a) $\nabla\cdot\vec{D}(\vec{r},\omega)=0$ $\quad \nabla\times\vec{E}(\vec{r},\omega)=i\omega\vec{B}(\vec{r},\omega)=i\omega\mu_0\vec{H}(\vec{r},\omega)$ $\quad \vec{B}=\mu_0\vec{H}+\vec{M}$

$\nabla\cdot\vec{H}(\vec{r},\omega)=0$ $\quad \nabla\times\vec{H}(\vec{r},\omega)=-i\omega\vec{D}(\vec{r},\omega)$

$\nabla\cdot\vec{D}(\vec{r},\omega)=\nabla\cdot[\varepsilon_0\varepsilon(\vec{r},\omega)\vec{E}(\vec{r},\omega)]=\varepsilon_0\varepsilon(\vec{r},\omega)\underline{\nabla\cdot\vec{E}(\vec{r},\omega)}+\varepsilon_0\vec{E}(\vec{r},\omega)\nabla\cdot\varepsilon(\vec{r},\omega)$

? $=0$

$$\nabla\cdot\vec{E}(\vec{r},\omega)=-\frac{\mathrm{grad}\,\varepsilon(\vec{r},\omega)}{\varepsilon(\vec{r},\omega)}\vec{E}(\vec{r},\omega)\leftarrow$$

b) $\nabla\times\nabla\times\vec{E}(\vec{r},\omega)=i\omega\mu_0\nabla\times\vec{H}(\vec{r},\omega)=\omega^2\mu_0\varepsilon_0\varepsilon(\vec{r},\omega)\vec{E}(\vec{r},\omega)$

$=\nabla\nabla\cdot\vec{E}(\vec{r},\omega)-\nabla^2\vec{E}(\vec{r},\omega)$ $\quad =\frac{\omega^2}{c^2}\varepsilon(\vec{r},\omega)\vec{E}(\vec{r},\omega)$

$$\nabla^2\vec{E}(\vec{r},\omega)+\frac{\omega^2}{c^2}\varepsilon(\vec{r},\omega)\vec{E}(\vec{r},\omega)=-\nabla\left[\frac{\nabla\varepsilon(\vec{r},\omega)}{\varepsilon(\vec{r},\omega)}\vec{E}(\vec{r},\omega)\right]$$

c) non-magnetic without source

$\nabla\cdot\vec{E}(\vec{r},\omega)$

$$\nabla^2\vec{E}(\vec{r},\omega)+\frac{\omega^2}{c^2}\varepsilon(\omega)E(\vec{r},\omega)=0$$

the physical consequence for light propagation of $\nabla\varepsilon(\vec{r},\omega)\neq0$

All field components couple

2. a) homogeneous waves $\quad k'//k''$

evanescent waves $\quad k'\perp k''$

$\alpha.\beta\neq0$ $\quad \alpha^2+\beta^2>k^2$

b) $-\mu_0\frac{\partial}{\partial t}\vec{H}(\vec{r},t)=\nabla\times\vec{E}(\vec{r},t)=\begin{bmatrix}\hat{x}&\hat{y}&\hat{z}\\\partial x&\partial y&\partial z\\E_x&0&0\end{bmatrix}=\frac{\partial E_x}{\partial z}\hat{y}-\frac{\partial E_x}{\partial y}\hat{z}$

$$-\mu_0\frac{\partial}{\partial t}\vec{H}(\vec{r},t)=\begin{bmatrix}0\\i\beta E_0\exp(-\alpha y+i\beta z)\exp(-i\omega t)\\\alpha E_0\exp(-\alpha y+i\beta z)\exp(-i\omega t)\end{bmatrix}$$

$$\vec{H}(\vec{r},t)=\begin{bmatrix}0\\-\frac{\beta}{\mu_0\omega}E_0\exp(-\alpha y+i\beta z)\exp(-i\omega t)\\\frac{i\alpha}{\mu_0\omega}E_0\exp(-\alpha y+i\beta z)\exp(-i\omega t)\end{bmatrix}\times$$

c) $\langle\vec{S}_r(\vec{r},t)\rangle=\frac{1}{2}R[\vec{E}(\vec{r},t)\times\vec{H}^*(\vec{r},t)]$ $\quad \begin{bmatrix}x&y&z\\E_x&0&0\\0&H_y&\end{bmatrix}$

$\langle\vec{S}_r(\vec{r},t)\rangle=-\frac{1}{2}E_0\exp(-\alpha y)\frac{\beta}{\omega\mu_0}E_0\,e(-\alpha y)$

