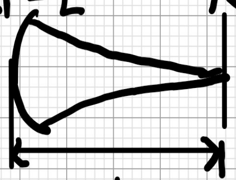
Problem 1 Jinsong Liu 206216

(a) $R_1 = L$ $R_2 = \infty$



$$g_1 = 1 - \frac{L}{R_1} = 0 \qquad g_1 \cdot g_2 = 0$$

$$g_2 = 1 - \frac{L}{R_2} = 1$$

$\therefore$ the resonator is stable

(b)

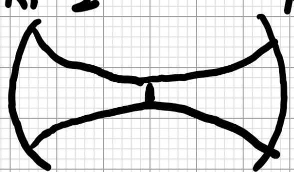
$$g_1 g_2 = 1 - \frac{L}{L+\Delta R_1} = \frac{\Delta R_1}{L+\Delta R_1}$$

if $\Delta R_1 > 0 \Rightarrow 0 < \frac{\Delta R_1}{L+\Delta R_1} < 1 \Rightarrow$ the resonator is stable

if $\Delta R_1 < 0 \Rightarrow \frac{\Delta R_1}{L+\Delta R_1} < 0$

$\Rightarrow$ the resonator is unstable.

(c) $R_1 = \frac{L}{2}$ $R_2 = \frac{L}{2}$



$$R_1 = R_2 = \frac{L}{2} = R$$

(d)

$$g_1 g_2 = \left(1 - \frac{\frac{L}{2}}{\frac{L}{2}+\Delta R_1}\right)\left(1 - \frac{\frac{L}{2}}{\frac{L}{2}+\Delta R_2}\right) = \frac{(\Delta R_1 - R)(\Delta R_2 - R)}{(R+\Delta R_1)(R+\Delta R_2)}$$

$\therefore g_1 g_2 > 0$

when $g_1 g_2 \leq 1$

$$(\Delta R_1 - R)(\Delta R_2 - R) \leq (R + \Delta R_1)(R + \Delta R_2)$$

$$2\Delta R_1 R + 2\Delta R_2 R \geq 0$$

$$(\Delta R_1 + \Delta R_2) \cdot R \geq 0$$

$\therefore$ if $\Delta R_1 + \Delta R_2 \geq 0$, the resonator can be stable.

Problem 2 Jinsong Liu 206216

(a)

$$\begin{cases} R_1 = R(L_1) = L_1 + \dfrac{z_R^2}{L_1} \\ R_2 = R(L_2) = L_2 + \dfrac{z_R^2}{L_2} \end{cases} \quad L = L_1 + L_2 \quad g_1 = 1 - \frac{L}{R_1} \quad g_2 = 1 - \frac{L}{R_2}$$

$$\Rightarrow \begin{cases} \dfrac{L}{1-g_1} = L_1 + \dfrac{z_R^2}{L_1} \\ \dfrac{L}{1-g_2} = L_2 + \dfrac{z_R^2}{L_2} \end{cases}$$

$$\therefore \left(\frac{L}{1-g_1} - L_1\right) \cdot L_1 = \left(\frac{L}{1-g_2} - L_2\right) \cdot L_2$$

$$\therefore \left(\frac{L}{1-g_1} + \frac{L}{1-g_2} - 2L\right) \cdot L_1 = \frac{L^2}{1-g_2} \cdot L^2$$

$$\therefore L_1 = g_2 \cdot (1-g_1) \frac{L}{g_1+g_2-2g_1g_2}$$

$$L_2 = g_1(1-g_2) \cdot \frac{L}{g_1+g_2-2g_1g_2}$$

(b) $$\frac{L}{1-g_1} = L_1 + \frac{z_R^2}{L_1}$$

$$\therefore z_R^2 = \left(\frac{L}{1-g_1} - L_1\right)\cdot L_1 = g_1 g_2 (1-g_1g_2)\frac{L^2}{(g_1+g_2-2g_1g_2)^2}$$

$$W_0 = \sqrt{\frac{\lambda\cdot z_R}{\pi}} = \sqrt{\frac{\lambda}{\pi}}\cdot\left[g_1g_2(1-g_1g_2)\frac{L^2}{(g_1+g_2-2g_1g_2)^2}\right]^{\frac{1}{4}}$$

