

# Problem 1

Jinsong Liu

20b21b

(a)

Solution:

$$\sigma_v = \sigma_0 \cdot \bar{F}(v) \quad \bar{F}(v) - \text{line shape function}$$

$$\bar{F}(v) = A \cdot \frac{\left(\frac{\Delta v}{2}\right)^2}{(v - v_{21})^2 + \left(\frac{\Delta v}{2}\right)^2} \quad A - \text{constant}$$

Before frequency shift:

$$\bar{F}(v) = A$$

$$\therefore \eta = \frac{\bar{F}'(v)}{\bar{F}(v)} = \frac{\frac{A}{5}}{A} = 20\%$$

After frequency shift:

$$\begin{aligned} \bar{F}'(v) &= A \cdot \frac{\left(\frac{\Delta v}{2}\right)^2}{(v - v_{21})^2 + \left(\frac{\Delta v}{2}\right)^2} \\ &= A \cdot \frac{1}{5} = \frac{A}{5} \end{aligned}$$

(b)

Solution:

$$\begin{aligned} \bar{F}(v) &= \frac{1}{T} \int_T \bar{F}(v) dt = \frac{1}{T} \int_{-T}^T A \frac{\left(\frac{\Delta v}{2}\right)^2}{(v - v_{21})^2 + \left(\frac{\Delta v}{2}\right)^2} dt \\ &= \frac{1}{10} \int_0^{10} A \frac{25}{100t^2 + 25} dt \end{aligned}$$

$$\bar{\eta} = \frac{\bar{F}(v)}{A} = 1.6\%$$

$$\begin{aligned} &= \frac{A}{10} \int_0^{10} \frac{1}{4t^2 + 1} dt \\ &= \frac{A}{20} [\tan^{-1}(20) - \tan^{-1}(0)] = 0.076A \end{aligned}$$

(C)

Jinsong Liu 2021b

solution:

$\therefore$  The laser is an inhomogeneously broadened laser

$$\therefore P(v) = B \cdot \exp \left[ -4 \ln 2 \left( \frac{v - v_{21}}{\Delta V_L} \right)^2 \right] \quad \Delta V_L = 1 \text{ GHz}$$

When the laser is a monochromatic laser

$$\dot{P}(v) = \delta(v - v_{21})$$

Suppose the average power of excitation laser is 1.

$$\int_{-\infty}^{\infty} \dot{P}(v) dv = \int_{-\infty}^{\infty} \delta(v - v_{21}) dv = 1$$

$$\int_{-\infty}^{\infty} P(v) dv = \int_{-\infty}^{\infty} B \cdot \exp \left[ -4 \ln 2 \left( \frac{v - v_{21}}{\Delta V_L} \right)^2 \right] dv = 1$$

from Gaussian integral  $\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$

$$\therefore B = \sqrt{\frac{4 \ln 2}{\pi \Delta V_L^2}}$$

from (a), we know that  $\sigma_v = \sigma_0 \cdot \bar{f}(v)$

$$\bar{f}(v) = A \cdot \frac{\left(\frac{\Delta v}{2}\right)^2}{(v - v_{21})^2 + \left(\frac{\Delta v}{2}\right)^2}$$

$$\therefore \int_{-\infty}^{\infty} \dot{P}(v) \bar{f}(v) dv = A$$

$$\int_{-\infty}^{\infty} P(v) \bar{f}(v) dv = A \cdot B \cdot \frac{\pi \Delta v}{2}$$

$$\eta = \frac{A \cdot B \cdot \frac{\pi \Delta V}{2}}{A} = B \cdot \frac{\pi \cdot \Delta V}{2} = \sqrt{\frac{4 \ln 2}{\pi (\Delta V_L)^2}} \frac{\pi \Delta V}{2} = \sqrt{\frac{4 \ln 2}{\pi (\Delta V_L)^2}} \frac{\Delta V}{2}$$

$$\Delta V = 10 \text{ MHz} \quad \Delta V_L = 1 \text{ GHz}$$

$$\therefore \eta \approx 0.0148 = 1.48\%$$

(d) Solution:

$$\Delta V_D = \frac{2V_{21}}{C} \cdot \sqrt{\frac{2k \ln 2}{m}} = 1.0 \text{ GHz}$$

(e) Solution:

$$\text{now } \bar{f}(v) = A \exp \left[ -4 \ln 2 \left( \frac{v - V_{21}}{\Delta V_D} \right)^2 \right]$$

$$\int_{-\infty}^{\infty} p(v) \bar{f}(v) dv = 1$$

$$\int_{-\infty}^{\infty} p(v) \bar{f}(v) dv = A \cdot B \int_{-\infty}^{\infty} \exp \left[ -4 \ln 2 \left( \frac{1}{\Delta V_L^2} + \frac{1}{\Delta V_D^2} \right) (v - V_{21})^2 \right] dv$$

$$= \sqrt{\frac{4 \ln 2}{\pi (\Delta V_L)^2}} \sqrt{\frac{\pi}{4 \ln 2}} \sqrt{\frac{1}{\frac{1}{\Delta V_L^2} + \frac{1}{\Delta V_D^2}}} = \sqrt{\frac{\Delta V_D^2}{\Delta V_L^2 + \Delta V_D^2}}$$

