Institute of
Applied Physics
Friedrich-Schiller-Universität Jena

Lens Design I

Lecture 8: Aberrations III

2024-06-06

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Preliminary Schedule - Lens Design I 2024

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1	04.04.	Basics	Zhang	Introduction, Zemax interface, menues, file handling, preferences, Editors, updates, windows, coordinates, System description, 3D geometry, aperture, field, wavelength
2	18.04.	Properties of optical systems I	Tang	Diameters, stop and pupil, vignetting, layouts, materials, glass catalogs, raytrace, ray fans and sampling, footprints
3	25.04.	Properties of optical systems II	Tang	Types of surfaces, cardinal elements, lens properties, Imaging, magnification, paraxial approximation and modelling, telecentricity, infinity object distance and afocal image, local/global coordinates
4	02.05.	Properties of optical systems III	Tang	Component reversal, system insertion, scaling of systems, aspheres, gratings and diffractive surfaces, gradient media, solves
5	16.05.	Advanced handling I	Tang	Miscellaneous, fold mirror, universal plot, slider, multiconfiguration, lens catalogs
6	23.05.	Aberrations I	Zhang	Representation of geometrical aberrations, spot diagram, transverse aberration diagrams, aberration expansions, primary aberrations
7	30.05.	Aberrations II	Zhang	Wave aberrations, Zernike polynomials, measurement of quality
8	06.06.	Aberrations III	Tang	Point spread function, optical transfer function
9	13.06.	Optimization I	Tang	Principles of nonlinear optimization, optimization in optical design, general process, optimization in Zemax
10	20.06.	Optimization II	Zhang	Initial systems, special issues, sensitivity of variables in optical systems, global optimization methods
11	27.06.	Correction I	Zhang	Symmetry principle, lens bending, correcting spherical aberration, coma, astigmatism, field curvature, chromatical correction
12	04.07.	Correction II	Zhang	Field lenses, stop position influence, retrofocus and telephoto setup, aspheres and higher orders, freeform systems, miscellaneous

Contents

1. Diffraction at an aperture
2. Ideal PSF
3. Real PSF and Strehl
4. OTF

Diffraction at the System Aperture

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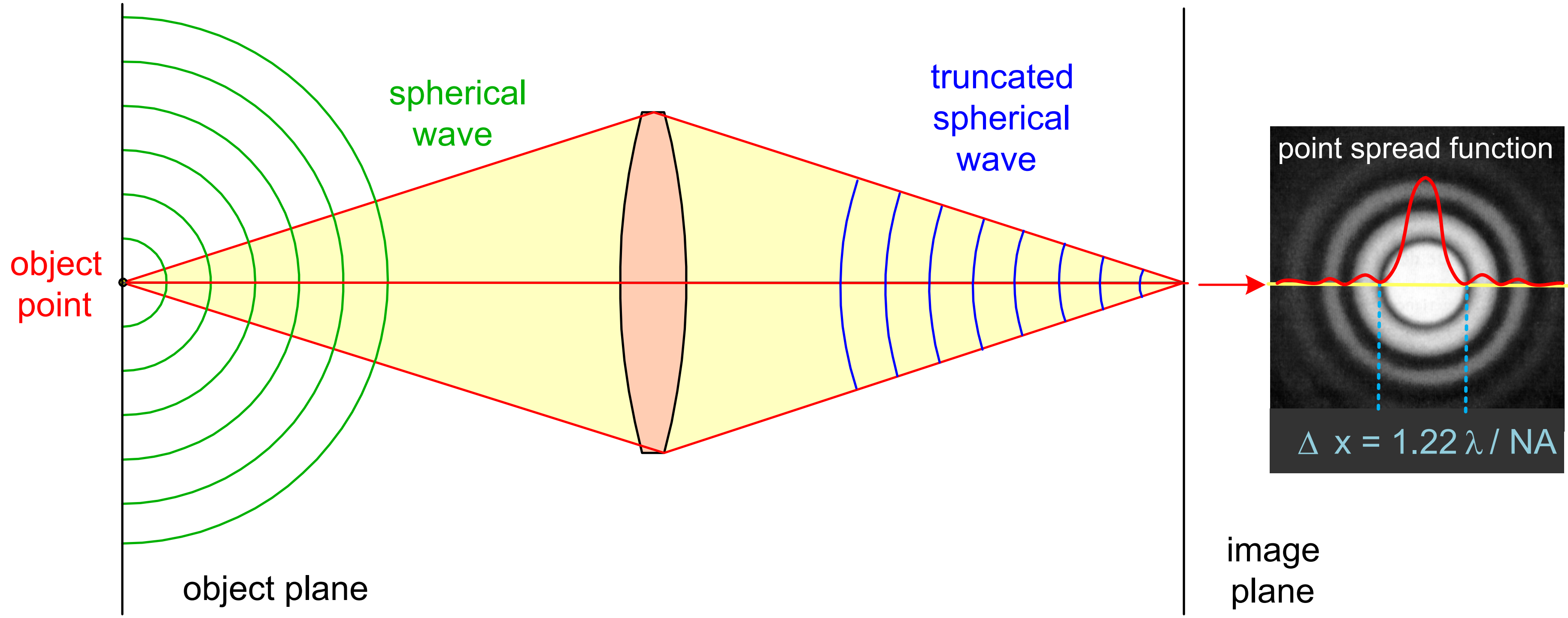
- Self luminous points: emission of spherical waves
- Optical system: only a limited solid angle is propagated, the truncaton of the spherical wave results in a finite angle light cone
- In the image space: uncomplete constructive interference of partial waves, the image point is spreaded
- The optical systems works as a low pass filter



