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Exam
FUNDAMENTALS OF MODERN OPTICS

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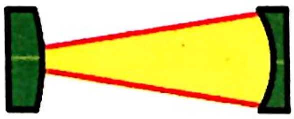
Exercise 1 — 8 Points

A resonator consists of two spherical mirrors: one of them is a concave mirror with radius $R_1 > 0$, the other one is a convex mirror with radius $R_2 < 0$, and the distance between them is d. Define the conditions for this resonator to be stable, and sketch a stable configuration. Do not forget to mark the positions of the mirrors and their respective centers of curvature in your sketch.



Exercise 2 — 10 Points

In an experiment you have an input Gaussian beam profile with width W_{in} and flat phase (phase curvature $R_{in} = \infty$). Your task is to obtain a beam profile with the same parameters (W_{in} and R_{in}), but at a distance L from the input position. You are allowed to use just a single lens. Calculate the focal length of the lens you need and its position.

Exercise 3 — 15 Points

Illumination of a cross grating produces a light distribution

$$u_0(x,y) = \frac{A}{4}\left(1+\cos\frac{2\pi}{a}x\right)\left(1+\cos\frac{2\pi}{a}y\right),$$

with period length $a = 1\,\mathrm{mm}$. This light field is now imaged by a $4f$-setup, where in the plane $z = 2f$ a slit with the filter function

$$p(x,y) = \begin{cases} 1 & ,|x| < D/2 \\ 0 & ,\text{elsewhere} \end{cases}$$

is applied. The focal length is $f = 1\,\mathrm{m}$ and the wavelength used is $\lambda = 1\,\mu\mathrm{m}$.
Calculate the field $u(x,y,4f)$ at the end of the $4f$-setup for a slit width $D = 1\,\mathrm{mm}$.

Exercise 4 — 10+5 Points

a) Consider a dielectric medium with response function

$$R(t) = \frac{f}{\sqrt{a^2-b^2}}\, e^{-bt} \sin\left(\sqrt{a^2-b^2}\, t\right),$$

where $a > b > 0$. Calculate the electric susceptibility $\chi(\omega)$.

b) A company producing optical instruments is looking for a new homogeneous, isotropic material which should have the optical property

$$\chi(\omega) = Ae^{-\frac{(\omega-\omega_0)^2}{B^2}} + i\, C\delta(\omega - \omega_0),$$

where $A = 0.542$, $B = 1.02 \cdot 10^{15}\, s^{-1}$, $C = 3.29$ and $\omega_0 = 4.71 \cdot 10^{15}\, s^{-1}$. Do you think their research can be successful? Explain with the help of the Kramers Kronig relations.

Exercise 5 **7 Points**

Consider an uniaxial crystal with refractive indices for the ordinary wave n_o and the extraordinary wave n_e. The crystal's optical axis is parallel to its surface. A monochromatic, circularly polarized wave is normally incident on the crystal. Compute the propagation lengths after which the light is linearly polarized.

Exercise 6 **10 Points**

Two pulses are propagating in a homogeneous plasma. They have a carrier frequency much larger than the plasma frequency. The corresponding wavelengths of the carrier waves are λ_1 and λ_2.
The signals are recorded by a detector that is located at a distance L from the source. Use the dielectric function of the plasma

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

to compute the time delay between the two pulses.

Exercise 7 **5+10 Points**

Consider an interface between two media, $n(x < 0) = n_1 = 2$ and $n(x > 0) = n_2 = 1$.

a) Compute the angle of incidence ϕ_{Itot} above which total internal reflection occurs.

b) If the second medium (n_2) is not infinite, but forms a layer of thickness d [$n(0 < x < d) = n_2 = 1$], and $n(x > d) = n_3 = 2$, do you still expect reflectivity $\rho = 1$ at the interface at $x = 0$? Compute the reflectivity for this layer (TE-polarization) to prove your answer.

Exercise 8 **10+10 Points**

Investigate the propagation of a 1-dimensional initial field distribution $u_0(x) = A\cos^2(x/\Lambda)$ in paraxial (Fresnel) approximation.

a) At which propagation distances z_T do we observe $|u(x, z_T)|^2 = |u_0(x)|^2$?

b) Now consider the above $u_0(x)$ with a finite aperture $a = N\pi\Lambda/2$, so that

$$\tilde{u}_0(x) = \begin{cases} u_0(x) & , |x| < a \\ 0 & , \text{elsewhere.} \end{cases}$$

Compute the far field intensity distribution of $\tilde{u}_0(x)$, and discuss the necessary propagation distances to apply Fraunhofer approximation with respect to N.