Examination to Optical Modeling and Design I
Jena July 16, 2008
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Answer all questions in your own words and with mathematics where needed.

1. What is the definition of the Fourier transformation between time t and angular frequency? (2P)

$$E(r,t) = FT^{-1}\tilde{E}(r,\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\tilde{E}(r,\omega)exp(\quad i\omega t)d\omega$$

$$E(r,\omega) = FT\,\tilde{E}(r,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\tilde{E}(r,t)exp(i\omega t)dt$$

2. Describe the contents of the convolution theorem. (3P)

Convolution theorem is that under suitable conditions, the Fourier transform of a convolution is the point-wise product of Fourier transform of each part.

In equation: $FT\{f * g\} = FT\{f\} \bullet FT\{g\}$

3. What is the shape of the autocorrelation function of a rectangular function (1D)?(2P)

autocorrelation function of rectangular is a triangle. (Tigran)

4. What are the Fourier transformations of a constant function, a rect function and a Gaussian function?(3P)

$$FT\{\Lambda\} = \Lambda\delta(\omega)$$

$$FT\{rect(t)\} - \frac{1}{\sqrt{2\pi}} sinc(\frac{\omega}{2\pi})$$

$$f(x) = e^{-ax^2} \qquad FT\{f(x)\} = \frac{1}{\sqrt{2a}} e^{-\frac{\omega^2}{4a^2}}$$

5. A cosine function should have a period of 5 μm. What is the maximum sampling distance in order to maintain complete information of the cosine?(3P)

I think it is about Nyquist theorem (Fmax=2*Fsignal).?

6. What are the three matter equations in the frequency domain for linear media?(3P)

$$\widetilde{j_c}(r,\omega) = \tilde{\sigma}\ \widetilde{E_c}(r,\omega)$$

$$\widetilde{D_c}(r,\omega) = \tilde{\varepsilon}(r,\omega)\widetilde{E_c}(r,\omega) = \varepsilon_0 \widetilde{\varepsilon_r}(r,\omega)\widetilde{E_c}(r,\omega)$$

$$\widetilde{B_c}(r,\omega) = \tilde{\mu}\ \widetilde{H_c}(r,\omega)$$

7. Transfer one of them into time domain. (3P)

$$j_c(r,t) = \sigma E_c(r,t)$$

$$D_c(r,t) = \varepsilon(r,t)E_c(r,t) = \varepsilon_0\varepsilon_r(r,t)E_c(r,t)$$

$$B_c(r,t) = \mu H_c(r,t)$$

不確定 不对吧....傅里叶逆变换后 应该是卷积了吧？

8. When do we speak about homogeneous, isotropic and non-dispersion media respectively?(3P)

For factors, $\sigma(r,\omega),\varepsilon(r,\omega),\mu(r,\omega)$

Homogeneous media: no dependency of position r(x,y,z)

Isotropic media: all matter parameter are scalar quantities and direction-dependency disappear.

(they are all scalar quantities and no dependency of directions)

Non-dispersive media: no dependency of frequency

9. Define a plane wave. In which type of medium are they solutions of Maxwell's equations?Discuss further conditions on the parameters of a plane wave, which makes them to a solution?(5P)

E=Eo*exp(-i*k*r)

(i前面有負號嗎?我怎麼記得沒有?形式什么都可以 有负号的)in should be defined linear media. **k** should suffice dispersion relation and **E** should suffice Maxwell equation.

plane waves are solution of Maxwell's equation for homogeneous, isotropic media. To be the solution of Maxwell's equation plane waves also should satisfy dispersion relation.(Tigran)

10. What is difference between homogeneous and inhomogeneous plane waves?(2P)

For homogeneous plane waves, the k' = 0. While, for inhomogeneous plane waves,

k' ≠0且(k⊥k'这里不需要吧?因为从kk'=0得出必定垂直如何k'/=0)(可是FMO課件上有寫k⊥k'耶,你推一下就知道了,Nainai...寫了k⊥k'會更清楚明瞭吧?maybe,就2分),, evanescent waves.

homogeneous plane wave: the real (k) and imaginary (k prime) part of complex wavevector are parallel
Inhomogeneous plane wave: they are not parallel(Tigran)

11. What is dispersion relation of plane waves?(2P)

Dispersive relation is $|k|^2 = \frac{\omega^2}{c^2} n^2(\omega)$, k and n are complex in general case

12. What is the message of spectrum-of-plane-waves decomposition of any electromagnetic field in homogeneous and isotropic media?Discuss the reasons?(5P)

Spatial Fourier transform means decomposition of the field into the plane waves with different wavevectors. That means if a plane wave is a solution in homogeneous and isotropic media also the superposition of them, which is the real field is a solution. This is true for Maxwell's equations in linear medium. (Tigran)

13. How many components of electric and magnetic field vectors are independent in homogeneous and isotropic media?Discuss the reasons.(5P)

There only 4 components are independent but others are expressed by those 4 (Ex, Ey, Kx, Ky) with Maxwell equations and dispersion relation.

14. In Optics light fields are typically described by complex amplitudes of harmonic fields. How they related with the real electric and magnetic fields?(3P)
我認為此題答案好像是...公式(3.82)跟公式(3.88)
我還是覺得是公 式(3.82)跟公式(3.88)耶

Equation 3.28 on page 22 from lecture script is the answer, I think.
但是3.28並沒有跟H有關聯啊....,H is the same style with E
real field equals to the half of the sum of complex field and its conjugate. Also you can find it in more detail in the script on page 22.(Tigran)