Institute of Applied Physics
Friedrich-Schiller-Universität Jena

Lens Design I

Lecture 6: Aberrations I

2024-05-23

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Preliminary Schedule - Lens Design I 2024

1	04.04.	Basics	Zhang	Introduction, Zemax interface, menues, file handling, preferences, Editors, updates, windows, coordinates, System description, 3D geometry, aperture, field, wavelength
2	18.04.	Properties of optical systems I	Tang	Diameters, stop and pupil, vignetting, layouts, materials, glass catalogs, raytrace, ray fans and sampling, footprints
3	25.04.	Properties of optical systems II	Tang	Types of surfaces, cardinal elements, lens properties, Imaging, magnification, paraxial approximation and modelling, telecentricity, infinity object distance and afocal image, local/global coordinates
4	02.05.	Properties of optical systems III	Tang	Component reversal, system insertion, scaling of systems, aspheres, gratings and diffractive surfaces, gradient media, solves
5	16.05.	Advanced handling I	Tang	Miscellaneous, fold mirror, universal plot, slider, multiconfiguration, lens catalogs
6	23.05.	Aberrations I	Zhang	Representation of geometrical aberrations, spot diagram, transverse aberration diagrams, aberration expansions, primary aberrations
7	30.05.	Aberrations II	Zhang	Wave aberrations, Zernike polynomials, measurement of quality
8	06.06.	Aberrations III	Tang	Point spread function, optical transfer function
9	13.06.	Optimization I	Tang	Principles of nonlinear optimization, optimization in optical design, general process, optimization in Zemax
10	20.06.	Optimization II	Zhang	Initial systems, special issues, sensitivity of variables in optical systems, global optimization methods
11	27.06.	Correction I	Zhang	Symmetry principle, lens bending, correcting spherical aberration, coma, astigmatism, field curvature, chromatical correction
12	04.07.	Correction II	Zhang	Field lenses, stop position influence, retrofocus and telephoto setup, aspheres and higher orders, freeform systems, miscellaneous

Contents

1. Representation of geometrical aberrations
2. Spot diagram
3. Transverse aberration diagrams
4. Aberration expansions
5. Primary aberrations

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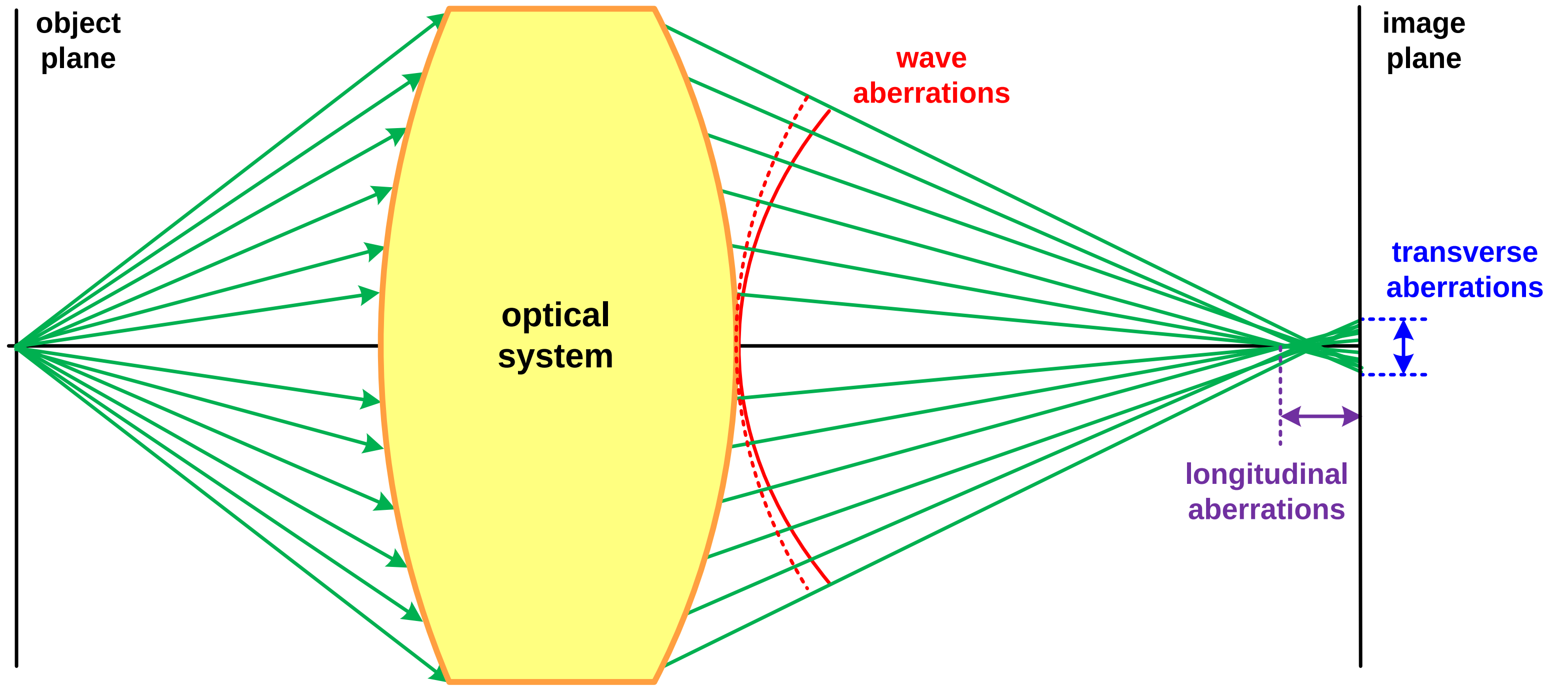
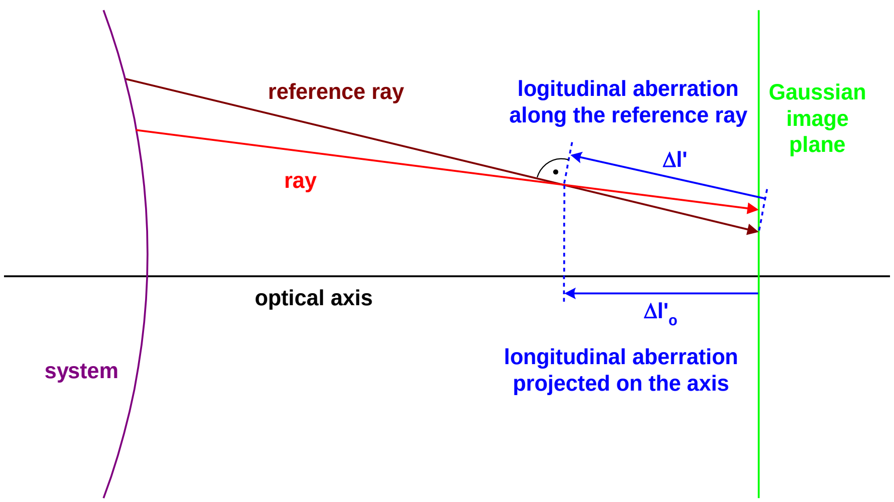
Optical Image Formation

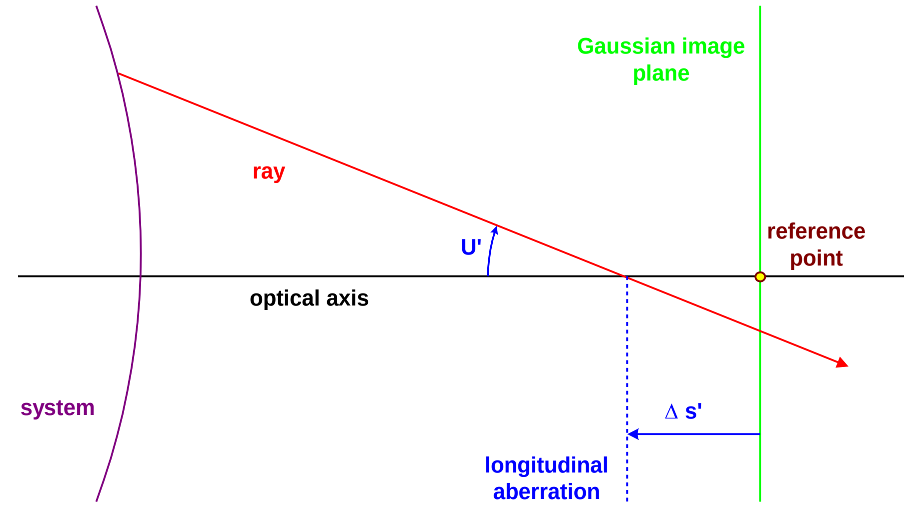
- Perfect optical image:
  All rays coming from one object point intersect in one image point
- Real system with aberrations:
  1. transverse aberrations in the image plane
  2. longitudinal aberrations from the image plane
  3. wave aberrations in the exit pupil



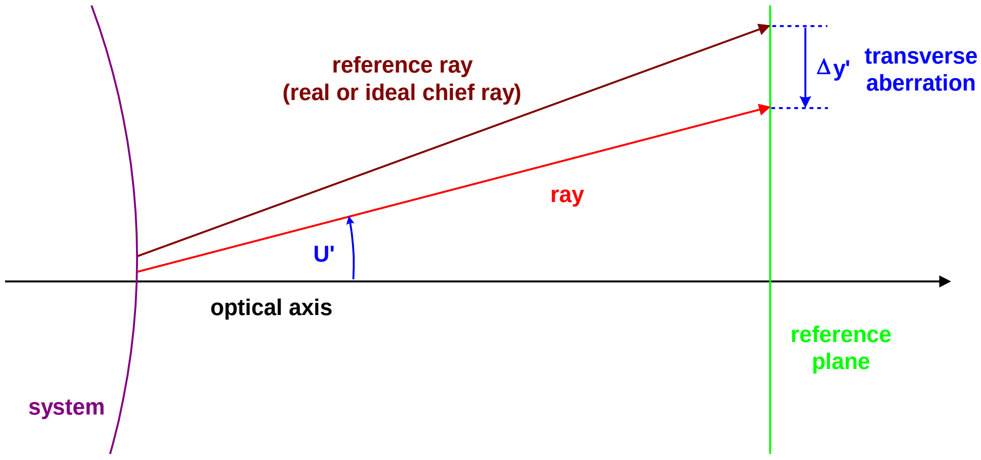
Representation of Geometrical Aberrations

- Longitudinal aberrations Δs



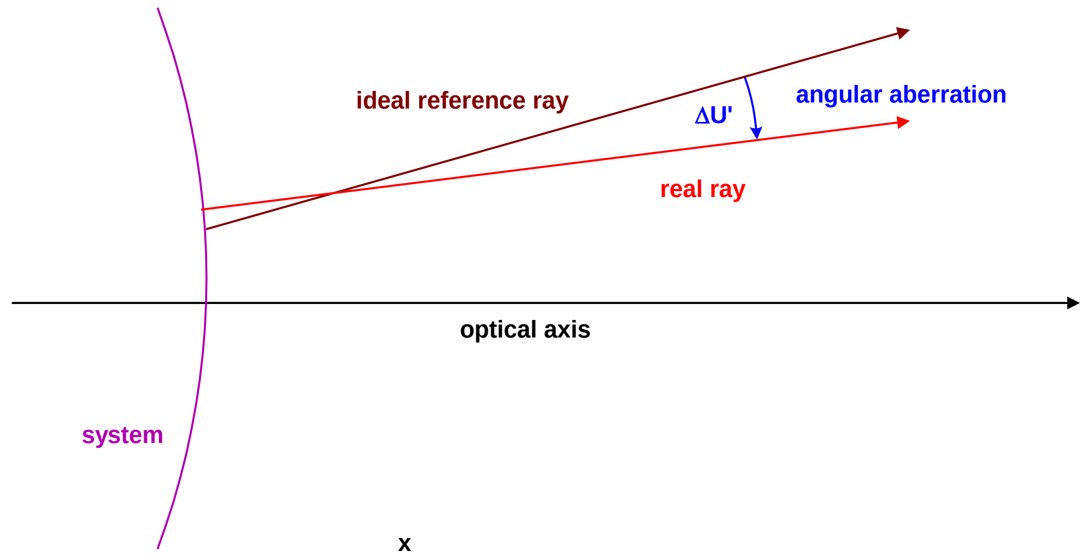


- Transverse aberrations Δy

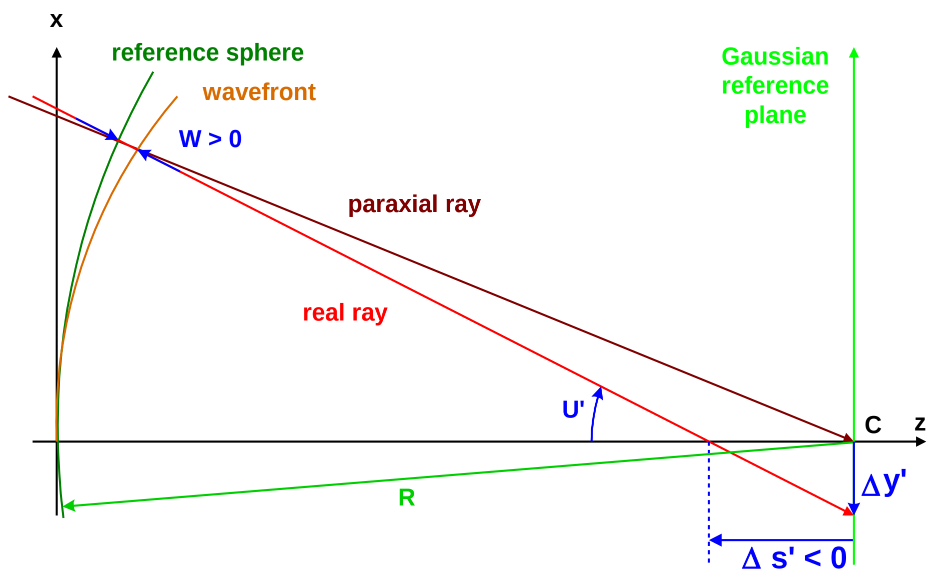


Representation of Geometrical Aberrations

- Angle aberrations Δu

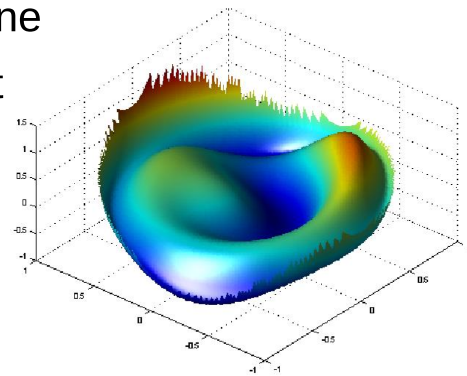


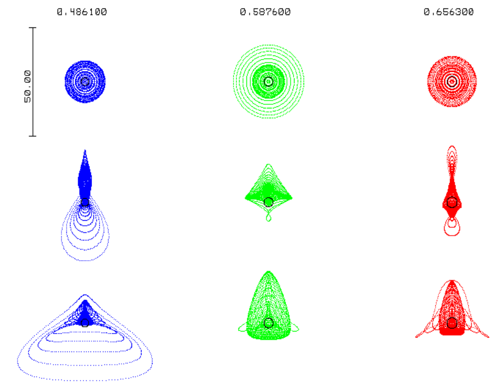
- Wave aberrations ΔW

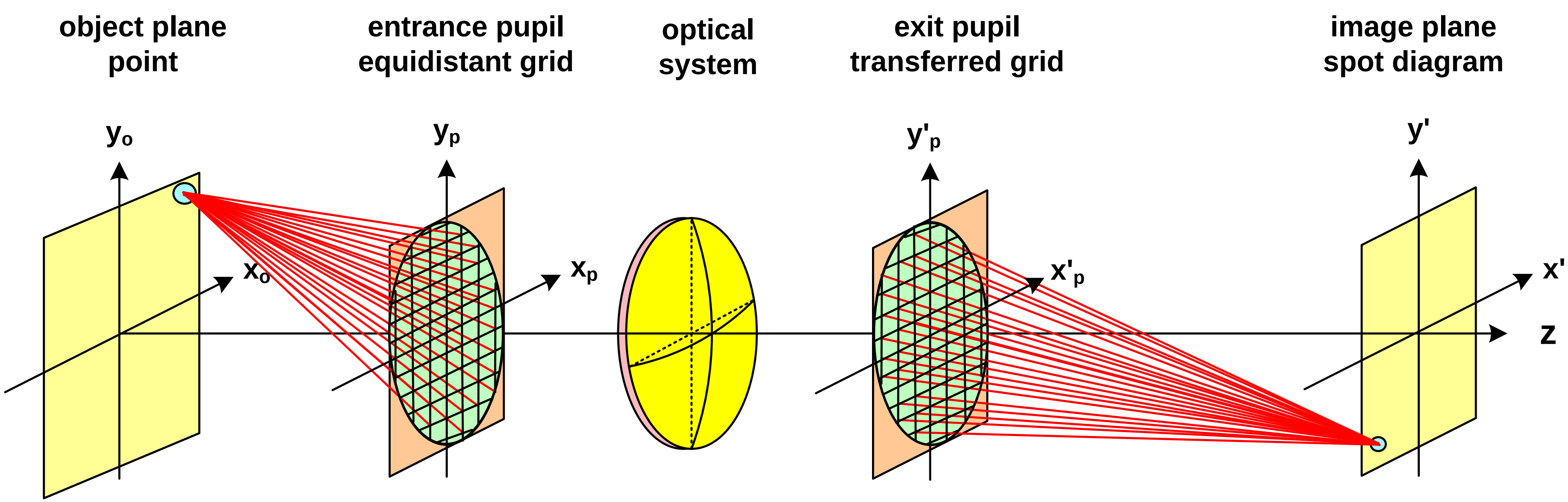


Spot Diagram

- All rays start in one point in the object plane
- The entrance pupil is sampled equidistant
- In the exit pupil, the transferred grid may be distorted
- In the image plane a spreaded spot diagram is generated







