



**Institute of
Applied Physics**

Friedrich-Schiller-Universität Jena

Optical System Design Fundamentals

Lecture 1: Optical Imaging Fundamentals

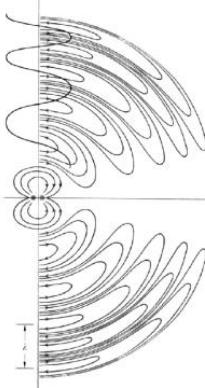
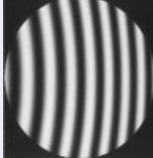
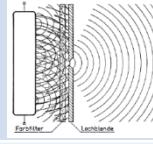
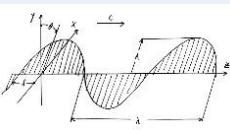
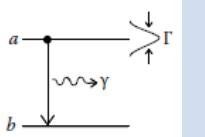
2024 / 05 / 07

Vladan Blahnik

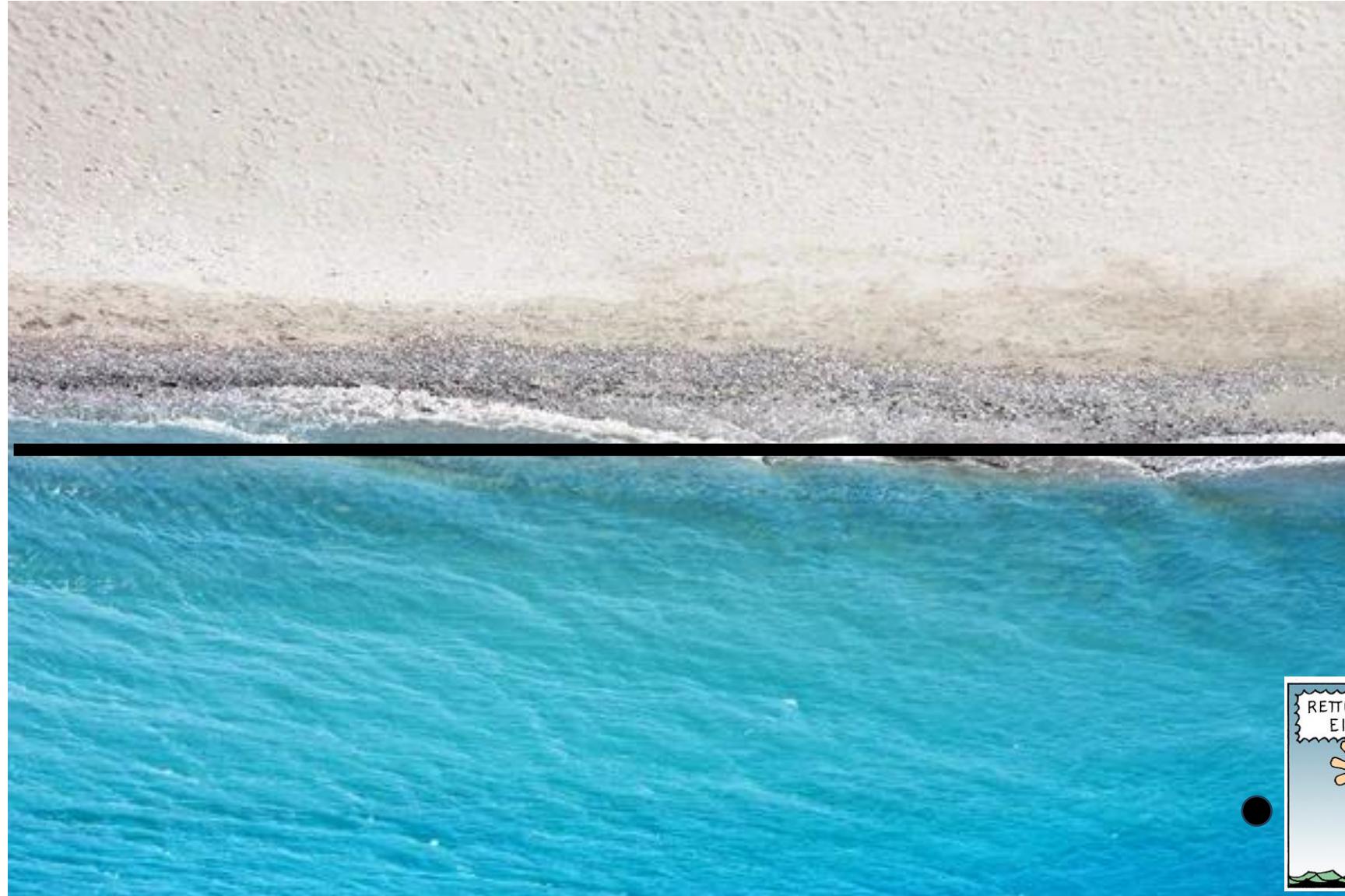
Preliminary Schedule - OSDF 2024

1	07.05.2024	Optical Imaging Fundamentals	Fermat principle, modeling light propagation with ray and wave model; diffraction; Snell's law, total internal reflection, light propagation in optical system, raytracing; ray and wave propagation in optical systems; pinhole camera; central perspective projection; étendue, f-number and numerical aperture, focal length; ideal projection laws; fish-eye imaging; anamorphic projection; perspective distortion, Panini projection; massive ray-tracing based imaging model	(S) (optic)
2	14.05.2024	The ideal image	Paraxial approximation, principal planes, entrance and exit pupil, paraxial imaging equations; Scheimpflug condition; paraxial matrix formulation; Delano diagram; Aplanatic imaging: Abbe sine condition, numerical aperture, sine scaled pupil for microscope imaging	(S)
3	21.05.2024	First order layout, system structures and properties	application of imaging equations; examples: layout of visual imaging system; layout of microscope lens and illumination system; telecentricity, "4f-setups"; tele and reversed tele (retrofocus) system; depth of field, bokeh; equivalence to multi-perspective 3D image acquisition; focusing methods of optical systems; afocal systems; connecting optical (sub-)systems: eye to afocal system; illumination system and its matching to projection optics); zoom systems; first order aberrations (focus and magnification error) and their compensation: lens breathing, aperture ramping; scaling laws of optical imaging	S
4	28.05.2024	Imaging model including diffraction	Gabor analytical signal; coherence function; degree of coherence; Rayleigh-Sommerfeld diffraction; optical system transfer as complex, linear operator; Fraunhofer-, Fresnel-approximation and high-NA field transfer wave-optical imaging equations for non-coherent, coherent and partially coherent imaging; isoplanatic patches, Hopkins Transfer Function for partially coherent imaging, optical transfer functions for non-coherent and coherent imaging; ideal wave-optical imaging equations; Fourier-Optics; spatial filtering; Abbe theory of image formation in microscope, phase contrast microscopy	S
5	04.06.2024	Wavefront deformation, Optical Transfer Function, Point Spread Function and performance criteria	wave aberrations, Zernike polynomials, measurement of system quality; point spread function (PSF), optical transfer function (OTF) and modulation transfer function (MTF); optical system performance criteria based on PSF and MTF: Strehl definition, Rayleigh and Marechal criteria, 2-point resolution, MTF-based criteria, coupling efficiency / insertion loss at photonics interface	S
6	11.06.2024	Aberrations: Classification, diagrams and identification in real images	measurement of wave front deformation: Shack-Hartmann sensor, Fizeau interferometer Symmetry considerations as basis for aberration theory; Seidel polynomial, chromatic aberrations; surface contributions and Pegel diagram, stop-shift-equation; Longitudinal and transverse aberrations, spot diagram; computing aberrations with Hamilton Eikonal Function; examples: defocus, plane parallel plate longitudinal and lateral chromatic aberration, Abbe diagram, dispersion fit; achromat and apochromat aberration in real images of (extended) objects	no
7	18.06.2024	Optimization process and correction principles	pre- and post-computer era correction methodology, merit function, system specification, initial setups, opto-mechanical constraints, damped-least-square optimization; symmetry principles, lens bending, aplanatic surface insertion, lens splitting, aspheres and freeforms	S
8	25.06.2024	Aberration correction and optical system structure	Petzval theorem, field flattening, achromat and apochromat, sensitivity analysis, diffractive elements; system structure of optical systems, e.g. photographic lenses, microscopic objectives, lithographic systems, eyepieces, scan systems, telescopes, endoscopes	(S)
9	02.07.2024	Tolerancing and straylight analysis	optical technology constraints, sensitivity analysis, statistical fundamentals, compensators, Monte-Carlo analysis, Design-To-Cost; false light; ghost analysis in optical systems; AR coating; opto-mechanical straylight prevention	S

Physical models of light and their relevance for optical imaging

Model		Relevant for:
Ray 	Simple model for the propagation of light (rays are perpendicular to wavefronts)	Calculation of aberrations/wave front deformation, ray tracing-based calculation of image intensity distributions
Wave 	Interference 	Anti-Reflection-Coating, Metrology (Optical Components, Systems), Image formation (from Pupil to Image Plane)
	Light propagation/ Diffraction 	Propagation of light in free space and in optical systems, Diffraction at (small) structures
	Polarization 	Anti-Aliasing Filter (birefringence), Polarization filter
Photon 	Quantum-theoretical models of light-matter-interaction	Statistics of noise at low intensity levels, Spectral distribution of light sources, Photoelectrical Effect

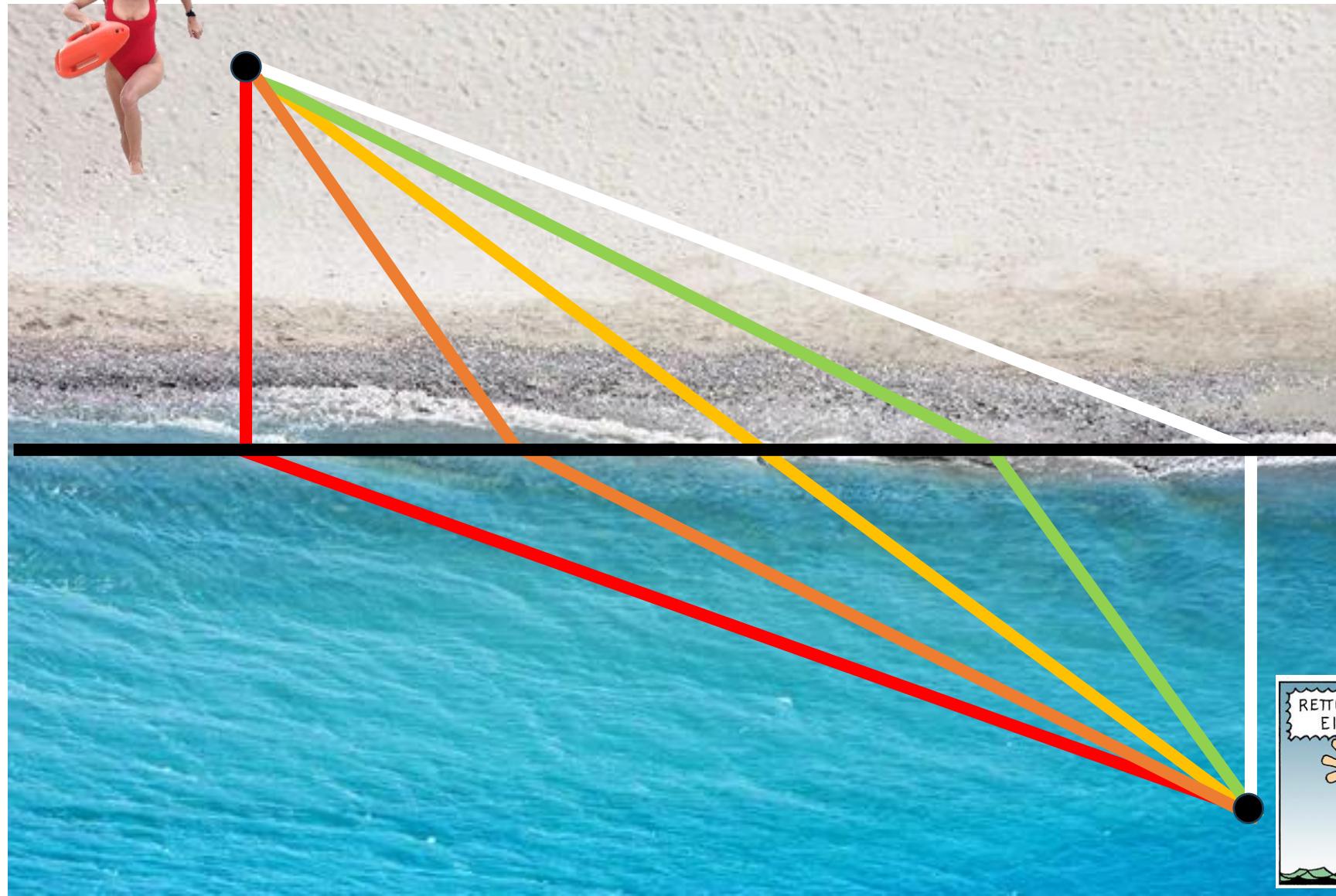
Fermat's principle



**Which path should
Baywatch take to get there
as fast as possible?**



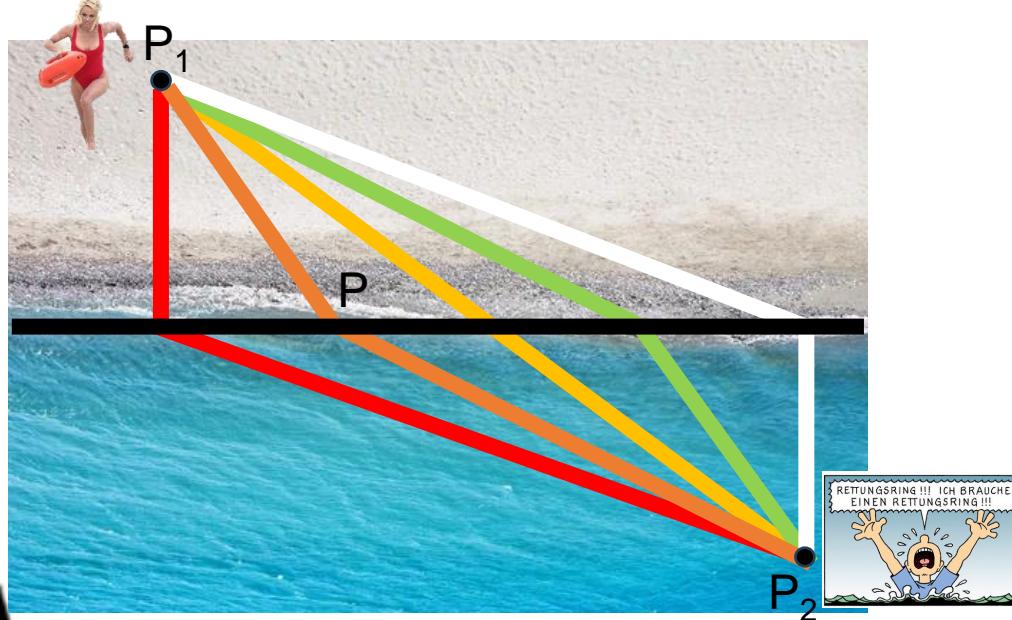
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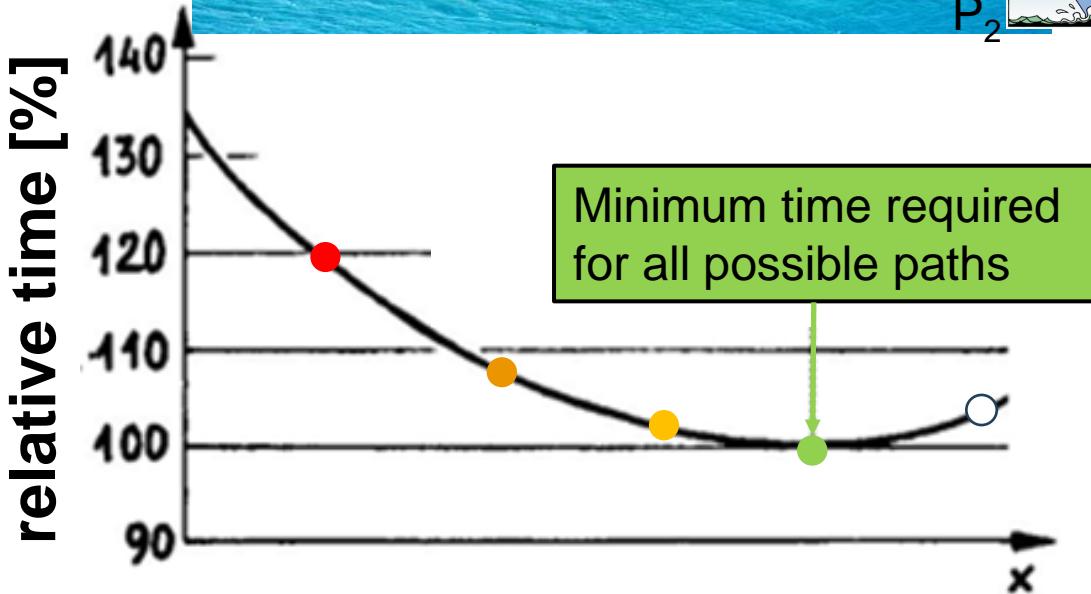


Fermat's principle



For a variable P we seek to find the minimum of

$$T(P) = \int_{P_1}^P c_1 dt + \int_P^{P_2} c_2 dt$$



There is a path along which the required time is optimal.

Can we obtain an explicit expression which direction to take? (without trying all possible paths)

Fermat principle:

The light takes the ray path, which corresponds to the shortest time of arrival.

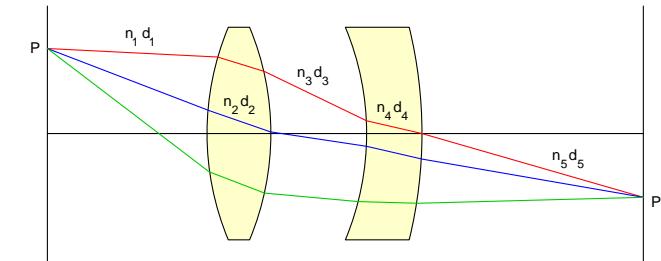
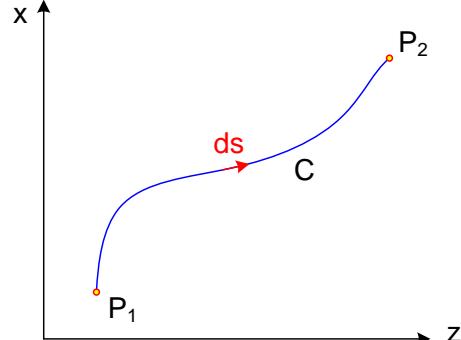
The optical path length OPL along a path $(x(s), y(s), z(s))$ is the path integral in the spatially varying medium of refractive index n :

$$OPL = \int_{P_1}^{P_2} n(x(s), y(s), z(s)) ds$$

In optical systems we usually have a succession of homogeneous media (lenses and air spaces):

$$OPL = \sum_{j=1}^N n_j d_j$$

with the distance between surface intersections $d_j = \sqrt{x_j^2 + y_j^2 + z_j^2}$.



The **realized path = the actual light ray** is a minimum and therefore the first derivatives vanish:

$$\delta OPL = 0$$

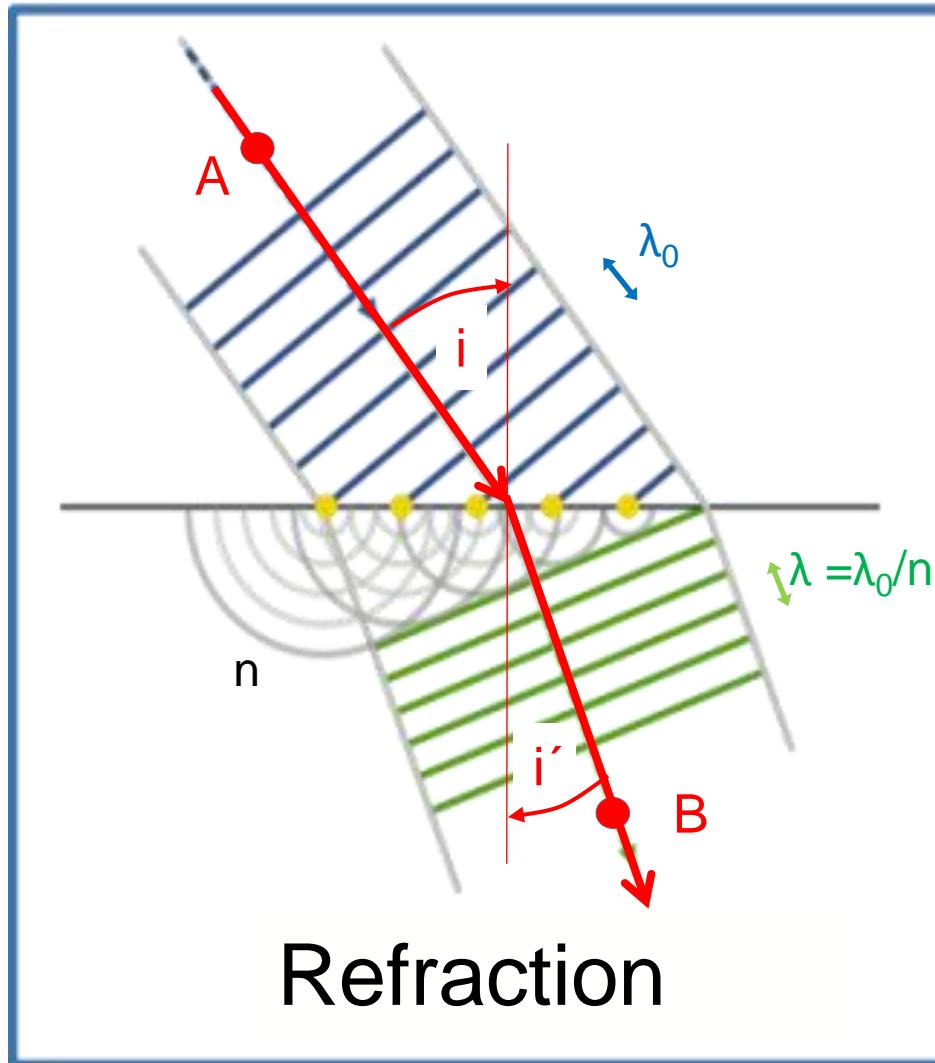
which for a suited parametrization of the paths with variables (u, v) means:

$$\frac{\partial OPL}{\partial u} = 0$$

$$\frac{\partial OPL}{\partial v} = 0$$

For the interface between media **Snell's Law** $n \sin i = n' \sin i'$ can be derived from Fermat's principle.

Refraction and Diffraction of Waves (Huygens, Fresnel)



Fermat's Principle:
„Light takes the fastest way between two points.“

Since in a medium of refractive index n light is slower by a factor of $1/n$, this connection is not a straight line.

Fermat formulated this principle in the year 1650. At this time it was mystical. This changed with the model of **wave-optical propagation** (Fresnel ca. 1800):

A wavefront at time of arrival impinges an elementary wave. The integral of all impinged elementary waves again defines a new wavefront.

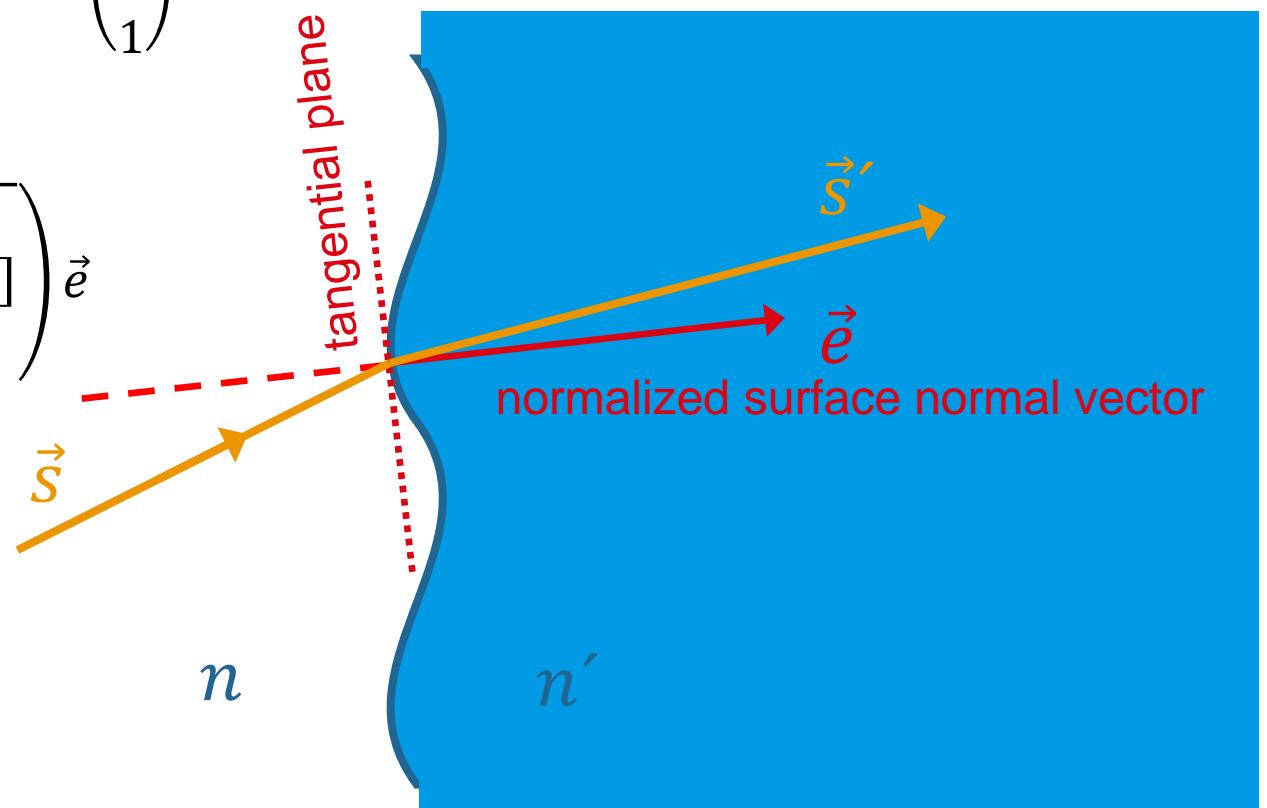
In case of a interface between media n and n' this wavefront (locally) changes its direction according to Snell's Law: $n \sin i = n' \sin i'$

General vectorial law of refraction (3D)

normalized ray
direction vector: $\vec{s} = \begin{pmatrix} 0 \\ \sin i \\ \cos i \end{pmatrix}$

normalized
surface
normal vector: $\vec{e} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$\vec{s}' = \frac{n}{n'} \vec{s} + \left(-\frac{n}{n'} \vec{s} \cdot \vec{e} + \sqrt{1 - \left(\frac{n}{n'}\right)^2 [1 - (\vec{s} \cdot \vec{e})^2]} \right) \vec{e}$$



General vectorial law of refraction (3D): Special case of refraction in yz-plane

normalized ray
direction vector: $\vec{s} = \begin{pmatrix} 0 \\ \sin i \\ \cos i \end{pmatrix}$

normalized
surface
normal vector: $\vec{e} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

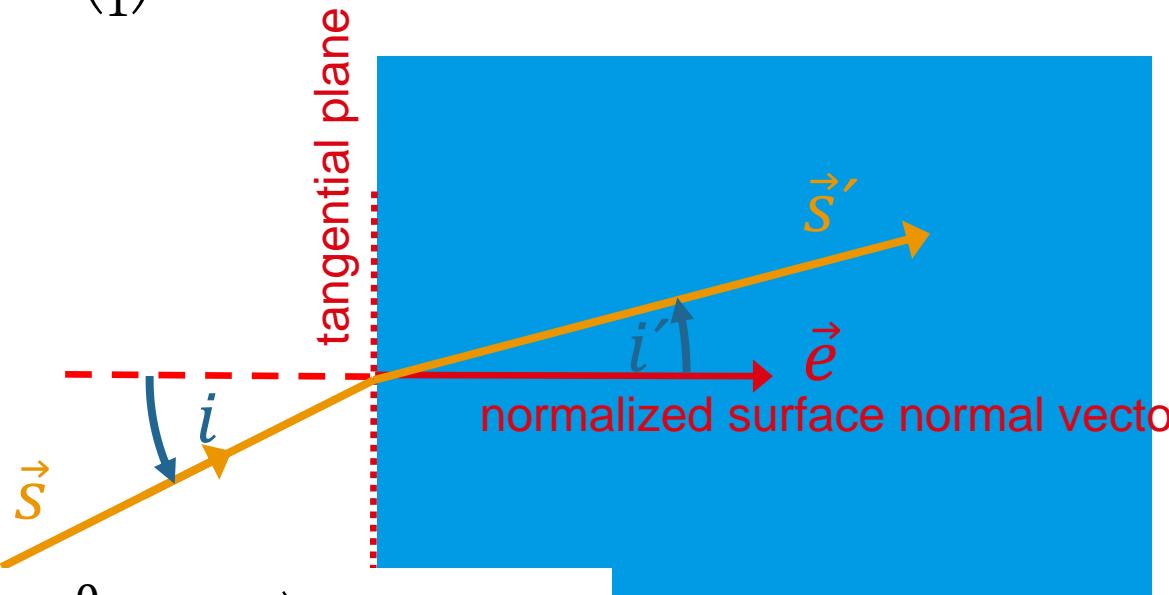
$$\vec{s}' = \frac{n}{n'} \vec{s} + \left(-\frac{n}{n'} \vec{s} \cdot \vec{e} + \sqrt{1 - \left(\frac{n}{n'}\right)^2 [1 - (\vec{s} \cdot \vec{e})^2]} \right) \vec{e}$$

$$= \frac{n}{n'} \begin{pmatrix} 0 \\ \sin i \\ \cos i \end{pmatrix} + \left(-\frac{n}{n'} \cos i + \sqrt{1 - \left(\frac{n}{n'}\right)^2 (\sin i)^2} \right) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

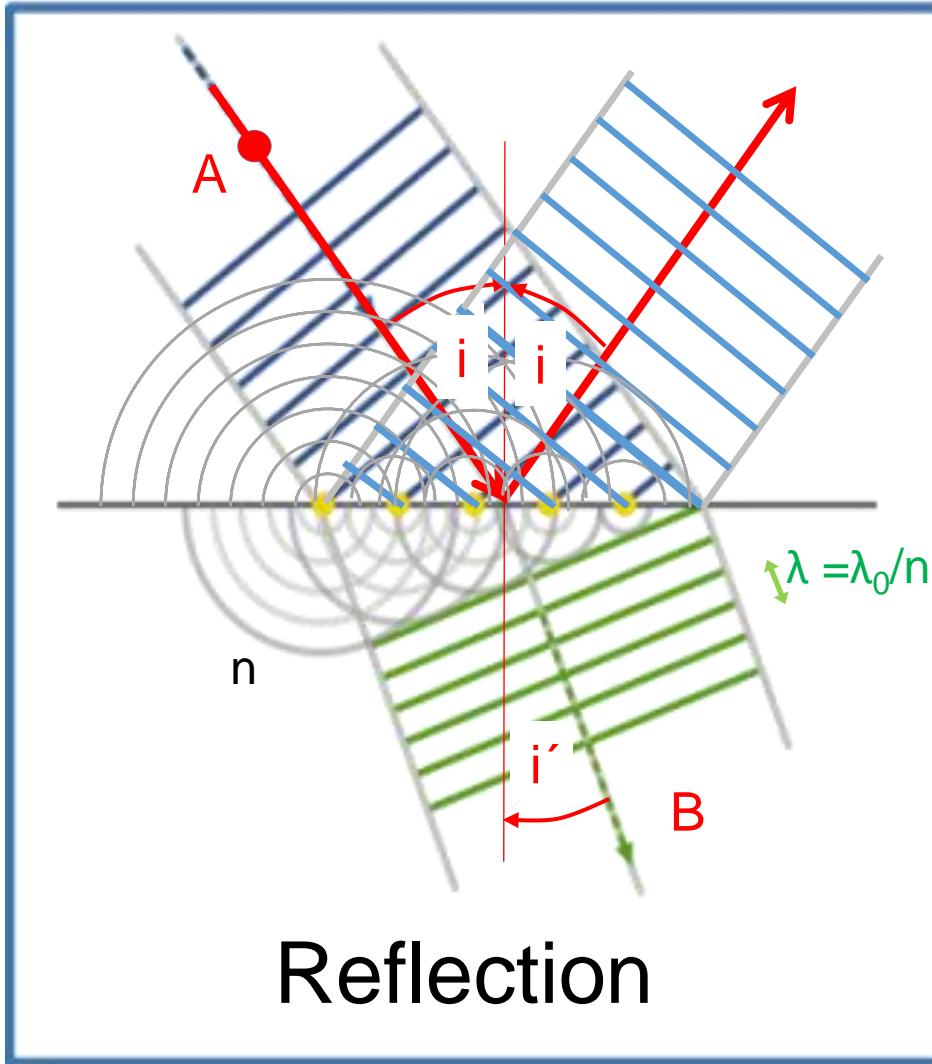
$$= \begin{pmatrix} 0 \\ \frac{n}{n'} \sin i \\ \frac{n}{n'} \cos i - \frac{n}{n'} \cos i + \sqrt{1 - \left(\frac{n}{n'}\right)^2 (\sin i)^2} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{n}{n'} \sin i \\ \sqrt{1 - \left(\frac{n}{n'}\right)^2 (\sin i)^2} \end{pmatrix} = \begin{pmatrix} 0 \\ \sin i' \\ \cos i' \end{pmatrix}$$

„direction cosine components“

y-component: $\frac{n}{n'} \sin i = \sin i'$



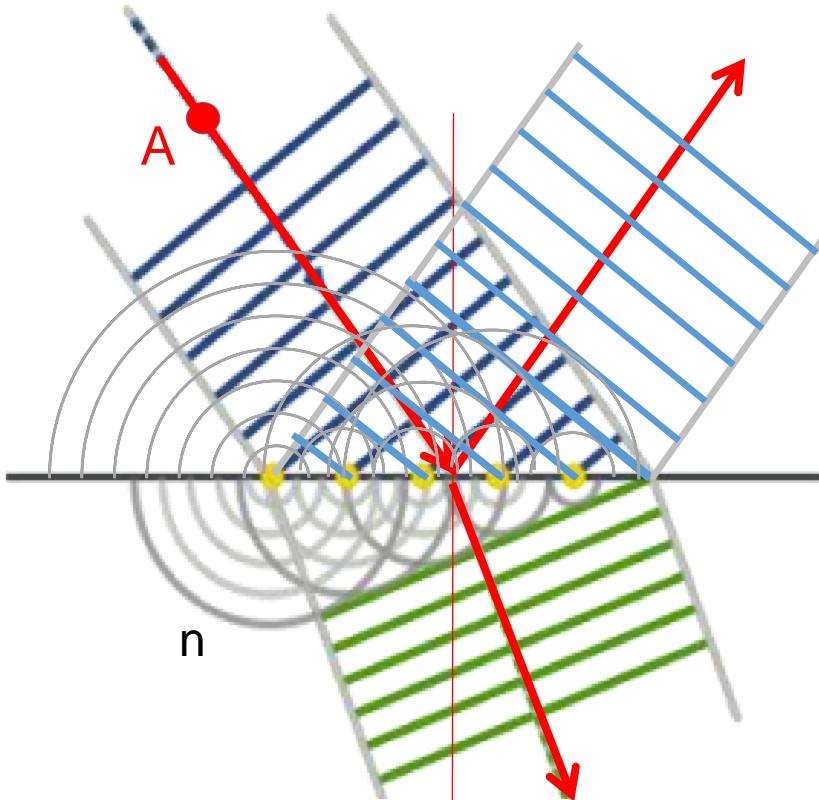
Refraction and Diffraction of Waves (Huygens, Fresnel)



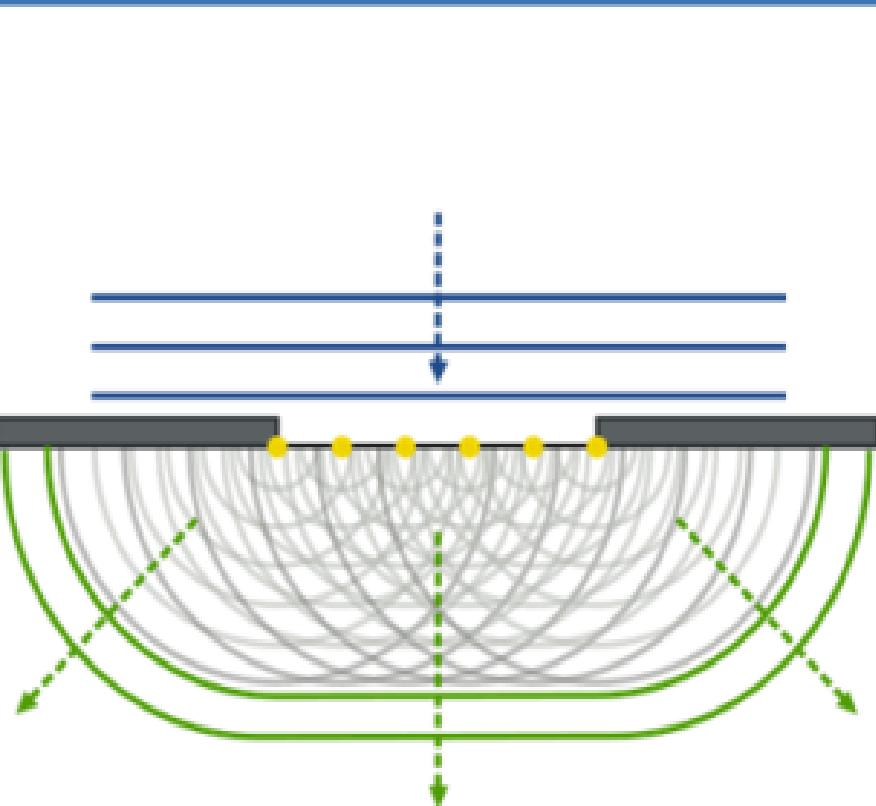
Law of Reflection: $i = -i'$

Law of reflection can be considered as special case of Snell's law.

Refraction, Reflection and Diffraction of Waves (Huygens, Fresnel)



Refraction
Reflection

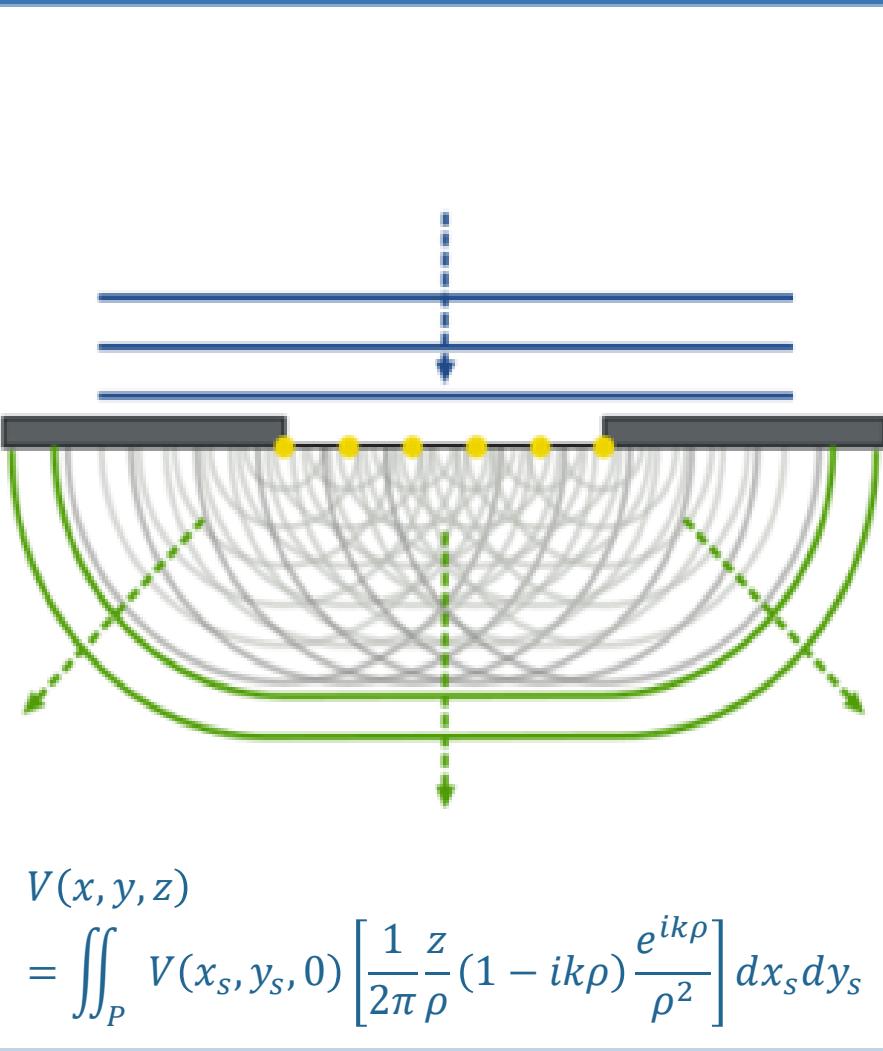
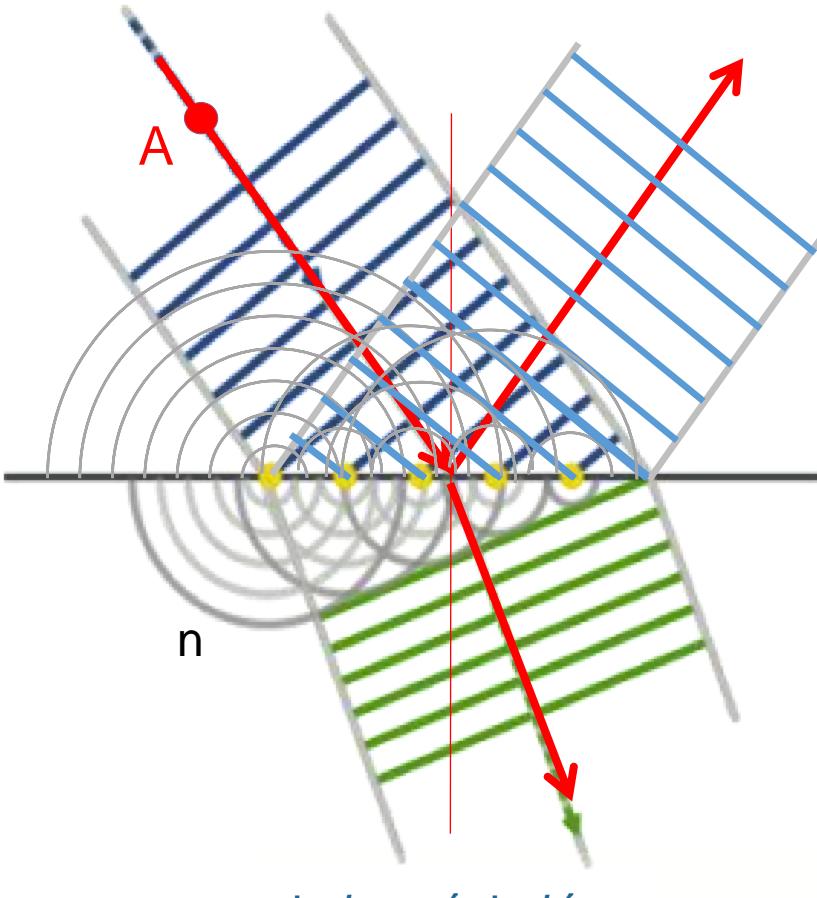


Diffraction

The same wave-model predicts that at boundaries, e.g., mechanical stops, light does not propagate along straight lines but is partly deflected along complicated distributions.

For this phenomenon called diffraction, there is no simple propagation law as for refraction and reflection.

Refraction, Reflection and Diffraction of Waves (Huygens, Fresnel)

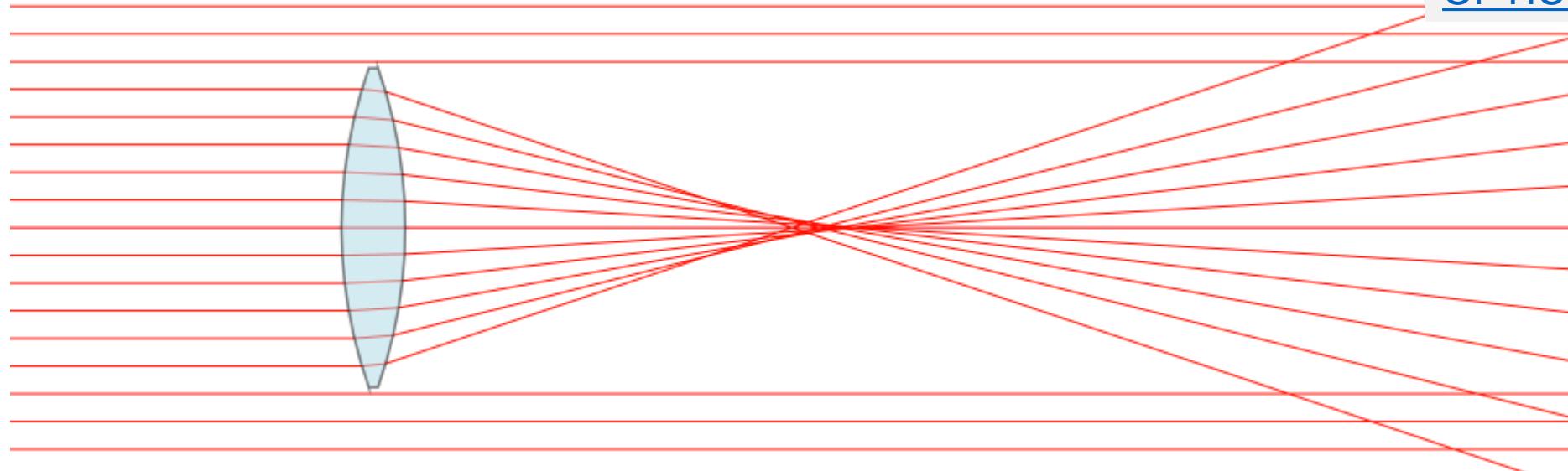


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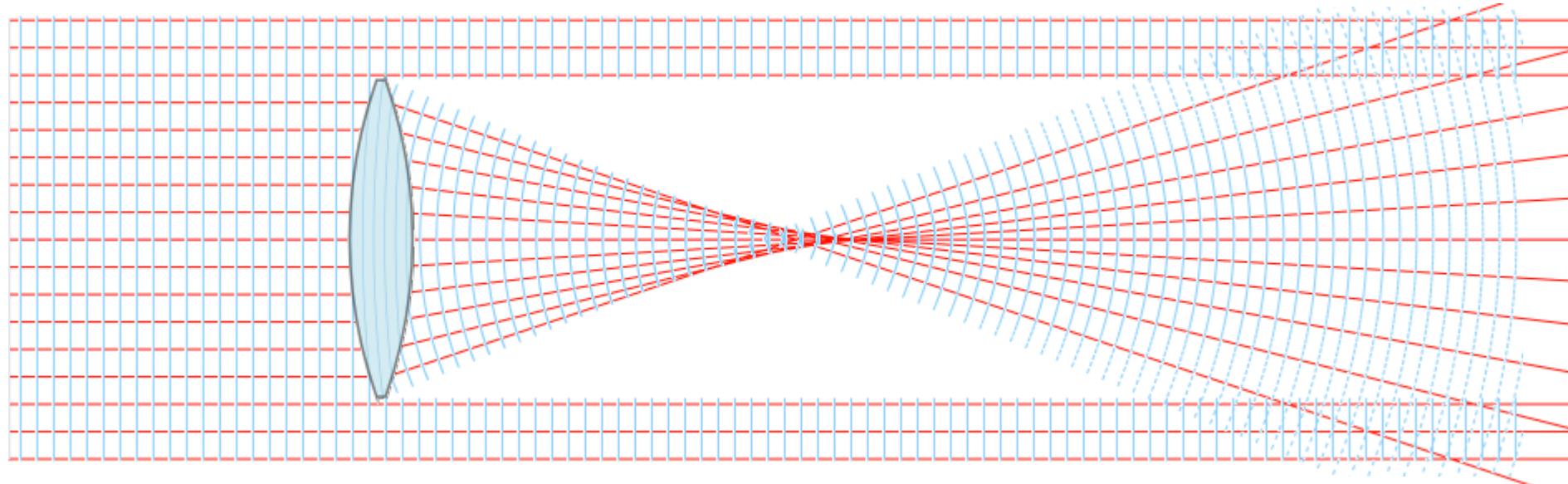
As for many imaging application diffraction is essential, a general imaging model requires (numerical) **integration**.

Raytracing through lens element



In Optical System Design we use the **convention** that light travels **from left to right**. However, the reverse light path is also true.

Light comes from “infinite distance”, that is the light beam is collimated. The portions of the beam which pass the lens converge to a focus and diverge behind the focus. Here the rays do not intersect perfectly in one point. This means that we have aberrations (the name of the aberration shown is “spherical aberration”).

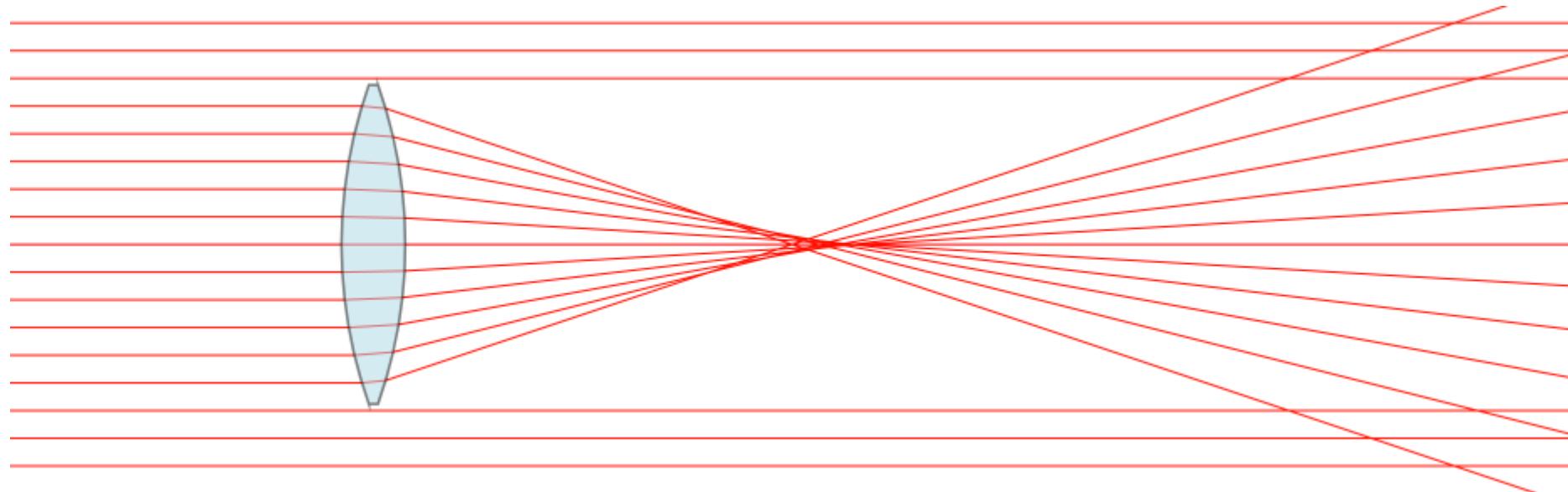


Rays are normal to the wavefront. In other words: Rays are local wavefront gradients.

