Institute of Applied Physics
Friedrich-Schiller-Universität Jena

Optical System Design Fundamentals

Lecture 3: First Order Layout and System Structures

2024 / 05 / 21

Vladan Blahnik

Preliminary Schedule - OSDF 2024

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1	07.05.2024	Optical Imaging Fundamentals	Fermat principle, modeling light propagation with ray and wave model; diffraction; Snell´s law, total internal reflection, light propagation in optical system, raytracing; ray and wave propagation in optical systems; pinhole camera; central perspective projection; étendue, f-number and numerical aperture, focal length; ideal projection laws; fish,eye imaging; anamorphic projection; perspective distortion, Panini projection; massive ray-tracing based imaging model	(S) (optic
2	14.05.2024	The ideal image	Paraxial approximation, principal planes, entrance and exit pupil, paraxial imaging equations; Scheimpflug condition; paraxial matrix formulation; Delano diagram; Aplanatic imaging; Abbe sine condition, numerical aperture, sine scaled pupil for microscope imaging	(S)
3	21.05.2024	First order layout, system structures and properties	application of imaging equations; examples: layout of visual imaging system; layout of microscope lens and illumination system; telecentricity, "4f-setups"; tele and reversed tele (retrofocus) system; depth of field, bokeh; equivalence to multi-perspective 3D image acquisition; focusing methods of optical systems; afocal systems; connecting optical (sub-)systems: eye to afocal system; illumination system and its matching to projection optics); zoom systems; first order aberrations (focus and magnification error) and their compensation: lens breathing, aperture ramping; scaling laws of optical imaging	S
4	28.05.2024	Imaging model including diffraction	Gabor analytical signal; coherence function; degree of coherence; Rayleigh-Sommerfeld diffraction; optical system transfer as complex, linear operator; Fraunhofer-, Fresnel-approximation and high-NA field transfer wave-optical imaging equations for non-coherent, coherent and partially coherent imaging; isoplanatic patches, Hopkins Transfer Function for partially coherent imaging, optical transfer functions for non-coherent and coherent imaging; ideal wave-optical imaging equations; Fourier-Optics; spatial filtering; Abbe theory of image formation in microscope, phase contrast microscopy	S
5	04.06.2024	Wavefront deformation, Optical Transfer Function, Point Spread Function and performance criteria	wave aberrations, Zernike polynomials, measurement of system quality; point spread function (PSF), optical transfer function (OTF) and modulation transfer function (MTF); optical system performance criteria based on PSF and MTF: Strehl definition, Rayleigh and Marechal criteria, 2-point resolution, MTF-based criteria, coupling efficiency / insertion loss at photonics interface	S
6	11.06.2024	Aberrations: Classification, diagrams and identification in real images	measurement of wave front deformation: Shack-Hartmann sensor, Fizeau interferometer Symmetry considerations as basis for aberration theory; Seidel polynomial, chromatic aberrations; surface contributions and Pegel diagram, stop-shift-equation; Longitudinal and transverse aberrations, spot diagram; computing aberrations with Hamilton Eikonal Function; examples: defocus, plane parallel plate longitudinal and lateral chromatic aberration, Abbe diagram, dispersion fit; achromat and apochromat aberration in real images of (extended) objects	no
7	18.06.2024	Optimization process and correction principles	pre- and post-computer era correction methodology, merit function, system specification, initial setups, opto-mechanical constraints, damped-least-square optimization; symmetry principles, lens bending, aplanatic surface insertation, lens splitting, aspheres and freeforms	S
8	25.06.2024	Aberration correction and optical system structure	Petzval theorem, field flattening, achromat and apochromat, sensitivity analysis, diffractive elements; system structure of optical systems, e.g. photographic lenses, microscopic objectives, lithographic systems, eyepieces, scan systems, telescopes, endoscopes	(S)
9	02.07.2024	Tolerancing and straylight analysis	optical technology constraints, sensitivity analysis, statistical fundamentals, compensators, Monte-Carlo analysis, Design-To-Cost; false light; ghost analysis in optical systems; AR coating; opto-mechanical straylight prevention	S

Paraxial optics: First order optical system analysis and synthesis

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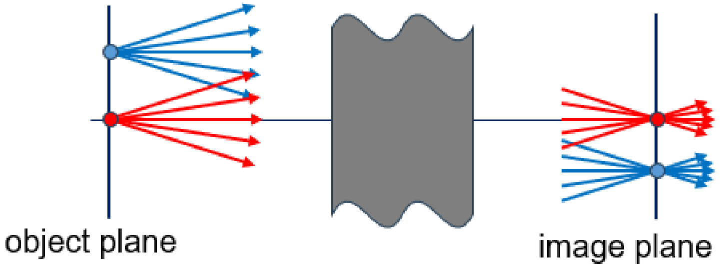
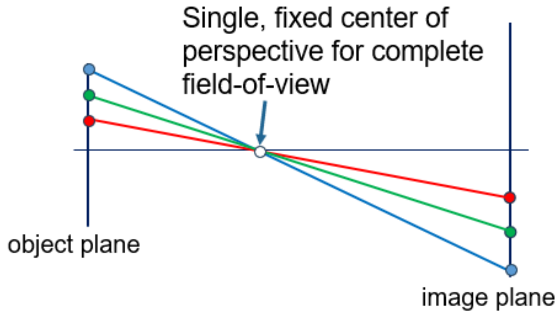
Paraxial ray tracing = determination of image position with (first order part) of system data directly:

$$-\frac{n_j}{s_j}+\frac{n_j{'}}{s_j{'}}=\frac{n_j{'}-n_j}{r_j}$$

$$s_{j+1} = s'_j - d_j \qquad n_{j+1} = n'_j \qquad u_{j+1} = u'_j \qquad h_j = h'_j$$

Paraxial image is ideal, i.e. images the entire object plane stigmatically and distortion-free onto image plane:

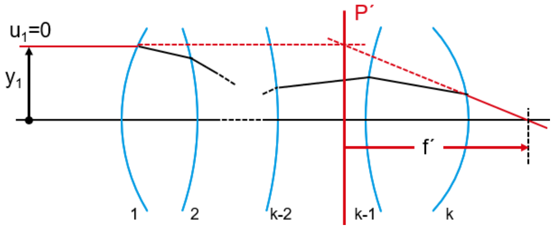




System quantities magnification and focal length and their (graphical) construction:

$$m=\frac{n_1}{n'_k}\prod_{j=1}^{k}\frac{s'_j}{s_j}=\frac{n_1 u_1}{n'_k u'_k}$$

$$f'=\frac{n_1}{n'_k}\left(s'_1\prod_{j=2}^{k}\frac{s'_j}{s_j}\right)$$



System as black box by entrance and exit pupil:

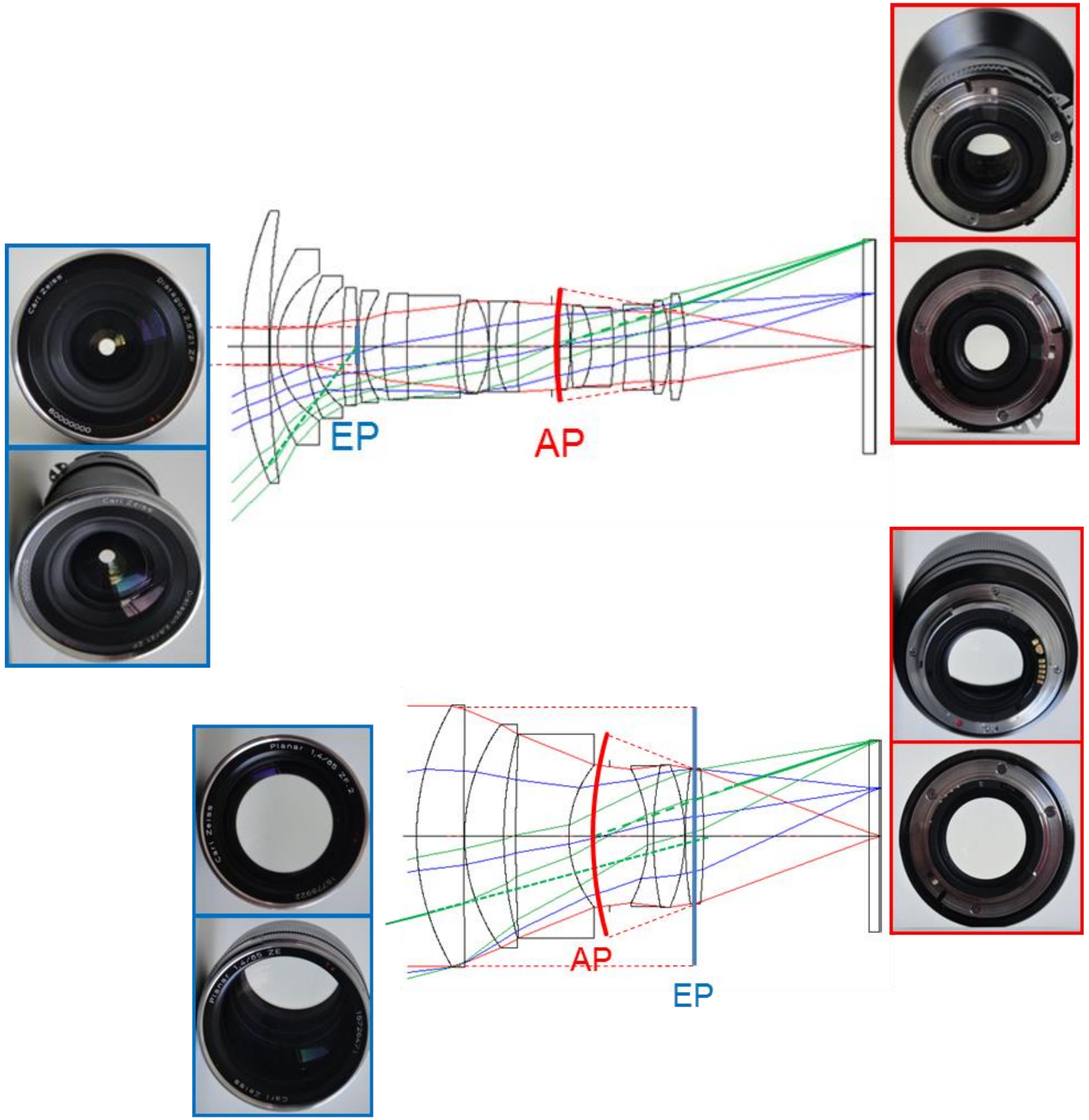
$$-\frac{1}{m_p s}+\frac{m_p}{s'}=\frac{1}{f'} \qquad m_p m=\frac{s'}{s}$$

Summation laws of refractive power / focal length:

$$\Phi=\Phi_1+\Phi_2-d\Phi_1\Phi_2 \qquad \frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

k surfaces: $\Phi=\dfrac{1}{h_1}\sum_{j=1}^{k}h_j\Phi_j$

Image formation described via entrance and exit pupil of optical systems



Imaging equation via entrance (EP) and exit (AP) pupil:

$$-\frac{1}{m_p s} + \frac{m_p}{s'} = \frac{1}{f'}$$

Pupil magnification:

$$m_p = \frac{\emptyset_{AP}}{\emptyset_{EP}}$$

Depth relation:

$$m_p m = \frac{s'}{s}$$

s is the distance between entrance pupil and object,
s´ between exit pupil and image.

This imaging equation apparently has fallen into oblivion in modern optics textbook literature.

It features:

1. The image position for a given object position
2. The chief ray angle in object and image plane
3. The actual relative size of the pupils

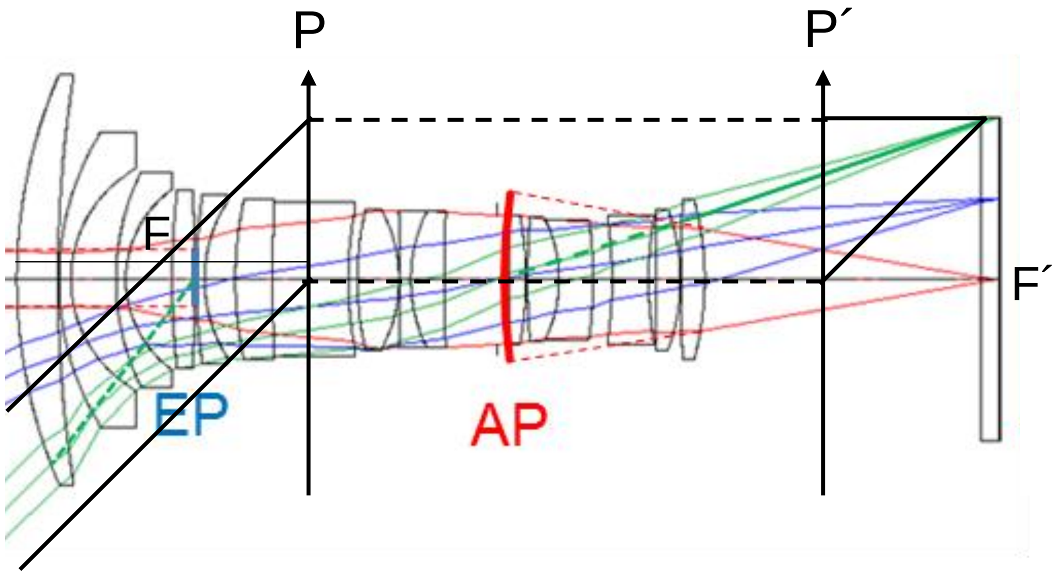
(2. and 3. are not described, when the system is modeled via its principal planes)

Entrance and exit pupil versus principal planes as system conjugates

Principal planes are suited to (graphically) determine object and image position.

Principal planes are **not suited to make any prediction of actual ray paths**. Necessarily for actual ray paths the **stop position must included**!

Here: object point at infinity!

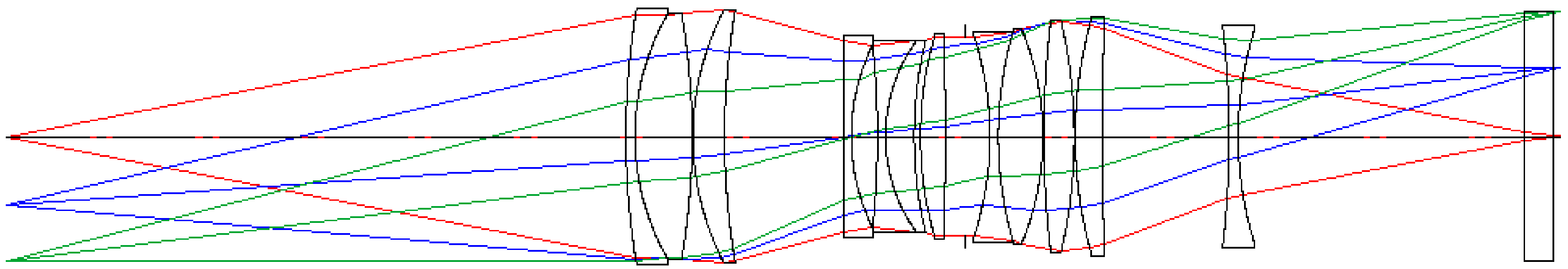


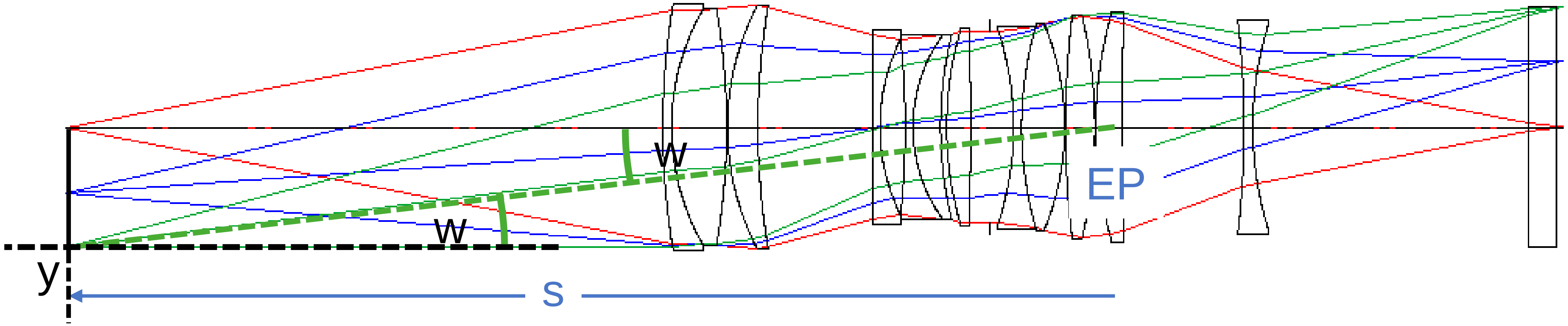
$$-\frac{1}{m_p s} + \frac{m_p}{s'} = \frac{1}{f'}$$

includes perspective imaging properties!

Principal planes and corresponding light paths according to paraxial construction principles (although we are far out of paraxial regime here)

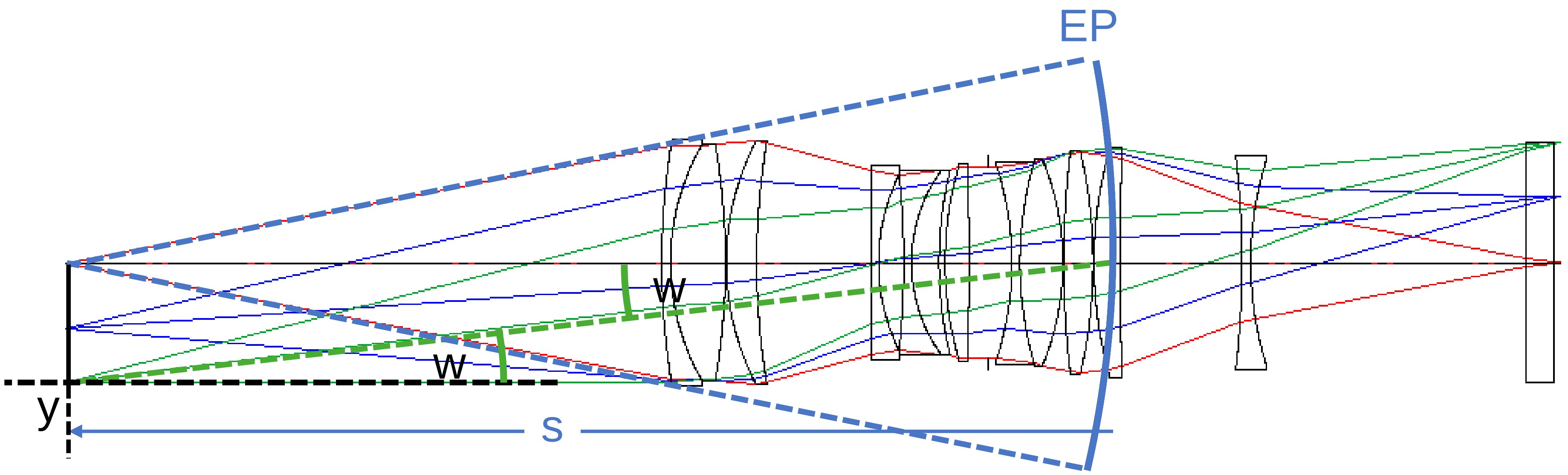
Chief ray angles / Telecentricity





Entrance pupil position at distance s from object plane = crossing point of **chief ray** coming from object space with optical axis

Chief ray angles / Telecentricity



Entrance pupil size = crossing point of marginal rays with entrance pupil sphere* coming from <u>object</u> space

*not plane, as optical systems are aplanatic; we come this later in this lecture

Chief ray angles / Telecentricity

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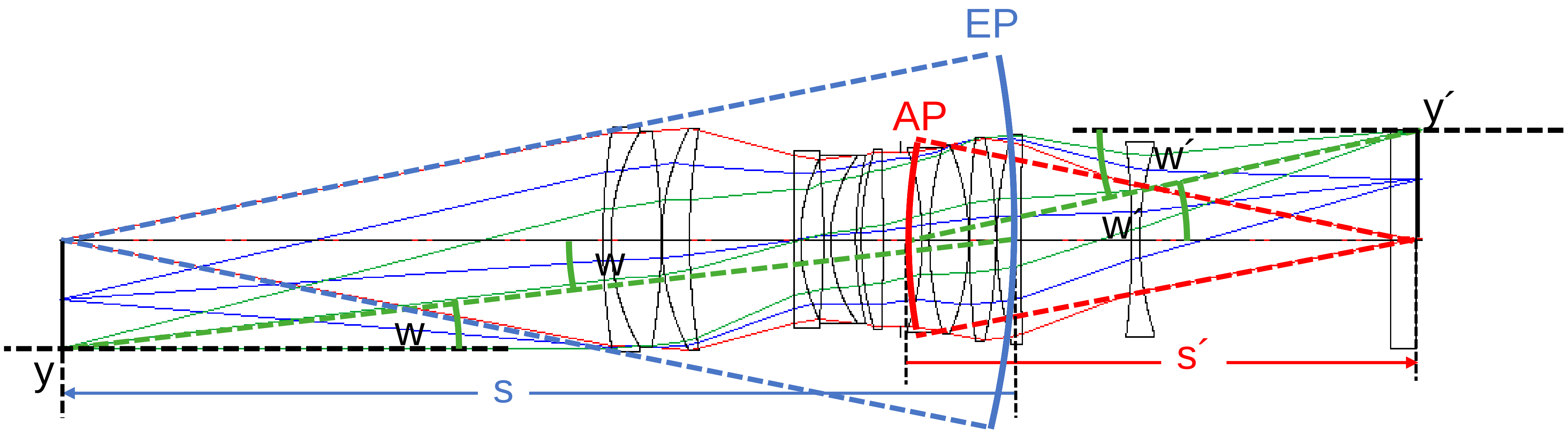
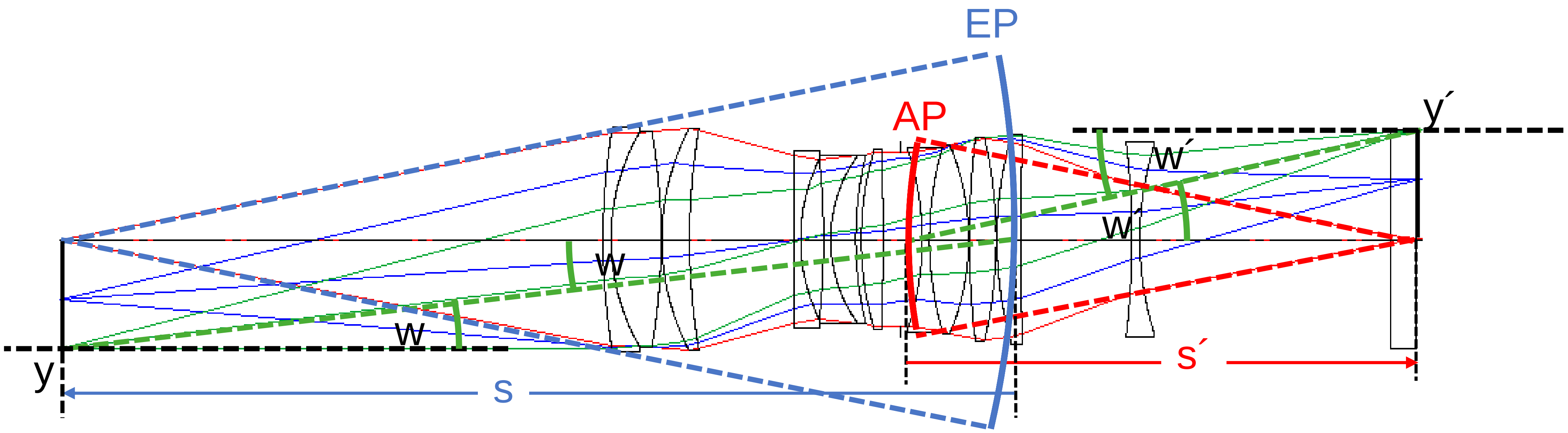


Exit pupil (AP) position and size: analog procedure coming from image space

EP and AP correspond to the perspective centers of the optical system. The path of the chief ray determines the perspective projection of the imaging system.



Chief ray angles w and w´ in object and image space respectively:

$$\tan w = \frac{y}{s}$$

object-side telecentric, if w=0 or s → ∞ ("entrance pupil at infinity"),

$$\tan w' = \frac{y'}{s'}$$

image-side telecentric, if w´=0 or s´ → ∞ ("entrance pupil at infinity").

Chief ray angles / Telecentricity

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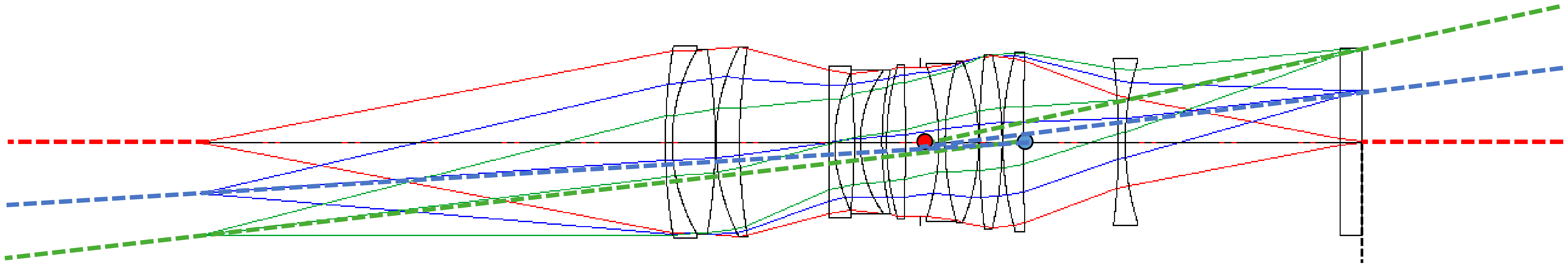
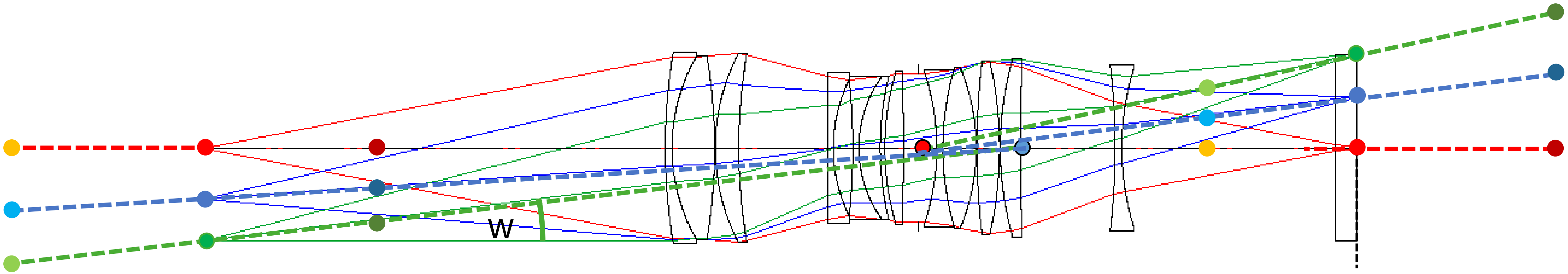


Chief ray paths via entrance and exit pupil.

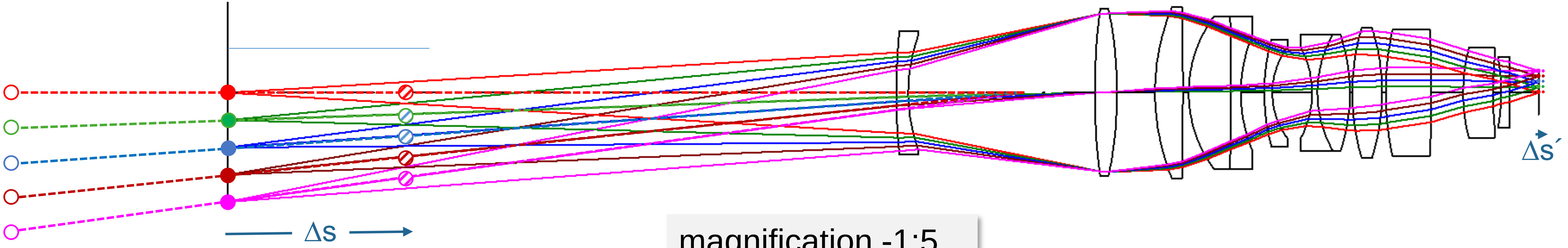
Chief ray angles / Telecentricity

EP and AP correspond to the perspective centers of the optical system. The path of the chief ray determines the perspective projection of the imaging system.



Conjugate point pairs lie along chief ray paths!

Historic lithography lens (1980)



magnification -1:5,
NA´=0.36,
y´=10mm, 436nm

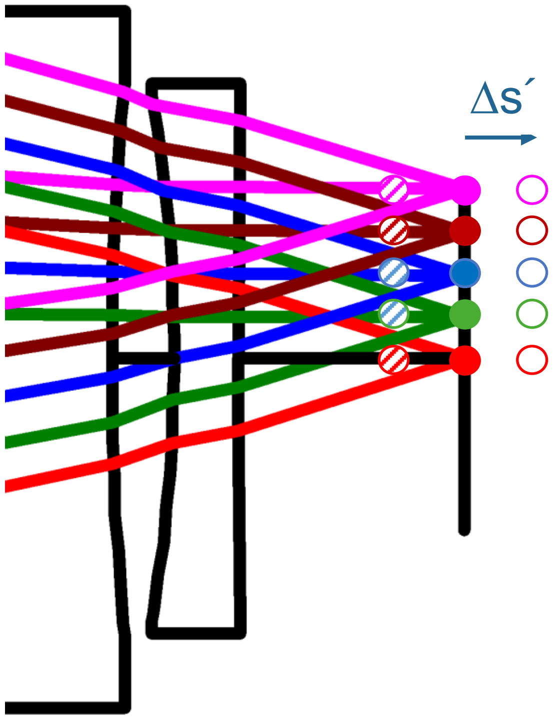


image-side telecentric

Magnification $m = -\frac{1}{5} = -0.2$

Depth relation $\frac{\Delta s'}{\Delta s} \approx m^2 = \frac{1}{25} = 0.04$

Telecentric and non-telecentric projection

a) object with pillars | b) image with conventional lens | c) image with telecentric lens

Ref: Haosheng Hu

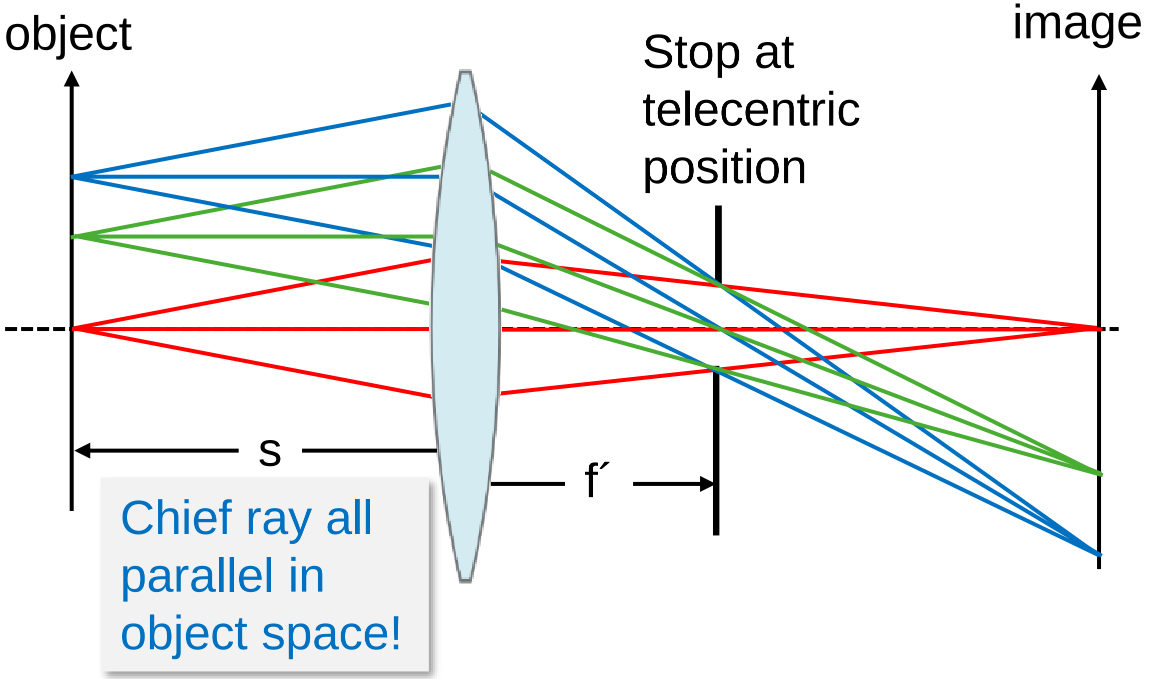
Telecentricity

Telecentricity realized if stop positioned in:

1. back focal plane: object-sided telecentric
2. front focal plane: image-sided telecentric
3. intermediate focal plane: both-sided telecentric

Telecentricity:

1. pupil at infinity
2. chief ray parallel to the optical axis



Telecentricity

Telecentricity realized if stop positioned in:
1. back focal plane: object-sided telecentric
2. front focal plane: image-sided telecentric
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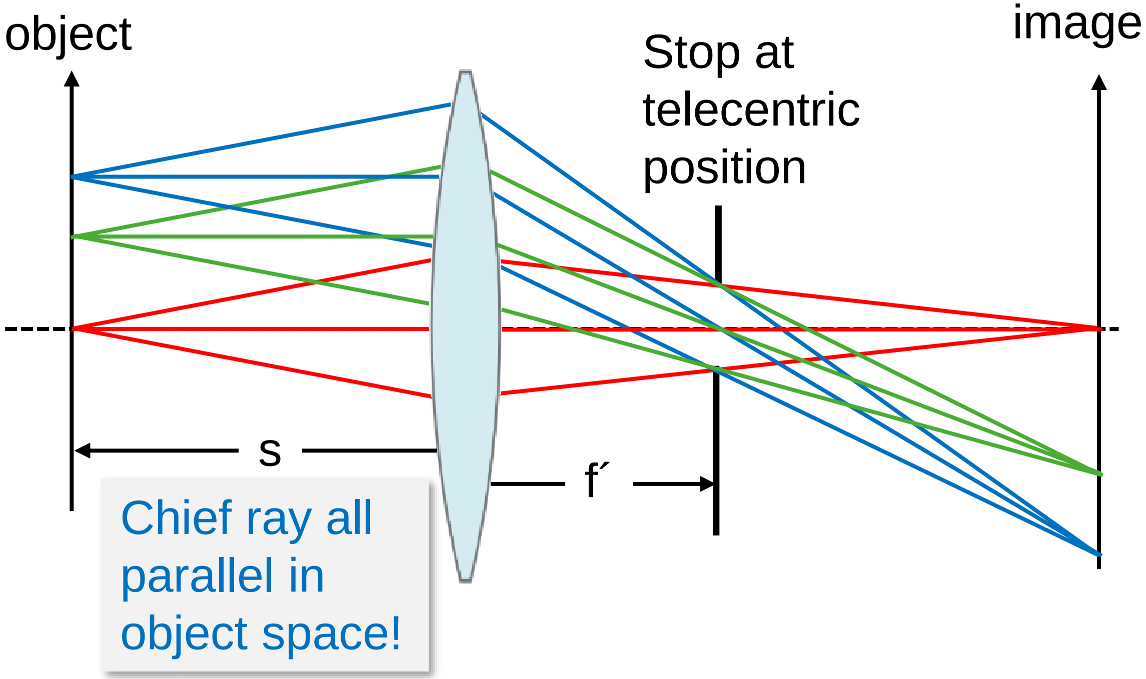
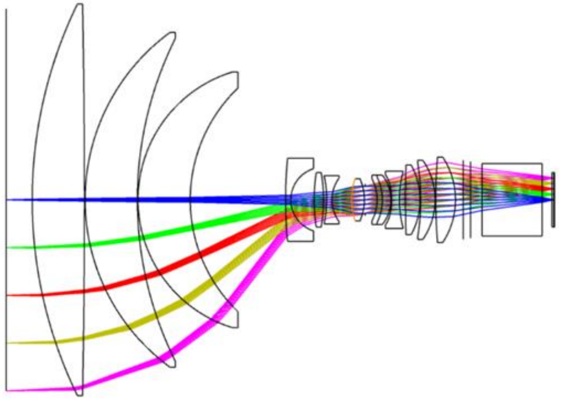


Telecentricity:
1. pupil at infinity
2. chief ray parallel to the optical axis

Problem in practical systems:
large diameters necessary for large fields

$$D > 2 \cdot (y_{\max} + f\ NA)$$

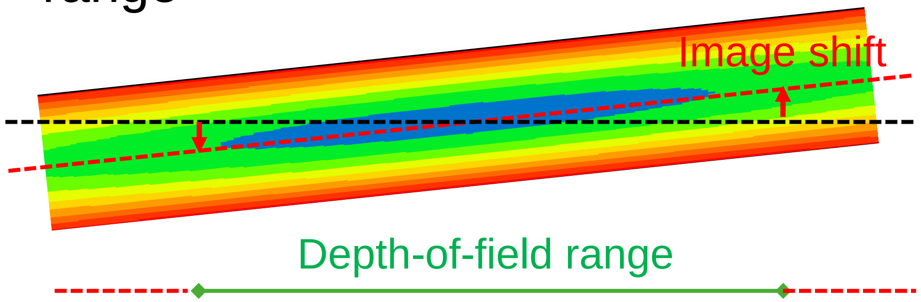
Telecentric lenses for industrial machine vision application



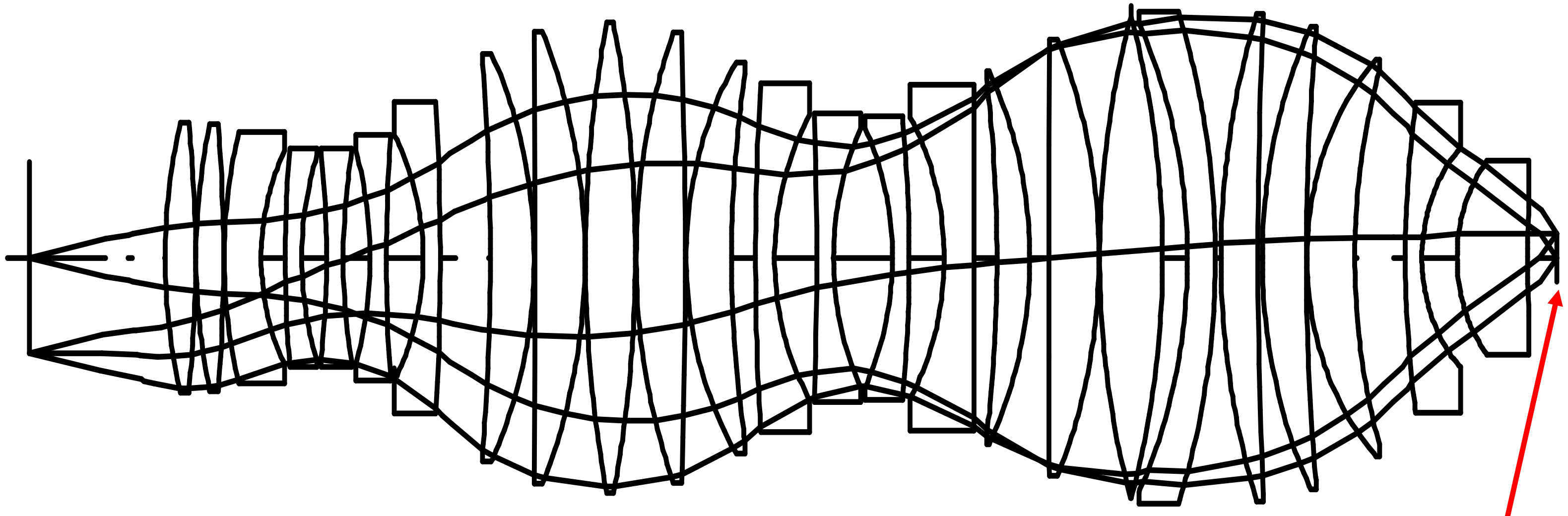
Example: Modern lithography lens double-side telecentric and „4f-model“

Required to be telecentric:

Image side:
Avoids image shift → overlay error within depth-of-field range



Object side:
Avoid polarization and vignetting effects for oblique incidence on mask structures

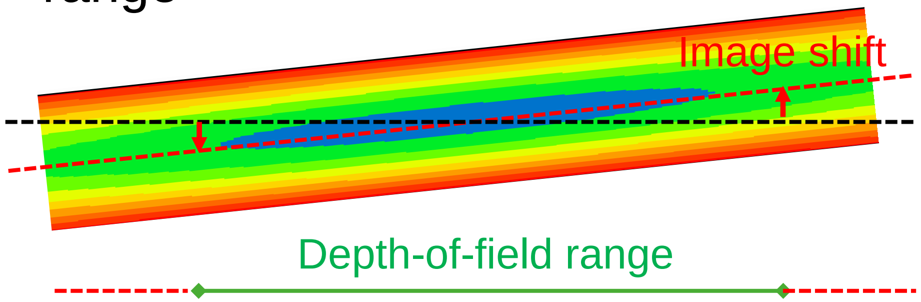


Requirement on telecentricity within a few Milliradiants [mrad]

Example: Modern lithography lens double-side telecentric and „4f-model“

Required to be telecentric:

Image side:
Avoids image shift → overlay error within depth-of-field range



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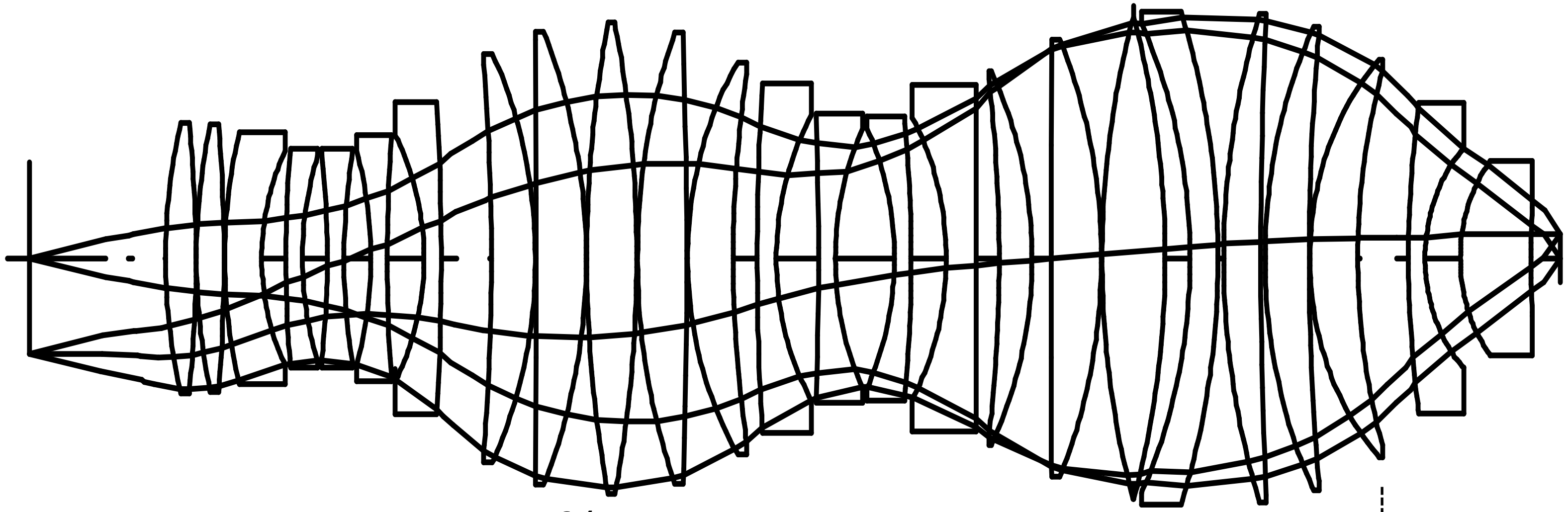
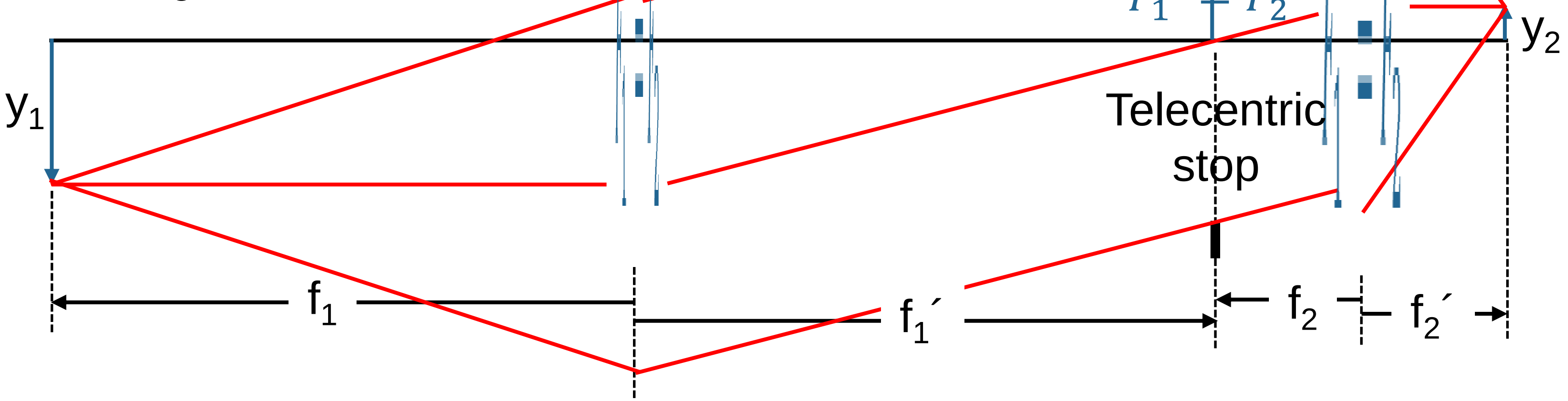


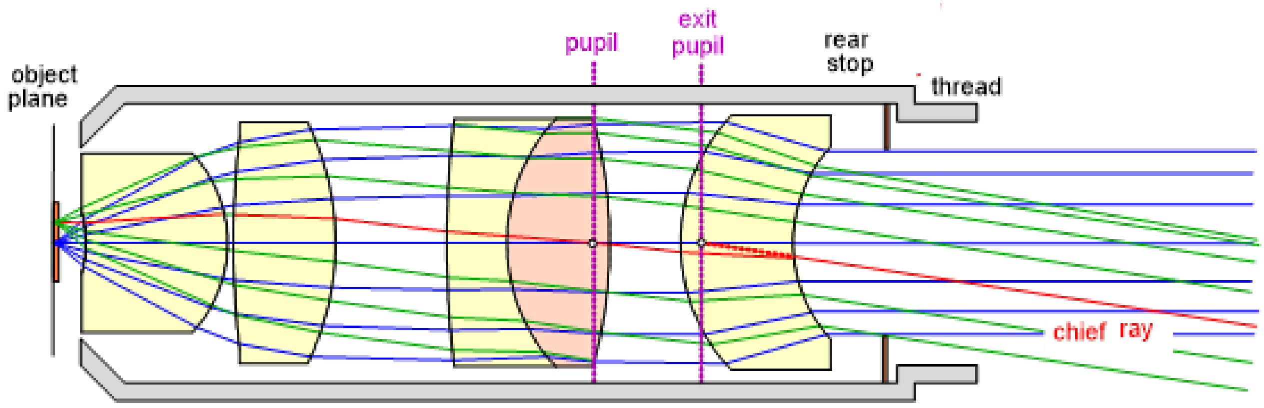
Magnification $m = \frac{y_2}{y_1} = \frac{f_2´}{f_1}$

$m = -\frac{1}{4}$ for latest tool generations

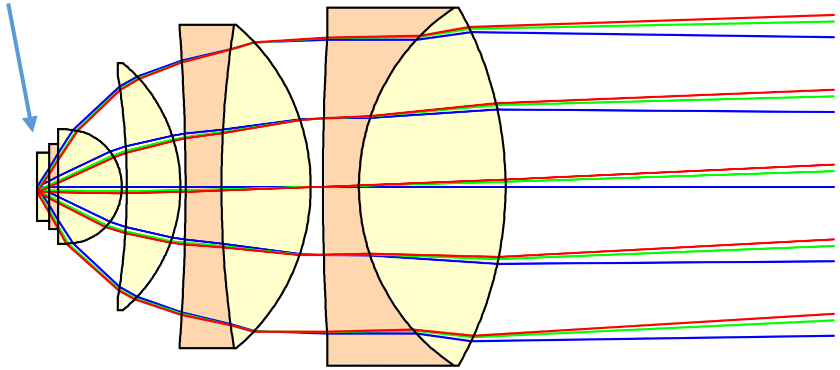


Further telecentric systems

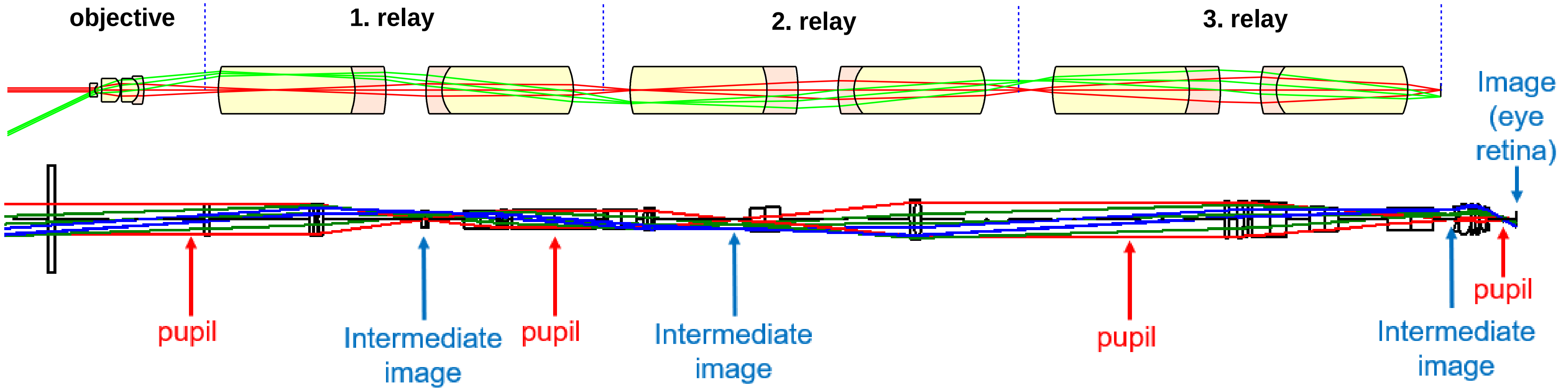
Microscopes
object-side telecentric (EP→∞)



Propagation through cover glasses at high NA:
Telecentricity critical



Endoscopes I Periscopes
cascading of relays
systems needs telecentric intermediate images



What are all those lens data?

camera lens

binocular

10 x 42

telescope

„80 / 400 refractor

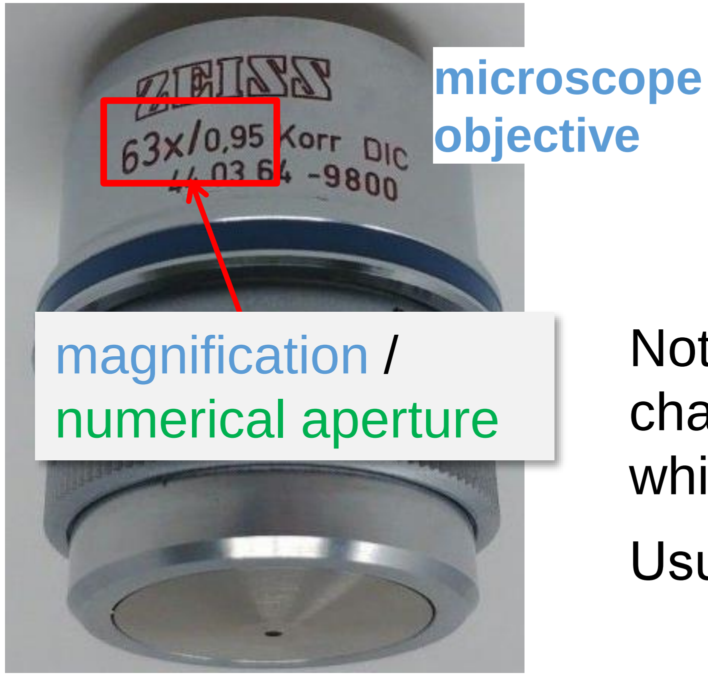
microscope objective

Eye glasses

Rifle scope

1.8-14 x 50

Relationship between characteristic data



There are relationships between those quantities:

f-number K and image side numerical aperture NA:

$$K = \frac{1}{2\,NA'}$$

Magnification m and focal length f: You need another parameter for the relationship, e.g., object distance s (to lens) or image distance s´ (from lens to image plane):

$$m = \frac{f}{f+s}$$

Note that there are some very relevant data which are not given in those characteristic data, e.g., the physical size of the field (object or image diameter) which the lens is capable to image.

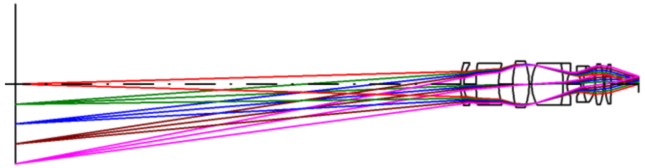
Usually, you must read the manufacturers description to find these data…

Relationship between characteristic data

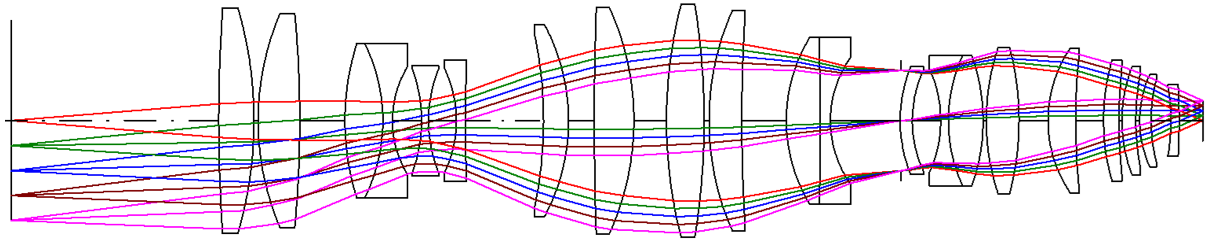
Historically, lithography lenses were a niche applications of photography and characterized by f-number and focal length.

This changed later on (in 1980s) towards the specification of microscopes by magnification and numerical aperture.

This is much more useful as those tools are used at one fixed magnification and there is no practical meaning of focal length.



1968:
S-Planar **1.6/25mm**
β=-1:10, NA0.31, y´=4mm, 436nm

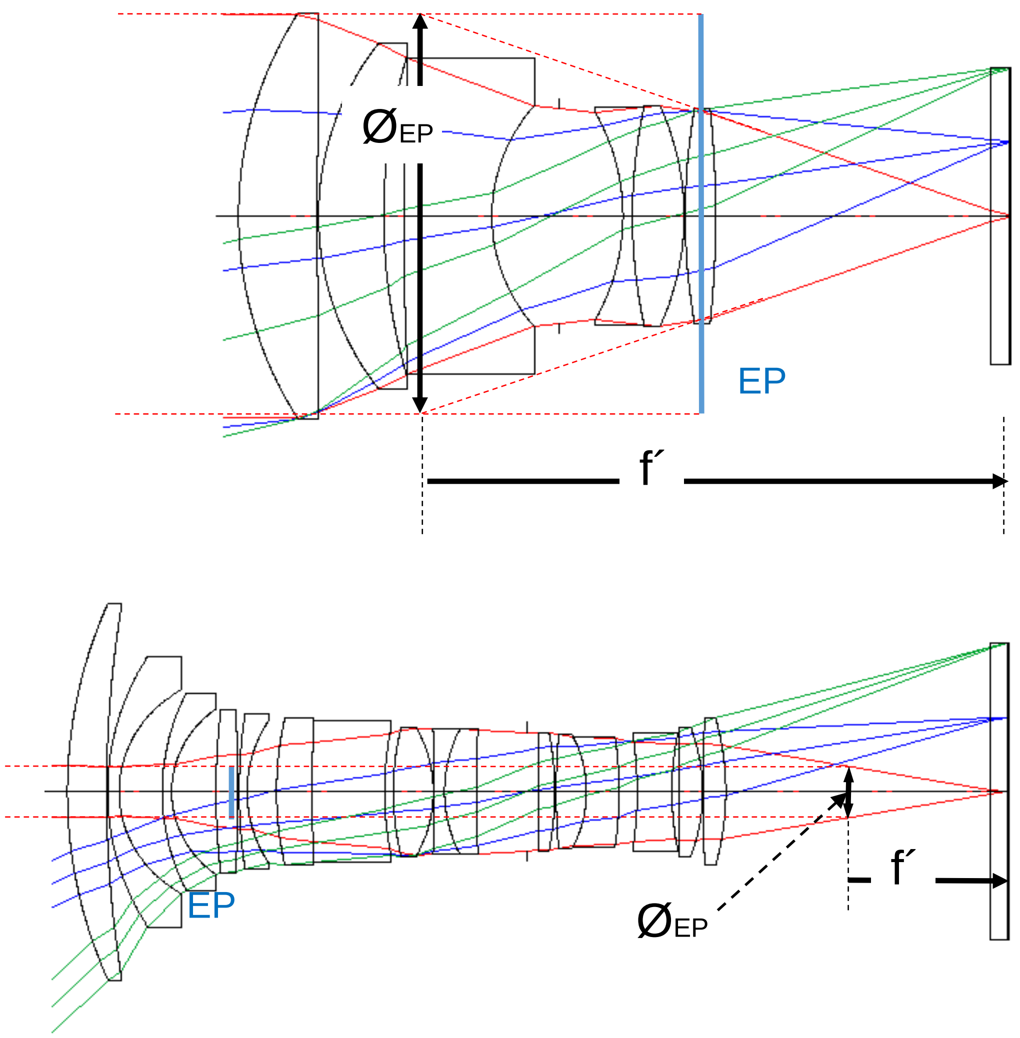


1985:
S-Planar **5x / NA0.40**,
y´=10mm, 365nm

Basic Lens Data: Focal length and f-number or NA

f-number / focal length [mm]

F-number and numerical aperture



f-number (f/#) K_0:

$$K_0 = \left. \frac{f'}{Ø_{EP}} \right|_{m=0}$$

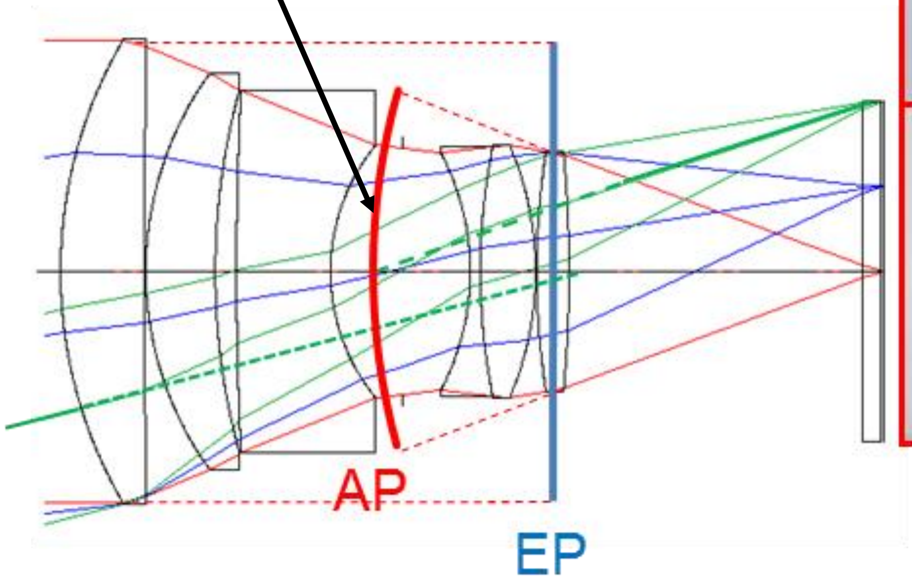
Working f-number and image-side numerical aperture (**NA´**) relation**:**

$$K = \frac{1}{2\mathrm{NA}'} := \frac{1}{2 sin u'_{\max}}$$

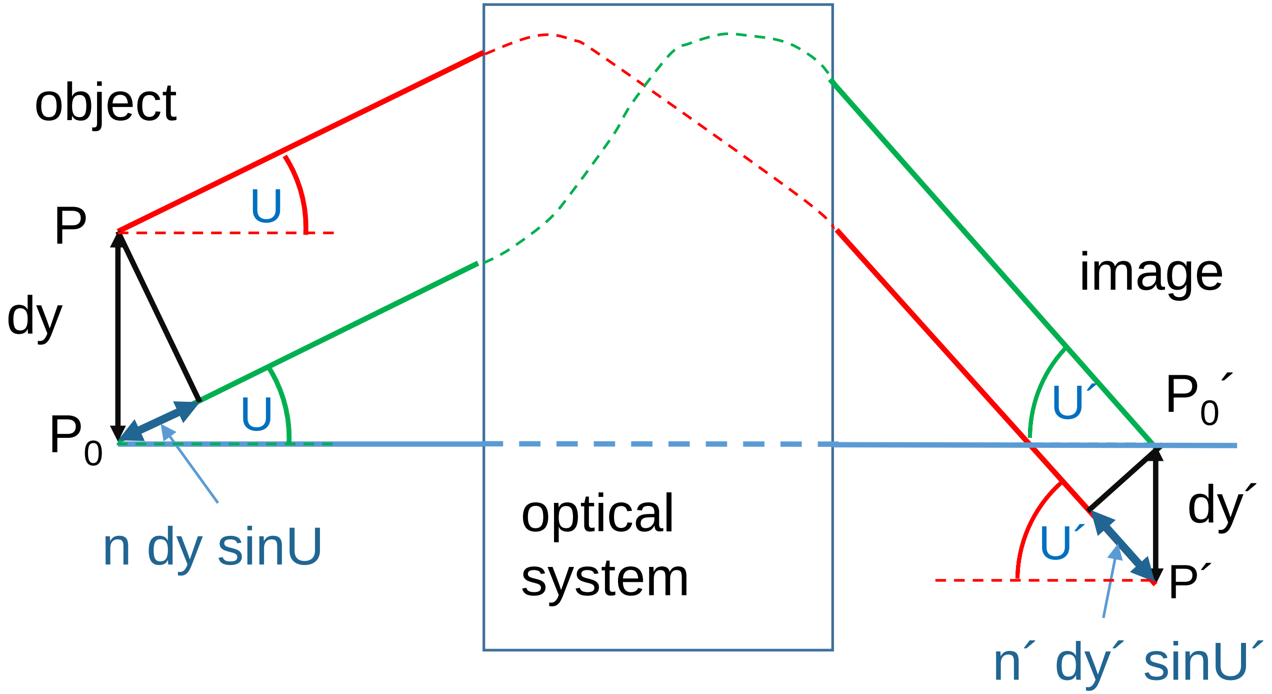
This is a direct relation!
f-number is related with **sine of angles**!

