Maxwell’s Equations

Poynting Vector

Normal Modes and Poynting Vector

Tutorial Problems

Normal Modes and Poynting Vector.

a). $\vec{H}(\vec{r}) = -\frac{i}{\omega\mu_0}\,\mathrm{rot}\,\vec{E}(\vec{r})$

$$= -\frac{i}{\omega\mu_0}\begin{vmatrix}\hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ 0 & E_y & 0\end{vmatrix} = -\frac{i}{\omega\mu_0}\begin{pmatrix}-\partial_z E_y \\ 0 \\ \partial_x E_y\end{pmatrix}$$

$$= -\frac{i}{\omega\mu_0}\begin{pmatrix}-(i\beta-\alpha)E_0\cos[(\beta+i\alpha)x]\,e^{(i\beta-\alpha)z} \\ 0 \\ -(\beta+i\alpha)E_0\sin[(\beta+i\alpha)x]\,e^{(i\beta-\alpha)z}\end{pmatrix} = \frac{i}{\omega\mu_0}\begin{pmatrix}(i\beta-\alpha)E_0\cos[(\beta+i\alpha)x]\,e^{(i\beta-\alpha)z} \\ 0 \\ (\beta+i\alpha)E_0\sin[(\beta+i\alpha)x]\,e^{(i\beta-\alpha)z}\end{pmatrix}$$

b). Wave equation: $\Delta\vec{E}(\vec{r}) + \frac{\omega^2}{c^2}\varepsilon(\omega)\,\vec{E}(\vec{r}) = 0$

$$\Delta\vec{E}(\vec{r}) = \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)E(\vec{r}) = \left[-(\beta+i\alpha)^2+(i\beta-\alpha)^2\right]E(\vec{r})$$

$$\therefore\ -(\beta+i\alpha)^2+(i\beta-\alpha)^2+\frac{\omega^2}{c^2}\varepsilon(\omega) = 0$$

$$2\alpha^2-2\beta^2-4i\beta\alpha = -\frac{\omega^2}{c^2}\varepsilon(\omega) = -\frac{\omega^2}{c^2}(\varepsilon'+i\varepsilon'')$$

$$\begin{cases}\beta^2-\alpha^2 = \frac{\omega^2}{2c^2}\varepsilon' \\ \beta\alpha = \frac{\omega^2}{4c^2}\varepsilon''\end{cases} \Rightarrow \begin{cases}\beta \approx \frac{\omega}{\sqrt{2}c}\sqrt{\varepsilon'(\omega)} \\ \alpha \approx \frac{1}{2\sqrt{2}}\frac{\omega}{c}\frac{\varepsilon''(\omega)}{\sqrt{\varepsilon'(\omega)}}\end{cases}$$

c) $\varepsilon'' = 0 \rightarrow \alpha = 0 \quad \beta = \frac{\omega}{\sqrt{2}c}\sqrt{\varepsilon'}$

$$E(\vec{r}) = E_0\cos\left(\frac{\omega}{\sqrt{2}c}\sqrt{\varepsilon'}\cdot x\right)\exp\left(i\frac{\omega}{\sqrt{2}c}\sqrt{\varepsilon'}\,z\right)\cdot\hat{y}$$

$$= E_0\cos\beta x\cdot e^{i\beta z}\,\hat{y}$$

$$E(\vec{r}) = E_0 \cos\beta x \, e^{i\beta z} \cdot \hat{y}$$

$$H(\vec{r}) = \frac{i}{w\mu_0} \begin{pmatrix} i\beta E_0 \cos\beta x \cdot e^{i\beta z} \\ 0 \\ \beta E_0 \sin\beta x \cdot e^{i\beta z} \end{pmatrix}$$

$$\langle S(\vec{r},t)\rangle = \frac{1}{2} Re \left[E \times H^* \right]$$

$$= \frac{1}{2} Re \left[\begin{pmatrix} 0 \\ E_0 \cos\beta x e^{i\beta z} \\ 0 \end{pmatrix} \times \frac{-i}{w\mu_0} \begin{pmatrix} -i\beta E_0 \cos\beta x \cdot e^{-i\beta z} \\ 0 \\ \beta E_0 \sin\beta x \, e^{-i\beta z} \end{pmatrix} \right]$$

$$= \frac{1}{2w\mu_0} Re \left[\begin{pmatrix} -i E_0^2 \sin\beta x \cos\beta x \\ 0 \\ \beta E_0^2 \cos^2\beta x \end{pmatrix} \right]$$

$$\langle S(\vec{r},t)\rangle = \frac{1}{2w\mu_0} \cdot \beta E_0^2 \cos^2\beta x \cdot \hat{z}$$

Beam Propagation

Tutorial Problems.