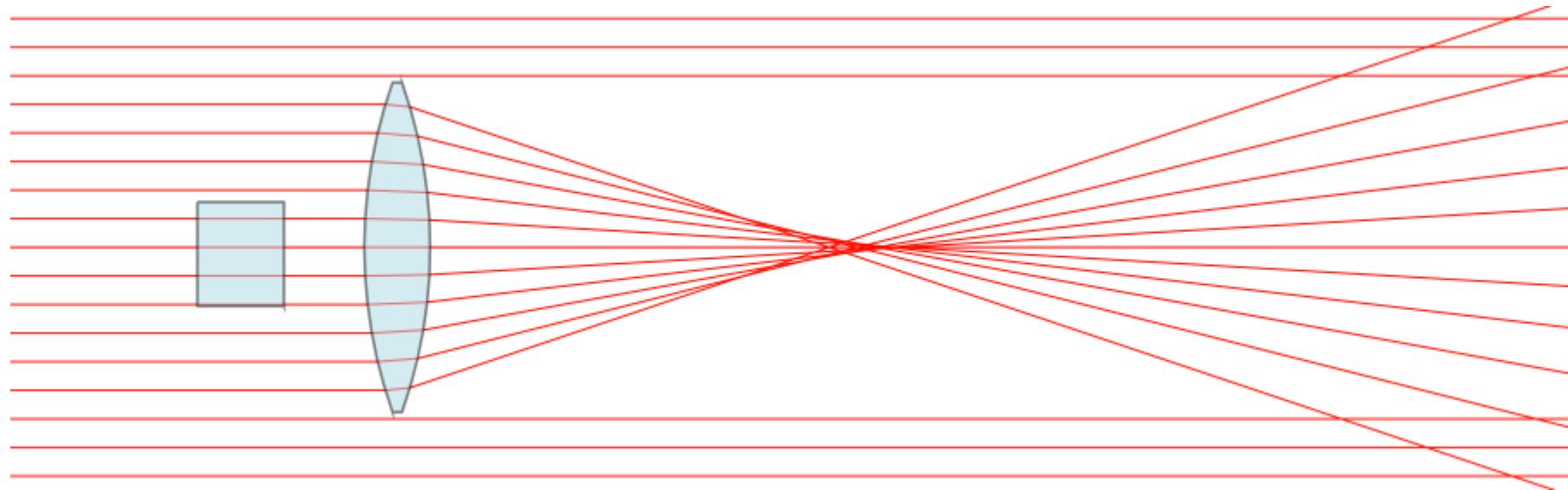
Collimated rays correspond to a plane wave, which is transformed by the lens to a spherical wave.

Due to the aberrations of the lens the converging lens is not perfectly spherical - its deviation to the perfect sphere around the lens focus is called wavefront deformation.

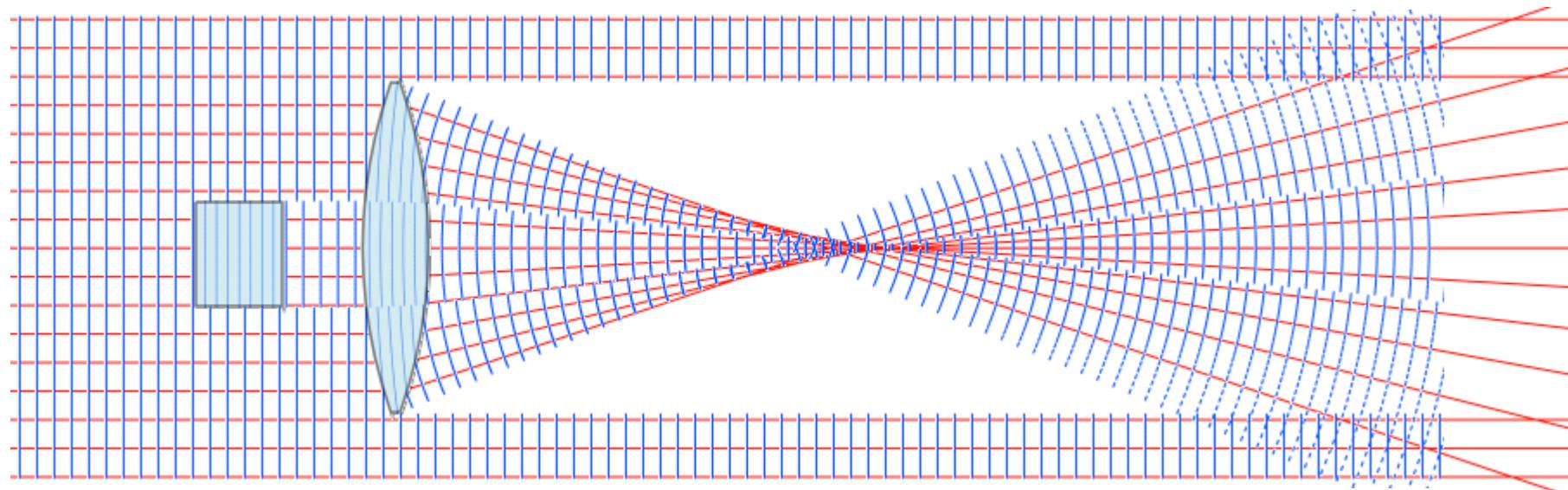
Raytracing through lens element



Raytracing through lens element



Now if we insert a small plane plate in front of the lens and look on the ray paths apparently nothing changes.

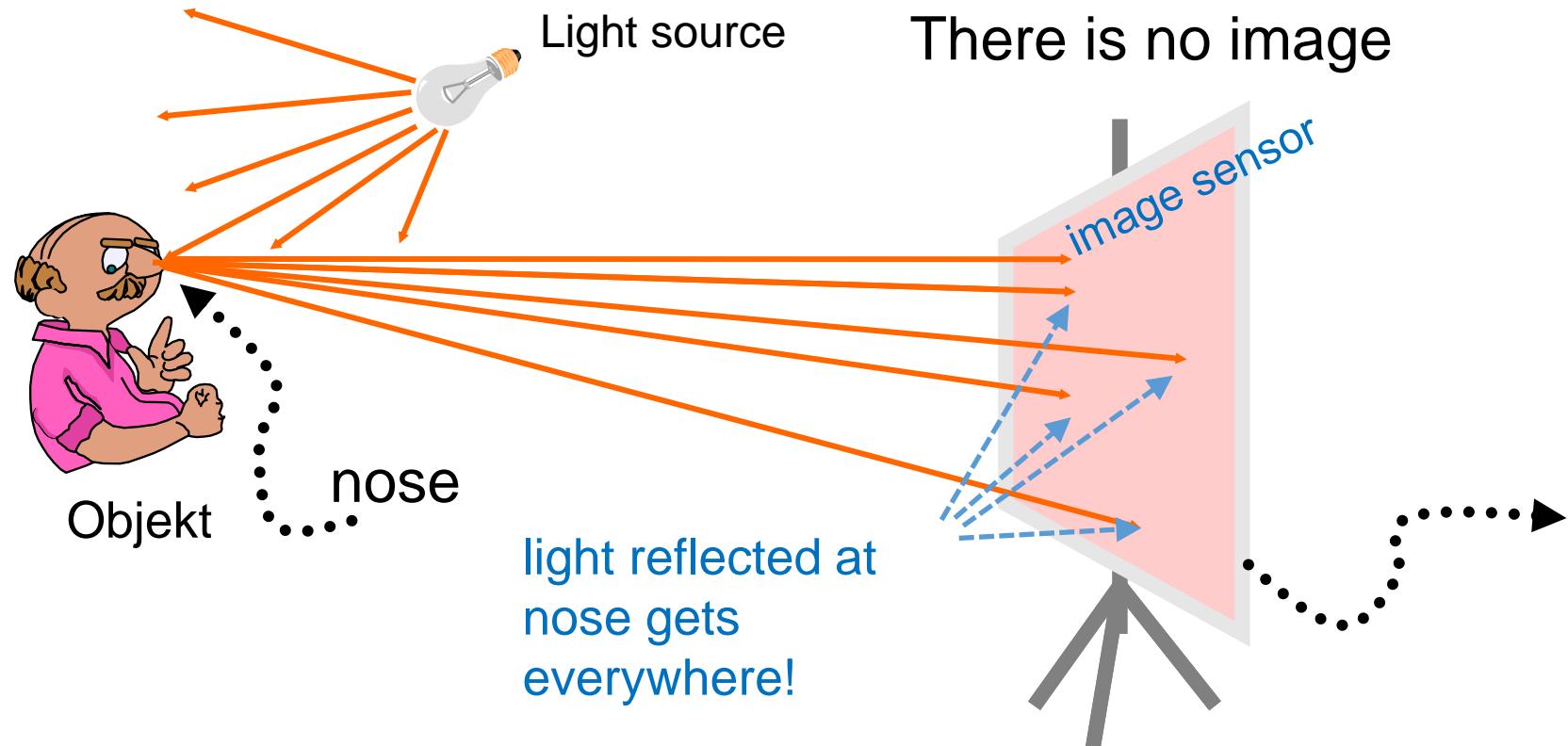


However, the wavefront portion passing the plane plate is delayed and therefore out of phase relative to the remaining converging wavefront behind the lens. This impacts the intensity distribution behind the lens.

In general, when we insert discontinuous objects into the system, we need to take the relative change of OPL into account (e.g., phase contrast microscopy).

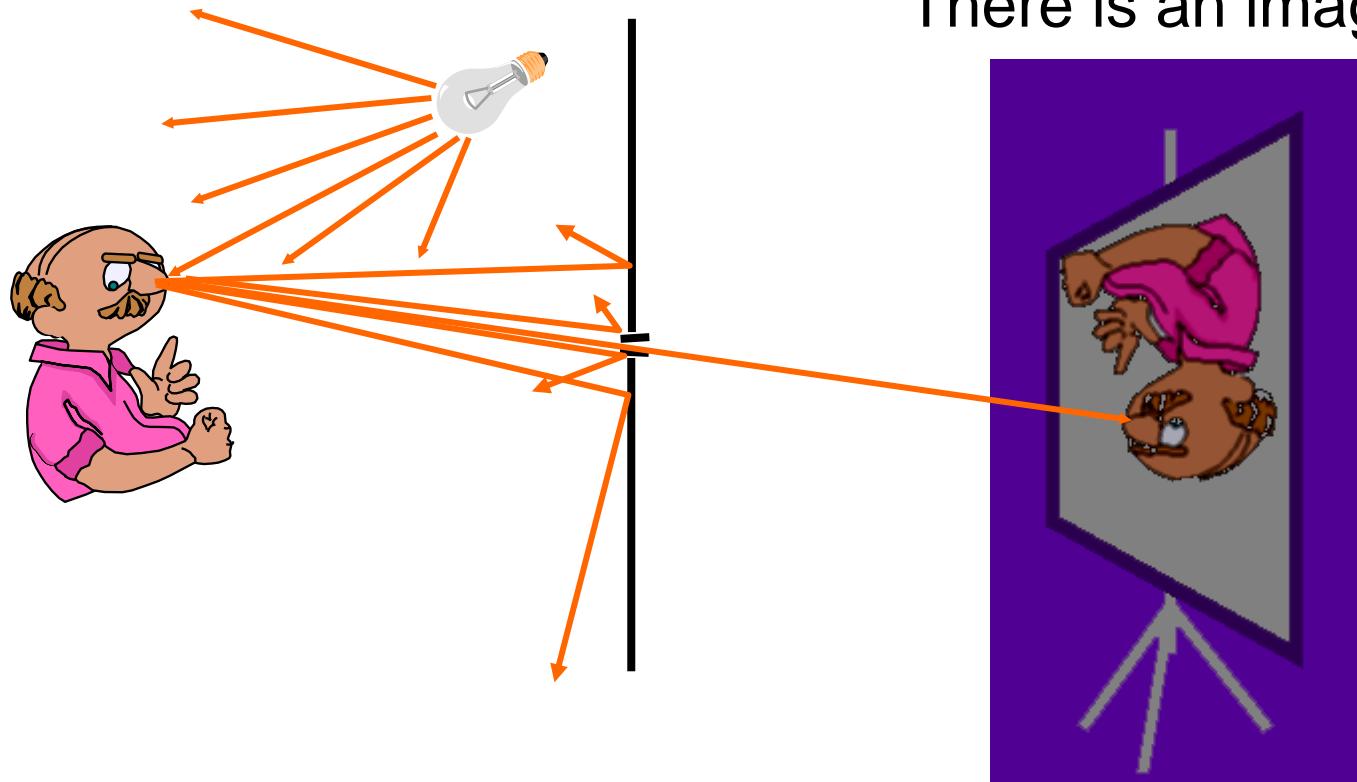
In many cases all optical elements of a system are continuous. Consequently, the wavefront in image space remains continuous, even close to focus. This fact to a good approximation justifies to look on ray aberration diagrams in lens design disregarding optical path differences.

Imaging problem: Imaging of an object point onto an image point



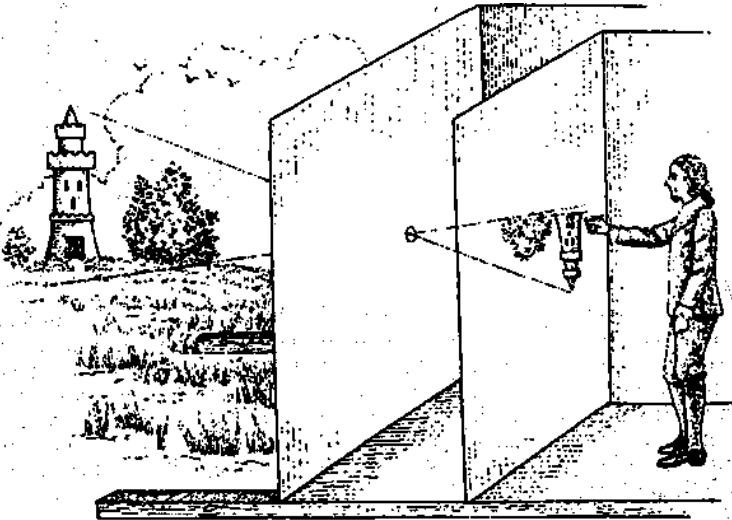
From every position of the object light goes everywhere
⇒ In the image plane the light from all those object points is superposed
⇒ Complete Blur, there is no image at all

Pinhole imaging



Pinhole creates an unique light path (“ray” along straight line).
On the image sensor, there is a unique object point for each pixel.

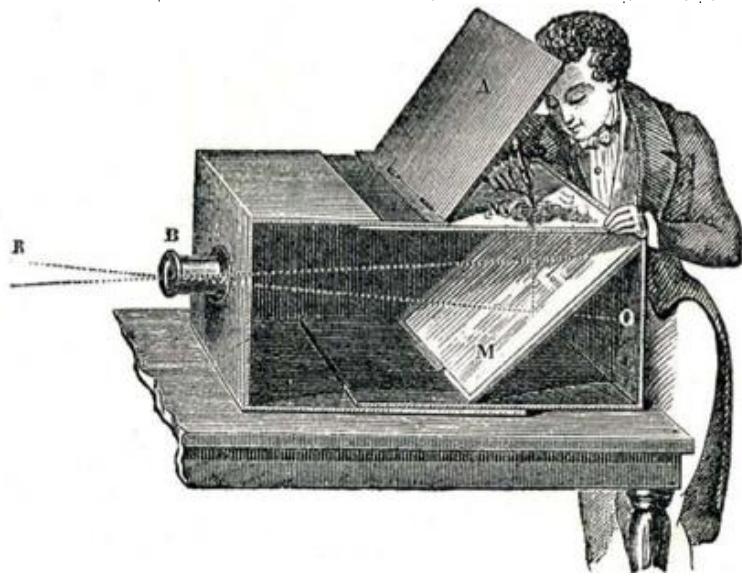
Camera obscura and the beginnings of photography



Used in year 1021 Ibn al-Haytham (arab scientist):

- required a small room
- pinhole or lens to project an image on the drawing screen

*camera obscura was probably already known by Aristoteles



In 1685 Johann Zahn reduced the size of the camera obscura to a portable box size

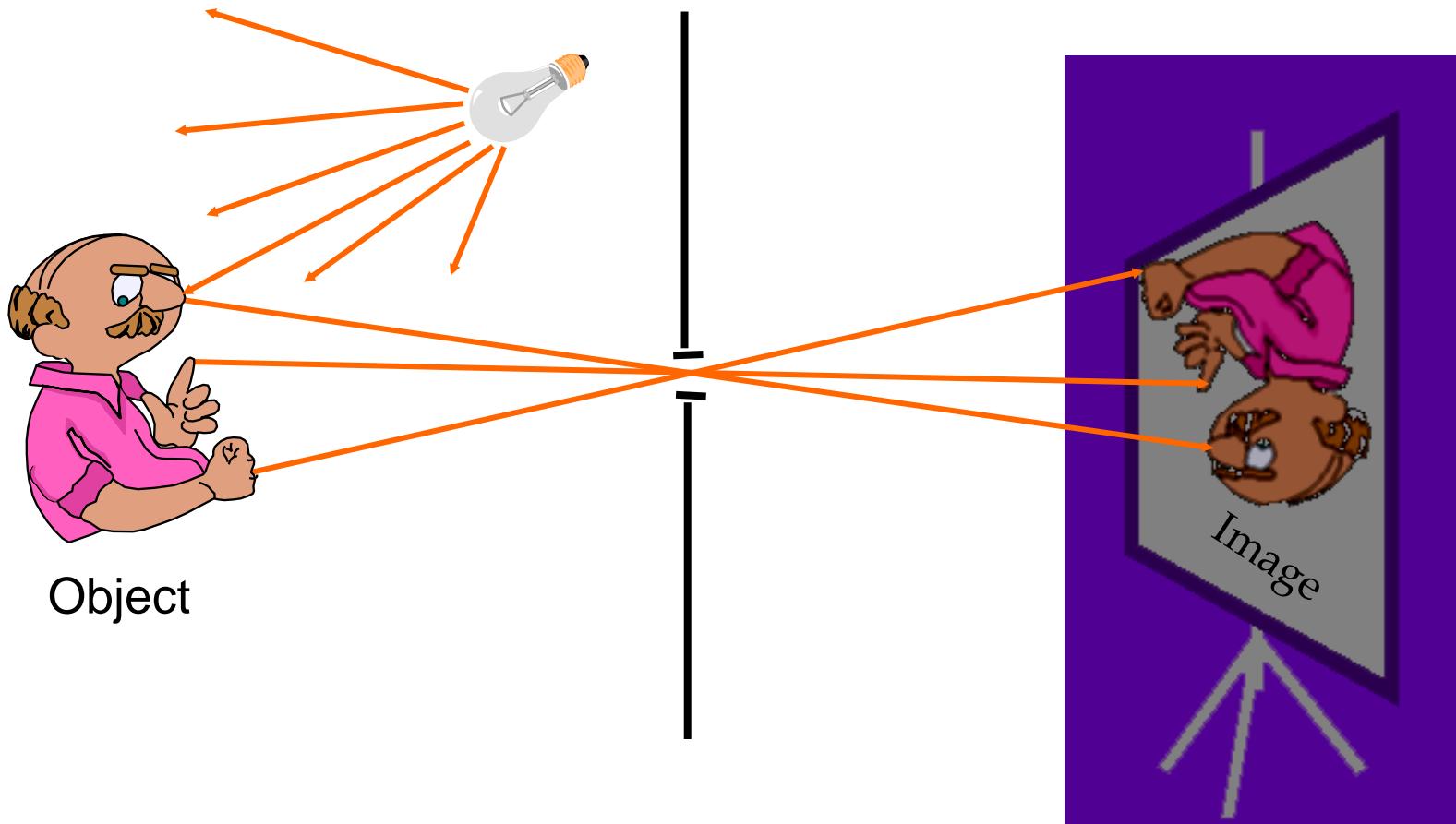


In 1838 Louis Daguerre made capturing images on film practicable.

Pinhole images

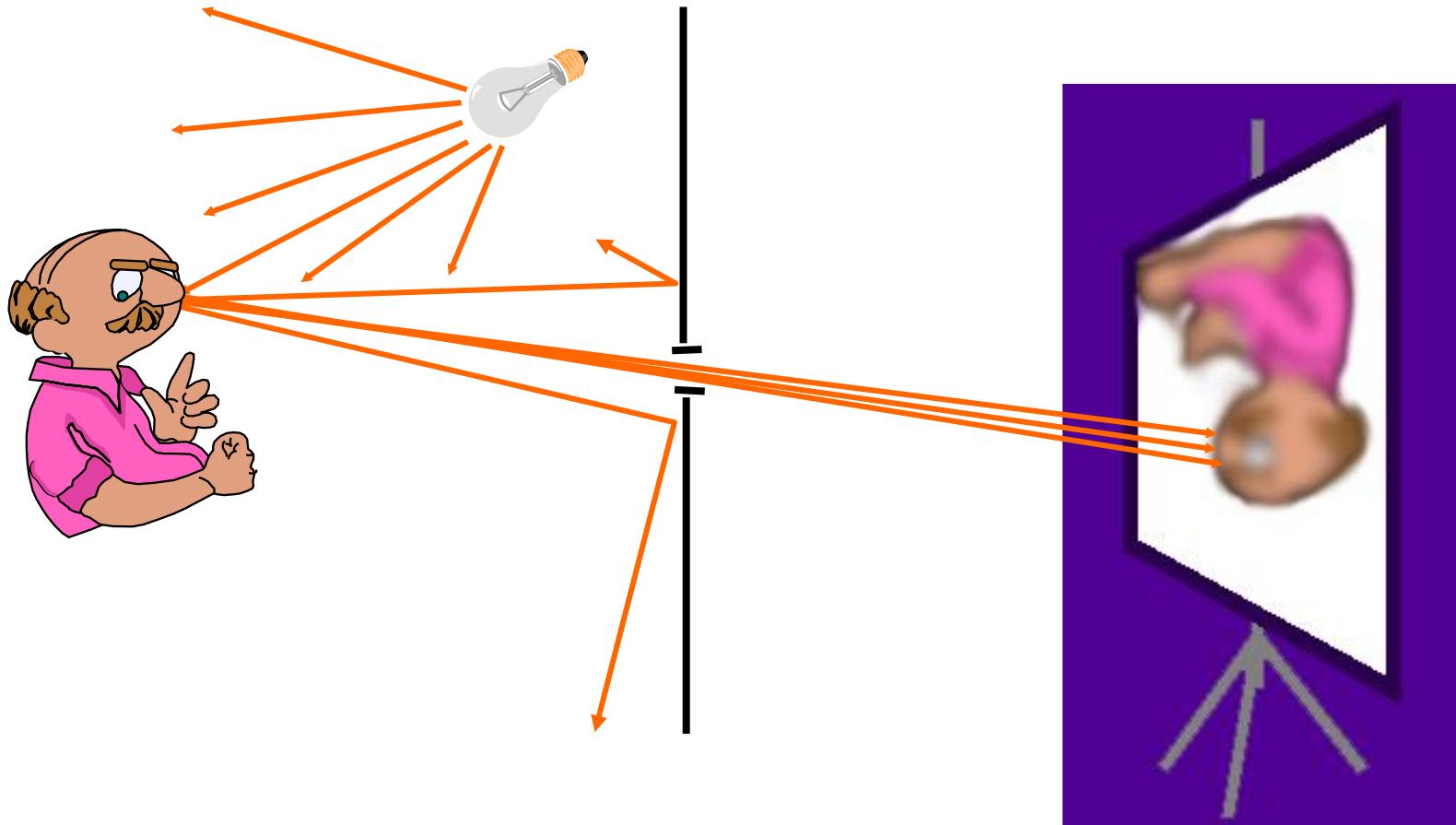


Imaging with a pinhole



But: Image is dark

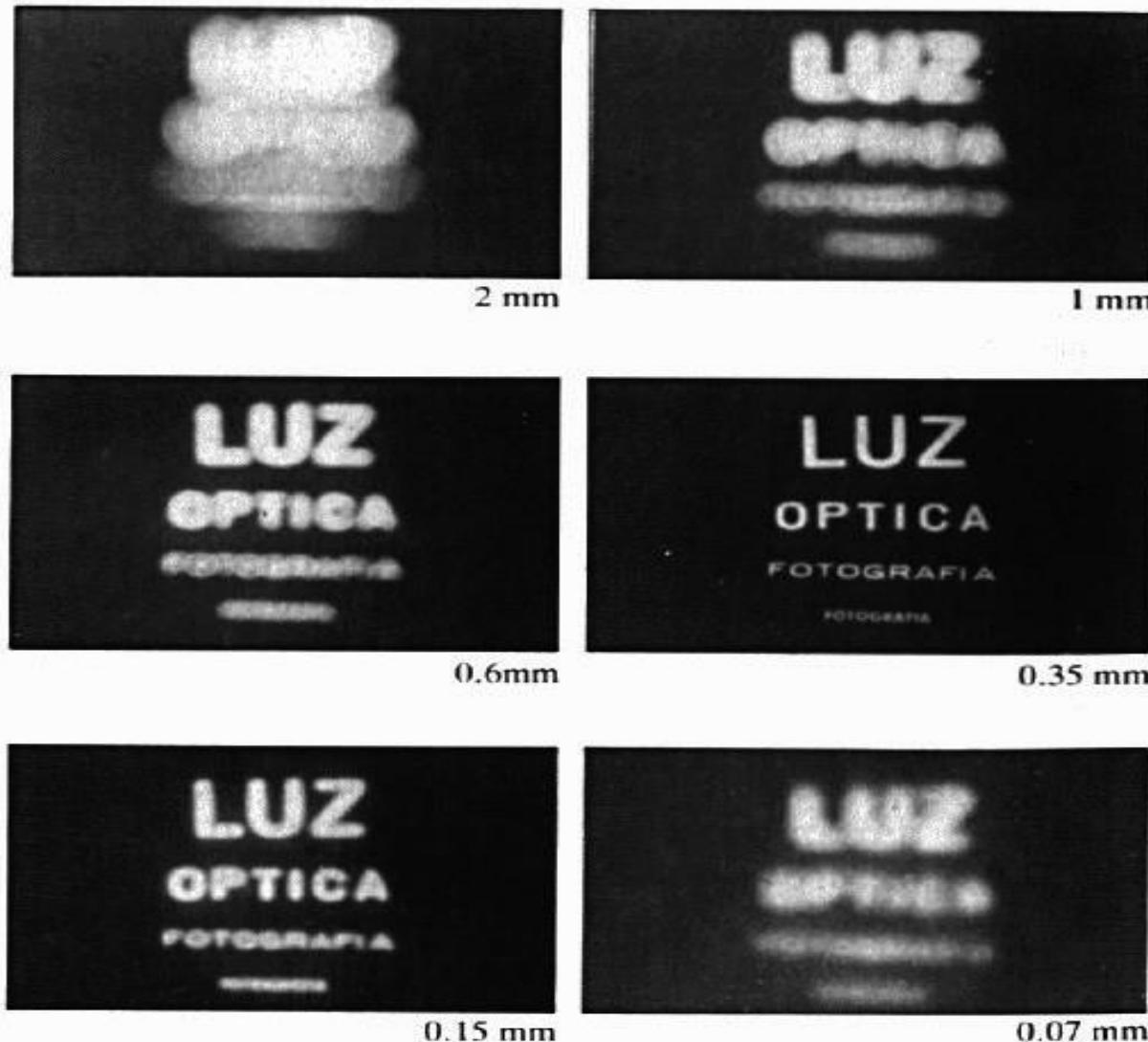
Imaging with a pinhole: More light with larger pinhole



The larger the aperture, the
brighter the image

But there is more blur in the image

Pinhole camera: Sharpness vs. Pinhole diameter

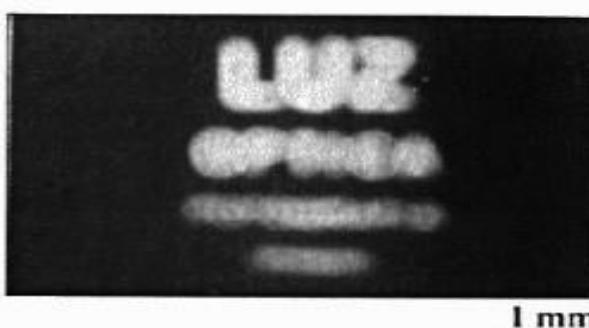


Pinhole camera: Sharpness vs. Pinhole diameter

„geometrical blur“ →



2 mm



1 mm



0.6 mm



0.35 mm

Optimal size for visible light:
 $\sqrt{f}/28$ (in millimeters) where f is “focal length” (here: distance pinhole - image)

optimum



0.15 mm



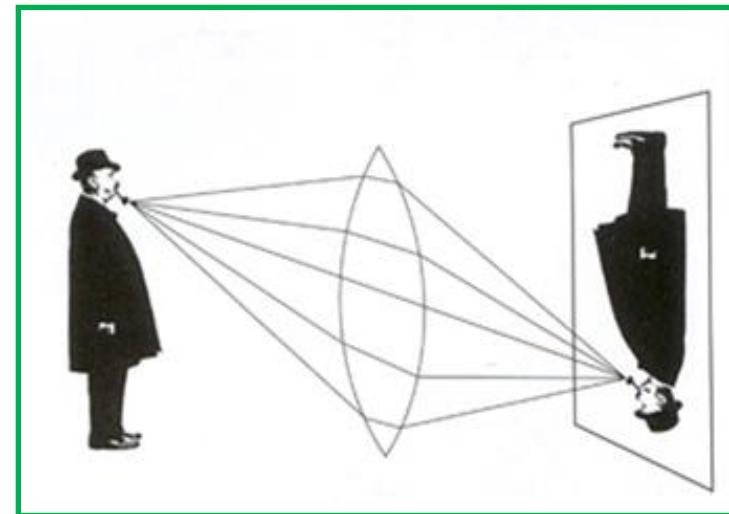
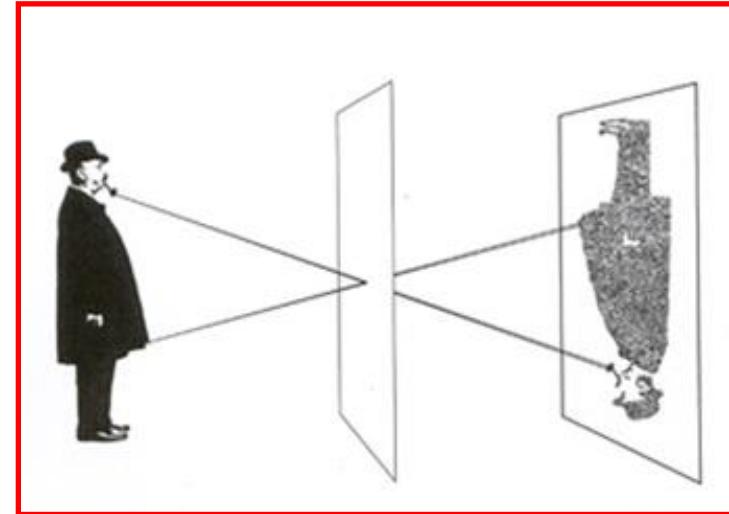
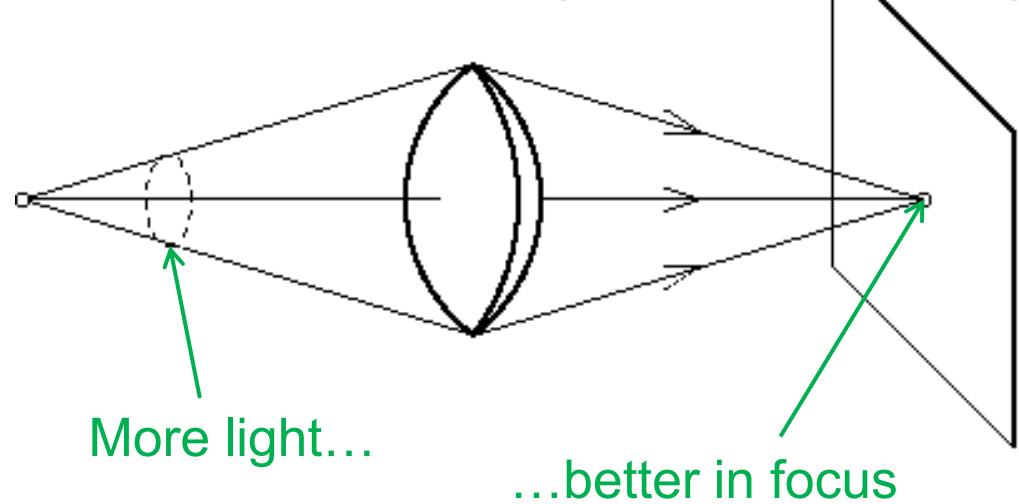
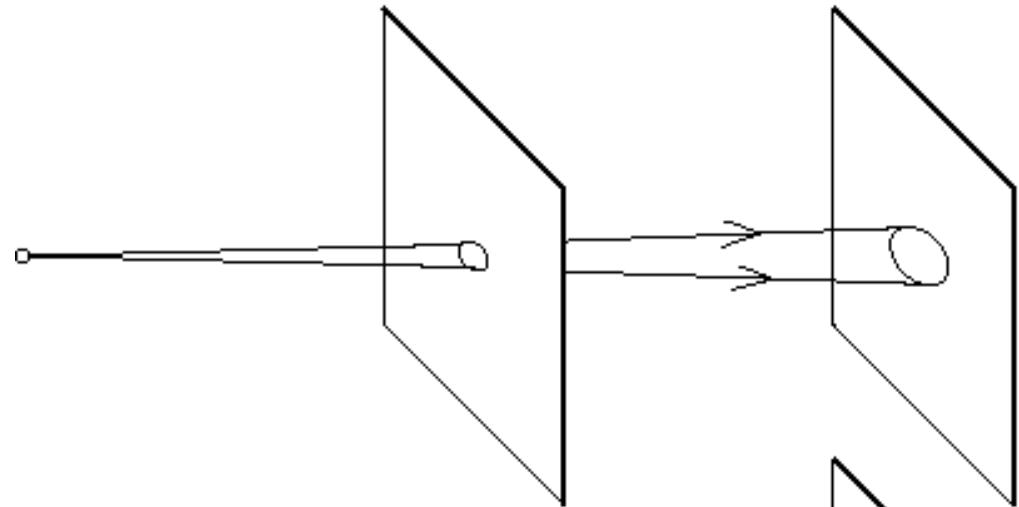
0.07 mm

Irradiance decreases with pinhole diameter:

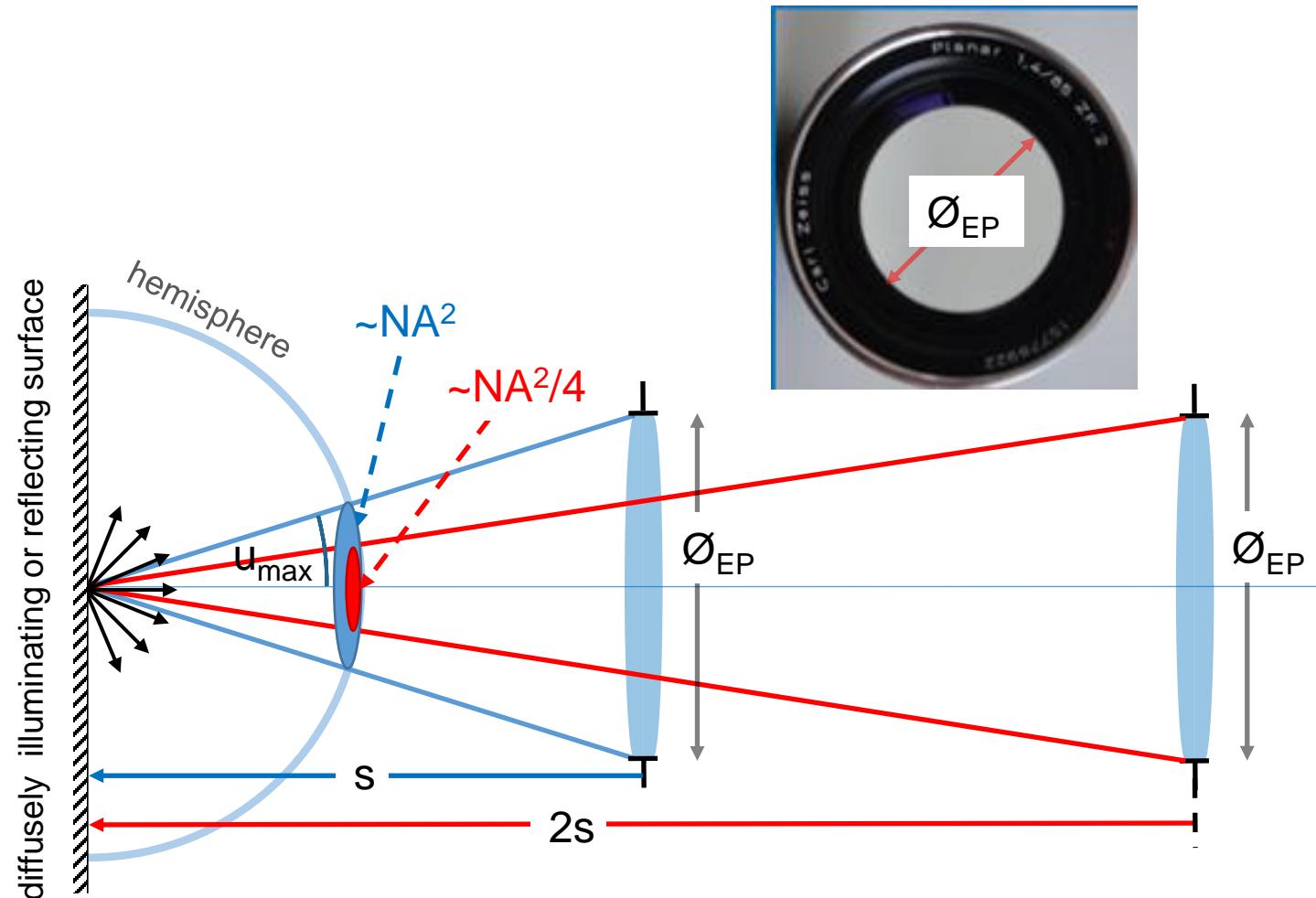
$$E' \sim 1/\varnothing_{loch}^2$$

„diffraction-limited“ ←

Replacement of pinhole by optical system for etendue



Radiant power entering an optical system (pupil contribution)



The radiant power entering the optical system is proportional to the **solid angle Ω** , given by the ratio of the entrance pupil area to the square of the distance from the object to the **entrance pupil**, i.e.

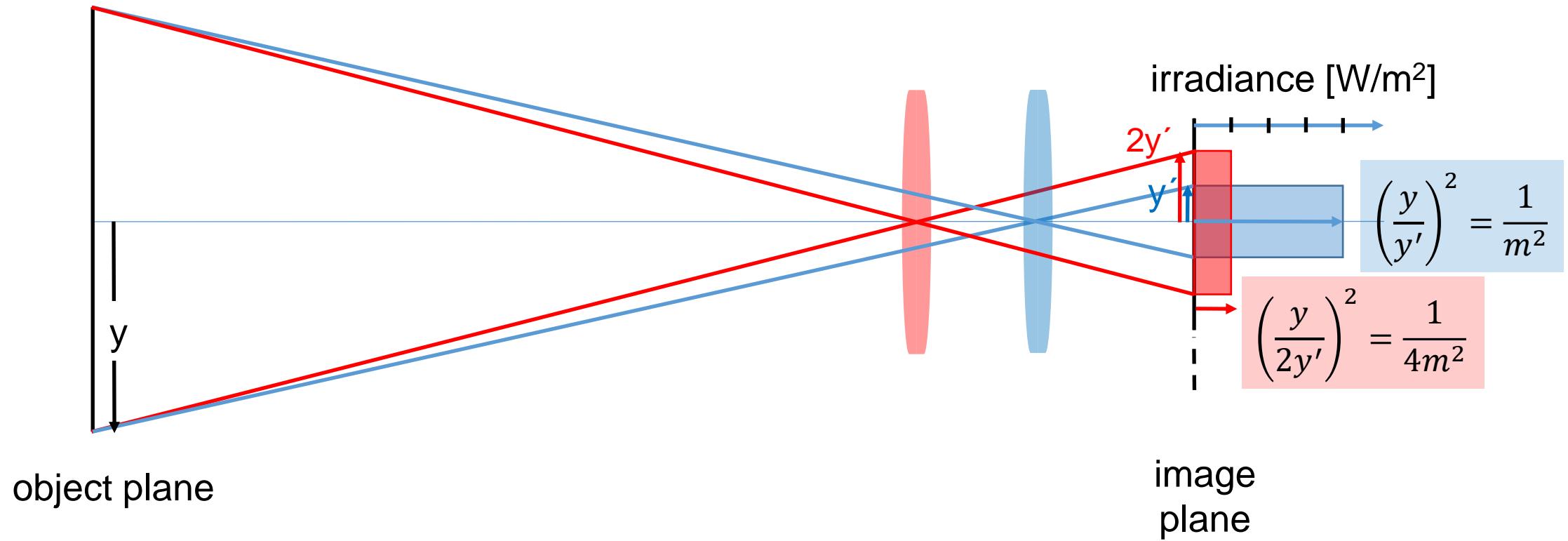
$$\Omega = \pi \frac{\varnothing_{EP}^2}{s^2} = \pi NA^2$$

Numerical aperture **NA**

$$NA = n \sin u_{max}$$

Field transfer of radiant power

Regarding the radiant power transferred via the field the power per surface element, or **irradiance** [W/m²] scales with the relative size of the transferred field area, that is $\frac{dF'}{dF} = \frac{dy'^2}{dy^2} = m^2$

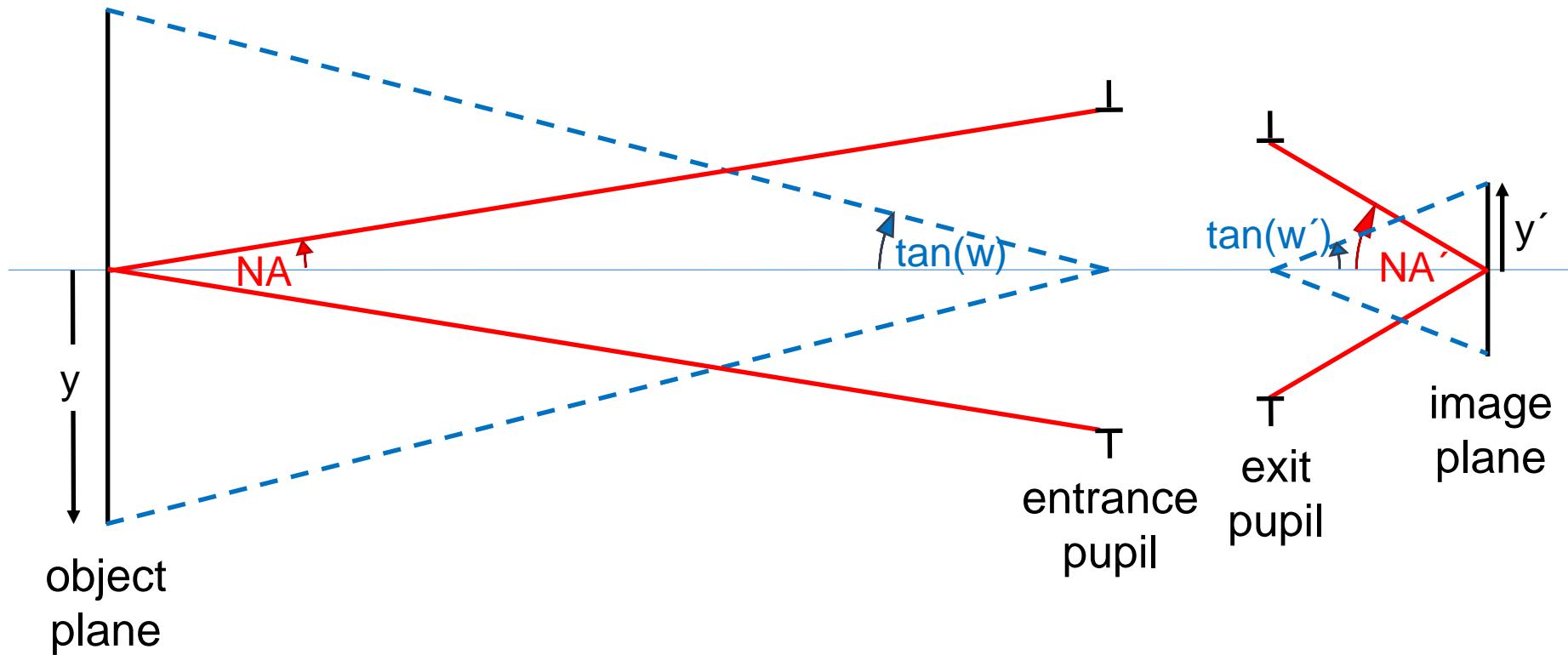


Combining these dependencies of radiant transfer via pupil and via field we find that the product of (local) field area multiplied with numerical aperture squared (proportional to solid angle) is conserved. This quantity

$$y^2 NA^2 = y'^2 NA'^2$$

Is called **étendue, throughput or AΩ-product**.

Formulated by Rudolf Clausius in 1864 in his famous Fundamentals of Thermodynamics.

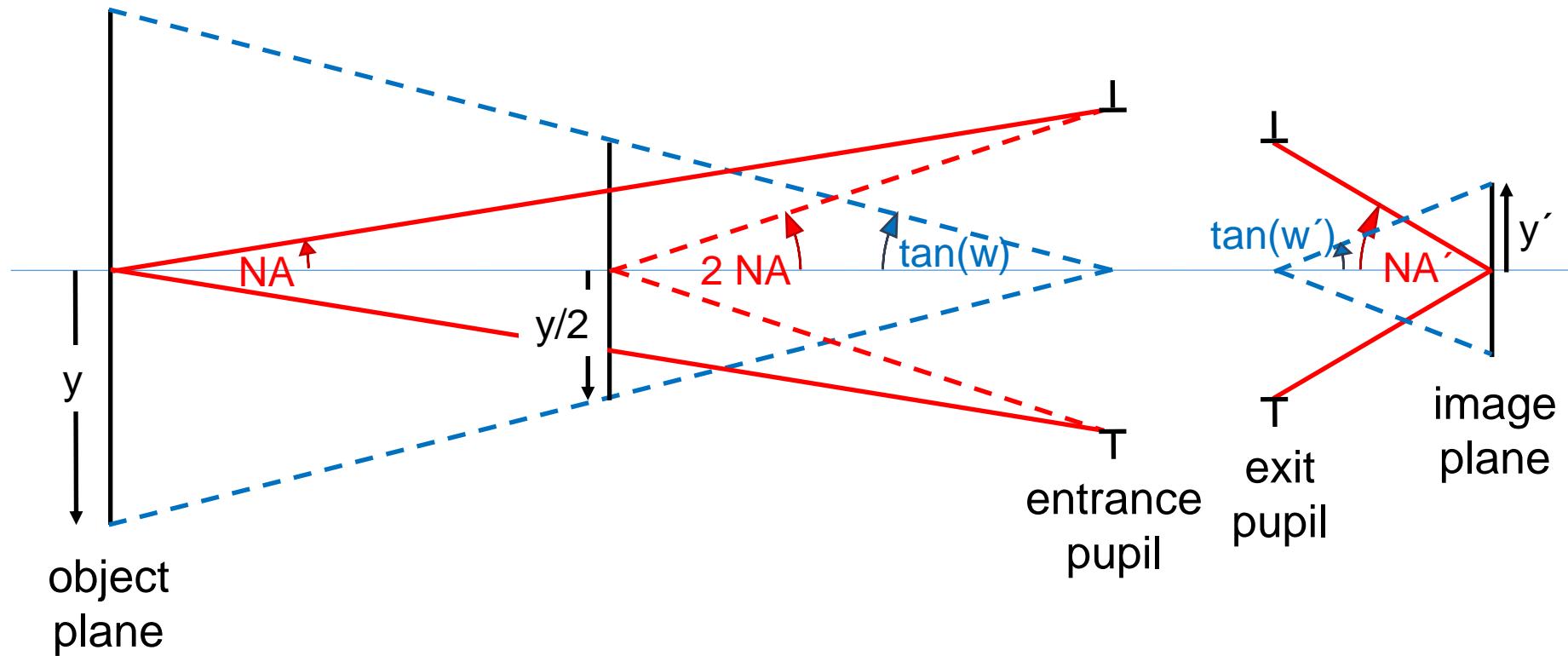


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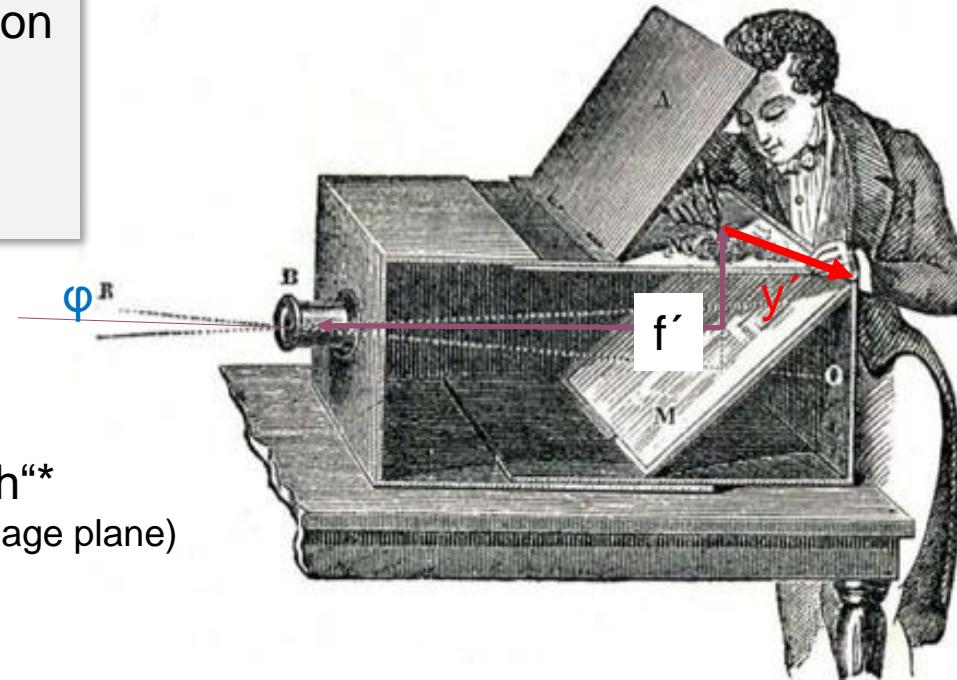
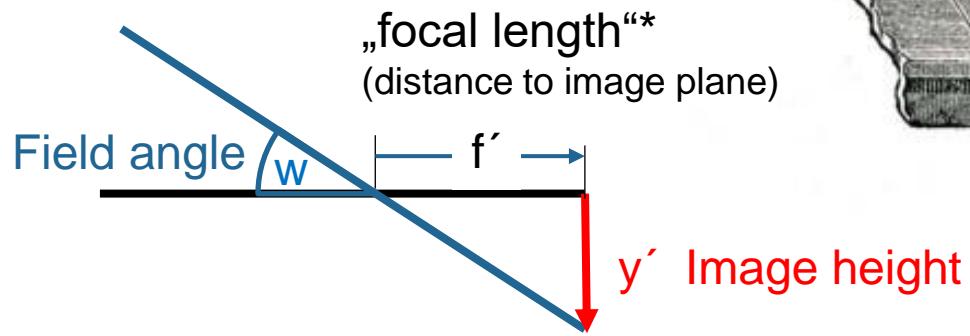
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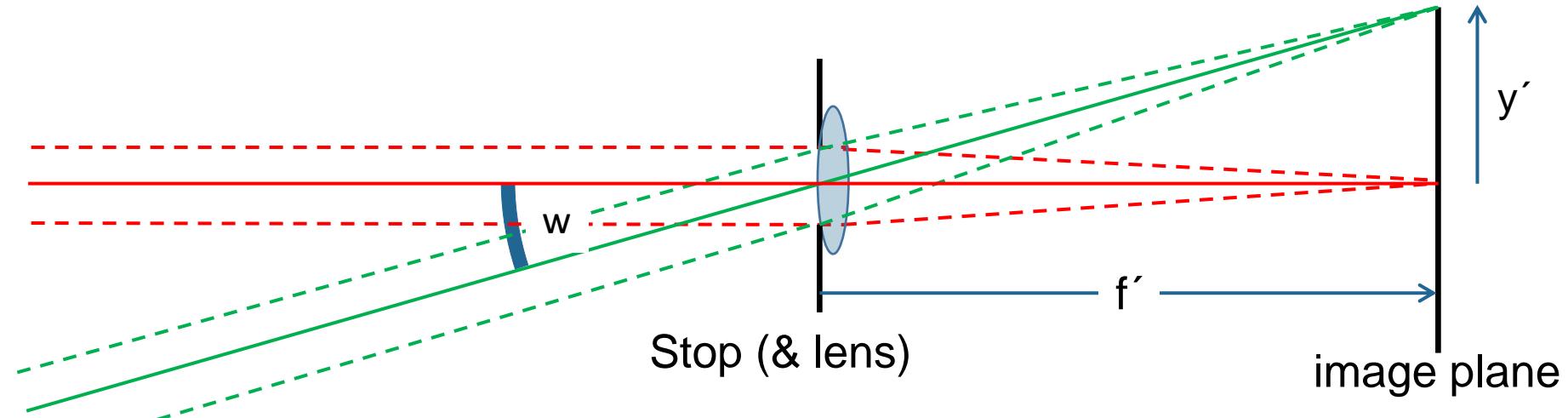
Rectilinear imaging with camera obscura

Rectilinear (gnomonic) Projection

$$\tan w = \frac{y'}{f'}$$



*: If a lens is used instead of the hole, this must have the focal length f for the image from infinity, so that the image is sharp in the image plane.



