Important formulations in FoMO

Maxwell's equations.

$$\boxed{\begin{aligned} &\begin{cases} \operatorname{rot}\vec{E}(\vec{r},t) = -\mu_0 \dfrac{\partial \vec{H}(\vec{r},t)}{\partial t} \\ \operatorname{rot}\vec{H}(\vec{r},t) = \vec{j}(\vec{r},t) + \dfrac{\partial \vec{D}(\vec{r},t)}{\partial t} \end{cases} \quad && \operatorname{div}\vec{D}(\vec{r},t) = \rho(\vec{r},t) \\ & && \operatorname{div}\vec{H}(\vec{r},t) = 0 \\ &\begin{cases} \operatorname{rot}\vec{E}(\vec{r},\omega) = i\omega\mu_0 \vec{H}(\vec{r},\omega) \\ \operatorname{rot}\vec{H}(\vec{r},\omega) = \vec{j}(\vec{r},\omega) - i\omega \vec{D}(\vec{r},\omega) \end{cases} && \operatorname{div}\vec{D}(\vec{r},\omega) = \rho(\vec{r},\omega) \\ & && \operatorname{div}\vec{H}(\vec{r},\omega) = 0 \end{aligned}}$$

$$\vec{D}(\vec{r},\omega) = \varepsilon_0 \vec{E}(\vec{r},\omega) + \varepsilon_0 \chi(r,\omega)\vec{E}(\vec{r},\omega) = \varepsilon_0 \varepsilon(r,\omega)\vec{E}(\vec{r},\omega)$$

$$\vec{j}(\vec{r},\omega) = \sigma(\omega)\vec{E}(\vec{r},\omega)$$

$$\boxed{\varepsilon(\omega) = 1 + \chi(\omega) + \frac{i}{\omega\varepsilon_0}\sigma(\omega)}$$

$$\boxed{\vec{P}_i(\vec{r},t) = \varepsilon_0 \sum_j \int_{-\infty}^{t} R_{ij}(\vec{r},t-t')\vec{E}_j(\vec{r},t')\,dt'}$$

$$\vec{P}_i(\vec{r},\omega) = \varepsilon_0 \sum_j \chi_{ij}(\vec{r},\omega)\vec{E}_j(\vec{r},\omega)$$

$$\Delta\vec{E}(\vec{r},\omega) + \frac{\omega^2}{c^2}\varepsilon(\vec{r},\omega)\vec{E}(\vec{r},\omega) = -\operatorname{grad}\left[\frac{\operatorname{grad}\varepsilon(\vec{r},\omega)}{\varepsilon(\vec{r},\omega)}\cdot\vec{E}(\vec{r},\omega)\right]$$

$$R(t) = \begin{cases} \dfrac{f}{\Omega}\exp\left(-\dfrac{g}{2}t\right)\sin\Omega t \\ 0 \end{cases} \longleftrightarrow \chi(\omega) = \frac{f}{\omega_0^2 - \omega^2 - ig\omega}$$

$$\boxed{\langle S(\vec{r},t)\rangle = \frac{1}{2}\operatorname{Re}\left[E \times H^*\right]}, \quad I = \left|\langle S(\vec{r},t)\rangle\right|$$

$$\Delta\vec{E}(\vec{r},\omega) + \frac{\omega^2}{c^2}\varepsilon(\omega)\vec{E}(\vec{r},\omega) = 0$$

$$\boxed{\begin{aligned} \vec{H}(\vec{r},\omega) &= -\frac{i}{\omega\mu_0}\operatorname{rot}\vec{E}(\vec{r},\omega) \\ \vec{E}(\vec{r},\omega) &= \frac{i}{\omega\varepsilon_0\varepsilon(\omega)}\operatorname{rot}\vec{H}(\vec{r},\omega) \end{aligned}}$$

1) $\varepsilon', \varepsilon'' > 0, \ \varepsilon'' \ll \varepsilon'$

$$n(\omega) \approx \sqrt{\varepsilon'(\omega)}, \quad k(\omega) \approx \frac{1}{2}\frac{\varepsilon''(\omega)}{\sqrt{\varepsilon'(\omega)}}$$

2) $\varepsilon' < 0, \ \varepsilon'' > 0, \ \varepsilon'' \ll |\varepsilon'|$

$$n(\omega) \approx \frac{1}{2}\frac{\varepsilon''(\omega)}{\sqrt{|\varepsilon'(\omega)|}}, \quad k(\omega) \approx \sqrt{|\varepsilon'(\omega)|}$$

Transfer function and Response function

$$H(\alpha,\beta;z) = \exp(i\sqrt{k^2-\alpha^2-\beta^2}\,z)$$

Fresnel

$$\boxed{\begin{aligned} H_F(\alpha,\beta;z) &= \exp(ikz)\exp\left(-i\frac{\alpha^2+\beta^2}{2k}\right) \\ h_F &= -\frac{ik}{2\pi z}\exp\left\{ikz\left[1+\frac{x^2+y^2}{2z^2}\right]\right\} \end{aligned}}$$

$|\alpha x|.|\alpha y| > 10\frac{\lambda}{n}$

$0.1 < N_F < 10$

Fraunhofer:

$$\boxed{\begin{aligned} u_{FR}(x,y,z_B) &= -i\frac{(2\pi)^2}{\lambda z_B}\exp(ikz_B)\,U_t\left(k\frac{x}{z_B},k\frac{y}{z_B}\right)\exp\left[i\frac{k}{2z_B}(x^2+y^2)\right] \\ I_{FR}(x,y,z_B) &\sim \frac{1}{(\lambda z_B)^2}\left|U_t\left(k\frac{x}{z_B},k\frac{y}{z_B}\right)\right| \qquad N_F \leq 0.1 \end{aligned}}$$

