Institute of Applied Physics
Friedrich-Schiller-Universität Jena

# Lens Design I

Lecture 4: Properties of optical systems III

2024-05-02

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# Preliminary Schedule - Lens Design I 2024

| | | | | |
|---|---|---|---|---|
| 1 | 04.04. | Basics | Zhang | Introduction, Zemax interface, menues, file handling, preferences, Editors, updates, windows, coordinates, System description, 3D geometry, aperture, field, wavelength |
| 2 | 18.04. | Properties of optical systems I | Tang | Diameters, stop and pupil, vignetting, layouts, materials, glass catalogs, raytrace, ray fans and sampling, footprints |
| 3 | 25.04. | Properties of optical systems II | Tang | Types of surfaces, cardinal elements, lens properties, Imaging, magnification, paraxial approximation and modelling, telecentricity, infinity object distance and afocal image, local/global coordinates |
| 4 | 02.05. | Properties of optical systems III | Tang | Component reversal, system insertion, scaling of systems, aspheres, gratings and diffractive surfaces, gradient media, solves |
| 5 | 16.05. | Advanced handling I | Tang | Miscellaneous, fold mirror, universal plot, slider, multiconfiguration, lens catalogs |
| 6 | 23.05. | Aberrations I | Zhang | Representation of geometrical aberrations, spot diagram, transverse aberration diagrams, aberration expansions, primary aberrations |
| 7 | 30.05. | Aberrations II | Zhang | Wave aberrations, Zernike polynomials, measurement of quality |
| 8 | 06.06. | Aberrations III | Tang | Point spread function, optical transfer function |
| 9 | 13.06. | Optimization I | Tang | Principles of nonlinear optimization, optimization in optical design, general process, optimization in Zemax |
| 10 | 20.06. | Optimization II | Zhang | Initial systems, special issues, sensitivity of variables in optical systems, global optimization methods |
| 11 | 27.06. | Correction I | Zhang | Symmetry principle, lens bending, correcting spherical aberration, coma, astigmatism, field curvature, chromatical correction |
| 12 | 04.07. | Correction II | Zhang | Field lenses, stop position influence, retrofocus and telephoto setup, aspheres and higher orders, freeform systems, miscellaneous |

# Contents Lecture

1. Aspheres
2. Gratings and diffractive surfaces
3. Gradient media
4. Solves in Zemax

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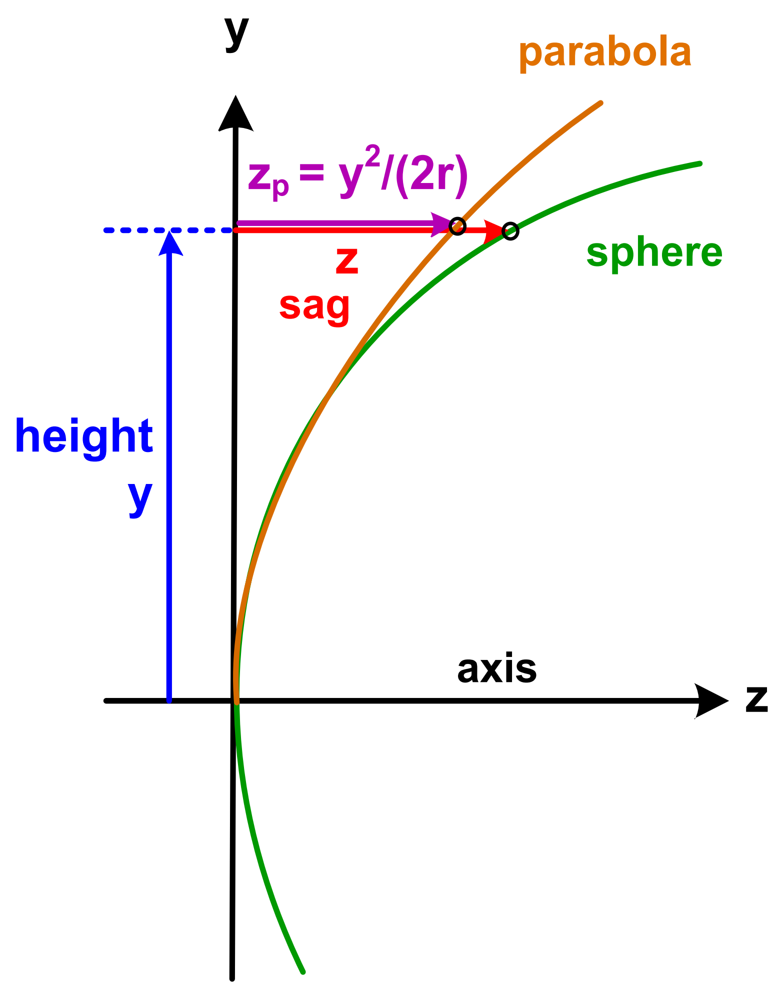
# Sag of a surface

- Sag z at height y for a spherical surface:

$$z = r - \sqrt{r^2 - y^2}$$

- Paraxial approximation: quadratic term

$$z_p \approx \frac{y^2}{2r}$$



# Conic sections

- Explicite surface equation, resolved to z

  Parameters: curvature c = 1 / R

  conic parameter κ

$$z = \frac{c\left(x^2 + y^2\right)}{1 + \sqrt{1 - (1+\kappa)c^2\left(x^2 + y^2\right)}}$$

- Influence of κ on the surface shape

| Parameter | Surface shape |
|---|---|
| κ = - 1 | paraboloid |
| κ < - 1 | hyperboloid |
| κ = 0 | sphere |
| κ > 0 | oblate ellipsoid (disc) |
| 0 > κ > - 1 | prolate ellipsoid (cigar ) |

- Relations with axis lengths a,b of conic sections

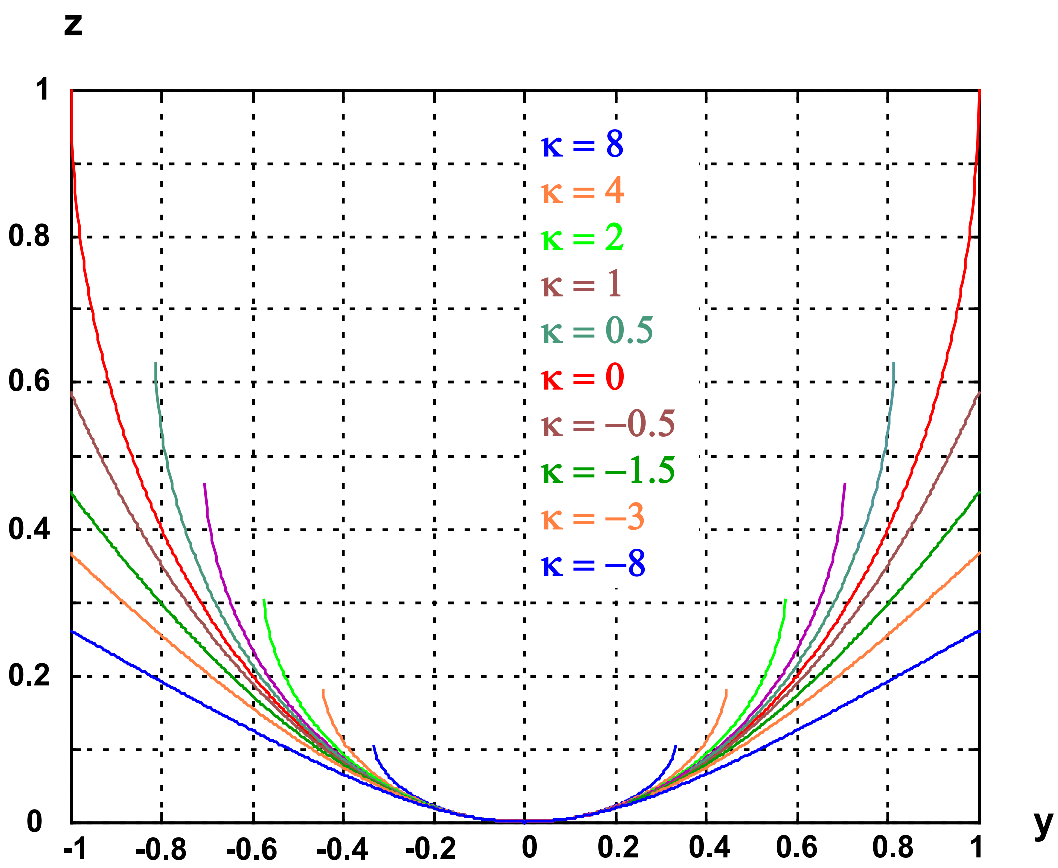
$$\kappa = \left(\frac{a}{b}\right)^2 - 1 \qquad c = \frac{b}{a^2} \qquad b = \frac{1}{\left|c(1+\kappa)\right|} \qquad a = \frac{1}{\left|c\sqrt{\left|1+\kappa\right|}\right|}$$