Hybrid model

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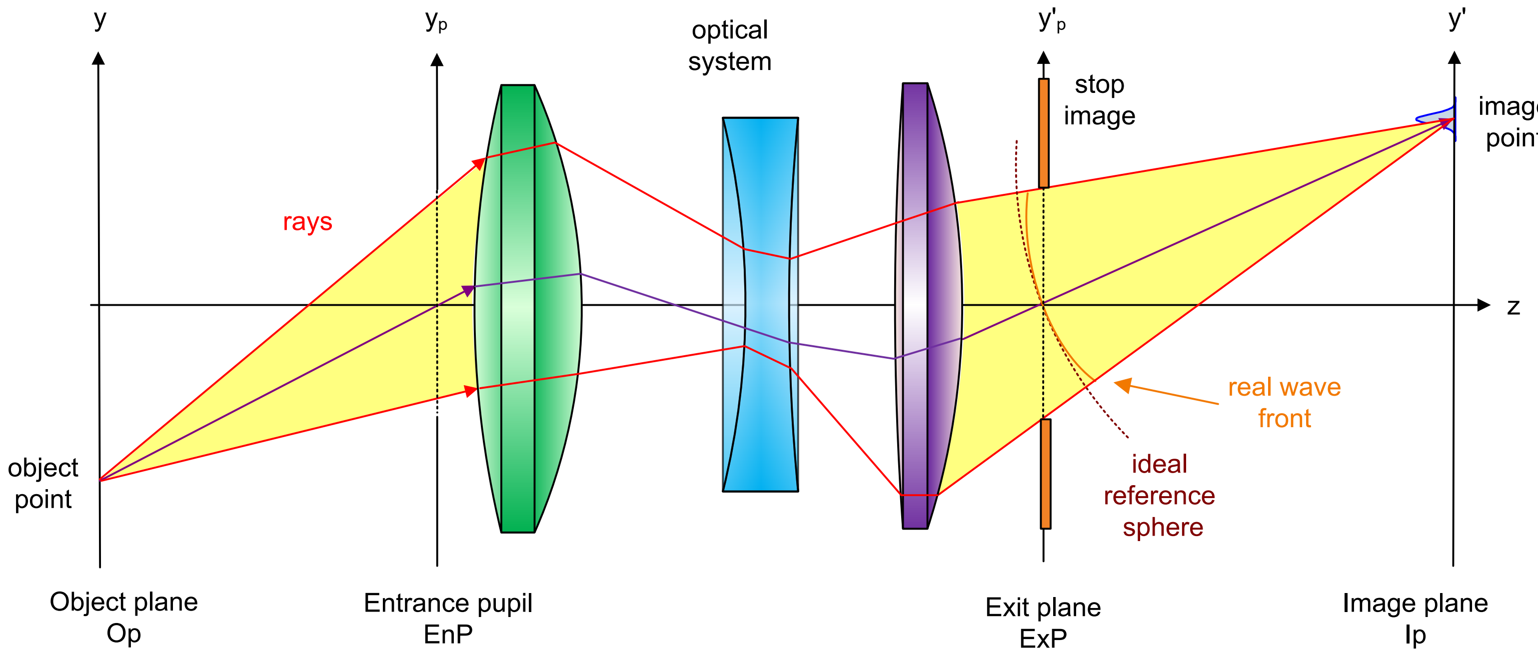
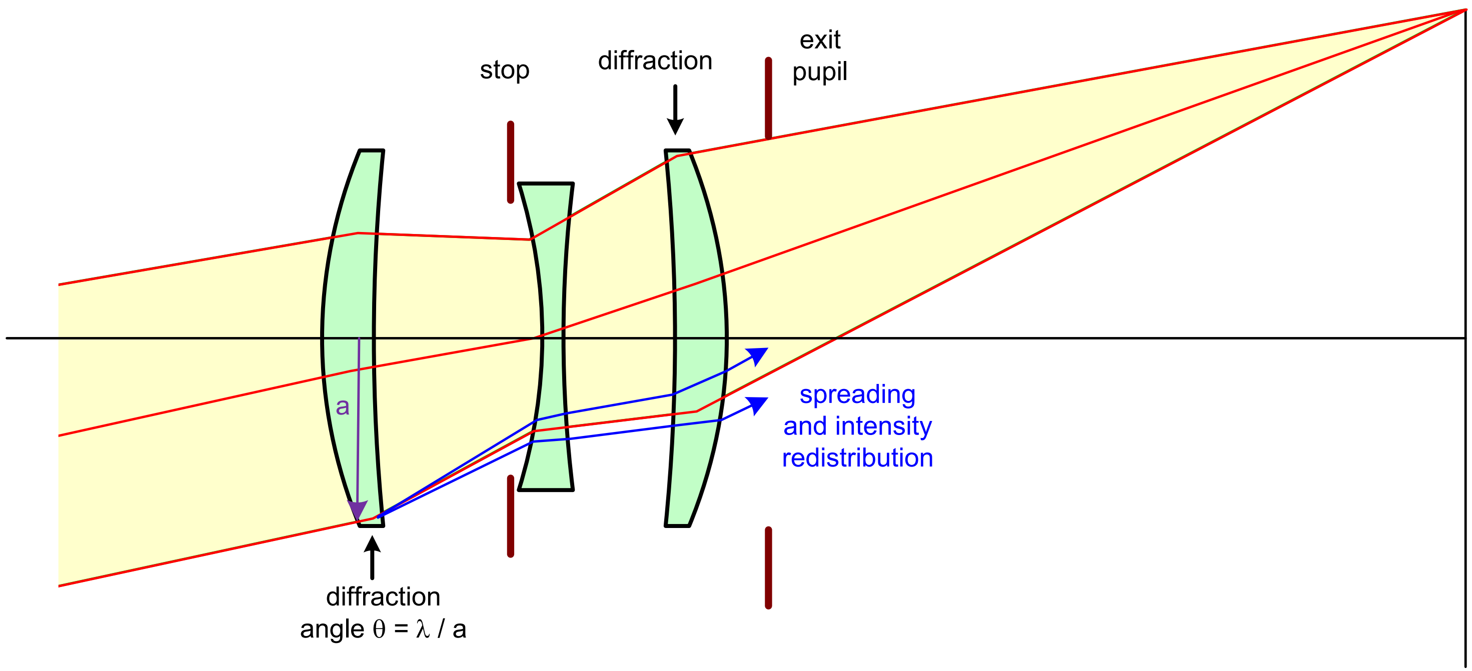
- Optical system: hybrid approach
  - Ray tracing until exit pupil
  - Neglecting diffraction effects until ExP
- Composition of the complex field in the ExP
  - Phase by OPD
  - Calculation of diffraction intergral
- Spherical wave in the ExP



Diffraction inside a system

- hybrid mode: rough appoximation
- In reality, every boundary of a system gives rise to edge diffraction
- In particular, long systems with small diameters in the front group changes the ExP field by propagating diffraction fiels contributions
- Due to practical experience the effects are rather small



Fraunhofer Point Spread Function

- Rayleigh-Sommerfeld diffraction integral,
  Mathematical formulation of the Huygens-principle

$$E_I(\vec{r}) = -\frac{i}{\lambda}\iint E(\vec{r}\,')\cdot\frac{e^{i\vec{k}|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}\cdot\cos\theta_d\,dx'dy'$$

- Fraunhofer approximation in the far field
  for small Fresnel number

$$N_F = \frac{r_p^2}{\lambda\cdot z} \approx 1$$

- Diffraction integral:

$$E(x',y') = \iint_{AP} \boxed{T\left(x_p,y_p\right)\cdot e^{2\pi i W\left(x_p,y_p\right)}} \cdot e^{\frac{2\pi i}{\lambda R_{AP}}\left(x_p x' + y_p y'\right)}\,dx_p dy_p$$

