Name (with capital letters): Yining Li
Matriculation number: 149824

Only to be used by the corrector!							
1	2	3	4	5a	5b	Σ	mark

Exam Structure of Matter Winter Term 2014 / 2015 (17.02.2015)

Please note: Please write your name and your matriculation number on each sheet! Please write your solution on the sheets provided and use only other sheets when necessary! Notes made with pencils or with colors others than blue or black will not be accepted!

Task 1 **6 + 2 Points**

a) A hydrogen atom is prepared in the state $|n=5, l=2, m=-2, s_z=1/2\rangle$. Into which states can it decay by emitting a single photon via a dipolar transition as discussed in the lecture? Calculate the energy of the emitted photons in terms of the Rydberg energy E_R !

b) How is the photon with the highest frequency polarized? Give a short reasoning!

(a) transition rule: $\Delta l = \pm 1$, $\Delta m = 0, \pm 1$, $\Delta S_z = 0$

energy of the emitted photons: $\Delta E = -E_R\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$, $E_R = 13.6\,eV$

possible transition state:

$n=4,\ l=1,\ m=-1, S_z=\frac{1}{2}, \Delta E = -13.6\left(\frac{1}{5^2} - \frac{1}{4^2}\right) eV = \frac{9}{400} E_R$

$n=3,\ l=1,\ m=-1, S_z=\frac{1}{2}, \Delta E = -13.6\left(\frac{1}{5^2} - \frac{1}{3^2}\right) eV = \frac{16}{225} E_R$

$n=2,\ l=1,\ m=-1, S_z=\frac{1}{2}, \Delta E = -13.6\left(\frac{1}{5^2} - \frac{1}{2^2}\right) eV = \frac{21}{100} E_R$

$n=1,\ l=1,\ m=-1, S_z=\frac{1}{2}, \Delta E = -13.6\left(\frac{1}{5^2} - 1\right) eV = \frac{24}{25} E_R$

(b) $\Delta E = \hbar w = -E_R\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$, when transition is from state $|n=5, l=2, m=-2, S_z=\frac{1}{2}\rangle$ to state $|n=1, l=1, m=-1, S_z=\frac{1}{2}\rangle$, ΔE is the biggest, which means the frequency is highest.

$$w = -\frac{E_R}{\hbar}\left(\frac{1}{n^2} - \frac{1}{m^2}\right) = -\frac{E_R}{\hbar}\left(\frac{1}{5^2} - \frac{1}{1^2}\right) = \frac{24 E_R}{25\hbar}$$

Task 2 **(2+2+2+2+2 Point)**

Estimate the energy between the ground state and first excited state.

a) H atom (E_R=13.6ev)

b) He^+ ion

c) Vibrational energy of H_2 molecule

d) Rotational energy of H_2 molecule

e) Draw diagram of black body radiation $u_w(w,T)$ for T1<T2.

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Task 3

3 + 6 + 3 + 2 Points

Consider a particle interacting with a δ-like potential $V(x) = V_0\,\delta(x)$ with $V_0 > 0$

a) Derive the transition condition $\lim_{\varepsilon\to 0}\left(\frac{d}{dx}\psi\Big|_{x=+\varepsilon} - \frac{d}{dx}\psi\Big|_{x=-\varepsilon}\right) = \frac{2m_e V_0}{\hbar^2}\psi(x=0)$.

b) Solve the stationary Schrödinger equation for energies $E > 0$. Consider solutions of the form $\psi(x) = \exp(ikx) + R\exp(-ikx)$ for $x < 0$ and $\psi(x) = T\exp(ikx)$ for $x > 0$ and compute k as well as R and T in terms of E and V_0.

c) In general, the probability flux density j is defined as

$$j = \frac{\hbar}{2im_e}\left(\psi^*\frac{\partial}{\partial x}\psi - \psi\frac{\partial}{\partial x}\psi^*\right).$$

Calculate the probability flux densities j_{in}, j_{refl}, j_{trans} of the incoming wavefunction $\exp(ikx)$, of the reflected wavefunction $R\exp(-ikx)$ and of the transmitted wavefunction $T\exp(ikx)$ for given R and T.

d) Calculate the reflectivity and transmittance $\rho = \left|\frac{j_{\text{refl}}}{j_{\text{in}}}\right|$ and $\tau = \left|\frac{j_{\text{trans}}}{j_{\text{in}}}\right|$ for given R and T ! What happens in the limiting case of $V_0 \to +\infty$?

