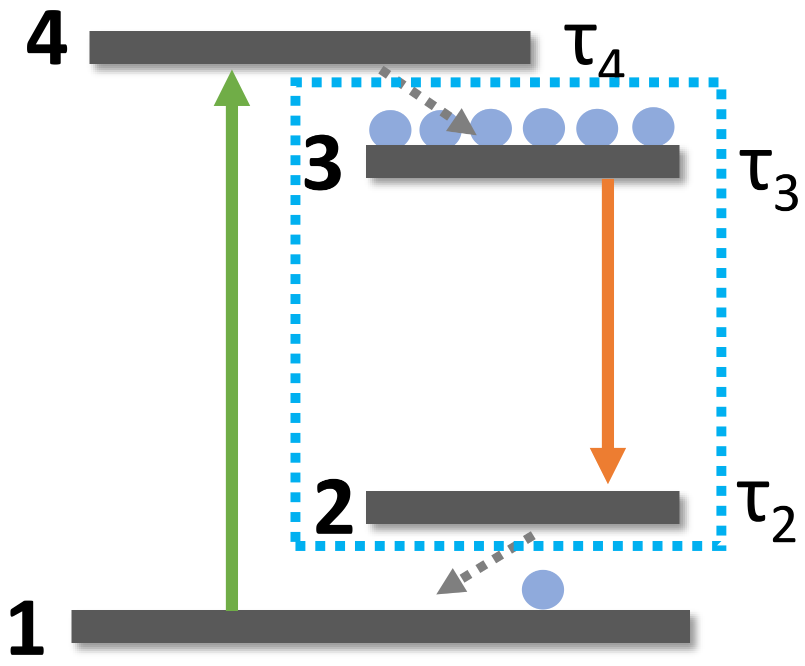
Question 1

a) Why is the inversion of the active medium in a laser clamped in the stationary regime? (1 point)

In the stationary regime (steady state) operation of a laser, gain is equal to loss.

This is the result of the various inputs and loss mechanisms in the laser system:

- Pumping rate and absorption
- useful output coupling
- Parasitic losses
- Gain saturation
- Etc.



In this steady state, the output power is constant. The rate of excitation and stimulated emission is constant and therefore the inverted population is constant (although the population is constantly being cycled through the energy level system)

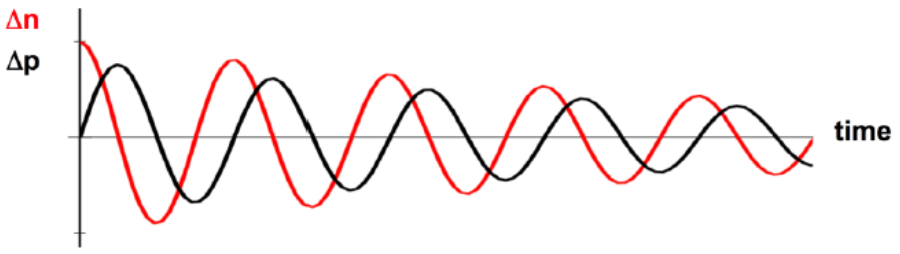
b) Which physical process leads to a damping of the relaxation oscillations? (1 point)

Spontaneous emission

Relaxation oscillations are the result of changes in inversion level and photon density in systems with relatively long upper state lifetimes. These oscillations take place with high frequencies in a perturbed system (e.g. turning up a pump source):

1. the inversion will be increased temporarily beyond the steady state, which will
2. lead to an increased photon flux due to stimulated emission. Then
3. The inversion temporarily falls below the steady state level

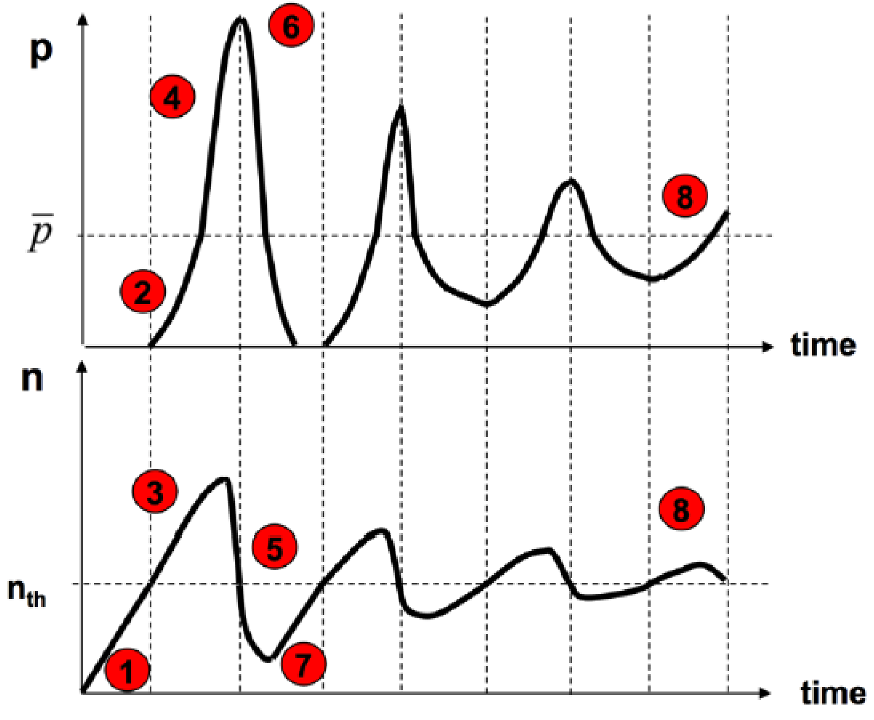
This oscillatory process continues until spontaneous emission dampens and extinguishes the effect.



c) What is spiking of a laser? (1 point)

Spiking is related to relaxation oscillations and has the same perturbative origin. However, in the case when the perturbation of inversion and photon densities is large (on the order of the steady state values), these oscillations are intense and no longer follow the equations of simple damped oscillators.

