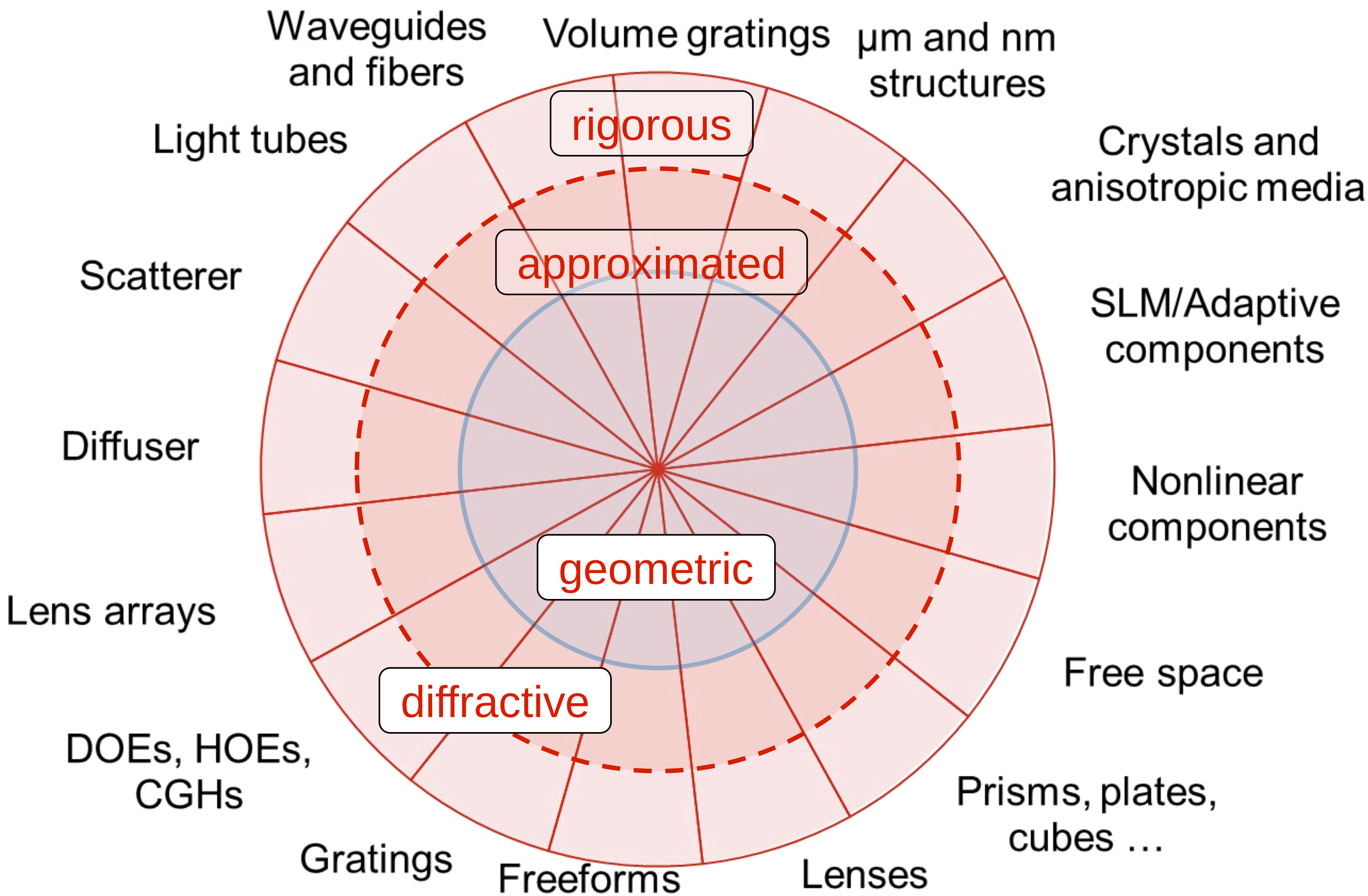
… Ray Tracing with Smart Rays

Smart rays know …

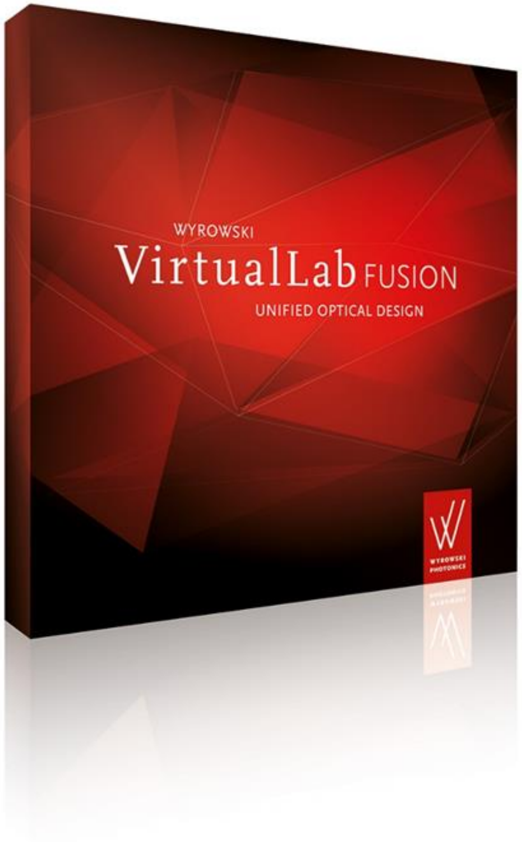
- … any electromagnetic field information at its position
- … its neighbors with respect to the wavefront
- … the correlation of the modes which they represent
- … if they are associated to a stationary or a pulsed field

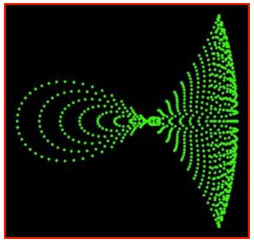
⇒ Solves Maxwell's equations in its geometric approximation as fast as ray tracing delivers ray information.

Unified Field Tracing

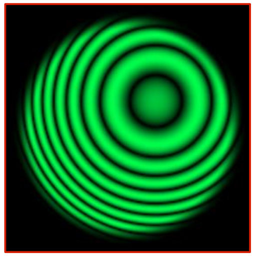


VirtualLab Fusion – Unified Optical Design

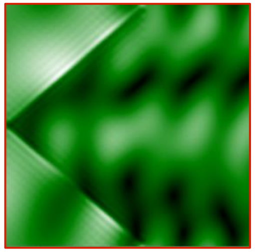




RAY TRACING

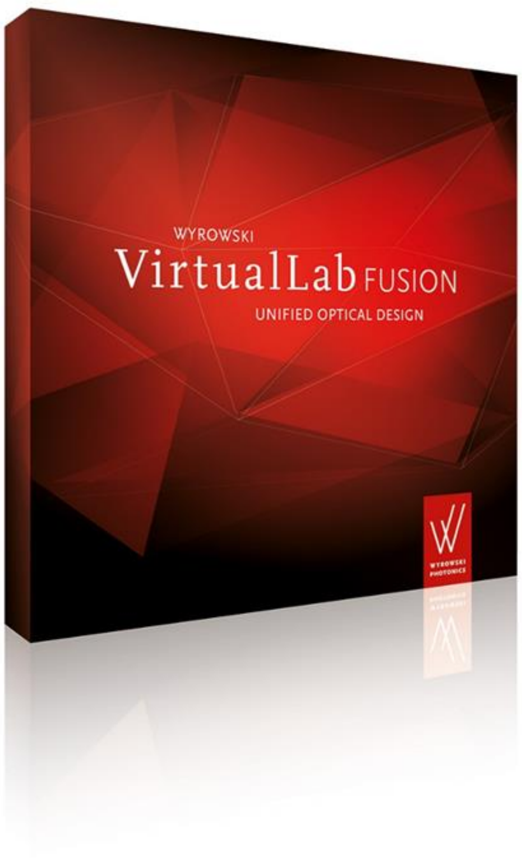


GEOMETRIC
FIELD TRACING

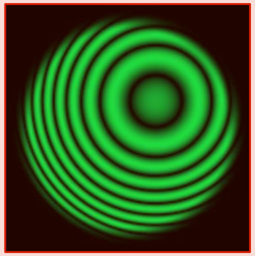


UNIFIED
FIELD TRACING

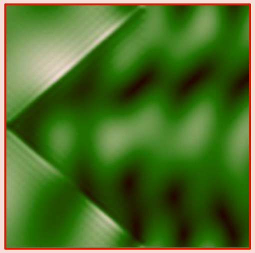
VirtualLab Fusion – Smart. Fast. Accurate.



Physical Optics



GEOMETRIC
FIELD TRACING

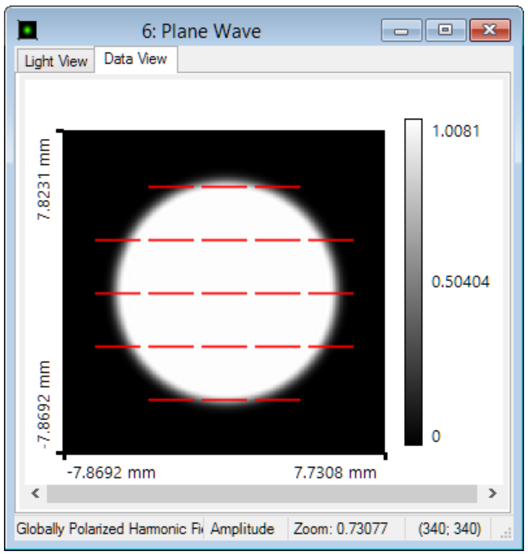


UNIFIED
FIELD TRACING

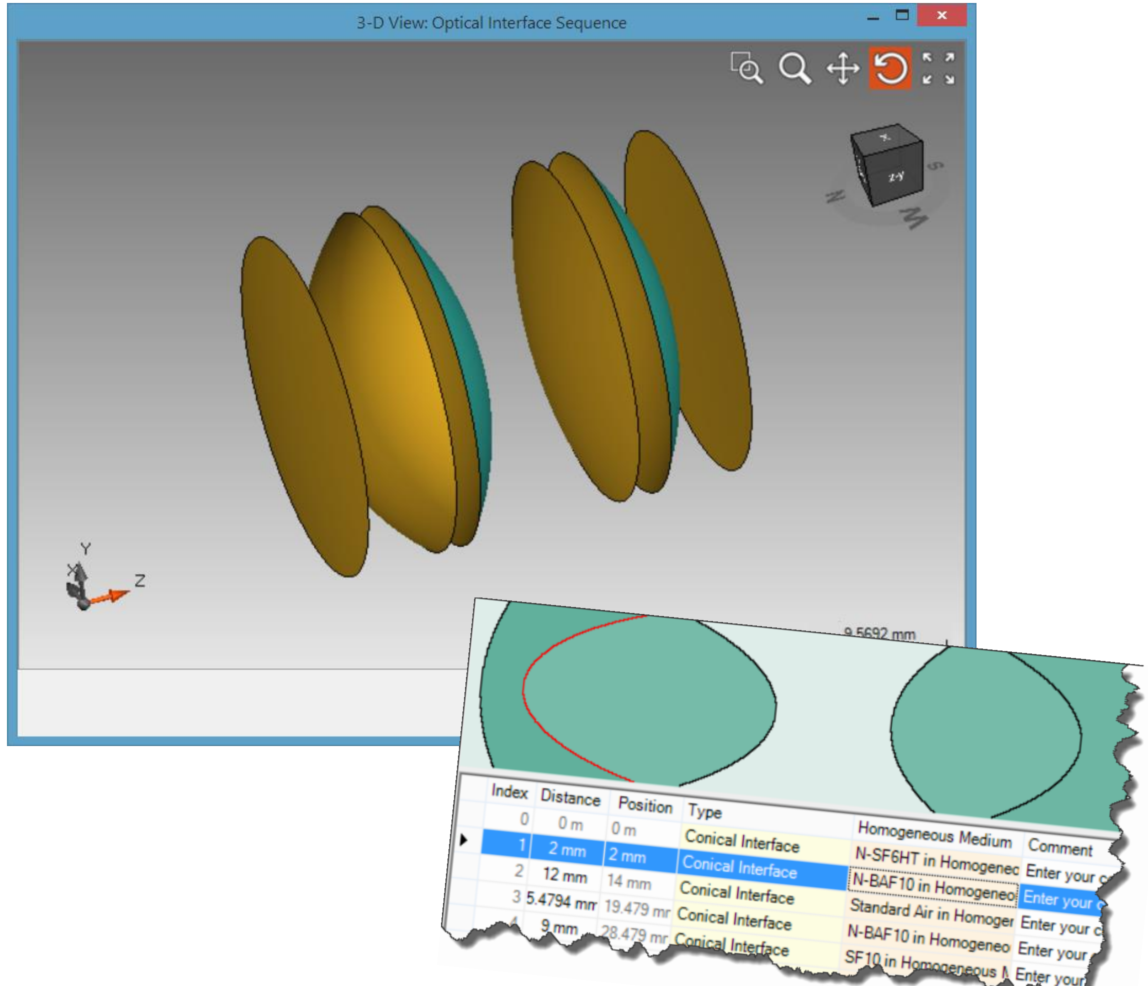
Fields in Focal Regions

Example Lens System

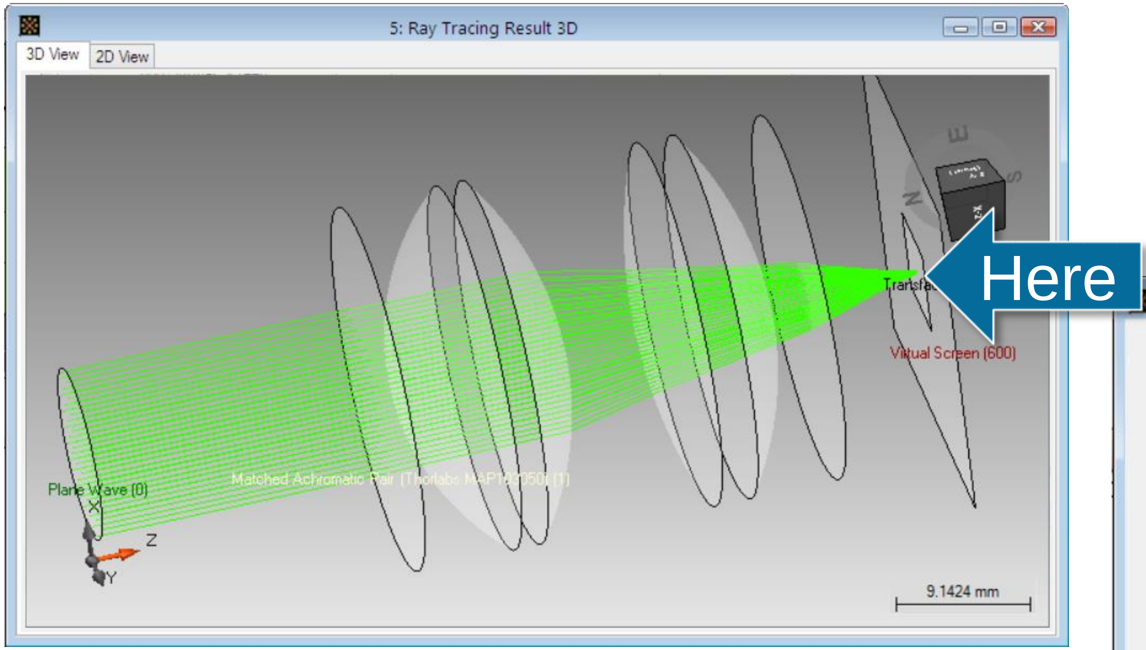
Lens System: Focusing

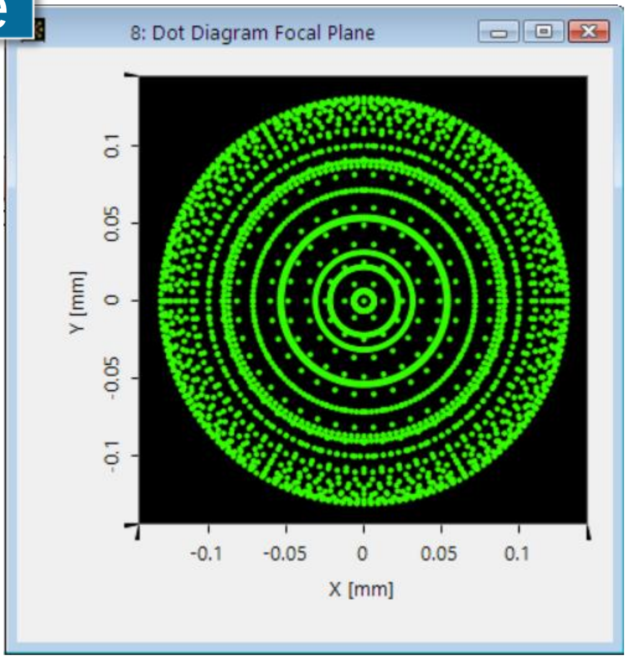


Plane wave
Diameter 10 mm
Linear polarization



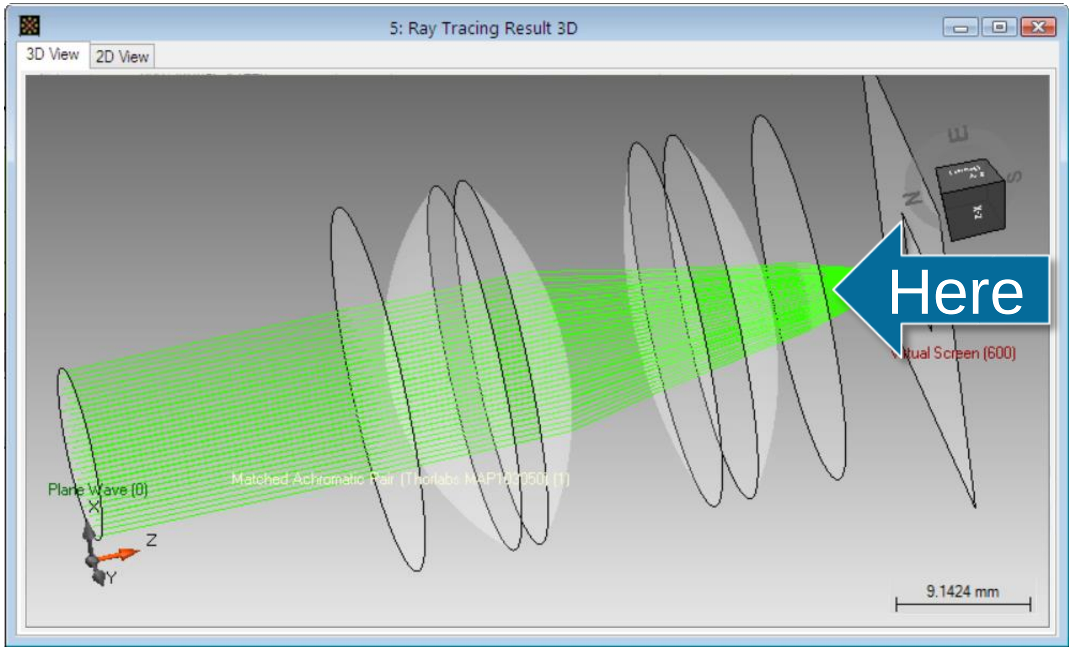
Lens System: Ray Tracing

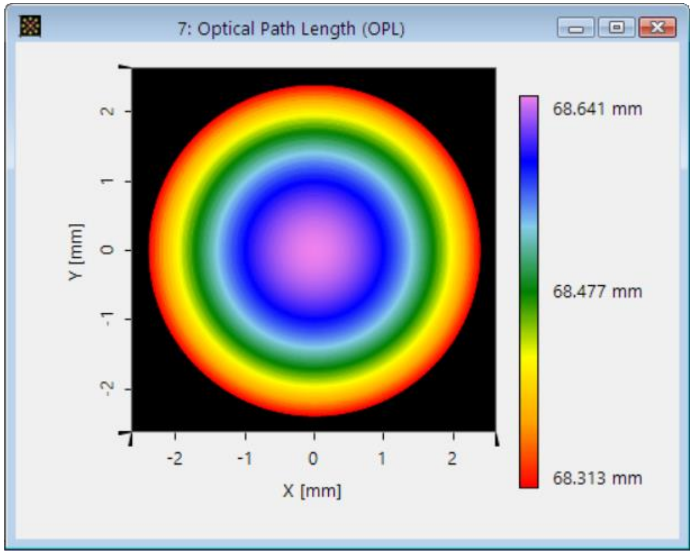




Dot diagram

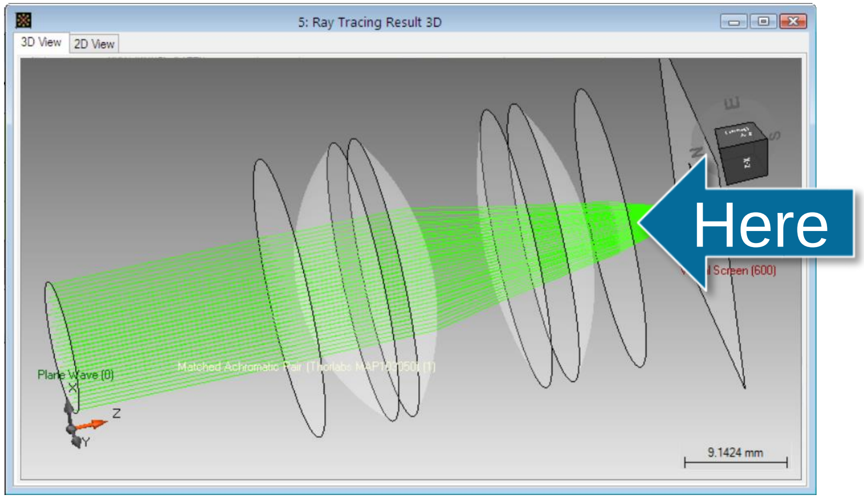
Lens System: Ray Tracing





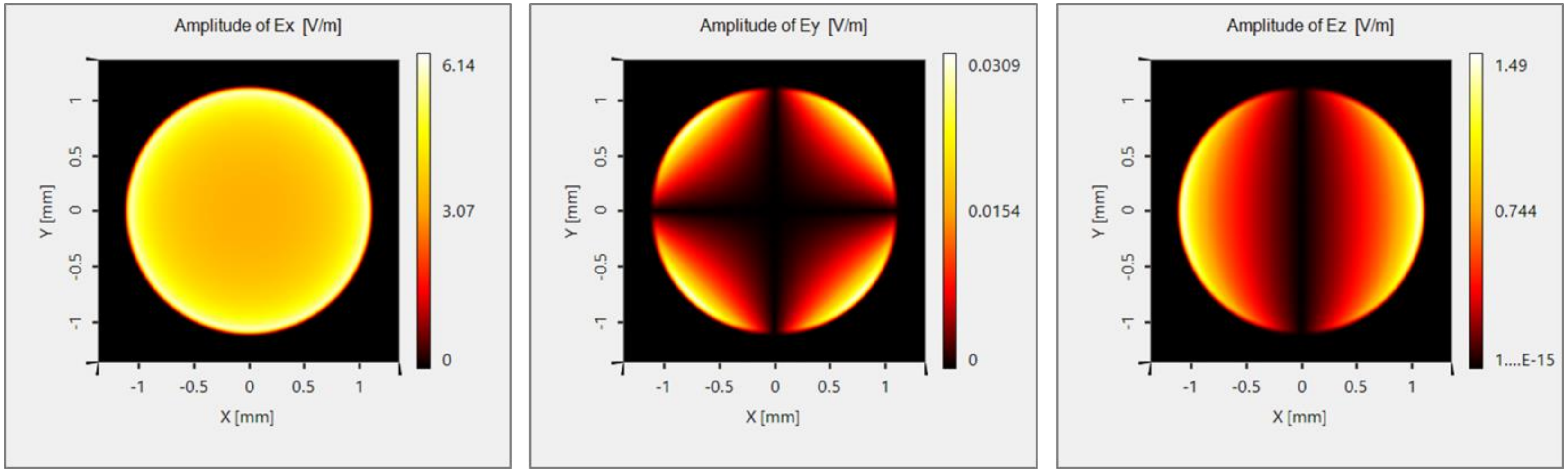
OPL behind last lens

Lens System: Geometric Field Tracing

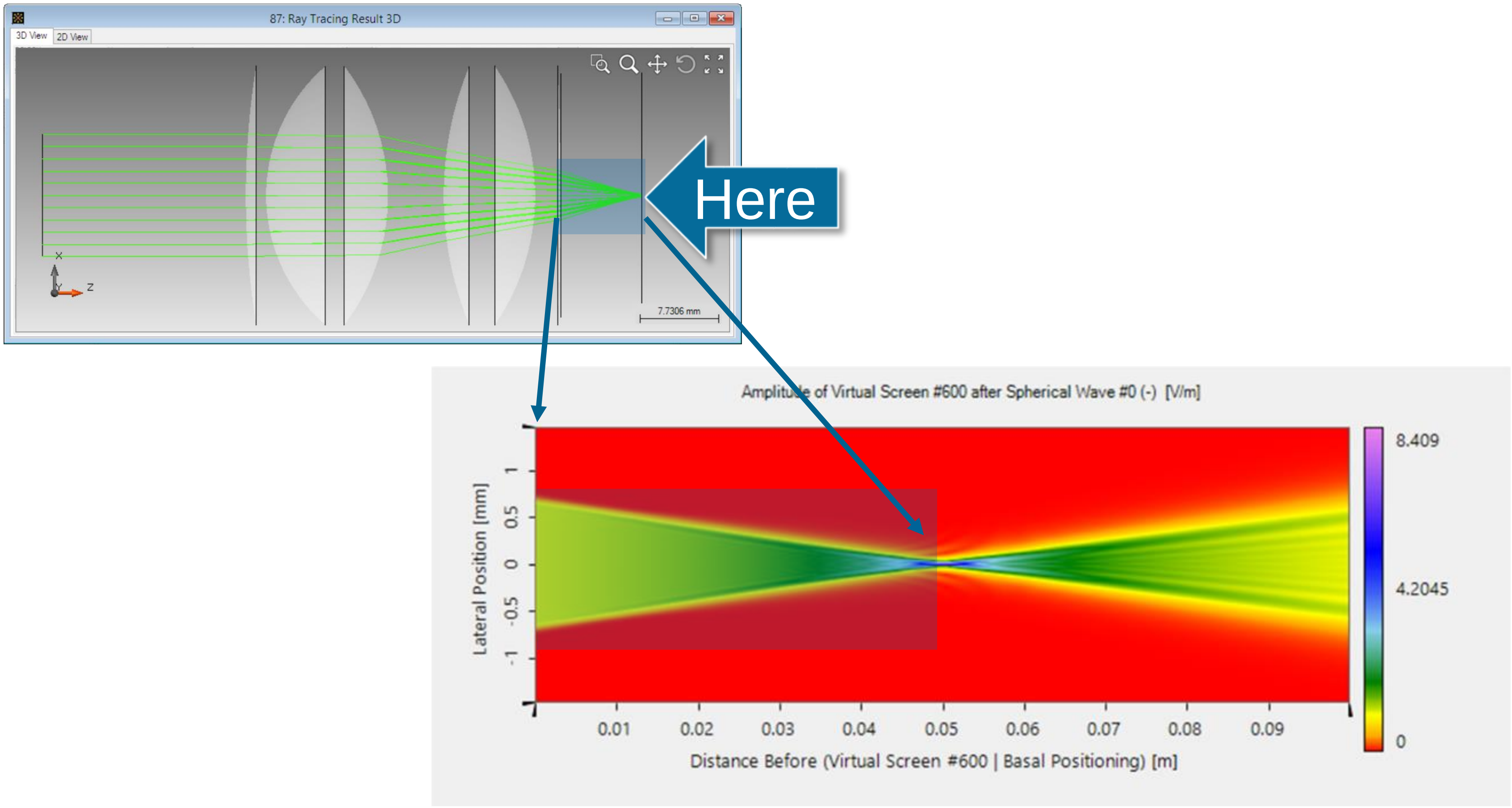


- Modeling from source to target plane behind last lens by geometric field tracing

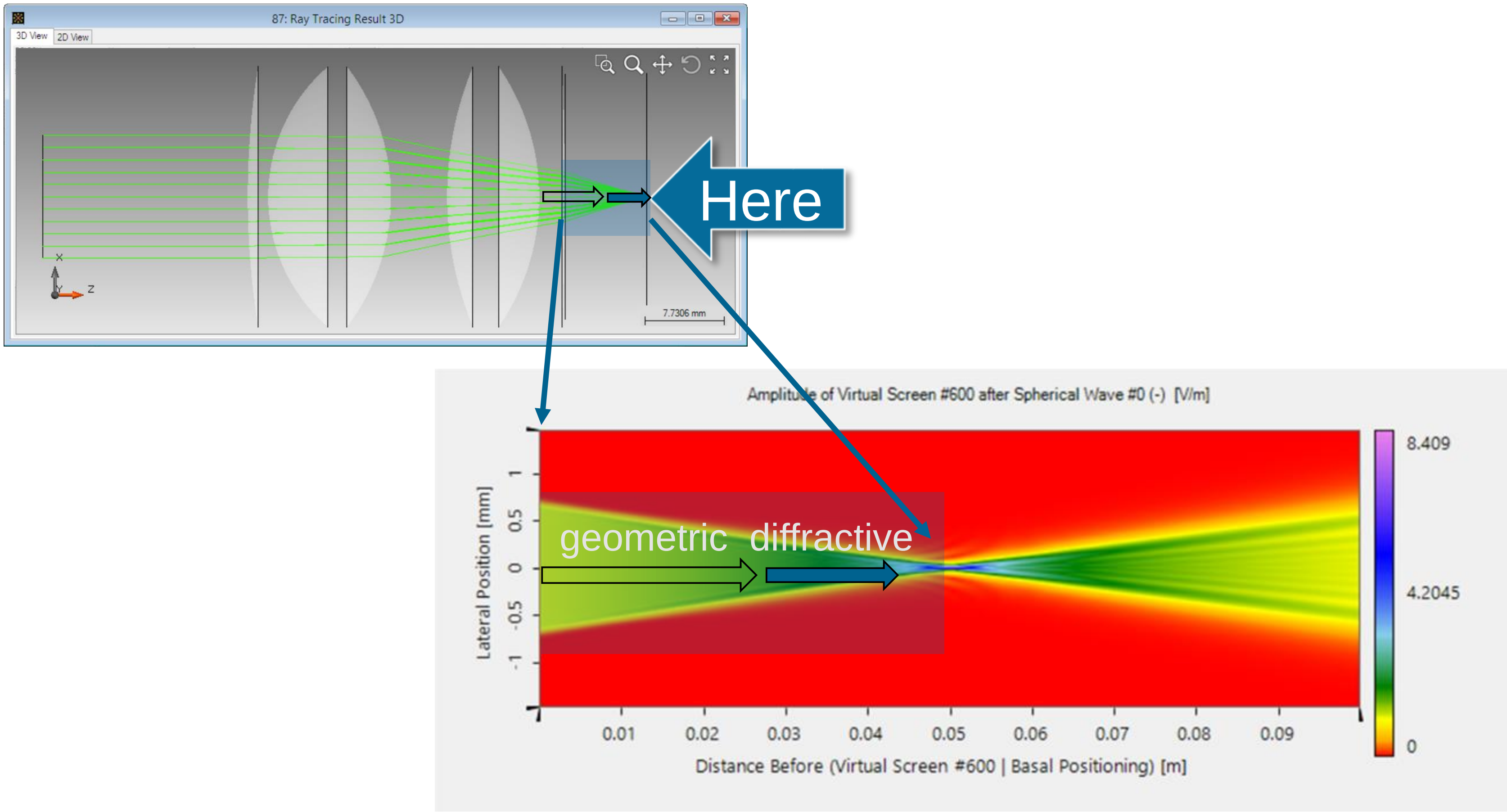
cpu time ≈ 1 sec



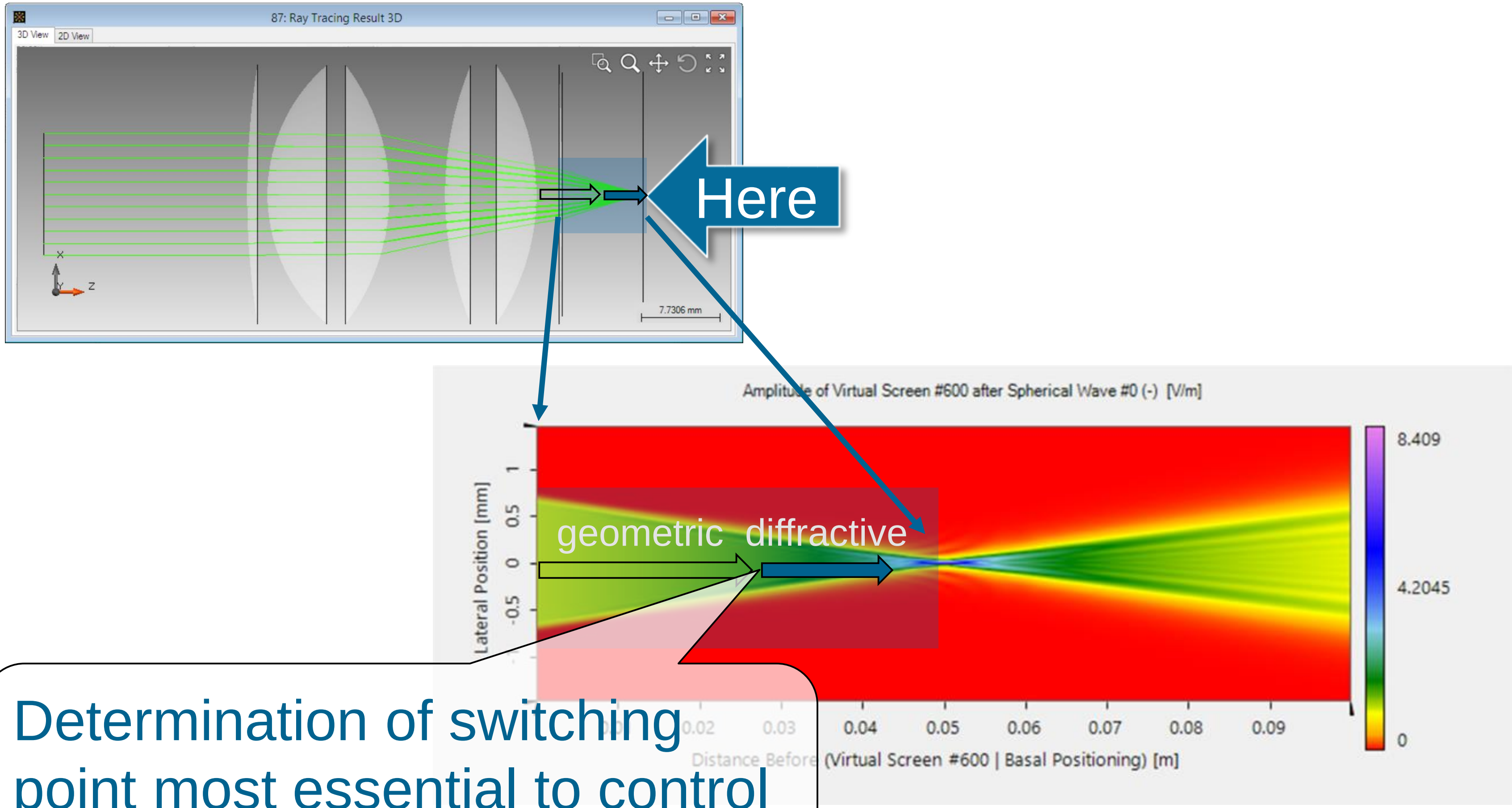
From Geometric to Diffractive Field Tracing



