Institute of Applied Physics

Friedrich-Schiller-Universität Jena

Metrology and Sensing

Lecture 11-1: Phase retrieval

2021-01-14

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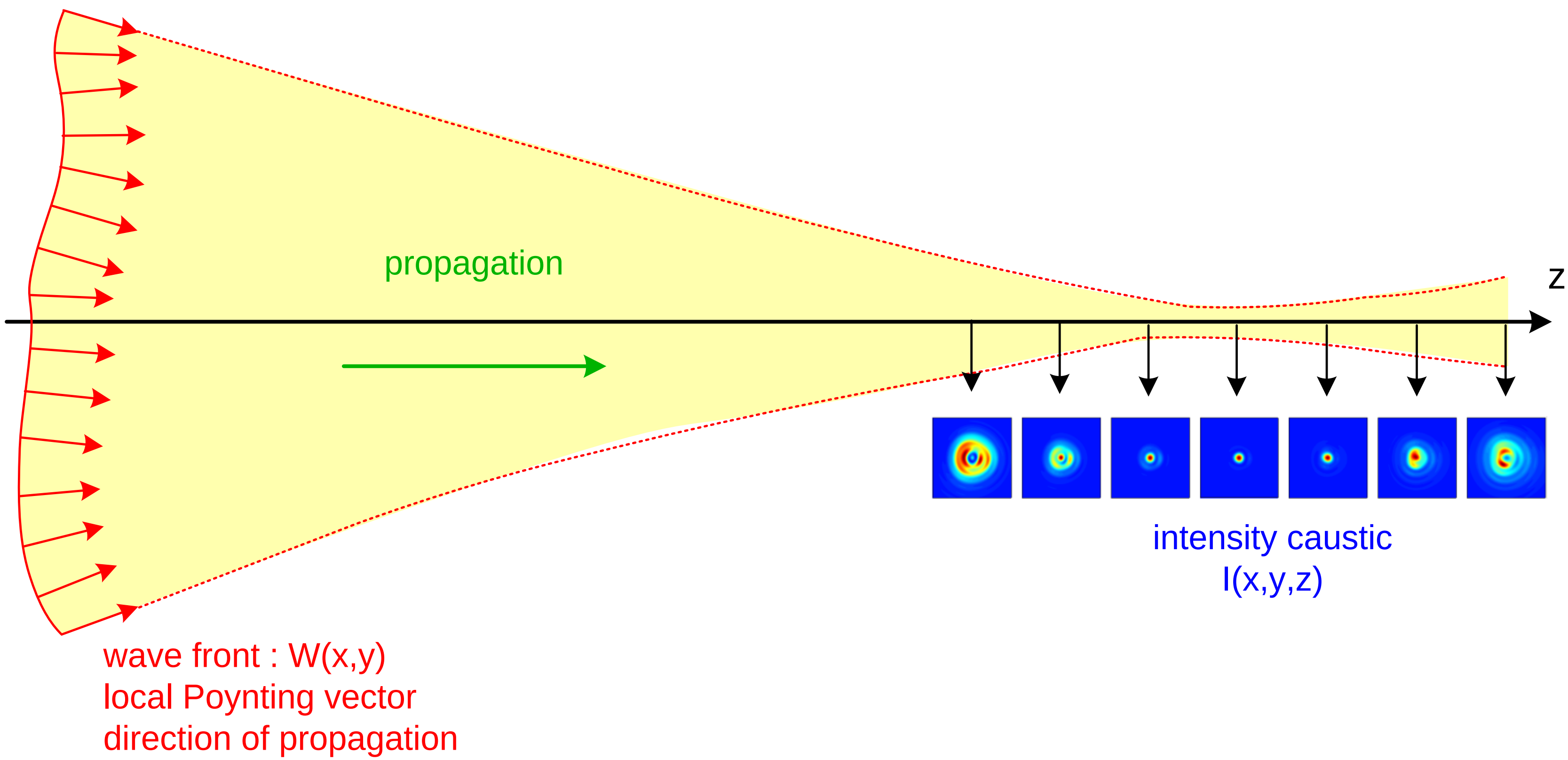
Content

