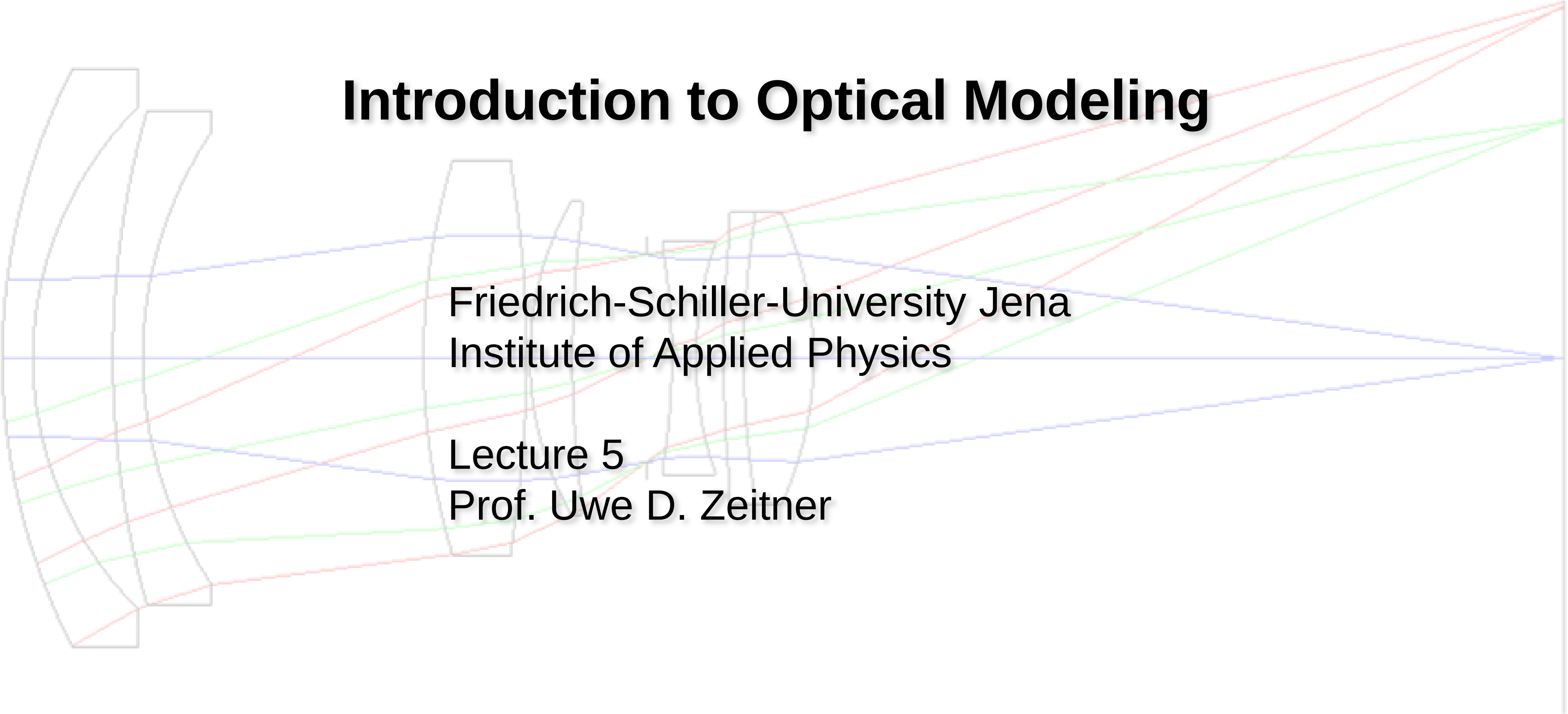
Introduction to Optical Modeling



Friedrich-Schiller-University Jena
Institute of Applied Physics

Lecture 5

Prof. Uwe D. Zeitner

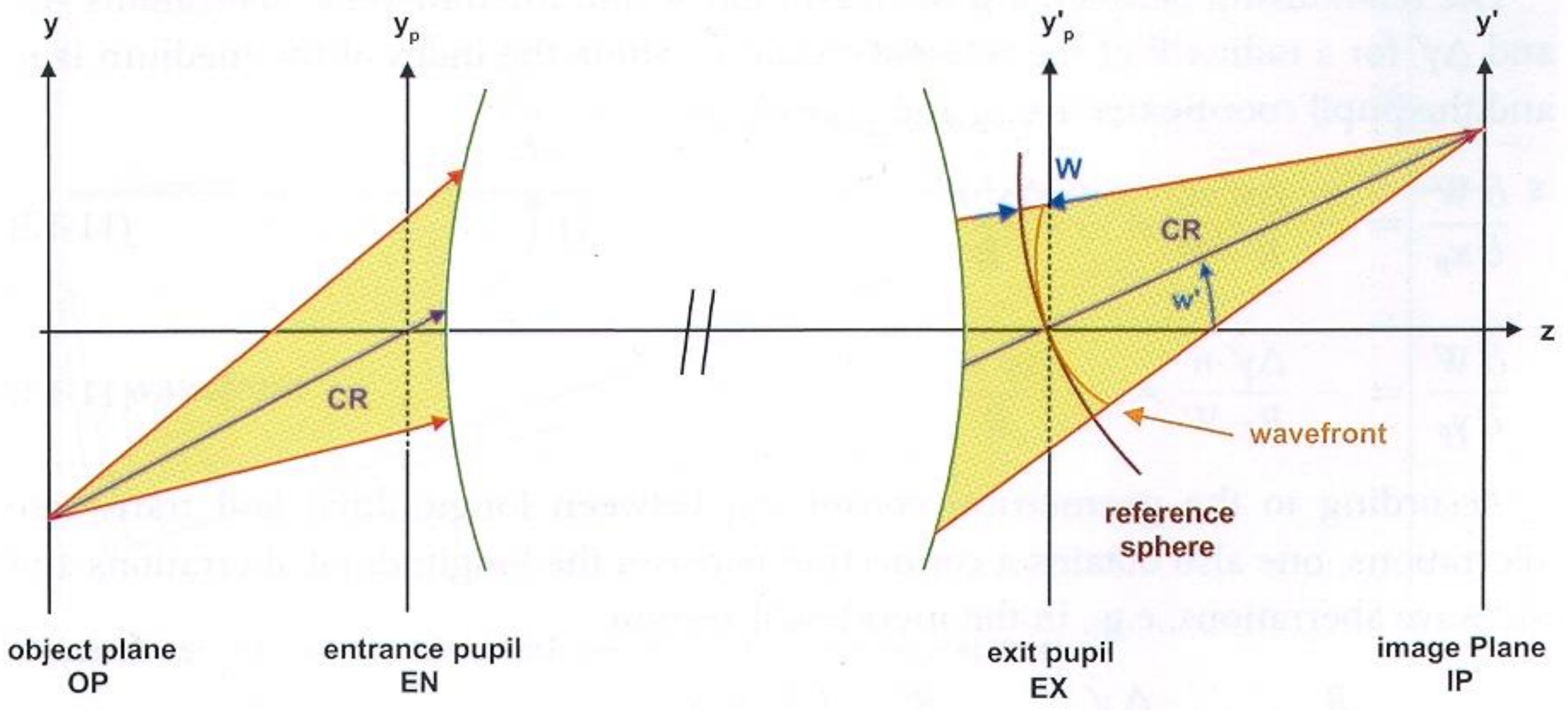
Course Overview

Part 1: Geometrical optics-based modeling and design (U.D. Zeitner)

1. Introduction
2. Paraxial approximation / Gaussian optics
3. ABCD-matrix formalism
4. Real lenses
5. Optical materials
   - glass types, dispersion
   - chromatic aberrations
6. **Imaging systems**
   - apertures/stops, entrance-/exit-pupil
   - **wavefront aberrations**

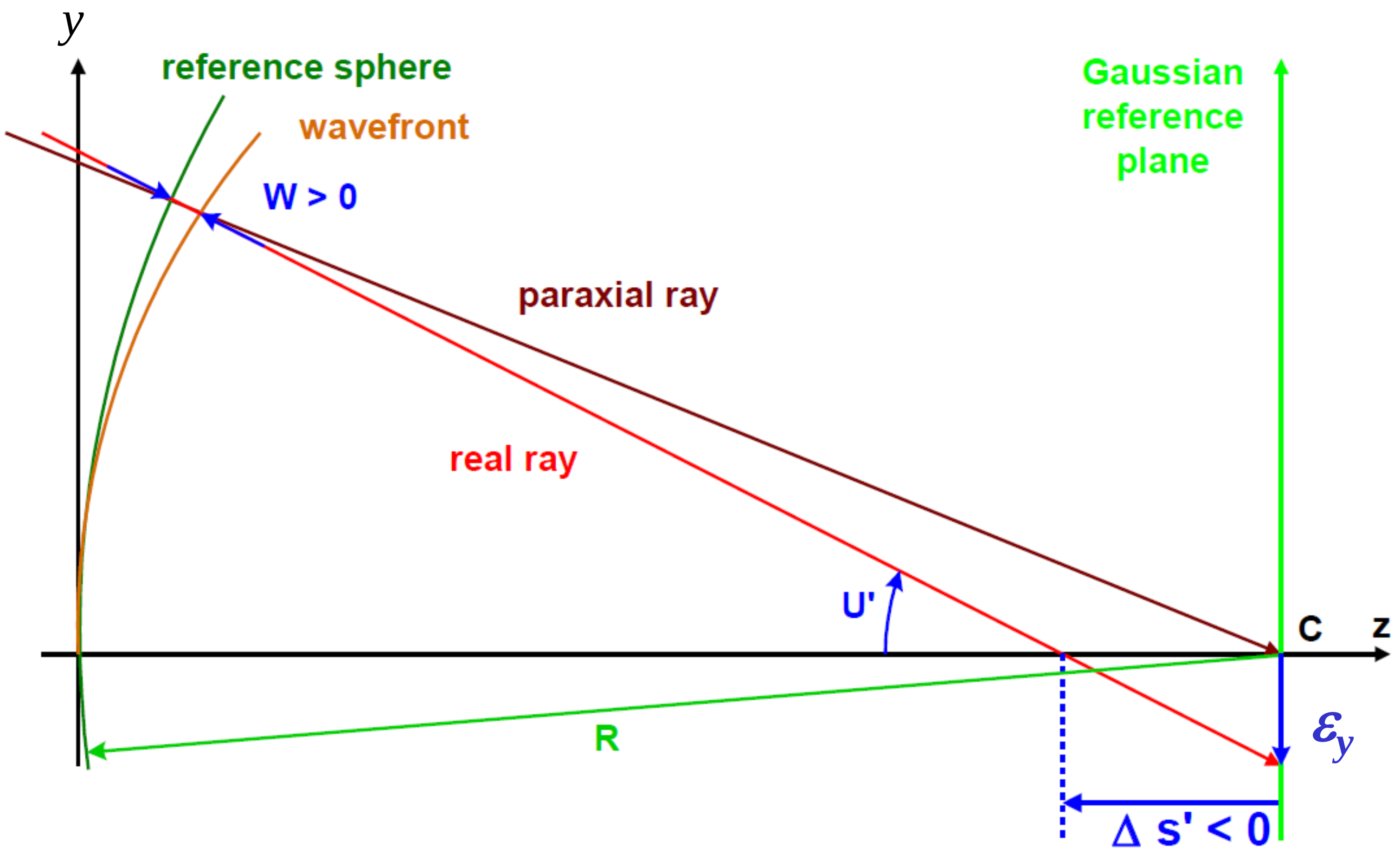
Part 2: Wave-optics based modeling (F. Wyrowski)

