Institute of Applied Physics

Friedrich-Schiller-Universität Jena

# Metrology and Sensing

Lecture 12-1: Optical Coherence Tomography

2021-02-02

Herbert Gross

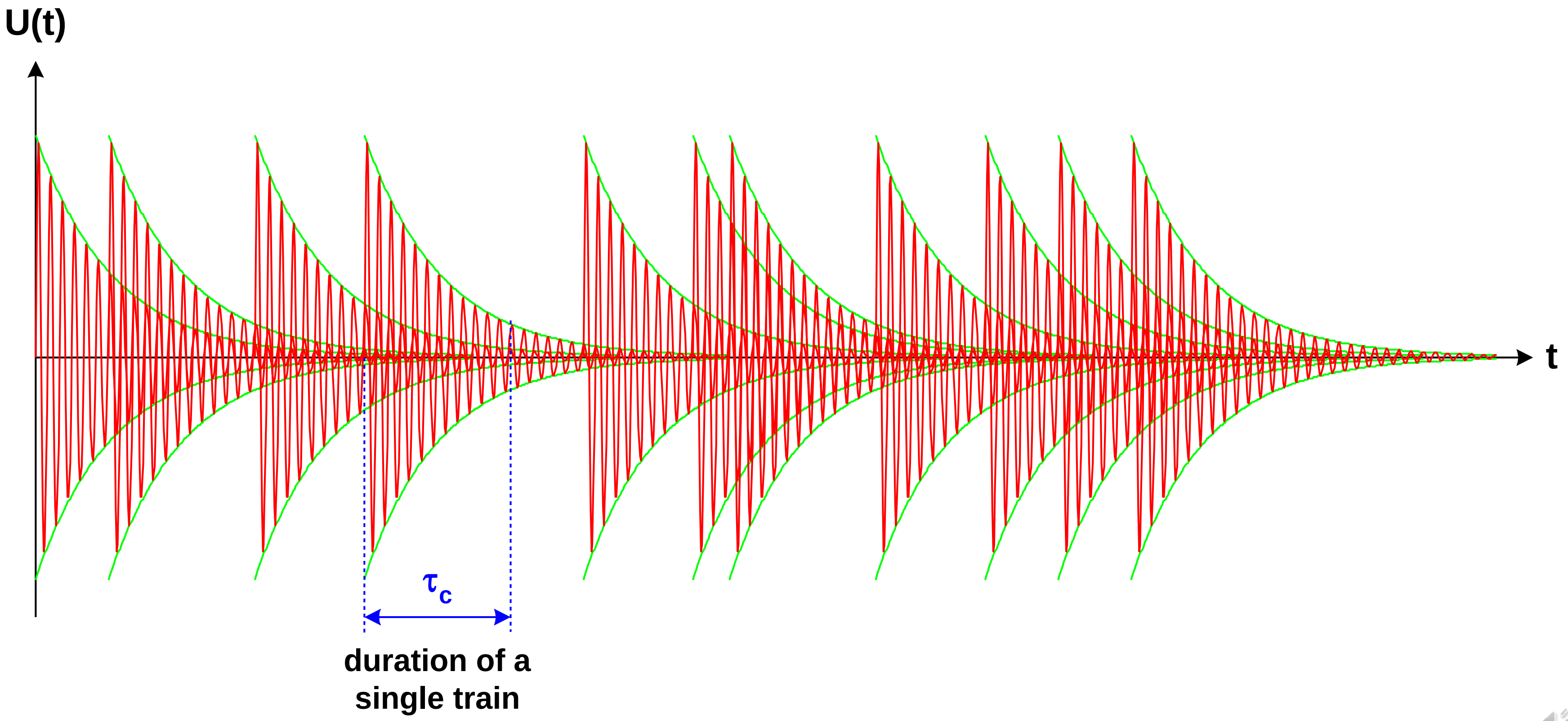
# Contents

- Temporal coherence
- Light sources
- Dispersion
- Scattering in biological tissue

# Temporal Coherence

Institute of Applied Physics
Friedrich-Schiller-Universität Jena

- Damping of light emission: wave train of finite length
- Starting times of wave trains: statistical



# Temporal Coherence

- Radiation of a single atom:
  Finite time $\Delta$t, wave train of finite length,
  No periodic function, representation as Fourier integral
  with spectral amplitude A($\nu$)

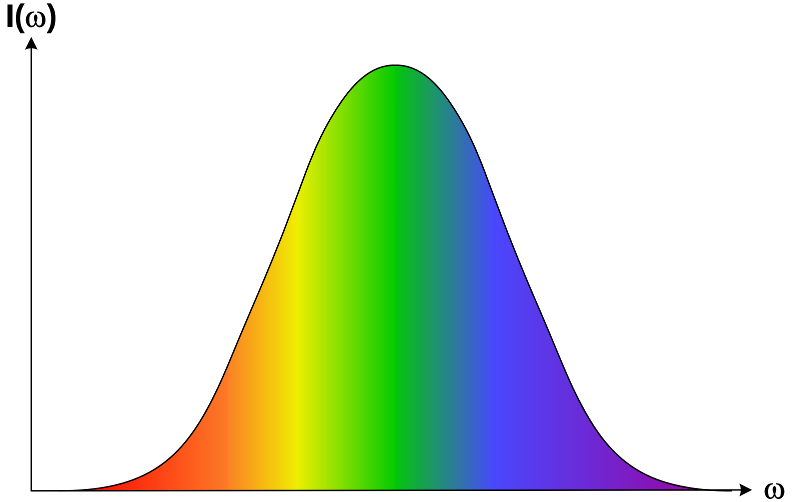
$$E(t) = \int A(\nu) \cdot e^{2\pi i \nu t} d\nu$$

- Example rectangular spectral distribution

$$A(\nu) = \frac{\sin(\pi \cdot \nu \cdot \Delta t)}{\pi \cdot \nu \cdot \Delta t}$$

- Finite time of duration: spectral broadening $\Delta\nu$,
  schematic drawing of spectral width

$$\Delta \nu = 1/\Delta t$$



# Time-Related Coherence Function



- Intensity of a multispectral field
  Integration of the power spectral density S($\nu$)

$$I = \int_0^\infty S(\nu) d\nu$$

- The temporal coherence function and the power spectral density are Fourier-inverse:
  Theorem of Wiener-Chintchin

$$S(\nu) = \int_{-\infty}^{\infty} \Gamma(\tau) e^{-2\pi i \nu \tau} d\tau$$

- The corresponding widths in time and spectrum are related by an uncertainty relation

$$\tau_c = \frac{1}{\Delta\nu}$$

- The Parceval theorem defines the coherence time as average of the normalized coherence function

