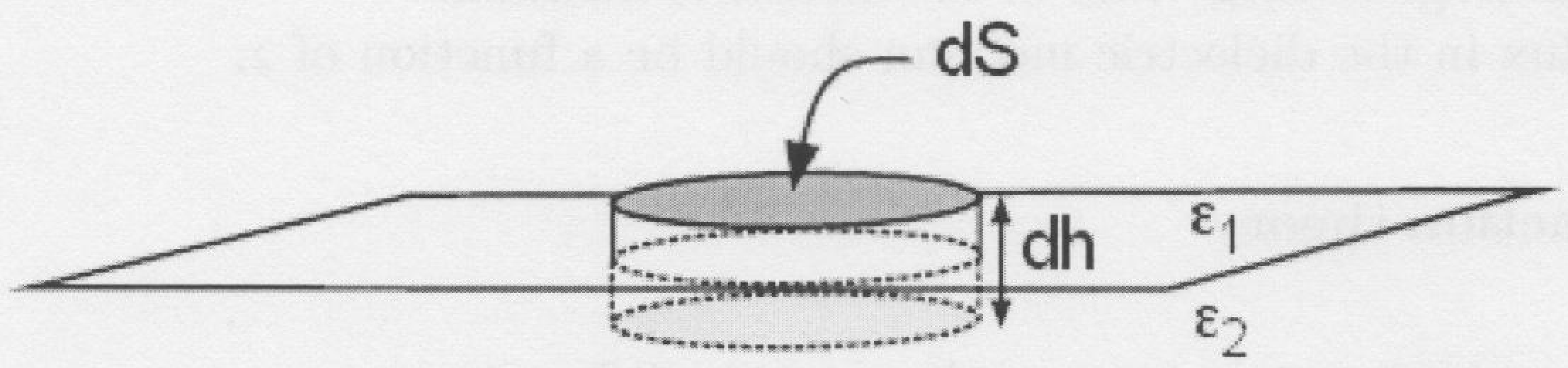
Fundamentals of Modern Optics WS 2012/13
Final Exam
to be written February 14.

Problem 1 – – Maxwell's equations $4 + 2 + 4 = 10$ **points**

a) Write down Maxwell's equations in time domain for a linear, dispersionless, isotropic, non-magnetizable, homogeneous dielectric medium in absence of charge and current densities ($\rho(\mathbf{r}, t) = 0$, $\mathbf{j}(\mathbf{r}, t) = \mathbf{0}$). Write down the constitutive relations for the dielectric flux density ($\mathbf{D}$) and the magnetic field ($\mathbf{H}$).

b) For the case of homogeneous media, derive the wave equations in frequency domain for the electric field ($\mathbf{E}$).

c) Consider the boundary between two homogeneous dielectric media characterized by their dielectric constants ϵ_1 and ϵ_2. Show that the normal component of the dielectric flux density ($\mathbf{D}$) is continuous at the boundary between these two different materials if that boundary contains no free surface charges. Deduce the relation between the normal electric fields across the boundary.
Hint: Consider a cylinder with a surface area (dS) much larger that its height (dh) that contains the bounding surface and use the divergence theorem.



Problem 2 – – Polarization Optics $1 + 1 + 1 + 2 = 6$ **points**

A beam of light propagates along the z-axis, such that its polarization vector is in the x-y-plane. Its polarization state can be described by the Jones vector $\mathbf{J} = \begin{bmatrix} J_x \\ J_y \end{bmatrix} = J_x\mathbf{e}_x + J_y\mathbf{e}_y$.

a) Write down the Jones-Vector for y-polarized light.

b) What is the polarization state of the light if, the Jones vector is $\mathbf{J} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$?

A linearly y-polarized beam of light propagates along the z-axis. It passes through a plate made of a uniaxial anisotropic crystal. The interface of the crystal is in the x-y-plane, the crystal axis $\mathbf{c}$ is in this plane, too. The thickness of the crystal is $d = \frac{\lambda}{4(n_e - n_o)}$.

c) How is such a crystal commonly called?

d) Which effect does it have on the polarization state of the beam?

Problem 3 – – Gaussian beams **6 points**

A lens of focal length f_1 is placed at a distance $d = f_1$ from the waist of a Gaussian beam. Use the matrix formulation to find the position of the waist and the Rayleigh range of the Gaussian beam after the lens.

