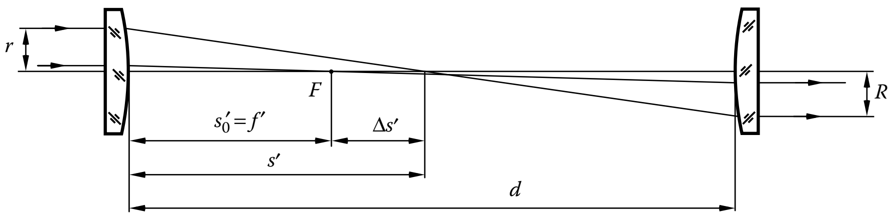

FIGURE 7.2 Basic layout of two aspheric lens laser beam shaping system.

where:
w_0 is the radius of the input Gaussian beam waist
n is the index of refraction of lenses

The first plano-convex lens focuses the laser beam and is responsible for introduction of the wave aberration in order to redistribute the beam irradiance, while the second plano-convex lens compensates for this wave aberration and collimates the beam so that the output beam has a flat phase front and flat-top irradiance profile.

Detailed theory of designing refractive beam shapers is presented in Chapter 6 [30] and in Reference [27]. Here, some important conclusions are used to develop the optimization methods for designing more complex laser beam shapers than were considered in Chapter 6. As a result of imposing the geometrical law of intensity [12] between the input and output apertures of a refractive beam shaper shown in Figure 7.2, one obtains the following ray mapping function [27] for an input Gaussian beam and output top-hat beam profile:

$$R^2 = R_{\max}^2 \left[\frac{1 - \exp(-2r^2 / w_0^2)}{1 - \exp(-2r_{\max}^2 / w_0^2)} \right] \tag{7.9}$$

Since r and R are radial coordinates that are always positive, one must be careful when taking the square root of Equation 7.9 to obtain a valid expression for R for either Keplerian or Galilean configuration of a refractive laser beam shaper. Using the same notation introduced by Shealy and Hoffnagle [27], the output radial coordinate, R, of a refractive beam shaper is given by the following result:

$$R = \epsilon R_{\max} \sqrt{\frac{1 - \exp[-2(r / w_0)^2]}{1 - \exp[-2(r_{\max} / w_0)^2]}} \tag{7.10}$$

where:
ϵ is defined by

$$\epsilon = \begin{cases} +1 & \text{Galilean configuration} \\ -1 & \text{Keplerian configuration} \end{cases} \tag{7.11}$$

As a result of the definition of ϵ, the radical in Equation 7.10 always refers to the positive square root. Equation 7.10 can be written in a more compact form as follows:

$$R = B\sqrt{1 - \exp\left[-2\left(\frac{r}{w_0}\right)^2\right]} \tag{7.12}$$

where:

$$B = \frac{\epsilon R_{\max}}{\sqrt{1 - \exp[-2(r_{\max}/w_0)^2]}} \tag{7.13}$$

From the geometry of the input and output rays shown in Figure 7.2, we can write the following expression:

$$\tan\alpha = \frac{r - R}{d} = \frac{r}{s'} \tag{7.14}$$

or

$$s' = \frac{rd}{r - R} \tag{7.15}$$

where:

α is the angle that a refracted ray from the first lens makes with the optical axis

Substituting Equation 7.12 for Keplerian beam shaper with $\epsilon = -1$ into Equation 7.15 gives

$$s' = \frac{d}{1 - (B/r)\sqrt{1 - \exp(-2r^2/w_0^2)}} \tag{7.16}$$

where it should be noted that for a Keplerian configuration we have $\epsilon = -1$, and thus, $B < 0$ from Equation 7.13. The location s_0' of the paraxial focus F of first plano-aspheric lens can be found by considering the limit as r approaches zero:

$$s_0' = \lim_{r\to 0} s' \tag{7.17}$$

To define this value, it is convenient to use the Taylor series expansion of the exponential function; the transformations give the result as follows:

$$s_0' = \frac{d}{1 - \sqrt{2}B/w_0} \tag{7.18}$$

Then, the longitudinal aberration (LA) $\Delta s'$ to be introduced by the first aspheric lens is given by

$$\Delta s' = s' - s_0' = \frac{d}{1 - (B/r)\sqrt{1 - \exp(-2r^2/w_0^2)}} - \frac{d}{1 - \sqrt{2}B/w_0} \tag{7.19}$$