# Aspherical shape of conic sections



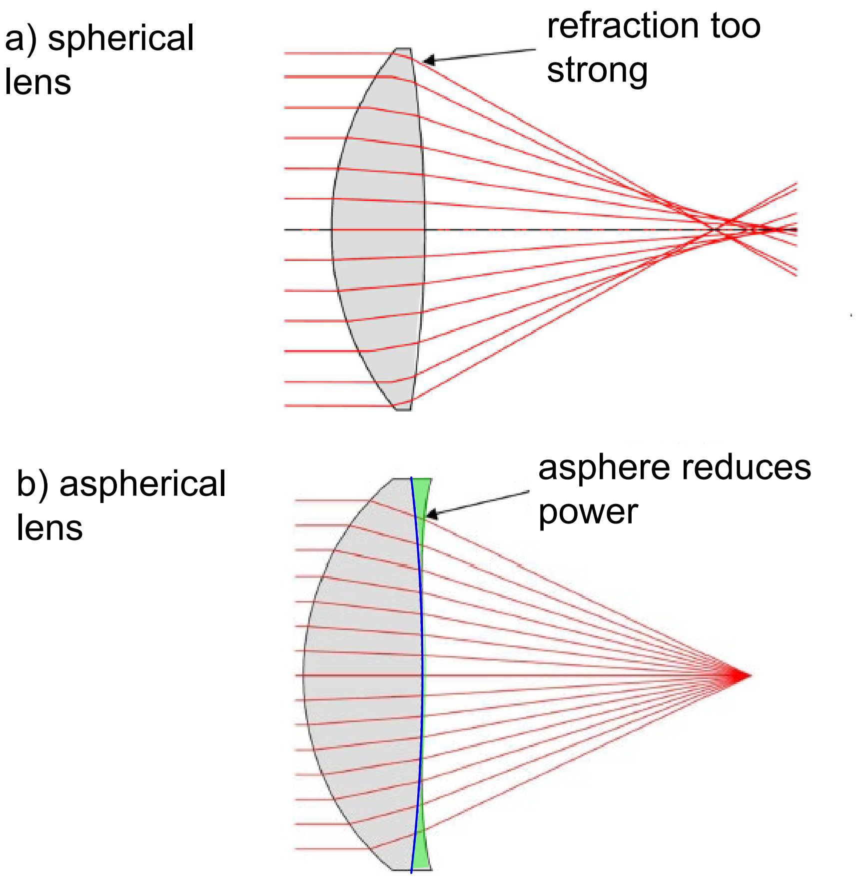
- Conic aspherical surface
- Variation of the conical parameter κ

$$z = \frac{cy^2}{1+\sqrt{1-(1+\kappa)\cdot c^2 y^2}}$$



# Aspherical Correction

- Correction of spherical aberration by an asphere




Ref: A. Herkommer

# Parabolic mirror

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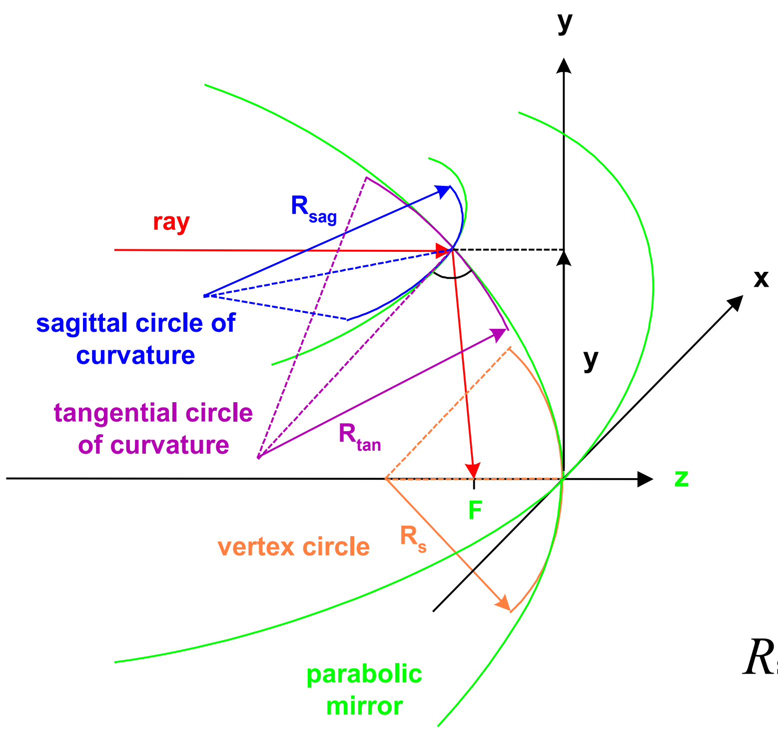
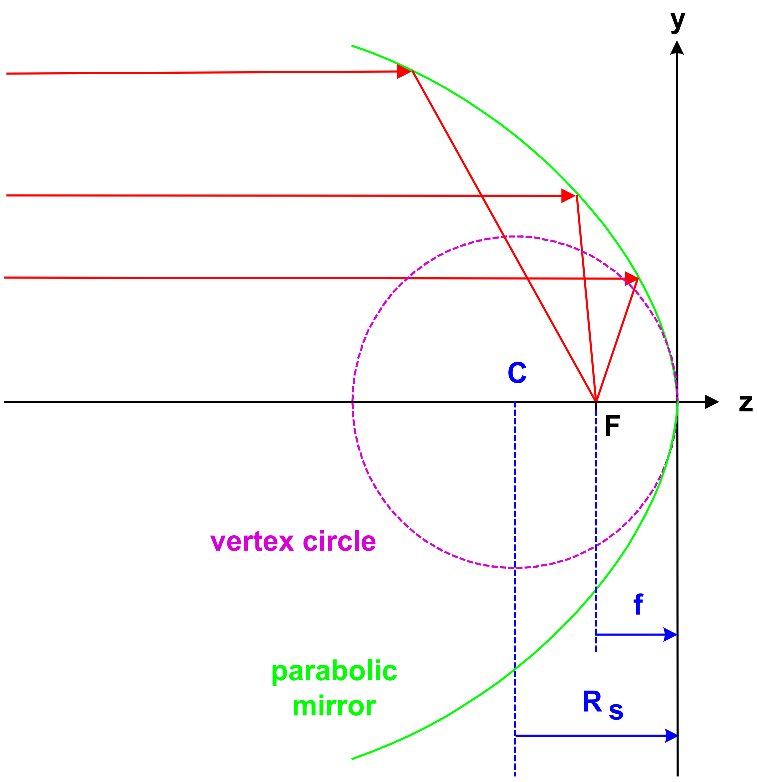
Equation

$$z = \frac{cy^2}{1+\sqrt{1-(1+\kappa)y^2c^2}}$$

c : curvature $1/R_s$

$\kappa$ : eccentricity ( = -1 )




radii of curvature :

$$R_{sag} = R_s \cdot \sqrt{1+\left(\frac{y}{R_s}\right)^2} \qquad R_{tan} = R_s \cdot \left[1+\left(\frac{y}{R_s}\right)^2\right]^{\frac{3}{2}}$$

