a) Consider two plane waves given as

$$\mathbf{E}_1(\mathbf{r},t) = E_0\cos(kz-\omega t+\delta_1)\hat{x} + E_0\sin(kz-\omega t+\delta_1)\hat{y} \quad \text{and}$$
$$\mathbf{E}_2(\mathbf{r},t) = E_0\cos(kz-\omega t+\delta_2)\hat{x} - E_0\sin(kz-\omega t+\delta_2)\hat{y}.$$

Derive and simplify the expression for the superposition of these waves. Describe in a couple of sentences which polarization have the initial plane waves $\mathbf{E}_1$, $\mathbf{E}_2$ and which polarization has the superimposed field.

b) Derive the expression for the magnetic field $\mathbf{H}$ of the superimposed field.

c) Calculate the instantaneous Poynting vector and the time-averaged Poynting vector of the superimposed field.

$$\cos(\tfrac{\pi}{2}+\alpha)$$
$$\cos(\alpha-\tfrac{\pi}{2})$$

(a) $\vec{E}=\vec{E}_1(\vec{r},t)+\vec{E}_2(\vec{r},t) = E_0[\cos(kz-wt+\delta_1)+\cos(kz-wt+\delta_2)]\hat{x}+E_0[\sin(kz-wt+\delta_1)-\sin(kz-wt+\delta_2)]\hat{y}$

$\cos\alpha = \cos(\frac{\alpha+\beta}{2}+\frac{\alpha-\beta}{2}) = \cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}-\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$\cos\beta = \cos(\frac{\alpha+\beta}{2}-\frac{\alpha-\beta}{2}) = \cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}+\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$\Rightarrow \cos\alpha+\cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\cos(kz-wt+\varphi_1)+\cos(kz-wt+\varphi_2) = 2\cos(kz-wt+\frac{\delta_1+\delta_2}{2})\cos(\frac{\delta_1-\delta_2}{2})$

$\sin\alpha = \sin(\frac{\alpha+\beta}{2}+\frac{\alpha-\beta}{2}) = \sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}+\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$\sin\beta = \sin(\frac{\alpha+\beta}{2}-\frac{\alpha-\beta}{2}) = \sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}-\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$\Rightarrow \sin\alpha-\sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$\sin(kz-wt+\delta_1)-\sin(kz-wt+\delta_2) = 2\cos(kz-wt+\frac{\delta_1+\delta_2}{2})\sin\frac{\delta_1-\delta_2}{2}$

$\Rightarrow \vec{E}_r = 2E_0[\cos\frac{\delta_1-\delta_2}{2}\cos(kz-wt+\frac{\delta_1+\delta_2}{2})\hat{x}+\sin\frac{\delta_1-\delta_2}{2}\cos(kz-wt+\frac{\delta_1+\delta_2}{2})\hat{y}]$

$\delta=(\varphi_x-\varphi_y)$ for E_1 and E_2 $\delta_1=\frac{\pi}{2}$ $\delta_2=-\frac{\pi}{2}$ thus circular Polarization

for $\vec{E}$ $\delta=\frac{\delta_1+\delta_2}{2}-\frac{\delta_1-\delta_2}{2}=0$ thus also linear Polarization

b) $\nabla\times\vec{E}=-\mu_0\frac{\partial\vec{H}}{\partial t}$

$$\nabla\times\vec{E}=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ E_x & E_y & 0\end{vmatrix} = -\frac{\partial}{\partial z}E_y\hat{x}+\frac{\partial}{\partial z}E_x\hat{y}+(\frac{\partial}{\partial x}E_y-\frac{\partial}{\partial y}E_x)\hat{z} = -\frac{\partial}{\partial z}E_y\hat{x}+\frac{\partial}{\partial z}E_x\hat{y}$$

$\frac{\partial}{\partial z}E_y = 2kE_0\sin\frac{\delta_1-\delta_2}{2}\sin(kz-wt+\frac{\delta_1+\delta_2}{2})$ $\quad \frac{\partial}{\partial z}E_x = -2kE_0\cos\frac{\delta_1-\delta_2}{2}\sin(kz-wt+\frac{\delta_1+\delta_2}{2})$

