Institute of Applied Physics
Friedrich-Schiller-Universität Jena

Laser Physics Seminar – Problem set 4

Problem 1 (4 points)

Answer the following questions with a brief explanation:

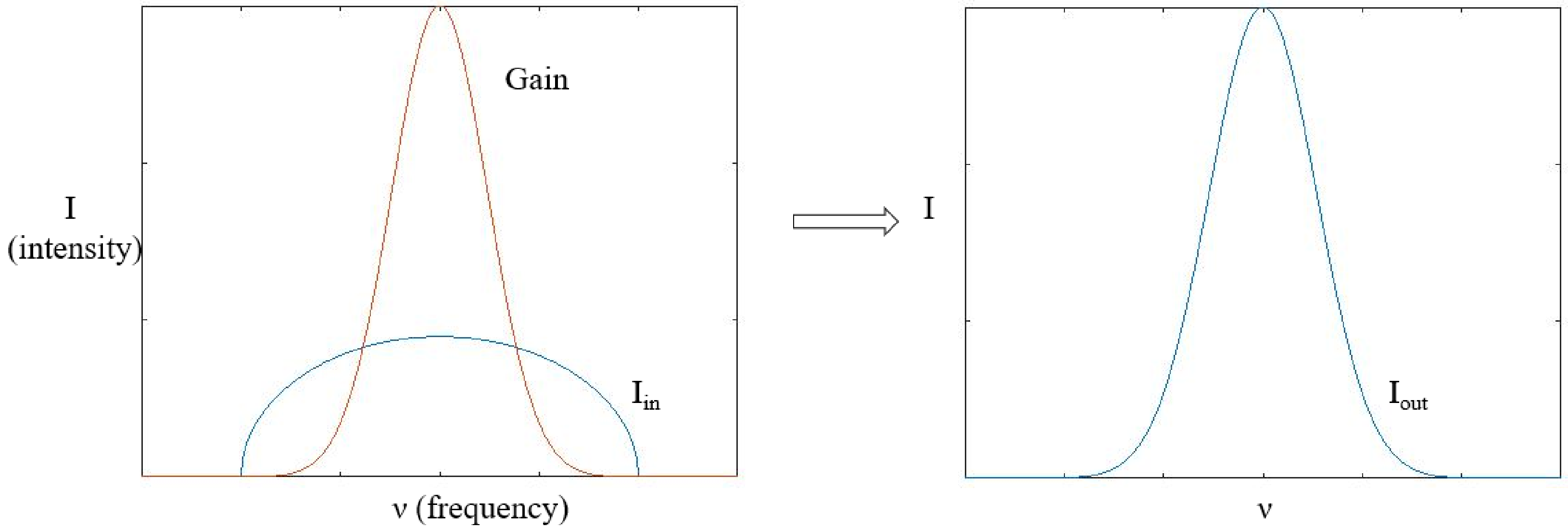
a) Explain the physical origin of gain narrowing. (1 point)

b) Why must the lifetime of the pump band in a three-level system be short? (1 point)

c) Why is it possible to achieve population inversion in a four-level system with low pump powers? (1 point)

d) How can you reduce the ASE content of the laser emission? (1 point)

a) A non-flat spectral characteristic of the cross section. Gain is λ-dependent, has a certain bandwidth



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b) Why must the lifetime of the pump band in a three-level system be short? (1 point)

c) Why is it possible to achieve population inversion in a four-level system with low pump powers? (1 point)

d) How can you reduce the ASE content of the laser emission? (1 point)

b) In order to avoid stimulated emission back to the ground state.

c) A single excited particle in level 2 is enough for population inversion.

d) Spectral filtering (ASE has usually a broad spectrum compared to the signal);
Spatial filtering (ASE has low spatial coherence);
Polarization filtering (ASE is not polarized)
And temporal gating for pulse laser.

Problem 3 (5 points)

The frequency-dependent amplification in a medium of thickness d is given by $G(\nu) = \exp[g(\nu)d]$

a) Considering a Gaussian broadening of the gain coefficient $g(\nu) = \sigma(\nu)n$ show that the FWHM gain bandwidth $\Delta\nu_G$ is reduced to:

$$\Delta\nu_G = \Delta\nu_g \cdot [lnG(\nu_0)]^{-\frac{1}{2}}$$

with n being the inversion density and $\Delta\nu_g$ the FWHM bandwidth of the gain coefficient. (1 point)

