Institute of Applied Physics
Friedrich-Schiller-Universität Jena

Metrology and Sensing

Lecture 6-1: Wavefront sensors

2020-12-08

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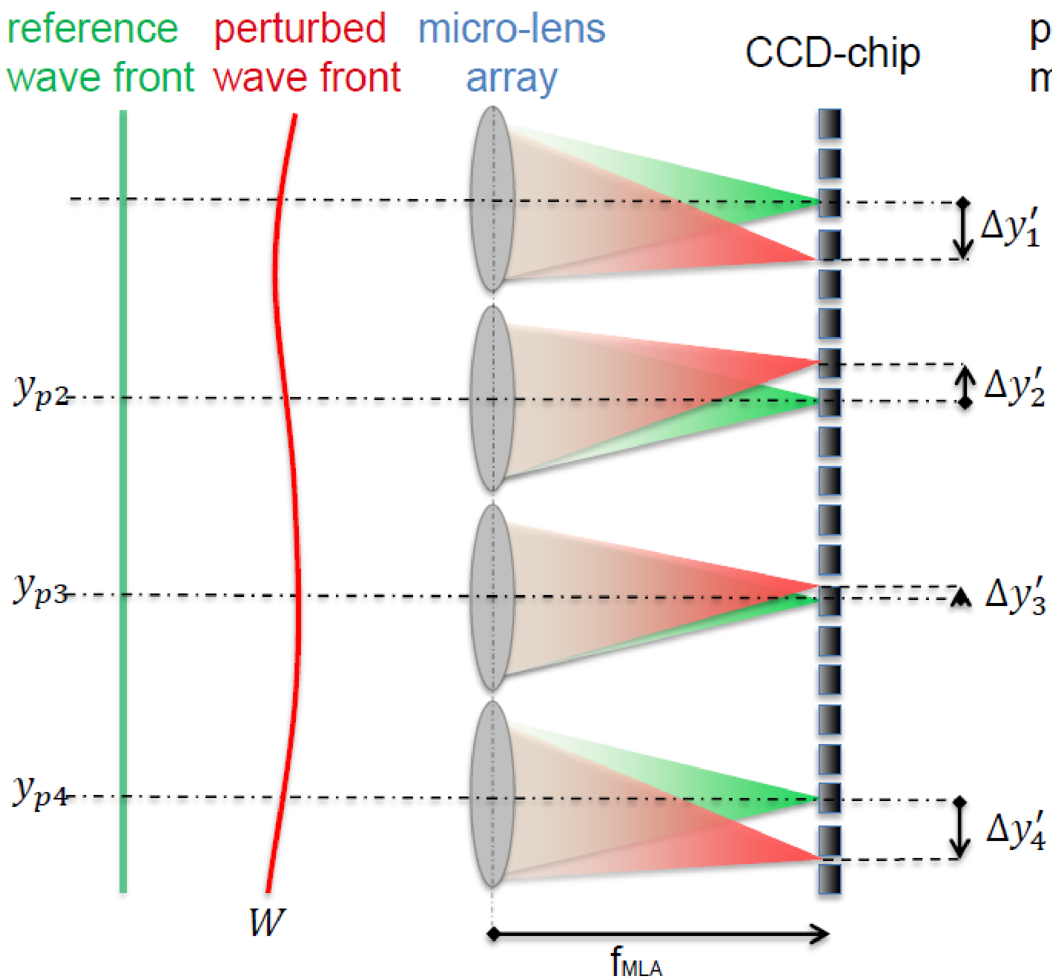
Content

Hartmann-Shack WFS:

- Principle
- Examples
- Properties

Hartmann-Shack Wavefront Sensor

- Basic principle

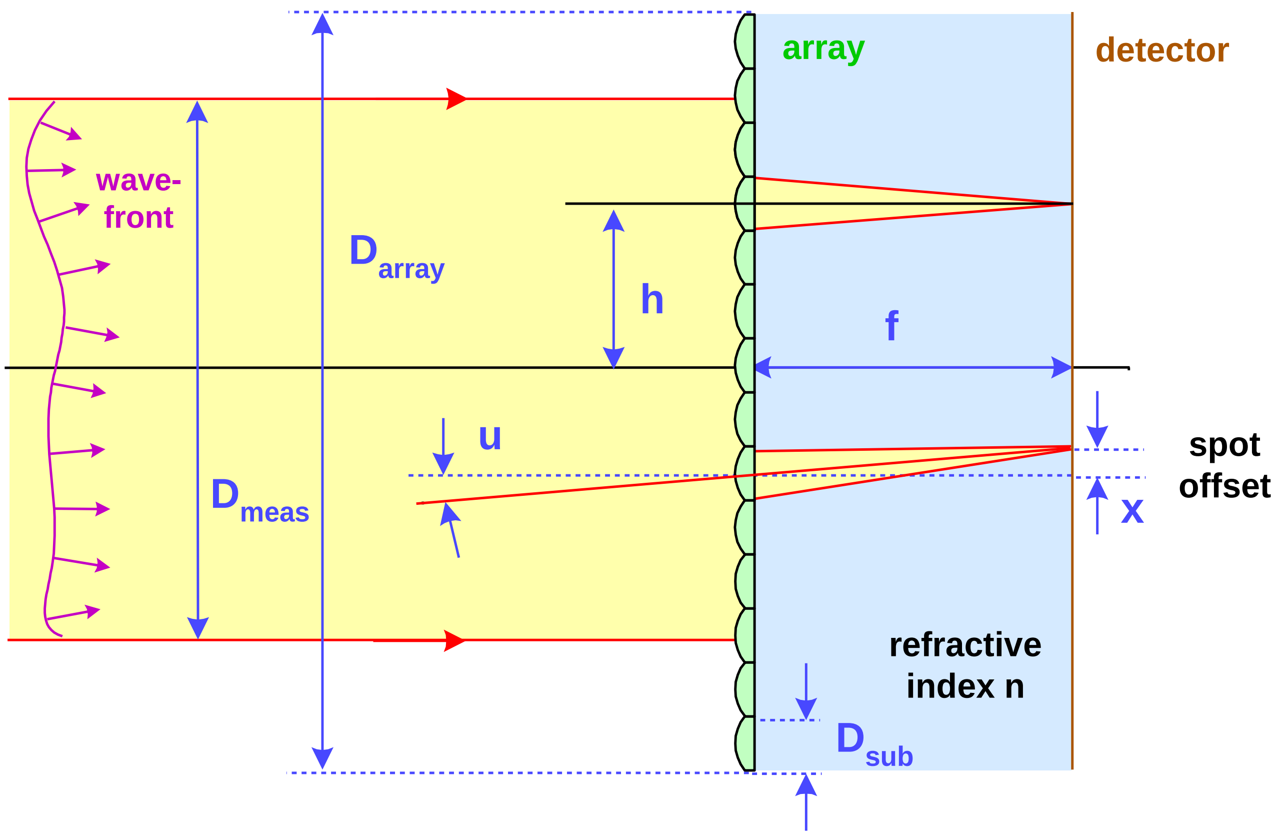


$$\left.\frac{\partial W(y_p)}{\partial y_p}\right|_{y_n} = \frac{-\Delta y'_n}{f_{MLA} - W} \approx \frac{-\Delta y'_n}{f_{MLA}}$$

Ref: S. Merx

Hartmann Shack Wavefront Sensor

- Lenslet array divides the wavefront into subapertures
- Every lenslet generates a simgle spot in the focal plane
- The averaged local tilt produces a transverse offset of the spot center
- Integration of the derivative matrix delivers the wave front W(x,y)



