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Metrology and Sensing

Lecture 11-2: Phase retrieval

2021-01-14

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Content

- Parameter
- Algorithm

Phase-Retrieval: Basic Parameter

- Necessary known basi cparameter for reconstruction:
  1. wavelength λ
  2. aperture in image space sin(u)
  3. pixel size of detector
  4.pinhole size in object space
  5. magnification m
  6. z-values of z-stack

- Critical data:
  1. pixel size
  2. size of pinhole (coherence, throughput)
  3. deconvolution parameter of algorithm
  4. background of intensity
  5. selection of z-planes

Mathematical Formulation

- Mathematical description for Image formation, Integral equation, inverse problem

$$I_{image}(x)=\int I_{psf}(x,x')\cdot I_{object}(x,x')\,dx'+I_{noise}(x)$$

- Approximation with isoplanatic range: Psf shift invariant, convolution computed with Fourier methods

$$I_{image}(x)=I_{psf}(x)*I_{object}(x)+I_{noise}$$

$$I_{image}(v)=I_{psf}(v)\cdot I_{object}(v)+I_{noise}$$

- Discretization: pixelized image delivers a linear system

$$I_{jk}^{(ima)}=\sum_{j'}\sum_{k'} I_{j-j',k-k'}^{(psf)}\cdot I_{j',k'}^{(obj)}+I_{jk}^{(noi)}$$

$$\vec{g}=\mathbf{A}\cdot\vec{x}+\vec{n}$$

- Solution via optimization due noise and constraints

$$\left|\mathbf{A}\cdot\vec{x}-\vec{b}\right|^2=\min$$

$$\Phi=\left|\mathbf{A}\cdot\vec{x}-\vec{b}\right|^2+\mu\cdot\left|\vec{x}\right|^2=\min$$