# Ellipsoid mirror

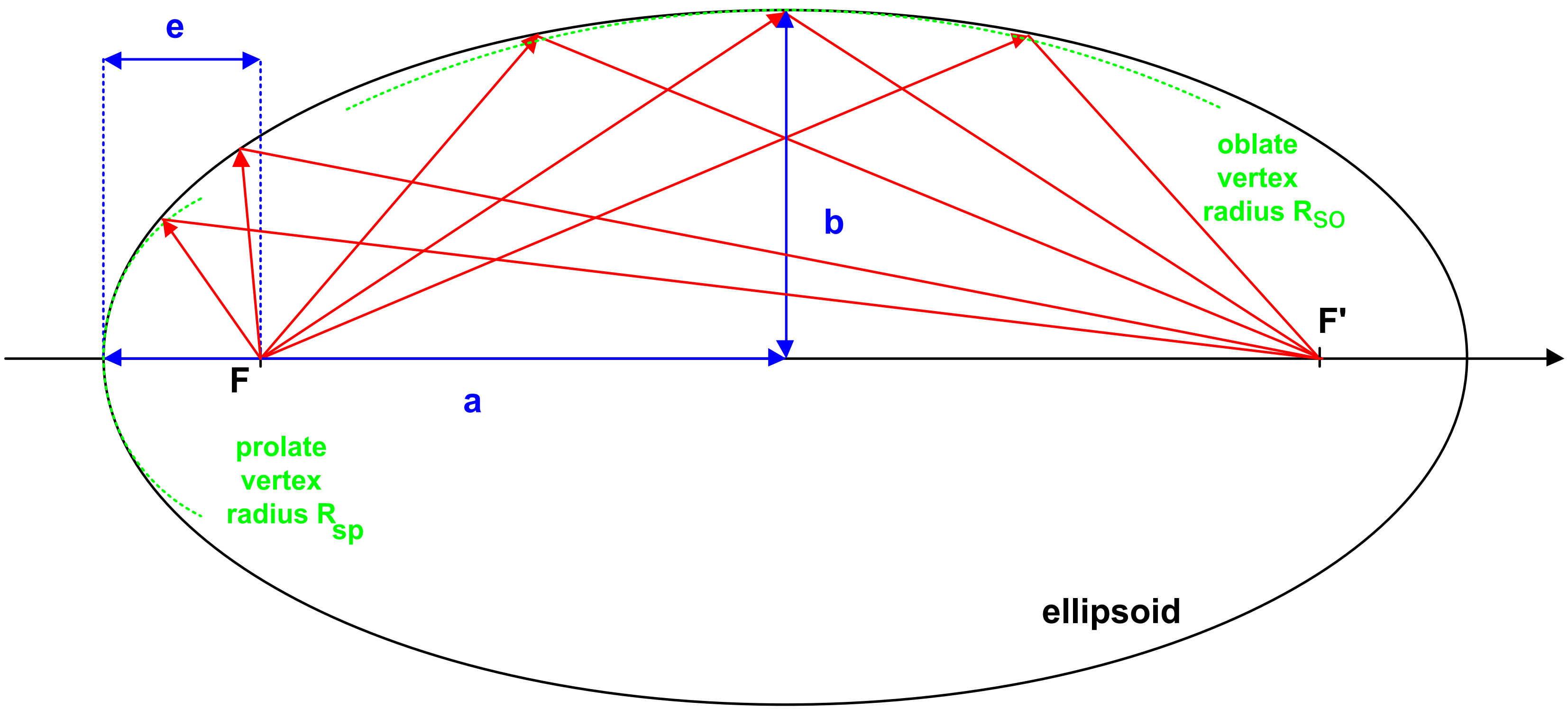


Equation

$$z = \frac{cy^2}{1+\sqrt{1-(1+\kappa)y^2c^2}}$$

c: curvature 1/R

κ: Eccentricity

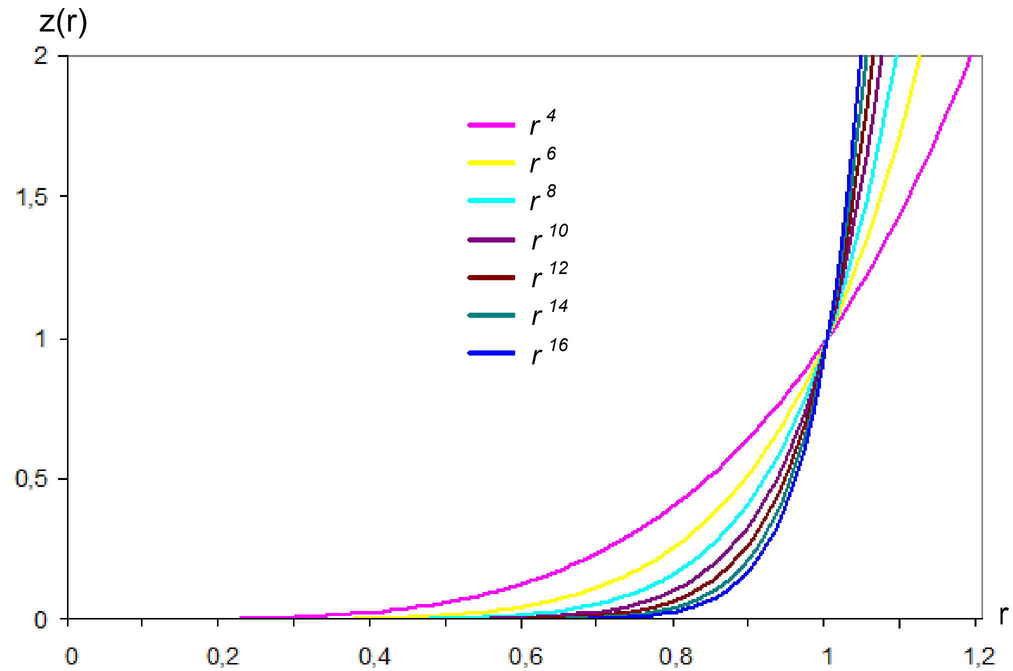


# Polynomial Aspherical Surface
# Standard rotational-symmetric description



- Basic form of a conic section superimposed by a Taylor expansion of z

$$z(\mathrm{r}) = \frac{cr^2}{1+\sqrt{1-(1+\kappa)c^2r^2}} + \sum_{m=0}^{M} a_m r^{2m+4}$$




r ... radial distance to optical axis

$$r = \sqrt{x^2 + y^2}$$

| | |
|---|---|
| c | curvature |
| $\kappa$ | conic constant |
| $a_m$ | aspherical coefficients |

Ref: K. Uhlendorf

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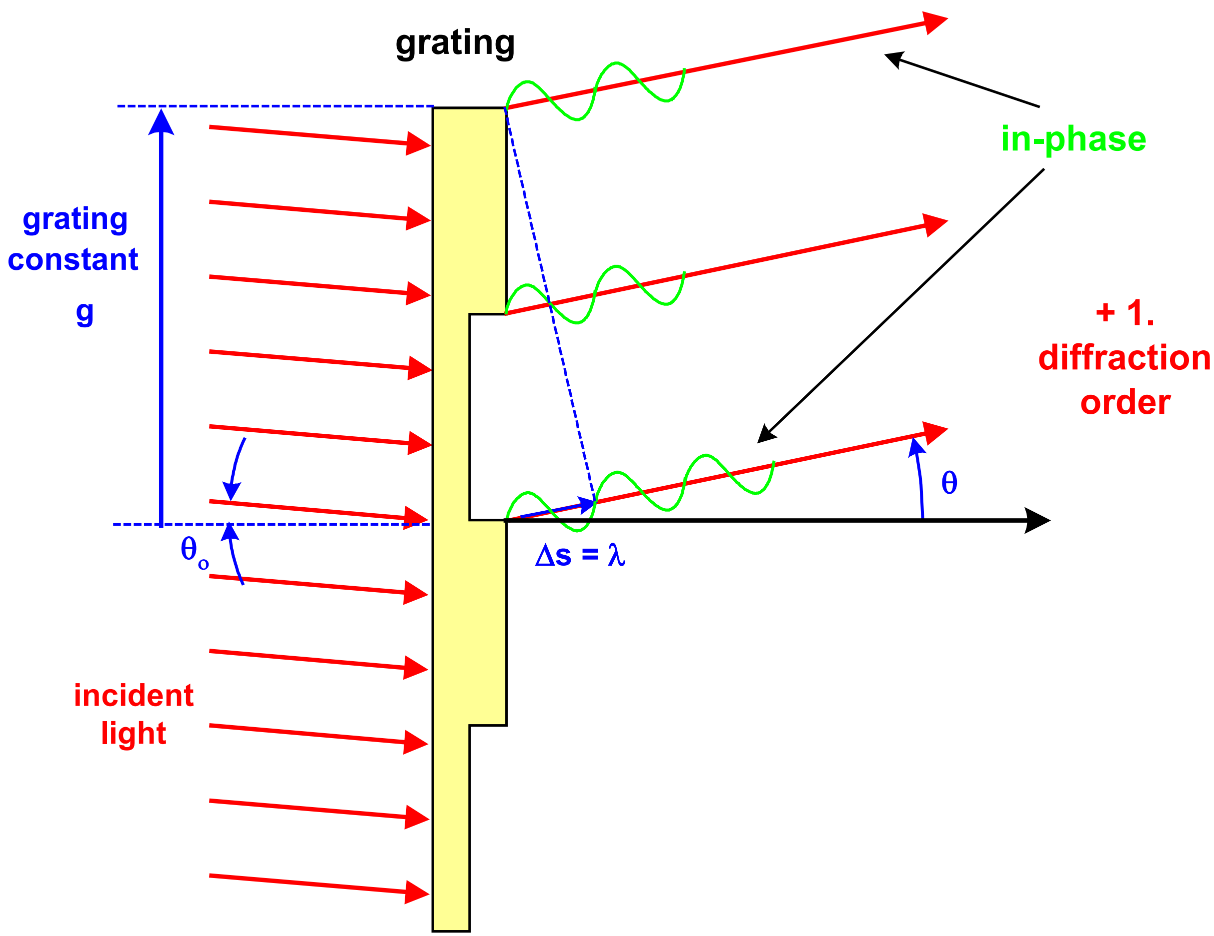
# Grating Diffraction

- Maximum intensity:
  constructive interference of the contributions of all periods

