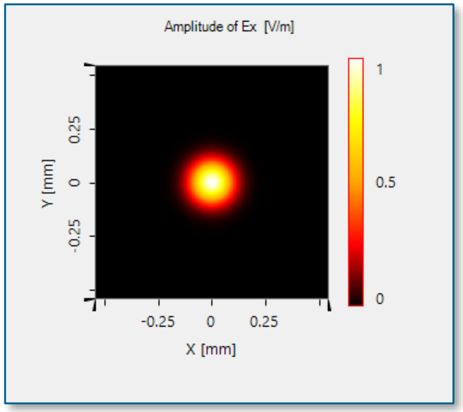
Seminar II – 20.01.2016 & 27.01.2016

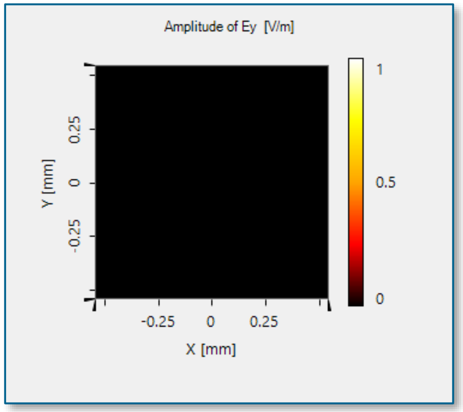
Introduction to Optical Modeling

Site Zhang site.zhang@uni-jena.de
Applied Computational Optics Group - ACOG
www.applied-computational-optics.org

