Introduction to Optical Modeling

Friedrich-Schiller-University Jena
Institute of Applied Physics

Lecture 4

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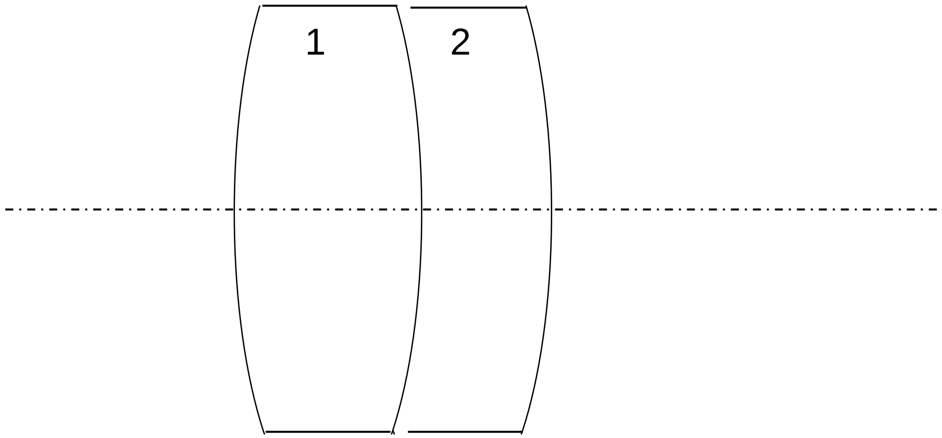
Achromatic Doublet

Example: desired focal length: $f' = 200\text{mm}$



Lens 1: BK7 $n_e = 1.518$
$\nu_e = 63.9$

Lens 2: SF6 $n_e = 1.805$
$\nu_e = 25.4$

symmetric lens

$\rightarrow$ $f'_1 = 120.5\text{mm}$ $\rightarrow$ $R_1 = 124.8\text{mm}$
$R_2 = -124.8\text{mm}$

$f'_2 = -303.1\text{mm}$ $\rightarrow$ $R_1 = -124.8\text{mm}$
$R_2 = -255.5\text{mm}$

Course Overview

Part 1: Geometrical optics-based modeling and design (U.D. Zeitner)

1. Introduction
2. Paraxial approximation / Gaussian optics
3. ABCD-matrix formalism
4. Real lenses
5. Optical materials
   - glass types, dispersion
   - chromatic aberrations
6. **Imaging systems**
   - **apertures/stops, entrance-/exit-pupil**
   - wavefront aberrations

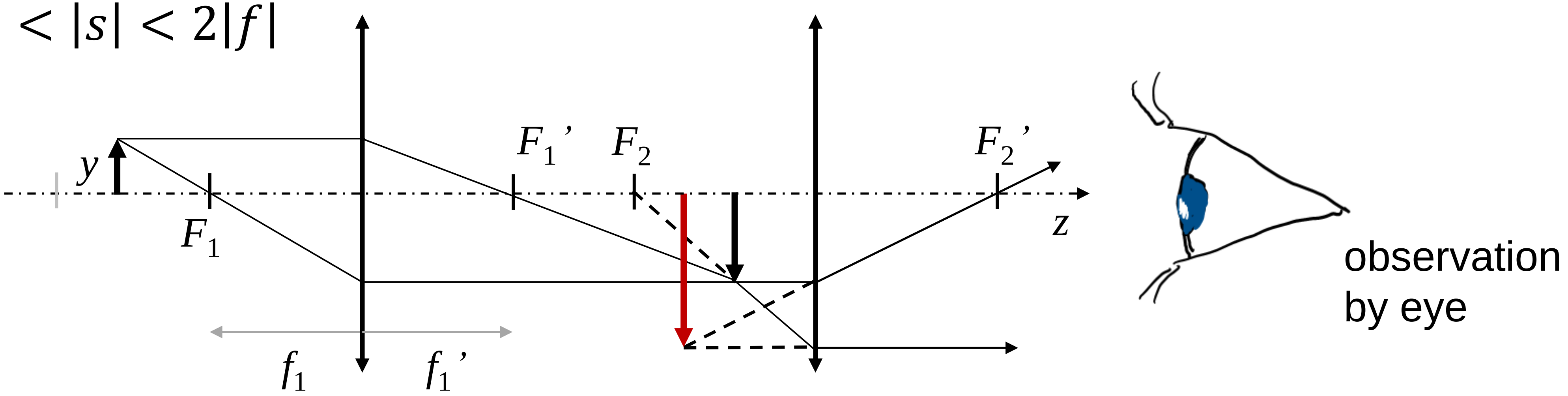
Part 2: Wave-optics based modeling (F. Wyrowski)