(a) According to the Schrödinger equation

$$-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial x^2}\psi + V(x)\psi = E\psi$$

$$\Rightarrow \lim_{\varepsilon\to 0}\left[\int_{-\varepsilon}^{+\varepsilon} -\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial x^2}\psi + \int_{-\varepsilon}^{+\varepsilon} V(x)\psi\right] = \lim_{\varepsilon\to 0}\int_{-\varepsilon}^{+\varepsilon} E\psi$$

$$-\frac{\hbar^2}{2m_e}\cdot\lim_{\varepsilon\to 0}\left[(\psi'(\varepsilon+) - \psi'(\varepsilon-)) + \int_{-\varepsilon}^{+\varepsilon} V_0\delta(x)\psi\right] = E\cdot\lim_{\varepsilon\to 0}\int_{-\varepsilon}^{+\varepsilon}\psi$$

$$-\frac{\hbar^2}{2m_e}\cdot\lim_{\varepsilon\to 0}(\psi'(\varepsilon+) - \psi'(\varepsilon-)) + V_0\psi(0) = 0$$

$$\lim_{\varepsilon\to 0}(\psi'(\varepsilon+) - \psi'(\varepsilon-)) = \frac{2m_e V_0}{\hbar^2}\psi(0)$$

(b) $\psi(x) = e^{ikx} + Re^{-ikx}$ for $x<0$

$\psi(x) = Te^{ikx}$ for $x>0$

boundary condition: $\psi(0_+) = \psi(0_-)$

from part a) $\lim_{\varepsilon\to 0}(\psi'(\varepsilon+) - \psi'(\varepsilon-)) = \frac{2m_e V_0}{\hbar^2}\psi(0)$

$$\Rightarrow \begin{cases} 1 + R = T \\ ikT - ik + ikR = \frac{2m_e V_0}{\hbar^2}T \end{cases}$$

$$\Rightarrow 2iRk = \frac{2m_e V_0}{\hbar^2}(1+R) \Rightarrow R = \frac{m_e V_0}{ik\hbar - m_e V_0}, \text{ so } T = 1 + R = \frac{ik\hbar}{ik\hbar - m_e V_0}$$

$$\begin{cases} R = \frac{m_e V_0}{ik\hbar - m_e V_0} \\ T = \frac{ik\hbar}{\dots} \end{cases}$$

see additional page

(e) $u_\omega(\omega,T) = \frac{\hbar\omega^3}{\pi^2 c^3} \frac{1}{e^{\hbar\omega/k_b T}}$

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1 + 1 + 1 + 3 Points

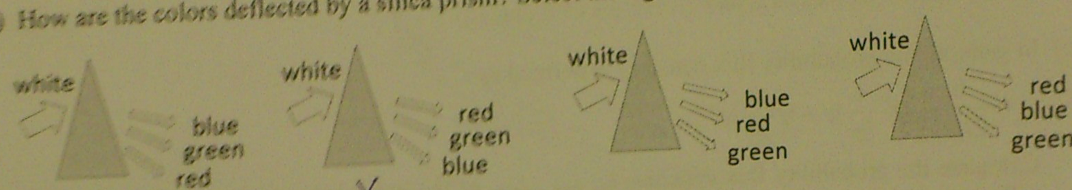
Task 4
Silica has normal dispersion in the visible spectral range.

a) What is the wavelength range of visible radiation?

b) What is "normal" dispersion?

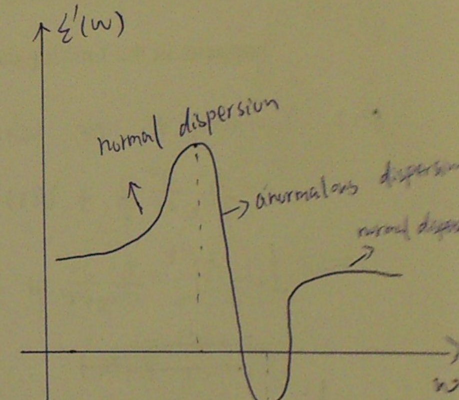
c) How are the colors deflected by a silica prism? Select the right configuration!

u_ω



d) Sketch the principal spectral dependence of the real part of ε(ω) of silica from the near infrared to the UV spectral range and explain why it is normal dispersive in the visible frequency range!

