



**Institute of  
Applied Physics**

Friedrich-Schiller-Universität Jena

# Lens Design I

---

Lecture 3: Properties of optical systems II

2024-04-25

Ziyao Tang

# Preliminary Schedule - Lens Design I 2024

1	04.04.	Basics	Zhang	Introduction, Zemax interface, menus, file handling, preferences, Editors, updates, windows, coordinates, System description, 3D geometry, aperture, field, wavelength
2	18.04.	Properties of optical systems I	Tang	Diameters, stop and pupil, vignetting, layouts, materials, glass catalogs, raytrace, ray fans and sampling, footprints
3	25.04.	Properties of optical systems II	Tang	Types of surfaces, cardinal elements, lens properties, Imaging, magnification, paraxial approximation and modelling, telecentricity, infinity object distance and afocal image, local/global coordinates
4	02.05.	Properties of optical systems III	Tang	Component reversal, system insertion, scaling of systems, aspheres, gratings and diffractive surfaces, gradient media, solves
5	16.05.	Advanced handling I	Tang	Miscellaneous, fold mirror, universal plot, slider, multiconfiguration, lens catalogs
6	23.05.	Aberrations I	Zhang	Representation of geometrical aberrations, spot diagram, transverse aberration diagrams, aberration expansions, primary aberrations
7	30.05.	Aberrations II	Zhang	Wave aberrations, Zernike polynomials, measurement of quality
8	06.06.	Aberrations III	Tang	Point spread function, optical transfer function
9	13.06.	Optimization I	Tang	Principles of nonlinear optimization, optimization in optical design, general process, optimization in Zemax
10	20.06.	Optimization II	Zhang	Initial systems, special issues, sensitivity of variables in optical systems, global optimization methods
11	27.06.	Correction I	Zhang	Symmetry principle, lens bending, correcting spherical aberration, coma, astigmatism, field curvature, chromatical correction
12	04.07.	Correction II	Zhang	Field lenses, stop position influence, retrofocus and telephoto setup, aspheres and higher orders, freeform systems, miscellaneous



1. Types of surfaces
2. Cardinal elements
3. Lens properties
4. Imaging
5. Special infinity cases
6. Paraxial approximation

- Special surface types
  - Data in Lens Data Editor or in Extra Data Editor
  - Gradient media are described as 'special surfaces'
  - Diffractive / micro structured surfaces described by simple ray tracing model in one order
- 
- |                       |   |
|-----------------------|---|
| ▪ Standard            | spherical and conic sections                  |
| ▪ Even asphere        | classical asphere                             |
| ▪ Paraxial            | ideal lens                                    |
| ▪ Paraxial XY         | ideal toric lens                              |
| ▪ Coordinate break    | change of coordinate system                   |
| ▪ Diffraction grating | line grating                                  |
| ▪ Gradient 1          | gradient medium                               |
| ▪ Toroidal            | cylindrical lens                              |
| ▪ Zernike Fringe sag  | surface as superposition of Zernike functions |
| ▪ Extended polynomial | generalized asphere                           |
| ▪ Black Box Lens      | hidden system, from vendors                   |
| ▪ ABCD                | paraxial segment                              |

# Notations of a lens



P principal point

S vertex of the surface

F focal point

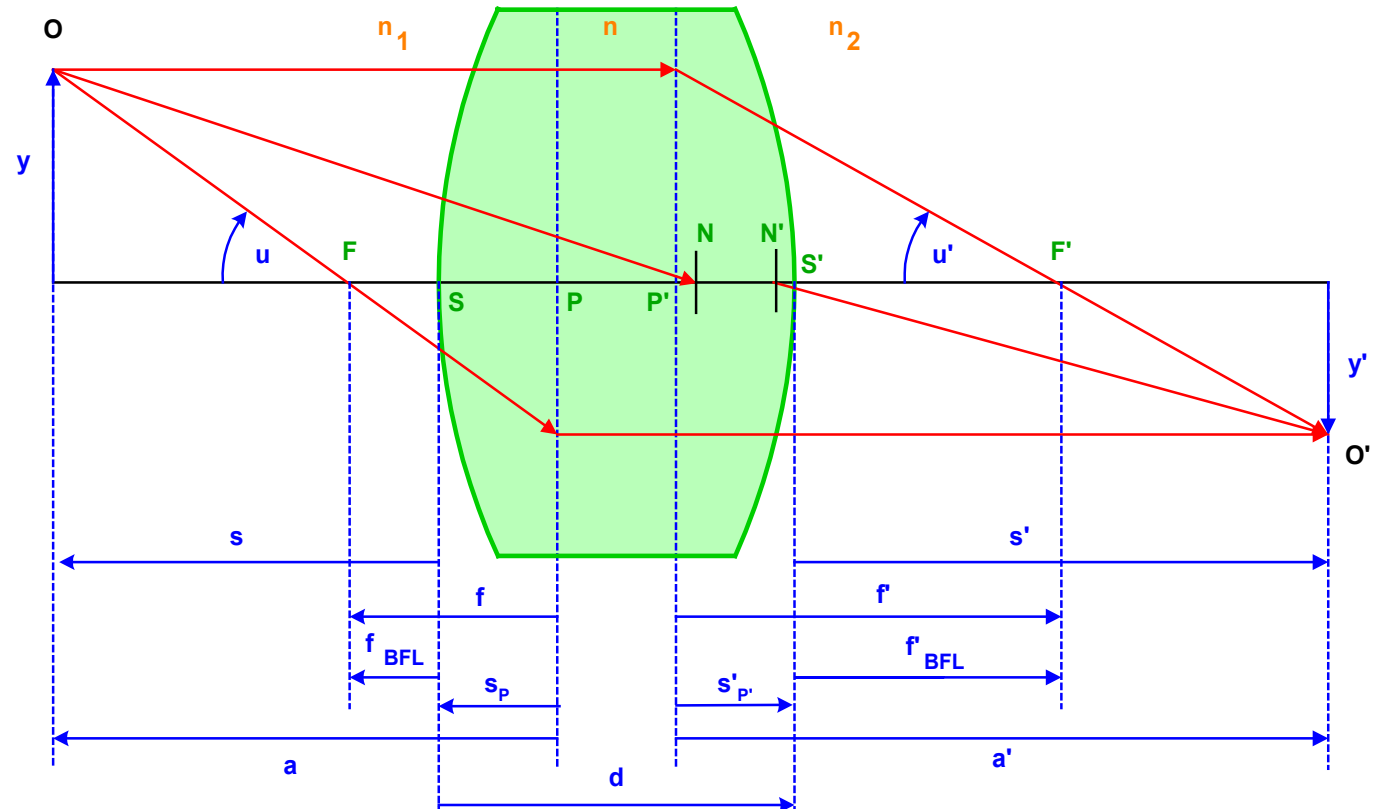
s intersection point  
of a ray with axis

f focal length PF

r radius of surface  
curvature

d thickness SS'

n refractive index





# Main properties of a lens

- Main notations and properties of a lens:

- radii of curvature  $r_1$  ,  $r_2$

- curvatures  $c$

- sign:  $r > 0$  : center of curvature located on the right side

- thickness  $d$  along the axis

- diameter  $D$

- index of refraction of lens material  $n$

$$c_1 = \frac{1}{r_1} \quad c_2 = \frac{1}{r_2}$$

- Focal length (paraxial)

