

Homework 02

In this homework we work on images, colormaps and sensor calibration.

```
1 begin
2     using ColorSchemes # for different colors maps
3     using Plots        # for heatmap
4     using TestImages   # for testimage
5     using ImageShow     # for better rendering of inline images in Pluto
6     using Colors # for Gray
7     using PlutoUI
8 end
```

my_show

```
my_show(arr::AbstractArray{<:Real}; set_one=false, set_zero=false)
```

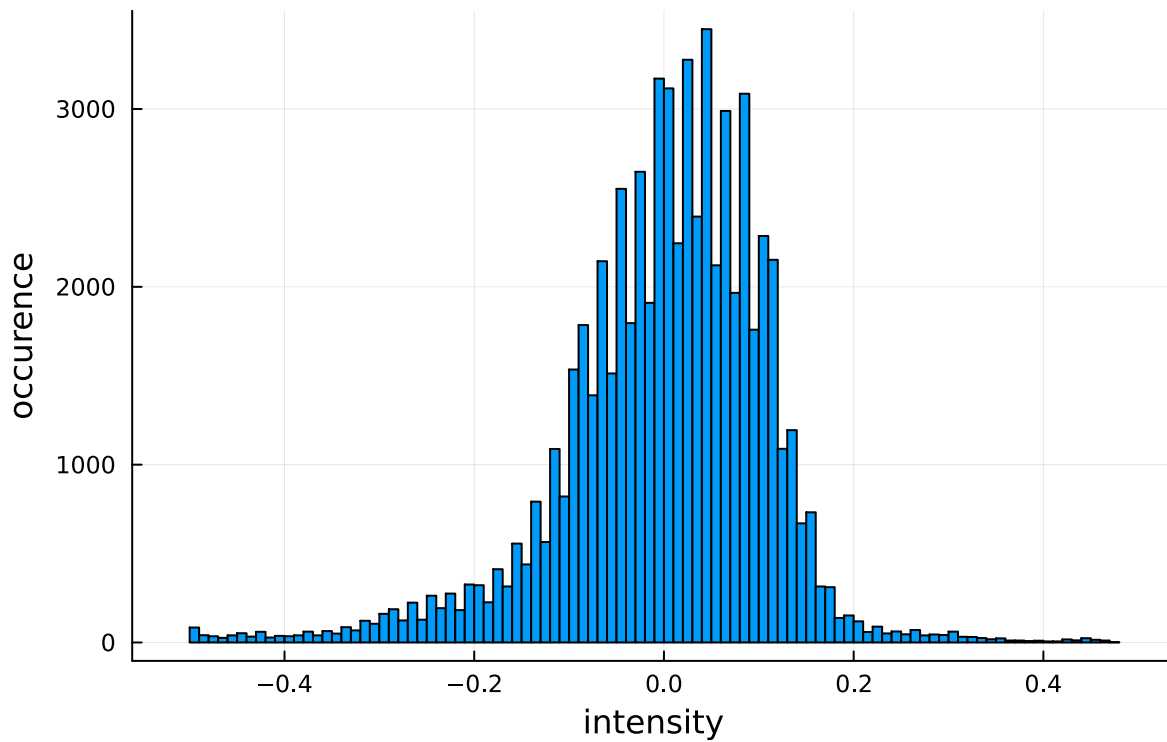
Displays a real valued array . Brightness encodes magnitude. Works within Jupyter and Pluto.

Keyword args

- `set_one=false` divides by the maximum to set maximum to 1
- `set_zero=false` subtracts the minimum to set minimum to 1

```
1 """
2     my_show(arr::AbstractArray{<:Real}; set_one=false, set_zero=false)
3 Displays a real valued array . Brightness encodes magnitude.
4 Works within Jupyter and Pluto.
5 ## Keyword args
6 * `set_one=false` divides by the maximum to set maximum to 1
7 * `set_zero=false` subtracts the minimum to set minimum to 1
8 """
9 function my_show(arr::AbstractArray{<:Real}; set_one=true, set_zero=false)
10     arr = set_zero ? arr .- minimum(arr) : arr
11     arr = set_one ? arr ./ maximum(arr) : arr
12     Gray.(arr)
13 end
```

Histogram Plot

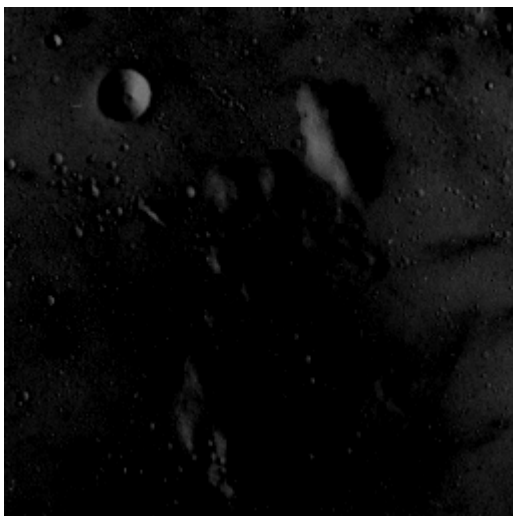


```
1 begin
2     # load a image and shift it to negative values as well
3     img = Float64.(testimage("moonsurface")) .- 0.5;
4     histogram(img[:, xlabel="intensity", ylabel="occurrence", title="Histogram
5     Plot", legend=nothing)
```

1. Colormaps

Very often in microscopy, we look at grayscale images. We already learnt that we can use `Gray`. (`img`) for that. However, that is only a very simple viewer. In general, there are much more sophisticated ones like `View5D.jl` or `Napari.jl`.