$=-\frac{\beta}{2\omega\mu_0}E_0^2\exp(-2\alpha y)\,\hat{e}_z$ $\quad \hat{z}E_xH_y\ dt$

d) $\vec{E}_r(\vec{r},t) = \frac{1}{2}[\vec{E}_c(\vec{r})\, e^{-i\omega t} + \vec{E}_c^*(\vec{r})\, e^{+i\omega t}]$

$= \frac{1}{2}\{ E_0 \exp(-\alpha y + i\beta z) e^{-i\omega t} + E_0 \exp(-\alpha y - i\beta z) e^{+i\omega t} \}$

$= \frac{1}{2} E_0 \exp(-2\alpha y) \cos(\beta z - \omega t)\, \vec{e}_x$

$\vec{H}_r(\vec{r},t) = \frac{1}{2} \begin{cases} 0 \\ \frac{\beta E_0}{2\omega\mu_0} \exp(2\alpha y) \cos(\beta z - \omega t) \\ \frac{i\alpha E_0}{\omega} \exp(-\alpha y + i\beta z) e^{-i\omega t} + -\frac{i\alpha E_0}{2\omega} \exp(-\alpha y - i\beta z + i\omega t) \end{cases}$

3.

a) $H(\alpha,\beta,z) = e^{ikz} = e^{i\sqrt{(\frac{2\pi}{\lambda})^2 - \alpha^2 - \beta^2}\, z}$ $\quad k = \frac{2\pi}{\lambda}$

$h(x,y,z) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} H(\alpha,\beta;z) \exp[i(\alpha x + \beta y)]\, d\alpha\, d\beta$

$U_0(\alpha,\beta) = \left(\frac{1}{2\pi}\right)^2 \iint_{-\infty}^{\infty} u_0(x,y) \exp[-i(\alpha x + \beta y)]\, dx\, dy$

$u_F(x,y,z) = \iint_{-\infty}^{\infty} H(\alpha,\beta,z) U_0(\alpha,\beta)\, e^{i\alpha x + i\beta y}\, d\alpha\, d\beta$

$= \iint_{-\infty}^{+\infty} h_F(x-x', y-y')\, u_0(x',y')\, dx' dy'$

b) $H(\alpha,\beta,z) = e^{i r(\alpha,\beta) z}$ $\quad r(\alpha,\beta) = \sqrt{k^2 - \alpha^2 - \beta^2}$

$= k^2 \sqrt{1 - \frac{\alpha^2+\beta^2}{k^2}} \approx k\left(1 - \frac{\alpha^2+\beta^2}{2k^2}\right)$

$= k - \frac{\alpha^2+\beta^2}{2k}$

$H_F = e^{ikz}\, e^{-i\frac{\alpha^2+\beta^2}{2k} z}$

c) $10 > N_F > 0.1$ $\quad N_F = \frac{a^2}{\lambda z}$ $\quad 2b >> \sqrt{2}a$

Fresnel $\quad N_F = \frac{(2b+\sqrt{2}a)^2}{\lambda z_B} \ll 1$ or < 0.1 $\quad a, b >> \lambda$

$N_F < \cancel{0.1}\ 1$ $\quad z_B > \frac{(2b+\sqrt{2}a)^2}{\lambda}$

Fraunhofer approximation

$U_0(x, z=0) = \begin{cases} 1 \\ 0 & \text{else} \end{cases}$

$u(x, z_B) \propto U_0\left(\frac{kx}{z_B}\right)$ $\quad \alpha = \frac{kx}{z_B}$

$$FT[U_0(x,z=0)](\alpha) \simeq \int_{-\infty}^{\infty} U_0(x,z=0)\, e^{-i\alpha x}\, dx$$

$$= \int_{-b-\frac{\sqrt{2}}{2}a}^{-b+\frac{\sqrt{2}}{2}a} e^{-i\alpha x}\, dx + \int_{b-\frac{\sqrt{2}}{2}a}^{b+\frac{\sqrt{2}}{2}a} e^{-i\alpha x}\, dx$$

