1: Radiation laws

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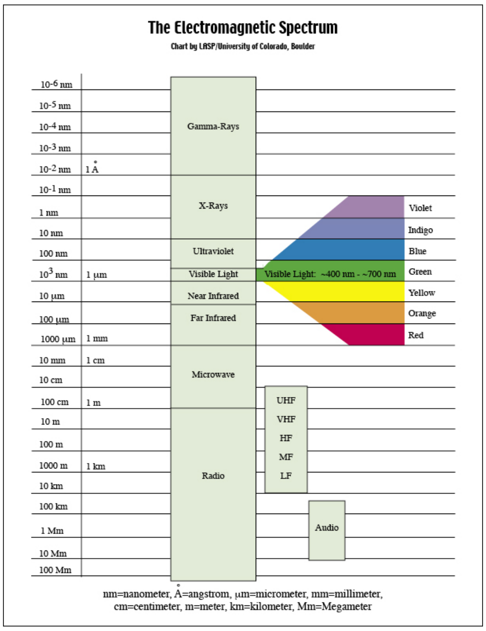
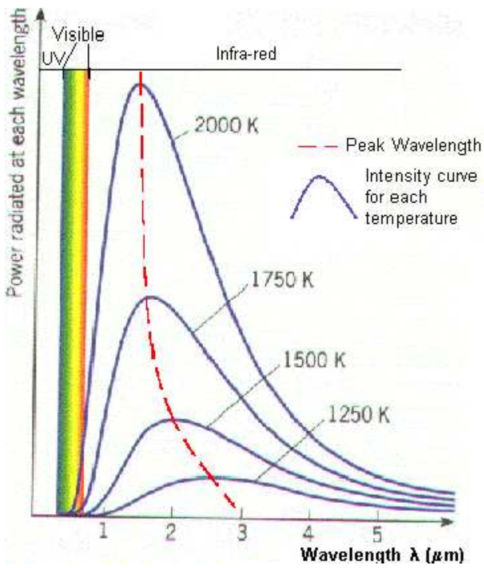
- Black body (b.b., imaginary):
  - body that absorbs 100 $\%$ of the radiation that reaches it
  - if its temperature is different from 0 it emits thermal radiation (E.M.)
  - if it is in thermal equilibrium it emits at every wavelength (λ)
  - the emitted energy depends on λ and T
- Practically, a rough cavity or enclosure with a small hole and uniform temperature T behaves as a b.b. (emits b.b. radiation)



- In the cavity the radiation is:
  - isotropic (its flux does not depend on the direction)
  - homogeneous (equal in every point)
  - equal in different cavities with equal T
- We define
  - **Spectral radiancy** $\tilde{E}(\lambda, T)$ emitted energy per unit λ, per unit area and per unit time with λ between λ and $\lambda + d\lambda$. It is a universal function (does not depend on the material of which it is made the cavity)
  - **Spectral energy density** $\tilde{\rho}(\lambda, T)$ energy per unit volume and per unit λ, with λ between λ and $\lambda + d\lambda$

$$\tilde{\rho}(\lambda, T) = \frac{4}{c}\tilde{E}(\lambda, T)$$

Spectral radiancy of the black body



W.L. $\tilde{\rho}(\lambda, T) = \lambda^{-5}\, f(\lambda T) \;=\; T^5\, f'(\lambda T)$ (T absolute)

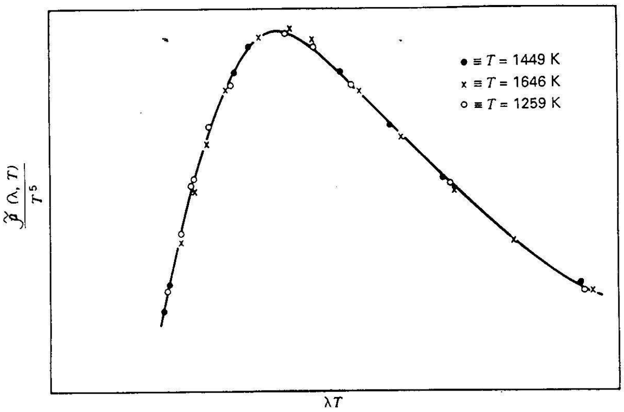


Figure 1-1. Experimental verification of (1.2) in the form of $\tilde{\rho}(\lambda, T)/T^5$ = a universal function of λT.

