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Series 8
Fundamentals of Modern Optics

to be returned on **05.01.2023**, at the beginning of the lecture

Task 1: Pulsed Beam (a=2, b=2, c=2, d=2 points)

Assume a transform-limited pulsed beam with a Gaussian-shaped intensity in both space and time:

$$I(x,y,t) = I_0 \exp[-2(x^2+y^2)/w_0^2] \exp(-2t^2/\tau_0^2),$$

where the pulse duration is $\tau_0 = 50\,\mathrm{ps}$ and the beam waist is $w_0 = 5\,\mathrm{mm}$. It has a central wavelength of 520 nm and travels in a medium with a group velocity dispersion of $\beta_2 = 0.05\,(\mathrm{ps})^2/\mathrm{m}$. The pulsed beam has a total energy of $E = 10\,\mathrm{mJ}$.

a) Calculate the peak intensity I_0 of the pulsed beam in units of $\mathrm{W/m^2}$.

b) Derive an equation for the decay of the peak intensity $I(z)$ of the pulse due to propagation. Write your answer in terms of the initial peak intensity I_0, the Rayleigh length of the Gaussian beam z_0, and the temporal dispersion length L_D.

c) For the parameters given above, which broadening mechanism is dominant? Spatial broadening due to diffraction or temporal broadening due to dispersion?

d) Approximate the pulse length τ_0 that would lead to equal relative broadening along x, y, and t directions. More precisely, what value of τ_0 results in $\frac{w(z)}{w_0} = \frac{\tau(z)}{\tau_0}$ for all z-values?

Task 2: Pulse compression (a=3, b=4, c=4, d=2 points)

A transform-limited Gaussian pulse given by

$$U(t) = B_0 \exp\left(-\frac{t^2}{\tau_0^2}\right),$$

can be compressed by transmitting it first through a quadratic phase modulator (QPM) and then through a chirp filter.

a) Using the QPM the pulse $U(t)$ is multiplied by a quadratic phase factor $\exp(i\zeta t^2)$ resulting in the chirping of the pulse. The resulting chirped pulse can be written as:

$$U_1(t) = B_{10} \exp\left(-(1-iC_1)\frac{t^2}{\tau_1^2}\right).$$

Find the chirp parameter C_1, the amplitude B_{10}, the pulse duration τ_1 and the spectral width ω_1 of the pulse (by spectral width we mean the value ω_1 at which the field envelope is reduced to $1/e$ times its maximum). *Hint:* You can use the fact that FT[$\exp(-\alpha t^2)$]= $\frac{1}{2\sqrt{\pi\alpha}}\exp(-\omega^2/4\alpha)$ for any complex number α with Re[α]> 0.

b) In order to make the chirped pulse transform-limited it is sent through a chirp filter with the envelope transfer function:

$$H_e(\omega) = \exp\left(\frac{-ib\omega^2}{4}\right).$$

The resulting pulse is given as:

$$U_2(t) = B_{20} \exp\left(-(1-iC_2)\frac{t^2}{\tau_2^2}\right).$$

Find the value of b such that the pulse $U_2(t)$ is transform-limited. *Hint*: The condition for the pulse to be transform-limited is $C_2 = 0$. Also remember that in the Fourier domain $U_2(\omega) = H_e(\omega)U_1(\omega)$.

c) Obtain the pulse duration τ_2, the spectral width ω_2, and the amplitude B_{20} of the pulse after the system, using the expression you found for the parameter b from the previous part.

d) Explain how the comparison of the QPM and the chirp filters can be done via examining their influence on the pulse.

Task 3: Pulse dispersion (a=2, b=2, c=3, d=4 points)

A pulse with central frequency ω_0 and bandwidth $\Delta\omega$ ($\Delta\omega \ll \omega_0$) is coupled into a dispersive fiber with the frequency-dependent refractive index:

$$n(\omega) = a_1 + a_2\omega^2 + a_3\omega^3 \qquad a_1, a_2, a_3 - \text{real constants}$$

a) While travelling through this fiber, a transform-limited initial pulse will in general undergo temporal broadening. Why?
b) Find the phase velocity v_{ph} at $\omega = \omega_0$ and group velocity v_{g} of the pulse and give short explanations about their physical meaning.
c) Express in terms of a_1, a_2 and a_3, the central frequency ω_0 of a pulse, for which the temporal broadening is minimized.
d) Show that the parabolic approximation for the dispersion relation $k(\omega)$ is justified in this fiber, under the condition that $\frac{1}{\omega_0}$ is negligible compared to $\frac{a_3}{a_2}$.
Hint: Use the Taylor expansion of $k(\omega)$ and see which terms are relevant. Keep in mind that the approximation must be valid for all frequencies of interest, i.e. for $\omega - \omega_0 \leq \Delta\omega$.

Task 4: Self evaluation

How did you so far study for FoMO Seminar tasks and Midterm? What methods were useful and successful that you would recommend to your fellow classmates? What methods were not? What methods would you like to try out in future?