Institute of Applied Physics
Friedrich-Schiller-Universität Jena

Laser Physics Problem sheet 2

02.05.2022 (05.05.2022)

Institute of Applied Physics, Abbe Center of Photonics,
Friedrich Schiller University Jena, Albert-Einstein-Str. 15, 07745 Jena, Germany

Problem 1 (7 points)

There are 4 basic laws to describe the behavior of an ideal gas:

- *Law of Boyle-Mariotte:* The pressure (P) is inversely proportional to volume (V) when the amount of moles (n) and the temperature (T) are constant:

$$PV = K_1(T, n)$$

- *Law of Gay-Lussac:* The volume is proportional to temperature at constant pressure and for a fixed amount of moles ($n = \frac{N}{N_A}$; with N the total number of gas atoms/molecules and N_A the Avogadro number):

$$V = K_2(P, n)T$$

- *Law of Avogadro:* The volume of a gas is directly proportional to the number of moles when measured at the same pressure and temperature:

$$V = K_3(P, T)n$$

- *Law of Amontons:* The pressure is proportional to the absolute temperature for a fixed amount of moles and in a fixed volume:

$$P = K_4(V, n)T$$

With this information answer the following questions:

a) Combining the laws above and assuming a constant number of moles n, demontrate that the ideal gas law is:

$$PV = RnT = k_B NT$$

where R is the ideal gas constant ($R = k_B N_A$, with k_B being the Boltzmann constant) (2 points)

We start with the 4 given equations:

1.) **Law of Boyle Mariotte:** $PV = K_1(T,n)$

2.) **Law of Gay- Lussac:** $V = K_2(P,n)T$

3.) **Law of Avogadro:** $V = K_3(P,T)n$

4.) **Law of Amontons:** $P = K_4(V,n)T$

Assumption: Constant amount of substance
$\Rightarrow n = \text{const}$

Divide 1.) by T: $$\frac{PV}{T} = \frac{K_1(T,n)}{T}$$

Multiply 2.) by P: $$\frac{PV}{T} = P\,K_2(P,n)$$

Combine 1.) and 2.): $$\underbrace{\frac{K_1(T,n)}{T}}_{\text{Only depends on } T} = \underbrace{P\,K_2(P,n)}_{\text{Only depends on } P} = K(n)$$

They must be constant to be always equal $$\boxed{\Rightarrow \frac{PV}{T} = K(n)}$$

With this information answer the following questions:

a) Combining the laws above and assuming a constant number of moles n, demontrate that the ideal gas law is:

$$PV = RnT = k_B NT$$

where R is the ideal gas constant ($R = k_B N_A$, with k_B being the Boltzmann constant) (2 points)

We start with the 4 given equations:

1.) Law of Boyle Mariotte: $PV = K_1(T, n)$

2.) Law of Gay- Lussac: $V = K_2(P, n)T$

3.) Law of Avogadro: $V = K_3(P, T)n$

4.) Law of Amontons: $P = K_4(V, n)T$

Assumption: Constant amount of substance
$\Rightarrow n = \text{const}$

$$\boxed{\frac{PV}{T} = K(n)}$$

Multiply 3.) by $\frac{P}{T}$: $\quad \frac{PV}{T} = n\boxed{\frac{P}{T} K_3(P,T)}$

Constant because $\frac{PV}{T} = \text{const}$

$$n = \frac{N}{N_\mathrm{A}}$$

$$\Rightarrow \frac{PV}{T} = Rn = \frac{R}{N_\mathrm{A}}(n \cdot N_\mathrm{A}) = k_\mathrm{B}\, N$$

$$\boxed{\Rightarrow \frac{PV}{T} = k_\mathrm{B}\, N\, T}$$

b) Demonstrate that the average speed of the gas atoms/molecules of mass m is given by:

$$\bar{v} = \sqrt{\frac{3k_BT}{m}}$$

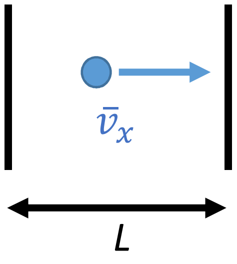
Note: In order to do this assume a cubic container of side L and calculate the pressure that the gas exerts on one of its sides. Then put this in relation to the ideal gas law. The gas pressure can be calculated from the force that an atom/molecule exerts on the wall when hitting it at a certain speed. (3 points)