Wave aberrations of an optical system



Wave aberrations for an optical system.

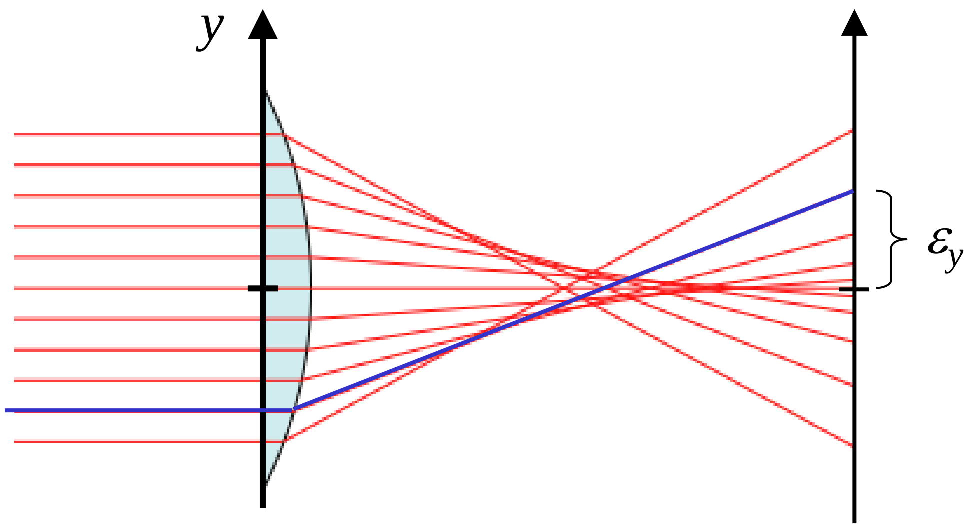
Relation between W and ε

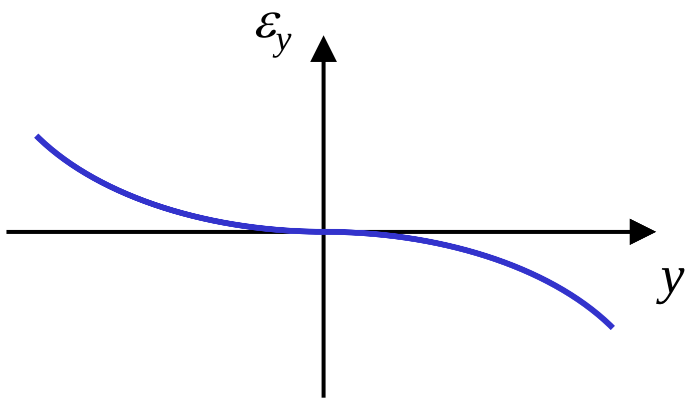


transversal ray aberration

longitudinal ray aberration

Ray Intercept Plot

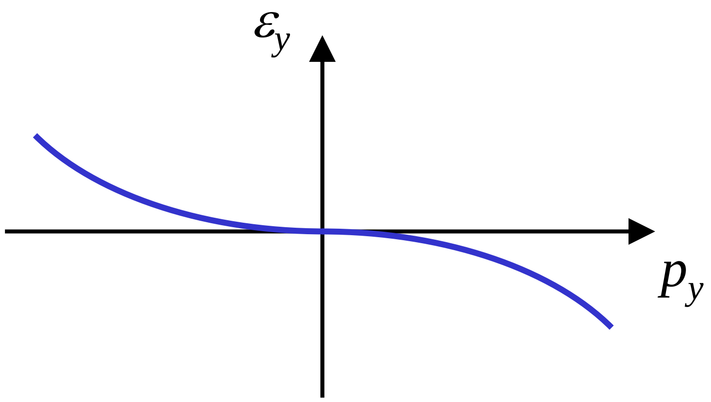




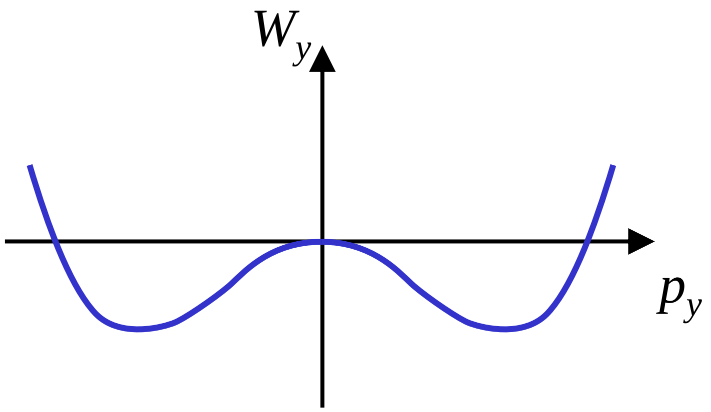
here: spherical aberrations

Ray Aberrations vs. Wavefront Aberrations

Example: spherical aberrations



transversal ray aberration ε
(in the image plane)

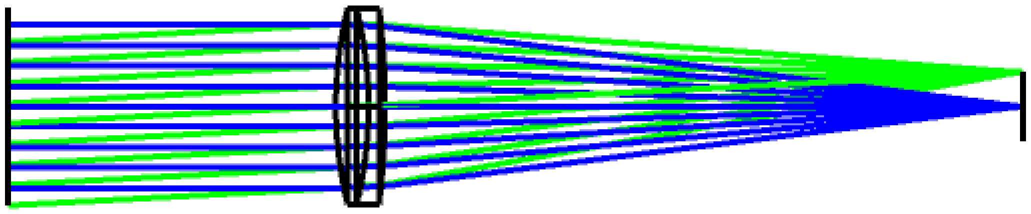


wave-front aberration W
(in the exit pupil)

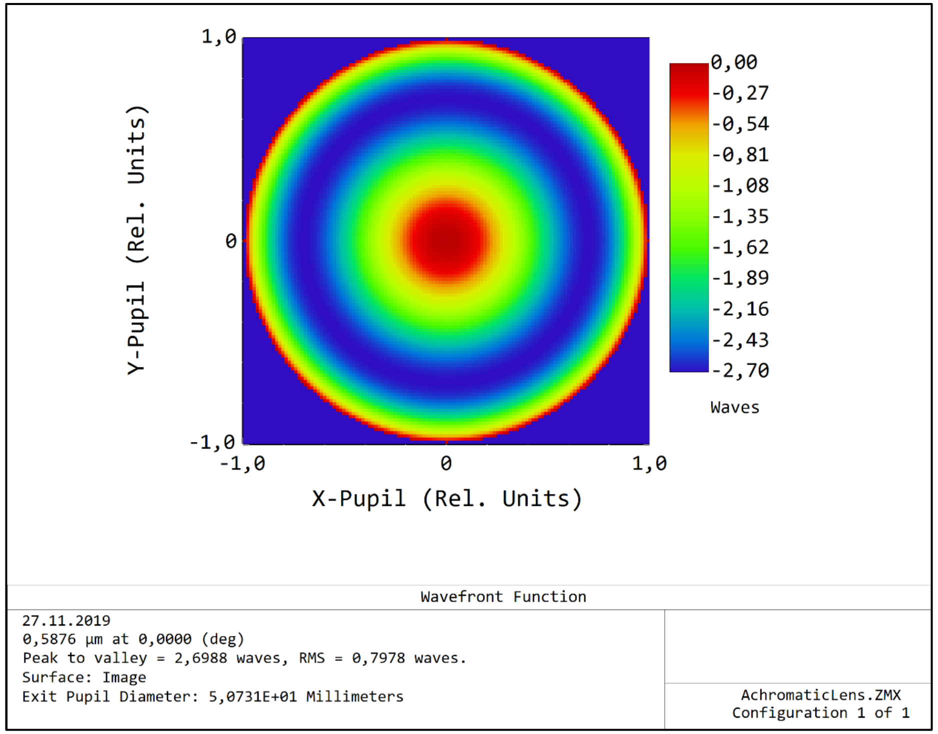
relation between both: $\epsilon_y \sim \frac{dW}{dy}$ $\quad \epsilon_x \sim \frac{dW}{dx}$

