Modeling Memorize

① Abbe sine condition

2021-5

Sine-Condition:

(a) What is the (Abbe) Sine-condition and what does it mean? (2P)

(b) What is the shape of the principal surfaces if the sine-condition is fulfilled? (1P)

(c) How are systems called fulfilling the sine-condition? (1P)

a) Lateral system magn: $m = \frac{\rho_i}{\rho_o} = \frac{\rho^{FS}}{\rho^{FOV}} > 0$

Lateral magn: $\frac{y_i}{y_o} = m$

Spot magn: $\frac{\rho_i^{spot}}{\rho_o^{spot}} = \underbrace{\frac{NA_o}{NA_i} = \frac{n_o \sin u_o}{n_i \sin u_i}}$

ASC: $\quad \| \quad m \Rightarrow \quad = m$

$m_p = \frac{\rho^{xp}}{\rho^{EP}}$, $\sin u_o = -\frac{\rho^{EP}}{\check{s}_o}$, $\sin u_i = -\frac{\rho^{xp}}{\check{s}_i} \Rightarrow m = \frac{1}{m_p} \frac{n_o}{n_i} \frac{\check{s}_i}{\check{s}_o}$

$\tan w_o = \frac{y_o}{s_o}$, $\tan w_i = \frac{y_i}{s_i} \Rightarrow m = \frac{s_i \tan w_i}{s_o \tan w_o} = \frac{\check{s}_i}{\check{s}_o} \frac{n_o}{n_i} \frac{1}{m_p}$

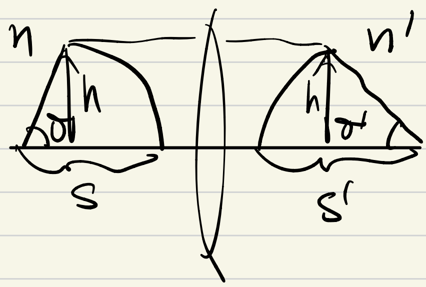
$\Rightarrow \frac{\check{s}_i}{\check{s}_o} = m m_p \frac{n_i}{n_o} \xrightarrow[\text{approx}]{\text{parax}} (\check{s} = s) \quad m = \frac{s_i}{s_o} \frac{n_o}{n_i} \frac{1}{m_p}$

$$\boxed{\text{when } n_o = n_i \Rightarrow m = \frac{\sin u_o}{\sin u_i} = \frac{\sin w_o}{\sin w_i}}$$

fullfill to produce sharp images.

b) Sphere

c) $\beta = \frac{y'}{y} = \frac{n \sin \sigma}{n' \sin \sigma'} \stackrel{?}{=} m = \frac{n \cdot s'}{n' \cdot s}$

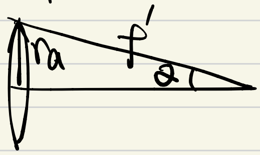


② Resolution Spot

2022-4 What limits the spot size of a beam focused by a lens in case the lens has no aberrations? How can this spot size be calculated? (2P)

$$\left.\begin{aligned} NA_o &= 0.61\frac{\lambda}{\rho_o^{spot}}, \quad \rho_o^{spot} = \frac{a}{2\pi}\frac{\lambda}{NA_o} \\ NA_i &= 0.61\frac{\lambda}{\rho_o^{spot}}\cdot\frac{1}{m}, \quad \rho_i^{spot} = \frac{a}{2\pi}\frac{\lambda}{NA_i} \end{aligned}\right\} RS$$

Diffraction limited spot size



$$\sin\alpha = \frac{r_a}{f'} \qquad \lambda = \frac{\lambda_0}{n}$$

$$r_0 = 0.61\frac{\lambda_0}{n\sin\alpha} = 0.61\frac{\lambda_0}{NA}$$

③ Lens Equation

Focal length :

$$f_i = s_i\frac{m_p}{m_p - m} \xrightarrow{ASC} \begin{cases} \dfrac{1}{f_i} = \dfrac{1}{s_i} - \dfrac{\check{s}_i}{s_i\check{s}_o}\dfrac{n_o}{n_i}\dfrac{1}{m_p} & \text{(Img side)} \\ \dfrac{1}{f_o} = \dfrac{1}{s_o} - \dfrac{\check{s}_o}{\check{s}_i s_o}\dfrac{n_i}{n_o}m_p^2 & \text{(Obj side)} \end{cases}$$

$$\xrightarrow[\text{approx}]{\text{paraxial}} \begin{matrix} s=\check{s} \\ f=\check{f} \end{matrix} \quad \begin{cases} \dfrac{1}{f_i} = \dfrac{1}{s_i} - \dfrac{1}{s_o}\dfrac{n_o}{n_i} \\ \dfrac{1}{f_o} = \dfrac{1}{s_o} - \dfrac{1}{s_i}\dfrac{n_i}{n_o}\cdot m_p^2 \end{cases} \Rightarrow \frac{f_i}{f_o} = -\frac{s_i}{s_o}\frac{m_p}{m}$$

