Fundamentals of Modern Optics
Exercise 8
08.12.2014

to be returned: 19.12.2014, at the beginning of the lecture

Problem 1 - Dispersion compensation (2+2 points)

A transform-limited Gaussian pulse with central frequency ω_0, where the envelope is defined as $E(t) = E_0 \exp\left(-\frac{(t-t_0)^2}{\tau^2}\right)$ is launched into a fiber. The pulse width is given by $\tau = 1\,\mathrm{ps}$. The fiber is characterized by a frequency dependence of the wavenumber k such that

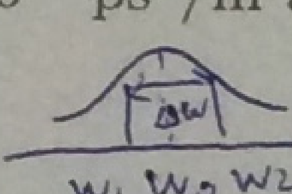
$$k(\omega) = k_0 + (1.5/c)(\omega - \omega_0) + \frac{\beta_2^{(1)}}{2}(\omega - \omega_0)^2,$$

where the group velocity dispersion of the fiber is $\beta_2^{(1)} = 10^{-1}\mathrm{ps}^2/\mathrm{m}$.

a) A pulse with a different frequency $\omega_1 = \omega_0 + \delta\omega$ is launched into the first fiber. Suppose that the 去谐 detuning is $\delta\omega = 1$ THz. Find the group index $n_{g,1}$of this pulse. If this pulse is launched 10 ps before the first pulse, when is it overtaken by the first?

b) After a fiber length of $L_1 = 10$ km, a second type of fiber is connected to the first. It has a group velocity dispersion of $\beta_2^{(2)} = -5 \cdot 10^{-1}\mathrm{ps}^2/\mathrm{m}$ and a length of L_2. How long does L_2 have to be in order to restore the initial pulse?

$k = k_0 + \frac{\omega-\omega_0}{v_{g0}} + \frac{D}{2}(\omega-\omega_0)^2 + \frac{\beta_3}{6}(\omega-\omega_0)^3$



$\Delta\omega \sim \frac{1}{\tau_0}$

$\frac{L_1}{v_{g1}(\omega)} + \frac{L_2}{v_{g2}(\omega)} = \text{const}$

$L_D^{(2)} = -\frac{\tau_0^2}{D}$ $\quad L_D^{(3)} \propto \frac{\tau_0^3}{\beta_3}$

$\tau_0 = \Delta T = \frac{L_D}{v_{g0}} - \frac{L_D}{v_{g2}}$

$\Rightarrow L_D = \frac{\tau_0^2}{D + \frac{\beta_3}{8\tau_0}}$

Problem 2 - Pulsed Beam (1+1+1+1 points)

$v_g = \frac{1}{v_{g0}} + D(\omega-\omega_0) + \frac{\beta_3}{2}(\omega-\omega_0)^2$

Assume that we have a transform-limited pulsed beam with Gaussian shape in both space and time with a pulse duration of $\tau_0 = 1\,\mathrm{ps}$ and a beam waist of $w_0 = 100\,\mu\mathrm{m}$. Measures of duration/waist describe the half-width at $1/e$ of the field envelope maximum. The beam has a carrier wavelength of $0.8\,\mu\mathrm{m}$ in a medium with a group velocity dispersion of $30 \times 10^3\,\mathrm{fs}^2\mathrm{m}^{-1}$. The pulse has an energy of $E = 1\,\mu\mathrm{J}$.

a) Calculate the peak intensity I_0 of the pulsed beam.

b) Find an equation for decay of the pulse's maximum intensity $I(z)$ due to propagation and find the length at which $I(z)$ has dropped to 1/100 of the peak intensity I_0.

c) For the parameters given above, which broadening mechanism is more influential? Spatial broadening due to diffraction or temporal broadening due to dispersion?

d) Approximate the pulse length τ_0 that would lead to equal broadening along x,y, and t-direction.

Problem 3 - Pulse compression (3+5+2+2 points)

Given is a transform limited Gaussian pulse

$$A_1(t) = A_{10}\exp(-t^2/\tau_1^2).$$

We can compress this pulse by transmitting it first through a quadratic phase modulator (QPM) and then a chirp filter.