Assumptions:

Pressure arises from impact on 1 wall

Elastic collisions $\Rightarrow$ On Impact: $v \rightarrow -v$

Particle moves in x-Direction:



Momentum on impact is transferred to wall: $\Delta p = m\,\Delta v = m\left(\overline{v_X} - (-v_X)\right) = 2m|v_X|$

Particle bounces back and forth $\Rightarrow$ Hits wall again after average time: $\Delta t = \frac{2L}{\bar{v}_x}$ $[s]$

Force exerted on wall (one Particle): $F_1 = ma = \frac{m\Delta v}{\Delta t} = \frac{\Delta p}{\Delta t} = \frac{m\bar{v}_x{}^2}{L}$

Force exerted on wall (N Particles): $\boxed{F_N = \frac{\Delta p}{\Delta t} = N \cdot \frac{m\bar{v}_x{}^2}{L}}$

b) Demonstrate that the average speed of the gas atoms/molecules of mass m is given by:

$$\bar{v} = \sqrt{\frac{3k_BT}{m}}$$

Note: In order to do this assume a cubic container of side L and calculate the pressure that the gas exerts on one of its sides. Then put this in relation to the ideal gas law. The gas pressure can be calculated from the force that an atom/molecule exerts on the wall when hitting it at a certain speed. (3 points)

Force exerted on wall (N Particles):

$$\boxed{F_N = \frac{\Delta p}{\Delta t} = N \cdot \frac{m\bar{v}_x{}^2}{L}}$$

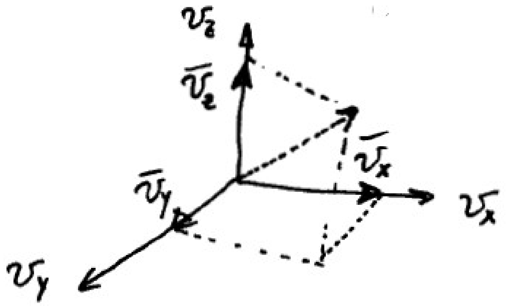
On average: velocities spread homogenously in 3D:

$$\bar{v}_x{}^2 = \frac{\bar{v}^2}{3}$$

Projection of a 45° vector in basis with orthogonal axes:

$$\bar{v}_x = \bar{v}_y = \bar{v}_z = \frac{\bar{v}}{\sqrt{3}}$$

$\bar{v}$ is the diagonal of a cube of side $\bar{v}_x = \bar{v}_y = \bar{v}_z$



With this we can write:

$$\boxed{F_N = \frac{\Delta p}{\Delta t} = N \cdot \frac{m\bar{v}^2}{3L}}$$

b) Demonstrate that the average speed of the gas atoms/molecules of mass m is given by:

$$\bar{v} = \sqrt{\frac{3k_B T}{m}}$$

Note: In order to do this assume a cubic container of side L and calculate the pressure that the gas exerts on one of its sides. Then put this in relation to the ideal gas law. The gas pressure can be calculated from the force that an atom/molecule exerts on the wall when hitting it at a certain speed. (3 points)

$$\boxed{F_N = \frac{\Delta p}{\Delta t} = N \cdot \frac{m\bar{v}^2}{3L}}$$

The Wall area is L^2:

$$\Rightarrow P = \frac{F_N}{L^2} = N \cdot \frac{m\bar{v}^2}{3L^3} = N \cdot \frac{m\bar{v}^2}{3V}$$

Volume of the box

Compare to ideal gas law:

$$PV = \frac{Nm\bar{v}^2}{3} \overset{!}{=} k_B NT$$

$$\boxed{\Rightarrow \bar{v} = \sqrt{\frac{3k_B T}{m}}}$$

c) Calculate the average time between collisions τ_{col} knowing that the average free-path between collisions is $l = \frac{V}{N\sigma}$, where $\sigma = \pi d^2$ is the collisional cross-section (with d being the diameter of an atom/molecule). Afterwards demonstrate that the pressure broadening is given by the formula given in the lecture: $\Delta\nu = \sqrt{\frac{3}{4k_B m T}} d^2 P$

