



**Institute of
Applied Physics**

Friedrich-Schiller-Universität Jena

Lens Design I

Lecture 9: Optimization I

2024-06-13

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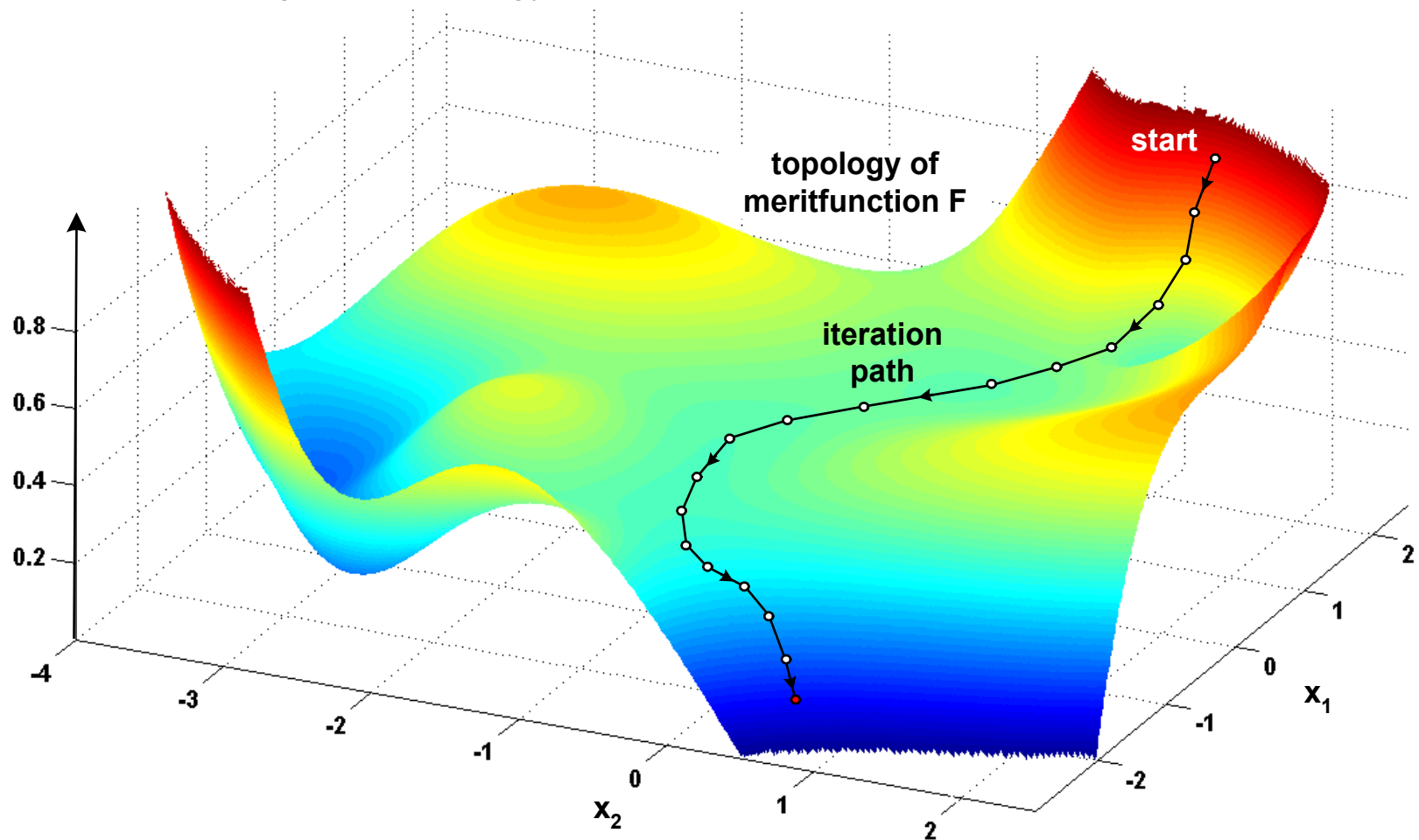
Preliminary Schedule - Lens Design I 2024