- Basic idea
- Theory
- Illumination

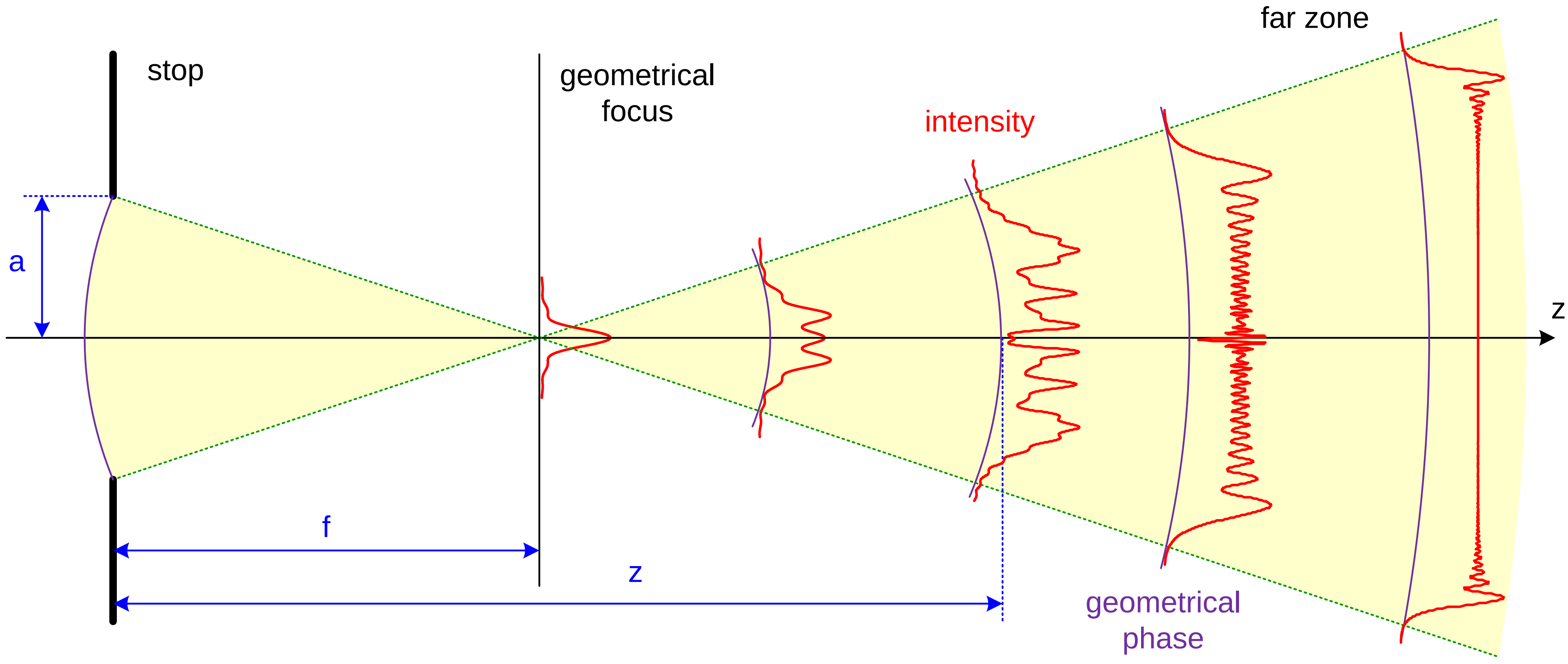
Principle of Psf Phase Retrieval

- Wave front W(x,y) determines local direction of propagation
- Propagation over distance z : change of transverse intensity distribution I(x,y,z)
- Intensity propagation contains amplitude and phase information
- Reconstruction of phase out of intensity measurements