$\Rightarrow -\mu_0\frac{\partial\vec{H}}{\partial t} = 2kE_0[\sin\frac{\delta_1-\delta_2}{2}\sin(kz-wt+\frac{\delta_1+\delta_2}{2})\hat{x}-\cos\frac{\delta_1-\delta_2}{2}\sin(kz-wt+\frac{\delta_1+\delta_2}{2})\hat{y}]$

$\Rightarrow \vec{H}_r(z,t) = \frac{2kE_0}{w\mu_0}[-\sin\frac{\delta_1-\delta_2}{2}\cos(kz-wt+\frac{\delta_1+\delta_2}{2})\hat{x}+\cos\frac{\delta_1-\delta_2}{2}\cos(kz-wt+\frac{\delta_1+\delta_2}{2})\hat{y}]$

(c) $$S(\vec{r},t)=\vec{E}_r\times\vec{H}_r = \begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ E_x & E_y & 0\\ H_x & H_y & 0\end{vmatrix} = (E_xH_y-E_yH_x)\hat{z}$$

$$S(\vec{r},t)=\frac{4E_0^2k}{\mu_0 w}[\cos^2\frac{\delta_1-\delta_2}{2}\cos^2(kz-wt+\frac{\delta_1+\delta_2}{2})+\sin^2(\frac{\delta_1-\delta_2}{2})\cos^2(kz-wt+\frac{\delta_1+\delta_2}{2})]$$

$$=\frac{4E_0^2k}{\mu_0 w}\cos^2(kz-wt+\frac{\delta_1+\delta_2}{2})$$

$$\langle S(\vec{r},t)\rangle = \frac{1}{2}Re[\vec{E}(r,t)\times\vec{H}^*(r,t)] = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}S(\vec{r},t)\,dt$$

$$\int_{-T}^{T}S(\vec{r},t)\,dt = \frac{4E_0^2k}{\mu_0 w}\int_{-T}^{T}\cos^2(kz-wt+\frac{\delta_1+\delta_2}{2})\,dt = \frac{2E_0^2k}{\mu_0 w}\int_{-T}^{T}[1+\cos(2kz-2wt+\delta_1+\delta_2)]\,dt$$

$$= \frac{2E_0^2 k}{\mu_0 w}\left(2T + \int_{-T}^{T} \cos(2kz - 2wt + \delta_1 + \delta_2)dt\right)$$

$$= \frac{2E_0^2 k}{\mu_0 w}\left[2T - \frac{1}{2w}\sin(2kz - 2wt + \delta_1 + \delta_2)\Big|_{-T}^{T}\right]$$

$$= \frac{2E_0^2 k}{\mu_0 w}\left[2T + \frac{1}{w}\cos(2kz + \delta_1 + \delta_2)\sin(2wT)\right]$$

$$\lim_{T\to\infty} \frac{2E_0^2 k}{\mu_0 w}\left[1 + \frac{\cos(2kz + \delta_1 + \delta_2)\sin(2wT)}{2Tw}\right] = \frac{2E_0^2 k}{\mu_0 w}$$

$$\vec{H}(z_i) = -\frac{2kE_0}{w\mu_0}\sin\frac{\delta_1 - \delta_2}{2} e^{i(kz + \frac{\delta_1+\delta_2}{2})}\hat{x} + \frac{2kE_0}{w\mu_0}\cos\frac{\delta_1 - \delta_2}{2} e^{i(kz + \frac{\delta_1+\delta_2}{2})}\hat{y}$$

$$\vec{E}(z) = 2E_0\cos\frac{\delta_1 - \delta_2}{2} e^{i(kz + \frac{\delta_1+\delta_2}{2})}\hat{x} + 2E_0\sin\frac{\delta_1 - \delta_2}{2} e^{i(kz + \frac{\delta_1+\delta_2}{2})}\hat{y}$$