1	04.04.	Basics	Zhang	Introduction, Zemax interface, menus, file handling, preferences, Editors, updates, windows, coordinates, System description, 3D geometry, aperture, field, wavelength
2	18.04.	Properties of optical systems I	Tang	Diameters, stop and pupil, vignetting, layouts, materials, glass catalogs, raytrace, ray fans and sampling, footprints
3	25.04.	Properties of optical systems II	Tang	Types of surfaces, cardinal elements, lens properties, Imaging, magnification, paraxial approximation and modelling, telecentricity, infinity object distance and afocal image, local/global coordinates
4	02.05.	Properties of optical systems III	Tang	Component reversal, system insertion, scaling of systems, aspheres, gratings and diffractive surfaces, gradient media, solves
5	16.05.	Advanced handling I	Tang	Miscellaneous, fold mirror, universal plot, slider, multiconfiguration, lens catalogs
6	23.05.	Aberrations I	Zhang	Representation of geometrical aberrations, spot diagram, transverse aberration diagrams, aberration expansions, primary aberrations
7	30.05.	Aberrations II	Zhang	Wave aberrations, Zernike polynomials, measurement of quality
8	06.06.	Aberrations III	Tang	Point spread function, optical transfer function
9	13.06.	Optimization I	Tang	Principles of nonlinear optimization, optimization in optical design, general process, optimization in Zemax
10	20.06.	Optimization II	Zhang	Initial systems, special issues, sensitivity of variables in optical systems, global optimization methods
11	27.06.	Correction I	Zhang	Symmetry principle, lens bending, correcting spherical aberration, coma, astigmatism, field curvature, chromatical correction
12	04.07.	Correction II	Zhang	Field lenses, stop position influence, retrofocus and telephoto setup, aspheres and higher orders, freeform systems, miscellaneous



1. Principles of nonlinear optimization
2. Optimization in optical design
3. General process
4. Optimization in Zemax

- Topology of the merit function in 2 dimensions
- Iterative down climbing in the topology



Where is the lowest point?



- Complex topology of the merit function
 - Many local minima
 - Merit function not smooth
 - Global minimum not known
 - Even higher dimension in optical system optimization



Mathematical description of the problem:

- n variable parameters
- m target values
- Jacobi system matrix of derivatives,
Influence of a parameter change on the
various target values,
sensitivity function
- Scalar merit function
- Gradient vector of topology
- Hesse matrix of 2nd derivatives

$$\vec{x}$$

$$\vec{f}(\vec{x})$$

$$J_{ij} = \frac{\partial f_i}{\partial x_j}$$

$$F(\vec{x}) = \sum_{i=1}^m w_i \cdot [y_i - f(\vec{x})]^2$$

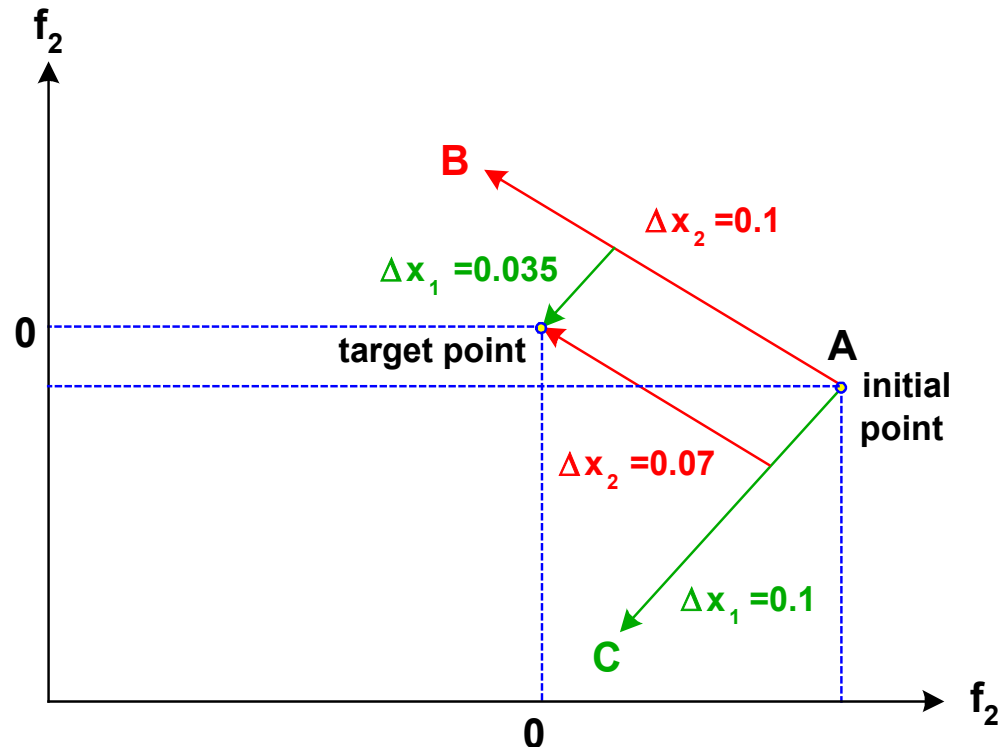
$$g_j = \frac{\partial F}{\partial x_j}$$

$$H_{jk} = \frac{\partial^2 F}{\partial x_j \partial x_k}$$



Optimization Principle for 2 Degrees of Freedom

- Aberration depends on two parameters
- Linearization of sensitivity, Jacobian matrix
Independent variation of parameters
- Vectorial nature of changes:
Size and direction of change
- Vectorial decomposition of an ideal
step of improvement,
linear interpolation
- Due to non-linearity:
change of Jacobian matrix,
next iteration gives better result