$$\tau_c = \int_{-\infty}^{\infty} \left|\gamma(\tau)\right|^2 d\tau$$

- The axial coherence length is the space equivalent of the coherence time

$$l_c = c \cdot \tau_c$$

Institute of Applied Physics
Friedrich-Schiller-Universität Jena

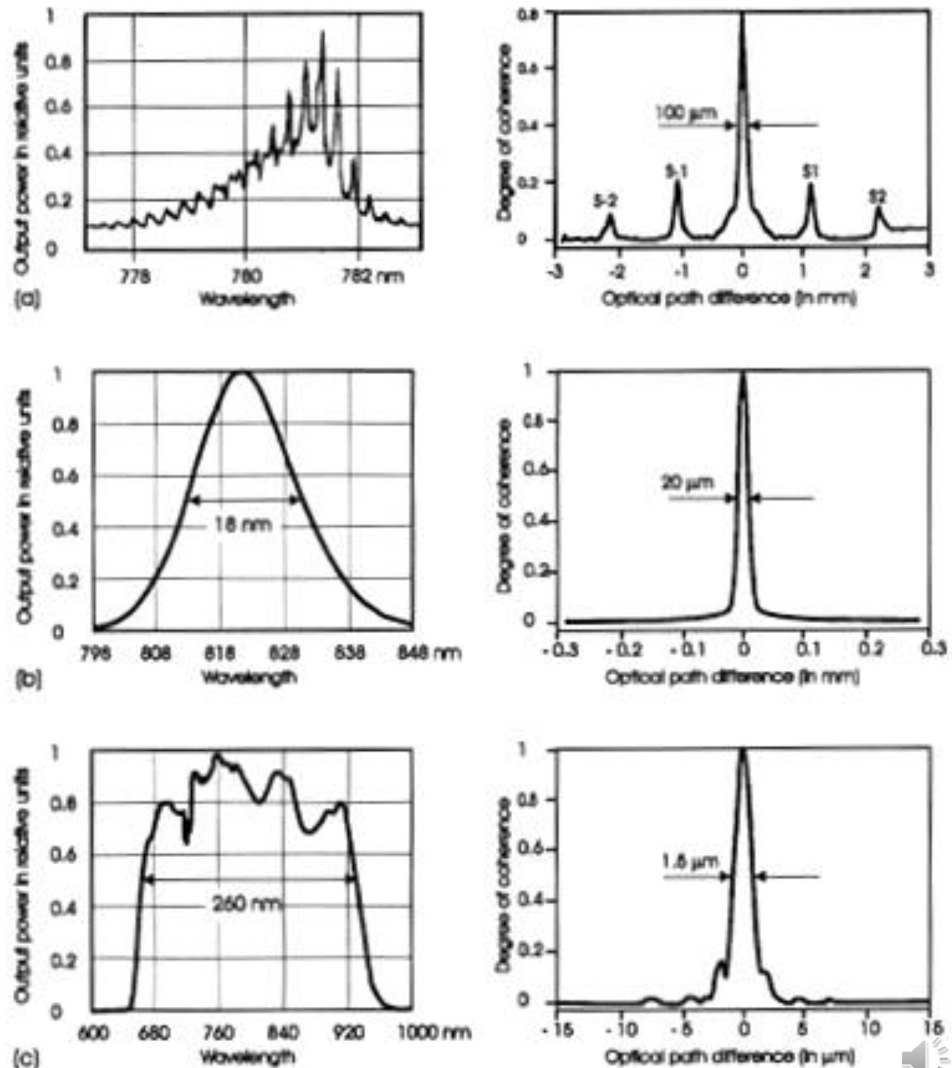
# OCT Sources and PCI Signal

- Left column:
  optical spectrum
- Right column:
  signal in the spatial domain

Multimode laser diode

Super Luminescent Diode

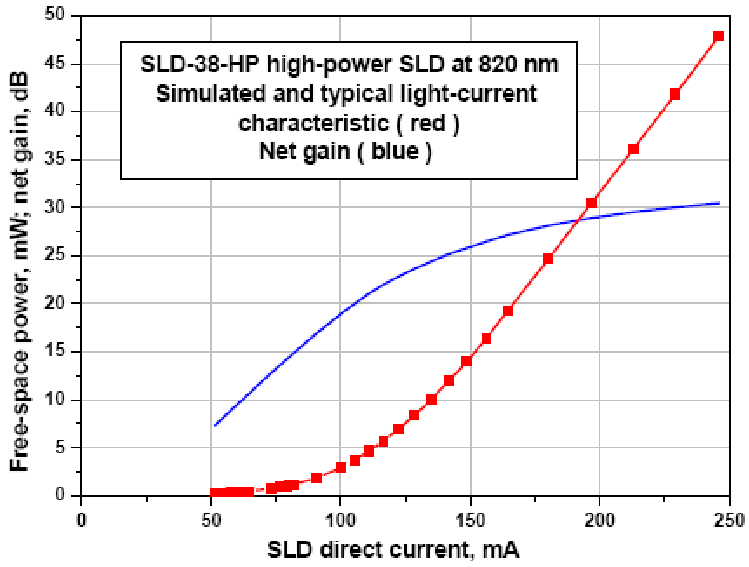
Ti-Saphir-Laser

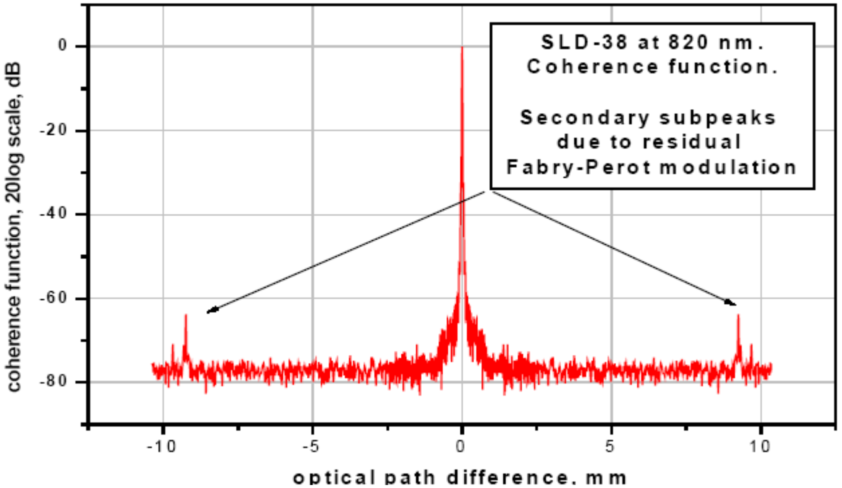


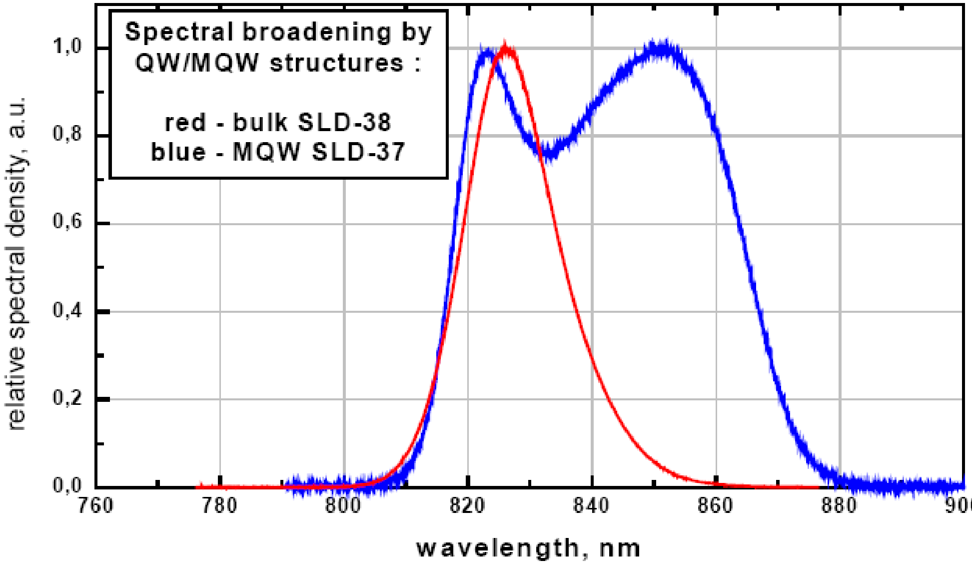

Ref: M. Kaschke

# OCT Light Sources






Ref: M. Kaschke

# Light Sources of OCT



- Typical light sources used for OCT

| No | Type of source | wavelength [nm] | axial resolution | remark |
|---|---|---|---|---|
| 1 | Superluminescent diode | 800-830 | 10 μm | |
| 2 | Swept laser source | 1050-1070 | | 2.8 kHz swept rate |
| 3 | Supercontinuum fiber laser | 450-1700 | | |
| 4 | Photonics crystal fiber (PCF) illuminated by a fs-Ti:Sapphire laser | 550-950 | < 1 μm | [3] |
| 6 | PCF source | 1300 | 2 μm | |
| 5 | Ti-Sapphire laser | 675-975 | 1 μm | [4] |

# GVD Dispersion

Institute of Applied Physics
Friedrich-Schiller-Universität Jena

- Dispersive material in OCT:
  - wavelength-dependent phase delay
  - group velocity dispersion
  - degradation of the axial resolution
  - the dispersion causes a distortion of the pulse shape during propagation

$$E(z,t)=E_0\cdot e^{i\cdot(\omega t-kz)}$$

$$\varphi=\omega\cdot t-k\cdot z$$

$$\varphi^{(2)}=-k''\cdot z=-\frac{\lambda^3 z}{2\pi c^2}\cdot\frac{d^2n}{d\lambda^2}$$

$$\Delta t=D\cdot z\cdot\Delta\lambda=\frac{\lambda^3\Delta\omega}{2\pi c^2}\cdot\sum_j z_j\cdot\frac{d^2n}{d\lambda^2}$$