$$\vec{E}(\vec{r}) \times \vec{H}^*(\vec{r}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ E_x & E_y & 0 \\ H_x^* & H_y^* & 0 \end{vmatrix} = (E_x H_y^* - E_y H_x^*)\hat{z}$$

$$= \frac{4E_0^2 k}{w\mu_0}\cos^2\frac{\delta_1 - \delta_2}{2} + \frac{4E_0^2 k}{w\mu_0}\sin^2\frac{\delta_1 - \delta_2}{2} = \frac{4E_0^2 k}{w\mu_0}$$

$$\langle S(\vec{r}, t)\rangle = \frac{1}{2}\mathrm{Re}[\vec{E}(\vec{r}) \times \vec{H}^*(\vec{r})] = \frac{2E_0^2 k}{w\mu_0}$$

The Jones formalism is a powerful technique for the treatment of polarized light. To take a look at it, let's consider a monochromatic plane wave in vacuum of the form $\mathbf{E}(r,t) = \mathbf{E_0}\, e^{i(kz-\omega t)}$, where the electric field is polarized in the (x,y)-plane, so $\mathbf{E_0} = (E_x, E_y, 0)$. We can write it in a form of so-called "Jones vector":

$$\mathbf{J}_{\text{in}} = \begin{pmatrix} E_x\, e^{i\varphi_x} \\ E_y\, e^{i\varphi_y} \end{pmatrix}.$$

Then light propagation through a polarizing optical element can be written as a linear transformation:

$$\mathbf{J}_{\text{out}} = \hat{\mathbf{T}} \cdot \mathbf{J}_{\text{in}}.$$

where $\hat{\mathbf{T}}$ denotes a "Jones matrix" of that element.

Recommendation: For more information about the Jones formalism we strongly recommend to read B.E.A. Saleh, M.C. Teich *"Fundamentals of Photonics"*, Chapter 6, pp.197-205.

a) Elements of the Jones vectors and Jones matrices depend on the choice of the coordinate system. Consider, the Jones matrix of an x-polarizer is given by $\hat{\mathbf{T}} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. Now we rotate the polarizer around z by an angle 2θ. Using matrix methods, define the Jones matrix in the new coordinate system.
Hint: Make use of the rotation matrix.

b) Is it possible to rotate the polarization direction of linearly polarized light by 90° using two linear polarizers? If yes, estimate in terms of intensity (excluding absorption), how large would be the total loss of such a system. How large would be the loss if one could use infinitely many linear polarizers? Explain your answers.

c) Given is an optical element characterized by Jones matrix $\hat{\mathbf{T}} = \frac{1}{2}\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$. Which polarization state should the light have to pass the element without change?

(a) $T' = R(\theta)\,T\,R(-\theta) \Rightarrow \hat{T}' = R(2\theta)\,\hat{T}\,R(-2\theta) = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos 2\theta & 0 \\ \sin 2\theta & 0 \end{bmatrix}\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} \cos^2 2\theta & \sin 2\theta\cos 2\theta \\ \sin 2\theta\cos 2\theta & \sin^2 2\theta \end{bmatrix}$$

b. x-polarized light $J_x = \hat{T} J_{in} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} E_x e^{i\varphi_x} \\ E_y e^{i\varphi_y} \end{bmatrix} = \begin{bmatrix} E_x e^{i\varphi_x} \\ 0 \end{bmatrix}$

Firstly rotating the polarized light at an angle of θ.

$$\Rightarrow J_\theta = R(\theta)\,T\,R(-\theta)\,J_x = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}\begin{bmatrix} E_x e^{i\varphi_x} \\ 0 \end{bmatrix} = \begin{bmatrix} E_x\cos^2\theta\, e^{i\varphi_x} \\ E_x\sin\theta\cos\theta\, e^{i\varphi_x} \end{bmatrix}$$

Then rotating the rotated light at an angle of $90° - \theta$. i.e. $\frac{\pi}{2}$ from the start.

