FRIEDRICH-SCHILLER-UNIVERSITÄT JENA
Institute of Condensed Matter Theory and Solid State Optics – PAF
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# Series VIII
# STRUCTURE OF MATTER

due on December 17th after the lecture

## Exercise 1 (Angular momentum operator) — 6 + 2 +2 points

In complete analogy to classical physics, in quantum mechanics the angular momentum operator is defined as $\mathbf{L} = \mathbf{r} \times \mathbf{p}$.

a) Calculate $L_x$, $L_y$, $L_z$ and $\mathbf{L}^2$ in spherical coordinates!

b) Determine the value of the commutator $\left[L_z, \mathbf{L}^2\right]$!

c) Show that two operators $\hat{A}$ and $\hat{B}$, which commute as $\left[\hat{A}, \hat{B}\right] = 0$ posses a joint system of eigenfunctions! Here we assume for simplicity that at least for one of the operators all eigenvalues are non-degenerate.

## Exercise 2 — 6 + 2 + 1 points

Stellar spectra exhibit a multitude of spectral lines, and by far not all of them are related to hydrogen. Some of them belong to the spectra of ionized Helium and Lithium. However, the spectra of $H$, $He^+$ and $Li^{2+}$ exhibit strong similarities.

a) Sketch and calculate the energy levels of $He^+$ and $Li^{2+}$ up to the Hydrogen level $n = 3$ and compare the energy levels with the corresponding energy levels of the Hydrogen atom.

b) Which spectral lines of $He^+$ and $Li^{2+}$ would coincide with the Lyman-$\alpha$ series of Hydrogen if the Rydberg constants of Hydrogen, $He^+$ and $Li^{2+}$ would be the same?

c) Let us assume that the reduced electron mass is the same for Hydrogen and $He^+$. Would we find some orbits with coinciding radii for the Hydrogen atom and the He-ion, respectively?

## Exercise 3 — 5 points

At $t = 0$ the electron in a hydrogen atom shall be described by the following state consisting of a superposition of eigenstates $\psi_{nlm}$

$$\psi(\mathbf{r}, t = 0) = \frac{1}{5}\left(3\psi_{100}(\mathbf{r}, 0) - 2\psi_{211}(\mathbf{r}, 0) + \sqrt{12}\psi_{21-1}(\mathbf{r}, 0)\right).$$

a) Calculate the expectation value of $H$ (in units of the Rydberg-energy $E_R$) as well as the expectation values of $\mathbf{L}^2$ and $\mathbf{L}_z$ at $t = 0$.

b) At $t = 0$ the energy is measured. What is the probability to find the energy of the $n = 2$ eigenstate $E_2$ in this measurement?