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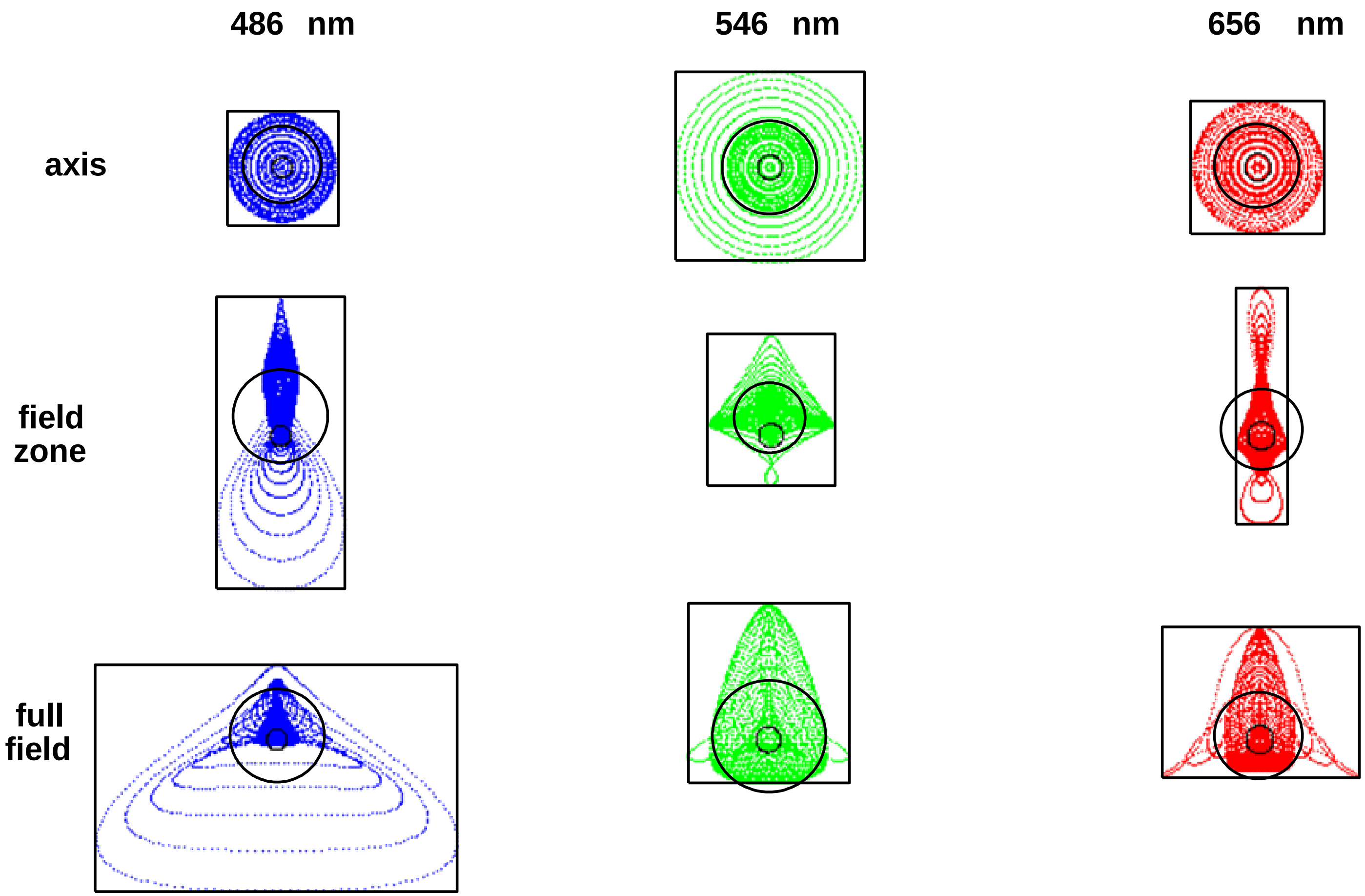
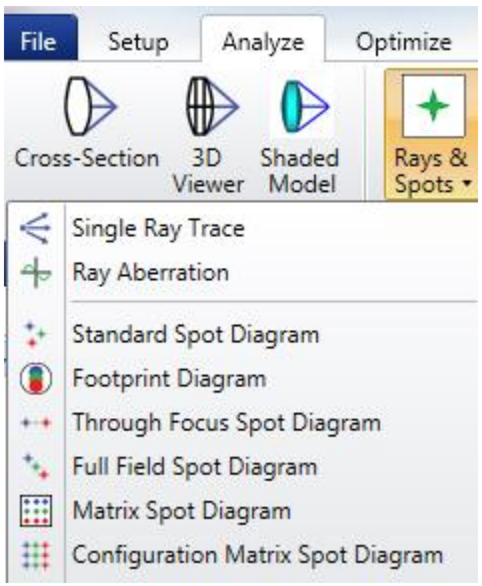
Spot Diagram

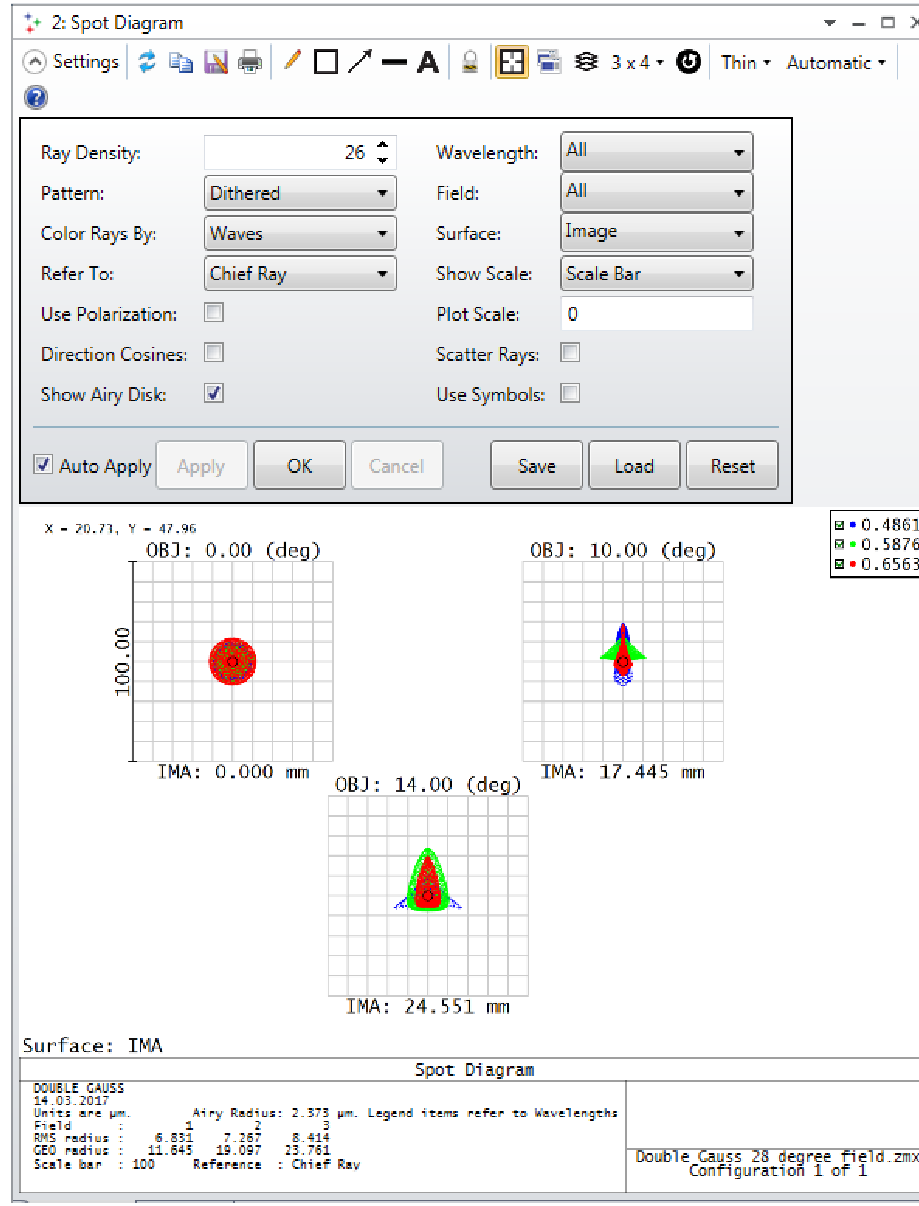
- Table with various values of:
  1. Field size
  2. Color
- Small circle:
  Airy diameter for comparison
- Large circle:
  Gaussian moment