From Geometric to Diffractive Field Tracing

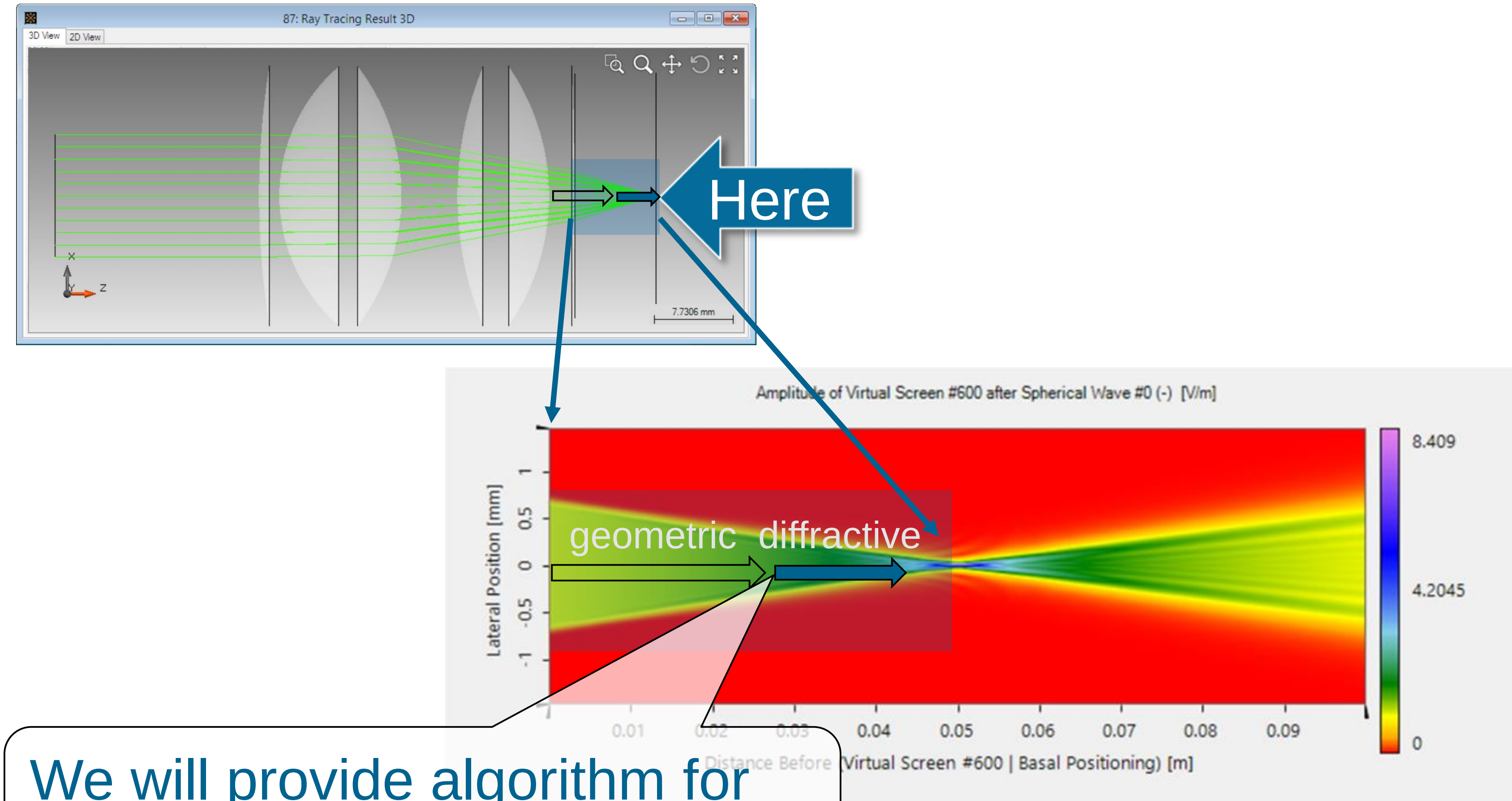


From Geometric to Diffractive Field Tracing



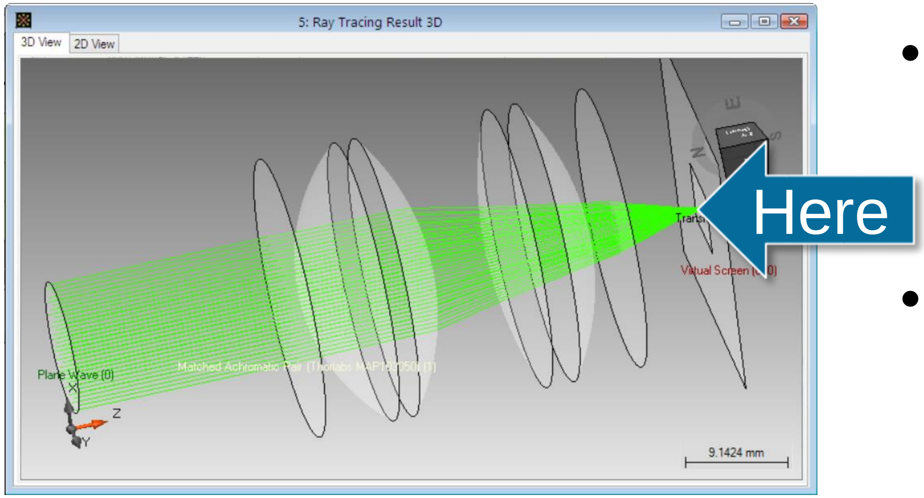
Determination of switching point most essential to control accuracy vs. numerical effort

From Geometric to Diffractive Field Tracing



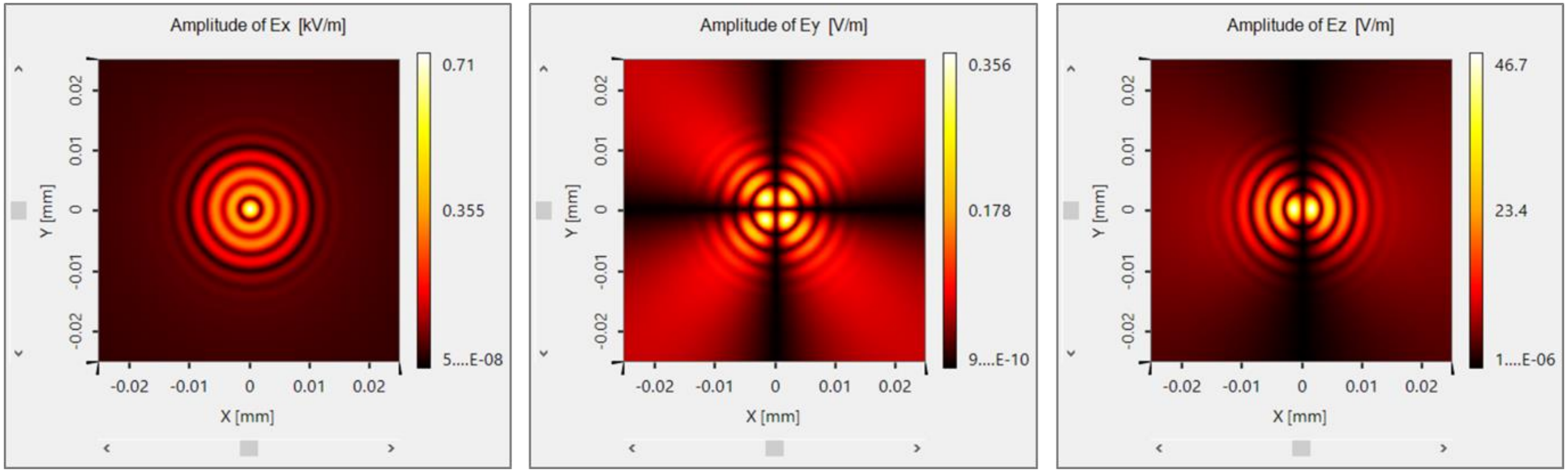
We will provide algorithm for automatic decision.

Lens System: Unified Field Tracing

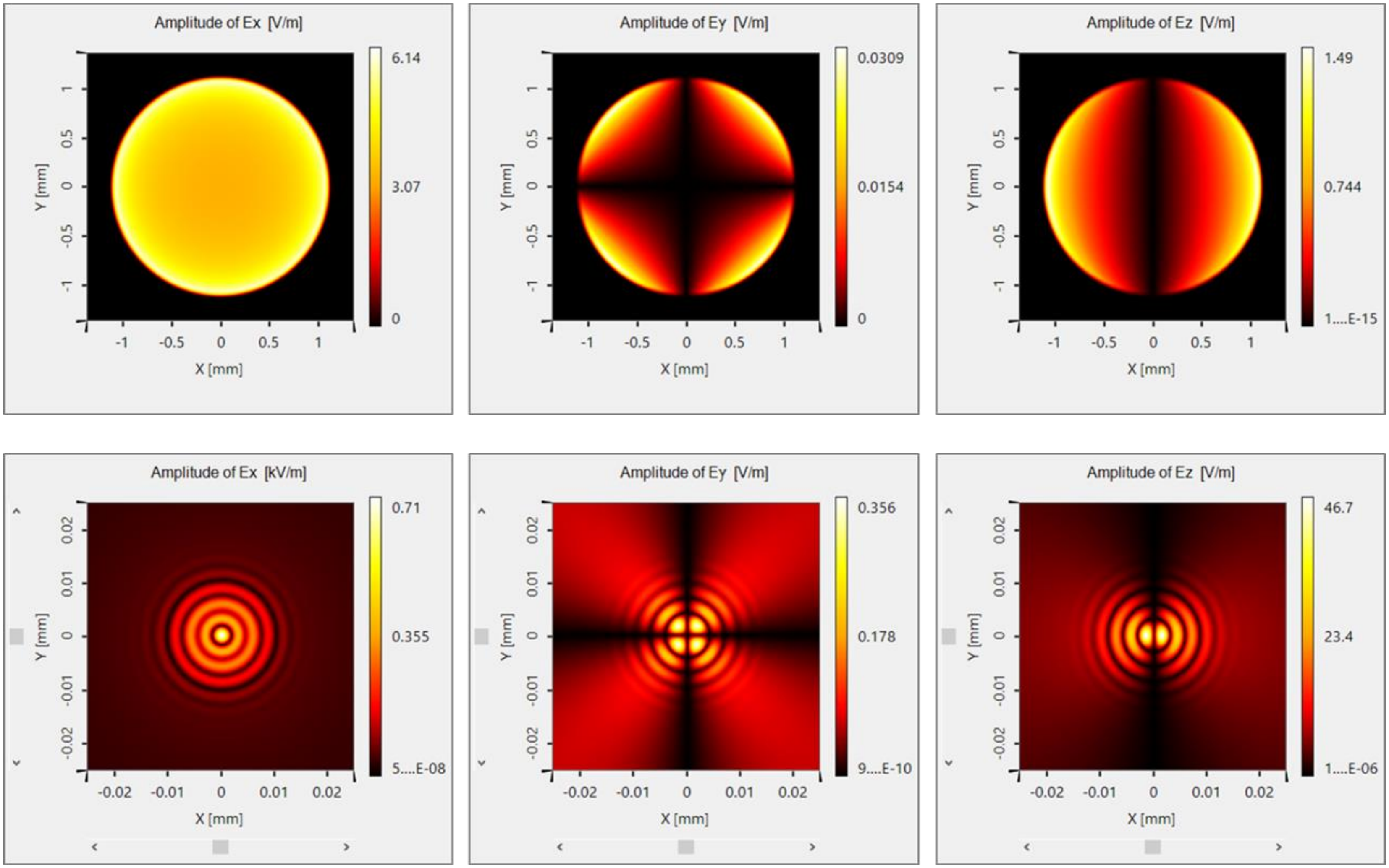


- Modeling from transition to focal plane by diffractive field tracing
- Inverse far field operator

cpu time ≈ 3 sec



Lens System: Unified Field Tracing



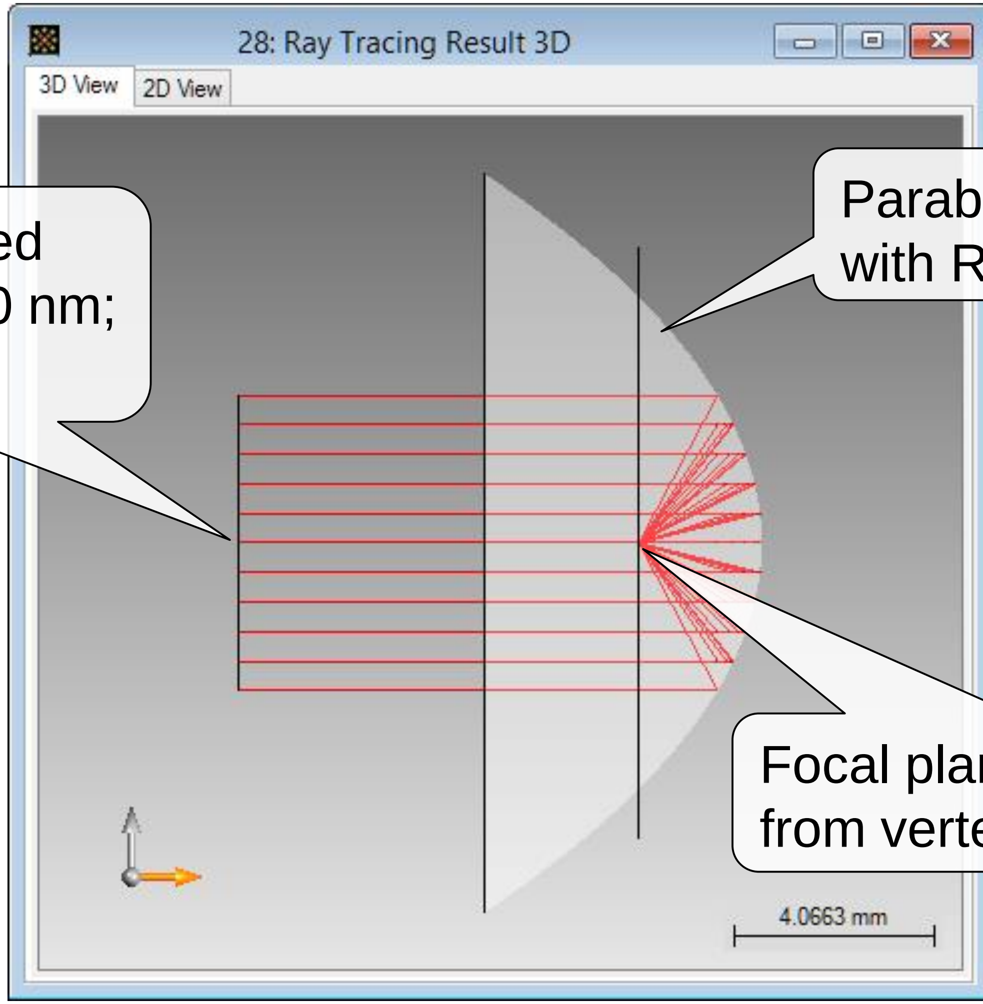
Fields in Focal Regions

Example Parabolic Mirror

Parabolic Mirror Modeling

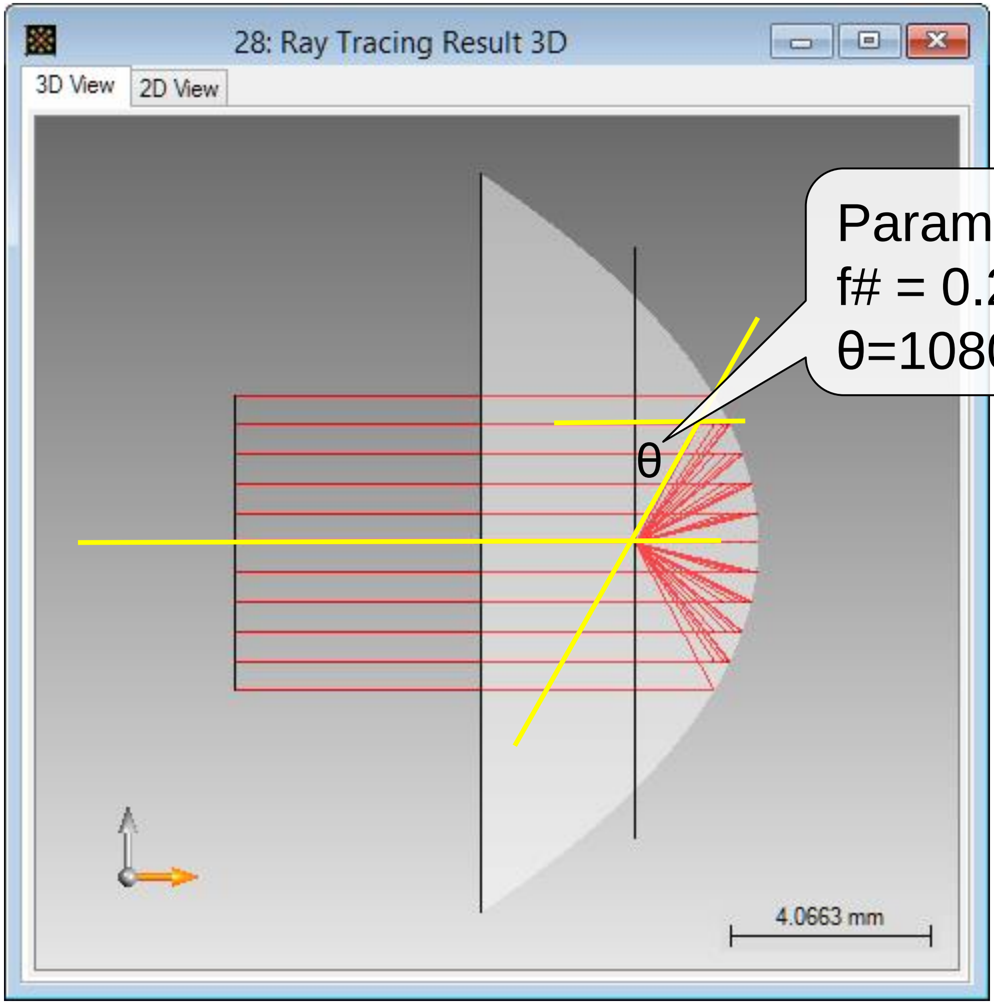
Linearly polarized plane wave; 800 nm; d = 5 mm

Parabolic silver mirror with R= - 5 mm.



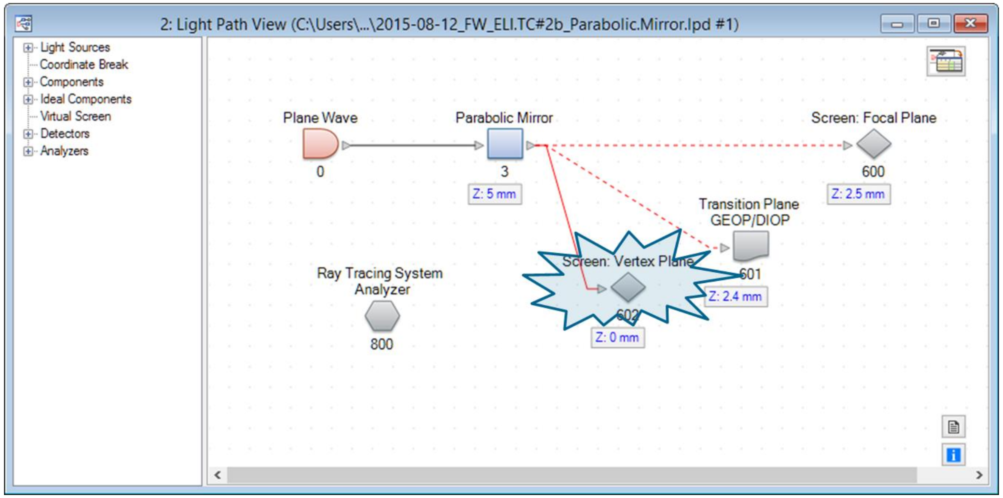
Focal plane at z = 2.5 mm from vertex of mirror.

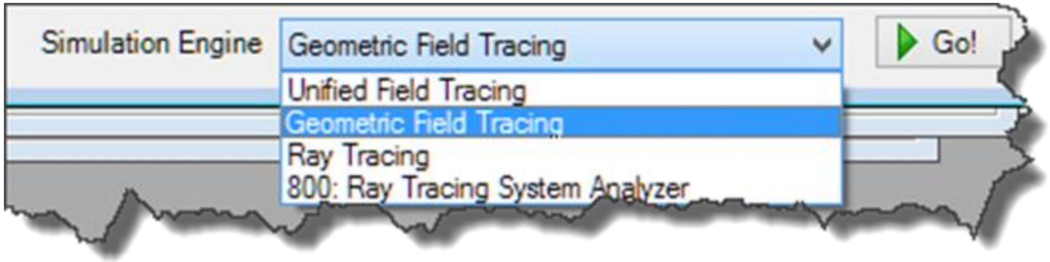
Modeling Situation: f# = 0.267



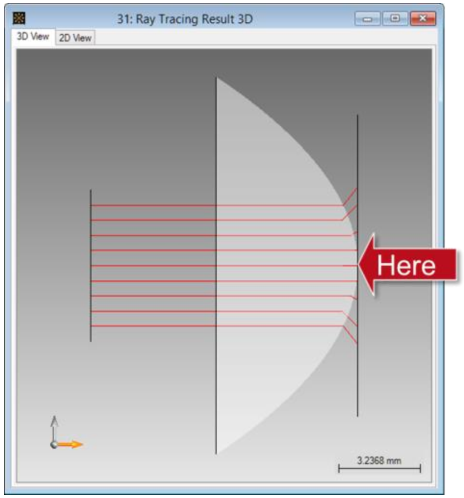
Parameters lead to f# = 0.267; angle θ=1080 mrad

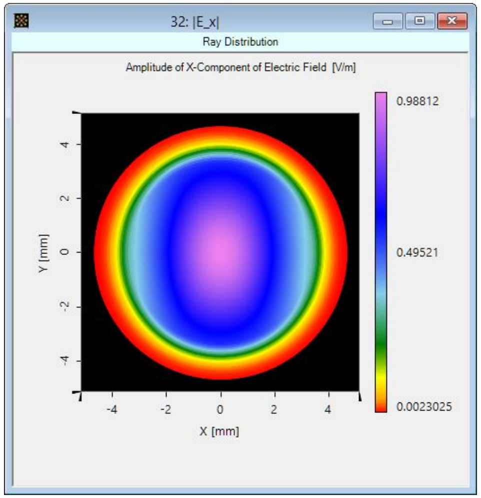
Field in Vertex Plane by Geometric Field Tracing