2.6 Imaging Systems

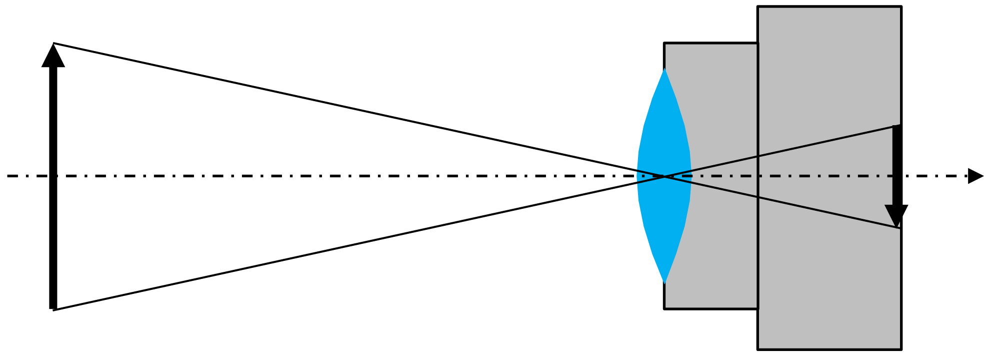
Examples

a) Microscope → generation of a magnified image

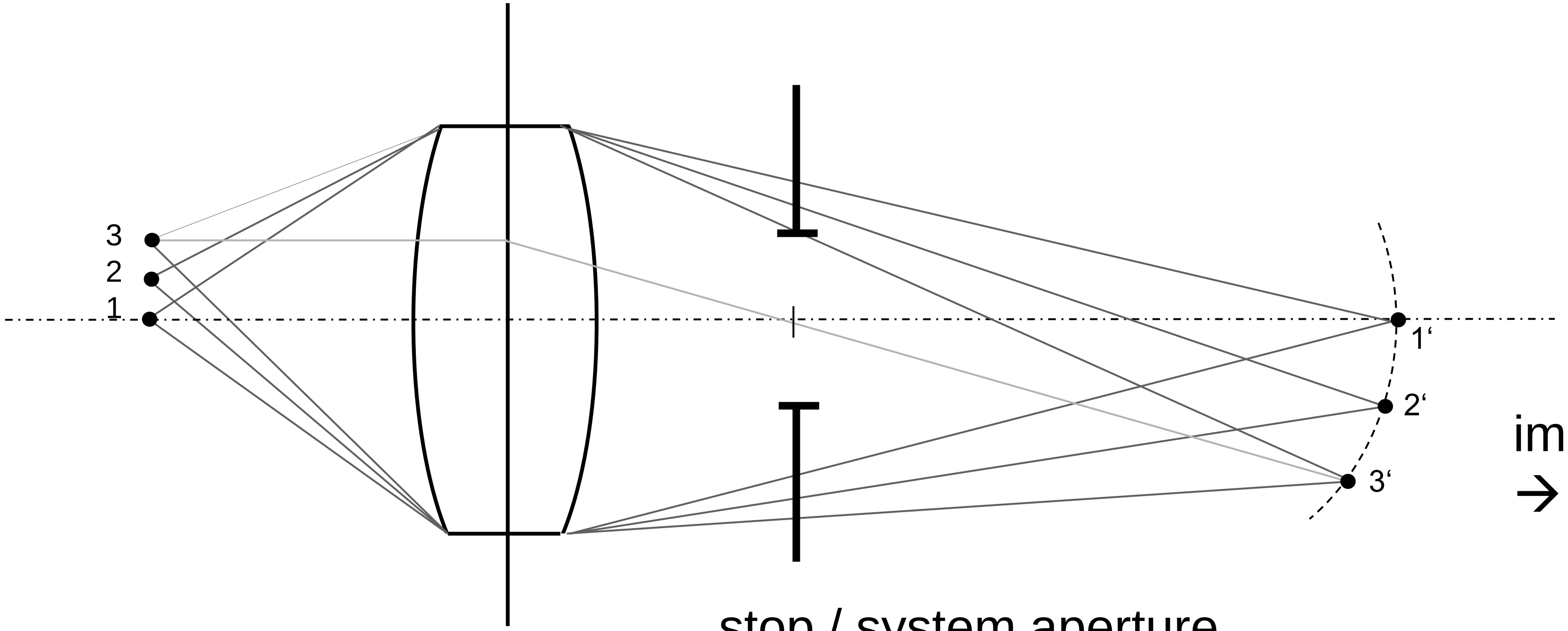
$$|f| < |s| < 2|f|$$



b) Camera → generation of a de-magnified image



Real imaging, extended field



stop / system aperture

image points not on a plane
→ field curvature

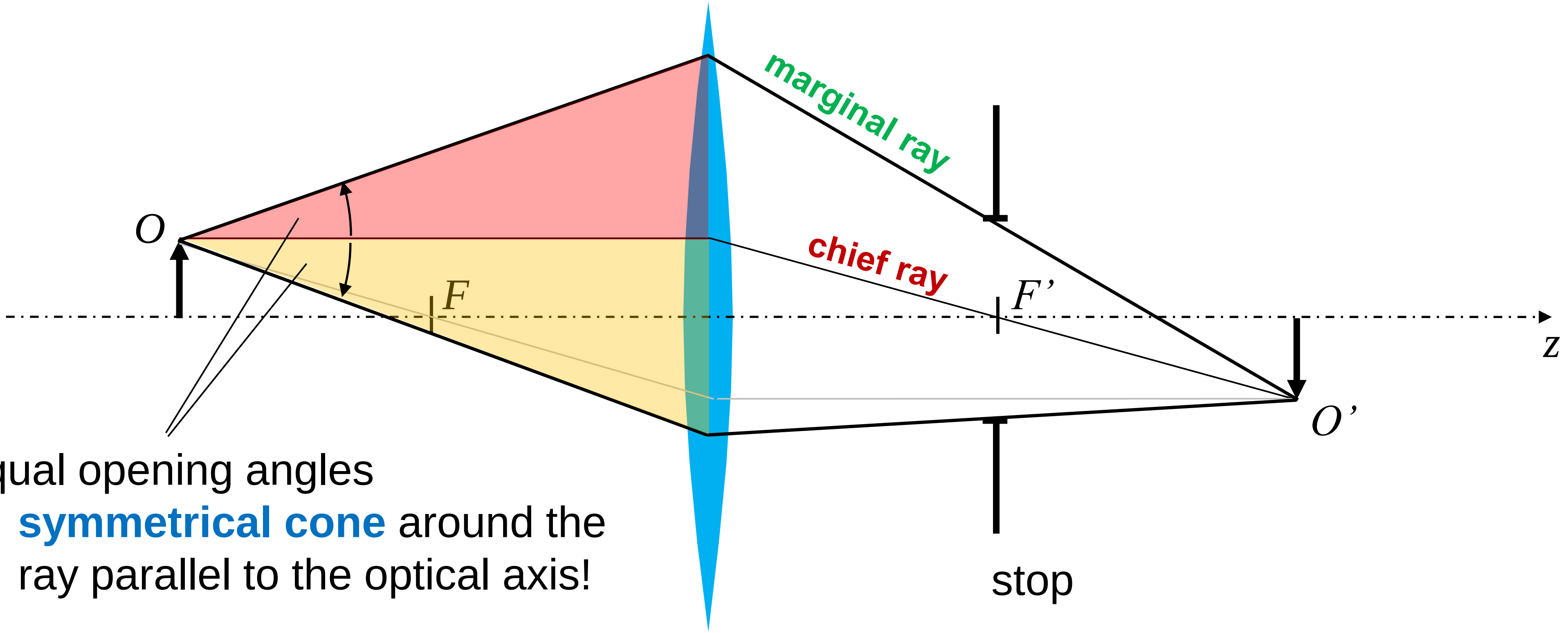
Important:

ray cones are always limited by the finite extend of an aperture
→ often one of the lenses in the system
→ can also be a separate diaphragm / aperture

} **stop** of the system

stop position has a substantial impact on the image characteristics

Special example: stop in the back focal plane



equal opening angles
→ **symmetrical cone** around the ray parallel to the optical axis!

