Task 1 Jinsong Liu

a)

Solution:

From the 2f-setup equation: 2+3

$$U(x,y,2f) = -i\frac{(2\pi)^2}{\lambda f}\exp(2ikf)\,U_0\left(\frac{k}{f}x, \frac{k}{f}y\right)$$

$$\therefore\ U^-(x) = -i\frac{2\pi}{\lambda f_1}\exp(2ikf_1)\,U_0\left(\frac{k}{f_1}x\right)$$

$$U^+(x) = U^-(x)\cdot H(x) = -i\frac{2\pi}{\lambda f_1}\exp(2ikf_1)\,U_0\left(\frac{k}{f_1}x\right)\cdot H(x)$$

$$U_1(x) = -i\frac{2\pi}{\lambda f_2}\exp(2ikf_2)\,U^+\left(\frac{k}{f_2}x\right)$$

$$= -i\frac{2\pi}{\lambda f_2}\exp(2ikf_2)\frac{1}{2\pi}\int_{-\infty}^{\infty} -i\frac{2\pi}{\lambda f_1}\exp(2ikf_1)\,U_0\left(\frac{k}{f_1}x'\right)H(x')\exp\left(-i\frac{k}{f_2}xx'\right)dx'$$

$$= -i\frac{1}{\lambda f_2}\exp(2ikf_2)\left[-i\frac{2\pi}{\lambda f_1}\exp(2ikf_1)\right]\int_{-\infty}^{\infty} U_0\left(\frac{k}{f_1}x'\right)H(x')\exp\left(-i\frac{k}{f_2}xx'\right)dx'$$

$$= -\frac{2\pi}{\lambda^2 f_1 f_2}\exp[2ik(f_1+f_2)]\int_{-\infty}^{\infty} U_0\left(\frac{k}{f_1}x'\right)H(x')\exp\left(-i\frac{k}{f_2}xx'\right)dx'$$

b)

Solution:

$$U_0(x) = e^{iA\cos(\alpha_0 x)} = 1 + iA\cos(\alpha_0 x)$$

$$U_0\left(\frac{k}{f_1}x\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty}[1+iA\cos(\alpha_0 x')]\exp\left(-i\frac{k}{f_1}xx'\right)dx'$$

$$= \delta(\alpha) + i\frac{A}{2}[\delta(\alpha+\alpha_0)+\delta(\alpha-\alpha_0)]$$

Let $C = -\frac{2\pi}{\lambda^2 f_1 f_2}\exp[2ik(f_1+f_2)]$ $\quad \varphi(\alpha) = \frac{A}{2}[\delta(\alpha+\alpha_0)+\delta(\alpha-\alpha_0)]$

$$U_1(x) = C\int_{-\infty}^{\infty} U_0(\alpha)\,H\left(\frac{f_1}{k}\alpha\right)\exp\left[i\frac{k}{f_2}xx'\right]dx'$$

$$= C\int_{-\infty}^{\infty} U_0(\alpha)\,H\left(\frac{f_1}{k}\alpha\right)\exp\left(i\frac{f_1}{f_2}x\alpha\right)\frac{f_1}{k}\,d\alpha$$

$$= \frac{Cf_1}{k}\int_{-\infty}^{\infty}[\delta(\alpha)+i\varphi(\alpha)]\,H\left(\frac{f_1}{k}\alpha\right)\exp\left(i\frac{f_1}{f_2}x\alpha\right)d\alpha$$

$$\because\ H(x) = \begin{cases}\exp(i\varphi_0) & -a\le x\le a\\ 1 & \text{else}\end{cases}$$

$$\therefore\ H\left(\frac{f_1}{k}\alpha\right) = \begin{cases}\exp(i\varphi_0) & -\frac{k}{f_1}a\le\alpha\le\frac{k}{f_1}a\\ 1 & \text{else}\end{cases}$$

In order to convert the phase modulation into an amplitude modulation, we need to multiply $\exp(i\varphi_0)$ with $\delta(\alpha)$ which is zero-frequency component. $\therefore \frac{k}{f_1}a < \alpha_0 = 2\pi/\Lambda$, $a < \frac{2\pi f_1}{k\Lambda}$. And $\varphi_0 = \pm\frac{\pi}{2}$, $\exp(i\varphi_0) = \pm i$, when we calculate the $|U_1(-x)|^2$, the phase modulation can convert into an amplitude modulation.