Entrance and exit pupil

Why did I draw a curved pupil?



Abbe's sine condition



Abbe's sine condition

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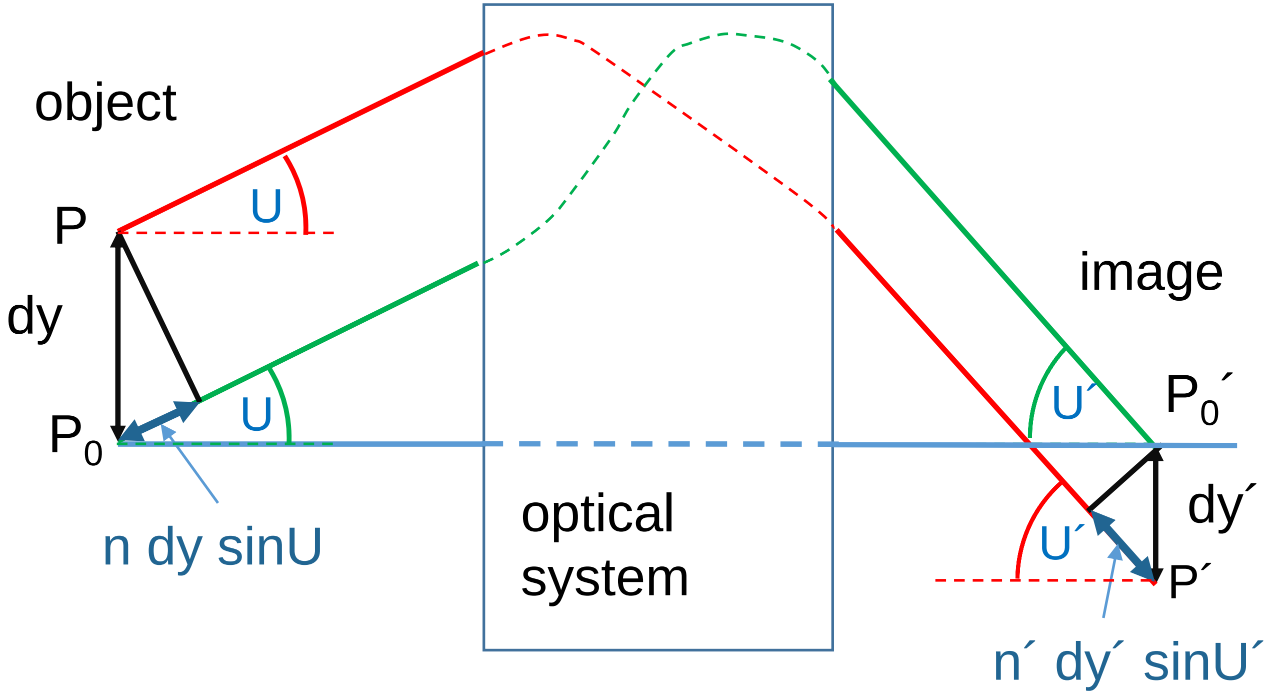


Abbe's sine condition:

$$m\,\frac{n´}{n}\,\sin U' = \sin U$$

Consequently, for $\mathrm{NA} = n\;\sin U_{max}$:

$$m\,NA´ = NA$$

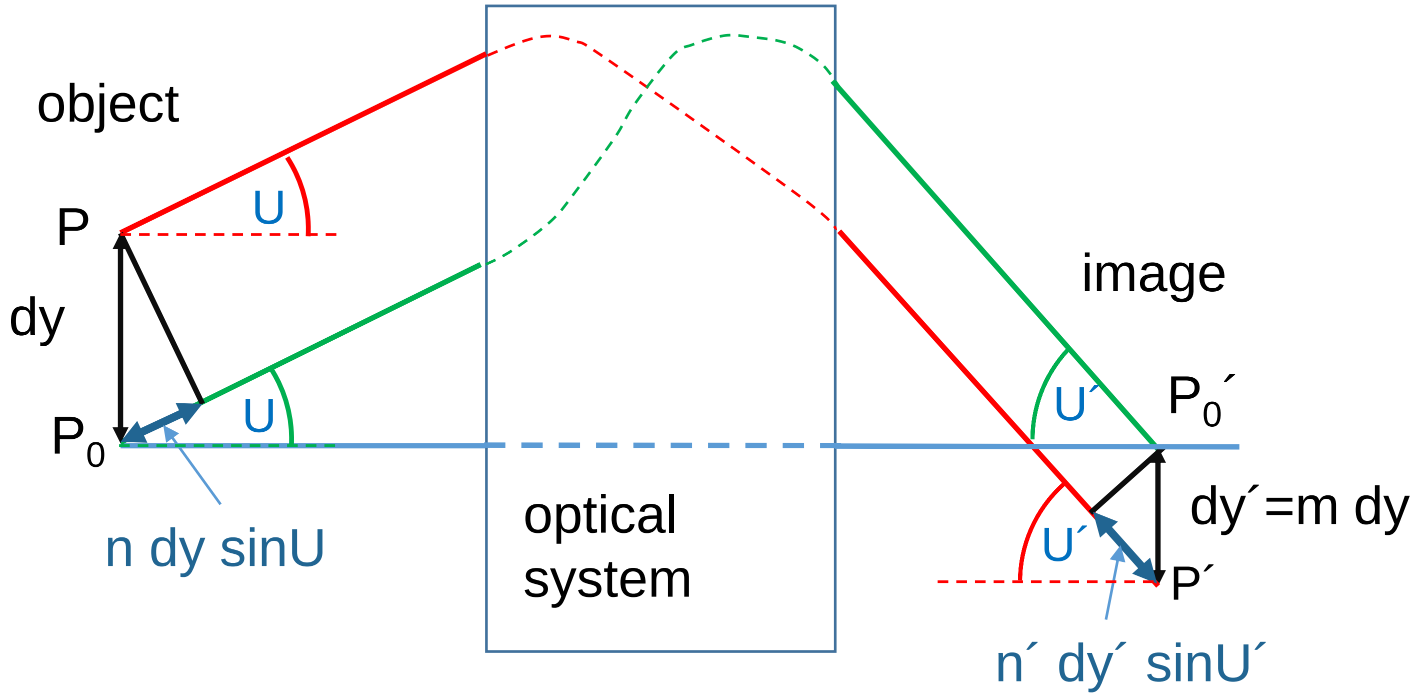
In accordance with local étendue conservation law.

Assume that the image on axis, between P_0 and $P_0´$ is stigmatic (or free of spherical aberration) even for large aperture angles U and U´ (denoted with large letter U to distinguish paraxial u). Then, according to Fermat´s principle, the optical path length is equal for any optical path between P_0 and $P_0´$ (green path and e.g., on-axis path).

To achieve a stigmatic image between object and image plane, oriented normal to the optical axis, the optical path length along a "directly adjacent" (infinitesimal distances dy and dy´ respectively) off-axis optical path (drawn red) must also be equal to the on-axis path length. Consequently, as the change of the aperture angles U, U´ is negligible over the distance dy, dy´:

$$\mathrm{n´\ dy´}\,\sin U' = n\,dy\,\sin U$$

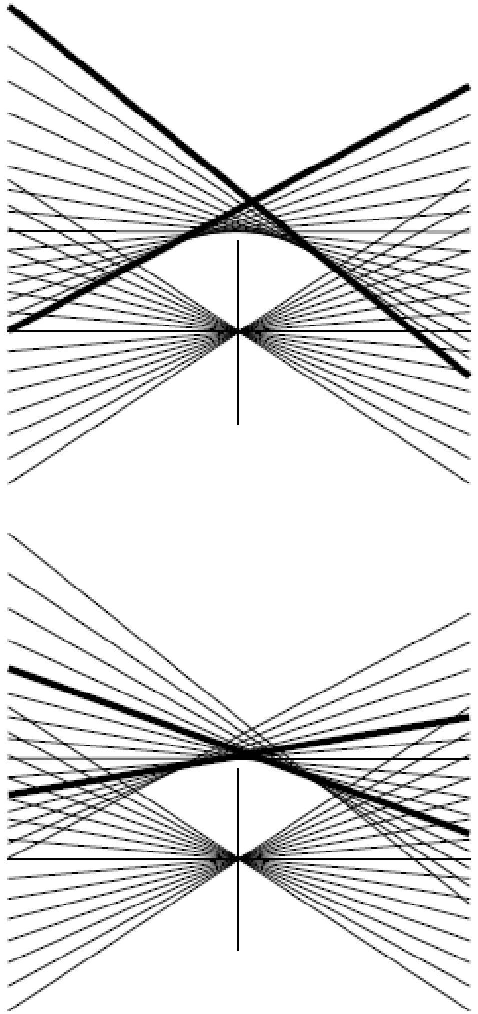
Abbe's sine condition



Abbe´s sine condition:

$$m \frac{n^{\prime}}{n} \sin U' = \sin U$$

If this condition is infringed, the magnification depends upon the ray height in the lens pupil → „coma"



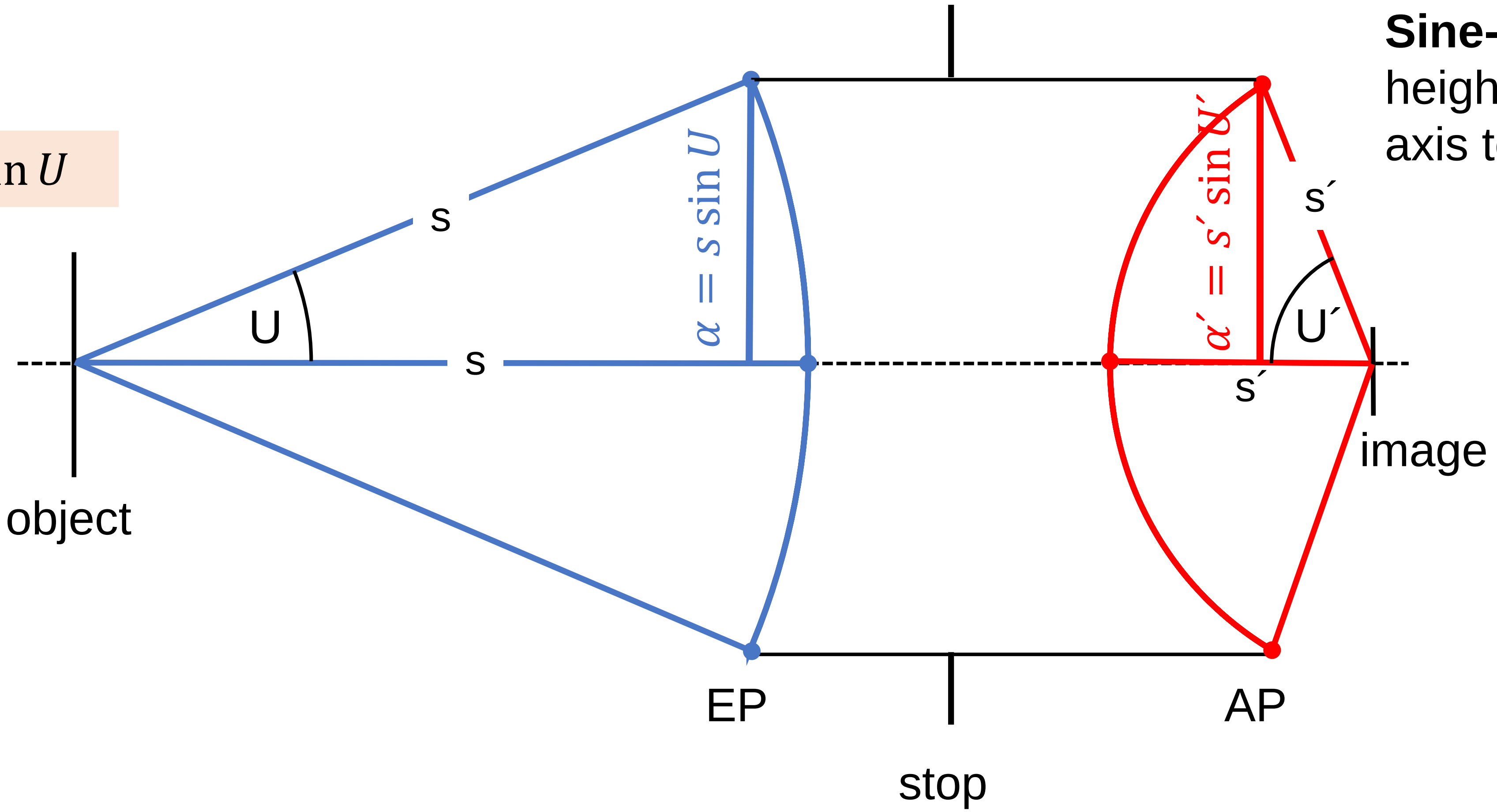
This aberration is **coma**

Sine condition geometrically:

Pupils are spherical

The pupils and principal surfaces are spherical and not plane…

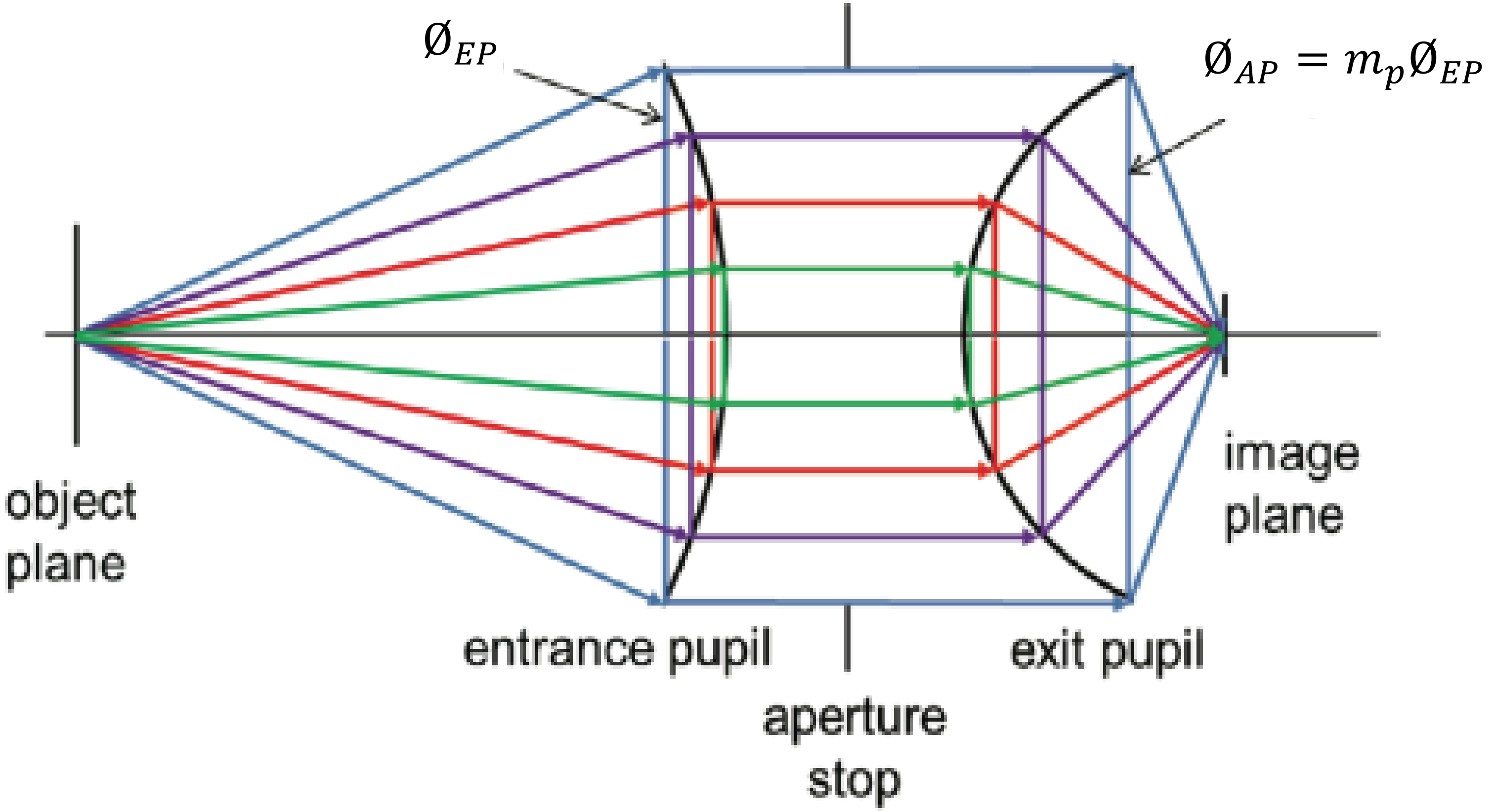
$$m \ \sin U' = \sin U$$



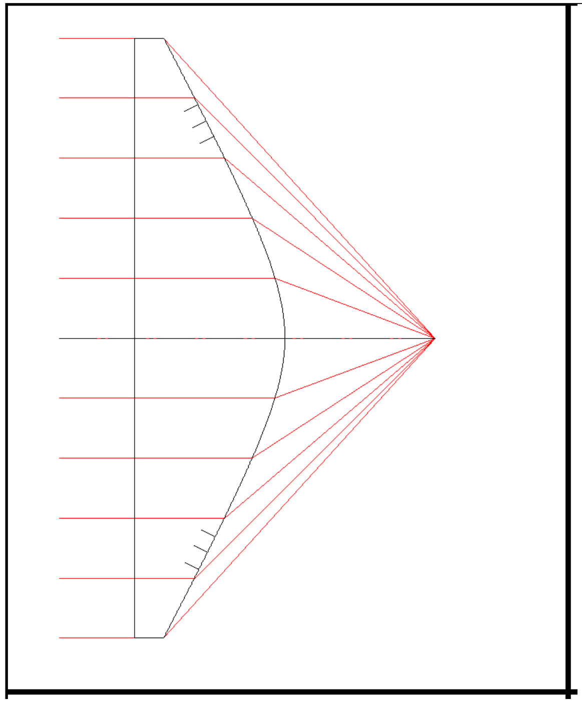
Sine-scaled pupil: height measured from axis to pupil sphere

Entrance and exit pupil diameters of sine-scaled pupil

$$\frac{\emptyset_{EP}}{s} = NA = m\,NA' = m\frac{\emptyset_{AP}}{s'}$$



Perfect imaging of an object point at infinite distance with a single lens element



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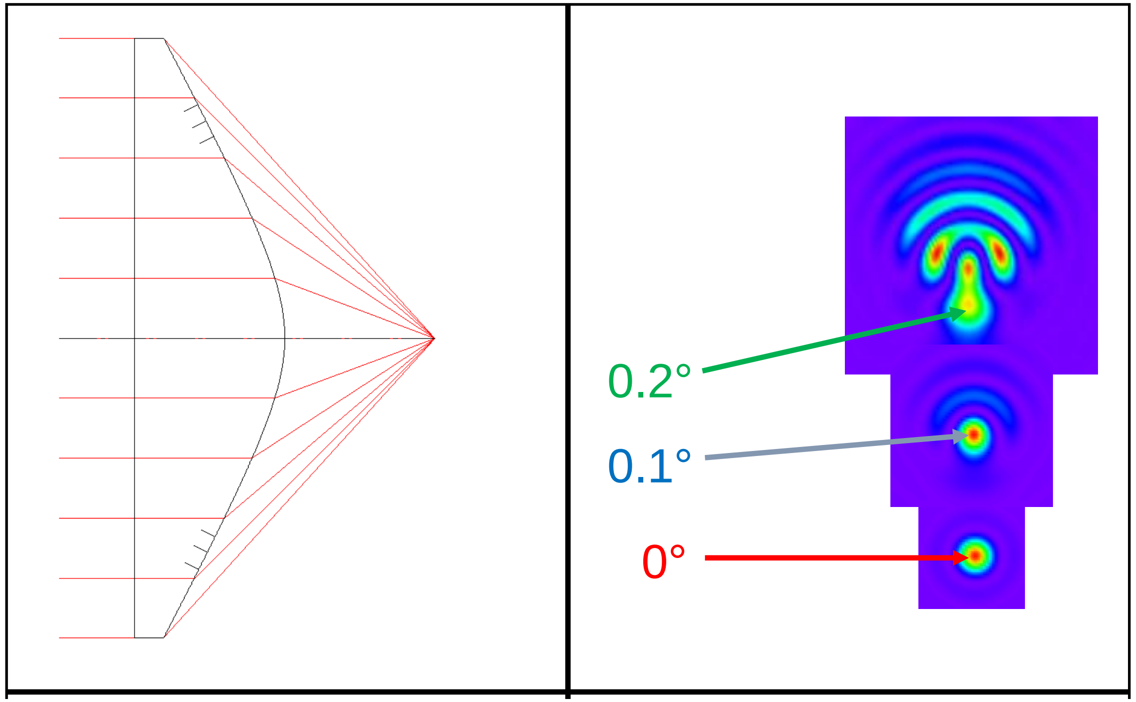

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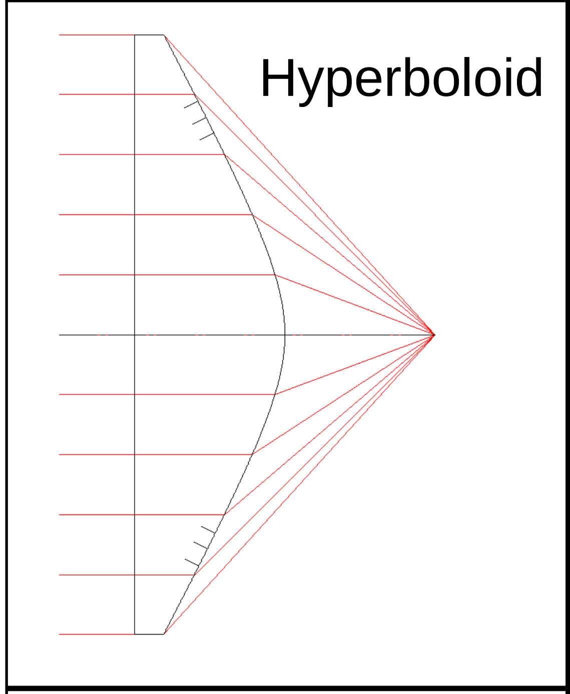
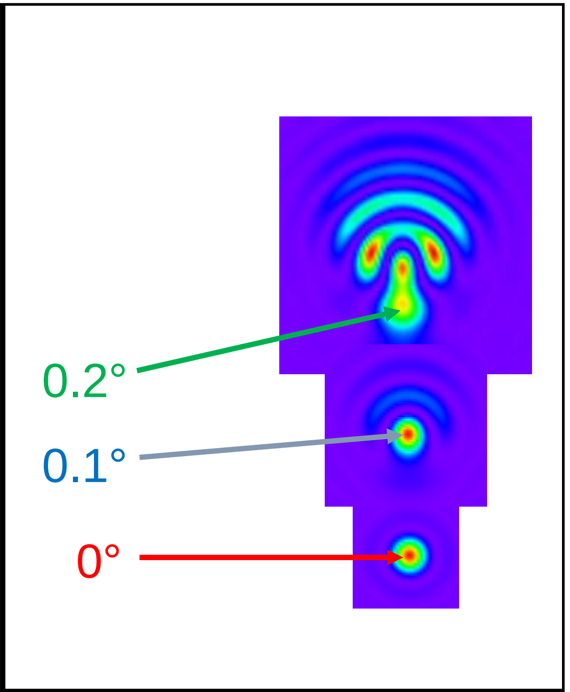
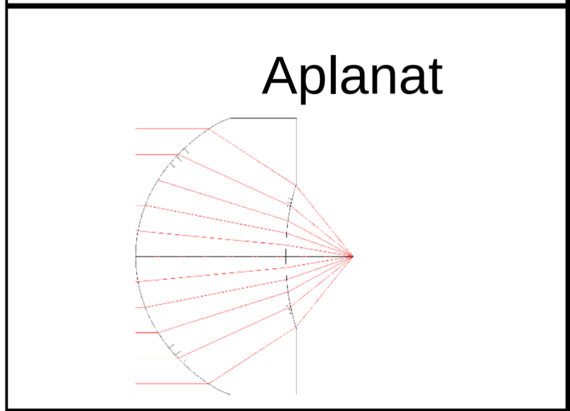
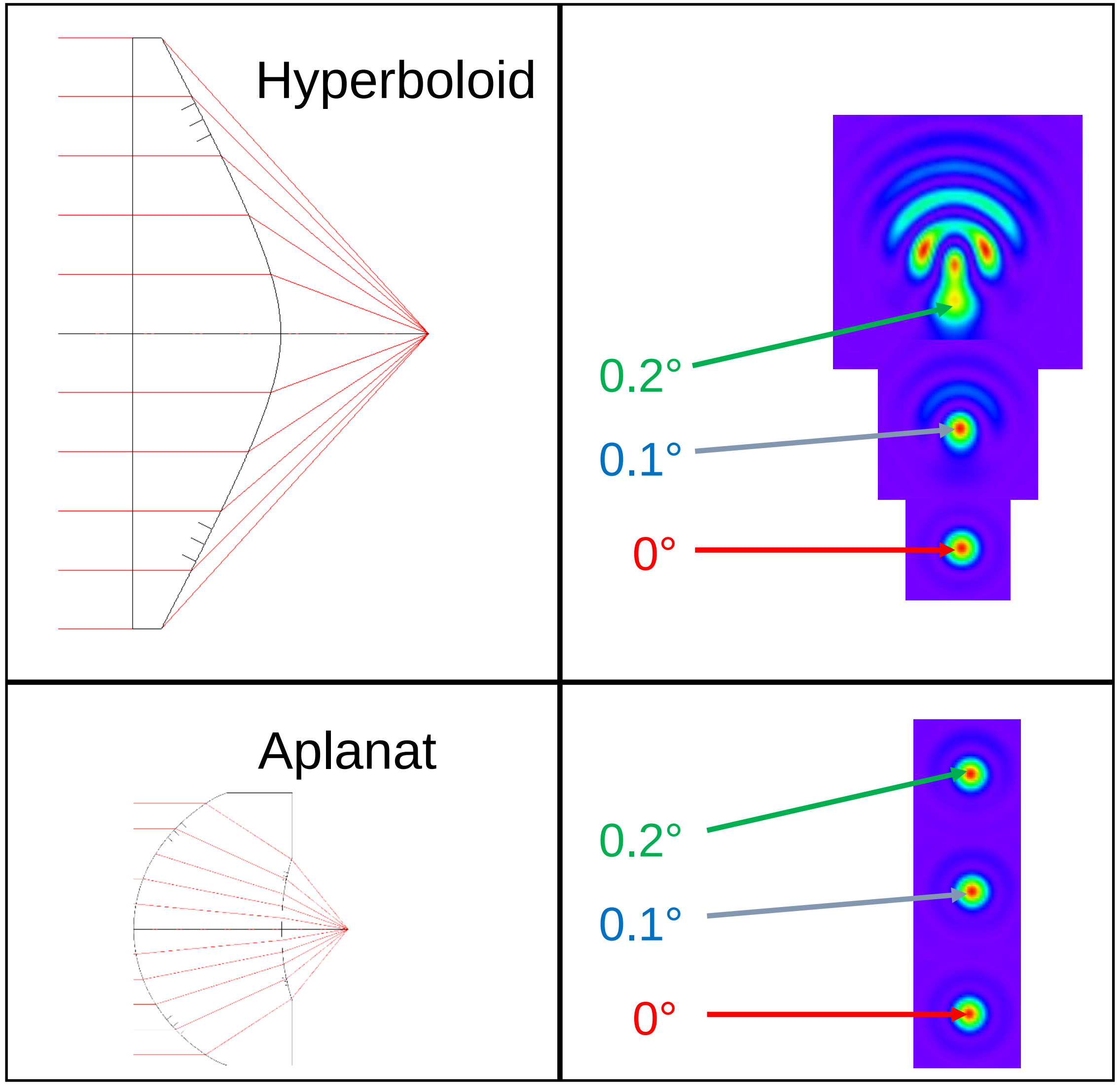






Perfect imaging of an object point at infinite distance with a single lens element



Perfect imaging of an object point at infinite distance with a single lens element

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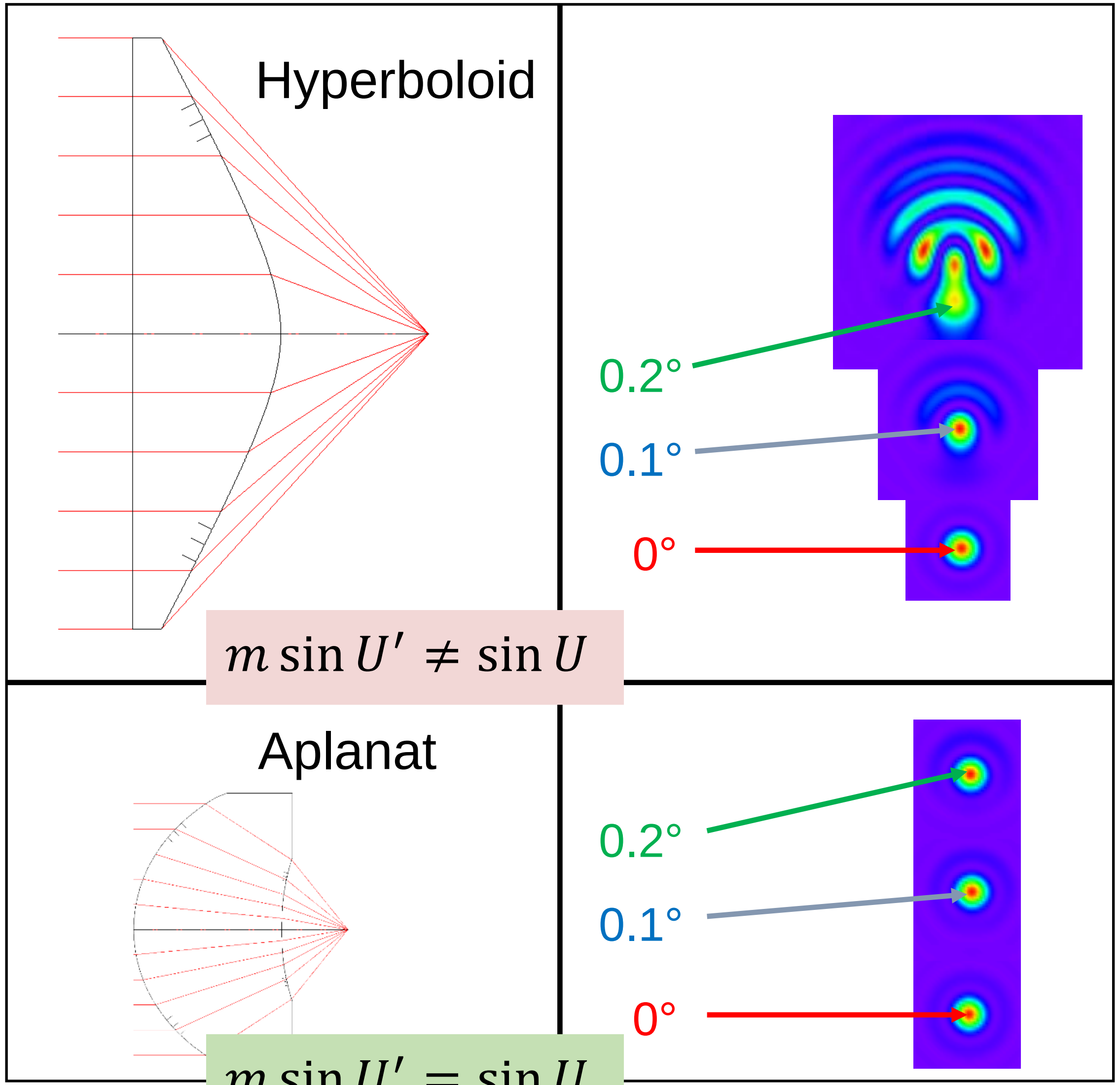
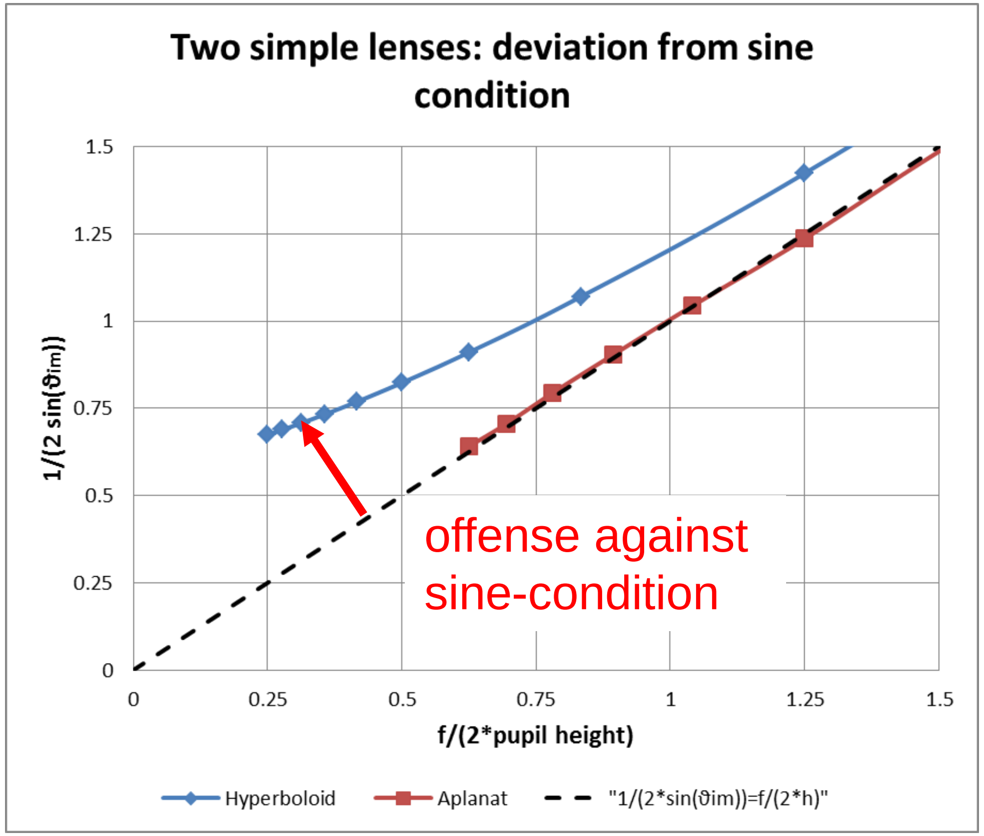


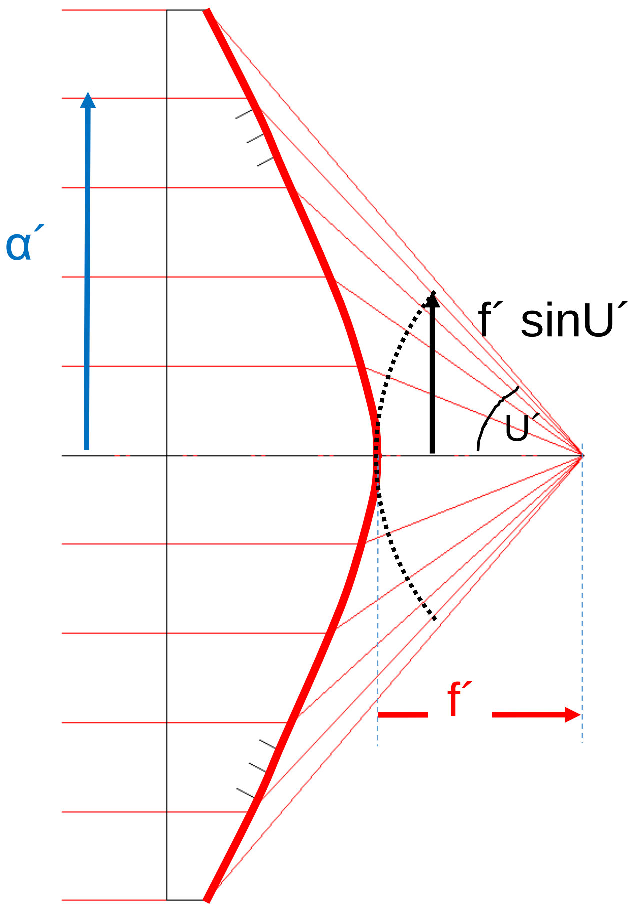
$$m \sin U' \neq \sin U$$

$$m \sin U' = \sin U$$



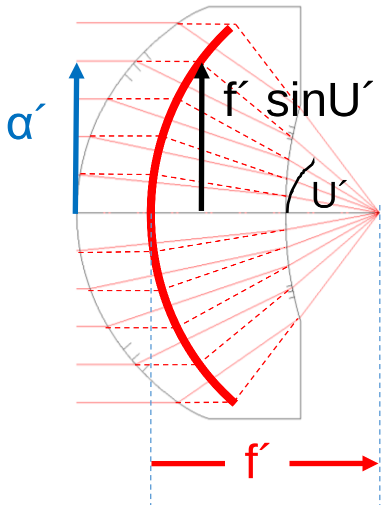
Such singlets are used in DVD-players („only one point image", but aplanatic design helps to reduce tolerances (tilt, lateral shift, …)).

Offense against sine condition and aplanatic lens



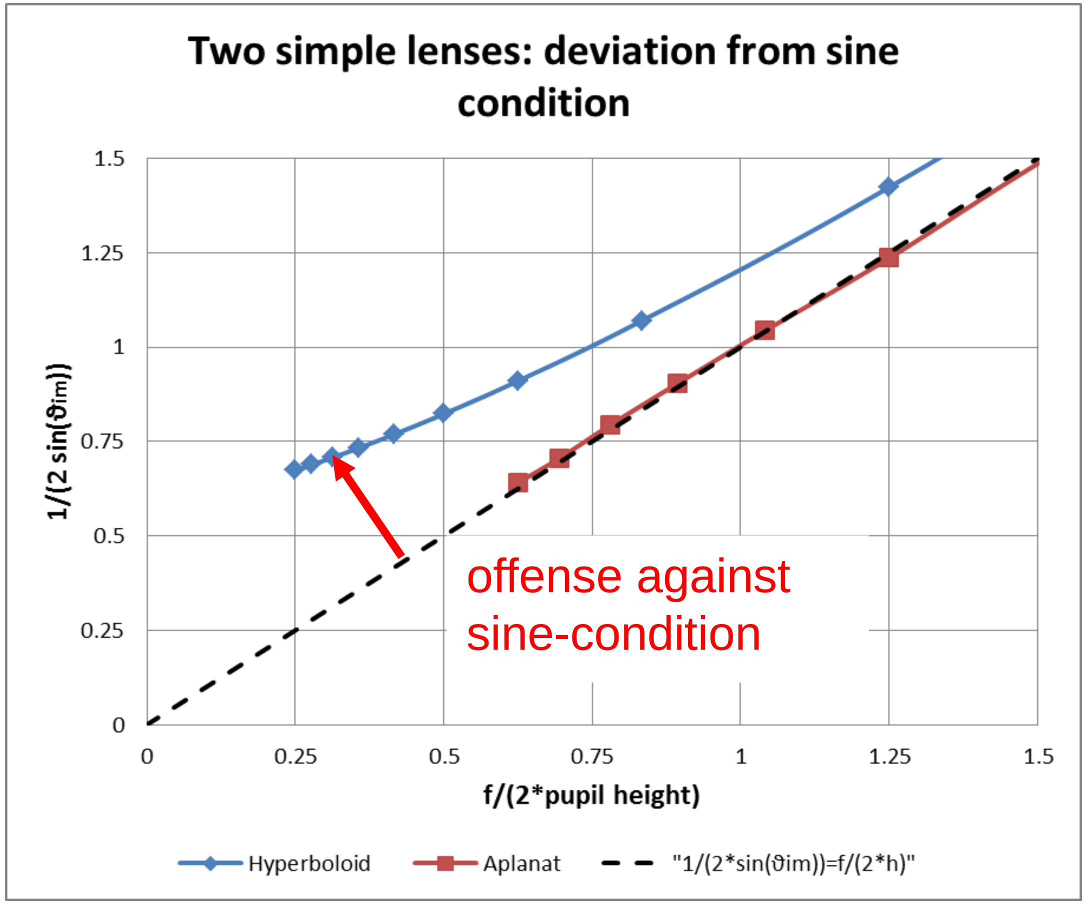
α´ ≠ f´ sinU´

Sine condition violated



α´ = f´ sinU´

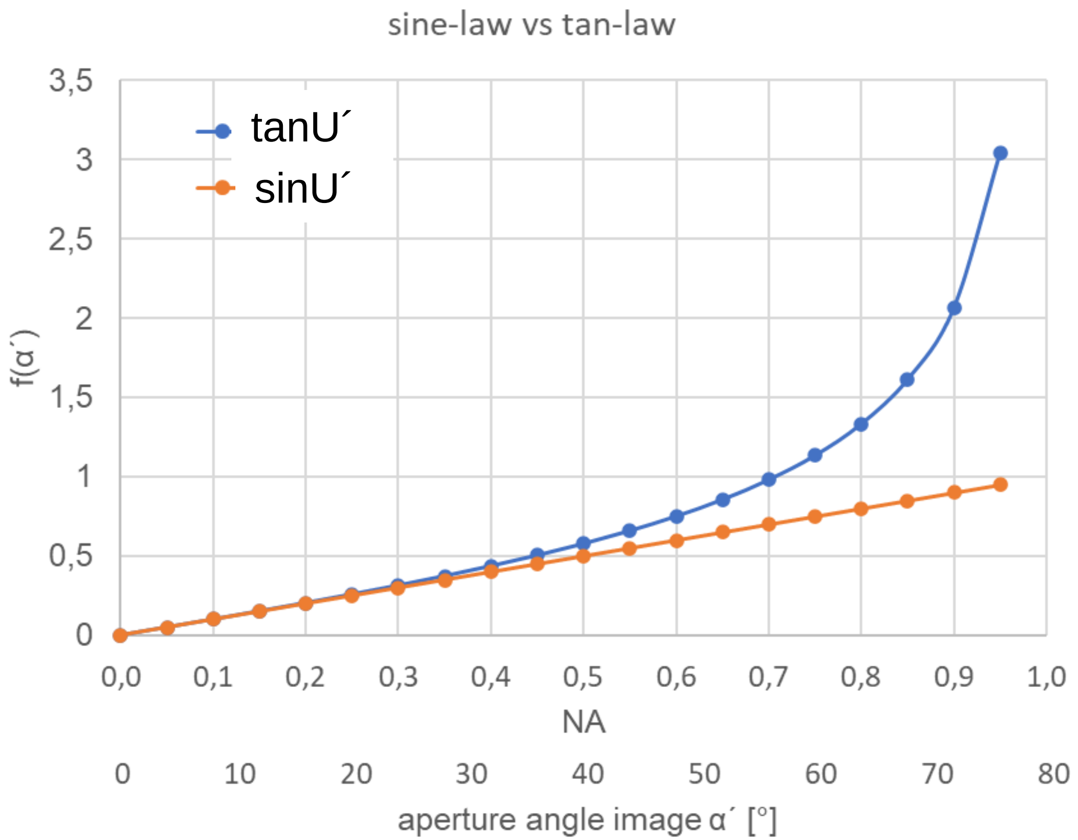
Sine condition fulfilled



Practical consequence of Sine Condition

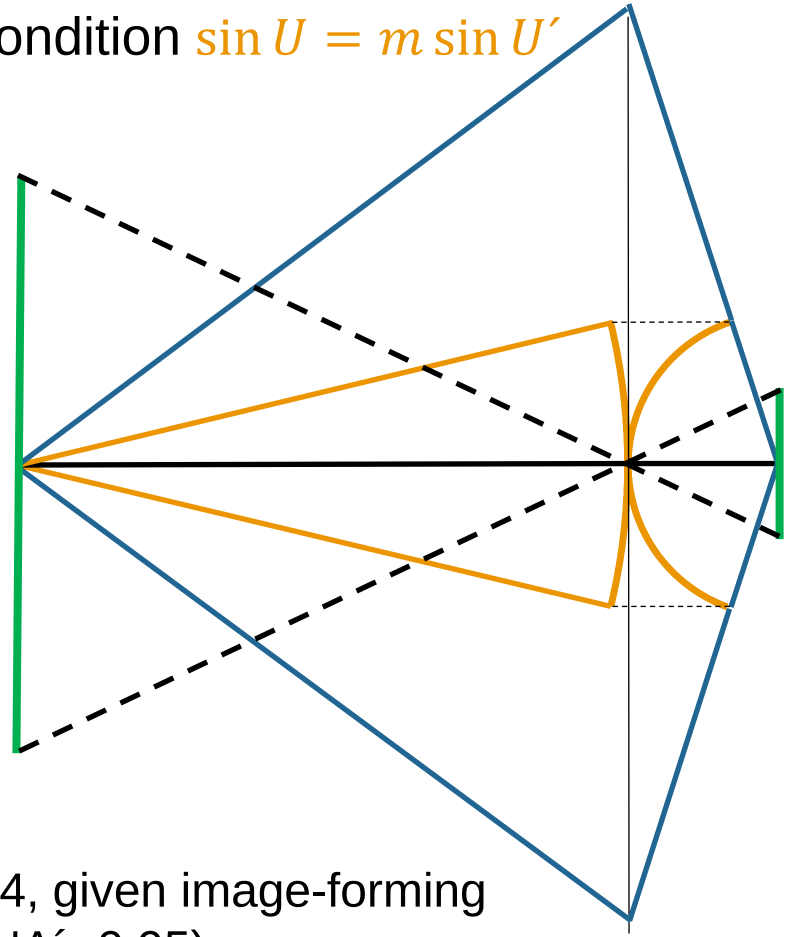
Scaling of lens size / diameters

sine vs tan



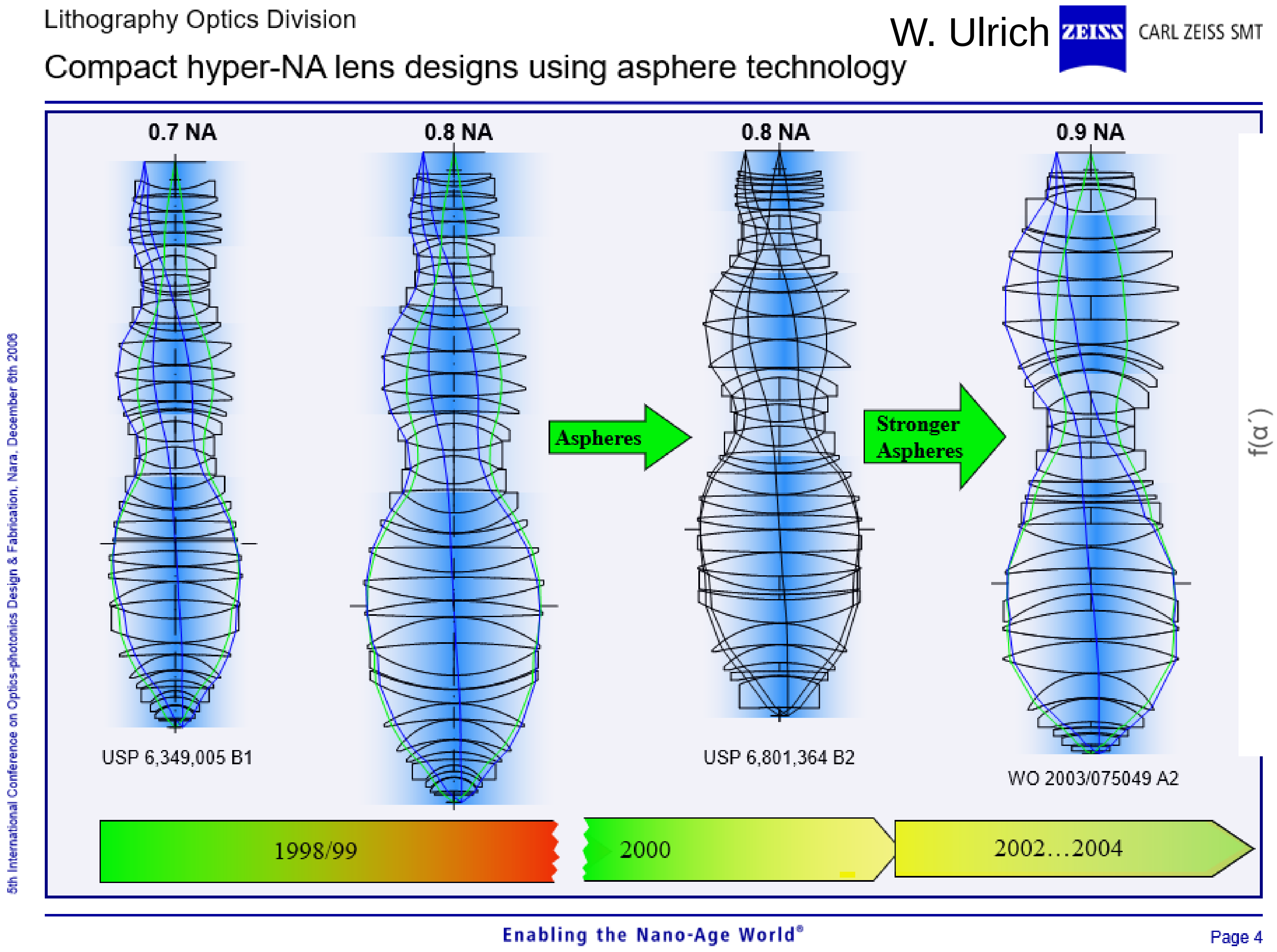
Schematics of system fulfilling

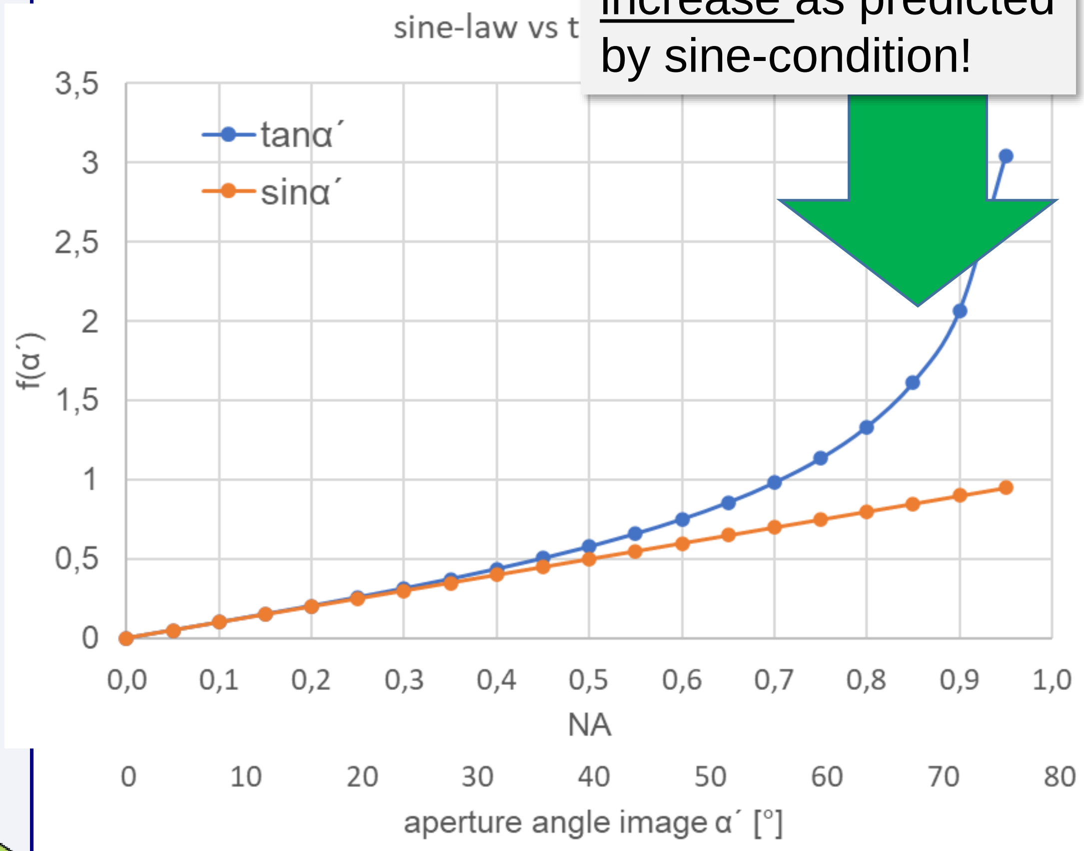
- “tangents-relation” $\tan U = m \tan U'$
- “Abbe sine-condition $\sin U = m \sin U'$



Example: m=-1/4, given image-forming angle: U´=72° (NA´=0.95)

Sine condition and lens diameters





Extreme high-NA: moderate diameter increase as predicted by sine-condition!

F-stops, irradiance and their scaling in camera lenses

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f-number (f/#) K_0:

$$K_0 = \left.\frac{f'}{Ø_{EP}}\right|_{m=0}$$

Camera lenses aperture definition refers to infinite object distance! (K_0 often increases towards close focus)

Working f-number and image-side numerical aperture (**NA´**) relation**:**

$$K = \frac{1}{2\mathrm{NA}'} := \frac{1}{2 sin u'_{\max}}$$

Relationship refers to <u>any focus distance</u>!
However, <u>K(s) depends on specific design</u>!

Irradiance in image plane:

$$E' \propto NA'^2 \propto \frac{1}{K^2}$$

Remember:
Etendue ∝ NA'^2 x (field area) is the relevant quantity for photographic exposure. For a given camera the field area defined by an image sensor is constant.

Due to this dependence the **f-stops** are scaled by √2 =1.4… (1.4, 2, 2.8, 4,…): With each f-stop the irradiance doubles.

Working f-number

f-number (refers to m=0 or s→∞ only!)

$$K_0 = \left.\frac{f'}{\text{Ø}_{EP}}\right|_{m=0}$$

working f-number (depends on magnification for a specific optical setup and focusing mechanism)

$$K(m) = \frac{1}{2\mathrm{NA}'(m)}$$

Imaging equation	$-\frac{1}{m_p s} + \frac{m_p}{s'} = \frac{1}{f'}$	I.E.
Imaging equation 2	$\frac{s'}{s} = m_p m$	I.E. 2
Sine Condition	$m \ \sin U' = \sin U$	S.C.

