



**Institute of  
Applied Physics**

Friedrich-Schiller-Universität Jena

# Lens Design I

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Lecture 12: Correction II

2024-07-04

Yueqian Zhang

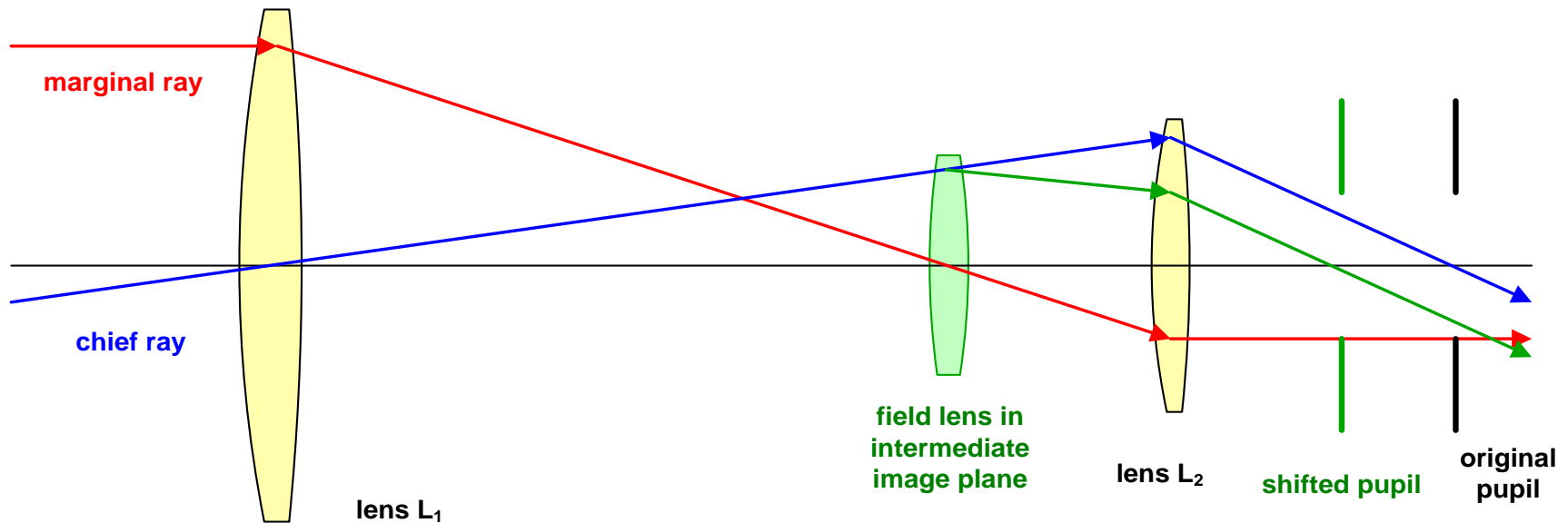


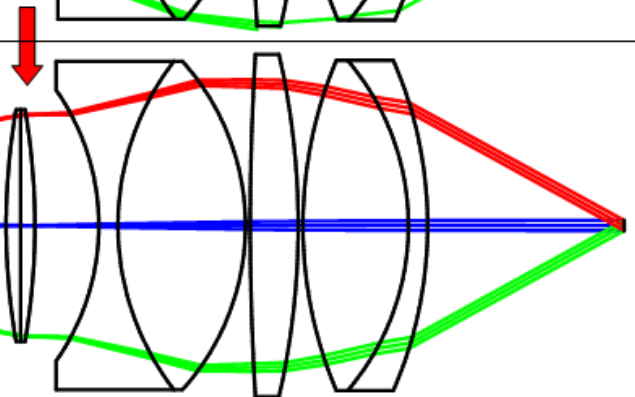
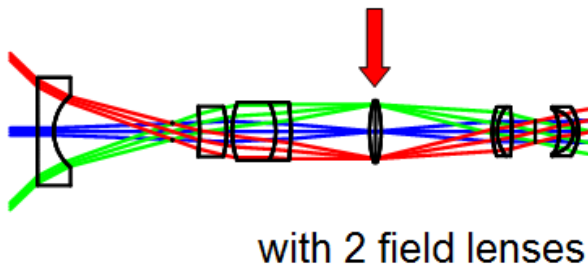
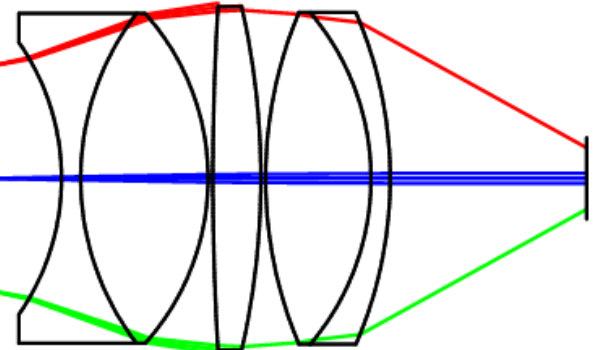
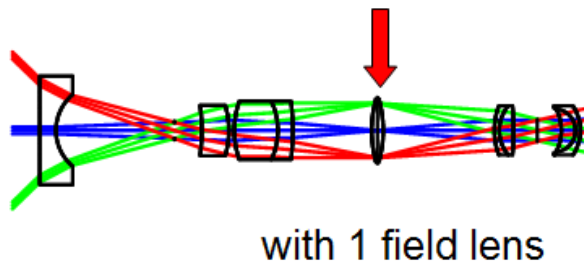
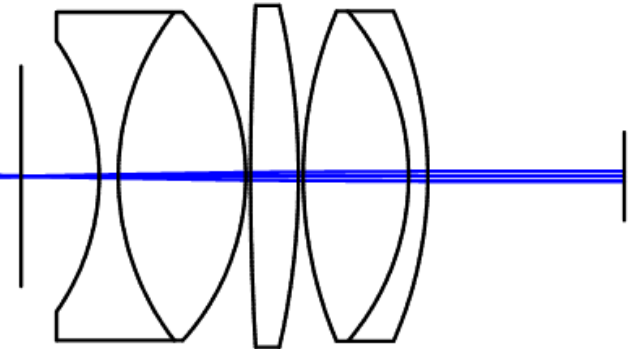
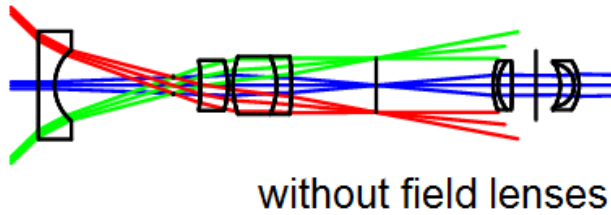
# Preliminary Schedule - Lens Design I 2024

1	04.04.	Basics	Zhang	Introduction, Zemax interface, menus, file handling, preferences, Editors, updates, windows, coordinates, System description, 3D geometry, aperture, field, wavelength
2	18.04.	Properties of optical systems I	Tang	Diameters, stop and pupil, vignetting, layouts, materials, glass catalogs, raytrace, ray fans and sampling, footprints
3	25.04.	Properties of optical systems II	Tang	Types of surfaces, cardinal elements, lens properties, Imaging, magnification, paraxial approximation and modelling, telecentricity, infinity object distance and afocal image, local/global coordinates
4	02.05.	Properties of optical systems III	Tang	Component reversal, system insertion, scaling of systems, aspheres, gratings and diffractive surfaces, gradient media, solves
5	16.05.	Advanced handling I	Tang	Miscellaneous, fold mirror, universal plot, slider, multiconfiguration, lens catalogs
6	23.05.	Aberrations I	Zhang	Representation of geometrical aberrations, spot diagram, transverse aberration diagrams, aberration expansions, primary aberrations
7	30.05.	Aberrations II	Zhang	Wave aberrations, Zernike polynomials, measurement of quality
8	06.06.	Aberrations III	Tang	Point spread function, optical transfer function
9	13.06.	Optimization I	Tang	Principles of nonlinear optimization, optimization in optical design, general process, optimization in Zemax
10	20.06.	Optimization II	Zhang	Initial systems, special issues, sensitivity of variables in optical systems, global optimization methods
11	27.06.	Correction I	Zhang	Symmetry principle, lens bending, correcting spherical aberration, coma, astigmatism, field curvature, chromatical correction
12	04.07.	Correction II	Zhang	Field lenses, stop position influence, retrofocus and telephoto setup, aspheres and higher orders, freeform systems, miscellaneous

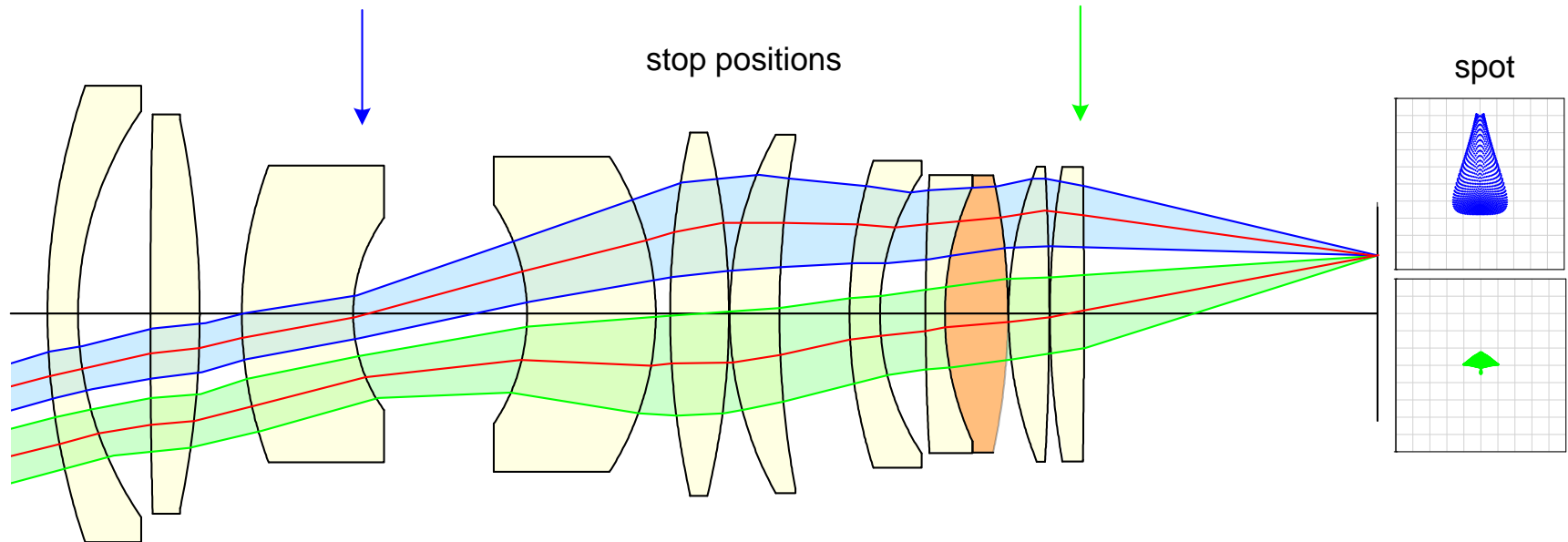
1. Field lenses
2. Stop position influence
3. Retrofocus and telephoto setup
4. Aspheres and higher orders
5. Freeform systems
6. Miscellaneous

