2nd Exam -*Structure of Matter*

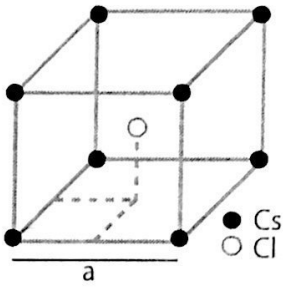
February 2, 2010

General remarks: *Please give adequate drawings where they can be helpful! Explain your solutions.*

1. Define and explain (no calculations!) *30 points*

a) Explain shortly sc, fcc, bcc and hcp. Name main differences between them. *8*
b) What is the first Brillouin zone and its physical meaning? *4*
c) Why is the momentum of phonons called quasi momentum? *2*
d) Which information on a crystal can you get using the structure factor in an x-ray experiment? (general case, no formulas) *3*
e) Which information on a crystal can you get using the atomic form factor in an x-ray experiment? (general case, no formulas) *3*
f) What is the difference between optical and acoustic phonons? *4*
g) What is the difference between Raman and Brillouin scattering? *2*
h) What is the effective mass of electrons? *2*
i) Explain shortly the difference in the energy bands of an isolator and a metal. *2*

2. CsCl *15 points*



a) Specify primitive basis vectors of the Bravais lattice as well as vectors of the atomic basis. *4*
b) Which Bravais lattice is this? *2*
c) Calculate the packing fraction in the model of hard spheres assuming same sizes of Cs and Cl. *6*
d) Calculate the basis of the reciprocal lattice and specify its Bravais type. *3*

3. Phonons *30 points*

a) Calculate the dispersion relation for a linear monatomic chain with force constant f between nearest neighbors (1-D). Start with forming the equation of motion. *8*
b) For the case of a) calculate the density of states. *6*
c) How does the dispersion relation from a) change if you close a chain out of N atoms to a circle? Give a physical interpretation. *6*
d) Explain the main idea of the Debye model. *6*
e) How many optical and acoustic phonon branches would you expect for CsCl? How many of each are transverse and longitudinal respectively? Give a short explanation ! *4*

4. Electrons *25 points*

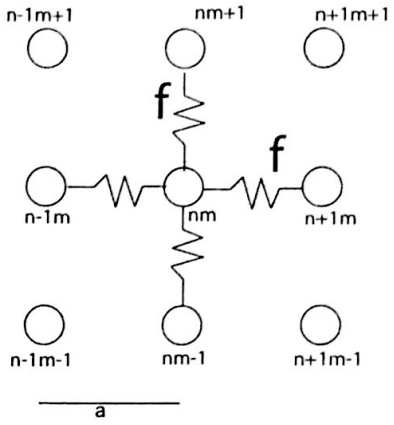
a) Give one example of a real structure where the model of a 2D free electron gas can be applied. *4*
b) Calculate the Dispersion relation for an ideal 2D free electron gas. Start with the Schrödinger equation: *5*

$$\mathcal{H}\Psi = \frac{\vec{p}^2}{2m}\Psi = E\Psi, \text{ where } \vec{p} = \frac{\hbar}{i}\begin{pmatrix} \frac{d}{dx} \\ \frac{d}{dy} \end{pmatrix}$$

c) Calculate the density of states for an ideal 2D free electron gas . *5*
d) Exlain how the result of c) will change, when you consider a small finite additional dimension? (no calculation) *5*
e) Explain in general the change in the occupation of states between zero and finite temperature. Use an appropriate sketch. (no calculation) *6*

Bonus: *10 points*

Calculate the phonon dispersion relation of a 2-D quadratic lattice with only nearest neighbour interaction (see picture). Start with forming the equation of motion.



Formulas:

$$\begin{aligned}
\cos(2x) &= 1 - 2\cdot\sin^2(x) \\
e^{ix} + e^{-ix} &= 2\cdot\cos(x) \\
\frac{d}{dx}\sin(x) &= \cos(x) \\
\frac{d}{dx}\cos(x) &= -\sin(x) \\
\frac{d}{dx}\text{acos}(x) &= -\frac{1}{\sqrt{1-x^2}} \\
\frac{d}{dx}\text{asin}(x) &= \frac{1}{\sqrt{1-x^2}}
\end{aligned}$$