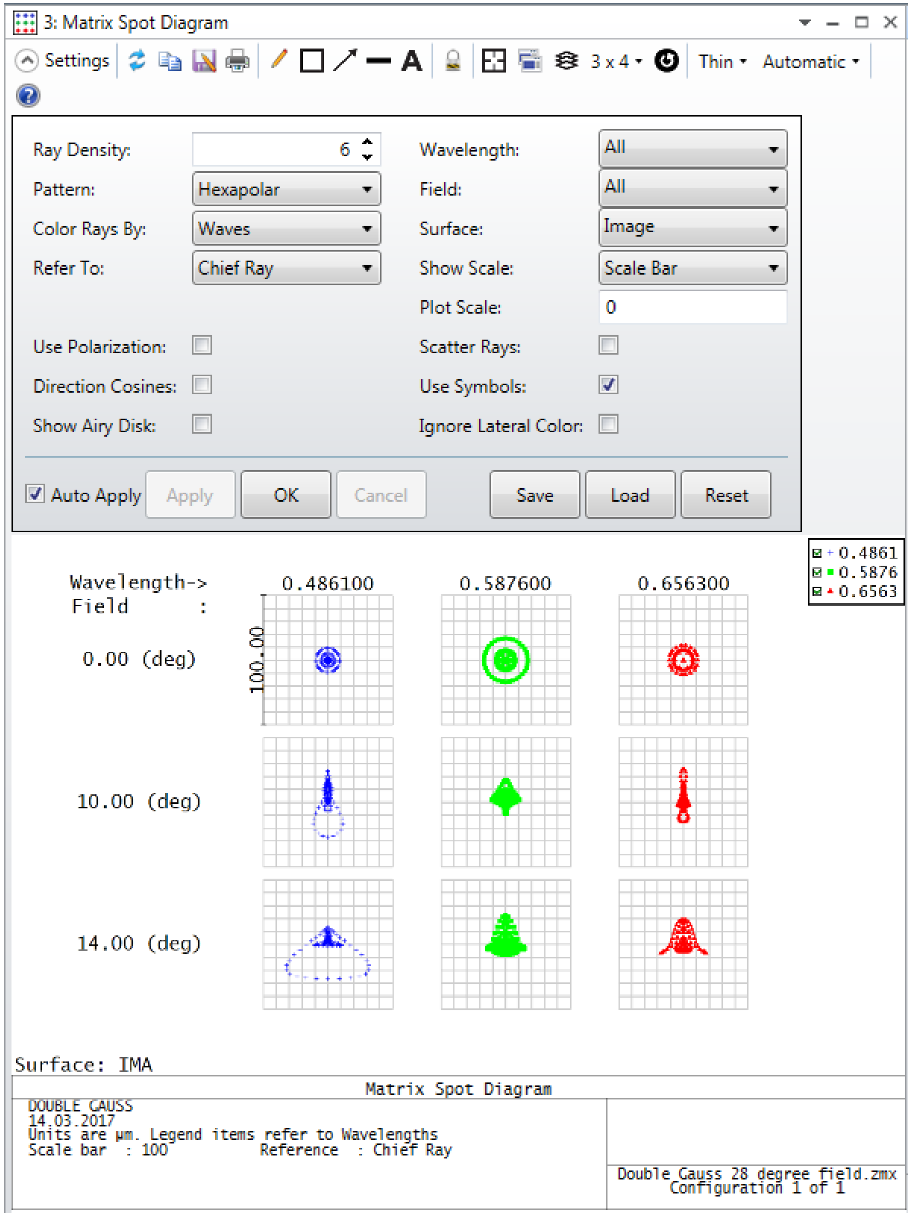
Spot Diagrams in Zemax

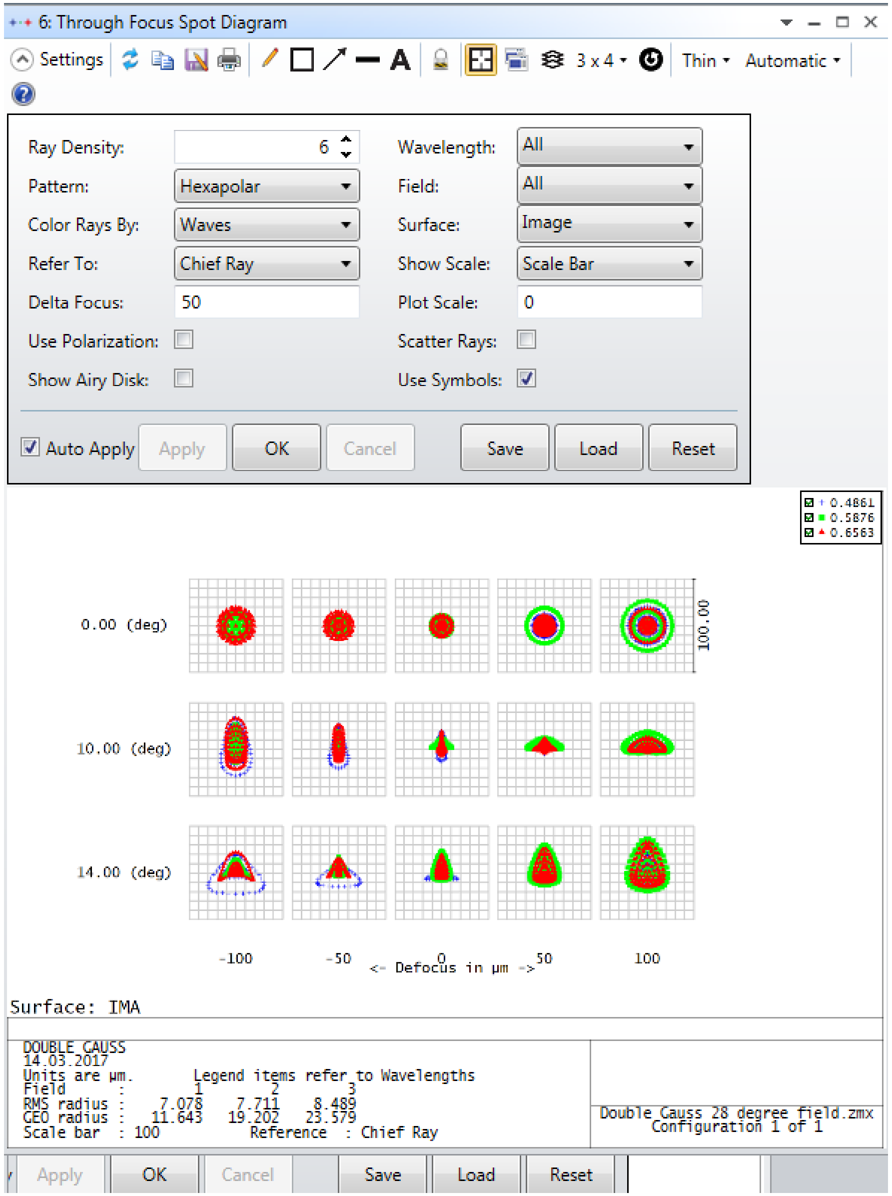
- Several options for representations



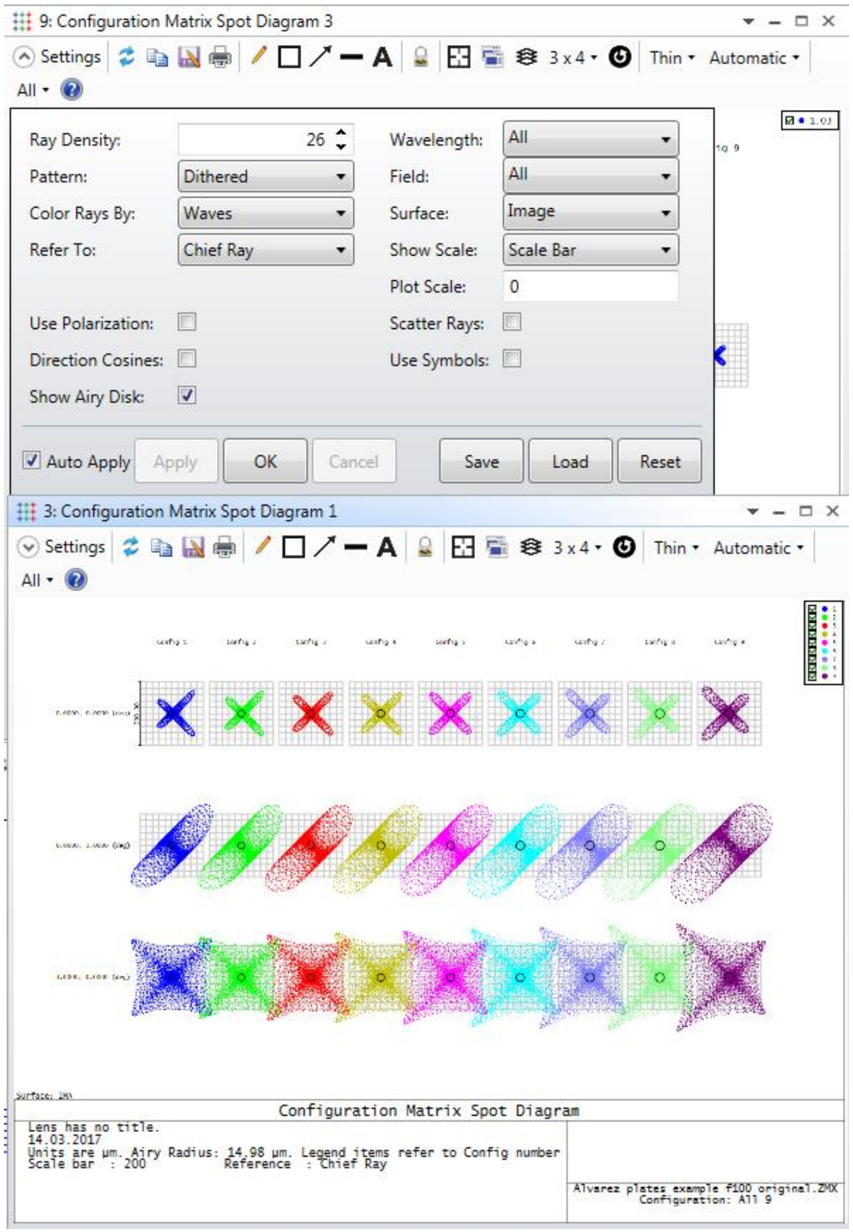


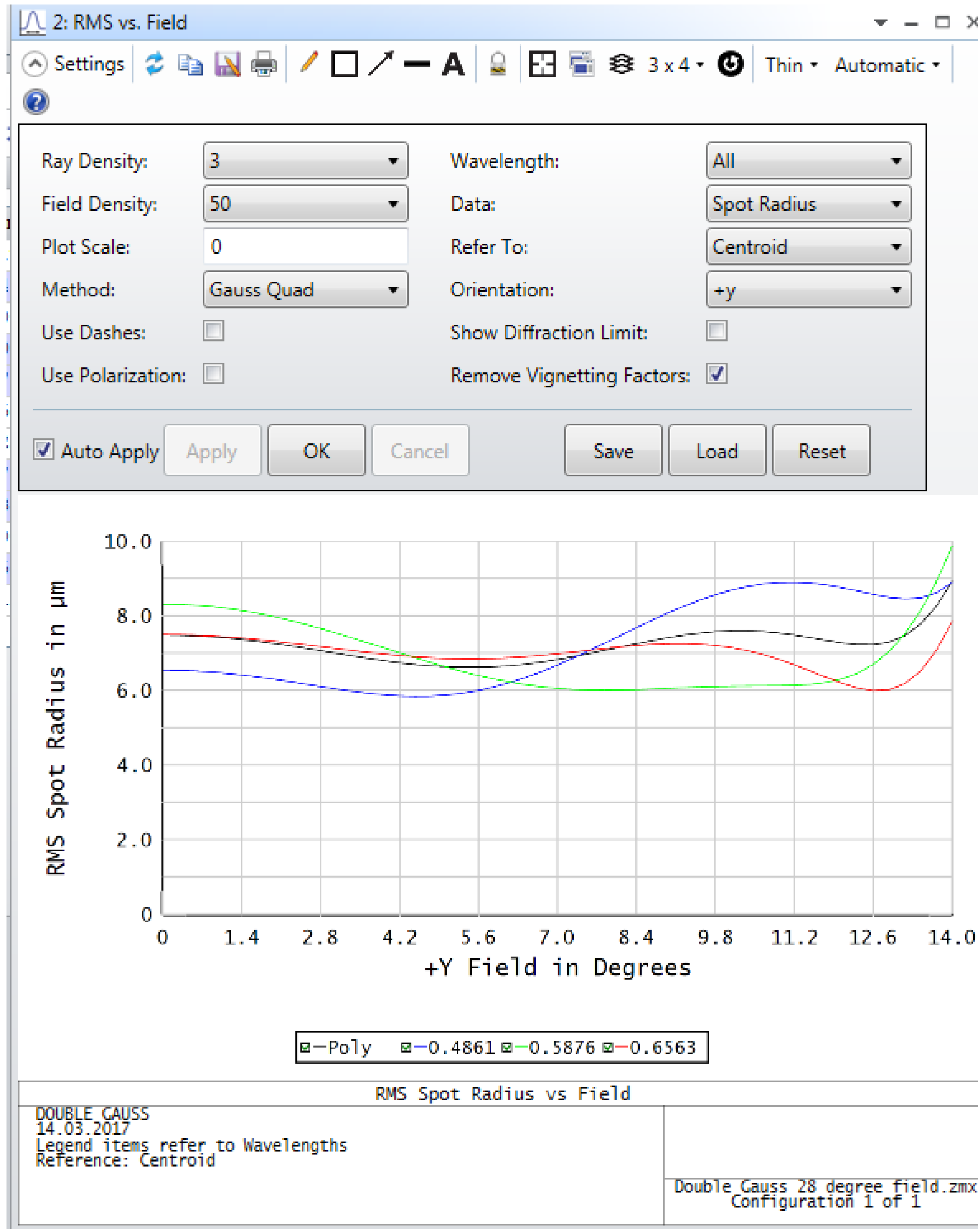
Spot Diagrams in Zemax





Spot Diagrams in Zemax





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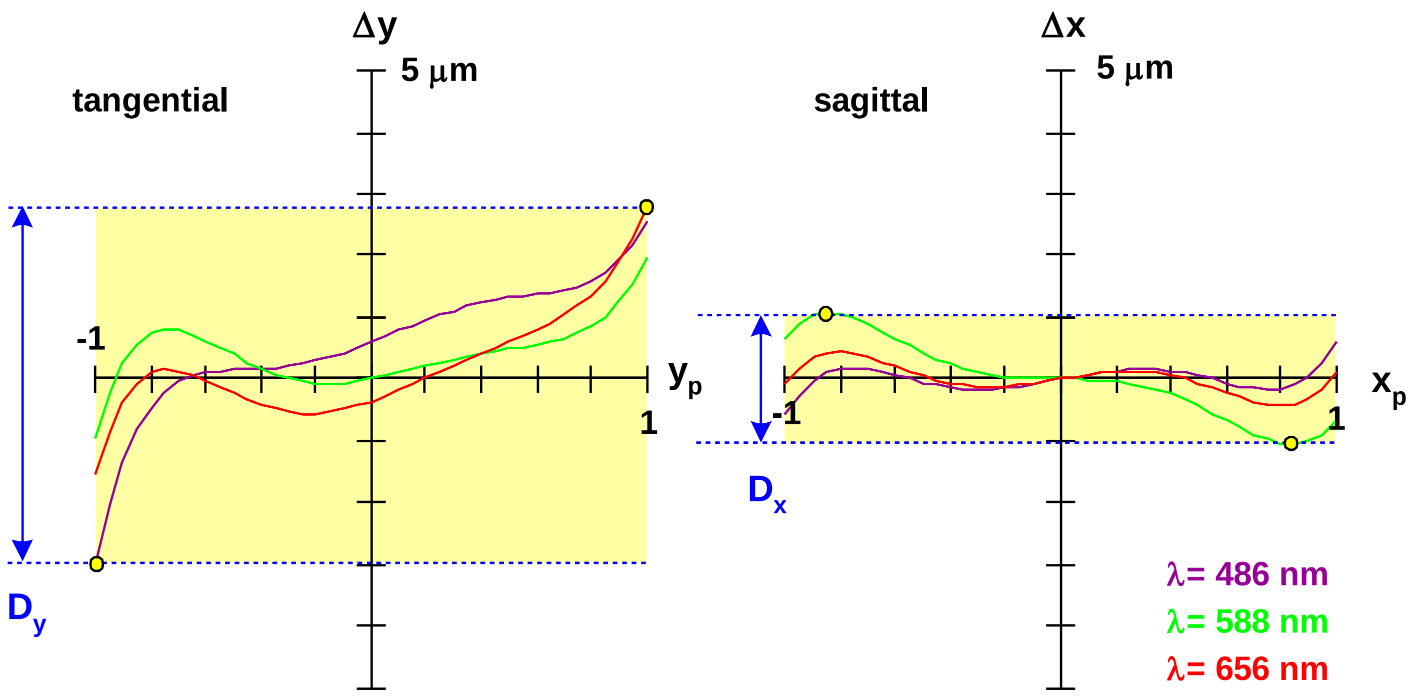
Transverse Aberrations

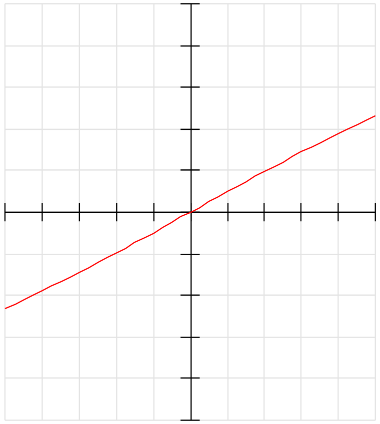
- Classical aberration curves
- Strong relation to spot diagram
- Usually only linear sampling along the x-, y-axis
  no information in the quadrant of the aperture



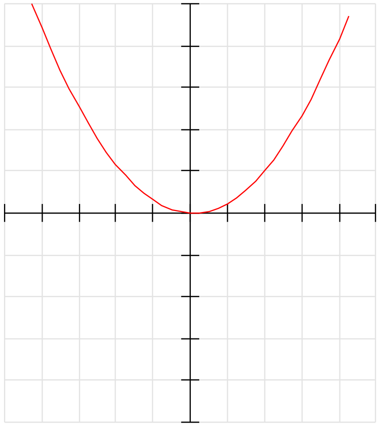
Transverse Aberrations

- Typical low order polynomial contributions for:
  Defocus, coma, spherical, lateral color
- This allows a quick classification of real curves

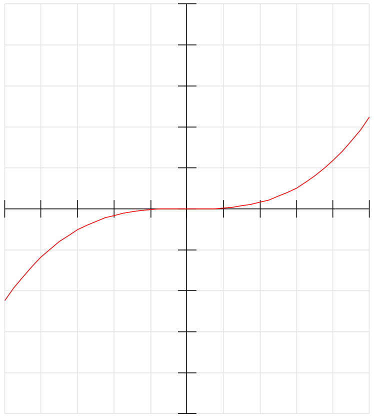
linear:
defocus



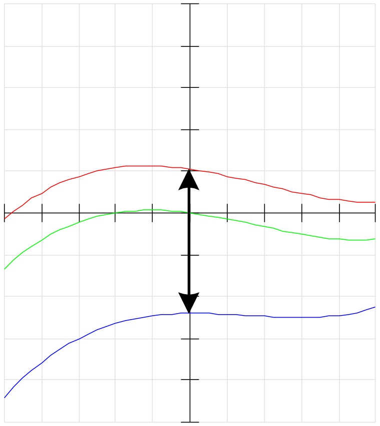
quadratic:
coma



cubic:
spherical

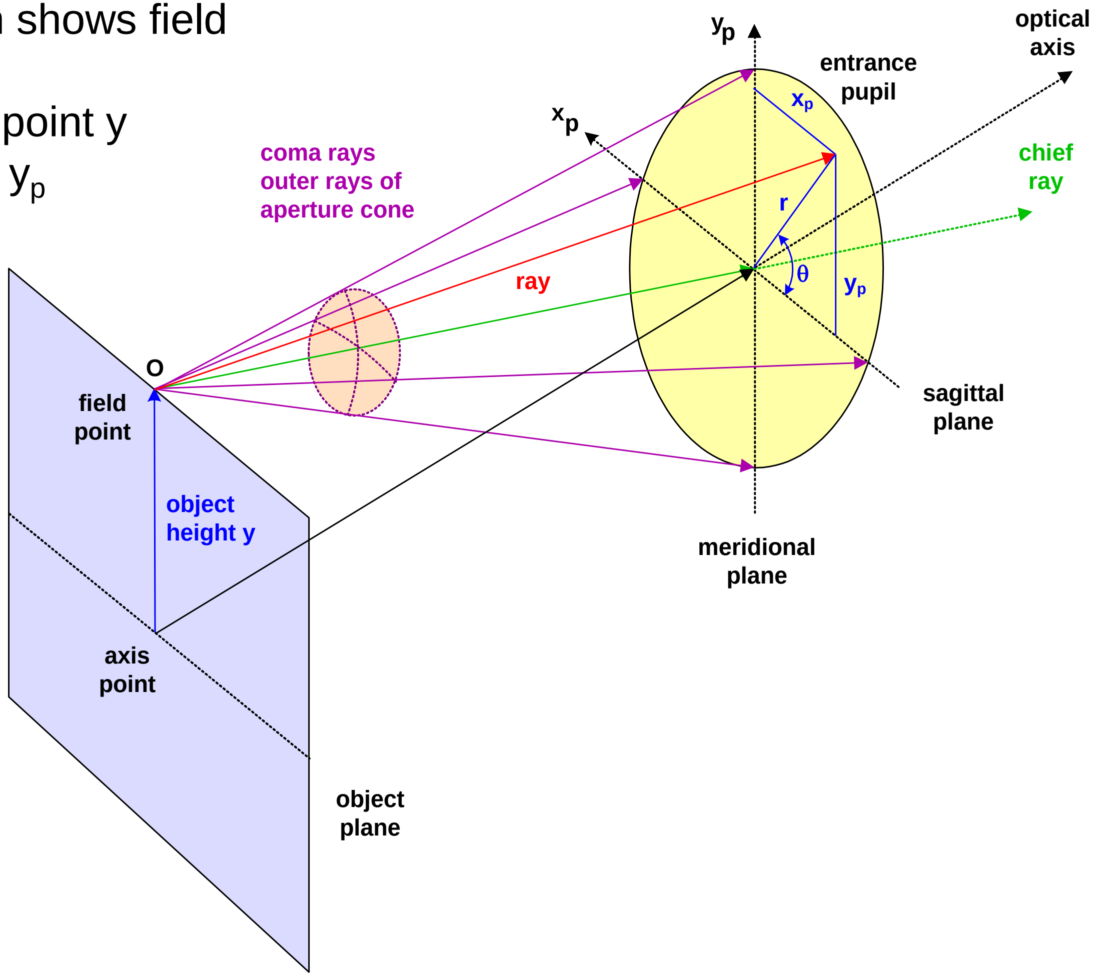


offset:
lateral color

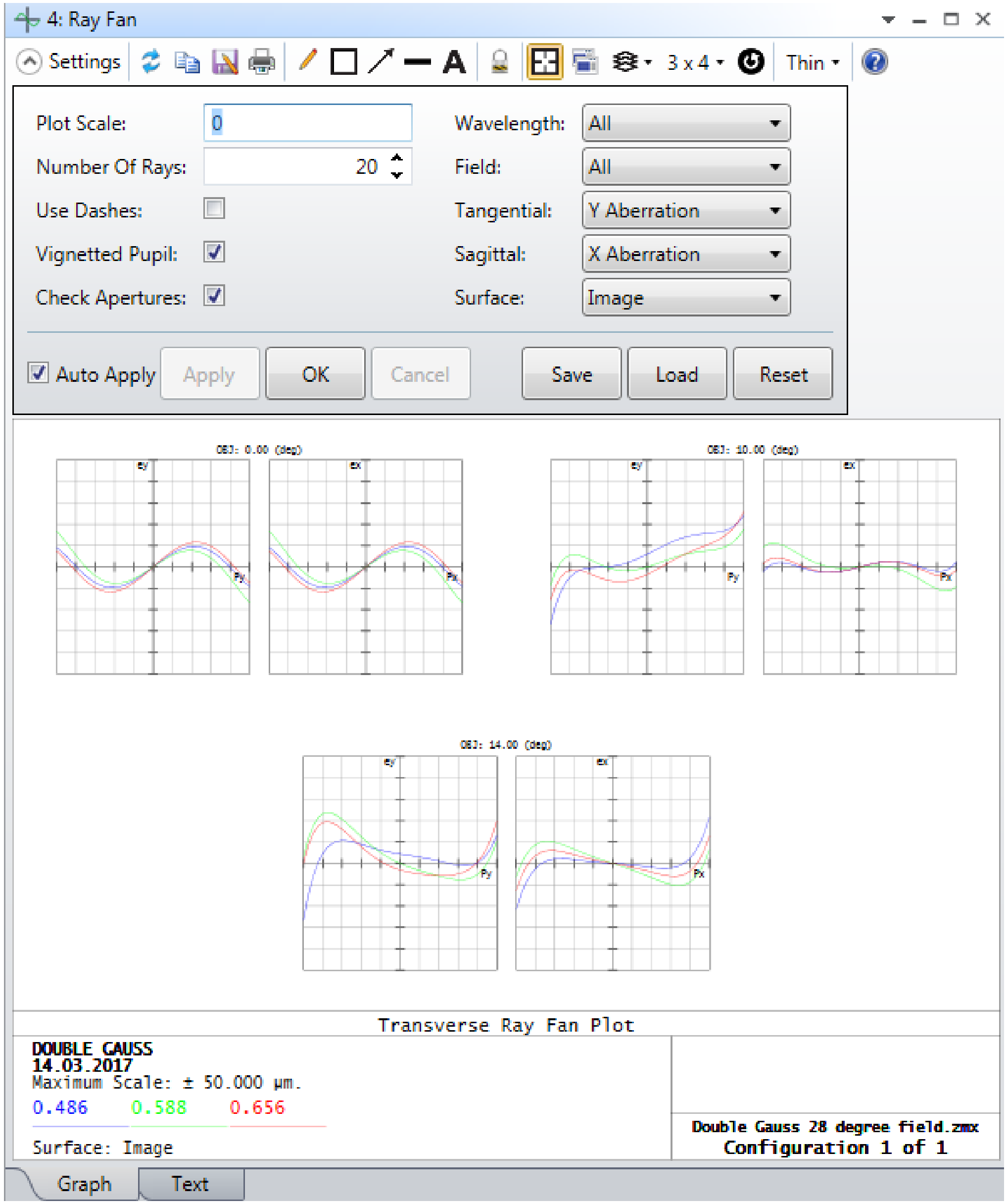


Polynomial Expansion of the Aberrations

- Paraxial optics: small field and aperture angles
  Aberrations occur for larger angle values
- Two-dimensional Taylor expansion shows field and aperture dependence
- Expansion for one meridional field point y
- Pupil: cartesian or polar grid in x_p / y_p