Spiking is related to relaxation oscillations and has the This can come from suddenly turning on/up a pump and in processes like Q-switching where the rate of stimulated emission is suddenly and drastically changed



d) What determines the quality factor of a laser cavity? (1 point)

From script:

Pumping rate and upper-state lifetime

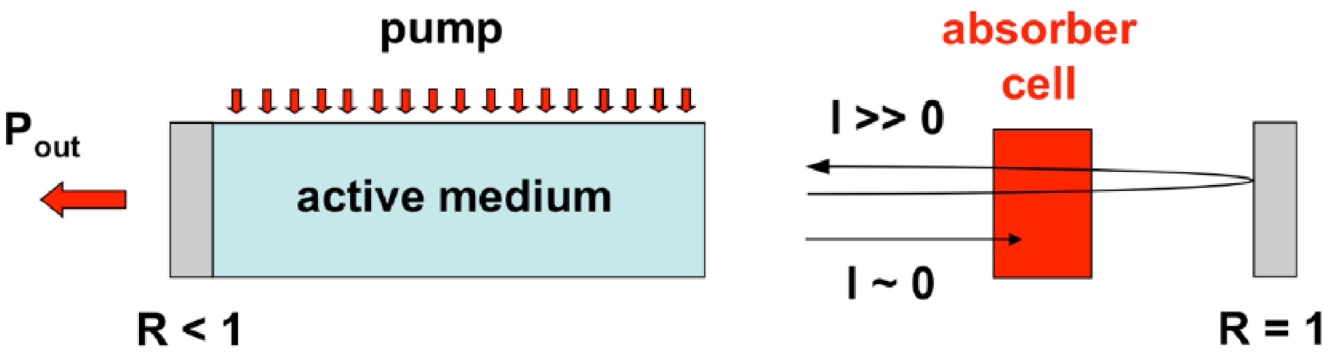
$$Q = \frac{Energy\ stored\ in - cavity}{Energy\ loss\ per\ round - trip}$$

Resonator losses, photon lifetime

Hence, a high-quality cavity implies low losses and a long photon lifetime τ_{ph}.

e) What determines the repetition rate in a passively Q-switched laser? (1 point)

Intensity of radiation incident on a saturable absorber, combined with the rate of inversion buildup (upper state lifetime and pump rate)



When the inversion and the resulting spontaneous emission reaches a level which saturates the absorber, the absorber will become temporarily transparent

f) What determines the minimum pulse duration in a Q-switched laser? (1 point)

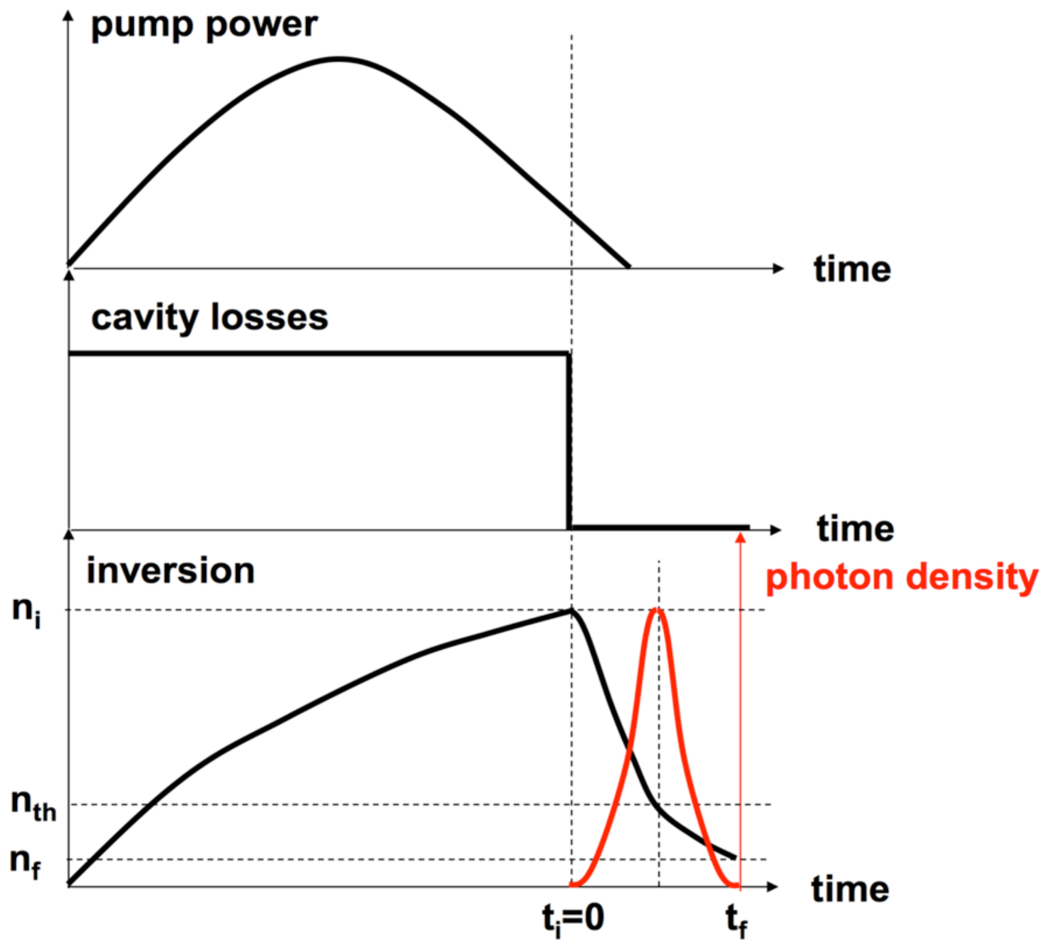
- Inversion level
  - Determines how quickly stimulated emission will increase the photon flux
  - Shapes the leading edge of a pulse (higher inversion = steeper leading edge)
- Photon lifetime in cavity
  - Determines the rate at which light leaves the cavity (trailing edge of pulse)
  - Higher lifetime = slower decay of trailing edge

