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Structure of matter: Homework to exercise3

ORDERS OF MAGNITUDE AND MEASUREMENT UNITS

Due on October 24th 2023 at noon

Please indicate your name on the solution sheets and send it to your seminar leader!

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1. Multiple-choice test: Please tick all box(es) with correct answer(s)!
   (correctly ticked box: +1/2 point; wrongly ticked box: -1/2 point)

The light wavelength 1064nm belongs to the	Ultraviolet spectral range	
	Visible spectral range	
	Infrared spectral range	✓
The mass moment of inertia of a diatomic molecule is typically of the order	10^{-27}kgm^2	
	10^{+47}kgm^2	
	10^{-47}kgm^2	✓

2. True or wrong? Make your decision!

$\mu = \frac{m_1 m_2}{m_1+m_2}$

(2 points): 1 point per correct decision, 0 points per wrong or no decision

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Assertion	true	wrong
The reduced mass of a system of two bodies is always smaller than each of the individual masses.	✓	
A naturally broadened emission line has a Lorentzian lineshape.	✓	

3. Calculate the following expressions (4 Points)

- $\sqrt{\varepsilon_0 \mu_0} = ?$ (ε_0 is the permittivity of free space, μ_0 is the permeability of free space)
- $\frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} = ?$ ($\hbar = \frac{h}{2\pi}$, e is the electron's charge; h is Planck's constant, and m_e the rest mass of an electron)
- $\frac{e^2}{4\pi\varepsilon_0 \hbar c} = ?$ (c is the velocity of light in vacuum)
- $\frac{h}{m_e c} = ?$

4. The interatomic potential between two neutral atoms may be approximated by:

$U(r)=4D\left[\left(\frac{\sigma}{r}\right)^{12}-\left(\frac{\sigma}{r}\right)^{6}\right]$ (D, σ are constants). Sketch that potential as a function of r in a graph. Find an expression for the distance r_0 where $U(r)$ has a local minimum. What are the measurement units of D and σ? (6 Points)

5. Imagine one liter of water uniformly spread over the surface of a sphere with the radius of the earth. Estimate the number of water molecules observed in 1 cm^2 of the surface of the sphere! (4 points)

6. A naturally broadened emission line has a central angular frequency of $10^{16}s^{-1}$ and a linewidth (FWHM) of $5*10^8s^{-1}$. Calculate the corresponding central wavelength λ as well as the corresponding linewidth $\Delta\lambda$ in wavelength units! (6 points)

3. ① $\sqrt{\varepsilon_0\mu_0} = \sqrt{8.86\times10^{-12}\,F/m \times 1.256\times10^{-6}\,Vs/(Am)} = \sqrt{11.12816\times10^{-18}}\cdot\sqrt{\frac{F\cdot V\cdot s}{m\cdot A\cdot m}}$

Farad: $1F=\frac{C}{V}$ coulomb $C = A\cdot s$ Then $\sqrt{\frac{F\cdot V\cdot s}{m\cdot A\cdot m}} = \sqrt{\frac{A\cdot s}{V\cdot m}\cdot\frac{V\cdot s}{A\cdot m}} = \frac{s}{m}$

Thus $\underline{\sqrt{\varepsilon_0\mu_0}\approx 3.336\times10^{-9}\,s/m}$ $\frac{1}{\sqrt{\varepsilon_0\mu_0}}\approx 2.998\times10^{8}\,m/s = c$

② $\frac{4\pi\varepsilon_0\hbar^2}{m_e\cdot e^2} = \frac{\varepsilon_0 h^2}{\pi m_e e^2} = \frac{8.86\times10^{-12}\frac{F}{m}\cdot 6.625^2\cdot10^{-68}\,W^2s^4}{3.14\cdot 9.108\times10^{-31}\,kg\cdot 1.602^2\cdot10^{-38}\,A^2\cdot s^2} \approx \frac{388.871}{73.397}\cdot\frac{F\cdot W^2\cdot s^2}{m\cdot kg\cdot A^2}\times10^{-11}$