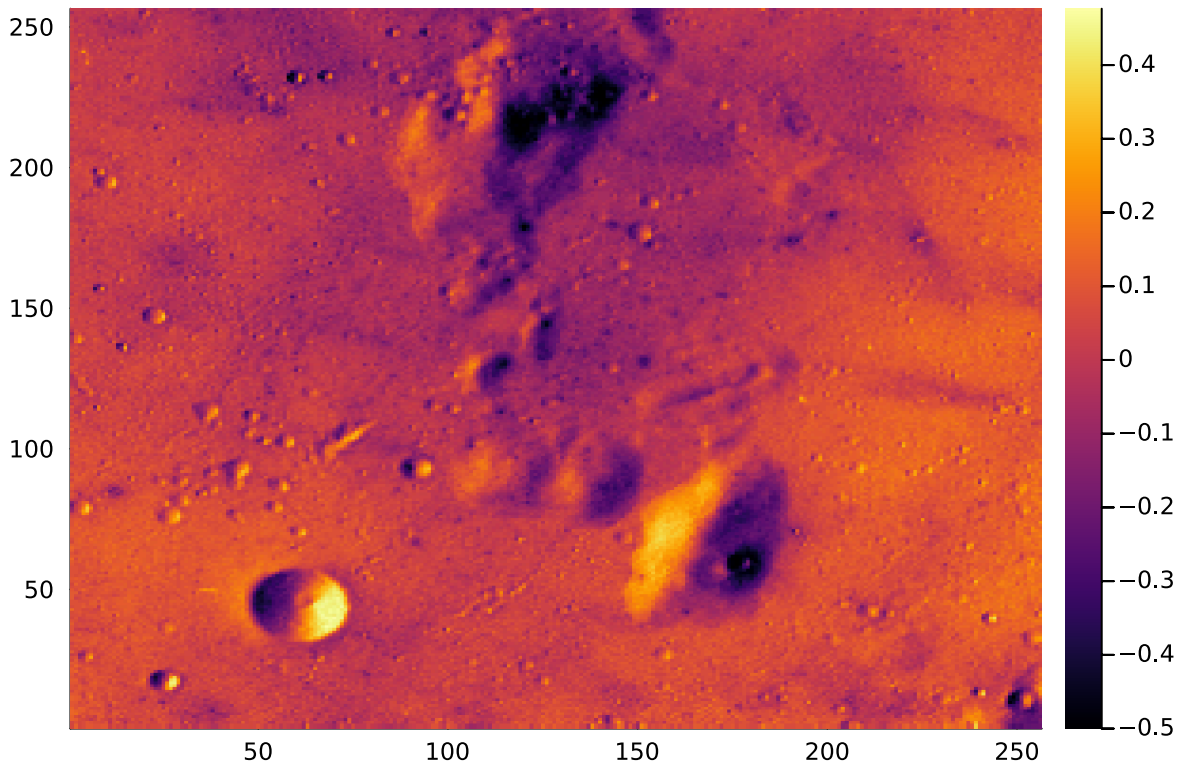
As you can see below, `Gray` fails pretty quickly and also does not make use of our human color vision.



```
1 Gray.(img)
```

1.1 Heatmap

`img` contains also negative values which `Gray` does not support by default. A nicer tool is `heatmap`.



```
1 heatmap(img)
```

1.2 Task - Equal Intensity Color Map

We now want to create a RGB color map which always has the same intensity.

Hence, $\text{green_value} + \text{red_value} + \text{blue_value} = \text{const}$ for all different inputs.

to_equal_intensity_tuple

```
to_equal_intensity_tuple(value::T) where T
```

Returns for a given scalar input a tuple of equal intensity. There is no unique solution, we only require that the sum is always 1. For example, below would be a valid solution.

Examples

```
julia> to_equal_intensity_tuple(1.0)
(0.04465819873852073, 0.33333333333333326, 0.6220084679281465)

julia> to_equal_intensity_tuple(0.0)
(0.044658198738520505, 0.3333333333333333, 0.6220084679281462)

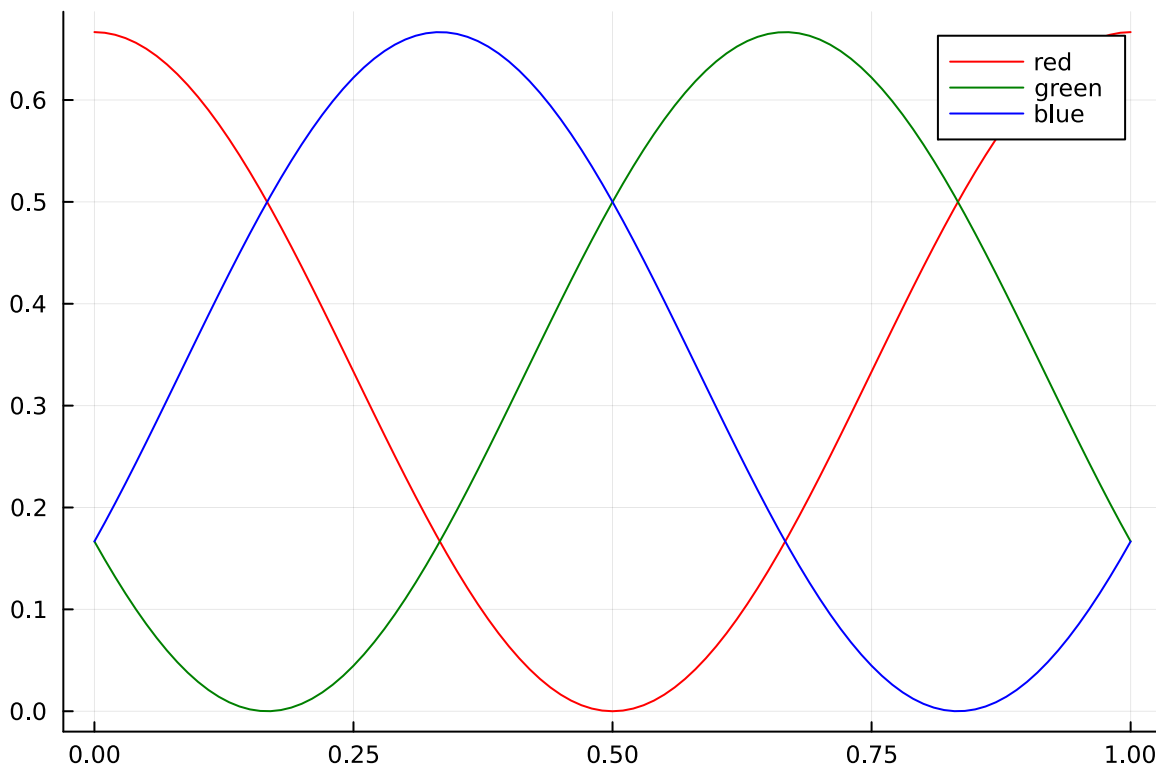
julia> to_equal_intensity_tuple(0.3f0)
(0.26402956f0, 0.6503522f0, 0.08561832f0)
```

```
1  """
2      to_equal_intensity_tuple(value::T) where T
3
4  Returns for a given scalar input a tuple of equal intensity.
5  There is no unique solution, we only require that the sum is always 1.
6  For example, below would be a valid solution.
7
8  ## Examples
9  ```julia-repl
10 julia> to_equal_intensity_tuple(1.0)
11 (0.04465819873852073, 0.33333333333333326, 0.6220084679281465)
12
13 julia> to_equal_intensity_tuple(0.0)
14 (0.044658198738520505, 0.3333333333333333, 0.6220084679281462)
15
16 julia> to_equal_intensity_tuple(0.3f0)
17 (0.26402956f0, 0.6503522f0, 0.08561832f0)
18 ```
19 """
20 function to_equal_intensity_tuple(x::T) where T
21     # with the function signature we can access T (which is the type of 'value')
22     # 0 has type Int
23     # 1.0 has type Float64
24     # 1.0f0 has type Float32
25     if x < 0 || x > 1
26         error("value must be within [0,1]")
27     end
28
29     # fix those three functions such that the property is achieved
30     # maybe think about periodic functions ;)
31     red_map(x) = (T(1) .+ cos(T(2π) .* x)) / T(3)
32
33     green_map(x) = (T(1) .+ cos(T(2π) .* x + T(2π/3))) / T(3)
34
35     blue_map(x) = (T(1) .+ cos(T(2π) .* x + T(4π/3))) / T(3)
36
37     return (red_map(x), green_map(x), blue_map(x)) # TODO, return the correct tuple!
38 end
```

