Institute of Applied Physics
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Laser Physics Seminar – Problem set 5

Problem 1 (5 points)

a) Can a medium exhibit homogeneous and ihnomogeneous broadening at the same time? Please, explain your answer. (1 point)

b) What is the precondition for optical amplification? (1 point)

c) What are regenerative amplifiers and why are they used? (1 point)

d) What is gain saturation? (1 point)

e) What is spectral hole burning and why does it occur? (1 point)

a) Yes, e.g. in He-Ne laser, there are two main broadening mechanisms: collision broadening (homogeneous) and Doppler broadening (inhomogeneous)

b) The excited state must have a higher population than the ground state (inversion)

c) A regenerative amplifier is an amplifier in which gain is increased by feeding the output signal back to the AM. They are used to effectively increase the gain

d) Gain saturation is a phenomenon that the gain of an active media is reducing when the input power is increasing

e) In an inhomogeneous gain medium, a selective decrease of the gain near the frequency of the incoming signal can be observed. Because in such media, the incoming light only interacts with a certain set of particles.

Problem 2 (4 points)

In this exercise the dependence of the Amplified Spontaneous Emission (ASE) on the geometrical shape of the active medium will be calculated.

a) We consider a medium with a Lorentzian frequency spectral profile of the gain coefficient $g(\nu)$, which peak value is $g(\nu_{21}) = 1000m^{-1}$ and which FWHM bandwidth is 4nm. If $g(\nu)$ is centered at a wavelength of $1\mu m$, calculate its value for incoming light with a wavelength of $999nm$. (1 point)

Consider a Lorentzian gain coefficient and frequency-wavelength transform:

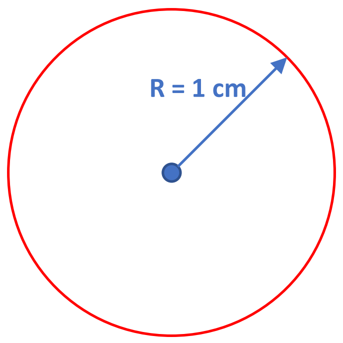
$$g(\nu) = g_0 \frac{\left(\frac{\Delta\nu_{FWHM}}{2}\right)^2}{(\nu-\nu_{21})^2 + \left(\frac{\Delta\nu_{FWHM}}{2}\right)^2} \qquad \Delta\nu = \frac{c \cdot \Delta\lambda}{\lambda_0^2}$$

Given:

$$\Delta\nu_{FWHM} = \frac{c \cdot \Delta\lambda_{FWHM}}{\lambda_0^2} \qquad \Delta\nu = \frac{c \cdot \Delta\lambda}{\lambda_0^2} \text{ with } \Delta\lambda = 1nm \qquad g_0 = 1000m^{-1}$$

$$g(999nm) = g_0 \frac{\left(\frac{\Delta\lambda_{FWHM}}{2}\right)^2}{(\Delta\lambda)^2 + \left(\frac{\Delta\lambda_{FWHM}}{2}\right)^2} = g_0 \frac{4}{1+4} = \frac{4}{5} g_0 = 800m^{-1}$$

b) A 2D-medium with the gain characteristics described in a) and circular shape with radius $R = 1cm$ is assumed. Now a single photon with $999nm$ wavelength is spontaneously emitted at the center of the medium. Calculate and plot the energy distribution of the emitted radiation depending of the angle φ. (1 point)