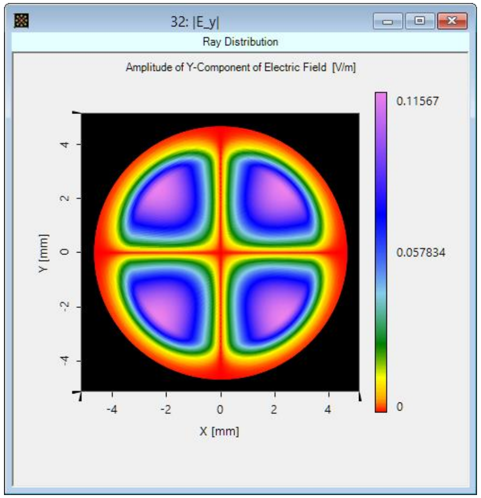


Field in Vertex Plane for x-Polarized Input Field

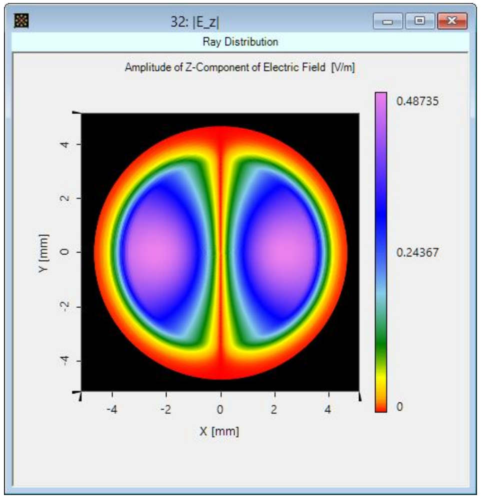




$|E_x|$

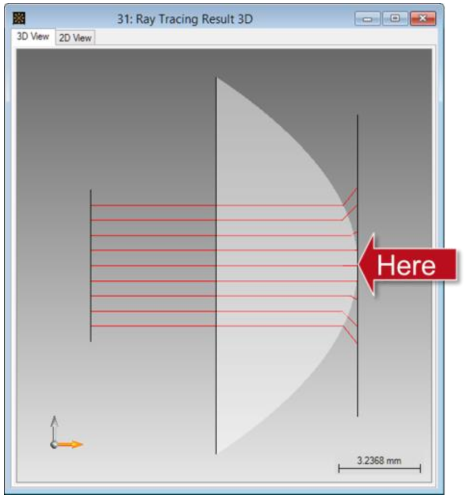


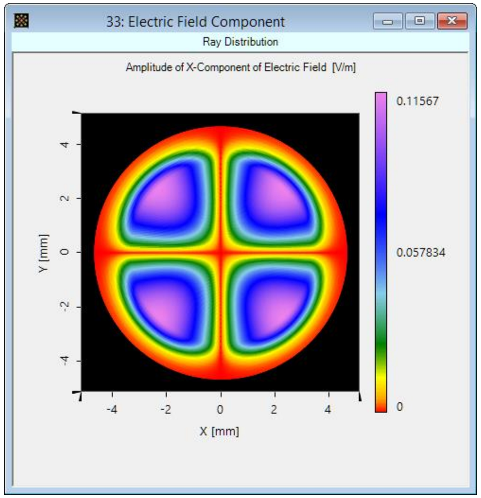
$|E_y|$



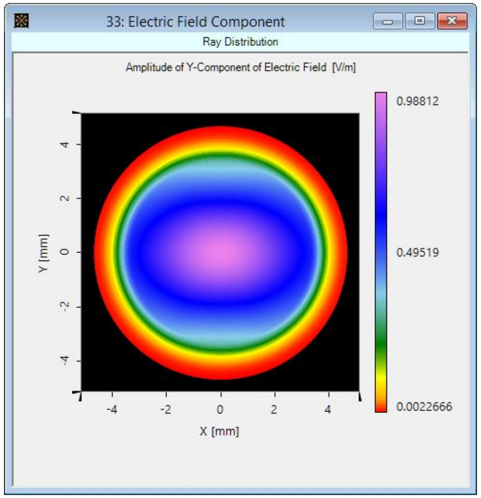
$|E_z|$

Field in Vertex Plane for y-Polarized Input Field

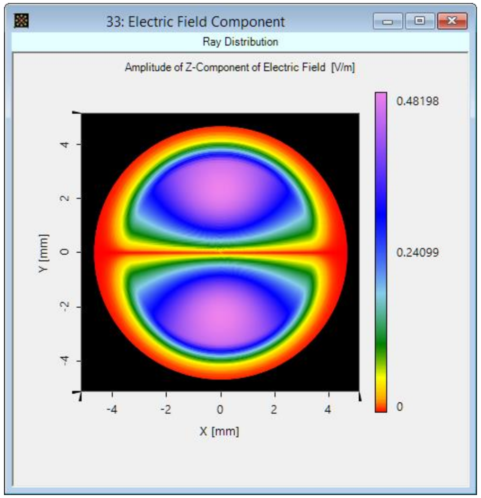




$|E_x|$

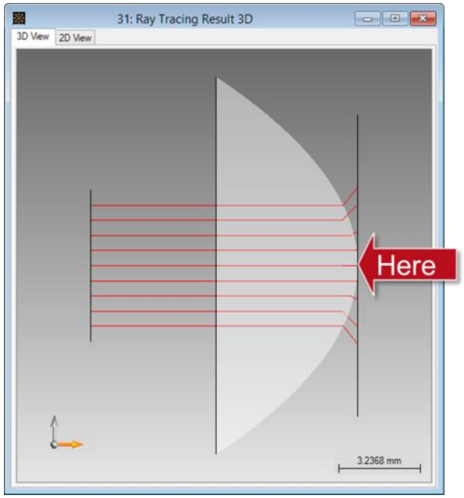


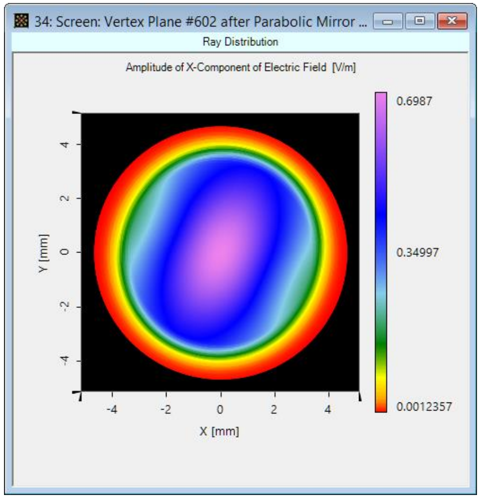
$|E_y|$



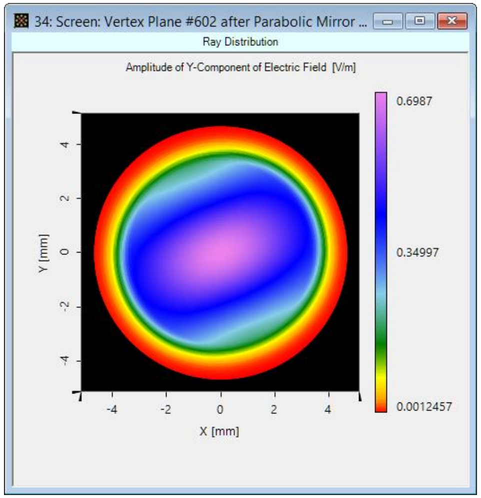
$|E_z|$

Field in Vertex Plane for 45°-Polarized Input Field

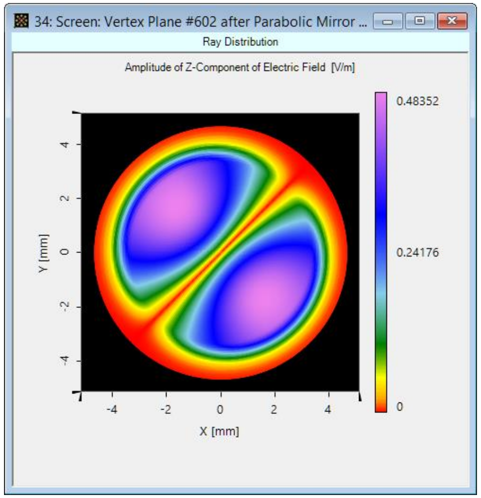




$|E_x|$

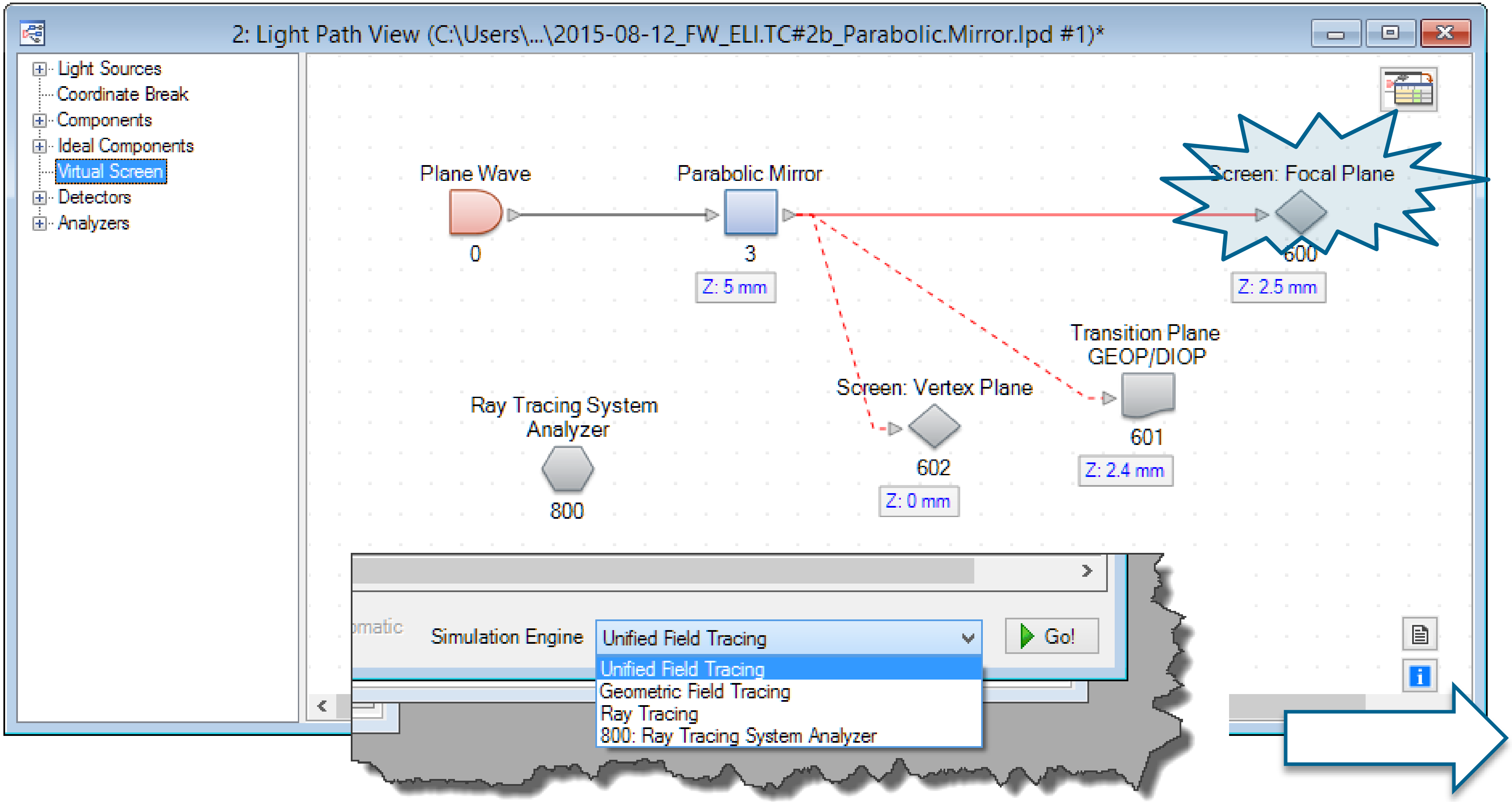


$|E_y|$

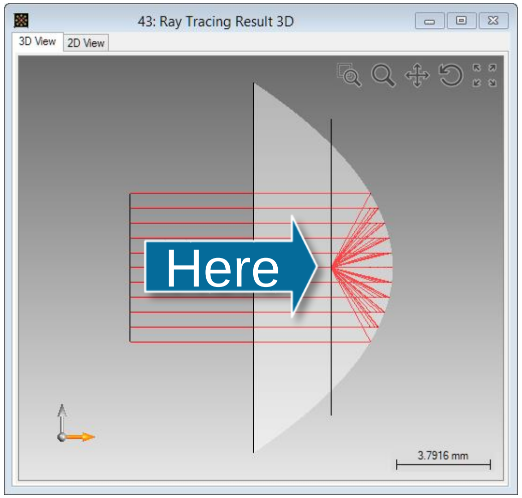


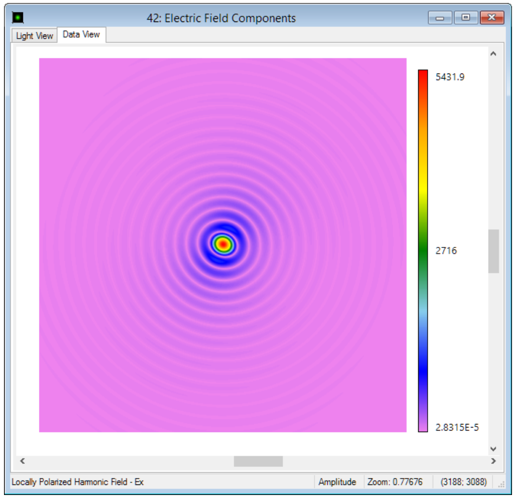
$|E_z|$

Field in Focal Plane by Unified Field Tracing

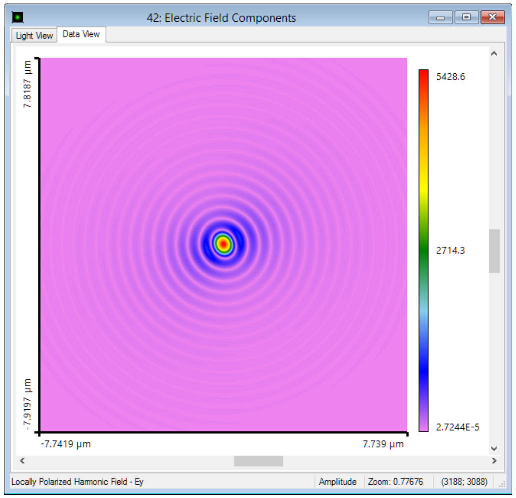


Field in Focal Plane for 45°-Polarized Input Field

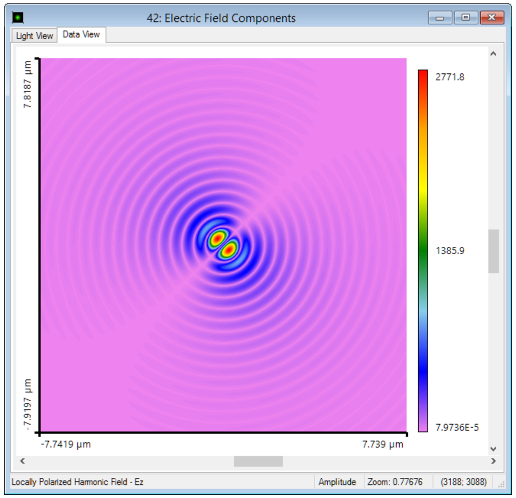




|E_x|

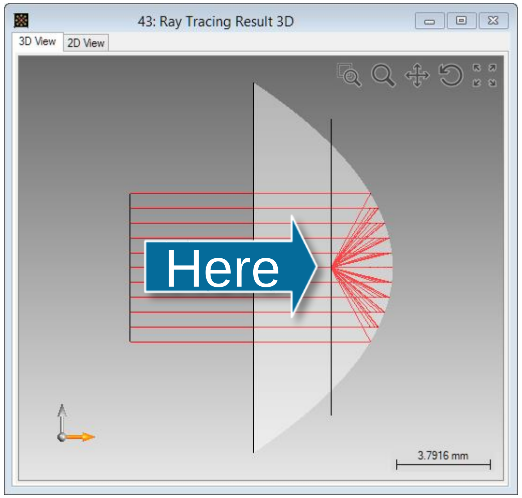


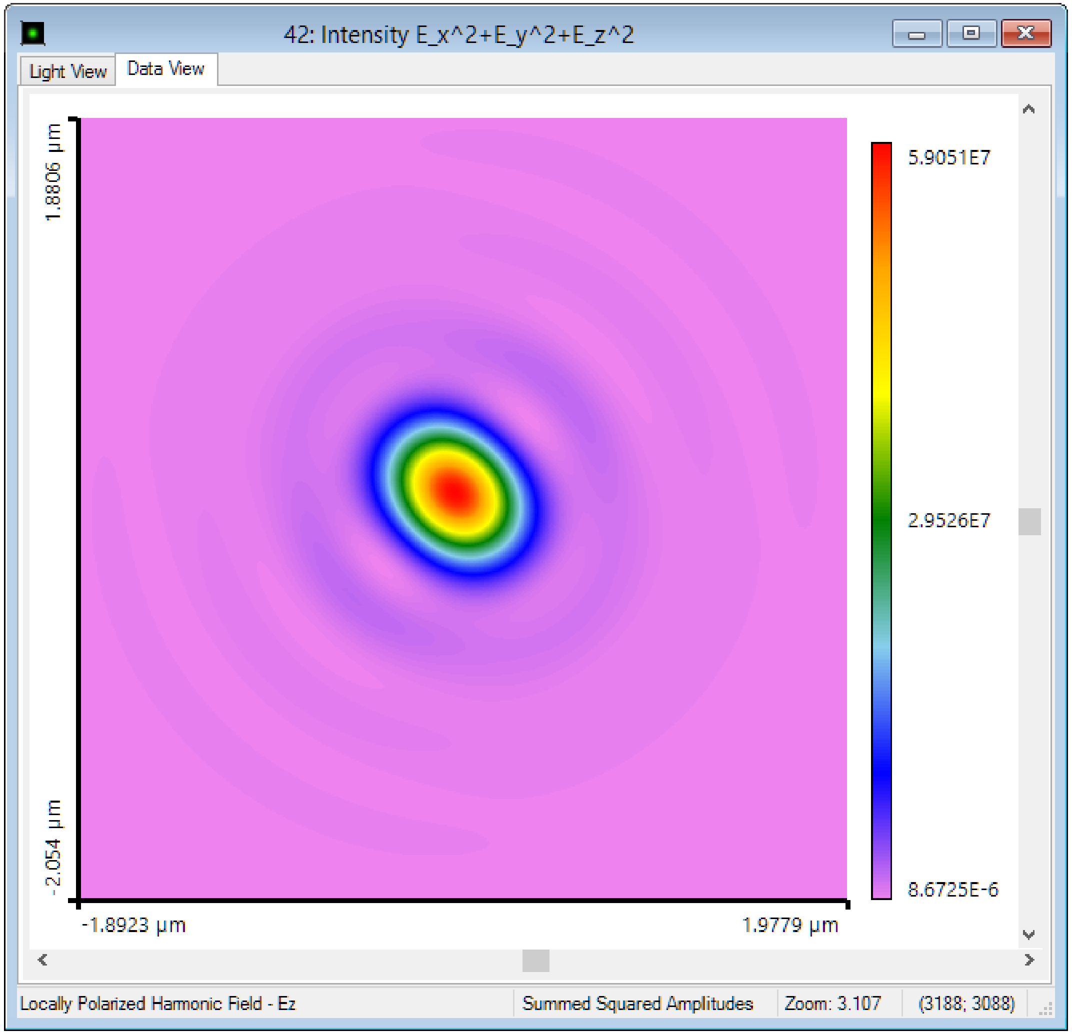
|E_y|



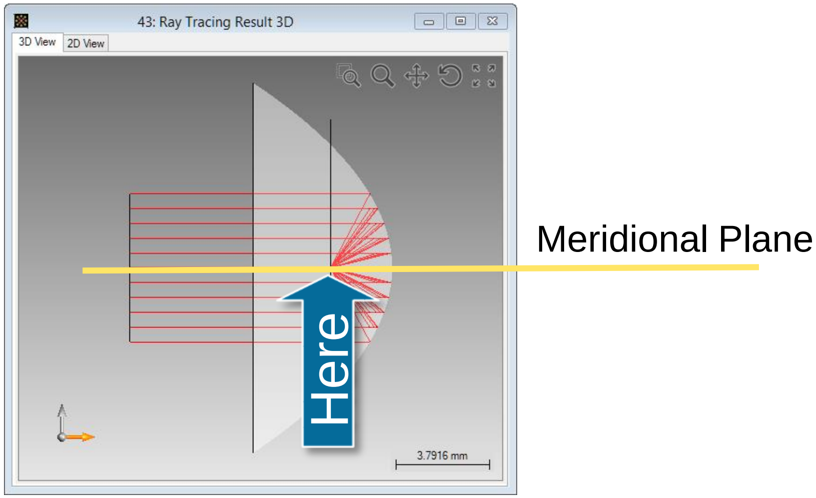
|E_z|

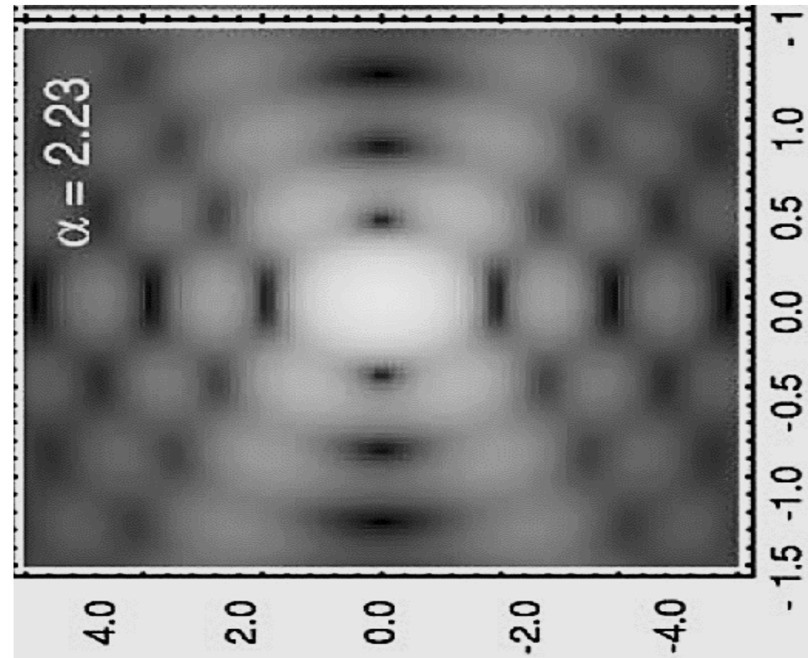
Intensity EE* in Focal Plane





Field in Meridional Plane





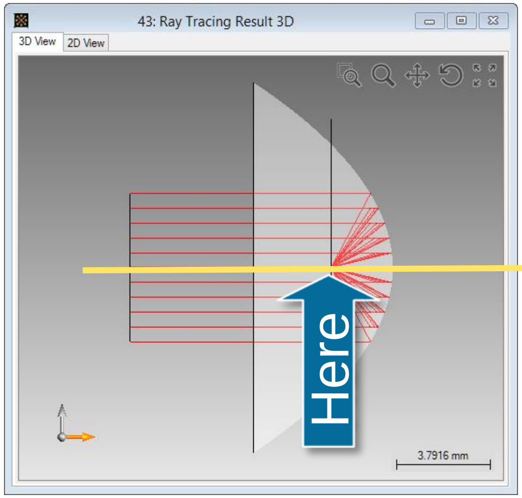
J. Opt. Soc. Am. A/Vol. 17, No. 11/November 2000 P. Varga and P. Török

Focusing of electromagnetic waves by paraboloid mirrors. II. Numerical results

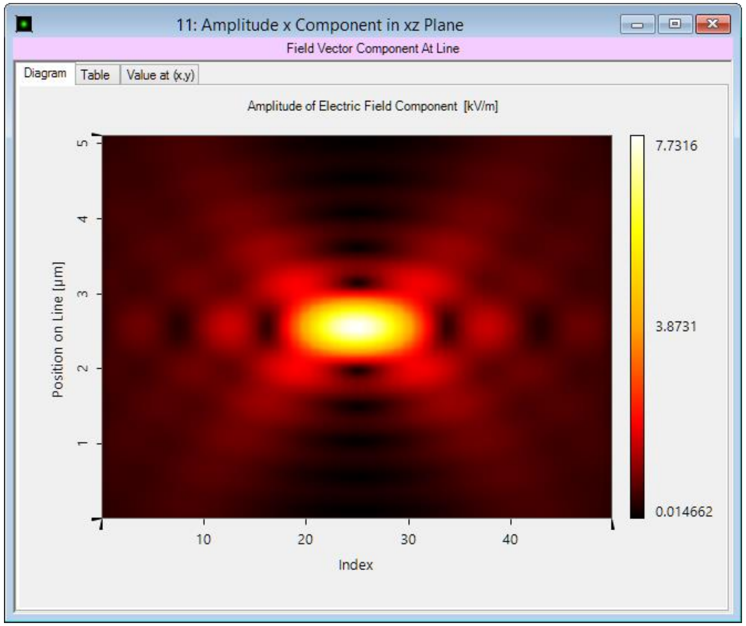
Peter Varga and Peter Török

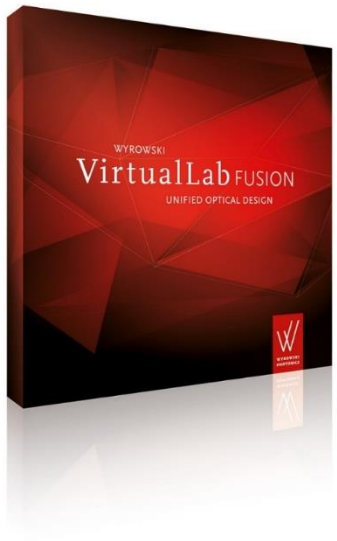
Department of Engineering Science, University of Oxford, Parks Road, Oxford OX1 3PJ, UK

Field in Meridional Plane



Meridional Plane





Comparison with Reference Paper

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Focusing of electromagnetic waves by paraboloid mirrors. II. Numerical results

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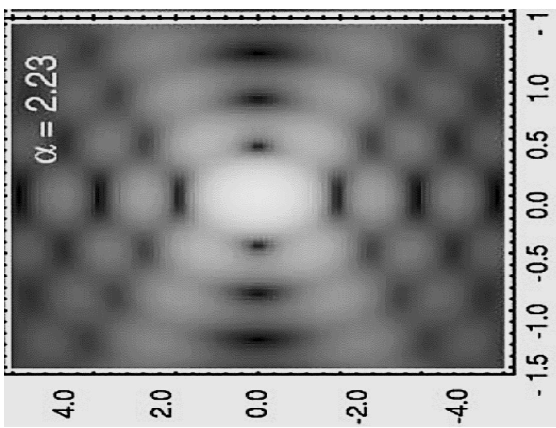
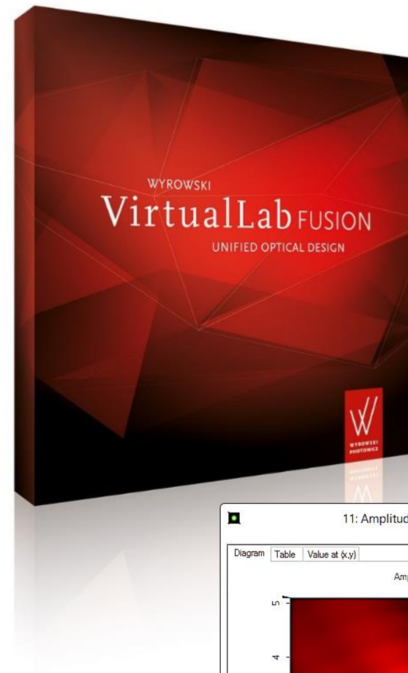
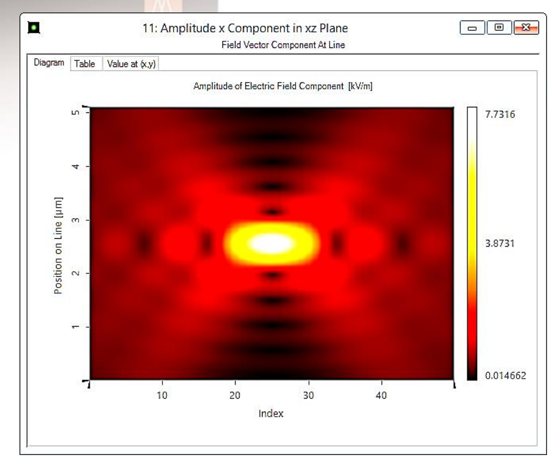


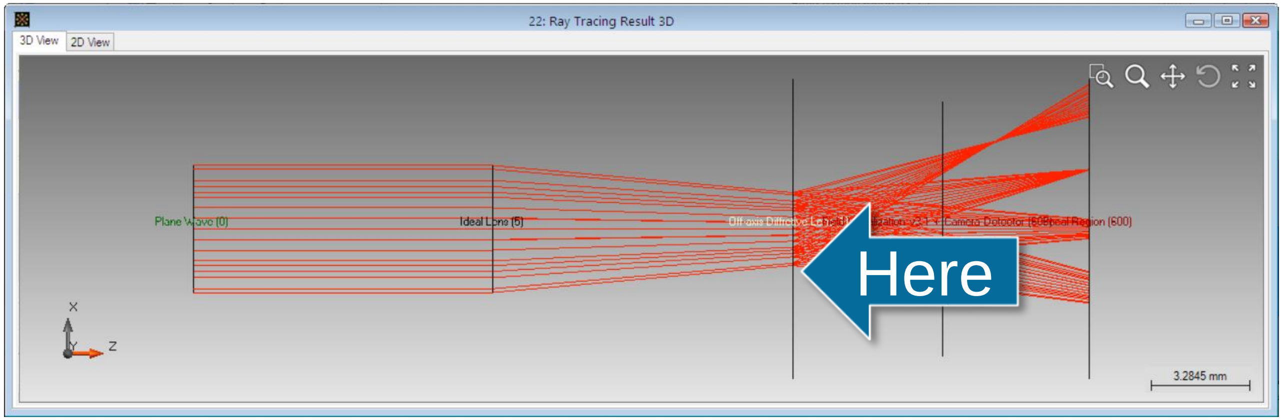




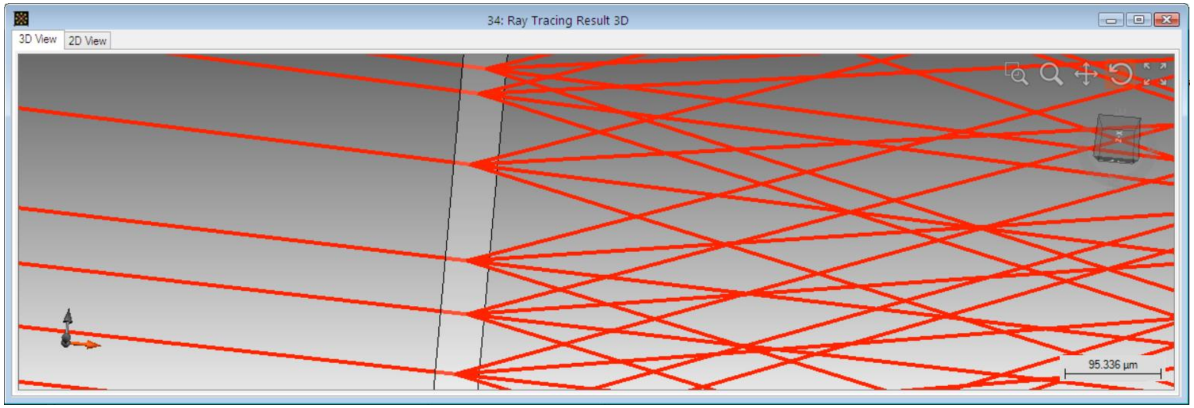
cpu time a few minutes

Example: Diffractive Lens

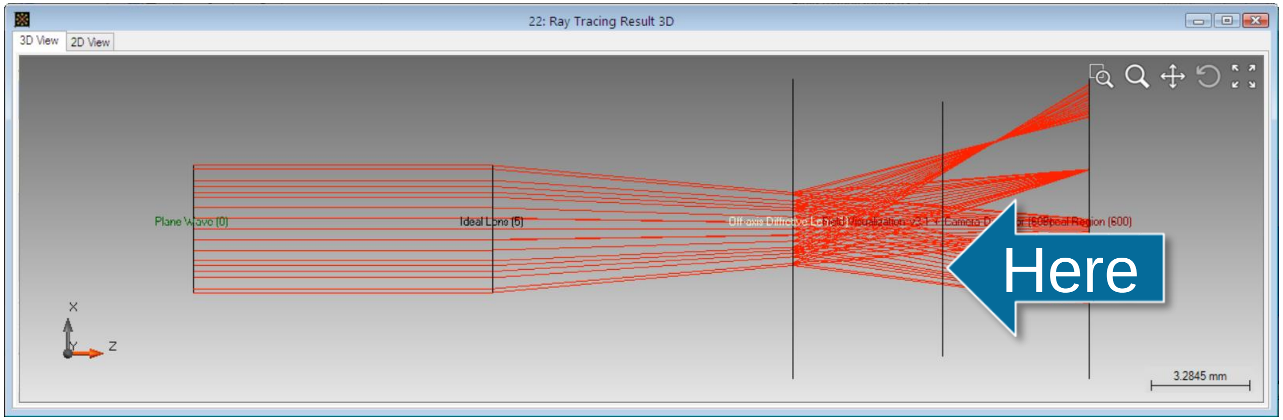
Diffractive Lens: Ray Tracing



Orders:
-1,0,1,2

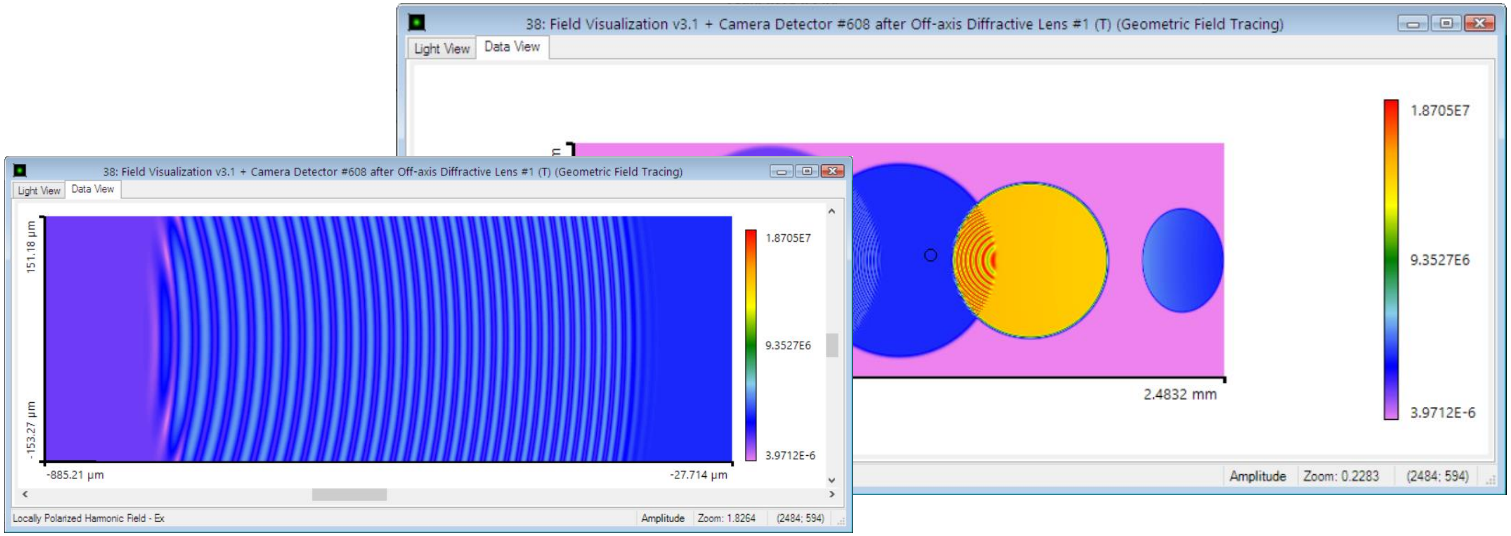


Diffractive Lens: Geometric Field Tracing

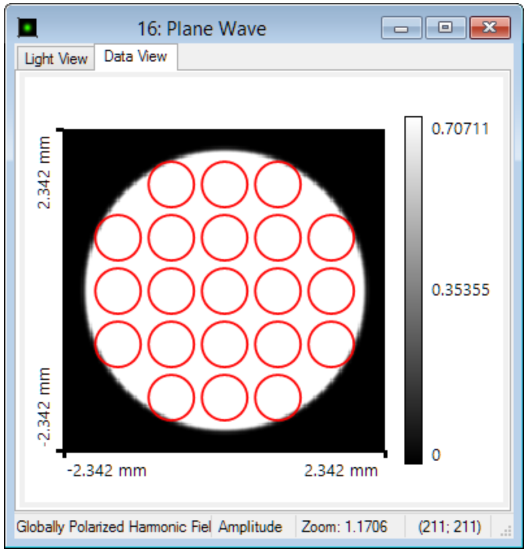


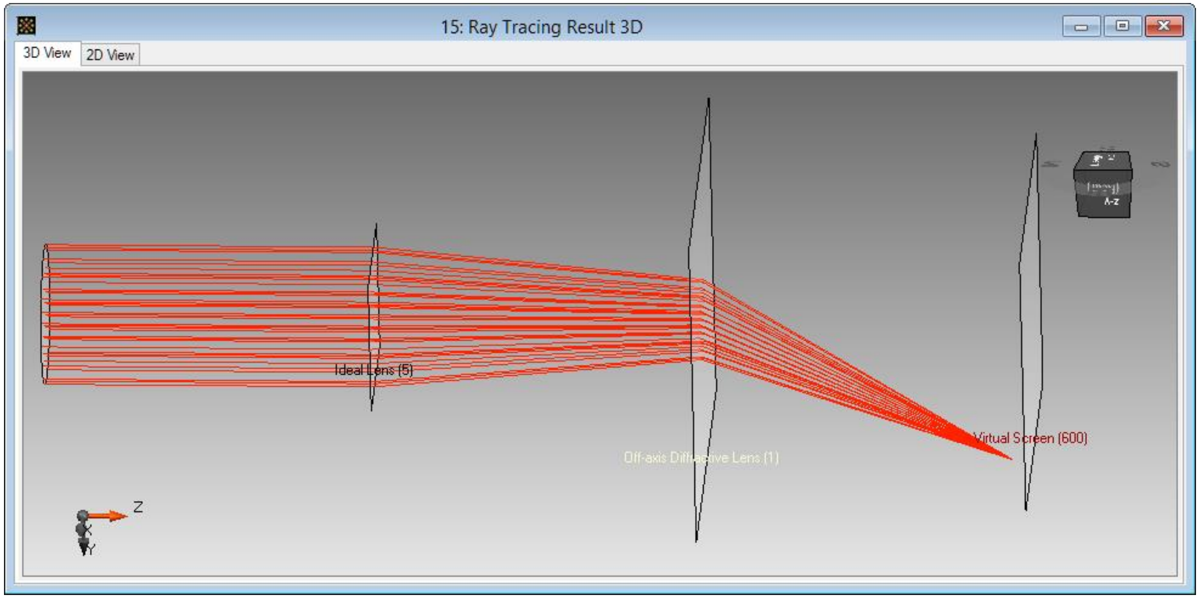
Orders:
-1,0,1,2

Amplitude $E_x(x,y)$



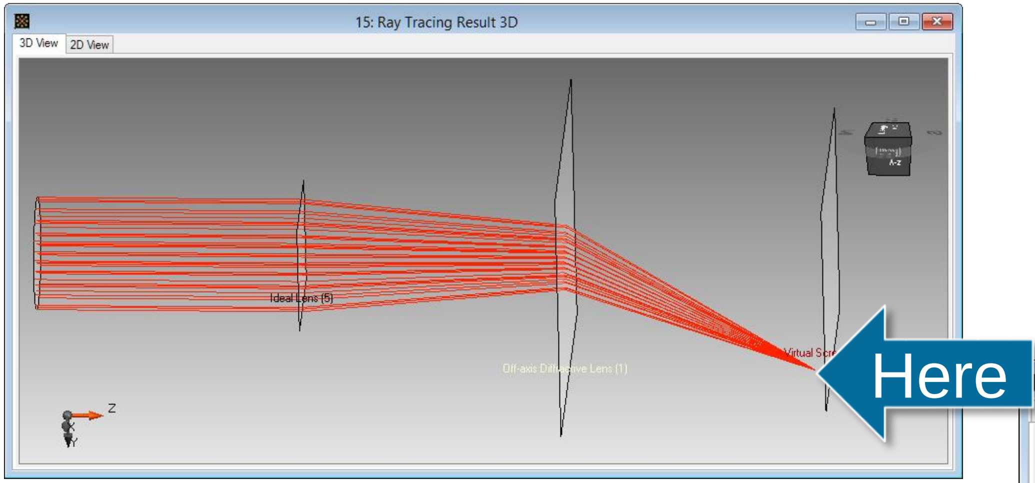
Off-axis Diffractive Lens





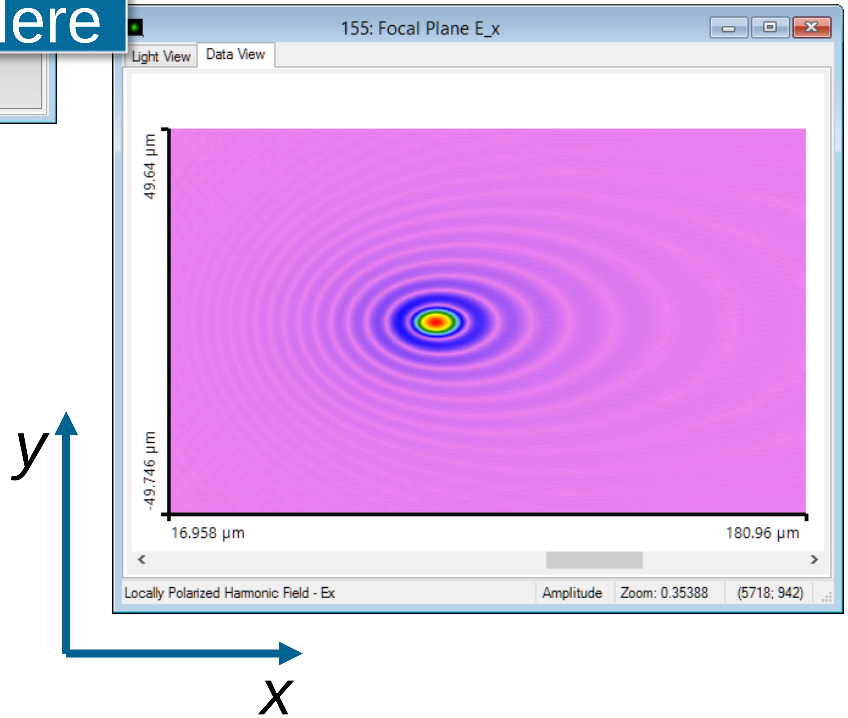
Plane wave
Diameter 4 mm
Circular polarization

