Problem 1

Jinsong Liu

(a)

$$\rho_\nu = \frac{I_\nu}{c} \qquad \phi_\nu = \frac{I_\nu}{h\nu}$$

For monochromatic light:

$$W_i = \int_0^\infty \phi_\nu \cdot \sigma_\nu \, \delta(\nu-\nu_0)\, d\nu = \phi_{\nu_0} \cdot \sigma_{\nu_0}$$

(b)

$\because$ σ_ν is much narrower than $\rho(\nu)/h\nu$

$$W_i = \int_0^\infty \frac{\rho_\nu c}{h\nu} \cdot \sigma_\nu \, d\nu$$

$$= \frac{\rho(\nu_{21})c}{h\nu} \cdot \int_0^\infty \sigma_\nu \, d\nu$$

$$= \frac{\rho(\nu_{21})c}{h\nu} \cdot \sigma$$

$$\because \sigma = \int_0^\infty \sigma_\nu \, d\nu = \frac{B_{12} \cdot h\nu_{21}}{c}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \quad B_{12} = B_{21} \quad A_{21} = 1/\tau_2$$

$$\therefore \sigma = \frac{\lambda^2}{8\pi\tau_2}$$

$$\therefore W_i = \frac{\rho(\nu_{21})c}{h\nu} \cdot \frac{\lambda^2}{8\pi\tau_2}$$

$$\because W_i = B_i \cdot \rho(\nu)$$

$$\therefore B_i = \frac{\lambda^3}{8\pi h\tau_2}$$

Problem 2

Jinsong Liu

(a)

the wave number $= \frac{1}{\lambda}$ [cm^{-1}] suppose $k = \frac{1}{\lambda}$

$\therefore$ the emission wavelength for all laser transitions:

$$\lambda_{ij} = \frac{1}{k_i - k_j} \Leftarrow \left(E = h\nu = hc/\lambda \ , \ h\nu_{ij} = E_i - E_j \right)$$

$k_i = 10930\ cm^{-1}, 10624\ cm^{-1}, 10327\ cm^{-1}$

$k_j = 785\ cm^{-1}, 612\ cm^{-1}, 565\ cm^{-1}, 0\ cm^{-1}$

$\therefore \lambda_{ij}$ has 12 values.

k_i [cm^{-1}]	k_j [cm^{-1}]	λ_{ij} [nm]
10930	0	914.9
10930	565	964.8
10930	612	969.2
10930	785	985.7
10624	0	941.3
10624	565	994.1
10624	612	998.8
10624	785	1016.4

k_i [cm^{-1}]	k_j [cm^{-1}]	λ_{ij} [nm]
10327	0	968.3
10327	565	1024.4
10327	612	1029.3
10327	785	1048.0

(b)

Because of the stark-effect, inhomogeneous broadening occurs in the cross-sections. The spontaneous emission of different excited particles also contributes to the broadening.

(c)

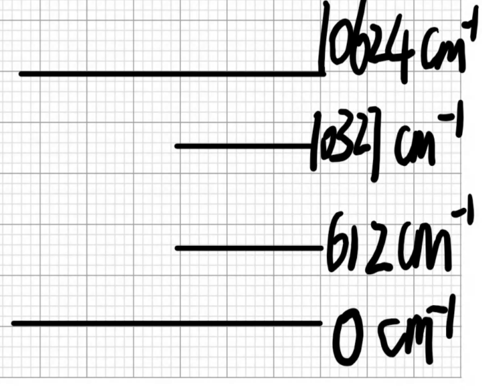
Because the transition energy doesn't transfer into the heat at the zero-phonon line.