Float32

```
1 typeof(Float32.(2π/3))
```

Maybe a visualization helps



```
1 begin
2   unzip(a) = map(x->getfield.(a, x), fieldnames(eltype(a)))
3   x = 0:0.01:1
4   r,g,b = unzip(to_equal_intensity_tuple.(x))
5   plot(x, r, color=:red, label="red")
6   plot!(x, g, color=:green, label="green")
7   plot!(x, b, color=:blue, label="blue")
8 end
```

1.2 Test

```
1 using PlutoTest
```

```
● all(to_equal_intensity_tuple(0.0f0) .≈ to_equal_intensity_tuple(1.0f0))
```

```
1 # cyclic
2 PlutoTest.@test all(to_equal_intensity_tuple(0f0) .≈ to_equal_intensity_tuple(1f0))
```

```
● to_equal_intensity_tuple(0.1) != to_equal_intensity_tuple(0.2)
```

```
1 PlutoTest.@test to_equal_intensity_tuple(0.1) != to_equal_intensity_tuple(0.2)
```

```
● 1 ≈ sum(to_equal_intensity_tuple(rand(0.0f0:1.0f0)))
```

```
1 # intensity is 1
2 PlutoTest.@test 1 ≈ sum(to_equal_intensity_tuple(rand(0f0:1f0)))
```

```

● all(1 .≈ sum.(to_equal_intensity_tuple.(rand(0.0:1.0, 100))))

1 # test for many values
2 PlutoTest.@test all(1 .≈ sum.(to_equal_intensity_tuple.(rand(0.0:1.0, 100))))

```

Check if the output is always $0 \leq \text{output} \leq 1$

Side Note: it took quite a while to engineer this line :D

```

bbb = true

1 bbb = foldl(&, foldl.(Ref{(acc, x) -> acc || 0<=x<=1}, to_equal_intensity_tuple.
  (rand(0.0:1.0, 100)), init=false))

```

```

● bbb

1 PlutoTest.@test bbb

```

Type Stability

The test might be wrong. Check what happens with expressions like $2\pi / 3$. Which type do they have? How can we convert them to different types?

```

● NTuple{3, Float32} == typeof(to_equal_intensity_tuple(rand(Float32.(0.0:1.0))))

1 PlutoTest.@test NTuple{3, Float32} == typeof(to_equal_intensity_tuple(rand(Float32.
  (0.0:1.0))))

```

```

● NTuple{3, Float64} == typeof(to_equal_intensity_tuple(rand(Float64.(0.0:1.0))))

1 PlutoTest.@test NTuple{3, Float64} == typeof(to_equal_intensity_tuple(rand(Float64.
  (0.0:1.0))))

```

1.3 Register Color Map

We are able to create valid outputs, but we still need to pass that to the colormap mechanism. We need to register the colormap so that we can load it with heatmap.

You don't need to know the details, but if you'd like understand, you find those in [ColorSchemes.jl](#).

```

create_equal_intensity_colormap (generic function with 1 method)

1 function create_equal_intensity_colormap()
2     values = range(0, 1, length=128)
3     return map(x -> RGB(x...), to_equal_intensity_tuple.(values))
4 end

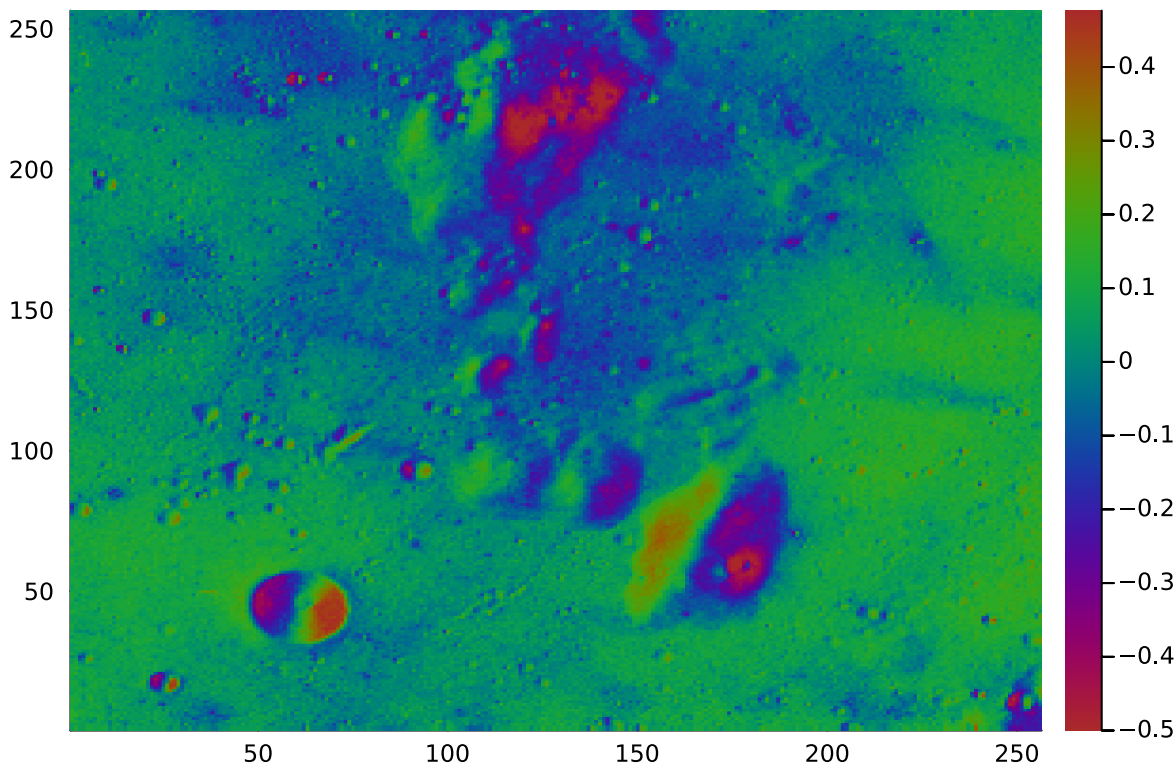
```



```

1 begin
2     my_equal_intensity_colormap = create_equal_intensity_colormap()
3     loadcolorscheme(:my_equal_intensity_colormap, my_equal_intensity_colormap)
4 end

