Institute of Applied Physics
Friedrich-Schiller-Universität Jena

Optical System Design Fundamentals

Lecture 7: Chromatic Aberrations and Seidel Aberration Contributions

2024 / 06 / 18

Vladan Blahnik

Preliminary Schedule - OSDF 2024

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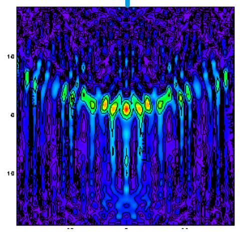
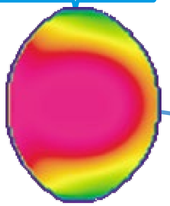
1	07.05.2024	Light propagation in optical systems	Fermat principle, modeling light propagation with ray and wave model; diffraction; Snell´s law, total internal reflection, light propagation in optical system, raytracing; ray and wave propagation in optical systems; pinhole camera; central perspective projection; étendue, f-number and numerical aperture, focal length; ideal projection laws; fish,eye imaging; anamorphic projection; perspective distortion, Panini projection; massive ray-tracing based imaging model	(S) (optional)
2	14.05.2024	The ideal image	Paraxial approximation, principal planes, entrance and exit pupil, paraxial imaging equations; Scheimpflug condition; paraxial matrix formulation; Delano diagram; Aplanatic imaging: Abbe sine condition, numerical aperture, sine scaled pupil for microscope imaging	S
3	21.05.2024	First order layout, system structures and properties	application of imaging equations; examples: layout of visual imaging system; layout of microscope lens and illumination system; telecentricity, "4f-setups"; tele and reversed tele (retrofocus) system; depth of field, bokeh; equivalence to multi-perspective 3D image acquisition; focusing methods of optical systems; afocal systems; connecting optical (sub-)systems: eye to afocal system; illumination system and its matching to projection optics); zoom systems; first order aberrations (focus and magnification error) and their compensation: lens breathing, aperture ramping; scaling laws of optical imaging	S
4	28.05.2024	Imaging model including diffraction	Gabor analytical signal; coherence function; degree of coherence; Rayleigh-Sommerfeld diffraction; optical system transfer as complex, linear operator; Fraunhofer-, Fresnel-approximation and high-NA field transfer wave-optical imaging equations for non-coherent, coherent and partially coherent imaging; isoplanatic patches, Hopkins Transfer Function for partially coherent imaging, ideal wave-optical imaging equations; Fourier-Optics; spatial filtering; Abbe theory of image formation in microscope, phase contrast microscopy	(S)
5	04.06.2024	Wavefront deformation, Optical Transfer Function, Point Spread Function and Image Simulation	Hopkins Transfer Function for partially coherent imaging, ideal wave-optical imaging equations; Fourier-Optics; spatial filtering; Abbe theory of image formation in microscope, phase contrast microscopy; optical transfer functions for non-coherent and coherent imaging; wave aberrations; point spread function (PSF), optical transfer function (OTF) and modulation transfer function (MTF); computing aberrations with Hamilton Eikonal Function; defocus wavefront deformation, PSF and MTF optical system performance criteria based on PSF and MTF: 2-point resolution, MTF-based criteria; Sampling theorems; sensor related definition of spatial frequency range; wave-optical vs geometrical computation of PSF	S
6	11.06.2024	Aberrations: Classification, diagrams and identification in real images	Zernike polynomials, measurement of system quality; measurement of wave front deformation: Shack-Hartmann sensor, Fizeau interferometer diagram, stop-shift-equation; Longitudinal and transverse aberrations, spot diagram; aberration in real images of (extended) objects	no
7	18.06.2024	Chromatic Aberrations, Aberration Contributions, System Optimization	coupling efficiency / insertion loss at photonics interface longitudinal and lateral chromatic aberration, Abbe diagram, dispersion fit; achromat and apochromat pre- and post-computer era correction methodology, merit function, system specification, initial setups, opto-mechanical constraints, damped-least-square optimization; Symmetry considerations as basis for aberration theory; Seidel polynomial, chromatic aberrations; surface contributions and Pegel symmetry principles, lens bending, aplanatic surface insertation, lens splitting, aspheres and freeforms	S
8	25.06.2024	Aberration correction and optical system structure	Petzval theorem, field flattening, achromat and apochromat, sensitivity analysis, diffractive elements; system structure of optical systems, e.g. photographic lenses, microscopic objectives, lithographic systems, eyepieces, scan systems, telescopes, endoscopes	S (changed)
9	02.07.2024	Tolerancing and straylight analysis	optical technology constraints, sensitivity analysis, statistical fundamentals, compensators, Monte-Carlo analysis, Design-To-Cost; false light; ghost analysis in optical systems; AR coating; opto-mechanical straylight prevention	**(S) tbd**

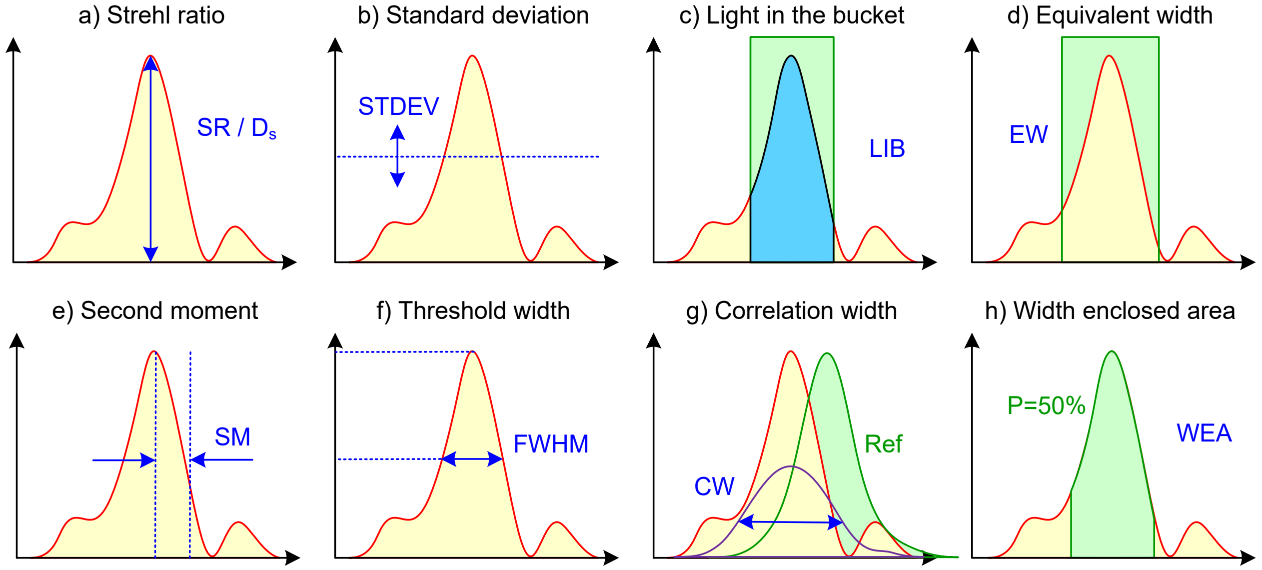
Quality Criteria based on Point Spread Function

$$\underbrace{PSF(x - m\xi, \lambda)}_{\text{monochromatic PSF}} = \left| \iint_{\text{lens pupil}} d\alpha d\beta \; \underbrace{L(\alpha, \lambda)}_{\text{lens pupil function}} \exp(-i2\pi w[\alpha \cdot (x - m\xi)]) \right|^2$$





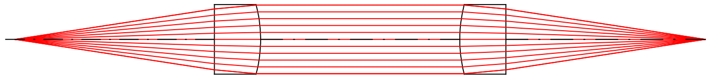
$$L(\alpha, \lambda) = \underbrace{\overbrace{L_0(\alpha, \lambda)}^{\epsilon\, \mathbb{R}}}_{\text{lens pupil shape, apodization}} \overbrace{\exp\left(i2\pi \underbrace{W(\alpha)}_{\text{wavefront deviation (units of } \lambda)}\right)}^{\epsilon\, \mathbb{C}}$$



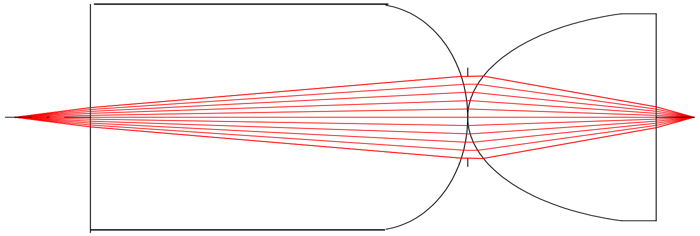
Usefulness of Criterium needs to be carefully considered **individually for the particular application**!

Examples of photonic coupling

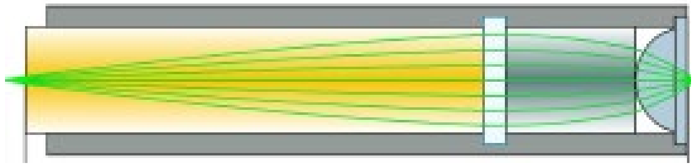
There is a large space of different technologies and solutions for coupling



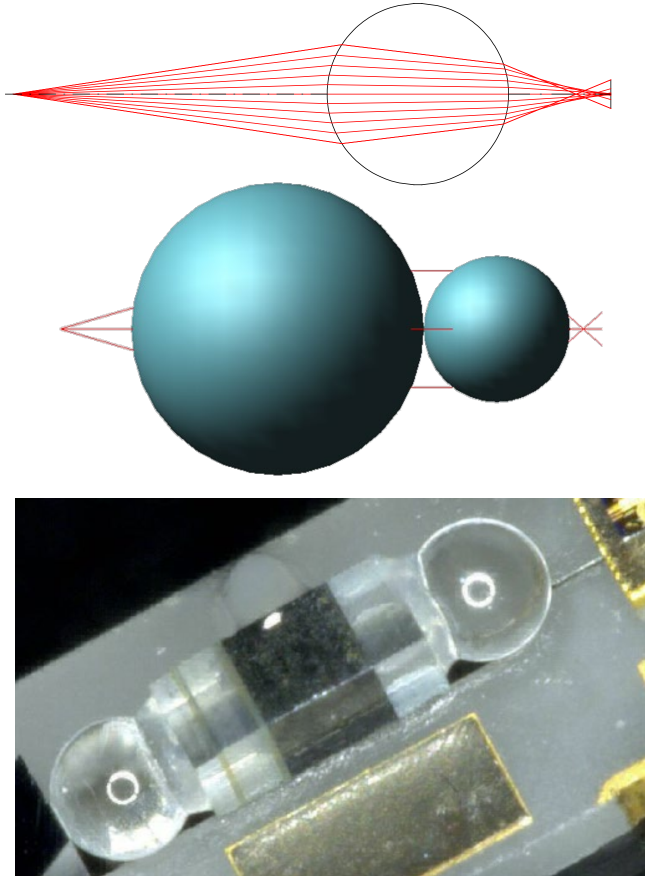
standard lenses



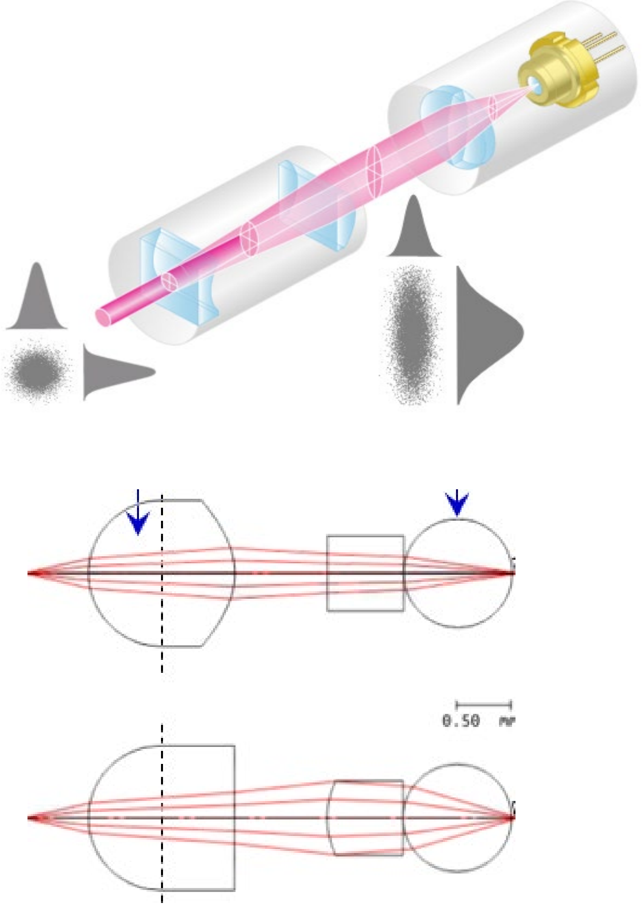
aspheres



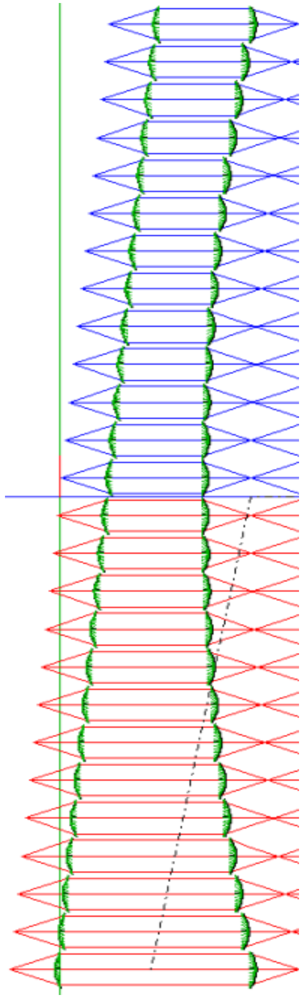
gradient-index media



ball lens(es)



cylindric lenses,
anamorphic setups



lens arrays

Coupling efficiency (Insertion loss)

Standard metric for photonic systems.

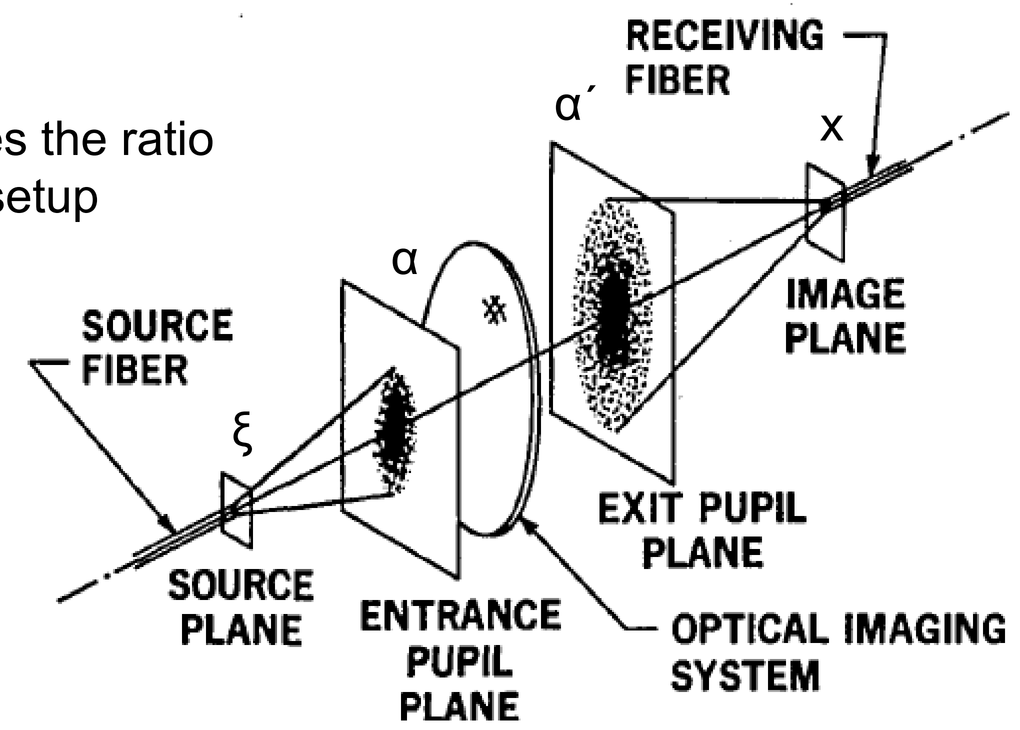
"Power-coupling efficiency" or just "coupling efficiency" describes the ratio of power of a source to a receiver transferred via some optical setup compared to the optimum power the receiver is able to receive.

$$\eta = \left| \int U_R(x)\, U_{S,T}^*(x)\, dx \right|^2$$

$0 \leq \eta \leq 1$

mode of receiver

complex **amplitude distribution at receiver position**



U_R and U_T are normalized such that
$\int U_R(x)\, U_R^*(x)\, dx = 1$ and $\int U_T(x)\, U_T^*(x)\, dx = 1$.

Expressed in dB:
"**insertion loss**": $10 \cdot \log_{10}(\eta)$

R. E. Wagner and W. J. Tomlinson (1982), **Coupling efficiency of optics in single-mode fiber Components,** APPLIED OPTICS, Vol. 21, No. 15.

Insertion Loss computation either in spatial coordinates or spatial frequencies

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Coupling efficiency:

Computation at spatial position coordinates / (receiver position, e.g. fiber exit):

Exit position x:

$$\eta = \left| \int U_R(x)\, U^*_{S,T}(x)\, dx \right|^2$$

Alternatively: pupil coordinates / spatial frequencies (interchanged integration):

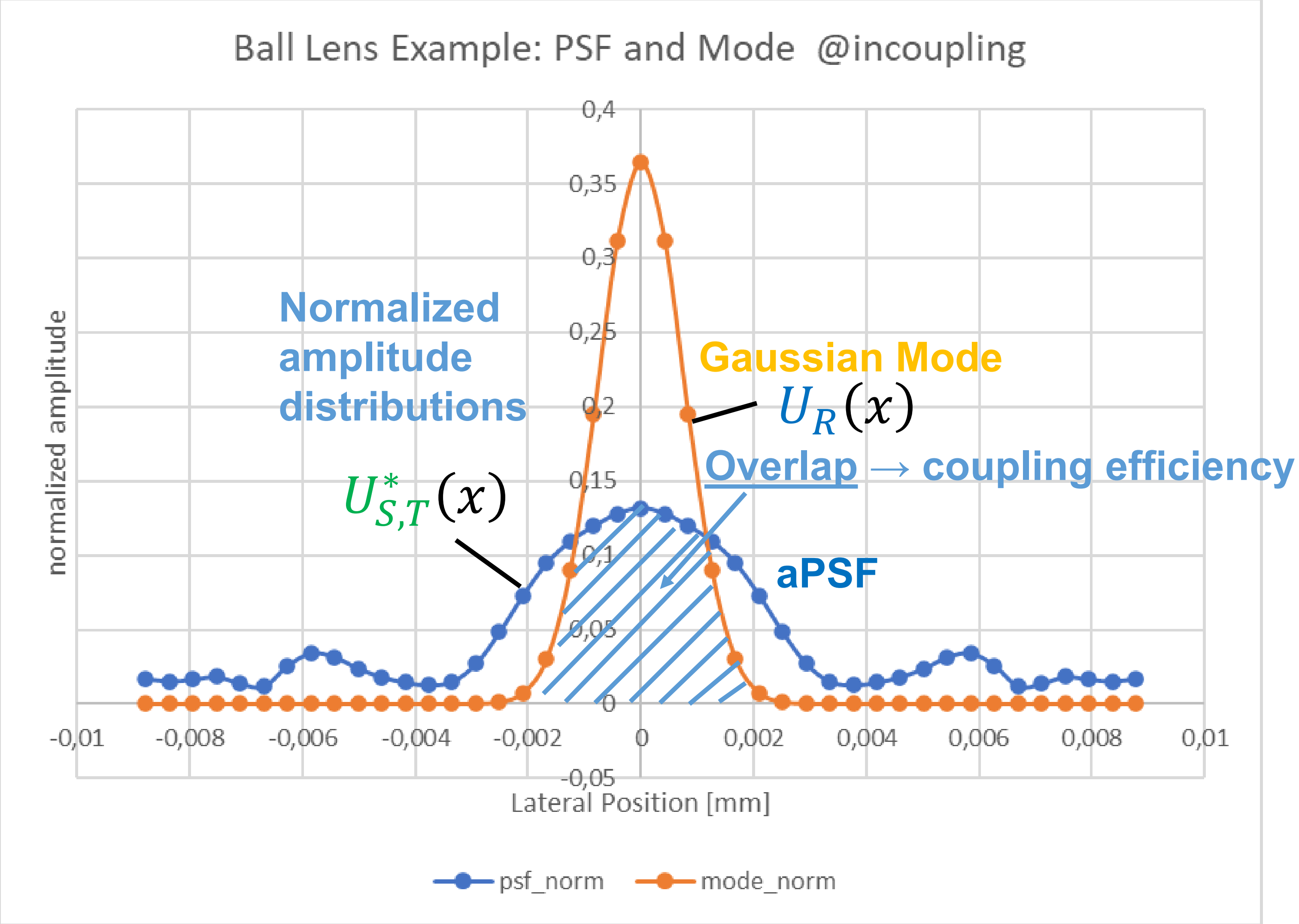
Exit pupil α´:

$$\eta = \left| \int_{\alpha'} L^*(\alpha')\, \overline{U}^*_S(\alpha')\, \overline{U}_R(\alpha')\, d\alpha' \right|^2$$

(coherent) optical transfer function

Angular source distribution (spectrum of spatial distribution)

Spectrum of receiver mode



Alternative, equivalent calculation of coupling efficiency

Inserting $U_{S,T}(x) = \int U_S(\xi)\, K_T(x-\xi) d\xi$ and $K_T(x-\xi) = \int L(\alpha')\exp\left(i2\pi \frac{NA'}{\lambda}\alpha' \cdot (x-\xi)\right) d\alpha$ in $\eta = \left|\int U_R(x)\, U_{S,T}^*(x)\, dx\right|^2$ we have:

$$\eta = \left| \int_x U_R(x) \int_\xi U_S^*(\xi) \int_{\alpha'} L^*(\alpha') exp\left(i2\pi \frac{NA'}{\lambda}\alpha' \cdot (x-\xi)\right) d\alpha'\, d\xi\, dx \right|^2$$

Changing the integration order to ξ, x, α we obtain:

$$\eta = \left| \int_{\alpha'} L^*(\alpha') \underbrace{\int_\xi U_S^*(\xi) exp\left(-i2\pi \frac{NA'}{\lambda}\alpha' \cdot \xi\right) d\xi}_{\bar{U}_S^*(\alpha')} \underbrace{\int_x U_R(x)\, exp\left(i2\pi \frac{NA'}{\lambda}\alpha' \cdot x\right) dx}_{\bar{U}_R(\alpha')}\, d\alpha' \right|^2$$

x and ξ integrations can be performed independently and leave amplitude spectrum in exit pupil (denoted with bar):

$$\eta = \left| \int_{\alpha'} L^*(\alpha')\, \bar{U}_S^*(\alpha')\, \bar{U}_R(\alpha')\, d\alpha' \right|^2$$

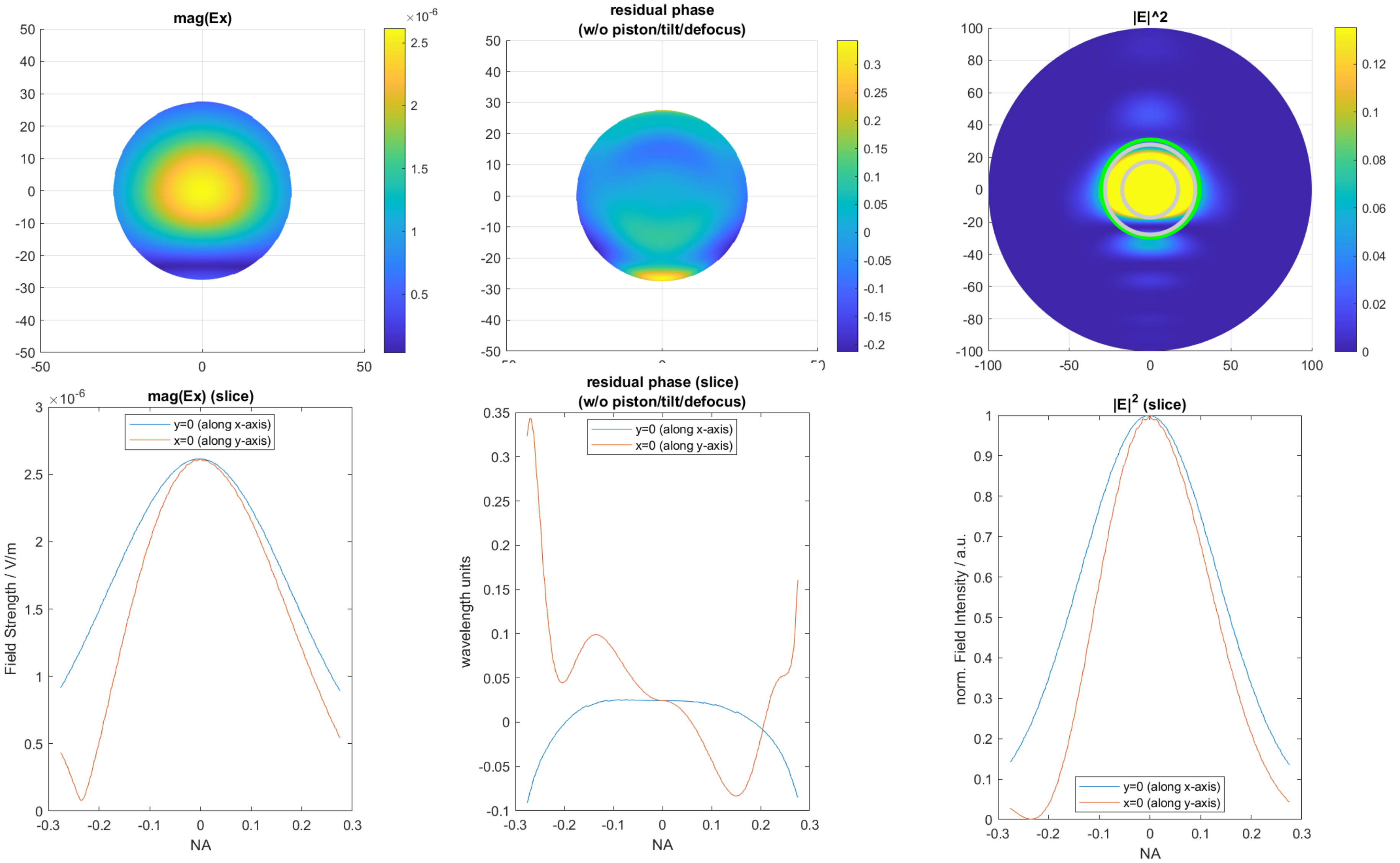
(coherent) optical transfer function

Angular source distribution
(spectrum of spatial distribution)

Spectrum of receiver mode

In this form the functions are given in angular space (or the exit pupil, which is equivalent).
Note that in this form the functions which describe source distribution, optical transfer and receiver amplitude response all separate!

Photonic light sources are often not just a perfect Gaussian beam
Example: Grating Coupler



Photonic source emission modes at exit:

Emission distribution depends upon numerous parameters: grating period, # grating lines, n_0, d_0,…

- Amplitude distribution in general not symmetric
- Phase deformation may be in the order of λ/2

Complete Optics – Photonics System must be optimized simultaneously!

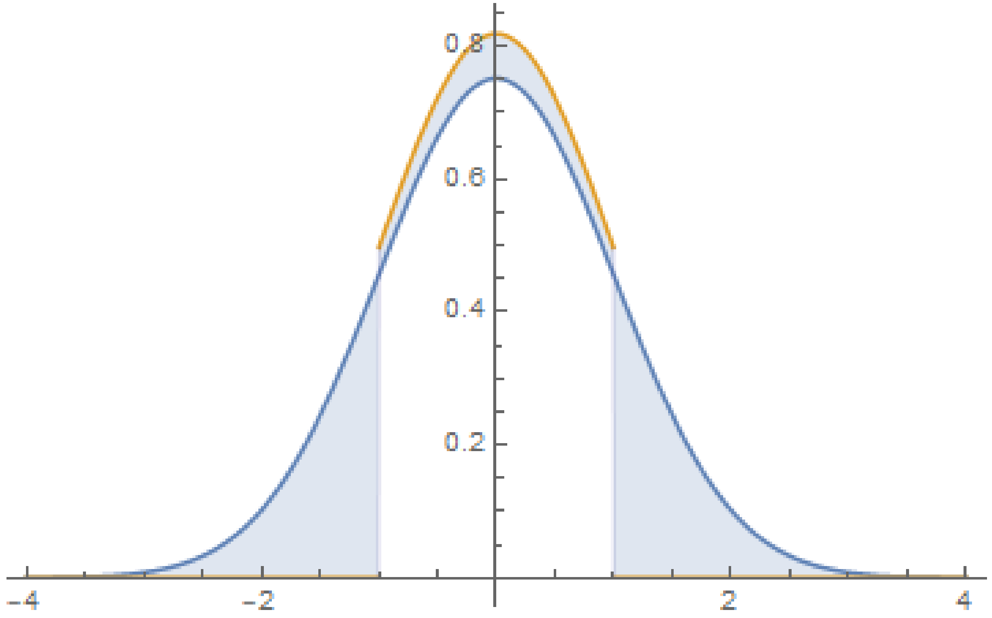
Coupling efficiency of Gaussian and truncated Gaussian

$$\eta = \left| \int U_R(x)\, U_{S,T}^*(x)\, dx \right|^2$$

$$U_R(x) = (\pi\, w^2)^{-1/4}\, exp\left(-\frac{x^2}{2\, w^2}\right)$$

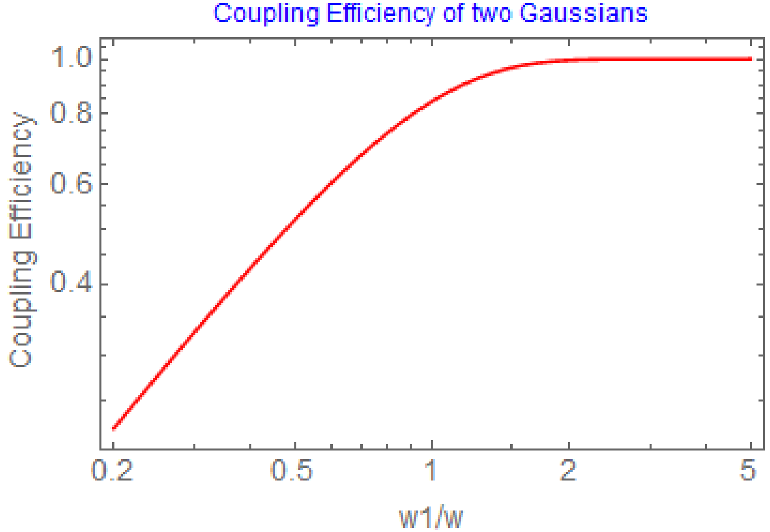
$$\eta = erf\left(\frac{w_{max}}{w}\right)$$

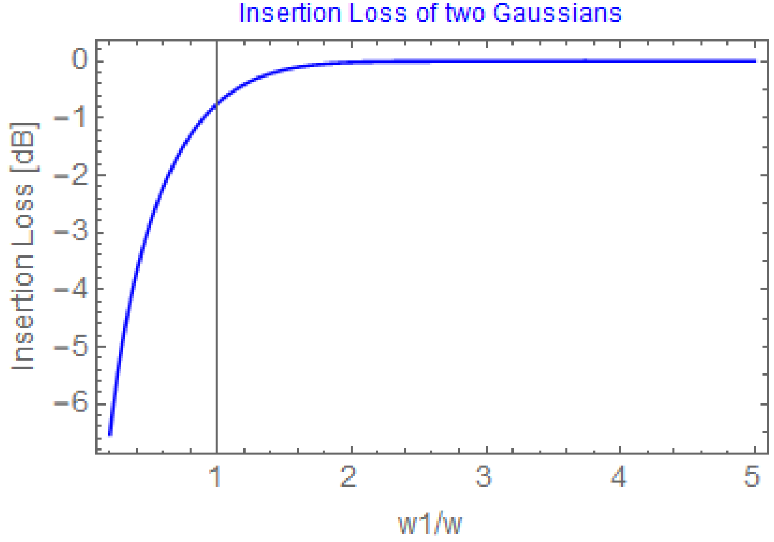
$$U_{S,T}^*(x) = \begin{cases} (\pi\, w^2)^{-1/4}\, exp\left(-\frac{x^2}{2\, w^2}\right), & x \le w_{max} \\ 0, & x > w_{max} \end{cases}$$



These losses are due to shape deviation only! (means: function is renormalized)

Light loss due to cut-off at aperture is additional loss.





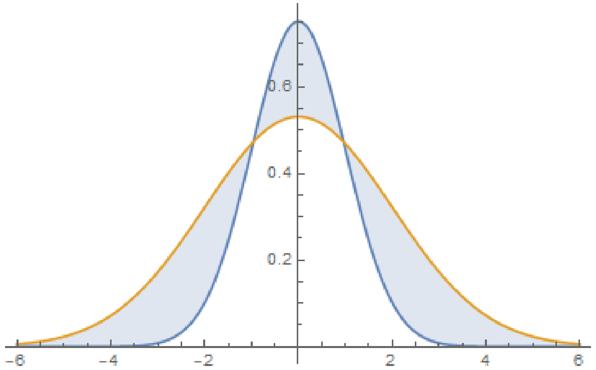
Further examples

$$\eta = \left| \int U_R(x)\, U_{S,T}^*(x)\, dx \right|^2$$

Gaussian - Gaussian

$$U_R(x) = (\pi\, w^2)^{-1/4}\, exp\left(-\frac{x^2}{2\, w^2}\right)$$

$$U_{S,T}^*(x) = (\pi\, w_1^2)^{-1/4}\, exp\left(-\frac{x^2}{2 w_1^2}\right)$$

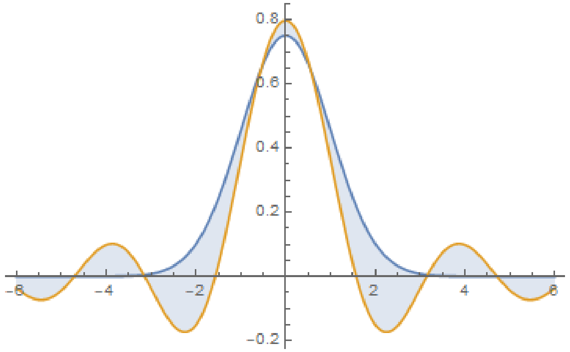


→ Homework Exercise

Gaussian - Sinc

$$U_R(x) = (\pi\, w^2)^{-1/4}\, exp\left(-\frac{x^2}{2\, w^2}\right)$$

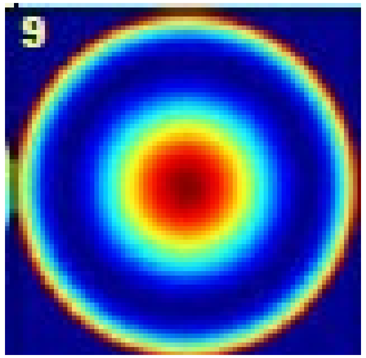
$$U_{S,T}^*(x) = \frac{sin(w_1\, x)}{\sqrt{\pi} w_1 x}$$



Gaussian and Gaussian with Spherical Aberration (Zernike Z9 [λ])

$$U_R(x) = (\pi\, w^2)^{-1/4}\, exp\left(-\frac{x^2}{2\, w^2}\right)$$

$$U_{S,T}^*(x) = (\pi\, w^2)^{-1/4}\, exp\left(-\frac{x^2}{2\, w^2}\right)\, exp\left(-i2\pi\, c_9(6r^6 - 6r^4 + 1)\right)$$



Classification of aberrations

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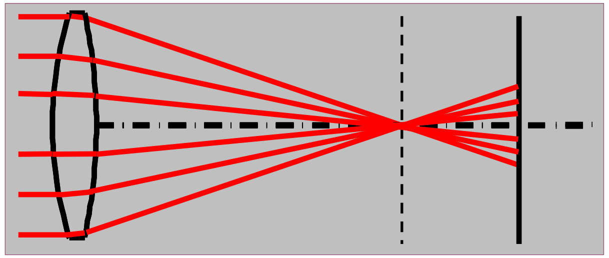
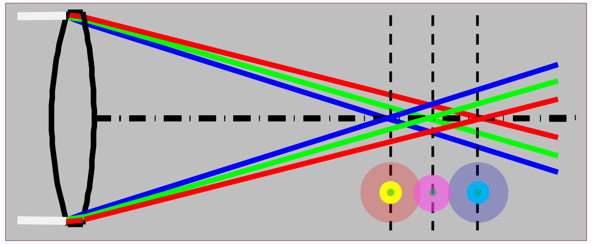
$$W(\alpha^2, \alpha \cdot \xi, \xi^2) = w_{200}\alpha^2 + w_{400}\,\alpha^4 + w_{310}\,\alpha^2\,\alpha \cdot \xi + w_{202}\,\alpha^2\,\xi^2 + w_{020}\,(\alpha \cdot \xi)^2 + w_{013}\,\alpha \cdot \xi\,\xi^2 + \cdots$$

1st order monochromatic:

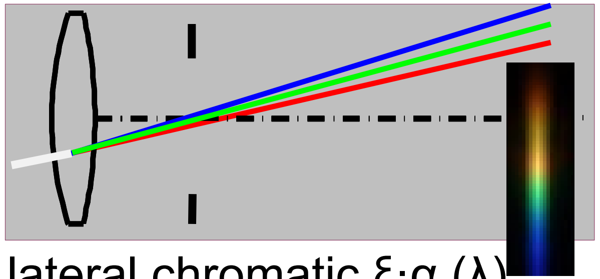


defocus α^2

1st order chromatic:

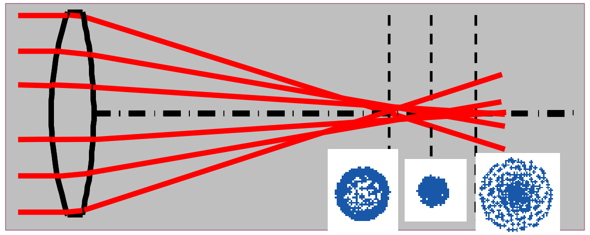


longitudinal chromatic α^2 (λ)

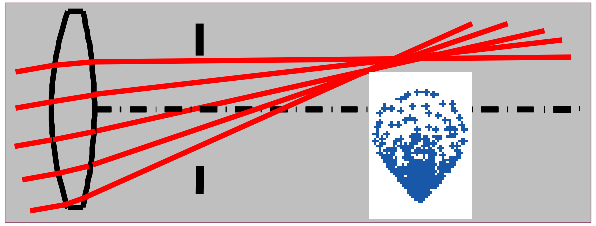


lateral chromatic $\xi \cdot \alpha$ (λ)

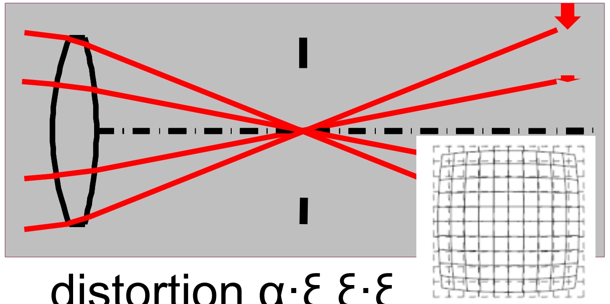
3rd order monochromatic:



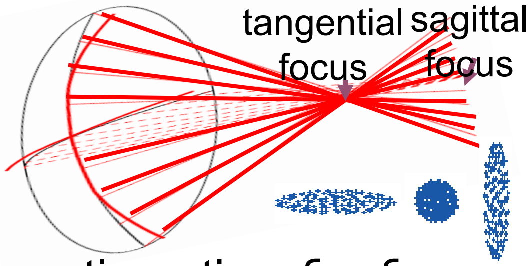
spherical aberration α^4



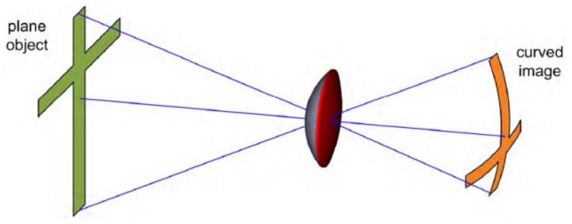
coma $\xi \cdot \alpha\ \alpha \cdot \alpha$



distortion $\alpha \cdot \xi\ \xi \cdot \xi$



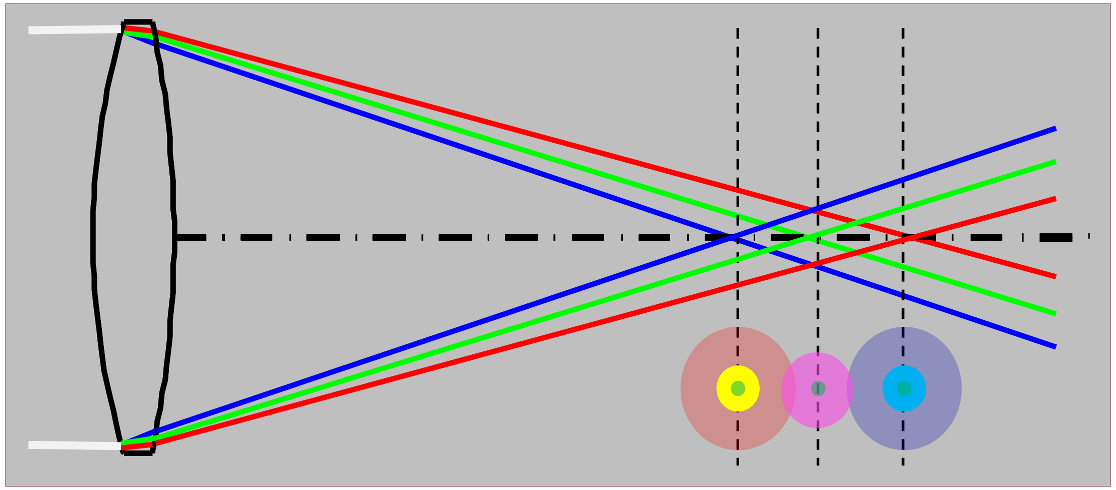
astigmatism $\xi \cdot \alpha\ \xi \cdot \alpha$



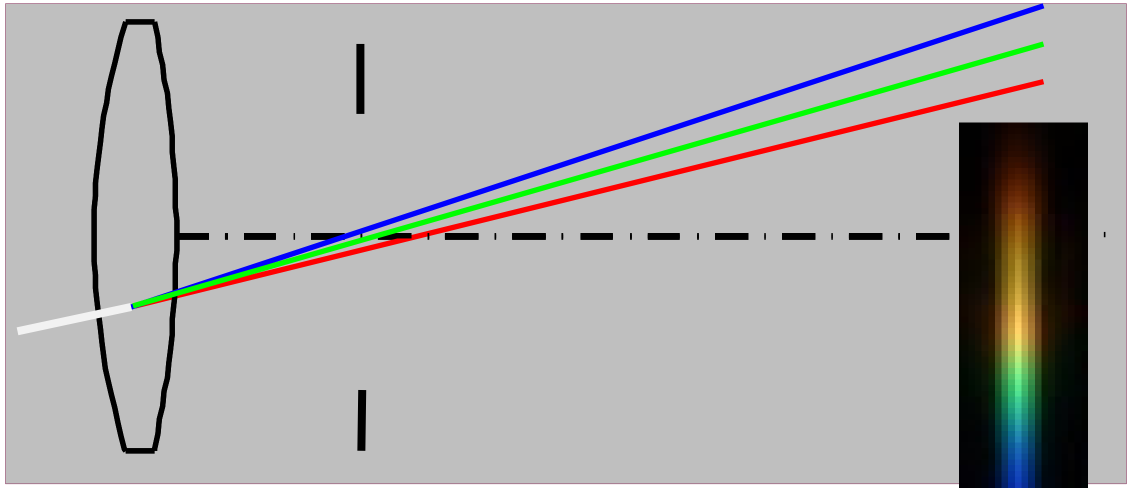
field curvature $\alpha \cdot \alpha\ \xi \cdot \xi$

5th order: 9 types
7th order: 14 types
…
n´th order (n+3)(n+5)/8-1

Classification of aberrations



longitudinal chromatic α^2 (λ)



lateral chromatic $\xi \cdot \alpha$ (λ)

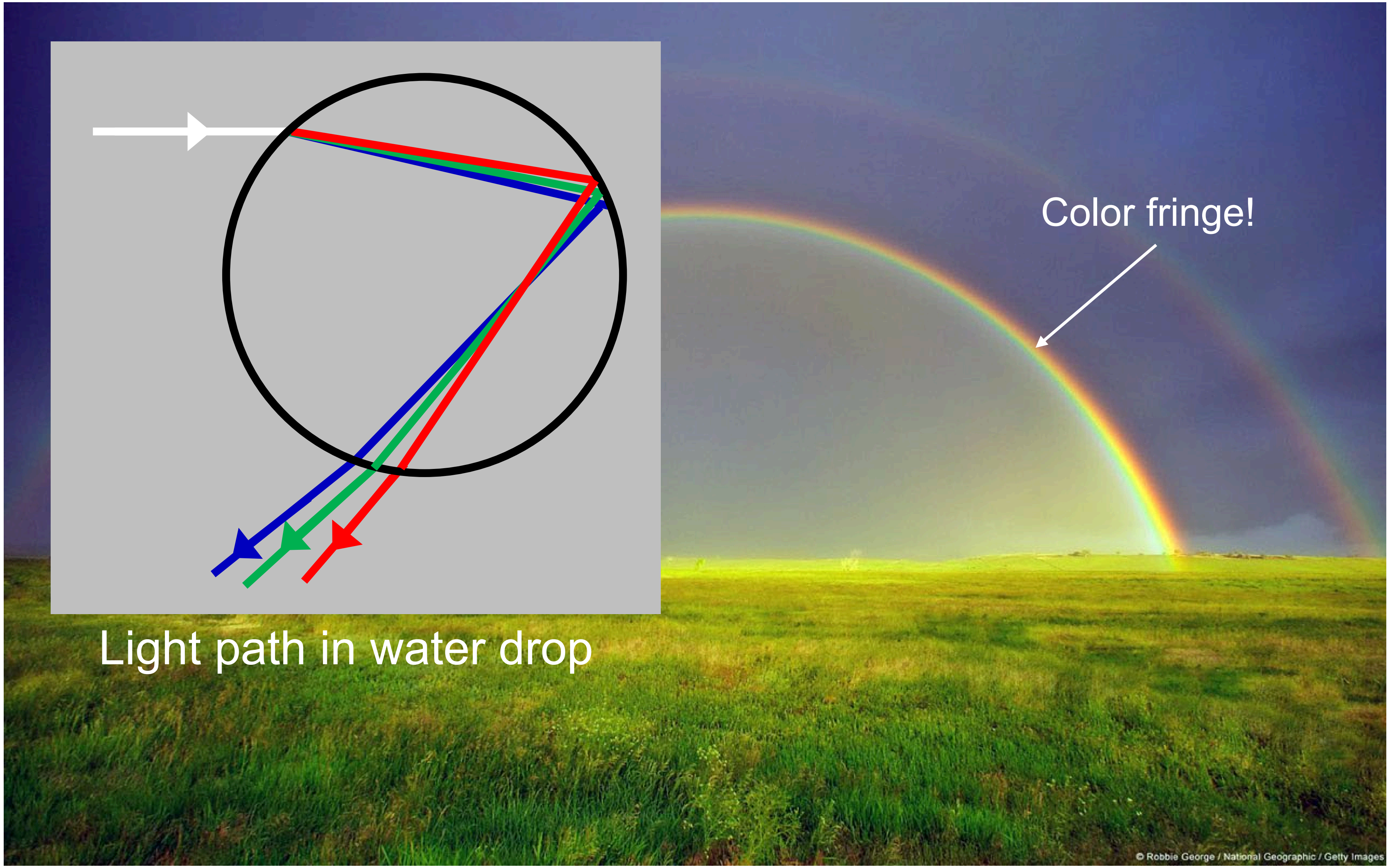
Higher order chromatic: all w_{jkl} depend on λ

$$W(\alpha^2, \alpha \cdot \xi, \xi^2) = \cdots + w_{400}\, \alpha^4 + w_{310}\, \alpha^2\ \alpha \cdot \xi + w_{202}\, \alpha^2\, \xi^2 + w_{020}\, (\alpha \cdot \xi)^2 + w_{013}\, \alpha \cdot \xi\, \xi^2 + \cdots$$

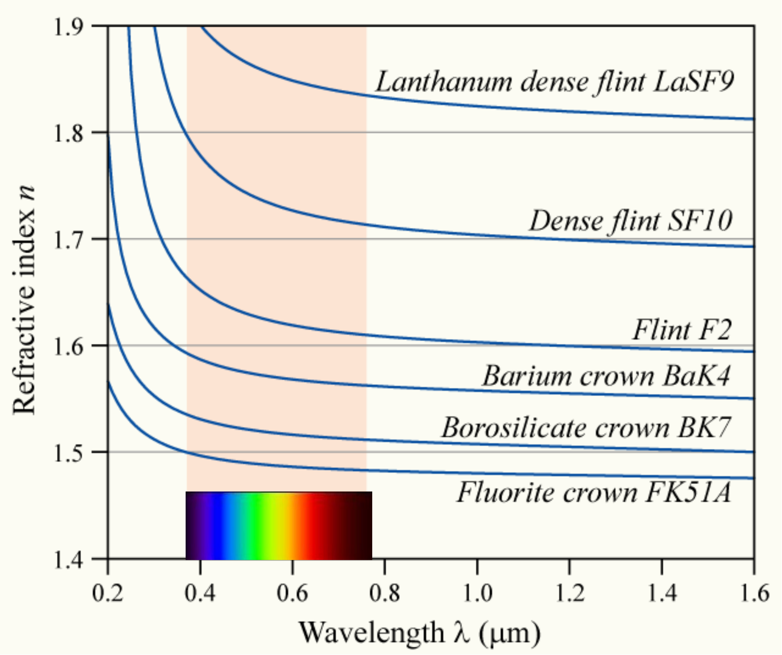
Chromatic aberration

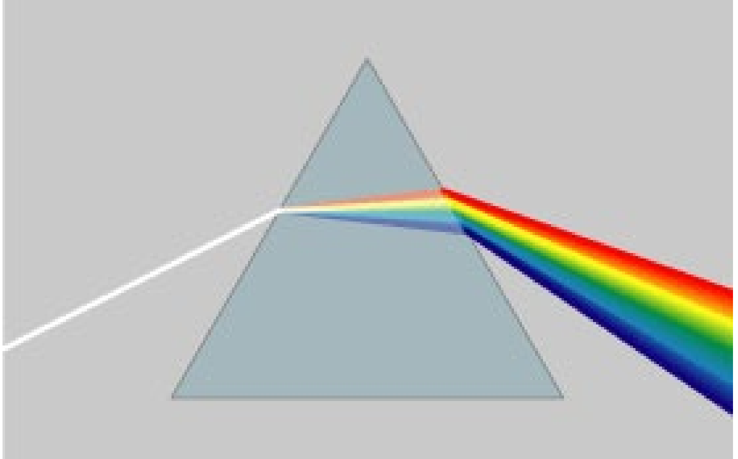
Rainbow

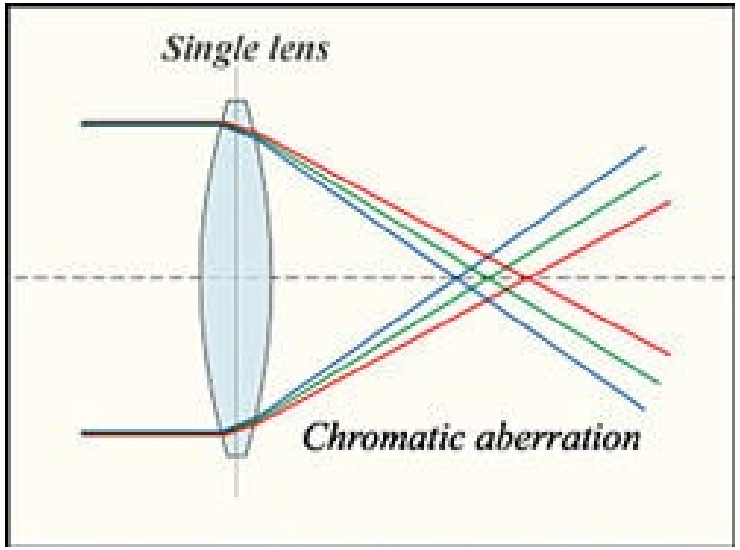
Rainbow


Color fringe!
Light path in water drop
© Robbie George / National Geographic / Getty Images

Dispersion and refractive index



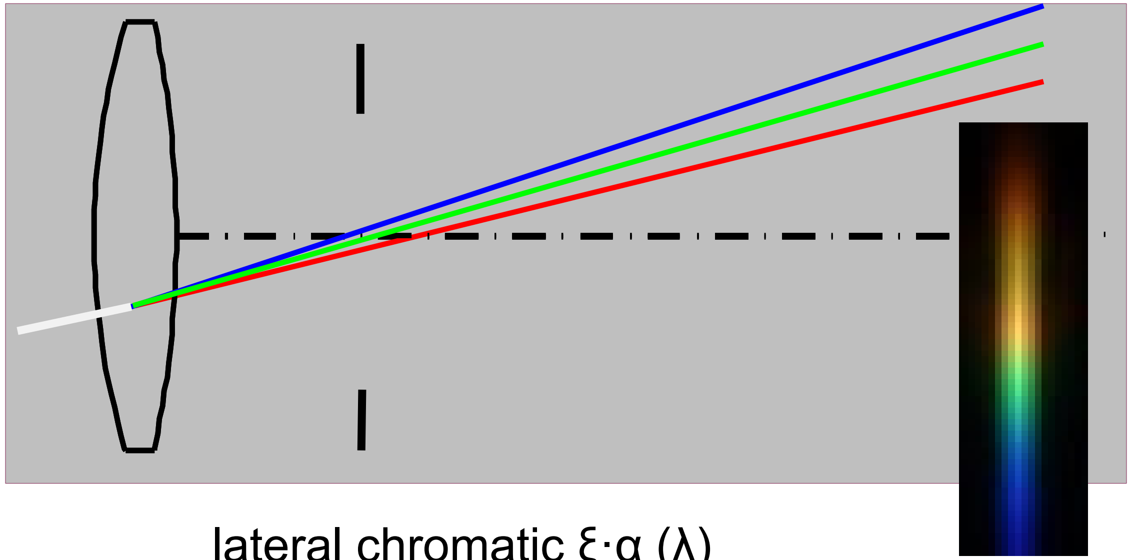




The refractive index of all glasses increases towards smaller wavelengths.

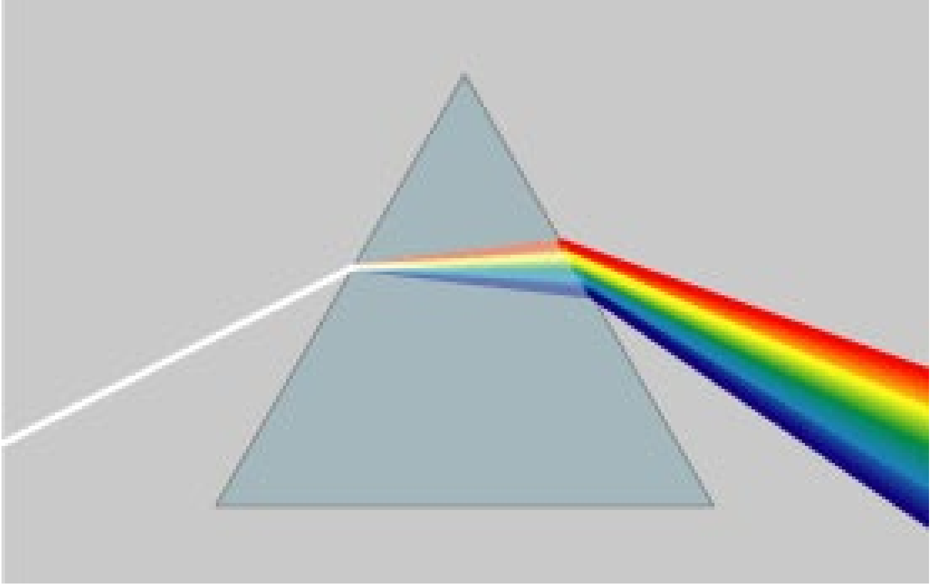
Therefore blue light is bend stronger than red light by lenses.

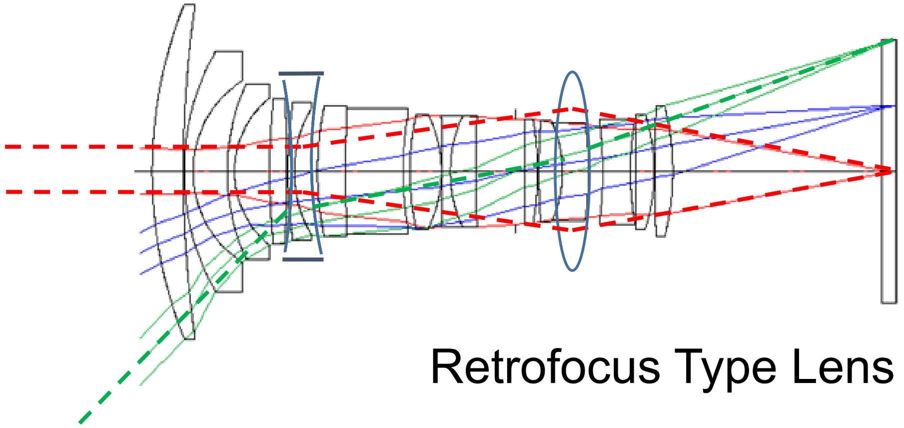
Classification of aberrations



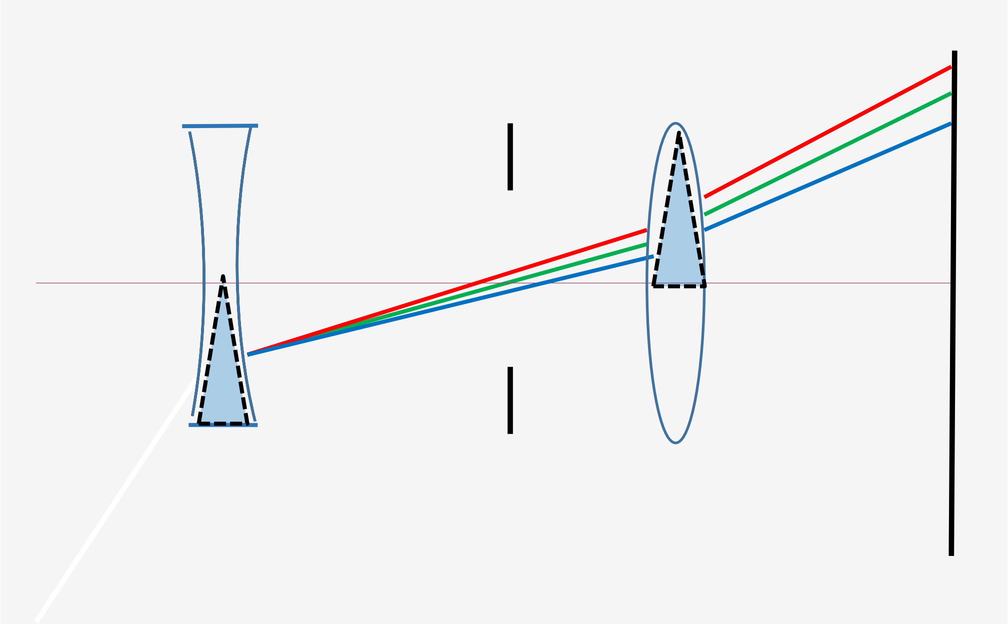
lateral chromatic ξ·α (λ)

Lateral chromatic aberration and retrofocus lens type



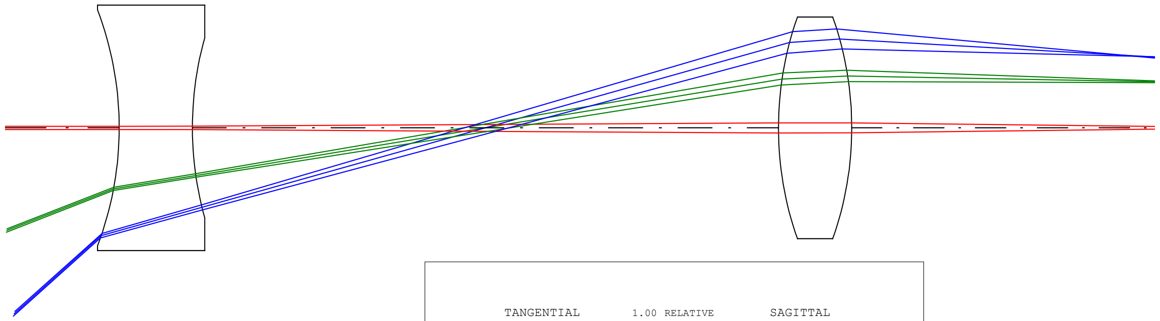


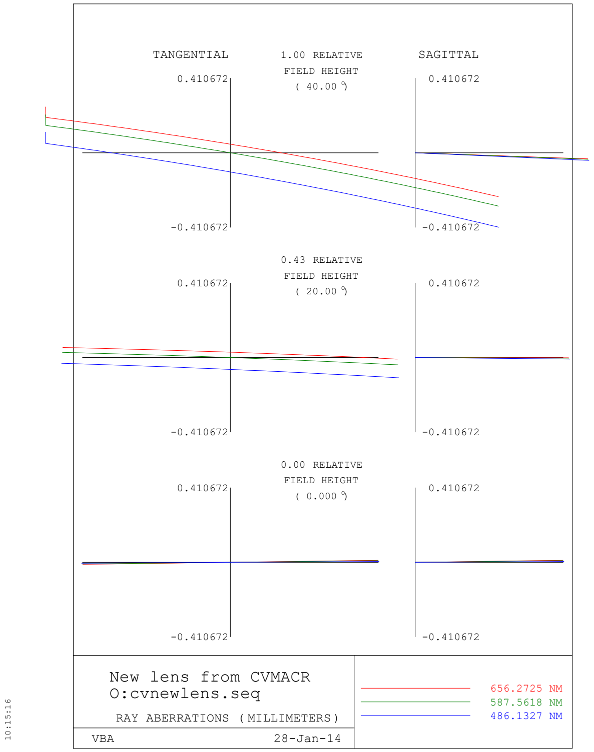
Retrofocus Type Lens



- Retrofocus lens structure „- +“ are typically (but not exclusively) wide-angle lenses for SLR-cameras, since this structure allows to leave the space for the viewfinder/AF mirror
- Lateral chromatic aberration is hard to correct for these lenses

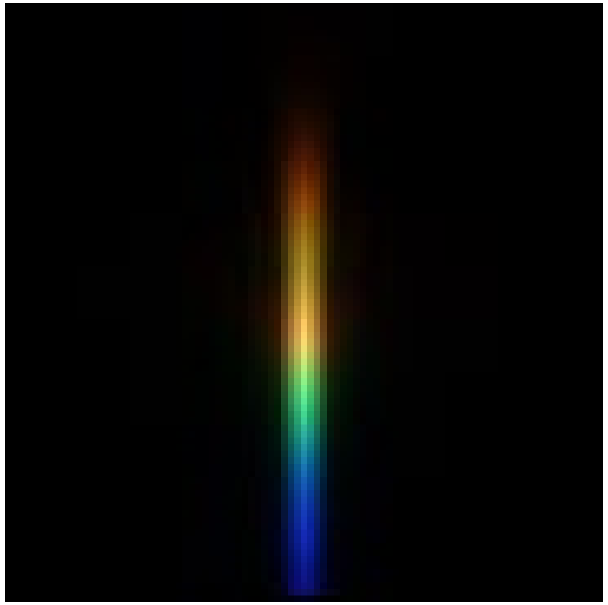
Lateral chromatic aberration and retrofocus lens type





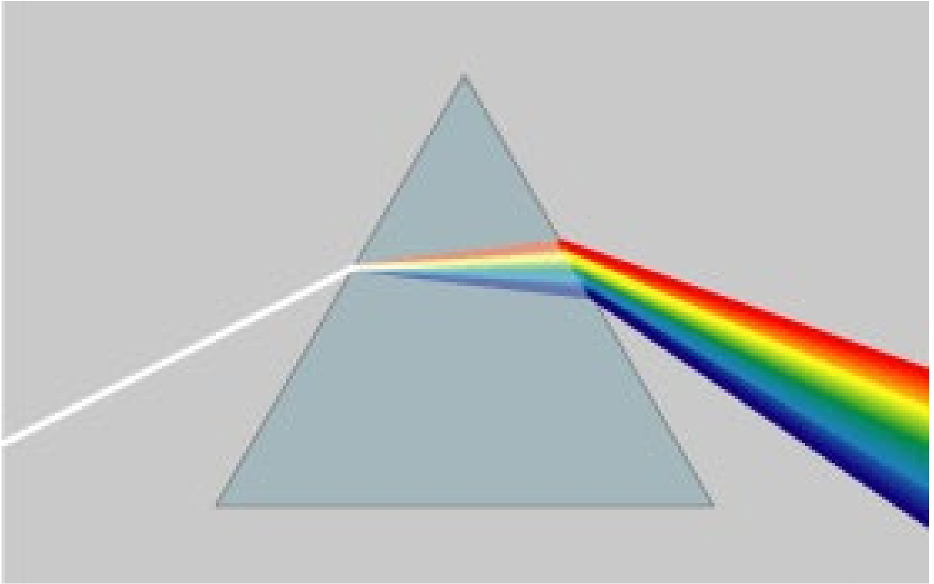
Ray aberration diagram

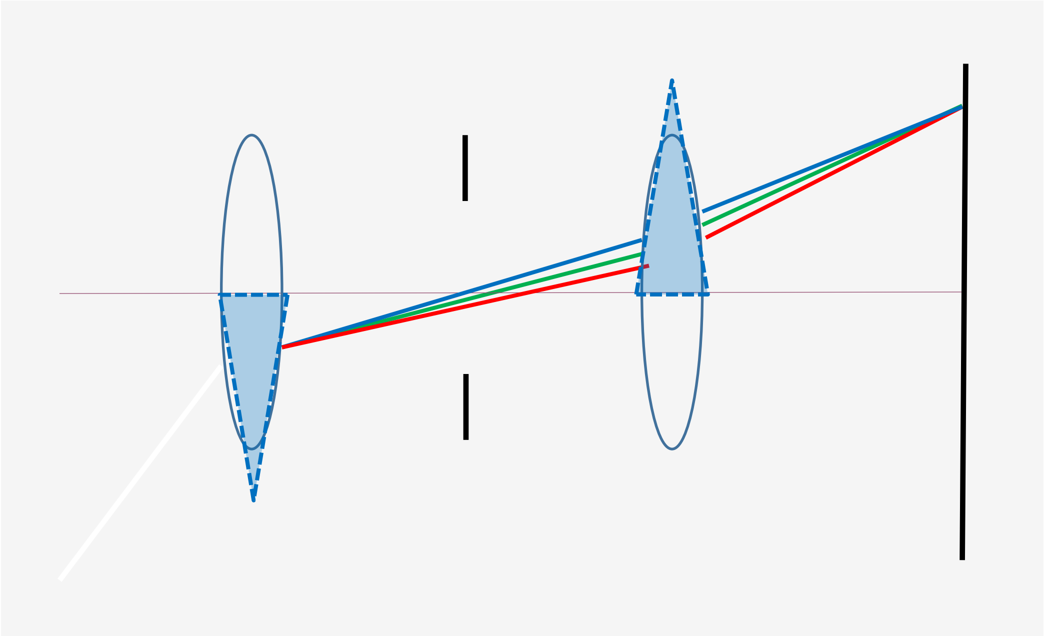
Spot diagram

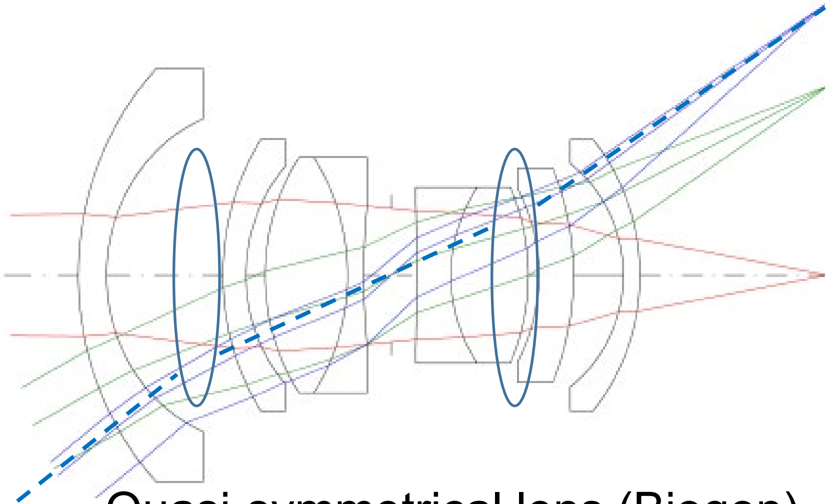


PSF

Lateral chromatic aberration and symmetrical lens type







Quasi-symmetrical lens (Biogon)

- Symmetrical lens structure
- Structure supports correction of lateral chromatic aberration (compensation)

Atomic Model of Dispersion

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- Atomic model for the refractive index: oscillator approach of atomic field interaction

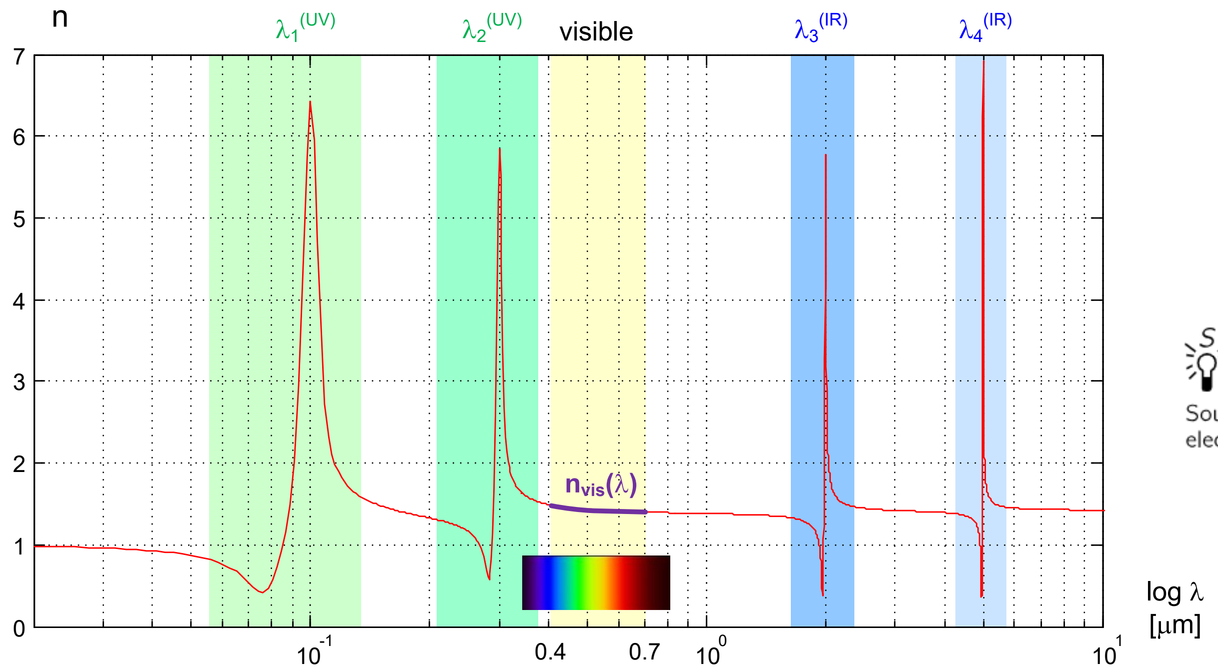
$$(n_r + i \cdot n_i)^2 = \frac{Ne^2}{2\pi \cdot c\varepsilon_0 m} \cdot \sum_j \frac{f_j \lambda^2 \lambda_j^2}{2\pi \cdot c \cdot (\lambda^2 - \lambda_j^2) + i\gamma_j \lambda \lambda_j^2}$$

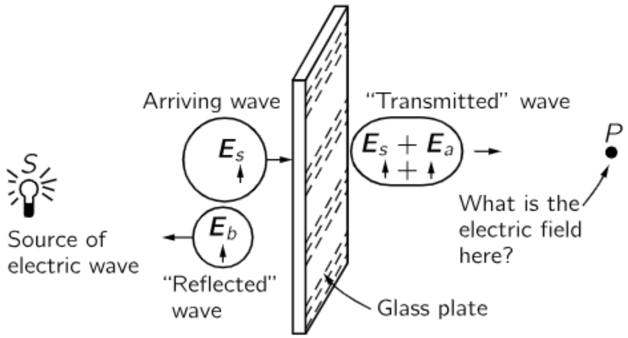
- Sellmeier dispersion formula: corresponding function

$$n^2 = A + \sum_j \frac{B_j \cdot \lambda^2}{\lambda^2 - C_j}$$

- Special case of coupled resonances: example quartz, degenerated oscillators

$$n^2 = A + \frac{B_0 \cdot \lambda^4}{(\lambda^2 - \lambda_o^2)^2} + \sum_{j=1} \frac{B_j \cdot \lambda^2}{\lambda^2 - C_j}$$





$$E_{\text{after plate}} = \underbrace{E_0 e^{i\omega(t-z/c)}}_{E_s} - \underbrace{\frac{i\omega(n-1)\,\Delta z}{c} E_0 e^{i\omega(t-z/c)}}_{E_a}$$

$$m\left(\frac{d^2x}{dt^2} + \omega_0^2 x\right) = q_e E_0 e^{i\omega t}$$

$$x = \frac{q_e E_0}{m(\omega_0^2 - \omega^2)} e^{i\omega t}.$$

$$n = 1 + \frac{N q_e^2}{2\epsilon_0 m(\omega_0^2 - \omega^2)}$$

Dispersion formulas

- Schott formula
  empirical

$$n = \sqrt{a_o + a_1\lambda^2 + a_2\lambda^{-2} + a_3\lambda^{-4} + a_4\lambda^{-6} + a_5\lambda^{-8}}$$

- Sellmeier
  Based on oscillator model

$$n(\lambda) = \sqrt{A + B\frac{\lambda^2}{\lambda^2 - \lambda_1^2} + C\frac{\lambda^2}{\lambda^2 - \lambda_2^2}}$$

very common

- Bausch-Lomb
  empirical

$$n(\lambda) = \sqrt{A + B\lambda^2 + C\lambda^4 + \frac{D}{\lambda^2} + \frac{E\lambda^2}{(\lambda^2 - \lambda_o^2) + \frac{F\lambda^2}{\lambda^2 - \lambda_o^2}}}$$

- Herzberger
  Based on oscillator model

$$n(\lambda) = a_o + a_1\lambda^2 + \frac{a_2}{\lambda^2 - \lambda_o^2} + \frac{a_3}{(\lambda^2 - \lambda_o^2)^2}$$

$$with\ \lambda_o = 0.168\mu m$$

- Hartmann
  Based on oscillator model

$$n(\lambda) = a_o + \frac{a_1}{a_3 - \lambda} + \frac{a_4}{a_5 - \lambda}$$

Optical glass data sheet (example SCHOTT): physical data (optical and environmental)

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https://www.schott.com/en-gb/products/optical-glass-p1000267/downloads

Datasheet

SCHOTT
glass made of ideas

SCHOTT N-BK7®
517642.251

n_d = 1.51680	ν_d = 64.17	$n_F - n_C$ = 0.008054
n_e = 1.51872	ν_e = 63.96	$n_{F'} - n_{C'}$ = 0.008110

Refractive Indices

	λ [nm]	
$n_{2325.4}$	2325.4	1.48921
$n_{1970.1}$	1970.1	1.49495
$n_{1529.6}$	1529.6	1.50091
$n_{1060.0}$	1060.0	1.50669
n_t	1014.0	1.50731
n_s	852.1	1.50980
n_r	706.5	1.51289
n_C	656.3	1.51432
$n_{C'}$	643.8	1.51472
$n_{632.8}$	632.8	1.51509
n_D	589.3	1.51673
n_d	587.6	1.51680
n_e	546.1	1.51872
n_F	486.1	1.52238
$n_{F'}$	480.0	1.52283
n_g	435.8	1.52668
n_h	404.7	1.53024
n_i	365.0	1.53627
$n_{334.1}$	334.1	1.54272
$n_{312.6}$	312.6	1.54862
$n_{296.7}$	296.7	
$n_{280.4}$	280.4	
$n_{248.3}$	248.3	

Internal Transmittance τ_i

λ [nm]	τ_i [10mm]	τ_i [25mm]
2500	0.67	0.36
2325	0.79	0.56
1970	0.93	0.84
1530	0.992	0.980
1060	0.999	0.997
700	0.998	0.996
660	0.998	0.994
620	0.998	0.994
580	0.998	0.995
546	0.998	0.996
500	0.998	0.994
460	0.997	0.993
436	0.997	0.992
420	0.997	0.993
405	0.997	0.993
400	0.997	0.992
390	0.996	0.989
380	0.993	0.983
370	0.991	0.977
365	0.988	0.971
350	0.967	0.92
334	0.91	0.78
320	0.77	0.52

...