The working f-number relation can be written more explicitly:

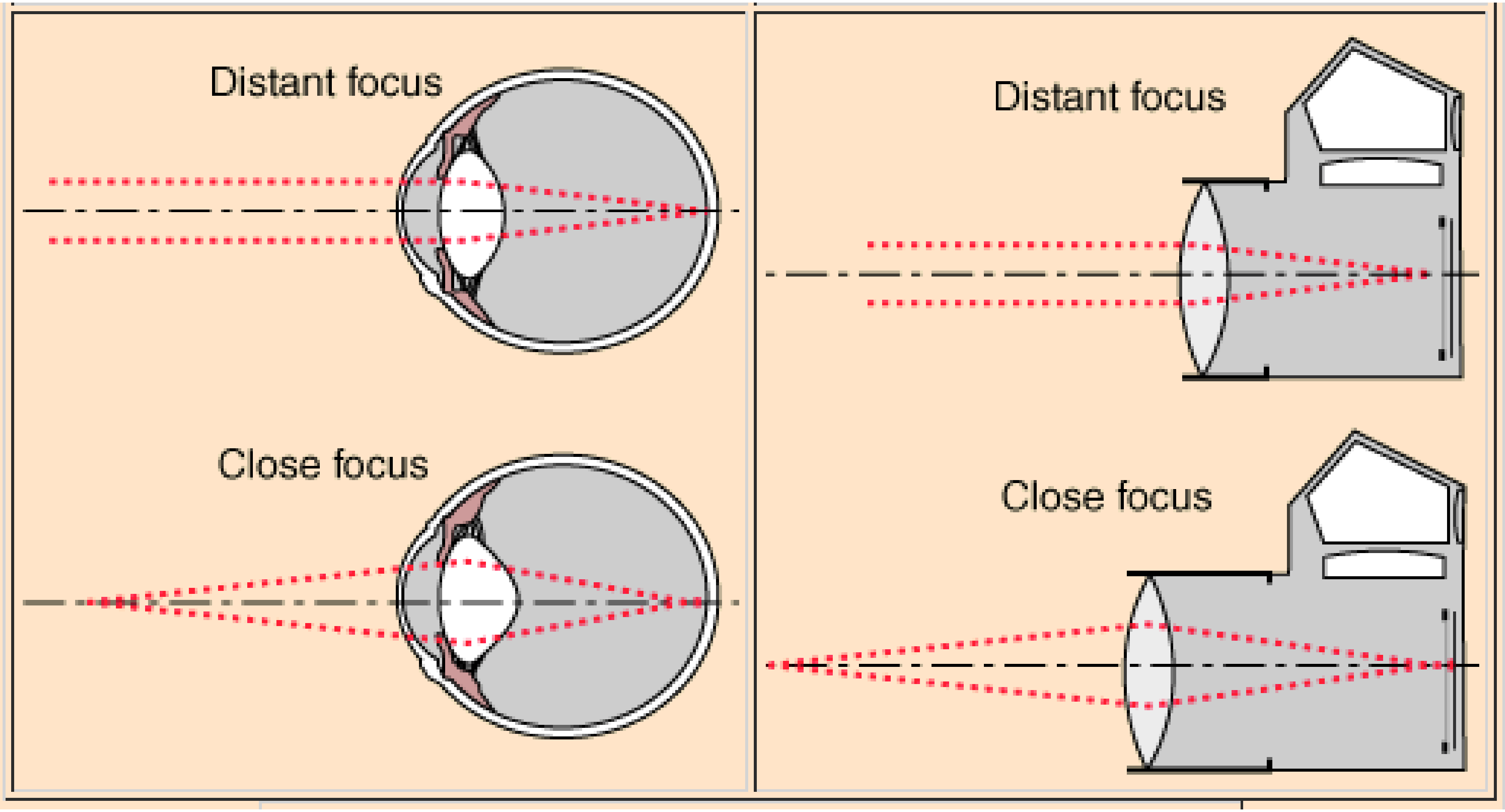
$$K = \frac{1}{2NA'} \overset{S.C.}{=} \frac{m}{2NA} = \frac{m}{\frac{\text{Ø}_{EP}}{s}} \overset{I.E.2}{=} \frac{s'}{m_p \text{Ø}_{EP}} \overset{I.E.}{=} \frac{(m_p - m)f'}{m_p \text{Ø}_{EP}} = \left(1 - \frac{m}{m_p}\right)\frac{f'}{\text{Ø}_{EP}}$$

Although this relation looks simpler, this is not really true: In general focal length f´, entrance pupil diameter Ø_{EP} and pupil magnification m_p all **depend via the specific focusing mechanism of the system and its optical power distribution on the magnification**!

$$K(m) = \left(1 - \frac{m}{m_p(m)}\right)\frac{f'(m)}{\text{Ø}_{EP}(m)}$$

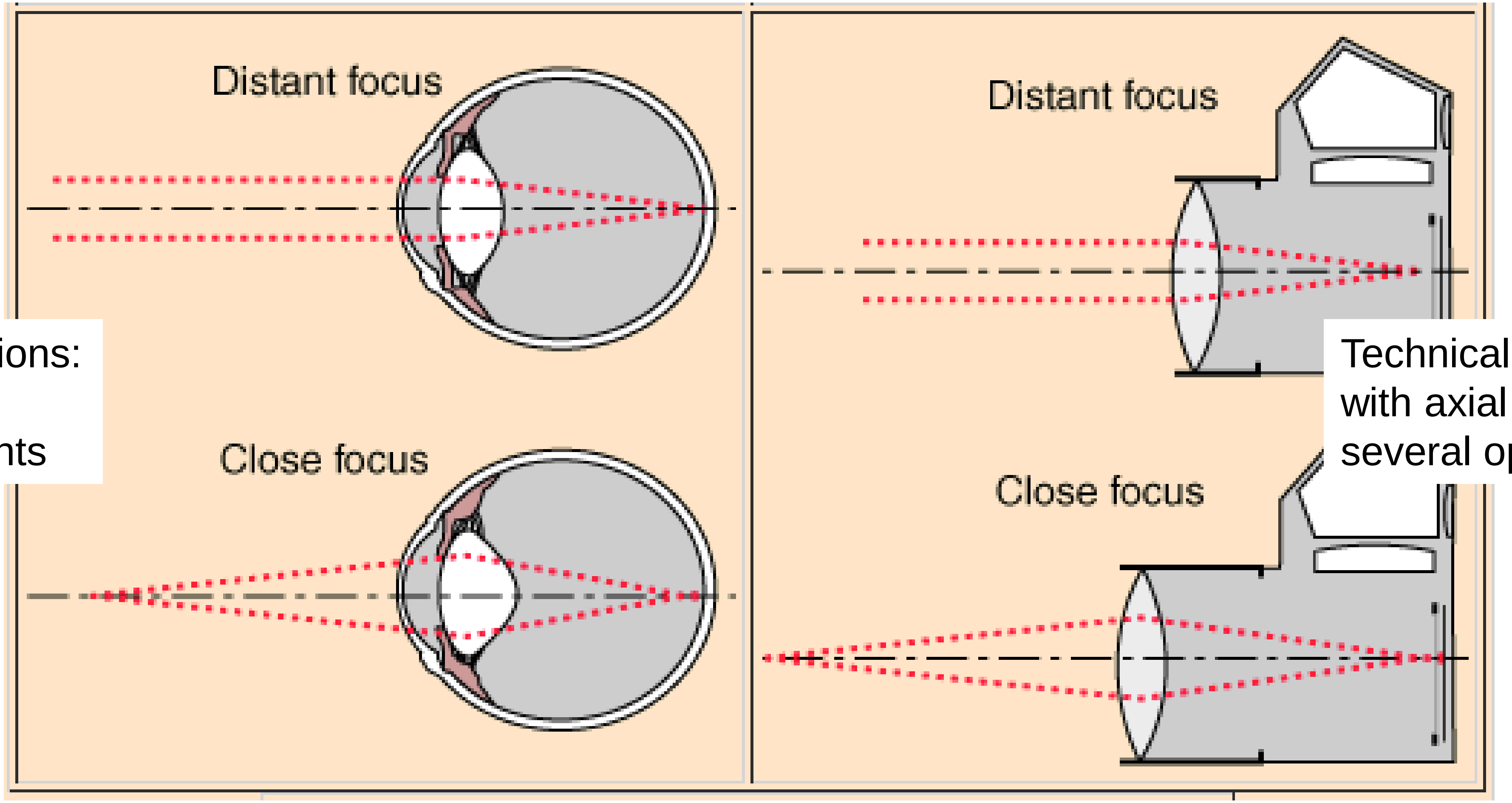
Focusing with human eye and camera

Variable refractive power ΔΦ | **Variable position Δs´**



Focusing with human eye and camera

Variable refractive power $\Delta\Phi$ | **Variable position $\Delta s'$**



Technical realizations:
- Liquid lenses
- Alvarez elements

Technical realization with axial shift of one or several optical groups

Axial shift principle by far more common in optical systems, however refractive power principle bears some interesting applications and technology evolution progresses.

Focusing by movement the complete lens

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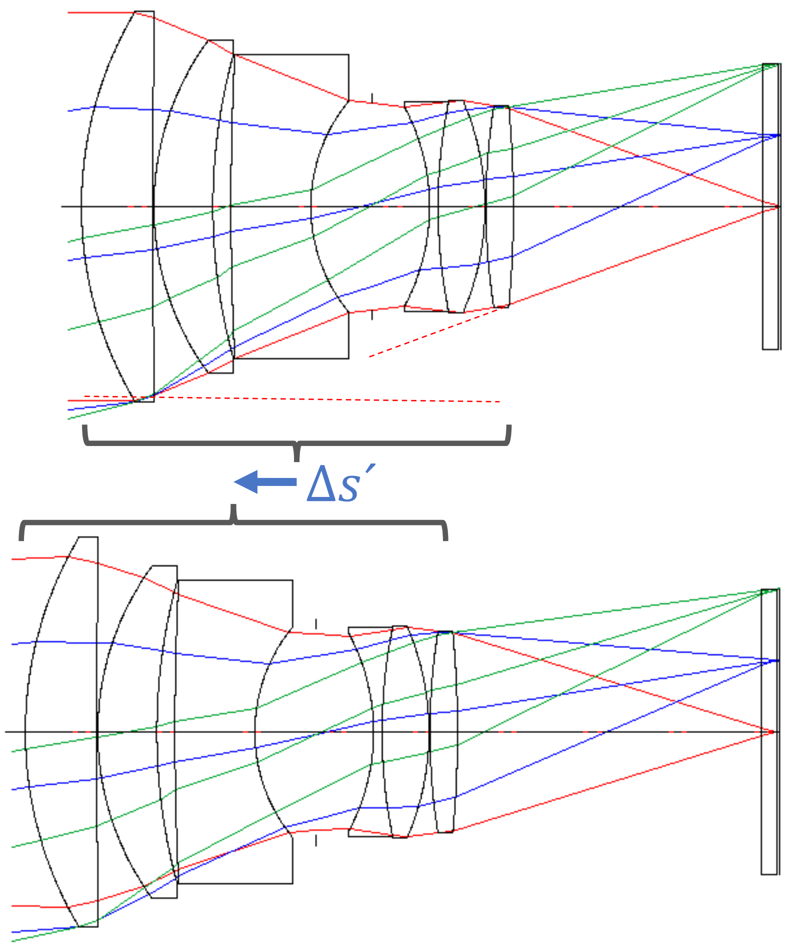
Large object distance



Small object distance

Inserting $\frac{s'}{s} = m_p m$ in $-\frac{1}{\beta_p s} + \frac{\beta_p}{s'} = \frac{1}{f'}$ and eliminating object distance s yields:

$$s' = (m_p - m) f'.$$

As for infinite object distance m=0, we have

$$s'(m=0) = s'_0 = m_p f'$$

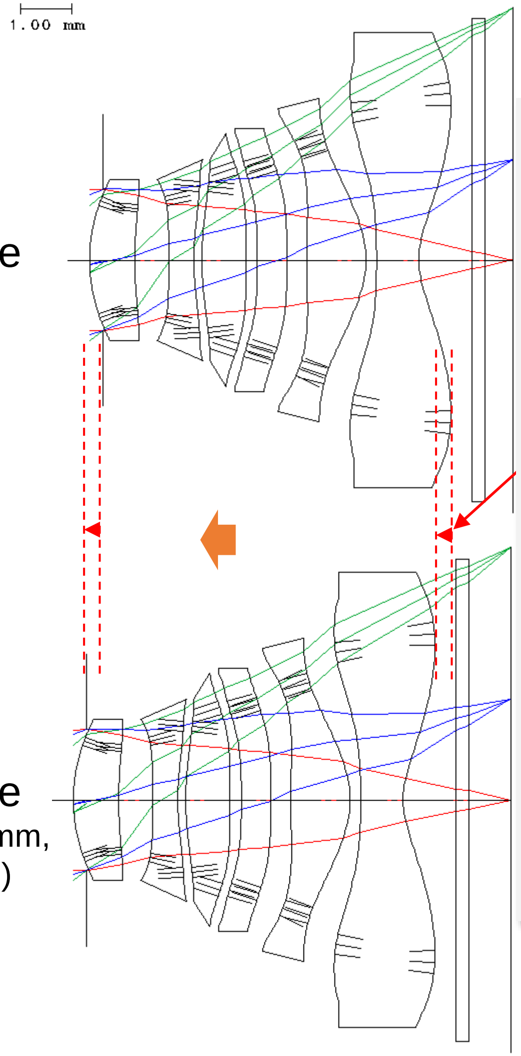
So the difference $\Delta s' = s'_0 - s'$ to a finite object distance s´ is

$$\boxed{\Delta s' = m\, f'}$$

A shift in the opposite direction by $-\Delta s'$ removes the defocus.

Focusing moving the complete lens: Smartphone Lens

Object distance infinity



Close distance (OO´=80mm, β_{Nah}=1:15)

Movement by
m f = 0.067 * 4.3mm = 0.28 mm

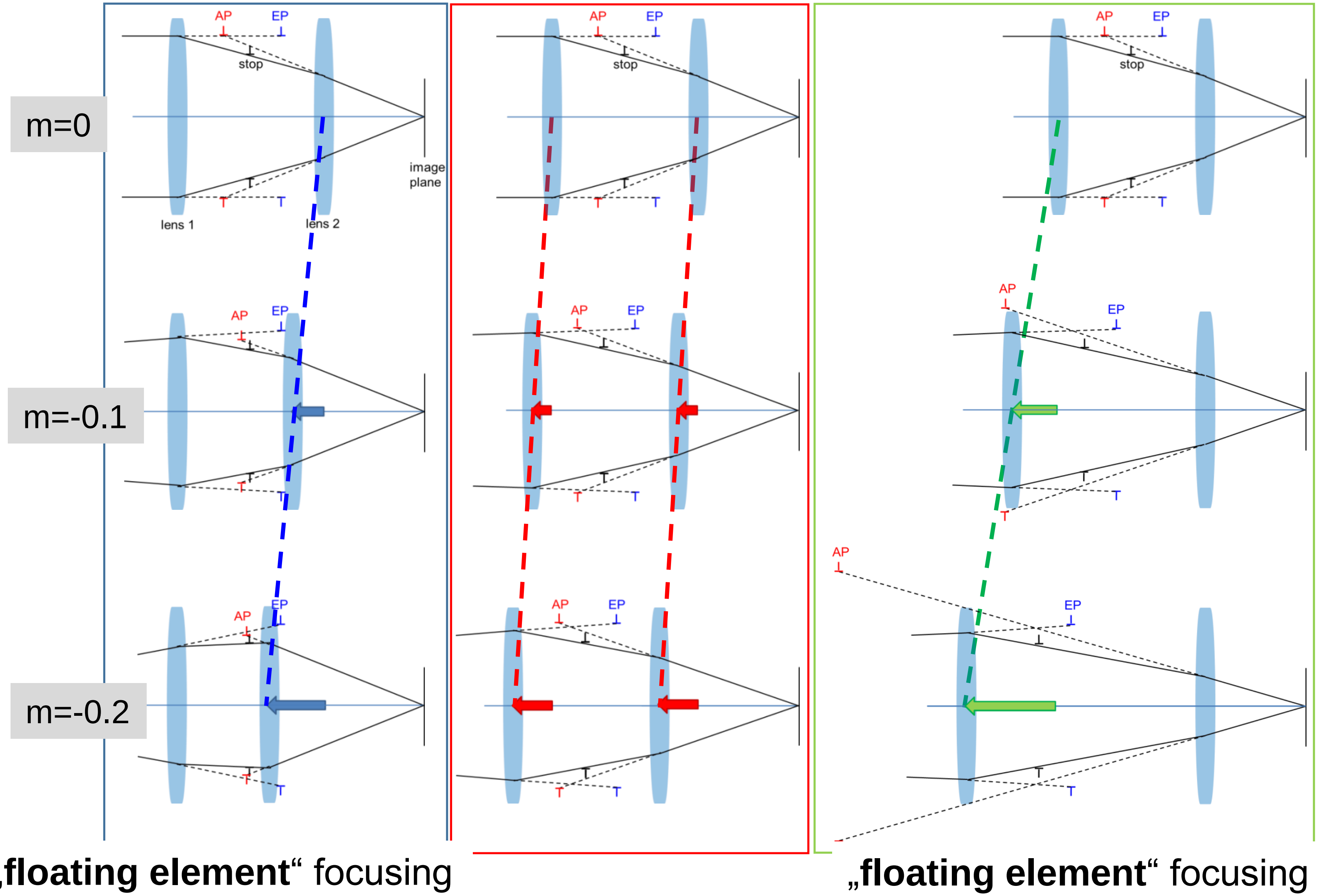
Object image distance OO´= inf
Magnification m = 0

The distance of lens movement is

Δs´ = m f

Minimum object image distance OO´= 80 mm
Magnification m = -0.067 = ca. 1/15

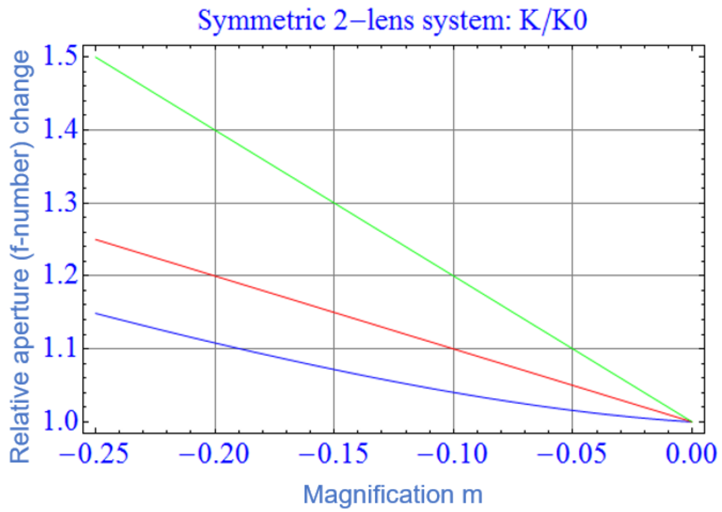
Two-lens element system with center stop: three focusing options



Focusing option with two system groups:

Differences in first order system data; following parameters vary differently with m:

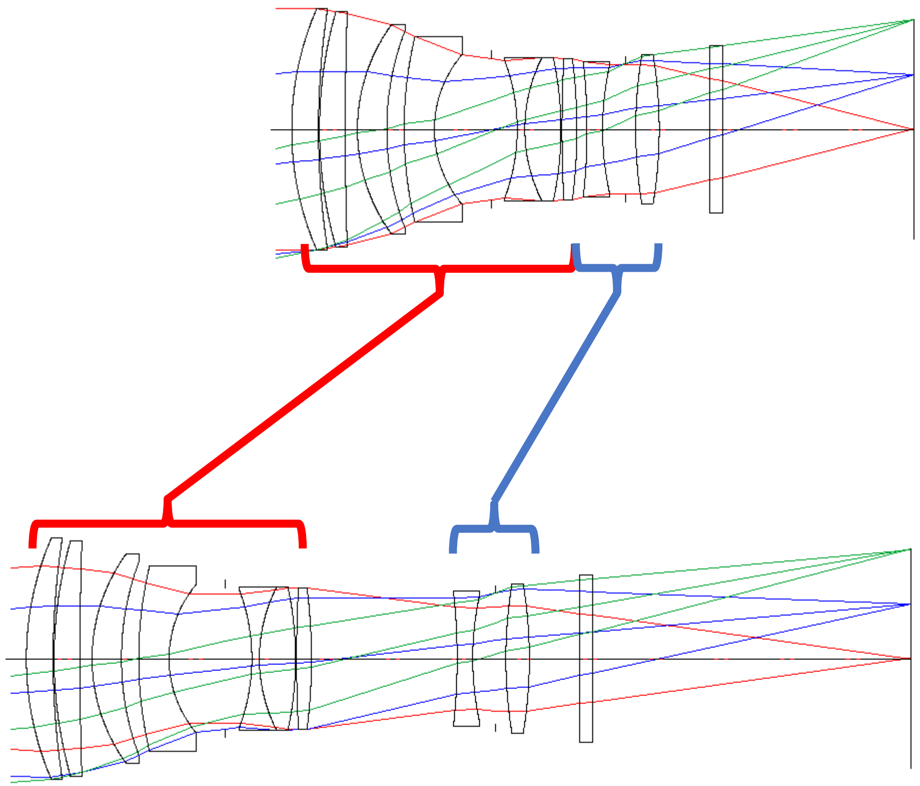
- EP, AP position
- system length
- f-number / NA´ (graph below)

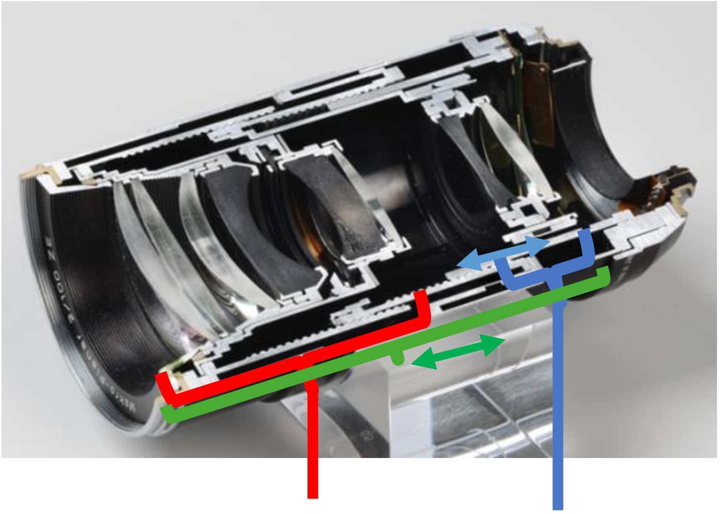


Example Photographic Macro Lens: Focusing with two “Floating Elements”*

Far distance:

Close distance:



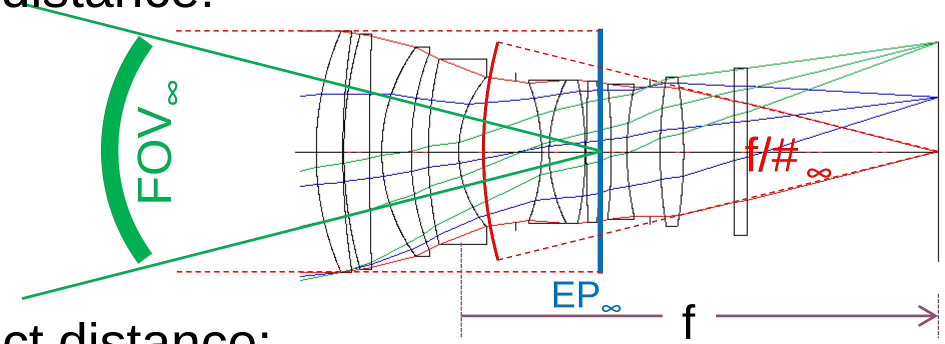


Moving lens groups coupled

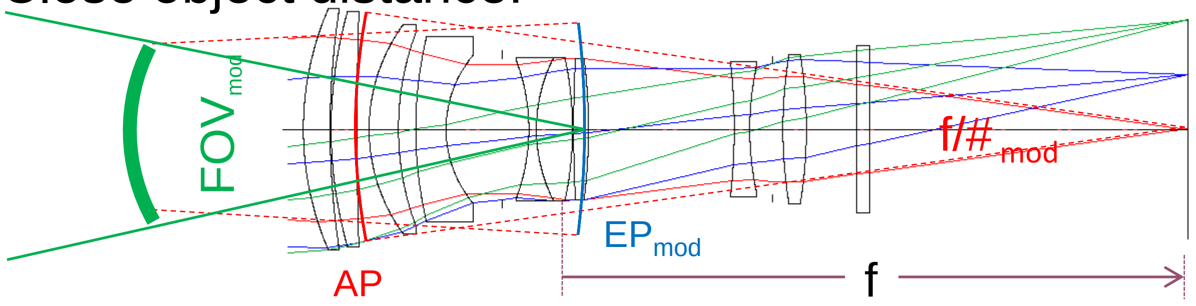
Instead of moving the entire lens forwards to focus on a nearby object, one or more lens groups are shifted. This enables better image performance over the entire distance range. The effort for the mechanical construction increases.

Dependence of FOV, f-stop and entrance pupil position on focus distance

Far object distance:



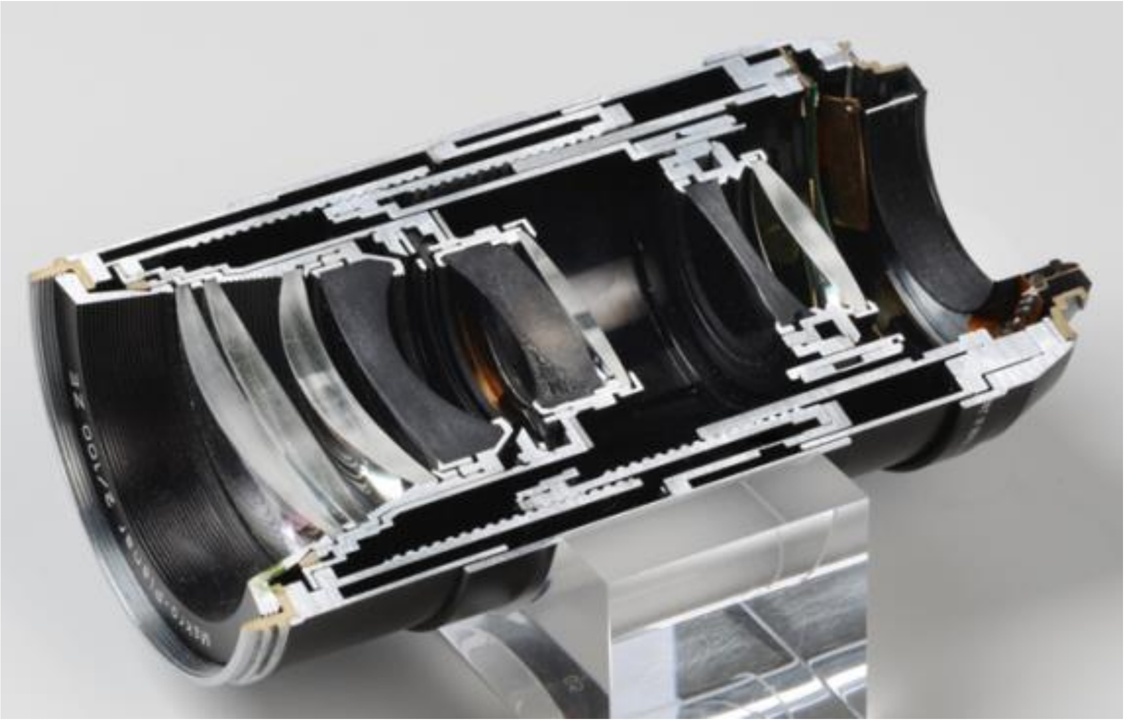
Close object distance:



lens magnification	f-#	entrance pupil position [mm]	diag. FOV [°]
0	2.05	-65.3	24.9
-0.5	3.64	-117.3	12.9

Real lens:

- FOV (and focal length f) varies with focusing distance
- perspective center = entrance pupil (≠ pinhole plane and varying with focusing distance)



Breathing: Changing Field-of-View with Focusing

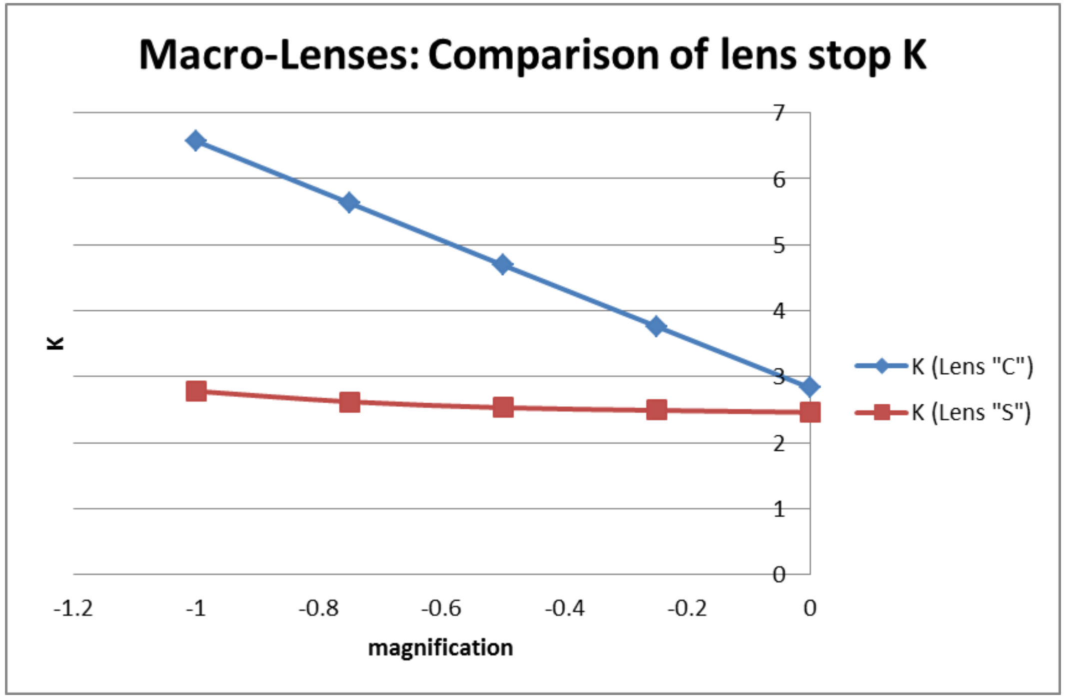
Interesting topic with relevant applications, e.g., photogrammetry or cinematography

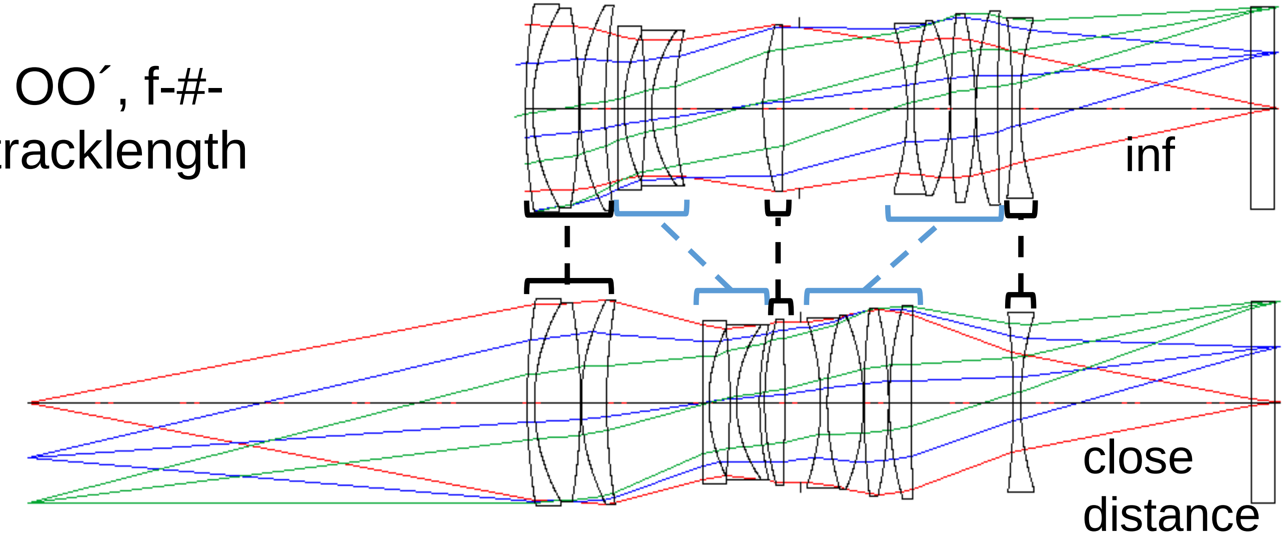
- Explore solution space with one or two moving groups;
- Systematic method to define focusing groups

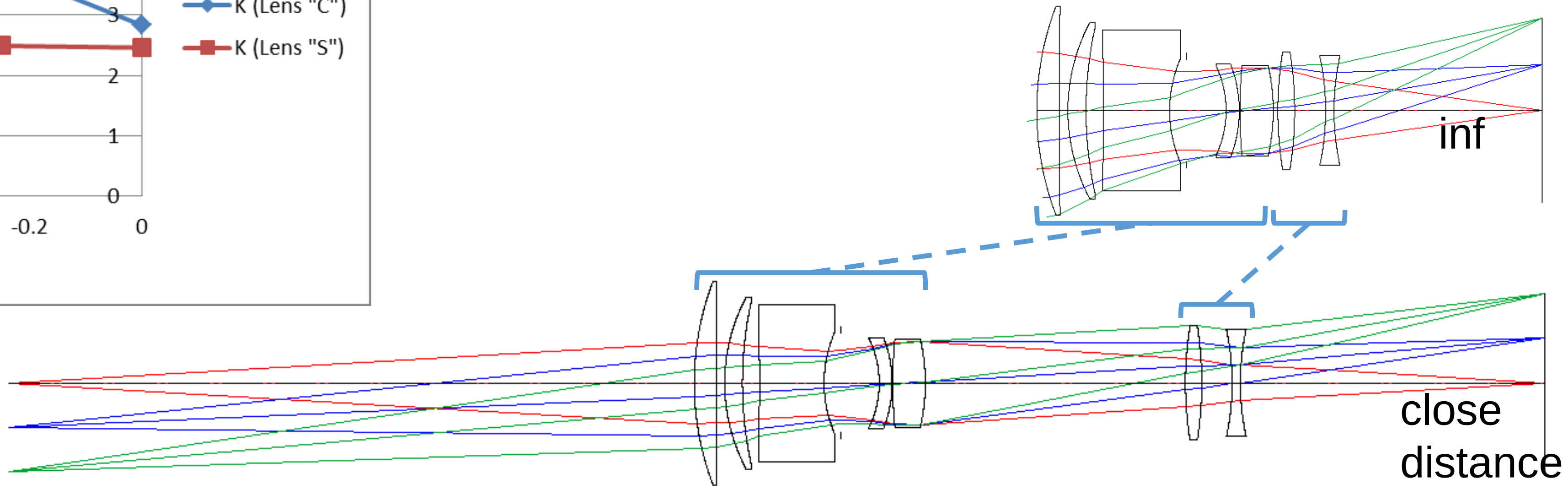
Goodsell, J., Blahnik, V., *"Method for Minimizing Lens Breathing with One Moving Group"*, In: Frontiers in Optics (2020); doi.org/10.1364/FIO.2020.FM1A.3

Alternative Macro Lens structures

Macro Lens structure (both 1:1 magn.):
Different focusing groups give raise to different OO´, f-#-variation, FOV-variation, constant vs. variable tracklength







Depth of field: Large vs small format camera

Same equivalent focal length (same field-of-view) and same f-number, but different image sensor sizes

smartphone

35mm format camera

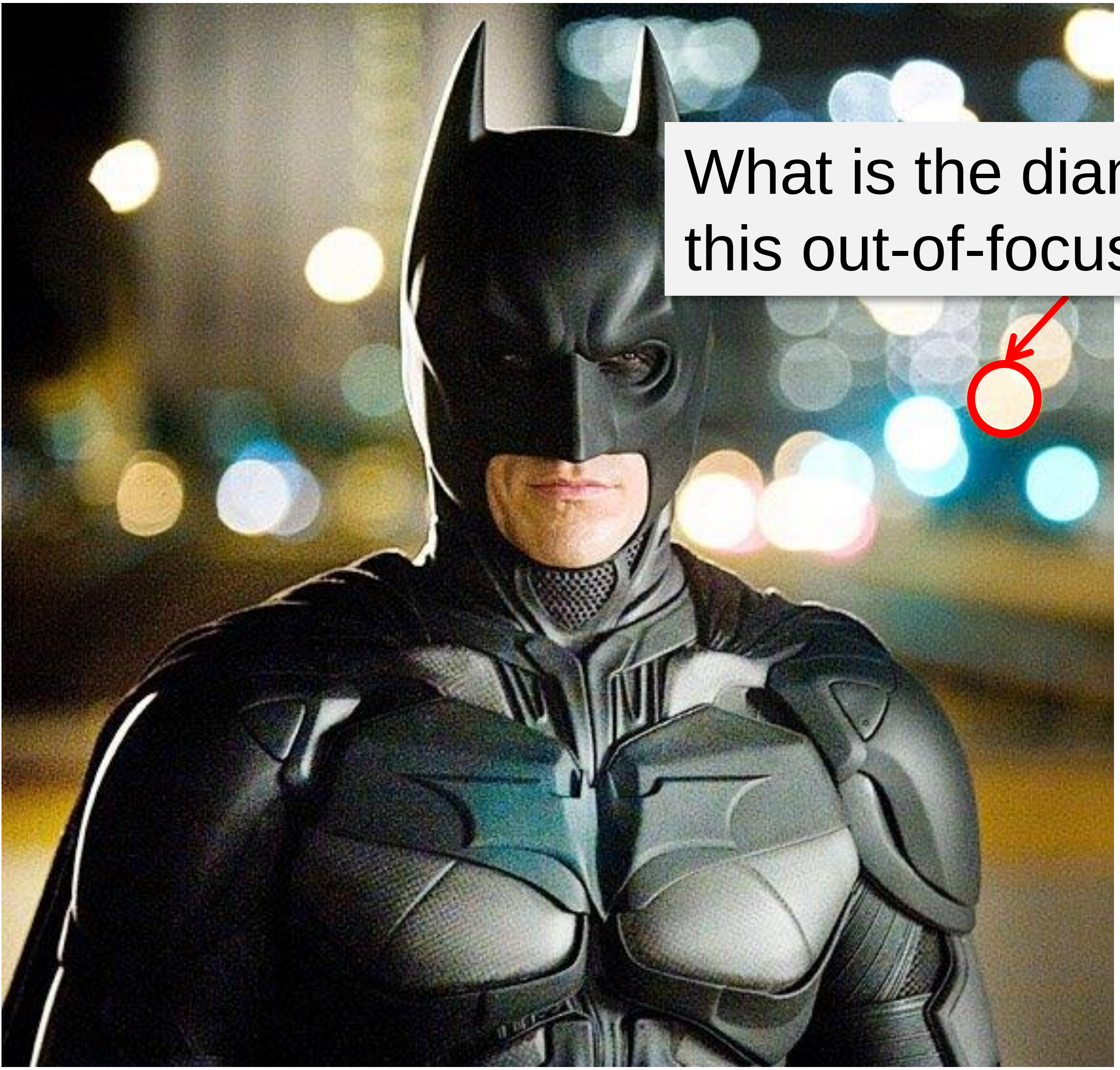
35mm-Format (36x24mm)

Smartphone image sensor Ø ca. 6mm

f = 4mm, diagonal field-of-view 75°, f/2.2 , image sensor Ø: 6mm

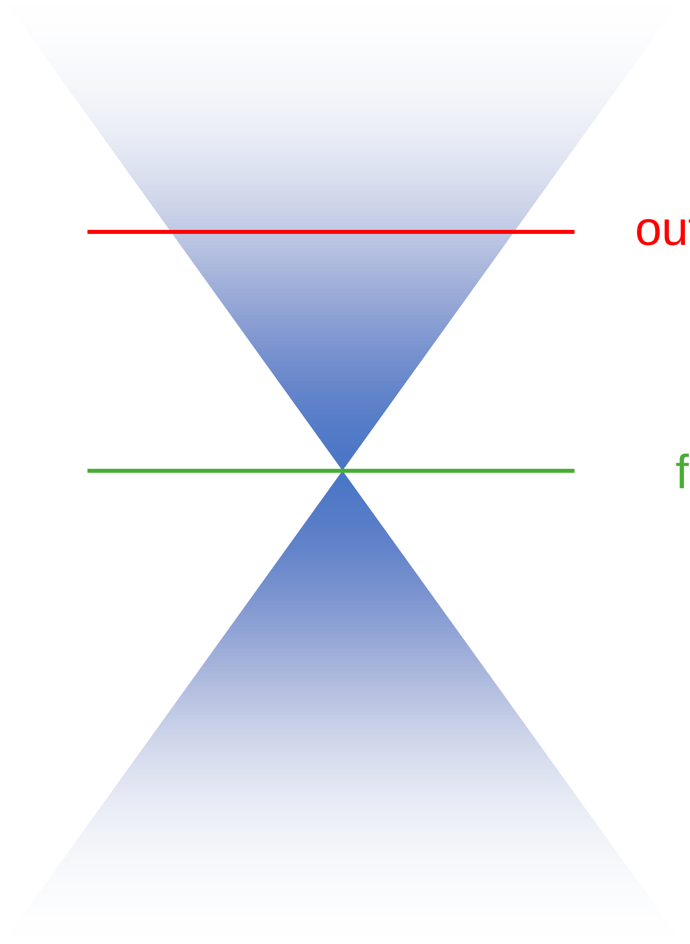
f = 28mm, diagonal field-of-view 75°, f/2.2 image sensor Ø: 43.3mm

Out-of-focus spot size



What is the diameter of this out-of-focus highlight?

Out-of-focus spot size



Out-of-focus spot size / Through-focus PSF (diffraction vs geometrical consideration)

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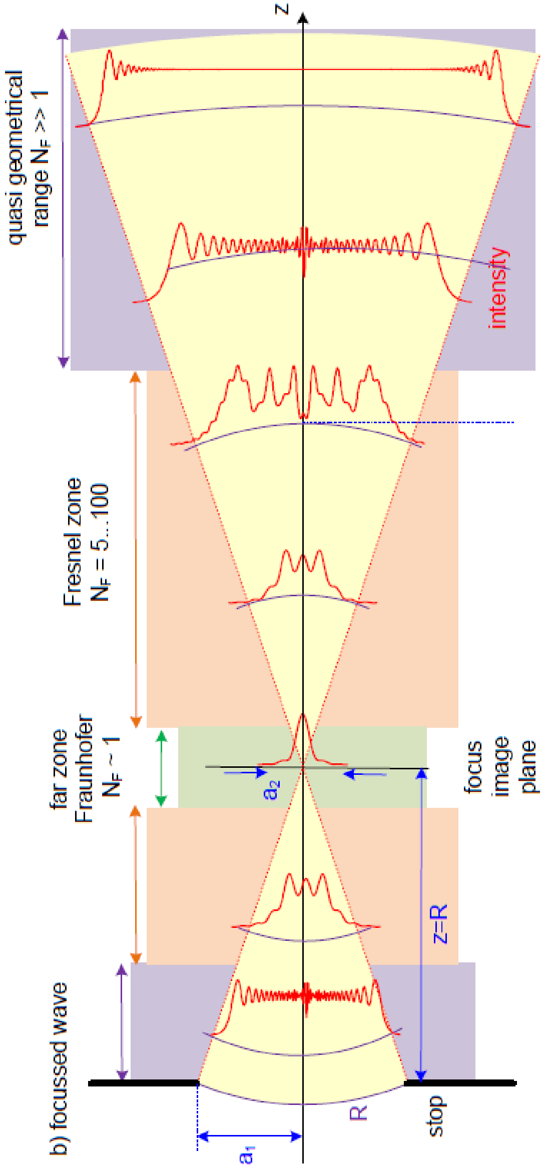
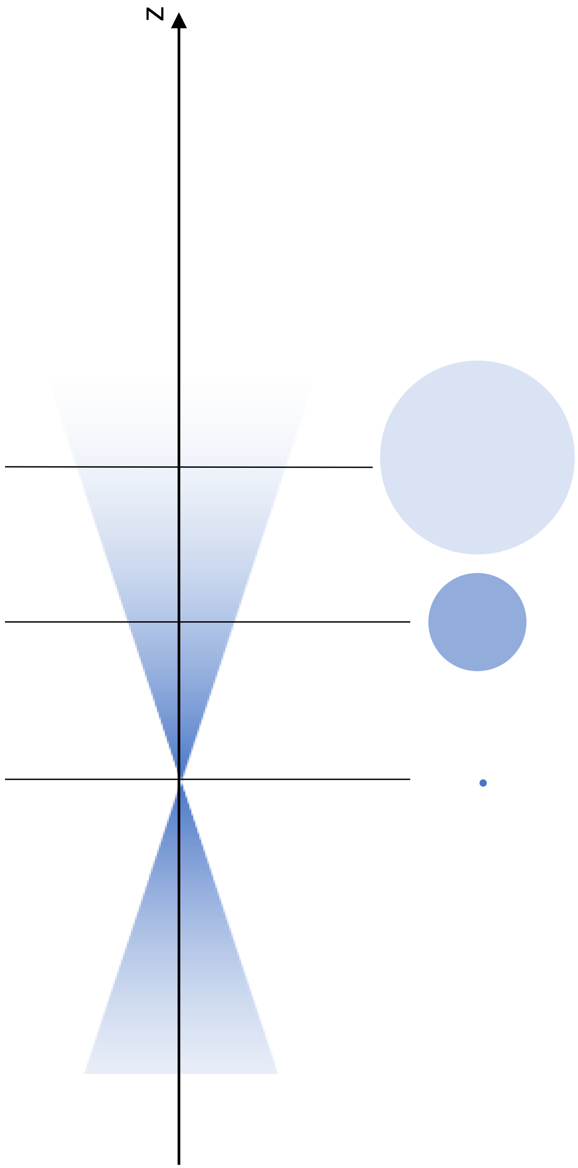
$$PSF(x,y,z) = \left| \iint d\alpha\, L_0(\alpha)\, exp\left(i2\pi W(\alpha)\right) exp\left(i2\pi \frac{z}{\lambda}\sqrt{1 - NA'^2\alpha^2}\right) exp\left(-i2\pi w\,(\alpha x + \beta y)\right) \right|^2$$

Diffraction

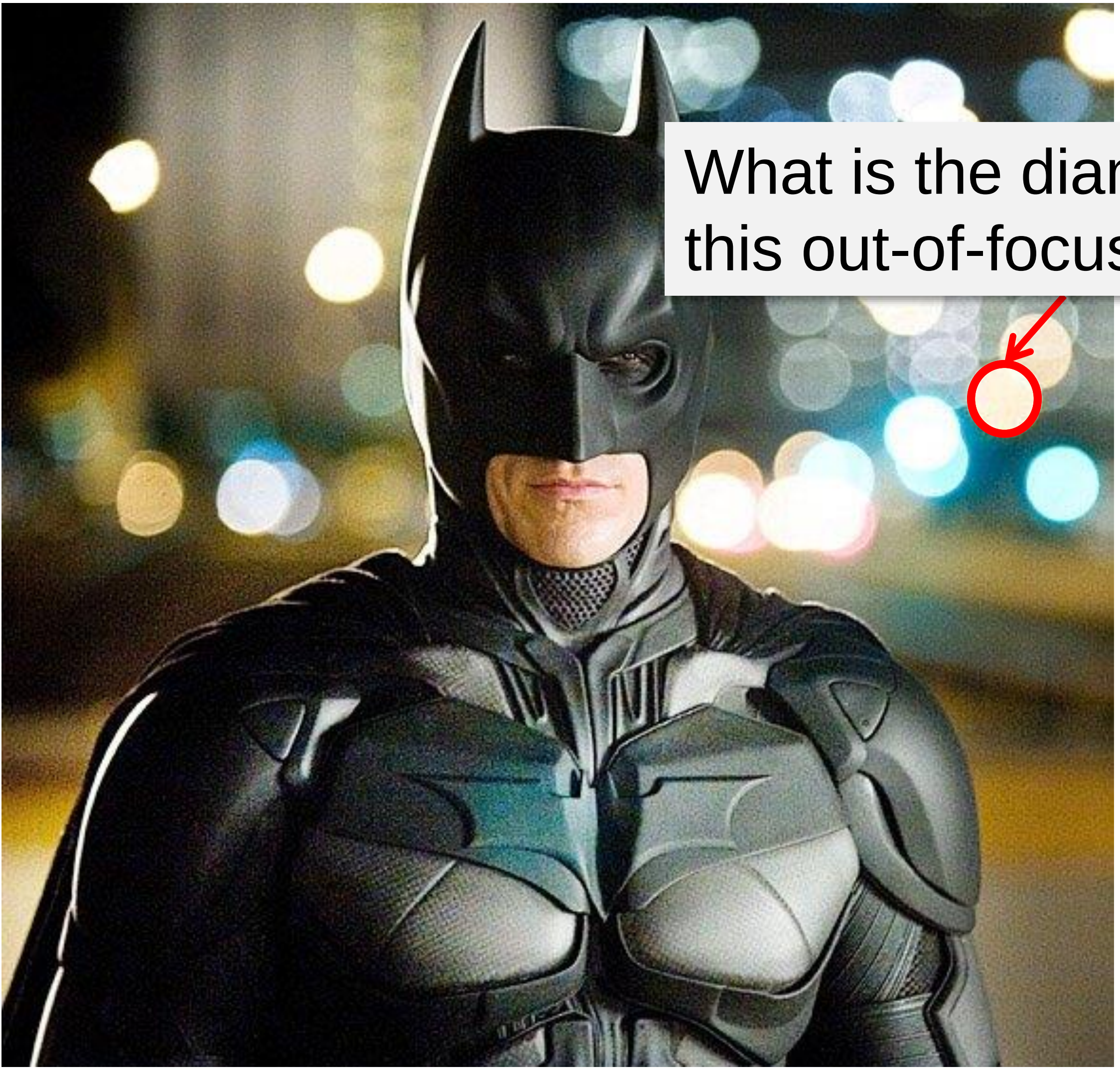




Homogeneous circular distributions

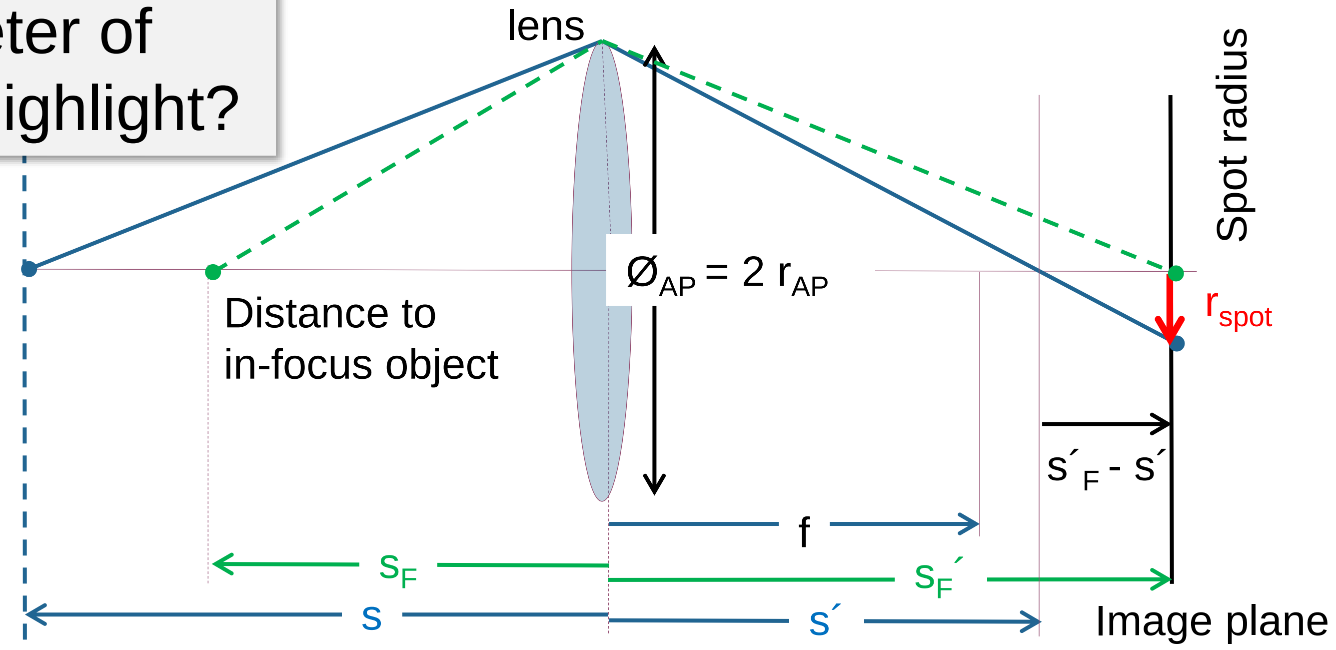
“Point” / delta-distribution δ(x)

Out-of-focus spot size



What is the diameter of this out-of-focus highlight?

Spot diameter $Ø_{spot}$ from this figure: applying similarity of triangles & lens makers equation $-\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$



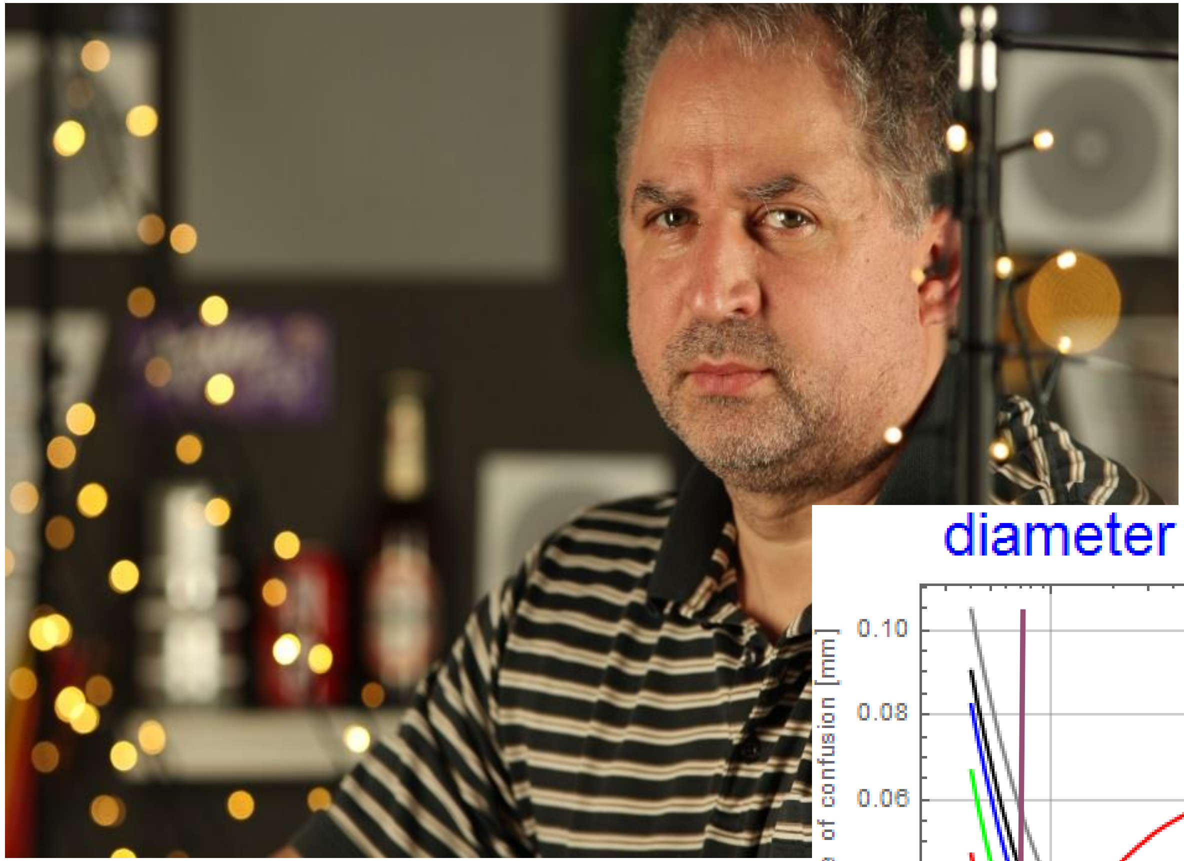
Focal length f
F-number K
Distance lens to in-focus object s_F
Distance lens to out-of-focus object s

$$Ø_{spot} = \frac{f^2}{K}\frac{s|s-s_F|}{(f+s)^2 s_F}$$

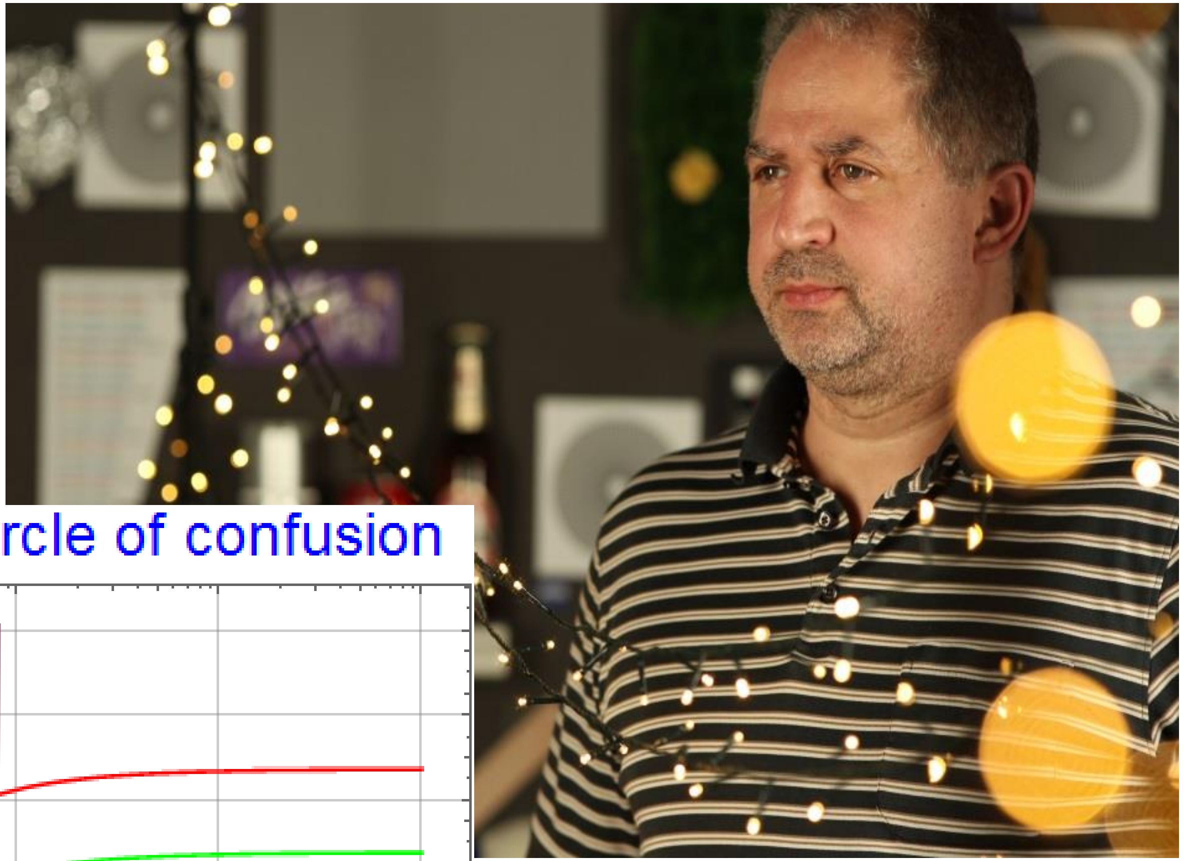
$$Ø_{spot} \approx \frac{f^2}{K}\frac{|s_F-s|}{s_F s}$$

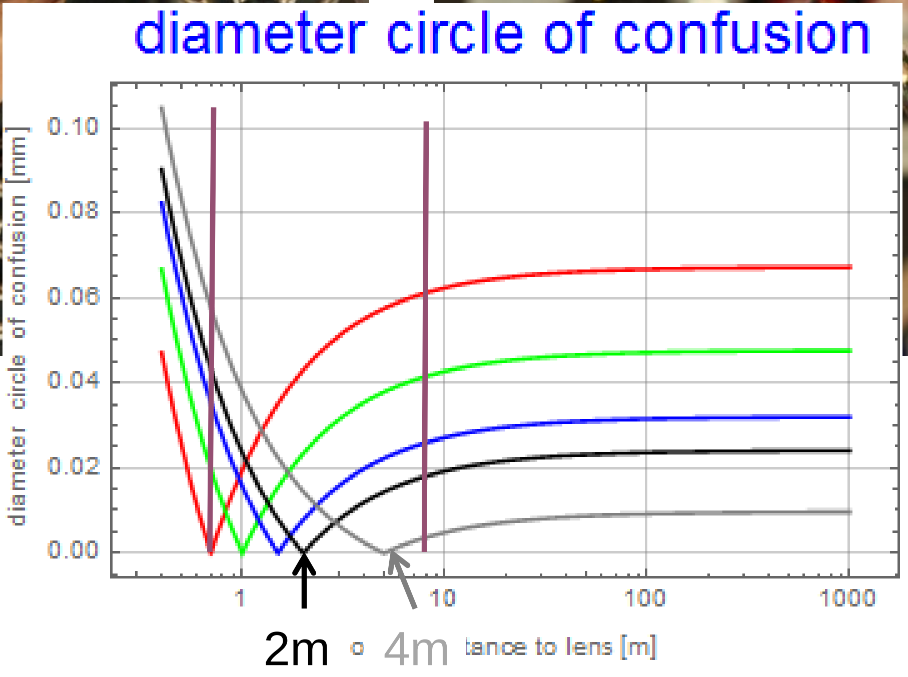
Circle of confusion diameter: Depth dependance

Object distance 2m



Object distance 4m





Pictures with same focal length and f-stop focusing at different distances.

$$\varnothing_{spot} \approx \frac{f^2}{K} \frac{|s_F - s|}{s_F s}$$

Threshold Sharpness (Circle of Confusion) Criterium and Depth-of-Field range

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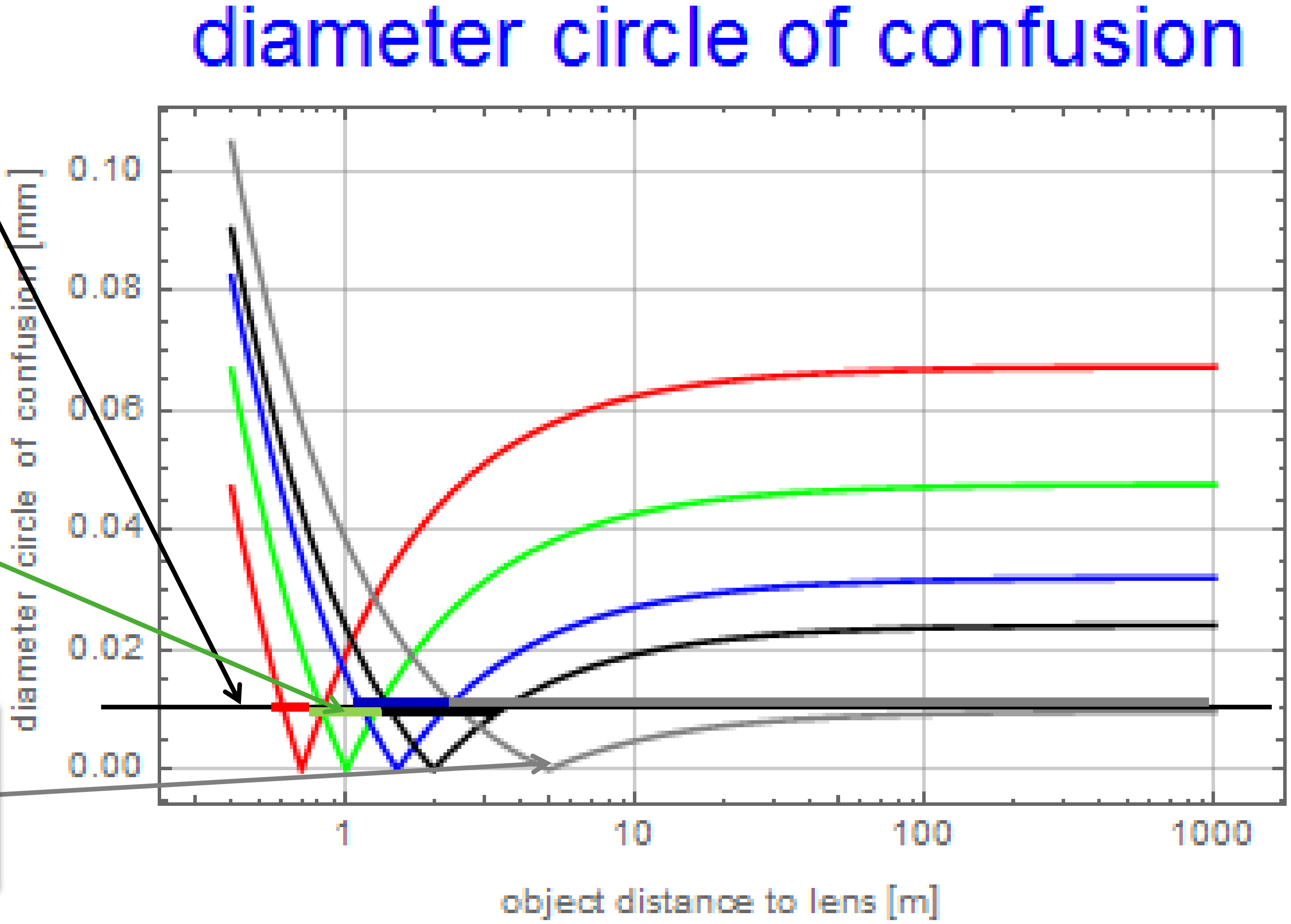
Threshold criterium scales with image sensor resolution („pixel criteria“)

Classic criterium in analogue film photography: 1/1500 of image diagonal.