Beam propagation.

a) $H(\alpha,\beta,z) = i\sqrt{k^2-\alpha^2-\beta^2}\cdot z$

b) $\alpha^2+\beta^2 \ll k^2$

paraxial approximation: $\sqrt{k^2-\alpha^2-\beta^2} = \sqrt{k^2-r^2} = f(r)$.

$f(0) = k$

$f'(r) = \frac{-2r}{2\sqrt{k^2-r^2}} = -\frac{r}{\sqrt{k^2-r^2}}$, $f'(0) = 0$

$f''(r) = -\frac{\sqrt{k^2-r^2}+r\cdot\frac{r}{\sqrt{k^2-r^2}}}{k^2-r^2}$, $f''(0) = -\frac{1}{k}$

$\therefore f(r) = f(0) + \frac{1}{2!}f''(0)r^2 = k - \frac{\alpha^2+\beta^2}{2k}$

$\therefore H_F(\alpha,\beta,z) = \exp(ikz)\exp(-i\frac{\alpha^2+\beta^2}{2k}z)$

c) $U_0(\alpha, z=0) = \frac{A\delta(\alpha)}{\cancel{2\pi}} + \frac{B}{\cancel{4\pi}}\left[\delta(\alpha-\frac{2\pi}{L}) + \delta(\alpha+\frac{2\pi}{L})\right]$

$U(\alpha, z) = \cancel{H(\alpha)} \, H(\alpha,z)\cdot U_0(\alpha,0)$

$= \frac{A\delta(\alpha)}{\cancel{2\pi}}\exp(i\sqrt{k^2-\alpha^2}\,z) + \frac{B}{\cancel{4\pi}}\left[\delta(\alpha-\frac{2\pi}{L})+\delta(\alpha+\frac{2\pi}{L})\right]\exp(i\sqrt{k^2-\alpha^2}\,z)$

$u(x,z) = FT^{-1}\left[U(\alpha,z)\right]$

$= \frac{A}{\cancel{2\pi}}e^{ikz} + \frac{B}{\cancel{4\pi}}\left[e^{i\sqrt{k^2-\frac{4\pi^2}{L^2}}\cdot z}\cdot e^{i\frac{2\pi}{L}x} + e^{i\sqrt{k^2-\frac{4\pi^2}{L^2}}\cdot z}\cdot e^{-i\frac{2\pi}{L}x}\right]$

$= \frac{A}{\cancel{2\pi}}e^{ikz} + \frac{B}{\cancel{2\pi}}e^{i\sqrt{k^2-\frac{4\pi^2}{L^2}}z}\cdot\cos\frac{2\pi}{L}x$

$= Ae^{ikz} + Be^{i\sqrt{k^2-\frac{4\pi^2}{L^2}}z}\cdot\cos\frac{2\pi}{L}x$

d) $u_0(x,z) = Ae^{ikz} + Be^{i\sqrt{k^2-\frac{4\pi^2}{L^2}}\cdot z} \cdot \cos\frac{2\pi}{L}x$

$$= e^{ikz}\left[A + B\exp\left[i\left(\sqrt{k^2-\frac{4\pi^2}{L^2}} - k\right)z\right]\cdot\cos\frac{2\pi}{L}x\right]$$

First reappearance:

$$\left(\sqrt{k^2-\frac{4\pi^2}{L^2}} - k\right)\cdot z = 2\pi$$

$$z = \frac{2\pi}{\sqrt{\frac{4\pi^2}{\lambda_0^2} - \frac{4\pi^2}{L^2}} - \frac{2\pi}{\lambda_0}}$$

$$z = \frac{1}{\sqrt{\frac{1}{\lambda_0^2} - \frac{1}{L^2}} - \frac{1}{\lambda_0}}$$

e) $\alpha^2 << k^2$ i.e. $\frac{2\pi}{L} << \frac{2\pi}{\lambda_0} \rightarrow L >> \lambda_0$

