Name: ______________________ Date of birth: ______________ Student ID No. ____________

Midterm Exam
"Fundamentals of modern optics"
WS 2015/16
to be written on December 14, 8:15 - 9:45 am

Problem 1 – Maxwell's equations **$3 + 5 + 3 = 11$ points**

a) Write down Maxwell's equations in time domain, with external sources, in a material, in its general form. Furthermore, write down the constitutive equations for auxiliary fields **D** and **H**, in time domain, where the material is homogeneous, dispersive, linear, isotropic, and non-magnetic.

b) Name and write the units (in SI) of all the variables and physical constants you used in part (a).

c) Given the assumptions of part (a), find the wave equation for the **H** field in the time domain.

Problem 2 – Normal Modes **$1 + 2 + 3 = 6$ points**

A plane electromagnetic wave has **B** given by

$$\mathbf{B}(x, y, z, t) = B_0 sin\left[(x + y)\frac{k}{\sqrt{2}} - \omega t\right]\hat{z}$$

where k is the wave number and $\hat{x}, \hat{y}$ and $\hat{z}$ are the cartesian unit vectors in x, y and z directions respectively.

a) At a given location, how many times $\mathbf{B}(x, y, z, t)$ become zero in one second for $\omega = 10^{14} rad/sec$?

b) Calculate the electric field $\mathbf{E}(x, y, z, t)$ corresponding to the above magnetic field.

c) Find the time averaged Poynting vector for this electromagnetic wave.

Problem 3 – Beam propagation **$2 + 2 + 2 + 2 + 2 = 10$ points**

A certain circularly symmetric object, infinite in extent, has amplitude transmittance

$$u_0 = 2\pi a J_0(2\pi a r) + 4\pi a J_0(4\pi a r)$$

where J_0 is a bessel function of the first kind, zero order, r is the radius in the two dimensional plane and a is a constant. This object is illuminated by a normally incident, unit amplitude plane wave of wavelength λ.

a) Calculate the spatial frequency spectrum $U_0(\rho; z = 0)$, where ρ is the spatial frequency in the radial direction given by $\rho = \sqrt{\alpha^2 + \beta^2}$

b) Write down the free space transfer function $H(\rho; z)$. Indicate propagating and evanescent wave regions.

c) Calculate the field $u(r, z)$.

d) At what distances behind this object will we find a field distribution that is of the same form as that of the object, upto a possible complex constant?

e) Please provide a condition on a with respect to wavelength λ for the periodic modulation to appear. Explain the physics behind it.

Problem 4 – Propagation of Gaussian Beams **$2 + 2 + 1 + 3 + 2 = 10$ points**

A Gaussian beam with the Rayleigh length $z_1 = kW_{01}^2/2$ is transmitted through a thin lens of focal length f. As marked in Figure , consider D_1 and D_2 to be the distances from the waist of the initial and transmitted beam, respectively, to the lens. W_{01} and W_{02} are the widths of the beams, respectively.

a) Explain the q-parameter, its connection with a Gaussian beam and why/how we use it.

b) Show that the distances of the waists (focii) of the incident and transmitted beams, D_1 and D_2, respectively, to the lens are related by

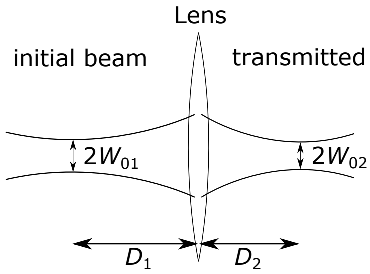
$$\frac{D_2}{f} = \frac{D_1/f - 1}{(D_1/f - 1)^2 + (z_1/f)^2} + 1 \quad .$$

Use the q-parameter method.

c) Find the dependency of the waist W_{02} on λ, f, z_1, and D_1.

d) We now want to make the location of the new waist W_{02} as distant as possible from the lens, i.e., we want to maximize D_2. For a given ratio z_1/f, show that the optimal value of D_1 is $D_1 = f + z_1$.

e) For this optimal case, determine the values of the distances D_2, the waist W_{02} of the transmitted beam and the corresponding magnification $M = W_{02}/W_{01}$ depending on λ, f, and z_1.



Problem 5 – Pulse propagation **$3 + 2 + 2 = 7$ points**

A pulse is propagating in a homogeneous material with the dielectric function given by

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \quad , \tag{1}$$

where the pulse's carrier frequency ω_0 is much larger than the plasma frequency ω_p.

a) Calculate the group velocity of the pulse in respect of ω_0.

b) The pulse is propagating towards a detector. Which frequencies arrive earlier, the ones higher than the carrier frequency or the lower ones? Prove with a (short) calculation.

c) Now consider a second pulse of the different carrier frequency $\omega_2 \gg \omega_p$, propagating in the same direction. Calculate the time delay between both pulses after a length L.

Problem 6 – Fraunhofer diffraction **$2 + 2 = 4$ points**

a) Name two different approximations that can be made in diffraction theory and shortly explain their meaning in two sentences.

b) Calculate the intensity of the diffracted field pattern $I(x, z_\mathrm{B}) = |u(x, z_\mathrm{B})|^2$ at $z = z_B$ in paraxial Fraunhofer approximation for one slit illuminated with a normally incident plane wave (just the diffraction pattern, no prefactors). The width of the slit is a ($a > \lambda$):

$$u_0(x, z = 0) = \begin{cases} 1, & \text{for } |x| \leq a/2 \\ 0, & \text{elsewhere.} \end{cases}$$

— **Useful formulas** —

$\nabla \times \nabla \times \mathbf{a} = \nabla(\nabla \cdot \mathbf{a}) - \Delta \mathbf{a}$

$\nabla \cdot (\nabla \times \mathbf{a}) = 0$

$\nabla \cdot (c\mathbf{a}) = \mathbf{a} \cdot \nabla c + c\, \nabla \cdot \mathbf{a}$

$\mathrm{FT}(J_0(2\pi r)) = \frac{1}{2\pi a}\delta(\rho - a)$

Gaussian q-parameter transform law:

$$q' = \frac{Aq + B}{Cq + D}$$

ABCD matrix for a thin lens:

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$