Depth-of-field = depth range with „sharp image“

„Hyperfocal distance: „Focus distance, where complete background is sharp („up to infinite object distance“)



Depth-of-Field range

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With a threshold for the out-of-focus spot diameter, $Ø_{spot} = Ø_{thres}$, we have the „sharpness condition“:

$$Ø_{thres} \geq Ø_{spot} = \frac{f^2}{K} \frac{|s - s_F|}{(f+s)s_F}$$

Solving for s gives the „depth-of-field“ in object space:

$$Ø_{thres} = \frac{\pm(s - s_F)}{(f+s)s_F} \frac{f^2}{K} \quad \Rightarrow K(f+s)s_F\, Ø_{thres} = \pm(s - s_F)f^2$$

$$\Rightarrow s(K s_F\, Ø_{thres} \mp f^2) = \mp s_F f^2 - K f s_F\, Ø_{thres}$$

$$\Rightarrow s(K s_F\, Ø_{thres} \mp f^2) = \mp s_F f^2 - K f s_F\, Ø_{thres}$$

$$\pm s = f s_F \left(\frac{\pm f + K\, Ø_{thres}}{\pm f - K\, s_F\, Ø_{thres}} \right) \approx \frac{s_F}{1 \pm \frac{K\, Ø_{thres}}{f^2}(s_F - f)}$$

The last equation is most often used in literature and deviates very little from our result.

classical definition of analog film camera for 36 x 24mm image format is 1/1500 of image diagonal: $\frac{\sqrt{36^2+24^2}\,mm}{1500} = \frac{43.2mm}{1500} \approx 30\mu m$).

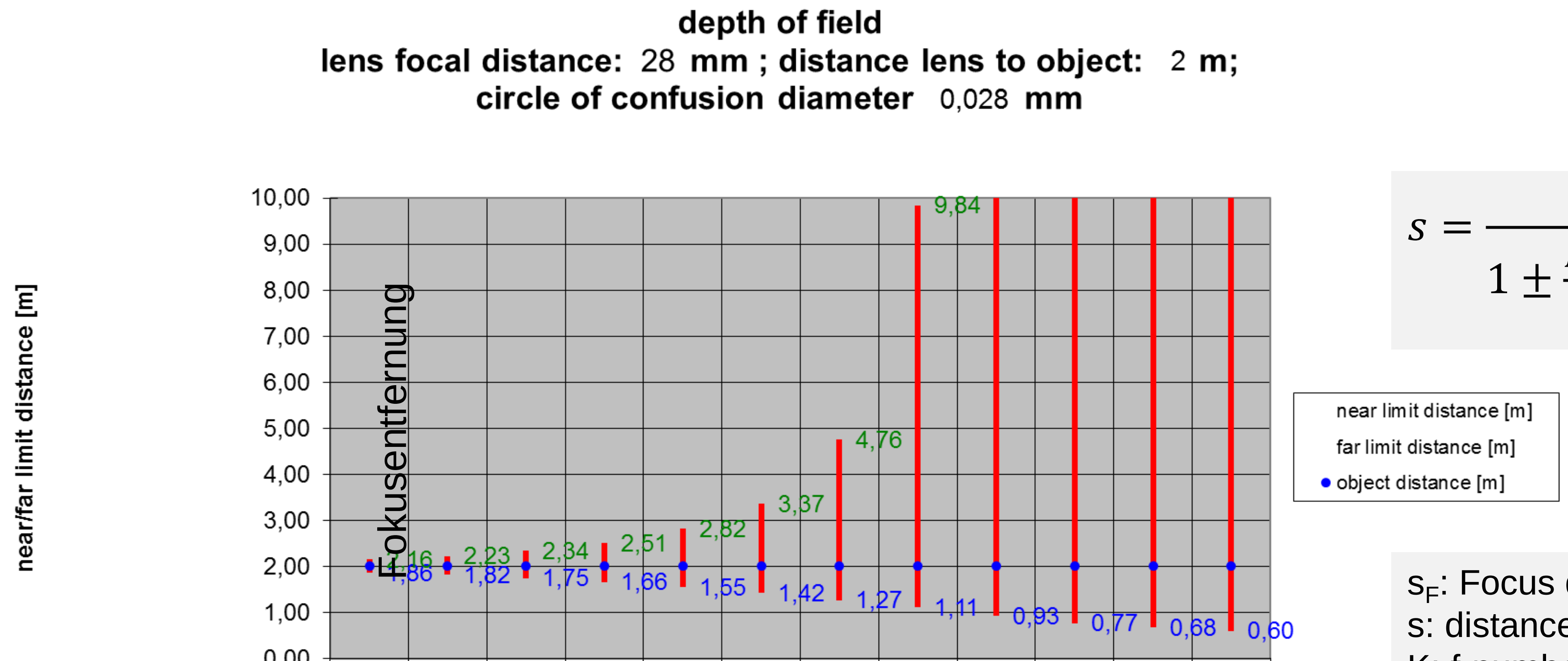
From close and far distance of depth-of-field range:

$$s_n = \frac{s_0}{1 + \frac{KØ_{thres}}{f^2}(s_0 - f)} \qquad s_f = \frac{s_0}{1 - \frac{KØ_{thres}}{f^2}(s_0 - f)}$$

We obtain for $s_F \to \infty$ the hyperfocal focusing distance:

$$s_{0,hyp} = \frac{f^2}{KØ_{thres}} + f$$

Depth-of-field calculator



	1	1,4	2	2,8	4	5,6	8	11	16	22	27	32
near limit distance [m]	1,86	1,82	1,75	1,66	1,55	1,42	1,27	1,11	0,93	0,77	0,68	0,60
far limit distance [m]	2,16	2,23	2,34	2,51	2,82	3,37	4,76	9,84	100,0	100,0	100,0	100,0
• object distance [m]	2,00	2,00	2,00	2,00	2,00	2,00	2,00	2,00	2,00	2,00	2,00	2,00

f#

Blendenzahl

$$s = \frac{s_F}{1 \pm \frac{K Ø_{thres}}{f^2}(s_F - f)}$$

s_F: Focus distance
s: distance where image is still sharp
K: f-number
f: focal length
$Ø_{thres}$: circle of confusion (e.g. (image diameter)/1500)

What is the relevant system parameter for background spot size?

$$\varnothing_{rel.spot,\infty} = \frac{\varnothing_{EP}}{\varnothing_{im}} m = \frac{\varnothing_{EP}}{\varnothing_{ob,Portrait}} = \frac{\varnothing_{EP}}{700mm}$$

Relative spot size of background bokeh only depends upon absolute size of the entrance pupil!

Portrait diameter of object field*:

$$\varnothing_{ob,Portrait} = 700\ mm = \frac{\varnothing_{im}}{m}$$

Let´s derive a practical equation which is independent on image format…

$\varnothing_{EP} = 1.5\text{mm}$

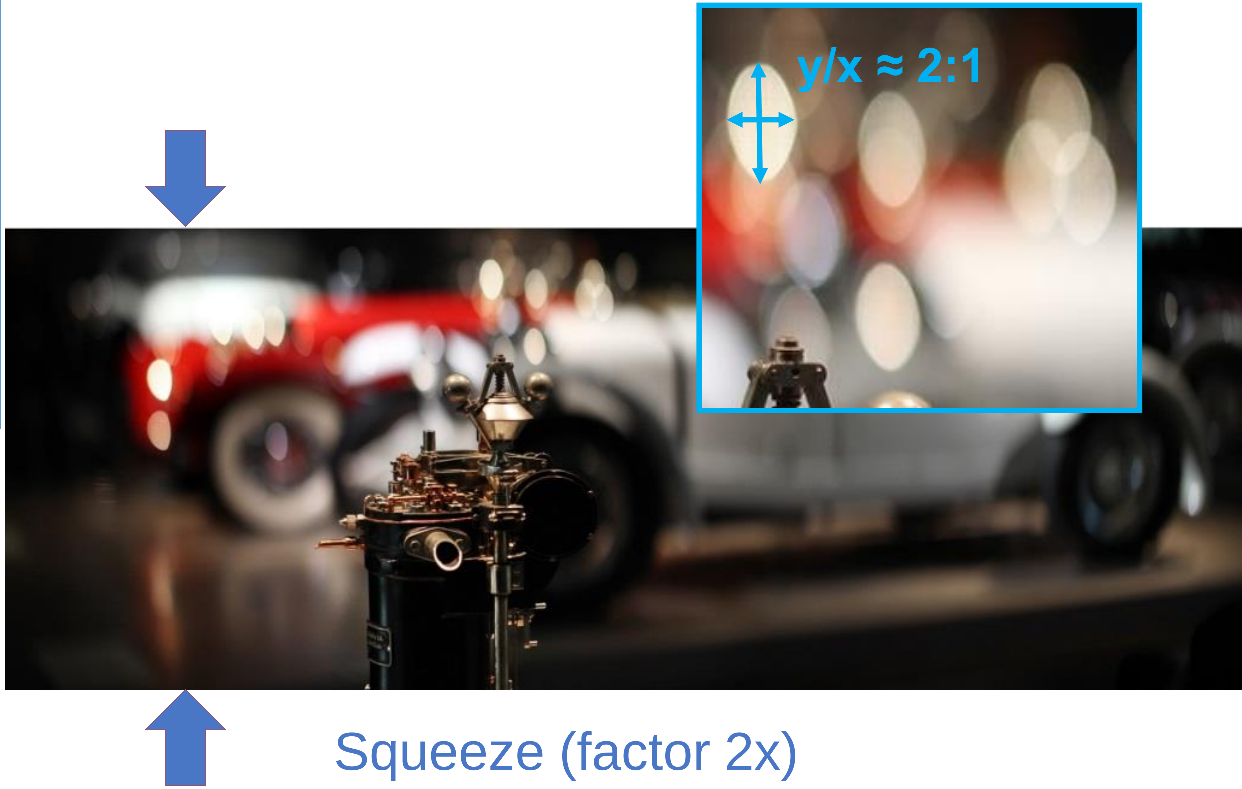
$\varnothing_{rel.spot,\infty} \approx$ **0.2%**

$\varnothing_{EP} = 150\text{mm}$

$\varnothing_{rel.spot,\infty} \approx$ **20%**

Anamorphic imaging in cinematography
Raw image and de-anamorph step

Andreas Bogenschütz, Carl Zeiss AG



Squeeze (factor 2x)

Implication: **Anamorphic lens** = different focal length horizontal and vertical

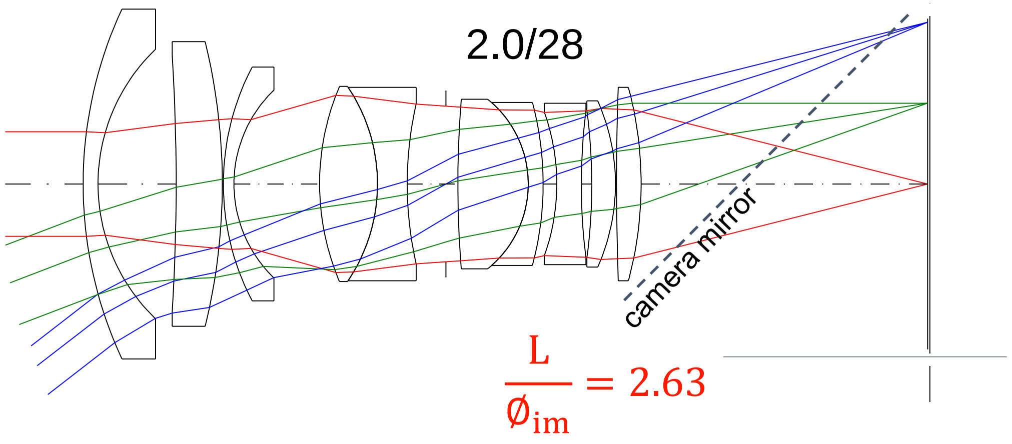
Out-of-focus spot is non-rotational symmetric, even after de-squeezing step! Follows from depth-of-field laws.

Structure of Optical Systems

Characterization of Optical System Structure via

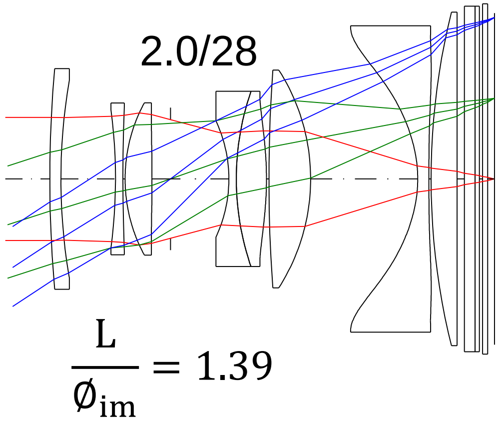
- Power distribution
- Pupil positions and pupil magnification (symmetry)
- …

Size & form factor comparison: Lens for DSLR versus view finder camera



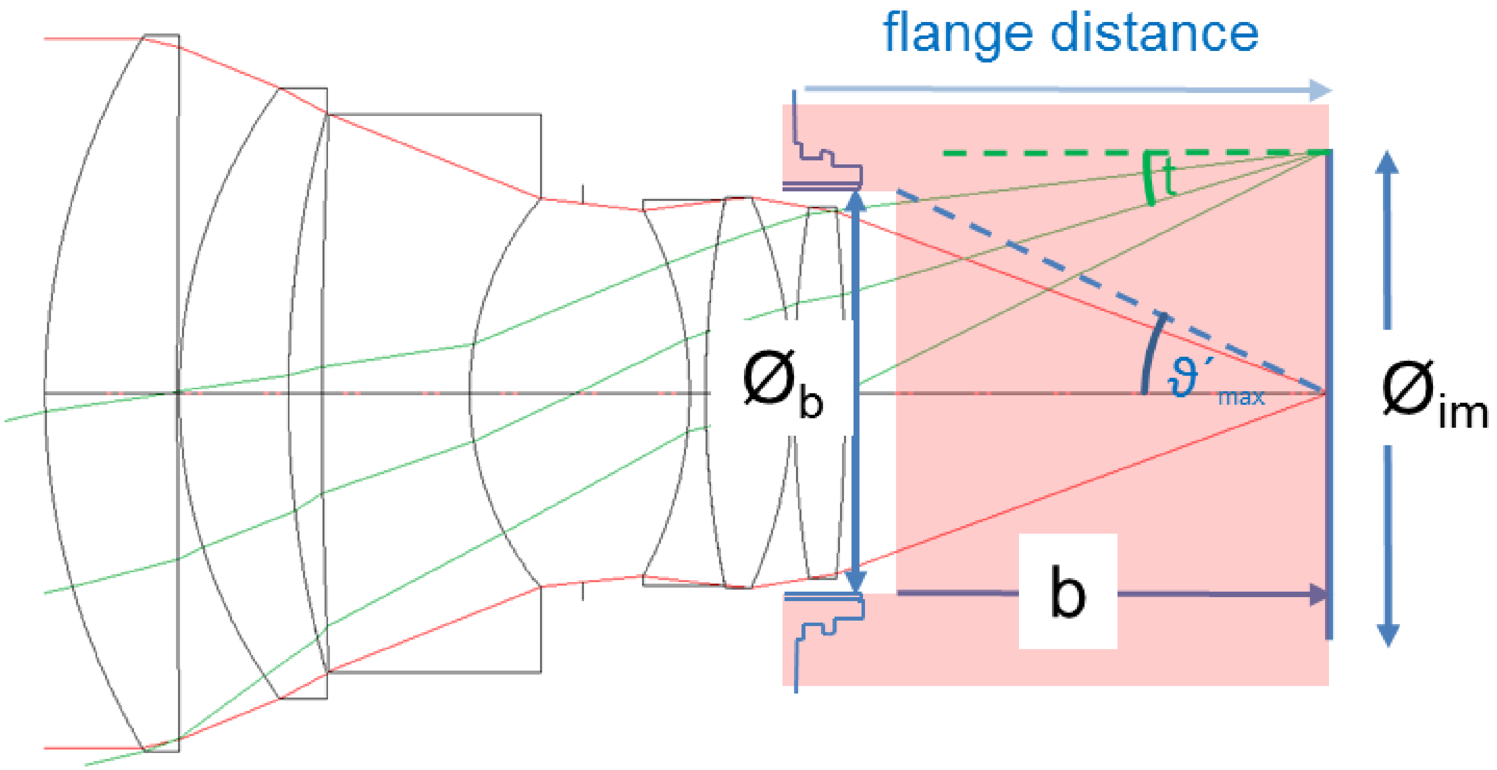
DSLR (digital single line reflex)

Very much different mechanical constraints!



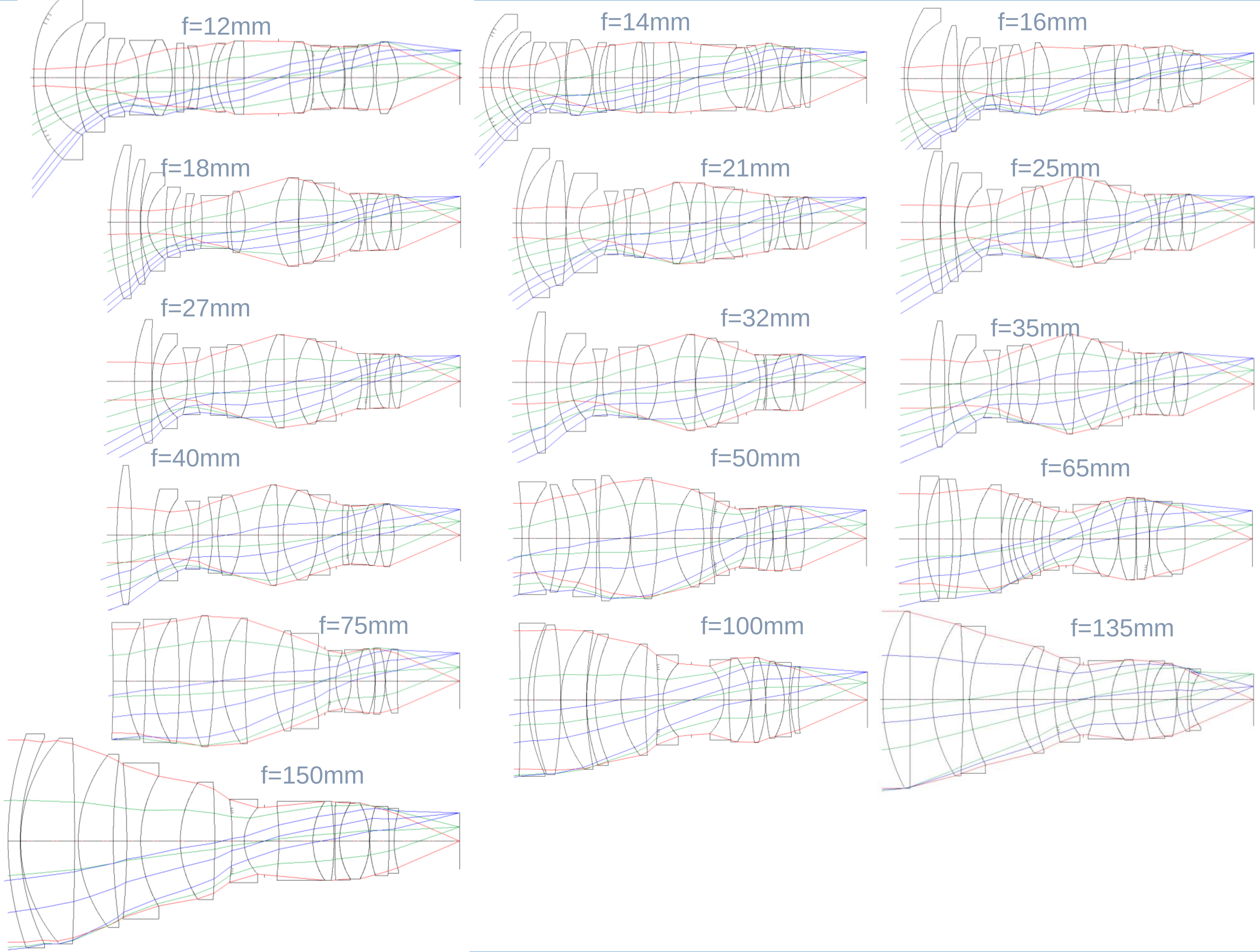
mirror-less system camera

Mechanical constraints and its impact on exit pupil position, max. aperture, vignetting



With Nikon's F-mount, for example, the image diagonal is $\sqrt{(24^2+36^2)}$ "mm"="43.2mm" , the minimum distance from the last lens to the image plane given by the camera mirror b > 38.4mm and the diameter of the inner ring on the bayonet $Ø_b$="ca. 36mm" (limits the diameter of the last lenses).

With this geometry, the f-number K cannot be less than approx. 1.2. There is necessarily a chief ray angle of more than approx. 14°. This corresponds to a minimum distance of the exit pupil from the image plane of about 85mm. The minimum telecentricity value also depends on the vignetting that is allowed at the edge of the field.


f=12mm
f=14mm
f=16mm
f=18mm
f=21mm
f=25mm
f=27mm
f=32mm
f=35mm
f=40mm
f=50mm
f=65mm
f=75mm
f=100mm
f=135mm
f=150mm

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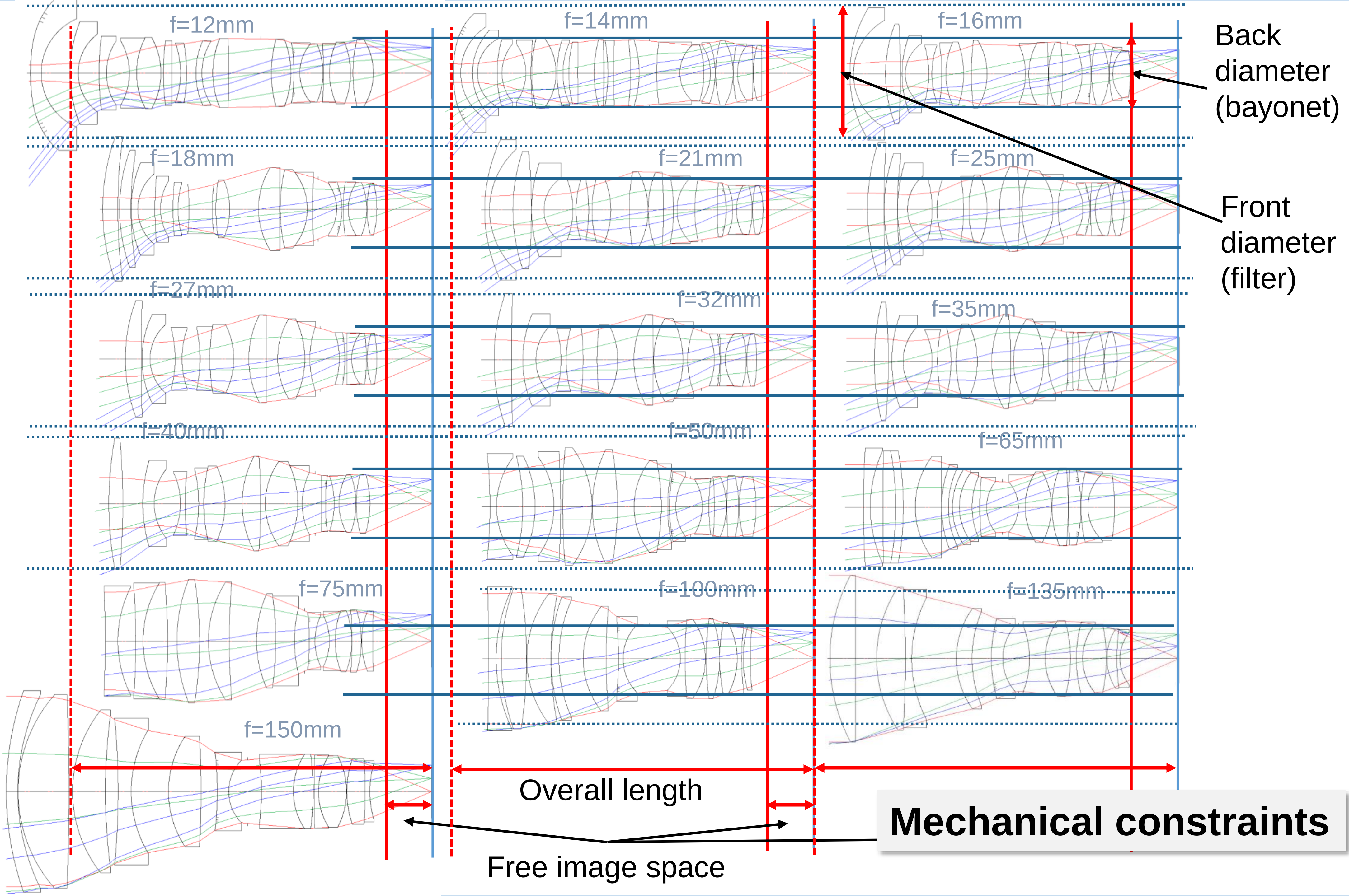
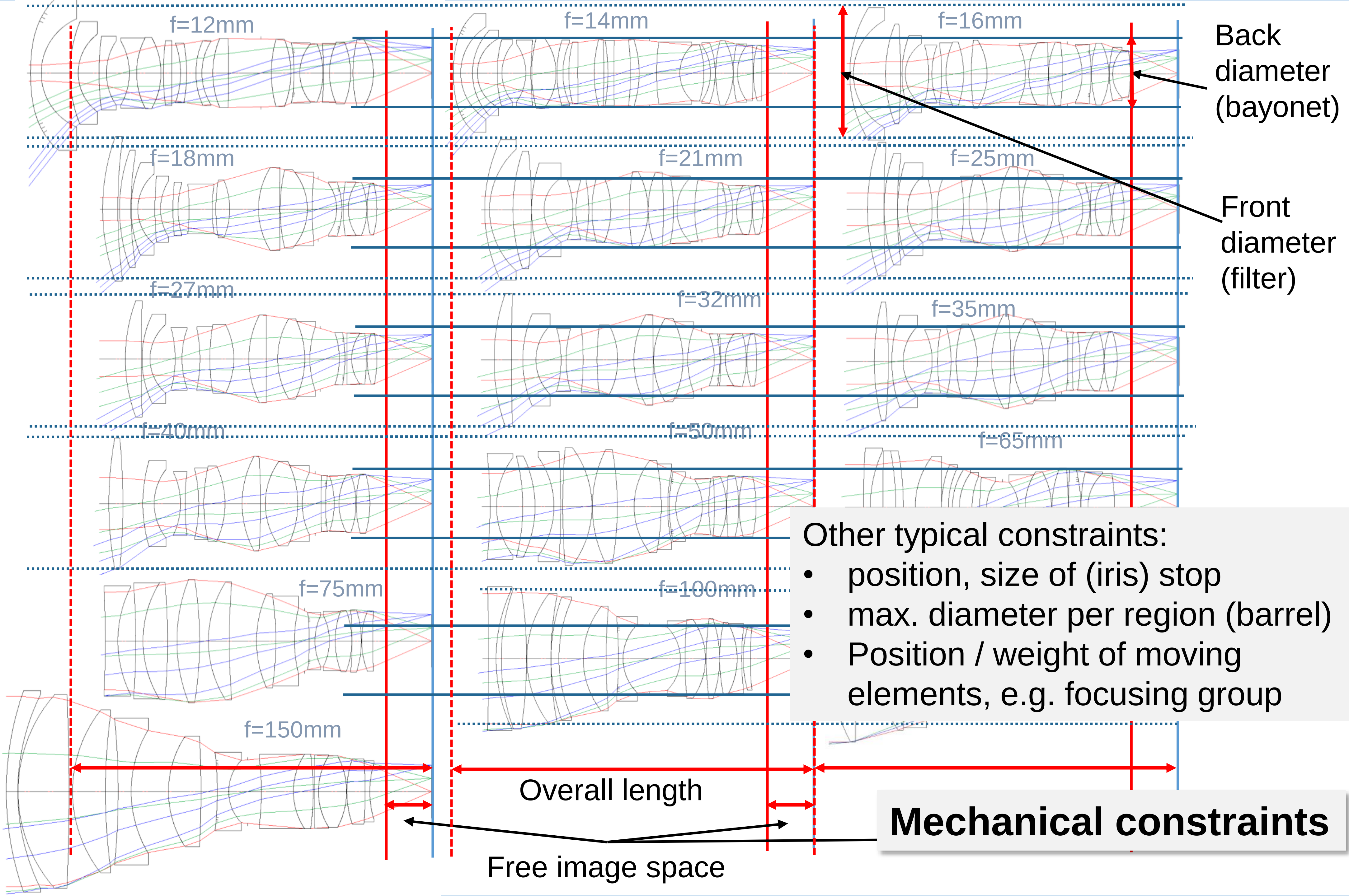


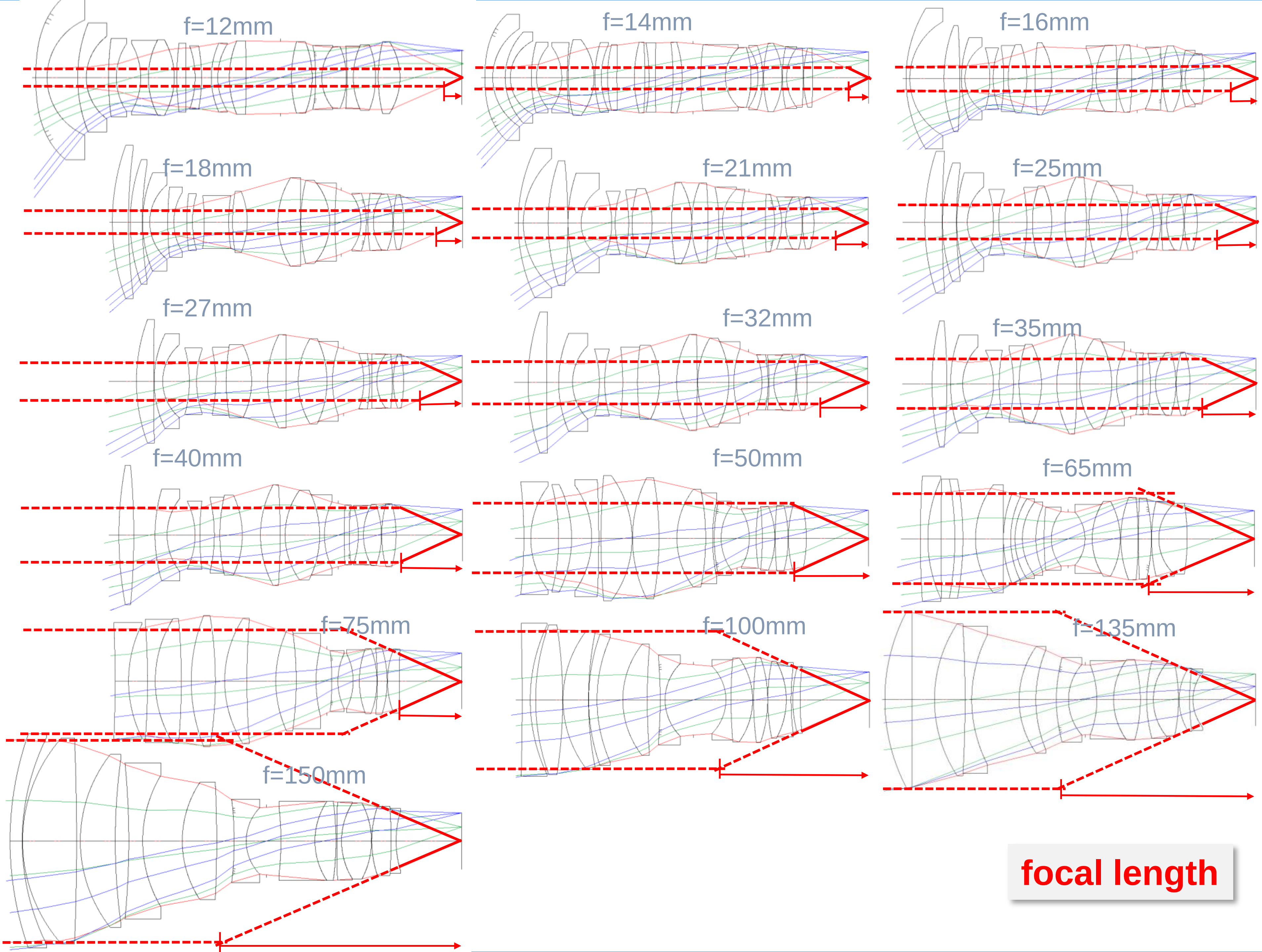
Mechanical constraints

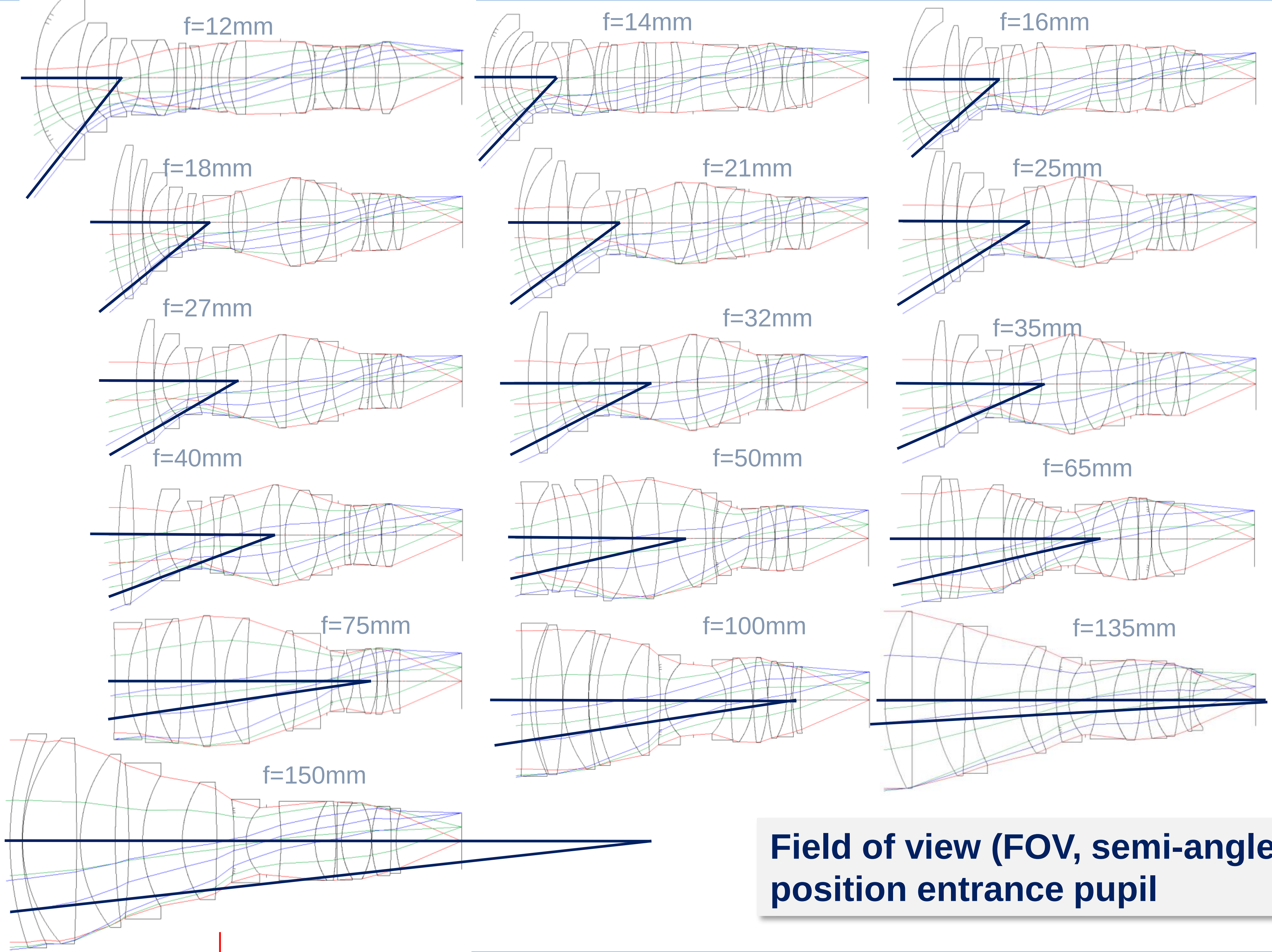


Other typical constraints:

- position, size of (iris) stop
- max. diameter per region (barrel)
- Position / weight of moving elements, e.g. focusing group

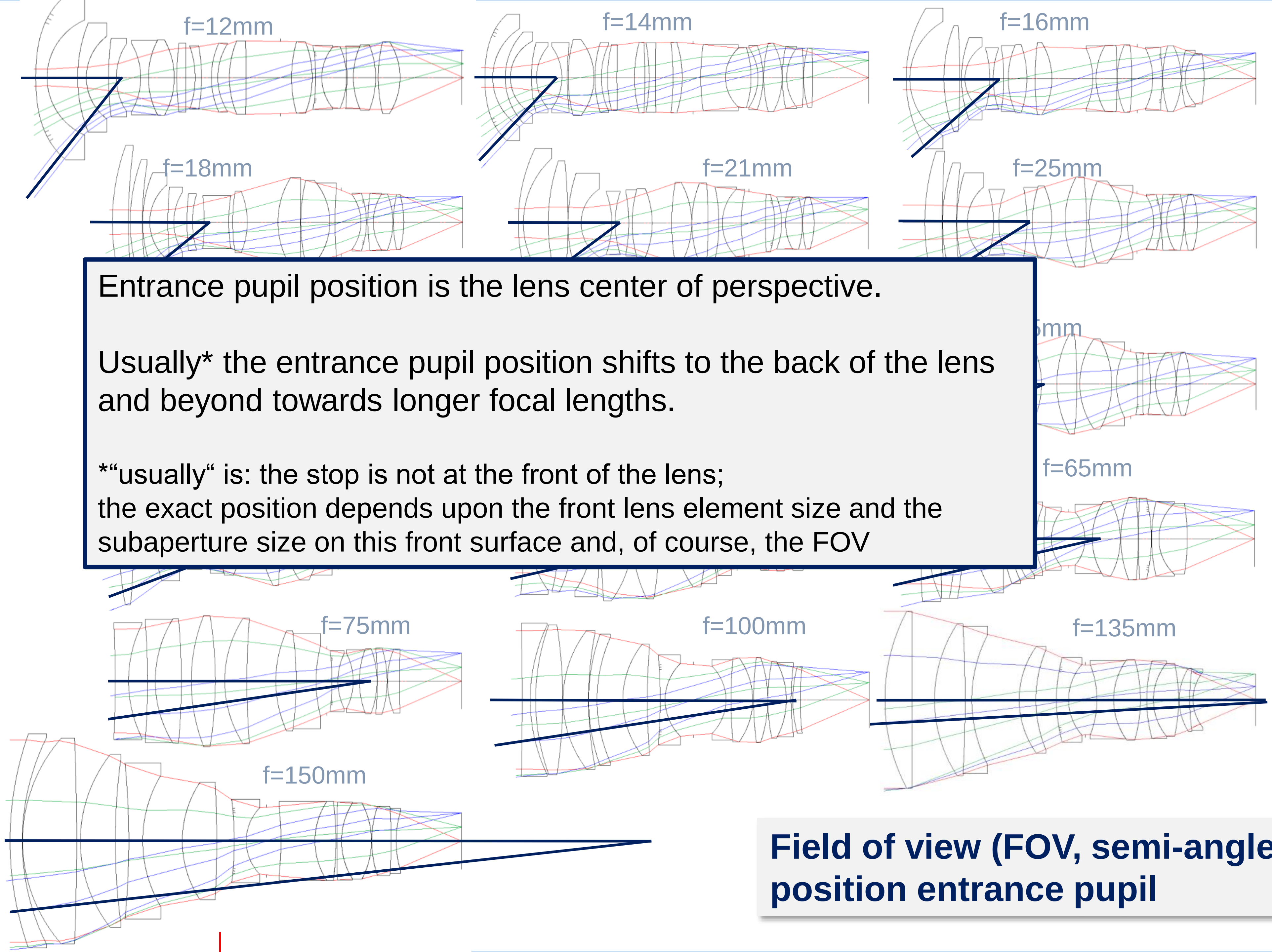
Mechanical constraints


f=12mm
f=14mm
f=16mm
f=18mm
f=21mm
f=25mm
f=27mm
f=32mm
f=35mm
f=40mm
f=50mm
f=65mm
f=75mm
f=100mm
f=135mm
f=150mm
focal length



Field of view (FOV, semi-angle) & position entrance pupil

Field of view (FOV, semi-angle) & position entrance pupil



Entrance pupil position is the lens center of perspective.

Usually* the entrance pupil position shifts to the back of the lens and beyond towards longer focal lengths.

*“usually“ is: the stop is not at the front of the lens;
the exact position depends upon the front lens element size and the subaperture size on this front surface and, of course, the FOV

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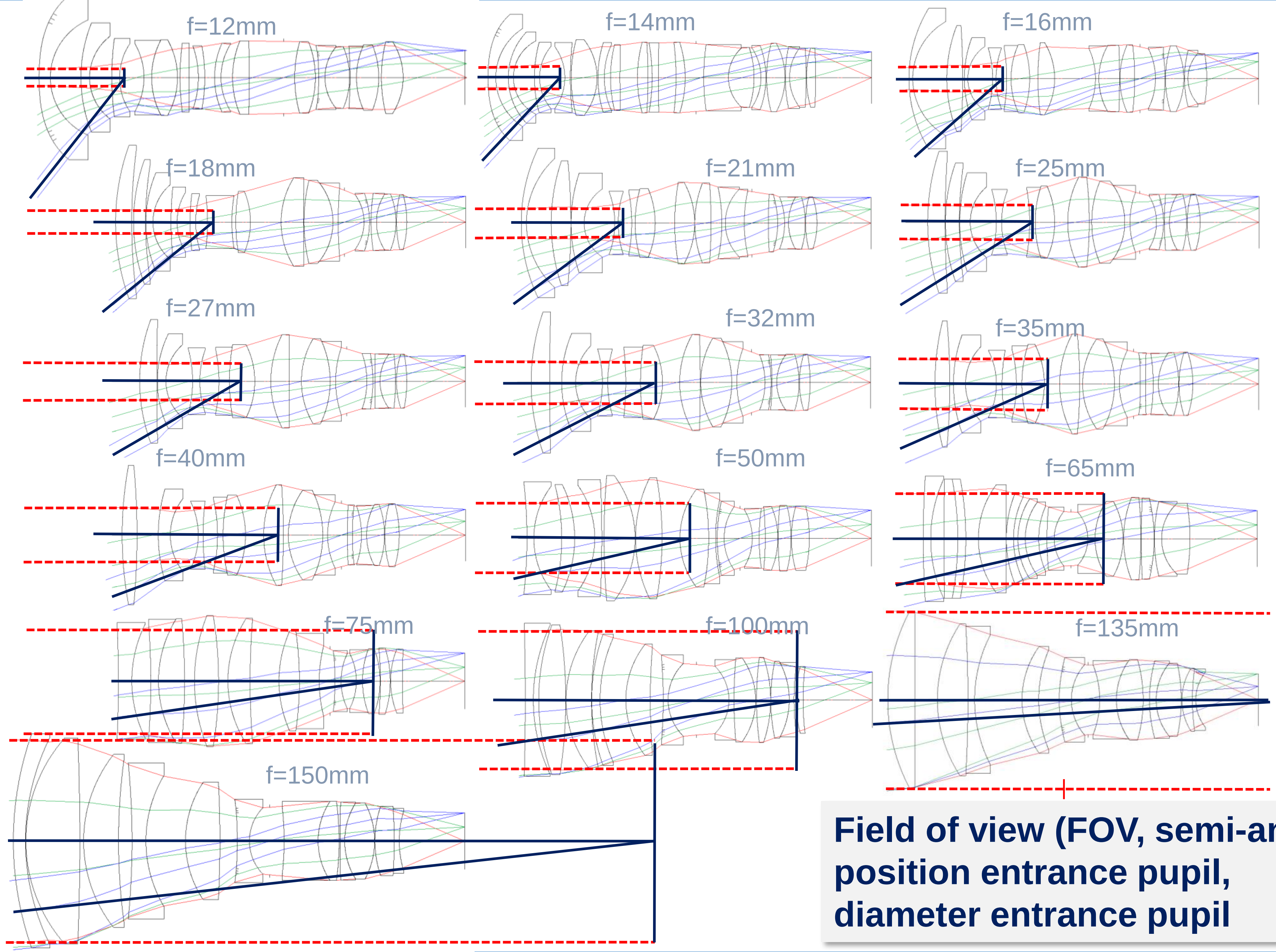

Field of view (FOV, semi-angle), position entrance pupil, diameter entrance pupil

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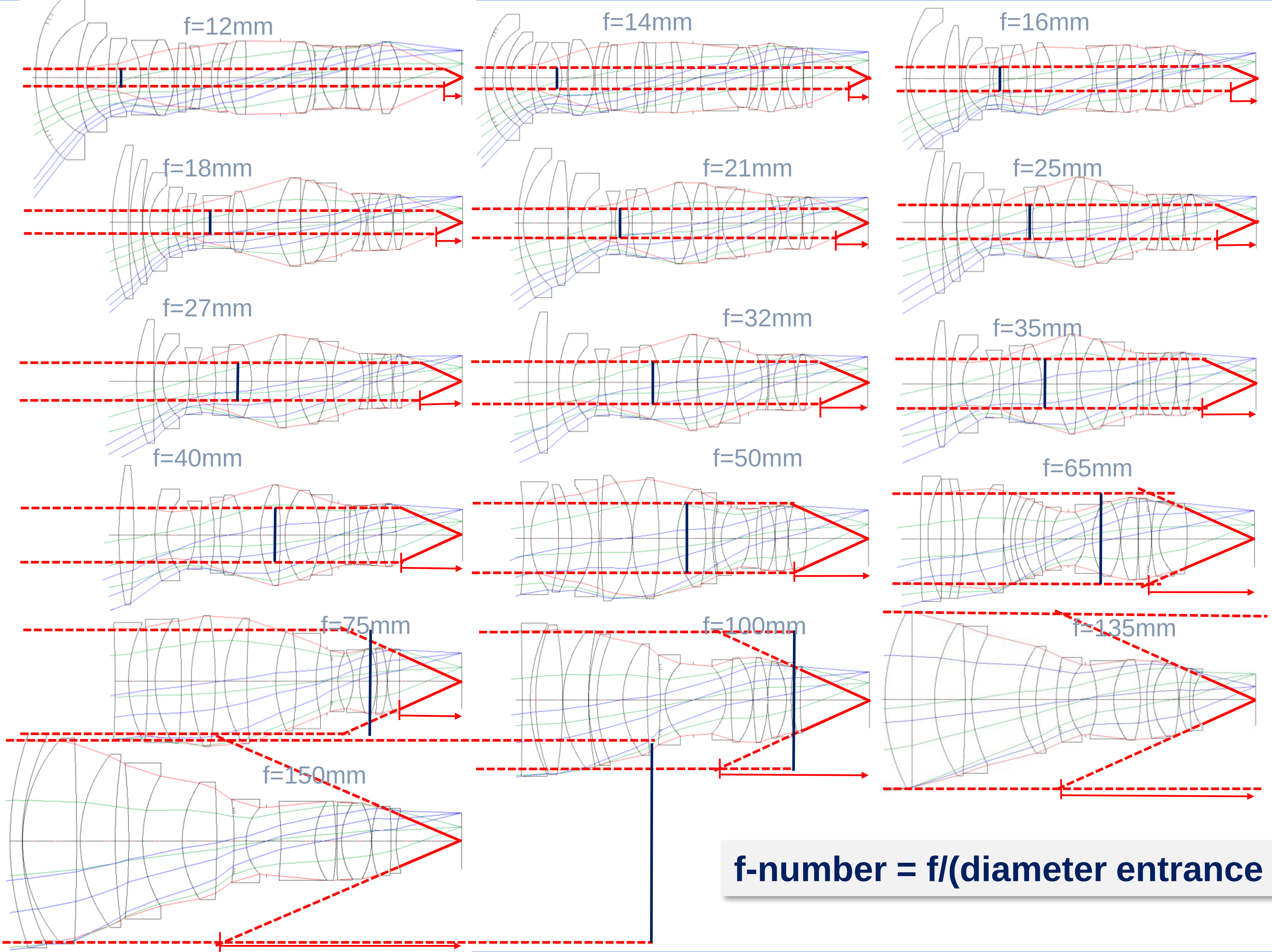

f-number = f/(diameter entrance pupil)

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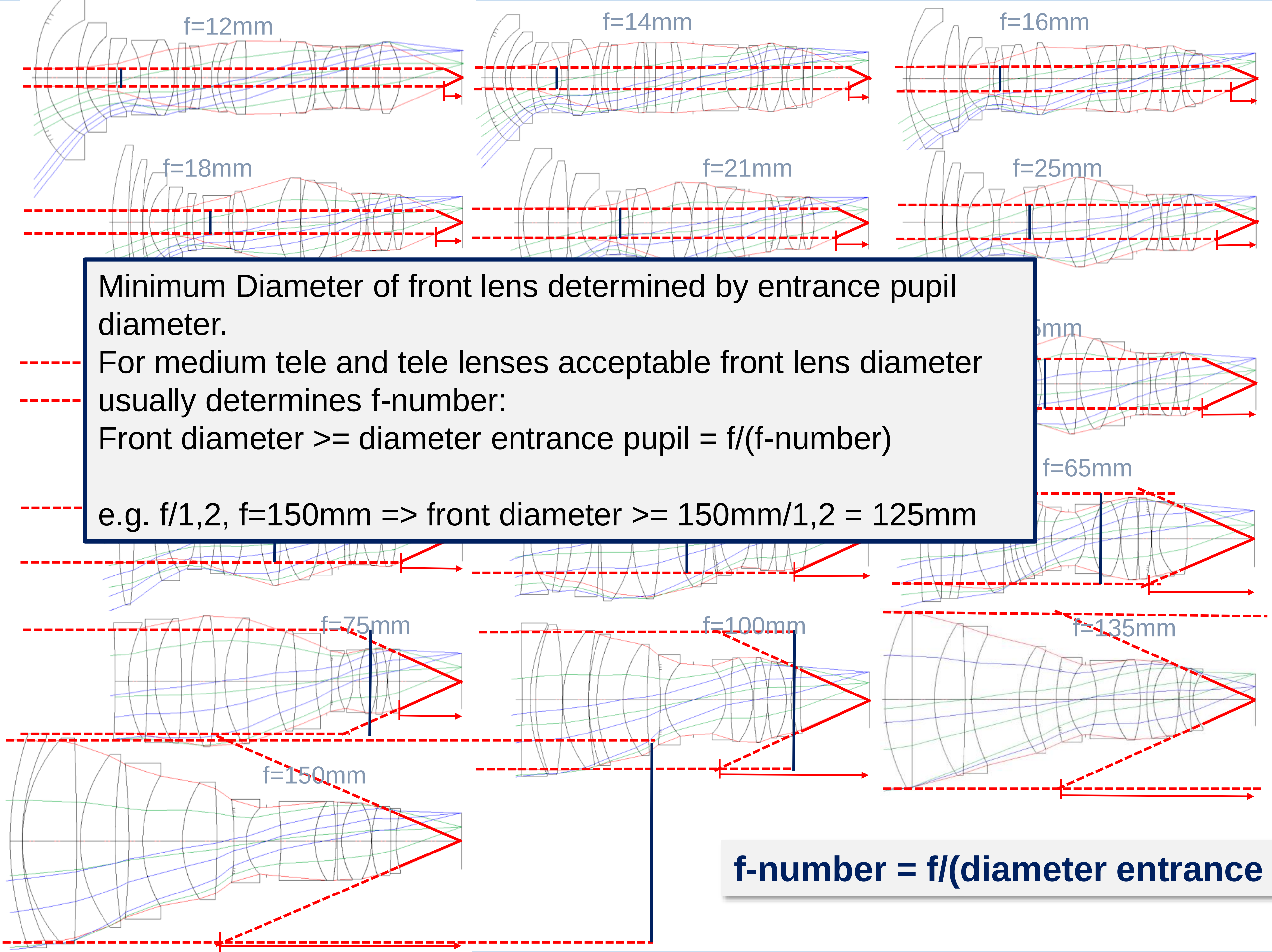
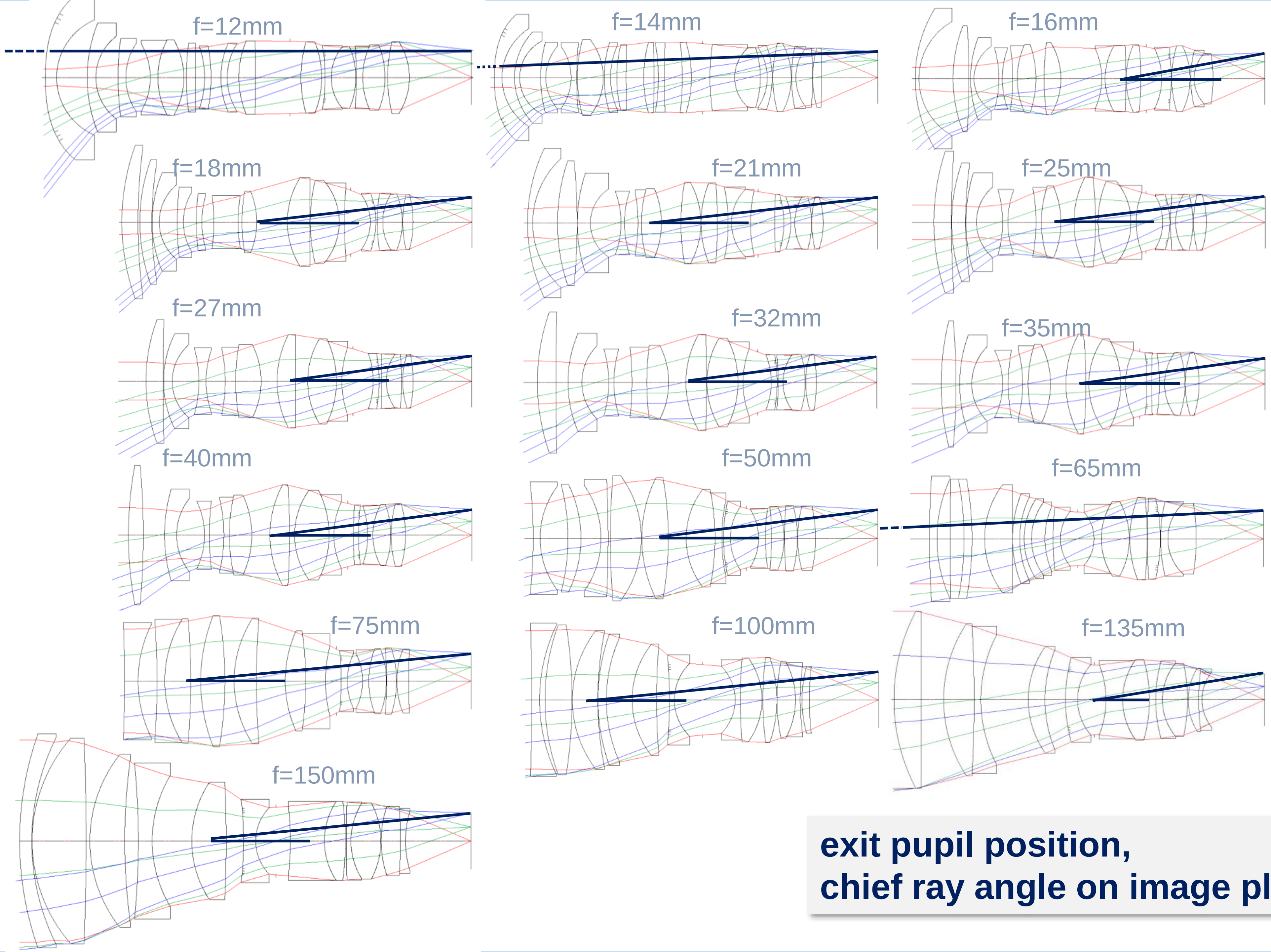