$-b-\frac{\sqrt{2}}{2}a < x < -b+\frac{\sqrt{2}}{2}a$

$b-\frac{\sqrt{2}}{2}a < x < b+\frac{\sqrt{2}}{2}a$

$$= \frac{i}{\alpha}\left[e^{-i\alpha x}\Big|_{-b-\frac{\sqrt{2}}{2}a}^{-b+\frac{\sqrt{2}}{2}a} + e^{-i\alpha x}\Big|_{b-\frac{\sqrt{2}}{2}a}^{b+\frac{\sqrt{2}}{2}a} \right] \Rightarrow \frac{i}{\alpha}\left(e^{i\alpha b} + e^{-i\alpha b}\right)\left(-e^{-\frac{\sqrt{2}}{2}a i} + e^{\frac{\sqrt{2}}{2}a i}\right)$$

$$= \frac{i}{\alpha} 2\cos(\alpha b)\left[-2i\sin\left(\alpha \frac{\sqrt{2}}{2} a\right)\right]$$

$$= 4\sqrt{2}\, a \cos(\alpha b)\ \mathrm{sinc}\left(\frac{\sqrt{2}}{2}\alpha a\right)$$

$$I(x, z_B) \simeq |u(x, z_B)|^2 \propto \mathrm{sinc}^2\left(\frac{\sqrt{2}}{2}\frac{kx}{z_B}a\right)\cos\left(\frac{kx}{z_B}b\right)$$

4. $$k(w) = k_0 + \frac{\partial k}{\partial w}\Big|_{w_0}(w-w_0) + \frac{1}{2}\frac{\partial^2 k(w)}{\partial w^2}\Big|_{w_0}(w-w_0)^2$$

only a narrow frequency range of the excited pulse.

$v_{ph} = \frac{w_0}{k}$ the velocity of the phasefronts for light at the central frequency $w = w_0$

$v_g = \frac{1}{\frac{\partial k}{\partial w}\big|_{w_0}}$ the velocity of the center of the pulse with central frequency at $w = w_0$

$D<0 \quad \frac{\partial^2 k}{\partial w^2}\Big|_{w_0} < 0$

$D>0 \quad \frac{\partial k_g}{\partial w}\Big|_{w_0} < 0$

red faster

$D<0 \quad \frac{\partial k_g}{\partial w}\Big|_{w_0} > 0$

blue faster

$$U_{FR}(x,y,z) = \frac{-i}{\lambda z}(2\pi)^2 \exp(ikz)\, U_0\left(\frac{kx}{z}, \frac{ky}{z}\right) \exp\left[\frac{ik}{2z}(x^2+y^2)\right]$$

5. a) $q_0 = -iz_0 \qquad z_0 = \frac{\pi w_0^2}{\lambda}$

b) $q_1 = q_0 = -iz_0 \qquad \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$

$$q_1' = \frac{Aq_1+B}{Cq_0+D} = \frac{-iz_0}{-\frac{z_0 i}{f}+1} = \frac{-iz_0 f}{f - z_0 i} = \frac{-iz_0 f(f+iz_0)}{f^2+z_0^2}$$

$$= \frac{-iz_0 f^2 - z_0^2 f}{f^2+z_0^2}$$

$$q_1 = q_1' + d = \frac{-iz_0 f^2 - z_0^2 f + df^2 + dz_0^2}{f^2+z_0^2} = -iz_1$$

$$d = \frac{fz_0^2}{f^2+z_0^2} \qquad z_1 = \frac{f^2 z_0}{f^2+z_0^2} \qquad w_1 = \sqrt{\frac{\lambda}{\pi} z_1}$$

if $z_0 \gg f$

$$z_0^2 + f^2 \approx z_0^2 \qquad z_1 = \frac{f^2}{z_0} \qquad w_1 = f\sqrt{\frac{\lambda}{\pi z_0}}$$

c. $q_g = q' + g \qquad$ let $d' - g = L$

$$q_L = \frac{q_g}{\frac{1}{n}} = n q_g = (q'+g)n = -iz_1'$$

$$g = \frac{1}{n}\frac{fz_0^2}{f^2+z_0^2} \qquad z_1' = \frac{f^2 z_0 n}{f^2+z_0^2} \qquad w_1' = f\sqrt{\frac{\lambda z_0 n}{\pi(f^2+z_0^2)}}$$

$d' = g$

$z_0 \gg f$

$$S = d - d' = f - \frac{f}{n} = \frac{(n-1)f}{n}$$

6. $u_1(x) = f\{u_0(x), H(x)\}$

1) $u^-(x) = -\frac{i2\pi}{\lambda d} \exp(2ikd)\, U_0(kx/d_1)$

$U_0(\alpha)$

2) $u^+(x) = u^-(x)\, p(x) = -i\frac{2\pi}{\lambda f_1} \exp(2ikd)\, U_0(kx/f_1)\, p(x)$