Given:

Average Free path: $l = \frac{V}{N \cdot \pi d^2}$

From a) : $\frac{N}{V} = \frac{P}{k_B \cdot T}$

From b) : $\overline{v} = \sqrt{\frac{3k_B T}{m}}$

$$\tau_{\text{collision}} = \frac{\text{mean free} - \text{path between collisions}}{\text{average speed of the atoms}}$$

$$\tau_{\text{col}} = \frac{l}{\overline{v}} = \frac{1}{\sigma \cdot \frac{N}{V}\sqrt{\frac{3k_B \cdot T}{m}}} = \frac{1}{\pi \cdot d^2 \cdot \frac{N}{V}\sqrt{\frac{3k_B \cdot T}{m}}} = \frac{1}{P \cdot \pi \cdot d^2 \sqrt{\frac{3}{m \cdot k_B \cdot T}}}$$

From lecture:

$$\Delta v_{\text{col}} = \frac{1}{2\pi \cdot \tau_{\text{col}}} = \frac{P \cdot \pi \cdot d^2 \sqrt{\frac{3}{m \cdot k_B \cdot T}}}{2\pi} = \boxed{\sqrt{\frac{3}{4 \cdot m \cdot k_B \cdot T}} \cdot P \cdot d^2}$$

Problem 2 (3 points)

Assume a 2-level system with degenerated energy levels. The degeneracy factors for the upper and lower levels are g_2 and g_1, respectively.

a) Demonstrate that the equation that governs the evolution of the photon flux along this system is:

$$\Phi_\nu(x) = \Phi_\nu(0) \cdot \exp\left[\left(N_2 - \frac{g_2}{g_1}N_1\right) \cdot F(\nu) \cdot \left(\frac{c}{2\pi\nu_0}\right)^2 \cdot \frac{A_{21}}{\Delta\nu} \cdot x\right],$$

where A_{21} is the Einstein coefficient.
Consider that the absorption and emission cross-sections ($\sigma_{a\nu}$ and $\sigma_{e\nu}$, respectively) are described by a Lorentz profile with bandwidth $\Delta\nu$ (i.e. homogeneous broadening) and that they are centered at the frequency ν_0. Additionally, consider that I_ν (i.e. the intensity per frequency interval at frequency ν) is spectrally much broader than the cross-sections. Neglect the spontaneous emission in your calculations. (2 points)

b) Demonstrate, using the equations obtained in the previous sections, that in spite of having inversion, there is no photon flux gain in this 2-level system. (1 point)

Hints: $B_{21} = \frac{g_1}{g_2}B_{12}$, $A_{21} = \frac{8\pi h\nu^3}{c^3}B_{21}$, $\rho(\nu) = \frac{I_\nu}{c}$, where $\rho(\nu)$ is the spectral energy density (per volume)

$\sigma_{i\nu} = \sigma_{i0} \cdot F(\nu) = \sigma_{i0}\frac{\left(\frac{\Delta\nu}{2}\right)^2}{(\nu-\nu_0)^2+\left(\frac{\Delta\nu}{2}\right)^2}$, $\int_{-\infty}^{\infty} F(\nu)d\nu = \frac{\pi\Delta\nu}{2}$

Problem 2 (3 points)

Assume a 2-level system with degenerated energy levels. The degeneracy factors for the upper and lower levels are g_2 and g_1, respectively.

a) Demonstrate that the equation that governs the evolution of the photon flux along this system is:

$$\Phi_\nu(x) = \Phi_\nu(0) \cdot \exp\left[\left(N_2 - \frac{g_2}{g_1}N_1\right) \cdot F(\nu) \cdot \left(\frac{c}{2\pi\nu_0}\right)^2 \cdot \frac{A_{21}}{\Delta\nu} \cdot x\right],$$

where A_{21} is the Einstein coefficient.
Consider that the absorption and emission cross-sections ($\sigma_{a\nu}$ and $\sigma_{e\nu}$, respectively) are described by a Lorentz profile with bandwidth $\Delta\nu$ (i.e. homogeneous broadening) and that they are centered at the frequency ν_0. Additionally, consider that I_ν (i.e. the intensity per frequency interval at frequency ν) is spectrally much broader than the cross-sections. Neglect the spontaneous emission in your calculations. (2 points)