Pulses:

$$\boxed{\begin{aligned} V(\bar{w};z) &= V_0(\bar{w})\exp\left[iz\frac{D}{2}\bar{w}^2\right] \\ \hat{h}_p(\tau;z) &= \sqrt{\frac{2}{-i\pi Dz}}\exp\left[-i\frac{\tau^2}{2Dz}\right] \end{aligned}}$$

Gaussian Beam:

$$v(x,y,z) = A_0\frac{1}{\sqrt{1+\left(\frac{z}{z_0}\right)^2}}\exp\left[-\frac{x^2+y^2}{w^2(z)}\right]\exp\left[i\frac{k}{2}\frac{x^2+y^2}{R(z)}\right]\exp(i\varphi(z))$$

$$\boxed{\begin{aligned} &A(z) = \frac{A_0}{\sqrt{1+\left(\frac{z}{z_0}\right)^2}},\quad R(z) = z+\frac{z_0^2}{z},\quad W(z) = W_0\sqrt{1+\left(\frac{z}{z_0}\right)^2},\ \tan\varphi = \frac{z}{z_0} \\ &z_0 = \frac{kw_0^2}{2},\quad q(z) = z-iz_0 = \frac{1}{q(z)} = \frac{1}{R(z)}+i\frac{\lambda_n}{\pi W^2(z)} \end{aligned}}$$

$$0 < g_1 g_2 < 1,\quad g_1 = \left(1+\frac{d}{R_1}\right),\quad g_2 = \left(1+\frac{d}{R_2}\right)$$

$R>0$ $\|)$ $\qquad$ $R<0$ $\|($

$$k(w) = k(w_0)+\frac{\partial k}{\partial w}\Big|_{w_0}(w-w_0)+\frac{1}{2}\frac{\partial^2 k}{\partial w^2}\Big|_{w_0}(w-w_0)^2$$

Gaussian Pulse

$$Z_0 = -\frac{T_0^2}{2D} \qquad T(z) = T_0\sqrt{1+\left(\frac{z}{z_0}\right)^2}, \quad C(z) = -\frac{z}{z_0\left(1+\left(\frac{z}{z_0}\right)^2\right)}$$

$$= -\frac{z_0}{R(z)}$$

Fourier Optics

$$\boxed{\begin{aligned} &u(x,y,2f) = -i\frac{(2\pi)^2}{\lambda f_1}\exp(2ikf_1)\,U_0\left(\frac{k}{f_1}x, \frac{k}{f_1}y\right) \\ &H_A(\alpha,\beta;4f) \sim p\left(\frac{f_1}{k}\alpha, \frac{f_1}{k}\beta\right);\ h_A(x,y) \sim P\left[-\frac{k}{f_2}x, -\frac{k}{f_2}y\right] \end{aligned}}$$

$$u(-x,-y,4f) \sim \iint_{-\infty}^{\infty} H_A(\alpha,\beta,4f)\,U_0(\alpha,\beta)\exp\left[i\left(\frac{f_1}{f_2}\alpha x + \frac{f_1}{f_2}\beta y\right)\right]d\alpha\, d\beta$$

$$\Delta r_{min} = \frac{1.22\lambda f}{nD}$$

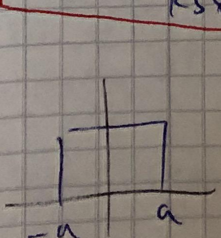
$$D^{(i)} = \{D_i \exp[i(k_i r - \omega t)]\}\hat{e}_i, \quad k_i = \frac{\omega}{c}n_i, \quad E_i = \frac{D_i}{\varepsilon_0\varepsilon_i}$$

$$\sum \frac{u_i^2}{n^2-\varepsilon_i} = \frac{1}{n^2}$$

$$u_1^2(n^2-\varepsilon_2)(n^2-\varepsilon_3)n^2 + u_2^2(n^2-\varepsilon_1)(n^2-\varepsilon_3)n^2 + u_3^2(n^2-\varepsilon_1)(n^2-\varepsilon_2)n^2 = (n^2-\varepsilon_1)(n^2-\varepsilon_2)(n^2-\varepsilon_3)$$

Single interface

$$\boxed{\begin{aligned} &R_{TE} = \frac{k_{sx}-k_{cx}}{k_{sx}+k_{cx}} \quad T_{TE} = \frac{2k_{sx}}{k_{sx}+k_{cx}}, \quad \rho_{TE} = R_{TE}^2, \quad \tau_{TE} = \frac{\Re(k_{cx})}{k_{sx}}T_{TE}^2 \\ &R_{TM} = \frac{k_{sx}\varepsilon_c - k_{cx}\varepsilon_s}{k_{sx}\varepsilon_c + k_{cx}\varepsilon_s} \quad T_{TM} = \frac{2k_{sx}\sqrt{\varepsilon_c\varepsilon_s}}{k_{sx}\varepsilon_c + k_{cx}\varepsilon_s}, \quad \rho_{TM} = R_{TM}^2, \quad \tau_{TM} = \frac{\Re(k_{cx})}{k_{sx}}|T_{TM}|^2 \end{aligned}}$$



— $\mathrm{sinc}(\alpha a)$

$$f(x-a) \to e^{i\omega\tau} \to e^{i\omega a}$$
$$\to e^{-i\alpha x} \to e^{-i\alpha a}$$

$$e^{i\alpha_0 x} \xleftrightarrow{\alpha - x} \delta(\alpha-\alpha_0)$$
$$e^{i\omega_0 t} \xleftrightarrow{\omega - t} \delta(\omega+\omega_0)$$