Wavefront Aberration Function

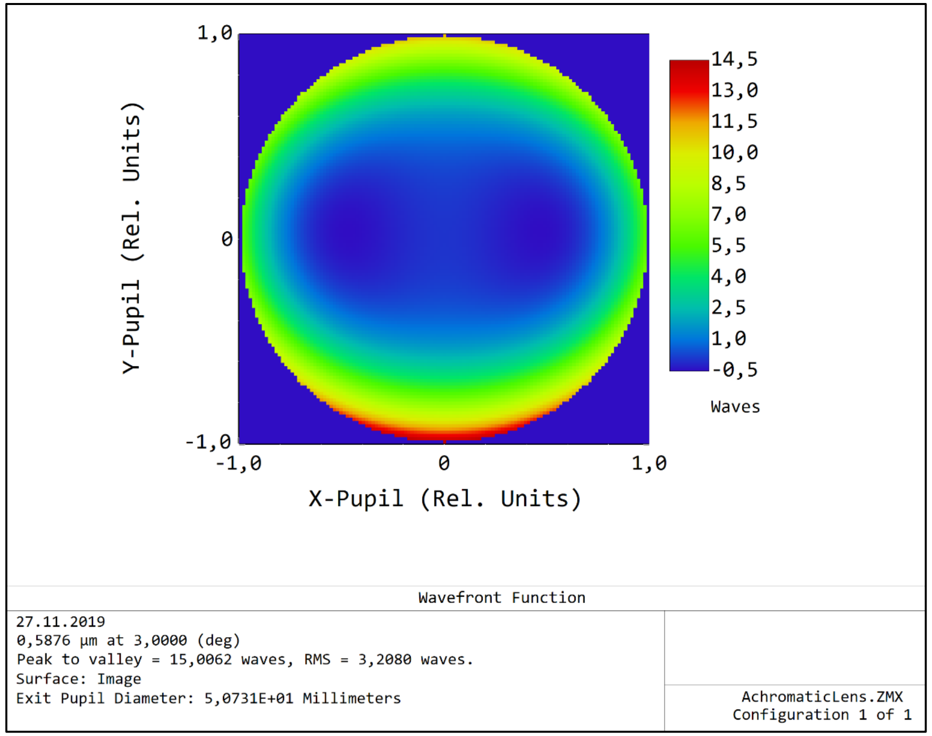
Example: achromatic doublet



β=0°



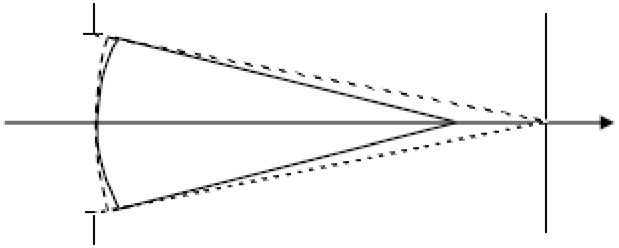
β=3°



First Order Aberrations

Defocus

$W_{020}\, r^2$



Longitudinal Focal Shift

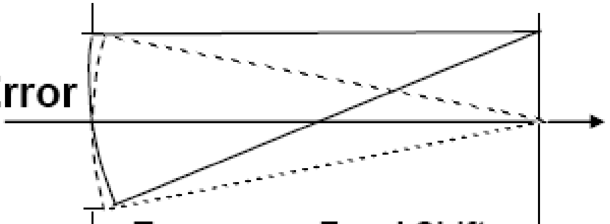
Simply changes the curvature.

Still a spherical wavefront!

Still a good image!

Lateral Magnification Error

$W_{111}\, \beta\, r \cos \psi$



Transverse Focal Shift

Changes the position of the center of curvature. Still a spherical wavefront! Points are still imaged into points and lines are still imaged into lines.

Polynomial expansion of $W(\beta, r, \psi)$

$$W(\beta, r, \psi) = \underbrace{W_{000}}_{\text{Piston Error}}$$

$$+ \underbrace{W_{200} \cdot \beta^2}_{\text{Piston error}} + \underbrace{W_{020} \cdot r^2}_{\text{Defocus}} + \underbrace{W_{111} \cdot \beta \cdot r \cdot \cos\psi}_{\text{Lateral Magnification Error}}$$

$$+ \underbrace{W_{400} \cdot \beta^4}_{\text{Piston Error}} + \underbrace{W_{040} \cdot r^4}_{\text{SA}} + \underbrace{W_{131} \cdot \beta \cdot r^3 \cdot \cos\psi}_{\text{Coma}} + \underbrace{W_{222} \cdot \beta^2 \cdot r^2 \cdot \cos^2 \psi}_{\text{Astigmatism}}$$

$$+ \underbrace{W_{220} \cdot \beta^2 \cdot r^2}_{\text{Field Curvature}} + \underbrace{W_{311} \cdot \beta^3 \cdot r \cdot \cos\psi}_{\text{Distortion}}$$

+ … aberrations of higher order

Polynomial expansion of $W(\beta, r, \psi)$

image location

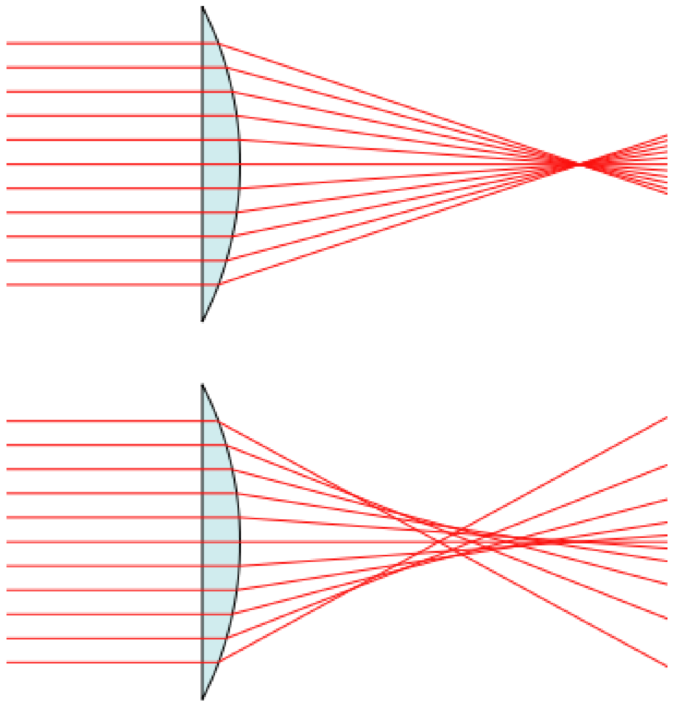
primary aberrations / Seidel aberrations

secondary aberrations

		field β →				
		β^0	β^1	β^2	β^3	β^4
aperture r ↓	r^1		Tilt $\beta r \cos\psi$		Distortion primary $\beta^3 r \cos\psi$	
	r^2	Defocus r^2		Astig. $\beta^2 r^2 \cos^2\psi$ Curvat. $\beta^2 r^2$		
	r^3		Coma $\beta r^3 \cos\psi$			
	r^4	Spherical primary r^4				
	r^5					

Spherical Aberration (~r^4)

Origin: different focal lengths for different ray heights

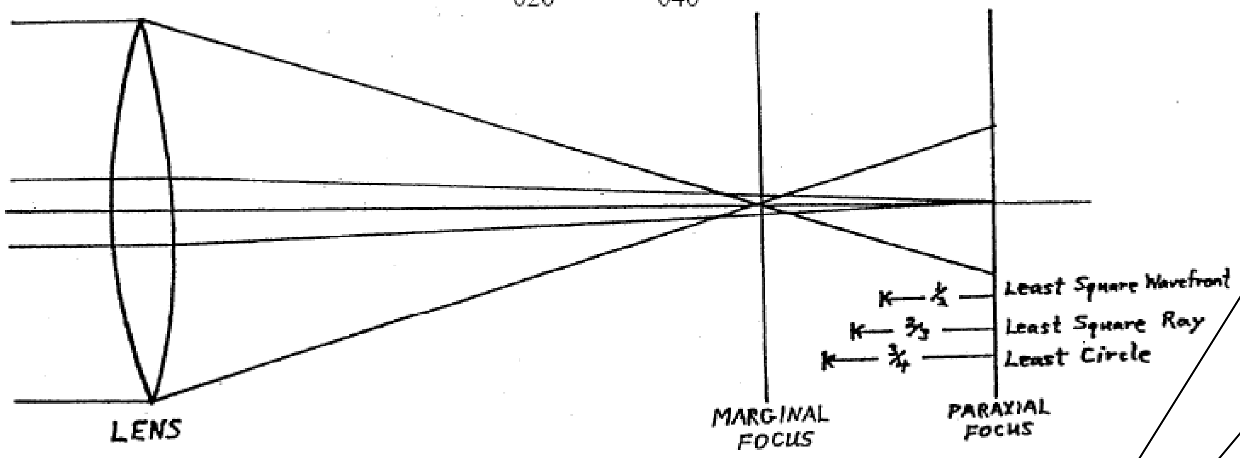


Spherical aberration. A perfect lens (top) focuses all incoming rays to a point on the optic axis. A real lens with spherical surfaces (bottom) suffers from spherical aberration: it focuses rays more tightly if they enter it far from the optic axis than if they enter closer to the axis. It therefore does not produce a perfect focal point.