Focal length f' has two meanings:

1. It is the distance from the image plane where a parallel beam entering the optical system is focused onto the image plane
2. Focal length determines (together with image sensor size) the field of view (the angular range which enters the lens)

f' : focal length

y' : image height

w : field angle object

$$\tan w = \frac{y'}{f'}$$

$2 \cdot w = \text{diagonal field of view} = \text{diagFOV}_{\max}$

Landscape Lens, Wollaston 1812

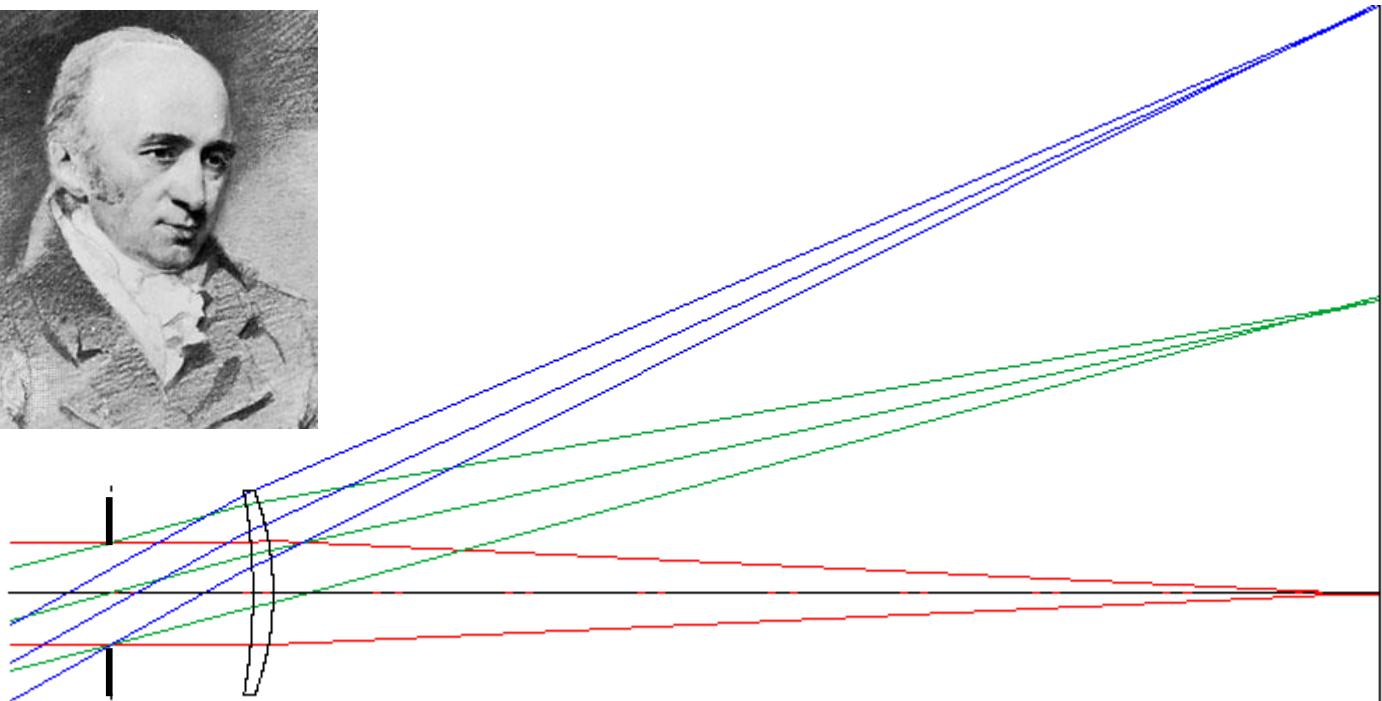
XIX. *On a Periscopic Camera Obscura and Microscope.* By
William Hyde Wollaston, M.D. Sec. R. S.

Read June 11, 1812.

ALTHOUGH the views, which I originally had of the advantage to be derived from the periscopic construction of spectacles,* naturally suggested to me a corresponding improvement in the *camera obscura*, by substituting a meniscus for the double convex lens, I have hitherto deferred making it known to others, except as a subject of occasional conversation.

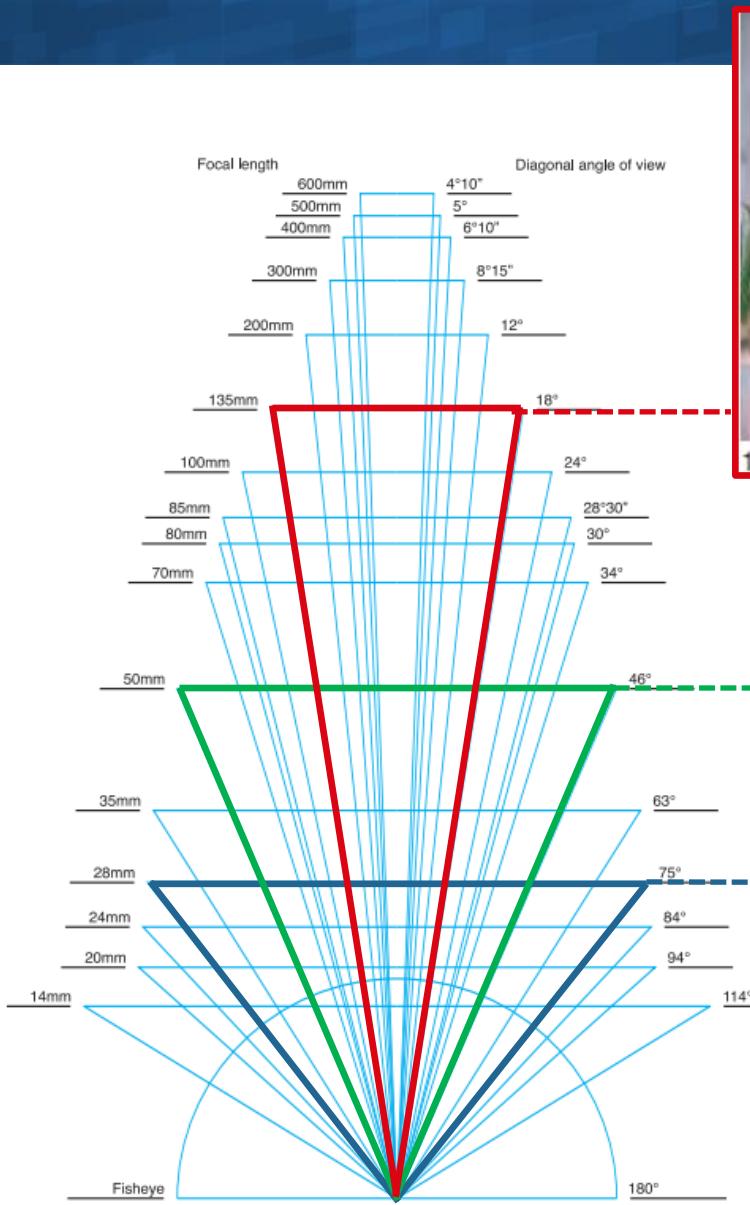
Since in vision with spectacles, as in common vision the

Philosophical Transactions of
the Royal Society of London,
Vol. 102 (1812), pp. 370-377



- image diagonal 580mm, focal length 548mm
- f/11, FOV 60° (full diagonal)
- meniscus-shaped lens („thin“) with curvature ratio 1:2 ($r_1=-288\text{mm}$, $r_2=-144\text{mm}$)
- stop position ca. 1/8 of focal length in front of lens
- glass ca. $n_d = 1.5028$, $v_d = 60.57$ (former „K6“ assumed)
- lens diameter (field stop) twice as large as aperture stop ($\varnothing_{\text{stop}}=50.8\text{mm}$, $\varnothing_{\text{lens}}=101.6\text{mm}$)

Focal length and field-of-view



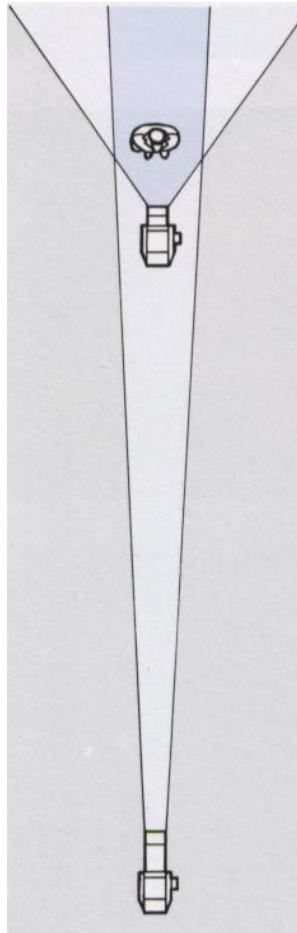
$$\tan w = \frac{y'}{f'}$$

$$2 \cdot w_{max} = \text{diagFOV}_{\max}$$

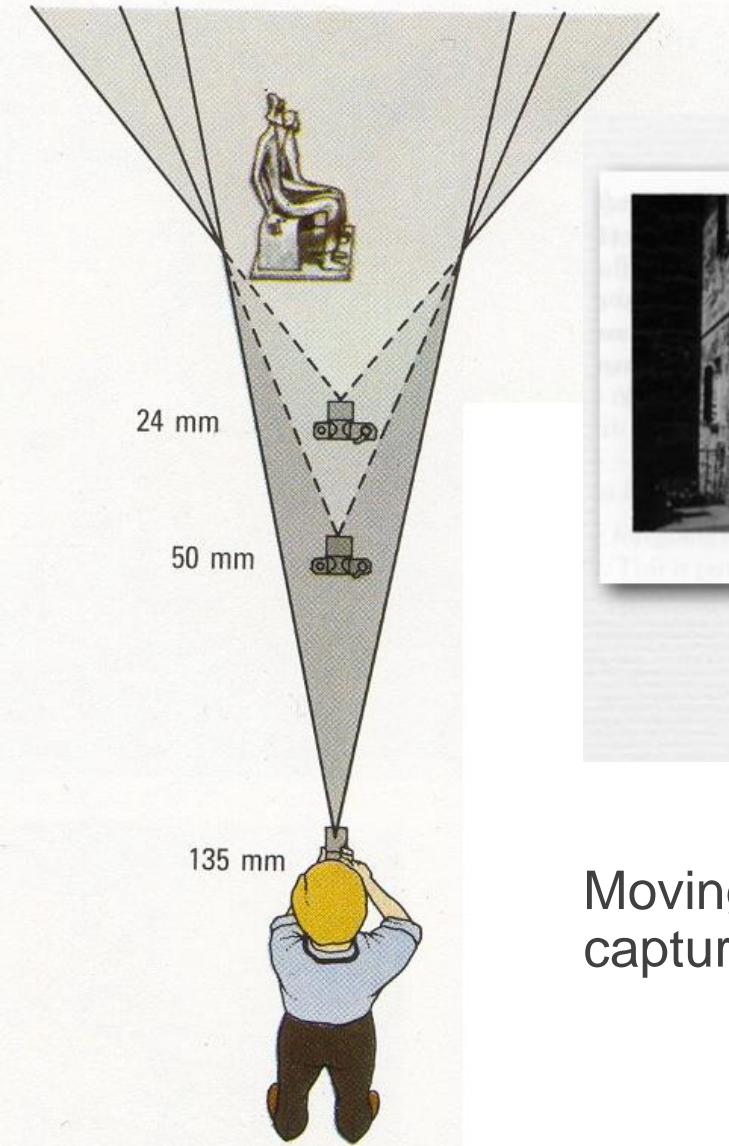
FOV vs Viewpoint

Focal length NOT ONLY changes subject size

Same size by moving the viewpoint, but different FOV (i.e., background)



Perspective and focal length



wide-angle

telephoto and
moved back

Moving the viewpoint correspondingly that the subject size is kept constant, the captured background is larger with increasing FOV.

Field-of-view and viewpoint



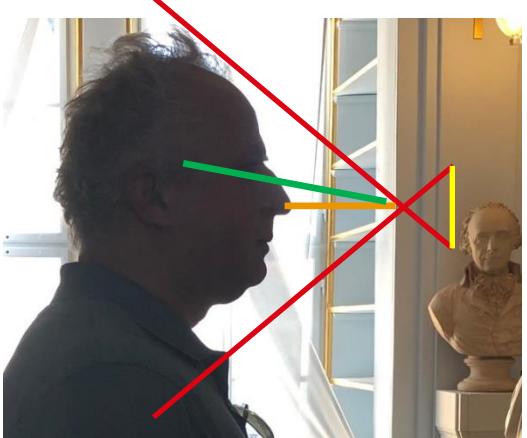
Wide angle



Standard



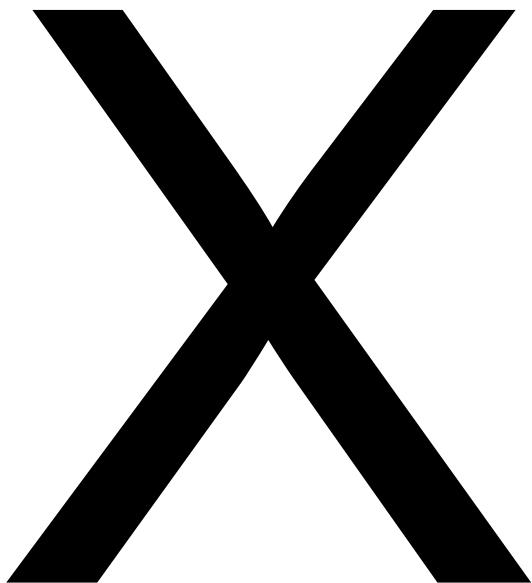
Telephoto



To image the same object frame, with a wide-angle system you must step closer to the object. Consequently, relative distances in depth are much larger compared to tele (right). Therefore, also relative magnifications are different in depths leading to perspective deformations.

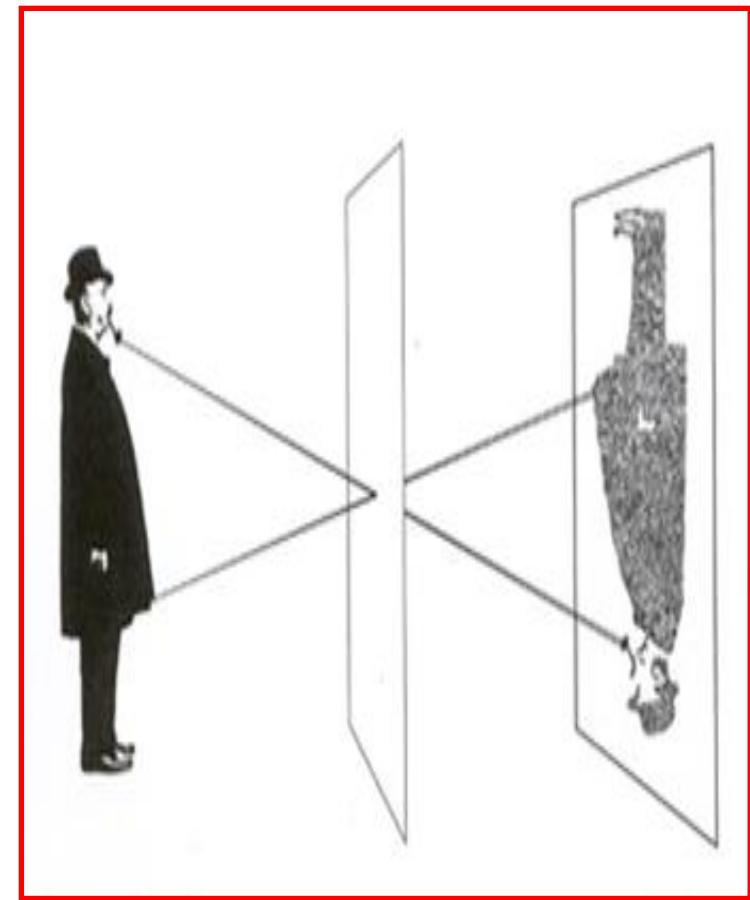


Which letter in the alphabet represents best the basic principle of optical image formation?



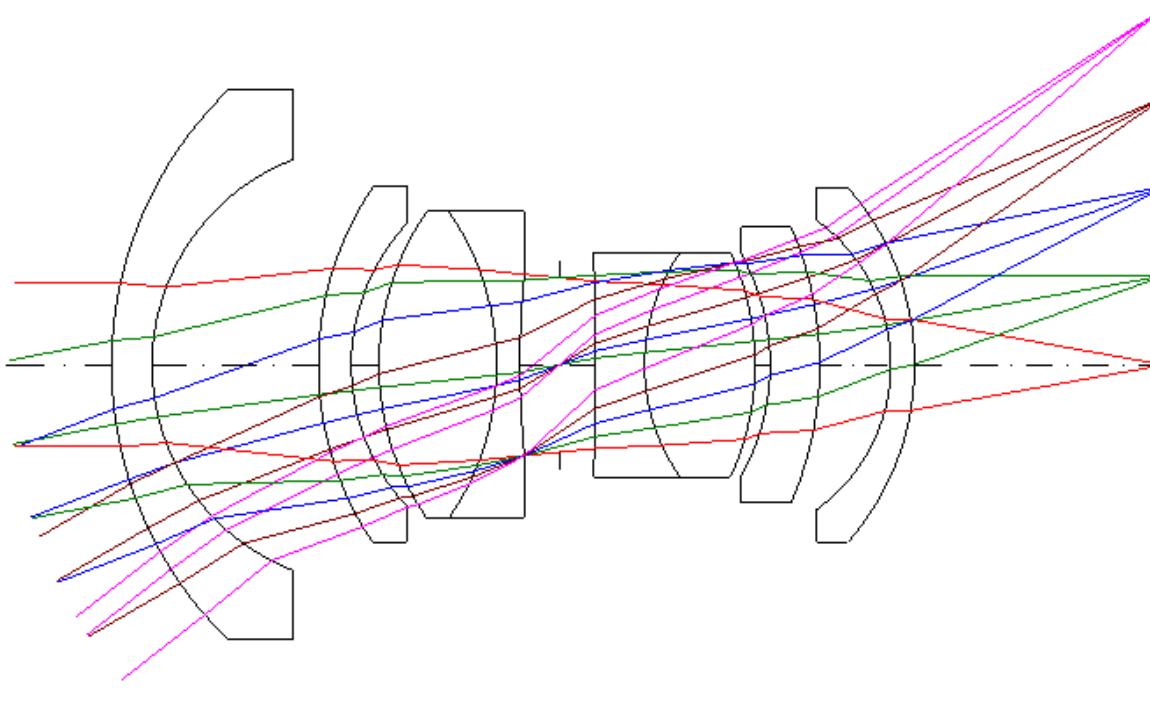
Optical Imaging

Optical Imaging

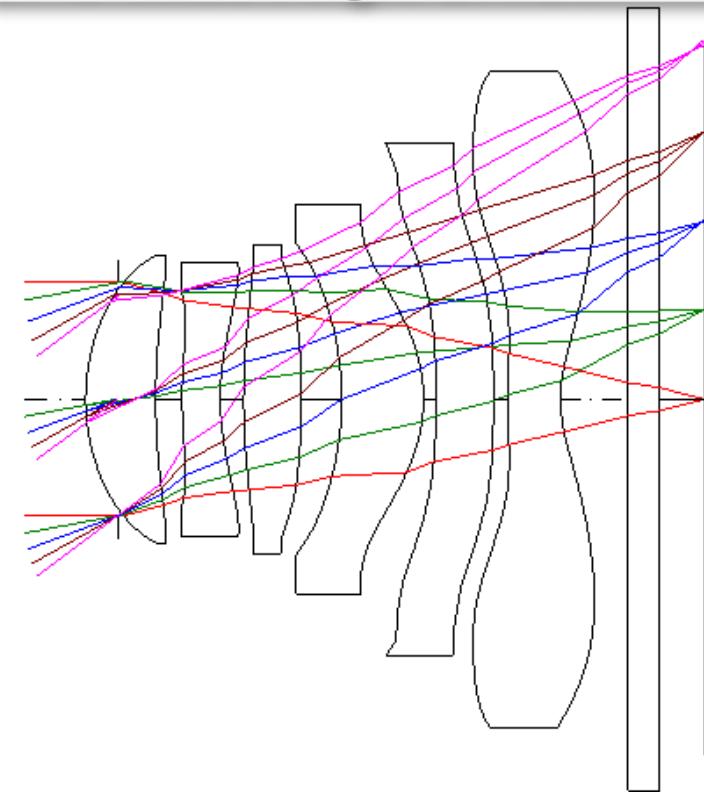


Compact camera lenses: Classic Biogon and modern Smartphone Lens

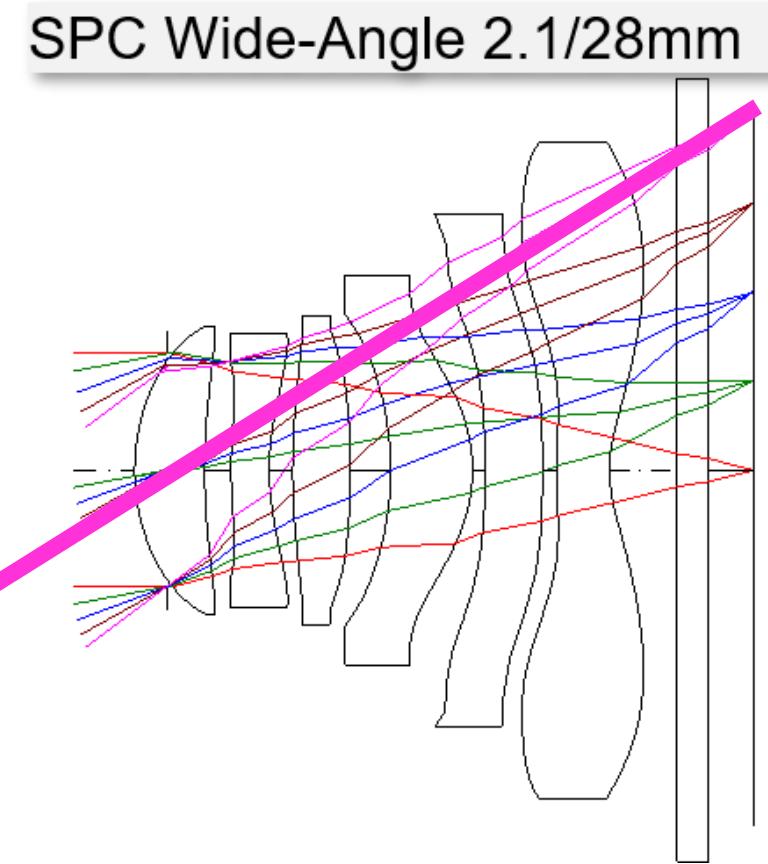
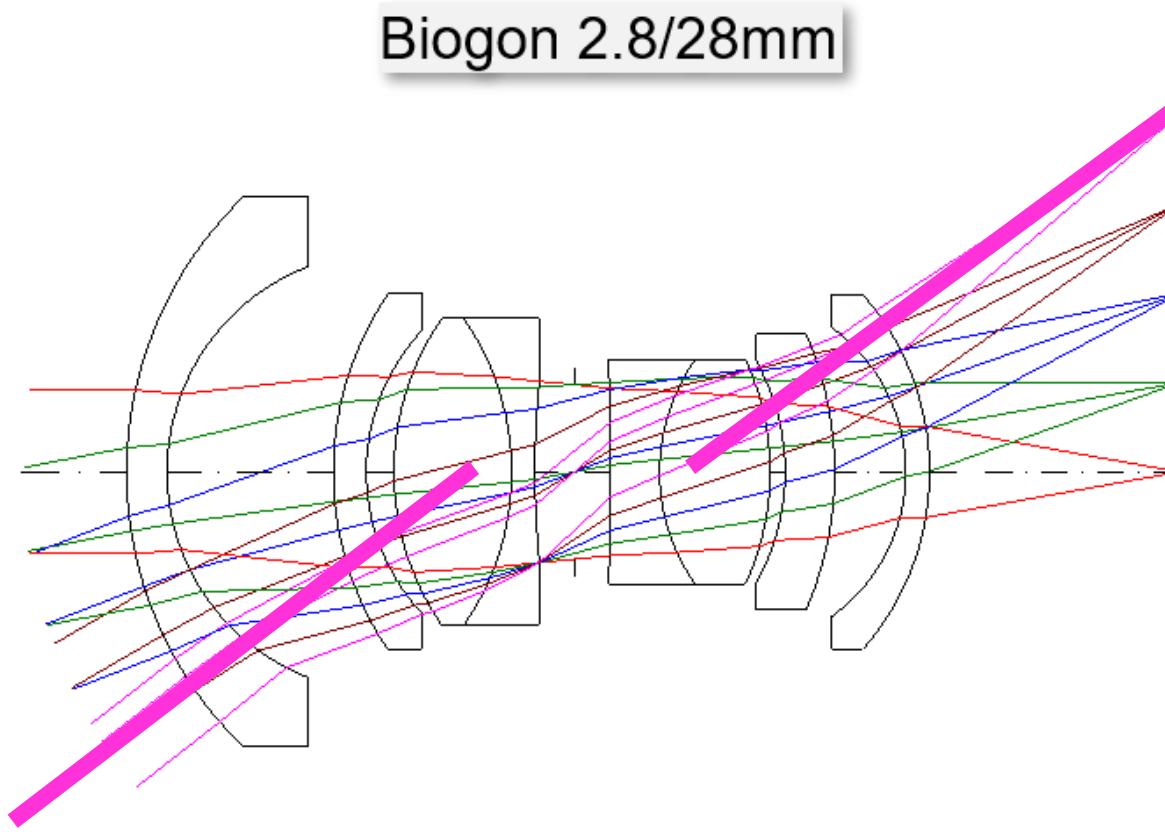
Biogon 2.8/28mm



SPC Wide-Angle 2.1/28mm

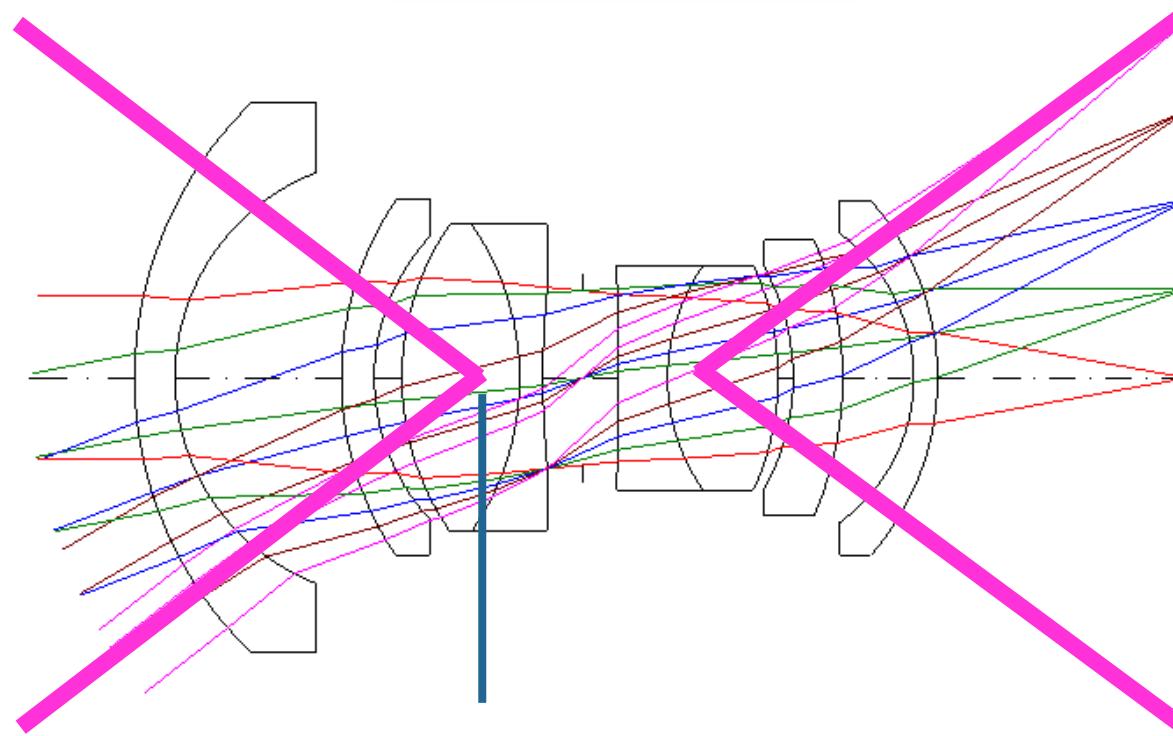


Compact camera lenses: Classic Biogon and modern Smartphone Lens

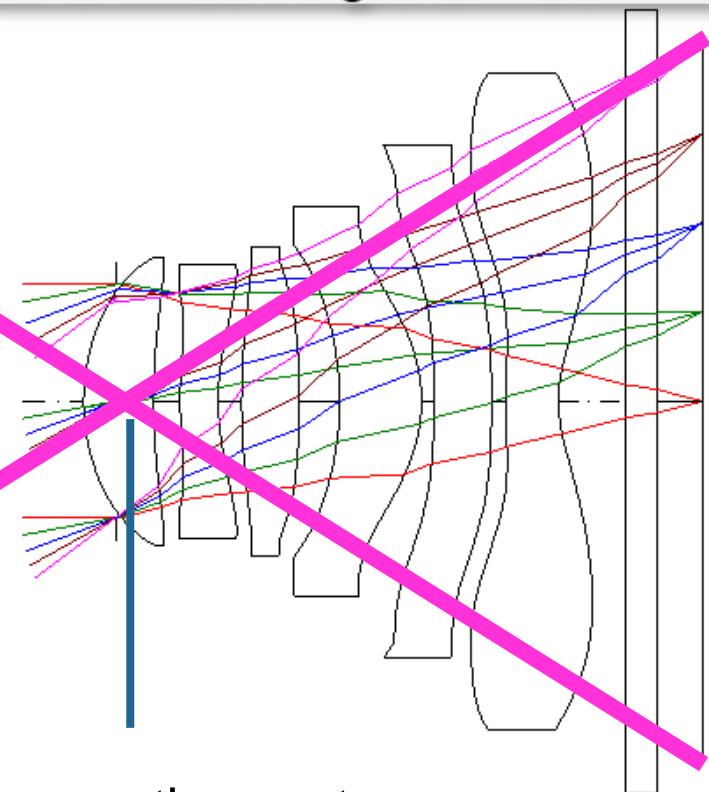


Compact camera lenses: Classic Biogon and modern Smartphone Lens

Biogon 2.8/28mm



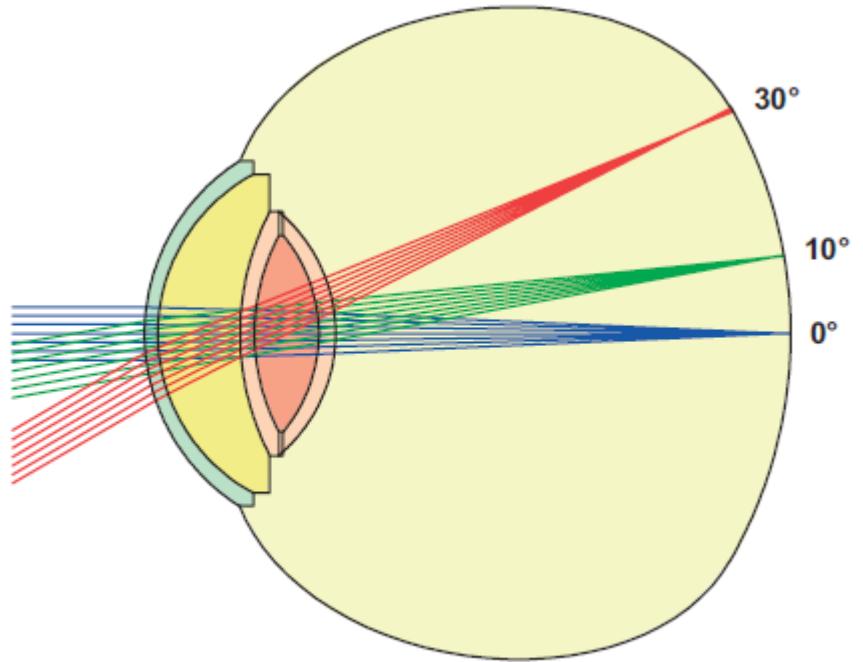
SPC Wide-Angle 2.1/28mm



Crossing point towards object space =

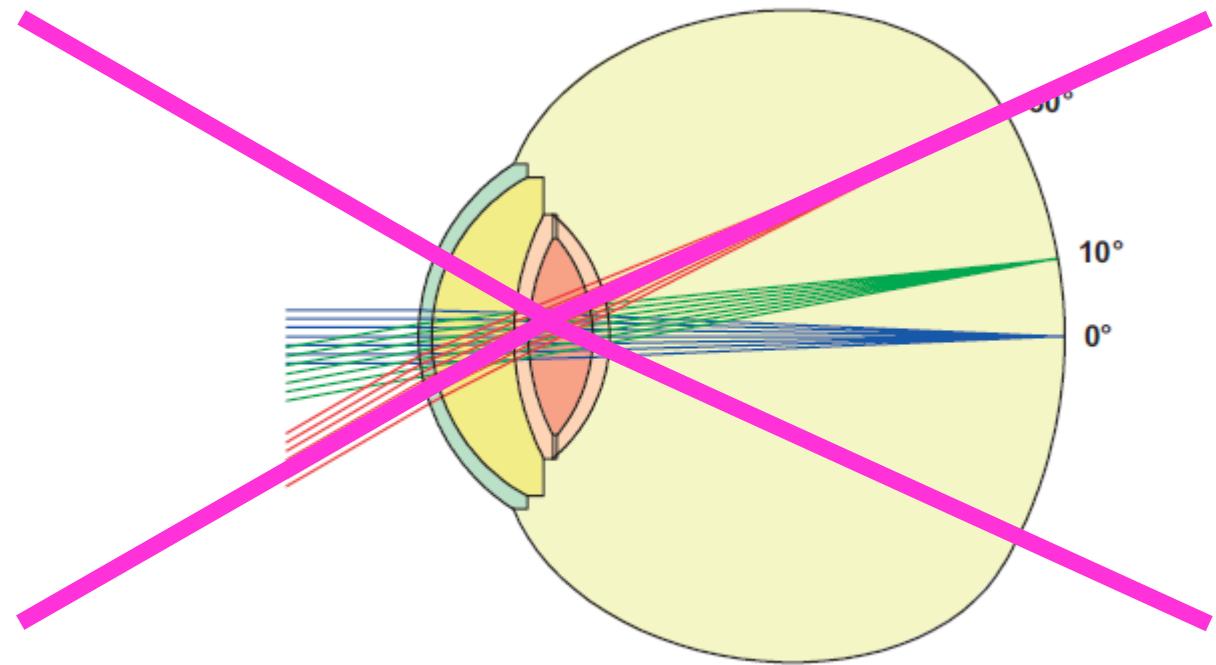
Perspective center

= Entrance Pupil



Optical model of the
human eye (Gullstrand)

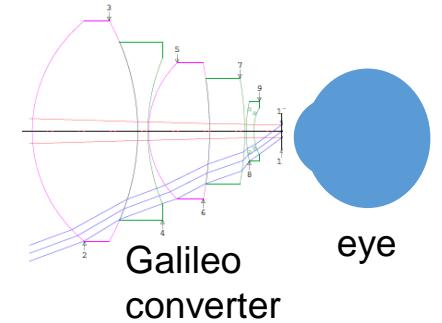
Human Eye



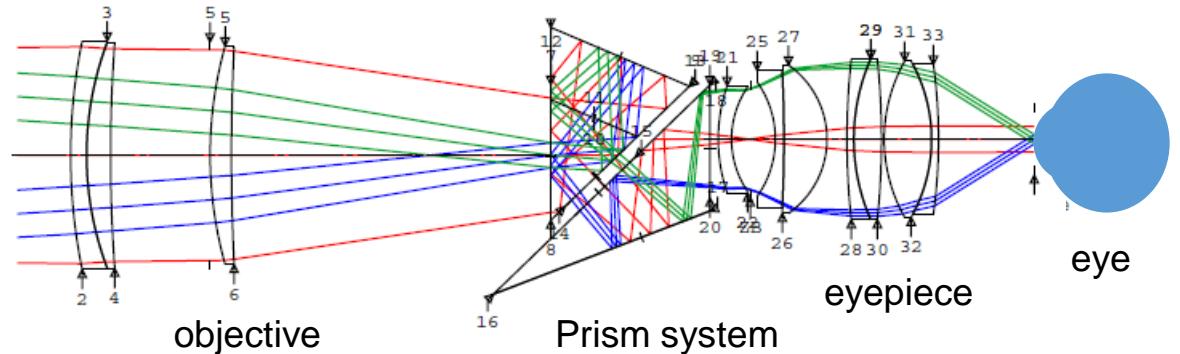
Optical model of the
human eye (Gullstrand)

Visual optical systems in combination with human eye

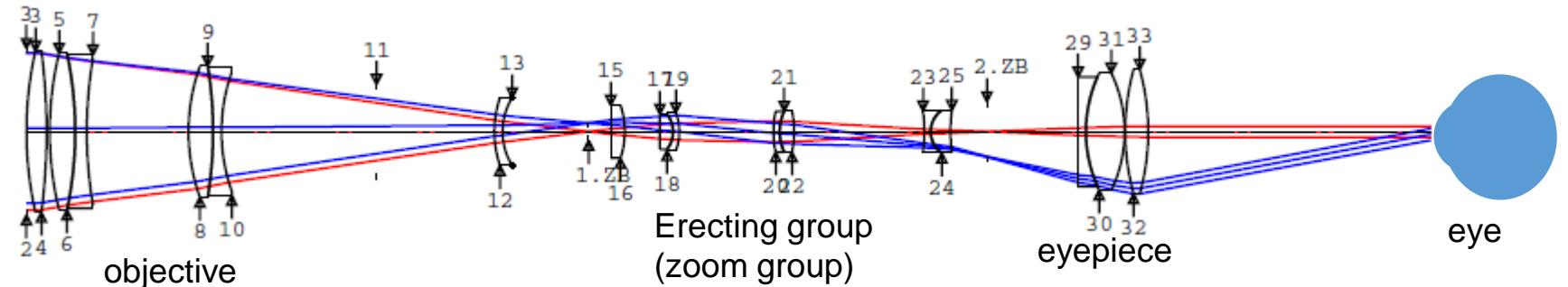
1. Opera glass



2. Binocular or spotting scope

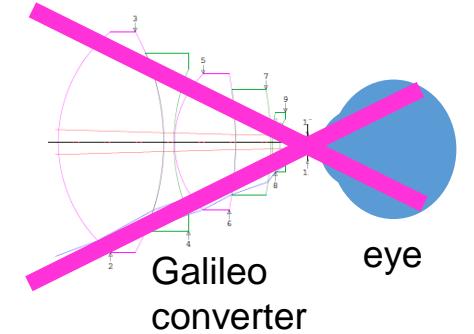


3. Rifle scope or „pirate spyglass“

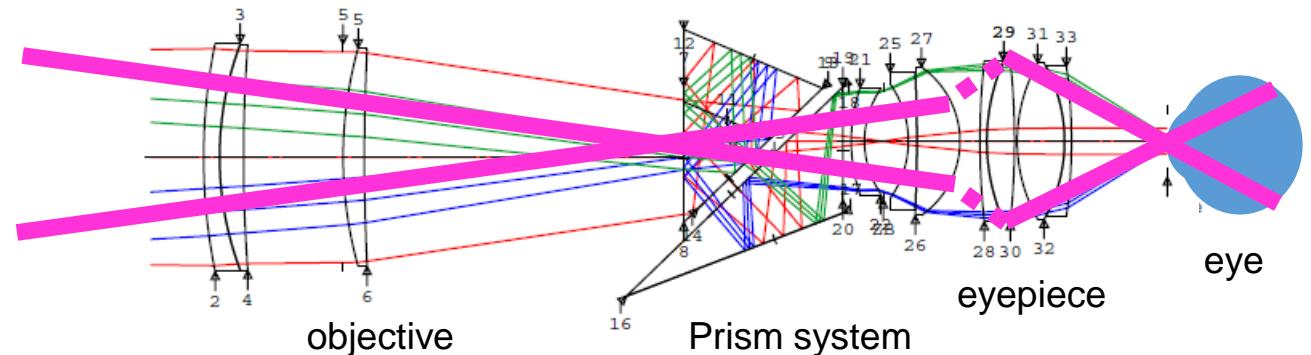


Visual optical systems in combination with human eye

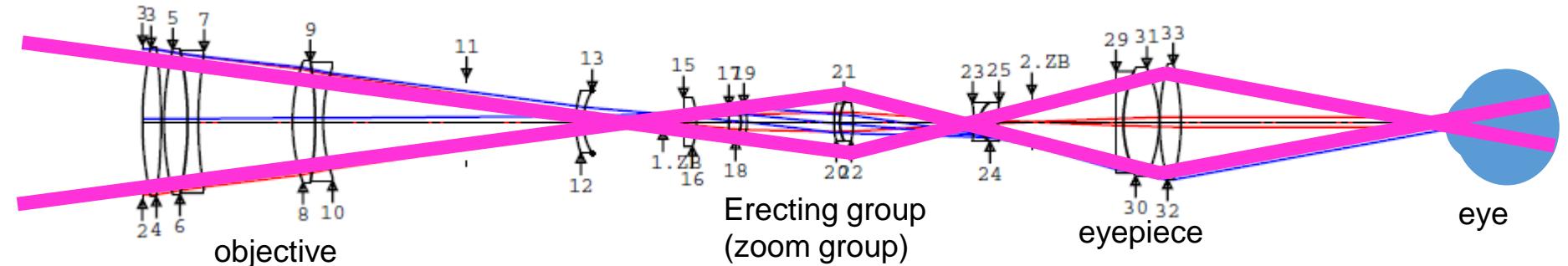
1. Opera glass

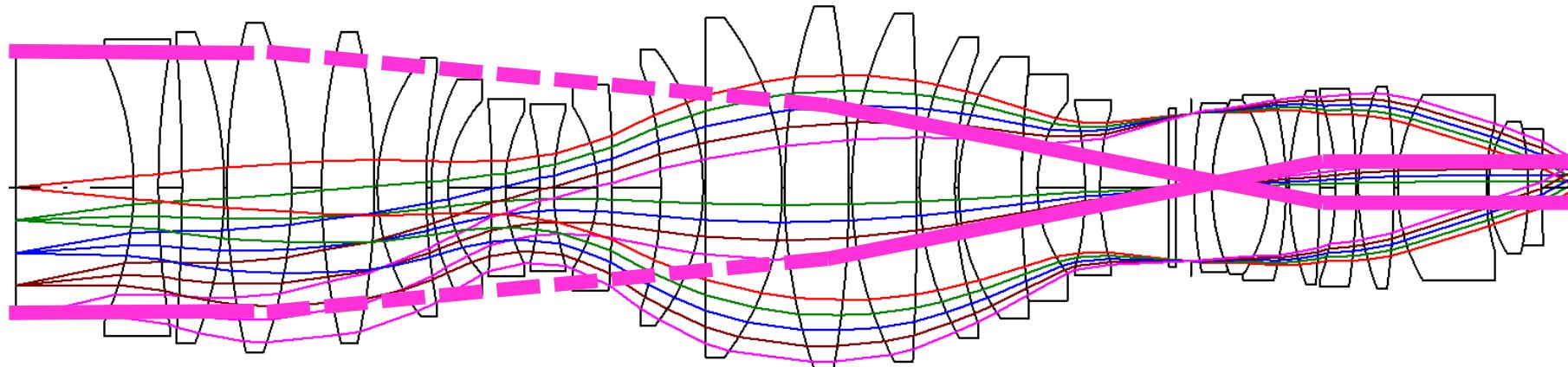


2. Binocular or spotting scope



3. Rifle scope or „pirate spyglass“

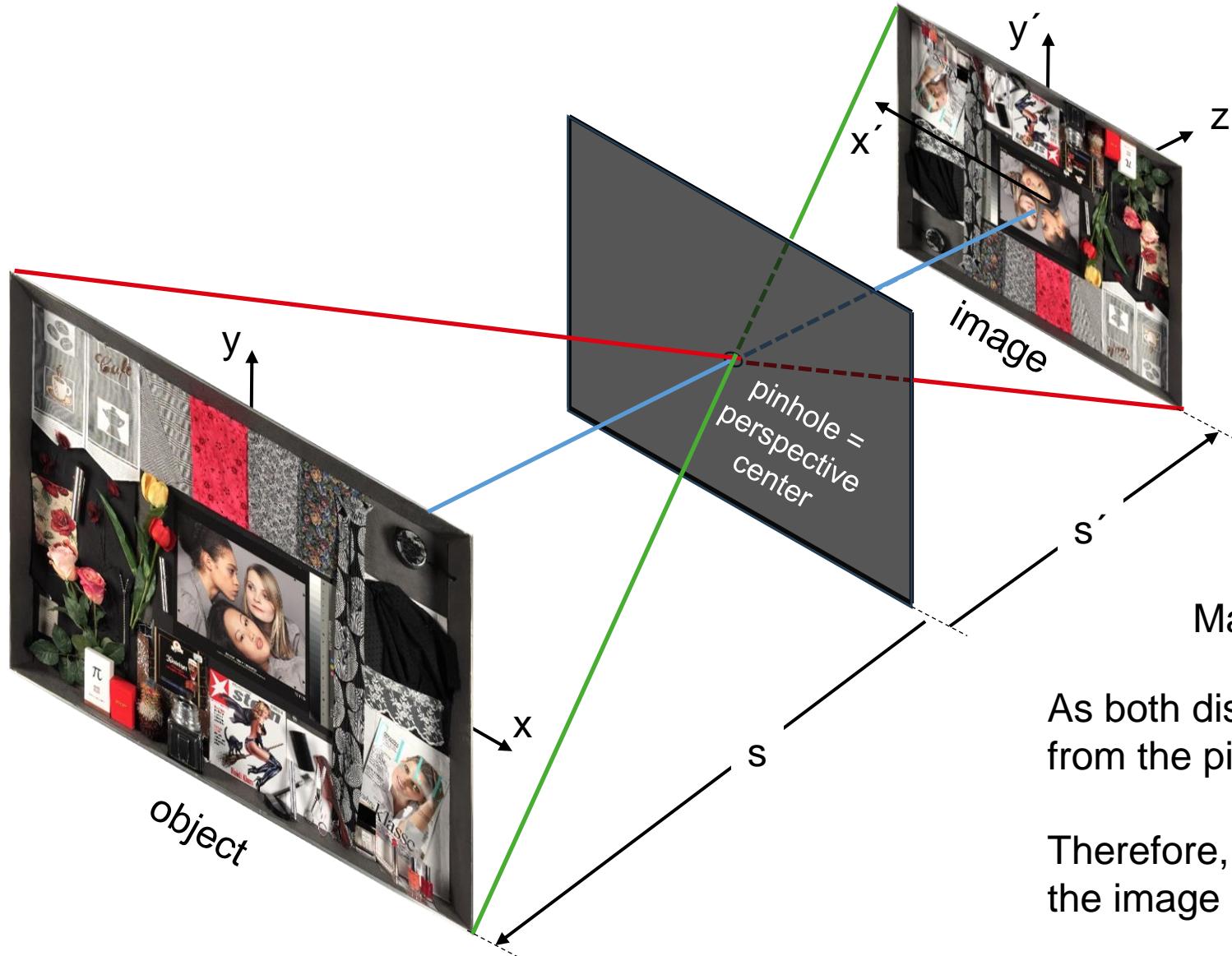




Crossing point of chief ray in stop plane.

Chief ray at object and image plane (almost) parallel to axis = object- and image-side telecentric
= entrance and exit pupil at infinity

Rectilinear imaging via pinhole



Magnification

$$m = \frac{x'}{x} = \frac{y'}{y} = \frac{s'}{s}$$

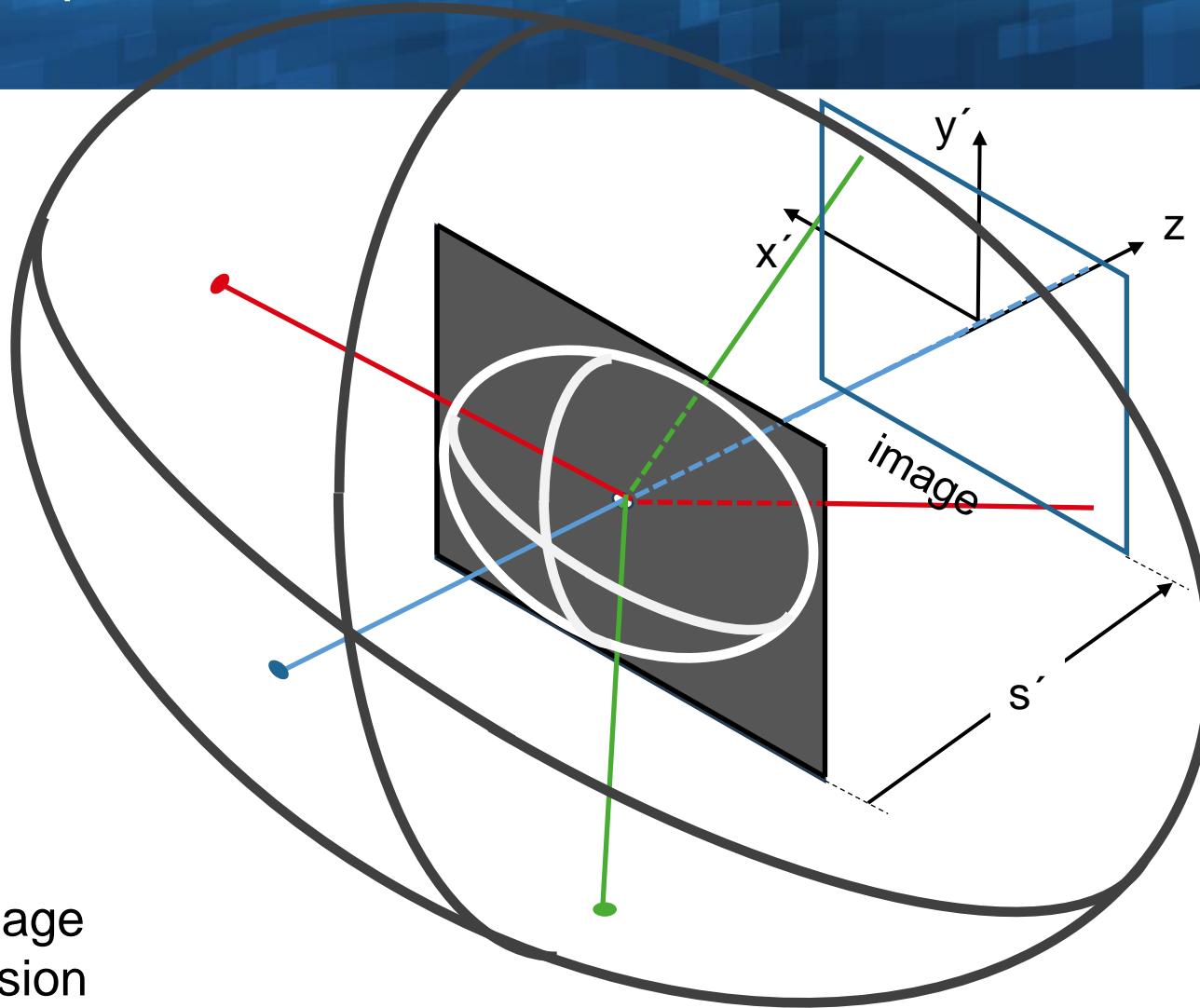
(for $x, y \neq 0$)

Magnification is constant.