Relative Partial Dispersion P

$P_{s,t}$	0.3098
$P_{C,s}$	0.5612
$P_{d,C}$	0.3076
$P_{e,d}$	0.2386
$P_{g,F}$	0.5349
$P_{i,h}$	0.7483

Relative Partial Dispersion P'

$P'_{s,t}$	0.3076
$P'_{C',s}$	0.6062
$P'_{d,C'}$	0.2566
$P'_{e,d}$	0.2370
$P'_{g,F'}$	0.4754
$P'_{i,h}$	0.7432

Deviation of Rel. Partial Disp. ΔP from the normal line

$\Delta P_{C,t}$	0.0216
$\Delta P_{C,s}$	0.0087
$\Delta P_{F,e}$	-0.0009
$\Delta P_{g,F}$	-0.0009
$\Delta P_{i,g}$	0.0035

Chemical Properties

CR	1

...

$n_{280.4}$	280.4
$n_{248.3}$	248.3

Constants of Dispersion Formula

B_1	1.039612120
B_2	0.231792344
B_3	1.010469450
C_1	0.006000699
C_2	0.0200179144
C_3	103.56065300

Constants of Formula for dn/dT

D_0	1.86E-06
D_1	1.31E-08
D_2	-1.37E-11
E_0	4.34E-07
E_1	6.27E-10
λ_{TK} [μm]	0.170

334	0.91	0.78
320	0.77	0.52
310	0.57	0.25
300	0.29	0.05
290	0.06	
280		
270		
260		
250		

Color Code

λ80 / λ5	33/29

Remarks

step 0.5 available

Temperature Coefficients of the Refractive Index

	$\Delta n_{rel}/\Delta T$ [10^{-6}/K]			$\Delta n_{abs}/\Delta T$ [10^{-6}/K]		
[°C]	1060.0	e	g	1060.0	e	g
-40/-20	2.4	2.9	3.3	0.3	0.8	1.2
+20/+40	2.4	3.0	3.5	1.1	1.6	2.1
+60/+80	2.5	3.1	3.7	1.5	2.1	2.7

Chemical Properties

CR	1
FR	0
SR	1
AR	2.3
PR	2.3

Other Properties

$\alpha_{-30/+70°C}$ [10^{-6}/K]	7.1
$\alpha_{+20/+300°C}$ [10^{-6}/K]	8.3
T_g [°C]	557
$T_{10^{13}}$ [°C]	557
$T_{10^{7.6}}$ [°C]	719
c_p [J/(g*K)]	0.858
λ [W/(m•K)]	1.114
AT [°C]	609
ρ [g/cm^3]	2.51
E [10^3 N/mm^2]	82
μ	0.206
K [10^{-6} mm^2/N]	2.76
$HK_{0.1/20}$	610
HG	3

Transmission of optical glass (SCHOTT 2014)
All 125 optical glasses contained in the graph

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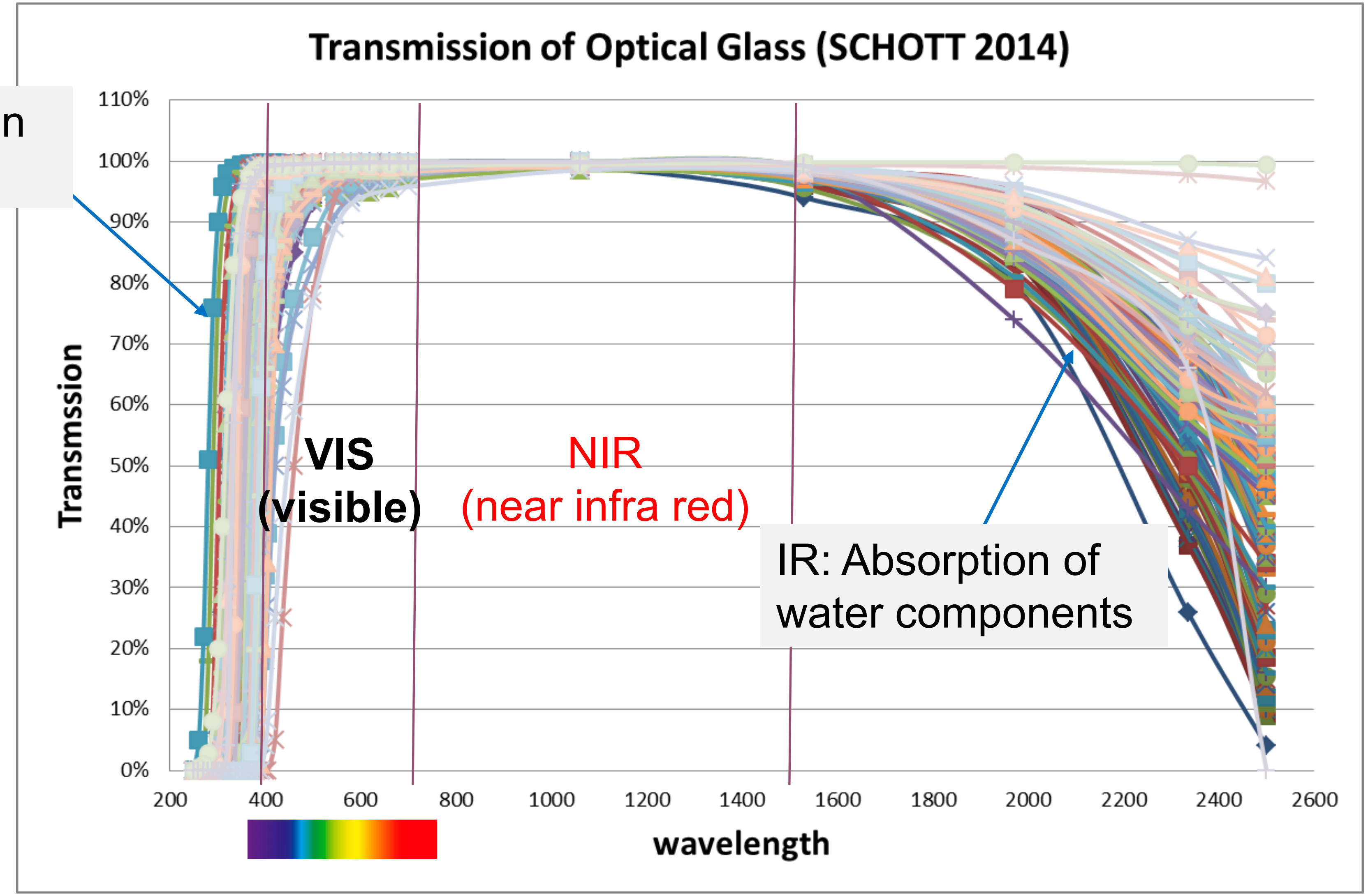
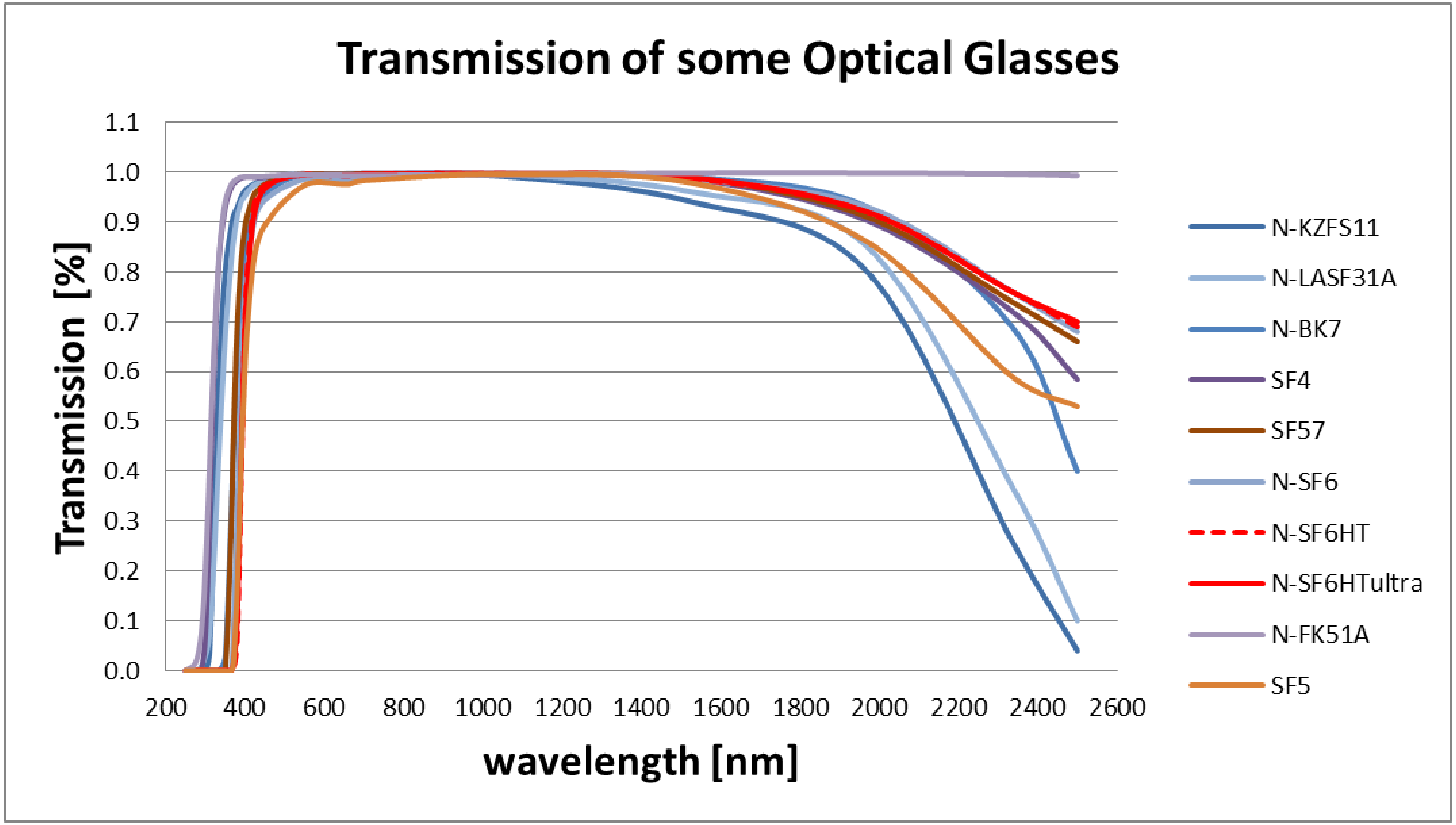
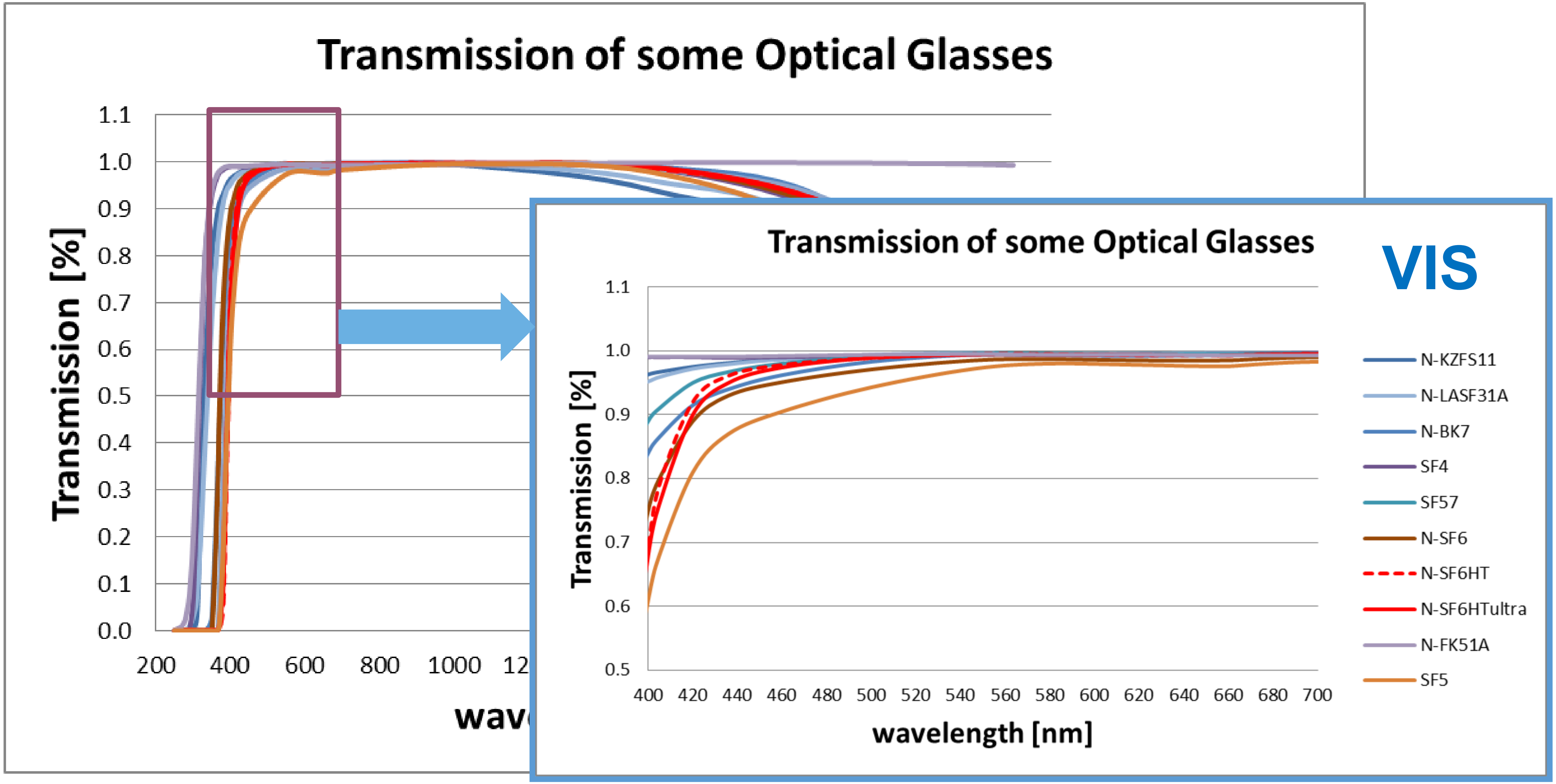
UV: Ionization of atoms



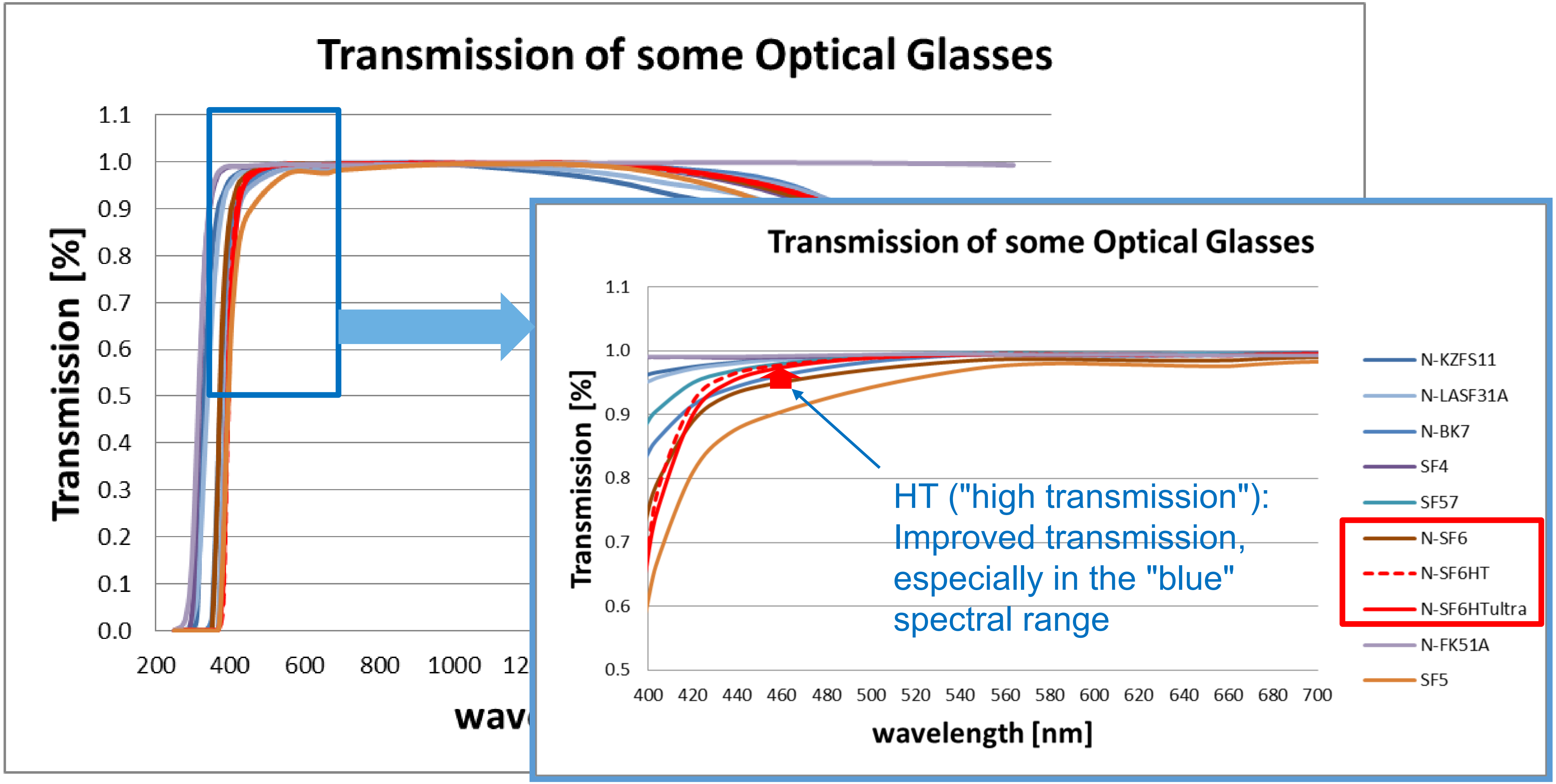
Transmission of some optical glasses (1/2)



Transmission of some optical glasses (2/2)
Visible spectrum

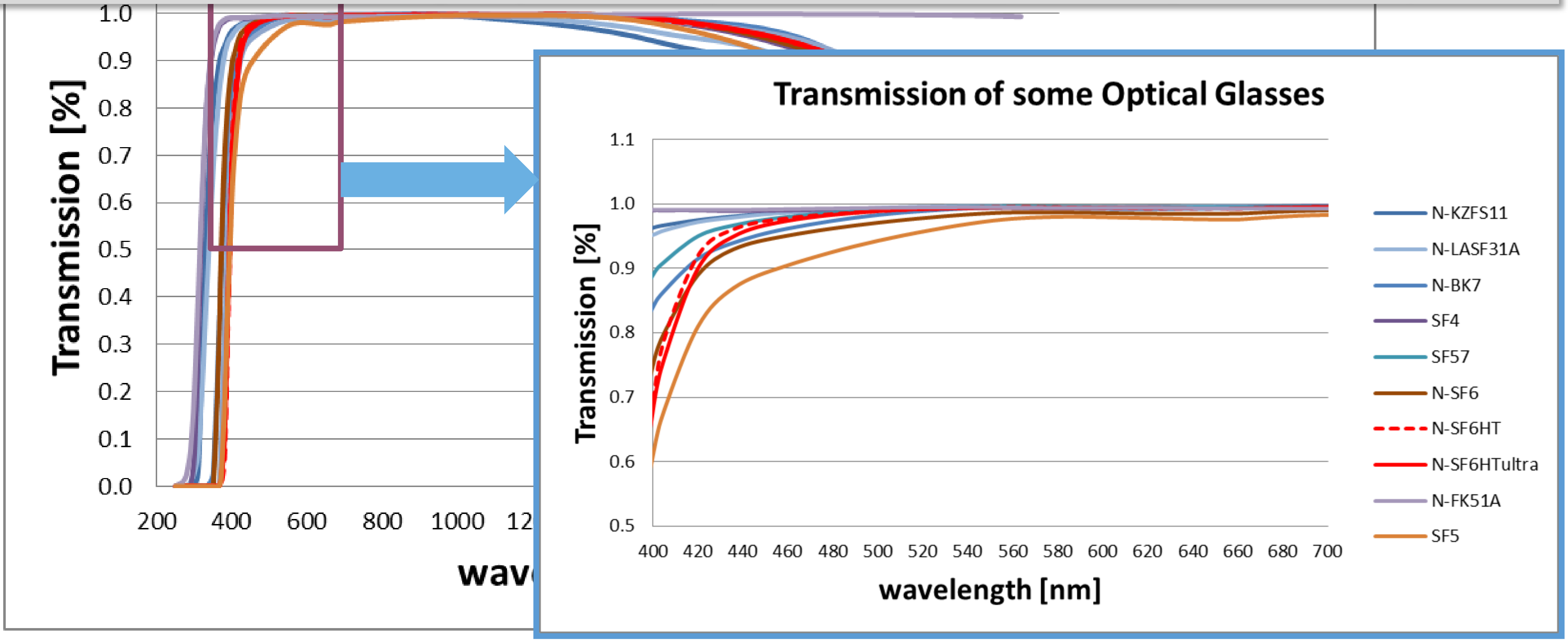


Transmission of some optical glasses (2/2)
Visible spectrum

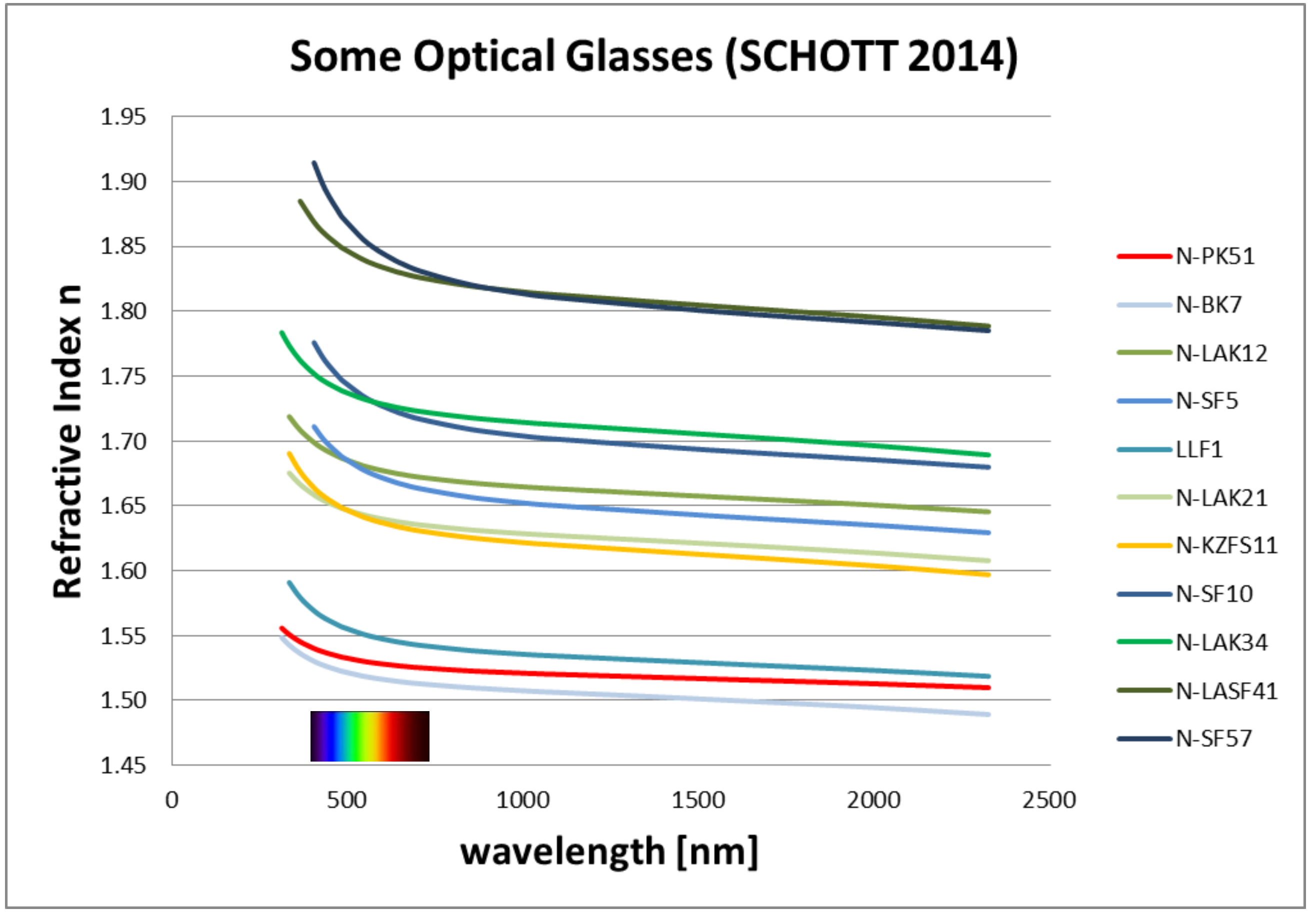


Transmission of some optical glasses (2/2) Visible spectrum

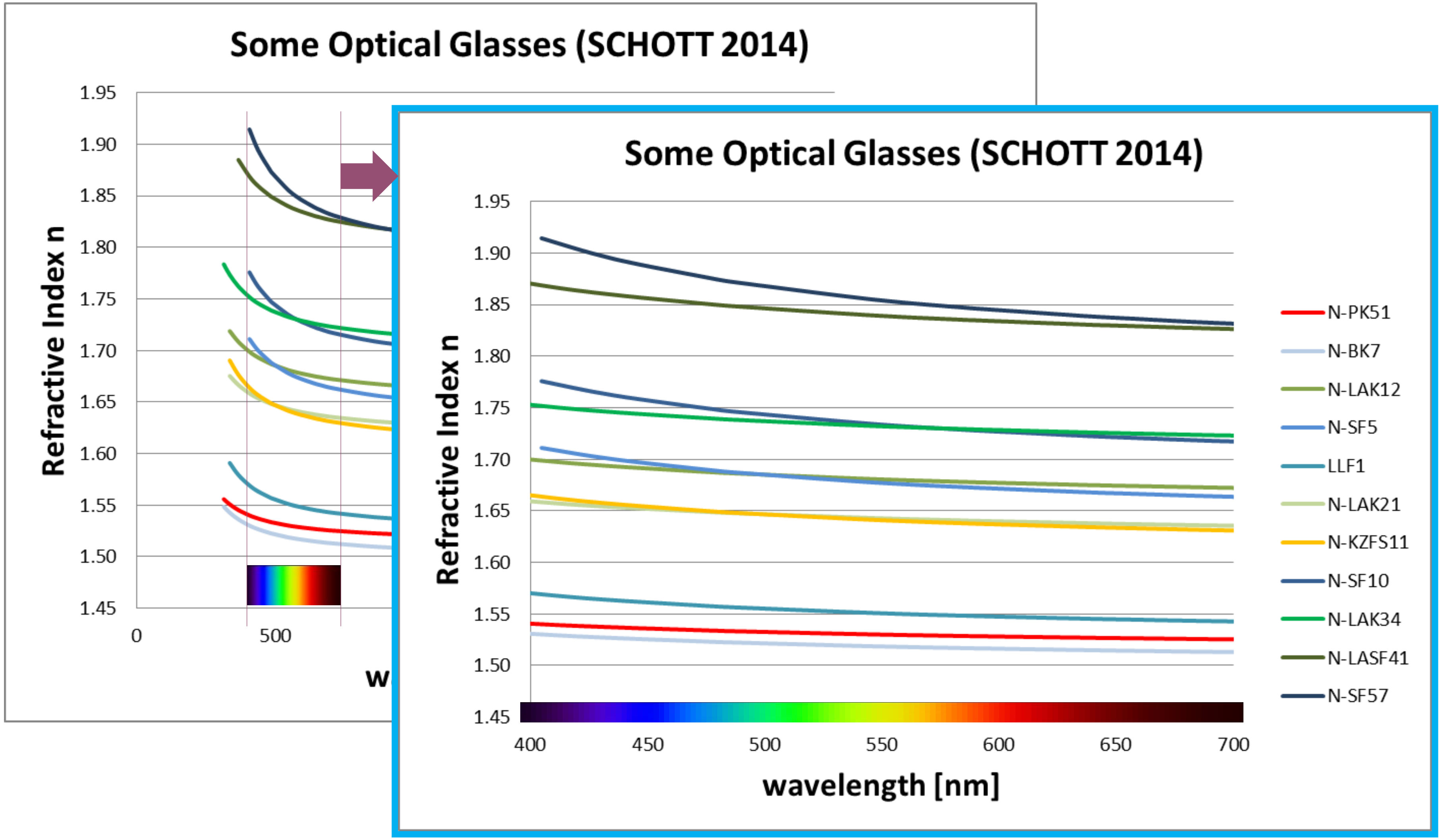
High-refractive glass usually absorbs more strongly in the blue region of the spectrum. Must be checked in the optical design, possibly compensated (e.g. “achromatic cement"), otherwise the color rendering is "yellowish" (yellow is the complementary color of blue).



Some Optical Glasses: n(λ)



Some Optical Glasses: n(λ)



Some Optical Glasses: n(λ)

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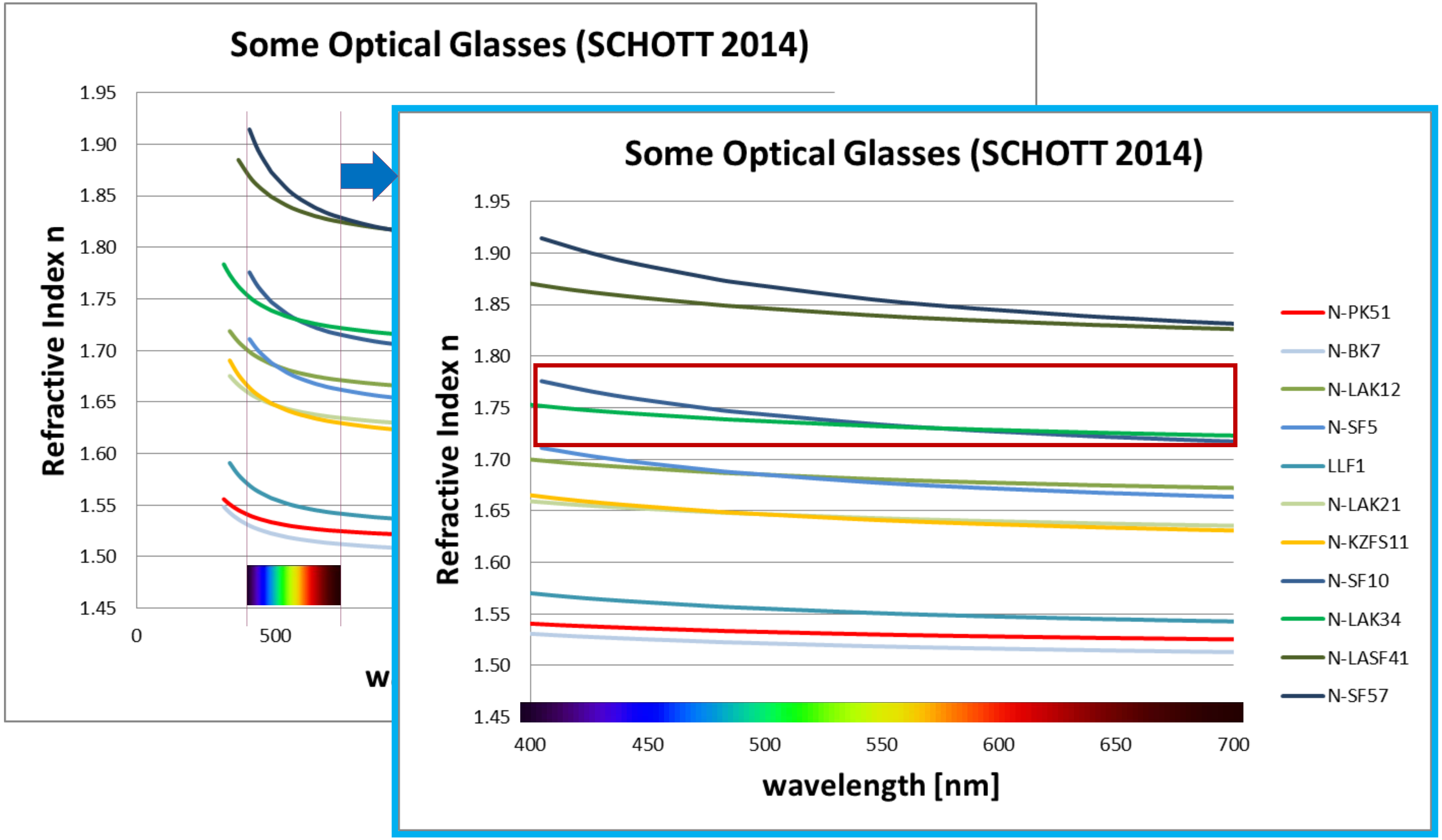
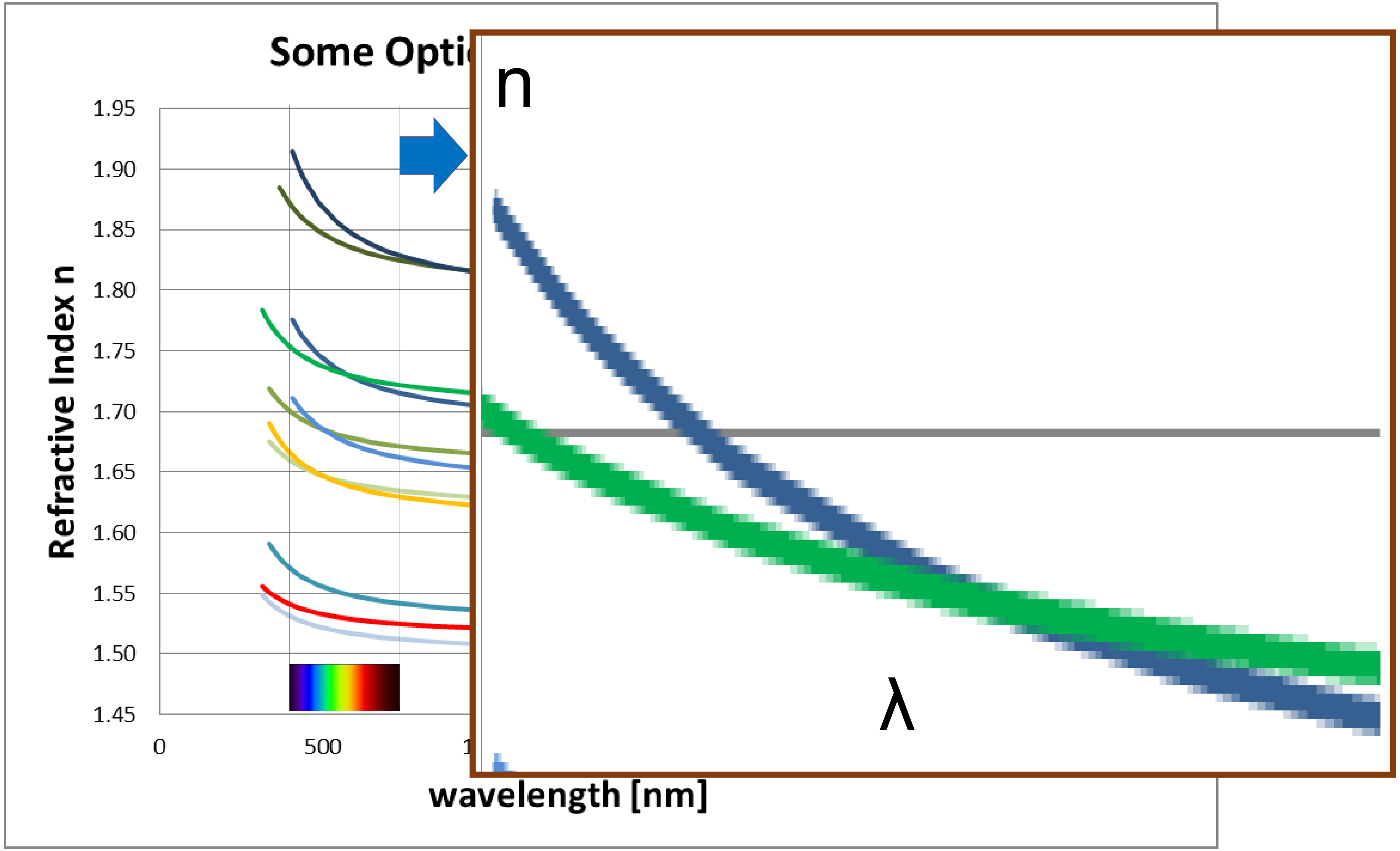


Some Optical Glasses: n(λ)



Definition of Abbe number ()

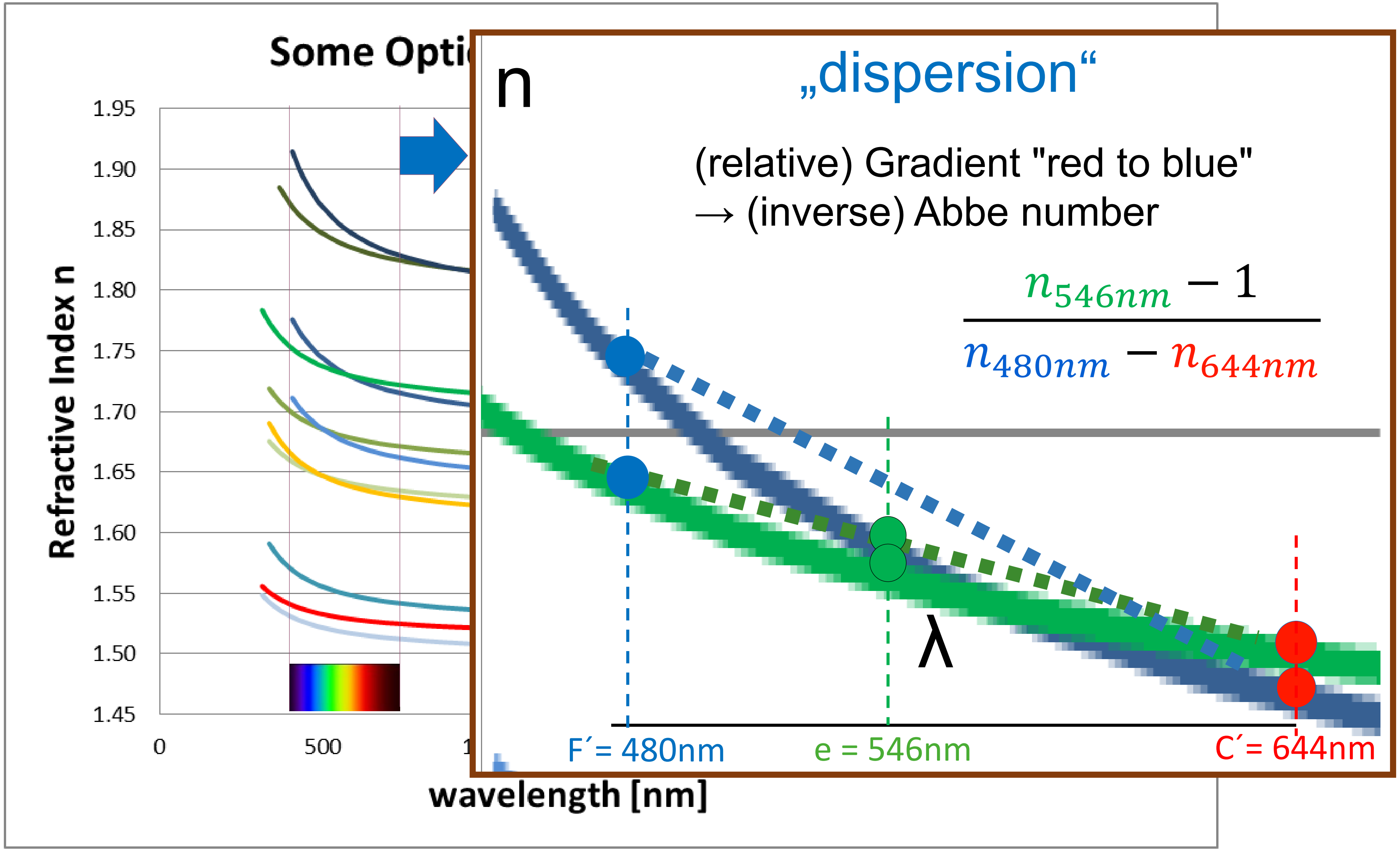
Abbe-number $\nu_e = \frac{n_e - 1}{n_{F'} - n_{C'}}$

with

F´= 480nm,

C´= 644nm and

e = 546nm.



Spectral lines in visible light used for Abbe number definitions

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Wavelength [nm]	Designation	Spectral Line
706.5	r	Red Helium Line
656.3	C	Red Hydrogen Line
643.8	C´	Red Cadmium Line
632.8		Helium-Neon-Gas-Laser
589.3	D	Yellow Sodium Line
587.6	d	Yellow Helium Line
546.1	e	Green Mercury Line
486.1	F	Blue Hydrogen Line
480	F´	Blue Cadmium line
435.8	g	Blue Mercury Line
404.7	h	Violet Mercury Line

$$\nu_d = \frac{n_d - 1}{n_F - n_C}$$

$$\nu_e = \frac{n_e - 1}{n_{F'} - n_{C'}}$$

ν_d common in e.g. photography,

ν_e in microscopy

Glass diagram ("Abbe diagram") SCHOTT 2022
n_d, ν_d

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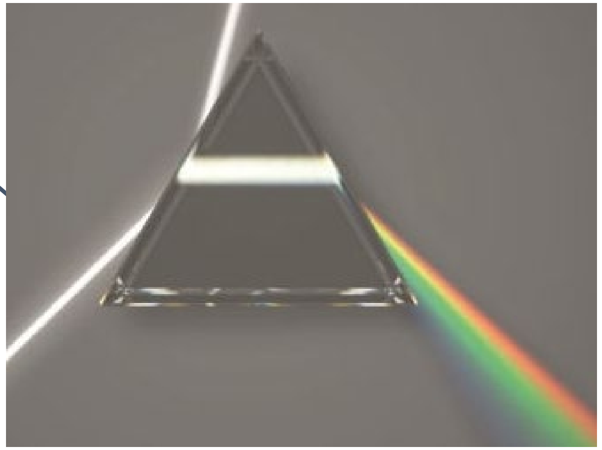
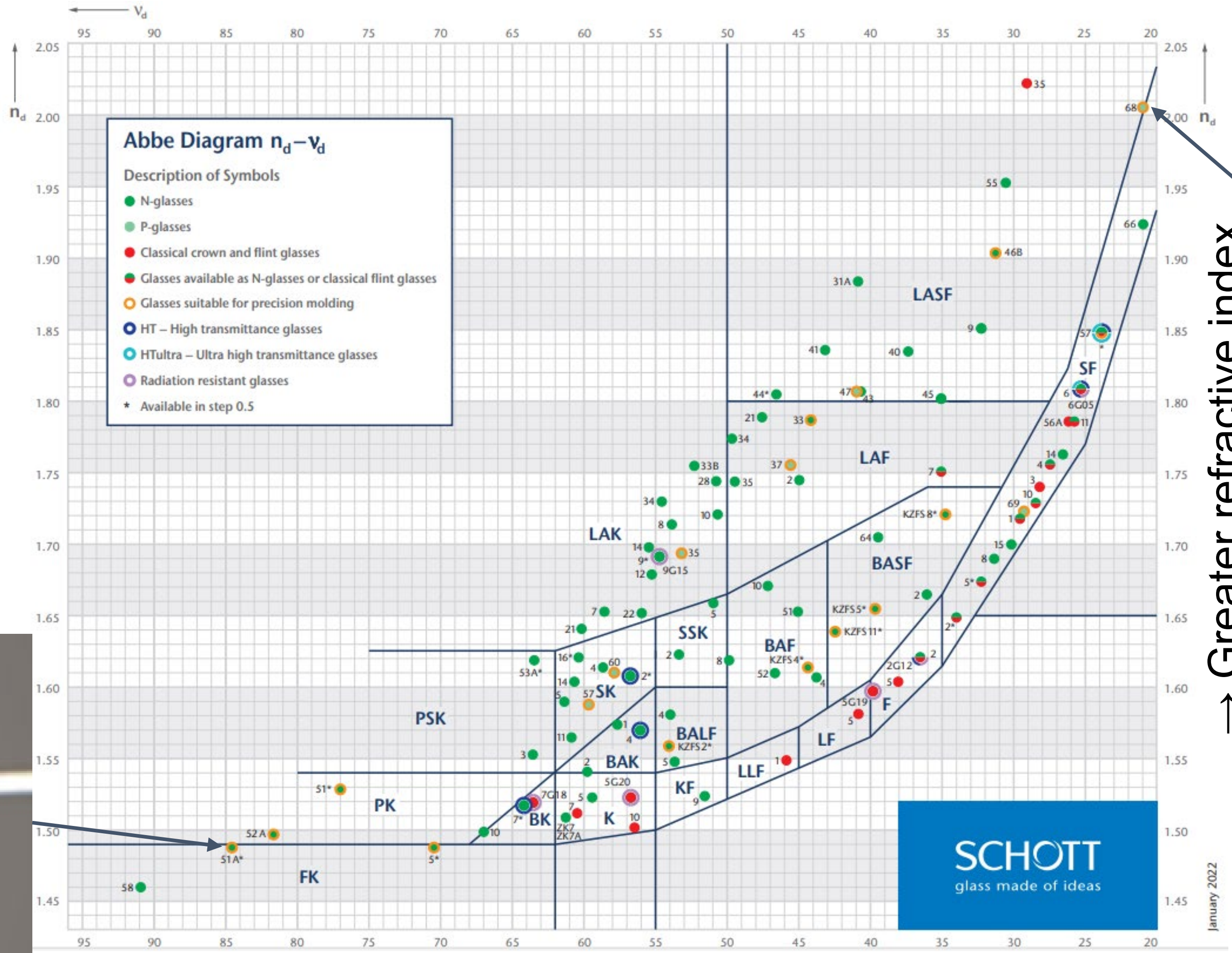
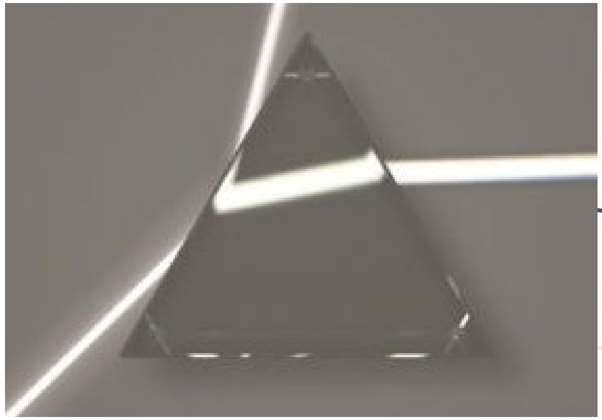




→ Greater refractive index



→ increasing dispersion

Glass diagram ("Abbe diagram") SCHOTT 2022
n_d, v_d

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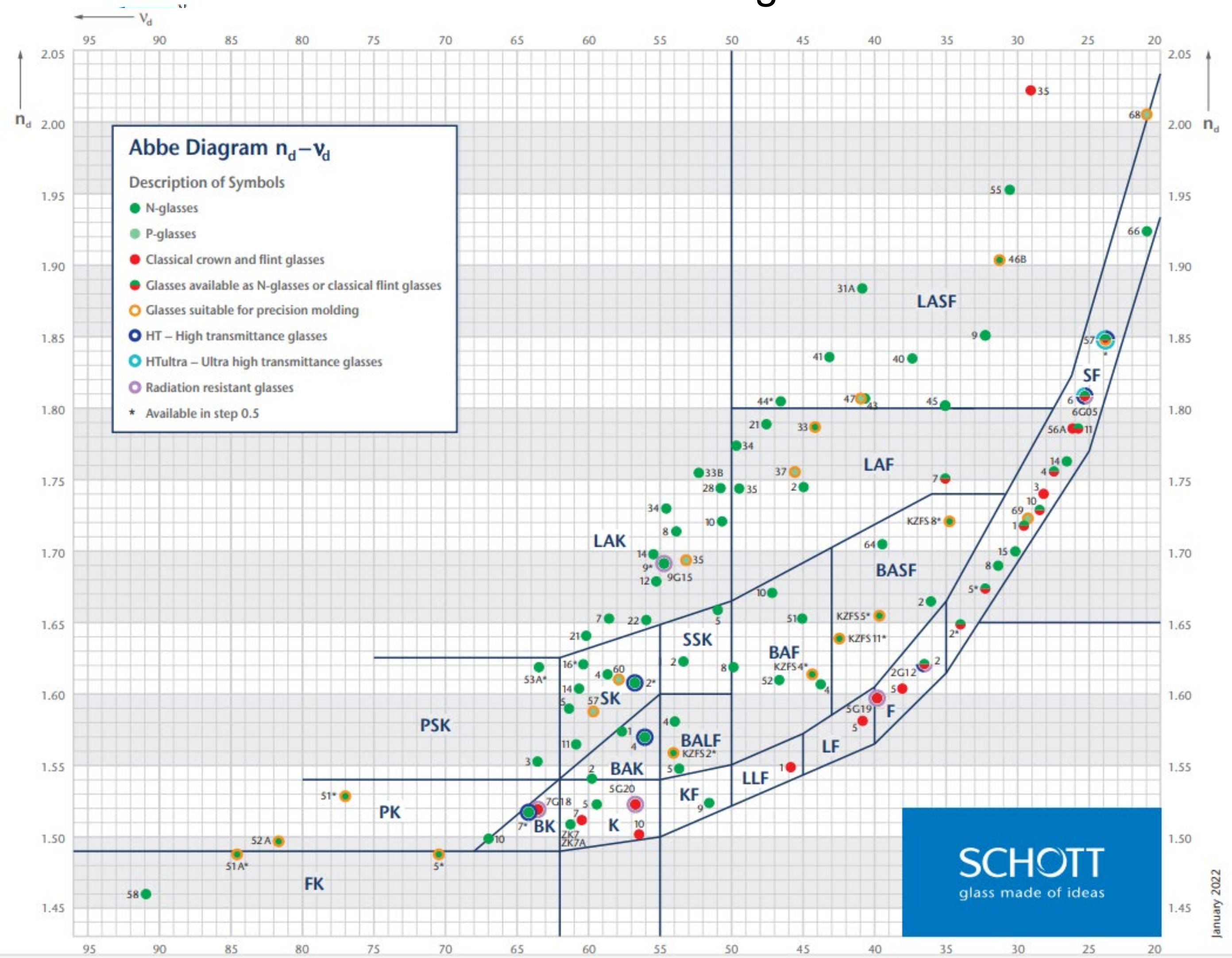
Glass names are different for different glass manufacturers.



FK	Fluorid Kron	fluoride crown
PF	Phosphat Kron	phosphate crown
PSK	Phosphat Schwerkron	phosphate crown
BK	Borsilikat Kron	borsilicate crown
BaK	Barium Kron	barium crown
SK	Schwerkron	heavy crown
K	Kron	crown
LaK	Lanthan Kron	lanthan crown
SSK	sehr schweres Kron	very heavy crown
KF	Kronflint	crown flint
LaSF	Lanthan Schwerflint	lanthan heavy flint
LaF	Lanthan Flint	lanthan flint
BaF	Barium Flint	barium flint
BaSF	Barium Schwerflint	barium heavy flint
LLF	leichtes Leichtflint	light light flint
LF	Leichtflint	light flint
F	Flint	flint
SF	Schwerflint	heavy flint
ZK	Zinkkron	zinc crown
KzSF	KurzFlint	short heavy flint

Crown and flint glass

Krongläser

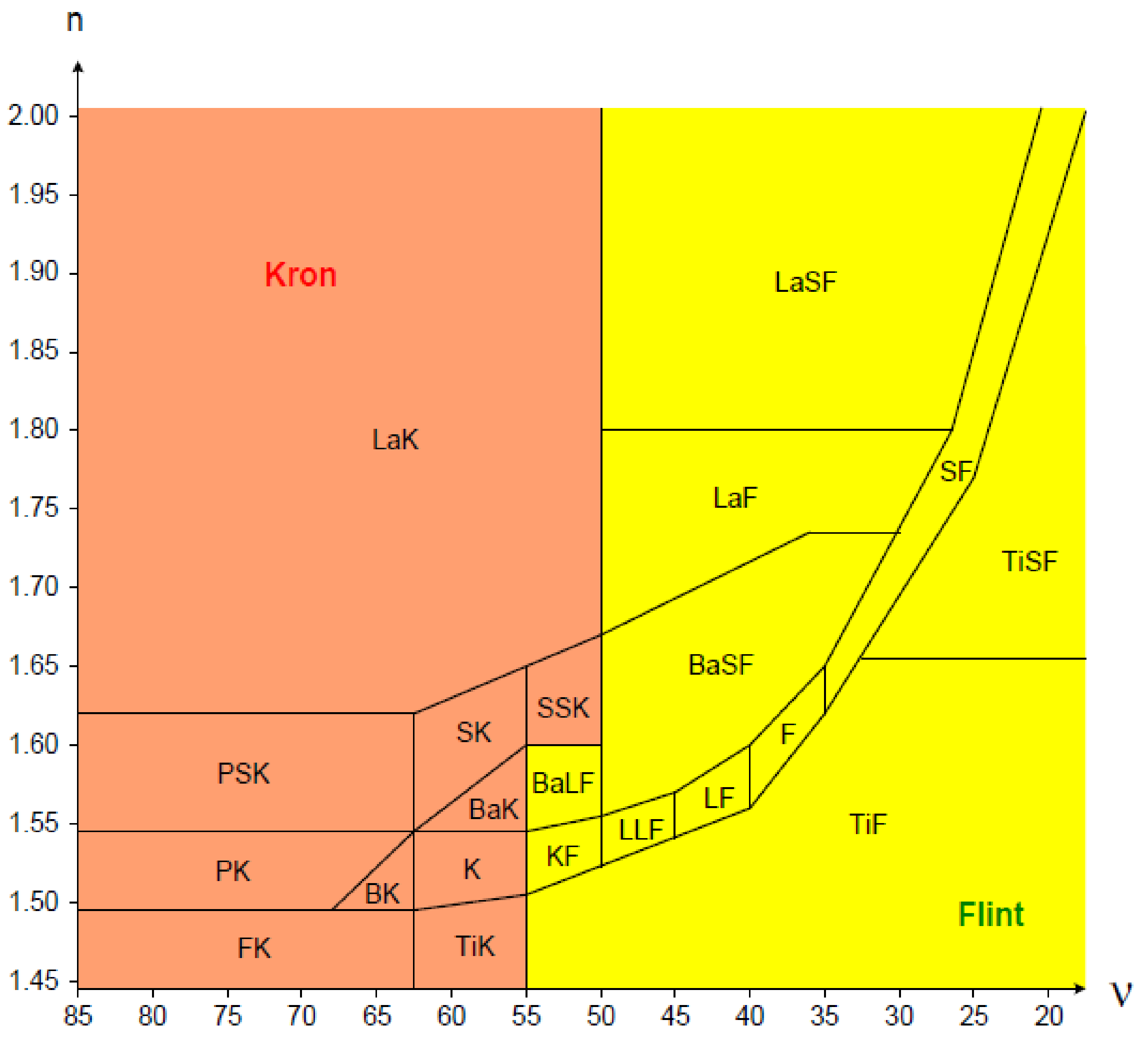
ν > 55 für n < 1.60

ν > 50 für n > 1.60

Flintgläser

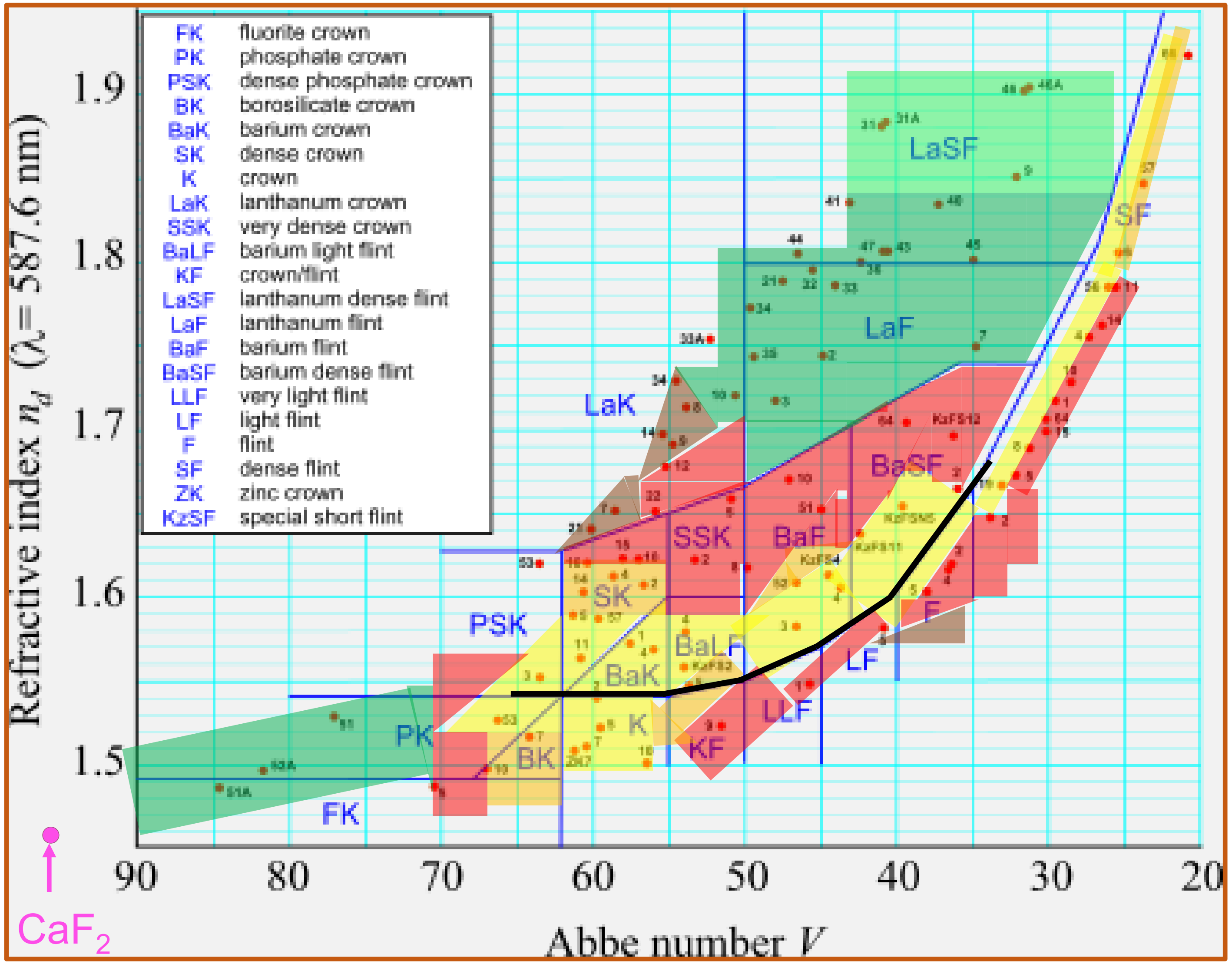
ν < 55 für n < 1.60

ν < 50 für n < 1.60



The name "crown"/"flint" probably dates back to the 13th century (Venetian glassblowers) to distinguish the only glasses known at that time:
Crown glass (alkali silicate),
Flint glass (lead glass)

History of optical glass (SCHOTT)



-1880 “iron line”

1880-1886

1887-1913

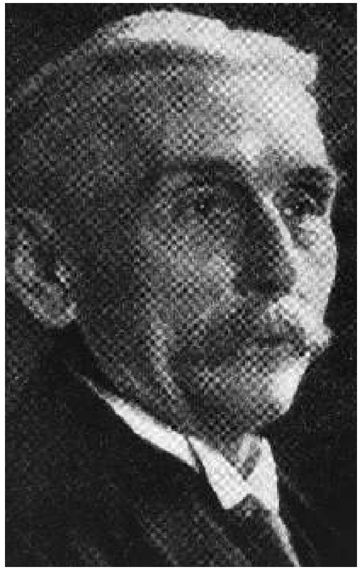
1914-1944

1945-1960

1961-1988

1990-

2000 -… „Lead Replacement “



Otto Schott

source:
C. Hofmann (1989), Feingerätetechnik 38

History number of optical glasses of SCHOTT
Years 1860 - 2015

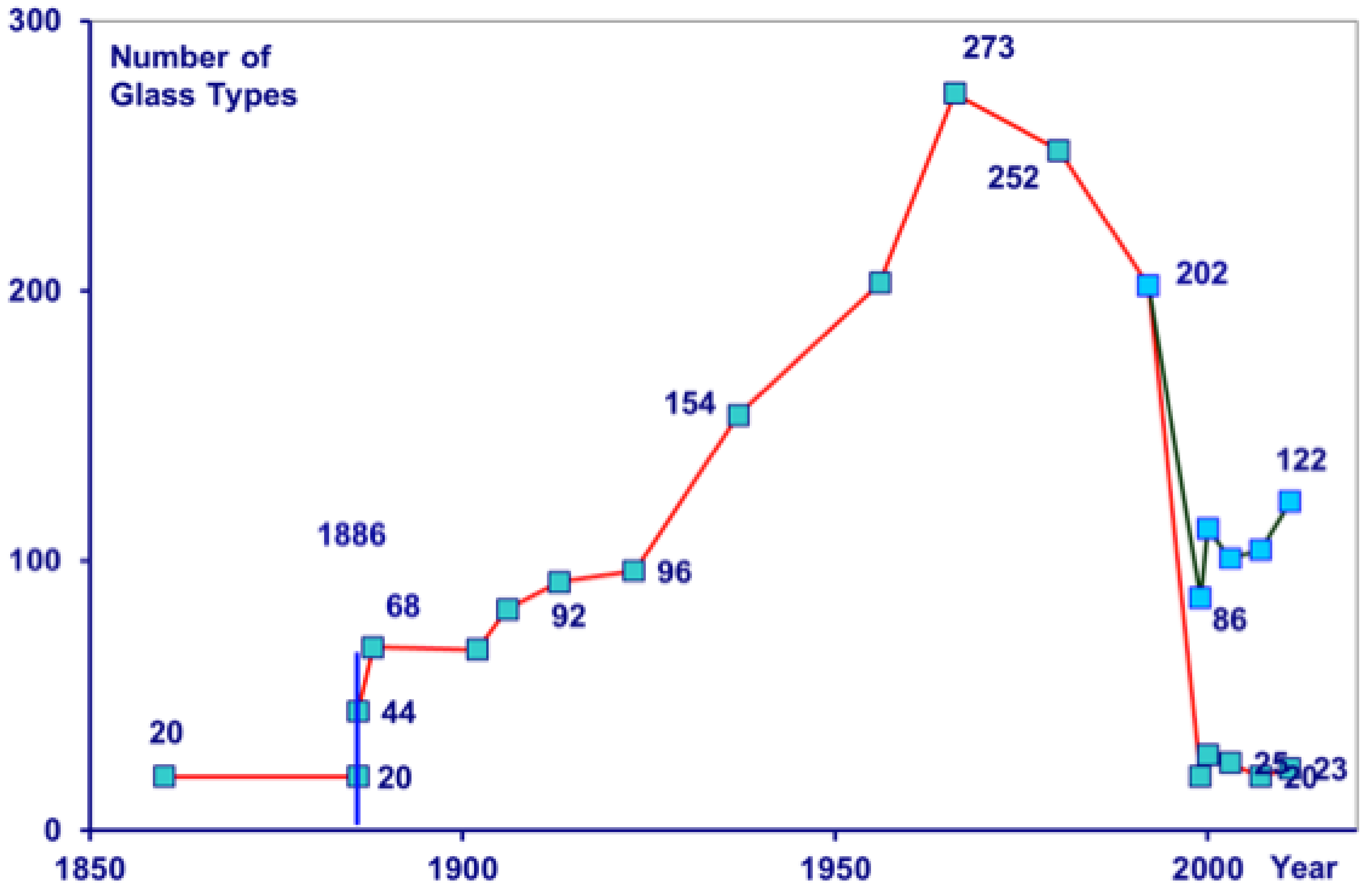
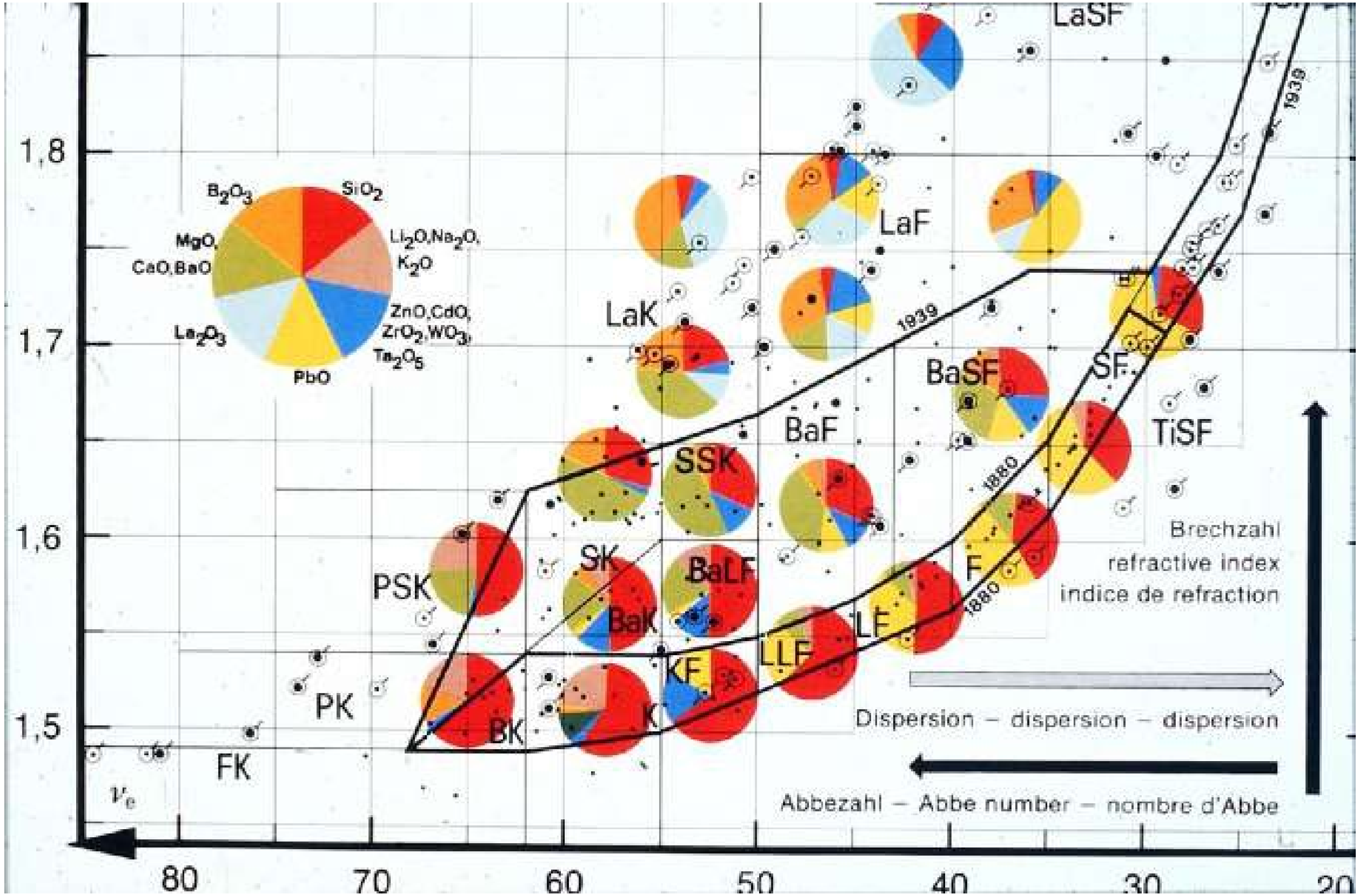


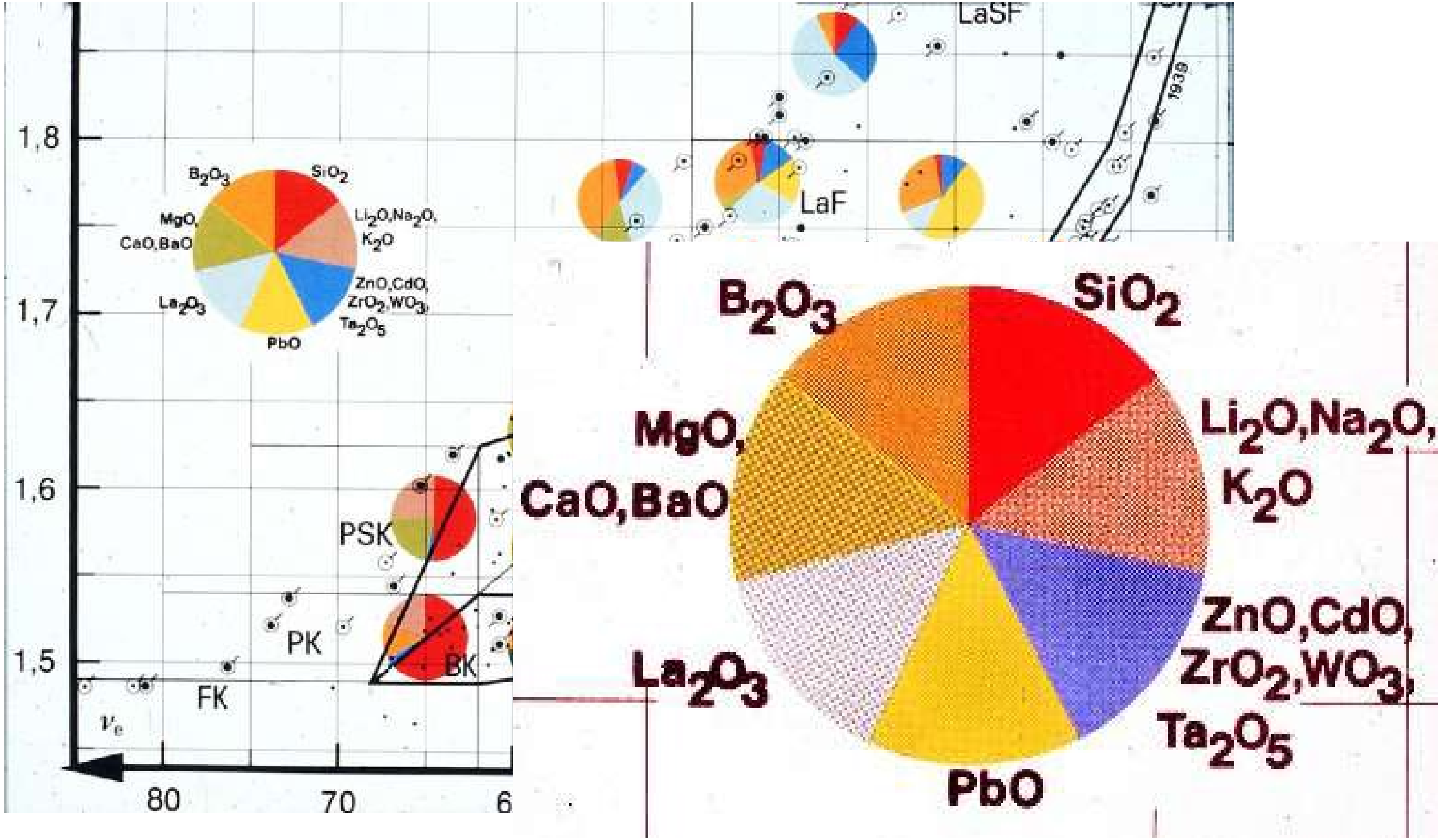
Figure 1.7 History of the optical glass types portfolio of SCHOTT. Red line: classical glass types, black line: total number. Note the sharp rise with the beginning of glass-type development and the sharp drop due to the change to lead- and arsenic-free glass types and economic restructuring.