(a) Visible range : 400 nm ~ 700 nm



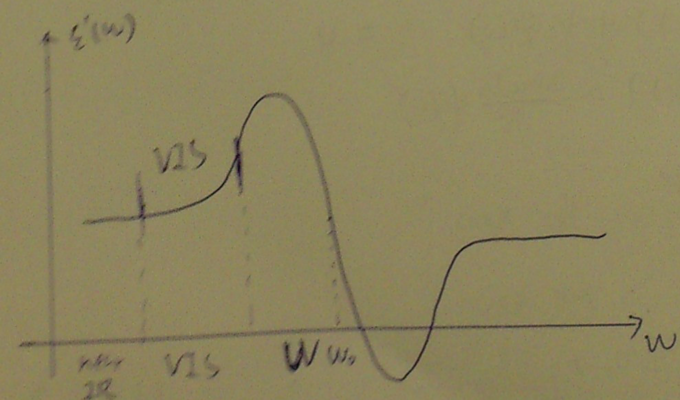
(b) $\varepsilon(\omega) = \varepsilon'(\omega) + \varepsilon''(\omega)$.

when $\frac{d\varepsilon'(\omega)}{d\omega} > 0$, this is normal dispersion

$\frac{d\varepsilon'(\omega)}{d\omega} < 0$, anormalous dispersion.

(c) the second is right.

(d)



from the figure, we know the frequency of visible range is smaller than the absorption frequency ω_0, so in VIS it is normal dispersion.

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You should solve either task 5a or task 5b. If you solve both only the points of the task where you have achieved the best result will be counted!

Task 5a)

1 + 2 + 4 + 3 Points

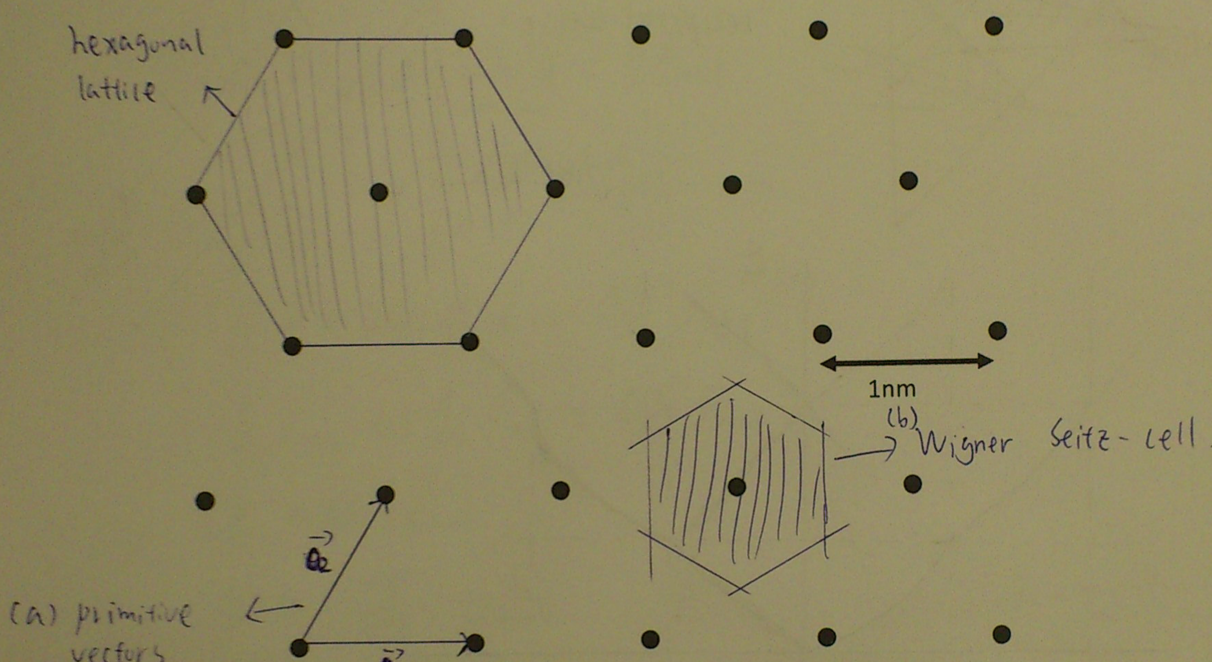
Given a two dimensional hexagonal lattice with a lattice constant of 1nm.

a) Draw the primitive lattice vectors!

b) Draw the Wigner-Seitz-cell of the lattice!

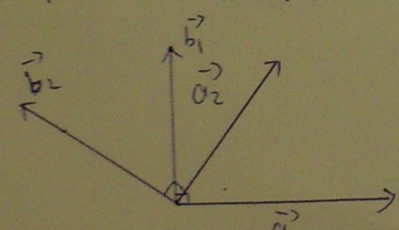
c) Draw in a new figure the vectors of the reciprocal lattice schematically and calculate their length!

d) Draw the reciprocal lattice and identify the first Brillouin zone!