Hint: $ln(1-x) \approx -x$

b) Now we consider a parabolic spectral distribution of the cross section $\sigma(\nu)$:

$$\sigma(\nu) = a - \sigma_0 \left[\frac{\nu - \nu_0}{\Delta\nu}\right]^2$$

(valid for $|\nu| < \nu_0 + \Delta\nu \cdot \sqrt{\frac{a}{\sigma_0}}$). Disregarding any saturation effects, calculate the gain factor $G(\nu)$ and its FWHM bandwidth $\Delta\nu_G$ for a single pass through the gain medium. (1 point)

c) We have an input signal with a Gaussian spectrum and a FWHM bandwidth of $\Delta\lambda_{in} = 4.5\,\text{nm}$ at a center wavelength of $\lambda = 1\,\mu\text{m}$. We want to use an active medium with a parabolic spectral distribution of the cross section $\sigma(\nu)$:

$$\sigma(\nu) = \sigma_0 \left[1 - \left(\frac{\nu - \nu_0}{\Delta\nu_g}\right)^2\right]$$

with $\sigma_0 = 1 \cdot 10^{-24}\,\text{m}^2$, $\Delta\nu_g = 1 \cdot 10^{12}\,\text{Hz}$ and inversion $n = 3 \cdot 10^{25}\,\text{ions/m}^3$ to filter it until the output FWHM bandwidth, centered at $\lambda = 1\,\mu\text{m}$, is $\Delta\lambda_{out} = 2\,\text{nm}$. Calculate the length of the active medium. (2 points)

d) What is the maximum spectral gain of the active medium in c)? (1 point)

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Gain is frequency dependent: $G(\nu) = \exp(g(\nu) \cdot d)$

Gain coefficient $g(\nu) = \sigma(\nu) \cdot n$

bandwidth of $G(\nu)$: $\Delta\nu_G$

$$G(\nu_0 \pm \underbrace{\tfrac{1}{2}\Delta\nu_G}_{\text{FWHM}}) \equiv \frac{1}{2}G(\nu_0)$$

mit $G(\nu) = \exp(\sigma(\nu) \cdot nd) = \exp\left\{nd \cdot \underbrace{\sigma_0 \cdot \exp\left[-4 \cdot \ln(2) \cdot \left(\frac{\nu - \nu_0}{\Delta\nu_g}\right)^2\right]}_{\text{Gauß Funktion}}\right\}$

$$G(\nu_0) = \exp(\sigma_0 \cdot nd)$$

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$G(\nu_0) = \exp(\sigma_0\cdot nd)$

$$\exp\left\{nd\cdot\sigma_0\cdot\exp\left[-4\cdot\ln(2)\cdot\left(\frac{\nu_0 \pm \frac{1}{2}\Delta\nu_G - \nu_0}{\Delta\nu_g}\right)^2\right]\right\} = \frac{1}{2}\exp(\sigma_0\cdot nd) \qquad |\ \ln$$

$$nd\cdot\sigma_0\cdot\exp\left[-\ln(2)\cdot\left(\frac{\Delta\nu_G}{\Delta\nu_g}\right)^2\right] = \sigma_0\cdot nd - \ln(2) \qquad |:\sigma_0\cdot nd \equiv ln\big(G(\nu_0)\big)$$

$$\exp\left[-\ln(2)\cdot\left(\frac{\Delta\nu_G}{\Delta\nu_g}\right)^2\right] = 1 - \frac{\ln(2)}{ln\big(G(\nu_0)\big)} \qquad |\ \ln$$

$$-\ln(2)\cdot\left(\frac{\Delta\nu_G}{\Delta\nu_g}\right)^2 = ln\left(1 - \frac{\ln(2)}{ln\big(G(\nu_0)\big)}\right) \approx -\frac{\ln(2)}{\ln\big(G(\nu_0)\big)}$$

$$\left(\frac{\Delta\nu_G}{\Delta\nu_g}\right)^2 = \frac{1}{\ln\big(G(\nu_0)\big)} \quad\Rightarrow\quad \underline{\Delta\nu_G = \Delta\nu_g\sqrt{\frac{1}{\ln\big(G(\nu_0)\big)}} = \Delta\nu_g\sqrt{\frac{1}{\sigma_0\cdot nd}}}$$

b) Now we consider a parabolic spectral distribution of the cross section $\sigma(\nu)$:

$$\sigma(\nu) = a - \sigma_0 \left[\frac{\nu - \nu_0}{\Delta\nu}\right]^2$$

(valid for $|\nu| < \nu_0 + \Delta\nu \cdot \sqrt{\frac{a}{\sigma_0}}$). Disregarding any saturation effects, calculate the gain factor $G(\nu)$ and its FWHM bandwidth $\Delta\nu_G$ for a single pass through the gain medium. (1 point)

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Now: parabolical spectral distribution of cross section $\sigma(\nu) = a - \sigma_0 \left(\frac{\nu-\nu_0}{\Delta\nu}\right)^2$ für $|\nu| < \nu_0 + \Delta\nu\sqrt{\frac{a}{\sigma_0}}$

$$G(\nu) = \exp(g(\nu)\cdot d) = \exp(\sigma(\nu)\cdot nd) = \exp\left(and - \sigma_0 nd\left(\frac{\nu-\nu_0}{\Delta\nu}\right)^2\right)$$

Single pass gain bandwidth $\Delta\nu_G$:

$$G\left(\nu_0 \pm \frac{\Delta\nu_G}{2}\right) \equiv \frac{1}{2}G(\nu_0) = \frac{1}{2}\exp(and) \qquad |\ \ln$$

$$and - \sigma_0 nd\left(\frac{\Delta\nu_G}{2\Delta\nu}\right)^2 = and - \ln(2) \quad \Rightarrow \quad \Delta\nu_G = 2\Delta\nu\sqrt{\frac{\ln(2)}{\sigma_0 nd}}$$

c) We have an input signal with a Gaussian spectrum and a FWHM bandwidth of $\Delta\lambda_{in} = 4.5\,\text{nm}$ at a center wavelength of $\lambda = 1\,\mu\text{m}$. We want to use an active medium with a parabolic spectral distribution of the cross section $\sigma(\nu)$:

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d) What is the maximum spectral gain of the active medium in c)? (1 point)

The gain is: $G = exp(\sigma_0 n L(1 - (\frac{\nu-\nu_0}{\Delta\nu_g}))^2)$

and the input spectrum is $I_{\nu_{in}} = I_0\ exp(-4ln(2)[\frac{\nu-\nu_0}{\Delta\nu_{in}}]^2)$

Thus the output spectrum will be:

$$I_{\nu_{out}} = I_{\nu_{in}} \cdot G = I_0\ exp\left(\sigma_0 n L(1 - \left(\frac{\nu-\nu_0}{\Delta\nu_g}\right)^2) - 4ln(2)\left[\frac{\nu-\nu_0}{\Delta\nu_{in}}\right]^2\right)$$

$$I_{\nu_{out}} = I_0 exp\left(\sigma_0 n L\left(1 - (\nu-\nu_0)^2 \cdot \left[\frac{1}{\Delta\nu_g^2} + \frac{4ln(2)}{\sigma_0 n L \Delta\nu_{in}^2}\right]\right)\right)$$

Thus the FWHM is:

$$I_0 \cdot exp\left(\sigma_0 n L\left(1 - (\nu-\nu_0)^2 \cdot \left(\frac{1}{\Delta\nu_g^2} + \frac{4ln(2)}{\sigma_0 n L \Delta\nu_{in}^2}\right)\right)\right) \stackrel{!}{=} \frac{I_0 e^{\sigma_0 n L}}{2}$$

With some calculation you recieve:

$$L = \frac{\nu_g^2}{\sigma_0 n} \cdot 4ln(2)\left(\frac{1}{\Delta\nu^2} - \frac{1}{\Delta\nu_{in}^2}\right) \approx 0.206m$$

Where was used:
$\Delta\nu_{in} = -\frac{c}{\lambda_{in}^2} \cdot \Delta\lambda_{in,FWHM} = 1.35 \cdot 10^{12} Hz$
$\Delta\nu = 6 \cdot 10^{11} Hz$
$\Delta\nu_g = 1 \cdot 10^{12} Hz$

And the maximuam spectral gain for the active medium is:

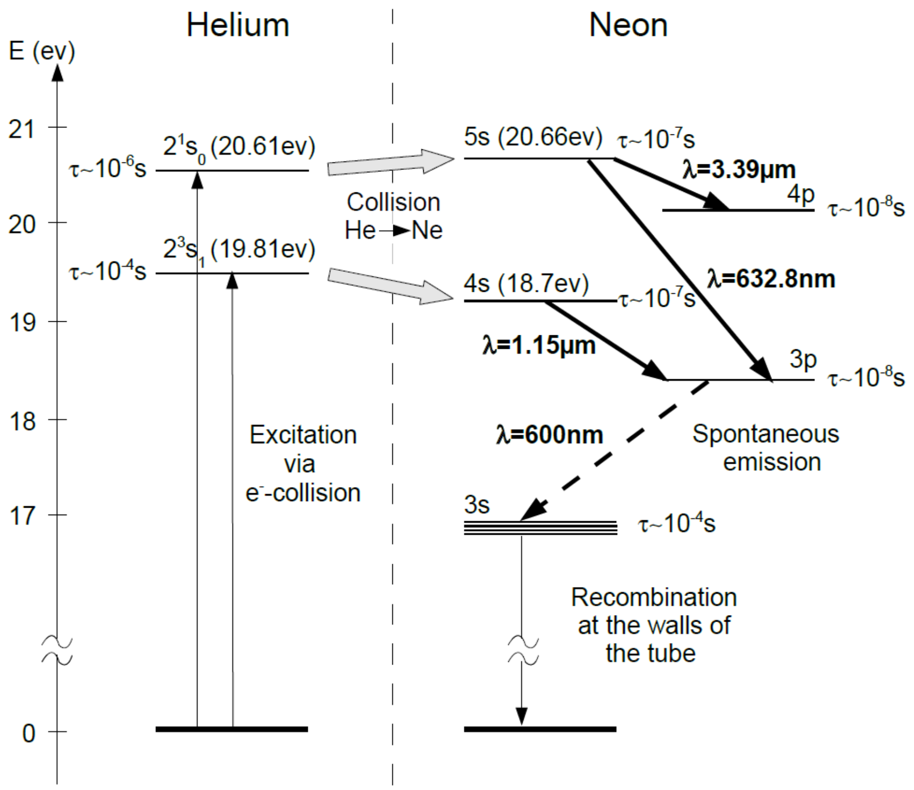
$$G = exp\left(\sigma_0 n L\right) \approx 483$$

Problem 2 (8 points)

The He-Ne laser is one of the most common type of lasers. It consists of a long and narrow discharge tube which is filled with Helium and Neon atoms (usually in a 10:1 ratio) with typical pressures of 1 mbar. The energy level diagram of the He-Ne laser is:

This type of lasers is pumped by applying an electric discharge of 1.5kV. Additionally only the Neon energy levels take part in the laser emission.

a) Based on the energy level diagram above, explain why the direct pumping of the Ne atoms would be inefficient. (2 points)



2 reasons for inefficient pumping:

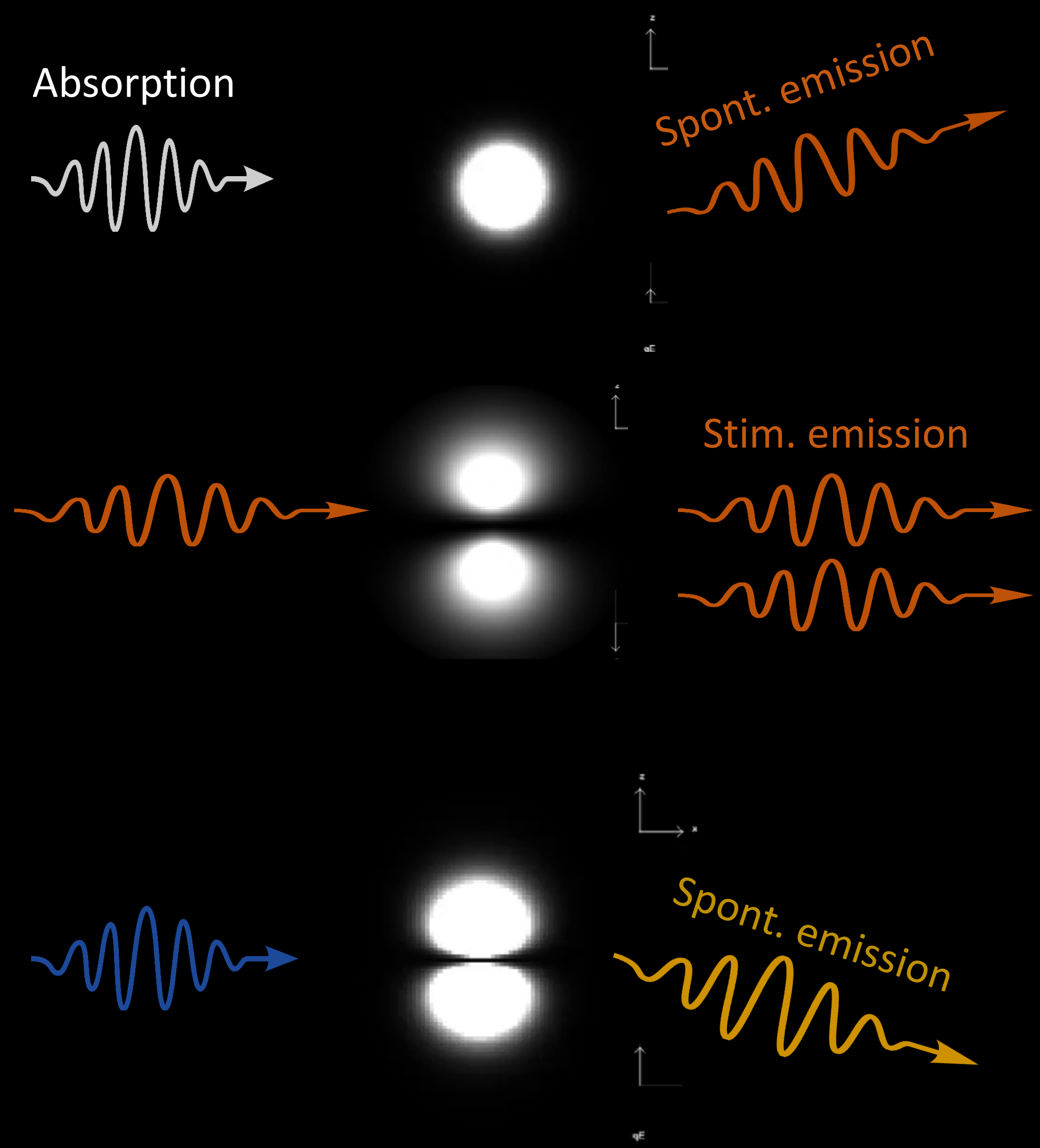
- **Short lifetime of the upper levels 5s, 4p, 4s and 3p in Neon: Hard to achieve inversion**
- **most of the energy levels in Neon will be populated by excitation via e^- collision**