Fresnel Diffraction

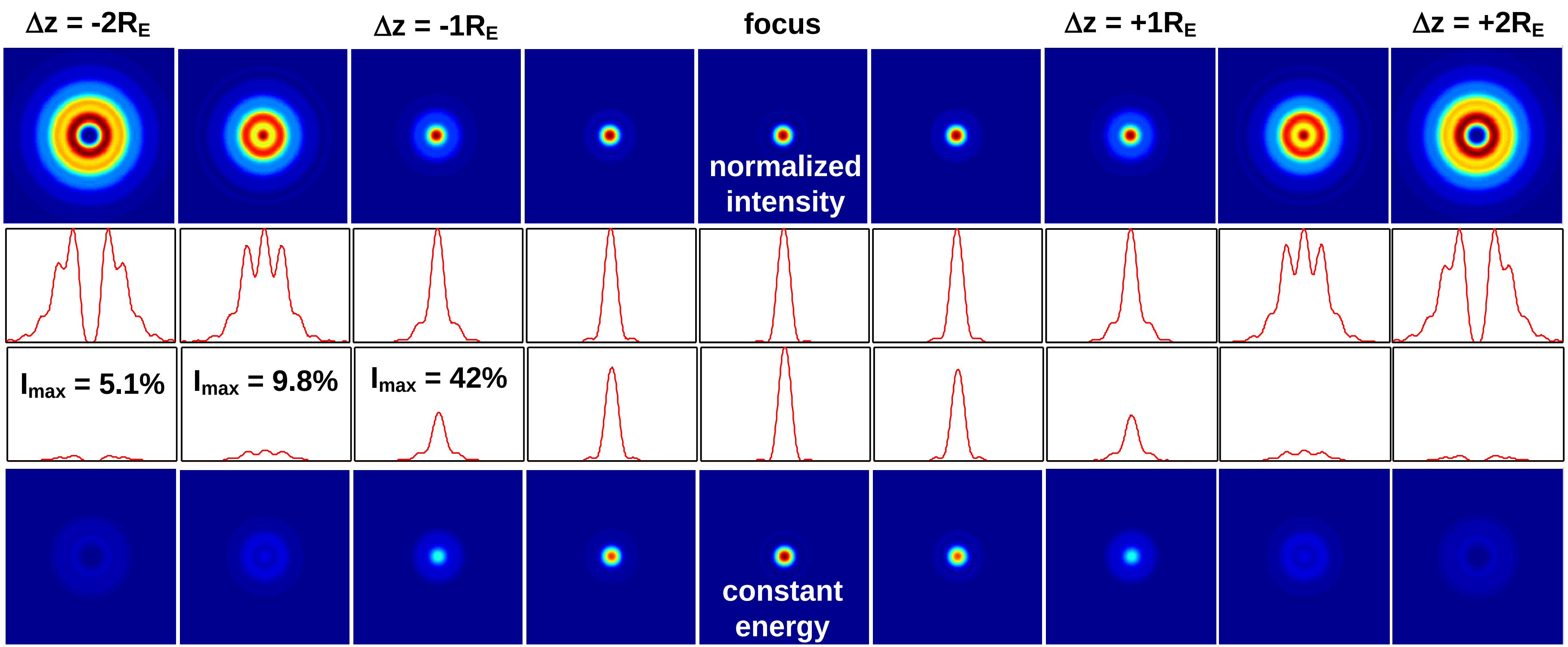
- Typical change of the intensity profile



- Near to the focal plane, the changes of intensity are more significant
- The best extraction of information is obtained from the transition far-near field range
- In particular the imapct of aberrations is nearly not seen in the far zone