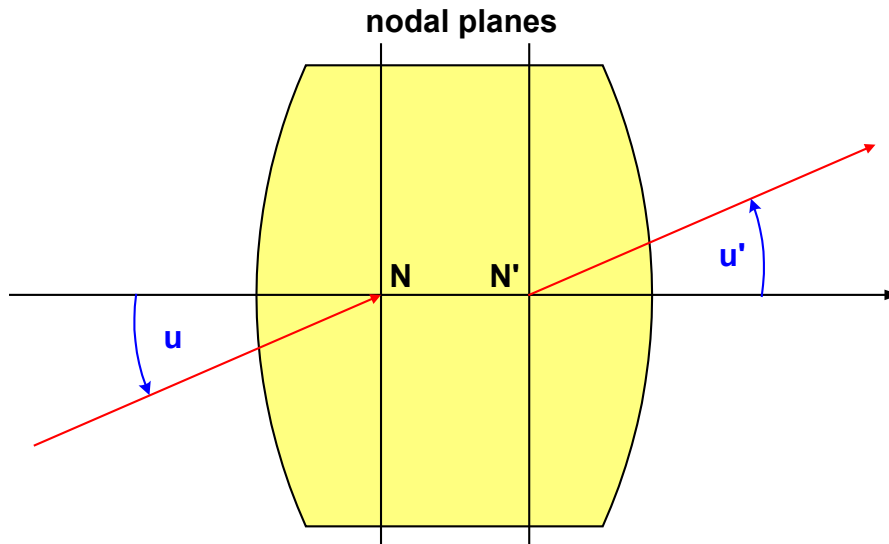
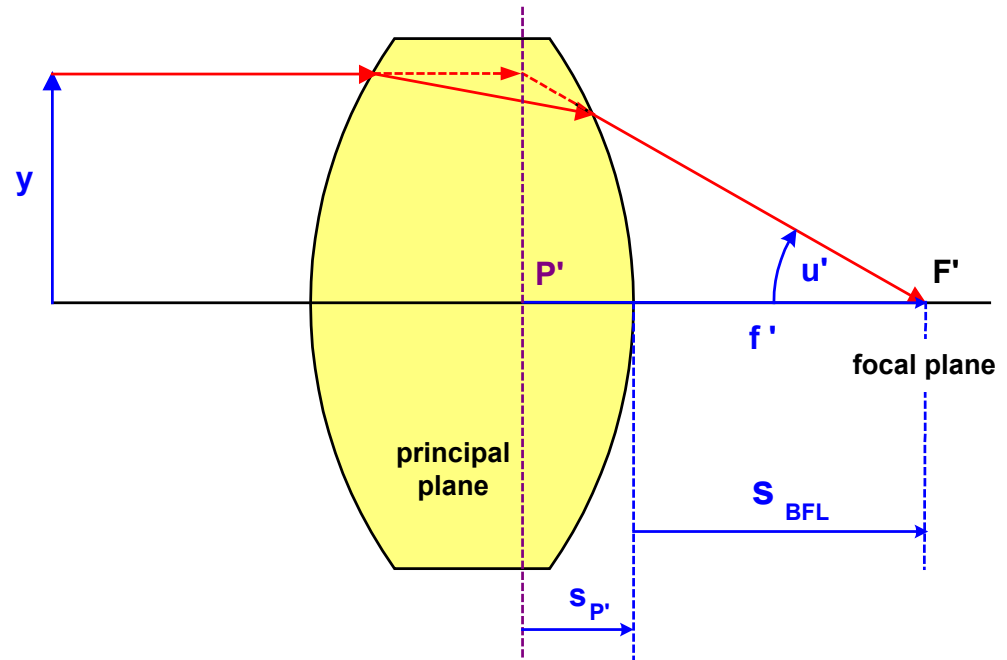
$$f = \frac{y_F'}{\tan u} \quad , \quad f' = \frac{y}{\tan u'}$$

- Optical power

$$F = -\frac{n}{f} = \frac{n'}{f'}$$

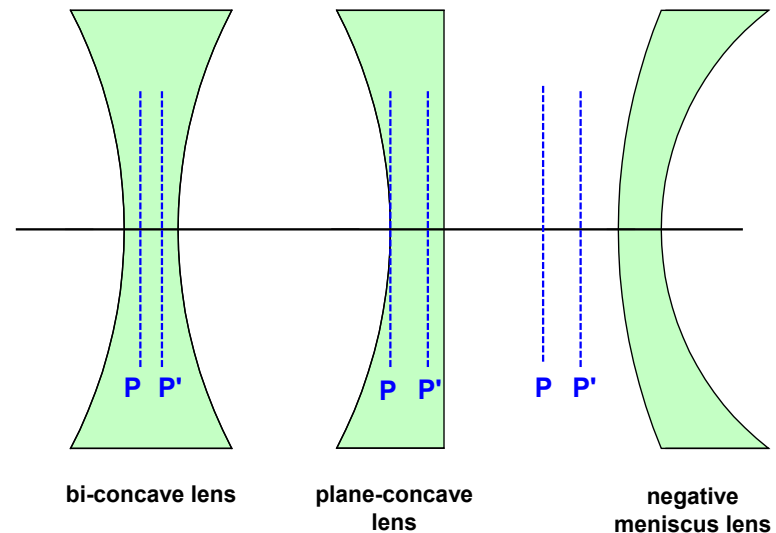
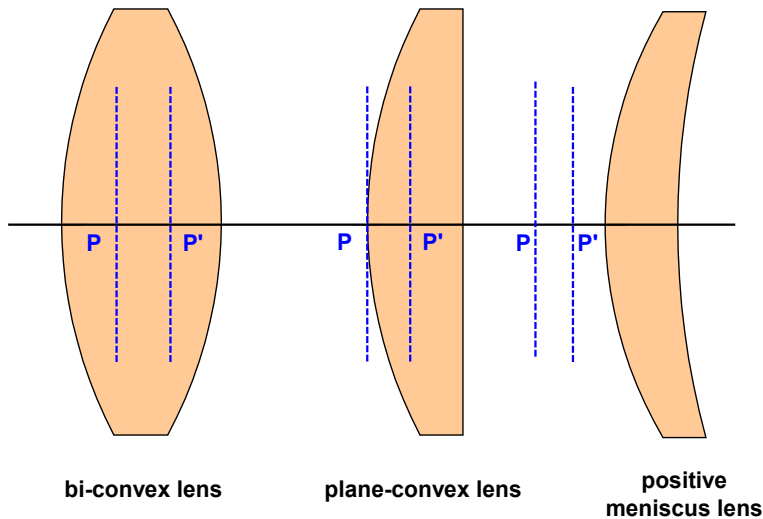
- Back focal length: focal length measured from the vertex point

- Focal points:
  1. incoming parallel ray intersects the axis in  $F'$
  2. ray through  $F$  leaves the lens parallel to the axis
- Principal plane  $P$ :  
location of apparent ray bending  
i.e. Lateral magnification=1



- Nodal points:  
Ray through  $N$  goes through  $N'$   
and preserves the direction  
i.e. Angular magnification=1

- Different shapes of singlet lenses:
  1. bi-, symmetric
  2. plane convex / concave, one surface plane
  3. Meniscus, both surface radii with the same sign
- Convex: bending outside  
Concave: hollow surface
- Principal planes  $P$ ,  $P'$ : outside for meniscus shaped lenses