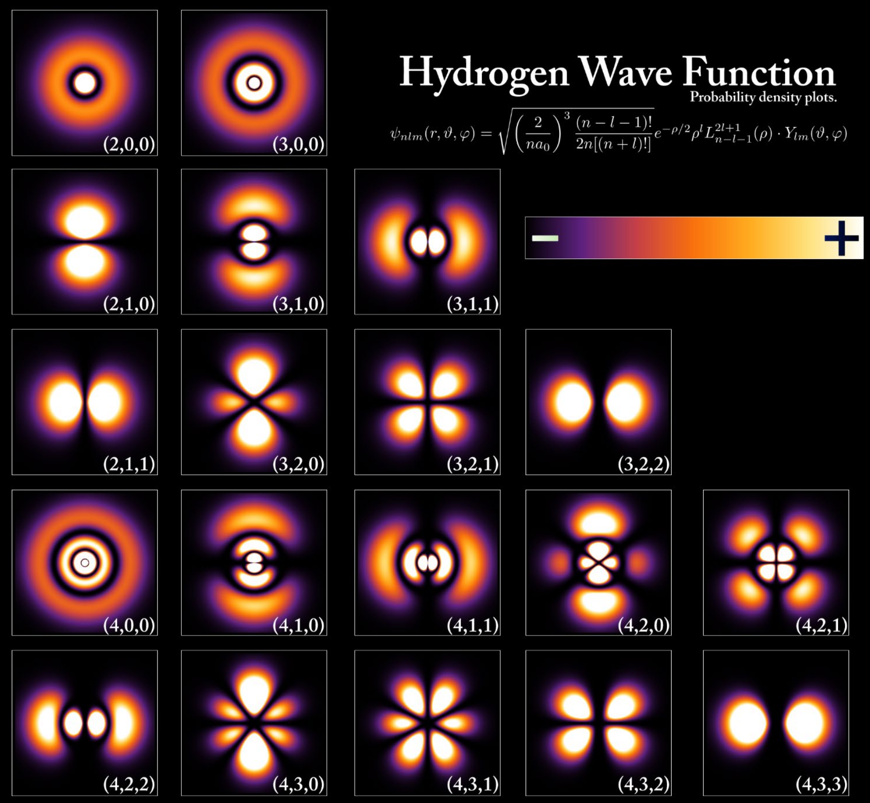
  ⇨ inversion in aiming levels is reduced (between 5s & 3p and 4s & 3p)

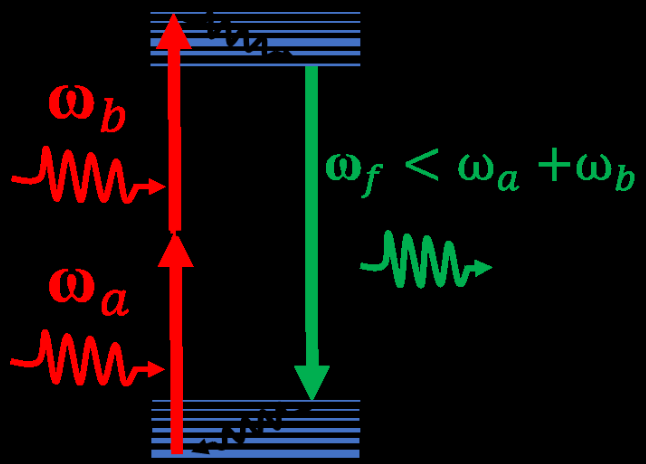
  ⇨ lower gain

  ⇨ lower output power

A picture explanation behind energy level diagrams:

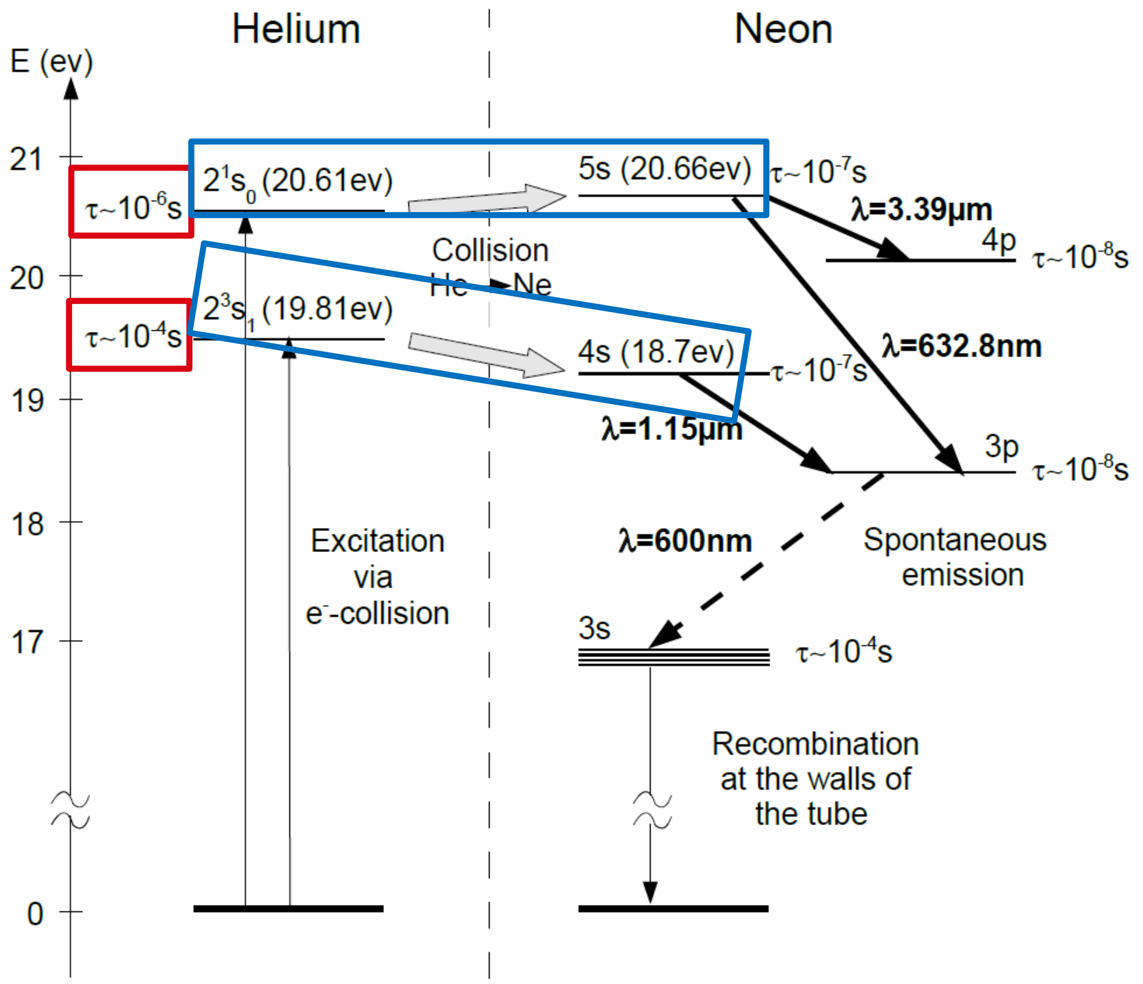






Energy level diagram

b) What is the mission of the He atoms? How do they contribute to solving the problems of pumping the Ne atoms? (2 points)



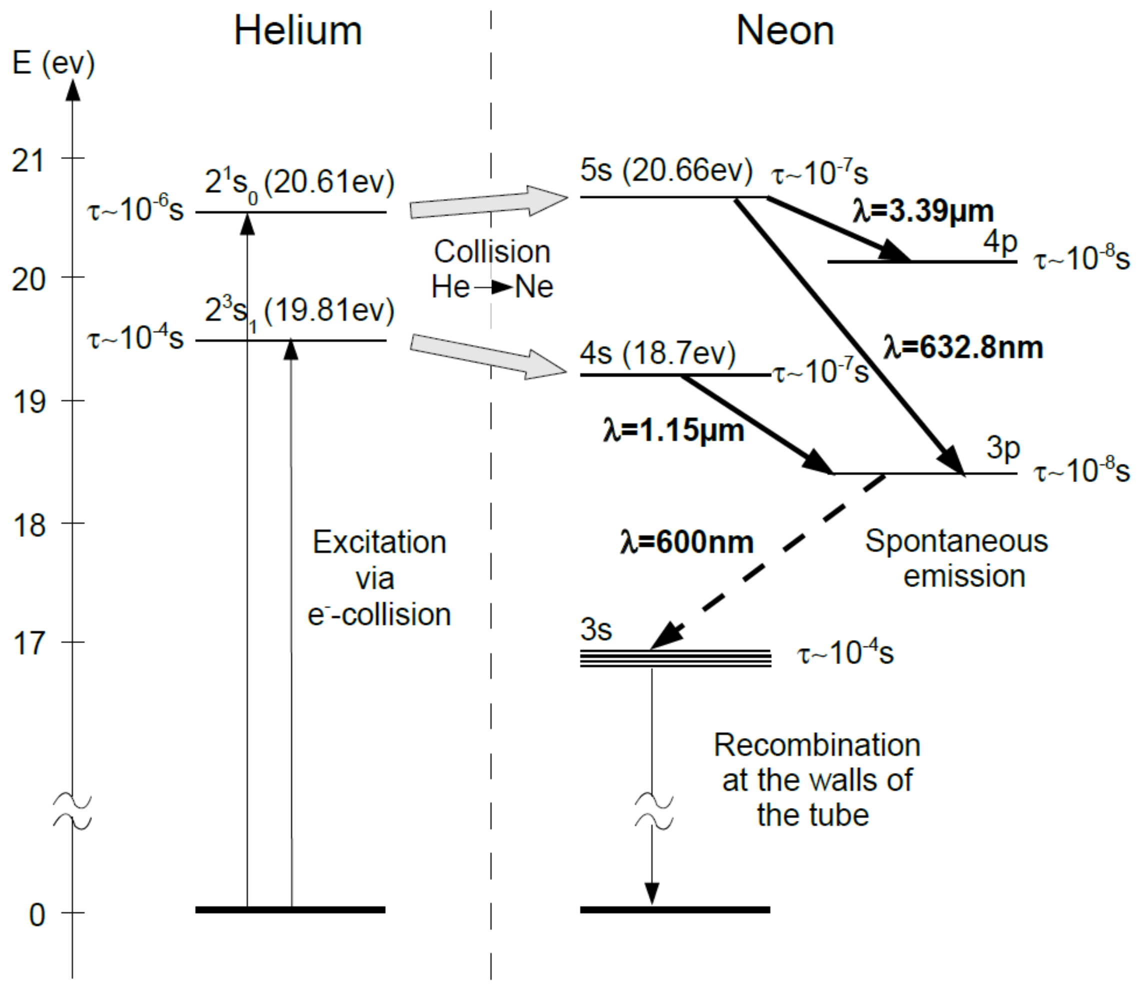
Mission of He atoms is two-fold:

- **Long lifetime of levels 2^1s_0 and 2^3s_1** (due to forbidden optical transitions to ground state and between each other)
  ⇨ accumulating significantly **more energy** than in Ne atoms
  ⇨ provide a continuous source of energy to Ne atoms

- **Levels 2^1s_0 (He) & 5s (Ne) and 2^3s_1 (He) & 4s (Ne) have roughly same energies**
  ⇨ allows for selective excitation of 5s and 4s of Ne
  ⇨ only levels 5s and 4s of Ne are populated

  ⇨ increase of inversion of 4-level system
  ⇨ more gain ⇨ more power

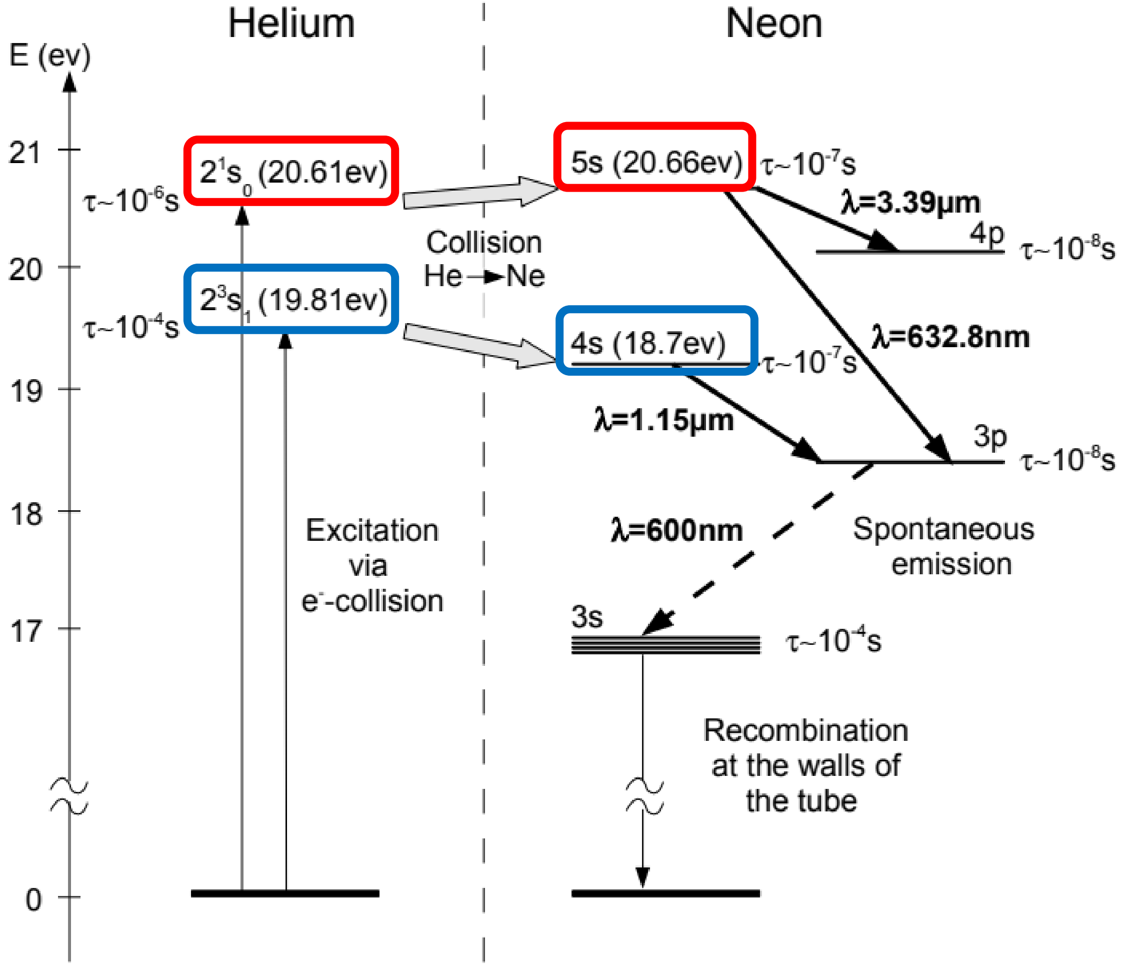
c) Why is the number of He atoms higher than the number of Ne atoms? (1 point)