As both distances s, s' are measured from the pinhole plane, $s < 0$ and $s' > 0$.

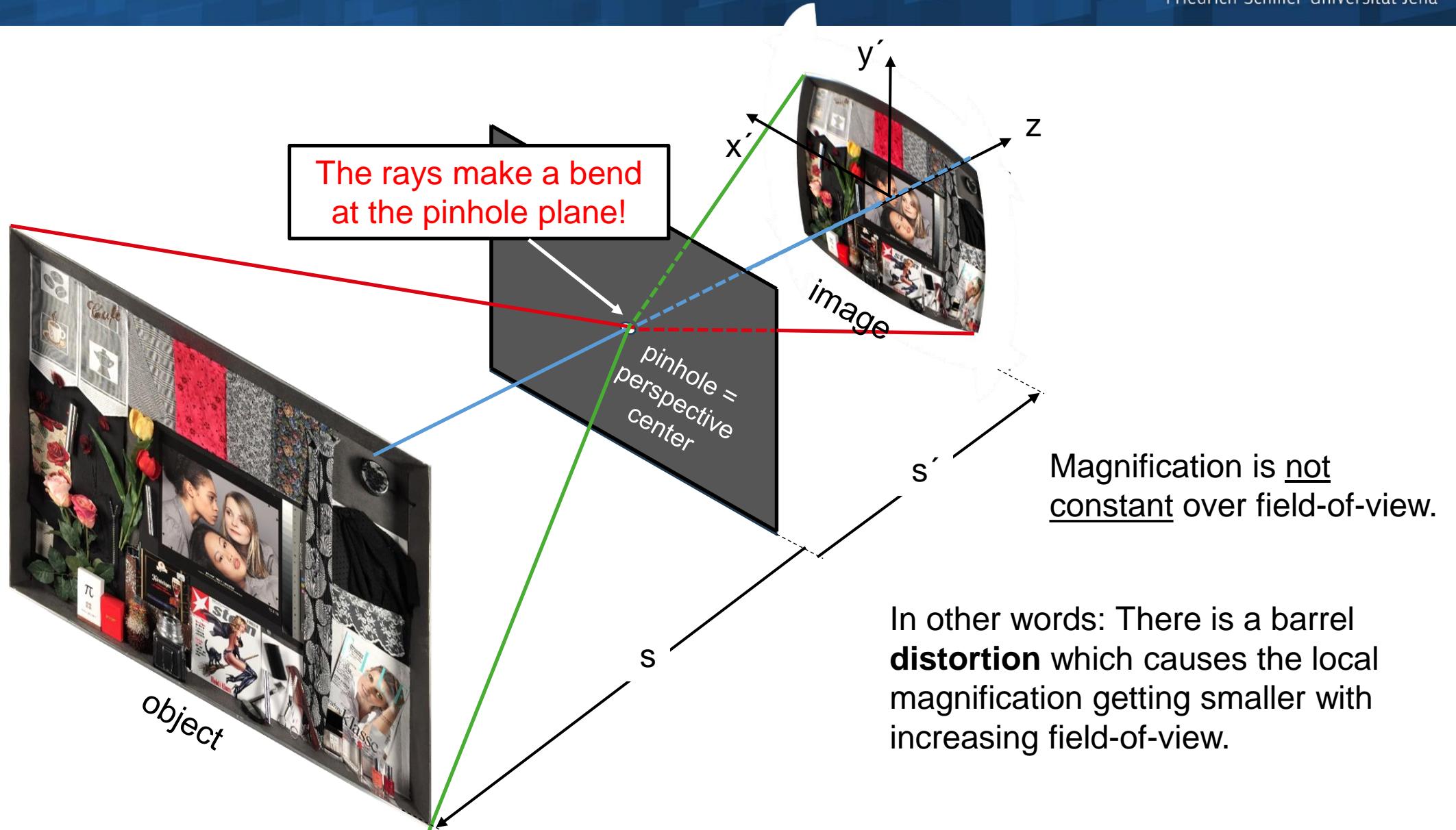
Therefore, m is negative. This means the image is inverted.

Fisheye imaging via pinhole

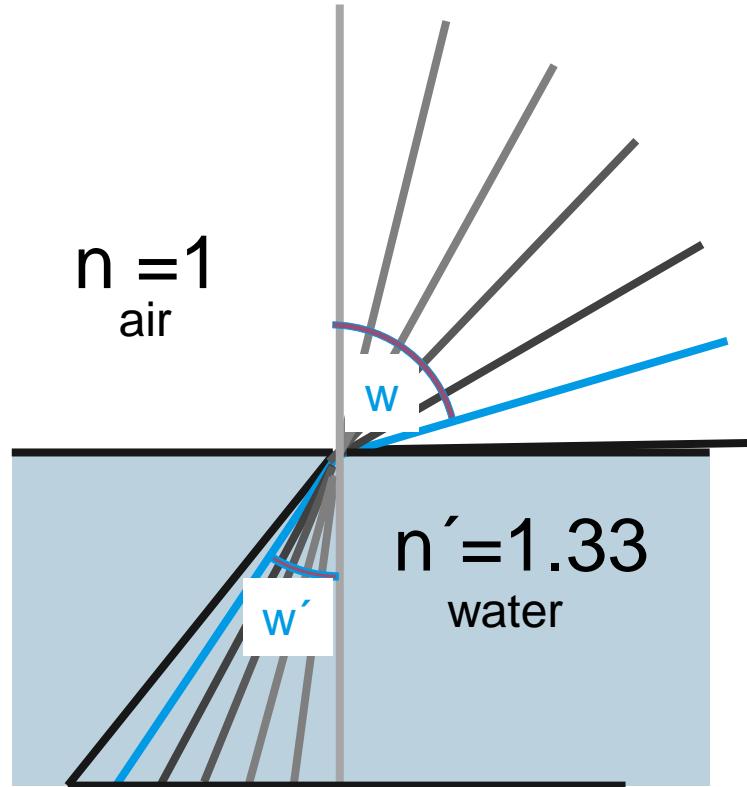


Imaging of object
hemisphere onto image
plane of finite extension

Fisheye imaging via pinhole



Fish-Eye Lens Pinhole Camera

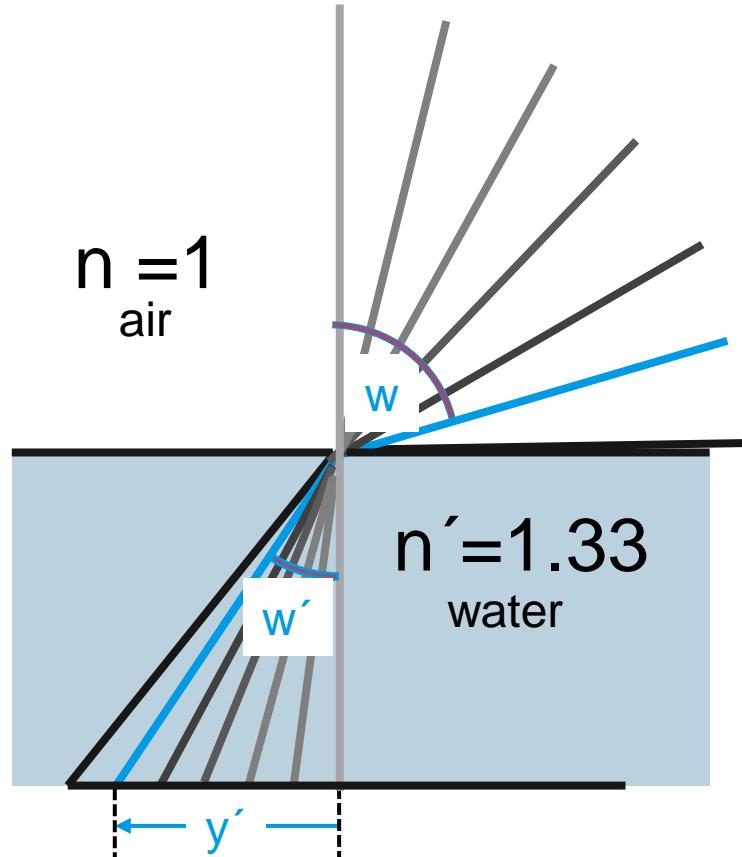


R. Wood (1906), Fish-Eye Views and Vision
Under Water, Philosophical Magazine

Snells Law:

$$n' \sin w' = n \sin w$$

Fish-Eye Lens Pinhole Camera

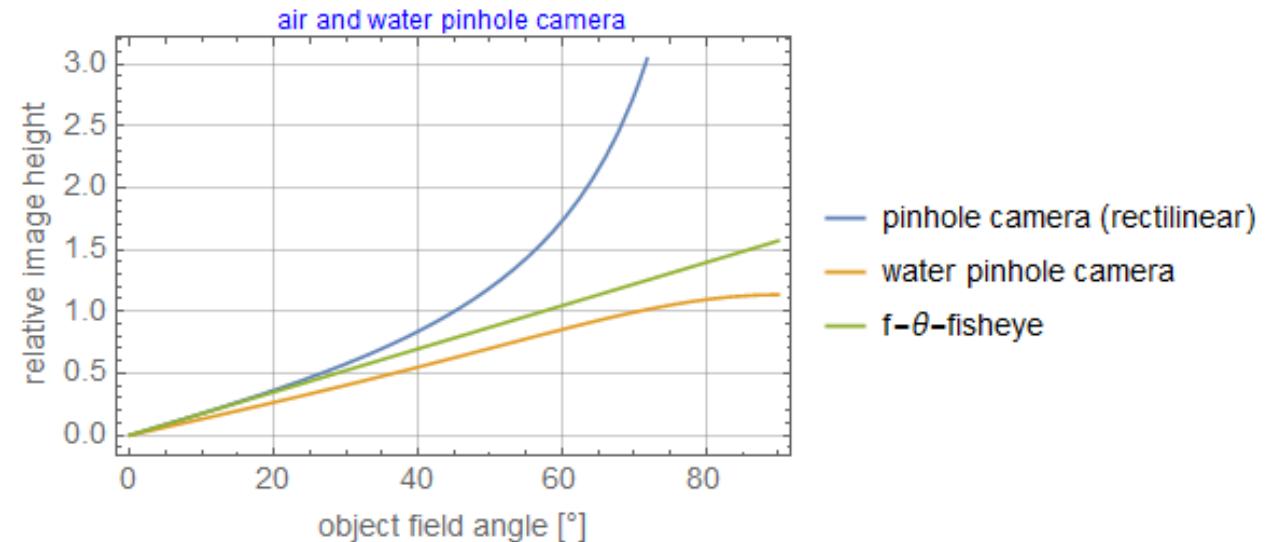


Projection Law:

$$y' = \frac{f'}{n} \frac{\sin w}{\sqrt{1 - \frac{\sin^2 w}{n^2}}}$$

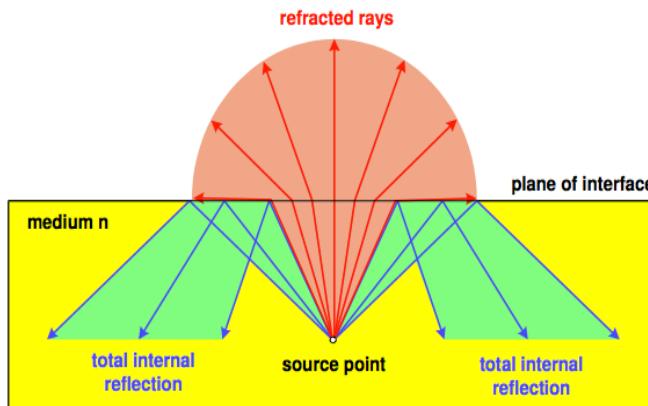
Special case:
Pinhole in air, $n=1$:

$$y' = f' \tan w$$

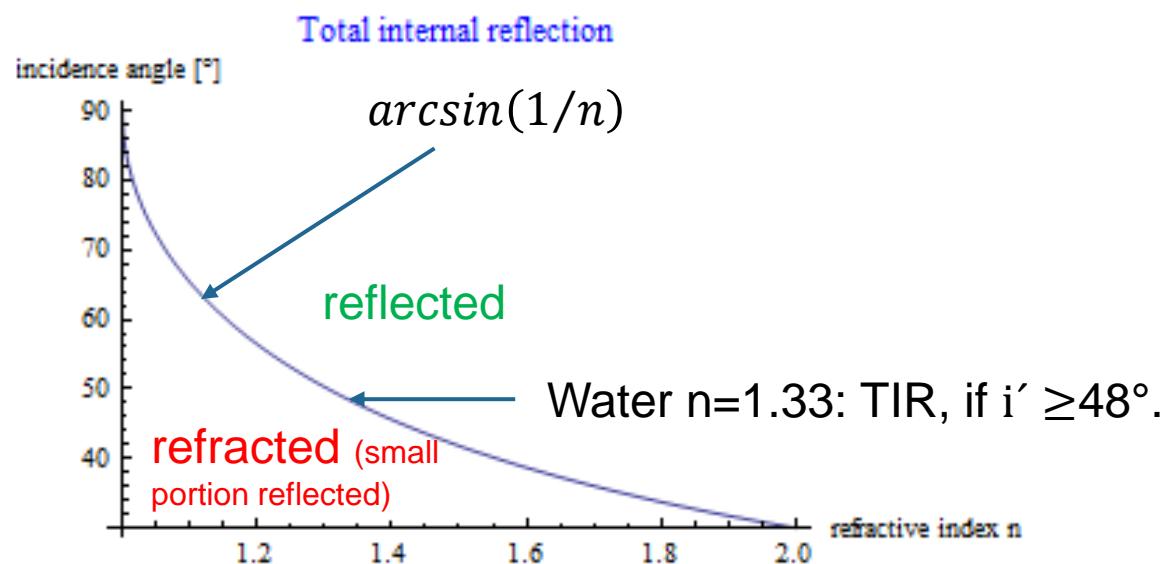


- demagnified image near center by factor $1/n$ (3/4 in water)
- Field-of-View in water limited by total internal reflection to 96° (corresponding to 180° in air)

Total internal reflection



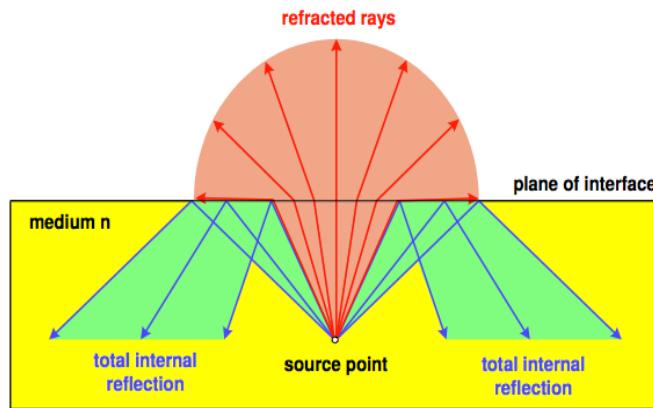
According to Snell's Law $n \sin i' = \sin i$ a ray coming from the optical denser medium n can not be transferred to air for a certain incidence angle $i' =$, since $\sin i$ can not be greater than 1. For this critical angle at $\sin i = 1$, namely $\sin i' = 1/n$ or $i' = \arcsin(1/n)$, and beyond all light is reflected at this interface. This is called „total internal reflection“ (TIR).



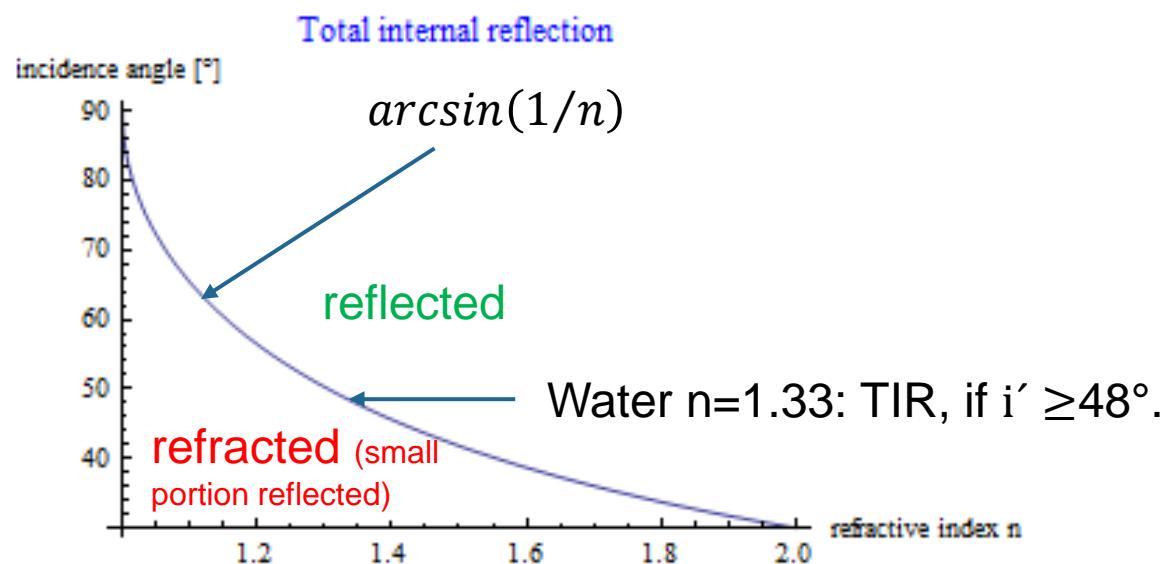
For refractive index of connecting medium $n=1$



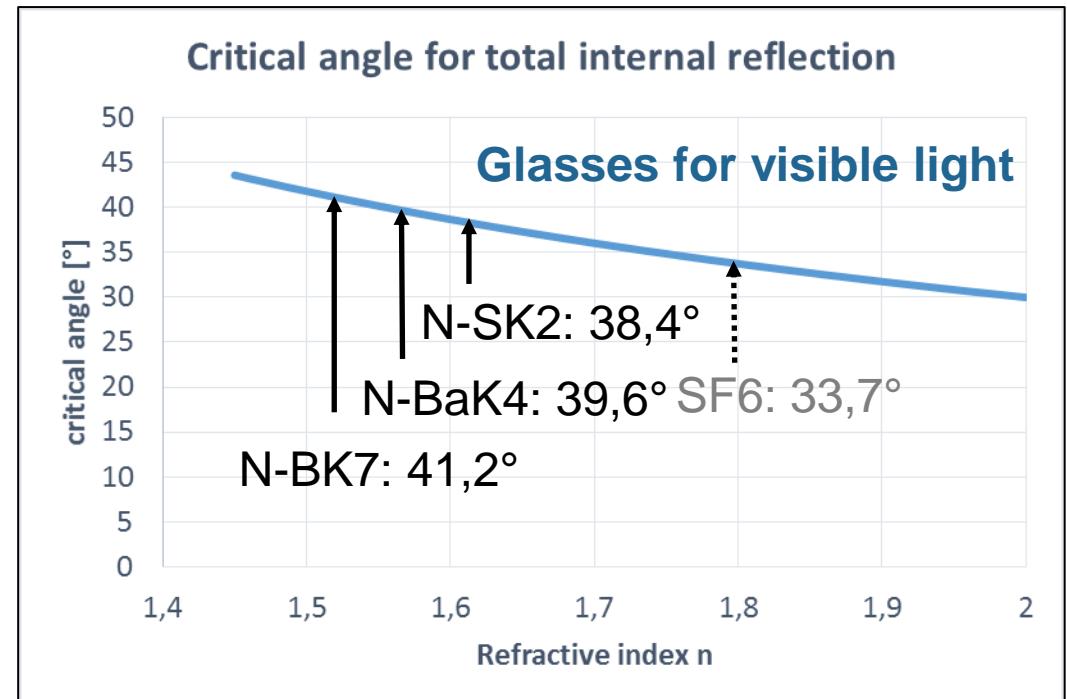
Total internal reflection



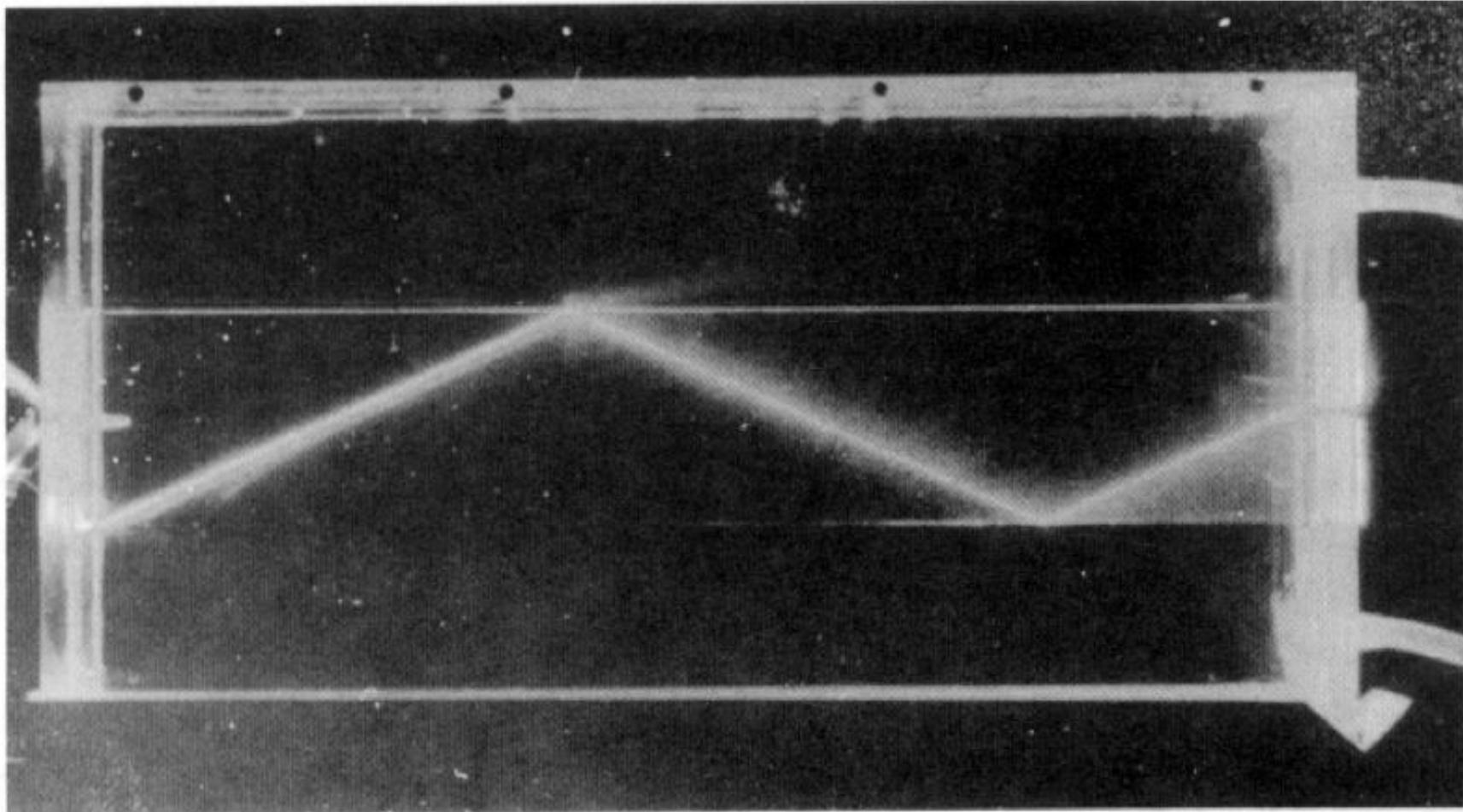
According to Snell's Law $n \sin i' = \sin i$ a ray coming from the optical denser medium n can not be transferred to air for a certain incidence angle $i' =$, since $\sin i$ can not be greater than 1. For this critical angle at $\sin i = 1$, namely $\sin i' = 1/n$ or $i' = \arcsin(1/n)$, and beyond all light is reflected at this interface. This is called „total internal reflection“ (TIR).



For refractive index of connecting medium $n=1$

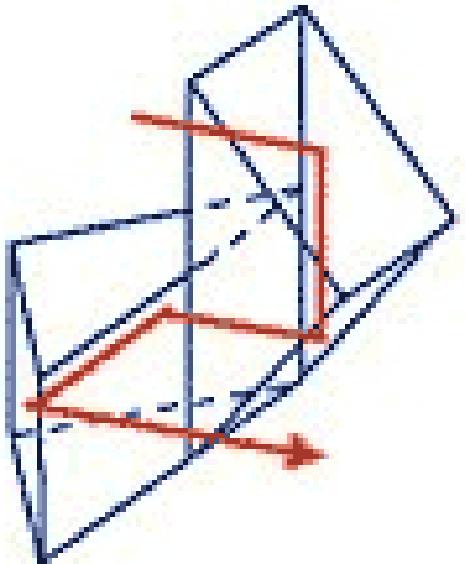


Total internal reflection in fiber glass cable

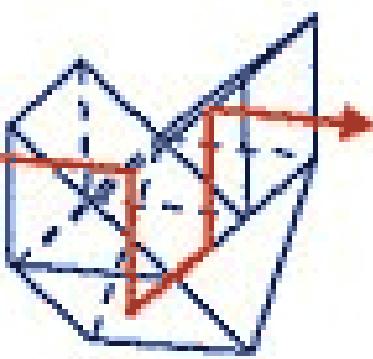


Ref.: M. Kaschke

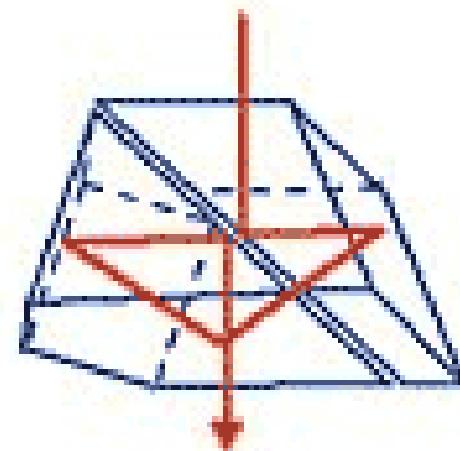
Total internal reflection in prisms for image inversion



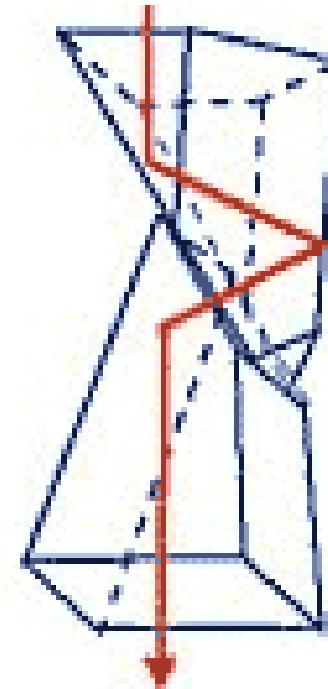
Porro 1



Porro 2



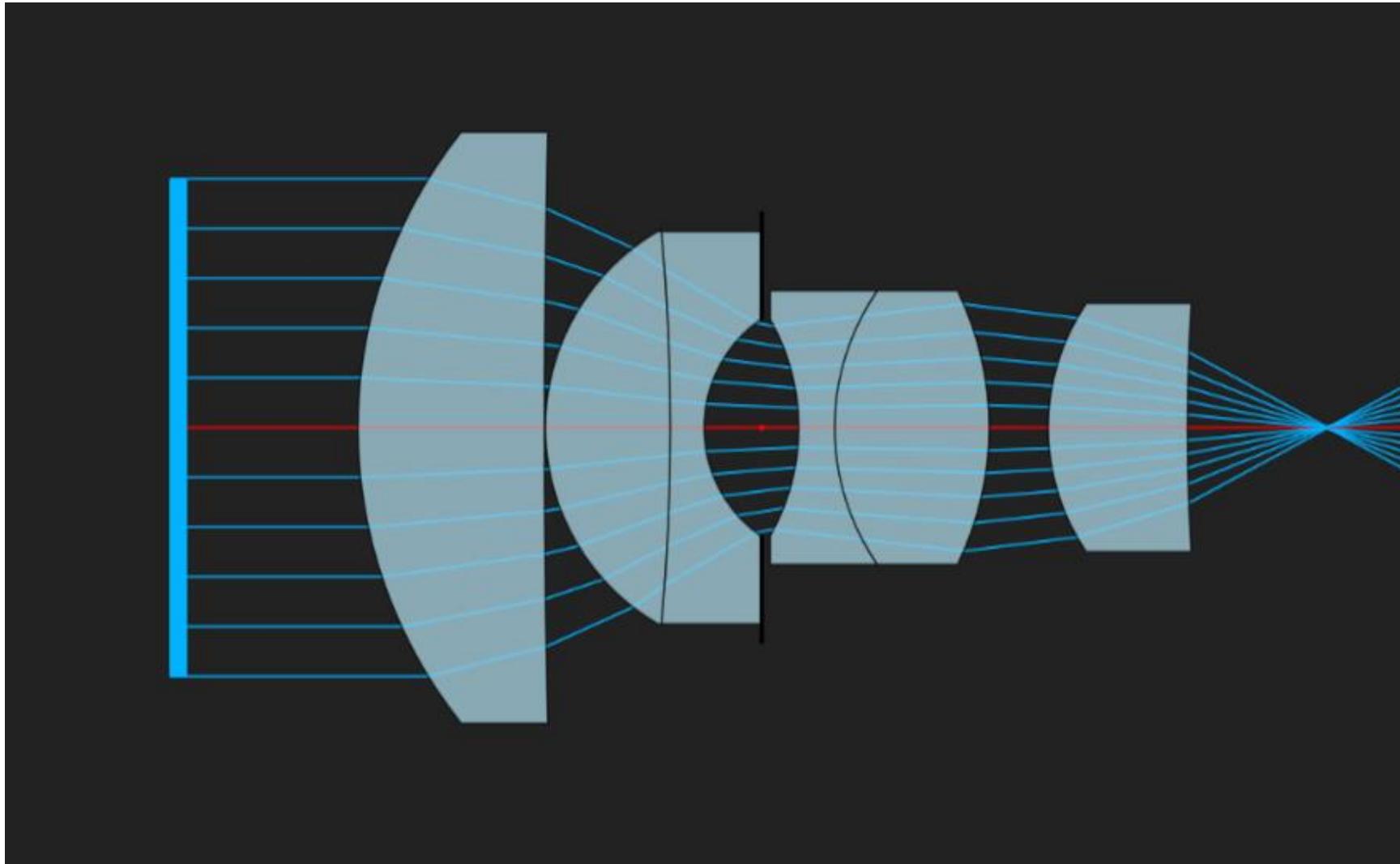
Schmidt-Pechan

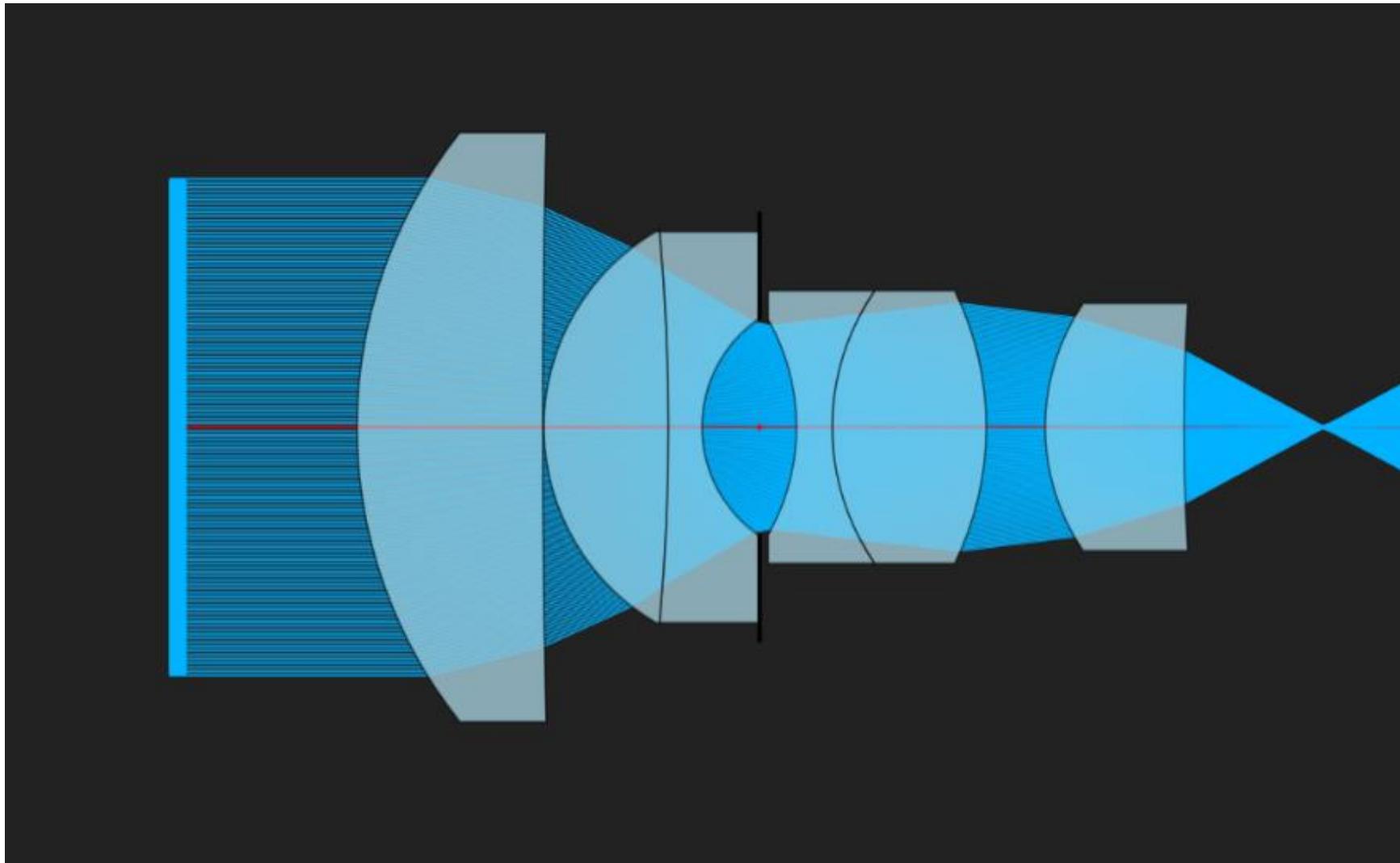


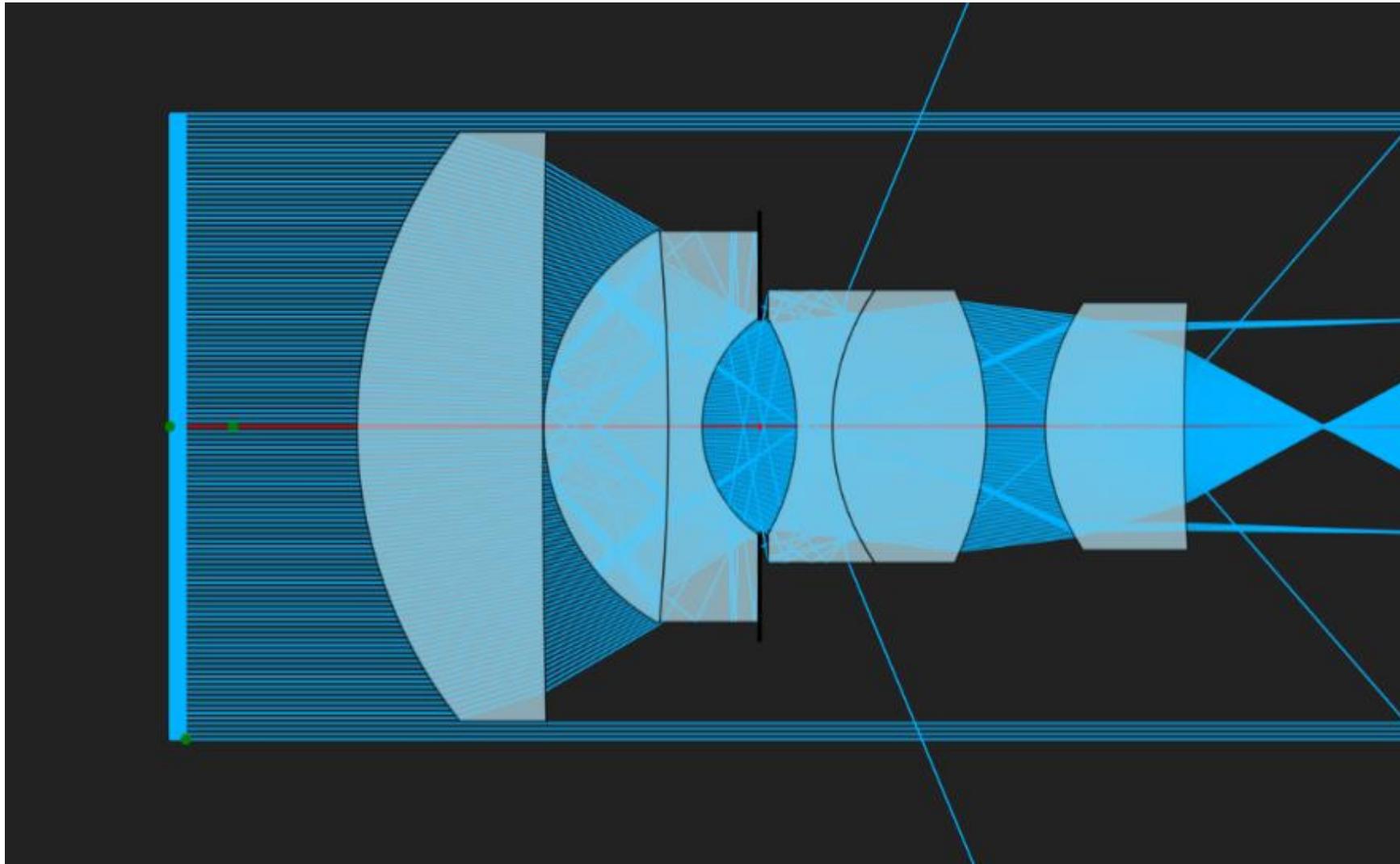
Abbe-König

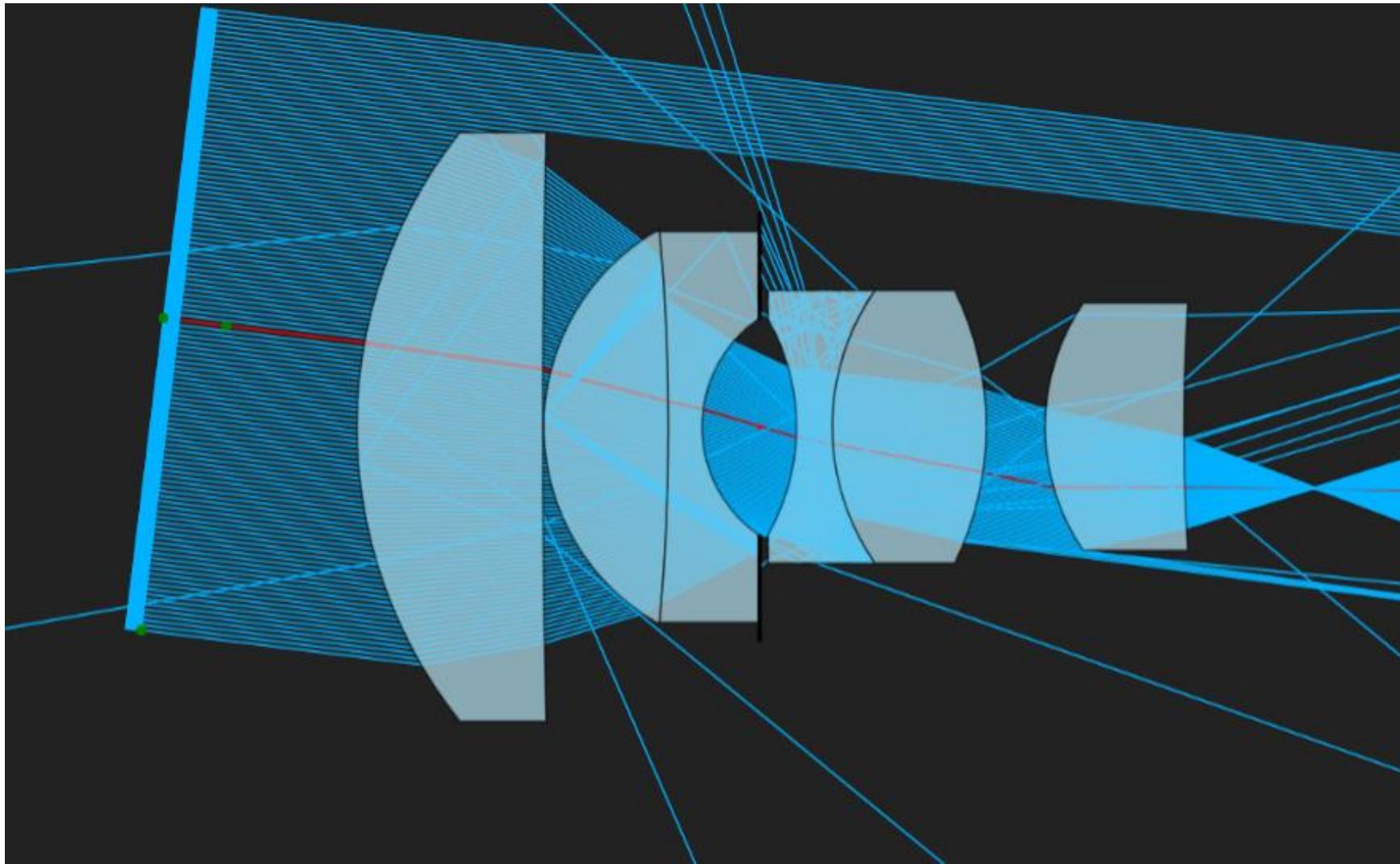
One of the surfaces
needs to have
reflection coating

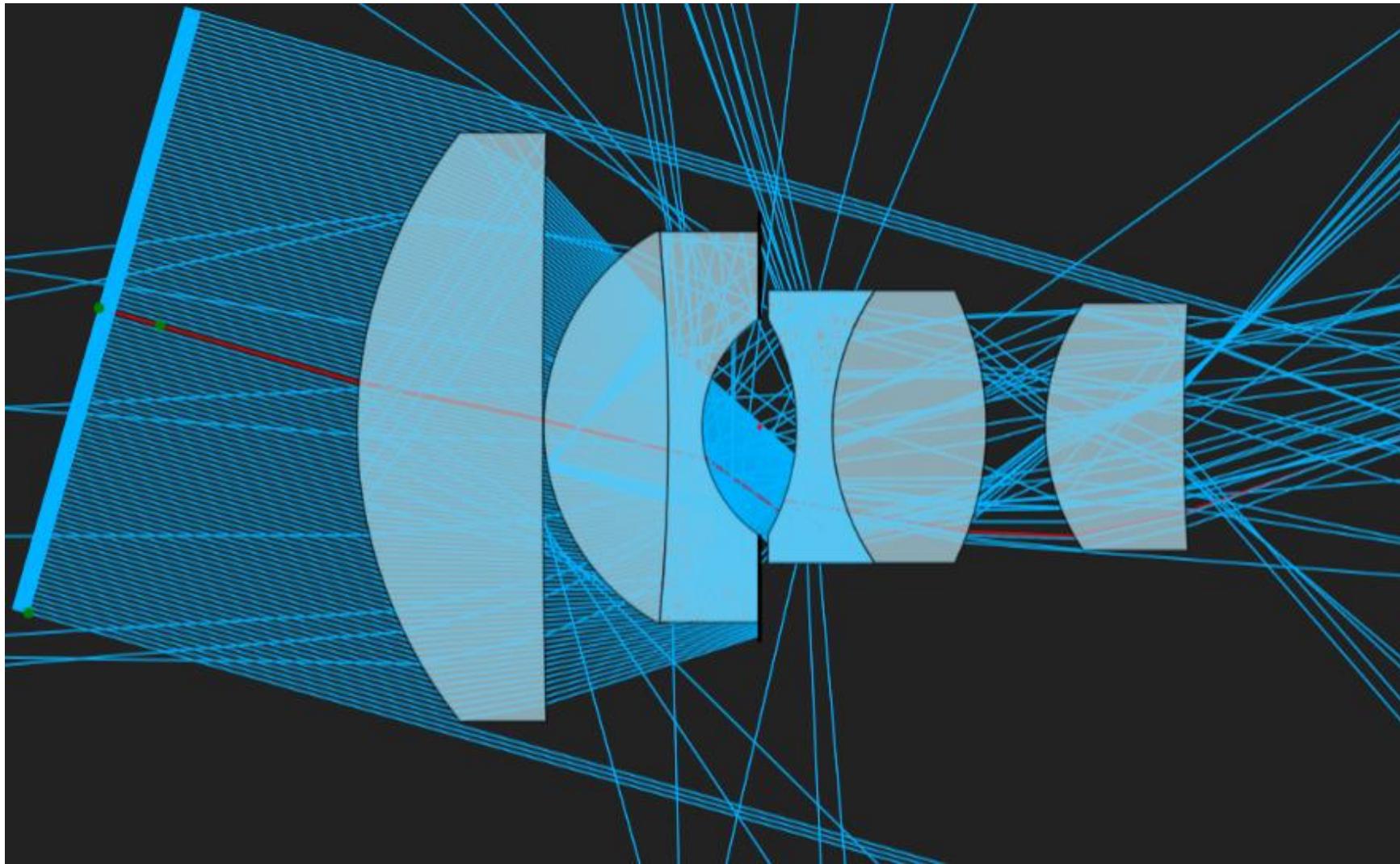
[OPTICO](#), Michael Wick

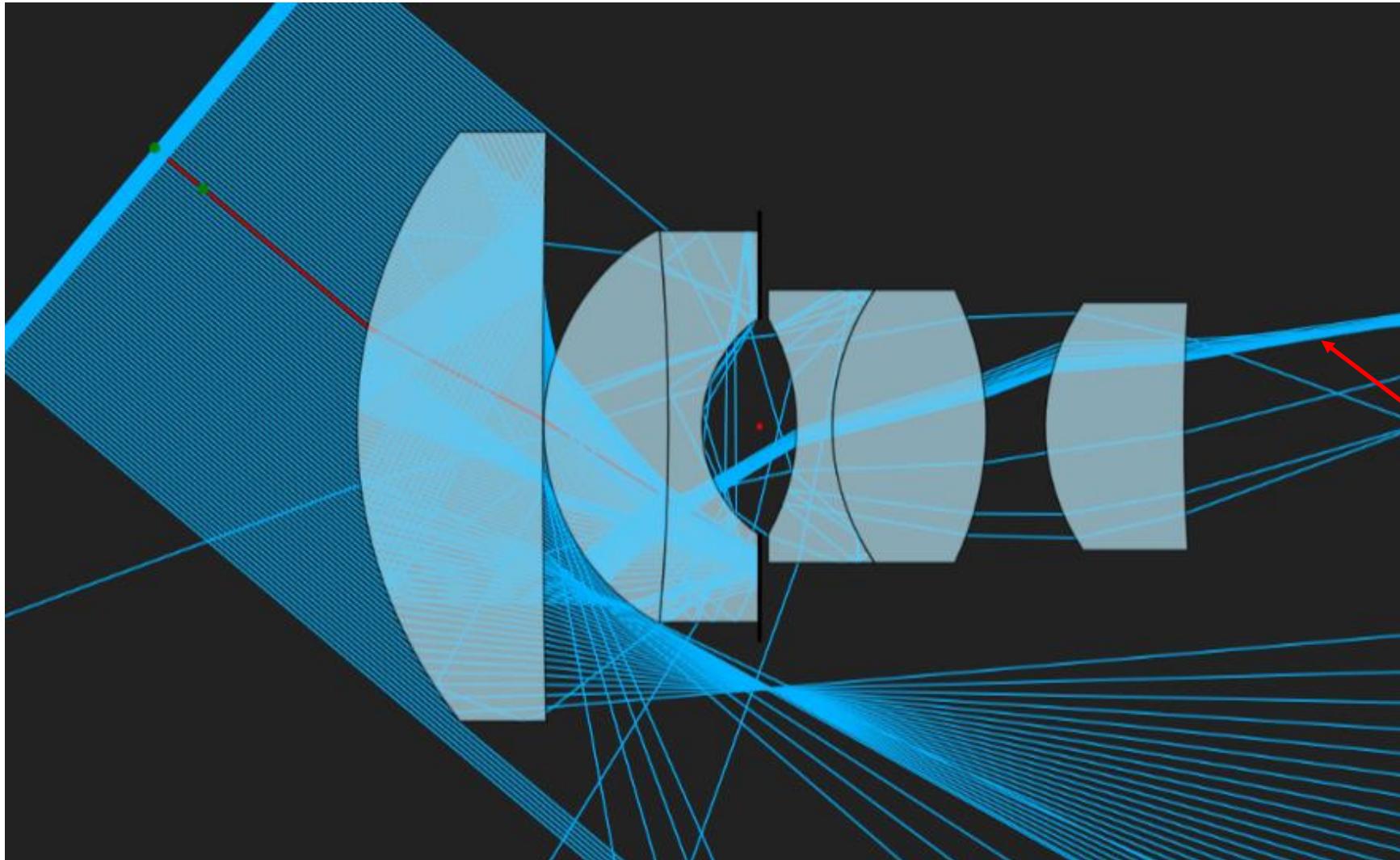










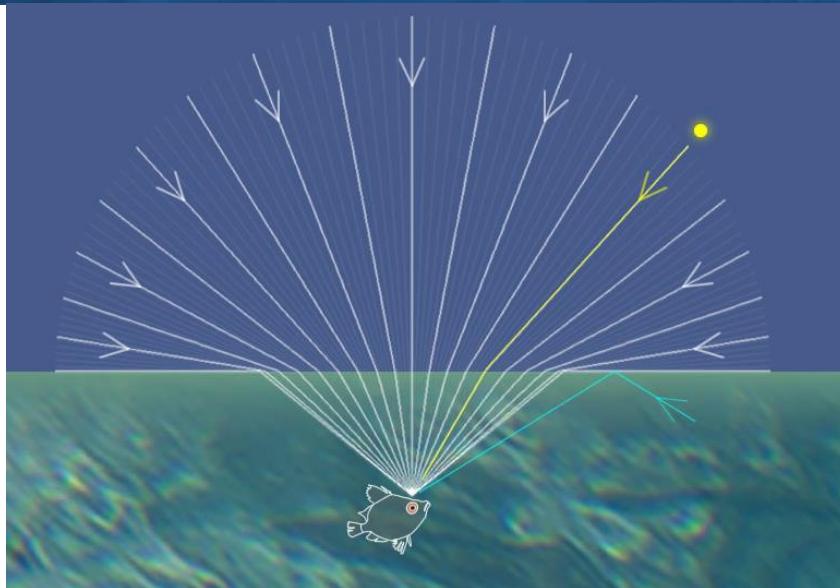


The lens is obviously not designed to capture such a large field angle.

However, light from this angle of field might still harm the quality of the image as straylight.

!