Diffractive Lens: Unified Field Tracing

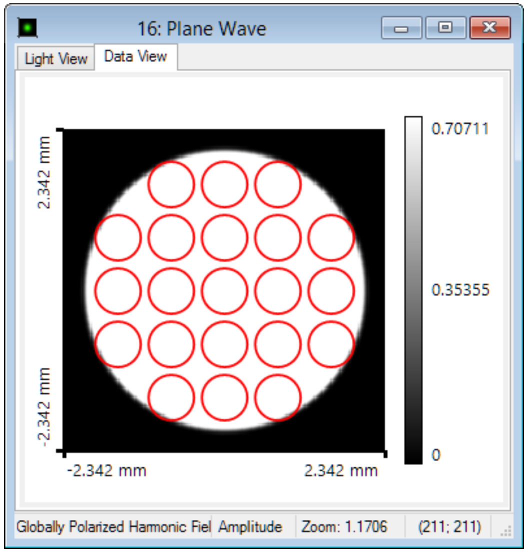


cpu time ≈ 3 sec

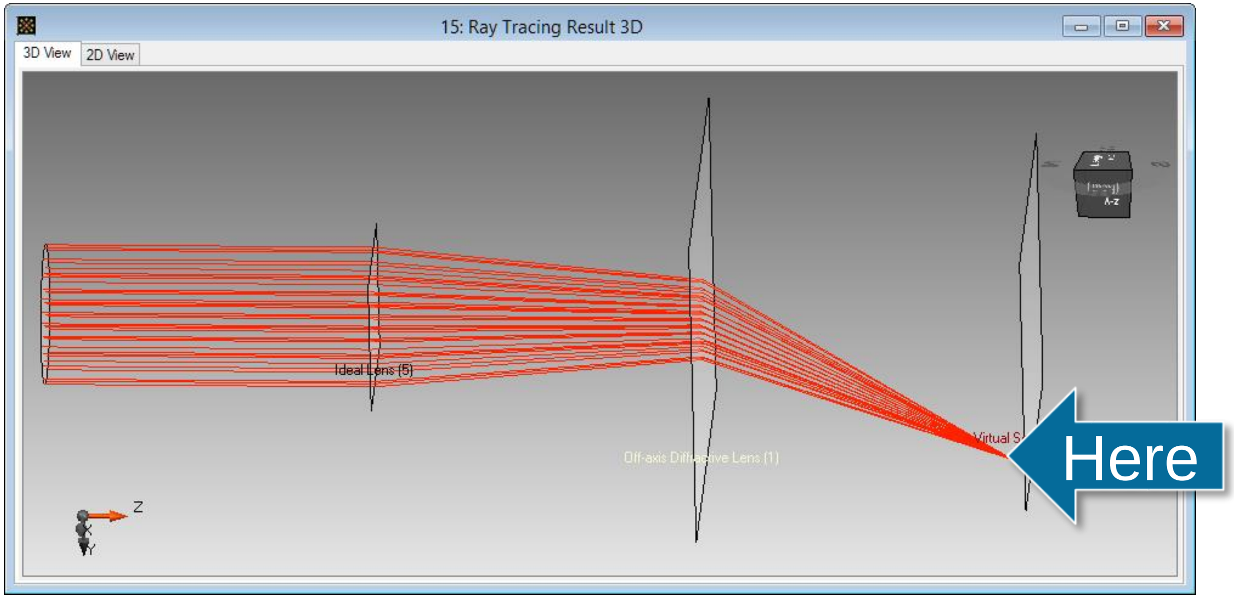
Amplitude $E_x(x,y)$



Diffractive Lens: Two Adjacent Spots

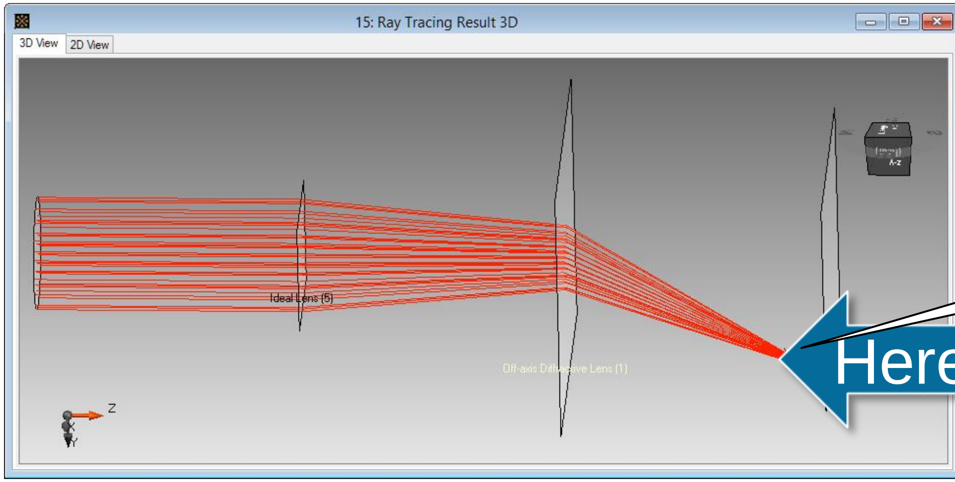


Plane wave
Diameter 4 mm
Circular polarization



Two spots at
X = 9 mm
X = 8.99 mm

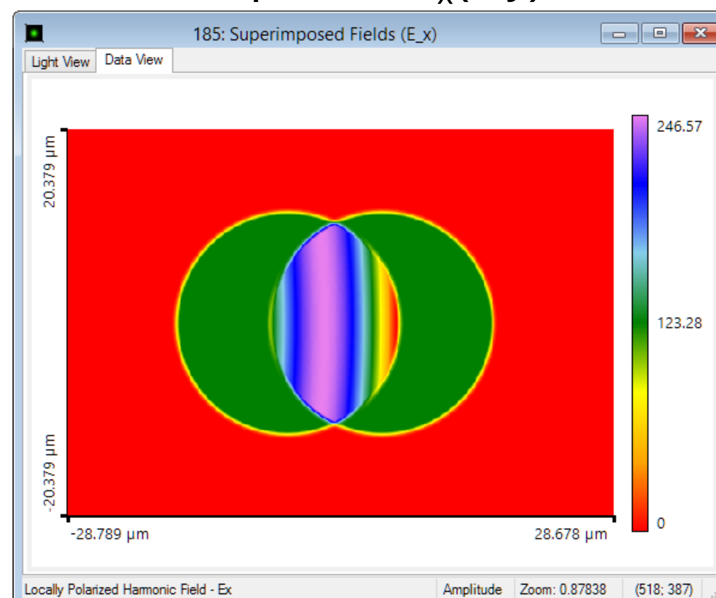
Diffractive Lens: Geometric Field Tracing



Here

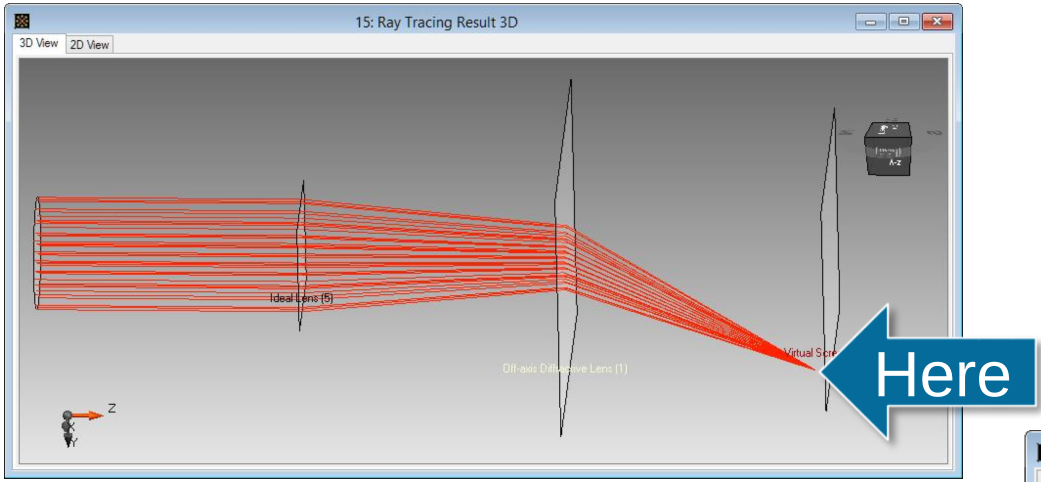
Plane 100 µm in front of focus

Amplitude $E_x(x,y)$

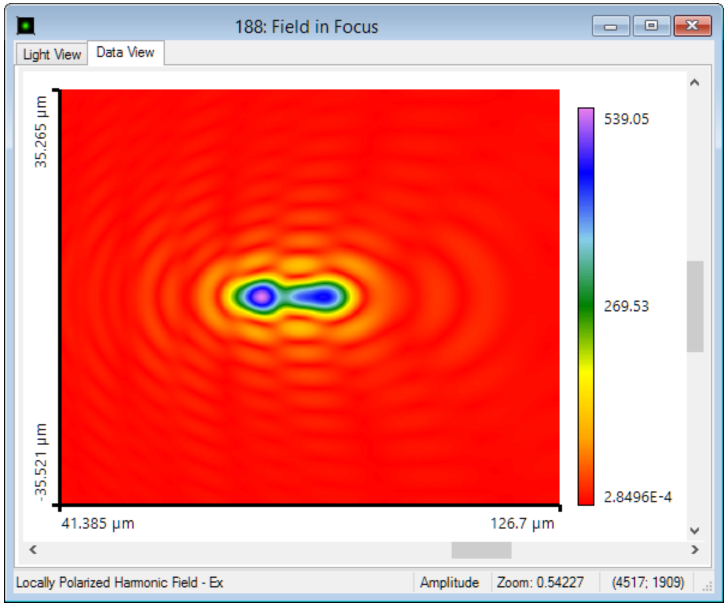


cpu time ≈ 1 sec

Diffractive Lens: Unified Field Tracing



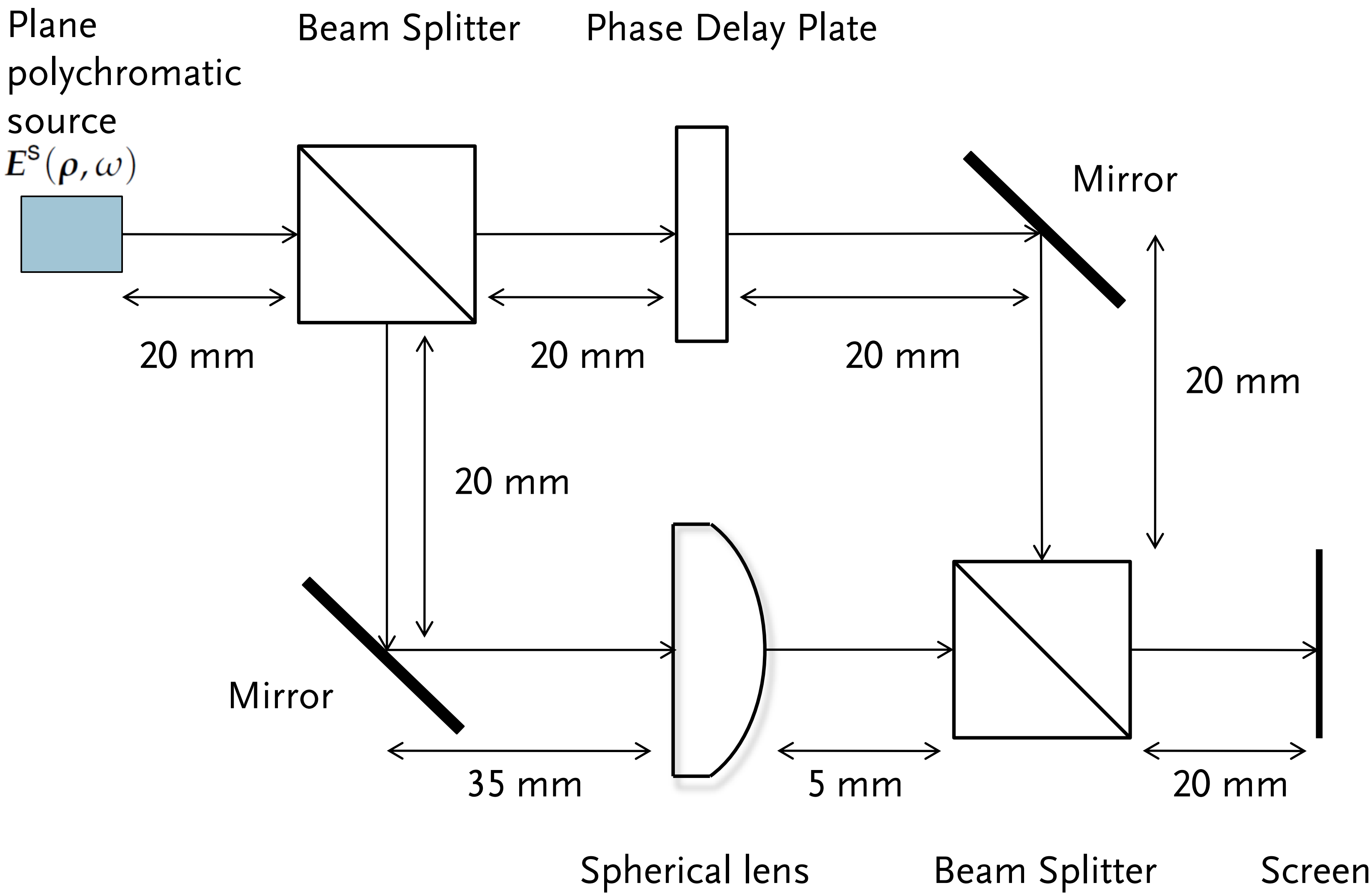
Amplitude $E_x(x,y)$



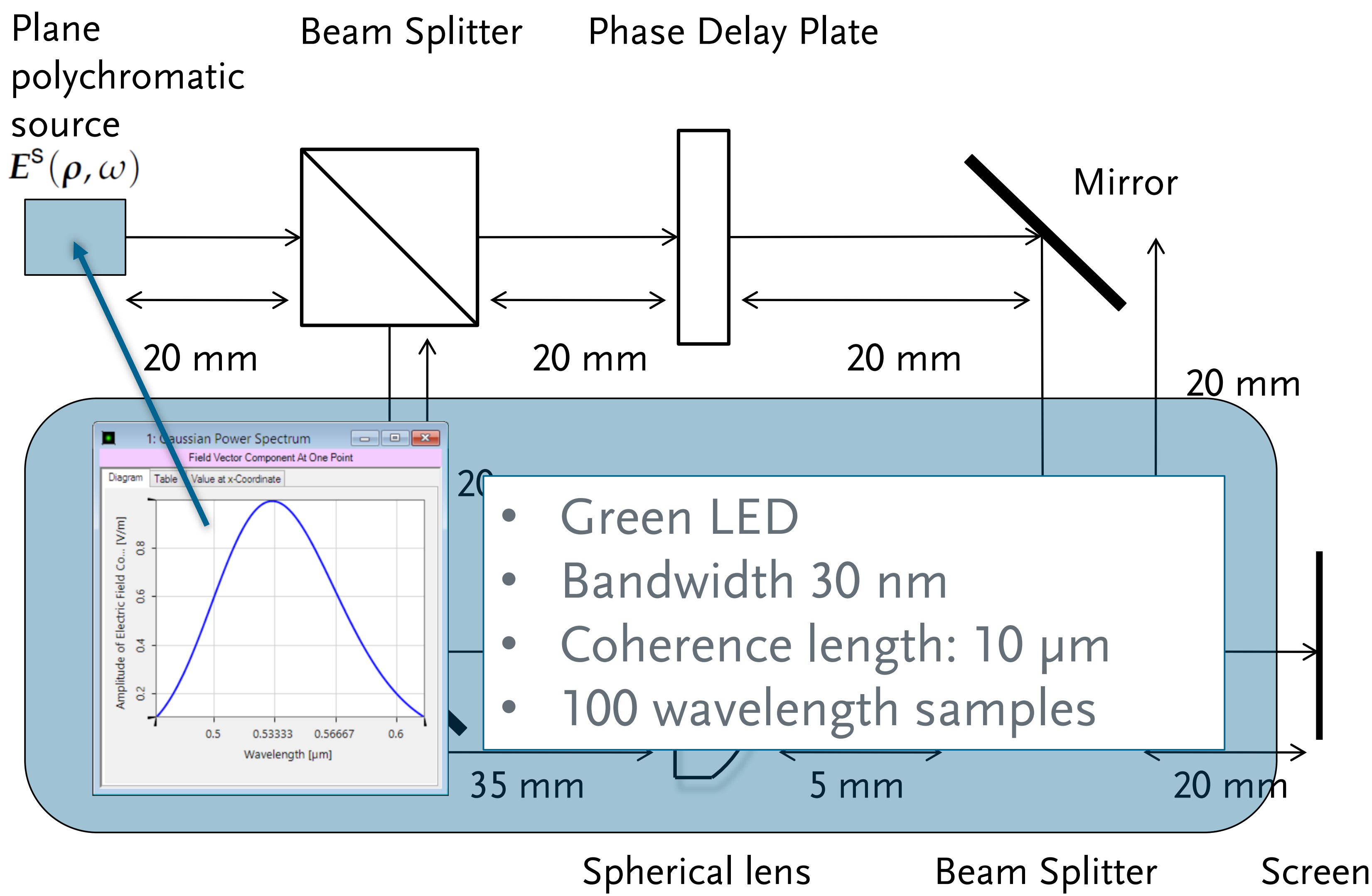
cpu time ≈ 3 sec

Example: Interferometer

Temporal Coherence Modeling

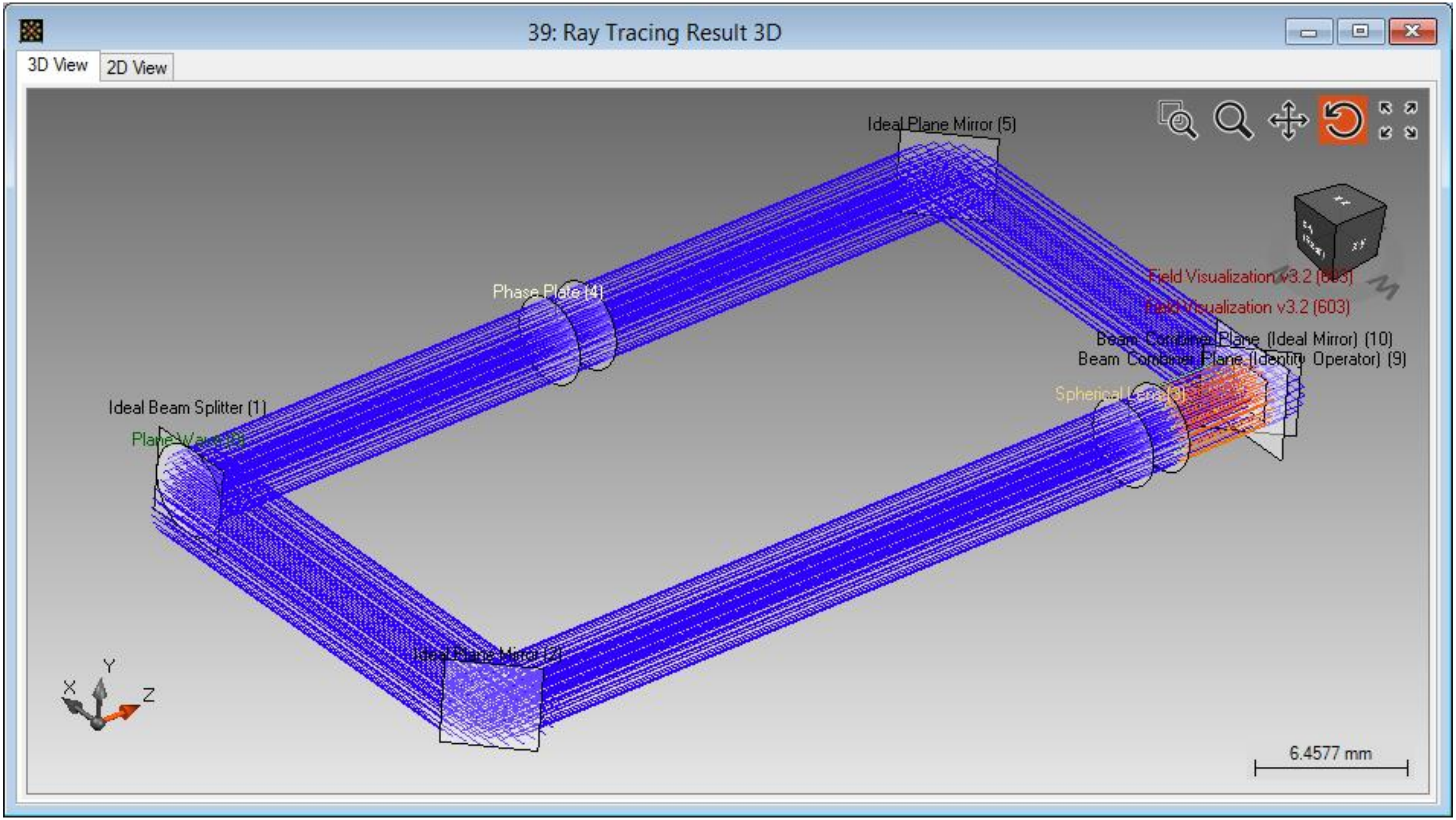


Temporal Coherence Modeling

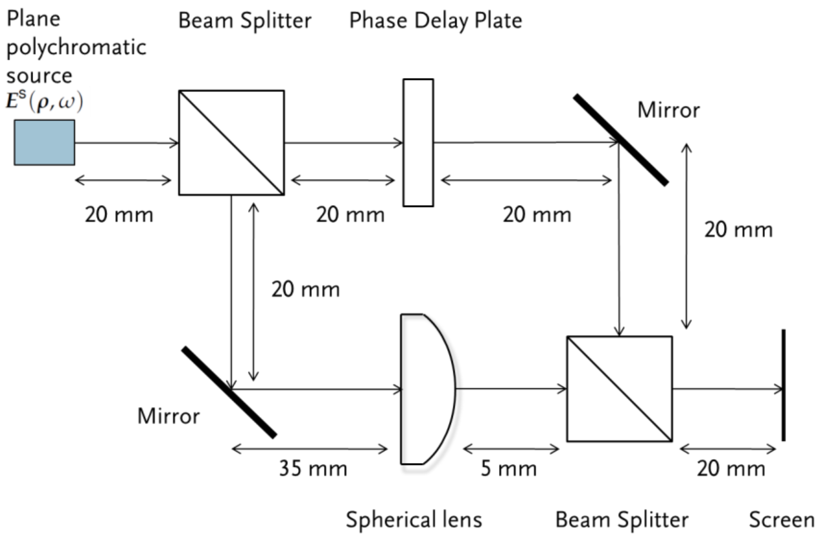


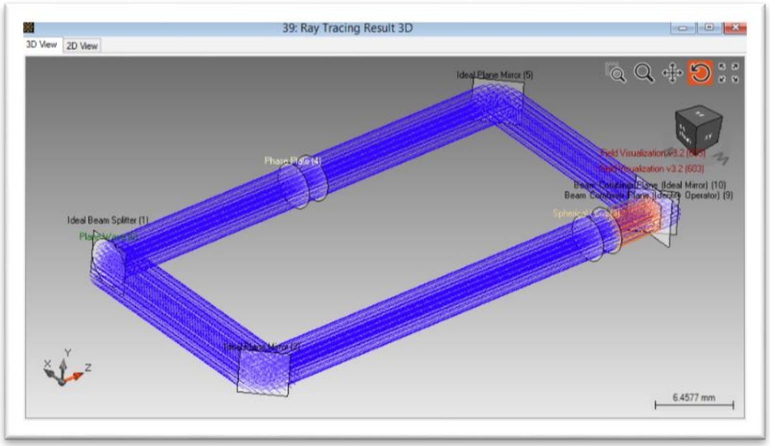
- Green LED
- Bandwidth 30 nm
- Coherence length: 10 µm
- 100 wavelength samples

3D Ray Tracing

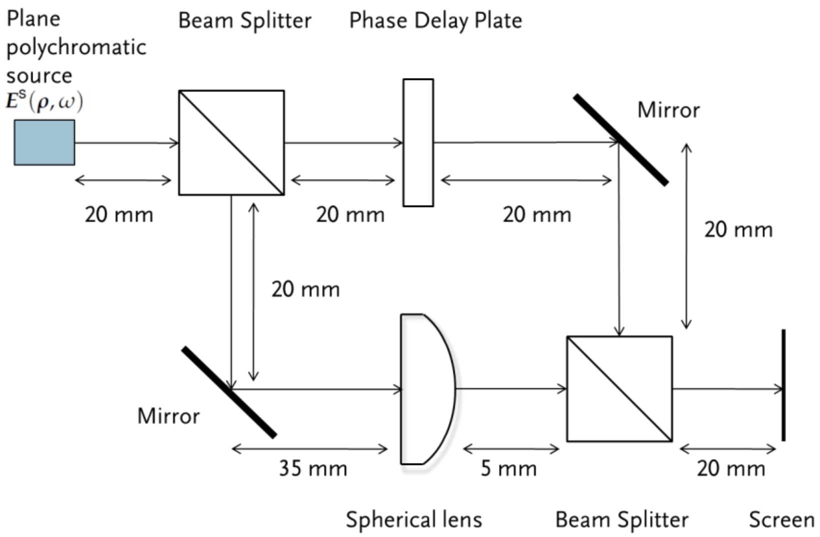


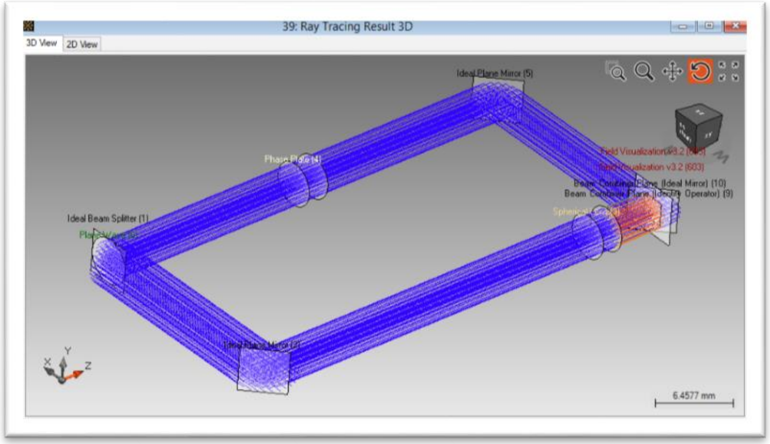
Modeling by Geometric Field Tracing

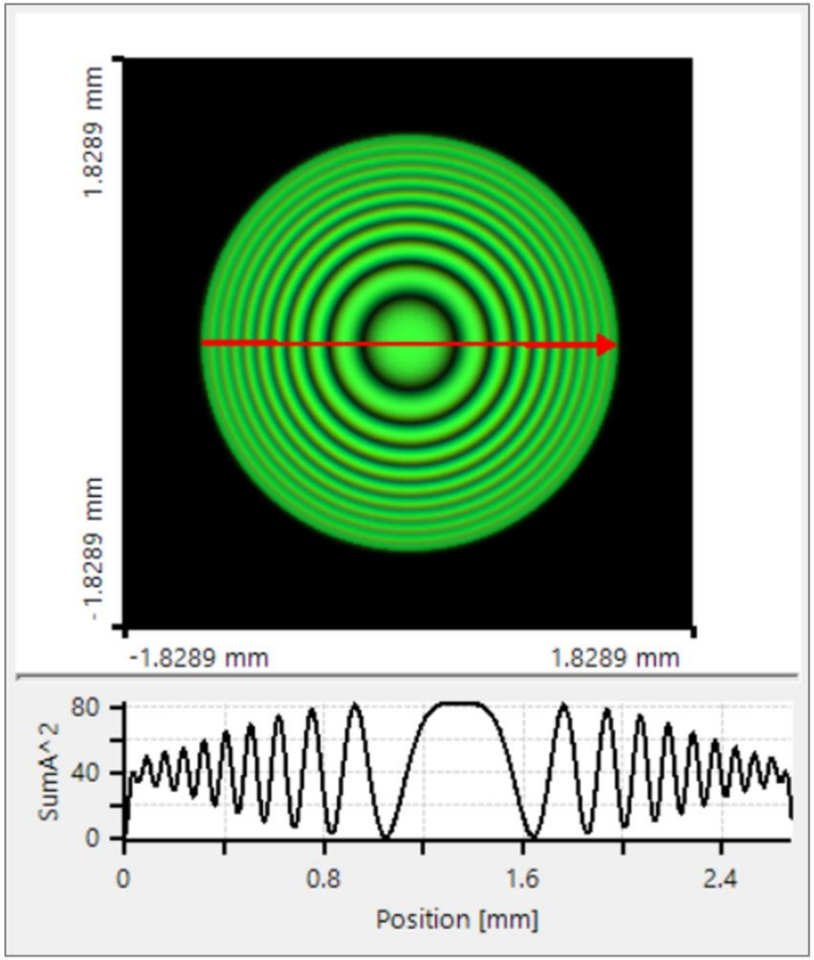




Modeling by Geometric Field Tracing



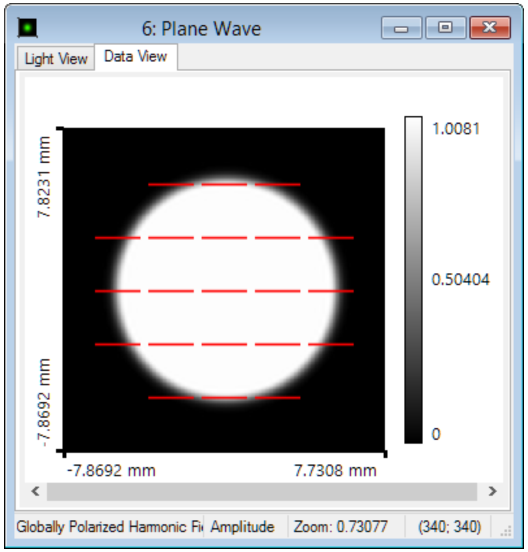


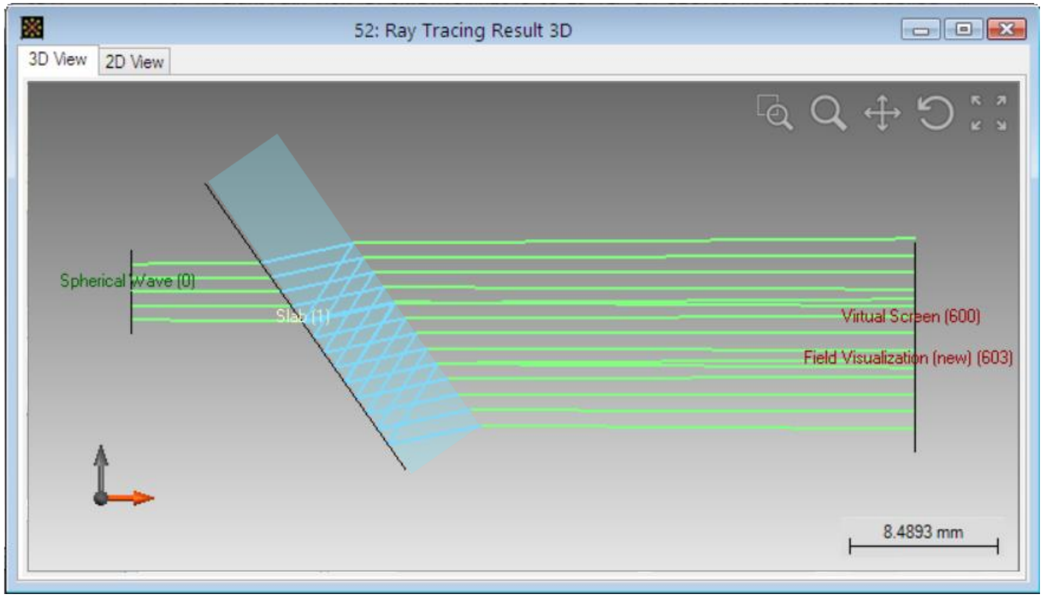


Result for geometric field tracing
(simulation time = 1 min)