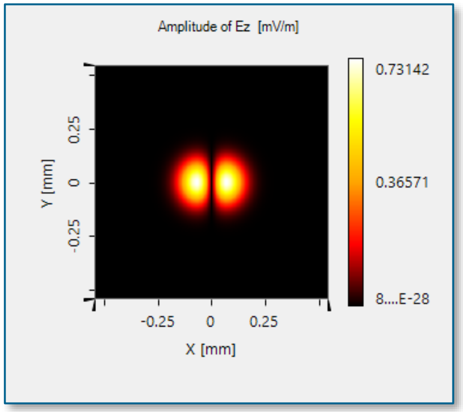
Warm-up: Gaussian Field Components



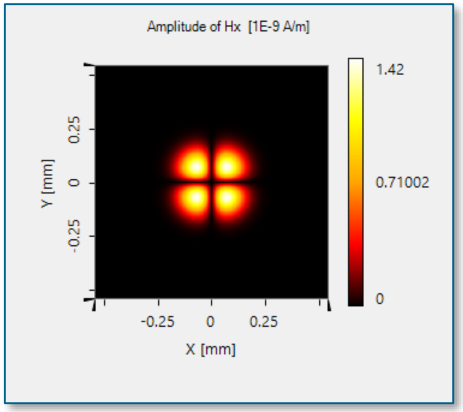
E_x



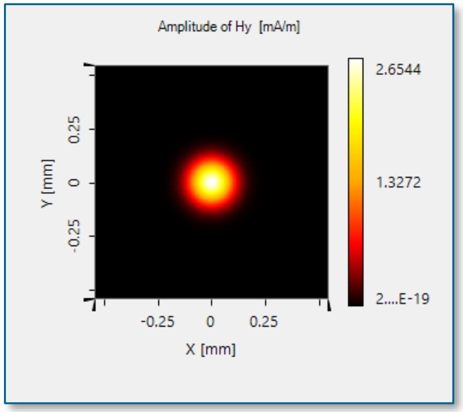
E_y



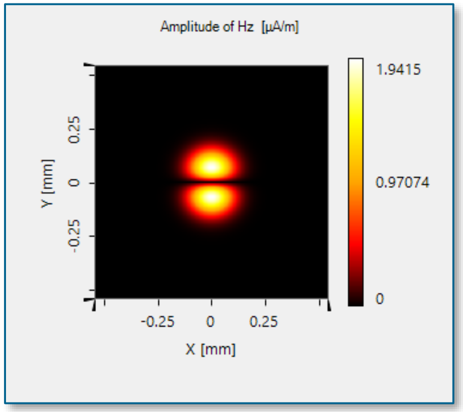
E_z



H_x



H_y



H_z

Polarization of A Harmonic Field

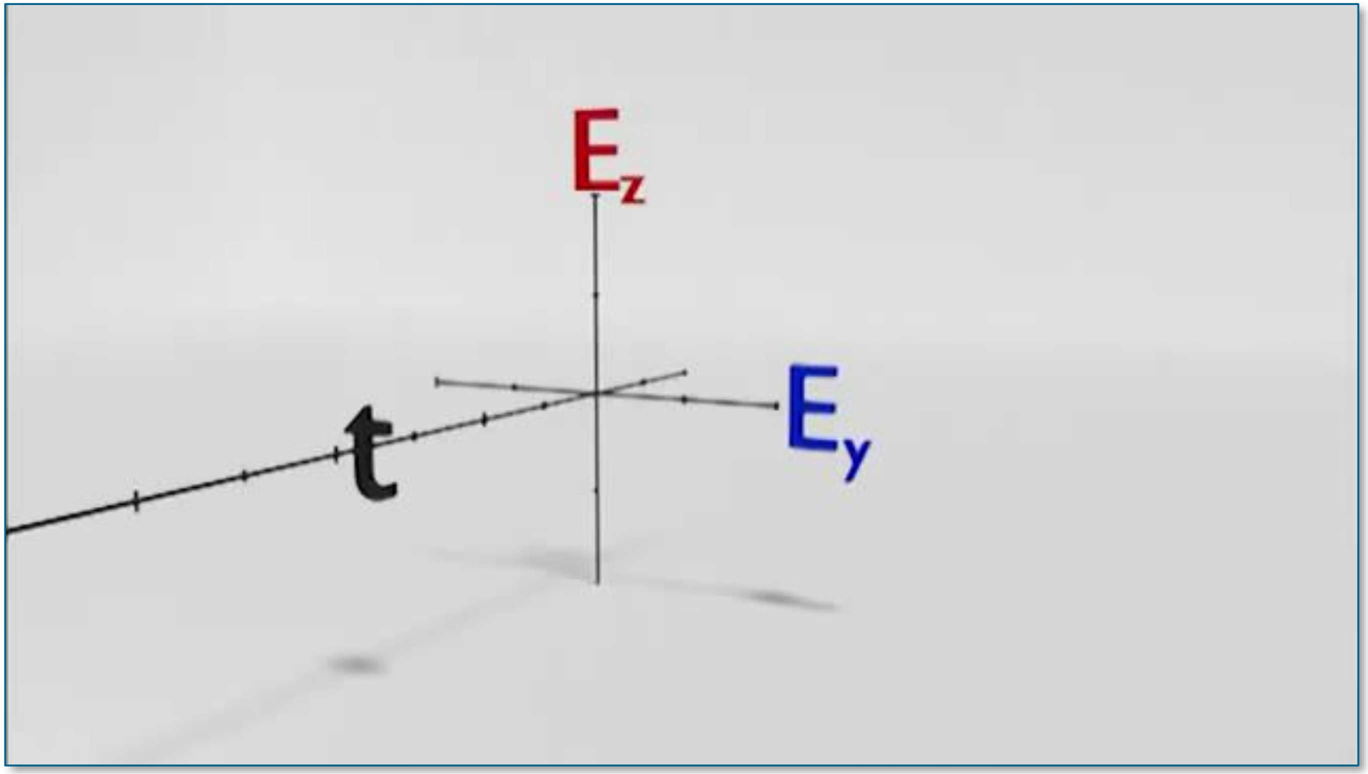
Polarization of Light

- In general we may define: a light field is fully polarized, if the tip of the electric field vector moves on a time independent loop in space at any position.
- It is very important to know: light is polarized even if the loop changes with the position.
- In contrast, there are partially polarized light and unpolarized light.
- Polarization is a 3D phenomenon of electromagnectic fields, because the vector of the electric field is a 3D vector.