- Field lens: in or near image planes
- Influences only the chief ray: pupil shifted
- Critical: conjugation to image plane, surface errors sharply seen

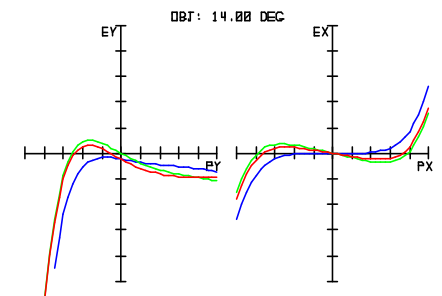
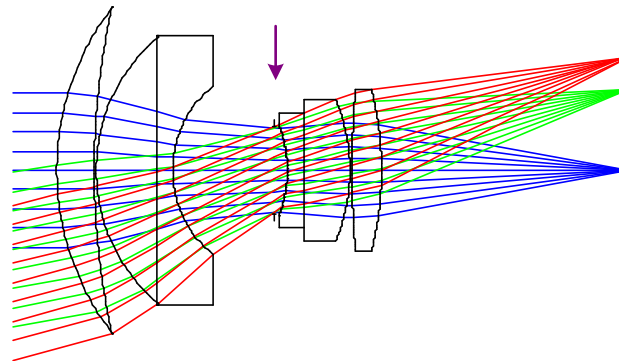
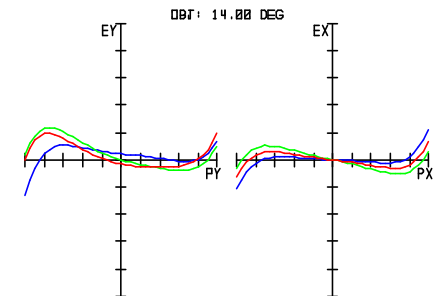
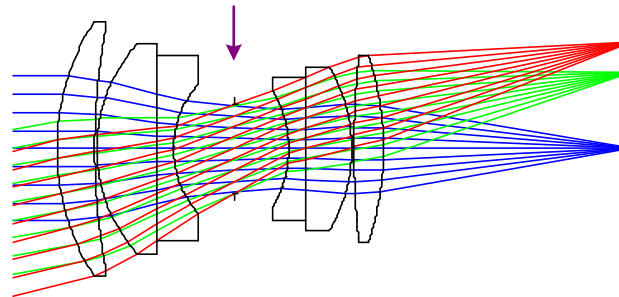
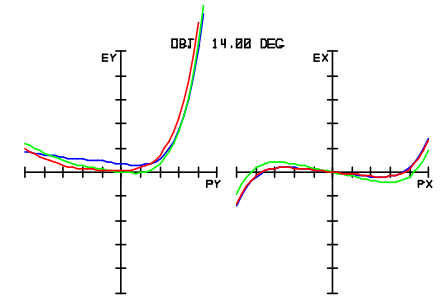
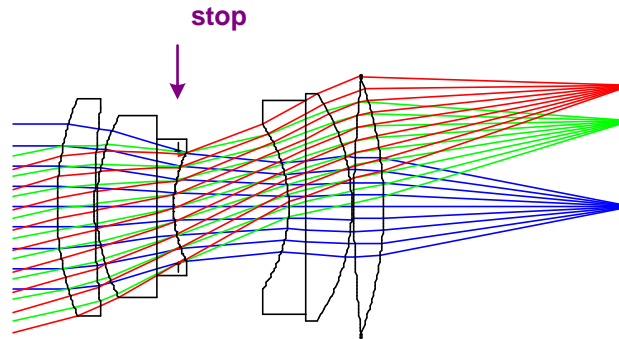




- Ray path of chief ray depends on stop position



- Example photographic lens
- Small axial shift of stop changes transverse aberrations
- In particular coma is strongly influenced



- Combination of a positive and a negative lens:  
Shift of the first principal plane in front of the system
- The intersection length is smaller than the focal length: reduction factor  $k$
- Typical values:  $k = 0.6 \dots 0.9$
- Focal lengths:

$$f_a = \frac{f' \cdot d}{f' \cdot (1 - k) + d}$$

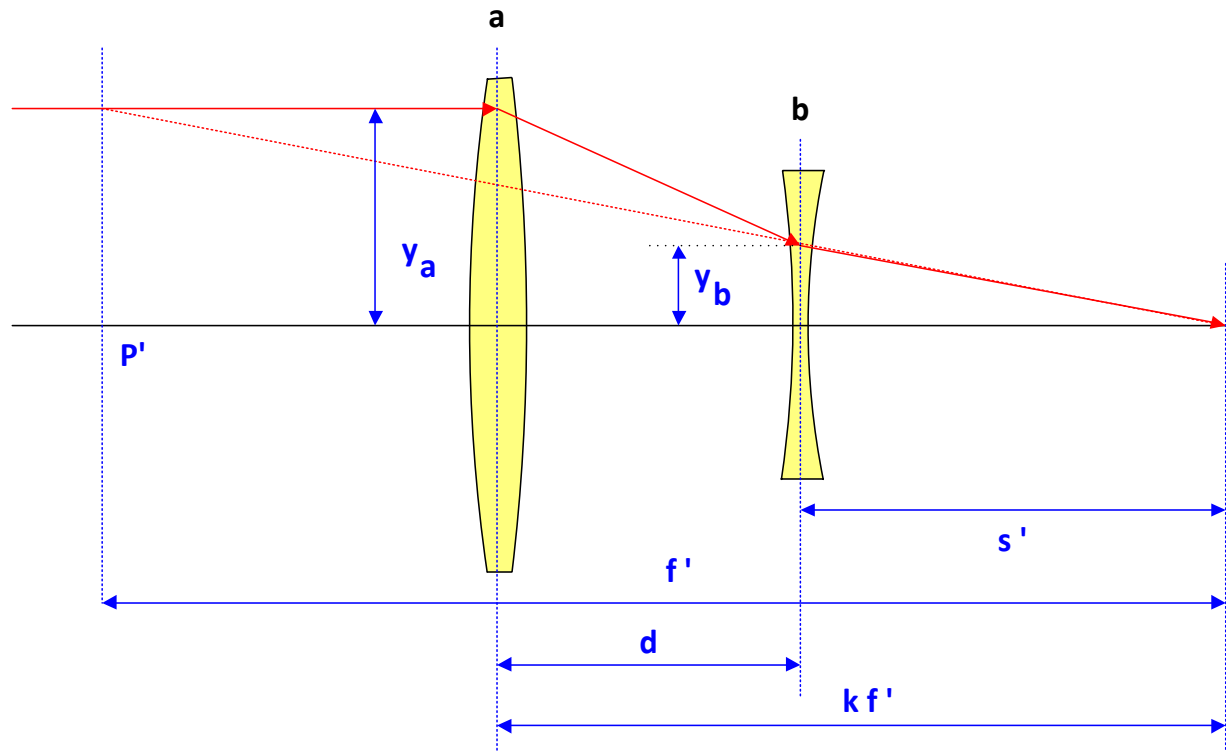
$$f_b = \frac{(f_a - d)(kf' - d)}{f_a - kf'}$$