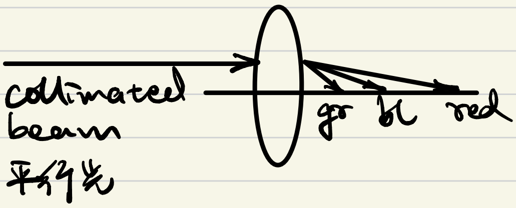
$$F/\# = -n_i\frac{f_i}{\check{f}_i}\frac{1}{2NA_i} \xrightarrow[\text{approx}]{\text{para}} F/\# = -\frac{n_i}{2NA_i}$$

Lens maker eq.

thin — $\dfrac{1}{f_i} = \dfrac{n'-n}{n}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$, $n=1$

(d→0) $\quad = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

④ Chromatic Aberration



failure of a lens to focus all colors to the same point.

caused by dispersion: the refractive index of the lens elements varies with the wavelength of light.

Correction

by using an achromatic doublet, with elements made of crown and flint glass.

$$\frac{\phi_1}{v_1}+\frac{\phi_2}{v_2}=0$$ (explain in ⑦)

⑤ ABCD Matrix

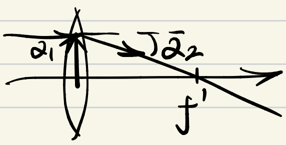
a) Free-space

$x_2 = x_1 + \Delta z \cdot \tan\alpha_1$

$\alpha_2 = \alpha_1$

$$\begin{pmatrix} 1 & \Delta z \\ 0 & 1 \end{pmatrix}$$

b) Thin Lens



$r_2 = r_1$

$\alpha_2 = -\frac{r_1}{f'} + \alpha_1$

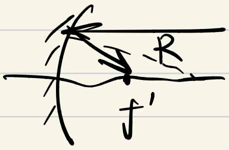
$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f'} & 1 \end{pmatrix}$$

c) Sphere interface

$$\begin{pmatrix} 1 & 0 \\ \frac{n_1-n_2}{n_2 \cdot R} & \frac{n_1}{n_2} \end{pmatrix}$$

$R \to \infty$ $$\begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix}$$

d) Curved mirror



$R = 2f$

$$\begin{pmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{pmatrix}$$

⑥ Lens Phase

$$\vec{E}(x_1, y_1, z_1) = \vec{E}_r(x_1, y_1, z_1) \cdot \underbrace{e^{-i\frac{k(x_1^2+y_1^2)}{2f'}}}_{\text{lens phase}} \qquad f' = oz$$

↑ on input plane ↑ on ref-sphere

⑦ Abbe Number

definition: measure of dispersion, dispersion ↑, ν ↓

$$\nu = \frac{n_\lambda - 1}{n_{\lambda_1} - n_{\lambda_2}}, \quad \nu_e = \frac{n_e - 1}{n_{F'} - n_{C'}}$$

(空气中)

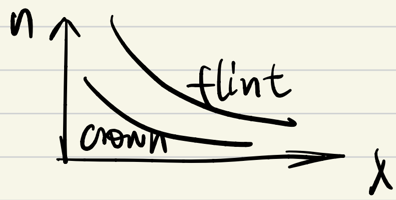
lens power: $\phi = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \nu(n_{F'} - n_{C'})\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

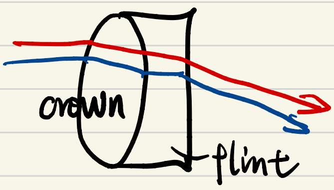
镜片

For achromatization: 消色差

$$\frac{\phi_1}{\nu_1} + \frac{\phi_2}{\nu_2} = 0 \Rightarrow \phi_1 = \frac{\nu_1 \cdot \phi_{total}}{\nu_1 - \nu_2} \rightarrow \text{Crown glass for pos. lens}$$

$$\phi_2 = \frac{-\nu_2 \cdot \phi_{total}}{\nu_1 - \nu_2} \rightarrow \text{Flint glass for neg. lens}$$





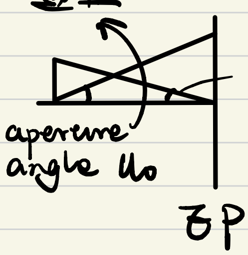
⑧ Aperture stop: limits the cone angle of the spherical field.

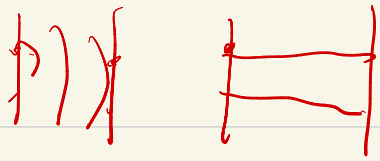
Entrance pupil: image of the aperture stop to the obj. side.

Chief ray: goes through center of EP.

