Task 1: Basic Metrology Principles

a) What is the meaning of 'confidence interval' of measured values?

b) Explain the meaning of the sensitivity and the resolution of an instrument.

a) A confidence interval is the mean of your estimate plus and minus the variation in that estimate. This is the range of values you expect your estimate to fall between if you redo your test, within a certain level of confidence.
The confidence interval for data which follows a standard normal distribution is:

$CI = \bar{x} \pm z^* \frac{s}{\sqrt{n}}$ $\bar{x}$ = the sample mean z^* = the critical value of the z distribution
s = the sample standard deviation $\sqrt{n}$ = the square root of the population size

b) Sensitivity: smallest signal the instrument can measure
Resolution: smallest amount of input signal change the instrument can detect reliably

Task 2 Sampling Theory

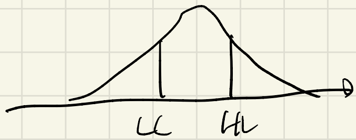
a) For an object of 2cm dimension, what is the minimum spatial resolution (in cm) when the object is digitized into an array of 512 samples?

b) How many harmonics will be present in the Fourier transform of the digitized object?

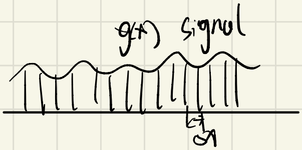
c) What is the lowest (but not DC) spatial frequency involved in the transform of the object described above and what is the highest?

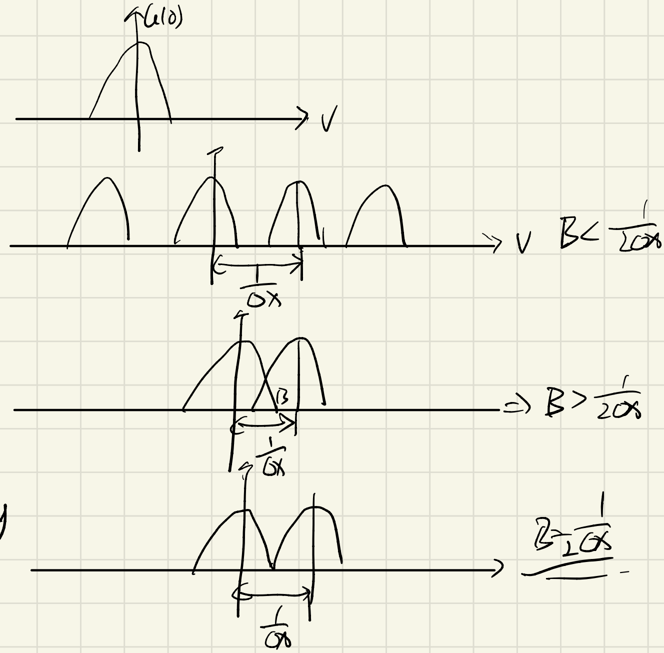
a) $D = N \cdot \Delta x \Rightarrow \Delta x = \frac{D}{N} = \frac{2\,cm}{512} = \frac{1}{256}$ cm

① $CI = \bar{x} \pm z\frac{s}{\sqrt{n}}$ $\quad LL < x < HL$



② a.





$g(x) \times \sum_{n}^{\infty} \delta(x-n)$

$G(f_x) \otimes \delta(f_x - n)$

$f_s = \frac{512}{2cm} = 256\ cm^{-1}$

$\Delta x = \frac{2cm}{512} = \frac{1}{256} cm$

$\Delta x = \frac{1}{f_s}$

highest frequency

$B = \frac{1}{2\Delta x} = 128\ cm^{-1}$

minimum spatial resolution

$x_{min} = \frac{1}{128} cm = 78.13\ mm$

(b) minimum frequence $v_{min} = \frac{1}{L} = \frac{1}{2cm}$

$N = \frac{B}{v_{min}} = 256$

(c) Lowest frequency $v_{min} = \frac{1}{2} = \frac{1}{2cm} = 0.5\ cm^{-1}$

Highest frequency $v_{max} = B = \frac{1}{2\Delta x} = 128 cm^{-1}$

1 harmonic need two information (frequency, Amplitude)