Fish-Eye Imaging: Perspective center in medium

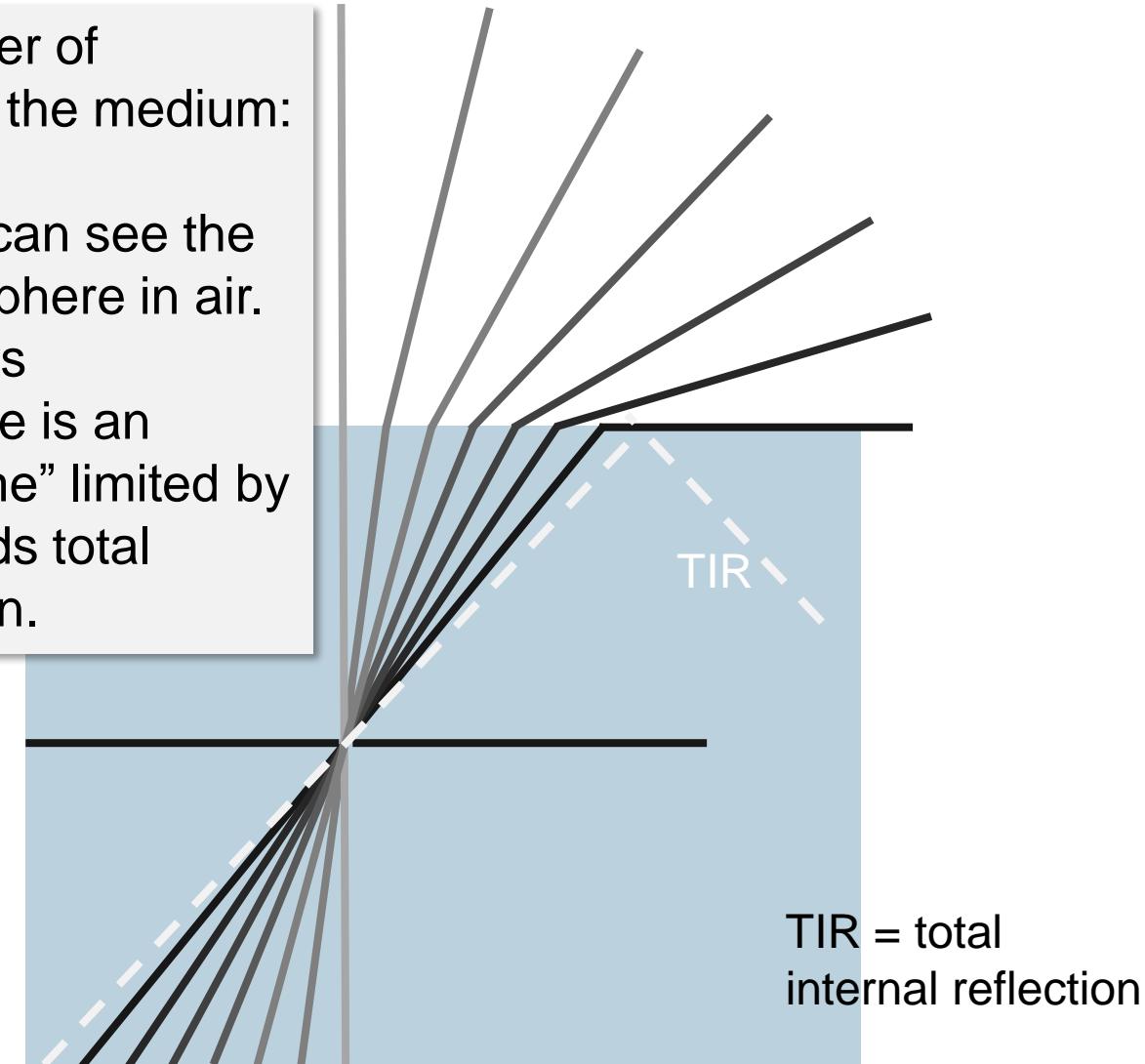


Perspective of a fish looking outside water



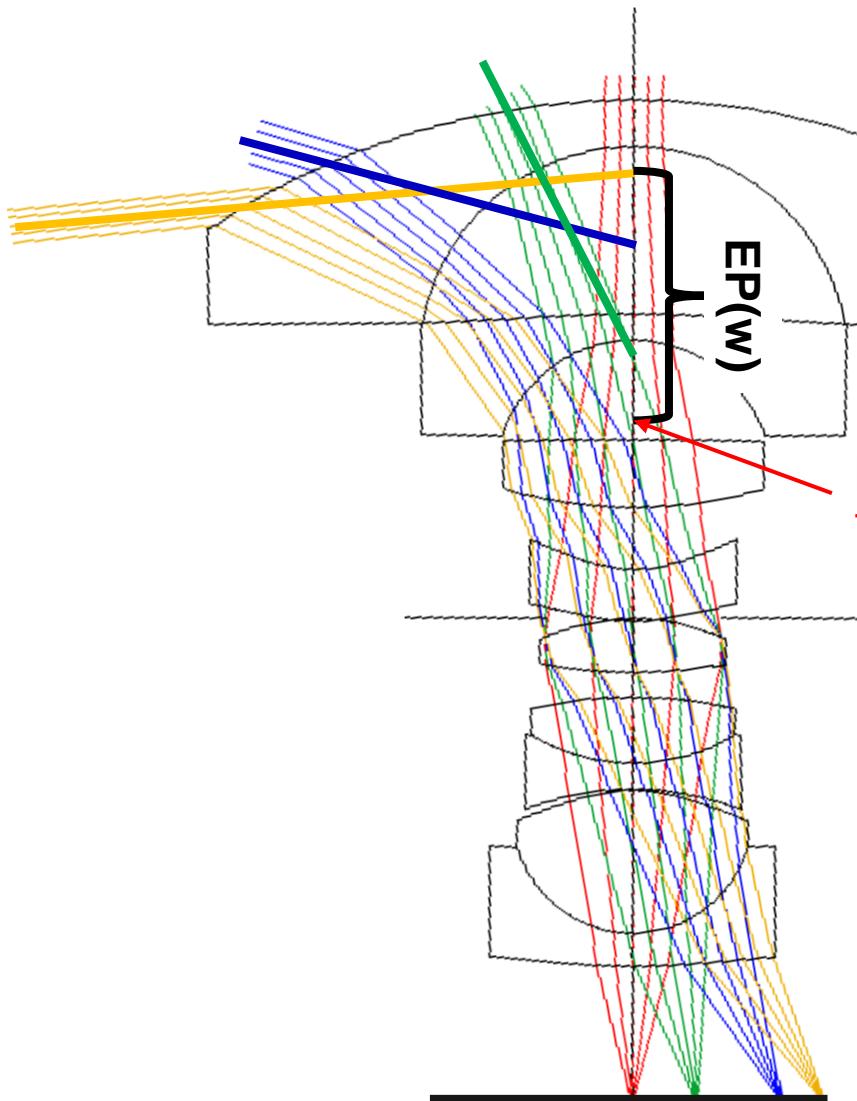
Moving the center of perspective into the medium:

The viewer still can see the complete hemisphere in air. From the viewers perspective there is an “acceptance cone” limited by the angle towards total internal reflection.



TIR = total internal reflection

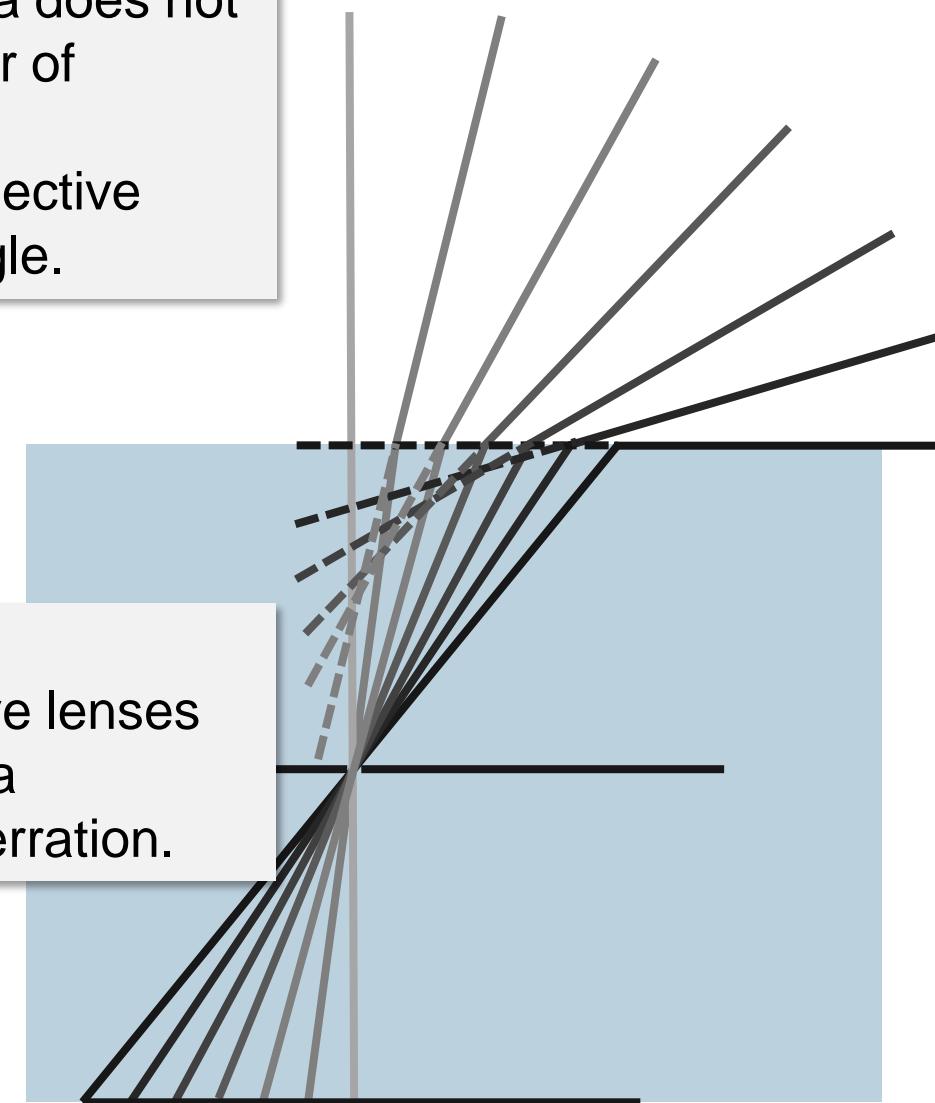
Fish-Eye Imaging: Perspective center in medium vs real fish-eye lens: Field-angle dependent center of perspective



This pinhole camera does not have a single center of perspective.
The center of perspective varies with field angle.

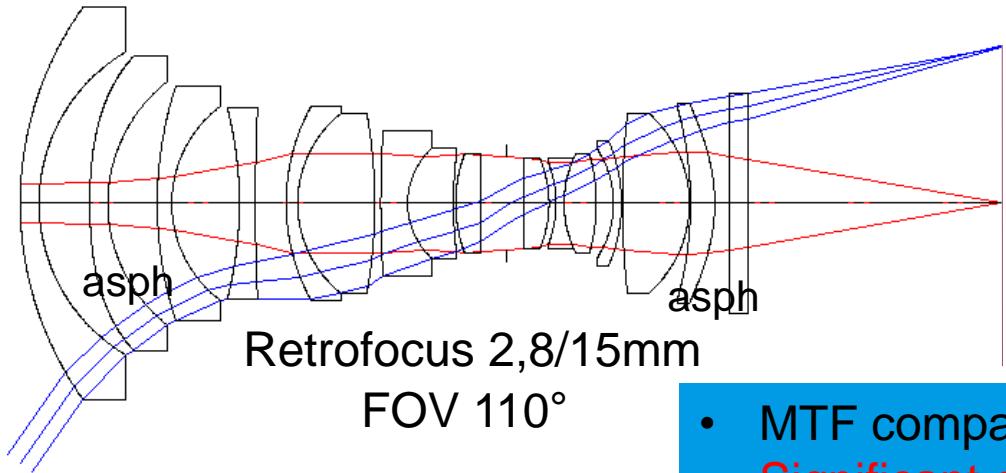
paraxial focus

This is called “pupil aberration”. Fish-eye lenses typically also have a significant pupil aberration.

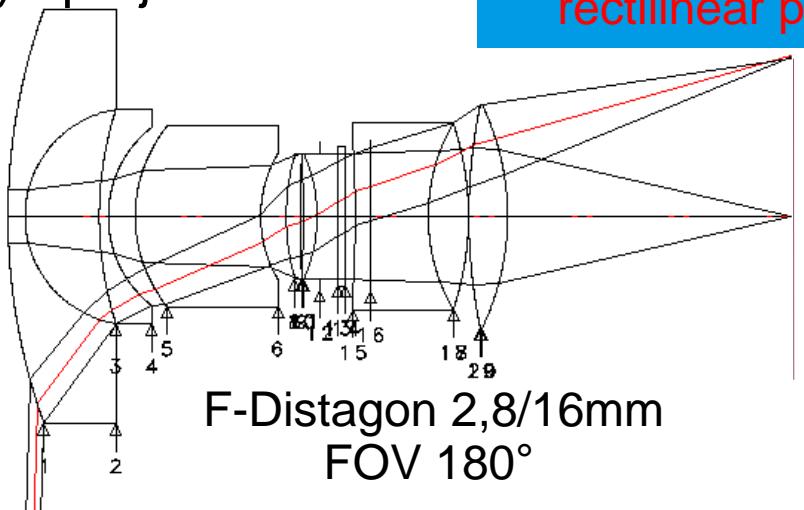


Extreme wide-angle lenses: rectilinear retrofocus vs fish-eye lens

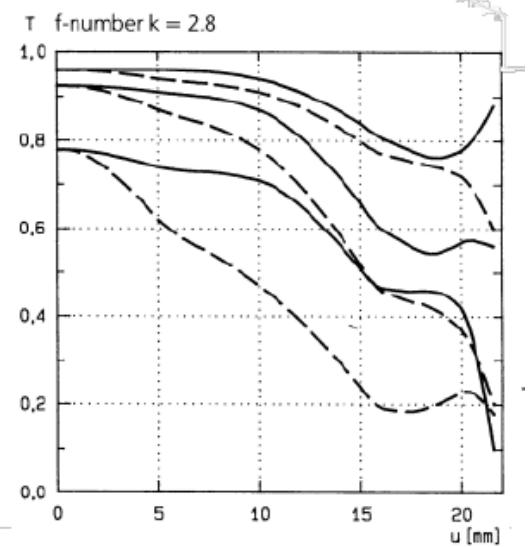
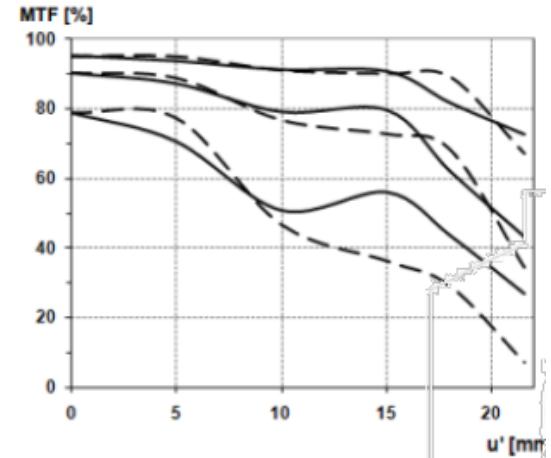
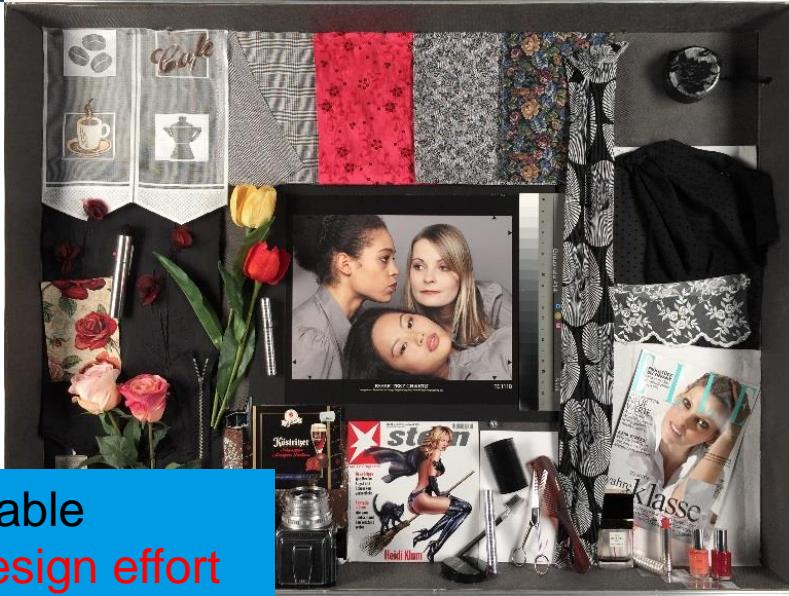
Rectilinear (“distortion-free”) projection:



Fish-eye projection:



- MTF comparable
- Significant design effort to correct distortion for rectilinear perspective!



Perspective deformation in rectilinear imaging



Hönliger/Nasse (2009)

The picture of this couple exhibits skewed “eggheads” while the balloon in the middle looks quite normal.

The shot was taken with a $f=15\text{mm}$ lens on 35mm format.

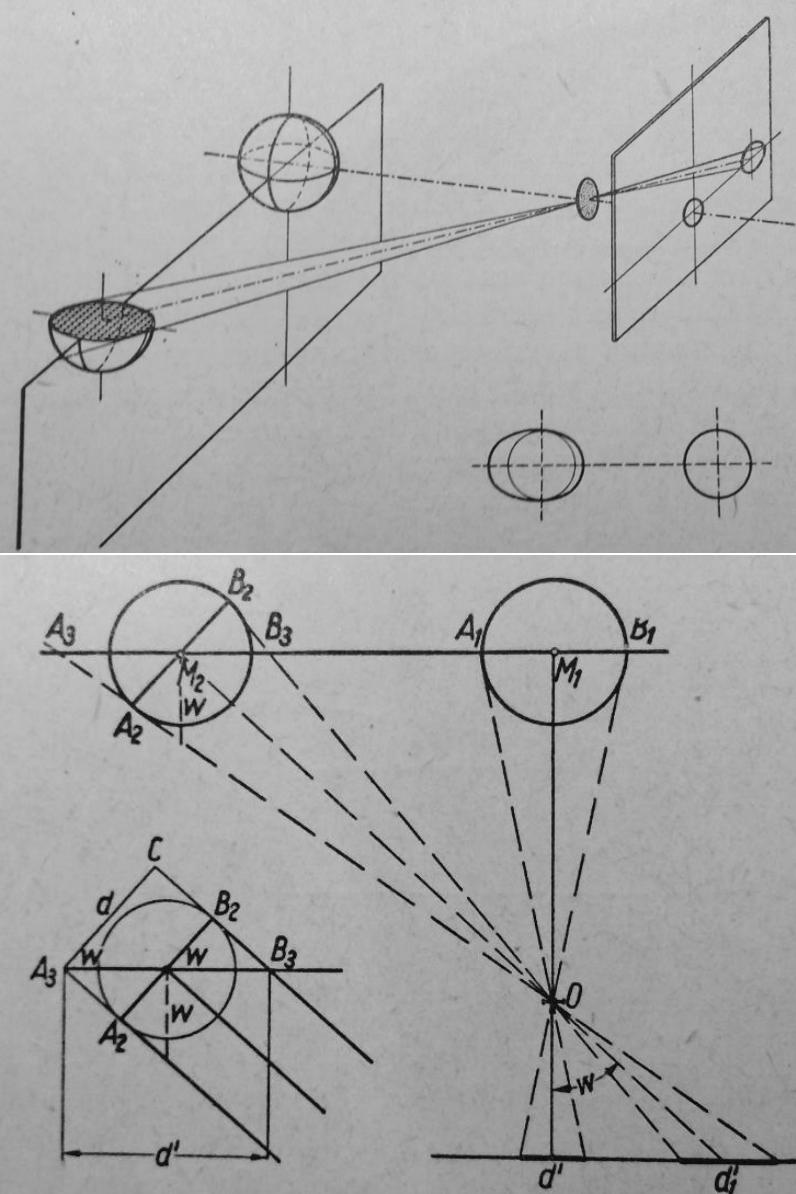
Perspective deformation in rectilinear imaging „Egghead effect“



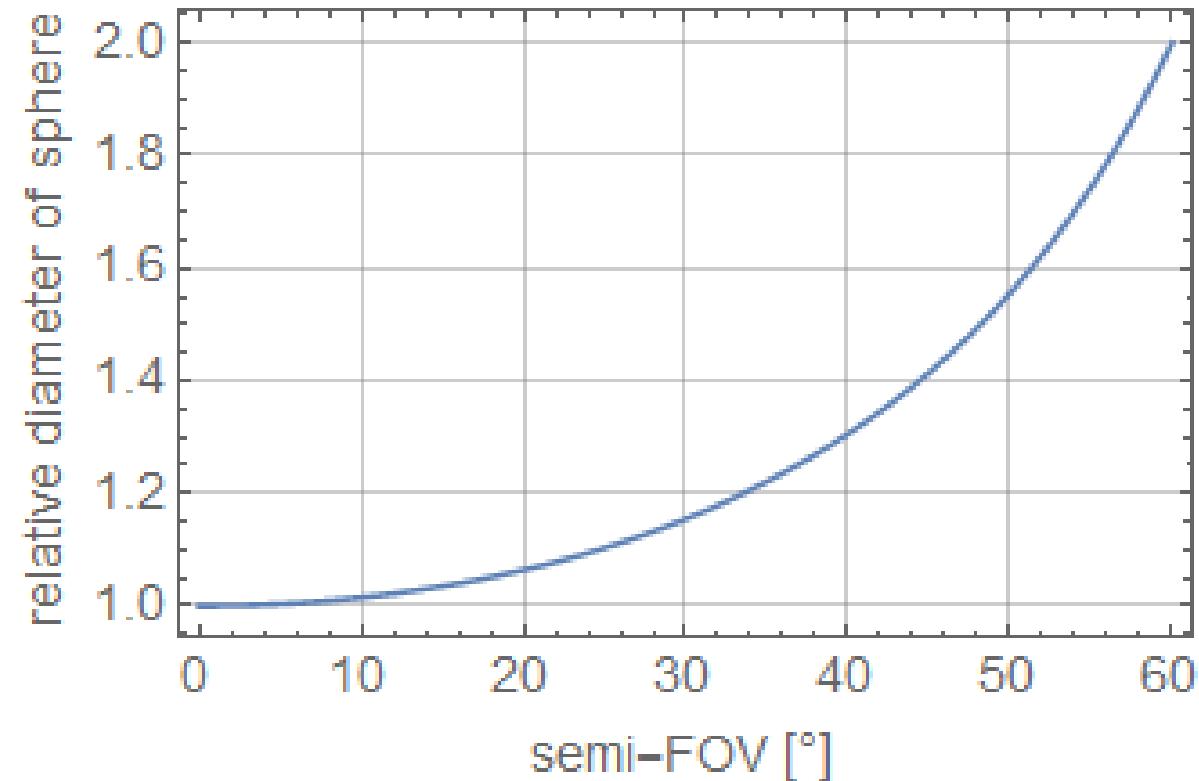
Perspective deformation in rectilinear imaging „Egghead effect“



Rectilinear imaging: Perspective projection of sphere



Perspective projection of sphere depending on lateral position:



Projected diameter
(far distance approximation):

$$d(w) = \frac{d_0}{\cos w}$$

Rectilinear wide-angle imaging and depth perception



Rectilinear wide-angle images give the impression of very large depth.

Often too much, since interesting distant details lose attention.

The sidewalls appear elongated and boring.

Rectilinear and Panini Perspective

rectilinear



Panini
(x-stereographic,
y-rectilinear)



Extreme wide-angle view with Panini projection



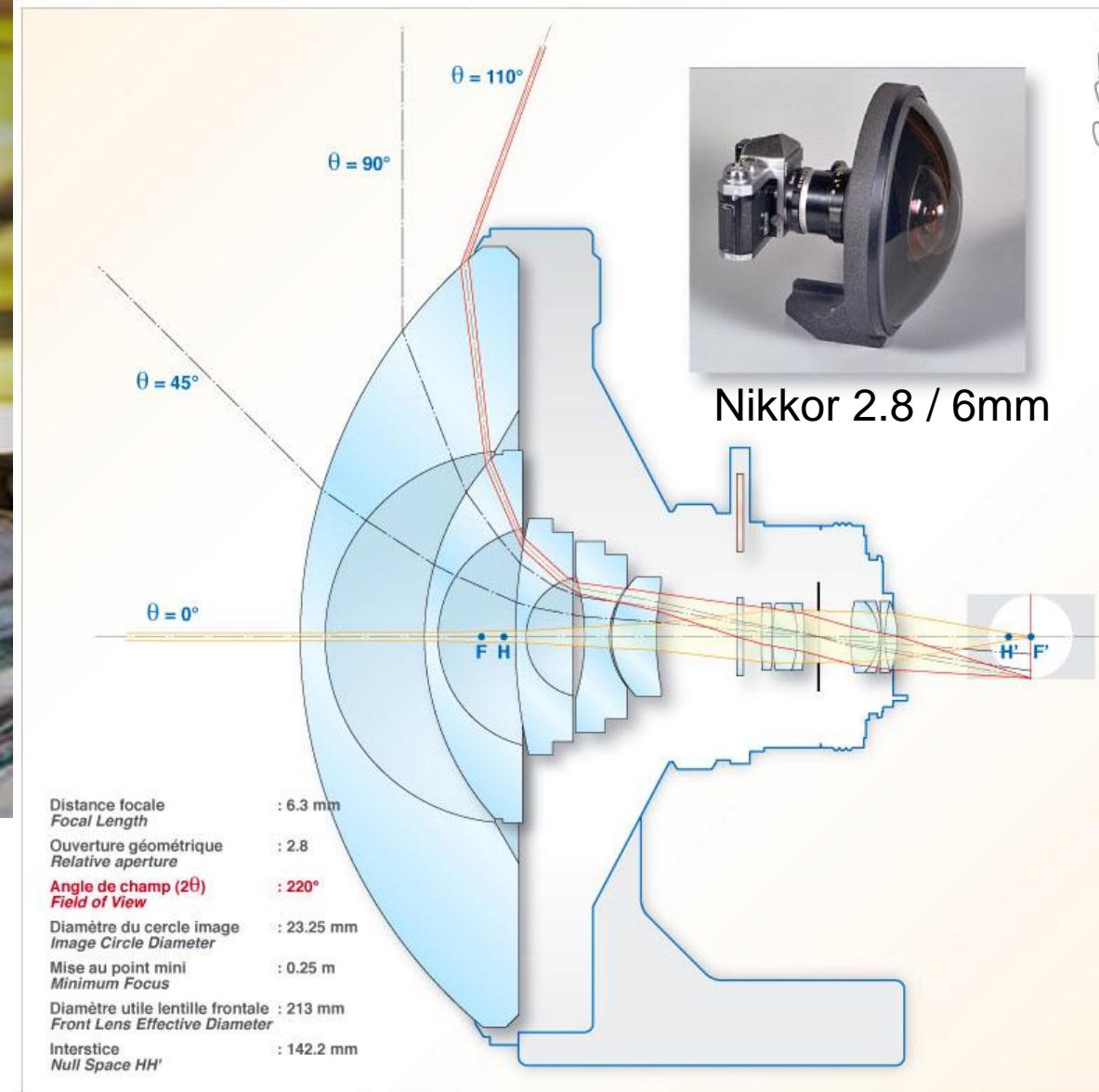
220° horizontal field-of-view, Panini projection

source: T. K. Sharpless

Hyper-Hemispherical Fisheye Lens, field-of-view 220°



Front negative meniscus lens elements enable extreme field-of-views over the complete hemisphere (180°) and beyond. Consequently, those lenses are retrofocus type and require compensation of asymmetric aberrations, e.g. lateral chromatic and coma.



1691-1765, born in Piacenza, Italy

- Practitioner of *vedutismo* (*landscape and architecture painting*)
- Professor of perspective at French Academy in Rome

Unfortunately, there are no documents of his teaching.
Recently reverse engineering studies of his arts have been performed.

Pannini: A New Projection for Rendering Wide Angle Perspective Images

Thomas K. Sharpless¹, Bruno Postle, and Daniel M. German²

¹tksharpless@gmail.com ²dmg@uvic.ca, Dept. of Computer Science, University of Victoria.

Abstract

The widely used rectilinear perspective projection cannot render realistic looking flat views with fields of view



Computer Graphics Scientists analyze paintings of 18th century artists

Surprising result: Scenes are not painted in rectilinear perspective!



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The widely used rectilinear perspective projection cannot render realistic looking flat views with fields of view



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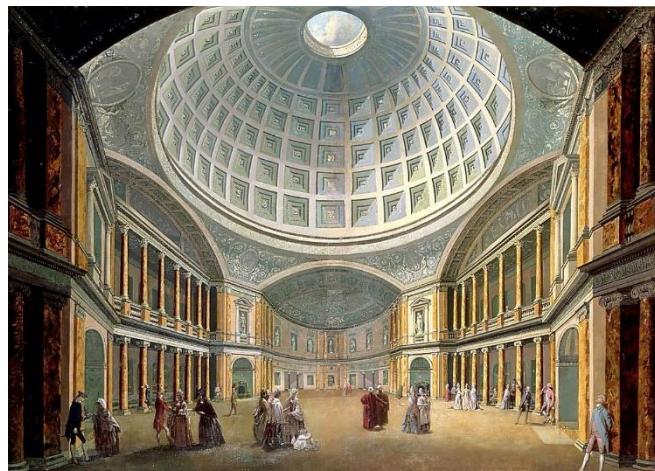
Surprising result: Scenes are not painted in rectilinear perspective!



T. K. Sharpless et al. / The Pannini Projection

Artist	View	Year	Unlikely	Best	FOV°	RMS%	RMS°	D	N
Pannini	Interior Pantheon	1732	rect,ster,cyl	orth	82.8	2.61	1.98	13787	17
	Interior Pantheon	1734	none	ster	57.0	0.57	0.33	1.30	12
	Sta Maria Maggiore,	1753	rect,ster	orth	73.9	1.58	1.09	715365	21
	Piazza San Pietro	1754	rect,ster,cyl	orth	112.8	1.53	1.46	6056	12
	Interior San Pietro	1756	rect,ster,cyl	orth	96.4	0.50	0.43	12.3	21
Hodge	Wyatt's London Pantheon	1772	rect,orth	cyl	171.5	0.57	0.97	2.08	26

Table 1: Horizontal projection analysis of vedutismo paintings. Key: FOV° = horizontal field of view in degrees; RMS% = root mean square error as % of view width; RMS° = same as angle in degrees, D = compression parameter of fitted Pannini projection, N = number of data points.



Art of Giovanni Paolo Panini (1691-1765)

St. Peters Dome



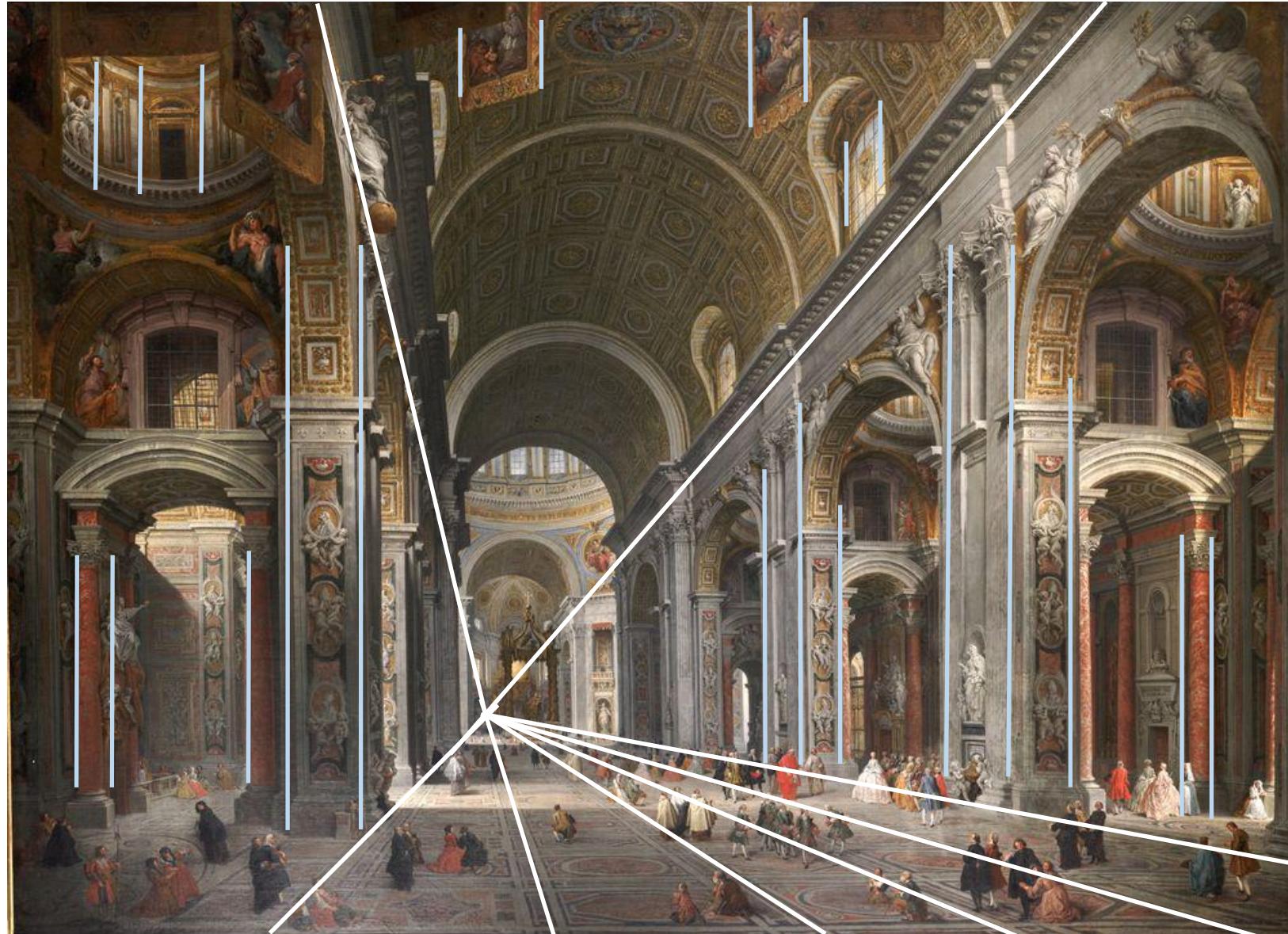
*The Interior of Saint Peter's with
the Visit of the Duc de Choiseul,
1756–57.*

Giovanni Paolo Panini (Italian,
1691–1765).

Oil on canvas; 164.3 x 223.5 cm.
The Boston Athenaeum,
purchase, 1834, UR12

Art of Giovanni Paolo Panini (1691-1765)

St. Peters Dome



Central Projection
Vertical lines are straight

Art of Panini vs photographic pictures

St. Peters Dom



Nowadays you can take photos there...
But why does the perspective in Panini's paintings
seem so much more pleasant than in the photos?

Artistic Proportioning: Painting of Panini versus Photography



Same place, same
field-of-view:

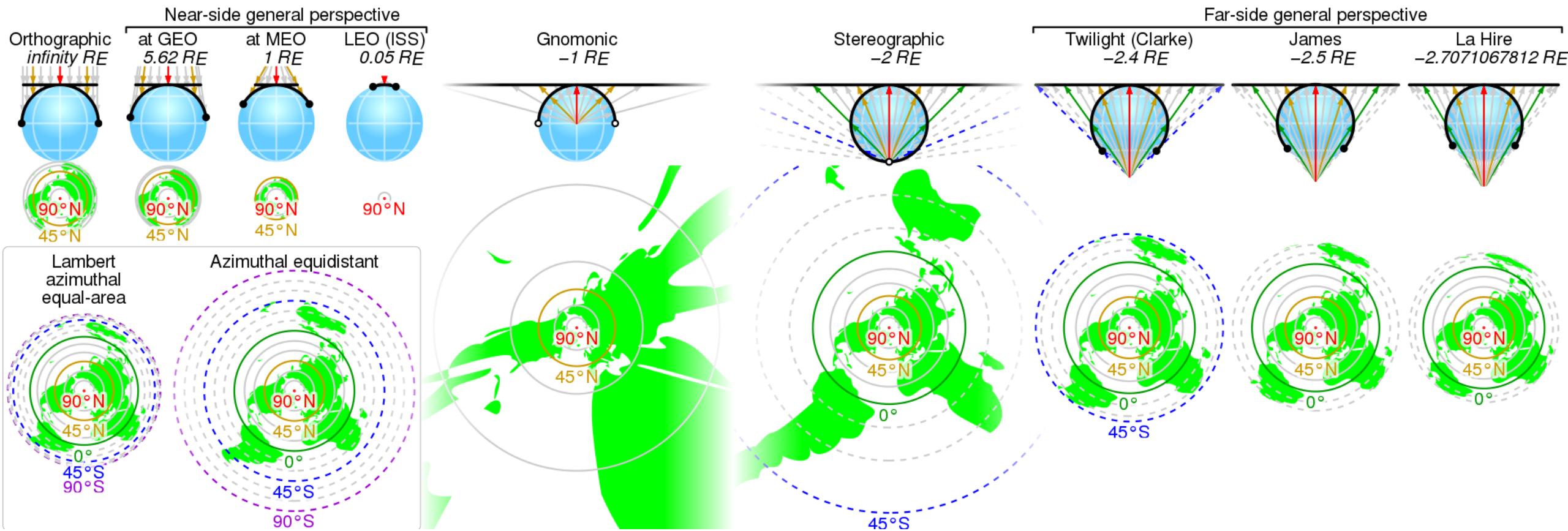
Panini (1754):
Piazza San Pietro

Horizontal field-
of-view ca. 120°



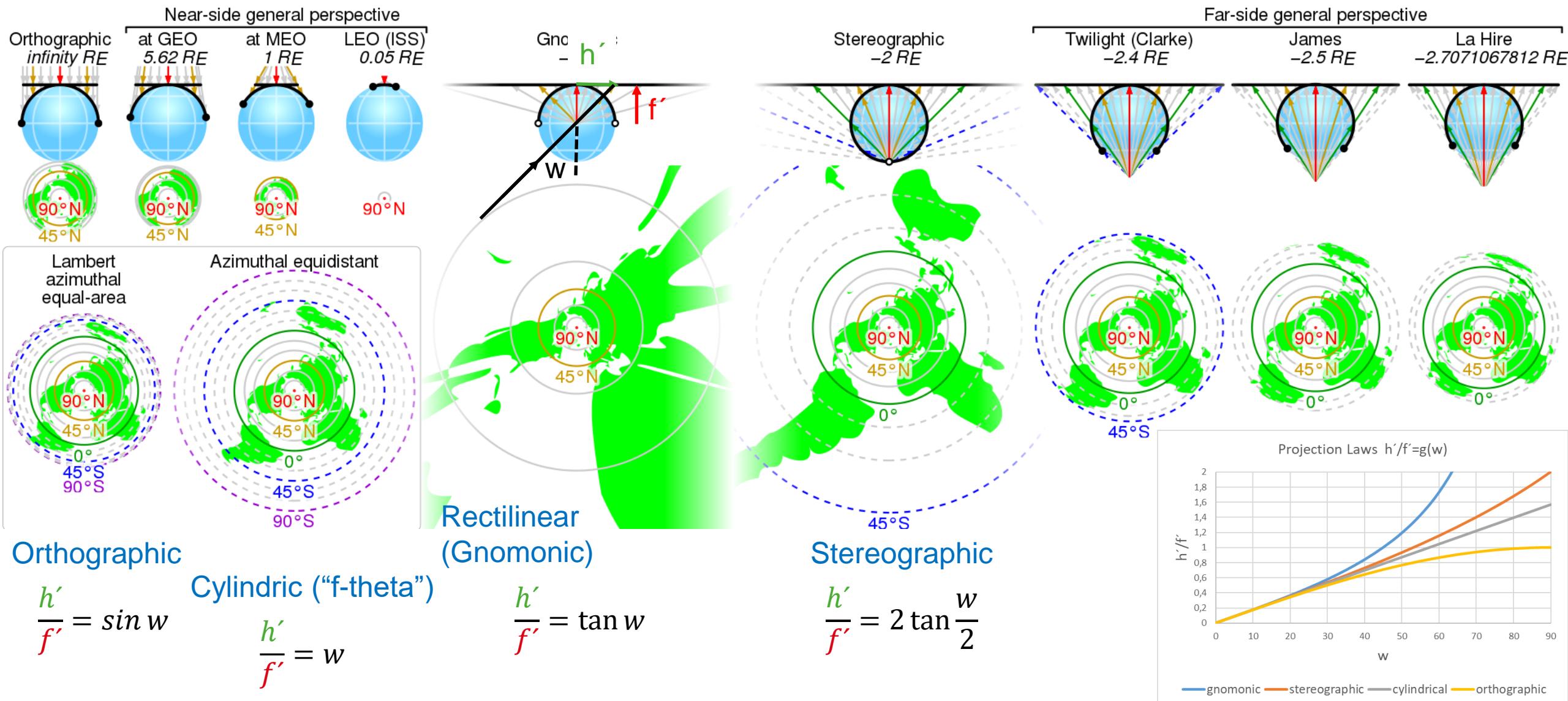
Panorama
picture of St
Peters Place

Cartography: Mapping surface of sphere onto plane



Rotational-symmetrical projection laws known from cartography.

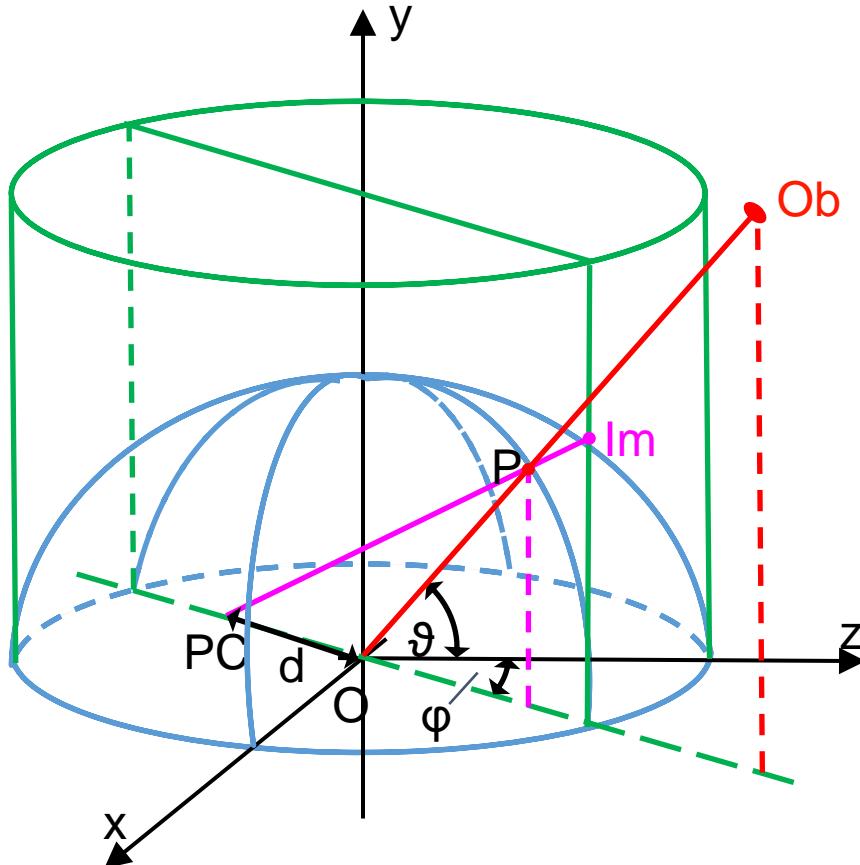
Cartography: Mapping surface of sphere onto plane



General Panini Perspective Transformation

(Formulation by Th. Sharpless)

Cylindrical coordinates are used to define the non-rotational-symmetric Panini projection. There is a free parameter **d** to adjust the “squeezing” along horizontal direction.



General Panini Perspective Transformation:

$$\frac{x'}{f'} = \sin \varphi \frac{d + 1}{d + \cos \varphi}$$

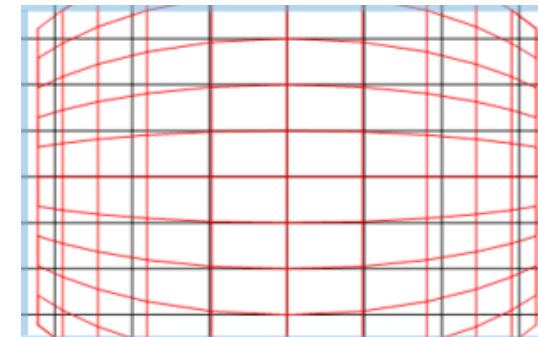
$$\frac{y'}{f'} = \tan \vartheta \frac{d + 1}{d + \cos \varphi}$$

horizontal

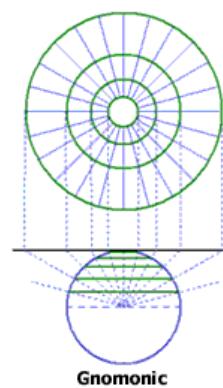
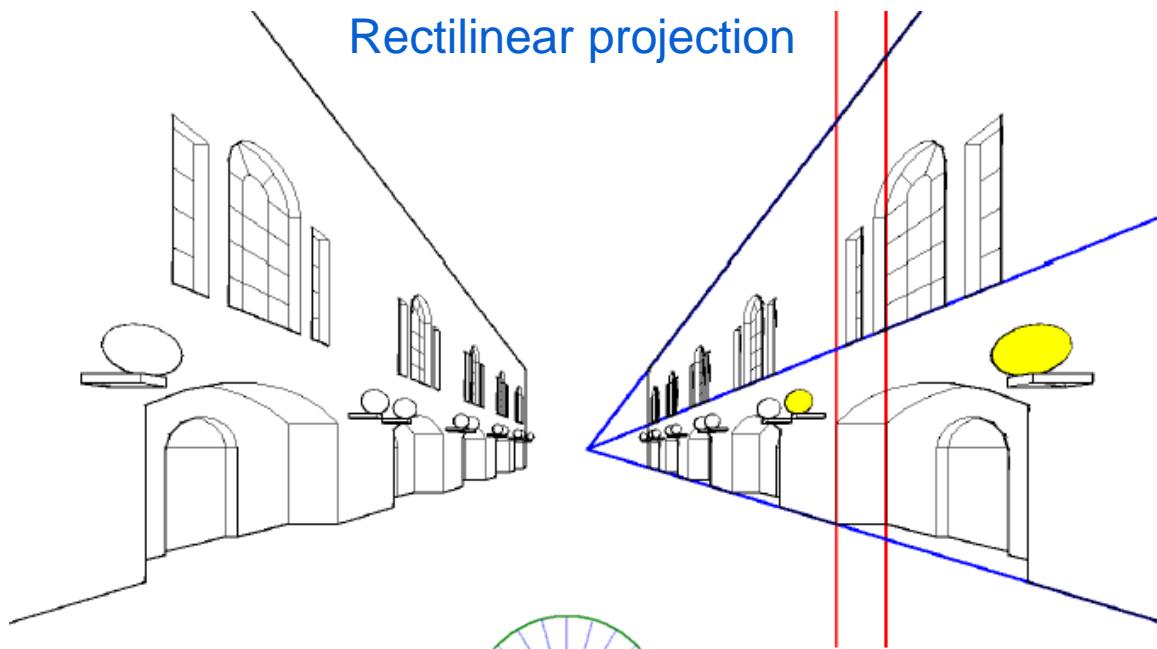
vertical

d	0	1	inf
h	$\tan \varphi$	$2 \tan \varphi / 2$	$\sin \varphi$
v	$\tan \vartheta / \cos \varphi$	$\frac{2 \tan \vartheta}{1 + \cos \varphi}$	$\tan \vartheta$

rectilinear rect./stereographic rect./orthographic

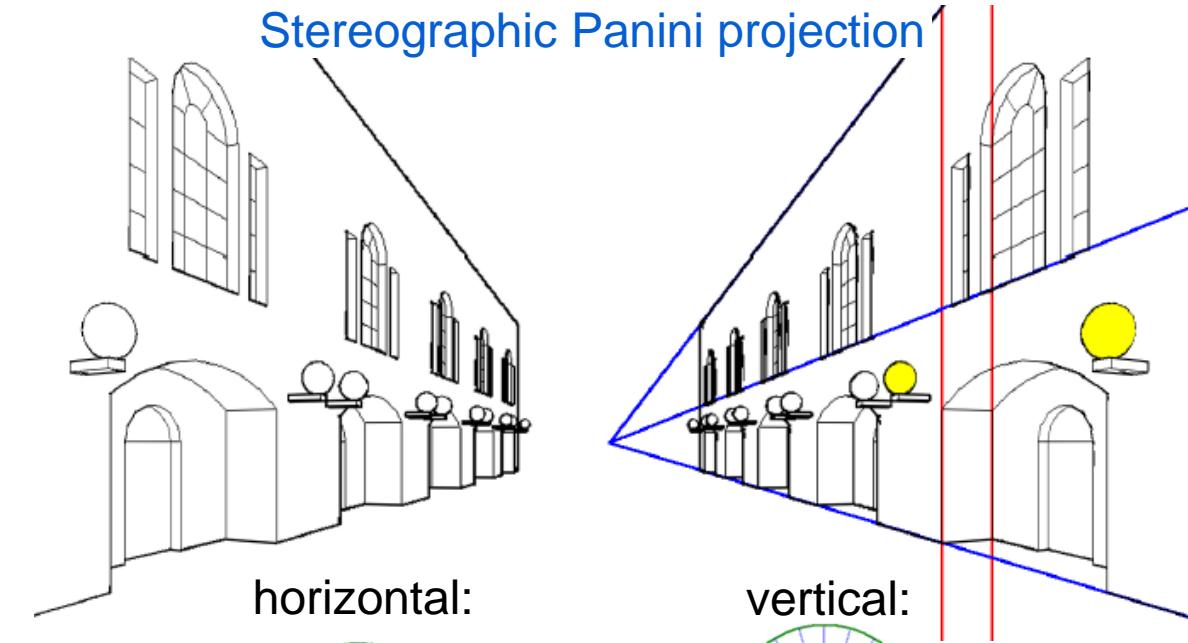


Rectilinear versus Panini perspective

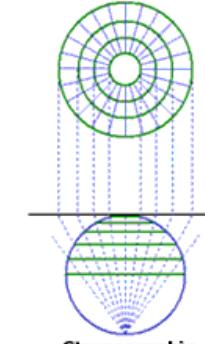


$$\frac{\sqrt{x'^2 + y'^2}}{f} = \tan w$$

source: T.K. Sharpless

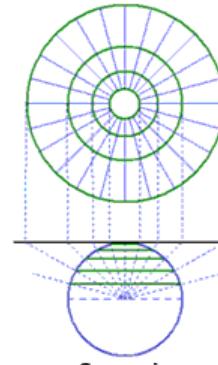


horizontal:



Stereographic

vertical:



Gnomonic

$$\frac{x'}{f} = 2 \tan \varphi / 2$$

$$\frac{y'}{f} = \frac{2 \tan \vartheta}{1 + \cos \varphi}$$

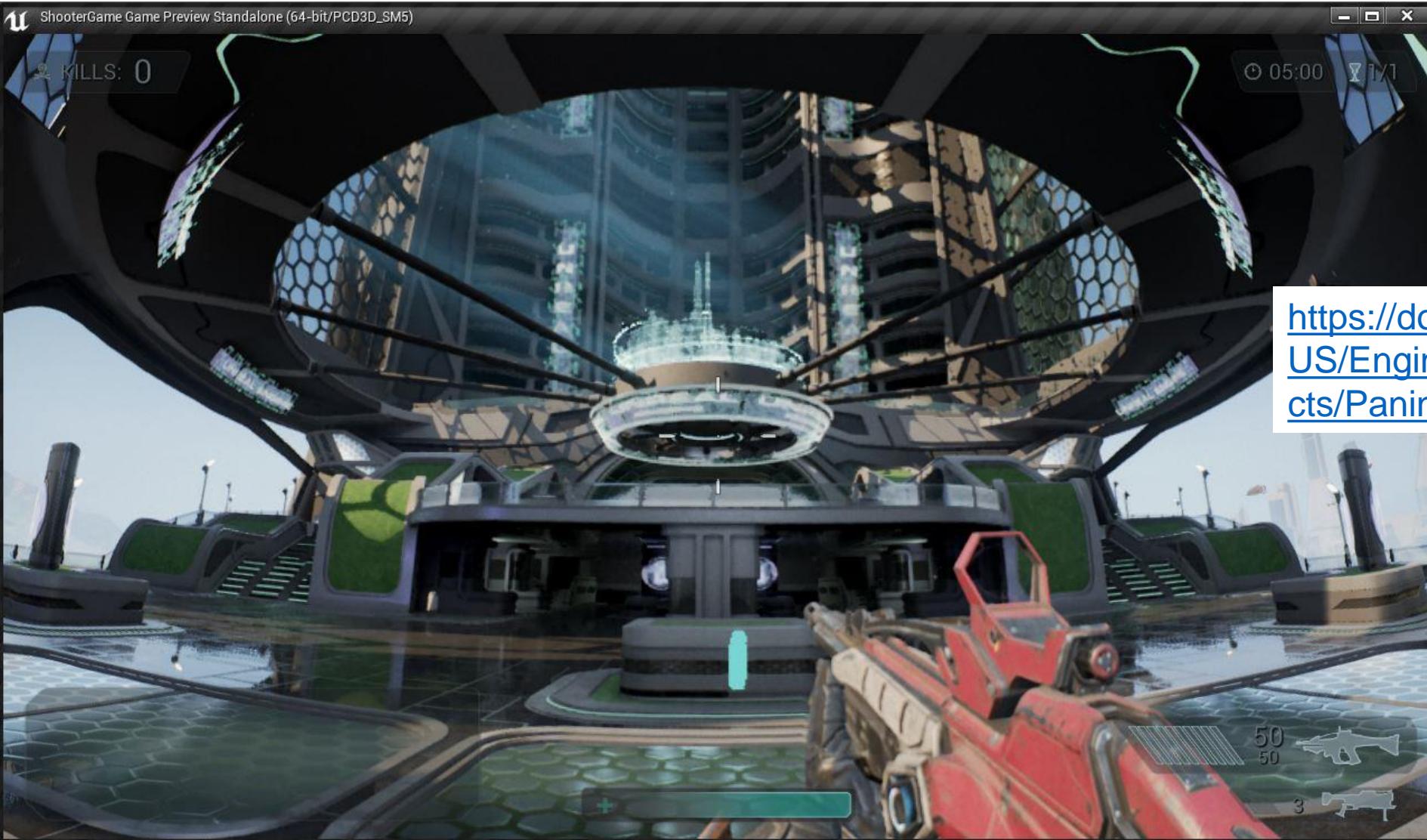
Panini Perspective (Unreal Engine)

<https://docs.unrealengine.com/en-US/Engine/Rendering/PostProcessEffects/PaniniProjection/index.html>



rectilinear

Panini Perspective (Unreal Engine)



Panini

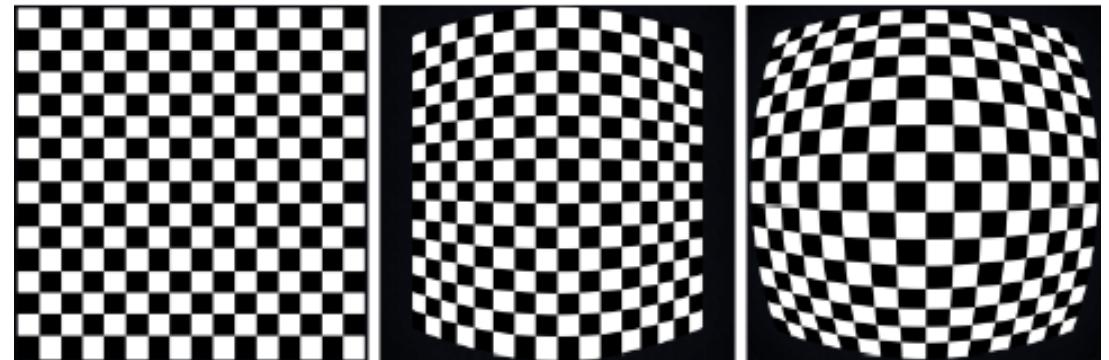
<https://docs.unrealengine.com/en-US/Engine/Rendering/PostProcessEffects/PaniniProjection/index.html>

Which of these wide FOV perspective projections looks most natural?

