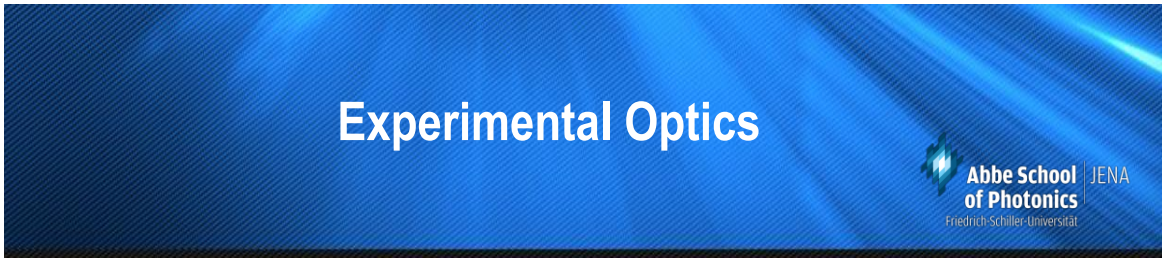

Contact person:
Roland Ackermann
Friedrich-Schiller-Universität Jena
Abbe School of Photonics
Max-Wien-Platz 1
07743 Jena, Germany
Phone : +49 3641 9-47821, e-mail : Roland.Ackermann@uni-jena.de

Last edition: Roland Ackermann, February 2015

Lab Title: Ultrasound velocity measurement

Group number	G1
Student name(s)	Evelina Eriksson
Name of TA	Ria Becker
Date of Lab	16.11.2013
Date of Final Report return	25.11.2013

Noninvasive Measurement Techniques
Department of Physics
Umeå University

Project Ac3
Sound Velocity Measurements Using Ultrasound

Evelina Eriksson Ria Becker

November 11, 2009

Supervisor: Ove Andersson

The velocity of sound was determined for three different gases (air, argon and nitrogen) in a transparent tube with two different methods. One was a time delay measurement, used also to measure the velocity of sound as a function of pressure. In agreement with the theory the result shows that the velocity is independent of the pressure. In the second measurement the velocity of sound in air was determined with the help of standing waves in the tube.

Contents

1 Introduction

Sound travels through an elastic medium as a wave. It's velocity dependents on the temperature and the properties of the material the sound is traveling through. There exist various possibilities for measurements of the velocity of sound.

The aim of this project was to measure the velocity of sound waves in three different gases (air, argon and nitrogen) as a function of its physical parameters using at least two different methods. Then, with the results, the degrees of freedom of the gas molecules were to be determined.

2 Theory

2.1 The Velocity of Sound

A sound wave is a longitudinal wave, the molecules of air are moving in the same direction as the wave itself. Sound passes through a media by compressing and expanding the distance between the atoms, transmitting energy between them. The velocity of sound through the media depends on the stiffness of the bonds between the particles and their weight.

From the equations of conservation of mass and momentum, under assumption of adiabatic condition, it can be deduced that the velocity of sound c_s is given by [1]

$$c_s = \sqrt{\gamma \frac{p}{\rho}}, \tag{1}$$

where p is the pressure, ρ is the density and γ is the adiabatic index. The adiabatic index is the ratio of specific heats of a gas at a constant pressure to a gas at a constant volume ($\gamma = C_p/C_v$, [1]).

Using the ideal gas law, $pV = nRT$ [2], equation (1) can be rewritten to

$$c_s = \sqrt{\gamma \frac{RT}{M}}, \tag{2}$$

where R is the universal gas constant, T the temperature and M the molar mass. The ideal gas law is an approximation, most accurate for monoatomic gases at relatively low pressures and high temperatures, but applicable also to gases of more complicated structure, for example air at room temperature and atmospheric pressure.

