

Problem 1

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(a)

$$\frac{dN_1}{dt} = -W_{13} N_1 + S_{31} N_3 - W_{12} N_1 + W_{21} N_2 + S_{21} N_2 \quad ①$$

$$\frac{dN_2}{dt} = W_{12} N_1 - W_{21} N_2 - S_{21} N_2 + S_{32} N_3 \quad ②$$

$$\frac{dN_3}{dt} = -S_{32} N_3 - S_{31} N_3 + W_{13} N_1 \quad ③$$

(b)

From equation ③ in (a) and

N_3 is 0 before starting the pump process

∴

$W_{13} N_1 = S_{32} N_3 + S_{31} N_3$ is the condition required

to keep $N_3 = 0$. The increasing particles must be equal to the decreasing particles of 3 to keep $N_3 = 0$.

(C)

$$\frac{dN_1}{dt} = -W_{13}N_1 - W_{12}N_1 + W_{21}N_2 + S_{21}N_2 + S_{31}N_3$$

$$\frac{dN_2}{dt} = -W_{21}N_2 + W_{12}N_1 - S_{21}N_2 + S_{32}N_3$$

$$W_{13} \cdot N_1 = (S_{31} + S_{32})N_3 \quad (N_3 \approx 0)$$

$$\therefore \eta = \frac{S_{32}N_3}{(S_{31} + S_{32})N_3} = \frac{S_{32}N_3}{W_{13} \cdot N_1}$$

$$\therefore S_{31} \cdot N_3 = (1-\eta) W_{13} \cdot N_1 \quad S_{32} \cdot N_3 = \eta \cdot W_{13} \cdot N_1$$

$$\therefore \frac{dN_1}{dt} = -W_{12}N_1 + W_{21}N_2 + S_{21}N_2 - \eta W_{13} \cdot N_1$$

$$\frac{dN_2}{dt} = -W_{21}N_2 + W_{12}N_1 - S_{21}N_2 + \eta W_{13} \cdot N_1$$

(d) The rate equations from c) are :

$$\frac{dN_1}{dt} = -W_{12}N_1 + W_{21}N_2 + S_{21}N_2 - \eta W_{13} \cdot N_1$$

$$\frac{dN_2}{dt} = -W_{21}N_2 + W_{12}N_1 - S_{21}N_2 + \eta W_{13} \cdot N_1$$

From the equations, we can get that

$$\frac{dN_1}{dt} = -\eta \cdot W_{13} \cdot N_1 \quad \frac{dN_2}{dt} = \eta \cdot W_{13} \cdot N_1$$

$$\therefore N_3 \approx 0$$

∴ All pumped particles come from N_1

$$\therefore \left. \frac{dN_1}{dt} \right|_{\text{pump}} = - \left. \frac{dN_2}{dt} \right|_{\text{pump}}$$

Problem 2

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(a)

$$\tau_{ph} = \tau_R / \gamma$$

$$\gamma_1 = -\ln(1-T_1) = -\ln R_1, \quad \gamma_2 = -\ln(1-T_2) = -\ln R_2$$

γ is the logarithmic cavity loss

$$\gamma_i = -\ln(1-L_i) \quad \gamma = \gamma_i + \gamma_1 + \gamma_2$$

From "Principles of Lasers", Orazio Svelto

(b) With $R_i = 1$, L is the round trip loss.

under circumstances: $0 < T_2 \ll 1$, $0 < L \ll 1$

when $x \ll 1$ occurs, $-x \approx \ln(1-x)$

$$\gamma_2 = -\ln(1-T_2) \approx T_2 \quad \gamma_i = -\ln(1-L) \approx L$$

Problem 3

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(a)

For a four-level system and $n \ll N_{\text{tot}}$

$$\frac{dn}{dt} = -\sigma \cdot c \cdot p \cdot n - \Gamma \cdot n + W_p \cdot N_{\text{tot}} \quad ①$$

$$\frac{dp}{dt} = \frac{p \cdot \left(\frac{n}{N_{\text{th}}} - 1 \right)}{\tau_{\text{ph}}} \quad (\text{neglect } S) \quad ②$$

$$n = \bar{n} + \Delta n \quad |\Delta n| \ll \bar{n} \quad ③$$

$$p = \bar{p} + \Delta p \quad |\Delta p| \ll \bar{p} \quad ④$$

insert ③, ④ into ①, ② and neglect $\Delta n \Delta p$

$$\bar{n} = N_{\text{th}}, \bar{p} = N_{\text{tot}}(W_p - W_{\text{th}})\tau_{\text{ph}}, W_{\text{th}} = \Gamma \frac{N_{\text{th}}}{N_{\text{tot}}}$$

