Problem 1 Jinsong Liu 206216

(a)

$$\frac{dN_1}{dt} = -W_{13} N_1 + S_{31} N_3 - W_{12} N_1 + W_{21} N_2 + S_{21} N_2 \quad ①$$

$$\frac{dN_2}{dt} = W_{12} N_1 - W_{21} N_2 - S_{21} N_2 + S_{32} N_3 \quad ②$$

$$\frac{dN_3}{dt} = -S_{32} N_3 - S_{31} N_3 + W_{13} N_1 \quad ③$$

(b)

From equation ③ in (a) and

N_3 is 0 before starting the pump process

$\therefore$

$W_{13} N_1 = S_{32} N_3 + S_{31} N_3$ is the condition required to keep $N_3 = 0$. The increasing particles must be equal to the decreasing particles of 3> to keep $N_3 = 0$.

(C)

$$\frac{dN_1}{dt} = -W_{13}N_1 - W_{12}N_1 + W_{21}N_2 + S_{21}N_2 + S_{31}\cdot N_3$$

$$\frac{dN_2}{dt} = -W_{21}N_2 + W_{12}N_1 - S_{21}N_2 + S_{32}N_3$$

$$W_{13}\cdot N_1 = (S_{31}+S_{32})N_3 \quad (N_3 \approx 0)$$

$$\therefore \eta = \frac{S_{32}N_3}{(S_{31}+S_{32})N_3} = \frac{S_{32}N_3}{W_{13}\cdot N_1}$$

$$\therefore S_{31}\cdot N_3 = (1-\eta)W_{13}\cdot N_1 \qquad S_{32}\cdot N_3 = \eta\cdot W_{13}\cdot N_1$$

$$\therefore \frac{dN_1}{dt} = -W_{12}N_1 + W_{21}N_2 + S_{21}N_2 - \eta W_{13}\cdot N_1$$

$$\frac{dN_2}{dt} = -W_{21}N_2 + W_{12}N_1 - S_{21}N_2 + \eta W_{13}\cdot N_1$$

(d) The rate equations from c) are :

$$\frac{dN_1}{dt} = -W_{12}N_1 + W_{21}N_2 + S_{21}N_2 - \eta W_{13} \cdot N_1$$

$$\frac{dN_2}{dt} = -W_{21}N_2 + W_{12}N_1 - S_{21}N_2 + \eta W_{13} \cdot N_1$$

From the equations, we can get that

$$\frac{dN_1}{dt} = -\eta \cdot W_{13} \cdot N_1 \qquad \frac{dN_2}{dt} = \eta \cdot W_{13} \cdot N_1$$

$\because N_3 \approx 0$

$\therefore$ All pumped particles come from N_1

$$\therefore \left.\frac{dN_1}{dt}\right|_{pump} = -\left.\frac{dN_2}{dt}\right|_{pump}$$

Problem 2 Jinsong Liu 206216

(a)

$\tau_{ph} = \tau_R / \gamma$

$\gamma_1 = -\ln(1-T_1) = -\ln R_1 \quad \gamma_2 = -\ln(1-T_2) = -\ln R_2$

γ is the logarithmic cavity loss

$\gamma_i = -\ln(1-L_i) \qquad \gamma = \gamma_i + \gamma_1 + \gamma_2$

From "Principles of Lasers", Orazio Svelto

(b) With $R_1 = 1$, L is the round trip loss.

under circumstances: $0 < T_2 \ll 1$, $0 < L \ll 1$

when $x \ll 1$ occurs, $-x \approx \ln(1-x)$

$\gamma_2 = -\ln(1-T_2) \approx T_2 \quad \gamma_i = -\ln(1-L) \approx L$

Problem 3 Jinsong Liu 206216

(a)

For a four-level system and $n \ll n_{tot}$

$$\frac{dn}{dt} = -\sigma \cdot c \cdot p \cdot n - \Gamma \cdot n + W_p \cdot n_{tot} \quad ①$$

$$\frac{dp}{dt} = \frac{p \cdot \left(\frac{n}{n_{th}} - 1\right)}{\tau_{ph}} \quad \text{(neglect } S\text{)} \quad ②$$

$$n = \bar{n} + \Delta n \qquad |\Delta n| \ll \bar{n} \quad ③$$

$$p = \bar{p} + \Delta p \qquad |\Delta p| \ll \bar{p} \quad ④$$

insert ③, ④ into ①, ② and neglect $\Delta n \Delta p$

$$\bar{n} = n_{th}, \quad \bar{p} = n_{tot}(W_p - W_{th})\tau_{ph}, \quad W_{th} = \Gamma \frac{n_{th}}{n_{tot}}$$