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a) The quadratic phase modulator simply multiplies the phase factors of $\exp(i\zeta t^2)$ to the pulse $A_1(t)$. This process results in chirping the pulse; consequently, after the QPM, the pulse has following chirped Gaussian pulse shape with chirp parameter of a_2,

$$A_2(t) = A_{20}\exp(-(1 - ia_2)t^2/\tau_2^2).$$

Find the chirp parameter (a_2), the pulse temporal width (τ_2), and the pulse spectral width (ω_2). Remember both τ and ω_2 are the half-width at 1/e of the field envelope maximum.

b) As you can see from a), the pulse is chirped after the QPM, and we want to make it transform limited by passing it through a chirp filter with the envelope transer function of

$$H_e(\omega) = \exp(-ib\omega^2/4).$$

If the resulting pulse is given as $A_3(t) = A_{30}\exp(-(1 - ia_3)t^2/\tau_3^2)$, in what value of b of the chirp filter, will the pulse A_3 becomes transform limited ($a_3 = 0$)? Remember in Fourier domain $A_3(\omega) = H_e(\omega)A_2(\omega)$.

c) Find the pulse temporal width (τ_3), the pulse spectral width (ω_3), and the pulse amplitude (A_{30}).

d) Compare the QPM and the chirp filter, explain how they affect the pulse.

to be returned: 09.01.2015, at the beginning

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Problem 1 Dispersion compensation

a) $w_1 = w_0 + \delta w$

group index : $n_{g,1} = \frac{c}{V_{g,1}} = c \cdot \frac{\partial k}{\partial w}\Big|_{w_1}$

$= c \cdot [\frac{1.5}{c} + 10^{-1} \cdot (w_1 - w_0)]$

$= 1.5 + 10^{-1} \cdot \delta w \cdot c$

$= 1.5 + 10^{-1} \times 10^{-24} \times 10^{12} \times 3 \times 10^{8}$

$= 1.5 + 3 \times 10^{-5}$ ✓

group velocity of ~~plase~~ pulse with frequency w_0 :

$$\frac{1}{V_{g,0}} = \frac{\partial k}{\partial w}\Big|_{w_0} = \frac{1.5}{c}$$

group velocity of ~~plase~~ pulse with frequency w_1 :

$$\frac{1}{V_{g,1}} = \frac{\partial k}{\partial w}\Big|_{w_1} = \frac{1.5}{c} + 10^{-1} \delta w$$

when the second ~~plase~~ pulse overtake the first plase, we have

$$V_{g,0} \cdot t = V_{g,1} (t + 10\ ps)$$

$$\frac{V_{g,0}}{V_{g,1}} \cdot t = t + 10 \times 10^{-12}\ s$$

$$\frac{c}{1.5} \times (\frac{1.5}{c} + 10^{-1} \times 10^{-24} \times 10^{12}) \cdot t = t + 10^{-11}$$

$$\Rightarrow \quad t = 5 \times 10^{-7} s$$ ✓

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b) $L_1 = 10\,km$

$D_1 = \frac{\partial^2 k}{\partial \omega^2}\Big|_{\omega_0} = 10^{-1}\ \frac{ps^2}{m}$

$D_2 = \frac{\partial^2 k'}{\partial \omega^2}\Big|_{\omega_0} = -5 \times 10^{-1}\ \frac{ps^2}{m}$

The transfer function in the first fiber in the co-moving frame

$\hat{H}_1(\bar{\omega}, \alpha, \beta, z) = e^{i\frac{z_1}{2}(D_1\bar{\omega}^2 - \frac{\alpha^4\beta^2}{k_0})}$ $\quad \bar{\omega} = \omega - \omega_0$

The transfer function in the second fiber in the co-moving frame

$\hat{H}_2(\bar{\omega}, \alpha, \beta, z) = e^{i\frac{z_2}{2}(D_2\bar{\omega}^2 \frac{\alpha^4\beta^2}{k_0}}$ $\quad \bar{\omega} = \omega - \omega_0$

In Fourier space the propagation of the initial spectrum $\hat{V}_0(\alpha, \beta, \bar{\omega})$ is described as follow:

$\hat{H}_2 \hat{H}_1 \hat{V}_0 = e^{i\frac{\bar{\omega}^2}{2}(z_1 D_1 + z_2 D_2)}\ e^{i\varphi(z_1, z_2, \alpha, \beta)} \cdot \hat{V}_0(\alpha, \beta, \bar{\omega})$

we can compensate the dispersion effect $z_1 D_1$ by the term $z_2 D_2$.

