Fundamentals of Modern Optics

Exercise 13

28.01.2015

not to be evaluated

Problem 1 - Optical layer (2+2+2+2 points)

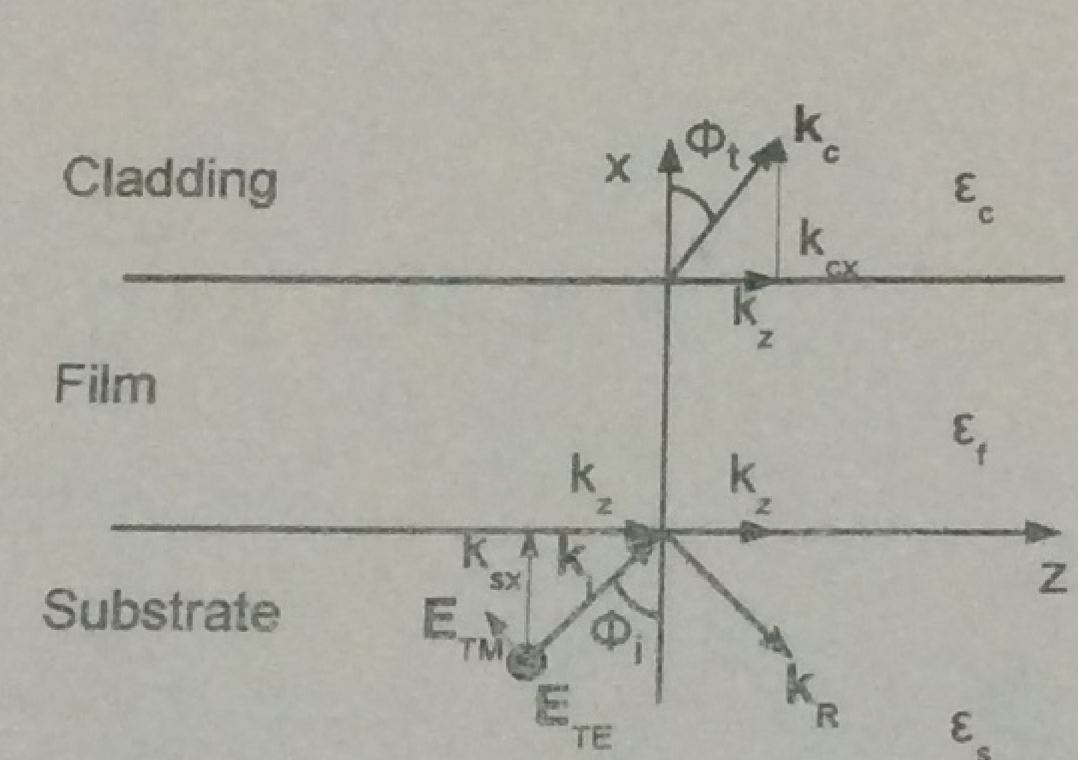
Let us consider a single optical layer as shown in the figure a. For simplicity, we consider light in TE polarization only.

a) What is the angle φ_t of the transmitted beam as a function of the angle of the incident beam φ_i? Assume the refractive indices of substrate $n_s = \sqrt{\varepsilon_s}$, film $n_f = \sqrt{\varepsilon_f}$, and cladding $n_c = \sqrt{\varepsilon_c}$, and the thickness of the layer d.

b) Compute the coefficients for reflection and transmission as functions of the angle of incidence φ_i.

c) Compute the reflectivity and transmissivity of the single layer.

d) Consider the special case of $k_{fx}d = d\sqrt{k_f^2 - k_z^2} = \pi/2$ ($\lambda/4$-layer) and compute the reflectivity. Now assume the incident light perpendicular to the layer ($\varphi_i = 0$). Give conditions for the refractive indices to obtain minimum or maximum reflectivity.



Problem 2 - Planar Waveguide (3+1+1+1 points)

The dispersion relation of guided modes in a symmetric planar waveguide of thickness d ($\varepsilon_{s,c} = 1, \varepsilon_f = \varepsilon$) can be determined from the poles of the reflection or transmission coefficients. Alternatively and physically very intuitive, we can understand the existence of guided modes as constructive interference of plane waves for which we have total reflection at the inner interfaces. The phase accumulated upon propagation over the distance d (two times) plus the phase accumulated upon total reflection (two times) has to fulfill

$$2\varphi_R + 2kd = 2m\pi\,, m \in \mathbb{N}.$$

a) Use this condition to derive the dispersion relation of guided waves. The reflection coefficient of a plane wave at a planar interface between two isotropic media (substrate 's' and cladding 'c') is

$$R = \frac{\alpha_s k_{sx} - \alpha_c k_{cx}}{\alpha_s k_{sx} + \alpha_c k_{cx}}, \quad k_{s,cx} = \sqrt{\frac{\omega^2}{c^2}\varepsilon_{s,c} - k_z^2}.$$

b) Specify the range of the effective index ($n_{\text{eff}} = k_z/k_0$) of the giuded modes.

c) Specify the cut-off thickness (where a mode is not guided anymore) for the fundamental mode in TE-polarisation.

d) Specify the thickness up to which the waveguide is single-mode.

Problem 3 - Fabry-Perot resonator (1+2+2 points)

The transmission for a symmetric, lossless Fabry-Perot resonator is given as:

$$\tau = \left\{1 + F\sin^2\left(\frac{\delta}{2}\right)\right\}^{-1}, \quad \text{where } F = \frac{4\varrho}{(1-\varrho)^2} \text{ and } \frac{\delta}{2} = kD + \varphi$$

with the reflectivity ϱ, the normal component of the wave vector inside the resonator k, the thickness of the resonator D and the phase jump φ upon reflection at the mirrors.

a) Define and calculate the finesse.

b) Make a qualitative sketch of the transmissivity as a function of the wavelength for 3 different reflectivities.

c) Specify the minimum transmissivity and calculate the spectral resolution of the resonator.