Source: P. Hartmann

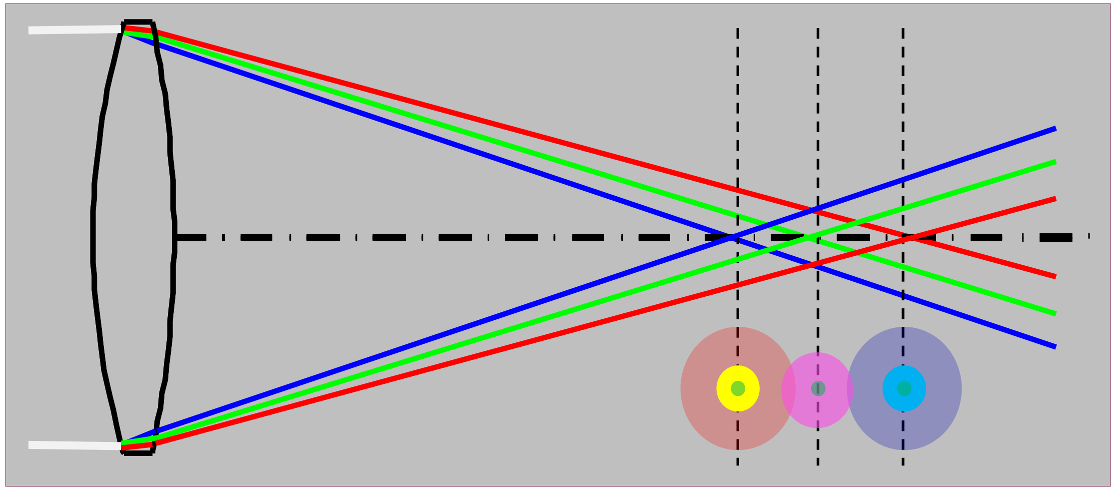
Optical Glass: Composition of materials „classical“ glasses incl. e.g. Pb



Optical Glass: Composition of materials



Classification of aberrations



longitudinal chromatic α^2 (λ)

Effect of glass dispersion on longitudinal chromatic aberration of single lens

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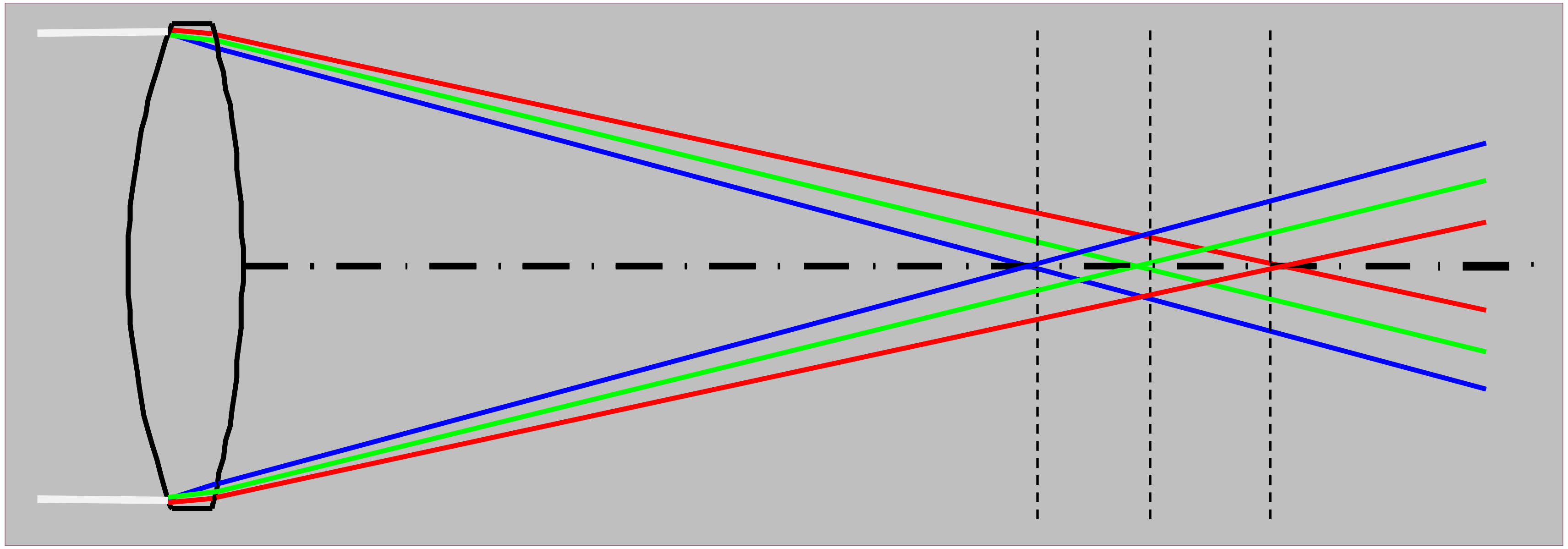


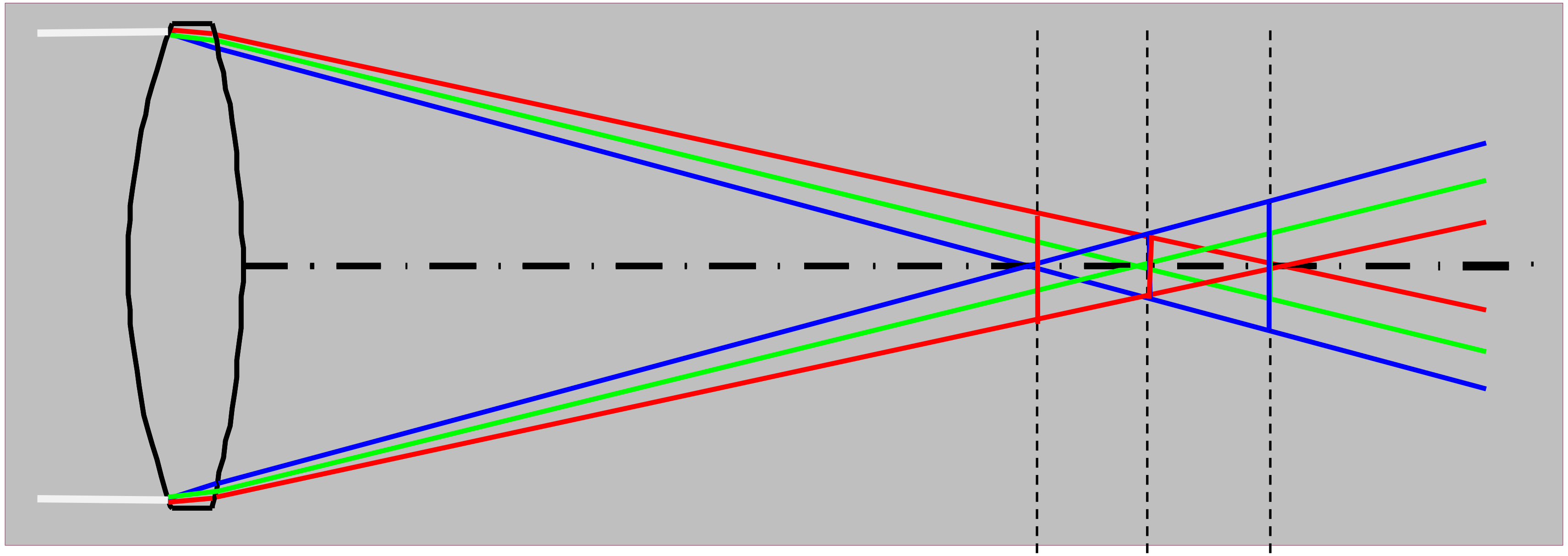
A single lens with the radii r_1, r_2 (or curvatures $c_1 = 1/r_1$, $c_2 = 1/r_2$), refractive index n and thickness d (the distance between the two principal planes P, P´) has the refractive power vs wavelength

$$\Phi(\lambda) = (n(\lambda) - 1)\left(\frac{1}{r_1} - \frac{1}{r_2} + \frac{(n(\lambda)-1)\, d(\lambda)}{n(\lambda)\, r_1 r_2}\right).$$

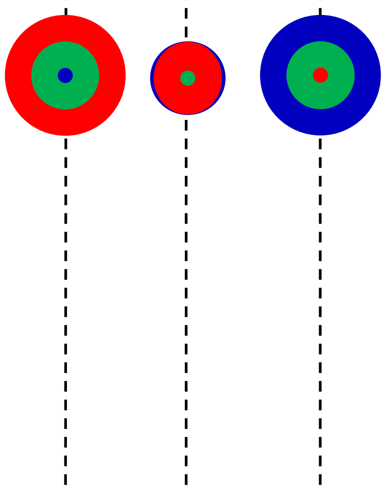
For a thin lens, the last term is neglected (notation for curvature difference $\Delta c = c_1 - c_2$):

$$\Phi(\lambda) = (n(\lambda) - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad \text{or} \quad \Phi(\lambda) = (n(\lambda) - 1)\Delta c.$$

Longitudinal chromatic aberration of single lens: primary spectrum



Spot sizes:



Longitudinal chromatic aberration of single lens: primary spectrum

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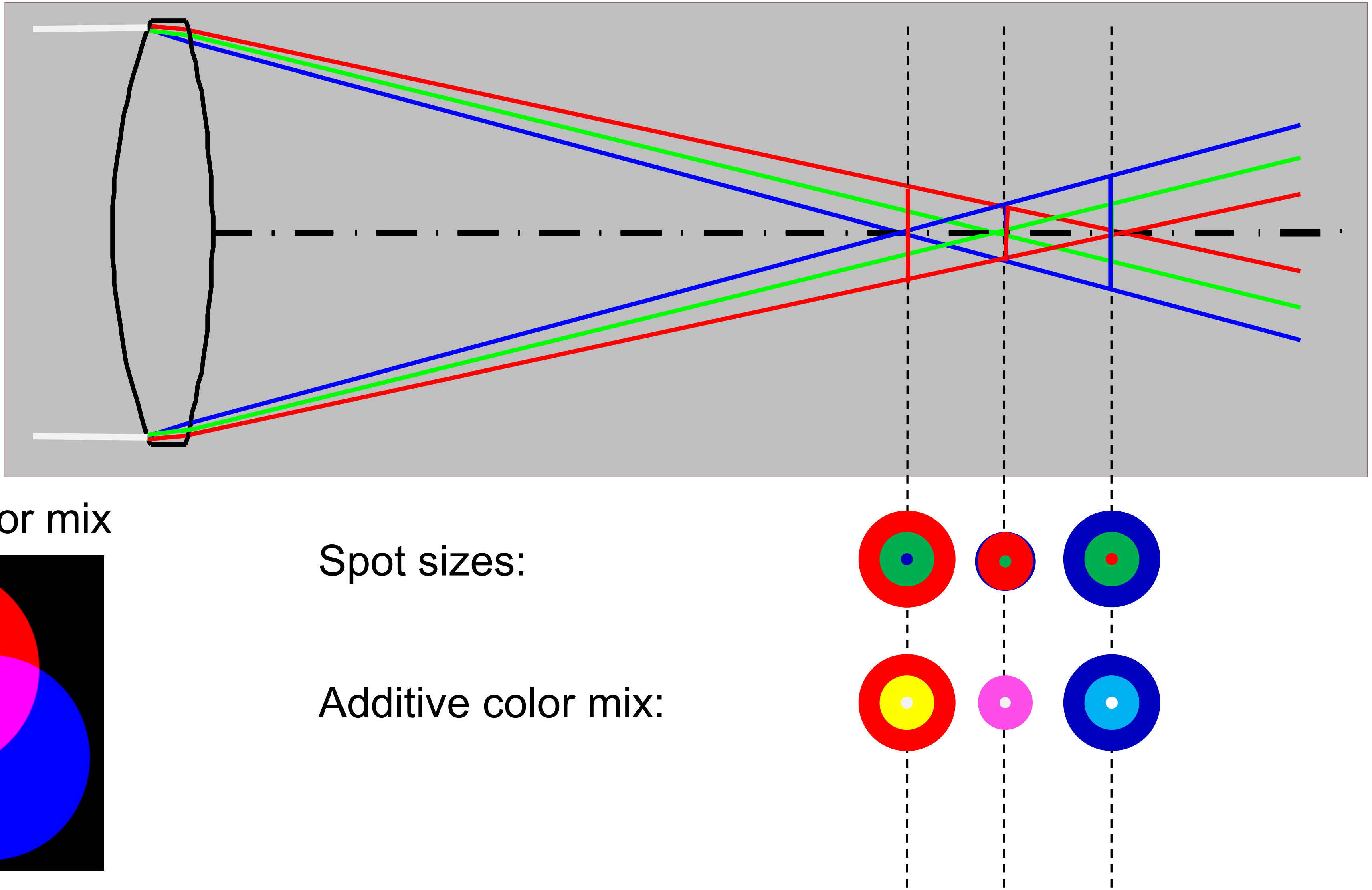
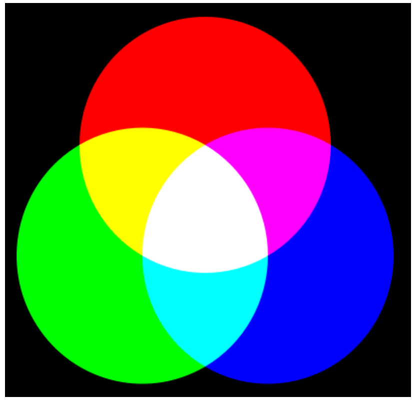


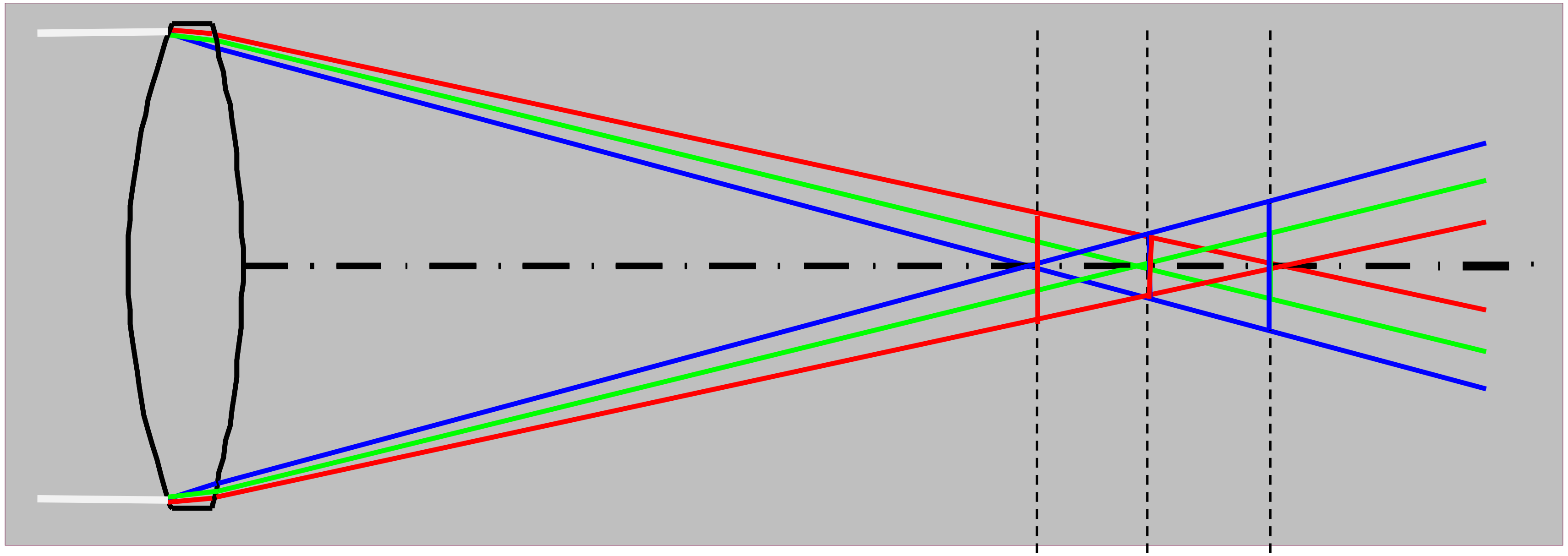
additive color mix



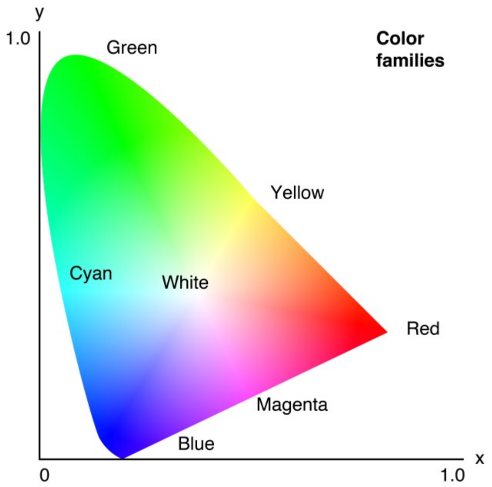
Spot sizes:

Additive color mix:

Longitudinal chromatic aberration of single lens: primary spectrum



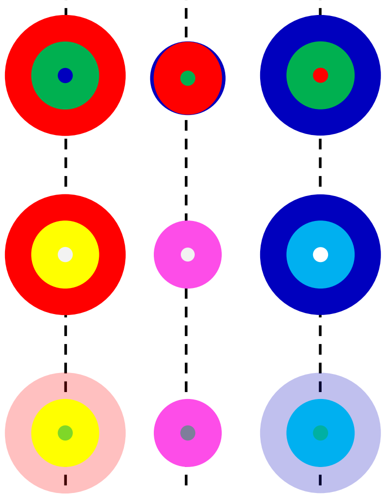
additive color mix



Spot sizes:

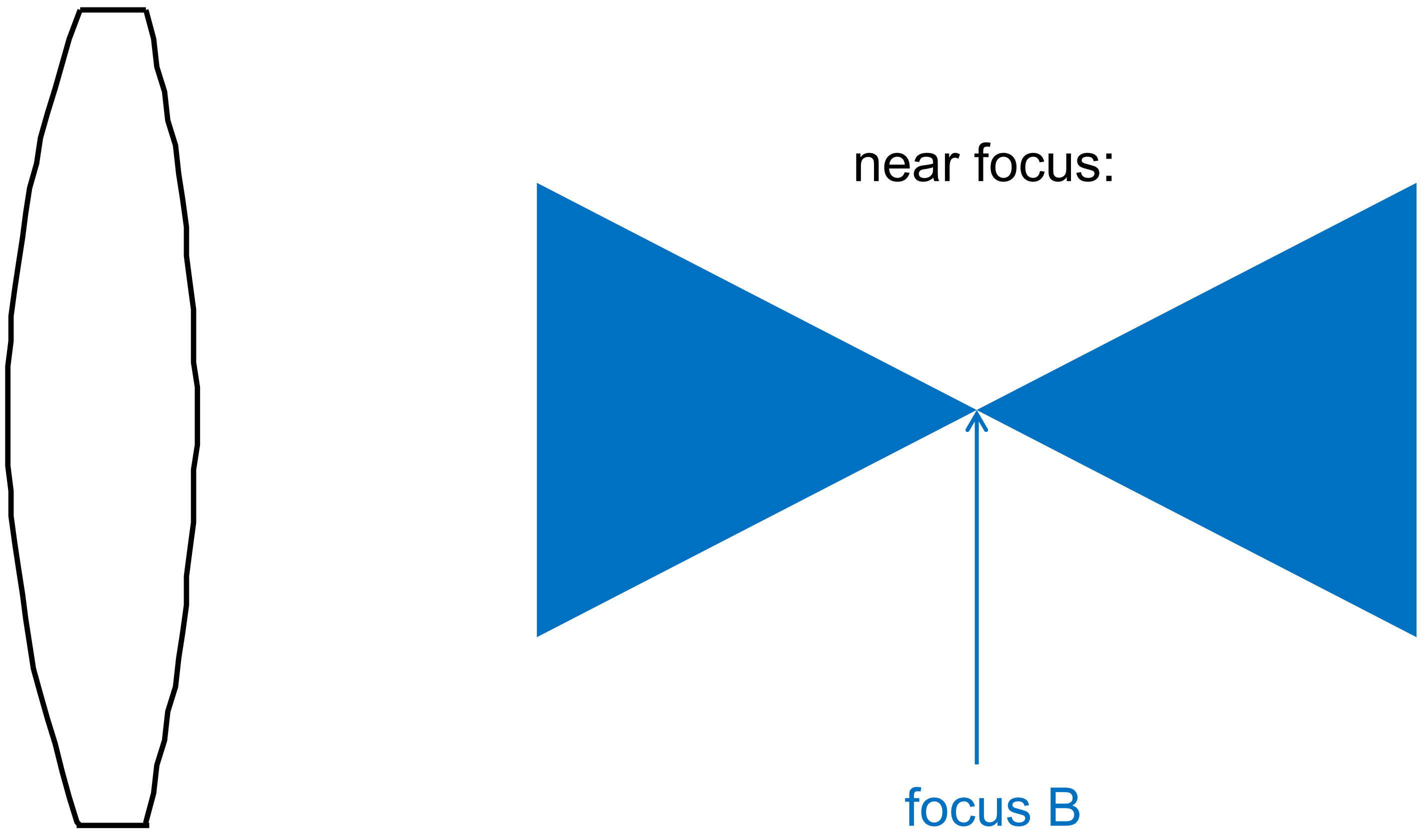
Additive color mix:

Color appearance including intensity weights:

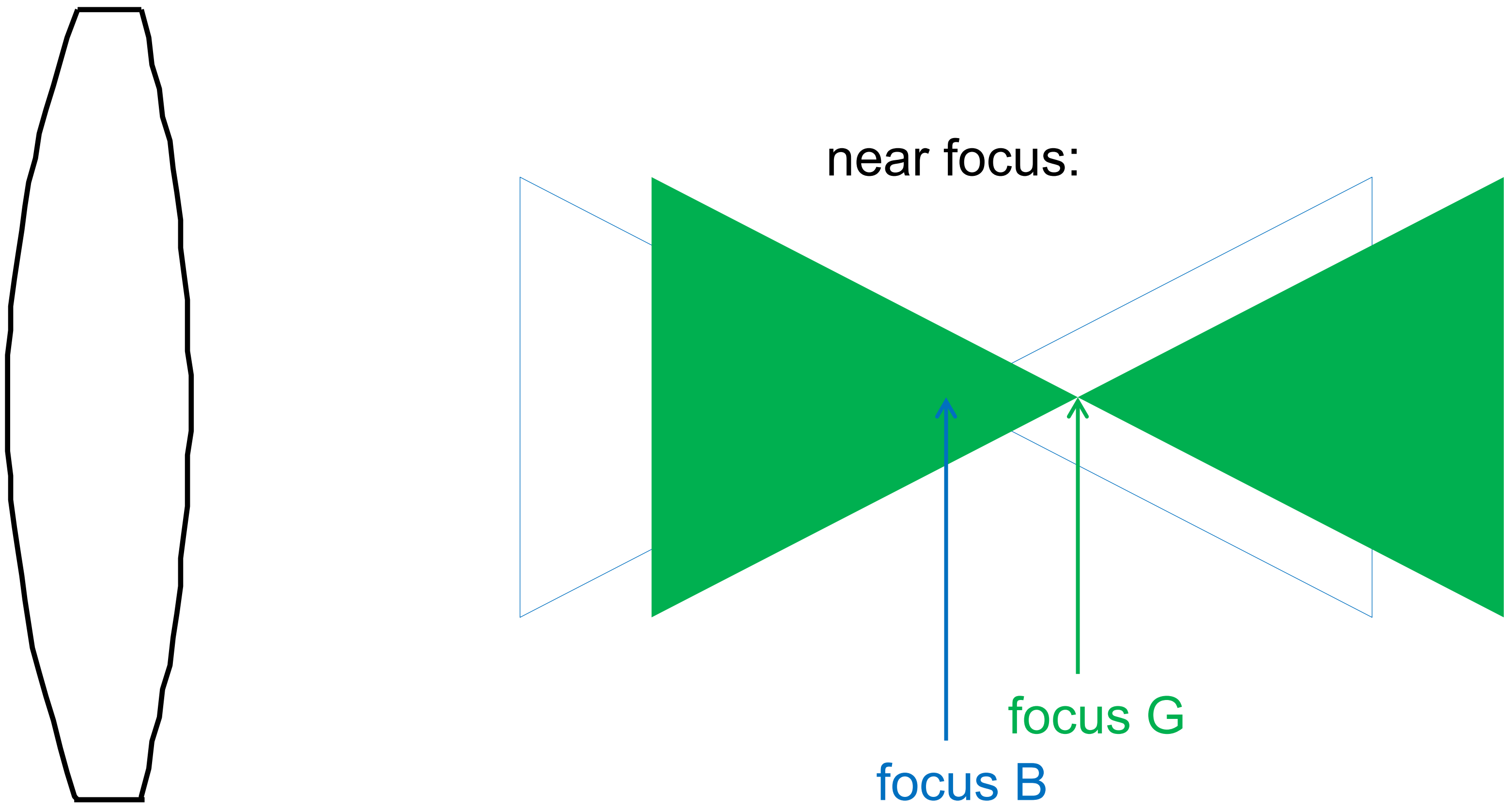


a larger spread of the spot comes together with smaller intensities & other therefore color weights!

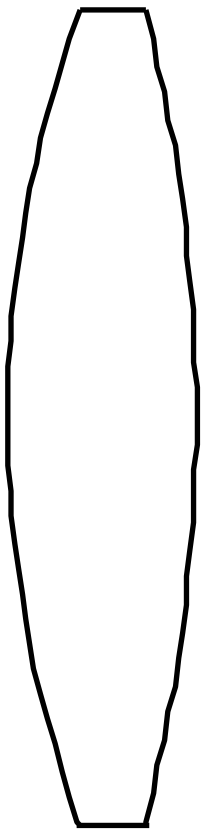
Through focus PSF with longitudinal chromatic aberration
Simplified picture

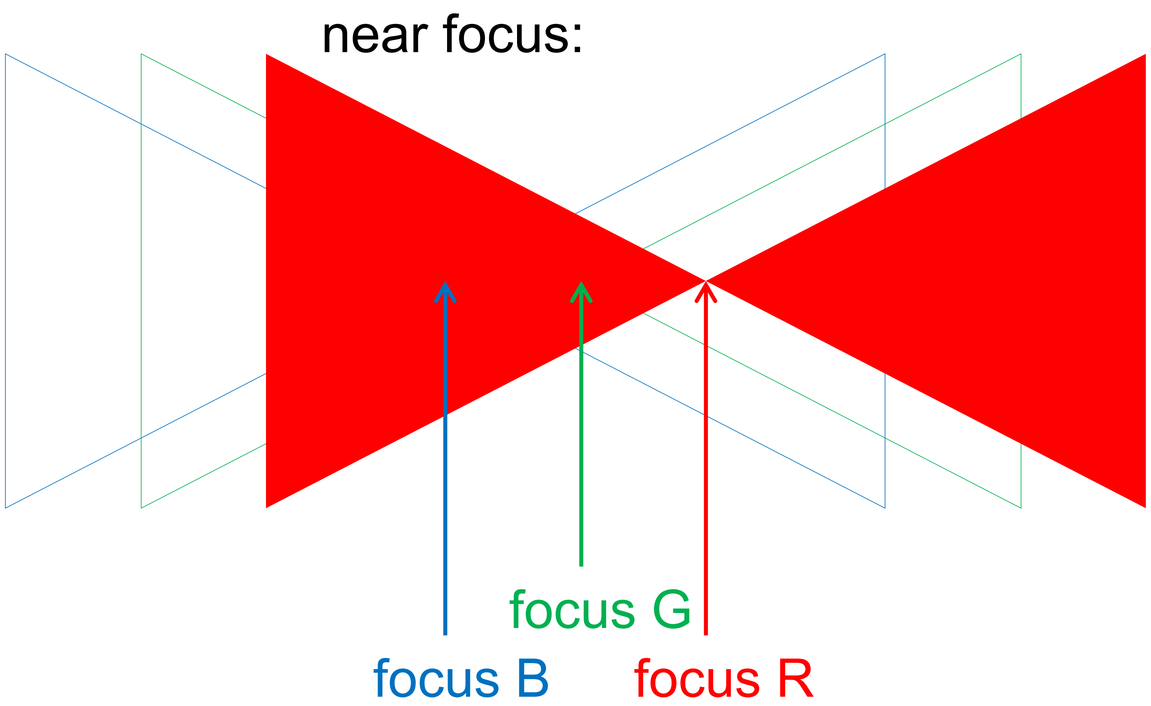


Through focus PSF with longitudinal chromatic aberration
Simplified picture

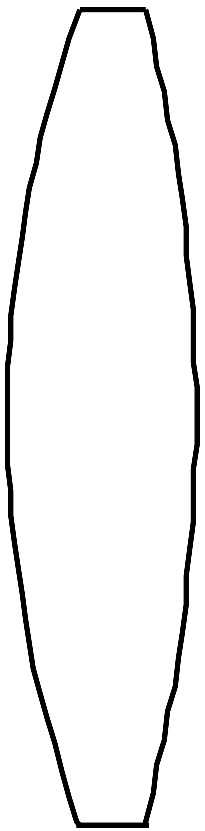


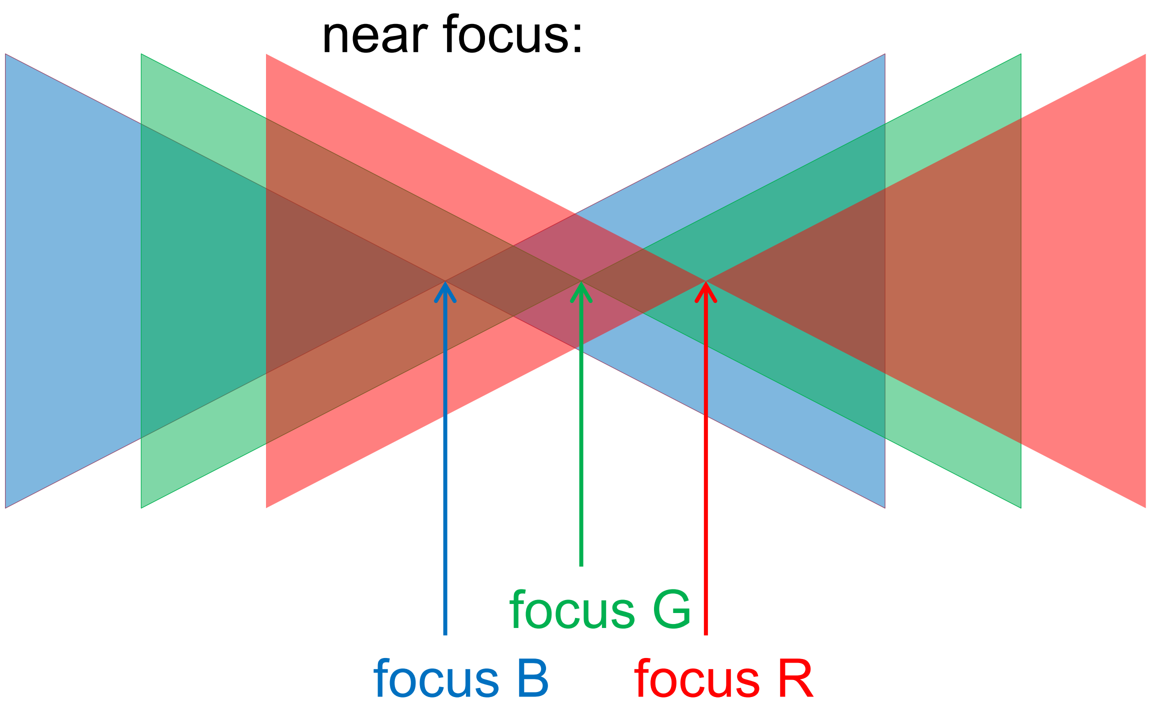
Through focus PSF with longitudinal chromatic aberration
Simplified picture



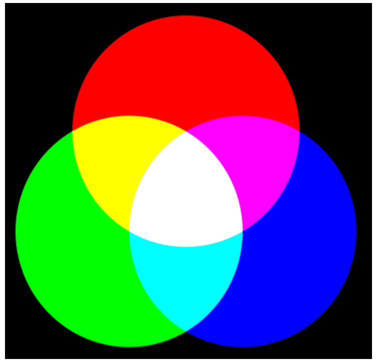


Through focus PSF with longitudinal chromatic aberration
Simplified picture

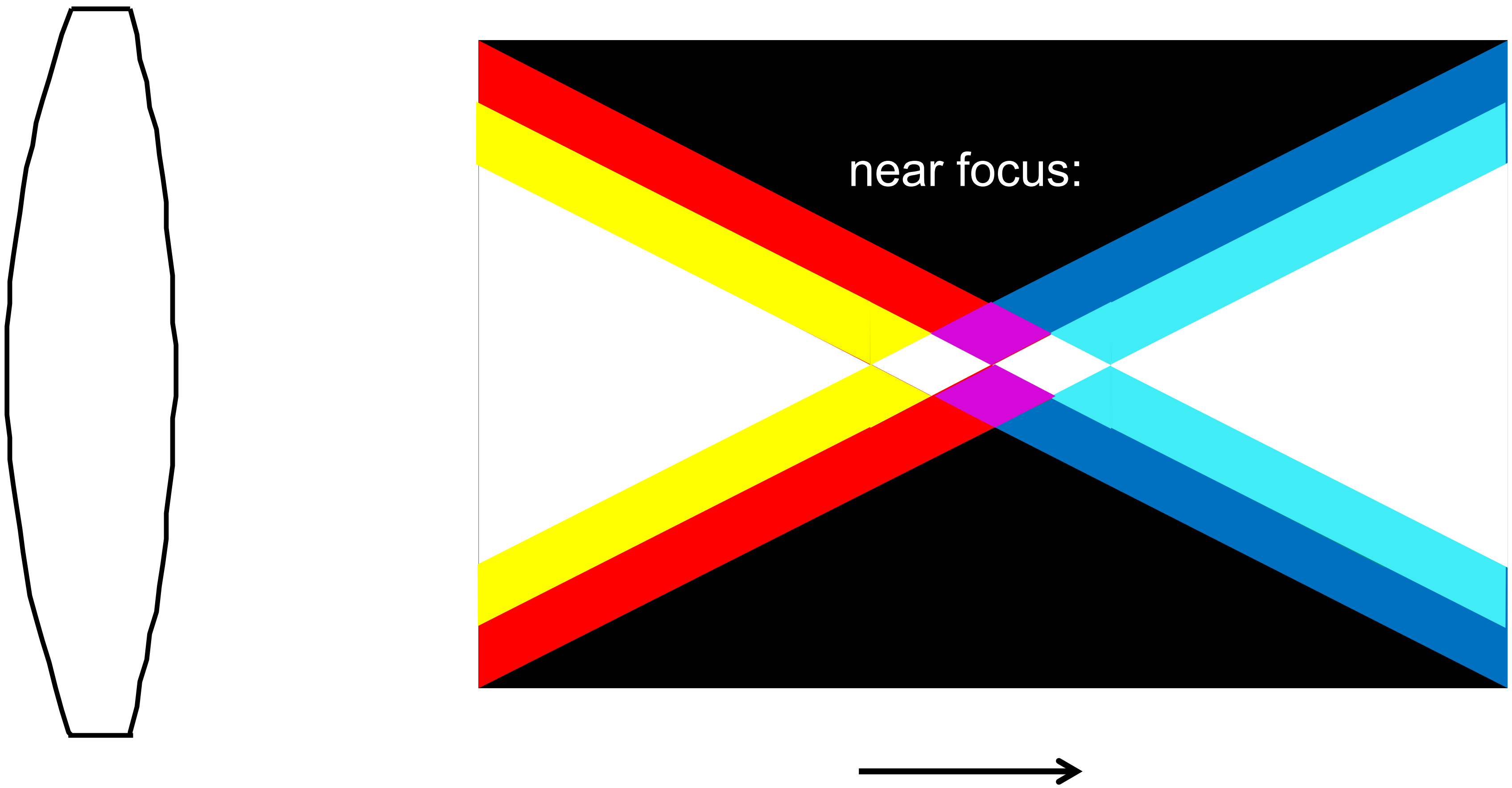




Additive color mix in those regions!

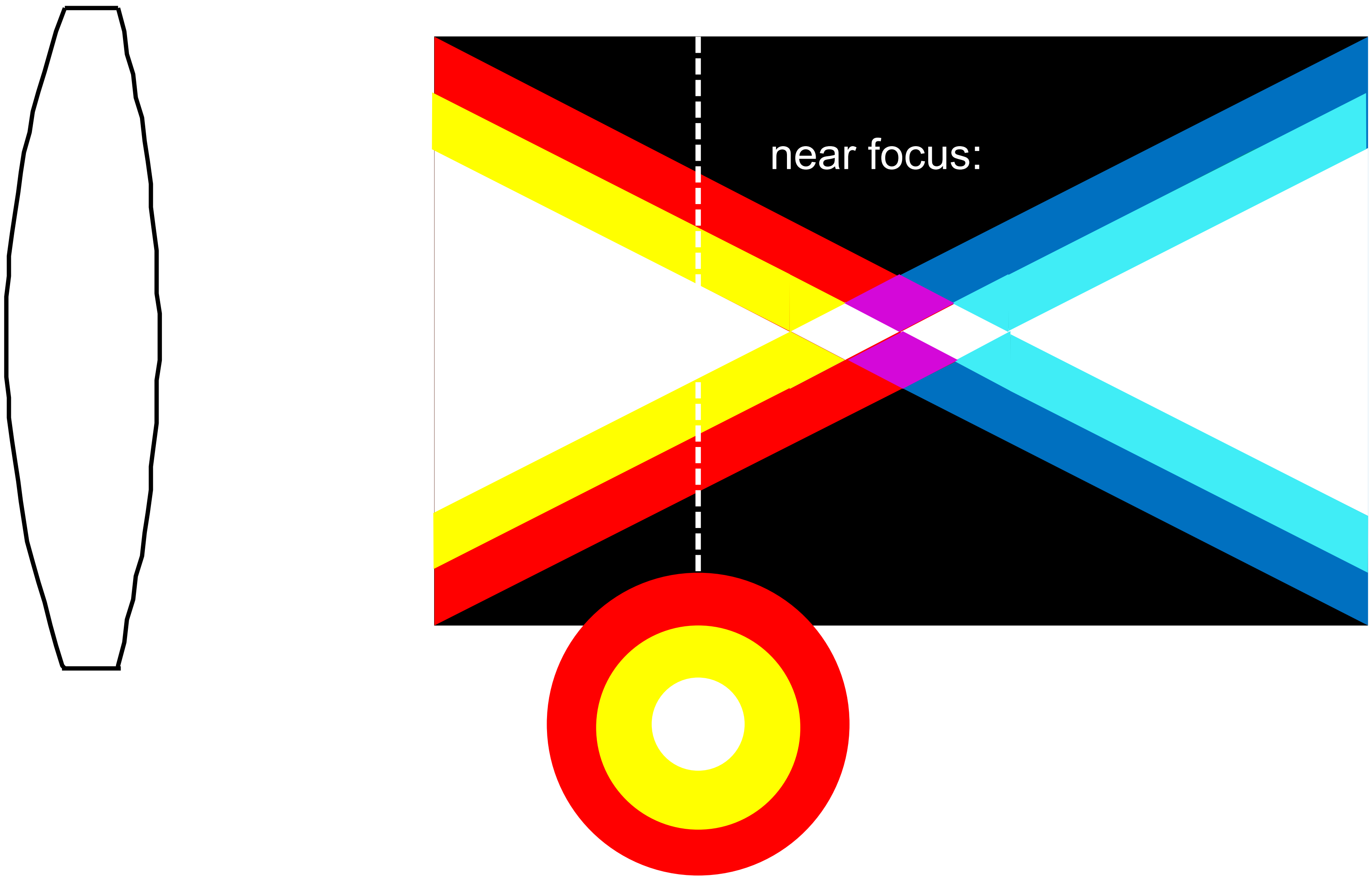


Through focus PSF with longitudinal chromatic aberration
Simplified picture



through focus

Through focus PSF with longitudinal chromatic aberration
Simplified picture



Through focus PSF with longitudinal chromatic aberration
Simplified picture

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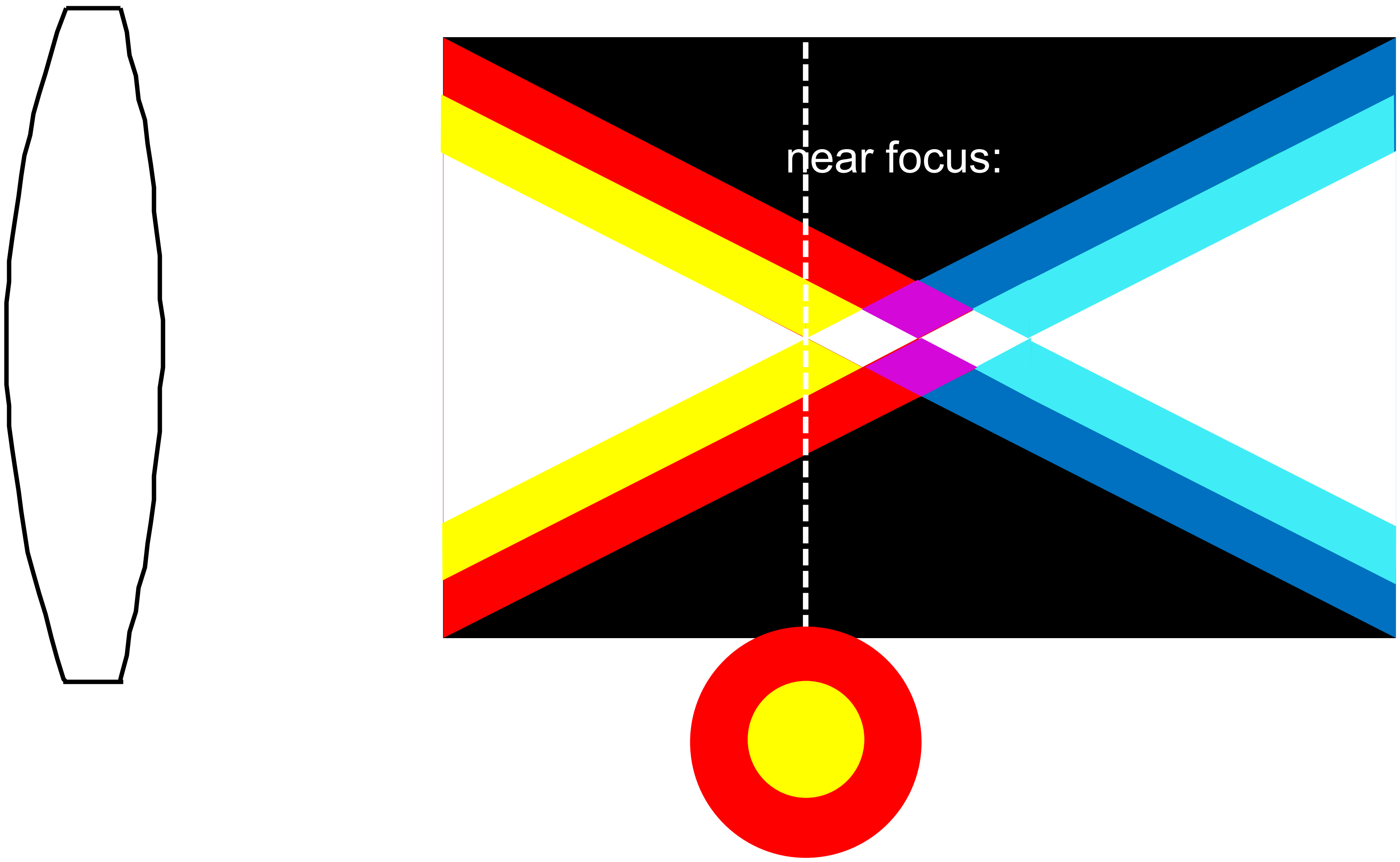

Through focus PSF with longitudinal chromatic aberration
Simplified picture

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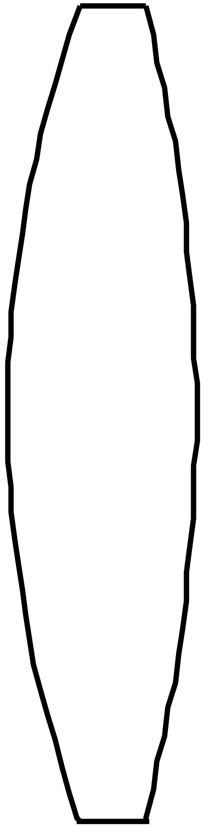
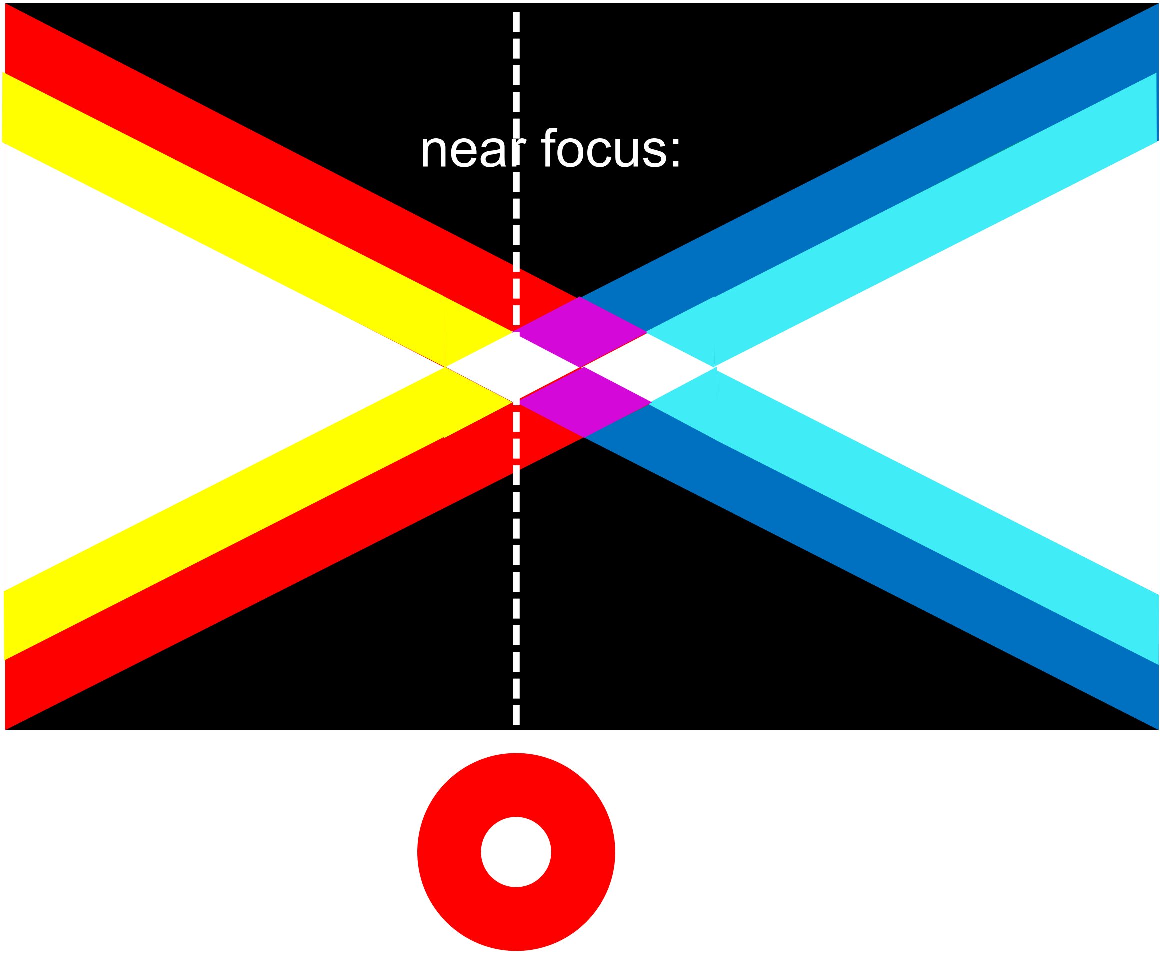
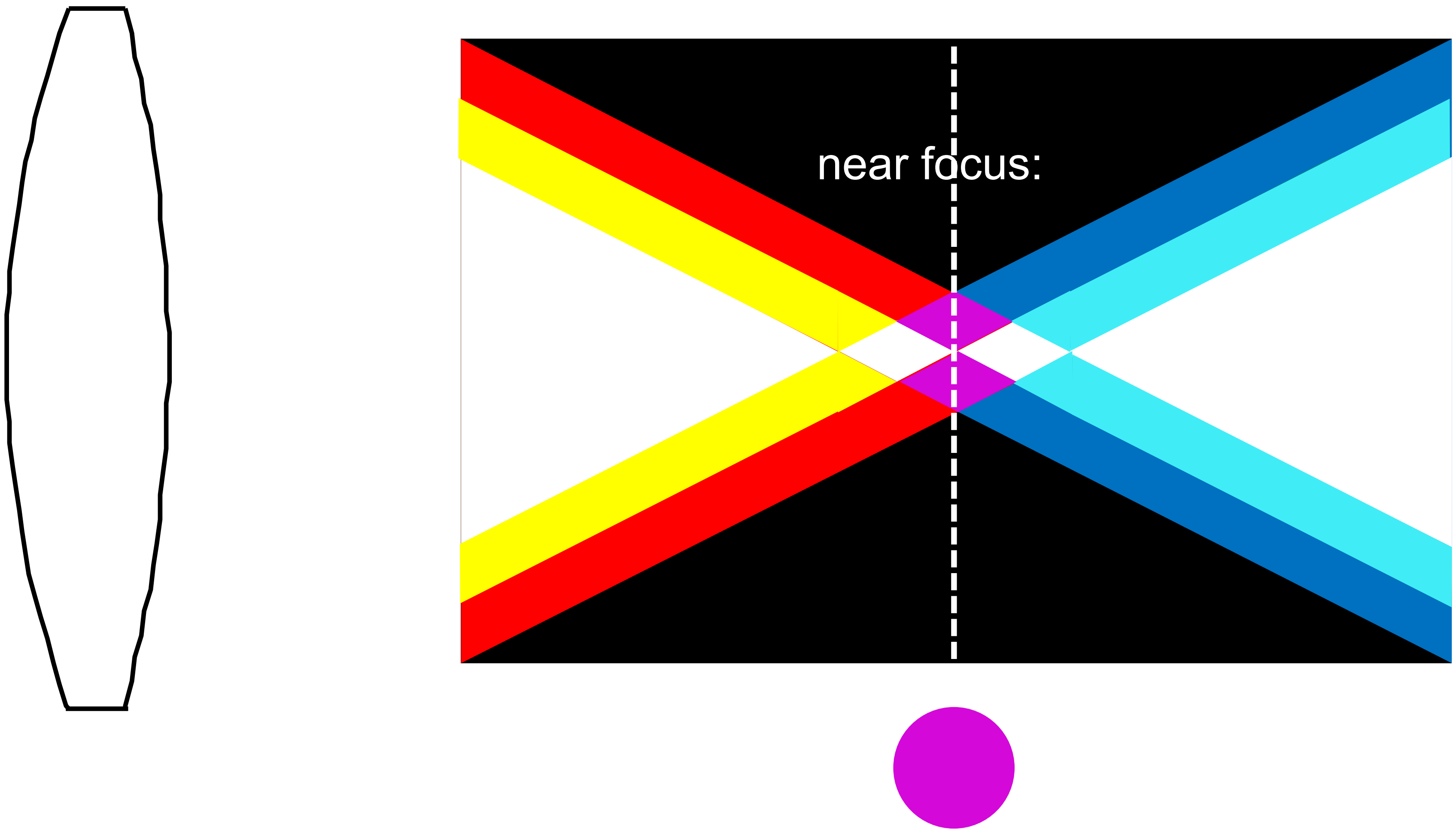
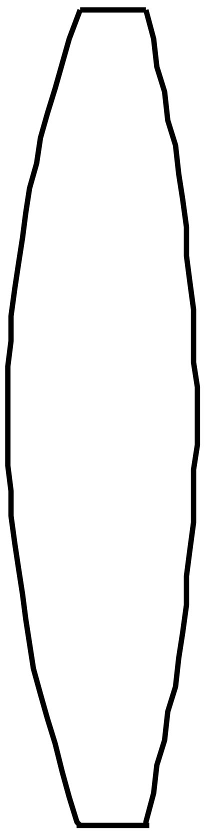


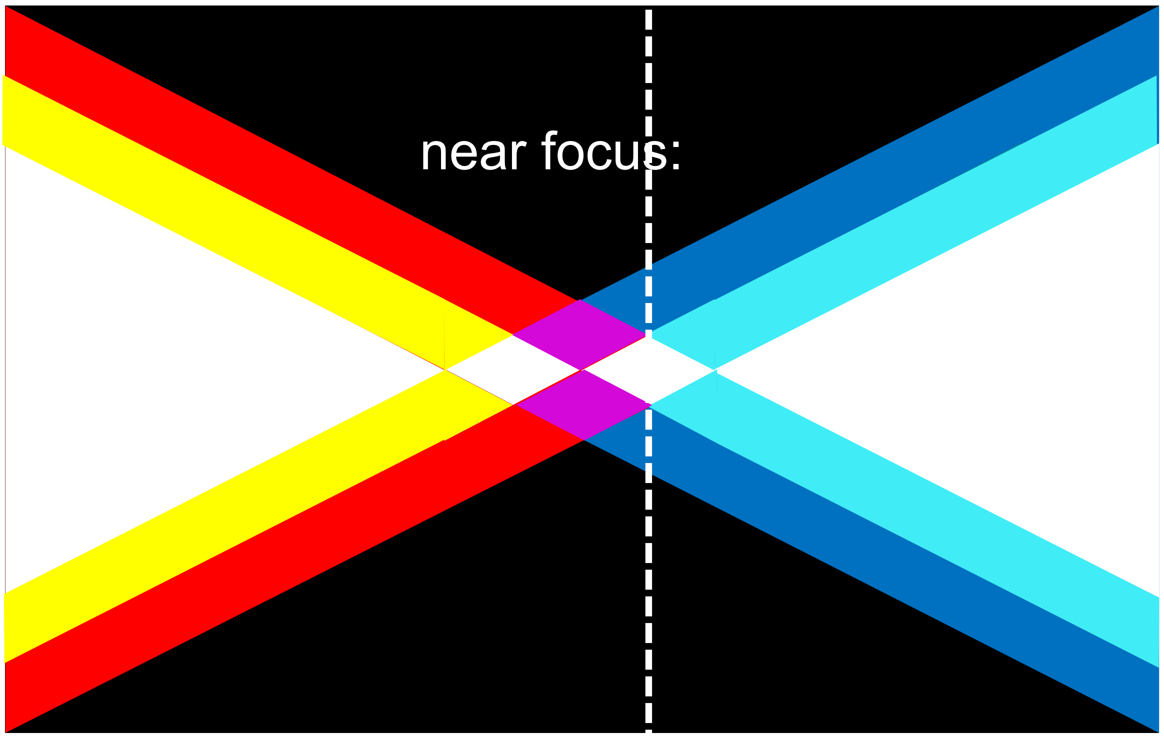


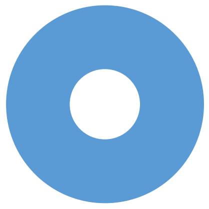
Through focus PSF with longitudinal chromatic aberration
Simplified picture



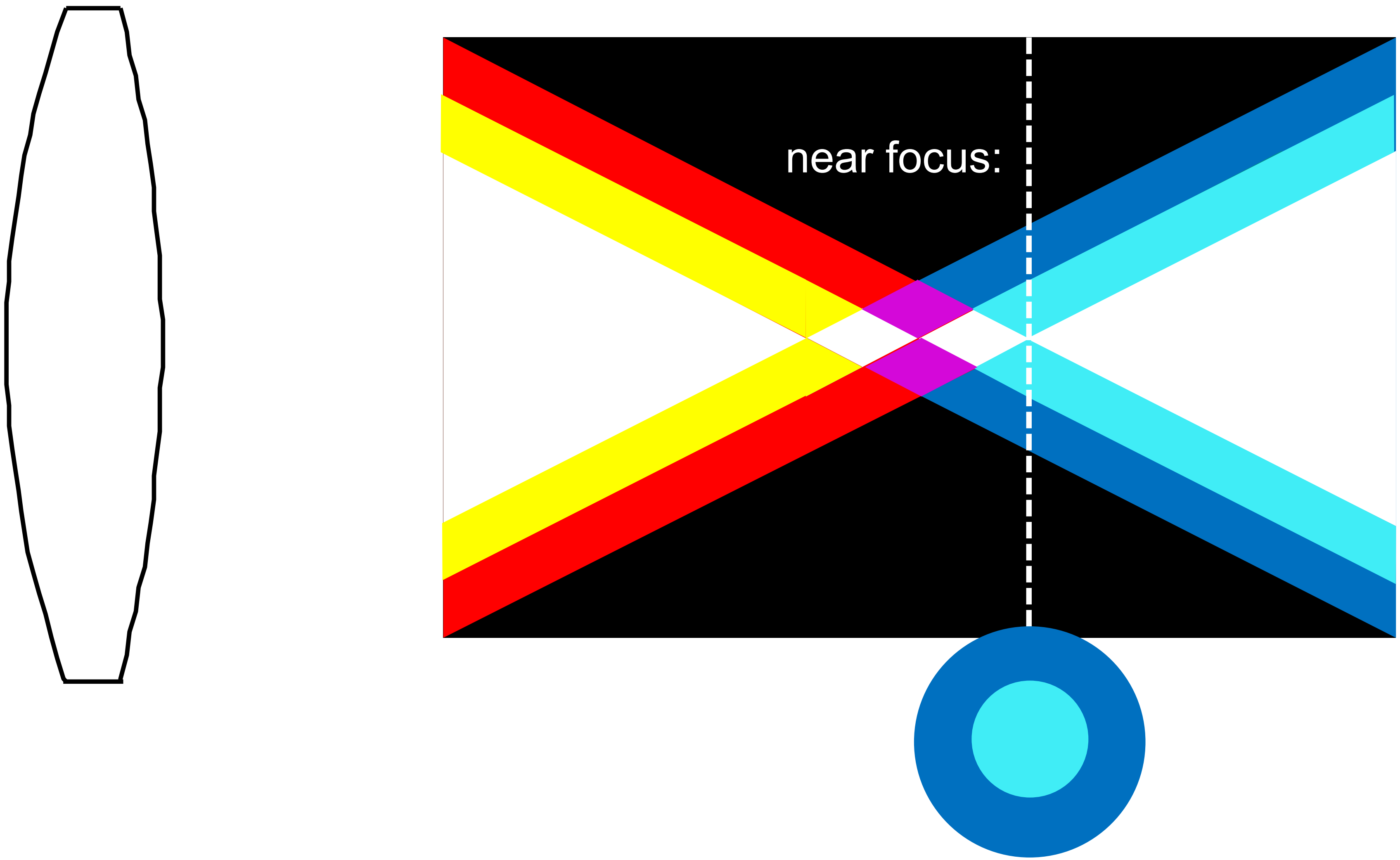
Through focus PSF with longitudinal chromatic aberration
Simplified picture







Through focus PSF with longitudinal chromatic aberration
Simplified picture



Through focus PSF with longitudinal chromatic aberration
Simplified picture

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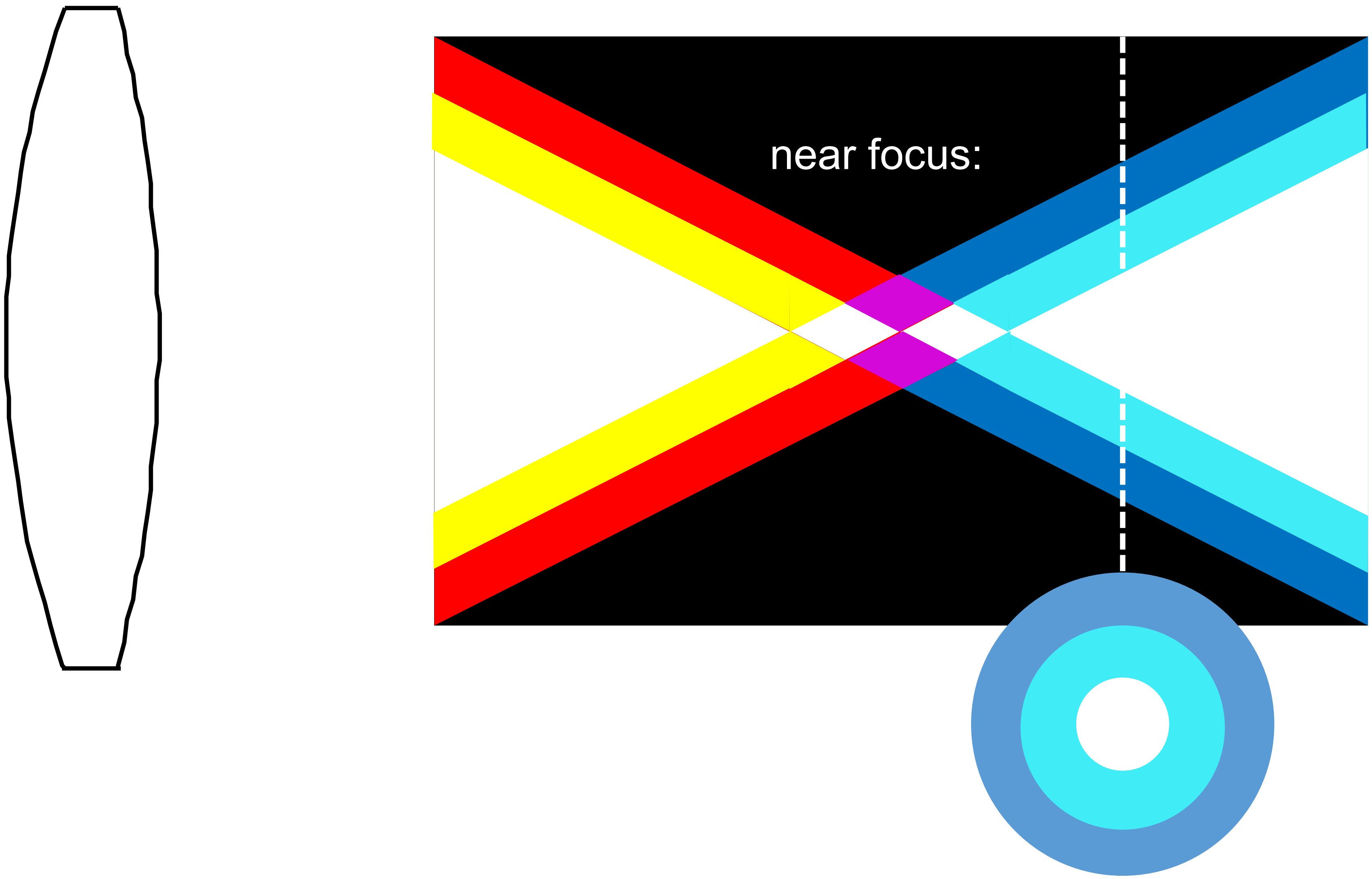
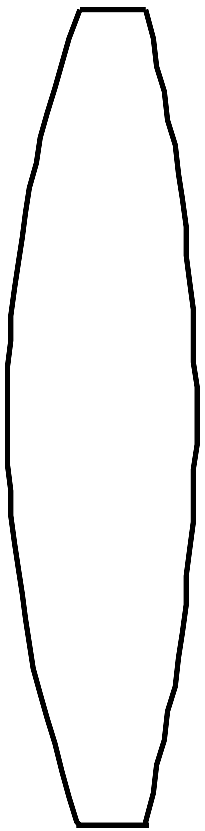
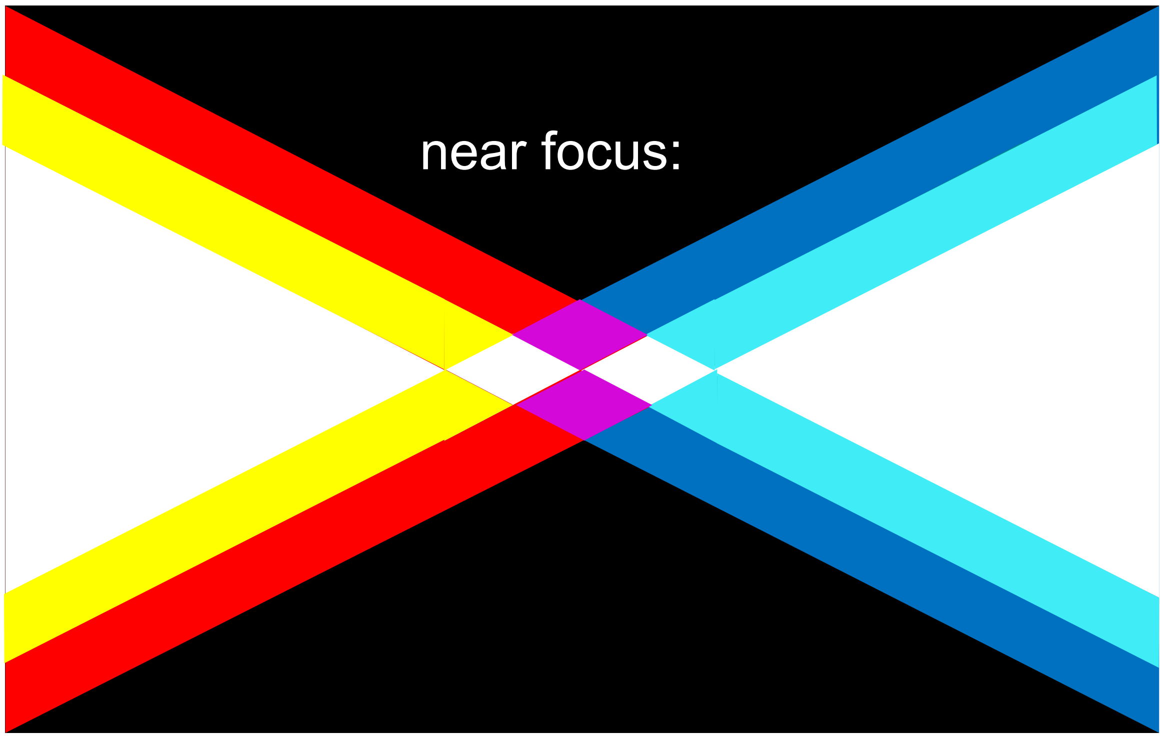


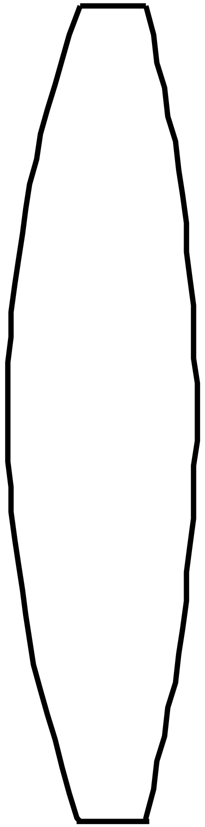
Through focus PSF with longitudinal chromatic aberration
Simplified picture

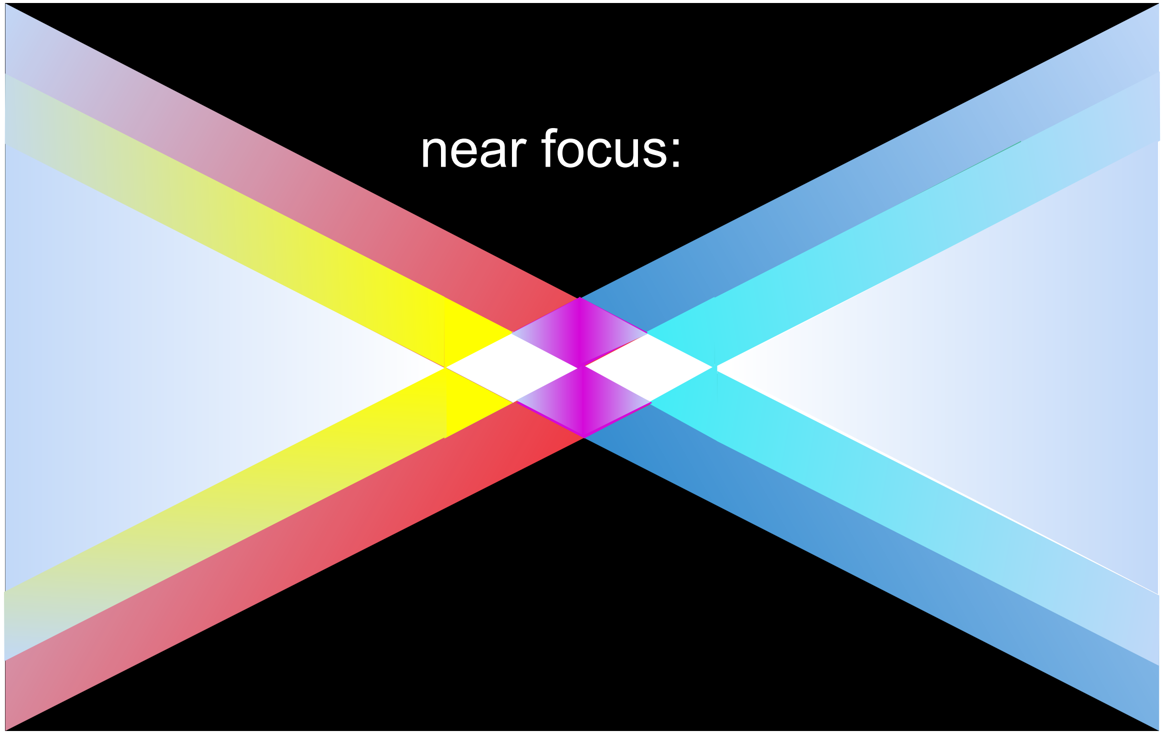




For this simplified picture we neglected that the light intensity reduces with image spot area.