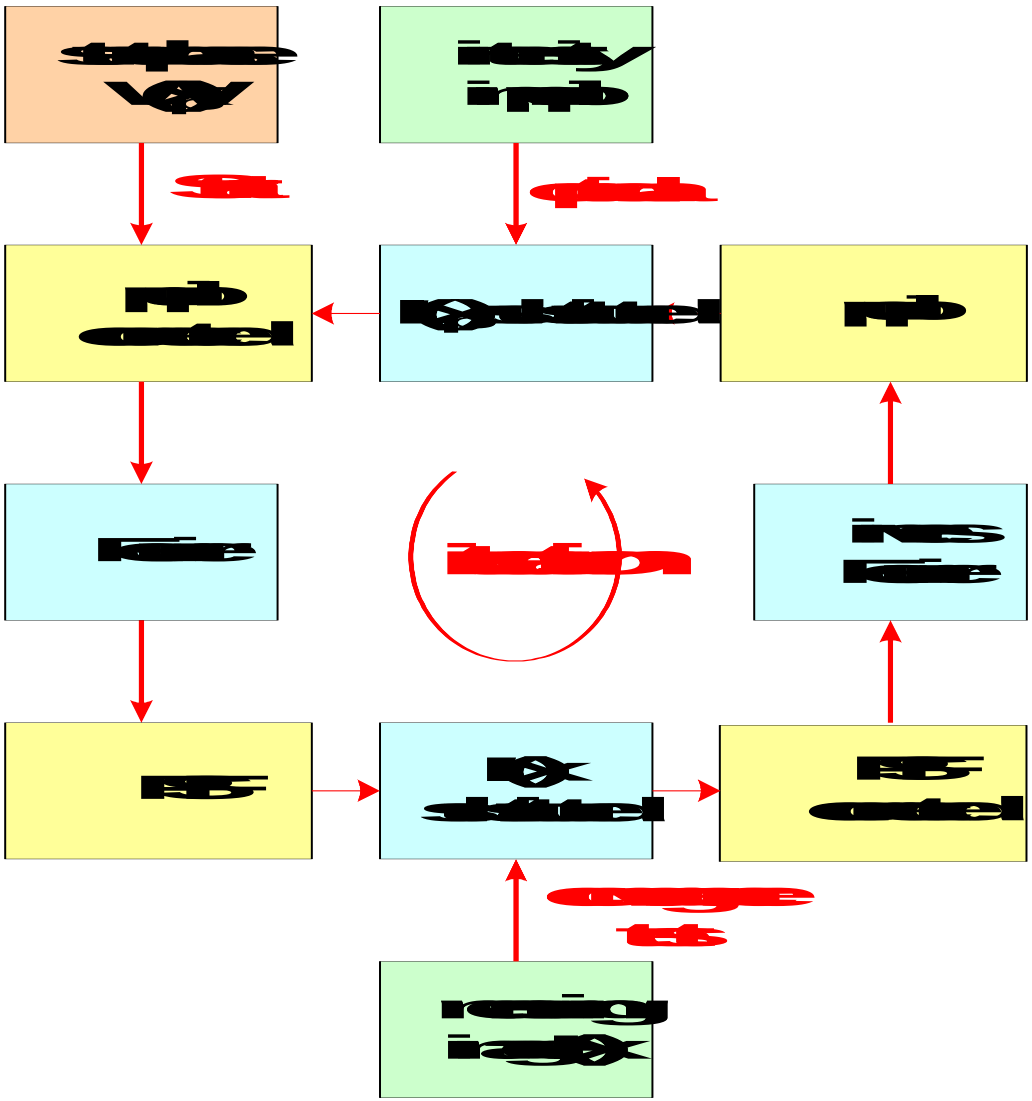
Gerchberg-Saxton-Algorithm

- Iterative reconstruction of the pupil phase with back-and-forth calculation between image and pupil:
  IFTA / Gerchberg-Saxton

- Substitution of known intensity

- Problems with convergence:
  Twin-image degeneration

- Modified algorithms:
  1. Fienup-acceleration
  2. Non-least-square
  3. Use of pupil intensity



Phase - Retrieval Algorithms

Possible numerical algorithms:

1. Fourier algorithm
2. NLSQ-algorithm with Zernike coefficients (modal)
3. Input-Output-algorithm according to Fienup
4. Yang-Gu-algorithm
5. Ping-Pong-algorithm
6. Gerchberg-Saxton-algorithm (error reduction)
7. Ferwerda-algorithm
8. Gradient methods

