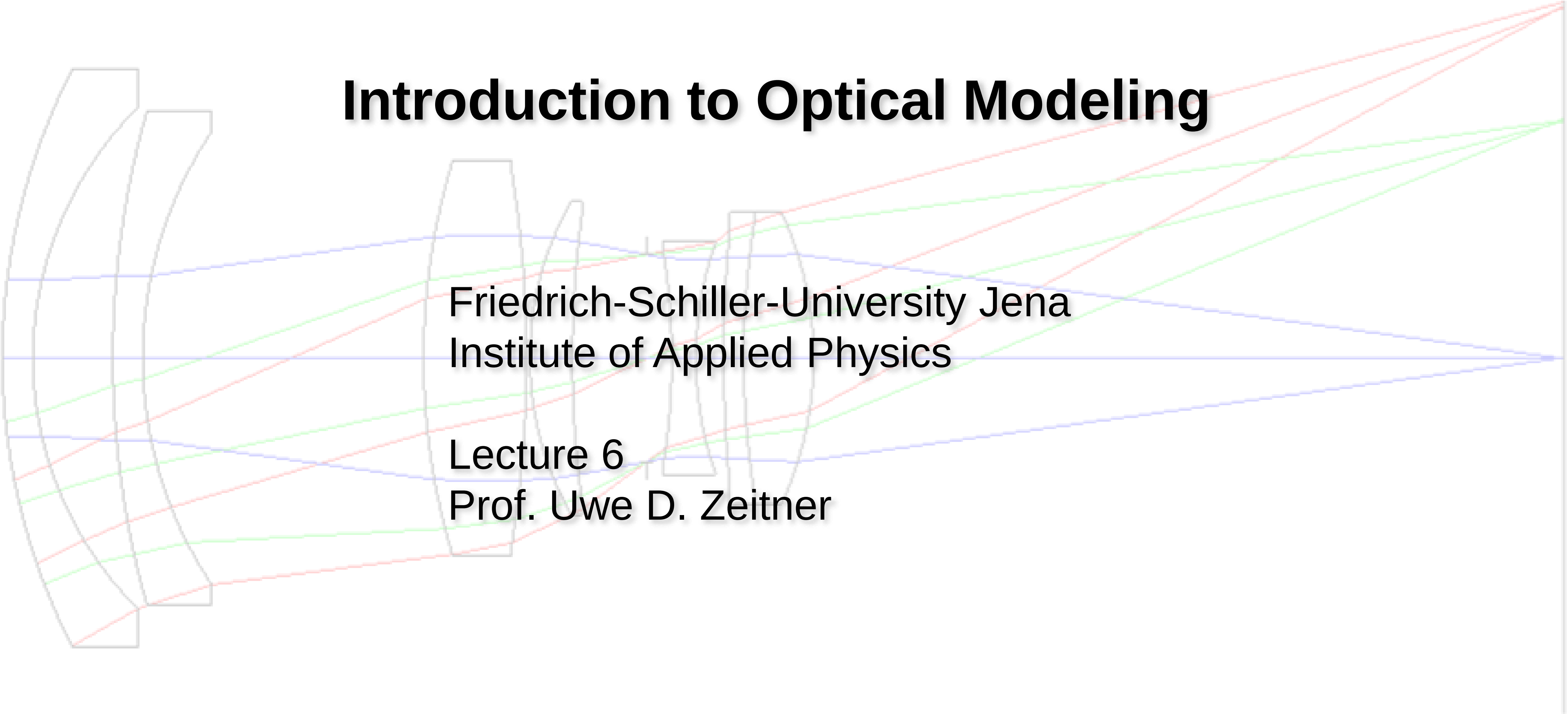


Introduction to Optical Modeling

Friedrich-Schiller-University Jena
Institute of Applied Physics

Lecture 6

Prof. Uwe D. Zeitner

Course Overview

Part 1: Geometrical optics-based modeling and design (U.D. Zeitner)

1. Introduction
2. Paraxial approximation / Gaussian optics
3. ABCD-matrix formalism
4. Real lenses
5. Optical materials
   - glass types, dispersion
   - chromatic aberrations
6. **Imaging systems**
   - apertures/stops, entrance-/exit-pupil
   - **wavefront aberrations**

Part 2: Wave-optics based modeling (F. Wyrowski)

Short Excursus: Energy in an Optical System

Definitions related to energy in the system:

Radiant power $\Phi = \int \vec{S} d\vec{A}$ [W]

- integral power of a source or a ray
- also called ***radiant flux*** or ***light flux***
- integral of pointing vector S through a surface element dA

Radiance $L = \frac{d^2\Phi}{\cos\theta \, d\Omega dA}$ [W / m² / sr]

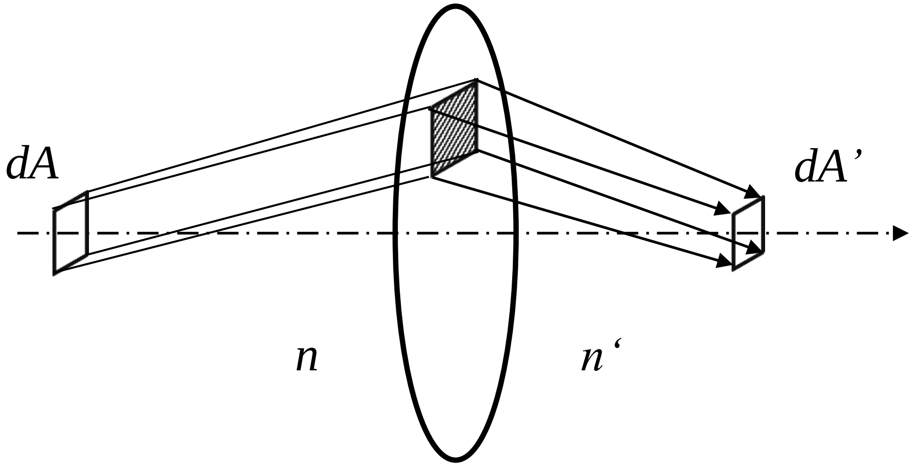
- **density of flux** on a surface element dA into a solid-angle element $d\Omega$
- if radiant power is not evenly distributed

θ ... angle between surface normal and direction of radiation
$d\Omega$ ... solid angle element
dA ... surface element

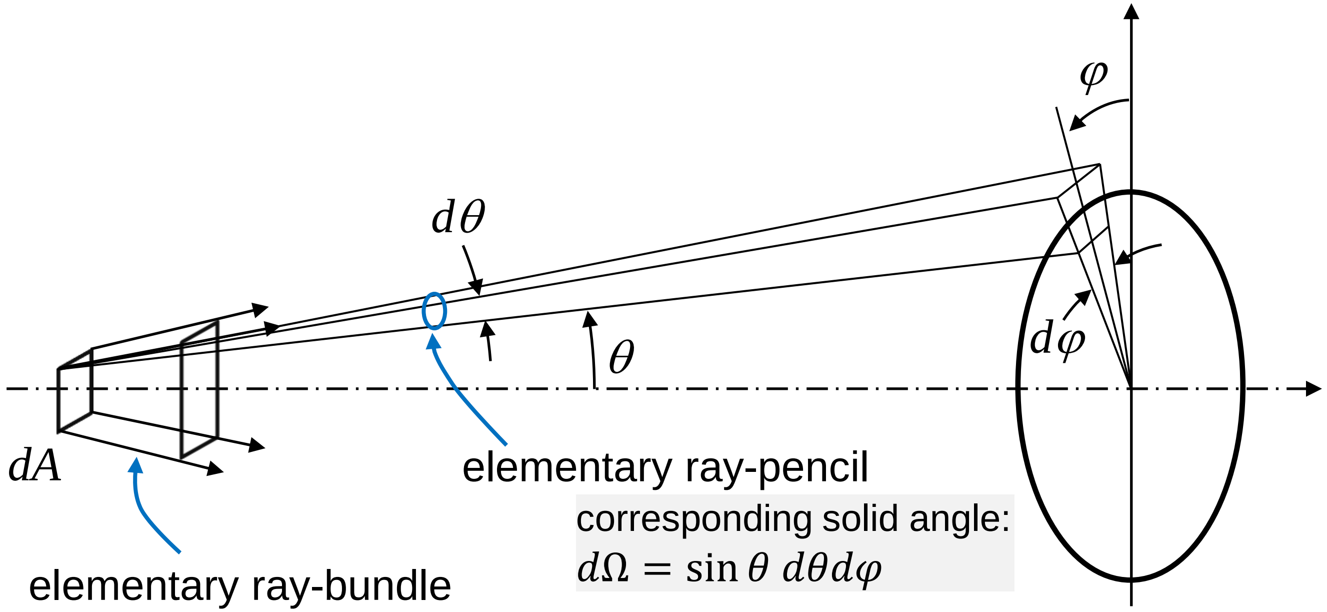
Short Excursus: Transport of Energy...

... through an optical imaging system

Imaging of a small surface element dA:



elementary ray-bundle in an optical system



elementary ray-pencil as part of the elementary ray-bundle

Conservation of energy: $\boxed{d^2\Phi = d^2\Phi'}$

- no reflection losses
- no other losses

→ radiant flux within the ray-bundle remains conserved

Short Excursus: Transport of Energy...

... through an optical imaging system

Conservation of energy: $\boxed{d^2\Phi = d^2\Phi'}$

- no reflection losses
- no other losses

→ radiant flux within the elementary ray-bundle remains conserved

flux density definition

➡ $L \cos\theta \; d\Omega dA = L' \cos\theta' \; d\Omega' dA'$

with $d\Omega = \sin\theta \; d\theta d\varphi$ and $\dfrac{d(\sin^2\theta)}{d\theta} = \dfrac{1}{2}\sin\theta\cos\theta$

➡ $L \; dA \; d\varphi \; d(\sin^2\theta) = L' \; dA' \; d\varphi' \; d(\sin^2\theta')$

axially symmetric system → no change of $d\varphi$ → $d\varphi = d\varphi'$

➡ $$\frac{L'}{L} = \frac{dA \; d(\sin^2\theta)}{dA' \; d(\sin^2\theta')}$$

Short Excursus: Transport of Energy...

... through an optical imaging system

Conservation of energy: $\boxed{d^2\Phi = d^2\Phi'}$

- no reflection losses
- no other losses

→ radiant flux within the elementary ray-bundle remains conserved

$$\frac{L'}{L} = \frac{dA\, d(\sin^2\theta)}{dA'\, d(\sin^2\theta')}$$

squared sine-condition: $$\underbrace{n^2 y^2}_{dA} \sin^2\theta = \underbrace{n'^2 y'^2}_{dA'} \sin^2\theta'$$

as we are not only considering an angle θ within the ray-bundle, but an angle interval it is allowed to introduce the differential:

$$n^2 dA\, d(\sin^2\theta) = n'^2 dA'\, d(\sin^2\theta')$$

$$\boxed{\frac{L'}{L} = \frac{n'^2}{n^2}}$$

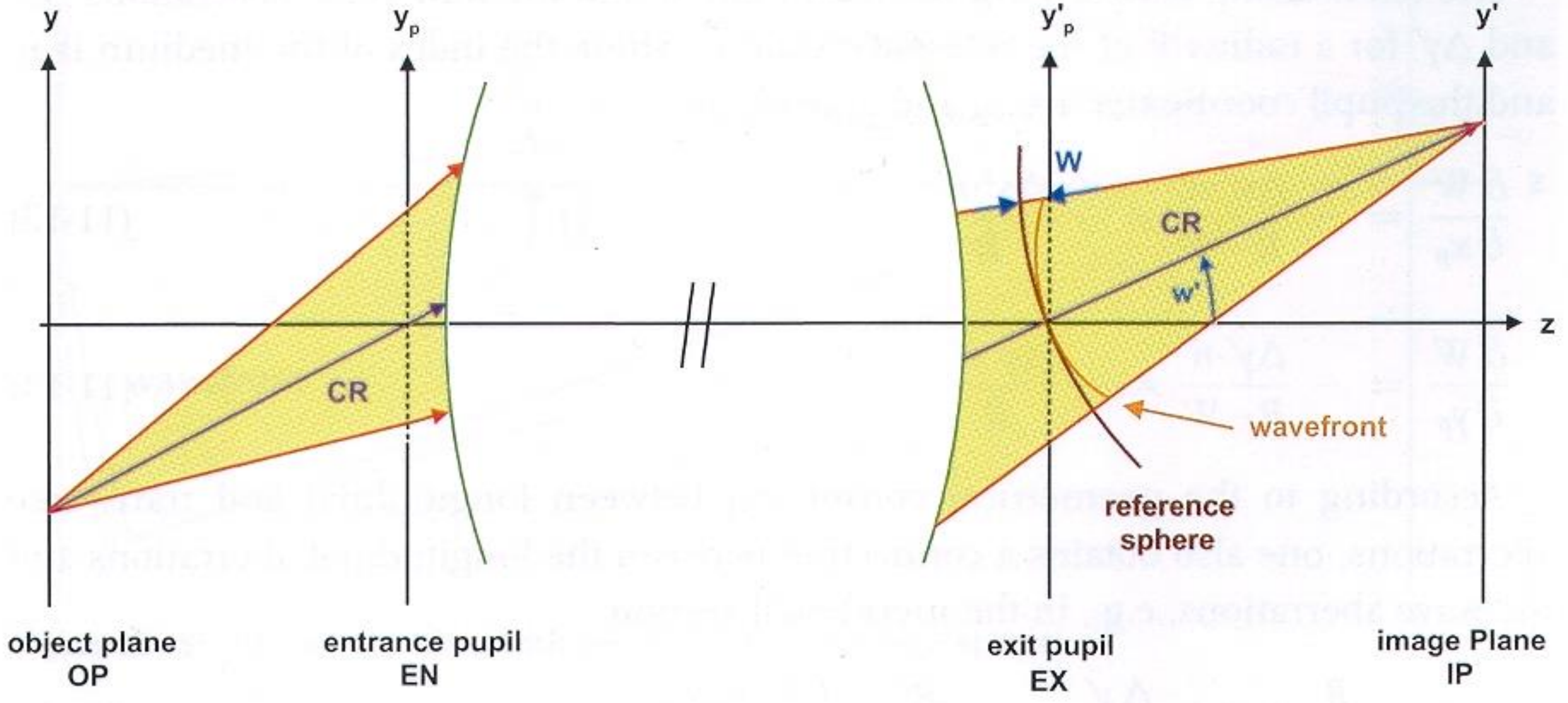
with $n=n'$

$$\boxed{L = L'}$$

conservation of ray density

→ it is not possible to increase the ray density of a beam by focusing!

