Examination

Introduction to Optical Modeling and Design

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Place and Date: Jena, 07.02.2018, 10 am

Answer all questions in your own words and with mathematics where needed for your argumentation.

1. Assume an optical set-up composed of a single lens (focal length $f_1' > 0$) placed at a distance d in front of a plane mirror. Calculate the ABCD-matrix for the full light path (lens – mirror – lens). (4P)
   What is the effective focal length of this system (hint: compare the matrix component C with the one of an ideal lens)? (1P)
   Make a sketch of the image formation for the case that $d = 0.5f_1'$ and an object distance $s = -2f_1'$ in front of the lens and correctly mark the relevant quantities (focal lengths, object/image distances from the lens). (4P)
   Revisar
   What is the exact image distance from the lens (in units of f_1')? (1P)
   Assume that the mirror act as the stop (limiting aperture) of the system. Construct the location of the exit pupil and give its position from the lens (in units of f_1'). (2P)
2. What is described by the Abbe-Number (which physical effect) and how is this number mathematically defined? (2P)
   What are the two main categories of glass materials distinguished by their Abbe-Numbers? (2P + 1P if the range is correctly given.) $\nu > 55$
   Make a sketch of $n(\lambda)$ in a λ-n-diagram for both categories. (2P)
3. What is the origin of the spherical aberration? (1P)
   Make a sketch to illustrate the effect. (1P)
   Sketch the OPD-diagram (optical-path-difference diagram). (1P)
   Give two methods suitable to minimize spherical aberration? (2P)?
4. What limits the spot size of a beam focused by a lens in case the lens has no aberrations? How can this spot size be calculated? (2P) $\frac{\lambda}{0.4\,NA}$
5. Assume $g(\omega) = \mathcal{F}_\omega f(t)$. Complete the equation

$$\mathcal{F}_\omega f(t - t_0) = ?,$$

with $t_0 \in \mathbb{R}$.(2P) Derive the equation starting from the definition of the Fourier transform $\mathcal{F}_\omega$. (3P)

6. The triangular function is defined by

$$\operatorname{tri}(x) \stackrel{\text{def}}{=} \operatorname{rect}(x) * \operatorname{rect}(x),$$

with the rectangular function $\operatorname{rect}(x)$ and the convolution $*$. Derive the Fourier transform of the triangular function, that is $g(k) = \mathcal{F}_k(\operatorname{tri}(x))$. (3P)

$+ k_0^2 \, \varepsilon_r(\omega)$

7. Express the Helmholtz' equation for an arbitrary field component $V(x, y, z, \omega)$. (2P) Its transformation into the k-domain reveals

$$\left(-k_x^2 - k_y^2 + \frac{\mathrm{d}^2}{\mathrm{d}^2 z}\right) \tilde{V}(\boldsymbol{\kappa}, z, \omega) + k_0^2 \tilde{n}^2(\omega) \tilde{V}(\boldsymbol{\kappa}, z, \omega) = 0.$$

What can be concluded for the z-dependency of $\tilde{V}(\boldsymbol{\kappa}, z, \omega)$? (4P) $E(\vec{r}, \omega) =$ $\delta(\omega - \omega_0)$

8. What is the form of the electric field vector $\boldsymbol{E}(\boldsymbol{r}, \omega)$ of a plane wave? $\vec{E} \exp\left(i\vec{k}\vec{r} - i\omega t\right)$ (2P) Discuss the difference between homogeneous and inhomogeneous plane waves? (2P) How depends the complex vector $\bar{\boldsymbol{k}} = \boldsymbol{k} + \mathrm{i}\boldsymbol{k}'$ for homogeneous plane waves from the complex refractive index $\tilde{n} = n + \mathrm{i}n'$? (2P)

9. Assume the field component $\tilde{H}_z(\boldsymbol{\kappa}, z_0, \omega)$ is known in the k domain. With which operation is it possible to obtain the field component in $z = z_0 + \Delta z$, that is $\tilde{H}_z(\boldsymbol{\kappa}, z_0 + \Delta z, \omega)$? (3P) $\exp\left(i k_z \Delta z\right)$

10. What is the definition of a paraxial field? (1P) Why it is allowed to assume, that a paraxial field is approximately transversal (3P)?

11. What is the definition of harmonic (also called monochromatic) fields? (2P) Can harmonic fields be unpolarized? (1P) What is the difference between globally (also uniformly) and locally polarized harmonic fields? Discuss for paraxial fields only. (3P)

$K_z \cdot z$