Friedrich-Schiller-Universität Jena
Institute of Condensed Matter Theory and Solid State Optics – PAF
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WS 2015/2016

Series I
Structure of Matter

due on October 27 after the lecture

Exercise 1 **3 Points**

Solve the following inhomogeneous system of linear equations using the Gaussian elimination algorithm:

$$x + y + z = 6; x + 2y - z = 2; 2x + y - z = 1.$$

Hint: In accordance with the Gaussian elimination algorithm the system can be modified until the main matrix reaches row echelon (or triangular) form where all elements below diagonals elements become zero.

Exercise 2 **3 + 3 Points**

Please consider the following matrix:

$$\hat{A} = \begin{pmatrix} 1 & 3 & -1 \\ 3 & 4 & -2 \\ -1 & -2 & 2 \end{pmatrix}.$$

a) Find the eigenvalues and a set of eigenvectors of this matrix. Verify that its eigenvectors are mutually orthogonal.
b) Is it possible to diagonalize this matrix by means of a basis transformation? Please construct the change-of-basis matrix $\hat{T}$ and give the diagonalized version of the matrix $\hat{A}$.

Exercise 3 **5 Points**

One of the most powerful models in physics is the coupled harmonical oscillators. The dynamics of this system is described by the following set of differential equations:

$$m\ddot{x}_1 = -k_1 x_1 - k(x_1 - x_2)$$
$$m\ddot{x}_2 = -k_2 x_2 - k(x_2 - x_1).$$

The solution can be found in the following general form: $x_{1,2}(t) = \varphi_{1,2} e^{-i\omega t}$. Substitute this ansatz in the differential equations, and derive the eigenvalue problem, where ω is the eigenvalue (eigenfrequency) and (φ_1, φ_2) the eigenvector. Please solve this eigenvalue problem and give the solutions for $x_{1,2}(t)$.

Exercise 4 **3 Points**

Show that a Hermitian matrix has real eigenvalues only. The matrix is Hermitian provided that $\hat{A}^\dagger = \hat{A}$, where ${A_{ij}}^\dagger = {A^*}_{ji}$, and * denotes complex conjugation.