- Grating equation

$$g \cdot \left( \sin\theta - \sin\theta_o \right) = m \cdot \lambda$$



# Ideale diffraction grating

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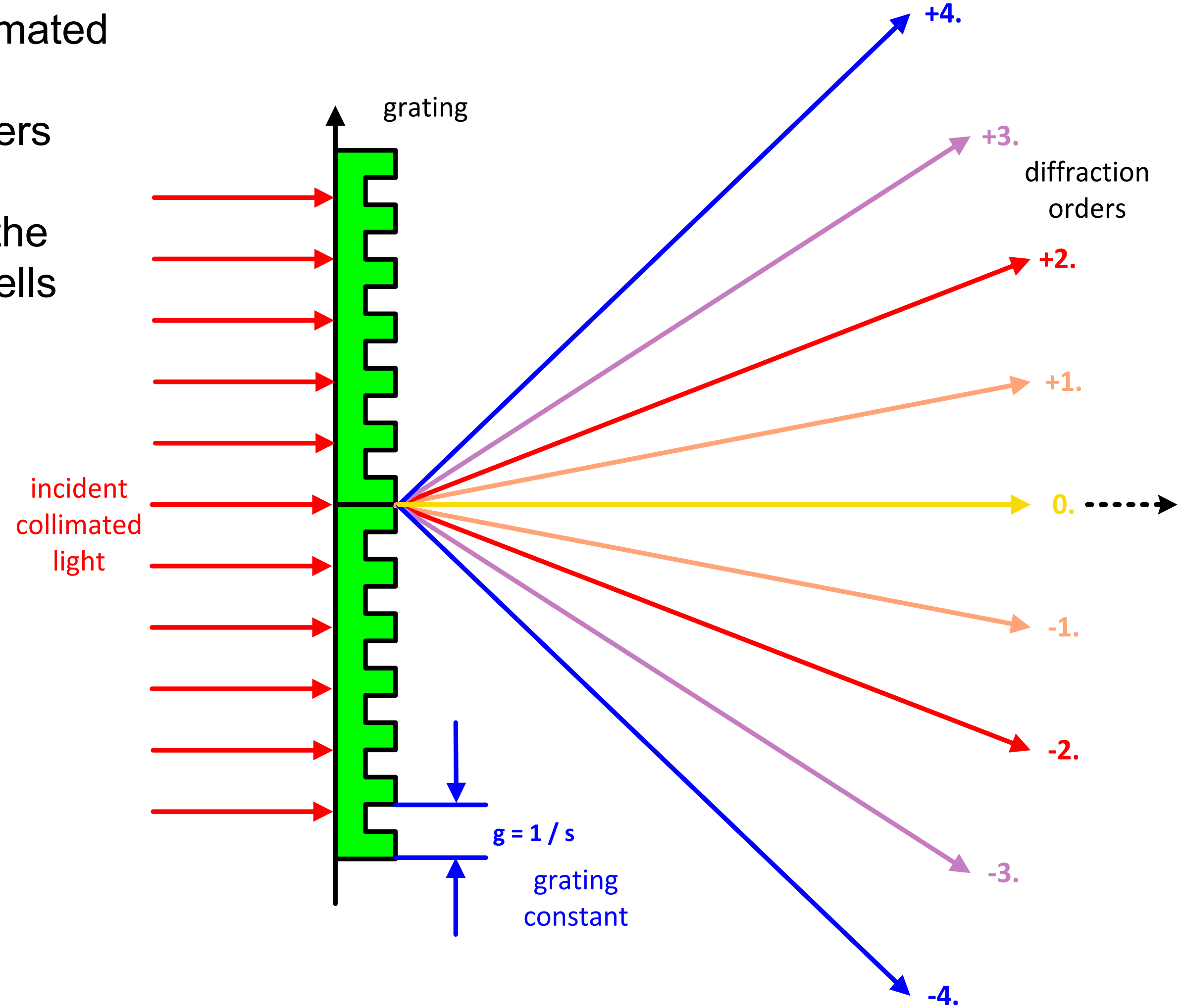
- Ideal diffraction grating: monochromatic incident collimated beam is decomposed into discrete sharp diffraction orders
- Constructive interference of the contributions of all periodic cells



# Finite width of real grating orders

- Interference function of a finite number N of periods

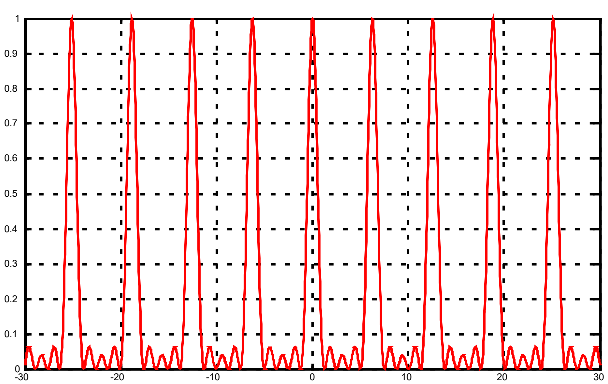
$$I = \frac{\sin^2\left(\dfrac{\pi \cdot g \cdot N \cdot \sin\theta}{\lambda}\right)}{\sin^2\left(\dfrac{\pi \cdot g \cdot \sin\theta}{\lambda}\right)}$$

- Finite angular width of every order depends on N

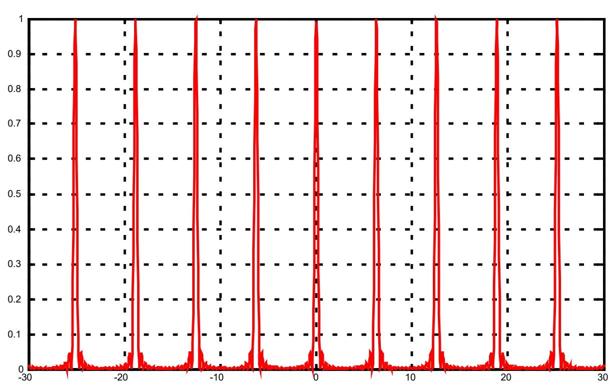
$$\sin\frac{\theta_{1/2}}{2} = \frac{\lambda}{4g \cdot N}$$

