Institute of
Applied Physics
Friedrich-Schiller-Universität Jena

Lens Design I

Lecture 7: Aberrations II

2024-05-30

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Preliminary Schedule - Lens Design I 2024

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1	04.04.	Basics	Zhang	Introduction, Zemax interface, menues, file handling, preferences, Editors, updates, windows, coordinates, System description, 3D geometry, aperture, field, wavelength
2	18.04.	Properties of optical systems I	Tang	Diameters, stop and pupil, vignetting, layouts, materials, glass catalogs, raytrace, ray fans and sampling, footprints
3	25.04.	Properties of optical systems II	Tang	Types of surfaces, cardinal elements, lens properties, Imaging, magnification, paraxial approximation and modelling, telecentricity, infinity object distance and afocal image, local/global coordinates
4	02.05.	Properties of optical systems III	Tang	Component reversal, system insertion, scaling of systems, aspheres, gratings and diffractive surfaces, gradient media, solves
5	16.05.	Advanced handling I	Tang	Miscellaneous, fold mirror, universal plot, slider, multiconfiguration, lens catalogs
6	23.05.	Aberrations I	Zhang	Representation of geometrical aberrations, spot diagram, transverse aberration diagrams, aberration expansions, primary aberrations
7	30.05.	Aberrations II	Zhang	Wave aberrations, Zernike polynomials, measurement of quality
8	06.06.	Aberrations III	Tang	Point spread function, optical transfer function
9	13.06.	Optimization I	Tang	Principles of nonlinear optimization, optimization in optical design, general process, optimization in Zemax
10	20.06.	Optimization II	Zhang	Initial systems, special issues, sensitivity of variables in optical systems, global optimization methods
11	27.06.	Correction I	Zhang	Symmetry principle, lens bending, correcting spherical aberration, coma, astigmatism, field curvature, chromatical correction
12	04.07.	Correction II	Zhang	Field lenses, stop position influence, retrofocus and telephoto setup, aspheres and higher orders, freeform systems, miscellaneous

Contents

Rays and Wavefronts

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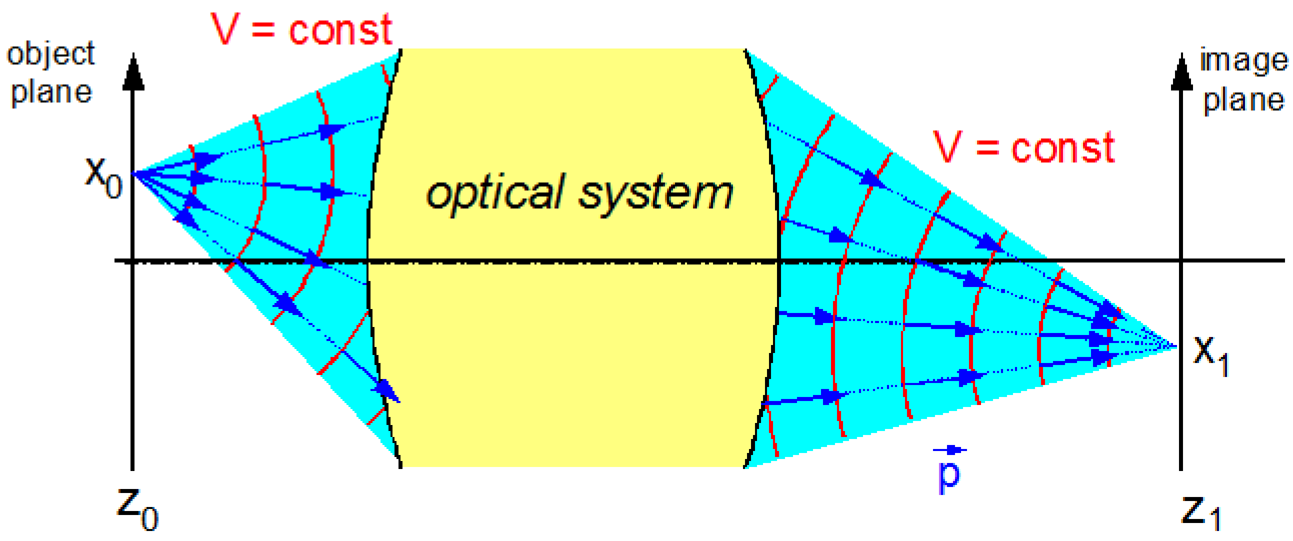
- Rays and Wavefront forms an orthotomic system
- Any closed path integral has zero value
- Corresponds to law of Malus and Fermats principle



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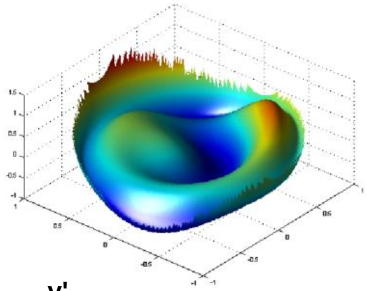
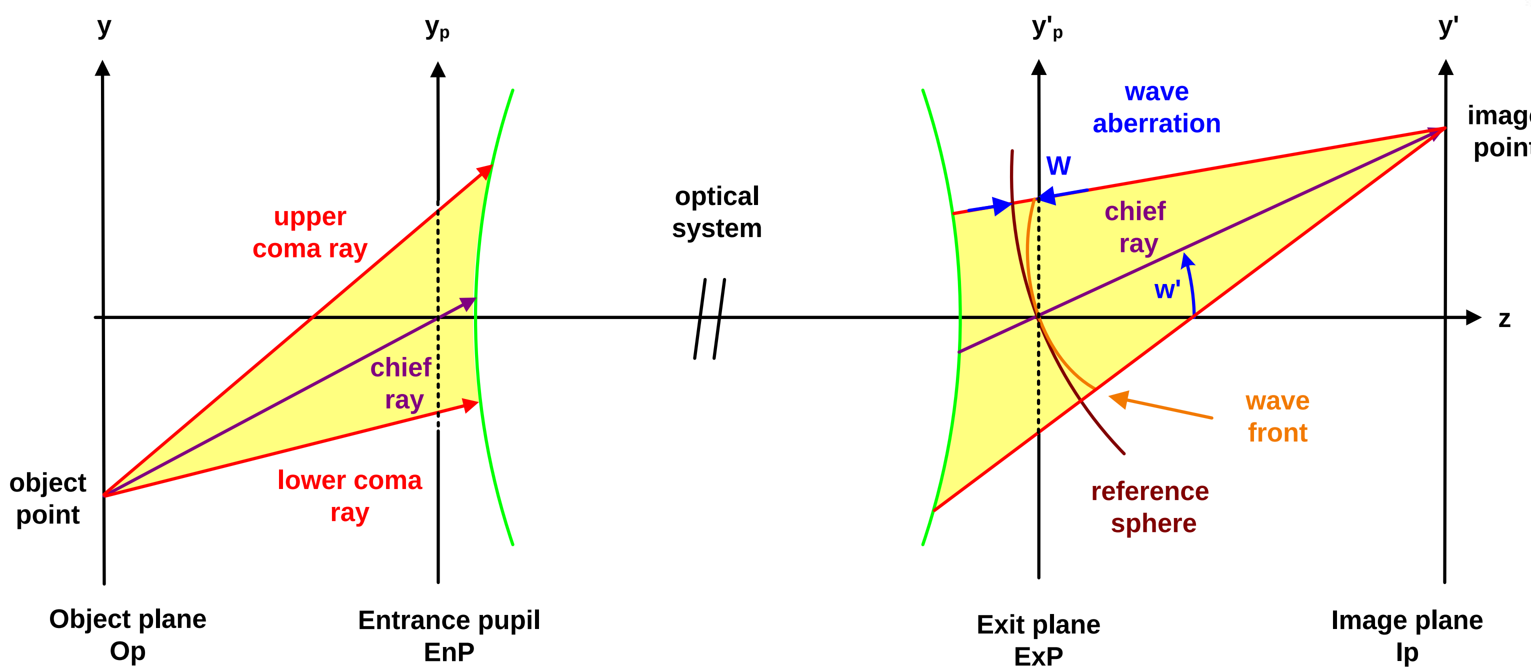
Wave Aberration in Optical Systems

