Problem 1 Jinsong Liu 206216

(a)

The physical origin of gain narrowing is gain can shrink the amplified spectrum towards the resonance frequency if: $g(\nu_0)\cdot l = \sigma(\nu_0)\cdot n\cdot l \gg 1$.

(b)

Because we need to avoid the transition of particles from 2> to 3> during pumping.

(c)

Because the essential condition for continuous population inversion is: $S_{10} > S_{21}$.

(d)

We can reduce the influence of ASE by spatial filtering.

Problem 2 Jinsong Liu 206216

(a)

$\because$ $g(\nu)$ has Gaussian profile

$$\therefore g(\nu) = g_0 \cdot \exp\left[-4 \cdot \ln 2 \left\{\frac{(\nu-\nu_0)}{\Delta\nu_g}\right\}^2\right]$$

g_0 is the peak value of $g(\nu)$.

$$\therefore G(\nu) = \exp[g(\nu)\cdot d] = \exp\left[g_0 \cdot \exp\left[-4 \cdot \ln 2 \left\{\frac{(\nu-\nu_0)}{\Delta\nu_g}\right\}^2\right]\cdot d\right]$$

$$\therefore G(\nu_0) = \exp[g(\nu_0)\cdot d] = \exp[g_0 \cdot d]$$

the FWHM of $G(\nu)$ is $\Delta\nu_G$

$$\therefore G(\nu_0 \pm \tfrac{1}{2}\Delta\nu_G) = \tfrac{1}{2} G(\nu_0)$$

$$G(\nu_0 \pm \tfrac{1}{2}\Delta\nu_G) = \exp\left[g_0 \cdot \exp\left[-4\ln 2 \left\{\frac{\pm\frac{1}{2}\Delta\nu_G}{\Delta\nu_g}\right\}^2\right]\cdot d\right]$$

$$\tfrac{1}{2} G(\nu_0) = \exp\left[g_0 \cdot \exp\left[-4 \cdot \ln 2 \cdot \tfrac{1}{4}\frac{\Delta\nu_G^2}{\Delta\nu_g^2}\right] d\right]$$

$$\ln G(\nu_0) - \ln 2 = g_0 \cdot d \cdot \exp\left[-\ln 2 \left(\frac{\Delta\nu_G}{\Delta\nu_g}\right)^2\right]$$

$$\ln G(\nu_0) - \ln 2 = \ln G(\nu_0) \cdot \exp\left[-\ln 2 \left(\frac{\Delta\nu_G}{\Delta\nu_g}\right)^2\right]$$

$$1 - \frac{\ln 2}{\ln G(\nu_0)} = \exp\left[-\ln 2 \left(\frac{\Delta\nu_G}{\Delta\nu_g}\right)^2\right]$$

$$\therefore \ln\left[1-\frac{\ln 2}{\ln G(\nu_0)}\right] = -\ln 2\left(\frac{\Delta\nu_G}{\Delta\nu_g}\right)^2$$

$$\because \ln(1-x) \approx -x$$

$$\therefore -\frac{\ln 2}{\ln G(\nu_0)} = -\ln 2\left(\frac{\Delta\nu_G}{\Delta\nu_g}\right)^2$$

$$\therefore \Delta\nu_g^2 = \Delta\nu_G^2 \cdot \ln G(\nu_0)$$

$$\therefore \Delta\nu_g = \Delta\nu_G \cdot \sqrt{\ln G(\nu_0)} \qquad \Delta\nu_G = \Delta\nu_g \cdot \frac{1}{\sqrt{\ln G(\nu_0)}}$$