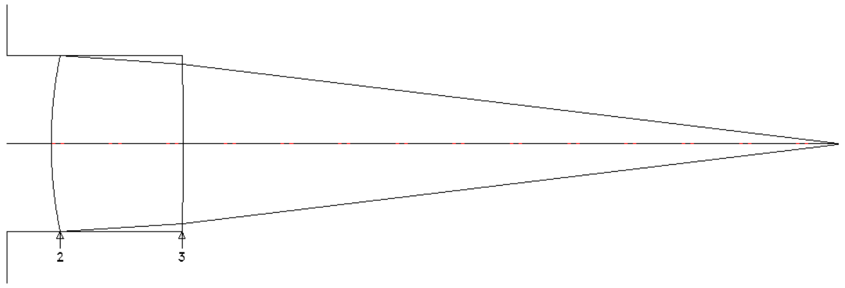
Through focus PSF with longitudinal chromatic aberration
Simplified picture



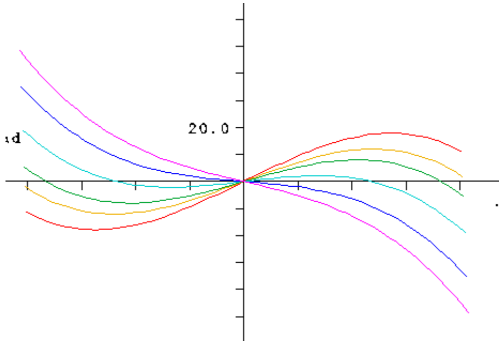


For this simplified picture we neglected that the light intensity reduces with image spot area.
The out-of-focus area becomes increasingly gray (shown above); the color transitions are more continuous with even much more hues (not shown above)!

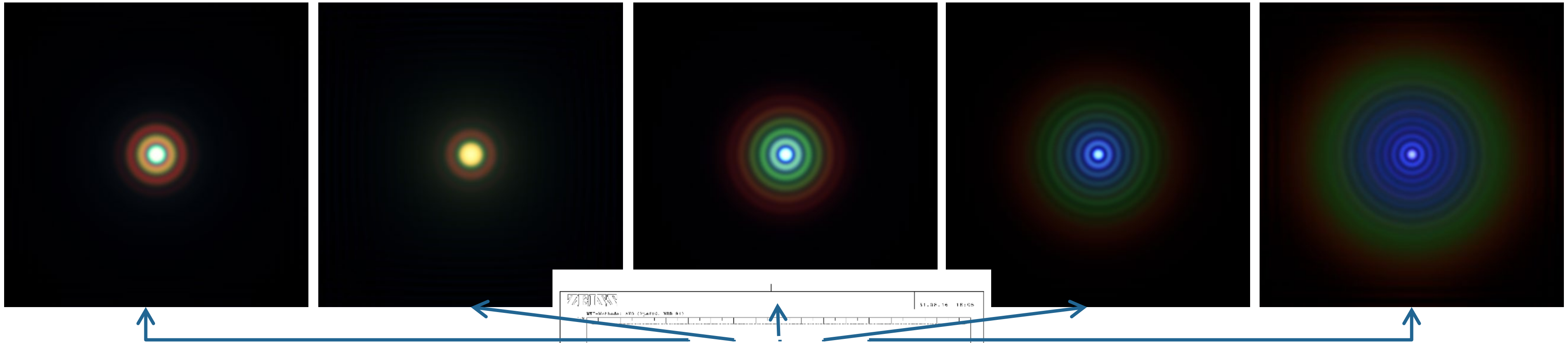
Through focus PSF with longitudinal chromatic aberration
Simulated PSF



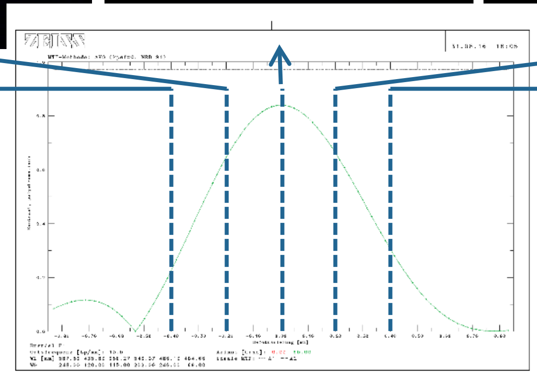
f/4-single lens (N-BK7)



Primary longitudinal aberration+ spherical aberration

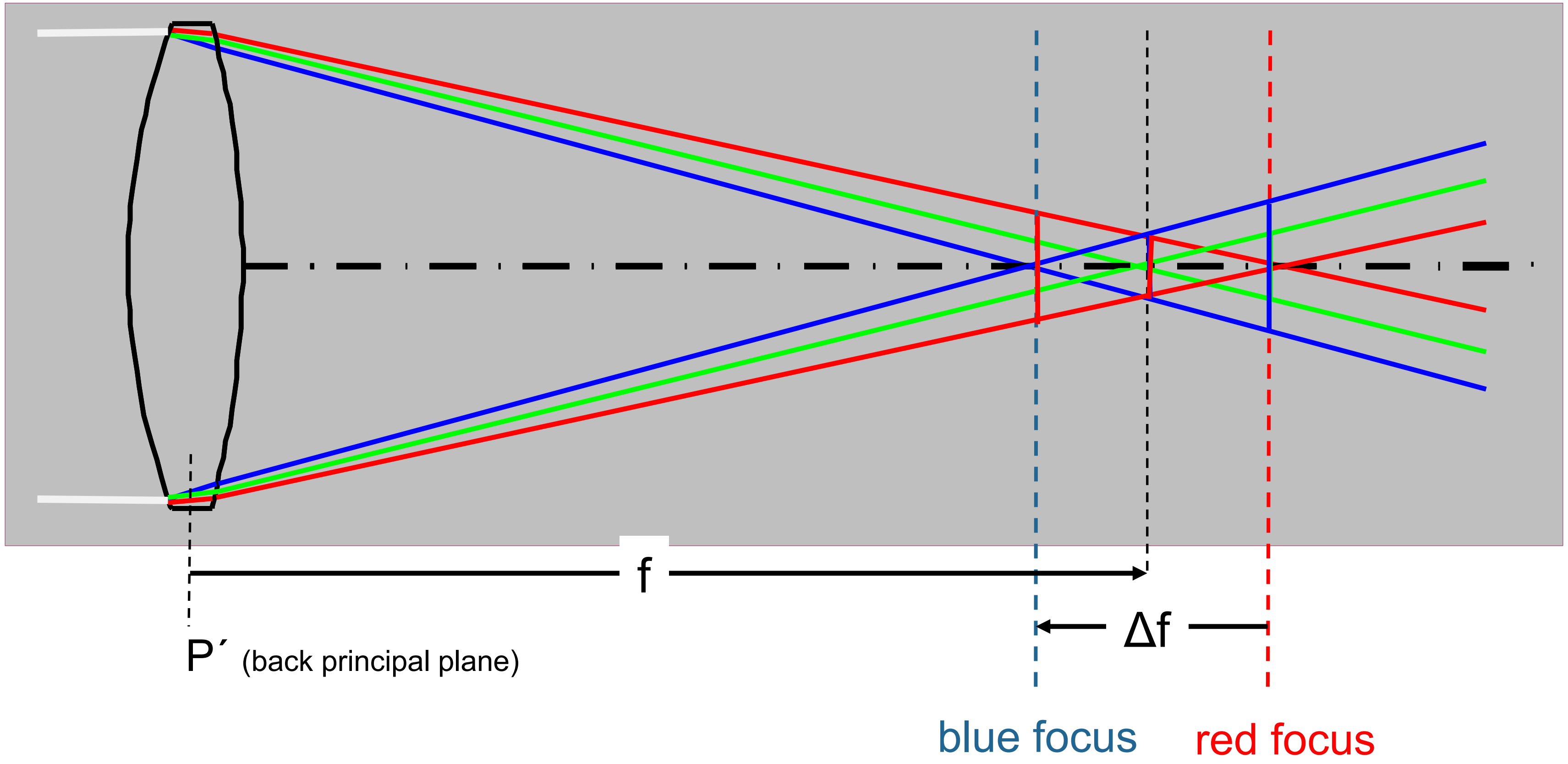


MTF through focus:



PSF through focus shows distinguished „color layers“ (red, orange, yellow, cyan, green, blue) shifted due to spherical aberration and superposed by wave-optical fringing.

Longitudinal chromatic aberration of single lens: primary spectrum



Longitudinal chromatic aberration of a single thin lens and definition of Abbe number

For two discrete wavelengths λ_1, wavelengths λ_2 the refractive power $\Phi(\lambda) = (n(\lambda) - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ becomes

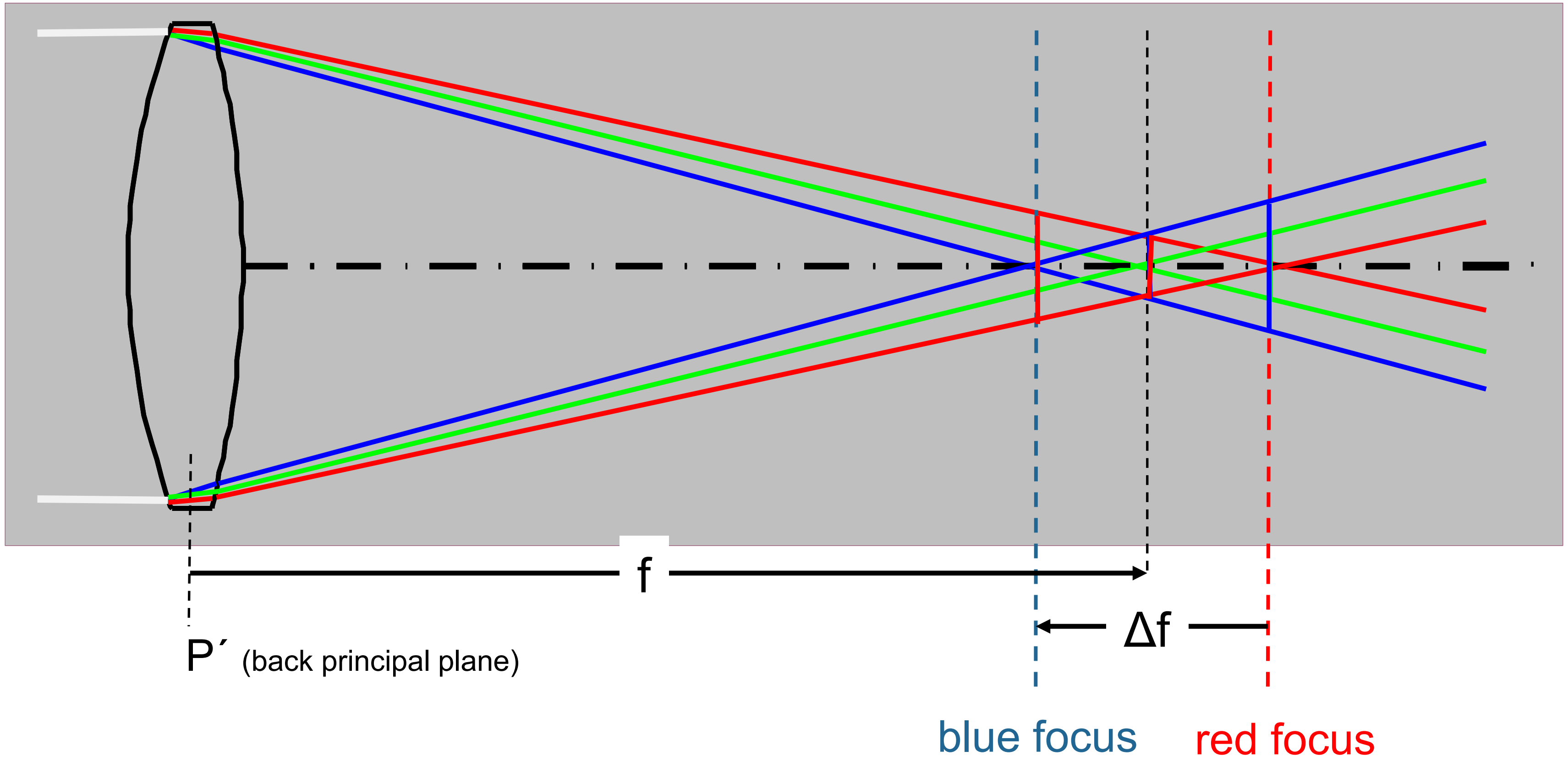
$$\Phi(\lambda_1) - \Phi(\lambda_2) = [n(\lambda_1) - n(\lambda_2)]\, \Delta c \;= \frac{n(\lambda_1) - n(\lambda_2)}{n(\lambda_3) - 1} \underbrace{(n(\lambda_3) - 1)\Delta c}_{=\Phi(\lambda_3)}$$

The relative refractive power deviation $\Delta\Phi/\Phi$ is therefore represented by the expression $\frac{[n(\lambda_1) - n(\lambda_2)]}{n(\lambda_3) - 1}$. The wavelength λ_3 is by definition between wavelengths λ_1 and λ_2. Camera lenses common **Abbe number** $\boldsymbol{\nu_d}$, where d the yellow Helium line at 587nm: difference between the spectral lines C = 656nm (red) and F = 486nm (blue): $\frac{1}{\nu_d} = \frac{n(F) - n(C)}{n(d) - 1}$ or $\nu_d = \frac{n_d - 1}{n_F - n_C}$.

So, relative refractive power differences directly scales with 1/v: $\frac{\Delta\Phi}{\Phi} = \frac{1}{\nu}$.

For the focal length, denoting $f(\lambda_1) = f_1$ etc:, we have $\frac{1}{f_1} - \frac{1}{f_2} = \frac{f_2 - f_1}{f_1 f_2} = \underbrace{\frac{n_1 - n_2}{n_3 - 1}}_{1/\nu} \underbrace{(n_3 - 1)\Delta c}_{1/f_2}$ one obtains the same form (but different sign) $f_1 - f_2 = -\frac{1}{\nu}\frac{f_1 f_2}{f_3} \approx -\frac{1}{\nu}\frac{1}{f}$ or $\frac{\Delta f}{f} = -\frac{1}{\nu}$

Longitudinal chromatic aberration of single lens: primary spectrum

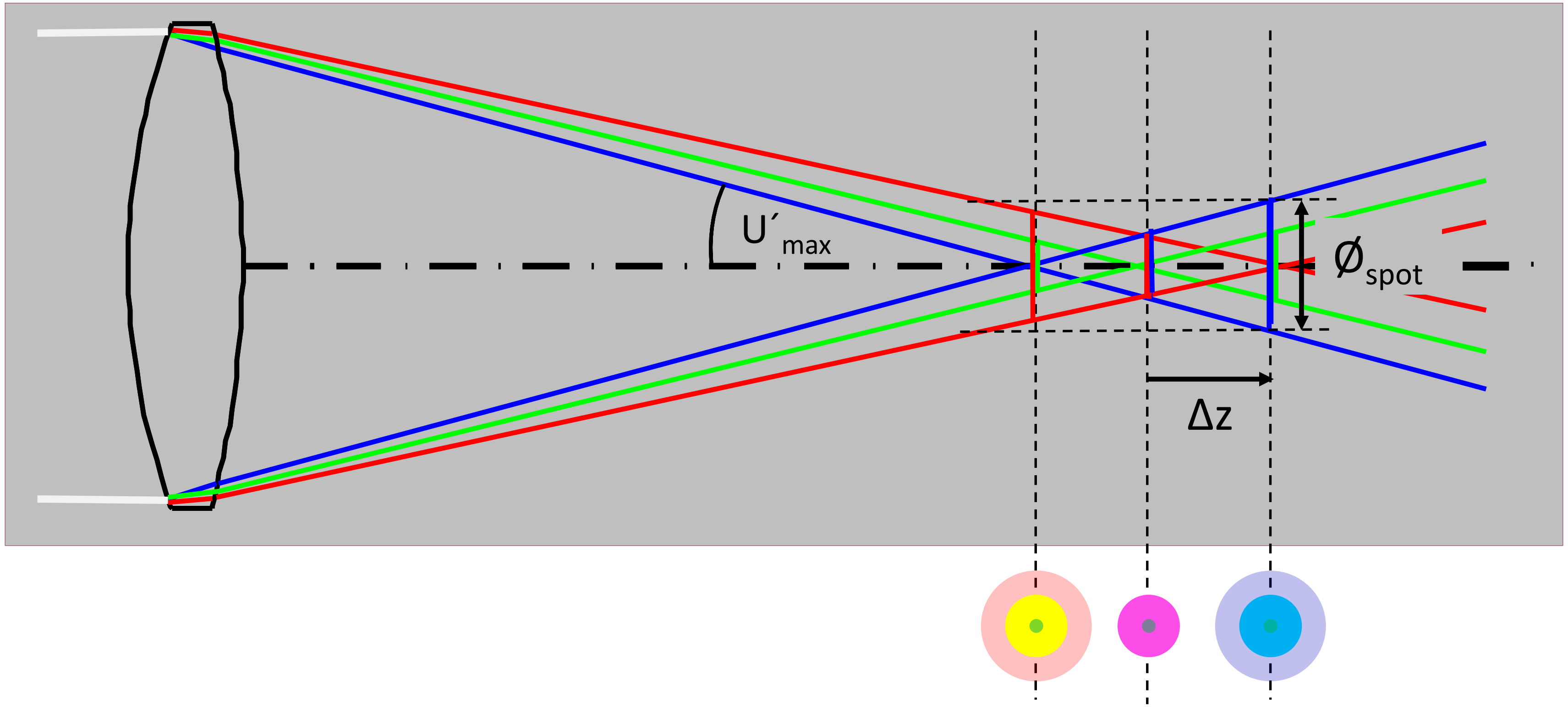


$$\frac{\Delta f}{f} = -\frac{1}{\nu}$$

Abbe numbers visible spectrum: ν = 20 … 95,

$\rightarrow \frac{\Delta f}{f} =$ **1% … 5%**.

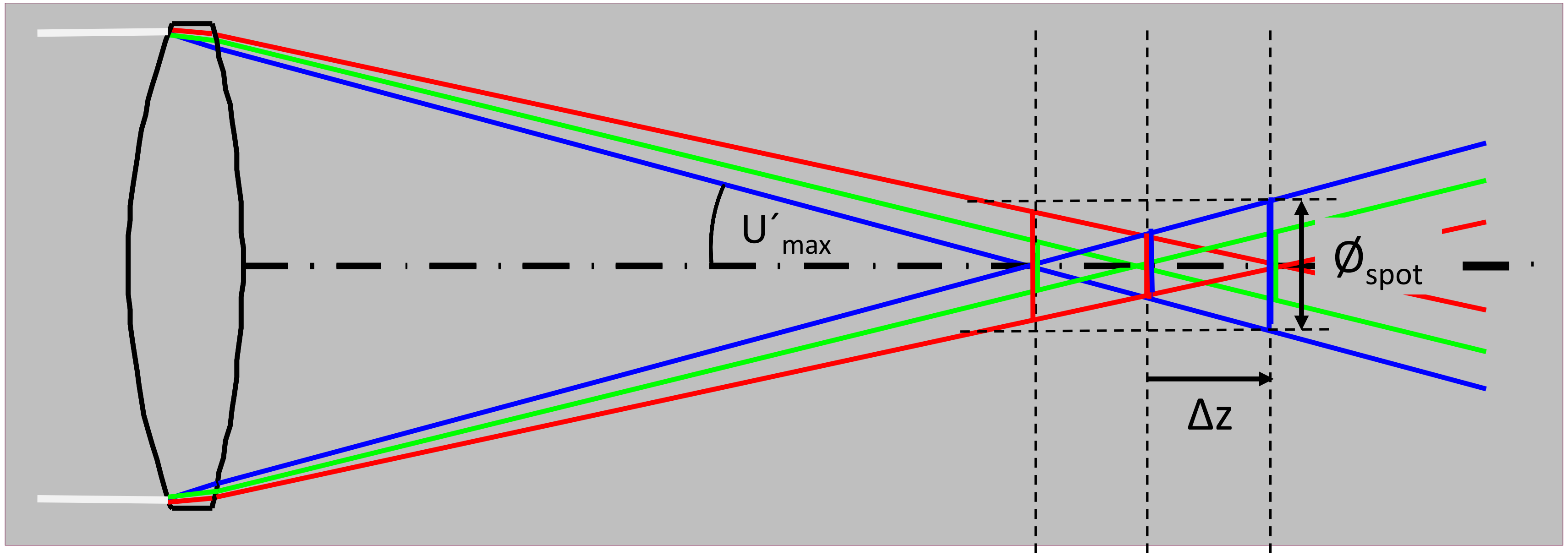
Longitudinal chromatic aberration of single lens: primary spectrum



The spot diameter (of the PSF) depends on the aperture (f-number defined as $K = \frac{1}{2NA'} = \frac{1}{2\sin U_{max}'}$)

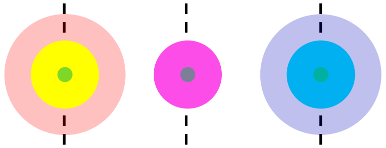
$$Ø_{spot} = 2\ |\Delta z|\ \tan\vartheta'_{\max} \approx 2NA'\ |\Delta z| = \frac{|\Delta z|}{K}$$

Longitudinal chromatic aberration of single lens: primary spectrum



For a single lens with a dispersion of Abbe-number $v_e = (n_e - 1)/(n_{F´} - n_{C´})$, the spectral lines referring to following wavelengths: C´= 643.8nm (red), e = 546.1nm (green), F´= 480nm (blue) the relative defocus is:

$$\frac{\Delta z}{f} = \frac{\Delta f}{f} = \frac{1}{\nu_e} \approx 1.5\% \ \ldots . \ 5\%$$



The spot diameter "at red focus" becomes:

$$\O_{spot} \approx \frac{|\Delta z|}{K} \approx (1.5\% \ \ldots \ 5\%) \ \frac{f}{K} = (1.5\% \ \ldots \ 5\%) \ \O_{EP}$$

Even for a telescope of moderate size, $Ø_{EP}$=50mm, the spot size is extremely large, ca. $Ø_{spot}$=1mm. (The Airy-spot diameter of a f/20-lens with f=1000mm is only 20µm, that is 50x smaller.)

Alternative Abbe diagram: n versus 100/v

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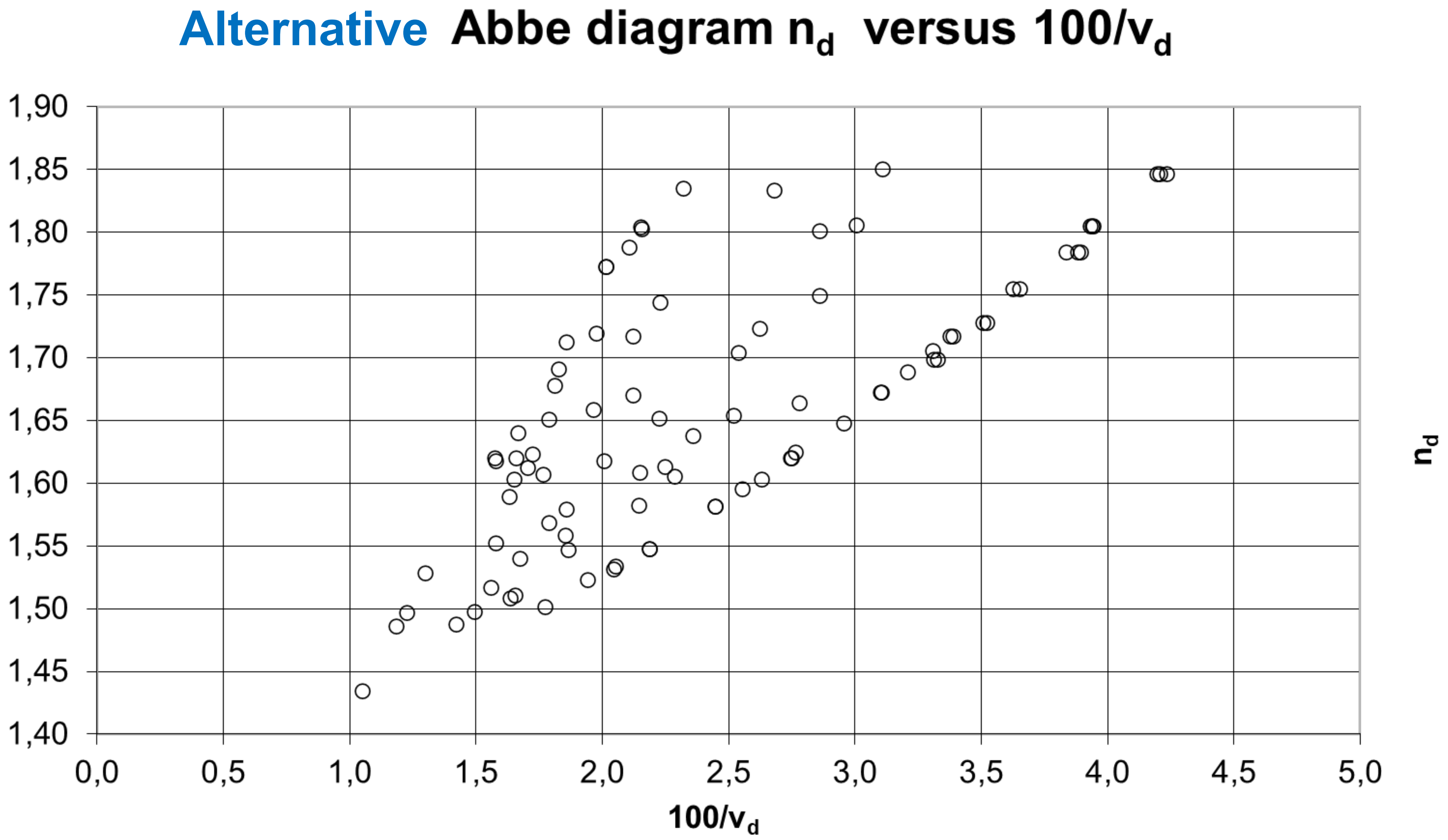
$$\frac{\Delta f}{f} = -\frac{1}{\nu}$$



Alternative Abbe diagram: n versus 100/v

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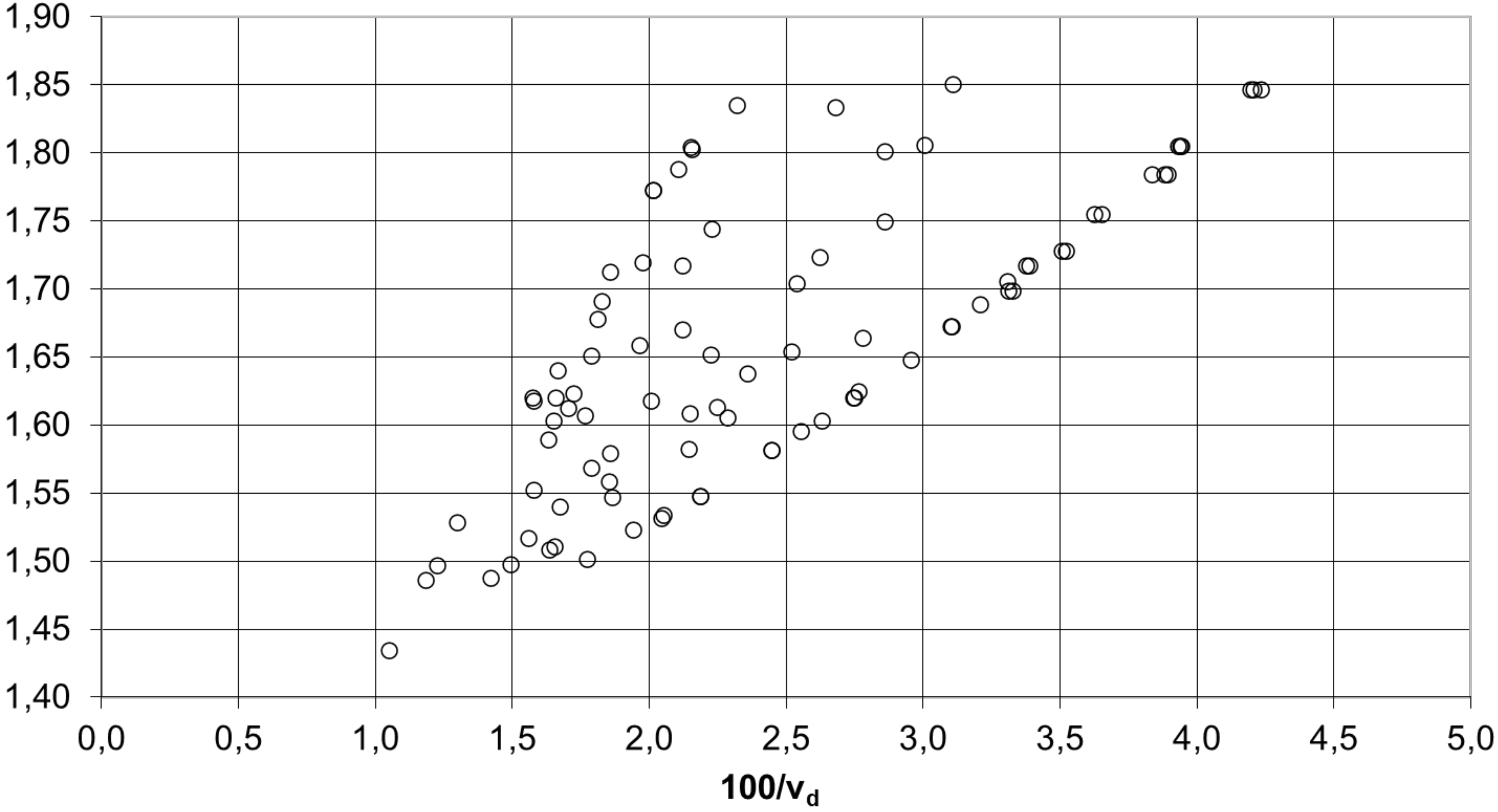
Alternative Abbe diagram n_d versus $100/v_d$

$$\frac{\Delta f}{f} = -\frac{1}{\nu}$$



Advantages of alternative diagram:

- Direct interpretation: dispersion parameter corresponds to relative longitudinal focus shift
- Linear scale for dispersion
- Contains dispersion value zero, allows practical comparison with IR
- Achromatic condition on linear scale

Alternative Abbe diagram: n versus 100/v; including optical plastics

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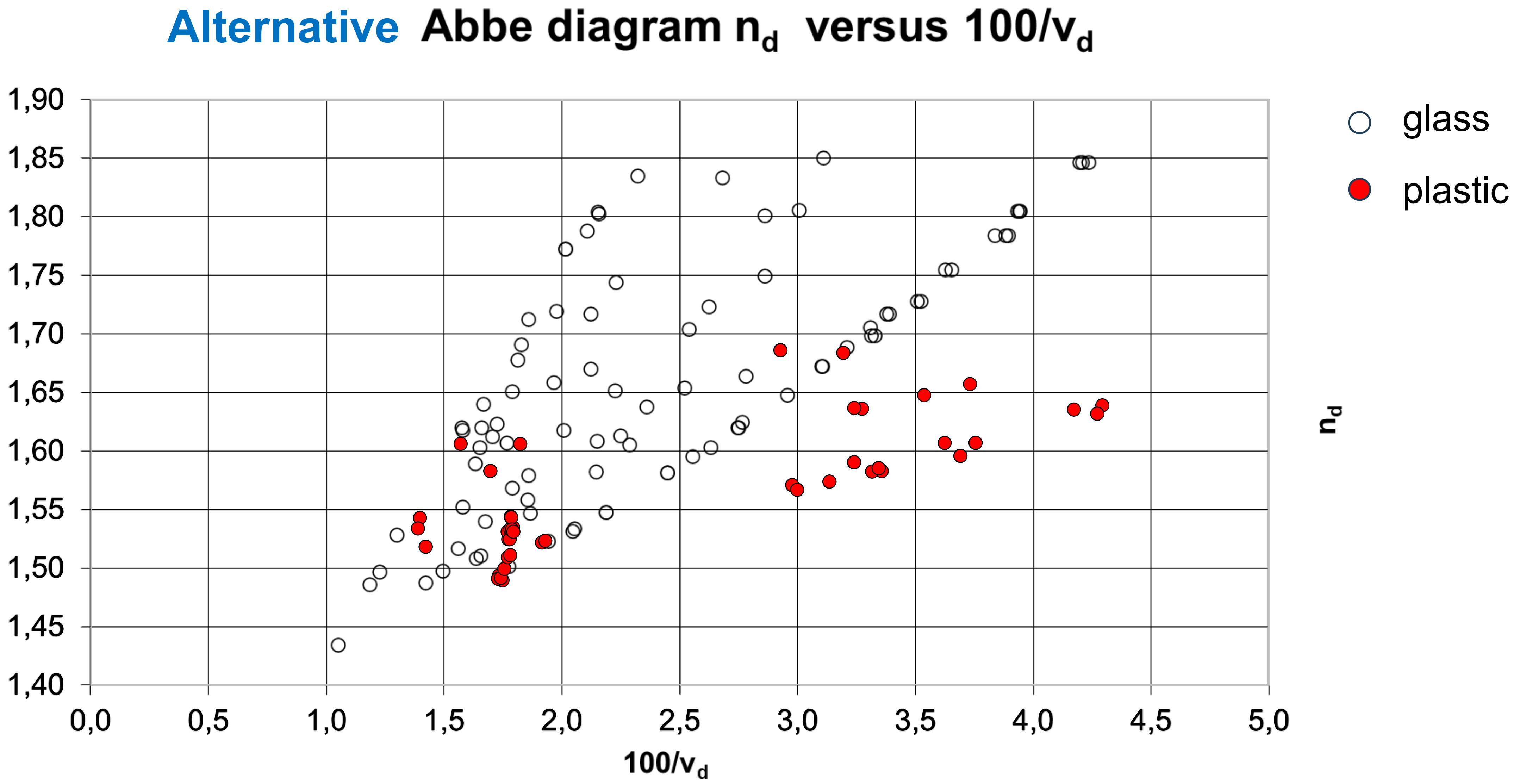

Alternative Abbe diagram: n versus 100/v

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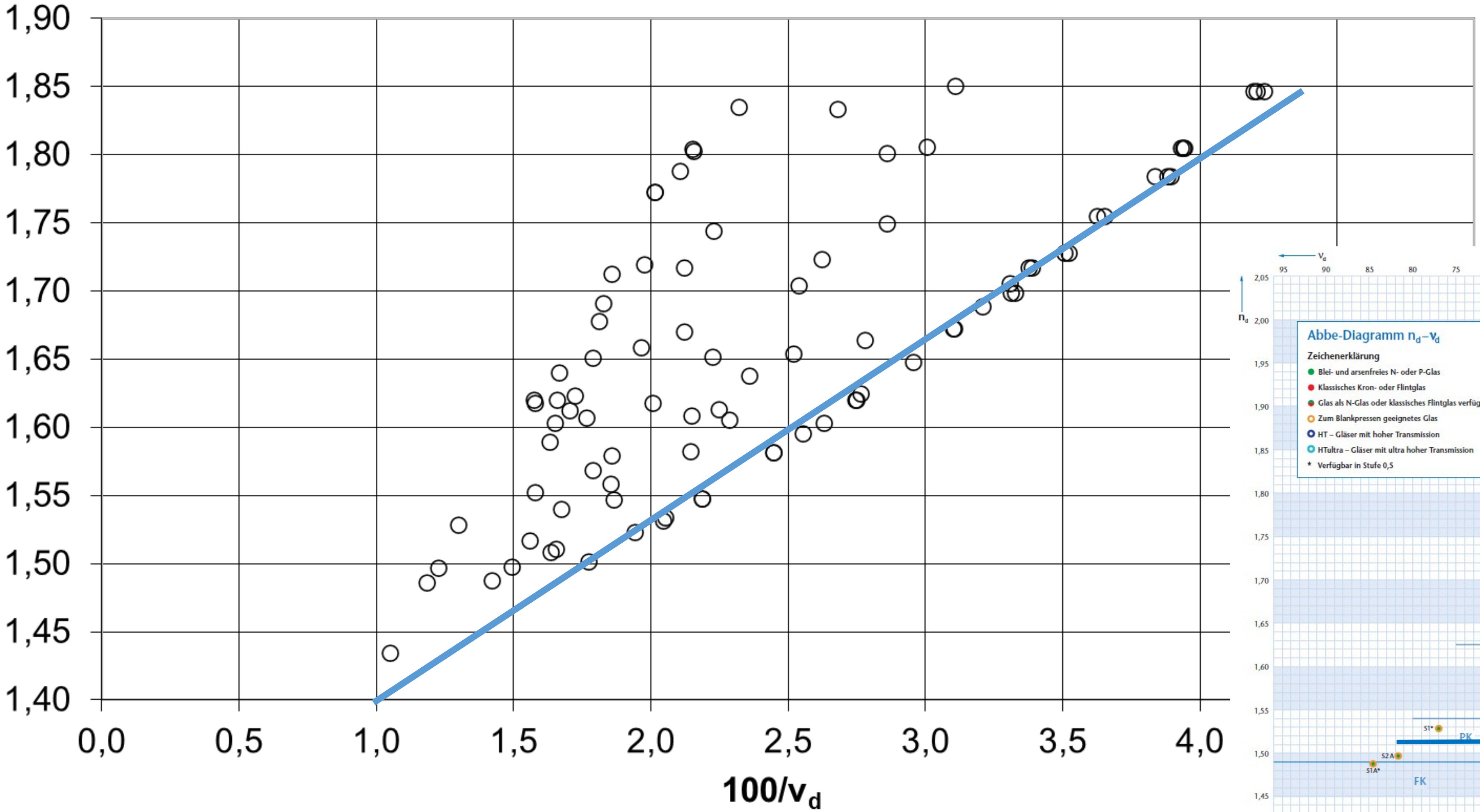
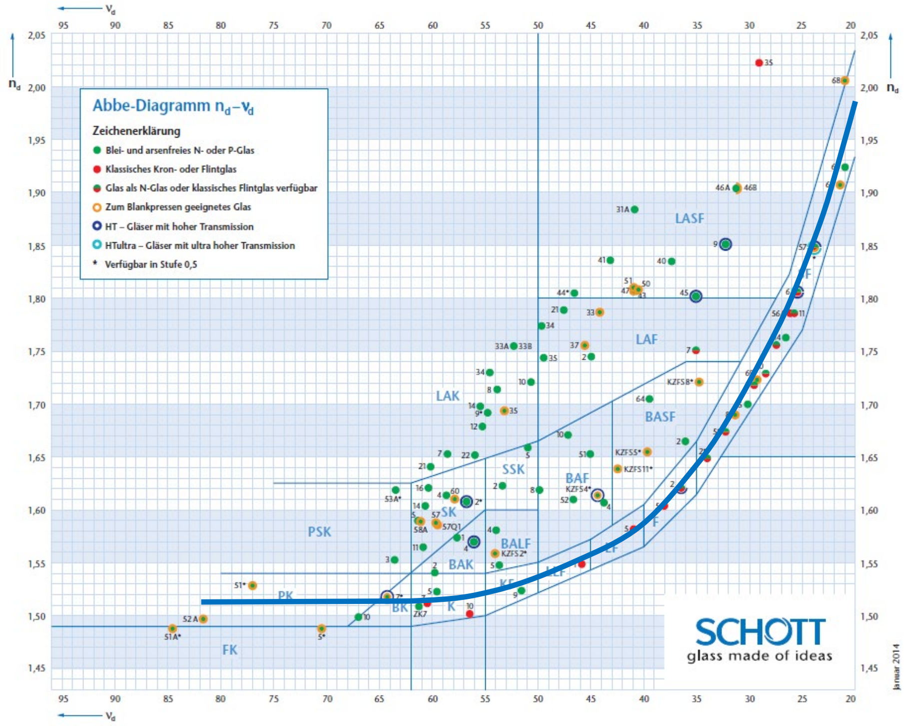
Alternative Abbe diagram n_d versus $100/v_d$



„Iron line“ in this representation is nearly linear!

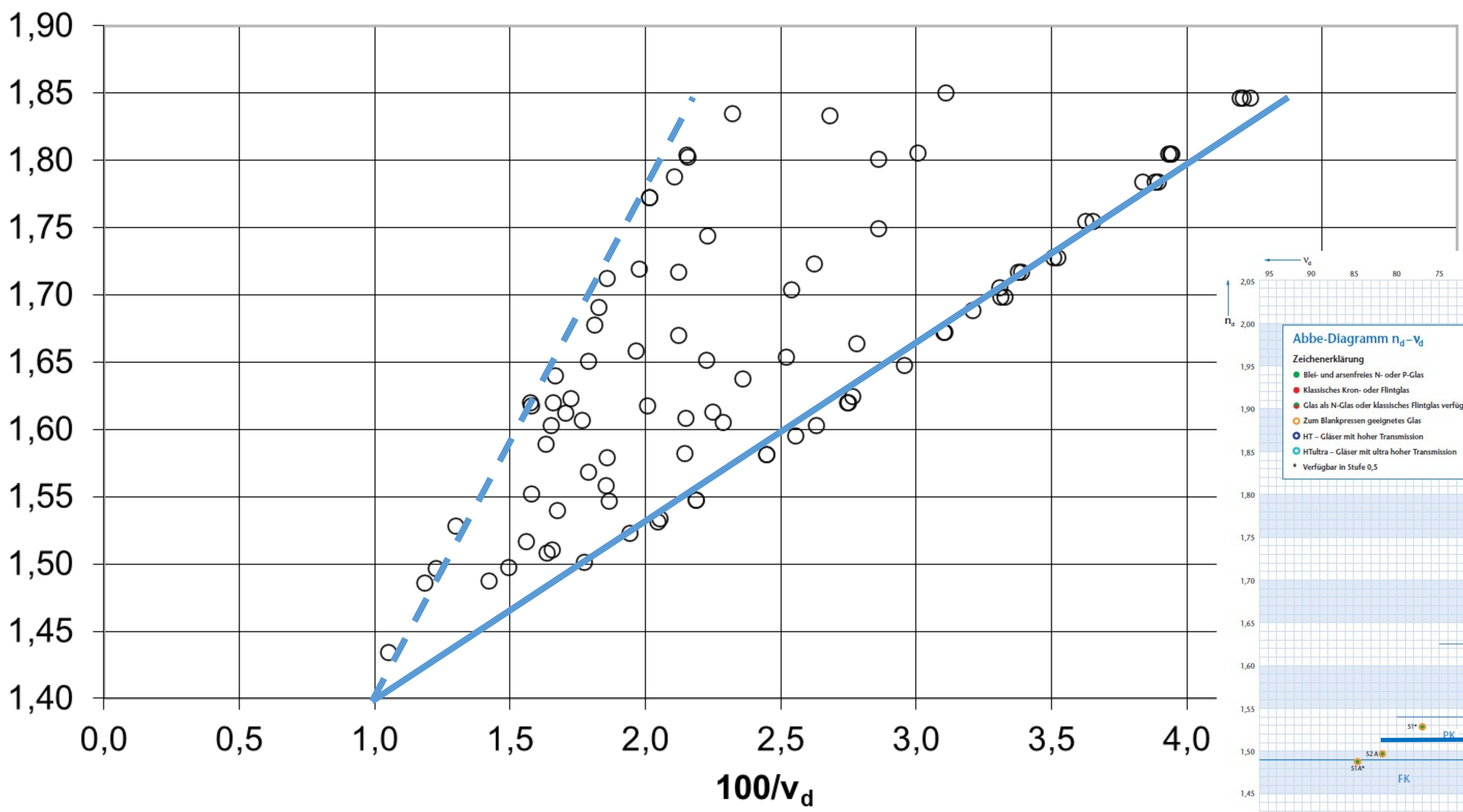
$$\frac{n}{100/\nu} = \frac{n\,\nu}{100} = \frac{0.4}{3}$$

$$n\,\nu = 133$$



Alternative Abbe diagram: n versus 100/v

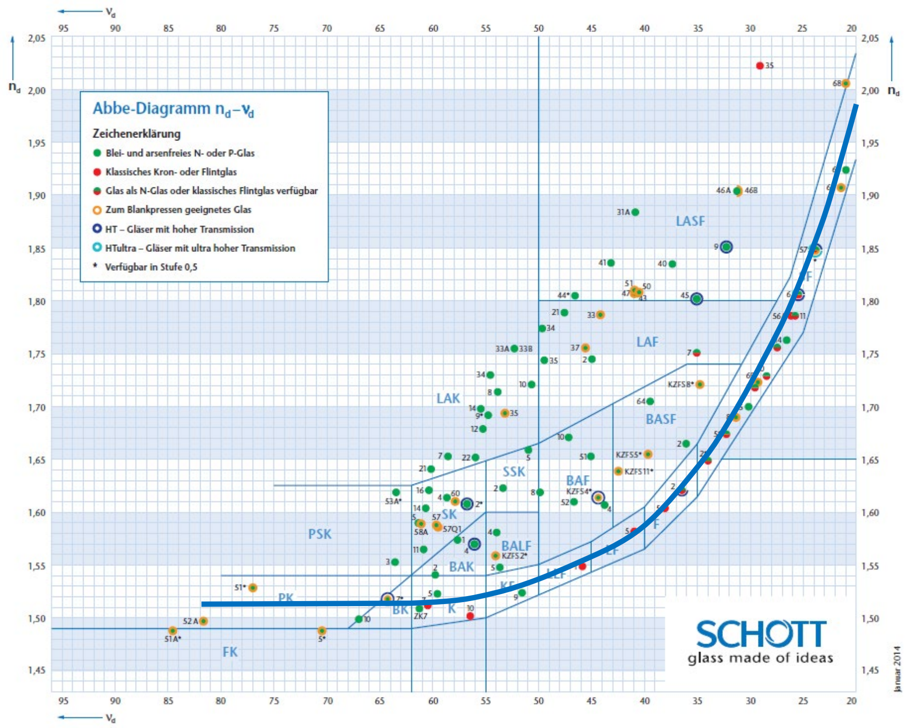
Alternative Abbe diagram n_d versus $100/v_d$



The current max envelop of „max. n“ versus Abbe-value is also nearly linear

$$\frac{n}{100/\nu} = \frac{n\,\nu}{100} = \frac{0.4}{1.1}$$

$$n\,\nu \approx 370$$



IR glasses overview

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		n(4µm)	ν(4µm, 3 - 5µm)	n(10µm)	ν(10µm, 8 - 12µm)	tmin [µm]	tmax [µm]	spectral band
Arsenic trisulfide	As2S3	2,4116	181,0	2,3816	46,7	0,6	13	NIR, SWIR, MWIR, LWIR
Barium fluoride	BaF2	1,4558	45,1	1,4014		0,15	12,5	UV, VIS, NIR, SWIR, MWIR, LWIR
Cadmium telluride	CdTe	2,6926	162,2	2,6751	150,6	0,9	16	SWIR, MWIR, LWIR
Calcium fluoride	CaF2	1,4096	21,7	1,3002		0,13	10	UV, VIS, NIR, SWIR, MWIR, LWIR
Gallium arsenide	GaAs	3,3036	140,9	3,2745	104,2	1	16	SWIR, MWIR, LWIR
GASIR1	Ge22As20Se58	2,5100	196,1	2,4944	119,6			
GASIR2	Ge20Sb15Se65	2,6039	170,6	2,5842	100,6			
Germanium	Ge	4,0245	101,8	4,0032	904,6	1,8	23	MWIR, LWIR
IG2	Ge33As12Se55	2,5129	201,7	2,4967	108,0	1	15	SWIR, MWIR, LWIR
IG3	Ge30As13Se32Te25	2,8034	152,8	2,7870	163,9	1	12,5	SWIR, MWIR, LWIR
IG4	Ge10As40Se50	2,6210	202,6	2,6084	176,0	1	12,5	SWIR, MWIR, LWIR
IG5	Ge28Sb12Se60	2,6226	180,3	2,6038	102,1	1	15	SWIR, MWIR, LWIR
IG6	As40Se60	2,7945	168,0	2,7775	161,6	1	13	SWIR, MWIR, LWIR
IRG100		2,6200	165,3	2,6002	103,2	1	12	SWIR, MWIR, LWIR
Lithium fluoride	LiF	1,3494	8,7	–	–	0,12	6	UV, VIS, NIR, SWIR, MWIR, LWIR
Magnesium oxide	MgO	1,6680		–	–	0,3	6	UV, VIS, NIR, SWIR, MWIR
Potassium bromide	KBr	1,5346	232,4	1,5242	64,7	0,23	25	UV, VIS, NIR, SWIR, MWIR, LWIR
Potassium chloride	KCl	1,4722	144,9	1,4564	30,0	0,21	20	UV, VIS, NIR, SWIR, MWIR, LWIR
Silicon	Si	3,4255	250,1	–	–	1,2	9	UV, VIS, NIR, SWIR, MWIR
Sodium chloride	NaCl	1,5217	97,5	1,4947	18,7	0,2	15	UV, VIS, NIR, SWIR, MWIR, LWIR
Thallium bromoiodide	KRS-5	2,3820	232,3	2,3707	165,1	0,6	40	NIR, SWIR, MWIR, LWIR
Zinc selenide	ZnSe	2,4331	177,6	2,4065	57,8	0,6	21	NIR, SWIR, MWIR, LWIR
Zinc sulfide	ZnS	2,2518	112,9	2,2001	22,7	0,2	15	UV, VIS, NIR, SWIR, MWIR, LWIR
FI-02				3,4650	253,0			MWIR, LWIR

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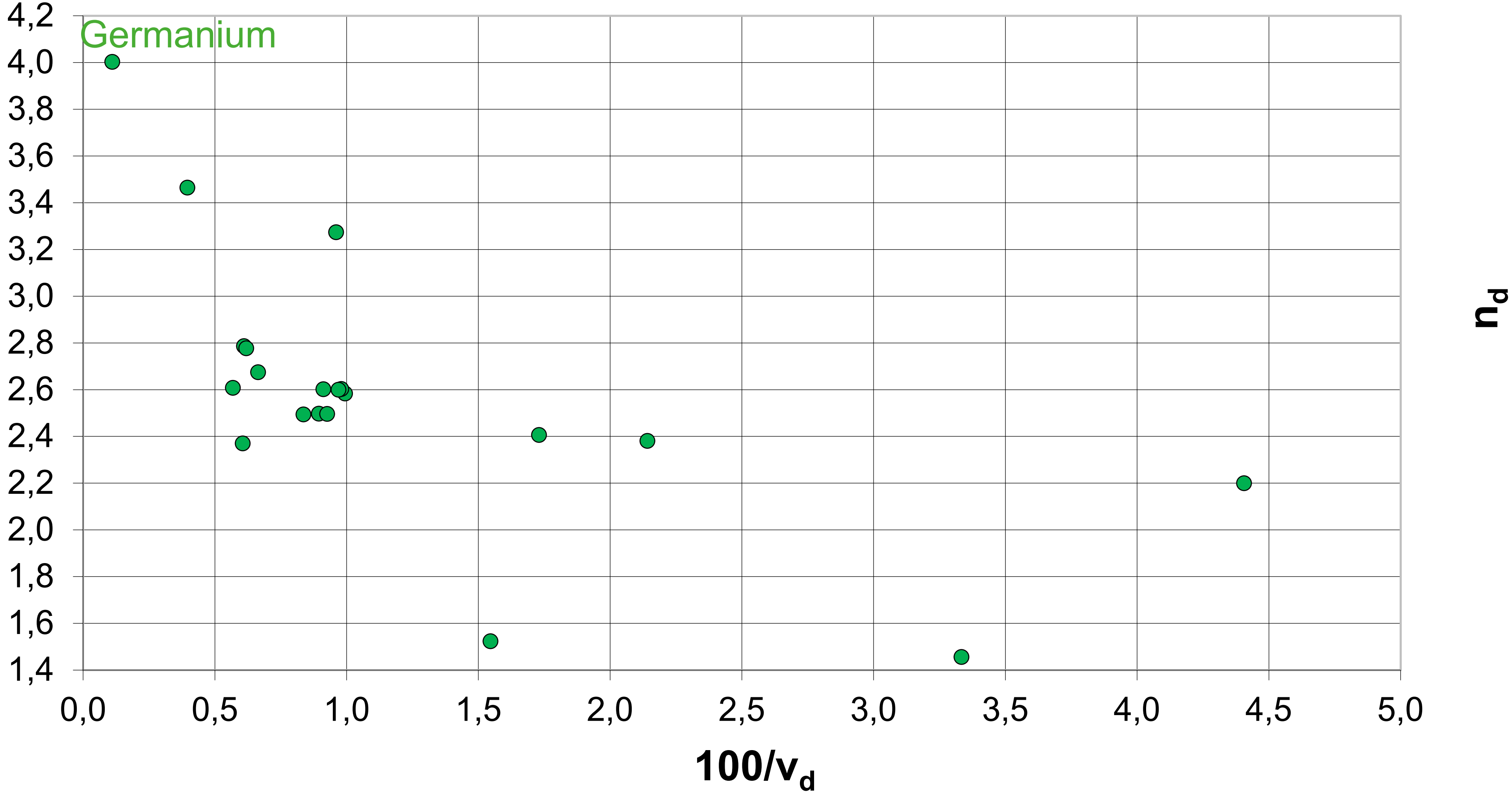
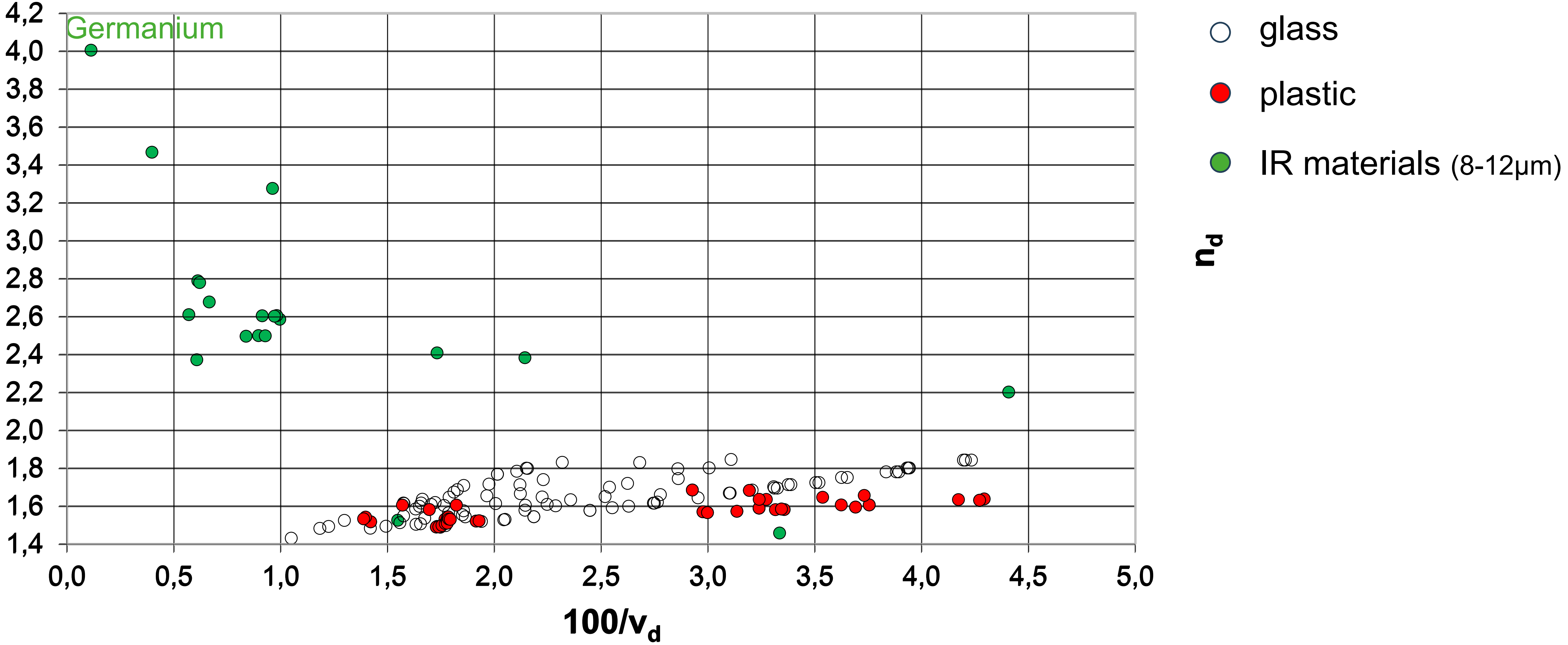
Alternative Abbe diagram n_d versus v_d



	100/v	n
AMTIR1	0,8945	2,4977
AMTIR3	0,9116	2,6023
Arsenic trisulfide	2,1413	2,3816
Cadmium telluride	0,6640	2,6751
Gallium arsenide	0,9597	3,2745
GASIR1	0,8361	2,4944
GASIR2	0,9940	2,5842
Germanium	0,1105	4,0032
IG2	0,9259	2,4967
IG3	0,6101	2,7870
IG4	0,5682	2,6084
IG5	0,9794	2,6038
IG6	0,6188	2,7775
IRG100	0,9690	2,6002
Potassium bromide	1,5456	1,5242
Potassium chloride	3,3333	1,4564
Sodium chloride	5,3476	1,4947
Thallium bromoiodide	0,6057	2,3707
Zinc selenide	1,7301	2,4065
Zinc sulfide	4,4053	2,2001
FI-02	0,3953	3,4650

Alternative Abbe diagram n_d versus $100/v_d$



Infrared Optics: Are we in a Lens design Wonderland?

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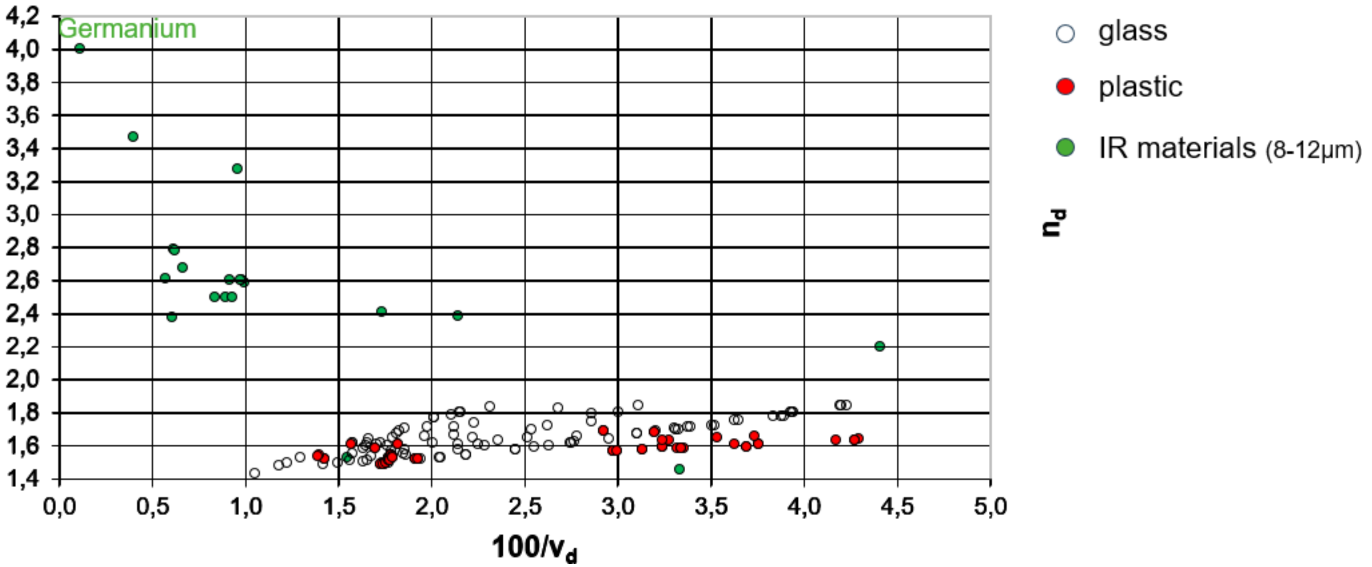
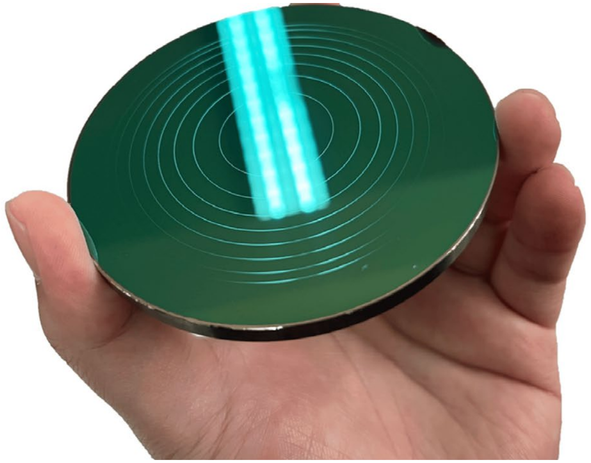
Alternative Abbe diagram n_d versus $100/v_d$



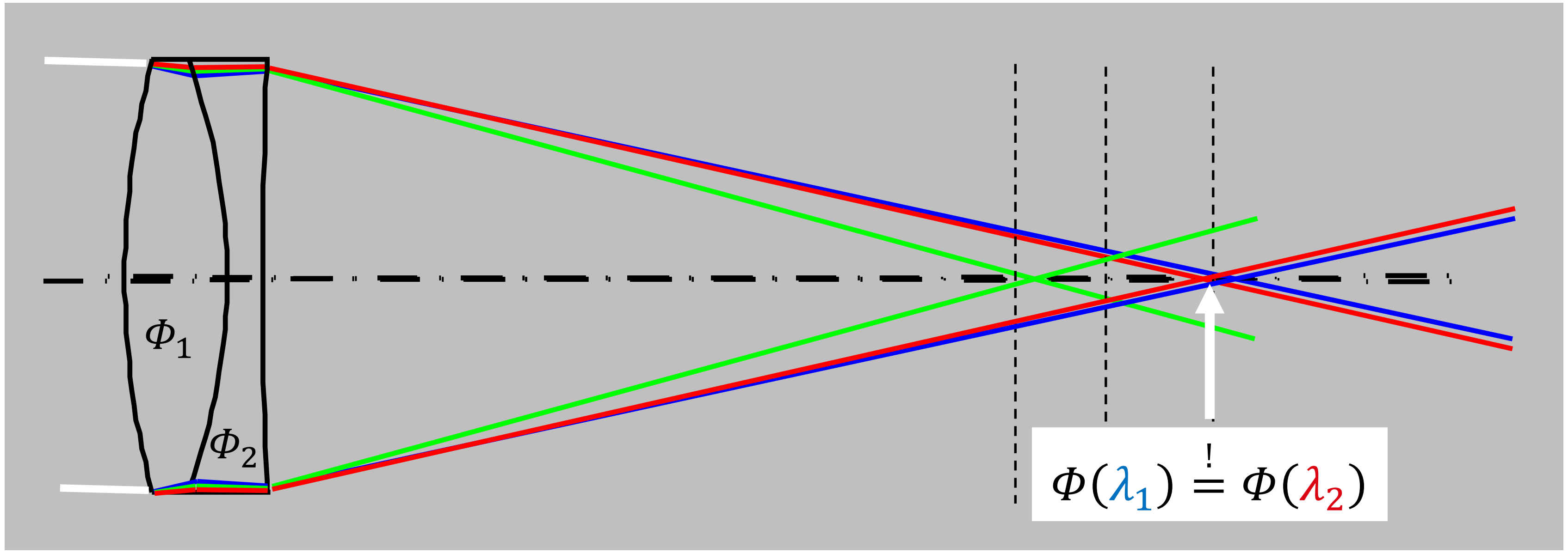


- Aberration correction possibilities in singular elements
  - ✓ High refractive index and Low dispersion crowns
  - ✓ Extraordinary lens Bending
  - ✓ Aspherical surfaces
  - ✓ Refractive/Diffractive Hybrids

} Diamond Turning Technology

G. Raju / V. Blahnik (2024), Survey of Thermal Infrared Zoom lenses in the 2010s and beyond

Achromatic condition



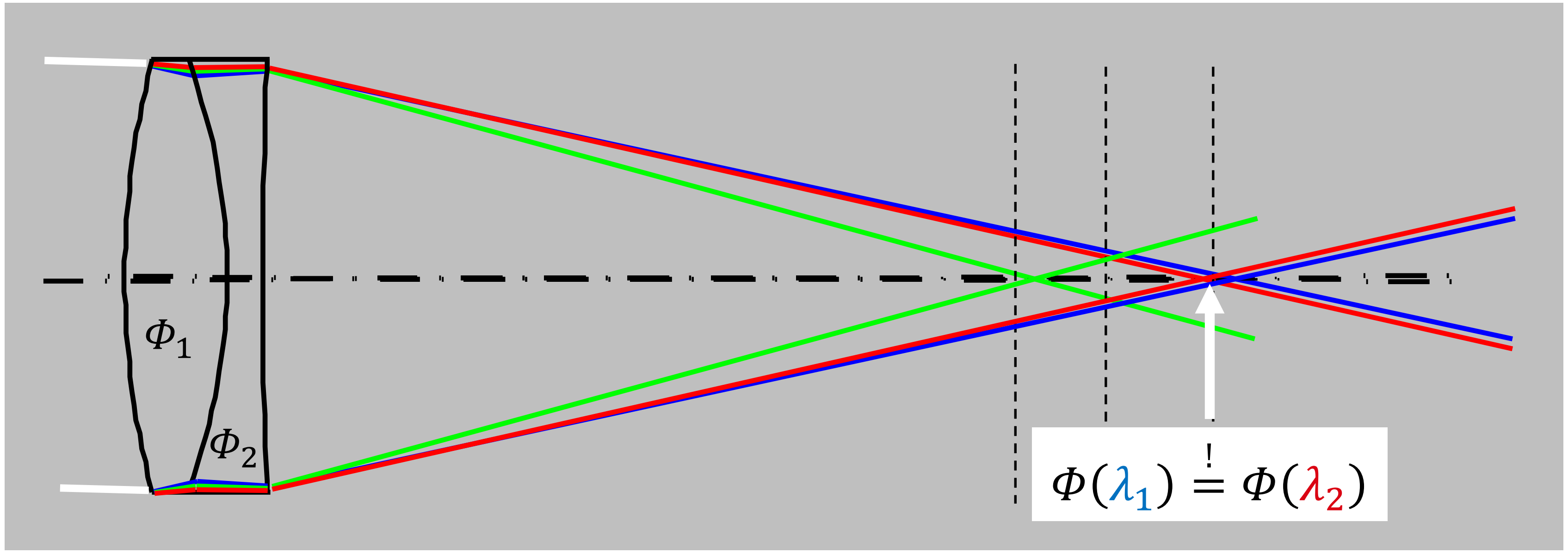
Refractive power of two lenses $\Phi = \Phi_1 + \Phi_2 - d\Phi_1\Phi_2$ with small relative distance: $\Phi = \Phi_1 + \Phi_2$.