2.2 The Adiabatic Index

Every gas molecule containing N atoms has $3N$ degrees of freedom [3]. These are classified into three groups: translational, rotational and vibrational. The translational are used to specify the position of the center of mass. Monoatomic gases have only translational degrees of freedom. The rotational degrees of freedom describe the rotational orientation of the molecule. Asymmetric molecules have three, while diatomic or linear molecules only have two, as rotation around it's own axis cannot be distinguished. The remaining degrees of freedom are vibrational. They describe all the possible ways to stretch and bend the atoms of the molecule with respect to each other. Diatomic molecules have one vibrational degree of freedom. Both the rotation and vibration of molecules vanish at low temperatures, when

the energy is no longer sufficient. Rotation appears at lower temperatures than vibration, but the actual temperatures differ between different gases.

According to the equipartition theorem, each square term in the total energy U per molecule of a gas, contributes to it with $1/2kT$, where k is the Boltzmann constant. This implies that for a monoatomic gas the total energy is $U = 3/2kT$. For a diatomic gas with rotational degrees of freedom follows $U = 5/2kT$ and with both rotational and vibrational degrees of freedom $U = 7/2kT$. The vibrational degree of freedom contributes with kT because it gives rise to two square terms in the expression of the total energy, one for kinetic and one for potential energy.

From these expressions of the total energy per molecule of the gas, by using the equations of heat capacity, the adiabatic index can be determined. For monoatomic gases it is 5/3. For diatomic gases with rotational degrees of freedom it is 7/3, and for diatomic gases with both rotational and vibrational degrees of freedom the adiabatic index is 9/7.

3 Experimental Realization

The velocity of sound was investigated in three different gases: air, argon and nitrogen. The measurements were performed at room temperature (20.5°C) with two different techniques and at different pressures.

The experimental setup consisted of a closed, transparent plastic tube with a combined piezoelectric ultrasonic receiver and transmitter at each end (see Fig. 1). The tube was connected to a vacuum pump and a pressure gauge. It had also a connection for gas tubes.

One of the receiver/transmitters was used as a receiver and the other as transmitter. By applying an alternating voltage, generated by the function generator, to the transmitter, it produced sound waves with the same frequency as the applied voltage. The signals from the transmitter and the receiver were both collected by an oscilloscope, which was used to analyze the time difference between the two signals caused by the distance the soundwave traveled through the pipe.

To exchange the gas inside the tube, one gas was pumped out of the system, which was then filled with another gas. This procedure was performed twice to ensure that the gas left in the pipe was pure.

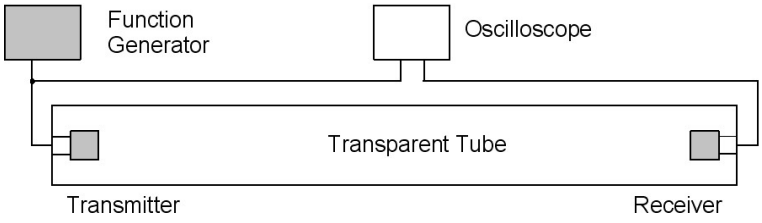

Figure 1: Experimental setup.

3.1 Time Delay Measurement

One method of determining the velocity of sound is the time delay measurement. Thereby the time delay between a pulse producing a sound wave in the transmitter and a pulse produced by that sound wave in the receiver is measured. To be able to distinguish the pulse clearly a square wave was used. A constant frequency of 20Hz of the signal from the function generator was applied in these measurements.

This method was used to obtain the velocity of sound as a function of pressure for the three gases: air, argon and nitrogen. The time delay was measured at different pressures, from about 40 kPa, where the signal of the receiver was strong enough to be distinguished, up to atmospheric pressure (about 100 kPa). The velocity of sound at each point of measurement could easily be determined using the equation [2]

$$c_s = \frac{L_0}{\Delta t} \tag{3}$$

where Δt is the time delay and $L_0 = (930 \pm 2)$ mm is the length between receiver and transmitter.

3.2 Resonance Detection

Resonance appears in the tube when it's length $L = (1000 \pm 10)$ mm is an multiple n (n is an integer) of half of the wavelength λ of the sound [2]:

$$L = n\frac{\lambda}{2}. \tag{4}$$

At this point a standing wave appears with a maximum amplitude. The frequencies at which resonance occur could be detected in the receiver signal, which then also has a maximum amplitude. The receiver used in this experiment had self resonance at 40 kHz, which means that the amplitude of the produced sound wave was largest, and therefore the amplifying effect was best observable around this frequency.