Question 2

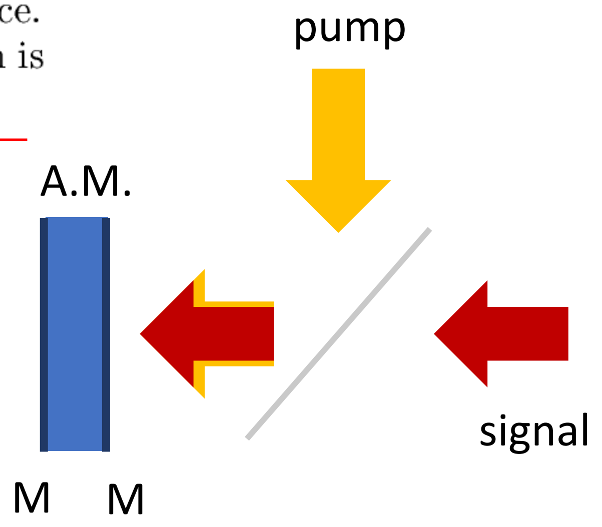
Problem 3 (5 points)

Assume a thin-disk active medium, i.e. an active medium with the shape of a round disk of diameter d and thickness w, where $w <<< d$. Both the pump and the laser signal are launched in the active medium through one of its flat sides of diameter d. This active medium is homogeneously pumped and it is employed to build a Q-switched laser. The surfaces of the active medium are polished. The system is designed so that the active medium reaches its maximum possible inversion directly before the Q-factor of the cavity is switched.

a) Briefly explain the operating principle of Q-switching. (1 point)

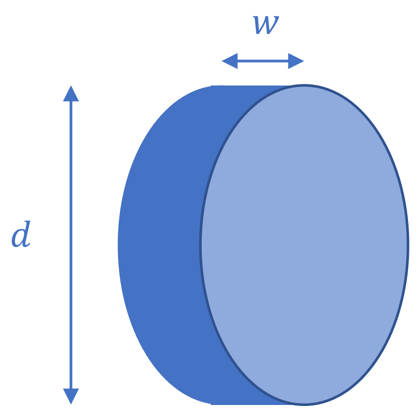


b) Calculate the maximum diameter of the disk before spontaneous lasing in the transversal direction takes place. The refractive index of the active medium is $n = 1.83$, the emission cross-section at the signal wavelength is $\sigma = 2 \cdot 10^{-20} cm^2$ and the doping concentration is $N = 7 \cdot 10^{20} Ions/cm^3$). (1 point)



Reflectance at the surfaces of the active medium (normal incidence)

$$R = \left|\frac{n_1 - n_2}{n_1 + n_2}\right|^2 = \left|\frac{1.83 - 1}{1.83 + 1}\right|^2 = 0.086$$



Condition for lasing

$$e^{gd} R(1-L) e^{gd} R(1-L) \geq 1$$

(L: other losses per one trip)

Assuming L = 0

$$e^{2gd} R^2 \geq 1$$

$$2gd \geq \ln\left(\frac{1}{R^2}\right)$$

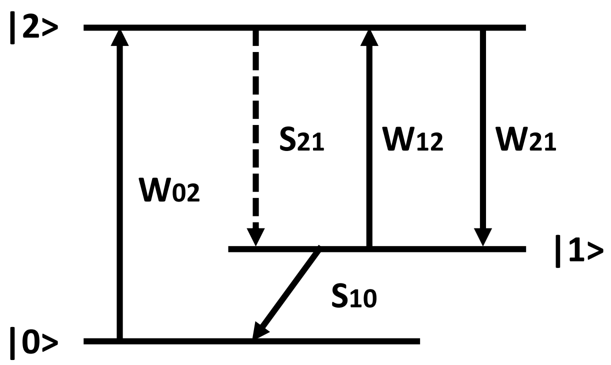
$$2\sigma n d \geq -2\ln(R)$$

$$d \geq -\frac{\ln(R)}{\sigma n}$$

Therefore, **before spontaneous lasing**:

$$d < -\frac{\ln(R)}{\sigma n_{max}}$$

inverse three-level system



$$S_{10} \gg S_{21}$$

$$n = n_2 - n_1 \approx n_2$$

Population inversion threshold:

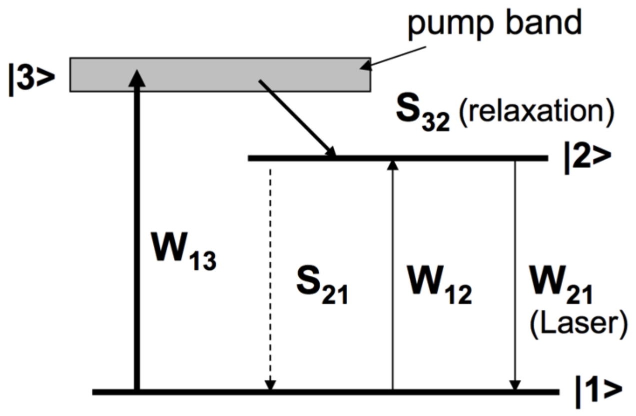
$$n = 0, when: n_2 = n_1 = 0$$

Max inversion population:

$$(n)_{max} = \frac{1}{2} n_{tot},$$

$$when: (n_2)_{max} = \frac{1}{2} n_{tot}$$

three-level system



$$S_{32} \gg W_{13}$$

$$n = n_2 - n_1$$

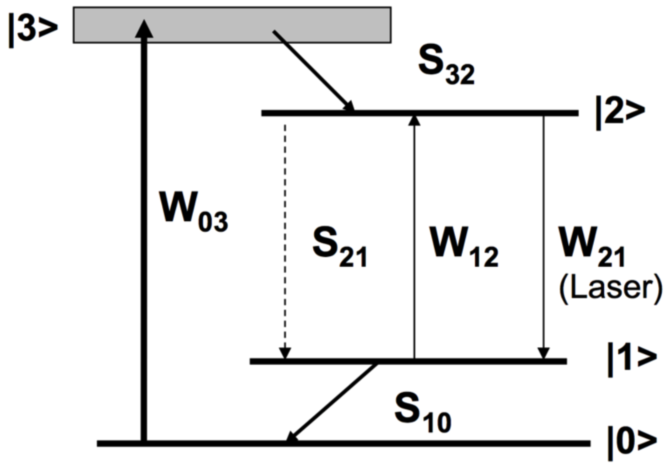
Population inversion threshold:

$$n = 0, when: n_2 = n_1 = \frac{1}{2} n_{tot}$$

Max inversion population:

$$(n)_{max} = n_{tot}, when: n_2 = n_{tot}$$

four-level system



$$S_{10} \gg S_{21}; S_{21} \ll S_{32}$$

$$n = n_2 - n_1 \approx n_2$$

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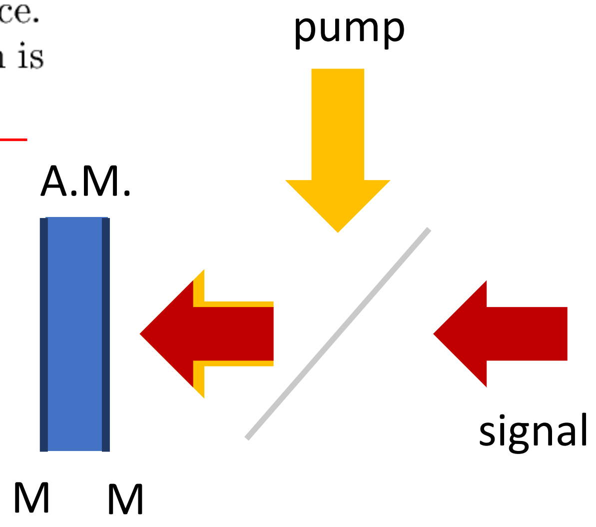

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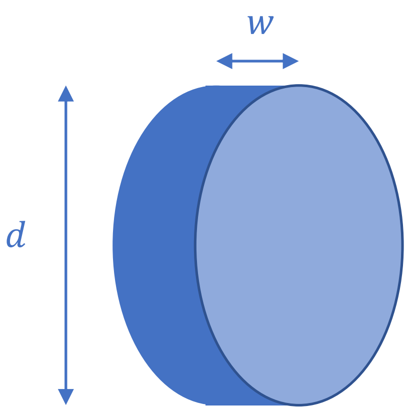

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$$e^{gd} R(1 - L) e^{gd} R(1 - L) \geq 1$$

(L: other losses per one trip)

Assuming L = 0

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$$2gd \geq \ln\left(\frac{1}{R^2}\right)$$

$$2\sigma n d \geq -2\ln(R)$$

$$d \geq -\frac{\ln(R)}{\sigma n}$$

Therefore, **before spontaneous lasing**:

$$d < -\frac{\ln(R)}{\sigma n_{max}}$$

In a 4-level system, or a 3-level system

$$n_{max} = n_{tot}$$

$$d < 1.75mm$$

In an inverse-3-level system

$$n_{max} = \frac{1}{2} n_{tot}$$

$$d < 3.5mm$$

c) How much energy can be stored in the active medium if it is a: 3-level system, a 4-level system or an inverse 3-level system? Assume that the thickness of the disk is $w = 100\mu m$ and that the pump wavelength is $976nm$. (1 point)

Stored energy in the active medium

$$E_{stored} = pump\ photon\ energy \cdot max\ inversion\ population$$

$$= h\nu_{pump} \cdot V n_{max}$$

Or max population in the upper laser level?

$$= h\frac{c}{\lambda_{pump}} \cdot \pi\left(\frac{d}{2}\right)^2 w n_{max}$$

In a 4-level system, or a 3-level system

$$n_{max} = n_{tot}$$

$$E_{stored} = h\frac{c}{\lambda_{pump}} \cdot \pi\left(\frac{d}{2}\right)^2 w n_{tot} = 34.3 \text{ mJ}$$