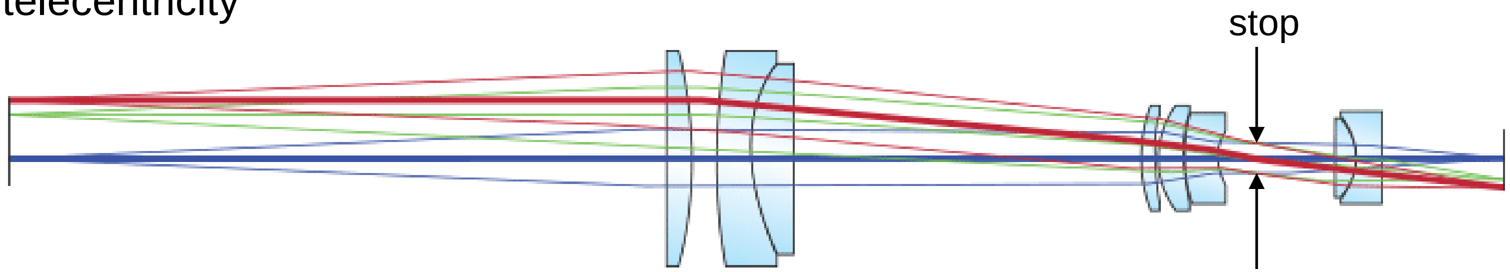
→ true for <u>each</u> object point

Property: object side telecentric imaging

special rays:
ray through center of stop: **chief ray**
ray at edge of stop: **marginal ray**