Minimum Diameter of front lens determined by entrance pupil diameter.
For medium tele and tele lenses acceptable front lens diameter usually determines f-number:
Front diameter >= diameter entrance pupil = f/(f-number)

e.g. f/1,2, f=150mm => front diameter >= 150mm/1,2 = 125mm

f-number = f/(diameter entrance pupil)



exit pupil position,
chief ray angle on image plane

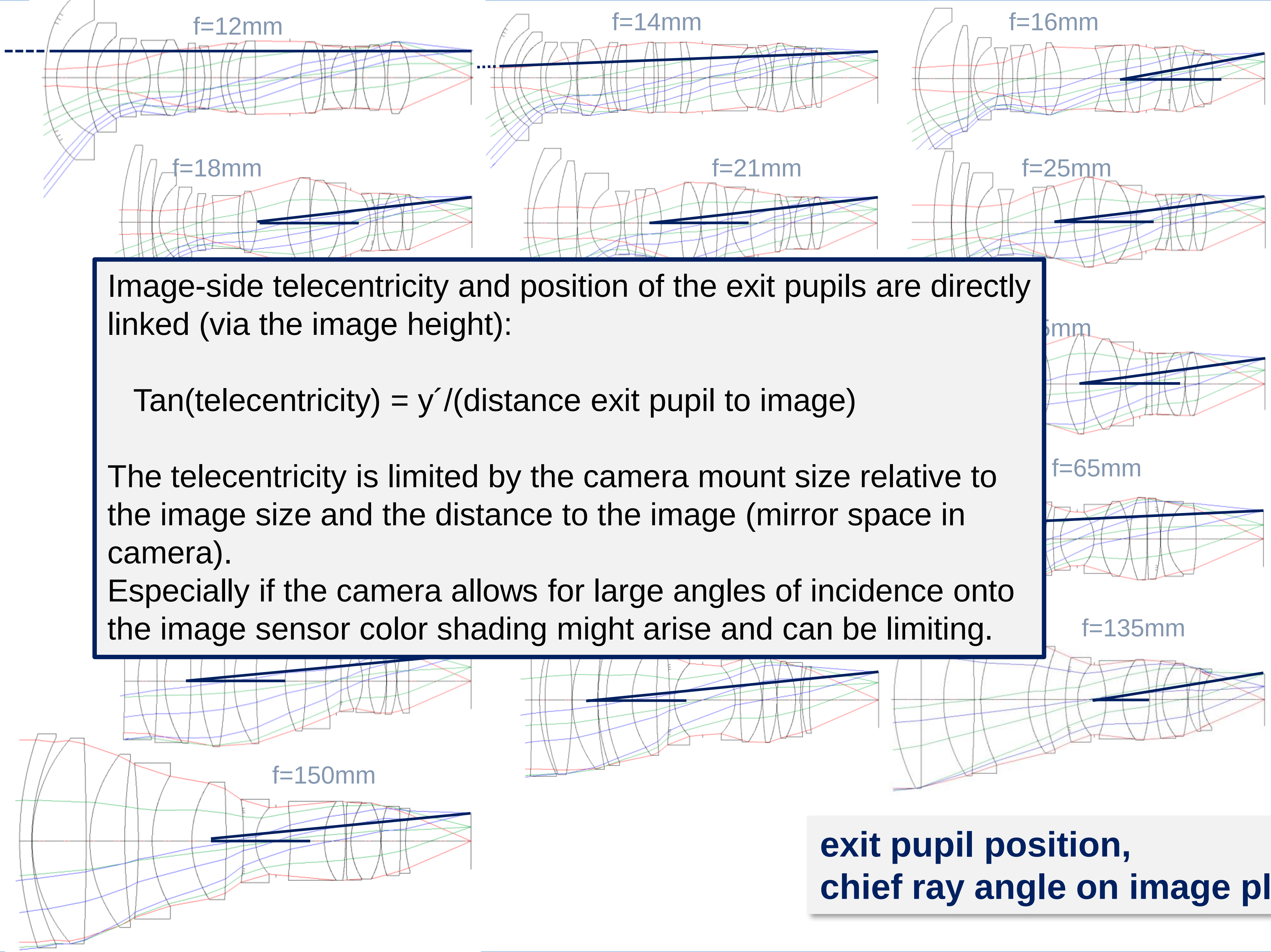


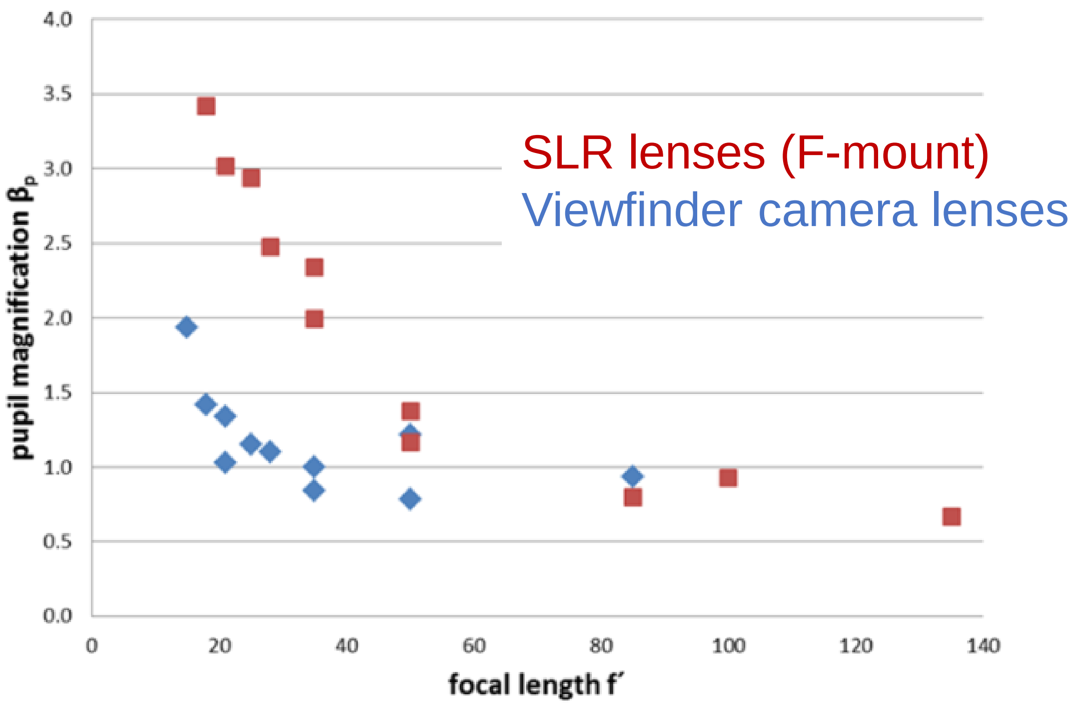
Image-side telecentricity and position of the exit pupils are directly linked (via the image height):

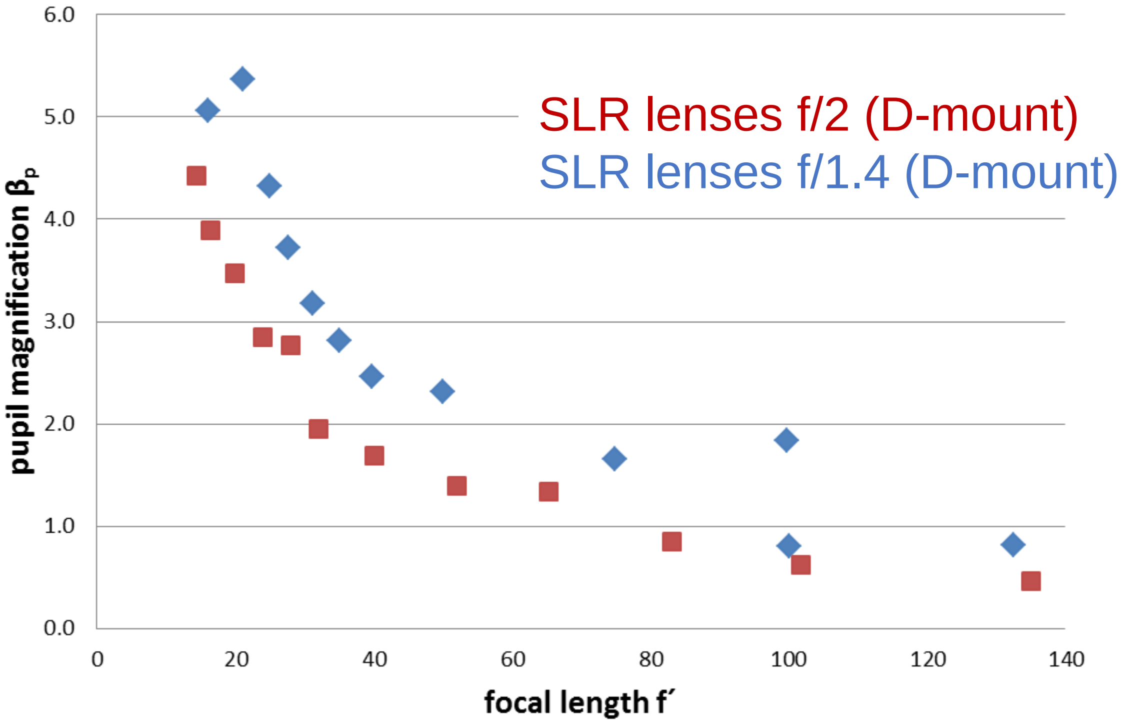
Tan(telecentricity) = y´/(distance exit pupil to image)

The telecentricity is limited by the camera mount size relative to the image size and the distance to the image (mirror space in camera).
Especially if the camera allows for large angles of incidence onto the image sensor color shading might arise and can be limiting.

exit pupil position,
chief ray angle on image plane

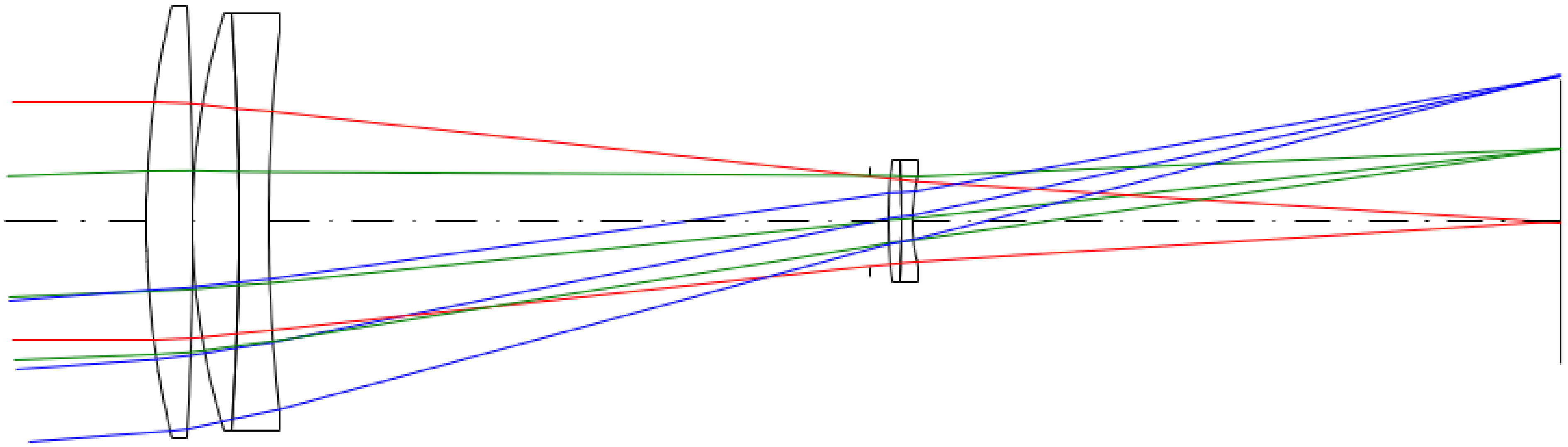
Systematics depending on system constraints, e.g., symmetry parameter





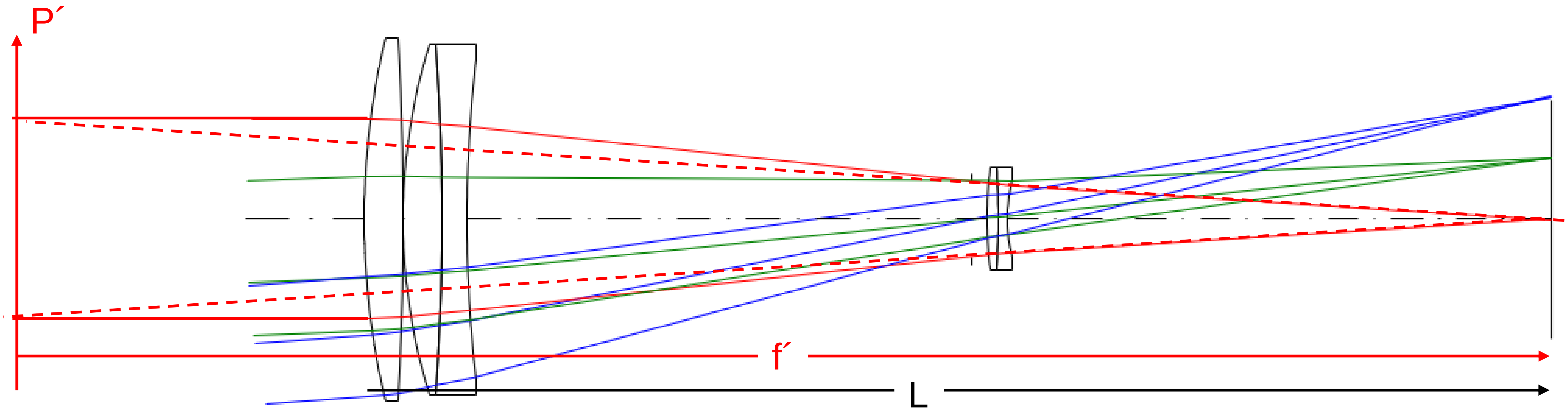
Dependence of m_p(f´) mainly a consequence of mechanical constraints in combination with system characteristics, e.g. focal length.
On the other hand mp is a useful parameter to characterize the system symmetry.

Tele-Tessar 8 / 500 mm (used on the moon surface!)

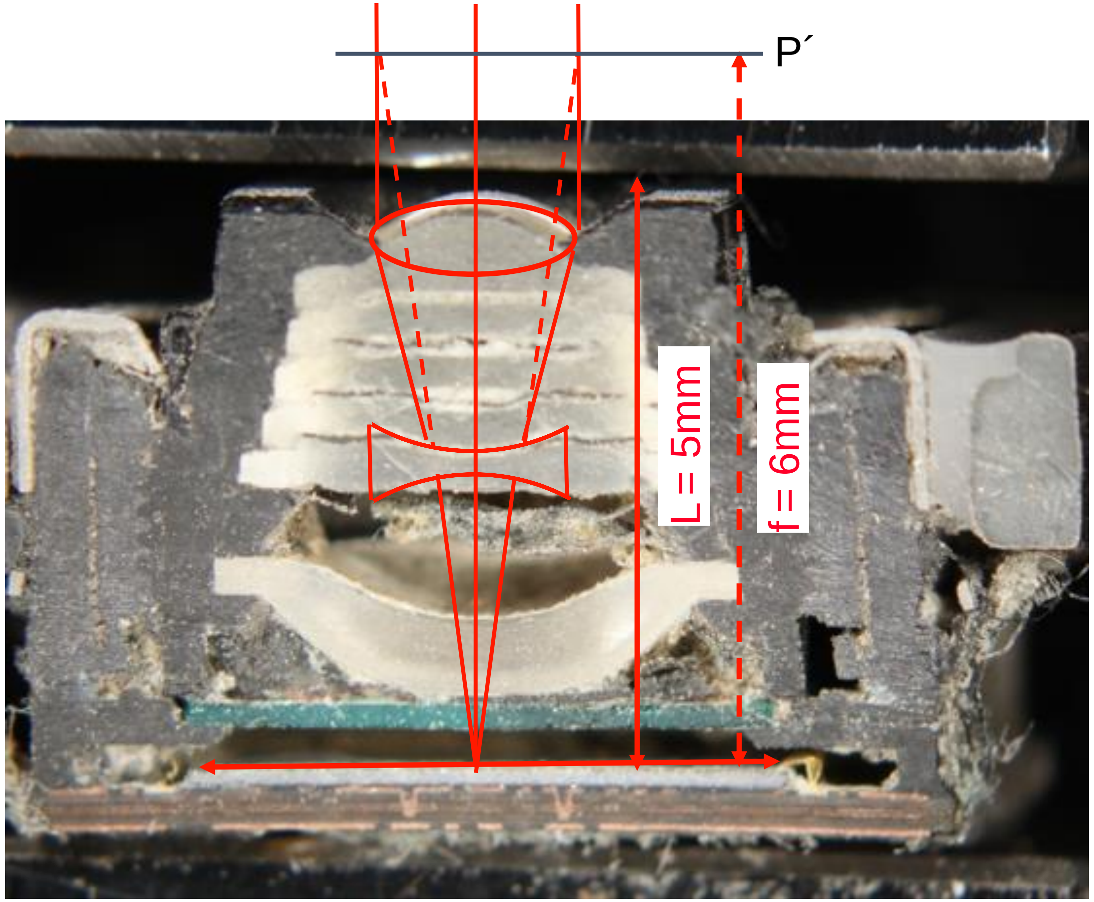


Telephoto Lens

Focal length f´ is longer than system length L (front surface to image)!
f´>L defines a **telephoto** lens.



“Tele Lens” defined by ratio focal length to track length – not by field-of-view

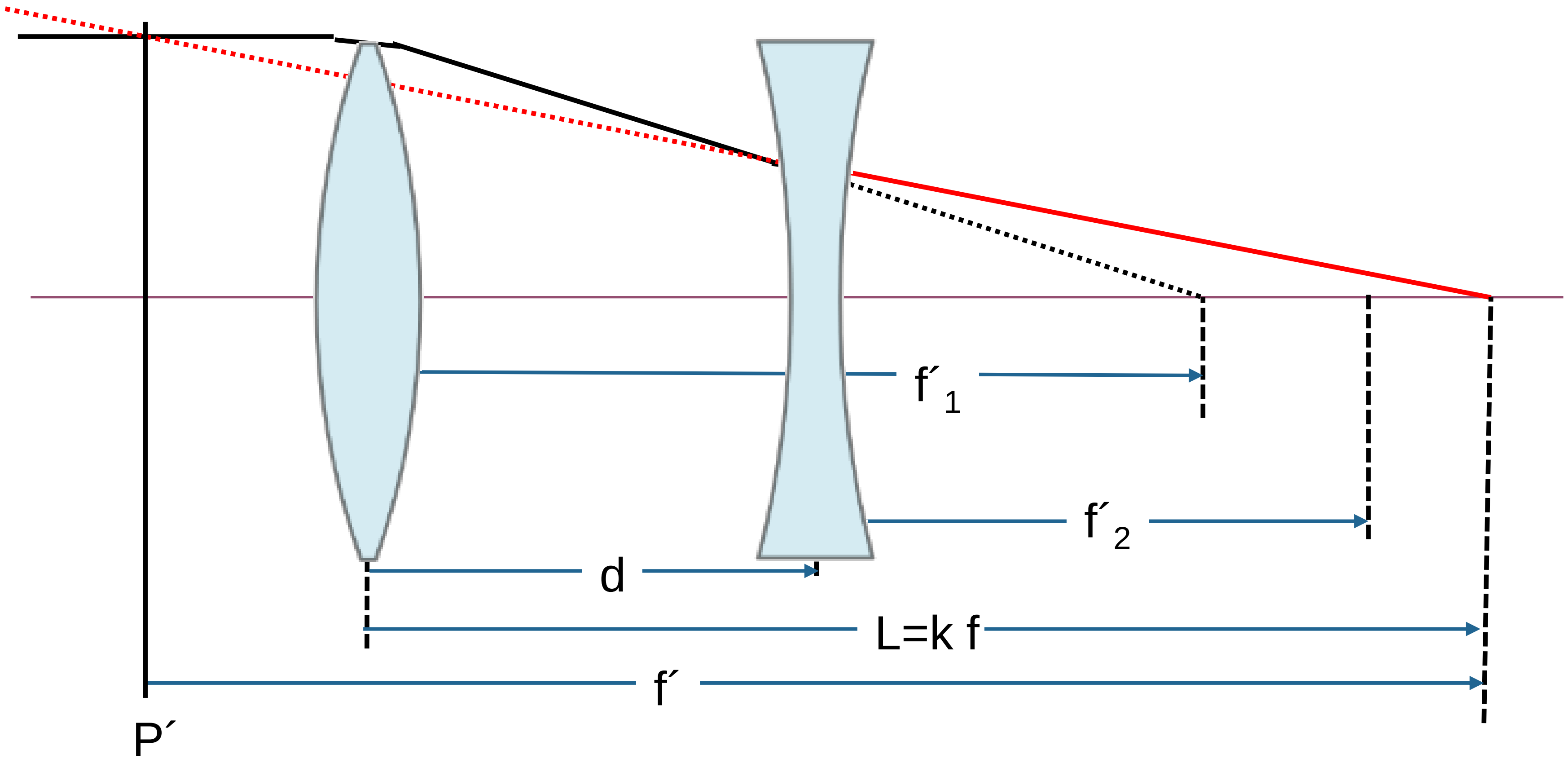


Field-of-view 40°

A photographer would call this a standard field-of-view.

Note: The actual definition of „telephoto“ is **L<f´**. Usually telephoto systems have narrow field-of-view. Therefore many people associate „tele“ as small FOV. However there are counter examples, e.g. smartphone camera lenses with

Telephoto System:
Reduction of system length below focal length



$$f_1 = \frac{f\,d}{f\,(1-k)+d} \qquad f_2 = \frac{d\,(d-k\,f)}{f(1-k)}$$

Tele system: Dependencies of refractive power, relative distance and tele factor

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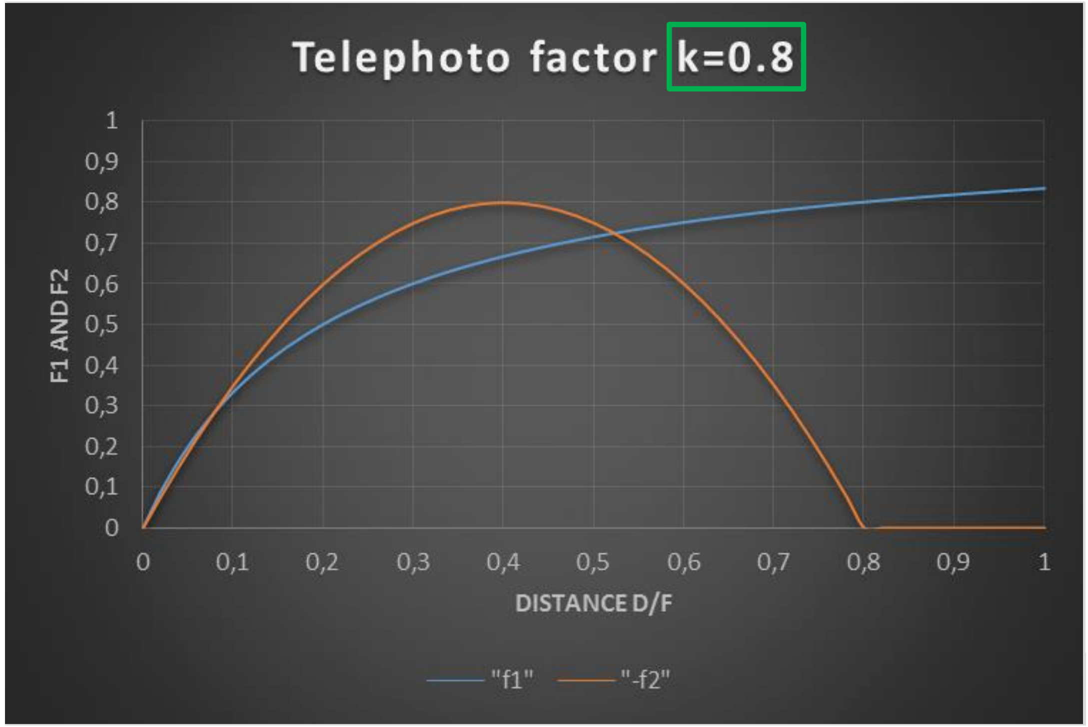

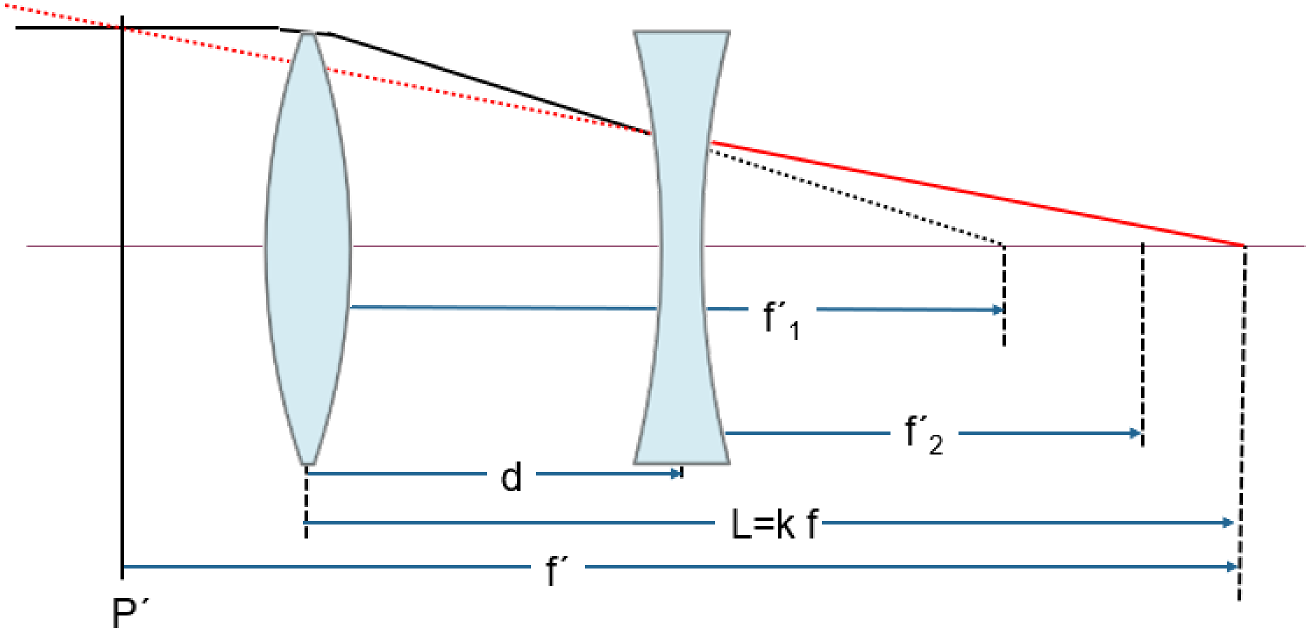

- The greater the lens distance, the greater the focal length of the positive lens at the front.
- The focal length of the negative lens is greatest when the negative lens is exactly in the middle between the positive and negative lenses.
- The smaller the focal length of a lens (or in the real system of the corresponding lens group), the more complex the optical design.

$$f_1 = \frac{f\ d}{f\ (1-k) + d}$$

$$f_2 = \frac{d\ (d - k\ f)}{f(1-k)}$$

Tele system: Dependencies of refractive power, relative distance and tele factor

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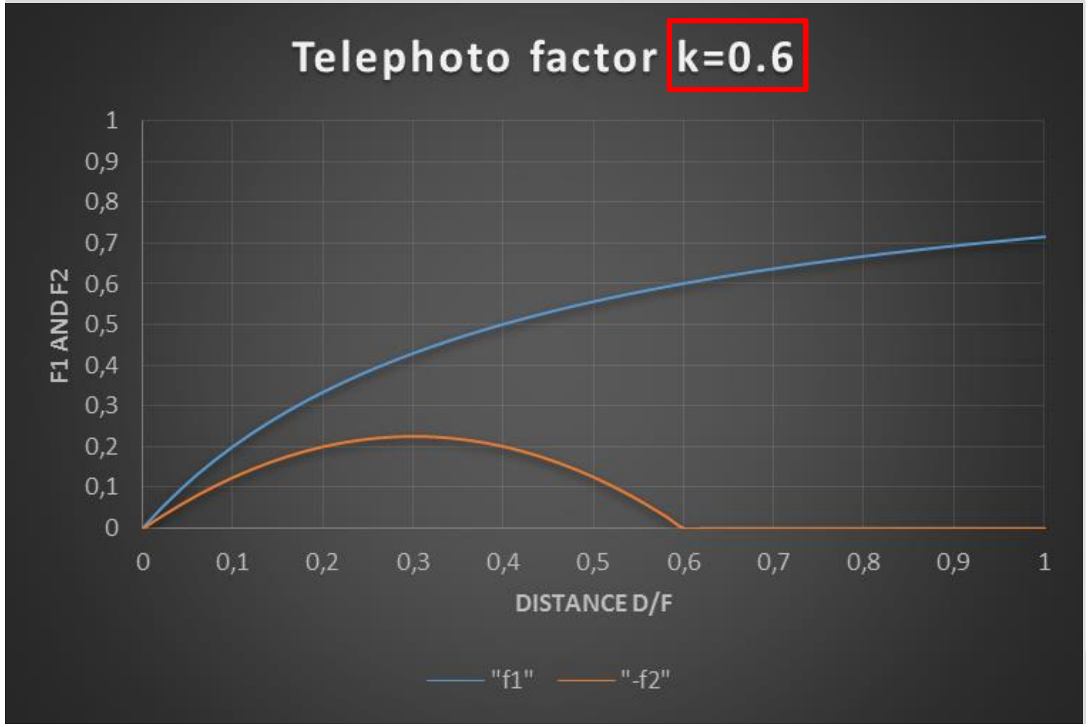
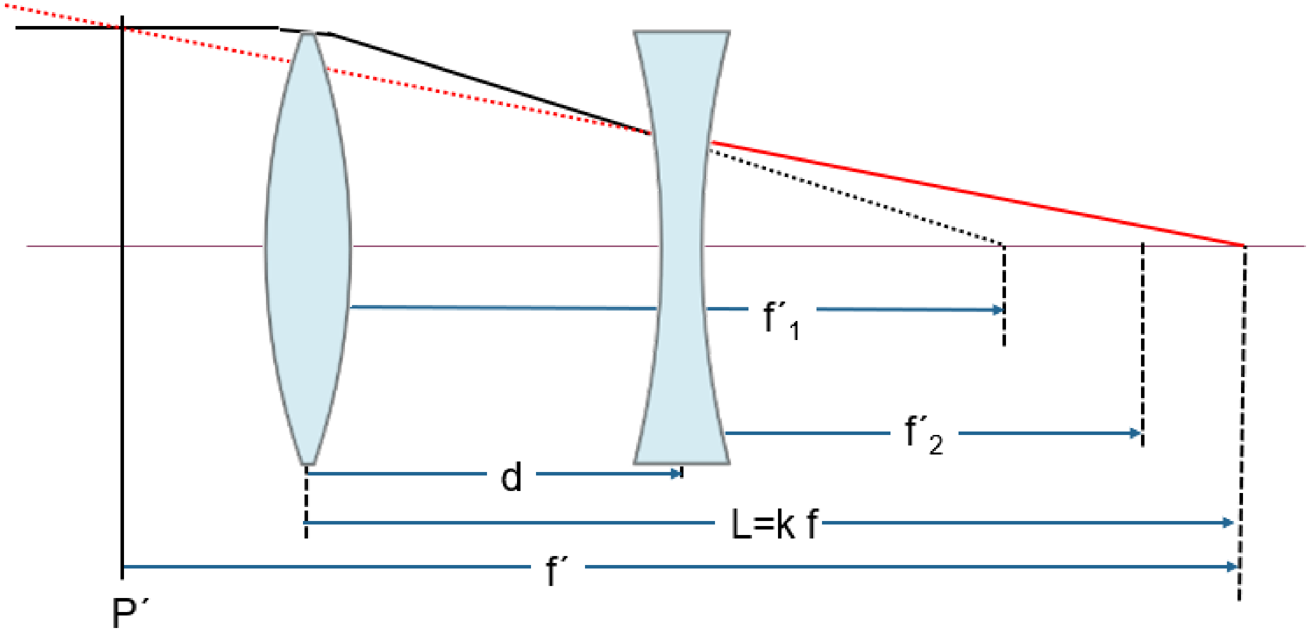


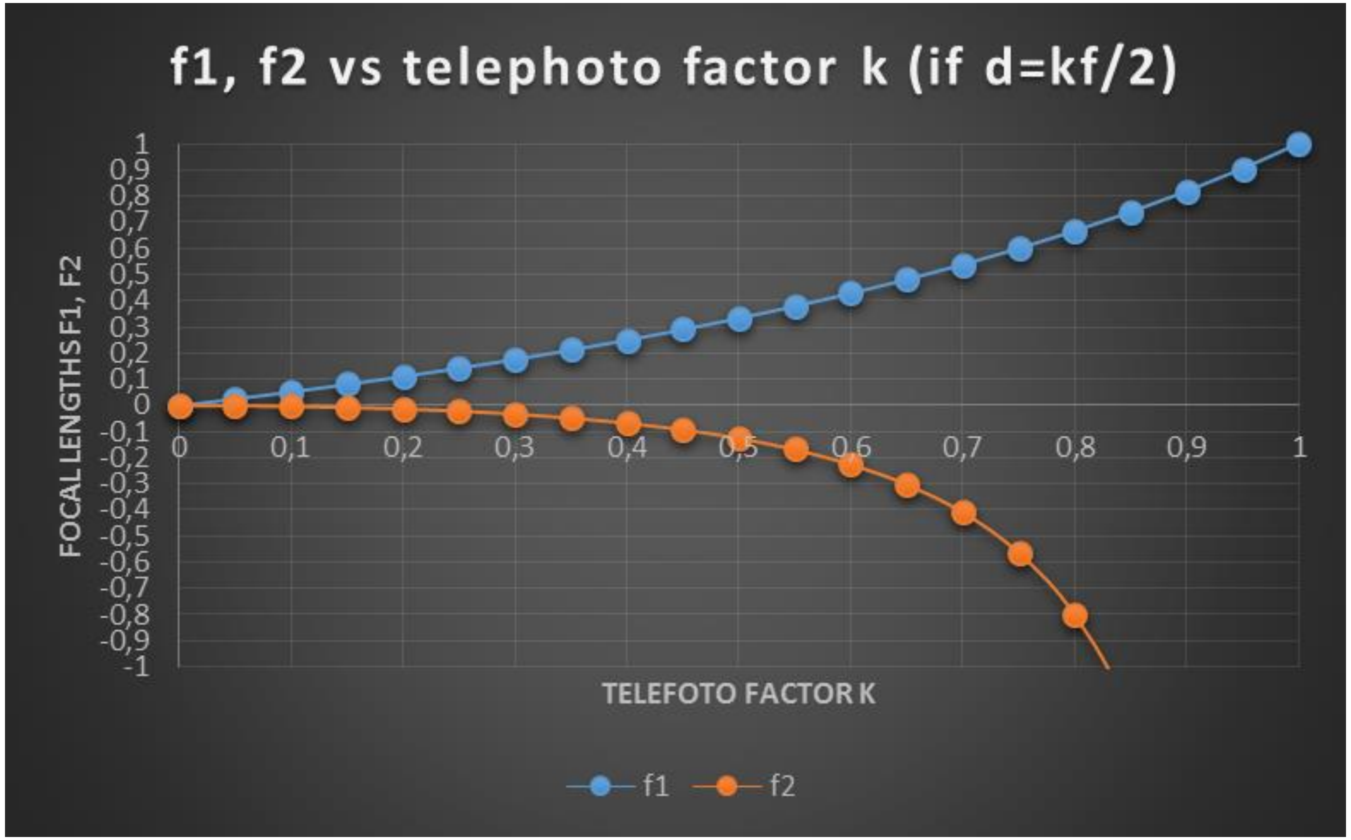


To achieve a smaller telephoto factor (here k=0.6 instead of 0.8), smaller lens focal lengths (i.e., larger refractive powers) are required, i.e. the complexity of the lens groups increases considerably.

$$f_1 = \frac{f\ d}{f\ (1-k) + d}$$

$$f_2 = \frac{d\ (d - k\ f)}{f(1-k)}$$

Tele system: Focal length vs tele factor
Layout with minimum power distribution



Reducing the overall length of a tele system requires considerably higher refractive powers (especially of the negative group). This also increases the complexity of the optical design considerably.

$$f_1 = \frac{f\,d}{f\,(1-k)+d} \qquad f_2 = \frac{d\,(d-k\,f)}{f(1-k)}$$

$$\frac{\partial f_2}{\partial d} = \frac{2d-k\,f}{f(1-k)} = 0,$$

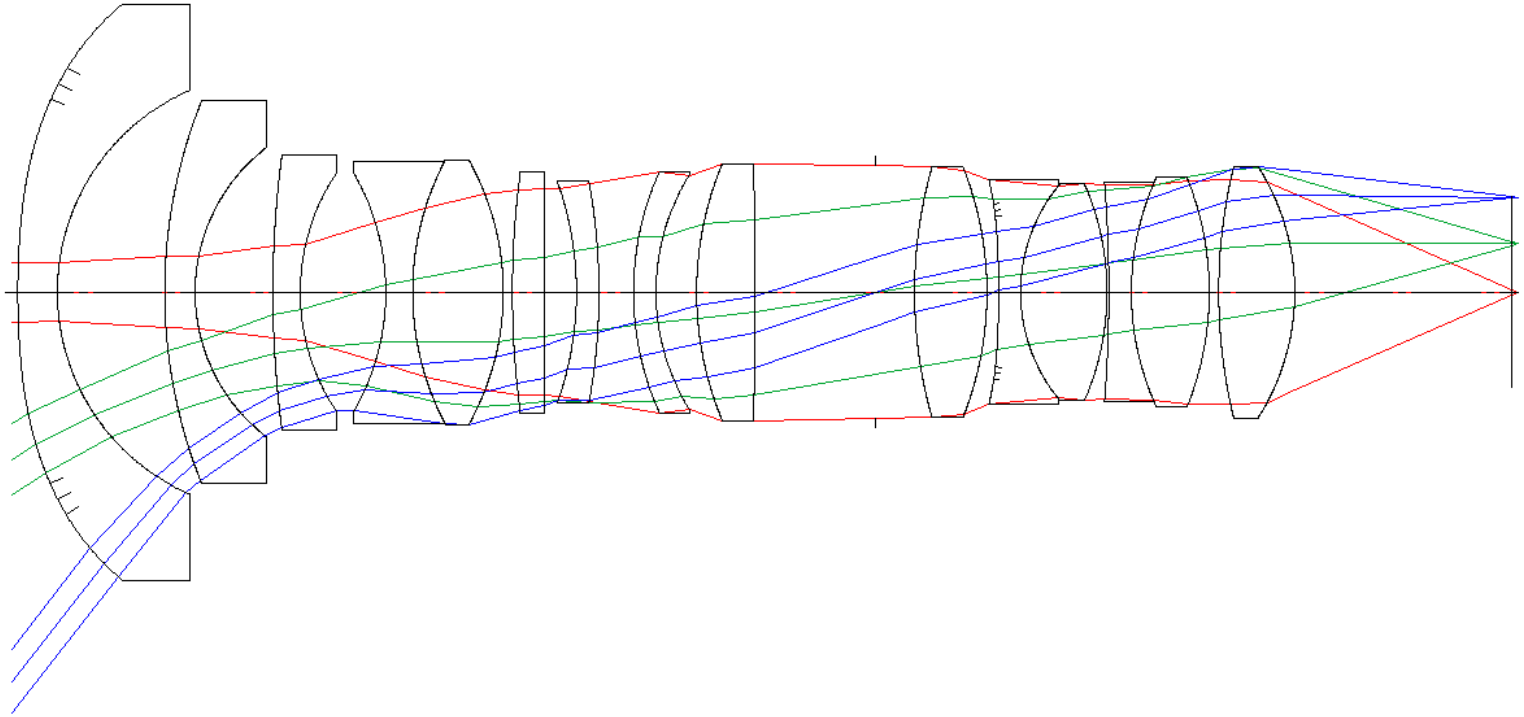
wenn $d = \frac{k\,f}{2}$ ist

The minimum refractive power of the negative lens is then

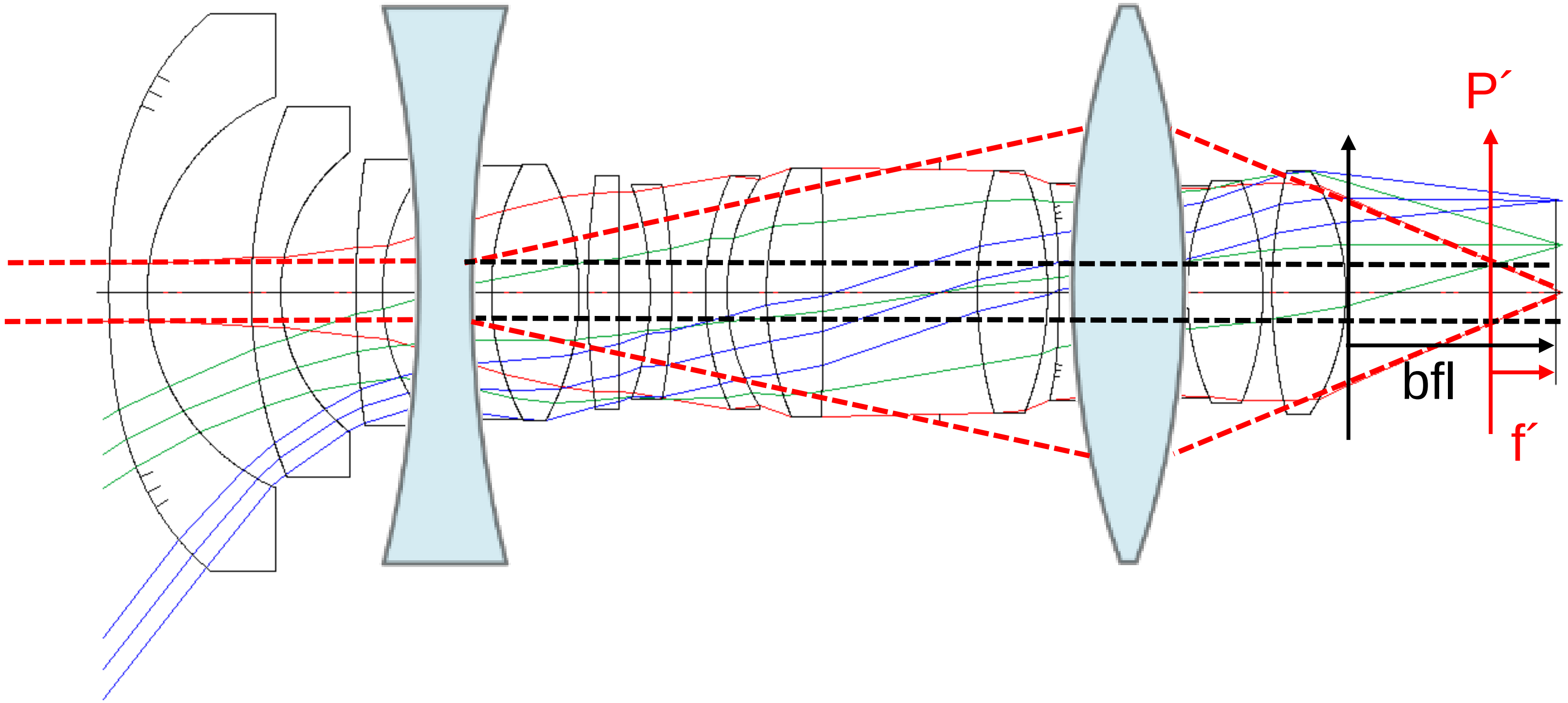
$$f_{1,max} = \frac{k\,f}{2-k}$$

$$f_{2,max} = -\frac{k^2\,f}{4(1-k)}$$

Inverted Telephoto = Retrofocus Lens



Inverted Telephoto = Retrofocus Lens



The distance between last lens surface and image plane is called „**back focal length**“ bfl.
A **retrofocus** lens realizes **f´<bfl** with an appropriate (-,+) power distribution.

What are all those lens data?

camera lens

binocular

10 x 42

telescope

„80 / 400 refractor“

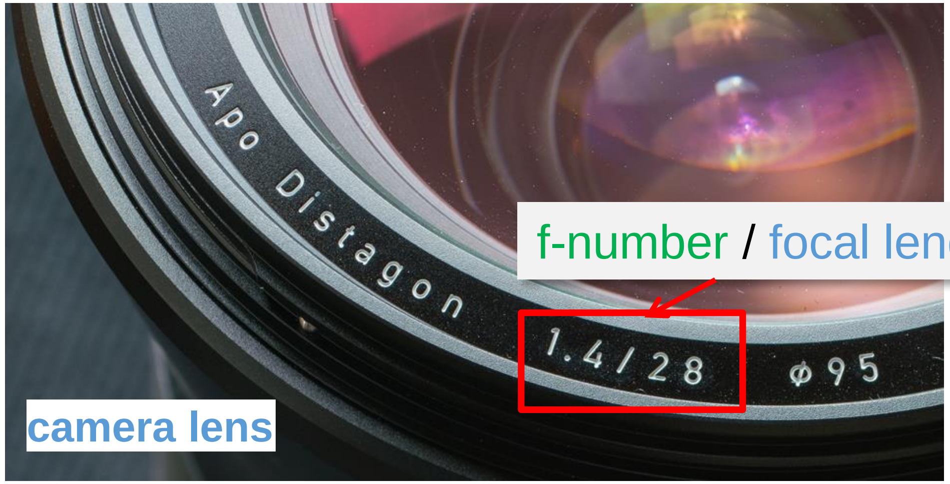
microscope objective

Eye glasses

Rifle scope

1.8-14 x 50

What are all those lens data?



f-number / focal length [mm]

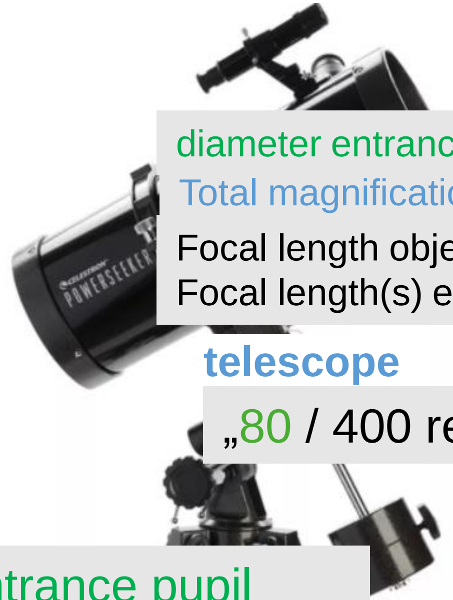
camera lens

binocular

10 x 42

Angular magnification („10x“)

diameter entrance pupil (42mm, „amount of light“)



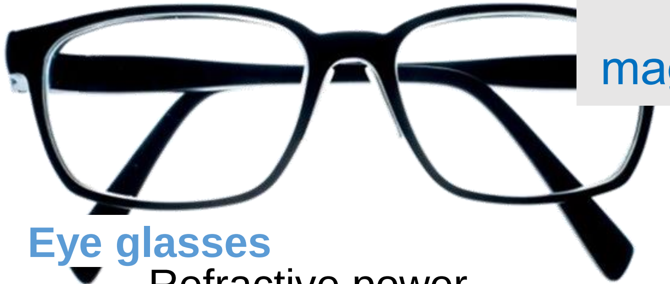
diameter entrance pupil
Total magnification (min,
Focal length objective
Focal length(s) eyepiece(s)

telescope

„80 / 400 refractor“

magnification / numerical aperture

microscope objective



Eye glasses
Refractive power („diopter“)

Angular magnification range (zoom)

diameter entrance pupil

1.8-14 x 50

angular magnification (conversion factor) („0.6x“, „2x“)

What are all those lens data?

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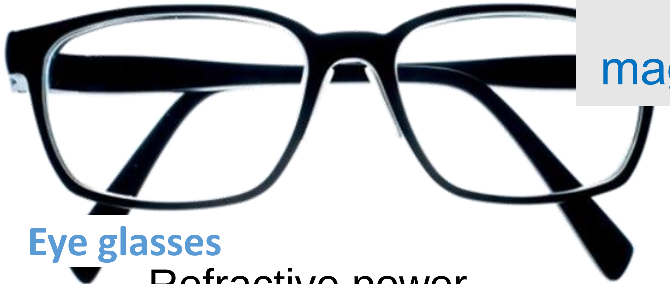
camera lens

magnification / numerical aperture

microscope objective



Eye glasses

Refractive power („diopter“)

focusing

binocular

10 x 42

Angular magnification („10x“)

diameter entrance pupil (42mm, „amount of light“)

diameter entrance pupil

Total magnification (min,

Focal length objective

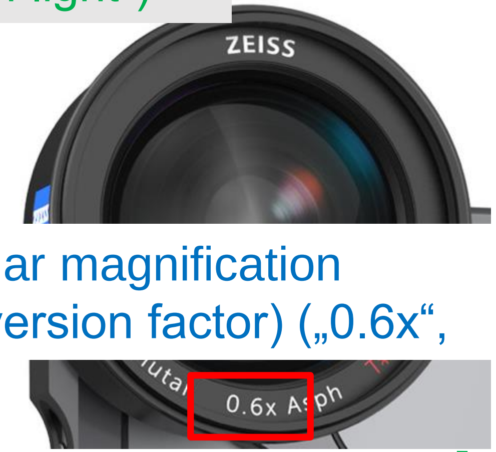
Focal length(s) eyepiece(s)

telescope

„80 / 400 refractor“

riflescope

1.8-14 x 50



angular magnification (conversion factor) („0.6x“, „2x“)

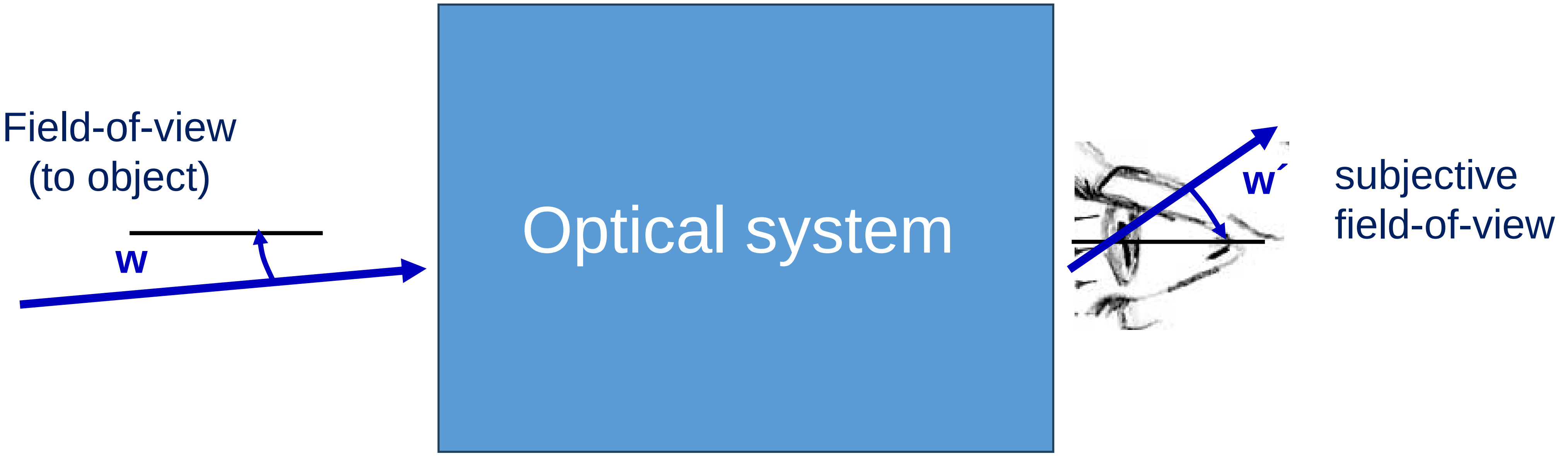
afocal → field-of-view conversion!

What are those lens data?: Binoculars

Optical system layout example:
Far-distance magnifying system with interface to human eye

Angular magnification:

$$\Gamma = \frac{\tan w'}{\tan w}$$



The ratio of the tangent of the viewing angle on the eyepiece (ocular) side to that on the object side is the angular magnification.

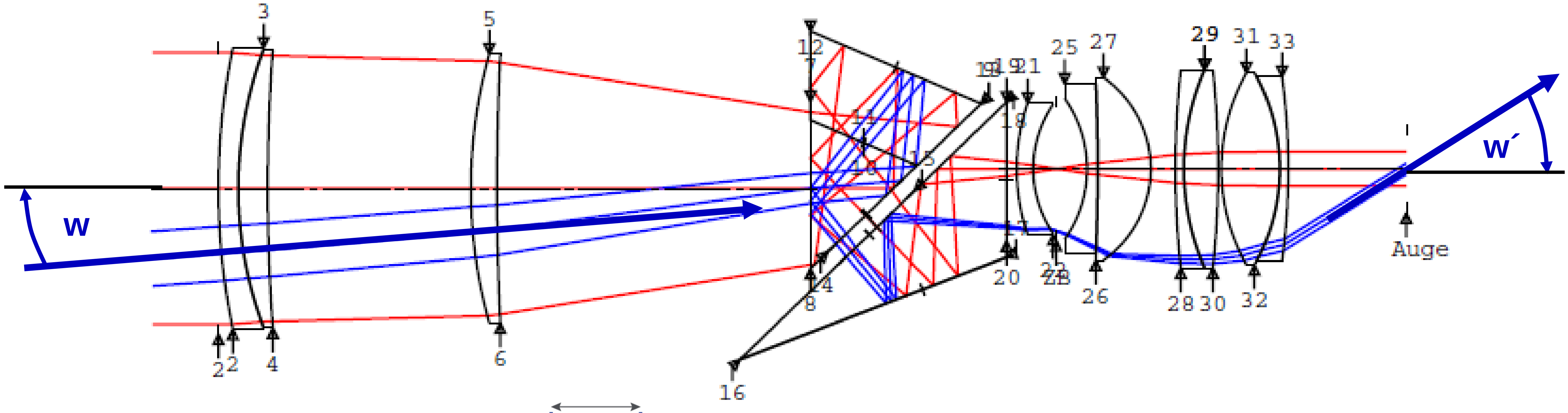
Angular magnification

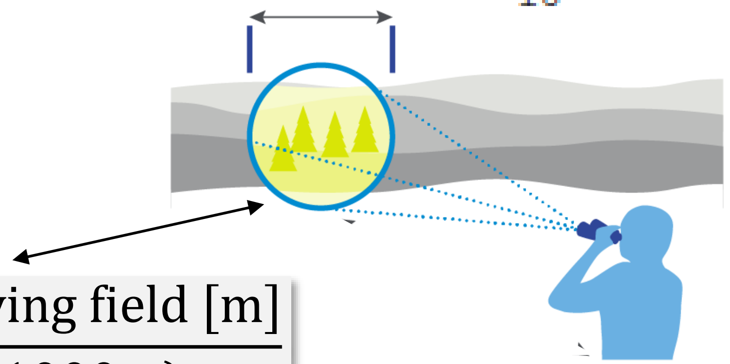
Angular magnification: $\Gamma = \dfrac{\tan w'}{\tan w}$

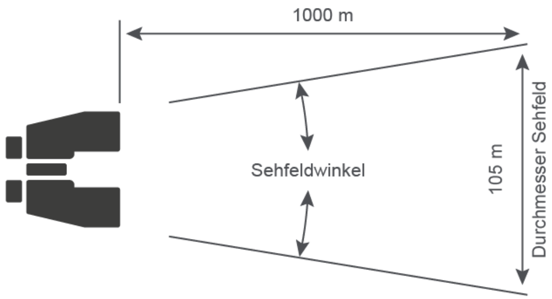
The ratio of the tangent of the viewing angle on the eyepiece (ocular) side to that on the object side is the angular magnification.

Optical Layout of Binocular

Optical Layout of Binocular







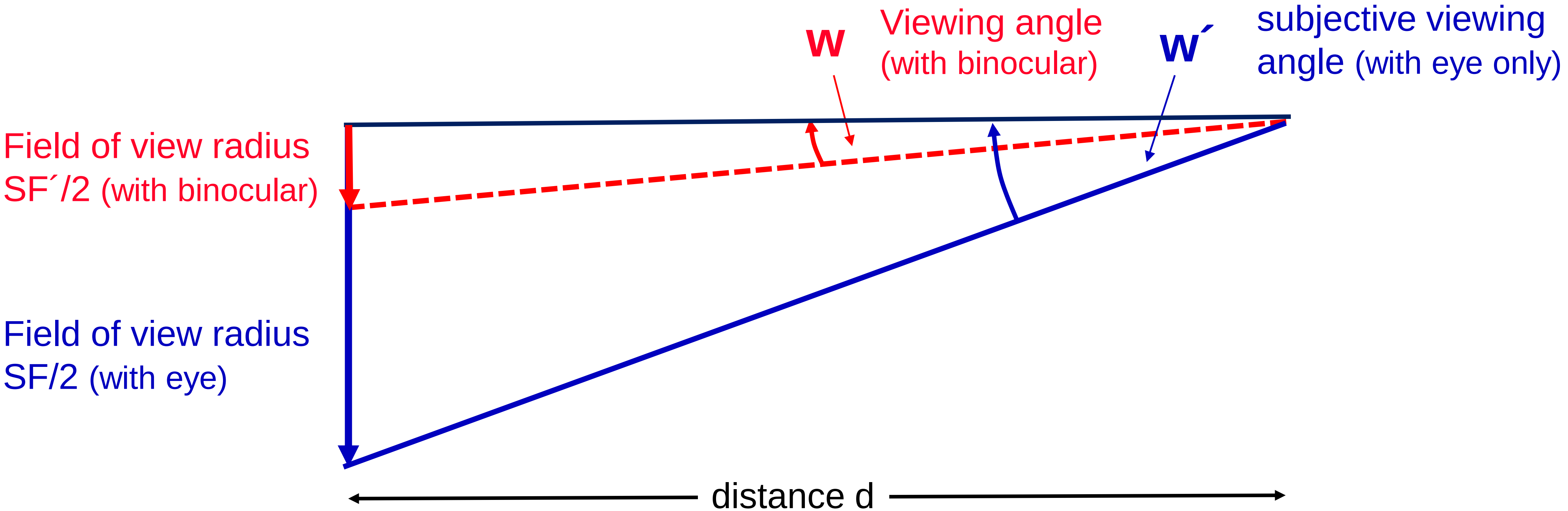
$$2 \cdot \tan w = \frac{\text{diameter viewing field [m]}}{\text{distance (1000m)}}$$

Magnification of an afocal system

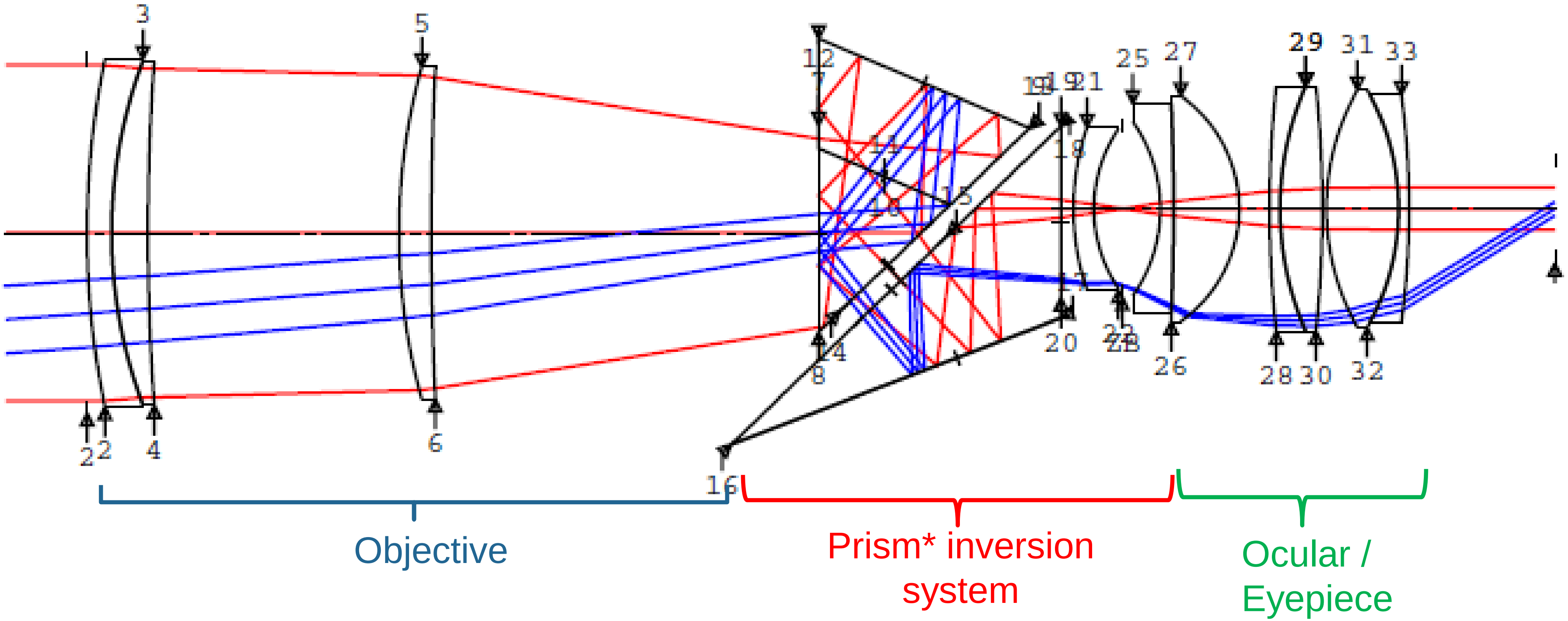
Angular magnification

$$\Gamma = \frac{\tan w'}{\tan w} = \frac{SF'/2d}{SF/2d} = \frac{SF'}{SF}$$

Magnification is the ratio of the field of view of binoculars to the naked eye at the same object distance.