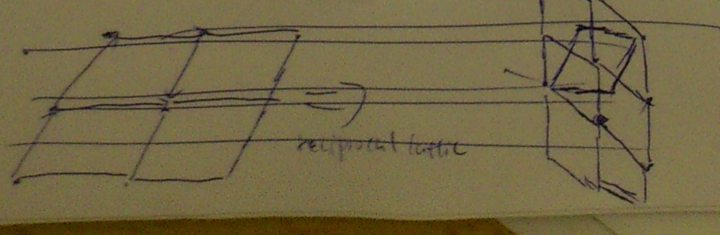
(c) real lattice: $\vec{R} = l_1\vec{a_1} + l_2\vec{a_2} + l_3\vec{a_3}$ we have $\vec{a_i}\cdot\vec{b_j} = 2\pi\delta_{ij}$

reciprocal lattice: $\vec{K} = h_1\vec{b_1} + h_2\vec{b_2} + h_3\vec{b_3}$.



the length of vector $\vec{b_1}$ and $\vec{b_2}$ is $2\pi\cdot nm^{-1}$

(d) Brillouin zone: Wigner Seite cell of the reciprocal lattice.



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Yining Li

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You should solve either task 5a or task 5b. If you solve both only the points of the task where you have achieved the best result will be counted!

Task 5b)

4 + 3 + 1 + 2 Points

Consider the optical response of a conventional metal by using the classical Drude model.

a) Derive an analytical expression for the complex dielectric function !

b) Draw the real and imaginary part of the dielectric function versus frequency for $\omega > 0$.

c) How does the plasma frequency depend on the concentration of free charges?

d) In which frequency range do metals become transparent and why?

(a) $$m\frac{d^2}{dt^2}\vec{d} + m\gamma\frac{d}{dt}\vec{d} + \underbrace{m\omega_0^2\vec{d}}_{\text{for metal, is } 0} = q\vec{E}_{real}$$

$$\vec{p} = q\vec{d}, \quad \vec{P}_{real} = N\vec{p}$$

$$\Rightarrow Nq\cdot\frac{d^2}{dt^2}\vec{d} + Nq\gamma\frac{d}{dt}\vec{d} = \frac{Nq^2}{m}\vec{P}_{real}$$

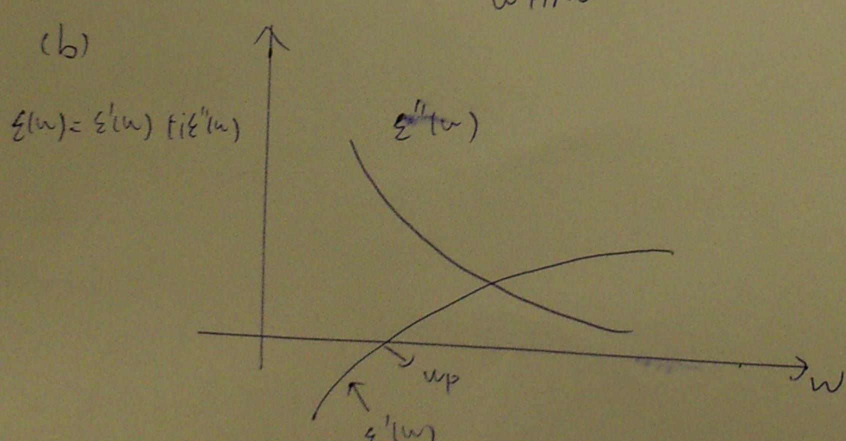
$$\frac{d^2}{dt}\vec{P}_{real} + \gamma\frac{d}{dt}\vec{P}_{real} = \frac{Nq^2}{m}\vec{P}_{real}$$

$$\vec{P}_{real} = \varepsilon_0\chi(\omega)\vec{E}_{real}$$

so $$\chi(\omega) = -\frac{Nq^2}{\varepsilon_0 m}\cdot\frac{1}{\omega^2+i\gamma\omega} \qquad \frac{Nq}{\varepsilon_0 m} = \omega_p^2 \Rightarrow \chi(\omega) = -\frac{\omega_p^2}{\omega^2+i\gamma\omega}$$

$$\varepsilon(\omega) = 1+\chi(\omega) = 1-\frac{\omega_p^2}{\omega^2+i\gamma\omega}$$

(b)



(c)

(d) for ~~ωp>0, ωp<0~~ $\omega_p < 0$