$$J_{\frac{\pi}{2}} = R(\tfrac{\pi}{2})\,T(R(-\tfrac{\pi}{2}))J_0 = \begin{bmatrix} \cos^2\frac{\pi}{2} & \sin\frac{\pi}{2}\cos\frac{\pi}{2} \\ \sin\frac{\pi}{2}\cos\frac{\pi}{2} & \sin^2\frac{\pi}{2} \end{bmatrix} \begin{bmatrix} E_x\cos^2\theta\, e^{i\varphi_x} \\ E_x\sin\theta\cos\theta\, e^{i\varphi_x} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} E_x\cos^2\theta\, e^{i\varphi_x} \\ E_x\sin\theta\cos\theta\, e^{i\varphi_x} \end{bmatrix} = \begin{bmatrix} 0 \\ E_x\sin\theta\cos\theta\, e^{i\varphi_x} \end{bmatrix}$$

$I = E_x^2$ $\quad I_{\frac{\pi}{2}} = E_x^2\sin^2\theta\cos^2\theta$ $\quad I_{Loss} = I - I_{\frac{\pi}{2}} = E_x^2(1-\sin^2\theta\cos^2\theta) = E_x^2(1-\frac{1}{4}\sin^2 2\theta)$

$\Rightarrow \sin^2 2\theta = 0 \rightarrow$ Maximum of Loss $I_{max} = E_x^2$

$\sin^2 2\theta = 1 \rightarrow$ Minimum of Loss $I_{min} = \frac{3}{4}E_x^2$

Rotating at an angle of $\theta \Rightarrow J_\theta = \begin{bmatrix} E_x\cos^2\theta\, e^{i\varphi_x} \\ E_x\cos\theta\sin\theta\, e^{i\varphi_x} \end{bmatrix} \Rightarrow I = E_x^2(\cos^4\theta + \sin^2\theta\cos^2\theta) = E_x^2\cos^2\theta = I_0\cos^2\theta$

Assume that we rotate the linear polarizers at $\frac{\pi}{2n}$ every time, totally n times

$\Rightarrow I = I_0\cos^{2n}(\frac{\pi}{2n})$ $\quad I_{loss} = I_0 - I = I_0[1-\cos^{2n}(\frac{\pi}{2n})]$

$n=1 \rightarrow$ maximum of loss : $I_{max} = I_0$

$n\to\infty \rightarrow$ minimum of loss : $I_{loss} = I_0 \lim_{n\to\infty}[1-\cos^{2n}(\frac{\pi}{2n})] = 0$.

(c) $J_{out} = \hat{T}J_{in} = \frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}\begin{bmatrix} E_x e^{i\varphi_x} \\ E_y e^{i\varphi_y} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}\begin{bmatrix} J_x \\ J_y \end{bmatrix} = \lambda\begin{bmatrix} J_x \\ J_y \end{bmatrix}$

$$\det(\hat{T} - \lambda\hat{I}) = \begin{vmatrix} \frac{1}{2}-\lambda & \frac{i}{2} \\ -\frac{i}{2} & \frac{1}{2}-\lambda \end{vmatrix} = (\tfrac{1}{2}-\lambda)^2 - \tfrac{1}{4} = 0 \Rightarrow \lambda_1 = 0 \text{ and } \lambda_2 = 1$$

$$\lambda = 0 \Rightarrow \begin{bmatrix} \frac{1}{2} & \frac{i}{2} \\ -\frac{i}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} J_x + iJ_y = 0 \\ -iJ_x + J_y = 0 \end{cases} \Rightarrow \begin{cases} J_x = 1 \\ J_y = i \end{cases} \Rightarrow J_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$\lambda = 1 \Rightarrow \begin{bmatrix} \frac{1}{2} & \frac{i}{2} \\ -\frac{i}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} J_x \\ J_y \end{bmatrix} \Rightarrow \begin{cases} J_x + iJ_y = 2J_x \\ -iJ_x + J_y = 2J_y \end{cases} \Rightarrow \begin{cases} J_x = 1 \\ J_y = -i \end{cases} \Rightarrow J_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$$

For $J_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{i0} \\ e^{i\frac{\pi}{2}} \end{bmatrix}$ $\quad \delta = \varphi_x - \varphi_y = -\frac{\pi}{2}$ $\quad E_x = E_y \Rightarrow$ circular polarization