Check of Caustic

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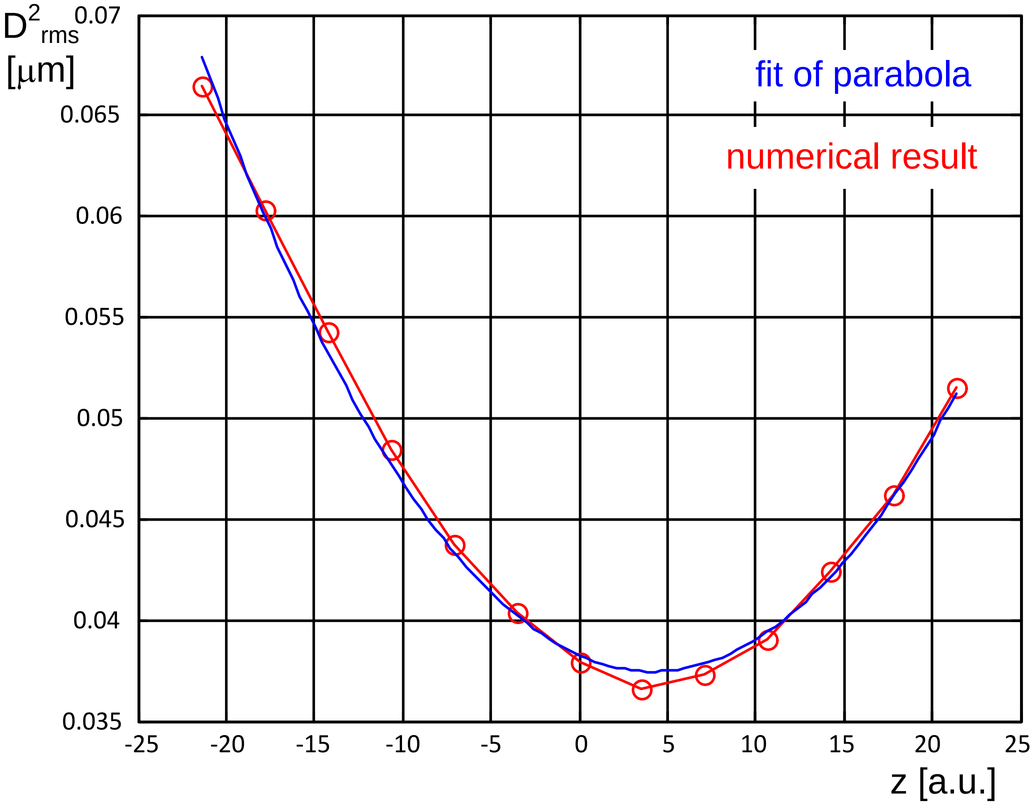
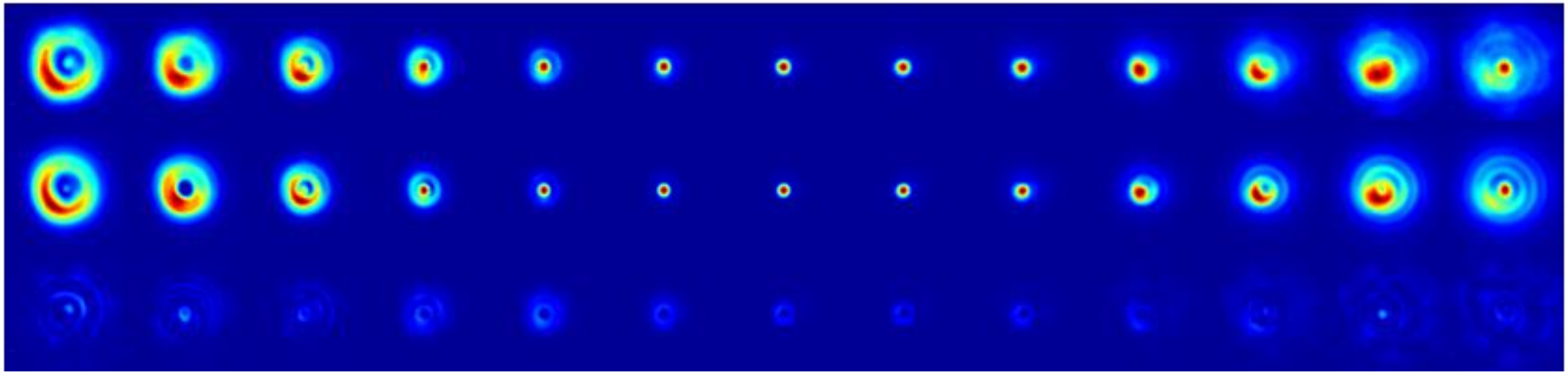
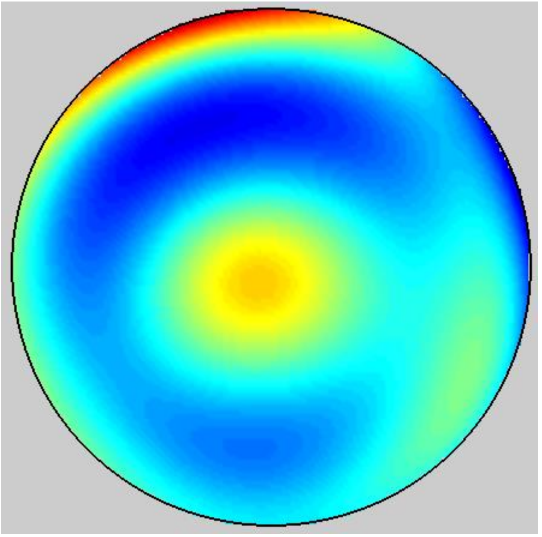
- Paraxial approximation:
  - quadratic curve of second moment of spot size
  - check of wrong data possible
  -deviations for larger aberrations