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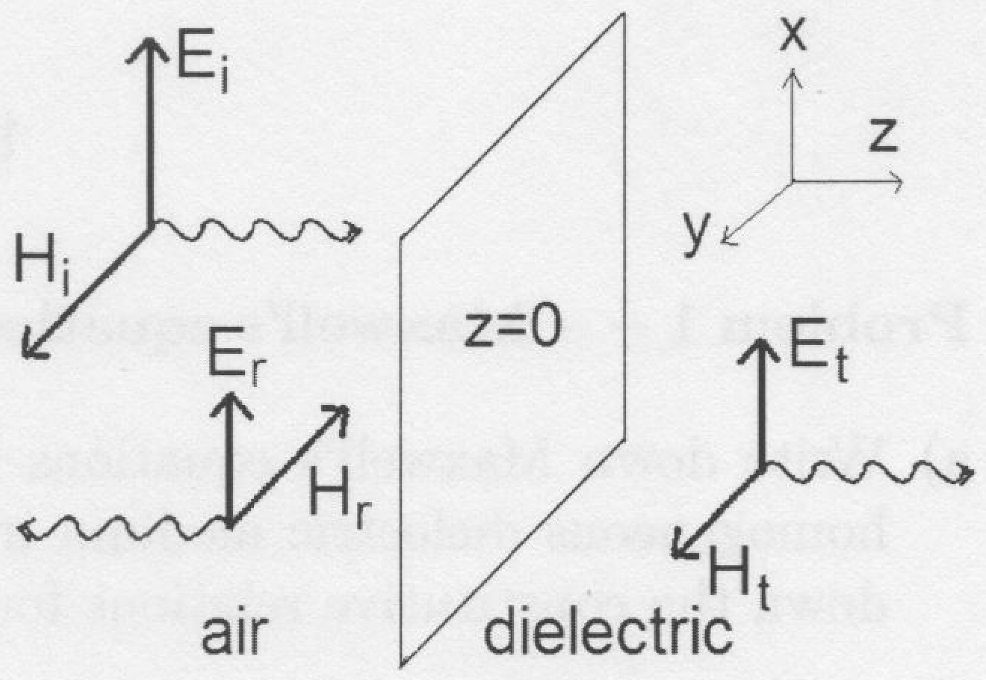
Problem 4 – – Normal modes $2+4+2+4 = 12$ points



A semi-infinite block of some dielectric (with relative permittivity of $\epsilon' + i\epsilon''$, if needed use $\sqrt{\epsilon' + i\epsilon''} = n + i\kappa$) is illuminated perpendicularly to its surface with a plane monochromatic wave of frequency ω from air. The electric and magnetic field of the incoming and the reflected and the transmitted plane waves have the form

$$\mathbf{E_i} = E_i e^{i(k_0 z)}\hat{\mathbf{x}} \quad , \quad \mathbf{E_r} = E_r e^{i(-k_0 z)}\hat{\mathbf{x}} \quad , \quad \mathbf{E_t} = E_t e^{i(k_1 z)}\hat{\mathbf{x}}$$

$$\mathbf{H_i} = \frac{k_0}{\omega\mu_0} E_i e^{i(k_0 z)}\hat{\mathbf{y}} \ \mathbf{H_r} = \frac{-k_0}{\omega\mu_0} E_r e^{i(-k_0 z)}\hat{\mathbf{y}} \ , \ \mathbf{H_t} = \frac{k_1}{\omega\mu_0} E_t e^{i(k_1 z)}\hat{\mathbf{y}}$$

respectively, where $k_0 = \omega/c$.

