Task 1 Jinsong Liu

a)

Solution:

From the 2f - setup equation:

$$U(x,y,2f) = -i\frac{(2\pi)^2}{\lambda f}\exp(2ikf)\,U_0(\tfrac{k}{f}x, \tfrac{k}{f}y)$$

$$\therefore\ U^-(x) = -i\frac{2\pi}{\lambda f_1}\exp(2ikf_1)\,U_0(\tfrac{k}{f_1}x)$$

$$U^+(x) = U^-(x)\cdot H(x) = -i\frac{2\pi}{\lambda f_1}\exp(2ikf_1)\,U_0(\tfrac{k}{f_1}x)\cdot H(x)$$

$$U_1(x) = -i\frac{2\pi}{\lambda f_2}\exp(2ikf_2)\,U^+(\tfrac{k}{f_2}x)$$

$$= -i\frac{2\pi}{\lambda f_2}\exp(2ikf_2)\frac{1}{2\pi}\int_{-\infty}^{\infty} -i\frac{2\pi}{\lambda f_1}\exp(2ikf_1)\,U_0(\tfrac{k}{f_1}x')\,H(x')\exp(-i\tfrac{k}{f_2}xx')\,dx'$$

$$= -i\frac{1}{\lambda f_2}\exp(2ikf_2)\Big[-i\frac{2\pi}{\lambda f_1}\exp(2ikf_1)\Big]\int_{-\infty}^{\infty} U_0(\tfrac{k}{f_1}x')\,H(x')\exp(-i\tfrac{k}{f_2}xx')\,dx'$$

$$= -\frac{2\pi}{\lambda^2 f_1 f_2}\exp[2ik(f_1+f_2)]\int_{-\infty}^{\infty} U_0(\tfrac{k}{f_1}x')\,H(x')\exp(-i\tfrac{k}{f_2}xx')\,dx'$$

b)

Solution:

$$U_0(x) = e^{iA\cos(\alpha_0 x)} = 1 + iA\cos(\alpha_0 x)$$

$$U_0(\tfrac{k}{f_1}x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}[1+iA\cos(\alpha_0 x')]\exp(-i\tfrac{k}{f_1}xx')\,dx'$$

$$= \delta(\alpha) + i\frac{A}{2}[\delta(\alpha+\alpha_0) + \delta(\alpha-\alpha_0)]$$

Let $C = -\frac{2\pi}{\lambda^2 f_1 f_2}\exp[2ik(f_1+f_2)]$ $\quad \varphi(\alpha) = \frac{A}{2}[\delta(\alpha+\alpha_0)+\delta(\alpha-\alpha_0)]$

$$U_1(x) = C\int_{-\infty}^{\infty} U_0(\alpha)\,H(\tfrac{f_1}{k}\alpha)\exp[i\tfrac{k}{f_2}xx']\,dx'$$

$$= C\int_{-\infty}^{\infty} U_0(\alpha)\,H(\tfrac{f_1}{k}\alpha)\exp(i\tfrac{f_1}{f_2}x\alpha)\frac{f_1}{k}\,d\alpha$$

$$= \frac{Cf_1}{k}\int_{-\infty}^{\infty}[\delta(\alpha)+i\varphi(\alpha)]\,H(\tfrac{f_1}{k}\alpha)\exp(i\tfrac{f_1}{f_2}x\alpha)\,d\alpha$$

$$\because\ H(x) = \begin{cases}\exp(i\varphi_0) & -a \le x \le a\\ 1 & \text{else}\end{cases}$$

$$\therefore\ H(\tfrac{f_1}{k}\alpha) = \begin{cases}\exp(i\varphi_0) & -\tfrac{k}{f_1}a \le \alpha \le \tfrac{k}{f_1}a\\ 1 & \text{else}\end{cases}$$

In order to convert the phase modulation into an amplitude modulation, we need to multiply $\exp(i\varphi_0)$ with $\delta(\alpha)$ which is zero-frequency component. $\therefore\ \frac{k}{f_1}a < \alpha_0 = 2\pi/\Lambda$, $a < \frac{2\pi f_1}{k\Lambda}$. And $\varphi_0 = \pm\frac{\pi}{2}$, $\exp(i\varphi_0) = \pm i$, when we calculate the $|U_1(-x)|^2$, the phase modulation can convert into an amplitude modulation.

