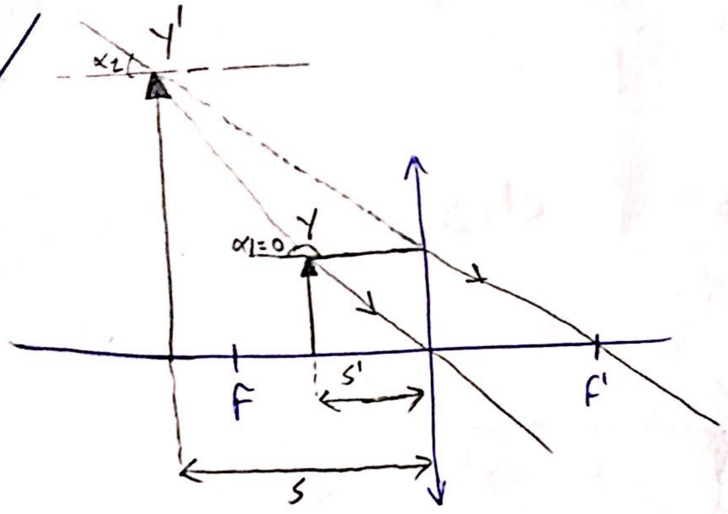
Ex/



$\Gamma = x$ ray coordinate

$\Gamma_2 = \Gamma_1 . M$

$\tan\alpha_2 = \frac{\Gamma_1}{f} + \alpha_1 \approx \alpha_2$

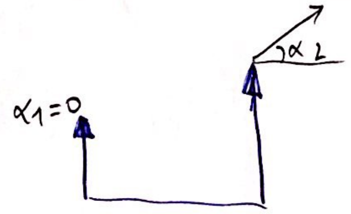
$$\begin{pmatrix} \Gamma_2 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} M & 0 \\ -\frac{1}{f} & \frac{1}{M} \end{pmatrix} \begin{pmatrix} \Gamma_1 \\ \alpha_1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & s' \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-\frac{s}{f} & s \\ \frac{-1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & s' \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1-\frac{s}{f} & \left(1-\frac{s}{f}\right)s' + s \\ -\frac{1}{f} & -\frac{s'}{f}+1 \end{pmatrix}$$

By means of magnification one can look at Γ_1 and $\alpha_1 = 0$ entering the system

So $\Gamma_1 = \Gamma_1 . M = \Gamma_1 \frac{s}{s'}$



$\tan\alpha_2 \approx \alpha_2 = -\frac{\Gamma_1}{f} + \frac{\alpha}{M}$

$\left(1-\frac{s}{f}\right)s' + s = 0$, $M = \frac{s}{s'} = 1-\frac{s}{f} = \frac{f-s}{f}$

$$\left(1-\frac{s}{f}\right)s' = -s \quad \rightarrow \quad s' - \frac{ss'}{f} = -s$$

$$\frac{ss'}{f} = s' + s$$

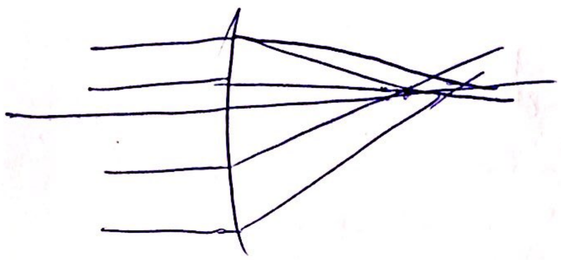
$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

Ex/

Spherical-aberration

$\sim r^4$

when rays passing the aperture far from the optical axis have focal lengths different from the paraxial rays



Ex/ 5 Classical Seidel Aberrations:

$$W(\beta, r, \psi) = \underbrace{W_{000}}_{\text{Piston Error}}$$

$$+ \underbrace{W_{200}\,\beta^2}_{\text{(Piston Er.)}} + \underbrace{W_{020}\,r^2}_{\text{(Defocus Err.)}} + \underbrace{W_{111}\,\beta \cdot r \cdot \cos\psi}_{\text{(Lateral Mag Err.)}}$$

$$+ \underbrace{W_{400}\,\beta^4}_{\text{(Piston)}} + \underbrace{W_{040}\,r^4}_{\text{(SA)}} + \underbrace{W_{131}\,\beta r^3 \cos\psi}_{\text{(Coma)}} + \underbrace{W_{222}\,\beta^2 r^2 \cos^2\psi}_{\text{(Astigmatism)}}$$

$$+ \underbrace{W_{220}\,\beta^2 r^2}_{\text{(Field Curvature)}} + \underbrace{W_{311}\,\beta^3 r \cos\psi}_{\text{(Distortion)}}$$