In an inverse-3-level system

$$n_{max} = \frac{1}{2} n_{tot}$$

$$E_{stored} = h\frac{c}{\lambda_{pump}} \cdot \pi\left(\frac{d}{2}\right)^2 w \frac{1}{2} n_{tot} = 17.15 \text{ mJ}$$

d) How much energy can you extract from the disk of section c), if it is a: 3-level system, a 4-level system or an inverse 3-level system? Assume that the signal wavelength is $1064nm$. (1 point)

Max extration energy in the active medium

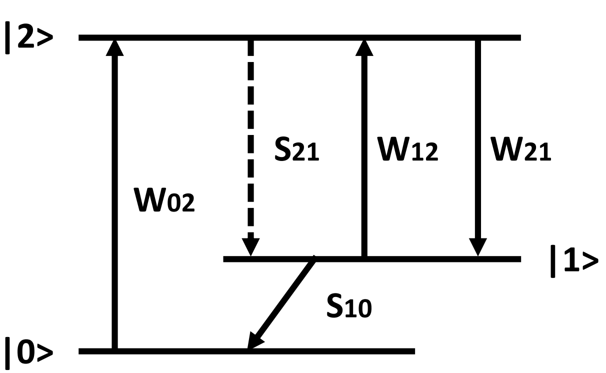
$$E_{extract} = signal\ photon\ energy \cdot extraction\ inversion\ population$$

$$= h\nu_{signal} \cdot Vn_{ext}$$

$$= h\frac{c}{\lambda_{signal}} \cdot \pi\left(\frac{d}{2}\right)^2 wn_{ext}$$

The max ion number that contributes to the amplification process

inverse three-level system



$S_{10} \gg S_{21}$

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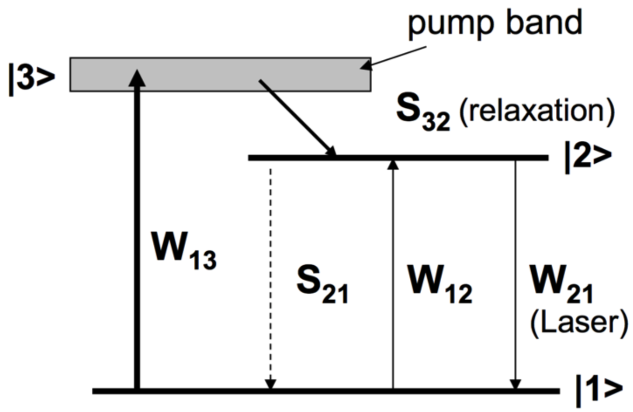
three-level system



$S_{32} \gg W_{13}$

$n = n_2 - n_1$

Population inversion threshold:

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Max inversion population:

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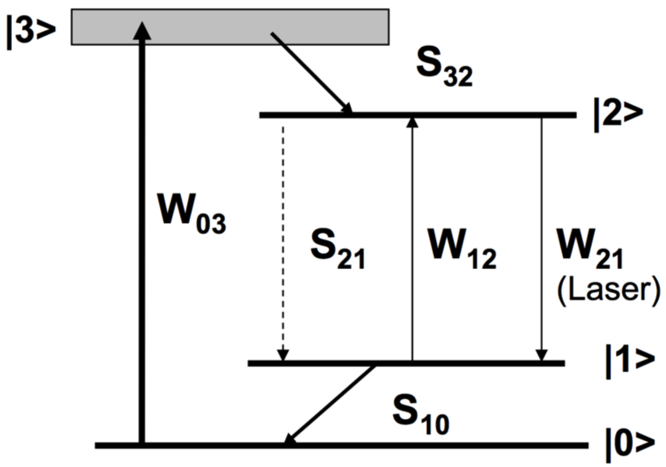
four-level system



$S_{10} \gg S_{21}; S_{21} \ll S_{32}$

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Population inversion threshold:

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Max inversion population:

$(n)_{max} = n_{tot}, when: n_2 = n_{tot}$

d) How much energy can you extract from the disk of section c), if it is a: 3-level system, a 4-level system or an inverse 3-level system? Assume that the signal wavelength is $1064nm$. (1 point)

Max extration energy in the active medium

$$E_{extract} = signal\ photon\ energy \cdot extraction\ inversion\ population$$

$$= h\nu_{signal} \cdot Vn_{ext}$$

$$= h\frac{c}{\lambda_{signal}} \cdot \pi\left(\frac{d}{2}\right)^2 wn_{ext}$$

In a 4-level system

$$n_{ext} = n_{tot}$$

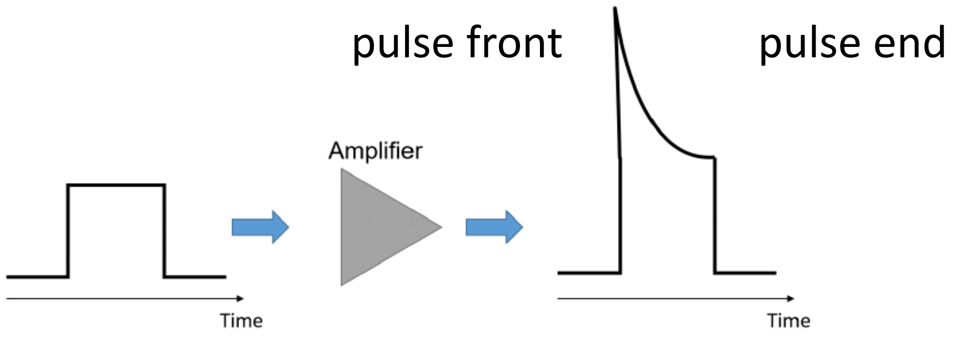
$$E_{extr} = h\frac{c}{\lambda_{signal}} \cdot \pi\left(\frac{d}{2}\right)^2 wn_{tot} = 31.4\ \mathrm{mJ}$$