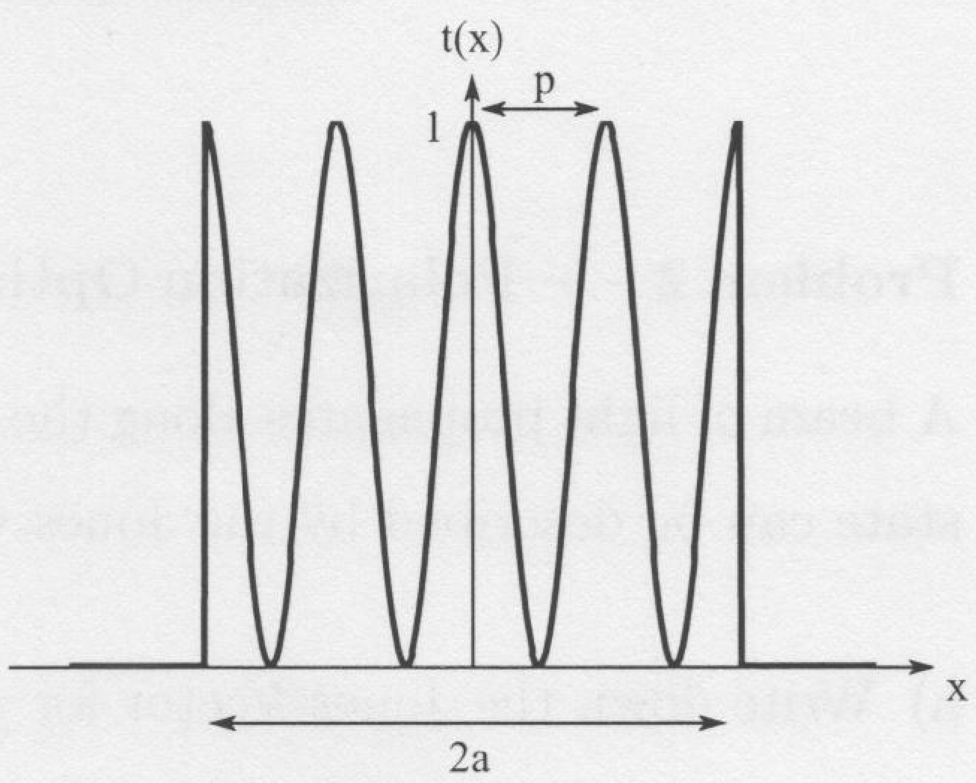
a) Write down the dispersion relation for a plane wave in the dielectric medium and calculate k_1.

b) Use the continuity of the tangential components of the electric and magnetic field at the interface between the two media to find E_t as a function of E_i.

c) Give a formula for the calculation of the time averaged energy flux in a medium.

d) Calculate the time averaged energy flux in the dielectric medium.
Hint: This energy flux in the dielectric medium should be a function of z.

Problem 5 – – Diffraction theory $4+6+2 = 12$ points



a) Name two different approximations that can be made in diffraction theory and shortly explain their meaning in two sentences.

b) Given is a one-dimensional cosine grating with period $p \gg \lambda$ that is shaded by a slit, so that only N grating periods are illuminated by a plane wave (see Figure). The transmission function of the aperture is thus given as

$$t(x) = \frac{1}{2}\left[1 + \cos\left(\frac{2\pi}{p}x\right)\right] \cdot \mathrm{rect}\left(\frac{x}{a}\right).$$

Calculate the far-field diffraction pattern.

c) You want to observe the far-field pattern in your tiny lab. How would you do it?

Problem 6 – – Pulse propagation $1+2+4+1 = 8$ points

The dispersion relation of a medium is given by the following expression:

$$k(\omega) = k_0 + k_1(\omega - \omega_0) + \frac{k_2}{2}(\omega - \omega_0)^2$$

where the central frequency $\omega_0 = 200 \cdot 10^{12}\mathrm{s}^{-1}$. The parameters are given as $k_1 = 3/c$, $k_2 = 0.01\mathrm{ps}^2/\mathrm{m}$. A pulse with a central frequency of $\omega = 300 \cdot 10^{12}\mathrm{s}^{-1}$ is now launched in this medium. Upon launching the pulse it has a Gaussian shape, a pulse duration of $\tau_0 = 100$ fs and a flat phase.

a) What is the dispersion coefficient D for this pulse?

b) Calculate the pulse's group index n_g (assuming $c = 3 \cdot 10^8$ m/s)

c) Which length does the pulse have after 1 m of propagation?

d) Is the pulse after 1 m up-chirped (long wavelength components arrive first), down-chirped (short wavelength components arrive first), or un-chirped (all wavelength components arrive simultaneously)?

Hint: The length of a pulse is given by $\tau(z) = \tau_0\sqrt{1 + z^2/L_D^2}$, where L_D is the dispersion length.