3. $\mathcal{F}\{\mathcal{F}[g(x,y)]\} = \iint_{-\infty}^{\infty} df_x\, df_y \{G(f_x,f_y)\}\, e^{-2\pi i(f_x x+f_y y)}$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} df_x\, df_y \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\xi\, d\eta\, g(\xi,\eta)\, e^{-2\pi i(\xi f_x+\eta f_y)}\right] e^{-2\pi i(f_x x+f_y y)}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\xi\, d\eta\, g(\xi,\eta) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} df_x\, df_y\, e^{-2\pi i[(\xi+x)f_x+(\eta+y)f_y]}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\xi\, d\eta\, g(\xi,\eta)\, \delta(\xi+x, \eta+y) = g(-x,-y)$$

$\mathcal{F}[g(x,y)h(x,y)] = \mathcal{F}[g(x,y)] * \mathcal{F}[h(x,y)] = G(f_x,f_y) * H(f_x,f_y)$

$\mathcal{F}^{-1}[\mathcal{F}(g(x,y)h(x,y))] = \mathcal{F}^{-1}[G(f_x,f_y) * H(f_x,f_y)]$

$g(x,y)h(x,y) = \mathcal{F}^{-1}\iint_{-\infty}^{\infty} G(\xi,\eta)\, H(f_x-\xi, f_y-\eta)\, d\xi\, d\eta$

$\mathcal{F}[g(x-a)] = G(f_x)e^{-2\pi i f_x a}$ $\quad \mathcal{F}^{-1}[G(f_x-a)] = g(x)e^{2\pi i x a}$

$\Rightarrow g(x,y)h(x,y) = h(x,y)\iint_{-\infty}^{\infty} G(\xi,\eta)\, e^{2\pi i(x\xi+y\eta)}\, d\xi\, d\eta = g(x,y)h(x,y)$

(4) $\mathrm{comb}(ax) \Rightarrow \frac{1}{|a|}\mathrm{comb}(\frac{f_x}{a})$ $\quad \mathrm{comb}(Xf_x) = \sum_{n=-\infty}^{\infty}\delta(Xf_x-n)$ $\quad \delta(ax) = \frac{1}{a}\delta(x)$

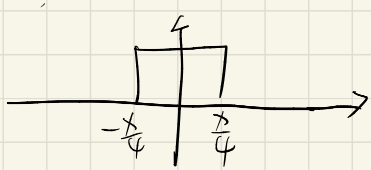
$P(f_x,f_y) = G(f_x,f_y)\cdot XY[\mathrm{comb}(Xf_x)\mathrm{comb}(Yf_y)]$

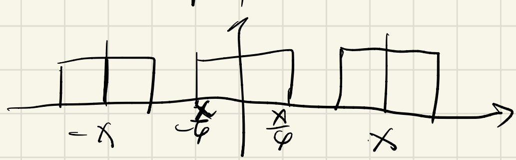
$$= G(f_x,f_y)\, XY \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\delta(Xf_x-n)\,\delta(Yf_y-m)]$$

$$= G(f_x,f_y)\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\delta(f_x-\tfrac{n}{X})\,\delta(f_y-\tfrac{m}{Y})]$$

$$= \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} G(\tfrac{n}{X},\tfrac{m}{Y})\,\delta(f_x-\tfrac{n}{X})\,\delta(f_y-\tfrac{m}{Y})$$

b) $\mathrm{rect}(2\frac{x}{X})$





$$F[\mathrm{rect}(ax)] = \frac{1}{a}\,\sin\left(\frac{F_x}{a}\right)$$

$$G(f_x, f_y) = F\left[\mathrm{rect}\left(\frac{2x}{X}\right)\right] = \frac{X}{2}\,\sin\left(\frac{X F_x}{2}\right)$$

$$P(f_x, f_y) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \frac{X}{2}\,\mathrm{sinc}\left(\frac{X}{2} \cdot \frac{n}{X}\right) \delta\left(f_x - \frac{n}{X}\right) = \sum\sum \frac{X}{2}\,\mathrm{sinc}\left(\frac{n}{2}\right)$$