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Midterm Exam

Fundamentals of Modern Optics

to be written on December 17, 8:15 am – 9:45 am

Problem 1: Maxwell's Equations — 4 + 2 + 3 = 9 points

a) Write down Maxwell's equations (MWE) in the time domain and in the frequency domain in a linear, homogeneous, isotropic, and dispersive medium without external charges but with currents.

b) Write down the relation between $\mathbf{D}$ and $\mathbf{E}$ in the time and frequency domain in this medium. Name additional functions used in the relations!

c) Derive the Helmholtz equation for the magnetic field $\mathbf{H}(\mathbf{r},\omega)$ from MWE in this medium. Assume that $\mathbf{j}(\mathbf{r},\omega)=\sigma(\omega)\,\mathbf{E}(\mathbf{r},\omega)$.

Problem 2: Poynting vector — 2 + 2 + 3 + 4 = 11 points

a) What is the general connection between the Poynting vector $\mathbf{S}(\mathbf{r},t)$ and the optical intensity $I(\mathbf{r})$?

Now consider the electric field

$$\mathbf{E}(z,t)=\mathbf{E}_0\exp\left[\mathrm{i}(\alpha z-\omega_0 t)\right]+\mathbf{E}_1\exp\left[\mathrm{i}(\beta z-\omega_0 t)\right],$$

where $\mathbf{E}_0=A_0\hat{\mathbf{e}}_x$, $\mathbf{E}_1=A_1\hat{\mathbf{e}}_y$. A_0, A_1, α, and β are real valued.

b) Calculate the magnetic field $\mathbf{H}(z,t)$ for the electric field defined above.

c) Calculate the time-averaged Poynting vector $\langle\mathbf{S}(\mathbf{r},t)\rangle$ and the intensity.

d) Calculate the intensity for the case $\mathbf{E}_1=A_1\hat{\mathbf{e}}_x$ (keep $\mathbf{E}_0=A_0\hat{\mathbf{e}}_x$). Explain the physical reason why you get a different result than in c).

Problem 3: Beam Propagation — 3 + 1 + 4 + 2 = 10 points

Consider the propagation of a monochromatic, scalar field $u(x,y,z)$ at wavelength λ along the z-direction starting from a given initial field distribution $u(x,y,z=0)=u_0(x,y)$.

a) Describe the steps necessary to calculate the field distribution $u(x,y,z)$ for any $z>0$. Name and define all functions and quantities that you use.

b) State the condition under which the paraxial approximation can be applied. How does the computation of $u(x,y,z)$ change in this case?

In the following we consider a slit with a width d and a linear phase profile $\varphi=\xi x$ in x-direction. Assume that the field after the slit can be written as

$$u(x,z=0)=u_0(x)=\begin{cases}\exp\left(\mathrm{i}\xi x\right) & \text{for } |x|<d/2,\\ 0 & \text{else.}\end{cases}$$

c) Calculate the Fourier transform $U_0(\alpha)$ of the initial field. Sketch the resulting spectrum and explain what effect the linear phase mask has on the spectrum.

d) Find the range of ξ such that the main spectral lobe (region from spectral maximum to the first zeros) is fully propagating assuming a wavelength of $\lambda=2d/3$.

Problem 4: Pulses — 3 + 2 + 1 + 2 = 8 points

The evolution equation for the slowly varying envelope $\tilde{v}(z,\tau)$ of a pulse with central frequency ω_0 in the co-moving frame is given by

$$\mathrm{i}\frac{\partial\tilde{v}(z,\tau)}{\partial z}-\frac{D}{2}\frac{\partial^2\tilde{v}(z,\tau)}{\partial\tau^2}=0.$$

a) Which approximations are applied to derive the equation above from Maxwell's equations? Name them and state under which conditions they are applicable.

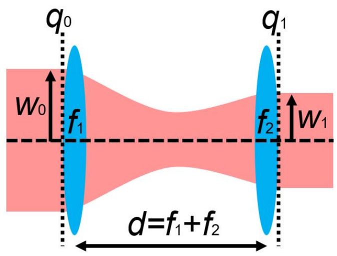
b) Define the co-moving frame τ and the dispersion D in the above equation and give their value in terms of the time t and the wavenumber $k(\omega)$.

c) What are the equivalents of $\partial/\partial\tau$ and D in the analogous beam diffraction equation?

d) By analogy to diffraction argue how $|\tilde{v}(z,\tau)|$ looks for very large z when the initial excitation is a temporally localized pulse $\tilde{v}_0(\tau)$.

Problem 5: Gaussian Beams — 1 + 3 + 2 + 2 = 8 points

A collimated Gaussian beam of width w_0 propagates first through a thin lens with a focal distance f_1 and then through a thin lens with a focal distance f_2. The two lenses are separated by a distance $d = f_1 + f_2$ (see figure).



a) Write the general expression of the q-parameter, which allows to describe the Gaussian beam propagation. Relate $1/q(z)$ to physical parameters of the Gaussian beam.

b) Calculate the ABCD matrix of the whole system. The general form of the ABCD matrix of a thin lens is:

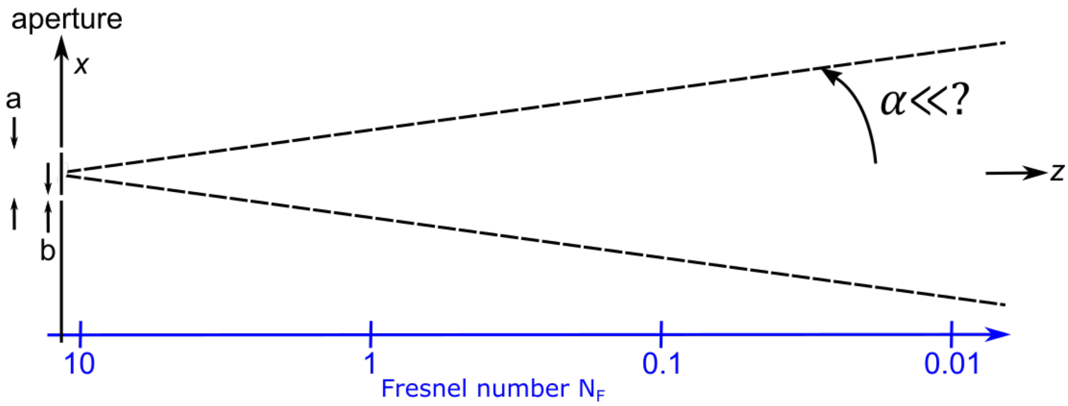
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}.$$

c) Calculate q_1 (see figure). Use that the Gaussian beam is still collimated after the system to simplify your calculation.

d) Calculate the beam width w_1 after the system.

Problem 6: Fraunhofer diffraction — 2 + 2 + 2 = 6 points

a) Write down the conditions where 1) the Fresnel approximation, 2) the paraxial Fraunhofer approximation, and 3) the non-paraxial Fraunhofer approximation are valid. Mark the regions where the different diffraction approximations are valid in the following figure.



b) We consider two one-dimensional slits located on the x-axis where the slits have a width of b and are separated by a distance of a. Calculate the resulting far-field intensity when the field directly after the aperture is

$$u(x, z=0) = \begin{cases} 1 & \text{for } |x| < b/2 \\ 1 & \text{for } a - b/2 < x < a + b/2 \\ 0 & \text{otherwise} \end{cases}.$$

Hint: You may leave out the prefactors.

c) When $a \gg b$, which of these dimensions defines the applicability of the Fresnel approximation and the Fraunhofer approximation, respectively? For both approximations, state the condition(s) in relation with the wavelength λ and the observation distance z_B.