- Sharp order direction only in the limit of $N \rightarrow \infty$

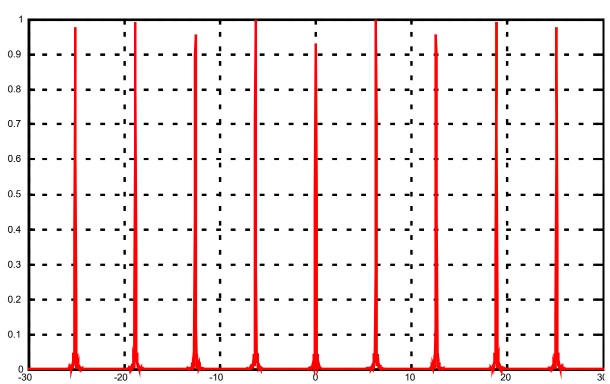
**N = 5**



**N = 15**

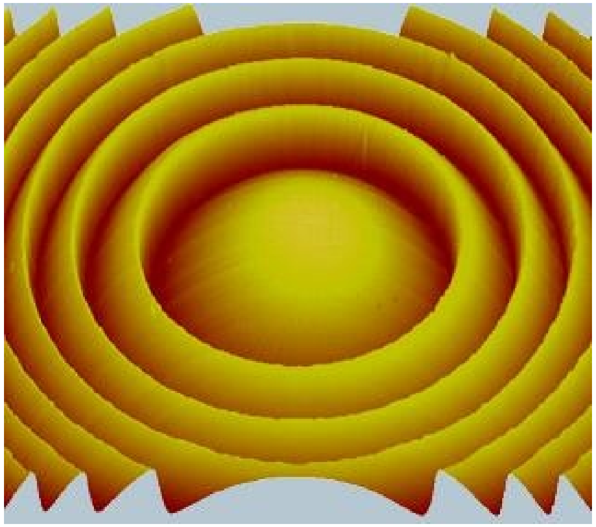


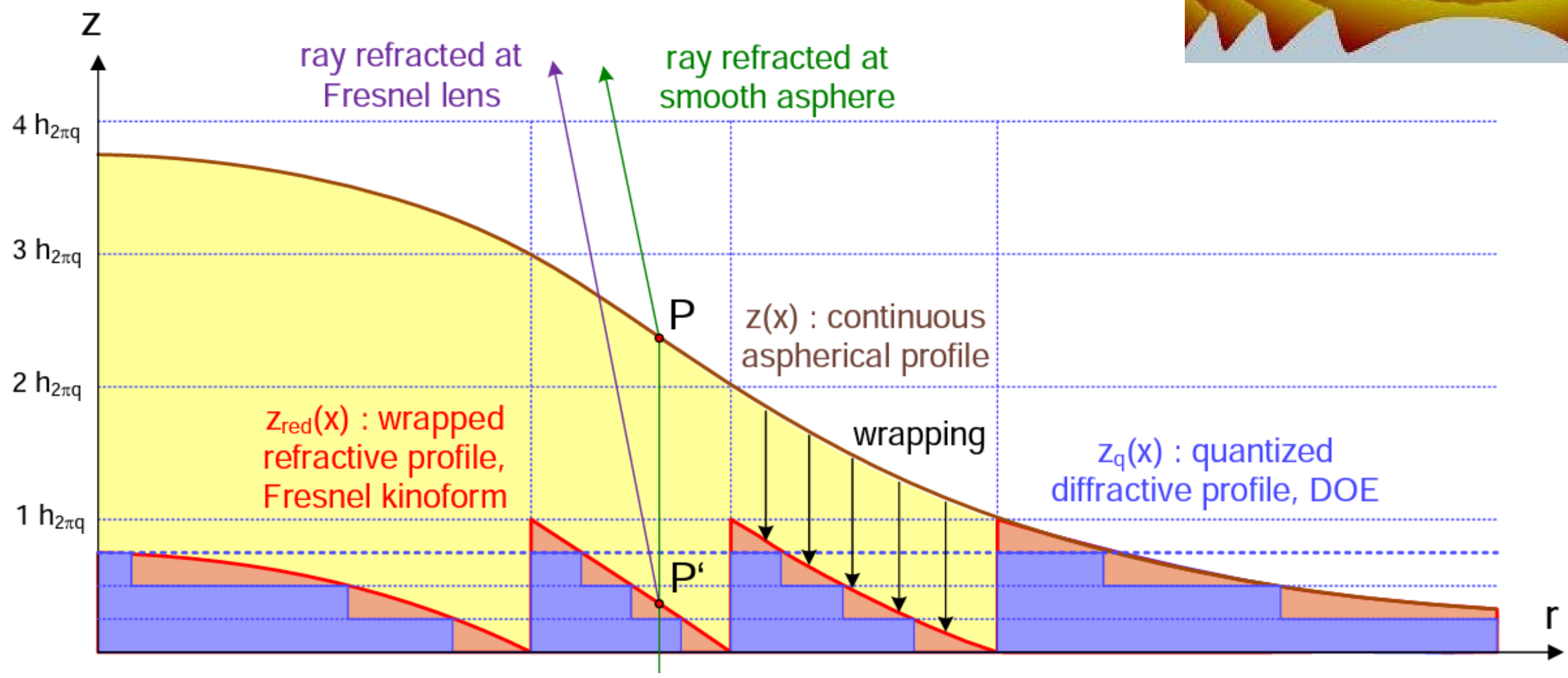
**N = 50**



# Diffractive Elements

- Original lens height profile h(x)
- Wrapping of the lens profile: $h_{red}(x)$ reduction on maximal height $h_{2\pi}$
- Digitalization of the reduced profile: $h_q(x)$



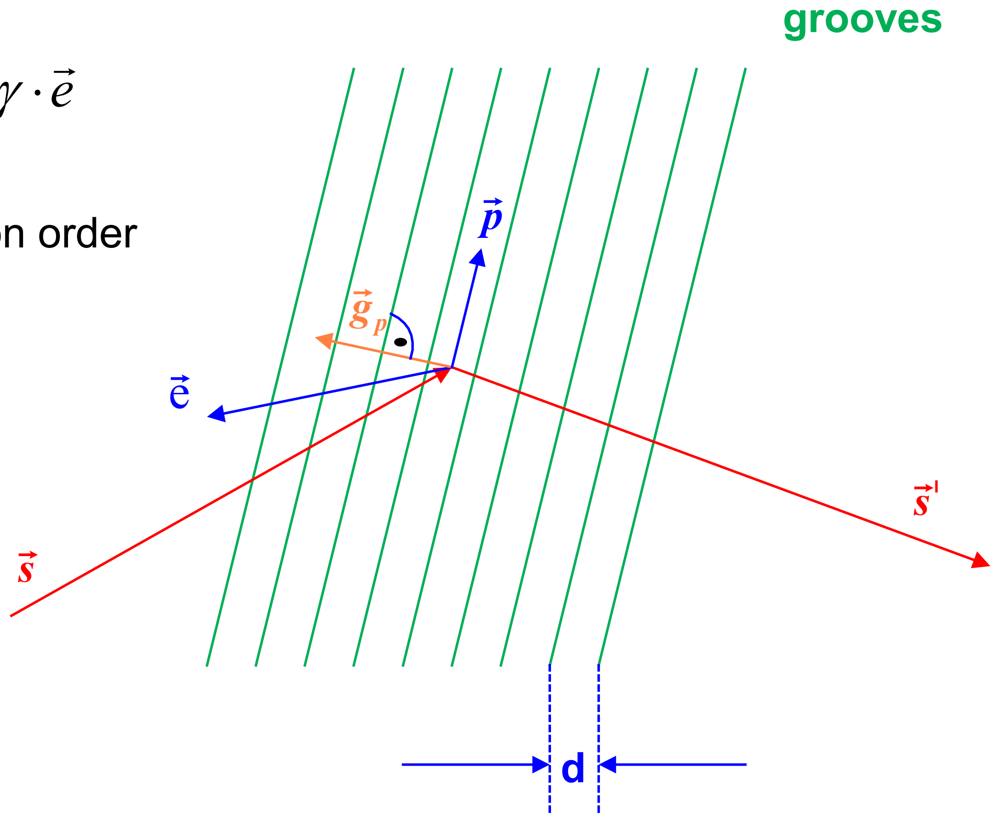


# Diffracting surfaces

- Surface with grating structure:
  new ray direction follows the grating equation
- Local approximation in the case of space-varying grating width

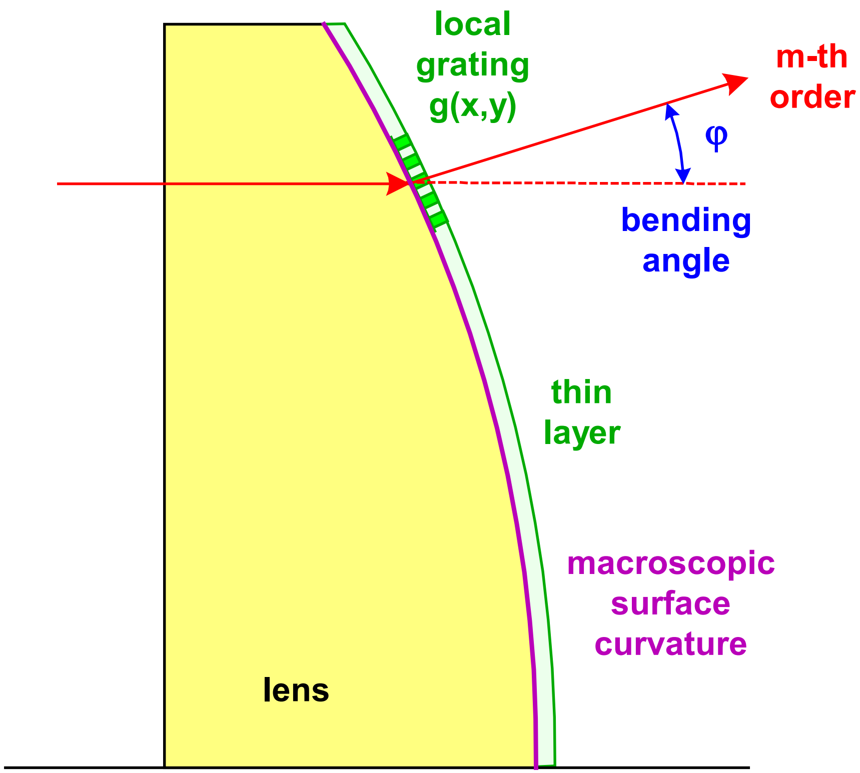
$$\vec{s}\,' = \frac{n}{n'} \cdot \vec{s} + \frac{m\lambda g}{n'd} \cdot \hat{\vec{g}} + \gamma \cdot \vec{e}$$

- Raytrace only into one desired diffraction order
- Notations:
  g : unit vector perpendicular to grooves
  d : local grating width
  m : diffraction order
  e : unit normal vector of surface
- Applications:
  - diffractive elements
  - line gratings
  - holographic components



# Diffracting surfaces

- Local micro-structured surface
- Location of ray bending :
  macroscopic carrier surface
- Direction of ray bending :
  local grating micro-structure
- Independent degrees of freedom:
  1. shape of substrate determines the point of the ray bending
  2. local grating constant determines the direction of the bended ray