Defocussed Perfect Psf

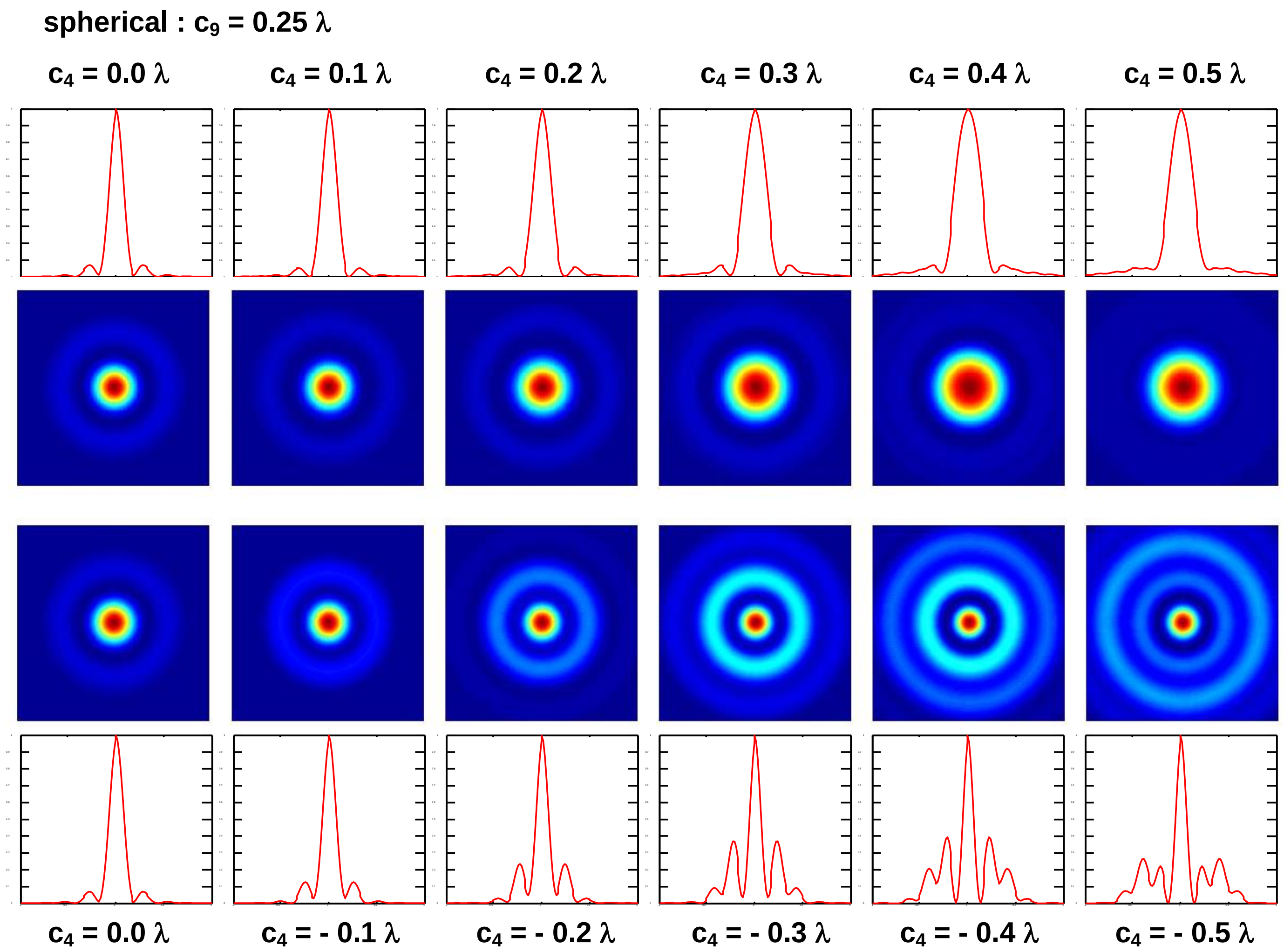
- Perfect point spread function with defocus
- Representation with constant energy: extreme large dynamic changes



Defocused Psf with Spherical Aberration

Asymmetry for different sign of defocus:

- Positive:
  extra focal,
  compact profile
  with broadened
  basis
- Negative:
  intra focal,
  ring structure



Transport of Intensity Equation

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- Paraxial wave equation

$$2ik_o n_o \frac{\partial E}{\partial z} = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + k_o^2 \left[n^2(x,z) - n_o^2 \right] E$$

- Separation of the complex field E into real amplitude A and phase W

$$E(x,y) = A(x,y) \cdot e^{ikW(x,y)}$$

- Separation of real and imaginary part results in two coupled equations

$$\frac{\partial A}{\partial z} = \frac{A}{2}\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} \right) + \frac{\partial A}{\partial x}\frac{\partial W}{\partial x} + \frac{\partial A}{\partial y}\frac{\partial W}{\partial y}$$

$$\frac{\partial W}{\partial z} = -\frac{1}{2Ak^2}\left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} \right) + \frac{1}{2}\left[\left(\frac{\partial W}{\partial x} \right)^2 + \left(\frac{\partial W}{\partial y} \right)^2 \right]$$

- Substituting the amplitude by the intensity

$$A = \sqrt{I}$$

- Transport of intensity equation (TIE) is obtained

$$\frac{\partial I}{\partial z} = -\nabla I \cdot \nabla W - I \cdot \nabla^2 W$$