```



```
1 heatmap(img, c=:my_equal_intensity_colormap)
```

1.4 Task

Now we want to create a colormap, which maps values between 0 and 0.5 to blue values and all values above 0.5 to red. Don't let you confuse by negativ and positive intensities. `loadcolor schemes` needs the values within the interval `[0, 1]` and will automatically scale it to the image.

to_negative_positive_tuple

```
to_negative_positive_tuple(value::T) where T
```

Examples

```
julia> to_negative_positive_tuple(0.0)
(0.0, 0.0, 1.0)
```

```
julia> to_negative_positive_tuple(0.25)
(0.5, 0.5, 1.0)
```

```
julia> to_negative_positive_tuple(0.5)
(1.0, 1.0, 1.0)
```

```
julia> to_negative_positive_tuple(0.75)
(1.0, 0.5, 0.5)
```

```
julia> to_negative_positive_tuple(1.0)
(1.0, 0.0, 0.0)
```

```
1  """
2      to_negative_positive_tuple(value::T) where T
3
4
5  ## Examples
6  ```
7  julia> to_negative_positive_tuple(0.0)
8  (0.0, 0.0, 1.0)
9
10 julia> to_negative_positive_tuple(0.25)
11 (0.5, 0.5, 1.0)
12
13 julia> to_negative_positive_tuple(0.5)
14 (1.0, 1.0, 1.0)
15
16 julia> to_negative_positive_tuple(0.75)
17 (1.0, 0.5, 0.5)
18
19 julia> to_negative_positive_tuple(1.0)
20 (1.0, 0.0, 0.0)
21 ```
22 """
23 function to_negative_positive_tuple(x::T) where T
24
25     if x < 0 || x > 1
26         error("value must be within [0,1]")
27     end
28
29     if x > 0.5
30         x_r = T(1)
31         x_g = T(2) * (T(1) - x)
32         x_b = T(2) * (T(1) - x)
33         return (x_r, x_g, x_b)
34     end
35
36     if x <= 0.5
37         x_r = T(2) * x
```



```

38     x_g = T(2) * x
39     x_b = T(1)
40     return (x_r, x_g, x_b)
41 end
42 end

```

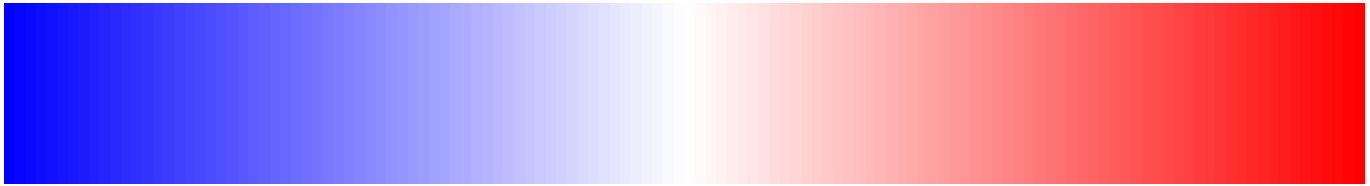
Try to show again a heatmap image. The procedure is exactly the same above, except that you need to exchange the core part of the color generation.

create_negative_positive_colormap (generic function with 1 method)

```

1 function create_negative_positive_colormap()
2     values = range(0, 1, length=128)
3     return map(x -> RGB(x...), to_negative_positive_tuple.(values))
4 end

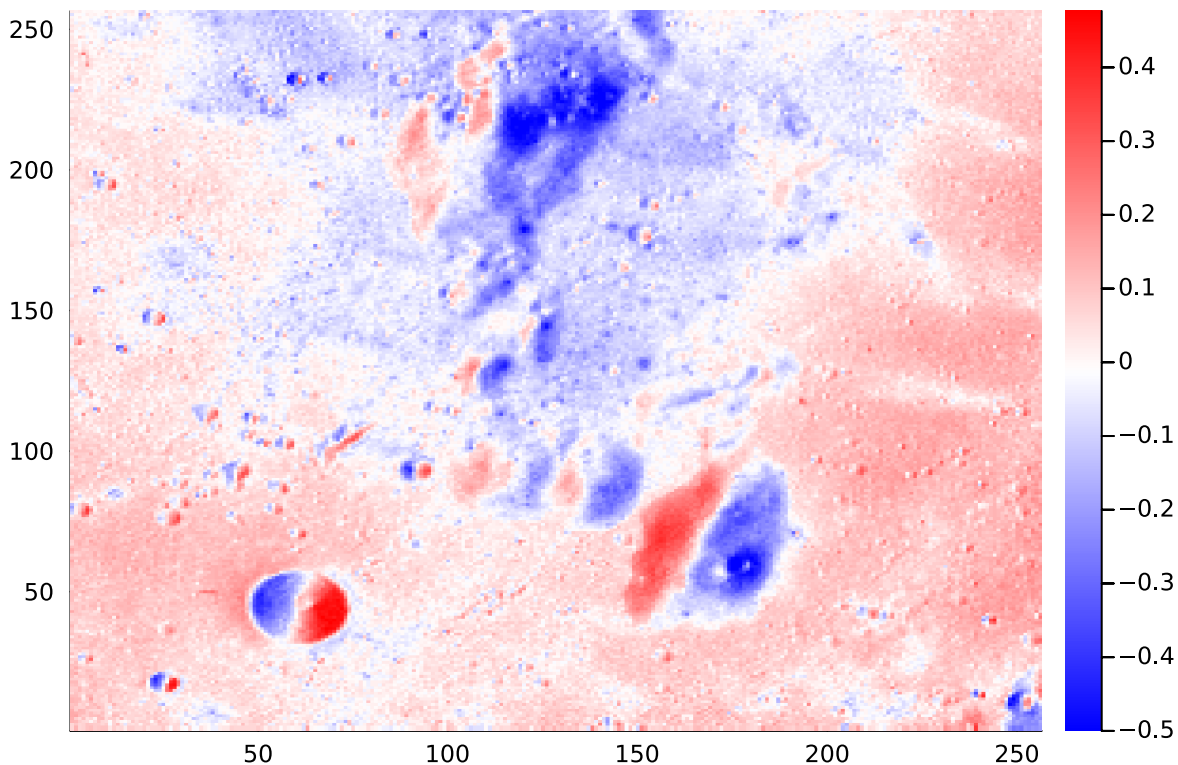
```



```

1 begin
2     my_negative_positive_colormap = create_negative_positive_colormap()
3     loadcolorscheme(:my_negative_positive_colormap, my_negative_positive_colormap)
4 end