# Raytracing in GRIN media

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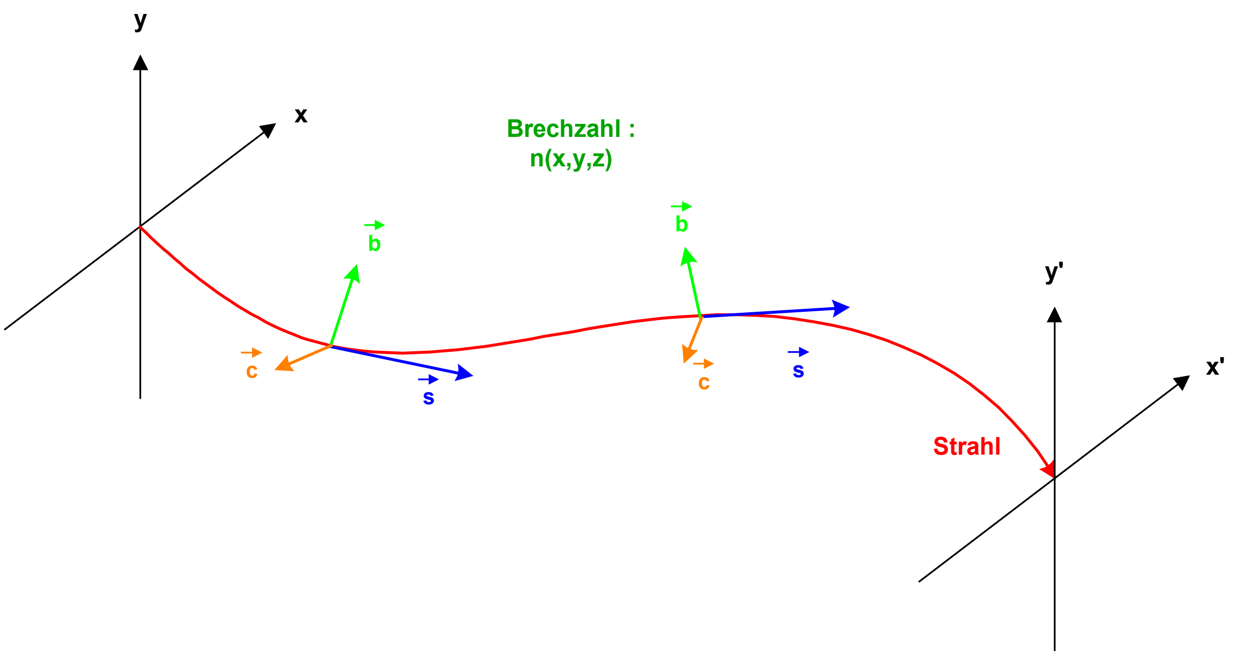
- Ray: in general curved line
- Numerical solution of Eikonal equation
- Step-based Runge-Kutta algorithm

  4th order expansion, adaptive step width
- Large computational times necessary for high accuracy

$$\frac{d^2\vec{r}}{dt^2} = n \cdot \nabla n = \vec{D} = \begin{pmatrix} n\frac{\partial n}{\partial x} \\ n\frac{\partial n}{\partial y} \\ n\frac{\partial n}{\partial z} \end{pmatrix}$$



# Description of GRIN media



- Analytical description of grin media by Taylor expansions of the function n(x,y,z)
- Separation of coordinates

$$n=n_{o,\lambda}+c_1 h+c_2 h^2+c_3 h^4+c_4 h^6+c_5 h^8+c_6 z+c_7 z^2+c_8 z^3+c_9 z^4$$
$$+c_{10} x+c_{11} x^2+c_{12} x^3+c_{13} y+c_{14} y^2+c_{15} y^3$$

- Circular symmetry, nested expansion with mixed terms

$$n=n_{o,\lambda}+c_1 h^2+c_2 h^4+c_3 h^6+c_4 h^8+z\left(c_5+c_6 h^2+c_7 h^4+c_8 h^6+c_9 h^8\right)$$
$$+z^2\left(c_{10}+c_{11} h^2+c_{12} h^4+c_{13} h^6+c_{14} h^8\right)+z^3\left(c_{15}+c_{16} h^2+c_{17} h^4+c_{18} h^6+c_{19} h^8\right)$$

- Circular symmetry only radial

$$n=n_{o,\lambda}\sqrt{1+c_2(c_1 h)^2+c_3(c_1 h)^4+c_4(c_1 h)^6+c_5(c_1 h)^8+c_6(c_1 h)^{10}}$$

- Only axial gradients

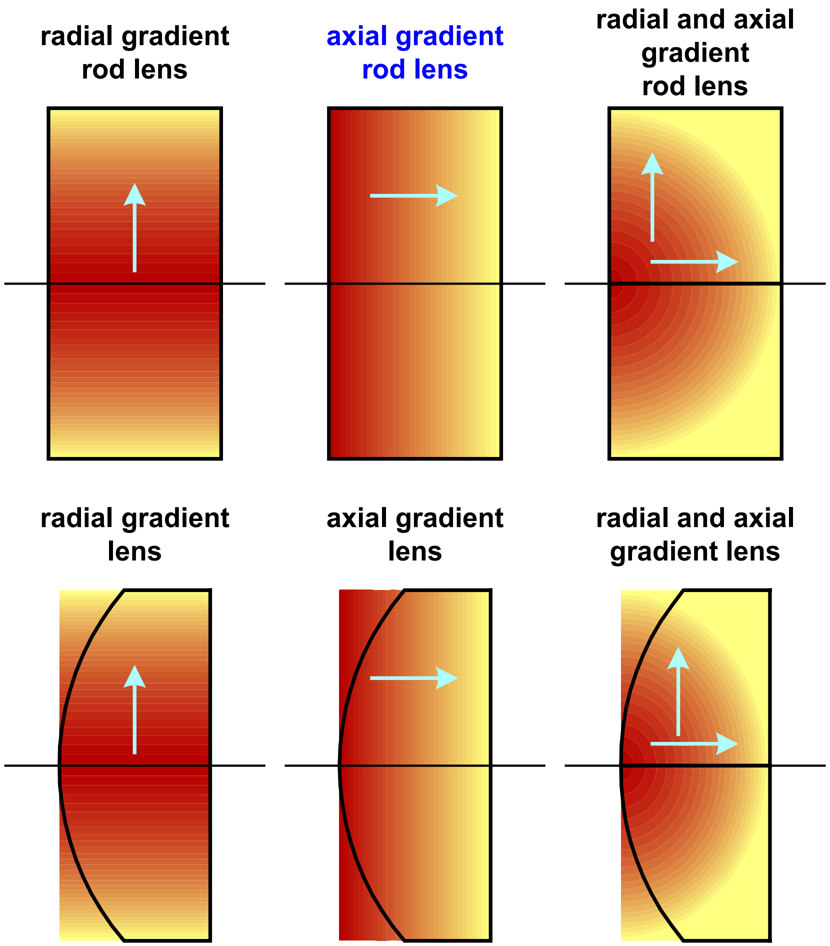
$$n=n_{o,\lambda}\sqrt{1+c_2(c_1 z)^2+c_3(c_1 z)^4+c_4(c_1 z)^6+c_5(c_1 z)^8}$$

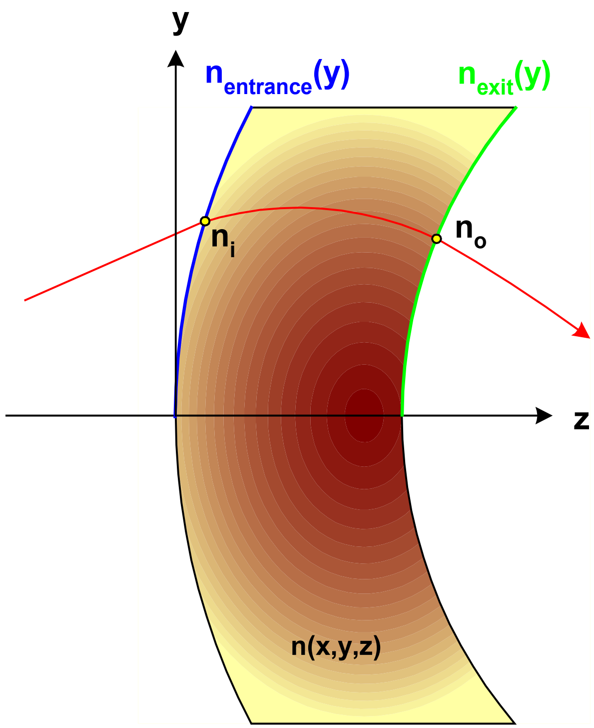
- Circular symmetry, separated, wavelength dependent

$$n=n_{o,\lambda}+c_{1,\lambda} h^2+c_{2,\lambda} h^4+c_{3,\lambda} h^6+c_{4,\lambda} h^8+c_{5,\lambda} z+c_{6,\lambda} z^2+c_{7,\lambda} z^3$$