Transverse Aberrations in Zemax



Polynomial Expansion of Aberrations

- Taylor expansion of the deviation:
  y' Image height — index k
  r_p Pupil height — index l
  θ Pupil azimuth angle — index m

$$\Delta y(y', r_p, \theta) = \sum_{k,l,m} a_{klm} \cdot y'^k \cdot r_p^l \cdot \cos^m \theta$$

- Symmetry invariance: selection of special combinations of exponent terms

- Number of terms: sum of indices in the exponent isum

i_{sum}	number of terms	Type of aberration
2	2	image location
4	5	primary aberrations, 3rd/4th order
6	9	secondary aberrations, 5th/6th order
8	14	higher order

- The order of the aperture function depends on the aberration type used:
  primary aberrations:
  - 3rd order in transverse aberration Δy
  - 4th order in wave aberration W

  Since the coupling relation

$$\Delta y = -R \cdot \frac{\partial W}{\partial x_p}$$

  changes the order by 1

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Polynomial Expansion of Aberrations

- Representation of 2-dimensional Taylor series vs field y and aperture r
- Selection rules: checkerboard filling of the matrix
- Constant sum of exponents according to the order

Image location

Primary aberrations / Seidel

Secondary aberrations

			Field y →					
			Spherical	Coma	Astigmatism			
			y^0	y^1	y^2	y^3	y^4	y^5
	Distortion	r^0		$y\cos\theta$ Tilt		$y^3\cos\theta$ Distortion primary		$y^5\cos\theta$ Distortion secondary
		r^1	r^1 Defocus		$y^2 r^1 \cos^2\theta$ $y^2 r^1$ Astig./Curvat.		$y^4 r^1 \cos^2\theta$ $y^4 r^1$	
Aper-ture r ↓		r^2		$y r^2 \cos\theta$ Coma primary		$y^3 r^2 \cos^3\theta$ $y^3 r^2 \cos\theta$		
		r^3	r^3 Spherical primary		$y^2 r^3 \cos^2\theta$ $y^2 r^3$			
		r^4		$y r^4 \cos\theta$ Coma secondary				
		r^5	r^5 Spherical secondary					

Primary Aberrations

- Expansion of the transverse aberration Δy on image height y and pupil height r
- Lowest order 3 of real aberrations: primary or Seidel aberrations
- Spherical aberration: S
  - no dependence on field, valid on axis
  - depends in 3rd order on apertur
- Coma: C
  - linear function of field y
  - depends in 2rd order on apertur with azimuthal variation
- Astigmatism: A
  - linear function of apertur with azimuthal variation
  - quadratic function of field size
- Image curvature (Petzval): P
  - linear dependence on apertur
  - quadratic function of field size
- Distortion: D
  - No dependence on apertur
  - depends in 3rd order on the field size

$$\Delta y = r^3 \cdot S + y \cdot r^2 \cdot \cos\theta \cdot C + y^2 \cdot r \cdot \cos^2\theta \cdot A + y^2 \cdot r \cdot P + y^3 \cdot D$$

Transverse Aberrations of Seidel

- Transverse deviations

$$\Delta x' = \frac{x'_p\left(x'^2_p + y'^2_p\right)s'^4}{2n'R'^3_p}S' - \frac{\left[2x'_p\left(x'x'_p + y'y'_p\right) + x'\left(x'^2_p + y'^2_p\right)\right]s'^3 s'_p}{2n'R'^3_p}C' + \frac{x'\left(x'x'_p + y'y'_p\right)s'^2 s'^2_p}{n'R'^3_p}A' + \frac{x'_p\left(x'^2_p + y'^2_p\right)s'^2 s'^2_p}{2n'R'^3_p}P' - \frac{x'\left(x'^2 + y'^2\right)s' s'^3_p}{2n'R'^3_p}D'$$

$$\Delta y' = \frac{y'_p\left(x'^2_p + y'^2_p\right)s'^4}{2n'R'^3_p}S' - \frac{\left[2y'_p\left(x'x'_p + y'y'_p\right) + y'\left(x'^2_p + y'^2_p\right)\right]s'^3 s'_p}{2n'R'^3_p}C' + \frac{y'\left(x'x'_p + y'y'_p\right)s'^2 s'^2_p}{n'R'^3_p}A' + \frac{y'_p\left(x'^2_p + y'^2_p\right)s'^2 s'^2_p}{2n'R'^3_p}P' - \frac{y'\left(x'^2 + y'^2\right)s' s'^3_p}{2n'R'^3_p}D'$$

- Sum of surface contributions

$$S' = \sum_{j=1}^{k} S_j$$

$$C' = \sum_{j=1}^{k} C_j$$

$$A' = \sum_{j=1}^{k} A_j$$

$$P' = \sum_{j=1}^{k} P_j$$

$$D' = \sum_{j=1}^{k} D_j$$

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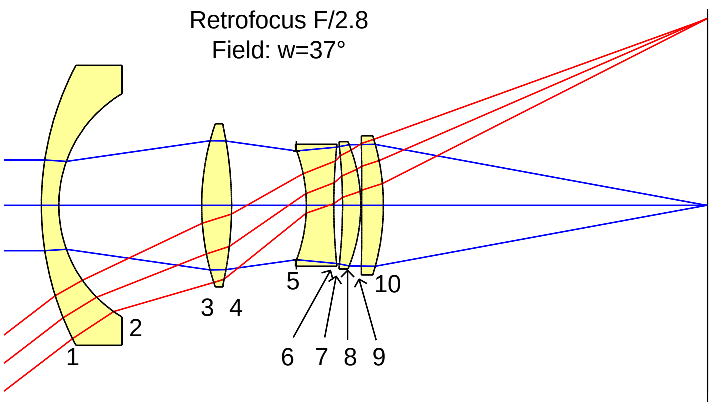
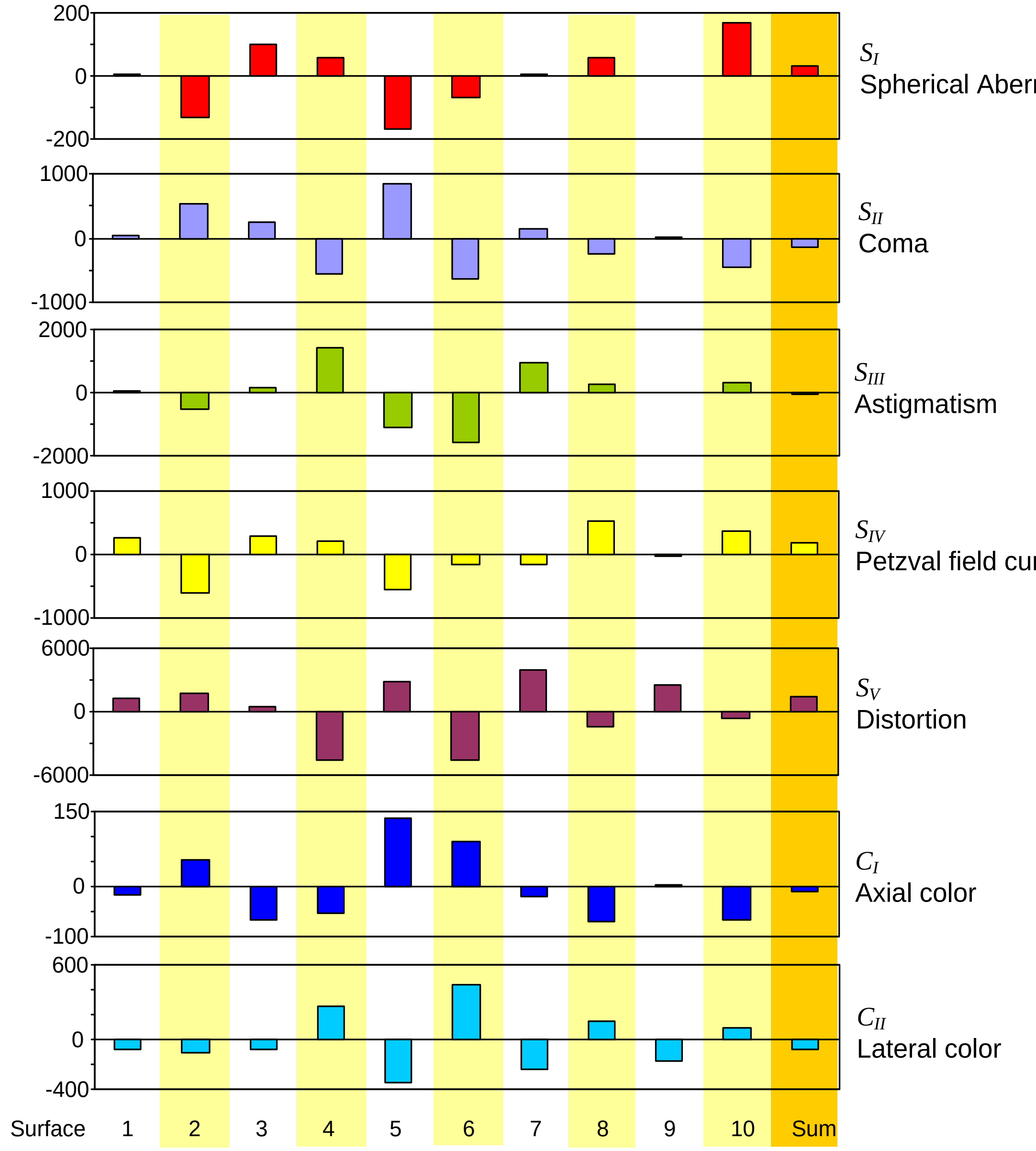
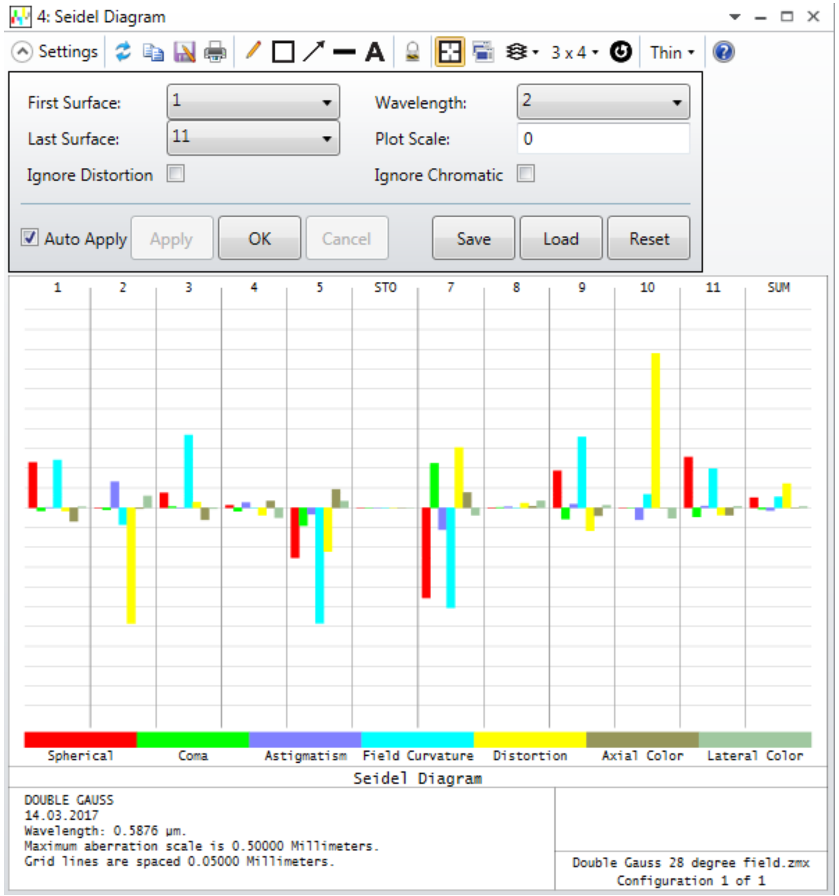
Surface Contributions: Example

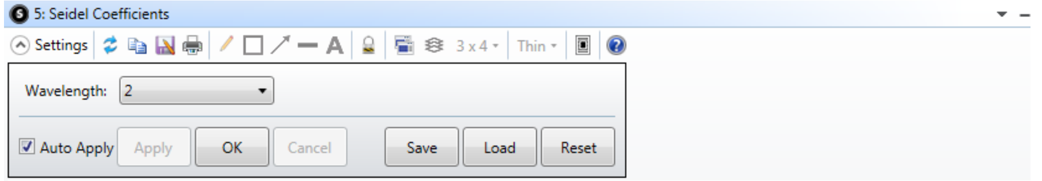
- Seidel aberrations: representation as sum of surface contributions possible
- Gives information on correction of a system
- Example: photographic lens





Seidel Aberrations in Zemax





5: Seidel Coefficients

Settings | 3 x 4 | Thin

Wavelength: 2

Auto Apply | Apply | OK | Cancel | Save | Load | Reset

Listing of Aberration Coefficient Data

File : C:\Gross\Zemax\Data collections\Samples Zemax\Sequential\Objectives\Double Gauss 28 degree field.zmx
Title: DOUBLE GAUSS
Date : 14.03.2017

Wavelength	:	0.5876 µm
Chief Ray Slope, Object Space	:	0.2493
Chief Ray Slope, Image Space	:	0.2292
Marginal Ray Slope, Object Space	:	0.0000
Marginal Ray Slope, Image Space	:	-0.1675
Petzval radius	:	-623.2905
Optical Invariant	:	4.1551

Seidel Aberration Coefficients:

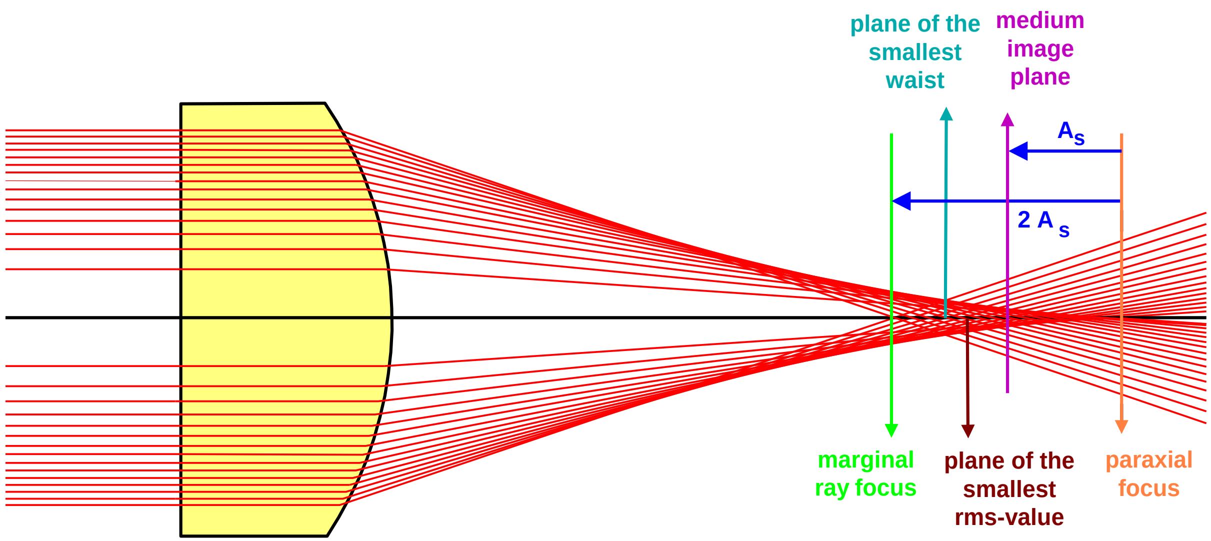
Surf	SPHA S1	COMA S2	ASTI S3	FCUR S4	DIST S5	CLA (CL)	CTR (CT)
1	0.114175	-0.008176	0.000586	0.120467	-0.008669	-0.034222	0.002451
2	0.000396	-0.005093	0.065502	-0.042772	-0.292349	-0.002298	0.029553
3	0.037615	0.002873	0.000219	0.183865	0.014061	-0.030572	-0.002335
4	0.006027	-0.008824	0.012918	-0.000000	-0.018911	0.017026	-0.024926
5	-0.126410	-0.045356	-0.016274	-0.291746	-0.110518	0.046478	0.016677
STO	-0.000000	0.000000	-0.000000	-0.000000	0.000000	0.000000	-0.000000
7	-0.227787	0.112367	-0.055431	-0.252955	0.152127	0.038302	-0.018894
8	0.000130	0.000581	0.002588	0.000000	0.011535	0.003927	0.017502
9	0.093297	-0.028806	0.008894	0.178746	-0.057936	-0.020104	0.006207
10	-0.000002	-0.000264	-0.030245	0.033653	0.389861	-0.000233	-0.026621
11	0.128073	-0.023063	0.004153	0.098441	-0.018475	-0.019011	0.003423
IMA	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
TOT	0.025514	-0.003761	-0.007089	0.027699	0.060727	-0.000706	0.003038

Seidel Aberration Coefficients in Waves:

Surf	W040	W131	W222	W220P	W311	W020	W111
1	24.288415	-6.957359	0.498230	51.254016	-7.376475	-29.120000	4.170677
2	0.084231	-4.333486	55.736947	-18.197852	-248.765515	-1.955190	50.294990

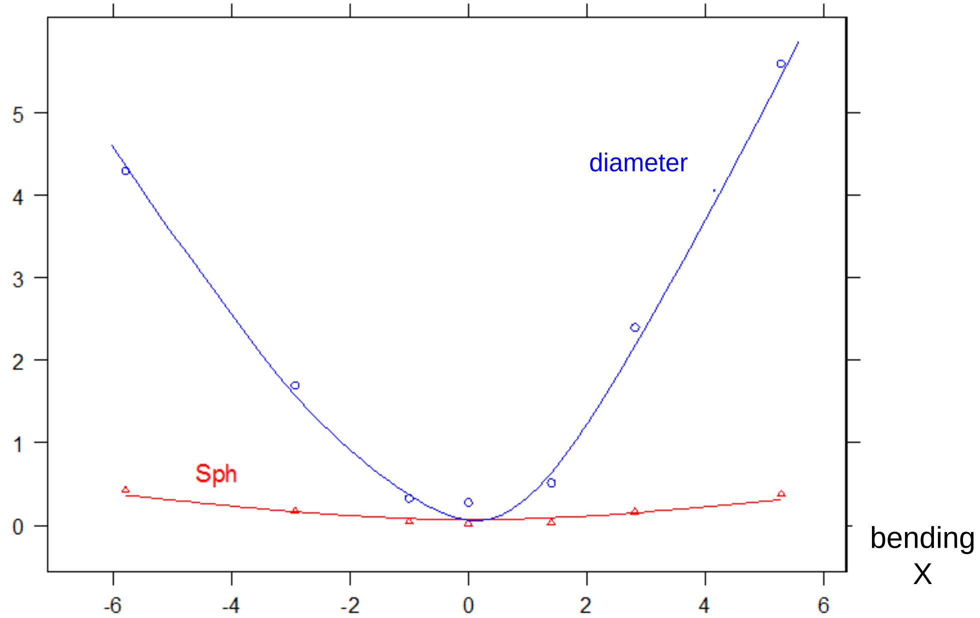
Spherical Aberration

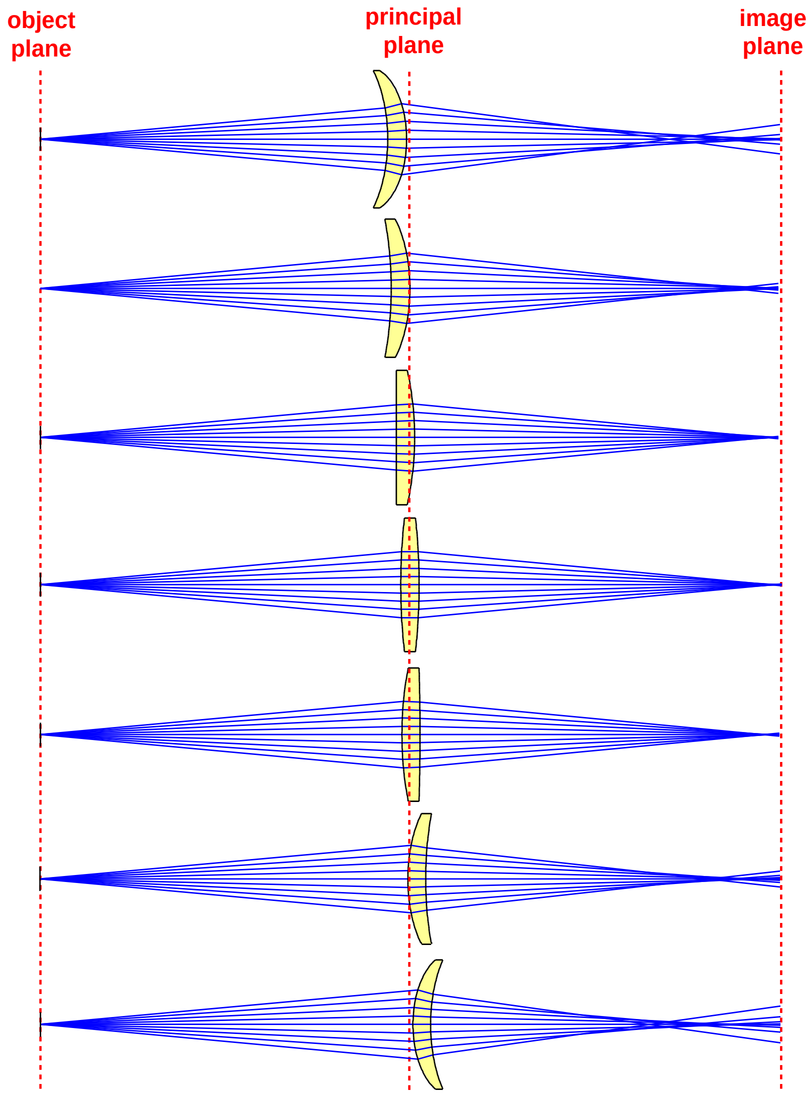
- Spherical aberration:
  On axis, circular symmetry
- Perfect focussing near axis: paraxial focus
- Real marginal rays: shorter intersection length (for single positive lens)
- Optimal image plane: circle of least rms value



Spherical Aberration: Lens Bending

- Spherical aberration and focal spot diameter as a function of the lens bending (for n=1.5)
- Optimal bending for incidence averaged incidence angles
- Minimum larger than zero: usually no complete correction possible





Lens Contributions of Seidel

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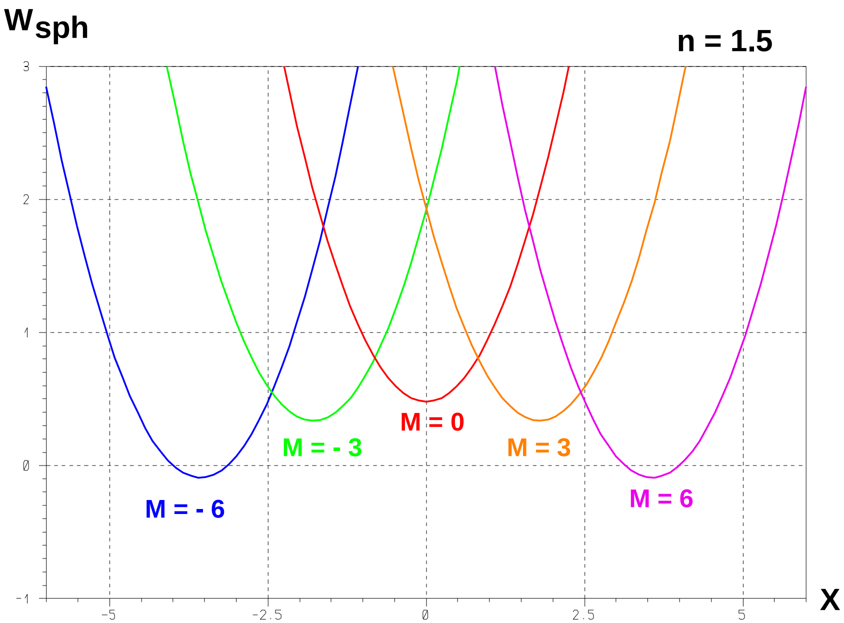
- Spherical aberration

$$S_{lens} = \frac{1}{32n(n-1)f^3}\left[\frac{n^3}{n-1} + \frac{n+2}{n-1}\cdot\left(X - \frac{2(n^2-1)}{n+2}\cdot M\right)^2 - \frac{n^2(n-1)}{n+2}\cdot M^2\right]$$

- Special impact on correction:
  1. Special quadratic dependence on bending X
     Minimum at $$X_{sph\min} = -\frac{2\left(n^2-1\right)}{n+2}M$$
  2. No correction for small n and M
  3. Correction for large
     n: infrared materials
     M: virtual imaging
     Limiting value $$M_{s=0}^2 = \frac{n\left(n+2\right)}{\left(n-1\right)^2}$$



Delano‘s Representation of Spherical Aberration

- Paraxial optics: Delano relation

$$n' \cdot q' \cdot U' = n \cdot q \cdot U + n \cdot i \cdot (Q' - Q)$$

- Real ray comparison:

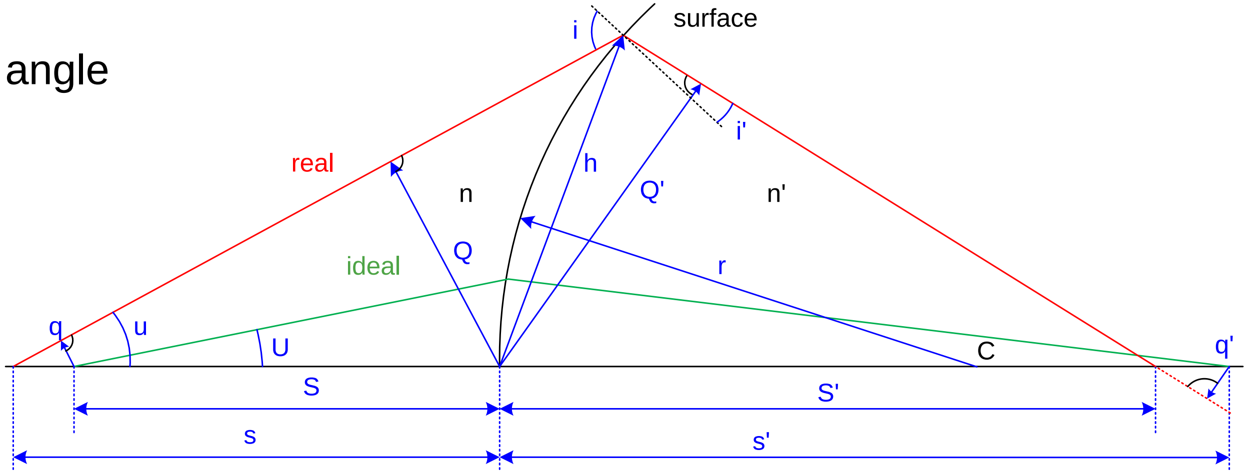
  Delano surface contribution

$$\Delta s'_{SPH} = \Delta s_{SPH} \cdot \frac{n_1 U_1 \sin u_1}{n'_k U'_k \sin u'_k} + \sum_j \frac{(Q - Q') \cdot i \cdot n_j}{n'_j U'_j \sin u'_j}$$

$$\Delta s'_{SPH} = \Delta s_{SPH} \cdot \frac{n_1 U_1 \sin u_1}{n'_k U'_k \sin u'_k} + \sum_j \frac{n_j}{n'_j} \cdot h \cdot \sin\frac{i' - i}{2} \cdot \frac{2i \cdot \sin\frac{i' - u}{2}}{U'_j \sin u'_j}$$

Surface contribution grows with
1. ratio of refractive indices
2. height of the marginal ray
3. Influence of ray bending angle

- Influence of ray bending angle



Aplanatic Surfaces

- Aplanatic surfaces: zero spherical aberration:

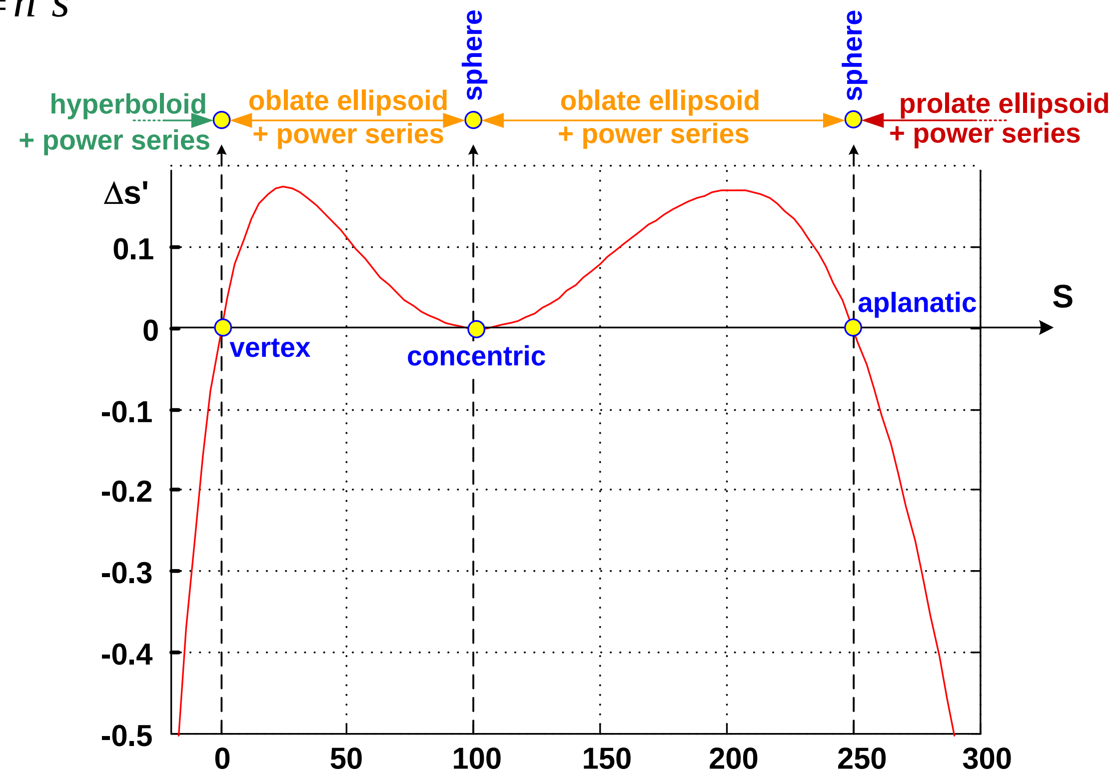
  1. Ray through vertex $s' = s = 0$
  2. concentric $s' = s \quad und \quad u = u'$
  3. Aplanatic $ns = n's'$

- Condition for aplanatic surface:

$$r = \frac{ns}{n+n'} = \frac{n's'}{n+n'} = \frac{ss'}{s+s'}$$

- Virtual image location

- Applications:
  1. Microscopic objective lens
  2. Interferometer objective lens