Napieralla, Jonah & Sundstedt, Veronica (2020). Ultrawide Field of View by Curvilinear Projection Methods. Journal of WSCG. 28. 163-167. 10.24132/JWSCG.2020.28.20.



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Applied Physics
Friedrich-Schiller-Universität Jena

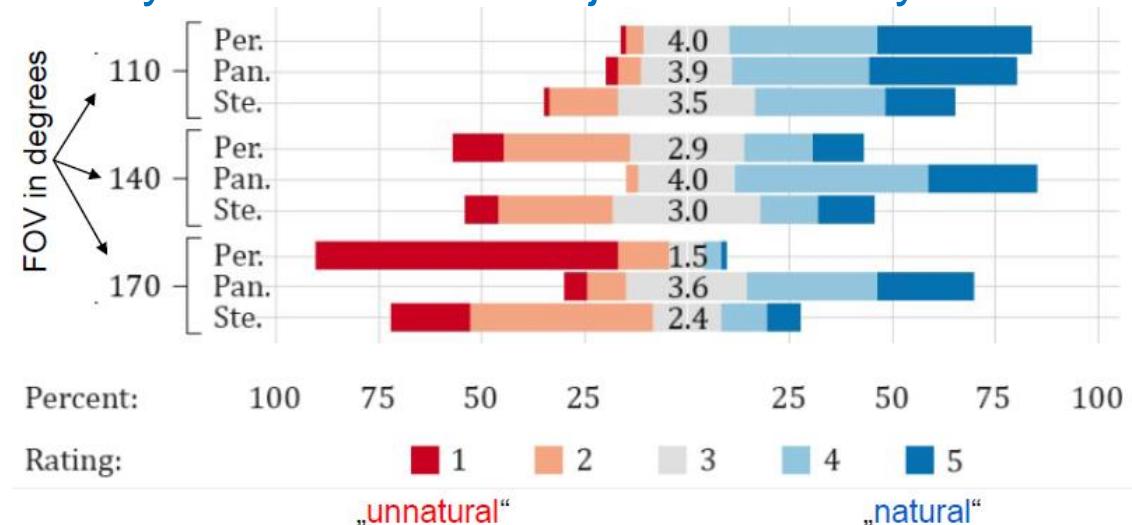


(a) Perspective

(b) Panini

(c) Stereographic

Study of Perceived Subjective Quality:



→ Panini projection superior for most scenes with increasing field-of-view.

A new type of wide-angle lens projecting in Panini perspective

(12) NACH DEM VERTRAG ÜBER DIE INTERNATIONALE ZUSAMMENARBEIT AUF DEM GEBIET DES PATENTWESENS (PCT) VERÖFFENTLICHTE INTERNATIONALE ANMELDUNG

(19) Weltorganisation für geistiges Eigentum
Internationales Büro

(43) Internationales Veröffentlichungsdatum
18. August 2022 (18.08.2022)



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(22) Internationales Anmeldedatum:
20. Januar 2022 (20.01.2022)

(25) Einreichungssprache: Deutsch

(26) Veröffentlichungssprache: Deutsch

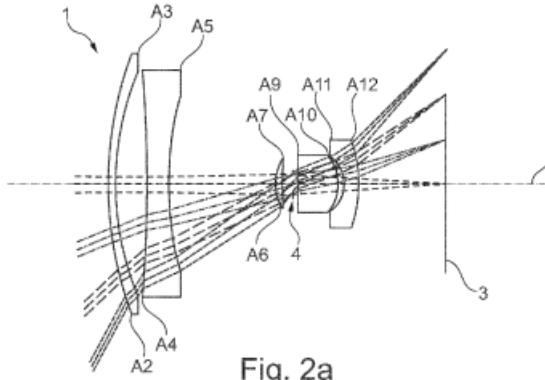
(30) Angaben zur Priorität:
10 2021 103 323.3
12. Februar 2021 (12.02.2021) DE

(71) Anmelder: CARL ZEISS AG [DE/DE]; Carl-Zeiss-Straße 22, 73447 Oberkochen (DE).

(72) Erfinder: BLAHNIK, Vladan; Keltensstraße 5, 73447 Oberkochen (DE).

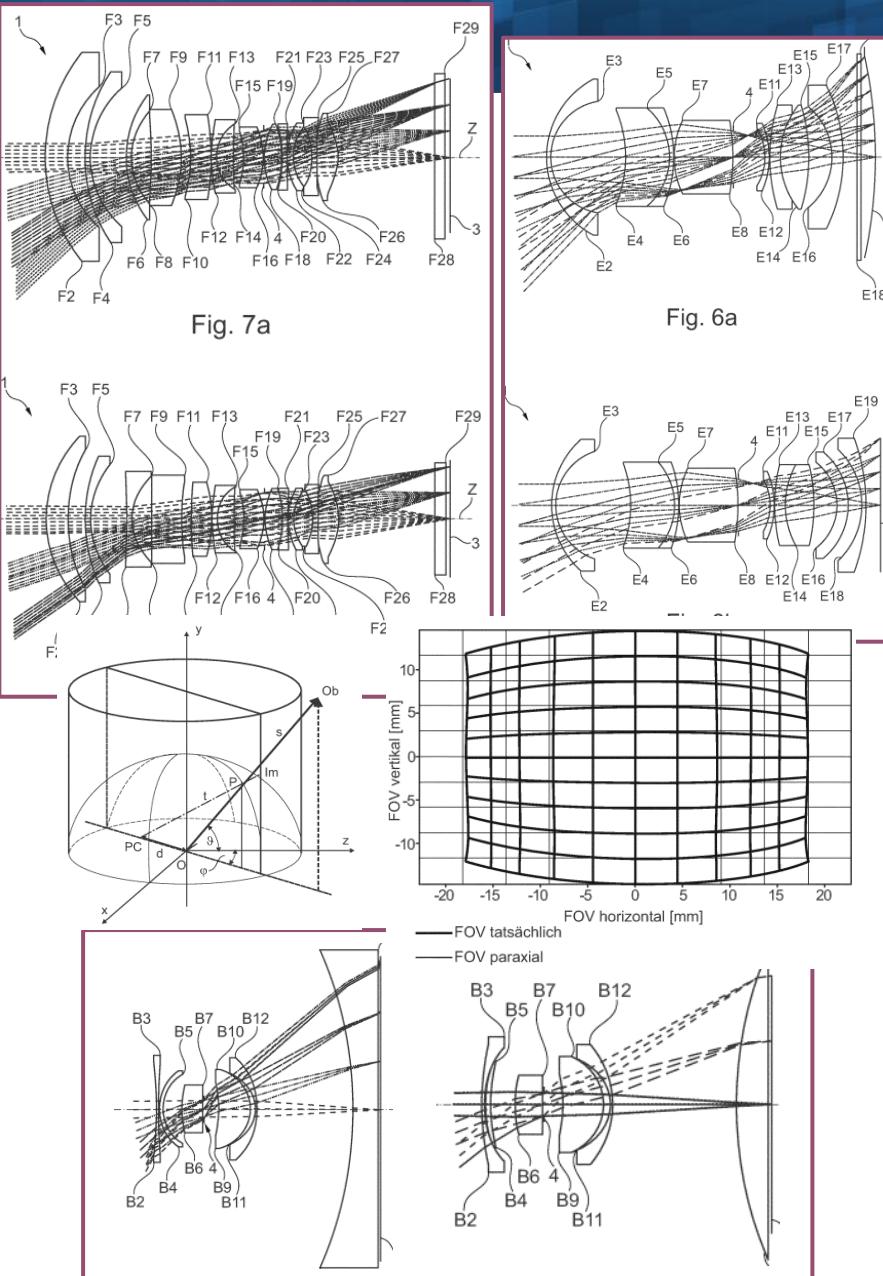
(54) Titel: PANNINI LENS AND IMAGING OPTICAL DEVICE

(54) Bezeichnung: PANNINI-OBJEKTIV UND ABBILDENDES OPTISCHES GERÄT



(57) Abstract: A lens (1) is specified which has image-forming optical elements (2) which have curved surfaces with a non-rotational effect and are arranged along an optical axis z. The image-forming optical elements (2) are configured and arranged in such a way that an image to be formed on an image surface (3) is a Panini projection having the following features: identical projection scale in the center of the image along every direction in the image surface (3), straight vertical projection of straight vertical lines over the entire image field and barrel distortion of the object space in a horizontal direction. An imaging optical device (10), a mobile terminal (100) and also a set comprising a lens (1) or an imaging optical device (10) and instructions for use (201) are additionally specified.

(57) Zusammenfassung: Es wird ein Objektiv (1) angegeben, das bildformende optische Elemente (2) mit nichtrotationsymmetrisch wirkenden gekrümmten Flächen aufweist, die entlang einer optischen Achse z angeordnet sind. Die bildformenden optischen Elemente



Patent (V. Blahnik) on optical designs of a new type of camera lens projecting in alternative perspective.

Requires cylindric, toric or freeform lens technology, which is currently becoming state-of-the-art.

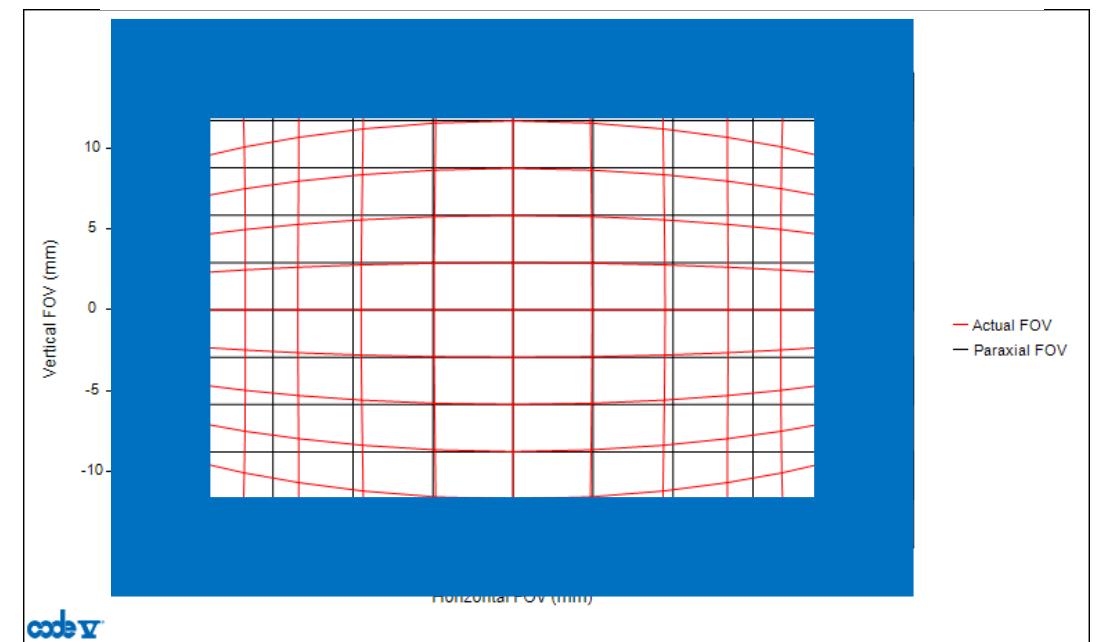
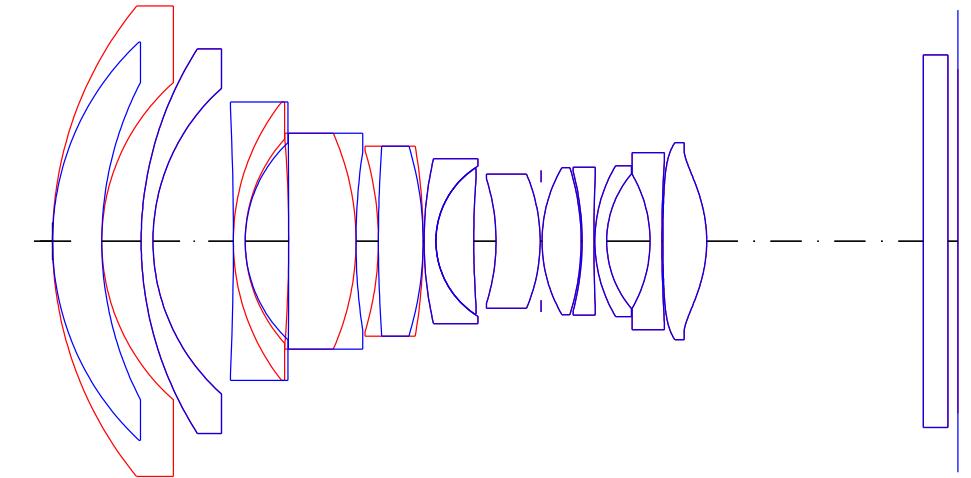
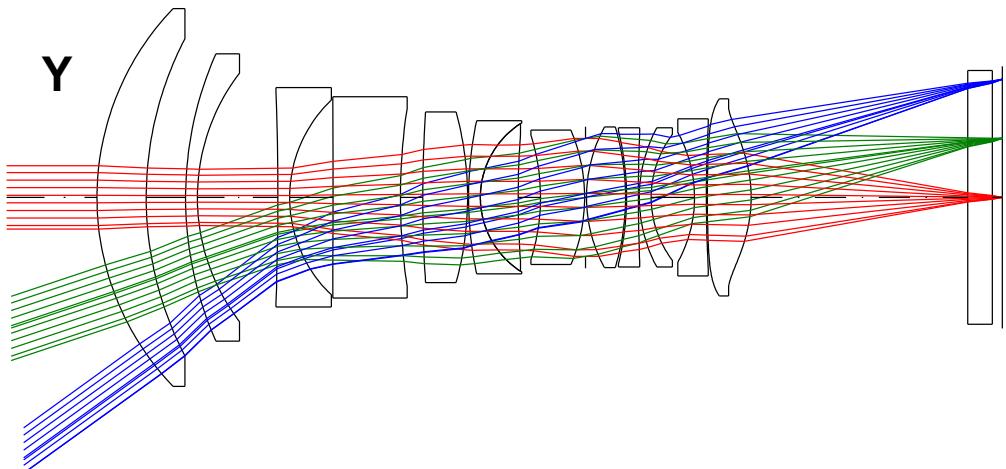
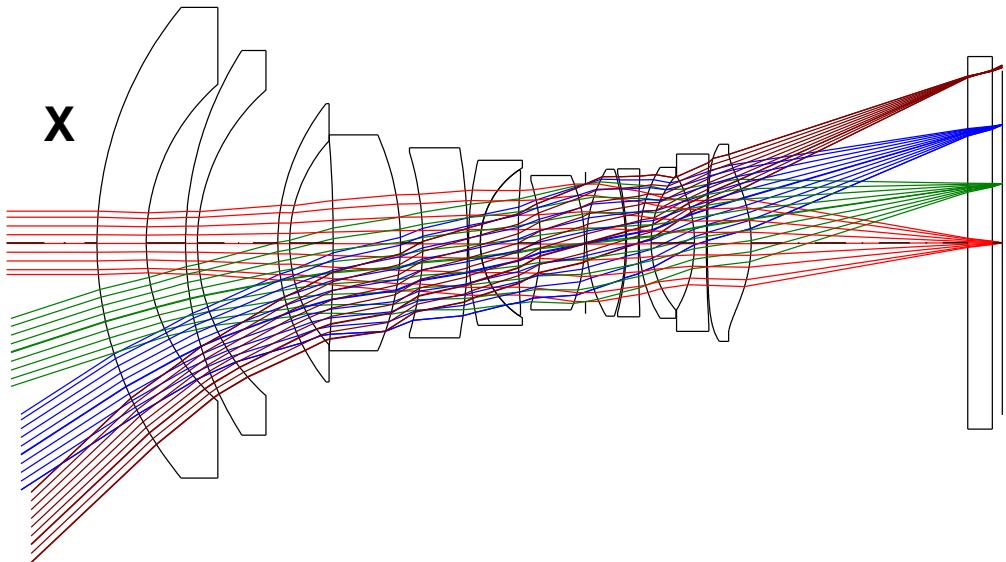
Cylindrical curved sensors are beneficial to improve optical performance, much simpler technology compared to spherically curved sensors, where there were many yet commercially not successful attempts.

Extensive collection of optical design examples for different application fields:

- Cinematography
- Professional photography
- Smartphone imaging
- Action cams
- Industrial

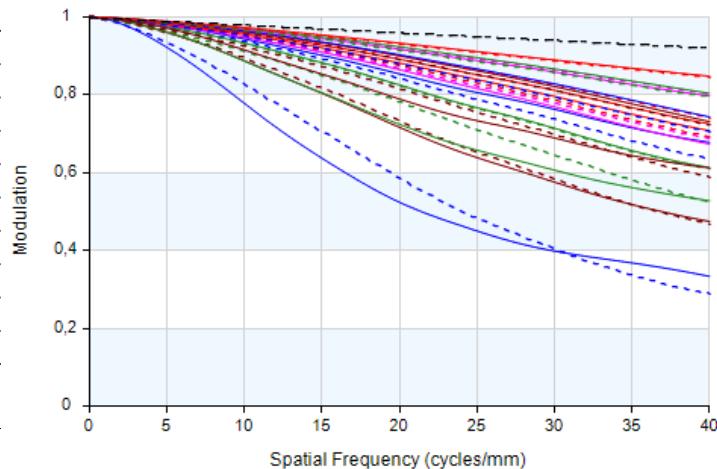
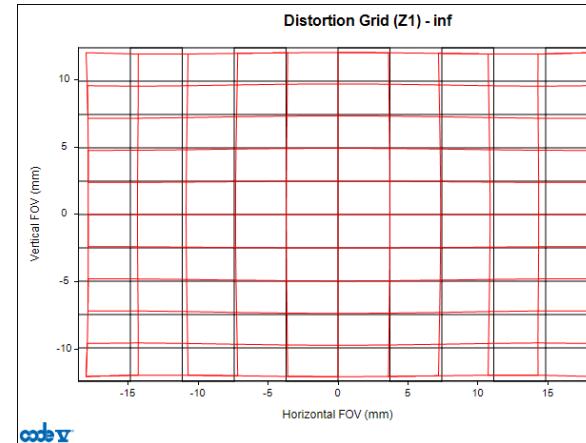
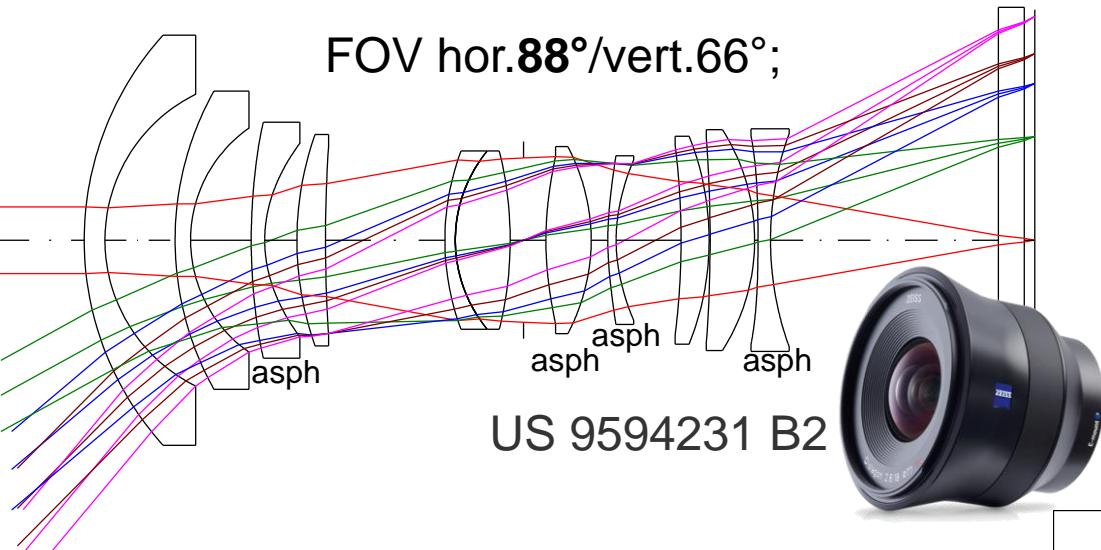
2.8/18mm Wide-Angle Lens Mirrorless Full Format (35mm) FOV hor.104° / vert.66°

(for comparison: rect. f=18mm:
hor.88°/vert.66°; rect. lens would need f=14mm)

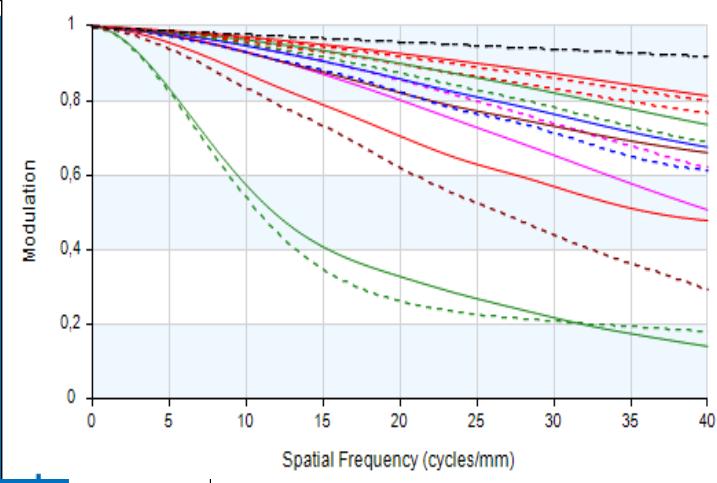
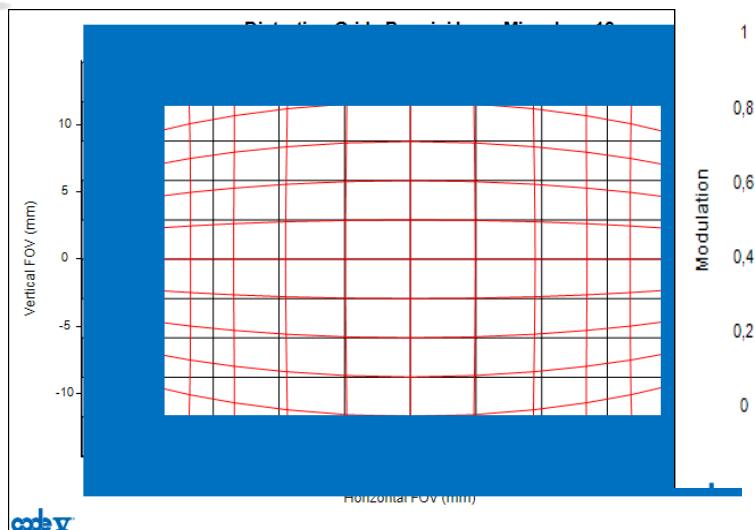
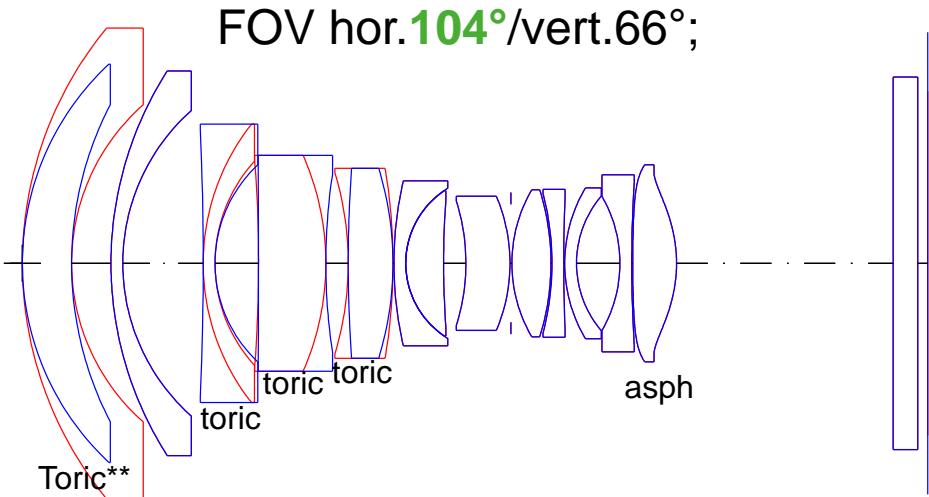


2.8/18mm Wide-Angle Lens Mirrorless Full Format (35mm) ZEISS Batis 2.8/18 and Panini Design*

Batis 2.8/18



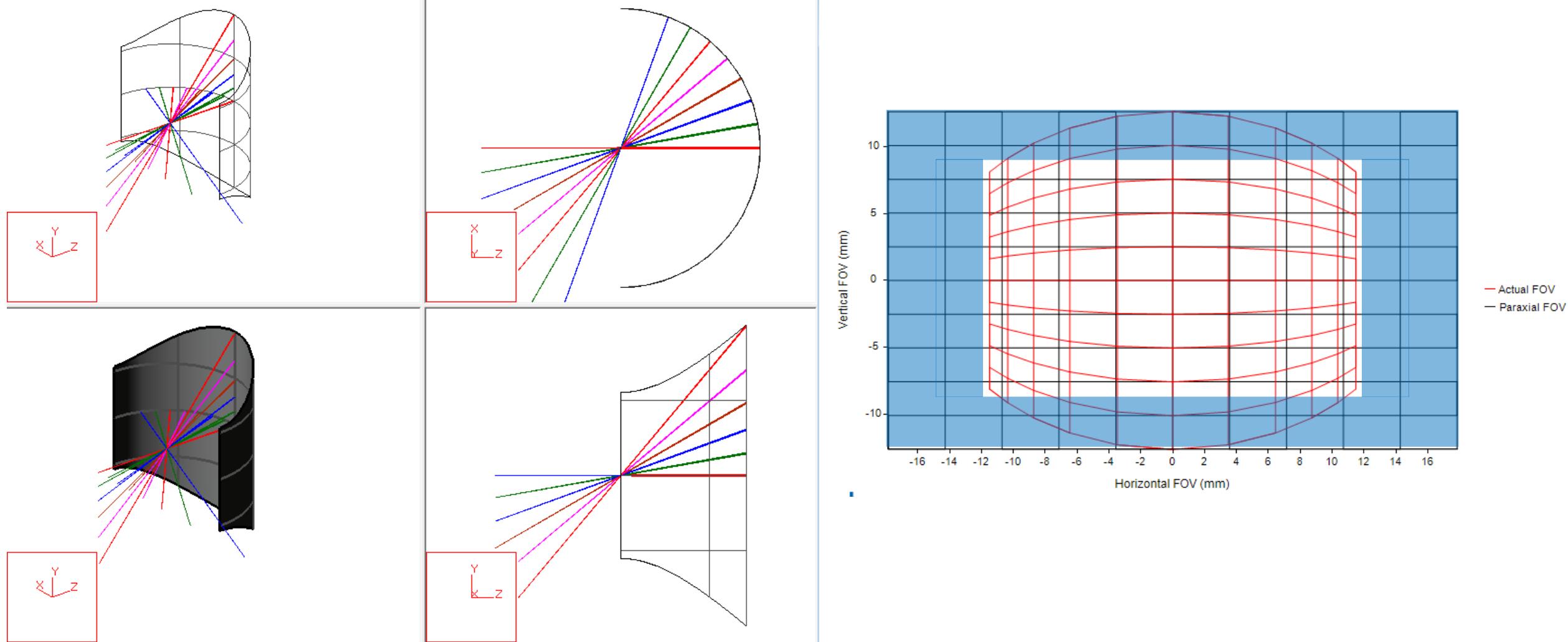
Panini 2.8/18



* Lens yet only optimized for infinite object distance

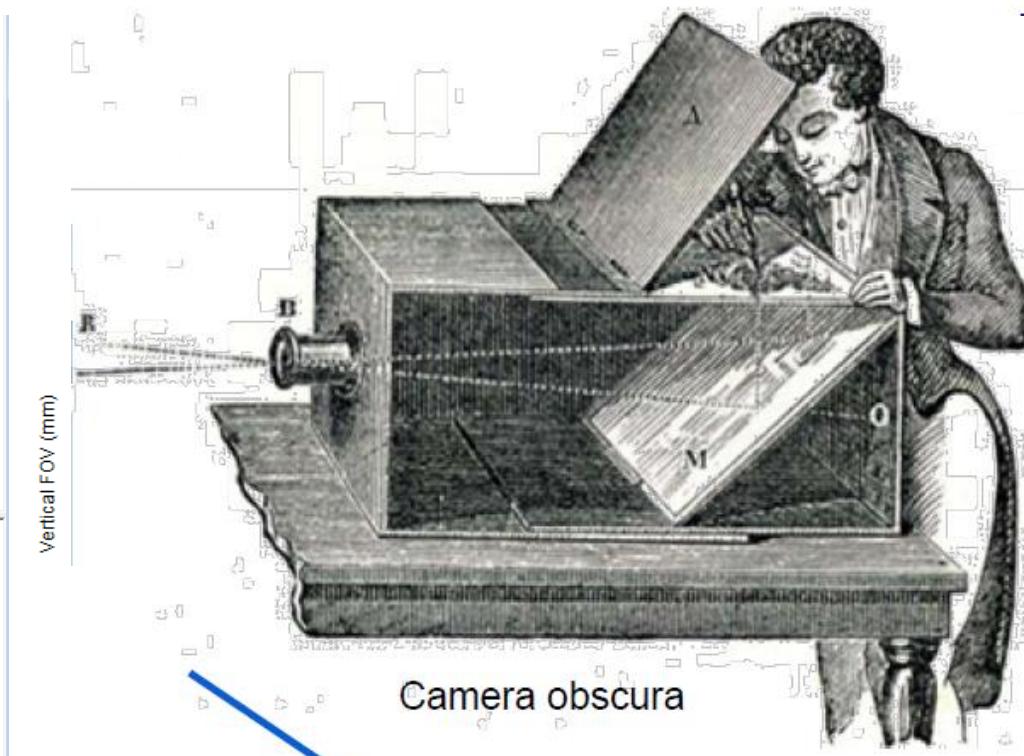
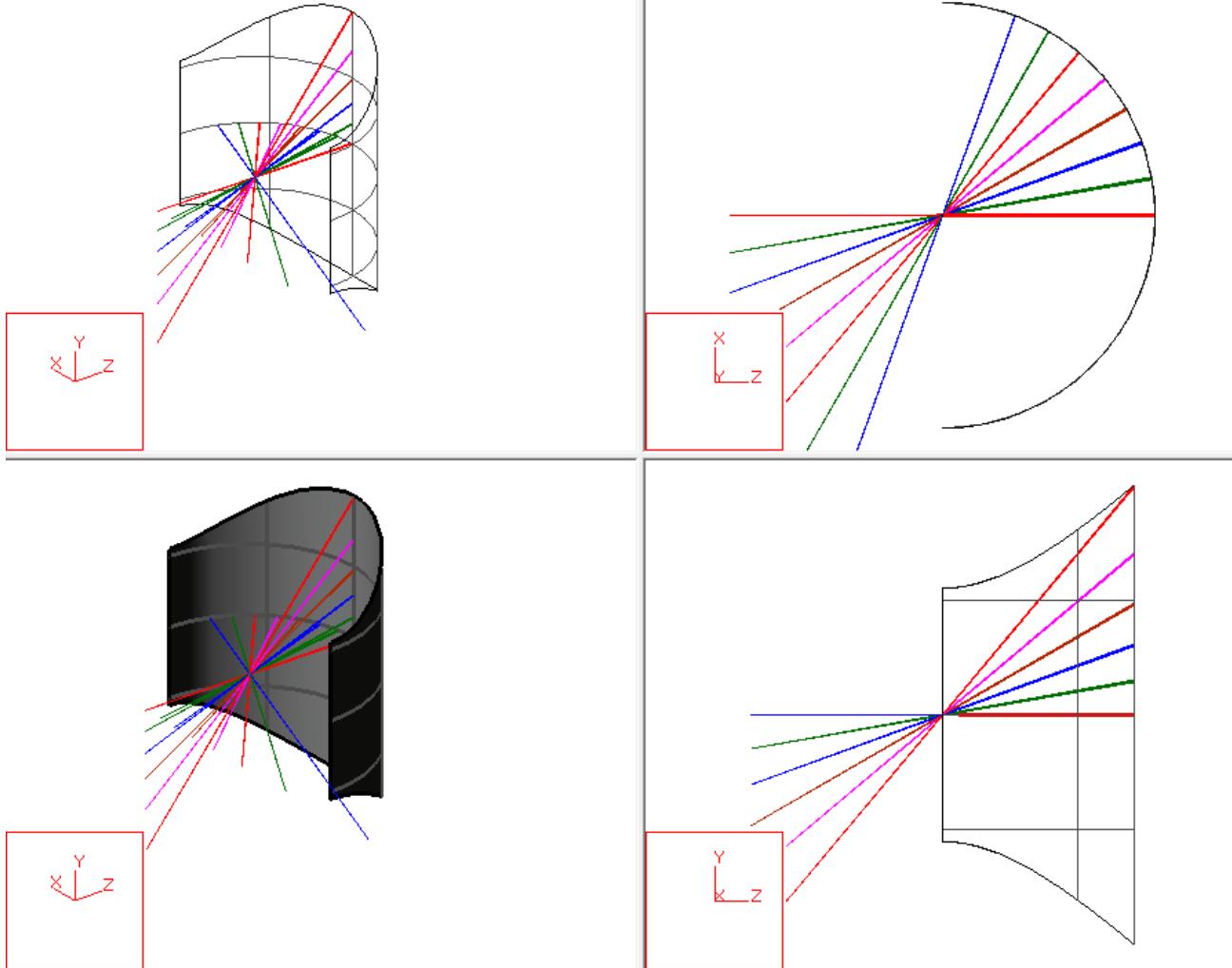
**Toric w/o higher order free form coefficients

Pinhole image on cylindric image surface

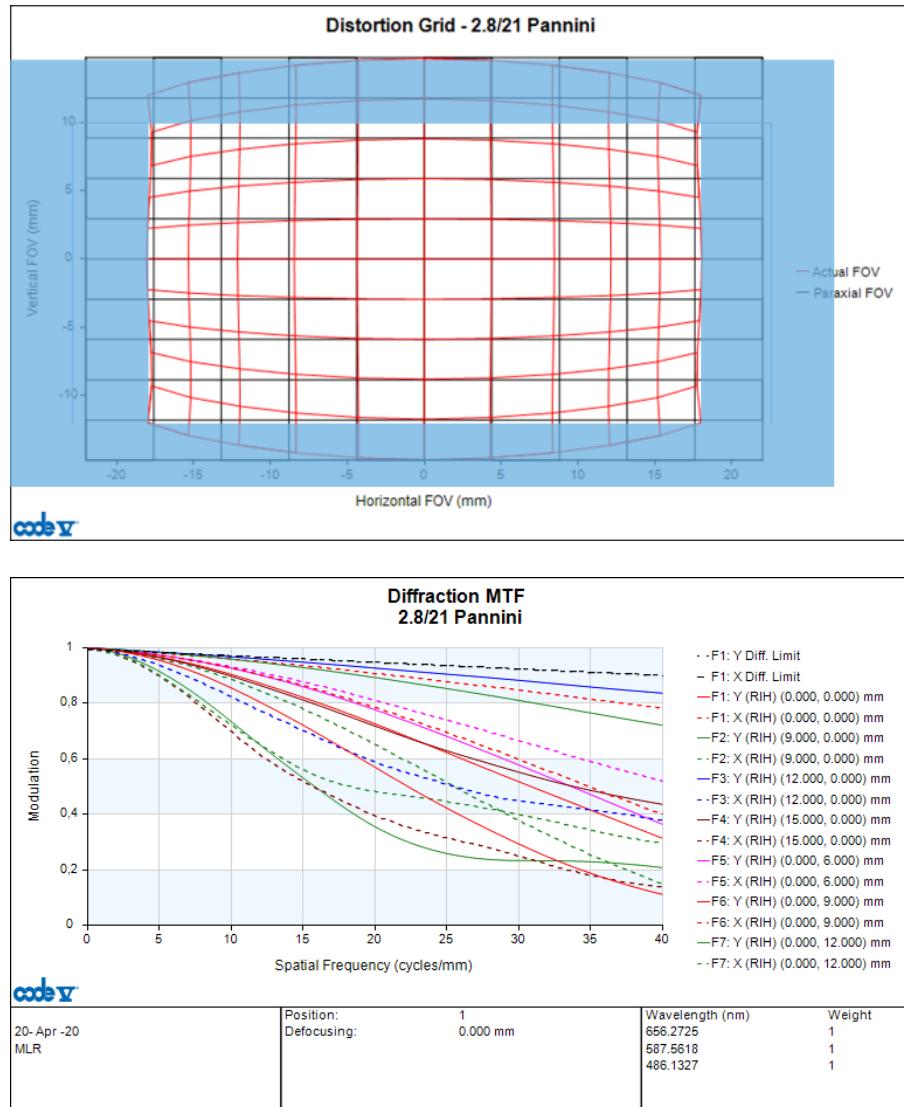
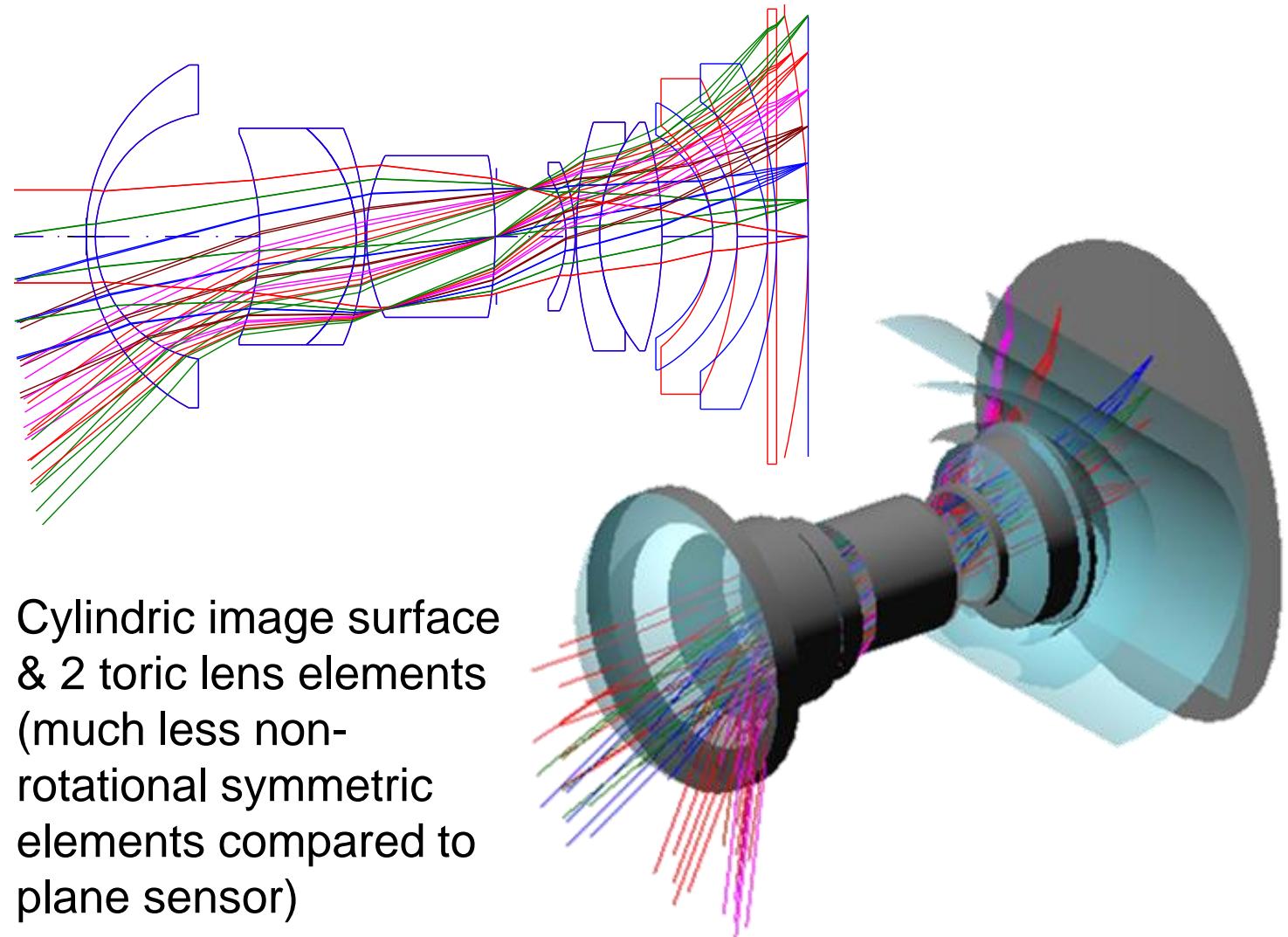


Pinhole image on cylindric image surface

Presumably perspective sketch on cylindric surface by Panini?

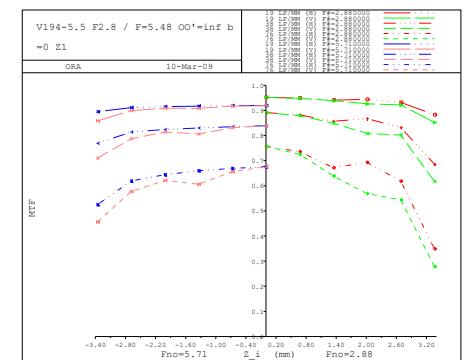
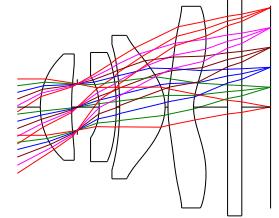
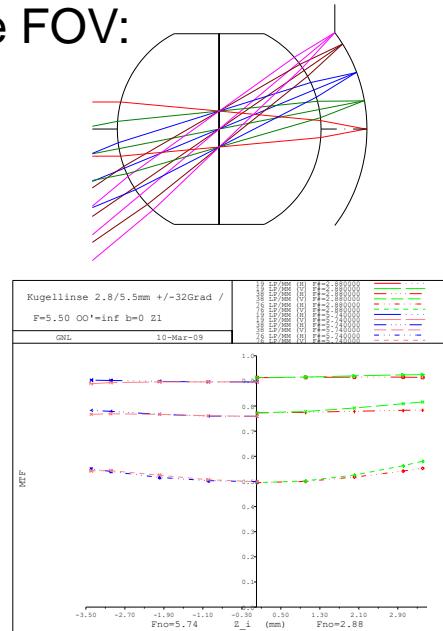


Panini lens camera system with cylindric image sensor

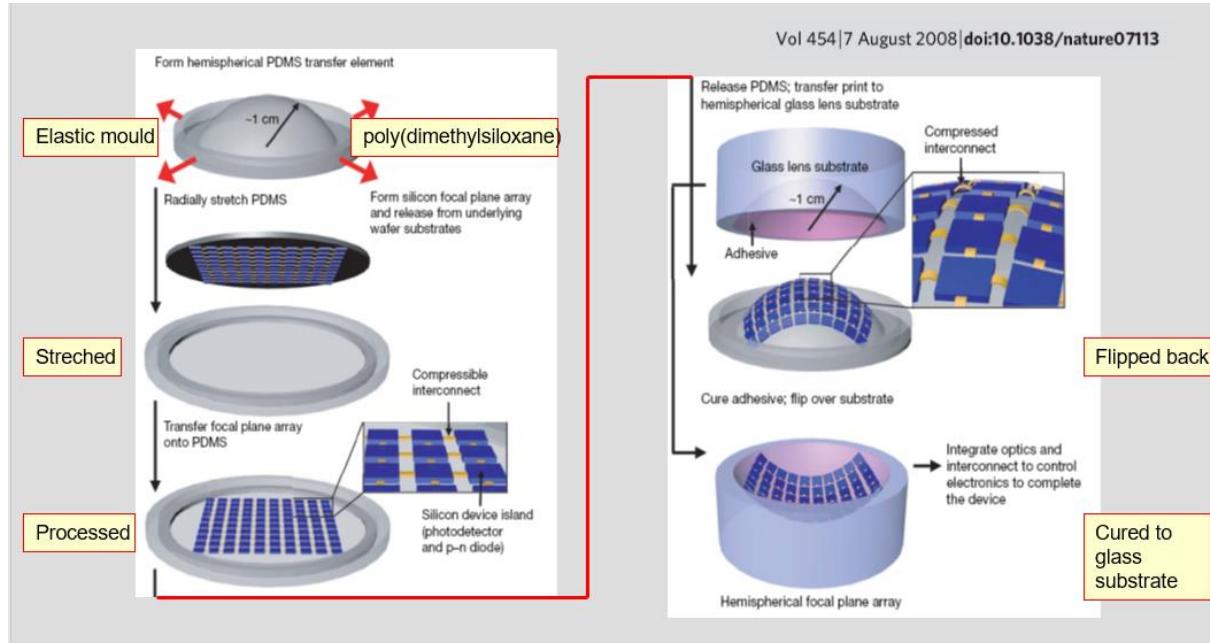
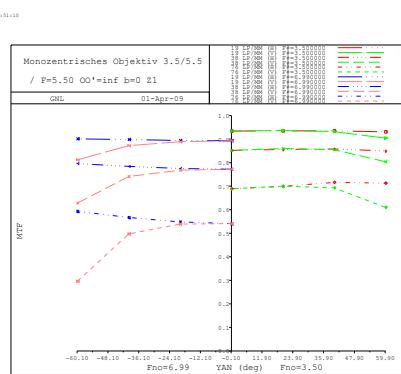
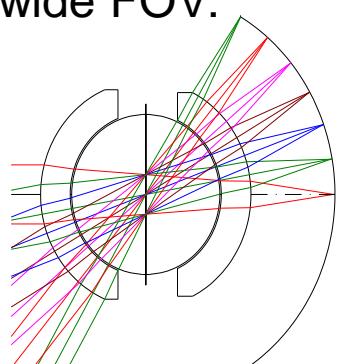


Studies on spherically curved image sensors (rotational symmetric lenses)

Wide FOV:



Extreme wide FOV:

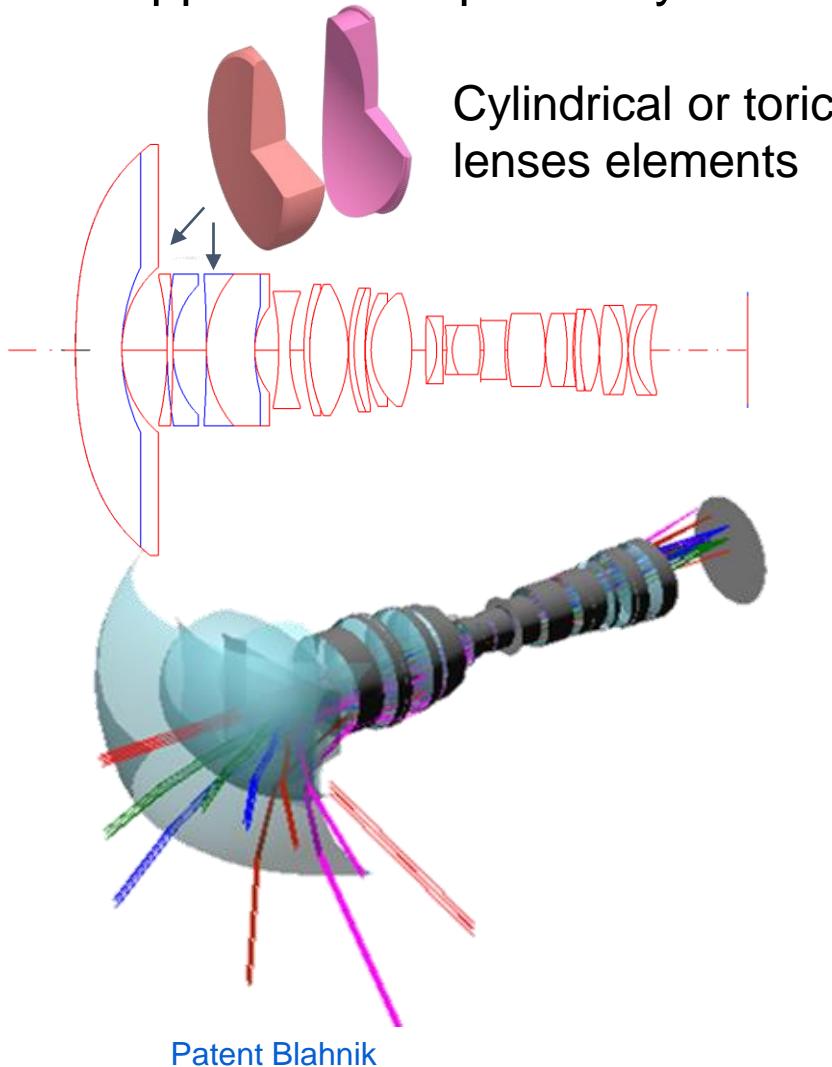


After more than one decade of different attempts (also by leading companies like SONY) spherically curved sensors are available but still not broadly established.

