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Problem 1

a)

Solution:

Maxwell's equations in time domain:

$$\mathrm{rot}\,\vec{E}(\vec{r},t) = -\frac{\partial \vec{B}(\vec{r},t)}{\partial t} \quad ① \qquad \mathrm{div}\,\vec{D}(\vec{r},t)=0 \quad ②$$

$$\mathrm{rot}\,\vec{H}(\vec{r},t) = \frac{\partial \vec{D}(\vec{r},t)}{\partial t} \quad ③ \qquad \mathrm{div}\,\vec{B}(\vec{r},t)=0 \quad ④$$

$\because \vec{D} = \varepsilon_0 \vec{E}(\vec{r},t) + \vec{P}(\vec{r},t)$

$\therefore$ the equation ②, ③ can be written as

$$\varepsilon_0\,\mathrm{div}\,\vec{E}(\vec{r},t) + \mathrm{div}\,P(\vec{r},t) = 0$$

$$\mathrm{rot}\,\vec{H}(\vec{r},t) = \varepsilon_0 \frac{\partial \vec{E}(\vec{r},t)}{\partial t} + \frac{\partial \vec{P}(\vec{r},t)}{\partial t}$$

In frequency domain:

① $\mathrm{rot}\,\vec{E}(\vec{r},\omega) = i\omega\mu_0 \vec{H}(\vec{r},\omega)$ ② $\varepsilon_0\,\mathrm{div}\,\vec{E}(\vec{r},\omega) + \mathrm{div}\,\vec{P}(\vec{r},\omega) = 0$

③ $\mathrm{rot}\,\vec{H}(\vec{r},\omega) = -i\omega\varepsilon_0 \vec{E}(\vec{r},\omega) - i\omega \vec{P}(\vec{r},\omega)$

b)

Solution:

$\because$ this medium is linear, isotropic, dispersive, non-magnetizable and inhomogeneous dielectric medium

$$\therefore \quad \vec{D}(\vec{r},\omega) = \varepsilon_0 \vec{E}(\vec{r},\omega) + \vec{P}(\vec{r},\omega)$$

$$= \varepsilon_0 \vec{E}(\vec{r},\omega) + \varepsilon_0 \chi(\vec{r},\omega)\vec{E}(\vec{r},\omega)$$

$$= \varepsilon_0 [1+\chi(\vec{r},\omega)]\vec{E}(\vec{r},\omega)$$

$$\because \vec{P}(\vec{r},t) = \varepsilon_0 \int_{-\infty}^{t} R(\vec{r},t-t')\vec{E}(\vec{r},t')\,dt' \qquad R(\vec{r},t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \chi(\vec{r},\omega)\exp(-i\omega t)\,d\omega$$

$$\vec{P}(\vec{r},\omega) = \varepsilon_0 \chi(\vec{r},\omega)\vec{E}(\vec{r},\omega)$$

$$\therefore \varepsilon(\vec{r},\omega) = 1+\chi(\vec{r},\omega) = 1+\int_{-\infty}^{t} R(\vec{r},t)\exp(i\omega t)\,dt$$

Problem 2 Jinsong Liu 206216

a)

Solution:

$$\langle \vec{S}(\vec{r},t)\rangle = \frac{1}{2}\mathrm{Re}[\vec{E}(\vec{r})\cdot\vec{H}^*(\vec{r})]$$

b) solution:

$$\vec{E}_r(\vec{r},t) = E_0\vec{e}_x e^{-k''z}\cos(k'z-\omega t) \quad \cancel{\vec{E}(\vec{r},t) = E_0 e^{-k''z}\frac{e^{i(k'z-\omega t)}+e^{i(k'z+\omega t)}}{2}\vec{e}_x} = \frac{1}{2}E_0 e^{-k''z}(e^{k'z}$$

