Name: ______________________ Date of birth: ______________ Student ID No. ______________

Final Exam of "Fundamentals of modern optics" WS 2015/16
to be written on February 15, 10:00 - 12:00 am

Problem 1 – Maxwell's equations (MWEs) **3 + 2 + 2 + 3 = 10 points**

a) Write down MWEs in frequency domain, with external sources, in its most general form in a material. Furthermore, write down the constitutive equations for auxiliary fields **D** and **H**, in frequency domain, where the material is inhomogeneous, dispersive, linear, isotropic, and magnetic.

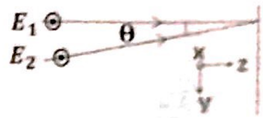
b) Find the wave equation for **E**, from the MWEs of part (a), for an inhomogeneous, dispersive, linear, isotropic, and non-magnetic material, in the presence of external sources.

c) Write down the MWEs of part (a) in their integral form, using the Stoke's and divergence theorems.

d) Derive the continuity equation connecting the charge density and the current density, from MWEs in (a). Write your final answer also in terms of the total charge in a volume and the total current escaping that volume.

Problem 2 – Normal Modes **2 + 2 + 1 + 3 = 8 points**

Consider two monochromatic plane waves with electric field amplitudes E_1 and E_2 polarized along the x direction in free space with wavelength λ. One plane wave is propagating along the z direction and the other in yz plane at an angle θ as shown in the figure.



a) Write down the space and time dependent electric field vector for both plane waves.

b) Find the intensity pattern due to interference of these two waves and plot it along the y direction (assume x=z=0 and E_1=$2E_2$).

c) Calculate the distance between two consecutive maxima (or minima) in y direction. What will be the effect of increasing angle θ on the observed pattern?

d) Define and calculate the time averaged Poynting vector.

Problem 3 – Diffraction theory **2 + 1 + 6 = 9 points**

a) Explain the condition shortly and write a mathematical inequality for which you can apply the following approximations: (i) Fresnel approximation. (ii) Fraunhofer approximation.

b) Can the Fraunhofer approximation be applied without the Fresnel approximation. Please explain your answer in two sentences.

c) Give formulas for calculating the observed diffraction patterns of an illumination function $u_0(x, y)$(in scaler approximation) for: (i) non-paraxial case. (ii) Fresnel approximation. (iii) Fraunhofer approximation.

Problem 4 – Pulses **1 + 2 + 3 = 6 points**

Consider a fiber of the length $L_1 = 1\,\mathrm{m}$ has the dispersion relation $k_1(\omega) = \alpha_1\omega_0 + \beta_1(\omega - \omega_0) + \frac{\gamma_1}{\omega_0}(\omega - \omega_0)^2$, with the frequency ω and the constants $\omega_0 = 2 \cdot 10^{15}\mathrm{rad/s}$, $\alpha_1 = \frac{3}{2c}$, $\beta_1 = \frac{3}{4c}$ and $\gamma_1 = \frac{3}{8c}$, where $c = 3 \cdot 10^8\mathrm{m/s}$ is the speed of light in vacuum.

a) What is the phase velocity of monochromatic light of frequency $\omega_1 = 3 \cdot 10^{15}\frac{\mathrm{rad}}{\mathrm{s}}$ in this fiber?

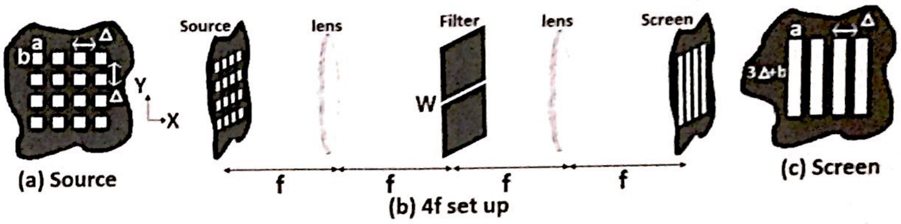
b) Now, we couple a Gaussian pulse into the fiber (center frequency $\omega_1 = 3 \cdot 10^{15}\frac{\mathrm{rad}}{\mathrm{s}}$ and FWHM 100 fs). Its maximum starts at $t_0 = 0$ sec at the beginning of the fiber. At which time does this maximum arrive at the end of the fiber?

c) Consider a second fiber with the dispersion relation $k_2(\omega) = \alpha_2\omega_0 + \beta_2(\omega - \omega_0) + \frac{\gamma_2}{\omega_0}(\omega - \omega_0)^2$, where $\alpha_2 = \frac{10}{c}$, $\beta_2 = \frac{2}{c}$ and $\gamma_1 = \frac{-4}{9c}$. It is merged to the end of the first fiber. We launch two Gaussian pulses with the center frequencies $\omega_1 = 3 \cdot 10^{15}\frac{\mathrm{rad}}{\mathrm{s}}$ and $\omega_2 = 4 \cdot 10^{15}\frac{\mathrm{rad}}{\mathrm{s}}$, respectively, and both with the FWHM of 100 fs at the same time into the first fiber. Calculate the length L_2 of the second fiber when both pulses maxima shall arrive at the end of the second fiber at the same time.