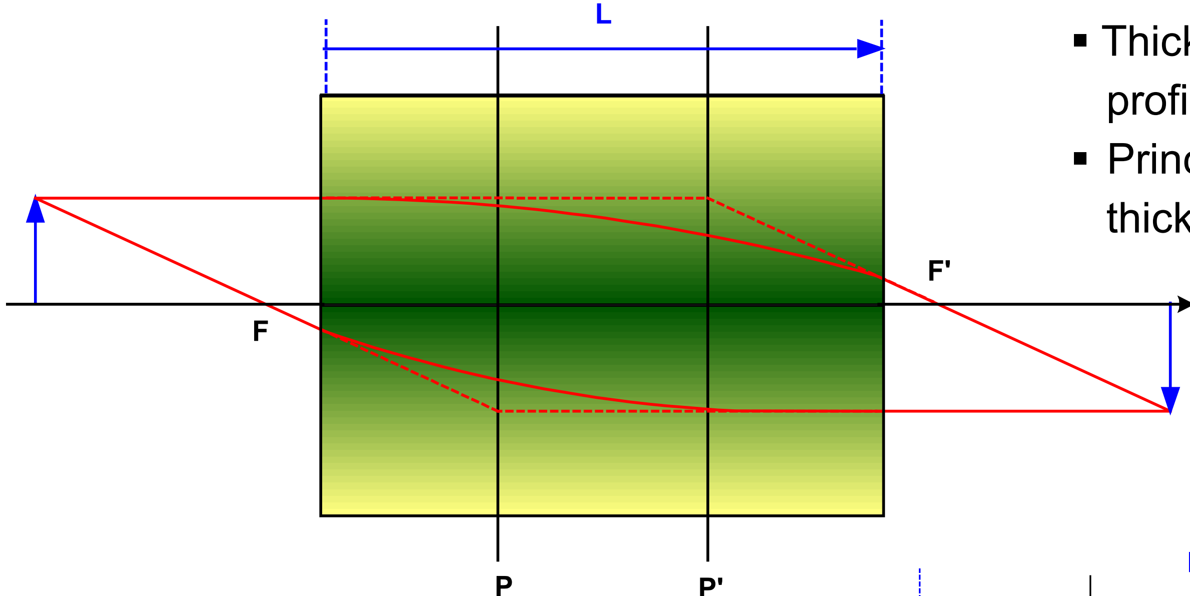
# Gradient Lens Types

- Curved ray path in inhomogeneous media
- Different types of profiles



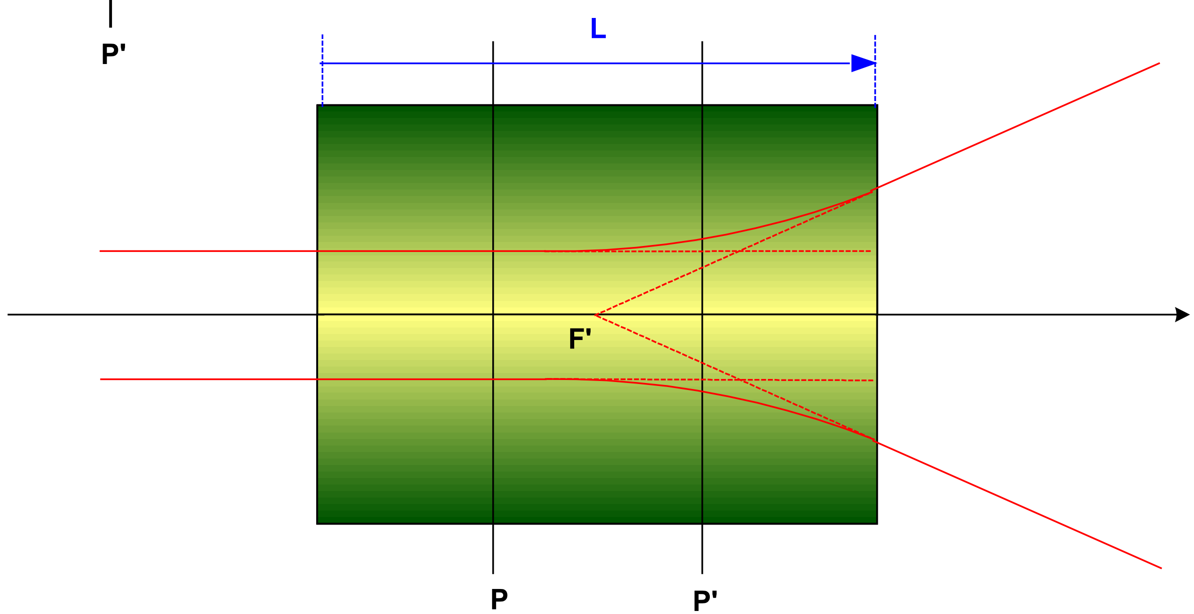
# Collecting radial selfoc lens



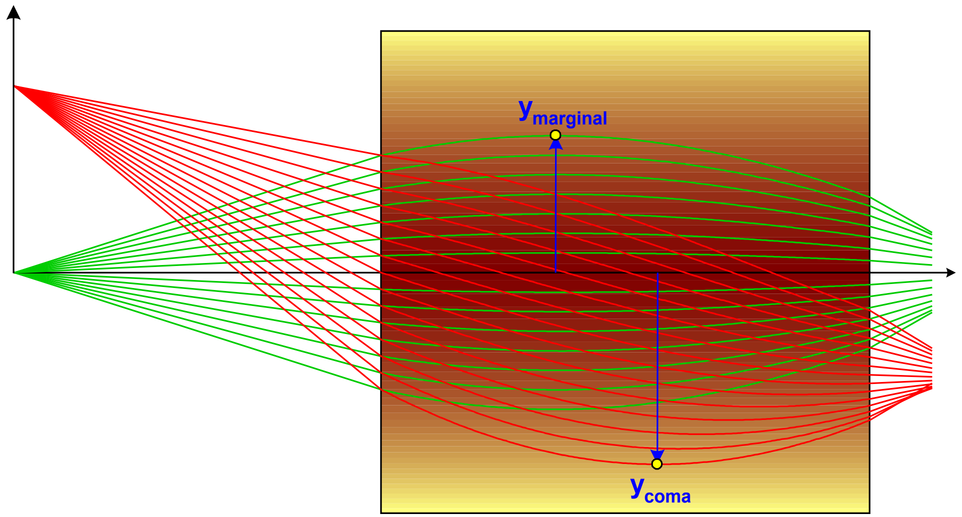

- Thick Wood lens with parabolic index profile
- Principal planes at 1/3 and 2/3 of thickness