Polarization of A Harmonic Field

- A harmonic (monochromatic) field which is defined, in frequency domain, as
$$\boldsymbol{E}(\boldsymbol{r},\omega)=\boldsymbol{E}(\boldsymbol{r})\delta(\omega-\omega_0)$$
with the complex amplitude vector
$$\boldsymbol{E}(\boldsymbol{r})=\begin{bmatrix}|E_x(\boldsymbol{r})|\exp[\mathrm{i}\varphi_x(\boldsymbol{r})]\\|E_y(\boldsymbol{r})|\exp[\mathrm{i}\varphi_y(\boldsymbol{r})]\\|E_z(\boldsymbol{r})|\exp[\mathrm{i}\varphi_z(\boldsymbol{r})]\end{bmatrix}$$
- Then the real field vector in time domain can be found as
$$\bar{\boldsymbol{E}}^{(\mathrm{r})}(\boldsymbol{r},t)=\Re\big[\boldsymbol{E}(\boldsymbol{r})\exp(-\mathrm{i}\omega_0 t)\big]=\begin{bmatrix}|E_x(\boldsymbol{r})|\cos[\varphi_x(\boldsymbol{r})-\omega_0 t]\\|E_y(\boldsymbol{r})|\cos[\varphi_y(\boldsymbol{r})-\omega_0 t]\\|E_z(\boldsymbol{r})|\cos[\varphi_z(\boldsymbol{r})-\omega_0 t]\end{bmatrix}$$
and it is always polarized! The movement of the electric field vector defines an ellipse in 3D space.

Polarization of A Harmonic Field



Polarization of A Harmonic Field

- A harmonic (monochromatic) field defined, in time domain, in the following form

$$\bar{\boldsymbol{E}}^{(\mathrm{r})}(\boldsymbol{r},t)=\Re\left[\boldsymbol{E}(\boldsymbol{r})\exp(-\mathrm{i}\omega_0 t)\right]=\begin{bmatrix}|E_x(\boldsymbol{r})|\cos[\varphi_x(\boldsymbol{r})-\omega_0 t]\\|E_y(\boldsymbol{r})|\cos[\varphi_y(\boldsymbol{r})-\omega_0 t]\\|E_z(\boldsymbol{r})|\cos[\varphi_z(\boldsymbol{r})-\omega_0 t]\end{bmatrix}$$

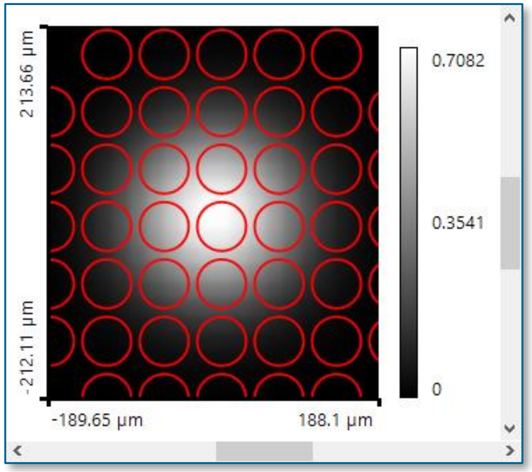
  is always polarized! The movement of the electric field vector defines an ellipse in 3D space. =>*Mathematica*
- Often we consider the projection of this ellipse onto one plane only. Then just two field components contribute to the curve.

Polarization of A Harmonic Field

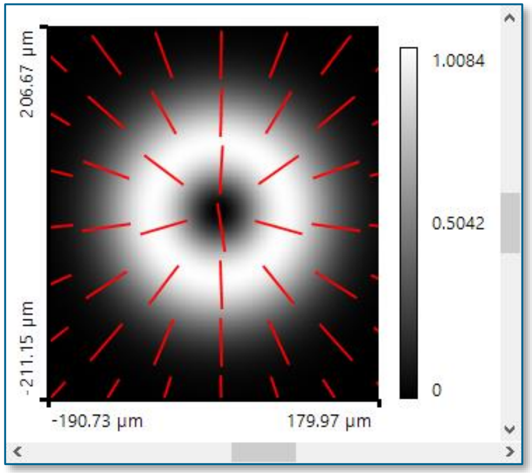
- Generally, the shape and the size of the ellipse depend on position. That leads to locally polarized light.
- Often globally polarized, paraxial fields are considered and they can be expressed as

$$\Big(E_x(\boldsymbol{\rho}), E_y(\boldsymbol{\rho})\Big) = \boldsymbol{J}\, U(\boldsymbol{\rho})$$