Wave aberrations of an optical system



Wave aberrations for an optical system.

Polynomial expansion of $W(\beta, r, \psi)$

$$W(\beta, r, \psi) = \underbrace{W_{000}}_{\text{Piston Error}}$$

$$+ \underbrace{W_{200} \cdot \beta^2}_{\text{Piston error}} + \underbrace{W_{020} \cdot r^2}_{\text{Defocus}} + \underbrace{W_{111} \cdot \beta \cdot r \cdot \cos\psi}_{\text{Lateral Magnification Error}}$$

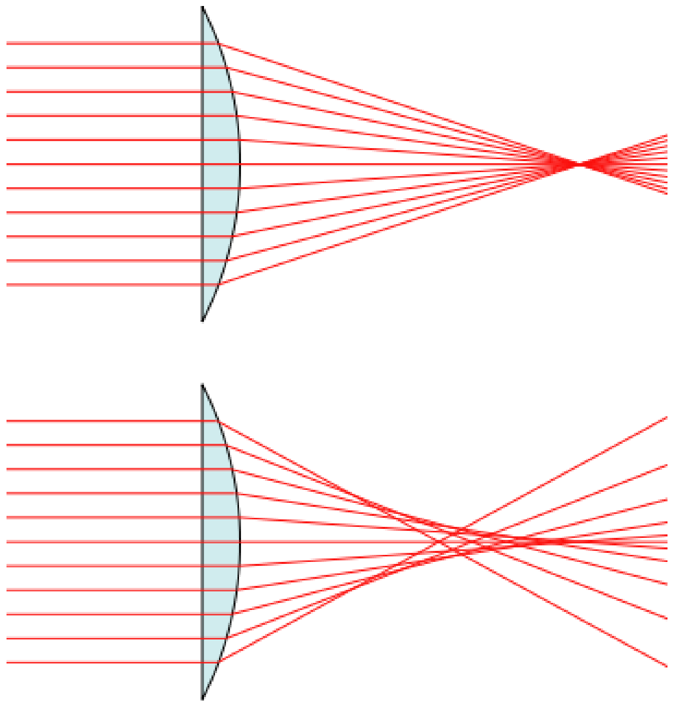
$$+ \underbrace{W_{400} \cdot \beta^4}_{\text{Piston Error}} + \underbrace{W_{040} \cdot r^4}_{\text{SA}} + \underbrace{W_{131} \cdot \beta \cdot r^3 \cdot \cos\psi}_{\text{Coma}} + \underbrace{W_{222} \cdot \beta^2 \cdot r^2 \cdot \cos^2\psi}_{\text{Astigmatism}}$$

$$+ \underbrace{W_{220} \cdot \beta^2 \cdot r^2}_{\text{Field Curvature}} + \underbrace{W_{311} \cdot \beta^3 \cdot r \cdot \cos\psi}_{\text{Distortion}}$$

+ … aberrations of higher order

Spherical Aberration (~r^4)

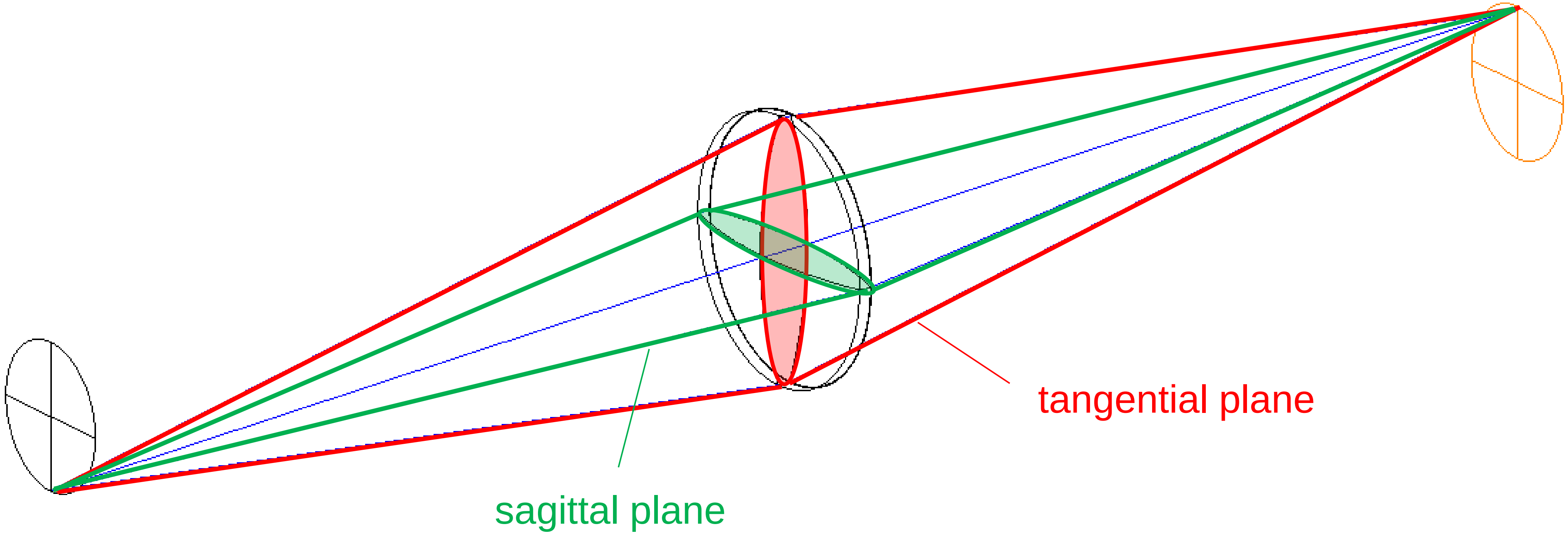
Origin: different focal lengths for different ray heights



Spherical aberration. A perfect lens (top) focuses all incoming rays to a point on the optic axis. A real lens with spherical surfaces (bottom) suffers from spherical aberration: it focuses rays more tightly if they enter it far from the optic axis than if they enter closer to the axis. It therefore does not produce a perfect focal point.

Tilted Ray Bundles

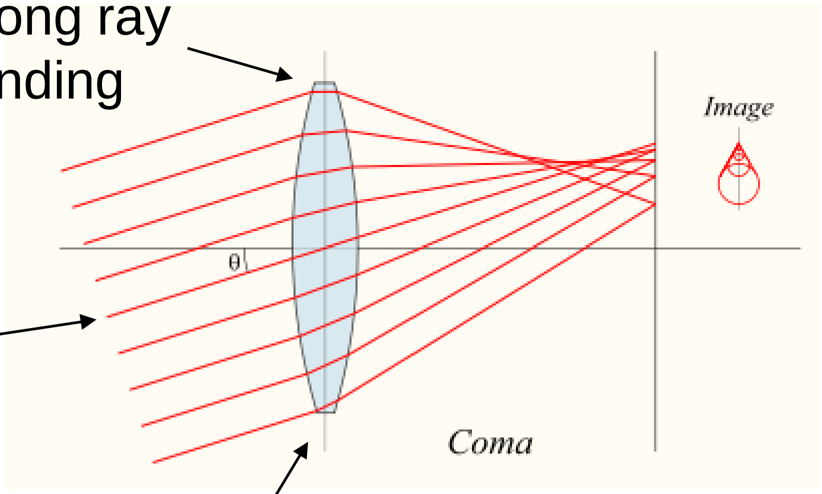
Definition of **tangential-plane** and **sagittal-plane**



Coma ($\sim \beta\, r^3 \cos\psi$)

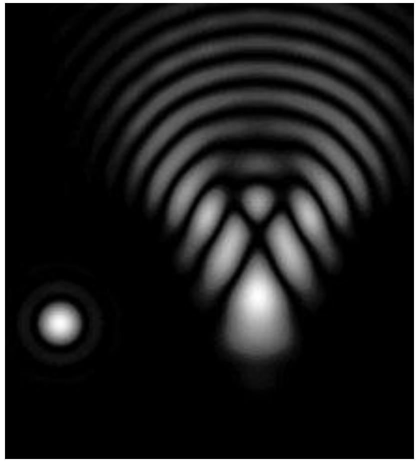
Origin: Non-symmetry of bundle around chief ray
→ **"non-symmetry error"**