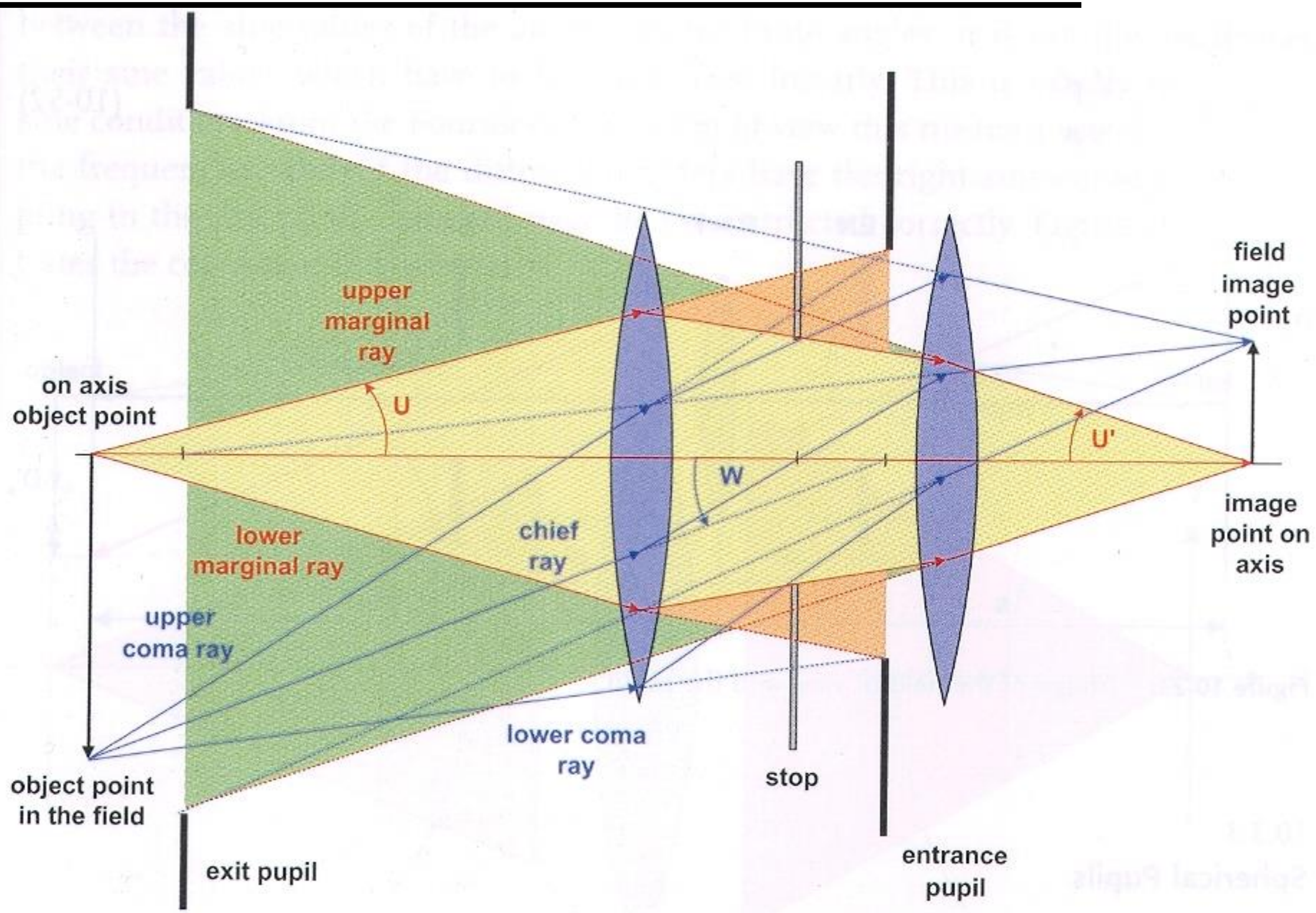
Telecentric Imaging

Object space telecentricity



source: edmund optics

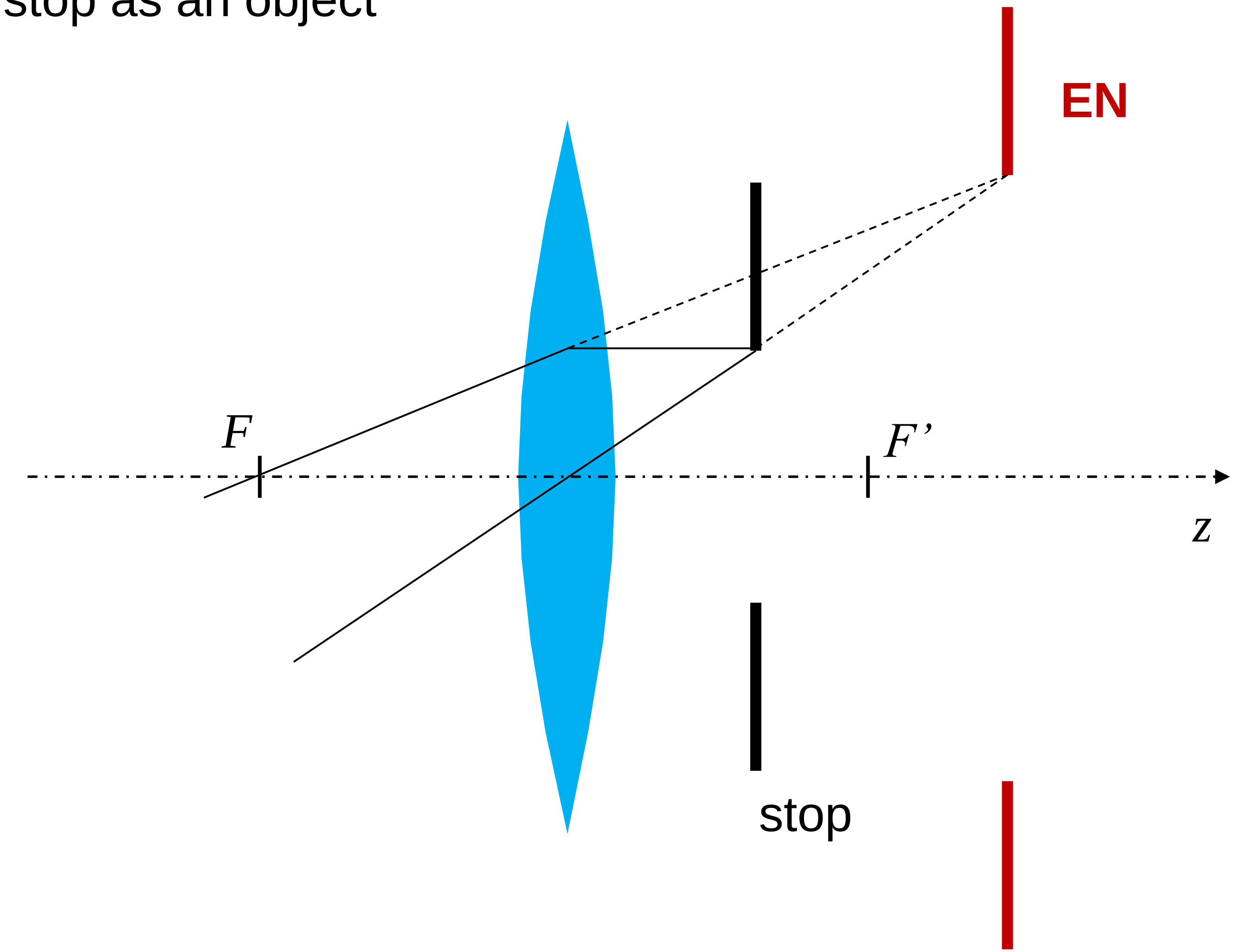
Definition of Stops, Entrance and Exit Pupil