3) $G(\alpha) = -\alpha^2 F(\alpha)$ $\quad F(\alpha) = U_0$ $\quad g(x) = \frac{d^2 f}{dx^2}$ $\quad \alpha = \frac{kx}{d}$

4) $u(x, y, 4f) = -\frac{(2\pi)^2}{\lambda^2 d^2} \exp(2ikd)\, U_+\left(\frac{k}{d}x, \frac{k}{d}y\right)$

$U_0(kx/d) -$

$\alpha = \frac{k}{d}$ $\quad \alpha = \frac{k^2 x}{d^2}$

(5)

$-k$

$\frac{\pi\theta}{2\lambda}$ P119 $\quad G(\alpha) = -\frac{k^2}{d^2} F(\alpha)$

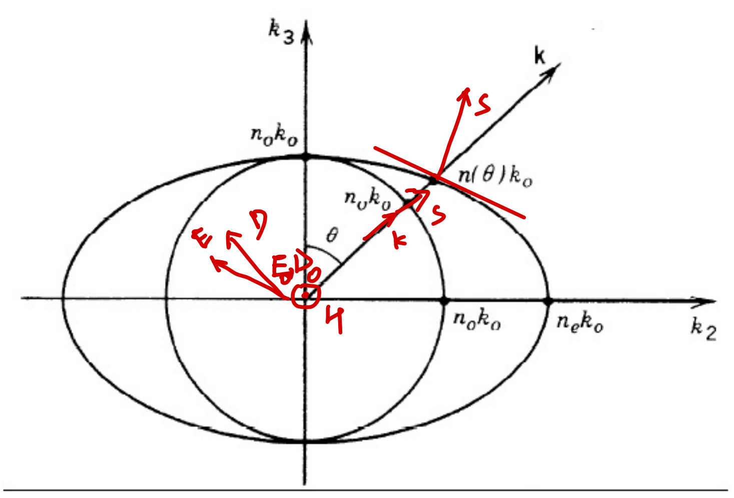
7. a) 1, 2.

b) ordinary: $\vec{D}$ perpendicular to optical axis and $\vec{k}$, $\vec{D} \perp \vec{k}$, $\vec{D} // \vec{E}$

extraordinary: $\vec{D}$ perpendicular to $\vec{k}$ and in the plane spanned by $\vec{k}$ and the optic axis.

$\vec{D} \perp \vec{k}$, $\vec{D} \not\parallel \vec{E}$ because $\vec{D_2} = \varepsilon_0 \varepsilon_{or} \vec{E_2}$ $\quad \vec{D_3} = \varepsilon_0 \varepsilon_e \vec{E_3}$

c)



Normal surfaces for a uniaxial crystal.

d) $\sum \frac{u_i^2}{n^2-\varepsilon_i} = \frac{1}{n^2}$ $\quad (1,1,1) \longrightarrow \left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

$n^2 \big(u_1(n^2-\varepsilon_2)(n^2-\varepsilon_3) + u_2(n^2-\varepsilon_1)(n^2-\varepsilon_3)$

$+ u_3(n^2-\varepsilon_1)(n^2-\varepsilon_2) \big) = (n^2-\varepsilon_1)(n^2-\varepsilon_2)(n^2-\varepsilon_3)$

$n^2 \big(\frac{\sqrt{3}}{3}(n^2-\sqrt{2})(n^2-\sqrt{3}) + \frac{\sqrt{3}}{3}(n^2-1)(n^2-\sqrt{3})$

$+ \frac{\sqrt{3}}{3}(n^2-1)(n^2-\sqrt{2}) \big) n^2 = (n^2-1)(n^2-\sqrt{2})(n^2-\sqrt{3})$

$\frac{\sqrt{3}}{3}(bc + ca + ab) n^2 = abc$

8. $E_{t1} = E_{t2}$ $\qquad \frac{E_{n1}}{E_{n2}} = \frac{\sqrt{n_2}}{\sqrt{n_1}}$

$E_{n1}\varepsilon_1 = E_{n2}\varepsilon_2$

$k_z \qquad \tan\varphi_t = \frac{E_{n1}}{E_{n2}} \qquad \frac{\tan\varphi_t}{\tan\varphi_i} = \frac{E_{n1}}{E_{n2}} = \frac{\sqrt{n_2}}{\sqrt{n_1}}$