<u>From a):</u>

$$g(999nm) = 800m^{-1}$$

<u>The gain factor in each direction is the same:</u>

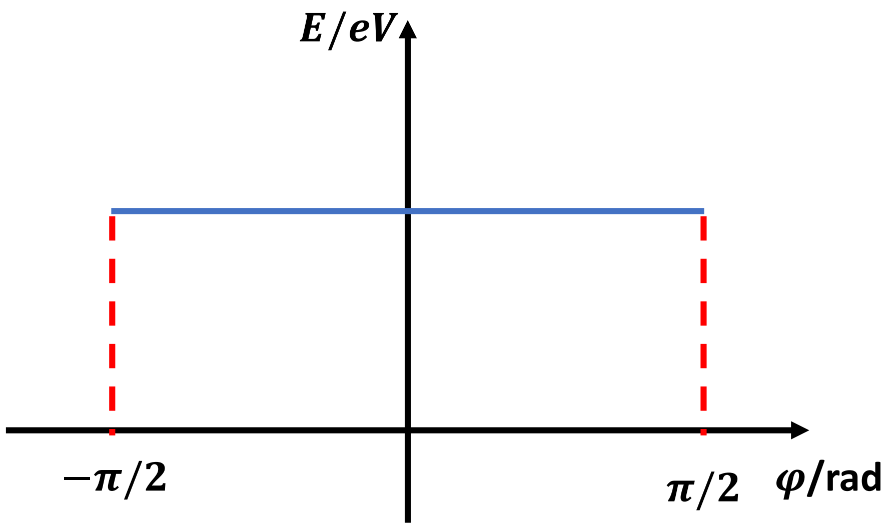
$$G = exp(g(999nm)R) = exp(800 \times 0.01) = 3 \times 10^3$$

<u>Each photon at 999nm has energy:</u>

$$E_0 = \frac{hc}{\lambda} = 1.24\ eV = 1.9867 \times 10^{-19} J$$

<u>After the amplification:</u>

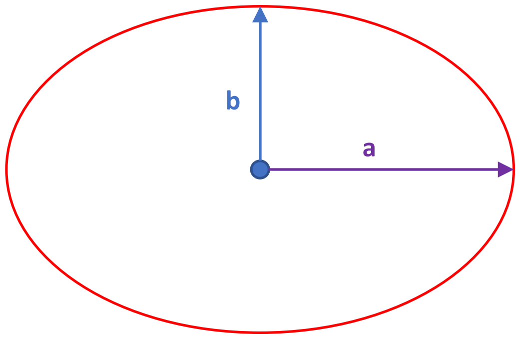
$$E = GE_0 = 3720\ eV = 5.9601 \times 10^{-19} J$$



c) Now we consider another 2D medium with the same gain characteristics as above and with an elliptical shape. The length of the semi-axes fulfill the following condition $a = 2b$. Additionally, the area of the ellipse is equal to the the area of the circle in b). Calculate and plot the energy distribution depending on the angle of emission φ. How is such an ASE-source called? (2 points)

The ellipse has the same area as the circle in b):

$$S = \pi ab = \pi R^2 \quad \Rightarrow \quad a = \sqrt{2}R \qquad b = \frac{R}{\sqrt{2}}$$



At any angle φ, the radius is:

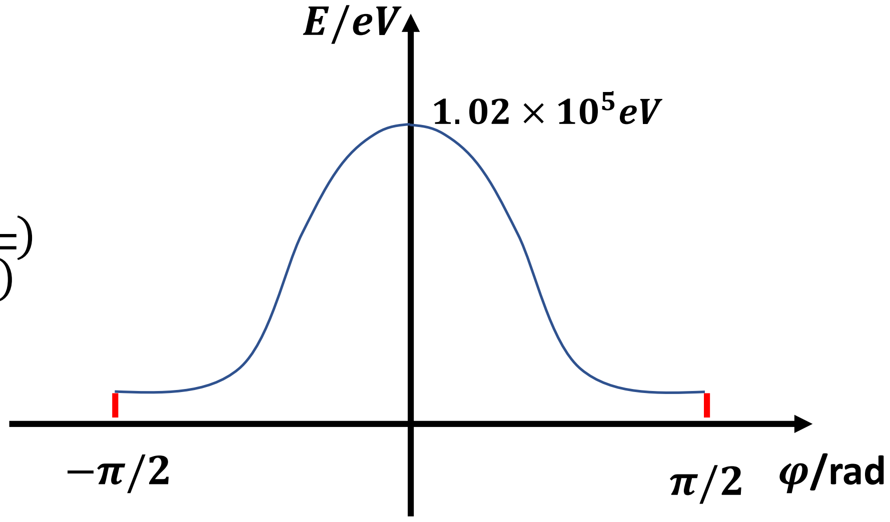
$$R(\varphi) = \frac{b}{\sqrt{1 - \frac{a^2 - b^2}{a^2} cos^2(\varphi)}} = \frac{1}{\sqrt{2}\sqrt{1 - 0.75cos^2(\varphi)}} cm$$

