Friedrich Schiller University Jena WS 2022/2023
Institute of Applied Physics
Prof. Dr. Thomas Pertsch
Anastasia Romashkina, Shreyas Ramakrishna, Mostafa Abasifard, Tina (Shiu Hei Lam), Pawan Kumar, Xiao Chen, Dr. Bayra Narantsatsralt

# Series 6

## Fundamentals of Modern Optics

to be returned on **01.12.2022**, at the beginning of the lecture

## Task 1: Response function of Fresnel approximation (5 points)

Consider the Fresnel approximation, under which the transfer function in the spatial frequency domain reads:

$$H_F(\alpha,\beta,z) = \exp(ikz)\exp\left[-i\frac{\alpha^2+\beta^2}{2k}z\right]$$

Derive the response function $h_F(x,y,z>z_0)$ in the spatial domain, as given in the lecture notes. Use the integral:

$$\int_{-\infty}^{+\infty} e^{-ix^2}\mathrm{d}x = \sqrt{\frac{\pi}{i}}.$$

## Task 2: Gaussian beam (2+2 points)

In the lecture we defined the Gaussian beam as

$$v(x,y,z) = A(z)\exp\left[-\frac{x^2+y^2}{w(z)^2}\right]\exp\left[ikz + i\frac{k}{2}\frac{x^2+y^2}{R(z)} + i\varphi(z)\right].$$

a) Derive a spherical wave in paraxial approximation and show that for which condition the wavefront of a Gaussian beam is the same as a wavefront of the spherical wave . *Hint: Neglect Guoy phase shift of the Gaussian beam.*

b) How far can a Gaussian beam with $\lambda = 630$ nm and $W_0 = 8$ mm stay collimated(we consider maximum 10% broadening after propagating $z_1$ from waist)?

## Task 3: Focusing a Gaussian Beam (4+2 points)

A collimated Gaussian beam of wavelength $\lambda$ with a waist $W_0$ (the waist is just behind the lens) is focused by a lens with a focal distance $f$, as shown in Figure 1. The Rayleigh length of the beam before the lens, $z_0 = \frac{\pi W_0^2}{\lambda}$, is much larger than $f$. The focused Gaussian beam after the lens would have the waist $W_1$ at the distance $d$ after the lens.

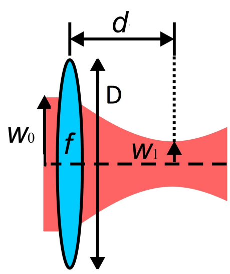


**Figure 1:** Focusing with a lens.

a) Use the ABCD matrix of the system to calculate $W_1$ and $d$ exactly. Then use the fact that $z_0 >> f$ to simplify your results.

b) How small can $2W_1$ be? In other words, how small can the focal spot after the lens be? Use the approximate result of (a) in the $z_0 >> f$ regime.
*Hint:* To make a statement about this, you have to make some assumptions. Firstly, you have to notice that for the calculation in (a) to be correct, you are assuming that the lens aperture $D$ is large enough to let a

substantial part of the Gaussian beam to pass through it. Let us say that $2W_0$ should be smaller than $D$ for a substantial part of the beam to pass through the lens. Moreover, for a thin lens, the size of the aperture is also limited based on its focal length. So let us assume that $D/2$ is smaller than $f$, such that the ratio $D/2f$ is always smaller than 1. Put all these statements together, to be able to find a limit on how small the size of the focused beam can be.