Friedrich-Schiller-Universität Jena
Institute of Condensed Matter Theory and Solid State Optics – PAF
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WS 2014/2015

Midterm Exam
Structure of Matter

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Exercise 1 **8 Points**

A certain knowledge of scales and dimensions of physical quantities is essential in all fields of physics.

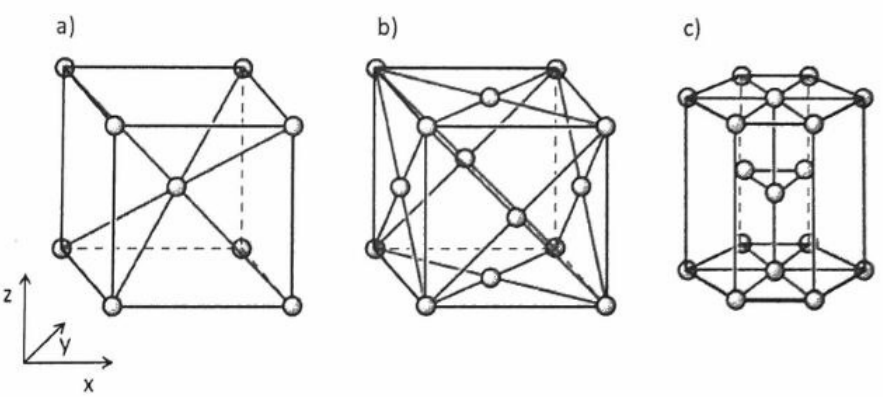
i) Please give an estimate value for the de Broglie wavelength of a resting football ($\mathbf{v} = 0$). What is a typical wavelength of IR, visible light and UV?

ii) Order the following colors with respect ~~to the wavelength~~. Start with the shortest wavelength: *yellow, red, blue, green.*
λ red yel. Green blue
630 530 630 530

iii) Which of the colors mentioned in ii) will create the highest voltage via the photoelectric effect and why?

iv) An electron and a football travel at $\mathbf{v} = 1\,ms^{-1}$. Which of them has the shorter de Broglie wavelength and why?

v) Which of the displayed lattices has a nonisotropic optical response? In which plane is the electric field always parallel ~~to~~ the induced polarization for all three crystals!



$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$

$\nabla \times E = -\frac{\partial B}{\partial t}$

$\nabla \cdot B = 0$

$\nabla \times B = \frac{1}{c^2}\frac{\partial}{\partial t}\vec{E} + \mu_0 \vec{J}$

Figure 1: Unit cells of different lattices

Exercise 2 **8 Points**

Consider the electric field of a plane wave propagating in vacuum given by $\mathbf{E}(\mathbf{r}, t) = \Re\mathbf{E}_0 \left\{e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\right\}$.

i) Under which conditions is this field divergence-free ($\operatorname{div} \mathbf{E} = 0$)?

ii) Please derive the magnetic field $\mathbf{B}(\mathbf{r}, t)$ linked to this electric field in the absence of external currents.

iii) Derive the expression for the time-averaged Poynting vector of this plane wave.

iv) Please derive the general wave equation for the $\mathbf{B}$-field from Maxwell's equations.

Please turn page

$\langle S \rangle = \frac{1}{2} \operatorname{Re}\{E_1 \times B_1^*\}$

Exercise 3 **12 Points**

In frequency domain, the response of a material to an external electric field is described by the complex dielectric function $\varepsilon(\omega)$. The respective dielectric functions of a dielectric material with a single resonance and a typical metal are given by

$$\varepsilon_{dielectric}(\omega) = 1 + \frac{f}{\omega_0^2 - \omega^2 - i\gamma\omega}, \qquad \varepsilon_{metal}(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}.$$

i) Please sketch $\varepsilon'(\omega)$ and $\varepsilon''(\omega)$ for both cases. Mark the regions of normal and anomalous dispersion and the region of strong absorption for the dielectric.

ii) Give the formula of the plasma frequency in a metal. What happens with $\varepsilon'_{metal}(\omega)$ in the vicinity of the plasma frequency? Is there anything similar in dielectrics?

iii) What happens to the field propagation for $\varepsilon'(\omega) < 0$?

Exercise 4 **5 Points**

i) Assume that the imaginary part of the dielectric function is given by $\varepsilon''(\omega) = A\delta(\omega - \omega_0)$ for positive frequencies $\omega > 0$.

(a) What does $\varepsilon''(\omega)$ look like for $\omega < 0$?

b) Calculate $\varepsilon'(\omega)$ for all frequencies.

ii) Name two properties of the response function $R(\tau)$ which are necessary to derive the Kramers-Kronig relations?

Exercise 5 **8 Points**

Consider a massive particle in a potential box

$$V(x) = \begin{cases} 0 & 0 \leq x \leq a, \\ \infty & \text{otherwise} \end{cases}$$

The particle shall be in a superposition composed of the ground state and the first excited state, so that its wavefunction at $t = 0$ can be written as

$$\Psi(x, t = 0) = c_1\psi_1(x) + c_2\psi_2(x),$$

where c_1 and c_2 are constants and $\psi_1(x)$ and $\psi_2(x)$ are the eigenfunctions corresponding to the ground state and first excited state, respectively. The average value of the energy of the particle is $\frac{\pi^2\hbar^2}{ma^2}$. What does this mean for the constants c_1 and c_2? Calculate the wavefunction $\Psi(x, t)$ for $t > 0$.