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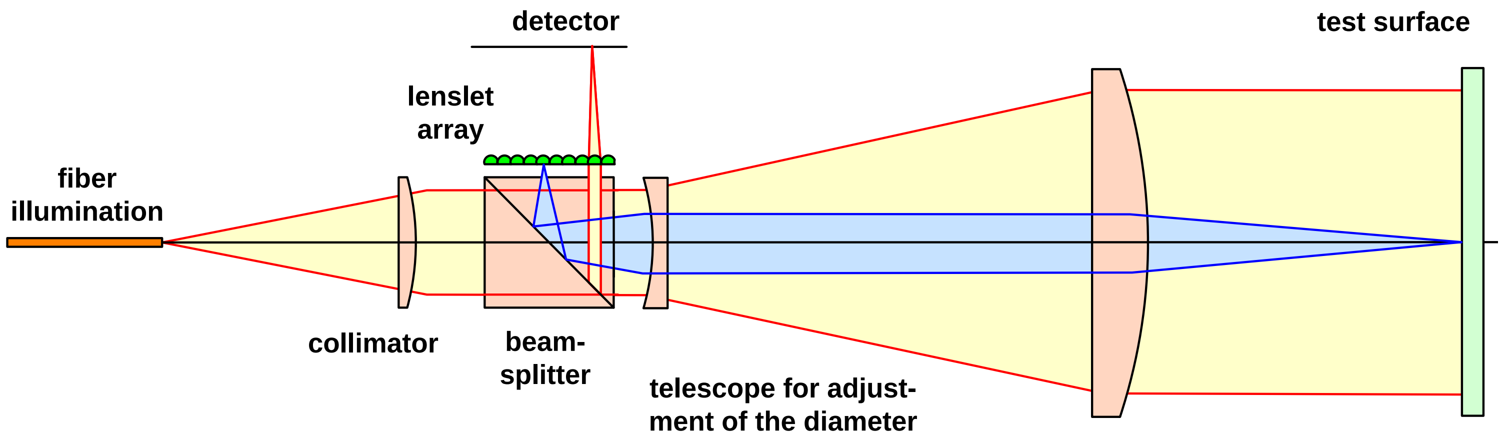
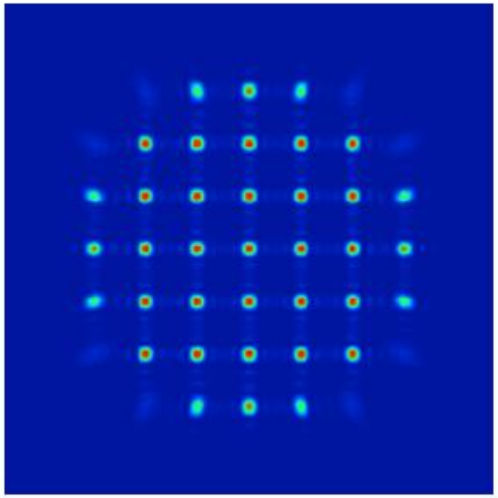
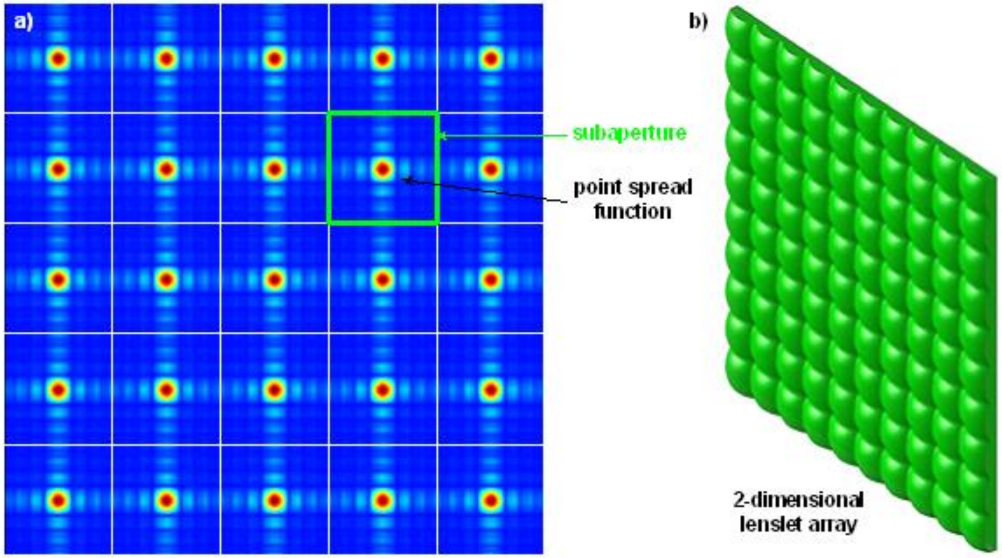
Hartmann Shack Wavefront Sensor

- Typical setup for component testing



- Lenslet array





Spot Pattern of a HS - WFS

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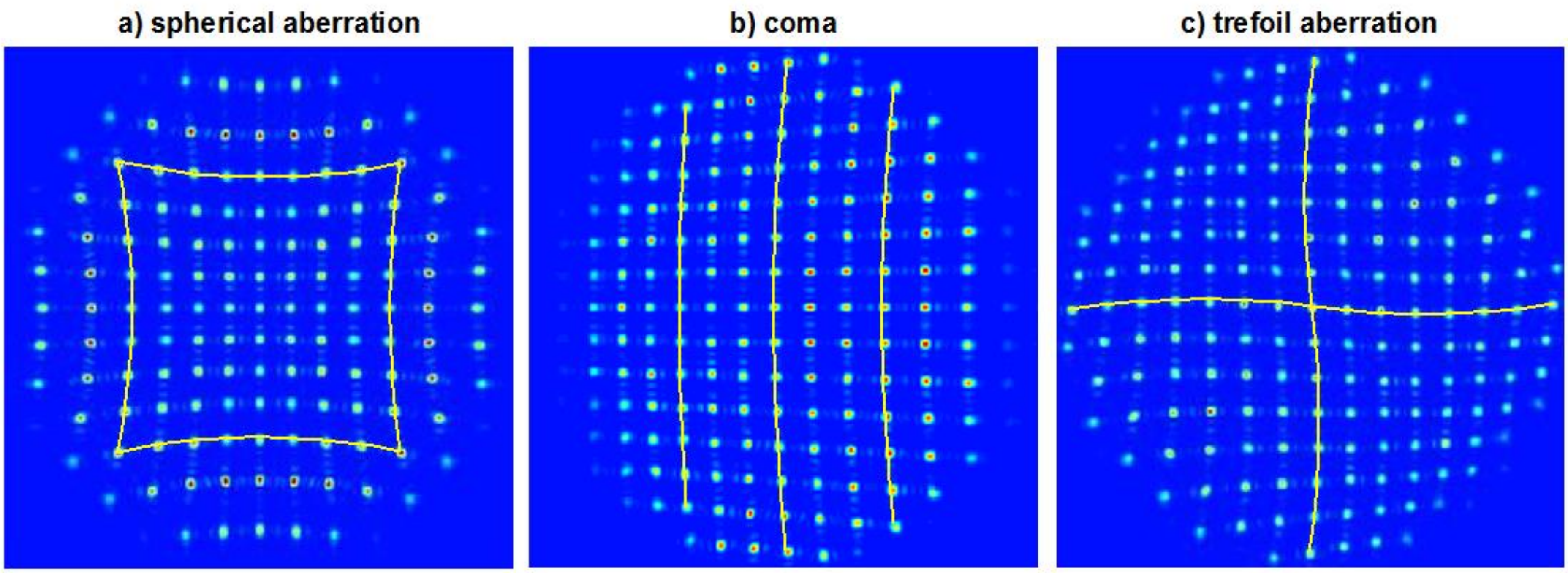
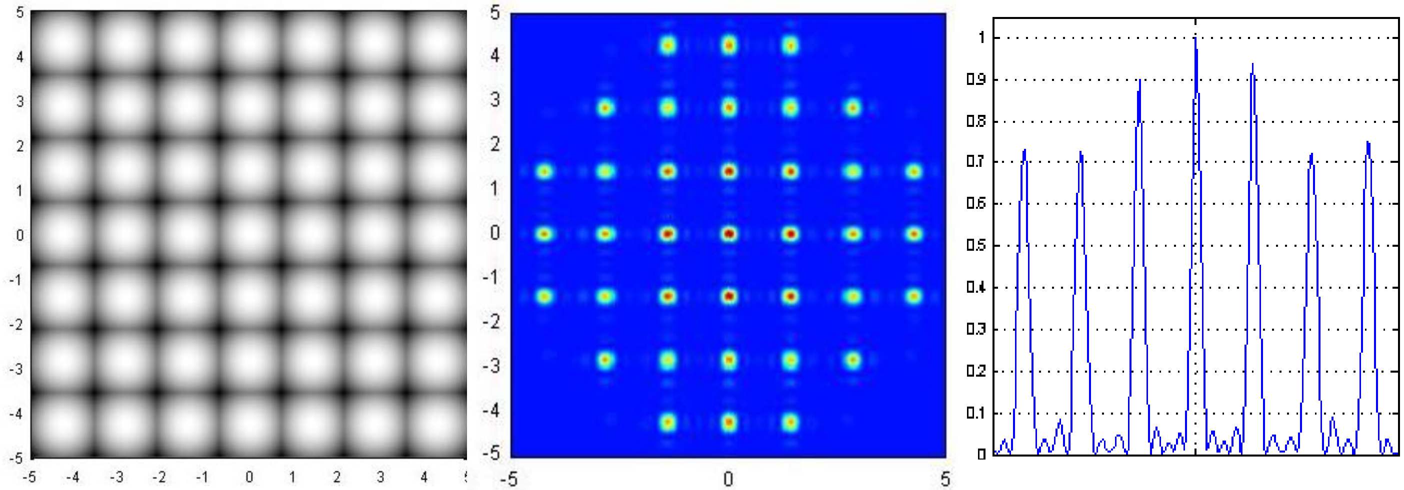
- Aberrations produce a distorted spot pattern
- Calibration of the setup for intrinsic residual errors
- Problem: correspondence of the spots to the subapertures