- Overall length

$$L = k \cdot f'$$

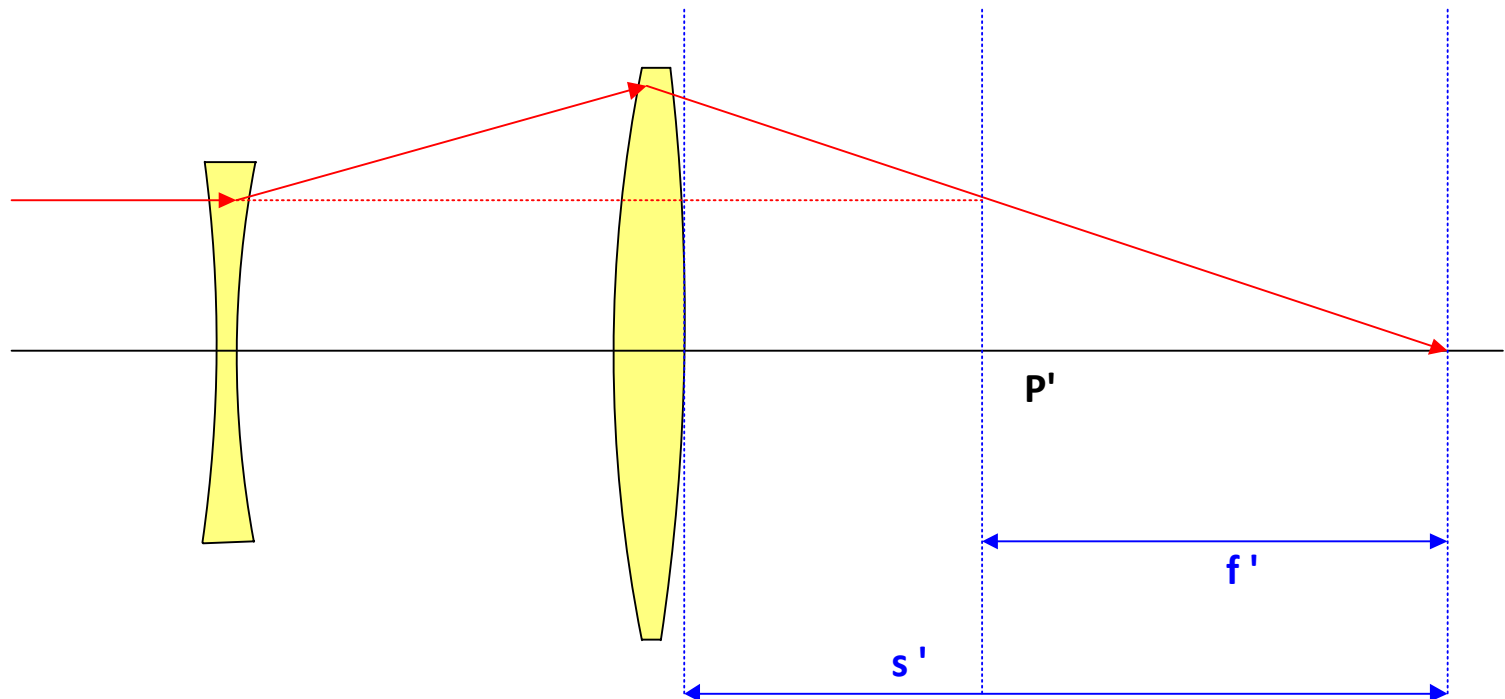
- Free intersection length

$$s_f = k \cdot f' - d$$





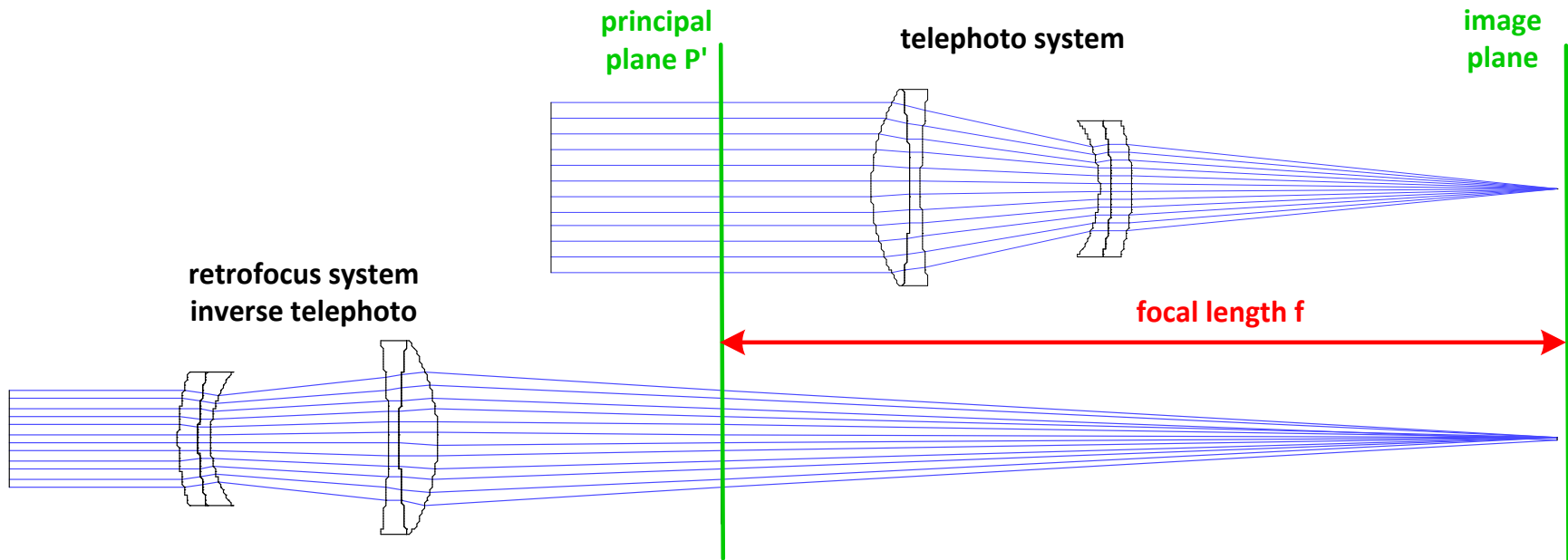
- Combination of a negative and a positive lens:  
Shift of the second principal plane behind the system
- The intersection length is larger than the focal length
- Application: systems for large free working distance
- Corresponds to an inverse telephoto system



# Telephoto and inverse Telephoto Principle

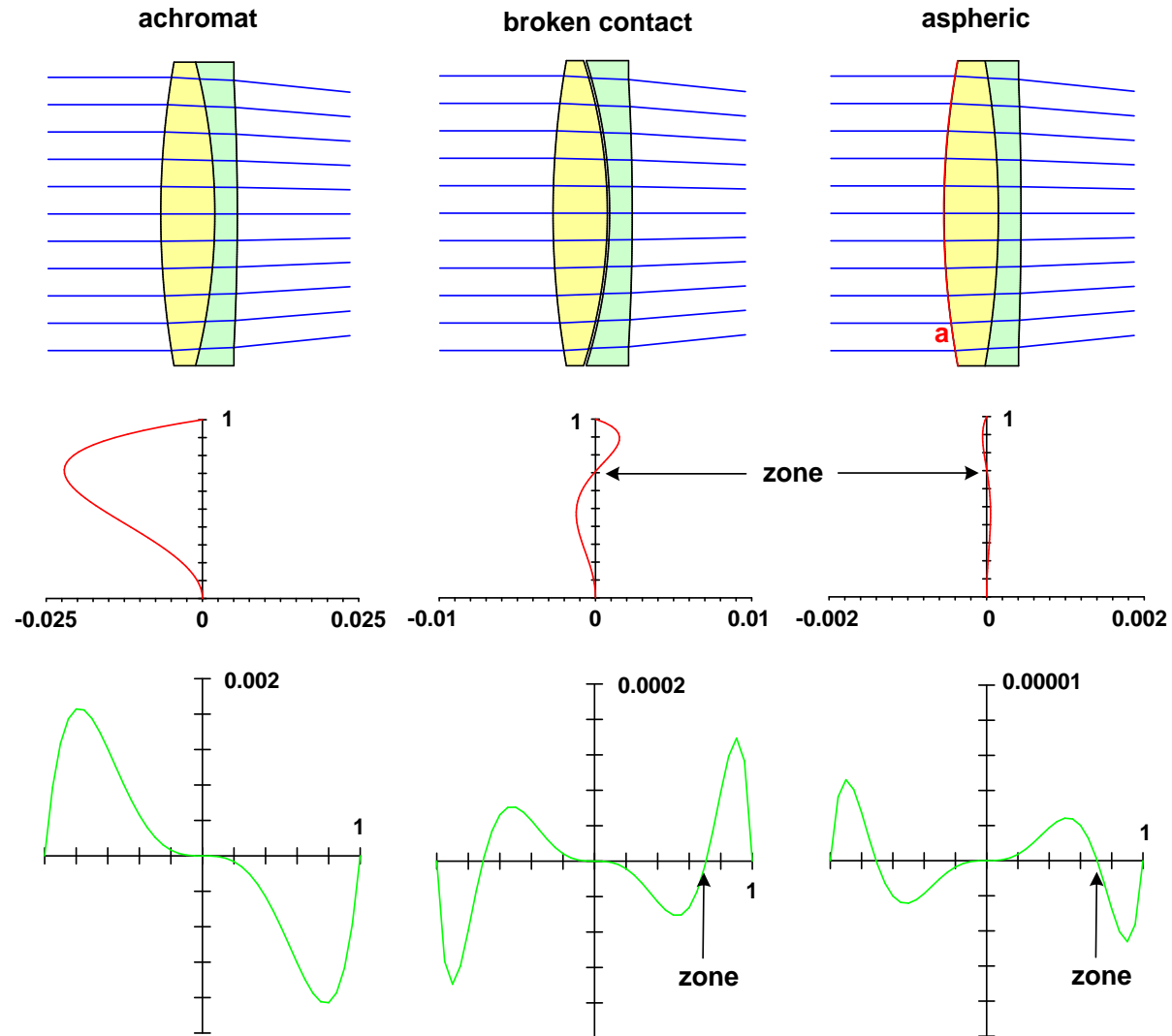
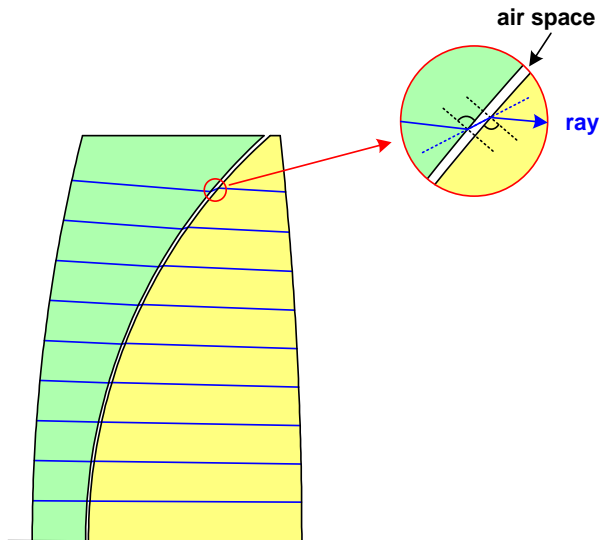


Retrofocus system results form a telephoto system by inversion



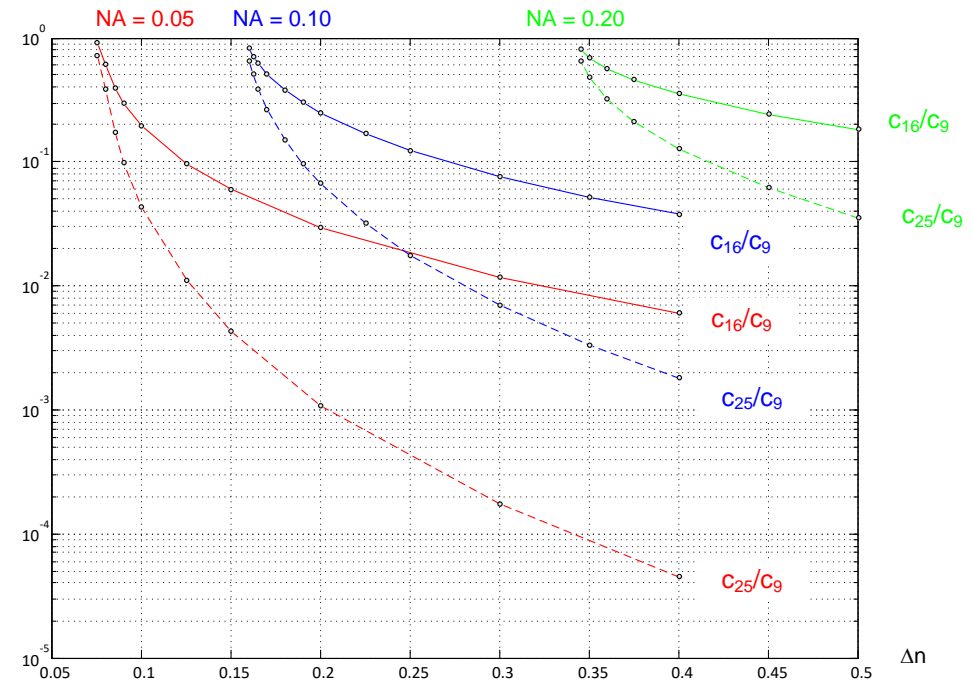
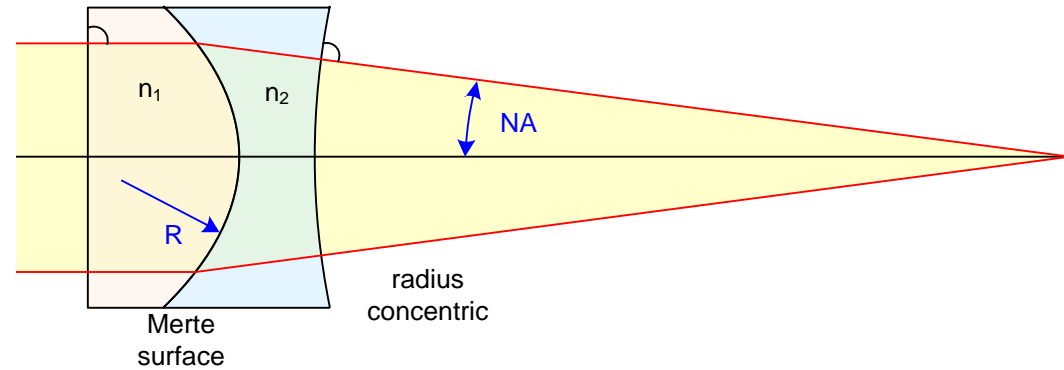
# Higher Order Aberrations: Achromate, Aspheres