Under Fresnell approximation.

$$U(\alpha, z) = A\delta(\alpha)\exp(ikz)\exp\left(-i\frac{\alpha^2}{2k}z\right) + \frac{B}{2}\left[\delta\left(\alpha-\frac{2\pi}{L}\right)+\delta\left(\alpha+\frac{2\pi}{L}\right)\right]\exp(ikz)\exp\left(-i\frac{\alpha^2}{2k}z\right)$$

$$u(x,z) = A\exp(ikz) + \frac{B\exp(ikz)}{2}\left[e^{-i\frac{\frac{4\pi^2}{L^2}}{2k}z}\cdot e^{i\frac{2\pi}{L}x} + e^{-i\frac{\frac{4\pi^2}{L^2}}{2k}z}\cdot e^{-i\frac{2\pi}{L}x}\right]$$

$$= A\exp(ikz) + B\exp(ikz)\exp\left(-i\frac{4\pi^2}{2kL^2}z\right)\cos\frac{2\pi x}{L}$$

$$= \exp(ikz)\left[A + B\exp\left(-i\frac{4\pi^2}{2kL^2}z\right)\cos\frac{2\pi x}{L}\right]$$

First reappearance:

$$\frac{4\pi^2}{2kL^2}\cdot z = 2\pi$$

$$z = \frac{2L^2}{\lambda_0}$$

Gaussian Beams

Tutorial problems

Gaussian Beams.

a). $q(z) = z - iz_0$ $\qquad R(z) = z + \frac{z_0^2}{z}$

$$\frac{1}{q(z)} = \frac{1}{z + \frac{z_0^2}{z}} + \frac{\frac{1}{z_0}}{1 + \left(\frac{z}{z_0}\right)^2} \qquad w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_0}\right)^2}$$

$$\therefore \frac{1}{q(z)} = \frac{1}{R(z)} + \frac{\lambda_n}{\pi w^2(z)}$$

b) $\hat{M} = M_{R_2} M_d M_{R_1}$

$$= \begin{pmatrix} 1 & 0 \\ \frac{n_1 - n_2}{n_1 r} & \frac{n_2}{n_1} \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{n_2 - n_1}{n_2 r} & \frac{n_1}{n_2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ \frac{n_1 - n_2}{n_1 r} & \frac{n_2}{n_1} \end{pmatrix} \begin{pmatrix} 1 - \frac{n_2 - n_1}{n_2 r} d & \frac{n_1}{n_2} d \\ -\frac{n_2 - n_1}{n_2 r} & \frac{n_1}{n_2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \frac{n_2 - n_1}{n_2 r} d & \frac{n_1}{n_2} d \\ \frac{(n_1 - n_2)}{n_1 r}\left(2 + \frac{n_1 - n_2}{n_2 r} d\right) \; \cancel{\frac{n_1 - n_2}{n_1 r}\left(2 + \frac{n_1 - n_2}{n_2 r}\right)} & \frac{n_1 - n_2}{n_2 r} d + 1 \end{pmatrix}$$

$$q = \frac{A q_1 + B}{C q_1 + D} = \frac{\left(1 - \frac{n_2 - n_1}{n_2 r}\right) q_1 + \frac{n_1}{n_2} d}{\frac{n_1 - n_2}{n_1 r}\left(2 + \frac{n_1 - n_2}{n_2 r} d\right) q_1 + \frac{n_1 - n_2}{n_2 r} d + 1}$$

c) when $d=0$. $\hat{M}_{Lens} = \begin{pmatrix} 1 & 0 \\ \frac{2(n_1-n_2)}{n_1 r} & 1 \end{pmatrix}$

set:

$q(0) = -iz_0$

$q(1) = L_1 - iz_0$

$$q^*_{R_2} = \hat{M}_{Lens}\, q(1) = \frac{q_1 + 0}{\frac{2(n_1-n_2)}{n_1 r} q_1 + 1}$$

$$= \frac{L_1 - iz_0}{\frac{2(n_1-n_2)}{n_1 r}(L_1 - iz_0) + 1} = \frac{L_1 - iz_0}{C\cdot(L_1 - iz_0) + 1} = \frac{L_1 - iz_0}{CL_1 + 1 - iCz_0}$$

$$= \frac{(L_1 - iz_0)\left[(CL_1+1) + iCz_0\right]}{(CL_1+1)^2 + C^2 z_0^2}$$

$$\mathrm{Re}\, q_{R_2} = \frac{C\cdot(L_1^2 + z_0^2) + L_1}{(CL_1+1)^2 + C^2 z_0^2}, \quad q_2 = q_{R_2} + L_2. \quad \mathrm{Re}\, q_2 = 0. \text{ if waist.}$$

$$\mathrm{Re}\, q_{R_2} + L_2 = \frac{C\cdot(L_1^2+z_0^2) + L_1 + L_2(CL_1+1)^2 + L_2 C^2 z_0^2}{(CL_1+1)^2 + C^2 z_0^2} = 0$$

$$L_2 = -\frac{C\cdot(L_1^2+z_0^2) + L_1}{(CL_1+1)^2 + C^2 z_0^2} \qquad C = \frac{2(n_1-n_2)}{n_1 r}$$

$$L_2 = \frac{\frac{2(n_2-n_1)}{n_1 r}(L_1^2 + z_0^2) - L_1}{\left(\frac{2(n_1-n_2)}{n_1 r} + 1\right)^2 + \frac{4(n_1-n_2)^2}{n_1^2 r^2} z_0^2}$$

Pulses

Tutorial Problems.

Pulses.

a) $k(w) = k(w_0) + \frac{\partial k}{\partial w}\Big|_{w_0}(w-w_0) + \frac{1}{2}\frac{\partial^2 k}{\partial w^2}\Big|_{w_0}(w-w_0)^2$

$\frac{1}{V_{ph}} = \frac{k(w_0)}{c}$, $\frac{1}{V_g} = \frac{\partial k}{\partial w}$, $\frac{\partial^2 k}{\partial w^2} = \frac{\partial}{\partial w}\left(\frac{1}{V_g}\right)$.

phase velocity — group velocity — group velocity dispersion

b) $\frac{\partial k}{\partial w} = \frac{\partial \frac{w}{c} n(w)}{\partial w} = \frac{1}{c}\left[n(w_0) + w_0 \frac{\partial n}{\partial w}\Big|_{w_0}\right] = \frac{1}{V_g}$

$$V_g = \frac{c}{(A+Bw_0^2) + w_0(2Bw_0)} = \frac{c}{A+3Bw_0^2}$$

$$V_{g1} = \frac{c}{A+3Bw_1^2}, \quad V_{g2} = \frac{c}{A+3Bw_2^2}$$

If the 2^{nd} pulse can catch the 1^{st}, $V_{g2} > V_{g1} \curvearrowright w_2 < w_1$

$$t = \frac{V_{g1}\Delta t}{(V_{g2} - V_{g1})} = \frac{\frac{c}{A+3Bw_1^2}\cdot \Delta t}{\left(\frac{c}{A+3Bw_2^2} - \frac{c}{A+3Bw_1^2}\right)}$$

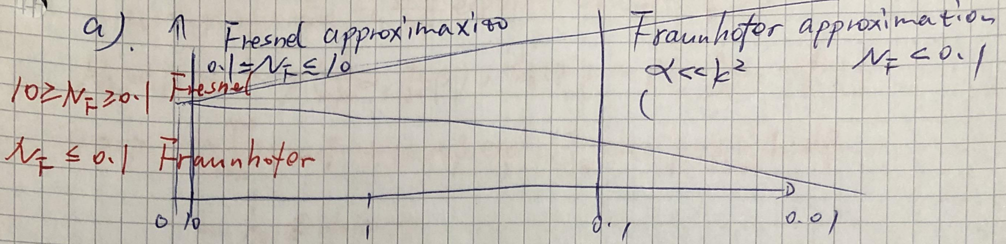
$$= \frac{\Delta t}{\frac{A+3Bw_1^2}{A+3Bw_2^2} - 1}$$

Fraunhofer diffraction

Tutorial problems.

Fraunhofer diffraction.

a).



I'm not sure.

1) $N_F \le 0.1$ 2) $N_F \le 10$, 3) $\alpha \le k$.

b) $rect(a) \sim sinc(\alpha a)$.