$$\begin{aligned} \frac{d(\Delta n)}{dt} &= -\sigma \cdot c \cdot (\bar{p} + \Delta p) \cdot (\bar{n} + \Delta n) - \Gamma(\bar{n} + \Delta n) + W_p \cdot N_{\text{tot}} \\ &= -\sigma \cdot c (\bar{p} \cdot \bar{n} + \bar{p} \Delta n + \bar{n} \Delta p) - \Gamma(\bar{n} + \Delta n) + W_p \cdot N_{\text{tot}} \\ &= -\sigma \cdot c \bar{p} \cdot \bar{n} - \sigma \cdot c \bar{p} \Delta n - \sigma \cdot c \bar{n} \Delta p - \Gamma \bar{n} - \Gamma \Delta n + W_p \cdot N_{\text{tot}} \\ &= -\sigma \cdot c \bar{p} \cdot \bar{n} - (\sigma \cdot c \bar{p} + \Gamma) \Delta n - \frac{\Delta p}{\tau_{\text{ph}}} - \Gamma \bar{n} + W_p \cdot N_{\text{tot}} \end{aligned}$$

↓

$$\begin{aligned}
 &= -\frac{n_{tot}}{\tau_{ph}} (N_b - N_m) \bar{\tau}_{ph} - (\sigma \cdot c \cdot \bar{p} + \Gamma) \Delta n - \frac{\Delta P}{\tau_{ph}} - \Gamma \bar{n} + W_p n_{tot} \\
 &= W_{th} n_{tot} - W_p n_{tot} - (\sigma \cdot c \cdot \bar{p} + \Gamma) \Delta n - \frac{\Delta P}{\tau_{ph}} - \Gamma \bar{n} + W_p n_{tot} \\
 &= -(\sigma \cdot c \cdot \bar{p} + \Gamma) \Delta n - \frac{\Delta P}{\tau_{ph}}
 \end{aligned}$$

$$\begin{aligned}
 \frac{d(\Delta p)}{dt} &= \frac{(\bar{p} + \Delta p) \left(\frac{\bar{n} + \Delta n}{n_m} - 1 \right)}{\tau_{ph}} \\
 &= \frac{(\bar{p} + \Delta p) \frac{\Delta n}{n_m}}{\tau_{ph}} \\
 &= \frac{\bar{p} \cdot \frac{\Delta n}{n_m}}{\tau_{ph}} = \sigma \cdot c \cdot \bar{p} \cdot \Delta n
 \end{aligned}$$

(b) For a three-level system

$$\frac{dn}{dt} = -2 \cdot \sigma \cdot c \cdot p \cdot n - \Gamma \cdot \left\{ n + n_{tot} \right\} + W_p (n_{tot} - n) \quad ①$$

$$\frac{dp}{dt} = \frac{P \cdot \left(\frac{n}{n_{th}} - 1 \right)}{\tau_{ph}} \quad ②$$

$$n = \bar{n} + \Delta n \quad ③$$

$$p = \bar{p} + \Delta p \quad ④$$

$$\sigma c n_{th} = \frac{1}{\tau_{ph}}$$

$$n_{tot} n_{th} = \Gamma \bar{n}$$

$$\begin{aligned} \frac{d(\Delta n)}{dt} &= -2 \cdot \sigma \cdot c \cdot (\bar{p} + \Delta p)(\bar{n} + \Delta \bar{n}) - \Gamma(\bar{n} + n_{tot} + \Delta n) \\ &\quad + W_p (n_{tot} - \bar{n} - \Delta n) \\ &= -2 \sigma \cdot c \cdot (\bar{p} \bar{n} + \Delta p \bar{n} + \Delta n \bar{p}) - \Gamma(\bar{n} + n_{tot} + \Delta n) \\ &\quad + W_p (n_{tot} - \bar{n} - \Delta n) \\ &= -2 n_{tot} (W_p - W_{th}) - 2 \frac{\Delta p}{\tau_{ph}} - 2 \sigma \cdot c \cdot \bar{p} \Delta n - \Gamma \bar{n} - \Gamma n_{tot} - \Gamma \Delta n \\ &\quad + W_p n_{tot} - W_p \bar{n} - W_p \Delta n \\ &= - \bar{p} / \tau_{ph} - 2 \frac{\Delta p}{\tau_{ph}} - (2 \sigma \cdot c \cdot \bar{p} + \Gamma + W_p) \Delta n - \Gamma n_{tot} \\ &\quad - (\sigma \cdot c \cdot \tau_{ph})^{-1} W_p \end{aligned}$$

(c)

physical meaning

δ : damping coefficient

ω : oscillation frequency

① If δ is much higher than ω , α_n and α_p will return to 0 without oscillation.

② If δ is much lower than ω , α_n and α_p will oscillate with slightly decreasing amplitudes.

(d)

The second can see relaxation oscillation,

$$\because \tau_2 = 100\mu s \gg 100ns$$

\therefore the second $\delta \ll$ the first δ