- Definition of optical path length in an optical system:
  Reference sphere around the ideal object point through the center of the pupil
- Chief ray serves as reference
  Difference of OPL : optical path difference OPD
- Practical calculation: discrete sampling of the pupil area,
  real wave surface represented as matrix





Relationships

- Concrete calculation of wave aberration: addition of discrete optical path lengths (OPL)

$$l_{OPL} = \int_{OE}^{AP} n \cdot d\vec{r}$$

- Reference on chief ray and reference sphere (optical path difference)

$$\Delta_{OPD}(x,y) = l_{OPL}(x,y) - l_{OPL}(0,0)$$

- Relation to transverse aberrations

$$\frac{\partial W}{\partial y_p} = -\frac{\Delta y'}{R-W} \approx -\frac{\Delta y'}{R}$$

- Conversion between longitudinal transverse and wave aberrations

$$\Delta s' = \frac{R}{y_p} \cdot \Delta y' = \frac{\Delta y'}{\sin u'} = -\frac{R^2}{y_p} \cdot \frac{\partial W(x_p, y_p)}{\partial y_p}$$

- Scaling of the phase / wave aberration:
  1. Phase angle in radiant
  2. Light path (OPL) in mm
  3. Light path scaled in λ

$$E(x) = A(x) \cdot e^{i \cdot \varphi(x)}$$

$$E(x) = A(x) \cdot e^{i \cdot k \, \Delta_{OPD}(x)}$$

$$E(x) = A(x) \cdot e^{2\pi i \cdot W(x)}$$

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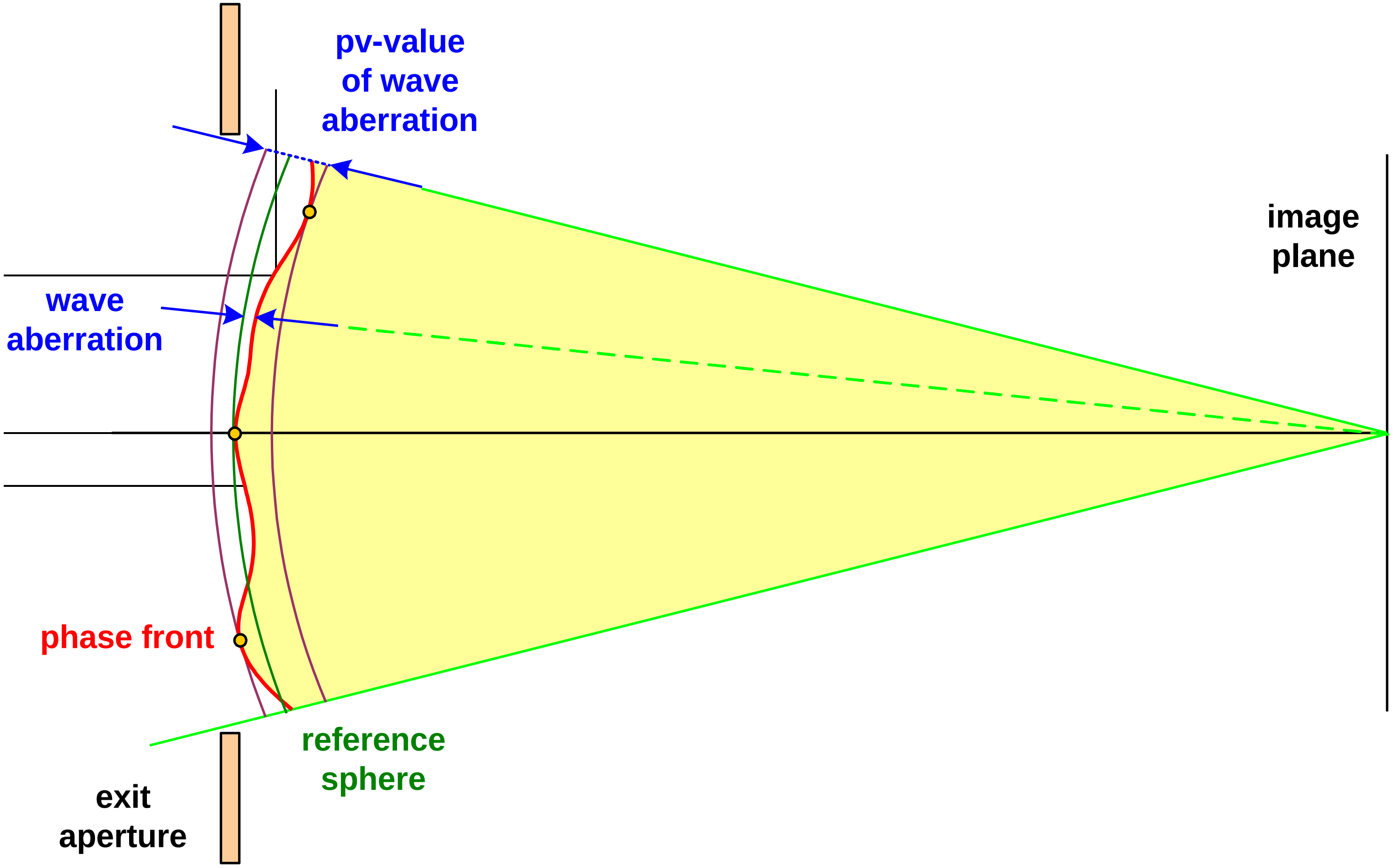
Wave Aberration