  Pupil amplitude/transmission/illumination $T(x_p,y_p)$
  Wave aberration $W(x_p,y_p)$
  Transition from exit pupil to image plane

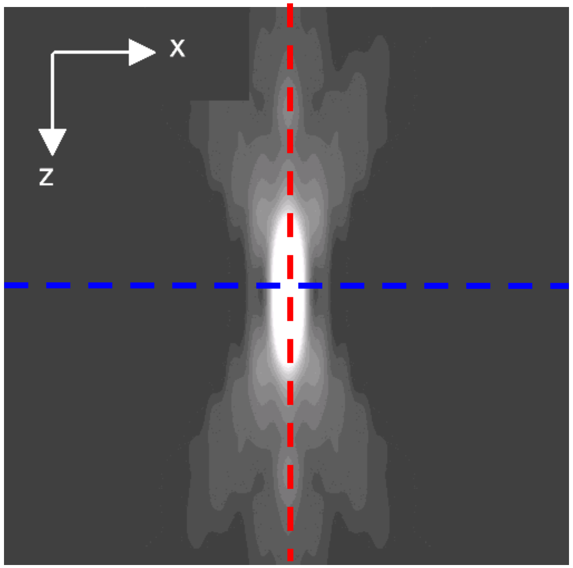
- Point spread function (PSF): Fourier transform of the complex pupil function $A(x_p,y_p)$

$$A(x_p,y_p) = T(x_p,y_p)\cdot e^{2\pi i W(x_p,y_p)}$$

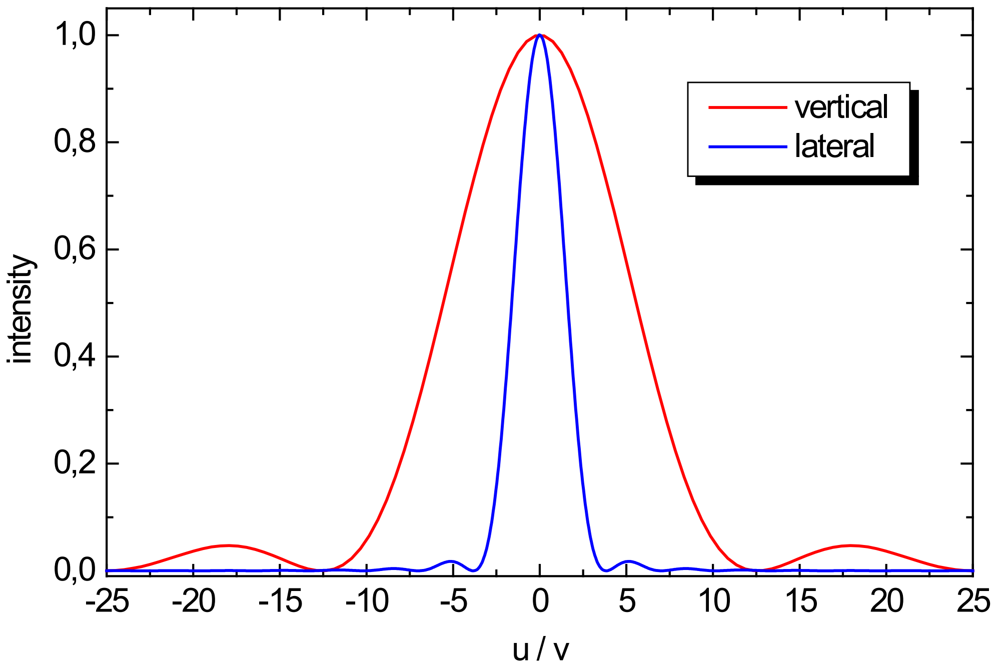
Perfect Point Spread Function

Circular homogeneous illuminated Aperture: intensity distribution

- transversal: Airy scale: $$D_{Airy} = \frac{1.22 \cdot \lambda}{NA}$$
- axial: sinc scale $$R_E = \frac{n \cdot \lambda}{NA^2}$$
- Resolution transversal better than axial: $\Delta x < \Delta z$



Ref: M. Kempe



$$I(0,v) = \left[\frac{2J_1(v)}{v}\right]^2 I_0$$

$$I(u,0) = \left[\frac{\sin(u/4)}{u/4}\right]^2 I_0$$

Scaled coordinates according to Wolf :

axial : $u = 2\pi z n / \lambda NA^2$

transversal : $v = 2\pi x / \lambda NA$

Perfect Lateral Point Spread Function: Airy

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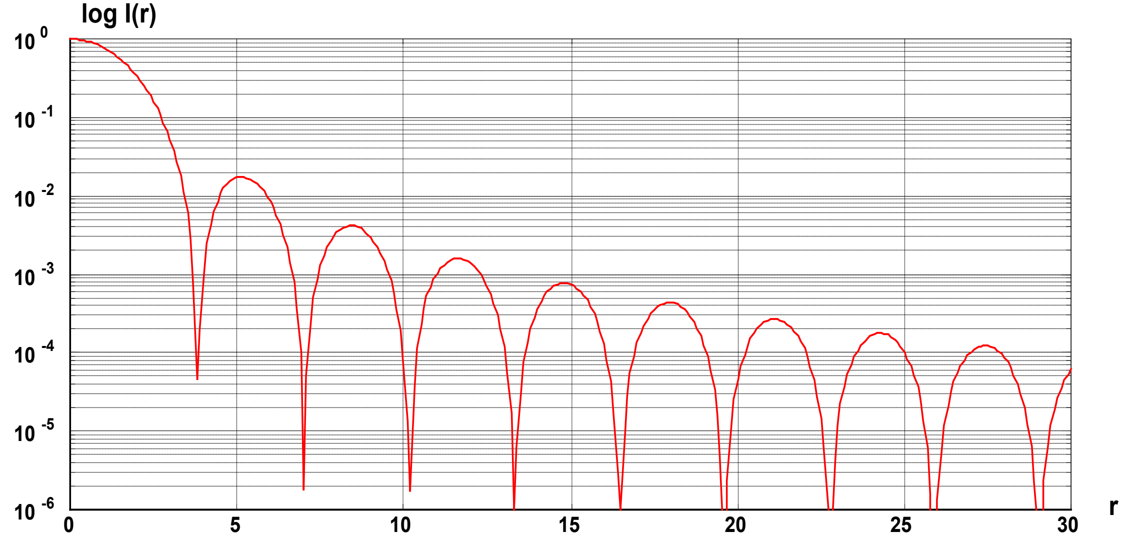
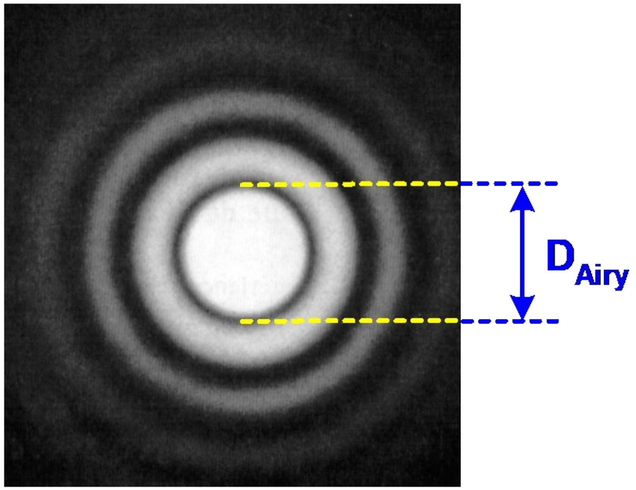
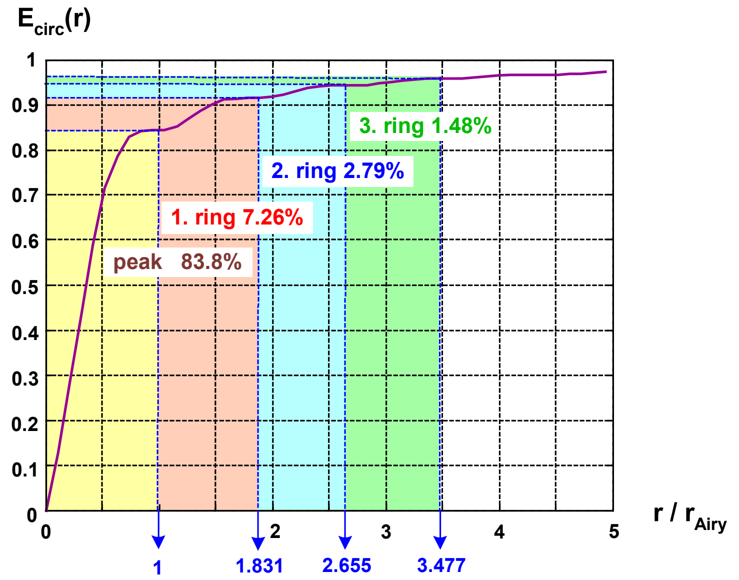
Airy distribution:

- Gray scale picture
- Zeros non-equidistant
- Logarithmic scale
- Encircled energy







Perfect Axial Point Spread Function

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- Axial distribution of intensity
  Corresponds to defocus

$$I(z) = I_0 \cdot \left(\frac{\sin(\bar{z})}{\bar{z}} \right)^2 = I_o \cdot \left(\frac{\sin u/4}{u/4} \right)^2$$

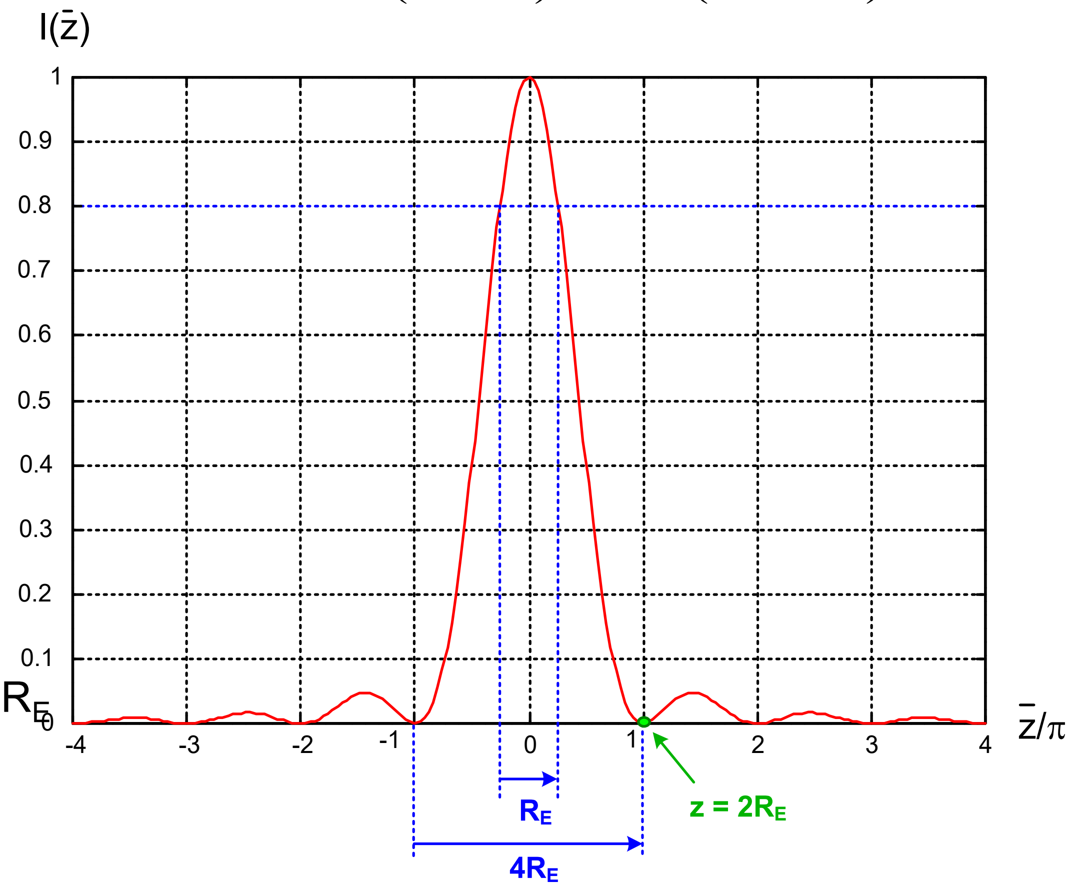
- Normalized axial coordinate

$$\bar{z} = \frac{\pi \, NA^2}{2 \cdot \lambda} \cdot z = \frac{u}{4}$$

- Scale for depth of focus :