4 Results

4.1 The Velocity of Sound as a Function of Pressure

First, the velocity of sound as a function of pressure was determined for three different gases by using the time delay measurement. The time delay was measured and the velocity of sound calculated using equation (3). The results can be seen in table 1 and figure 2. Only the calculated value for nitrogen agrees with the tabulated value. The results for air and argon are too small. One reason for that could be that the begin of the pulse was not exactly found.
In agreement with the theory the velocity of sound is not dependent on the pressure. However, the amplitude of the signal was growing with increasing pressure. This is caused by the fact, that with increasing pressure the number of particles which transmit the sound through the tube is also increasing. Therefore the higher is the amplitude of the receiving signal.

		c_s (m/s)	
	Δt (ms)	Measurement	Literature [4]
Air	2.81±0.02	331 ± 3	343
Argon	2.99±0.02	311 ± 3	318
Nitrogen	2.76±0.02	337 ± 3	334

Table 1: Results for the velocity of sound at room temperature using the method of time delay (error calculation see appendix A).

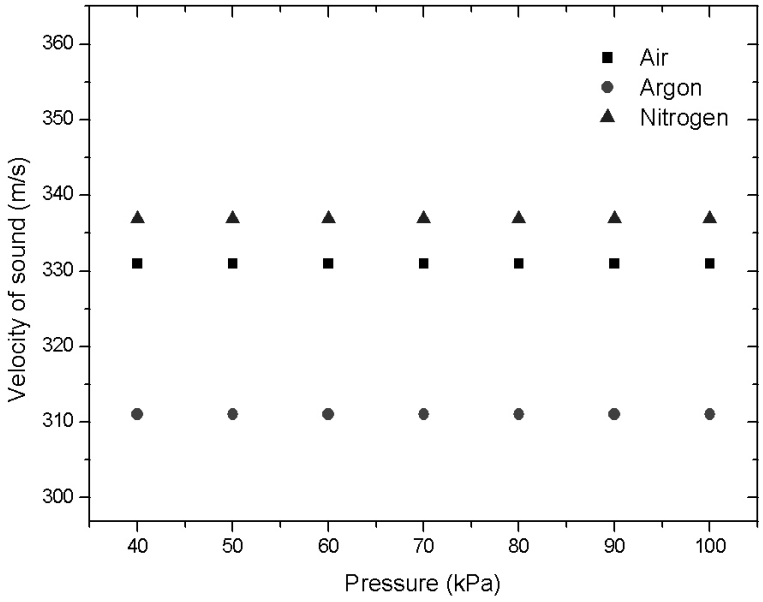

Figure 2: Velocity of sound as a function of pressure for air, argon and nitrogen at room temperature.

The velocity of sound in air at standard atmospheric pressure was also detected with the aid of the resonance detection. To be able to calculate the wavelength associated to the frequency at which resonance appears, the number of half wavelengths, which fit into the tube, must be obtained (see Eq. (4)). This can be done by determining the wavelengths for two resonances λ_0 and λ_i, with $n_i = n_0 + i$. By using $c = \lambda f$ [2] and equation (4) for n_0 follows

$$n_0 = \frac{i f_0}{f_i - f_0} \tag{5}$$

and for the velocity of sound c_s

$$c_s = 2L \, \frac{f_i - f_0}{i}. \tag{6}$$

The measurements and results can be seen in table 2. This gives an average value of

$$c_s = (315 \pm 20) \text{ m/s}$$

whereas the error is the standard deviation. This result does not agree with the tabulated value of 343m/s [4]. The large error is caused by the fact that the frequency could only be adjusted in steps of 10 Hz. Another fact was that the signal was not very stable because of background noises and therefore it was difficult to find the frequency with the maximum amplitude.

i	f_i (kHz)	c_s (m/s)
1	39.70 ± 0.01	280.0 ± 43
2	39.90 ± 0.01	340.0 ± 23
3	40.05 ± 0.01	326.7 ± 17
4	40.20 ± 0.01	320.0 ± 13
5	40.35 ± 0.01	316.0 ± 11
6	40.51 ± 0.01	316.7 ± 10
7	40.64 ± 0.01	308.6 ± 9

Table 2: Results for the velocity of sound using the method of resonance detection ($f_0 = (39.56 \pm 0.01)$ kHz, $L = (100 \pm 1)$ mm, error calculation see appendix A).

