Task 1 Jinsong Liu

Solution:

ABCD matrix: $\begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/f_1 & 1 \end{bmatrix}\begin{bmatrix} 1 & f_1 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1-\frac{d_1}{f_1} & f_1 \\ -\frac{1}{f_1} & 0 \end{bmatrix}$$

~~$q_1=-iz_1$~~ $q_1=-iz_1$ is a purely imaginary number

$\therefore \frac{f_1^2-iz_0f_1+iz_0d_1}{iz_0}$ is a purely imaginary number

$\therefore d_1-f_1=0 \quad d_1=f_1$

$\because \frac{1}{q}=\frac{1}{R}+i\frac{\lambda}{\pi W^2}$

$\therefore \frac{1}{q_1}=-\frac{1}{iz_1}=i\frac{\lambda}{\pi W_1^2}=\frac{iz_0}{f_1^2}$

$\therefore W_1^2=\frac{\lambda f_1}{\pi z_0}$

$\because \frac{1}{q_0}=-\frac{1}{iz_0}=i\frac{\lambda}{\pi W_0^2}$

$\therefore \frac{1}{z_0}=\frac{\lambda}{\pi W_0^2}$

$z_0=\frac{\pi W_0^2}{\lambda}$

$W_1^2=\frac{\lambda^2 f_1^2}{\pi^2 W_0^2}$

$W_1=\frac{\lambda f_1}{\pi W_0}$

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b) $q_0 \to q_2$

2

ABCD matrix

$\begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/f_2 & 1 \end{bmatrix}\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/f_1 & 1 \end{bmatrix}\begin{bmatrix} 1 & f_1 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1-\frac{d_2}{f_2}-\frac{1}{f_1}[d(1-\frac{d_2}{f_2})+d_2] & 1-\frac{d_2}{f_2} \\ -\frac{1}{f_2}-\frac{1}{f_1}(-\frac{d}{f_2}+1) & -\frac{f_1}{f_2} \end{bmatrix}$$

$q_2=\frac{Aq_0+B}{Cq_0+D}$

$\because d=f_1+f_2$

$\therefore q_2=\frac{[redacted]\,1-\frac{d_2}{f_2}-iz_0\{1-\frac{d_2}{f_2}-\frac{1}{f_1}[d(1-\frac{d_2}{f_2})+d_2]\}}{-\frac{f_1}{f_2}}$

$\because q_2=-iz_2$ is a purely imaginary number

$\therefore 1-\frac{d_2}{f_2}=0 \quad d_2=f_2$

$\therefore q_2=\frac{\frac{f_2}{f_1}iz_0}{-\frac{f_1}{f_2}}$

$=-i\frac{f_2^2}{f_1^2}\frac{\pi W_0^2}{\lambda}$

$W_2=\sqrt{\frac{-Im\{q_2\}\lambda}{\pi}}=\frac{f_2}{f_1}W_0$

c)

$W_1\approx 2.19\times 10^{-6}m$

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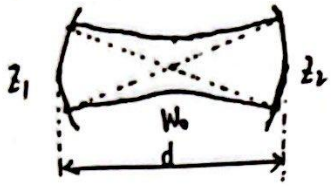
$W_2=\frac{150}{200}\times 15\ mm=11.25mm$

Task 2

Solution :

a)

0.5



$|R_1| = |R_2| = d$

3+1

Task 3

a)

Solution: $w_1 = w_0 - \delta w$

$$n_{g1} = c \cdot \frac{\partial k}{\partial w}\Big|_{w_1}$$

$$= c\left[\frac{1.5}{c} + 0.15(w - w_0)\right]$$

$$= 1.5 + 0.15c \cdot \delta w$$

$$= \cancel{1.5 + 0.15 \times 3 \times 10^{8} \times 10^{12}}$$

$$= 1.5 + 0.15 \times 10^{-24} \times 3 \times 10^{8} \times 10^{12}$$

$$= 1.5 + 4.5 \times 10^{-5}$$

$$n_{g0} = 1.5$$

$$\frac{1}{V_{g0}} = \frac{\partial k}{\partial w}\Big|_{w_0} = 1.5/c \qquad \frac{1}{V_{g1}} = \frac{\partial k}{\partial w}\Big|_{w_1} = 1.5/c + 0.15\,\delta w$$

$$= 0.5 \times 10^{-8} + 0.15 \times 10^{-12} \text{ s/m}$$

If the second pulse overtake the first one

$$V_{g0} \cdot t = V_{g1}(t + 20\text{ns})$$

$$\frac{c}{1.5} \cdot \left(\frac{1.5}{c} + 0.15 \times 10^{-12}\right) t = t + 20 \times 10^{-9}$$

$$t = \frac{20 \times 10^{-9}}{3 \times 10^{-5}} \text{ s} \approx 6.67 \times 10^{-4} \text{ s}$$

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b)

solution:

$$D_1 = \frac{\partial^2 k}{\partial w^2}\Big|_{w_0} = 0.15 \frac{(ps)^2}{m} \qquad \tilde{H}_{1p}(\bar{w}; z) = \exp\left[iz\frac{D_1}{2}\bar{w}^2\right]$$

$$D_2 = \frac{\partial^2 k}{\partial w^2}\Big|_{w_0} = -0.3 \frac{(ps)^2}{m} \qquad \tilde{H}_{2p}(\bar{w}; z) = \exp\left[iz\frac{D_2}{2}\bar{w}^2\right]$$

If we want the pulse to be restored

$$z_1\frac{D_1}{2} + z_2\frac{D_2}{2} = 0$$

$$\therefore z_1 D_1 + z_2 D_2 = 0$$

$\because z_1 = 5\text{km}$

$$\therefore z_2 = \frac{5 \times 0.15 \times 10^3}{0.3} \text{ m}$$

$$z_2 = 2.5\text{km}$$

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