Getting rid of **Spherical Aberration (~r^4)**

by balancing with defocus

$$W = W_{020} r^2 + W_{040} r^4$$



There are several criteria for the "best" focus:

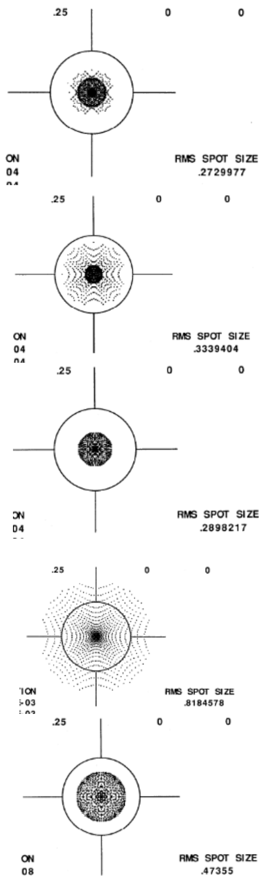
Least square ray criteria - $W_{020} = -\frac{4}{3} W_{040}$

Least square wavefront error - - - - - - - - - - - - - - - - - - - $W_{020} = -W_{040}$

Least circle (smallest circle containing all rays) - - - - - - $W_{020} = -\frac{3}{2} W_{040}$

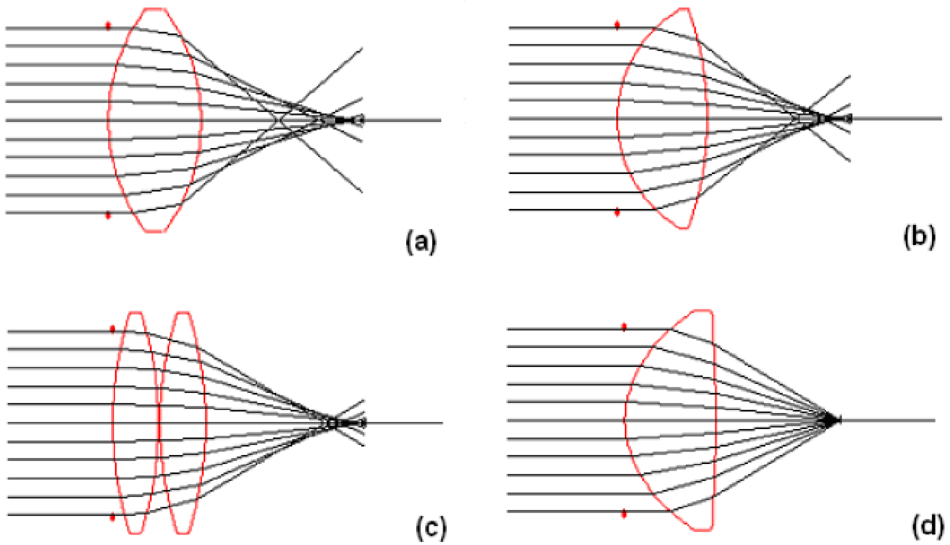
Paraxial focus - $W_{020} = 0$

Marginal focus - $W_{020} = -2 W_{040}$



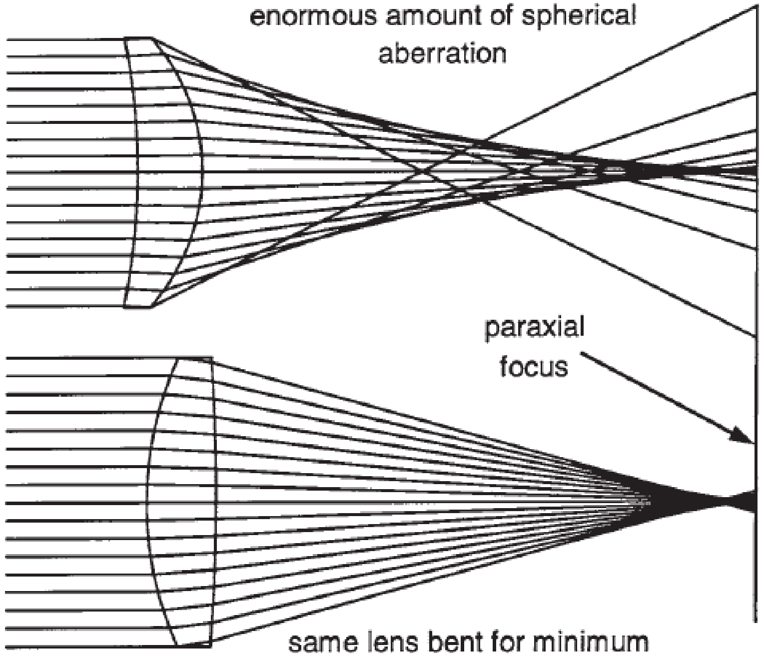
Getting rid of **Spherical Aberration (~r^4)**

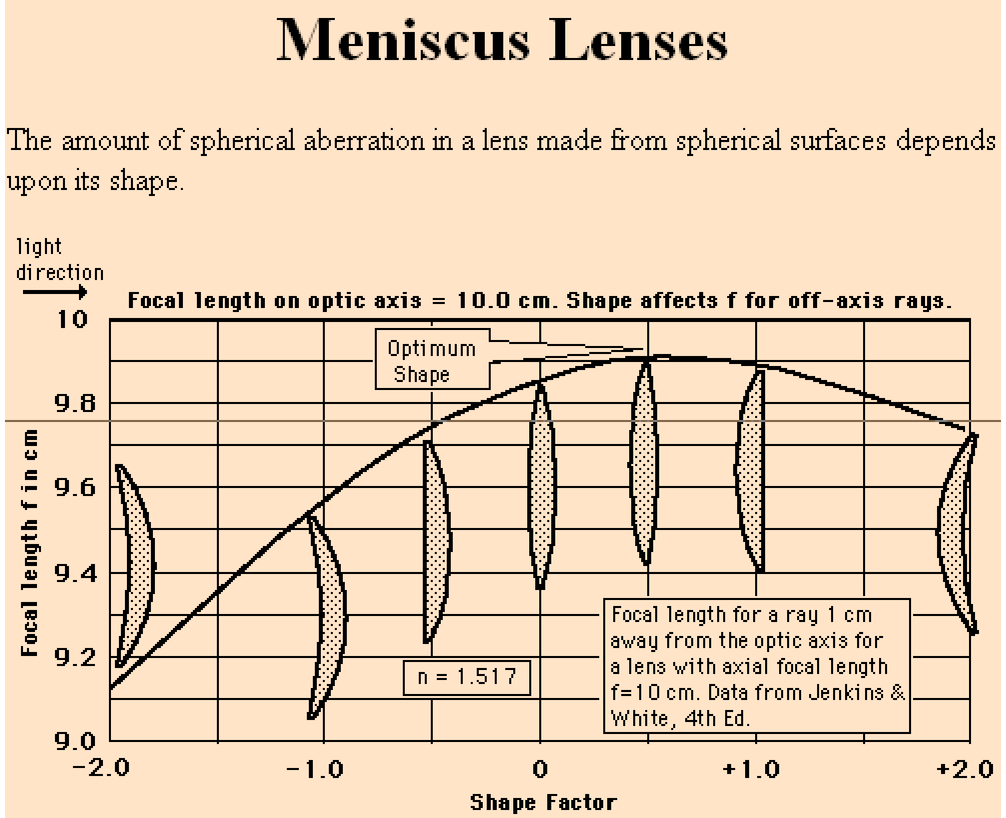
- Lens bending (b)
- Lens splitting (c)
- High refractive index
- Aspheric lenses (d)



