PROJECT REPORT: FRACTURE FIXATION FEA SIMULATION

TECHNICAL REPORT

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ABSTRACT

This report details the implementation and evaluation of a finite element analysis simulation for fracture fixation. We present a comprehensive study examining the mechanical behavior of bone-implant systems under various loading conditions. Our methodology incorporates detailed geometric modeling, material property characterization, and boundary condition specifications to accurately simulate the biomechanical environment.

Keywords FEA · Fracture Fixation

1 Introduction

In fracture repair processes, as splints or internal fixators gradually reduce load-bearing, the fracture region begins to regain stress-bearing capacity. This project simulates how decreasing splint stiffness (or increasing fracture surface bonding strength) in a "bone+splint" system affects stress redistribution during healing, helping to understand the biomechanical phenomenon of "stress transfer".

2 FEA Modeling Stress Redistribution During Fracture Healing

2.1 Problem Definition

- **Type**: Non-homogeneous elastic body under load with time-varying material properties (elastic modulus)
- **Model**: 2D "bone+splint" system with either:
  - Gradually recovering fracture surface stiffness, or
  - Progressively decreasing splint stiffness
- **Features**: Simulation of stress transfer from splint to bone tissue

2.2 Mathematical Model

2.2.1 Mechanical Framework

Linear elasticity theory with time-dependent parameters:

$$\boldsymbol{\sigma}^{(t)} = \mathbf{D}^{(t)}\boldsymbol{\varepsilon}$$

where $\mathbf{D}^{(t)}$ evolves with healing progression.

2.2.2 Material Evolution

$$\begin{aligned}\text{Bone tissue}: E_{bone} &\approx 17 \text{ GPa (constant)}\\ \text{Splint}: &E_{splint}^{(t)} \text{ (may decrease)}\\ \text{Callus}: &E_{callus}^{(t)} \text{ (increases with healing)}\end{aligned}$$

2.2.3 Fracture Interface

Bonding coefficient $\beta(t)$ represents healing progress:

$$\beta(t) \in [0, 1] \quad \text{(0 = unhealed, 1 = fully bonded)}$$

2.3 Finite Element Implementation

2.3.1 CST Element Formulation

Displacement field for triangular elements:

$$\mathbf{u} = \begin{bmatrix} u(x, y) \\ v(x, y) \end{bmatrix} = \mathbf{N}\mathbf{d}^e$$

Strain-displacement relationship:

$$\boldsymbol{\varepsilon} = \mathbf{B}\mathbf{d}^e$$

where $\mathbf{B}$ is the strain-displacement matrix.

2.3.2 Time-Stepping Algorithm

1. Initialize material properties $E_{callus}(t_0)$, $E_{splint}(t_0)$
2. Compute element stiffness matrices:
$$\mathbf{K}^{e(t)} = tA\mathbf{B}^T\mathbf{D}^{(t)}\mathbf{B}$$
3. Assemble global system:
$$\mathbf{K}^{(t)}\mathbf{d}^{(t)} = \mathbf{F}^{(t)}$$
4. Update material properties for next time step

2.4 Expected Outcomes

- Quantification of stress shielding effects
- Optimal load transfer protocols
- Time-dependent stress distribution visualizations
- Correlation between mechanical environment and healing rates

2.5 Results

2.6 Discussion

3 Conclusion

[Kour and Saabne, 2014]

References

George Kour and Raid Saabne. Real-time segmentation of on-line handwritten arabic script. In *Frontiers in Handwriting Recognition (ICFHR), 2014 14th International Conference on*, pages 417–422. IEEE, 2014.