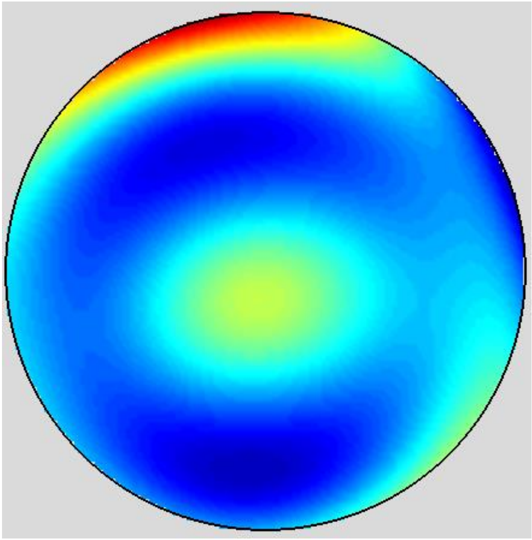


Example Phase Retrieval

- Evaluation of real data psf-stack

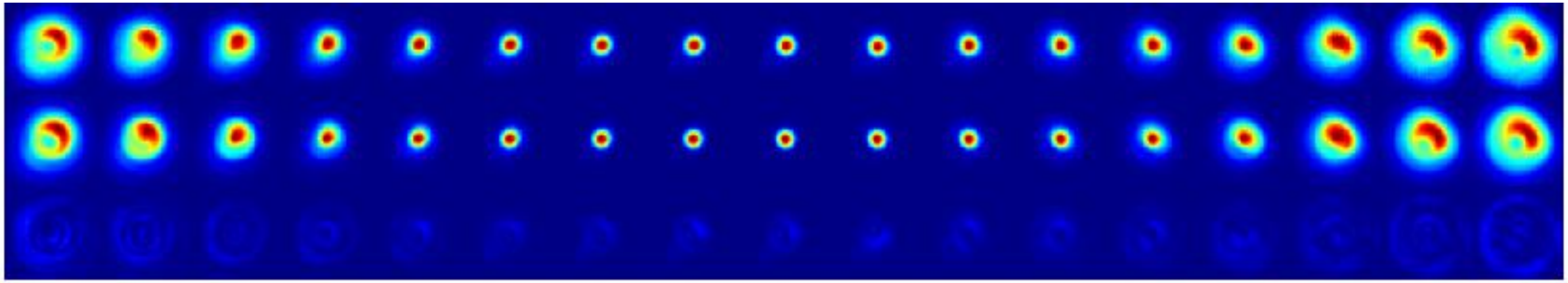




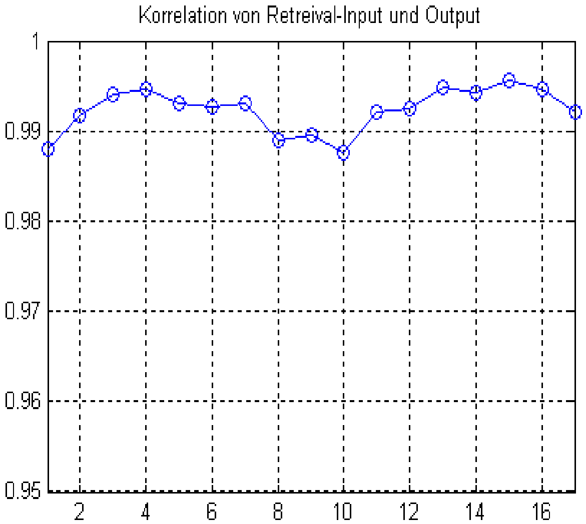


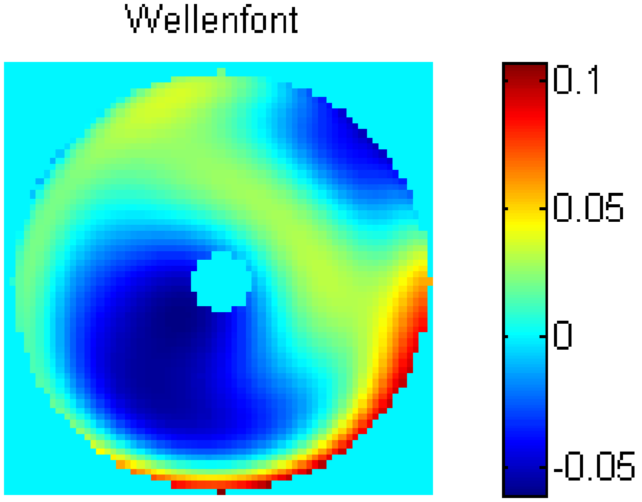