Getting rid of **Spherical Aberration ($\sim r^4$)**

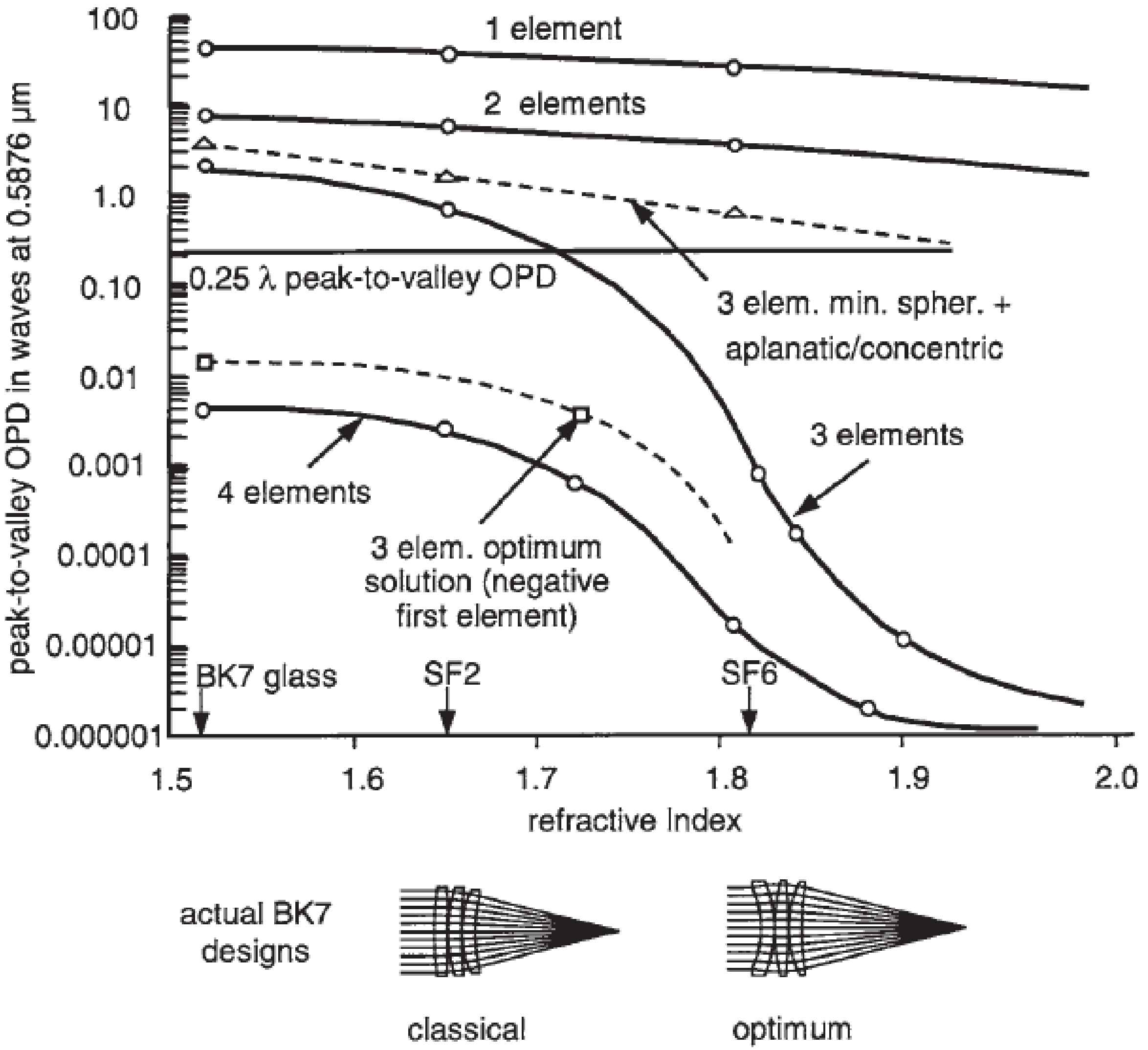
Effect of lens bending



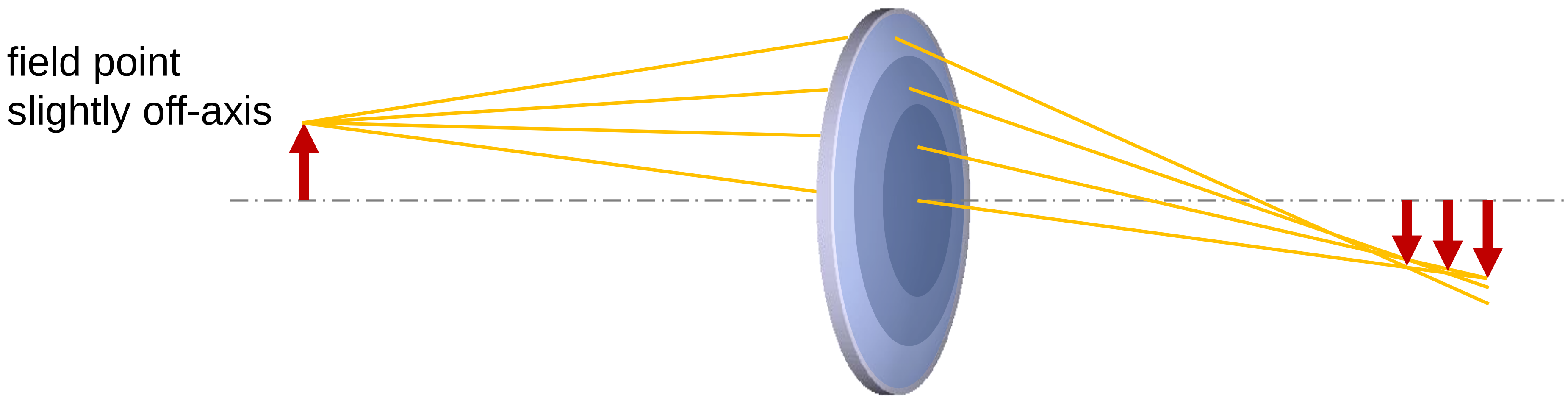


Getting rid of **Spherical Aberration (~r^4)**

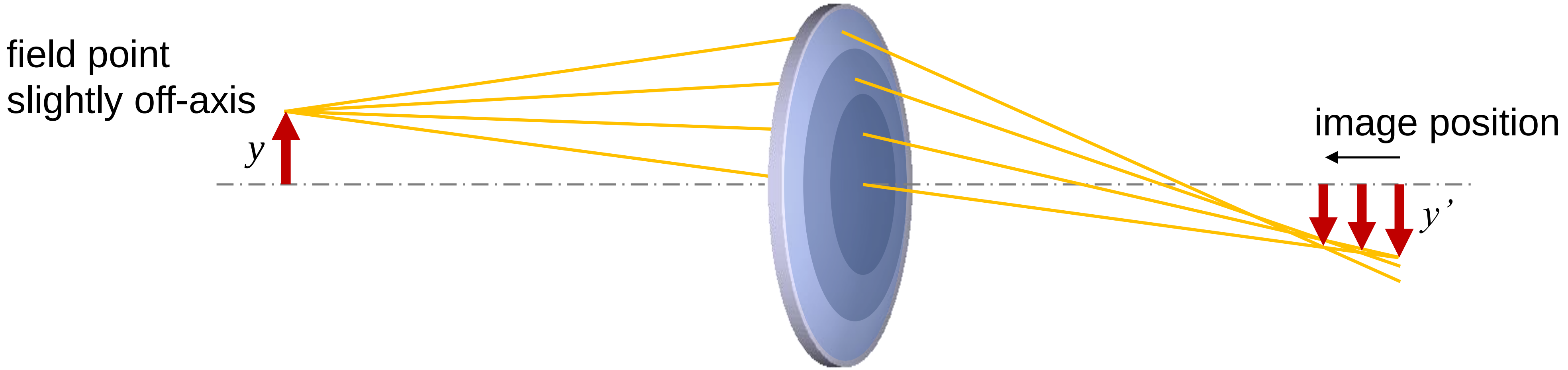
Effect of material choice and # of elements



Impact of Aberrations on the Image Scale



Impact of Aberrations on the Image Scale

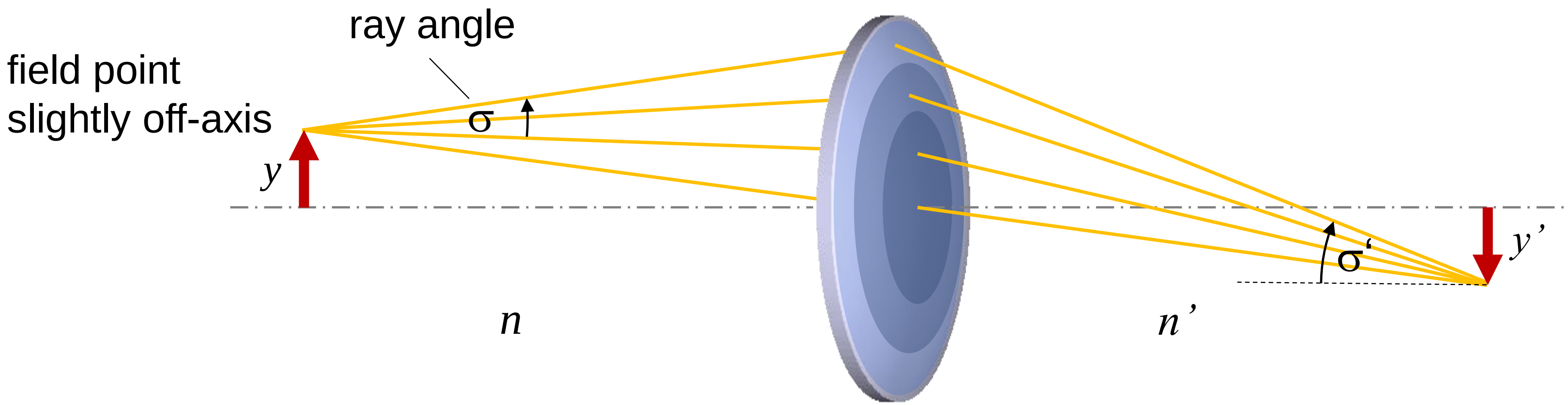


→ images generated by different lens zones appear in different distances to the lens due to aberrations

→ **lateral magnification** of imaging **depends on ray angle** of the image generating rays

Condition for “Aberration-Minimized” Imaging...

…of **small objects** with **large ray angles**



Lateral magnification needs to be **independent from ray angle σ**

Solution: Abbe‘s Sine-Condition

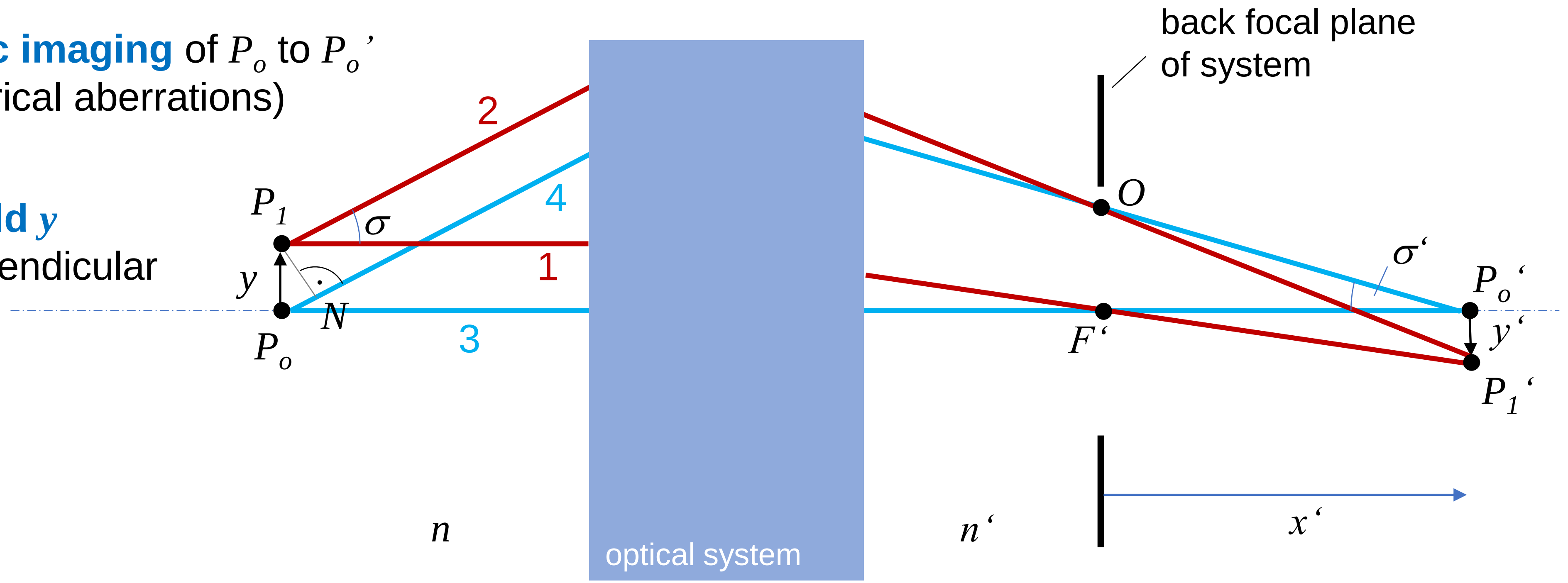
$$\boxed{\beta = \frac{y'}{y} = \frac{n \cdot \sin(\sigma)}{n' \cdot \sin(\sigma')}}$$

→ condition for **aberration-free imaging** of a **small object field** located at the optical axis with **large ray angles**

Derivation of the Sine-Condition

stigmatic imaging of P_o to P_o'
(no spherical aberrations)

small field y
axis-perpendicular



Fermat‘s principle

condition for point-to-point imaging of P_o and P_1 :