- Dispersion:
  k not linear changing with $\nu$ / $\omega$

$$k(\nu)=\frac{2\pi\cdot n(\nu)\cdot\nu}{c_o}\qquad k(\omega)=\frac{2\pi\cdot n(\lambda)}{\lambda}=\frac{\omega\cdot n(\omega)}{c_0}$$

- Group velocity
  1st derivative

$$\frac{1}{v_{gr}}=\frac{1}{2\pi}\cdot\frac{dk}{d\nu}$$

- Group velocity dispersion
  2nd derivative

$$D_\lambda=\frac{d}{d\lambda}\left(\frac{1}{v_{gr}}\right)=-\frac{2\pi c}{\lambda^2}\cdot\frac{d^2k}{d\lambda^2}=-\frac{\lambda}{c}\frac{d^2n}{d\lambda^2}$$

$$D_\nu=\frac{1}{2\pi}\cdot\frac{d^2k}{d\nu^2}=\frac{d}{d\nu}\left(\frac{1}{v_{gr}}\right)=\frac{\lambda^3}{c^2}\frac{d^2n}{d\lambda^2}$$

# GVD Dispersion



- Dispersion relation

$$k_0 = k(\omega_0) = \frac{2\pi \cdot n(\lambda_0)}{\lambda_0}$$

- Expansion

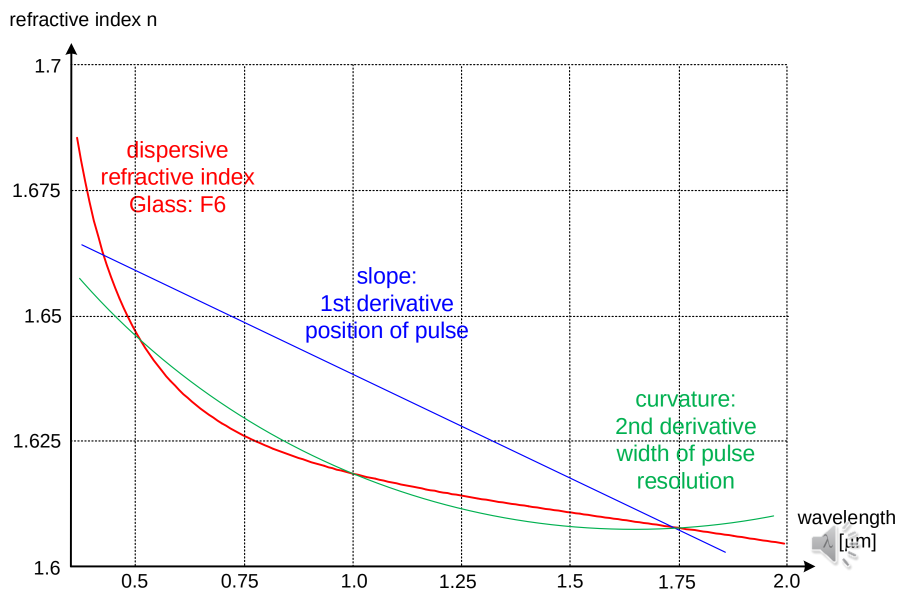
$$k_S(\omega) = k(\omega_0) + \left.\frac{\partial k(\omega)}{\partial \omega}\right|_{\omega=\omega_0} \cdot (\omega - \omega_0) + \frac{1}{2}\left.\frac{\partial^2 k(\omega)}{\partial \omega^2}\right|_{\omega=\omega_0} \cdot (\omega - \omega_0)^2$$

$$= k(\omega_0) + k'(\omega_0) \cdot (\omega - \omega_0) + \frac{1}{2} k''(\omega_0) \cdot (\omega - \omega_0)^2$$

- Rearrangement with $\lambda$ as variable
introduction of group velocity and GVD
(group velocity dispersion)

$$\frac{dk}{d\omega} = \frac{d\lambda}{d\omega} \cdot \frac{dk}{d\lambda} = -\frac{\lambda^2}{c}\frac{d}{d\lambda}\left(\frac{n(\lambda)}{\lambda}\right) = \frac{1}{c}\left(n - \lambda \cdot \frac{dn}{d\lambda}\right) = \frac{1}{v_g}$$

$$\frac{d^2 k}{d\omega^2} = \frac{d\lambda}{d\omega} \cdot \frac{dk'}{d\lambda} = \frac{\lambda^3}{2\pi c^2}\frac{d^2 n(\lambda)}{d\lambda^2} = \frac{1}{2\pi} D_v$$