- Interpretation:
  - change of intensity during propagation I(z) for every lateral position x,y
  - the phase modulates the change, the intensity acts as a weighting

Propagation of Intensity

- Transport of intensity equation couples phase and intensity

$$k \cdot \frac{\partial I(x,y,z)}{\partial z} = -\nabla\left[I(x,y,z)\cdot\nabla W(x,y)\right]$$

- Solution with z-variation of the intensity delivers start phase at z = 0

- Determine phase from intensity distribution.
  - Inverse propagation problem : ill posed
  - Boundary condition : measured z-stack I(x,y,z)

- Algorithm for numerical solution
  - IFTA / Gerchberg Saxton (error reduction)
  - Acceleration (conjugate gradients, Fienup,...)
  - Modal non least square-methods
  - Extended Zernike method

- Applications:
  - Calculation of diffractive components for given illumination distribution
  - Wave front reconstruction
  - Phase microscopy

Properties of Phase Retrieval

- Determine phase from intensity distribution
  - Inverse propagation problem: ill posed
  - Boundary condition:
    redundant information to overcome problems with noise, discretization, errors,...

- Imaging Fourier transfer theory $$I_{image}(x) = I_{psf}(x) * I_{object}(x) + I_{noise}$$

  Approach 1:
  Object known, system (psf) evaluated, metrology of lenses

  $$I_{image}(x) = I_{psf/transfer}(x) * I_{object/\mathrm{illumination}}(x) + I_{noise}$$

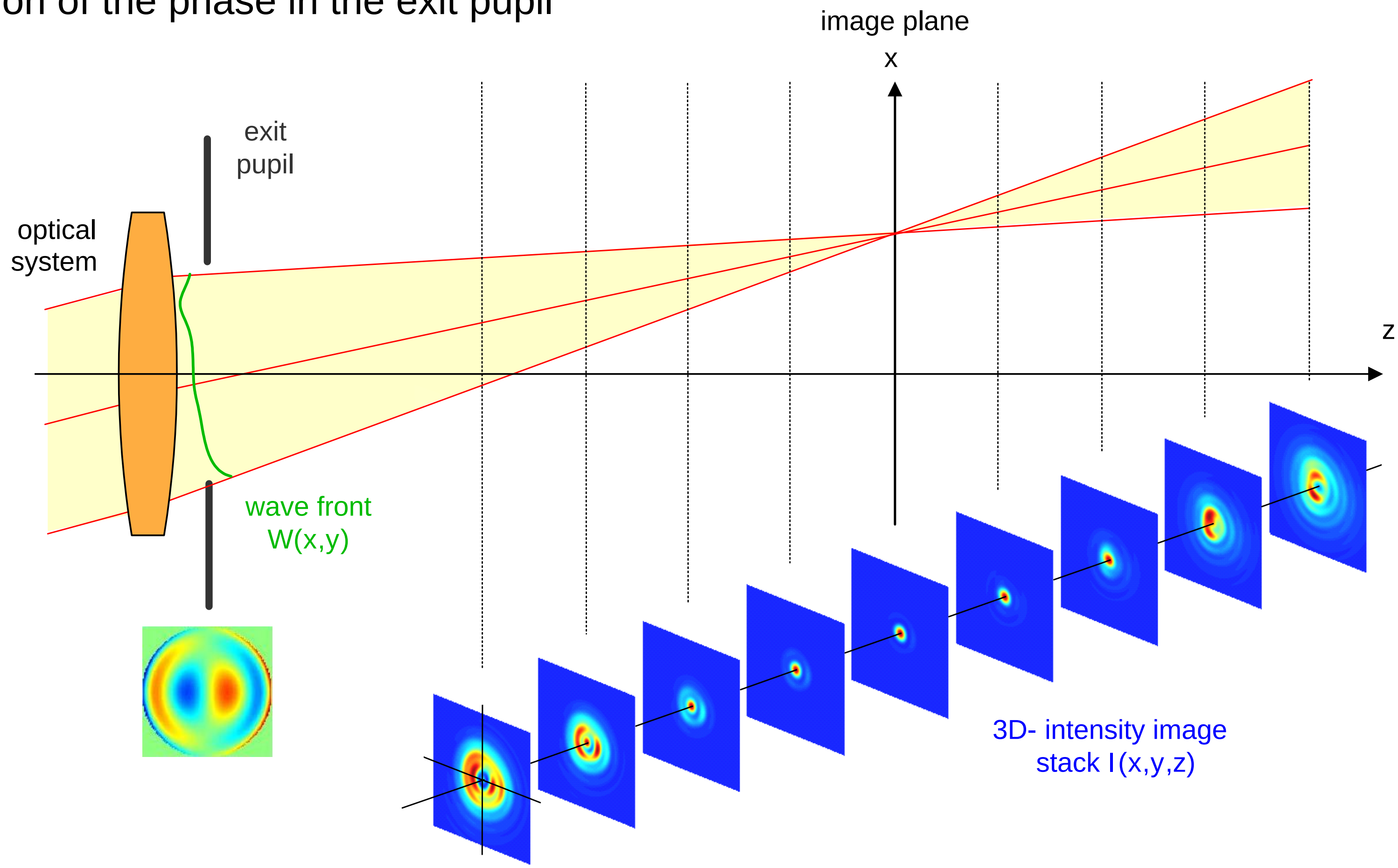
  Approach 2:
  System known, object/illumination evaluated, metrology of beams