```



```

1 heatmap(img, c=:my_negative_positive_colormap)

```

1.4 Test

```

● to_negative_positive_tuple(0.0) == (0.0, 0.0, 1)

```

```

1 PlutoTest.@test to_negative_positive_tuple(0.0) == (0.0, 0.0, 1)

```

● `to_negative_positive_tuple(1.0) == (1.0, 0.0, 0.0)`

```
1 PlutoTest.@test to_negative_positive_tuple(1.0) == (1.0, 0.0, 0.0)
```

● `to_negative_positive_tuple(0.5) == (1.0, 1.0, 1.0)`

```
1 PlutoTest.@test to_negative_positive_tuple(0.5) == (1.0, 1.0, 1.0)
```

● `to_negative_positive_tuple(0.75) == (1.0, 0.5, 0.5)`

```
1 PlutoTest.@test to_negative_positive_tuple(0.75) == (1.0, 0.5, 0.5)
```

● `to_negative_positive_tuple(0.25) == (0.5, 0.5, 1)`

```
1 PlutoTest.@test to_negative_positive_tuple(0.25) == (0.5, 0.5, 1)
```

● `typeof(to_negative_positive_tuple(0.25f0)) == Tuple{Float32, Float32, Float32}`

```
1 PlutoTest.@test typeof(to_negative_positive_tuple(0.25f0)) == Tuple{Float32,
Float32, Float32}
```

● `typeof(to_negative_positive_tuple(0.0)) == Tuple{Float64, Float64, Float64}`

```
1 PlutoTest.@test typeof(to_negative_positive_tuple(0.0)) == Tuple{Float64, Float64,
Float64}
```

2 Sensor Simulation

In this part, we want to simulate a sensor. Later, we try to calibrate our artificial sensor.

In short: Each pixel in the sensor measures a number of photons (`n_photons`) which is affected by Poisson noise. Afterwards, the measured value is altered by additive Gauss noise with a certain standard deviation σ . This value is then converted to ADU units (analog to digital units) with a certain gain (linear conversion factor). Additionally, each sensor has an offset which is finally added.

```
1 begin
2     using PoissonRandom # for poisson noise ('pois_rand')
3     using Statistics # for mean, var
4 end
```

2.1 Task

Fill in the missing parts.

simulate_pixel

```
simulate_pixel(n_photons; read_σ=5, offset=0.5, gain=0.1)
```

In this function we simulate a single pixel. It transforms a photon number to a digital unit (ADU).

The input is a integer number of photons which is then altered with poisson noise (use `pois_rand` for that). This number is changed with additive Gauss noise (see HW01). Finally, the result of this operation is multiplied by the linear `gain`. At the end, we add `offset`.

- `n_photons`: number of photons
- `read_σ`: additive read noise of the sensor (applied before `gain`)
- `gain`: how much digital output per photon
- `offset`: how much digital offset

Example Outputs - can vary in your case!

```
julia> simulate_pixel(10)
0.8570431305838304

julia> simulate_pixel(10)
2.060753446522642

julia> simulate_pixel(10)
1.020461460534897

julia> simulate_pixel(100)
8.96276245560446

julia> simulate_pixel.(ones((3,3)))
3×3 Matrix{Float64}:
 0.396078  0.96849  -0.0387851
 1.09982   0.210089  0.685605
 0.606973  1.42573   -0.675297
```

```
1 """
2     simulate_pixel(n_photons; read_σ=5, offset=0.5, gain=0.1)
3
4 In this function we simulate a single pixel.
5 It transforms a photon number to a digital unit (ADU).
6
7 The input is a integer number of photons which is then altered with poisson noise
8 (use `pois_rand` for that).
9 This number is changed with additive Gauss noise (see HW01).
10 Finally, the result of this operation is multiplied by the linear `gain`.
11 At the end, we add `offset`.
12
13
14 * `n_photons`: number of photons
15 * `read_σ`: additive read noise of the sensor (applied before `gain`)
16 * `gain`: how much digital output per photon
17 * `offset`: how much digital offset
18
```

```

19
20 ## Example Outputs - can vary in your case!
21 ```julia
22 julia> simulate_pixel(10)
23 0.8570431305838304
24
25 julia> simulate_pixel(10)
26 2.060753446522642
27
28 julia> simulate_pixel(10)
29 1.020461460534897
30
31 julia> simulate_pixel(100)
32 8.96276245560446
33
34 julia> simulate_pixel.(ones((3,3)))
35 3×3 Matrix{Float64}:
36  0.396078  0.96849  -0.0387851
37  1.09982   0.210089  0.685605
38  0.606973  1.42573  -0.675297
39 ```
40 """
41 function simulate_pixel(n_photons; read_σ=5, offset=0.5, gain=0.1)
42     poisson_noise = pois_rand.(n_photons)
43     gaussian_noise = read_σ .* randn(size(n_photons))
44     noise = poisson_noise .+ gaussian_noise
45     gained = noise .* gain
46     signal = gained .+ offset
47     return signal
48 end

```

```
single_output = 11.36955308442801
```

```
1 single_output = simulate_pixel(100)
```

```
123.33030000000014
```

```
1 mean(simulate_pixel.(ones((1000,)) * 123, read_σ=0, offset=0.0123, gain=1))
```

```
► [110.7, 135.3]
```

```
1 [123 - 12.3, 123 + 12.3]
```

```
116.06178078078086
```

```
1 var(simulate_pixel.(ones((1000,)) * 123, read_σ=0, offset=0.0123, gain=1))
```

```
► [110.7, 135.3]
```

```
1 [123 - 12.3, 123 + 12.3]
```

```
12.364908822646608
```

```
1 std(simulate_pixel.(zeros((1000,)) * 123, read_σ=12.3, offset=0, gain=1))
```

```
► [11.07, 13.53]
```

```
1 [12.3 - 1.23, 12.3 + 1.23]
```

```
535.0860767446889
```

```
1 std(simulate_pixel.(zeros((1000,)) * 123, read_σ=12.3, offset=0, gain=42))
```

```
► [464.94, 568.26]
```

```
1 [516.6 - 51.66, 516.6+51.66]
```