Optical Layout of Binocular: 3 Optics Groups



*einige wenige Ferngläser verwenden Spiegel- oder Hybridsysteme, z. B. Zeiss Jena 8x16, Leica Amplivid

Optical Layout of Binocular: Intermediate Image

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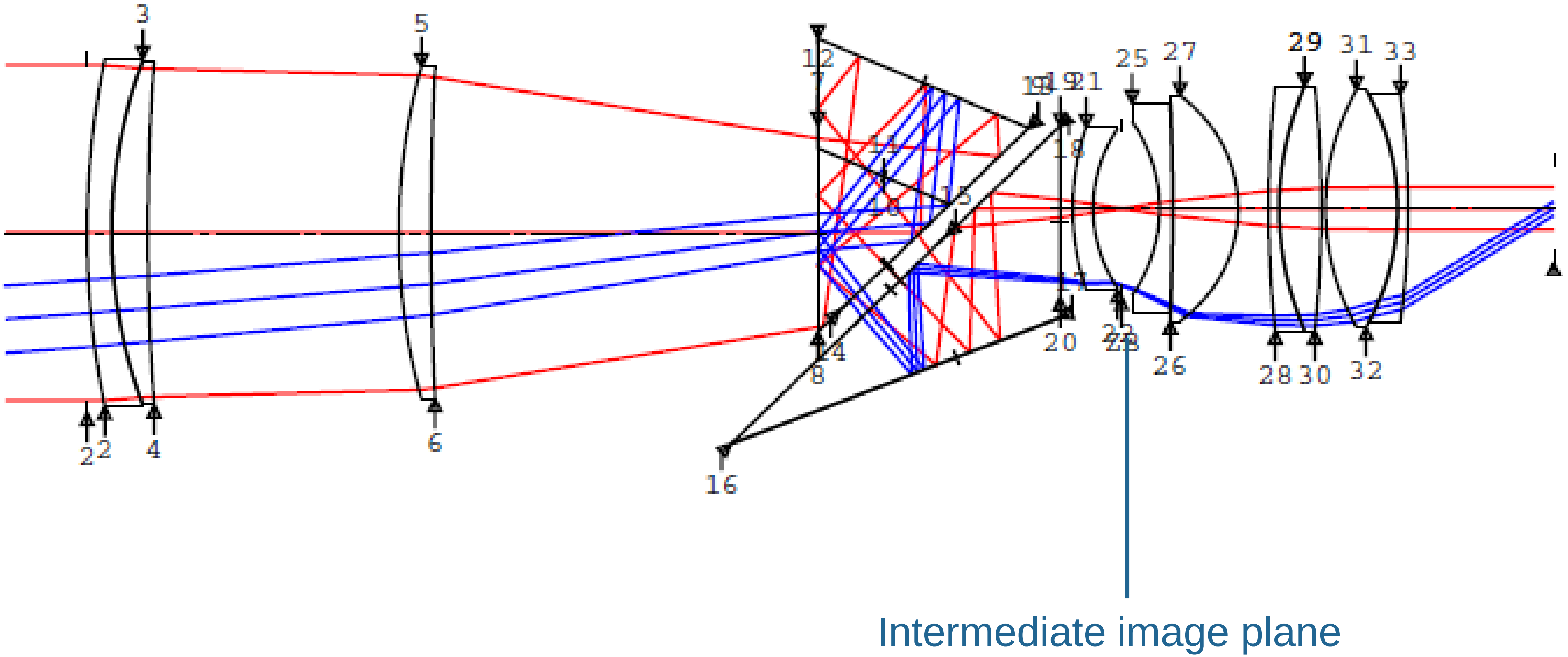
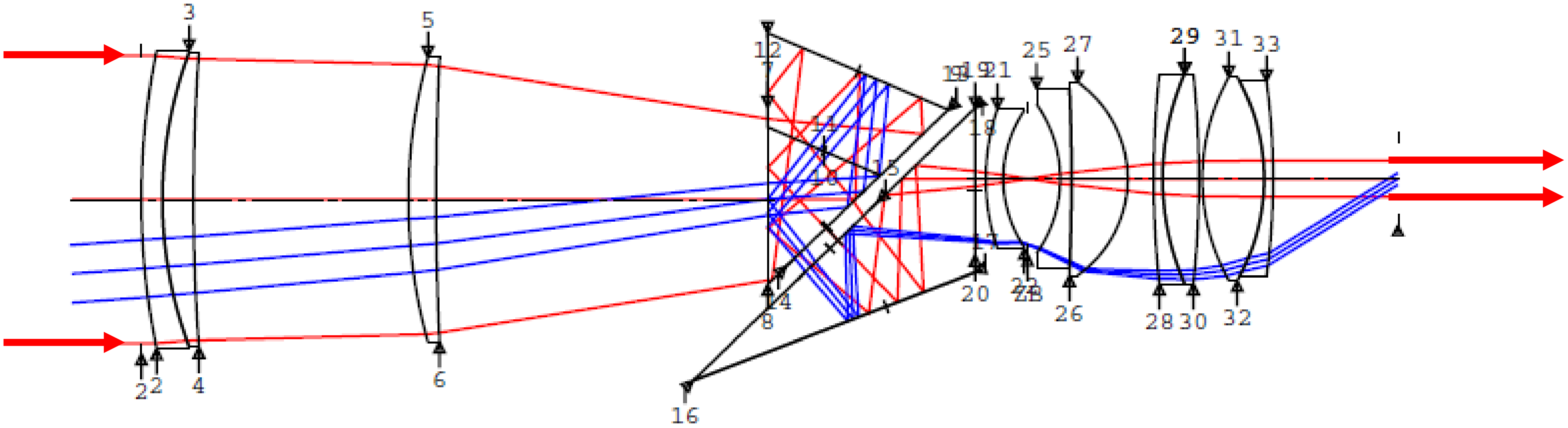


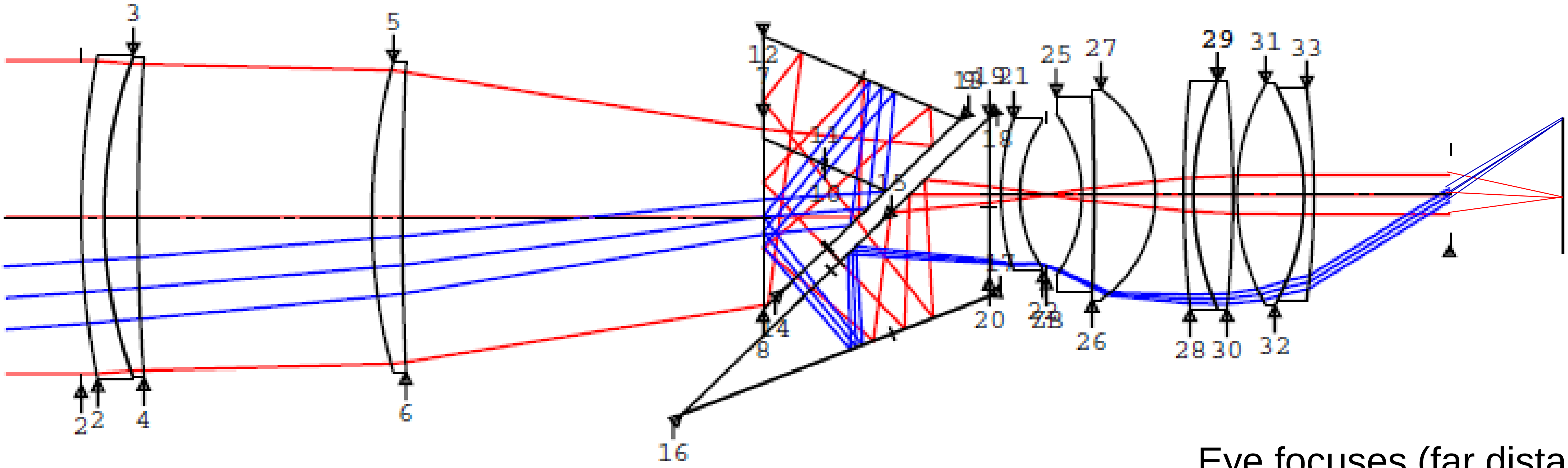
Afocal system



Afocal (“not focusing”): Parallel incoming rays of light emerge in parallel.

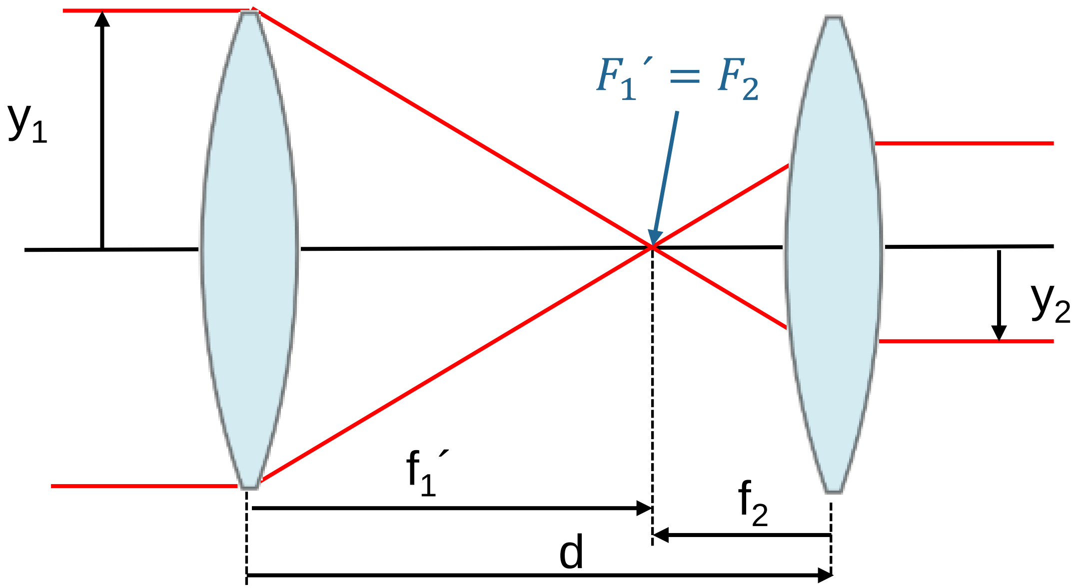
An afocal system therefore does not change the focusing of the object, **only the (angular) magnification**.

Afocal system: Focusing with Eye



Eye focuses (far distant) object on retina

Structure of an afocal system



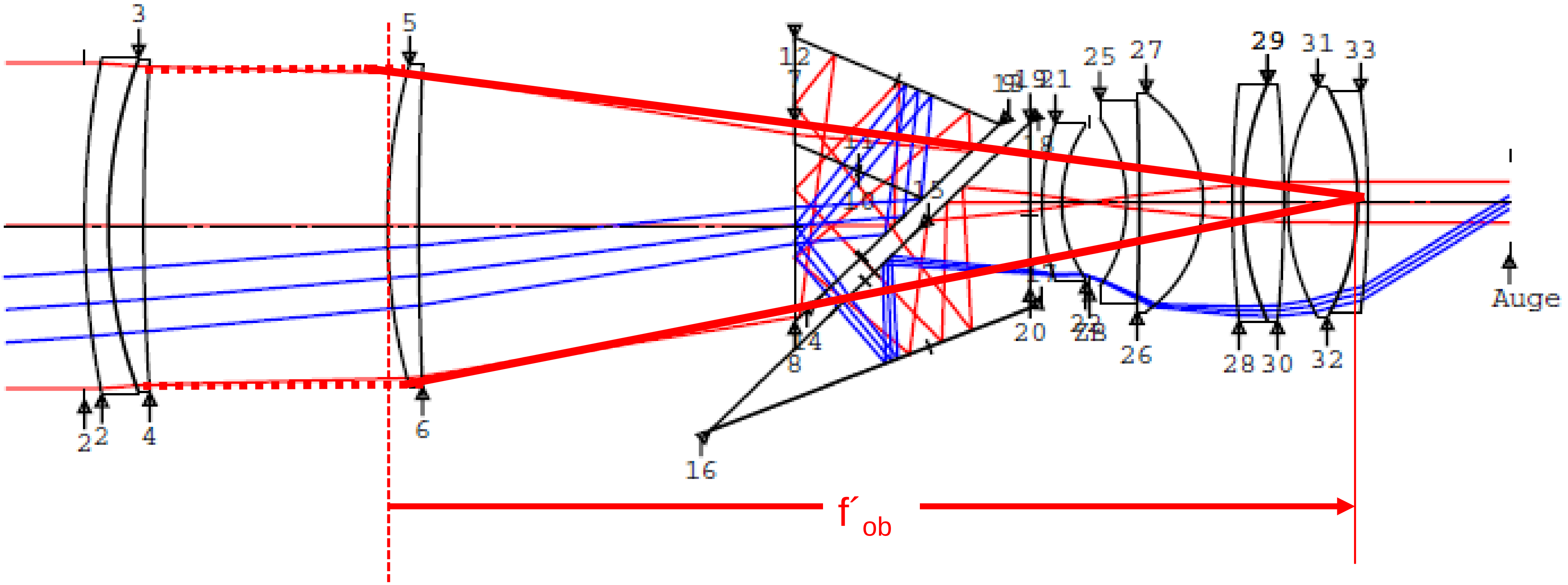
Required distance for afocal system:

$$d = f_1' + f_2' \qquad (*)$$

(*) follows from equation of refractive power of two subsystems for f´ → ∞:

$$\frac{1}{f'} = \frac{1}{f_1'} + \frac{1}{f_2'} - \frac{d}{f_1' f_2'} \stackrel{!}{=} 0$$

Focal length of Objective



P´ob
(back principal plane of objective)

Focal length of Eyepiece

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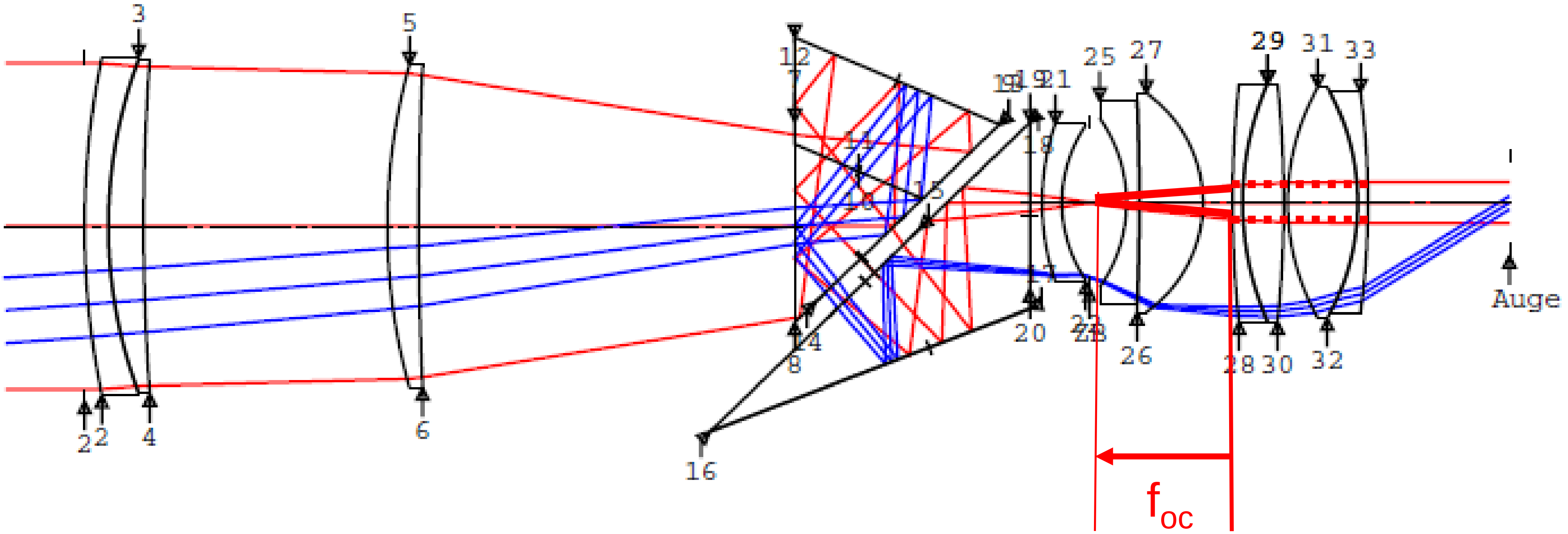
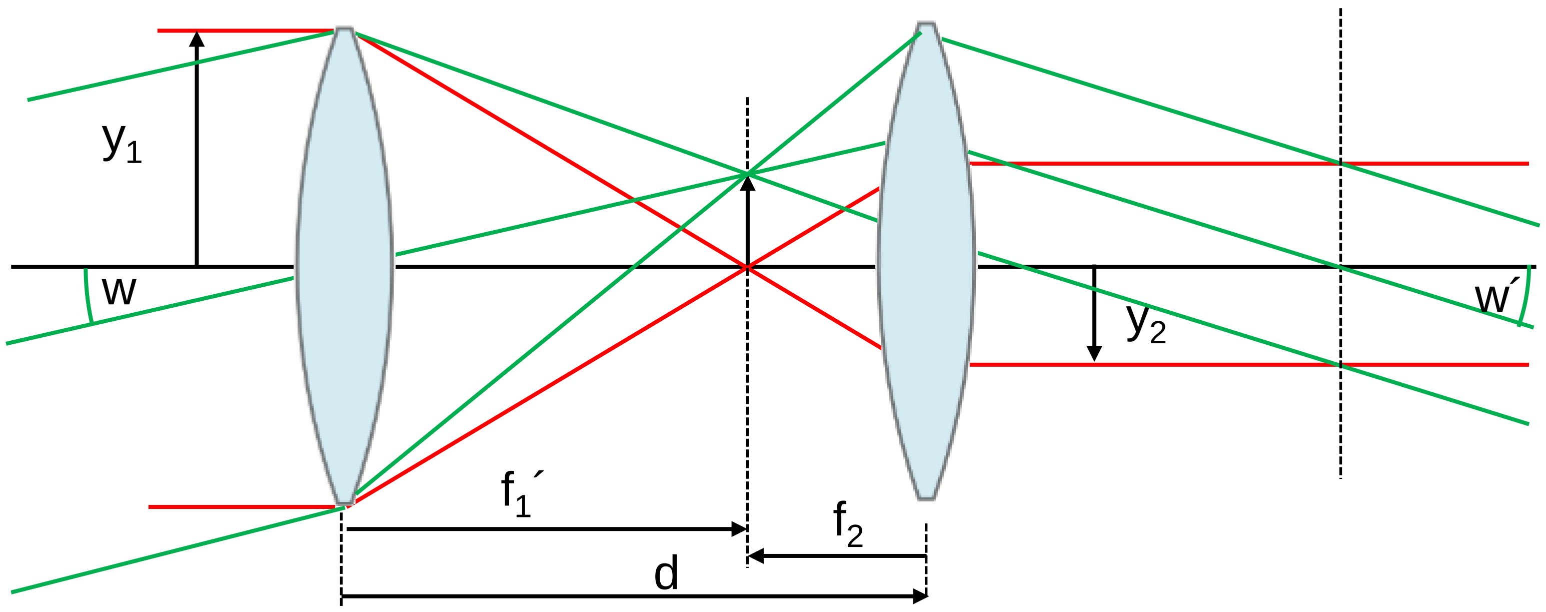


P_{oc}
(front principal plane of ocular)

Structure of an afocal system



In the intermediate image we have for the first subsystem:

$$y_1{}' = f_1{}' \tan(w)$$

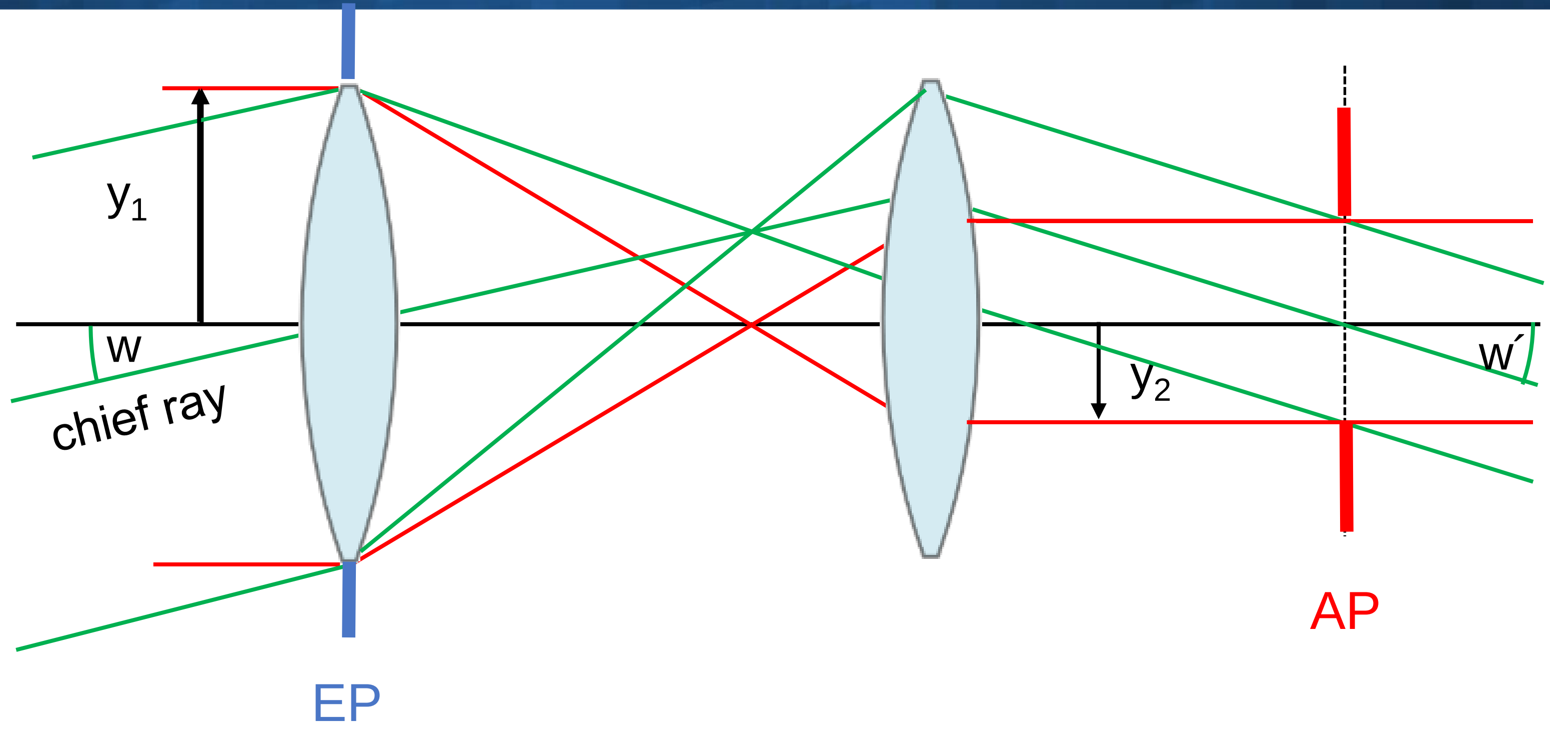
and correspondingly for the second subsystem

$$y_2 = f_2 \tan(w')$$

So, with $y_1{}' = y_2$:

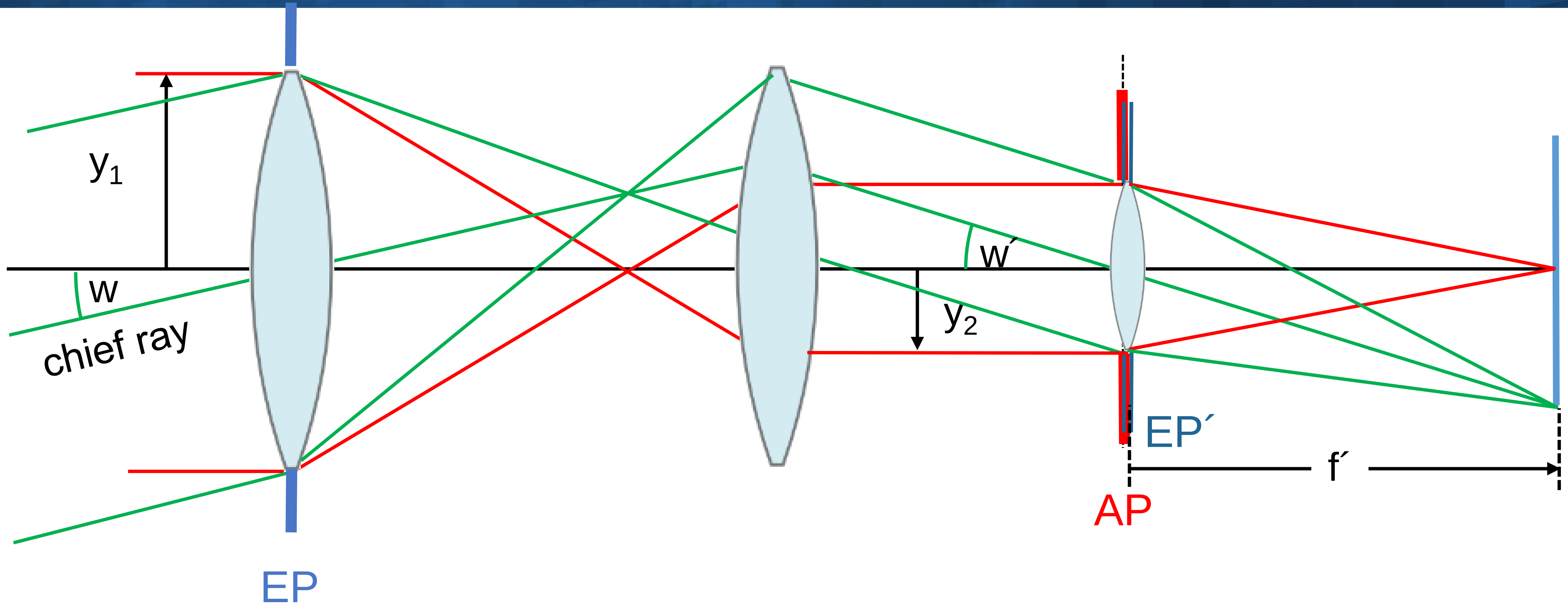
$$\boxed{f_1{}' \tan(w) = f_2 \tan(w')}$$

Structure of an afocal system



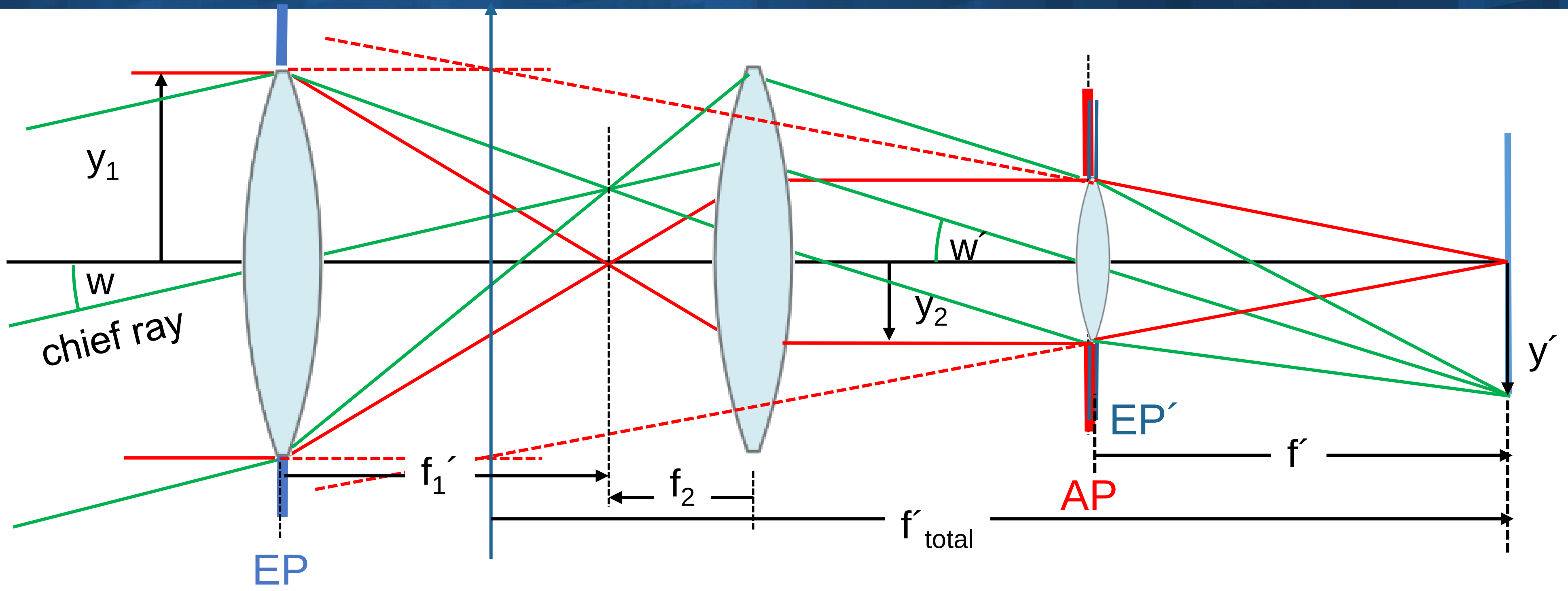
From the crossing points of the chief ray with the optical axis we can directly identify the position of entrance (EP) and exit pupil (AP).

Attaching focusing system at exit pupil: Pupil matching



At the position of the exit pupil (AP) we place a focusing system (e.g. a camera lens or the human eye) with focal length f´.
As we have chosen the entrance pupil EP´ of the focusing system of same size as the exit pupil of the afocal system AP and at exactly the same position the **pupils are matched**.

Attaching focusing system at exit pupil: Pupil matching



Afocal system: $f_1{}'\tan(w) = f_2\tan(w')$ [AS]

Focusing system: $\tan(w') = \dfrac{y'}{f'}$ [FS]

Total system (focusing + afocal): $\tan(w) = \dfrac{y'}{f_{total}{}'}$ [TS]

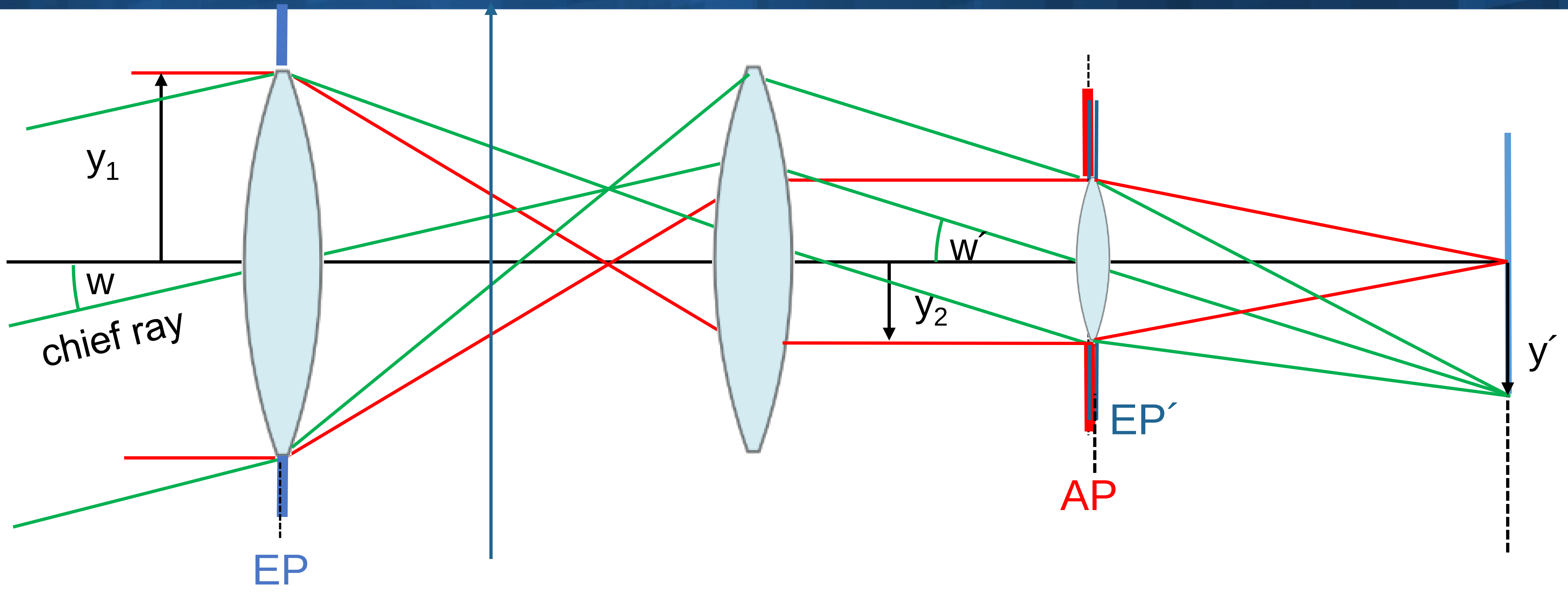
Thus, we find the relation between f'_{total} and f'

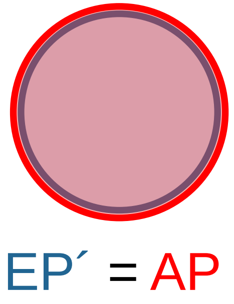
$$\frac{y'}{f_{total}{}'} = \tan(w) = \frac{f_2}{f_1{}'}\tan(w') = \frac{f_2}{f_1{}'}\frac{y'}{f'}$$, thus:

$$f' = \Gamma\, f_{total}{}'$$

Angular magnification: $\Gamma = \dfrac{\tan w'}{\tan w} = \dfrac{f_2}{f_1{}'}$

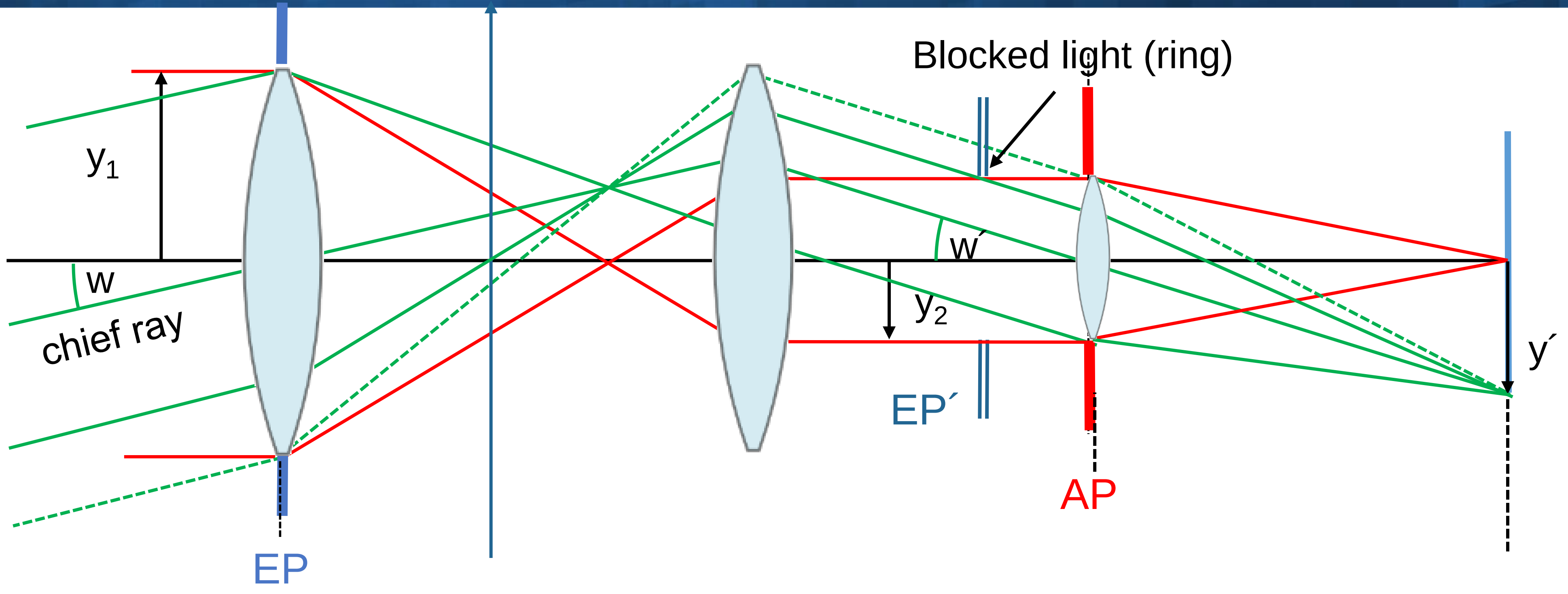
Attaching focusing system at exit pupil: Pupil matching

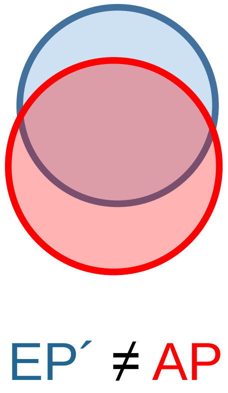




Pupils matched!

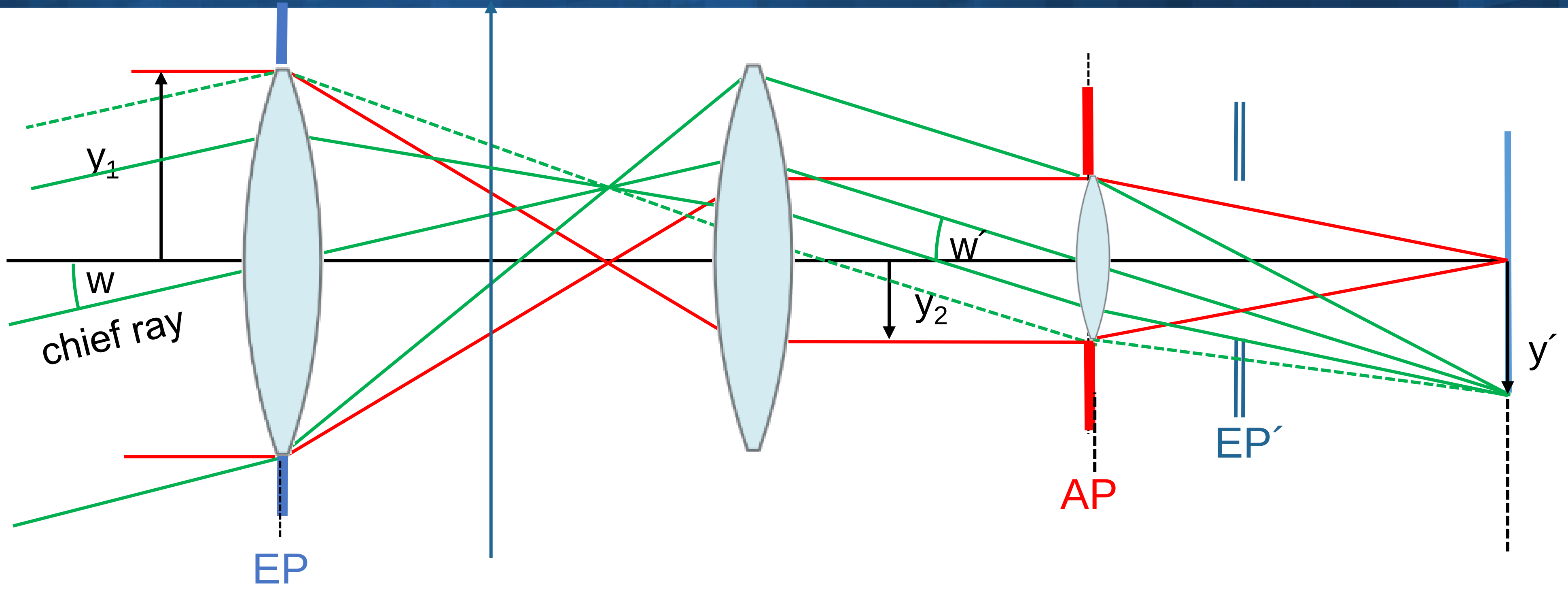
Attaching focusing system at exit pupil: Pupil matching

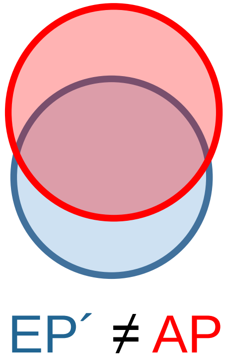




Entrance pupils out-of-focus!
Vignetting in image.

Attaching focusing system at exit pupil: Pupil matching





Entrance pupils out-of-focus!
Vignetting in image.

Attaching focusing system at exit pupil: Pupil matching

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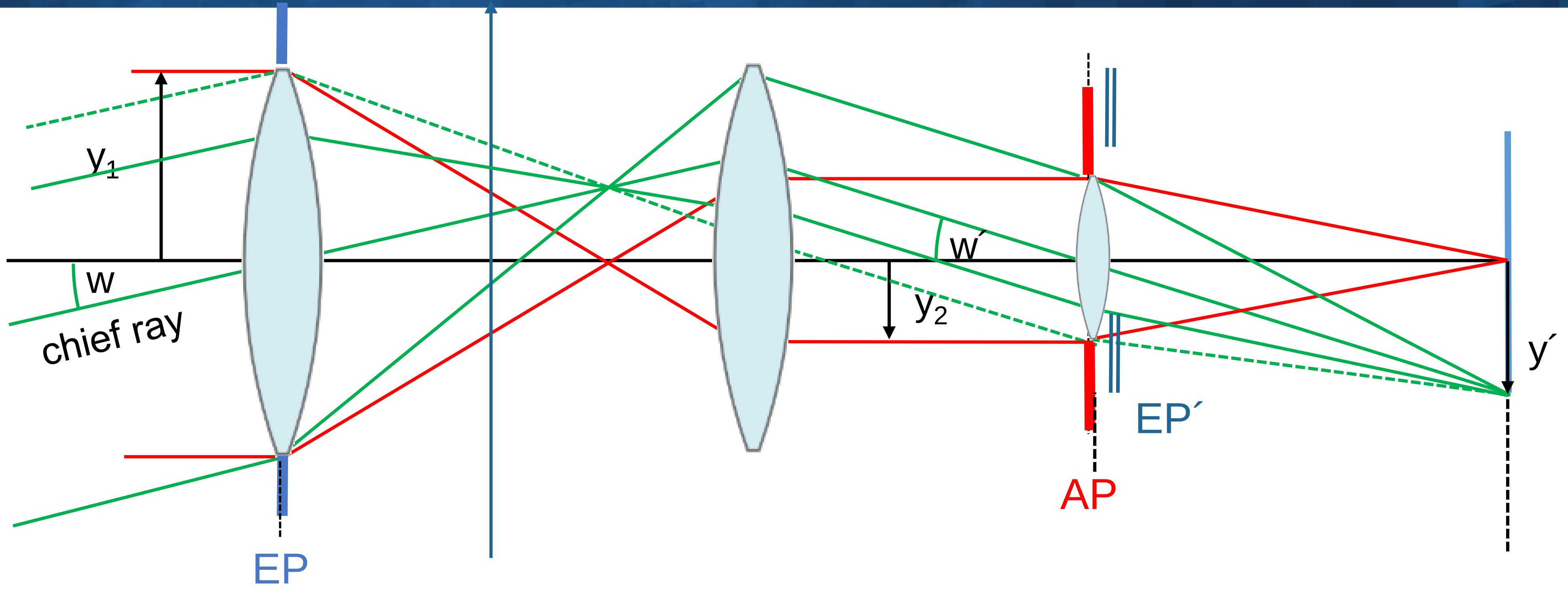
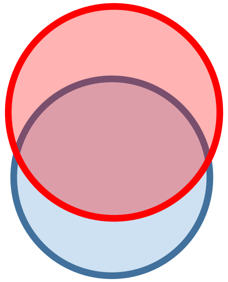
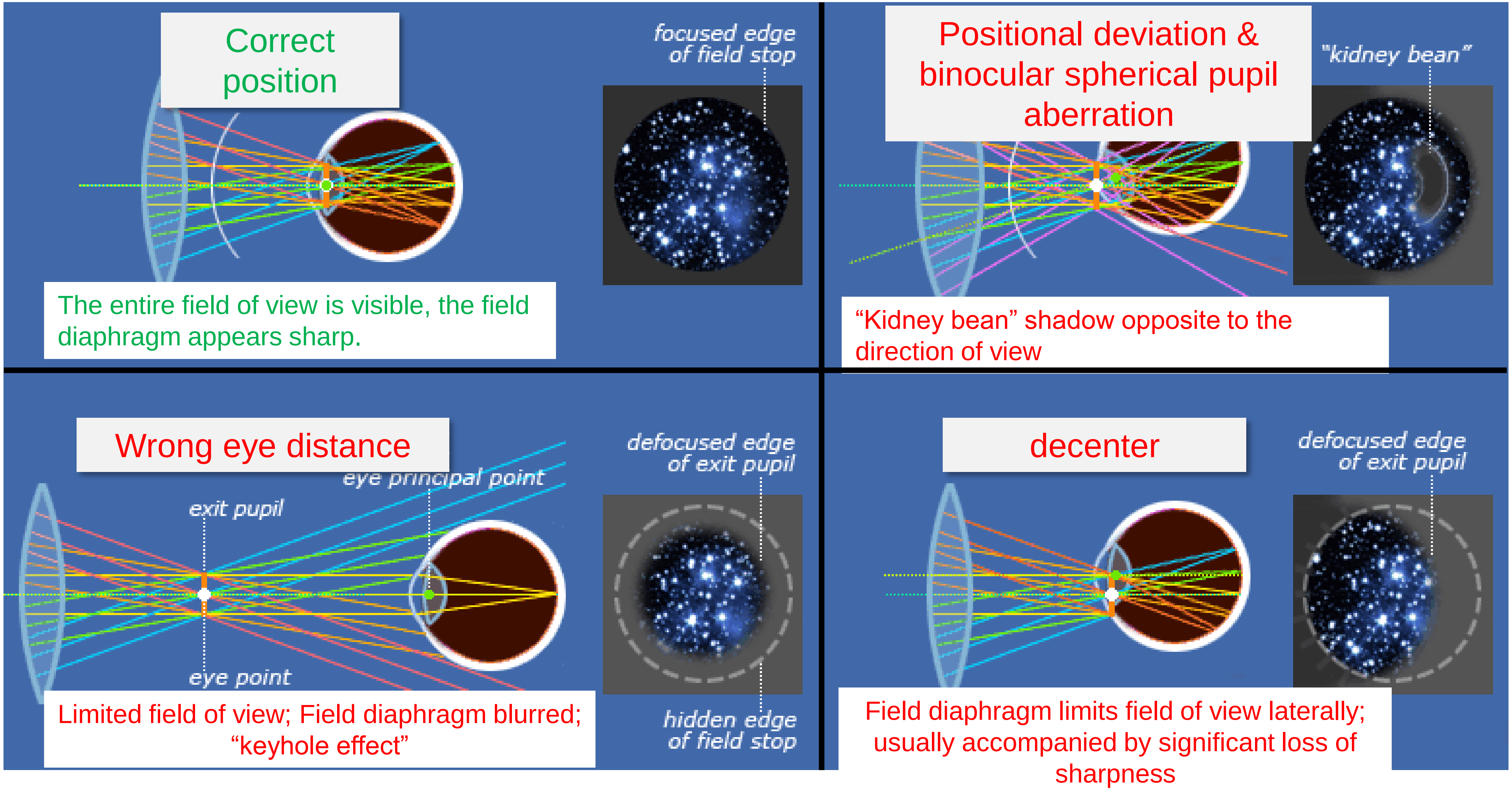




Entrance pupils out-of-focus!
Vignetting in image.

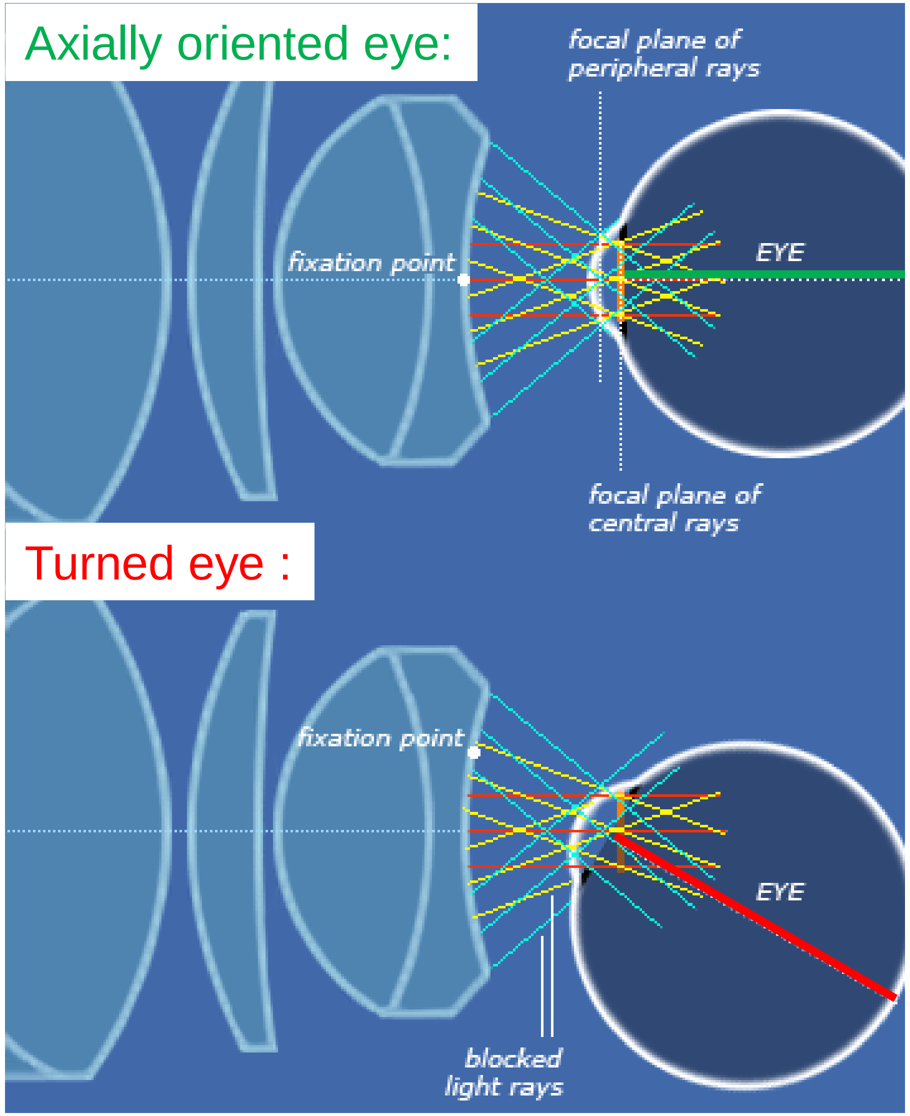
Effect of pupil mismatch or eyepiece aberrations

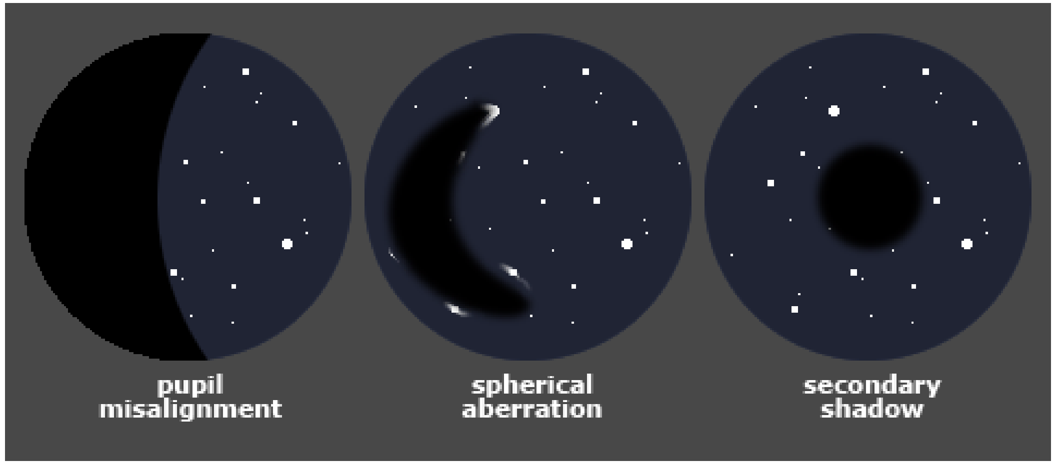


Eye rotation and viewing through an eyepiece

Axially oriented eye:

Turned eye :



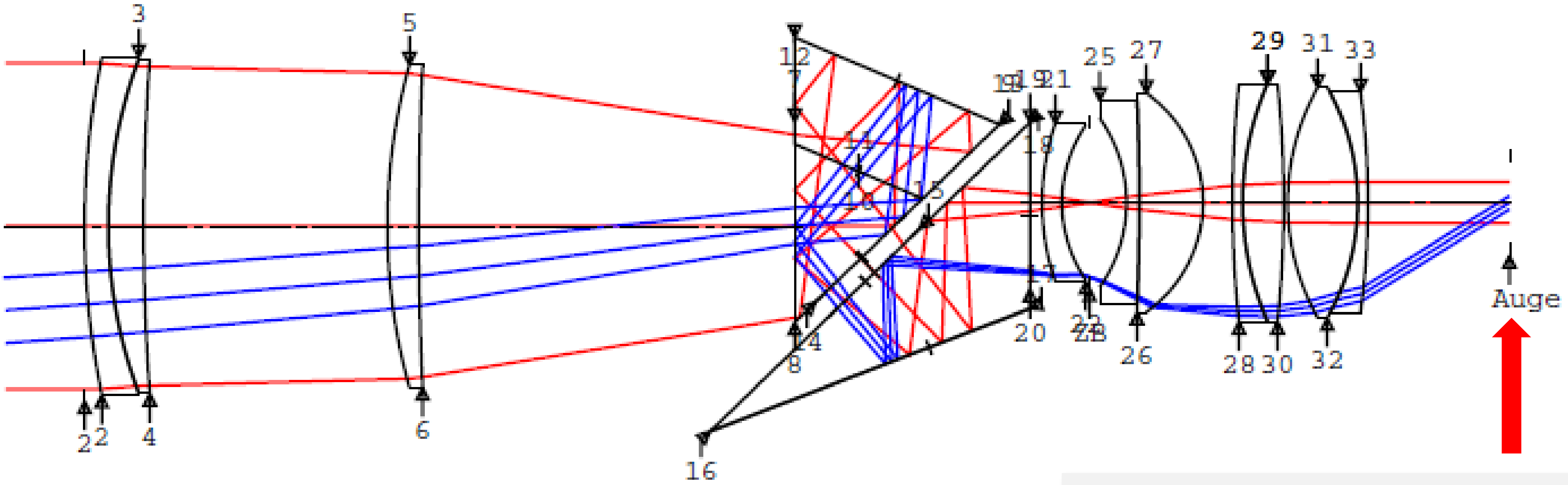


As soon as you have selected an image section by swiveling the binoculars, the binoculars rest and the eye observes local areas (foveal vision) in this section by rotating.

With the rotated eye, the binocular to eye system is relatively tilted, i.e. there are two optical axes.

Aberrations of the binoculars appear differently depending on the tilt direction of the eye.

Entrance and exit pupil



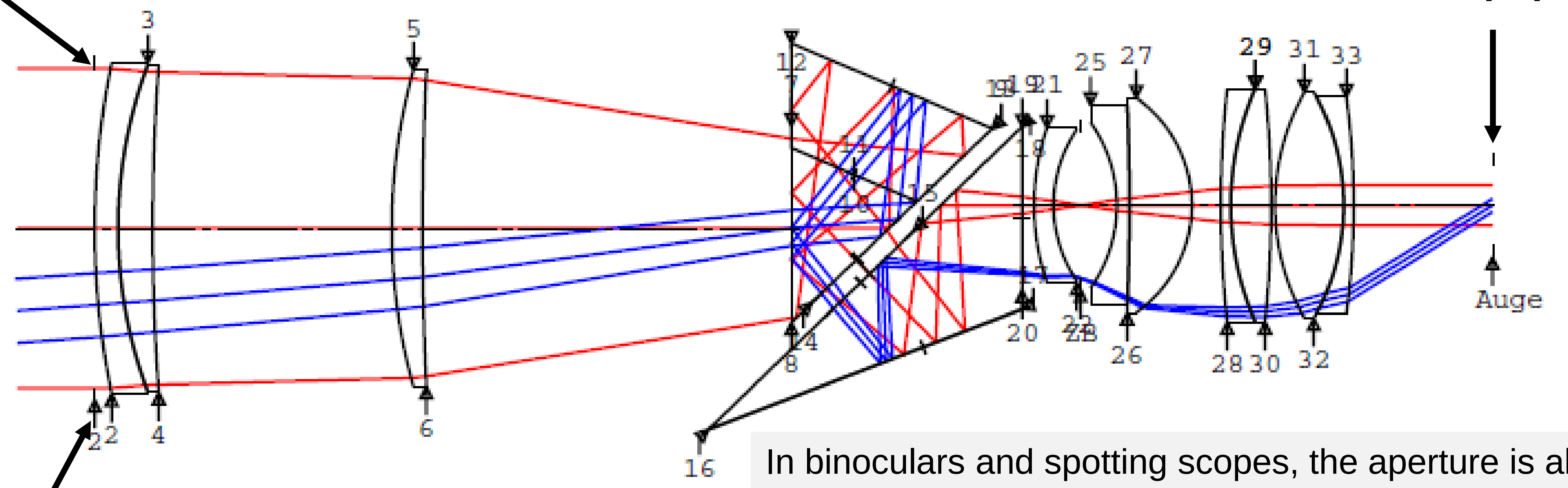
Here we need a real* exit pupil! We place the pupil of our eye there and thus obtain an image on the retina.

*: with photographic lenses, the exit pupil is always a virtual image of the aperture. There, the position of the AP characterizes the telecentricity (main beam angle to the perpendicular of the image plane) in the image plane: tan(telecentricity angle) = image height/(distance AP to image plane).