To fulfill the **achromatic condition** $\Phi(\lambda_1) = \Phi(\lambda_2)$, e.g. “same focus at blue and red wavelength” we obtain directly from $\Phi(\lambda_1) - \Phi(\lambda_2) = \frac{\Phi}{\nu}$:

$$0 \overset{!}{=} \Phi(\lambda_1) - \Phi(\lambda_2) = \Phi_1(\lambda_1) + \Phi_2(\lambda_1) - \Phi_1(\lambda_2) - \Phi_2(\lambda_2) = \big(\Phi_1(\lambda_1) - \Phi_1(\lambda_2)\big) + \big(\Phi_2(\lambda_1) - \Phi_2(\lambda_2)\big)$$

$$= \frac{\Phi_1}{\nu_1} + \frac{\Phi_2}{\nu_2}$$

Achromatic condition



With this relationship $0 = \frac{\Phi_1}{\nu_1} + \frac{\Phi_2}{\nu_2}$ and replacing $\Phi = \Phi_1 + \Phi_2$ we get the refractive powers Φ_1 and Φ_2 of the achromat as:

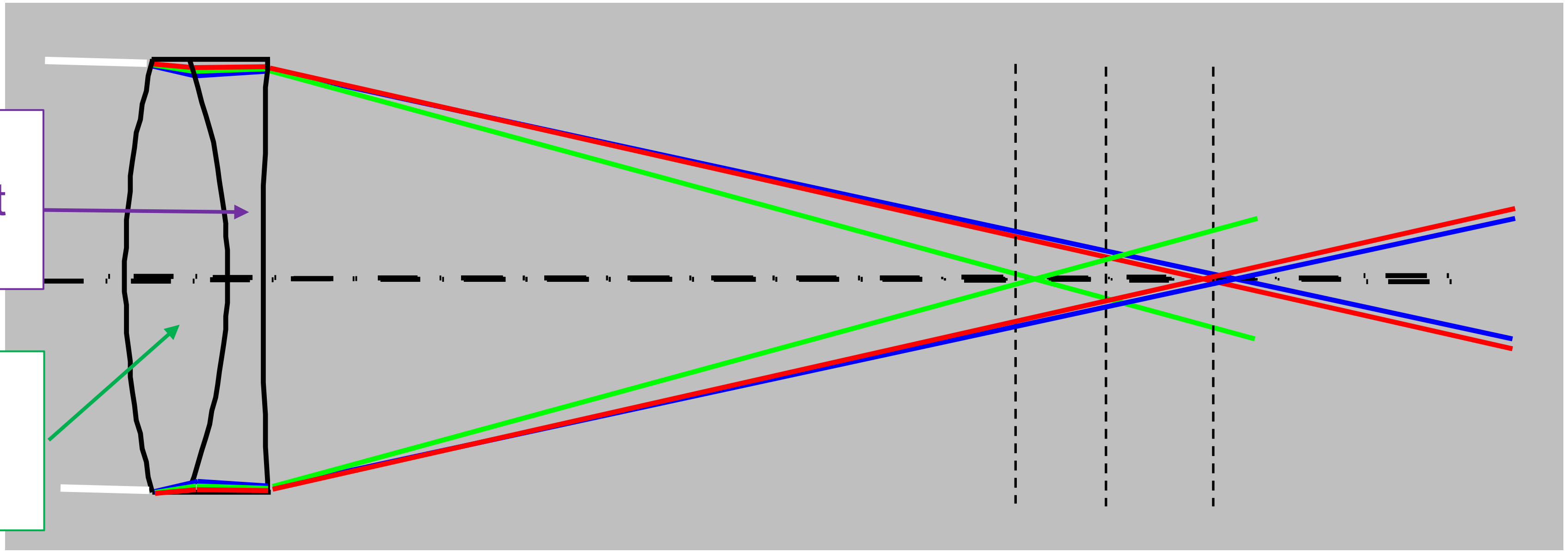
$$\Phi_1 = \Phi \, \frac{1}{1 - \frac{\nu_2}{\nu_1}},$$

$$\Phi_2 = \Phi - \Phi_1 = \Phi \left(1 - \frac{1}{1 - \frac{\nu_2}{\nu_1}}\right).$$

Longitudinal chromatic aberration of achromat

- : high dispersion („flint glass“)

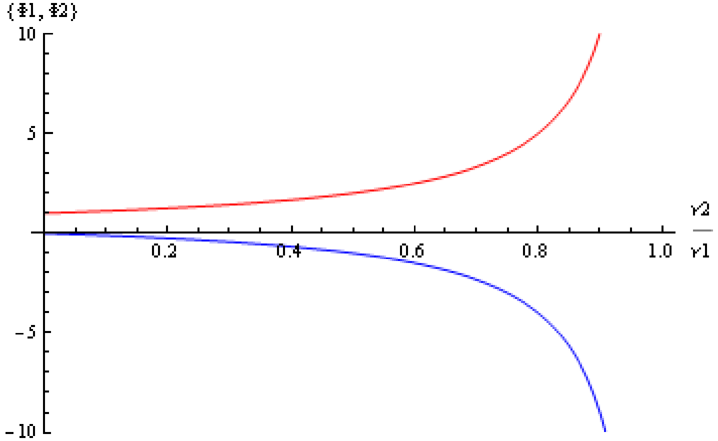
+ : small dispersion („crown glass“)



The total power of both elements is „+“: $\Phi = \Phi_1 + \Phi_2$.

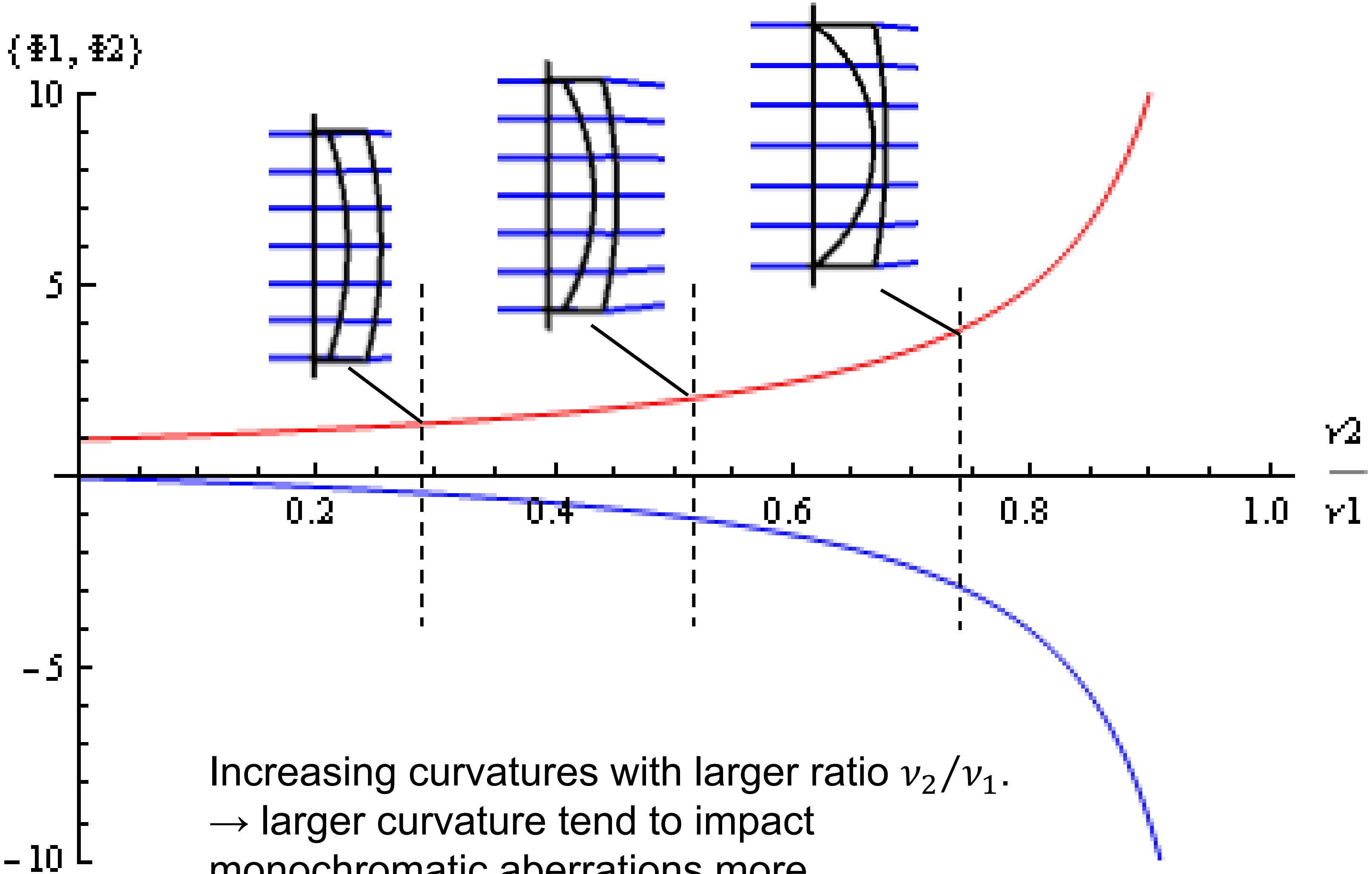
$$\Phi_1 = \Phi \frac{1}{1 - [\nu_2/\nu_1]}$$

$$\Phi_2 = \Phi \frac{1}{1 - \frac{1}{[\nu_2/\nu_1]}}$$



- an achromat consist of a **“+” and “-”** element, “+” has higher dispersion than “-”
- required refractive power per element depends on the **ratio** of the Abbe-numbers
- The **larger the difference** between the Abbe-numbers the **smaller the power** per element (lower curvatures, which is good to avoid other aberrations)

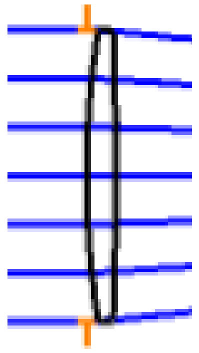
Longitudinal chromatic aberration of achromat



$$\Phi_1 = \Phi \frac{1}{1-[\nu_2/\nu_1]}$$

$$\Phi_2 = \Phi \frac{1}{1-\frac{1}{[\nu_2/\nu_1]}}$$

Singlet with same power:



Increasing curvatures with larger ratio ν_2/ν_1.
→ larger curvature tend to impact monochromatic aberrations more

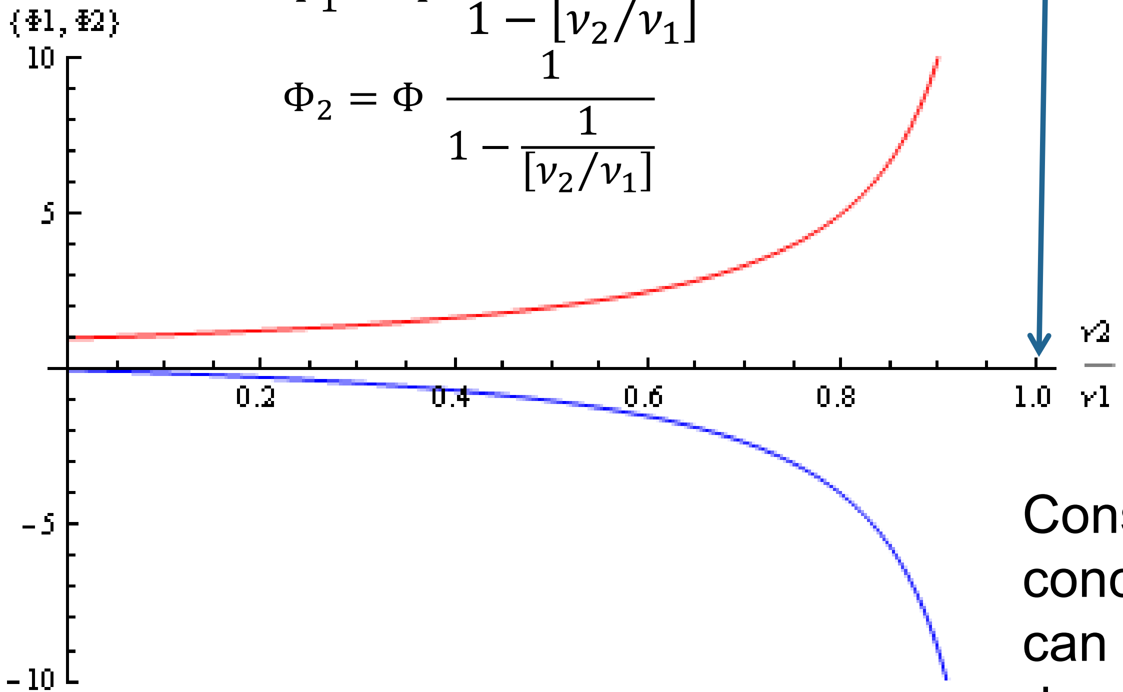
Longitudinal chromatic aberration of achromat

Isaac Newton believed that all glasses have the same relative dispersion.

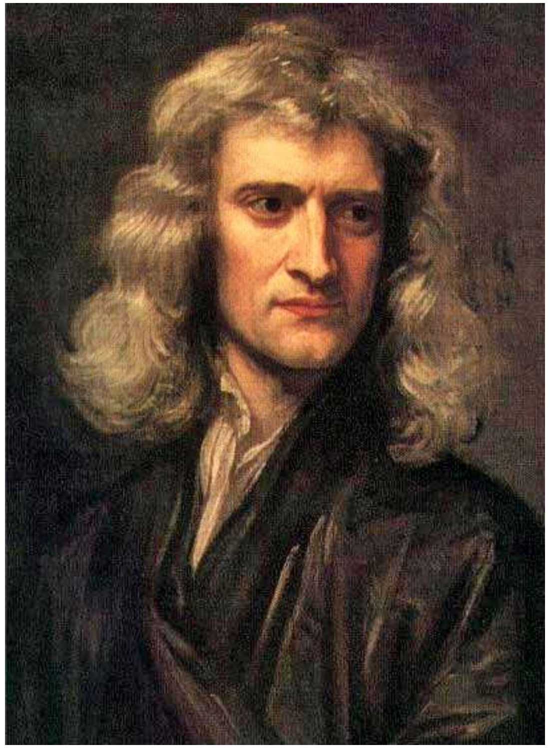
Then we have $v_2/v_1=1$ and there is no solution.

$$\Phi_1 = \Phi \frac{1}{1-[\nu_2/\nu_1]}$$

$$\Phi_2 = \Phi \frac{1}{1-\frac{1}{[\nu_2/\nu_1]}}$$



Consequently Newton came to the conclusion that achromatic lenses can not be made. So he started to develop (excellent) reflecting telescopes…



Isaac Newton
(1643-1727)

Galilei Telescope (1610)

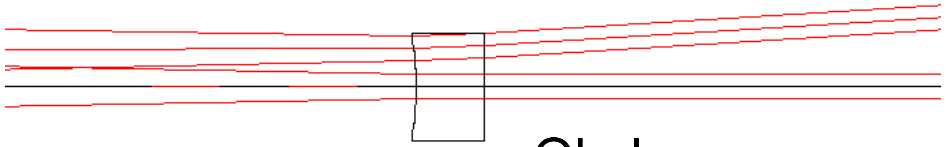
Negative lens: Upright image, but no pupil imaging → very small field

Telescope ~ 980 mm, $Ø_{EP}$ = 37 mm

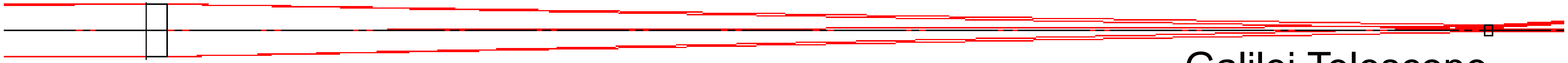
Eyepiece ~ 50 mm

→ angular magnification 20x

Field of view approx. 4.5° in the eye / 0.2° in the sky



Okular

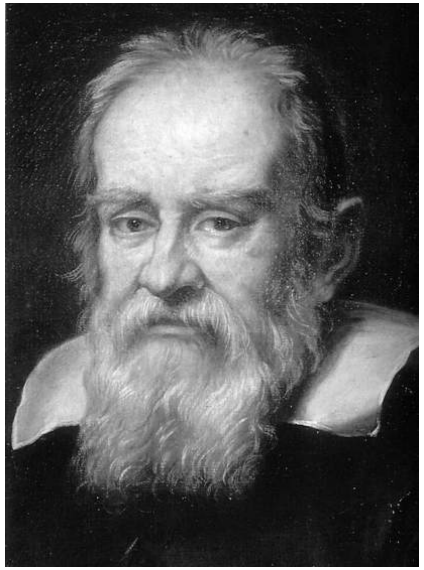


Galilei Telescope

For more than 150 years since the invention of the telescope in ca. 1608 chromatic aberration was limiting the usage. It was necessary to reduce the aperture to reduce color fringes to an acceptable level – at the expense of light.

(In other words: the f-number = (focal length) / (front lens diameter*) was required to be large.)

Hans Lippershey (1570-1619)



Galileo Galilei (1564-1642)

Johannes Kepler (1571-1630)

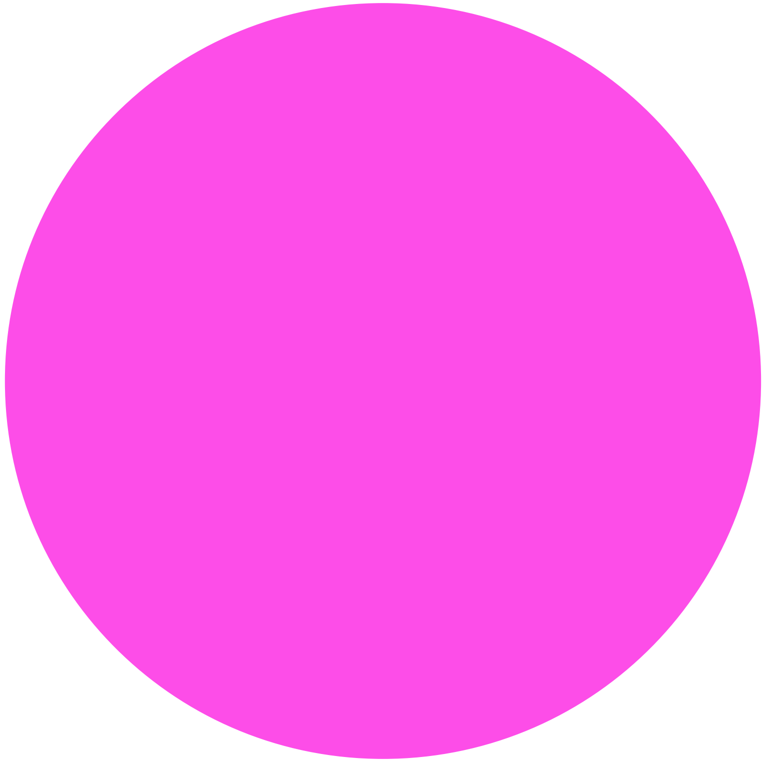
Telescopes enabled astronomy, the quantitative exploration of the movement of the planets and stars.

*in general: entrance pupil diameter

Chromatic spot size: Singlet vs Achromat

Singlet

$$\varnothing_{spot} = (0{,}015 \ldots 0{,}05)\ \varnothing_{EP}$$



Spot diameter 1mm

Even for a compact telescope with $\varnothing_{EP}$=50mm entrance pupil diameter, this spot diameter is extremely large, approx. $\varnothing_{spot}$=1mm.

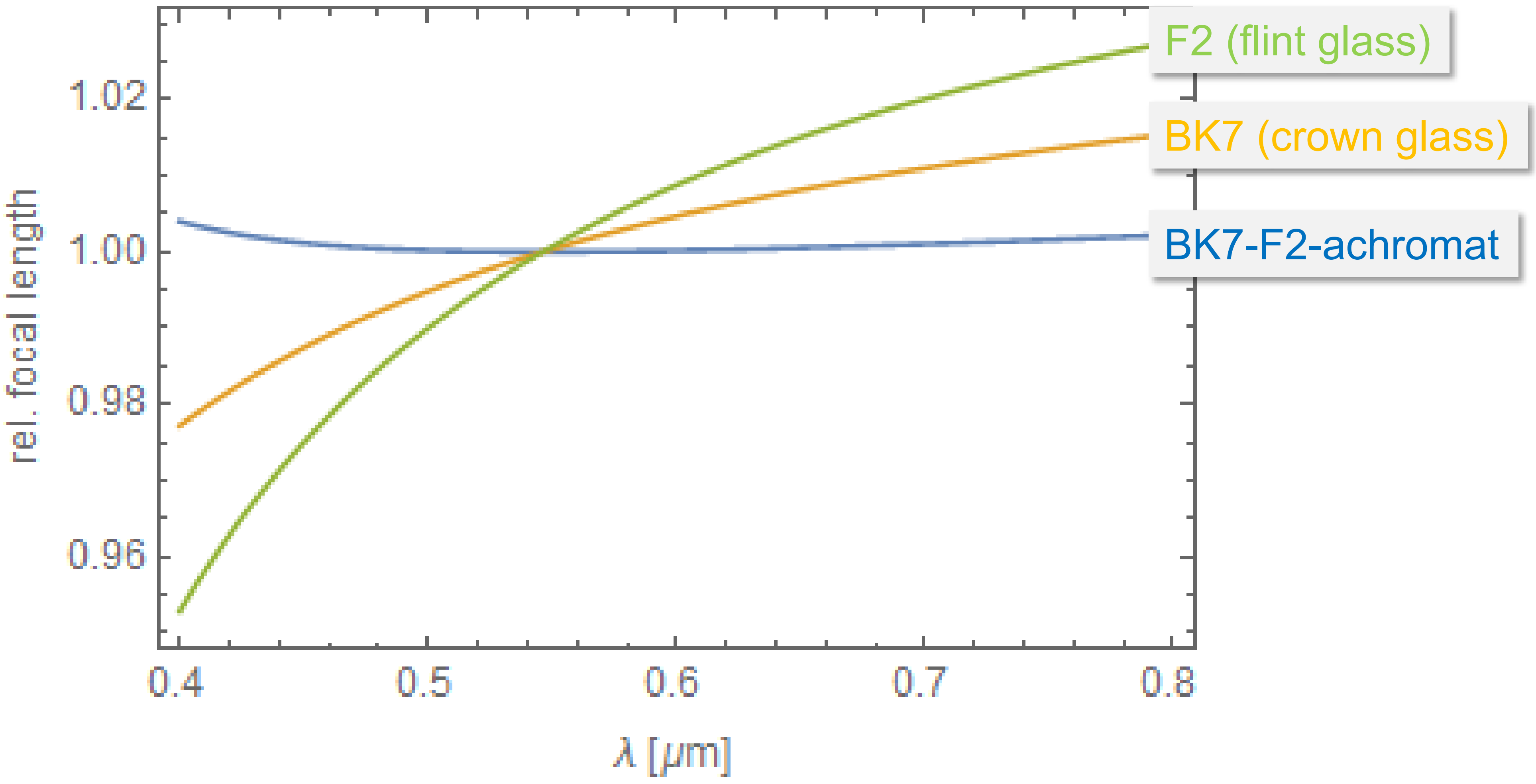
Achromat

$$\varnothing_{spot} \approx 0{,}0005\ \varnothing_{EP}$$

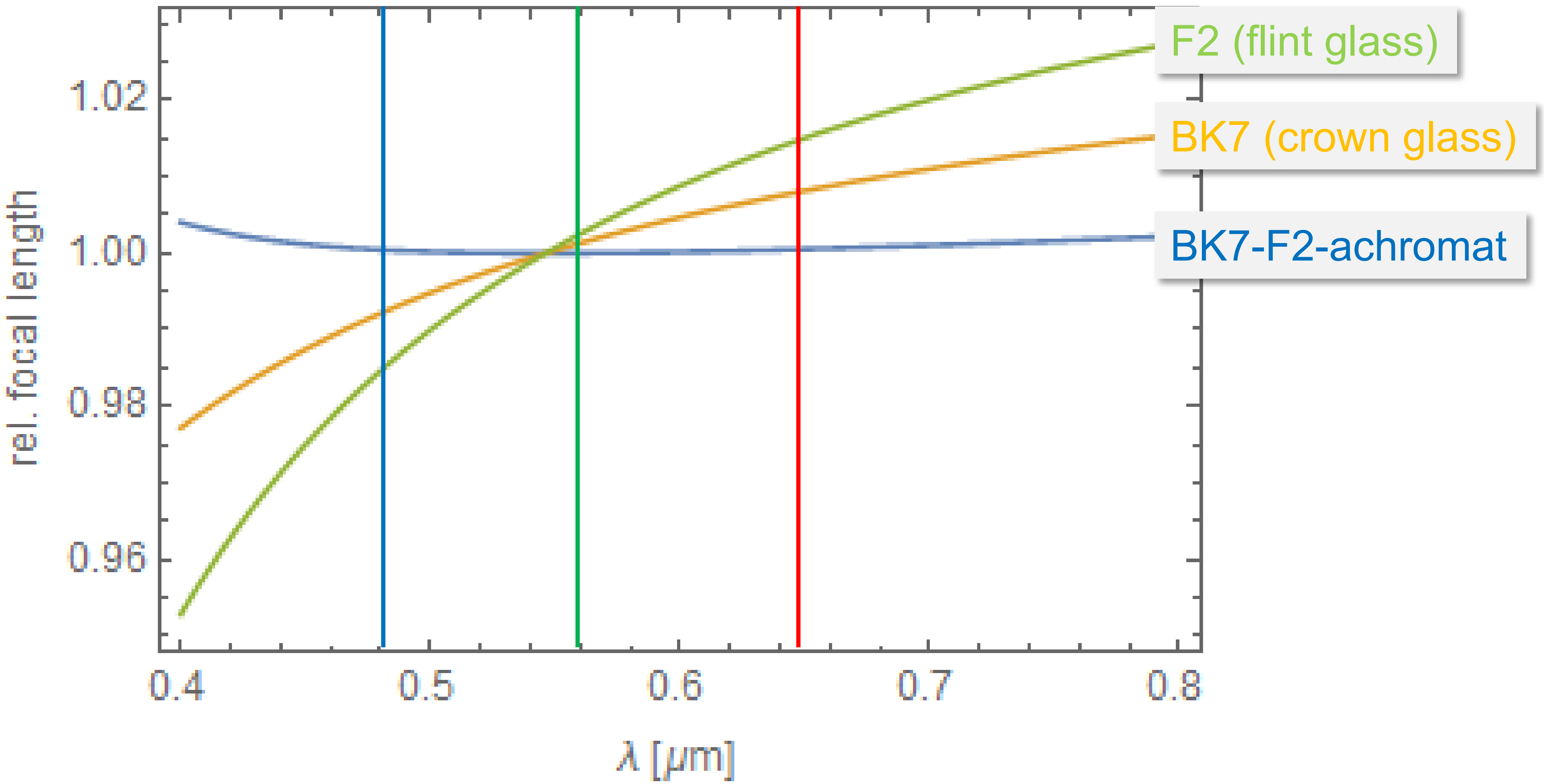
Spot diameter <30µm

The Airy-Spot diameter of an f/20 lens with focal length f=1000mm is only 20µm, so it is 50x smaller

Comparison of focal shift vs. wavelength: Simple lens vs. achromat

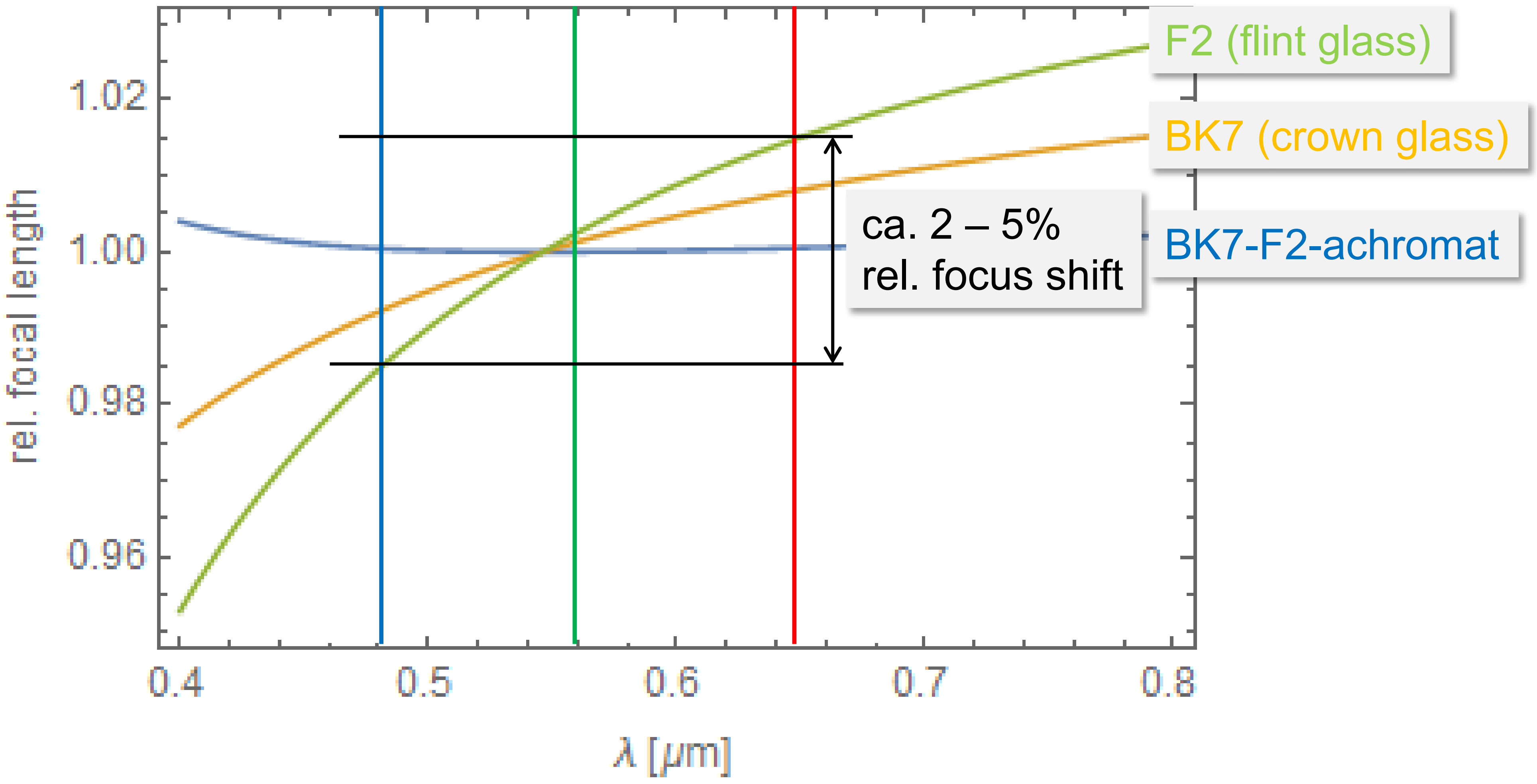


Comparison of focal shift vs. wavelength: Simple lens vs. achromat

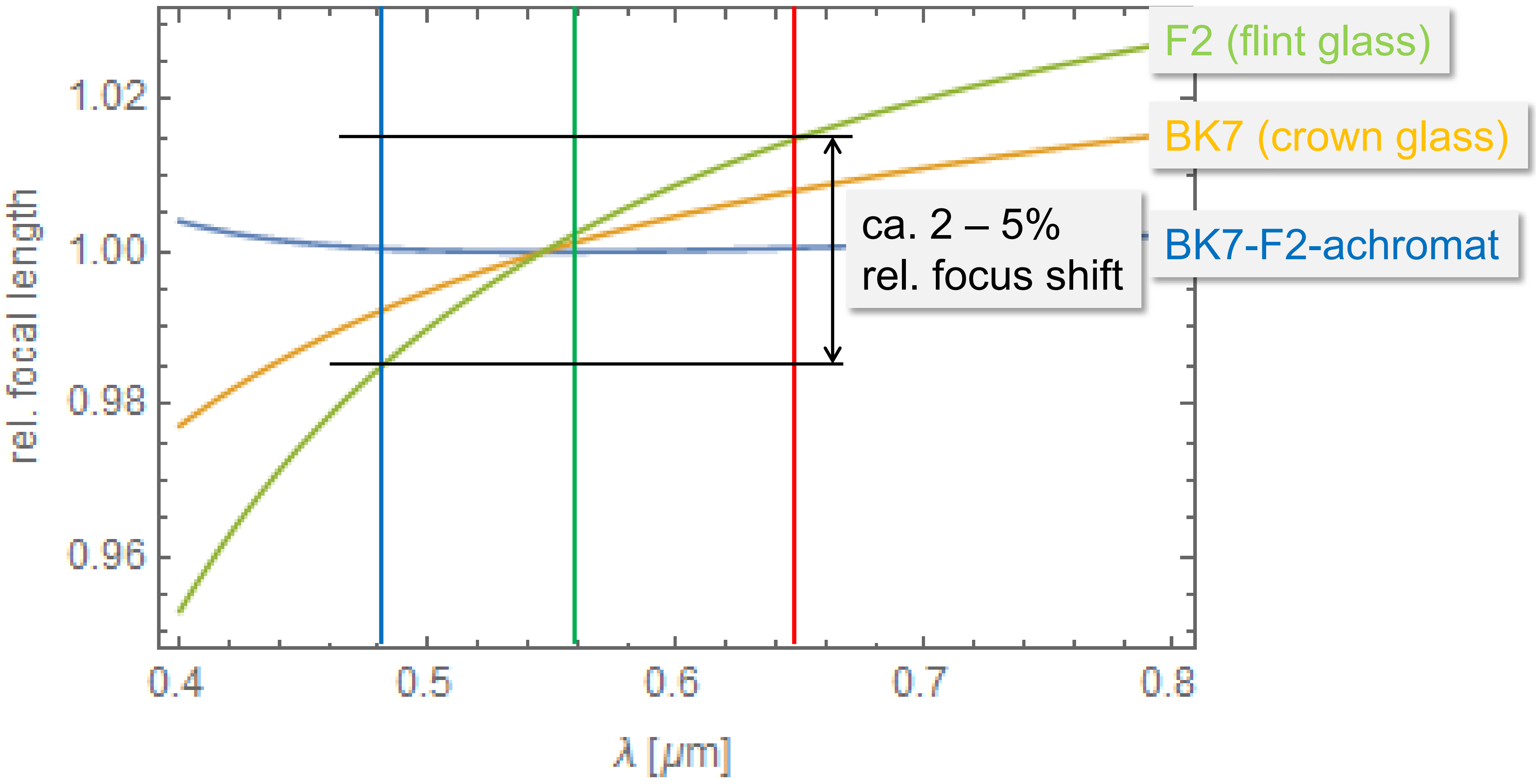


„mean wavelengths RGB“ (simplified consideration, in general we need to integrate over spectral regions)

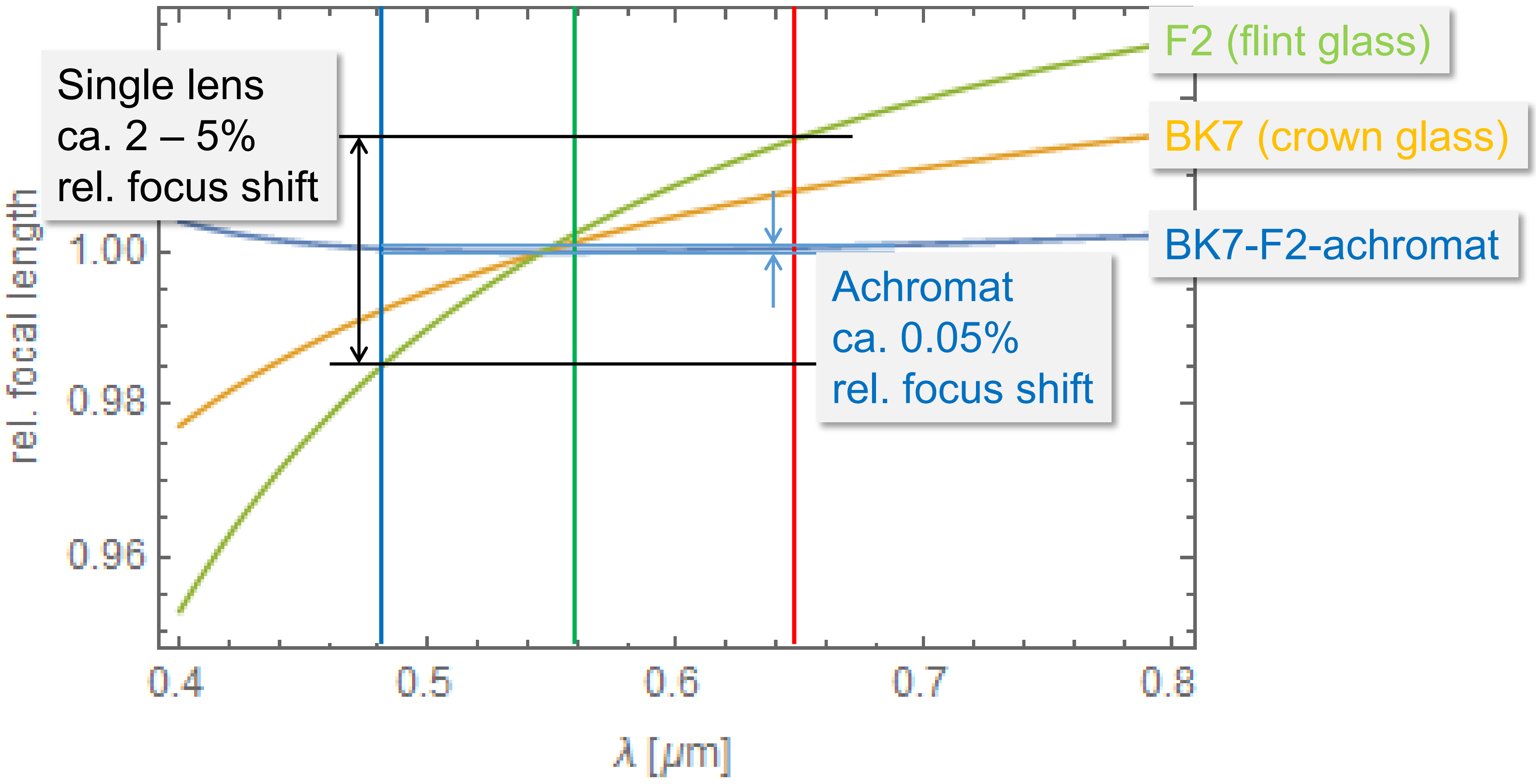
Comparison of focal shift vs. wavelength: Simple lens vs. achromat



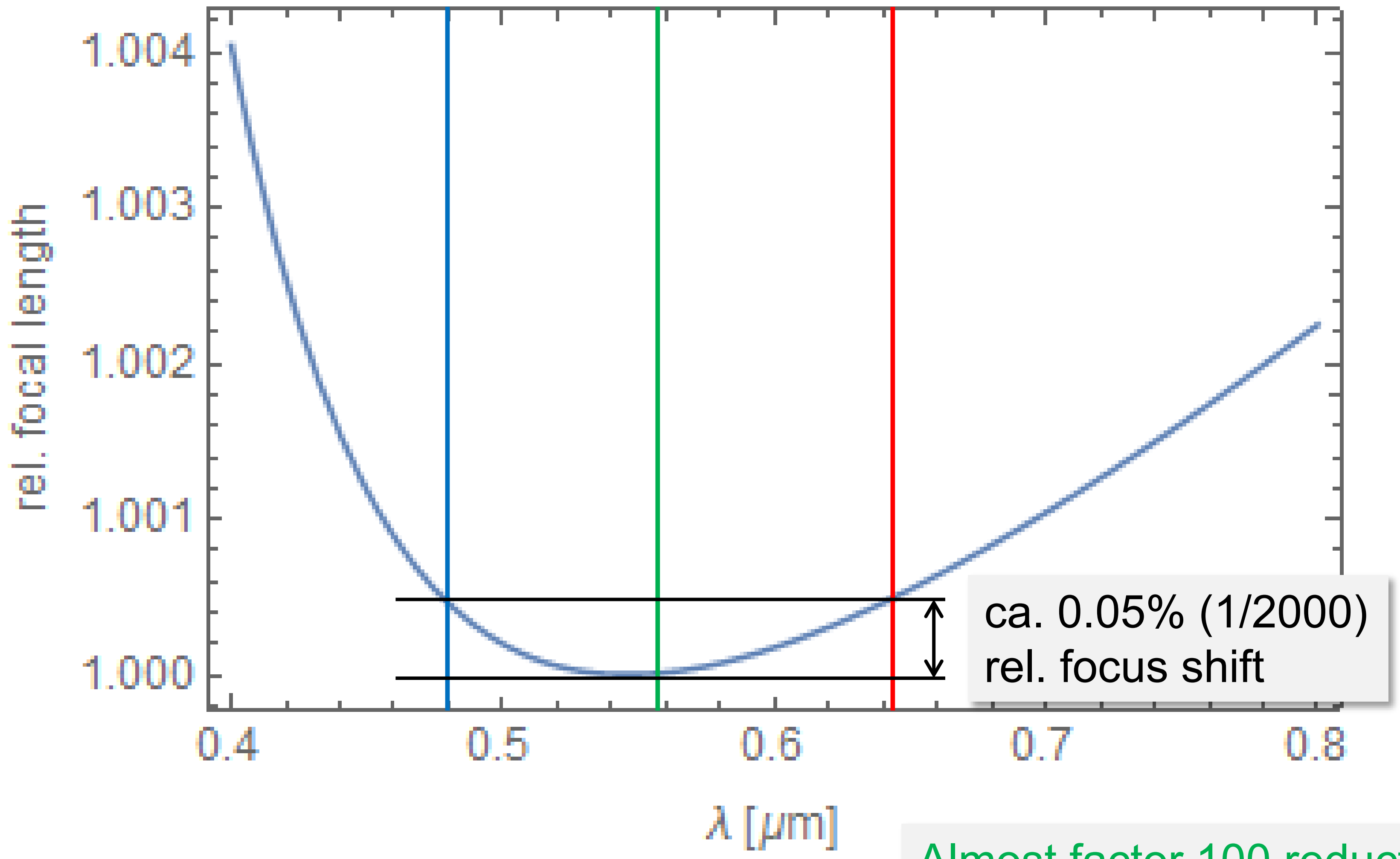
Comparison of focal shift vs. wavelength: Simple lens vs. achromat



Comparison of focal shift vs. wavelength: Simple lens vs. achromat



Achromat: rel. focal length vs. wavelength
Residual chromatic aberration is called „secondary spectrum“



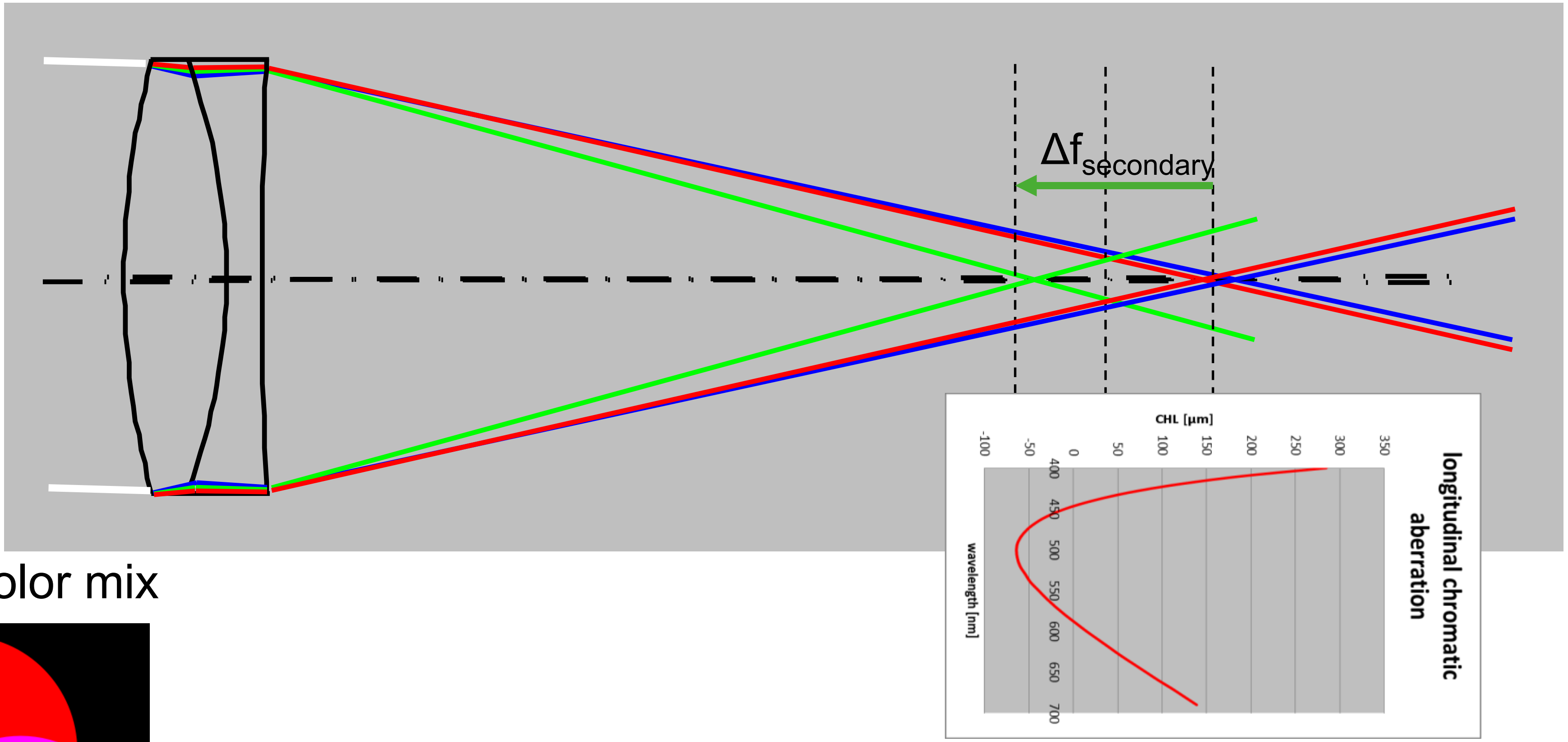
Almost factor 100 reduction of max. focus shift

Achromat as a company logo

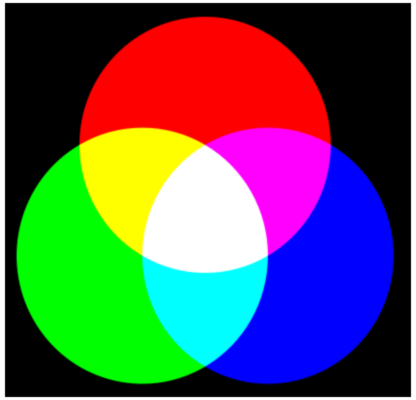
CARL ZEISS
JENA

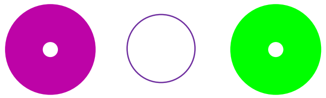
Part of it survived in modern logo: ZEISS

Longitudinal chromatic aberration of achromat



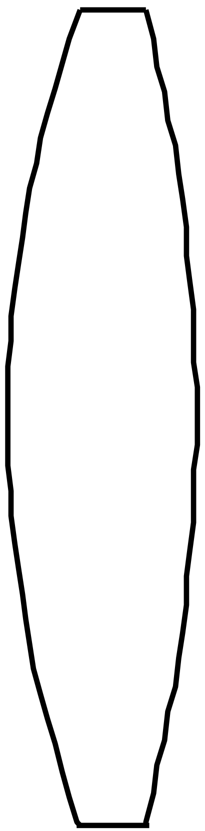
additive color mix

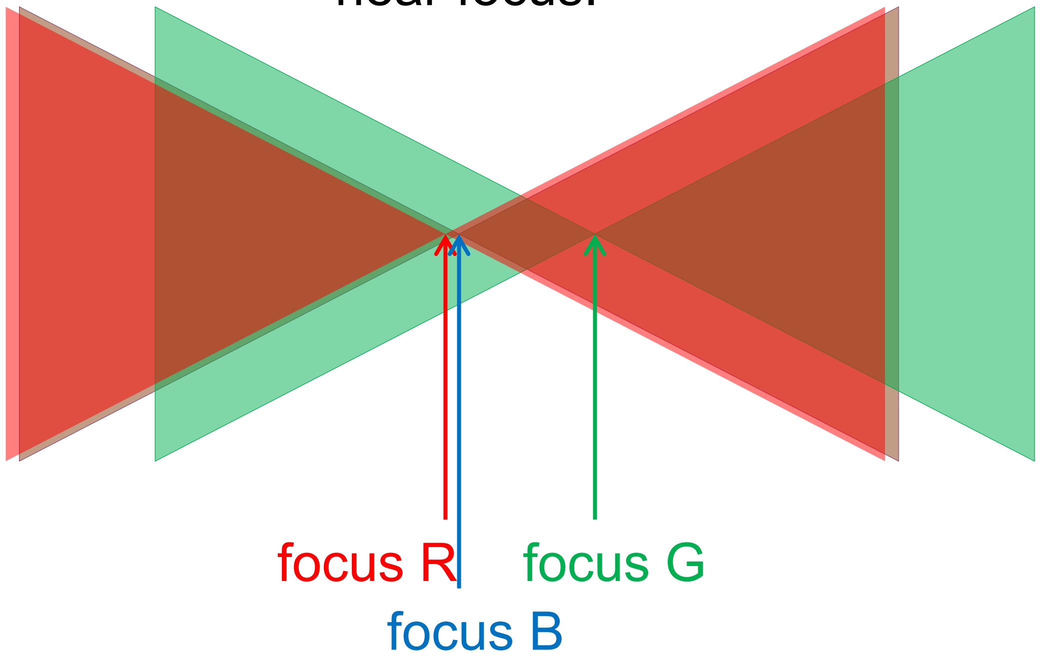


Color appearance: 

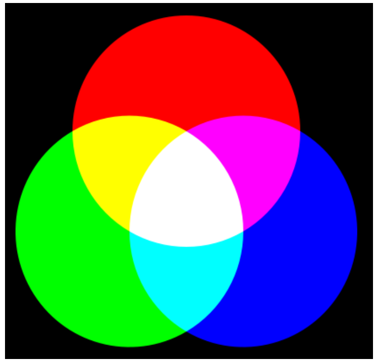
The size of the circle of confusion depends on the aperture

Through focus PSF with longitudinal chromatic aberration
Simplified picture

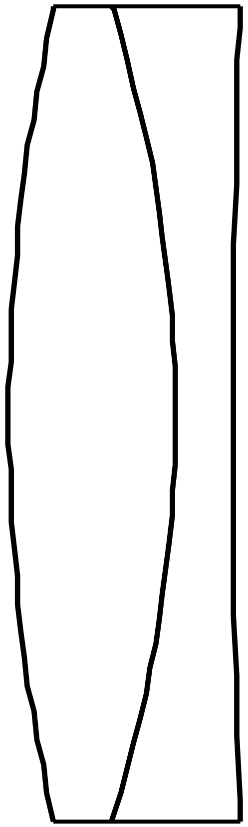


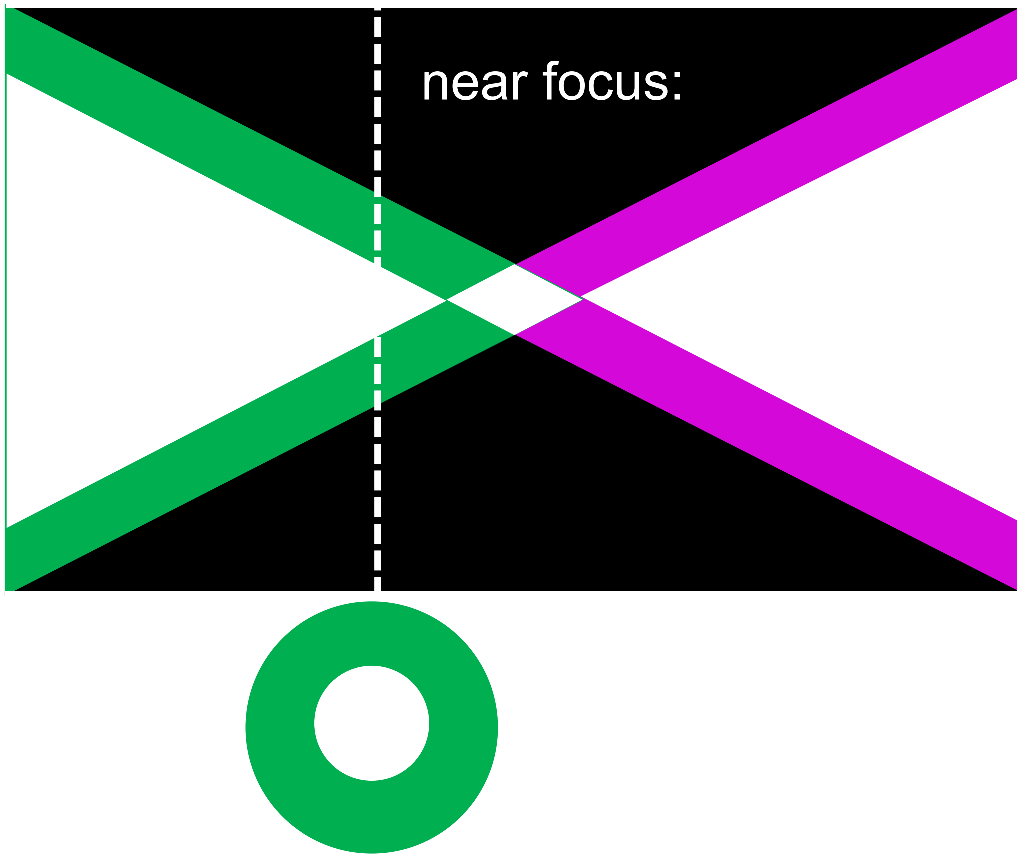


Additive color mix in those regions!

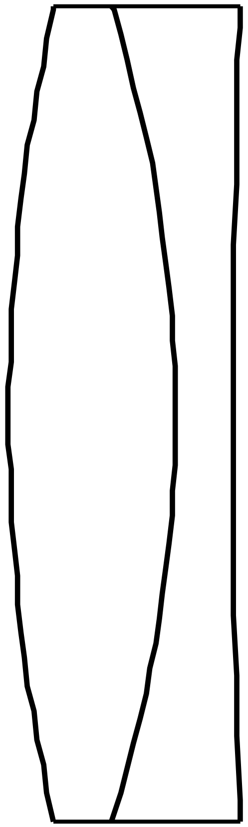


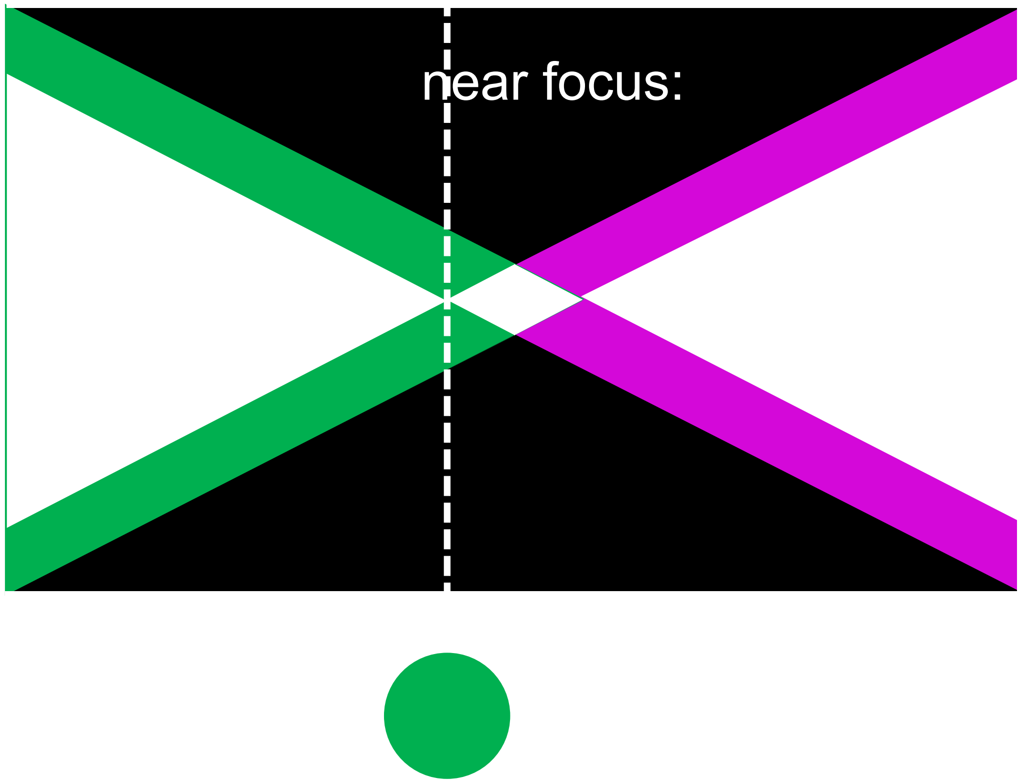
Through focus PSF with longitudinal chromatic aberration
Simplified picture, secondary spectrum



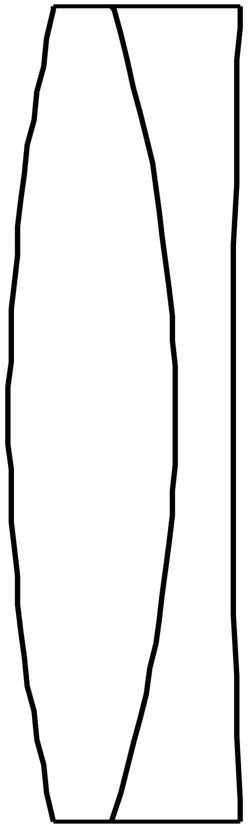


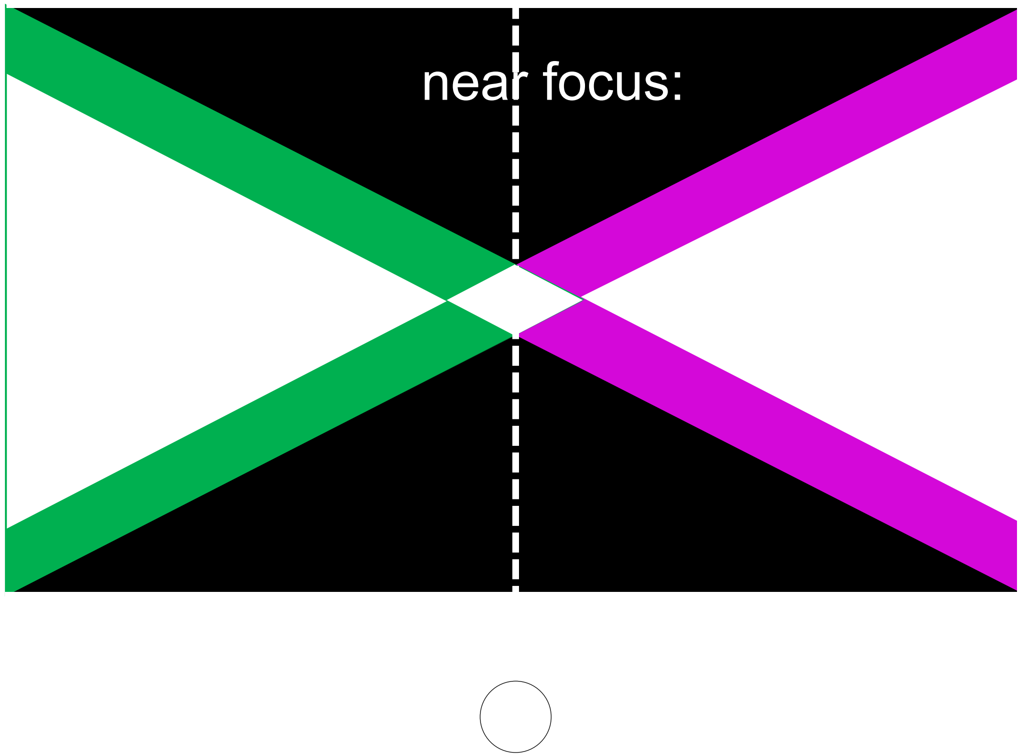
Through focus PSF with longitudinal chromatic aberration
Simplified picture, secondary spectrum





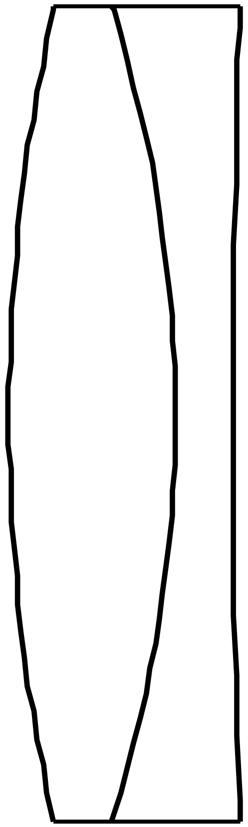
Through focus PSF with longitudinal chromatic aberration Simplified picture, secondary spectrum

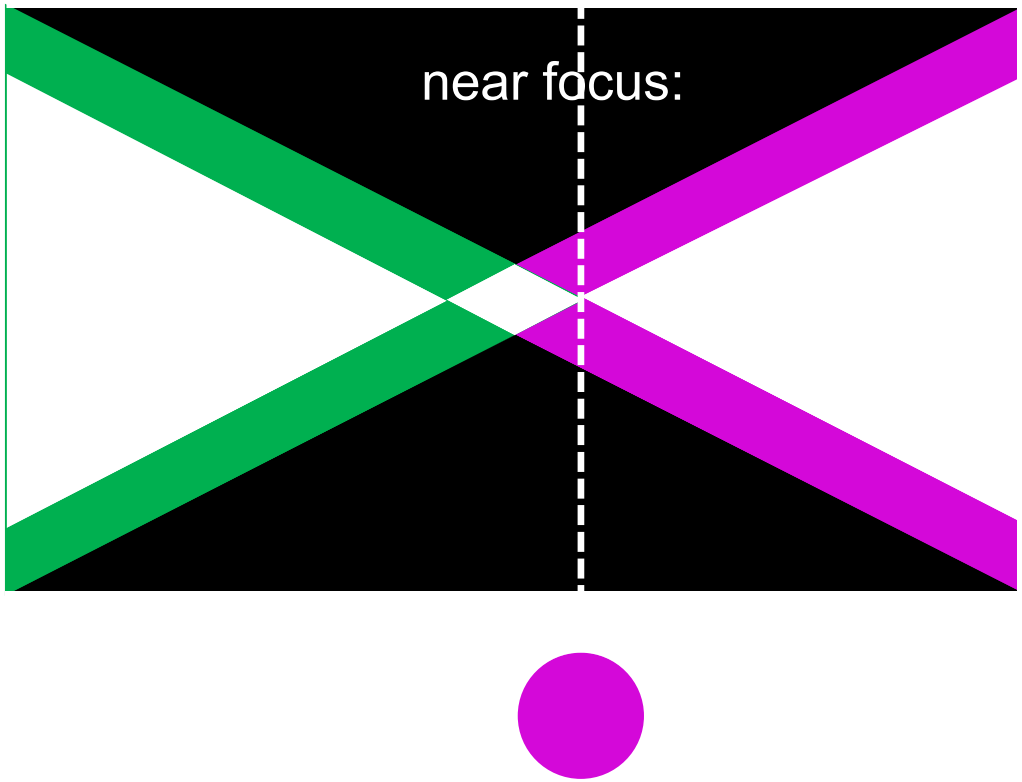




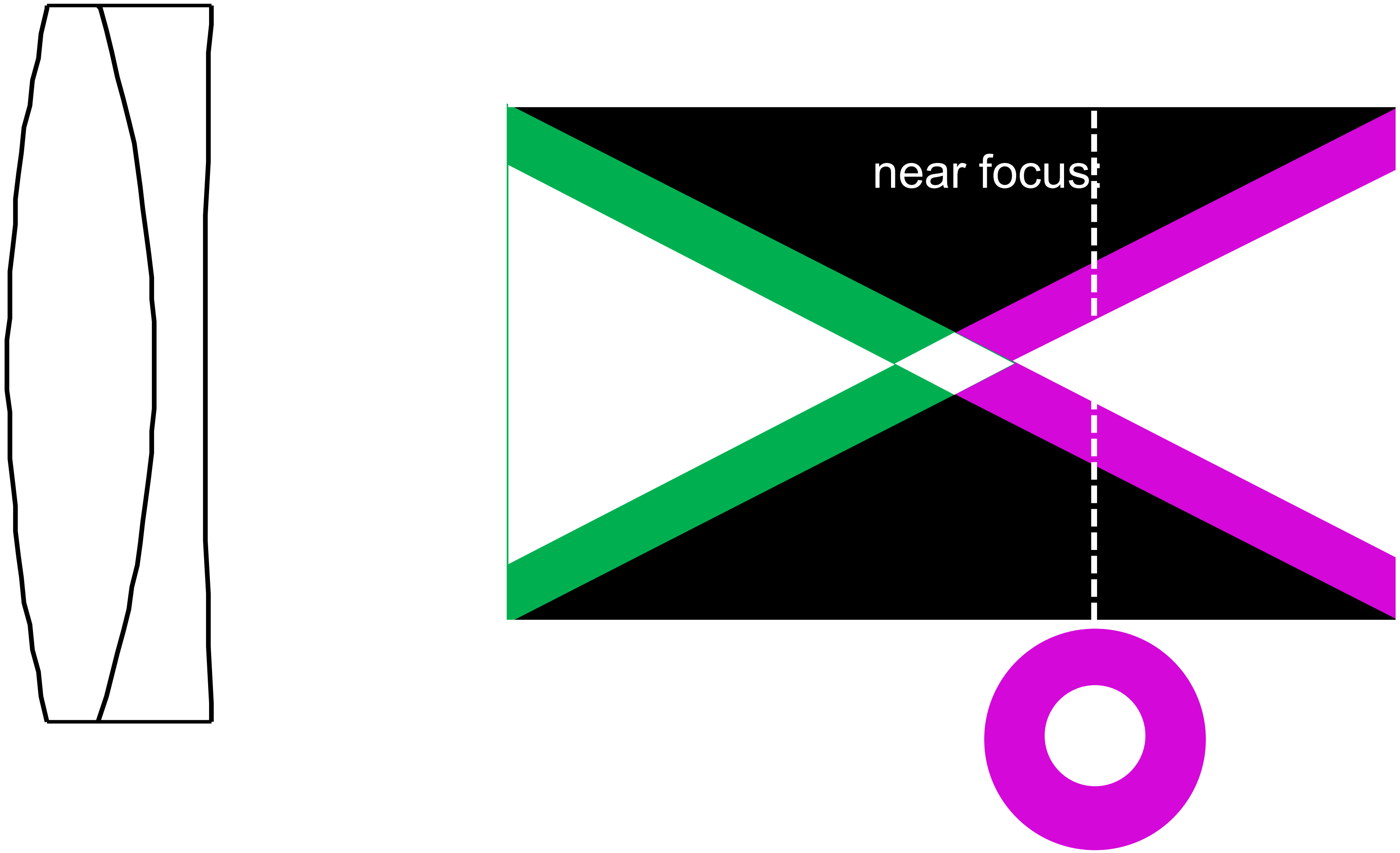
neutral white in best focus!