$\therefore rect(\frac{b}{z}) \sim sinc(\alpha \frac{b}{z}) \sim sinc(\frac{kb}{2z})$

shift theorem: $f(x-a) \rightarrow \exp(-i\alpha a)$

$f(x-\frac{a}{2}) \longrightarrow \exp(-i\frac{k}{z}\cdot\frac{a}{2})$.

$f(x+\frac{a}{2}) \longrightarrow \exp(i\frac{k}{z}\cdot\frac{a}{2})$.

~~$\therefore U(x)$~~

$\therefore u(x,z) = sinc(\frac{kb}{2z})\cdot\left[\exp(-i\frac{k}{z}\frac{a}{2}) + \exp(i\frac{k}{z}\frac{a}{2})\right]$

$= 2 sinc(\frac{kb}{2z})\cdot\cos(\frac{ka}{2z})$.

$I = |u(x,z)|^2 = 4 sinc^2(\frac{kb}{2z})\cos^2(\frac{ka}{2z})$ not precise

~~b)~~ c) a defines Fraunhofer and b defines Fresnell.

Fraunhofer: $\frac{a}{\lambda}\cdot\frac{a}{z_B} \le 0.1 \curvearrowright a^2 \lesssim 0.1\lambda z_B$

Fresnel: ~~$\frac{b}{\lambda}\cdot\frac{b}{z_B} \lesssim 10 \curvearrowright b^2 \lesssim 10\lambda z_B$~~

$b > 10\lambda$

Paraxial Approximation: $|\delta x|, |\delta y| > 10\lambda_n$

Fourier Optics

Tutorial Problems.

Fourier Optics

$$u(x,y,2f) = -i\frac{|2\pi|^2}{\lambda f}\exp(i k f)\, U_0\left(\frac{k}{f_1}x, \frac{k}{f_1}y\right)$$

$$H_4(\alpha,\beta,4f) \sim p\left(\frac{f_1}{k}\alpha, \frac{f_1}{k}\beta\right);\ h_4(x,y) \sim P\left[-\frac{k}{f_2}x, -\frac{k}{f_2}y\right]$$

$$u(-x,-y,4f) \sim \iint_{-\infty}^{\infty} H_4(\alpha,\beta,4f)\, U_0(\alpha,\beta) \exp\left[i\left(\frac{f_1}{f_2}x\alpha + \frac{f_1}{f_2}y\beta\right)\right] d\alpha\, d\beta$$

$$\sim h_4 \otimes u_0(x,y).$$

a) $u(x,y,2f) \sim U_0\left(\frac{k}{f_1}x, \frac{k}{f_1}y\right).$

$$T(\alpha) = FT[t(x)] = \frac{1}{2}\delta(\alpha) + \frac{1}{4}\left[\delta\left(\alpha - \frac{2\pi}{d}\right) + \delta\left(\alpha + \frac{2\pi}{d}\right)\right]$$

$$\alpha = \frac{k}{f_1}x = \frac{2\pi}{\lambda f_1}x$$

$$U_0\left(\frac{k}{f_1}x, f_1\right) = \cancel{\frac{1}{2}\delta(\alpha)\ \frac{1}{2}\delta\left(\frac{2\pi}{d}\right) + \frac{1}{4}}$$

$$= \frac{1}{2}\delta\left(\frac{2\pi}{\lambda f_1}x\right) + \frac{1}{4}\left[\delta\left(\frac{2\pi x}{\lambda f_1} - \frac{2\pi}{d}\right) + \delta\left(\frac{2\pi x}{\lambda f_1} + \frac{2\pi}{d}\right)\right]$$

peaks: $x_0 = 0$, $\boxed{x_1 = \frac{\lambda f_1}{d} = 0.2\,cm}$ $\boxed{x_2 = -\frac{\lambda f_1}{d} = -0.2\,cm}$

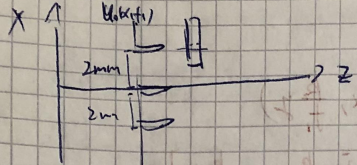
b) shift.