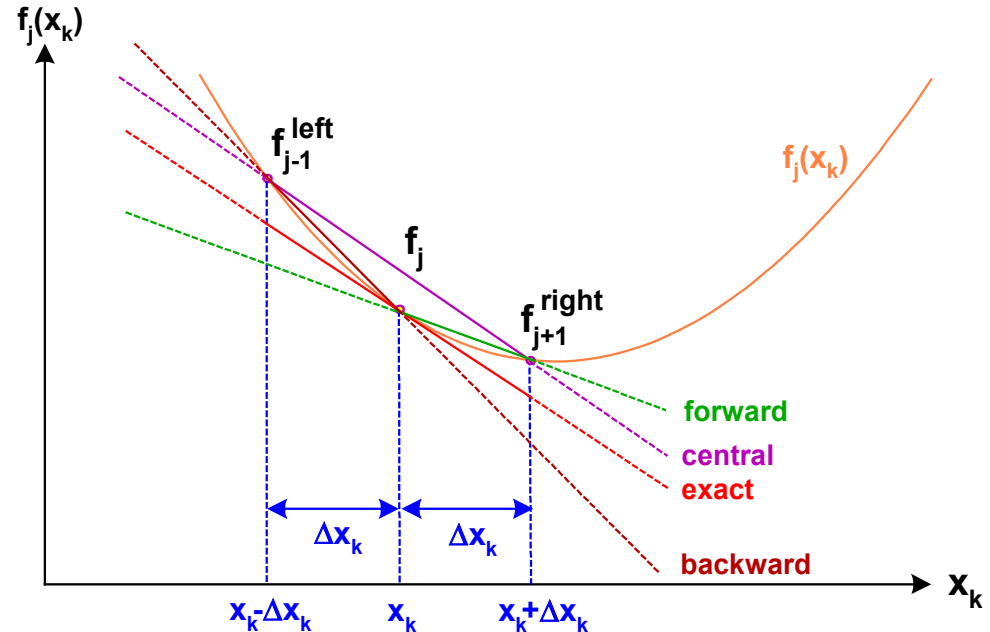


Calculation of Derivatives

- Derivative vector in merit function topology:
Necessary for gradient-based methods
- Numerical calculation by finite differences
- Possibilities and accuracy

$$g_{jk} = \frac{\partial f_j(\vec{x})}{\partial x_k} = \nabla_{x_k} f_j(\vec{x})$$

$$g_{jk} = \frac{f_j^{\text{right}} - f_j}{\Delta x_k}$$



- Linearized environment around working point
Taylor expansion of the target function

$$\vec{f} = \vec{f}_0 + \underline{J} \cdot \vec{x}$$

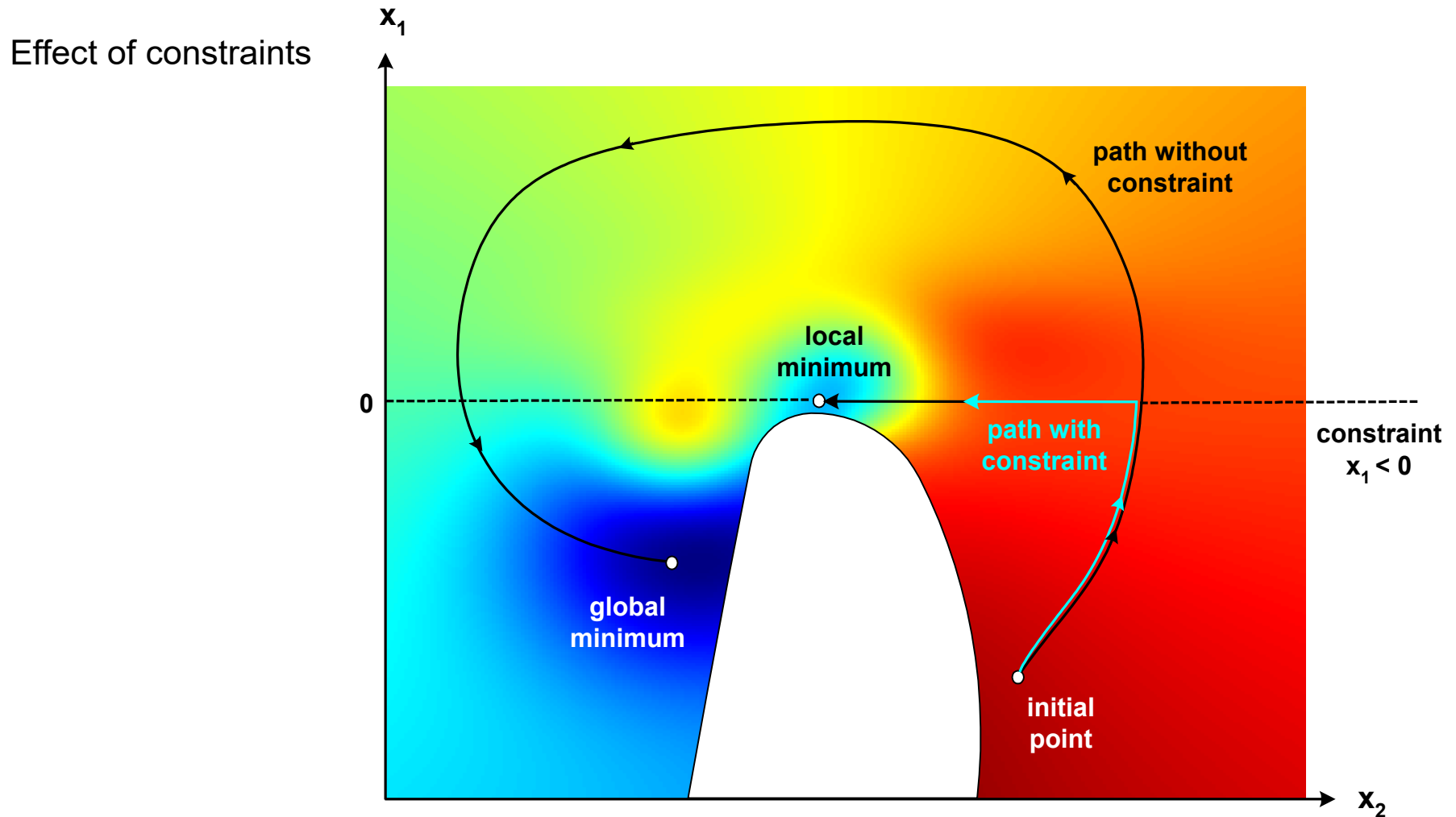
- Quadratical approximation of the merit function