with the constant Jones vector $\boldsymbol{J} = (J_x, J_y)$



globally polarized light with circular polarization



Locally polarized light with radial polarization

Plane Waves and Related Concepts

Plane Wave Definition

- In the lecture, we define a plane wave in the form
$$\boldsymbol{E}(\boldsymbol{r}) = \breve{\boldsymbol{E}} \exp(\mathrm{i}\breve{\boldsymbol{k}} \cdot \boldsymbol{r})$$
with the complex wave vector
$$\breve{\boldsymbol{k}} = \boldsymbol{k} + \mathrm{i}\boldsymbol{k}' = k\hat{\boldsymbol{k}} + \mathrm{i}k'\hat{\boldsymbol{k}}'$$
and, a plane wave is a solution to Maxwell‘s equation given the dispersion relation
$$\breve{\boldsymbol{k}}(\omega) \cdot \breve{\boldsymbol{k}}(\omega) = \|\breve{\boldsymbol{k}}(\omega)\|^2 = \breve{k}_x^2 + \breve{k}_y^2 + \breve{k}_z^2 \stackrel{!}{=} k_0^2 \breve{n}^2(\omega) = k_0^2 \big(n(\omega) + \mathrm{i}n'(\omega)\big)^2$$
is fulfilled.
- We distinguish two type of plane waves
  - Homogeneous plane wave: $\boldsymbol{k} \parallel \boldsymbol{k}'$
  - Inhomogeneous plane wave: $\boldsymbol{k} \nparallel \boldsymbol{k}'$

Inhomogeneous Plane Waves

- Specification of wavevector: Type I ...
- Specification of wavevector: Type II
  - The concept of plane wave decomposition is often used and it is defined via the inverse Fourier transform as

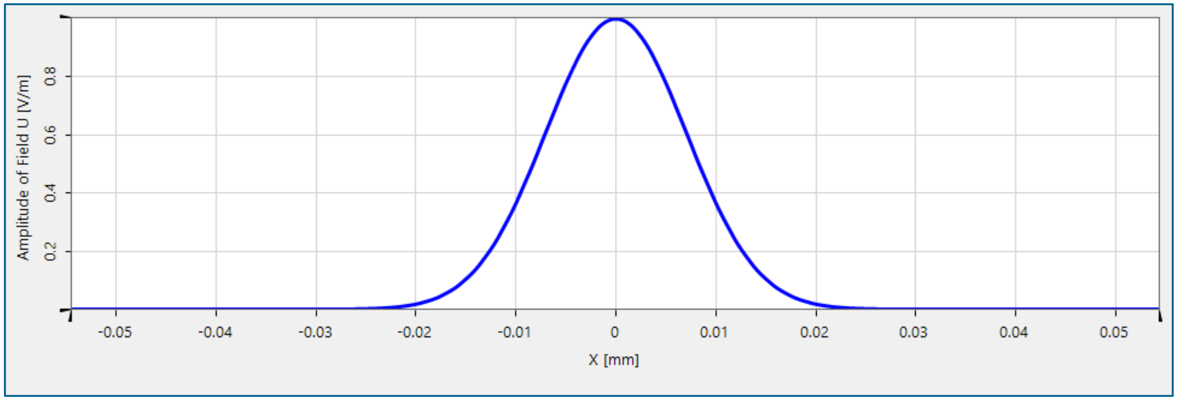
Fourier transform kernel – form of plane wave

$$\underbrace{E_x(x,y,z)}_{\text{field in spatial domain}} = \frac{1}{2\pi}\int\int_{-\infty}^{\infty} \underbrace{\hat{E}_x(k_x,k_y,z)}_{\text{angular spectrum}} \boxed{\mathrm{e}^{\mathrm{i}(k_x x + k_y y)}}\,\mathrm{d}k_x\,\mathrm{d}k_y$$