## ■ Splitted achromate



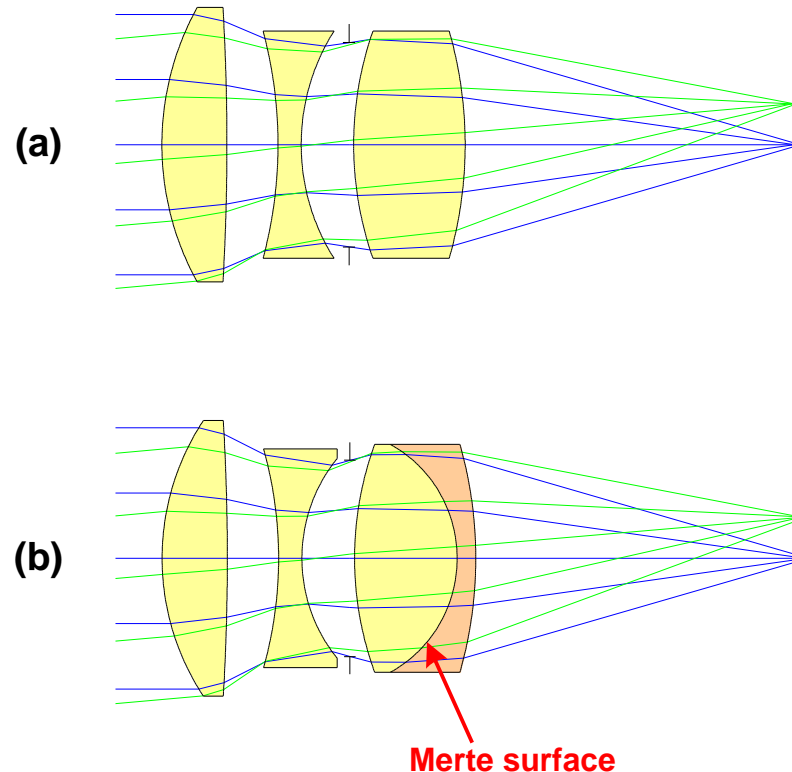
## ■ Aspherical surface

- Small difference in refractive index
- Growing higher order contributions

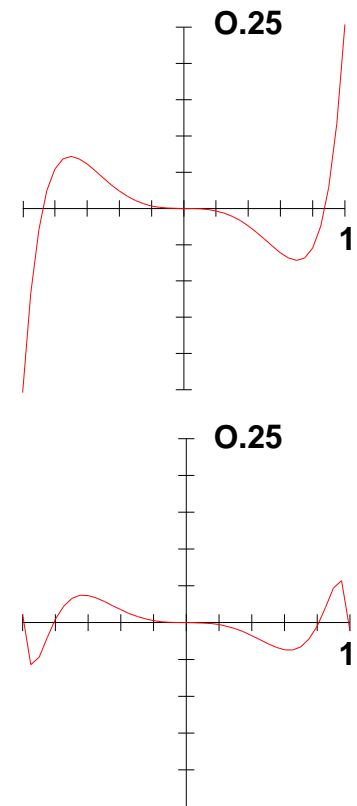


# Higher Order Aberrations: Merte Surface

- Merte surface:
  - low index step
  - strong bending
  - mainly higher aberrations generated



**Transverse  
spherical aberration**



# Polynomial Aspherical Surface

## Standard rotational-symmetric description



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- Basic form of a conic section superimposed by a Taylor expansion of  $z$

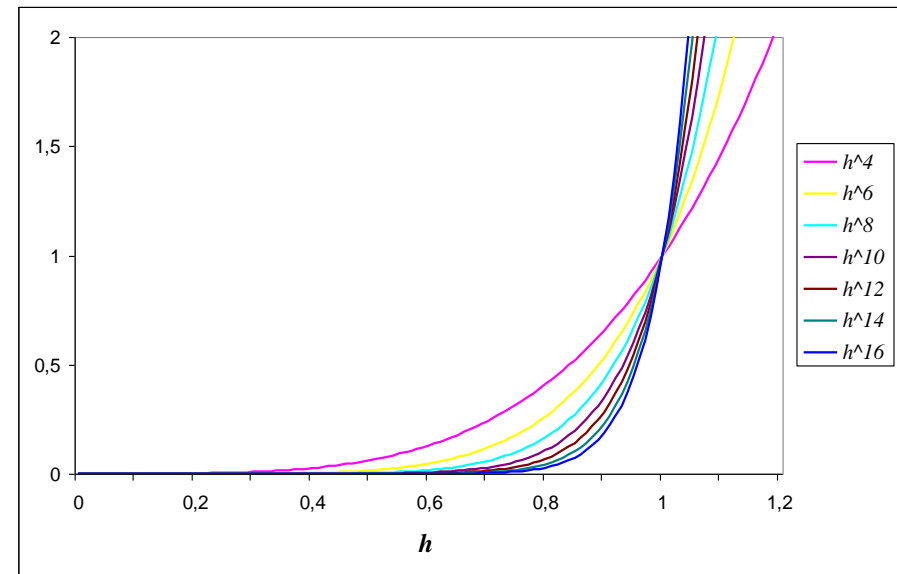
$$z(h) = \frac{\rho h^2}{1 + \sqrt{1 - (1 + c)\rho^2 h^2}} + \sum_{m=0}^M a_m h^{2m+4}$$

$h$  ... Radial distance to optical axis

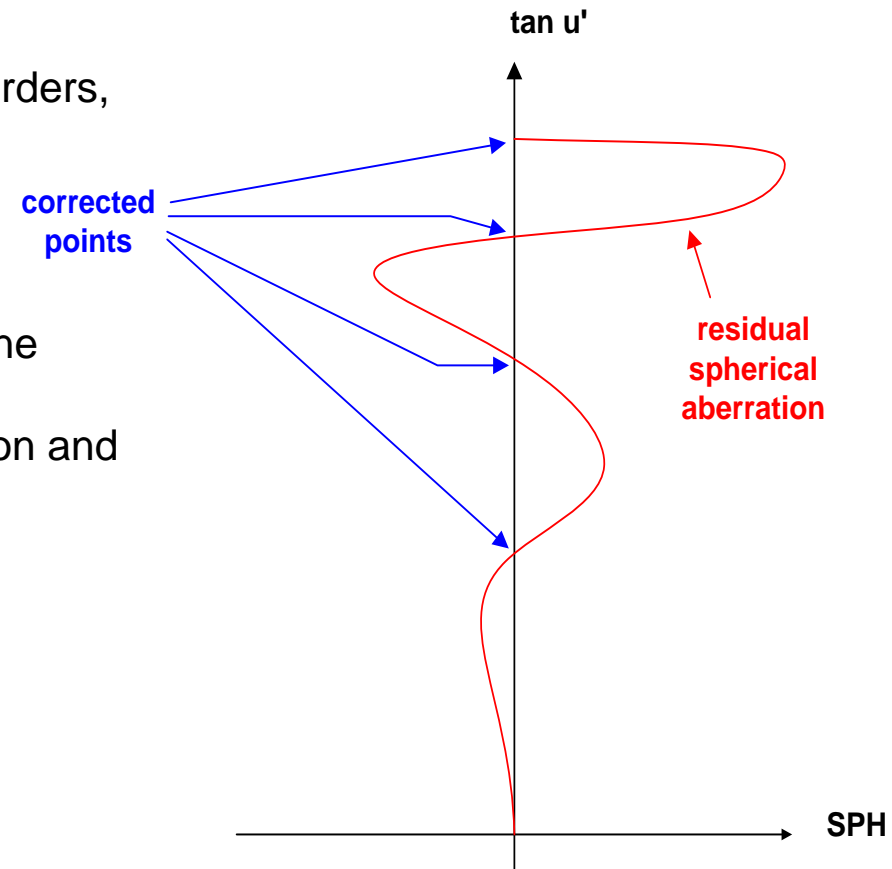
$\rho$  ... Curvature

$c$  ... Conic constant

$a_m$  ... Apherical coefficients



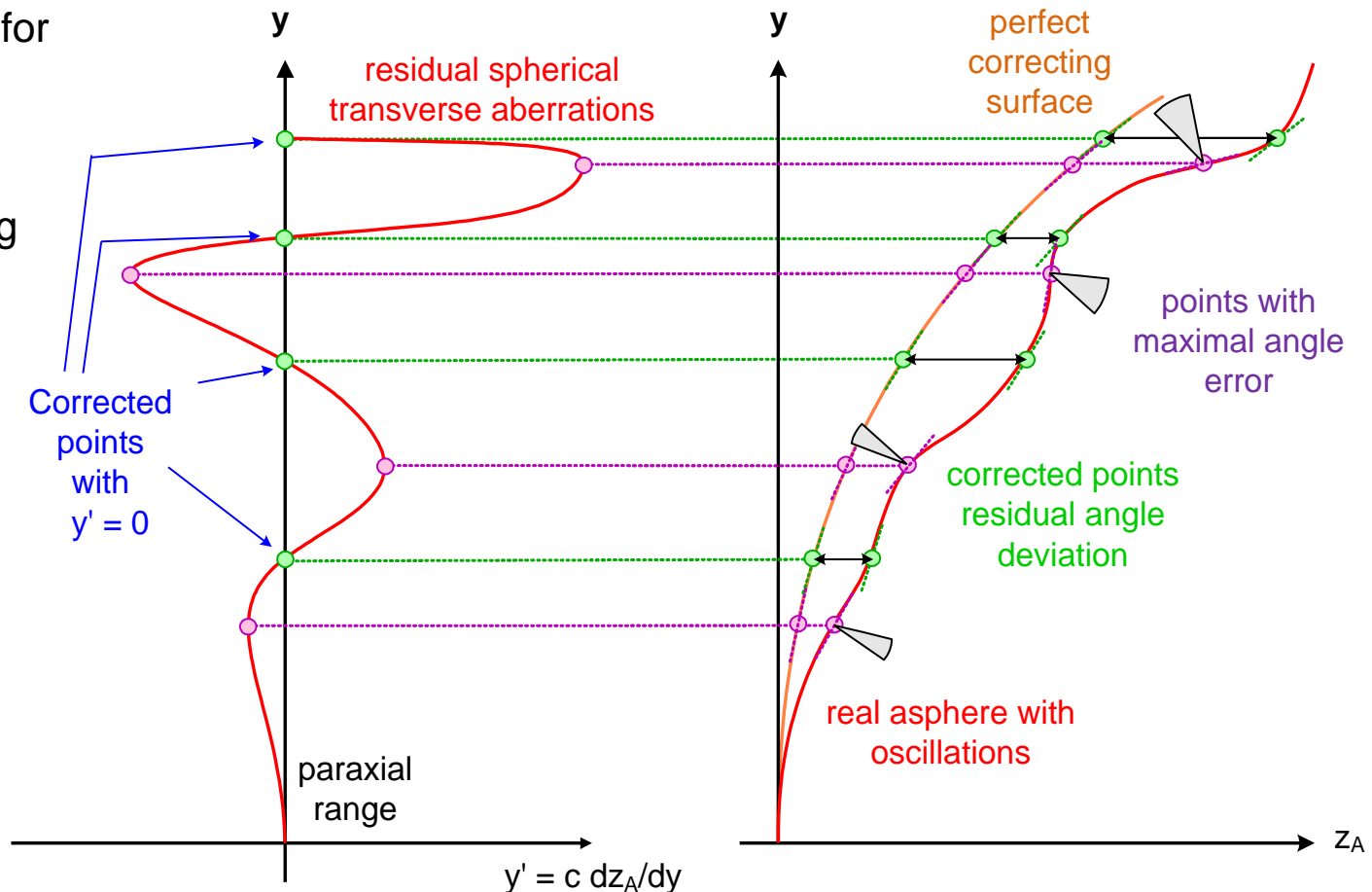
- Additional degrees of freedom for correction
- Exact correction of spherical aberration for a finite number of aperture rays
- Strong asphere: many coefficients with high orders, large oscillative residual deviations in zones
- Location of aspherical surfaces:
  1. spherical aberration: near pupil
  2. distortion and astigmatism: near image plane
- Use of more than 1 asphere: critical, interaction and correlation of higher orders