Array Signal

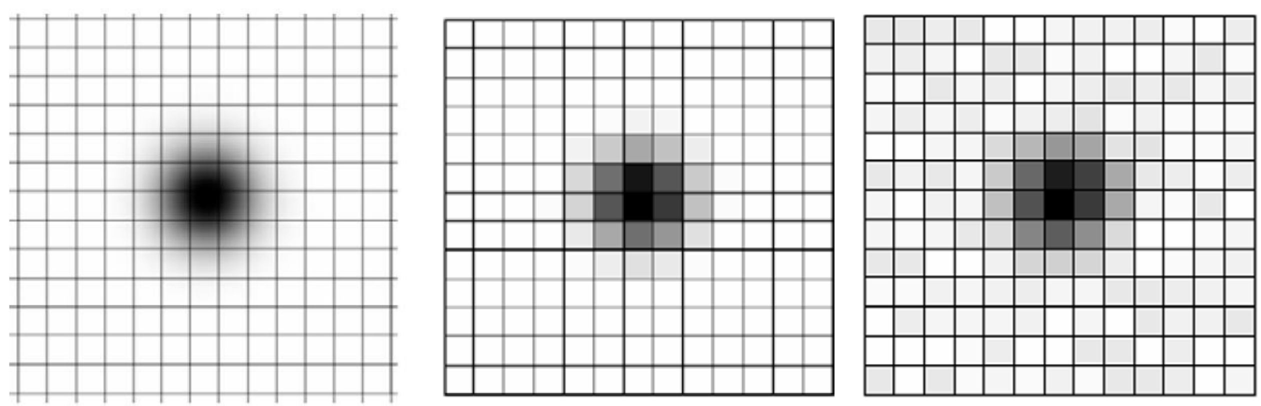
- Lenslet array ideal signal

array of phase | spot pattern | cross section of the spot pattern



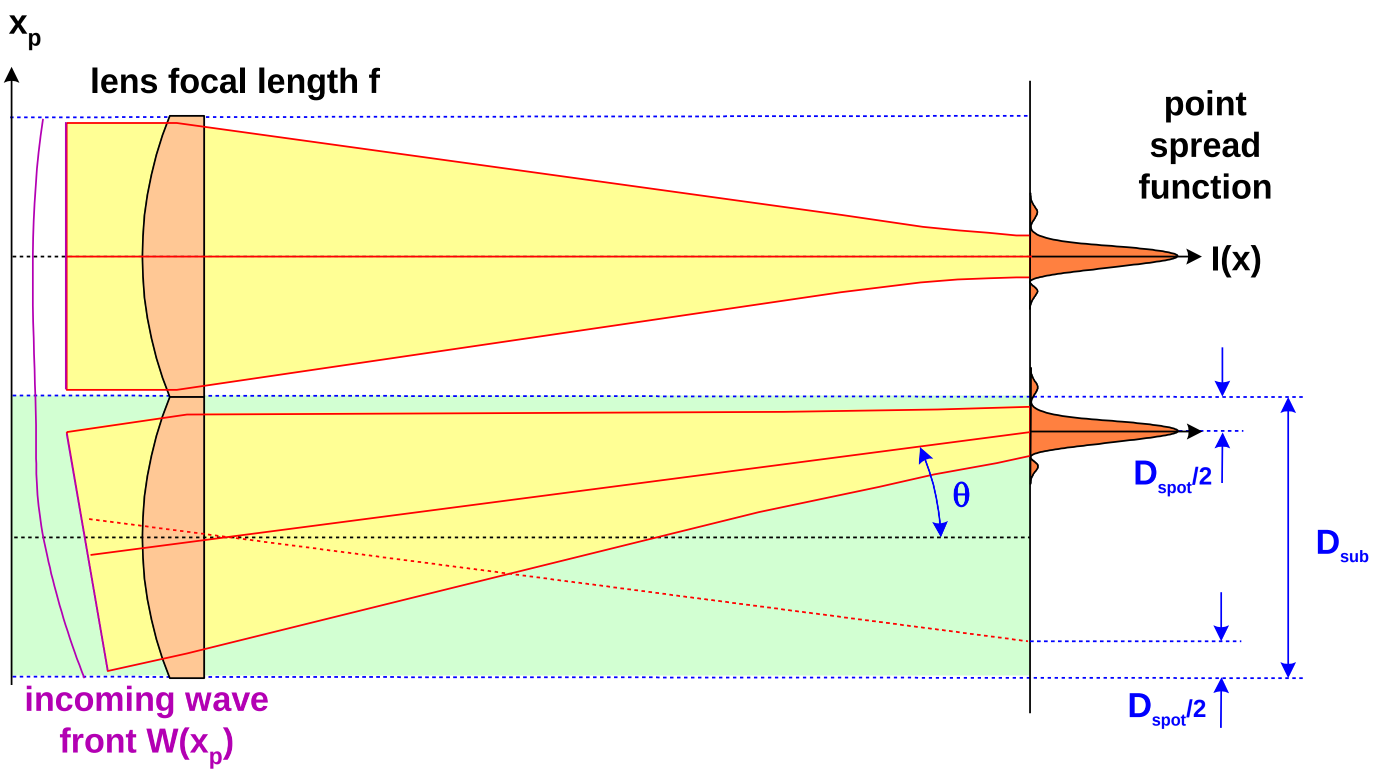
- Real signal:
  1. discretization
  2. quantization
  3. noise