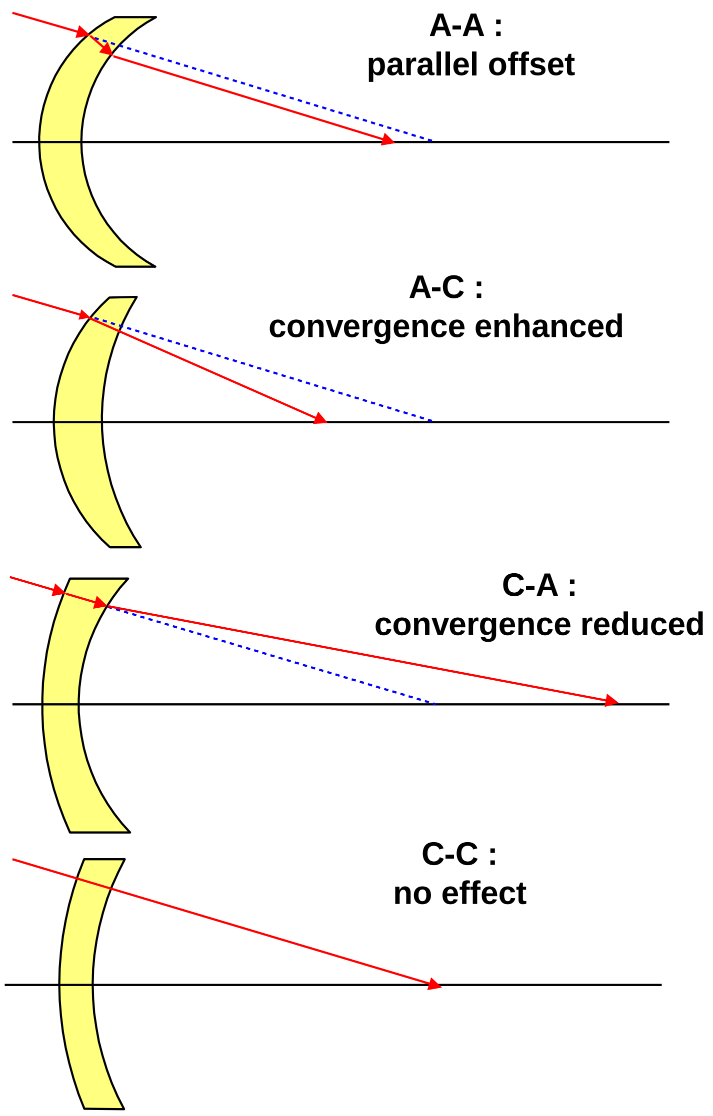


Aplanatic Lenses

- Aplanatic lenses

- Combination of one concentric and one aplanatic surface:
  zero contribution of the whole lens to spherical aberration

- Not useful:
  1. aplanatic-aplanatic
  2. concentric-concentric
  bended plane parallel plate,
  nearly vanishing effect on rays



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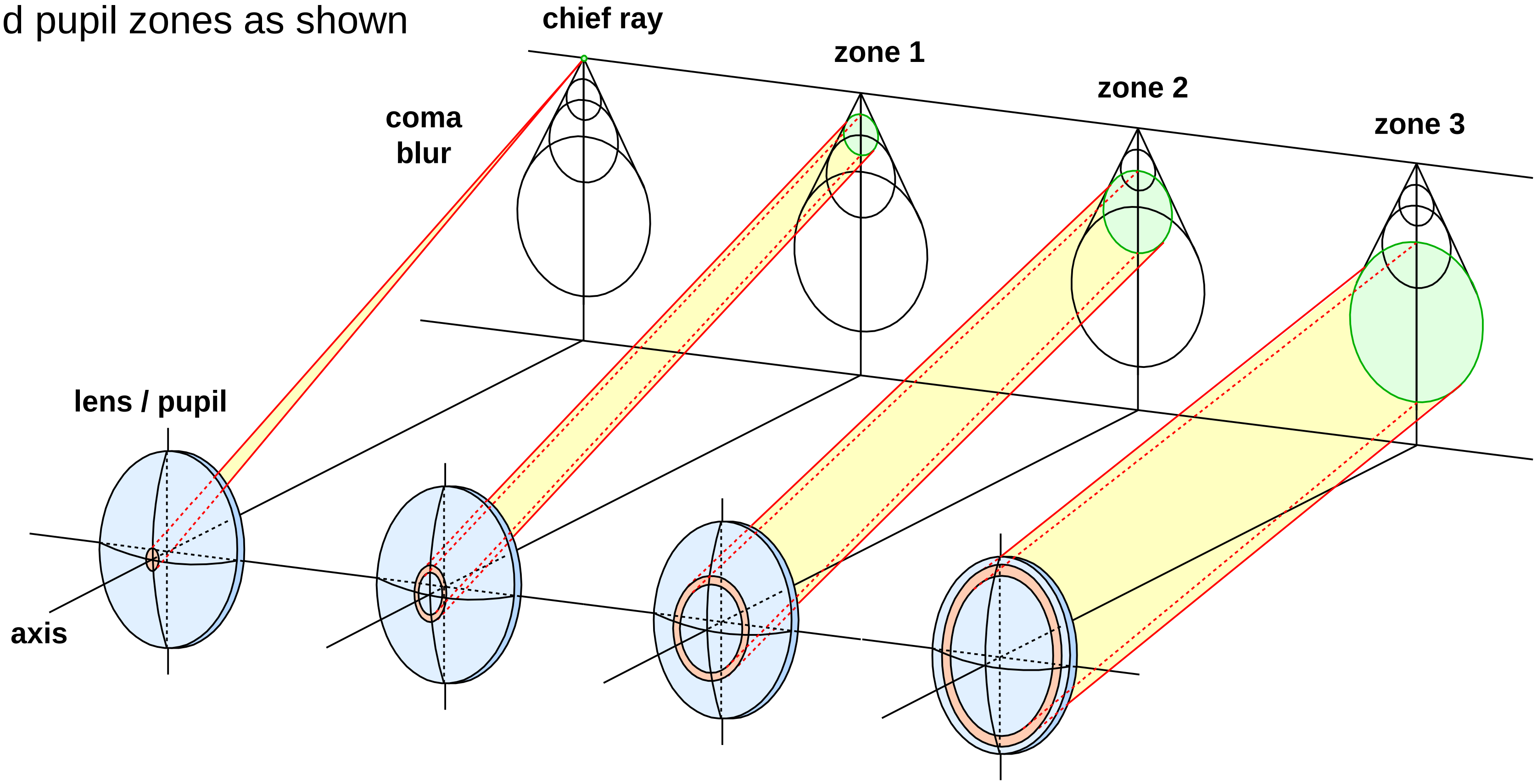
Blurred Coma Spot

- Coma aberration: for oblique bundels and finite aperture due to asymmetry
- Primary effect: coma grows linear with field size y
- Systems with large field of view: coma hard to correct
- Relation of spot circles and pupil zones as shown



Lens Bending and Natural Stop Position

- The lens contribution of coma is given by if the stop is located at the lens

$$C_{lens} = \frac{1}{4ns' f^2} \cdot \left[\frac{n+1}{n-1} X - (2n+1) M \right]$$

- Therefore the coma can be corrected by bending the lens
- The optimal bending is given by and corrects the 3rd order coma complelty

$$X = \frac{(2n+1)(n-1)}{n+1} \cdot M$$

- The stop shift equation for coma is given by

$$S_{II}^{\ *} = S_{II} + \delta E \cdot S_I$$

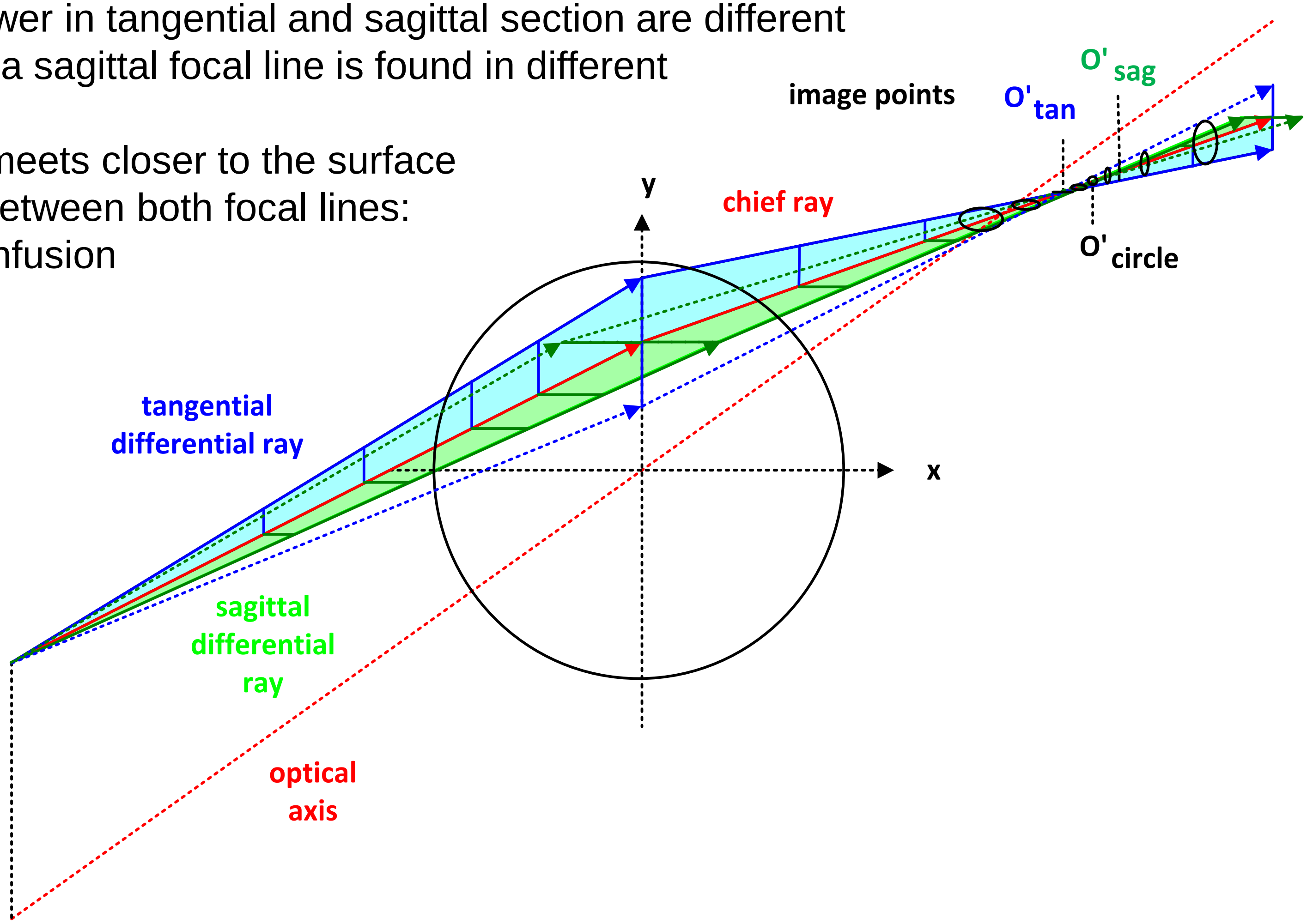
  with the normalized ratio of the chief ray height to the marginal ray height

$$\delta E = \frac{\bar{h}_{new} - \bar{h}_{old}}{h}$$

- If the spherical aberration S_I is not corrected, there is a natural stop position with vanishing coma
- If the spherical aberration is corrected (for example by an aspheric surface), the coma doesn't change with the stop position

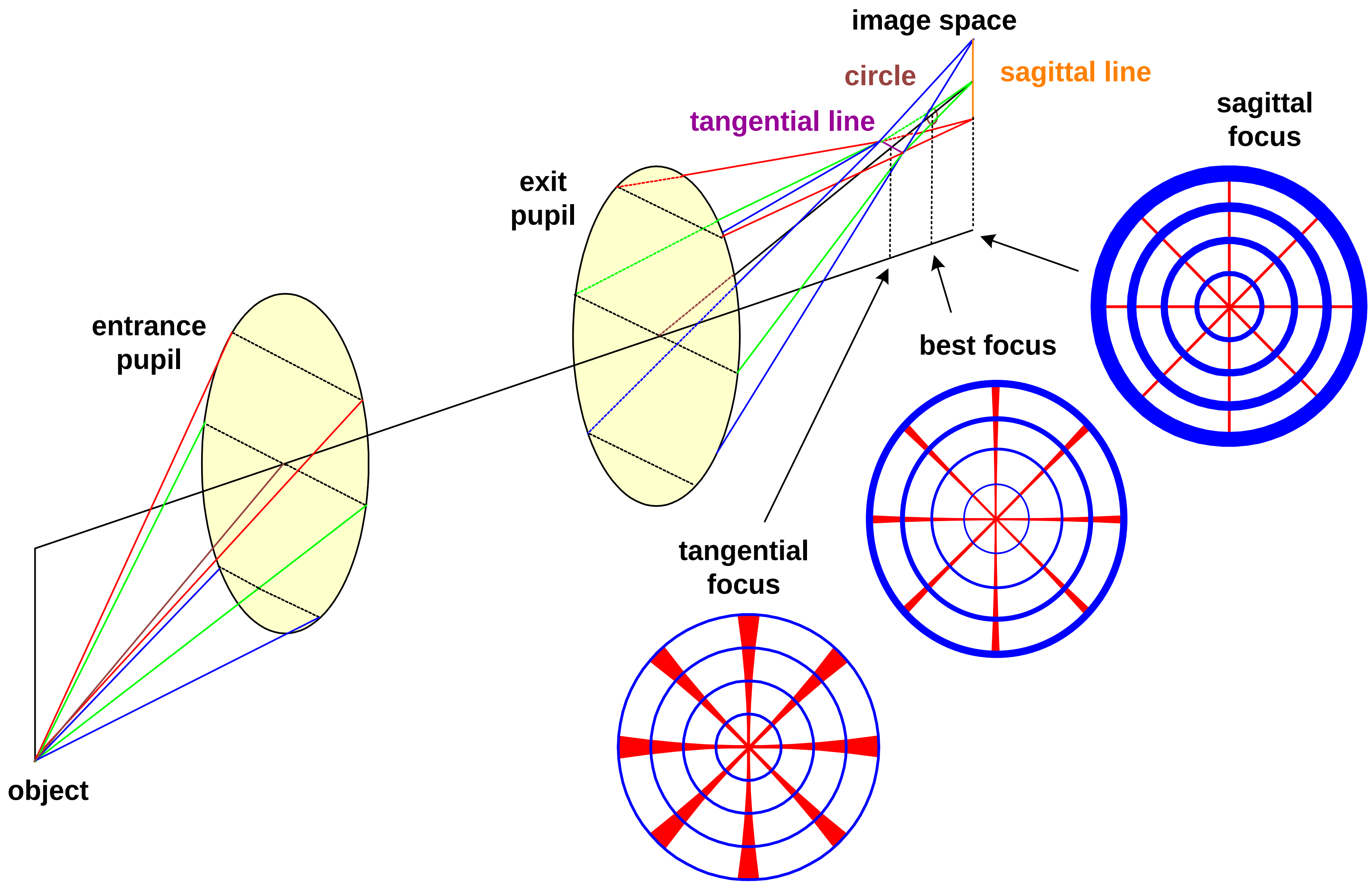
Astigmatism

- Reason for astigmatism:
  chief ray passes a surface under an oblique angle,
  the refractive power in tangential and sagittal section are different
- A tangential and a sagittal focal line is found in different distances
- Tangential rays meets closer to the surface
- In the midpoint between both focal lines:
  circle of least confusion



