Friedrich Schiller University Jena WS 2022/2023
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Series 5

Fundamentals of Modern Optics

to be returned on **24.11.2022**, at the beginning of the lecture

Task 1: Poynting vector (2+2 points)

a) What is the general connection between the Poynting vector $\mathbf{S}(\mathbf{r},t)$ and the optical intensity $I(\mathbf{r})$?

Now consider the electric field $\mathbf{E}(z,t) = \mathbf{E}_0 \exp[\mathrm{i}(\alpha z - \omega_0 t)] + \mathbf{E}_1 \exp[\mathrm{i}(\beta z - \omega_0 t)]$,
where $\mathbf{E}_0 = A_0 \hat{\mathbf{e}}_x$ and $\mathbf{E}_1 = A_1 \hat{\mathbf{e}}_y$. A_0, A_1, α, and β are real valued.

b) Calculate the time-averaged Poynting vector $\langle \mathbf{S}(\mathbf{r},t)\rangle$ and the intensity.

Task 2: Evanescent and propagating waves in diffraction theory (2+2+2 points)

Consider an external source of electric field in vacuum, oscillating with the frequency ω, with a Gaussian amplitude distribution of $u_0(x,y)$ at the $z=0$ plane, in Cartesian coordinates (neglect the polarization):

$$u_0(x,y) = A \exp\left[-(x^2+y^2)/W^2\right] \quad \text{with} \quad W > 0.$$

a) Compute its spatial frequency spectrum $U_0\left(k_x,k_y\right)$.
b) Consider $U_0\left(k_x,k_y\right)$ as a function of $k_r = \sqrt{k_x^2+k_y^2}$. What is the ratio $U_0(k_r=k_0)/U_0(k_r=0)$? This should only be a function of W and λ. Now consider the specific case of $W=\lambda/\pi$ and plot $U_0(k_r)/U_0(k_r=0)$ as a function of k_r. Have your horizontal axis in units of $k_0 = 2\pi/\lambda$, where λ is the vacuum wavelength. What is the value of $U_0(k_r=k_0)/U_0(k_r=0)$ on this plot?
c) We know from the diffraction theory that the spatial frequency spectrum at the plane of $z=z_0$ is $U\left(k_x,k_y;z_0\right) = U_0\left(k_x,k_y\right)\exp\left[ik_z\left(k_x,k_y\right)z_0\right]$, with $k_z\left(k_x,k_y\right) = \sqrt{k_0^2-k_x^2-k_y^2}$. Plot the real and imaginary part of $k_z\left(k_x,k_y\right)$ in the same plot as part (b). Specify the $\left(k_x,k_y\right)$ ranges of $U_0\left(k_x,k_y\right)$ that contribute to propagating and evanescent waves along the z-direction.

Task 3: Talbot Effect, with and without the Fresnel approximation (5+2+5+3 * points)

Assume an initial field $f(x,z=0)$ (with full translational symmetry in the y-direction), which is periodic along the x-direction with a period of a, such that $f(x+a,z=0) = f(x,z=0)$. We want to calculate the field $f(x,z)$ after propagation along the z-direction, in vacuum, where the vacuum wavelength of the field is λ. If we treat this diffraction problem in the Fresnel (paraxial) approximation, we will find that after a certain length L_T the initial field reappears except for an extra phase, such that

$$f(x,z=L_T) = f(x,z=0)\exp(ikL_T + i2\pi m_l) \quad \text{with } m_l \in \mathbb{Z}$$

This is known as the Talbot effect and L_T is known as the Talbot length.

a) Find the expression for L_T under the assumptions specified above. Hint: You do not need to know the specific expression for the function $f(x)$. Express $f(x)$ as a Fourier series, and then follow through with the standard approach for calculating beam diffraction. Keep in mind that we assume the paraxial approximation to be valid.
b) Which colour light field should be used to have the Talbot length as 25.71 m, given the period to be $a = 3$ mm?

The Talbot effect always holds true in the Fresnel approximation. In contrast, if the Fresnel approximation is not valid, for example when the period a is comparable to the wavelength λ, the Talbot effect does not necessarily take place. However, it can still occur for certain field patterns.

c) Show that for an initial field distribution of the form

$$f(x, z = 0) = A\cos(x2\pi/a_1)\cos(x2\pi/a_2)$$

the Talbot effect still takes place outside the paraxial regime and calculate the Talbot length. Find the value of L_T for the wavelength of $\lambda = 800$ nm and periods $a_1 = 4\mu\text{m}, a_2 = 5\mu\text{m}$.

d*) Consider now an initial field, which is formed as the superposition of three periodic components

$$f(x, z = 0) = A_1\cos(x2\pi/a_1) + A_2\cos(x2\pi/a_2) + A_3\cos(x2\pi/a_3).$$

Show that the Talbot effect in this case will only take place if a certain relation between λ, a_1, a_2, a_3 is satisfied and find this relation.