$$= \sqrt{\frac{1.0^2}{1 + 1.0^2}} \approx 0.73$$

$$\therefore \eta = 0.73$$

# Problem 2

Jinsong Liu 206216

(a) Solution:

$$\begin{cases} \alpha = N \cdot \sigma_\nu \\ N = 1 \times 10^{19} \text{ cm}^{-3} = 1 \times 10^{25} \text{ m}^{-3} \\ \sigma_\nu = 2.3 \times 10^{-24} \cdot \left[ 1 - \left( \frac{\nu - \nu_0}{\frac{1}{2} \Delta \nu} \right)^2 \right] \text{ m}^2 \end{cases}$$

∴ the peak wavelength is 976nm

$$C = \lambda \nu \quad \Delta \lambda = 10 \text{ nm}$$

$$\therefore \nu_0 = 3.07 \times 10^{14} \text{ Hz}$$

$$\Delta \nu = 3.15 \times 10^2 \text{ Hz}$$

$$\therefore \alpha = 2.3 \cdot \left[ 1 - \left( \frac{\nu - 3.07 \times 10^{14} \text{ Hz}}{1.57 \times 10^{12} \text{ Hz}} \right)^2 \right]$$

(b) Solution:

$$\frac{I_\nu(x)}{I_\nu(0)} = \exp(-\alpha_\nu \cdot x)$$

For laser pump diode

$$\frac{I_\nu(x)}{I_\nu(0)} = \frac{\int \delta(\nu - \nu_0) e^{(-\alpha_\nu \cdot l)} d\nu}{\int \delta(\nu - \nu_0) d\nu} = e^{-2.3} \approx 0.1$$

$$\eta = 0.9$$

$$\lambda = \frac{C}{\nu}$$

$$\Rightarrow \ln \lambda = \ln C - \ln \nu$$

$$\Rightarrow \frac{1}{\lambda} d\lambda = -\frac{1}{\nu} d\nu$$

$$\Rightarrow \left| \frac{\Delta \nu}{\nu} \right| = \left| \frac{\Delta \lambda}{\lambda} \right|$$

1C) Solution:

Jinsong Lin 20b21b

for LED (suppose the value of rectangular spectrum is 1)

$$\begin{aligned}
 \frac{I_{\nu}(\omega)}{I_{\nu}(\omega)} &= \frac{\int_{\nu}(\omega)}{1 \cdot \Delta\nu_{LED}} \\
 &= \frac{1}{\Delta\nu_{LED}} \left[ \int_{\nu_0 - \frac{\Delta\nu}{2}}^{\nu_0 + \frac{\Delta\nu}{2}} \exp(-0.7) d\nu + \int_{\nu_0 + \frac{\Delta\nu}{2}}^{\nu_0 + \frac{\Delta\nu}{2}} \exp(-2.3) d\nu \right. \\
 &\quad \left. + \int_{\nu_0 + \frac{\Delta\nu}{2}}^{\nu_0 + \frac{\Delta\nu}{2}} \exp(-0.7) d\nu \right] \\
 &= \frac{1}{\Delta\nu_{LED}} \left[ \int_{\nu_0 - \frac{\Delta\nu}{2}}^{\nu_0 + \frac{\Delta\nu}{2}} \exp(-d\nu \cdot 1) d\nu + \Delta\nu_{LED} - \Delta\nu \right] \\
 &= \frac{1}{\Delta\nu_{LED}} \left[ \int_{\nu_0 - \frac{\Delta\nu}{2}}^{\nu_0 + \frac{\Delta\nu}{2}} \exp\left[-23 \cdot \left(1 - \left(\frac{\nu - \nu_0}{\Delta\nu/2}\right)^2\right)\right] d\nu + \Delta\nu_{LED} - \Delta\nu \right] \\
 &= \frac{1}{\Delta\nu_{LED}} \left[ \int_{-\frac{\Delta\nu}{2}}^{\frac{\Delta\nu}{2}} \exp\left[-23 \cdot \left(1 - \left(\frac{\nu}{\Delta\nu/2}\right)^2\right)\right] d\nu + \Delta\nu_{LED} - \Delta\nu \right] \\
 &= \frac{1}{\Delta\nu_{LED}} \left[ \exp(-2.3) \frac{\bar{\pi} \Delta\nu \operatorname{erfi}(\sqrt{2.3})}{2 \sqrt{2.3}} + \Delta\nu_{LED} - \Delta\nu \right] \\
 &\approx 85.6\%
 \end{aligned}$$

$$\therefore \eta = 14.4\%$$

(d) Solution:

Jinsong Liu 206216

Advantage:

High pump efficiency

Disadvantage:

Expensive

(e) Solution:

Laser:

$$P_{\text{output}} = \frac{P_{\text{input}} \cdot \eta_1}{E_{\text{pump}}} \cdot E_{\text{signal}} = P_{\text{in}} \cdot \eta_1 \cdot \frac{\lambda_{\text{pump}}}{\lambda_{\text{output}}} = 850 \mu\text{W}$$

