Homework 9

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May 12, 2024

1 Problem 1

Let $A = (\neg p \vee q) \wedge (r \to \neg q)$, $B = p \to \neg r$. We need to prove that $A \to B \equiv T$

$$
\begin{aligned}
&A \to B = (\neg p \vee q) \wedge (r \to \neg q) \to (p \to \neg r) \\
&= \neg((\neg p \vee q) \wedge (r \to \neg q)) \vee (p \to \neg r) \\
&= (\neg(\neg p \vee q) \vee \neg(r \to \neg q)) \vee (p \to \neg r) \\
&= ((p \wedge \neg q) \vee (r \wedge q)) \vee (p \to \neg r) \\
&= (p \wedge \neg q) \vee (r \wedge q) \vee (\neg p \vee \neg r) \\
&= (p \wedge \neg q) \vee (r \wedge q) \vee \neg p \vee \neg r \\
&= (p \wedge \neg q) \vee \neg p \vee (r \wedge q) \vee \neg r \\
&= \neg q \vee \neg p \vee q \vee \neg r \\
&= T
\end{aligned}
$$

2 Problem 2

Let $A = (p \to q \vee r) \wedge (q \to s) \wedge (r \to \neg p)$, $B = p \to s$. We need to prove that $A \wedge \neg B \equiv F$

$$
\begin{aligned}
&A \wedge \neg B = (p \to q \vee r) \wedge (q \to s) \wedge (r \to \neg p) \wedge \neg(p \to s) \\
&= (\neg p \vee (q \vee r)) \wedge (\neg p \vee s) \wedge (\neg r \vee \neg p) \wedge \neg(\neg p \vee s) \\
&= (\neg p \vee q \vee r) \wedge (\neg p \vee s) \wedge (\neg r \vee \neg p) \wedge (p \wedge \neg s) \\
&= (\neg p \vee q \vee r) \wedge (\neg p \vee s) \wedge (\neg r \vee \neg p) \wedge p \wedge \neg s \\
&= (\neg p \vee q \vee r) \wedge (p \wedge s) \wedge (\neg r \vee \neg p) \wedge \neg s \\
&= (\neg p \vee q \vee r) \wedge (\neg r \vee \neg p) \wedge p \wedge (s \wedge \neg s) \\
&= (\neg p \vee q \vee r) \wedge (\neg r \vee \neg p) \wedge F \\
&= F
\end{aligned}
$$

3 Problem 3

Let p be proposition "It rains", let q be proposition "it is foggy", let r be proposition "The sailing race will be held", and let t be proposition "The life saving demonstration will go on", and let u be proposition "The trophy will be awarded".

Building Arguments:
(1): $\neg u$ Premise
(2): $r \implies u$ Premise
(3): $\neg r$ Modus Tollens of (1) and (2)
(4): $(\neg p \vee \neg q) \implies (r \wedge t)$ Premise
(5): $\neg(r \wedge t) \implies \neg(\neg p \vee \neg q)$ Contrapositive of (4)
(6): $\neg r \vee \neg t \implies (p \wedge q)$ Equivalence of (5)
(7): $\neg r \vee \neg t$ Addition of (3)
(8): $p \wedge q$ Modus Ponens of (6) and (7)
(9): p Simplification of (8)

4 Problem 4

Premises: $p \wedge q \rightarrow r$, $\neg r \vee s$, $p \rightarrow \neg s$

Conclusion: $p \rightarrow \neg q$

(1): $p \wedge q \rightarrow r$ Premise
(2): $\neg r \vee s$ Premise
(3): $p \rightarrow \neg s$ Premise
(4): $\neg p \vee \neg s$ Equivalence of (3)
(5): $\neg p \vee \neg r$ Resolution of (2) and (4)
(6): $p \rightarrow \neg r$ Equivalence of (5)
(7): $r \rightarrow \neg p$ Contrapositive of (6)
(8): $(p \wedge q) \rightarrow \neg p$ Rule of Inference from (1) and (7)
(9): $\neg(p \wedge q) \vee \neg p$ Equivalence of (8)
(10): $\neg p \vee \neg q \vee \neg p$ De Morgan's Law of (9)
(11): $\neg p \vee \neg p \vee \neg q$ Commutative Law of (10)
(12): $\neg p \vee \neg q$ Idempotent Law of (11)
(13): $p \rightarrow \neg q$ Equivalence of (12)

5 Problem 5

Premises: $p_1 \to (q_1 \to r_1)$, $p_2 \to (q_2 \to r_2)$, $p_3 \to (q_3 \to r_3)$, $\cdots$, $p_n \to (q_n \to r_n)$, $q_1 \wedge q_2 \wedge q_3 \wedge \cdots \wedge q_n$

Conclusion: $(p_1 \to r_1) \wedge (p_2 \to r_2) \wedge (p_3 \to r_3) \wedge \cdots \wedge (p_n \to r_n)$

(1): $p_i \to (q_i \to r_i)$ Premise
(2): $q_1 \wedge q_2 \wedge q_3 \wedge \cdots \wedge q_n$ Premise
(3): q_i Simplification of (2)
(4): $\neg p_i \vee r_i$ Equivalence of (1), (3)
(5): $p_i \to r_i$ Equivalence of (4)
(6): $(p_1 \to r_1) \wedge (p_2 \to r_2) \wedge (p_3 \to r_3) \wedge \cdots \wedge (p_n \to r_n)$ Conjunction of (5)

6 Problem 6

(a): True
(b): False
(c): True
(d): True

7 Problem 7

Let p be proposition "A is the suspect", q be proposition "A visited the victim's room", r be proposition "A leave before 2 am", s be proposition "The hotel staff saw A".

Building Arguments:
(1): $(q \wedge \neg r) \to p$ Premise
(2): q Premise
(3): $r \to s$ Premise
(4): $\neg s$ Premise
(5): $\neg s \to \neg r$ Contrapositive of (3)
(6): $\neg r$ Modus Ponens of (4) and (5)
(7): $q \wedge \neg r$ Conjunction of (2) and (6)
(8): p Modus Ponens of (1) and (7)
So we have proved A is the suspect.