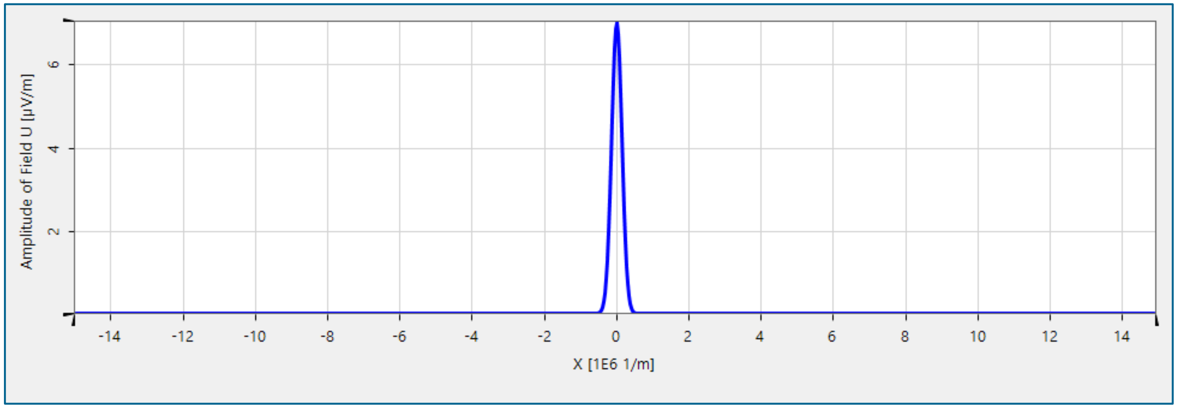
  - The x- and y-components of the wavevector must be real-valued.
  - They can have any values.

Angular Spectrum Analysis

- Relation between fields in both domains
  - Consider a real-valued refractive index n.



field in spatial domain



angular spectrum

Angular Spectrum Analysis

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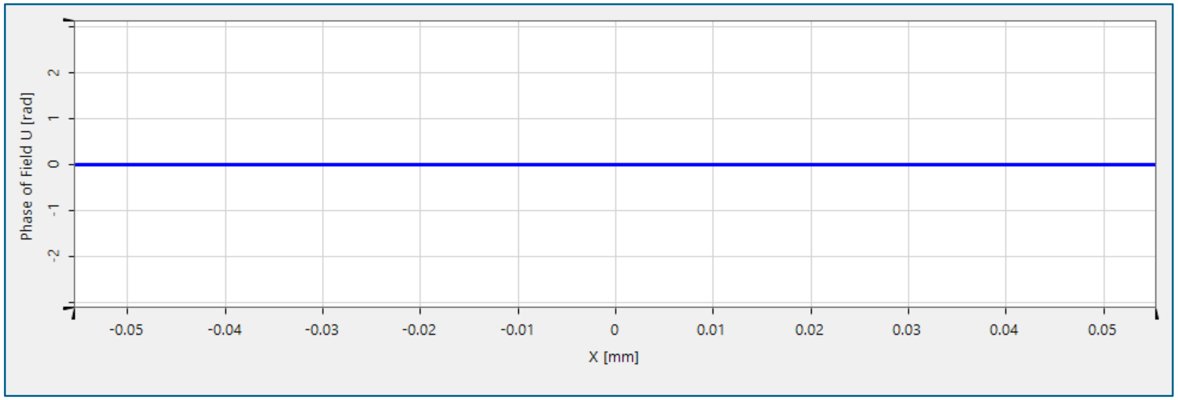
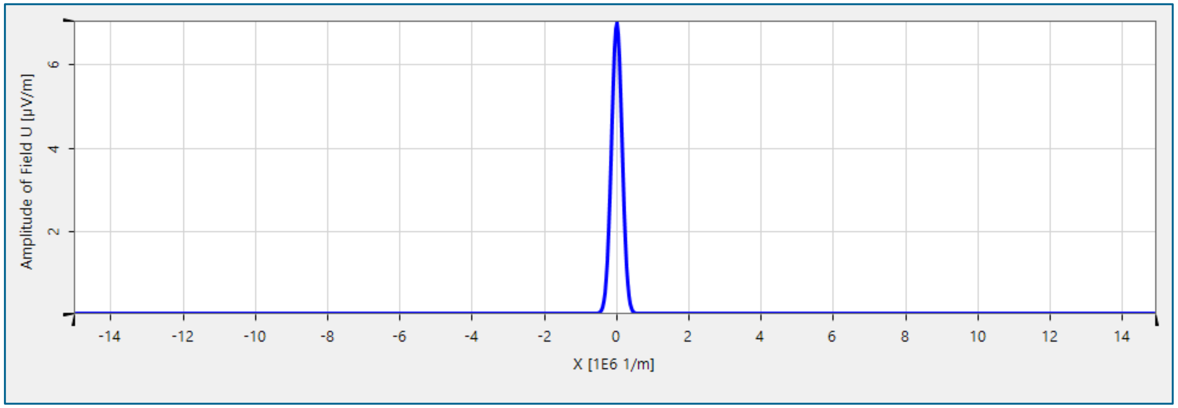