In a 3-level system, and an inverse-3-level system

$$n_{ext} = \frac{1}{2}n_{tot}$$

$$E_{extr} = h\frac{c}{\lambda_{signal}} \cdot \pi\left(\frac{d}{2}\right)^2 w\frac{1}{2}n_{tot} = 15.7\ \mathrm{mJ}$$

e) The short pulses generated by this Q-switched laser are actively shaped to a rectangular temporal profile and, afterwards, they are further amplified to higher energies with an additional amplifier. After amplification, the temporal profile of the formerly rectangular pulses is shown in the following picture:



Explain why the temporal pulse shape is distorted after amplification. (1 point)

Before the pulse arrives, with enough pump power, the active medium in the amplifier is fully inversed (plenty of ions in the upper laser level).

When the pulse front arrives, it sees a large gain.

As the input pulse power is high, during amplification process, the pulse extracts a lot of energy from the active medium and fast depletes the inversion level. (gain saturation)

Thus, when the following part of the pulse arrives, it sees lower inversion level of the active medium, the gain is lower. Therefore, the power of the amplified pulse decreases along time.

How to explain the exp decay?

Question 3

Problem 3 (8 points)

Pulse energy scaling is a crucial requirement in various industrial applications, such as shock peening and lift-off processes. The short pulse durations in these applications is instrumental in mitigating thermal issues and enhancing process accuracy. In light of this, our objective is to design an actively Q-switched fiber laser using an acousto-optic modulator. For this design, we consider a $1m$-long, Yb-doped fiber, with an upper level lifetime of $\tau_2 = 0.8ms$ and an absorption cross-section of $\sigma_0 = 2.3e - 24m^2$ at the pump wavelength. The fiber has a doping concentration of $n_{tot} = 5e25m^{-3}$ and an active region diameter of $20\mu m$. It is efficiently pumped with a diode laser emitting at $976nm$. The emission wavelength of the Q-switched laser is $1030nm$. The resonator configuration consists of a $2m$-long cavity with 4% losses at each side of the fiber, where one side of the fiber acts as the output coupler. (consider a single-trip fiber loss $L = 5\%$ and quantum efficiency of 95%). In this problem we will consider that the pump process (during the low-Q state) brings the inversion from an initial value $n_0 = 0$ to a final value n_i.

a) In a Q-switched laser system, there is an inherent trade-off between pulse energy and pulse duration. Explain the relationship between these two parameters and discuss the factors that influence this relationship. Provide an example to illustrate the concept. (2 points)

- Pulse energy and pulse duration are inversely related: as pulse energy increases, pulse duration decreases, and vice versa.
- This relationship is a result of the energy conservation principle and the dynamics of the Q-switching process.
- The pulse duration is primarily determined by the Q-switching mechanism, which involves the **rapid release of stored energy in the gain medium**.
- Higher pump power increases energy accumulation and pulse energy, but it also leads to shorter pulse durations.
- Opening the Q-switch depletes the population inversion in the gain medium, leading to a fast rise in laser output power and generating a short-duration pulse known as the "spike" or "nanosecond" pulse.
- **Pulse energy is directly related to the amount of energy stored in the gain medium before the Q-switching event.**
- Factors influencing the pulse energy and duration relationship include **pump power, gain medium properties, and the Q-switching technique** used.
- Gain medium properties, such as **gain bandwidth**, can affect the pulse energy and duration relationship.
- Different Q-switching techniques have their own dynamics and limitations that influence the pulse energy and duration relationship.

b) When evaluating the potential damage mechanism of the fiber facets, it is essential to consider both the maximum fluence ($170 J/cm^2$) and the maximum peak intensity ($10 GW/cm^2$). The question at hand is two-fold: first, what is the maximum pump power that can be applied to the system before encountering damage? And second, which of these two factors, fluence or peak intensity, imposes the primary limitation on Q-switched fiber lasers?

Hint: Take into account that for a Gaussian pulse the peak power is determined by the following expression:

$$P_{peak} = I_{max} \cdot A$$

$$I_{max} = \frac{(1 - R_{oc}) c h \nu_s p_{max}}{2 n_{Yb}}$$

where A is the area of the fiber core, R_{oc} is the reflectivity of the outcopler mirror, ν_s is the frequency of the signal photons, $n_{Yb} = 1.45$ is the refractive index of the Yb-doped fiber and p is the photon density. (3 points)