Combination + and − lens

Achromatic Aberrations

$$\frac{\phi_1}{\nu_1} + \frac{\phi_2}{\nu_2} = 0$$

$$\Phi_1 = A_1(n_1 - 1) = (n_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Ex/ Maxwell's Eqn in frequency domain

$$\nabla \times \bar{E}(r,w) = iw B(r,w) = iw\mu_0 H(r,w)$$

$$\nabla \times H(r,w) = j(r,w) - iwD(r,w)$$

$$\nabla \times D(r,w) = \rho(r,w)$$

$$\nabla \cdot B(r,w) = 0$$

$$D(r,w) = \varepsilon_0 E(r,w)\,\varepsilon(r,w) = \varepsilon_0 \varepsilon(r,w) E(r,w)$$

$$|k| = \frac{w}{c} n = \frac{w}{c}\sqrt{\varepsilon}$$

Ex/7. mathematical expression of complex field vector of an harmonic (monochromatic) electromagnetic field in freq. and time domain. What is the corresponding real field in time domain?

$$E(r,t) = E(r,w)e^{-iw_0 t} \qquad \text{with } k^2 = k_x^2 + k_y^2 + k_z^2 = \frac{w^2}{c^2} n_a^2$$

$$r = (x, y, z)$$

$$E(r,w) = \begin{bmatrix} E_x(r)\, e^{i\varphi_x} \\ E_y(r)\, e^{i\varphi_y} \\ E_z(r)\, e^{i\varphi_z} \end{bmatrix}$$

$$E(r,t) = Re\left(E(r,t)\right) = \begin{bmatrix} |E_x(r,w_0)| \cos(\varphi_x - w_0 t) \\ |E_y(r,w_0)| \cos(\varphi_y - w_0 t) \\ |E_z(r,w)| \cos(\varphi_z - w_0 t) \end{bmatrix}$$

Ex

8\. Complex amplitude expression of the electric field

a) vector of an electromagnetic plane wave.

$$E(r,w) = E(w) \cdot e^{-ikr} \qquad \vec{k} = \vec{k}' + i\vec{k}'' = k'\hat{k}' + k''\hat{k}''$$

b) Different between homogeneous and inhomogeneous plane waves?

$\mathcal{E}(w) \rightarrow \mathcal{E}(r,w)$ so Helmolz eqn. isn't

from wave equation <u>homogenous is</u> for $\hat{k}'' \parallel \hat{k}'$ or $k''=0$

<u>inhomogeneous</u> for $\vec{k} \not\parallel k'$ doesn't have unique direction

homogenoous waves:

* We distinguish between two basic type of plane waves

1) <u>homogeneous plane waves:</u>

$k' = 0$ or $\vec{k} \parallel \vec{k}$ and hence

$\hat{s} = \hat{k}' = \hat{k}$ ~~$\hat{k} \neq \hat{k}_0 (n \pm i\kappa)\hat{s}$~~

~~$\hat{k} = \hat{k}(\hat{k}_s + i\hat{k}_s)\hat{s}$~~ ~~$\hat{s}(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$~~

2) <u>Inhomogeneous plane waves</u>

$\vec{k} \not\parallel \vec{k}'$ and hence $\hat{k} \neq \hat{k}'$

Ex/7. mathematical expression of complex field vector of an harmonic (monochromatic) electromagnetic field in freq. and time domain. What is the corresponding real field in time domain?

