

# Homework 8

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## 1 Problem 1

The truth table for  $p \rightarrow \neg q \vee r \rightarrow (\neg r \rightarrow s \wedge p)$  is shown below:

$q$	$r$	$s$	$p$	$p \rightarrow \neg q \vee r \rightarrow (\neg r \rightarrow s \wedge p)$
F	F	F	F	T
F	F	F	T	T
F	F	T	F	T
F	F	T	T	T
F	T	F	F	T
F	T	F	T	T
F	T	T	F	T
F	T	T	T	T
T	F	F	F	F
T	F	F	T	T
T	F	T	F	T
T	F	T	T	T
T	T	F	F	T
T	T	F	T	T
T	T	T	F	T
T	T	T	T	T

## 2 Problem 2

(1): The truth table for the formula  $((p \vee q) \wedge (r \vee s)) \rightarrow (((p \rightarrow q) \vee (p \rightarrow r)) \wedge ((q \rightarrow p) \vee (q \rightarrow p)))$  is:

$p$	$q$	$r$	$s$	$((p \vee q) \wedge (r \vee s)) \rightarrow (((p \rightarrow q) \vee (p \rightarrow r)) \wedge ((q \rightarrow p) \vee (q \rightarrow r)))$
F	F	F	F	T
F	F	F	T	T
F	F	T	F	T
F	F	T	T	T
F	T	F	F	T
F	T	F	T	F
F	T	T	F	F
F	T	T	T	F
T	F	F	F	T
T	F	F	T	F
T	F	T	F	T
T	F	T	T	T
T	T	F	F	T
T	T	F	T	T
T	T	T	F	T
T	T	T	T	T

So the formula is a contingency.

(2): The truth table for the formula  $(\neg(p \leftrightarrow q) \rightarrow ((p \wedge \neg q) \vee (\neg p \wedge q))) \vee r$  is:

$p$	$q$	$r$	$\neg(p \leftrightarrow q) \rightarrow ((p \wedge \neg q) \vee (\neg p \wedge q)) \vee r$
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

So the formula is a tautology.

(3): The truth table for the formula  $((p \rightarrow r) \wedge (q \rightarrow s) \wedge (p \vee q)) \rightarrow (r \vee s)$  is:

$p$	$q$	$r$	$s$	$((p \rightarrow r) \wedge (q \rightarrow s) \wedge (p \vee q)) \rightarrow (r \vee s)$
F	F	F	F	T
F	F	F	T	T
F	F	T	F	T
F	F	T	T	T
F	T	F	F	T
F	T	F	T	T
F	T	T	F	T
F	T	T	T	T
T	F	F	F	T
T	F	F	T	T
T	F	T	F	T
T	F	T	T	T
T	T	F	F	T
T	T	F	T	T
T	T	T	F	T
T	T	T	T	T

So the formula is a tautology.

### 3 Problem 3

- (1):  $(B \vee A) \rightarrow C$   
(2):  $((A \rightarrow C) \wedge (\neg A)) \rightarrow ((\neg B) \vee (\neg C))$

### 4 Problem 4

(1): A: "x\*\*y is valid Python", B: "x is a numeric number", C: "y is a numeric number". The logical notation is:  $A \leftrightarrow (B \wedge C)$ .

(2): A: "x + y is valid Python", B: "x is a numeric number", C: "y is a numeric number", D: "x is a list", E: "y is a list". The logical notation is:  $A \leftrightarrow ((B \wedge C) \vee (D \wedge E))$ .

(3): A: "x \* y is valid Python", B: "x is a numeric number", C: "y is a numeric number", D: "x is a list", E: "y is a list". The logical notation is:  $A \leftrightarrow ((B \wedge C) \vee (D \wedge C) \vee (E \wedge B))$ .

(4): A: "x \* y is a list", B: "x \* y is valid Python", C: "x is a numeric num-

ber", D: "y is a numeric number". The logical notation is:  $(B \wedge (\neg(C \wedge D))) \rightarrow A$ .

(5): A: "x + y is valid Python", B: "x \*\* y is valid Python", C: "x is a list".  
The logical notation is:  $(A \wedge B) \rightarrow (\neg C)$ .

## 5 Problem 5

(1):  $(p \rightarrow \neg p) \rightarrow (p \rightarrow q)$

(2):  $(p \rightarrow (\neg p \rightarrow p)) \rightarrow q$

## 6 Problem 6

(1): To prove that  $A \oplus B \leftrightarrow \neg(A \leftrightarrow B)$  is a tautology, let's consider the truth table for  $A \oplus B$  and  $\neg(A \leftrightarrow B)$ :

$A$	$B$	$A \oplus B$	$\neg(A \leftrightarrow B)$
F	F	F	F
F	T	T	T
T	F	T	T
T	T	F	F

So  $A \oplus B \leftrightarrow \neg(A \leftrightarrow B)$  is a tautology.

(2): A = "You may contact me by phone", B = "You may contact me by email",  
C = "I am available for an on-site interview on October 8th in Minneapolis", D  
= "I am available for an on-site interview on October 8th in Hong Kong". The  
logical notation is:  $(A \vee B) \wedge (C \oplus D)$ .