Hints: $B_{21} = \frac{g_1}{g_2}B_{12}$, $A_{21} = \frac{8\pi h\nu^3}{c^3}B_{21}$, $\rho(\nu) = \frac{I_\nu}{c}$, where $\rho(\nu)$ is the spectral energy density (per volume)

$\sigma_{i\nu} = \sigma_{i0} \cdot F(\nu) = \sigma_{i0}\frac{\left(\frac{\Delta\nu}{2}\right)^2}{(\nu-\nu_0)^2+\left(\frac{\Delta\nu}{2}\right)^2}$, $\int_{-\infty}^{\infty} F(\nu)d\nu = \frac{\pi\Delta\nu}{2}$

$$\boxed{\begin{gathered}\alpha_{\mathrm{a},\nu} = N_1 \cdot \sigma_{a,\nu} \\ \alpha_{\mathrm{e},\nu} = N_2 \cdot \sigma_{e,\nu}\end{gathered}}$$

<u>From the lecture/seminar :</u>

	Change of photon flux per unit length	**Photon flux evolution**
due to absorption:	$d\,\phi_\nu(x)\big\vert_{\mathrm{abs}} = -\alpha_{\mathrm{a},\nu} \cdot \phi_\nu \cdot dx$	$\phi_\nu(x)\big\vert_{\mathrm{abs}} = \phi_\nu(0) \cdot e^{-N_1\sigma_{\mathrm{a},\nu}\cdot x}$
due to induced emission:	$d\,\phi_\nu(x)\big\vert_{\mathrm{ind}} = +\alpha_{\mathrm{e},\nu} \cdot \phi_\nu \cdot dx$	$\phi_\nu(x)\big\vert_{\mathrm{ind}} = \phi_\nu(0) \cdot e^{N_2\cdot\sigma_{\mathrm{e},\nu}\cdot x}$

Total photon flux evolution $$\phi_\nu(x)\Big|_{\text{total}} = \phi_\nu(0)\cdot e^{(N_2\cdot\sigma_{e,\nu}-N_1\cdot\sigma_{a,\nu})\cdot x}$$ (eq. 1)

<u>Now we consider the relation between cross-section and Einstein coefficient</u>

I_ν much broader than σ_ν

Power consumption $$dP = -dV\cdot\int\alpha_{a,\nu}\cdot I_\nu\cdot \mathrm{d}\nu = -dV\cdot I_\nu\cdot\int N_1\cdot\sigma_{a,\nu}\cdot \mathrm{d}\nu$$ (eq. 2a)

Power emission $$dP = dV\cdot\int\alpha_{\mathrm{e},\nu}\cdot I_\nu\cdot \mathrm{d}\nu = dV\cdot I_\nu\cdot\int N_2\cdot\sigma_{\mathrm{e},\nu}\cdot \mathrm{d}\nu$$ (eq. 2b)

<u>Addtionally we have,</u>

$F(\nu)$: Lorentzian lineshape

$$\boxed{\sigma_{a/e} = \int_{-\infty}^{\infty}\sigma_{a/e,\nu}\cdot \mathrm{d}\nu = \int_{-\infty}^{\infty}\sigma_{a/e,0}\cdot F(\nu)\mathrm{d}\nu = \sigma_{\mathrm{a/e},0}\cdot\frac{\pi\Delta\nu}{2}}$$ (eq. 3)

Given in task:

Substitute (eq.3) into (eq.2a)

Power consumption $$\mathrm{d}P = -\mathrm{d}V \cdot \int \alpha_{a,\nu} \cdot I_\nu \cdot \mathrm{d}\nu = -dV \cdot I_\nu \cdot N_1 \cdot \sigma_{a,0} \cdot \frac{\pi\Delta\nu}{2} \qquad \text{(eq. 3a)}$$

From the lecture we have,

Population density change $$\left.\frac{d\,N_1}{dt}\right|_{\text{absorption}} = -B_{12}\,\rho(\nu)\,N_1 = -B_{12} \cdot \frac{I_\nu}{c} \cdot N_1$$