  $$I_{image}(x) = I_{psf/transfer}(x) * I_{\mathrm{object/illumination}}(x) + I_{noise}$$

- Applications:
  - Calculation of diffractive components for given illumination distribution (1)
  - Wave front reconstruction (1)
  - Phase imaging / microscopy (2)
  - Laser beam quality metrology (2)

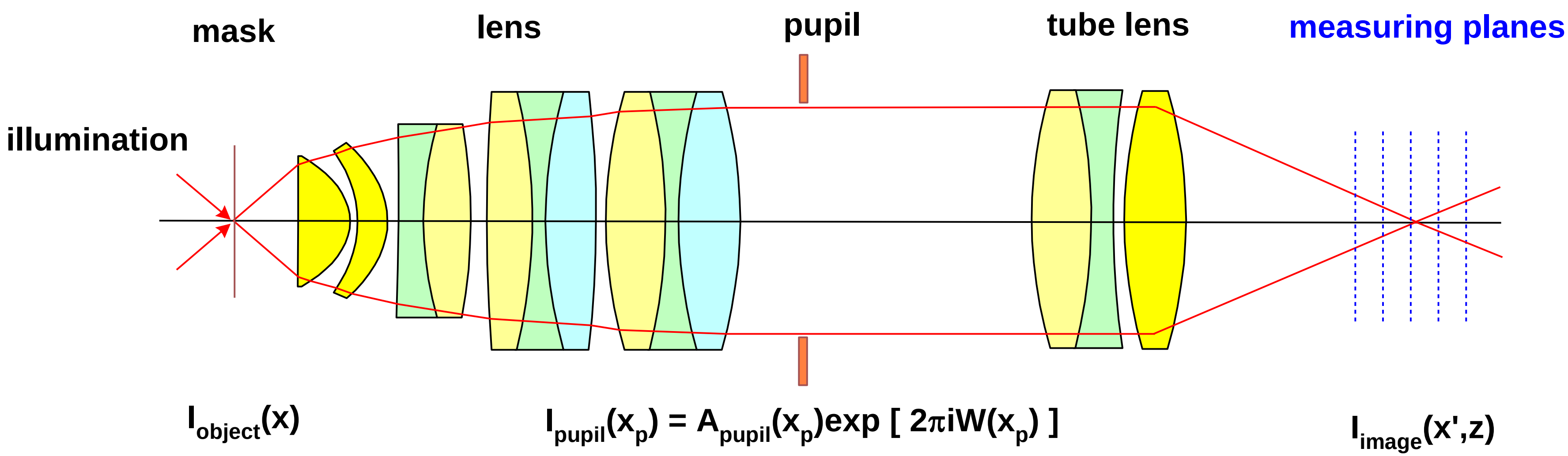
Phase Retrieval

- Principle of phase retrieval for metrology of optical systems
- Measurement of intensity caustic z-stack
- Reconstruction of the phase in the exit pupil