Through focus PSF with longitudinal chromatic aberration
Simplified picture, secondary spectrum

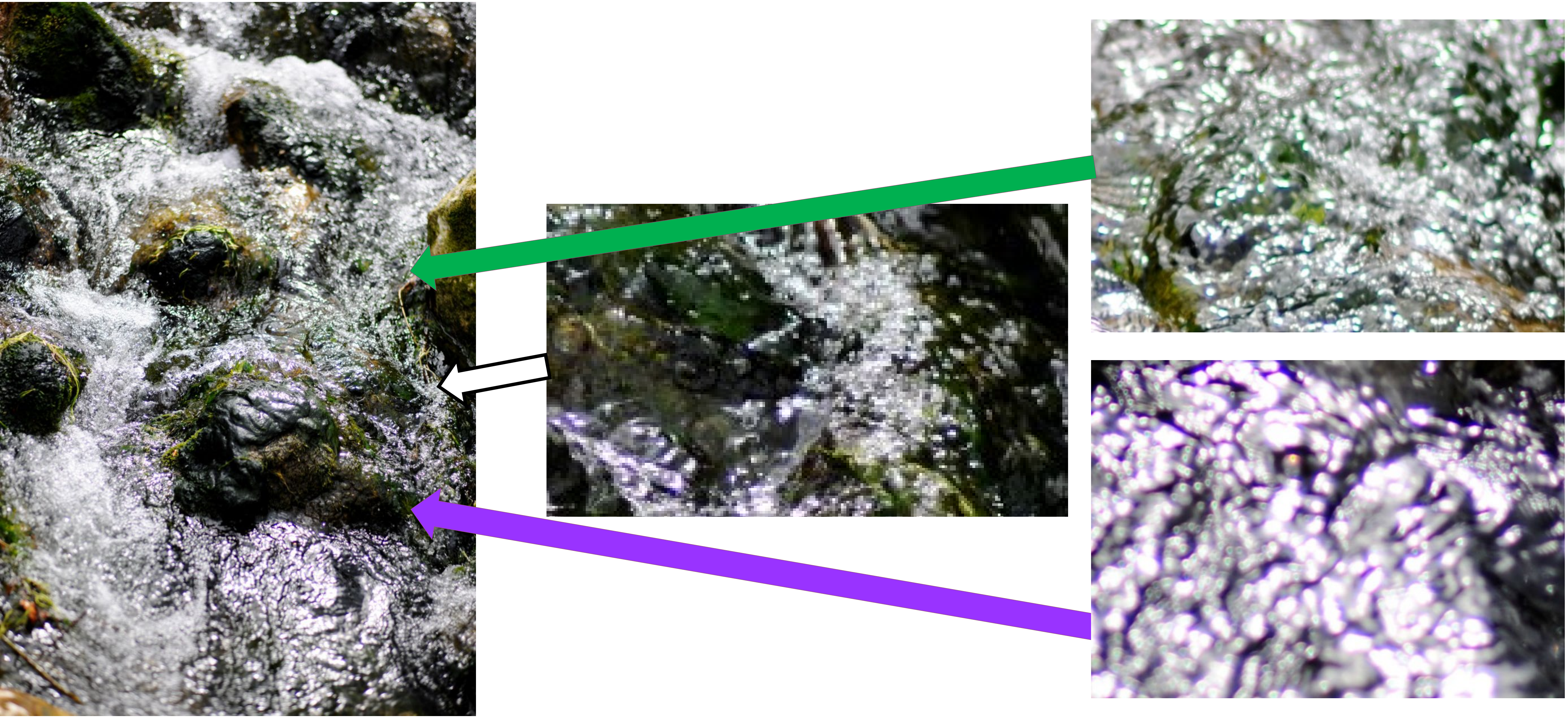




Through focus PSF with longitudinal chromatic aberration
Simplified picture, secondary spectrum



Longitudinal chromatic aberration of an achromat



Longitudinal chromatic aberration of an achromat

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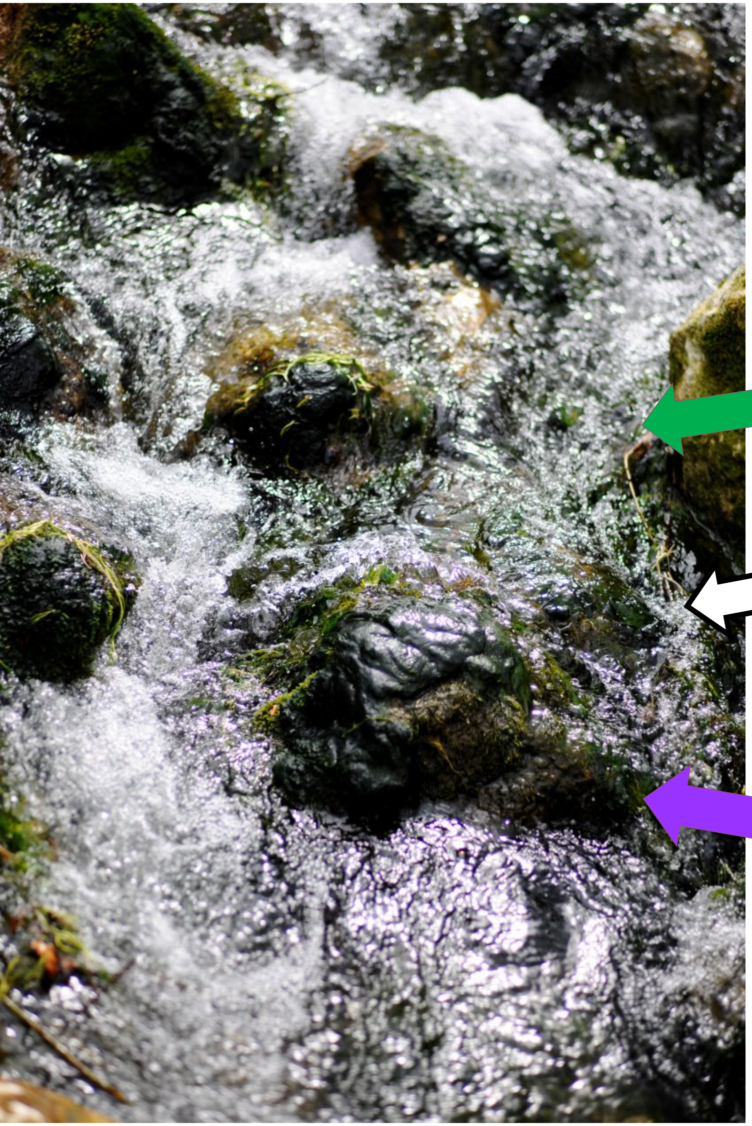
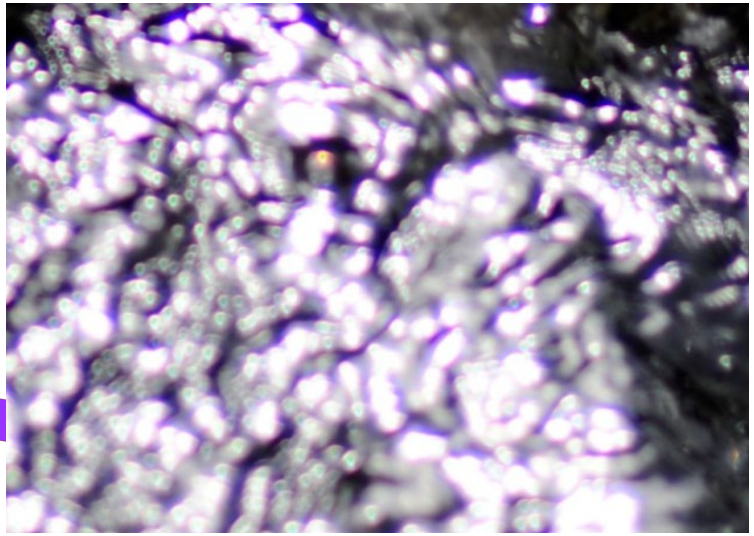
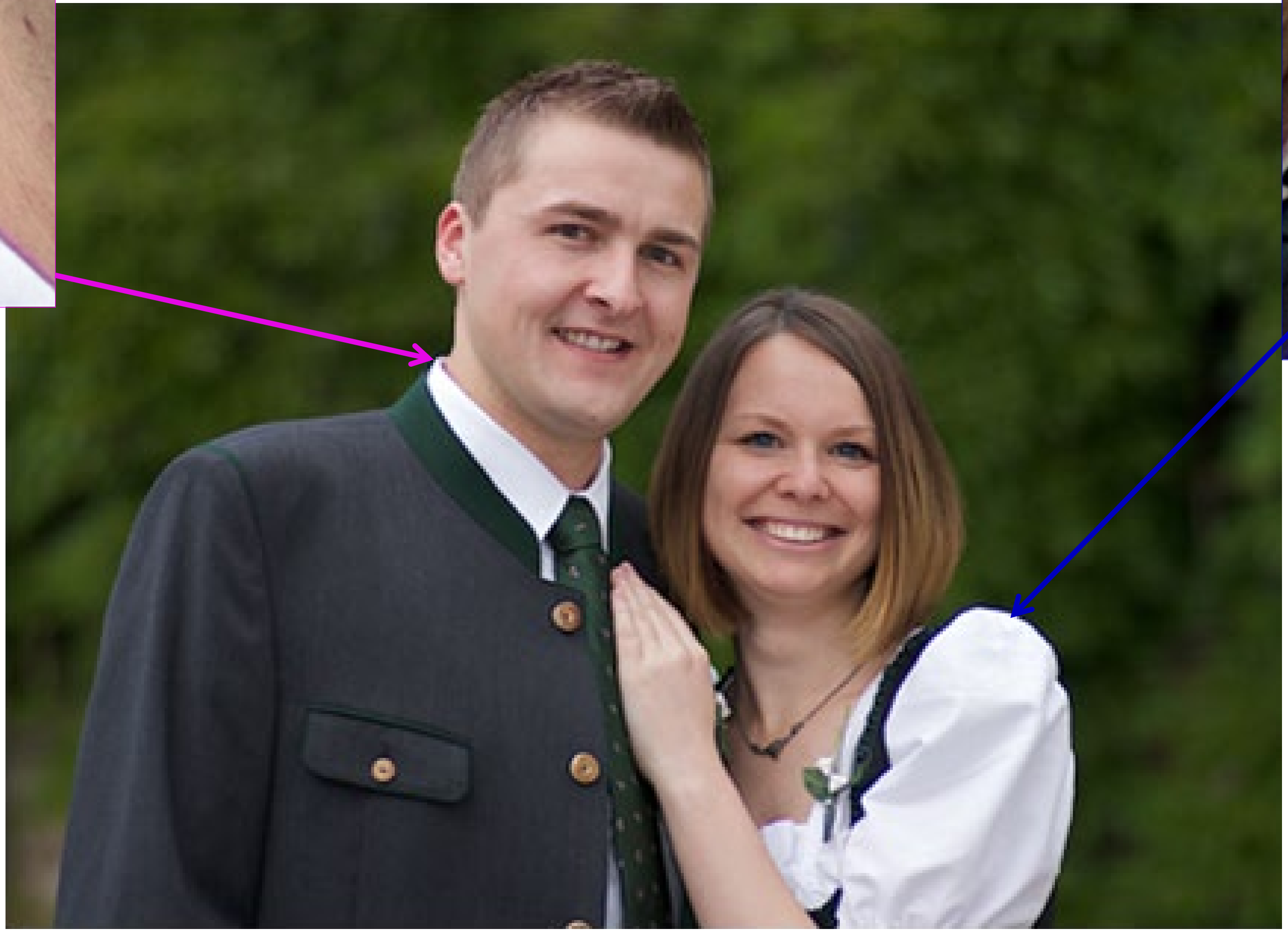


Green and purple fringes appear.

However we will shortly see that the explanation is not as straightforward, as we here should look on **edge spread functions**, not point spread functions!

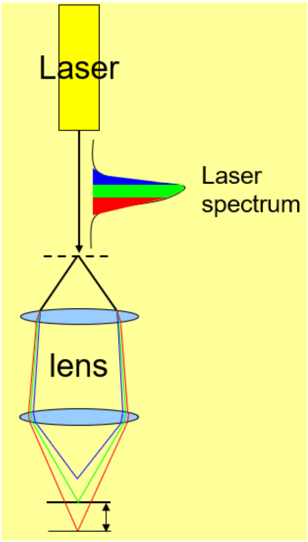


Color fringes in images



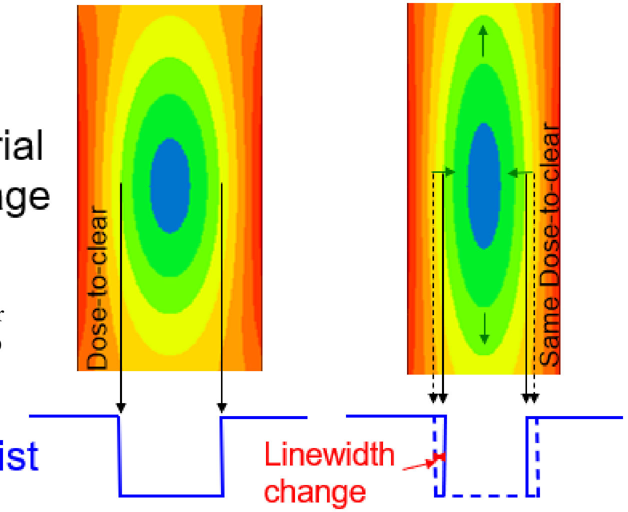
Impact of chromatic aberrations on imaging

Example Lithography: Fading (blurring) effect



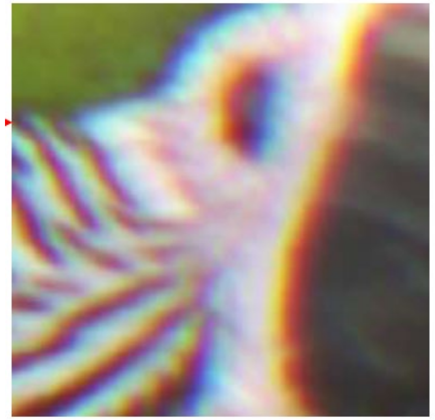
$$\Delta F = \underbrace{\frac{dF}{d\lambda}}_{\text{chromatic defocus by lens}} \cdot \underbrace{\Delta\lambda}_{\text{deviation to center wavelength } \lambda-\lambda_0}$$

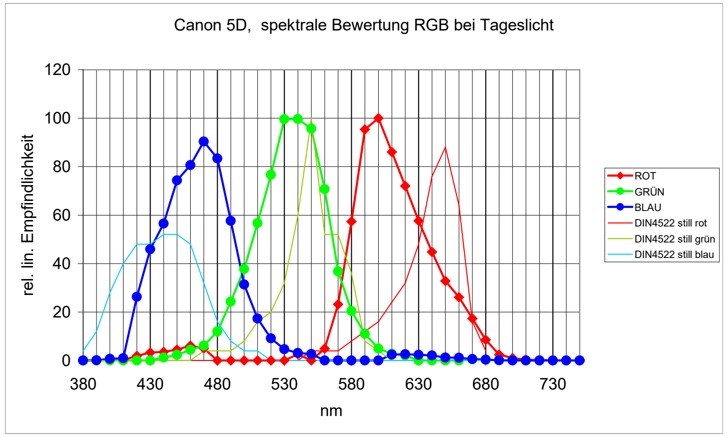
Aerial image



resist

Example Photography: Spatial separation in RGB and fading (blurring) effect





Spectral sensor response both shifts between RGB channels and blurs per channel

Greek **χρωμα** (chroma) → color

Often chromatic aberrations are associated with <u>color</u> fringes in images.
However this is not necessarily the case: e.g. in optical lithography there is only a “greyscale image” (resist). Nevertheless chromatic aberration of the projection lens is highly relevant, e.g. for optical proximity matching: a laser bandwidth variation in conjunction with lens chromatic aberration leads to feature dependent linewidth deviation (optical proximity error).

Edge spread function for chromatic aberration evaluation

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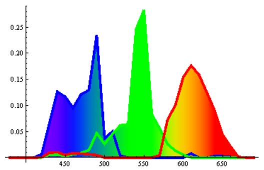
spectral response



spectral light source distribution

monochromatic PSF

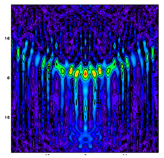
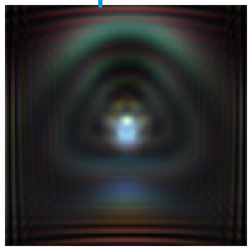


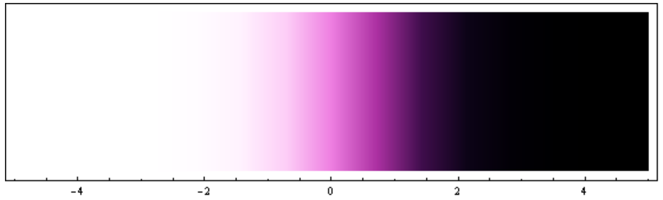
image intensity

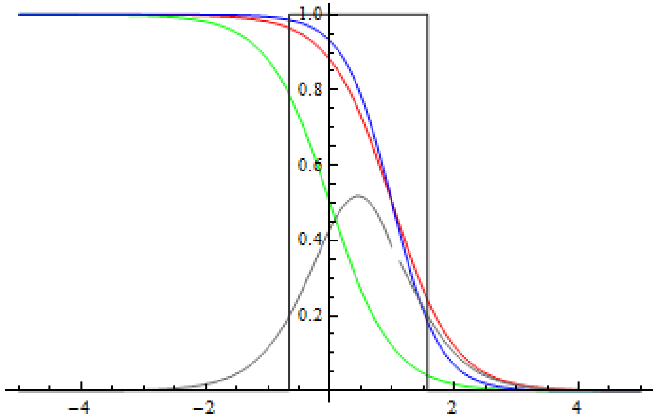
object

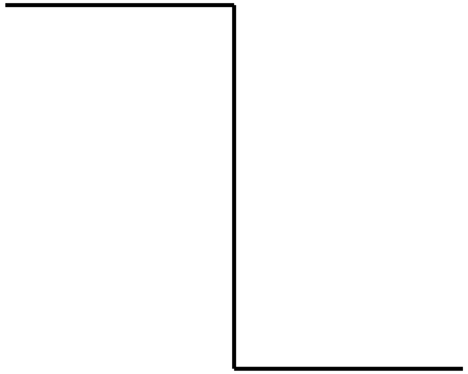
$$\underbrace{I_{\{R,G,B\}}(x)}_{} = \iint_{\text{object surface}} d\xi d\eta\ \underbrace{I_{ob}(\xi)}_{} \int_{\text{wavelength}} \underbrace{d\lambda\ s_{\{R,G,B\}}(\lambda) q(\lambda)\ PSF(x - m\xi, \lambda)}_{}$$

photorealistic PSF









The spectral weight $g_i(\lambda) := s_i(\lambda) q(\lambda) t(\lambda)$ with

$s_i(\lambda)$ spectral sensor response
$q(\lambda)$ spectral source (scene) distribution
$t(\lambda)$ spectral lens transmission

must be normalized, $\int d\lambda\, g_{\mathrm{i}}(\lambda) = 1$ for all color channels i=(R,G,B).

Relative spectral sensitivity of the eye

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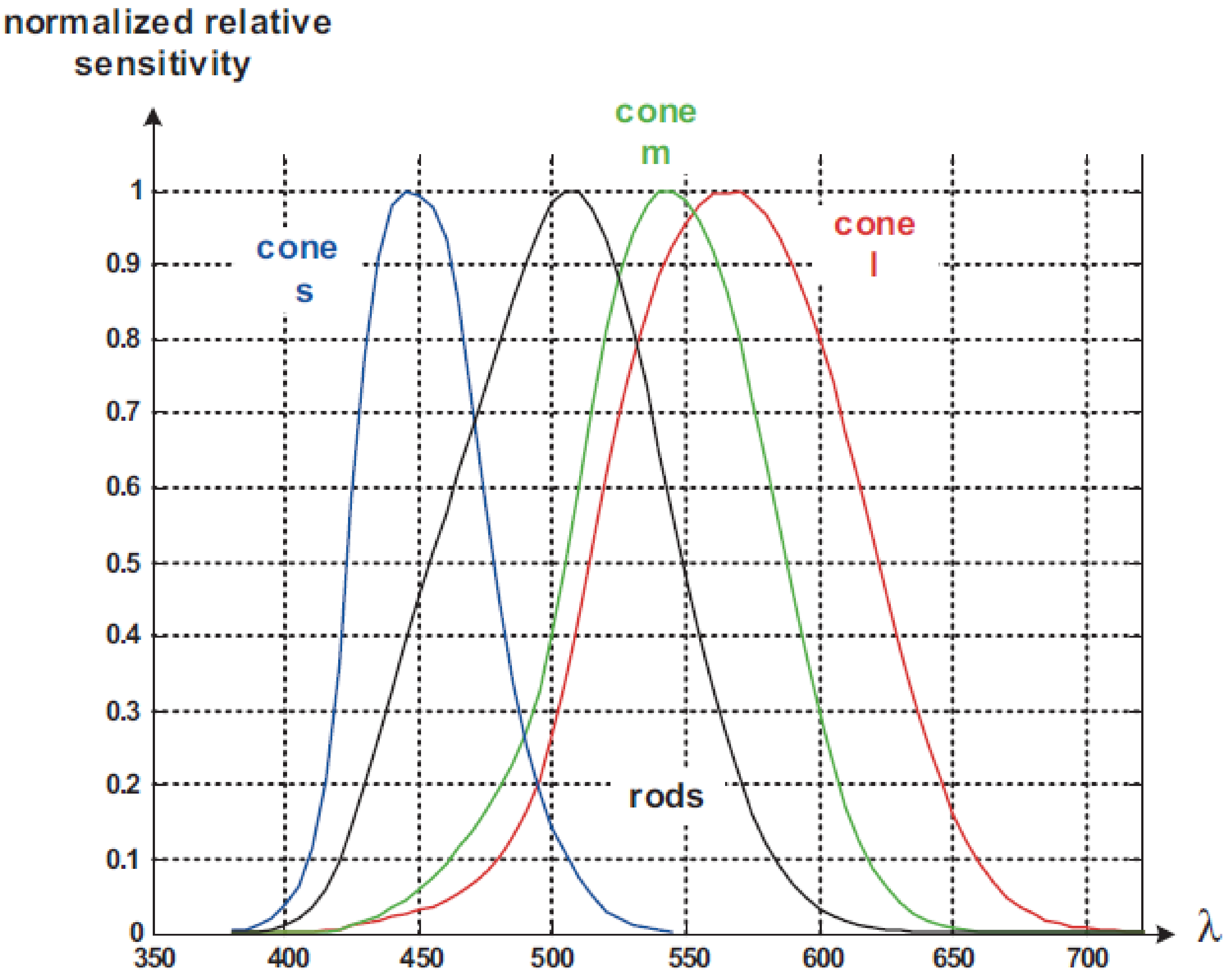
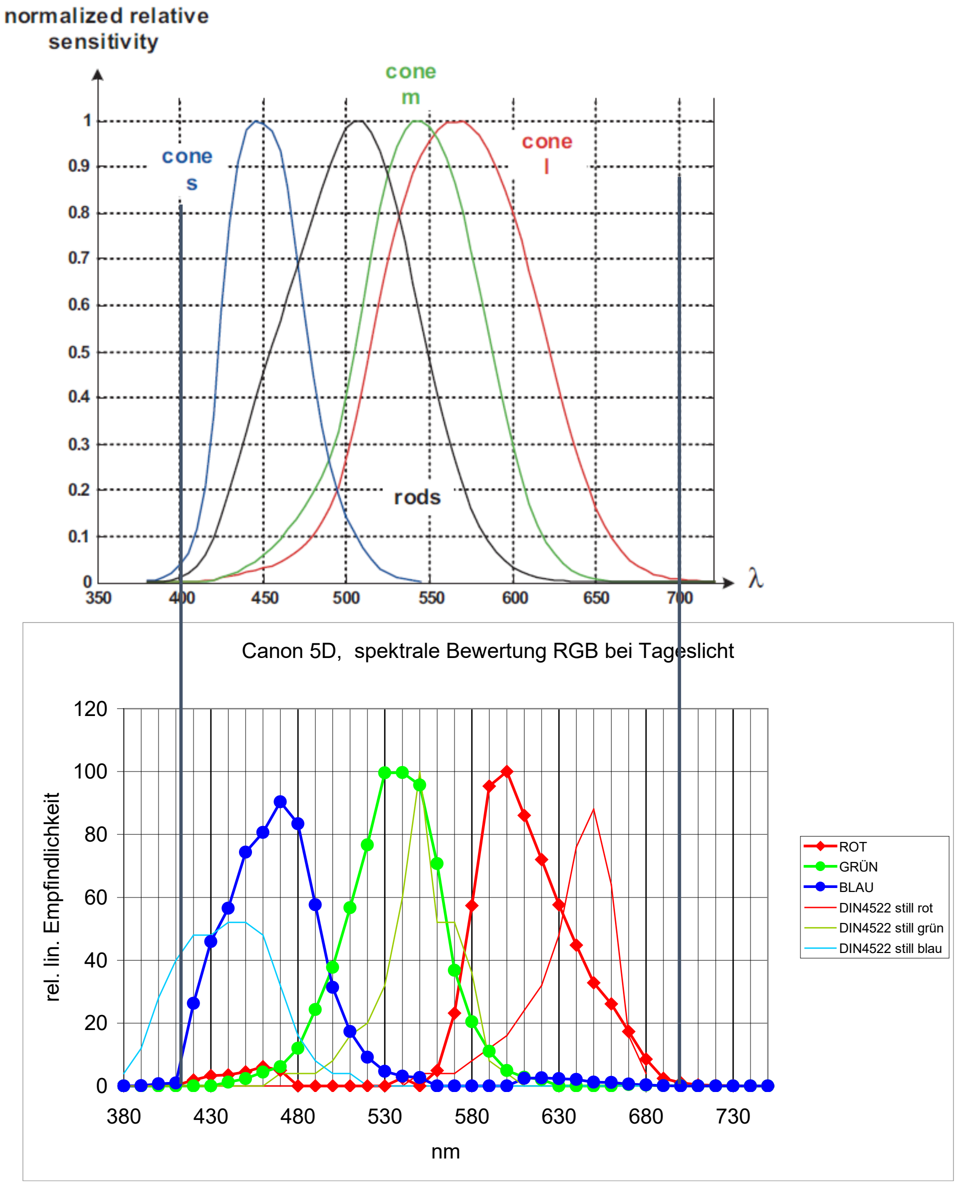


Night vision (scotopic) with rods

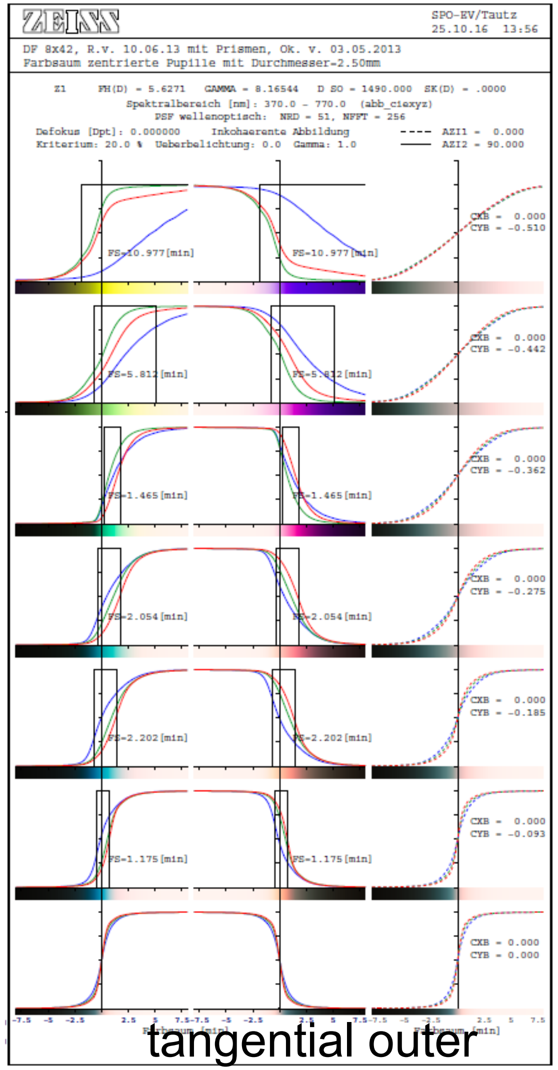
Daytime vision (photopic) with cones (3 types give the sensory impression "color")

Relative spectral sensitivity eye vs. camera sensor



The spectral sensitivity range of the eye and digital camera is similar:
approx. 410 - 680nm.
In the eye, the green and red sensitivity ranges are closer together than in digital camera sensors.

Color fringing at edges and color fringing width



Increasing angle of field

tangential outer

tangential inner

sagittal

Color fringing at edges and color fringing width

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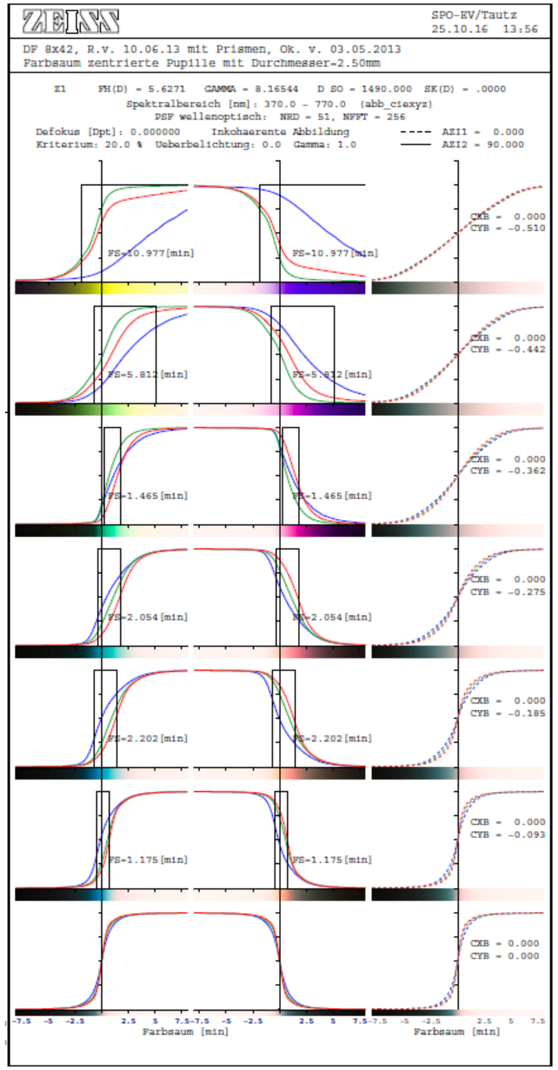
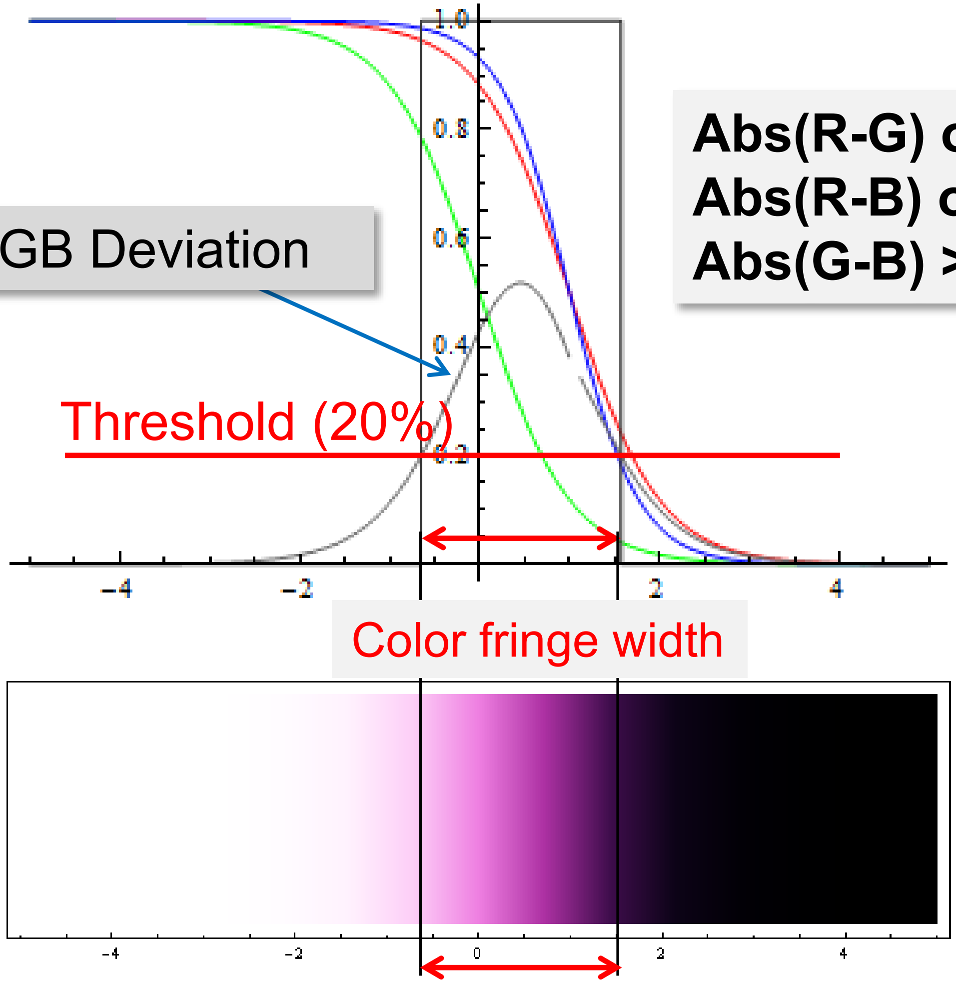


The simulation of the images of edges provides color fringing widths.
Criterion:

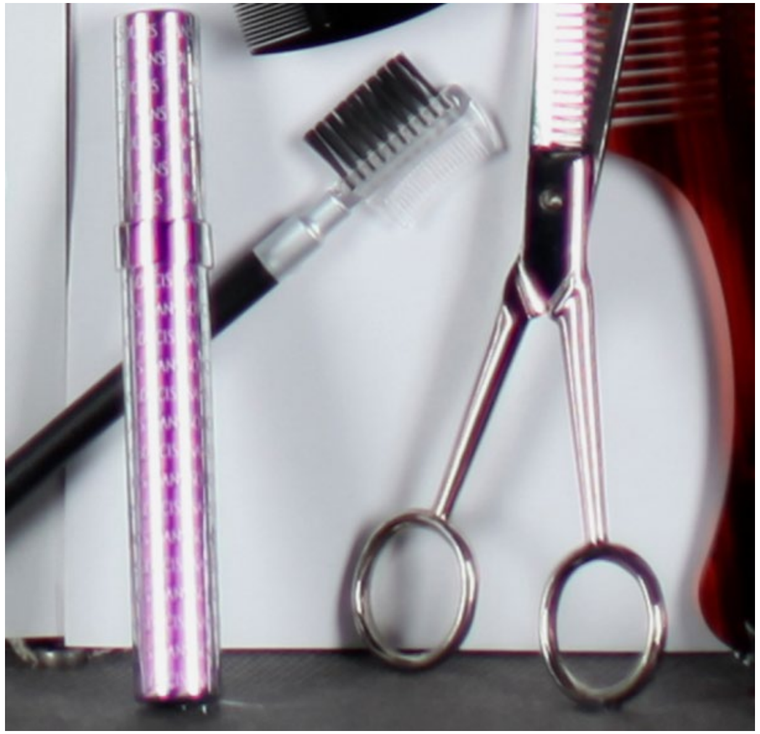
Abs(R-G) or
Abs(R-B) or
Abs(G-B) > 20%

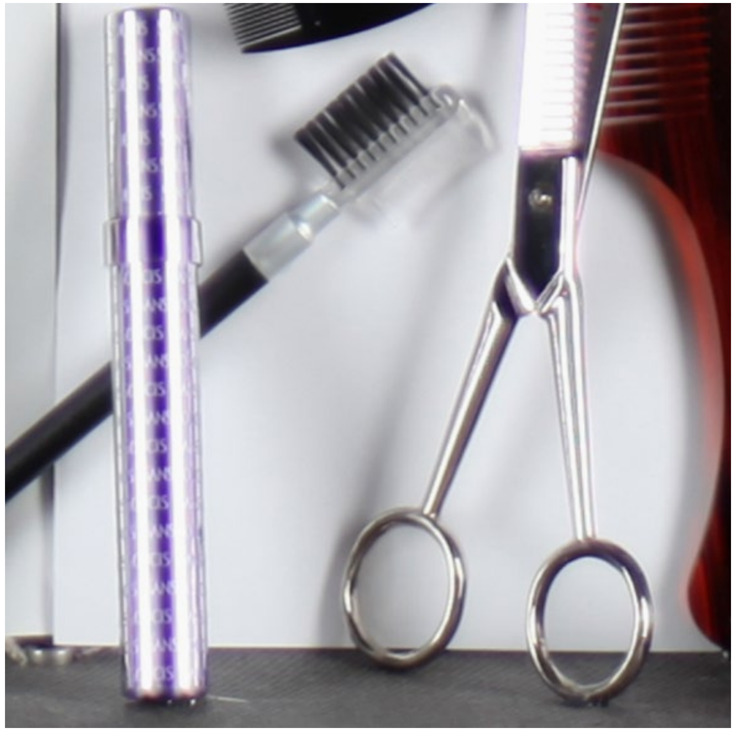


Color fringes and exposure

It is very important to include exposure in imaging simulations.

Color fringes are significantly enhanced near bright sources or strongly reflecting regions.





Overexposure

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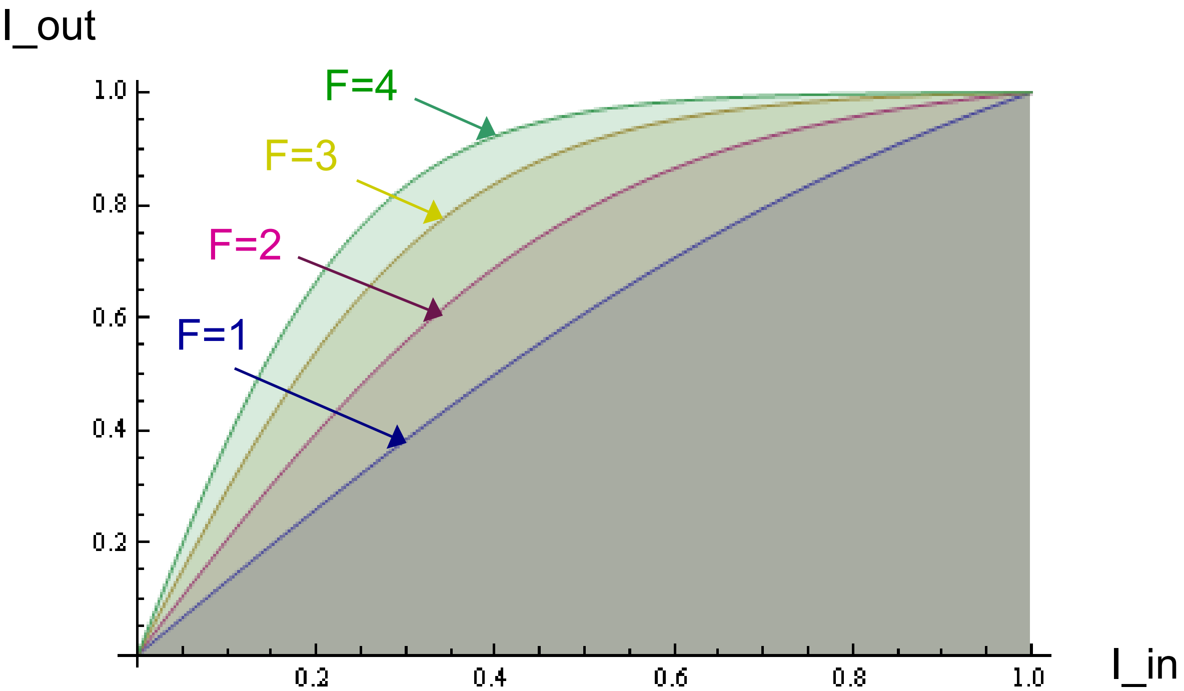
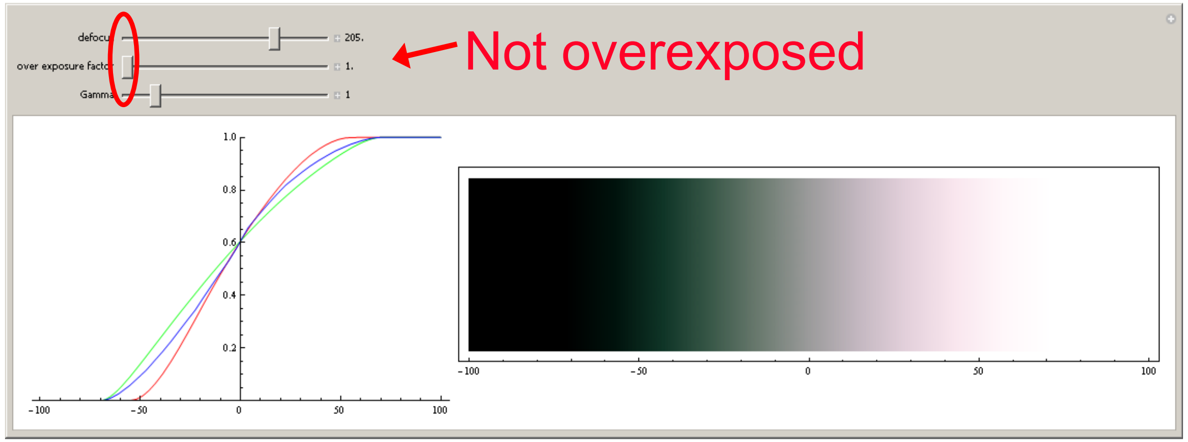
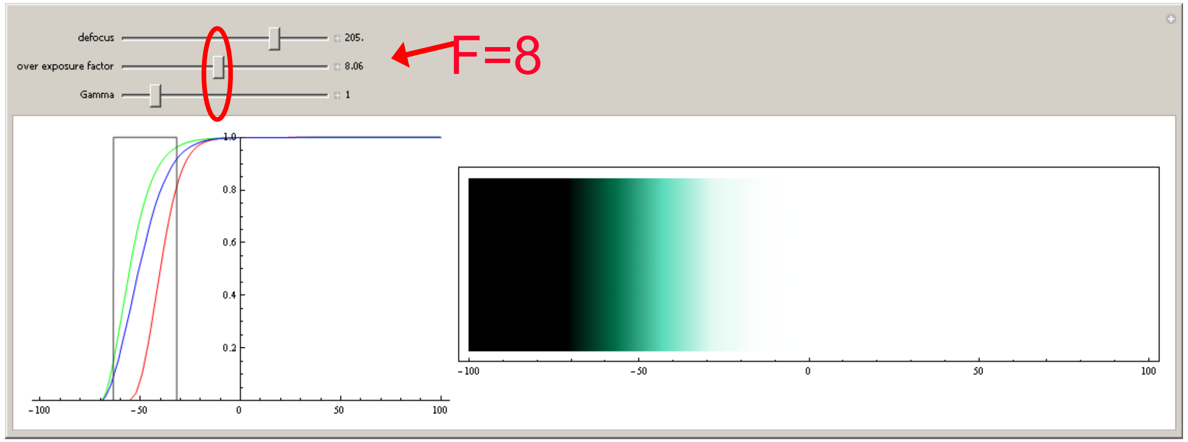
$$I \rightarrow \frac{\tanh\left(F \cdot \frac{I}{I_{max}}\right)}{\tanh(F)} \cdot I_{max}$$

Simplified model of (over)exposure is determined by the opto-electronic conversion function (OECF).

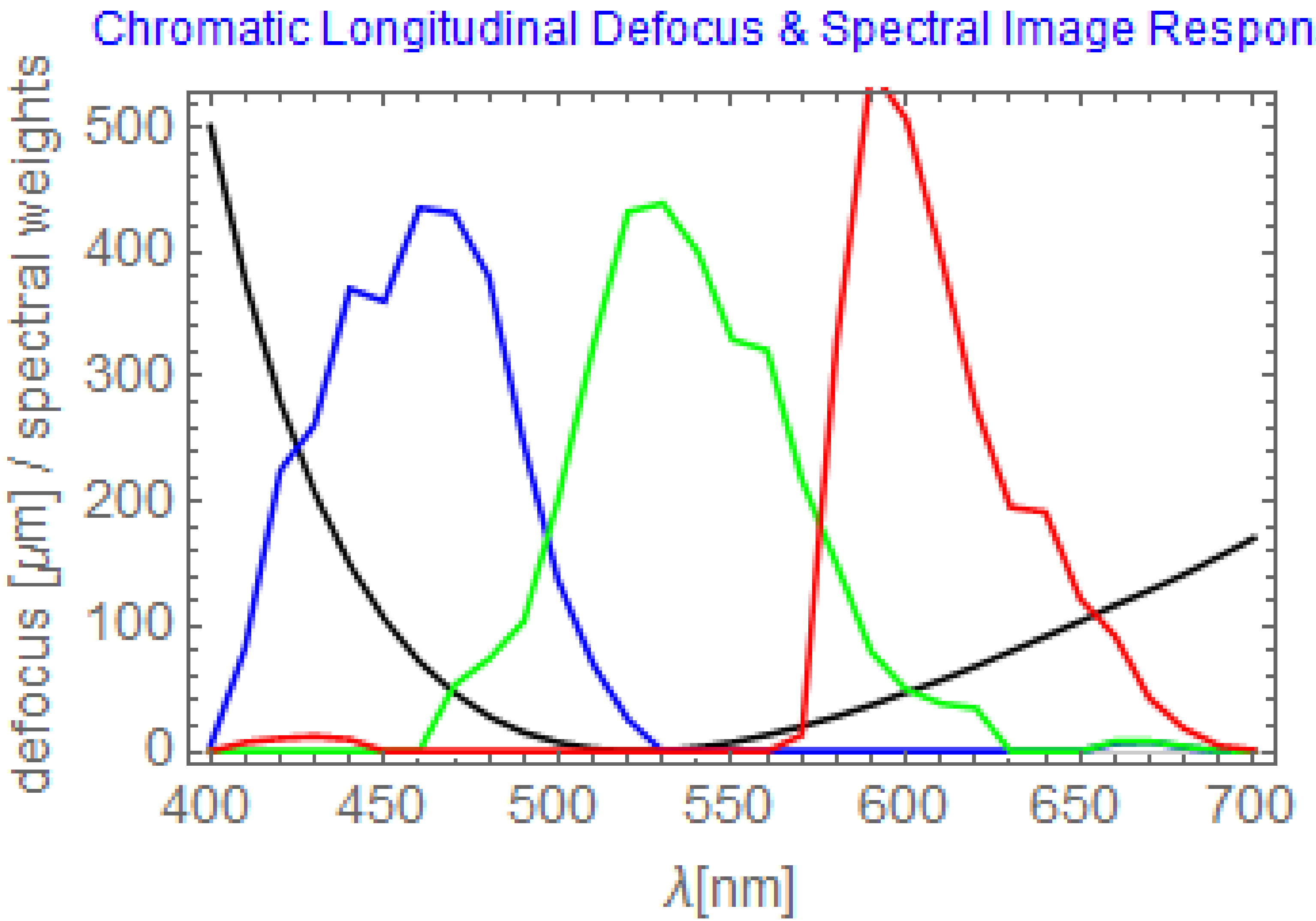
I_{max} = maximum intensity level, e. g. for 8bit = 255
I = actual intensity value
F = over exposure factor



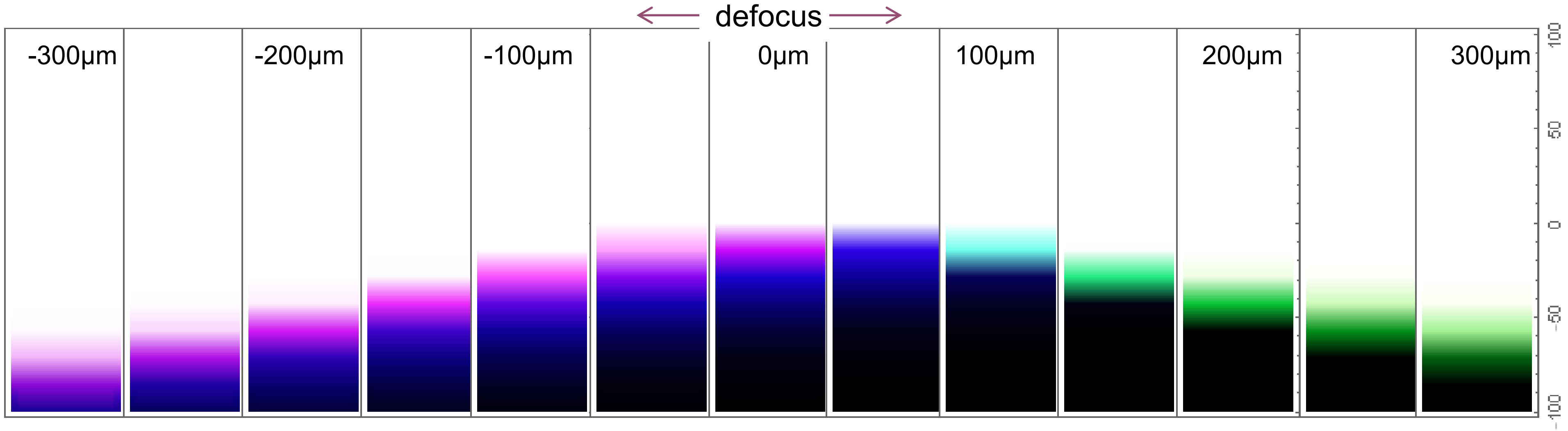


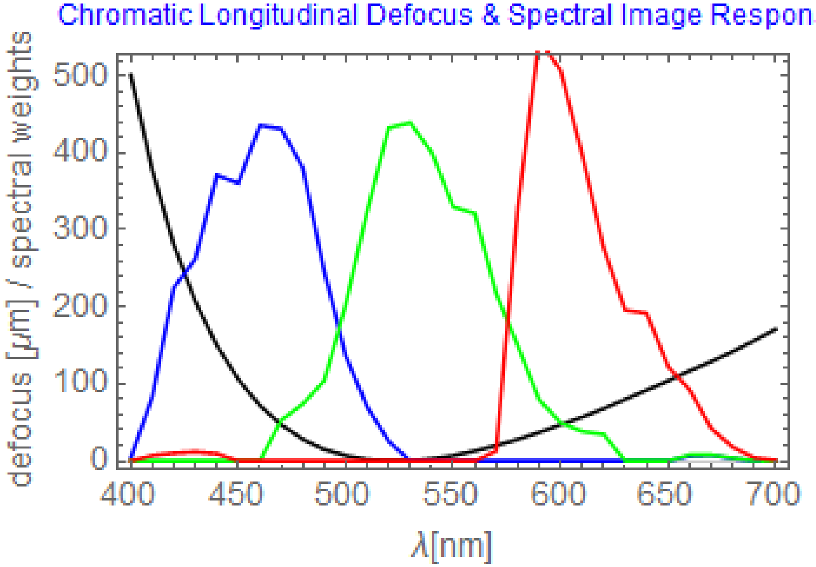


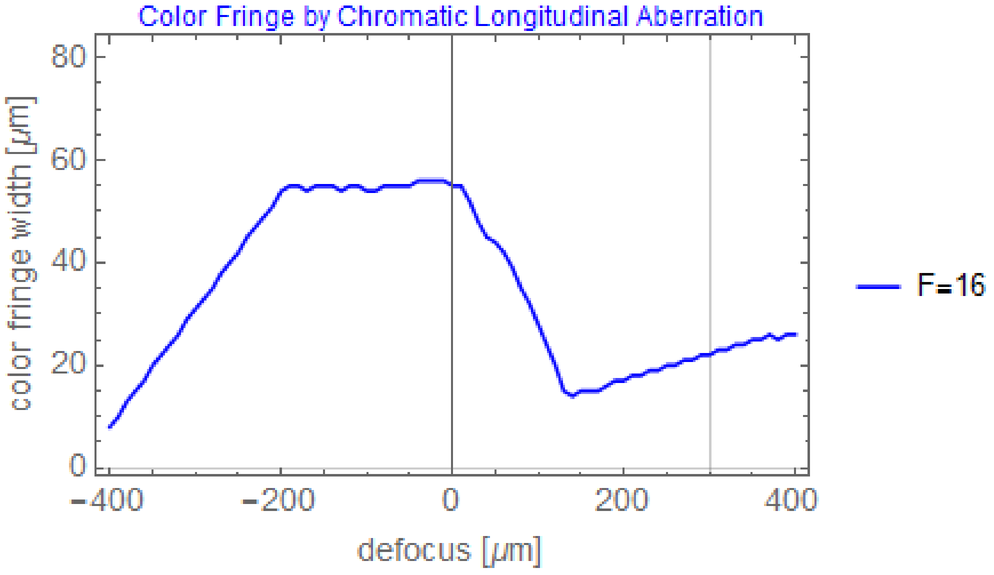
Longitudinal chromatic aberration and color fringing
Example: CHL(λ) and spectral response distribution



Longitudinal chromatic error and color fringing







Longitudinal chromatic error and color fringing: Normal exposure

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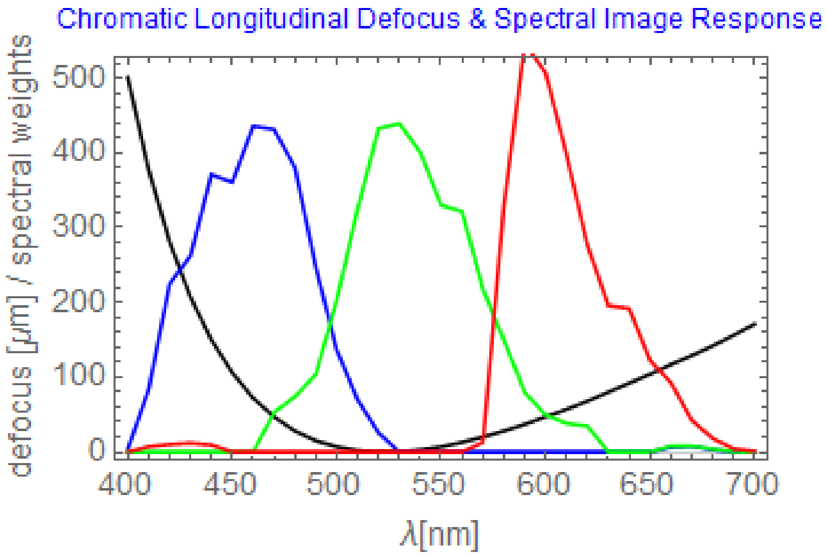
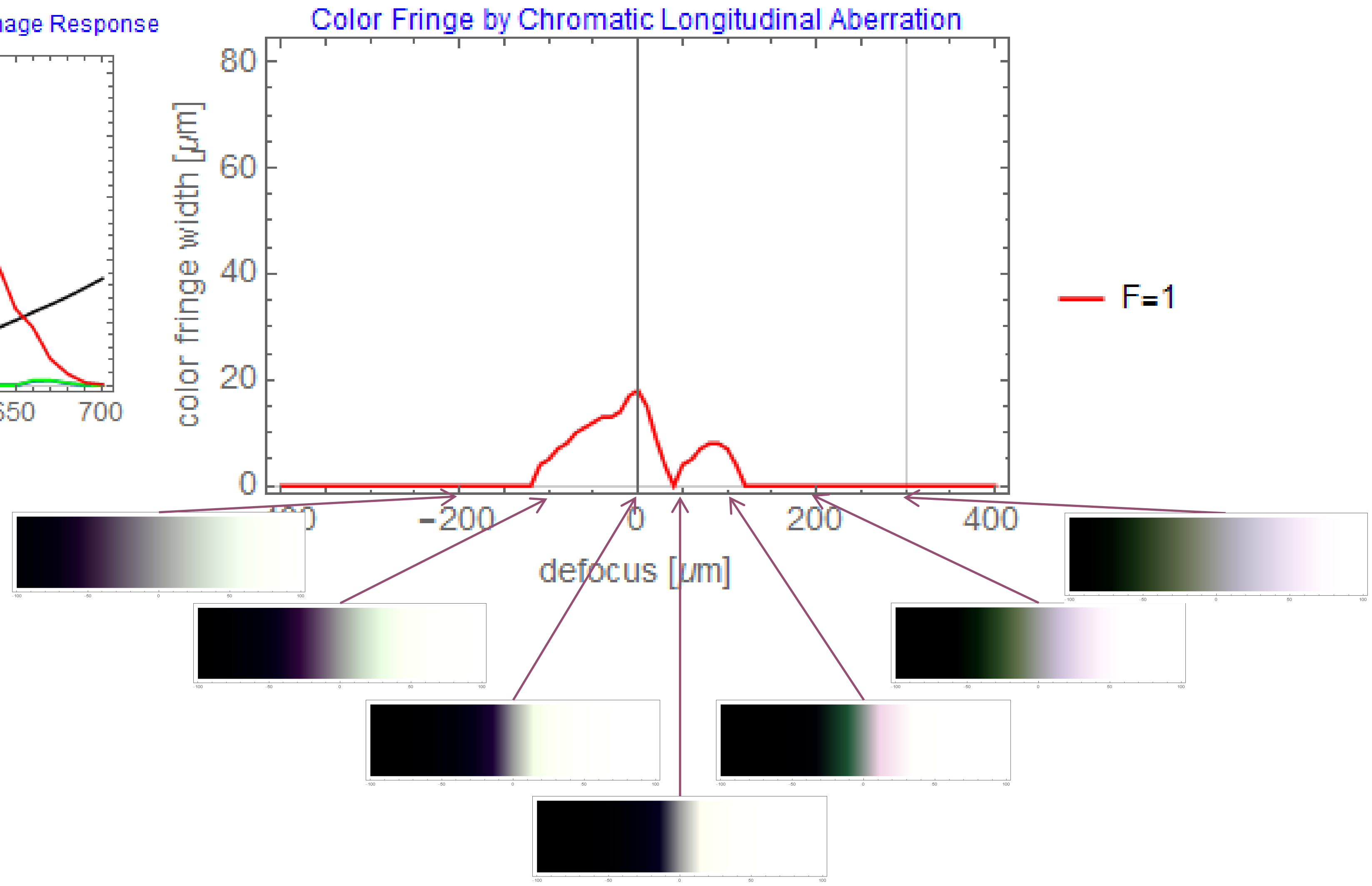
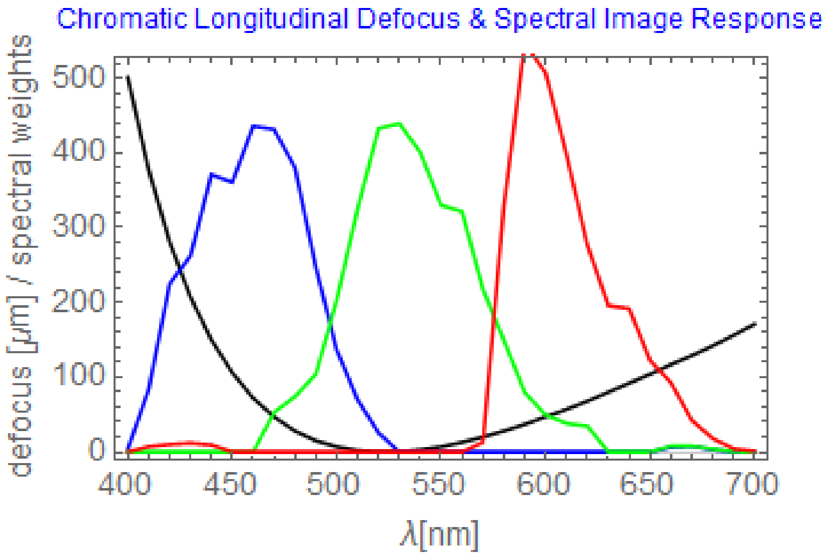
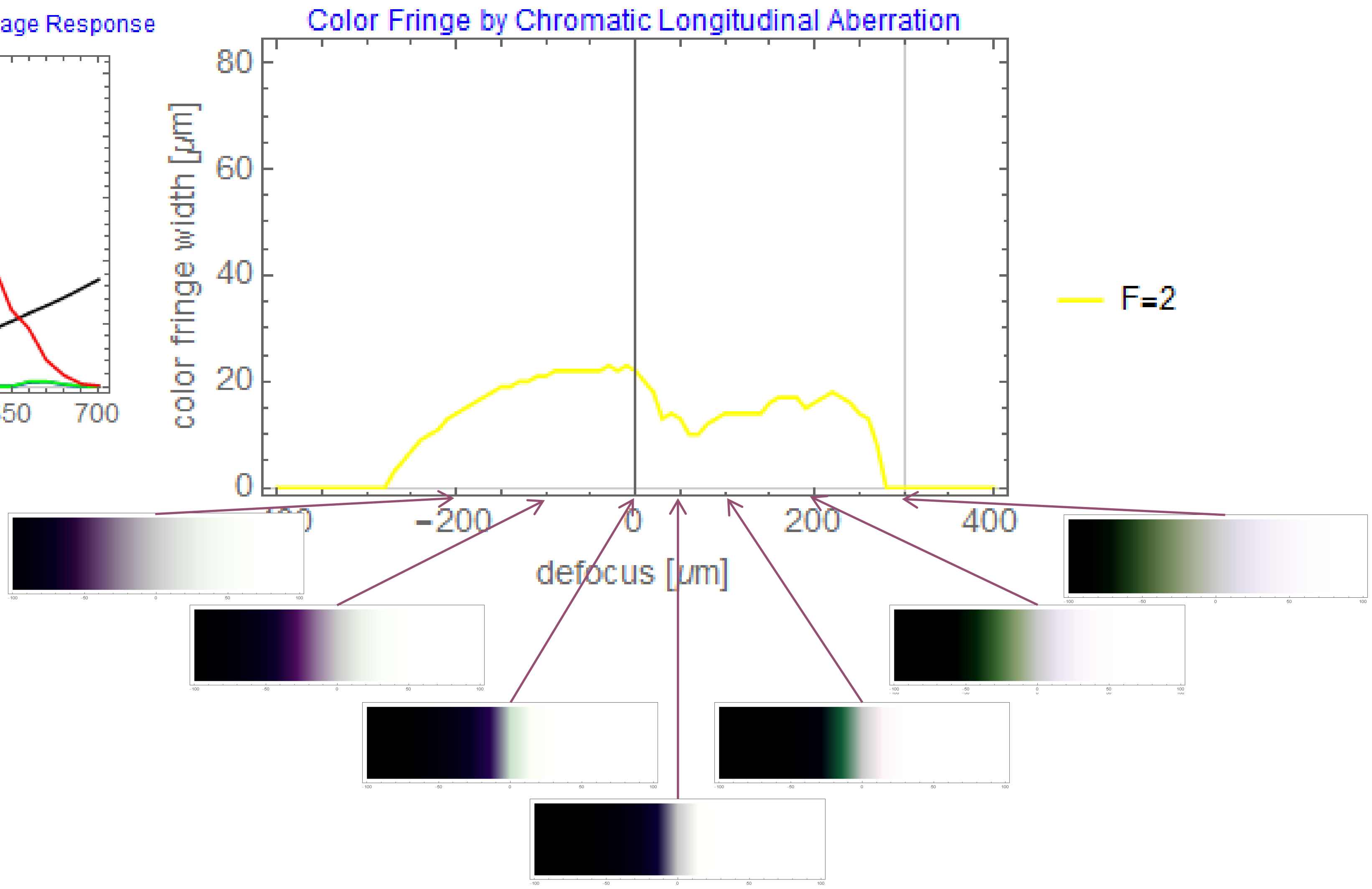


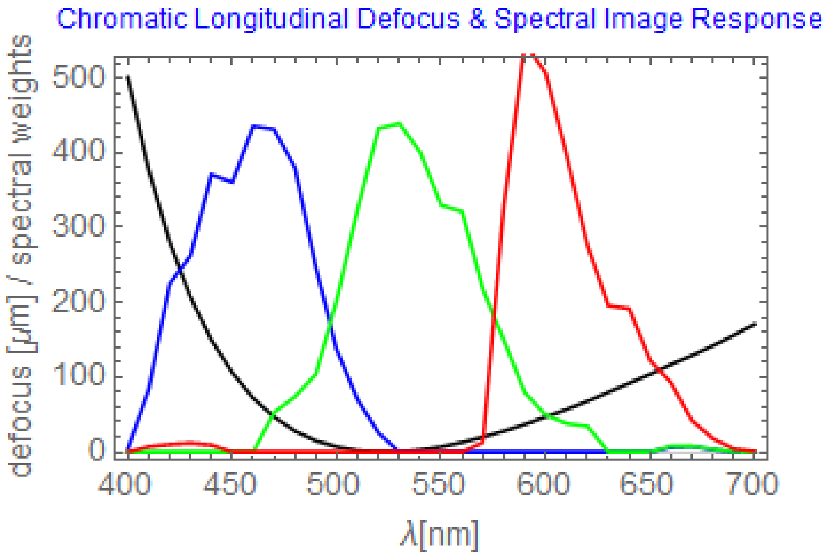


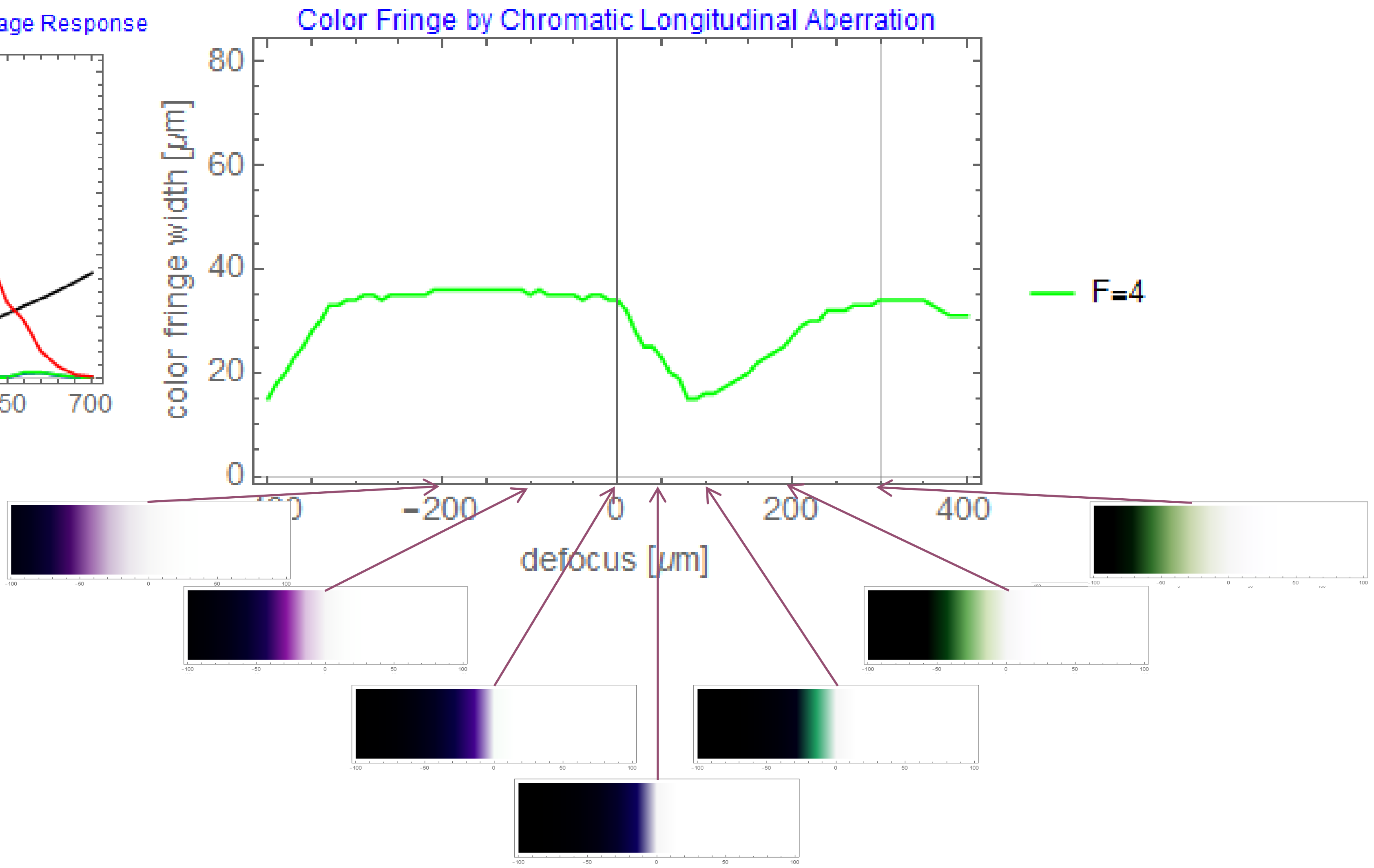
Longitudinal chromatic error and color fringing: 2x overexposed