# Aspheres: Correction of Higher Order



- Correction at discrete sampling
- Large deviations between sampling points
- Larger oscillations for higher orders
- Better description: slope, defines ray bending





- New orthogonalization and normalization using Jacobi-polynomials  $Q_m$

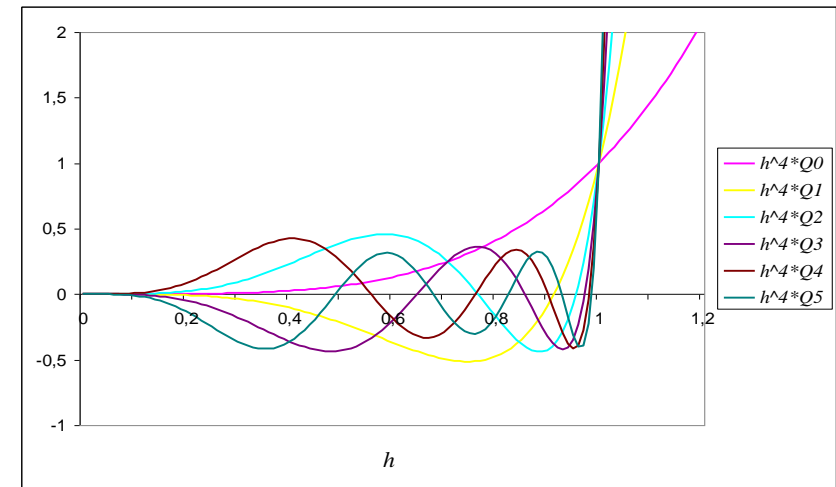
$$z(h) = \frac{\rho h^2}{1 + \sqrt{1 - (1 + c)\rho^2 h^2}} + \left(h / h_{\text{max}}\right)^4 \sum_{m=0}^M a_m Q_m \left(\left(h / h_{\text{max}}\right)^2\right)$$

requires normalization radius  $h_{\text{max}}$

(1:1 conversion to standard aspheres possible)

- Mean square slope

$$\sum_{m=0}^M a_m / (m + 5)$$



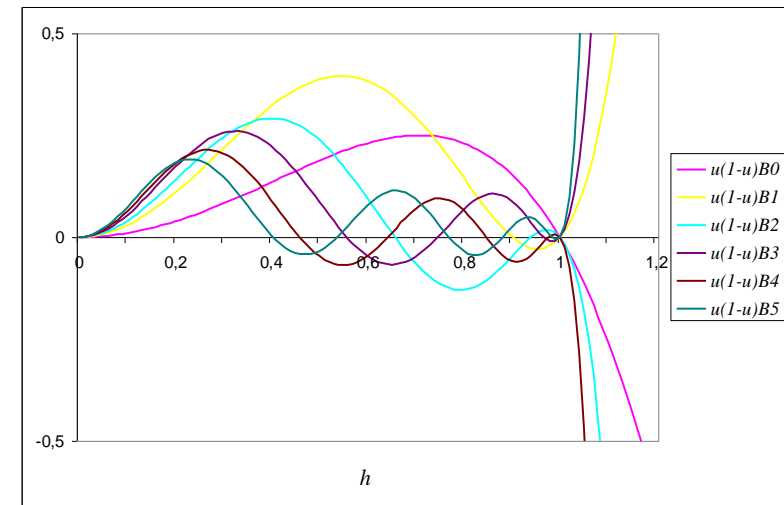
- Limit gradients by special choice of the scalar product

$$z(h) = \frac{\rho_0 h^2}{1 + \sqrt{1 - \rho_0^2 h^2}} + \frac{u(1-u)}{\sqrt{1 - \rho_0^2 h^2}} \sum_{m=0}^M a_m B_m(u) \quad \text{mit} \quad u := (h / h_{\max})^2$$

(1:1 conversion to standard aspheres not possible)

- Mean square slope

$$\left(1 / h_{\max}\right)^2 \sum_{m=0}^M a_m^2$$

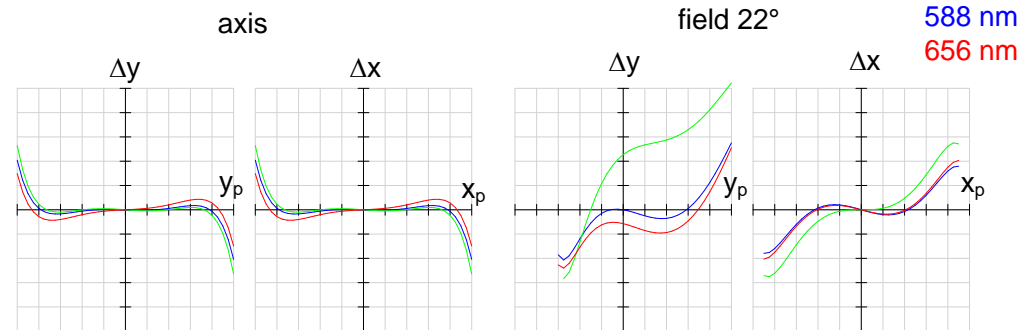
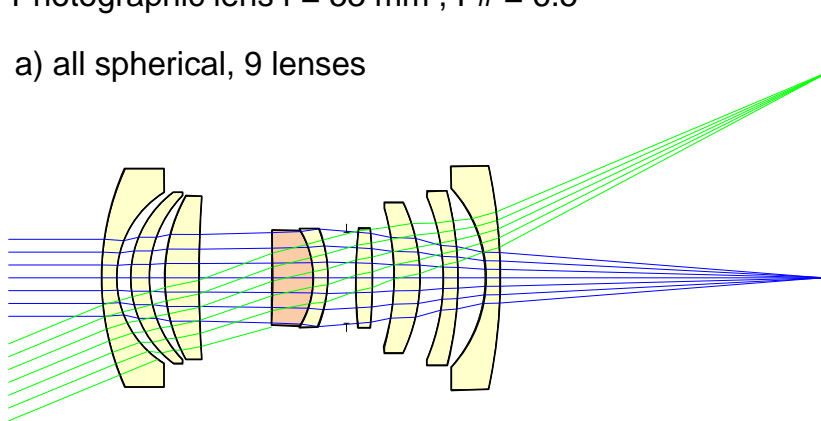


# Reducing the Number of Lenses with Aspheres

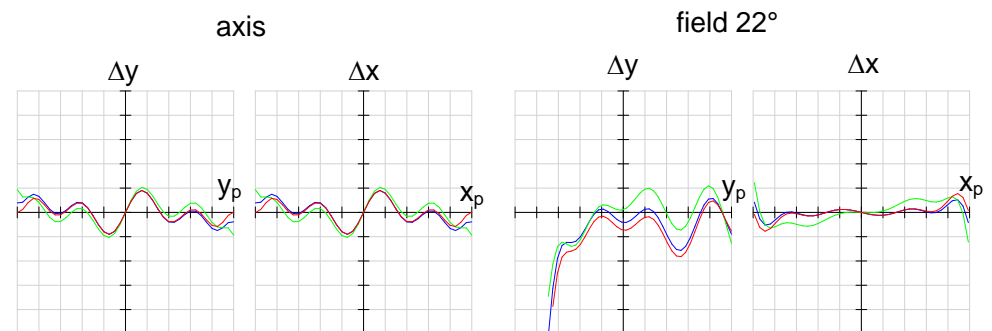
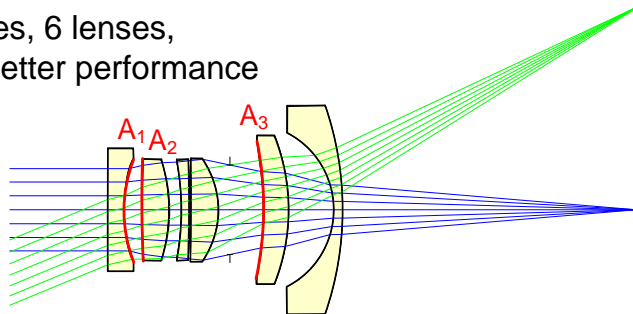
- Example photographic zoom lens
- Equivalent performance
- 9 lenses reduced to 6 lenses
- Overall length reduced

Photographic lens  $f = 53 \text{ mm}$  ,  $F\# = 6.5$

a) all spherical, 9 lenses

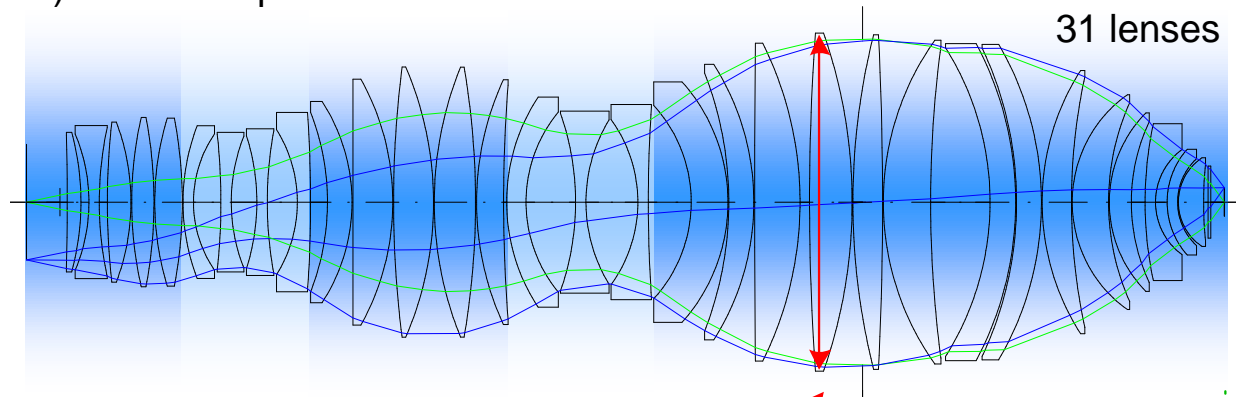


b) 3 aspheres, 6 lenses,  
shorter, better performance

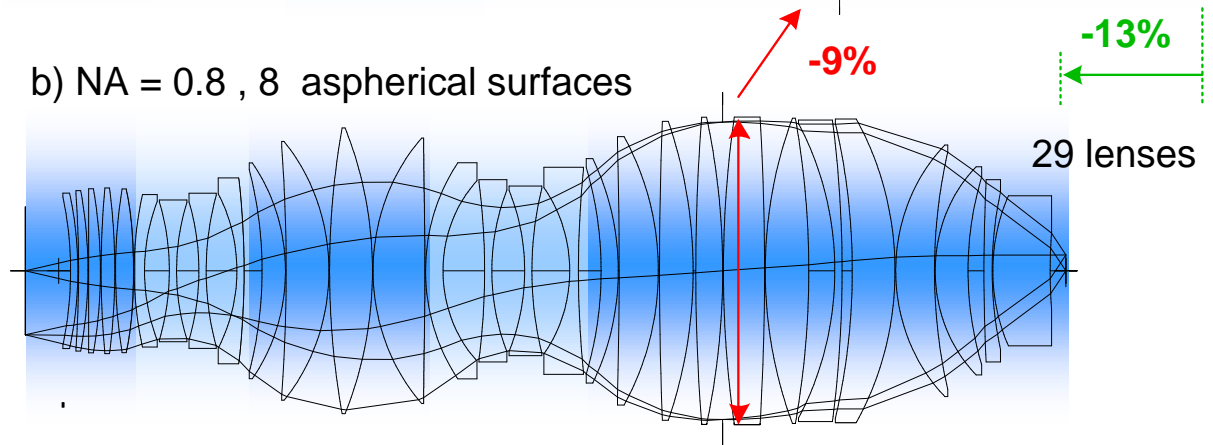


- Considerable reduction of length and diameter by aspherical surfaces
- Performance equivalent
- 2 lenses removable