$\Delta s = (n_{glass} - 1)\, w = 100.5\,\mu m$

$\Delta\varphi = k \cdot \Delta s = \frac{2\pi}{\lambda} \cdot \Delta s = 201\pi = \pi.$ $\curvearrowright$ $p(x,y) = \begin{cases} e^{i\pi} = -1 & 1mm < x < 3mm \\ 1 & \text{elsewhere.} \end{cases}$

$$\cancel{H_4(\alpha, 4f) \sim p\left(\frac{f_1}{k}\alpha\right) = \begin{cases} -1 & 1 < \frac{f_1}{k}\alpha < 3 \\ 1 & \text{elsewhere.} \end{cases}}$$

$$\cancel{T_0(\alpha) = \frac{1}{2}\delta(\alpha) + \frac{1}{4}\left[\delta\left(\alpha - \frac{2\pi}{d}\right) + \delta\left(\alpha + \frac{2\pi}{d}\right)\right]}$$



$\cancel{U_0^+(x_1}$

$\therefore\ U_0^+(\frac{k}{f_1}x, f_1) = \frac{1}{2}\delta(\frac{2\pi}{\lambda f_1}x) + \frac{1}{4}\left[-\delta(\frac{2\pi x}{\lambda f_1} - \frac{2\pi}{d}) + \delta(\frac{2\pi x}{\lambda f_1} + \frac{2\pi}{d})\right] = u_0(x, 2f_1)$

$u(x, 2f_1) = \frac{1}{2}\delta(\frac{2\pi}{\lambda f_1}x) + \frac{1}{4}\left[\delta(\frac{2\pi x}{\lambda f_1} + \frac{2\pi}{d}) - \delta(\frac{2\pi x}{\lambda f_1} - \frac{2\pi}{d})\right] = u_1$

$u(x, 4f_1) \sim U_1(\frac{k}{f_2}x)$

$U_1(\frac{k}{f_2}x) = \int_{-\infty}^{\infty} \frac{1}{2}\delta(\frac{2\pi}{\lambda f_1}x') \cdot \exp(-i\frac{k}{f_2}x x')\,dx' + \frac{1}{4}\int_{-\infty}^{\infty}\left[\delta(\frac{2\pi x'}{\lambda f_1} + \frac{2\pi}{d}) - \delta(\frac{2\pi x'}{\lambda f_1} - \frac{2\pi}{d})\right]\exp(-i\frac{k}{f_2}x x')\,dx'$

$= \frac{1}{2} + \frac{1}{4}\left[\exp(i\frac{f_1}{f_2}\frac{2\pi}{d}x) - \exp(-i\frac{f_1}{f_2}\frac{2\pi}{d}x)\right]$

$= \frac{1}{2} + \frac{i}{2}\sin(\frac{f_1}{f_2}\frac{2\pi}{d}x)$. $\quad f_1/f_2 = 2$.

$\therefore\ u(x, f_1+f_2) = \frac{1}{2} + \frac{i}{2}\sin(\frac{4\pi}{d}x)$.

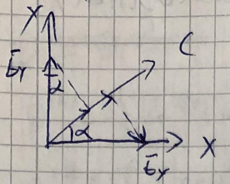
c) $I = u \cdot u^* = \frac{1}{4} + \frac{1}{4}\sin^2(\frac{4\pi x}{d})$

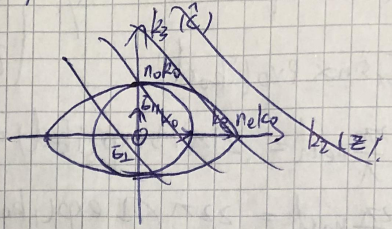
Anisotropy 1

Tutorial Problem

Anisotropy 1.

a).

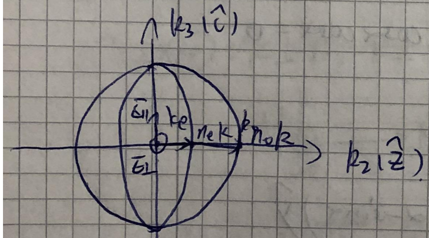




$$X: \begin{cases} E_{\parallel} = E_x \cos\alpha \exp(i k_e z) \\ E_{\perp} = E_x \sin\alpha \exp(i k_o z) \end{cases} \qquad Y: \begin{cases} E_{\parallel} = E_y \sin\alpha \exp(i k_e z) \\ E_{\perp} = E_y \cos\alpha \exp(i k_o z) \end{cases}$$

$$k_o = k \cdot n_o = \frac{2\pi}{\lambda} \cdot n_o = 13.82\ (\mu m^{-1})$$

$$k_e = k \cdot n_e = \frac{2\pi}{\lambda} \cdot n_e = 13.19\ (\mu m^{-1})$$



$$E = E_x\left(\cos\alpha \exp(i k_e z) + \sin\alpha \exp(i k_o z)\right)\hat{x} + E_y\left(\sin\alpha \exp(i k_e z) + \cos\alpha \exp(i k_o z)\right)\hat{y}$$

~~It is~~ a $\Delta\varphi = k \cdot \Delta S = (k_o - k_e) \cdot \Delta S = \pi$.