strong ray bending



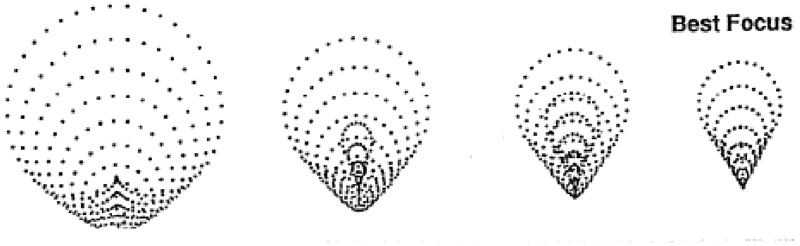
chief ray

weak ray bending

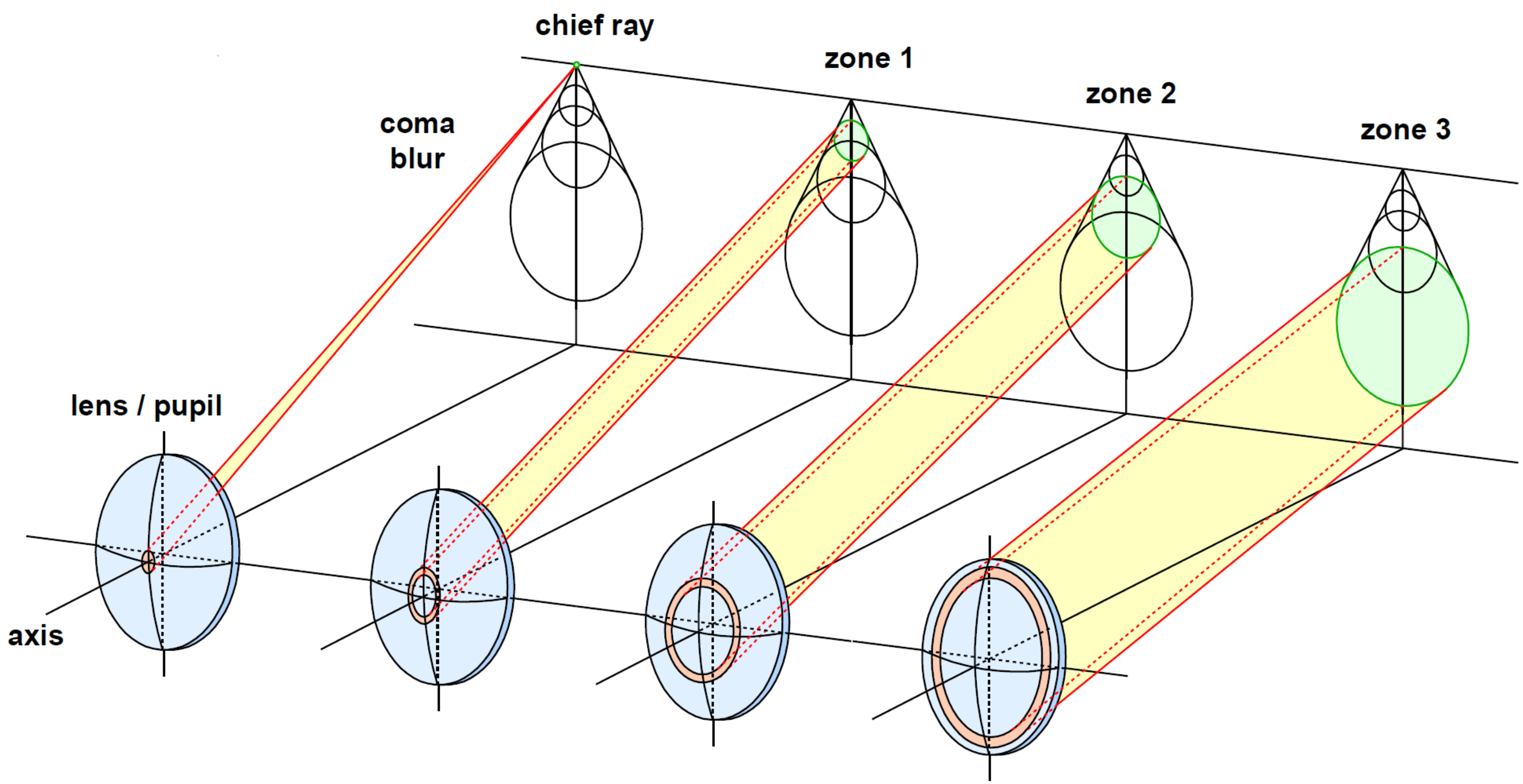


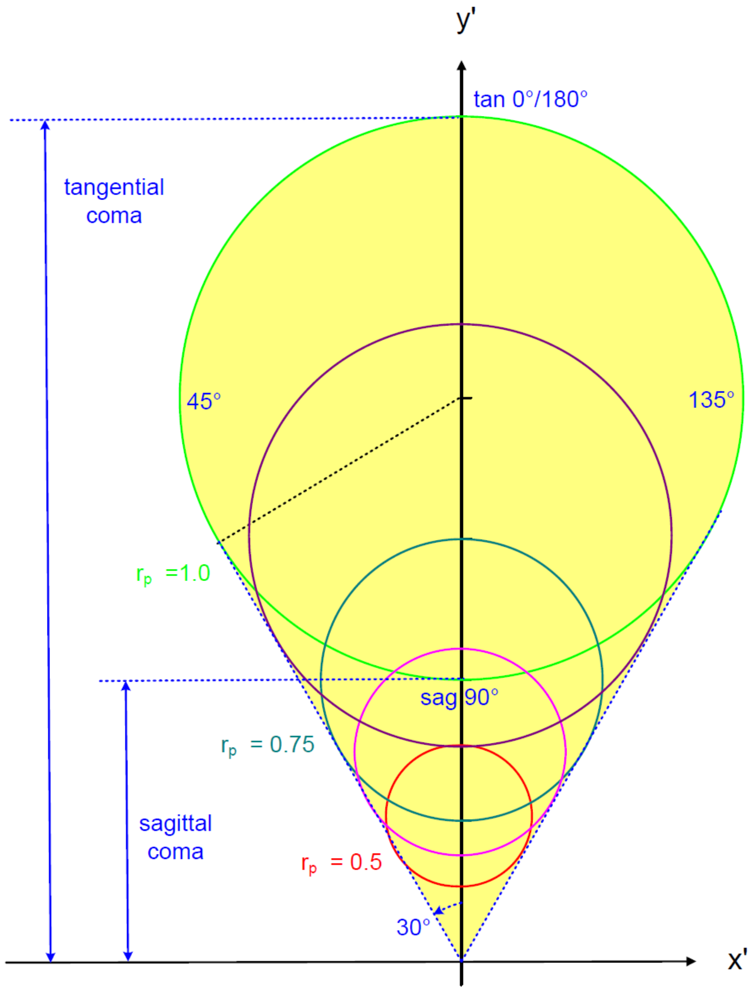
diffraction limited spot

spot perturbed by coma



Coma Figure

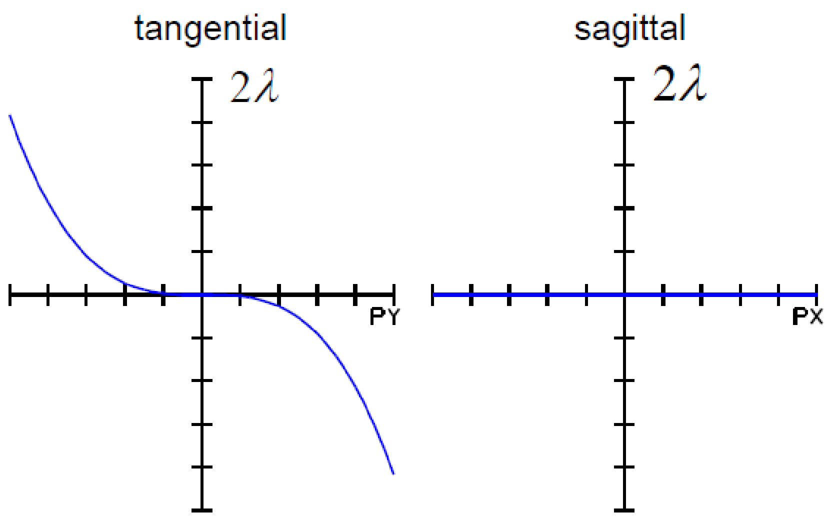


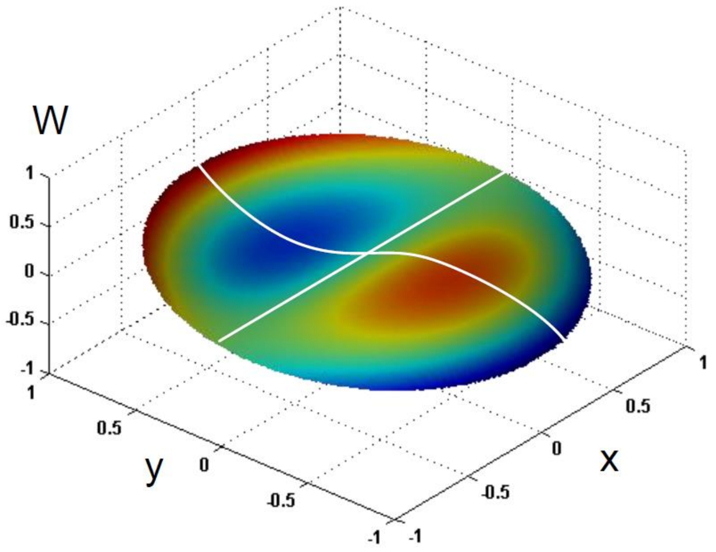


- each circular zone in the pupil generates a circle in the image plane
- diameter and position of circles vary with r