# GVD Dispersion



- Numerical stable calculation with Sellmeier formula
- Refractive index

$$n(\lambda)=\sqrt{1+\sum_{j=1}^{3}\frac{K_j\cdot\lambda^2}{\lambda^2-L_j}}$$

- Derivatives

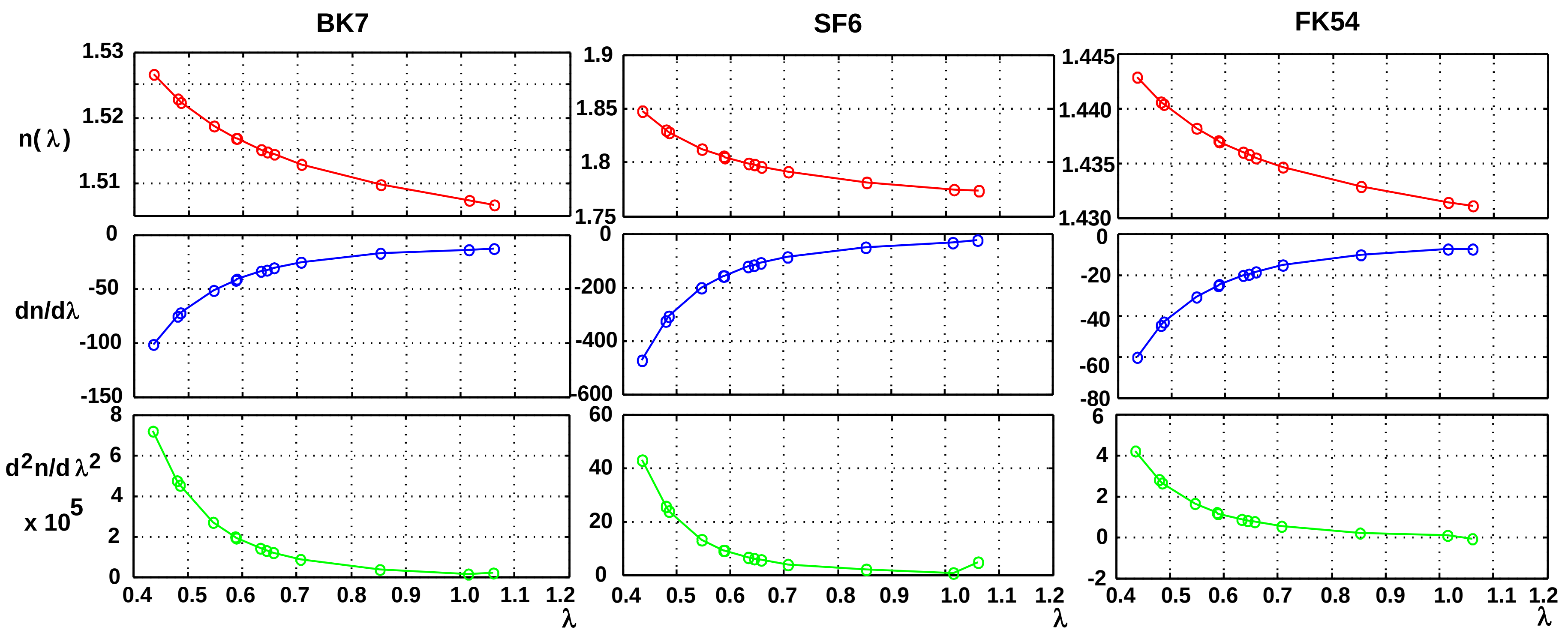
$$\frac{dn}{d\lambda}=-\frac{1}{n}\cdot\sum_j\frac{K_jL_j\lambda}{\left(\lambda^2-L_j\right)^2}$$

$$\frac{d^2n}{d\lambda^2}=-\frac{1}{n^3}\cdot\left[\sum_j\frac{K_jL_j\lambda}{\left(\lambda^2-L_j\right)^2}\right]^2+\frac{1}{n}\cdot\sum_j\frac{K_jL_j\cdot\left(L_j+3\lambda^2\right)}{\left(\lambda^2-L_j\right)^3}$$

# GVD - Group Velocity Dispersion

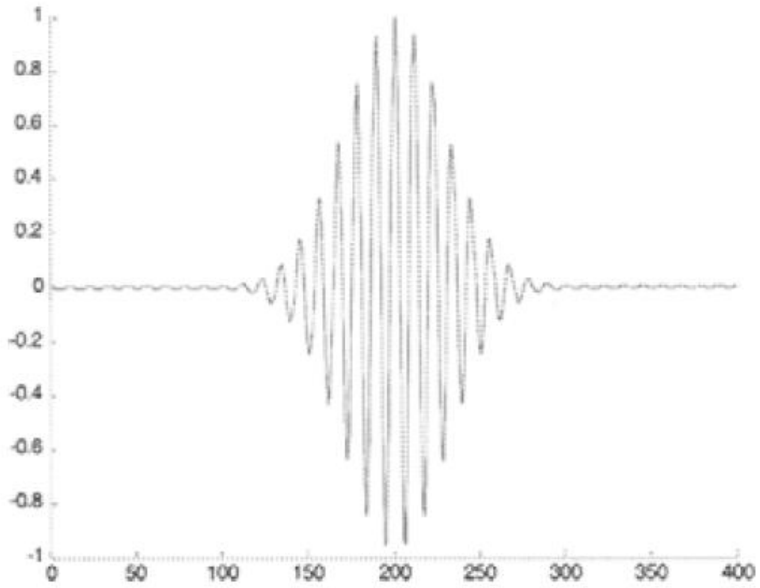


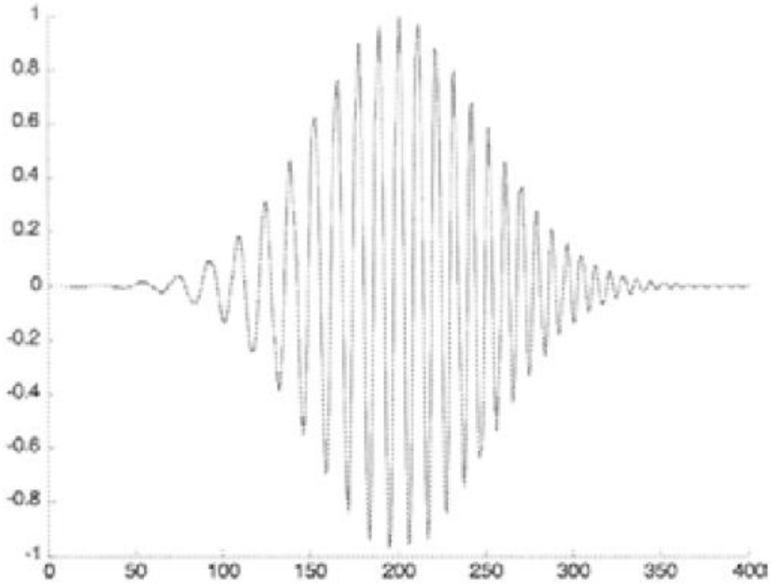
- Example:
  n, n' and n'' for three types of glasses



# Dispersion

- Pulse transmission through dispersive medium
  1. input pulse
  2. after propagation with dispersion

