Name: ______________________ Date of birth: ______________ Student ID No. ____________

Final Exam of "Fundamentals of Modern Optics", Winter semester 2017/18
to be written on February 16, 10:00 - 12:00

Problem 1 – Maxwell's Equations **$2 + 2 + 2 + 2 = 8$ points**

a) Write down the Maxwell's equations in a material in *frequency* domain in general form.

b) Write down the names and units of measure for 4 *physical vector* fields *of your choice* in Maxwell's equations.

c) Derive the wave equation in *frequency* domain for the magnetic field $\mathbf{H}(\mathbf{r},\omega)$ in a dispersive, isotropic, homogeneous medium with sources $\rho(\mathbf{r},\omega), \mathbf{j}(\mathbf{r},\omega) \neq 0$.

d) Specify under which conditions in the *source-free* case the electric field $\mathbf{E}(\mathbf{r},\omega)$ will be divergence-free, i.e. the following condition will be fulfilled:

$$\nabla \cdot \mathbf{E}(\mathbf{r},\omega) = 0.$$

Problem 2 – Normal Modes **$2 + 3 + 1 + 2 = 8$ points**

Two monochromatic waves with frequency ω are traveling in a homogeneous, isotropic medium, have electric field vectors $\boldsymbol{E_1} = E_0 e^{ikz}(\cos\theta\hat{\boldsymbol{x}} + \sin\theta\hat{\boldsymbol{y}})$ and $\boldsymbol{E_2} = E_0 e^{ikz} e^{i\phi}\hat{\boldsymbol{x}}$

a) Derive an expression for the corresponding magnetic field vector $\boldsymbol{H_t}$ when the two waves are superimposed.

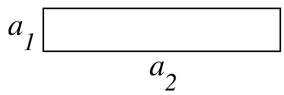
b) Calculate a time averaged Poynting vector $\boldsymbol{S_1}$ and $\boldsymbol{S_2}$ for each wave as well as the total time averaged Poynting vector $\boldsymbol{S_t}$.

c) For what conditions the total time averaged Poynting vector $\boldsymbol{S_t}$ of the two waves is the sum of their corresponding Poynting vectors?

d) If $k = \beta + i\alpha$ use the dispersion relation for plane wave and derive an expression for β and α in terms of the complex susceptibility χ of a weekly absorbing medium in which $\Re[1+\chi] \gg \Im[\chi]$.

Problem 3 – Diffraction **$2 + 2 + 2 + 2 = 8$ points**

Consider a rectangular aperture with sides a_1, a_2 illuminated by the normal plane wave with wavelength λ. The image plane is located at the distanse L.



a) Specify the conditions on a_1, a_2, λ and L for which the diffraction pattern in the image plane can be calculated using: (i) Fresnel approximation; (ii) paraxial Fraunhofer approximation; (iii) non-paraxial Fraunhofer approximation.

b) Calculate the *paraxial Fraunhofer* diffraction pattern (up to a constant factor).

c) Assume that we have N such apertures equally spaced along a line in the apperture plane with distances $b > max(a_1, a_2)$. Specify the positions of local maxima of the diffraction pattern (diffraction orders).

d) Now assume the Fresnel approximation and calculate the spatial frequency (angular) spectrum of the electric field $U(\alpha,\beta,L)$ in the image plane for a single aperture.

Problem 4 – Pulse propagation **$2 + 3 + 2 = 7$ points**

a) Which approximations allow to apply results from beam propagation (spatial dynamics) to pulse propagation (temporal dynamics)? Keep it short!

b) A dispersive material has a refractive index given by $n(\omega) = \alpha + \beta\omega^2 + \gamma\omega^3$. An unchirped Gaussian pulse with a center frequency of ω_0 enters this material. What is the average phase velocity? What is the group velocity of the pulse?

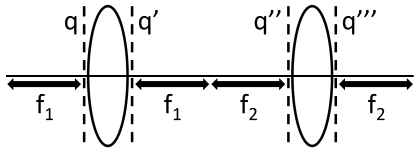
c) The duration of the pulse follows $T(z) = T_0\sqrt{1 + (z/z_0)^2}$ with $z_0 = -T_0^2/(2D)$, where D is the group velocity dispersion. Calculate the distance after which the pulse duration has doubled! How does this distance depend on α, β and γ?

Turn the page.