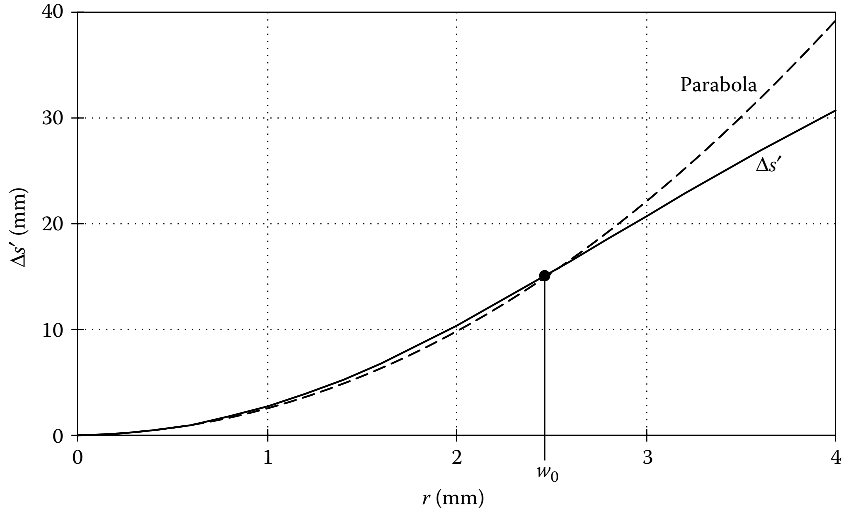

FIGURE 7.3 Analysis of LA: solid curve represents aberration; dashed curve represents approximate parabola.

The character of the aberration function $\Delta s'$ can be understood by evaluating Equation 7.19 for the 1-to-1K example [27] with the parameters given by Equation 7.8. Then, the distance s'_0 to the paraxial focus is equal to 43.81 mm, and the LA $\Delta s'$ as the function of the input beam radius r is shown in Figure 7.3.

In contrast, Shealy and Hoffnagle evaluated the axial curvature of the first plano-aspheric lens of 1-to-1K beam shaper by using the intensity law of geometrical optics to be equal to -0.042656mm^{-1} or an axial distance of 50.767 mm from the vertex of the first lens with the optical axis to the axial focal point [31,32]. Therefore, in contrast to imaging applications, the paraxial focus is less important in beam shaping applications than the overall ray mapping function that describes how the input rays are mapped to the output aperture as required by the intensity law for beam shapers.

There are several important conclusions. First, it is important to note that the LA required to transform an input Gaussian beam into a flat-top beam is reached over several tens of millimeters, which corresponds to a very strong wave aberration, up to 100λ, existing between the components of this laser beam shaping system. The aberration is positive, which means that it is not possible to introduce this level of aberration by a simple spherical surface, since the positive spherical lenses have negative spherical aberration [33]. Therefore, the surface profile of the laser beam shaper must be aspheric. Another important feature is that in the range of input beam radii from 0 to w_0, the function of the LA can be approximated by parabolic function with very high precision, which is shown in Figure 7.3, where the approximate parabola is shown by the dashed line. This parabola and function of LA are crossing at the point equal to the waist radius w_0. Majority of the energy of the Gaussian laser beam, about 87%, is concentrated within a circle of radius of w_0. Therefore, any calculations based on parabolic approximation of the aberration function would be valid for evaluating the performance of a laser beam shaper system.

Expanding Equation 7.19 to terms including r^2 then gives an explicit expression for spherical aberration. The function of the LA can be written as follows:

$$\mathrm{LA} = \Delta s' \approx Ar^2 \tag{7.20}$$

where:
A is given by

$$A = \frac{Bd}{\sqrt{2}(w_0 - \sqrt{2}B)^2} \tag{7.21}$$

As we will see, a similar mathematical expression has been used to describe the LA in the third-order aberration theory. Therefore, the third-order aberration theory can be used to design laser beam shapers.

7.3.1.1.2 Third-Order Aberrations of Two-Lens System

The theory of the third-order aberrations is a powerful tool for analysis of optical systems and developing an initial design that can be used as a starting point for further optimization. For the systems when the aberrations can be approximated by the third-order series approximation, this theory allows calculating of the system parameters being close to global minimum solution. Hence, the further optimization process is very short and gives optimum system parameter values after few iterations. There are adequate descriptions in the literature of several approaches of using third-order approximation of aberrations of optical system on the basis of Seidel sums, where we are using the formulations given in the literature [33–35]. An essential feature of this theory is in calculating the third-order coefficients of the aberration series expansion by using paraxial values of the parameters and specifications of an optical system. Now, this approach is convenient for using in the designing process.

A beam shaping effect is always achieved through introducing certain aberration. In the particular case of the 1-to-1K beam shaping system, the transformation of its irradiance distribution is achieved through introducing just spherical aberration by the first lens. Since these systems work typically in narrow angular field, it is sufficient to consider relationships for spherical aberration only.