Longitudinal chromatic error and color fringing: 4x overexposed





Longitudinal chromatic error and color fringing: 8x overexposed

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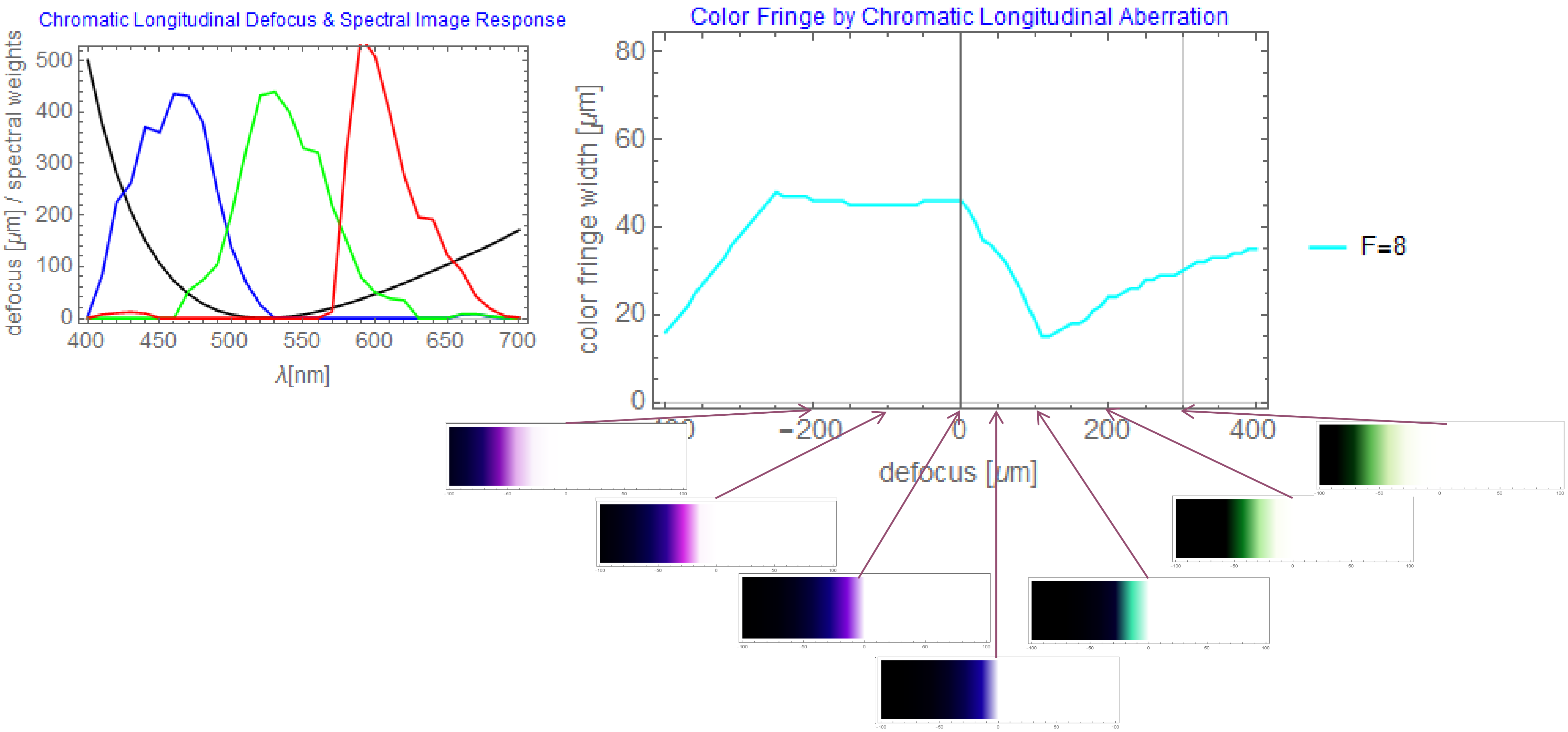
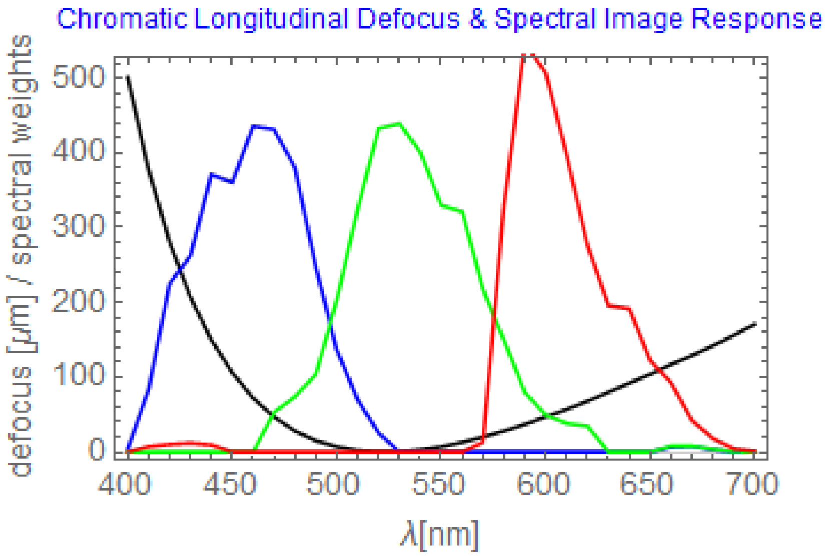
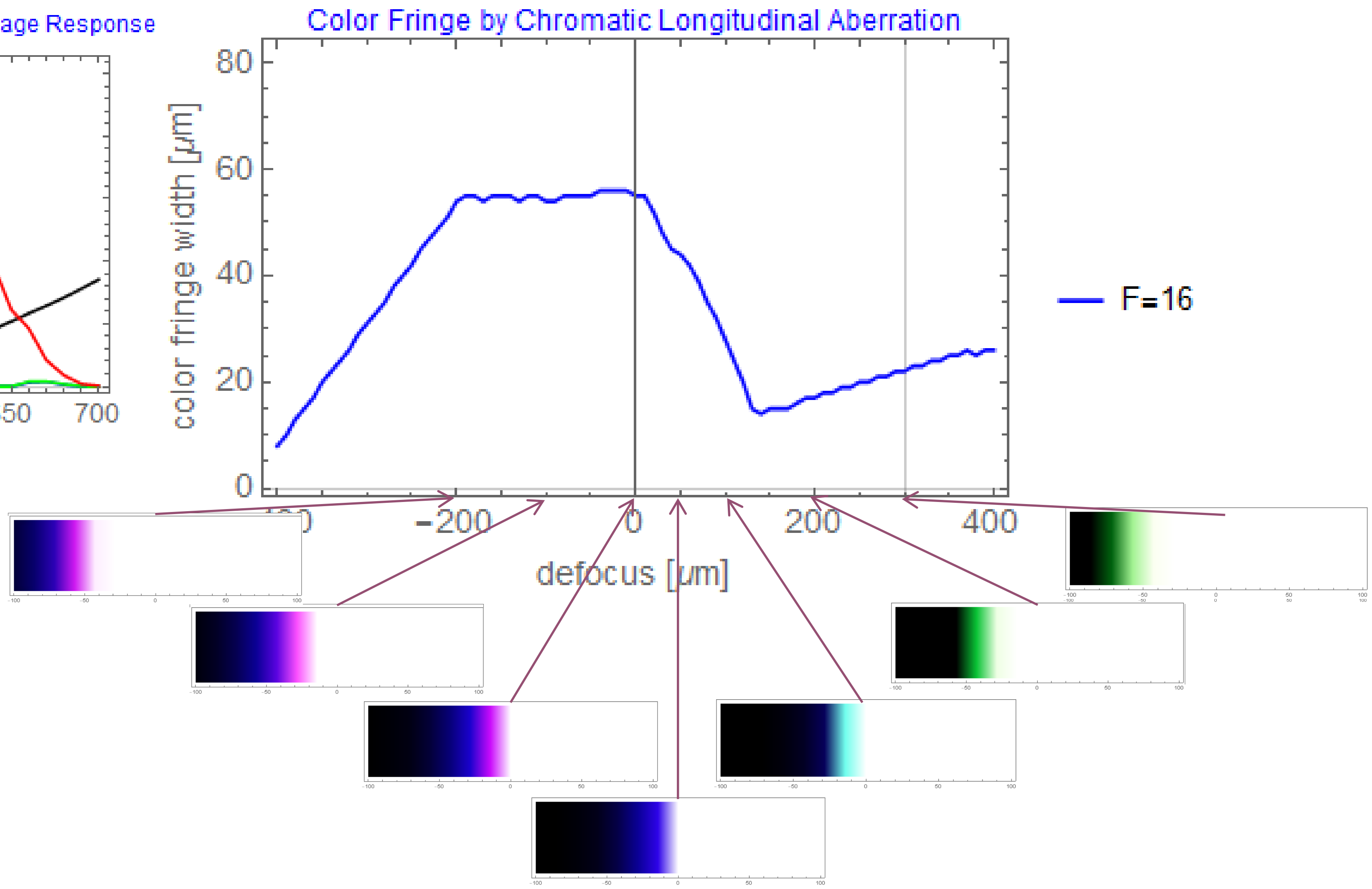


Longitudinal chromatic error and color fringing: 16x overexposed





Longitudinal chromatic error and color fringing: 1000x overexposed

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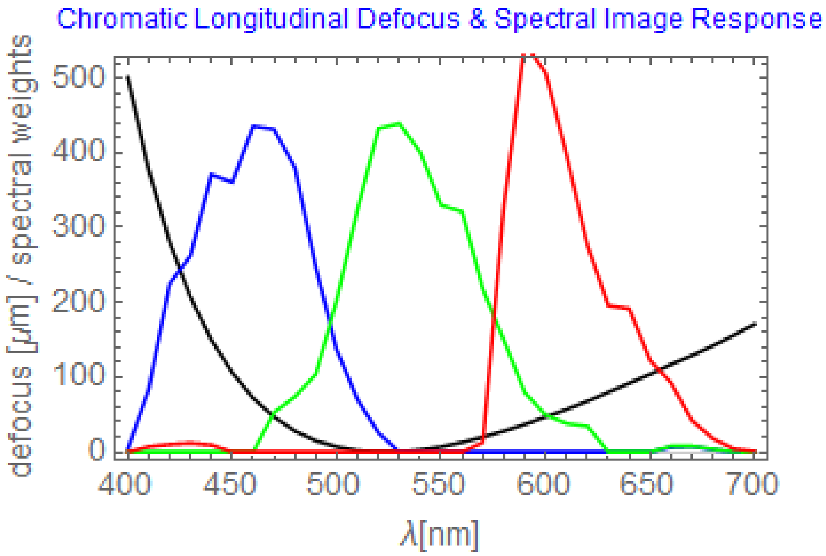
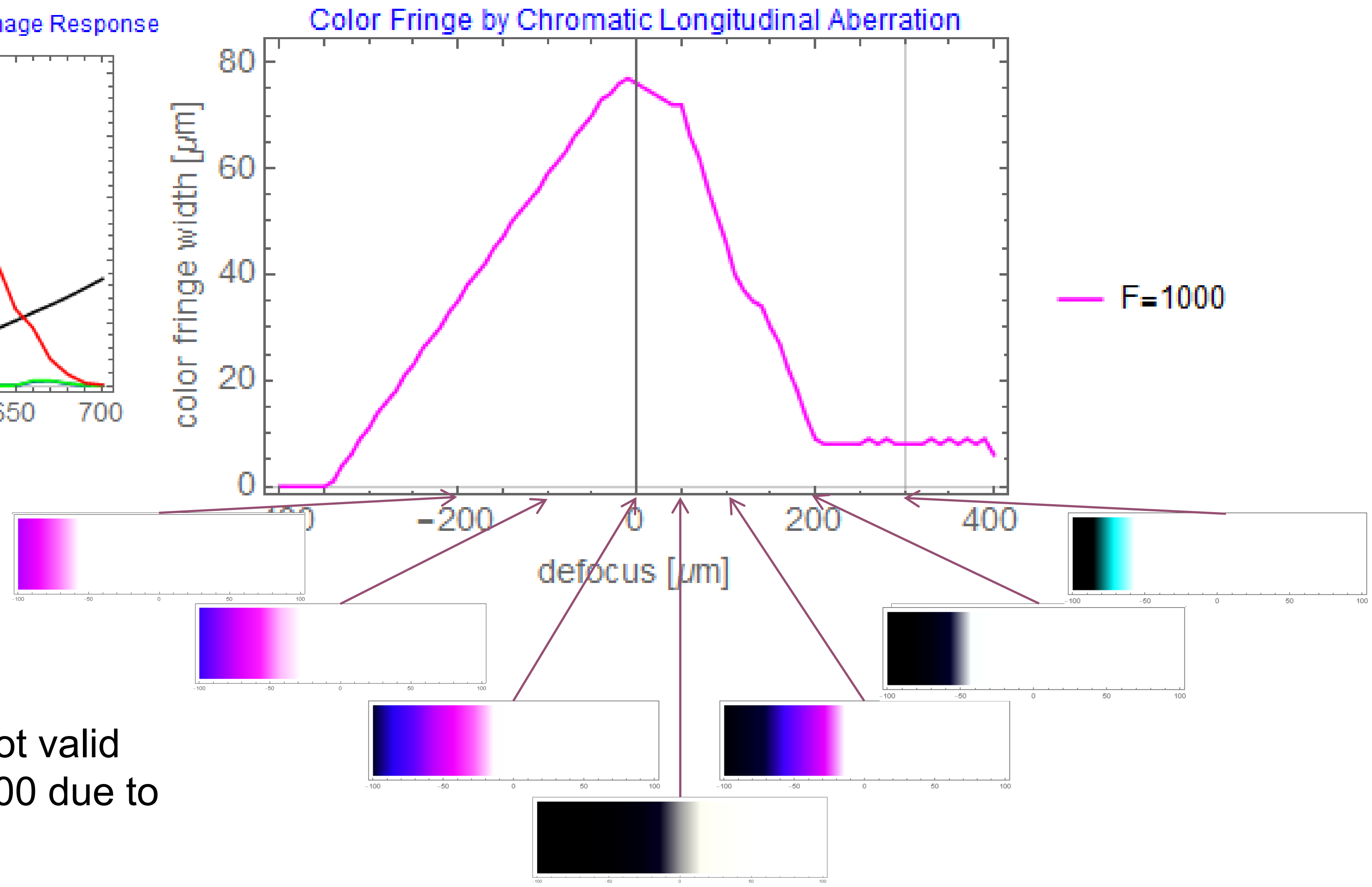
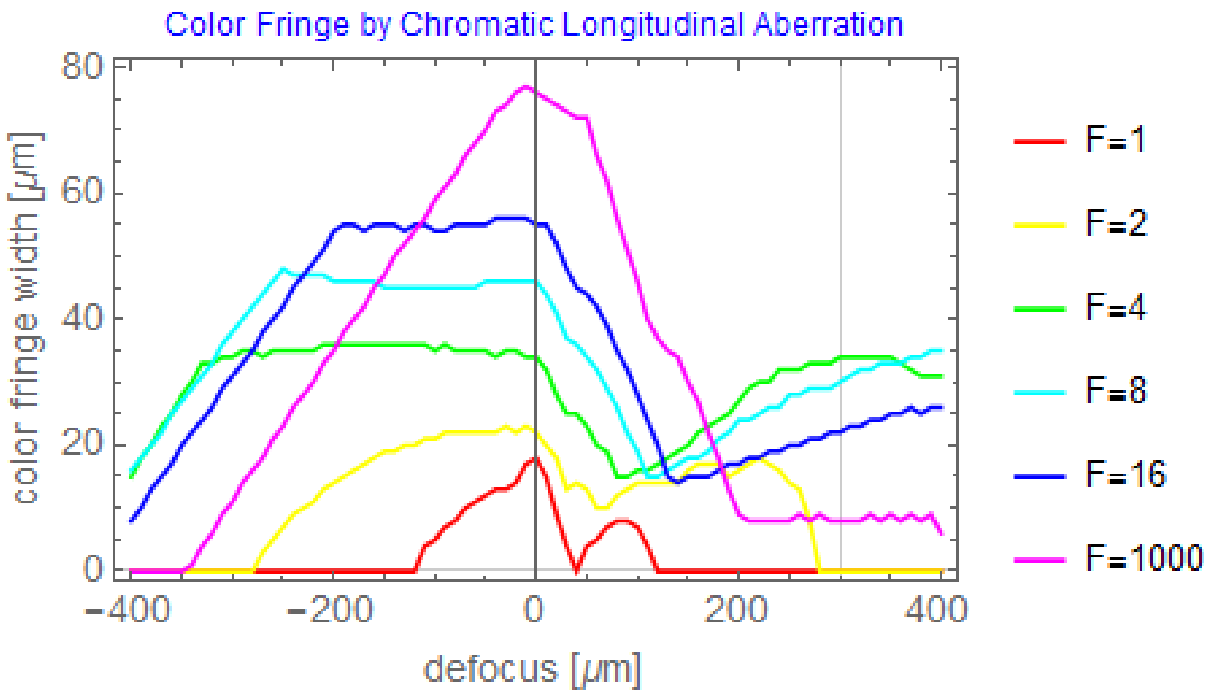


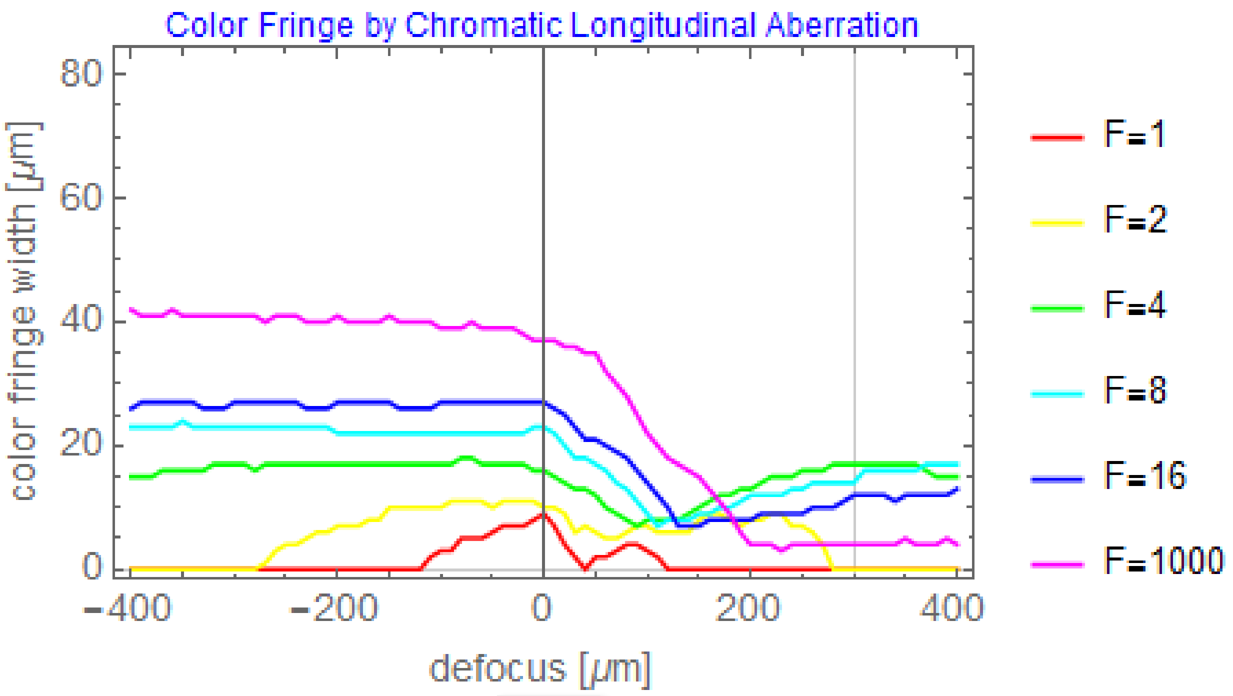


Note: color fringe vs focus not valid outside range 0 < defocus 100 due to limited evaluation range

Color fringe width and lens aperture



f/2

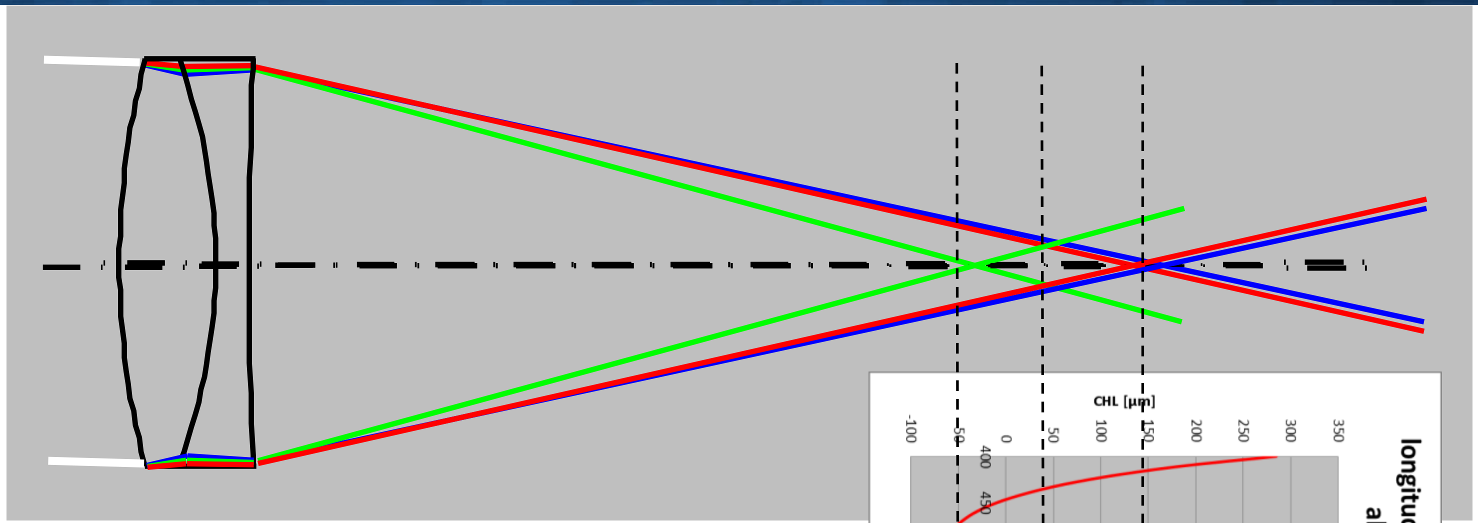


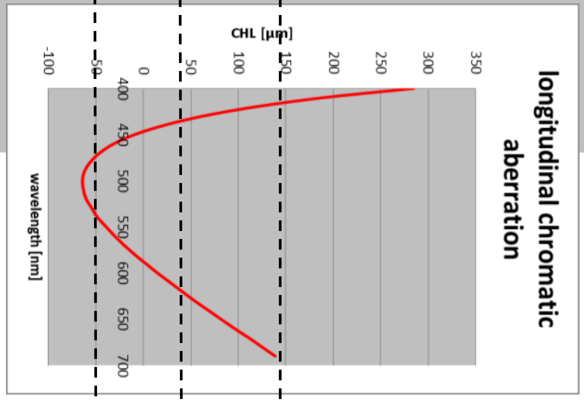
f/4

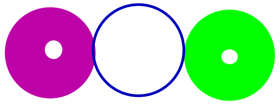
Rules of thumb:
Color fringing due to longitudinal chromatic aberration increases linearly with increasing aperture
The depth ranges for which color fringing can be seen increases with the degree of overexposure

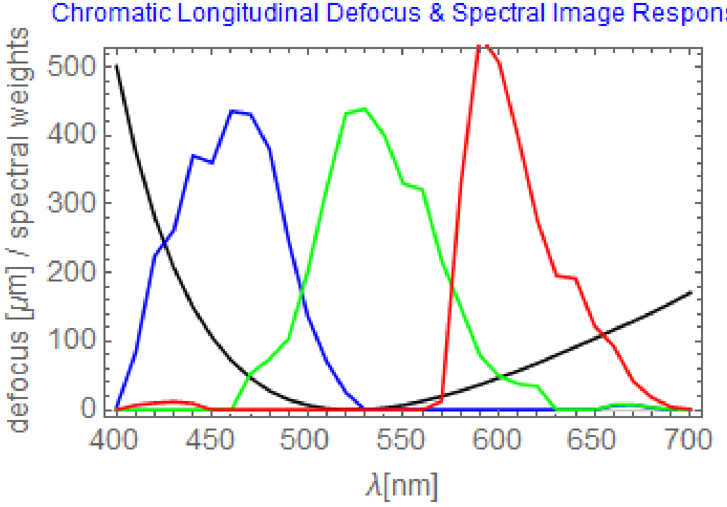
* however no simple proportionality relation

Longitudinal chromatic aberration of an achromat

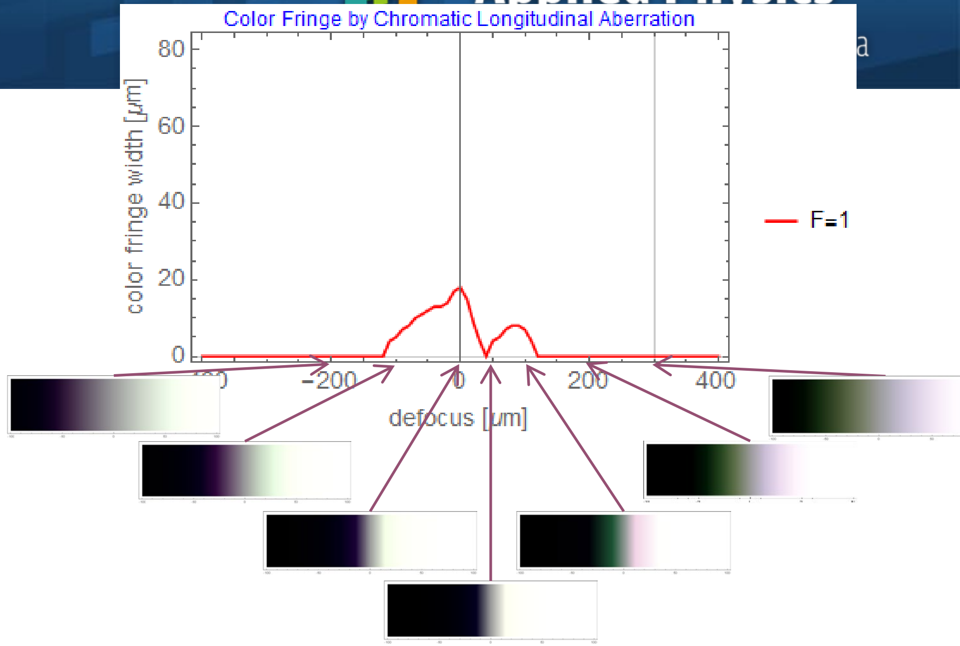


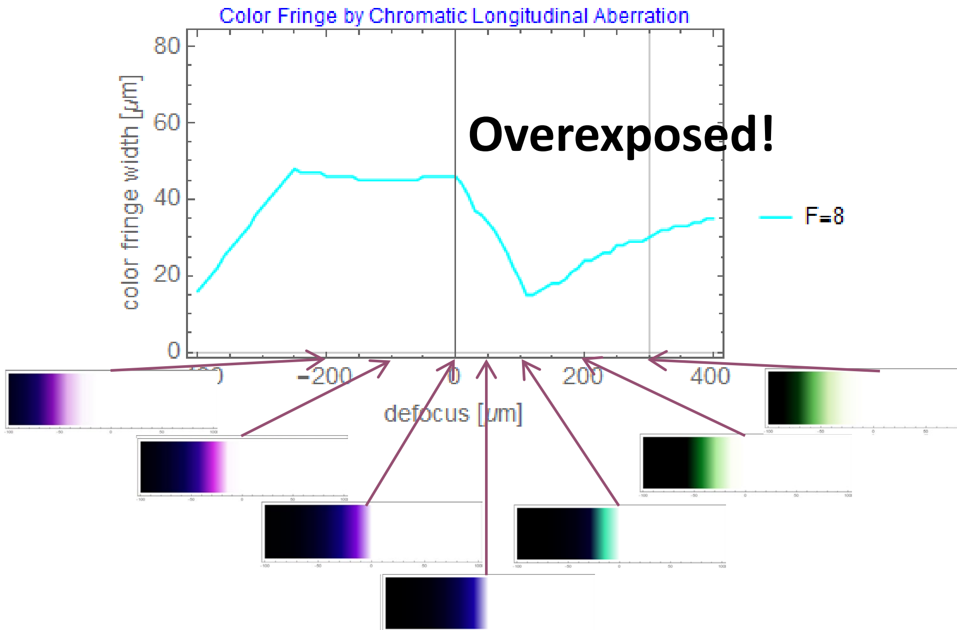






Long. chromatic aberration significantly depends upon exposure!





Longitudinal chromatic aberration curve: influence on colors and color fringe width

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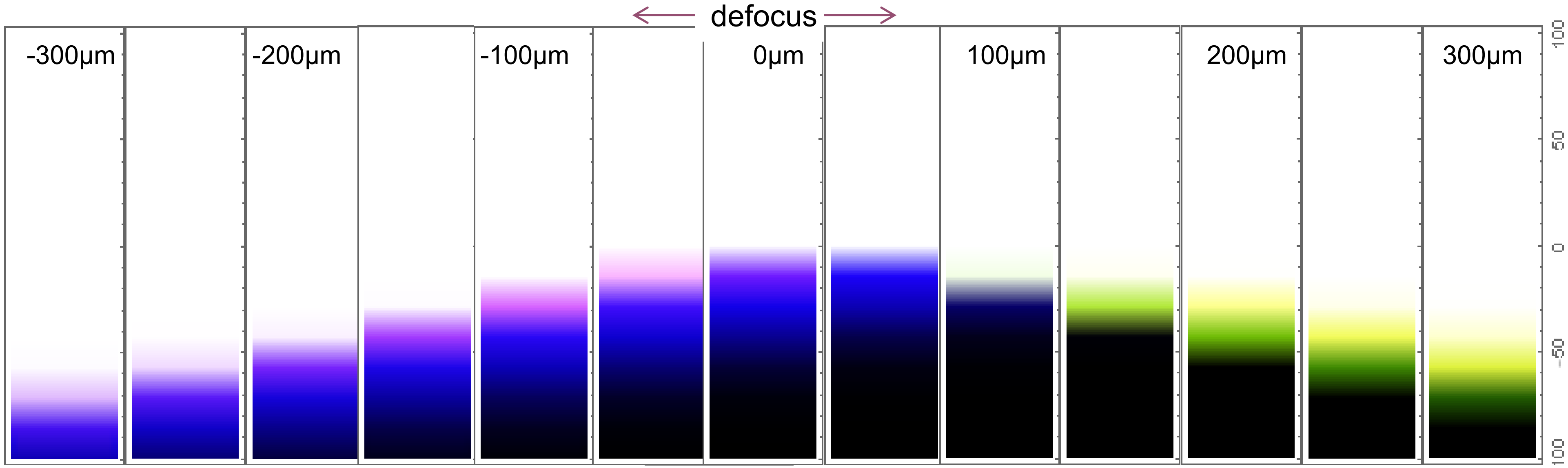
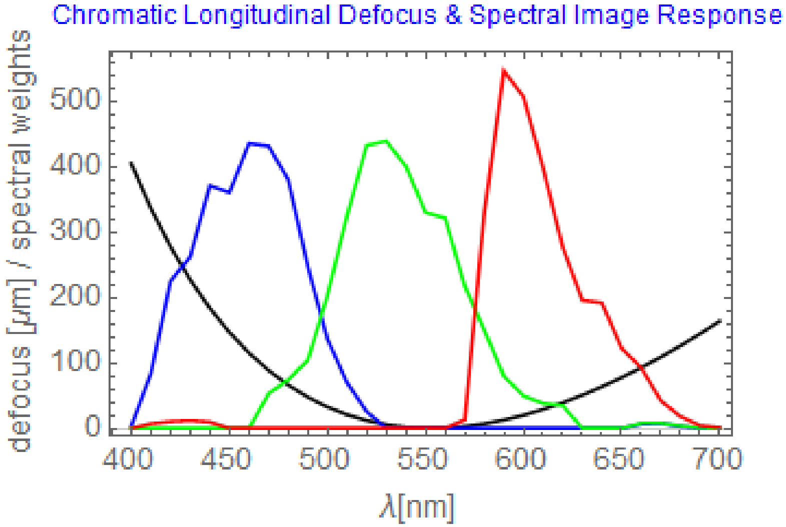
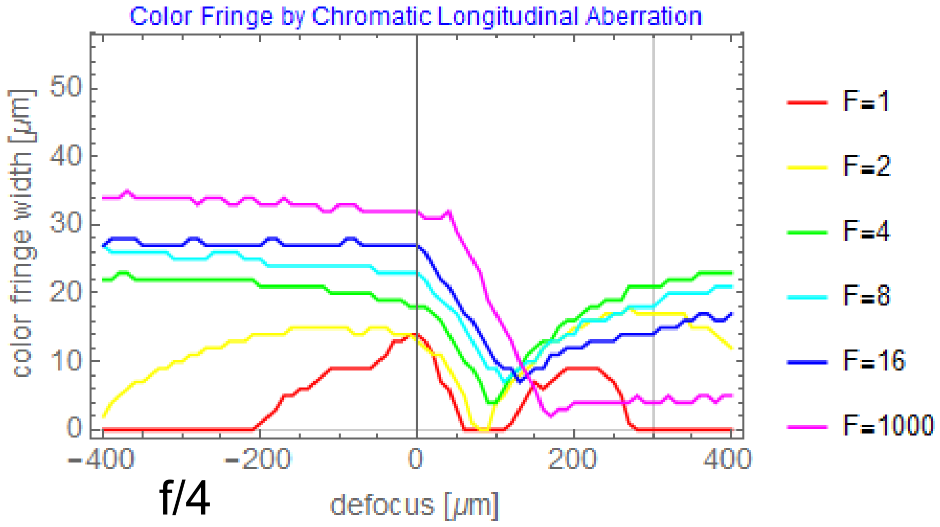
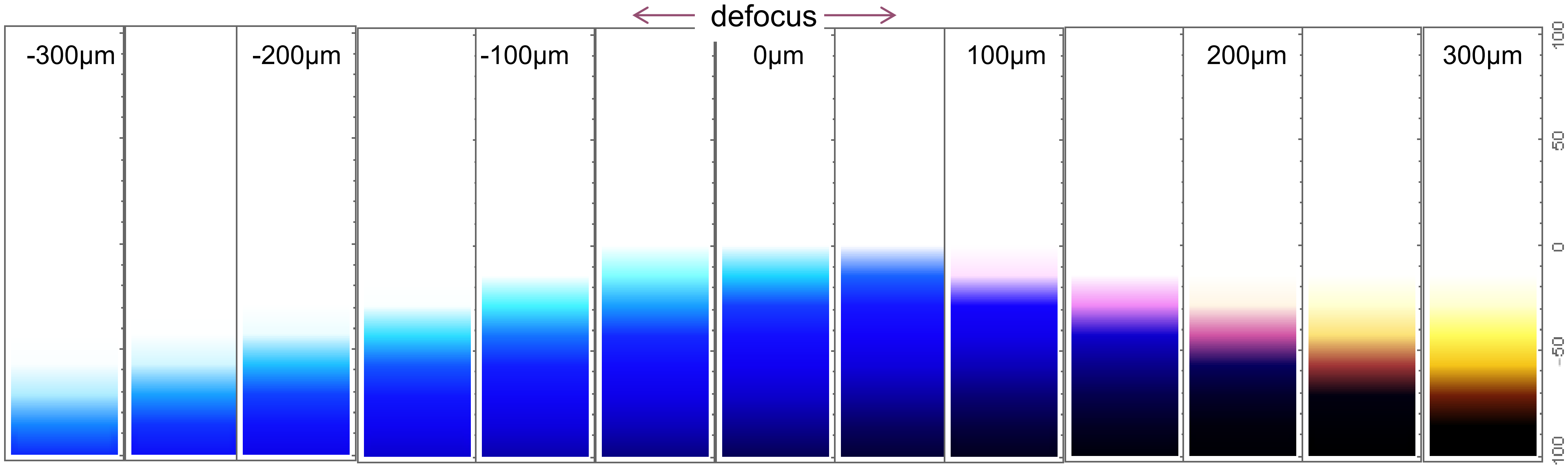


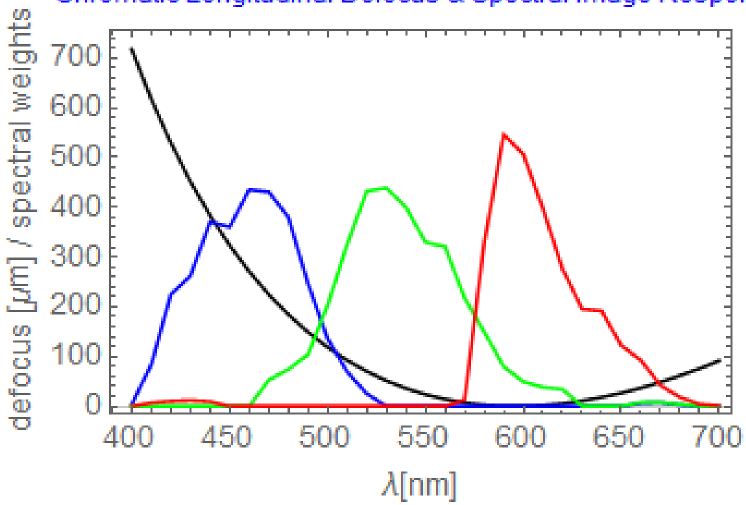


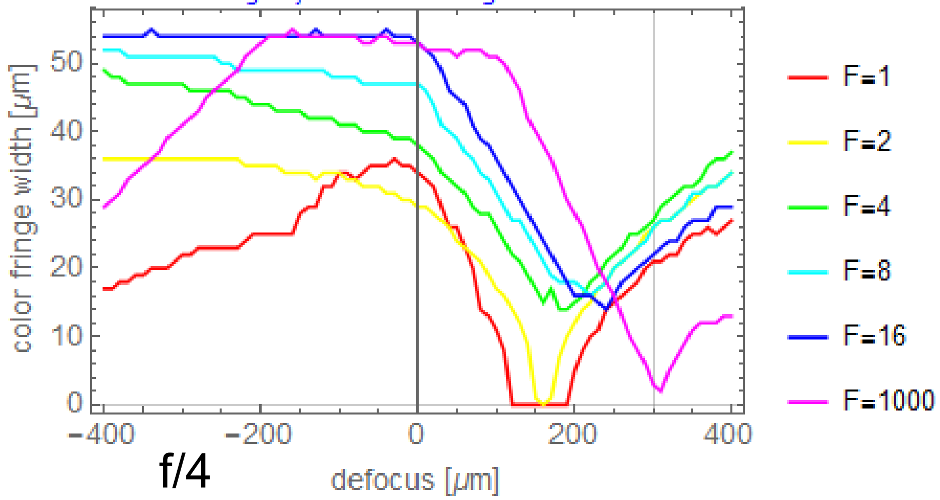


f/4

Longitudinal chromatic aberration curve: influence on colors and color fringe width







Longitudinal chromatic aberration curve: influence on colors and color fringe width

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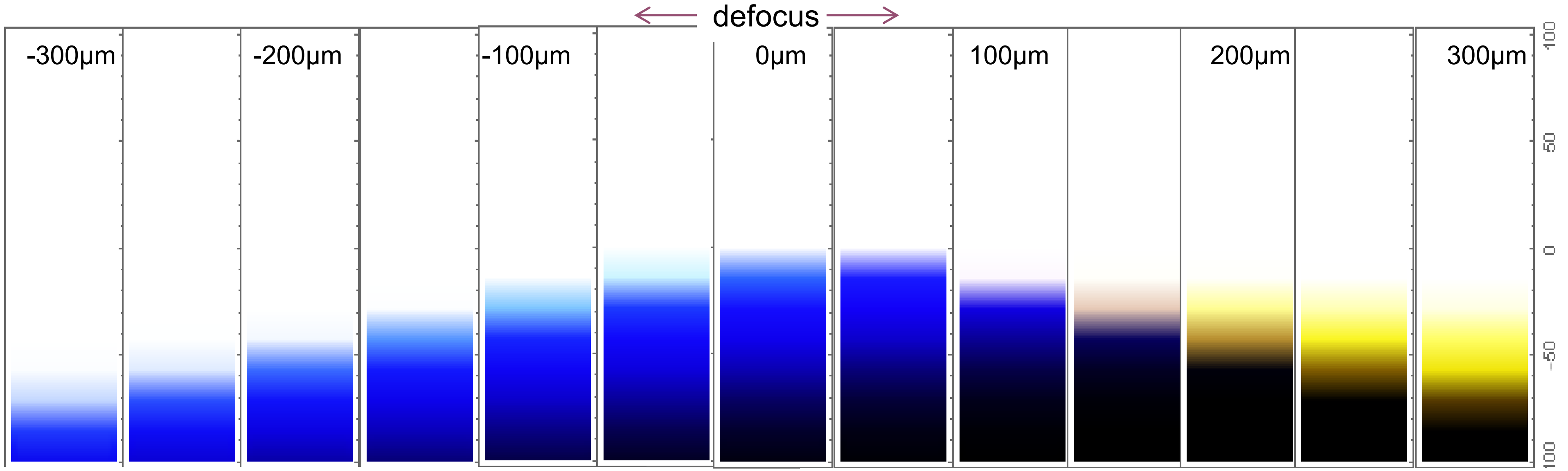

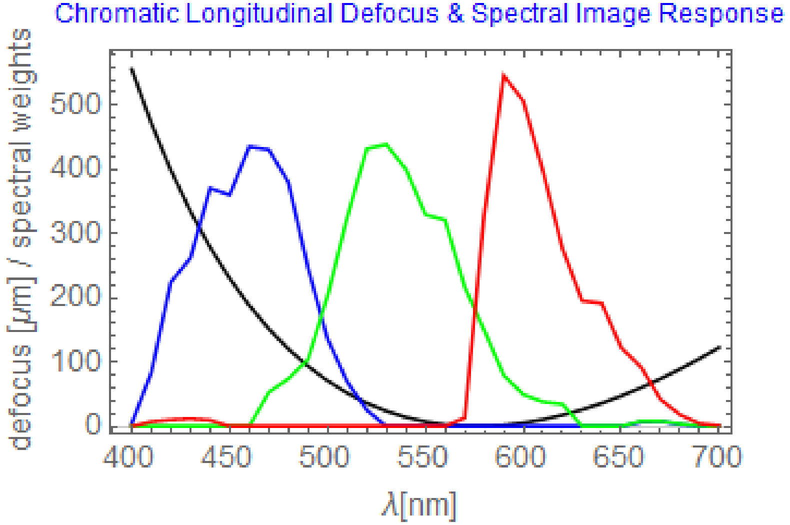

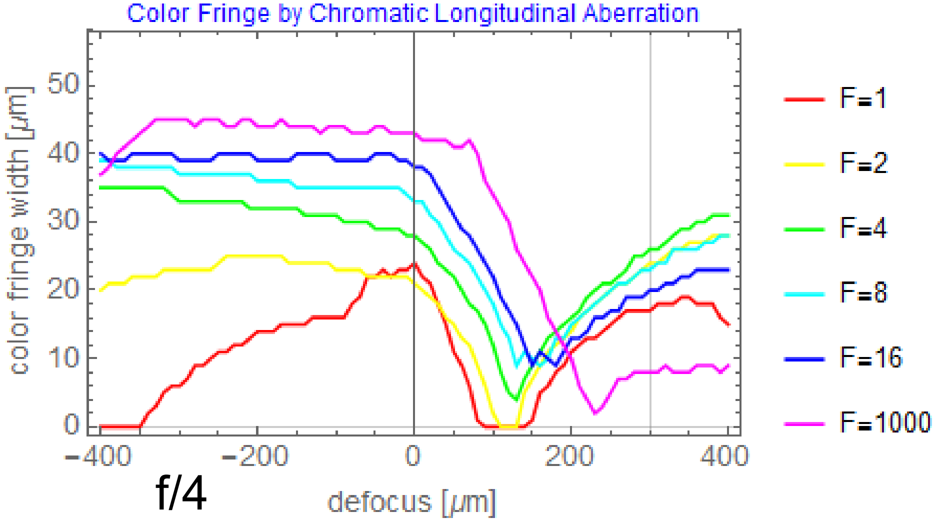

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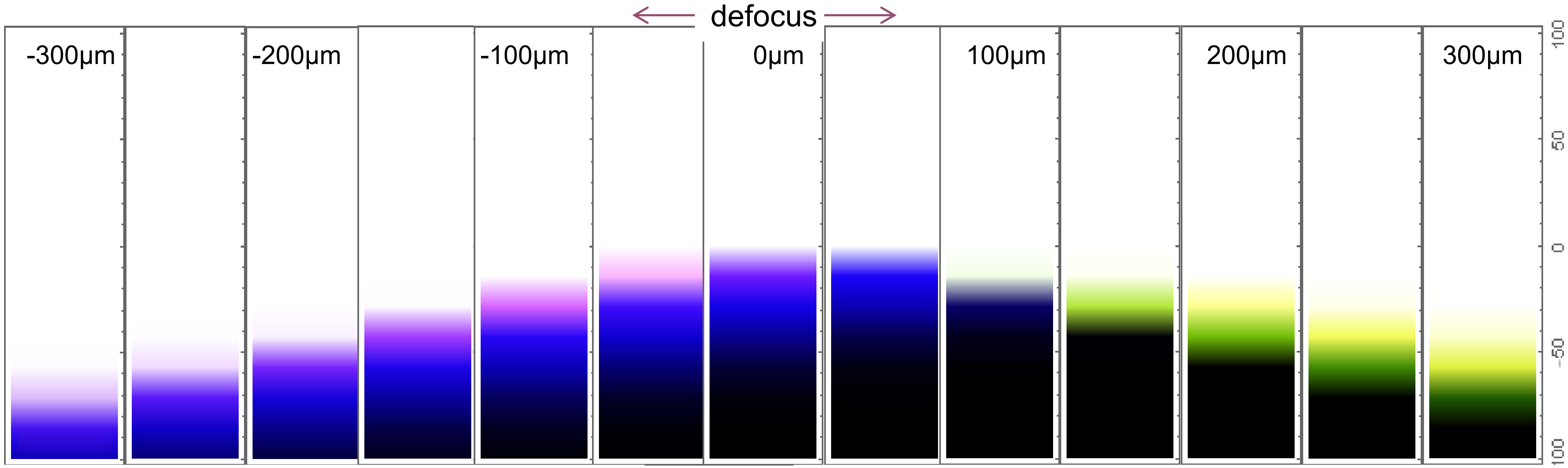
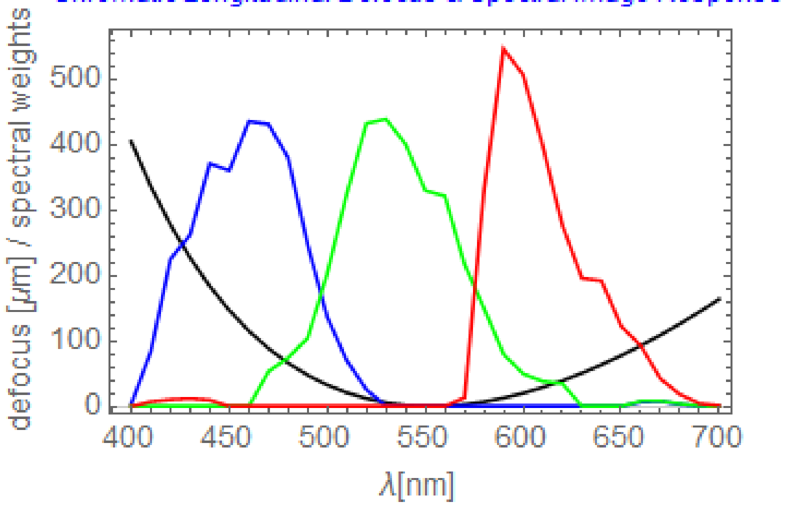
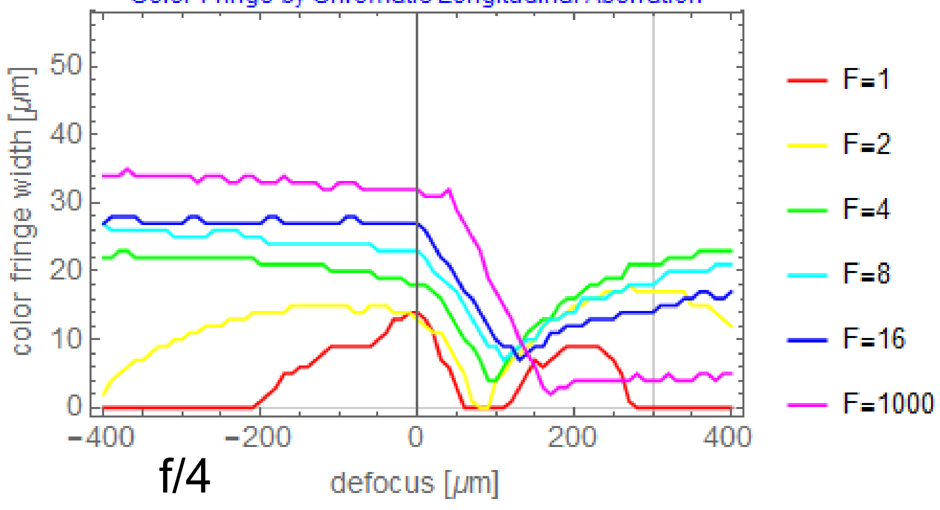
Longitudinal chromatic aberration curve: influence on colors and color fringe width







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Longitudinal chromatic aberration curve: influence on colors and color fringe width

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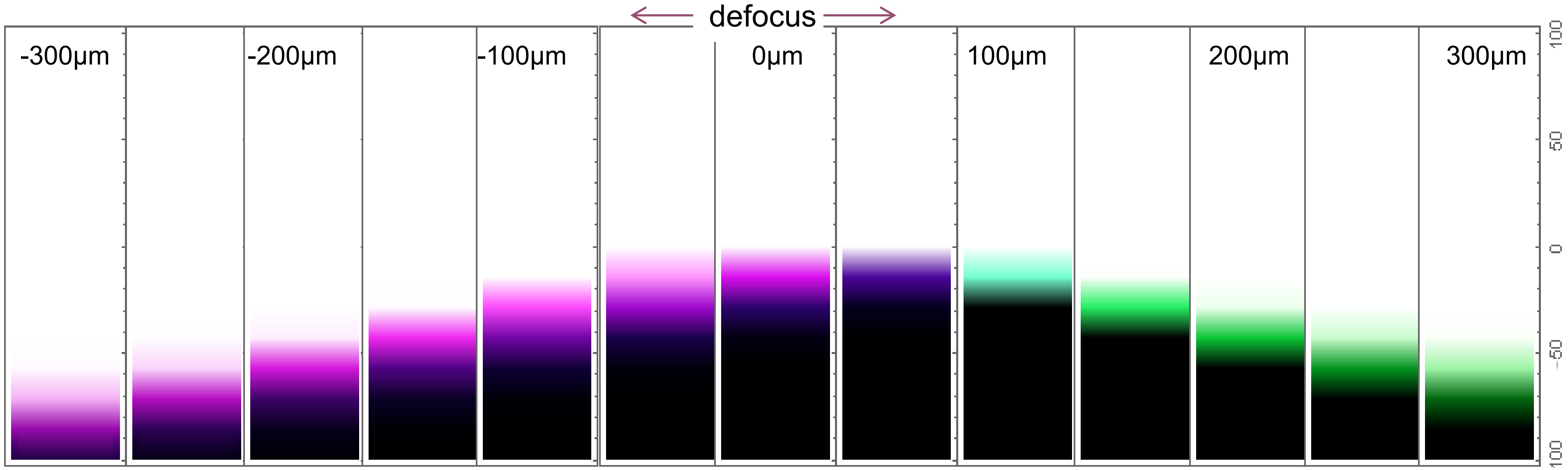
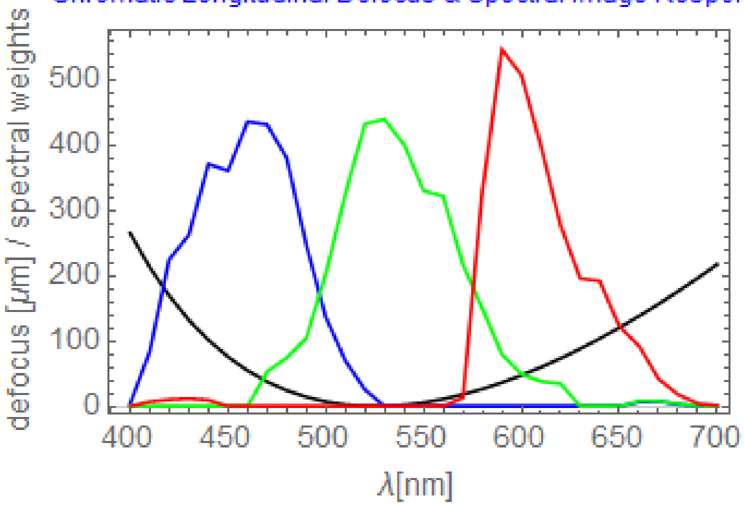
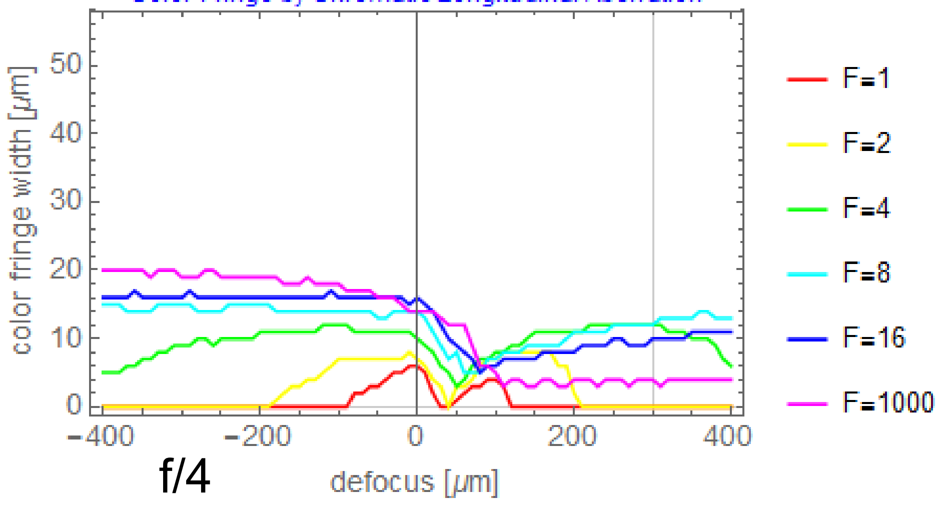
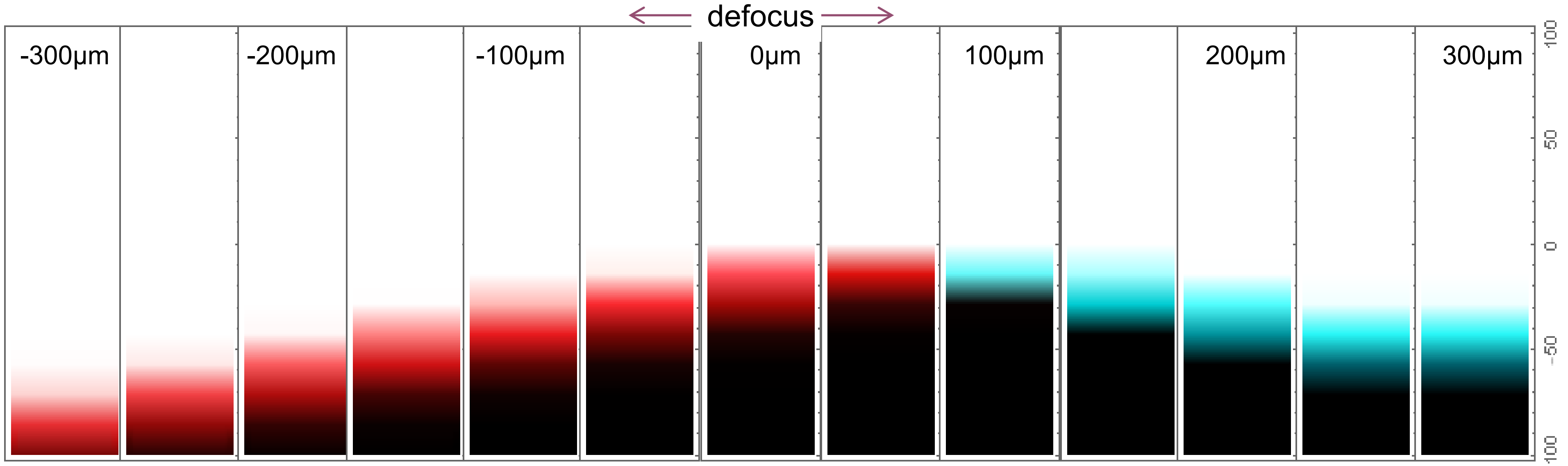


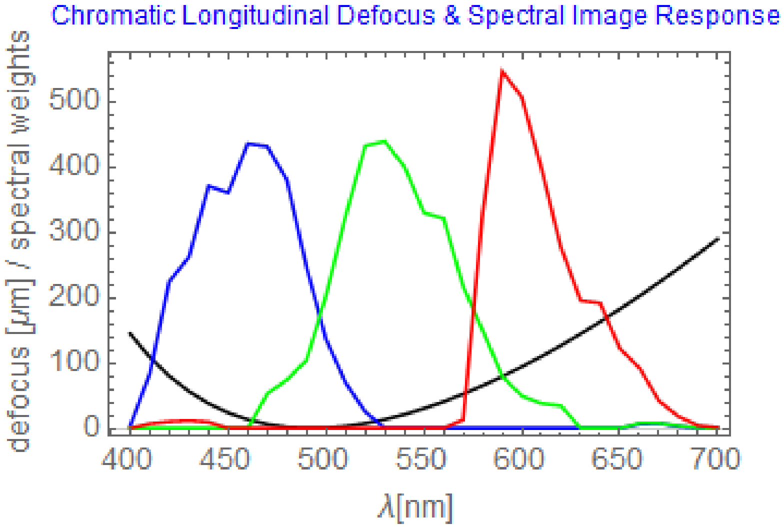


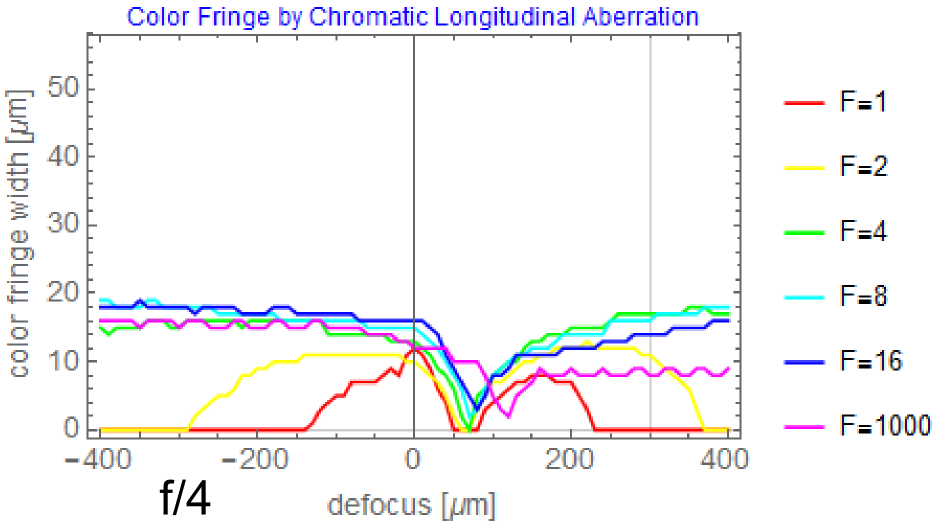


f/4

Longitudinal chromatic aberration curve: influence on colors and color fringe width

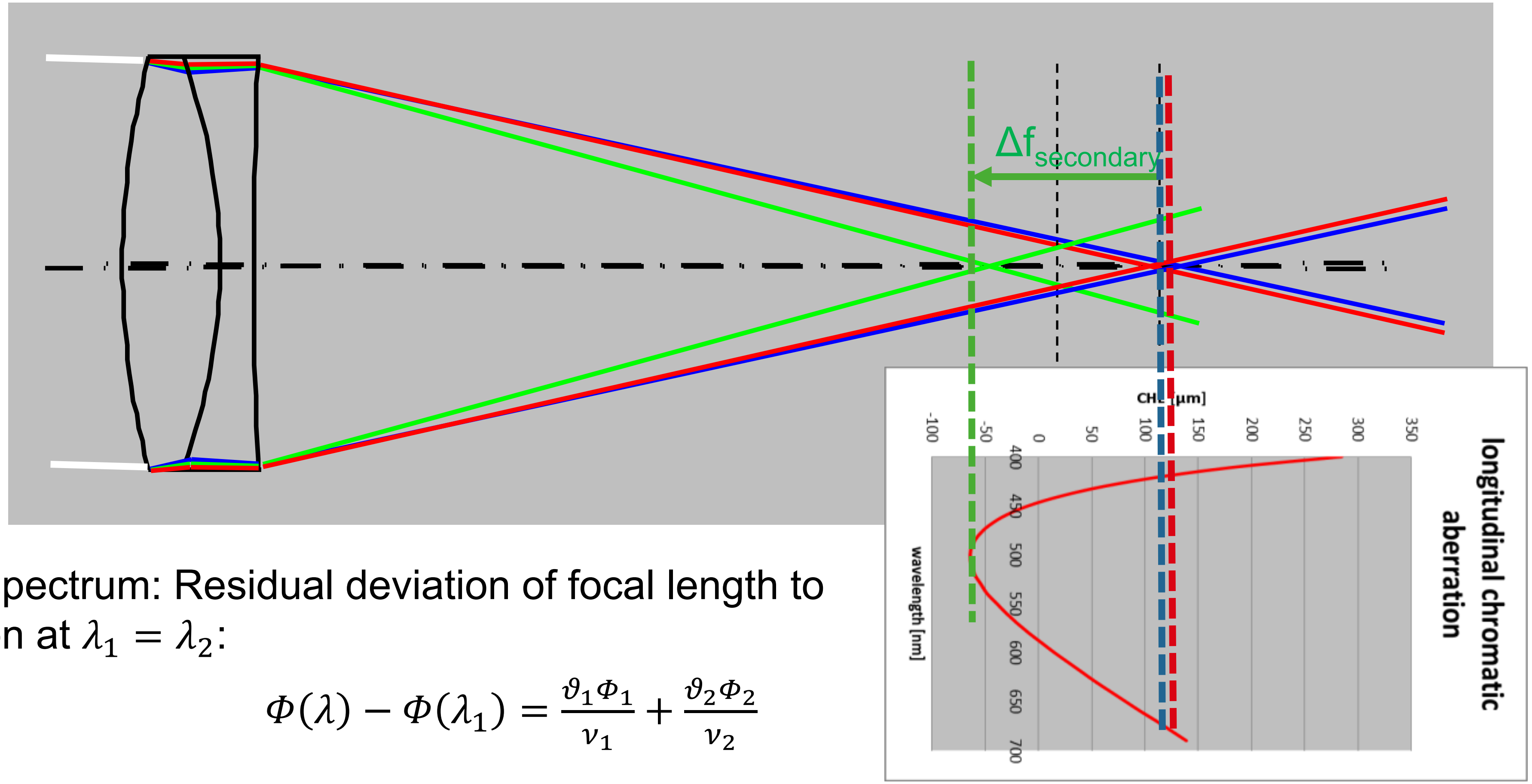






f/4

Longitudinal chromatic aberration of achromat



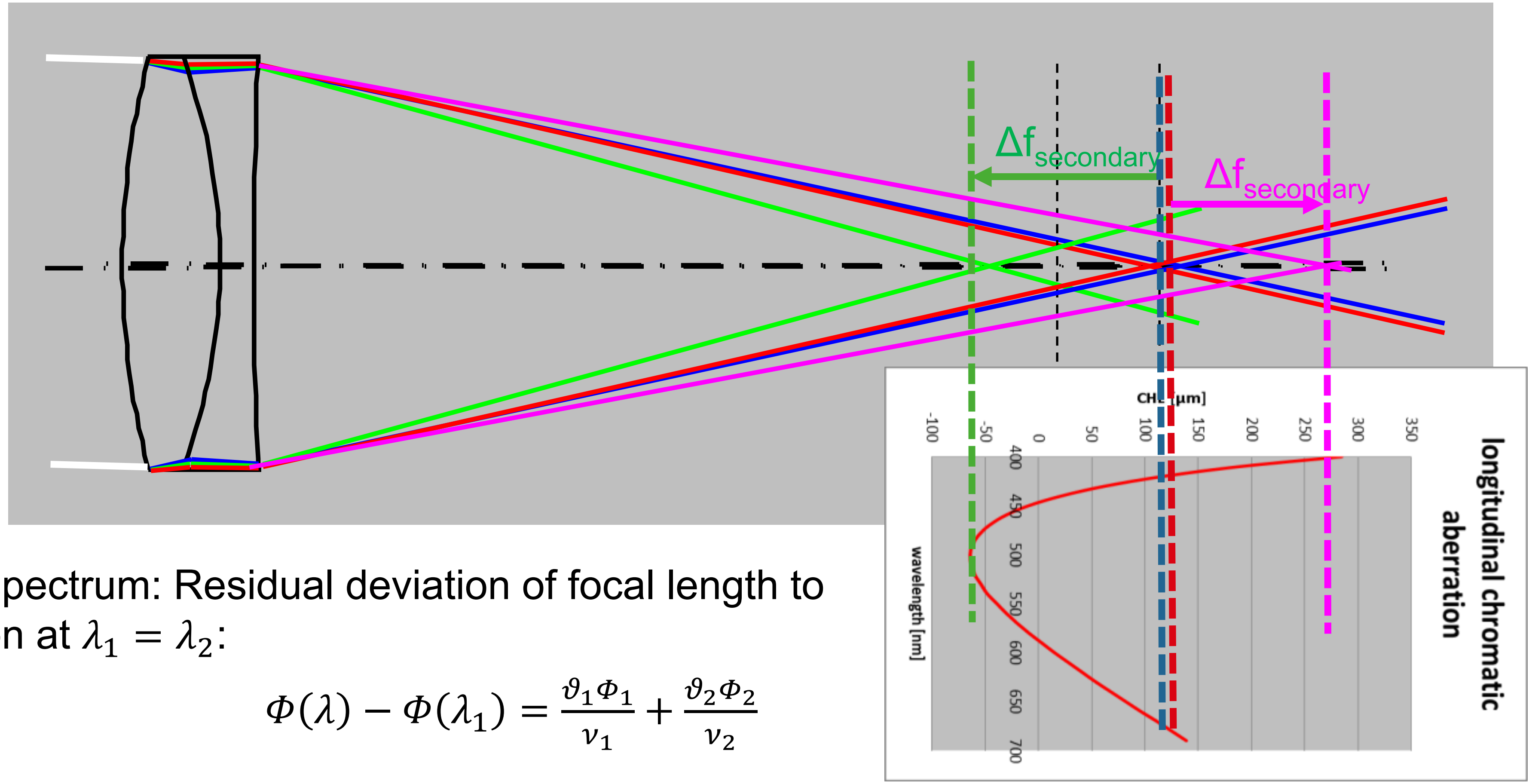
Secondary spectrum: Residual deviation of focal length to focus position at $\lambda_1 = \lambda_2$:

$$\Phi(\lambda) - \Phi(\lambda_1) = \frac{\vartheta_1 \Phi_1}{\nu_1} + \frac{\vartheta_2 \Phi_2}{\nu_2}$$

Expressed by another local slope ϑ_j, e.g. $\vartheta_{g,F} = \frac{n_g - n_F}{n_F - n_C}$, the **relative partial dispersion**.

Secondary spectrum $\Phi(\lambda) - \Phi(\lambda_1) = \frac{\vartheta_1 \Phi}{\nu_1 - \nu_2} - \frac{\vartheta_2 \Phi}{\nu_1 - \nu_2} = \frac{\vartheta_1 - \vartheta_2}{\nu_1 - \nu_2} \Phi$ or $f(\lambda) - f(\lambda_1) = -\frac{\vartheta_1 - \vartheta_2}{\nu_1 - \nu_2} f$.

Longitudinal chromatic aberration of achromat



$$\vartheta_{g,F} = \frac{n_g - n_F}{n_F - n_C}$$

$$\nu_d = \frac{n_d - 1}{n_F - n_C}$$

"another Abbe number":

$$\frac{\vartheta_{g,F}}{\nu_d} = \frac{n_g - n_F}{n_d - 1}$$

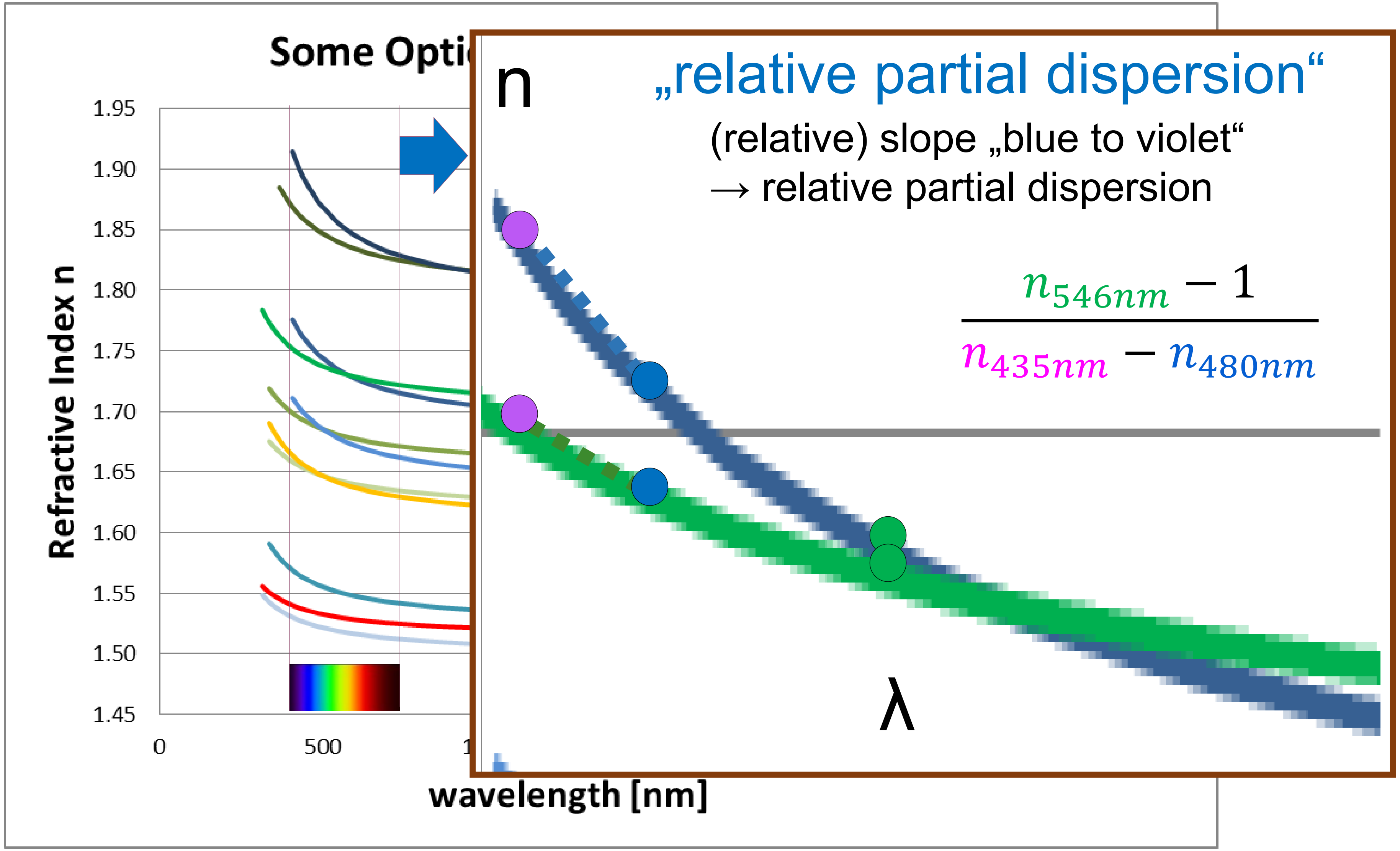
Secondary spectrum: Residual deviation of focal length to focus position at $\lambda_1 = \lambda_2$:

$$\Phi(\lambda) - \Phi(\lambda_1) = \frac{\vartheta_1 \Phi_1}{\nu_1} + \frac{\vartheta_2 \Phi_2}{\nu_2}$$

Expressed by another local slope ϑ_j, e.g. $\vartheta_{g,F} = \frac{n_g - n_F}{n_F - n_C}$, the **relative partial dispersion**.