$$F = \frac{E_{out}}{A_{eff}}$$

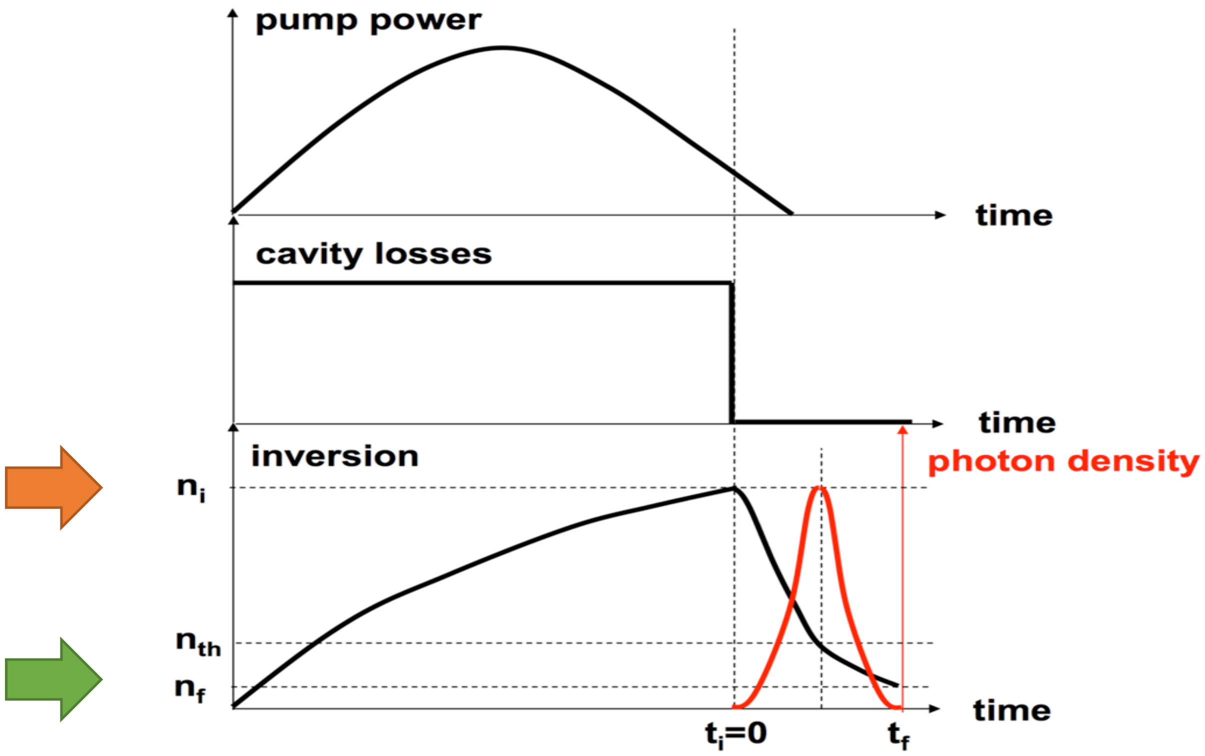
170J/cm2

$$E_{out} = F * A_{eff} == 133.5 \text{ uJ}$$

$\left(A_{eff} = {}^{A_{real}}/_{2} \right.$ For the Gaussian beam profile)

$$E_{\text{out}} = V(n_{\text{i}} - n_{\text{f}}) \cdot h \cdot \nu_{\text{Laser}}$$

We need to calculate the inversion of the fiber during the q-switching.



Steady state condition

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -\Gamma \cdot n_{\mathrm{i}} + W_{\mathrm{p}}(n_{\mathrm{tot}} - n_{\mathrm{i}}) = 0$$

$$\rightarrow\; n_{\mathrm{i}} = \frac{W_{\mathrm{p}} \cdot n_{\mathrm{tot}}}{\Gamma + W_{\mathrm{p}}} = \frac{W_{\mathrm{p}} \cdot n_{\mathrm{tot}}}{\frac{1}{\tau_2} + W_{\mathrm{p}}}$$

Eq. 1: Initial inversion

$$P_{\mathrm{pump}} = n_{\mathrm{tot}} \cdot V \cdot h \cdot \nu_{\mathrm{p}} \cdot \frac{W_{\mathrm{p}}}{\eta}$$

$$\rightarrow\; W_{\mathrm{p}} = \frac{\eta \cdot P_{\mathrm{pump}}}{n_{\mathrm{tot}} \cdot V \cdot h \cdot \nu_{\mathrm{p}}}$$

Eq. 3: Pump power

Integrate to find the accumulated photon density as the inversion is reduced from n_i to n_f

$$\int_{p_{\mathrm{i}}}^{p_{\mathrm{f}}} \mathrm{d}p = \int_{n_{\mathrm{i}}}^{n_{\mathrm{f}}} \left(\frac{n_{\mathrm{YAG}}}{\sigma \cdot c \cdot n \cdot \tau_{\mathrm{ph}}} - 1 \right) \mathrm{d}n$$

$$p_{\mathrm{f}} - p_{\mathrm{i}} = \left[\frac{n_{\mathrm{YAG}} \cdot \ln(n)}{\sigma \cdot c \cdot \tau_{\mathrm{ph}}} - n \right]_{n_{\mathrm{i}}}^{n_{\mathrm{f}}}$$

we're given $p_{\mathrm{i}} = p_{\mathrm{f}} = 0$

$$\frac{n_{\mathrm{YAG}} \cdot \ln\left(\frac{n_{\mathrm{f}}}{n_{\mathrm{i}}}\right)}{\sigma \cdot c \cdot \tau_{\mathrm{ph}}} - (n_{\mathrm{f}} - n_{\mathrm{i}}) = 0$$

re-arrange

$$n_{\mathrm{f}} = n_{\mathrm{i}} e^{\frac{\sigma \cdot c \cdot \tau_{\mathrm{ph}}}{n_{\mathrm{Yb}}} \cdot (n_{\mathrm{f}} - n_{\mathrm{i}})}$$