P_1 to P_1‘: $\quad OPL(1) = OPL(2) \qquad (1)$

P_o to P_o‘: $\quad OPL(3) = OPL(4) \qquad (2)$

plane wave condition

optical paths to F‘ :

$$OPL(1) - n' \cdot \overline{F'P_1{}'} \overset{!}{=} OPL(3) - n' \cdot x' \qquad (3)$$

optical paths to O :

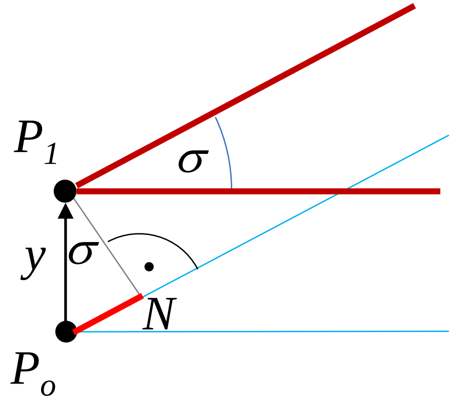
$$OPL(2) - n' \cdot \overline{OP_1{}'} = OPL(4) - n' \cdot \overline{OP_o{}'} - n \cdot \overline{P_oN} \qquad (4)$$

Derivation of the Sine-Condition

equation 4: $OPL(2) - n' \cdot \overline{OP_1{}'} = OPL(4) - n' \cdot \overline{OP_o{}'} - n \cdot \underline{\overline{P_o N}}$

$$\downarrow$$

$$y \cdot n \cdot \sin(\sigma)$$



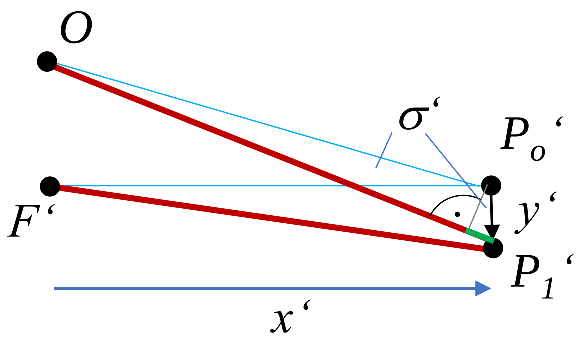
(1): $OPL(1) = OPL(2)$

(2): $OPL(3) = OPL(4)$

(3): $OPL(1) - n' \cdot \overline{F'P_1{}'} = OPL(3) - n' \cdot x'$

$$\sigma) = \underbrace{OPL(4) - OPL(2)}_{} + n' \cdot \left(\overline{OP_1{}'} - \overline{OP_o{}'}\right)$$

Eq. (1), (2) → (3): $= n'x' - n' \cdot \overline{F'P_1{}'}$



$$y \cdot n \cdot \sin(\sigma) = n' \cdot \left(\underline{\overline{OP_1{}'} - \overline{OP_o{}'}} + x' - \underline{\overline{F'P_1{}'}}\right)$$

$$\downarrow \qquad\qquad \downarrow$$

$$y' \cdot \sin(\sigma')$$

$$= \sqrt{x'^2 + y'^2} = x' \cdot \sqrt{1 + \left(\frac{y'}{x'}\right)^2} \approx \underline{x'} \cdot \left(1 + \frac{y'^2}{2x'^2} + \cdots\right)$$

small y' → Taylor-expansion around $y' = 0$
neglecting higher order terms in y'^2

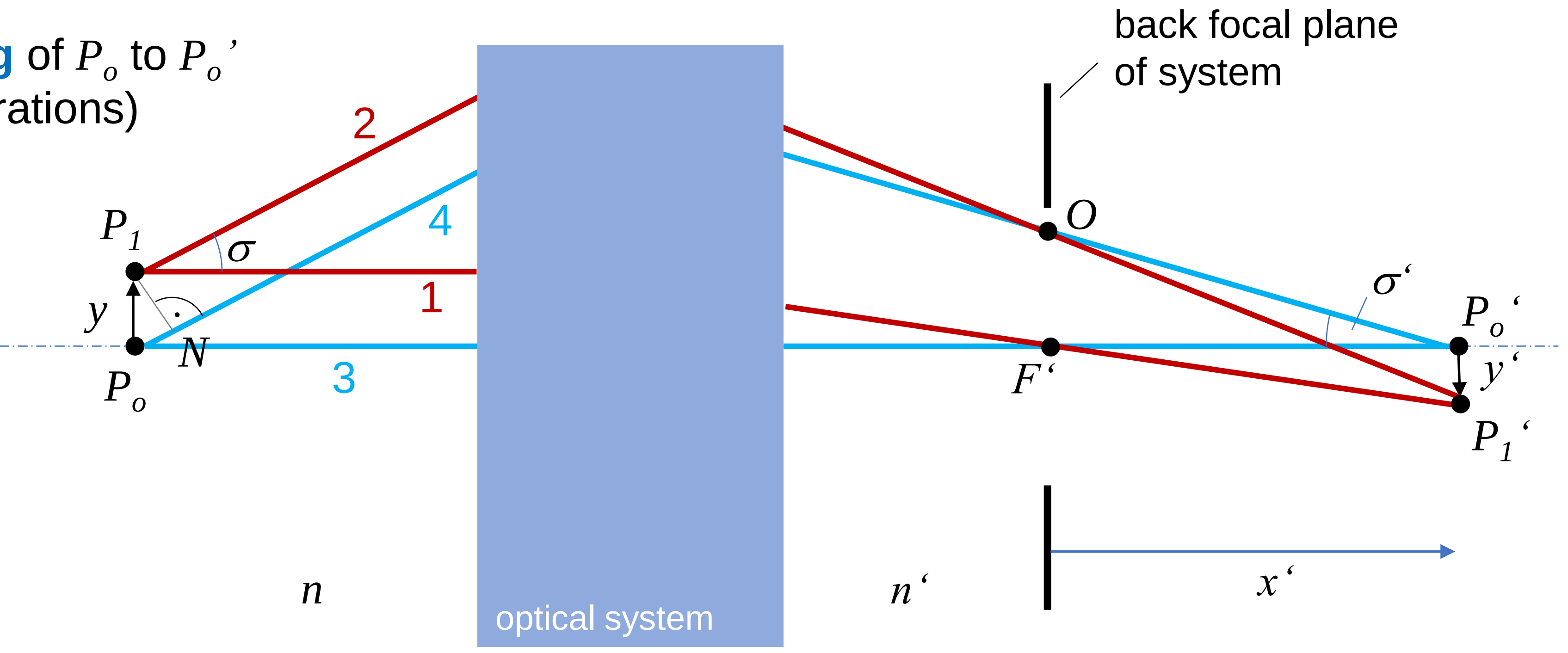
$$\boxed{y \cdot n \cdot \sin(\sigma) = y' \cdot n' \cdot \sin(\sigma')}$$

Abbe‘s Sine-Condition

Derivation of the Sine-Condition

stigmatic imaging of P_o to P_o'
(no spherical aberrations)

small field y
axis-perpendicular



$$\boxed{y \cdot n \cdot \sin(\sigma) = y' \cdot n' \cdot \sin(\sigma')}$$

Abbe‘s Sine-Condition

→ must be fulfilled by an optical system for aberration-free imaging of a small field near the optical axis with large ray angles
→ **aplanatic** system

Remember: Helmholtz-Lagrange invariant (paraxial imaging) $y \cdot n \cdot \sigma = y' \cdot n' \cdot \sigma'$

→ Sine-condition is the generalization for large field angles

Principal Surfaces in Aplanatic Systems

Fulfillment of the sine-condition in **aplanatic systems** can be easily checked via the specific shape of the **principal surfaces** for large ray-angles.

Principal surfaces: (small object heights)

Sine-condition: $$\beta = \frac{y'}{y} = \frac{n \cdot \sin(\sigma)}{n' \cdot \sin(\sigma')}$$

from sketch: $$\sin\sigma = \frac{h}{l} \quad ; \quad \sin\sigma' = \frac{h}{l'}$$