  Rayleigh length

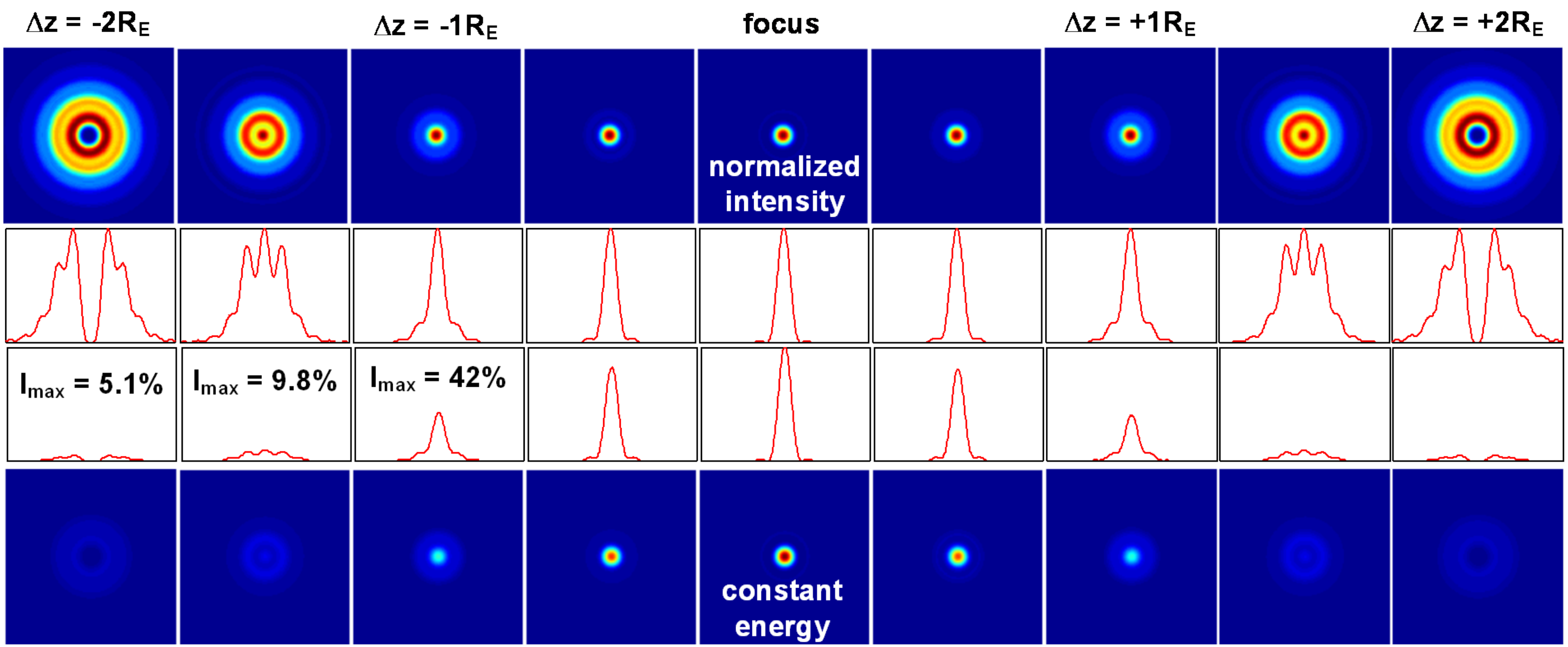
$$R_E = \frac{\lambda}{n' \sin^2 u'}$$

- Zero crossing points:
  equidistant and symmetric,
  Distance zeros around image plane $4R_E$



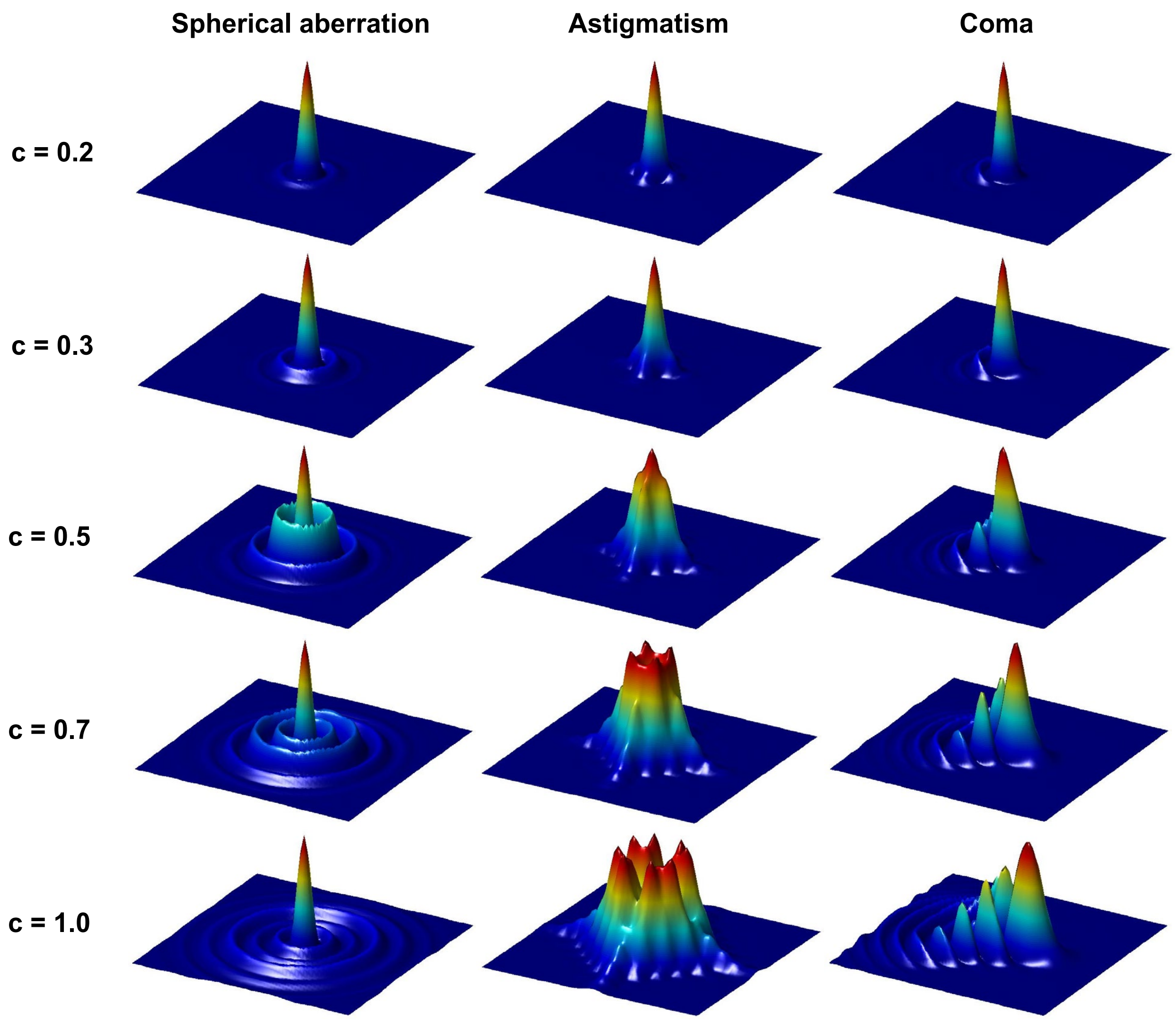
Defocussed Perfect Psf

- Perfect point spread function with defocus
- Representation with constant energy: extreme large dynamic changes



Psf with Aberrations

- Zernike coefficients c in λ
- Spherical aberration,
  Circular symmetry
- Astigmatism,
  Split of two azimuths
- Coma,
  Asymmetric