Task 2 Jinsong Liu

a)

Solution:

$$FT[g(x')] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{2}[1+\cos(\frac{2\pi}{d}x')]\exp(-i\frac{k}{f}xx')\,dx'$$

$$= \frac{1}{2}\delta(\alpha) + \frac{1}{4}[\delta(\alpha+\frac{2\pi}{d}) + \delta(\alpha-\frac{2\pi}{d})]$$

$$FT[n(x',y')] = \frac{1}{(2\pi)^2}\iint_{-\infty}^{\infty}\exp[-\frac{x'^2}{W^2} - \frac{(y'-\frac{R}{5})^2}{l^2}]\exp[-i(\alpha x'+\beta y')]\,dx'dy'$$

$$= \frac{W\cdot l}{4\pi}\exp[-\frac{W^2\alpha^2+l^2\beta^2}{4}]\cdot\exp(-i\beta\frac{R}{5})$$

$$FT[m(x',y')] = \frac{lW}{4\pi}\exp[-\frac{l^2\alpha^2+W^2\beta^2}{4}]\cdot\exp(i\beta\frac{R}{2})$$

$$FT[e_l(x',y')] = \frac{W^2}{4\pi}\exp[-\frac{W^2\alpha^2+W^2\beta^2}{4}]\cdot\exp(-i\alpha\frac{R}{2})\cdot\exp(-i\beta\frac{R}{2})$$

$$FT[e_r(x',y')] = \frac{W^2}{4\pi}\exp[-\frac{W^2\alpha^2+W^2\beta^2}{4}]\exp(i\alpha\frac{R}{2})\exp(-i\beta\frac{R}{2})$$

2

$\because$ $R > l > w \gg d$

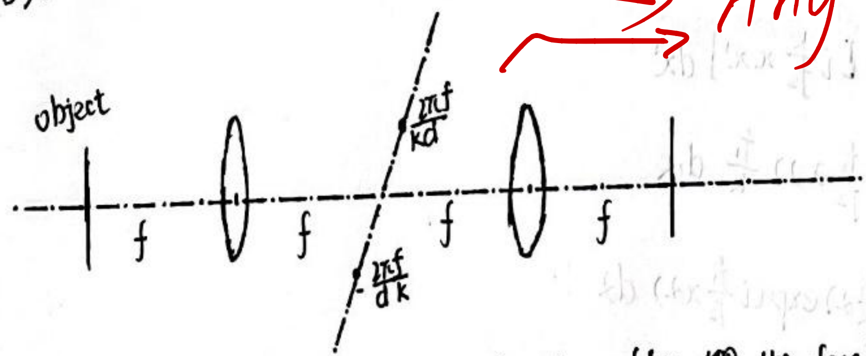
$\therefore$ $\frac{2\pi}{d} \gg \frac{1}{W} > \frac{1}{l}$

So the $\delta(\alpha+\frac{2\pi}{d})$ and $\delta(\alpha-\frac{2\pi}{d})$ will not make too much influence over the Gausian spectrum.

To remove the prison. we can add filter at $\alpha = 0, \pm\frac{2\pi}{d}$.
$(x=0, \pm\frac{2\pi f}{kd})$

Any real filter should have a width

b):



The initial optical field is the object, after the focal length of the first lens, it does a Fourier transform and generates the field in between. The final optical field is inverse Fourier transformation of the field between 2 lenses.

2

C)

Solution:

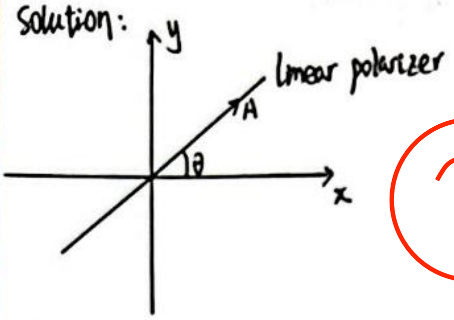
∵ The prison has zero-frequency component which is involved in the Gaussian spectrum.

To minimize the effect, we can multiply a very small number with $\delta(d)$.

Task 3

a)

Solution:



2

incident: $J_{in} = \begin{bmatrix} A_1 \\ B_1 \end{bmatrix}$

$A = A_1 \cos\theta + B_1 \sin\theta$

After the polarizer:

$J_{out} = \begin{bmatrix} A_2 \\ B_2 \end{bmatrix}$

$A_2 = A\cos\theta = A_1 \cos^2\theta + B_1 \cos\theta \sin\theta$

$B_2 = A\sin\theta = A_1 \cos\theta \sin\theta + B_1 \sin^2\theta$

$\therefore T_\theta = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$

b)

Solution:

incident: $J_{in} = \begin{bmatrix} A_1 \\ B_1 \end{bmatrix}$

2

From the problem, we can get

$T J_{in} = \lambda J_{in}$

$\therefore (T-\lambda) J_{in} = 0$

$\det(T-\lambda) = \begin{vmatrix} \frac{1-2\lambda}{2} & \frac{i}{2} \\ \frac{-i}{2} & \frac{1-2\lambda}{2} \end{vmatrix} = 0$

$\therefore (1-2\lambda)^2 - 1 = 0$

$\lambda = 0, 1$ ($\lambda = 0$ is wrong)

when $\lambda = 1$

$\begin{cases} A_1 + iB_1 = 2A_1 \\ -iA_1 + B_1 = 2B_1 \end{cases}$

$\therefore A_1 = 1 \quad B_1 = -i$

The normalized Jones vector is $J = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$

This state is right circular polaried light.