Construction of EN

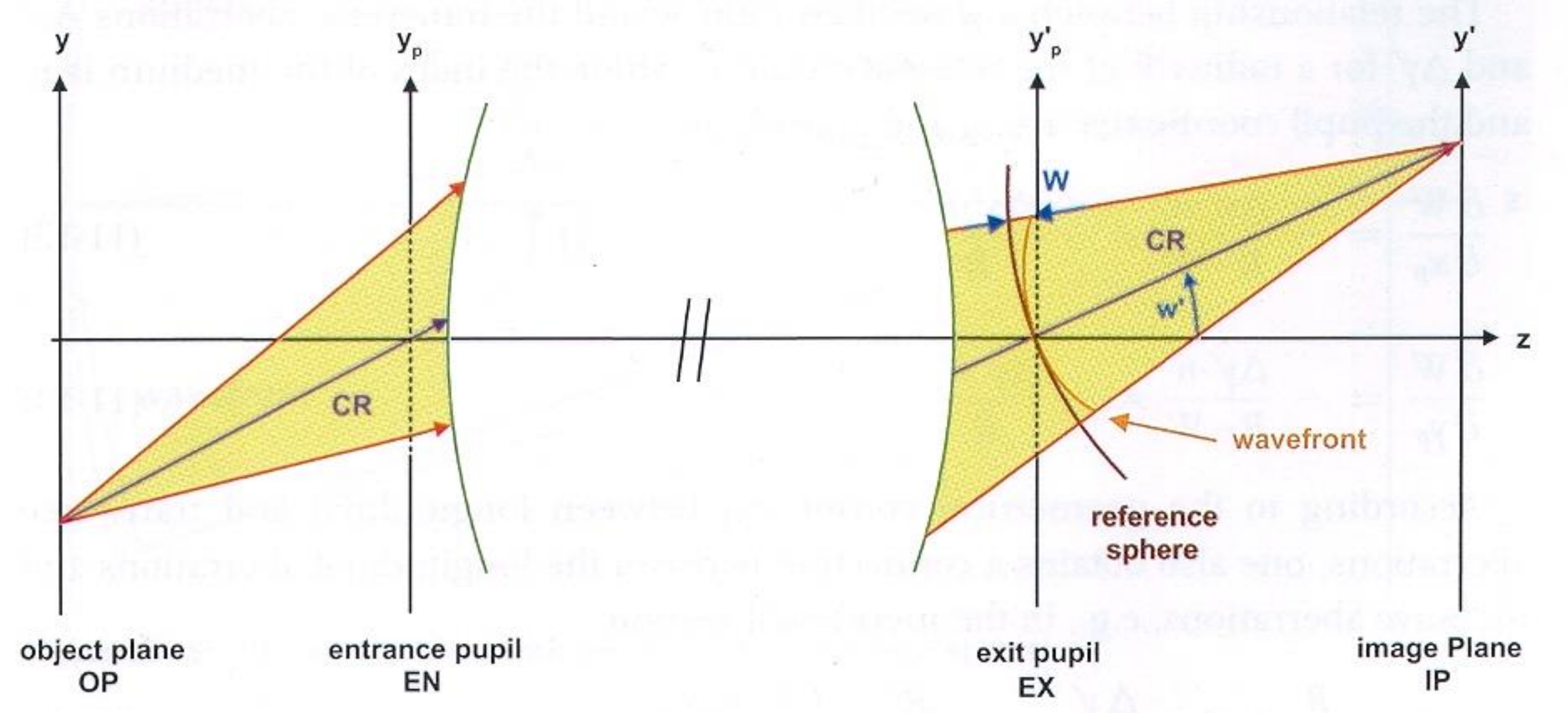
(one example, method)

treat the stop as an object



entrance pupil (EN):
stop as seen from the object space

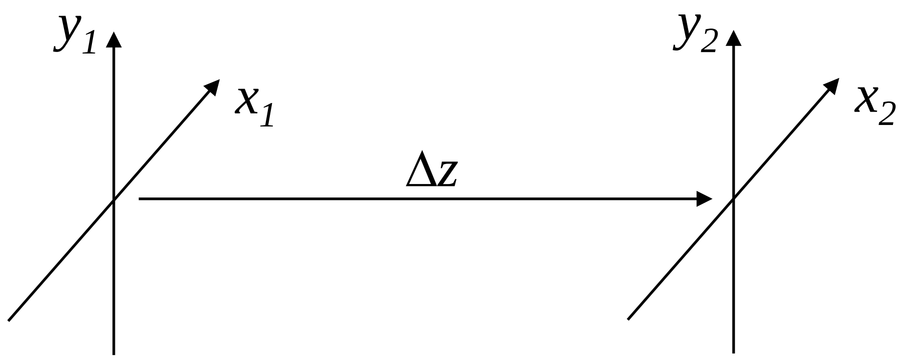
Wave aberrations of an optical system



Wave aberrations for an optical system.