original | discretized and quantized | with noise

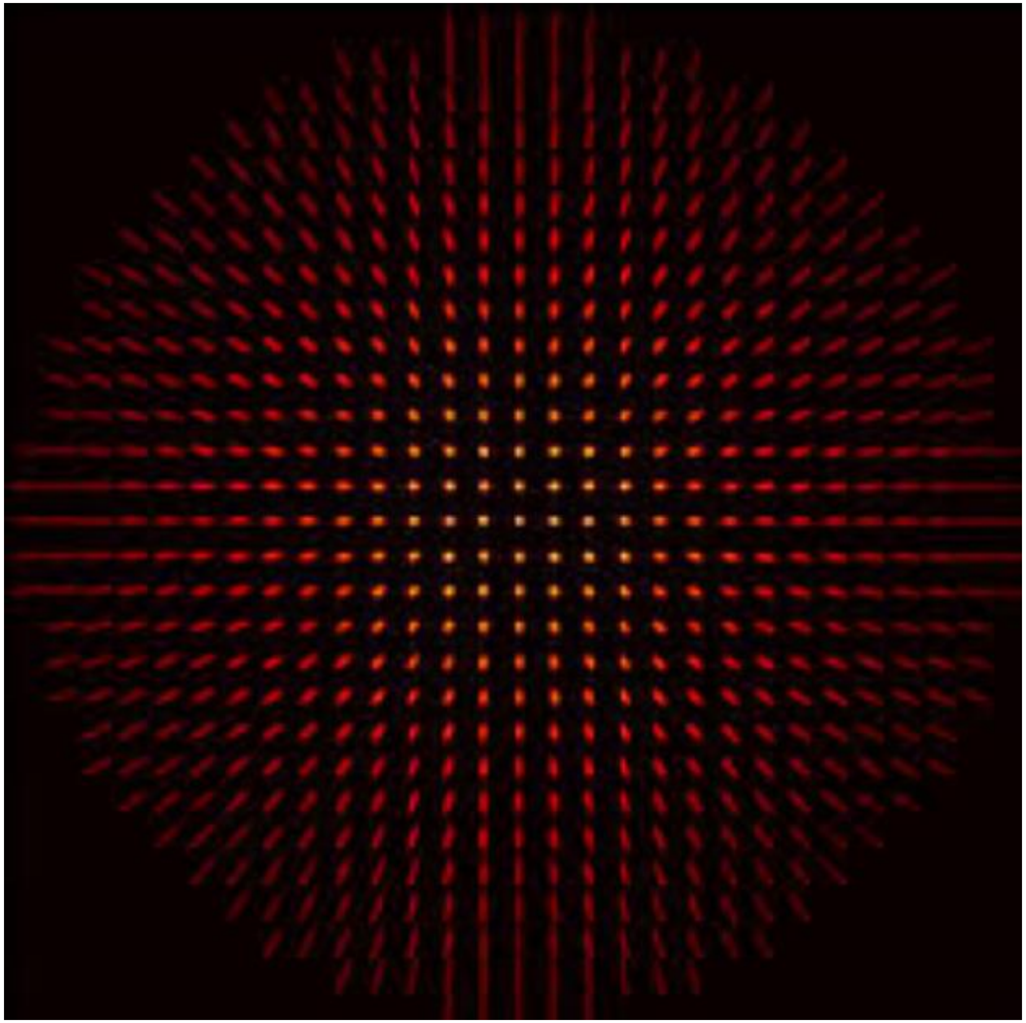


HS - WFS : Size of Sub-Apertur

- Dynamic range: ratio of spot diameter to size of sub-aperture
- Averaging of wavefront slope inside sub-aperture



- Real spot pattern:
  - broadening of spots
  - real boundary definition
  - signal to noise ratio
  - separation of spots
  - correspondence of spots to subapertures



HS-WFS

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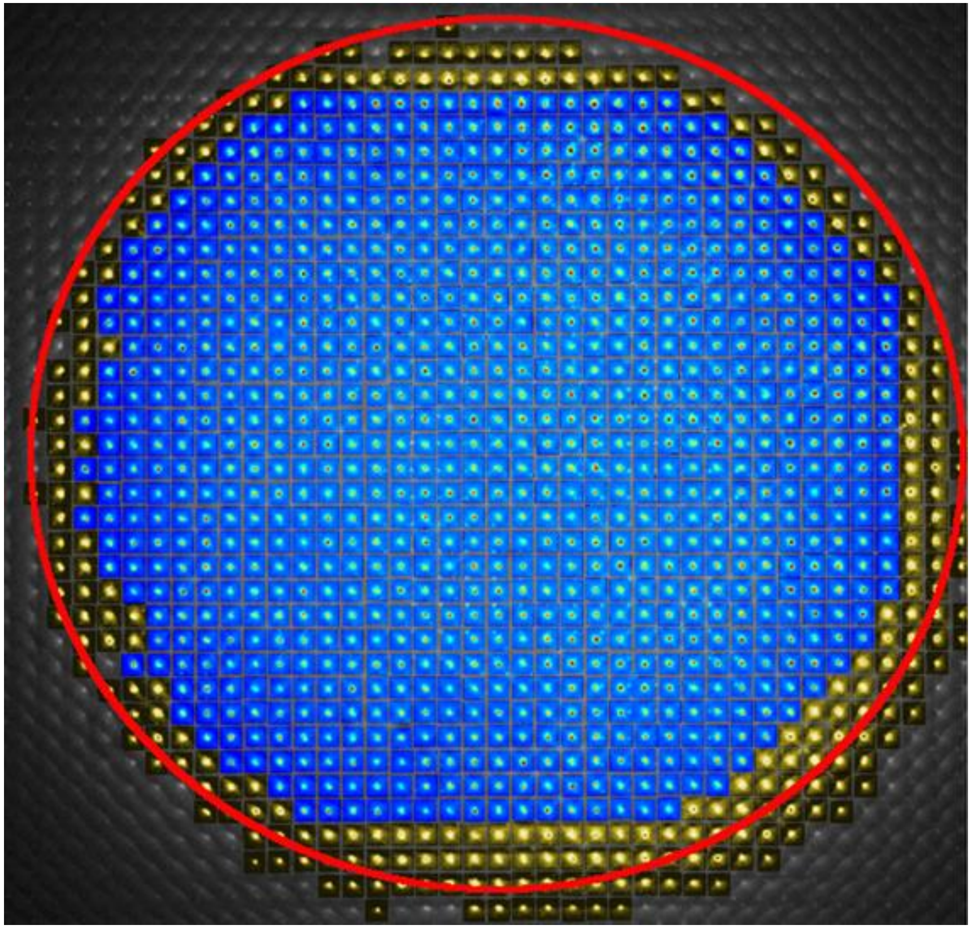
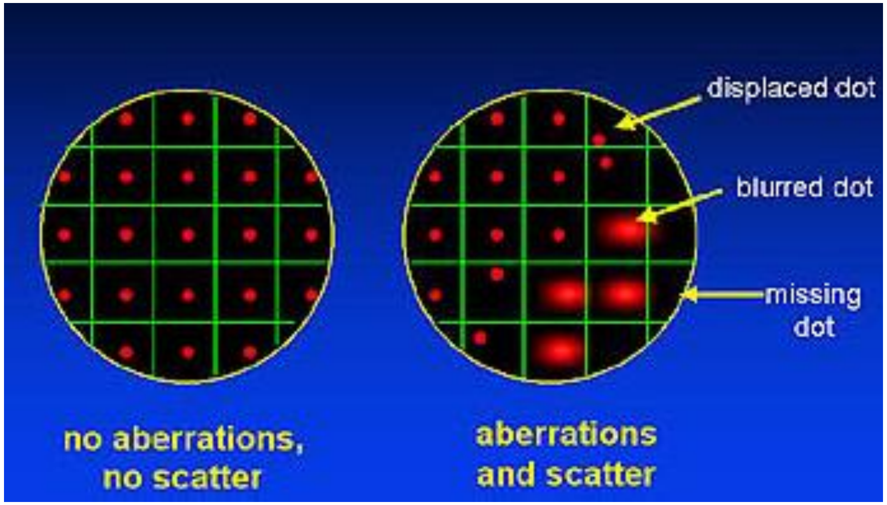
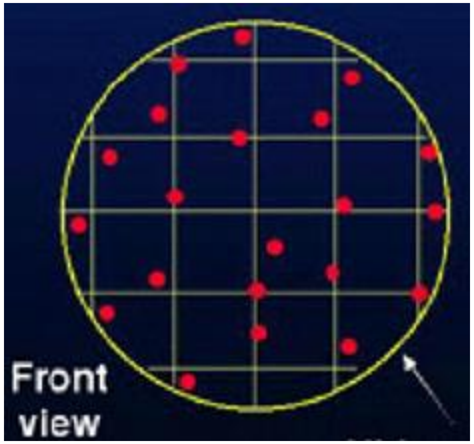
- Finding the boundary
- Special problem for determining Zernike polynomials