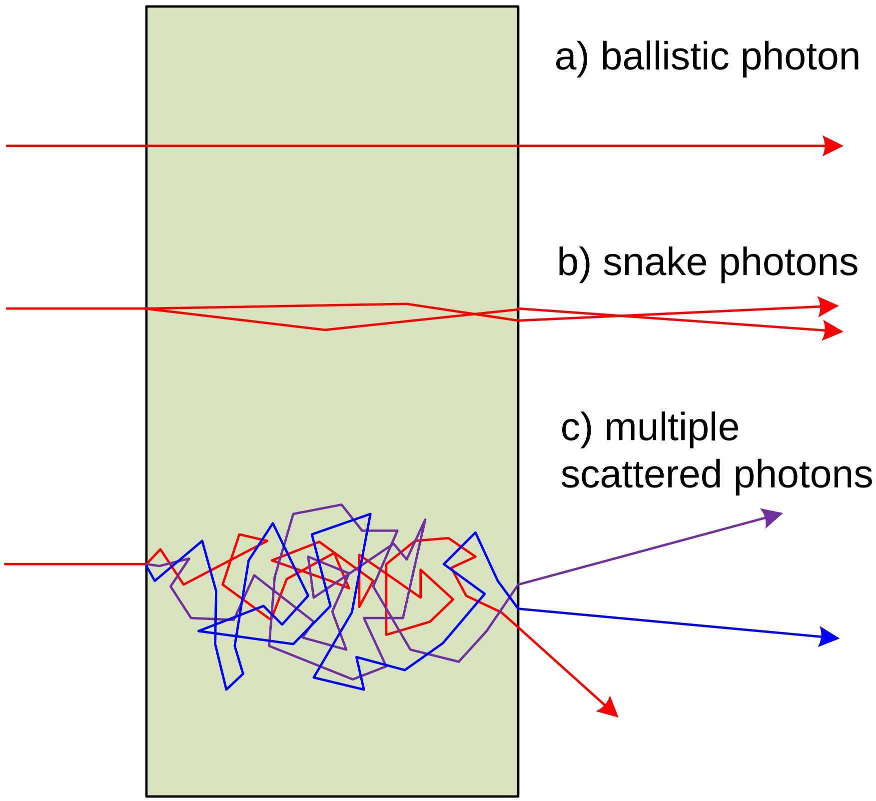



Ref: M. Brezinsky

# Scattering in Turbid Media

- Different strengths of interaction
- Behavior depends on density of scattering centers
- Changed time of flight of the light photons