2.1 Test

```
● 0.0123 ≈ simulate_pixel(0, read_σ = 0, offset = 0.0123, gain = 1)
```

```
1 # check offset
2 PlutoTest.@test 0.0123 ≈ simulate_pixel(0, read_σ=0, offset=0.0123, gain=1)
```

```
● ≈(123, mean(((x→simulate_pixel.(x; read_σ = 0, offset = 0.0123, gain = 1))).(ones((100,)) * 123)))
```

```
1 # check mean poisson
2 PlutoTest.@test 123 ≈ mean((x -> simulate_pixel.(x; read_σ=0, offset=0.0123, gain=1)).(ones((100,)) * 123)) rtol=0.1
```

```
● ≈(123, var(((x→simulate_pixel.(x; read_σ = 0, offset = 0.0123, gain = 1))).(ones((100,)) * 123)))
```

```
1 # check variance poisson
2 PlutoTest.@test 123 ≈ var((x -> simulate_pixel.(x; read_σ=0, offset=0.0123, gain=1)).(ones((100,)) * 123)) rtol=0.1
```

```
● ≈(12.3, std(((x→simulate_pixel.(x; read_σ = 0, offset = 0.0123, gain = 1))).(ones((100,)) * 123)))
```

```
1 # check readnoise
2 PlutoTest.@test 12.3 ≈ std((x -> simulate_pixel.(x; read_σ=0, offset=0.0123, gain=1)).(ones((100,)) * 123)) rtol=0.1
```

```
● ≈(42 * 12.3, std(((x→simulate_pixel.(x; read_σ = 12.3, offset = 0, gain = 42))).(zeros((1000,)) * 123)))
```

```
1 # check gain
2 PlutoTest.@test 42 * 12.3 ≈ std((x -> simulate_pixel.(x; read_σ=12.3, offset=0, gain=42)).(zeros((1000,))*123)) rtol=0.1
```

2.2 Variance Mean Projection

Now we want to do a Variance Mean Projection to fit the values of offset and gain since we often don't know them for real sensors.

For this test, you take a sample and in practice you defocus your microscope heavily. Via that trick, you have an image which covers all values between very dark and very bright. Hence, we expect that our image sensors both measures a large dynamic range.

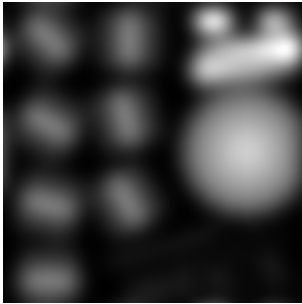
```
1 using FourierTools
```

```

1 # that's already done for you
2 begin
3     ideal = resample(Float32.(testimage("resolution_test_512"))[50:450, 50:450],
4         (150, 150)) # random amount of photons in the interval 0:500
5     x_cords = range(-10, 10, length=size(ideal, 1))
6     gauss = Float32.(exp.(-(x_cords.^2 .+ x_cords'.^2)))
7     gauss ./= sum(gauss)
8     img_blurry = conv_psf(ideal, gauss)
9     img_blurry .-= minimum(img_blurry)
10    img_blurry ./= maximum(img_blurry)
11    img_blurry = round.(Ref{Int}, img_blurry .* 1000) # round to integers
12    Gray.(img_blurry ./ 500)
13 end;

```

That's an ideal image, where we have both very bright and very dark regions

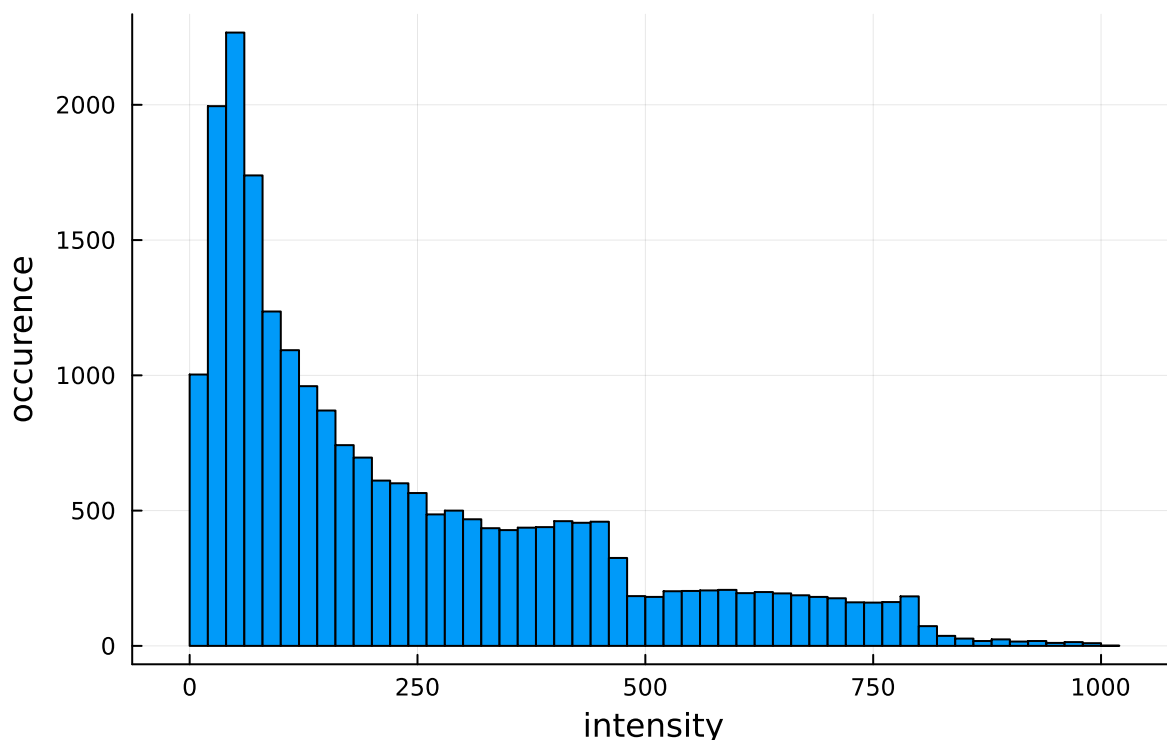


```

1 # our blurry sample which emits 1000 photons in maximum
2 my_show(img_blurry)

```

Histogram Plot



```

1 histogram(img_blurry[:,], xlabel="intensity", ylabel="occurrence", title="Histogram
  Plot", legend=nothing)

