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Series 12
Fundamentals of Modern Optics

to be returned on **02.02.2023**, at the beginning of the lecture

Task 1: Optical Waveplates (3+2+2+3 points)

A slide of a transparent, uniaxial, anisotropic crystal with the refractive indices n_e and n_o and with thickness d is oriented such that the surface normal is along $\mathbf{e}_z$ and the crystal axis is along $\mathbf{c} = \cos(\alpha)\mathbf{e}_x + \sin(\alpha)\mathbf{e}_y$. An x-polarized plane wave of wavelength λ, with a vacuum wavevector $\mathbf{k} = (2\pi/\lambda)\mathbf{e}_z$, is excited at the beginning of the slide and propagates through the crystal. Consider the lossless propagation and neglect the Fresnel reflection at both interfaces.

a) Decompose the incident field into the normal modes (ordinary and extraordinary waves) of the anisotropic medium and write the dispersion relation for both of them.
Hint: For the decomposition use the crystal coordinate system.

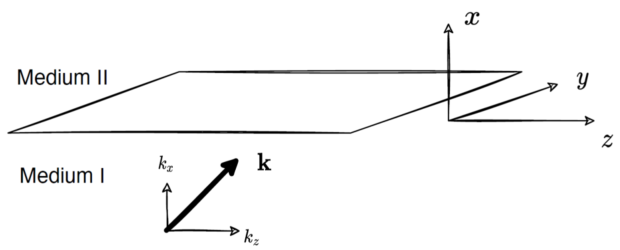
b) Calculate the relation between the wavevector $\boldsymbol{k}$ and the corresponding Poynting vector $\boldsymbol{S}$ for both normal modes. Show that they are parallel.

c) Calculate the electric field in laboratory coordinates after propagating through the slide. What is its polarization state?

d) We choose the thickness d of the slide such that $(n_e - n_o)d{=}\lambda/2$. Calculate and describe the impact of the crystal onto the polarization state of the plane wave directly after the slide as a function of the crystal rotation angle α. If we place a linear polarizer after this so-called half-wave plate and rotate the crystal, which device do we get?

Task 2: Optical interfaces (general) (1 + 2 + 2 = 5 points)

The symmetries and invariances are very useful for the solutions and simplifications of different physical problems. Consider an infinite interface between two media with different relative permittivity ε_r:



a) Consider a general wave-equation:

$$\nabla \times \nabla \times \bar{\mathbf{E}}(\mathbf{r}, t) - \frac{\omega^2}{c^2}\bar{\mathbf{E}}(\mathbf{r}, \omega) = i\omega\mu_0\bar{\mathbf{j}}(\bar{r}, \omega) + \mu_0\omega^2\bar{\mathbf{P}}(\bar{r}, \omega)$$

Which simplification of the $\nabla \times \nabla \times \bar{\mathbf{E}}(\mathbf{r}, \omega)$ relation can be obtained from translation invariance of the infinite flat interface between two media (1P).

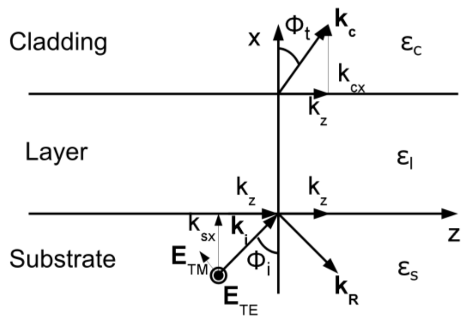
b) Give a short explanation of the $\mathbf{E} = \mathbf{E}_{TM} + \mathbf{E}_{TE}$ decomposition of an arbitrary E-field (1P). What are the advantages of this decomposition for the situation described in a) (1P)?

c) Give a short explanation of the "continuity of field" and the "continuity of wave vector" on an interface. For which fields and polarization components these conditions are satisfied (2P)?

Task 3: Optical layer (1+3+3*+2* points)

Let us consider a single optical layer with thickness d that is embedded between a substrate and a cladding material as shown below. The refractive indices of the layer, substrate, and cladding materials are $n_1 = \sqrt{\varepsilon_1}$,

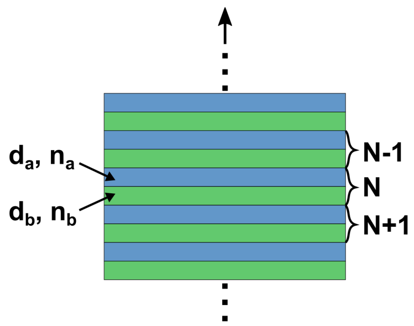
$n_s = \sqrt{\varepsilon_s}$, and $n_c = \sqrt{\varepsilon_c}$, respectively. For simplicity, we consider light in TE polarization only. The incident beam makes an incident angle of φ_i in the substrate.



a) Calculate the angle φ_t of the transmitted beam as a function of the incident angle φ_i.
b) Compute the coefficients of reflection and transmission as functions of the incident angle φ_i.
c) Compute the reflectivity and transmissivity of the single layer, and show that they add up to 1. For simplicity, assume $\varepsilon_l > \varepsilon_s \sin^2(\varphi_i)$ and $\varepsilon_c > \varepsilon_s \sin^2(\varphi_i)$.
d) Consider the special case of a $\lambda/4$-layer, i.e. $k_{l,x}d = d\sqrt{k_l^2 - k_z^2} = \pi/2$, and calculate its reflectivity. Now assume the incident light is perpendicular to the layer ($\varphi_i = 0$) and find the condition for the refractive indices to obtain minimum reflection.

Task 4: Stratified Media (4*+2*+2* points)

A beam of light with free-space wave number $k_0 = \omega/c$ is passing through a periodic stack of alternating layers. The odd layers have a refractive index n_a and a thickness d_a, the even layers have a refractive index n_b and a thickness d_b as shown below. For the case of an infinite periodic stack, the fields in the structure follow the Bloch theorem, which states that this structure supports Bloch modes of wave number K. For such a mode the field at the beginning of the double layer N is connected to the field at the beginning of the double layer $N+1$ through $E_{N+1} = \exp(iK\Lambda)E_N$ with $\Lambda = d_a + d_b$. For simplicity consider the case of normal incidence $\varphi_i = 0$.



a) Derive the dispersion relation for K as a function of n_a, n_b, d_a, d_b, and k_0.
b) Determine under which condition we get a propagating or a decaying Bloch mode.
c) For the case of decaying Bloch modes find the frequencies for which the decay becomes strongest. For simplicity assume $n_a d_a = n_b d_b$.