To increase probability of collisions of e^- with He atoms

⇨ increase pump efficiency

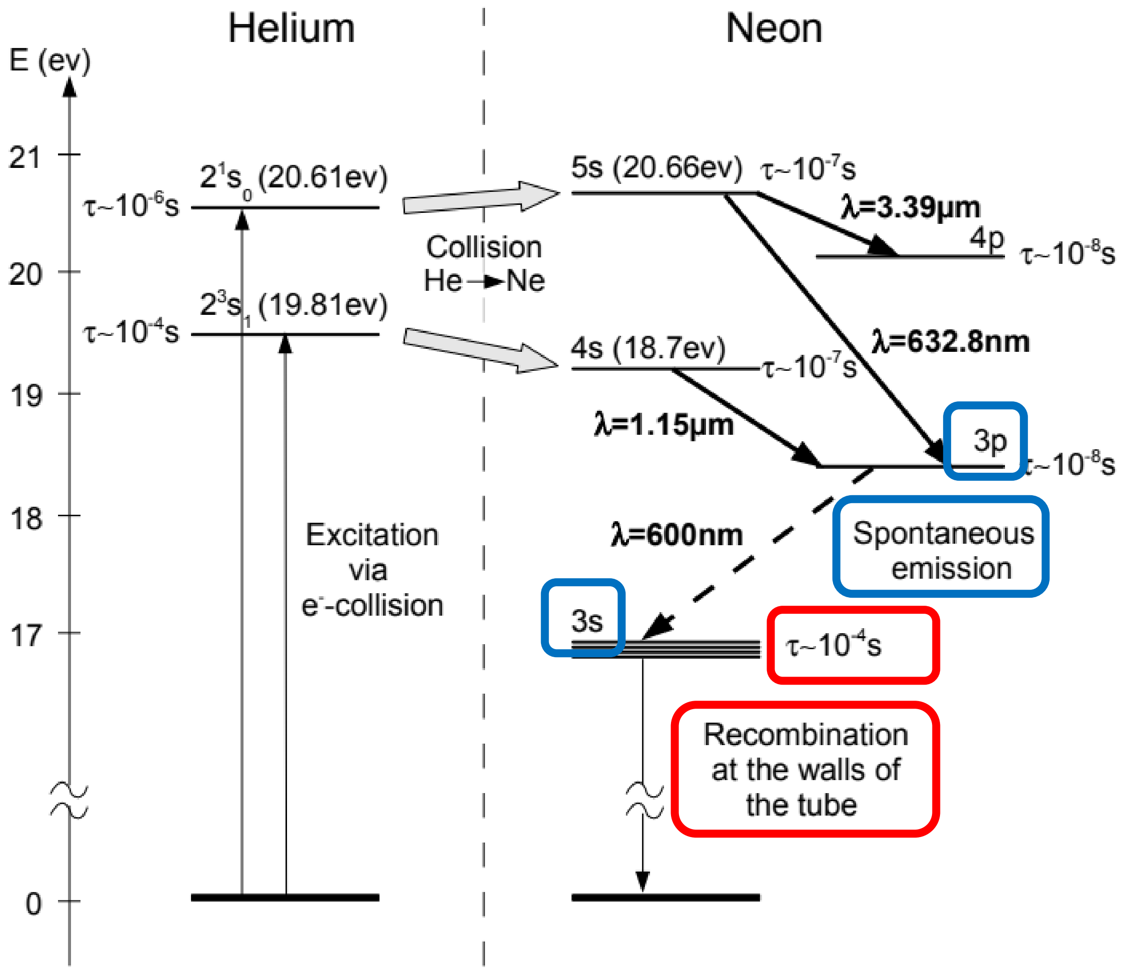
d) As can be seen, the $5s$ level of the Ne atom is higher than the 2^1s_0 level of the He atom. Where does the extra energy come from that allows the energy transfer between these two levels?. In the same way, the 2^3s_1 level of He is higher than the $4s$ level of Ne. Where does the excess energy go in the energy transfer process? (1 point)



Extra energy comes from **kinetic energy** of He atoms

Excess energy is transferred into **kinetic energy** of Ne atoms

e) Why does the gain of this laser medium increase with decreasing tube diameters? (1 point)



3s (long lifetime of 10^{-4} s) decays due collision with tube walls

- Problem sheet 3:

$$\text{colission time} = \frac{\text{mean free path betw. collisions}}{\text{av. speed of atoms}}$$

- decrease tube diameter to decrease time between collision
- fast depopulation of 3s

Low population of 3s increases inversion (gain) by two effects:

- more ground state Ne atoms are available for excitation to the upper laser level again -> high population of upper laser level
- Less re-absorption of spontaneously emitted photons from 3p→3s to re-populate level 3p -> low population of lower laser level

f) What is the dominant broadening mechanism in this type of laser? why? (1 point)

From Problem sheet 3: **Collision broadening**

$$\Delta v_{col} = \sqrt{\frac{3}{4 \cdot m \cdot k_B \cdot T}} \cdot P \cdot d^2$$

$$\Delta v_{col} \approx \text{ MHz}$$

Do some searching to find typical parameters in He-Ne:

- P = 1 mbar = 100 Pa
- T = 300 K
- m_{Ne} = 20 u = 20 x 1.66e-27 kg
- d_{Ne} = ~150 pm
- v_{trans} = c/633 nm

From Problem sheet 3: **Doppler broadening**

$$\Delta v_d = \frac{2 \cdot v_{trans}}{c} \sqrt{\frac{2 \cdot k_B \cdot T \cdot \ln(2)}{m}}$$

$$\Delta v_d \approx \text{ GHz}$$

Due to low pressure within the gas tube Doppler broadening is dominant broadening mechanism