For $J_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{i0} \\ e^{-i\frac{\pi}{2}} \end{bmatrix}$ $\quad \delta = \varphi_x - \varphi_y = \frac{\pi}{2}$ $\quad E_x = E_y \Rightarrow$ circular Polarization

Thus the light should have circular polarization

Normal modes of a polarization system are the waves which polarization states remain unchanged after transmission through the polarization system. These modes are characterized by the Jones vectors satisfying the eigenproblem

$$\hat{\mathbf{T}}\mathbf{J} = a\mathbf{J},$$

where a is a constant, meaning that the normal modes a the eigenvectors of the Jones matrix.

a) Show that the normal modes of the linear polarizer with

$$\hat{\mathbf{T}} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

are linearly polarized waves.

b) Show that the normal modes of the wave retarder with

$$\hat{\mathbf{T}} = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\Gamma} \end{pmatrix}$$

are linearly polarized waves. As can be seen from the Jones matrix, the wave retarder delays the y-component by a phase Γ, while the x-component remains unchanged, which is why this element is called "retarder".

c) Show that the normal modes of the polarization rotator with

$$\hat{\mathbf{T}} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

are right and left circularly polarized waves.

(a) $\det(\hat{T} - aI) = \begin{bmatrix} 1-a & 0 \\ 0 & -a \end{bmatrix} = 0 \Rightarrow a(1-a) = 0 \Rightarrow a_1 = 1 \quad a_2 = 0$

for $a_2 = 0 \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} J_x \\ J_y \end{bmatrix} = 0\begin{bmatrix} J_x \\ J_y \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow J_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ e^{i0} \end{bmatrix}$

for $a_1 = 1 \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} J_x \\ J_y \end{bmatrix} \Rightarrow J_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} e^{i0} \\ 0 \end{bmatrix}$

Thus the normal modes are linear waves

(b) $\det(\hat{T} - aI) = \begin{vmatrix} 1-a & 0 \\ 0 & e^{-i\Gamma} - a \end{vmatrix} = (1-a)(e^{-i\Gamma} - a) = 0 \Rightarrow \begin{matrix} a_1 = 1 \\ a_2 = e^{-i\Gamma} \end{matrix}$

for $a_1 = 1 \quad \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\Gamma} \end{bmatrix}\begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} J_x \\ J_y \end{bmatrix} \Rightarrow J_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

for $a_2 = e^{-i\Gamma} \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\Gamma} \end{bmatrix}\begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} e^{-i\Gamma} J_x \\ e^{-i\Gamma} J_y \end{bmatrix} = J_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Thus they are linear polarized waves

(c) $\det(\hat{T} - aI) = \begin{vmatrix} \cos\theta - a & -\sin\theta \\ \sin\theta & \cos\theta - a \end{vmatrix} = (\cos\theta - a)^2 + \sin^2\theta = 0 \Rightarrow a^2 - 2a\cos\theta + 1 = 0$

$a_1 = \dfrac{2\cos\theta + 2\sqrt{\cos^2\theta - 1}}{2} = \cos\theta + i\sin\theta = e^{i\theta} \qquad a_2 = e^{-i\theta}$

for $a_1 = e^{i\theta} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} e^{i\theta} J_x \\ e^{i\theta} J_y \end{bmatrix} \Rightarrow \begin{cases} \cos\theta J_x - \sin\theta J_y = e^{i\theta} J_x \\ \sin\theta J_x + \cos\theta J_y = e^{i\theta} J_y \end{cases} \Rightarrow J_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$

for $a_2 = e^{-i\theta} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} e^{-i\theta} J_x \\ e^{-i\theta} J_y \end{bmatrix} \Rightarrow \begin{cases} J_x\cos\theta - J_y\sin\theta = e^{-i\theta} J_x \\ J_x\sin\theta + J_y\cos\theta = e^{-i\theta} J_y \end{cases} \Rightarrow J_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$

Thus they are right and left circular polarized waves.