PSF Phase Retrieval

- Phase retrieval method
- Image z-stack



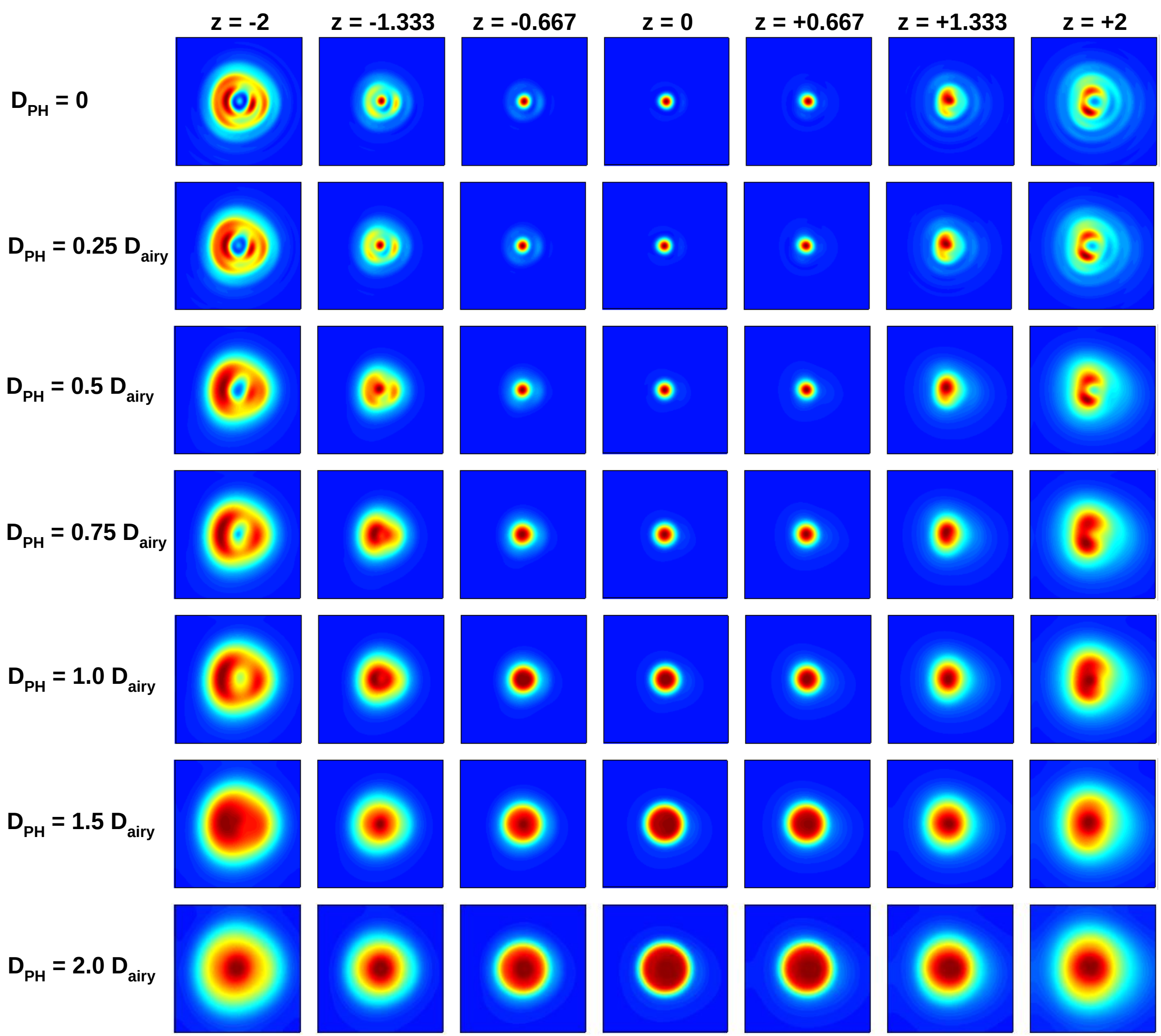
- Correlation of image
- Phase in pupil





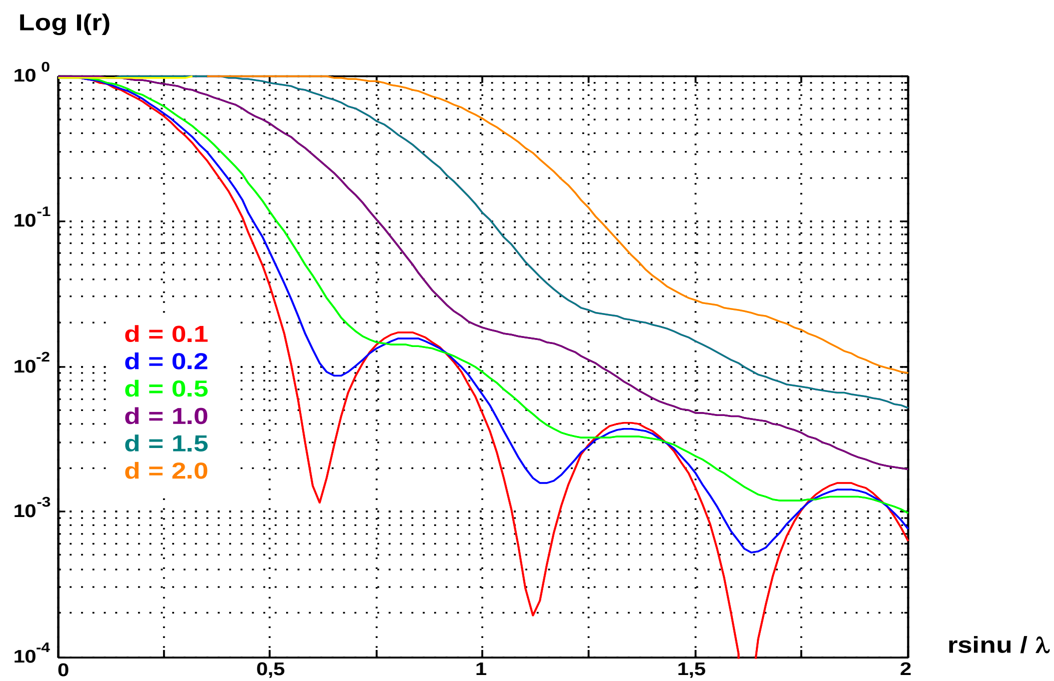
Finite Size of Pinhole

- Image with finite size of the pinhole:
  convolution
- Characteristic diffraction structures hidden with growing size D_{ph}
- Deconvolution necessary for D_{ph} >0.3...0.4 D_{airy}
- Pinholes larger than 4 D_{airy} are not feasible