field in spatial domain



angular spectrum

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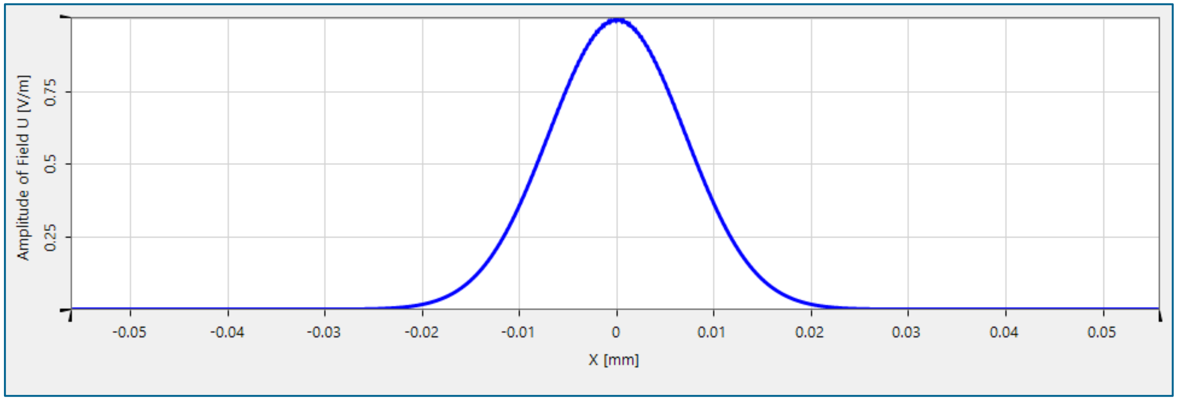
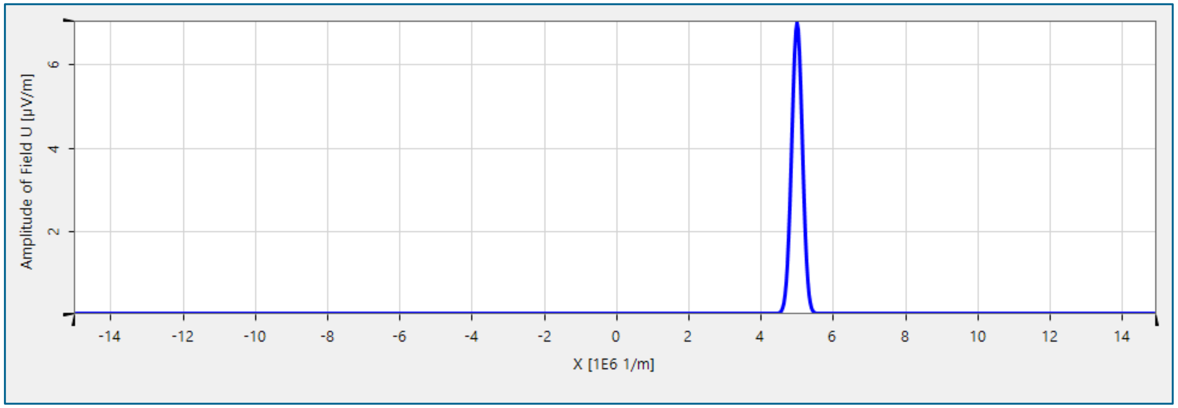


