Structure of Matter: Written Test 2022

1. Multiple-choice test: Please tick all box(es) with correct answer(s)!
   Correctly ticked box = 1/2point; wrongly ticked box = -1/2 point; empty box = zero points

1.1 A photon with an energy of 2eV belongs to the	a) visible (VIS) spectral range	
	b) ultraviolet (UV) spectral range	
	c) infrared (IR) spectral range	
1.2 UV photons have photon energies	a) larger than VIS photons	
	b) larger than IR photons	
	c) larger than 3eV	
1.3 An electron confined in a box potential	a) has discrete stationary energy levels	
	b) obeys the Bloch theorem	
	c) is a fermion	
1.4 Imagine a single-particle wavefunction Ψ normalized according to: $\int_V \lvert\Psi\rvert^2 dV = 1$ with V – volume. This wavefunction	a) has no measurement unit	
	b) is given in $m^{-3/2}$	
	c) is given in m^{-3}	
1.5 Non-linear optical phenomena	a) include frequency-conversion processes	
	b) are observed at large light intensities	
	c) are of no practical relevance in laser optics	
1.6 The selection rules for Raman scattering	a) are identical to those valid for IR absorption	
	b) exclude Raman activity in any crystals	
	c) exclude Raman activity in any quantum system with inversion symmetry	
1.7 The atomic state ${}^1D_9$	a) is possible	
	b) is impossible	
1.8 Solids may be	a) crystalline	
	b) amorphous	
	c) electrically conducting	

2. True or wrong? – make your decision!
   Correct choice = 1 point; wrong or no choice = zero points

Assertion:	true	wrong
a) $\left[\hat{x}, \hat{p}_y\right] = i\hbar$		
b) $\left[U(r), \hat{L}_z\right] = 0$		
c) $\left[\hat{x}^2, \hat{p}_y^2\right] = 0$		
d) $\left[\hat{x}^2, \hat{p}_x^2\right] = \hbar$		
e) $\left[\hat{L}_x, \hat{L}_z\right] = i\hbar y$		

3. Imagine a one-dimensional harmonic oscillator in the eigenstate with the quantum number n=50. As a result of an electric dipole-allowed quantum transition, the oscillator is transferred to another eigenstate with the quantum number m. Indicate the values of all oscillator strength $f_{m,n=50}$ that are different from zero! (3 points)

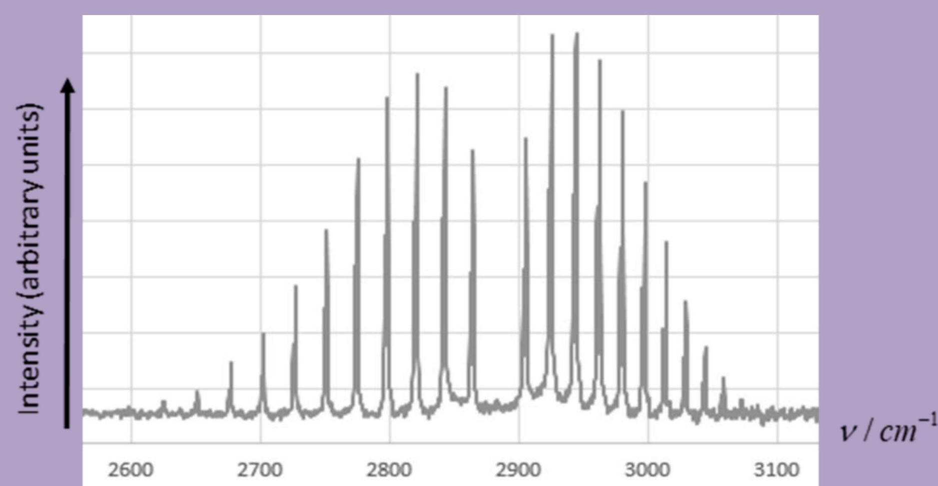
4. Assume a resting hydrogen atom in an excited state with the principal quantum number n=2. Let the hydrogen atom return to its ground state by emitting a single photon. Because of the recoil effect, after the emission of the photon, the atom is no more at rest. Making use of a <u>non-relativistic approach</u>, please:
    - Write down the equation for energy conservation in this process! (1 point)
    - Write down the equation for momentum conservation in this process! (1 point)
    - Estimate the velocity of the hydrogen atom after emission of the photon! (4 points)
    - Indicate the wavelength of the photon and the corresponding spectral series! (0.5 points)

5. Imagine a hydrogen atom in the excited quantum state $|n,l,m\rangle = |2,1,1\rangle$. In spherical coordinates, the wavefunction of the electron in that state may be written as:

$$\psi(r,\varphi,\theta) = \frac{1}{8\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{\frac{3}{2}} \frac{r}{a_0} e^{-\frac{r}{2a_0}} \sin\theta e^{i\varphi}$$

. a_0 is the Bohr's radius. Consider the z-coordinate of this electron and calculate its variance $\operatorname{var}(z) = \langle z^2\rangle - \langle z\rangle^2$ in this quantum state! (10 points)

6. The figure below sketches the middle infrared spectrum of a gas of diatomic molecules held at a certain temperature.



a) Indicate the resonance wavenumber ν_0 corresponding to pure vibration of the molecule. Indicate the wavenumber regions where you observe the P- and R-branches of the spectrum (1.5 point)

b) What is the rotational constant B, provided that the spacing between adjacent rotation lines is approximately 20cm^{-1}? (0.5 point)

c) from these data, estimate the mass moment of inertia of a single molecule (2 points)

d) estimate the temperature of the gas (2 points)

e) guess the molecule (1 point)