Cartesian oval representation of freeform optics in illumination systems

D. Michaelis,* P. Schreiber, and A. Bräuer

Fraunhofer Institute for Applied Optics and Precision Engineering, Albert-Einstein-Strasse 7, 07745 Jena, Germany
**Corresponding author: dirk.michaelis@iof.fraunhofer.de*

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The geometrical method for constructing optical surfaces for illumination purpose developed by Oliker and co-workers [*Trends in Nonlinear Analysis* (Springer, 2003)] is generalized in order to obtain freeform designs in arbitrary optical systems. The freeform is created by a set of primitive surface elements, which are generalized Cartesian ovals adapted to the given optical system. Those primitives are determined by Hamiltonian theory of ray optics. The potential of this approach is demonstrated by some examples, e.g., freeform lenses with collimating front elements.
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Especially for illumination and beam shaping, the conventional rotationally symmetric optics cannot satisfactorily meet all application requirements. For the efficient generation of complex illumination patterns refractive or reflective freeform surfaces without any symmetry may provide the required degrees of freedom. Very frequently, numerical optimizations using conventional ray-tracer software fails for the freeform design due to the lack of an adequate surface representation, a huge number of free variables and the rather involved merit functions with a wealth of local optima. In contrast to this, direct design methods provide a mathematical or geometrical strategy to certainly find desired solutions, which can be also used for conventional optimizations as an initial guess afterward. The simultaneous multiple surface design method [1] is a prominent example of a direct method. It provides a geometrical way of constructing two (or N) freeforms, which generally transform two (or N) input wavefronts into two (or N) output wavefronts. The application of transformation optics could develop to a further powerful alternative for freeform design [2]. Another type of a direct method is the reformulation of the illumination problem to obtain partial differential equations (PDEs) for the freeform surfaces [3–5]. By solving these equations usually a given input wavefront (e.g., a source point or collimated light) can be converted into an arbitrary output wavefront. Thus, an arbitrary complex illumination pattern can be created. The solutions are in principle identical to point-to-point mappings [6], which are nontrivial tasks in 3D [3,5]. For sources with a pronounced extension, as well as with ray divergence, the inevitable "smearing out" effect can be reduced by anticipating the corresponding blurring in the illumination task [7]. Besides rather complicated numerical schemes for solving such PDEs, geometrical ideas can be used to compute the corresponding freeform surfaces [3].

For the task of creating an arbitrary illumination pattern in the far-field by means of a single reflector and a small, pointlike source, Oliker and co-workers developed the method of "Supporting Paraboloids" [3], where the freeform is represented by a discrete set of paraboloid segments with appropriate parameters. The convergence of the proposed algorithm for determining the paraboloid parameters has been rigorously established.

In this Letter, we generalize the method for designing such freeform elements in given optical systems. Thus, this approach opens a simple way for designing various types of optics ranging from freeform lenses up to freeforms in complex systems. Disregarding mathematical aspects, Fig. 1 illustrates some basic ideas of the design method from an intuitive point of view. For illustration purposes we begin with the simple situation of shaping the light from a pointlike source at F_0 inside a medium (refractive index n_1) at a given target outside the medium (air) by means of a single refracting surface. The desired spatial intensity pattern at the target is discretized $I(\vec{r}) \approx \sum_{i,k} P_{i,k}^{\mathrm{des}} \delta(\vec{r} - \vec{F}_{i,k})$ in a set of power values $P_{i,k}^{\mathrm{des}}$ at discrete target points $\vec{F}_{i,k}$. An ordinary Cartesian oval (CO) is an optical surface that brings light from a source point F_0 to a single target point $\vec{F}_{i,k}$, i.e., the points F_0 and

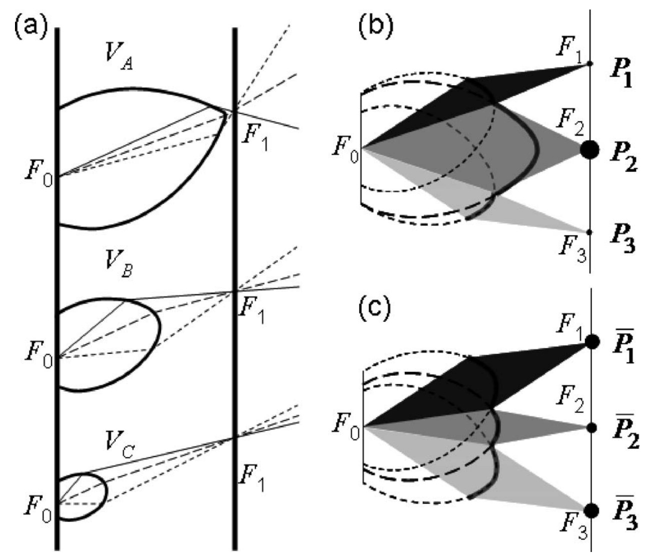

Fig. 1. Scheme of freeform design. The freeform is constructed by segments of primitive surfaces. Here ordinary COs are utilized for illustration, (a) COs with the same foci but decreasing path length parameter V_a to V_c, (b), (c) simple illustration of modification of the discrete power distribution P_i by a three-segment optics made from three COs due to changing the path length parameter of the middle CO.