$\Rightarrow z_1 D_1 + z_2 D_2 = 0 \quad \Rightarrow z_2 = -\frac{z_1 D_1}{D_2}$

$z_1 = L_1 = 10\,km \quad \Rightarrow L_2 = z_2 = \frac{1}{5} \times 10\,km = 2\,km$

So $L_2 = 2\,km$

✓

2/2

$\frac{L_1}{V_{g_1}(\omega)} + \frac{L_2}{V_{g_2}(\omega)} = const$

$k = k_0 + \frac{\omega - \omega_0}{V_{g_0}} + \frac{D}{2}(\omega - \omega_0)^2$

$V_g = \frac{1}{V_{g_0}} + D(\omega - \omega_0)$

$L_1[\frac{1}{V_{g_0}} + D_1(\omega - \omega_0)] + L_2 \cdot [\frac{1}{V_{g_0}} + D_2(\omega - \omega_0)] = \frac{L_1 + L_2}{V_{g_0}} + (\omega - \omega_0)[\underbrace{L_1 D_1 + L_2 D_2}_{=0}] = const$

Problem 2 - Pulsed Beam

$\tau_0 = 1\,ps$, $w_0 = 100\times10^{-6}\,m$ $\lambda_0 = 0.8\times10^{-6}\,m$

$D = 30\times10^3\ fs^2\cdot m^{-1} = 30\times10^3\times10^{-30}\ s^2/m$, $E = 10^{-6}J$

a) $U(\vec r, t) = A_0 e^{-\frac{t^2}{\tau_0^2}} e^{-\frac{x^2+y^2}{w_0^2}} e^{i(w_0 t + kz)}$

$|U(\alpha,\beta,w)|^2 = A_0^2 e^{-\frac{2t^2}{\tau_0^2}} \cdot e^{-2\times\frac{x^2+y^2}{w_0^2}}$

$E = \iiint |U(\alpha,\beta,w)|^2\, dx\,dy\,dt$

$= \iiint A_0^2 e^{-2(\frac{t^2}{\tau_0^2}+\frac{x^2+y^2}{w_0^2})}\, dx\,dy\,dt$

$= A_0^2 \cdot \sqrt{\frac{\pi}{2}}\cdot w_0 \cdot \sqrt{\frac{\pi}{2}}\cdot w_0 \cdot \sqrt{\frac{\pi}{2}}\cdot \tau_0$

So $I_0 = |A_0|^2 = \frac{2\sqrt{2}E}{\pi\sqrt{\pi}\,w_0^2\tau_0} = \frac{2\sqrt{2}\times10^{-6}}{\pi\sqrt{\pi}\times(100\times10^{-6})^2\times10^{-12}}\ J/(m^2\cdot s)$

$= 5.08\times10^{13}\ J/(m^2\cdot s)$ ✓ 1/1

$E = I_0 (\frac{\pi}{2})^{\frac{3}{2}} w_0^2 \tau_0 = I(z)(\frac{\pi}{2})^{\frac{3}{2}} w^2(z)\tau(z)$, $w = w_0\sqrt{1+(\frac{z}{z_0})^2}$ $z_0 = \frac{\pi w_0^2}{\lambda} = 0.04\,m$

$\tau = \tau_0\sqrt{1+(\frac{2z}{z_D})^2}$ $z_D = \frac{\tau_0^2}{D_2} = 33.3\,m$ } important!

b) $A(z) = A_0 \cdot \sqrt[4]{\frac{1}{1+(\frac{z}{z_1})^2}} \cdot \sqrt{\frac{1}{1+(\frac{z}{z_0})^2}}\ e^{-\frac{t^2}{T(z)^2}} e^{-\frac{x^2+y^2}{w(z)^2}}$

$T(z) = \tau_0\sqrt{1+(\frac{z}{z_1})^2}$, $w(z) = w_0\sqrt{1+(\frac{z}{z_0})^2}$

$z_1 = -\frac{1}{2}\cdot\frac{\tau_0^2}{D} = -\frac{1}{2}\times\frac{10^{-24}}{30\times10^3\times10^{-30}}\ m = -16.67\ m$

$z_0 = \frac{\pi}{\lambda_0} w_0^2 = \frac{\pi\times(100\times10^{-6})^2}{0.8\times10^{-6}}\ m = 3.92\times10^{-2}\ m$

The pulse has maximum intensity in center:

$\cancel{I_{max}(z) = |A(z)|^2 = A_0^2 \sqrt{1+(\frac{z}{z_1})^2}\,(1+(\frac{z}{z_0})^2) \to A_0 \sqrt{}}$

$\cancel{\frac{I_{max}(z)}{I_0} = \frac{1}{100} \Rightarrow \sqrt{1+(\frac{z}{z_1})}\,(1+(\frac{z}{z_0})^2) \frac{1}{100}}$

$$I_{max}(z) = |A(z)|^2 = A_0^2 \left(\sqrt[4]{\frac{1}{1+(\frac{z}{z_1})^2}} \cdot \sqrt{\frac{1}{1+(\frac{z}{z_0})^2}}\right)^2 = A_0^2 \sqrt{\frac{1}{1+(\frac{z}{z_1})^2}} \cdot \left(\frac{1}{1+(\frac{z}{z_0})^2}\right)$$

$$\frac{I_{max}(z)}{I_0} = \frac{1}{100} \Rightarrow \sqrt{\frac{1}{1+(\frac{z}{z_1})^2}} \cdot \frac{1}{1+(\frac{z}{z_0})^2} = \frac{1}{100}, \text{ where } z_0 = 3.92\times10^{-2}\text{ m}$$

$$z_1 = -16.67\text{ m}$$

use mathematica to solve the equation.

$$z = \pm 0.390676 \text{ m}$$

1/1

$z_0 \ll z \ll L_D$ $\quad 1+(\frac{z}{z_0})^2 \approx 100 \Rightarrow \frac{z}{z_0} \approx 10 \quad z \approx 10 z_0 \approx 0.4\text{ m}$

c) $T(z) = T_0\sqrt{1+(\frac{z}{z_1})^2}$

$W(z) = W_0\sqrt{1+(\frac{z}{z_0})^2}$

$\frac{T(z)}{T_0} = \sqrt{2}$ at $z = |z_1| = 16.67$ m

$\frac{W(z)}{W_0} = \sqrt{2}$ at $z = |z_0| \approx 3.92\times10^{-2}$ m

$|z_1| > z_0 \Rightarrow$ the broadening of pulse due to diffraction is more influential than that due to dispersion.

1/1

d) equal broadening $\Rightarrow |z_0| = |z_1|$

$$\Rightarrow \frac{\pi}{\lambda_0} W_0^2 = \frac{1}{2}\frac{T_0'^2}{D}$$

$$T_0' = \sqrt{\frac{2\pi}{\lambda_0}D} \cdot W_0 = \sqrt{\frac{2\pi\times30\times10^3\times10^{-30}}{0.8\times10^{-6}}} \cdot 100\times10^{-6}\text{ s} \approx 4.85\times10^{-14}\text{ s} = 48.5\text{ fs}$$

1/1

$e^{i\xi t^2}$ $\quad He = e^{-i\frac{bw^2}{4}}$

$A_1(t) = A_{10} e^{-\frac{t^2}{\tau_1^2}} \to \boxed{QPM} \to \boxed{chirp}$

- Problem 3. - Pulse compression

a) $A_1(t) = A_{10}\, e^{-\frac{t^2}{\tau_1^2}}$

$A_2(t) = A_{20}\, e^{-\frac{t^2}{\tau_2^2}}\, e^{\frac{i a_2 t^2}{\tau_2^2}} = A_{10}\, e^{-\frac{t^2}{\tau_1^2}}\, e^{i\xi t^2}$

$$\Rightarrow \begin{cases} A_{10} = A_{20} \checkmark \\ \tau_1 = \tau_2 \checkmark \\ \frac{a_2}{\tau_2^2} = \xi \Rightarrow a_2 = \xi \tau_1^2 \checkmark \end{cases}$$

b) Make a Fourier transform

$$A_2(w) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A_{20}\, e^{-(1-ia_2)\frac{t^2}{\tau_2^2}}\, e^{iwt}\, dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} A_{20}\cdot e^{-\left[\frac{(1-ia_2)}{\tau_2^2}\cdot t^2 - iwt + \left(\frac{jw}{2\times\sqrt{\frac{1-ia_2}{\tau_2^2}}}\right)^2\right]}\, e^{\left(\frac{jw}{2\sqrt{\frac{1-ia_2}{\tau_2^2}}}\right)^2} dt$$