4.2 Degrees of Freedom of the Gas Molecules

From equation (2) follows for the adiabatic index

$$\gamma = \frac{Mc^2}{RT}. \tag{7}$$

By using this equation and the results from section 4.1 the adiabatic indices of the different gases could be calculated. The results are listed in table 3. Then the degrees of freedom of the gas molecules could be obtained (see section 2.2). Therefore the molecules in air and nitrogen have both, rotational and vibrational degrees and argon is monoatomic.
But if the calculated values are compared with the tabulated values of the adiabatic index, it can be seen that they do not agree for all three gases. This is caused by the fact that the values of the velocity of sound do not agree with the tabulated ones. Therefore the molecules in air and nitrogen should only have a rotating degree of freedom.

		Measurement		Theory [4]	
	M (kg/mol) [4]	c_s (m/s)	γ	c_s (m/s)	γ
air	0.029	331±3	1.30±0.02	343	1.40
argon	0.040	337±3	1.86±0.03	318	1.66
nitrogen	0.028	311±3	1.11±0.02	334	1.37

Table 3: Calculation of the adiabatic index γ for three different gases (error calculation see appendix A). For the molecular mass of air an average value based on the composition 71% N_2, 21%O_2 (M=0.032 kg/mol) and 1%Ar was used, $T = 273.65$ K and $R = 8.32$ Jmol^{-1}K^{-1}.

5 Conclusion

In this project the velocity of sound in air, argon and nitrogen was to be analyzed in a transparent tube. The velocity was determined as a function of pressure for the three different gases by using a time delay measurement. The results of the velocity of sound are in air $c_{s,air} = (331 \pm 3)$ m/s, in argon $c_{s,Ar} = (337 \pm 3)$ m/s and in nitrogen $c_{s,N_2} = (311 \pm 3)$ m/s. In agreement with the theory the velocity of sound is not dependent on the pressure.
The velocity of sound in air was also detected with the aid of the resonance detection technique. Thereby standing waves and resonance were produced in the tube. The result $c_{s,air} = (315 \pm 20)$ m/s agrees not with the value found in the literature. The error was relatively large caused by background noise and the fact that it was not possible to find the

exact frequency at which resonance occurs.

Although the time delay measurement was easier and faster to perform and the deviations are relatively small, the results obtained by this method do not completely agree with the tabulated values. The resonance detection technique had in this experiment a high error. This could be prevented by using a different function generator with which frequencies can be adjusted better. A lower background noise could also improve the results.

References

[1] NASA, **Speed of Sound**, `http://www.grc.nasa.gov/WWW/K-12/airplane/sound.html`

[2] C. Nordling, J. Östermann, **Physics Handbook** for Science and Engineering, Edition 8:3, Studentlitteratur, 2006

[3] **Heat Capacity Ratio for Gases**, `http://www.chemistry.mcmaster.ca/~ayers/chem2PA3/labs/2PA35.pdf`

[4] Wikipedia, `http://www.wikipedia.de`

A Error Calculation

For the error calculation the general formula for propagation of errors for a function $f(x, y, ...)$ was used:

$$\Delta f = \left|\frac{\partial f}{\partial x}\right| \Delta x + \left|\frac{\partial f}{\partial y}\right| \Delta y + \dots \tag{8}$$

For the velocity of sound c_s determined by using the time delay (Eq. (3)) follows

$$\Delta c_s = c_s \left(\frac{\Delta L_0}{L_0} + \frac{\Delta(\Delta t)}{\Delta t}\right) \tag{9}$$

and by using the resonance detection (Eq. (6)) follows

$$\Delta c_s = c_s \left(\frac{\Delta L}{L} + \frac{\Delta f_0 + \Delta f_i}{f_i - f_0}\right). \tag{10}$$

For the adiabatic index γ (Eq.(7)) follows

$$\Delta\gamma = 2\gamma \frac{\Delta c_s}{c_s}. \tag{11}$$