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$\vec{F}_{i,k}$ are the two foci or two perfect conjugate points of the CO [8]. It is a surface of fourth order and is given by a constant optical path length $n_1\|\vec{r}_{\mathrm{CO}} - \vec{F}_0\| + \|\vec{F}_{i,k} - \vec{r}_{\mathrm{CO}}\| = V_{i,k}$ between the two foci F_0 and $\vec{F}_{i,k}$ [see Fig. 1(a)]. In order to transfer light to all discrete target points $\vec{F}_{i,k}$ the entire freeform has to be assembled by segments of those COs where one focus is always at F_0 but the other focus location passes through the entire target points $\vec{F}_{i,k}$ [see Figs. 1(b) and 1(c)]. Therefore, the focus parameters of all surface segments are fixed. But even for fixed focal parameters, there is still one free parameter left for each segment—the optical path length $V_{i,k}$, which is a measure for the element thickness. A change of these parameters leads to a change of the power distribution at the target [compare Figs. 1(b) and 1(c)], because here the entire element is the union of all subelements. Decreasing the path length parameter of a single segment [middle segment in Fig. 1(b)] decreases the power in the corresponding focus [focus $\vec{F}_2$ with power P_2 in Fig. 1(c)] but increases the power of the neighboring segments [focus $\vec{F}_1$ and $\vec{F}_2$ in Fig. 1(c)]. Because the power is a monotonic function of the corresponding path length $V_{i,k}$ the design method "Supporting Paraboloids" [3] can be easily adapted to this situation. Roughly speaking, one begins with an easy reachable situation where one segment collects more power $P_{i0,k0}$ than the corresponding desired value $P^{\mathrm{des}}_{i0,k0}$ and for all other segments $P_{i,k} \leq P^{\mathrm{des}}_{i,k}$ holds. The path length parameter of the (i_0,k_0)-segment is always kept at a fixed initial value $V_{i0,k0}$. Now, all other parameters $V_{i,k}$ are iteratively increased while the relation $P_{i,k} \leq P^{\mathrm{des}}_{i,k}$ is maintained [3].

The generalization of the treatment to design a beam shaping freeform in a given optical system is in principle straight forward. One has to determine the whole family of primitives, i.e., generalized COs with focus and path length parameters, which takes into account all effects of the entire optical system. Segments of these primitives are used to build up the freeform. From now on we restrict ourselves to the simplest situation where the freeform is the last surface of an arbitrary optical system. Figure 2 illustrates the generalized CO as a surface with a desired optical path $V_0 + n_N\|r_N - r_D\| + V_{\mathrm{end}} = V_{i,k}$. V_0 is the optical path length from the source point to a dummy plane, $n_N\|r_N - r_D\|$ is the optical path from the dummy plane to the CO (here the N-th surface S_N). In order to reach light collimation with a ray direction $\vec{k}$ the additional path length term reads as $V_{\mathrm{end}} = -\vec{k}\vec{r}_N$ (plane phase). For light focusing at a focus $\vec{F}_{i,k}$ the relation $V_{\mathrm{end}} = \|\vec{F}_{i,k} - \vec{r}_N\|$ holds. Usually, rather involved expressions result for the N-th surface. But, in order to practically carry out the design procedure one needs a quasi-analytical, explicit expression for the COs, which is additionally suitable to easily perform ray tracing, i.e., one needs a parametric surface representation $\vec{r}_N = \vec{r}_N(p_1, q_1)$. p_1, q_1 are the transverse components of the ray vector $\vec{g}_1$ in front of the first optics surface, where $g_1/n_1 = (p_1, q_1, m_1)^T/n_1$ is the corresponding unit ray direction from the source point $\vec{r}_0$. Ray tracing from $\vec{F}_0$ to $\vec{r}_N$ is quasi-analytically and straight forward with such a representation. It can be ea-