$$= \frac{1}{2\pi}\cdot A_{20}\cdot e^{-\frac{w^2\tau_2^2}{4(1-ia_2)}}\cdot\int_{-\infty}^{\infty} e^{-\left(\frac{\sqrt{1-ia_2}}{\tau_2}t - \frac{iw}{2\sqrt{\frac{1-ia_2}{\tau_2^2}}}\right)^2} dt$$

$$= \frac{1}{2\sqrt{\pi}} A_{20} \frac{\tau_2}{\sqrt{1-ia_2}}\cdot e^{\frac{-w^2\tau_2^2(1+ia_2)}{4(1+a_2^2)}}$$

$$A_2(w_2) = \frac{1}{2\sqrt{\pi}}\cdot A_{20}\cdot\frac{\tau_2}{\sqrt{1-ia_2}}\cdot e^{-1} \Rightarrow w_2^2 = \frac{4(1+a_2^2)}{\tau_2^2} \Rightarrow w_2 = \frac{2}{\tau_2}\sqrt{1+a_2^2}$$
$$= \frac{2}{\tau_1}\sqrt{1+\xi^2\tau_1^4} \checkmark$$

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c) $A_3(t) = A_{30}\, e^{-(1-ia_3)\frac{t^2}{\tau_3^2}} \Rightarrow A_3(w) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A_3(t)\, e^{iwt} dt$

$$= \frac{1}{2\sqrt{\pi}} A_{30} \frac{\tau_3}{\sqrt{1-ia_3}}\cdot e^{\frac{-w^2\tau_3^2(1+ia_3)}{4(1+a_3^2)}}$$

$$A_3(w) = He(w)\, A_2(w) = \frac{1}{2\sqrt{\pi}} A_{20}\cdot\frac{\tau_2}{\sqrt{1-ia_2}}\cdot e^{-\frac{w^2\tau_2^2(1+ia_2)}{4(1+a_2^2)}}\cdot e^{-\frac{ibw^2}{4}}$$ 2

So $\frac{a_3 w^2\tau_3^2}{4(1+a_3^2)} = \frac{a_2 w^2 \tau_2^2}{4(1+a_2^2)} + \frac{bw^2}{4}$ with $a_3 = 0$

3/5

$\Rightarrow b(1+a_2^2) + a_2\tau_2^2 = 0 \Rightarrow b = \frac{-a_2\tau_2^2}{1+a_2^2} = \frac{-\xi\tau_1^4}{1+\xi^2\tau_1^4} \checkmark$

$\tau_3 = \frac{\tau_1^2}{1+a_2^2} \Rightarrow \tau_3 = \frac{\tau_1}{\sqrt{1+(\xi\tau_1^2)^2}}$

but for all the points you should show more details (some students solved this on 2 pages...)

to be returned: 09.01.2015, at the beginning ...

... for diffraction

c) $A_{30}\frac{z_3}{\sqrt{1-ia_3}} = A_{20}\frac{z_2}{\sqrt{1-ia_2}}$ with $a_3=0$

$$\Rightarrow A_{30} = A_{20}\frac{z_2}{z_3\sqrt{1-ia_2}} = A_{10}\frac{z_1}{z_3\sqrt{1-iSz_1^2}}$$

$$\frac{-w^2z_3^2}{4(1+a_3^2)} = \frac{-w^2z_2^2}{4(1+a_2^2)} \Rightarrow z_3 = \frac{z_2}{\sqrt{1+a_2^2}} = \frac{z_1}{\sqrt{1+S^2z_1^4}}$$

So $A_{30} = A_{20}\cdot\frac{z_2}{\sqrt{1-ia_2}}\times\frac{\sqrt{1+a_2^2}}{z_2} = A_{20}\sqrt{1+ia_2} = A_{10}\sqrt{1+iSz_1^2}$

$$e^{-\frac{w_3^2z_3^2(1+ia_3)}{4(1+a_3^2)}} = e^{-1} \Rightarrow w_3 = \sqrt{\frac{4(1+a_3^2)}{z_3^2}} = \frac{2\sqrt{1+S^2z_1^4}}{z_1}$$

2/2

d) QPM: It adds a chirp, the Intensity peak and the temporal width don't change. But the spectral width becomes wider. (In time domain)

chirp filter: It adds a chirp, the Intensity peak and the spectral width don't change. But the temporal width becomes wider. (In frequency domain)

2/2