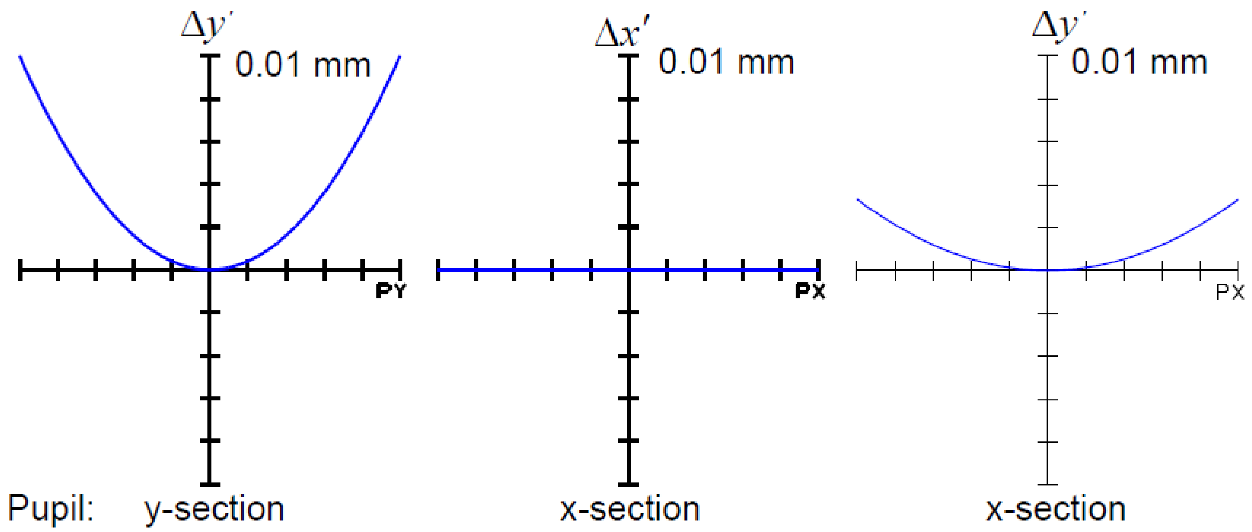
Indicate Coma

wave aberration W



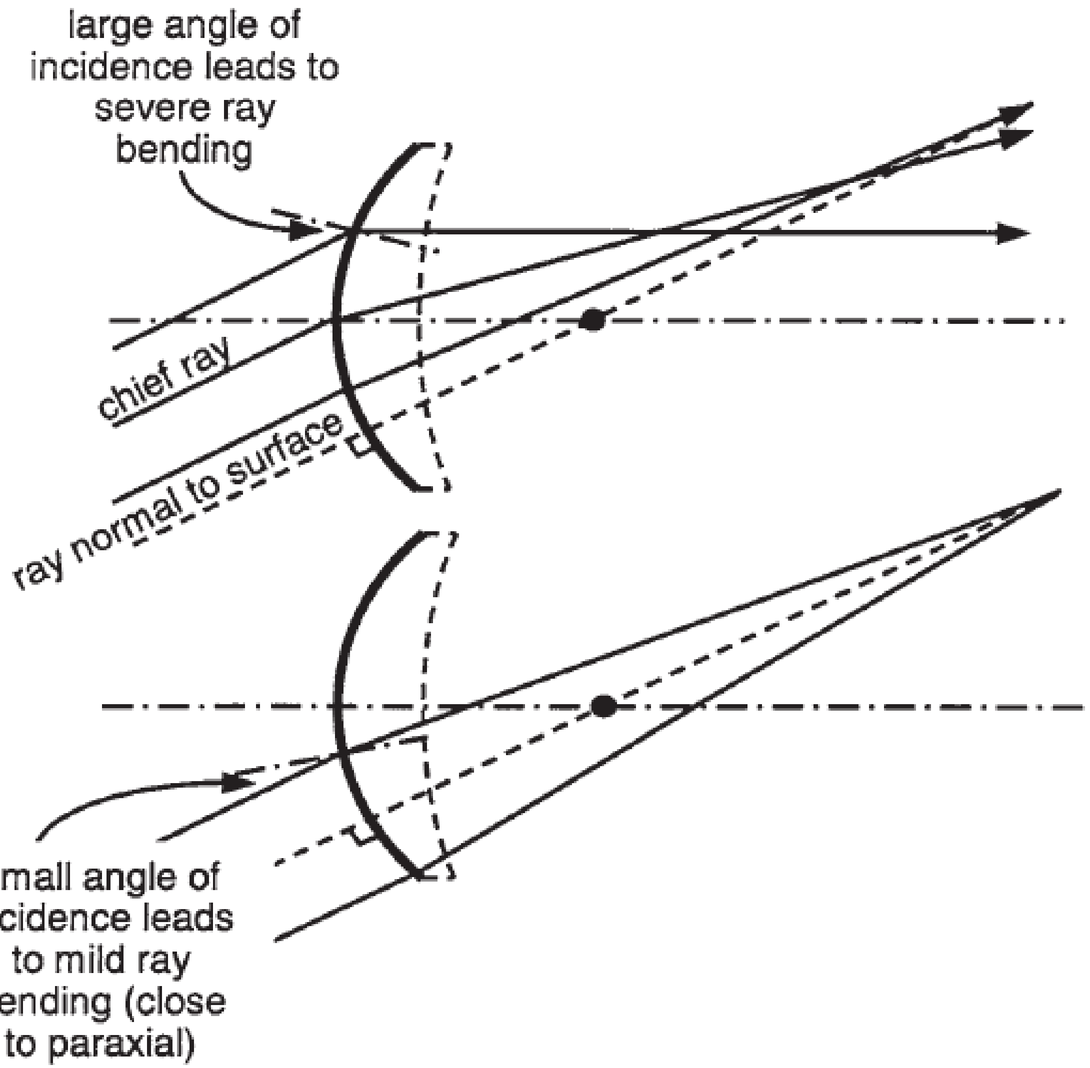


transverse ray aberration ε



Getting Rid of Coma

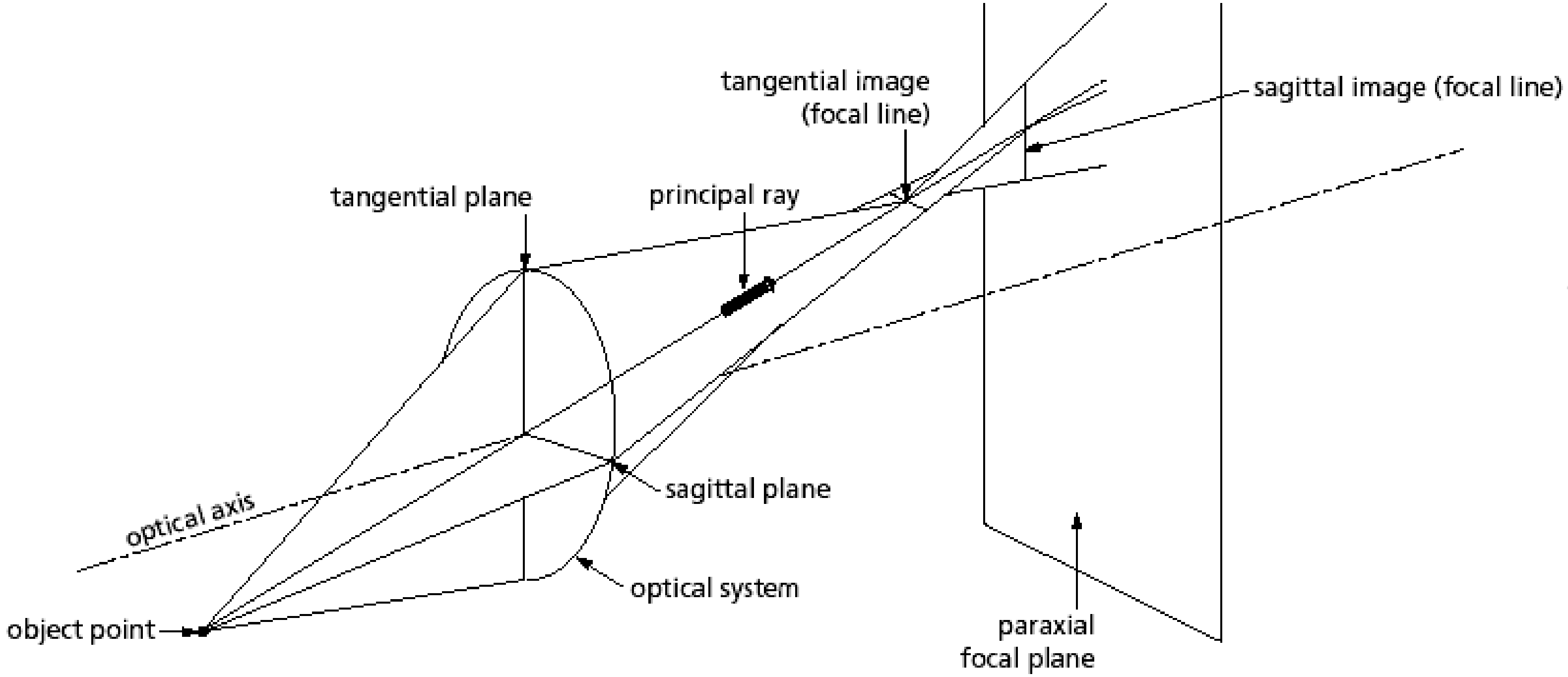
→ **Move stop!**



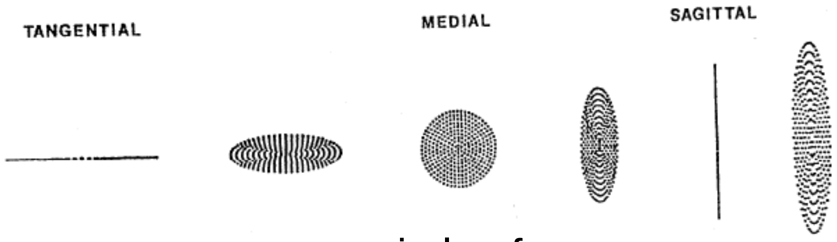
Astigmatism (~ β² r² cos² ψ)

Origin: different optical powers in x and y due to oblique incidence / projection

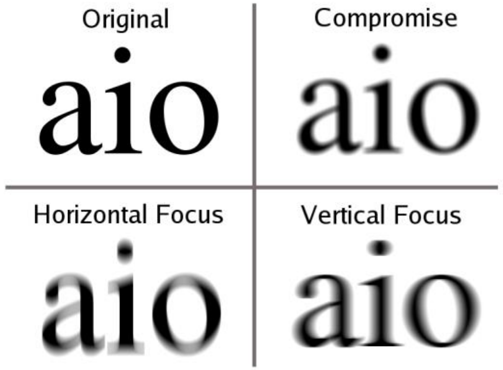
→ effective ROC in tangential plane: $R_t = R \cdot \cos\theta$ θ … incidence angle



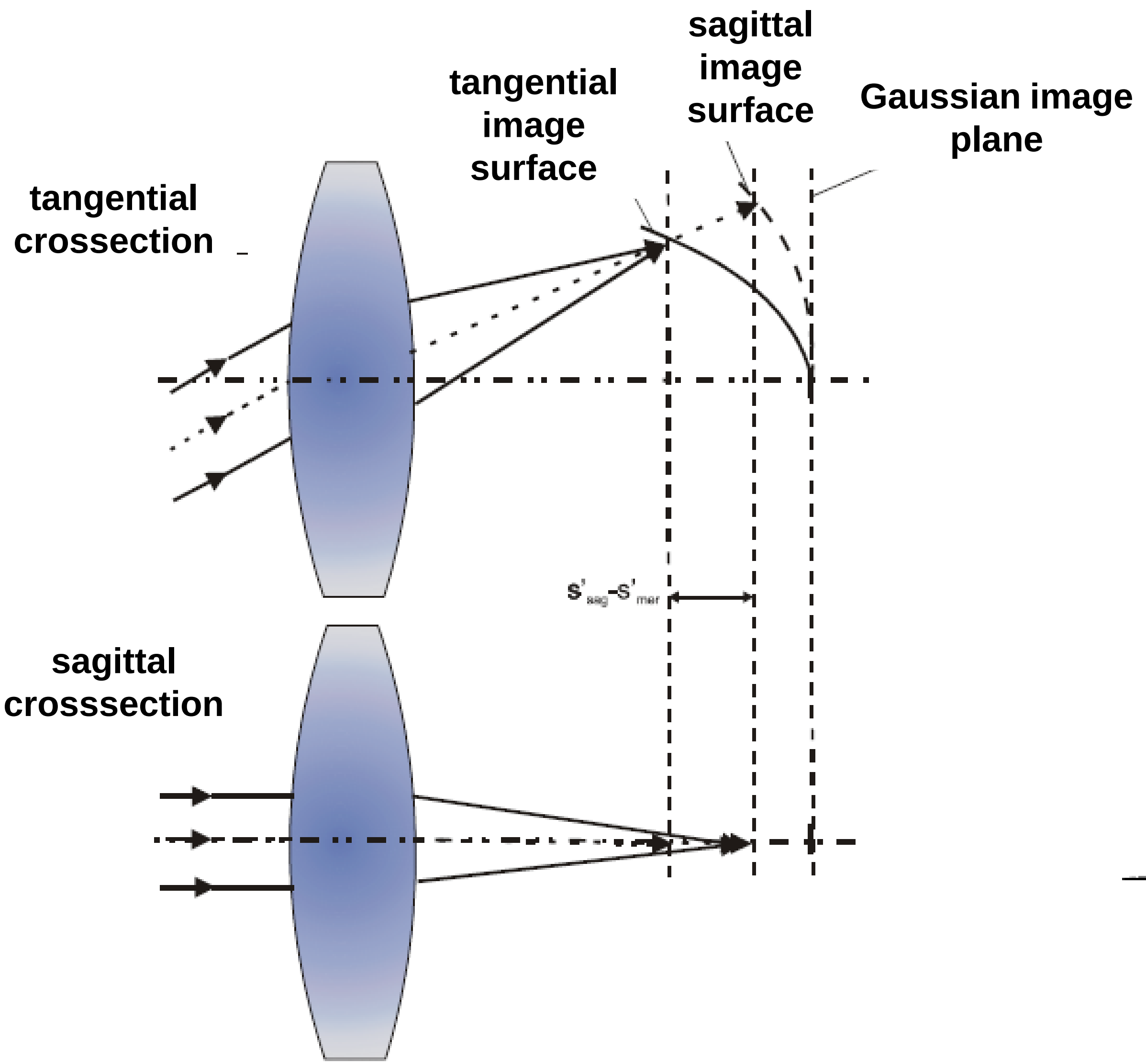
focus geometry



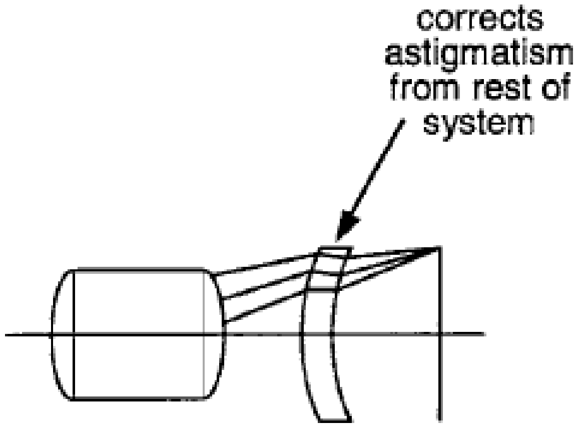
circle of
least confusion



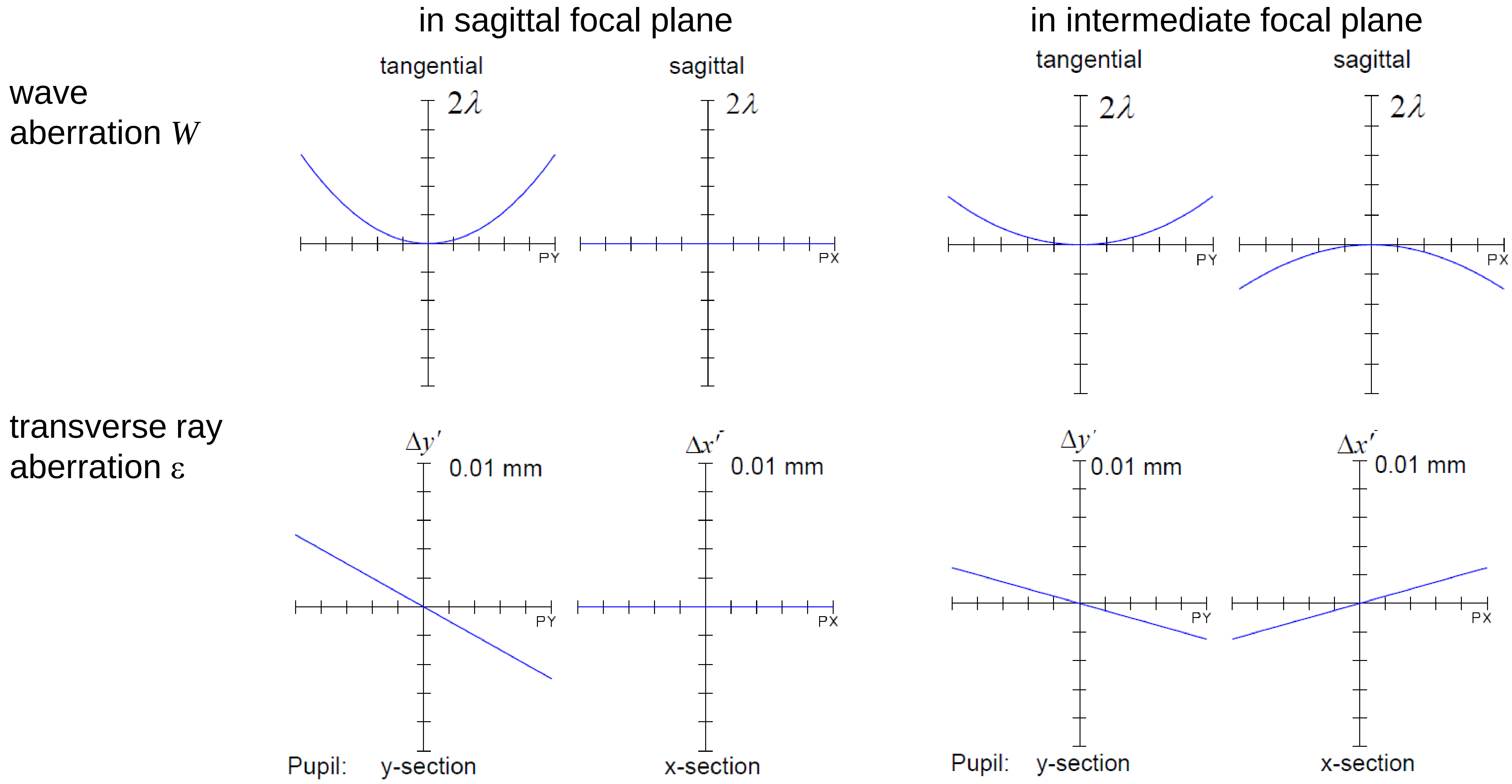
Astigmatism (~ $\beta^2 r^2 \cos^2 \psi$)