- Definition of the peak valley value



Wave Aberrations

- Mean quadratic wave deviation (W_{Rms} , root mean square)

$$W_{rms} = \sqrt{\langle W^2 \rangle - \langle W \rangle^2} = \sqrt{\frac{1}{A_{ExP}} \iint \left[W(x_p, y_p) - W_{mean}(x_p, y_p) \right]^2 dx_p dy_p}$$

with pupil area

$$A_{ExP} = \iint dx dy$$

- Peak valley value W_{pv} : largest difference

$$W_{pv} = \max \left[W_{\max}(x_p, y_p) - W_{\min}(x_p, y_p) \right]$$

- General case with apodization:
  weighting of local phase errors with intensity, relevance for psf formation

$$W_{rms} = \sqrt{\frac{1}{A_{ExP}^{(w)}} \iint I_{ExP}(x_p, y_p) \cdot \left[W(x_p, y_p) - W_{mean}^{(w)}(x_p, y_p) \right]^2 dx_p dy_p}$$

Criteria of Rayleigh and Marechal

- Rayleigh criterion:
  1. maximum of wave aberration: $W_{pv} < \lambda/4$
  2. beginning of destructive interference of partial waves
  3. limit for being diffraction limited (definition)
  4. as a PV-criterion rather conservative: maximum value only in 1 point of the pupil
  5. different limiting values for aberration shapes and definitions (Seidel, Zernike,...)

- Marechal criterion:
  1. Rayleigh crierion corresponds to $W_{rms} < \lambda/14$ in case of defocus

$$W_{rms}^{Rayleigh} \leq \frac{\lambda}{\sqrt{192}} = \frac{\lambda}{13.856} \approx \frac{\lambda}{14}$$

  2. generalization of $W_{rms} < \lambda/14$ for all shapes of wave fronts
  3. corresponds to Strehl ratio $D_s > 0.80$ (in case of defocus)
  4. more useful as PV-criterion of Rayleigh

Wave Aberrations

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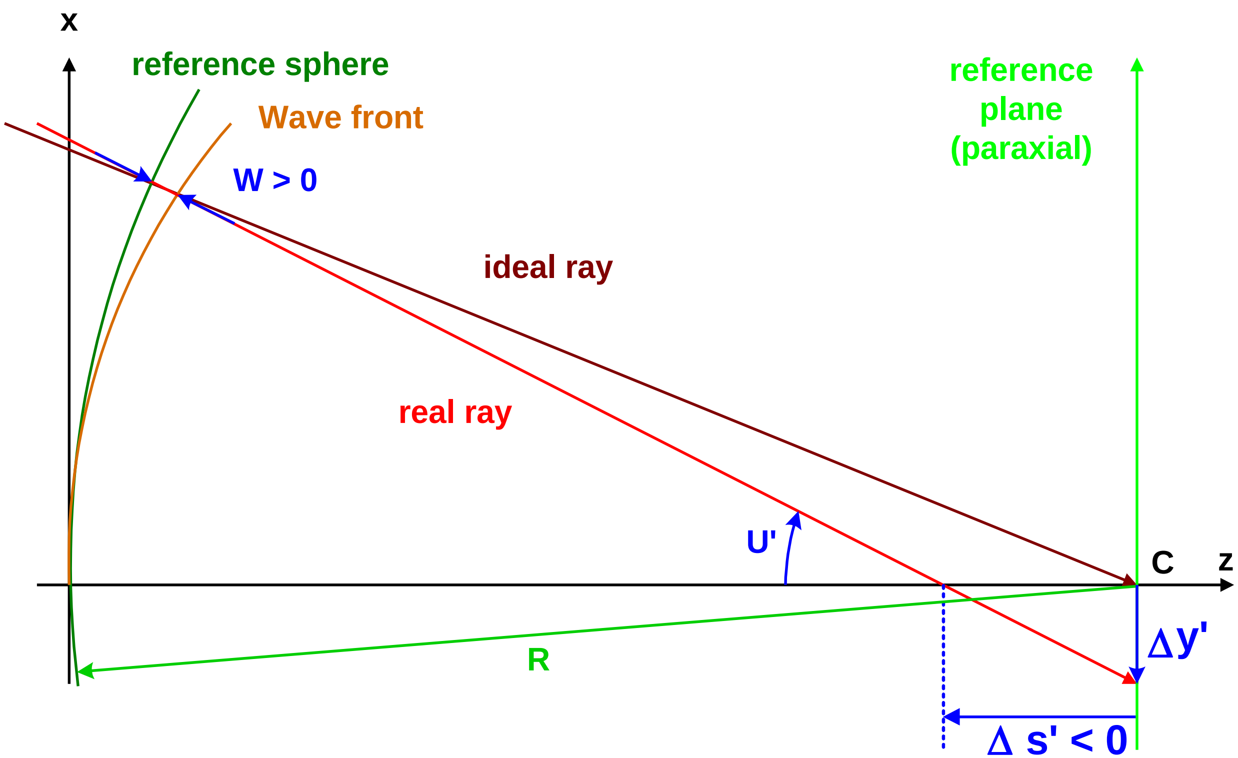
- Wave aberration: relative to reference sphere
- Choice of offset value: vanishing mean

$$\langle W(x,y)\rangle = \frac{1}{F_{ExP}} \iint W(x,y)\,dx\,dy = 0$$

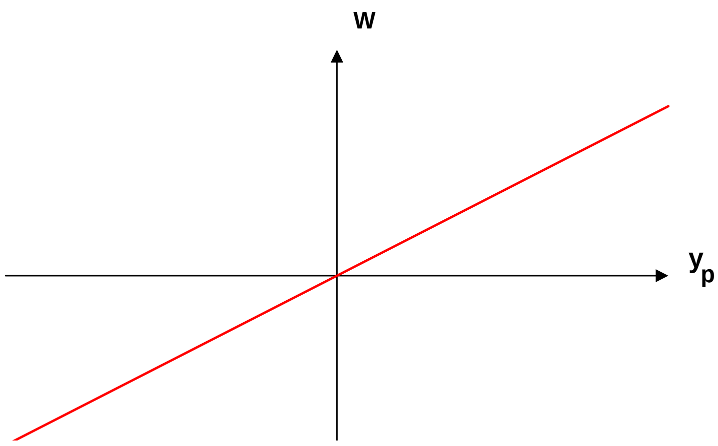
- Sign of W :
  - W > 0 : stronger convergence intersection : s < 0
  - W < 0 : stronger divergence intersection : s < 0