Example: Light Guide Plates

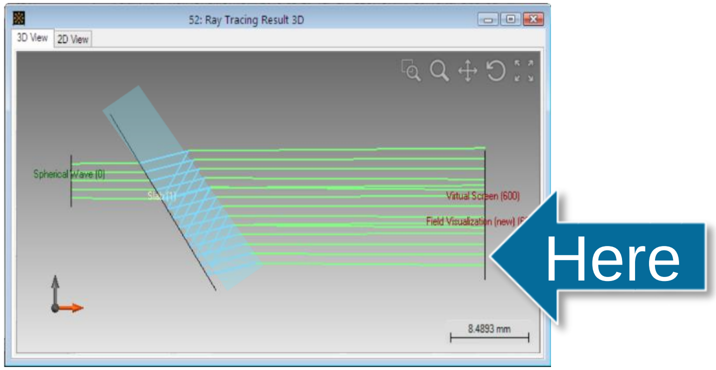
Waveguide Plate



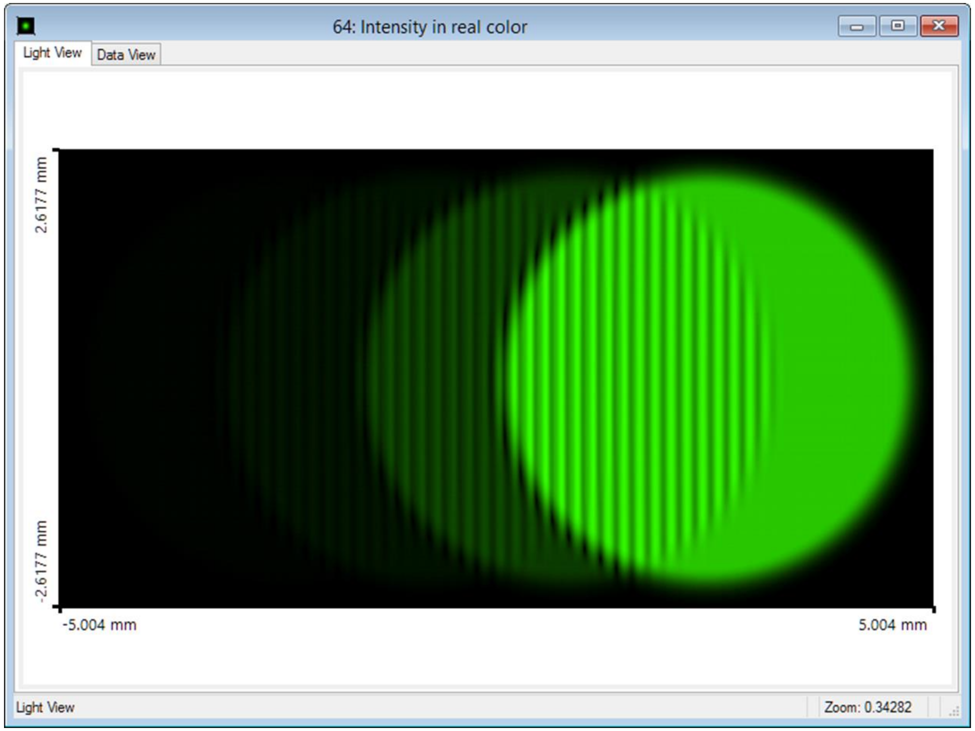


Spherical wave
Diameter 4 mm
R= 500 mm
Linear polarization

Waveguide Plate: Geometric Field Tracing

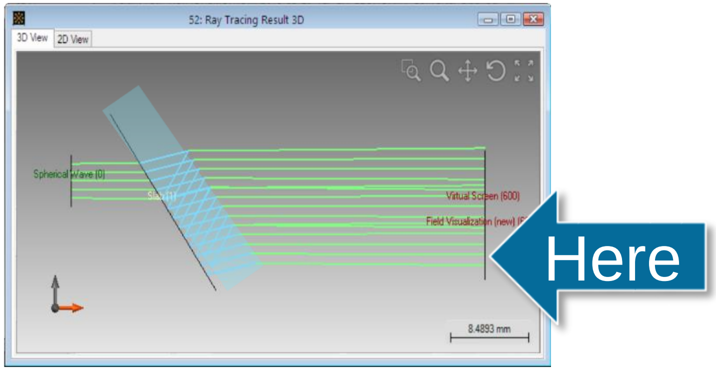


Intensity

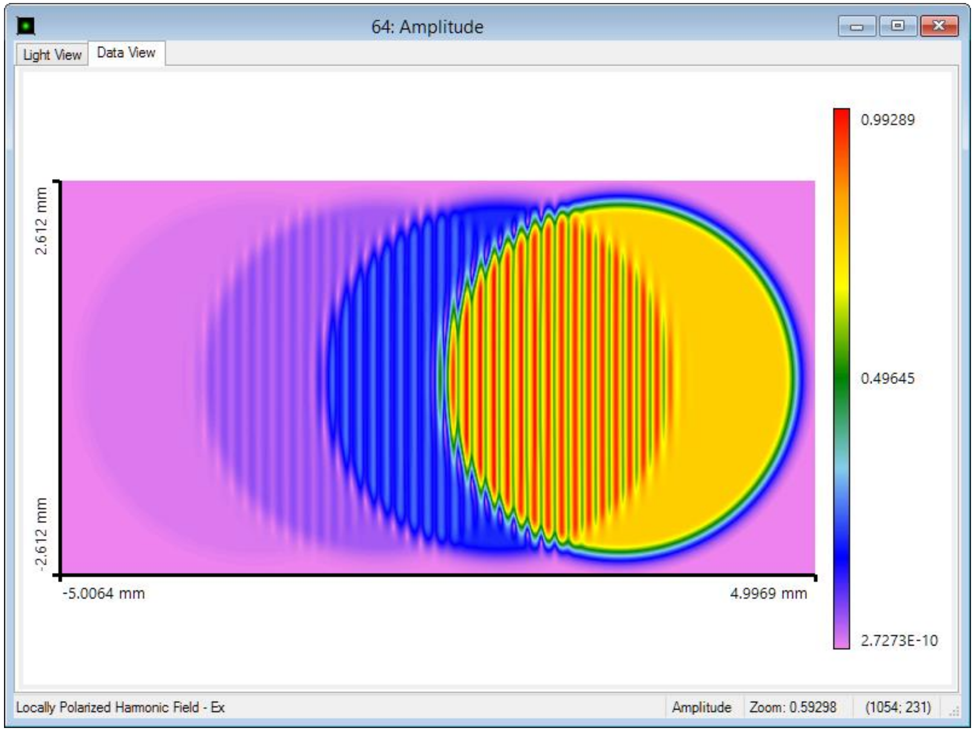


cpu time ≈ 3 sec

Waveguide Plate: Geometric Field Tracing

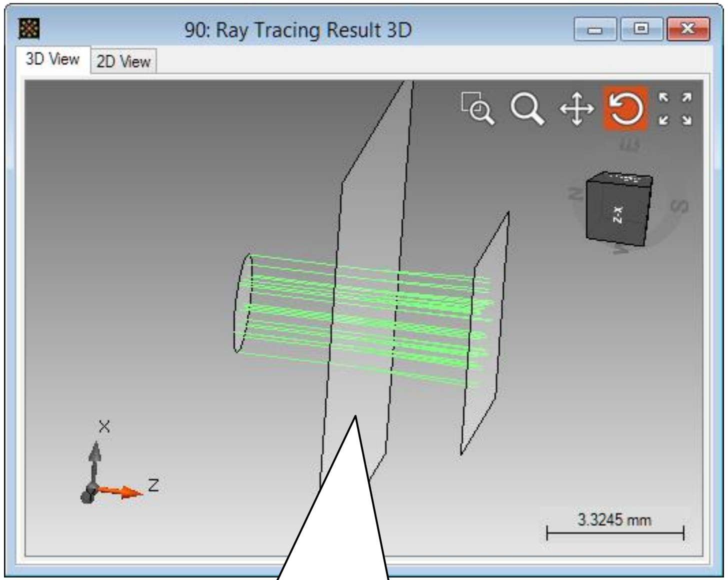


Amplitude $E_x(x,y)$



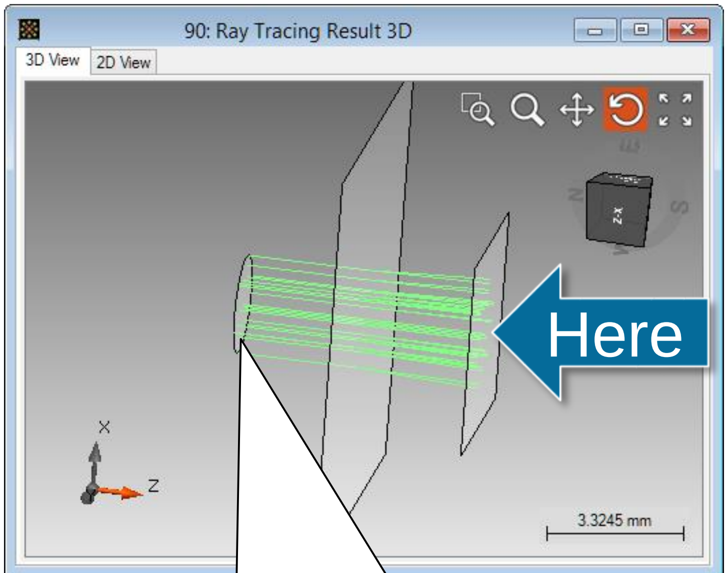
cpu time ≈ 3 sec

Thin Film: Geometric Field Tracing



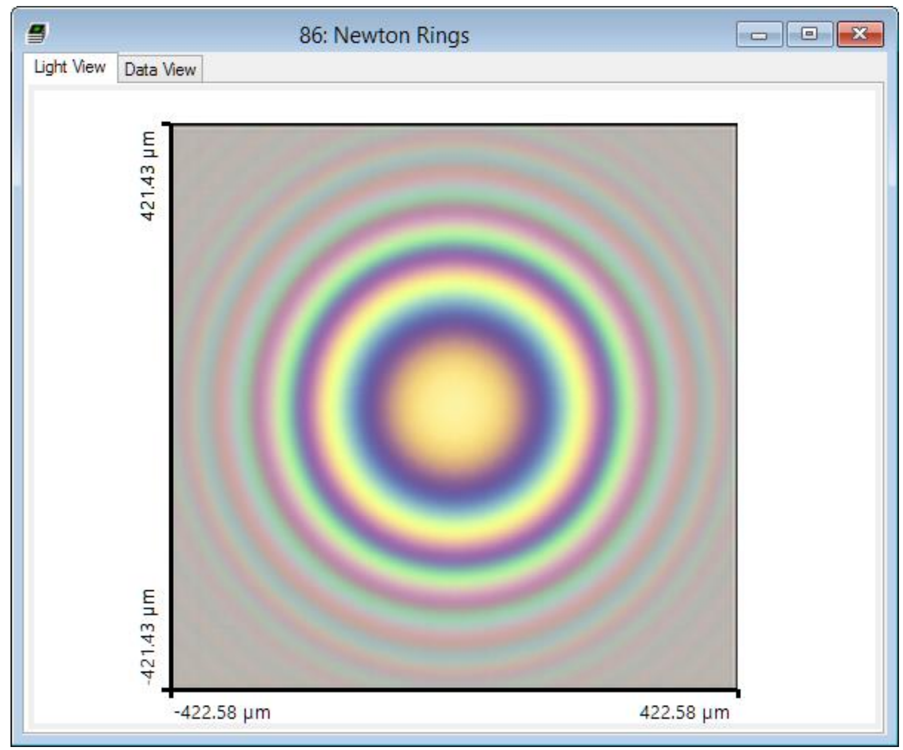
Thin film (index 3)
100 nm thick, curved

Thin Film: Geometric Field Tracing



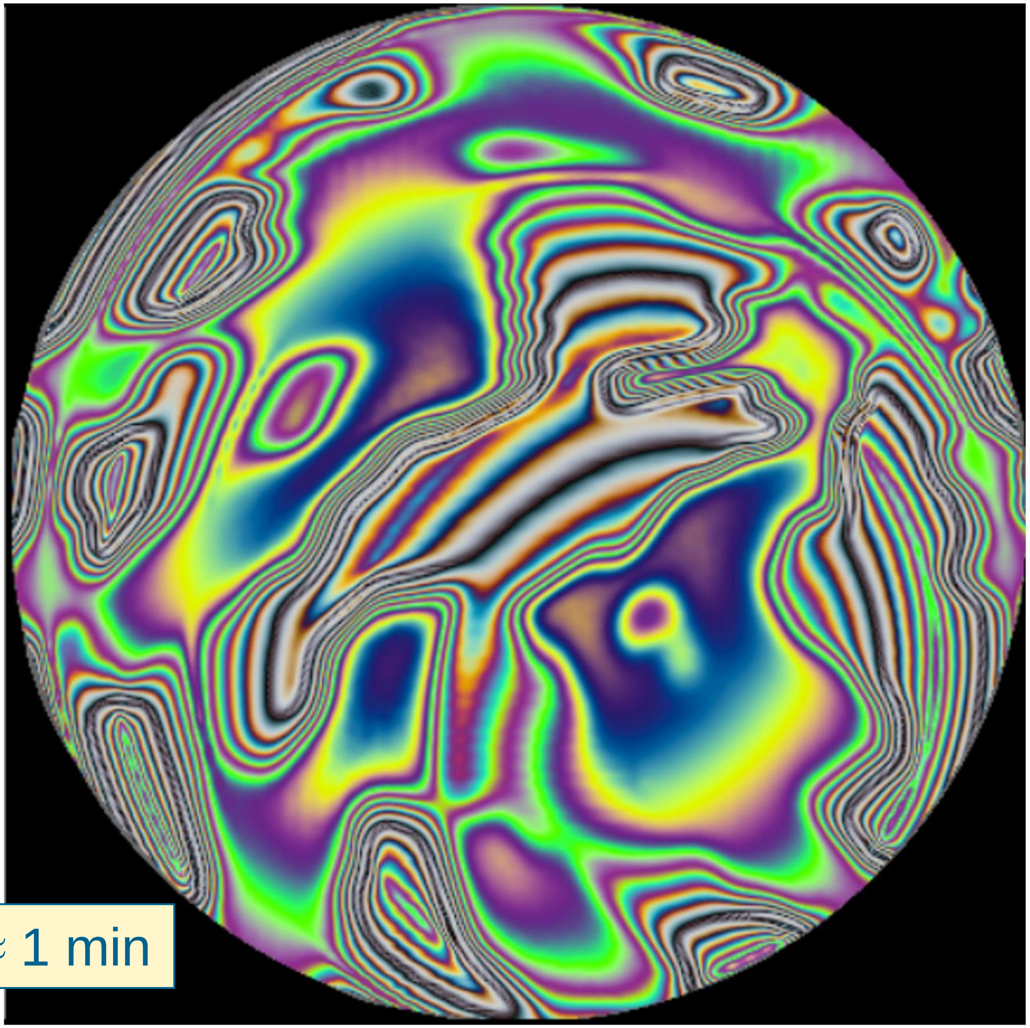
Visible spectrum (uniform)
24 wavelengths

Intensity



cpu time ≈ 18 sec

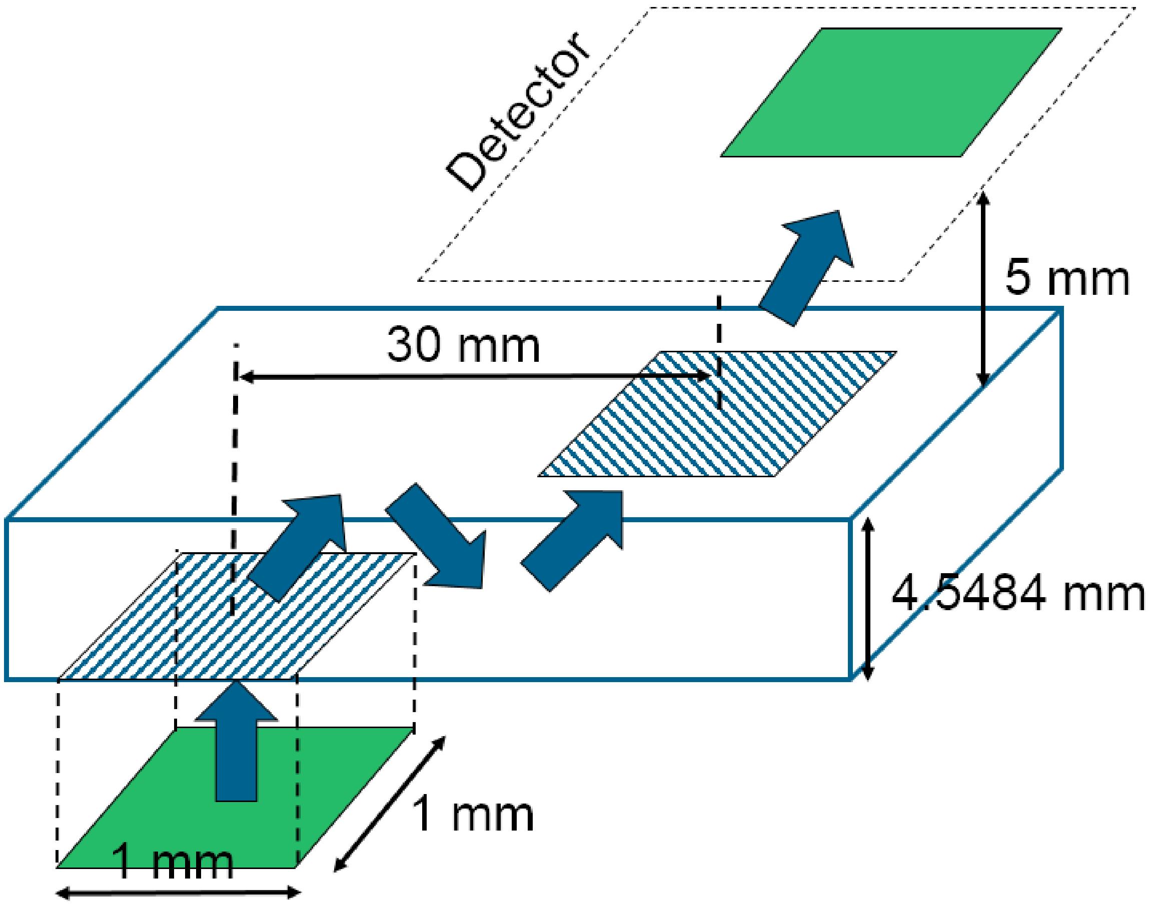
Thin Film: Geometric Field Tracing (RGB)



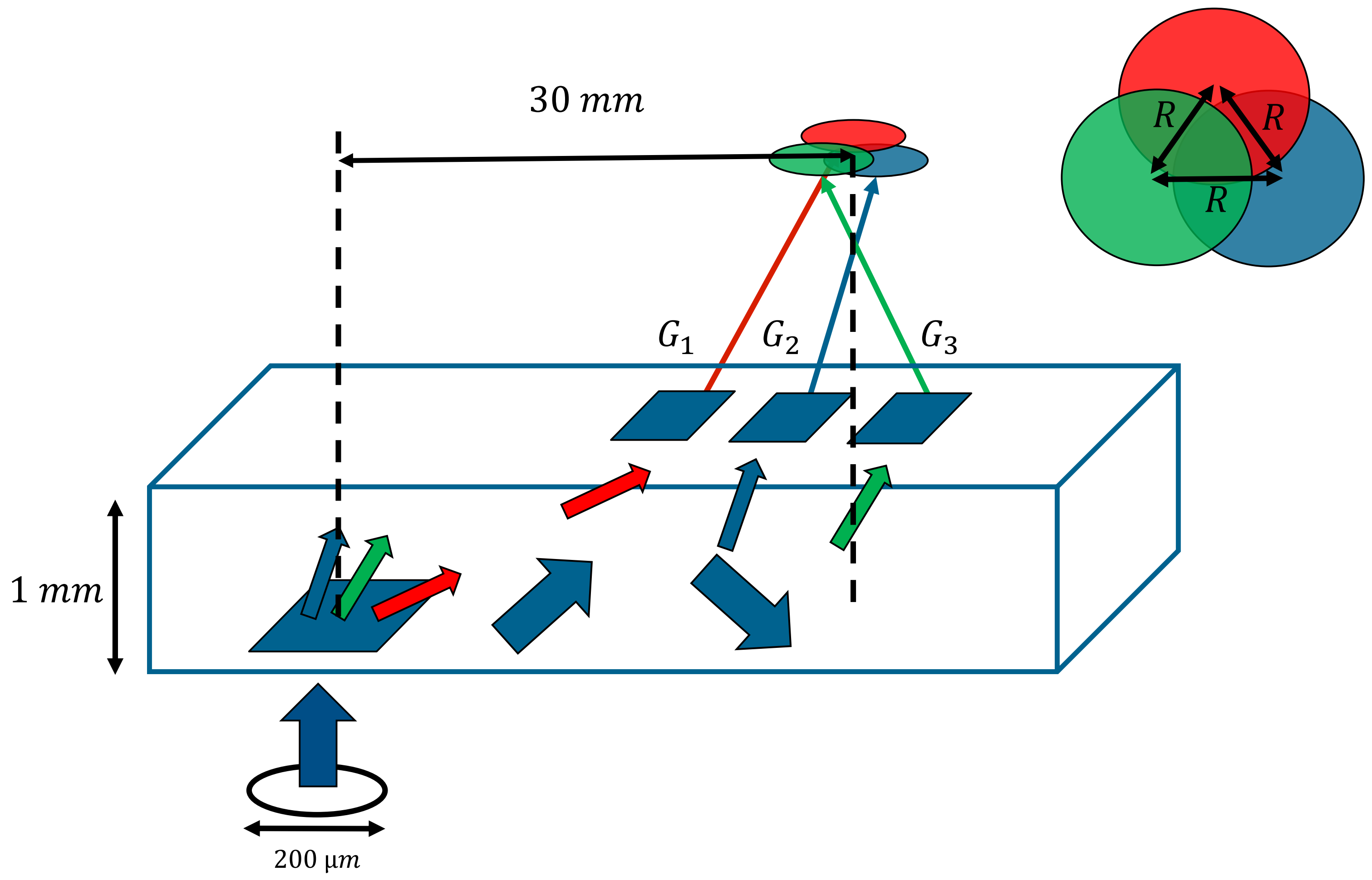
cpu time ≈ 1 min

Example: Light Guide Plates with Gratings

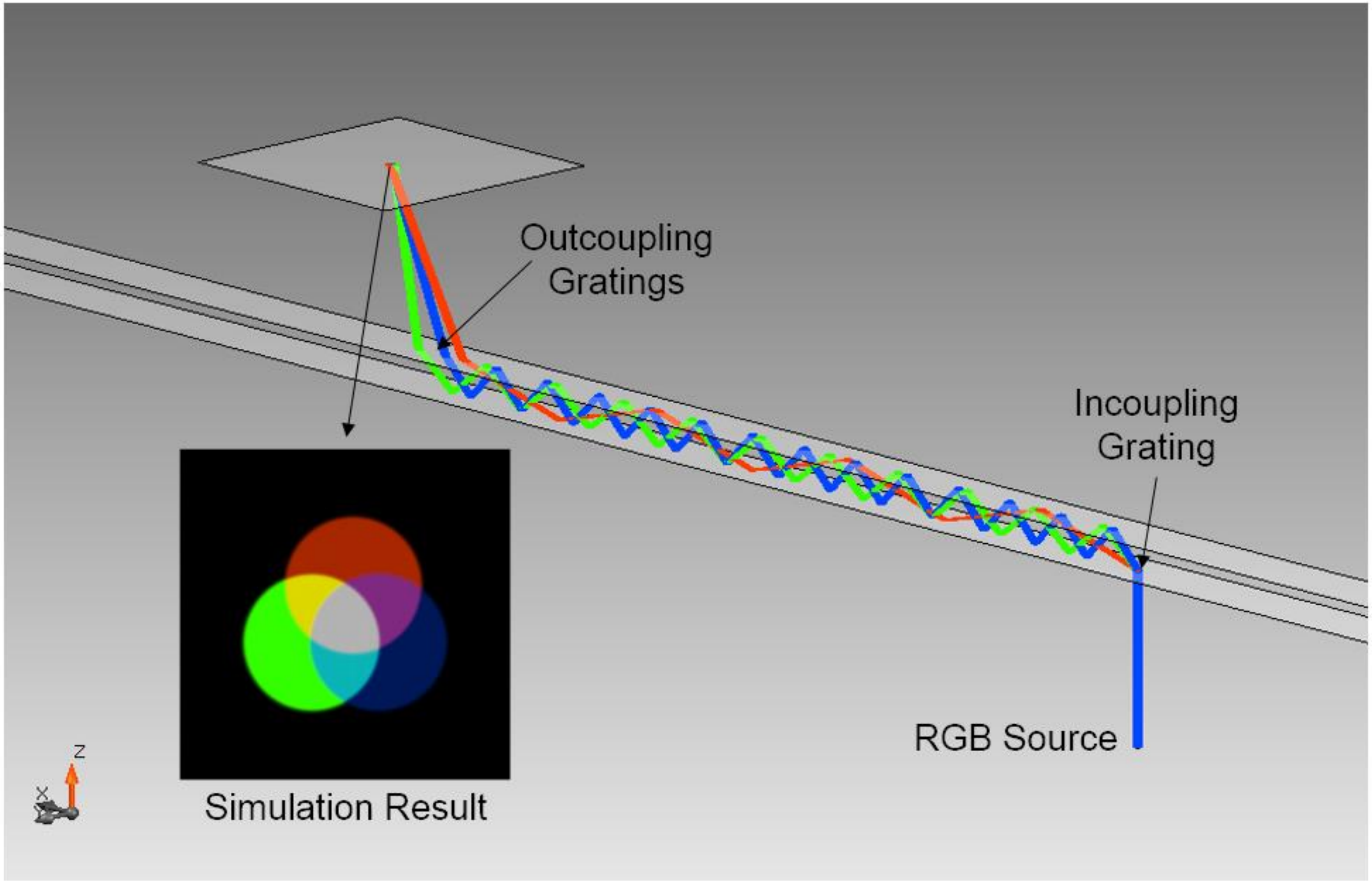
Waveguide Optics



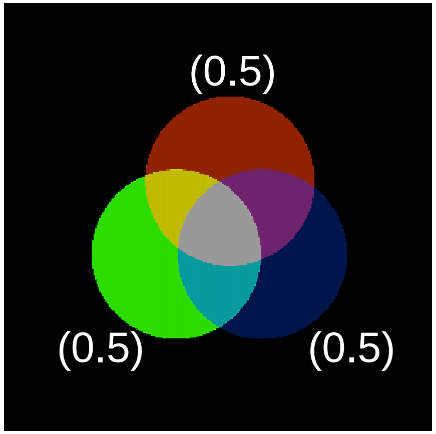
Waveguide Optics: Color Mixing

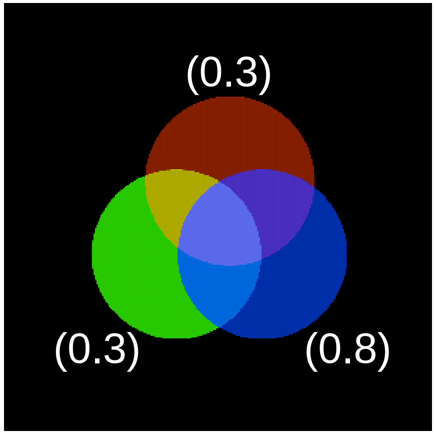


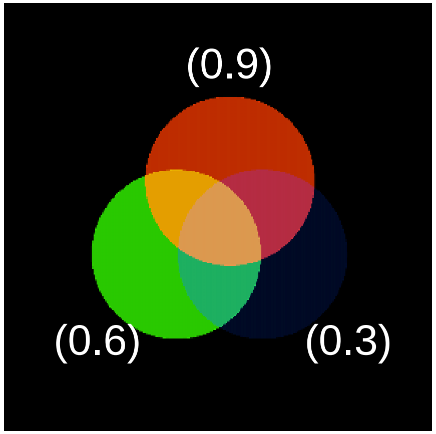
Result 3D View

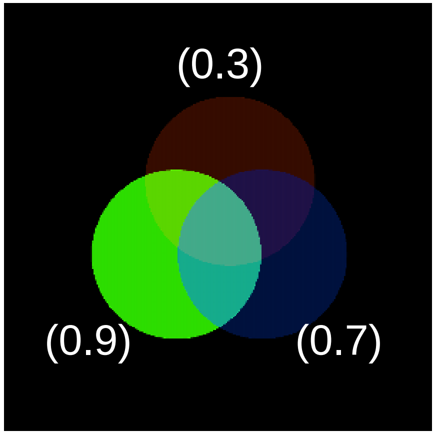


Varying the Outcoupling Efficiencies

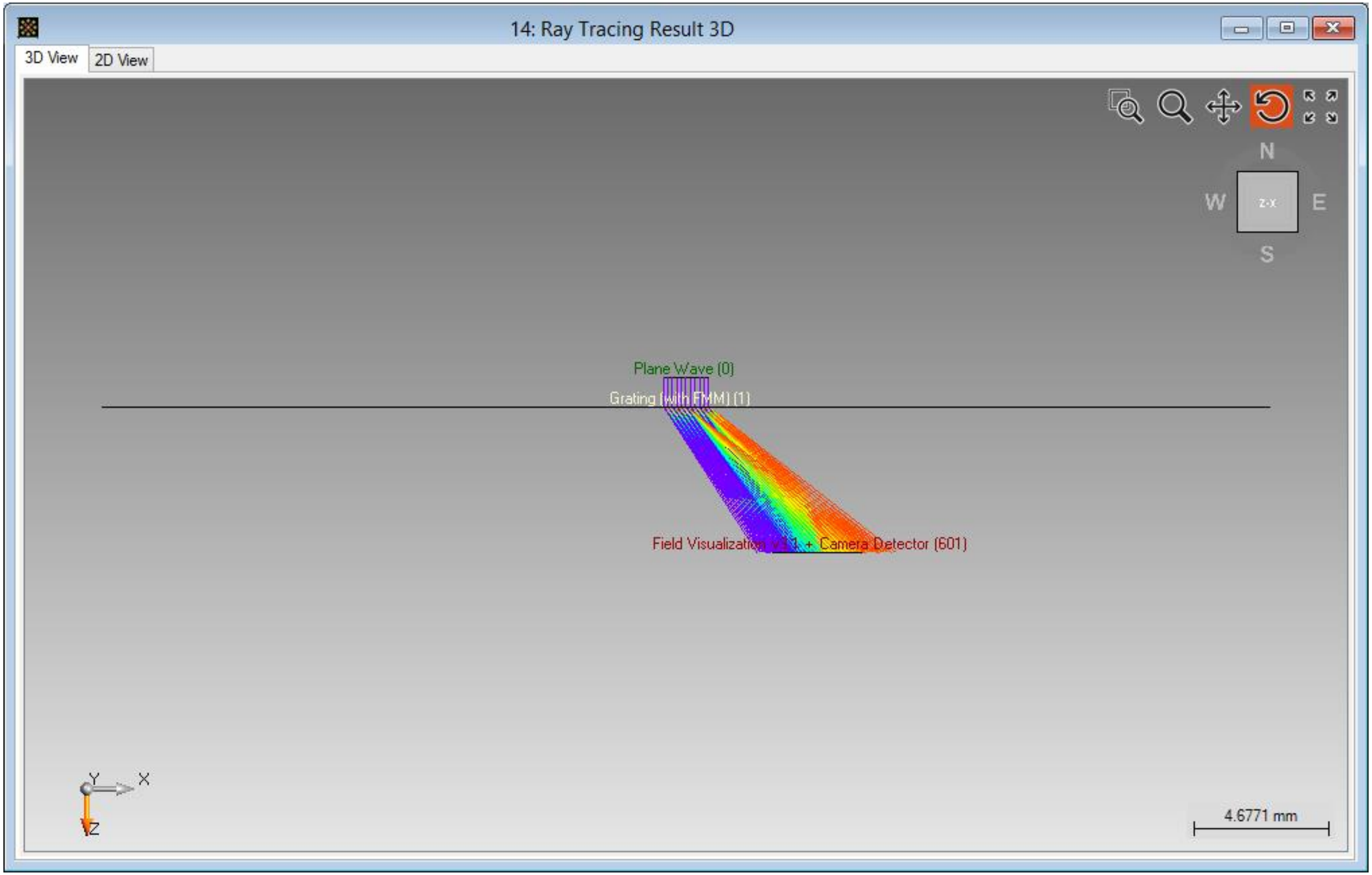




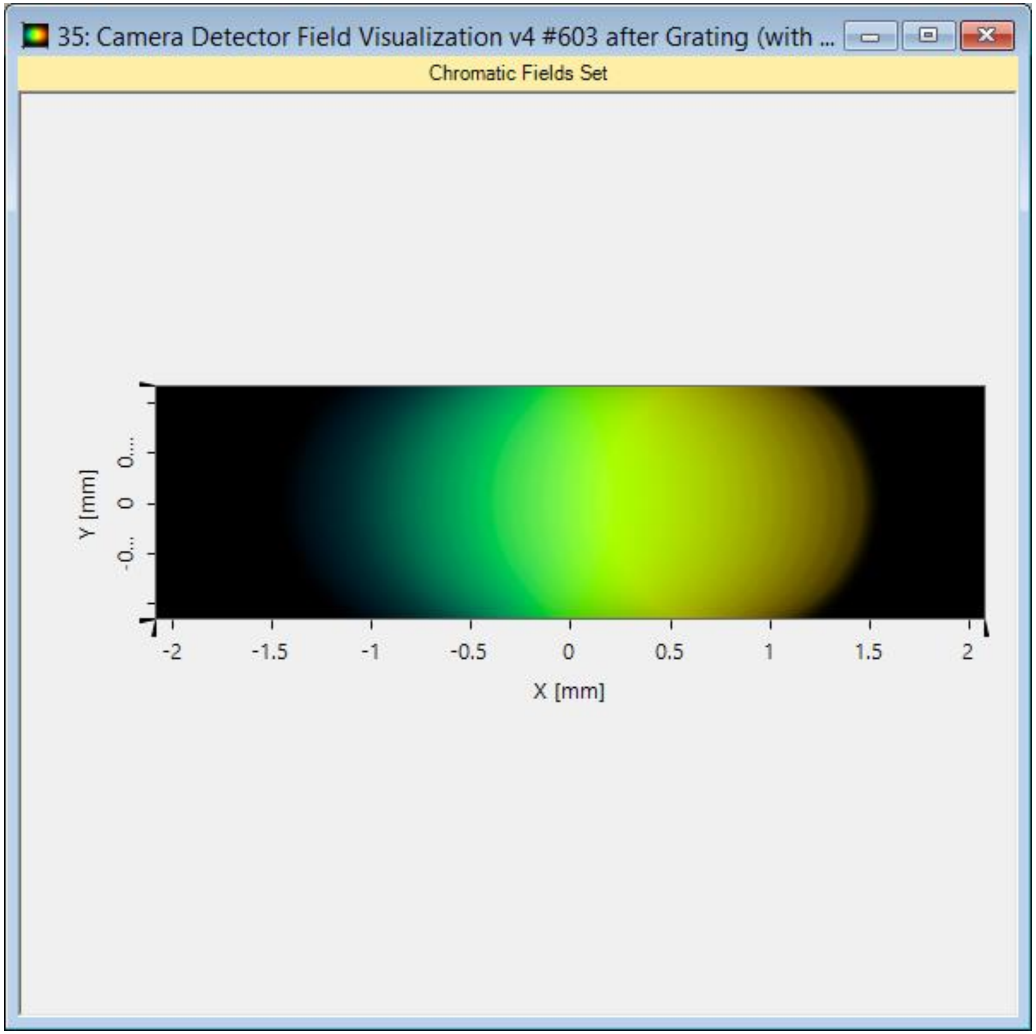




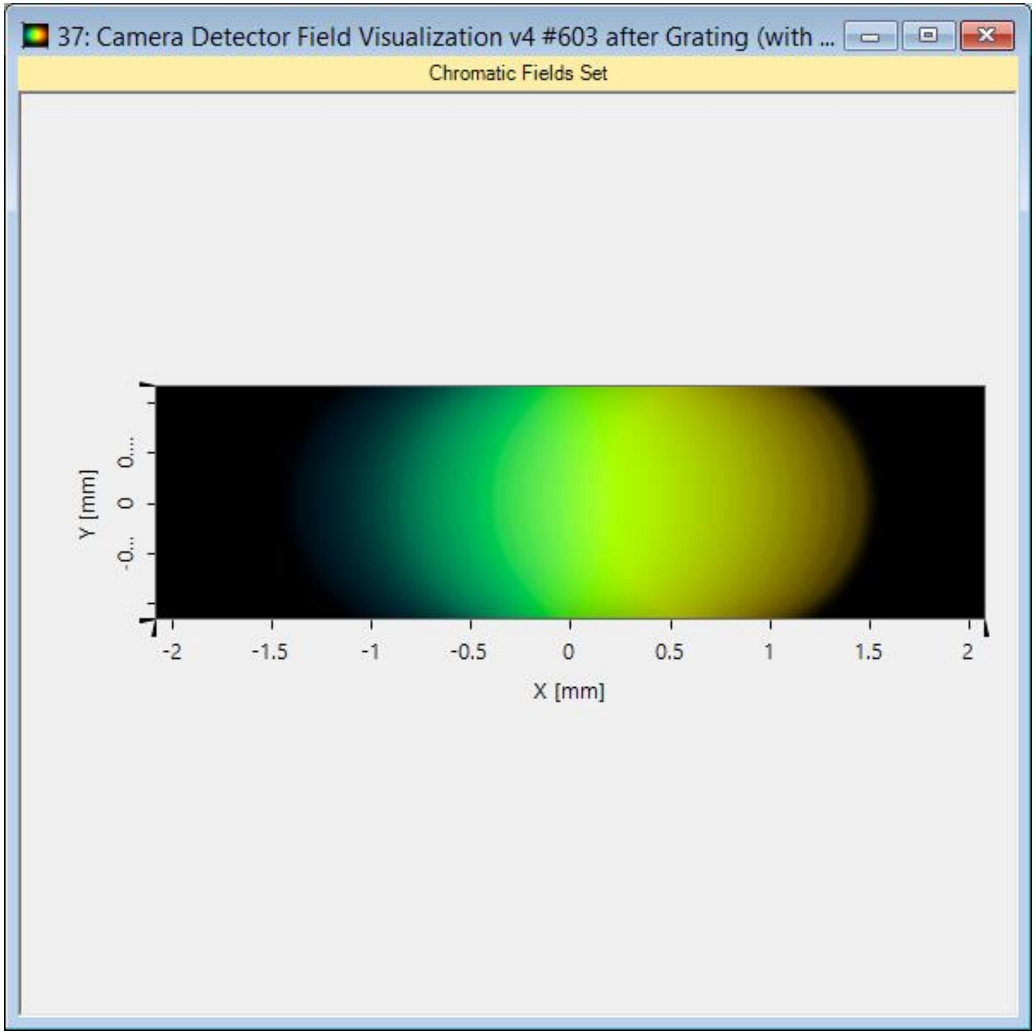
Setup of System (Including 3D Ray Tracing)