```

2.2 Task - Apply to image

Now apply the function `simulate_pixel` to `img_blurry`. Use the automatic broadcasting of Julia for that.

```

simulated_img =
150x150 Matrix{Float64}:
 2.99083  3.50343  1.89752  2.89792  ...  7.65918  5.49777  3.98531  2.11414
 5.13804  3.12868  2.63481  4.21995  ...  7.20419  4.96713  4.085   2.98867
 3.04647  3.40469  4.12338  4.32181  ...  8.94944  7.24168  4.97596  4.86995
 4.45162  5.29623  2.37724  3.67055  ... 12.492   8.80551  7.68381  6.81927
 5.87267  4.12868  3.50683  4.81572  ...  9.82668 10.982   8.11079  7.0015
 5.52869  4.98652  5.29048  3.90158  ... 15.2217 11.3741 10.9411  7.27603
 5.79101  6.42081  5.58361  4.27117  ... 16.9599 13.8018  9.90212  9.64191
 ⋮
 4.3552   4.24614  5.71966  6.39614  ...  2.67628  1.80562  2.25036  2.61056
 3.97601  4.28687  4.29555  6.99228  ...  2.83758  4.17655  2.79016  3.22355
 4.0145   3.8391   3.70017  5.22811  ...  3.26172  2.60752  2.79034  3.53797
 2.82407  4.22781  4.25234  6.72661  ...  1.98373  2.68762  2.99539  3.23128
 1.49067  2.81211  2.60929  3.58231  ...  3.56308  3.75088  3.04687  2.70705
 3.23915  2.62102  4.48338  4.12253  ...  6.02629  5.0067   5.11273  3.697

```

```

1 simulated_img = simulate_pixel.(img_blurry) # TODO, fix that line such that it uses
  'simulate_pixel'. Use broadcasting mechanism

```

In practice we would need to take a few images (like 10)

Hence we repeat the image 10 times digitally

The output is an array with `size(imgs) == (150, 150, 10)`

Apply the same trick (broadcasting) for a 2D image to this 3D image

```

1 begin
2     imgs = repeat(img_blurry, outer=(1, 1, 10));
3     images = simulate_pixel.(imgs; offset, read_σ, gain);
4 end;

```

2.3 Task - Calculate mean and variance

Calculate the mean and variance of those images, but only along the third dimension! Either use a library (Statistics) or write it yourself :)

```

arr_test = 2x1x2 Array{Int64, 3}:
 [:, :, 1] =
    1
    2

 [:, :, 2] =
    3
    2

```

```

1 arr_test = [1; 2;;; 3; 2]

```

In this case, the mean along the third dimension would be

```

2x1x1 Array{Float64, 3}:
[:, :, 1] =
 2.0
 2.0

```

```

1 [2.0; 2.0;;;]

```

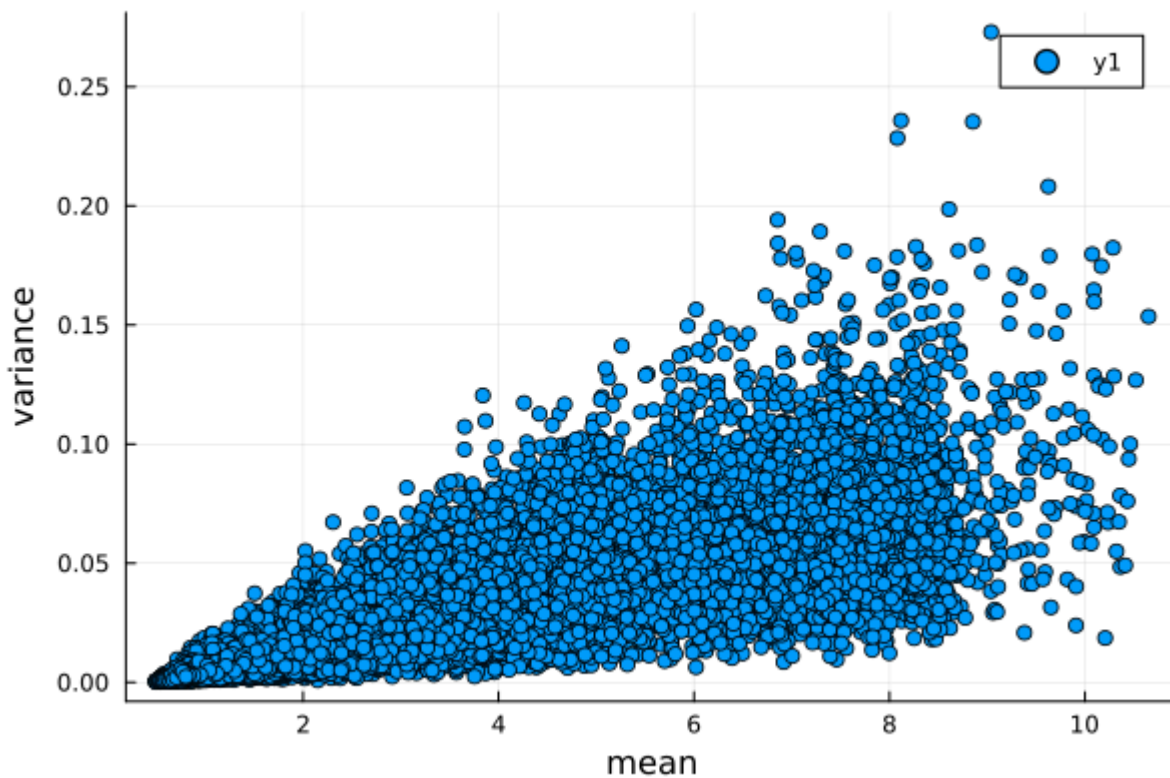
► [0.79168, 0.793474, 0.908173, 0.882048, 1.02107, 1.05338, 1.13815, 1.29924, 1.37484, 1.5145

```
1 begin
2   # mean
3    $\mu\_arr$  = mean(images, dims=3) # calculate the mean of 'images' along dim 3
4    $\mu$  =  $\mu\_arr[:]$ 
5 end
```

► [0.00417168, 0.0027955, 0.00638866, 0.00176376, 0.00300359, 0.00399284, 0.00677027, 0.01030

```
1 begin
2   # variance
3   variance_arr = var(images, dims=3) # calculate the variance of 'images' along dim
4   variance = variance_arr[:]
5 end
```

Finally, we produce a scatter plot



```
1 scatter( $\mu$ , variance, xlabel="mean", ylabel="variance")
```


2.4 Fitting

Having our noisy dataset, we want to **extract the gain** of the detector since the gain translate from a photon count to an eletrical signal (ADU).

Why does plotting the **variance** σ^2 over the **mean** μ allows to fit the gain? We know, that in a Poisson distribution the expected mean μ is always equal to the variance σ^2 .

$$\sigma^2 = \mu$$

If we now take 10 captures of the same image, we always expect the same mean μ for a certain pixel. Of course, those are 10 evaluation of a random measurement process, therefore we observe fluctuations. Hence, for each pixel we can calculate the mean and the variance.