Power consumption $$dP = -\mathrm{d}V \cdot B_{12} \cdot \frac{I_\nu}{c} \cdot N_1 \cdot h\nu_0 \qquad \text{(eq. 3b)}$$

absorption process /unit time and volume (energy unit $h\nu$)

Combine (eq. 3a) and (eq. 3b)

$$dP = -\cancel{dV} \cdot B_{12} \cdot \frac{\cancel{I_\nu}}{c} \cdot \cancel{N_1} \cdot h\nu_0 = -\cancel{dV} \cdot \cancel{I_\nu} \cdot \cancel{N_1} \cdot \sigma_{a,0} \cdot \frac{\pi\Delta\nu}{2}$$

$$\sigma_{a,0} = \mathrm{B}_{12} \cdot \frac{2h\nu_0}{\pi c\Delta\nu} = \mathrm{B}_{21}\frac{g_2}{g_1} \cdot \frac{2h\nu_0}{\pi c\Delta\nu} = \frac{g_2}{g_1}\left(\frac{c}{2\pi\nu_0}\right)^2 \frac{A_{21}}{\Delta\nu} \qquad \text{(eq. 4)}$$

$$B_{12} = \frac{g_2}{g_1}B_{21} \qquad A_{21} = \frac{8\pi h\nu^3}{c^3}B_{21}$$

Similarly in induced emission:

Combine lecture and (eq. 2b)

Power emission

$$dP = \cancel{d}V \cdot B_{21} \cdot \cancel{\frac{I_\nu}{c}} \cdot \cancel{N_2} \cdot h\nu_0 = \cancel{-}dV \cdot \cancel{I_\nu} \cdot \cancel{N_2} \cdot \sigma_{e,0} \cdot \frac{\pi\Delta\nu}{2}$$

$$\sigma_{e,0} = B_{21}\frac{2h\nu_0}{\pi c\Delta\nu} = \left(\frac{c}{2\pi\nu_0}\right)^2 \frac{A_{21}}{\Delta\nu} \qquad \text{(eq. 5)}$$

$$A_{21} = \frac{8\pi h\nu^3}{c^3} B_{21}$$

Introduce the (eq. 4) and (eq. 5) into (eq. 1)

$$\phi_\nu(x)\Big|_{\text{total}} = \phi_\nu(0) \cdot e^{(N_2 \cdot \sigma_{e,\nu} - N_1 \cdot \sigma_{a,\nu}) \cdot x} \qquad \sigma_{a/e,\nu} = \sigma_{a/e,0} \cdot F(\nu)$$

$$\boxed{\phi_\nu(x)\Big|_{\text{total}} = \phi_\nu(0) \cdot e^{\left(N_2 - N_1\frac{g_2}{g_1}\right) F(\nu) \left(\frac{c}{2\pi\nu_0}\right)^2 \frac{A_{21}}{\Delta\nu} \cdot x}}$$

b) Demonstrate, using the equations obtained in the previous sections, that in spite of having inversion, there is no photon flux gain in this 2-level system. (1 point)

Hints: $B_{21} = \frac{g_1}{g_2} B_{12}$, $A_{21} = \frac{8\pi h \nu^3}{c^3} B_{21}$, $\rho(\nu) = \frac{I_\nu}{c}$, where $\rho(\nu)$ is the spectral energy density (per volume)

$\sigma_{i\nu} = \sigma_{i0} \cdot F(\nu) = \sigma_{i0} \frac{\left(\frac{\Delta\nu}{2}\right)^2}{(\nu-\nu_0)^2 + \left(\frac{\Delta\nu}{2}\right)^2}$, $\int_{-\infty}^{\infty} F(\nu) d\nu = \frac{\pi \Delta\nu}{2}$

From solution 1a:

$$\phi_\nu(x)\Big|_{\text{total}} = \phi_\nu(0) \cdot \exp\left(\left(N_2 - N_1 \frac{g_2}{g_1}\right) F(\nu) \left(\frac{c}{2\pi\nu_0}\right)^2 \frac{A_{21}}{\Delta\nu} \cdot x\right)$$