Entrance and exit pupil

Stop*

Exit pupil



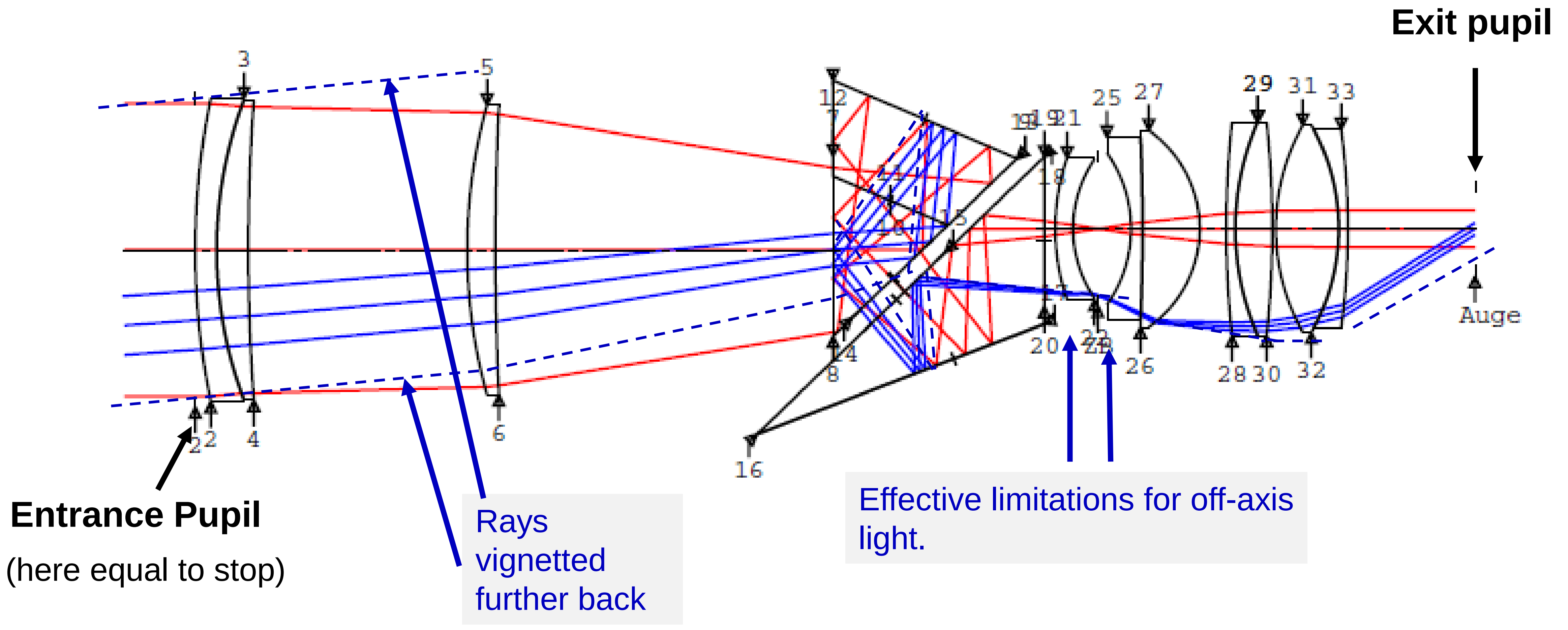
Entrance Pupil

(here equal to stop)

In binoculars and spotting scopes, the aperture is almost always in front of the first objective lens* and therefore corresponds to the entrance pupil (EP = “the aperture imaged into the object space”).
This is why the entrance pupil diameter is often simply referred to as the objective lens diameter.

*: würde man die Blende weiter nach hinten legen, dann würde der Durchmesser des Objektivs größer werden.

Entrance and exit pupil



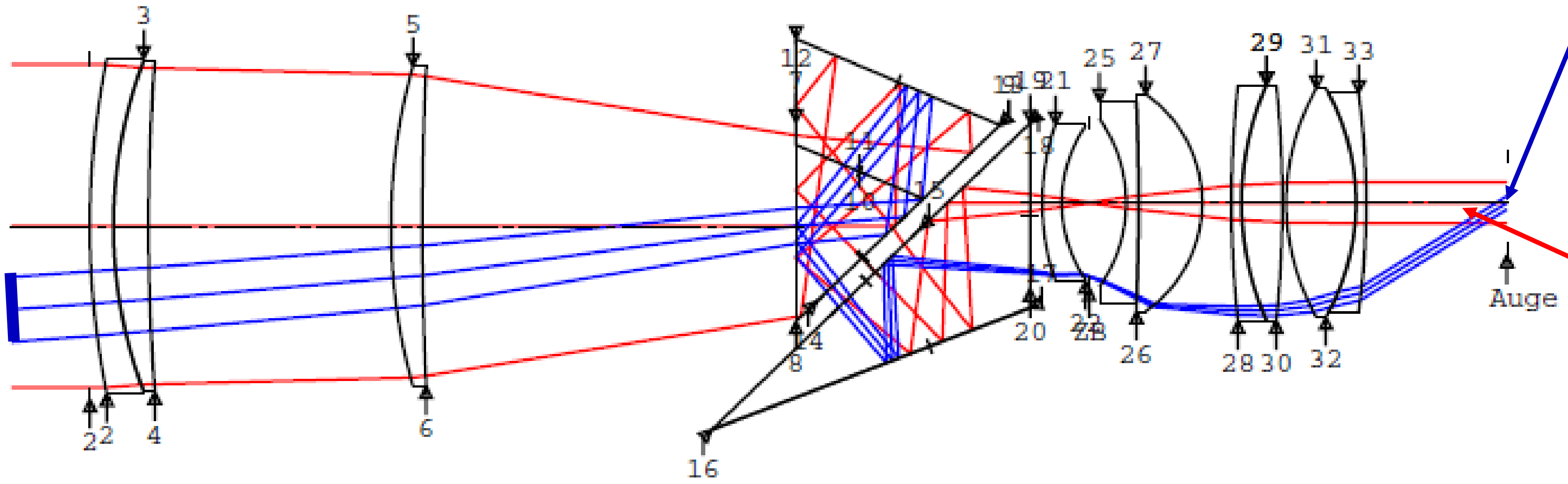
Exit pupil of binoculars

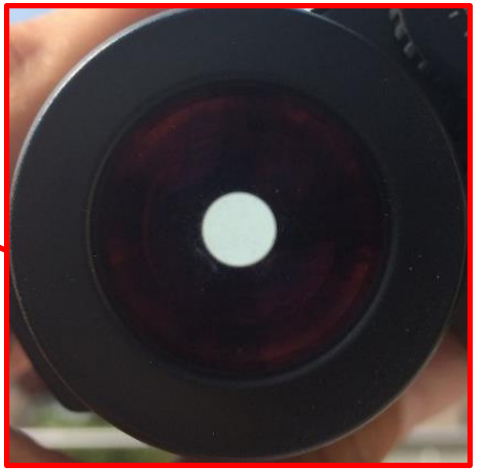
The exit pupil can be seen when looking into the binoculars from the eyepiece side. The exit pupil of the binoculars is round when viewed directly, and smaller and “cat's eye-shaped” when viewed at an angle. You can see that the exit pupil is outside the binoculars if you turn them slightly.

Useful light

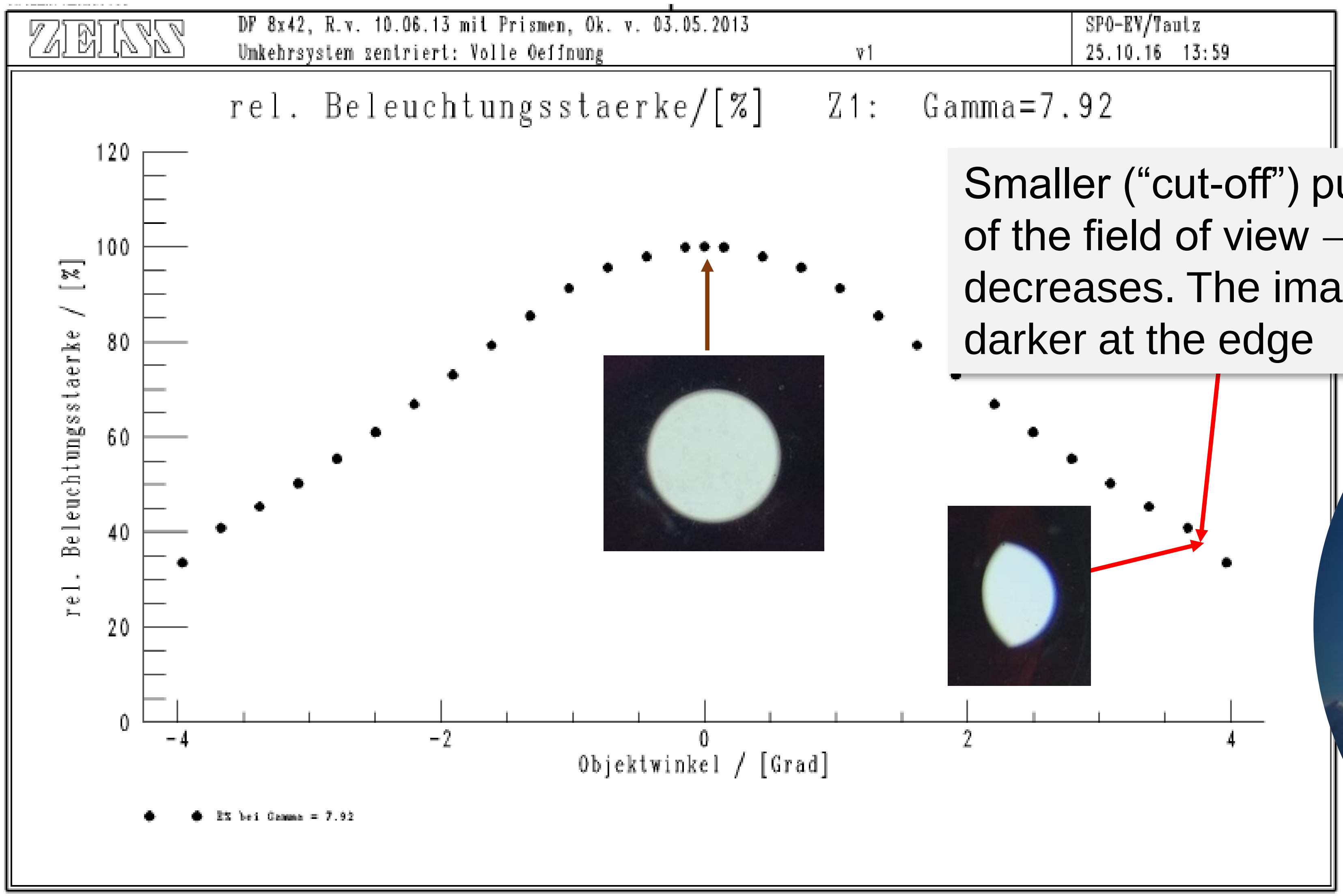
At higher viewing angles, less useful light reaches the eye due to the trimming at the field lenses/mount edges (usually on the eyepiece side).The bundle of rays is truncated and takes on the shape of a “cat's eye”. Due to the smaller bundle area, a lower image intensity reaches the edge of the field of view than its center: the “relative illuminance” decreases towards the edge of the field.

Useful light

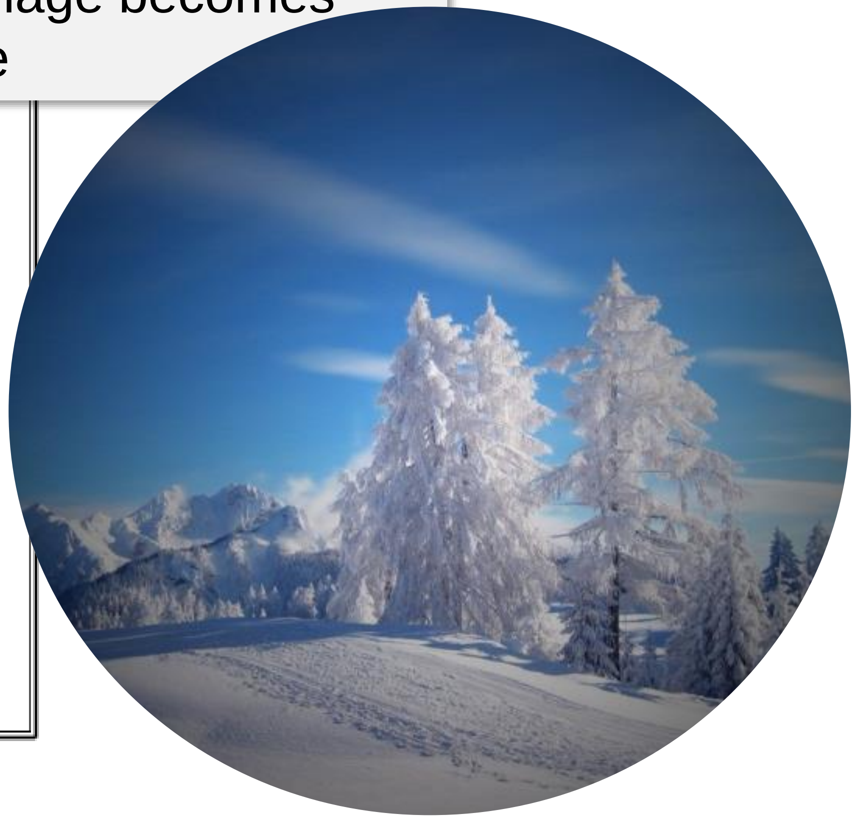




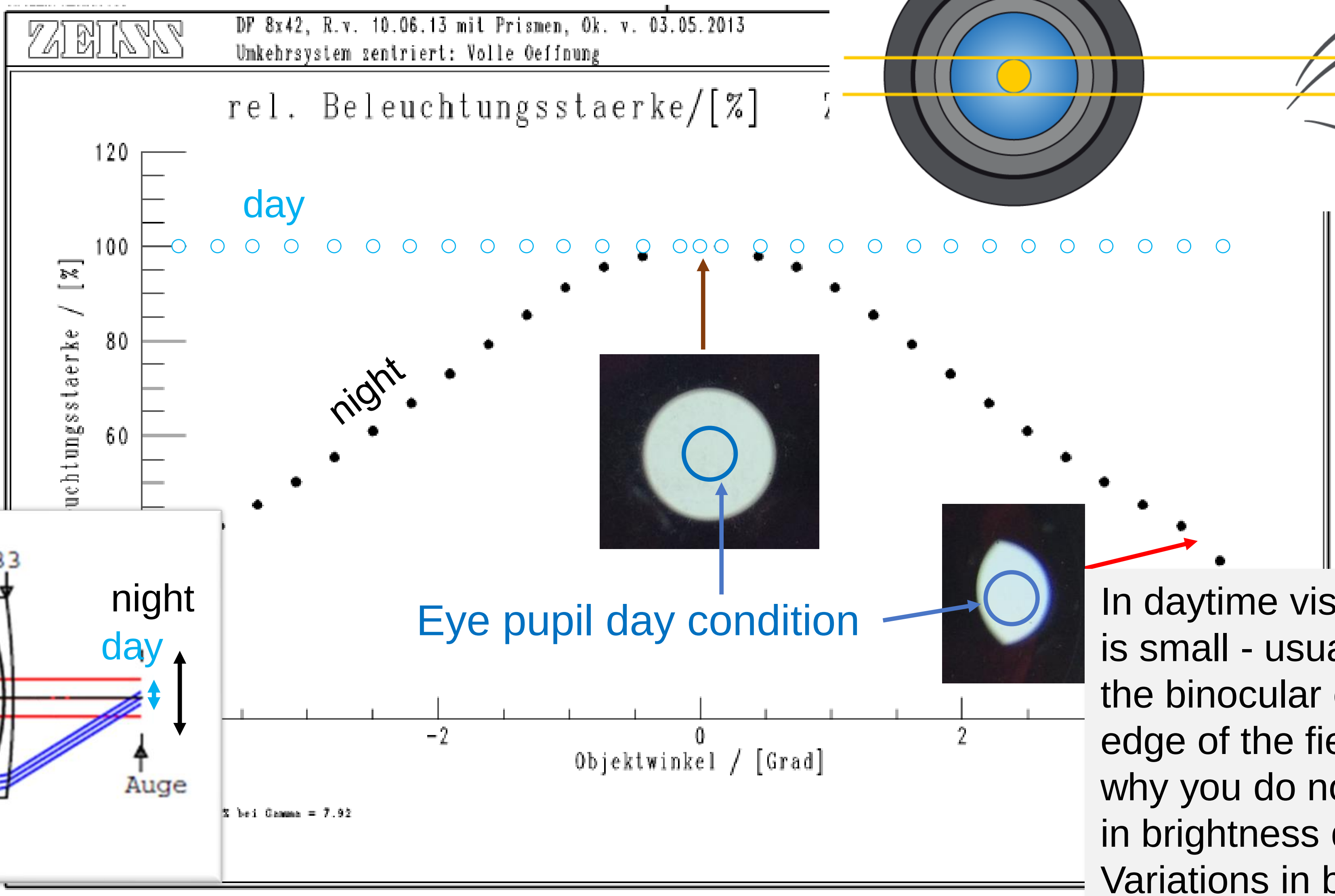
Relative illuminance

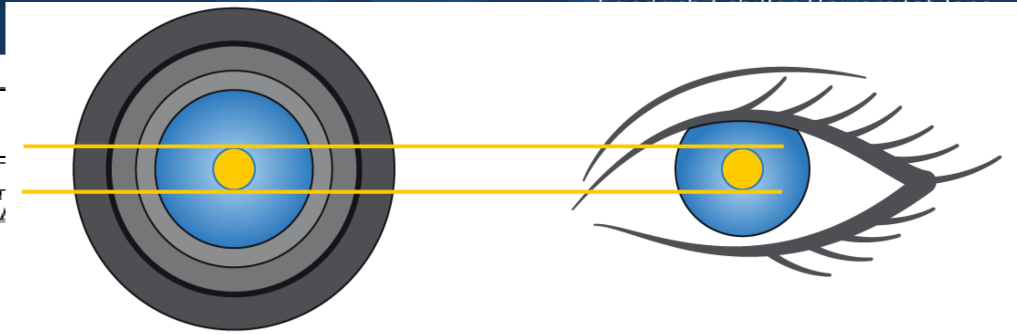


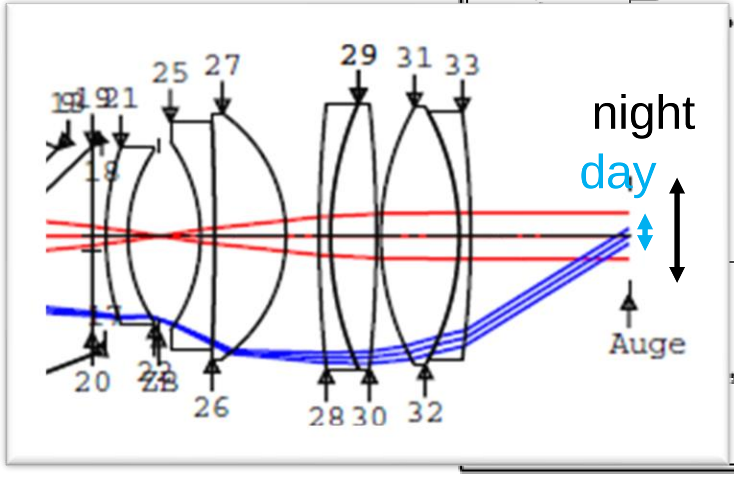
Smaller ("cut-off") pupil at the edge of the field of view → Illuminance decreases. The image becomes darker at the edge



Relative illuminance: Daytime conditions







In daytime vision, the eye pupil is small - usually smaller than the binocular exit pupil at the edge of the field of view. This is why you do not perceive a drop in brightness during the day. Variations in brightness increase with twilight vision.

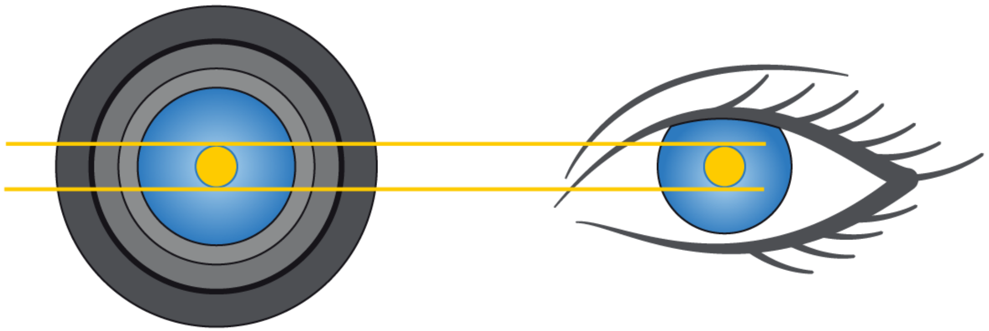
Eye pupil as effective stop

The pupil of the eye can become smaller than the exit pupil provided by the binoculars (→ day vision)

$$L \sim AP_{\mathrm{eff}}^2$$

$$AP_{\mathrm{eff}} = \min\left(AP_{\mathrm{binocular}}, EP_{\mathrm{eye}}\right)$$

Luminous flux is proportional to the area of the effective exit pupil, i.e. “the smaller of the two, whether the exit pupil of the binoculars or the eye pupil”.
For example, with night vision (6mm diameter eye pupil) to day vision (2mm) we have a higher luminous flux in the ratio $(6\text{mm}/2\text{mm})^2 = 9$ (if the binoculars provide at least 6mm exit pupil).



Limiting aperture: Binocular exit pupil or eye entrance pupil

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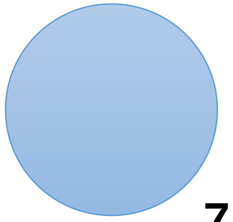
Exit pupil of binocular

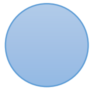
8 x 56



7 mm

$$\varnothing_{AP} = \varnothing_{EP}/\Gamma$$

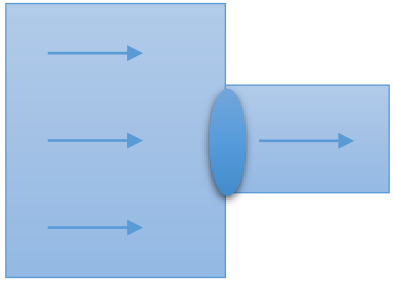
8 x 20

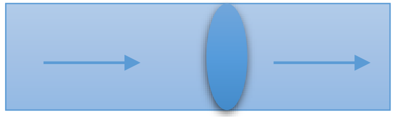


2,5 mm

What use is the large entrance pupil?

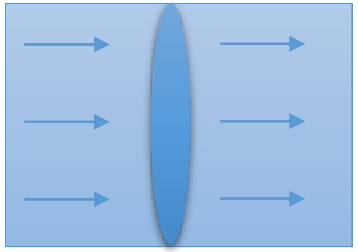
Day Eye pupil
z.B. 2,5 mm

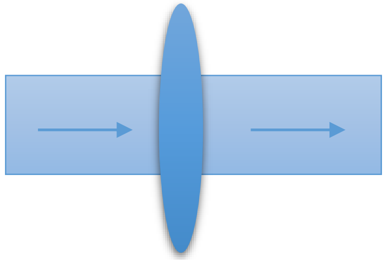




During the day, the large aperture remains unused.
(The only advantage of the larger aperture is that it is more ergonomic, as the eye can be decentered.)

Night Eye pupil
z.B. 7 mm





At night more light enters the eye.
(Here a factor $(7/2.5)^2=7.84$)

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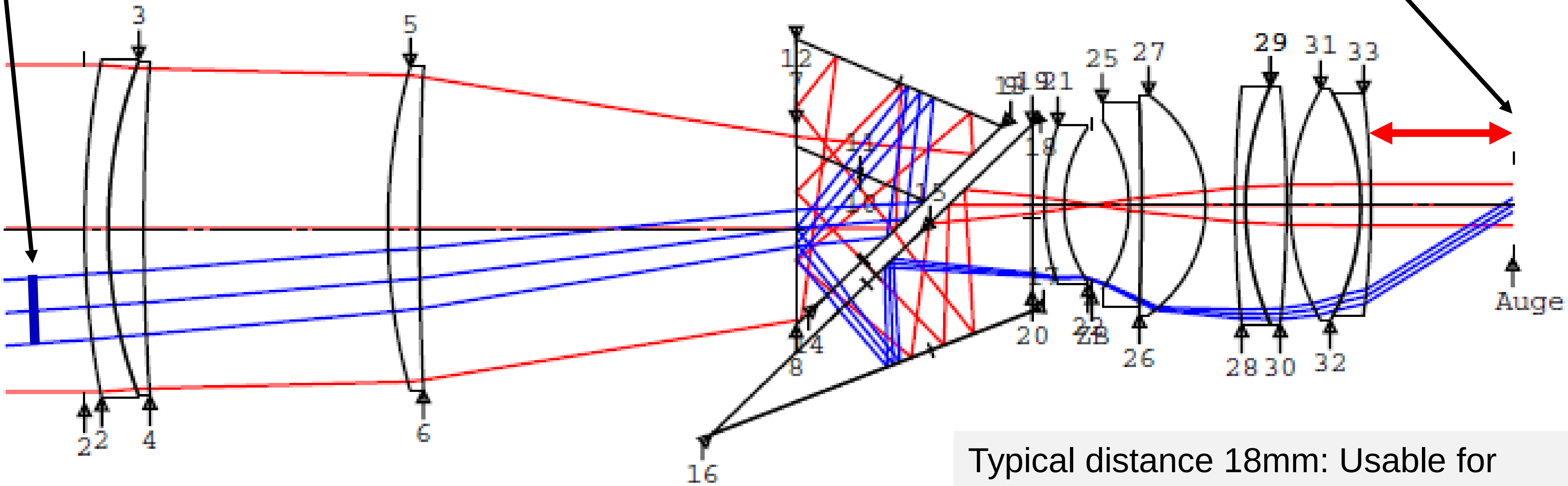
Eye relief

EP of ray bundle

AP of ray bundle

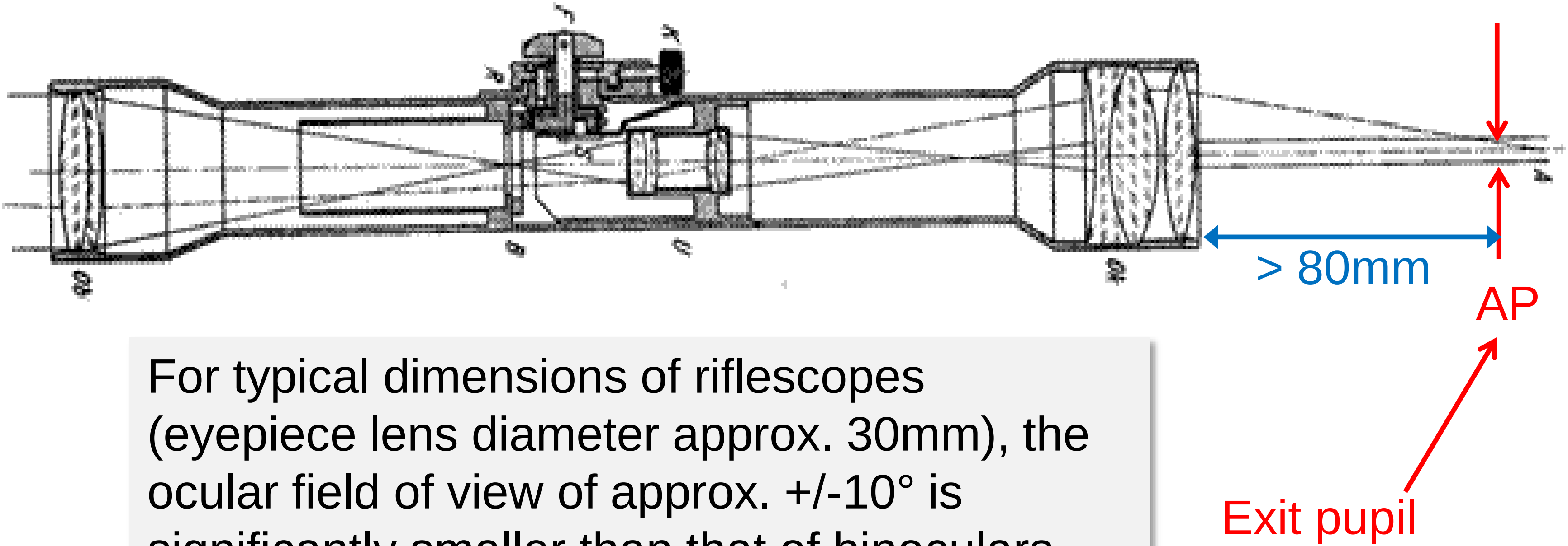


The intersection of the central beam of the off-axis bundle with the optical axis gives the position of the exit pupil (to which the eye is guided). The bundles of all angles of vision should meet at this one point (this is the task of the optical designer to achieve this)

Typical distance 18mm: Usable for spectacle wearers
Larger distance would increase eyepiece diameter

Exit pupil far outside the system (example rifle scope)

Due to the recoil of the weapon, the exit pupil must be more than 80 mm away from the eyepiece to avoid injury to the eye.



For typical dimensions of riflescopes (eyepiece lens diameter approx. 30mm), the ocular field of view of approx. +/-10° is significantly smaller than that of binoculars (+/-30° or more).

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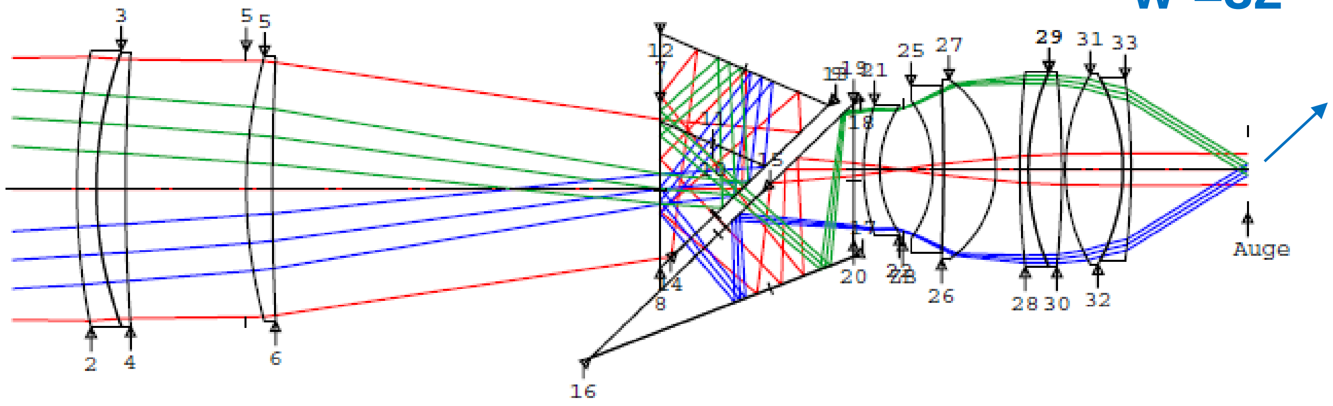
Example: Binocular and Rifle Scope both at 8x angular magnification

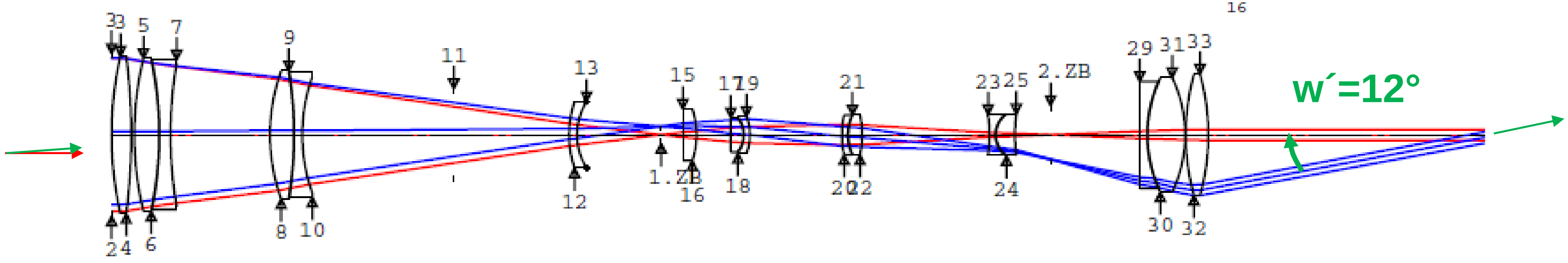
w´=32°

$$View\ Field'[m] = 1000m \cdot 2 \cdot \tan w'/\Gamma$$

Binocular $\textbf{View field}'[\mathbf{m}] = \mathbf{156m}$

Rifle Scope $\textbf{View field}'[\mathbf{m}] = \mathbf{53m}$





w´=12°

Pupil position at expense of field-of-view

Adapter solutions

Adapter solutions

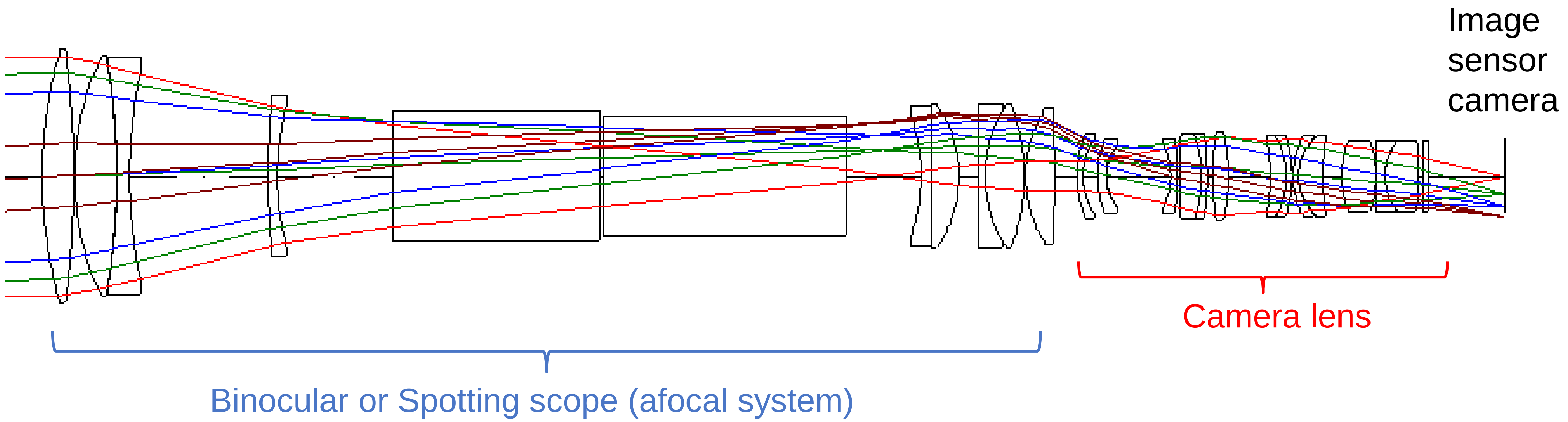
Relevant data for pupil matching:

- $\varnothing_{AP}$
- w'
- Eye relief*

- $\varnothing_{EP}$
- w
- Position EP

*Eye relief = distance last lens to exit pupil

Example: Camera adapted binocular via camera lens

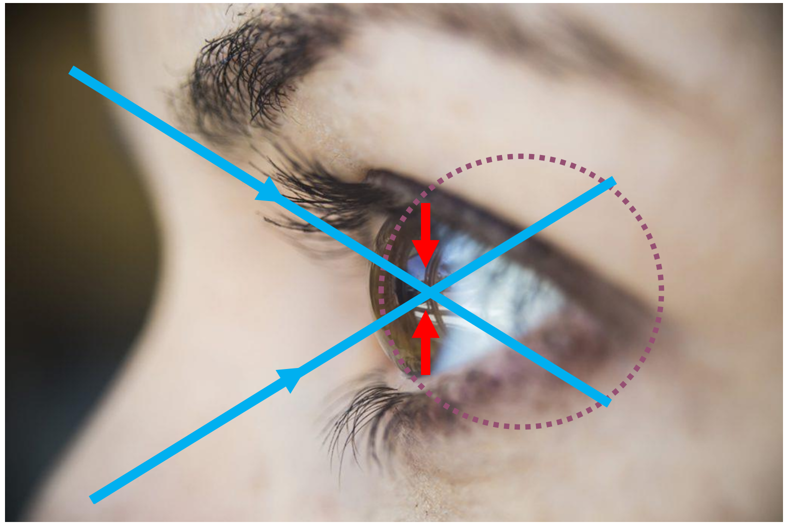


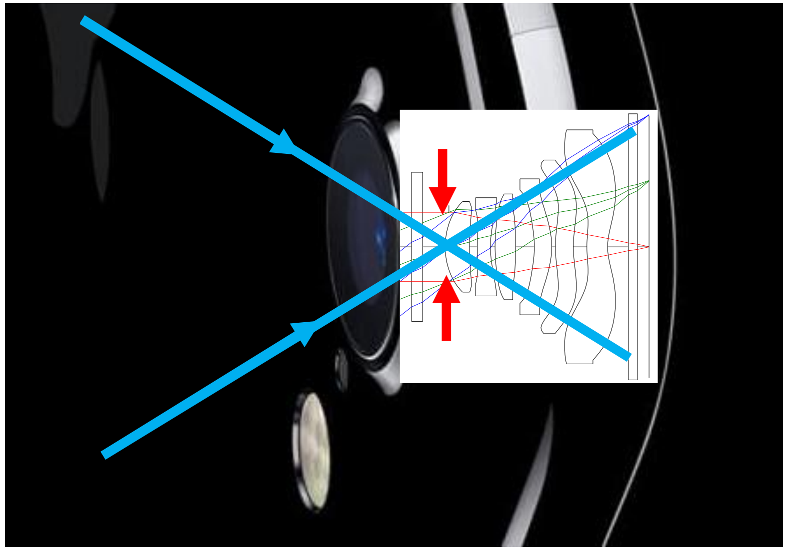
Connecting mobile phone or tablet cameras with optical systems

„star tracking“

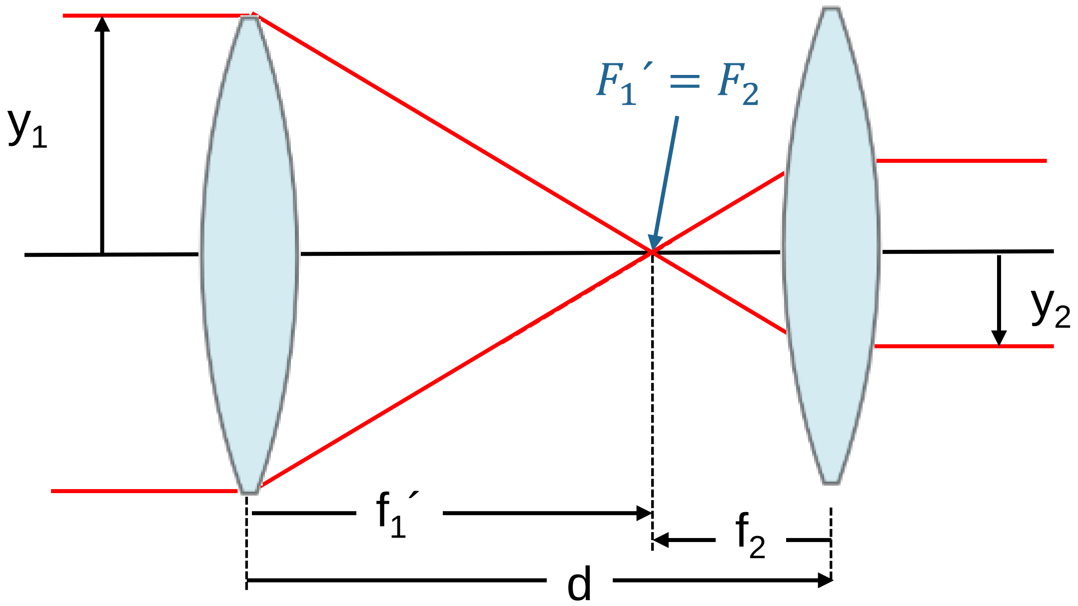
Adaption binocular and smartphone camera

- The typical **angle of view** of cell phone lenses **75°** (full diagonal) is similar to the subjective angle of view of many eyepieces (microscope, telescope, spotting scope, binoculars, ...)
- The typical **entrance pupil diameter** of cell phone lenses of 1.5-**2mm** is similar to that of the eye during the day
- The entrance pupil of cell phone lenses is at the front, making it easily **accessible**
- This means that no additional optics are required to connect spotting scopes/binoculars to cell phone cameras





Structure of an afocal system



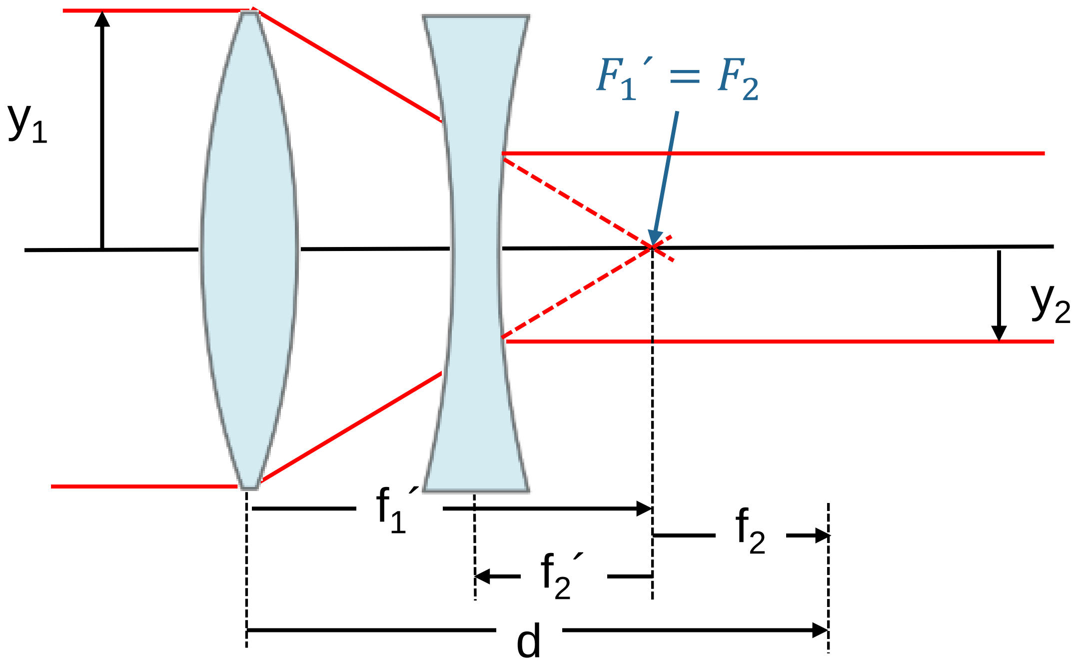
Required distance for afocal system:

$$d = f_1' + f_2' \qquad (*)$$

(*) follows from equation of refractive power of two subsystems for f´ → ∞:

$$\frac{1}{f'} = \frac{1}{f_1'} + \frac{1}{f_2'} - \frac{d}{f_1' f_2'} \overset{!}{=} 0$$

Structure of an afocal system

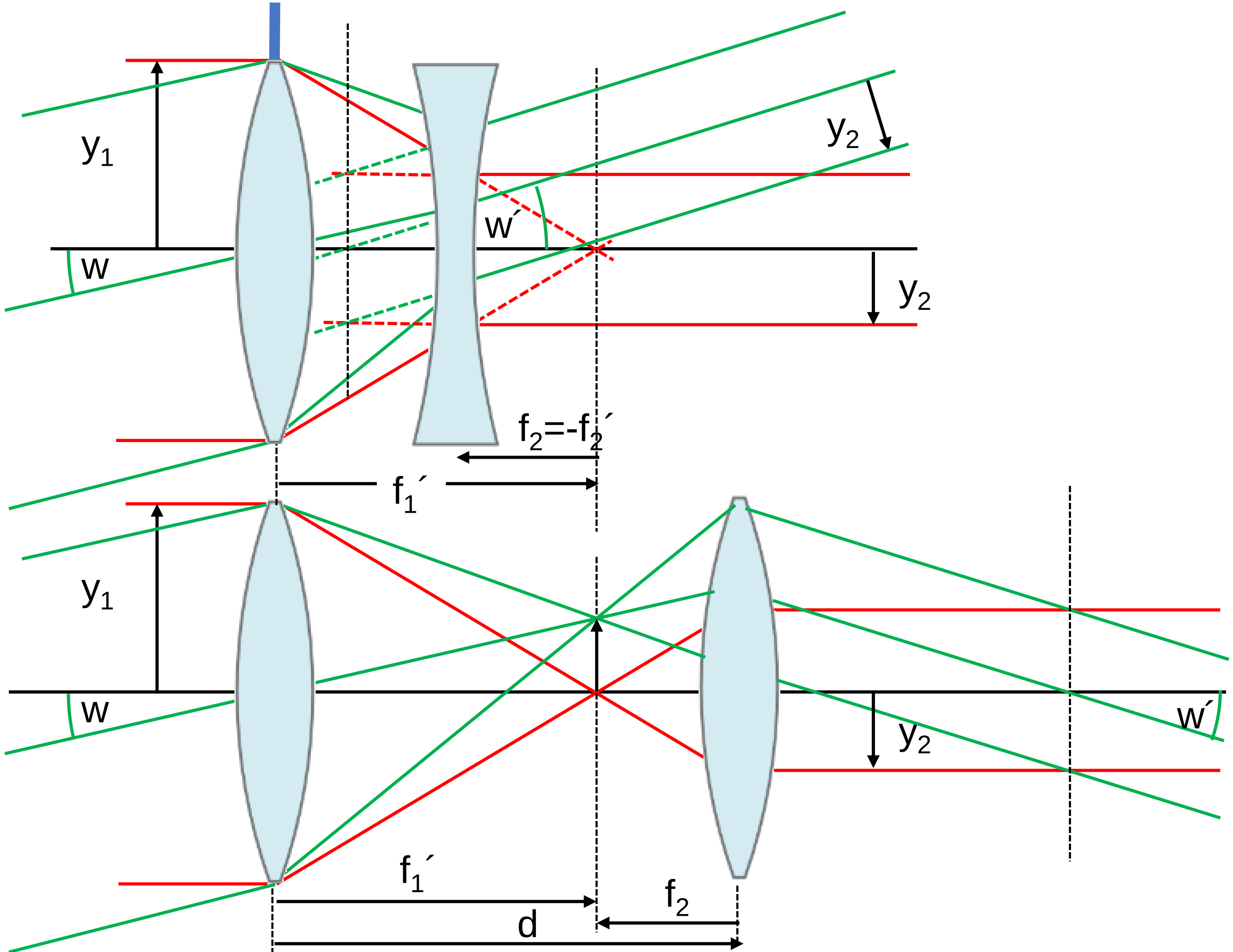


Same condition for afocal system,

$$d = f_1{}' + f_2 \qquad (*)$$

Now fulfilled with a negative focal length $f_2{}'$.

Alternative setup with same absolute value of magnification

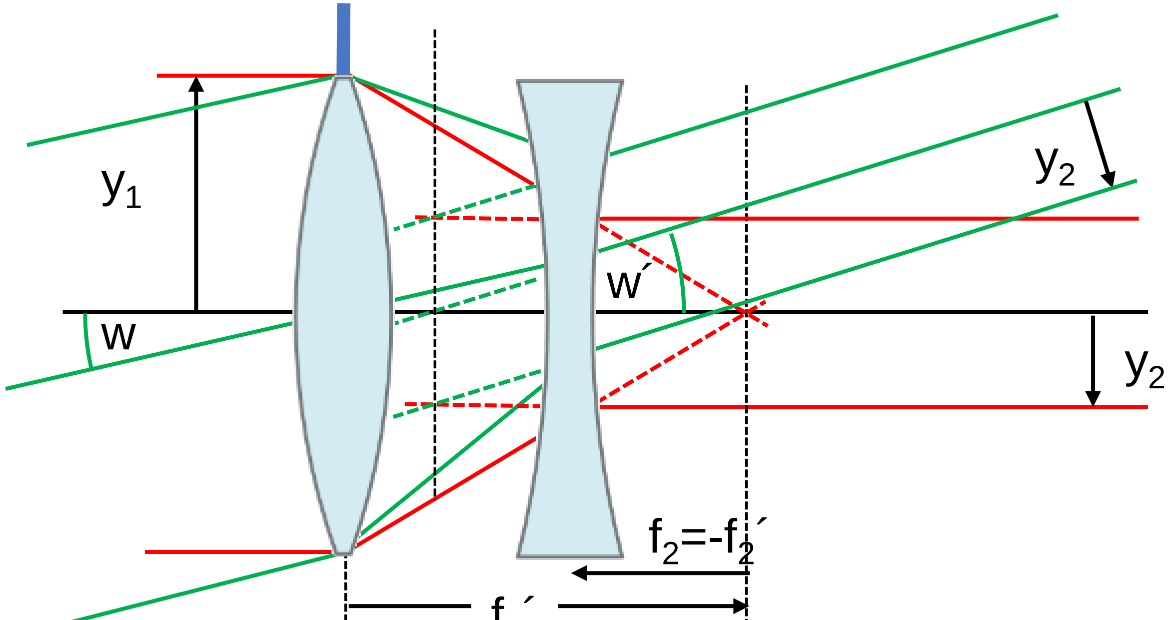


According to $\Gamma = \frac{\tan w'}{\tan w} = \frac{f_2}{f_1'}$

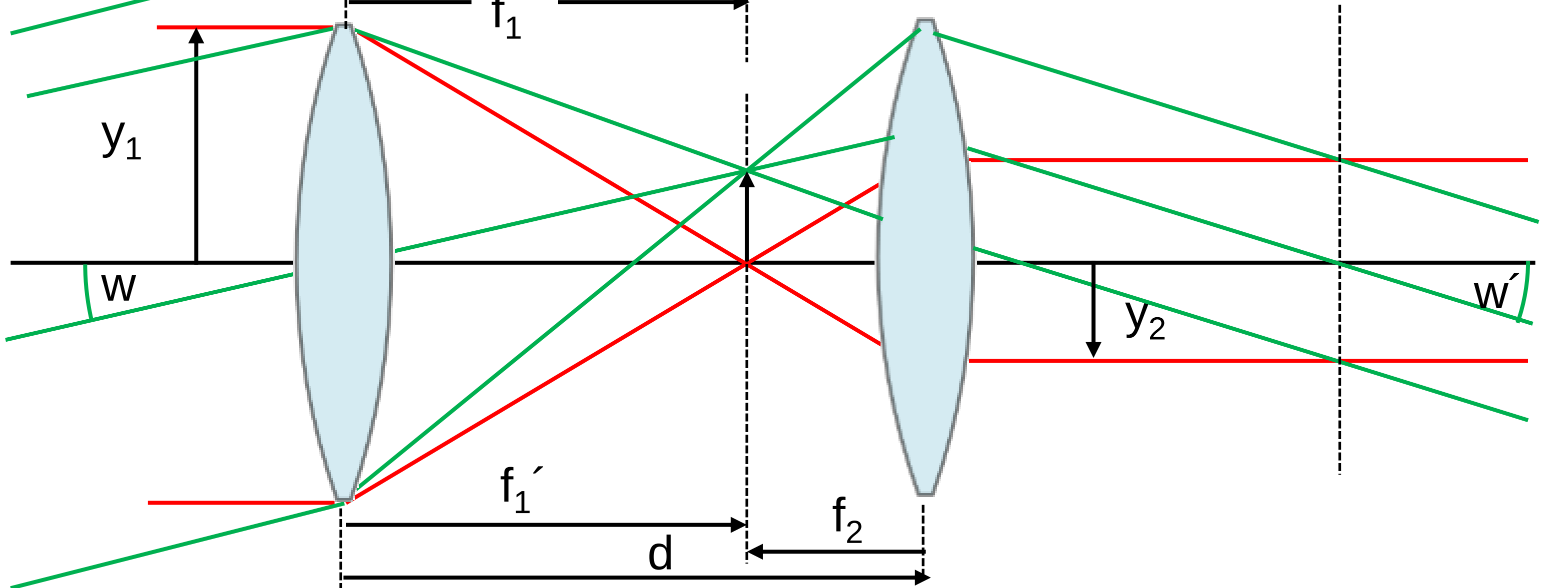
With the sign change of f_2 the absolute value of angular magnification is unchanged and also abs(w´).

However with the sign change of Γ the image orientation changes.

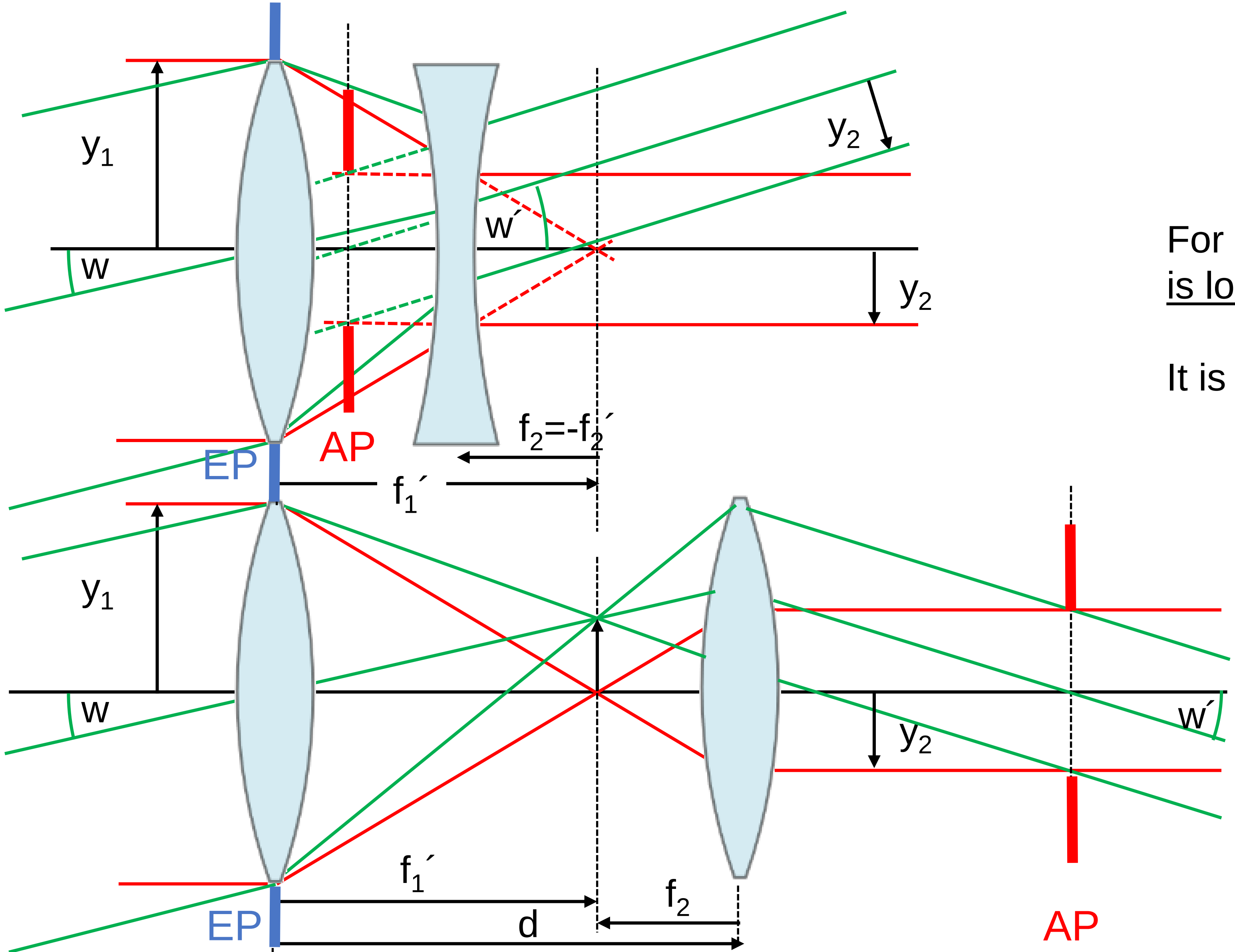
Galilei and Kepler afocal system layout



Galilei Layout



Kepler Layout



For the <u>shown</u> Galilei setup the <u>exit pupil (AP)</u> <u>is located to the left</u> of the back negative lens.

It is **not well accessible**.

Position of entrance and exit pupil (Galilei setup)

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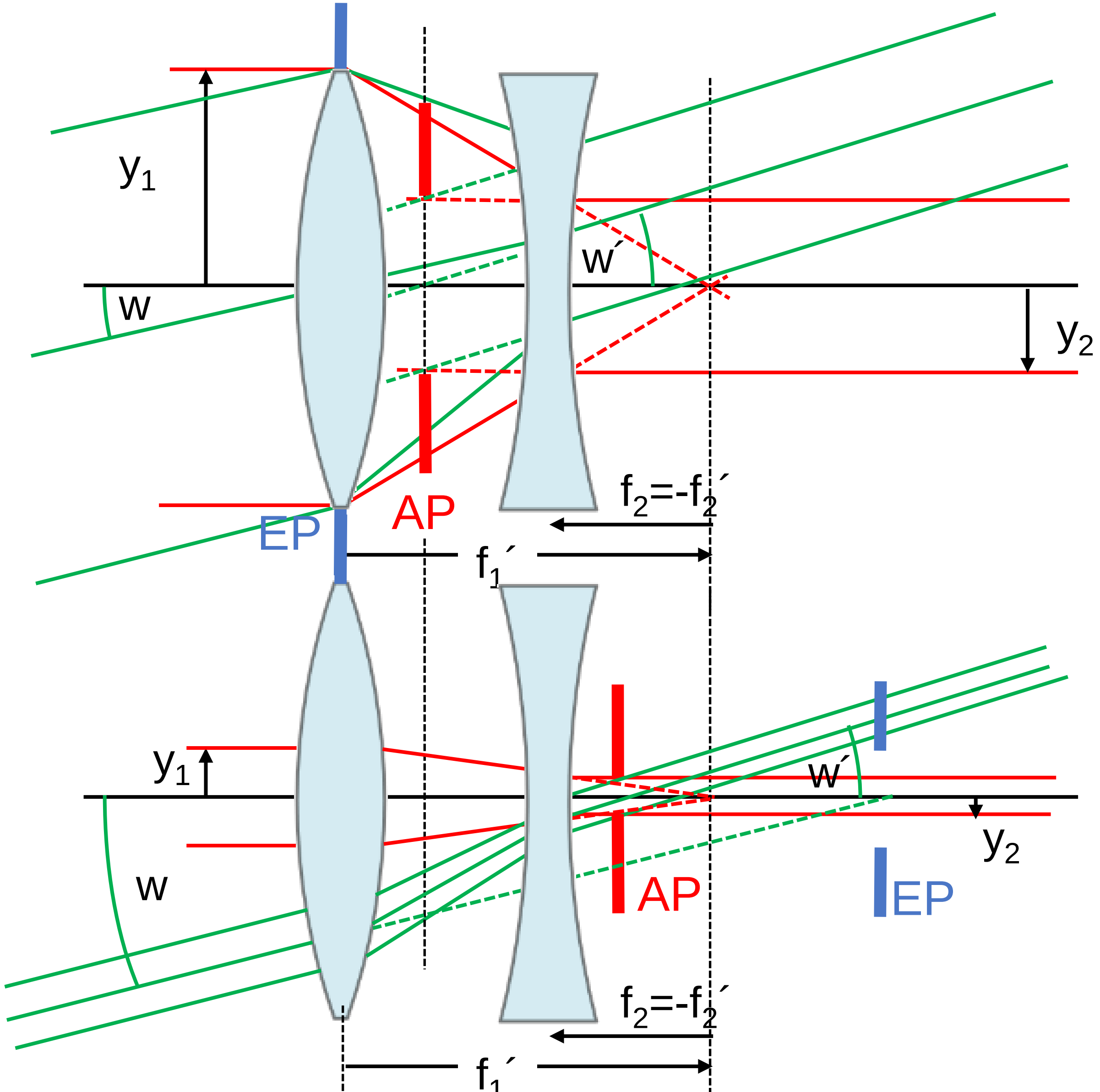
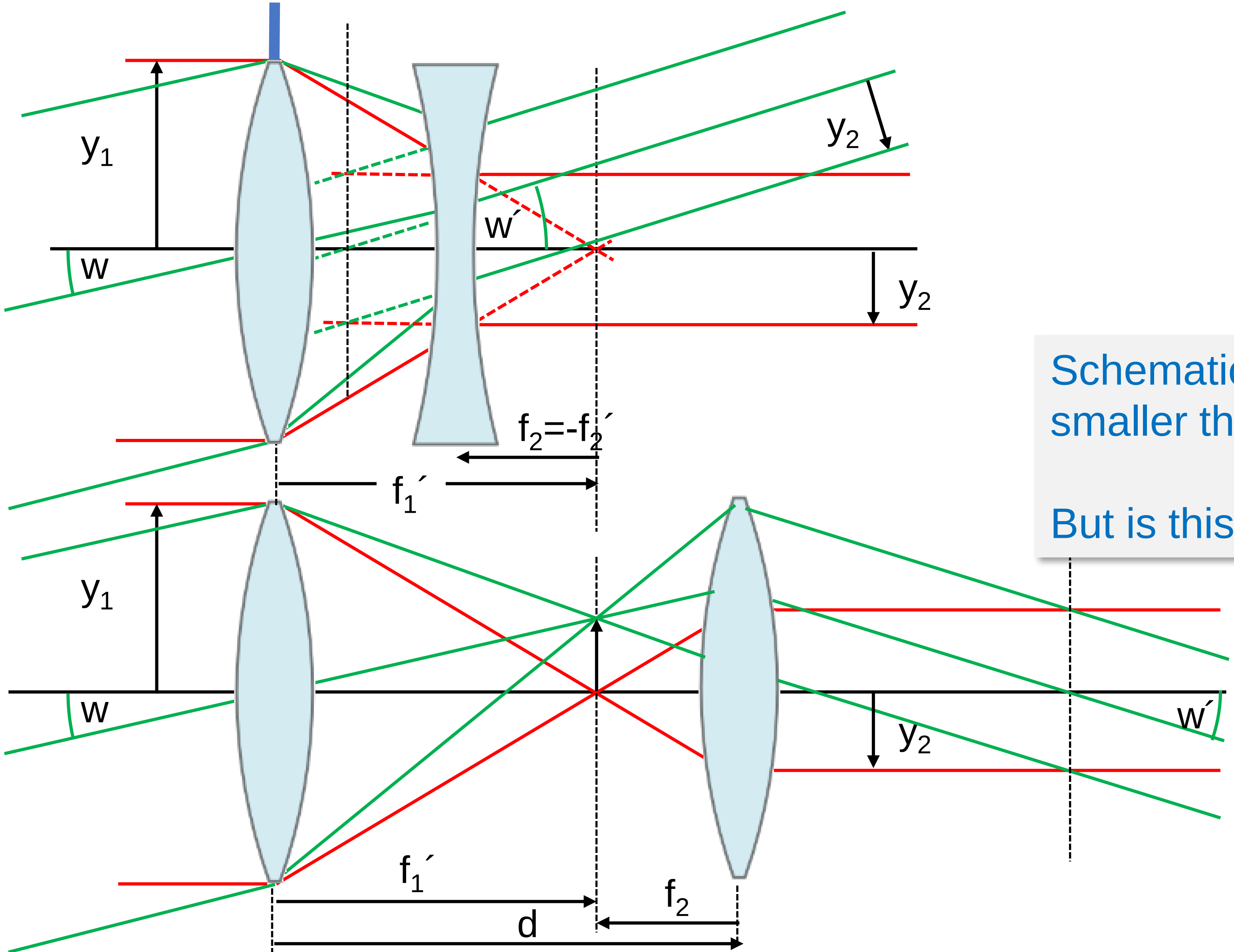


We must however note, that the location of the exit pupil is not universal for this setup.