(b)

$$G(\nu) = \exp[g(\nu)\cdot d] = \exp[\sigma(\nu)\cdot n\cdot d]$$

$$= \exp\left[\left(a-\sigma_0\left[\frac{\nu-\nu_0}{\Delta\nu_0}\right]^2\right)\cdot n\cdot d\right] \quad ①$$

$$\because \sigma(\nu) = a-\sigma_0\left[\frac{\nu-\nu_0}{\Delta\nu}\right]^2$$

is valid for $|\nu| < \nu_0 + \Delta\nu\cdot\sqrt{\frac{a}{\sigma_0}}$

From equation ①, we can get:

$$G(\nu_0) = \exp[a\cdot n\cdot d]$$

$$G(\nu_0 \pm \tfrac{1}{2}\Delta\nu_G) = \exp\left[n\cdot d\cdot\left(a-\sigma_0\left[\frac{\pm\frac{1}{2}\Delta\nu_G}{\Delta\nu}\right]^2\right)\right]$$

$$G(\nu_0 \pm \tfrac{1}{2}\Delta\nu_G) = \tfrac{1}{2}G(\nu_0)$$

$$\frac{1}{2}\exp(a\cdot n\cdot d) = \exp\left[n\cdot d\left(a - \sigma_0\left[\frac{\Delta\nu_G}{2\Delta\nu_\sigma}\right]^2\right)\right]$$

$$a\cdot n\cdot d - \ln 2 = a\cdot n\cdot d - \sigma_0\cdot n\cdot d\left[\frac{\Delta\nu_G}{2\Delta\nu_\sigma}\right]^2$$

$$\ln 2 = \sigma_0\cdot n\cdot d\left[\frac{\Delta\nu_G}{2\Delta\nu_\sigma}\right]^2$$

$$\therefore \frac{\ln 2}{\sigma_0\cdot n\cdot d}(2\Delta\nu_\sigma)^2 = \Delta\nu_G^2$$

$$\therefore \Delta\nu_G = 2\Delta\nu_\sigma\cdot\sqrt{\frac{\ln 2}{\sigma_0\cdot n\cdot d}}$$

(C)

$$G(\nu) = \exp[g(\nu)\cdot d] = \exp[\sigma_\nu\cdot n\cdot d]$$

$$\because \sigma(\nu) = \sigma_0\left[1 - \left(\frac{\nu-\nu_0}{\Delta\nu_g}\right)^2\right]$$

$$\therefore G(\nu) = \exp\left[\sigma_0\cdot n\cdot d\cdot\left[1-\left(\frac{\nu-\nu_0}{\Delta\nu_g}\right)^2\right]\right]$$

$\because$ the input signal has a Gaussian spectrum

$$\therefore I_{in}(\nu) = I_0\cdot\exp\left[-4\ln 2\left\{\frac{\nu-\nu_0}{\Delta\nu_{in}}\right\}^2\right]$$

$$I_{out}(\nu) = I_{in}(\nu)\cdot G(\nu)$$

$$= I_0\exp\left\{\sigma_0\cdot n\cdot d\left[1-\left(\frac{\nu-\nu_0}{\Delta\nu_g}\right)^2\right] - 4\ln 2\left(\frac{\nu-\nu_0}{\Delta\nu_{in}}\right)^2\right\}$$

$\because c = \lambda \cdot \nu \qquad \nu = \frac{c}{\lambda} \qquad \ln \nu = \ln c - \ln \lambda$

$$\frac{1}{\nu} d\nu = -\frac{1}{\lambda} d\lambda$$

$$\therefore \left|\frac{\Delta \nu}{\nu}\right| = \left|\frac{\Delta \lambda}{\lambda}\right|$$