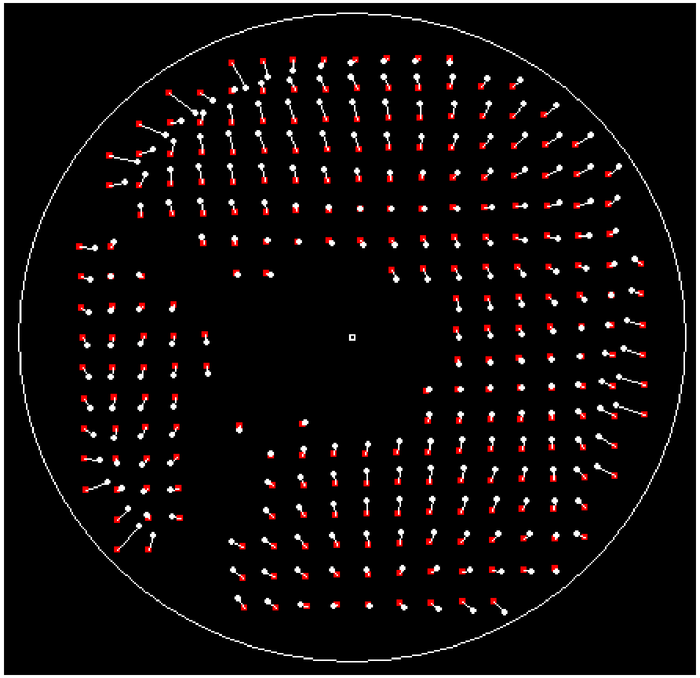


HS-WFS

- Assignment of spots
  - dynamic range limitation
  - integrability can solve the problem
  - practical help: shift arrows
- Pitfalls:
  - large broadening
  - overlapp of spots
  - missing spots
  - clear assignment of spots to sub-aperture

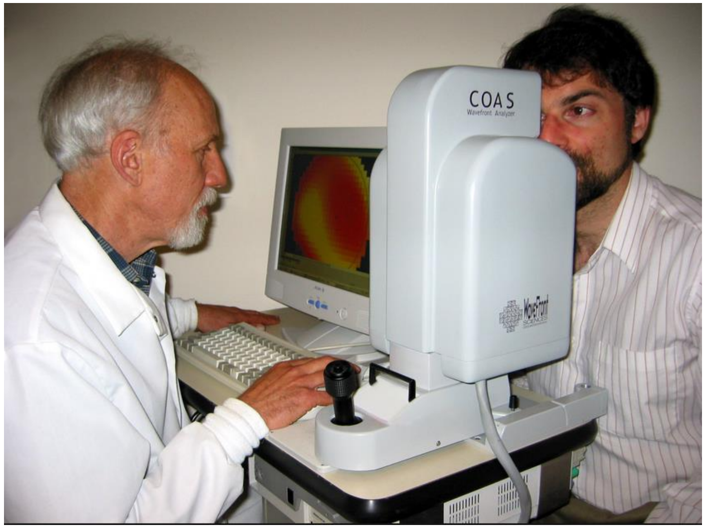






Measurement of the Human Eye by HS-WFS

- More or less motivating product advertisements



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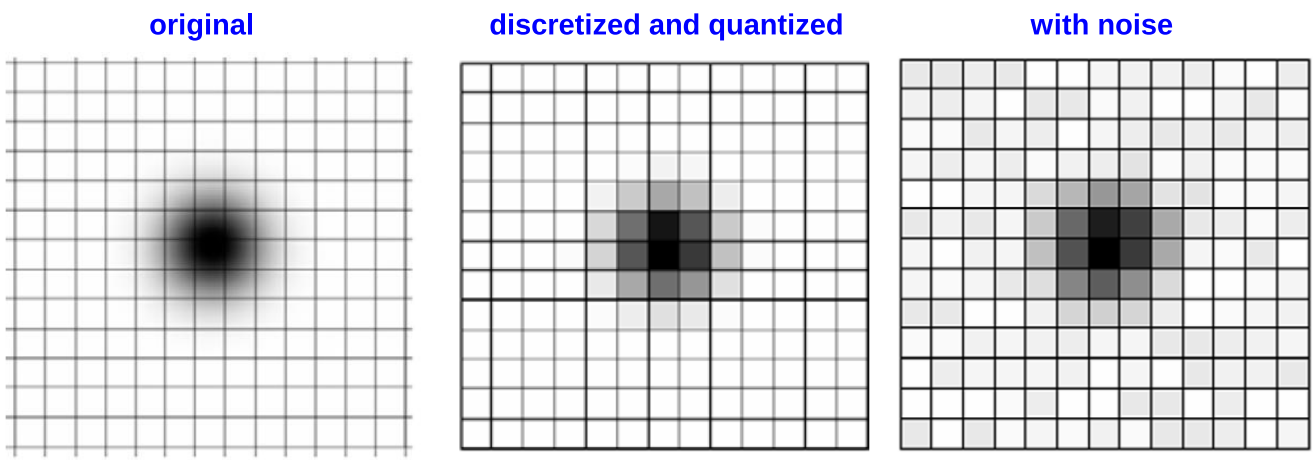
Real Measurement of a HS-WFS

- Problem in practice: exact determination of the spot centroid:
  - noise
  - discretization
  - quantization
  - broadening by partial coherence
  - broadening by local curvature
  - error by centroid affecting coma
  - error by partly illuminated pixels



Parametrization of a HS-WFS

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Layout parametrization:

- Fresnel number

$$N_F = \frac{D_{meas}^2}{4\lambda f N_{sub}^2 \eta^2} = \frac{D_{sub}^2}{4\lambda f}$$

