Problem 1 Jinsong Liu 206216

(a)

$$\because W_p = \frac{\eta P_p}{h\nu_p A d (n_{tot} - n)}$$

$$\therefore P_p = h\nu_p A d (n_{tot} - n) \frac{W_p}{\eta}$$

$$P_{th} = h\nu_p A d (n_{tot} - n_{th}) \frac{W_{th}}{\eta}$$

$$W_{th} = \Gamma \frac{n_{th}}{n_{tot} - n_{th}}$$

$$\therefore P_{th} = h\nu_p A d (n_{tot} - n_{th}) \cdot \frac{\Gamma n_{th}}{\eta} (n_{tot} - n_{th})^{-1}$$

$$= h\nu_p A d \frac{\Gamma n_{th}}{\eta}$$

$$= h\nu_s A d \frac{\Gamma n_{th}}{\eta \eta_{stokes}}$$

$$= A d \frac{I_s \frac{1}{c\tau_{ph}}}{\eta \eta_{stokes}}$$

$$= 2 A d \frac{1}{c\tau_R} \frac{I_s \frac{\tau_R}{\tau_{ph}}}{2\eta \eta_{stokes}}$$

$\because\ d/c = \frac{1}{2}\tau_R \quad \tau_R/\tau_{ph} = -\ln(1-L) - \ln(1-T)$

$$\therefore\ P_{th} = \frac{-\ln[(1-L)(1-T)]\ A \cdot I_{sat}}{2\eta\eta_{stokes}}$$

$$P_{out} = \eta \cdot \frac{\nu_s}{\nu_p} \cdot T \cdot -\ln[(1-L)(1-T)]^{-1} \cdot (P_p - P_{th})$$

$$= \frac{\eta \cdot \eta_{stokes} \cdot T}{-\ln[(1-L)(1-T)]} \cdot (P_p - P_{th})$$

$$\therefore\ \sigma_s = \frac{\eta \cdot \eta_{stokes} \cdot T}{-\ln[(1-L)(1-T)]}$$

(b) $P_l = \sigma_s \cdot (P_p - P_{th})$

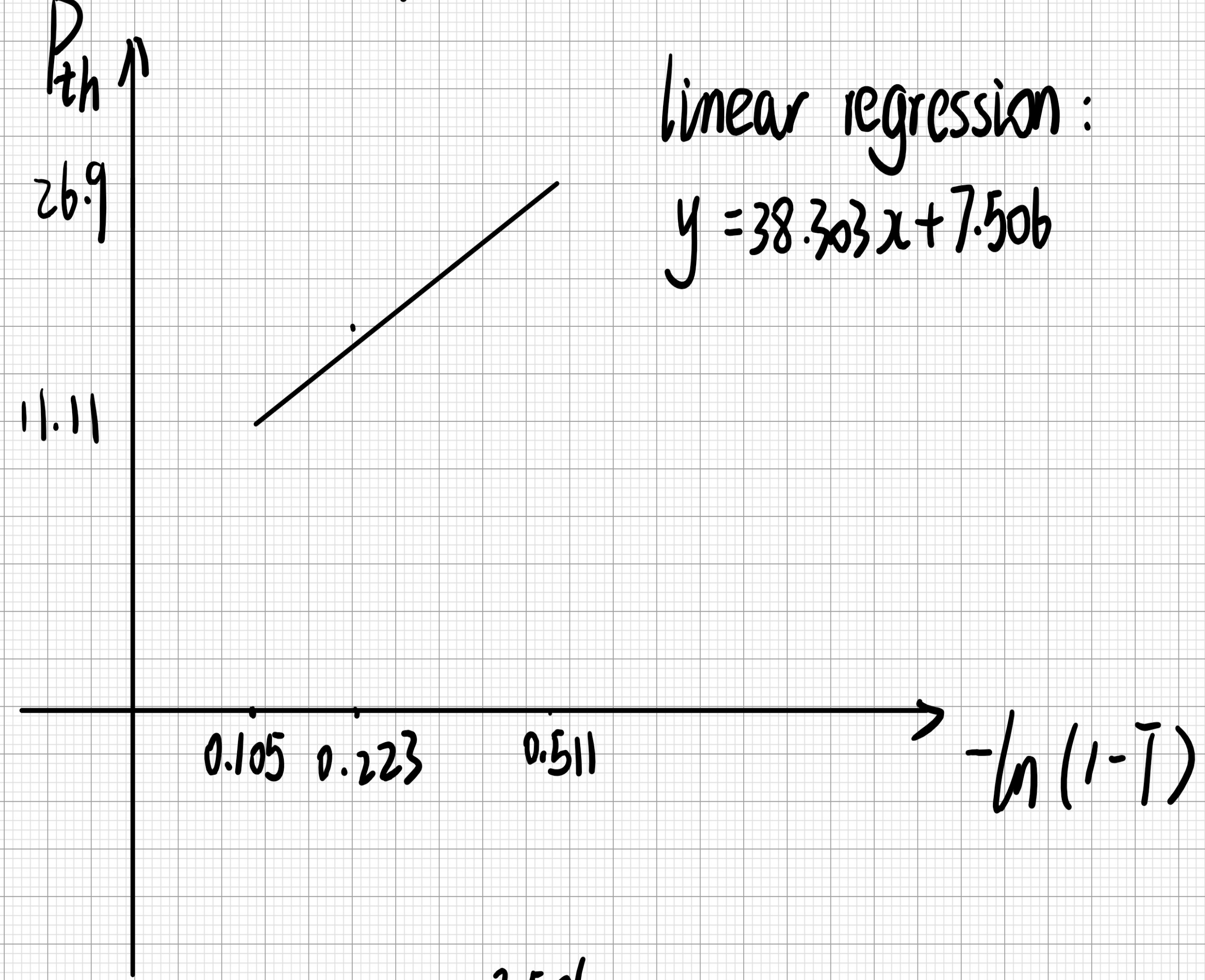
From the data sheet and linear regression calculation we can get:

$T = 0.1 \quad P_l = 0.18 P_p - 2 \quad P_{th} = \frac{100}{9} \approx 11.11$

$T = 0.2 \quad P_l = 0.255 P_p - 4.25 \quad P_{th} = \frac{50}{3} \approx 16.67$

$T = 0.4 \quad P_l = 0.29 P_p - 7.8 \quad P_{th} \approx 26.90$

$$P_{th} = \frac{-\ln[(1-L)(1-T)] \cdot A I_{sat}}{2\eta\eta_{stokes}} = -\ln(1-T)\frac{A I_{sat}}{2\eta\eta_{stokes}} - \ln(1-L)\frac{A I_{sat}}{2\eta\eta_{sto}}$$



linear regression:

$y = 38.303x + 7.506$

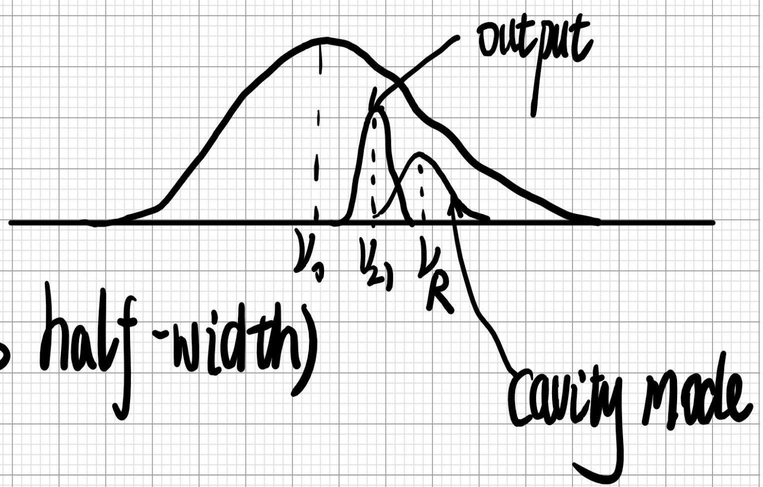
let $y=0$ $\quad x = \frac{7.506}{38.303} = 0.196 = -\ln(1-L)$

$\therefore L \approx 0.178 = 17.8\%$

Problem 2 Jinsong Liu 206216

(a)

$$P(\nu) \sim \frac{\left(\frac{2\delta\nu_R}{2}\right)^2}{(\nu-\nu_R)^2+\left(\frac{2\delta\nu_R}{2}\right)^2}$$



$\delta\nu_R = (4\pi\tau_{ph})^{-1}$ ($\delta\nu_R$ is half-width)

(b) The power P is the intensity equal to that of the stimulated emission of all excited modes in the cavity. Only if the desired mode is separated from the large number of undesired ones, would the rather monochromatic radiation stand out clearly against the much wider frequency distribution of spontaneous emission.

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(c)

He-Ne: $\tau_R = \frac{L}{\gamma c}$ $\delta\nu_R = 1/2\pi\tau_R$

$L = 20\,cm$ $\gamma = -\ln R_2 \approx 0.01$ $\lambda = 633nm$ $\nu_{21} = 4.7\times10^{14}Hz$

$$\tau_R \approx \frac{20\times10^{-2}}{0.01\times3\times10^8} \approx 6\times10^{-8}s$$

$\delta\nu_R \approx 2.65\times10^6\ Hz$ $P = 1\,mW$

$$\Delta\nu_{21} = \frac{2\pi h\nu_{21}(\delta\nu_R)^2}{P} = 0.01\ Hz$$

GaAs:

$$R = \left|\frac{n_1-n_2}{n_1+n_2}\right|^2 = \left(\frac{2.5}{4.5}\right)^2 \approx 0.3$$

$\gamma = -\ln R \approx 1.2$ $L = 300\mu m$

$$\tau_R = nL/\gamma c = \frac{3.5\times300\times10^{-6}}{1.2\times3\times10^8} = 2.92\times10^{-12}s$$

$= 2.92\,ps$

$\delta\nu_R = 5.5\times10^{10}Hz$

$$\Delta\nu_{21} = \frac{2\pi h\nu_{21}(\delta\nu_R)^2}{P} = 4.3MHz$$

Problem 3 Jinsong Liu 206216

(a)

$$\frac{dN_2}{dt} = W_{02} N_0 - S_{21} N_2$$

$$\frac{dN_1}{dt} = S_{21} N_2 + W_{01} N_0 - S_{10} N_1 - W_{10} N_1$$

$$\frac{dN_0}{dt} = -W_{02} N_0 - W_{01} N_0 + W_{10} N_1 + S_{10} N_1$$