Coma ray: off-axis field point ⟶ edge of EP

Marginal ray: obj. center ⟶ edge of EP.





⑨ SPW、Spectrum of plane wave propagation:

Wave General situation: each value in input contributes to all values in output.
→ Pointwise : pointwise modeling of components can be rigorous (k domain) 频域计算
从时间到空间转换 or a good approximation (x domain). 空域近似、
When NA gets larger, the wavefront phase gets strong and the integral operator can be approximated by a pointwise operation.

⑩ Wave optics. (connection)

Ray optics is represented by position $\vec{r}$ and direction $\vec{s}$, while physical optics by $\vec{E}$ and $\vec{H}$.

We identify a subject in physical optics to generalize the concept of ray optics to EM fields by dealing with geometrical laws relating to the propagation of the 'amplitude vector' $\vec{E}$ and $\vec{H}$.

⑪ TEA. Thin element approximation. ?

Neglect refraction at interface. OPL calculation only.

Application : at any position of input & output plane

$$\Delta\varphi^{TEA}(\rho) = \Delta\varphi^{in} + \Delta\varphi^{out} = k_0 h(\rho)(n^{in} - n^{out}) + \Delta\varphi_0$$

Assumption: $\Delta\varphi(\rho) \propto h(\rho)$ height profile $\qquad \Delta\varphi_0 = k_0 \Delta h\, n^{out}$

High accuracy for smooth surface.
-the paraxial approximation is valid for thin surface profile only.

⑫ Wavefront phase

$$s_\perp(\vec{\rho}) = \frac{1}{k_0 n} \nabla_\perp \cdot \psi(\vec{\rho}), \quad s_z = \sqrt{1 - s_x^2 - s_y^2}$$

planar wf: $\psi(\vec{r}) = k_0 n \hat{s} \cdot \vec{r}$, $\hat{s}_\perp(\vec{\rho}) = (s_x, s_y)$

spherical wf: $\psi(\vec{r}) = \mathrm{sign}(z)\sqrt{x^2+y^2+z^2}$, $\hat{s}_\perp^{wf}(\vec{\rho}) = \mathrm{sign}(z)\frac{(x,y)}{r}$, $\hat{s}^{spherical}(\vec{r}) = \mathrm{sign}(z)\hat{r}$

Lens system in paraxial approximation
the quadratic wavefront response of a paraxial ideal lens:

pl wf : $\psi^{lens}(\rho) = \frac{1}{2} k_0 \frac{n_0}{f_0} \|\rho\|^2$

sph wf : $\psi^{lens}(\rho) = -\frac{1}{2} k_0 \frac{n_i}{f_i} \|\rho\|^2$

⑬ Maxwell EQ

a) T domain MEQ

$$\begin{cases}\nabla\times\vec{E}(\vec{r},t) = -\frac{\partial}{\partial t}\vec{B}(\vec{r},t)\\ \nabla\times\vec{H}(\vec{r},t) = \vec{j}(\vec{r},t) + \frac{\partial}{\partial t}\vec{D}(\vec{r},t)\\ \nabla\cdot\vec{D}(\vec{r},t) = \rho(\vec{r},t)\\ \nabla\cdot\vec{B}(\vec{r},t) = 0\end{cases}$$

$\vec{E}$ electric field
$\vec{H}$ magnetic field
$\vec{D}$ elec~ displacement
$\vec{B}$ magn~ induction

$[\bar{E}] = Vm^{-1}$
$[E] = Vm^{-1}s$

单位 in F/T

b) F domain MEQ

$$\begin{cases}\nabla\times\vec{E}(\vec{r},\omega) = i\omega\vec{B}(\vec{r},\omega)\\ \nabla\times\vec{H}(\vec{r},\omega) = j(\vec{r},\omega) - i\omega\vec{D}(\vec{r},\omega)\\ \nabla\cdot\vec{D}(\vec{r},\omega) = \rho(\vec{r},\omega)\\ \nabla\cdot\vec{B}(\vec{r},\omega) = 0\end{cases}$$

c) Linear matter eq.

$$\begin{cases}\vec{j}(\vec{r},\omega) = \sigma(\vec{r},\omega)\vec{E}(\vec{r},\omega)\\ \vec{D}(\vec{r},\omega) = \varepsilon(\vec{r},\omega)\vec{E} = \varepsilon_0\varepsilon_r(\vec{r},\omega)\vec{E}\\ \vec{B}(\vec{r},\omega) = \mu(\vec{r},\omega)\vec{H}\end{cases}$$

$\vec{j}$ current density
$\vec{\sigma}$ elec~ conductivity
$\vec{\varepsilon}$ elec~ permittivity
$\vec{\mu}$ magn~ permittivity

d) Restrictions:

1) $\rho = 0$ 各向同性
2) Isotropic media: matter quantities 're scaler functions instead of tensors.
3) $\vec{\mu}(\vec{r},\omega) = \mu_0$ 均匀介质
4) Homogeneous media: ~ 're not dependent of the location $\vec{r}$.
5) Non-dispersive: ~'re not dependent of the frequency ω.

$$\Rightarrow\begin{cases}\nabla\times\vec{E}(\vec{r},\omega) = i\omega\mu_0\vec{H}(\vec{r},\omega)\\ \nabla\times\vec{H}(\vec{r},\omega) = -i\omega\varepsilon_0\check{\varepsilon}_r(\vec{r},\omega)\vec{E}(\vec{r},\omega)\\ \nabla\cdot(\varepsilon_r(\vec{r},\omega)\vec{E}(\vec{r},\omega)) = 0\\ \nabla\cdot\vec{H}(\vec{r},\omega) = 0\end{cases}$$

generalised permittivity: $\check{\varepsilon}_r(\omega)$

e) In homogeneous media

$$\begin{cases}\nabla\times\vec{E}(\vec{r},\omega) = i\omega\mu_0\vec{H}(\vec{r},\omega)\\ \nabla\times\vec{H}(\vec{r},\omega) = -i\omega\varepsilon_0\check{\varepsilon}_r(\omega)\vec{E}(\vec{r},\omega)\\ \nabla\cdot\vec{E}(\vec{r},\omega) = 0\\ \nabla\cdot\vec{H}(\vec{r},\omega) = 0\end{cases}$$

$\nabla\cdot\vec{E}(\vec{r},\omega) = 0 \longrightarrow$ only two components can be selected independently (E_x, E_y)

f) Wave number in vacuum

$$k_0 = \frac{\omega}{c} = \frac{2\pi\nu}{c}$$

$$\nu = \frac{\omega}{2\pi} \qquad \lambda_0 = \frac{2\pi}{k_0} = \frac{c}{\nu}$$

⑭ Aberration

<Perfect> All rays coming from one object point intersect in one image point.

<Real> 1) transverse ~ in the image plane
2) longitudinal ~ from the image plane } geom. representation of wave aberr
3) wave ~ in the exit pupil

$$\Delta W = \underbrace{W_{000}}_{\text{Piston Error}} + \underbrace{W_{200}\beta^2}_{\text{P.E}} + \underbrace{W_{020}\rho^2}_{\text{Defocus}} + \underbrace{W_{111}\beta\rho\cos\theta}_{\text{Lateral Magnification Error}} + \underbrace{W_{400}\beta^4}_{\text{P.E.}} + \underbrace{W_{040}\rho^4}_{\text{Spherical aberration}}$$

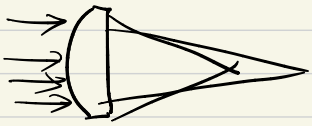
$$+ \underbrace{W_{131}\beta\rho^3\cos\theta}_{\text{Coma 慧差}} + \underbrace{W_{222}\beta^2\rho^2\cos^2\theta}_{\text{Astigmatism 散光}} + \underbrace{\underbrace{W_{220}\beta^2\rho^2}_{\text{Field Curvature 像场弯曲}} + \underbrace{W_{311}\beta^3\rho\cos\theta}_{\text{Distortion 畸变}}}_{\text{strong dependence on field}}$$

a) Spherical Aberr / $W_{040}\rho^4$ / pupil size

Problem:
image spot is blurred for spherical surface.

Correction:
①using aspherical surface which has a form of a conic section.

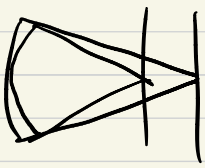
Reason: ②③lens bending / splitting ④high refractive index



outer rays strongly refracted → focused earlier
inner ~ weakly ~ → ~ further away

b) Defocus / $W_{020}\rho^2$ / pupil size

Problem:



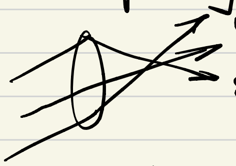
longitudinal shift

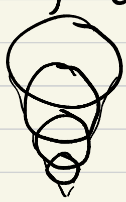
change curvature

c) Coma / $W_{131}\beta\rho^3\cos\theta$ / pupil & field

Problem:
off-axis image points show 'tail of comet'.

non-symmetry of bundle around chief ray





correction — more stop ?

d) Astigmatism / $W_{222}\rho^2 r^2\cos^2\theta$ / pupil & field

e) Field curvature / $W_{220}\rho^2 r^2$ / pupil & field

f) Distortion / $W_{311}\rho^3 r\cos\theta$ / field
Problem:
sharp everywhere
have sharp but bended edges
no distortion along central lines

Reason: differcne transversal maganifications.

15 Paraxial: analytical solution via ABCD matrices.
Nonparaxial: iterative solution by sequence of surface designs
or parametric optimization.