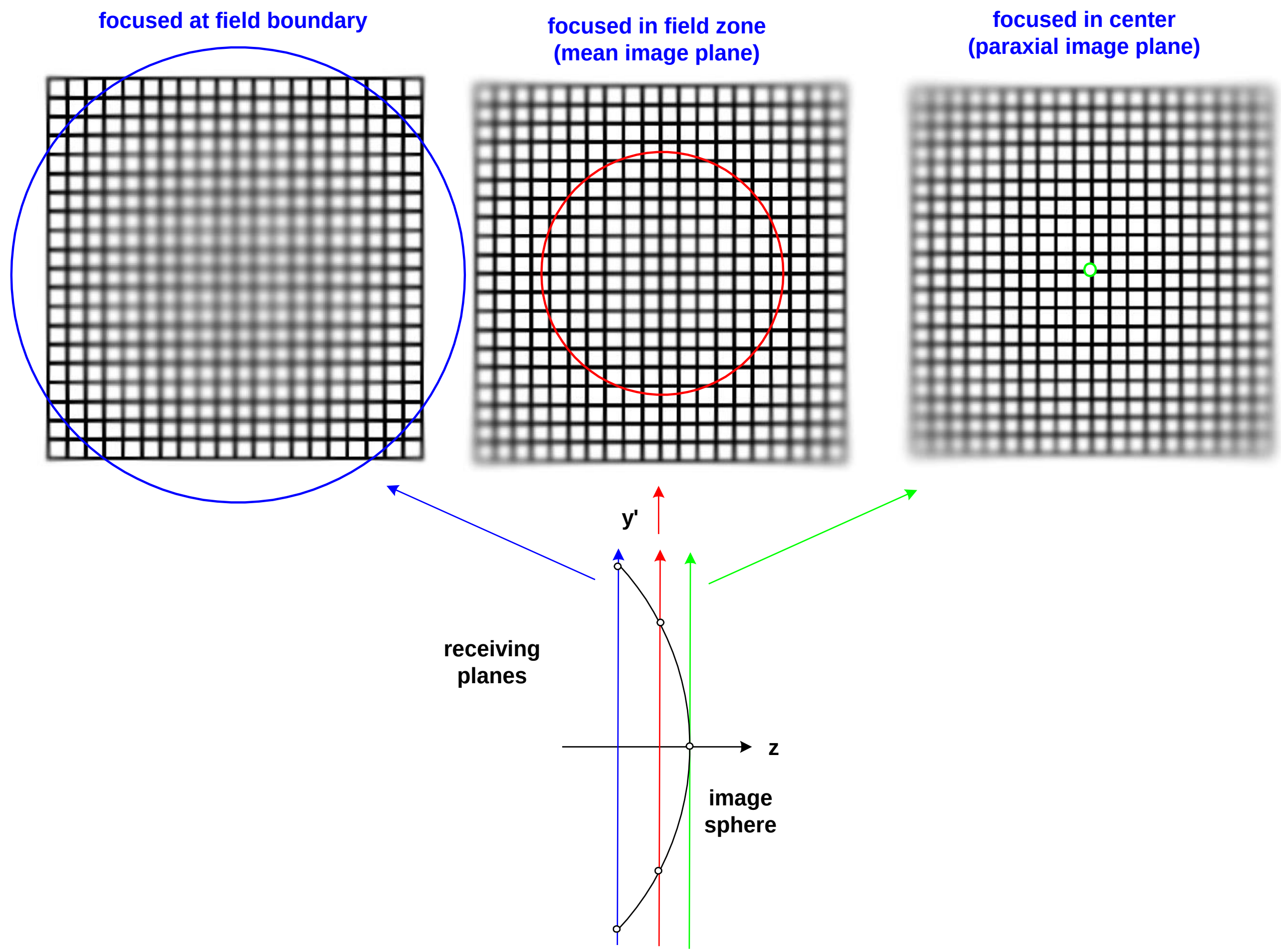
Astigmatism

Imaging of a polar grid in different planes



Field Curvature

- Focussing into different planes of a system with field curvature
- Sharp imaged zone changes from centre to margin of the image field

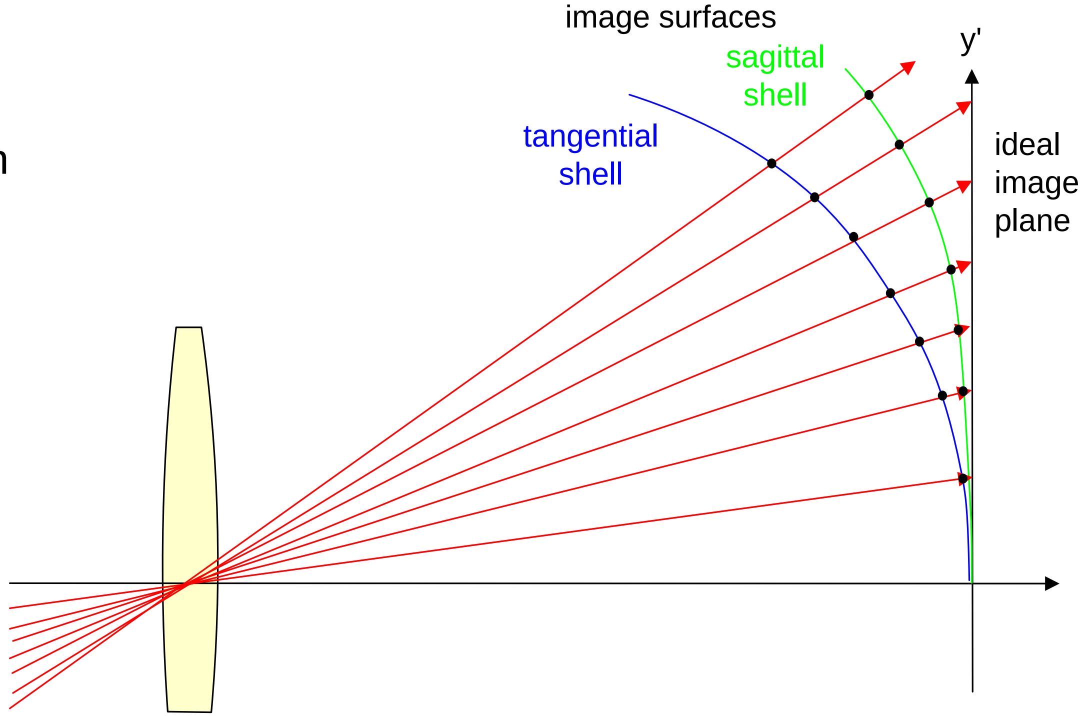


Field Curvature and Image Shells

- Imaging with astigmatism:
  Tangential and sagittal image shell depending on the azimuth
- Difference between the image shells: astigmatism
- Astigmatism corrected:
  It remains a curved image shell,
  Bended field: also called Petzval curvature
- System with astigmatism:
  Petzval sphere is not an optimal
  surface with good imaging resolution
- Law of Petzval: curvature given by:

$$\frac{1}{r_p} = -n' \cdot \sum_k \frac{1}{n_k \cdot f_k}$$

- No effect of bending on curvature,
  important: distribution of lens
  powers and indices



Distortion Example: 10%

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- Image with sharp but bended edges/lines
- No distortion along central directions

Ref : H. Zügge

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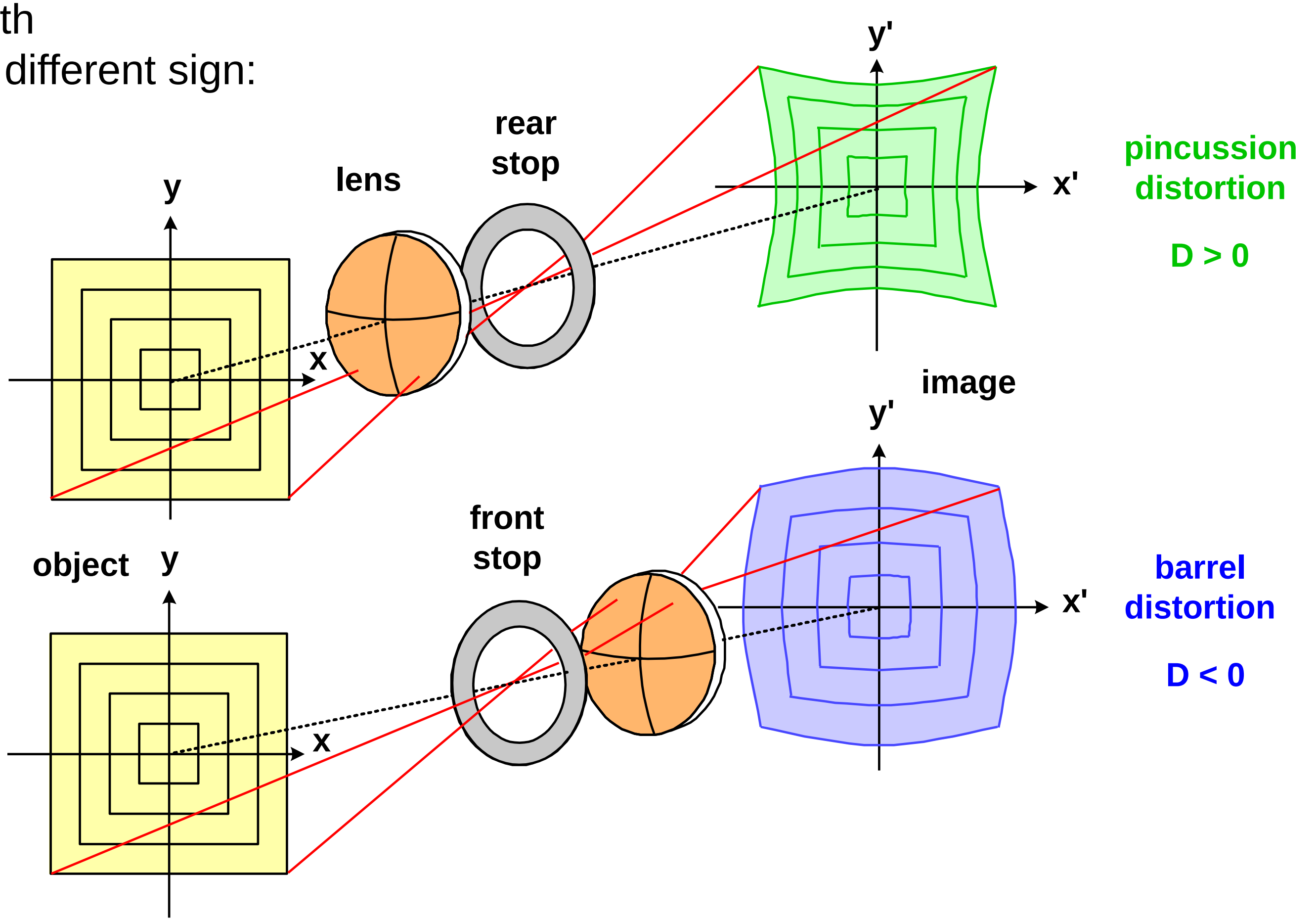
Distortion

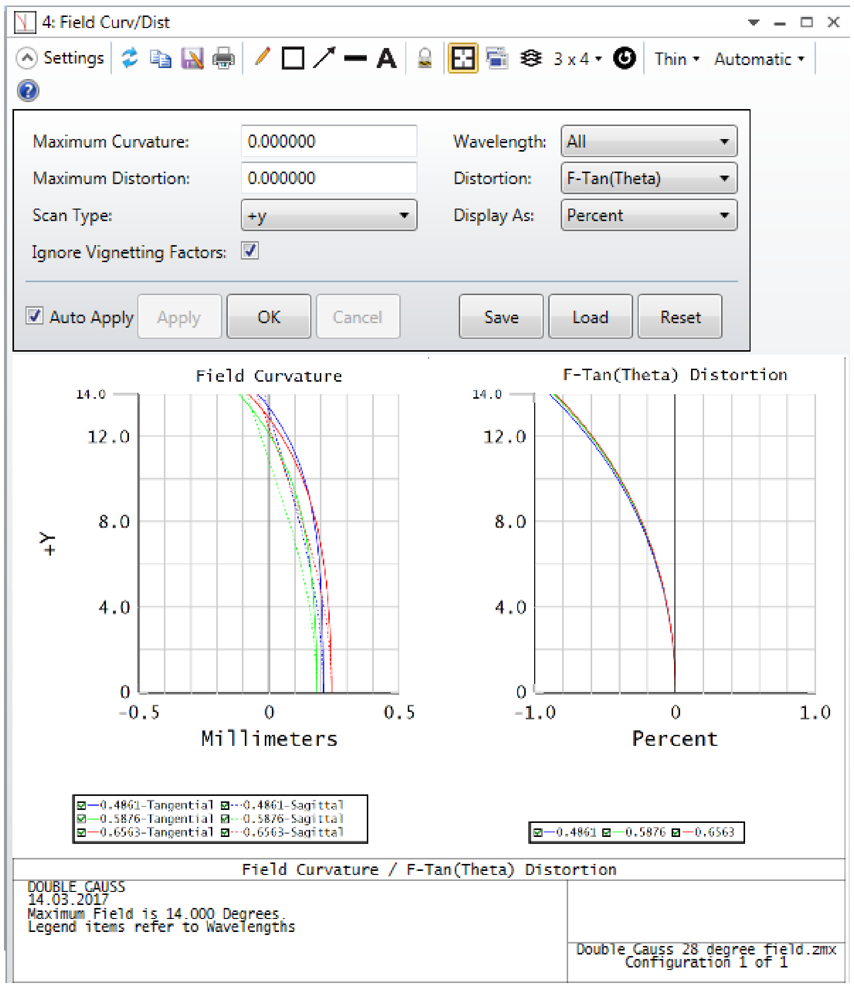
- Purely geometrical deviations without any blurr
- Distortion corresponds to spherical aberration of the chief ray
- Important is the location of the stop:
  defines the chief ray path
- Two primary types with different sign:
  1. barrel, D < 0
     front stop
  2. pincushion, D > 0
     rear stop
- Definition of local magnification changes

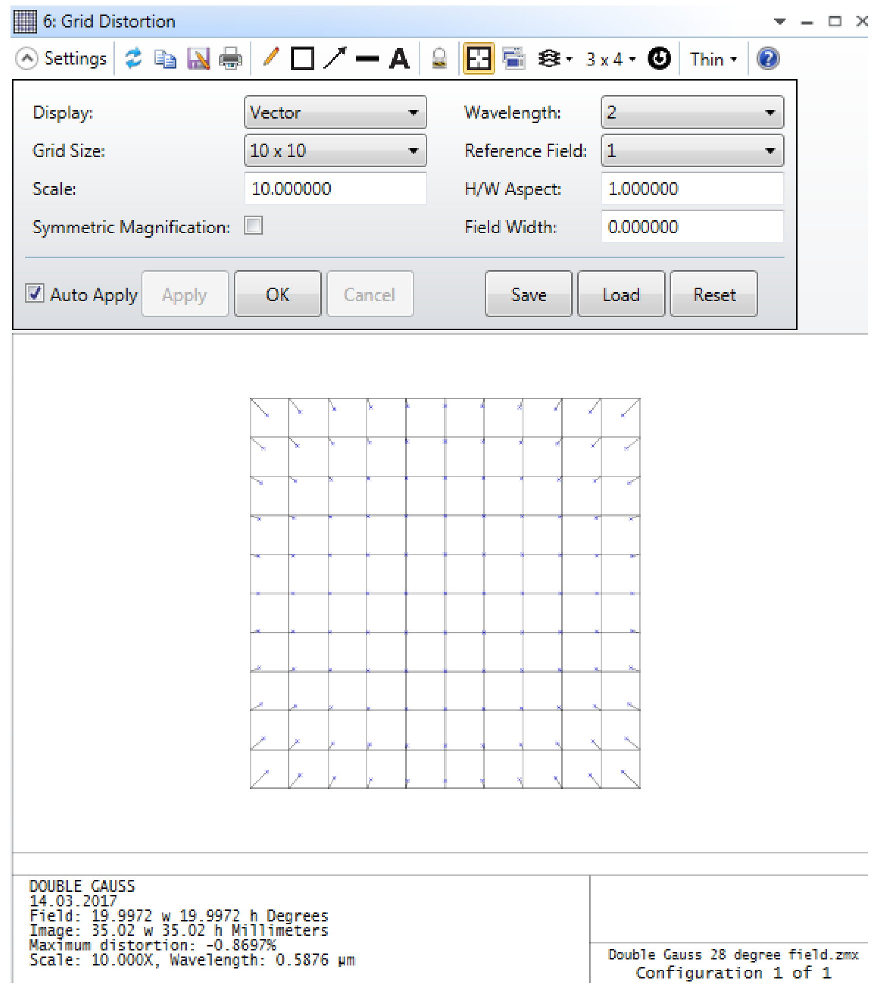
$$D = \frac{y'_{real} - y'_{ideal}}{y'_{ideal}}$$