From Maxwell's equation in Frequency domain:

$$\vec{H}(\vec{r},\omega) = \frac{1}{i\omega\mu_0}\,\mathrm{rot}\,\vec{E}(\vec{r},\omega)$$

$$\cancel{\vec{E}(\vec{r},\omega)}\ \vec{E}(\vec{r},t) = E_0 e^{-k''z}\frac{e^{i(k'z-\omega t)}+e^{-i(k'z-\omega t)}}{2}\vec{e}_x$$

$$= \frac{1}{2}E_0 e^{-k''z}(e^{ik'z}\cdot e^{-i\omega t}+e^{-ik'z}\cdot e^{i\omega t})\vec{e}_x$$

$$= \frac{1}{2}E_0 e^{-k''z}e^{ik'z}e^{-i\omega t}\vec{e}_x + \frac{1}{2}E_0 e^{-k''z}e^{-ik'z}e^{i\omega t}\vec{e}_x$$

$$\vec{E}(\vec{r},\omega) = \frac{1}{2}E_0 e^{-k''z}e^{ik'z}\cdot\frac{1}{2\pi}\delta(\omega-\omega')\vec{e}_x + \frac{1}{2}E_0 e^{-k''z}e^{-ik'z}\frac{1}{2\pi}\delta(\omega+\omega')\vec{e}_x$$

$$\therefore \vec{H}(\vec{r},\omega) = -\frac{i}{\omega\mu_0}\,\mathrm{rot}\,\vec{E}(\vec{r},\omega)$$

$$= -\frac{i}{\omega\mu_0}\cdot\frac{1}{4\pi}E_0(-k''+ik')e^{(-k''+ik')z}\delta(\omega-\omega')\vec{e}_y - \frac{i}{\omega\mu_0}\cdot\frac{E_0}{4\pi}(-k''-ik')e^{(-k''-ik')z}\delta(\omega+\omega')\vec{e}_y$$

$$\vec{H}(\vec{r},t) = -\frac{i}{\omega'\mu_0}\cdot\frac{1}{4\pi}E_0(-k''+ik')e^{(-k''+ik')z}e^{i\omega' t}\vec{e}_y - \frac{i}{\omega'\mu_0}\frac{E_0}{4\pi}(-k''-ik')e^{(-k''-ik')z}e^{-i\omega' t}\vec{e}_y$$

$$\cancel{= -\frac{iE_0}{2\pi\omega'\mu_0}[(-k''+ik')e^{(-k''+ik')z}+(-k''-ik')e^{(-k''-ik')z}]}$$

$$= -\frac{iE_0}{4\pi\omega'\mu_0}\cdot[(-k''+ik')e^{(-k''+ik')z}e^{i\omega' t}+(-k''-ik')e^{(-k''-ik')z}e^{-i\omega' t}]\vec{e}_y$$

Problem 3

Jinsong Liu

a)

Solution: $\vec{E}(\vec{r},t)=\vec{E}_0 \exp[i(\vec{k}\cdot\vec{r}-wt)] = \vec{E}_0 e^{i\vec{k}\cdot\vec{r}} e^{-iwt} = E_0 e^{i\vec{k}'\cdot\vec{r}} e^{-\vec{k}''\cdot\vec{r}} e^{-iwt}$

Homogeneous wave:

① $\vec{k}'\vec{r}$ = constant ② $\vec{k}''\cdot\vec{r}$ = constant plane ① ⊥ plane ②

inhomogeneous wave:

evanescent waves:

$k'' \neq 0$

b)

c)

Solution:

$\varepsilon(w) = \hat{n}^2(w) = \cancel{\varepsilon'(w)+i\varepsilon''(w)}\ \varepsilon'(w)+i\varepsilon''(w)$

$(n+iK)^2 = \cancel{\varepsilon'(w)+i\varepsilon''}\ \varepsilon'(w)+i\varepsilon''(w)$

$n^2+2inK-K^2 = \varepsilon'(w)+i\varepsilon''(w)$

$$\begin{cases} n^2-K^2 = \varepsilon'(w) \\ 2nK = \varepsilon''(w) \end{cases}$$

Problem 4 Jinsong Liu 206216

a)

Solution:

- general transfer function: $\exp(i\sqrt{k^2-\alpha^2-\beta^2}\, z)$

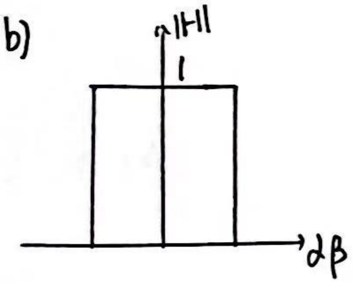
~~$\exp(i\sqrt{k^2-\alpha^2-\beta^2}\, z)$~~