$$\beta = \frac{n \cdot l'}{n' \cdot l}$$

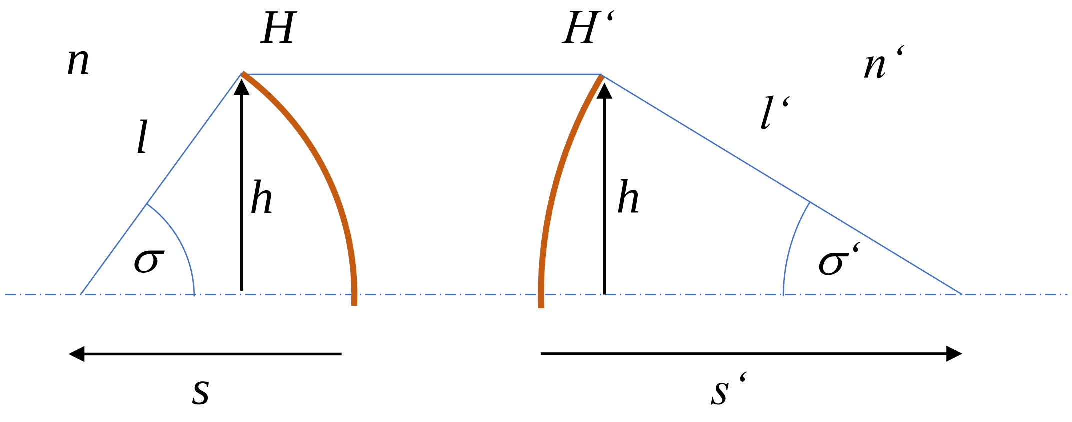


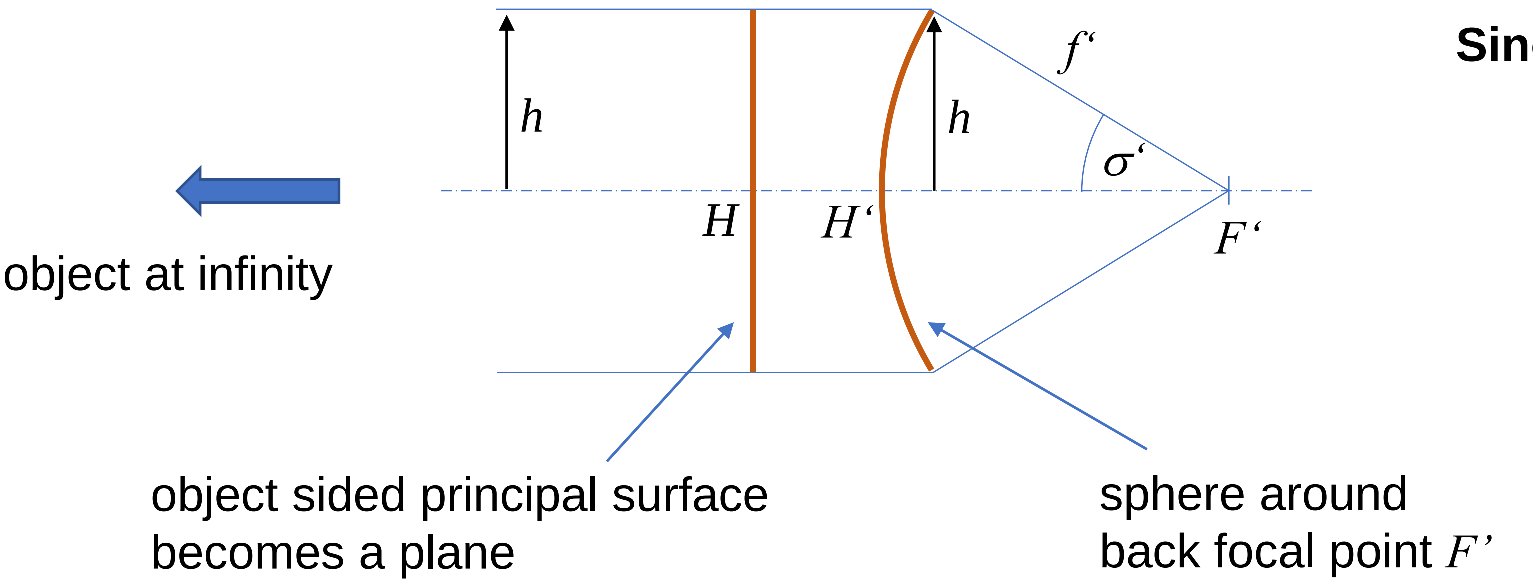
image scale from general imaging equation: $$m = \frac{n \cdot s'}{n' \cdot s}$$

requirement: $m \overset{!}{=} \beta$

$\Rightarrow s = l$ and $s' = l'$ for all σ

in **aplanatic systems** the **principal surfaces** are **spheres** around object/image points

Principal Surfaces, Infinite Object Distance



Sine-Condition:

$$\boxed{\sin(\sigma') = \frac{h}{f'}}$$

for all ray hights h

Consequence for maximal aperture ratio:

$\sin(\sigma') \leq 1$

→ under consideration of the sine-condition the aperture ratio of a lens can not arbitrarily increased

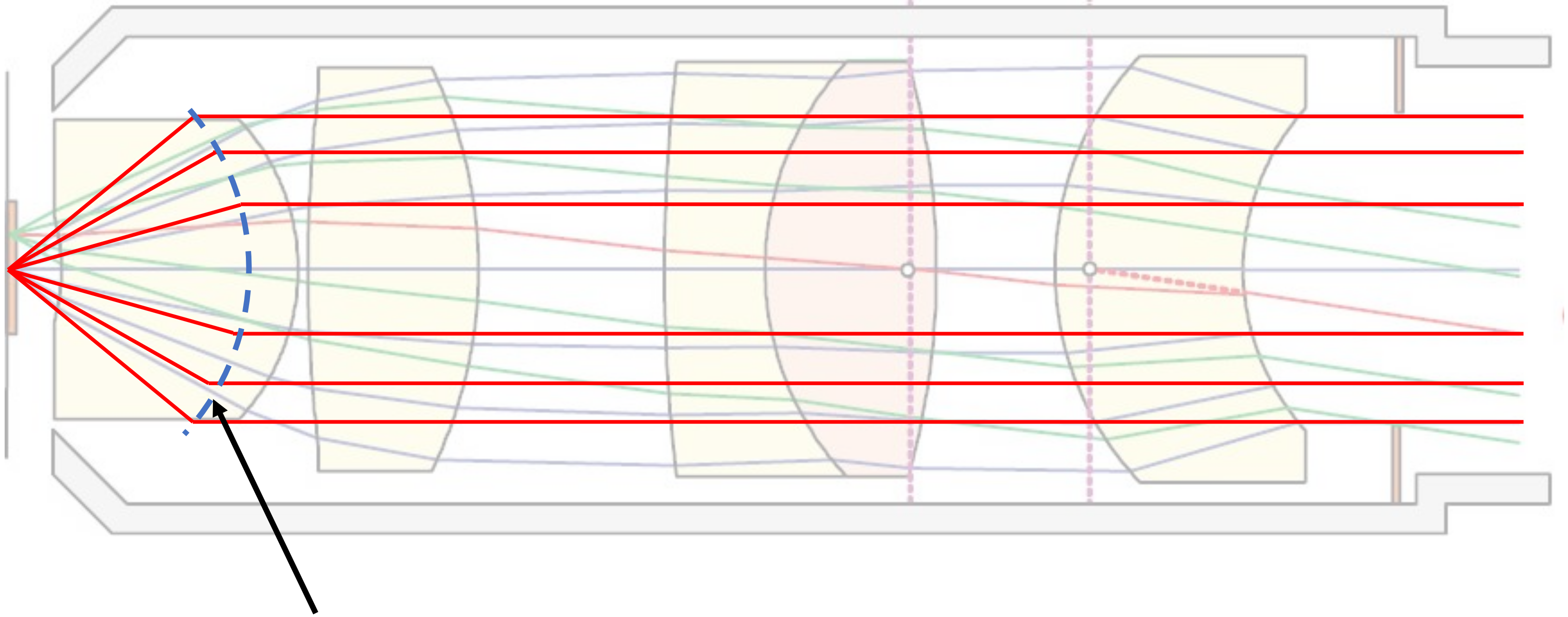
→ max. aperture: $2h = D \leq 2f'$

→ for $D = 2f'$ principal surface is a half-sphere

Application: Microscope Objective

Microscope: **magnified imaging of small fields with large numerical aperture**

→ large field angles σ



principal surface = sphere around object point

→ Sine-condition is fulfilled

Objective design from Y. Zhang, H. Gross, Adv. Opt. Tech. 8, 313 (2019)

Application: Mirror Systems for X-Ray Imaging

wavelength λ = 0.001 – 10nm → no materials for refractive optics
no normal incidence mirrors

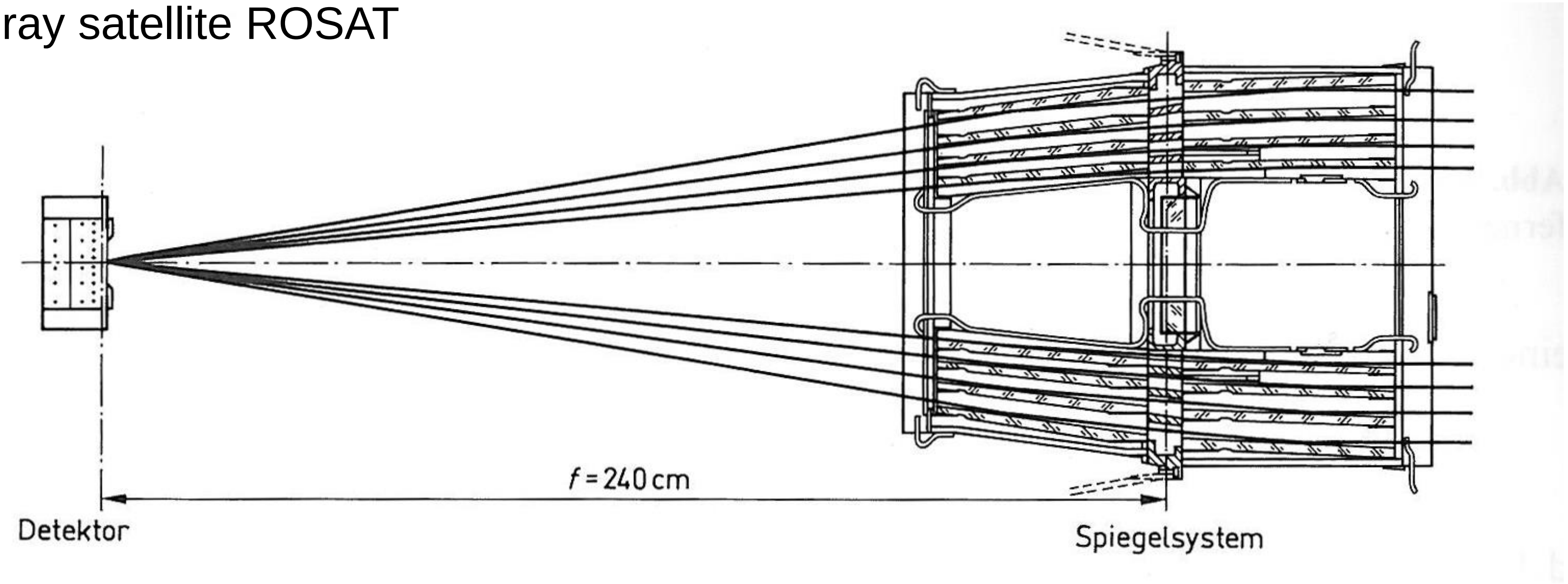
Way out: real part of refractive index in medium < 1

$$n_{vacuum} = 1$$
$$n_{medium} = 1 - \delta + i\beta$$

→ **external total reflection** for grazing incidence

example: carbon $\delta = 4.9 \cdot 10^{-5}$
$\beta = 5.71 \cdot 10^{-7}$

Application example: X-ray satellite ROSAT



Beam path of ROSAT's mirror system
4 nested Wolter-mirrors of equal focal length to obtain a large collecting area