$$F(\vec{x}) = F(\vec{x}_0) + \underline{J} \cdot \Delta \vec{x} + \frac{1}{2} \cdot \Delta \vec{x} \cdot \underline{H} \cdot \Delta \vec{x}$$

- Solution by lineare Algebra
system matrix \underline{A}
cases depending on the numbers
of n / m

$$\underline{A}^+ = \begin{cases} \underline{A}^{-1} & \text{if } m = n \\ \left(\underline{A}^T \underline{A} \right)^{-1} \cdot \underline{A}^T & \text{if } m > n \text{ (under determined)} \\ \underline{A}^T \cdot \left(\underline{A} \underline{A}^T \right)^{-1} & \text{if } m < n \text{ (over determined)} \end{cases}$$

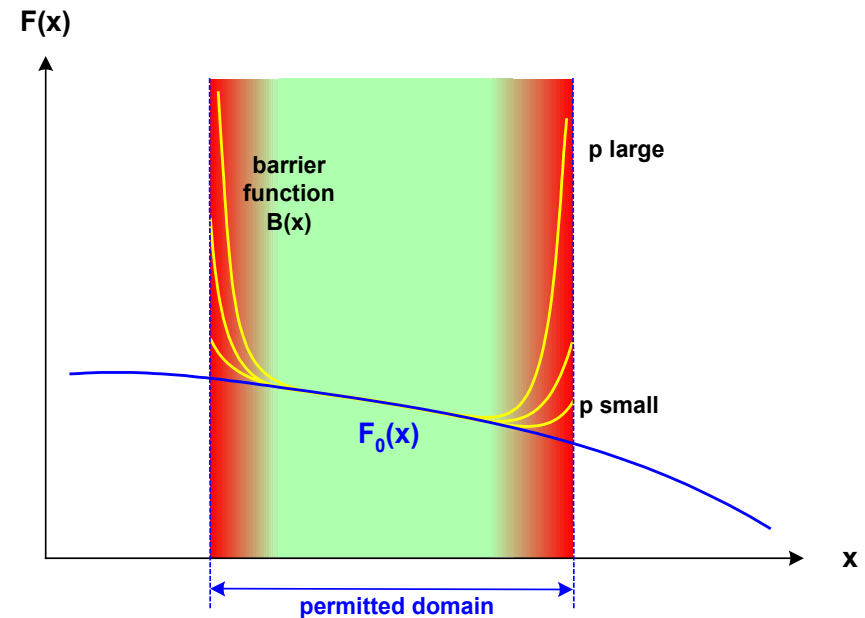
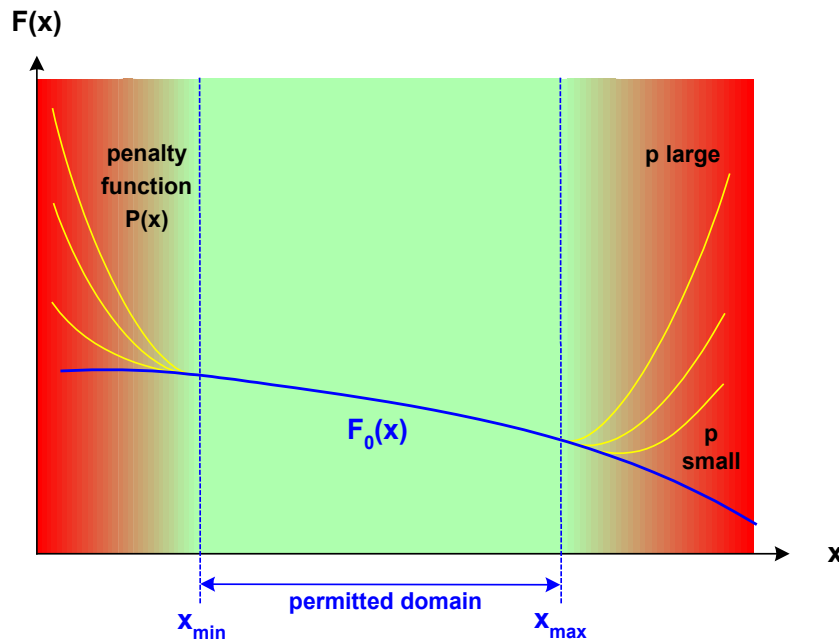
- Iterative numerical solution:
Strategy: optimization of
 - direction of improvement step
 - size of improvement step