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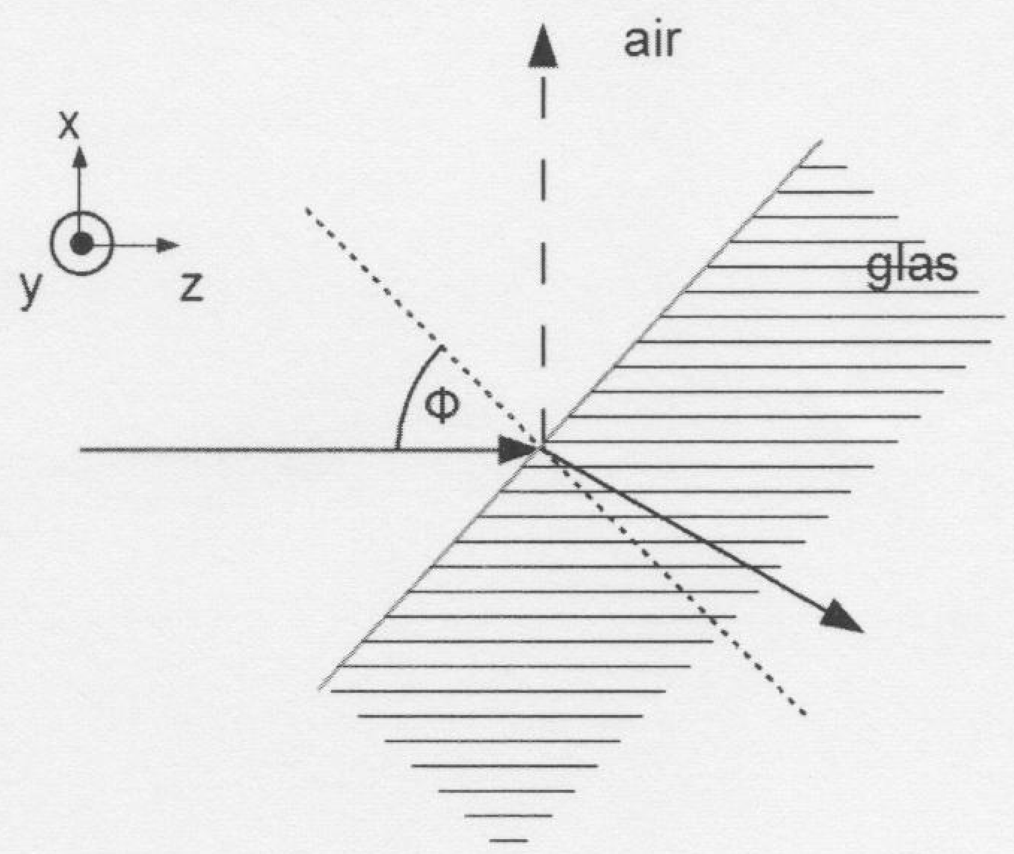
Problem 7 – – Interfaces **2 + 2 + 2 = 6** points

A beam of light is incident onto the surface of a plane glass block. Its surface is oriented at an angle of $\phi = 45°$ with respect to the incident beam and reflects a part of the light due to Fresnel reflection (see Figure).



a) Along which axis does the light have to be polarized in order to be TE-polarized? When is it TM-polarized?

b) Is more light reflected in TM or TE polarization? What led you to this conclusion?

c) Assume that the polarization case is the one with lower reflection. To which angle $\tilde{\phi}$ do we have to rotate the glass plate to achieve zero reflection, if the refractive index is assumed to be $\sqrt{3} \approx 1.73$?

— Useful formulas —

$$\int e^{iax} e^{ikx} \, \mathrm{d}x = 2\pi\delta(k+a)$$

$$\int \delta(x-a) f(x) \, \mathrm{d}x = f(a)$$

$$\cos\phi = \frac{1}{2}\left(e^{i\phi} + e^{-i\phi}\right)$$

$$\sin\phi = \frac{1}{2i}\left(e^{i\phi} - e^{-i\phi}\right)$$

$$\nabla \times \nabla \times \mathbf{a} = \nabla(\nabla \cdot \mathbf{a}) - \Delta\mathbf{a}$$

$$\nabla \cdot (\nabla \times \mathbf{a}) = 0$$

Gaussian q parameter:

$$\frac{1}{q(z)} = \frac{1}{R(z)} + i\frac{2}{k\,w^2(z)}$$

free space propagation:

$$q(z) = q_0 + z$$

Gaussian q-parameter transform law:

$$q' = \frac{Aq+B}{Cq+D}$$

ABCD matrix for a thin lens:

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$