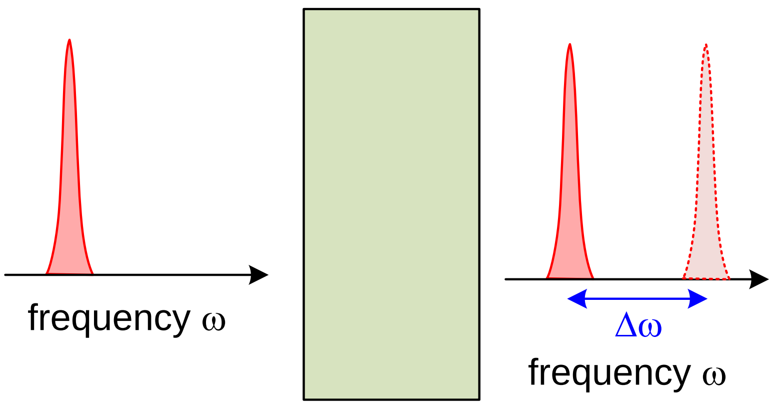



Ref: M. Gu

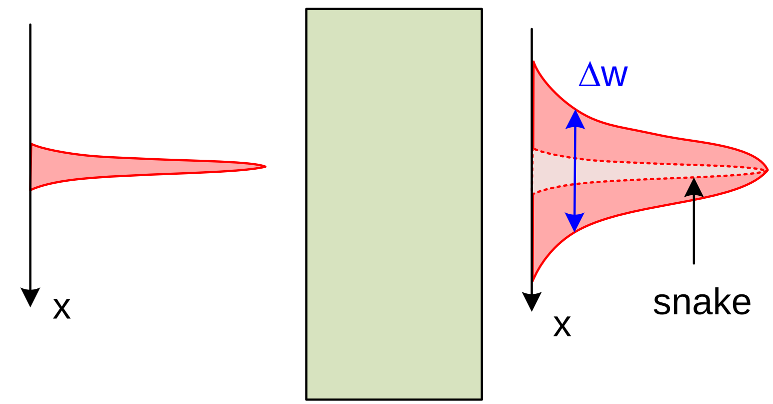
# Scattering in Turbid Media

- Change of light properties

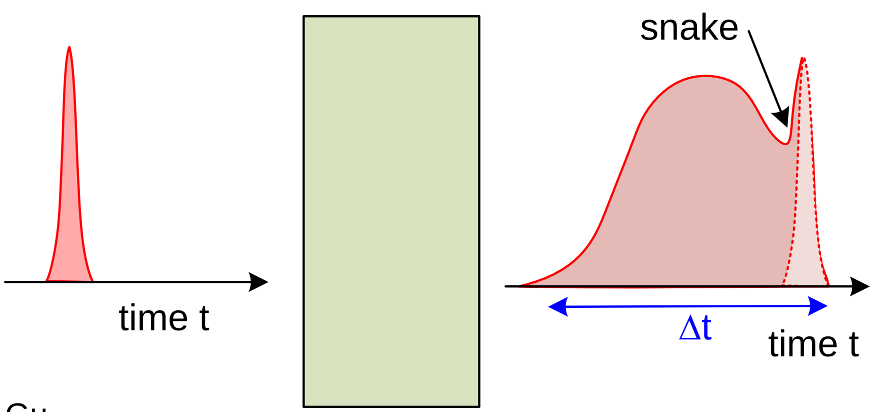
a) spectral shift



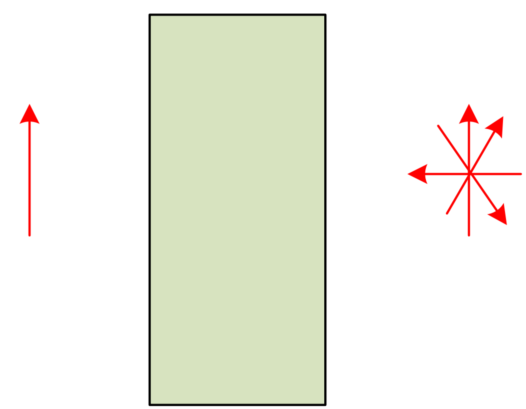

b) spatial broadening




c) temporal broadening




d) polarization

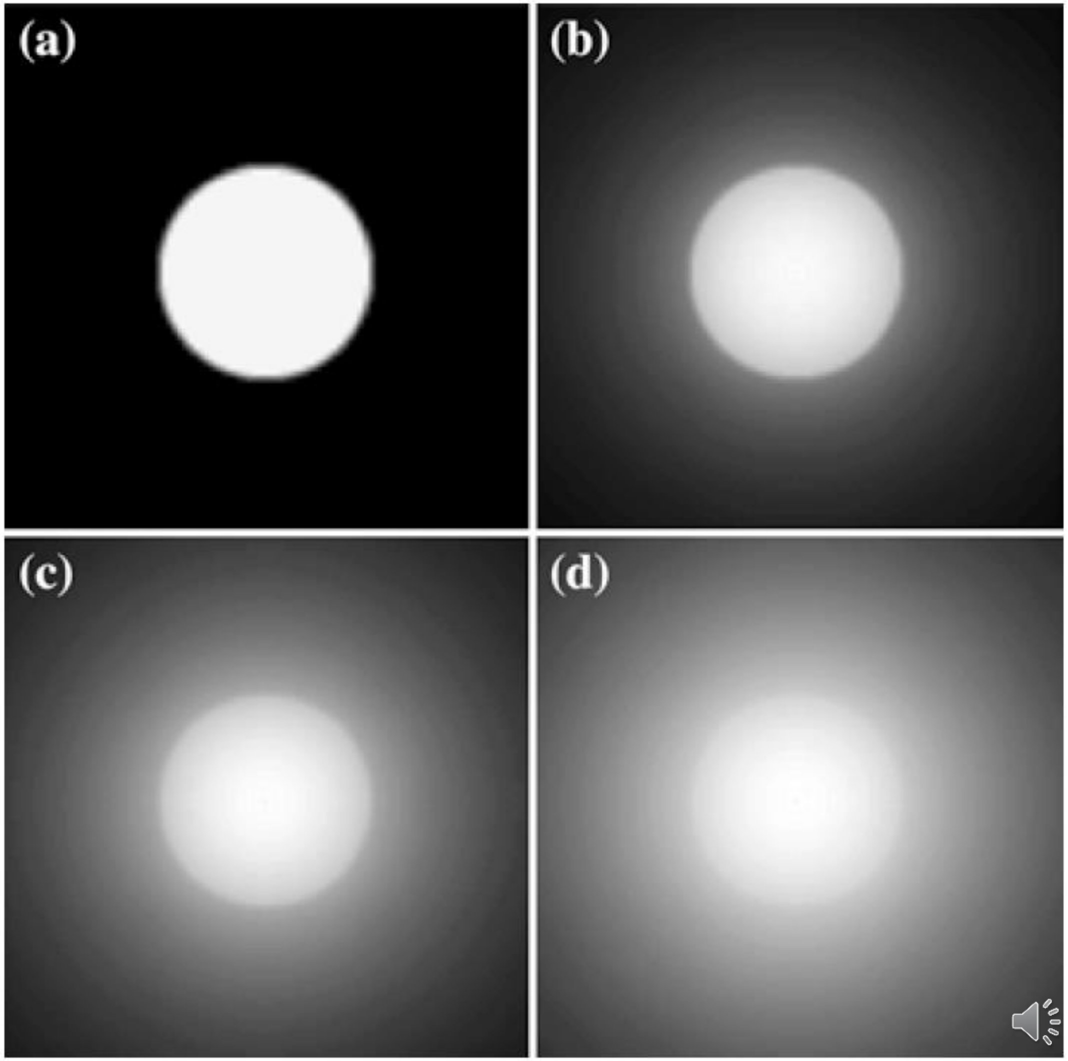


Ref: M. Gu

# Scattering in Turbid Media



- Imaging of a circular disc through a turbid medium with growing scattering strength a)...d)




Ref: M. Gu

# Scattering in Tissue



- Description of the light propagation in tissue:

  1. Coefficient of absorption $\mu_a$
     Loss of energy on the path.

  2. Coefficient of scattering $\mu_s$
     Probability of directional change per unit length of the path

  3. Phase function p(q)
     Mean angle distribution of the scattering process.
     Frequently used model: Henyey-Greenstein

- The sum of both coefficients is called the total extinction coefficient

$$\mu_t = \mu_a + \mu_s$$

# Henyey-Greenstein Scattering Model

Institute of Applied Physics
Friedrich-Schiller-Universität Jena

- Henyey-Greenstein model for human tissue
  Phase function

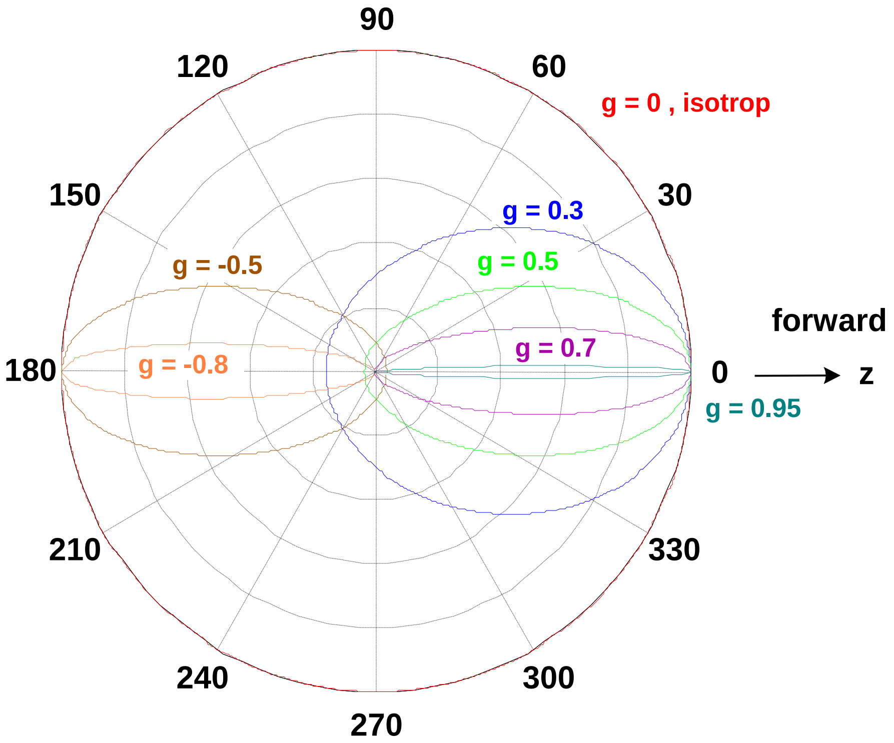
$$p_{HG}(\theta,g)=\frac{1}{4\pi}\cdot\frac{1-g^2}{\left(1+g^2-2g\cos\theta\right)^{3/2}}$$

- Asymmetry parameter g:
  Relates forward / backward scattering
  g = 0 : isotropic
  g = 1 : only forward
  g = -1: only backward