Photon flux gain:

$$\phi_\nu(x)\Big|_{\text{total}} > \phi_\nu(0) \Longrightarrow \exp\left(\left(N_2 - N_1 \frac{g_2}{g_1}\right) F(\nu) \left(\frac{c}{2\pi\nu_0}\right)^2 \frac{A_{21}}{\Delta\nu} \cdot x\right) > 1$$

$$\underbrace{\left(N_2 - N_1 \frac{g_2}{g_1}\right)}_{?} \underbrace{F(\nu) \left(\frac{c}{2\pi\nu_0}\right)^2 \frac{A_{21}}{\Delta\nu} \cdot x}_{\text{Always} > 0} > 0$$

Consider the global rate of population in upper level in Thermal equilibrium:

Steady state

$$\frac{dN_2}{dt} = \left.\frac{dN_2}{dt}\right|_{abs} + \left.\frac{dN_2}{dt}\right|_{ind} + \left.\frac{dN_2}{dt}\right|_{spon} = (B_{12}N_1 - B_{21}N_2)\rho(\nu) - A_{21}N_2 = 0 \quad \Longrightarrow \quad B_{12}\rho(\nu)N_1 = (\rho(\nu)B_{21} + A_{21})N_2$$

$$\frac{N_2}{N_1} = \frac{B_{12}\rho(\nu)}{B_{21}\rho(\nu) + A_{21}} = \frac{\frac{g_2}{g_1}B_{21}\rho(\nu)}{B_{21}\rho(\nu) + \frac{8\pi h\nu^3}{c^3}B_{21}} = \frac{\frac{g_2}{g_1}}{1 + \frac{8\pi h\nu^3}{c^2 I_\nu}}$$

$B_{21} = \frac{g_1}{g_2}B_{12}$, $A_{21} = \frac{8\pi h\nu^3}{c^3}B_{21}$

Therefore:

$$\left(N_2 - N_1\frac{g_2}{g_1}\right) = \frac{\frac{g_2}{g_1}N_1}{1 + \frac{8\pi h\nu_0^3}{c^2 I_\nu}} - N_1\frac{g_2}{g_1} < 0 \quad \Longrightarrow \quad \phi_\nu(x) < \phi_\nu(0)$$

No gain possible in a 2 level system!

Problem 3 (5 points)

In the lecture we have modelled the interaction between the photons and the laser-active atoms in an active medium with the help of a well-defined area associated to each of these atoms: the cross-section.

a) For a given wavelength within the absorption bandwidth of the active medium, draw schematically the cross-sections of several laser-active atoms contained in a surface element dA of the medium for two different cases: an homogeneously-broadened medium and an inhomogeneously-broadened one. (1 point)

b) Explain qualitatively how the individual cross-sections of the laser ions change with the wavelength of the photons both for an homogeneously-broadened and for an inhomogenoeusly-broadened active medium. (1 point)

c) Now imagine an homogeneously-broadened medium with a surface of $1cm^2$ and a thickness of $1\mu m$. If the absorption cross-section of the atoms is $2 \cdot 10^{-21} m^2$ at $633nm$ and the laser-active ion concentration is $1.5 \cdot 10^{26} m^{-3}$, what is the probability of a photon being absorbed when hitting this active medium?. (1 point)
Hint: for the calculation of the probability consider the active medium as being two-dimensional with a surface of $1cm^2$.

d) Disregarding the effect of spontaneous emission and assuming that the distribution of laser-active atoms in the medium is homogeneous, what is the maximum number of photons that can be absorbed if we illuminate the medium homogeneously during 1 second with $1\mu W$ of light at $633nm$? and if we illuminate it during one second with $100W$ of light at $633nm$? Explain your results. (2 points)

Problem 3 (5 points)

In the lecture we have modelled the interaction between the photons and the laser-active atoms in an active medium with the help of a well-defined area associated to each of these atoms: the cross-section.

a) For a given wavelength within the absorption bandwidth of the active medium, draw schematically the cross-sections of several laser-active atoms contained in a surface element dA of the medium for two different cases: an homogeneously-broadened medium and an inhomogeneously-broadened one. (1 point)

Recap of the lecture: What is cross-section?