Tilt of Wavefront

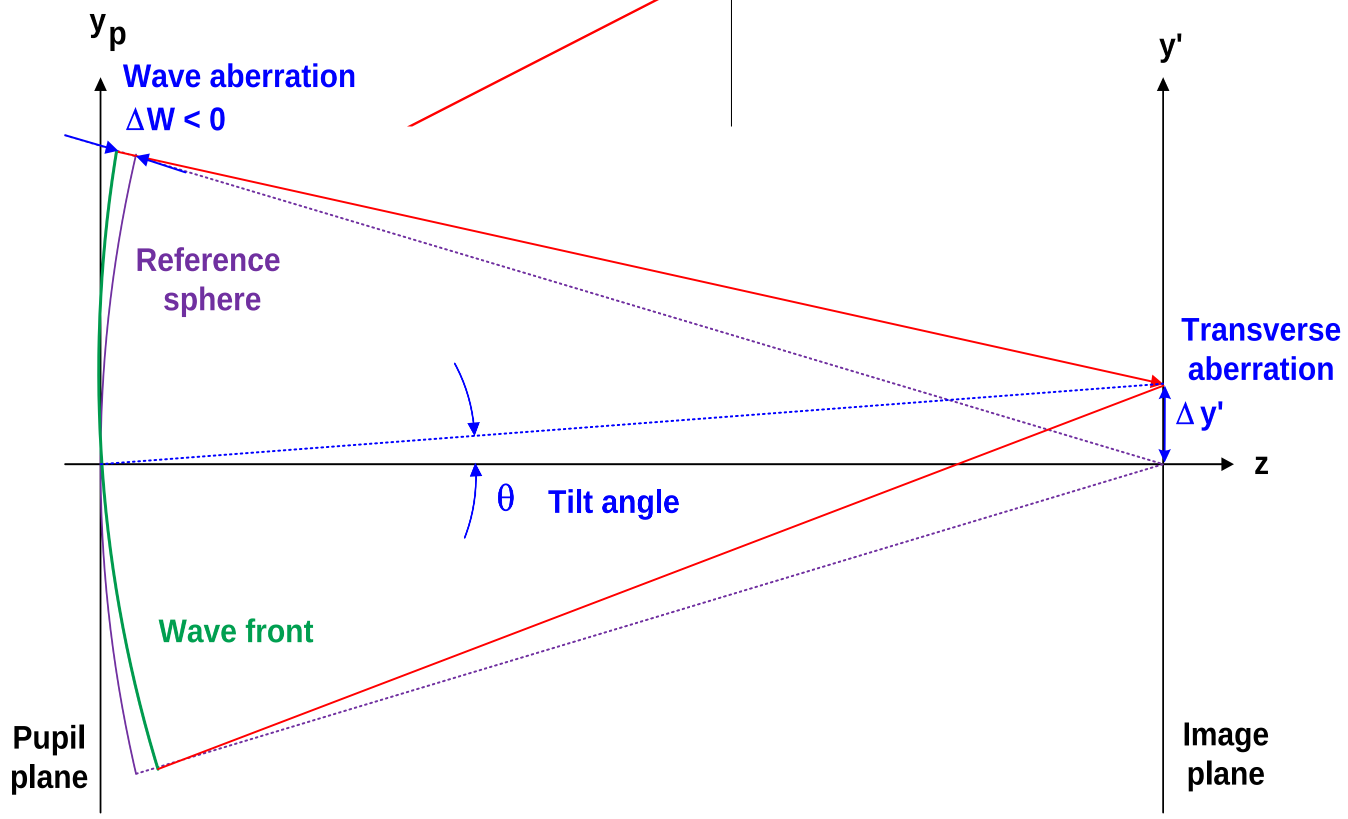
- Change of reference sphere:
  tilt by angle θ
  linear in y_p

$$\Delta W_{tilt} = n \cdot y_p \cdot \theta$$



- Wave aberration
  due to transverse
  aberration $\Delta y'$

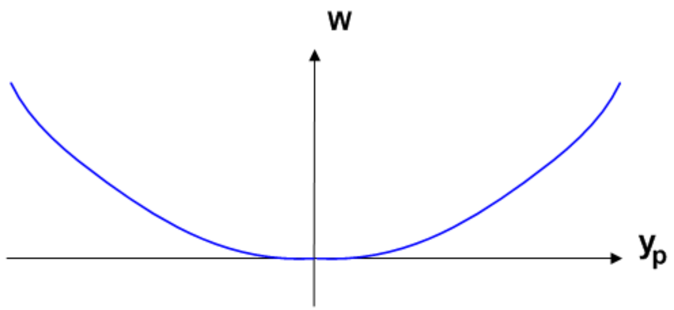
$$\Delta W_{tilt} = -\frac{y_p}{R_{Ref}} \cdot \Delta y'$$

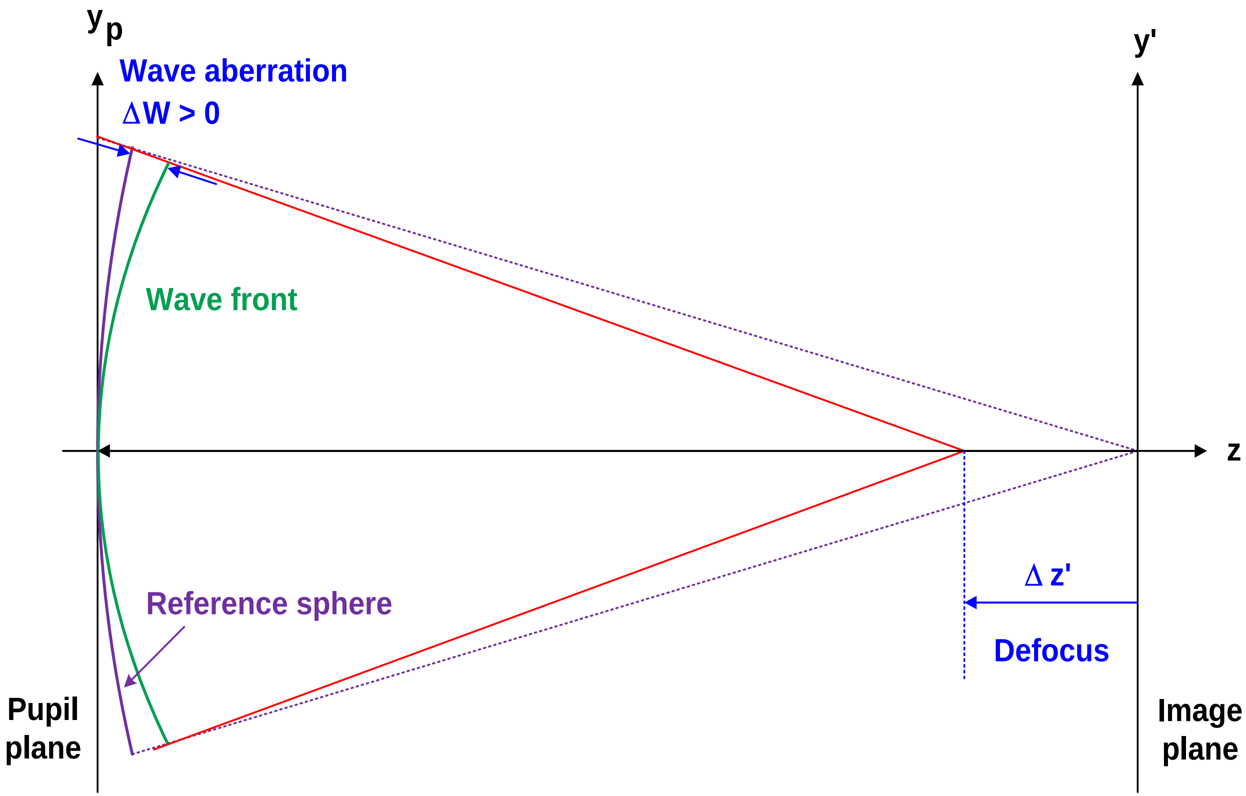


Defocussing of Wavefront

Paraxial defocussing by Δz:
Change of wavefront

$$\Delta W_{Def} = -\frac{n \cdot r_p^2}{2R_{ref}^2} \cdot \Delta z' = -\frac{1}{2} n \cdot \Delta z' \cdot \sin^2 u$$