Fresnel Diffraction

Calculation of field propagation between two <u>planes</u> with distance Δz



$\vec{E}(x_1, y_1, z_1)$ ... field at the input plane of the system
$\vec{E}(x_2, y_2, z_2)$ ... field at the output plane of the system
$\Delta z = z_2 - z_1$... plane-to-plane distance

Fresnel-diffraction formula:

$$\vec{E}(x_2, y_2, z_2) = \frac{e^{ik\Delta z}}{i\lambda\Delta z} \underline{e^{i\frac{k}{2\Delta z}(x_2^2+y_2^2)}} \frac{1}{4\pi^2} \iint \left[\vec{E}(x_1, y_1, z_1) \cdot \underline{e^{i\frac{k}{2\Delta z}(x_1^2+y_1^2)}}\right] \cdot \underline{e^{-i\frac{k}{\Delta z}(x_1 x_2 + y_1 y_2)}} dx_1 dy_1$$

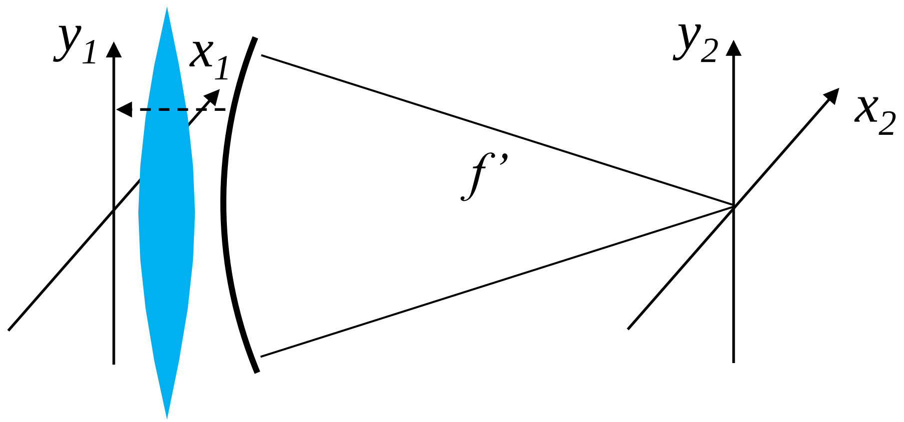
‘spherical’ phase with radius Δz

‘spherical’ phase with radius Δz

Fourier-transformation

Propagation into the focal plane

From the reference sphere onto the plane



$\vec{E}(x_1, y_1, z_1)$ ... field on input plane
$\vec{E}_r(x_1, y_1, z_1)$ ... field on reference sphere
$\Delta z = f'$

$$\vec{E}(x_1, y_1, z_1) = \vec{E}_r(x_1, y_1, z_1) \cdot \underbrace{e^{-i\frac{k\left(x_1^2+y_1^2\right)}{2f'}}}_{\text{lens phase corresponding to } f'}$$

Propagation into the focal plane (insert $\vec{E}$ into Fresnel-integral):

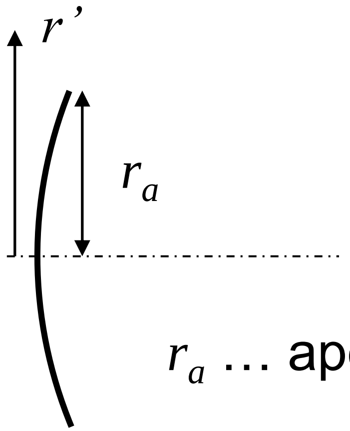
$$\vec{E}(x_2, y_2, z_2) = \frac{e^{ikf'}}{i\lambda f'} e^{i\frac{k}{2f'}\left(x_2^2+y_2^2\right)} \frac{1}{4\pi^2} \iint \vec{E}_r(x_1, y_1, z_1) \cdot e^{-i\frac{k}{f'}\left(x_1 x_2 + y_1 y_2\right)} dx_1 dy_1$$

➡ field in **focal plane** is the **Fourier-transformed** field on the **reference sphere**

Ideal spherical wave of finite extent

→ Aperture

field on reference sphere: $\vec{E}_r(x_1, y_1, z_1) = \mathrm{circ}\left(\dfrac{r'}{r_a}\right)$