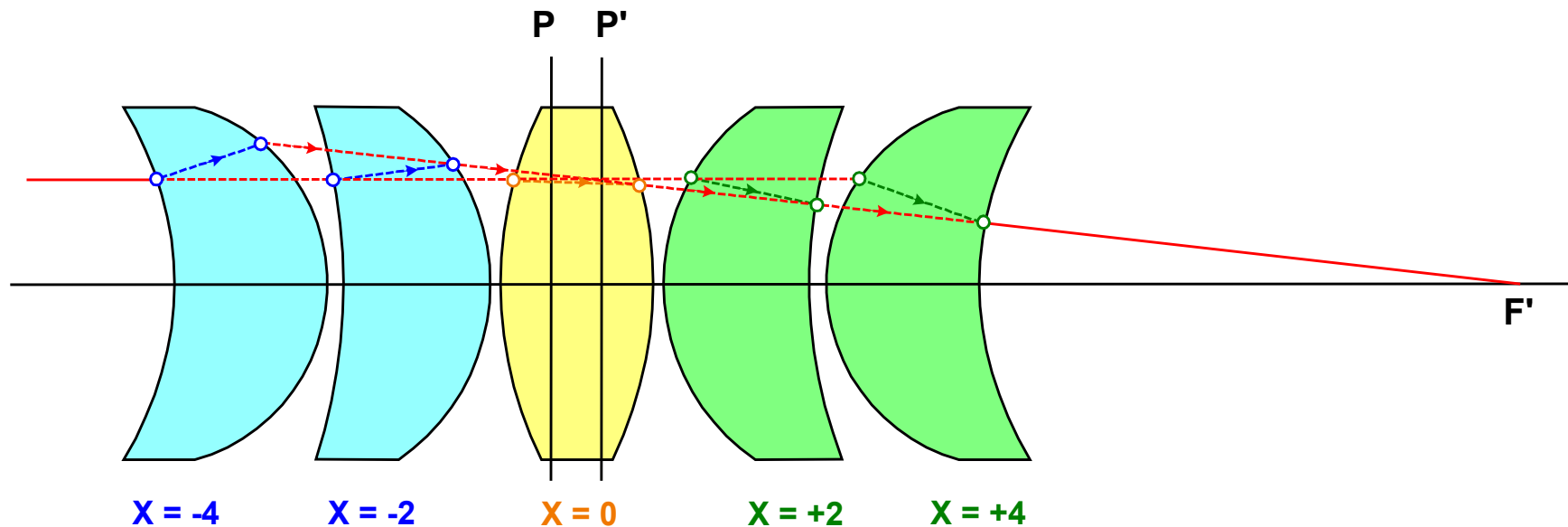




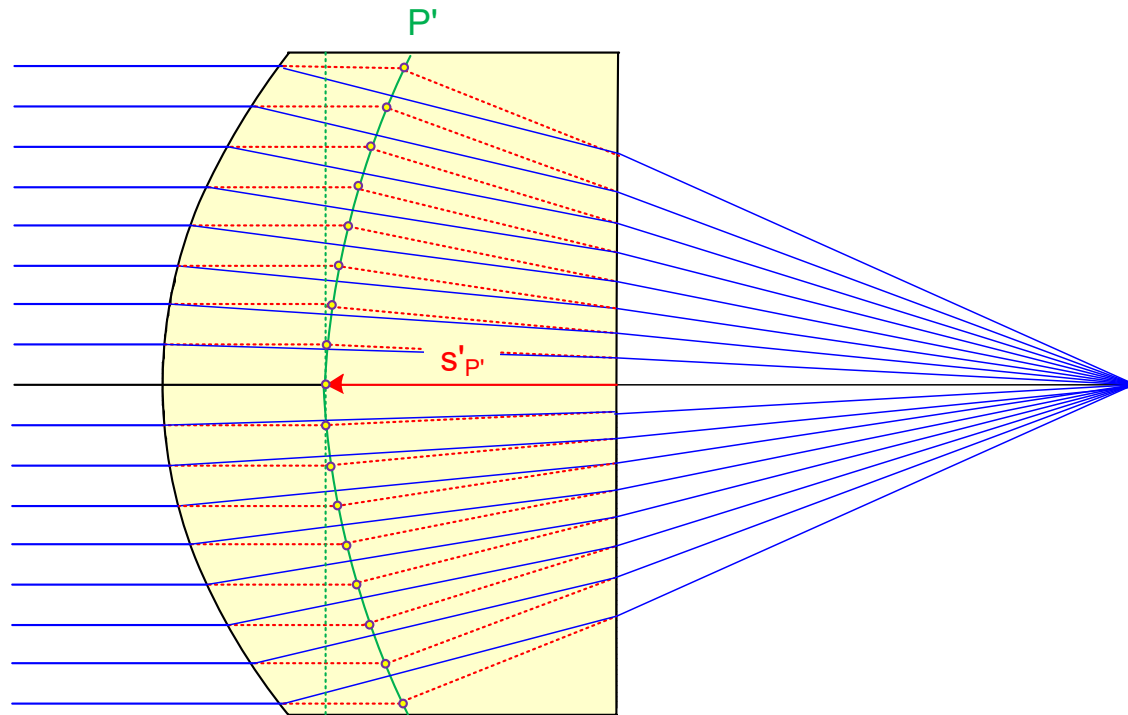
# Lens bending und shift of principal plane

- Ray path at a lens of constant focal length and different bending
- Quantitative parameter of description  $X$ :
- The ray angle inside the lens changes
- The ray incidence angles at the surfaces changes strongly
- The principal planes move  
For invariant location of  $P$ ,  $P'$  the position of the lens moves