Secondary spectrum $\Phi(\lambda) - \Phi(\lambda_1) = \frac{\vartheta_1 \Phi}{\nu_1 - \nu_2} - \frac{\vartheta_2 \Phi}{\nu_1 - \nu_2} = \frac{\vartheta_1 - \vartheta_2}{\nu_1 - \nu_2}\Phi$ or $f(\lambda) - f(\lambda_1) = -\frac{\vartheta_1 - \vartheta_2}{\nu_1 - \nu_2} f$.

Optical Glasses
Refractive index vs wavelength in visible spectrum



„relative partial dispersion“

(relative) slope „blue to violet“
→ relative partial dispersion

$$\frac{n_{546nm} - 1}{n_{435nm} - n_{480nm}}$$

Optical Glass with normal and anomalous partial dispersion

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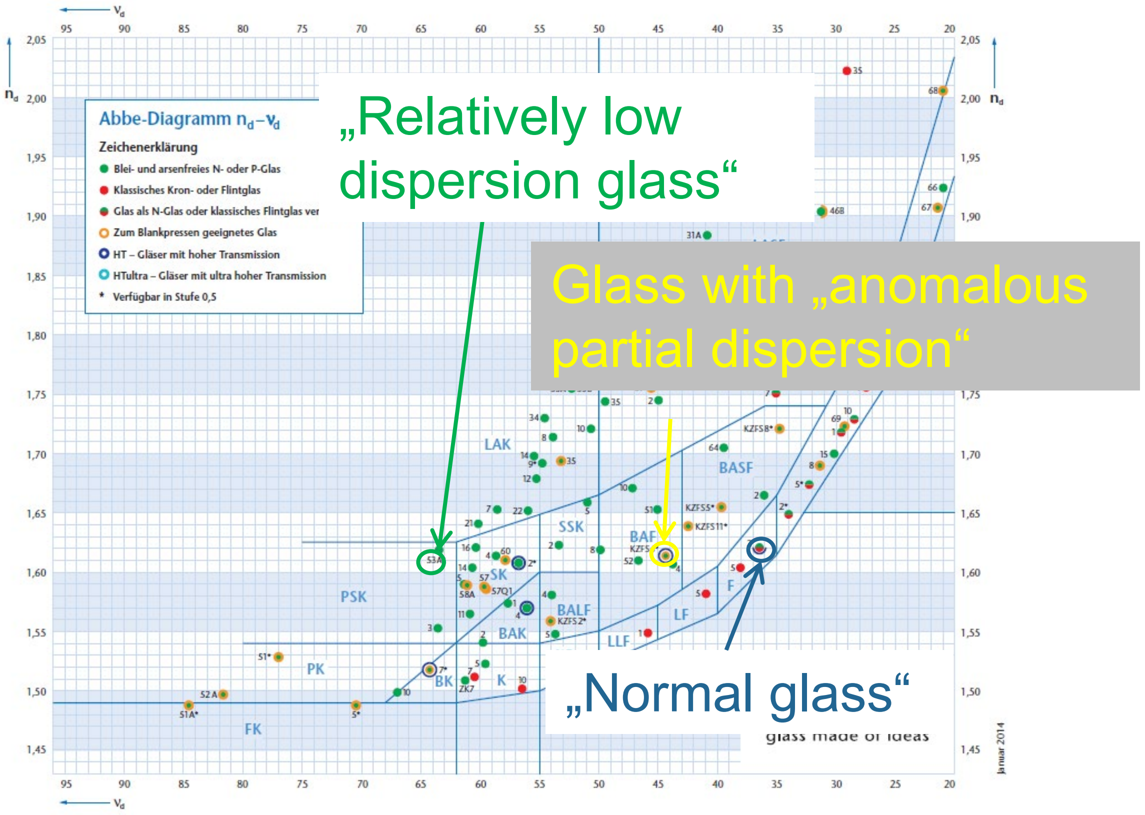
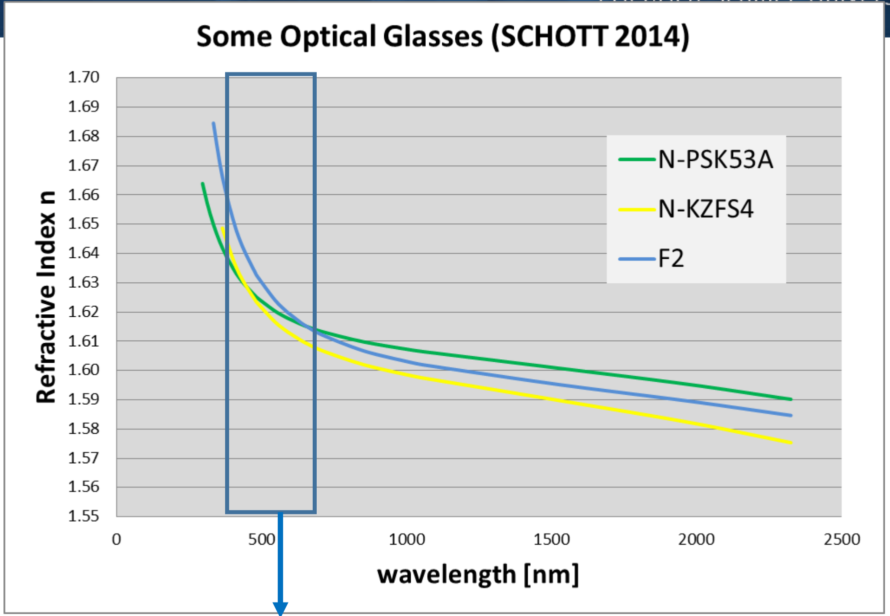
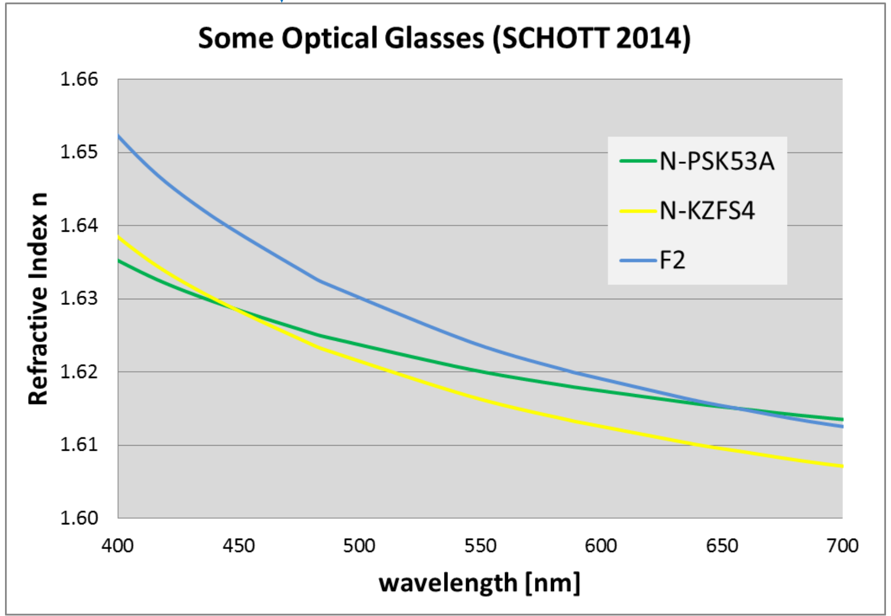






Example: 3 different glasses with about equal refracrive index n, and different dispersion and partial dispersion

Longitudinal chromatic aberration: secondary spectrum

Furthermore, it follows that significantly different Abbe numbers would reduce the focus deviations if the partial dispersions of the two glasses were similar. Unfortunately, however, the glasses do not behave in this way: for almost all glasses, the partial dispersion is proportional to the Abbe number, so there is a linear relationship

$$\vartheta(\nu) \approx a + b \cdot \nu,$$

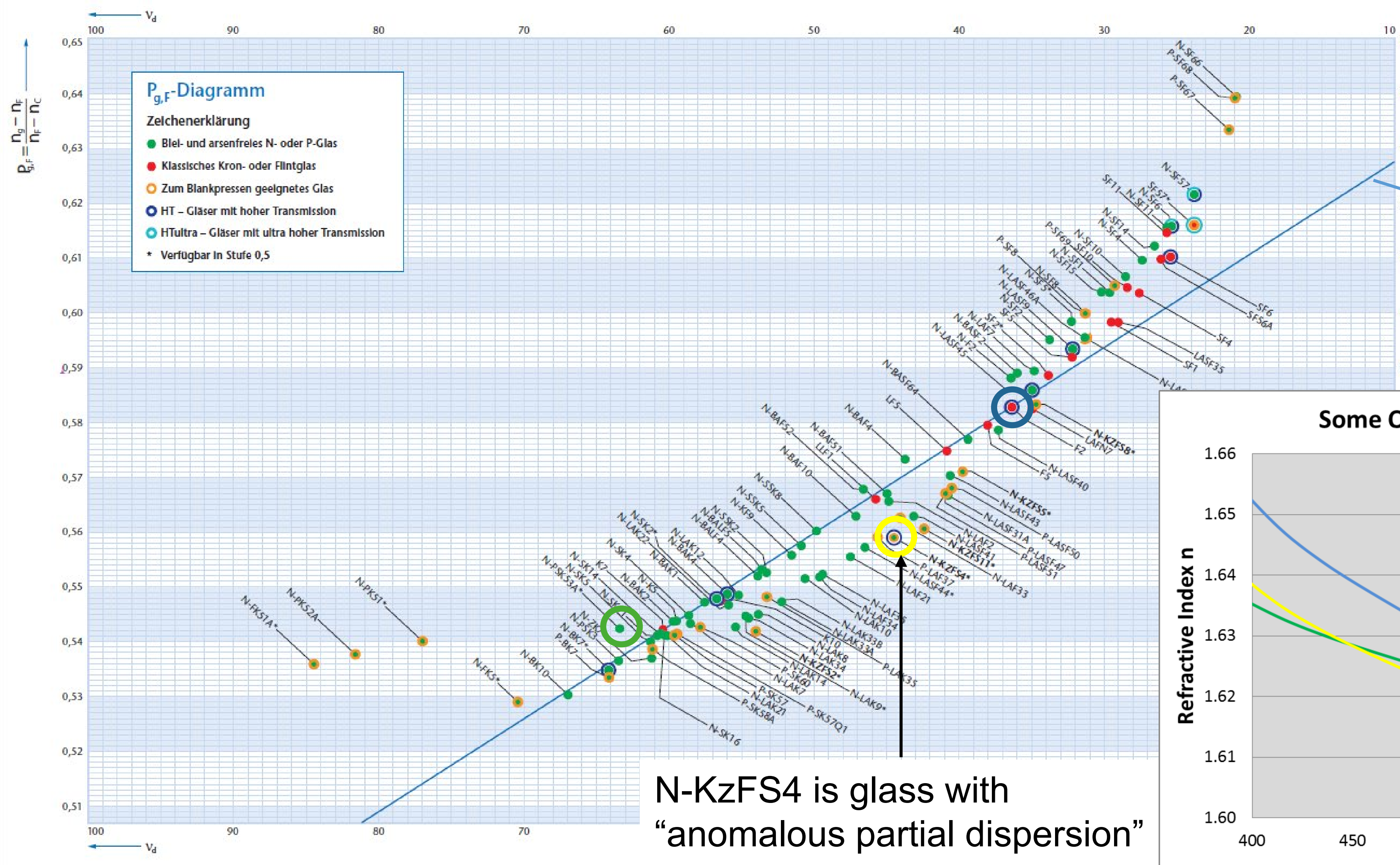
where the coefficients a and b depend on the wavelength pair under consideration. This means that the relative deviation from the nominal focal length is always the same for these glasses on this so-called "**normal line**":

$$f(\lambda) - f(\lambda_1) = -\frac{\vartheta_1 - \vartheta_2}{\nu_1 - \nu_2} f = -\frac{(a + b \cdot \nu_1) - (a + b \cdot \nu_2)}{\nu_1 - \nu_2} f = -bf$$

i.e. practically independent of the combination of the two glasses.* "Practically the same" actually means that the deviations of the secondary spectrum can be about a factor of 2-3. Combinations with glasses with anomalous partial dispersion, on the other hand, can result in secondary spectra that are more than an order of magnitude smaller.

Fortunately, there are exceptions: these are some fluorine-crown or phosphate-crown glasses (bottom left in the partial dispersion versus Abbe number diagrams) such as calcium fluoride, Ohara: S-FPL53, S-FPL51, Schott: N-PK51, N-PK52A, N-FK51A) and to a lesser extent special short flint glass (Schott: e.g. N-KzFS11, Ohara somewhat less favorable L-LAH83).

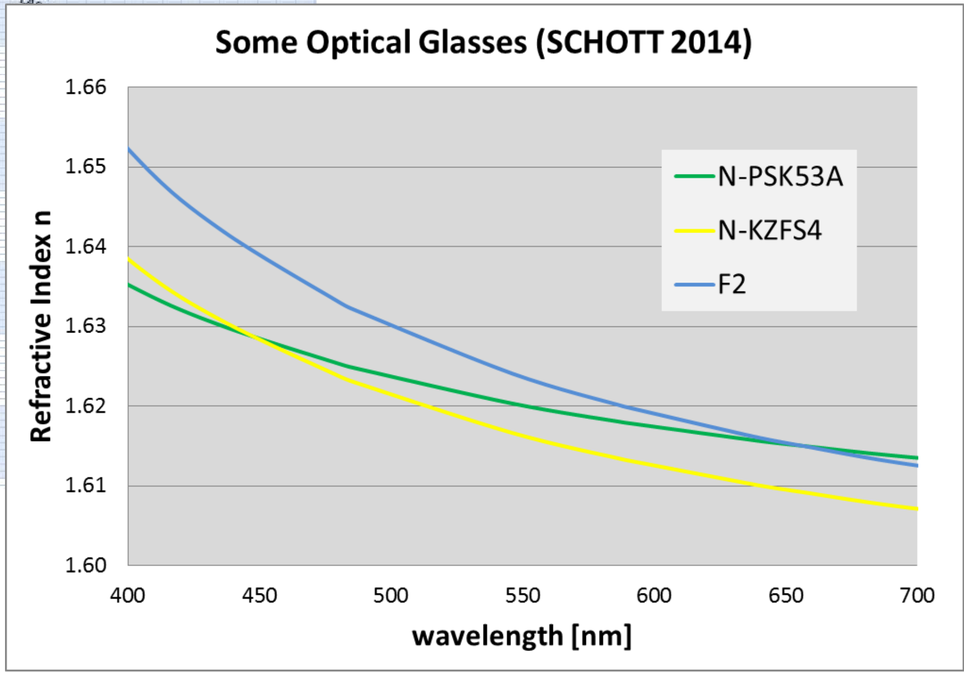
Relative partial dispersion diagram



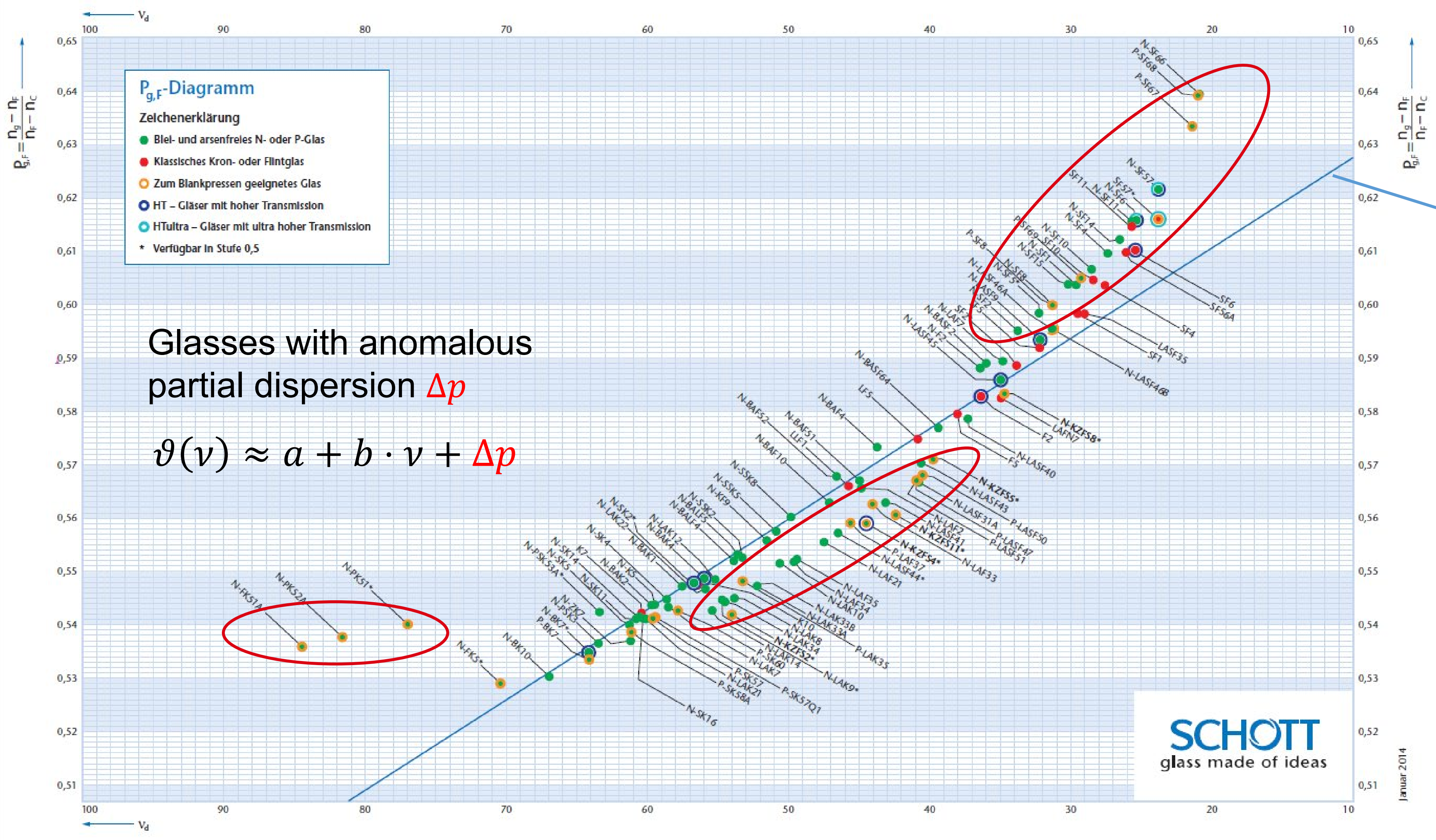
“normal line”:

$$\vartheta(\nu) \approx a + b \cdot \nu$$

N-KzFS4 is glass with “anomalous partial dispersion”



Relative partial dispersion diagram



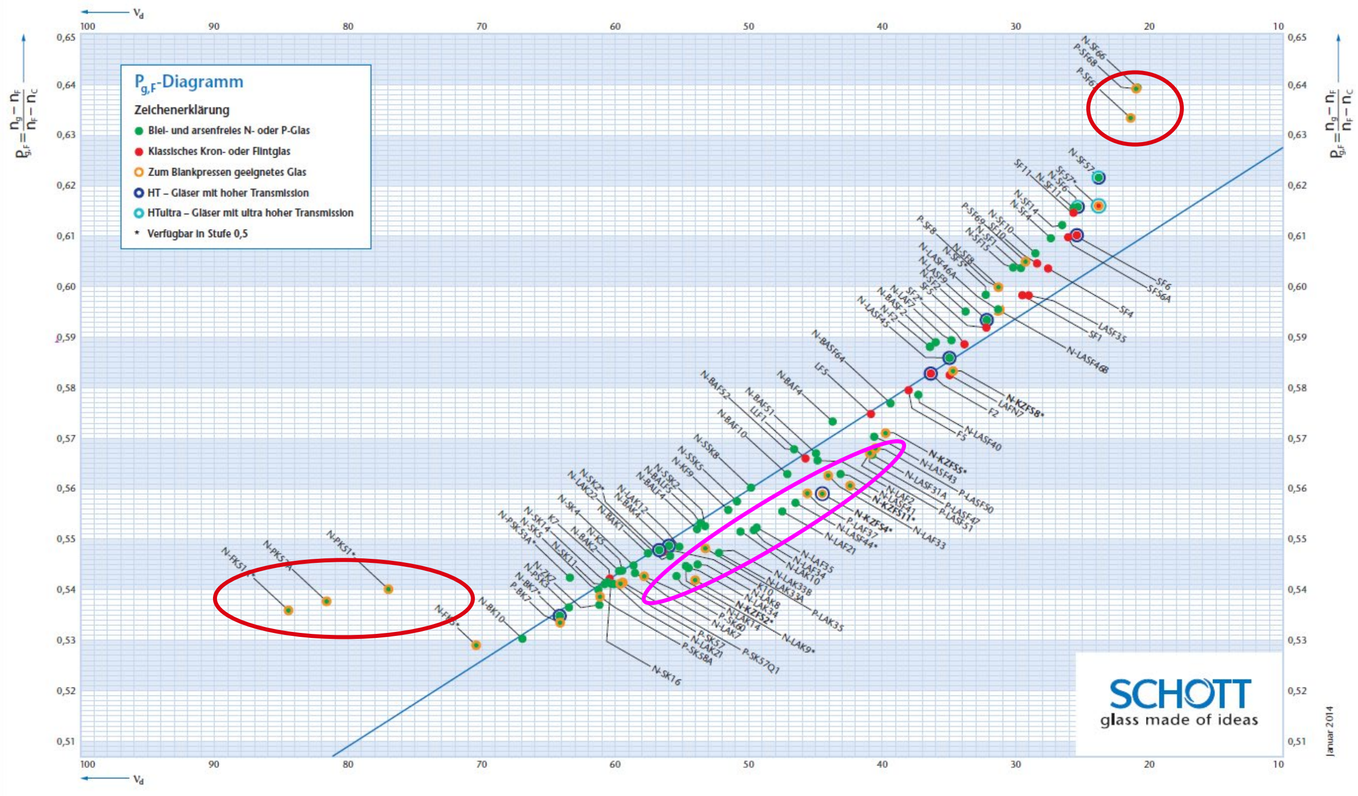
Glasses with anomalous partial dispersion Δp

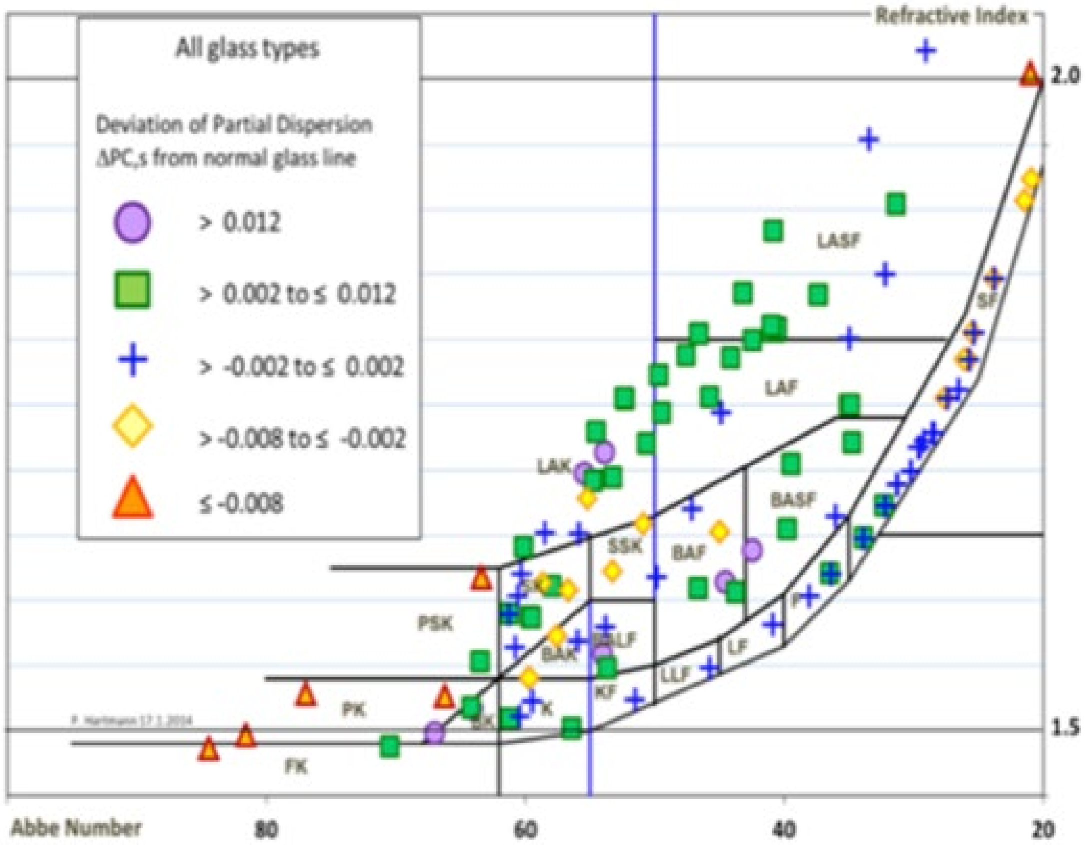
$$\vartheta(\nu) \approx a + b \cdot \nu + \Delta p$$

"normal line":

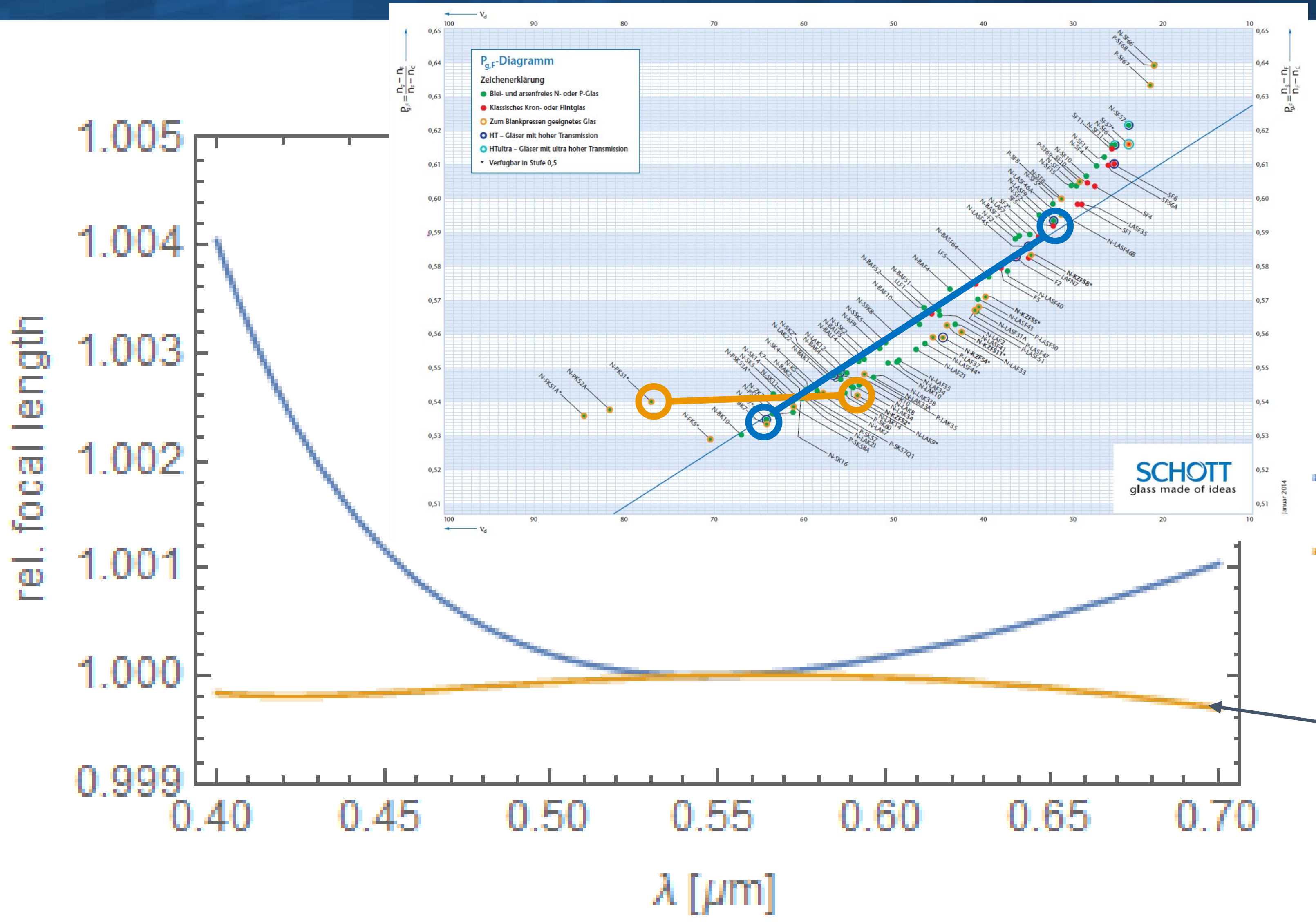
$$\vartheta(\nu) \approx a + b \cdot \nu$$

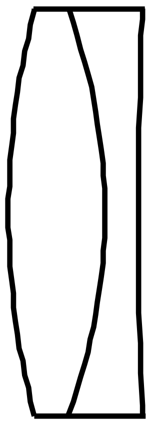
Glasses with anomalous partial dispersion





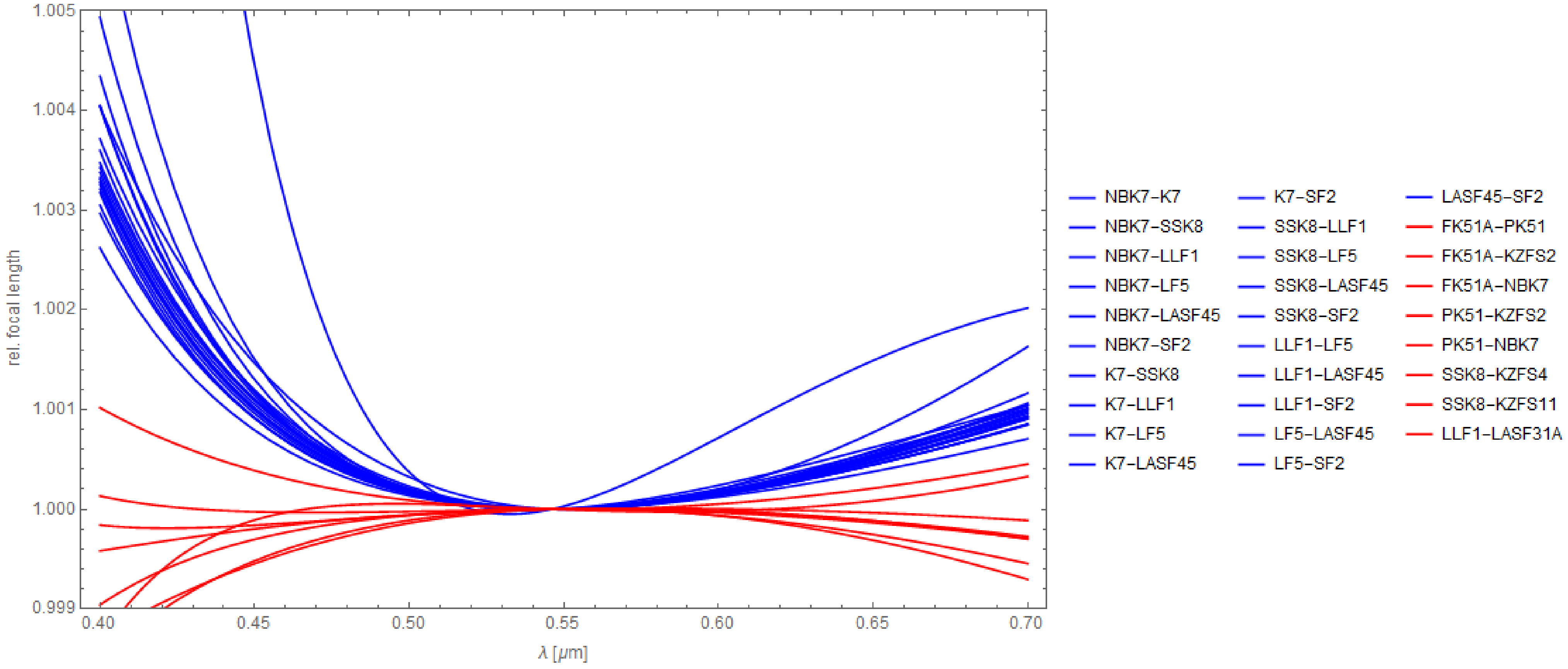
Achromat and Apochromat
Comparison of two doublets

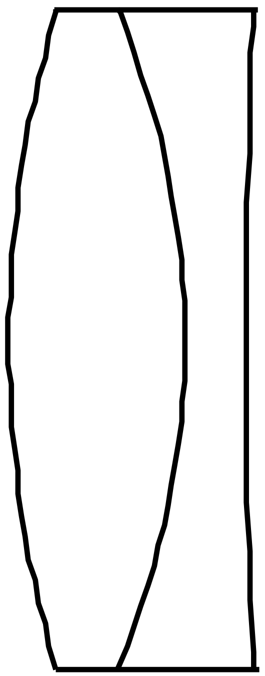




Glass combination with "anomalous partial dispersion"

Achromat and Apochromat
Comparison of duplets with different glass combinations





The achromats marked in red contain “special glasses” deviating from the normal line, the blue ones do not.

Asking for a perfect achromat „Achromat complement“ and residual spectrum

We ask for a perfect achromat $\Phi_1(\lambda) + \Phi_2(\lambda) \overset{!}{=} \Phi_0$ for **all wavelengths λ**!

For a given lens element 1 with refractive index $n_1(\lambda)$ and power $\Phi_1=(n_1(\lambda)-1)\cdot\Delta c_1$ a straight forward calculation gives the required refractive index $n_2(\lambda)$ which compensates all dispersion:

$$n_2(\lambda) = -a\ n_1(\lambda) + b$$

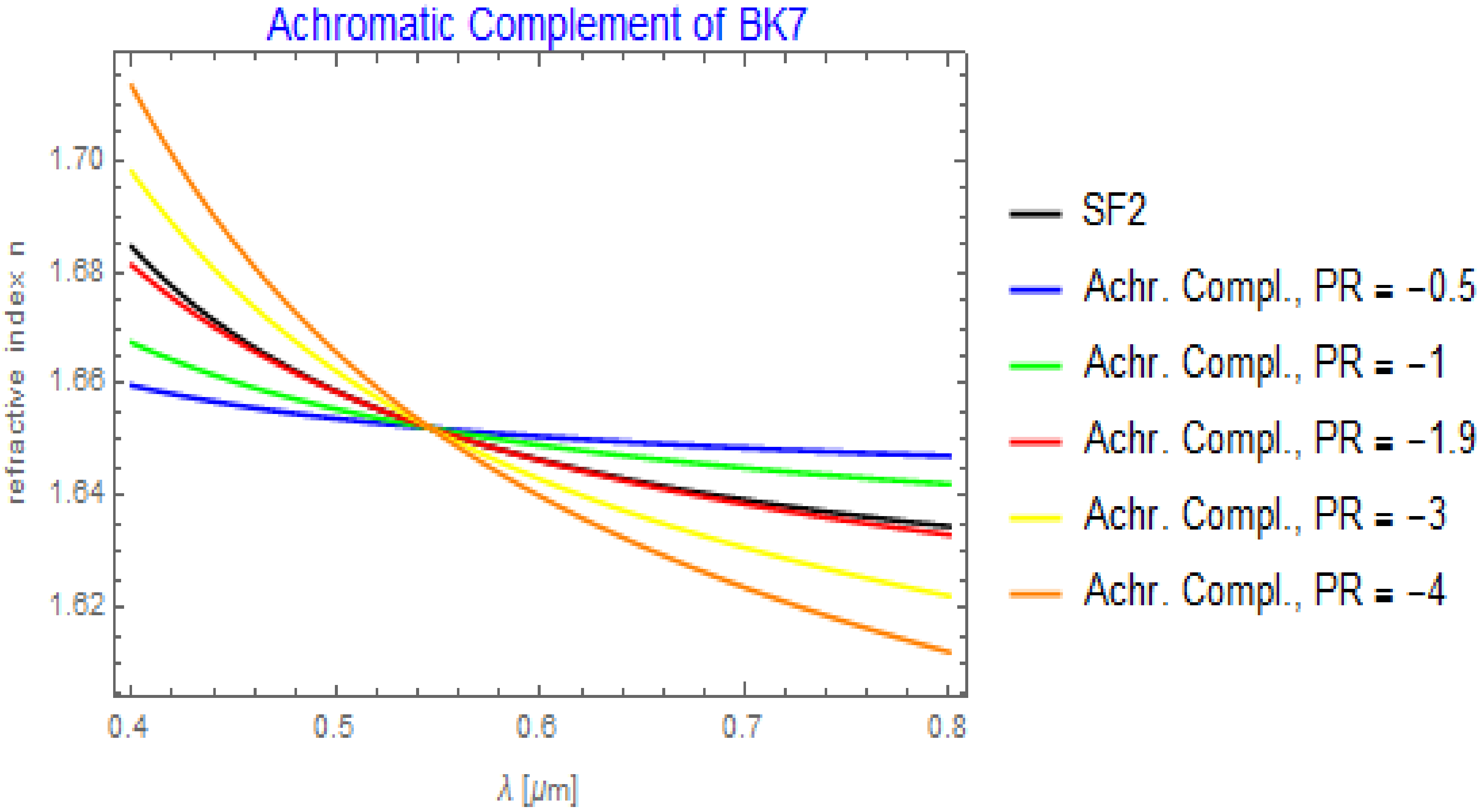
$$a = \frac{(n_{2,0}-1)\Phi_{1,0}}{(n_{1,0}-1)\Phi_{2,0}}$$

$$b = \frac{(n_{2,0}-1)\Phi_{1,0}}{(n_{1,0}-1)\Phi_{2,0}} + 1 + (n_{2,0}-1)\left(\frac{\Phi_{1,0}}{\Phi_{2,0}} + 1\right)$$

- Linear relationship
- Power ratio $\boldsymbol{\Phi}_{10}/\ \boldsymbol{\Phi}_{20}$ is a free parameter
- Power ratio must be negative (“+/-” or “-/+”)

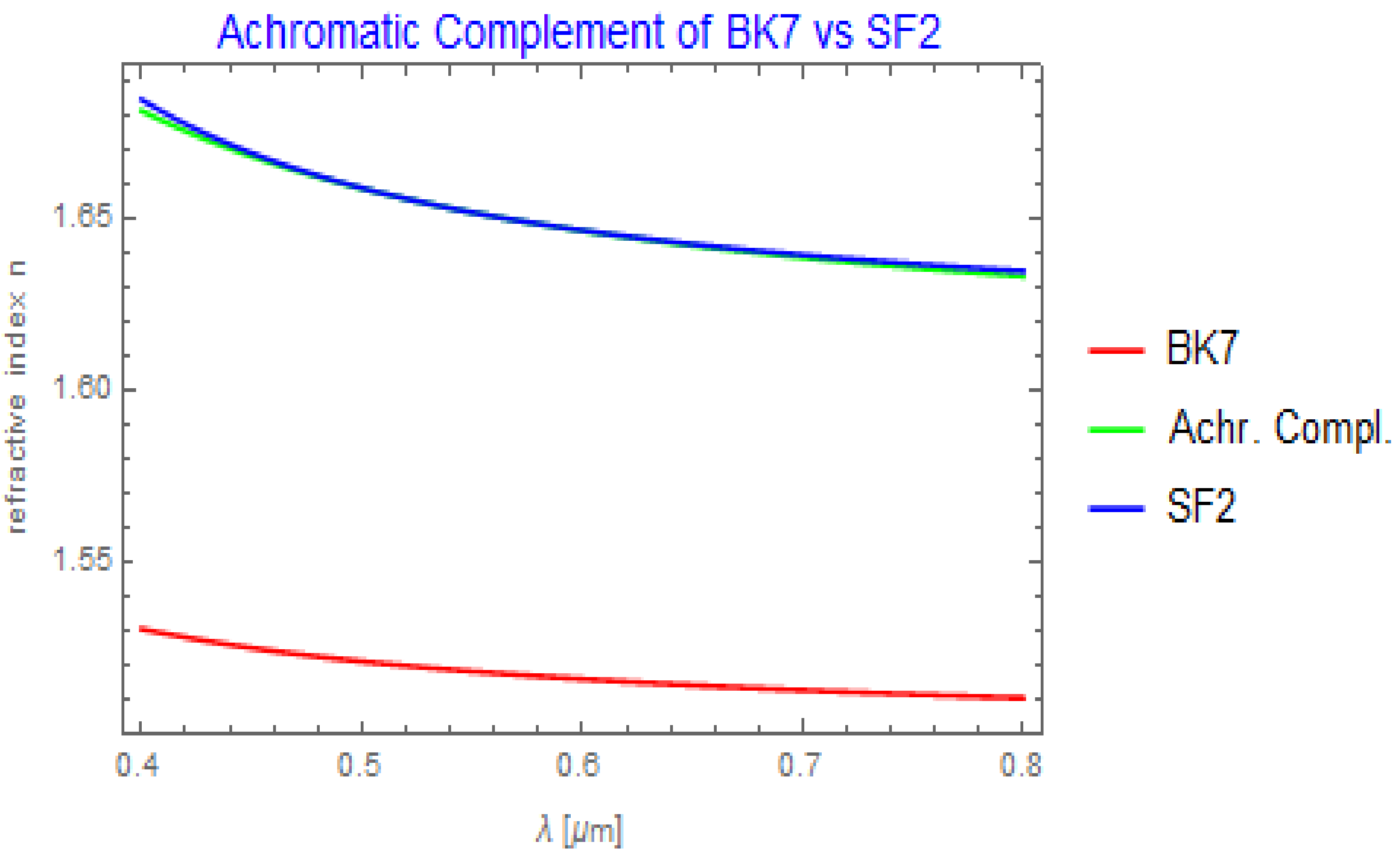
The index 0 denotes the wavelength λ_0 (e.g. 587nm) for n or $\boldsymbol{\Phi}$.

Asking for a perfect achromat „Achromat complement“ and residual spectrum

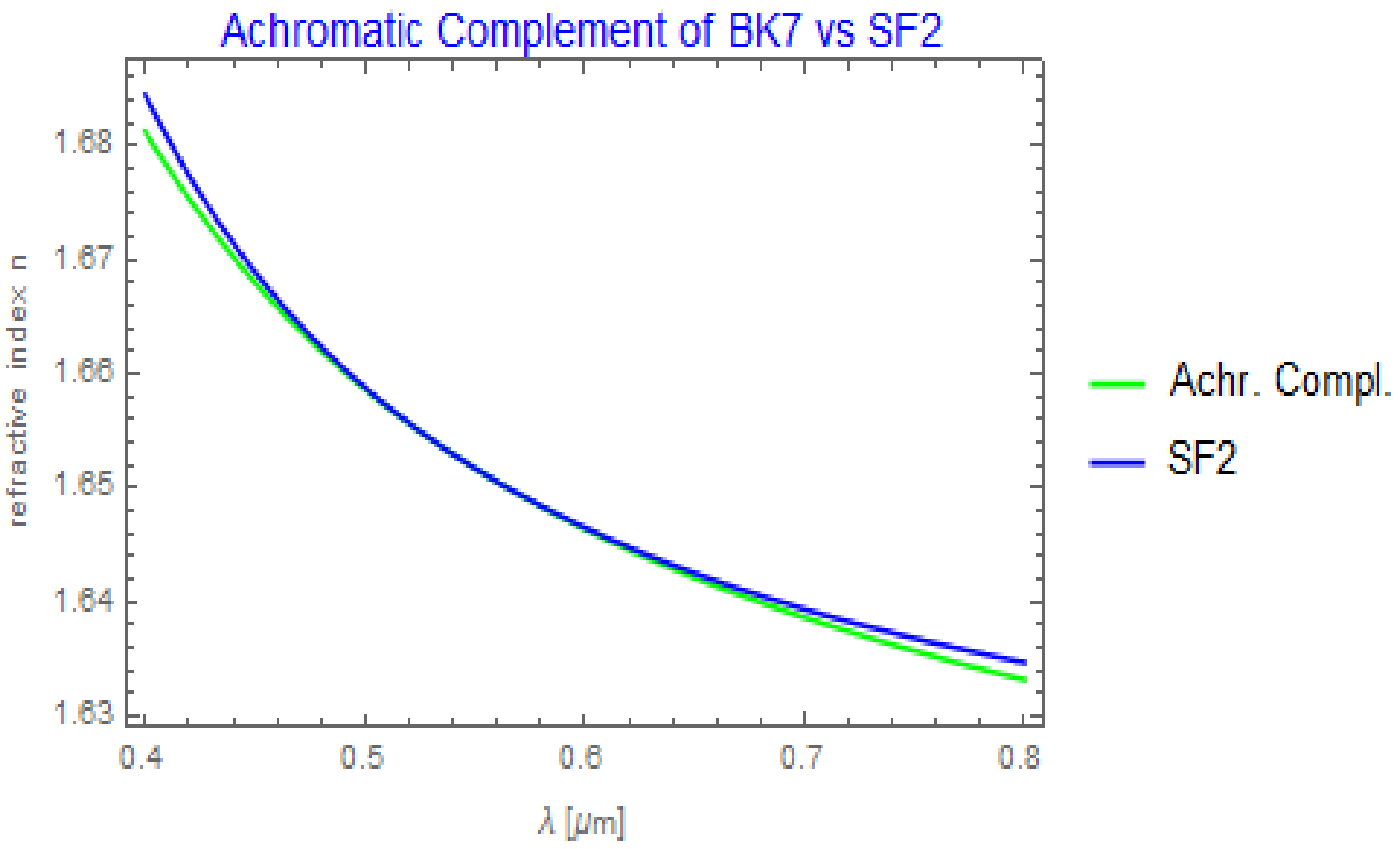


Hypothetical glasses with n=1.65 @λ_0=546nm (and SF2), which would give a perfect achromat in combination with BK7.

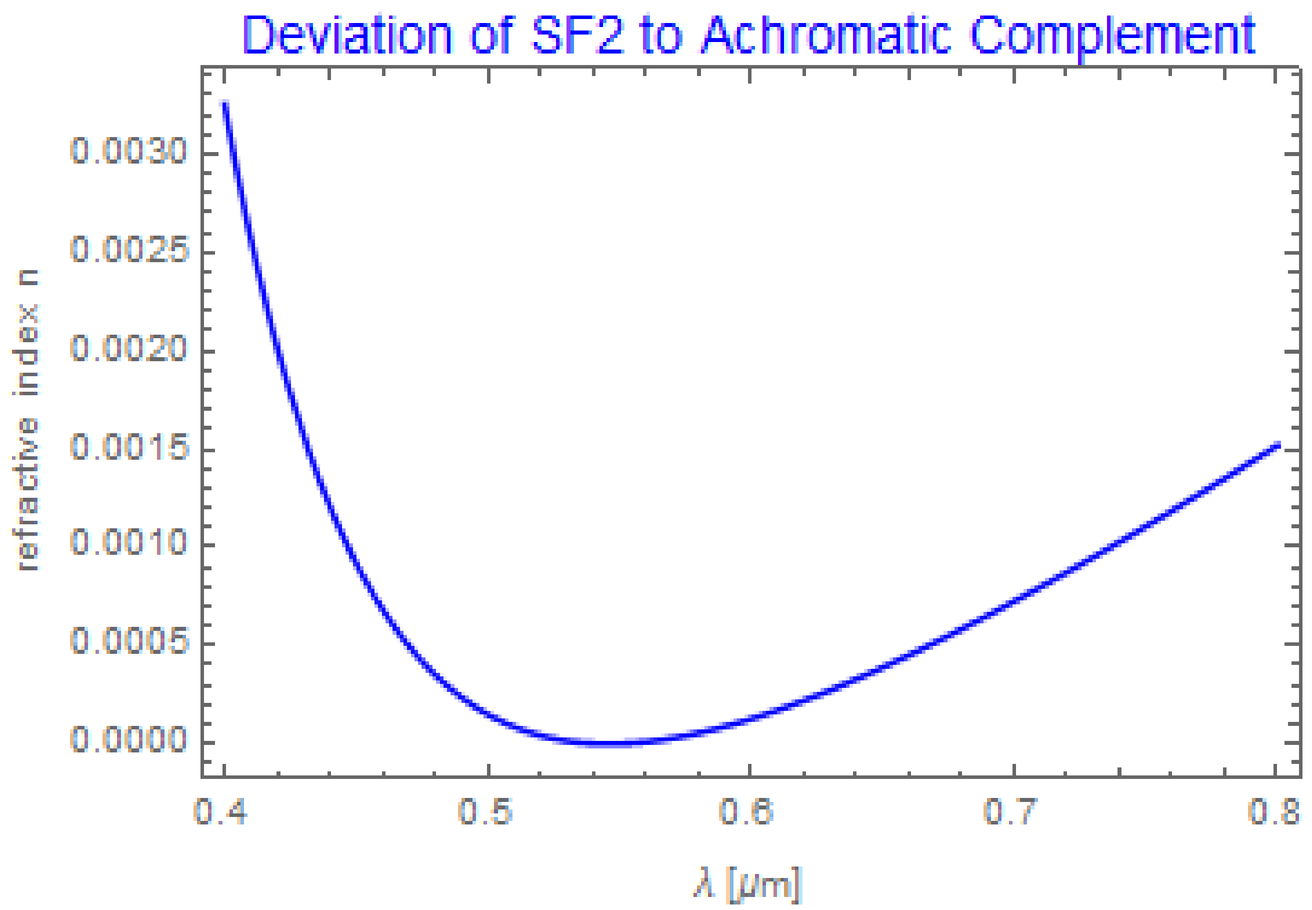
Asking for a perfect achromat
„Achromat complement“ and residual spectrum



Asking for a perfect achromat
„Achromat complement“ and residual spectrum



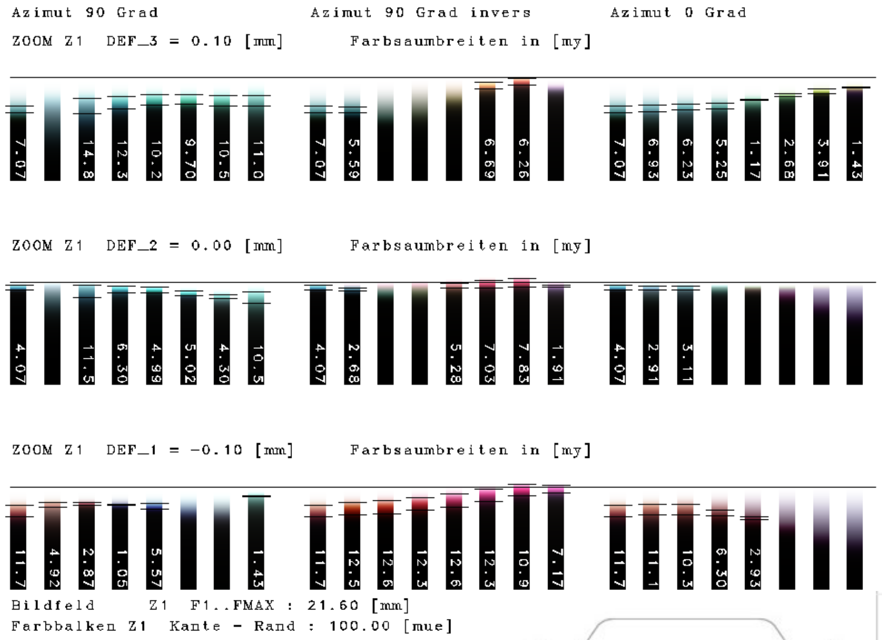
Asking for a perfect achromat
„Achromat complement“ and residual spectrum



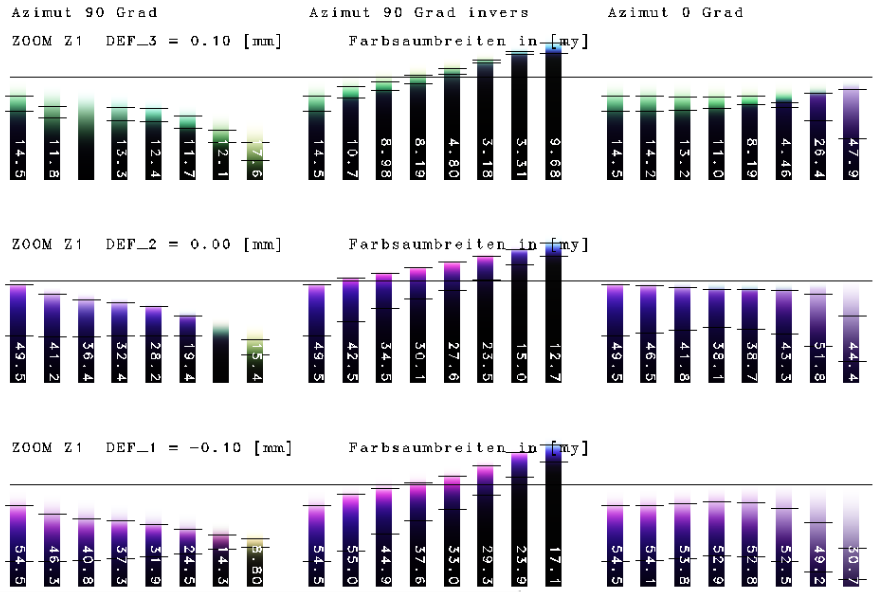
The curvature of n(λ) of SF2 is too large.
In other words: dn/dλ should be smaller to give a better achromat in combination with BK7.

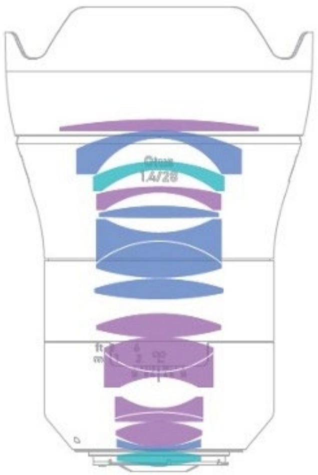
Achromat and Apochromat

“Apochromat”



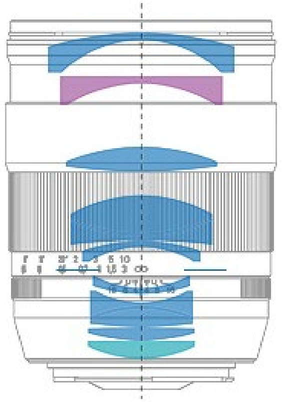
“Achromat”





Linse mit asphärischer Optikfläche

Linse aus Sonderglas mit anomaler Teildispersion



Lens mark chromatic aberration quality, e.g. “achromat”, “apochromat”, “super apochromat” …

There is no formal norm standard.

The distinction should be made by physical-optical criteria, e.g. color fringe width.

To achieve small color fringe widths usually “special glasses” with anomalous partial dispersion are required.

Material price

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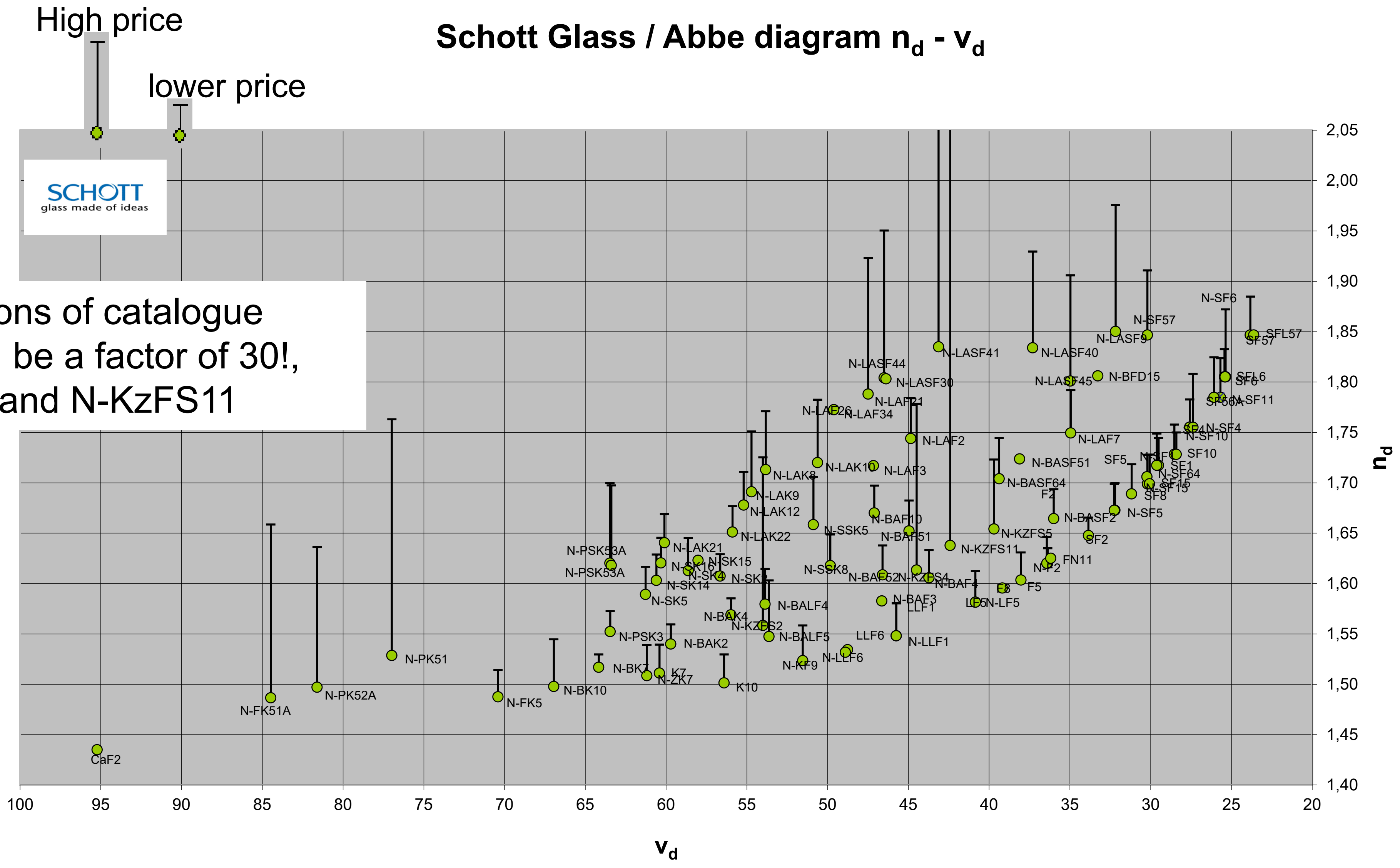
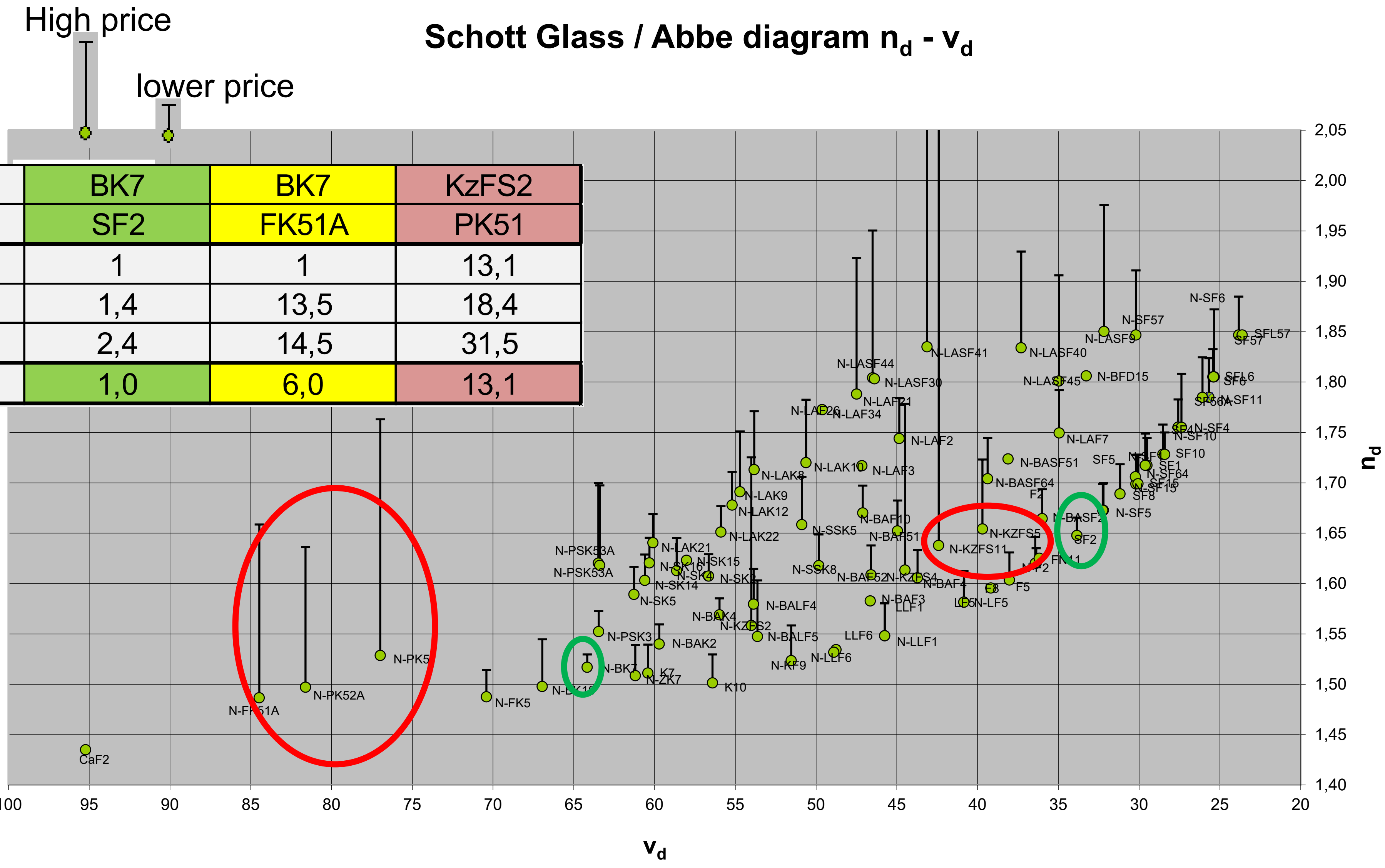
Cost variations of catalogue glasses can be a factor of 30!, e.g. N-BK7 and N-KzFS11

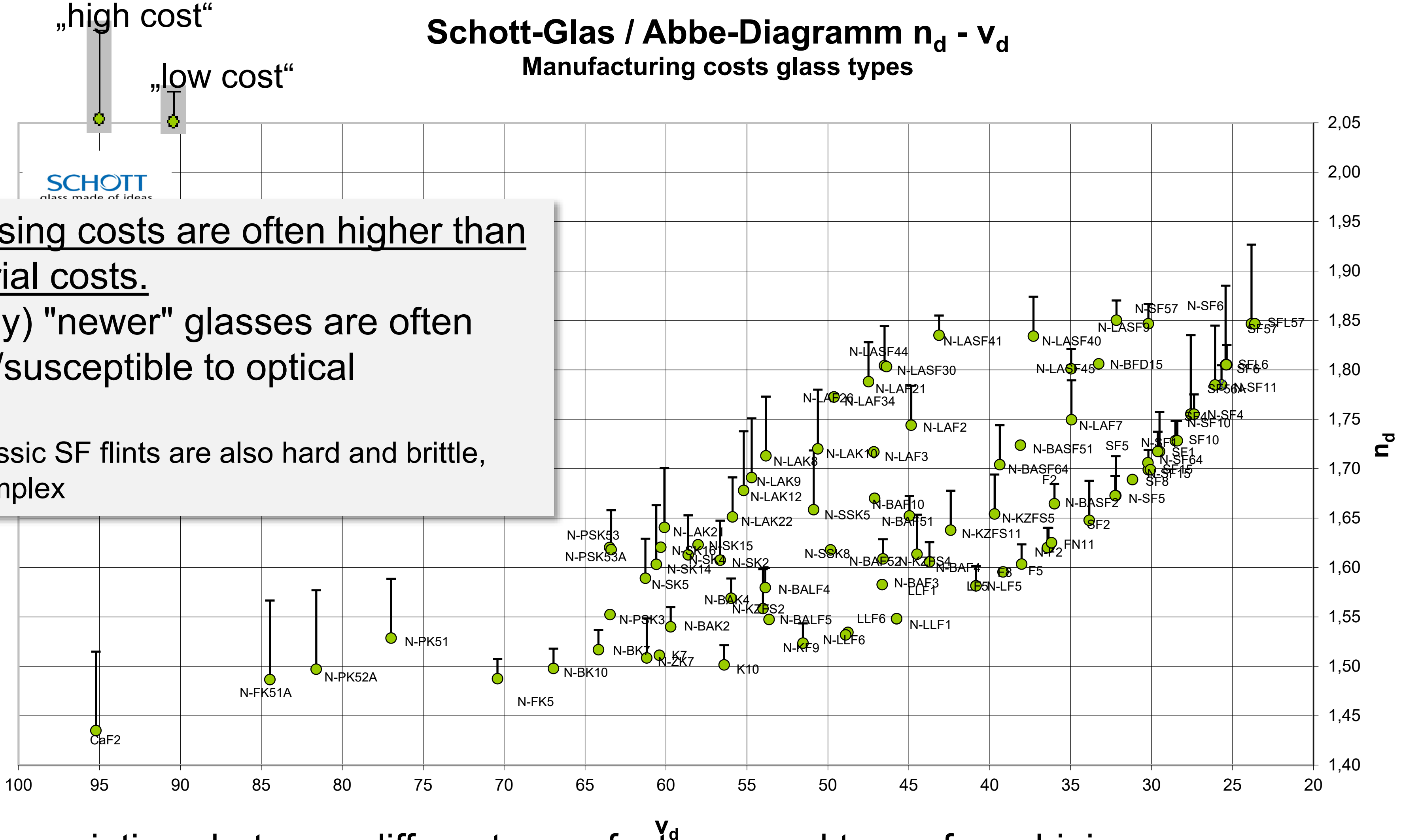


Material price

Achromat	BK7	BK7	KzFS2
	SF2	FK51A	PK51
	1	1	13,1
	1,4	13,5	18,4
Summe	2,4	14,5	31,5
rel. Summe	1,0	6,0	13,1



Optical processing: Cost*



Optical processing costs are often higher than the pure material costs.

The (historically) "newer" glasses are often more complex/susceptible to optical processing.

Exception: e.g. classic SF flints are also hard and brittle, therefore more complex

* Strong variations between different manufacturers and type of machining processes possible!

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Summary / Take-Aways

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- Important quality measure for photonic devices or coupling of optics to photonics (e.g. glass fibers, grating couplers etc) is the coupling efficiency (value between 0 and 1) or insertion loss (given in dB on a logarithmic scale) $\eta = \left|\int U_R(x)\, U_{S,T}^*(x)\, dx\right|^2$
- Lens materials dispersion curve n(λ) is for a given spectral range, e.g. visible range a monotonically increasing function with decreasing λ
- a linear approximation of the dispersion curve of an optical material is given by the Abbe number, e.g. $\nu_d = \frac{n_d - 1}{n_F - n_C}$ and represented in the (n_d, ν_d) Abbe diagram
- for a single lens the relative focal length change between two wavelengths (e.g. F- and C-line respectively for ν_d) is $\frac{\Delta f}{f} = -\frac{1}{\nu}$
- Based on $\frac{\Delta f}{f} = -\frac{1}{\nu}$ an alternative Abbe diagram (n,100/v) instead (n, v) features the advantages of a more direct interpretation on the impact of dispersion, dispersion is characterized on a linear instead of a nonlinear and allows a much more practical comparison with IR glasses in the same diagram
- As dispersion is in general different for different glass types an achromat can be constructed, defined by having equal refractive power at two distinct wavelengths with the equation $0 = \frac{\Phi_1}{\nu_1} + \frac{\Phi_2}{\nu_2}$

Summary / Take-Aways

- Secondary spectrum: Residual deviation of focal length to focus position at $\lambda_1 = \lambda_2$ is defined as $\Phi(\lambda) - \Phi(\lambda_1) = \frac{\vartheta_1 \Phi_1}{\nu_1} + \frac{\vartheta_2 \Phi_2}{\nu_2}$, where ϑ_j is the partial dispersion and $\frac{\vartheta_j}{\nu_j}$ characterizes another Abbe-number also referring to λ_1 now to a wavelength λ_3, different to λ_2.
- Actually as for most glasses, the partial dispersion is about proportional to the Abbe number, that is a linear relationship $\vartheta(\nu) \approx a + b \cdot \nu$, that is these lie all on the „normal line" of a (ϑ,ν)-diagram the secondray spectrum of all these combinations of „normal glasses" gives the same secondary spectrum $f(\lambda) - f(\lambda_1) = -\frac{\vartheta_1 - \vartheta_2}{\nu_1 - \nu_2} f = -\frac{(a+b\cdot\nu_1)-(a+b\cdot\nu_2)}{\nu_1 - \nu_2} f = -bf$
- There are a few exceptions of glasses with "anomalous partial dispersion": they can be found apart from the normal line in the (ϑ,ν)-diagram; those glasses are required to design „apochromats" with a residual chromatic aberration superior to achromats
- Actually the secondary spectrum of an achromat in the visible spectrum gives rise to reduction of the relative focal length change by about two orders of magnitude; usage of glasses with anomalous partial dispersion can further reduce the residual chromatic aberration by another order of magnitude

Summary / Take-Aways

- The actual chromatic performance of optical systems should be evaluated by physical optical simulation e.g. as edge spread functions considering also a relevant depth range and also varying the exposure parameter (for applications where overexposure is possible)
- The criterium color fringe width is a useful measure to quantify chromatic aberration