Task 2 Jinsong Liu

a)

Solution:

$$FT[g(x')] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{2}\left[1+\cos\left(\frac{2\pi}{d}x'\right)\right] \exp\left(-i\frac{k}{f}xx'\right) dx'$$

$$= \frac{1}{2}\delta(\alpha) + \frac{1}{4}\left[\delta\left(\alpha+\frac{2\pi}{d}\right)+\delta\left(\alpha-\frac{2\pi}{d}\right)\right]$$

$$FT[n(x',y')] = \frac{1}{(2\pi)^2}\iint_{-\infty}^{\infty} \exp\left[-\frac{x'^2}{W^2}-\frac{(y'-\frac{R}{5})^2}{l^2}\right]\exp[-i(\alpha x'+\beta y')]\,dx'dy'$$

$$= \frac{W\cdot l}{4\pi}\exp\left[-\frac{W^2\alpha^2+l^2\beta^2}{4}\right]\cdot\exp\left(-i\beta\frac{R}{5}\right)$$

$$FT[m(x',y')] = \frac{lW}{4\pi}\exp\left[-\frac{l^2\alpha^2+W^2\beta^2}{4}\right]\cdot\exp\left(i\beta\frac{R}{2}\right)$$

$$FT[e_l(x',y')] = \frac{W^2}{4\pi}\exp\left[-\frac{W^2\alpha^2+W^2\beta^2}{4}\right]\cdot\exp\left(-i\alpha\frac{R}{2}\right)\cdot\exp\left(-i\beta\frac{R}{2}\right)$$

$$FT[e_r(x',y')] = \frac{W^2}{4\pi}\exp\left[-\frac{W^2\alpha^2+W^2\beta^2}{4}\right]\exp\left(i\alpha\frac{R}{2}\right)\exp\left(-i\beta\frac{R}{2}\right)$$