Problem 5 – Imaging optics

$3 + 4 + 3 = 10$ **points**

Assume, a source with 16 bright illuminating rectangles, each of size $a \times b$. They are arranged in x and y direction with period Δ (shown in figure a).



a) Please write the amplitude transmittance function of the source (figure a).

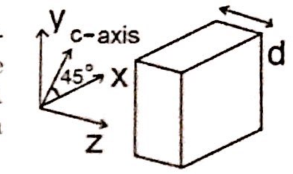
b) Calculate the diffraction pattern at 2f distance just before the filter (shown in figure b).

c) Suppose one wishes to obtain the diffraction pattern on the screen as shown in the figure c. What should the range of aperture width W in the filter be, to realize this pattern?

Problem 6 – Anisotropy

$2 + 1 + 3 = 6$ **points**

A layer of a uniaxial crystal of thickness $d = 5\mu m$ is shown in the figure. The extraordinary crystal axis is in the x-y plane and makes a 45° angle with the x and y axis. The ordinary and extraordinary refractive indices are $n_o = 2.2$ and $n_e = 2.15$, respectively. A plane wave with an electric field of $\boldsymbol{E} = E_0 e^{i\frac{2\pi}{\lambda}z}(\hat{x} + i\hat{y})$ is incident on this layer from one side, where $\lambda = 1\mu m$ is the wavelength of the wave in free space.



a) What are the two eigenmodes of the crystal propagating in the z direction. Specify the direction of electric field (in terms of $\hat{x}$ and $\hat{y}$) and the magnitude of k-vector for each eigenmode.

b) Decompose the input electric field polarization into the two eigenmodes of part (a).

c) Calculate the electric field polarization at the other side of this crystal layer. You can ignore the reflections from the crystal surfaces; it is the final polarization state that matters to us.

Problem 7 – Interface

$1 + 2 + 2 = 5$ **points**

The reflection coefficient of a TE mode field, with incident angle of θ_1 and refracted angle of θ_2, from a media 1 (n_1) into a media 2 (n_2) is described by the following Fresnel equation,

$$r_{TE} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}.$$

a) Assume $n_1 > n_2$. What is the relation for the critical angle θ_c, after which we have total internal reflection?

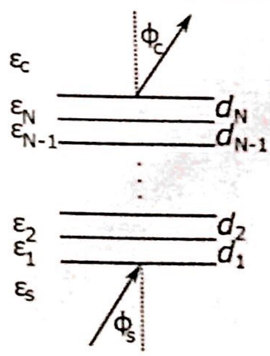
b) Show that for $\theta_1 > \theta_c$ we get $|r_{TE}| = 1$.

c) Find the value of the extra phase that the reflected wave acquires for the limiting case of $\theta_1 = \pi/2$.

Problem 8 – Multilayer system

$2 + 2 + 4 = 8$ **points**

Consider a multilayer system of N isotropic layers with the permittivities ϵ_i and the thicknesses d_i ($i = 1, 2, ..., N$). In front of and behind this multilayer system, there are the substrate (ϵ_s) and the cladding (ϵ_c), respectively.



a) Let the layer permittivities be real and hold the following relation: $\epsilon_s < \epsilon_1 < \epsilon_2 < ... < \epsilon_i < ... < \epsilon_N < \epsilon_c$. An incoming monochromatic plane wave hits the interface to the multilayer system with an angle of ϕ_s with respect to the interface normal (see figure). Find the the angle ϕ_c of the refracted wave after the multilayer stack.

b) Explain shortly the matrix method and its use for the multilayer system. Give also a short mathematical description.

c) Derive the matrix for a single layer, for a TE-polarized monochromatic plane wave.

Maybe useful formulas: $\nabla \times \nabla \times \mathrm{a} = \nabla(\nabla \cdot \mathrm{a}) - \Delta \mathrm{a}$, $\nabla \cdot (\nabla \times \mathrm{a}) = 0$.