It is a half wave plate.

b).

$$x:\begin{cases} E_{\parallel} = E_x \cos\alpha \exp(ik_e d) \\ E_{\perp} = E_x \sin\alpha \exp(ik_o d). \end{cases} \qquad y:\begin{cases} E_{\parallel} = E_y \sin\alpha \exp(ik_e d) \\ E_{\perp} = E_y \cos\alpha \exp(ik_e d). \end{cases}$$

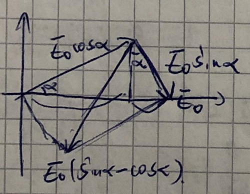
$$k_e d = \frac{2\pi}{\lambda} n_e d = 21\pi \wedge \exp(ik_e d) = -1$$

$$k_o d = \frac{2\pi}{\lambda} n_o d = 22\pi \wedge \exp(ik_o d) = 1$$

$$\therefore \quad x:\begin{cases} E_{\parallel} = -E_x \cos\alpha \\ E_{\perp} = E_x \sin\alpha \end{cases} \qquad y:\begin{cases} E_{\parallel} = -E_y \sin\alpha \\ E_{\perp} = E_y \cos\alpha. \end{cases}$$

$$E_{out} = E_x(\sin\alpha - \cos\alpha)\hat{x} + E_y(\cos\alpha - \sin\alpha)\hat{y}$$

c) $E_{out} = E_0(\sin\alpha - \cos\alpha)\hat{x}$



if $E_{out} = E_0\hat{y}$

$$E_0 \sin\alpha \sin\alpha \cancel{+} - E_0 \cos\alpha \cos\alpha = 0$$

$$\sin^2\alpha = \cos^2\alpha.$$

$$\alpha = 45^\circ$$

d) $E_{out} = E_0(\sin\alpha - \cos\alpha)\hat{x} + iE_0(\cos\alpha - \sin\alpha)\hat{y}$

$$= E_0\hat{x} - iE_0\hat{y}$$

$$\begin{cases} \sin\alpha - \cos\alpha = 1 \longrightarrow \begin{matrix} \sin\alpha = 1 \\ \cos\alpha = 0 \end{matrix} \longrightarrow \alpha = 90^\circ \\ \cancel{\cos\alpha - \sin\alpha = 1} \end{cases}$$

Anistropy 2

Tutorial problems.

Anisotropy 2.

a) ~~$D_1 = D_1 \exp ex$~~

$D^{(a)} = \{D_1 \exp[i(\vec{k}_a \vec{r} - wt)]\} \hat{e}_1 \qquad k_a^2 = \frac{w^2}{c^2} n_a^2$

$D^{(b)} = \{D_2 \exp[i(\vec{k}_b \vec{r} - wt)]\} \hat{e}_2 \qquad k_b^2 = \frac{w^2}{c^2} n_b^2$

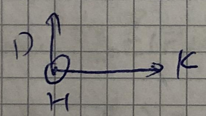
b) Start from Maxwell's equations for plane wave.

① $k \cdot D = 0$ ③ $k \times E = w\mu_0 H$ and ⑤ $\langle S \rangle = E \times H$

② $k \cdot H = 0$ ④ $k \times H = -wD$

~~④ $k \perp D$, +~~

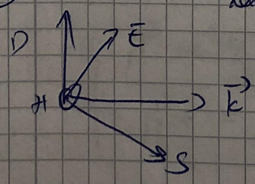
①②④: $k \perp D$, $k \perp H$, $D \perp H$ → $H \perp \overline{DK}$



③: $H \perp \overline{Ek}$ and $H \perp \overline{DK}$ → E, D, k are coplane.

⑤: $S \perp \overline{EH}$. Generally, $E \nparallel D$, ∴ $S \nparallel K$.

→ S, E, k are coplane.



c)