Imaging Using a Paraboloid

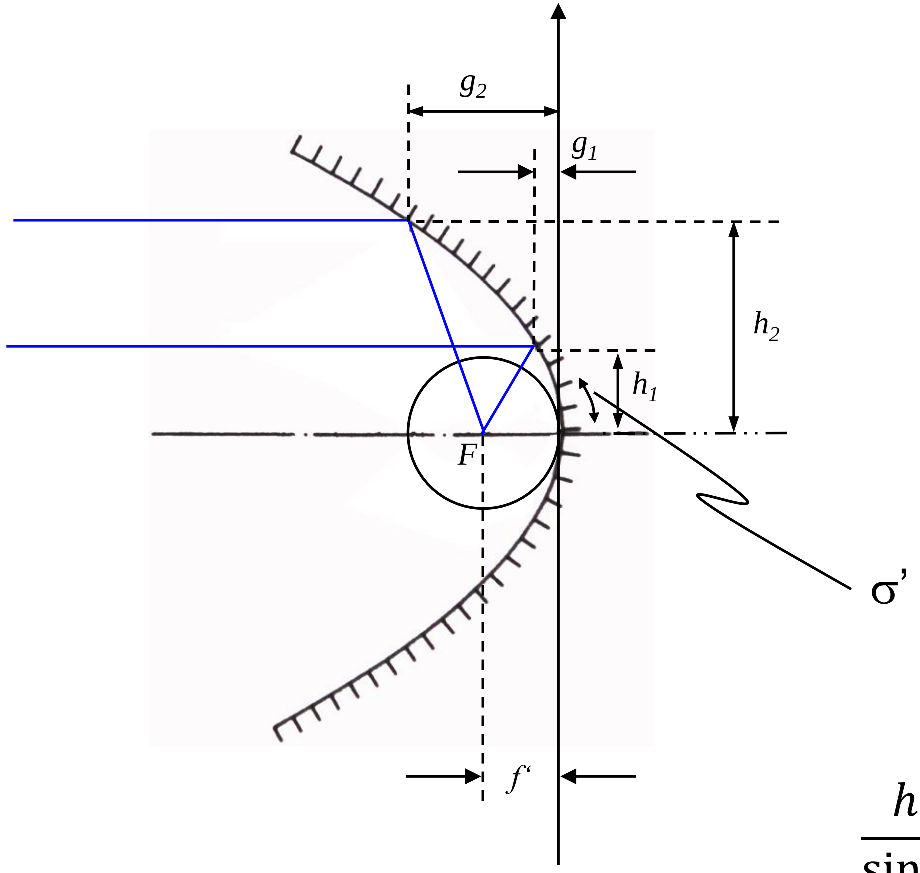
sigmatic imaging of an object point at infinity

equation of a parabola:

$$h^2 = 4f'g$$

Sine-condition:

$$\boxed{f' = \frac{h}{\sin(\sigma')}}$$



Geometry:

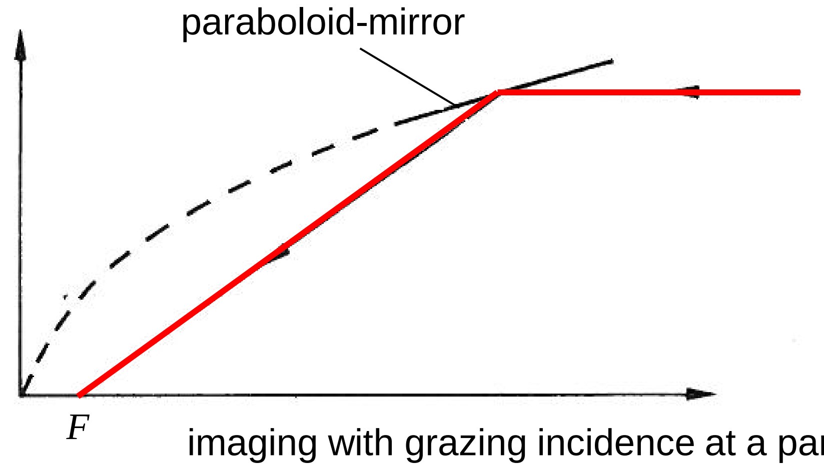
$$\sin\sigma' = \frac{h_1}{\sqrt{(f' - g_1)^2 + h_1^2}}$$

$$\frac{h_1}{\sin\sigma'} = \sqrt{(f' - g_1)^2 + 4f'g} = f' + g$$

→ violation of sine-condition by sag-height *g* !

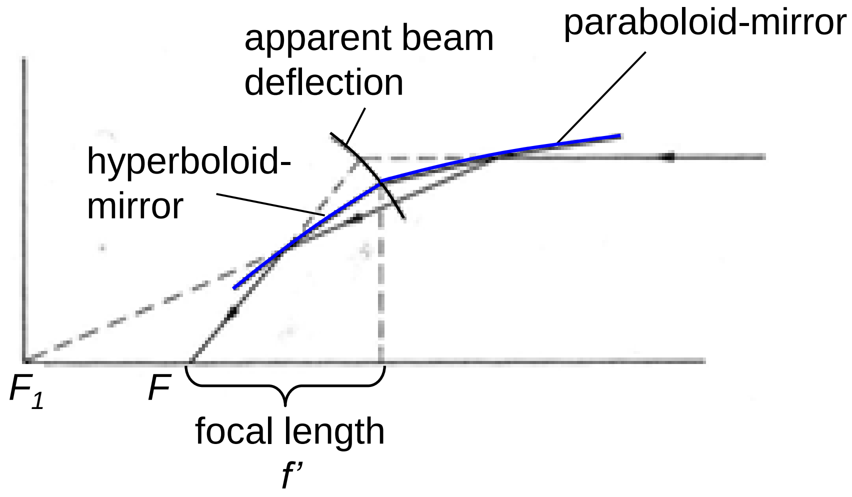
Principle of the Wolter-Telescope

Sine-condition: principal surface needs to be a sphere with radius R around the focal point



imaging with grazing incidence at a paraboloid mirror

→ strong violation of sine-condition, as beam deflection happens nearly along the beam direction and not perpendicular to it



Solution: Wolter-telescope

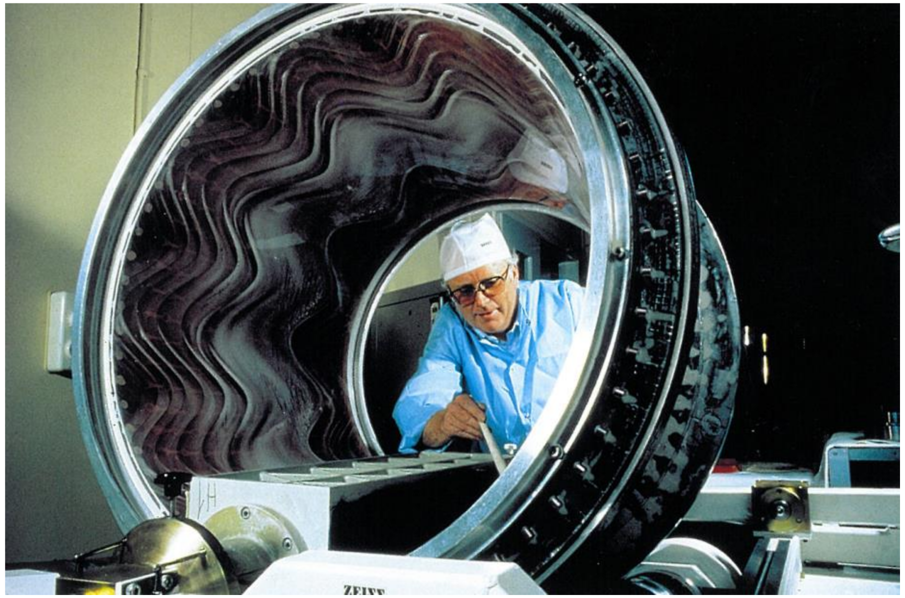
→ combination of a paraboloid and a hyperboloid

→ back-side focal point of hyperboloid F_1 coincides with the focal point of the paraboloid

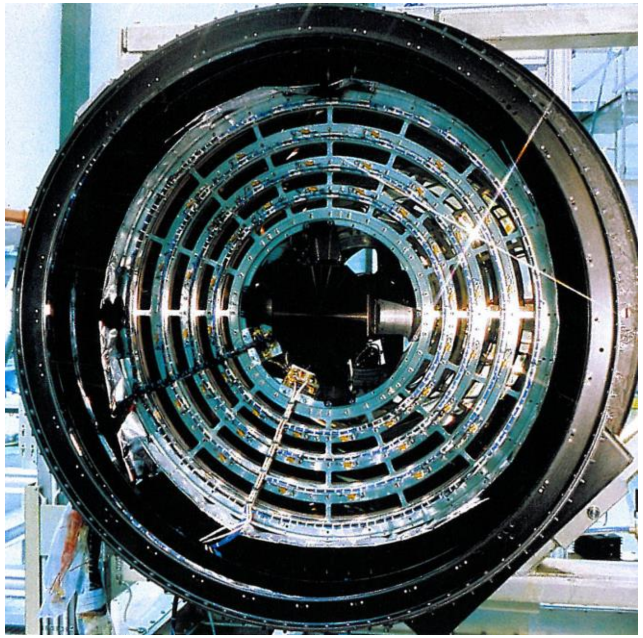
→ surface of apparent beam deflection (principal surface) corresponds to the edge between the two mirrors → paraboloidal surface around focal point F

X-Ray Satellite ROSAT

Pre-developments of ROSAT lead to Wolter-telescopes with 32cm opening, used and tested at different space missions.



As “smoothest mirror in the world” the ROSAT mirrors were mentioned in the “Guinness-book of records”



Front view of the ROSAT mirror system.
The circular grating is stabilizing the nested mirror shells

Summary

- **Abbe’s Sine-Condition:**
  Condition to an optical system in order to obtain an **aberration free imaging of small objects near the optical axis with large ray angles**.

$$\boxed{\frac{y'}{y} = \frac{n \cdot \sin(\sigma)}{n' \cdot \sin(\sigma')}}$$

- Systems satisfying the sine-condition are called **aplanatic systems**

- **Principal surfaces** of aplanatic Systems **are spheres** around object and image point, respectively

- Application examples:
  - microscope lenses
  - mirror systems