Written test „Structure of Matter" 2018, December 4th

-Mid term test-

solution proposals

Name:

Matriculation number:

1. Multiple-choice test: Please tick the box(es) with the correct answer(s)!
(correctly ticked box: +1/2 point; wrongly ticked box: -1/2 point)

A CO_2 laser generates light at a wavelength of 10.6μm. This wavelength belongs to the	Ultraviolet spectral range	
	Visible spectral range	
	Infrared spectral range	x
Vibrations of nuclei in molecules and solids cause light absorption in the	Ultraviolet spectral range	
	Visible spectral range	
	Infrared spectral range	x
The Paschen series in the emission spectrum of hydrogen atoms is observed in the	Ultraviolet spectral range	
	Visible spectral range	
	Infrared spectral range	x
Doppler broadening of spectral lines in gases results in a linshape that is	Gaussian	x
	Lorentzian	
	triangular	
The Einstein coefficient A_{21} responsible for spontaneous light emission is given in	s^{-1}	x
	W/m^2	
	m/s	
With increasing temperature, the light intensity emitted by a heated black body will	increase	x
	decrease	
	remain constant	
Electrons belong to the class of	fermions	x
	bosons	
	Neither of them	
Slater determinants describe the wavefunction of systems of non-interacting	fermions	x
	bosons	
	Neither of them	

	$\varepsilon_0 = 8.86*10^{-12}$F/m
	$\mu_0=1.256*10^{-6}$Vs/(Am)
Plancks constant:	$h=6.625*10^{-34}Ws^2$
elementary charge:	$e=1.602*10^{-19}$As
electron rest mass:	$m_e=9.108*10^{-31}$kg
proton rest mass:	$m_p=1.672*10^{-27}$kg
Boltzmanns constant:	$k_B=1.38*10^{-23}$Ws/K

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2. Quantum mechanical treatment of the one - dimensional harmonic oscillator (decide whether the statement is true or wrong and tick the appropriate box):

Statement	true	wrong
The harmonic oscillator has equidistant energy levels.	x	
All wavefunctions of the harmonic oscillator are even functions of x.		x
In the ground state, $<x^2>=0$		x
In the ground state, $<x^3>=0$	x	
Electric dipole-allowed quantum transitions occur only between adjacent energy levels	x	

(5 points): 1 point per correct decision, 0 points per wrong or no decision

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3. Assume an electron confined in a rectangular potential box of length L = 1nm with impermeable walls (one-dimensional case). What is the transition matrix element x_{jk} and the corresponding oscillator strength for the transition $k = 2 \rightarrow j = 4$?

(3 points)

matrix element = 0 because both wavefunctions are odd with respect to symmetry centre, so their product is even, while x is odd. Once $x_{kj}=0$, the oscillator strength must be zero, too.

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4. Calculate the relation between the ground state energies of an electron in the hydrogen atom and in the Li^{2+} (Z = 3, mass number = 7) ion!

(4 points)

$$\frac{E_{Li^{2+}}}{E_H} = Z_{Li^{2+}}^2 \frac{\mu_{Li^{2+}}}{\mu_H} = 9\frac{\mu_{Li^{2+}}}{\mu_H} = 63\frac{m_e + m_n}{m_e + 7m_n} = 63\frac{1+\frac{m_n}{m_e}}{1+7\frac{m_n}{m_e}} \approx 63\frac{1+1836}{1+7*1836} \approx 9.004$$

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5. Calculate the commutation relation $\left[\hat{x},\hat{p}_x^2\right]$ (x - Cartesian coordinate; p-momentum)!

(6 points)

$$\left[\hat{x},\hat{p}_x^2\right]=2i\hbar\hat{p}_x=2\hbar^2\frac{\partial}{\partial x}$$

$a)$ direct calculation using $\hat{p}_x=-i\hbar\dfrac{\partial}{\partial x}$

$$b)\ \left[\hat{x},\hat{p}_x^2\right]=\hat{x}\hat{p}_x^2-\hat{p}_x^2\hat{x}=\hat{x}\hat{p}_x^2-\hat{p}_x^2\hat{x}+\hat{p}_x\hat{x}\hat{p}_x-\hat{p}_x\hat{x}\hat{p}_x=\hat{p}_x\hat{x}\hat{p}_x-\hat{p}_x^2\hat{x}+\hat{x}\hat{p}_x^2-\hat{p}_x\hat{x}\hat{p}_x=$$

$$\hat{p}_x\left(\hat{x}\hat{p}_x-\hat{p}_x\hat{x}\right)+\left(\hat{x}\hat{p}_x-\hat{p}_x\hat{x}\right)\hat{p}_x=\hat{p}_x\left(i\hbar\right)+\left(i\hbar\right)\hat{p}_x=2i\hbar\hat{p}_x$$

$$c)\ \left[\hat{x},\hat{p}_x^2\right]=\hat{x}\hat{p}_x^2-\hat{p}_x^2\hat{x}=\left(i\hbar+\hat{p}_x\hat{x}\right)\hat{p}_x-\hat{p}_x\left(\hat{x}\hat{p}_x-i\hbar\right)=2i\hbar\hat{p}_x$$

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6. Consider the hydrogen state as given by n=2, l=1, m=0. Its wavefunction is:

$$\left|210\right\rangle = \frac{1}{4\sqrt{2\pi}a_0^{\frac{3}{2}}}\frac{r}{a_0}e^{-\frac{r}{2a_0}}\cos\theta$$ with a_0 – Bohr’s radius

a) What is the total energy of the electron in this state (in Ry)? (2 points)

b) Calculate the expectation value $\left\langle \frac{1}{r} \right\rangle$ in this state by direct integration in spherical coordinates! (8 points)

c) Indicate the expectation value of the potential energy (in Ry) in this state! (2 points)

$$E_{ground\ state} = -\frac{Ry}{n^2} = -\frac{Ry}{4}$$

$$\left|210\right\rangle = \frac{1}{4\sqrt{2\pi}a_0^{\frac{3}{2}}}\frac{r}{a_0}e^{-\frac{r}{2a_0}}\cos\theta$$

$$\Rightarrow \left\langle \frac{1}{r} \right\rangle = \frac{1}{32\pi a_0^5}\underbrace{\underbrace{\int_0^\infty r^3 e^{-\frac{r}{a_0}}dr}_{6a_0^4}\underbrace{\int_0^{2\pi} d\varphi}_{2\pi}\underbrace{\int_0^\pi \cos^2\theta\sin\theta d\theta}_{\frac{2}{3}}}_{8\pi a_0^4} = \frac{1}{4a_0} \Rightarrow$$

$$\Rightarrow \left\langle U(r) \right\rangle = -\frac{e^2}{4\pi\varepsilon_0}\left\langle \frac{1}{r} \right\rangle = -\frac{e^2}{16\pi\varepsilon_0 a_0} = -\frac{Ry}{2}$$