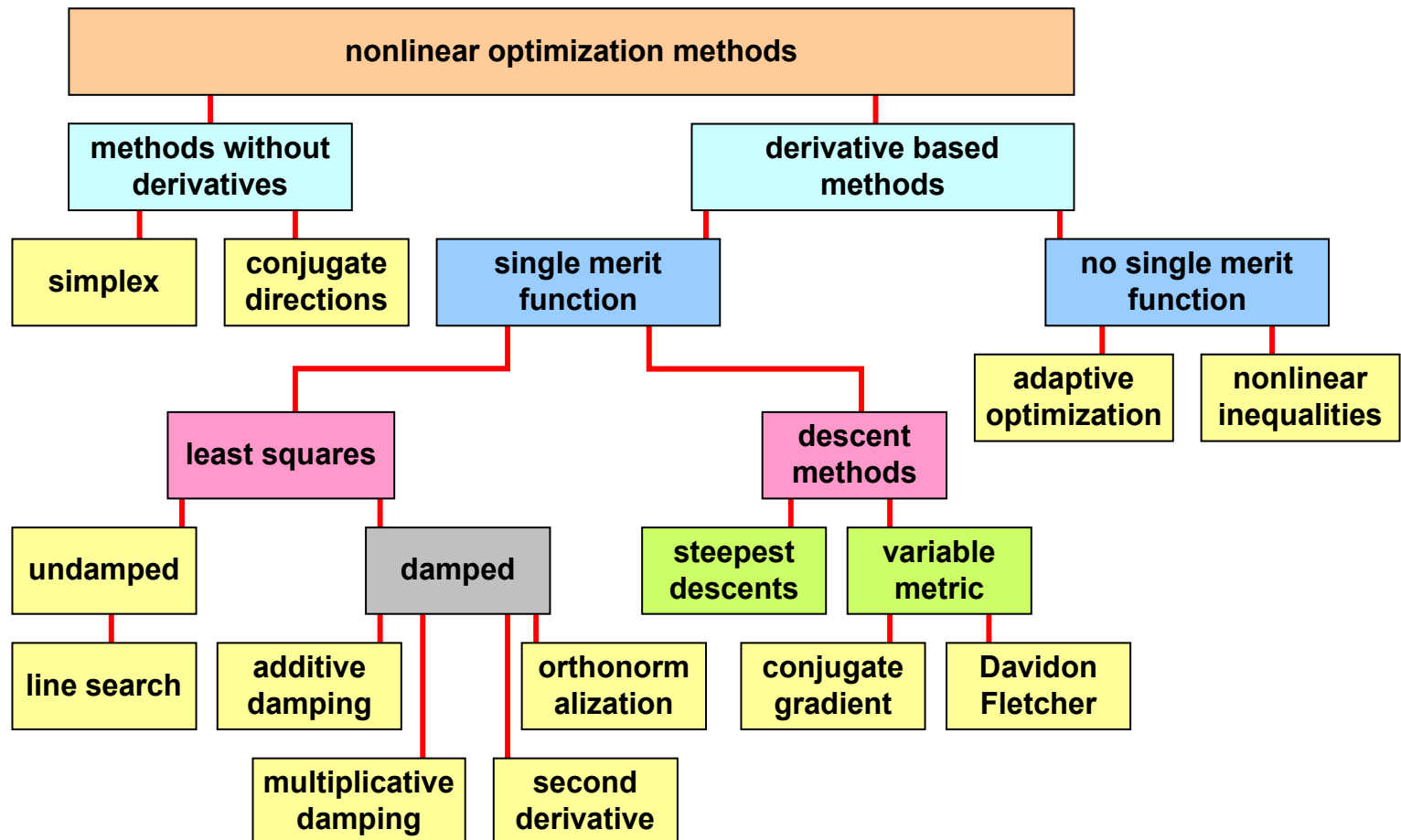


Boundary Conditions and Constraints

- Types of constraints
 - Equation, rigid coupling, pick up
 - One-sided limitation, inequality
 - Double-sided limitation, interval
- Numerical realizations :
 - Lagrange multiplier
 - Penalty function
 - Barriere function
 - Regular variable, soft-constraint



