Examination

Introduction to Optical Modeling and Design

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Place and Date: Jena, 24.02.2021, 10:00-12:00, Online

Answer all questions in your own words and with mathematics where needed for your argumentation.

1. Assume a ball-lens (=sphere of glass) of radius R with refractive index n_s surrounded by air ($n = 1$). Use the ABCD-matrix formalism to calculate the value of the refractive index n_s in order to have a collimated beam entering the lens on one side being focused exactly onto the opposite surface of the ball-lens. (4P)

2. Make a sketch of a single-lens imaging set-up with the limiting aperture (stop) being located at distance $f'/2$ in front of the lens. Construct the exit pupil of the system (location and size)! (4P)

3. Optical Materials:
   (a) Sketch a diagram showing the differences in the wavelength dependency of the refractive index for Crown- and Flint-glasses. (1P)
   (b) How is the Abbe-number defined? (clearly define the quantities in the equation – e.g. using the diagram of 3a) (2P)
   (c) Which condition needs to be fulfilled to obtain an achromatic doublet? (clearly define the quantities in the equation) (2P)

4. What effect is described by the astigmatism aberration? Make a sketch to illustrate the effect. (3P)

5. Sine-Condition:
   (a) What is the (Abbe) Sine-condition and what does it mean? (2P)
   (b) What is the shape of the principal surfaces if the sine-condition is fulfilled? (1P)
   (c) How are systems called fulfilling the sine-condition? (1P)

6. What is meant by the term "diffraction limited spot size"? Which shape should the wavefront in the exit pupil have in this case? How can this spot size be calculated? (3P)

7. One of the material equations in frequency domain is given by $\boldsymbol{D}(\boldsymbol{r},\omega) = \epsilon(\boldsymbol{r},\omega)\,\boldsymbol{E}(\boldsymbol{r},\omega)$. Transform this equation into the time-space domain for dispersive and non-dispersive media. (3P)

8. A field component $V(\boldsymbol{\rho}, z_0)$ is known at position z_0. Provide the spectrum-of-plane-waves (SPW) operator in its mathematical form to express $V(\boldsymbol{\rho}, z \geq z_0)$. (2P) Does this formula include any approximations? (1P)

9. Propagation integrals:

   (a) What are major steps to obtain the Rayleigh integral from the SPW operator. (2P)

   (b) What is changed in the SPW operator formula to get the generalized Debye integral? (1P) When is this step justified? (1P) What is assumed in addition to obtain the classical Debye integral? (1P)

   (c) What is changed in the SPW operator formula to get the generalized far-field integral? (1P) When is this step justified? (1P) What is assumed in addition to obtain the classical far-field integral? (1P)

   (d) How to come from the far-field integral to the Fraunhofer diffraction integral? (2P)

   Make your major arguments clear. Detailed mathematical derivations are not needed!

10. How to calculate $E_z(\boldsymbol{\rho}, z_0)$ from $E_x(\boldsymbol{\rho}, z_0)$ and $E_y(\boldsymbol{\rho}, z_0)$? (2P)

11. The Gaussian beam is mathematically expressed by

$$\begin{aligned} V(\boldsymbol{\rho}, z) =& V_0\,\frac{w(0)}{w(z)}\exp\left\{-\mathrm{i}\arctan\left(\frac{z}{z_\mathrm{R}}\right)\right\}\exp(\mathrm{i}k_0 n z) \\ &\times\exp\left(-\mathrm{i}\frac{k_0 n}{2R(z)}\boldsymbol{\rho}^2\right)\exp\left(-\frac{\boldsymbol{\rho}^2}{w^2(z)}\right), \end{aligned}$$

   with $w(z) = w(0)\sqrt{1+\frac{z^2}{z_\mathrm{R}^2}}$ and $R(z) = z + \frac{z_\mathrm{R}^2}{z}$.

   (a) Does this result include any approximation? Explain? (2P)

   (b) Derive the divergence of the Gaussian beam with waist radius $w(0)$. (2P)