Correction by applying menisc lenses

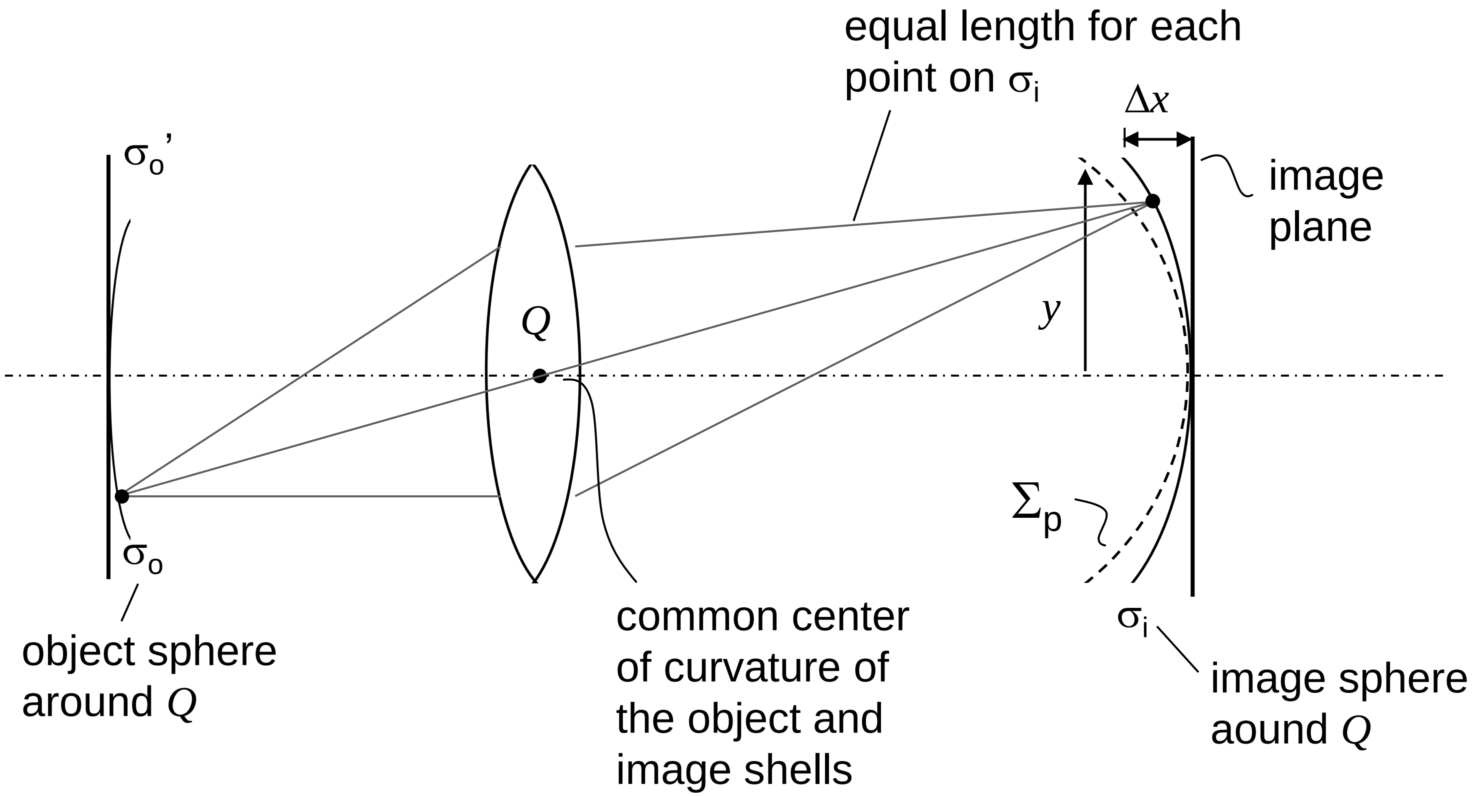


Indicate Astigmatism



Field Curvature (~ β^2 r²)

Origin: natural image surface is spherical, not planar
→ "Petzval curvature"

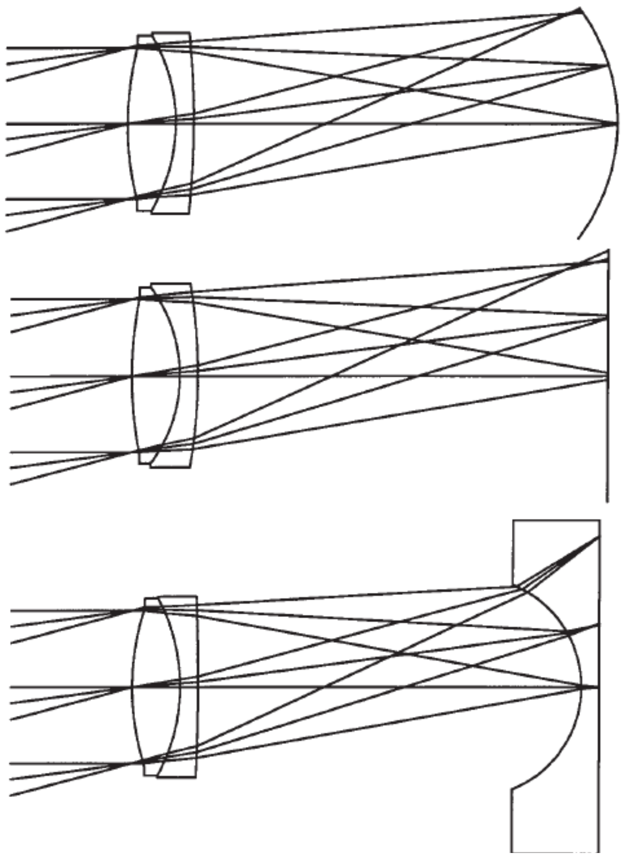


$$\frac{1}{s'} - \frac{1}{s} = \frac{1}{f'}$$

Σ_p … Petzval surface

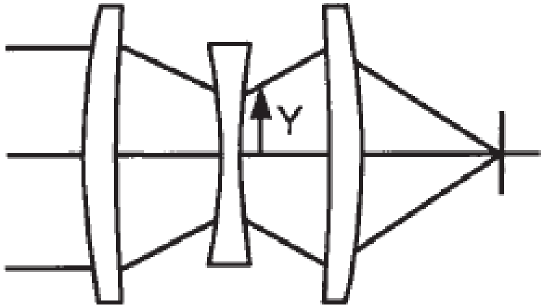
Josef Max Petzval (1807 – 1891), Hungarian Mathematician

Field Curvature (~ $\beta^2 r^2$)



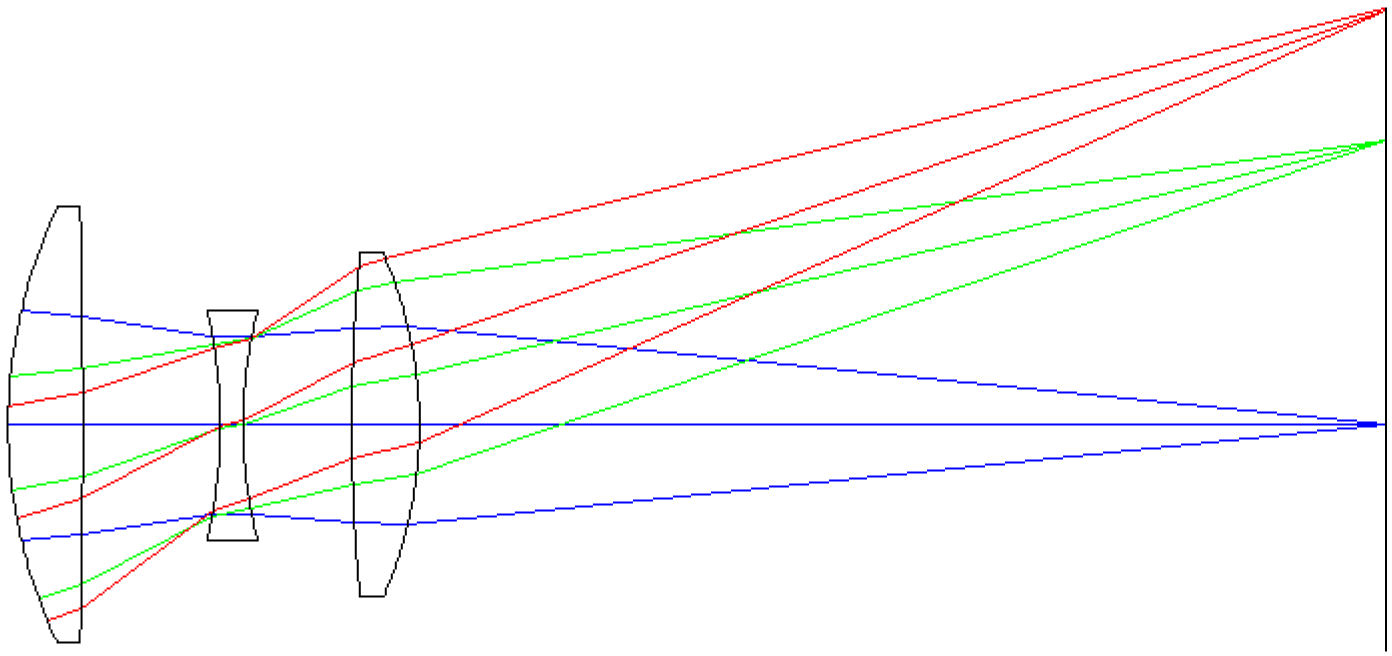
lens with field flattener
(Petzval lens)

- Make Petzval sum equal zero!
- Balance with astigmatism!



Cooke Triplet

The Cooke Triplet



Cooke triplet lenses

Cooke triplet is a well-know lens form that provides good imaging performance over a field of view of +/- 20-25 degrees. Many consumer grade film cameras use lenses of this type.

New Achromate

Achromate: typically corrected for axial chromatic aberration

condition of achromasie: $$\frac{\Phi_1}{\nu_1} + \frac{\Phi_2}{\nu_2} = 0$$

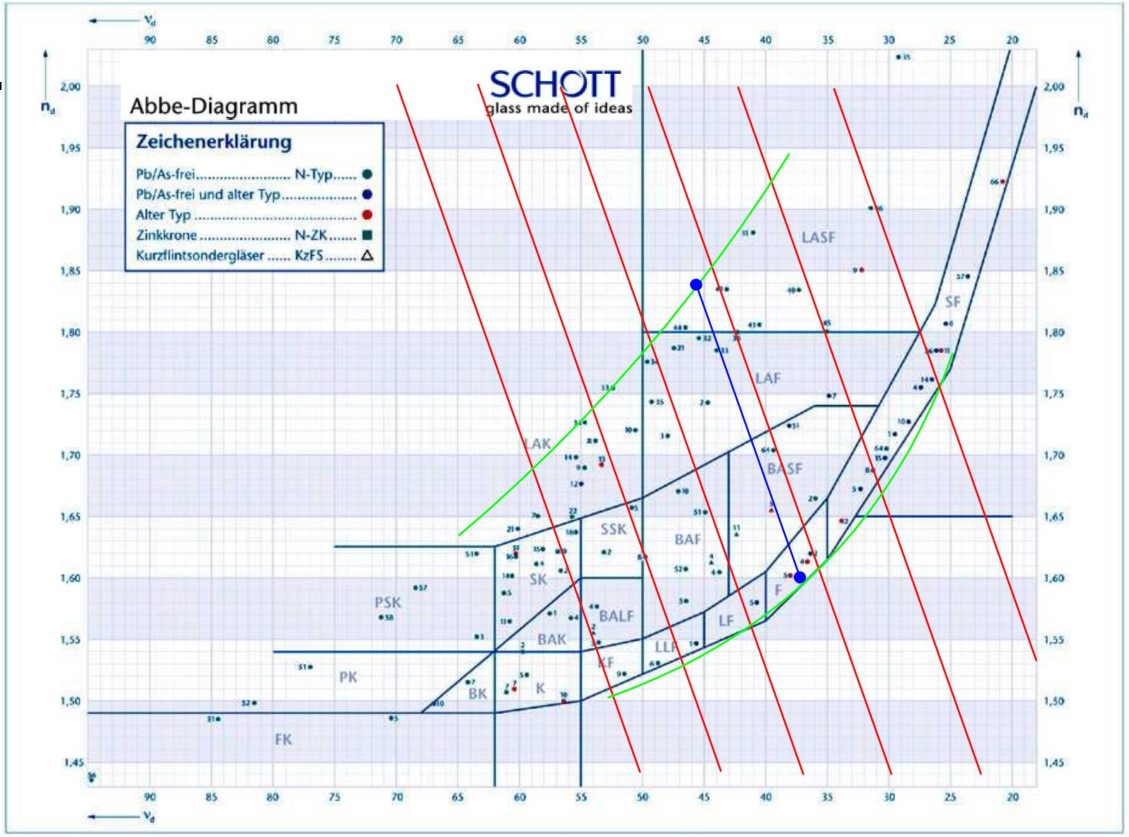
Correction for field curvature:

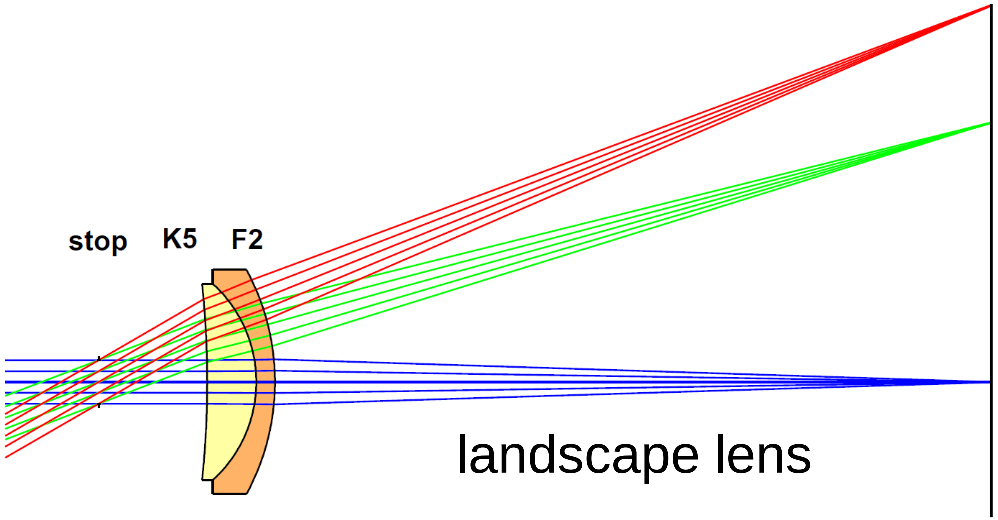
make Petzval-sum zero $$\frac{\Phi_1}{n_1} + \frac{\Phi_2}{n_2} = 0$$