$$E(r,t) = E(r,w)e^{-iw_0 t}$$

with $k^2 = k_x^2 + k_y^2 + k_z^2 = \frac{w^2}{c^2} n^2$

$r = (x, y, z)$

$$E(r,w) = \begin{bmatrix} E_x(r)\, e^{i\varphi_x} \\ E_y(r)\, e^{i\varphi_y} \\ E_z(r)\, e^{i\varphi_z} \end{bmatrix}$$

$$E(r,t) = Re\left(E(r,t)\right) = \begin{bmatrix} |E_x(r,w_0)| \cos(\varphi_x - w_0 t) \\ |E_y(r,w_0)| \cos(\varphi_y - w_0 t) \\ |E_z(r,w)| \cos(\varphi_z - w_0 t) \end{bmatrix}$$

Ex/8

c) What is the dependency of the complex k-vector $\hat{k} = \vec{k} + i\vec{k}'$ for homo. plane waves, when we know the refractive index $\check{n} = n + in'$

$\vec{k} = (k + ik')\hat{s}$, $\hat{s} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$

dispersion relation: $\hat{k}^2 = k_0^2\,\check{n}^2(w)$

insert $\vec{\lfloor}$ $\hat{k} = (k + ik')\hat{s}$ $\rightarrow$ for homo. plane waves

$$(k + ik')^2\hat{s}^2 = k_0^2(n + in')^2$$

$k = k_0 n$ and $k' = k_0 n'$ and thus;

$\left(k = \frac{w}{c} n\right)$ and $\left(k' = \frac{w}{c} n'\right)$

$$\check{k} = k_0\check{n}.\hat{s} = k_0(n + in)\hat{s} = \frac{w}{c}(n + in)\hat{s}$$

as a necessary condition for homogeneous plane waves

from that we can easily conclude that the plane waves which constitude k- domain discussions must be inhomogeneous in any media with $n' \neq 0$

d) How would you prove that homogeneous plane waves are always of this form?

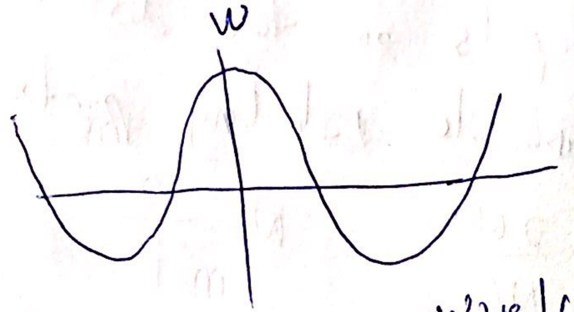
They are eigenstates of freespace propagation and can go through the system unchanged can be seen from plane wave propagation operator

e) Plane wave with $Re(\breve{k}) = 2k_0 n$ and media with $n' = 0$. Is this plane wave homo or inhomo? What is the decay distance and wavelength for this plane wave?

Ex/ the Wave-front-Aberration function:

distance of the real WF from the ideal reference sphere

OPD: Wavefront deformation. Represents the aberrations by introduced the optical system

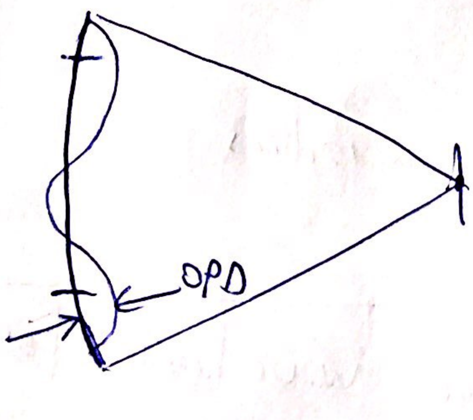


wavefront aberration function

$W = W(\rho, r, \varphi)$

Normalized field height — Normalized pupil height — Azimuth point of ρ

- Can be calculated by tracing rays from EN to EX



Reference sphere in exit pupil

Ex/ Minimum focal spot size with no aberrations

optical resolution ≡ Ability of an imaging system to resolve detail in the object that is being imaged

- resolution depends on the distance between two distinguishable radiating points.

Abbe resolution limit : $\Delta x = \frac{0.61\lambda}{NA}$ $NA = \frac{D}{f}$

$$\lambda = \frac{\lambda_0}{n}$$

$$r = \frac{1.22\lambda}{2n\sin\theta} = \frac{0.61\lambda}{NA}$$

r: min distance between resolvable points

θ: half angle of the light that enters objective.