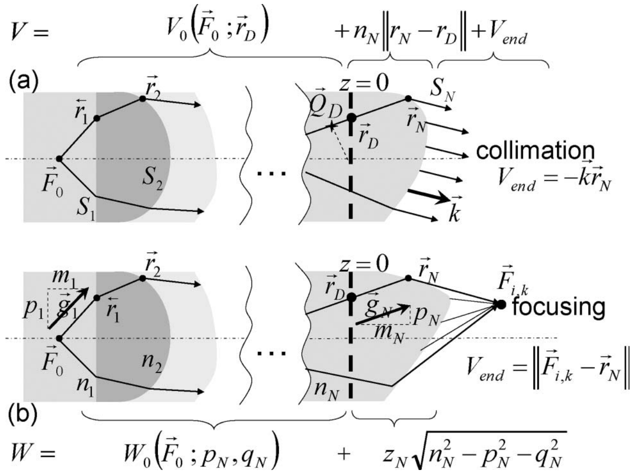

Fig. 2. Scheme for determination of the family of COs as primitives for the construction of freeforms. An arbitrary system (here N front surfaces S_1–S_{N-1} separating N-regions with different refractive indices n_i) is described by the optical path length V_0 or the mixed characteristics W_0 from the source point F_0 to a dummy plane at $z = 0$ (bold dashed line). By adding the additional path length $n_N\|\vec{r}_D - \vec{r}_N\|$ or mixed characteristics $z_N m_N$ up to the CO (S_N) correspondingly and considering the final path length V_{end} for (a) collimation or (b) focusing the CO—surface is obtained parametrically. The ray intersection at the surface S_i is denoted by $\vec{r}_i$ and the ray vectors in the regions with the index n_i are called $\vec{g}_i = (p_i, q_i, m_i)^T$ with $\|\vec{g}_i\| = n_i$.

sily obtained using Hamiltonian theory of ray optics [9]. To this end the path length $V(\vec{F}_0, \vec{r}_N)$ between $\vec{F}_0$ and $\vec{r}_N$ (so called point characteristics) is converted to the mixed characteristics by means of a Legendre transformation $V(\vec{r}_0, \vec{r}_N) = W(\vec{r}_0, p_N, q_N) + x_N p_N + y_N q_N$, where $(x_N, y_N, 0)^T = -\nabla_\perp(W)$ with $\nabla_\perp = (\partial_p, \partial_q, 0)$ holds. The mixed characteristics $W_0(\vec{r}_0, p_N, q_N)$ is determined up to the dummy plane $z = 0$. The remaining part of W up to the CO surface is a simple linear function in the z-coordinate $W = W_0 + z_N\sqrt{n_N^2 - p_N^2 - q_N^2} = W_0 + z_N m_N$. Using these relations, parametric representations for the desired COs result. For light collimation [see Fig. 2(a)] one obtains

$$\vec{r}_N(p_N, q_N) = -\nabla_\perp W_0 + \vec{g}_N \frac{V_{i,k} - W_0 - \langle \vec{k} - \vec{g}_N | \nabla_\perp W_0 \rangle}{n_N^2 - \langle \vec{k} | \vec{g}_N \rangle}.$$

In the case of light focusing [see Fig. 2(b)] at a focus $\vec{F}_{i,k}$ a quadratic equation for the z-coordinate

$$\begin{aligned} & z_N^2 \frac{n_N^2 - n_N^4}{m_N^2} \\ & - 2z_N^1 \left\{ \frac{\langle \vec{g}_N | \vec{F}_{i,k} + (1 - n_N^2)\nabla_\perp W_0 \rangle - n_N^2 (V_{i,k} - W_0)}{m_N} \right\} \\ & + \|\nabla_\perp W_0 + \vec{F}_{i,k}\|^2 - (V_{i,k} - W_0 + \langle \vec{g}_N | \nabla_\perp W_0 \rangle)^2 \\ & = 0 \end{aligned}$$

results, leading to the parametric equation for the CO surface $\vec{r}_N(p_N, q_N) = -\nabla_\perp W_0 + \vec{g}_N z_N / m_N$.

