Optical Metrology and Sensing – Seminar 1

Task 1: Basic Metrology Principles

a) What is the meaning of 'confidence interval' of measured values?

b) Explain the meaning of the sensitivity and the resolution of an instrument.

Task 2 Sampling Theory

a) For an object of 2cm dimension, what is the minimum spatial resolution (in cm) when the object is digitized into an array of 512 samples?

b) How many harmonics will be present in the Fourier transform of the digitized object?

c) What is the lowest (but not DC) spatial frequency involved in the transform of the object described above and what is the highest?

Task 3: Fourier Theory

Prove the following Fourier transform theorems: Note $*$ symbol stand for convolution

a) $\mathcal{F}\{\mathcal{F}\{g(x,y)\}\} = \mathcal{F}^{-1}\mathcal{F}^{-1}\{g(x,y)\} = g(-x,-y)$ at all points of continuity of g

b) $\mathcal{F}\{g(x,y)h(x,y)\} = \mathcal{F}\{g(x,y)\} * \mathcal{F}\{h(x,y)\}$

Task 4: Fourier Theory

The expression

$$p(x,y) = g(x,y) * [comb(\frac{x}{X})comb(\frac{y}{Y})]$$

Defines the periodic function with period X in the x direction and Y in the y direction.

a) Show that the Fourier transform of p can be written

$$P(f_X, f_Y) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} G(\frac{n}{X}, \frac{m}{Y})\delta(f_X - \frac{n}{X}, f_Y - \frac{m}{Y})$$

Where G is the fourier transform of g

b) Sketch the function p(x,y) when,

$$g(x,y) = rect(2\frac{x}{X})rect(2\frac{y}{Y})$$

And find the corresponding Fourier transform P(fx,fy)

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Task 5: Two-Beam Interference

Consider two monochromatic plane waves propagating in x-direction

$$\vec{E}_1 = \vec{e}_1\sqrt{I_1}\exp\big(i(kx+\varphi_1)\big)$$
$$\vec{E}_2 = \vec{e}_2\sqrt{I_2}\exp\big(i(kx+\varphi_2)\big)$$

with real-valued intensities I_1, I_2 and complex-valued polarization vectors $\vec{e}_1$, $\vec{e}_2$ such that $\vec{e}_1 \neq \vec{e}_1^*$, $\vec{e}_2 \neq \vec{e}_2^*$. Assume $|\vec{e}_1| = |\vec{e}_2| = 1$.

a) Show that the intensity distribution $I_{\text{tot}} = \vec{E}_{\text{tot}} \cdot (\vec{E}_{\text{tot}})^*$ for superimposed fields $\vec{E}_1$ and $\vec{E}_2$ can be written as:

$$I_{\text{tot}} = I_1 + I_2 + 2\sqrt{I_1 I_2} \cdot Real\left\{\vec{e}_1\vec{e}_2^*\exp\big(i(\varphi_1-\varphi_2)\big)\right\}$$

b) Use the result from a) to prove that circular polarized plane waves with opposite handedness do not interfere.