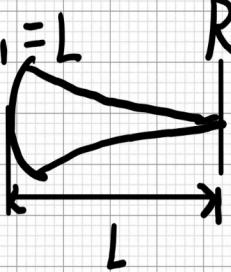


Problem 1

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20bzlb

(a) $R_1 = L$ $R_2 = \infty$



$$g_1 = 1 - \frac{L}{R_1} = 0$$

$$g_1 \cdot g_2 = 0$$

$$g_2 = 1 - \frac{L}{R_2} = 1$$

\therefore the resonator is stable

(b) $g_1 g_2 = 1 - \frac{L}{L + \Delta R_1} = \frac{\Delta R_1}{L + \Delta R_1}$

if $\Delta R_1 > 0 \Rightarrow 0 < \frac{\Delta R_1}{L + \Delta R_1} < 1 \Rightarrow$ the resonator is stable

if $\Delta R_1 < 0 \Rightarrow \frac{\Delta R_1}{L + \Delta R_1} < 0$

\Rightarrow the resonator is unstable.

(c) $R_1 = \frac{L}{2}$ $R_2 = \frac{L}{2}$

$$R_1 = R_2 = \frac{L}{2} = R$$

(d) $g_1 g_2 = \left(1 - \frac{\frac{L}{2}}{\frac{L}{2} + \Delta R_1}\right) \left(1 - \frac{\frac{L}{2}}{\frac{L}{2} + \Delta R_2}\right) = \frac{(\Delta R_1 - R)(\Delta R_2 - R)}{(R + \Delta R_1)(R + \Delta R_2)}$

$\therefore g_1 g_2 > 0$

when $g_1 g_2 \leq 1$

$$(\Delta R_1 - R)(\Delta R_2 - R) \leq (R + \Delta R_1)(R + \Delta R_2)$$

$$2\Delta R_1 R + 2\Delta R_2 R \geq 0$$

$$(\Delta R_1 + \Delta R_2) \cdot R \geq 0$$

\therefore if $\Delta R_1 + \Delta R_2 \geq 0$, the resonator can be stable.

Problem 2

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(a)

$$\left\{ \begin{array}{l} R_1 = R(L_1) = L_1 + \frac{Z_R^2}{L_1} \\ R_2 = R(L_2) = L_2 + \frac{Z_R^2}{L_2} \end{array} \right.$$

$$L = L_1 + L_2$$

$$g_1 = 1 - \frac{L}{R_1}$$

$$g_2 = 1 - \frac{L}{R_2}$$

$$\Rightarrow \left\{ \begin{array}{l} \frac{L}{1-g_1} = L_1 + \frac{Z_R^2}{L_1} \\ \frac{L}{1-g_2} = L_2 + \frac{Z_R^2}{L_2} \end{array} \right.$$

$$\therefore \left(\frac{L}{1-g_1} - L_1 \right) \cdot L_1 = \left(\frac{L}{1-g_2} - L_2 \right) \cdot L_2$$

$$\therefore \left(\frac{L}{1-g_1} + \frac{L}{1-g_2} - 2L \right) \cdot L_1 = \frac{L^2}{1-g_2} \cdot L^2$$

$$\therefore L_1 = g_2 \cdot (1-g_1) \frac{L}{g_1+g_2-2g_1g_2}$$

$$L_2 = g_1 \cdot (1-g_2) \cdot \frac{L}{g_1+g_2-2g_1g_2}$$

$$(b) \frac{L}{1-g_1} = L_1 + \frac{Z_R^2}{L_1}$$

$$\therefore Z_R^2 = \left(\frac{L}{1-g_1} - L_1 \right) \cdot L_1 = g_1 g_2 (1-g_1 g_2) \frac{L^2}{(g_1 + g_2 - 2g_1 g_2)^2}$$

$$W_0 = \sqrt{\frac{\lambda \cdot Z_R}{\pi}} = \sqrt{\frac{\lambda}{\pi}} \cdot \left[g_1 g_2 (1-g_1 g_2) \frac{L^2}{(g_1 + g_2 - 2g_1 g_2)^2} \right]^{\frac{1}{4}}$$