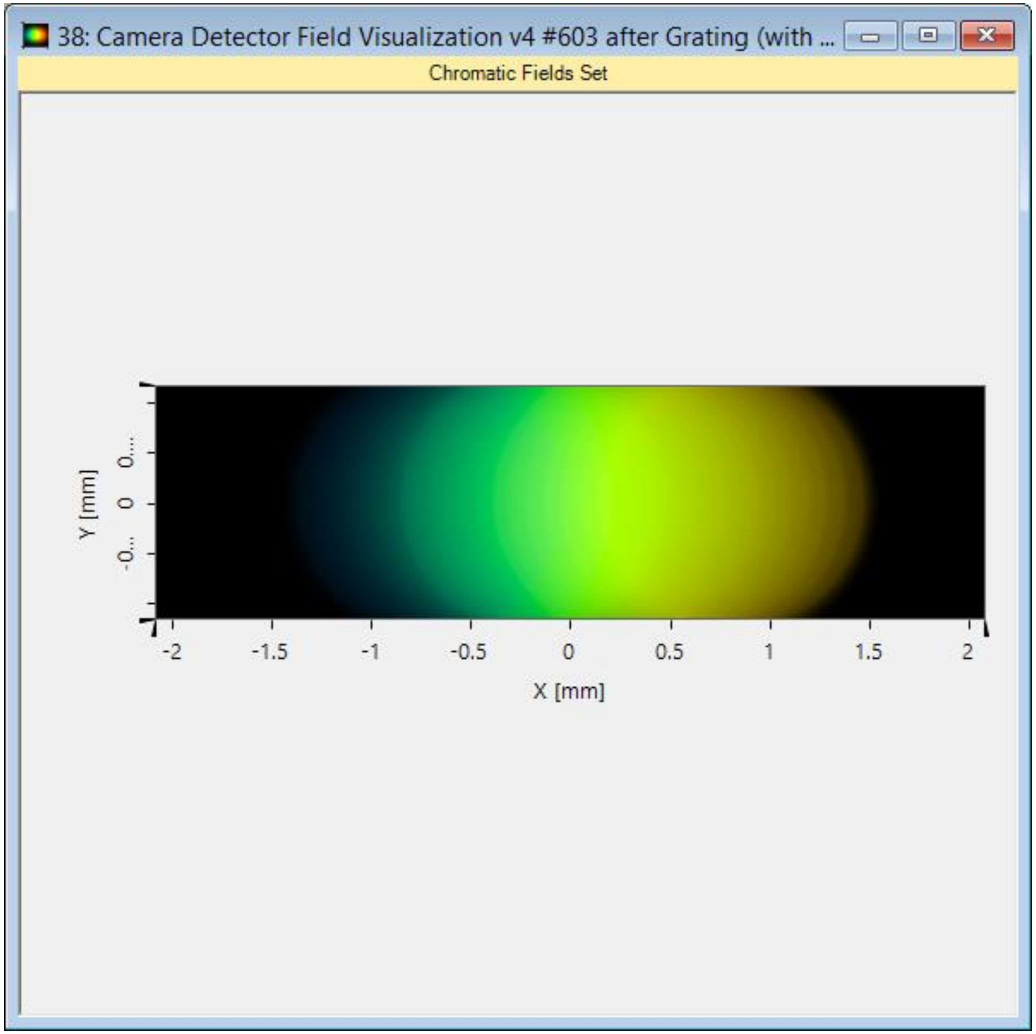
Color vs. Polarization (Linear 0°)



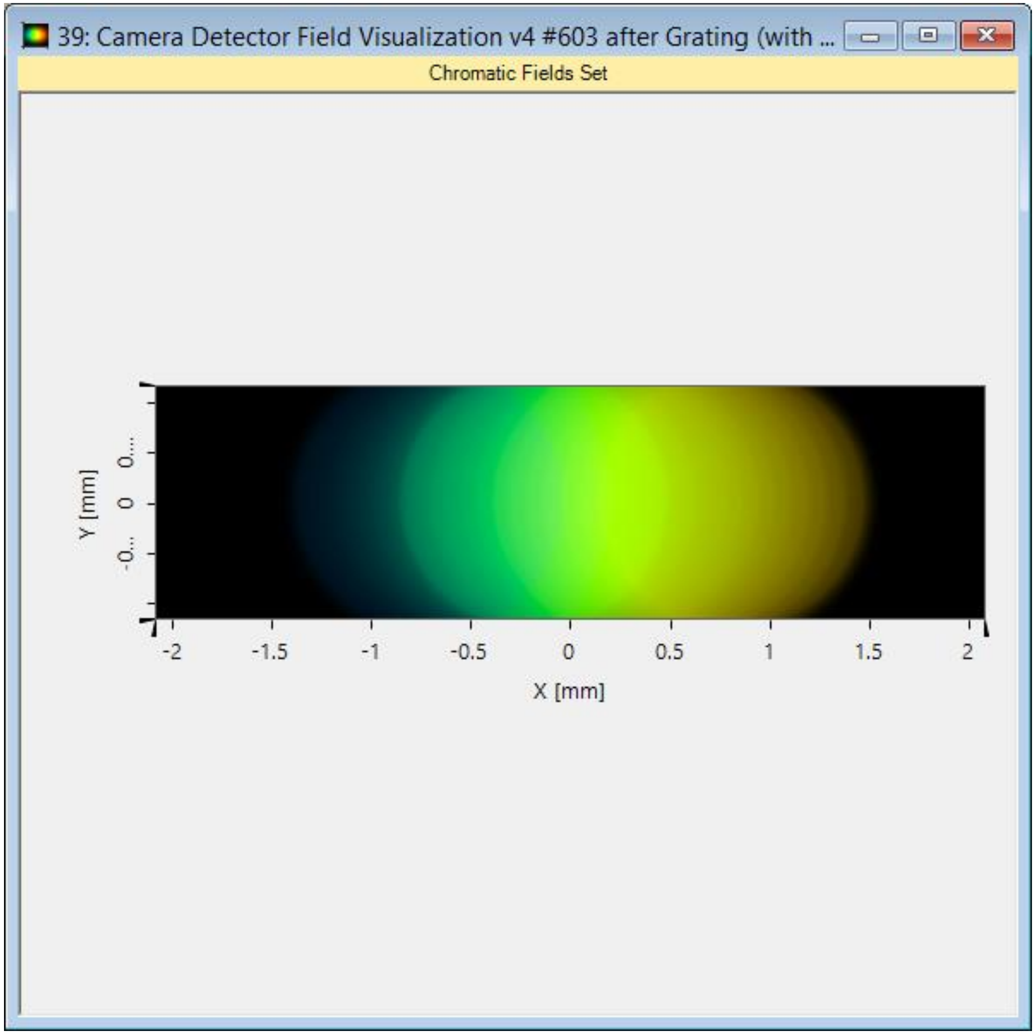
Color vs. Polarization (Linear 30°)



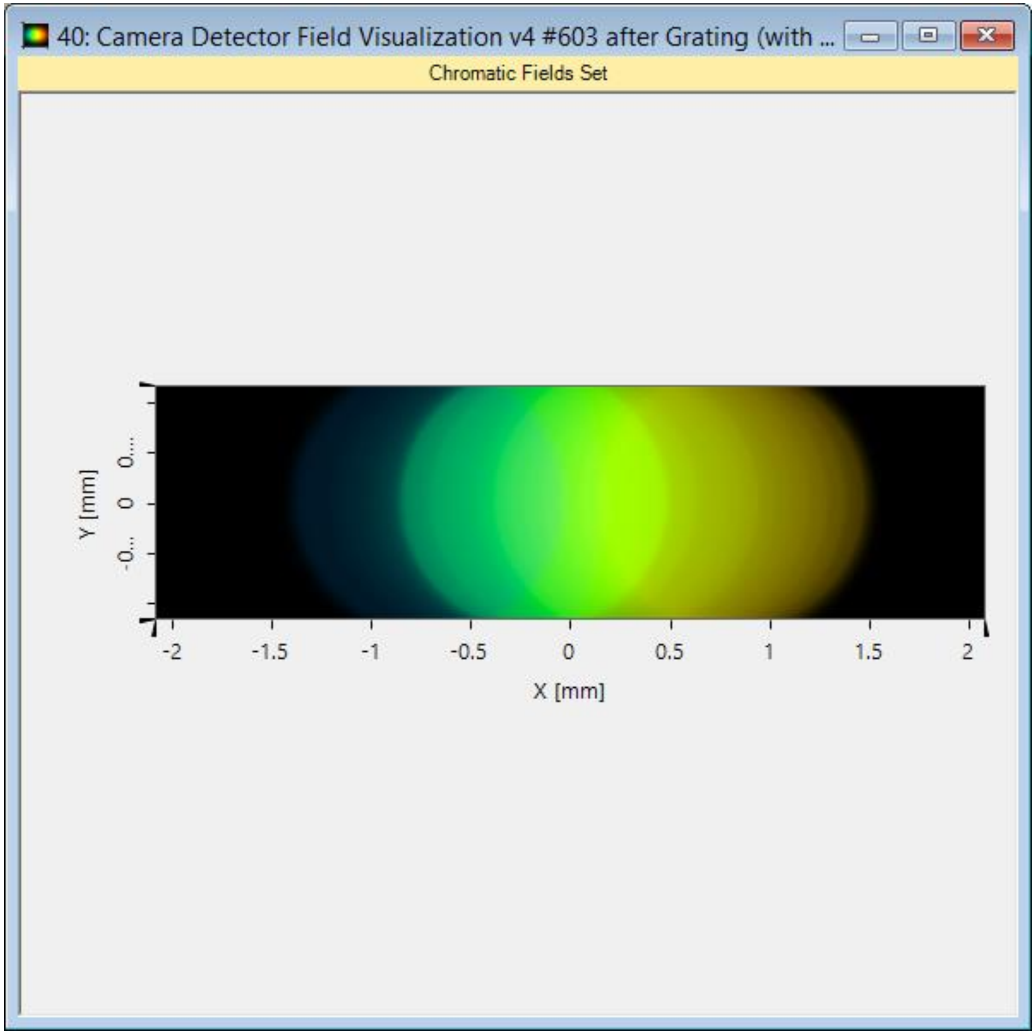
Color vs. Polarization (Linear 45°)



Color vs. Polarization (Linear 60°)

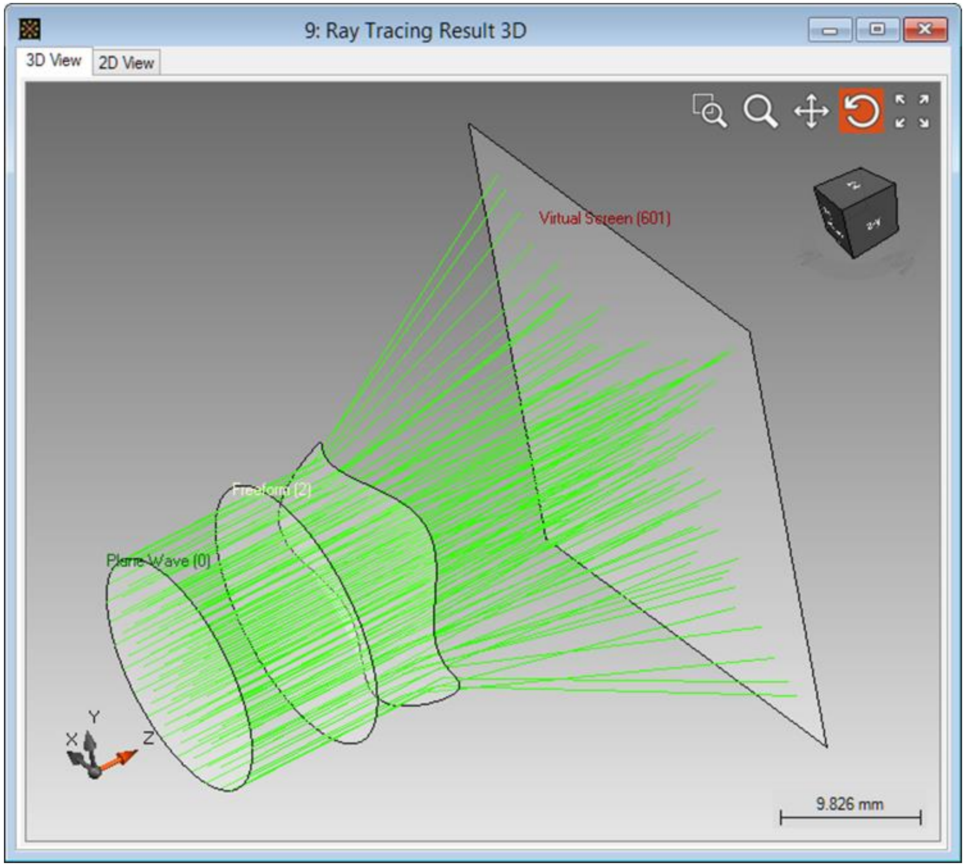


Color vs. Polarization (Linear 90°)

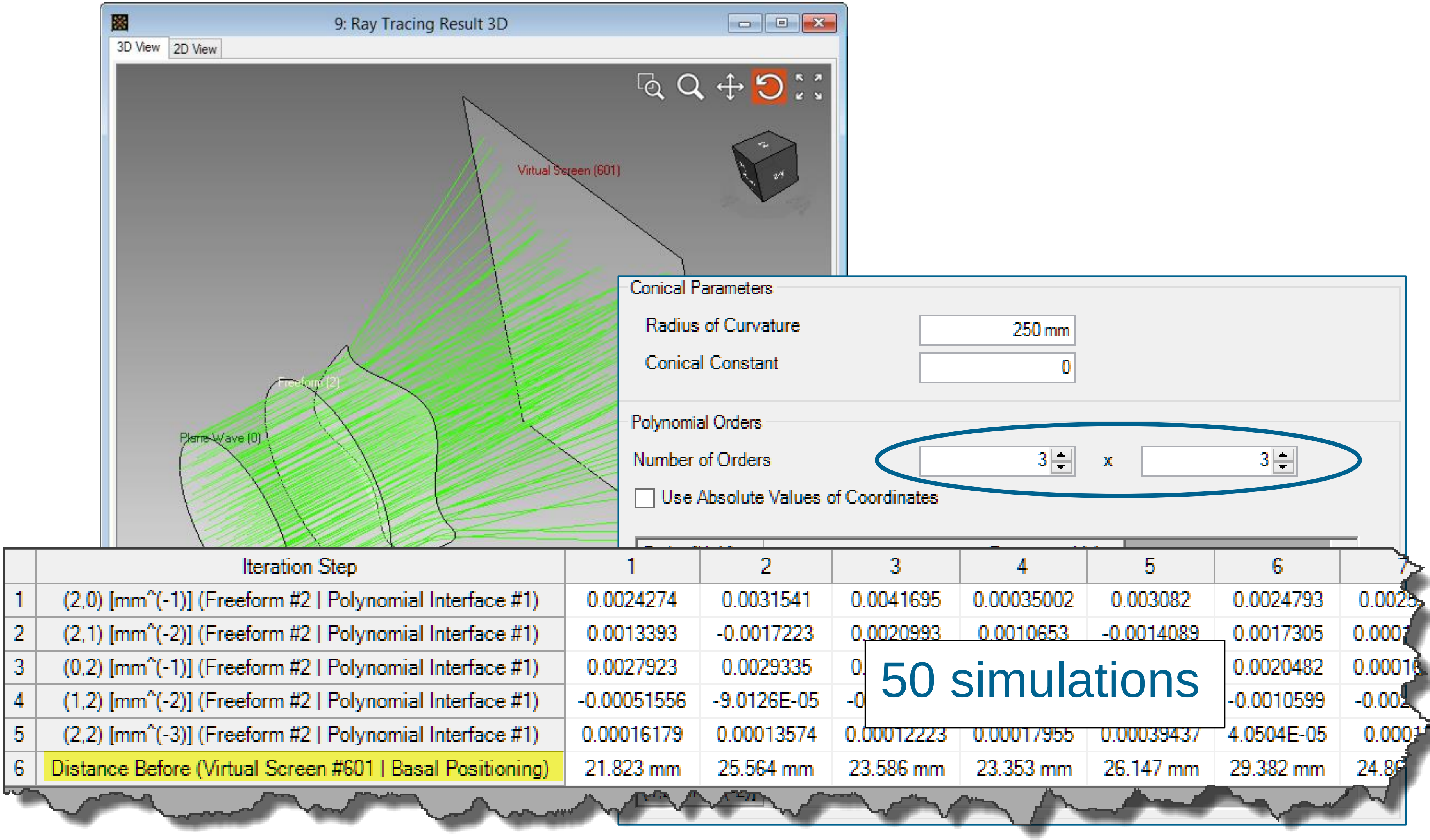


Example: Freeform Surface

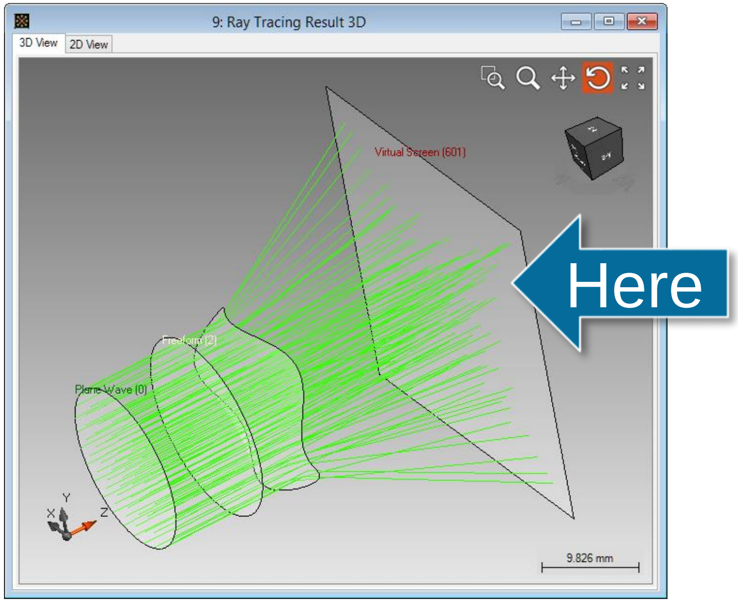
Polynomial Surface



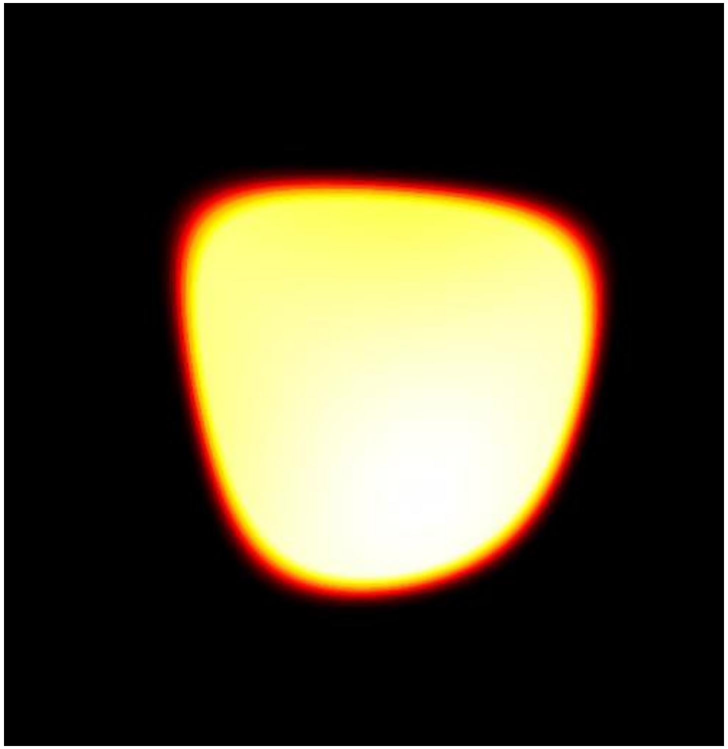
Polynomial Surface



Freeform: Geometric Field Tracing

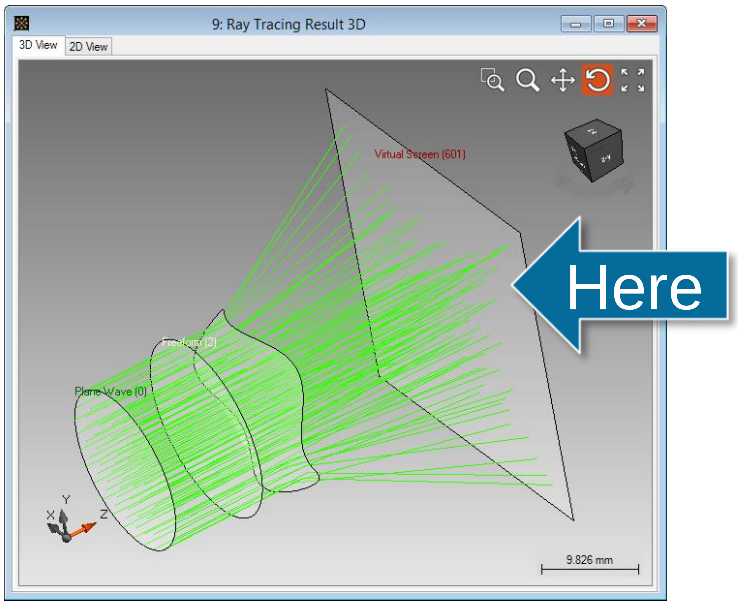


Amplitude $E_x(x,y)$

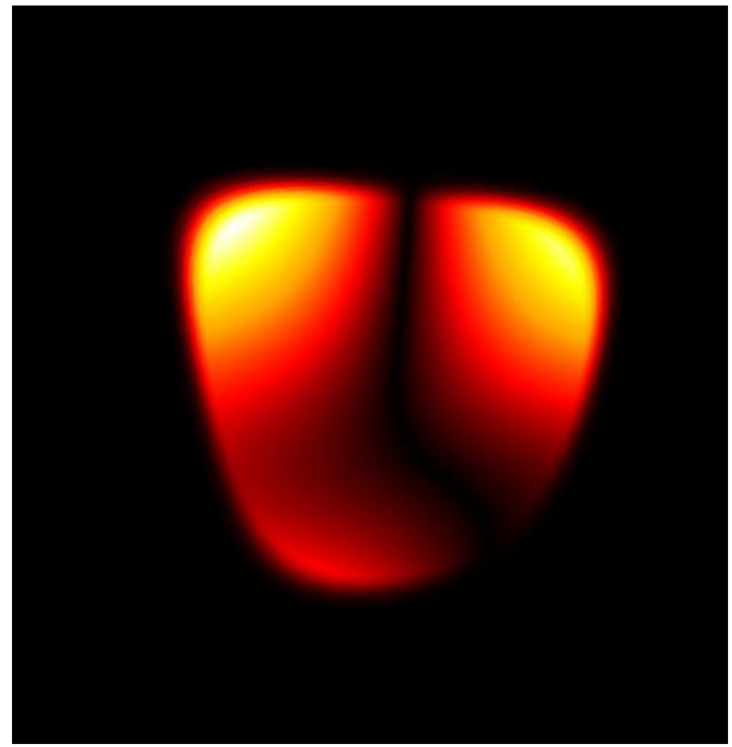


cpu time per simulation < 1 sec

Freeform: Geometric Field Tracing



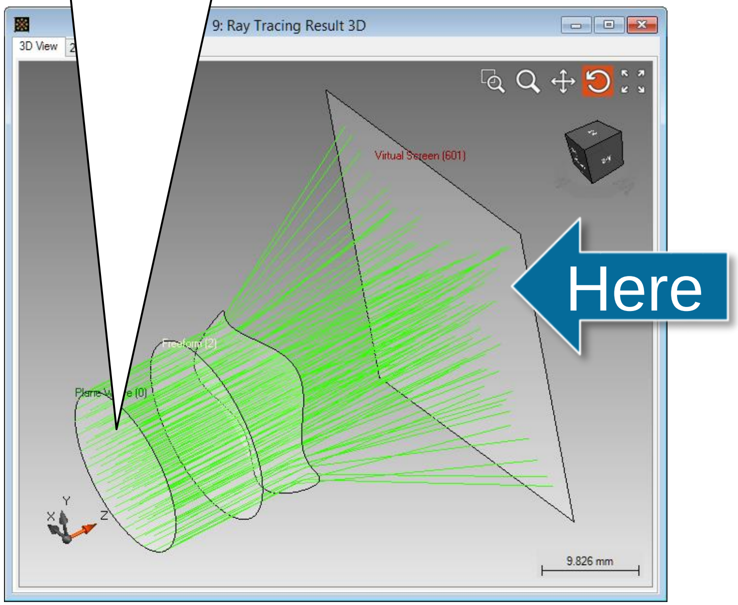
Amplitude $E_z(x,y)$



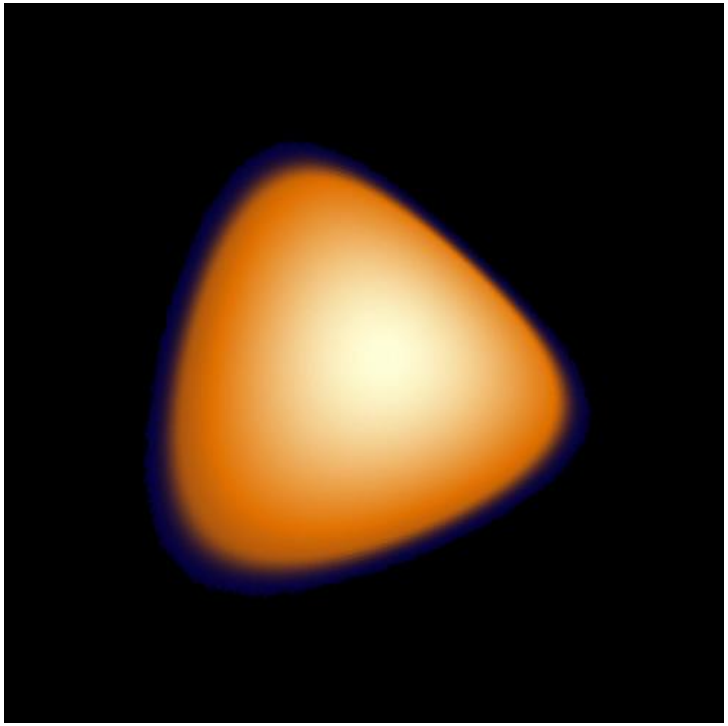
cpu time per simulation < 1 sec

Freeform: Geometric Field Tracing

Input: Gaussian beam
Diameter 10 mm



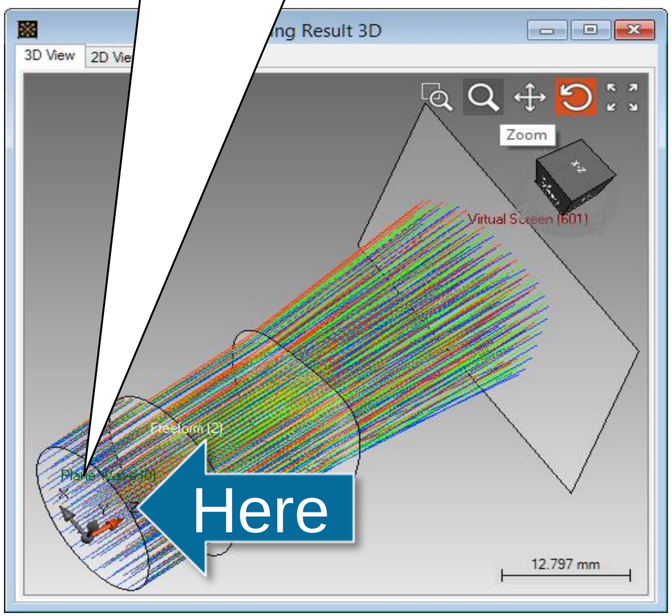
Amplitude $E_x(x,y)$



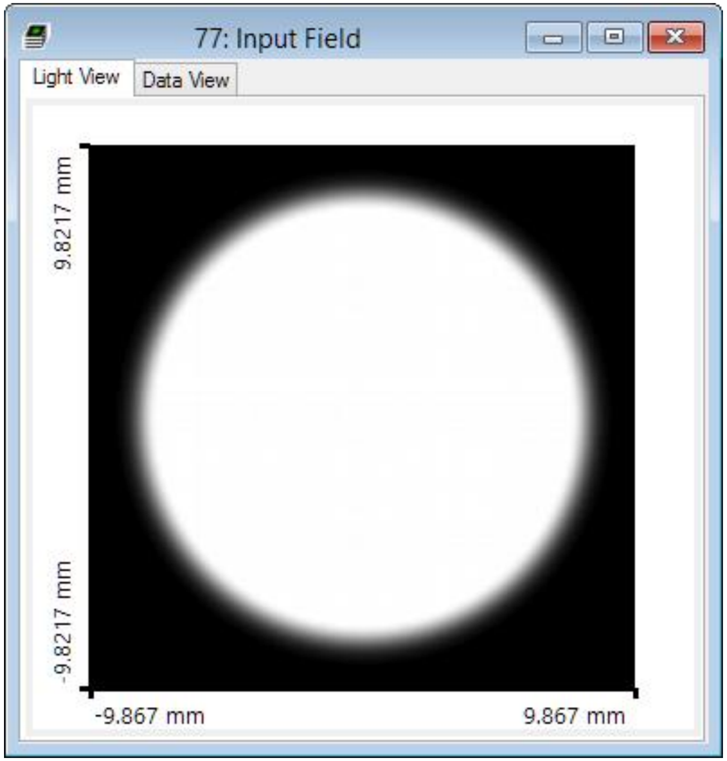
cpu time per simulation < 1 sec

Freeform: Geometric Field Tracing

Input: Plane waves
RGB (white, identical k)