## 7 Problem 7

$$A_1 = p \wedge q \wedge r$$

$$A_2 = p \wedge q \wedge \neg r$$

$$A_3 = p \wedge \neg q \wedge r$$

$$A_4 = p \wedge \neg q \wedge \neg r$$

$$A_5 = \neg p \wedge q \wedge r$$

$$A_6 = \neg p \wedge q \wedge \neg r$$

$$\begin{aligned}
A_7 &= \neg p \wedge \neg q \wedge r \\
A_8 &= \neg p \wedge \neg q \wedge \neg r \\
A &= A_2 \vee A_4 \vee A_6 \vee A_7 \vee A_8
\end{aligned}$$

## 8 Problem 8

$$A = (p \wedge q) \vee (\neg p \wedge q \wedge r), B = (p \vee (q \wedge r)) \wedge (q \vee (\neg p \wedge r))$$

$$\begin{aligned}
A &\equiv (p \wedge q) \vee (\neg p \wedge q \wedge r) \\
&\equiv (p \vee (\neg p \wedge q \wedge r)) \wedge (q \vee (\neg p \wedge q \wedge r)) \\
&\equiv (p \vee \neg p) \wedge (p \vee (q \wedge r)) \wedge (q \vee (\neg p \wedge r)) \\
&\equiv (p \vee (q \wedge r)) \wedge (q \vee (\neg p \wedge r)) \equiv B
\end{aligned}$$

## 9 Problem 9

(1): To show that  $(p \rightarrow q) \wedge (r \rightarrow s) \rightarrow ((p \wedge r) \rightarrow (q \wedge s))$  is a tautology, we are going to rewrite this to T:

$$\begin{aligned}
&(p \rightarrow q) \wedge (r \rightarrow s) \rightarrow ((p \wedge r) \rightarrow (q \wedge s)) \leftrightarrow (\neg p \vee q) \wedge (\neg r \vee s) \rightarrow \neg(p \wedge r) \vee (q \wedge s) \\
&\leftrightarrow (\neg p \vee q) \wedge (\neg r \vee s) \rightarrow (\neg p \vee \neg r) \vee (q \wedge s) \\
&\leftrightarrow (\neg p \vee q) \wedge (\neg r \vee s) \rightarrow (\neg p \vee \neg r \vee q) \wedge (\neg p \vee \neg r \vee s) \\
&\leftrightarrow (\neg p \vee q) \wedge (\neg r \vee s) \rightarrow ((\neg p \vee q) \wedge (\neg r \vee s)) \vee (\neg p \vee \neg r) \\
&\leftrightarrow \neg((\neg p \vee q) \wedge (\neg r \vee s)) \vee ((\neg p \vee q) \wedge (\neg r \vee s)) \vee (\neg p \vee \neg r) \\
&\leftrightarrow T \vee (\neg p \vee \neg r) \equiv T
\end{aligned}$$

(2): To show that  $(p \rightarrow q) \wedge (r \rightarrow s) \rightarrow ((p \wedge r) \rightarrow (q \wedge s))$  is a tautology, we are going to rewrite this to T:

$$\begin{aligned}
&((p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow r \\
&\leftrightarrow ((p \vee q) \wedge (\neg p \vee r) \wedge (\neg q \vee r)) \rightarrow r \\
&\leftrightarrow \neg((p \vee q) \wedge (\neg p \vee r) \wedge (\neg q \vee r)) \vee r \\
&\leftrightarrow \neg(p \vee q) \vee \neg(\neg p \vee r) \vee \neg(\neg q \vee r) \vee r \\
&\leftrightarrow (\neg p \wedge \neg q) \vee (\neg \neg p \wedge \neg r) \vee (\neg \neg q \wedge \neg r) \vee r \\
&\leftrightarrow (\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee (q \wedge \neg r) \vee r \\
&\leftrightarrow (\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee ((q \vee r) \wedge (\neg r \vee r)) \\
&\leftrightarrow (\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee ((q \vee r) \wedge T) \\
&\leftrightarrow (\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee q \vee r \\
&\leftrightarrow (\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee r \vee q \\
&\leftrightarrow (\neg p \wedge \neg q) \vee ((p \vee r) \wedge (\neg r \vee r)) \vee q \\
&\leftrightarrow (\neg p \wedge \neg q) \vee ((p \vee r) \wedge T) \vee q \\
&\leftrightarrow (\neg p \wedge \neg q) \vee p \vee r \vee q \\
&\leftrightarrow ((\neg p \vee p) \wedge (\neg q \vee p)) \vee r \vee q \\
&\leftrightarrow (T \wedge (\neg q \vee p)) \vee r \vee q
\end{aligned}$$

$$\begin{aligned}
&\leftrightarrow \neg q \vee p \vee r \vee q \\
&\leftrightarrow \neg q \vee q \vee p \vee r \\
&\leftrightarrow T \vee p \vee r \\
&\leftrightarrow T
\end{aligned}$$

## 10 Problem 10

You should ask: "What path would the other type of person tell me is the correct one?" And then always choose the opposite one to go.

P: The man is a knight, R: The way he told me is to turn left. Q: The correct way is to turn left. And here's the truth table for the problem:

P	Q	R
T	T	F
T	F	T
F	T	F
F	F	T