(c) when the resonator is concentric symmetric resonator

$$g_1 = g_2 = 1 - \frac{L}{R}$$

$$\therefore Z_R^2 = g^2 (1-g^2) \cdot \frac{L^2}{(2g-2g^2)^2} = \frac{1+g}{1-g} \cdot \frac{L^2}{4} = \frac{1}{4} [-(L-R)^2 + R^2]$$

$$\text{when } L=2R, Z_R^2=0$$

$$\Rightarrow W_0 = \sqrt{\frac{\lambda Z_R}{\pi}} = 0$$

\therefore the waist is the smallest

when $L=R$, Z_R^2 gets the maximum value.

\Rightarrow the waist is the biggest.

$$(d) W(z) = W_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2} = \sqrt{\frac{\lambda}{\pi}} \cdot \sqrt{z_R + \frac{z^2}{z_R}}$$

$$\therefore R(z) = z + \frac{z_R^2}{z}$$

$$\therefore W(z) = \sqrt{\frac{\lambda}{\pi}} \cdot \sqrt{\frac{z \cdot R(z)}{z_R}}$$

$$\therefore W(L_1) = \sqrt{\frac{\lambda \cdot L}{\pi}} \cdot 4 \sqrt{\frac{g_2}{g_1(1 - g_1 g_2)}}$$

$$W(L_2) = \sqrt{\frac{\lambda \cdot L}{\pi}} \cdot 4 \sqrt{\frac{g_1}{g_2(1 - g_1 g_2)}}$$

(e) The transversal length of the mirrors, because different wavelengths have different diffraction losses.

Problem 3

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(a)

the ABCD matrix for one round trip:

$$\begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{2}{R} & 1 \end{bmatrix} \begin{bmatrix} 1 & L-L_0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & n_0 \end{bmatrix} \begin{bmatrix} 1 & L_0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & L_0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{n_0} \end{bmatrix} \begin{bmatrix} 1 & L_0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2(L-L_0) + \frac{2}{n_0} L_0 \\ -\frac{2}{R} & 1 - \frac{4 L_0}{R n_0} \end{bmatrix}$$

(b)

$$g_1 = 1 - \frac{L}{R} = 1 - \frac{109}{200} = 0.455$$

$$g_2 = 1$$

$$0 < g_1 g_2 < 1$$

\therefore The resonator is stable

(c)

$$\begin{bmatrix} r' \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & l \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ r'_0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & l \\ -\frac{1}{f} & 1 - \frac{l}{f} \end{bmatrix} \begin{bmatrix} 0 \\ r'_0 \end{bmatrix} = \begin{bmatrix} r'_0 \cdot l \\ r'_0 (1 - \frac{l}{f}) \end{bmatrix}$$

$$\therefore r'_0 \cdot (1 - \frac{l}{f}) = 0 \quad \therefore l = f$$

(c)

- ∴ the optical system is a telescope system
∴ the incident beam and output beam are parallel to the optical axis.

$$\begin{bmatrix} r_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & f_1 + f_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix} \begin{bmatrix} r_0 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} -\frac{f_2}{f_1} & f_1 + f_2 \\ 0 & -\frac{f_1}{f_2} \end{bmatrix} \begin{bmatrix} r_0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{f_2}{f_1} r_0 \\ 0 \end{bmatrix}$$

$$\therefore r_1 = -\frac{f_2}{f_1} r_0$$

$$\text{magnification: } -\frac{f_2}{f_1}$$

(e)

$$1 \text{ inch} = 25.4 \text{ mm}$$

$$2 \text{ inch} = 50.8 \text{ mm}$$

$$\because \sin \theta_m \approx \tan \theta_m$$

∴ when aperture diameter is 1 inch, the beam diameter is $30 \times 0.22 \times 2 = 13.2 \text{ mm}$. When aperture diameter is 2 inch,

$$\text{the beam diameter is } 100 \times 0.22 \times 2 = 44 \text{ mm}$$