Condition for simultaneous correction of chromatic aberration and field curvature:

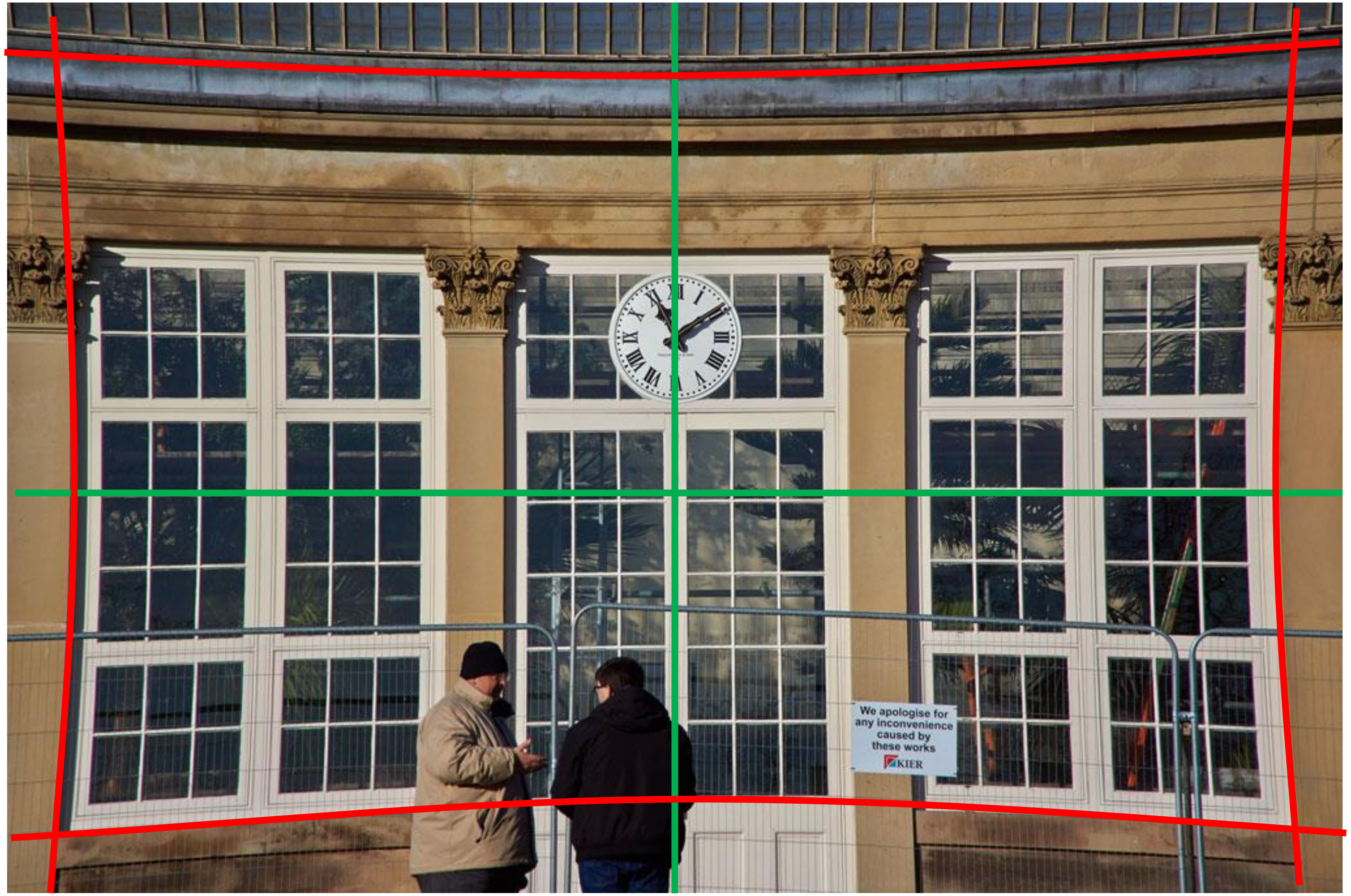
$$\Rightarrow \boxed{\frac{\nu_1}{\nu_2} = \frac{n_1}{n_2}}$$

→ find two glasses on one straight line in the glass diagram





Distortion



- image is **sharp** everywhere!
- **equality** to object is **not given**

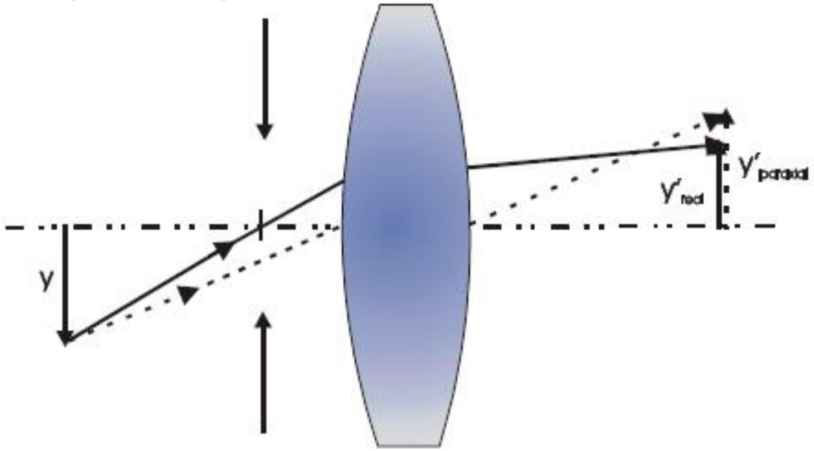
→ image with sharp but bended edges
→ no distortion along central lines

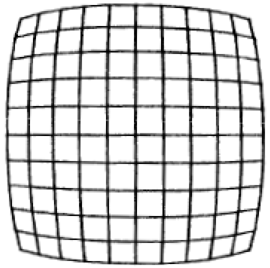
Ref.: www.ephotozine.com

Distortion (~β³ r cos ψ)

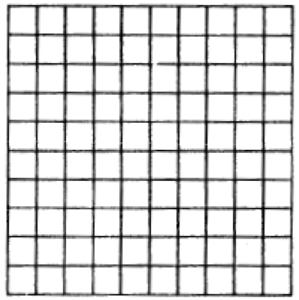
→ deformation of image scale due to **different transversal magnifications for each field point**

system aperture



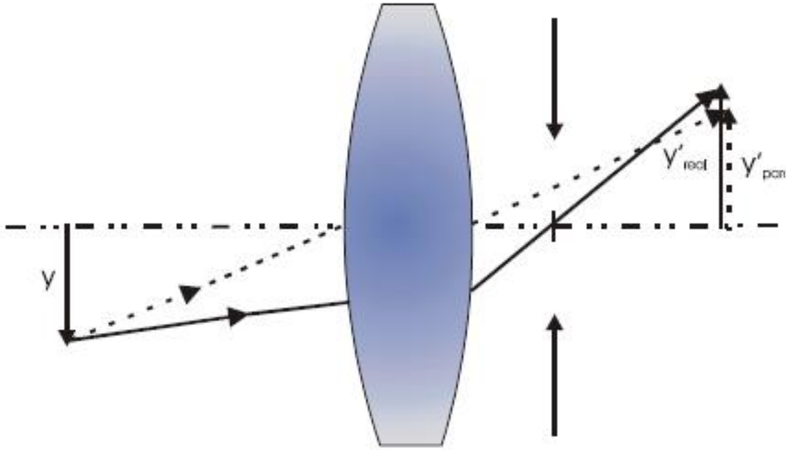


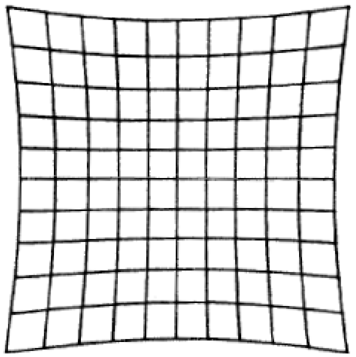
Positive (Barrel) Distortion



No Distortion

system aperture





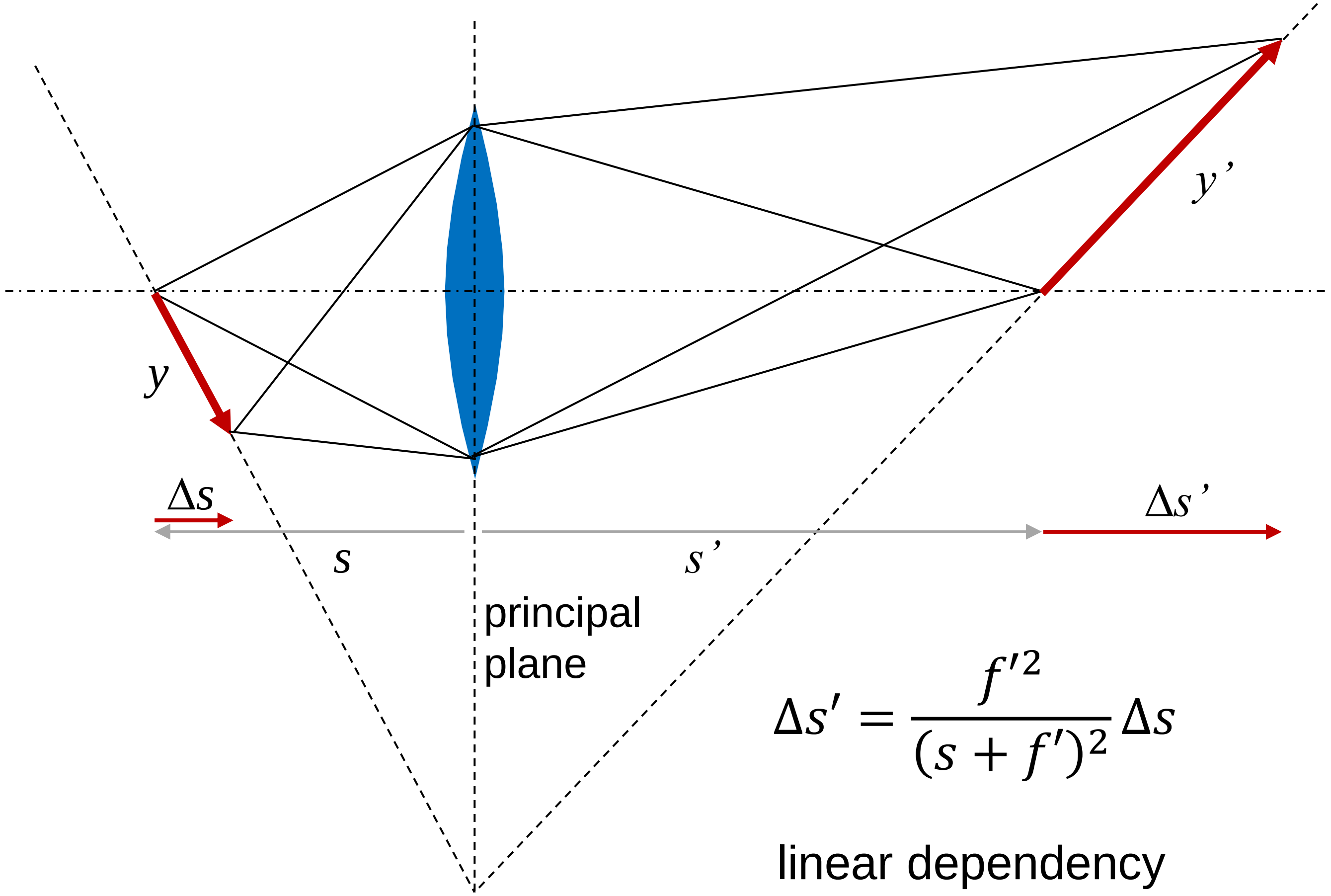
Negative (Pincushion) Distortion

→ function of the stop position

A Special Kind of Distortion: Imaging of Tilted Objects

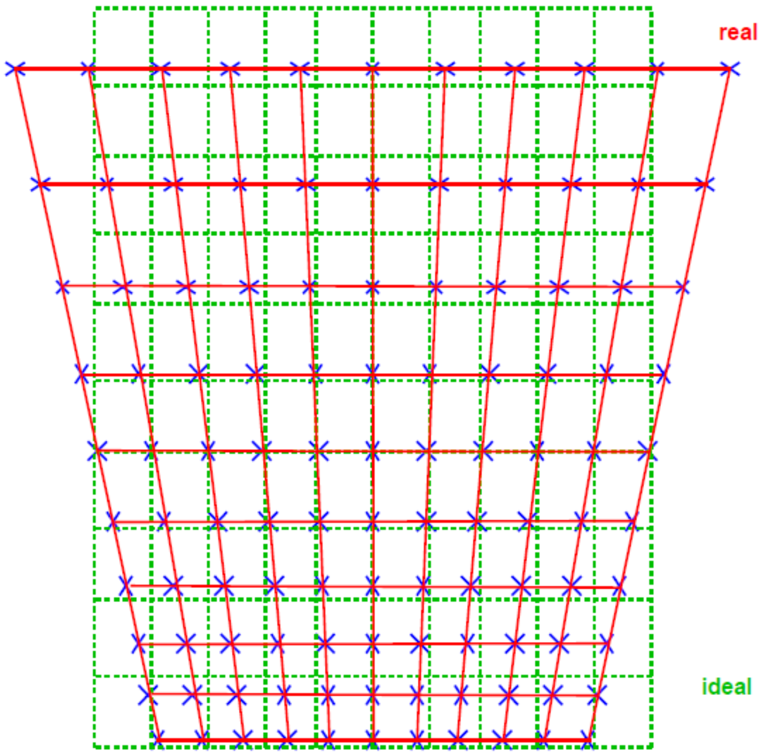
remember: imaging equation $$\frac{1}{s'} - \frac{1}{s} = \frac{1}{f'}$$