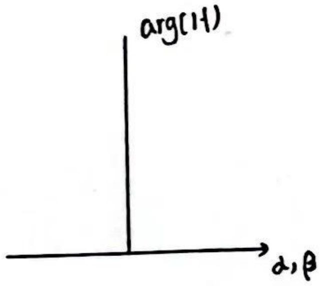
Fresnel approximation: $\alpha^2+\beta^2 \ll k^2$

$$\exp(i\sqrt{k^2-\alpha^2-\beta^2}\, z) = \exp\left(ik\sqrt{1-\frac{\alpha^2+\beta^2}{k^2}}\, z\right)$$

$$\approx \exp(ikz)\exp\left(-i\frac{\alpha^2+\beta^2}{2k} z\right)$$

b)





c)

Solution:

50μm

Problem 5:

a)

Solution: $W(z) = W_0\sqrt{1+\left(\frac{z}{z_0}\right)^2}$

b)

$R(z) = z\left(1+\frac{z_0^2}{z^2}\right)$

$z\uparrow \quad w(z)\uparrow$

$\lim\limits_{z\to\infty} R(z) = z$

c)

$q = z - iz_0$

d)

$q_0 = -iz_0$

$q_{L1} = \frac{q_0}{-\frac{1}{f_1}q_0+1}$

$q_{L2} = q_{L1} + d$

$q_1 = \frac{q_{L2}}{-\frac{1}{f_2}q_{L2}+1}$

Problem 6 Jinsong Liu

a)

$$u_0(x, y) \xrightarrow{\text{Fourier transformation}} U_0(\alpha, \beta) \xrightarrow{\text{multiply transfer function}} U_0(\alpha, \beta; z) \xrightarrow{\text{Fourier inverse transformation}} U(x, y, z)$$

Problem 7 Jinsong Liu 206216

b)

Beam diffraction	Pulse propagation
x^2+y^2	τ
(α, β)	$\bar{\omega}$
$\frac{\partial}{\partial x}, \frac{\partial}{\partial y}$	$\frac{\partial}{\partial \tau}$
W_0 at $z=0$, $W(z)$	$z=0 \to T_0$, $T(z)$
$\frac{1}{k}$	D
$L_B = \frac{2\pi}{\lambda} W_0^2$	$L_D = -\frac{T_0^2}{2D}$
spatial frequency	temporal
$i\frac{\partial \tilde{U}(x,y,z)}{\partial z} + \frac{1}{2k_0}\Delta^2 \tilde{U}(x,y,z) = 0$	$i\frac{\partial \tilde{V}(z,\tau)}{\partial z} - \frac{D}{2}\frac{\partial^2}{\partial \tau^2}\tilde{V}(z,\tau) = 0$

a)

~~group~~ group velocity: the velocity of the wavefront propagation

phase velocity: the velocity of the phase change

$$\frac{1}{V_{ph}} = \frac{k_0}{w_0} = \frac{n(w_0)}{c}$$

$$\frac{1}{V_{gr}} = \frac{\partial k}{\partial w} \leftarrow k = \frac{w}{c} n(w)$$

$$\frac{1}{V_{gr}} = \frac{1}{c}\left[n(w) + w\frac{\partial n(w)}{\partial w}\right]$$

$$\text{GVD: } \frac{\partial^2 k}{\partial w^2} = -\frac{1}{V_g^2}\frac{\partial V_g}{\partial w}$$

c)

$$U(x,y,z) = A_0\sqrt{\frac{T_0}{T(z)}}\exp\left(-\frac{\tau^2}{T^2(z)}\right)\exp\left(-iC(z)\frac{\tau^2}{T_0^2}\right)\exp(i\varphi)\, e^{i(kz-wt)}$$

$$T(z) = T_0\sqrt{1+\left(\frac{z^2}{z_0^2}\right)}$$

$$C(z) = \frac{T_0^2}{2DR(z)}$$

$$R(z) = \frac{z^2+z_0^2}{z}$$