Zernike Polynomials

- Expansion of the wave aberration on a circular area

$$W(r,\varphi)=\sum_n \sum_{m=-n}^{n} c_{nm} Z_n^m(r,\varphi)$$

$$c_{nm}=\frac{2(n+1)}{\pi(1+\delta_{m0})}\cdot\int_0^1\int_0^{2\pi} W(r,\varphi) Z_n^{m*}(r,\varphi) d\varphi r dr$$

- Zernike polynomials in cylindrical coordinates:
  Radial function R(r), index n
  Azimuthal function φ, index m

$$Z_n^m(r,\varphi)=R_n^m(r)\cdot\begin{cases}\sin m\varphi & \text{für } m>0\\ \cos m\varphi & \text{für } m<0\\ 1 & \text{für } m=0\end{cases}$$

- Orthonormality

$$\int_0^1\int_0^{2\pi} Z_n^m(r,\varphi) Z_{n'}^{m'*}(r,\varphi)\, d\varphi r dr=\frac{\pi\cdot(1+\delta_{m0})}{2(n+1)}\cdot\delta_{nn'}\delta_{mm'}$$

- Advantages:
  1. Minimal properties due to W_{rms}
  2. Decoupling, fast computation
  3. Direct relation to primary aberrations for low orders

- Problems:
  1. Computation on discrete grids
  2. Non circular pupils
  3. Different conventions concerning indeces, scaling, coordinate system

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Zernike Polynomials: Different Nomenclatures

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1. Fringe - representation
   - CodeV, Zemax, interferometric test of surfaces
   - Standardization of the boundary to ± 1
   - no additional prefactors in the polynomial
   - Indexing accordint to m (Azimuth), quadratic number terms have circular symmetry
   - coordinate system invariant in azimuth

2. Standard - representation
   - CodeV, Zemax, Born / Wolf
   - Standardization of rms-value on ± 1 (with prefactors), easy to calculate Strehl ratio
   - coordinate system invariant in azimuth

3. Original - Nijboer - representation
   - Expansion:

$$W(r,\varphi) = a_{00} + \frac{1}{\sqrt{2}} \sum_{n=0}^{k} a_{0n} R_n^0 + \sum_{n=0}^{k} \sum_{\substack{m=1 \\ n-m \\ gerade}}^{n} a_{nm} R_n^m \cos(m\varphi) + \sum_{n=0}^{k} \sum_{\substack{m=1 \\ n-m \\ gerade}}^{n} b_{nm} R_n^m \sin(m\varphi)$$

   - Standardization of rms-value on ± 1
   - coordinate system rotates in azimuth according to field point

Zernike Polynomials

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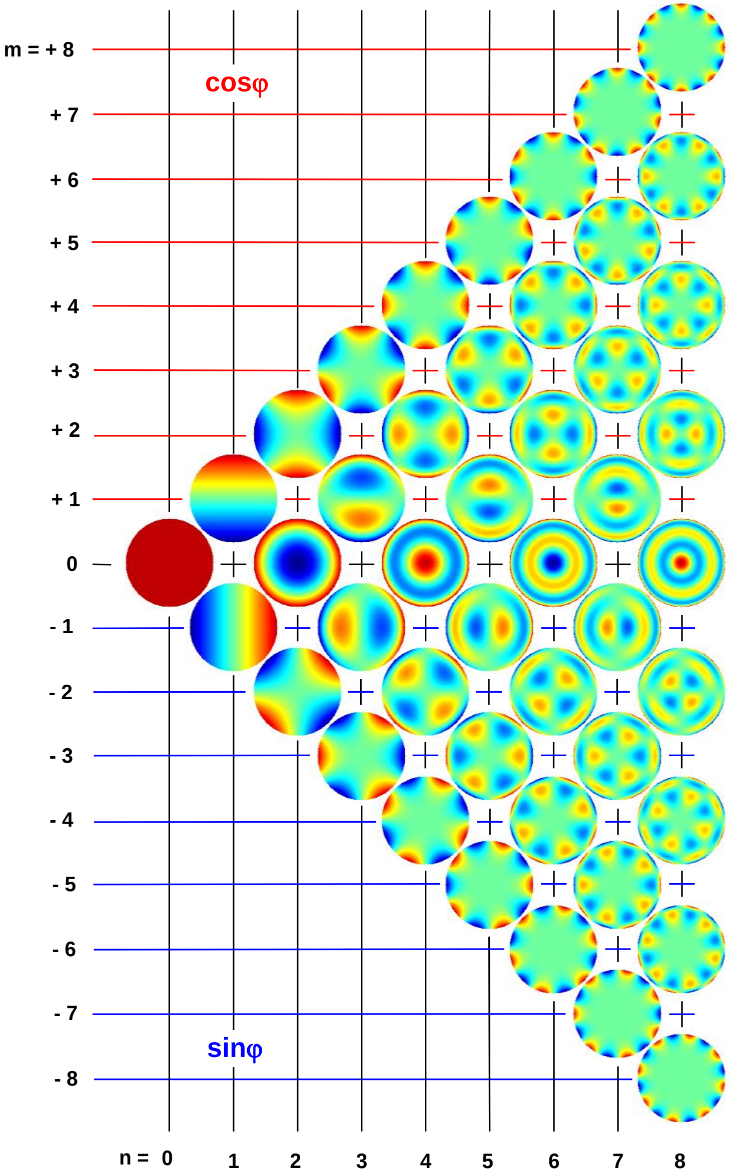
- Zernike polynomials orders by indices:
  n : radial
  m : azimuthal, sin/cos
- Orthonormal function on unit circle

$$Z_n^m(r,\varphi) = R_n^m(r) \cdot \begin{cases} \sin m\varphi & \text{für } m>0 \\ \cos m\varphi & \text{für } m<0 \\ 1 & \text{für } m=0 \end{cases}$$

- Expansion of wave aberration surface

$$W(r,\varphi) = \sum_n \sum_{m=-n}^{n} c_{nm} Z_n^m(r,\varphi)$$

- Direct relation to primary aberration types
- Direct measurement by interferometry
- Orthogonality perturbed:
  1. apodization
  2. discretization
  3. real non-circular boundary