Local working optimization algorithms





Optimization Merit Function in Optical Design

- Goal of optimization:
Find the system layout which meets the required performance targets according of the specification
- Formulation of performance criteria must be done for:
 - Apertur rays
 - Field points
 - Wavelengths
 - Optional several zoom or scan positions
- Selection of performance criteria depends on the application:
 - Ray aberrations
 - Spot diameter
 - Wavefornt description by Zernike coefficients, rms value
 - Strehl ratio, Point spread function
 - Contrast values for selected spatial frequencies
 - Uniformity of illumination
- Usual scenario:
Number of requirements and targets quite larger than degrees of freedom,
Therefore only solution with compromise possible

- Merit function:
Weighted sum of deviations from target values
- Formulation of target values:
 1. fixed numbers
 2. one-sided interval (e.g. maximum value)
 3. interval
- Problems:
 1. linear dependence of variables
 2. internal contradiction of requirements
 3. initial value far off from final solution
- Types of constraints:
 1. exact condition (hard requirements)
 2. soft constraints: weighted target
- Finding initial system setup:
 1. modification of similar known solution
 2. Literature and patents
 3. Intuition and experience

$$\Phi = \sum_{j=1,m} g_j \cdot \left(f_j^{ist} - f_j^{soll} \right)^2$$

- Characterization and description of the system delivers free variable parameters of the system:
 - Radii
 - Thickness of lenses, air distances
 - Tilt and decenter
 - Free diameter of components
 - Material parameter, refractive indices and dispersion
 - Aspherical coefficients
 - Parameter of diffractive components
 - Coefficients of gradient media
- General experience:
 - Radii as parameter very effective
 - Benefit of thickness and distances only weak
 - Material parameter can only be changes discrete

Constraints in the optimization of optical systems:

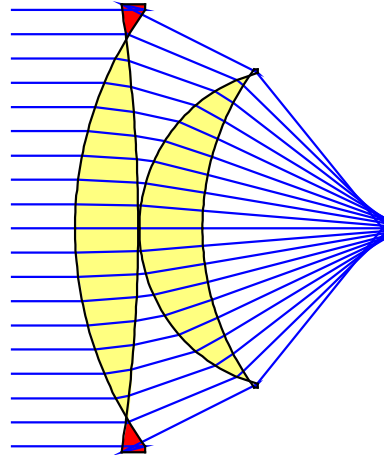
1. Discrete standardized radii (tools, metrology)
2. Total track
3. Discrete choice of glasses
4. Edge thickness of lenses (handling)
5. Center thickness of lenses (stability)
6. Coupling of distances (zoom systems, forced symmetry,...)
7. Focal length, magnification, working distance
8. Image location, pupil location
9. Avoiding ghost images (no concentric surfaces)
10. Use of given components (vendor catalog, availability, costs)

