Structure of Matter: Written Test 2022

1. Multiple-choice test: Please tick all box(es) with correct answer(s)!
   Correctly ticked box = 1/2point; wrongly ticked box = -1/2 point; empty box = zero points

1.1 A photon with an energy of 2eV belongs to the	a) visible (VIS) spectral range	✓
	b) ultraviolet (UV) spectral range	
	c) infrared (IR) spectral range	
1.2 UV photons have photon energies	a) larger than VIS photons	✓
	b) larger than IR photons	✓
	c) larger than 3eV	✓
1.3 An electron confined in a box potential	a) has discrete stationary energy levels	✓
	b) obeys the Bloch theorem	✓
	c) is a fermion	✓
1.4 Imagine a single-particle wavefunction Ψ normalized according to: $\int_V \lvert\Psi\rvert^2 dV = 1$ with V – volume. This wavefunction	a) has no measurement unit	
	b) is given in $m^{-3/2}$	
	c) is given in m^{-3}	✓
1.5 Non-linear optical phenomena	a) include frequency-conversion processes	✓
	b) are observed at large light intensities	✓
	c) are of no practical relevance in laser optics	
1.6 The selection rules for Raman scattering	a) ~~are identical to those valid for IR absorption~~	
	b) exclude Raman activity in any crystals	
	c) exclude Raman activity in any quantum system with inversion symmetry	
1.7 The atomic state $^1D_9$	a) is possible	
	b) is impossible	
1.8 Solids may be	a) crystalline	✓
	b) amorphous	✓
	c) electrically conducting	✓

2. True or wrong? – make your decision!
   Correct choice = 1 point; wrong or no choice = zero points

Assertion:	true	wrong
a) $\left[\hat{x}, \hat{p}_y\right] = i\hbar$		·
b) $\left[U(r), \hat{L}_z\right] = 0$	·	
c) $\left[\hat{x}^2, \hat{p}_y^2\right] = 0$	·	
d) $\left[\hat{x}^2, \hat{p}_x^2\right] = \hbar$		·
e) $\left[\hat{L}_x, \hat{L}_z\right] = i\hbar y$		·

3. Imagine a one-dimensional harmonic oscillator in the eigenstate with the quantum number n=50. As a result of an electric dipole-allowed quantum transition, the oscillator is transferred to another eigenstate with the quantum number m. Indicate the values of all oscillator strength $f_{m,n=50}$ that are different from zero! (3 points)

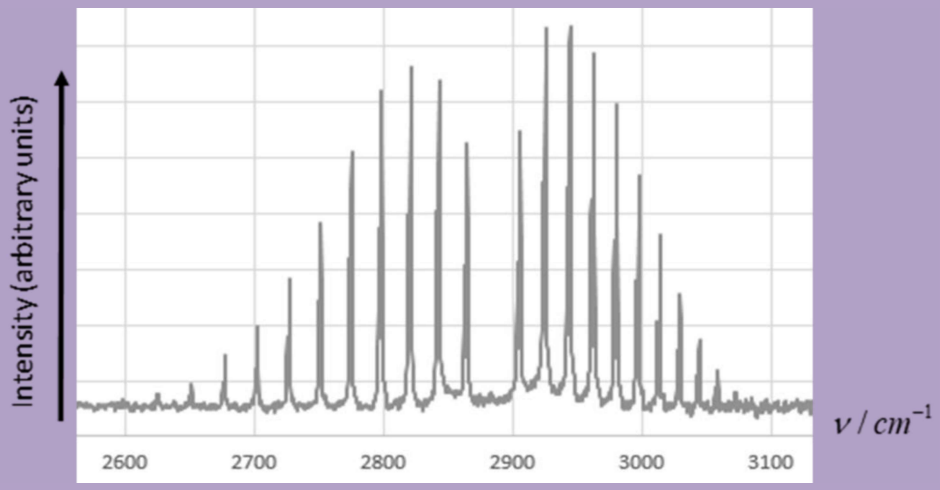
4. Assume a resting hydrogen atom in an excited state with the principal quantum number n=2. Let the hydrogen atom return to its ground state by emitting a single photon. Because of the recoil effect, after the emission of the photon, the atom is no more at rest. Making use of a non-relativistic approach, please:
   - Write down the equation for energy conservation in this process! (1 point)
   - Write down the equation for momentum conservation in this process! (1 point)
   - Estimate the velocity of the hydrogen atom after emission of the photon! (4 points)
   - Indicate the wavelength of the photon and the corresponding spectral series! (0.5 points)

5. Imagine a hydrogen atom in the excited quantum state $|n,l,m\rangle = |2,1,1\rangle$. In spherical coordinates, the wavefunction of the electron in that state may be written as:

$$\psi(r,\varphi,\theta) = \frac{1}{8\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{\frac{3}{2}} \frac{r}{a_0} e^{-\frac{r}{2a_0}} \sin\theta e^{i\varphi}$$

. a_0 is the Bohr's radius. Consider the z-coordinate of this electron and calculate its variance $\mathrm{var}(z) = \langle z^2 \rangle - \langle z \rangle^2$ in this quantum state! (10 points)

6. The figure below sketches the middle infrared spectrum of a gas of diatomic molecules held at a certain temperature.



a) Indicate the resonance wavenumber ν_0 corresponding to pure vibration of the molecule. Indicate the wavenumber regions where you observe the P- and R-branches of the spectrum (1.5 point)

b) What is the rotational constant B, provided that the spacing between adjacent rotation lines is approximately 20cm^{-1}? (0.5 point)

c) from these data, estimate the mass moment of inertia of a single molecule (2 points)

d) estimate the temperature of the gas (2 points)

e) guess the molecule (1 point)