Homework 13

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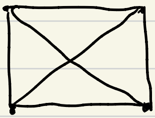
(这周电脑坏了，没发用 LaTex 打。麻烦助教忍受一下我丑陋的手写作业)

1.



2. According to Euler's Formula, we have $r = e - v + 2$

Because $d(v) = 3$, we have $2e = 3v$

Assume that all regions are made of 6 edges or more

Then $6r \cdot \frac{1}{2} \le e \Rightarrow r \le \frac{e}{3}$

So $e - v + 2 \le \frac{1}{3}e$

$\Rightarrow \frac{1}{2}v + 2 \le \frac{1}{2}v$ which is a contradiction

So in this way we have proved that there must exist a face with at most 5 edges.

3. We start at the vertex a:

$a \to b \to c \to d \to a$ 18

$a \to b \to d \to c \to a$ 20

$a \to c \to b \to d \to a$ 16

$a \to c \to d \to b \to a$ 20

$a \to d \to b \to c \to a$ 16

$a \to d \to c \to b \to a$ 18

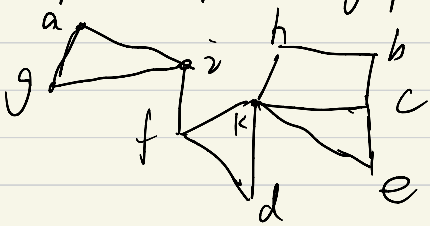
So for the traveling salesperson problem, we have two paths:

$a \to c \to b \to d \to a$

$a \to d \to b \to c \to a$

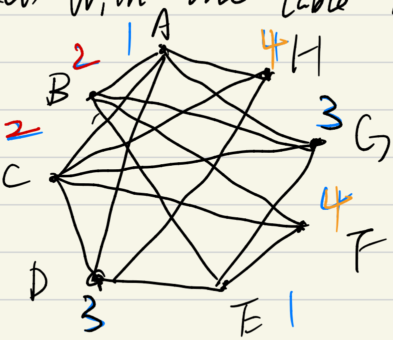
4. G is nonplanar

H is planar graph and we can redraw it like this:



k is nonplanar

5. (a) With the table below. we draw the graph



(b) Consider the subgraph $\{A, H, C, D\}$, it is K_4. so chromatic number must be no less than 4

And we can see from the graph that we use 4 colors to color the graph. So chromatic number is 4

(c) ~~Because~~ the chromatic number is 4, we need 4 aquarium.

6. $k=0$, $S_0 = \phi$, $L_0(u) = 0$

$L_0(a) = L_0(b) = L_0(c) = L_0(d) = L_0(e) = L_0(f) = L_0(h) = L_0(g) = L_0(i) = L_0(v) = \infty$

$k=1$, $u_i = u \to S_1 = \{u\}$

$L_0(u) + w(u,a) = 2 < L_0(a) \to L_1(a) = 2$

$L_0(u) + w(u,c) = 6 < L_0(c) \to L_1(c) = 6$

$L_0(u) + w(u,d) = 1 < L_0(d) \to L_1(d) = 1$

$k=2 \quad u := d \to S_2 = \{u, d\}$

$L_1(d) + w(d,e) = 4 < L_1(e) \to L_2(e) = 4$

$L_1(d) + w(d,v) = 10 < L_1(v) \to L_2(v) = 10$

$L_1(d) + w(d,g) = 3 < L_1(g) \to L_2(g) = 3$

$k=3: \quad u := a \to S_3 = \{u, d, a\}$

$L_2(a) + w(a,b) = 3 < L_2(b) \to L_3(b) = 3$

$k=4 \quad u := b, g \to S_4 = \{u, d, a, b, g\}$

$L_3(g) + w(g,i) = 12 < L_3(i) \to L_4(i) = 12$

$L_3(b) + w(b,f) = 12 < L_3(f) \to L_4(f) = 12$

$k=5 \quad u := e \to S_5 = \{u, d, a, b, g, e\}$

$L_4(e) + w(c,e) = 5 < L_4(c) \to L_5(c) = 5$

$L_4(e) + w(e,f) = 8 < L_4(f) \to L_5(f) = 8$

$k=6 \quad u := c \to S_6 = \{u, d, a, b, g, e, c\}$

$L_5(c) + w(c,f) = 7 < L_5(f) \to L_6(f) = 7$

$k=7 \quad u := f \to S_7 = \{u, d, a, b, g, e, c, f\}$

$L_6(f) + w(f,b) = 8 < L_6(h) \to L_7(h) = 8$

$k=8: \quad u := h \to S_8 = \{u, d, a, b, g, e, c, f, h\}$

$L_7(h) + w(h,v) = 9 < L_7(v) \to L_8(v) = 9$

$k=9: \quad u := v \to S_9 = \{u, d, a, b, g, e, c, f, h, v\}$

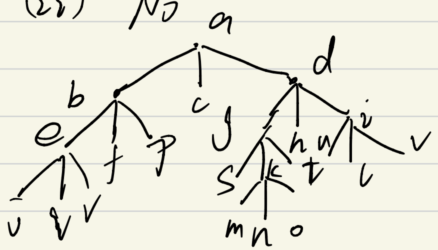
$L_8(v) + w(v,i) = 11 < L_8(i) \to L_9(i) = 11$

$k=10: \quad u := i \to S_{10} = \{u, d, a, b, g, e, c, f, h, v, i\}$

7. (a)

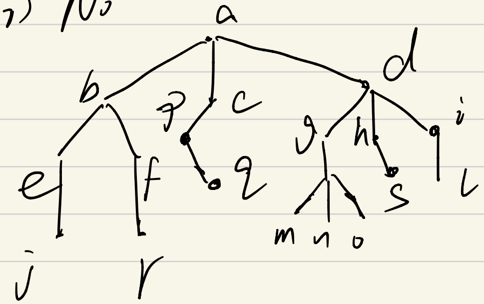
(i) For $m \geq 3$, T is a m-ary tree.

(ii) No



We can add 7 edges to make it a full 3-ary tree

(iii) No



We can add 4 edges to make it balanced

(b) Let $n = 2^k$. Now consider the tree with $n-1$ processors, denoted by the vertices of tree. Number of internal vertices of tree is $i = \frac{(n-1)-1}{2} = 2^{k-1} - 1$ and the number of leaves is $(n-1) - i = 2^{k-1}$

Each processor at a leaf can add 2 numbers and communicate the sum to its parent. All the leaves can then count the sum of $l = 2^{k-1} = \frac{n}{2}$ pairs. Thus, we can group the n numbers into $\frac{n}{2}$ pairs and calculate theirs sums and communicate them to the above level of internel vertices. Keep this process and we will finish after $k = \log_2 n$ steps.

So $\log_2 n$ (or $\log n$ in CS) steps are needed

8. "$\Rightarrow$"

Because G is a tree, so for each two vertices, there exists only one path.

Let's assume there exists an edge e, between u, v, is not a bridge. Then there exists another path besides e between u, v, which is a contradiction.

"$\Leftarrow$"

Because every edge in the graph is a bridge, then there is no cycle in G, and there exists only one path between every two vertices.

Thus G is a tree.

9. (a)

$$((((a*b)-c)\div(d+(e*f)))*g)+((h*i)\div(j*(k-l)))$$

(b)

$$+ * \div - * a b c + d * e f g \div * h i * j - k l$$

(c)

$$a b * c - d e f * + \div g * h i * j k l - * \div +$$