Eq. 2: Final inversion

$$E_{\mathrm{out}} = V(n_{\mathrm{i}} - n_{\mathrm{f}}) \cdot h \cdot \nu_{\mathrm{Laser}}$$

- By inserting the equations 1 and 2 inside the (*), we can solve three equations ad three unknowns, to calculate P_{pump} inside eq 3.
- The final results will be:
- $n_i = 4.8e25$
- $n_f = 4.3e23$
- $P_{pump} = 225\ mW$ Maximum pump power, before the fiber get damaged

We need to calculate the peak power of the pulse with calulated pump, and compare to the damage threshold

$$I_{max} = \frac{F_{threshold}}{0.94 * \tau_{pulse}} \overset{?}{==} 60 \text{ GW/cm^2}$$

$$I_{\max} = \frac{(1 - R_{OC})ch\nu_{\mathrm{s}}p_{\max}}{2n_{\mathrm{Yb}}}$$

We need to calculate the max photon flux first:

Formulas from lectures

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -\sigma \cdot \frac{c}{n_{YAG}} \cdot p \cdot n \quad + \left(\begin{array}{c} \text{here we neglect the} \\ \text{pump since the emission} \\ \text{process is very fast} \end{array} \right)$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = p \cdot \left(\sigma \frac{c}{n_{\mathrm{YAG}}} \cdot n - \frac{1}{\tau_{\mathrm{ph}}} \right)$$

Combine these to express *p* in terms of *n*

$$\frac{\mathrm{d}p}{\mathrm{d}n} = \left(\frac{\mathrm{d}p}{\mathrm{d}t} \right) \cdot \frac{1}{\left(\frac{\mathrm{d}n}{\mathrm{d}t} \right)} = \cancel{p} \cdot \left(\sigma \frac{c}{n_{\mathrm{YAG}}} \cdot n - \frac{1}{\tau_{\mathrm{ph}}} \right) \cdot \left(-\frac{n_{\mathrm{YAG}}}{\sigma \cdot c \cdot \cancel{p} \cdot n} \right)$$

$$\frac{\mathrm{d}p}{\mathrm{d}n} = \frac{n_{\mathrm{YAG}}}{\sigma \cdot c \cdot n \cdot \tau_{\mathrm{ph}}} - 1$$

Therefore, we know that

$$p_{\mathrm{f}} - p_{\mathrm{i}} = \left[\frac{n_{\mathrm{Yb}} \cdot \ln(n)}{\sigma \cdot \mathrm{c} \cdot \tau_{\mathrm{ph}}} - n\right]_{n_{\mathrm{i}}}^{n_{\mathrm{f}}}$$

When $n = n_{th}$ this leads to the max. photon density

$$p_{\max} = \left[\frac{n_{\mathrm{YAG}} \cdot \ln(n)}{\sigma \cdot c \cdot \tau_{\mathrm{ph}}} - n\right]_{\mathrm{n_i}}^{n_{\mathrm{th}}} = n_{\mathrm{th}} \ln\left(\frac{n_{\mathrm{th}}}{n_{\mathrm{i}}}\right) - (n_{\mathrm{th}} - n_{\mathrm{i}})$$

We need to calculate the threshold inversion

$$n_{\mathrm{th}} = \frac{1}{\sigma \cdot c \cdot \tau_{\mathrm{ph}}} \longrightarrow n_{\mathrm{th}} = 1.25e24$$

$$\tau_{\mathrm{ph}} = -\frac{\tau_{\mathrm{R}}}{\ln(R_1 R_2 R_{OC}(1-Loss)^2)} = 1.15\ ns$$

$$\tau_{\mathrm{R}} = \frac{2\ * L_{\mathrm{Resonator}}}{c} = \frac{2\ * (1 + \frac{L_{\mathrm{fiber}}}{n_{\mathrm{Yb}}})}{c} = 11.2\ ns$$

$$p_{\mathrm{max}} = 1.88\ \mathrm{e}24$$

$$I_{\mathrm{max}} = 3.6 \frac{GW}{cm2}$$

The fiber will not damage by peak intensity

c) A good approximation of the pulse duration for a Q-switched laser with fully energy extraction is given by:

$$\tau_{pulse} \approx \frac{r}{r-1-ln(r)}\tau_{ph}$$

Where r is the initial inversion ratio($r = n_i/n_{th}$), n_{th} is the threshold inversion and τ_{ph} is the photon lifetime. How much does the pulse duration change by using a 10% OC instead of 4%. (2 points)

- From previous section we have calculated the initial and final inversion:

$$r = \frac{n_i}{n_{th}} = 3.85 \quad and \quad \tau_{pulse} = 2.96\, ns$$

- With 10% output coupler we eill have:

$$r = \frac{n_i}{n_{th}} = 4.16 \quad and \quad \tau_{pulse} = 3.06\, ns$$

- The pulse duration increases

d) What are the consequences for your laser if the pulse repetition period becomes longer than the fluorescence lifetime of the upper laserr level? What can you do to mitigate these detrimental effects? (1 point)

- In this situation, the upper laser level will not have sufficient time to fully decay before the next pulse arrives.
- The incomplete decay of the upper laser level will result in residual population inversion, which can adversely affect laser performance.
- The laser may experience reduced efficiency and diminished output power.
- The laser may exhibit higher levels of spontaneous emission, leading to increased noise and reduced signal-to-noise ratio.

Mitigation solution

- Increase the pulse repetition rate
- Use pulsed pumping
- Consider mode-locking