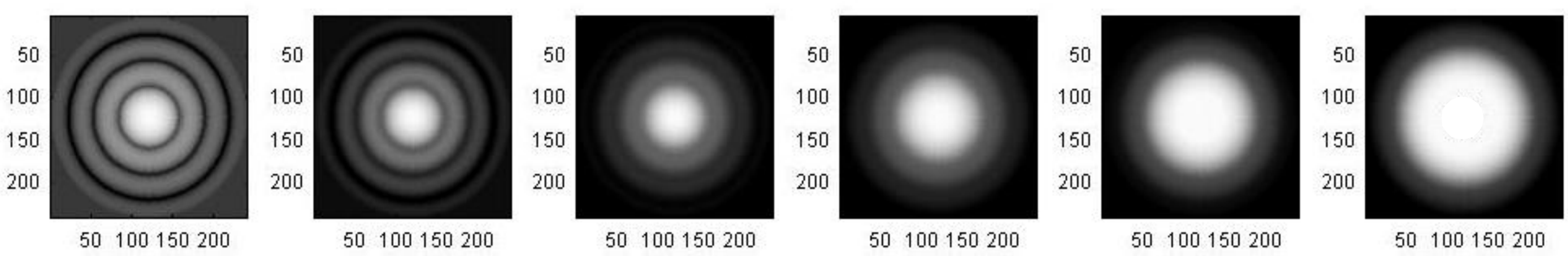


Incoherent Image of a Pinhole

- Logarithm of intensity
- Diffraction ripples disappear with growing diameter d



d = 0.1 d = 0.2 d = 0.5 d = 1.0 d = 1.5 d = 2.0



Deconvolution - Algorithms

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- Incoherent imaging with noise

$$I_{image}(x) = I_{psf}(x) * I_{object}(x) + I_{noise}$$

- Wiener deconvolution with fixed Tikhonov regularization

$$I_{object}(\nu) = \frac{I^{*}_{psf}(\nu) \cdot I_{image}(\nu)}{\left|I_{psf}(\nu)\right|^2 + \mu}$$

- Wiener deconvolution with variable Tikhonov regularization

$$I_{object}(\nu) = \frac{I^{*}_{psf}(\nu) \cdot I_{image}(\nu)}{\left|I_{psf}(\nu)\right|^2 + \frac{P_{noise}}{P_{object}}}$$