field in spatial domain



angular spectrum

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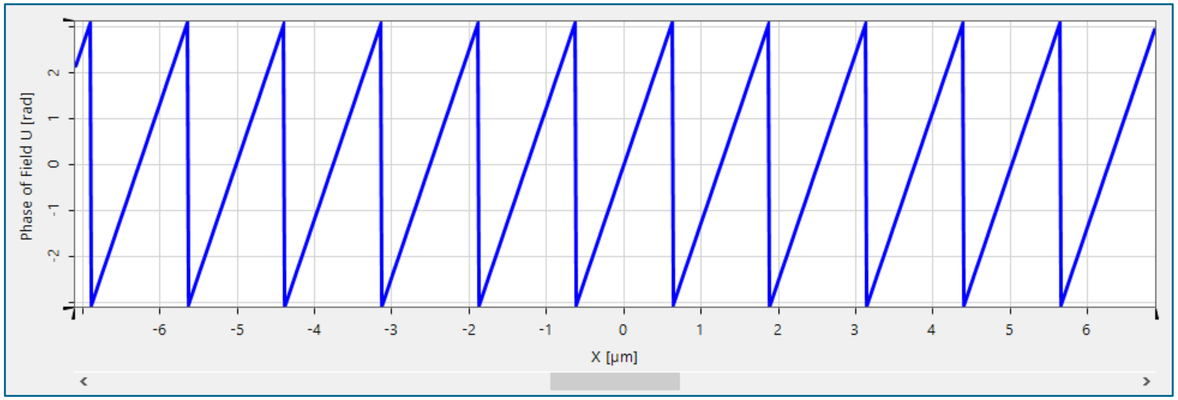
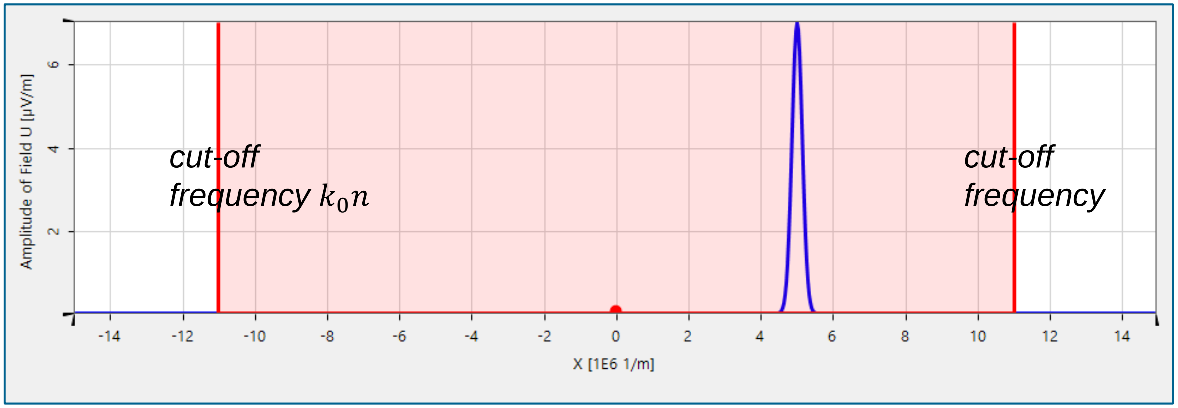