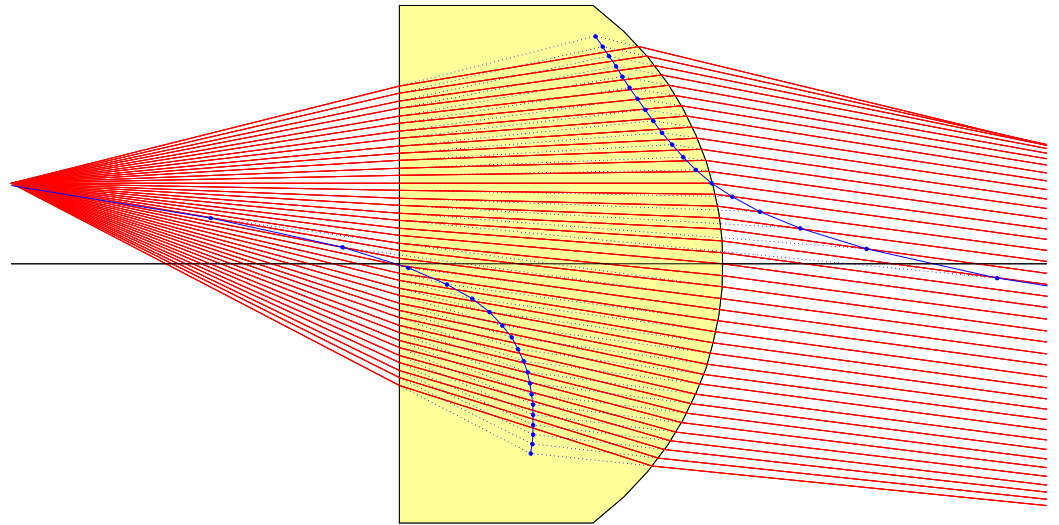
$$X = \frac{R_1 + R_2}{R_2 - R_1}$$



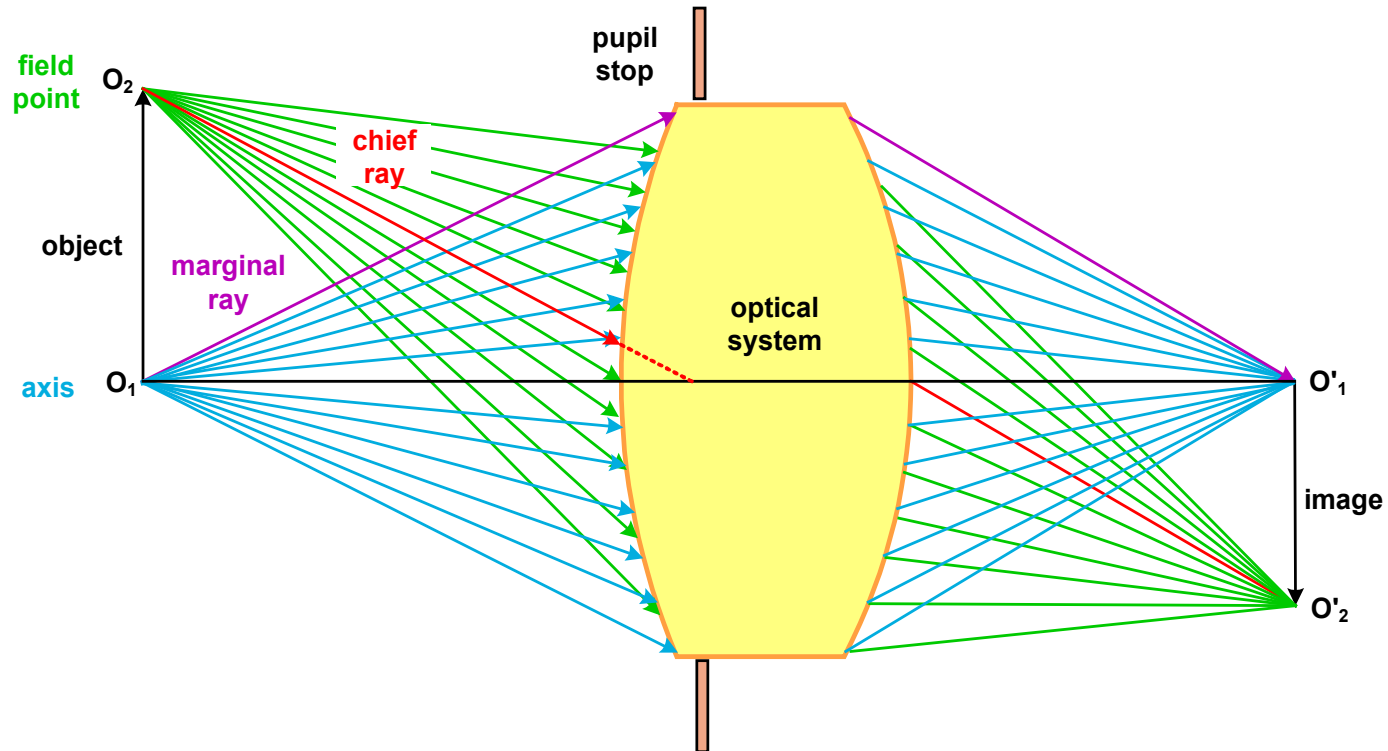
- Real lenses:  
The surface with the principal points as apparent ray bending points is a curved shell
- The ideal principal plane exists only in the paraxial approximation



- The principal planes in paraxial optics are defined as the locations of the apparent ray bending of a lens or system
- In the case of a system with corrected sine conditions, these surfaces are spheres
- Sine condition and pupil spheres are also limited for off-axis points near to the optical axis
- For object points far from the axis, the apparent locations are complicated surfaces, which may consist of two branches



- Optical Image formation:  
All ray emerging from one object point meet in the perfect image point
- Region near axis:  
gaussian imaging  
ideal, paraxial
- Image field size:  
Chief ray
- Aperture/size of  
light cone:  
marginal ray  
defined by pupil  
stop





# Formulas for surface and lens imaging

- Single surface imaging equation

$$\frac{n'}{s'} - \frac{n}{s} = \frac{n' - n}{r} = \frac{1}{f'}$$

- Thin lens in air focal length

$$\frac{1}{f'} = (n - 1) \cdot \left( \frac{1}{r_1} - \frac{1}{r_2} \right)$$

- Thin lens in air with one plane surface, focal length

$$f' = \frac{r}{n - 1}$$

- Thin symmetrical bi-lens

$$f' = \frac{r}{2 \cdot (n - 1)}$$

- Thick lens in air focal length

$$\frac{1}{f'} = (n - 1) \cdot \left( \frac{1}{r_1} - \frac{1}{r_2} \right) + \frac{(n - 1)^2 d}{n \cdot r_1 r_2}$$

# Imaging equation

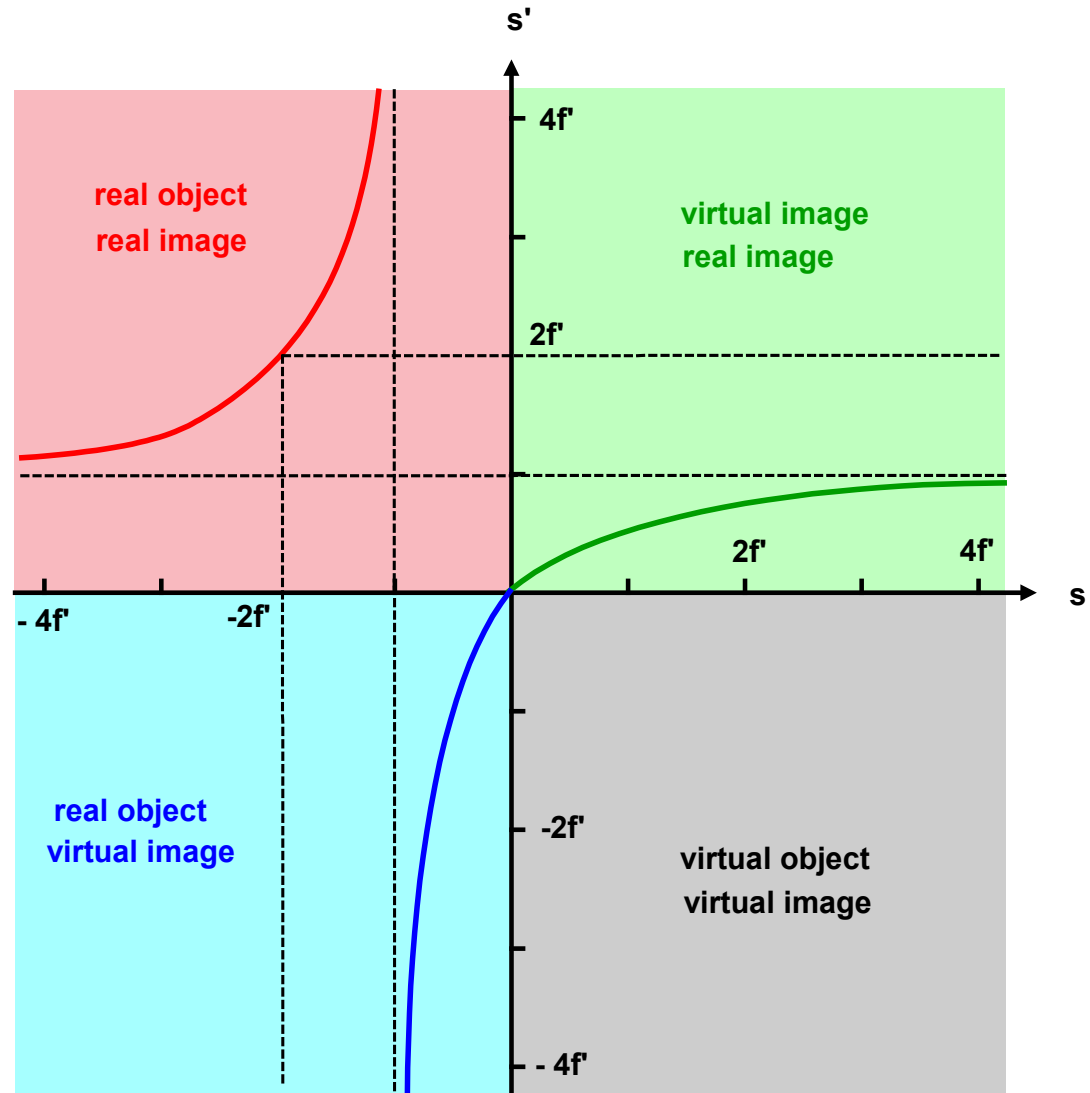
- Imaging by a lens in air:  
lens makers formula