Cylindrically curved sensor are significantly easier to realize.

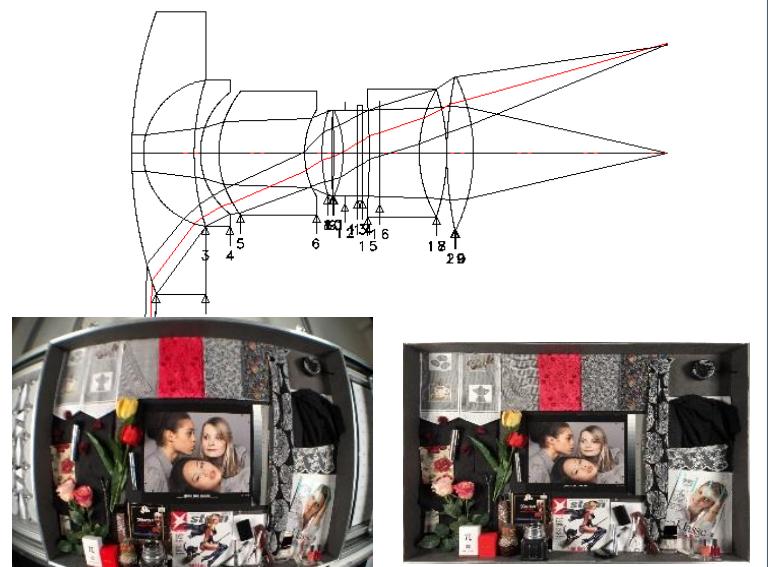
Classical optical design vs digital optical co-optimization

Approach 1: Optics only



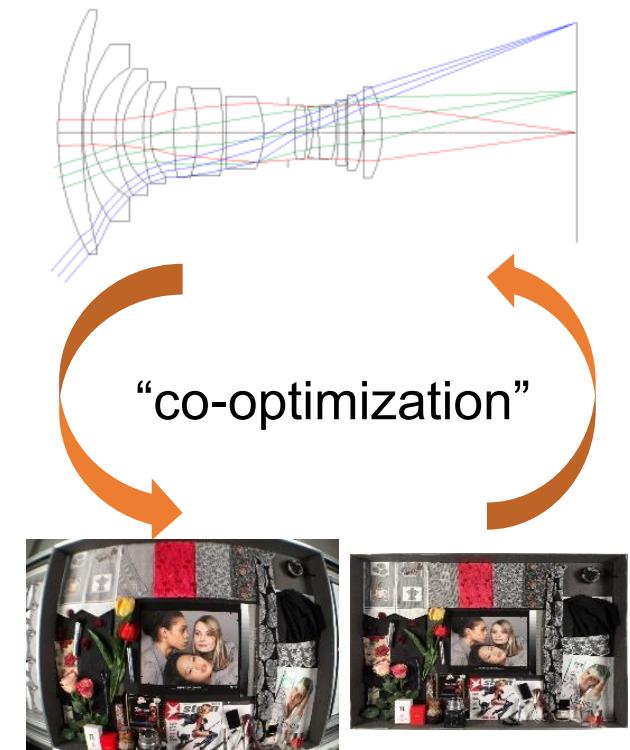
Approach 2:

- simple lens & software
- Panini perspective
- Performance with undesired resolution loss?



• “Th. Sharpless demonstrations and pictures”

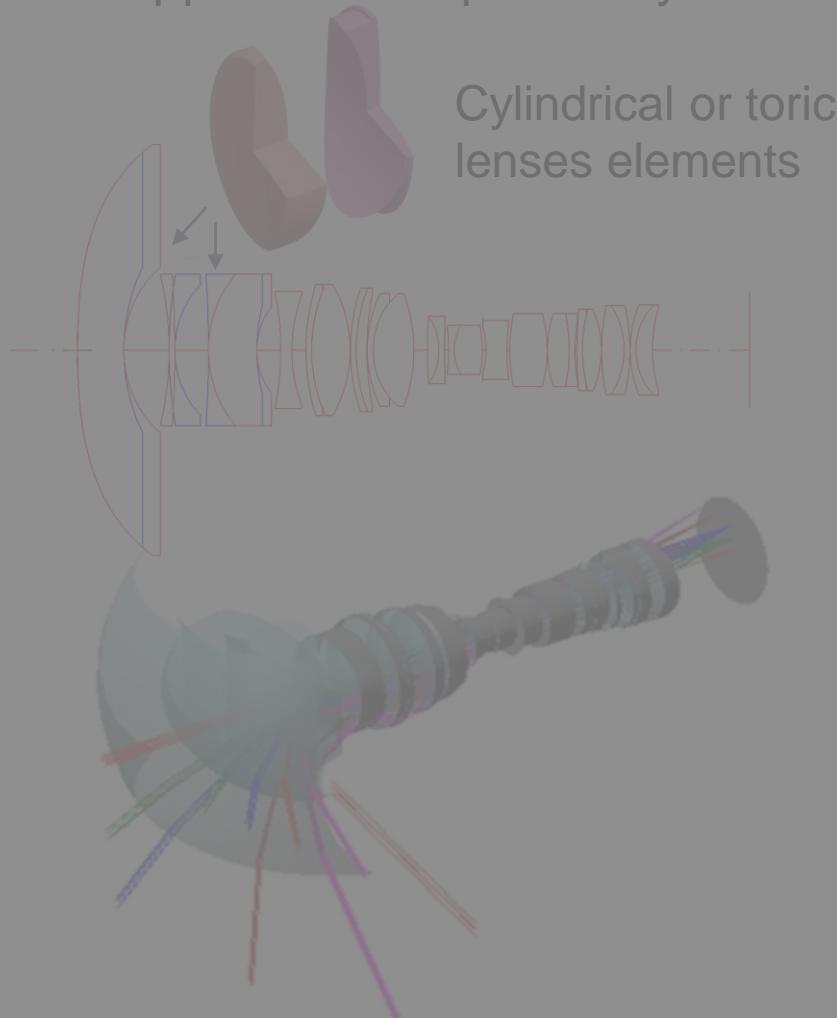
Approach 3: Co-optimize system to reduce undesired resolution loss



digital transf., control MTF,
shading, chromatic aberration etc

Classical optical design vs digital optical co-optimization

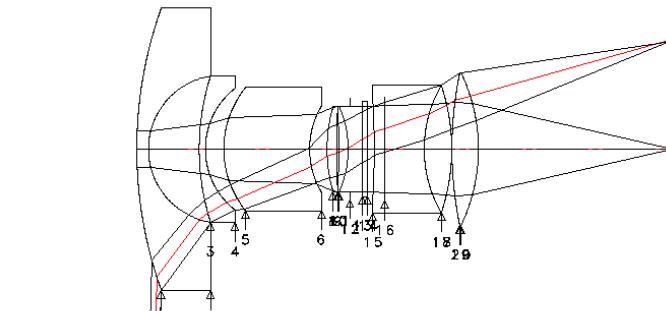
Approach 1: Optics only



Patent Blahnik

Approach 2:

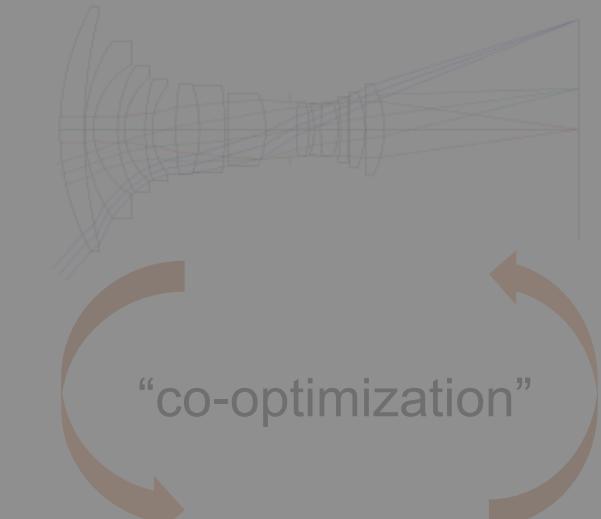
- simple lens & software
Panini perspective
Performance with undesired resolution loss?



digital transformation

- “Th. Sharpless demonstrations and pictures”

Approach 3: Co-optimize system to reduce undesired resolution loss



digital transf., control MTF,
shading, chromatic aberration etc



Picture with fisheye lens.

Task:
Transform to rectilinear perspective.



Reverse mapping of
fisheye view to
rectilinear perspective.

Corner regions are
strongly magnified.



Cropping to represent
image on rectangular
display.

Corner regions get lost.

Digitally transformed Panini view

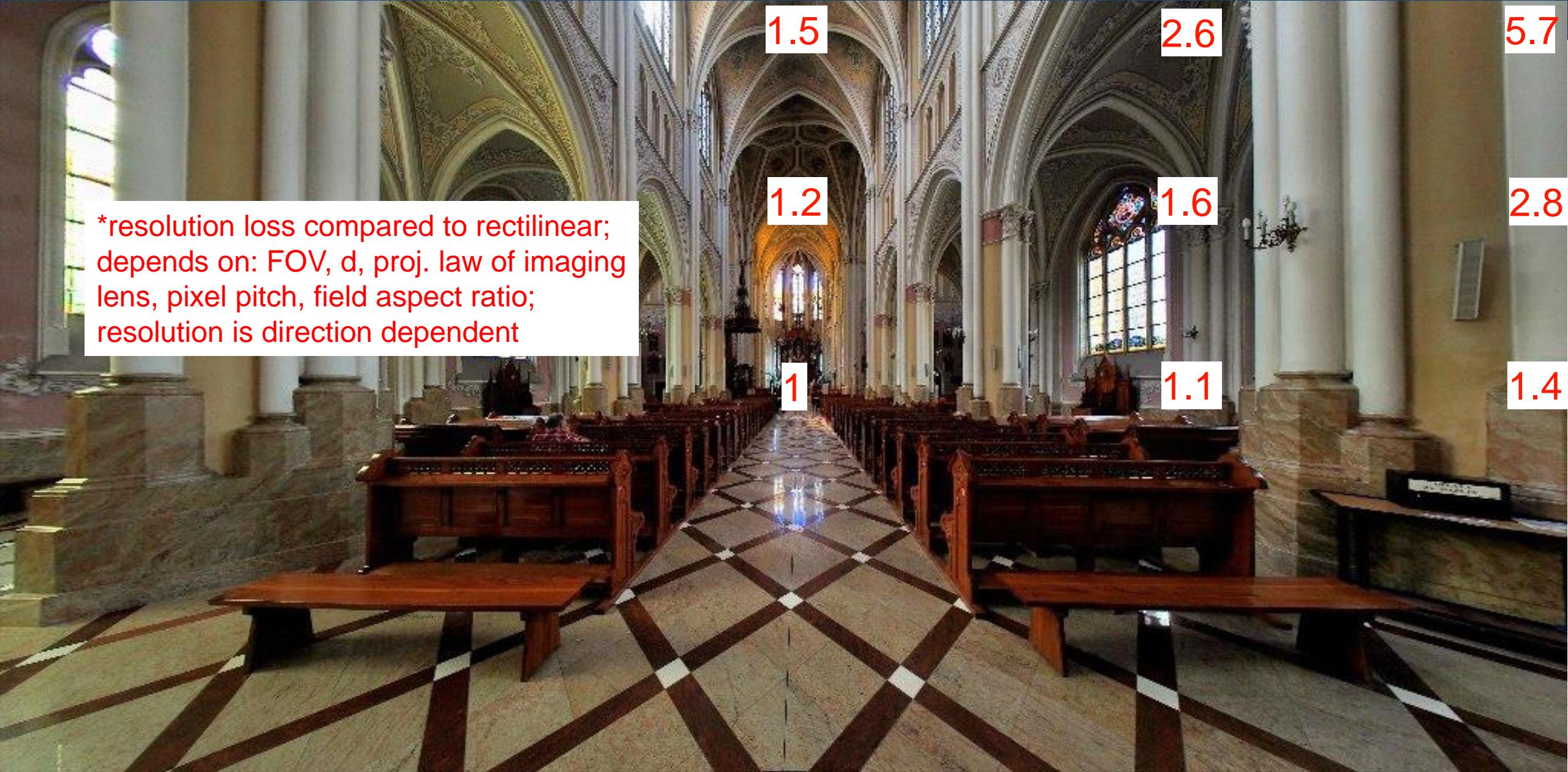


Digitally transformed Panini view: Relative resolution loss*

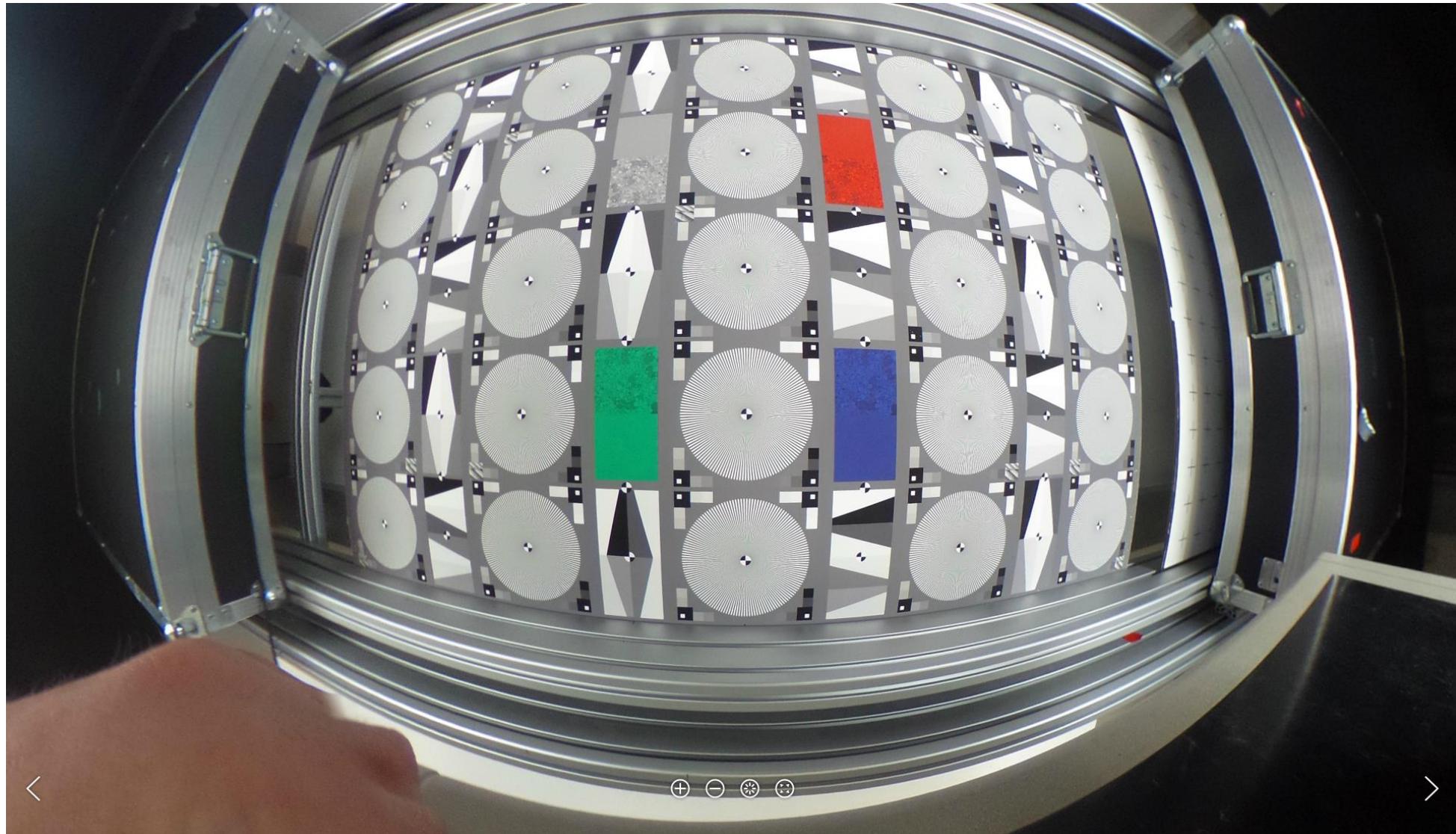
*Depends on: FOV, d, proj. law of imaging lens, pixel pitch, field aspect ratio; resolution direction dependent



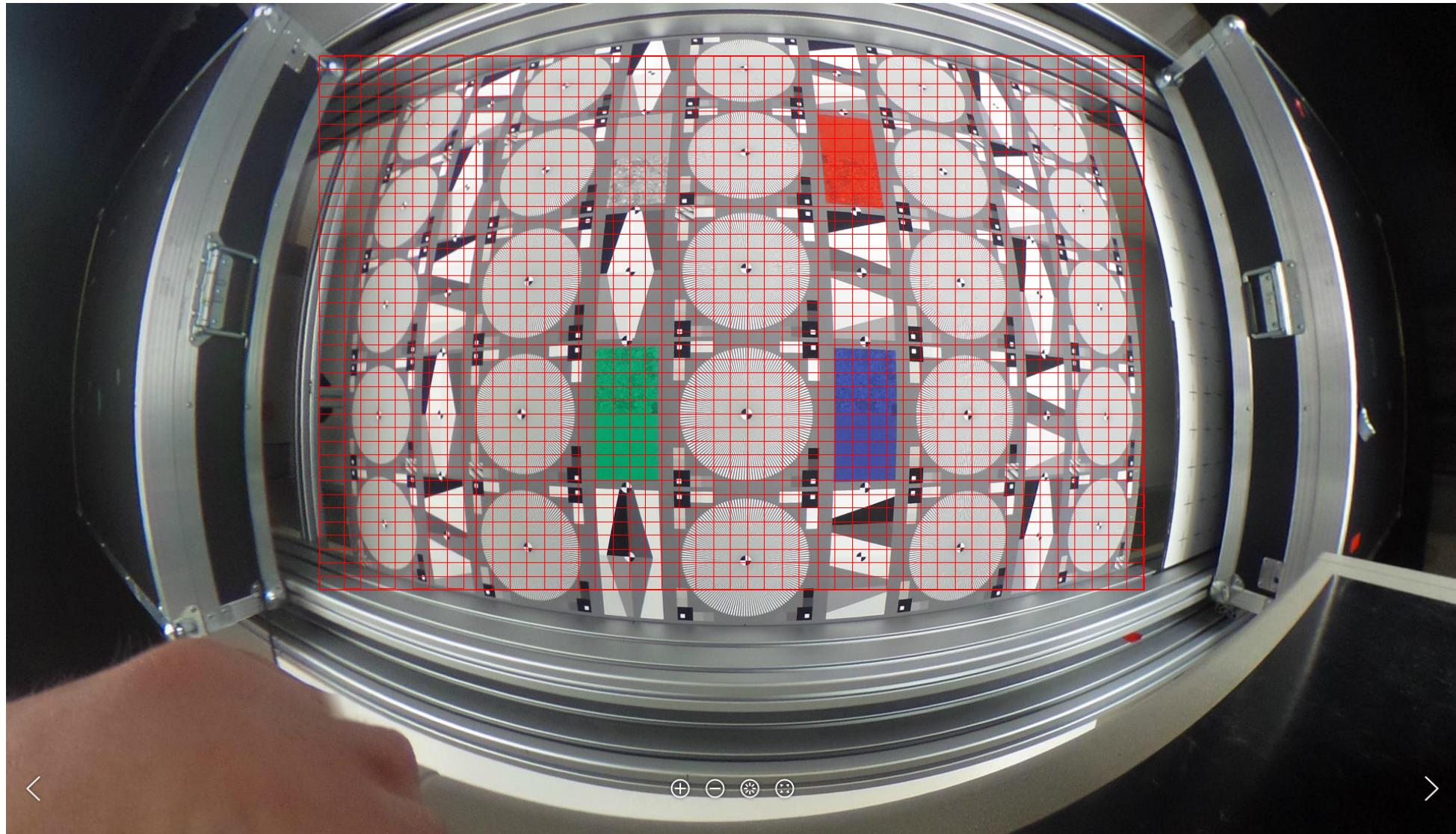
Institute of
Applied Physics



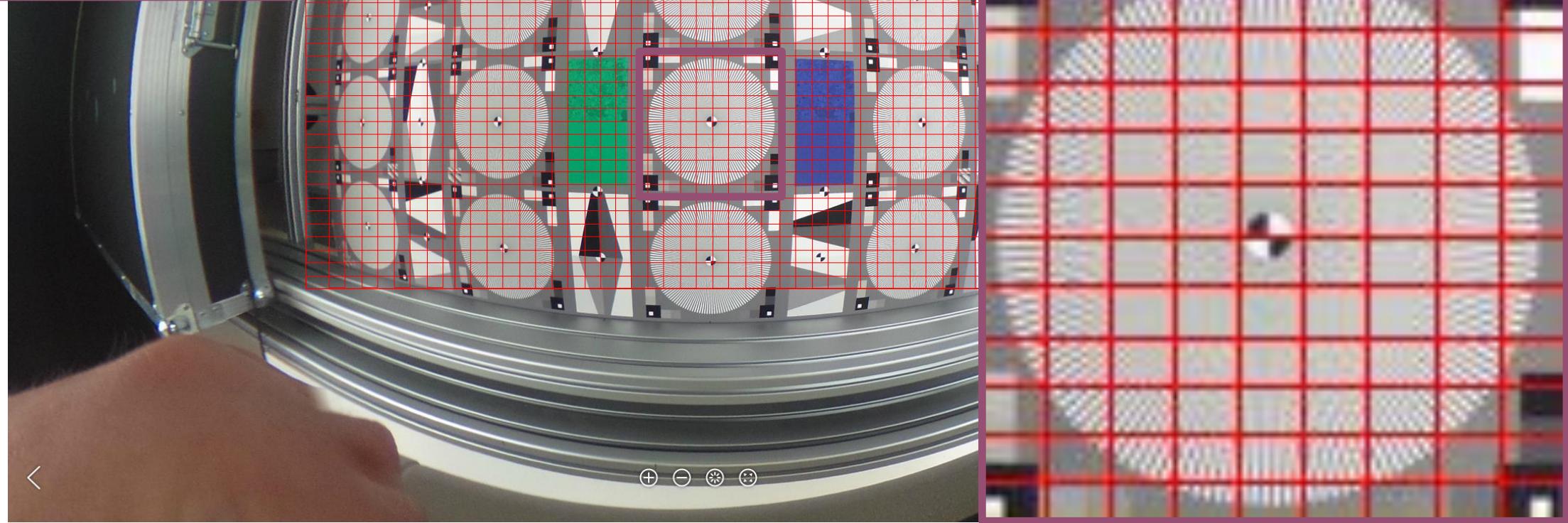
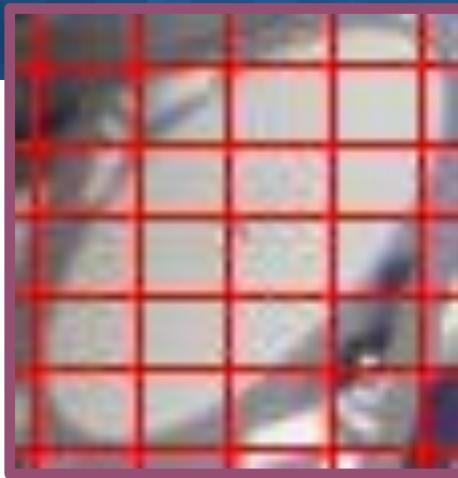
On field-dependance of resolution of distorted image



On field-dependance of resolution of distorted image



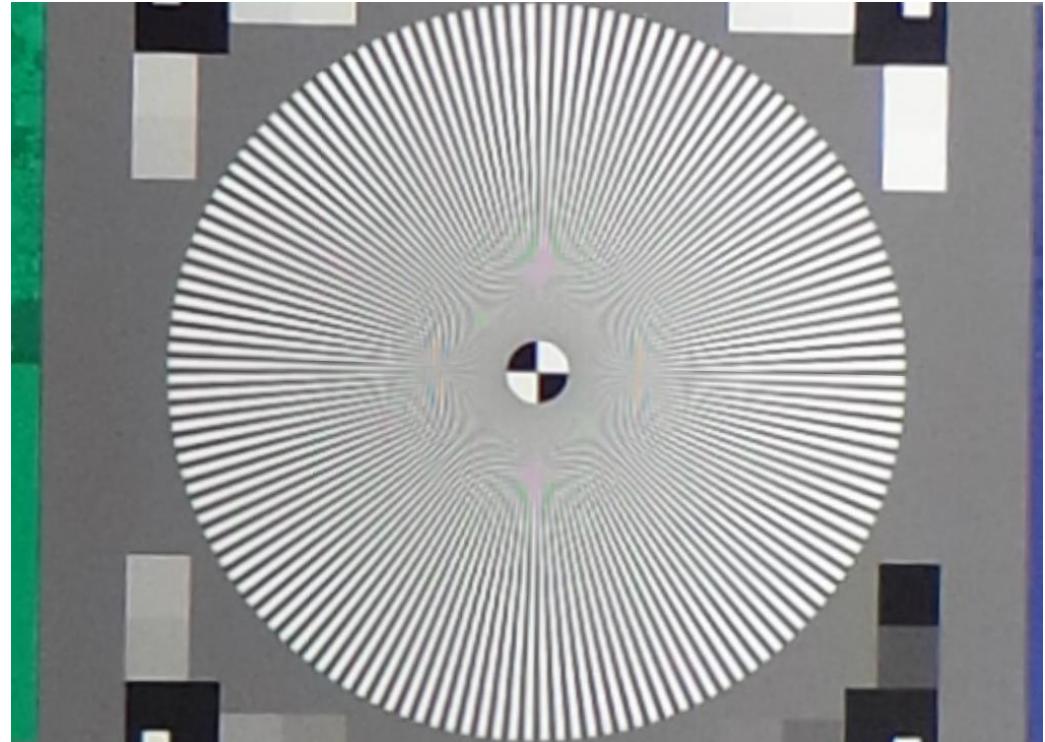
On field-dependance of resolution of distorted image



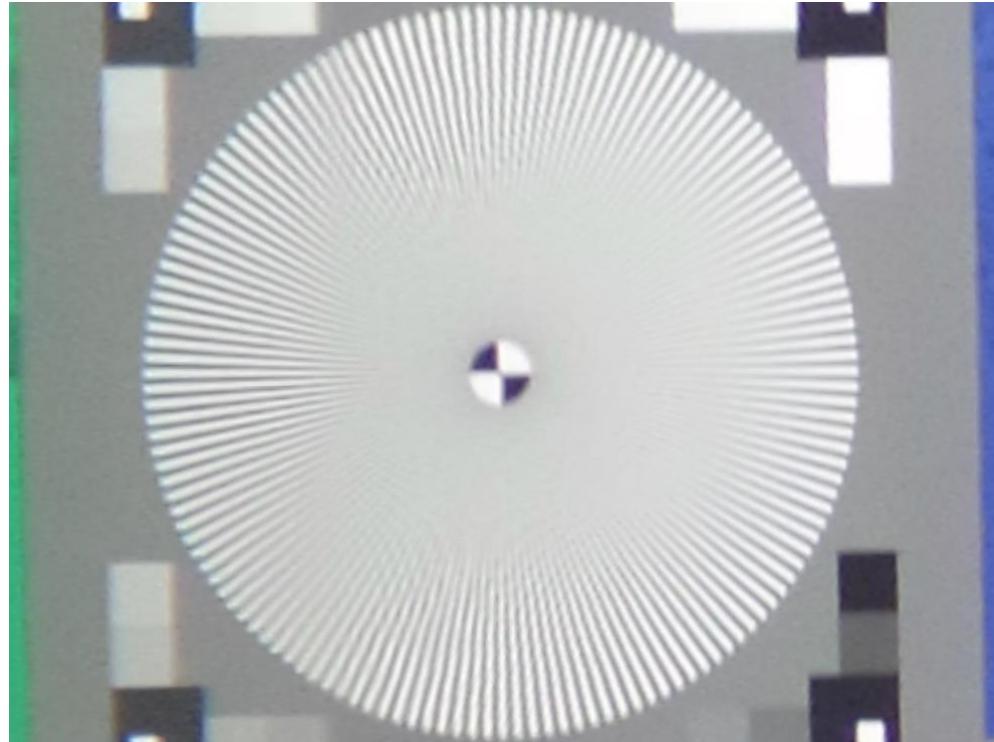
Compact 360° Cameras and azimuthal resolution variation
2 Fisheye Lenses (FOV > 180°) directed in opposite directions



Ricoh Theta S



0°

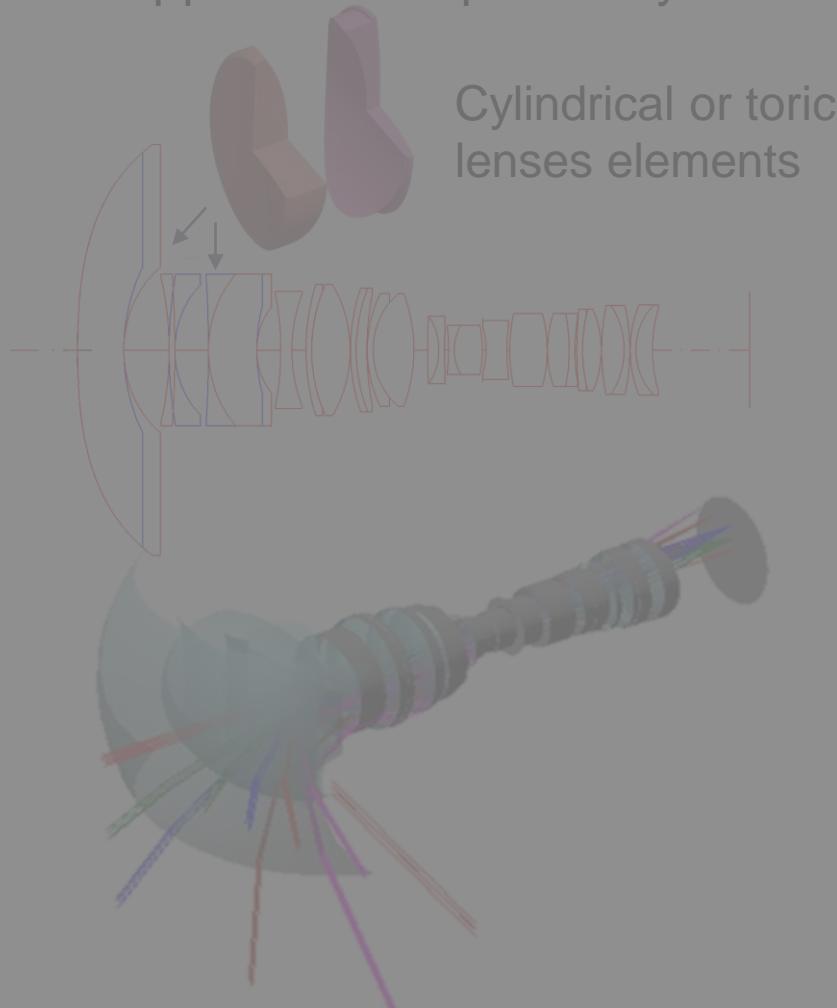


70°

Significant resolution and contrast loss towards “side view”.

Classical optical design vs digital optical co-optimization

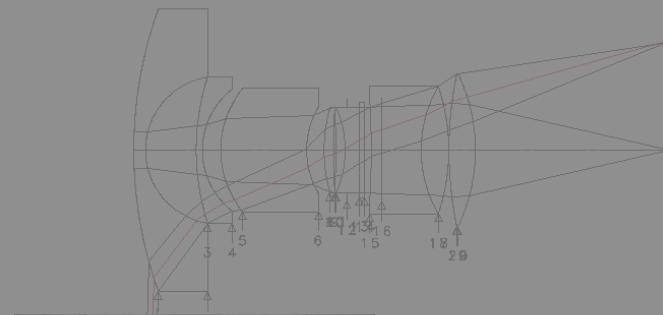
Approach 1: Optics only



Patent Blahnik

Approach 2:

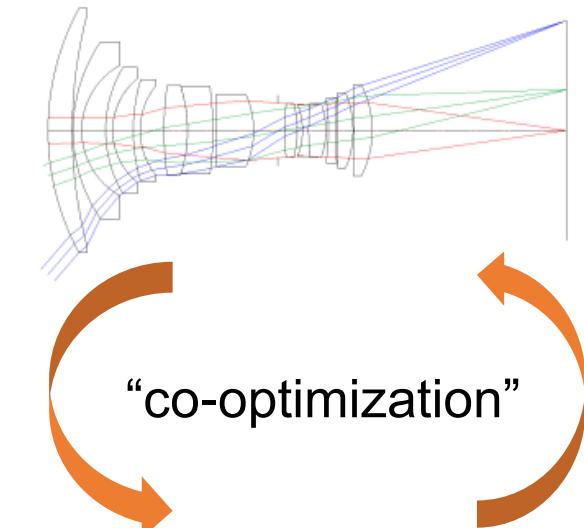
- simple lens & software
- Panini perspective
Performance with undesired resolution loss?



digital transformation

- “Th. Sharpless demonstrations and pictures”

Approach 3: Co-optimize system to reduce undesired resolution loss



digital transf., control MTF,
shading, chromatic aberration etc

Smartphone Camera Optics

Ultra-wide-angle (ca. 120° FOV) lenses become standard in high-end smartphones in year 2020.
Even higher FOVs might be available in near future.

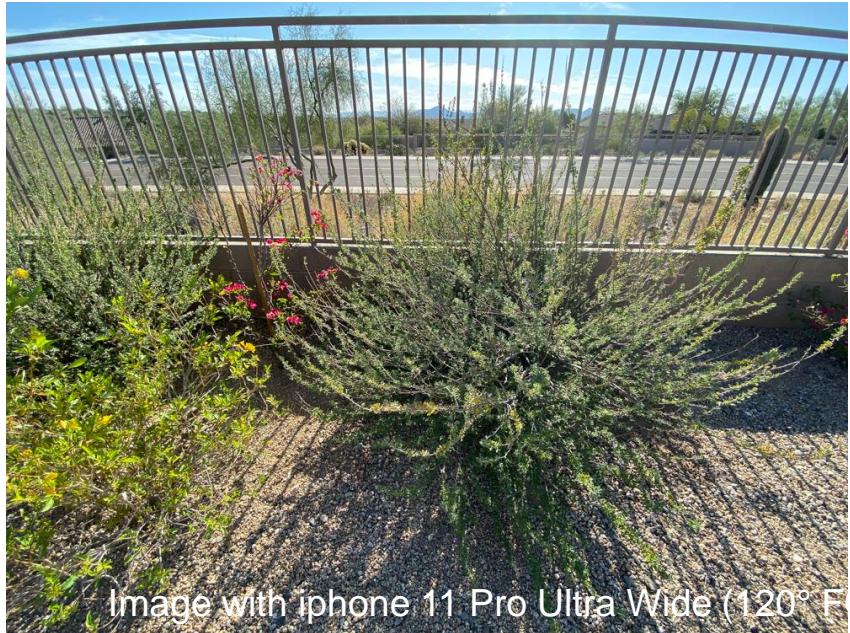
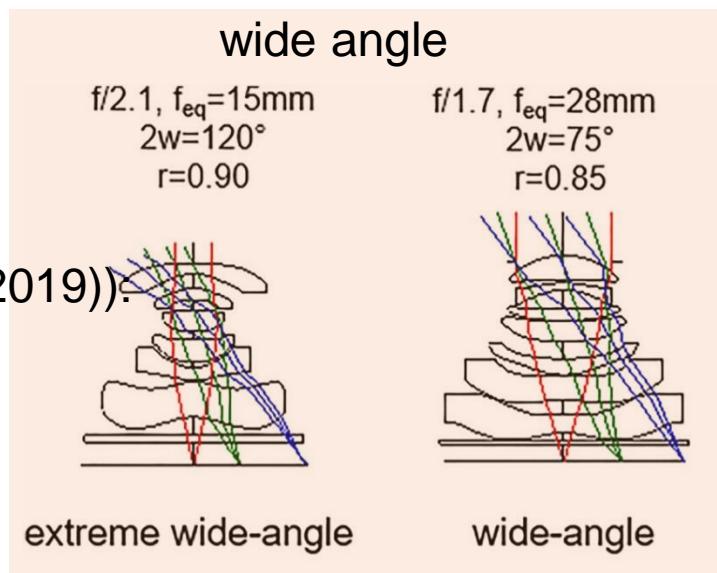
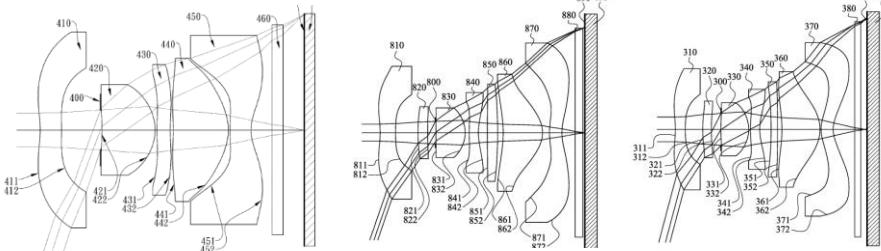


Image with iPhone 11 Pro Ultra Wide (120° F)

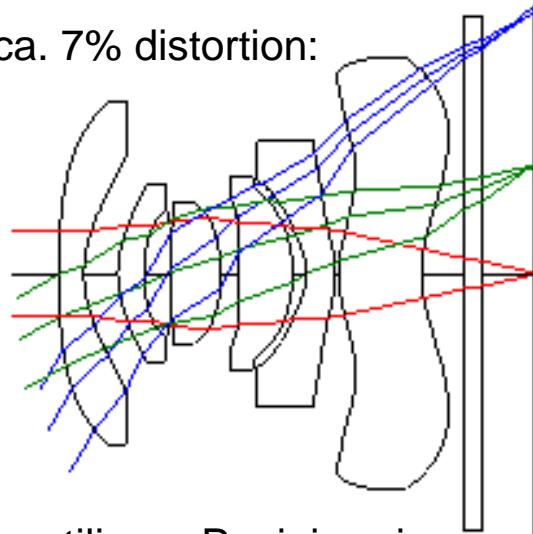
Recent patents exceeding 150° FOV (Langan (2019)):



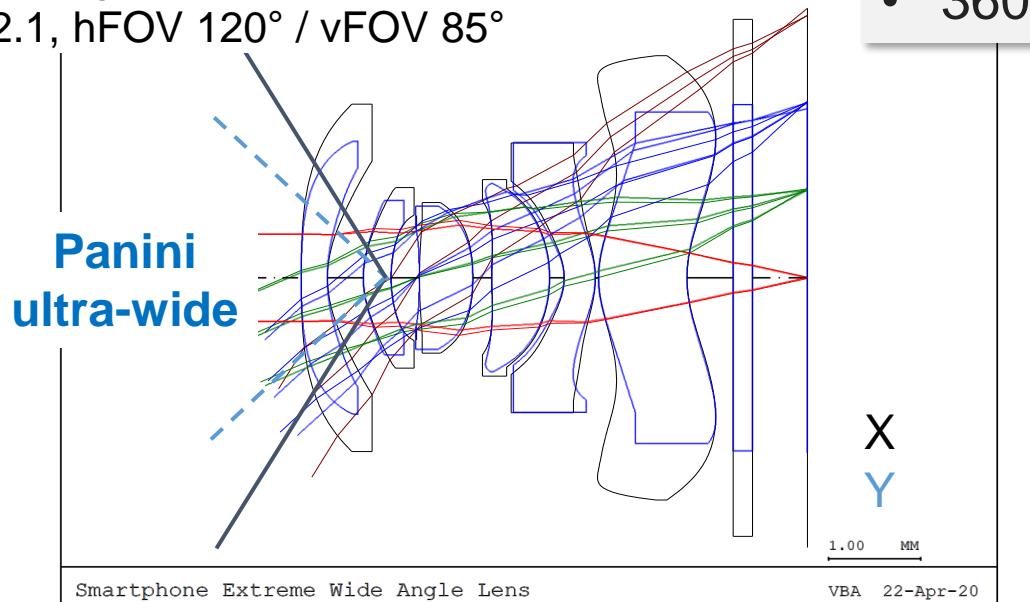
Towards wide screen Panini Panorama (Single-Shot)

Rectilinear with ca. 7% distortion:
f/2.1, FOV 120°

**Standard
ultra-wide**



Stereographic – rectilinear Panini-proj.
f/2.1, hFOV 120° / vFOV 85°



Applications:

- One-Shot Panorama,
- Round-Table Video Conference
- 360-Widescreen-ActionCam

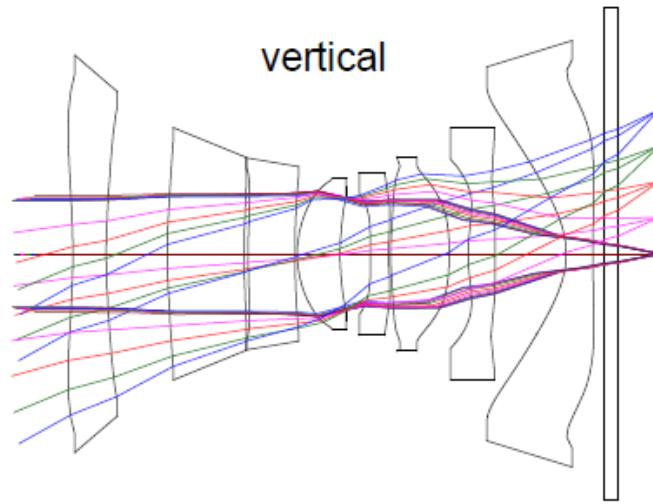
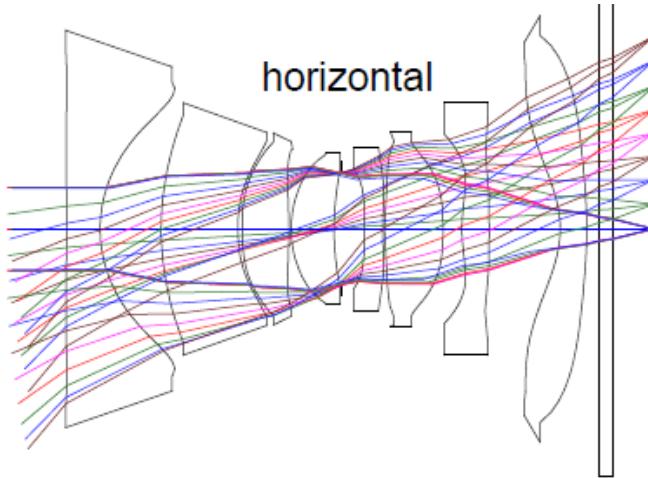
Display of
uncropped
image from 4:3
image sensor



Panini Projection
in widescreen 2:1
image format

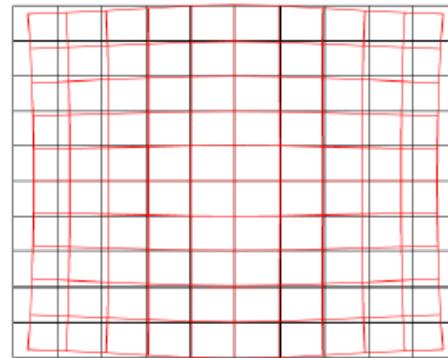


Digital Co-Optimization Approach

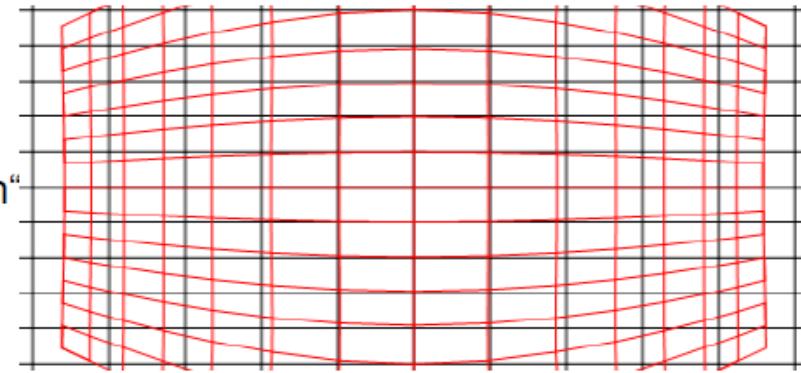


Anamorphic system, larger f in vertical direction

Example: $f_v/f_h = 1.5$



„de-anamorph“
&
Pannini

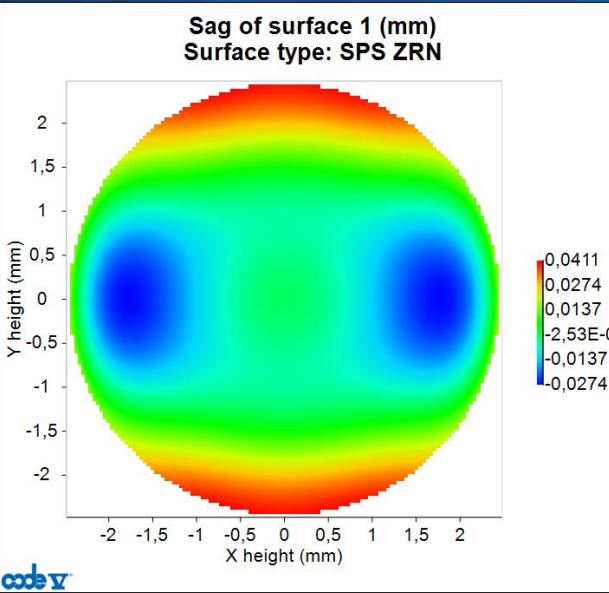


For given example the design significantly relaxes considerably allowing for an anamorphic magnification (kept variable for optimization).

When the anamorphic deformation is removed digitally the shown system projects in Panini perspective.

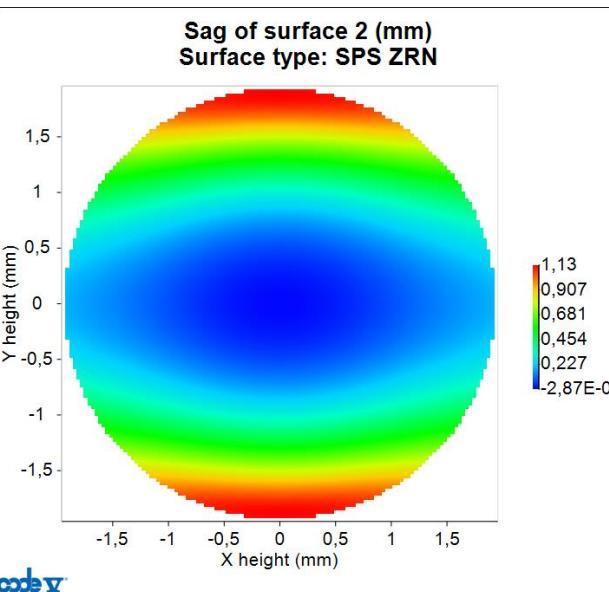
Procedure could be generalized to a digital co-optimization scheme.

Freeform surface shape on first lens element

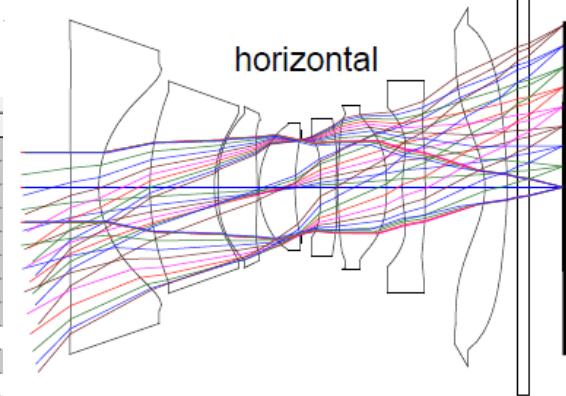


Surface 1

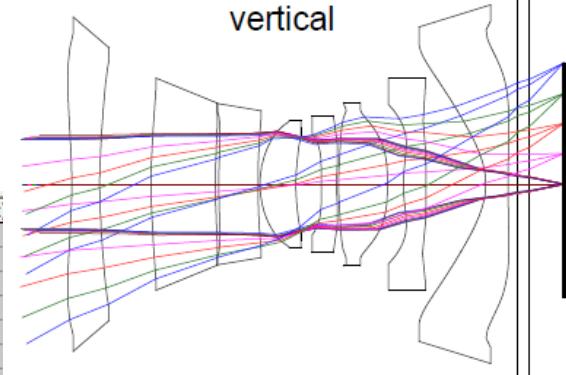
Max Radial Order	$\sin(3t)$	$\sin(2t)$	$\sin(t)$	Azimuth Independent	$\cos(t)$	$\cos(2t)$	$\cos(3t)$	$\cos(4t)$	$\cos(5t)$	$\cos(6t)$	$\cos(7t)$	$\cos(8t)$
R**0 (Piston)				-0.1563								
R**1 (Tilt)			0.0000		0.0000							
R**2 (Power)		0.0000		-0.1307		-0.0312						
R**3	0.0000		0.0000		0.0000		0.0000					
R**4		0.0000		0.0203		0.0107		-0.0027				
R**5	0.0000		0.0000		0.0000		0.0000	0.0000				
R**6		0.0000		-0.0038		0.0078		0.0049		0.0031		
R**7	0.0000		0.0000		0.0000		0.0000	0.0000		0.0000		
R**8		0.0000		0.0013		-0.0010		0.0002		0.0004		-0.0001
R**9	0.0000		0.0000		0.0000		0.0000	0.0000		0.0000		
R**10		0.0000	-0.0002		7.1591e-05		4.8498e-05		-0.0001		-5.5283e-05	
End Of Data												



Surface 2



horizontal

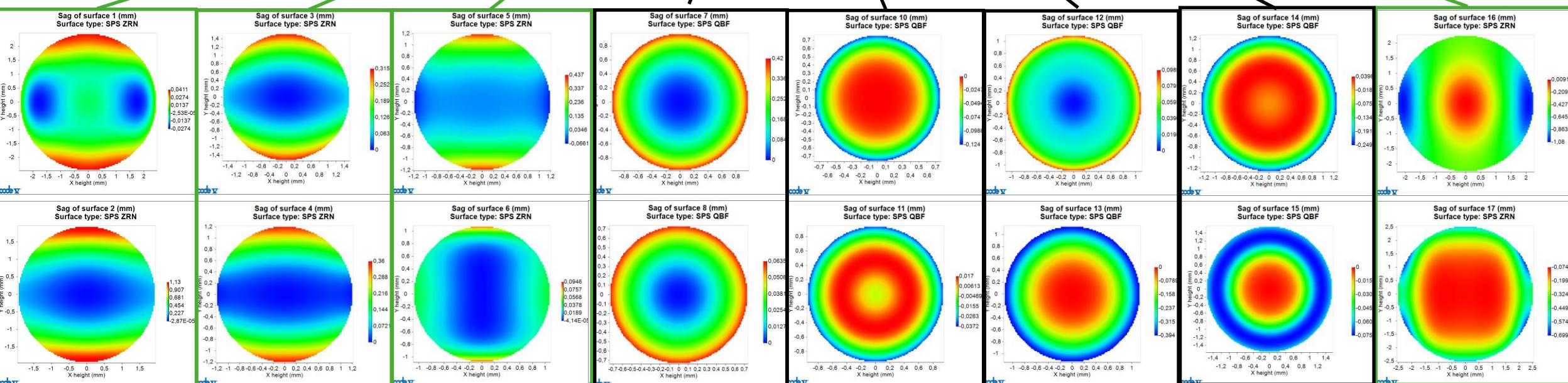
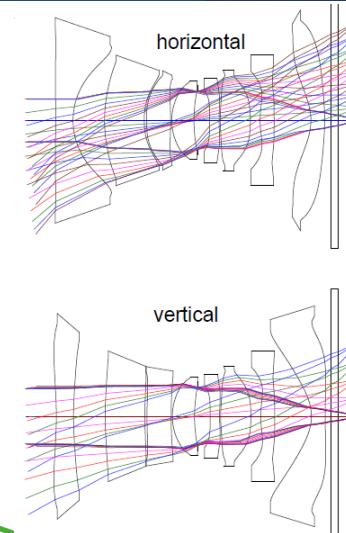
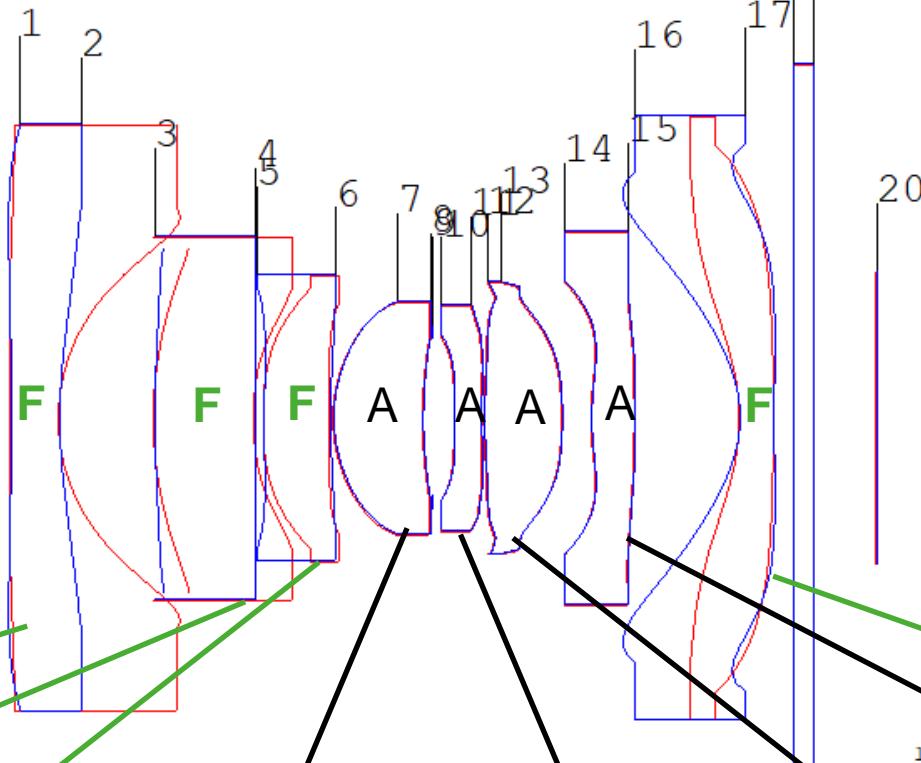


vertical

Freeform surface of even azimuthal order ($\cos(2m\varphi)$ terms)

Optical Surface Symmetry

A asphere rotational symmetric
F freeform azimuthal even



Anamorphic imaging

Anamorphic lens: original image on camera sensor

Andreas Bogenschütz, Carl Zeiss AG



Anamorphic lens

Andreas Bogenschütz, Carl Zeiss AG



ARRI/ZEISS MasterAnamorphic 60

Reversal of anamorphic image squeeze.
2x horizontal stretch.