Further Aberration Representations in Zemax

- Astigmatism and distortion



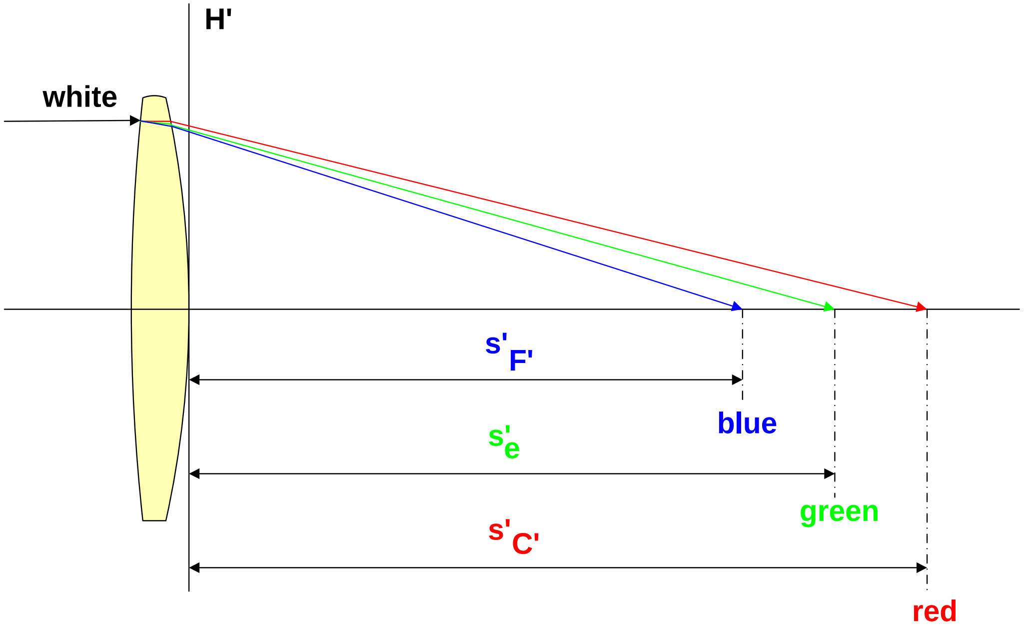


Axial Chromatical Aberration

- Axial chromatical aberration:
  Higher refractive index in the blue results in a shorter intersection length for a single lens
- The colored images are defocussed along the axis
- Definition of the error: change in image location / intersection length

$$\Delta s'_{CHL} = s'_{F'} - s'_{C'}$$

- Correction needs several glasses with different dispersion



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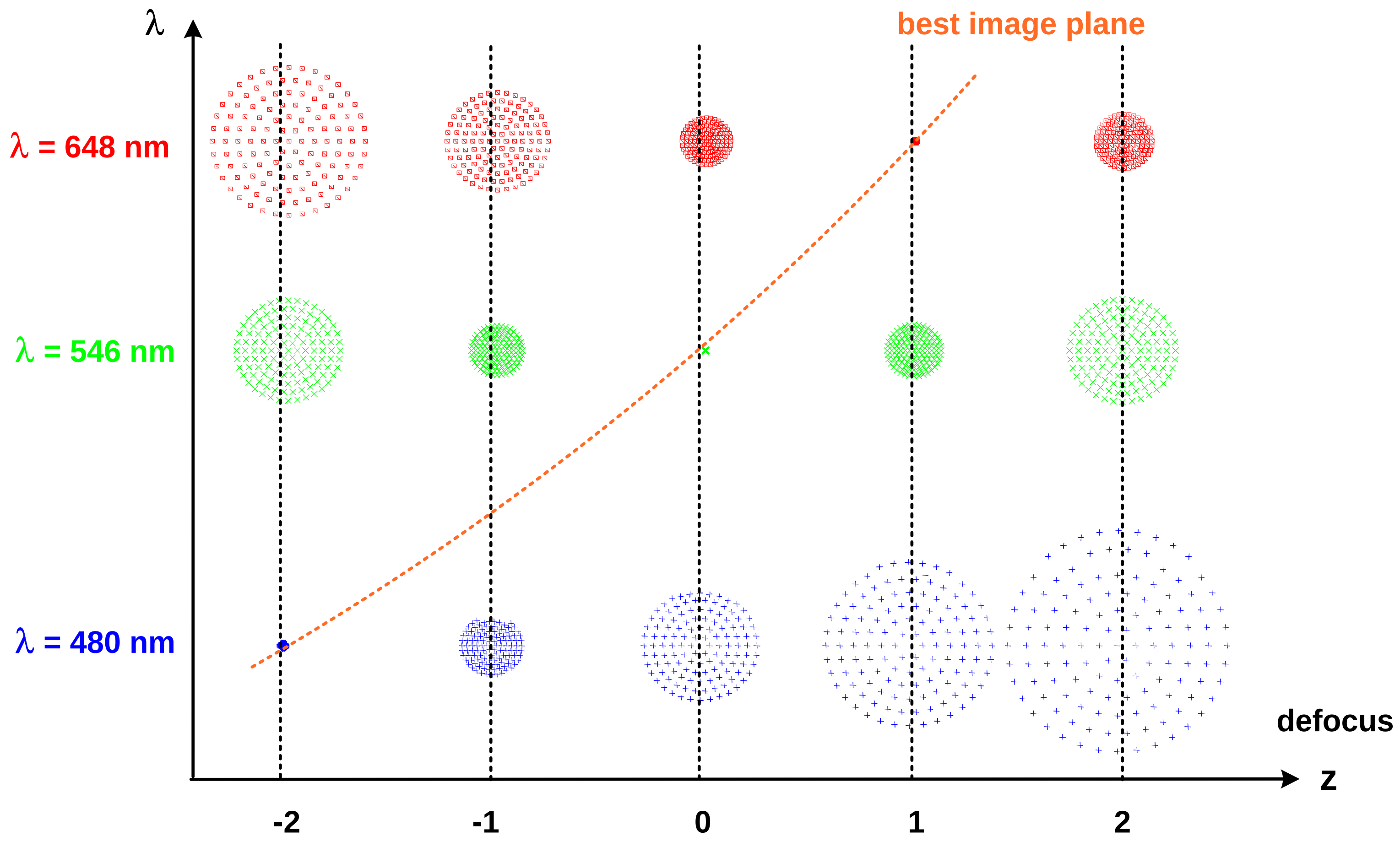
Axial Chromatical Aberration

- Longitudinal chromatical aberration for a single lens
- Best image plane changes with wavelength



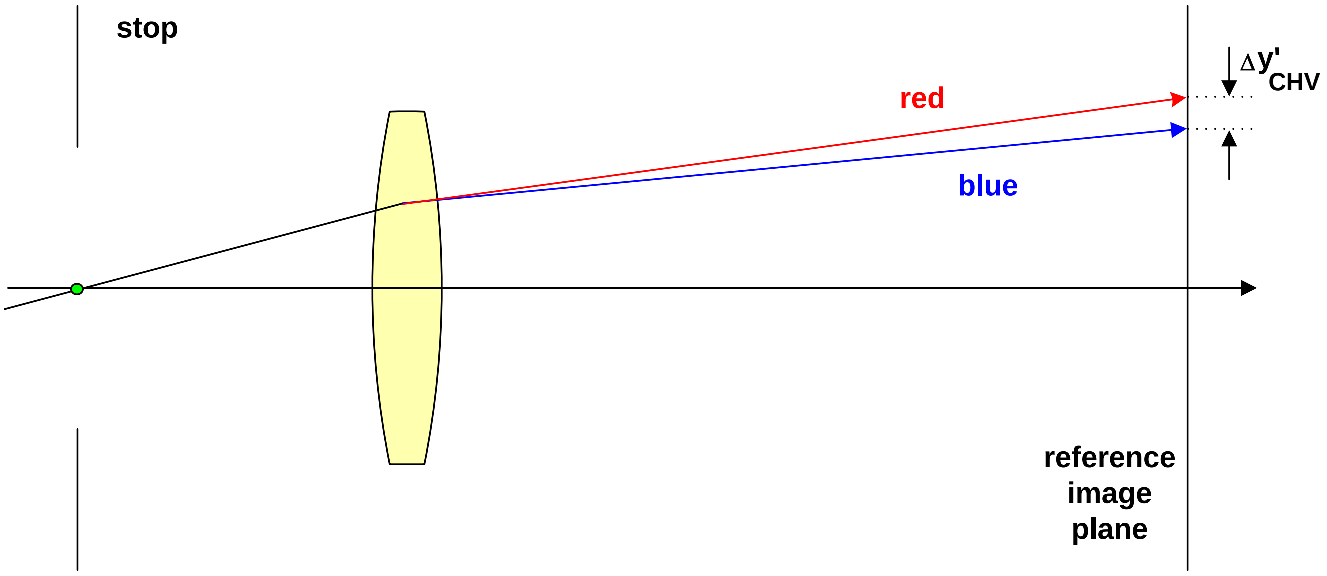
Ref : H. Zügge

Chromatic Variation of Magnification

- Lateral chromatical aberration:
  Higher refractive index in the blue results in a stronger ray bending of the chief ray for a single lens
- The colored images have different size,
  the magnification is wavelength dependent
- Definition of the error: change in image height/magnification
- Correction needs several glasses with different dispersion
- The aberration strongly depends on the stop position

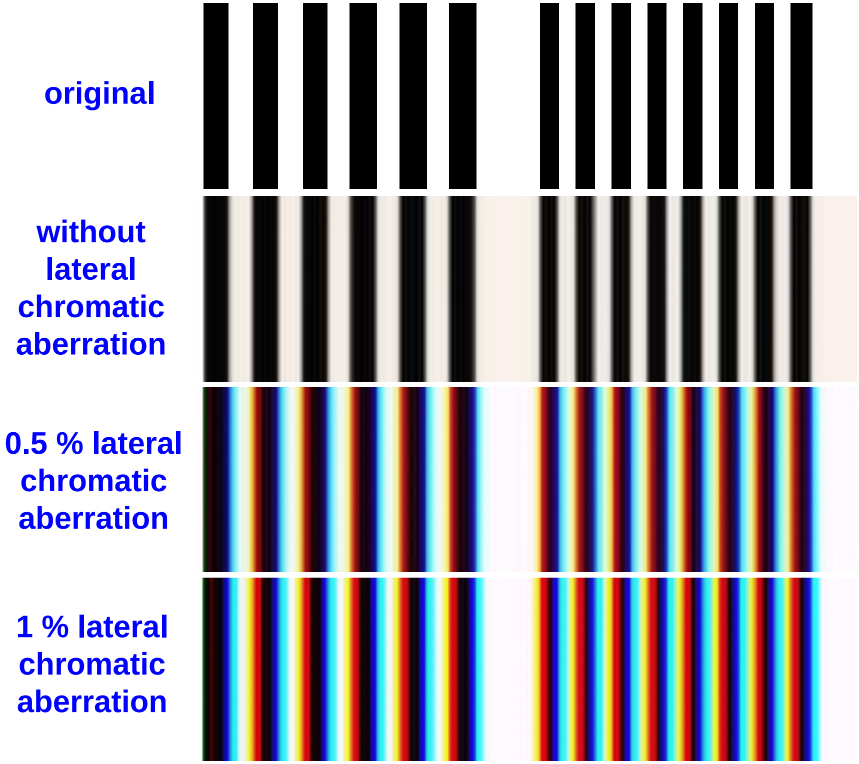
$$\Delta y'_{CHV} = y'_{F'} - y'_{C'}$$

$$\Delta \overline{y}'_{CHV} = \frac{y'_{F'} - y'_{C'}}{y'_e}$$



Chromatic Variation of Magnification

- Impression of CHV in real images
- Typical colored fringes blue/red at edges visible
- Color sequence depends on sign of CHV



Further Aberration Representations in Zemax

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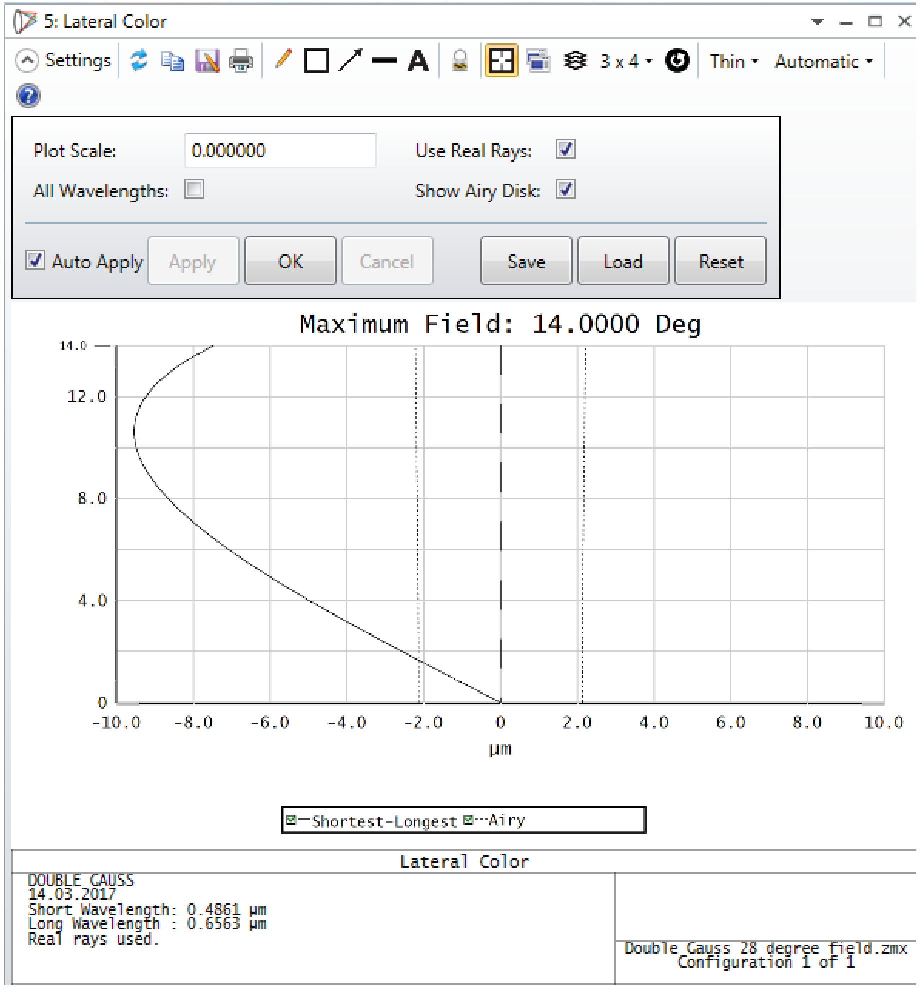
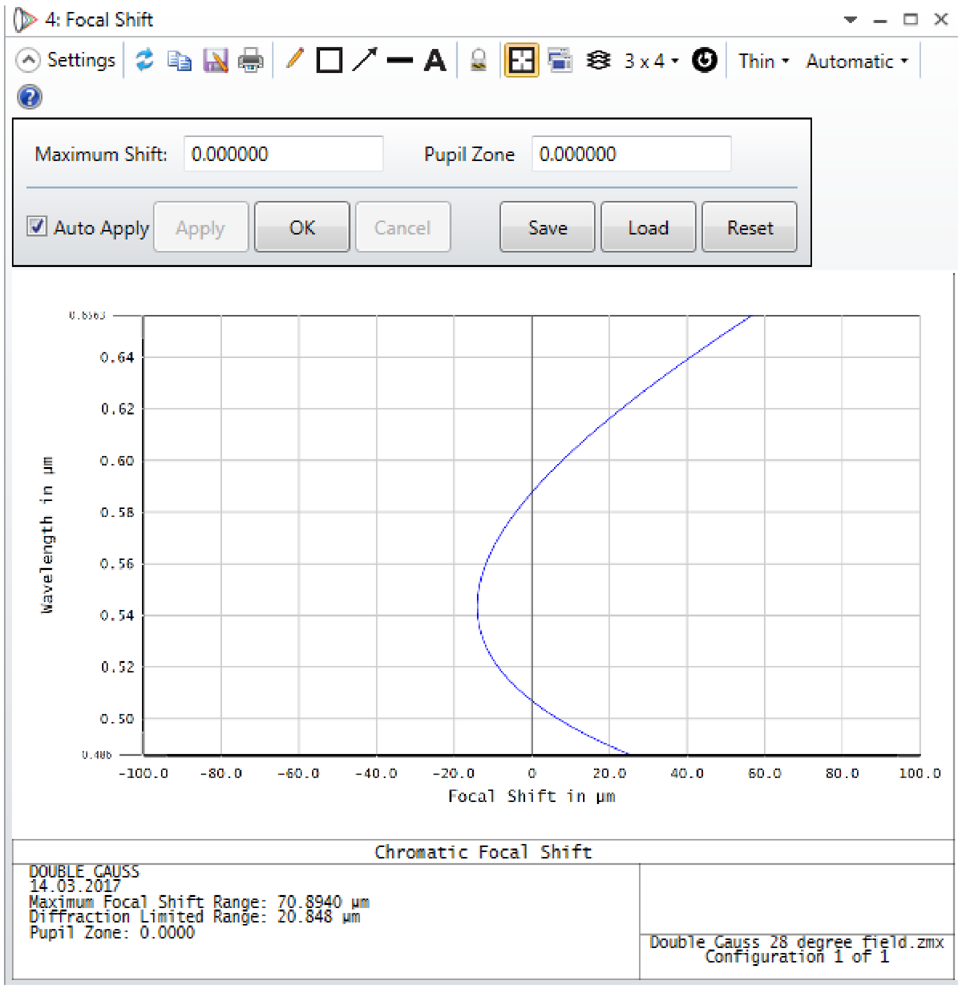
- Lateral color

Chromatical focal shift





Excercise I: Lens Bending

Most of the aberrations change with the bending of a lens.
a) Establish a lens with focal length f = 100 mm made of BK7 with thickness 5 mm for 587.6 nm for an object space numerical aperture of NA = 0.07. The object distance is 170 mm. The object field has a diameter of 30 mm. The stop is located at the lens.

b) Generate a universal plot for coma, spherical aberration, astigmatism and Petzval curvature, if the curvature of the first lens surface is varied between -0.03 ... +0.05. Explain the results.

c) Now modify the setup by placing the system stop 30 mm in front of the lens. What is changing?

d) It is obvious that the stop position and the bending have influence on the aberrations. Therefore, it makes sense to look at the combined effect. For getting this, generate a 2D universal plot, where the stop position and the bending are changed. Formulate the second thickness as a pickup to keep the object distance constant and change the distance between object and stop from 100 ...170 mm. Plot the 2D-dependence for spherical aberration, coma and astigmatism. Interpret the results. What is the optimal bending for a distance of 100 mm? Is it possible to correct all three aberrations simultaneously?

Coma

- Coma deviation, elimination of the azimuthal dependence: circle equation

- Diameter of the circle and position variiation with r_p^2
  Every zone of the circlegenerates a circle in the image plane

- All cricels together form a comet-like shape

- The chief ray intersection point is at the tip of the cone

- The transverse extension of the cone shape has a ratio of 2:3
  the meridional extension is enlarged and gives a poorer resolution