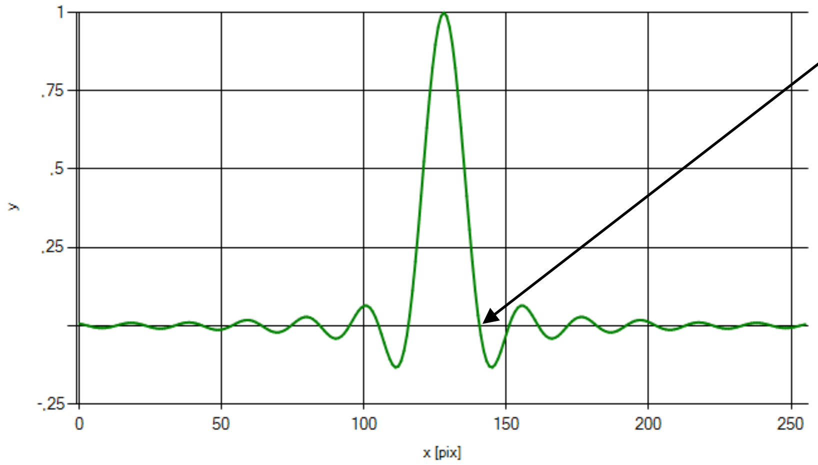


$$FT\left\{\mathrm{circ}\left(\frac{r'}{r_a}\right)\right\} = 2\pi r_a^2 \cdot \frac{\mathrm{J}_1\left(\frac{kr}{f} r_a\right)}{\frac{kr}{f} r_a}$$

J_1 … Bessel-Fct. of first kind

r_a … aperture radius

Airy-pattern:

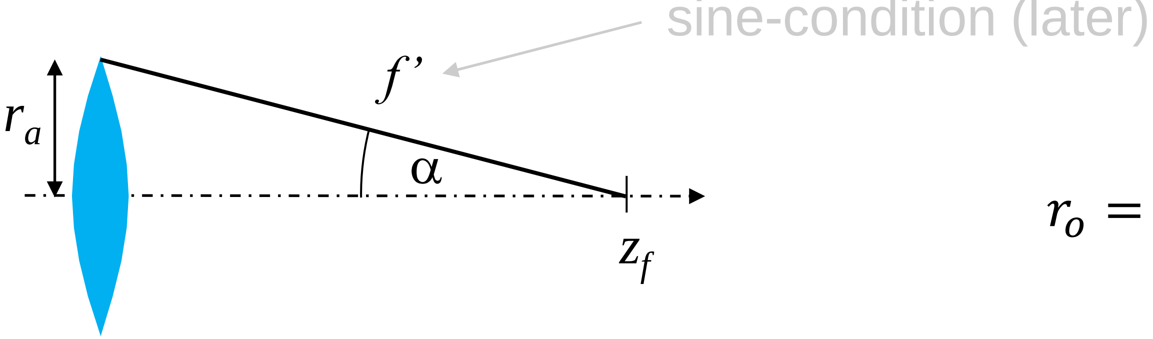


radius r_o of central spot

for $\mathrm{J}_1(\pi x) = 0$
at $x = 1.22$

Diffraction limited spot size

→ **diffraction at finite aperture**

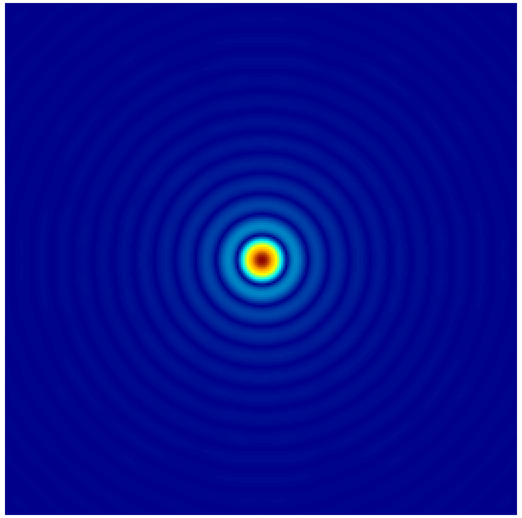


$$r_o = 0.61 \cdot \frac{\lambda \cdot f'}{r_a}$$

some relations: $\sin\alpha = \frac{r_a}{f'}$ $\lambda = \frac{\lambda_o}{n}$

$$\boxed{r_o = 0.61 \cdot \frac{\lambda_o}{n \cdot \sin\alpha} = 0.61 \cdot \frac{\lambda_o}{NA}}$$

Abbe's formula



diffraction limited focal spot

Spot size according to Ernst Abbe

memorial stone at Fürstengraben (near main building of University Jena)