We can define spectral radiancy and spectral energy density of energy per unit ν, $E(\nu, T)$ y $\rho(\nu, T)$ (e.m. radiation $\lambda = \frac{c}{\nu}$)

Also

$$\rho(\nu, T) = \frac{4}{c} E(\nu, T)$$

Energy per unit volume in corresponding intervals of ν or λ are equal

$$\rho(\nu, T)|d\nu| = \tilde{\rho}(\lambda, T)|d\lambda|$$

$$\rho(\nu, T) = \tilde{\rho}(\lambda, T)|\frac{d\lambda}{d\nu}|$$

$$\rho(\nu, T) = \frac{c}{\nu^2}\tilde{\rho}(\lambda, T)$$

W.L. (alternative as a function of ν)

$$\rho(\nu,T) = \nu^3 g(\frac{\nu}{T}) = T^3 g'(\frac{\nu}{T})$$

(equivalent to the former one, now as a function of ν) The maximum of $\tilde{\rho}(\lambda,T)$ (and $\tilde{E}(\lambda,T)$) is given by the **Wien's displacement law**:

$$\lambda_{max}(T) = \frac{b}{T}$$

$b = 0.2898\ cmK$ is the constant of Wien
The maximum of $\rho(\nu,T)$ (and $E(\nu,T)$):

$$\nu_{max}(T) = 5.89\ 10^{10}\ \frac{Hz}{K} T$$

$\lambda_{max}\ \nu_{max} \neq c$!
Wien gives $g(\frac{\nu}{T}) = Ce^{-\frac{\beta\nu}{T}}$ (it fits well at high frecuencies)

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Stefan or Stefan-Boltzmann Law

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We define **energy density, or total radiated energy per unit volume**

$$\rho(T) = \int_0^\infty\ \tilde{\rho}(\lambda,T)d\lambda = \int_0^\infty\ \rho(\nu,T)d\nu$$

Stefan-Boltzmann Law $\rho(T) = a\,T^4$ $a = 7.56\ 10^{-16}\frac{J}{m^3K^4}$

Total Radiancy

$$E(T) = \int_0^\infty\ \tilde{E}(\lambda,T)d\lambda = \int_0^\infty\ E(\nu,T)d\nu$$

Stefan-Boltzmann Law $E(T) = \sigma\,T^4$

$\sigma = 5.67\ 10^{-8}\frac{J}{m^2sK^4}$ (Stefan-Boltzmann constant)

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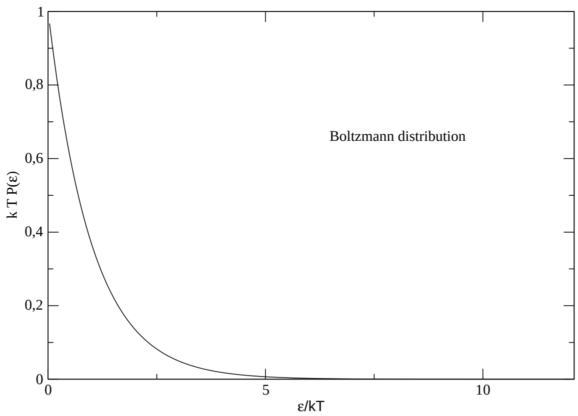
Rayleigh and Jeans derive $\rho_{R-J}(\nu, T) = \frac{8\ \pi\ \nu^2}{c^3} k\ T$

$$k = k_B = 1.381\ 10^{-23} \frac{J}{K}\ =\ 8.617\ 10^{-5} \frac{eV}{K},$$

using

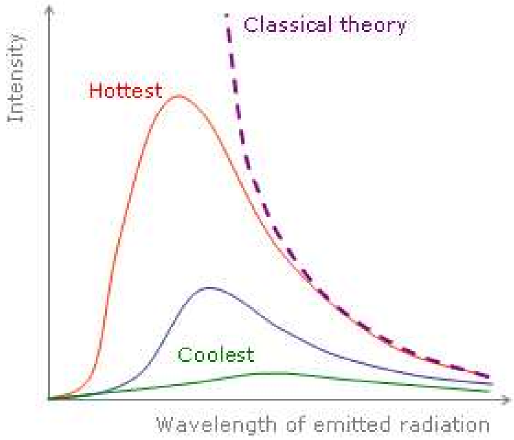
1. the classical law of equipartition of energy (Boltzmann's distribution) $\bar{\mathcal{E}}\ =\ k\ T$ (by degree of freedom, system in thermodynamic equilibrium)
2. number of modes (charged mechanical oscillators) per unit volume and per unit frecuency, with frequency between ν and $\nu + d\nu$ for e.m. radiation in a cavity $\rightarrow$ $\frac{8\ \pi\ \nu^2}{c^3}$

- Reproduces experimental data for low ν
- Fails for high ν
- $\rho(T)$ becomes infinite $\rightarrow$ **ultraviolet catastrophe**



Boltzmann Distribution

$$P(\mathcal{E}) = \frac{e^{-\frac{\mathcal{E}}{kT}}}{kT}$$



- - - Rayleigh-Jeans

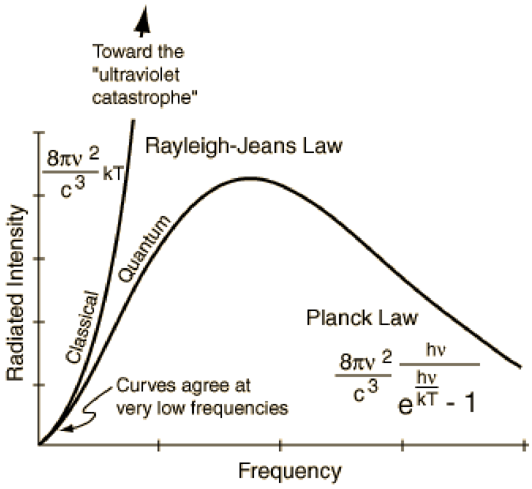
Planck Theory

$$\rho_{Planck}(\nu, T) = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{\frac{h\nu}{kT}} - 1}$$