a) NA = 0.8 spherical



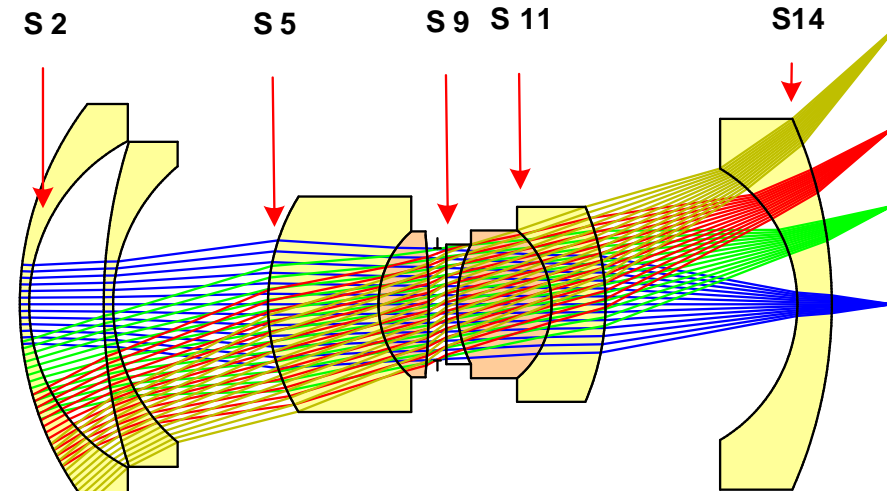
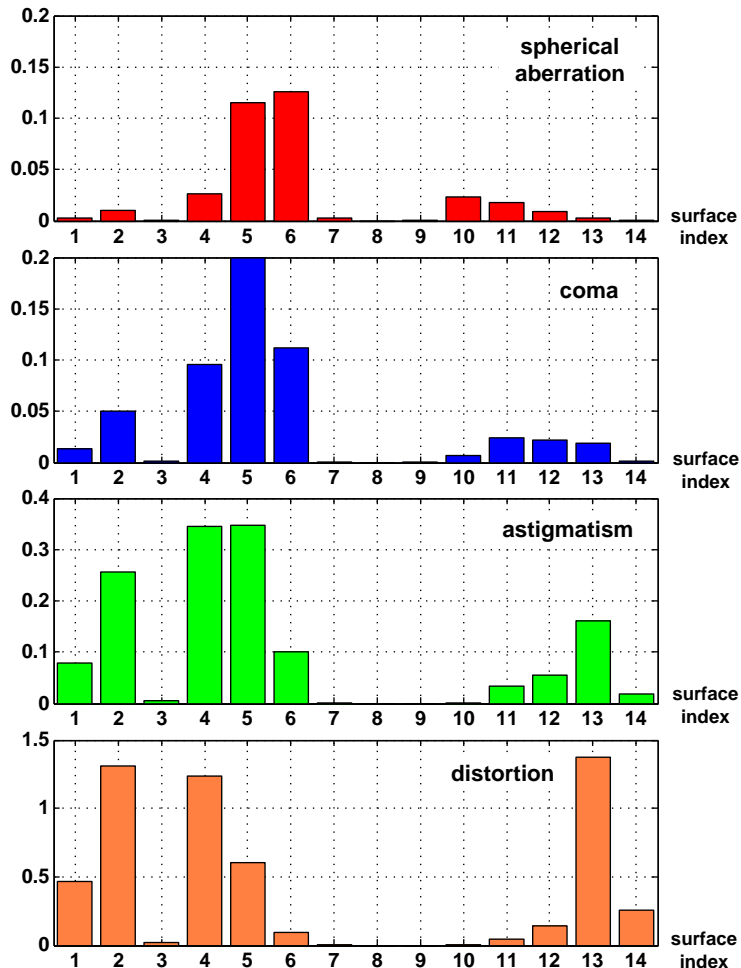
b) NA = 0.8 , 8 aspherical surfaces



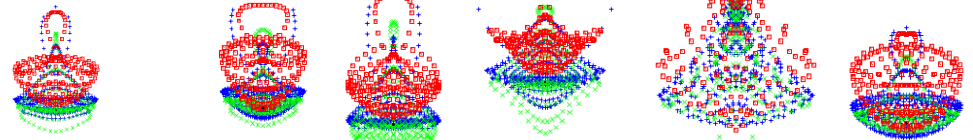


# Aspherization of a Camera Lens

- Selection of one aspherical surface in a photographic lens



spherical system: 197 nm    surface 2: 196 nm    surface 5: 185 nm    surface 9: 187 nm    surface 11: 278 nm    surface 14: 178 nm





# Freeform Systems: Surface Representations

- Extended polynomials  
classical non-orthogonal monomial representation
- Zernike surface  
Only useful for circular pupils and low orders
- Splines  
Localized description, hard to optimize, good for manufacturing characterization
- Generalized Forbes polynomials  
Promising new approach, two types, strong relation to tolerancing
- Radial basis functions  
Non-orthogonal local description approach, good for local effect description
- Wavelets  
Not preferred for smooth surfaces, only feasible for tolerancing
- Fourier representation  
Classical description without assumptions, but not adapted to aberrations
- Smooth vs segmented, faceted, steps, non-Fermat surfaces  
Real world is still more complicated



# Freeform Systems: Equations of Description

- Extended polynomials in x,y:

$$z(x, y) = \frac{c_x x^2 + c_y y^2}{1 + \sqrt{1 - (1 - \kappa_x) c_x^2 x^2 - (1 - \kappa_x) c_x^2 x^2}} + \sum_{n,m=2} a_{n,m} \cdot x^n y^m$$

- Zernike expansion

$$z(x, y) = \frac{c_x x^2 + c_y y^2}{1 + \sqrt{1 - (1 - \kappa_x) c_x^2 x^2 - (1 - \kappa_x) c_x^2 x^2}} + \sum_{j=0} c_j \cdot Z_j(x, y)$$

- Extended Forbes asphere

- Expansion in other orthogonal polynomial systems:  
Legendre, Chebychev, ...

$$z(x, y) = \frac{c r^2}{1 + \sqrt{1 - c^2 r^2}} + \frac{\frac{r^2}{a^2} \cdot \left(1 - \frac{r^2}{a^2}\right)}{\sqrt{1 - c^2 r^2}} \cdot \sum_{n=0} a_n^0 Q_n^0\left(\frac{r^2}{a^2}\right) + \sum_{m=1} \left(\frac{r}{a}\right)^m \sum_{n=0} [a_n^m \cos(m\theta) + b_n^m \sin(m\theta)] \cdot Q_n^m\left(\frac{r^2}{a^2}\right)$$

- Fourier expansion

$$z(x, y) = \frac{c_x x^2 + c_y y^2}{1 + \sqrt{1 - (1 + \kappa_x) c_x^2 x^2 - (1 + \kappa_y) c_y^2 y^2}} + \sum_{n,m} B_{nm} \cdot \text{Re} \left[ e^{-ik_{xm}x - ik_{ym}y} \right]$$

- Expansion into non-orthogonal local shifted Gaussian functions (RBF)

$$z(x, y) = \frac{c_x x^2 + c_y y^2}{1 + \sqrt{1 - (1 + \kappa_x) c_x^2 x^2 - (1 + \kappa_y) c_y^2 y^2}} + \sum_{n,m} a_{nm} \cdot e^{-\left(\frac{x-x_n}{w_x}\right)^2 - \left(\frac{y-y_n}{w_y}\right)^2}$$

- Cubic spline, locally in patch j,k defined as polynomials of order 3

$$z_{j,k}(x, y) = \sum_{m=0}^3 \sum_{n=0}^3 a_{jkmn} \cdot (x - x_j)^m \cdot (y - y_k)^n$$

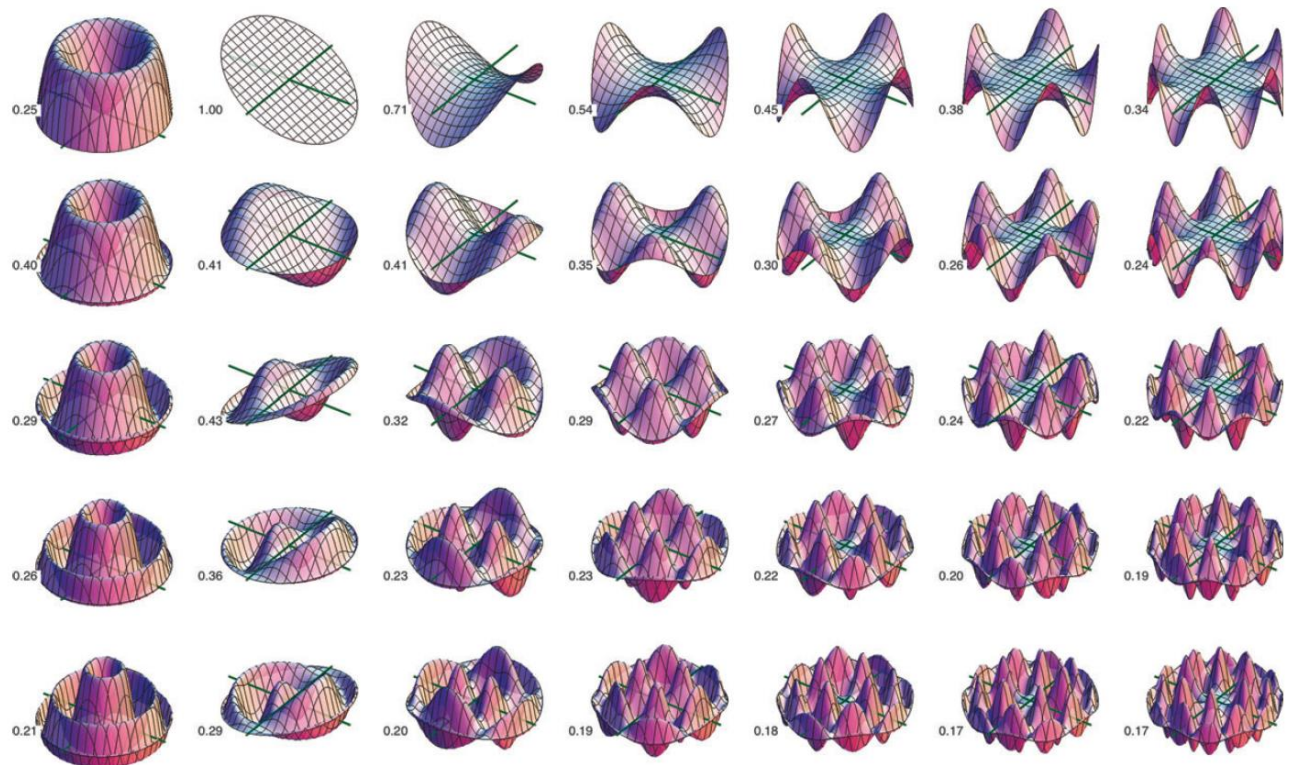




# Freeform Systems: Forbes Surfaces

- Generalized approach for orthogonal surface decomposition
- Slope orthogonality is guaranteed and is related to tolerancing

$$z(x, y) = \frac{cr^2}{1 + \sqrt{1 - c^2 r^2}} + \frac{r^2}{a^2} \cdot \left(1 - \frac{r^2}{a^2}\right) \cdot \sum_{n=0} a_n^0 Q_n^0 \left( \frac{r^2}{a^2} \right) + \sum_{m=1} \left( \frac{r}{a} \right)^m \sum_{n=0} \left[ a_n^m \cos(m\theta) + b_n^m \sin(m\theta) \right] \cdot Q_n^m \left( \frac{r^2}{a^2} \right)$$