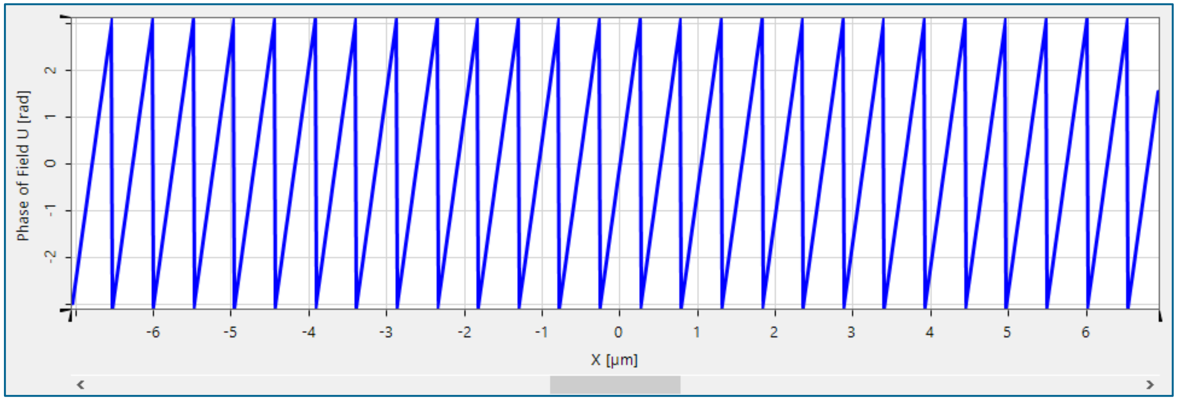
field in spatial domain



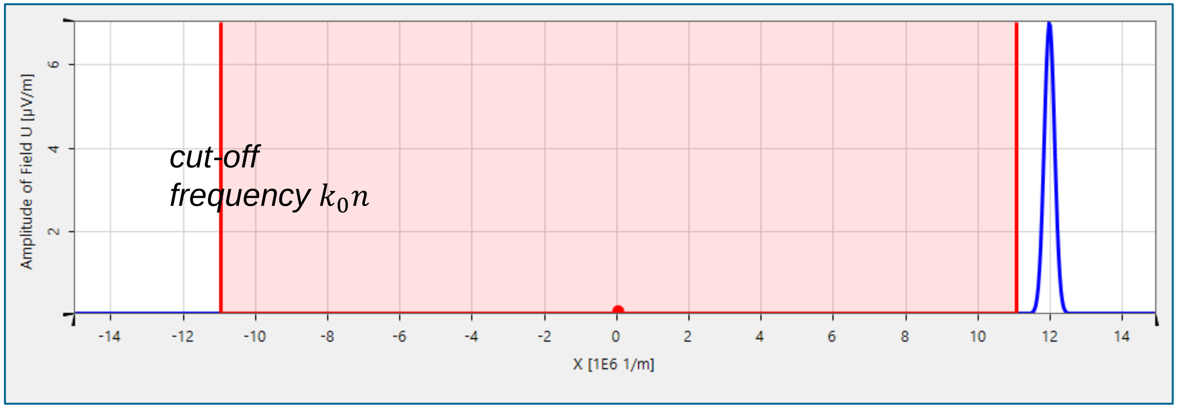
angular spectrum

Angular Spectrum Analysis

- Relation between fields in both domains
  - Consider a Gaussian field and real-valued refractive index.



field in spatial domain

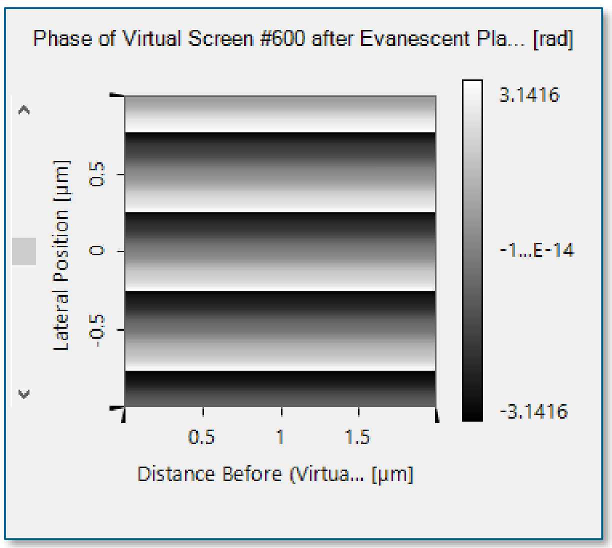


Evanescent Wave!

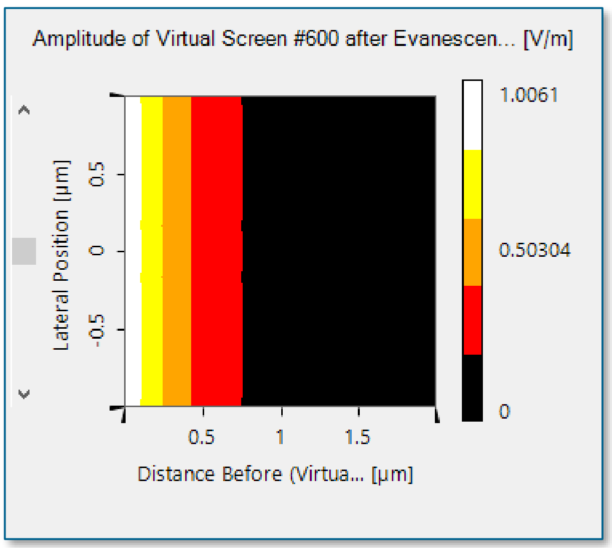
angular spectrum

Evanescent Plane Wave

- Construction of an evanescent wave in VirtualLab.
- Two planes of importance
  - Planes of constant phase (wavefront): $\hat{\boldsymbol{k}} \cdot \boldsymbol{r} = \mathrm{const}$
  - Planes of constant amplitude: $\hat{\boldsymbol{k}}' \cdot \boldsymbol{r} = \mathrm{const}$



Planes of constant phase



Planes of constant amplitude