Angular radius of the Airy disc

$\theta = 1.22\frac{\lambda}{D}$ θ: Angular resolution in radians

$k_0 = \frac{2\pi}{\lambda} = \frac{\omega}{c}$

e) $Re(k) = 2k_0 n$

$2k_0 n = k_0 n \cdot \frac{?}{2}$

$= \frac{2\pi}{(2\lambda)} n$

$\tilde{k} = k\hat{k} + ik'\hat{k}'$

$\tilde{k} = 2k_0 n\hat{k} + ik'\hat{k}'$

$e^{i\tilde{k}\cdot\bar{r}} = e^{i2k_0 n\hat{k}\cdot\bar{r}} \cdot e^{-k'\hat{k}'\cdot\bar{r}}$

$n' = 0 \rightarrow \tilde{k} = k\hat{k} + ik'\hat{k}' = k_0(n\hat{n} + i\underbrace{n'}_{0}\hat{n})$

$\tilde{k} = k\hat{k} + ik'\hat{k}' = k_0 n\hat{n}$

$\hookrightarrow k' = 0$

$\tilde{k} = k\hat{k} = 2k_0 n\hat{k} = k_0 n\hat{n}$

~~$|k|^2 = k^2 = 2k_0^2 n^2 = k_0^2 n^2$~~

$k = (k_x, k_y)$

$(k\hat{k} + i\underbrace{k'}_{0}\hat{k})^2 = \vec{k}^2 = k_x^2 + k_y^2 + k_z^2 = k_0^2\tilde{n}^2 = k_0^2(n\hat{n} + in'\hat{n}')^2 = k_0^2 n^2 \hat{S}^2$

$k_z = k_0\sqrt{n^2 - \frac{k_x^2 + k_y^2}{k_0}} = k_0\sqrt{n^2 - f^2(k)} = \begin{cases} k_0\sqrt{n^2 - f^2(k)} & n > f(k) \\ ik_0\sqrt{f^2 - n^2} & n < f(k) \end{cases}$

$e^{ik_z \Delta z}$

$k_z^2 = Re^2(\vec{k}) - f(k)k_0^2 = (4k_0^2 n^2) - f(k)k_0$

$k_z = \sqrt{4k_0^2 n^2 - k_0^2 f(k)} = k_0\sqrt{4n^2 - f^2(k)}$

- It is homogenous since $n' = 0 \rightarrow k' = 0 \Rightarrow \vec{k} = k\hat{k} = k\hat{S}$.
- $-1 = ik_z z_d$

$f > n$? $z_d = \frac{i}{\quad}$

$$\mathbf{k}\cdot\bar{E} = 0$$

$$(k_x, k_y, k_z)\circ(E_x, E_y, E_z)$$

$$\hookrightarrow E_z = -\frac{k_x E_x + k_y E_y}{k_z}$$

$E_x(x,y,z_0)$, $E_y(x,y,z_0)$ $\rightarrow$ $E_z(x,y,z_0)$ $\rightarrow$ $(E_x, E_y, E_z)(x,y,z_0)$

$\downarrow$

r, ω

$\downarrow$ $\uparrow r, \omega$

r, ρ

E_z

2014

1

thin lens $\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$

Prop $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$

Mag $\begin{pmatrix} M & 0 \\ 0 & 1/M \end{pmatrix}$

change of index $\begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix}$

$$M = \begin{pmatrix} 1 & 0 \\ -1/f_2 & 1 \end{pmatrix}\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1/f_1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1+d \\ -1/f_2 & -\frac{d}{f_2}+1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-\frac{(1+d)}{f_1} & 1+d \\ -\frac{1}{f_2}-\frac{1}{f_1}\left(1-\frac{d}{f_2}\right) & -\frac{d}{f_2}+1 \end{pmatrix}$$

$$-\frac{1}{f_{eff}} = -\left[\frac{1}{f_2}+\frac{1}{f_1}\left(1-\frac{d}{f_2}\right)\right]$$

$$\boxed{\frac{1}{f_{eff}} = \frac{1}{f_2}+\frac{1}{f_1}-\frac{d}{f_1 f_2}}$$