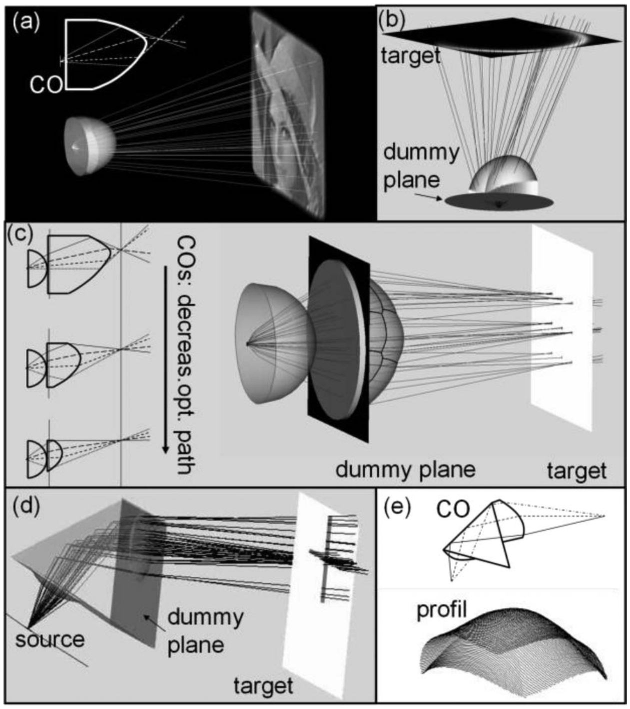

Fig. 3. Examples of illumination freeforms with CO-representation. (a) Freeform lens with differentiable profile and flat entrance face converting light from a Lambertian source (NA ~ 0.996) into the Lena picture with $15° \times 15°$ output divergence; discretization, ~20,000 CO-primitives; inset, cross section of a typical CO-primitive. (b) Nonsteady differentiable freeform lens creating a hemicycle with an angular radius of 19° and a width of 4°; source, Lambertian with NA ~ 0.996. (c) Telecentric spot generator consisting of a collimating lens and a steady, segmented splitting optics where the whole system is constructed by nine COs; the three cross sections show typical CO examples with different path length parameters; source, Lambertian with NA ~ 0.996. (d) Cross-line generation due to light collimation, total internal reflection inside a prism, and freeform light shaping; source, isotropic with $\mathrm{NA} = 0.5$, (e) Typical example of a CO-primitive and the freeform profile corresponding to (d).

These generalized COs are used as primitives for the freeform representation. The system's mixed characteristics W_0 as well as the parametrization in the ray components of the source region p_N, $q_N = f_{p,q}(\vec{r}_0, p_1, q_1)$ can be treated analytically for simple systems. For example, in case of a freeform lens (two surfaces) with a flat entrance face $p_2 = p_1, q_2 = q_1$ and $W_0 = D\sqrt{1 - p_1^2 - q_1^2}$ hold. Here, D is the distance between the source and the entrance face, which is identical to the dummy plane.

Figures 3(a) and 3(b) show two examples applying such primitives. In Fig. 3(a), a freeform lens is designed that converts ~95% of the light of a Lambertian source (divergence angle ~85°) into a rectangular illumination pattern showing the Lena picture with $15° \times 15°$ output divergence. About 20,000 COs are used for the target discretization. An interpolation between the sampling points leads to a continuous and differentiable lens profile. Figure 3(b) illustrates that nondifferentiable profiles can be easily designed, which are much harder to obtain with PDE methods. Here, again ~95% of a Lambertian source is transformed into a hemicycle with an angular radius of 19° and a width of 4°. The discontinuity in the derivative is due to the topological difference of the input and output light distribution. It emerges because neighboring freeform elements at both sides of the discontinuity lighten distant target elements at the two ends of the hemicycle.

For more complex front optics a fully analytical CO representation is generally not possible. In those cases, a conventional ray tracer is utilized to determine the system's mixed characteristics W_0 as well as the dependence on the ray components in the source region p_1, q_1. A discrete set of rays is traced through the systems and the required relations are interpolated to obtain a quasi-analytic CO representation. Figures 3(c) and 3(d) present two corresponding examples. The first example is a telecentric spot generator consisting of a collimating lens and a segmented splitting optics. The segmented optics exhibits a steady but nondifferentiable profile. The divergence of a Lambertian light source is reduced by the first element and then decomposed into spots of equal power, where the chief rays of all spots are approximately parallel to each other. The segments are certain members of the generalized COs [see three examples at the left in Fig. 3(c)], which fully take into account the effect of the collimating front optics. Thus, the generated spots are diffraction-limited. Finally, Figs. 3(d) and 3(e) show an example of the generation of a cross-line. Source light with an NA of 0.5 is roughly collimated at the first lens, deflected by total internal reflection inside a prism, and finally beam-shaped by a freeform. The profile of the freeform surface is depicted in Fig. 3(e).

In conclusion, generalized COs are used for freeform design in illumination systems. Desired parametric CO representations are obtained by Hamiltonian theory of ray optics. This approach offers the possibility for a simple geometrical construction of freeforms in complex optical systems.

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