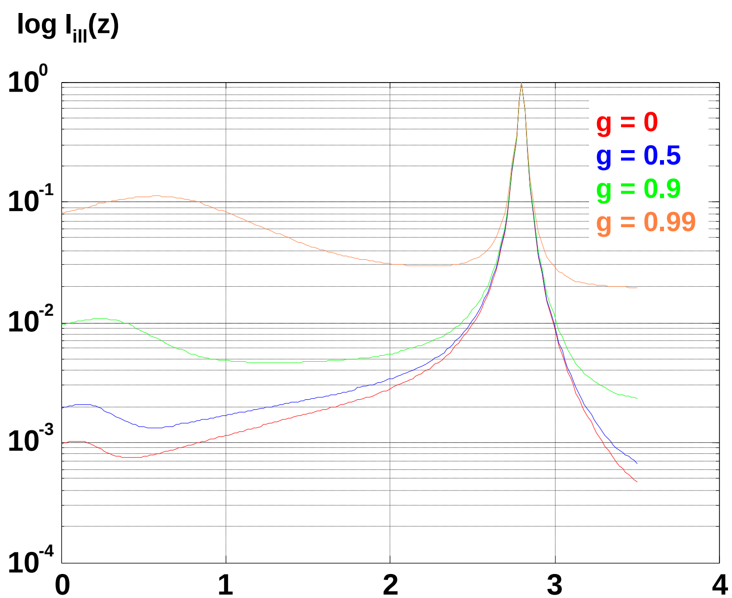
- Rms value of angle spreading

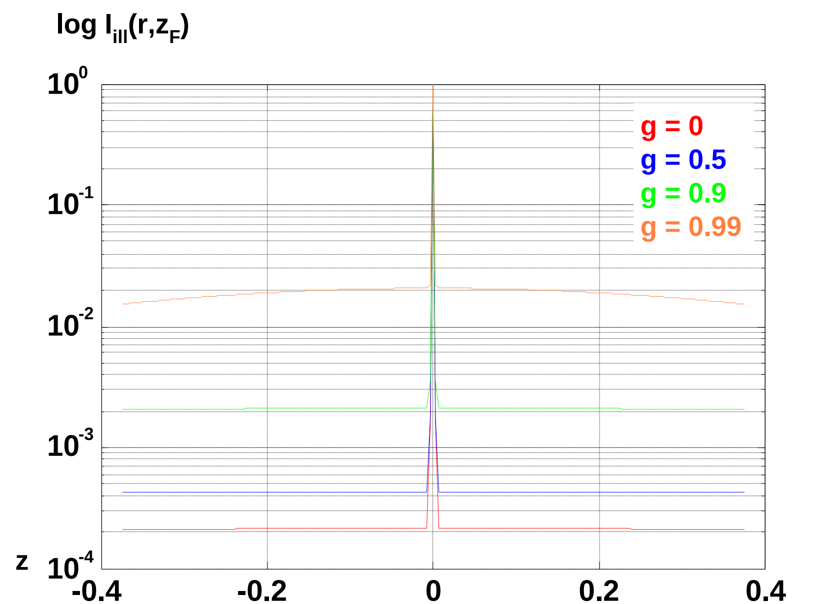
$$\theta_{rms}=\sqrt{2(1-g)}$$

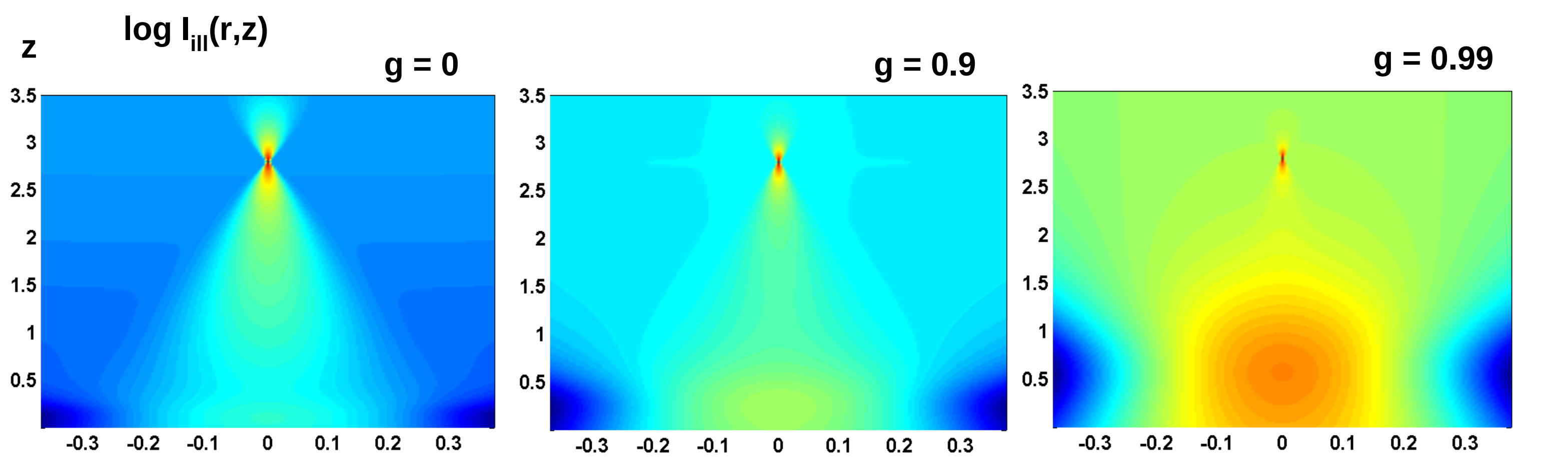
- Typical for human tissue:
  g = 0.7 ... 0.9