$$n(r) = n_0 - n_2 \cdot r^2$$

- $n_2 > 0$ : collecting lens
- $n_2 < 0$ : negative lens



# Gradient Lenses



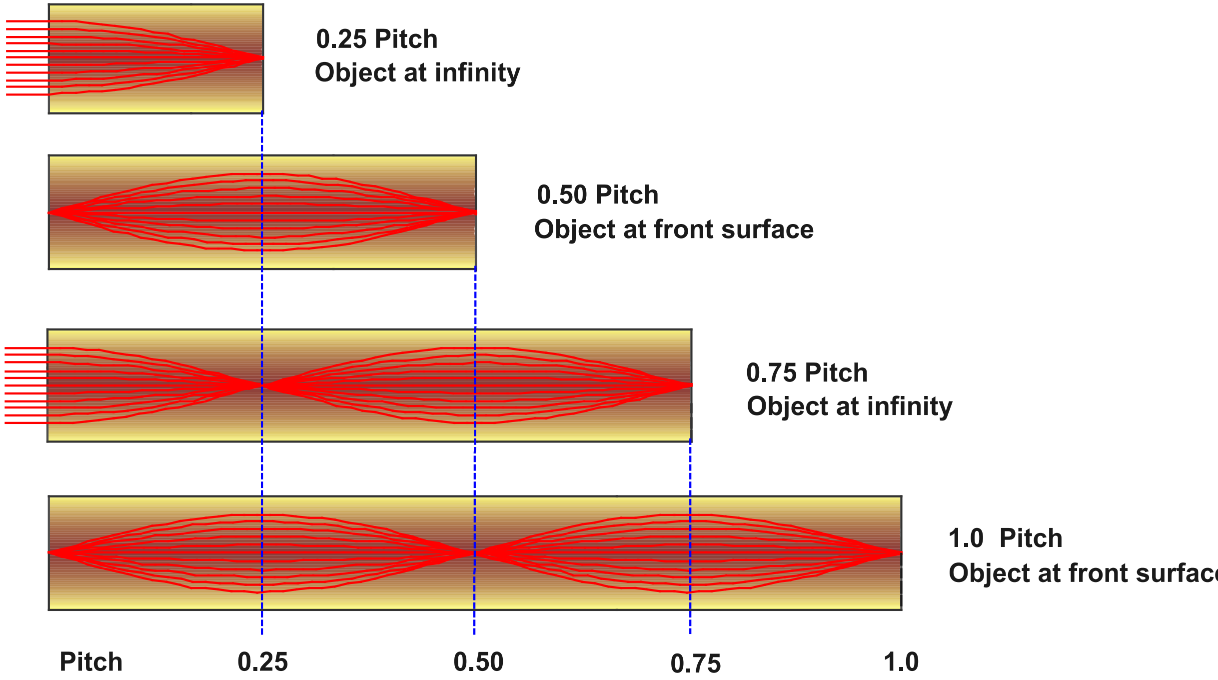

- Types of lenses with parabolic profile

$$n(r) = n_0 - n_2 \cdot r^2 = n_0 \cdot \left(1 - n_r \cdot r^2\right)$$

$$= n_0 \cdot \left(1 - \frac{1}{2} A \cdot r^2\right)$$

- Pitch length

$$p = 2\pi \cdot \sqrt{\frac{n_0}{2n_2}} = \frac{2\pi}{\sqrt{2n_r}}$$



# Description of Grin Media in Zemax



- Gradient 1 $$n = n_0 + n_{r2} r^2 + n_{r1} r \,,$$
- Gradient 2 $$n^2 = n_0 + n_{r2} r^2 + n_{r4} r^4 + n_{r6} r^6 + n_{r8} r^8 + n_{r10} r^{10} + n_{r12} r^{12}$$
- Gradient 3 $$n = n_0 + n_{r2} r^2 + n_{r4} r^4 + n_{r6} r^6 + n_{z1} z + n_{z2} z^2 + n_{z3} z^3$$
- Gradient 4 $$n = n_0 + n_{x1} x + n_{x2} x^2 + n_{y1} y + n_{y2} y^2 + n_{z1} z + n_{z2} z^2$$
- Gradient 5 $$z = \frac{cr^2}{1 + \sqrt{1 - (1+k)c^2 r^2}} + x\tan(\alpha) + y\tan(\beta)$$
- Gradient 6 with dispersion $$n = n_0 + n_1 r^2 + n_2 r^4 + n_3 r^6 + n_4 r^8$$ $$n_x = A_x + B_x \lambda^2 + \frac{C_x}{\lambda^2} + \frac{D_x}{\lambda^4}$$
- Gradient 7 spherical shells $$n = n_0 + \alpha(r - R) + \beta(r - R)^2 \text{, where}$$ $$r = \frac{R}{|R|}\sqrt{x^2 + y^2 + (R - z)^2} \,.$$

# Description of Grin Media in Zemax

- GRADIUM

$$n = \sum_{i=0}^{11} n_i \left( \frac{z + \Delta z}{z_{max}} \right)^i$$

- Gradient 9

  iso-index lines as

  z-surfaces

$$z = \frac{cr^2}{1 + \sqrt{1 - (1+k)c^2 r^2}} + x\tan(\alpha) + y\tan(\beta)$$

$$n = n_0 \left[ 1.0 - \frac{A}{2} r^2 \right] \qquad A(\lambda) = \left[ K_0 + \frac{K_1}{\lambda^2} + \frac{K_2}{\lambda^4} \right]^2$$

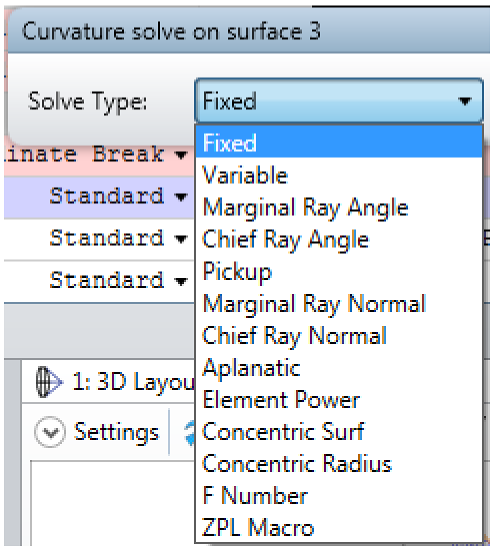
- Gradient 10

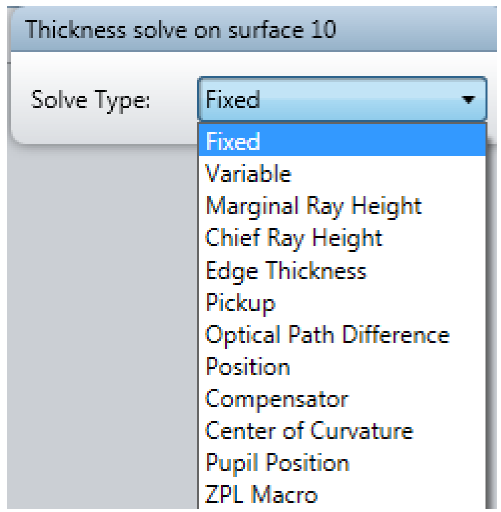
$$n = n_0 + n_{y1} y_a + n_{y2} y_a^2 + n_{y3} y_a^3 + n_{y4} y_a^4 + n_{y5} y_a^5 + n_{y6} y_a^6$$

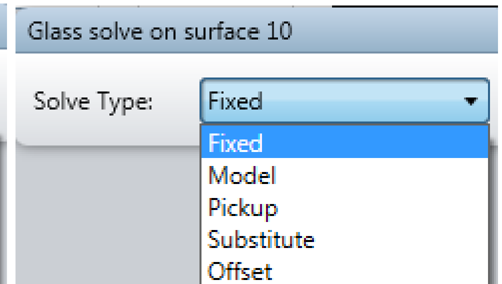
- Grid gradient

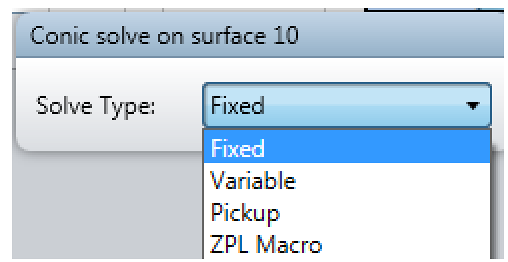
# Solves

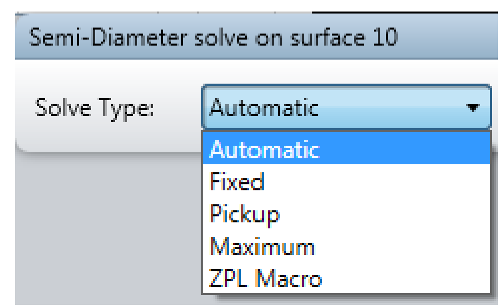
- Open different menus with a click in the corresponding editor cell
- Solves can be chosen individually
- Individual data for every surface in this menu



# Solves

- Examples for solves:

1. last radius forces given image side NA
2. get symmetry of system parts
3. multiple used system parts
4. moving lenses with constant system length
5. bending of a lens with constant focal length
6. non-negative edge thickness of a lens
7. bending angle of a mirror (i'=i)
8. decenter/tilt of a component with return

# Exercise 1: Aspherical Singlet



Aspheres are suitable for correction of spherical aberration, although the performance for finite field sizes is critical. A problem is the conventional Taylor expansion representation of aspheres, which is not orthogonal and therefore sometimes hard to optimize.

a) Establish an imaging setup with a magnification of m = 0.2, a wavelength $\lambda$ = 0.55 $\mu$m, a numerical aperture of NA = 0.6 with a setup, which has an overall length not larger than 50 mm with a single spherical lens made of the plastics material PMMA.

b) Now define the second surface to be aspherical. How many coefficients are necessary to obtain a diffraction limited performance on axis ?

c) What is the largest field size, which guarantees a performance not worse than a factor of two in comparison to the axis point.

d) Now try to install an asphere on both sides of the lens. Can the field behavior be improved ?

# Exercise 2: Grin lens

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Establish a grin rod focussing Wood lens. A component of diameter D = 8 mm, a length L = 25 mm and a refractive index no = 1.5 on axis has circular symmetric a radial gradient profile with quadratic coefficient nr2 = -0.0046262. The lens should be illuminated by a centered collimated beam with wavelength $\lambda$ = 500 nm and diameter 4 mm.

a) Determine the paraxial pitch length of the lens. It is defined by the length of the periodically sine-wave path of the marginal ray.

b) Cut the lens exactly in the focal point and determine the spot diameter in this location. Compare the diameter with the diffraction limited Airy diameter. Why is the spot so large ? What is the smallest spot radius rms value and where is this optimal focus position ?

c) Compare the marginal ray path in the paraxial approximation with the real ray. What is the size of the numerical aperture in the focal point ?

d) Now give the lens a length of 60 mm. Introduce a finite field angle of 25°, the front surface of the lens should be the stop location. Discuss the ray path in the lens drawing considering the diameter and the field ray bundle.