$$\frac{1}{s'} - \frac{1}{s} = \frac{1}{f}$$

- Magnification

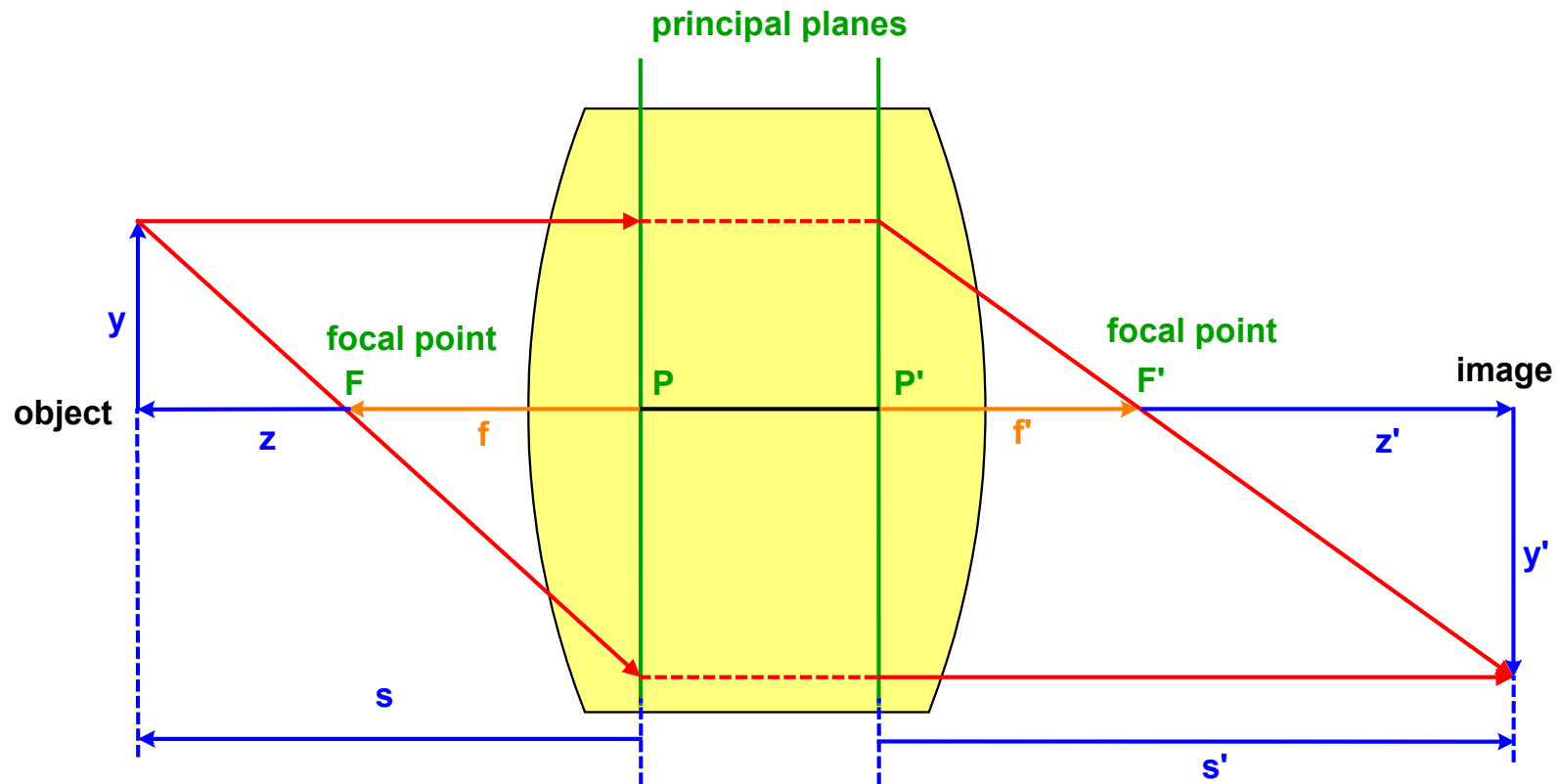
$$m = \frac{s'}{s}$$

- Real imaging:  
 $s < 0$  ,  $s' > 0$
- Intersection lengths  $s$ ,  $s'$   
measured with respect to the  
principal planes  $P$ ,  $P'$



- Lateral magnification for finite imaging
- Scaling of image size

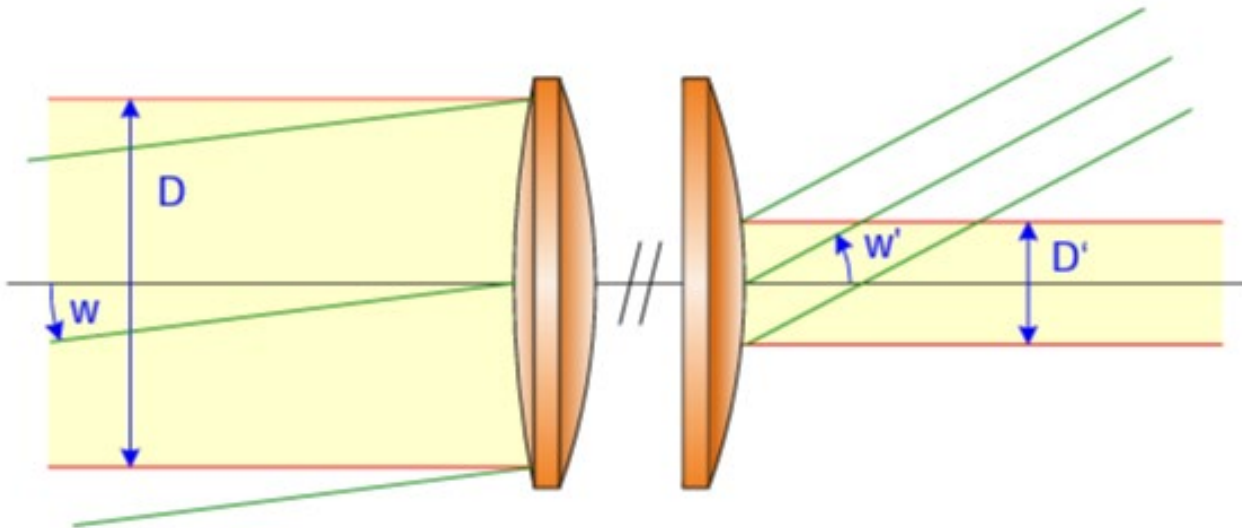
$$m = \frac{y'}{y} = - \frac{f \cdot \tan u}{f' \cdot \tan u'}$$



# Angle Magnification

- Afocal systems with object/image in infinity
- Definition with field angle  $w$   
angular magnification