Since the detector multiplies the measured photons with a certain gain, we do not measure the photons but the digital value. Hence the **measured mean** is

$$\hat{\mu} = \text{gain} \cdot \mu = \text{gain} \cdot \sigma^2$$

where μ would be the **expected photon number**.

However, the **measured variance** is

$$\hat{\sigma}^2 = \sum_i (\hat{\mu} - \text{gain} \cdot x_i)^2 = \text{gain}^2 \sigma^2$$

where σ^2 would be the variance of the photon number.

$$\hat{\sigma}^2 = \text{gain} \cdot \hat{\mu} + \text{offset}$$

The slope of our variance over mean is then

$$\frac{\Delta \hat{\sigma}^2}{\Delta \hat{\mu}} = \text{gain}$$

Dividing $\hat{\mu}$ by **gain** returns the measured photon number.

```
1 md" ## 2.4 Fitting
2 Having our noisy dataset, we want to **extract the `gain`** of the detector since
  the gain translate from a photon count to an eletrical signal (ADU).
3
4 Why does plotting the **variance**  $\sigma^2$  over the **mean**  $\mu$  allows to fit
  the gain?
5 We know, that in a Poisson distribution the expected mean  $\mu$  is always equal to
  the variance  $\sigma^2$ .
6
7  $\sigma^2 = \mu$ 
8
9 If we now take 10 captures of the same image, we always expect the same mean  $\mu$ 
  for a certain pixel. Of course, those are 10 evaluation of a random measurement
  process, therefore we observe fluctuations. Hence, for each pixel we can calculate
  the mean and the variance.
```

10

11

```

12 Since the detector multiplies the measured photons with a certain gain, we do not
13 measure the photons but the digital value.
14 Hence the measured mean is
15
16  $\hat{\mu} = \text{gain} \cdot \mu = \text{gain} \cdot \sigma^2$ 
17
18 where  $\mu$  would be the expected photon number.
19
20 However, the measured variance is
21
22 
$$\hat{\sigma}^2 = \sum_i (\hat{\mu} - \text{gain} \cdot x_i)^2 = \text{gain}^2 \sigma^2$$

23
24 where  $\sigma^2$  would be the variance of the photon number.
25
26  $\hat{\sigma}^2 = \text{gain} \cdot \hat{\mu} + \text{offset}$ 
27
28 The slope of our variance over mean is then
29
30 
$$\frac{\Delta \hat{\sigma}^2}{\Delta \hat{\mu}} = \text{gain}$$

31
32 Dividing  $\hat{\mu}$  by  $\text{gain}$  returns the measured photon number.

```

```

1 # for fitting
2 using LsqFit

```

2.5 LsqFit.jl

We now want to fit a linear function with a slope and a offset on our data. Try to find out [here](#) how that works

```
model (generic function with 1 method)
```

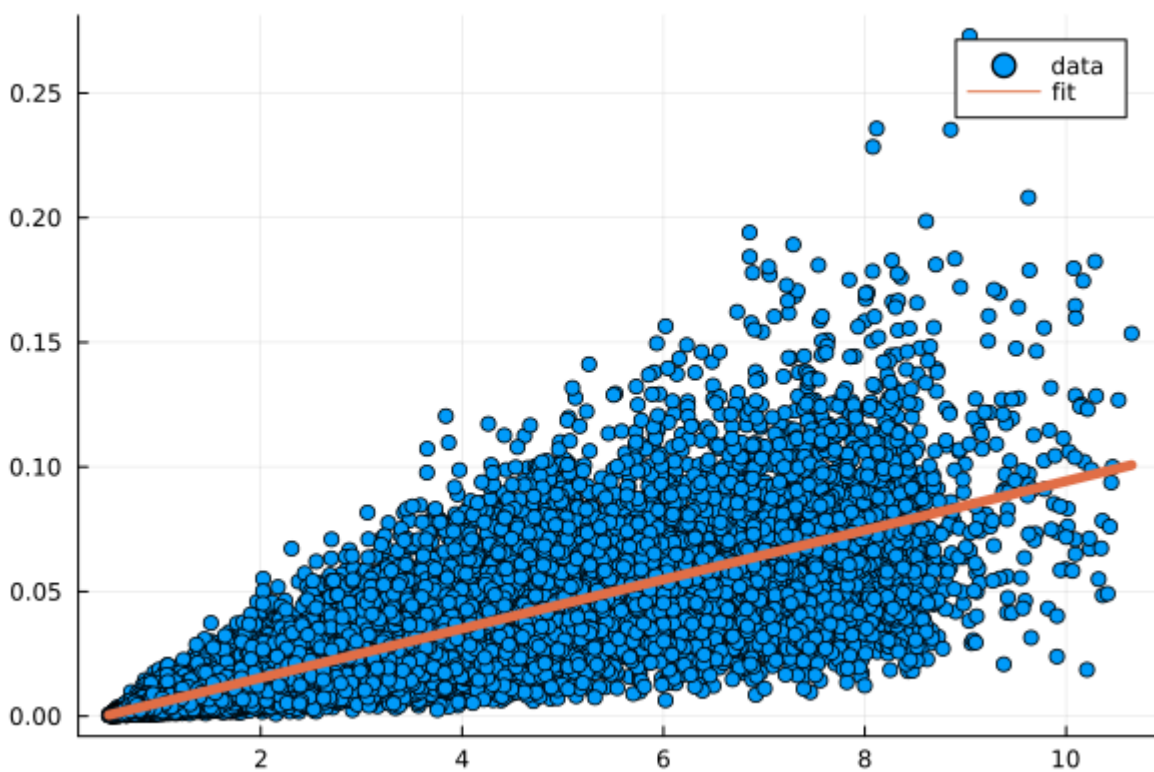
```
1 @. model(x, p) = p[1] .* x .+ p[2] #p[1] is gain, p[2] offset
```

```
p0 = ▶ [0.5, 0.5]
```

```
1 p0 = [0.5, 0.5] # TODO (maybe you need to convert it to Float32, maybe not), initial guess
```

```
fit =
▶ LsqFitResult([0.00987107, -0.00439468], [-0.000751632, 0.000642253, -0.0018187, 0.00254832,
```

```
1 fit = curve_fit(model, μ, variance, p0) # TODO
```



```

1 begin
2   scatter( $\mu$ , variance, label="data")
3   plot!( $\mu$ , model( $\mu$ , fit.param), linewidth=5, label="fit")
4 end

```

Final Check

If you slide those parameters, you change the simulated values.

offset = 0.5

read_σ = 2

gain = 0.01

The final solution is gain = $0.009871 \pm 4.7\text{e-}5$

```

1 md"The final solution is gain = $(round(fit.param[1], sigdigits=4)) ±
  $(round(stderror(fit)[1], sigdigits=2))"

```