- **cross-section defines probability that absorption/emission occurring**

For absorption:

$$\sigma_{\mathrm{abs}} = \frac{B_{12} h\nu_{21}}{c}$$

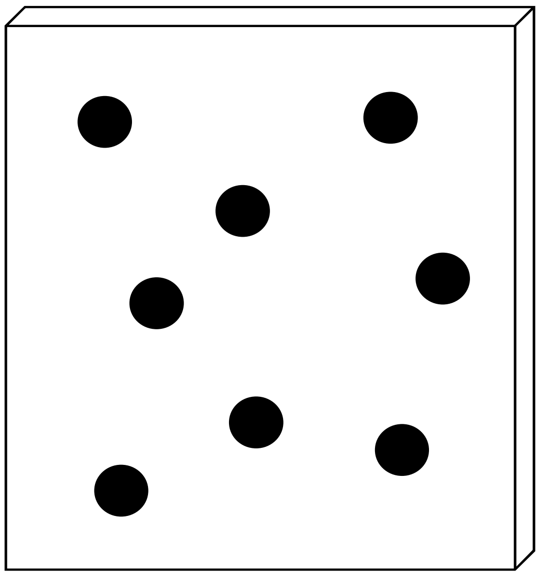
- **Cross-sections are <u>related</u> but <u>not equivalent</u> to Einstein coefficients**

Problem 2 (5 Points)

In the lecture we have modelled the interaction between the photons and the laser-active atoms in an active medium with the help of a well-defined area associated to each of these atoms: the cross-section.

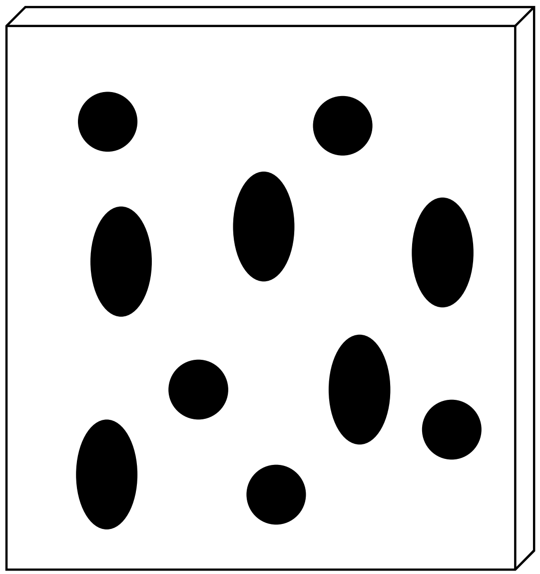
a) For a given wavelength within the absorption bandwidth of the active medium, draw schematically the cross-sections of several laser-active atoms contained in a surface element dA of the medium for two different cases: an homogeneously-broadened medium and an inhomogeneously-broadened one. (1 Point)

Homogenously-broadened medium



Here all cross-sections are equal for all the atoms for any given λ

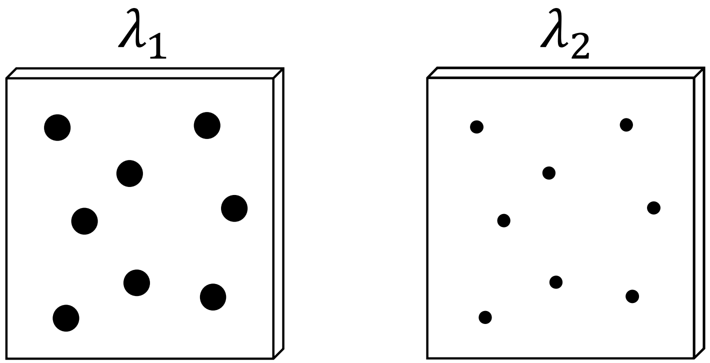
Inhomogenously-broadened medium



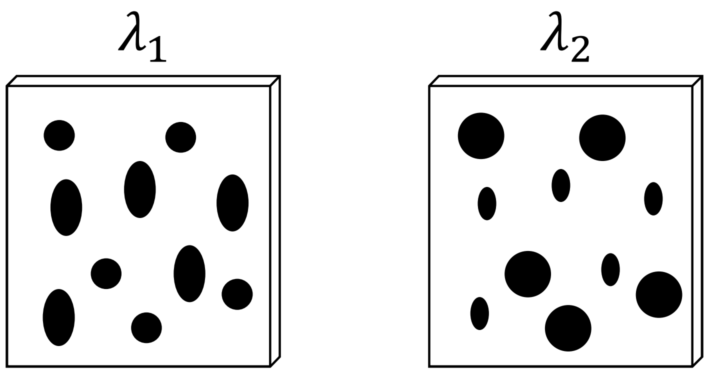
Here the cross-sections for the different atoms for a given λ is different

b) Explain qualitatively how the individual cross-sections of the laser ions change with the wavelength of the photons both for an homogeneously-broadened and for an inhomogenoeusly-broadened active medium. (1 Point)