LED:

$$P_{\text{output}} = \frac{P_{\text{input}} \cdot \eta_2}{E_{\text{pump}}} \cdot E_{\text{signal}} = P_{\text{in}} \cdot \eta_2 \cdot \frac{\lambda_{\text{pump}}}{\lambda_{\text{output}}} = 137 \mu\text{W}$$

Problem 3

Jinsong Liu 20b21b

(a) Solution:

∴  $R$  is the reflectivity (power) of the beam splitter,

$M_1$  and  $M_2$  are the (power) reflectivities of the mirrors.

$$\begin{aligned} E_{\text{out}} &= \overline{E_{\text{in}} \sqrt{(1-R) \cdot M_1 \cdot R} \cdot \exp(-ik \cdot 2L_1)} + \overline{\overline{E_{\text{in}} \sqrt{(1-R) \cdot M_2 \cdot R} \exp(-ik \cdot 2L_2)}} \\ &= \overline{E_{\text{in}} \sqrt{(1-R) \cdot R} \left[ \sqrt{M_1} \exp(-2ikL_1) + \sqrt{M_2} \exp(-2ikL_2) \right]} \end{aligned}$$

$$I_{\text{out}} = \langle \vec{S}(\vec{r}, t) \rangle = \frac{1}{2} \operatorname{Re} [\vec{E}(\vec{r}) \times \vec{H}^*(\vec{r})]$$

From Maxwell's equations, (in frequency domain)

$$\operatorname{rot} \vec{\vec{E}}(\vec{r}, \omega) = i\omega \mu_0 \vec{H}(\vec{r}, \omega) \Rightarrow \vec{H}(\vec{r}, \omega) = -\frac{i}{\omega \mu_0} \operatorname{rot} \vec{\vec{E}}(\vec{r}, \omega)$$

$$\therefore \langle \vec{S}(\vec{r}, t) \rangle = \frac{1}{2} \operatorname{Re} [\vec{E}(\vec{r}) \times \frac{i}{\omega \mu_0} \operatorname{rot} \vec{\vec{E}}(\vec{r})^*]$$

$$= \frac{1}{2} \operatorname{Re} \left[ \frac{i}{\omega \mu_0} \cdot (-ik) \vec{E}(\vec{r}) \vec{E}^*(\vec{r}) \right]$$

$$= \frac{1}{2} \operatorname{Re} \left[ \frac{1}{c \mu_0} \vec{E}(\vec{r}) \cdot \vec{E}^*(\vec{r}) \right]$$

$$= \frac{1}{2c \mu_0} |\vec{E}(\vec{r})|^2 = \frac{\sqrt{\epsilon_0}}{2\sqrt{\mu_0}} |\vec{E}(\vec{r})|^2$$

$$\therefore I_{\text{out}} = \frac{\sqrt{\epsilon_0}}{2\sqrt{\mu_0}} \vec{E}_{\text{in}} \cdot \vec{E}_{\text{in}}^*$$

$$\begin{aligned}
 I_{\text{out}} &= \frac{\sqrt{\epsilon_0}}{2\sqrt{\mu_0}} E_{\text{in}}^2 (1-R) R \left[ (\sqrt{M_1} \exp(2ikL_1) + \sqrt{M_2} \exp(2ikL_2)) \cdot (\sqrt{M_1} \exp(-2ikL_1) + \sqrt{M_2} \exp(-2ikL_2)) \right] \\
 &= \frac{\sqrt{\epsilon_0}}{2\sqrt{\mu_0}} E_{\text{in}}^2 (1-R) \cdot R \left[ M_1 + M_2 + \sqrt{M_1 M_2} \left\{ \exp[2ik(L_1 - L_2)] + \exp[-2ik(L_1 - L_2)] \right\} \right] \\
 &= \frac{\sqrt{\epsilon_0}}{2\sqrt{\mu_0}} E_{\text{in}}^2 (1-R) \cdot R \left[ M_1 + M_2 + 2\sqrt{M_1 M_2} \cos(2k(L_1 - L_2)) \right]
 \end{aligned}$$

(b) Solution:

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

$$\begin{cases} I_{\max} = \frac{\sqrt{\epsilon_0}}{2\sqrt{\mu_0}} E_{\text{in}}^2 (1-R) R [M_1 + M_2 + 2\sqrt{M_1 M_2}] \\ I_{\min} = \frac{\sqrt{\epsilon_0}}{2\sqrt{\mu_0}} E_{\text{in}}^2 (1-R) R [M_1 + M_2 - 2\sqrt{M_1 M_2}] \end{cases}$$

$$\therefore V = \frac{2\sqrt{M_1 M_2}}{M_1 + M_2}$$

(c) Solution:

$$\bar{E}_{in} = \frac{E_0}{2} \exp \left[ -2 \ln 2 \frac{t^2}{\tau^2} + i \omega_0 t \right] + C.C.$$

$$\bar{E}_{out} =$$