(d) For a four-level system:

the maximum value of absorption occurs at 941.3nm.
($k_i = 10624\ cm^{-1}$, $k_j = 0\ cm^{-1}$)

Meanwhile, the maximum value of emission occurs at 1029.3 nm ($k_i = 10327\ cm^{-1}$, $k_j = 612\ cm^{-1}$)

In summary, the four-level system is:

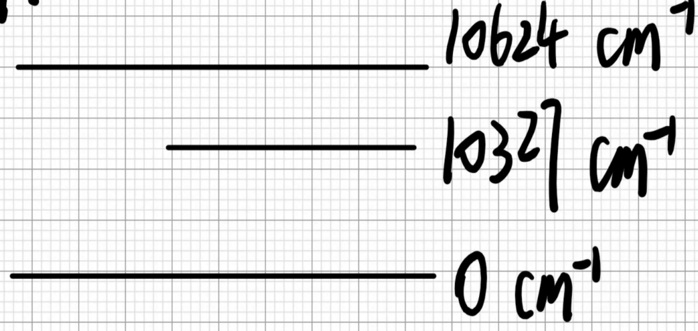


(e) For a three-level system:

the maximum value of absorption occurs at 941.3 nm ($k_i = 10624\ cm^{-1}$, $k_j = 0\ cm^{-1}$)

the emission between $k_i = 10327\ cm^{-1}$ and $0\ cm^{-1}$ is the zero-phonon line.

$\therefore$ the three-level system:



(f) For an inverse three-level system:

Problem 3 Jinsong Liu 206216

(a)

Gain saturation occurs when the increasing intensity leads to the decrease of the gain. The physical origin is the absorption particles (cross-sections) are saturable.

(b) The differential equation is:

$$\frac{dI(z)}{dz} = \frac{g_0}{1+\frac{I(z)}{I_{sat}}} I(z) = g_{sat}(z) I(z)$$

$$\therefore \frac{dI(z)}{I(z)}\left(1+\frac{I(z)}{I_{sat}}\right) = g_0 \, dz$$

$$\frac{dI(z)}{I(z)} + \frac{dI(z)}{I_{sat}} = g_0 \, dz$$

Assume that $I(z=0) = I_{in}$ $I(z=L) = I_{out}$

$$\int_{I_{in}}^{I_{out}} \frac{1}{I(z)} + \frac{1}{I_{sat}} \, dI(z) = \int_{z=0}^{z=L} g_0 \cdot dz$$

$$\ln(I_{out}) - \ln(I_{in}) + \frac{I_{out}-I_{in}}{I_{sat}} = g_0 \cdot L$$

$$\ln\left(\frac{I_{out}}{I_{in}}\right) + \frac{I_{out} - I_{in}}{I_{sat}} = g_0 \cdot L$$

$$\exp\left[\ln\left(\frac{I_{out}}{I_{in}}\right) + \frac{I_{out} - I_{in}}{I_{sat}}\right] = \exp(g_0 \cdot L)$$

$$\frac{I_{out}}{I_{in}} \cdot \exp\left(\frac{I_{out} - I_{in}}{I_{sat}}\right) = \exp(g_0 \cdot L)$$

$$\therefore\ G \cdot \exp\left(\frac{I_{out} - I_{in}}{I_{sat}}\right) = G_0$$

$$G = G_0 \cdot \exp\left(-\frac{I_{out} - I_{in}}{I_{sat}}\right)$$

(c)

$$L_{dB} = 10 \lg \frac{I_{out}}{I_{in}}$$

$L_1 = 10\,dB$	$L_2 = 9\,dB$
$I_{in} = 1\,W/cm^2$	$I_{in}' = 2\,W/cm^2$
$I_{out} = 10\,W/cm^2$	$I_{out}' = 15.9\,W/cm^2$
$G_1 = G_0 e^{-\frac{I_{out} - I_{in}}{I_{sat}}}$	$G_2 = G_0 e^{-\frac{I_{out}' - I_{in}'}{I_{sat}}}$
$G_1 = 10$	$G_2 = 7.95$

$\therefore I_{sat} \approx 21.4\ W/m^2$ $\qquad G_0 \approx 15.22$

(d) $I_{extr1} = 10 - 1\,W/m^2 = 9\,W/m^2$

$I_{extr2} = 15.9 - 2\,W/m^2 = 13.9\,W/m^2$

When I_{in} is $2W/m^2$, the extracted intensity is bigger.

$\because I_{in} < I_{saturation}$

$\therefore$ the extracted intensity increase with the increasing input intensity.

(e) $$\frac{dI(z)}{dz} = g(z)I(z) - \alpha I(z) = \left[\frac{g_0}{1+\frac{I(z)}{I_{sat}}} - \alpha\right] I(z)$$

(f)