# Freeform Systems: Exact Tailoring

- History:
  - exact solutions of Fermat-principle for one wavelength and only a few field points corresponding to the number of surfaces
  - development of algorithms for illumination tailoring
  - mostly methods are applicable for illumination and imaging
- Dimension:
  - 2D is much easier / 3D is complicated and often not unique
- SMS-method of Minano
  - construction of the surfaces ray by ray with simple procedure
  - approved method in illumination and imaging
- Ries tailoring
  - method used since longer time
  - exact algorithm not known
- Oliker-Method for illumination
  - approximation of smooth surface by sequence of parabolic arcs
- Reality:
  - due to finite size of source and broadband applications tailored methods are only useful for finding a good starting system for optimization



# Freeform Systems: Optimization

- Optimization of systems with freeform surfaces:
  - huge number of degrees of freedom
  - large differences in convergence according to surface representation
  - local vs global influence functions
  - definition of performance and formulation of merit function is complicated and cumbersome
- Classical system matrix for local defined splines is ill conditioned
- Starting systems:
  - still more important as in conventional optics
  - only a few well known systems published
  - larger archive for starting systems not available until now
  - own experience usually is poor
- Best location of FFF surfaces inside the system:
  - still more important as in the case of circular symmetric aspheres
  - no criteria known until now



# Freeform Systems: Applications

- General purpose:
  - freeform surfaces are useful for compact systems with small size
  - due to high performance requirements in imaging systems and limited technological accuracy most of the applications are in illumination systems
  - mirror systems are developed first in astronomical systems with complicated symmetry-free geometry to avoid central obscuration
- HMD  
Head mounted device with extreme size constraints
- HUD  
Head up display, only few surfaces allowed
- Schiefspiegler
  - astronomical systems without central obscuration
  - EUV mirror systems for next generation lithography systems
- Illumination systems  
Various applications, smooth and segmented

# Astigmatism of Oblique Mirrors

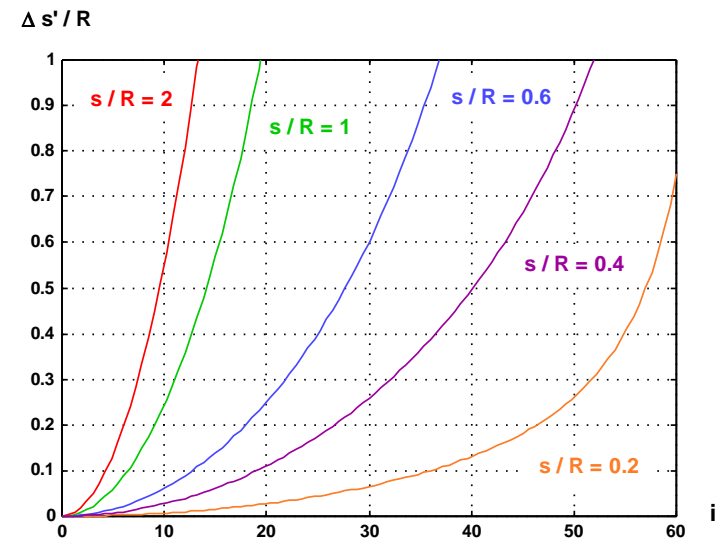
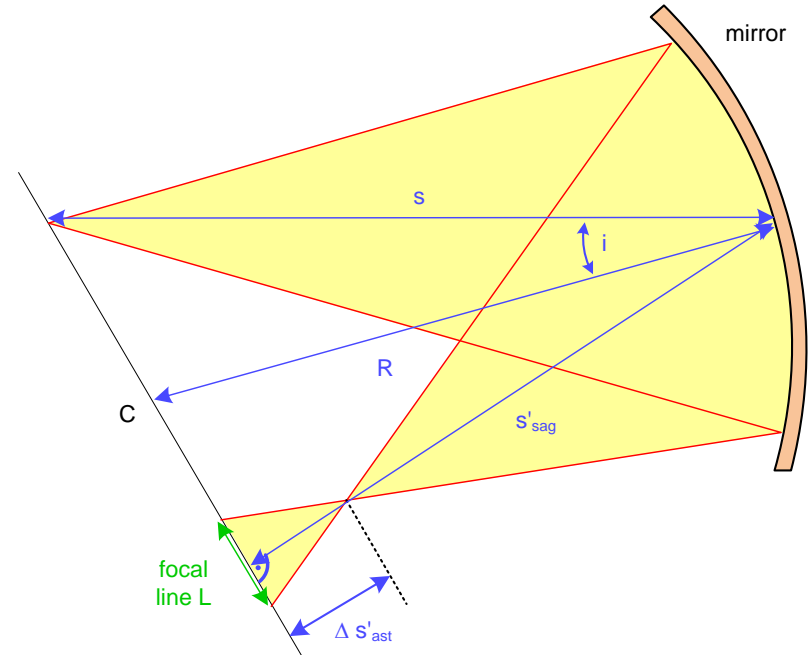
- Mirror with finite incidence angle:  
effective focal lengths

$$f_{\tan} = \frac{R \cdot \cos i}{2} \quad f_{\text{sag}} = \frac{R}{2 \cos i}$$

- Mirror introduces astigmatism

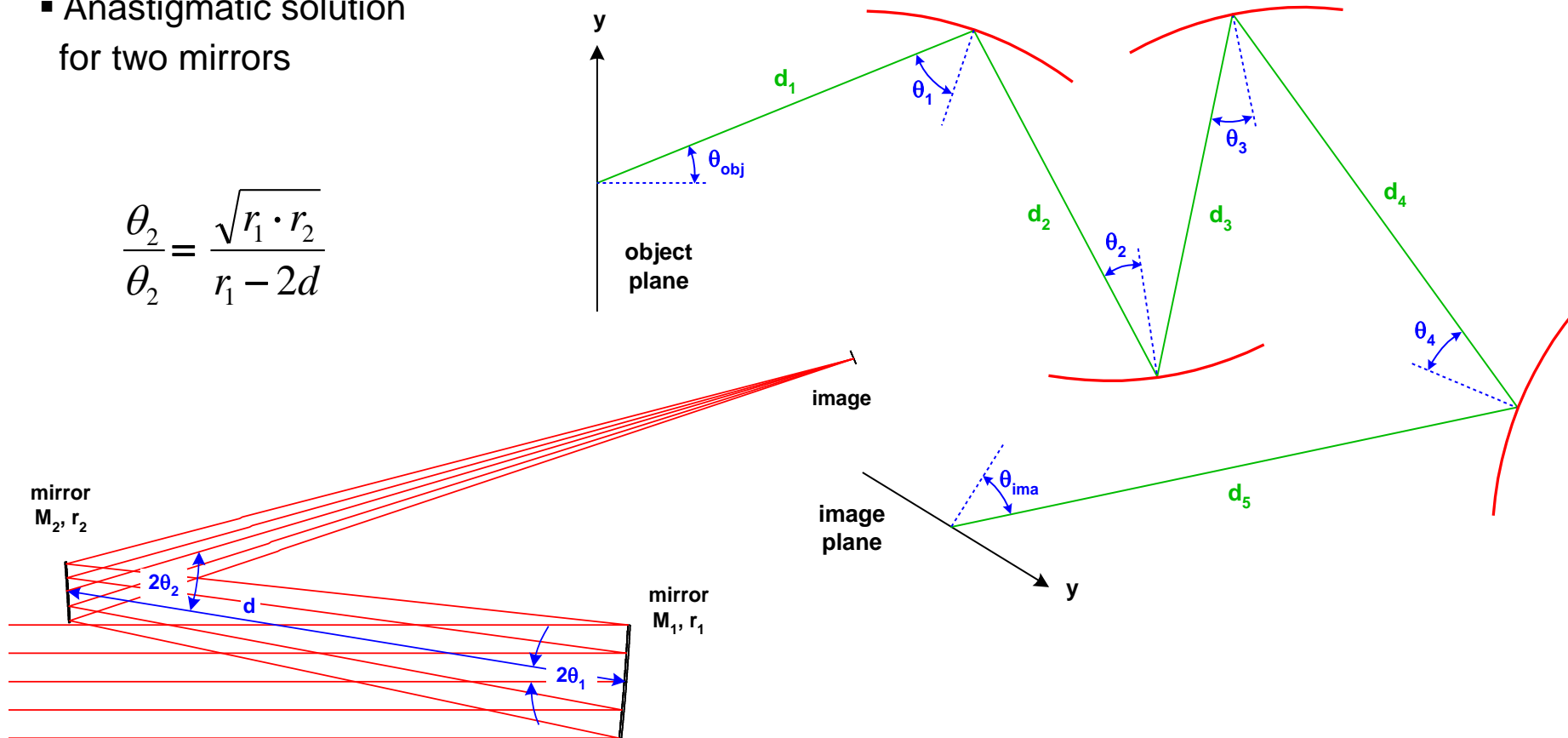
$$\Delta s'_{\text{ast}} = \frac{s^2 \cdot R \cdot \sin^2 i}{2 \cos i \cdot \left(s - \frac{R \cos i}{2}\right) \cdot \left(s - \frac{R}{2 \cos i}\right)}$$

