Task 1 Jinsong Liu

Solution:

ABCD matrix: $\begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f_1 & 1 \end{bmatrix} \begin{bmatrix} 1 & f_1 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1-\frac{d_1}{f_1} & f_1 \\ -\frac{1}{f_1} & 0 \end{bmatrix}$$

$\cancel{q_1=-iz_1}$ $q_1 = -iz_1$ is a purely imaginary number

$\therefore \dfrac{f_1^2 - iz_0 f_1 + iz_0 d_1}{iz_0}$ is a purely imaginary number

$$\therefore d_1 - f_1 = 0 \quad d_1 = f_1$$

$$\because \frac{1}{q} = \frac{1}{R} + i\frac{\lambda}{\pi W^2}$$

$$\therefore \frac{1}{q_1} = -\frac{1}{iz_1} = i\frac{\lambda}{\pi W_1^2} = \frac{iz_0}{f_1^2}$$

$$\therefore W_1^2 = \frac{\lambda f_1}{\pi z_0}$$

$$\because \frac{1}{q_0} = -\frac{1}{iz_0} = i\frac{\lambda}{\pi W_0^2}$$

$$\therefore \frac{1}{z_0} = \frac{\lambda}{\pi W_0^2}$$

$$z_0 = \frac{\pi W_0^2}{\lambda}$$

$$W_1^2 = \frac{\lambda^2 f_1^2}{\pi^2 W_0^2}$$

$$W_1 = \frac{\lambda f_1}{\pi W_0}$$

b) $q_0 \to q_2$

ABCD matrix

$$\begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f_1 & 1 \end{bmatrix} \begin{bmatrix} 1 & f_1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-\frac{d_2}{f_2} - \frac{1}{f_1}\left[d\left(1-\frac{d_2}{f_2}\right)+d_2\right] & 1-\frac{d_2}{f_2} \\ -\frac{1}{f_2} - \frac{1}{f_1}\left(-\frac{d}{f_2}+1\right) & -\frac{f_1}{f_2} \end{bmatrix}$$

$$q_2 = \frac{Aq_0 + B}{Cq_0 + D}$$

$$\because d = f_1 + f_2$$

$$\therefore q_2 = \frac{1-\frac{d_2}{f_2} - iz_0\left\{1-\frac{d_2}{f_2} - \frac{1}{f_1}\left[d\left(1-\frac{d_2}{f_2}\right)+d_2\right]\right\}}{-\frac{f_1}{f_2}}$$

$\because q_2 = -iz_2$ is a purely imaginary number

$$\therefore 1-\frac{d_2}{f_2} = 0 \quad d_2 = f_2$$

$$\therefore q_2 = \frac{\frac{f_2}{f_1} iz_0}{-\frac{f_1}{f_2}}$$

$$= -i\frac{f_2^2}{f_1^2}\frac{\pi W_0^2}{\lambda}$$

$$W_2 = \sqrt{\frac{-\mathrm{Im}\{q_2\}\lambda}{\pi}} = \frac{f_2}{f_1} W_0$$

c)

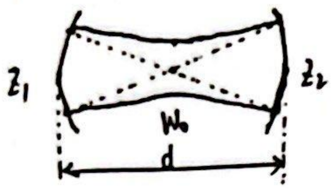
$$W_1 \approx 2.19 \times 10^{-6}\,\mathrm{m}$$

$$W_2 = \frac{150}{200} \times 15\,\mathrm{mm} = 11.25\,\mathrm{mm}$$

Task 2

Solution :

a)



$|R_1| = |R_2| = d$

Task 3

a)

Solution: $\omega_1 = \omega_0 - \delta\omega$

$$n_{g1} = c \cdot \frac{\partial k}{\partial \omega}\Big|_{\omega_1}$$

$$= c\left[\frac{1.5}{c} + 0.15(\omega - \omega_0)\right]$$

$$= 1.5 + 0.15c \cdot \delta\omega$$

$$= \cancel{1.5 + 0.15 \times 3 \times 10^{8} \times 10^{12}}$$

$$= 1.5 + 0.15 \times 10^{-24} \times 3 \times 10^{8} \times 10^{12}$$

$$= 1.5 + 4.5 \times 10^{-5}$$

$$n_{g0} = 1.5$$

$$\frac{1}{V_{g0}} = \frac{\partial k}{\partial \omega}\Big|_{\omega_0} = 1.5/c \qquad \frac{1}{V_{g1}} = \frac{\partial k}{\partial \omega}\Big|_{\omega_1} = 1.5/c + 0.15\,\delta\omega$$

$$= 0.5 \times 10^{-8} + 0.15 \times 10^{-12} \ \text{s/m}$$

If the second pulse overtake the first one

$$V_{g0} \cdot t = V_{g1}(t + 20\text{ns})$$

$$\frac{c}{1.5}\left(\frac{1.5}{c} + 0.15 \times 10^{-12}\right)t = t + 20 \times 10^{-9}$$

$$t = \frac{20 \times 10^{-9}}{3 \times 10^{-5}}\ \text{s} \approx 6.67 \times 10^{-4}\ \text{s}$$

b)

solution:

$$D_1 = \frac{\partial^2 k}{\partial \omega^2}\Big|_{\omega_0} = 0.15\,\frac{(\text{ps})^2}{\text{m}} \qquad \tilde{H}_{1p}(\bar{\omega}; z) = \exp\left[iz\frac{D_1}{2}\bar{\omega}^2\right]$$

$$D_2 = \frac{\partial^2 k}{\partial \omega^2}\Big|_{\omega_0} = -0.3\,\frac{(\text{ps})^2}{\text{m}} \qquad \tilde{H}_{2p}(\bar{\omega}; z) = \exp\left[iz\frac{D_2}{2}\bar{\omega}^2\right]$$

If we want the pulse to be restored

$$z_1\frac{D_1}{2} + z_2\frac{D_2}{2} = 0$$

$$\therefore z_1 D_1 + z_2 D_2 = 0$$

$$\because z_1 = 5\text{km}$$

$$\therefore z_2 = \frac{5 \times 0.15 \times 10^3}{0.3}\ \text{m}$$

$$z_2 = 2.5\text{km}$$