# Propagation in Tissue with Gaussian Beams




weak scattering

medium scattering

strong scattering

Institute of Applied Physics
Friedrich-Schiller-Universität Jena

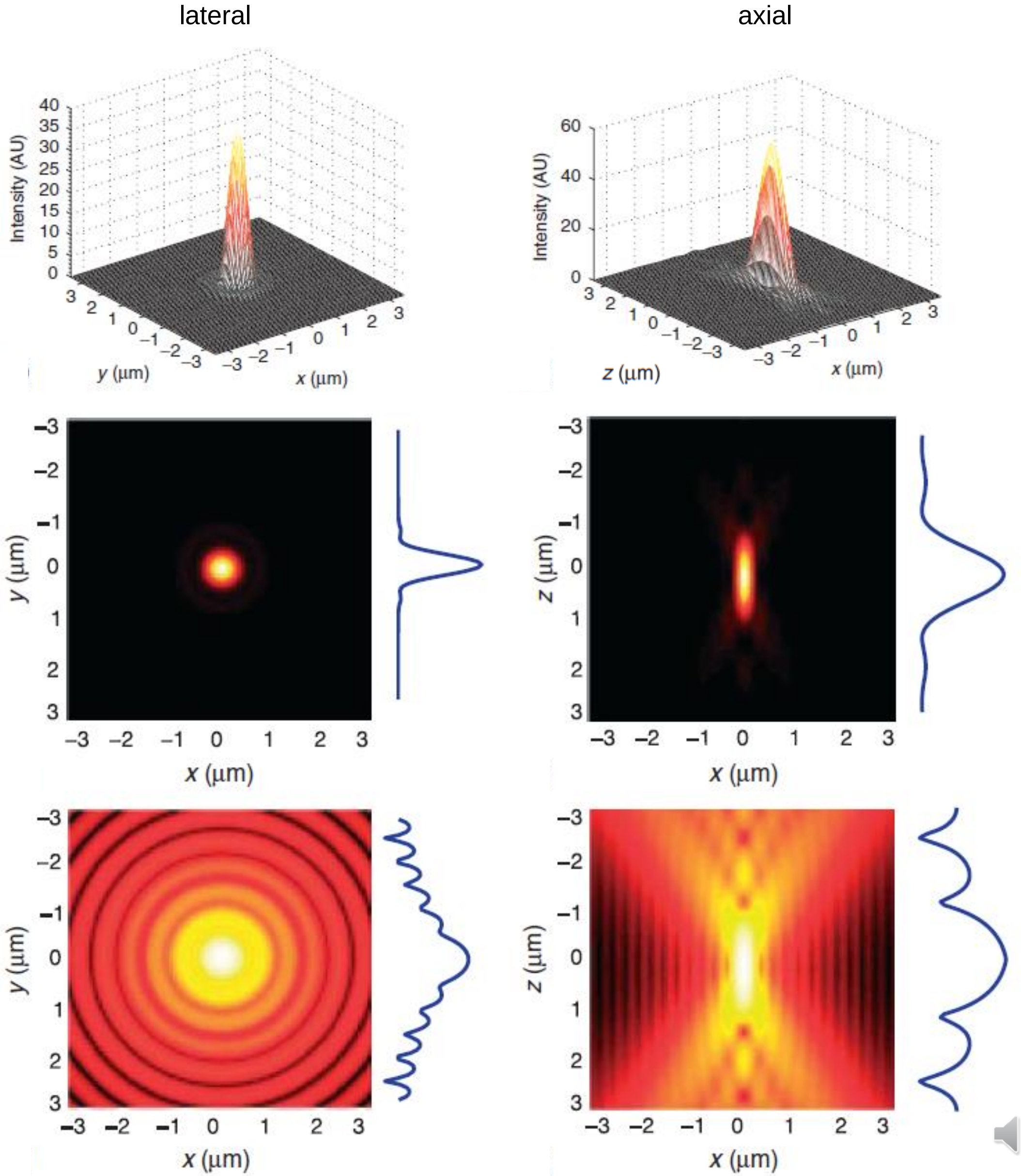
# Lateral and Axial Resolution

- Intensity distributions
- Aberration-free Airy pattern:
  lateral resolution

$$D_{Airy} = \frac{1.22 \cdot \lambda}{NA}$$

axial resolution

$$R_E = \frac{n \cdot \lambda}{NA^2}$$



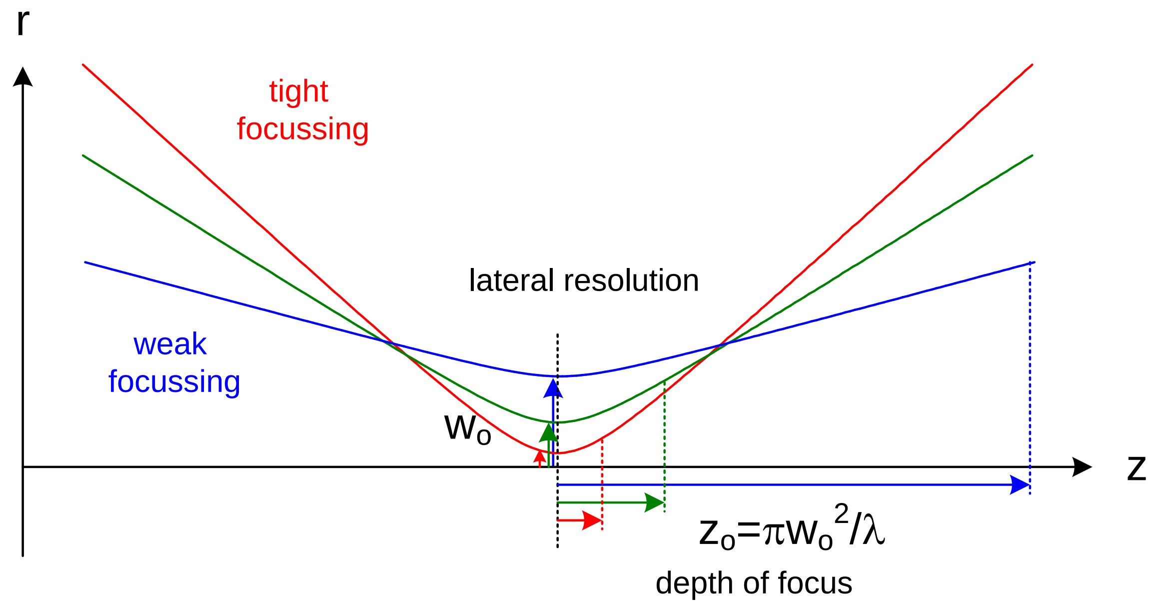

Ref: U. Kubitschek

Institute of Applied Physics
Friedrich-Schiller-Universität Jena

# Lateral Resolution vs Depth of Focus

$$z_o = \frac{\pi w_o^2}{\lambda} = \frac{w_0}{\theta_0} = \frac{\lambda_0}{\pi \theta_0^2}$$

- Gaussian beam as example:
  Lateral resolution $w_o$ coupled to depth of focus $z_o$
- Increase of depth resolution : tight focussing
- Measurement dilemma:
  measurement of deep bore holes with large divergence impossible,
  large depth of focus only for bad lateral resolution
- Imaging dilemma:
  large spreaded light cone gathers light from different depth in volume imaging,
  bad contrast conditions

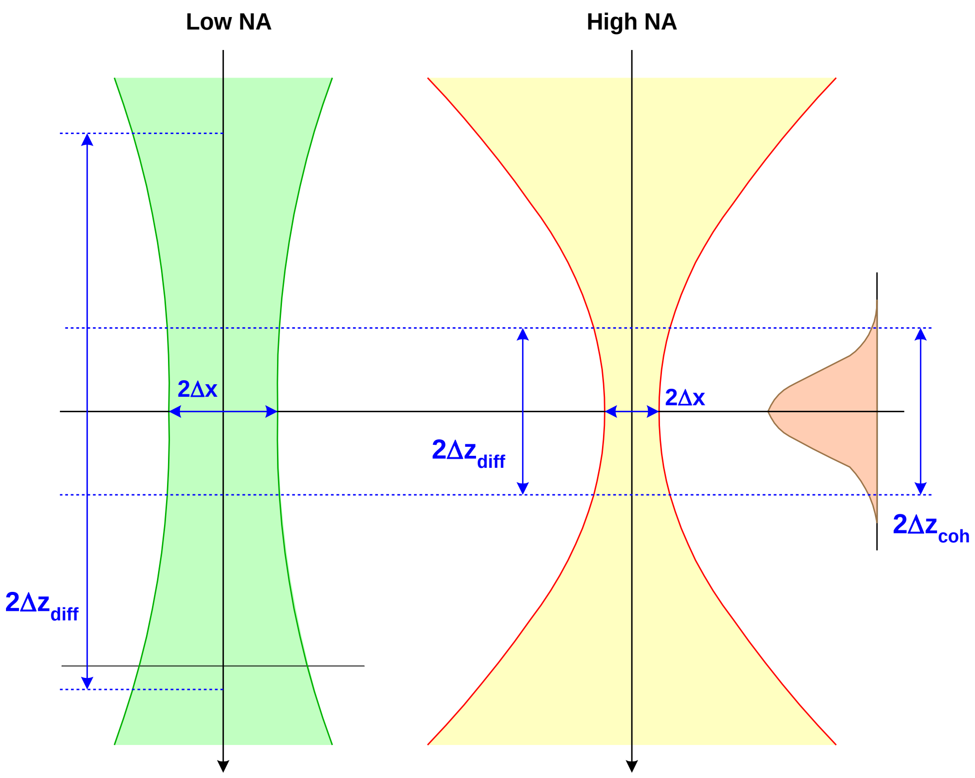


# Resolution in OCT

Institute of Applied Physics
Friedrich-Schiller-Universität Jena

1. Axial resolution limited by spectral bandwidth

$$\Delta z_{coh} = \frac{2\ln 2}{\pi} \cdot \frac{\lambda^2}{\Delta\lambda} \approx 0.4413 \frac{\lambda^2}{\Delta\lambda}$$




2. Lateral resolution: diffraction limited, improvement by confocal setup
3. Usually low NA