Problem 5 – Gaussian beam propagation **$2+4+2=8$ points**

a) Gaussian beams are described by their amplitude, beam width, phase curvature and Gouy phase. Which of these quantities are described in the q parameter formalism in Gaussian optics?

A lens of focal length f_1 is placed at a distance $d = f_1$ from the waist of a Gaussian beam. A second lens of focal length f_2 is placed after the first one at a distance $d = f_1 + f_2$ to form a telescope.



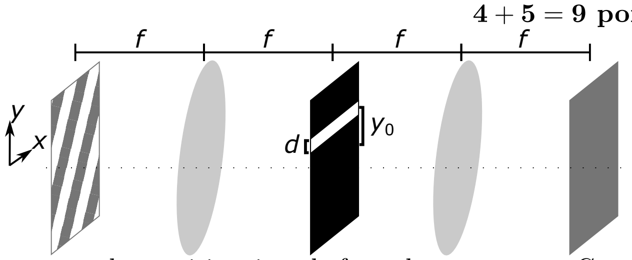
b) Use the q-parameter formalism to show that the waist of the Gaussian beam beyond the telescope is located at a distance f_2 after the second lens.

c) Calculate the magnification of this 4f setup as the relation of the sizes of the output waist and the input waist dependent on f_1 and f_2.

Helpful formulas: $$q' = \frac{Aq+B}{Cq+D} \quad , \quad M_{\text{lens}} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

Problem 6 – Fourier optics **$4+5=9$ points**

Assuming a $4f$-setup as in the figure with the initial field distribution $u_0(x,y) = \cos(\frac{x-y}{p}2\pi)$. Between the two lenses, there is an aperture consisting of a slit parallel to the x-axis with opening size d and its center at the y-position y_0 ($d < y_0$).



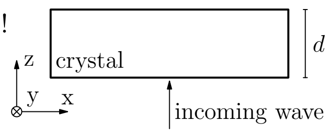
a) Calculate the field distribution after the first $2f$ setup at the position just before the aperture. Consider monochromatic light at the wavelength λ.

b) Explain the functionality of the aperture depending on y_0 and calculate the respective field distributions at the image plane.

Problem 7 – Anisotropy **$2+2+3+2=9$ points**

a) What is the optical axis of a uniaxial crystal? Give a definition in one sentence!

A uniaxial crystal has refractive indices $n_1 = n_2 = 1.4$ and $n_3 = 1.5$. A light wave of wavelength $\lambda = 600$ nm is incident normal to the crystal surface (see figure).



b) What are the two normal modes and their polarization states in the crystal if the optical axis is oriented in y-direction?

c) Consider that the incoming light is x-polarized and the light behind the crystal should be y-polarized. How has the optical axis to be oriented so that this is achieved at the smallest crystal thickness d? Calculate this thickness!

d) Consider that the optical axis lies in the x-z-plane and forms a 45° angle with the incident wave. Sketch and explain how for x- and y-polarized light the $\mathbf{k}$ vector and the Poynting vector are oriented!

Problem 8 – Interfaces/Multilayer **$2+2+5=9$ points**

Consider a system of two homogeneous media with the refractive indices n_1 and n_2, respectively. They shall have a planar interface. A plane wave hits the interface under an angle of θ and it propagates further in the second medium with an angle ϕ.

a) Which law connects the two angles θ and ϕ? Calculate ϕ with respect to n_1, n_2 and θ.

b) How does n_2 and n_1 have to relate to get Total Internal Reflection at the interface for the critical angle θ?

Consider now an interface between a homogeneous medium and an infinite multilayer system with alternating layers of the refractive indices n_1 and n_2 and the thicknesses d_1 and d_2, respectively.

c) Consider a TE polarized monochromatic plane wave with the wavenumber k_0 and normal incidence to the interface. Calculate the relation of d_1 and d_2 to block the light.
Hint: The propagation matrix for a TE polarized plane wave at normal incidence for layer i is

$$\hat{\mathbf{m}}_{\mathrm{i}} = \begin{pmatrix} \cos(k_0 n_{\mathrm{i}} d_{\mathrm{i}}) & \frac{1}{k_0 n_{\mathrm{i}}}\sin(k_0 n_{\mathrm{i}} d_{\mathrm{i}}) \\ -k_0 n_i \sin(k_0 n_{\mathrm{i}} d_{\mathrm{i}}) & \cos(k_0 n_{\mathrm{i}} d_{\mathrm{i}}) \end{pmatrix} \quad .$$