Principal Approach

- Given object $I_{object}(x)$
- Known illumination, usually incoherent
- Known measured image intensities $I_{image}(x',z)$ for several z-values
- To be calculated: transfer function of the system / pupil function
- Relationship: convolution



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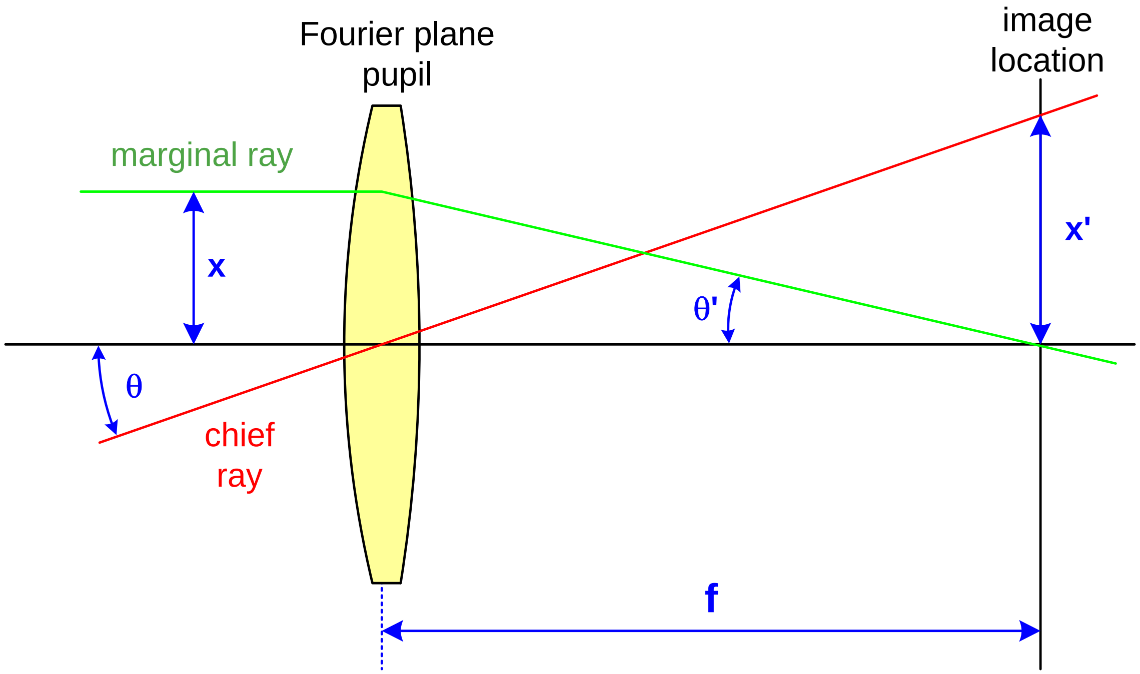
Phase Space: 90° -Rotation

- Transition pupil-image plane: 90° rotation in phase space
- Planes Fourier inverse
- Marginal ray: space coordinate x ---> angle θ'
- Chief ray: angle θ ---> space coordinate x'