- Parametric behavior of scales astigmatism

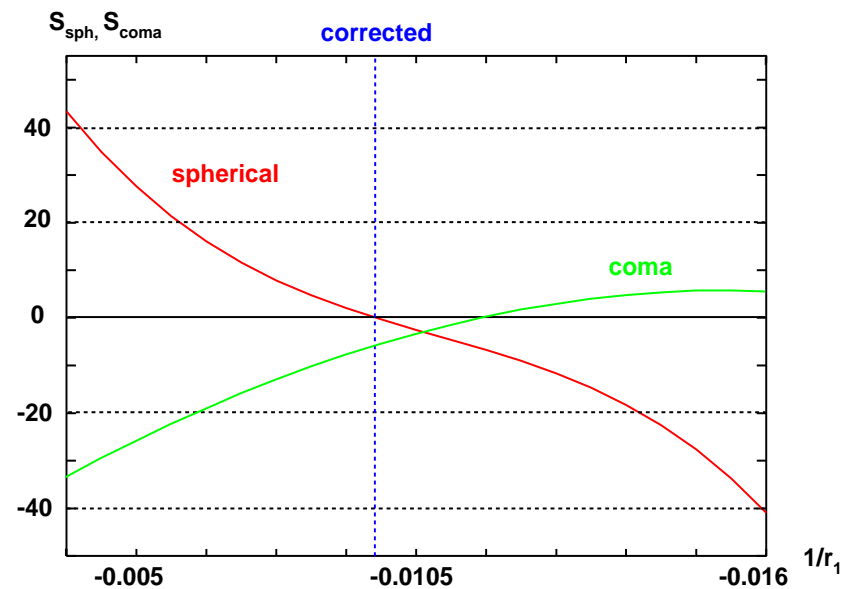
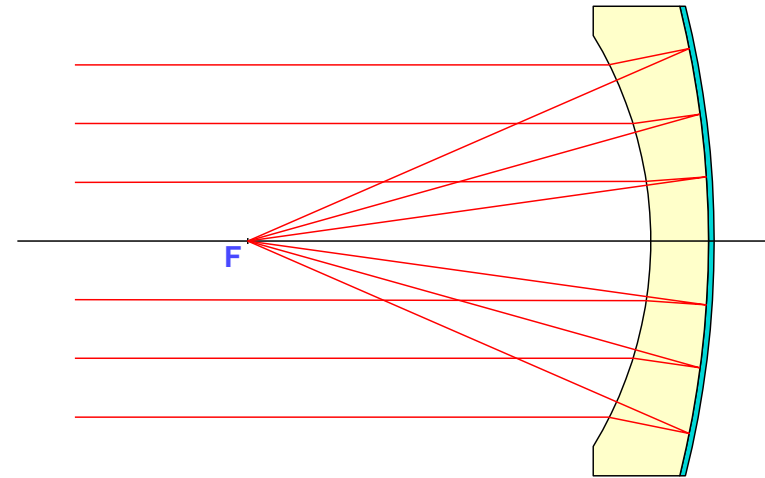


- Telescopes with tilted elements
- Anastigmatic solution for two mirrors

$$\frac{\theta_2}{\theta_1} = \frac{\sqrt{r_1 \cdot r_2}}{r_1 - 2d}$$

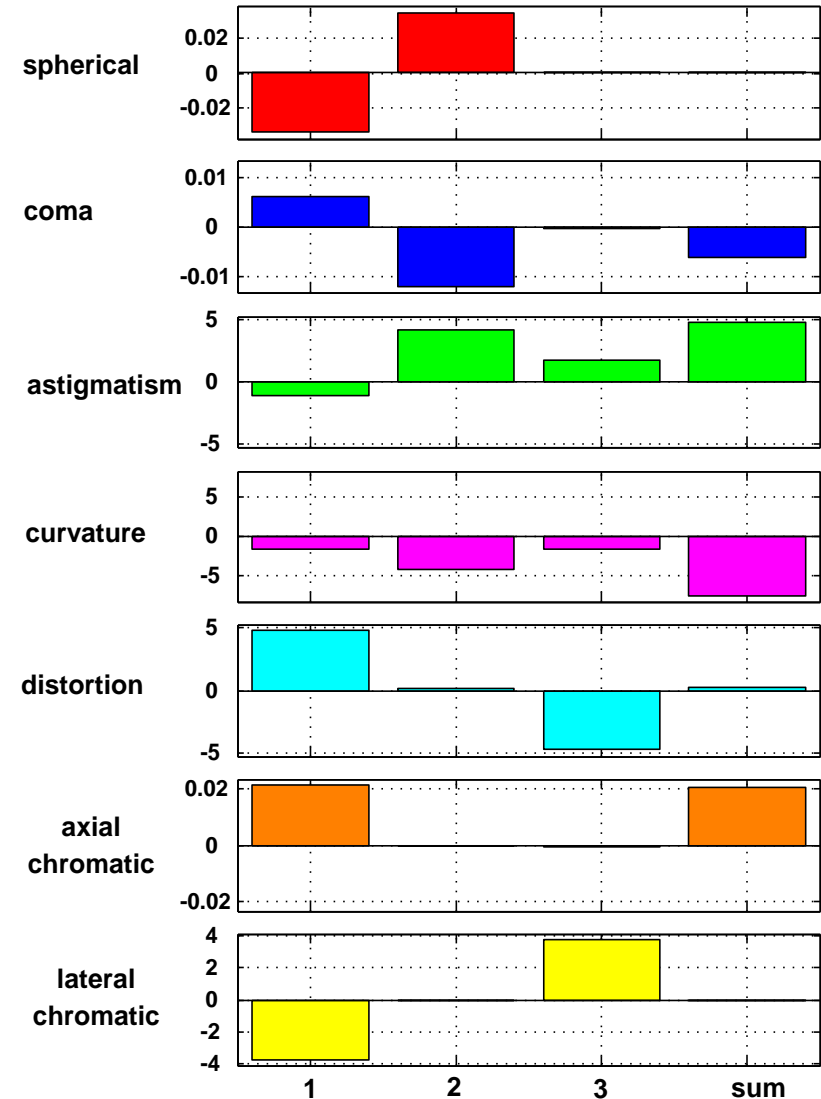
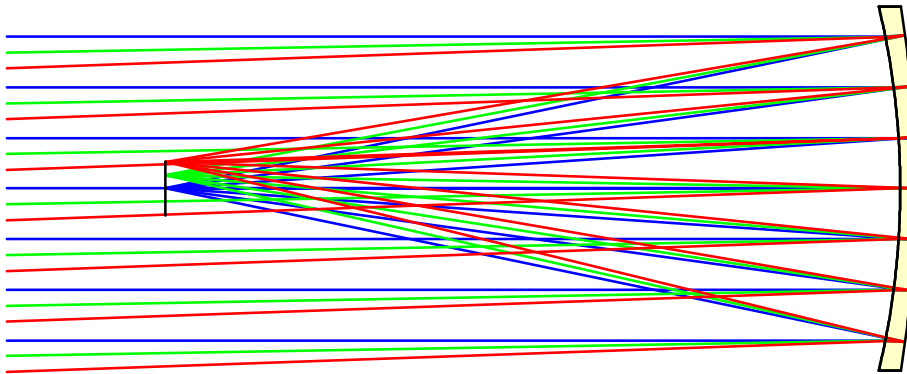


- Principle:  
Backside mirror, catadioptric lens
- Advantages:  
Mirror can be made spherical  
Refractive surface corrects spherical  
System can be made nearly aplanatic



# Mangin Mirror

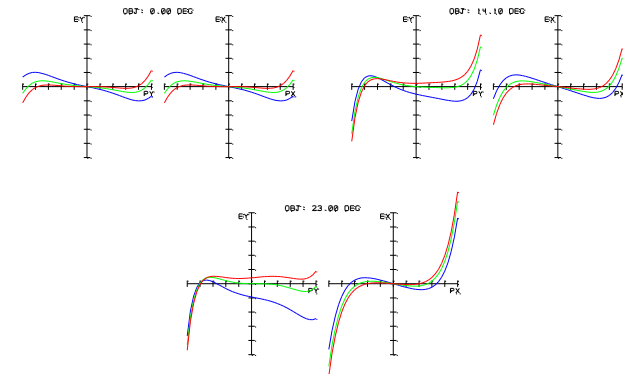
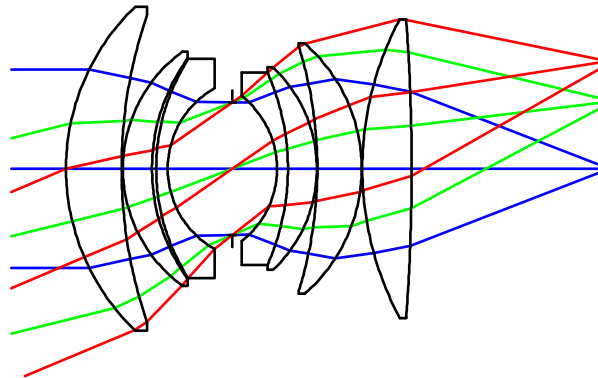
- Seidel surface contributions of a real lens:  
Spherical correction perfect  
Residual axial chromatic unavoidable



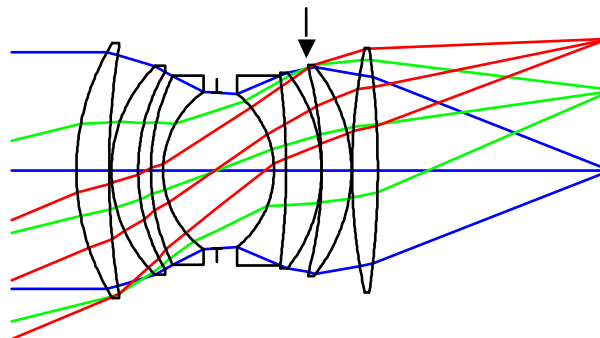
# Aberrations Limited by Vignetting

- Clipping of outer coma rays by vignetting
- Consequences:
  - reduced brightness
  - anisotropic resolution

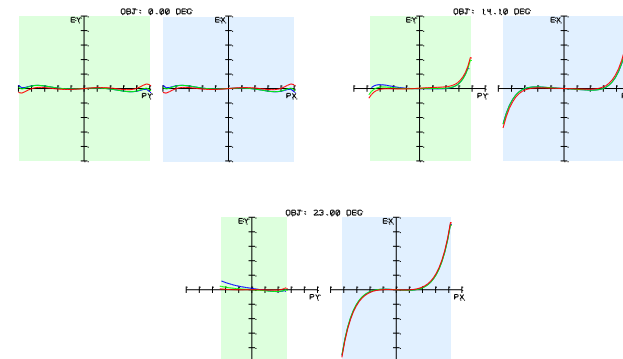
without vignettierung



with vignettierung



tangential / sagittal





# Exercise: Telephoto Lens



A telephoto lens is a system with a focal length, that is considerably larger than the overall length of the system. The general approach is to combine a positive lens group with a negative group.

- a) Establish a telephoto lens system with paraxial lenses for an incoming collimated ray bundle of diameter 10 mm. The numerical aperture after the first group should be  $NA = 0.1$  and  $NA = 0.05$  in the image space for  $\lambda = 587$  nm. The total focal length should be 3 times larger than the free working distance. Determine the focal length of the system and of both lens groups.
- b) Look for achromates for both groups, rescale the negative achromate, if the desired focal length is not available. Re-adjust the distances of the system by requiring the image sided numerical aperture. Is the system diffraction limited in performance ? Explain the result.
- c) Now the wavelength is changed to dFC and a field points of  $10^\circ$  and  $7^\circ$  are introduced. The stop is located at the first surface. The focal length and the free working distance are kept constant. Now optimize the second lens group and allow also for larger diameter. Is the performance diffraction limited ? Finally optimize also the radii of the first lens group. What is the obtained performance ? What happens, if the second group is now reverted back ? What is the remaining dominant aberration type ?
- d) Calculate the polychromatic modulation transfer function of the system on axis. What is the contrast at the spatial frequency 20 LP/mm ? Calculate the MTF behavior versus defocussing for this frequency at the central wavelength 587 nm. Determine the maximum defocussing values, that delivers a contrast not smaller than 50%.

# Time is Over



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