$$\begin{aligned}
\frac{d(\Delta n)}{dt} &= -\sigma \cdot c \cdot (\bar{p} + \Delta p) \cdot (\bar{n} + \Delta n) - \Gamma(\bar{n} + \Delta n) + W_p \cdot n_{tot} \\
&= -\sigma \cdot c(\bar{p} \cdot \bar{n} + \bar{p}\Delta n + \bar{n}\Delta p) - \Gamma(\bar{n} + \Delta n) + W_p \cdot n_{tot} \\
&= -\sigma \cdot c\bar{p} \cdot \bar{n} - \sigma \cdot c\bar{p}\Delta n - \sigma \cdot c\bar{n}\Delta p - \Gamma\bar{n} - \Gamma\Delta n + W_p \cdot n_{tot} \\
&= -\sigma \cdot c\,\bar{p} \cdot \bar{n} - (\sigma \cdot c\bar{p} + \Gamma)\Delta n - \frac{\Delta p}{\tau_{pn}} - \Gamma\bar{n} + W_p \cdot n_{tot}
\end{aligned}$$

$\downarrow$

$$= -\frac{n_{tot}}{\tau_{ph}}(W_p - W_{th})\tau_{ph} - (\sigma \cdot c \bar{p} + \Gamma)\Delta n - \frac{\Delta p}{\tau_{ph}} - \Gamma \bar{n} + W_p n_{tot}$$

$$= W_{th} n_{tot} - W_p n_{tot} - (\sigma \cdot c \cdot \bar{p} + \Gamma)\Delta n - \frac{\Delta p}{\tau_{ph}} - \Gamma \bar{n} + W_p n_{tot}$$

$$= -(\sigma \cdot c \cdot \bar{p} + \Gamma)\Delta n - \frac{\Delta p}{\tau_{ph}}$$

$$\frac{d(\Delta \bar{p})}{dt} = \frac{(\bar{p} + \Delta p)\left(\frac{\bar{n} + \Delta n}{n_{th}} - 1\right)}{\tau_{ph}}$$

$$= \frac{(\bar{p} + \Delta p)\frac{\Delta n}{n_{th}}}{\tau_{ph}}$$

$$= \frac{\bar{p} \cdot \frac{\Delta n}{n_{th}}}{\tau_{ph}} = \sigma \cdot c \cdot \bar{p} \cdot \Delta n$$

(b) For a three-level system

$$\frac{dn}{dt} = -2 \cdot \sigma \cdot c \cdot p \cdot n - \Gamma \cdot \left\{ n + n_{tot} \right\} + W_p \left(n_{tot} - n \right) \quad ①$$

$$\frac{dp}{dt} = \frac{p \cdot \left(\frac{n}{n_{th}} - 1 \right)}{\tau_{ph}} \quad ②$$

$$\sigma c \, n_{th} = \frac{1}{\tau_{ph}}$$

$$n = \bar{n} + \Delta n \quad ③$$

$$n_{tot} W_{th} = \Gamma \bar{n}$$

$$p = \bar{p} + \Delta p \quad ④$$

$$\frac{d(\Delta n)}{dt} = -2 \cdot \sigma \cdot c \cdot (\bar{p} + \Delta p)(\bar{n} + \Delta \bar{n}) - \Gamma \left(\bar{n} + n_{tot} + \Delta n \right)$$

$$+ W_p \left(n_{tot} - \bar{n} - \Delta n \right)$$

$$= -2\sigma \cdot c \cdot (\bar{p}\bar{n} + \Delta p \bar{n} + \Delta n \bar{p}) - \Gamma(\bar{n} + n_{tot} + \Delta n)$$

$$+ W_p \left(n_{tot} - \bar{n} - \Delta n \right)$$

$$= -2 n_{tot} (W_p - W_{th}) - 2 \frac{\Delta p}{\tau_{ph}} - 2\sigma \cdot c \cdot \bar{p} \Delta n - \Gamma \bar{n} - \Gamma n_{tot} - \Gamma \Delta n$$

$$+ W_p n_{tot} - W_p \bar{n} - W_p \Delta n$$

$$= - \bar{p} / \tau_{ph} - 2 \frac{\Delta p}{\tau_{ph}} - (2\sigma \cdot c \cdot \bar{p} + \Gamma + W_p) \Delta n - \Gamma n_{tot}$$

$$- (\sigma \cdot c \cdot \tau_{pn})^{-1} W_p$$

(c)

physical meaning

δ : damping coefficient

ω : oscillation frequency

① If δ is much higher than ω, Δn and Δp will return to 0 without oscillation.

② If δ is much lower than ω, Δn and Δp will oscillate with slightly decreasing amplitudes.

(d)

The second can see relaxation oscillation,

$\because \tau_2 = 100\mu s \gg 100 ns$

$\therefore$ the second $\delta \ll$ the first δ