

$$P = \epsilon_0 \chi$$

## Exam STRUCTURE OF MATTER

February 21

5 + 5 + 3 + 5 + 3 Points

### Exercise 1

Glasses usually show a weak dispersion in the optical frequency range because resonances are located far off this domain. Please consider a typical glass with a dielectric function characterized by two Lorentz-type resonances:

$$\epsilon(\omega) = 1 + \frac{f_p}{\omega_p^2 - \omega^2 - i g_p \omega} + \frac{f_e}{\omega_e^2 - \omega^2 - i g_e \omega}$$

$$e^{-i\omega\tau}$$

- Please explain the origin of both resonances and give the typical orders of magnitude of the resonance frequencies. Describe the physical meaning of all parameters.
- Sketch the real and imaginary part of  $\epsilon(\omega)$ . Mark the frequency ranges with strong absorption, normal and anomalous dispersion.
- In time domain, one can relate the polarization  $P(t)$  and the electric field  $E(t)$  via the response function  $R(\tau)$ . Write down the general relation between  $P(t)$  and  $E(t)$  for a linear, isotropic, homogeneous dielectric medium. Give also the general relations between the response function  $R(\tau)$  and the dielectric function  $\epsilon(\omega) = 1 + \chi(\omega)$ .
- Show that  $\int_0^\infty e^{-\frac{\gamma}{2}\tau} \sin(\Omega\tau) e^{i\omega\tau} d\tau = \frac{f}{\omega_0^2 - \omega^2 - i\gamma\omega}$ , where  $\Omega = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}}$ .
- By using d), write down the response function  $R(\tau)$  for the dielectric function  $\epsilon(\omega)$  given above.

$$e^{-ax} = -\frac{1}{a}$$

### Exercise 2

4 + 8 + 4 + 4 Points

Consider an electron moving in a  $\delta$ -like potential  $V(x) = V_0 \delta(x)$ .

- Derive the transition condition  $\lim_{\epsilon \rightarrow 0} \frac{d}{dx} \psi|_{x=\epsilon} - \frac{d}{dx} \psi|_{x=-\epsilon} = \frac{2m_0 V_0}{\hbar^2} \psi(x=0)$ .
- Solve the stationary Schrödinger equation for energies  $E > 0$ . Consider solutions of the form

$$\psi(x) = \exp(ikx) + R \exp(-ikx) \text{ for } x < 0, \quad \psi(x) = T \exp(ikx) \text{ for } x > 0,$$

and compute  $k$  as well as  $R$  and  $T$ .

- In general, the probability flux density  $j$  is defined as

$$j = \frac{\hbar}{2m_0 i} \left( \psi^* \frac{\partial}{\partial x} \psi - \psi \frac{\partial}{\partial x} \psi^* \right).$$

Calculate the probability flux densities  $j_{in}$ ,  $j_{refl}$  and  $j_{trans}$  of the incoming wavefunction  $\exp(ikx)$ , the reflected wavefunction  $R \exp(-ikx)$ , and the transmitted wavefunction  $T \exp(ikx)$ , respectively.

- Calculate the reflectivity and transmissivity  $\rho = \left| \frac{j_{refl}}{j_{in}} \right|$  and  $\tau = \left| \frac{j_{trans}}{j_{in}} \right|$  and give a short interpretation of your results.

(p.t.o)

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**Exercise 3****3 + 12 Points**

Consider transitions between different electronic energy levels in the Hydrogen atom.

- The emission or absorption of photons is mostly related to dipole allowed transitions. What are the selection rules for dipole allowed transitions?
- Explain what happens with the electronic energy levels in the presence of an external static magnetic field  $B$ . Sketch a term scheme showing all dipole allowed transition between the  $n = 3$  and  $n = 2$  energy levels in this case ( $n$  is the principal quantum number).

**Exercise 4****12 + 12 Points**

The atoms in a molecule can vibrate about their bonds, which gives the molecule vibrational energy in addition to its electronic energy. Qualitative features of optical transitions between the vibrational-electronic (vibronic) levels of a molecule can be understood by considering a simple diatomic molecule, which has only one vibrational degree of freedom.

- Based on this simple diatomic molecule, explain the main idea of the Franck-Condon principle. Sketch qualitatively the energy configuration diagram of the molecule in the electronic ground state and in an excited state.
- Please sketch the typical emission and absorption spectra of the optical transitions between the electronic ground state and an excited state of the molecule taking into account the vibrational degree of freedom. Why the emission spectrum of the molecule is approximately the 'mirror' of the absorption spectrum when reflected about the center frequency  $\hbar\omega_0$  (zero phonon line)? How is the center frequency  $\omega_0$  defined?

**Exercise 5****4 + 6 + 4 + 6 Points**

- Which Bravais lattice do you find if you take the reciprocal lattice of the reciprocal lattice?
- Show that the two following formulations of the Bloch theorem are equivalent:
  - $\psi(\mathbf{r} + \mathbf{R}) = \psi(\mathbf{r}) \exp(i\mathbf{k} \cdot \mathbf{R})$  for every  $\mathbf{R}$  in the Bravais lattice.
  - $\psi(\mathbf{r}) = u(\mathbf{r}) \exp(i\mathbf{k} \cdot \mathbf{r})$ , where  $u(\mathbf{r} + \mathbf{R}) = u(\mathbf{r})$  for every  $\mathbf{R}$  in the Bravais lattice.
- Compare the two band structure diagrams for GaAs and germanium below. What is the main qualitative difference with respect to optical interband transitions?
- Explain how you would use optical absorption measurements to determine whether a semiconductor has a direct or indirect band gap?