$h = 6.63\ 10^{-34}\ J\ s$ (Planck constant)

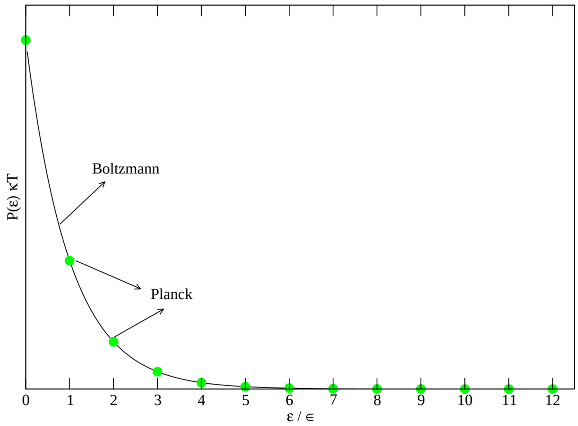
- If $\frac{h\,\nu}{k\,T} >> 1$ $\quad \rho_{Planck}(\nu, T) \simeq \frac{8\pi\,h}{c^3}\nu^3 e^{-\frac{h\nu}{kT}}$
  coincides with $\rho_{Wien}(\nu, T)$ with
  $C = \frac{8\pi h}{c^3}$ and $\beta = \frac{h}{k}$
- If $\frac{h\,\nu}{k\,T} << 1$ $\quad \rho_{Planck}(\nu, T) \rightarrow \rho_{R-J}(\nu, T)$
- $\rho_{Planck}(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\,\nu}{e^{\frac{h\nu}{kT}}-1} \rightarrow \bar{\mathcal{E}} = \frac{h\,\nu}{e^{\frac{h\nu}{kT}}-1}$
  $\bar{\mathcal{E}}$ depends on ν

$$\lim_{\nu\to\infty} \bar{\mathcal{E}} = 0$$



$$\rho(T) = \int_0^\infty \rho(\nu, T) d\nu = \frac{8\pi^5 k^4}{15 h^3 c^3} T^4$$

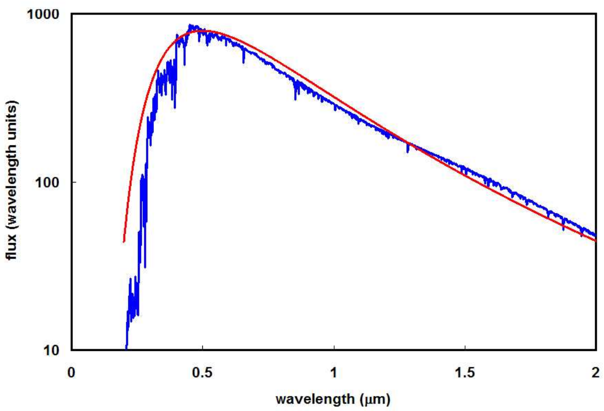
coincides with Stefan-Boltzmann's law and ν_{max} corresponds to Wien's displacement law



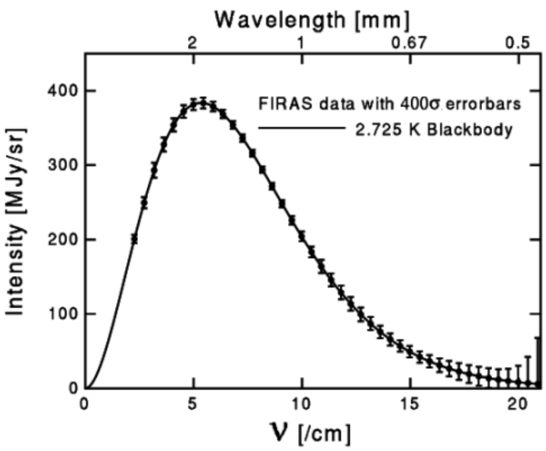
$$\mathcal{E} \;=\; n\,\epsilon \qquad n \;=\; 0, 1, 2....$$

$$\bar{\mathcal{E}} \;=\; \sum_{\mathcal{E}} \mathcal{E}\,P(\mathcal{E}) = \frac{\sum_{n=0}^{\infty} n\,\epsilon\,e^{-\frac{n\,\epsilon}{kT}}}{\sum_{m=0}^{\infty} e^{-\frac{m\,\epsilon}{kT}}} \;=\; \frac{\epsilon}{e^{\frac{\epsilon}{kT}} - 1}$$

$\epsilon \;=\; h\,\nu$ quantum of energy (photons)



Like many other astronomical objects, the sun radiates energy almost as though it were a true blackbody (the output of a 5800K blackbody is in red, that of the sun in blue.



Microwave Cosmic Background Radiation

$\nu \;\; c/cm$

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