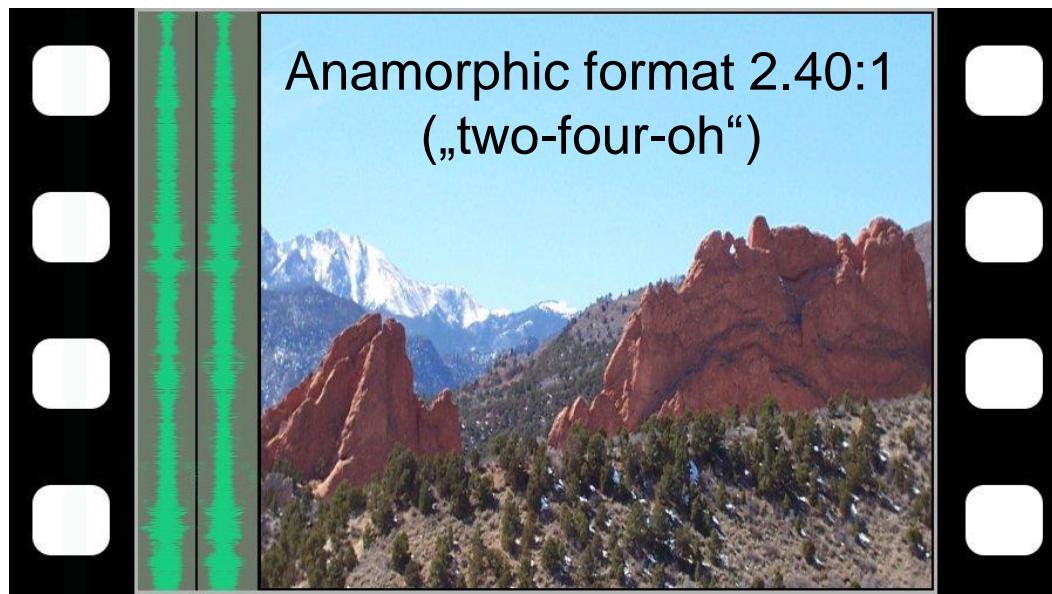
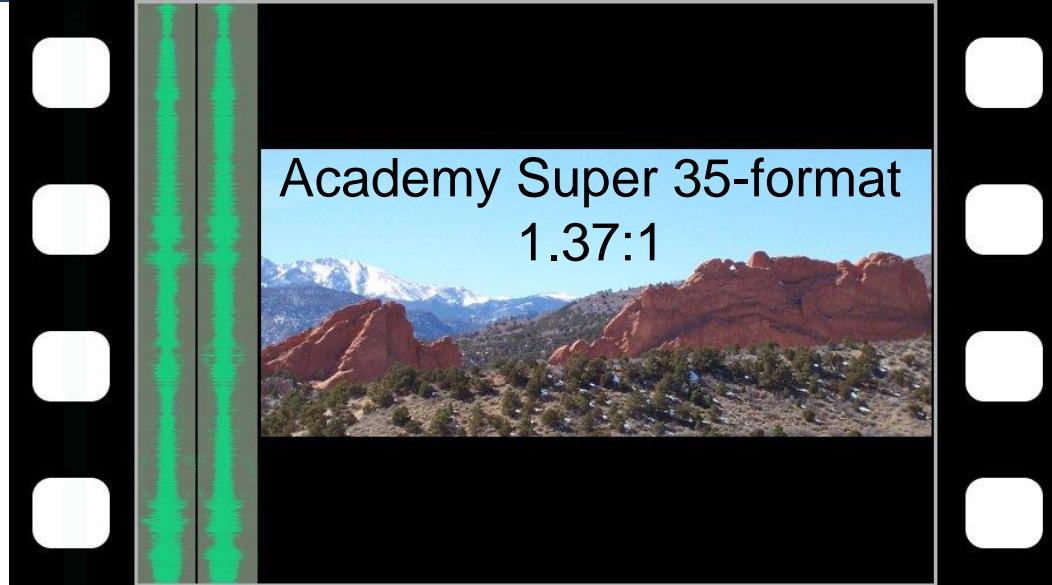
Spherical lens



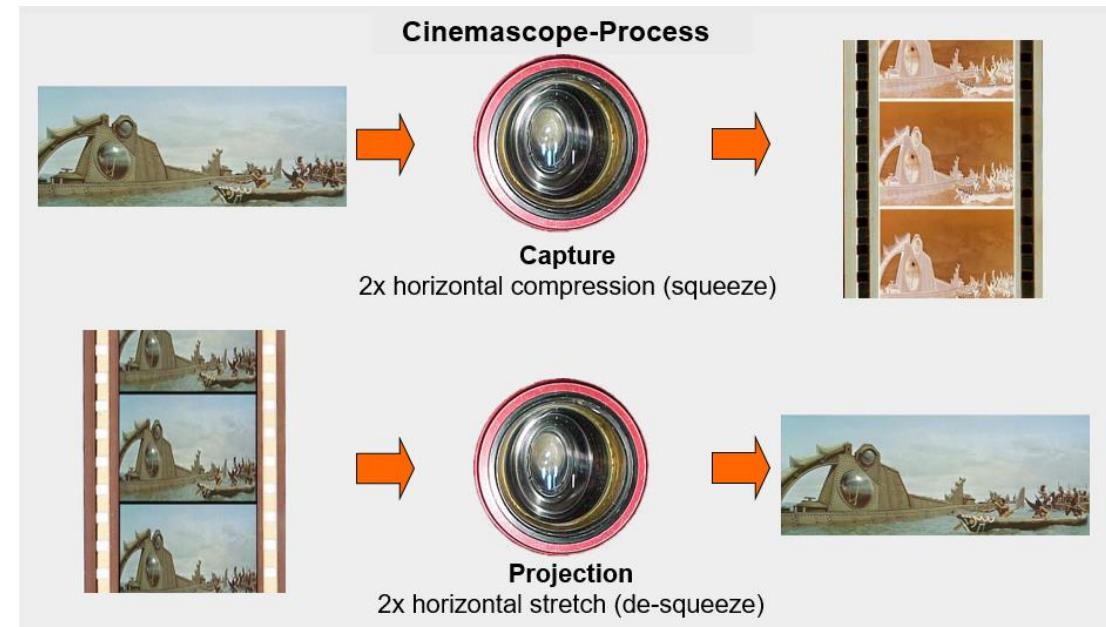
Andreas Bogenschütz, Carl Zeiss AG

ARRI/ZEISS MasterPrime 32

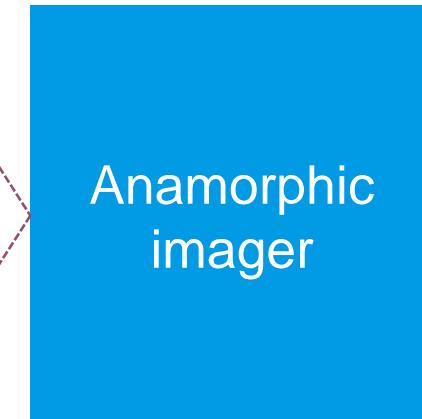
Origin of anamorphic film: Optimized shooting of widescreen format on 4-perforated film



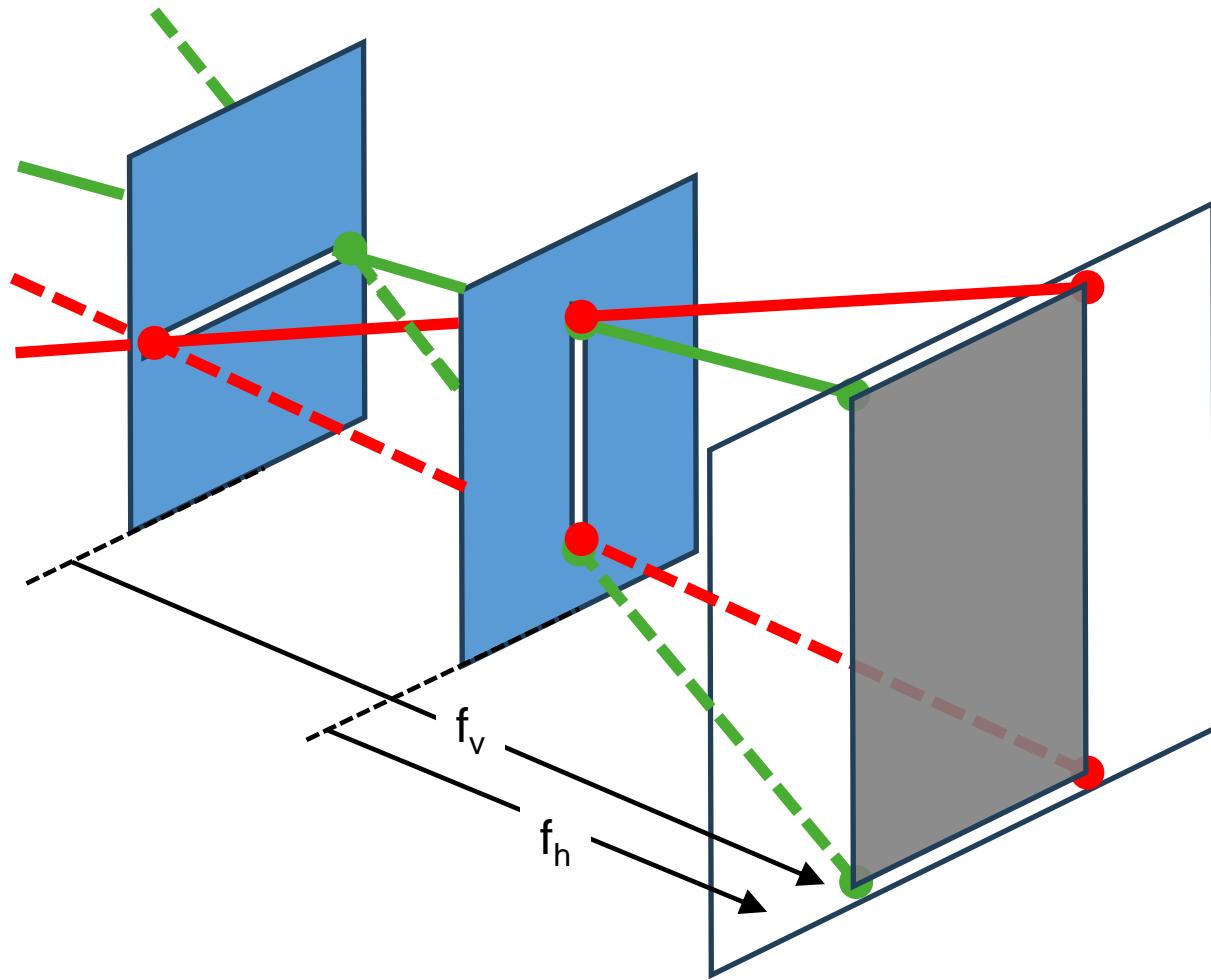
1953-67: 20th Century Fox promoted the
widescreen format CINEMASCOPE



Anamorphic Imaging without lenses?

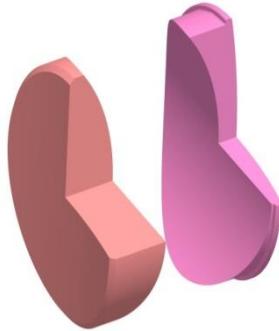


Anamorphic pinhole camera

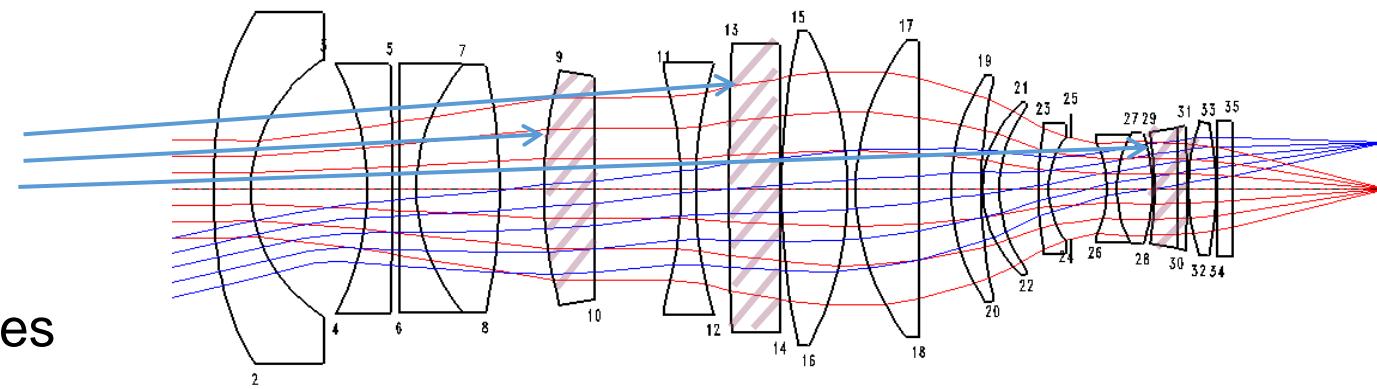


Anamorphic imaging without lenses

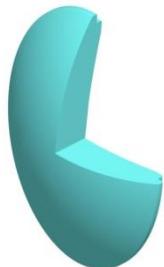
Anamorphic Lens



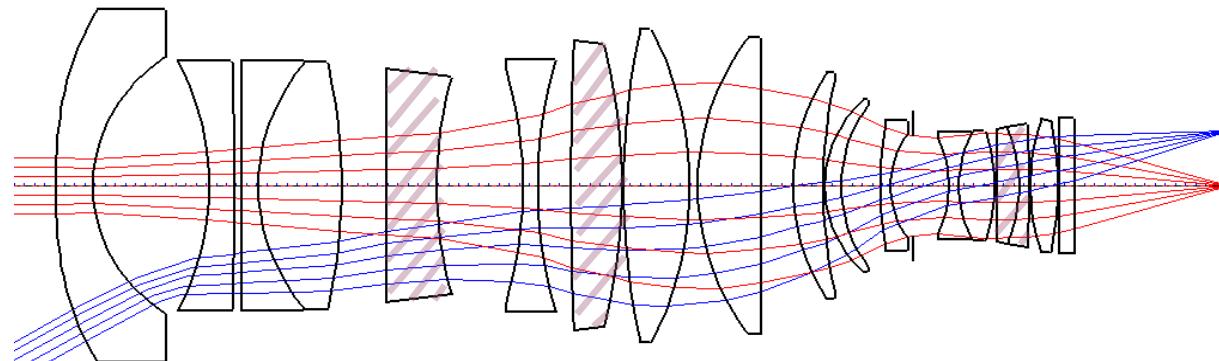
Cylindrical lenses



x-direction

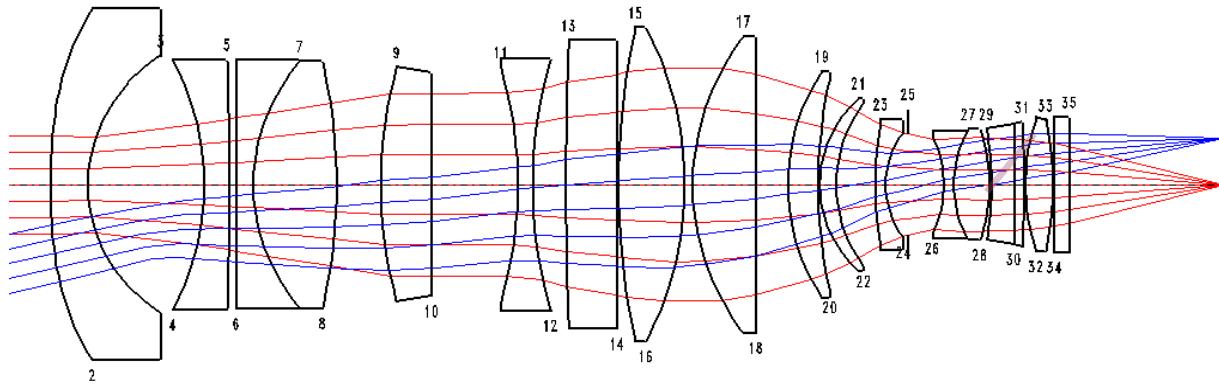


Spherical lenses

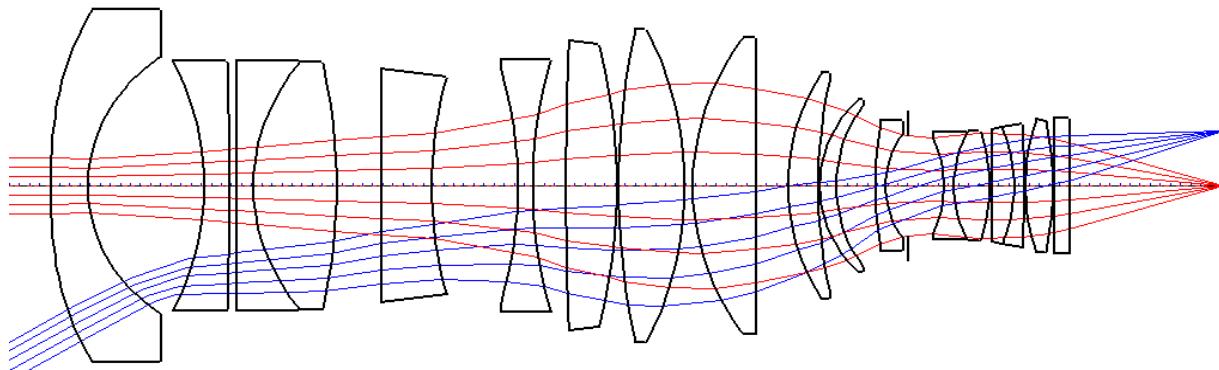


y-direction

Anamorphic Lens

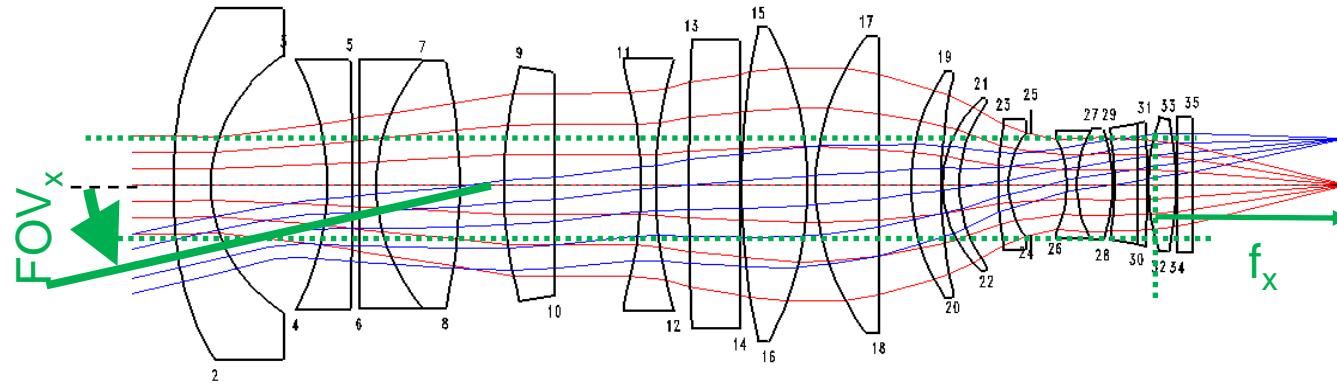


x-direction

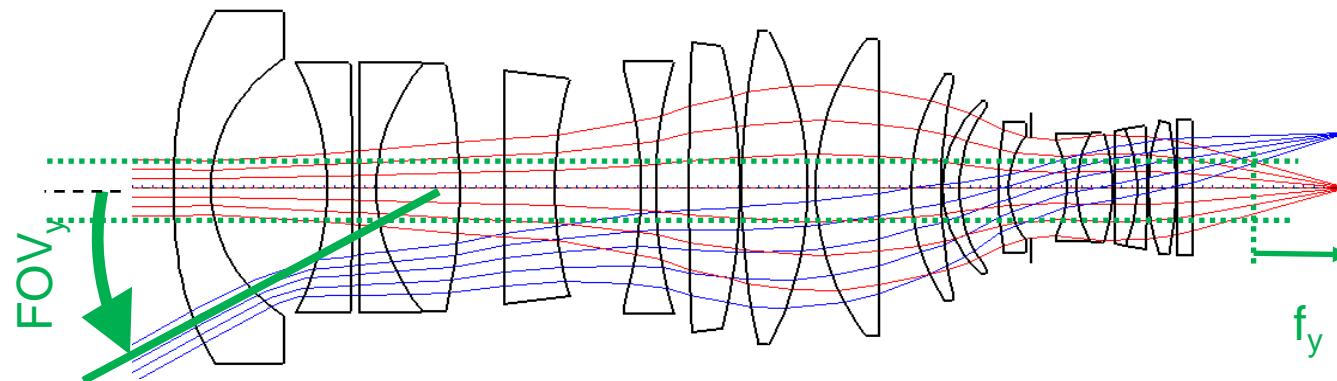


y-direction

Anamorphic Lens

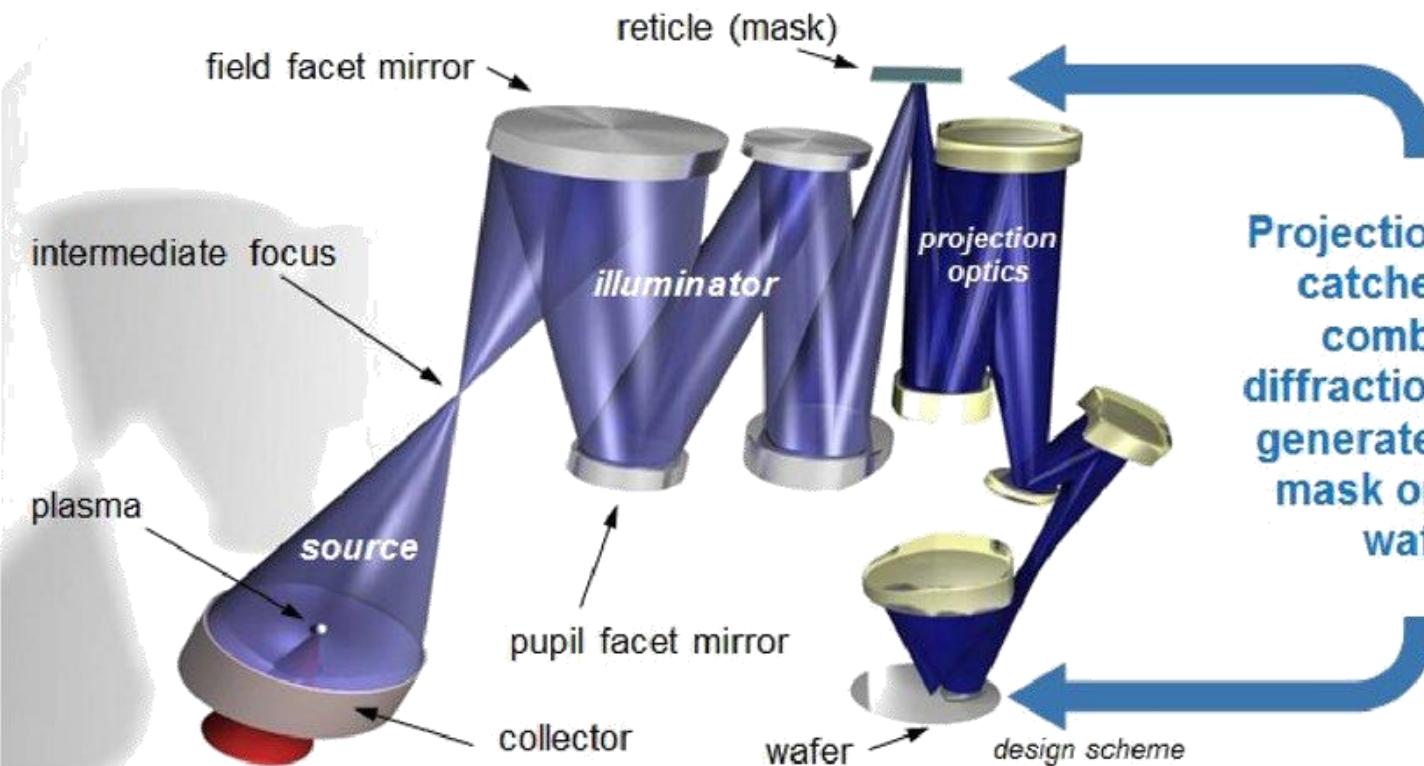


x-direction

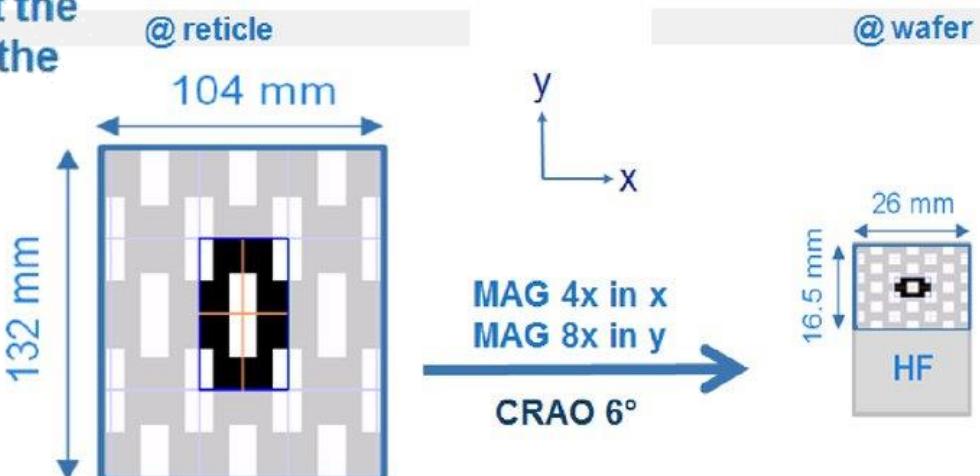
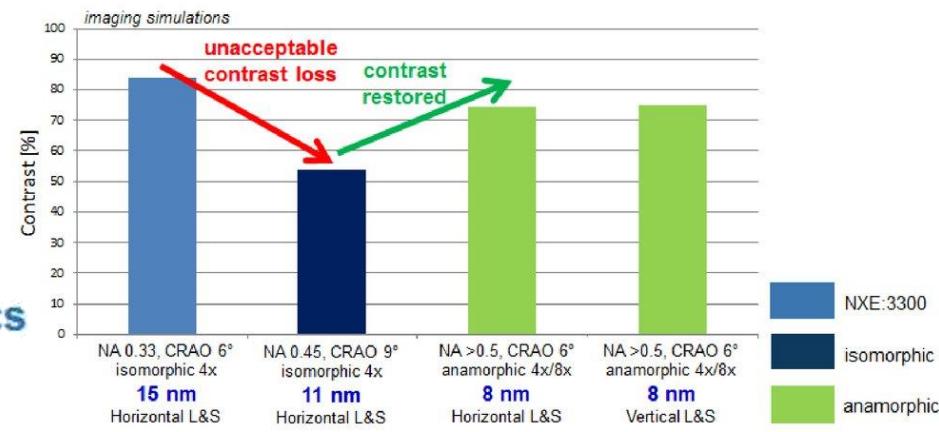


y-direction

Anamorphic high-NA EUV lithography optics



Projection optics
catches and
combines
diffraction orders
generated at the
mask onto the
wafer

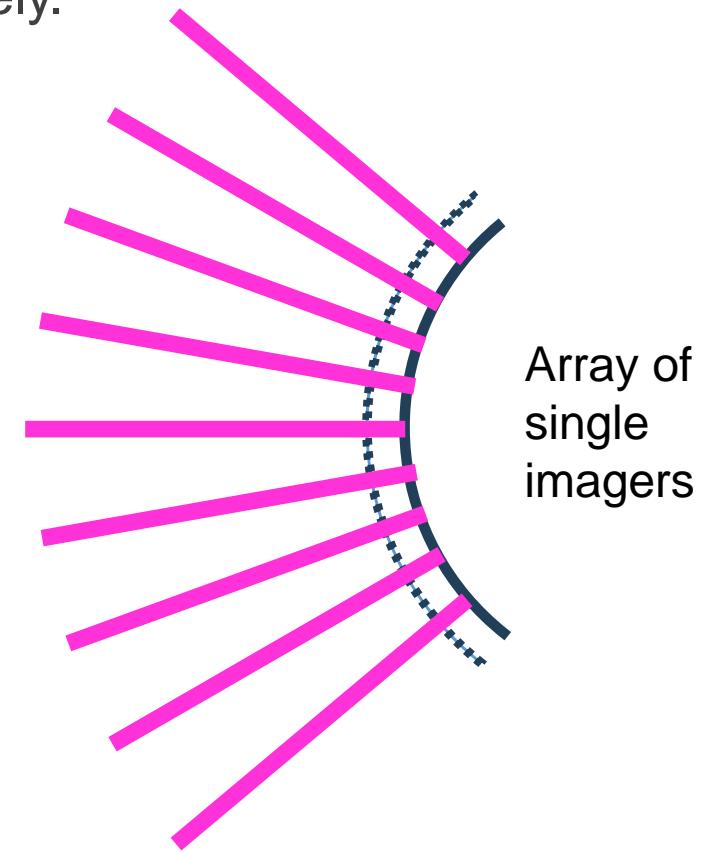
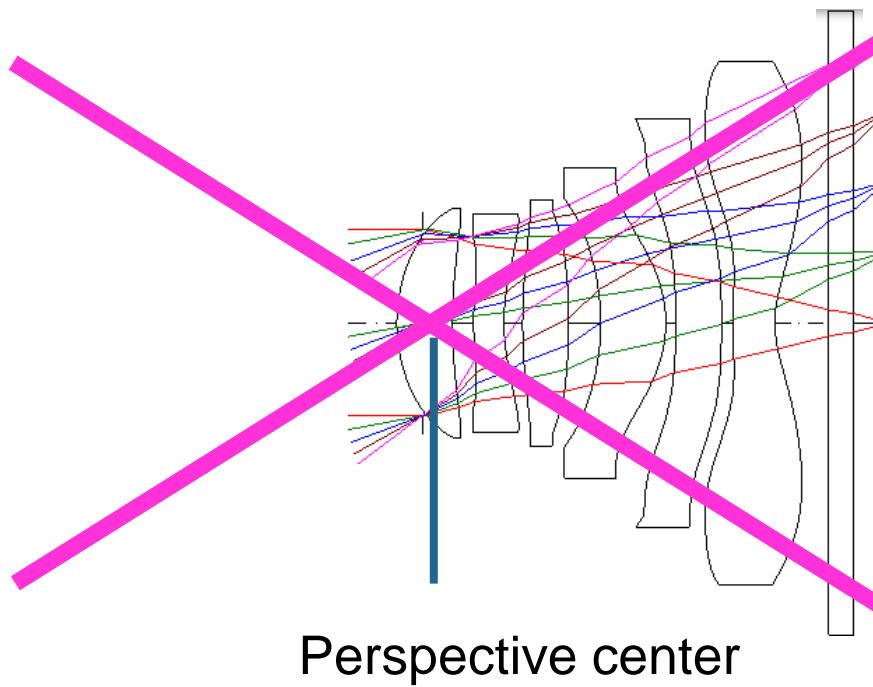


Alternative imaging architectures

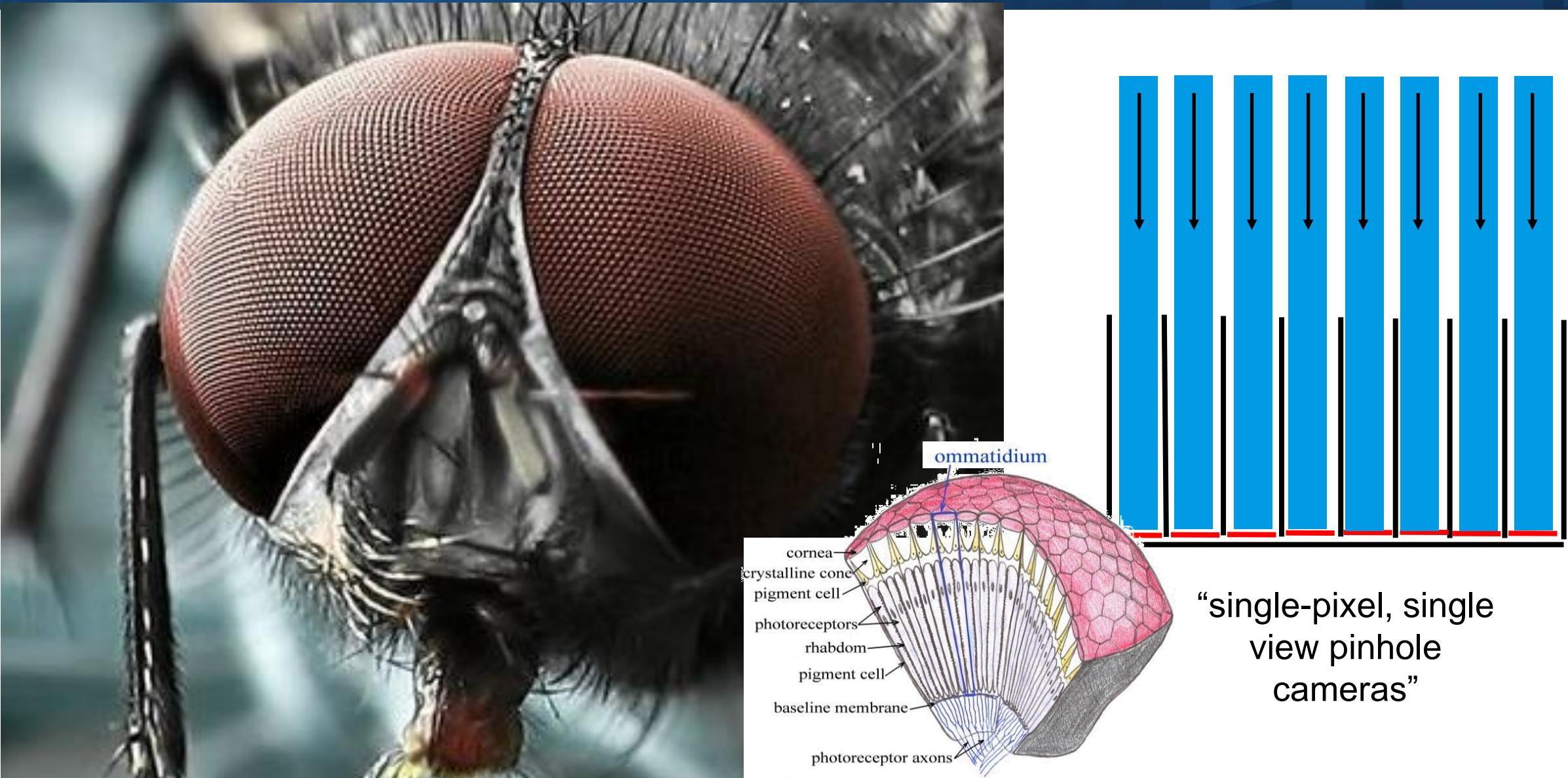
Central projection basic architecture of optical systems

By far most optical systems image according to central projection!

However, there are alternative architectures, many being evolved in recent years, e.g., systems with parallelized channels or “elementary imagers” respectively.

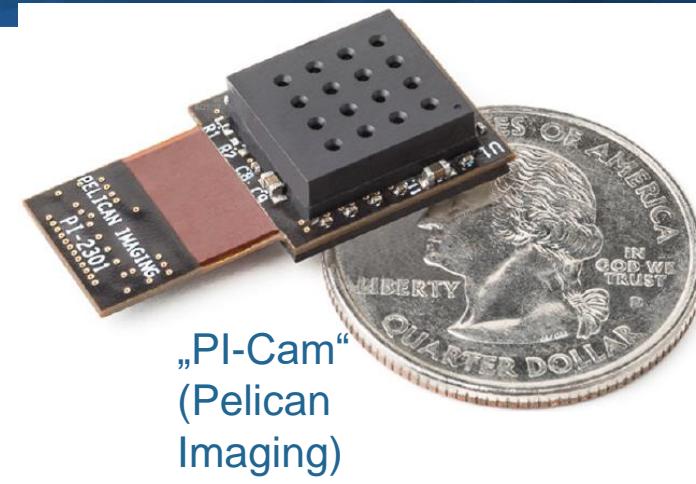
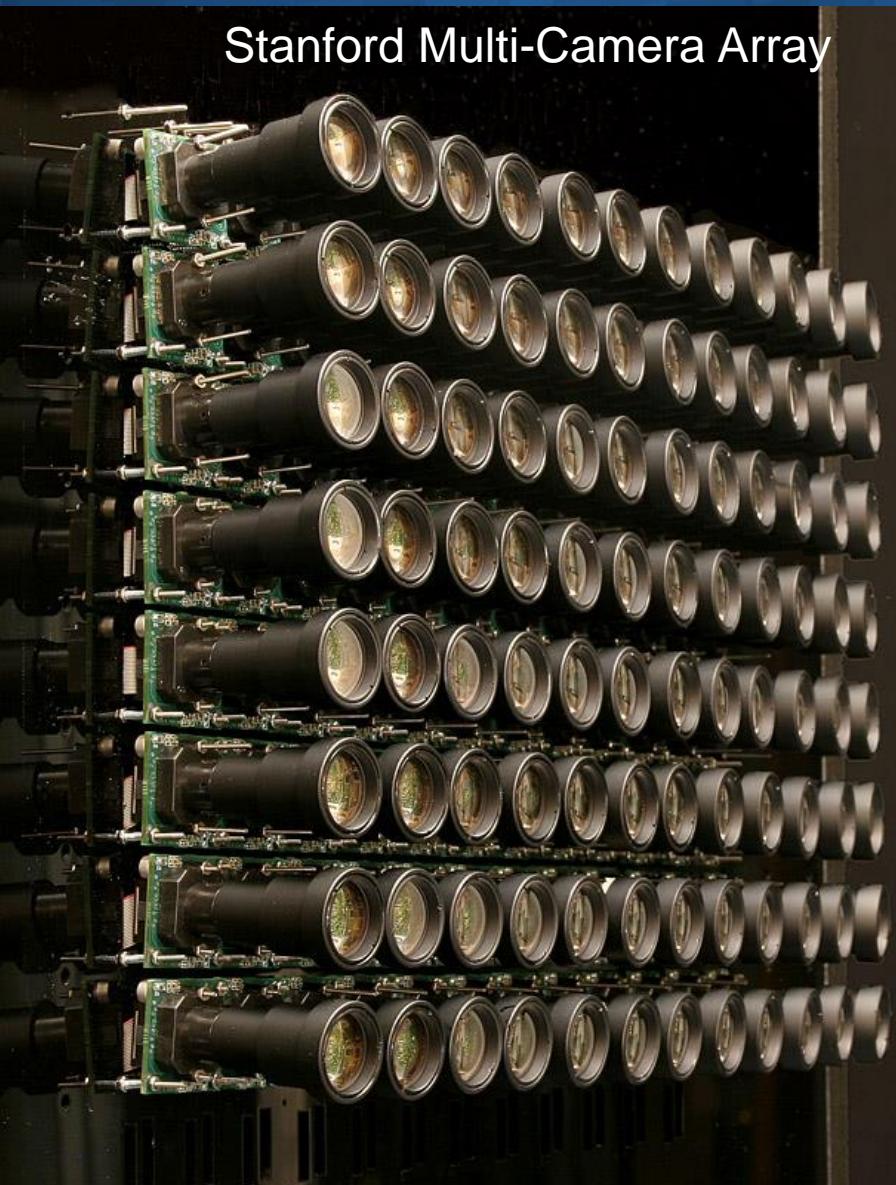


Fly's Eye Imaging



Multi-View Imaging

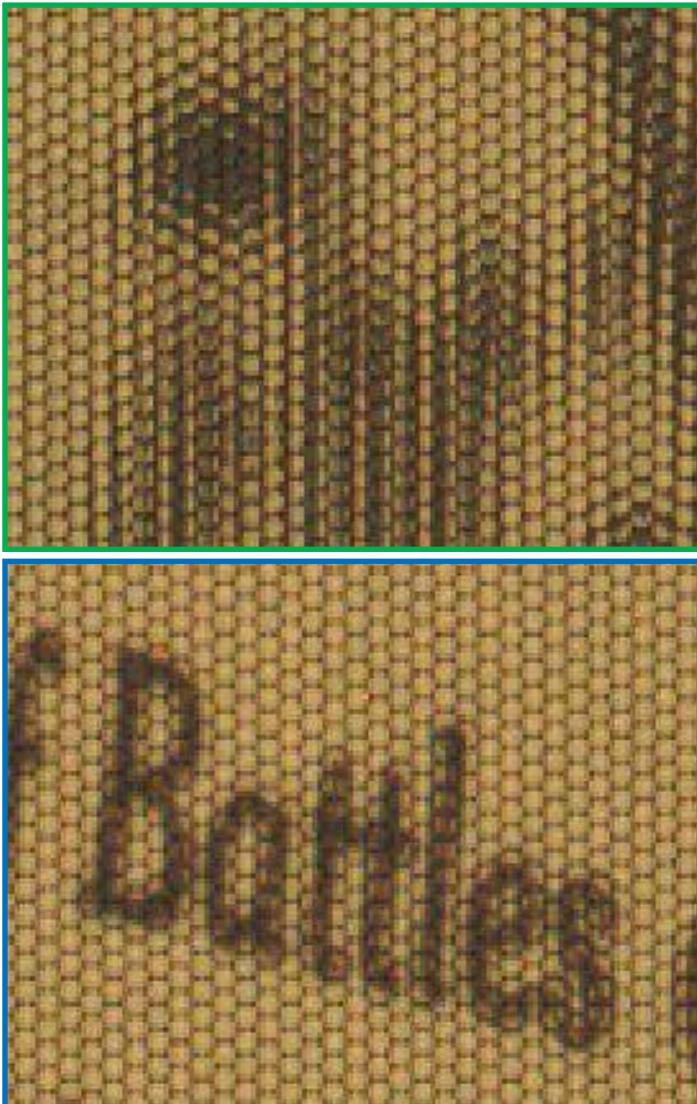
Stanford Multi-Camera Array



Lightfield Imaging raw images



Lytro



Raytrix



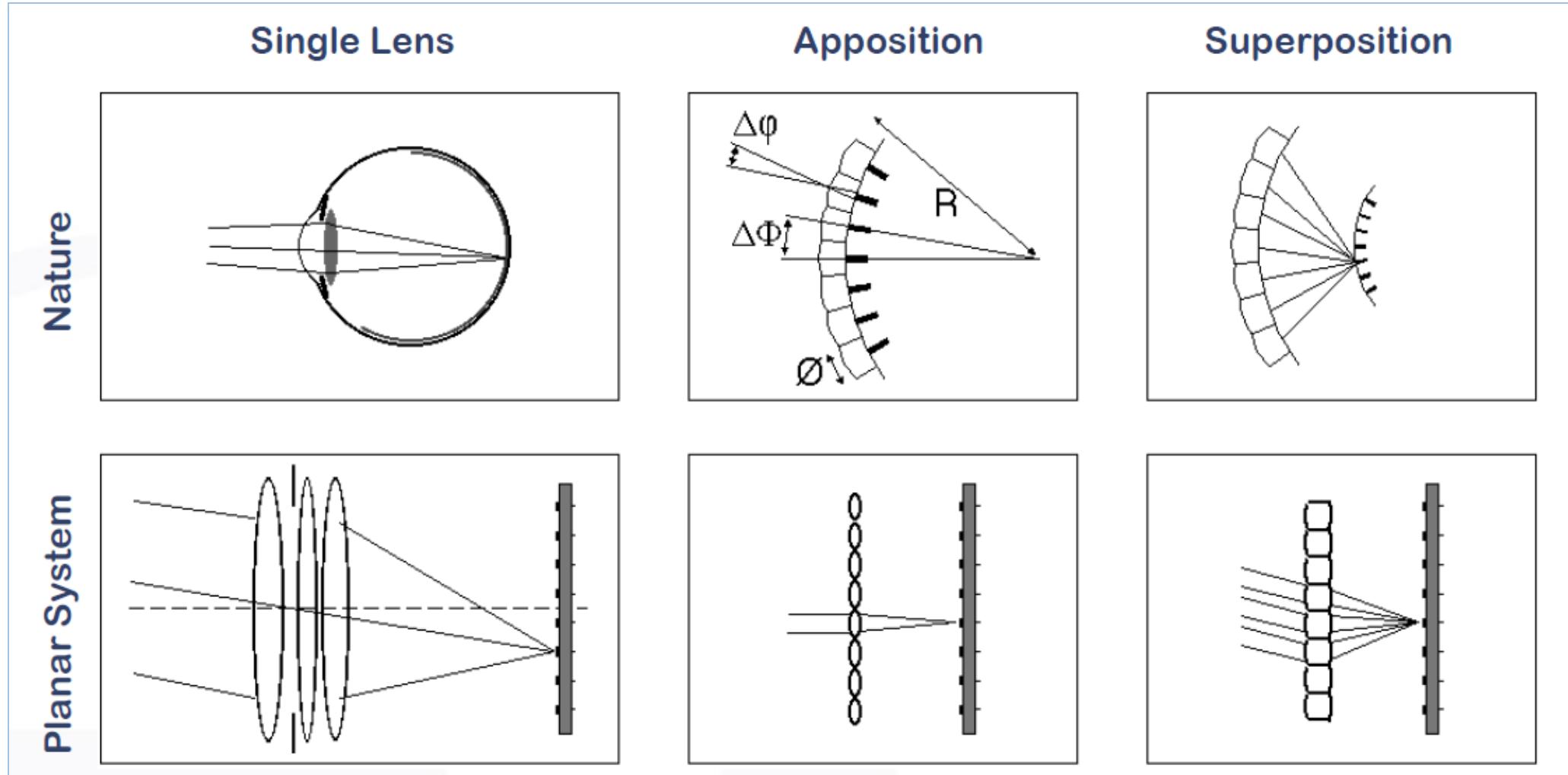
Arrays - Synthetic Aperture Systems

Radio telescope: VLA - New Mexico

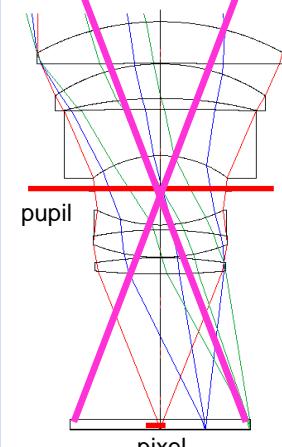
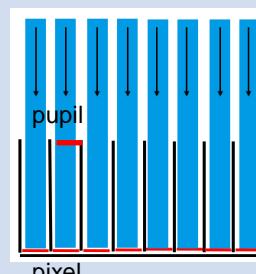


Arrays - Synthetic Aperture Systems

Principles



Central projection vs Multi Array Architecture

	Central projection	Multi array
principle		
etendue	good possible	limited (small aperture)
resolution	good possible	limited (aperture size * (# apertures determines total sensitive area))
homogeneity	usually requires additional size	good
Flatness of optical system	“long”, as focal length usually similar or longer as aperture size	very flat possible

Some optical systems combine / compromise both principles:
 Parallelized imaging subsystems, which are central projection systems with limited field-of-view, pixel number.

- Laws of reflection and refraction at interfaces between media can be derived from Fermat's principle, that light takes the path of shortest time. The mechanism of Fermat's principle can be understood by Huygens-Fresnel wave model
- Rays are normal vectors of the wavefront
- Most optical systems image a scene via central projection – the center of projection corresponds to the pinhole of a pinhole camera
- In optical systems the center of perspective corresponds to the entrance pupil, which in turn is the image of the lens stop towards object space
- In order to increase the amount of light in the image, which scales with étendue = field area \times $NA^2 = FOV^2 \times EP^2$, the pinhole is replaced by an optical system transferring light into a solid angle defined by the entrance pupil surface area and the object distance to EP
- An alternative imaging architecture is fly's eyes imaging featuring parallelized single-view cameras; advantageous: thin layout; disadvantageous: compromised étendue & resolution
- We will see in the next lecture that paraxial imaging is ideal between plane object and image plane; however in practice the desired "ideal image" projection law may be different - it depends on the practical problem

- “Distortion-free lenses”, regarding an object plane, inevitably lead to distorted 3D images: This is called “perspective distortion”
- An universal “optimum projection law” does not exist, as it is scene dependent – however 18th century Italian landscape artists around G. Panini knew and applied superior perspectives compared to rectilinear as delivered by camera obscura
- Those ideas are meanwhile applied in Computer Graphics, e.g. 3D games graphics engines, however have been widely unknown in the optics community
- The design of wide-angle lenses providing Panini perspective projections require non-rotational symmetric optical elements and benefit from increased degrees of freedom from these elements (freeform better than toric better than cylindrical)
- Cylindrical shaped image fields are beneficial for those design-types and are easier to realize
- Anamorphic imaging also requires non-rotational symmetric elements (cylinder, toric, freeform)
- Pinhole imaging models for rectilinear, fish-eye, Panini and anamorphic imaging can be explicitly given; the corresponding central projections may have varying locations of the center of perspective as a function of field-angle or field azimuth

- in a digital imaging chain modifications of the image can be realized via image processing; here optical designers need to consider the option of relaxing optical design requirements in favor of e.g. simpler, cost-optimized optical systems
- digital correction may have side-effects, e.g. correcting distortion digitally changes irradiance distribution in the images as well as resolution (or contrast) distribution
- a systematic co-optimization helps to control all requirements systematically while optimizing for the complete digital imaging chain