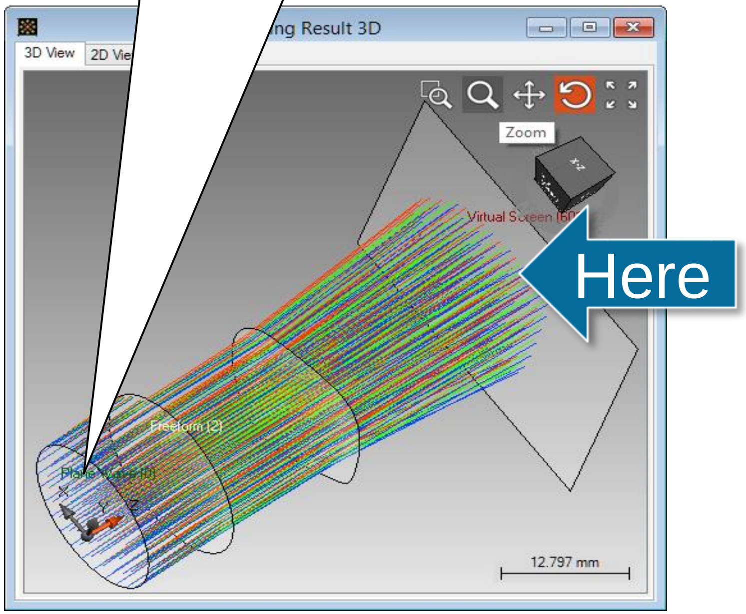


Intensity

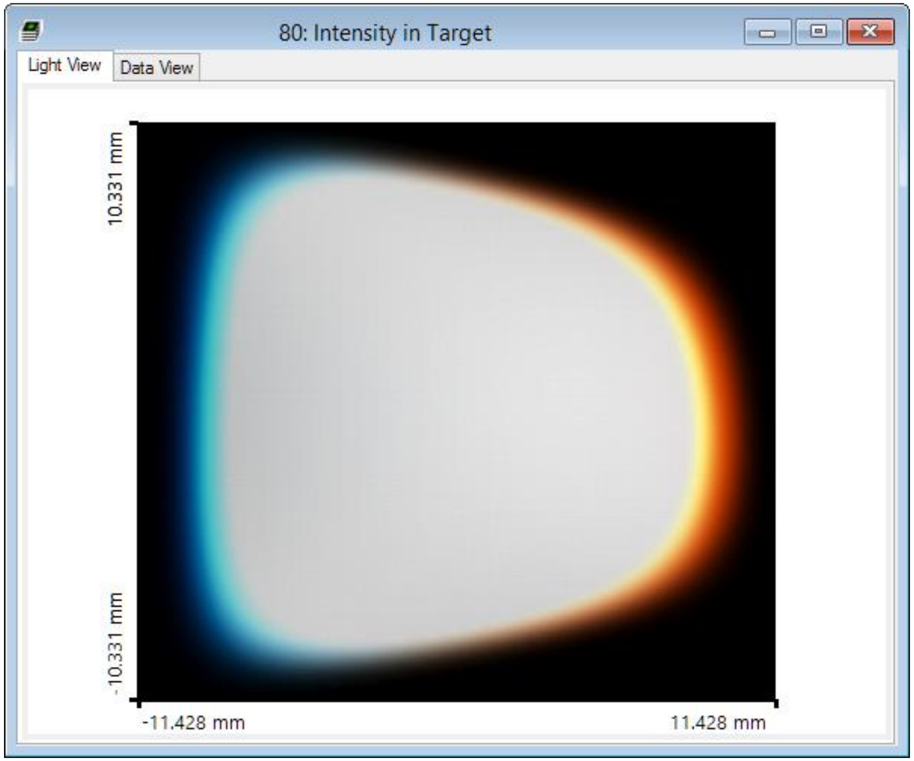


Freeform: Geometric Field Tracing

Input: Plane waves
RGB (white, identical k)

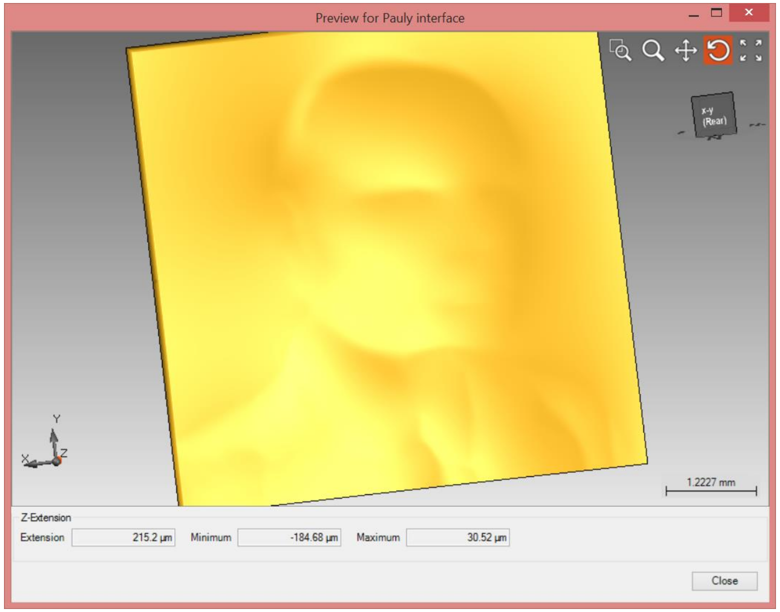


Intensity



cpu time ≈ 1 sec

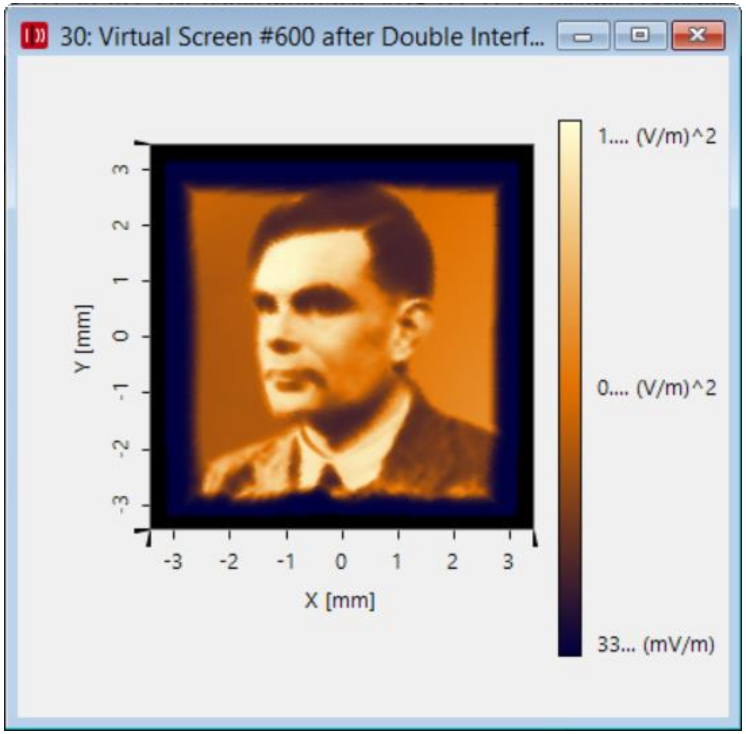
Example: Freeform Light Shaping



Yuliy Schwartzburg, Romain Testuz, Andrea Tagliasacchi, Mark Pauly

École Polytechnique Fédérale de Lausanne, Computer Graphics and Geometry Laboratory

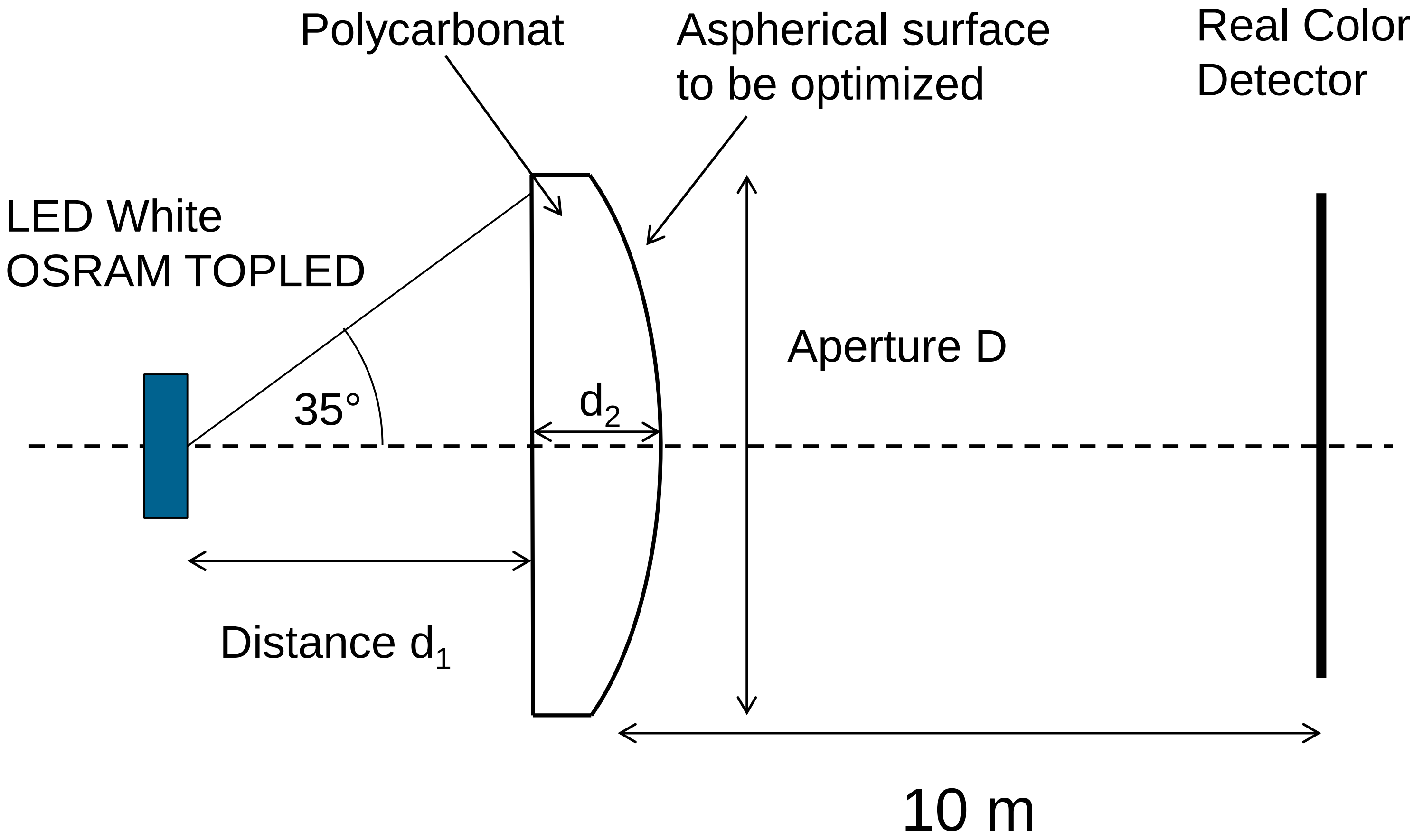
High-contrast Computational Caustic Design



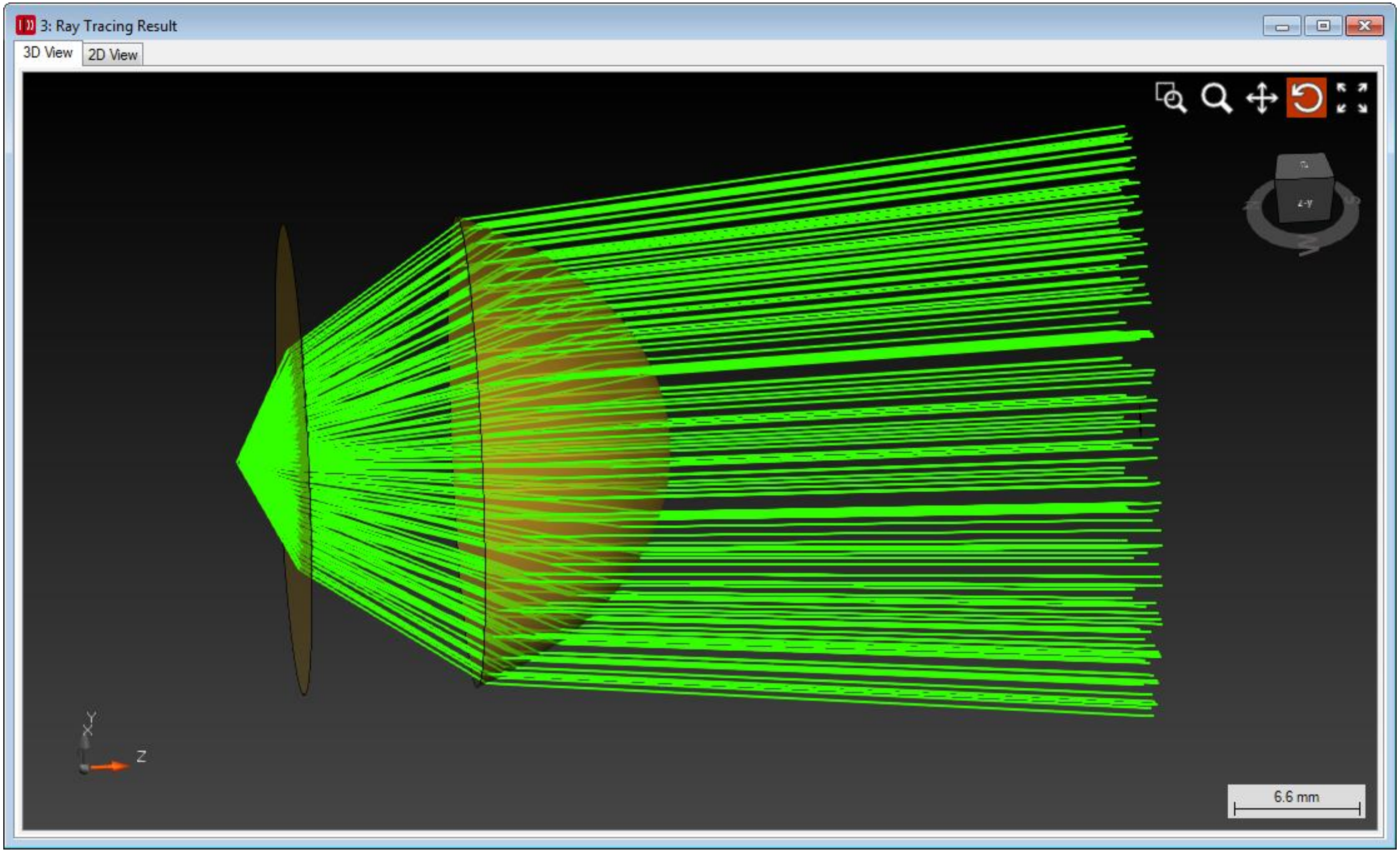
2015-02-20_VLF_FW_Pauly.Interface_2015-02-21_1_DesignED.Meshes

Example: LED Collimation

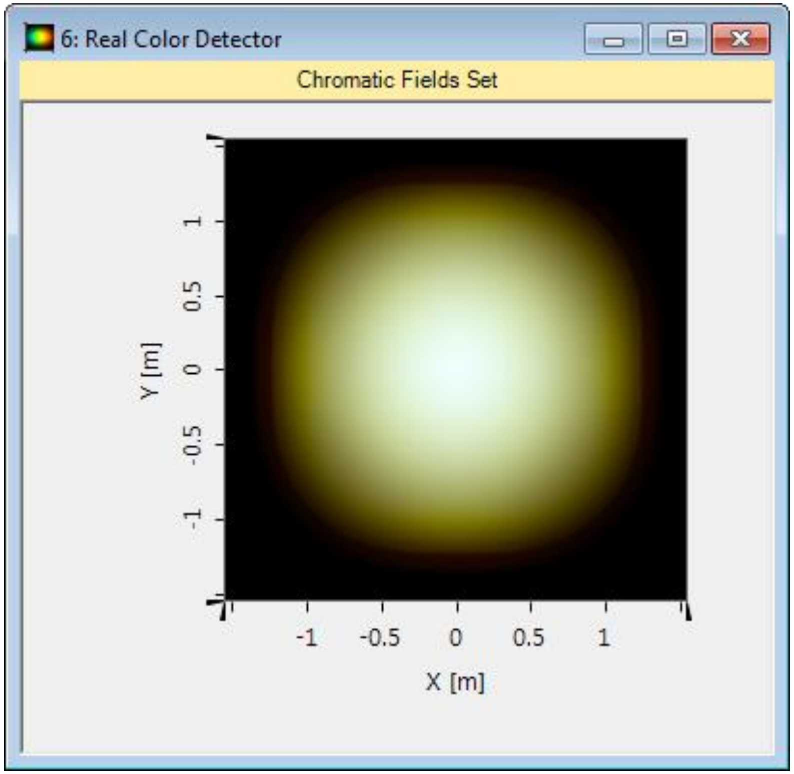
Modeling Task



Simulation Result



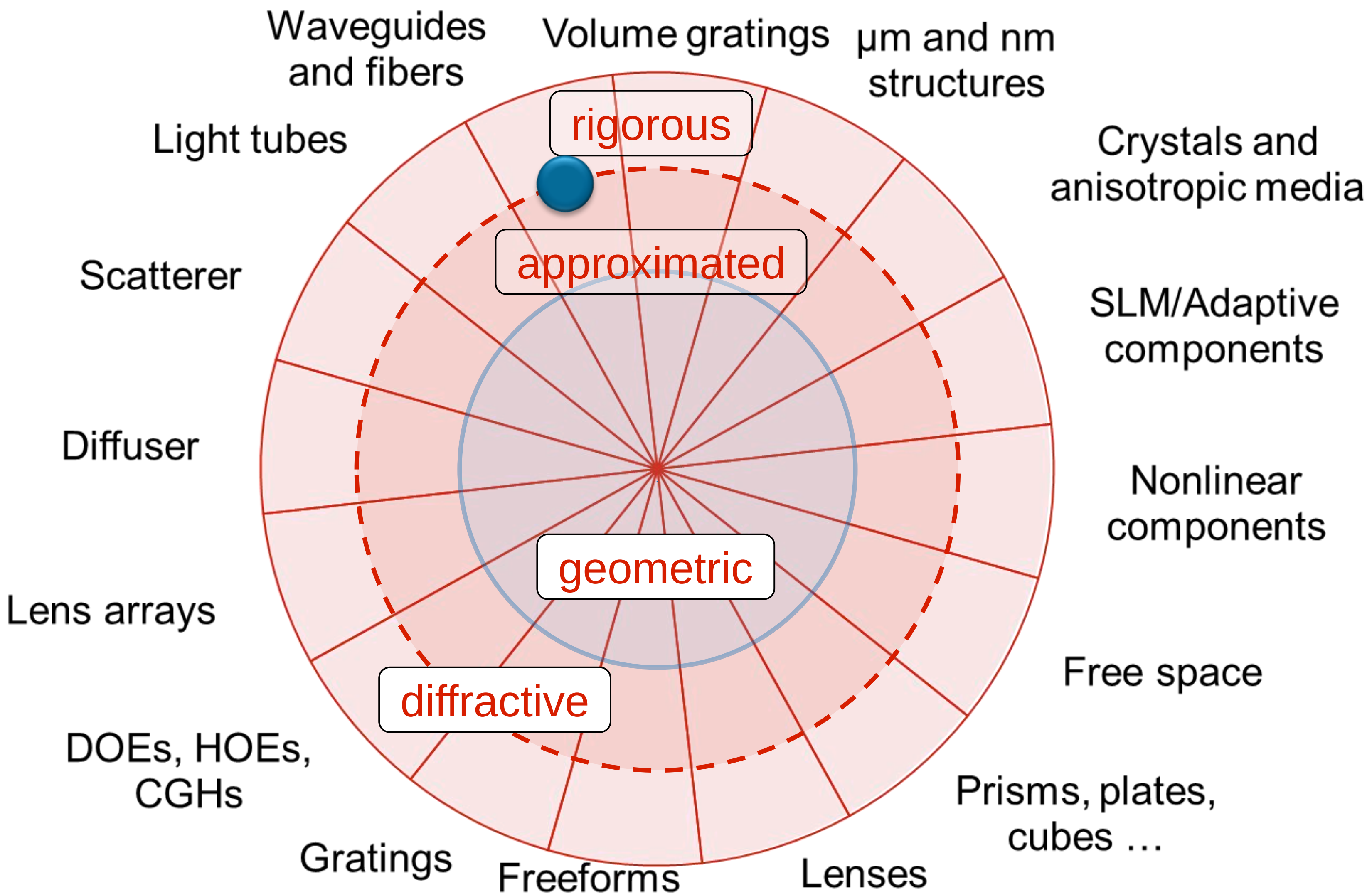
Simulation Results



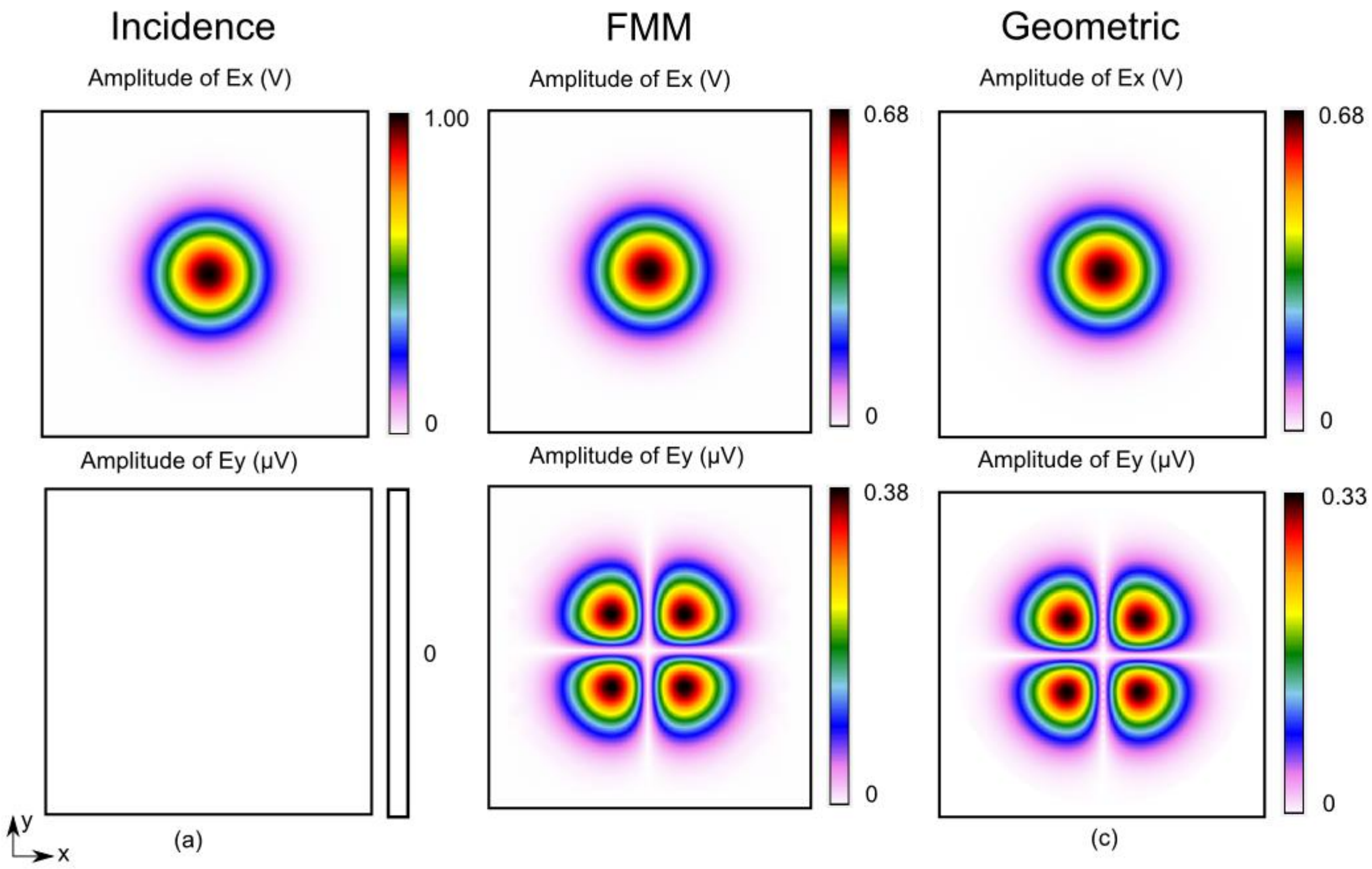
Real color distribution
300 x 300 pixel

Detector Resolution	Geometric Field Tracing		Monte-Carlo Ray Tracing	
	Number of Rays	Time	Number of Rays	Time
100 x 100	62000	18 s	10000000	20-25 s
300 x 300	62000	42 s	90000000	3-4 min
800 x 800	62000	3,5 min		

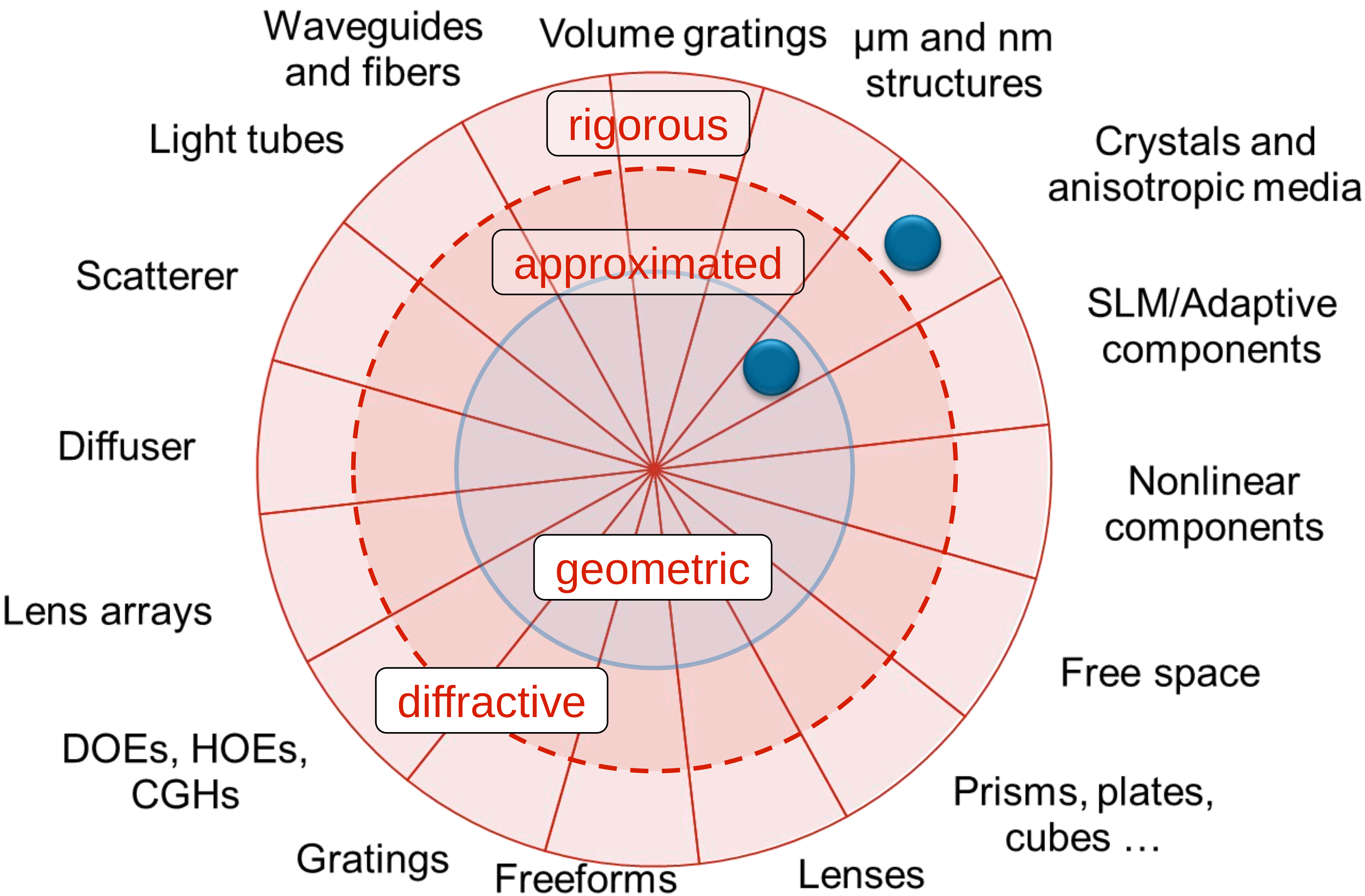
Unified Field Tracing



Propagation through Fiber



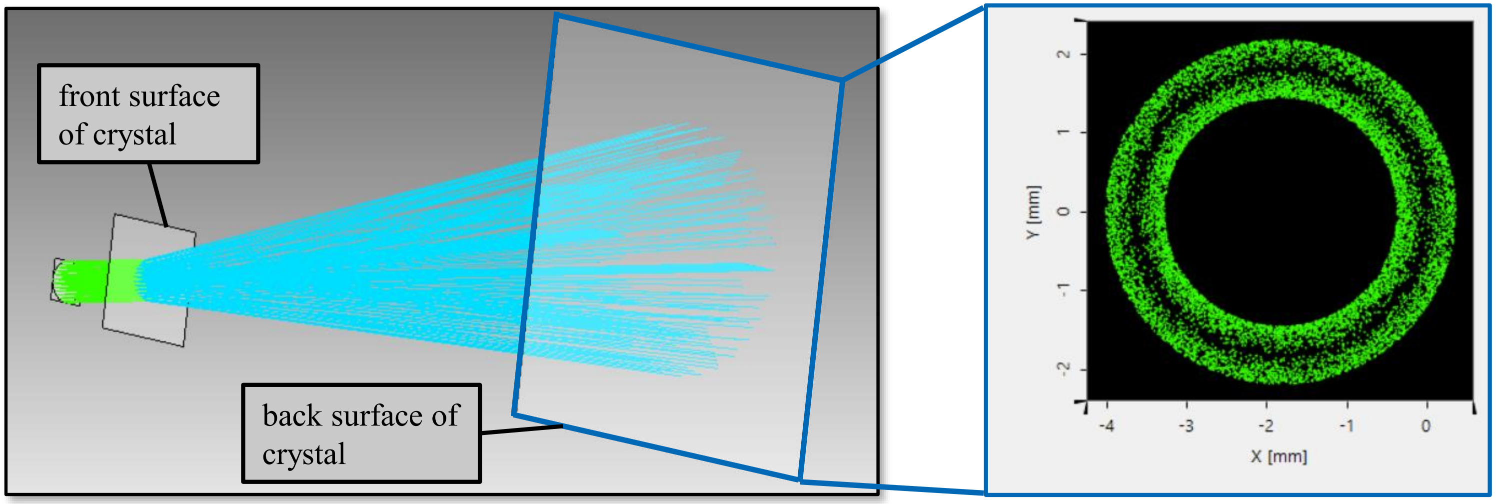
Unified Field Tracing



Conical Refraction Demonstration

- Setup and ray tracing

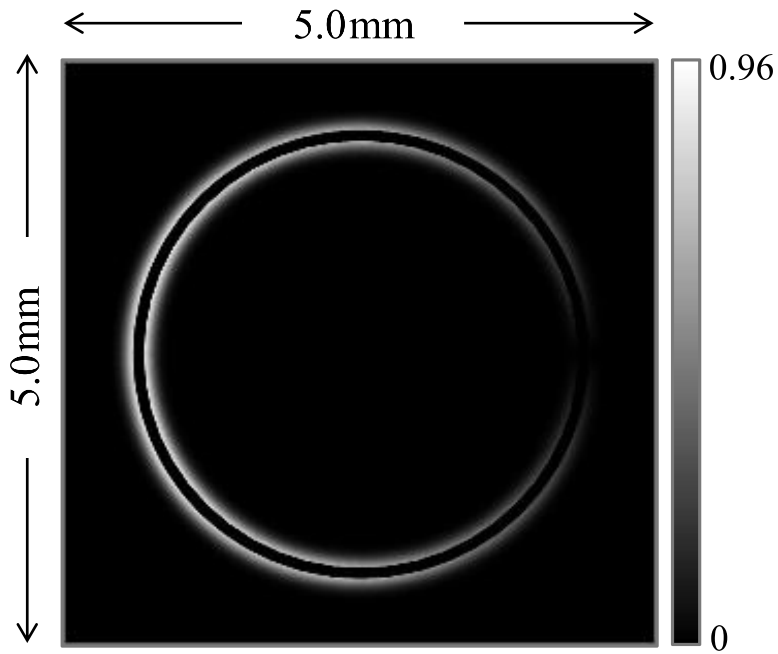
dot-diagram
with random rays



  - Incidence: weakly focused Gaussian – circularly polarized
  - Biaxial crystal: Naphthalene with n_1=1.525, n_2=1.722, n_3=1.945 @ 532 nm
  - Incidence along one optic axis of the crystal

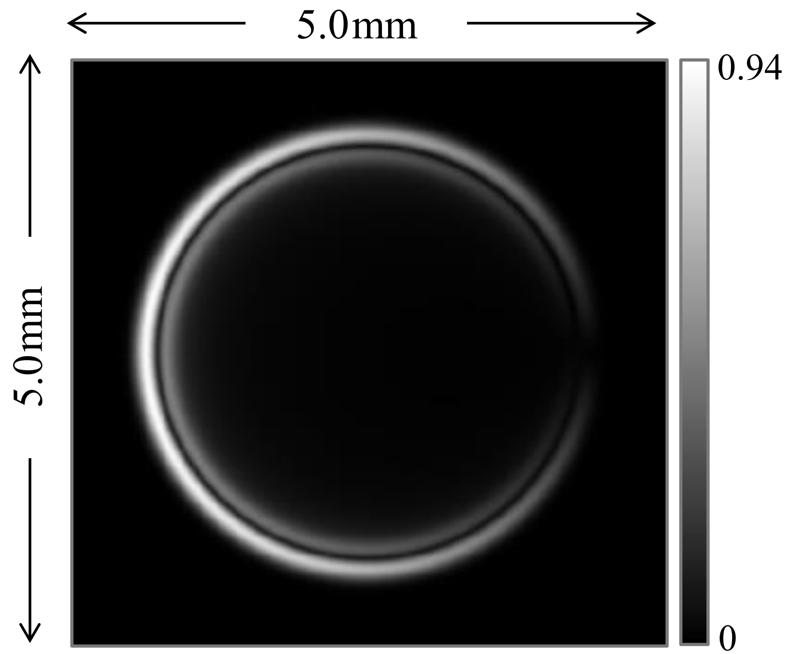
Conical Refraction Demonstration

- Field tracing



geometric field tracing

amplitude of E_x behind the biaxial crystal.