Comparison Geometrical Spot – Wave-Optical Psf

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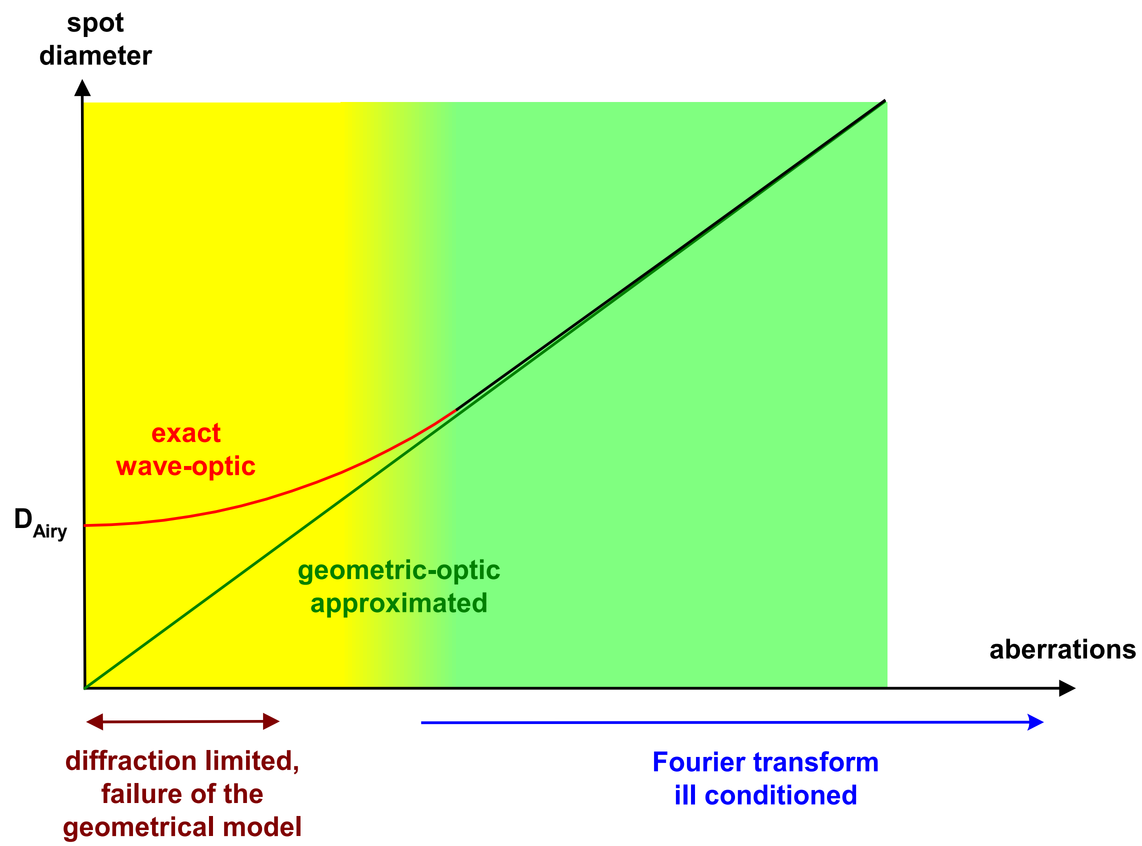
- Large aberrations:
  Waveoptical calculation shows bad conditioning
- Wave aberrations small: diffraction limited, geometrical spot too small and wrong
- Approximation for the intermediate range:

$$D_{Spot} = \sqrt{D_{Airy}^2 + D_{Geo}^2}$$



Strehl Ratio

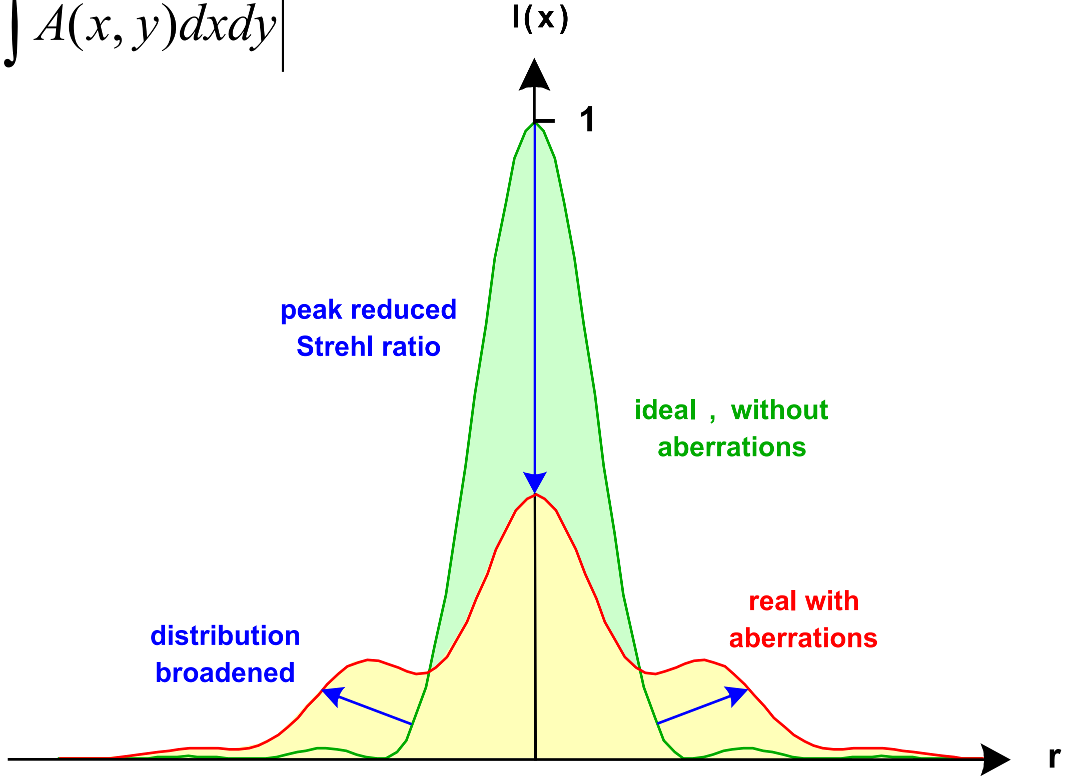
- Important citerion for diffraction limited systems:
  Strehl ratio (Strehl definition)
  Ratio of real peak intensity (with aberrations) referenced on ideal peak intensity

$$D_S = \frac{I_{PSF}^{(real)}(0,0)}{I_{PSF}^{(ideal)}(0,0)} \qquad D_S = \frac{\left| \iint A(x,y) e^{2\pi i W(x,y)} dxdy \right|^2}{\left| \iint A(x,y) dxdy \right|^2}$$