Then the gain factor is:

$$G(\varphi) = exp\big(g(999nm)R(\varphi)\big) = exp(\frac{8}{\sqrt{2}\sqrt{1 - 0.75cos^2(\varphi)}})$$

The energy is:

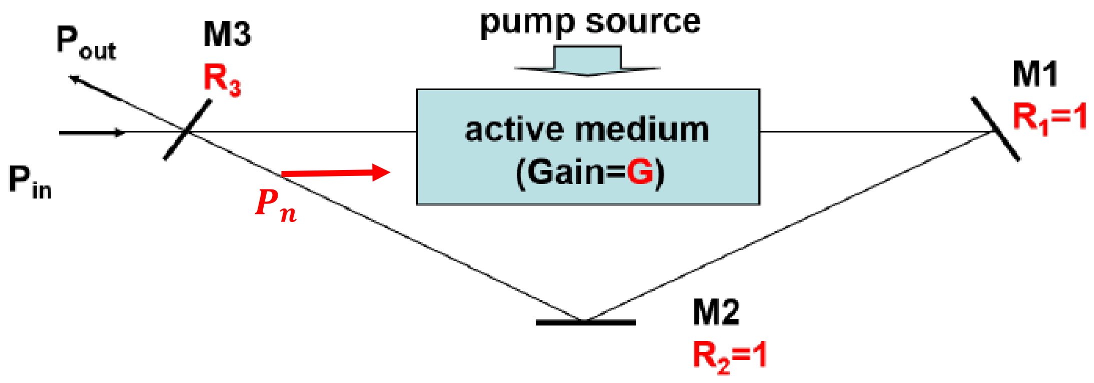
$$E(\varphi) = E_0 G(\varphi) = 1.24 \times exp\left(\frac{8}{\sqrt{2}\sqrt{1 - 0.75cos^2(\varphi)}}\right) eV$$



Superradiant source

Problem 3 (8 points)

Regenerative amplifiers are used to increase the amplification of a signal when using low-gain active media. In the lecture we have seen one configuration of regenerative amplifiers typically used for CW signals:



a) Calculate an expression for the gain of the system in steady state ($G_s = P_{out}/P_{in}$) disregarding saturation. (2 points)

At each round, the regenrative power is:

$$P_n = (GR_3)^{n-1}(1 - R_3)P_{in}$$

The output power is then:

$$P_{out} = G(1 - R_3)\sum_{n=1}^{\infty} P_n = G(1 - R_3)\frac{(1 - R_3)P_{in}}{1 - R_3 G} \qquad \text{If } R_3 G < 1$$

The gain factor:

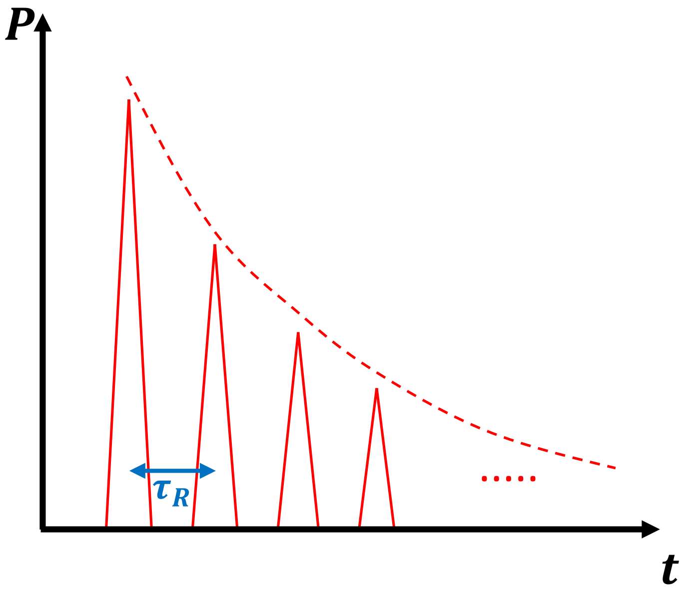
$$G_s = \frac{P_{out}}{P_{in}} = \frac{(1 - R_3)^2 G}{1 - R_3 G}$$

b) If the gain of the active medium is 1.1, what is the maximum value of the reflectivity of the outcoupling mirror (R_3) so that this configuration can work as a regenerative amplifier? (1 point)