In an homogenously-broadened medium all the atoms have always the same cross-sections for any given wavelength. So when the wavelength changes the cross-sections of all the atoms change equally



In an in inhomogenously-broadened medium the cross-sections of the individual atoms change differently depending on which class the atoms belong to



c) Now imagine an homogeneously-broadened medium with a surface of 1 cm^2 and a thickness of 1 μm. If the absorption cross-section of the atoms is $2 \cdot 10^{-21}$ m^2 at 633 nm and the laser-active ion concentration is $1.5 \cdot 10^{26}$ m^{-3}, what is the probability of a photon being absorbed when hitting this active medium? (1 Point)
Hint: for the calculation of the probability consider the active medium as being two-dimensional with a surface of 1 cm^2.

$$Volume\ V = (10^{-2}m)^2 \cdot 10^{-6}m = 10^{-10}m^3$$

$$number\ laser\ active\ ions\ N_{ions} = V \cdot n_{concentration} = 10^{-10}m^3 \cdot 1.5 \cdot 10^{26}\frac{ions}{m^3} = 1.5 \cdot 10^{16}$$

The total area covered by the cross-sections of the atoms at 633nm is

$$A_{abs} = N_{ions} \cdot \sigma_a = 1.5 \cdot 10^{16} \cdot 2 \cdot 10^{-21}\ m^2 = 3 \cdot 10^{-5}\ m^2$$

Which represents $\frac{3 \cdot 10^{-5}\ m^2}{(10^{-2}\ m)^2} = 0.3$ of the total area → 30% probability for absorbtion

d) Disregarding the effect of spontaneous emission and assuming that the distribution of laser-active atoms in the medium is homogeneous, what is the maximum number of photons that can be absorbed if we illuminate the medium homogeneously during 1 second with 1 μW of light at 633 nm? and if we illuminate it during one second with 100 W of light at 633 nm? Explain your results. (2 Points)

With 1µW at 633nm during 1 second,

the total number of photons is:

$$n_{photons} = \frac{P \cdot t}{h \cdot \frac{c}{\lambda}} = \frac{1\mu W \cdot 1s \cdot 633nm}{6.62 \cdot 10^{-34} Js \cdot 3 \cdot 10^{8} \frac{m}{s}} = 3.184 \cdot 10^{12}$$

Since $n_{photons} << n_{atoms}$ ($1.5 \cdot 10^{16}$, see c) then the effect of stimulated emission will be negligible, i.e. the probability of absorption (= 30%, see c) will always be much higher than the probability of emission. Thus

$$n_{absorbed\ photons} = n_{photons} \cdot P_{abs} = 3.184 \cdot 10^{12} \cdot 0.3 = 9.5 \cdot 10^{11}$$

d) Disregarding the effect of spontaneous emission and assuming that the distribution of laser-active atoms in the medium is homogeneous, what is the maximum number of photons that can be absorbed if we illuminate the medium homogeneously during 1 second with 1 μW of light at 633 nm? and if we illuminate it during one second with 100 W of light at 633 nm? Explain your results. (2 Points)

However, if now we illuminate the active medium with 100W, then the total number of photons will be

$$n_{photons} = \frac{P \cdot t}{h \cdot \frac{c}{\lambda}} = \frac{100W \cdot 1s \cdot 633nm}{6.62 \cdot 10^{-34} Js \cdot 3 \cdot 10^{8} \frac{m}{s}} = 3.184 \cdot 10^{20} \gg n_{atoms}$$

The effect of stimulated emission cannot be neglected anymore, which means that only 50% of the atoms can be excited (at most) in the upper energy level (two level system) → saturation of absorption

$$n_{absorbed\ photons} = \frac{n_{atoms}}{2} = 7.5 \cdot 10^{15}$$