Watt: $1W=\frac{J}{s}=\frac{N\cdot m}{s}=\frac{kg\cdot m^2}{s^3}$ $1F=\frac{C}{V}=\frac{A\cdot s}{V}$ $V=\frac{J}{C}=\frac{N\cdot m}{A\cdot s}=\frac{kg\,m^2}{A\cdot s^3}\Rightarrow 1F=\frac{C^2}{J}=\frac{A^2s^2}{J}$

then $\frac{F\cdot W^2\cdot s^2}{m\cdot kg\cdot A^2} = \frac{\cancel{A^2}\cdot s^2\cdot J^2\cdot \cancel{s^2}}{J\cdot m\cdot kg\cdot \cancel{A^2}\cdot \cancel{s^2}} = \frac{s^2\cdot J}{m\cdot kg} = \frac{s^2\cdot kg\cdot m^2}{m\cdot kg\cdot s^2} = m$

Thus: $\frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} = \underline{5.298\times10^{-11}\,m}$

③ $\frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{1.602^2\times10^{-38}\,C^2}{4\pi\times8.86\times10^{-12}\,F/m\times\frac{6.625\times10^{-34}\,W\cdot s^2}{2\pi}\times2.998\times10^{8}\,m/s} = \frac{1.602^2\cdot10^{-38}}{2\times8.86\times10^{-12}\cdot6.625\times10^{-34}\cdot2.998\times10^{8}}\cdot\frac{C^2\cdot m\cdot s}{F\cdot W\cdot s^2\cdot m}$

$\underline{=7.292\times10^{-3}}$

$\frac{C^2\cdot \cancel{m}\cdot \cancel{s}}{\frac{C}{V}\cdot\frac{J}{\cancel{s}}\cdot s^2\cdot\cancel{m}} = \frac{C\cdot V}{J} = 1$

$J = N\cdot m = \frac{kg\cdot m^2}{s^2} = C\cdot V = A\cdot s\cdot V \Rightarrow V=\frac{kg\cdot m}{A\cdot s^3}$

$N=\frac{kg\,m}{s^2}$

④ $\frac{h}{m_e\cdot c} = \frac{6.625\times10^{-34}\,W\cdot s^2}{9.108\times10^{-31}\,kg\cdot 2.998\times10^{8}\,m/s} = 2.426\times10^{-12}\cdot\frac{J\cdot s^2}{kg\cdot m} = 2.426\times10^{-12}\cdot\frac{kg\cdot m^2\cdot s^2}{kg\cdot m\cdot s^2} = \underline{2.426\times10^{-12}\,m}$

4. $U(r)=4D\left[\left(\frac{\sigma}{r}\right)^{12}-\left(\frac{\sigma}{r}\right)^{6}\right]$ obviously $r\to\infty$ $U(r)\to0$ $r\to0$ $U(r)\to\infty$

And if $\left(\frac{\sigma}{r}\right)^{12}-\left(\frac{\sigma}{r}\right)^{6}=0$, $U(r)=0 \Rightarrow \left(\frac{\sigma}{r}\right)^{6}=1 \Rightarrow r=\sigma$

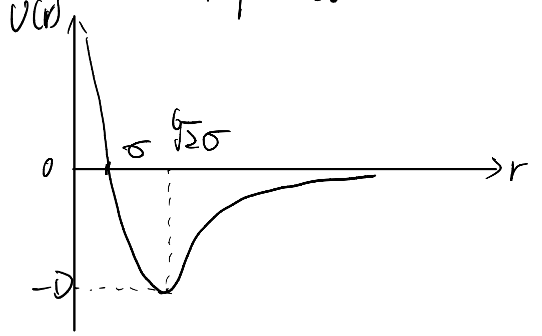
And when $U'(r_0)=0$, $U(r)$ has a local minimum