<u>Keeping the same lens setup</u> we could make the <u>exit pupil accessible as shown below</u>.

With the new position of the exit pupil also the size of the entrance pupil changes, as

Galilei and Kepler afocal system layout



Galilei Layout

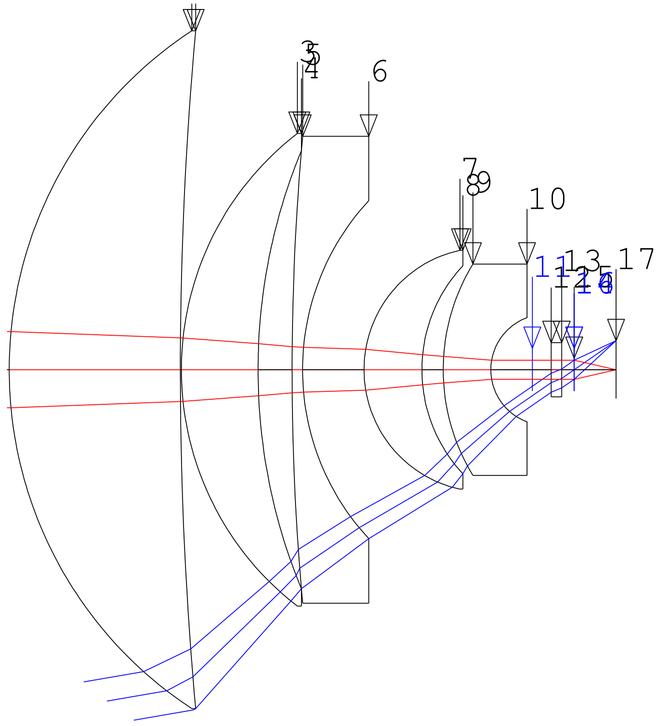
Schematic layout suggests that Galilei must be smaller than Kepler, lengths smaller by $2f_2$.

But is this always true?

Kepler Layout

Galilei is not necessarily smaller than Kepler!

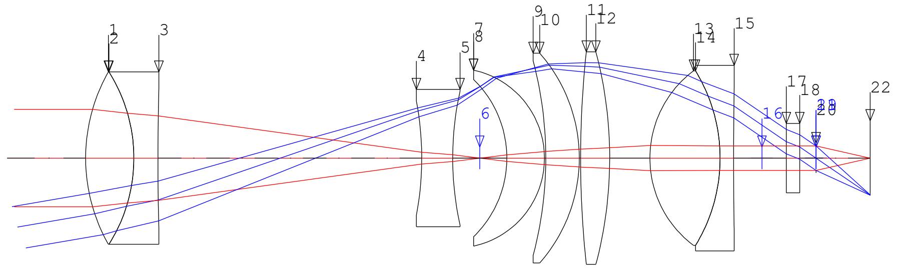
Galilei Typ



- Track length: 46.1mm
- Max. diameter : 64.5mm

Both systems are 4x!

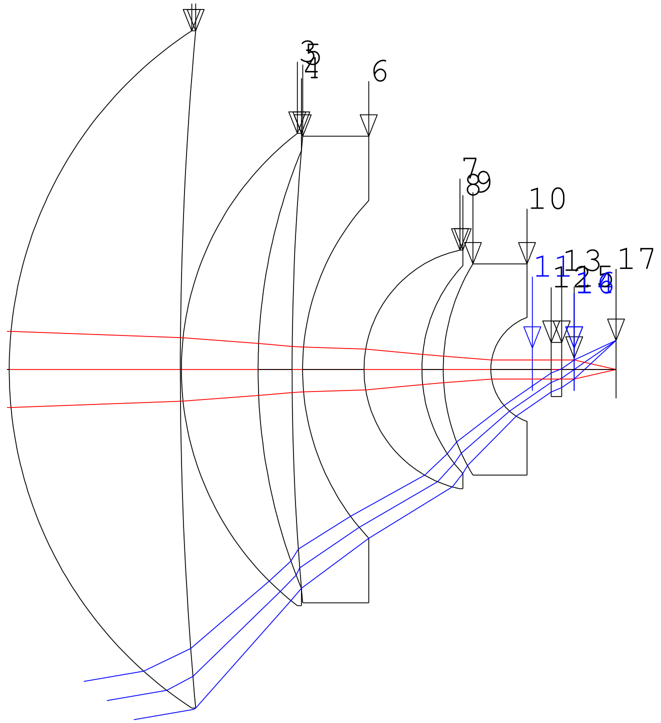
Kepler Typ



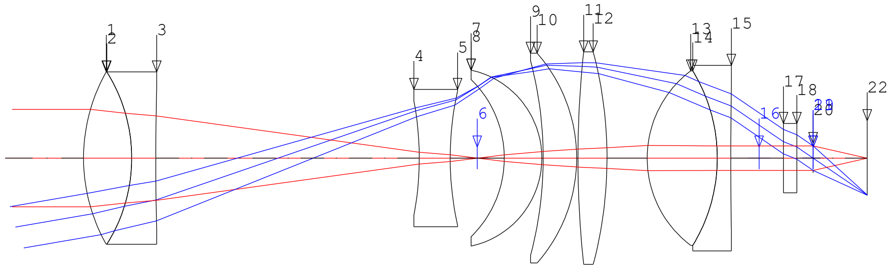
- Track length: 48mm
- Max. diameter : 14.1mm

Galilei is not necessarily smaller than Kepler!

Galilei Typ



Kepler Typ



Both systems are 4x!

The higher the magnification the more beneficial is Kepler.

…but with Kepler we need to invert the image again!

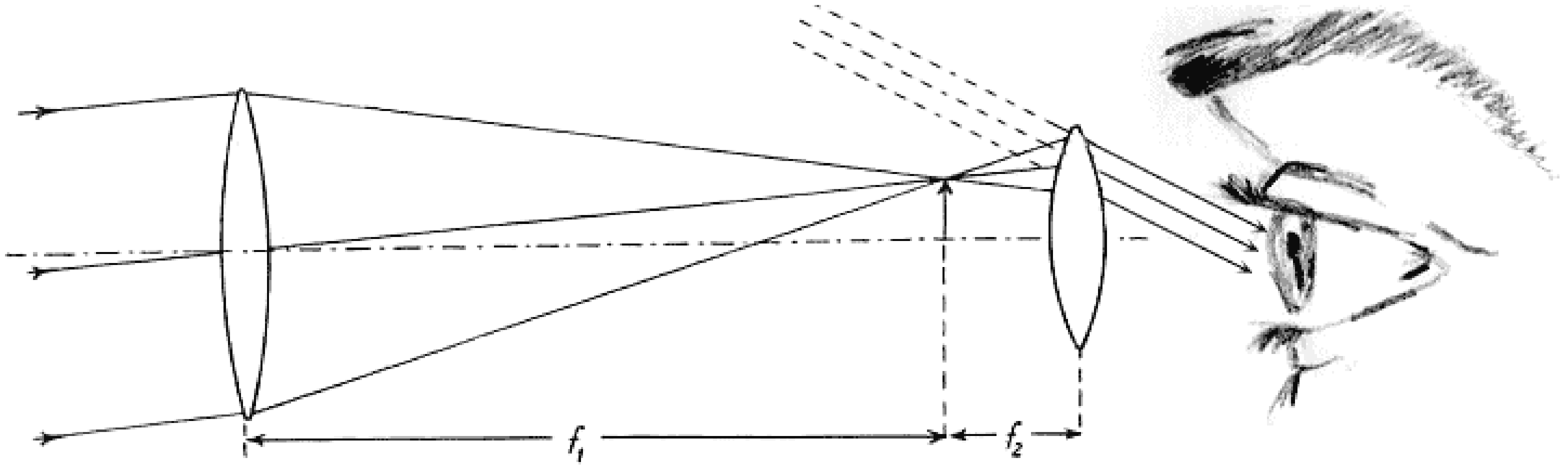
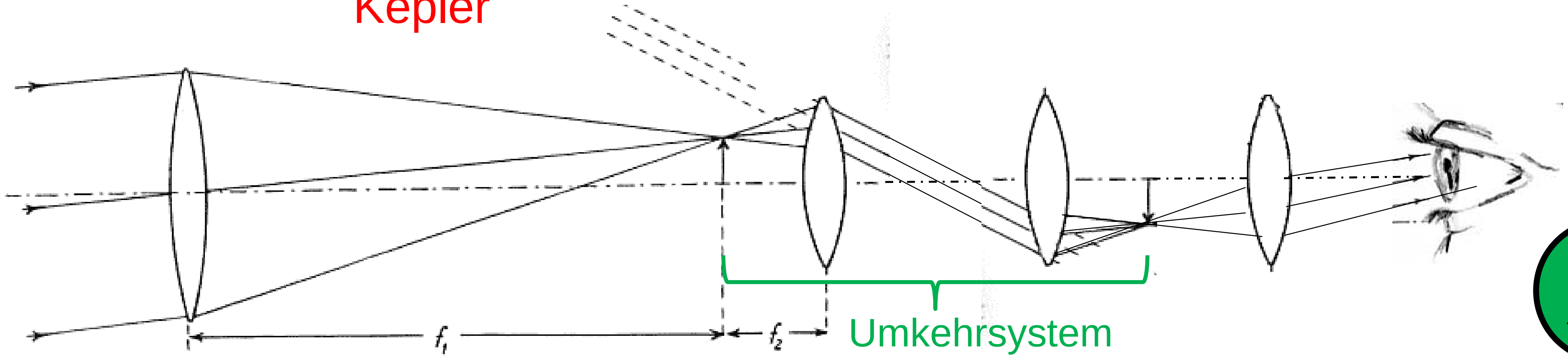


Image inversion!

Kepler

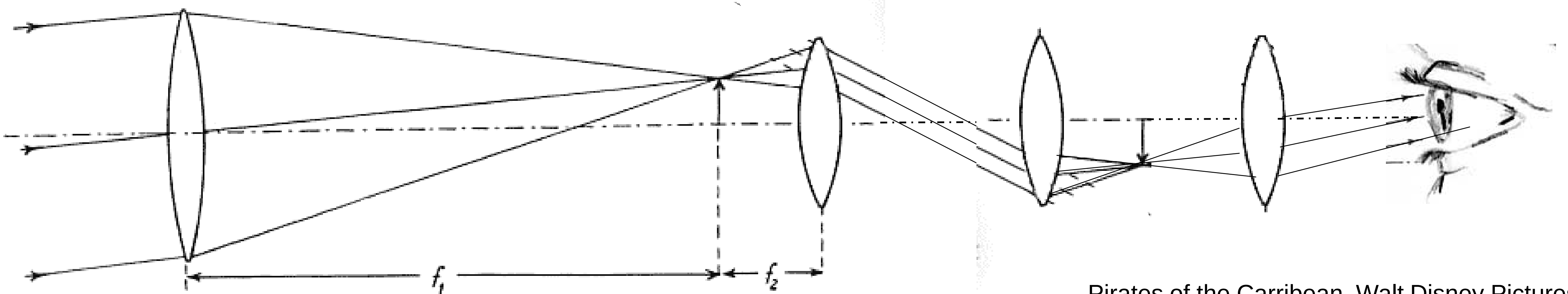


Kepler with inversion system

no inversion!

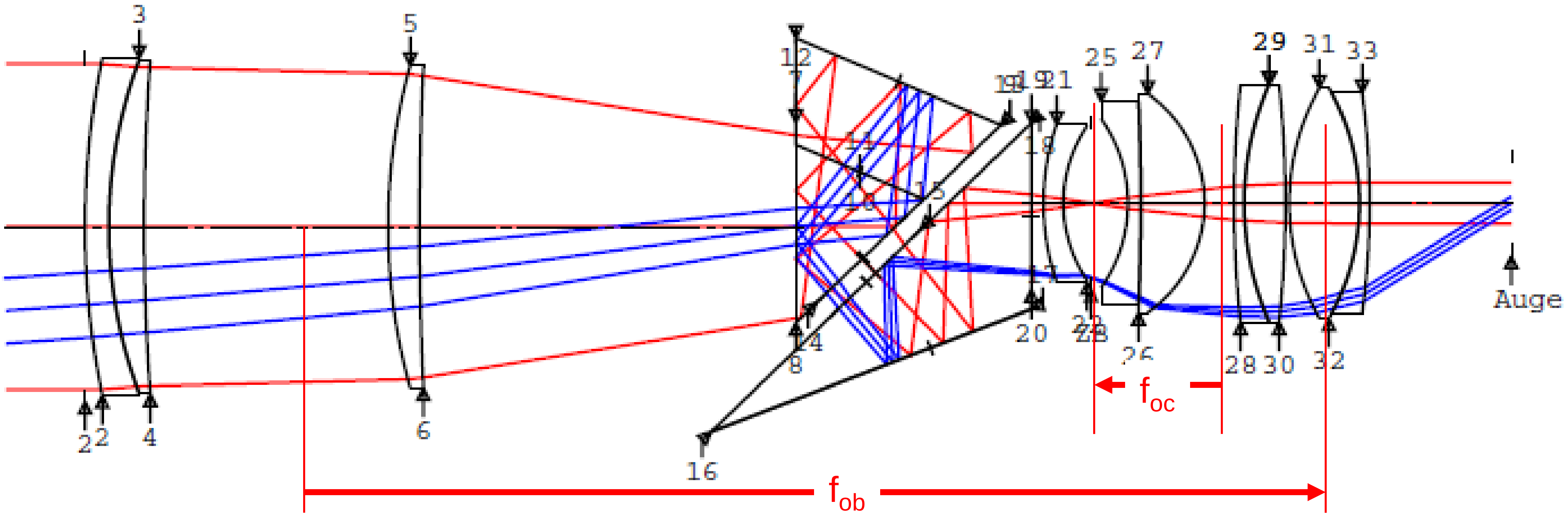
…this system gets quite long…

Pirates of the Carribean, Walt Disney Pictures

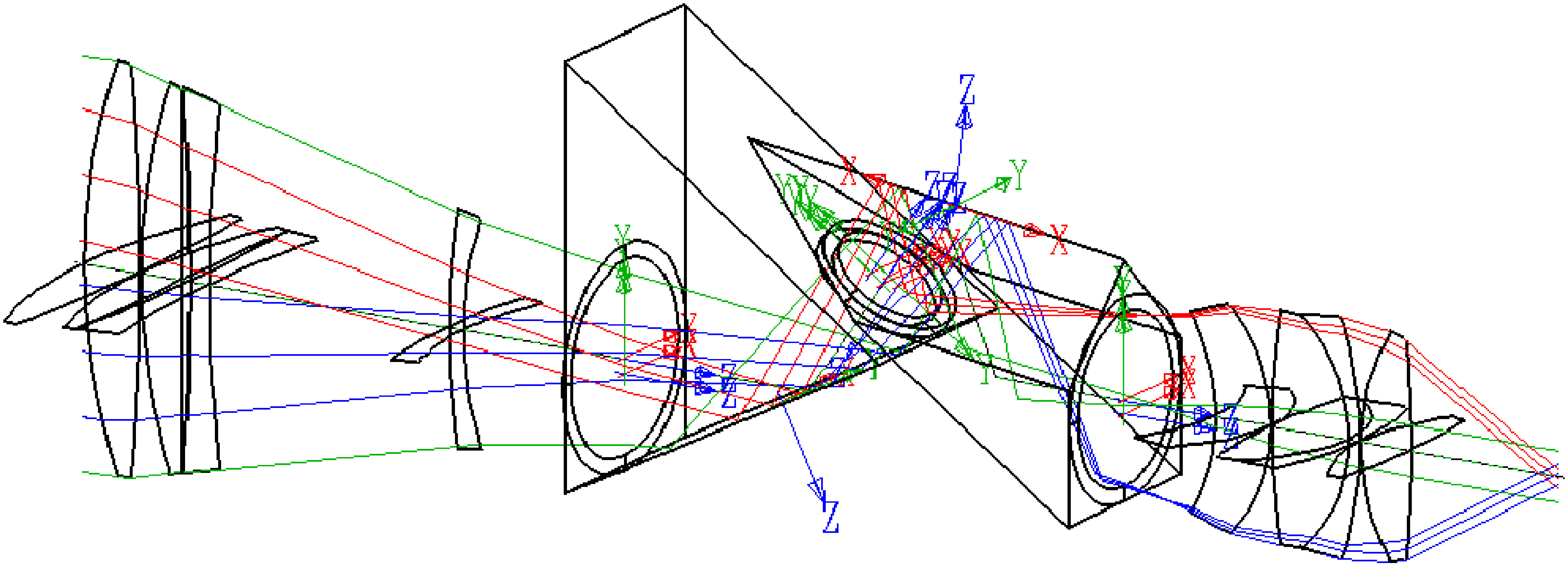


Angular magnification and objective to eyepiece focal length

Image inversion Prism system
saves considerably axial space!



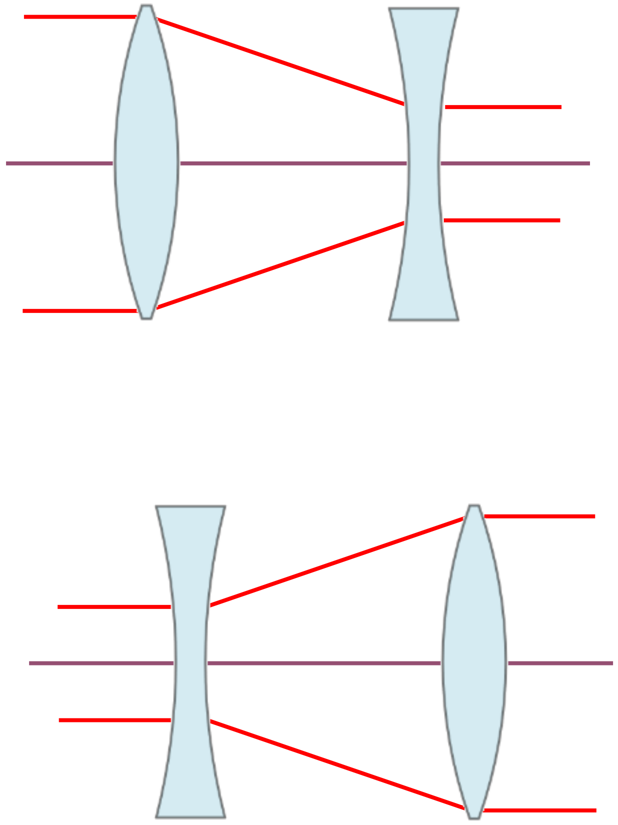
Binocular lens layout: 3D plot



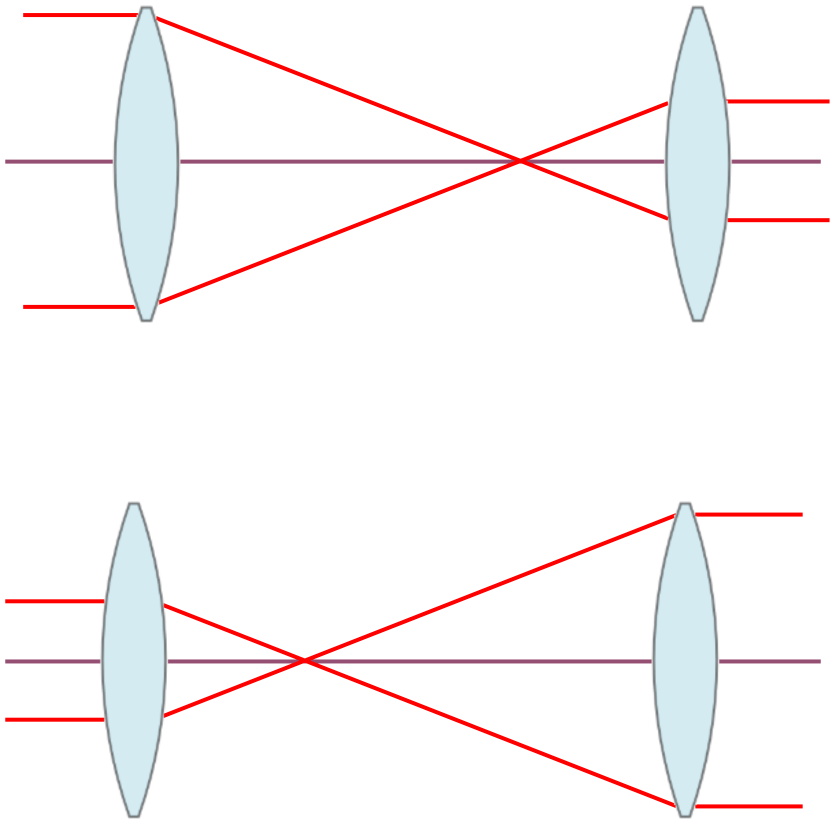
Positive and negative angular magnification of afocal systems

Tele

Wide-angle

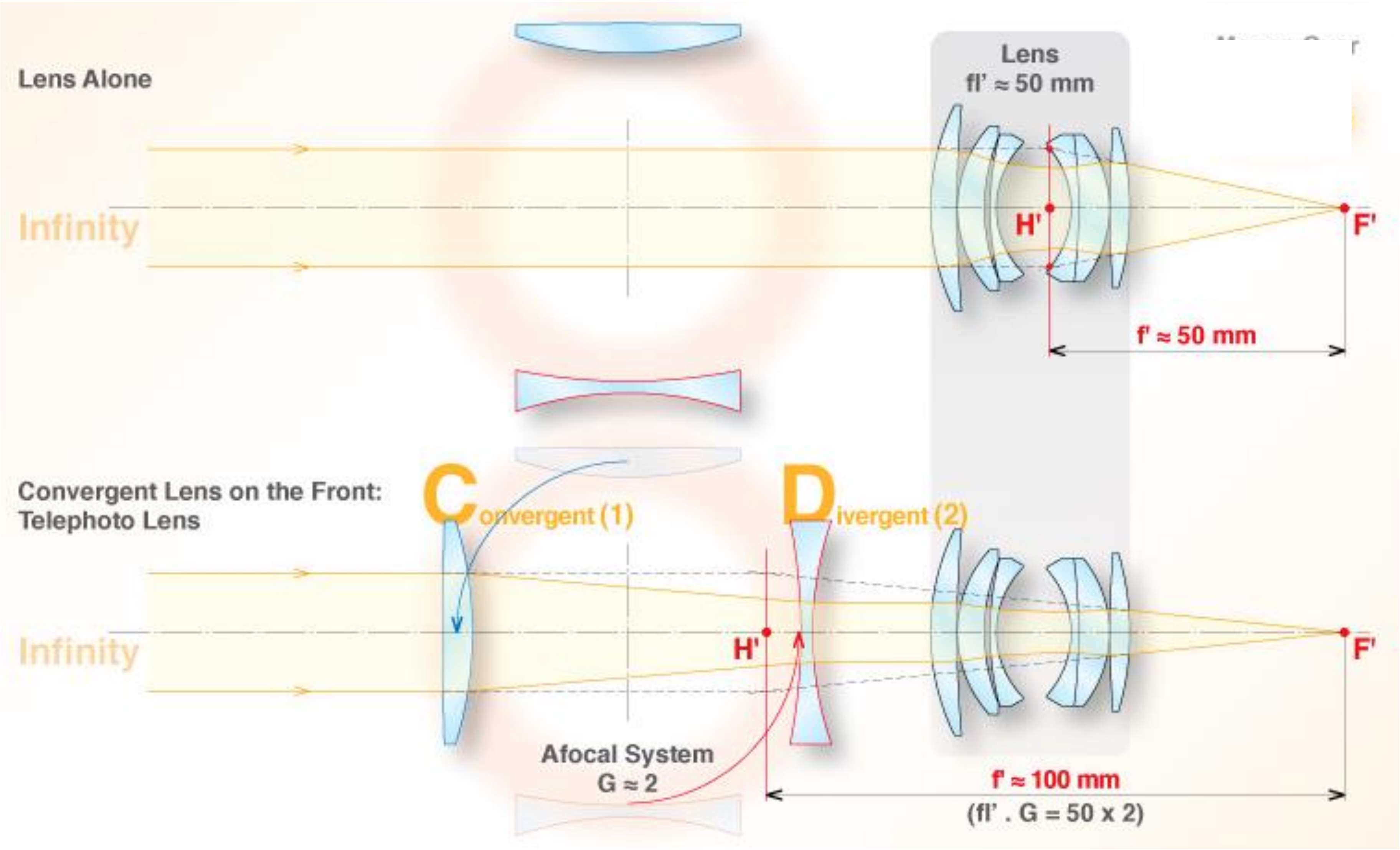


Galilei

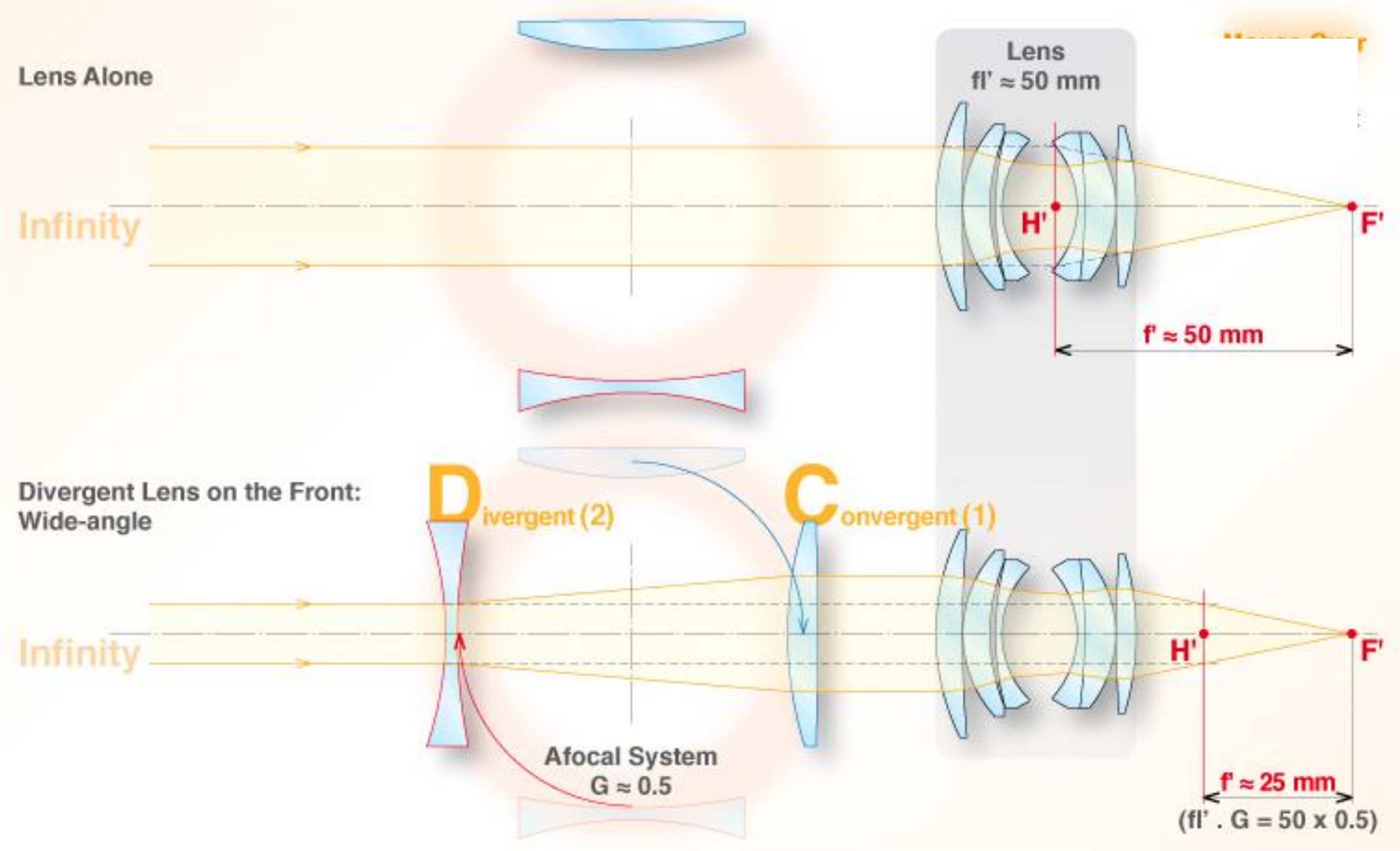


Kepler „ + / + „

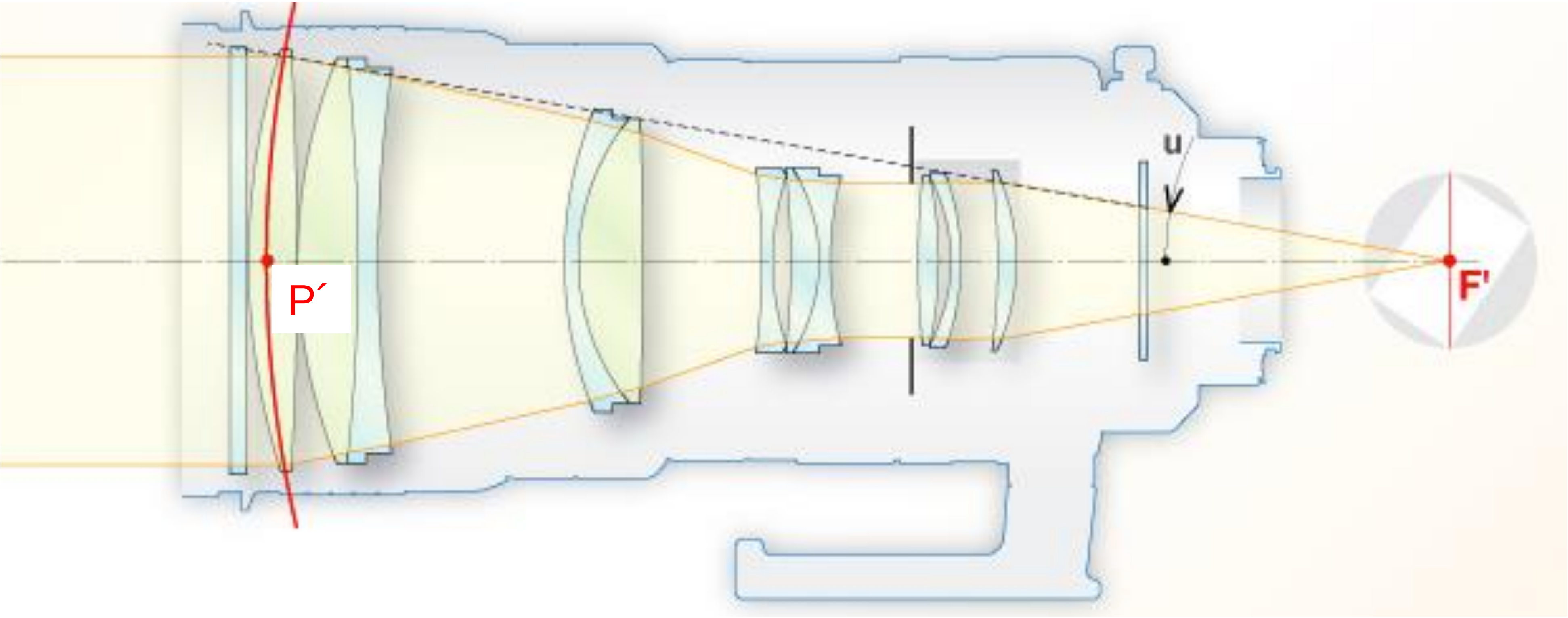
Afocal front attachment lens: Tele



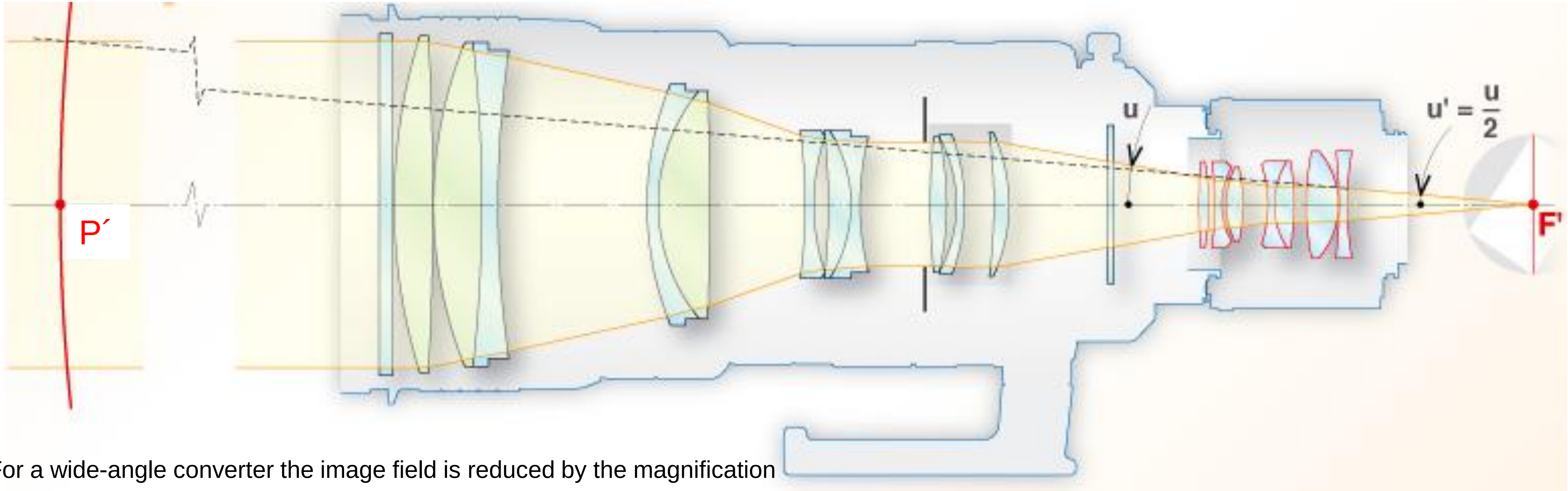
Afocal front attachment lens: Wide-Angle



Rear lens converter



Rear lens converter



* For a wide-angle converter the image field is reduced by the magnification

For exchangeable lenses rear lens converters exist.

Advantage is: they are relatively small (especially compared to large tele lenses).

Disadvantage: the lens aperture is reduced by the amount of magnification.*

Varifocal lens (Zoom lens)

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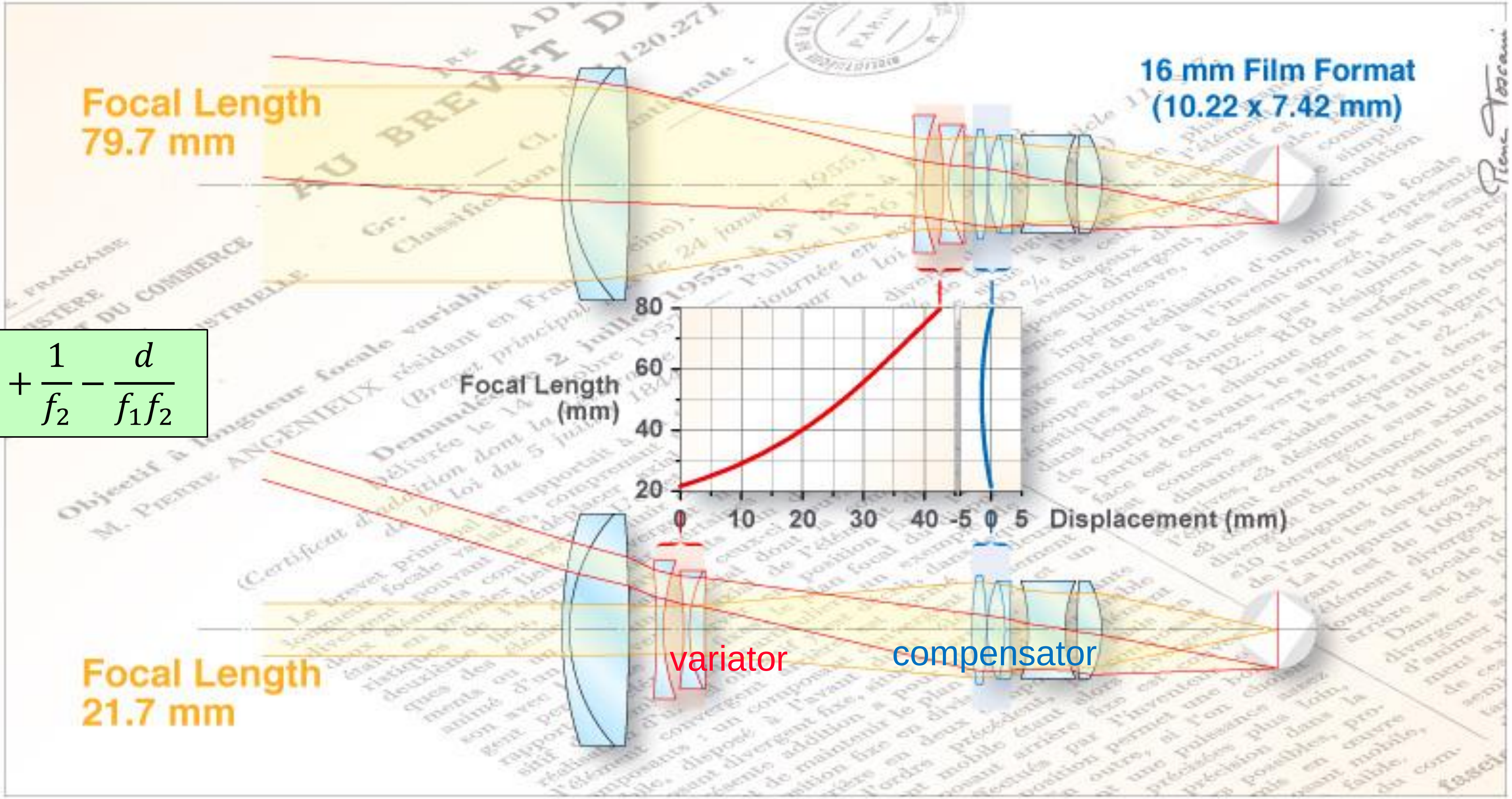
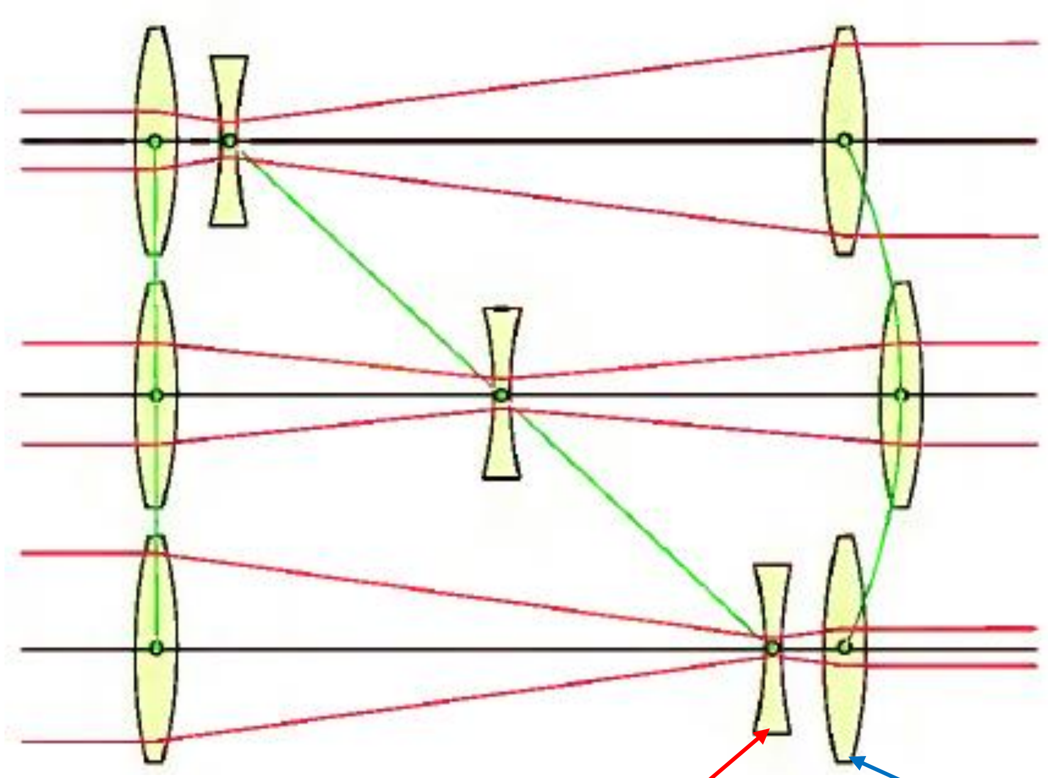
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$



Afocal Zoom System (3 groups)



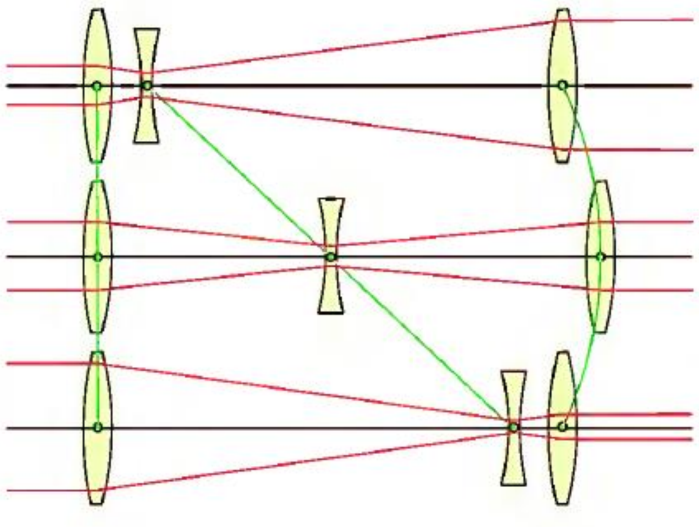
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

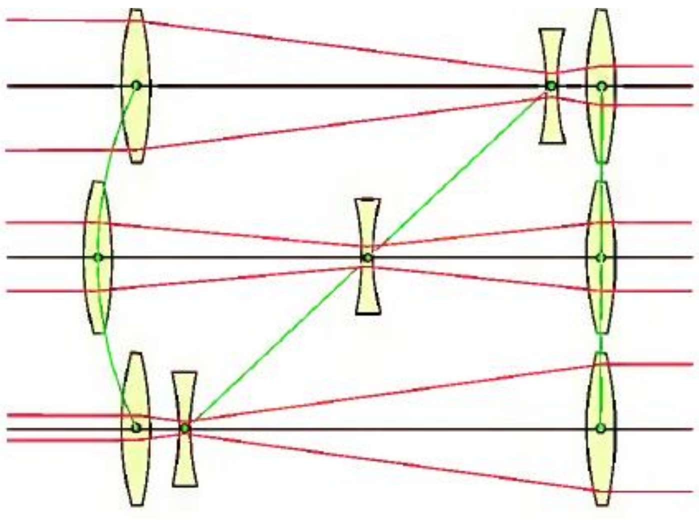
Variator (varies magnification)

Compensator (compensates deviation from parallel ray path exit)

Afocal Zoom System (3 groups)

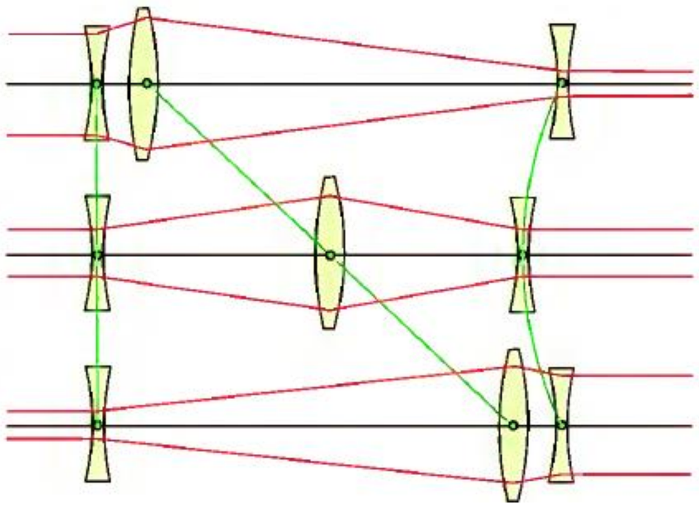
PNP (1st fixed)

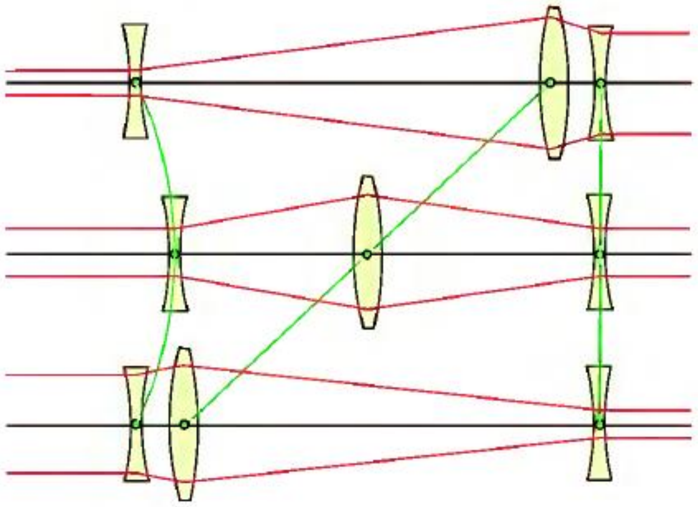




PNP (last fixed)

NPN (1st fixed)





NPN (1st fixed)

Another matching topic: Illumination systems for Microscopy…

Institute of Applied Physics
Friedrich-Schiller-Universität Jena

Zeitschrift für wissenschaftliche Mikroskopie in 1893

Ein neues Beleuchtungsverfahren für mikrophotographische Zwecke.

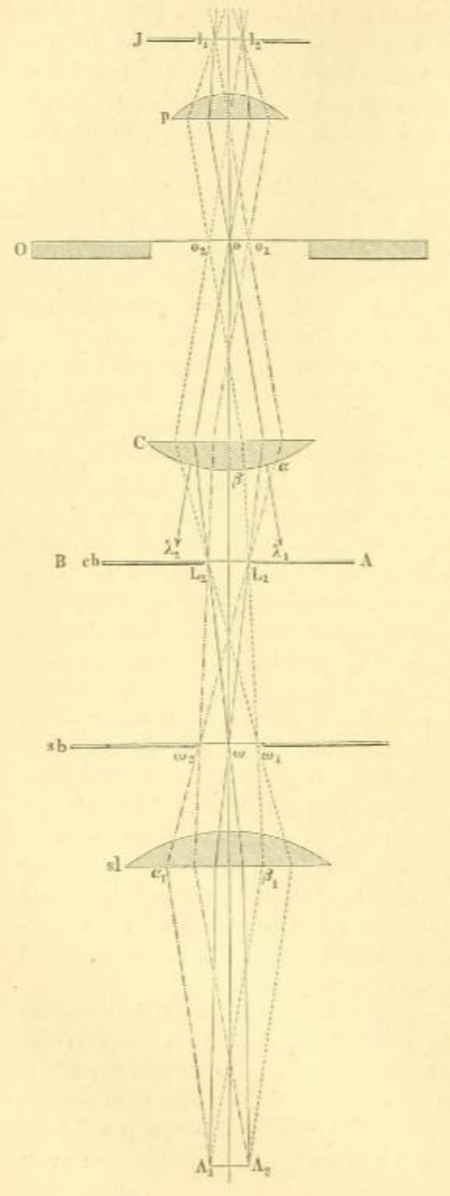
Von

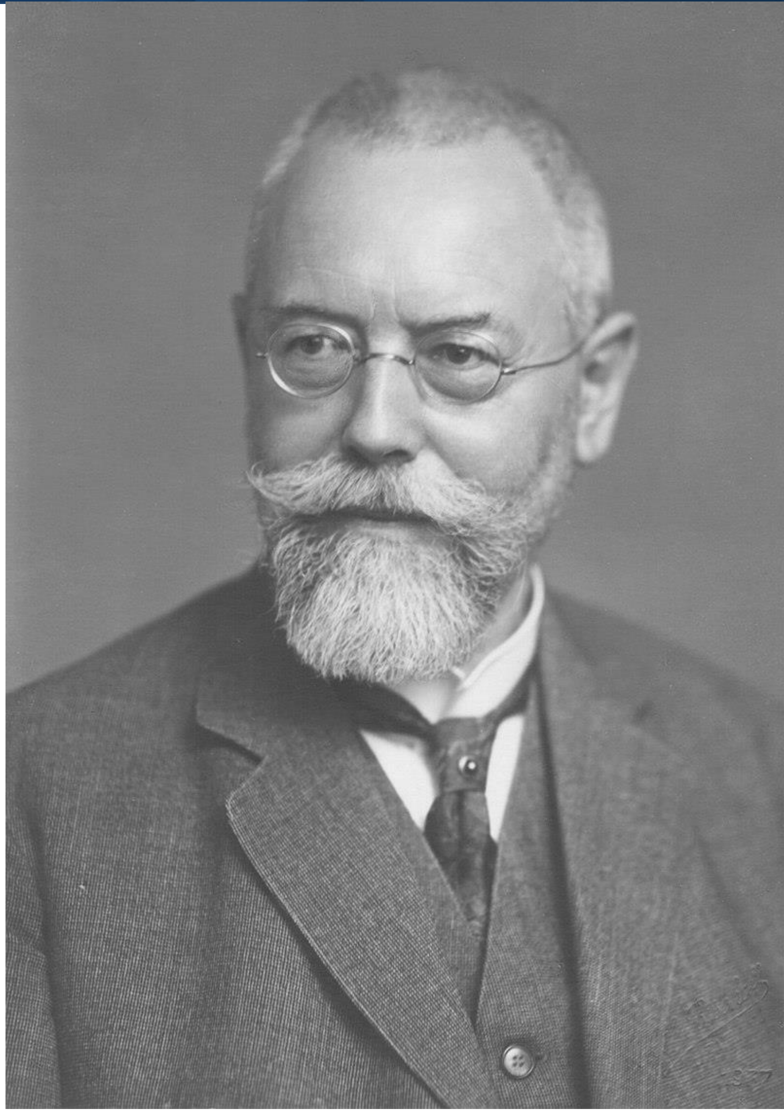
Dr. August Köhler
in Giessen.

Hierzu ein Holzschnitt.

Die Vorrichtungen, welche man verwendet, um bei Projectionsapparaten irgend welcher Art die zu projicirenden Objecte mit durchfallendem Licht zu beleuchten, das von einer begrenzten künstlichen Lichtquelle geliefert wird, lassen sich in zwei Gruppen eintheilen: die einen sind so construirt, dass die bei dem Abbildungsvorgang entstehenden Bilder der Lichtquelle in die Objectebene und die ihr zugeordneten Ebenen fallen, bei den anderen treten sie in von der Objectebene entfernten Querschnitten der Achse auf und bilden die Austrittspupille des Projectionssystems.

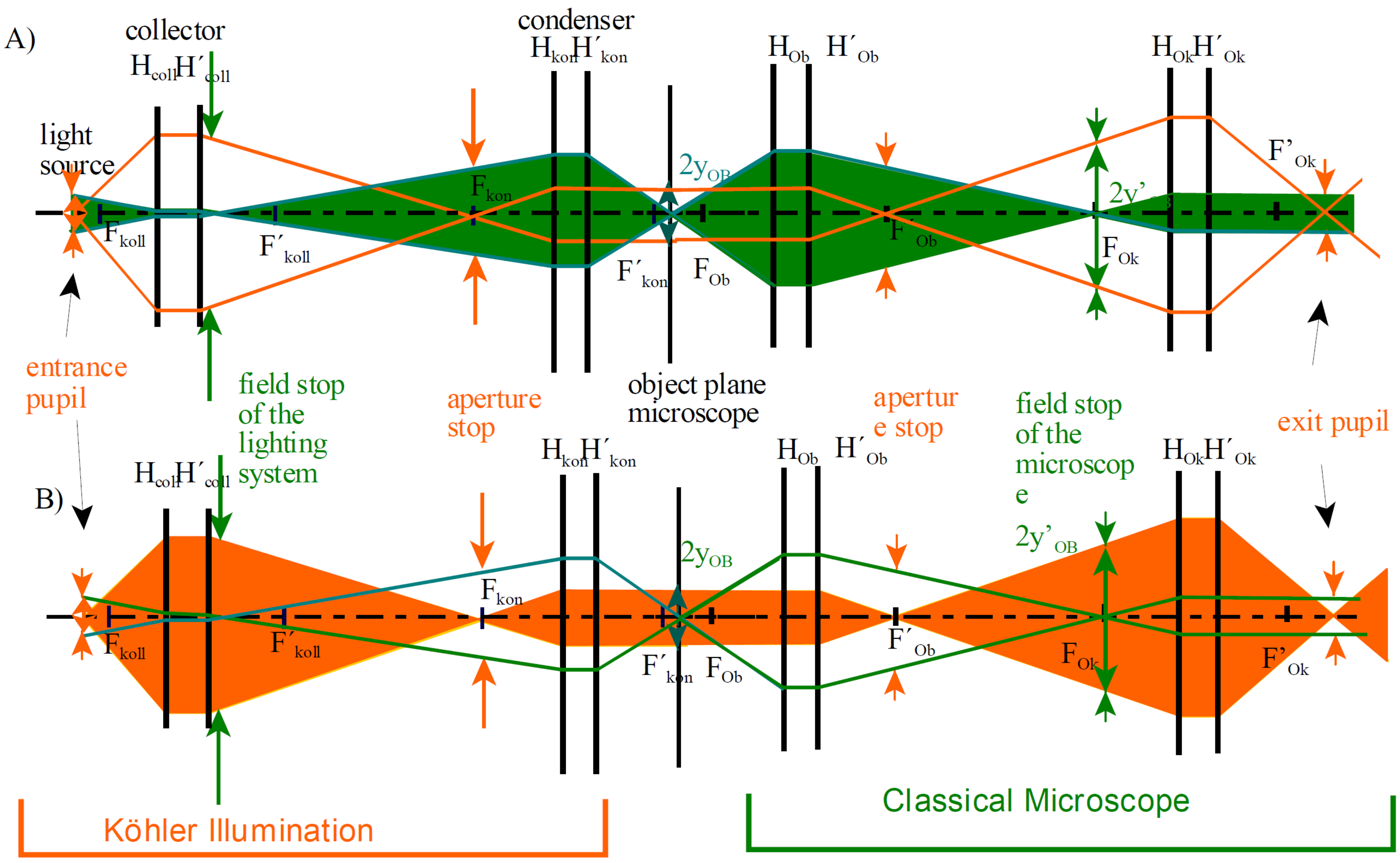
Man will bei solchen Apparaten hauptsächlich drei Bedingungen erfüllen, von denen je nach dem besonderen Zweck allerdings die eine oder die andere gegen die übrigen in den Hintergrund treten kann. Die Beleuchtungsvorrichtung muss nämlich erstens gestatten, die numerische Apertur und die Einfallsrichtung der beleuchtenden Strahlenkegel innerhalb möglichst weiter Grenzen zu variiren, zweitens soll nur der abzubildende Theil des Objects beleuchtet sein, damit störende Reflexe an den Linsenfassungen etc. thunlichst vermieden werden, und drittens muss, besonders für photographische Zwecke, die Beleuchtung dieses Theils der Objectebene eine ganz gleichmässige sein, da für das Auge kaum merkliche Helligkeitsunterschiede im photographischen Bild äusserst störend hervortreten können.





August Köhler (1866-1948)

Optical System for Microscopy



Summary / Take-Away´s

- for larger apertures paraxial imaging is generalized via **Abbe's Sine Condition** $m \sin U' = \sin U$ to **aplanatic** imaging: the pupil scales with the sine of aperture angles, meaning geometrically that **principal planes, as well as entrance and exit pupils are spheres** with center of curvature at object and image point respectively
- almost all practical optical systems fulfill the Sine-Condition: consequently the pupil should be **sine-scaled** (forget about any tan-relations concerning the lens pupil you sometimes find in the literature)
- optical systems is **telecentric** if the chief ray enter the object or image plane perpendicular; can be realized by placing the stop in a focal plane
- Combining different optical systems, object and image planes are well suited **interfaces** as well as pupil planes, e.g. the eye piece of a microscope with the human eye; the pupils must match
- if pupils don´t match, e.g., if their sizes or positions are different, light loss results; for example if pupils are axially shifted **vignetting** occurs
- **Afocal systems** transform parallel rays to parallel rays, fulfill the condition $d=f_1'+f_2'$, having an angular magnification $\Gamma = \frac{\tan(w')}{\tan(w)}$
- a **Galilei telescope** (+,-) has no intermediate image and therefore no image inversion, whereas a **Kepler** telescope (+,+) has one intermediate image and inverts the image
- Size and structure of optical systems usually critically depends on **mechanical** (and performance) **constraints**; when comparing different systems also compare the constraints, otherwise you may compare apples and pies!

Literature (Lecture 3)

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