- Fill factor

$$\eta = \frac{D_{meas}}{D_{array}}$$

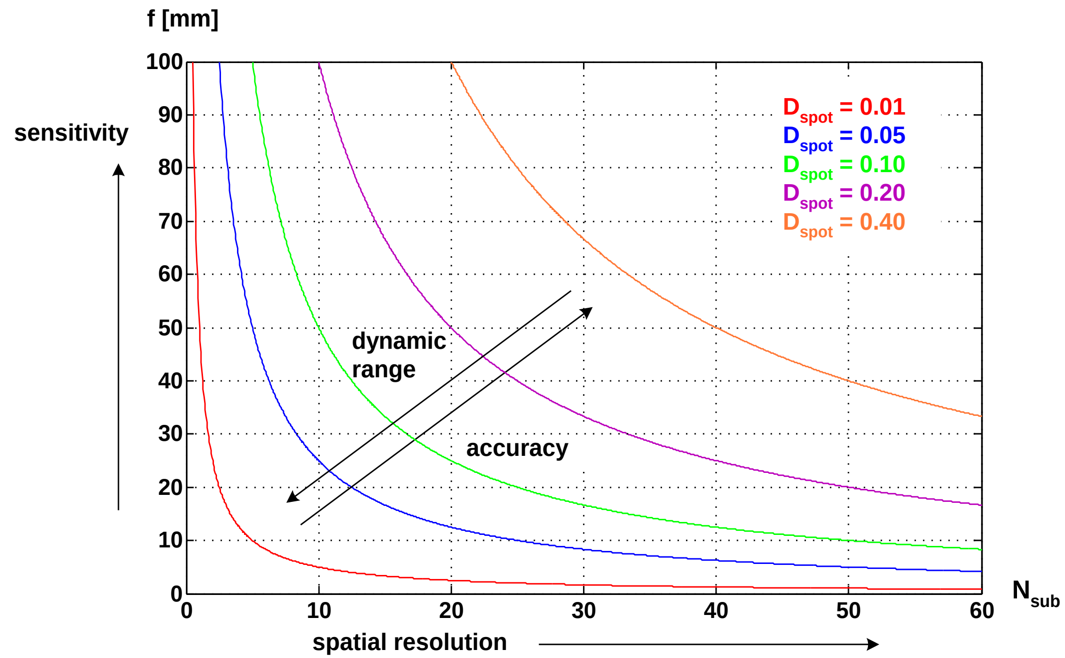
- Spot size

$$\frac{D_{spot}}{D_{sub}} = \frac{1}{2N_F}$$

$$D_{spot} = \frac{2\lambda f}{D_{sub}} = \frac{D_{sub}}{2N_F} = \frac{2\lambda f N_{sub}\eta}{D_{meas}}$$

- Accuracy:

$$\theta_{min} = \frac{k \cdot P}{f} \cdot \frac{m_{rel}}{\Gamma} \qquad \theta_{max} = h \cdot \frac{D_{sub}}{2f \cdot \Gamma^2}$$



P	pixel size sensor
k	relative sub-pixel accuracy
m_{rel}	magnification of relay lens
Γ	angle magnification of additional telescope

Properties of a Hartmann-Shack - Sensor

- Wavefront is averaged over one lens subaperture
- Fresnel number determines the relative spotsize

$$N_F = \frac{\varnothing_{sub}^2}{4\lambda \cdot f}$$

- Resolution with pixel size p and number of sub-apertures N

$$W_{\min} = \frac{4p\lambda \cdot N}{\varnothing_{mess}} \cdot N_f$$

- Relation / assignment of spots to subapertures
- Reconstruction of wavefront from gradients

$$\Delta x = -\frac{f}{n} \cdot \frac{\partial W}{\partial x}$$

- Measurement of centroids with sub-pixel accuracy
- Problems with partially illuminated lenses
- No trouble with spectral width, polarization, coherence

Errors in the HS - Wavefrontsensor

- Tilted sensor plane
- Rotated sensor in the azimuth
- Scattering of focal lengths of the lenslets
- Average of slope inside the subaperture area
- Errors in the wavefront reconstruction algorithms
- Coma of lenses
- Wrong focal length due to dispersion for different wavelength
- Sensor plane not exactly matched with focal plane
- Partly illuminated lenslets
- Electronical noise
- Zernike errors due to bad known normalization radius / edge of pupil
- Geometrical distortions of the array
- Truncation of spot by the corresponding subaperture / cross talk
- Discrete finite number of pixels
- Quantization of signal on the detector

Dynamic Range due to Local Curvature

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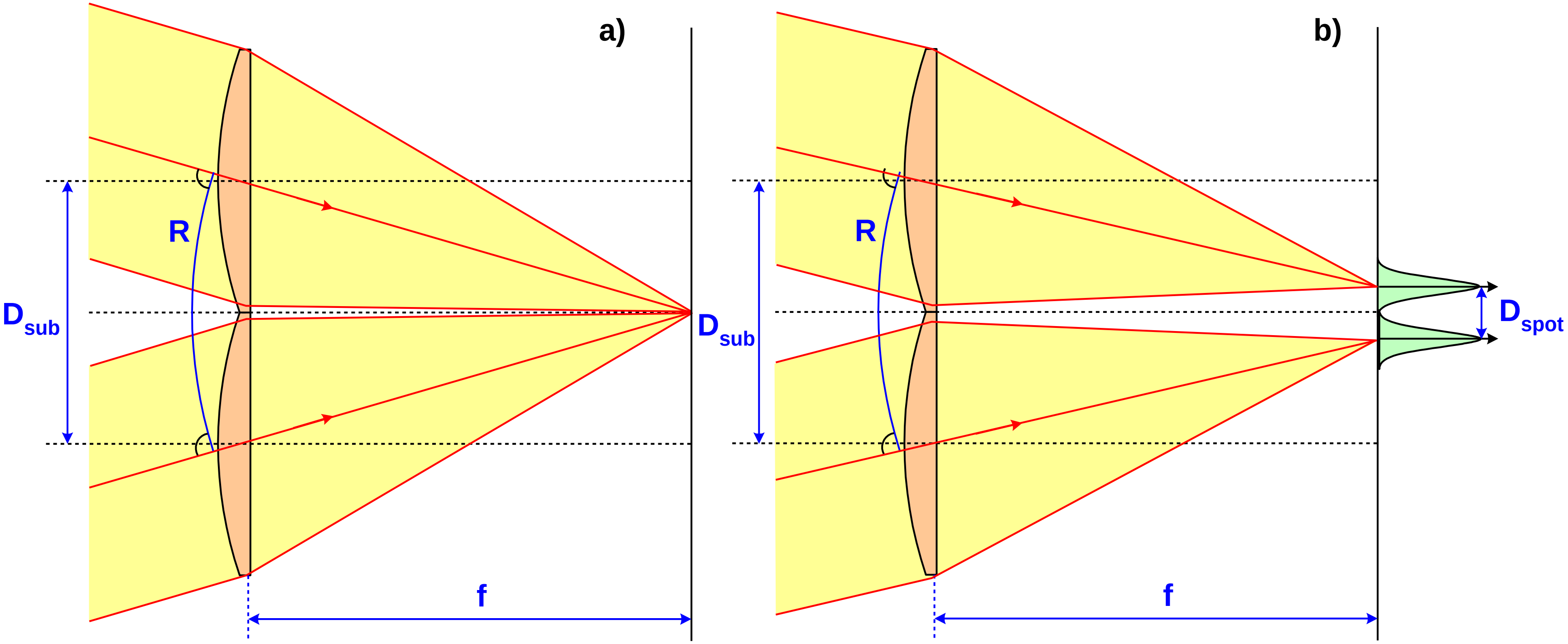
- Theoretical largest curvature: R = f
- Real size of point spread function:

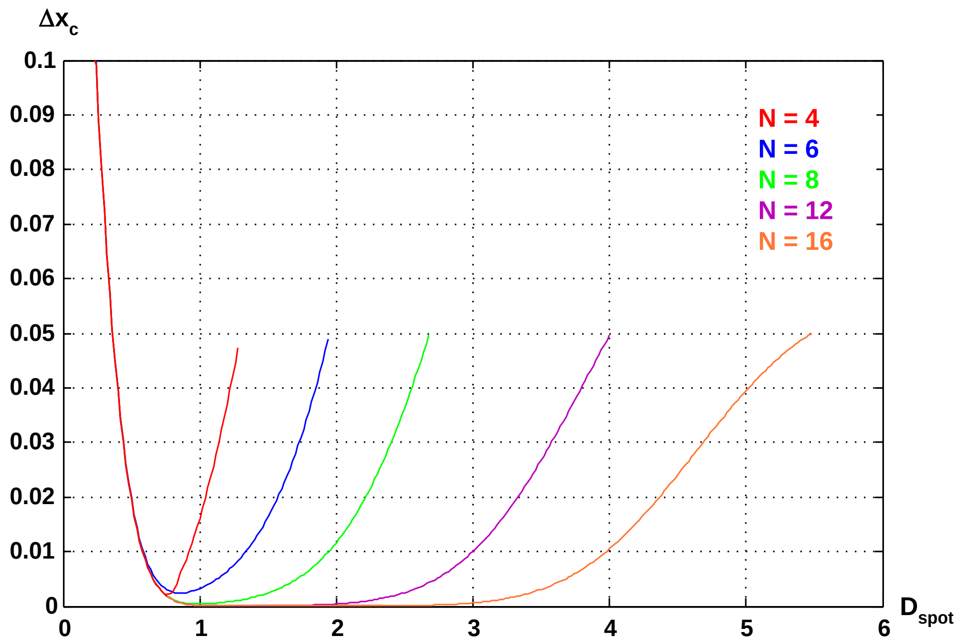
$$R_{\min} = \frac{f}{1 - \frac{1}{2N_F}}$$

- Larger curvature: cross talk generates errors



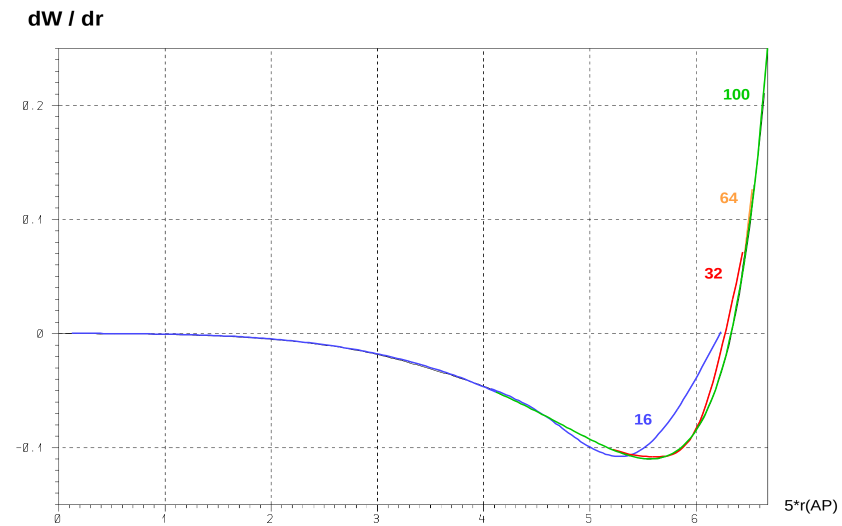
HS-WFS : Discretization Errors

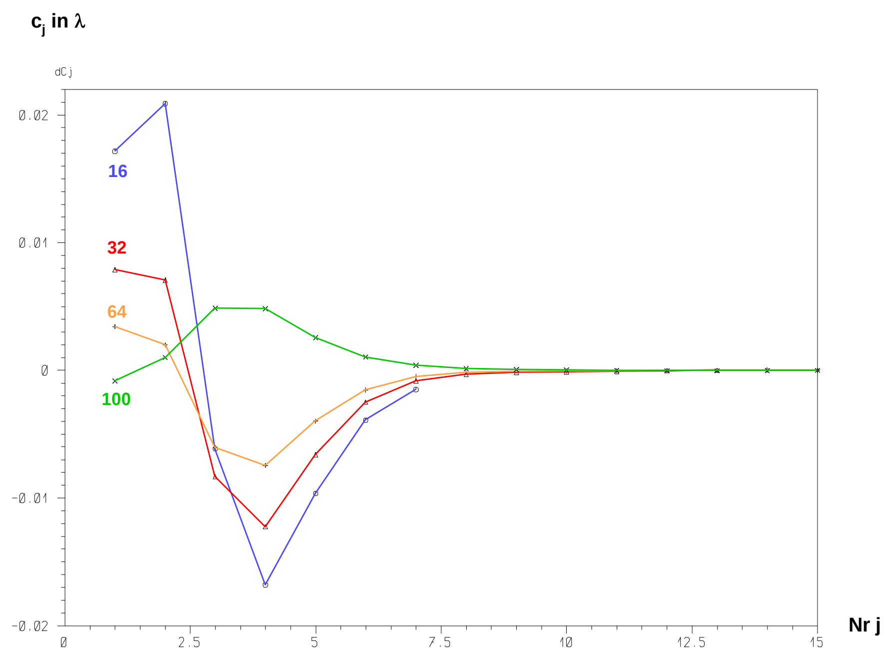
- Signal errors due to finite pixel size discretization of the point spread function
  N: number of pixels per sub-aperture

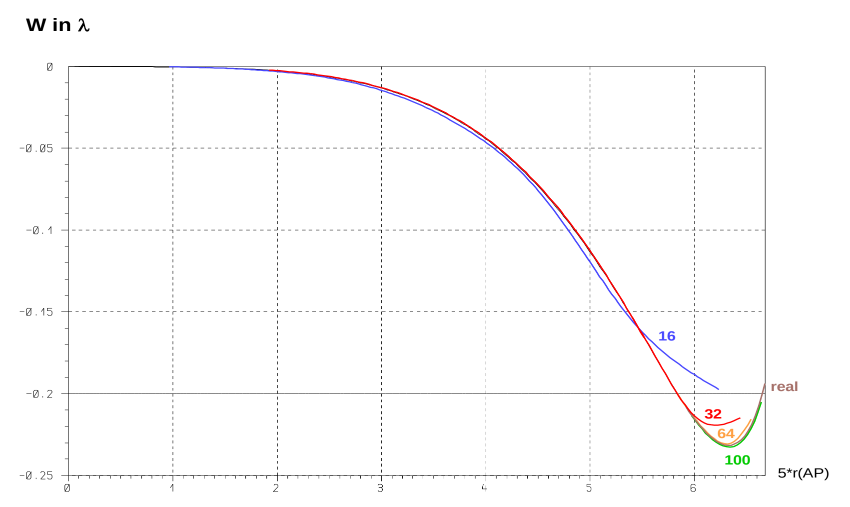


Averaging of Subapertures: Example

- Determination of wavefront of a microscopic lens
- Number ns of subapertures (linear):
  16 , 32 , 64 , 100
- Calculated:
  1. gradient of wavefront
  2. reconstructed wavefront
  3. errors Zernikes
- Errors due to averaging and shifted center of the subaperture