- D_S takes values between 0...1
  D_S = 1 is perfect

- Critical in use: the complete information is reduced to only one number

- The criterion is useful for 'good' systems with values D_s > 0.5



Optical Transfer Function: Definition

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- Normalized optical transfer function (OTF) in frequency space

$$H_{OTF}(v_x, v_y) = \frac{\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} \left|g(x_p, y_p)\right|^2 \cdot e^{-2\pi i\left(x_p v_x + y_p v_y\right)} dx_p dy_p}{\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} \left|g(x_p, y_p)\right|^2 dx_p dy_p}$$

- Fourier transform of the Psf-intensity

$$H_{OTF}(v_x, v_y) = \hat{F}\left[I_{PSF}(x, y)\right]$$

- Absolute value of OTF: modulation transfer function (MTF)

$$H_{MTF}(v) = \left|H_{OTF}(v)\right|$$

- MTF is numerically identical to contrast of the image of a sine grating at the corresponding spatial frequency

Optical Transfer Function of a Perfect System

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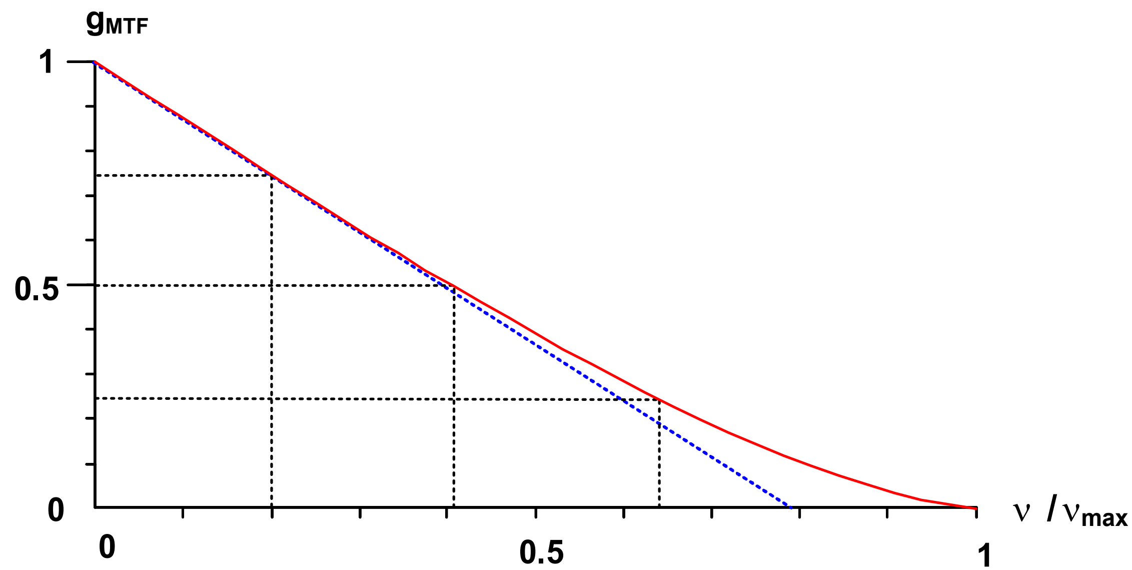
- Cut-off frequency:

$$\nu_G = 2\nu_0 = \frac{2na}{\lambda f} = \frac{2n \sin u'}{\lambda}$$



- Analytical representation
  - no aberration
  - No apodization
  - Circular symmetric pupil

$$H_{MTF}(\nu) = \frac{2}{\pi}\left[\arccos\left(\frac{\nu}{2\nu_0}\right) - \left(\frac{\nu}{2\nu_0}\right)\sqrt{1-\left(\frac{\nu}{2\nu_0}\right)^2}\,\right]$$

- Separation of the complex OTF function into:
  - absolute value: modulation transfer MTF
  - phase value: phase transfer function PTF

$$H_{OTF}(\nu_x,\nu_y) = H_{MTF}(\nu_x,\nu_y)\cdot e^{iH_{PTF}(\nu_x,\nu_y)}$$

Contrast / Visibility

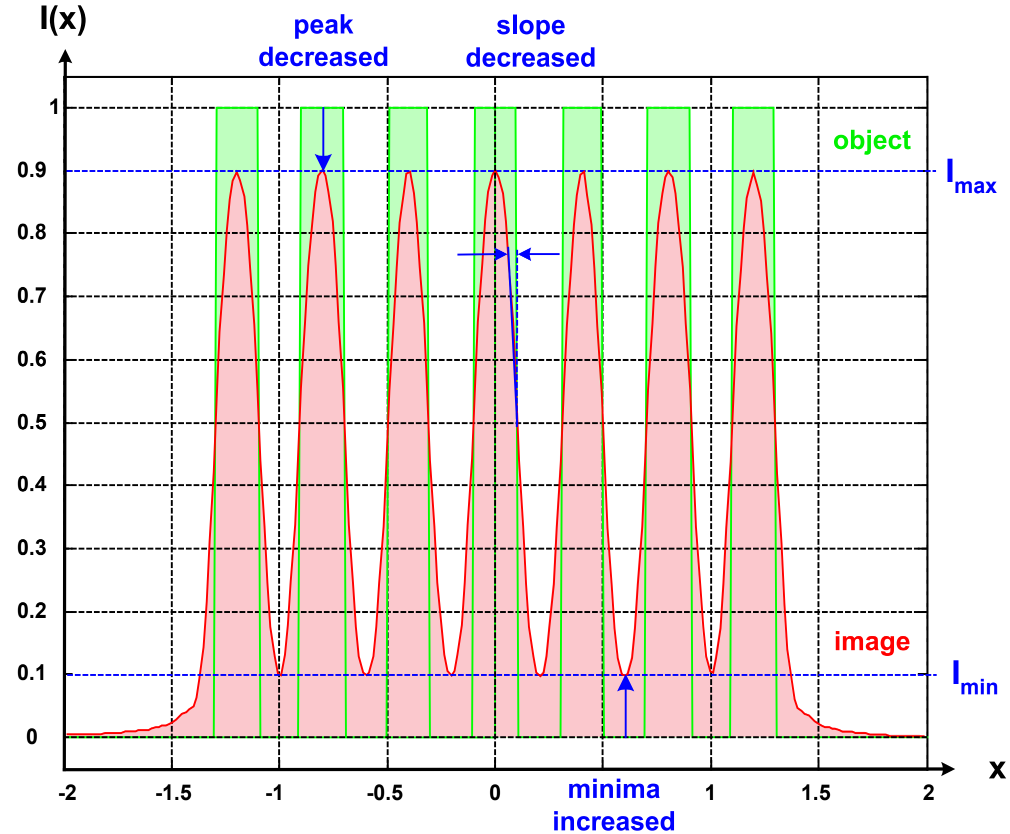
- The MTF-value corresponds to the intensity contrast of an imaged sin grating
- Visibility