image scale: $$m = \frac{s'}{s}$$



$$\Delta s' = \frac{f'^2}{(s+f')^2}\Delta s$$

linear dependency

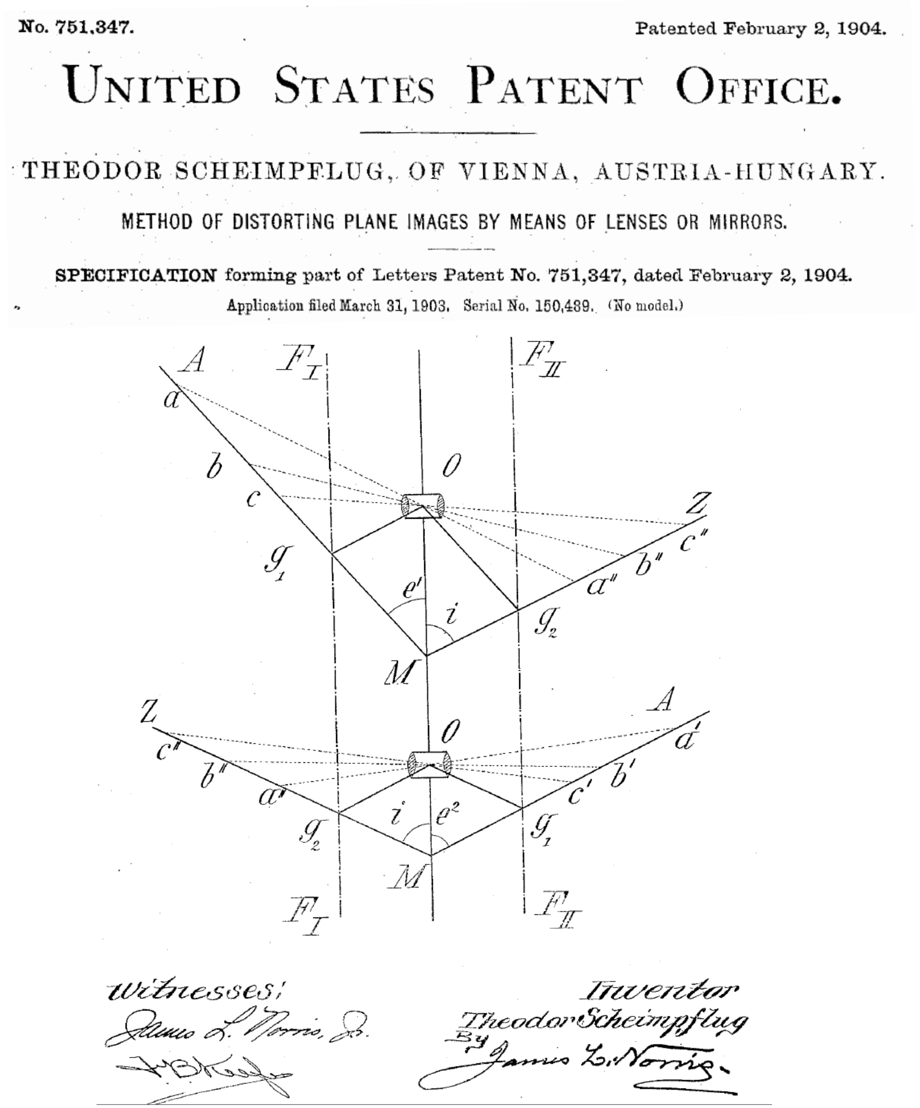


→ **keystone distortion**

Scheimpflug Imaging

(Theodor Scheimpflug, 1865 – 1911)

→ imaging of tilted objects


No. 751,347. Patented February 2, 1904.

UNITED STATES PATENT OFFICE.

THEODOR SCHEIMPFLUG, OF VIENNA, AUSTRIA-HUNGARY.

METHOD OF DISTORTING PLANE IMAGES BY MEANS OF LENSES OR MIRRORS.

SPECIFICATION forming part of Letters Patent No. 751,347, dated February 2, 1904.

Application filed March 31, 1903. Serial No. 150,489. (No model.)

Witnesses:

Inventor

Theodor Scheimpflug

By James L. Norris

Initial application:

correction of perspective distortions in areal photography

Today's examples:

- special effects in portrait photography
- imaging of small objects

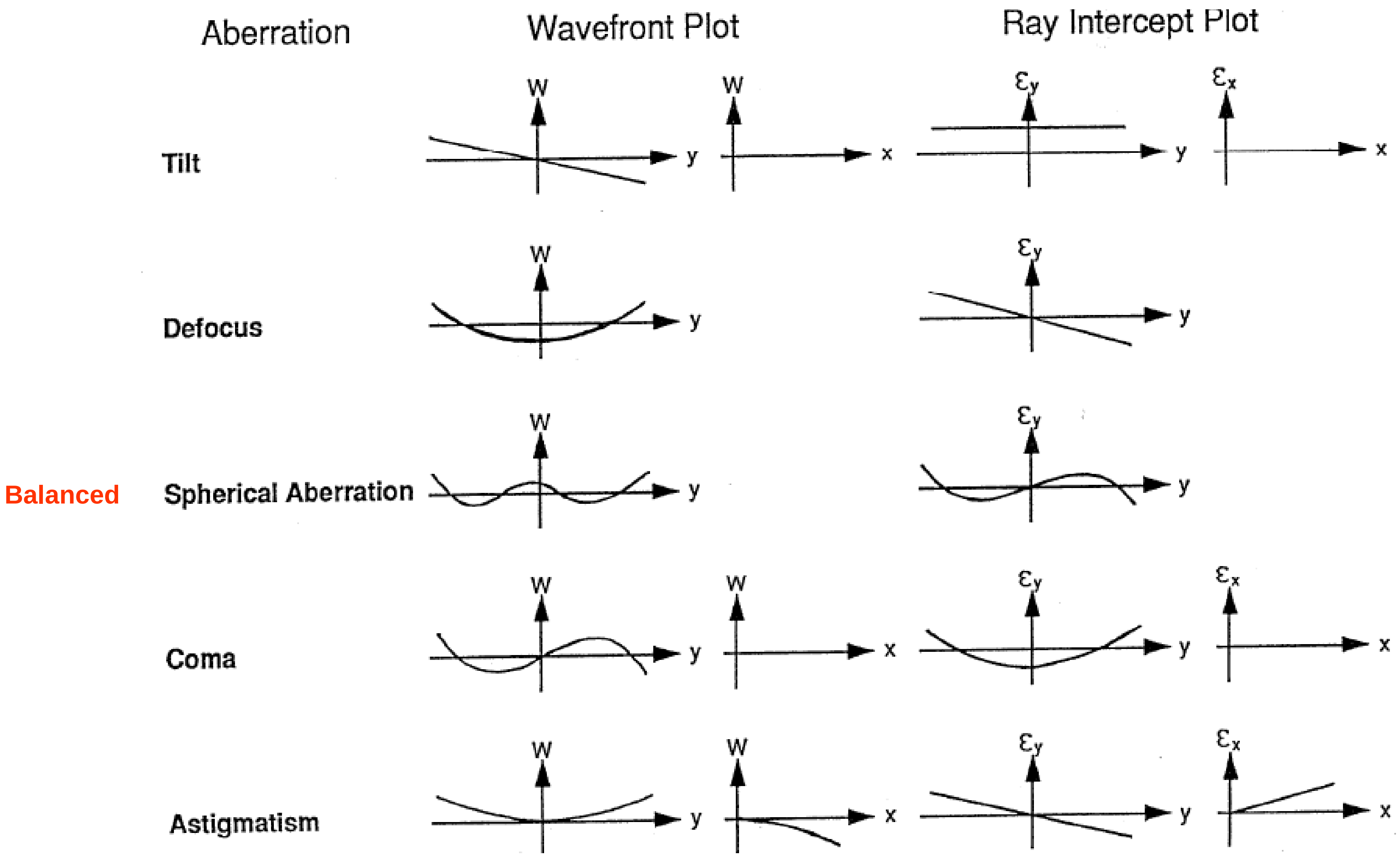

source: wikipedia


source: wikipedia

Summary of wavefront aberrations

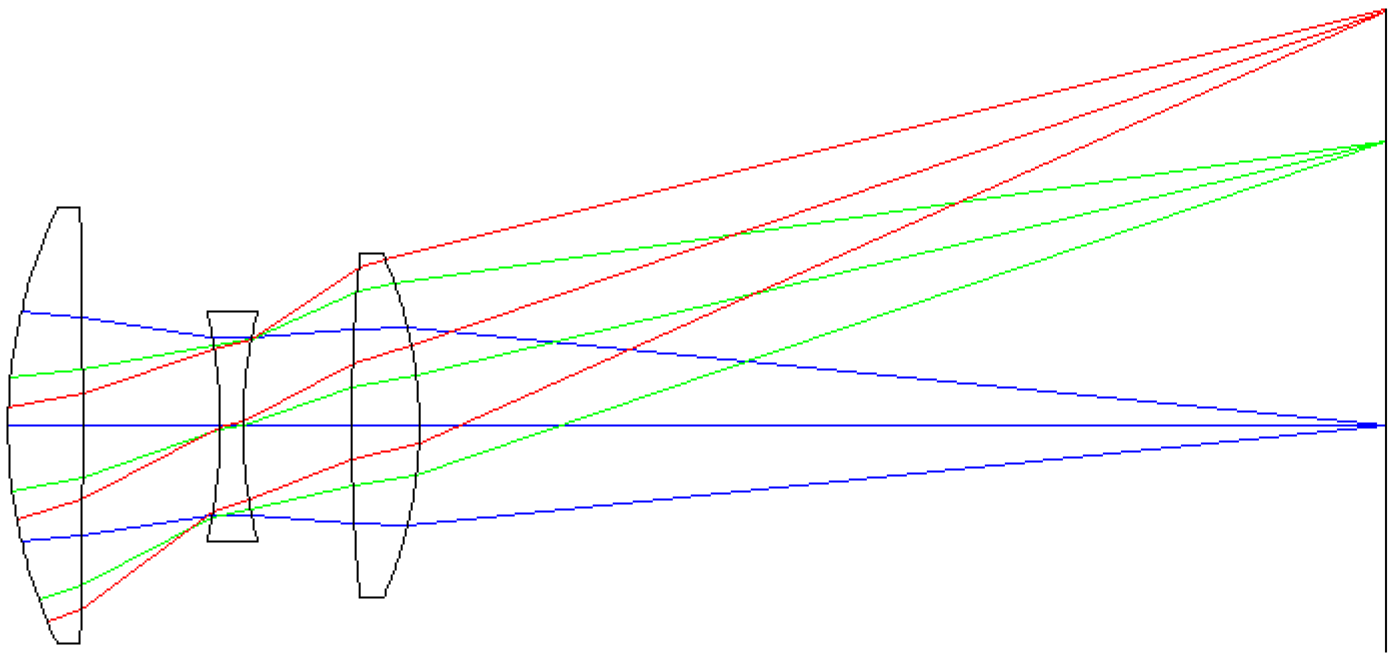
Longitudinal color	$f(\lambda)$	Varying focus with wavelength
Lateral color	$\beta(\lambda)$	Varying magnification with wavelength
Defocus	$\sim r^2$	Longitudinal focal shift
Tilt	$\sim \beta\ r \cos\ \psi$	Transverse focal shift, magnification error
Spherical	$\sim r^4$	Varying focus with radius in pupil plane
Coma	$\sim \beta\ r^3 \cos\ \psi$	Varying magnification and focus with radius in pupil
Astigmatism	$\sim \beta^2\ r^2 \cos^2\psi$	Varying focus with azimuthal angle in pupil
Field curvature	$\sim \beta^2\ r^2$	Varying focus with field
Distortion	$\sim \beta^3\ r \cos\ \psi$	Varying magnification with field