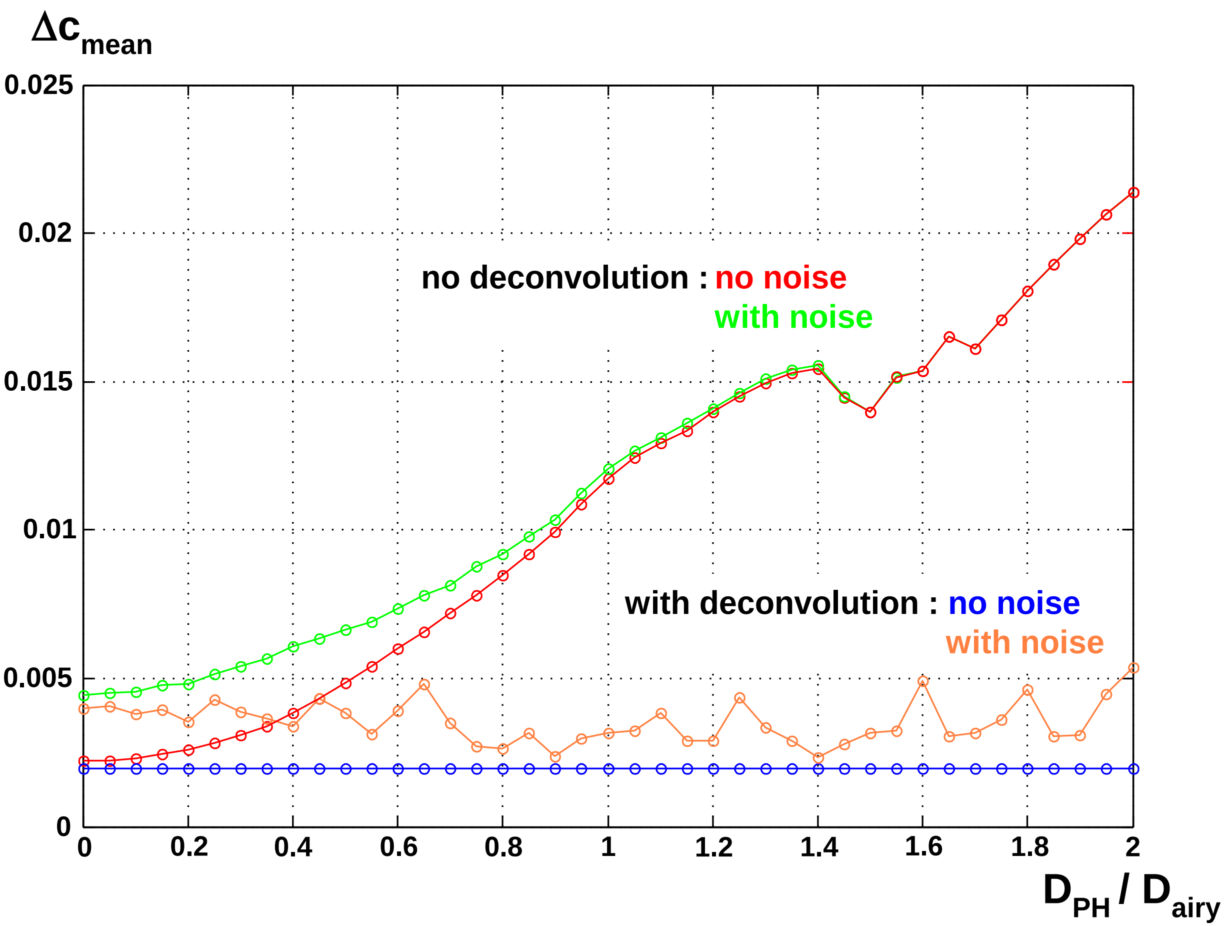
- Lucy-Richardson deconvolution

$$I^{(k+1)}_{obj}(x,y) = I^{(k)}_{obj}(x,y) \cdot \left[I_{psf}(-x,-y) * \frac{I_{image}(x,y)}{I_{psf}(x,y) * I^{(k)}_{obj}(x,y)} \right]$$

- Wavelet-Deconvolution

Error in Phase Retrieval without Deconvolution

- Error of results, if no deconvolution is performed
- Error increases with pinhole size
- Deconvolution seems to be necessary for pinholes larger than 0.4 D_{airy}



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Variable Pinhole Deconvolution: Criteria

- Criteria for pinhole deconvolution:

1. minimal rms of stack
   sensitive for noise

$$I_{rms} = \sqrt{\frac{\sum_j \left[I_j^{(model)} - I_j^{(ist)} \right]^2}{n_x \cdot n_y \cdot n_z}}$$

2. Maximum correlation of the stacks
   robust, but less significant

$$K = \frac{\iiint I_{model} \cdot I_{ist} \, dx\,dy\,dz}{\sqrt{\iiint I_{model}^2 \, dx\,dy\,dz} \cdot \sqrt{\iiint I_{ist}^2 \, dx\,dy\,dz}}$$

3. Minimum entropy
   best results

$$S = -\sum_{m,n} I_{mn} \cdot \ln I_{mn}$$

Iteration Phase Retrieval for finite sizes of Pinhole

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- Possibilities with / without deconvolution