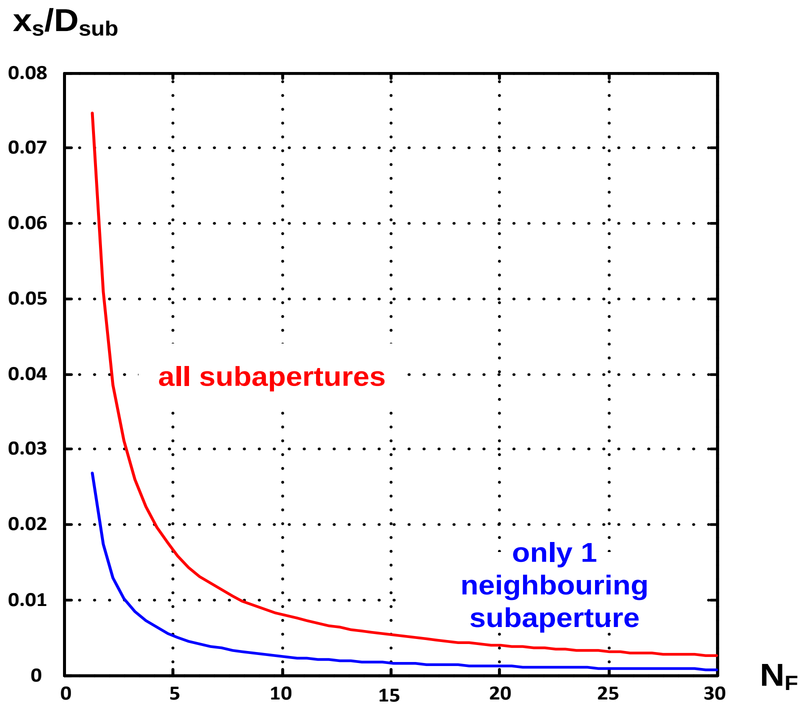


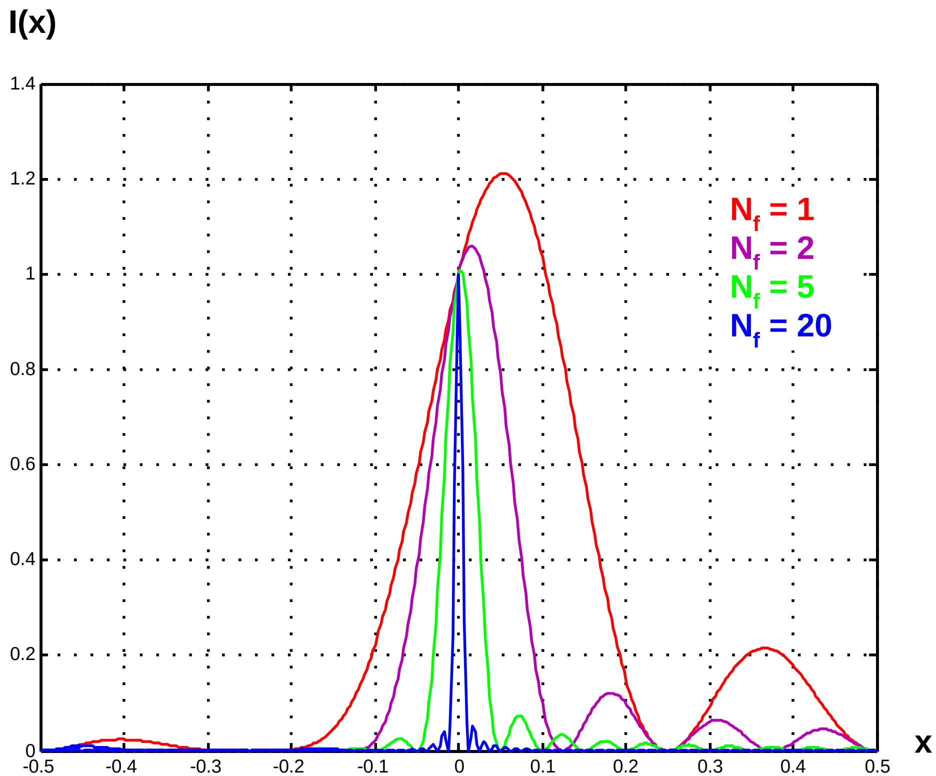


Fresnel Number and Crosstalk

- Relative size of the spot in a HS WFS: determined by Fresnel number
- Small NF: large PSF, crosstalk of neighbouring apertures
- Larger error of centroid calculation for subapertures at the edge

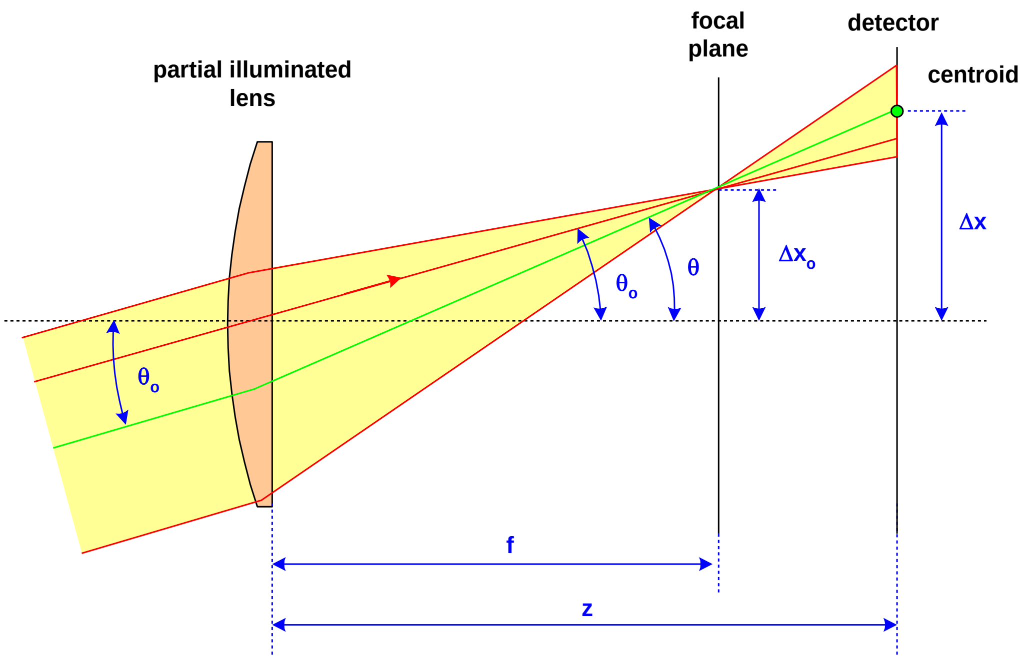
$$\frac{D_{spot}}{D_{sub}} = \frac{2\lambda \cdot f}{D_{sub}^2} = \frac{1}{2N_F}$$

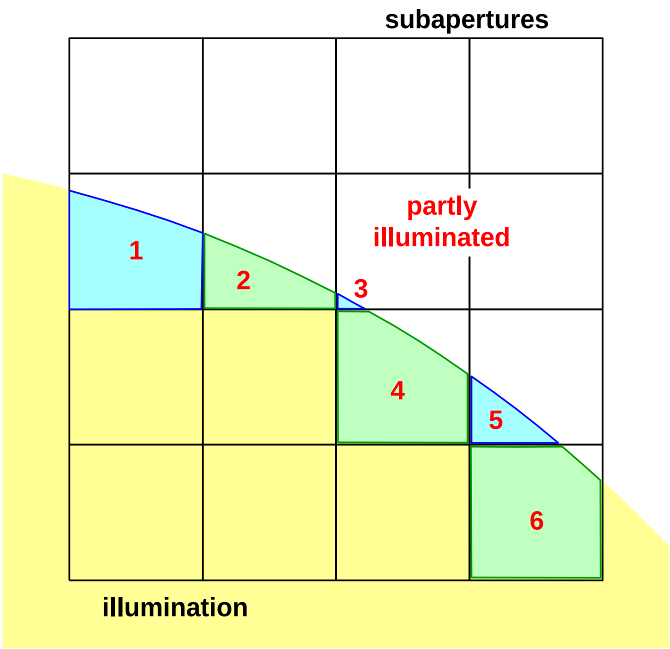




HS-WFS : Partly Illuminated Sub-Apertures

- Partly illuminated sub-aperture: change of centroid and error of signal
- Wrong signal for constant phase plateaus





- Example
  Change of point spread function due to partly illumination