Calculation of Zernike Polynomials

- Assumptions:
1. Pupil circular
2. Illumination homogeneous
3. Neglectible discretization effects /sampling, boundary)

- Direct computation by double integral:
1. Time consuming
2. Errors due to discrete boundary shape
3. Wrong for non circular areas
4. Independent coefficients

$$c_j = \frac{1}{\pi} \int_0^1 \int_0^{2\pi} W(r,\varphi) Z_j{}^*(r,\varphi)\, d\varphi\, r dr$$

- LSQ-fit computation:
1. Fast, all coefficients c_j simultaneously
2. Better total approximation
3. Non stable for different numbers of coefficients, if number too low
- Stable for non circular shape of pupil

$$\sum_{i=1} \left[W_i - \sum_{j=1}^{N} c_j Z_j(r_i) \right]^2 = \min$$

$$\vec{c} = \left(\underline{Z}^T \underline{Z} \right)^{-1} \underline{Z}^T \vec{W}$$

Zernike Polynomials: Explicite Formulas

n	m	Polar coordinates	Cartesian coordinates	Interpretation
0	0	1	1	piston
1	1	$r\sin\varphi$	x	tilt in x
1	-1	$r\cos\varphi$	y	tilt in y
2	2	$r^2\sin 2\varphi$	$2xy$	Astigmatism 45°
2	0	$2r^2-1$	$2x^2+2y^2-1$	defocussing
2	-2	$r^2\cos 2\varphi$	y^2-x^2	Astigmatism 0°
3	3	$r^3\sin 3\varphi$	$3xy^2-x^3$	trefoil 30°
3	1	$\left(3r^3-2r\right)\sin\varphi$	$3x^3-2x+3xy^2$	coma x
3	-1	$\left(3r^3-2r\right)\cos\varphi$	$3y^3-2y+3x^2y$	coma y
3	-3	$r^3\cos 3\varphi$	y^3-3x^2y	trefoil 0°
4	4	$r^4\sin 4\varphi$	$4xy^3-4x^3y$	Four sheet 22.5°
4	2	$\left(4r^4-3r^2\right)\sin 2\varphi$	$8xy^3+8x^3y-6xy$	Secondary astigmatism
4	0	$6r^4-6r^2+1$	$6x^4+6y^4+12x^2y^2-6x^2-6y^2+1$	Spherical aberration
4	-2	$\left(4r^4-3r^2\right)\cos 2\varphi$	$4y^4-4x^4+3x^2-3y^2-4x^2y^2$	Secondary astigmatism
4	-4	$r^4\cos 4\varphi$	$y^4+x^4-6x^2y^2$	Four sheet 0°

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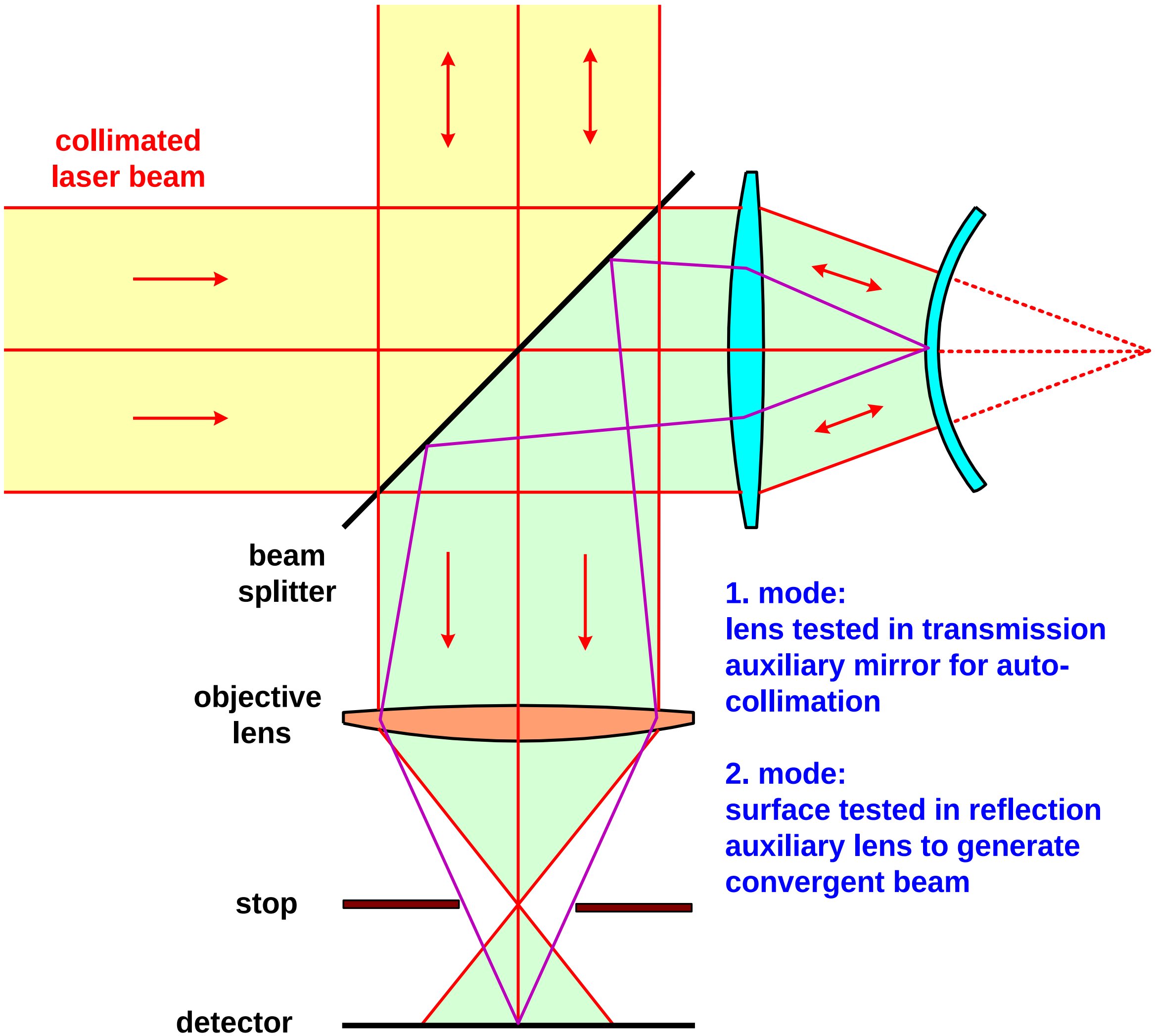
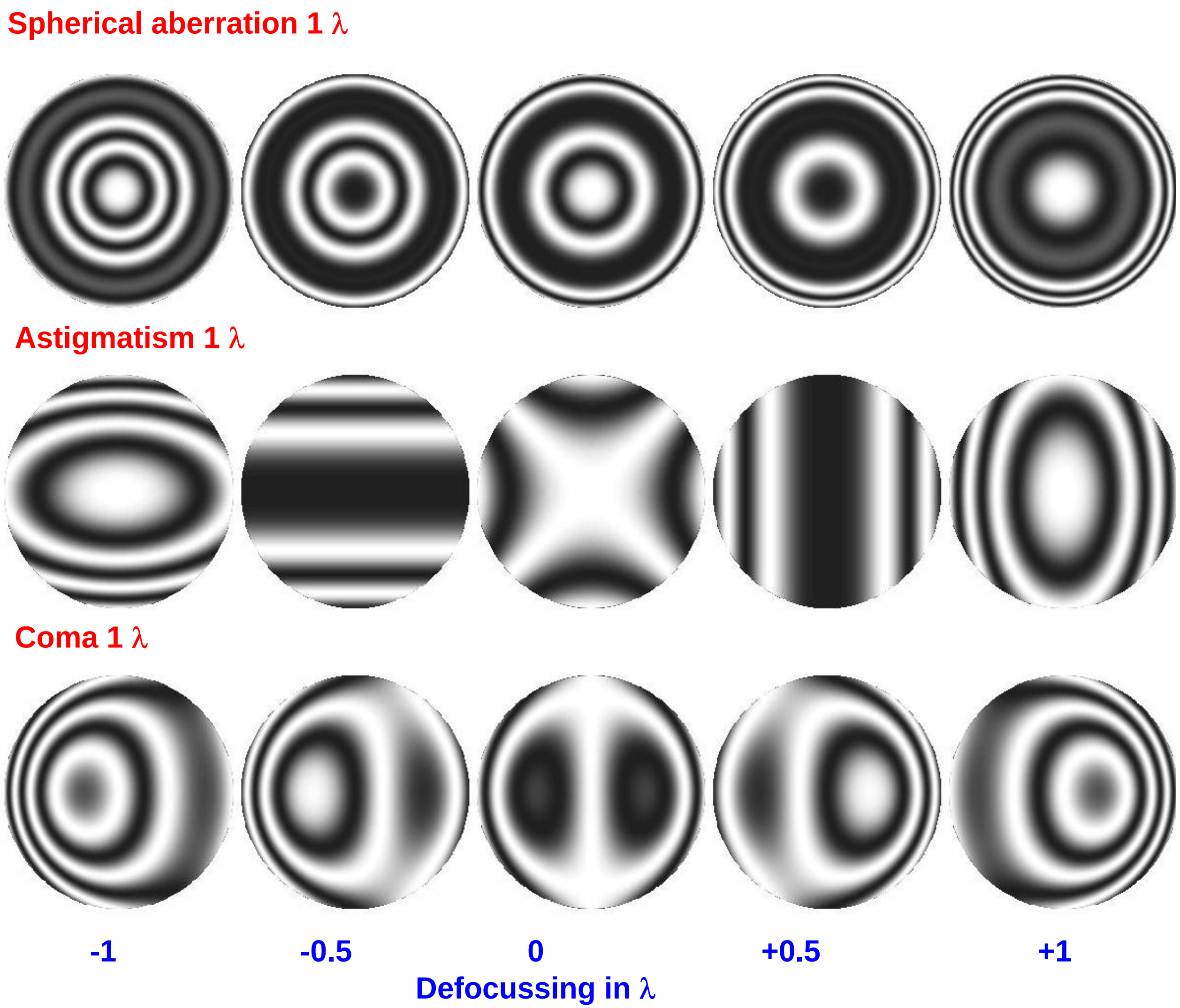
Testing with Twyman-Green Interferometer