$\because \lambda = 1\mu m \quad \begin{cases} \Delta\lambda_{in} = 4.5\,nm \\ \Delta\lambda_{out} = 2\,nm \end{cases} \quad \begin{matrix} \therefore \Delta \nu_{in} = 1.35 \times 10^{12}\,Hz \\ \therefore \Delta \nu_{out} = 6 \times 10^{11}\,Hz \end{matrix}$

$$I_{out}(\nu_0) = I_0 \exp(\sigma_0 \cdot n \cdot d)$$

$$I_{out}(\nu \pm \tfrac{1}{2}\Delta\nu_{out}) = I_0 \exp\left\{\sigma_0 \cdot n \cdot d \cdot \left(1 - \frac{\Delta\nu_{out}^2}{4\Delta\nu_g^2}\right) - 4\ln 2 \cdot \frac{\Delta\nu_{out}^2}{4\Delta\nu_{in}^2}\right\}$$

$$\frac{1}{2} I_{out}(\nu_0) = I_{out}(\nu \pm \tfrac{1}{2}\Delta\nu_{out})$$

$$\therefore \frac{1}{2} I_0 \exp(\sigma_0 \cdot n \cdot d) = I_0 \exp\left\{\sigma_0 \cdot n \cdot d \cdot \left(1 - \frac{\Delta\nu_{out}^2}{4\Delta\nu_g^2}\right) - 4\ln 2 \cdot \frac{\Delta\nu_{out}^2}{4\Delta\nu_{in}^2}\right\}$$

$$\sigma_0 \cdot n \cdot d - \ln 2 = \sigma_0 \cdot n \cdot d \left(1 - \frac{\Delta\nu_{out}^2}{4\Delta\nu_g^2}\right) - 4\ln 2 \cdot \frac{\Delta\nu_{out}^2}{4\Delta\nu_{in}^2}$$

$$-\ln 2 = -\sigma_0 \cdot n \cdot d \frac{\Delta\nu_{out}^2}{4\Delta\nu_g^2} - 4\ln 2 \cdot \frac{\Delta\nu_{out}^2}{4\Delta\nu_{in}^2}$$

$$\frac{4\ln 2}{\Delta\nu_{out}^2} = \sigma_0 \cdot n \cdot d \frac{1}{\Delta\nu_g^2} + 4 \cdot \ln 2 \frac{1}{\Delta\nu_{in}^2}$$

$$\sigma_0 \cdot n \cdot d \frac{1}{\Delta\nu_g^2} = 4\ln 2 \frac{1}{\Delta\nu_{out}^2} - 4\ln 2 \frac{1}{\Delta\nu_{in}^2}$$

$\therefore \quad d = \frac{\Delta\nu_g^2}{\sigma_0 n} \cdot 4\ln 2 \left(\frac{1}{\Delta\nu_{out}^2} - \frac{1}{\Delta\nu_{in}^2} \right)$

$\therefore \quad d \approx 0.206\,m$

(d)

$\because \quad G(\nu) = \exp\left[\sigma_0 \cdot n d \left[1 - \left(\frac{\nu - \nu_0}{\Delta\nu_g} \right)^2 \right] \right]$

$\therefore$ the maximum value of $G(\nu)$ is:

$G(\nu_0) = \exp(\sigma_0 \cdot n \cdot d) \approx 482.992$

Problem 3 Jinsong Liu 206216

(a)

Ne atoms have 5 energy-levels, the life time of 3p, 4s, 4p and 5s is much shorter than 3s. So it's really difficult to realize population inversion.

(b)

Mission: ① The laser process in a He-Ne laser starts with collision of electrons from the electrical discharge with the helium atoms, which excites helium from the ground state to the 2^3S_1 and 2^1S_0 metastable excited states.

② Collision of the excited helium atoms with the ground-state neon atoms results in transfer of energy to the neon atoms. He atoms can transfer energy to 5s and 4s of Ne atoms.

(c)

Because the laser starts from the collision of the electrons and He atoms. The more the He atoms exist, the higher pump efficiency is.

(d)

From energy conservation, the extra energy comes from the kinetic energy of He. And the excess energy becomes the kinetic energy of Ne.

(e)

$\because$ It can increase the proability of collisions of He atoms with Ne atoms.

(f)

From the manual of He-Ne laser experiment, we know that normally a He-Ne laser is working at 632.816nm with a very narrow gain bandwidth of a few GHz, which is dominated by Doppler broadening.