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

- The maximum value of the intensity is not identical to the contrast value since the minimal value is finite too

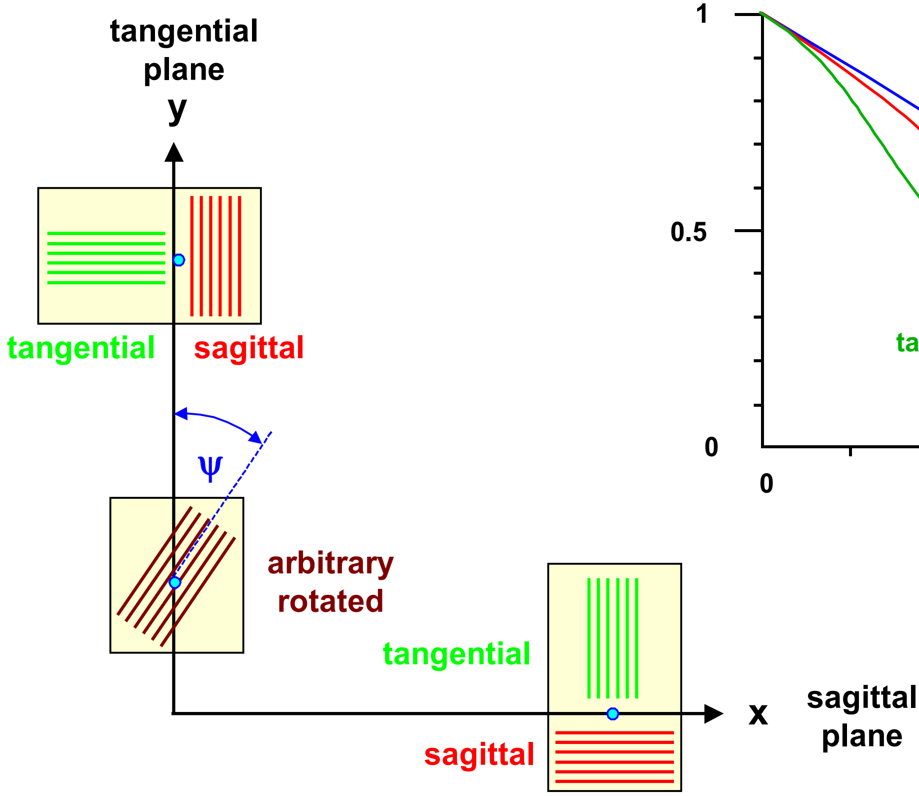
- Concrete values:

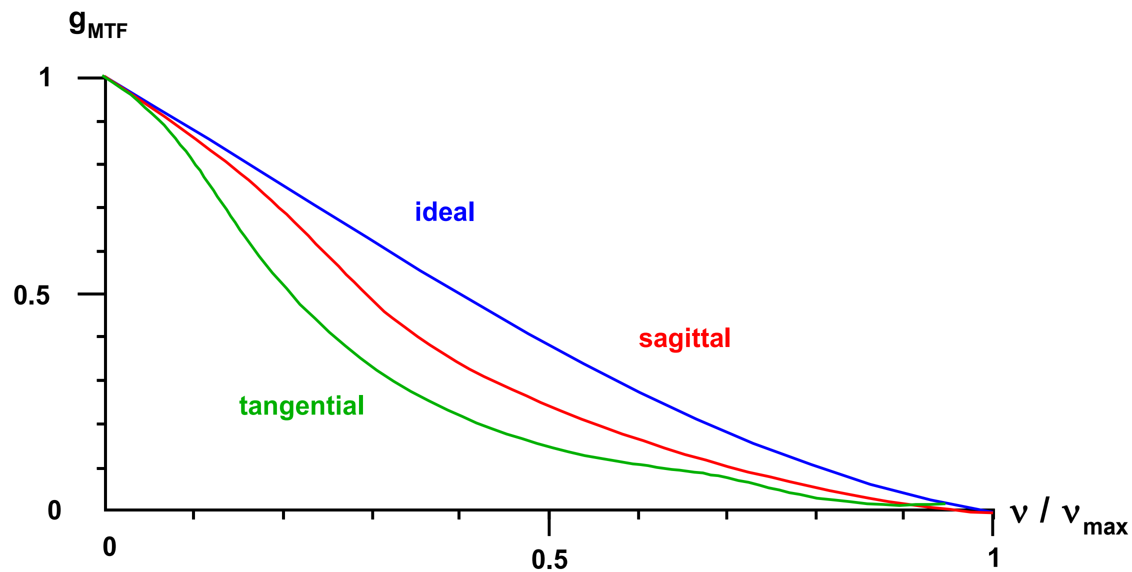
ΔI	I_{max}	V
0.010	0.990	0.980
0.020	0.980	0.961
0.050	0.950	0.905
0.100	0.900	0.818
0.111	0.889	0.800
0.150	0.850	0.739
0.200	0.800	0.667
0.300	0.700	0.538



Sagittal and Tangential MTF

- Due to the asymmetric geometry of the psf for finite field sizes, the MTF depends on the azimuthal orientation of the object structure
- Generally, two MTF curves are considered for sagittal/tangential oriented object structures





Exercise 1: PSF Calculation

Establish a symmetrical biconvex lens with a focal length of f = 100 mm made of SF1 for a wavelength of λ= 1μm. The diameter of the incoming beam should be 15 mm.

a) Calculate the spot diagram and the cross section of a point spread function (128 sampling points). Discuss the sizes of these two representations. Calculate and compare the Strehl ratio in the exact and the Marechal approximation.

b) Fix the plot window size of the PSF to 80 μm and produce a plot with the normalized PSF in the best plane and the two planes defocussed by +0.5 mm and -0.5 mm respectively. Discuss the result.

c) If now the aperture is enlarged to a diameter of 35 mm and the final distance is re-optimized, the PSF with 128 points calculation begins to show sampling problems. How can this be seen in the cross section representation ? How can this be seen in the 2D-plot ?

Exercise 2: Transfer function

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a) Load the Cooke triplet 40° from the sample files of Zemax. Restrict to the center wavelength and reduce the field to 12° at one off axis field point only. Locate the stop 20 mm in front of the system. What is the residual power transmitted in the field position ? Fix the vignetting in the field menu. Calculate the distortion.

b) What is the Airy radius in the field point ? Estimate the diameter at 10 % intensity of the PSF in the field in x and y respectively. Calculate the modulation transfer function and discuss the three curves. Estimate the contrast in the case, that the PSF diameter is approximately of half the width of a grating period.