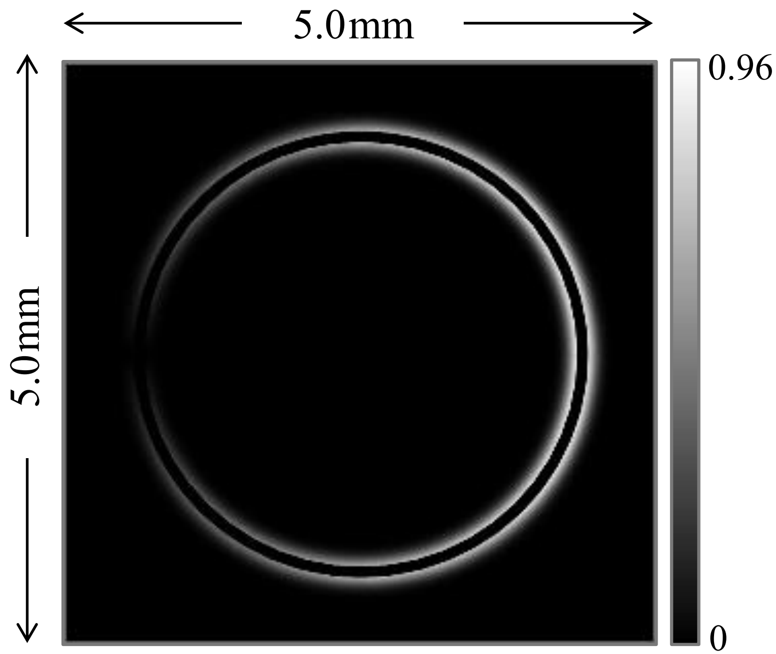


diffractive field tracing

amplitude of E_x behind the biaxial crystal

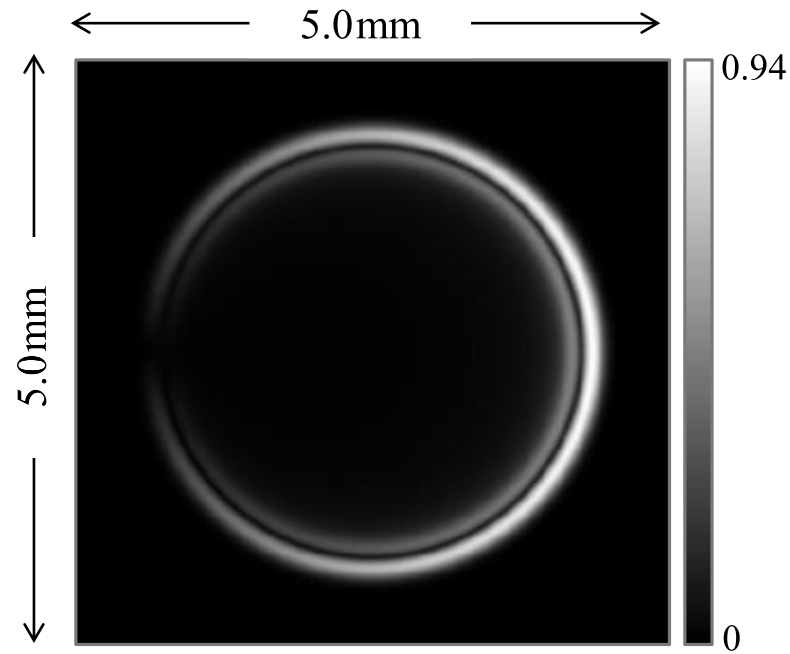
Conical Refraction Demonstration

- Field tracing



geometric field tracing

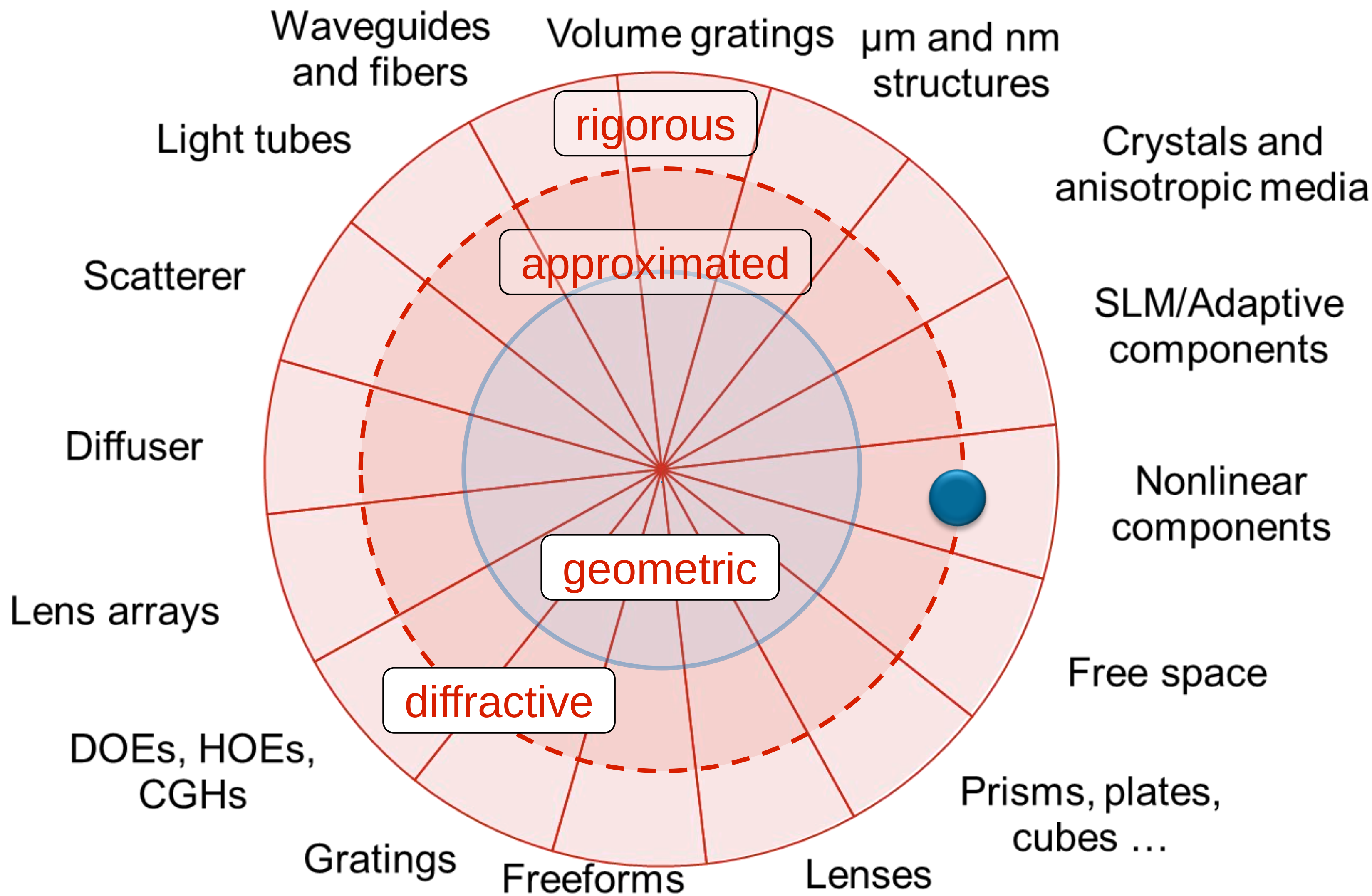
amplitude of E_y behind the biaxial crystal.



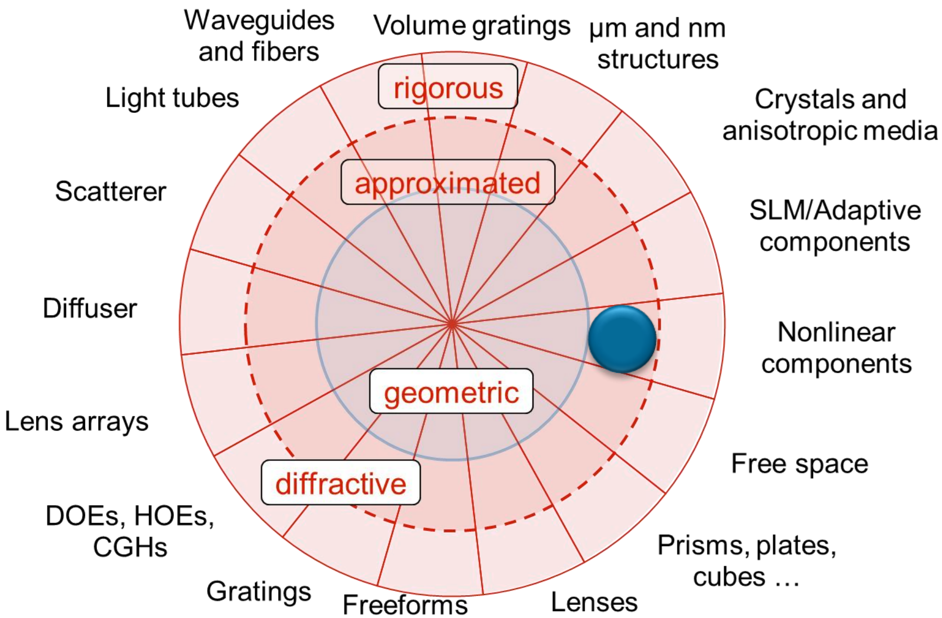
diffractive field tracing

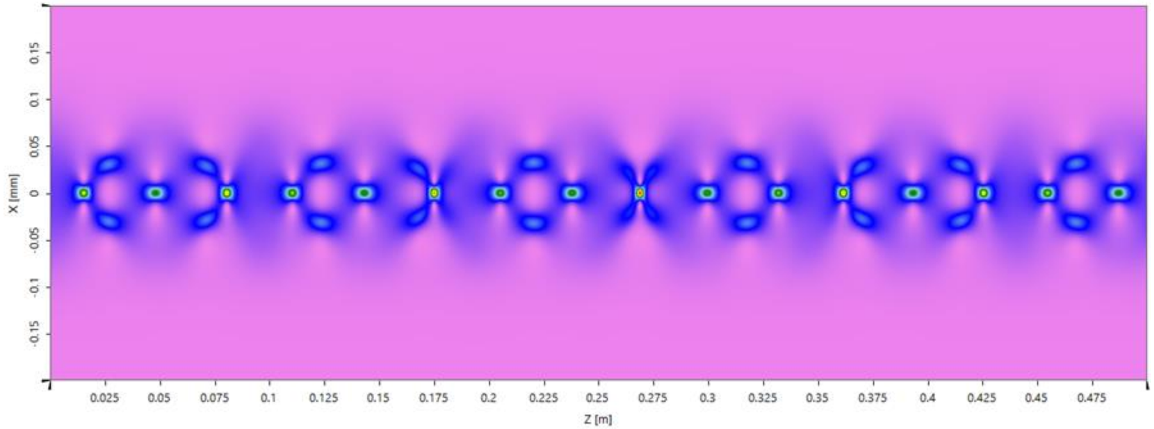
amplitude of E_y behind the biaxial crystal

Unified Field Tracing



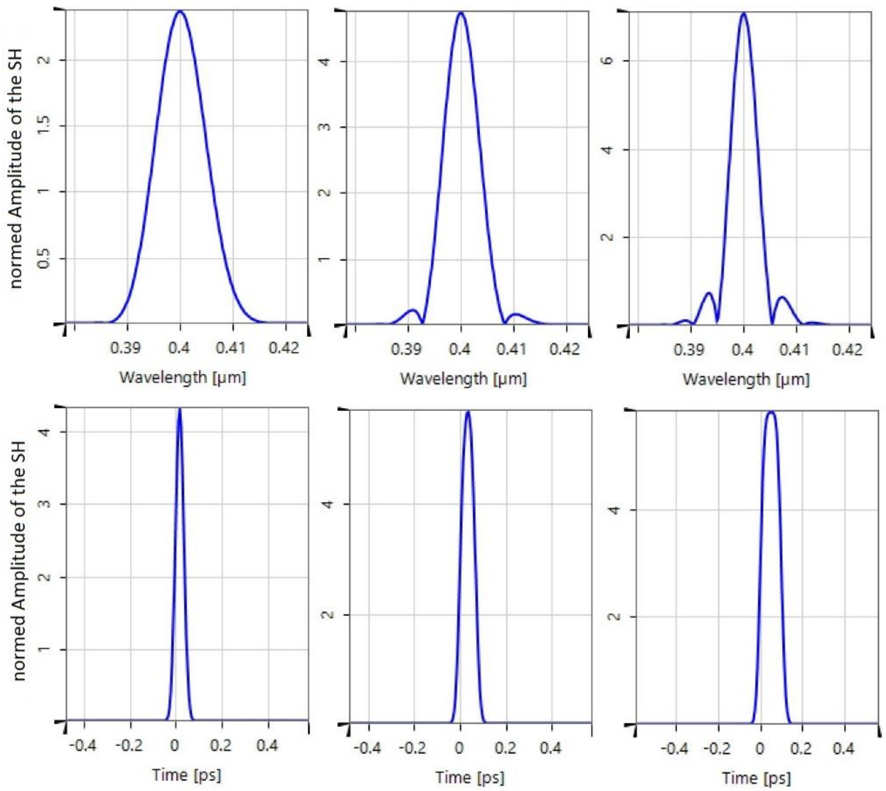
Example: Spatial Solitons by Kerr Effect





Phase Matched Case – Pulse Broadening

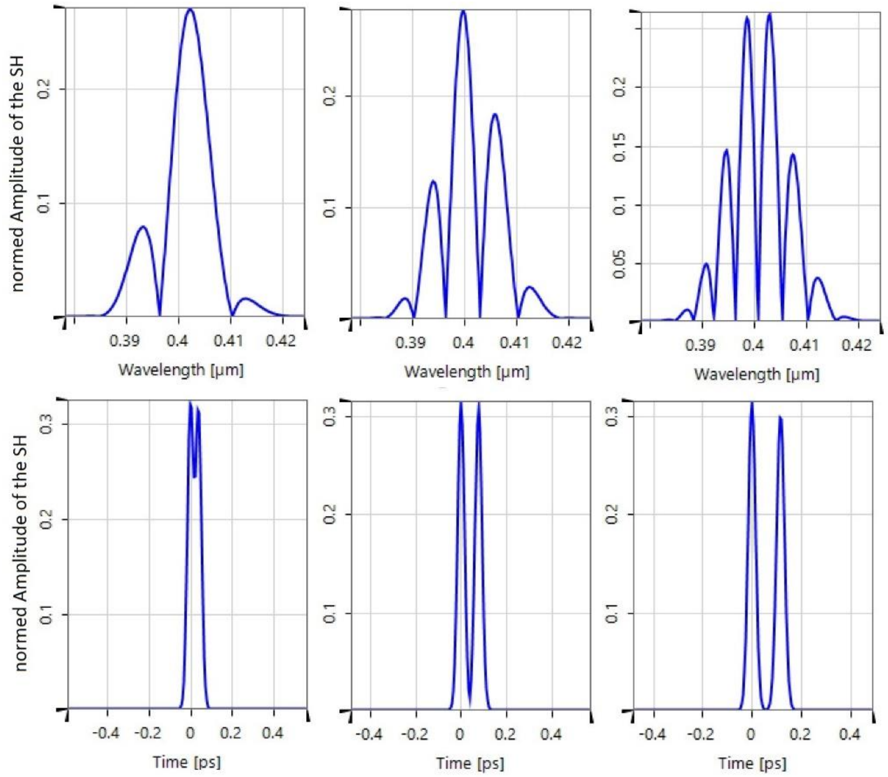
Normed SH in BBO, 800nm, incident Angle: 29.17° (phasematched) at various crystal lengths



0.2 mm   0.4 mm   0.6 mm

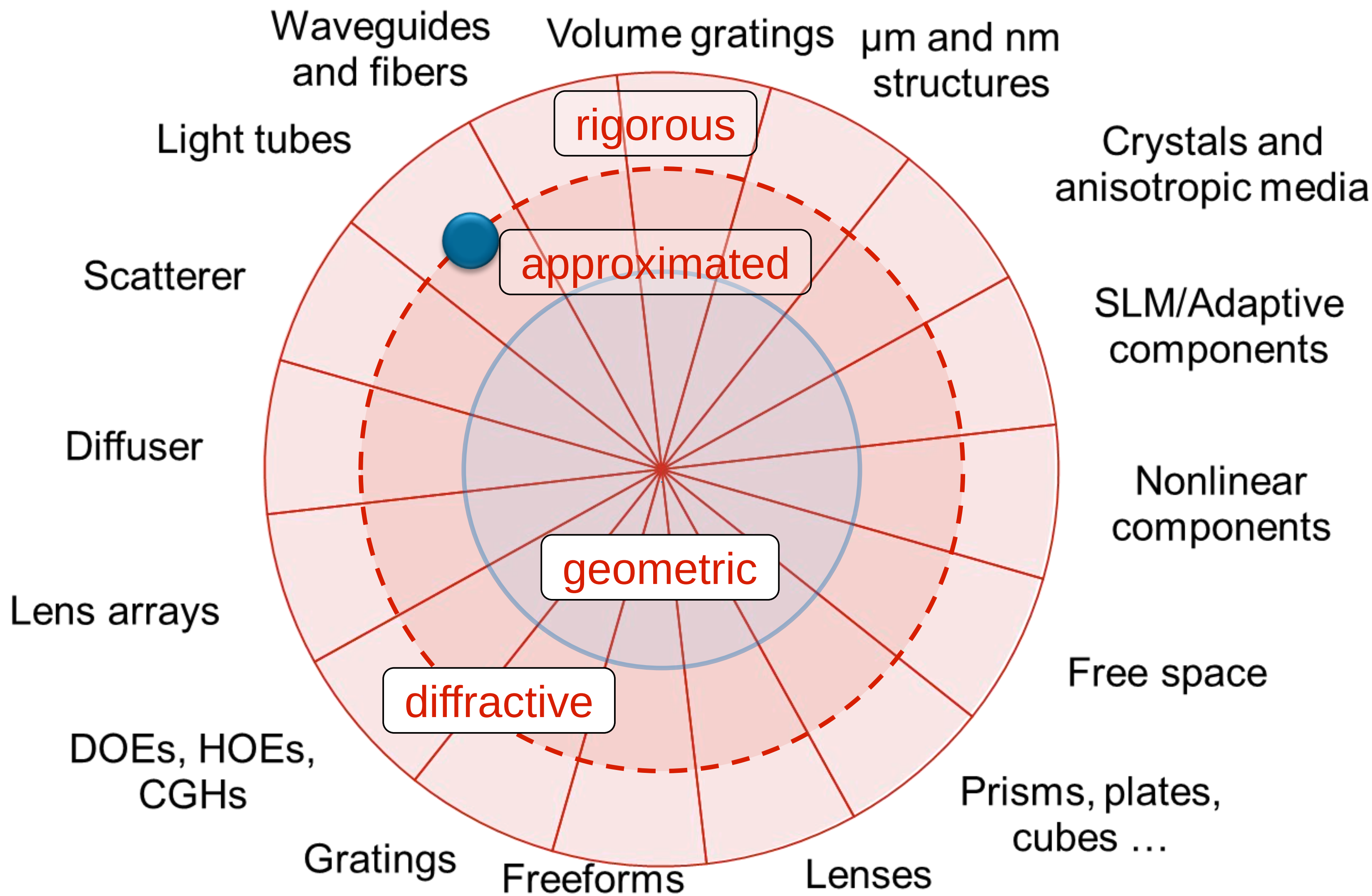
No Phase Matching – SH Split in Time

Normed SH in BBO, 800nm, incident Angle: 26° (non-phase matched) at various crystal lengths

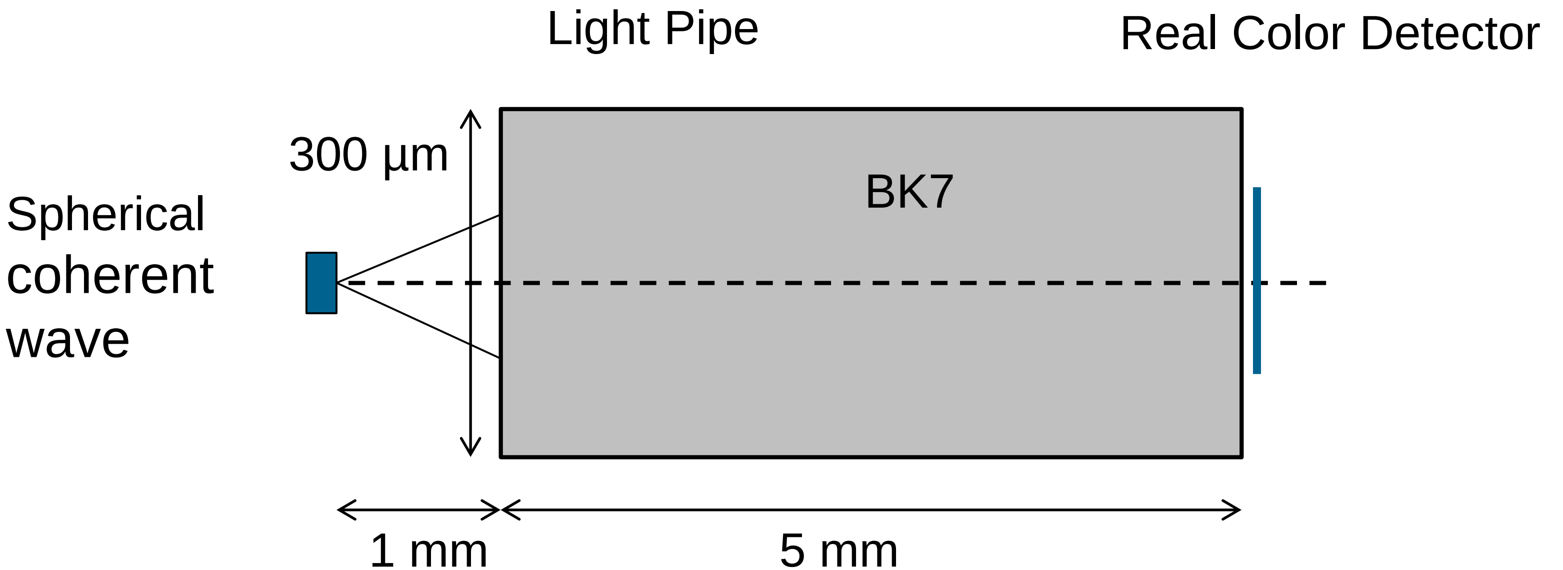


0.2 mm 0.4 mm 0.6 mm

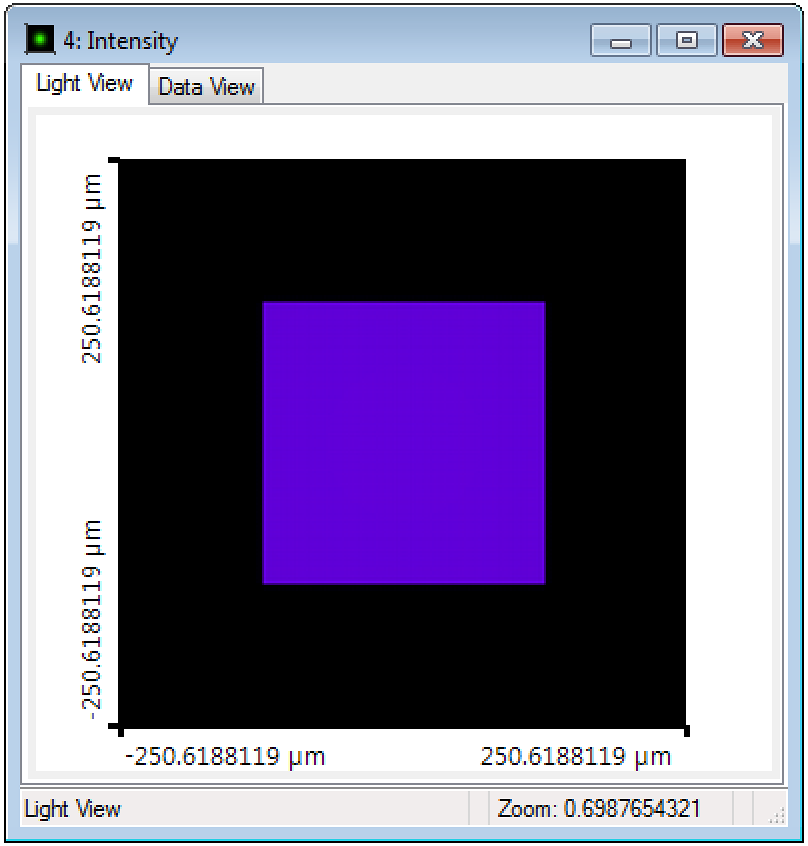
Field Tracing Powerful for Pulse Modeling



Modeling Task



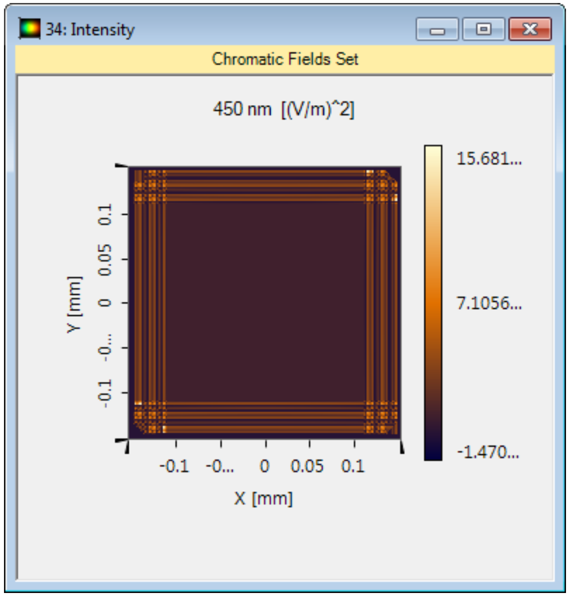
Incident Field



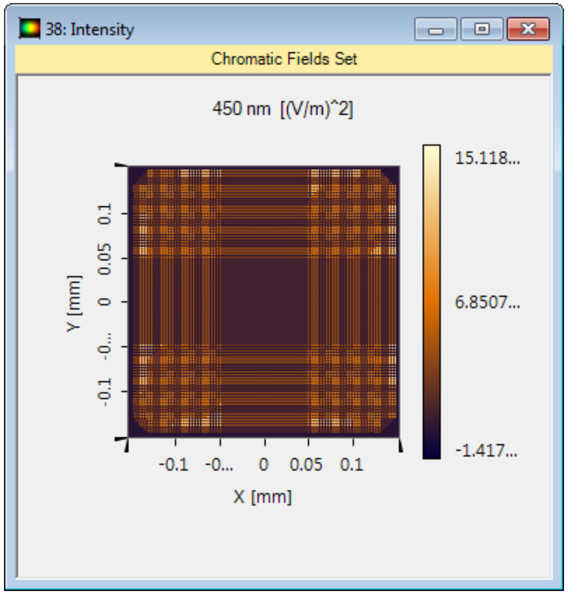
- Spherical coherent wave
- Wavelength: 450 nm
- Field size: 250 µm
- NA: 0.125

Simulation Results

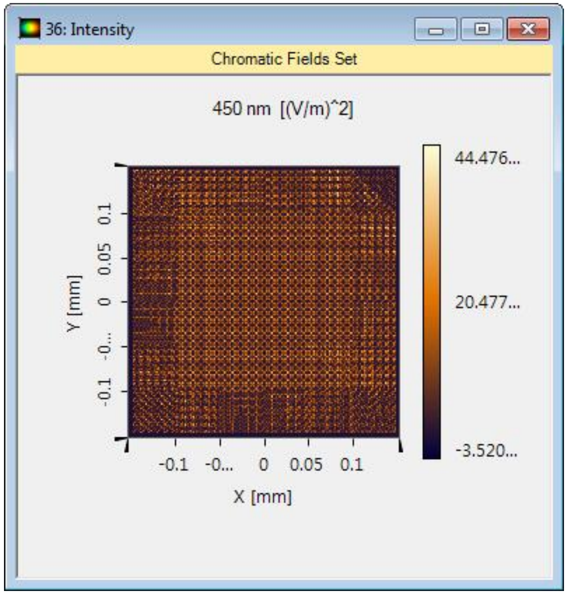
Intensity in Light Pipe



0.5 mm

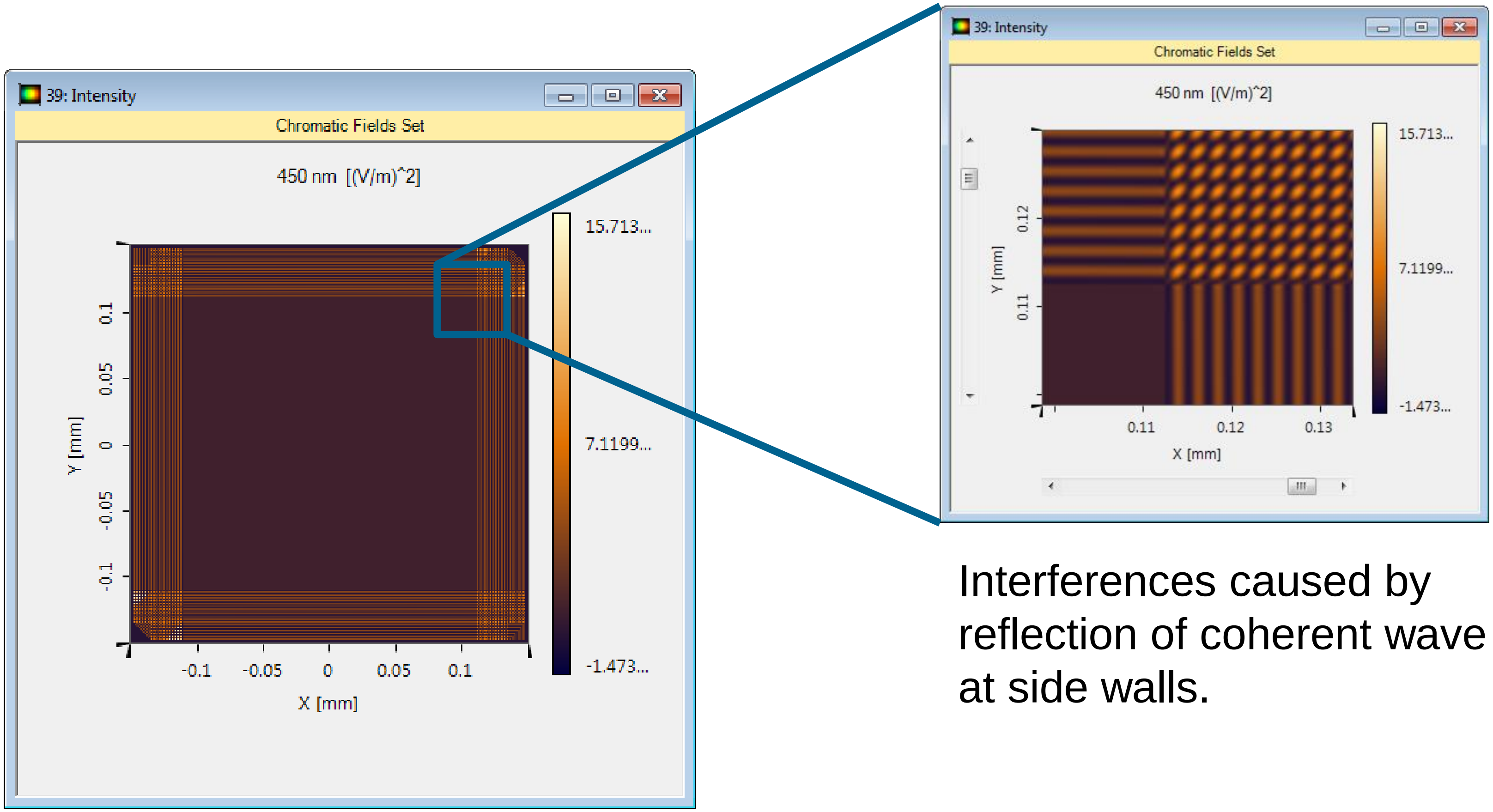


1 mm



5 mm

Simulation Results



Interferences caused by reflection of coherent wave at side walls.

Ray and Field Tracing