The condition is:

$$\boldsymbol{R_3 G < 1} \quad \Rightarrow \quad \boldsymbol{R_3 < 0.91}$$

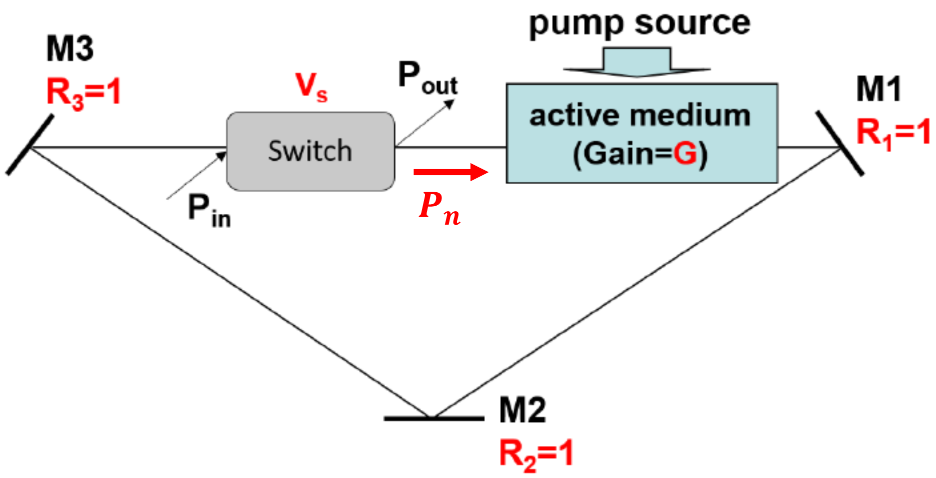
c) Explain why the configuration above does not work properly when amplifying pulsed signals. (1 point)



If a pulse is coupled in the cavity, the pulse train would be obtained at the output port

In this train, each pulse will be seperated from the previous one by the round-trip time and its peak power will also decrease

d) For pulsed signals the usual configuration of a regenerative amplifier employs an optical switch, as shown in the figure.



$$P_n = (GV_s)^{n-1} V_s P_{in}$$

<u>After N_R round trip:</u>

$$P_{out} = P_{N_R+1} = (GV_s)^{N_R} V_s P_{in}$$

<u>The gain is:</u>

$$G_S = \frac{P_{out}}{P_{in}} = (GV_s)^{N_R} V_s$$

Hereby, when the switch is open, a single pulse is coupled in the cavity. Then the switch is closed and the pulse circulates inside of the cavity a certain number of roud-trips (N_R). Afterwards, the switch is opened again and the amplified pulse is coupled out of the cavity. Additionally, assume that the switch is the only element in the cavity that introduces losses (V_s), which are independent of the propagation direction of the beams. Under these circumstances, calculate the amplification factor of the system ($G_s = P_{out}/P_{in}$) as a function of the number of round-trips. Neglect the impact of saturation. (2 points)

e) Which condition is required for the configuration in d) to provide net gain (i.e. $G_s > 1$)? Discuss the reason behind the difference with the CW case. (1 point)

From d), the gain is:

$$G_S = \frac{P_{out}}{P_{in}} = (GV_s)^{N_R} V_s$$

To provide net gain:

$$G_S > 1 \quad \Rightarrow \quad G > \frac{1}{V_s^{1+\frac{1}{N_R}}} \approx \frac{1}{V_s}$$

If N_R is very large

For the CW case, from b):

$$G < \frac{1}{R_3}$$

The CW system operates in steady-state whereas the pulsed system works in a transient that builds up the pulses

f) What would the impact of gain saturation be in the regenerative amplifier for pulsed operation? Explain your answer. (1 point)

It will limit the number of round trips. This will significantly reduce the total gain of the system