$$\Gamma = \frac{\tan w'}{\tan w} = \frac{nh}{n'h'}$$



- Relation with finite-distance magnification

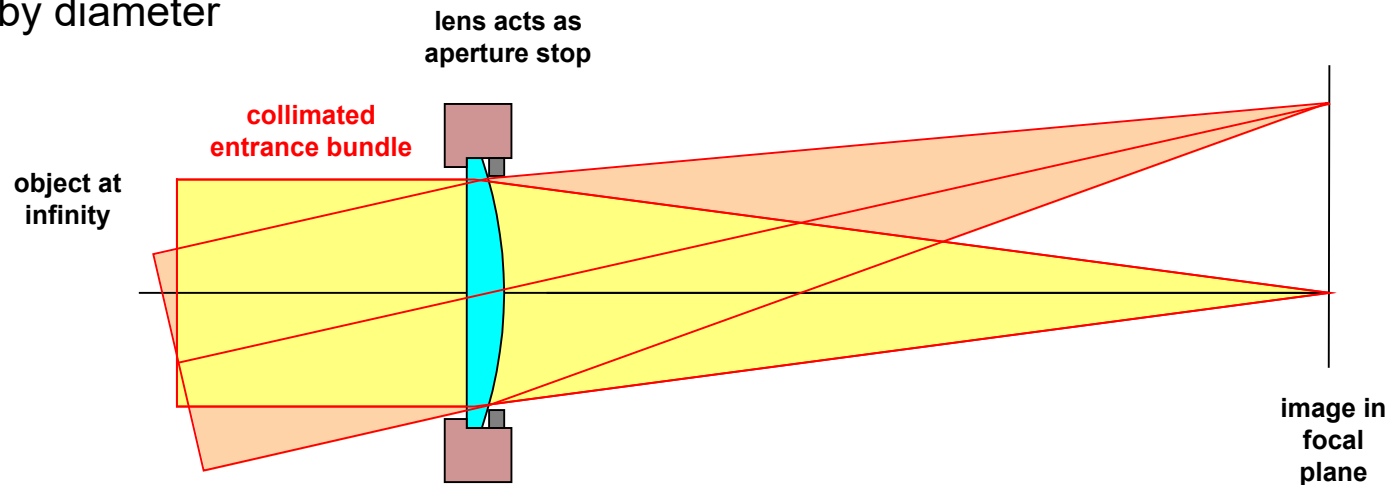
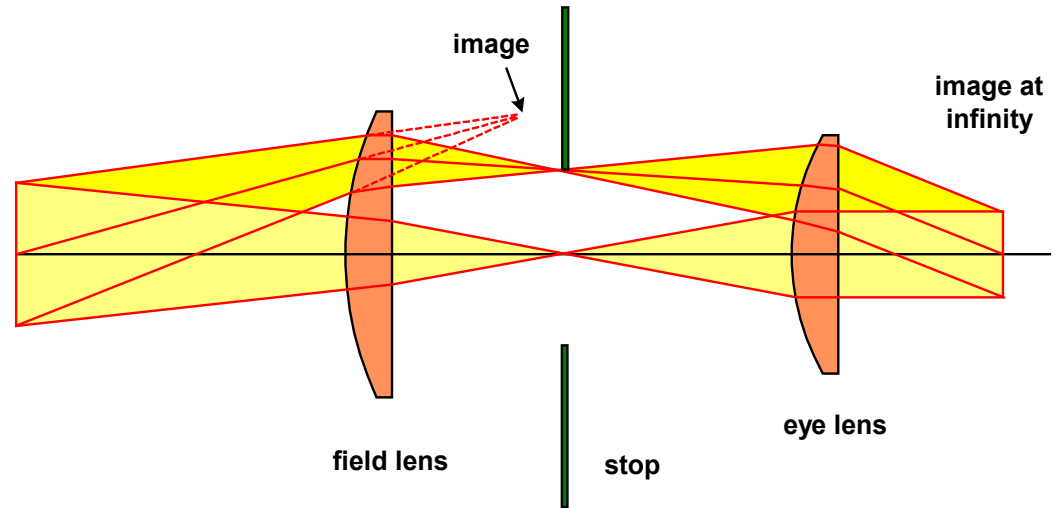
$$m \cdot \Gamma = -\frac{f}{f'}$$



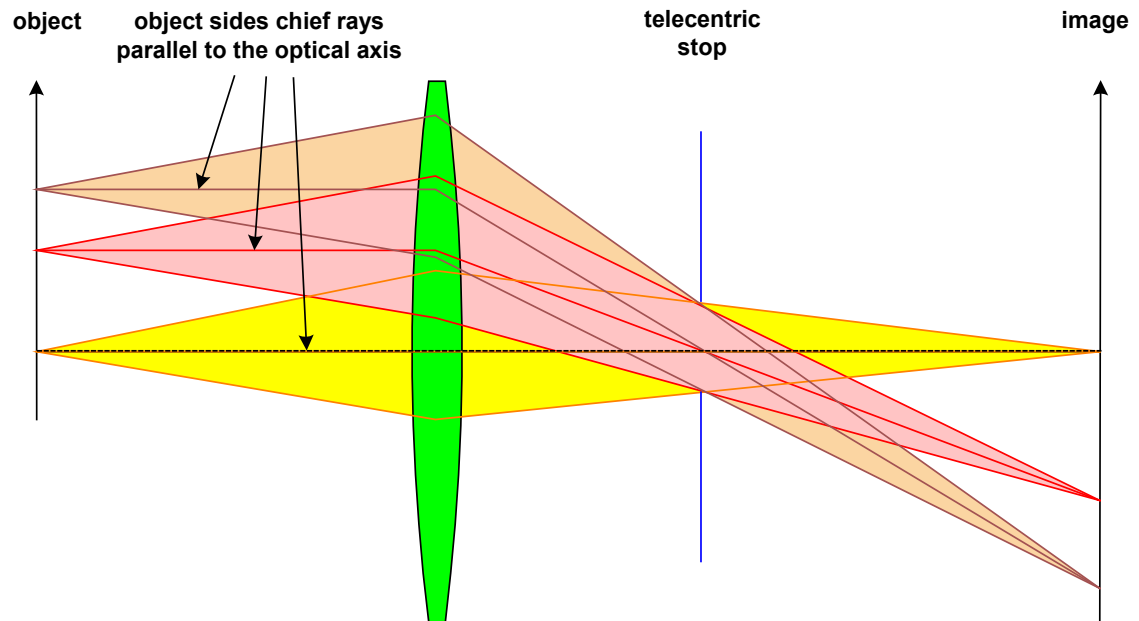
# Object or field at infinity



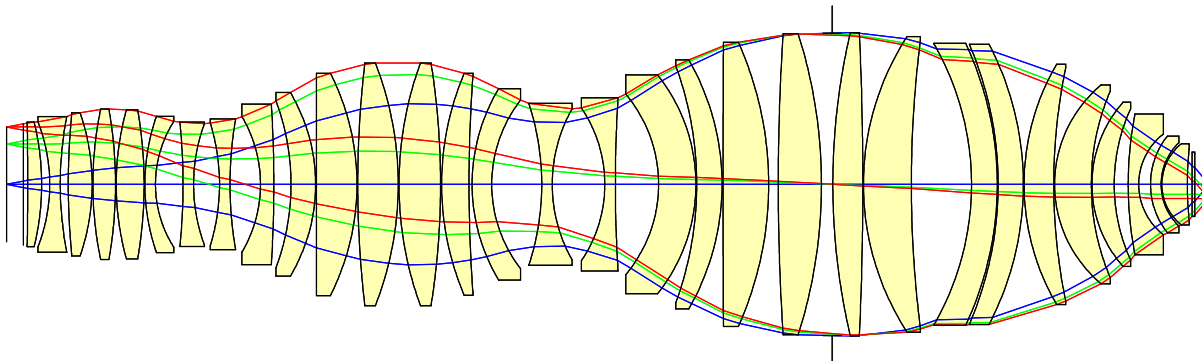
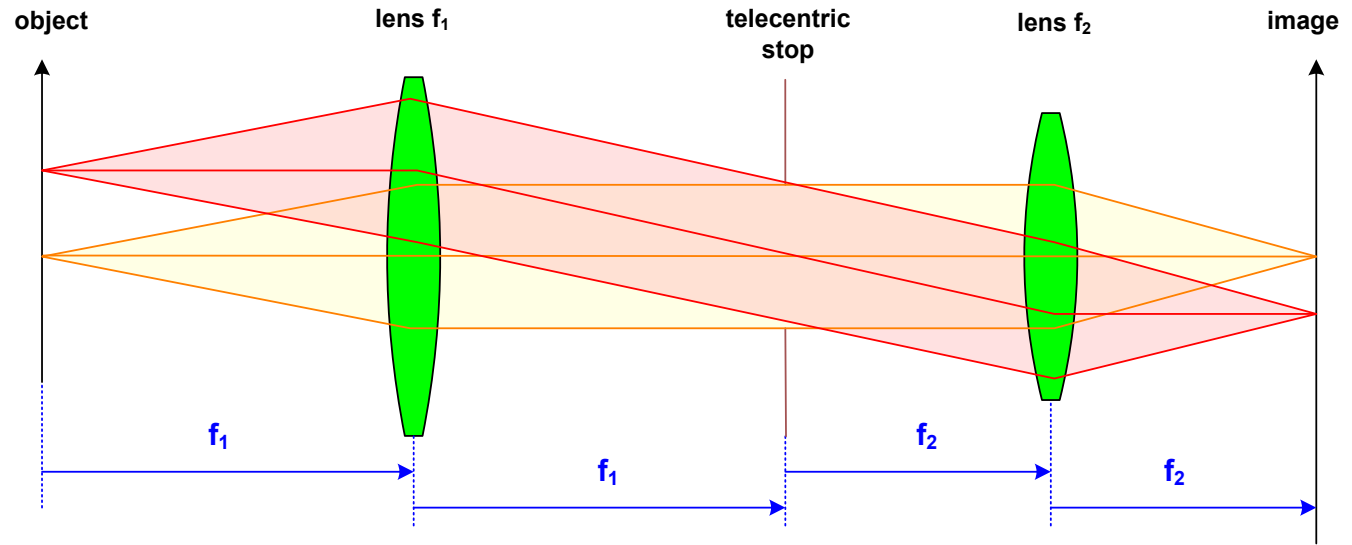
- Image in infinity:
  - collimated exit ray bundle
  - realized in binoculars
- Object in infinity
  - input ray bundle collimated
  - realized in telescopes
  - aperture defined by diameter not by angle