- Short common path, sensible setup
- Two different operation modes for reflection or transmission
- Always factor of 2 between detected wave and component under test

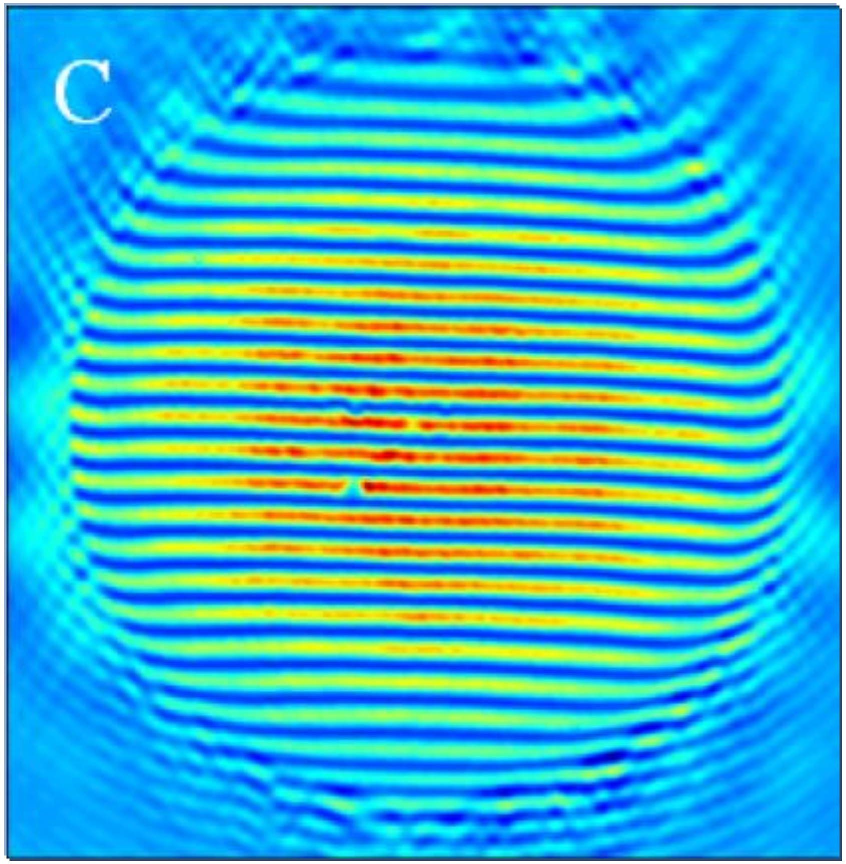


Interferograms of Primary Aberrations



Interferogram - Definition of Boundary

- Critical definition of the interferogram boundary and the Zernike normalization radius in reality



Exercise I: Wave Aberrations and Zernike Coefficients

Load the system data from Moodle. It is a microscopic lens with high NA = 1.28 from the book of Laikin.

a) Show the rms wave aberrations as a function of the defocussing. Discuss the results

b) Show the rms wave aberration as a function of the field for all wavelengths. Is the system diffraction limited ?

c) Calculate the Zernike coefficients for the primary wavelength on axis and for the maximum field size. What kind of aberration limits the performance in the field ?

d) Calculate the Zernikes on axis behind the first three components and in the image. What can be seen for the changes and the compensation effects in the spherical aberration coefficients ?