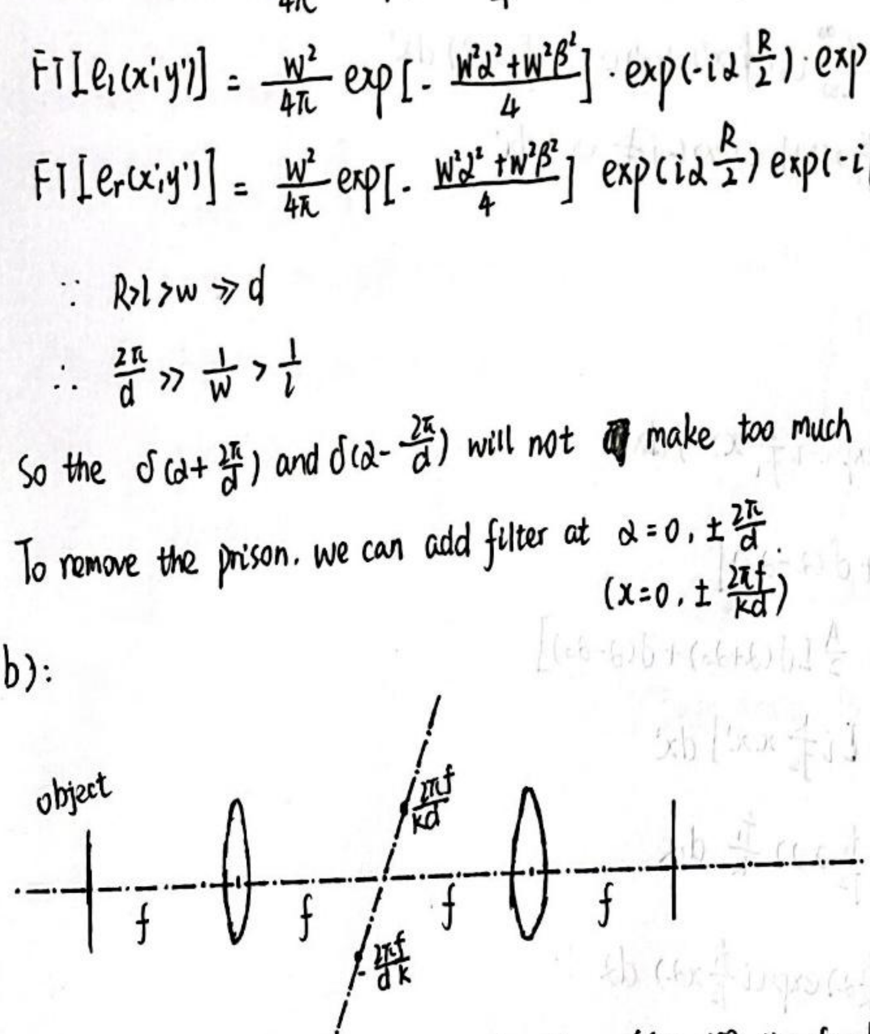
$\because R > l > w \gg d$

$\therefore \frac{2\pi}{d} \gg \frac{1}{W} > \frac{1}{l}$

So the $\delta(\alpha+\frac{2\pi}{d})$ and $\delta(\alpha-\frac{2\pi}{d})$ will not make too much influence over the Gausian spectrum.

To remove the prison, we can add filter at $\alpha = 0, \pm\frac{2\pi}{d}$.
$(x = 0, \pm\frac{2\pi f}{kd})$

b):



The initial optical field is the object, after the focal length of the first lens, it does a Fourier transform and generates the field in between. The final optical field is inverse Fourier transformation of the field between 2 lenses.

c)

Solution:

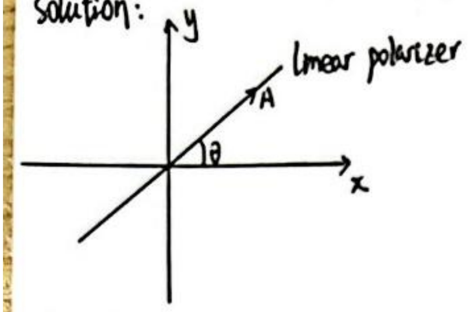
$\because$ The Prison has zero-frequency component which is involved in the Gaussian spectrum.

To minimize the effect, we can multiply a very small number with $\delta(x)$.

Task 3

a)

Solution:



incident: $J_{in} = \begin{bmatrix} A_1 \\ B_1 \end{bmatrix}$

$$A = A_1 \cos\theta + B_1 \sin\theta$$

After the polarizer:

$$J_{out} = \begin{bmatrix} A_2 \\ B_2 \end{bmatrix}$$

$$A_2 = A\cos\theta = A_1 \cos^2\theta + B_1 \cos\theta \sin\theta$$

$$B_2 = A\sin\theta = A_1 \cos\theta \sin\theta + B_1 \sin^2\theta$$

$$\therefore T_\theta = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

b)

Solution:

incident: $J_{in} = \begin{bmatrix} A_1 \\ B_1 \end{bmatrix}$

From the problem, we can get

$$T J_{in} = \lambda J_{in}$$

$$\therefore (T-\lambda) J_{in} = 0$$

$$\det(T-\lambda) = \begin{vmatrix} \frac{1-2\lambda}{2} & \frac{i}{2} \\ \frac{-i}{2} & \frac{1-2\lambda}{2} \end{vmatrix} = 0$$

$$\therefore (1-2\lambda)^2 - 1 = 0$$

$$\lambda = 0, 1 \quad (\lambda = 0 \text{ is wrong})$$

when $\lambda = 1$

$$\begin{cases} A_1 + iB_1 = 2A_1 \\ -iA_1 + B_1 = 2B_1 \end{cases}$$

$$\therefore A_1 = 1 \quad B_1 = -i$$

The normalized Jones vector is $J = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$

This state is right circular polaried light.