Lack of Constraints in Optimization

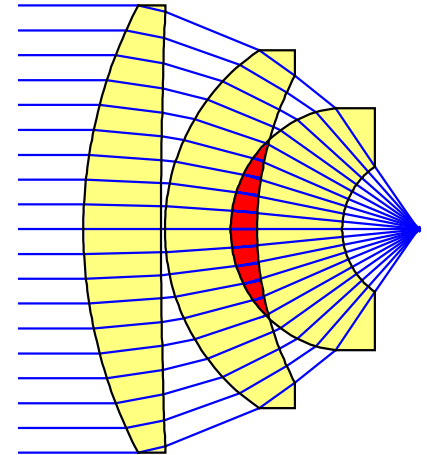


Illustration of not useful results due to non-sufficient constraints

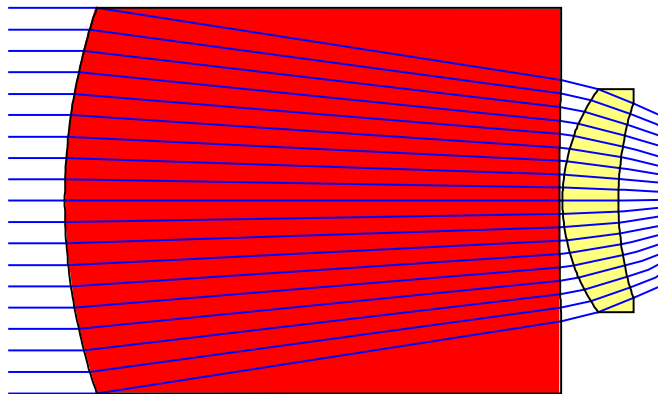
negative edge thickness



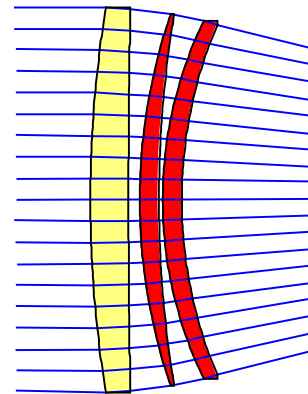
negative air distance



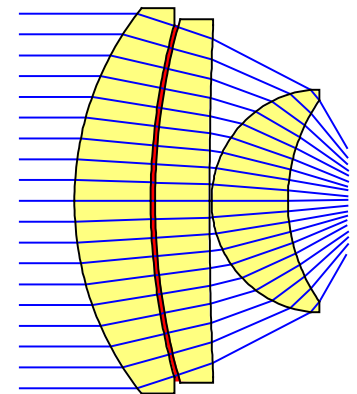
lens thickness to large



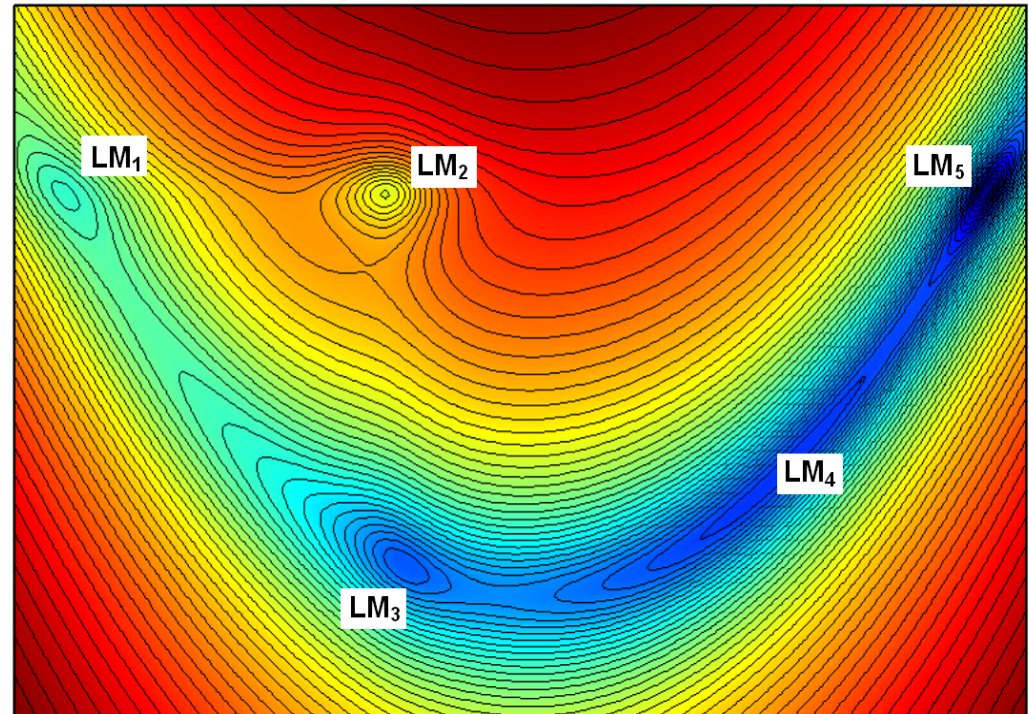
lens stability to small



air space to small



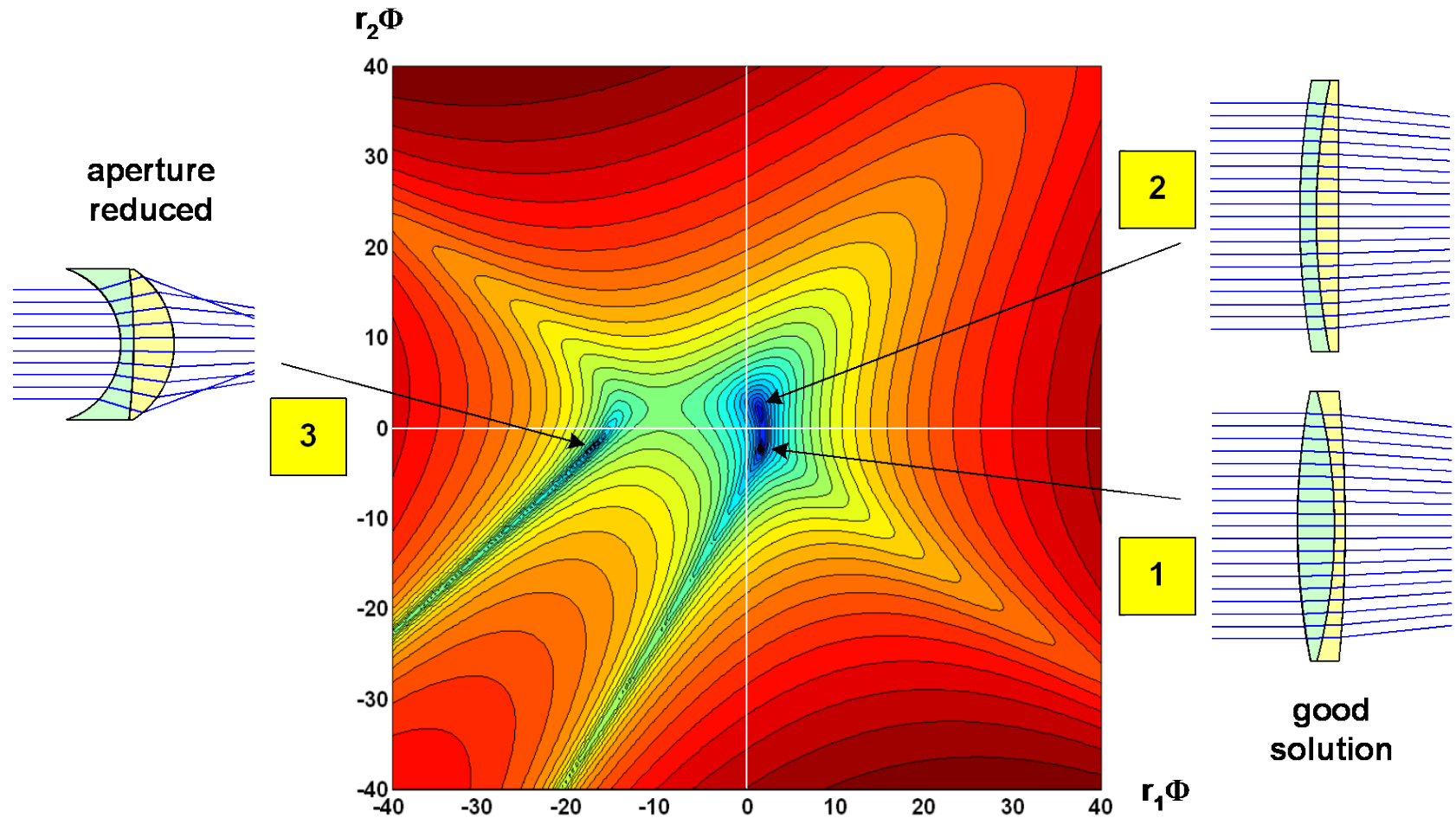
- Typical in optics:
Twisted valleys in the topology
- Selection of local minima





Optimization Landscape of an Achromate

- Typical merit function of an achromate
- Three solutions, only two are useful



1. Paraxial layout:
 - specification data, magnification, aperture, pupil position, image location
 - distribution of refractive powers
 - locations of components
 - system size diameter / length
 - mechanical constraints
 - choice of materials for correcting color and field curvature
2. Correction/consideration of Seidel primary aberrations of 3rd order for ideal thin lenses, fixation of number of lenses
3. Insertion of finite thickness of components with remaining ray directions
4. Check of higher order aberrations
5. Final correction, fine tuning of compromise
6. Tolerancing, manufactability, cost, sensitivity, adjustment concepts



Strategy of Correction and Optimization

Usefull options for accelerating a stagnated optimization:

- split a lens
- increase refractive index of positive lenses
- make surface with large spherical surface contribution aspherical
- Change stop position
- break cemented components
- use glasses with anomalous partial dispersion



Optimize a single lens with the data $\lambda = 546.07$ nm, object in the distance 100 mm from the lens on axis only, focal length $f = 45$ mm and numerical aperture $NA = 0.07$ in the object space. The lens should be made of the Schott glass N-K5 and has a thickness of 5 mm.

- a) Optimize the singlet with different starting systems. Does it always work well? Is the optimized lens diffraction limited in its performance ?
- b) One possibility to improve the result is to use an aspherical lens. The first approach is to use the rear surface with a conic constant to allow the program a conic section as solution. Is this sufficient to get a diffraction limited solution ?
- c) Now enlarge the numerical aperture by a factor of two. Re-optimize the system. What about the diffraction limited performance ? Use an aspherical coefficient of 4th order to improve the system. What is the result ?
- d) Now introduce a finite object size of diameter 10 mm. What is the dominant aberration for the off-axis field points ? Can the system be made diffraction limited by re-optimization, for example with more aspherical constants ? What can be done to get a better performance ?