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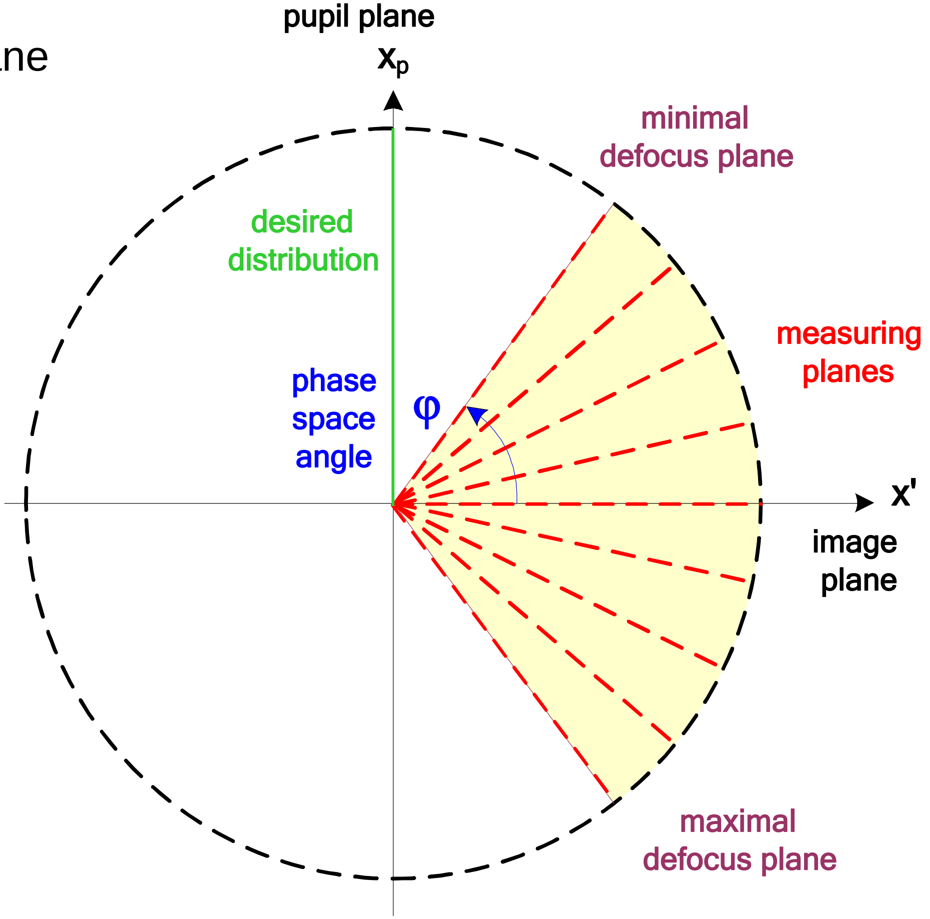
Phase Space Interpretation

- Known measurement of intensity in defocussed planes:
  - Several rotated planes in phase space
  - Information in and near the spatial domain
- Calculation of distribution in the Fourier plane
- Wave equation is valid
- Principle : Tomography



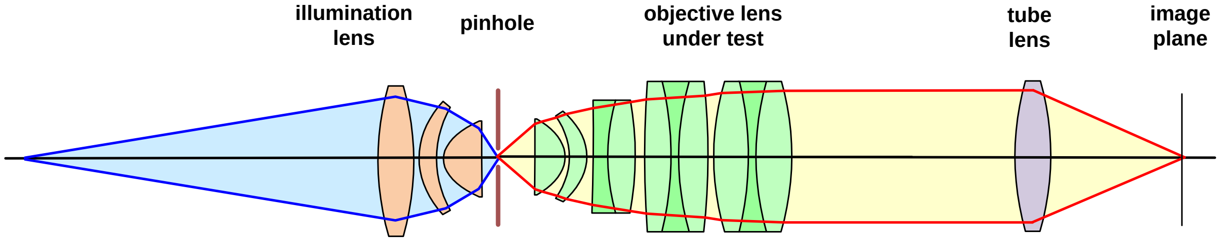
Phase Retrieval Illumination Setups

- Pinhole object
  - Deconvolution may be necessary
  - Illumination incoherent
  - Problems with pinhole

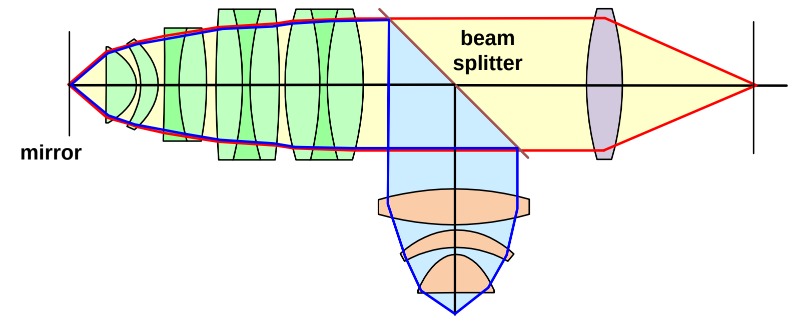
- Epi-illumination
  - Double pass with plane mirror
  - Only symmetrical aberrations
  - No field coma seen

- Trans-illumination with objective
  - Laser source, coherent
  - Calibration necessary
  - Speckle problem

a) Pinhole and trans illumination



b) Epi illumination and plane mirror



c) Calibrated illumination lens