Wavefront and Ray Intercept Plots



Common Camera Design Forms

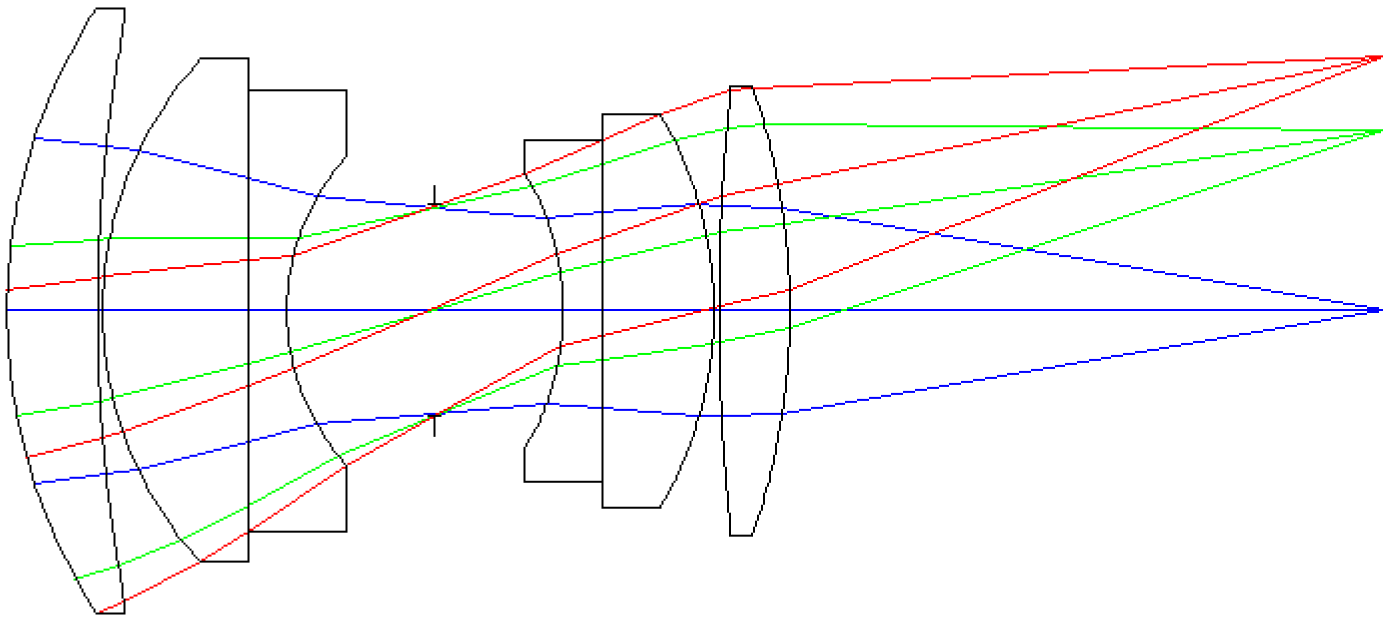
Cooke Triplet lens:



Achromats and Apochromats provide improved performance on-axis only. To achieve good performance both on- and off-axis, more complex lens forms are required. Cooke triplet is a well-know lens form that provides good imaging performance over a field of view of +/- 20-25 degrees. Many consumer grade film cameras use lenses of this type.

Common Camera Design Forms

Double Gaussian lens:



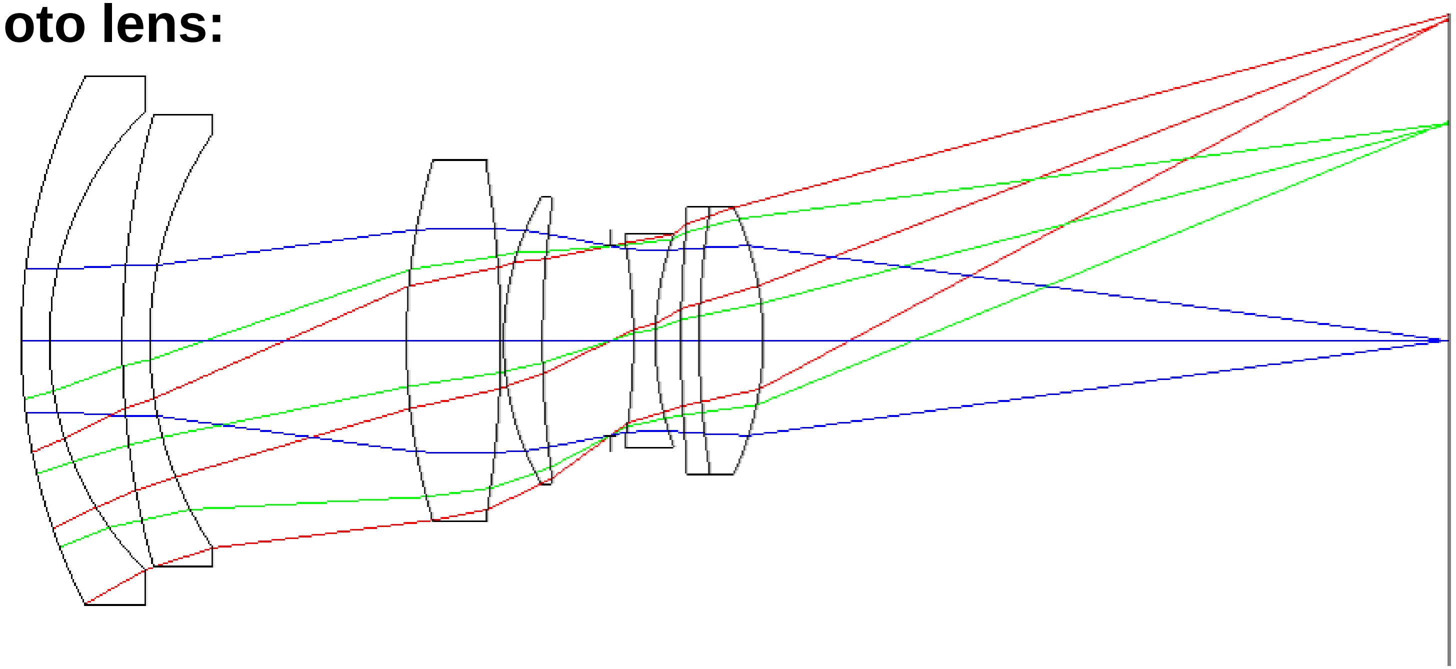
To achieve higher image quality and to increase the relative aperture (i.e, lowering the f/#) over a Cooke triplet, a lens form known as "Double Gaussian" is used. The double Gaussian design uses two cemented doublets and two companion singlets.

This lens form offers excellent performance over a significant field of view, and the relative aperture can be as low as F/1.2.

Double Gaussian lenses are used in many SLR lenses, and C-mount lenses for electronic cameras.

Common Camera Design Forms

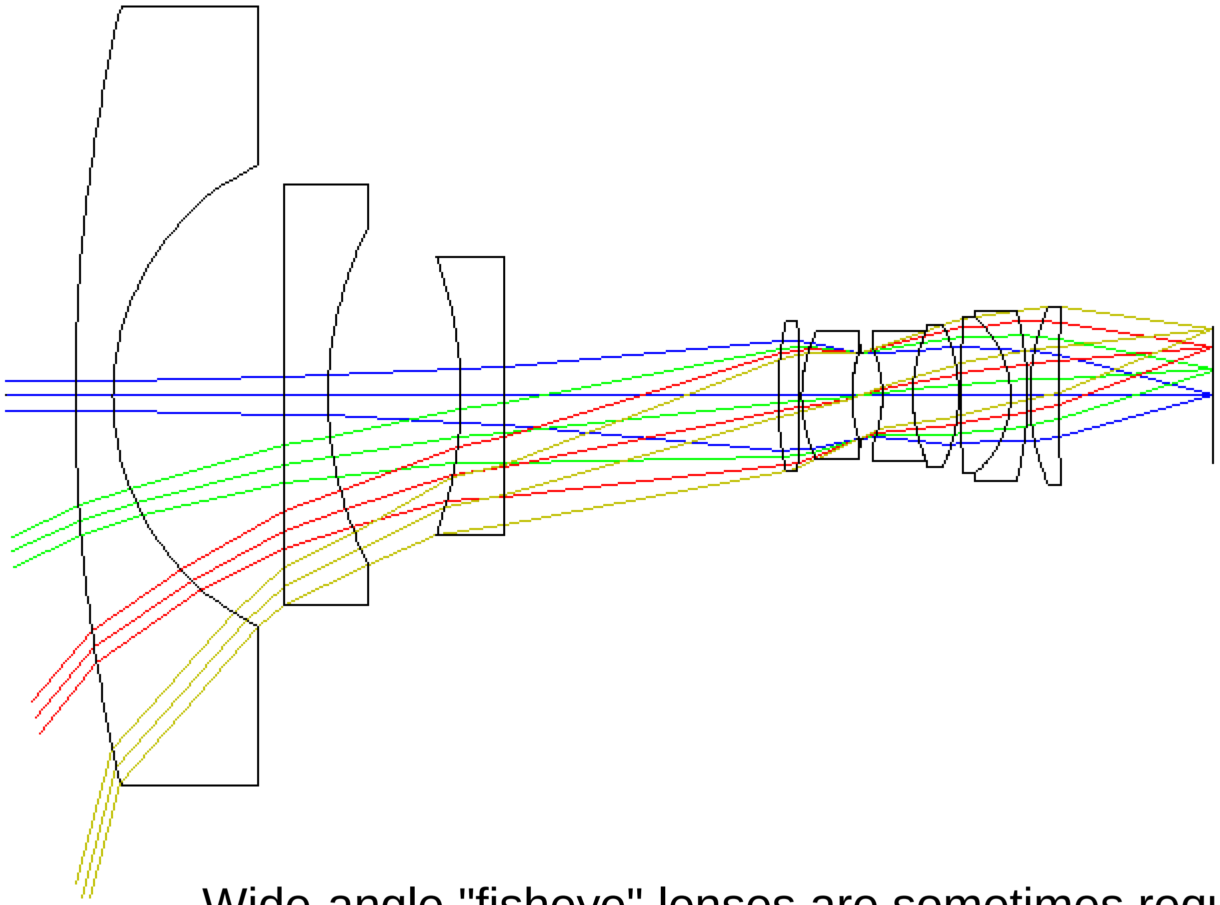
Reverse telephoto lens:



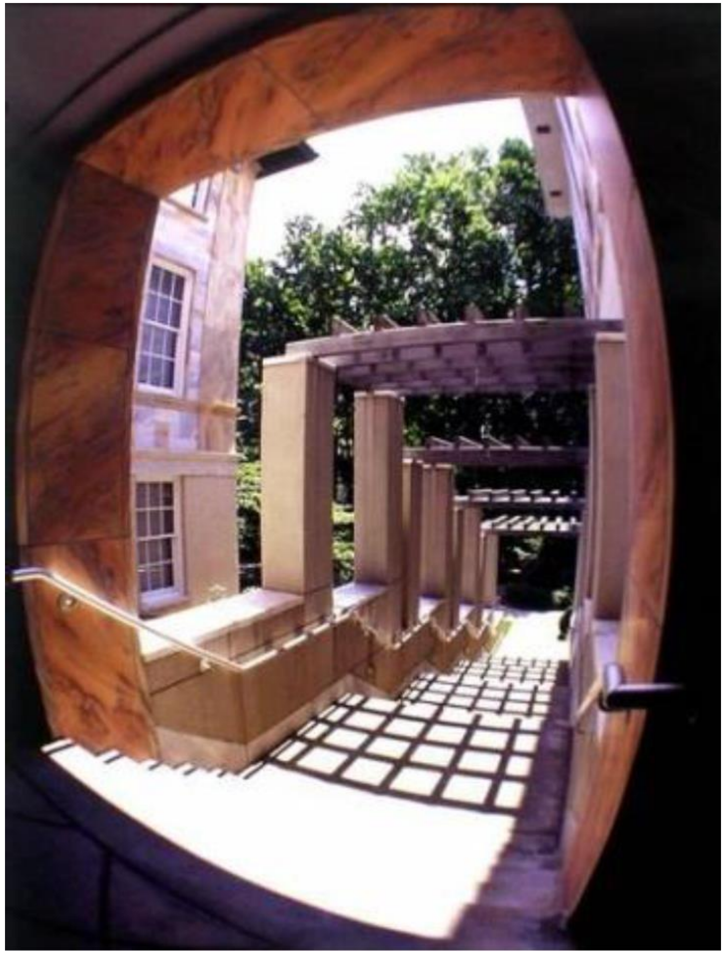
To provide more field of view coverage, a reverse telephoto lens type is often used. The front lens group has negative power which reduces the input field of view. The second group is positive and it does the focus. With this configuration, the field of view can be increased to +/-35 degrees. The other advantage of this configuration is that the system back focal length can be longer than the effective focal length. This property makes this design form very attractive to short focal length lenses commonly seen on digital cameras.

Common Camera Design Forms

Wide-angle “fisheye” lenses:



typical strong barrel distortion



Wide-angle "fisheye" lenses are sometimes required for security and surveillance applications. These lenses require significant number of components. It is also worth noting that the distortion of such lenses can be very significant.