- Special stop positions:
  1. stop in back focal plane: object sided telecentricity
  2. stop in front focal plane: image sided telecentricity
  3. stop in intermediate focal plane: both-sided telecentricity
- Telecentricity:
  1. pupil in infinity
  2. chief ray parallel to the optical axis



- Double telecentric system: stop in intermediate focus
- Realization in lithographic projection systems





# Telecentricity, Infinity Object and Afocal Image

## 1. Telecentric object space

- Set in menu General / Aperture
- Means entrance pupil in infinity
- Chief ray is forced to be parallel to axis
- Fixation of stop position is obsolete
- Object distance must be finite
- Field cannot be given as angle

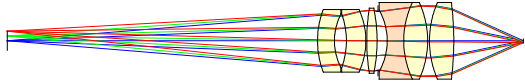
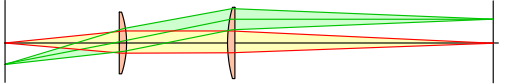
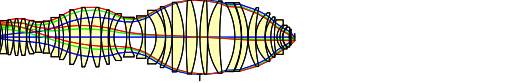
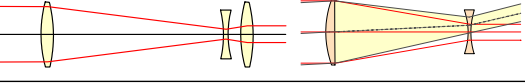
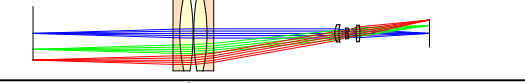
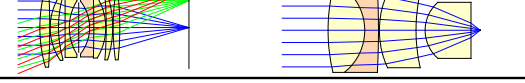
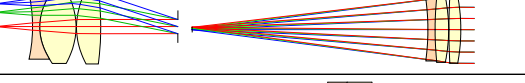
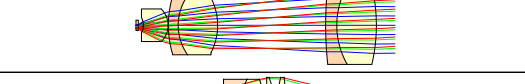
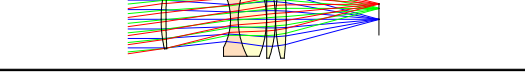
## 2. Infinity distant object

- Aperture cannot be NA
- Object size cannot be height
- Cannot be combined with telecentricity

## 3. Afocal image location

- Set in menu General / Aperture
- Aberrations are considered in the angle domain
- Allows for a plane wave reference
- Spot automatically scaled in mrad

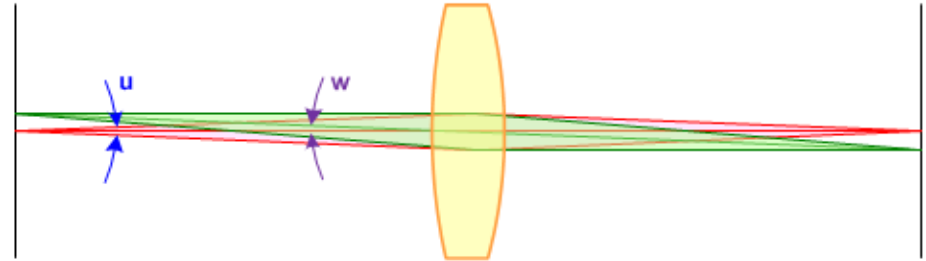
- Systematic of all infinity cases
- Physically impossible:
  1. object and entrance pupil in infinity
  2. image and exit pupil in infinity

case	object	image	entrance pupil	exit pupil	example	sample layout
1	finite	finite	finite	finite	relay	
2	finite	finite	finite	infinity image telecentric	metrology lens	
3	finite	finite	infinity object telecentric	infinity image telecentric	lithographic projection lens 4f-system	
4	infinity	infinity	finite	finite	afocal zoom telescopes beam expander	
5	finite	finite	infinity	finite	metrology lens	
6	infinity	finite	finite	finite	camera lens focussing lens	
7	finite	infinity	finite	finite	eyepiece collimator	
8	finite	infinity	infinity object telecentric	finite	microscopic lens	
9	infinity	finite	finite	infinity image telecentric	infinity metrology lens	
10	infinity	finite	infinity	finite	impossible	
11	finite	infinity	finite	infinity	impossible	
12	infinity	infinity	infinity	finite	impossible	
13	infinity	finite	infinity	infinity	impossible	
14	infinity	infinity	finite	infinity	impossible	
15	finite	infinity	infinity	infinity	impossible	
16	infinity	infinity	infinity	infinity	impossible	

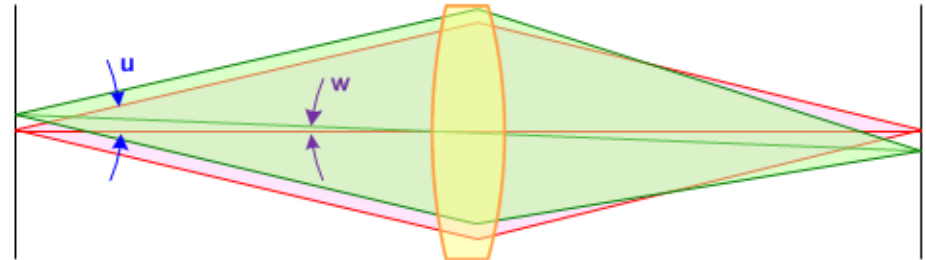
# Paraxial Approximation

- Paraxiality is given for small angles relative to the optical axis for all rays
- Large numerical aperture angle  $u$  violates the paraxiality, spherical aberration occurs
- Large field angles  $w$  violates the paraxiality, coma, astigmatism, distortion, field curvature occurs

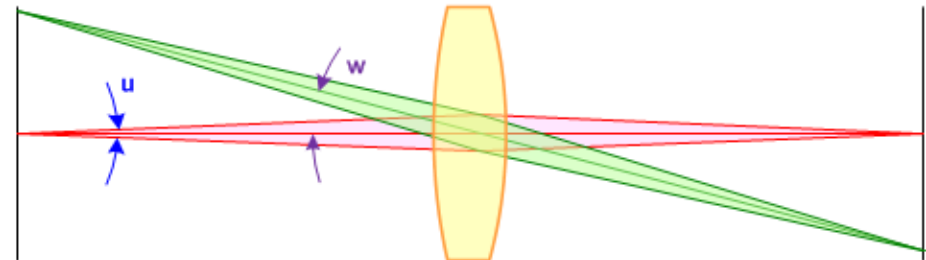
a) paraxial, small aperture / small field



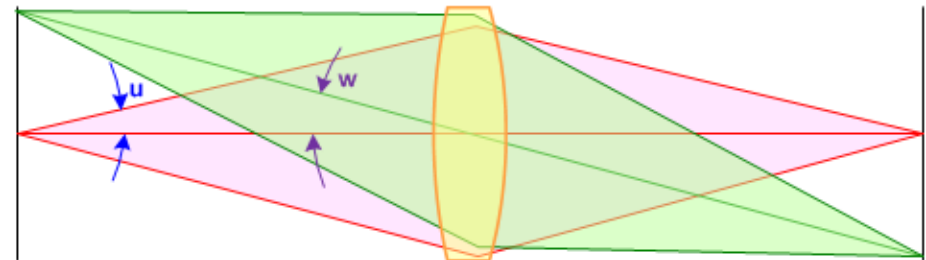
b) non-paraxial, large aperture / small field



c) non-paraxial, small aperture / large field



d) non-paraxial, large aperture / large field



Paraxial approximation:

- Small angles of rays at every surface
- Small incidence angles allows for a linearization of the law of refraction
- All optical imaging conditions become linear (Gaussian optics), calculation with ABCD matrix calculus is possible
- No aberrations occur in optical systems
- There are no truncation effects due to transverse finite sized components
- Serves as a reference for ideal system conditions
- Is the fundament for many system properties (focal length, principal plane, magnification,...)
- The sag of optical surfaces (difference in z between vertex plane and real surface intersection point) can be neglected
- All waves are plane or spherical (parabolic)
- The phase factor of spherical waves is quadratic

$$n \cdot i = n' \cdot i'$$

$$E(x) = E_0 \cdot e^{-\frac{i \pi x^2}{\lambda R}}$$



# Paraxial approximation

- Law of refraction

$$n \cdot \sin I = n' \cdot \sin I'$$

- Taylor expansion

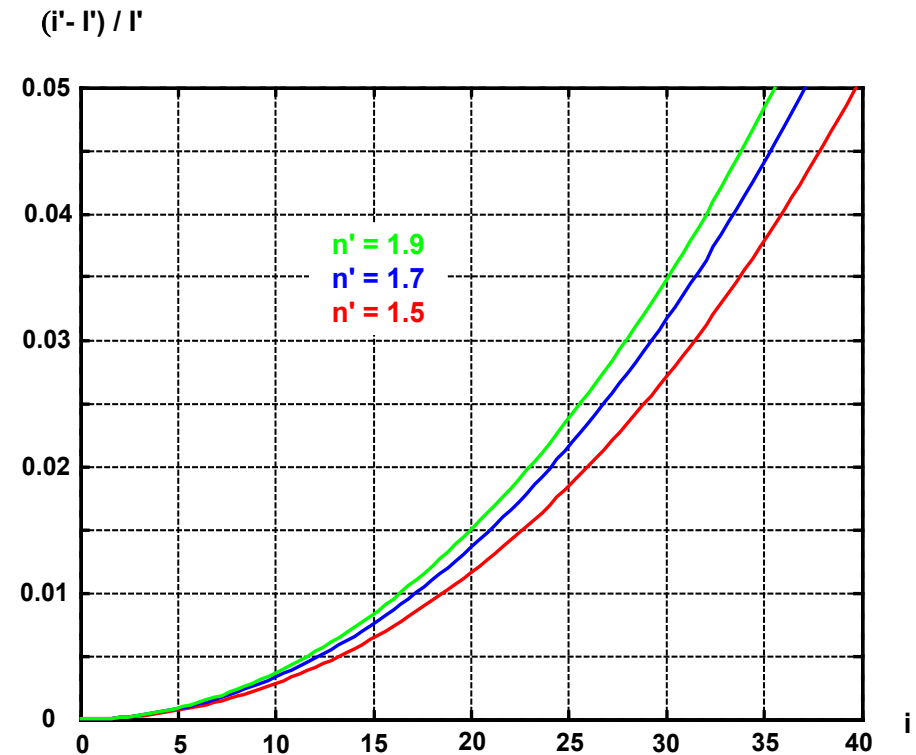
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

- Linear formulation of the law of refraction

$$n \cdot i = n' \cdot i'$$

- Error of the paraxial approximation

$$\varepsilon = \frac{i' - I'}{I'} = \frac{\frac{n \cdot i}{n'}}{\arcsin\left(\frac{n \cdot \sin i}{n'}\right)} - 1$$

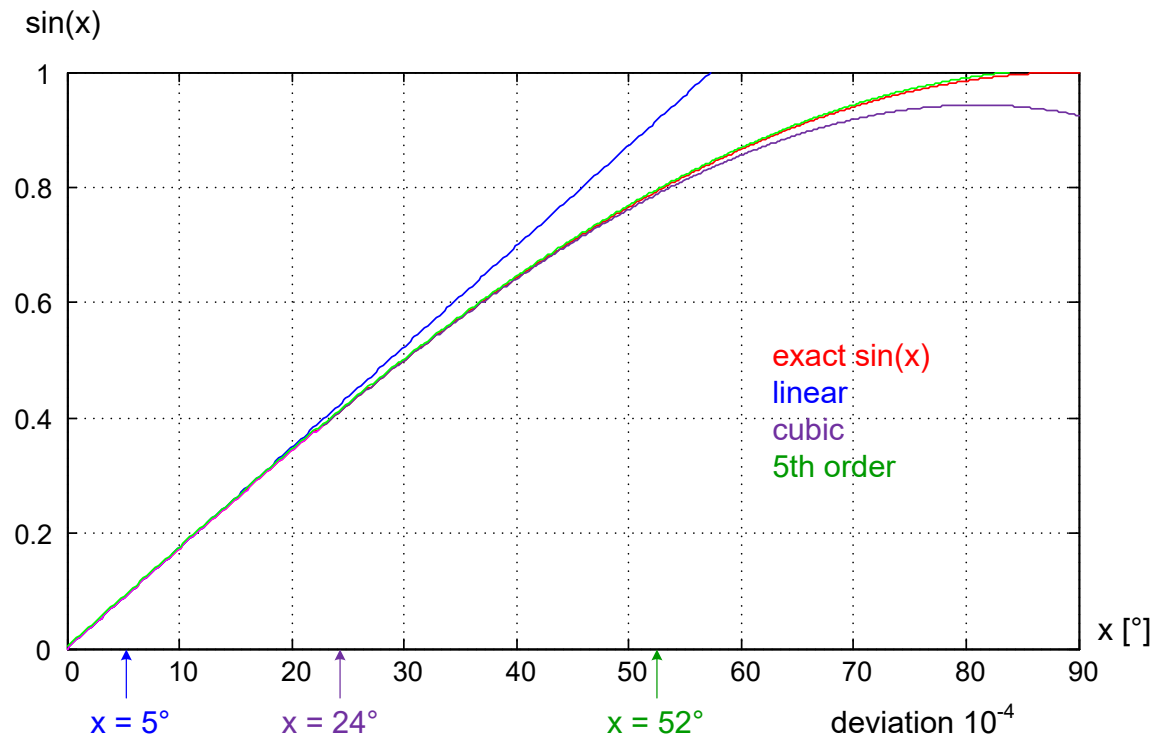






# Paraxial approximation

- Taylor expansion of the sin-function
- Definition of allowed error  $10^{-4}$
- Deviation of the various approximations:
  - linear:  $5^\circ$
  - cubic:  $24^\circ$
  - 5th order:  $52^\circ$



## 1. Coordinate reference

- Fixation of reference in menu: General / Misc
- Every surface vertex can be defined as global reference
- Helpful in constructing 3D-system geometries

## 2. Scale System

- In menu Tools / Miscellaneous / Scale
- Helpful in exploding/imploding all length scales
- Application: rescale patent systems
- Alternative option in menu Tools / Miscellaneous / Make focal, desired f realized

## 3. Add folding mirror

- Help command in menu Tools / Coordinates / Add fold mirror
- Automatically inserted coordinate break surface

## 4. Make double pass

- Help command in menu Tools / Miscellaneous / Make double pass
- Folding mirror and reversed system automatically generated



A system with an ellipsoidal mirror should be installed. For this task, the following steps should be performed:

- a) A source with wavelength  $\lambda = 1.064 \mu\text{m}$  and numerical aperture  $\text{NA} = 0.1$  is imaged by a spherical mirror in a 1:1 setup with a mirror radius of 20 mm
- b) The image distance is enlarged to 40 mm. The radius of the mirror and the conical constant are optimized for this geometry. According to the theory, an ellipsoidal mirror images one point perfect into another point.
- c) The coordinate system is rotated by  $60^\circ$  directly after the object. For a proper layout, the subaperture of the mirror which is used should be explicitly defined. Make a shaded model layout with this setup.  
What is the bending angle of the central ray at the mirror? Determine the shape and the approximate x/y-aspect ratio of the illuminated area on the mirror.