FRIEDRICH SCHILLER UNIVERSITY JENA WS 2022/2023
Institute of Applied Physics
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Series 7

FUNDAMENTALS OF MODERN OPTICS

to be returned on **15.12.2022**, at the beginning of the lecture

Task 1: Two lenses system (4+2+1 points)

Consider a system of two thin lenses with focal lengths of f_1 and f_2, respectively, and a distance $d = f_1 + f_2$ between them (see Fig. 1). Assume you have a Gaussian beam of waist W_0 and wavelength λ, where the waist is positioned at a distance f_1 before the first lens, as shown Fig. 1. This beam will be focused at a distance d_1 after the first lens, with a beam waist of W_1, and again refocused at some distance d_2 after the second lens, where the waist size will be W_2.

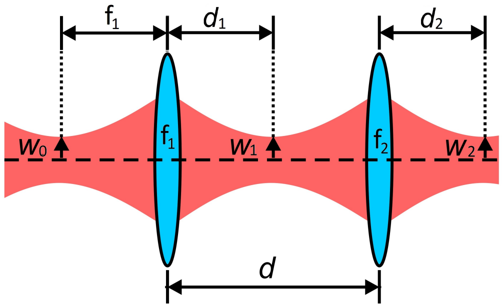

Figure 1: 2 lenses system.

a) Calculate the ABCD matrix describing the propagation of the Gaussian beam from the waist W_0 to distance d_1 after the first lens. Find d_1 and W_1.
b) Find d_2 and W_2.
c) Estimate W_1 and W_2, when $f_1 = 200\,\text{mm}$, $f_2 = 150\,\text{mm}$, $W_0 = 15\,\text{mm}$, and $\lambda = 515\,\text{nm}$.

Task 2: Laser Cavity (1+2+1+1* points)

A key element of a laser is an optical cavity. In a simplified version, it consists of two spherical mirrors separated by a distance d. In this task, consider a Gaussian beam in a symmetrical confocal cavity.

a) Make a sketch of the symmetrical confocal cavity. Sketch the spatial profile of the beam and the position of the waist w_0. Mark the distance d and radii of mirrors R_1 and R_2.
b) Derive the equations defining the waist w_0 of the Gaussian beam in the cavity. Also derive the expressions for the widths at the positions of the mirrors, w_1 and w_2.
c) Calculate the values of the beam widths: w_0, w_1 and w_2. Consider the distance between the mirrors equal to 40 cm and the use of a Argon laser with a wavelength of 515 nm.
*d) Is the cavity stable or unstable? Explain your answer.

Task 3: Dispersion compensation (3+1 points)

A transform-limited Gaussian pulse with central frequency ω_0 is coupled into a fiber. Its envelope is given by

$$U(t) = A\,\exp(-t^2/\tau^2),$$

where the pulse duration is $\tau = 10\,\mathrm{ps}$. The dispersion of the fiber is characterized by the frequency-dependent wavenumber k:

$$k(\omega) = k_0 + (1.5/c)\,(\omega - \omega_0) + (\beta_2/2)\;(\omega - \omega_0)^2\,,$$

where the group velocity dispersion of the fiber is $\beta_2 = 0.15\,(\mathrm{ps})^2/\mathrm{m}$.

a) A second pulse with a different frequency $\omega_1 = \omega_0 - \delta\omega$ is coupled into the same fiber after a delay of $T = 20\,\mathrm{ns}$. The detuning is $\delta\omega = 1\,\mathrm{THz}$. Calculate the group indices $n_{\mathrm{g}0}$ and $n_{\mathrm{g}1}$ of both pulses. Will the second pulse overtake the first one? If so, when will it happen?

b) Consider the scenario with just the first pulse. After $L_1 = 5\,\mathrm{km}$ of the fiber, we connect a second fiber with group velocity dispersion $\beta_2 = -0.3\,(\mathrm{ps})^2/\mathrm{m}$ and length L_2. After which propagation distance in the second fiber will the initial pulse be restored?