Exercise II: Aplanatic Lens

Consider a collimated incoming beam with wavelength 500 nm and diameter 10 mm. This bundle should be focussed by a perfect lens of focal length f = 50 mm.

a) Place an aplanatic-concentric lens shortly behind the ideal lens with the material SF57. What is the resulting numerical aperture in the image space ? Show at least two different methods to find the best image position.

b) Show that the spherical aberration of this setup is exactly zero for all orders.

c) Aplanatic means, that the linear coma vanishes and the imaging is free of coma for a small but finite field size. Show this property by using a small field of 2° for the current system. What is the largest present aberration ?

Delano‘s Representation of Spherical Aberration

- Paraxial optics: Delano relation

$$n' \cdot q' \cdot U' = n \cdot q \cdot U + n \cdot i \cdot (Q' - Q)$$

- Real ray comparison:

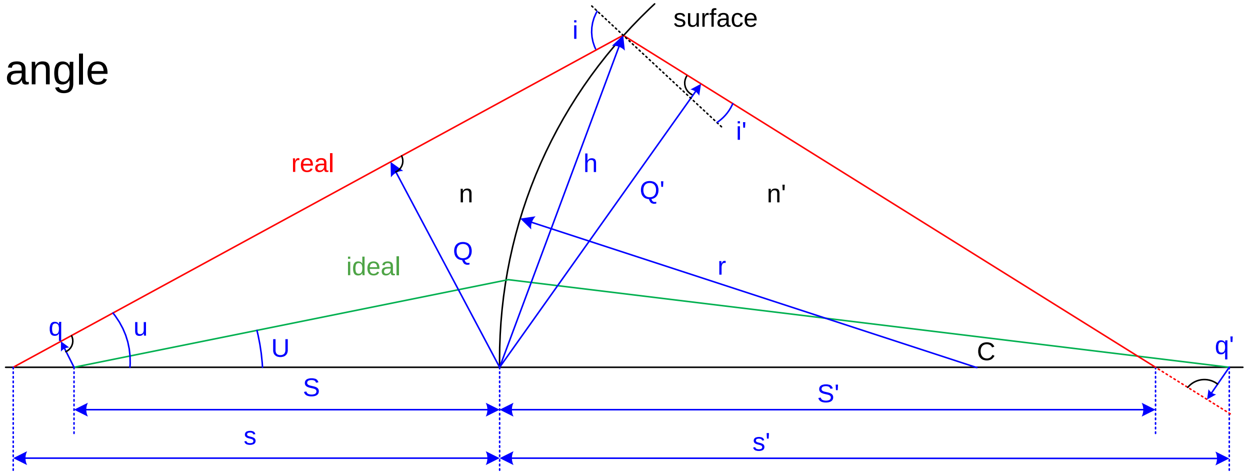
  Delano surface contribution

$$\Delta s'_{SPH} = \Delta s_{SPH} \cdot \frac{n_1 U_1 \sin u_1}{n'_k U'_k \sin u'_k} + \sum_j \frac{(Q - Q') \cdot i \cdot n_j}{n'_j U'_j \sin u'_j}$$

$$\Delta s'_{SPH} = \Delta s_{SPH} \cdot \frac{n_1 U_1 \sin u_1}{n'_k U'_k \sin u'_k} + \sum_j \frac{n_j}{n'_j} \cdot h \cdot \sin \frac{i' - i}{2} \cdot \frac{2i \cdot \sin \frac{i' - u}{2}}{U'_j \sin u'_j}$$

Surface contribution grows with
1. ratio of refractive indices
2. height of the marginal ray
3. Influence of ray bending angle

- Influence of ray bending angle



Aplanatic Surfaces

- Aplanatic surfaces: zero spherical aberration:

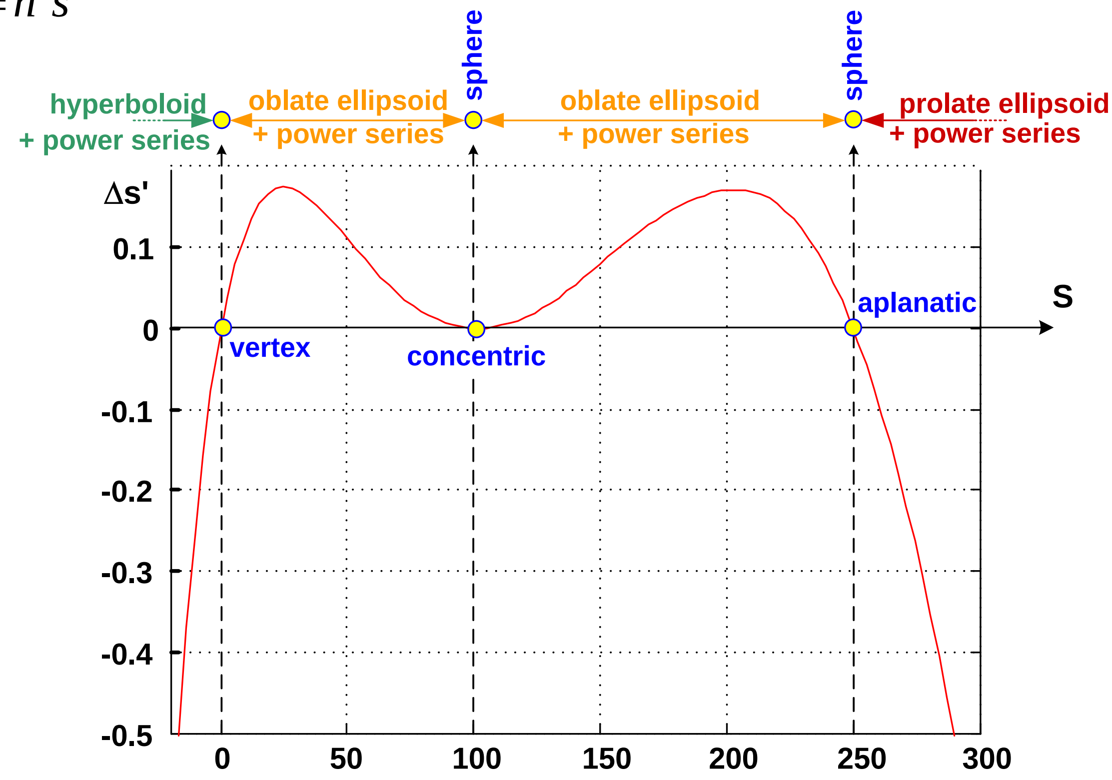
1. Ray through vertex $s' = s = 0$
2. concentric $s' = s \quad und \quad u = u'$
3. Aplanatic $ns = n's'$

- Condition for aplanatic surface:

$$r = \frac{ns}{n+n'} = \frac{n's'}{n+n'} = \frac{ss'}{s+s'}$$

- Virtual image location

- Applications:
  1. Microscopic objective lens
  2. Interferometer objective lens



Aplanatic Lenses

- Aplanatic lenses

- Combination of one concentric and one aplanatic surface:
  zero contribution of the whole lens to spherical aberration

- Not useful:
  1. aplanatic-aplanatic
  2. concentric-concentric
  bended plane parallel plate,
  nearly vanishing effect on rays