$U'(r) = 4D\sigma^6 \left[\frac{\sigma^6}{r^{12}} - \frac{1}{r^6} \right]'$ $\quad \left(\frac{\sigma^6}{r^{12}} - \frac{1}{r^6} \right)' = \frac{-12\sigma^6}{r^{13}} + \frac{6}{r^7}$

$\frac{6}{r^7} - \frac{12\sigma^6}{r^{13}} = 0 \Rightarrow r^6 = 2\sigma^6 \Rightarrow \underline{r_0 = \sqrt[6]{2}\sigma}$ ✓ 6m

$U(r_0) = 4D\left[\left(\frac{\sigma}{\sqrt[6]{2}\sigma}\right)^{12} - \left(\frac{\sigma}{\sqrt[6]{2}\sigma}\right)^6 \right] = -D$

Then the Graph can be sketched like:



σ has the same measurement unit with r, which is m

D has the same measurement unit with $U(r)$, which is J.

And $J = \frac{kg \cdot m}{s^2}$ in SI base units

5. Assume the density of water is 1kg/L, the a liter of water weigh 1kg.

And a molecule of water cosists of one oxygen atom and two hydrogen atoms.

An Oxygen atom consists of eight protons, eight neutrons and eight electrons. a hydrogen atom consists of one proton and one electron.

So, totally, in a molecule of water, there are ten protons, eight neutrons and ten electrons

Then mass of a proton is $m_p = 1.672 \times 10^{-27}$ kg, and the mass of electron is $m_e = 9.108 \times 10^{-31}$ kg

and the mass a neutron is $m_n = 1.675 \times 10^{-27}$ kg

Thus the mass a molecule of water is $m_w = 10 \cdot 1.672 \times 10^{-27} kg + 8 \cdot 1.675 \times 10^{-27} kg + 10 \cdot 9.108 \times 10^{-31} kg$

$m_w \approx 3.012 \times 10^{-26}$ kg

Thus, the amount of a liter of water is $N = \frac{1kg}{3.012 \times 10^{-26} kg} = 3.32 \times 10^{25}$ ✓ 4m

The radius of earth is $R = 6378.1 km = 6.3781 \times 10^8 cm$

Assume earth is a perfect sphere, then the Area of the surface of earth: ✓

$A = 4\pi R^2 = 5.11 \times 10^{18} cm^2$

Thus, the numeber of water molecules in $1 cm^2$ is $\rho = \frac{N}{A} = \frac{3.32 \times 10^{25}}{5.11 \times 10^{18} cm^2} \approx \underline{6.497 \times 10^6 cm^{-2}}$ ✓

6. $\omega = \frac{2\pi}{T} = \frac{2\pi c}{\lambda} \Rightarrow \lambda = \frac{2\pi c}{\omega} = \frac{2 \times 3.14 \times 2.998 \times 10^8 m/s}{10^{16} s^{-1}} = 1.883 \times 10^{-7} m = \underline{188.3 nm}$ 2

$\lambda_1 = \frac{2\pi c}{\omega - \frac{FWHM}{2}}$ $\quad \lambda_2 = \frac{2\pi c}{\omega + \frac{FWHM}{2}}$ $\quad \Delta\lambda = \lambda_1 - \lambda_2 = \frac{2\pi c}{\omega - \frac{FWHM}{2}} - \frac{2\pi c}{\omega + \frac{FWHM}{2}} = \frac{2\pi c \cdot FWHM}{\omega^2 - \left(\frac{FWHM}{2}\right)^2}$

because $\left(\frac{FWHM}{2}\right)^2 \ll FWHM < \omega$ $\quad$ Slow $\left|\frac{\Delta\lambda}{\lambda}\right| = \left|\frac{\Delta\omega}{\omega}\right| \rightarrow$ Differentiate $\quad$ 4m

$\Delta\lambda \approx \frac{2\pi c \cdot FWHM}{\omega^2} = \frac{2\pi \cdot 2.998 \times 10^8 m \cdot s^{-1} \cdot 5 \times 10^8 s^{-1}}{10^{32} s^{-2}} = \underline{9.414 \times 10^{-15} m}$ ✓ 2