(c) when the resonator is concentric symmetric resonator

$$g_1 = g_2 = 1-\frac{L}{R}$$

$$\therefore z_R^2 = g^2(1-g^2)\cdot\frac{L^2}{(2g-2g^2)^2} = \frac{1+g}{1-g}\cdot\frac{L^2}{4} = \frac{1}{4}[-(L-R)^2+R^2]$$

when $L=2R$, $z_R^2 = 0$

$$\Rightarrow W_0 = \sqrt{\frac{\lambda z_R}{\pi}} = 0$$

$\therefore$ the waist is the smallest

when $L=R$, z_R^2 gets the maximum value.

$\Rightarrow$ the waist is the biggest.

(d) $W(z) = W_0\sqrt{1+\left(\frac{z}{z_R}\right)^2} = \sqrt{\frac{\lambda}{\pi}} \cdot \sqrt{z_R + \frac{z^2}{z_R}}$

$\because R(z) = z + \frac{z_R^2}{z}$

$\therefore W(z) = \sqrt{\frac{\lambda}{\pi}} \cdot \sqrt{\frac{z \cdot R(z)}{z_R}}$

$\therefore W(L_1) = \sqrt{\frac{\lambda \cdot L}{\pi}} \cdot \sqrt[4]{\frac{g_2}{g_1(1-g_1 g_2)}}$

$W(L_2) = \sqrt{\frac{\lambda \cdot L}{\pi}} \cdot \sqrt[4]{\frac{g_1}{g_2(1-g_1 g_2)}}$

(e) The transversal length of the mirrors, because different wavelengths have different diffraction losses.

Problem 3 Jinsong Liu 206216

(a)

the ABCD matrix for one round trip:

$$\begin{bmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{bmatrix} \begin{bmatrix} 1 & L-L_0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & n_0 \end{bmatrix} \begin{bmatrix} 1 & L_0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & L_0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{n_0} \end{bmatrix} \begin{bmatrix} 1 & 1-L_0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2(L-L_0)+\frac{2}{n_0}L_0 \\ -\frac{2}{R} & 1-\frac{4L_0}{Rn_0} \end{bmatrix}$$

(b)

$$g_1 = 1-\frac{L}{R} = 1-\frac{109}{200} = 0.455$$

$$g_2 = 1$$

$$0 < g_1 g_2 < 1$$

$\therefore$ The resonator is stable

(c)

$$\begin{bmatrix} r_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & l \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ r_0' \end{bmatrix}$$

$$= \begin{bmatrix} 1 & l \\ -\frac{1}{f} & 1-\frac{l}{f} \end{bmatrix} \begin{bmatrix} 0 \\ r_0' \end{bmatrix} = \begin{bmatrix} r_0' \cdot l \\ r_0'(1-\frac{l}{f}) \end{bmatrix}$$

$$\therefore r_0' \cdot (1-\frac{l}{f}) = 0 \qquad \therefore l = f$$

(d)

$\because$ the optical system is a telescope system

$\therefore$ the incident beam and output beam are parallel to the optical axis.

$$\begin{bmatrix} r_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & f_1+f_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix} \begin{bmatrix} r_0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{f_2}{f_1} & f_1+f_2 \\ 0 & -\frac{f_1}{f_2} \end{bmatrix} \begin{bmatrix} r_0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{f_2}{f_1} r_0 \\ 0 \end{bmatrix}$$

$\therefore r_1 = -\frac{f_2}{f_1} r_0$

magnification: $-\frac{f_2}{f_1}$

(e) 1 inch = 25.4 mm 2 inch = 50.8 mm

$\because \sin\theta_m \approx \tan\theta_m$

$\therefore$ when aperture diameter is 1 inch, the beam diameter is $30 \times 0.22 \times 2 = 13.2$ mm. when aperture diameter is 2 inch, the beam diameter is $100 \times 0.22 \times 2 = 44$ mm