According to the third-order aberration theory [36–38], for an infinitely remote object the transverse spherical aberration $\Delta y'_{\mathrm{III}}$ of an optical system with focal length f' is written as

$$\Delta y'_{\mathrm{III}} = -\left[\frac{f'}{2r}\right] S_{\mathrm{I}} \tag{7.22}$$

where:
r is the height of a ray
S_{I} is the sum of the first Seidel coefficient

Then the LA is given by

$$\Delta s'_{\mathrm{III}} = \frac{f'}{r}\Delta y'_{\mathrm{III}} = -\left[\frac{(f')^2}{2r^2}\right] S_{\mathrm{I}} \tag{7.23}$$

The Seidel coefficients have to be calculated for each surface of an optical system by using special formulas described in Refs. [33–35] for the particular surface parameters: curvature radius or focal length, refractive index, air gap or lens thickness, conic constant for the second-order surfaces. In the case of the system illustrated in Figure 7.2, we assume that the convex surfaces are aspheric. In terms of the third-order aberration theory, they can be presented as the second-order surfaces. Then the Seidel coefficients S_{I1} and S_{I2} for correspondingly first and second aspheric surfaces can be written as

$$S_{I1} = \frac{r^4(n^2 + k_1)}{f_1'^3(n-1)^2} \tag{7.24}$$

$$S_{I2} = -\frac{R^4(n^2 + k_2)}{f_2'^3(n-1)^2} \tag{7.25}$$

where:
- n is the refractive index of the material of lenses
- k_1 and k_2 are the conic constants of the second-order surfaces

Based on these relationships, we now calculate the basic parameters of the beam shaping optical system.

7.3.1.1.3 Parameters of Second-Order Aspheric Lens

The expression for S_{I1} is used to evaluate the parameters of the first lens, which introduces the spherical aberration sufficient to create the necessary ray mapping function. Combining Equations 7.23 and 7.24, we obtain the third-order LA of the first lens:

$$\Delta s'_{III1} = -\frac{(n^2 + k_1)}{2f_1'(n-1)^2} r^2 \tag{7.26}$$

Clearly, the aberration of the first aspheric lens represents a parabolic function analogous to Equation 7.20 and can be used to approximate the ray mapping function. Now, we assume

$$\Delta s' = \Delta s'_{III1} \tag{7.27}$$

and take into account Equation 7.26. Then, we obtain the following expression for the conic constant of the first aspheric surface, k_1:

$$k_1 = \frac{-2\Delta s' f_1'(n-1)^2}{r^2} - n^2 \tag{7.28}$$

As discussed above and shown in Figure 7.3, it is convenient to consider the LA $\Delta s'_{w0}$ corresponding to the ray height at the beam waist of radius w_0. Taking this into account and also noticing that for a plano-convex lens,

$$f_1' = s_0' \tag{7.29}$$

then we can write a final expression for the conic constant as

$$k_1 = \frac{-2\Delta s'_{w0} s'_0 (n-1)^2}{w_0^2} - n^2 \tag{7.30}$$

Since the first lens is plano-convex, the vertex radius r_{c1} of its aspheric surface is

$$r_{c1} = f_1'(n-1) = s_0'(n-1) \tag{7.31}$$

The second basic condition of a laser beam shaper is for there to be no wave aberration present in the plane wavefront as it leaves the system. This means that the spherical aberration of the complete laser beam shaper system must be equal to zero. The corresponding condition of the third-order aberration theory implies that the total sum S_{I} of first Seidel coefficients for all optical surfaces is equal to zero, which can be expressed by the following relationship:

$$S_{\mathrm{I}} = S_{\mathrm{I}1} + S_{\mathrm{I}2} = 0 \tag{7.32}$$

This expression is convenient to calculate the aspheric parameters, conic constant k_2 and vertex radius r_{c2}, for the second lens. Combining Equations 7.24, 7.25, and 7.32 gives

$$k_2 = \frac{s_0' k_1 + n^2 d}{s_0' - d} \tag{7.33}$$

$$r_{c2} = (n-1)(d - s_0') \tag{7.34}$$

Thus, all parameters of the plano-aspheric lens pair beam shaping system are defined.

7.3.1.2 Example of Designing the Beam Shaper

Now, we carry out the calculations of the lens parameters on the example of a beam shaper analogous to the 1-to-1K system described by Shealy and Hoffnagle [27]. The initial data are assumed to be given by Equation 7.8, where n = 1.46071 for fused silica when λ = 532 nm.

7.3.1.2.1 Calculations of the Parameters for Second-Order Aspheres

Using Equations 7.13, 7.18, 7.19, 7.30, 7.31, 7.33, and 7.34 to calculate the second-order parameters gives the following results:

$$s_0' = 43.81\,\mathrm{mm} \qquad \Delta s'_{\omega 0} = 14.02\,\mathrm{mm} \tag{7.35}$$

$$r_{c1} = -20.182\,\mathrm{mm} \qquad k_1 = -48.71 \tag{7.36}$$

$$r_{c2} = 48.925\,\mathrm{mm} \qquad k_2 = 17.08 \tag{7.37}$$

where the radii signs are consistent with the optics sign convention [33]. The optical system layout is shown in Figure 7.4, where the design data in the form adopted in practice of optical system designing are given in Table 7.1.